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ESSAYS ON FINANCIAL POLICY AND MACROECONOMICS

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Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

I warrant that this authorisation does not, to the best of my belief, infringe the rights of any third party.

I declare that my thesis consists of approximately 31,500 words.

Statement of Conjoint Work

I confirm that chapter 3 was jointly co-authored with Ms Juanita González-Uribe, Assistant Professor at the Department of Finance of the London School of Economics and Political Science. Each of us contributed equally to the creation, development and writing of the chapter.

Following a confidentiality agreement with the Central Bank of Colombia, I was the sole coauthor authorised to access the dataset throughout the development of our research. A such, I carried out all the statistical and econometric calculations contained in the chapter.

Abstract

This thesis delves into the nature and effects of financial and monetary policy design. It comprises three chapters, each of which studies this topic from a different perspective and with a focus on different frictions.

The first chapter derives theoretically the optimal monetary policy for a small open economy characterized by the incompleteness of the domestic financial market. In the model, a tight relationship between the consumption of different agents and the aggregate debt-to-GDP ratio emerges in equilibrium. Optimal monetary policy is the result of a tradeoff between the stabilization of this ratio (risk-sharing) and the traditional policy objectives of price and output gap stabilization. Quantitative simulations suggest that price and output stabilization dominate debt-to-GDP stabilization in the optimal policy rule. Unlike the case of a closed economy, the policy objective of improving on risk-sharing is excessively costly from the point of view of the policymaker when the economy is open.

The second chapter studies theoretical issues related to the unsecured consumer credit market, normally characterized by incompleteness and poor protection of property rights. The chapter describes a theoretical mechanism by which the interest rate to a given borrower is affected by the default choices of other agents. Individual agents ignore the effect of their own default choices on aggregate credit conditions, and thus the volume of unsecured credit in a decentralized market will be generally inefficient. The chapter employs simulations of the model to conclude that recently observed credit booms in Latin America may be inefficient.

The third chapter analyzes empirically the relationship between information sharing and credit outcomes in the unsecured consumer credit market, focusing on the potential ex-post disciplinary effect and the ex-ante informational hold-up of long-lived negative information about borrowers. The chapter exploits a natural experiment in Colombia created by Law 1266/2008, whereby detailed information about past defaults that were exogenously "sufficiently old" was erased from Private Credit Bureaus (in what follows, PCB). Using a Differences-in-Differences (DD) specification, it is found that *after* old negative information is erased, there is a significant increase in the size and maturity of new loans, no significant changes in interest rates, and an increase in subsequent default rates for the treatment group, relative to the control group. Overall, these results are consistent with both ex-post disciplinary effects, and ex-ante information hold-up from long-lived negative information in PCB. Specifically, consistent with the hold-up theories, the chapter finds that most of the increase in the value of loans comes from outside banks. In addition, consistent with the disciplinary role of information sharing, the chapter finds evidence of a relative increase in the frequency of default for new loans after negative information is erased.

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This thesis is dedicated to her, to our shadows in a faraway, sunny, summer afternoon.

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1. Foreign Currency Debt and Optimal Monetary Policy: Is There a Role for the Exchange Rate in Completing Markets?

This chapter aims to characterize the optimal monetary and exchange rate policies that a small open economy should follow when there is an incomplete domestic financial market in which financial instruments (assets and liabilities) are denominated in foreign currency.

The importance of the issues raised by the pervasiveness of foreign currency financial instruments in an economy with market incompleteness is illustrated by the recent experience of household debt in several Eastern European economies such as Serbia and, most especially, Hungary (see Szpunar and Glogowski[76, 2012]). Since the beginning of the past decade, and perhaps due to lower interest rates and to the expectations created by a tight channel for the nominal exchange rate, Hungarian households started a process of rapid accumulation of both assets and liabilities denominated in currencies other than the Hungarian Forint (HUF), principally Swiss Francs (CHF).

The ratio of foreign currency household debt to GDP in Hungary increased from virtually zero in 1999 to slightly above 29% at the beginning of 2009. According to Balás and Nagy[4, 2010], close to 90% of this debt was denominated in CHF, and only 7% was denominated in Euros. Since then, borrowing in foreign currency has ground to a halt and the outstanding balances have unravelled quickly in a context of financial turbulence for households, nominal depreciation and extreme measures taken by Hungarian authorities with the aim of limiting issuance of this type of liabilities (see Balogh et al[3, 2013]). At the beginning of 2013, payments on about 20% of mortgages denominated in foreign currency in Hungary were overdue (see WESP[80, 2013]); the government was publicly discussing with the financial system the possibility to introduce differentiated exchange rates for households making prepayments of debt in foreign currency.

The struggle of Hungarian households and of the Hungarian government is indicative of the importance of monetary and exchange rate strategies in a context where financial contracts in foreign currency are pervasive, in particular taking into account (among others) the effect of inflation and the nominal exchange rate on the financial health of domestic agents.

This chapter characterizes optimal monetary and exchange rate policies for an open economy where financial claims are denominated in foreign currency. The first section of the chapter builds a New Keynesian model of the small open economy which is standard in the literature except for the following two key features. The first consists of abandoning the representative agent framework in the domestic economy with the aim of introducing a domestic financial market. This market is incomplete in the sense that *domestic households* will only be able to lend and borrow to each other in a nominal, non-contingent debt instrument[1]. This form of incompleteness will necessarily require some form of incompleteness in the international financial market as well (that is, in the market where households *of different countries* will exchange financial claims)[2]. The second key non-standard feature of the model is that the financial instrument traded by households of the domestic economy is denominated in foreign currency. As was the case for Hungary in recent years, the denomination of financial instruments will imply that the nominal exchange rate plays a crucial role in the degree to which the households of the domestic economy share risk. Importantly, the only driving force of the model is shocks to the productivity of labour.

Naturally, part of the strategy of monetary policy may include the decision to allow domestic agents to take positions among themselves in foreign currency in the first place. In other words, it is clear that the issues raised by the pervasiveness of foreign currency debt would disappear had the policymaker forbidden households to take this positions. This chapter will not consider the decision of the policymaker with regard to regulating ex-ante the ability of households to trade financial instruments in foreign currency, but will rather focus on optimal policy *once* financial contracts denominated in foreign currency have become a dominant feature in the financial system of the economy[3].

Beyond the effect of the nominal exchange rate on risk-sharing across domestic agents, monetary policy is affected by the introduction of these new elements in several ways. In a

[1] In particular, the model assumes an overlapping generations (OLG) structure with different generations of households that will borrow and lend from each other as a consequence of the fact that individual income is characterized by a standard life-cycle pattern. The incompleteness of the domestic financial markets will have the well-known implication that risk-sharing across generations is potentially suboptimal (from the point of view of a benevolent policymaker). The potential effects of monetary and exchange rate policies on risk-sharing across the generations of the model will be a key part of the story told by the model.

[2] As will be seen, the model presented below will assume the most extreme form of international market incompleteness, financial autarky. This implies that domestic households will not be able to share risk with foreign households. In such a way, the model 'stacks the cards' against risk sharing by domestic households. This assumption, however, goes against the results of the model as eliminating it would only weaken the incentive of optimal monetary policy to be proactive in the promotion of risk-sharing across households if this had already been achieved by other means.

[3] For an account of a risk-sharing argument to explain the high pervasiveness of foreign currency financial contracts in economies that recently achieved price stability, see Rappoport[67, 2009].

closed economy setting, for example, Sheedy[74, 2013] has demonstrated that the introduction of market incompleteness in an economy with heterogeneous agents may potentially render the strategy of inflation targeting suboptimal when compared to a Nominal GDP targeting rule. The intuition for this result is straightforward and, given its importance to understand the results of this chapter, it merits a brief consideration.

The basic model by Sheedy[74, 2013] considers an endowment economy with flexible prices, where the only existing friction is the financial market incompleteness, where households can trade only a nominal, non-contingent bond. This incompleteness implies that ex-post, aggregate risk is potentially unevenly shared across households. For instance, when inflation is constant (say, because of the Central Bank following an Inflation Targeting rule), creditors are relatively isolated from any productivity shock (they hold a non-contingent bond in real terms) whereas debtors are overly exposed to risk (their capacity to fulfil financial promises is affected by changes in income). From the point of view of ex-ante efficiency and given that households are risk-averse, a benevolent policymaker would like to devise a mechanism to transfer wealth from creditors to debtors after a particularly bad shock[4]. That mechanism is inflation, which changes the real burden of debt for debtors. The wealth transfers induced by inflation (which are not arbitrary but specifically engineered as part of an optimal monetary policy strategy) improve on risk-sharing and increase welfare from an ex-ante perspective.

The combination of market incompleteness with debt instruments denominated in foreign currency introduces additional considerations for a policymaker concerned about ex-ante risk-sharing, price stability and output gap stability. First, on the risk-sharing front, nominal depreciation emerges as an additional mechanism to transfer wealth across households. However, the ability of monetary policy to steer the nominal exchange rate and inflation in potentially different directions is related to its ability to control the real exchange rate, the terms of trade and output. In principle, a negative productivity shock should trigger nominal appreciations (and vice versa) in order to reduce the value of outstanding debt liabilities, and thus transfer wealth from creditors to debtors. However, the desire of the policymaker to improve on risk-sharing this way clashes with its objective to stabilize output and prices. In this sense, there is a trade-off for monetary policy between risk-sharing and the standard macroeconomic objectives, which is essentially a quantitative matter.

Optimal monetary and exchange rate policies are characterized analytically in the second section of this chapter by means of the Linear-Quadratic method. This is common to a great portion of the literature on optimal policy in New Keynesian models. From this characterization, the model provides a natural extension to the parametric condition studied by Cole and Obstfeld[28, 1991]. In particular, it is found that the same condition studied

[4] In general, from an ex-ante perspective the policymaker would like to transfer wealth from low marginal utility-households to high marginal utility-households after any aggregate shock. A negative (positive) productivity shock will imply a relatively low (high) consumption -high (low) marginal utility- for debtors.

by those authors (namely, unitary elasticity of substitution across goods produced in different countries) is a sufficient condition for full risk-sharing across domestic households independently of monetary policy.

The third section of the chapter studies the response of key variables of the model to a negative productivity shock under the optimal policy rule compared to a set of alternative policy regimes[5]: Producer Price Index (PPI) Inflation Targeting, Consumer Price Index (CPI) Inflation Targeting and an Exchange Rate Peg. For the calibration considered, the optimal monetary policy rule ascribes most of the weight in the abovementioned trade-off to the standard macroeconomic objectives of price and output stabilization. To this purpose, the policymaker will therefore mostly sacrifice risk-sharing considerations, which indicates that the objective of improving on risk-sharing is excessively costly from the point of view of the policymaker. In addition, some risk sharing takes place automatically if the calibration is close to the parametric condition of Cole and Obstfeld[28, 1991], which reduces the incentive of the policymaker to sacrifice the stability of inflation and output in favour of risk-sharing. As a consequence, the optimal policy rule is found to be closest to PPI Inflation Targeting than to Consumer Price Index Inflation Targeting or an Exchange Rate Peg, due to the ability of the former to replicate the flexible price equilibrium allocation.

Despite optimal policy mostly sacrificing risk-sharing considerations, the nominal exchange rate will still play an important role in creating ex-post redistributions of wealth. The fourth section of the chapter calculates the relative welfare losses imposed on households by an Exchange Rate Peg. It is found that a Peg can be significantly more harmful in terms of welfare in the overlapping generations model of Section 1.1 than in standard, representative agent models of the open economy, the reason being the need for the exchange rate to respond actively to productivity shock in a context where risk is unevenly shared across the small open economy.

The model of this chapter combines distinct elements from two separate areas of the literature. The first strand focuses on the study of optimal policy in New Keynesian models of the small open economy. The review by Corsetti, Dedola and Leduc[29, 2011] summarizes the issues at hand in characterizing optimal monetary policy in this setting[6]. A seminal work that inspires the spirit of this chapter is Galí and Monacelli[38, 2005], who derive optimal policy for a small open economy under internationally complete markets. Di Paoli[62, 2009] abandons this last assumption to study optimal policy under different international financial market structures, whereas Benigno and Benigno[5, 2006] abandon the small open economy setting to explore the international coordination aspects of monetary policy. All these papers assume that, at the national (domestic) level, either the economy is populated

[5] Following Galí and Monacelli[38, 2005].

[6] Key papers among this literature include Obstfeld and Rogoff[57, 1995], McCallum and Nelson[53, 2001], Clarida, Galí and Gertler[27, 2001], Corsetti and Pesenti[30, 2001] and De Fiore and Liu[36, 2005].

by a representative agent or by a continuum of identical households with perfect risk-sharing across its members. This chapter abandons this assumption to postulate instead the existence of a domestic financial market and imperfect risk-sharing at the national level.

The second strand of the literature is relatively younger, its main feature being the abandonment of the representative agent assumption to study optimal monetary policy under incomplete markets in a closed economy, New Keynesian setting. Besides the work by Sheedy[74, 2013] mentioned above, Pescatori[64, 2007] studies optimal policy as it relates to ex-post wealth redistribution across rich and poor individuals. Finally, Schmitt-Grohé and Uribe[72, 2004] introduce nominal rigidities into the otherwise classical framework of incomplete markets and optimal fiscal policy of Chari and Kehoe[24, 1999]. They discover that optimal monetary policy will not perform a great deal of ex-post wealth redistribution between the government and households as this would imply extremely volatile inflation. The results of this chapter are similar to theirs in the sense of monetary policy being relatively passive to risk-sharing considerations in order to avoid inflation variability, but in this chapter tax smoothing plays no role, redistribution is made across households and the key to the desirability of risk-sharing is risk aversion on the side of the latter.

This chapter therefore bridges the gap between these two strands of the literature by borrowing the open economy insights of the former and combining them with the domestic incomplete markets of the latter. The resulting framework is expanded to include foreign currency denominated financial claims and in such way to most closely resemble the environment of those group of economies (discussed above) most affected by the penetration of foreign currency debt in incomplete markets.

1.1. The Model

The model follows closely the open economy structure of Corsetti, Dedola and Leduc[29, 2011] and the incomplete markets framework of Sheedy[74, 2013]. The model postulates a world economy populated by a continuum of households of measure 1. A fraction n of these individuals reside in country H (Home), and the remaining fraction $1 - n$ reside in country F (Foreign). There is an international bond market in which the only financial instrument available is a nominal, one period, non-contingent bond denominated in the currency of country F. While the Home economy will be characterized by a generational structure that gives rise to a domestic financial market, the Foreign economy will be modeled as a standard, representative agent economy.

1.1.1. Households

1.1.1.1. Home Households

The Home economy is populated by a continuum of households of measure n. Each of these households lives for three periods. In the first period, the household is young, and his choice variables are indexed by y. In the second period, the household is middle-aged (m), and in the third period the household is old (o). At a given time, three generations (or cohorts) exist, each of which has a measure $\frac{n}{3}$. It is thus assumed that, at each period, a new cohort of young households is born in a measure that exactly replaces the measure of old households that die, in such a way that the demographic structure of the population stays invariant over time. The problem faced by every new generation of young households is given by:

$$\max_{\{C_{i,t},H_{i,t}\}} U_t \equiv \left\{ \ln C_{y,t} - \alpha_y^{-\eta} \frac{H_{y,t}^{1+\eta}}{1+\eta} \right\} + \beta E_t \left\{ \ln C_{m,t+1} - \alpha_m^{-\eta} \frac{H_{m,t+1}^{1+\eta}}{1+\eta} \right\} + \beta^2 E_t \left\{ \ln C_{o,t+2} - \alpha_o^{-\eta} \frac{H_{o,t+2}^{1+\eta}}{1+\eta} \right\} \quad (1.1.1)$$

where $C_{i,t}$ represents the consumption at time t of a basket of Home and Foreign goods by a household at period i of his life, $H_{i,t}$ his individual supply of labour, and $\eta > 0$. The set of parameters α_i is related to the disutility of work for generation i. These parameters are specified in such a way that, in equilibrium, the profile of total income over the lifetime of an individual resembles a traditional life cycle pattern (i.e., relatively low income when young and old, and relatively high income when middle-aged). The consumption basket is defined by the following CES-type aggregator:

$$C_{i,t} = \left[a_H^{\frac{1}{\phi}} \left(C_{i,t}^H \right)^{\frac{\phi-1}{\phi}} + (1-a_H)^{\frac{1}{\phi}} \left(C_{i,t}^F \right)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \quad (1.1.2)$$

with $C_{i,t}^j$ denoting the consumption of goods produced in country j (in what follows, "j goods") at time t by a household in period i of his life. The parameter a_H captures the degree of "home bias" in consumption, and it can also be interpreted as a measure of the "openness" of the economy. The parameter ϕ represents the elasticity of substitution across H and F goods. The consumption of H and F goods is itself a CES-type aggregate of infinite varieties produced in the respective country with a common elasticity of substitution

ϵ, as follows:

$$
\begin{aligned}
C_{i,t}^{H} &= \left[\int_{0}^{n} \left(\frac{1}{n} \right)^{\frac{1}{\epsilon}} C_{i,t}^{H}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} \\
C_{i,t}^{F} &= \left[\int_{n}^{1} \left(\frac{1}{1-n} \right)^{\frac{1}{\epsilon}} C_{i,t}^{F}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}
\end{aligned}
\tag{1.1.3}
$$

where $C_{i,t}^{x}(j)$ denotes consumption of variety j produced in country x, and $\epsilon > \phi$ is assumed[7]. The price of the consumption basket, or equivalently, the Consumer Price Index (CPI) of the Home economy, is given, following standard results from CES-type aggregators, by:

$$
P_t = \left[a_H \left(P_t^H \right)^{1-\phi} + (1-a_H) \left(P_t^F \right)^{1-\phi} \right]^{\frac{1}{1-\phi}} \tag{1.1.4}
$$

where P_t^j represents the nominal price of a good produced in country j at time t measured in currency units of the Home economy, and P_t^x is the price of a composite good produced in country x, defined by:

$$
\begin{aligned}
P_t^H &= \left[\left(\frac{1}{n} \right) \int_{0}^{n} P_t^H(j)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}} \\
P_t^F &= \left[\left(\frac{1}{1-n} \right) \int_{n}^{1} P_t^F(j)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}
\end{aligned}
\tag{1.1.5}
$$

P_t^H and P_t^F will be referred to in what follows as the Producer Price Indices (PPI) of the Home and Foreign economies, respectively, as they measure the price level of those goods produced within a given country. Finally, the allocation of the composite good produced in

[7]This assumption implies that varieties produced within a country are more similar (more substitutable) than goods produced in different countries. See Corsetti, Dedola and Leduc[29, 2011].

each country among each variety j is:

$$C_{i,t}^{H}(j) = a_H \left(\frac{1}{n}\right) \left[\frac{P_t^H}{P_t^H(j)}\right]^{\epsilon} \left(\frac{P_t}{P_t^H}\right)^{\phi} C_{i,t} \quad (1.1.6)$$

$$C_{i,t}^{F}(j) = (1-a_H) \left(\frac{1}{1-n}\right) \left[\frac{P_t^F}{P_t^F(j)}\right]^{\epsilon} \left(\frac{P_t}{P_t^F}\right)^{\phi} C_{i,t} \quad (1.1.7)$$

The budget constraints faced by a household at each of the three stages of his life, expressed in units of the Home currency, are given by:

$$P_t C_{y,t} + Q_t B_{y,t} e_t + \frac{M_{y,t}}{1+i_t} \leq W_{y,t} H_{y,t} + \alpha_y P_t J_t - P_t T_{y,t} \quad (1.1.8)$$

$$P_t C_{m,t} + Q_t B_{m,t} e_t + \frac{M_{m,t}}{1+i_t} \leq W_{m,t} H_{m,t} + \alpha_m P_t J_t - P_t T_{m,t} + B_{y,t-1} e_t + M_{y,t-1} \quad (1.1.9)$$

$$P_t C_{o,t} \leq W_{o,t} H_{o,t} + \alpha_o P_t J_t - P_t T_{o,t} + B_{m,t-1} e_t + M_{m,t-1} \quad (1.1.10)$$

Q_t is the price of a nominal, one period, non-contingent bond at time t. As this price is measured in currency units of country F currency, the nominal exchange rate e_t is also part of the budget constraint. One unit of this bond purchased at t promises the bearer the payment of one unit of foreign currency at $t+1$. The quantity of bonds purchased by a household is denoted by B. Note that the nominal exchange rate e_t is an important determinant of the burden of debt to be paid (or collected) by the middle-aged and old generations at the beginning of each period. There is a Home Central Bank that produces money, M. Households can deposit their holdings of money at the Central Bank at the riskless nominal interest rate i. Individuals are ex-ante homogeneous in the sense of having the same preferences, the same life cycle evolution of their endowment, and the same (zero) initial wealth. $W_{i,t}$ is the nominal wage and α_i also represents the proportion of total profits received by individuals of generation i[8]. Finally, J_t is the aggregate profits of firms in country H and $T_{i,t}$ represents lump-sum levied charged on generation i by the government (in units of the composite good C). For future reference, let C_t^H and C_t^F denote aggregate

[8]These parameters α_i coincide with the parameters of the disutility of labour in (1.1.1). It will be shown this structure implies constancy of the shares of aggregate income received by each generation.

home economy consumption of home and foreign goods respectively, defined as:

$$nC_t^H = \frac{n}{3}C_{y,t}^H + \frac{n}{3}C_{m,t}^H + \frac{n}{3}C_{o,t}^H \quad (1.1.11)$$
$$nC_t^F = \frac{n}{3}C_{y,t}^F + \frac{n}{3}C_{m,t}^F + \frac{n}{3}C_{o,t}^F \quad (1.1.12)$$

and aggregate consumption as:

$$nC_t = \frac{n}{3}C_{y,t} + \frac{n}{3}C_{m,t} + \frac{n}{3}C_{o,t} \quad (1.1.13)$$

Optimality Conditions The necessary first order conditions of the optimization problem (1.1.1) subject to (1.1.8)-(1.1.10) are reduced to the following set of Euler and intratemporal equations:

$$\frac{\beta}{Q_t}E_t\left[\frac{e_{t+1}}{e_t}\frac{P_t}{P_{t+1}}\left(\frac{C_{y,t}}{C_{m,t+1}}\right)\right] = 1 \quad (1.1.14)$$
$$\frac{\beta}{Q_t}E_t\left[\frac{e_{t+1}}{e_t}\frac{P_t}{P_{t+1}}\left(\frac{C_{m,t}}{C_{o,t+1}}\right)\right] = 1 \quad (1.1.15)$$
$$\beta E_t\left[\frac{P_t}{P_{t+1}}\left(\frac{C_{y,t}}{C_{m,t+1}}\right)\right] = \frac{1}{1+i_t} \quad (1.1.16)$$
$$C_{i,t}\left(\frac{\alpha_i}{H_{i,t}}\right)^{-\eta} = w_{i,t} \quad (1.1.17)$$

where $w_{i,t} = {}^{W_{i,t}}/_{P_t}$ denotes the real wage (expressed in units of the composite good).

1.1.1.2. Foreign Households

The Foreign economy is populated by a continuum of identical, infinitely-lived households of measure $1-n$. The problem of the representative household of this economy is given by:

$$\max_{\{C_t^*, H_t^*\}} U_t^* = E_t\sum_{\tau=0}^{\infty}\beta^\tau\left[\ln C_{t+\tau}^* - \frac{\left(H_{t+\tau}^*\right)^{1+\eta}}{1+\eta}\right] \quad (1.1.18)$$

where '$*$' will refer in what follows to variables of the foreign economy. C_t^* is a basket of goods produced in the Home and the Foreign country analogous to (1.1.2) with a bias parameter a_H^*. This parameter indicates the preference of the foreign households for H goods and represents a measure of the "foreign bias" of the Foreign economy. The expressions for the consumption aggregators, CPI, PPI and optimal consumption allocation of the Foreign economy are analogous to (1.1.4)-(1.1.7) replacing respective terms with C_t^{H*}, C_t^{F*}, $C_t^{H*}(j)$, $C_t^{F*}(j)$, P_t^*, P_t^{H*}, P_t^{F*}, $P_t^{H*}(j)$, $P_t^{F*}(j)$, where price levels are denominated in units of the foreign currency.

The budget constraint of the representative household of the foreign economy (in units of the foreign currency) is given by:

$$P_t^* C_t^* + Q_t B_t^* + \frac{M_t^*}{1+i_t^*} = W_t^* H_t^* + P_t^* J_t^* - P_t^* T_t^* + M_{t-1}^* + B_{t-1}^* \qquad (1.1.19)$$

where the nominal exchange rate is not included as long as the prices and quantities of bonds/securities are denominated in the currency of the foreign country. There is also a Foreign Central Bank who produces money M^* and riskless deposits for money at the nominal interest rate i^*. The remaining variables have the analogous interpretation as in the Home economy.

Optimality Conditions The necessary first order conditions of the optimization problem of the foreign household (1.1.18) subject to the sequence of constraints (1.1.19) are given by:

$$\beta E_t \left[\frac{P_t^*}{P_{t+1}^*} \left(\frac{C_t^*}{C_{t+1}^*} \right) \right] = Q_t \qquad (1.1.20)$$

$$\beta E_t \left[\frac{P_t^*}{P_{t+1}^*} \left(\frac{C_t^*}{C_{t+1}^*} \right) \right] = \frac{1}{1+i_t^*} \qquad (1.1.21)$$

$$C_t^* \left(H_t^* \right)^\eta = w_t^* \qquad (1.1.22)$$

where $w_t^* = W_t^*/P_t^*$ is the real wage of the foreign economy.

1.1.1.3. Interest Rate Parity

From the optimality conditions of households across the world it is possible to derive a version of the Uncovered Interest Rate Parity Condition (UIP). From (1.1.20) and (1.1.21), it is the case that:

$$Q_t = \frac{1}{1+i_t^*} \tag{1.1.23}$$

From (1.1.14) and (1.1.16), using (1.1.23), the UIP condition is:

$$(1+i_t^*)\, E_t\left[\frac{e_{t+1}}{e_t}\frac{P_t}{P_{t+1}}\left(\frac{C_{y,t}}{C_{m,t+1}}\right)\right] = E_t\left[\frac{P_t}{P_{t+1}}\left(\frac{C_{y,t}}{C_{m,t+1}}\right)\right](1+i_t) \tag{1.1.24}$$

1.1.2. Terms of Trade, Real Exchange Rate and Demand Functions

Let S_t denote the terms of trade of the Home economy, defined as the relative price of foreign goods in terms of home goods:

$$S_t \equiv \frac{P_t^F}{P_t^H} \tag{1.1.25}$$

Let Ω_t denote the real exchange rate of the home economy, defined as the ratio between the CPI of the foreign and the home economies, expressed in the same currency:

$$\Omega_t \equiv \frac{e_t P_t^*}{P_t} \tag{1.1.26}$$

In what follows, a law of one price for individual varieties produced in both countries is assumed[9]. For goods produced in country x:

$$P_t^x(j) = e_t P_t^{x*}(j) \tag{1.1.27}$$

It is straightforward to demonstrate that (1.1.27) implies:

$$e_t P_t^{H*} \;=\; P_t^H \qquad e_t P_t^{F*} = P_t^F \tag{1.1.28}$$

The real exchange rate is therefore rewritten as:

$$\Omega_t \equiv \left[\frac{a_H^*\left(P_t^H\right)^{1-\phi} + (1-a_H^*)\left(P_t^F\right)^{1-\phi}}{a_H\left(P_t^H\right)^{1-\phi} + (1-a_H)\left(P_t^F\right)^{1-\phi}}\right]^{\frac{1}{1-\phi}} \tag{1.1.29}$$

[9]This result can also be endogenously derived assuming producer-currency pricing on the side of firms, given the common elasticity of substitution ϵ across varieties in local aggregators.

The real exchange rate collapses to 1 if $a_H = a_H^*$. As will be seen, the ability of monetary policy to alter real outcomes in the Home economy is tightly linked to its ability to control the real exchange rate. Therefore, for the rest of the chapter it is assumed that $a_H \neq a_H^*$[10].

1.1.2.1. Demand Functions

For the specification of the problems of firms, it will be useful to calculate the aggregate demand for a given variety produced in a given country. For varieties produced at Home and in the Foreign economy respectively, these will be given by:

$$
\begin{aligned}
Y_t^{H,d}(j) &= nC_t^H(j) + (1-n)\,C_t^{H*}(j) \\
Y_t^{F,d}(j) &= nC_t^F(j) + (1-n)\,C_t^{F*}(j)
\end{aligned}
$$

Using (1.1.6), (1.1.7) and (1.1.11)-(1.1.13), these can be rewritten as:

$$
Y_t^{H,d}(j) = \left(\frac{P_t}{P_t^H}\right)^{\phi}\left[\frac{P_t^H}{P_t^H(j)}\right]^{\epsilon} Y_t^d \qquad (1.1.30)
$$

$$
Y_t^{F,d}(j) = \left(\frac{P_t}{P_t^F}\right)^{\phi}\left[\frac{P_t^F}{P_t^F(j)}\right]^{\epsilon} Y_t^{*d} \qquad (1.1.31)
$$

where $Y_t^d = a_H C_t + \left(\frac{1-n}{n}\right) a_H^* \Omega_t^{\phi} C_t^*$ is the total demand faced by the Home economy and $Y_t^{*d} = \left(\frac{n}{1-n}\right)(1-a_H)\,C_t + (1-a_H^*)\,C_t^*\Omega_t^{\phi}$ is the total demand faced by the Foreign economy.

1.1.3. Firms

1.1.3.1. Home Firms

A typical firm in the Home economy operates in a monopolistically competitive environment, producing a differentiated good (j) with the following linear technology in labour:

$$
Y_t^H(j) = A_t N_t(j) \qquad (1.1.32)
$$

[10] Notice this assumption need not imply the existence of home bias, which is observed when $a_H > 1/2$.

A_t represents the total productivity of labour and constitutes the only exogenous stochastic process of the economy. Following the literature on wage stickiness in New Keynesian models (in particular Erceg et al[34, 2000]), the demand for labour from different generations is aggregated using a Cobb-Douglas specification[11]:

$$N_t(j) = AN_{y,t}(j)^{\frac{\alpha_y}{3}} N_{m,t}(j)^{\frac{\alpha_m}{3}} N_{o,t}(j)^{\frac{\alpha_o}{3}} \tag{1.1.33}$$

with $A = \left[\left(\frac{\alpha_y}{3}\right)^{\frac{\alpha_y}{3}} \left(\frac{\alpha_m}{3}\right)^{\frac{\alpha_m}{3}} \left(\frac{\alpha_o}{3}\right)^{\frac{\alpha_o}{3}}\right]^{-1}$, and where $N_{i,t}(j)$ is the employment of hours of labour by individuals of generation i. Firms receive a proportional wage bill subsidy τ on labour costs. The firm solves a problem of allocating labour from different generations analogous to the one faced by the households when allocating components of a composite consumption basket. The cost-minimizing generational labour demand functions are given by:

$$\frac{\alpha_i}{3}\frac{w_t}{w_{i,t}}N_t(j) = N_{i,t}(j) \qquad i = y, m, o. \tag{1.1.34}$$

where $w_t = w_{y,t}^{\frac{\alpha_y}{3}} w_{m,t}^{\frac{\alpha_m}{3}} w_{o,t}^{\frac{\alpha_o}{3}}$ is the real wage index of the Home economy. The problem of the home firm is to maximise the present value of lifetime instantaneous profits in real terms:

$$\begin{aligned} J_t^H(j) = & \frac{P_t^H(j)\, Y_t^H(j)}{P_t} - (1-\tau) \times \\ & \left[w_{y,t}N_{y,t}(j) + w_{m,t}N_{m,t}(j) + w_{o,t}N_{o,t}(j)\right] \end{aligned} \tag{1.1.35}$$

The absence of intertermporal elements makes the cost minimization problem essentially static. Using (1.1.32), (1.1.34) and (1.1.30), the problem of firms can be redefined as:

$$\max_{P_t^H(j)} J_t^H(j) = \left\{ \frac{P_t^H(j)}{P_t}\left(\frac{P_t}{P_t^H}\right)^{\phi}\left[\frac{P_t^H}{P_t^H(j)}\right]^{\epsilon} - \right. \tag{1.1.36}$$

$$\left. (1-\tau)\, x_t \left(\frac{P_t}{P_t^H}\right)^{\phi}\left[\frac{P_t^H}{P_t^H(j)}\right]^{\epsilon} \right\} Y_t^d \tag{1.1.37}$$

where $x_t = \frac{w_t}{A_t}$ is the aggregate marginal cost and P_t, x_t, P_t^H and Y_t^d are taken as given. The solution of this problem depends on the price formation mechanisms of the economy.

[11] Erceg et al[34, 2000] employ a general CES aggregator for heterogeneous labour; the specification used in this chapter borrows the idea of using consumption-style aggregators.

Price Stickiness: Different Information Sets It is assumed that all firms set prices in the currency of the producer country (referred to in the literature as Producer Currency Pricing, PCP). This chapter considers a form of price rigidity in which different firms have random access to a different set of information. In particular, a fraction $1-\kappa$ of firms in the Home economy sets an optimal price with information up-to-date at the moment of making choices (that is, P_t, x_t, P_t^H and Y_t^d are observed at the moment of solving problem (1.1.36)). The remaining fraction κ sets an optimal price at t with the information set of period $t-1$, and must therefore rely on forecasts of the relevant variables P_t, x_t, P_t^H and Y_t^d . Notice that all firms are allowed to change prices between time periods.

The first order condition of problem (1.1.36) is common for all firms operating under full information. The optimal price will be therefore common among this group and equal to:

$$\hat{P}_t^H = x_t \mu (1-\tau) P_t \tag{1.1.38}$$

where $\mu = \frac{\epsilon}{\epsilon - 1}$, with $\mu(1-\tau)$ being the "gross effective markup" (net of the wage subsidy) charged by firms over marginal cost in nominal terms.

All firms with lagged information, on the other hand, solve the following problem:

$$\max_{P_t^H(j)} E_{t-1} J_t^H(j) \tag{1.1.39}$$

where $P_t^H(j) = E_{t-1}\left[P_t^H(j)\right]$. The first order condition of this problem is:

$$E_{t-1}\left\{\left(\frac{\check{P}_t^H}{P_t^H}\right)^{-\epsilon}\left(\frac{P_t}{P_t^H}\right)^{\phi} Y_t^d \left[\frac{\check{P}_t^H}{P_t} - \mu(1-\tau) x_t\right]\right\} = 0 \tag{1.1.40}$$

where $\check{P}_t^H$ is the common price chosen by these firms. From the definition of the PPI (1.1.5):

$$\left(P_t^H\right)^{1-\epsilon} = \left(\frac{1}{n}\right)\int_0^n P_t^H(j)^{1-\epsilon} dj = (1-\kappa)\left(\hat{P}_t^H\right)^{1-\epsilon} + \kappa\left(\check{P}_t^H\right)^{1-\epsilon} \tag{1.1.41}$$

Letting $\widehat{p}_t^H = \frac{\hat{P}_t^H}{P_t^H}$ and $\check{p}_t^H = \frac{\check{P}_t^H}{P_t^H}$, we can relate the prices set by firms belonging to different groups as follows:

$$\begin{aligned} 1 &= (1-\kappa)\left(\hat{p}_t^H\right)^{1-\epsilon} + \kappa\left(\check{p}_t^H\right)^{1-\epsilon} \\ \hat{p}_t^H &= \left[\frac{1}{1-\kappa} - \frac{\kappa}{1-\kappa}\left(\check{p}_t^H\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \end{aligned} \tag{1.1.42}$$

These results allow the pricing equations to be rewritten as follows for the relative prices under full information:

$$\hat{p}_t^H = x_t\mu(1-\tau)\frac{P_t}{P_t^H} \tag{1.1.43}$$

and for the relative prices under outdated information:

$$E_{t-1}\left\{\left(\check{p}_t^H\right)^{-\epsilon}\left(\frac{P_t}{P_t^H}\right)^{\phi-1} Y_t^d\left(\check{p}_t^H - \left[\frac{1}{1-\kappa} - \frac{\kappa}{1-\kappa}\left(\check{p}_t^H\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}\right)\right\} = 0 \tag{1.1.44}$$

Importantly, (1.1.43) and (1.1.44) depend on the ratio between the CPI and the PPI at Home, which is related (as will be shown) to the terms of trade, S_t. This observation will result in the Phillips curve of the Home economy depending on its terms of trade.

1.1.3.2. Foreign Firms

A typical firm in the Foreign economy operates in a monopolistically competitive environment, but (for simplicity) it is assumed that it is not subject to any form of price stickiness. The production function of a Foreign firm is given by:

$$Y_t^{F*}(j) = N_t^*(j) \tag{1.1.45}$$

where the total productivity of labour has been set to 1. Foreign firms use only one type of labour, supplied by the representative household of the foreign economy. Following an analogous procedure as in the previous section, the relative price set by all firms in the

Foreign economy is given by:

$$\hat{p}_t^{F*} = x_t^* \mu (1-\tau) \frac{P_t^*}{P_t^{F*}} = 1 \tag{1.1.46}$$

where $x_t^* = w_t^*$, $\hat{p}_t^{F*} = \frac{\hat{P}_t^{F*}}{P_t^{F*}}$ and the remaining variables have an analogous interpretation to those in the Home economy.

1.1.4. Governments

The only role of the governments in both the Home and Foreign economies is to transfer lump-sum taxes levied on households to firms as wage subsidies with the sole purpose of eliminating market power inefficiencies from the side of firms that are not perfectly competitive. This means that the wage bill subsidy rate τ will be engineered in such a way that the gross effective markup is one: $\tau = \epsilon^{-1}$. The budget constraint of the Home government is given by:

$$nT_t = \frac{n}{3}T_{y,t} + \frac{n}{3}T_{m,t} + \frac{n}{3}T_{o,t} = \epsilon^{-1} \int_0^n \left[w_{y,t}N_{y,t}(j) + w_{m,t}N_{m,t}(j) + w_{o,t}N_{o,t}(j) \right] dj \tag{1.1.47}$$

Assume that the proportion of aggregate government revenue coming from each generation is equal to the disutility parameter α_i: $T_{i,t} = \alpha_i T_t$. For the Foreign government,

$$(1-n)\,T_t = \epsilon^{-1} \int_n^1 w_t^* N_t^*(j)\, dj \tag{1.1.48}$$

1.1.5. Aggregate Equilibrium Conditions

The components described so far allow the construction of a simple condition that relates aggregate demand to aggregate supply in each of the two economies of the model. For the case of the Home economy, condition (1.1.30) implies that an individual variety market clearing condition can be written as:

$$Y_t^H(j) = \left(\frac{P_t}{P_t^H} \right)^{\phi} \left[\frac{P_t^H}{P_t^H(j)} \right]^{\epsilon} Y_t^d \tag{1.1.49}$$

for all j. In what follows, let $nY_t = \int_0^n \frac{P_t^H(j)Y_t^H(j)}{P_t}dj$ denote the real GDP in the Home country in terms of the composite good. Then, by (1.1.49), the aggregate equilibrium condition of the Home economy is:

$$Y_t = Y_t^d \left(\frac{P_t}{P_t^H}\right)^{\phi-1} \tag{1.1.50}$$

where we have used (1.1.5). The left hand side of expression (1.1.50) corresponds to the total supply of the economy. This can be transformed into an aggregate production function of the Home economy using the equilibrium conditions of the labour markets:

$$\int_0^n N_{i,t}(j)\,dj = \frac{n}{3}H_{i,t} \tag{1.1.51}$$

Letting $nN_t = \int_0^n N_t(j)\,dj$ denote the aggregate demand for labour of the Home economy, and using (1.1.32) and (1.1.50) in (1.1.49), the aggregate production function of the economy is obtained:

$$Y_t = A_t N_t \left(\frac{P_t}{P_t^H}\right)^{-1} \bigstar_t \tag{1.1.52}$$

where $\bigstar_t = \left\{(1-\kappa)\left[\frac{1}{1-\kappa} - \frac{\kappa}{1-\kappa}\left(\check{p}_t^H\right)^{1-\epsilon}\right]^{\frac{-\epsilon}{1-\epsilon}} + \kappa\left(\check{p}_t^H\right)^{-\epsilon}\right\}^{-1}$ captures the distorsions created by price dispersion in the Home economy.

Finally, for the Foreign economy, letting $(1-n)\,Y_t^* = \int_n^1 \frac{P_t^{F*}(j)Y_t^{F*}(j)}{P_t^*}dj$ denote the foreign real GDP, the equilibrium conditions imply:

$$Y_t^* = Y_t^{*d}\Omega_t^{-\phi}\left(\frac{P_t^*}{P_t^{F*}}\right)^{\phi-1} \tag{1.1.53}$$

and the aggregate production function is obtained following a similar procedure as above (the foreign labour market equilibrium condition being $\int_n^1 N_t^{F*}(j)\,dj = (1-n)\,H_t^*$):

$$Y_t^* = H_t^*\left(\frac{P_t^*}{P_t^{F*}}\right)^{-1} \tag{1.1.54}$$

1.1.5.1. Non-Financial Household Income

The structure of the model described so far implies that the equilibrium non-financial income of a given generation at Home is a constant share of aggregate nominal GDP. Using (1.1.34), (1.1.51), and the definition of aggregate profits at Home $nJ_t^H = \int_0^n J_t^H(j)\,dj$:

$$W_{i,t}H_{i,t} + \alpha_i P_t J_t - P_t T_{i,t} \quad = \quad \alpha_i P_t Y_t \tag{1.1.55}$$

Following a similar case, the total nominal non-financial income of the foreign household is equal to $P_t^* Y_t^*$. As discussed above, the parameters α_y, α_m, and α_o are set such that the profile of total non-financial income over the lifetime of an individual resembles a traditional life cycle pattern. Following Sheedy[74, 2013], this pattern is reduced to a structural parameter γ that relates the set of α_i as follows:

$$\alpha_y = 1 - \beta\gamma \qquad \alpha_m = 1 + (1+\beta)\,\gamma \qquad \alpha_o = 1 - \gamma \tag{1.1.56}$$

with $0 \leq \gamma \leq 1$ representing the slope of the life-cycle income pattern. Total non-financial income will be therefore maximum while middle-aged and minimum while old. The structure of the Home economy can be reduced to a simple representative economy framework by setting $\gamma = 0$. In that case, all generations are exactly the same ex-ante and ex-post, and there will be no trade in the domestic financial market. The domestic financial market will emerge as a result of different generations having different incomes $(\gamma > 0)$, different propensities to save and/or borrow and a desire to smooth consumption across their lifetime.

1.1.6. Marginal Cost and the Phillips Curve

The Phillips curve of the Home economy can be derived from the first order conditions of households and from the pricing equations. From (1.1.17):

$$C_{i,t}^{\frac{1}{1+\eta}} \left(w_t N_t\right)^{\frac{\eta}{1+\eta}} \quad = \quad w_{i,t} \tag{1.1.57}$$

Using the definition of the real wage and the aggregate production function (1.1.52), the real marginal cost can be rewritten as:

$$x_t = \frac{C_{y,t}^{\frac{\alpha_y}{3}} C_{m,t}^{\frac{\alpha_m}{3}} C_{o,t}^{\frac{\alpha_o}{3}}}{(A_t)^{1+\eta}\left[\frac{Y_t}{\bigstar_t}\left(\frac{P_t}{P_t^H}\right)\right]^{-\eta}} \tag{1.1.58}$$

Using (1.1.42), (1.1.58) and $\mu(1-\tau) = 1$, the pricing equation (1.1.43) can be reexpressed as:

$$\left[\frac{1}{1-\kappa} - \frac{\kappa}{1-\kappa}\left(\check{p}_t^H\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} = \frac{C_{y,t}^{\frac{\alpha_y}{3}} C_{m,t}^{\frac{\alpha_m}{3}} C_{o,t}^{\frac{\alpha_o}{3}}}{(A_t)^{1+\eta}\left[\frac{Y_t}{\bigstar_t}\left(\frac{P_t}{P_t^H}\right)\right]^{-\eta}} \frac{P_t}{P_t^H} \tag{1.1.59}$$

Equation (1.1.59) represents the non-linear Phillips curve of the Home economy. The left hand side of this expression includes elements related to the degree of price stickiness and to inflation, whereas the right hand side relates to the real side of the economy (recall the ratio between P_t and P_t^H is related to the terms of trade, a real variable). For the foreign economy, the analogous expression is:

$$1 = \frac{C_t^{*\sigma}}{\left[Y_t^{F*}\left(\frac{P_t^*}{P_t^{F*}}\right)\right]^{-\eta}} \frac{P_t^*}{P_t^{F*}} \tag{1.1.60}$$

1.1.7. The Equilibrium of a Small Open Home Economy under Financial Autarky

The system that characterizes the world equilibrium is composed of equations (1.1.8)-(1.1.10), (1.1.14)-(1.1.16), (1.1.19), (1.1.21), (1.1.44), (1.1.50), (1.1.53), (1.1.59), (1.1.60), and a market clearing condition for the international bond market:

$$\frac{n}{3}\left(B_{y,t} + B_{m,t}\right) + (1-n) B_t^* = 0 \tag{1.1.61}$$

using (1.1.23) to replace Q_t everywhere. Following di Paoli[62, 2009], the Home economy can be reduced to a small open economy by taking the limit of the system of equations when $n \rightarrow 0$ and $a_H^* \rightarrow 0$. For simplicity and to preserve international trade, the additional assumption of $\left(\frac{1-n}{n}\right) a_H^* \rightarrow 1 - a_H$ is imposed.

It is also assumed that the Foreign economy is in a steady state with zero inflation and zero initial wealth, which in equilibrium implies $B^* = 0$ for all t. The latter implies the following financial autarky condition:

$$B_{y,t} = -B_{m,t} = B_t \tag{1.1.62}$$

which indicates that any resources borrowed by the young generation must be lent by the middle-aged generation. It is in this sense that the financial market is domestic, under the assumption of financial autarky in international markets. The latter implies that the only possibility for domestic (that is, Home economy) households to share risk is through a domestic financial market. International risk sharing is not allowed in the model[12]. Adding up the set of budget constraints of the Home economy (1.1.8)-(1.1.10), using (1.1.55) and (1.1.56), the following simple trade balance condition is derived.

$$C_t = Y_t \tag{1.1.63}$$

A similar condition for the Foreign economy ($C^* = Y^*$) is derived directly from (1.1.53) using $a_H^* \to 0$ and $P^* = P^{F*}$.

The condition $a_H^* \to 0$ also implies the following relationships between the terms of trade and the real exchange rate and the ratio between the CPI and the PPI in the Home economy:

$$\Omega_t = \left[a_H S_t^{\phi-1} + (1-a_H)\right]^{\frac{1}{\phi-1}} \tag{1.1.64}$$

$$\frac{P_t}{P_t^H} = \left[a_H + (1-a_H)\, S_t^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{1.1.65}$$

Equation (1.1.63) can be combined with (1.1.50), (1.1.64), (1.1.65) and the trade balance condition in the Foreign economy to obtain:

$$\left[a_H S_t^{\phi-1} + (1-a_H)\right]^{\frac{\phi}{\phi-1}} \Delta_t = S_t^{1-\phi} \tag{1.1.66}$$

where $\Delta_t = \frac{Y^*}{Y_t}$ is a measure of relative incomes across countries. Equation (1.1.66)

[12] In a world without financial autarky, domestic households could potentially trade financial instruments and share risk with households in other countries. Thus, as mentioned before, the assumption of financial autarky "stacks the cards" against the ability of Home households to share risk and insure against productivity shocks. As will be seen in Section 1.2, the policymaker does not have a strong incentive to improve on risk-sharing relative to the decentralized equilibrium. Therefore, if anything, the assumption of financial autarky goes against the results of the chapter, insofar as eliminating it would reduce further the incentive of the policymaker to take action to improve risk-sharing across households of the Home economy, as households would be able to share risk by other means (with the reset of the world). As a consequence, introducing some possibility of international risk sharing would only strengthen the conclusions of the simulations of optimal monetary policy presented below.

determines the terms of trade of the Home economy.

These observations permit the reduction of the system that characterizes the equilibrium of the Home economy to the following equations (lower-case variables have been scaled by Y_t):

$$\beta E_t \left[\frac{R_{t+1}}{G_{t+1}} \left(\frac{c_{y,t}}{c_{m,t+1}} \right) \right] = 1 \tag{1.1.67}$$

$$\beta E_t \left[\frac{R_{t+1}}{G_{t+1}} \left(\frac{c_{m,t}}{c_{o,t+1}} \right) \right] = 1 \tag{1.1.68}$$

$$\beta E_t \left[\frac{I_t}{\Pi_{t+1} G_{t+1}} \left(\frac{c_{y,t}}{c_{m,t+1}} \right) \right] = 1 \tag{1.1.69}$$

$$c_{y,t} + l_t = 1 - \beta\gamma \tag{1.1.70}$$

$$c_{m,t} - l_t = 1 + (1+\beta)\gamma + d_t \tag{1.1.71}$$

$$c_{o,t} = 1 - \gamma - d_t \tag{1.1.72}$$

$$\left[a_H S_t^{\phi-1} + (1-a_H) \right]^{\frac{\phi}{\phi-1}} \Delta_t = S_t^{1-\phi} \tag{1.1.73}$$

$$E_{t-1} \left\{ \left(\check{p}_t^H \right)^{-\epsilon} Y_t^H \left(\check{p}_t^H - \left[\frac{1}{1-\kappa} - \frac{\kappa}{1-\kappa} \left(\check{p}_t^H \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \right) \right\} = 0 \tag{1.1.74}$$

$$\bigstar_t^{-\eta} (A_t)^{-\eta-1} Y_t^{(1+\eta)} c_{y,t}^{\frac{\alpha_y}{3}} c_{m,t}^{\frac{\alpha_m}{3}} c_{o,t}^{\frac{\alpha_o}{3}} \times \tag{1.1.75}$$

$$\left[a_H + (1-a_H) S_t^{1-\phi} \right]^{\frac{1+\eta}{1-\phi}} = \left[\frac{1}{1-\kappa} - \frac{\kappa}{1-\kappa} \left(\check{p}_t^H \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}$$

$$\left\{ (1-\kappa) \left[\frac{1}{1-\kappa} - \frac{\kappa}{1-\kappa} \left(\check{p}_t^H \right)^{1-\epsilon} \right]^{\frac{-\epsilon}{1-\epsilon}} + \kappa \left(\check{p}_t^H \right)^{-\epsilon} \right\}^{-1} = \bigstar_t \tag{1.1.76}$$

$$R_t = \frac{\bar{I}^* \xi_t}{\Pi_t} \tag{1.1.77}$$

where $c_{y,t} \equiv \frac{C_{y,t}}{Y_t}$, $c_{m,t} \equiv \frac{C_{m,t}}{Y_t}$, $c_{o,t} \equiv \frac{C_{o,t}}{Y_t}$, $G_t = \frac{Y_t}{Y_{t-1}}$, $\xi_t = \frac{e_t}{e_{t-1}}$, $\Pi_t = \frac{P_t}{P_{t-1}}$, $l_t = \frac{Q_t B_t e_t}{P_t Y_t}$,

$d_t = \frac{B_{t-1}e_t}{P_t Y_t}$, $I_t = 1 + i_t$ and $\bar{I}^*$ is the gross steady state nominal interest rate of the Foreign economy. In this setting, l_t denotes the end-of-period debt-to-GDP ratio, whereas d_t denotes the ratio between debt maturing at the beginning of time t and GDP. This beginning-of-period debt-to-GDP ratio will be the focus of the analysis of optimal monetary policy undertaken below, for it is intuitively the variable through which redistributions of wealth ex-post across borrowers and lenders operate. Equation (1.1.77) describes the ex-post real interest rate of the nominal, non-contingent bond traded within the domestic financial market of the Home economy[13]. As mentioned above, changes in this ex-post real interest rate (caused by changes in inflation or nominal depreciation) will create ex-post redistributions of wealth across lenders and borrowers, and therefore will be potentially important in the degree of risk-sharing across generations within the Home economy.

The system has the following endogenous variables: $c_{y,t}$, $c_{m,t}$, $c_{o,t}$, d_t , l_t, ξ_t , Π_t, I_t, R_t, $\bigstar_t$, $\check{p}_t^H$, S_t and Y_t. To this system of equations it is necessary to append the definition $d_t = l_{t-1}\frac{R_t}{G_t}$ and the policy rule. Following the optimal monetary policy literature, the policy rule will be provided by the solution to the optimal monetary policy problem in the form of a targeting rule for a set of key variables.

1.1.7.1. The Equilibrium of the Loglinearized System

Following standard practice, the non-linear system of equations (1.1.67)-(1.1.77) will be solved after being reexpressed in the form of logarithmic deviations from a deterministic steady state. In what follows, let $\tilde{x}_t = \ln\left(x_t/\bar{x}\right)$ denote the logarithmic deviation of variable x from its deterministic steady state value $\bar{x}$. The following proposition characterizes the deterministic steady state of the Home economy.

Proposition 1. *Assuming $\bar{A} = 1$ and $\bar{\Delta} = 1$, there exists a symmetric steady state where $\bar{c}_y = 1$, $\bar{c}_m = 1$, $\bar{c}_o = 1$, $\bar{l} = -\beta\gamma$, $\bar{d} = -\gamma$, $\beta\left(1+\bar{r}\right) = 1$, $\bar{S} = 1$, $\bar{Y} = 1$, $\bar{\check{p}}^H = \bar{\bigstar} = 1$ and $\bar{\xi} = \bar{\Pi}$. Proof: See Appendix A.1.*

[13] It is important to distinguish between the ex-post real interest rate of the economy, R, and the ex-ante real interest rate, which is related to the expectation of R and is crucial in the intertemporal allocation of consumption as indicated in equations (1.1.67)-(1.1.68).

The loglinearized system can be written as follows:

$$
\begin{aligned}
E_t\tilde{R}_{t+1}+\tilde{c}_{y,t}-E_t\tilde{G}_{t+1}-E_t\tilde{c}_{m,t+1} &= 0 && (1.1.78)\\
E_t\tilde{R}_{t+1}+\tilde{c}_{m,t}-E_t\tilde{G}_{t+1}-E_t\tilde{c}_{o,t+1} &= 0 && (1.1.79)\\
\tilde{I}_t &= E_t\tilde{\xi}_{t+1} && (1.1.80)\\
\tilde{c}_{y,t} &= \beta\gamma\tilde{I}_t && (1.1.81)\\
\tilde{c}_{m,t}+\beta\gamma\tilde{I}_t &= -\gamma\tilde{I}_{t-1}-\gamma\left(\tilde{R}_t-\tilde{G}_t\right) && (1.1.82)\\
\tilde{c}_{o,t} &= \gamma\tilde{I}_{t-1}+\gamma\left(\tilde{R}_t-\tilde{G}_t\right) && (1.1.83)\\
\tilde{d}_t &= \tilde{I}_{t-1}+\tilde{R}_t-\tilde{G}_t && (1.1.84)\\
\tilde{S}_t &= -\psi\tilde{Y}_t && (1.1.85)\\
-\frac{\kappa}{1-\kappa}\left(E_{t-1}\tilde{\Pi}_t^H-\tilde{\Pi}_t^H\right) &= (1-\eta)\left(\tilde{Y}_t-\tilde{A}_t\right) && (1.1.86)\\
&\quad+\left(\frac{\alpha_y}{3}\tilde{c}_{y,t}+\frac{\alpha_m}{3}\tilde{c}_{m,t}+\frac{\alpha_o}{3}\tilde{c}_{o,t}\right)\\
&\quad+(1-\eta)\left(1-a_H\right)\tilde{S}_t\\
\tilde{\bigstar}_t &= 0 && (1.1.87)\\
\tilde{R}_t &= -a_H\psi\left(\tilde{Y}_t-\tilde{Y}_{t-1}\right) && (1.1.88)
\end{aligned}
$$

with $\psi=\left[1-\phi\left(1+a_H\right)\right]^{-1}$. The following set of observations are in order.

First, equation (1.1.85) pins down the log-deviation of the terms of trade from its steady state only from the log-deviation of GDP. This relationship comes directly from equation (1.1.73).

Second, equation (1.1.88) reveals that the evolution of GDP also determines the log-deviation of the real interest rate. This equation is derived from the definition of the real interest rate (which in log-deviation corresponds to $\tilde{R}_t=\tilde{\Xi}_t-\tilde{\Pi}_t$), using equation (1.1.85) and the definitions of the log-deviations of S_t and Π_t:

$$\tilde{S}_t-\tilde{S}_{t-1}\equiv\tilde{\xi}_t-\tilde{\Pi}_t^H \qquad \tilde{\Pi}_t=a_H\tilde{\Pi}_t^H+\left(1-a_H\right)\tilde{\xi}_t$$

with $\Pi_t^H={}^{P_t^H}/_{P_{t-1}^H}$ being the PPI inflation rate of the Home economy. In other words, when expressed in loglinear deviations from the deterministic steady state, changes in real GDP are related to changes in the terms of trade that in turn create changes in the real interest rate through changes in the nominal depreciation and PPI inflation of the Home economy.

Third, equation (1.1.86) corresponds to the Phillips curve of the Home economy, and is derived as a loglinear approximation of the pricing equation (1.1.59) around the deterministic steady state, using the fact that $\tilde{\tilde{p}}_t^H = E_{t-1}\tilde{\Pi}_t^H - \tilde{\Pi}_t^H$.

Finally, equation (1.1.80) is the Uncovered Interest Rate Parity Condition in log-deviation form.

1.1.7.2. The Natural Allocation and Risk-Sharing

The system of equations (1.1.78)-(1.1.88) is easily solved taking into account that the subsystem composed by equations (1.1.78)-(1.1.83) coincides exactly with the system of equations of a closed economy described in Sheedy[74, 2013]. The solution of this subsystem is therefore established immediately from the following proposition.

Proposition 2. *The solution of the system of equations (1.1.78)-(1.1.88) is given by:*

$$E_t\tilde{d}_{t+1} = \lambda\tilde{d}_t \tag{1.1.89}$$

$$\tilde{R}_t = \tilde{d}_t + \frac{\tilde{d}_{t-1}}{\theta} + \tilde{G}_t \tag{1.1.90}$$

$$\tilde{c}_{y,t} = -\frac{\gamma\beta}{\theta}\tilde{d}_t \qquad \tilde{c}_{m,t} = -\gamma\left(1-\frac{\beta}{\theta}\right)\tilde{d}_t \qquad \tilde{c}_{o,t} = \gamma\tilde{d}_t \tag{1.1.91}$$

with θ and λ denoting combinations of structural parameters of the economy described in Appendix A.2.

Proof: Sheedy[74, 2013].

A particular type of equilibrium allocation that will be useful is given by studying a hypothetical small open Home economy where markets are complete (that is, where there is full risk sharing) and prices are flexible ($\kappa = 0$). This allocation will be referred to as the "natural" allocation of the small open economy, and is characterized by the following proposition.

Proposition 3. *The natural allocation of the small open economy subsystem of equations (1.1.78)-(1.1.88) is given by:*

$$\tilde{d}_t^n = 0$$

$$\tilde{R}_t^n = \tilde{G}_t^n$$

$$\tilde{c}^n_{y,t} = \tilde{c}^n_{m,t} = \tilde{c}^n_{o,t} = 0$$

where "n" denotes the "natural" allocation.

Proof: Sheedy[74, 2013].

Equation (1.1.91) indicates that the fluctuations of consumption across the different generations of the Home economy are tightly linked in equilibrium to the fluctuations of the debt-to-GDP ratio, $\tilde{d}_t$. Proposition 3 indicates that, under complete markets at Home, $\tilde{d}_t$ is fully stabilized at zero. Therefore, the degree of risk-sharing across generations (that is, the degree in which the decentralized equilibrium replicates the complete markets allocation) depends crucially on the degree in which the debt-to-GDP ratio is stabilized. From equation (1.1.90), the stabilization of $\tilde{d}_t$ is related in equilibrium to the degree to which the real interest rate responds (in the same direction) to changes in the growth rate of the economy. That is, a recessionary shock should trigger a fall in the real interest rate, and vice versa. From equation (1.1.85), this latter response is related to the endogenous reaction of output fluctuations to shocks to labour productivity, $\tilde{A}_t$.

Considering only the objective of replicating the complete markets allocation and improving on risk-sharing across generations, the required response of the real interest rate to productivity shocks also highlights the role of the nominal exchange rate in completing markets. Given PPI inflation, equation (1.1.77) indicates that the fall in the real interest rate required after a recessionary shock in order to help replicate the complete markets allocation can be brought about only through a nominal appreciation, which in turn implies an increase in S_t. This observation lends a key role to the nominal exchange rate in risk-sharing in a world where financial transactions are denominated in foreign currency: given PPI inflation, nominal appreciation reduces the real burden of debt and redistributes wealth from the creditor generation to the debtor generation. The quantitative relevance of this role will be explored in detail below.

The model thus implies a trade-off for monetary policy in the face of technological shocks: the desire by the policymaker to steer the economy towards a more stable debt-to-GDP ratio for more risk-sharing across generations potentially requires a more volatile output and (due to the Phillips curve) more volatile inflation, which generally will also be part of the objective of the policymaker. This trade-off will depend on the specific set of parameter values chosen and is therefore a quantitative matter.

Using the results from Propositions 2 and 3, defining the output gap of the Home economy as $\hat{Y}_t \equiv \tilde{Y}_t - \tilde{Y}^n_t = \tilde{Y}_t - \frac{\tilde{A}_t}{1-(1-a_H)\psi}$ and the terms of trade gap as $\hat{S}_t = \tilde{S}_t - \tilde{S}^n_t = \tilde{S}_t + \frac{\psi}{1-(1-a_H)\psi}\tilde{A}_t$, the system of equations that characterizes the equilibrium of the Home economy can be reduced to:

$$
\begin{aligned}
E_t \tilde{d}_{t+1} &= \lambda \tilde{d}_t & (1.1.92)\\
(1+a_H\psi)\left(\hat{Y}_t - \hat{Y}_{t-1}\right) &= -\tilde{d}_t - \frac{\tilde{d}_{t-1}}{\theta} - \frac{1+a_H\psi}{1-(1-a_H)\psi} \times & (1.1.93)\\
&\quad \left(\tilde{A}_t - \tilde{A}_{t-1}\right) \\
-\frac{\kappa}{1-\kappa}\left(E_{t-1}\tilde{\Pi}_t^H - \tilde{\Pi}_t^H\right) &= (1+\eta)\hat{Y}_t - \frac{\zeta}{\theta}\tilde{d}_t + (1+\eta)(1-a_H)\hat{S}_t & (1.1.94)\\
\hat{S}_t &= -\psi \hat{Y}_t & (1.1.95)
\end{aligned}
$$

where $\zeta = \left[\frac{1-\beta\gamma}{3}\beta\gamma + \gamma(\theta-\beta)\frac{1+\gamma(1+\beta)}{3} - \gamma\theta\frac{1-\gamma}{3}\right]$. Given the policy rule and a stochastic process for $\tilde{A}_t$, this system of equations provides the solution for the endogenous variables $\tilde{d}_t$, $\hat{Y}_t$, $\hat{S}_t$ and $\tilde{\Pi}_t^H$. This system of equations will constitute the set of constraints on a policymaker seeking to establish and implement an optimal monetary policy strategy. The following section is devoted to the analysis of the problem of the policymaker.

1.2. Optimal Monetary Policy

The Optimal Monetary Policy strategy will result from a benevolent policymaker/Central Bank who attempts to maximize the following welfare function, which comprises the weighted sum of utilities of every generation living in the Home economy at all times:

$$W_o = E_0\left[\frac{1}{3}\sum_{t=-2}^{\infty}\beta^t U_t\right] \tag{1.2.1}$$

subject to the system of equations (1.1.92)-(1.1.95)[14]. This optimization problem will be solved using the common approach in the New Keynesian literature developed by Rotemberg and Woodford[69, 1998] and Benigno and Woodford[7, 2004], which consists in constructing a second order approximation of the welfare function using the original system of non-linear equilibrium conditions. Appendix A.3 demonstrates that the problem of the

[14] The weight attached by the policymaker to each generation is $1/3$. With logaritmic utility, these set of weights support the complete markets equilibrium as the solution to the optimization problem of a hypothetical social planner that has access to a full set of state-contingent fiscal instruments that allows state-contingent transfers across generations (see Sheedy[74, 2013]).

policymaker is approximately equivalent to the minimization of the following loss function:

$$
\begin{aligned}
\mathcal{L}_0 \quad = \quad & E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} \frac{\chi}{\theta^2} \tilde{d}_t^2 + \frac{(1+\eta)}{2} \hat{Y}_t^2 + \frac{\epsilon}{2} \left(\frac{\kappa}{1-\kappa} \right) \left(\tilde{\Pi}_t^H - E_{t-1} \tilde{\Pi}_t^H \right)^2 \right. \qquad (1.2.2) \\
& + (1-a_H) \hat{S}_t + (1+\eta)(1-a_H) \hat{Y}_t \hat{S}_t \\
& + \frac{1}{2} (1-a_H) \left[(\phi+\eta)(1-a_H) + (1-\phi) \right] \hat{S}_t^2 \\
& \left. - \frac{\psi (1-a_H) a_H (1-\phi)}{1-(1-a_H)\psi} \hat{S}_t \tilde{A}_t \right\}
\end{aligned}
$$

where χ is a combination of structural parameters of the economy (see Appendix A.2). The policymaker picks optimal sequences for $\tilde{d}_t$, $\hat{Y}_t$, $\hat{S}_t$ and $\tilde{\Pi}_t^H$ subject to the system of linear constraints (1.1.92)-(1.1.95) given by the first order approximation to the system of non-linear equilibrium conditions. Appendix A.4 shows that the solution to this optimization problem is reduced to the following system of linear equations:

$$
\begin{aligned}
\Theta_\pi \left(\tilde{\Pi}_t^H - E_{t-1} \tilde{\Pi}_t^H \right) \quad &= \quad -\frac{\Theta_d}{\theta} \left(\tilde{d}_t - E_{t-1} \tilde{d}_t \right) - \frac{\Theta_A \psi}{1-(1-a_H)\psi} \times \qquad (1.2.3) \\
& \quad \left(\tilde{A}_t - E_{t-1} \tilde{A}_t \right) \\
\frac{\kappa}{1-\kappa} \left(\tilde{\Pi}_t^H - E_{t-1} \tilde{\Pi}_t^H \right) \quad &= \quad (1+\eta)(1-\psi+\psi a_H) \left(\hat{Y}_t - E_{t-1} \hat{Y}_t \right) \qquad (1.2.4) \\
& \quad -\frac{\zeta}{\theta} \left(\tilde{d}_t - E_{t-1} \tilde{d}_t \right) \\
(1+a_H\psi) \left(\hat{Y}_t - E_{t-1} \hat{Y}_t \right) \quad &= \quad -\left(\tilde{d}_t - E_{t-1} \tilde{d}_t \right) - \frac{1+a_H\psi}{1-(1-a_H)\psi} \times \qquad (1.2.5) \\
& \quad \left(\tilde{A}_t - E_{t-1} \tilde{A}_t \right)
\end{aligned}
$$

where Θ_π, Θ_d and Θ_A are combinations of structural parameters of the economy (see Appendix A.2). Equation (1.2.3) corresponds to the first order condition of the optimization problem. This condition encapsulates the intuition on optimal policy and risk-sharing that has been discussed throughout the chapter. For the calibration employed below, Θ_π, Θ_d and Θ_A are all positive, and $\psi < 0$. Thus, given PPI inflation, the optimal response to a negative productivity shock is to reduce the debt-to-GDP ratio. This is precisely the risk-sharing objective pursued by the policymaker. The linear system of equations provides an analytical solution for the unanticipated responses of the output gap $(\hat{Y}_t - E_{t-1}\hat{Y}_t)$, the debt-to-GDP ratio $(\tilde{d}_t - E_{t-1}\tilde{d}_t)$ and PPI inflation $(\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H)$ as a function of the only driving process of the model, the unanticipated innovation in labour productivity $(\tilde{A}_t - E_{t-1}\tilde{A}_t)$. It is also possible to calculate the response of the terms of trade, the real return and the nominal exchange rate. The unanticipated response of the terms of trade follows directly from (1.1.95):

$$\hat{S}_t - E_{t-1}\hat{S}_t = -\psi\left(\hat{Y}_t - E_{t-1}\hat{Y}_t\right) \tag{1.2.6}$$

For the real return, recall the definitions of the real return in (1.1.88) and the output gap $\tilde{Y}_t = \hat{Y}_t + \frac{\tilde{A}_t}{1-(1-a_H)\psi}$:

$$\tilde{R}_t = -a_H\psi\left(\hat{Y}_t - \hat{Y}_{t-1}\right) - \frac{a_H\psi}{1-(1-a_H)\psi}\left(\tilde{A}_t - \tilde{A}_{t-1}\right) \tag{1.2.7}$$

The unanticipated response of the nominal exchange rate, key in determining the degree of risk-sharing ex-post across generations, is calculated as the residual expression resulting from the combination of the terms of trade response and the Home inflation response:

$$\tilde{\xi}_t - E_{t-1}\tilde{\xi}_t = \left(\hat{S}_t - E_{t-1}\hat{S}_t\right) + \left(\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H\right) - \frac{\psi}{1-(1-a_H)\psi}\left(\tilde{A}_t - E_{t-1}\tilde{A}_t\right) \tag{1.2.8}$$

1.2.1. Cole and Obstfeld[28, 1991] extended

Before proceeding to the calculation of the set of unanticipated responses, the model admits an extension of a parametric condition first discussed in Cole and Obstfeld[28, 1991], by which the Home and Foreign economies fully share risk despite being unable to trade financial claims in a world under financial autarky. This result was derived in the context of a model with two economies that feature a representative agent and operate under financial autarky.

The following proposition generalizes the result by showing that the same parametric condition guarantees full risk sharing both across countries and across generations within the Home economy.

Proposition 4. *If the parametric condition $\phi = 1$ is imposed (and countries are assumed to have initial zero net foreign assets), the following are true:*

The optimal monetary policy problem rule achieves:

$$\hat{Y}_t - E_{t-1}\hat{Y}_t = \tilde{d}_t - E_{t-1}\tilde{d}_t = \tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H = 0 \tag{1.2.9}$$

There is full risk-sharing across countries:

$$\frac{\Omega_t}{\Omega_{t+1}}\frac{C_t^*}{C_{t+1}^*} = \frac{C_t}{C_{t+1}} \tag{1.2.10}$$

There is full risk-sharing across generations:

$$\tilde{c}_{y,t} - \tilde{c}_{m,t+1} = \tilde{c}_{m,t} - \tilde{c}_{o,t+1} \tag{1.2.11}$$

Proof:

If $\phi = 1$, $\psi = -a_H^{-1}$ and $\Theta_A = 0$ (see Appendix A.2). Thus, equation (1.2.5) implies $\tilde{d}_t - E_{t-1}\tilde{d}_t = 0$ (irrespective of monetary policy, the unanticipated response of the debt-to-GDP ratio is zero) and the first order condition (1.2.3) implies that the policymaker chooses $\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H = 0$. The Phillips curve (1.2.4) implies $\hat{Y}_t - E_{t-1}\hat{Y}_t = 0$ $\square$.

Under the assumption of zero initial net assets and given that the Foreign economy is assumed to be in steady state, Equation (1.2.10) implies: $\tilde{C}_t = \tilde{\Omega}_t = a_H\tilde{S}_t = -a_H\psi\tilde{Y}_t$, where the second equality comes from the loglinear first-order approximation to (1.1.64) and the third from (1.1.85). If $\phi = 1$, $\psi = -a_H^{-1}$ and the international risk sharing condition collapses to $\tilde{C}_t = \tilde{Y}_t$, which is equation (1.1.63) and holds in the equilibrium of the model $\square$.

From the first part of this proposition, $\tilde{d}_t = E_{t-1}\tilde{d}_t = \lambda d_{t-1}$, where the last equality comes from the equilibrium condition (1.1.92). Given $|\lambda| < 1$, $\tilde{d}_t = 0$. The proposition follows from the equilibrium allocation of consumption given in (1.1.91) $\square$.

Intuitively, the condition $\phi = 1$ guarantees full risk-sharing across generations because it guarantees that the output gap (and therefore the real interest rate) will react to productivity innovations in a way that is consistent with the full stabilization of the debt-to-GDP ratio in equation (1.1.90). Proposition 4 shows that the condition $\phi = 1$ has generally stronger implications than considered in standard, representative agent models of the open economy, by providing full risk-sharing and allowing the policymaker to achieve full macroeconomic stabilization, defined in this context as a situation in which no inflation, output gap or debt fluctuations occur.

1.3. Alternative Policy Regimes

This section calculates the set of unanticipated responses that solve the system of equations (1.2.3)-(1.2.5) for a particular baseline calibration in order to explore the effect of some key parameters and to compare the optimal policy rule with alternative policy regimes. The set of alternative policy regimes that will be studied in this chapter follows the seminal work by Galí and Monacelli[38, 2005]:

1. Producer Price Index Inflation Targeting (PPI-IT): A regime of PPI-IT would seek to have:

$$\tilde{\Pi}_t^H = 0$$

for all t. A commitment to follow this policy would imply that the surprise component of domestic inflation is set to zero:

$$\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H = 0 \tag{1.3.1}$$

The responses of variables can be obtained by solving the equilibrium system given by (1.2.4), (1.2.5) and (1.3.1).

2. Consumer Price Index Inflation Targeting (CPI-IT): A regime of CPI-IT would seek to have:

$$\tilde{\Pi}_t = 0$$

From the definition of CPI inflation, $\tilde{\Pi}_t = a_H\tilde{\Pi}_t^H + (1-a_H)\tilde{\xi}_t$. Thus, the policy prescribes:

$$\tilde{\xi}_t \quad = \quad -\left(\frac{a_H}{1-a_H}\right)\tilde{\Pi}_t^H$$

From this equation, it is apparent that a policy regime of CPI-IT postulates a particular relationship between the nominal exchange rate and PPI inflation. This observation will be relevant when discussing the intuition for the suboptimality of CPI-IT below. From (1.2.8):

$$\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H = -(1-a_H)\left(\hat{S}_t - E_{t-1}\hat{S}_t\right) + (1-a_H)\frac{\psi}{1-(1-a_H)\psi}\left(\tilde{A}_t - E_{t-1}\tilde{A}_t\right)$$

and substituting this into (1.2.6):

$$\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H = (1-a_H)\psi\left(\hat{Y}_t - E_{t-1}\hat{Y}_t\right) + (1-a_H)\frac{\psi}{1-(1-a_H)\psi}\left(\tilde{A}_t - E_{t-1}\tilde{A}_t\right) \tag{1.3.2}$$

The responses of variables are now obtained by solving the equilibrium system given by (1.2.4), (1.2.5) and (1.3.2).

3. Exchange Rate Peg: A fixed exchange rate regime would seek to have:

$$\tilde{\xi}_t = 0$$

Therefore, from (1.2.8):

$$\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H = -\left(\hat{S}_t - E_{t-1}\hat{S}_t\right) + \frac{\psi}{1-(1-a_H)\,\psi}\left(\tilde{A}_t - E_{t-1}\tilde{A}_t\right)$$

and from (1.2.6):

$$\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H = \psi\left(\hat{Y}_t^H - E_{t-1}\hat{Y}_t^H\right) + \frac{\psi}{1-(1-a_H)\,\psi}\left(\tilde{A}_t - E_{t-1}\tilde{A}_t\right) \quad (1.3.3)$$

The responses of variables are now obtained by solving the equilibrium system given by (1.2.4), (1.2.5) and (1.3.3).

The set of baseline parameters employed in this exercise is described below.

1.3.1. Calibration

Table 1.1 presents the set of baseline structural parameters employed in the calculations of section 1.3.2. The baseline values of a subset of the parameters have been borrowed from different papers on open economy models under complete markets and representative agents. This subset includes the parameters ϵ, ϕ and a_H. The baseline value for parameter η has been chosen so that the Frisch elasticity of labour supply is 0.4, in line with recent estimations for the US economy (Reichling and Wahleng[68, 2005]).

The parameters β and γ have been calibrated, to follow the factual motivation described in the introduction, to target some key moments of the Hungarian economy. To perform this calibration, one period of the model is taken to represent 10 years in the data, to make it consistent with the generational interpretation of the physical environment of the model. In particular, β has been chosen to mach a steady state real interest rate of 7%, which is the average real interest rate of mortgage loans denominated in Swiss Francs (CHF) in Hungary during the period 2005-2010. The slope of the life-cycle income γ has been calibrated to match the steady state debt-to-GDP ratio. Given the model's focus on consumption and private debt, γ is chosen to target a ratio of total household debt denominated in foreign currency to private comsumption of 58%, observed at the peak of the penetration of foreign currency debt in Hungary in the first quarter of 2009.

The parameter κ (the fraction of firms that update prices with outdated information) is also subject to the time convention of 10 years in the data corresponding to one period in the model. The inverse of $1-\kappa$ relates to the average duration of a spell of time without a given firm updating the information it uses to set prices (see Sheedy[74, 2013]). This spell is taken to be two years and a half for the baseline calibration. Finally, it is assumed in what follows that the stochastic process $\tilde{A}_t$ is white noise, which implies that $E_{t-1}\tilde{A}_t = 0$.

Table 1.1.: Baseline parameters

Parameter	Interpretation	Value	Target/Source
η	Inverse Frisch elasticity	2.5	Frisch elasticity = 0.4 (Reichling and Wahleng[68, 2005])
ϵ	Elasticity of substitution across varieties	10	Benigno and Woodford[8, 2005]
β	Discount factor	0.59	Real Rate on CHF Loans =7% (Hungary)
γ	Slope of life-cycle income	0.29	FC Debt/Consumption = 58% (Hungary)
ϕ	Elasticity of substitution across H and F	1.5	di Paoli[62, 2009]
a_H	Home Bias	0.7	di Paoli[62, 2009]
κ	Fraction of firms that update prices with outdated information	0.2	Information update every 2.5 years

1.3.2. Calculation of Responses

Figures 1 and 2 show the responses of the endogenous unanticipated components of macroeconomic variables ($\hat{Y}_t - E_{t-1}\hat{Y}_t$, $\tilde{d}_t - E_{t-1}\tilde{d}_t$, $\hat{S}_t - E_{t-1}\hat{S}_t$ $\tilde{\Pi}^H_t - E_{t-1}\tilde{\Pi}^H_t$, $\tilde{\xi}_t - E_{t-1}\tilde{\xi}_t$ and $\tilde{R}_t - E_{t-1}\tilde{R}_t$) to a negative one percent shock to labour productivity for different values of ϕ and κ. These results are discussed in turn.

1.3.2.1. The effect of ϕ

Figure 1 shows the unanticipated response of the set of variables to a negative one percent shock to labour productivity as a function of the elasticity of substitution ϕ, where ϕ is set in the range from 1 to ϵ. The qualitative responses are robust to changes in ϕ (for both the optimal monetary policy rule and the alternative policy regimes).

A negative shock to productivity triggers a fall in the real interest rate in an attempt by the policymaker to stabilize the debt-to-GDP ratio and redistribute wealth from creditors to debtors with the goal of improving ex-ante risk-sharing. This fall in the real interest rate is brought about by a nominal appreciation. The reaction of the nominal exchange rate lies at the heart of the effort of improving on risk-sharing. In doing so, however, the policymaker does not achieve a significant improvement in risk-sharing across generations, as is evident from the fact that the debt-to-GDP ratio is not significantly more stable than in alternative regimes.

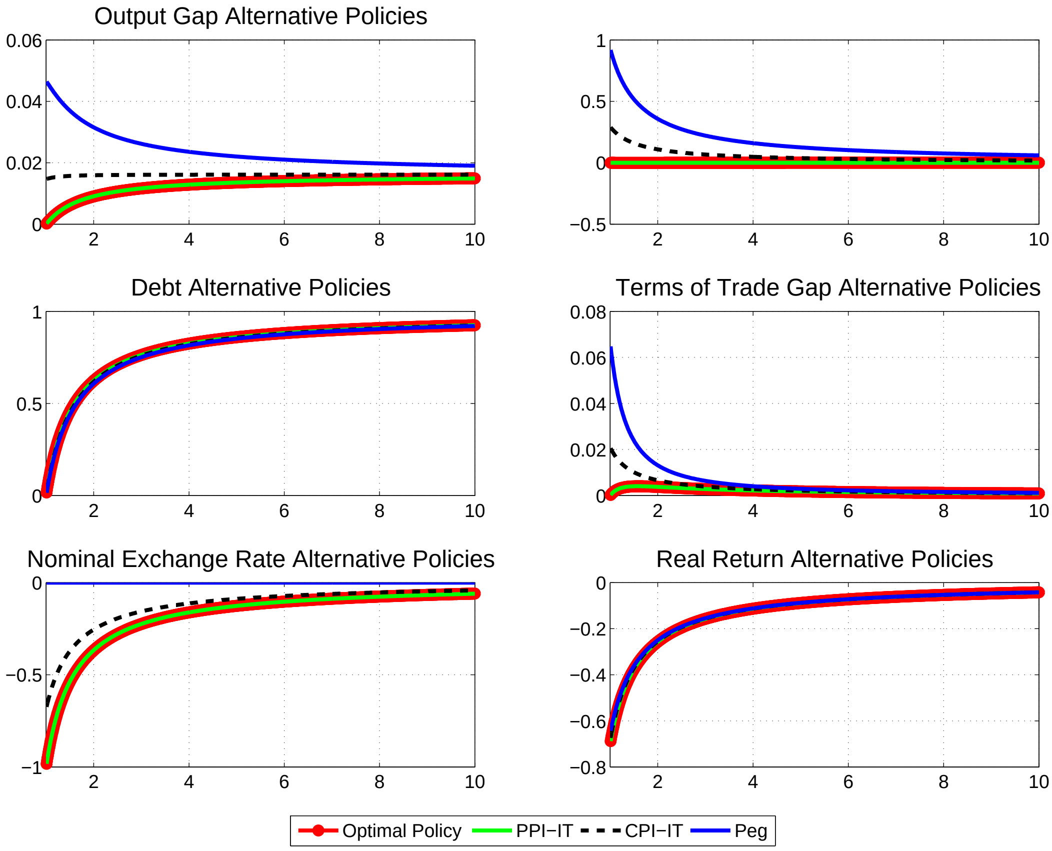
The fall in the real interest rate brought about by an increase in the output gap (see equation (1.1.88)) necessarily implies a (relatively small) positive reaction of inflation through the Phillips curve. The volatility of both the output gap and inflation is significantly smaller under the optimal policy rule than under alternative regimes. It is in this sense that the results indicate the policymaker prefers to concentrate on the stabilization of standard macroeconomic variables (the output gap and inflation) at the expense of not being active enough in boosting risk-sharing. The policymaker faces a trade-off between standard macroeconomic objectives and risk-sharing, and the latter proves to be very costly to undertake under the baseline calibration. In other words, the objective of stabilizing the debt-to-GDP ratio would require a significant surprise in terms output and inflation, which the policymaker finds suboptimal.

For this reason, the optimal policy rule is closest to PPI-IT than to any other alternative regime: the stabilization of PPI inflation is approximately equivalent to the implementation of the flexible-price equilibrium in a context where only technology shocks drive economic fluctuations. The stabilization of PPI inflation gives priority to standard macroeconomic objectives (output gap and inflation volatility) over risk-sharing considerations.

The suboptimality of CPI-IT and of an exchange rate peg is precisely related to the fact that these regimes create excessive volatility in output and inflation. Under the exchange rate peg, the fall in the real interest rate requires a very strong positive response of inflation, which is related to a strong positive response of output. CPI-IT allows some nominal appreciation and therefore reduces the response of inflation, but the dynamic behavior imposed by CPI-IT on the nominal exchange rate interferes with the role of the latter in contributing to risk-sharing and creates higher volatility of inflation and output.

The effect of the elasticity of substitution ϕ on these responses can be understood from the results described in Proposition 4 and equation (1.1.95). When $\phi \to 1$, Proposition 4 indicates that the optimal policy rule achieves full stabilization of output, inflation and the debt-to-GDP ratio. The stabilization of the latter implies that the real return reacts negatively by the exact amount needed to neutralize the effect of shocks to productivity. This negative response is related to a strong nominal appreciation. On the other hand, taking the limit of the economy when $\phi \to \infty$, equations (1.1.88) and (1.1.95) imply $\hat{S}_t \to 0$ and $\tilde{R}_t \to 0$. When Home and Foreign goods are perfect substitutes ($\phi \to \infty$), the policymaker does not have any "traction" over the terms of trade or the real interest rate of the economy. Therefore, as ϕ increases, the reaction of the output gap and inflation

Figure 1.3.1.: The effect of ϕ



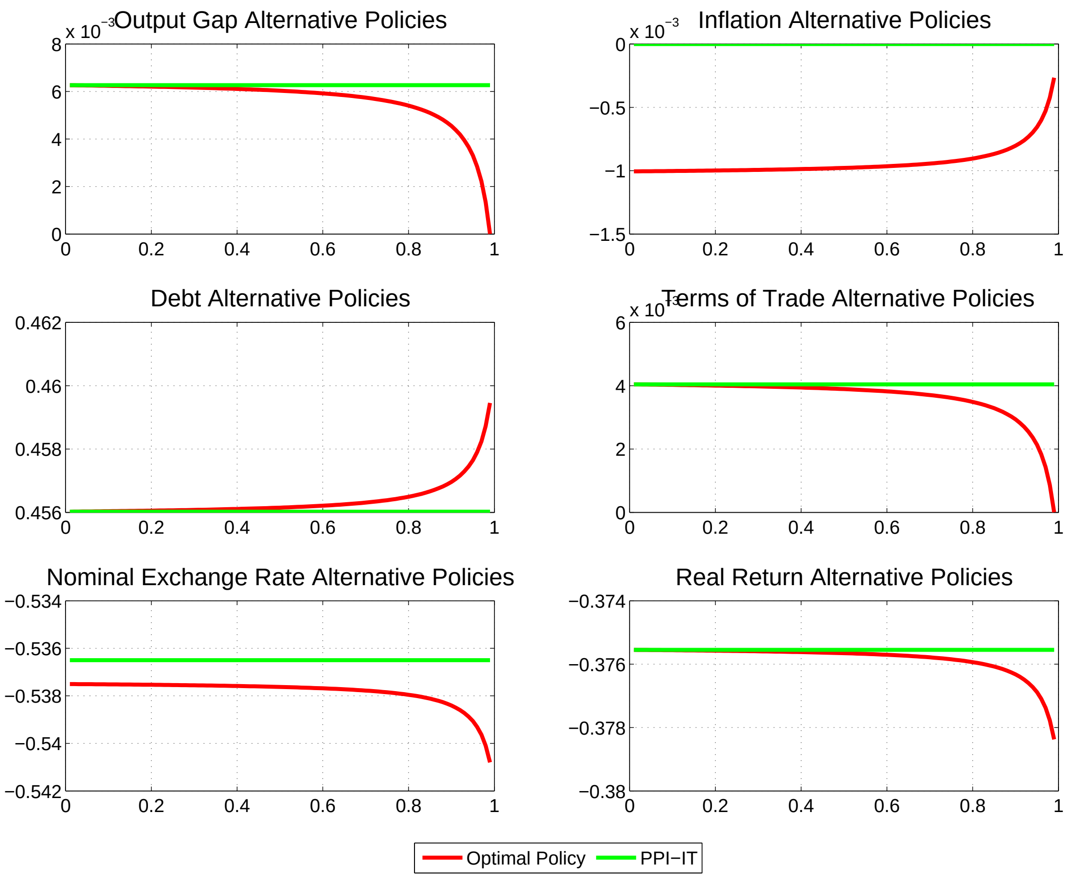
under the optimal policy rule has to be stronger to generate a smaller fall in the real interest rate, as long as monetary policy is less "powerful". Interestingly, for $\phi > 1$, a fall in the elasticity of substitution ϕ has two effects: it increases the power of monetary policy on key variables (the terms of trade and the real return in particular) but reduces the incentive of the policymaker to take action thanks to the results of Proposition 4. On the other hand, an increase in ϕ reduces the power of monetary policy in a context where a more decisive action from it is required.

1.3.2.2. The effect of κ

Figure 2 calculates the unanticipated response of the set of variables to a negative one percent shock to labour productivity as a function of the "information updating" parameter κ, where $\kappa \in (0, 1)$. To highlight the differences between the optimal policy rule and PPI-IT (not perceived at the scale of Figure 1), Figure 2 focuses only on these two policy regimes. Similar to the previous case, the qualitative responses are robust to changes in κ for both the optimal monetary policy rule and all the alternative policy regimes.

The intuitive interpretation of the responses in Figure 2 is the same as that described for

Figure 1.3.2.: The effect of κ



the case of ϕ in Figure 1, that is, the optimal policy engineers a fall in the real interest rate and a nominal appreciation in order to limit the response of the debt-to-GDP ratio, and to achieve this, a positive reaction of the output gap and PPI inflation are required. The latter two are almost negligible from a quantitative point of view, and indicate that regardless of the value of κ, the optimal policy rule achieves a high degree of stability of both variables.

Given ϕ, an increase in κ (which makes the economy more rigid and the Phillips curve flatter) reduces the inflationary cost of a given output gap response. However, the cost of improving on risk-sharing becomes larger as the rigidity of the economy increases. As κ increases, the policymaker finds it less desirable to stabilize the real interest rate and the debt-to-GDP ratio compared to the objective of output gap stabilization, which gains prominence naturally as the greater rigidity of the economy impairs its natural ability to stabilize itself after a negative productivity shock.

The following section focuses again on the remaining alternative regimes and offers a quantitative assessment of the degree of suboptimality created by CPI-IT and an Exchange Rate Peg (which are furthest away from the optimal policy rule).

1.4. Welfare Losses

The welfare losses attached to each alternative policy regime can be calculated as the unconditional expectation of the loss function, $E\left(L\right)$. To calculate the components of the loss function, it is useful to write:

$$\tilde{d}_t = \lambda \tilde{d}_{t-1} + \left(\tilde{d}_t - E_{t-1}\tilde{d}_t\right)$$

Using the the standard result for the mean and variance of a stationary AR(1) process:

$$E\left(\tilde{d}_t\right) = 0 \qquad E\left(\tilde{d}_t^2\right) = \frac{V\left(\tilde{d}_t - E_{t-1}\tilde{d}_t\right)}{1-\lambda^2}$$

For the variance of output, we can follow a similar strategy to equation (1.1.94).
As $E_{t-1}\hat{Y}_t = -\frac{\zeta}{(1+\eta)(1-\psi+\psi a_H)}\lambda \tilde{d}_{t-1}$, by definition:

$$\begin{aligned} \hat{Y}_t &= -\frac{\zeta\lambda}{\left(1+\eta\right)\left(1-\psi+\psi a_H\right)}\tilde{d}_{t-1} + \left(\hat{Y}_t - E_{t-1}\hat{Y}_t\right) \\ E\left(\hat{Y}_t\right)^2 &= \left[\frac{\zeta\lambda}{\left(1+\eta\right)\left(1-\psi+\psi a_H\right)}\right]^2 \frac{V\left(\tilde{d}_t - E_{t-1}\tilde{d}_t\right)}{1-\lambda^2} + V\left(\hat{Y}_t - E_{t-1}\hat{Y}_t\right) \end{aligned}$$

As $E\left(\tilde{d}_t\right) = 0$, $E\left(\hat{Y}_t\right) = 0$ and thus:

$$E\left(\tilde{A}_t\hat{Y}_t\right) = cov\left(\tilde{A}_t, \hat{Y}_t\right) = cov\left(\tilde{A}_t - E_{t-1}\tilde{A}_t, \hat{Y}_t - E_{t-1}\hat{Y}_t\right)$$

The welfare criterion is thus:

$$
\begin{aligned}
E(L) &= \sum_{t=0}^{\infty}\beta^t\left\{\frac{1}{2}\frac{\chi}{\theta^2}\frac{V\left(\tilde{d}^s\right)}{1-\lambda^2}+\frac{\epsilon}{2}\frac{\kappa}{1-\kappa}V\left(\tilde{\Pi}^{H,s}\right)\right.\\
&+\frac{\delta}{2}\left[\frac{\zeta\lambda}{(1+\eta)(1-\psi+\psi a_H)}\right]^2\frac{V\left(\tilde{d}^s\right)}{1-\lambda^2}+\frac{\delta}{2}V\left(\hat{Y}_t^s\right)+ \qquad (1.4.1)\\
&\left.\frac{\varrho\psi}{1-(1-a_H)\psi}cov\left(\tilde{A}_t,\hat{Y}_t^S\right)\right\}\\
&= \frac{1}{1-\beta}\left\{\frac{1}{2}\frac{\chi}{\theta^2}\frac{V\left(\tilde{d}^s\right)}{1-\lambda^2}+\frac{\epsilon}{2}\frac{\kappa}{1-\kappa}V\left(\tilde{\Pi}^{H,s}\right)\right.\\
&+\frac{\delta}{2}\left[\frac{\zeta\lambda}{(1+\eta)(1-\psi+\psi a_H)}\right]^2\frac{V\left(\tilde{d}^s\right)}{1-\lambda^2}\\
&\left.+\frac{\delta}{2}V\left(\hat{Y}_t^s\right)+\frac{\varrho\psi}{1-(1-a_H)\psi}cov\left(\tilde{A}_t,\hat{Y}_t^S\right)\right\}
\end{aligned}
$$

where $x_t^s = x_t - E_{t-1}x_t$ for a given variable x. Evaluating the expected loss requires a particular value of $V\left(\tilde{A}_t\right)$ to be specified. But given the interest of this chapter in relative welfare losses (compared to alternative policy regimes and other models), no value for this parameter is provided and, instead, expression (1.4.1) is scaled over $V\left(\tilde{A}_t\right)$, in which form it can be calculated directly from the structural parameters of the economy.

Table 1.2 compares the welfare losses (relative to the variance of the shock $V\left(\tilde{A}_t\right)$) of the regimes of CPI-IT and the Exchange Rate Peg for different values of κ, ϕ and two different sets of models derived from different parametric conditions: firstly, the "OLG" columns correspond to the baseline calibration shown above for the overlapping generations structure presented. Secondly, the "Representative Agent" columns correspond to a calculation of the welfare criterion under the condition $\gamma = 0$. As discussed above, this condition reduces the model to a standard, representative agent open economy by eliminating the life-cycle pattern of income and therefore the need for domestic financial markets.

The entries in the table are interpreted as permanent reductions in consumption under a given regime relative to steady state scaled by $V\left(\tilde{A}_t\right)$. The tables suggest that the exchange rate peg is generally more costly than the regime of CPI-IT. Besides, as ϕ increases from 1.5 (table 1.2a) to 6 (table 1.2b), the losses from both regimes become much larger than in a representative agent model. This allows to conclude that, given the role of the nominal exchange rate in completing markets in the overlapping generations model, the losses of a Peg and of a regime of CPI-IT are much larger compared to the ones calculated previously for models of the small open economy based on the representative agent assumption (see Galí and Monacelli[38, 2005]). Therefore, despite the relatively limited involvement of optimal monetary policy in risk-sharing (compared to standard macroeconomic objectives), the

Table 1.2.: Welfare Losses

(a) $\phi = 1.5$

	OLG		Representative Agent	
	CPI-IT	Peg	CPI-IT	Peg
$\kappa = 0.2$	3.41	5.15	3.41	5.16
$\kappa = 0.66$	9.79	13.74	9.83	13.82

(b) $\phi = 6$

$\phi = 6$	OLG		Representative Agent	
	CPI-IT	Peg	CPI-IT	Peg
$\kappa = 0.2$	7.50	10.20	3.80	5.89
$\kappa = 0.66$	17.60	25.33	14.25	21.58

nominal exchange rate plays a role in ex-post redistributions of wealth that are quantitatively important from the point of view of the welfare losses of households in the Home economy.

1.5. Concluding Comments

This chapter has characterized optimal monetary and exchange rate policies for a small open economy under incomplete markets at the local level and financial instruments denominated in foreign currency. Several conclusions arise from this effort.

The main finding of the chapter is that the risk-sharing considerations which arise from market incompleteness introduce a new trade-off for monetary policy under financial autarky. After any productivity shock, the variations in the real exchange rate and the real return required to replicate the complete markets allocation imply excessive volatility in the traditional macroeconomic objectives of output and inflation. Under the calibration considered, optimal policy resolves this trade-off in favour of the traditional objectives. Consequently, the optimal policy is closest to Producer Price Index (PPI) Inflation Targeting than to the more standard Consumer Price Index (CPI) Inflation Targeting or the more extreme Exchange Rate Peg, as it is closest to the flexible price allocation (the traditional aim of a broad range of New Keynesian optimal policy models). The cost of this strategy is an excessively volatile debt-to-GDP ratio and therefore imperfect risk-sharing across the different generations of the economy.

This result does not imply, however, total passivity of the optimal rule to risk-sharing/financial considerations. In particular, the optimal policy rule prescribes that the nominal exchange rate should appreciate after a negative productivity shock (and vice versa) so that the real burden of outstanding liabilities in foreign currency fall after a bad shock and wealth is transferred from the creditor generation to the debtor generation. Indeed, although the optimal policy is relatively passive when it comes to risk-sharing, being excessively passive (as implied by an Exchange Rate Peg) creates significant welfare losses on households, that significantly exceed those calculated by the literature under representative agent frameworks. The ability of the nominal exchange rate to react to shocks in a specific fashion is therefore crucial, as far as household welfare is concerned.

2. Flows of Information and Inefficiency in the Unsecured Consumer Credit Market: A Simple Framework

During the last decade, several Latin American economies have experienced a dramatic surge in the amount of consumer credit taken by households[1]. This trend is summarized in Figure 2.0.1 with quarterly data for Argentina, Brazil, Chile, Colombia, Mexico and Peru. For all these countries, the ratio of consumer credit to GDP (shown in the left panel of the figure) rose rapidly since 2002, in some instances reaching the 10% mark towards the end of the decade. An equivalent way to present this phenomenon (right panel) is to calculate indices and setting the starting value of the ratio for each country to 1. In this case, by the end of the decade most countries were above or around the mark of 2, which effectively means that their penetration of consumer credit doubled during these years[2].

A quick glance at available similar data for other groups of countries suggests that the growth of consumer credit experienced in Latin America is very rapid and, to a large extent, uncommon. For example, the amount of consumer credit in Brazil or Colombia grew from 4% to 9% of GDP in the space of less than 6 years, whereas in the United States the same increase took about 15 years[3]. A similar conclusion can be reached by studying data of other developed countries, such as the United Kingdom[4]. With regard to countries of

[1]Consumer Credit combines all forms of borrowing by households excluding housing or mortgage-related credit. These forms include (but are not limited to) credit card lines, car loans, overdrafts and personal loans.

[2]The only exception to this rule is Argentina. However, taking into account that the recession in Argentina ended around the end of 2002 and thus setting 2003:I as the base quarter (first quarter with positive GDP growth after the recession), the country ends up with an index of 3.73 in 2011:III, an almost fourfold increase in the ratio in just 8 years.

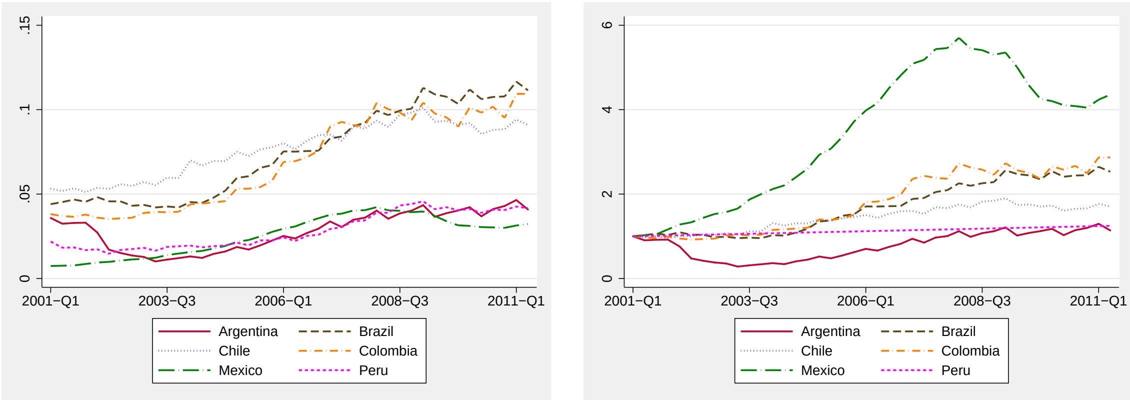
[3]Datasource: Federal Reserve. The sum of "Commercial Loans by All Banks" (Table H.8) and "Gross Consumer Accounts Receivable (excluding Pools of Securitized Consumer Assets) by Finance Companies" (G.20) divided by Nominal GDP grew from 4.09% in 1949 to 9.05% at the end of 1964.

[4]Datasource: Bank of England. The "Total Outstanding Amount of Sterling Net Unsecured Lending to Individuals" (series code LPQAVHH) hit 4.07% of GDP in June 1982 and reached 9.29% only in 2000.

a similar level of development than Latin America, BIS[16, 2009] (and references therein) largely confirms the rarity of such dramatic increases in penetration of consumer credit across countries[5].

Figure 2.0.1.: Consumer Credit in Latin America, 2001:I-2011:III

Consumer Credit / GDP   Consumer Credit / GDP (2001:I=1)



The speed and exceptionality of this trend raises at least two questions. First, why has borrowing expanded so quickly in Latin America? Potential explanations already put forward include financial innovations (see Livshits et al.[50, 2011]) and changes in the legal protection of creditors along the lines of Djankov et al.[32, 2007]. This chapter focuses on a second, closely related, question: is the growth of consumer credit in Latin America excessive from a social perspective? In other words, does it lead to a form of misallocation or constrained inefficiency that should be corrected by means of changes to the credit policy?

A search for an answer to the latter question may start with the recent literature on the efficiency of credit booms (see, among others, Benigno et al.[6, 2010], Bianchi[15, 2011], Lorenzoni[51, 2008] and Uribe[77, 2006]). These papers use the interaction between price indices and collateral constraints to shed light on the role of pecuniary externalities. However, their insights are of limited use to study the abovementioned trends. The reason for this is that consumer credit in less developed financial systems is mostly unsecured in nature, non-collateralized, and not subject to any formal procedure for individuals' bankruptcy filing[6]. By and large, creditors do not have the legal power to seize any asset or income in

[5] A recent exception is the case of the Republic of Korea, see Park[63, 2009]

[6] Reliable estimates of recovery rates for unsecured consumer loans in Latin America are nonexistent. A weak upper bound to these recovery rates might be given by some other forms of secured credit, like housing loans. In the latter case, recovery rates are generally estimated to be very low (see Obermann[56, 2006]). It is reasonable to deduce from this data that the recovery rates of unsecured consumer credit are close to zero.

the event of default, thus rendering collateral constraints inappropriate for understanding unsecured consumer credit.

Unsecured consumer credit is perhaps better captured by the existing tools of the theory of sovereign debt, in which default triggers non-pecuniary penalties on borrowers, such as exclusion of capital markets (Eaton and Gersovitz[33, 1981]) or the transmission of bad signals to other agents (Sandleris[70, 2008]). Recent examples to model unsecured consumer credit following this thread include Chatterjee et al.[26, 2007] and Livshits et al.[50, 2011]. However, so far there has not been any attempt to study the question of constrained inefficiency in the consumer credit market using theoretical frameworks akin to those of sovereign borrowing outside the toolkit of collateral constraints.

This chapter tackles precisely that gap in the literature. Within a simple framework similar to Livshits[50, 2011], it proposes a theoretical mechanism through which the individual conditions of access to unsecured consumer credit for one agent (in particular, the interest rate) are affected by the default choices of other agents in similar markets. The key behind this mechanism is the role played by aggregate variables, in particular by the aggregate default rate, in the problem of a lender facing the choice of whether to extend unsecured credit to a potential borrower and, if so, at which interest rate. Lenders give a crucial importance to the aggregate default rate in their decision process because the income prospects are imperfectly observable and, crucially, correlated across borrowers[7][8].

Given this observation, the model presented below seeks to capture the theoretical effect that the default choice of one consumer might have on the conditions of access (interest rate) of other consumers in the economy. An atomistic consumer who defaults will naturally take into account the consequences of his choice on his own payoffs (including a reputational cost, for example) but will ignore its effect on the borrowing possibilities of other agents. This form of externality through the flow of information across financial transactions will have welfare implications: the volume of unsecured credit in a decentralized financial market

[7] There is survey evidence that indicates the weight given to aggregate variables by bank loan officers in their decision process. See, for example, the results of surveys conducted on bank loan officers by the Federal Reserve (http://www.federalreserve.gov/boarddocs/SnLoanSurvey/), the European Central Bank (http://www.ecb.europa.eu/stats/money/surveys/lend/html/index.en.html), the Bank of Japan (http://www.boj.or.jp/en/statistics/dl/loan/loos/index.htm/), the Bank of England (http://www.bankofengland.co.uk/publications/other/monetary/creditconditions.htm), the Central Bank of Chile (http://www.bcentral.cl/estadisticas-economicas/credito-bancario/index.htm) and the Central Bank of Colombia (http://www.banrep.gov.co/informes-economicos/ine_enc_sit-cred-col_cp.html). Only in the case of Chile the aggregate default rate is included explicitly among the set of possible answers. The discussion by Park[63, 2009] about the abovementioned case of the Republic of Korea is also indicative of the role of a low starting default rate as a main driver of a rapid growth of credit at the beginning of a credit boom.

[8] Income can be correlated across borrowers for a number of reasons. If individual income is the combination of an idiosyncratic component and an aggregate component, then this correlation follows naturally.

will be generally inefficient, and a benevolent policymaker would choose optimally a different level of borrowing ex-ante to control the probability of this externality to materialize[9].

Interestingly, the direction of the inefficiency that arises in the model is not as clear-cut as might be deduced from this discussion. The set of forces described so far leads to the conclusion that the decentralized equilibrium would feature overborrowing, as a policymaker would desire to have less borrowing in order to cap from above the probability of default. However, the policymaker might also wish to have more borrowing because, given a default (or repayment) choice, a positive reputational signal can be transferred from one consumer to the next when borrowing is higher. The tension between these two forces will create an ambiguity regarding the optimality of decentralized unsecured consumer borrowing.

Notice that these externalities require only a flow of information across different financial transactions. Thus, besides the literature on unsecured consumer credit, sovereign debt and efficiency of credit booms that has already been cited, this chapter is also related to the research agenda on the link between information flows and constrained inefficiency. In particular, it draws economic intuition from old ideas about the effect of information revelation on the insurance possibilities of risk-averse agents in uncertain environments. Classic references are Hirshleifer[44, 1971] and the seminal paper on the efficiency of Rational Expectations Equilibria by Laffont[48, 1985]. More recently, Lepetyuk and Stoltenberg[49, 2012] have exploited this insight for the case of revelations of changes in policy targets. Albeit different in nature, the work by Morris and Shin[55, 2002] is also an example of how, under some conditions, more public information of a certain nature can be detrimental for welfare.

The rest of the chapter is organized in three sections as follows. Section 2.1 is devoted to presenting the basic, symmetric-information version of the model, together with the inefficiency result and its economic intuition. Section 2.2 presents simulations of the basic model under specific parameters and functional forms, in order to explore the effect of changes in the economic environment on the size and direction of inefficiency. This extension of the model reinforces the conclusions obtained with the basic version, while at the same time shedding light on the gap between first- and second-best allocations. Some reflections as concluding comments are presented in Section 2.3.

[9] Up to this point, it is apparent that this inefficiency mechanism has the potential to apply to more general situations than simply the unsecured consumer credit market. However, this is not necessarily the case as long as, if collateralization is permitted, borrowers might "buy themselves out" of the effects of the externality by simply posting more, or better, collateral. Thus, for the mechanism to work it is necessary to keep the focus in an unsecured credit market where the posting of collateral after default of other agents is not allowed.

2.1. The Basic Model

This section is devoted to describe the simplest possible framework that could capture the effect of information flows across financial transactions on the optimality of decentralized borrowing. After presenting the basic asumptions and ingredients of the model, the section describes the problem of each group of economic agents and the decentralized equilibrium. This decentralized 'benchmark' is then compared to the solution of a benevolent policy-maker that faces the same informational constraints of the decentralized economy. Some simulations of the model are presented in the next section as a means to shed light on the effect of some parameters on the gap between the decentralized and the policymaker solutions.

2.1.1. Structure, Timing, Assumptions, Information

There are two separate financial markets. In what follows, these markets will be respectively labeled Market A and Market B. Each of these two markets is composed of a risk-averse Borrower and a risk-neutral Bank, different across markets. There are thus four agents, and Borrowers and Banks will be labeled according to the market in which they participate: Borrower A and Bank A in Market A; Borrower B and Bank B in Market B. It is assumed that agents from one market cannot participate in the other market. In this way, there will be no flow of real resources across markets. As will be specified below, only information will flow between Markets A and B.

Each Market has two periods. In the first period, the respective Borrower (who has no endowment) borrows from the Bank in order to consume. This borrowing takes the form of the sale of an unsecured discount bond. The absence of a full set of state-contingent assets makes both markets incomplete by construction[10]. In the second and final period, the Borrower receives an income which is random from the point of view of period 1, and having observed income, decides whether to repay or default. Importantly, it is assumed that Market B opens *after* the closure of Market A. In other words, the first period of Market B occurs immediately *after* the second period of Market A. In a sense, the model uses time as a device to explore the information flows across financial markets, which are the main focus of this chapter.

The following key assumptions are made:

A1: There is free entry in the banking industry of each of the two financial markets.

[10]This form of incompleteness is imposed without considering the problem of optimal contracting in the context of the model.

A2: Banks are assumed to finance loans to (more precisely, purchases of bonds from) Borrowers by taking "deposits" from an exogenous, perfectly elastic wholesale supply of funds at an interest rate of zero[11].

Assumptions A1 and A2 will greatly help to facilitate the description and solution of the model via the simplification of the problem of Banks.

A3: The income of either of the two Borrowers is neither directly observable nor verifiable by either Bank or by the other Borrower. Naturally, each Borrower will observe his own income once it realizes in the second period of his respective Market (and will decide to repay or default accordingly), but this information will not be directly revealed to anyone else[12].

A4: The income of Borrower A is correlated with the income of Borrower B. As suggested before, this assumption can be justified along the lines of aggregate shocks affecting both Borrowers at the same time in a specific way.

A5: No partial default is allowed. The repayment of debts will entail the full payment of obligations, and default will entail no payment from the Borrower to the Bank.

Finally, the information structure of the model is relatively simple, but crucial nonetheless. Given that there is not any history before the first period of Market A, the only information that Bank A and Borrower A have is the unconditional probability distribution function (PDF) of future income of the latter. Thus, there is no information asymmetry between Borrower and Bank ex-ante. The agents in Market B, however, have an informational advantage over agents in Market A. In addition to knowing unconditional distributions, they observe all decisions taken in Market A (i.e.: borrowing and default/repayment by Borrower A). This information is promptly used to update the probability of default of Borrower B, given the correlation of income across the two borrowers.

2.1.2. The Problem of the Borrowers

Borrower $i \in \{A, B\}$ faces the following standard, time-separable, optimization problem:

$$V^i \equiv \max_{c_1^i, c_2^i} U^i \equiv u\left(c_1^i\right) + \beta E\left[u\left(c_2^i\right) \mid \Im^i\right] \tag{2.1.1}$$

[11]Assuming a strictly positive cost of funds would complicate the mathematics of the model without adding significant intuition. However, one of the strategies that the policymaker might use to steer the decentralized equilibrium might be the control of this exogenous cost of funds, as will be explored below.

[12]In a similar spirit, the time structure of the model implies that Borrowers (who live in separate periods of time) cannot collude to reveal the realization of income to one another.

where superscripts denote the particular Borrower and subscripts denote life periods. Following usual notation, c denotes consumption, $\beta < 1$ is the discount factor and u is a instantaneous utility function that satisfies the Inada conditions together with standard conditions on concavity, continuity and differentiability. Notice that the expectation operator E is taken conditional on the common information set of Borrower i and the respective Bank i, which is denoted by $\Im^i$. Consumption in each period is given by:

$$c_1^i = q^i \ell^i \tag{2.1.2}$$

$$c_2^i = \begin{cases} y^i - \ell^i & \text{if Repayment } \left(d^i = 0\right) \\ y^i - \gamma y^i & \text{if Default } \left(d^i = 1\right) \end{cases} \tag{2.1.3}$$

Given that Borrower i does not have any endowment in his first period, c_1^i will correspond to the proceeds of the sale of an unsecured discount bond with face value ℓ^i at price q^i. This form of debt contract is contingent only to the extent that full (and only full) default is permitted in the second period after the realization of income y^i. This choice is captured by the indicator variable d^i: in case of repayment ($d^i = 0$), Borrower i consumes his income net of the payment of the face value of the bond; in case of default ($d^i = 1$), Borrower i will have to give up a consumption cost (penalty) given by a fraction $\gamma \in (0, 1)$ of income.

The cost of default can be interpreted as a simplified, reduced-form version of the set of losses that a borrower faces after default in a more complicated, multiple-period setting: e.g. temporary exclusion from financial markets, negative flags in credit history records[13]. Importantly, as credit is unsecured, the penalty γy^i is not transferred to Bank i in the event of default. Borrower i decides to default in the second period only if this choice delivers a higher consumption than repayment. Clearly:

$$d^i = \begin{cases} 0 & \text{(Repayment) if } y^i \geq \ell^i/\gamma \\ 1 & \text{(Default) if } y^i < \ell^i/\gamma \end{cases} \tag{2.1.4}$$

The information sets of the Borrowers are given by $\Im^A = \{\varnothing\}$ and $\Im^B = \left\{\varnothing, \ell^A, d^A\right\}$, where the empty set $\varnothing$ includes information about the unconditional distribution of income, as specified next. The only flow between Market A and Market B is precisely the flow of information on ℓ^A and d^A.

[13] See Chaterjee et al.[26, 2007][25, 2009]. Chapter 3 of this thesis explores empirically the effects of negative flags in credit history on the credit outcomes of individual borrowers.

2.1.2.1. Income Distribution

Assumption A4 can be crystallised by postulating that the random variables y^A and y^B are distributed according to a joint density function $f\left(y^A, y^B\right)$. This joint density is known by all agents in advance. Let $F\left(y^A, y^B\right)$ denote the cumulative distribution function associated to the joint distribution f. Both $y^A$and y^B are also assumed to lie between a minimum income y_L and a maximum income y_H, with $y_H > y_L$. In other words, the support of the joint cumulative distribution function $F\left(y^A, y^B\right)$ is given by the space $[y_L, y_H] \times [y_L, y_H]$.

The functions given by $g\left(y^A\right) = \int_{y_L}^{y_H} f\left(y^A, y^B\right) dy^B$ and $G\left(y^A\right) = F\left(y^A, y_H\right)$will denote respectively the marginal density and cumulative distribution function of y^A. Similarly, $h\left(y^B\right) = \int_{y_L}^{y_H} f\left(y^A, y^B\right) dy^A$ and $H\left(y^B\right) = F\left(y_H, y^B\right)$ will represent respectively the marginal density and cumulative distribution function of y^B. The set of functions F, G, and H are assumed to be continuously differentiable within the support $[y_L, y_H] \times [y_L, y_H]$.

2.1.3. The Problem of the Banks

During its first period, Bank i sets an interest rate (or equivalently, a price on the bond q^i) that guarantees zero expected profit on the particular financial transaction with Borrower i. This is a direct implication of both assumptions A1 (free entry) and risk-neutrality on the side of Banks. Thus, denoting with π^i the probability of repayment of Borrower i and recalling A2 (zero cost of wholesale funds), the following zero-profit condition will characterize the equilibrium behavior of Banks:

$$\begin{aligned} \pi^i \ell^i + \left[1 - \pi^i\right] 0 &= q^i \ell^i \\ \pi^i &= q^i \end{aligned} \tag{2.1.5}$$

With probability π^i Bank i will be repaid the face value of the discount bond on hold, whereas with probability $\left(1 - \pi^i\right)$it will receive nothing. This second period-expected income must coincide, in equilibrium, with the amount of wholesale funds taken to finance the purchase of the bond in period 1, as the interest rate on these funds is assumed to be zero. This condition collapses to (2.1.5), that equates the equilibrium price of the bond issued by Borrower i to his own probability of repayment π^i.

The probability of repayment of Borrower i, π^i, is calculated by Bank i using rationally all available information. This information will not include in any case the realization of income, but does include the structure of the Borrowers' problem together with $\Im^i$. Using

(2.1.4), π^i will therefore be a function of ℓ^i and $\Im^i$ as follows:

$$\pi^i = \Pr\left(y^i \geq \frac{\ell^i}{\gamma} \mid \Im^i\right) \tag{2.1.6}$$

2.1.4. Decentralized Equilibrium

Using (2.1.5) and (2.1.6), and opening up the expectations term in (2.1.1) taking into account the default choice in (2.1.4), the problem of Borrower i can be rewritten as follows:

$$\begin{aligned} V^i \equiv & \max_{\ell^i} u\left(\Pr\left(y^i \geq \frac{\ell^i}{\gamma} \mid \Im^i\right) \ell^i\right) \\ & +\beta\left\{\Pr\left(y^i \geq \frac{\ell^i}{\gamma} \mid \Im^i\right) E\left[u\left(y^i - \ell^i\right) \mid y^i \geq \frac{\ell^i}{\gamma}; \Im^i\right]\right\} \\ & +\beta\left\{\Pr\left(y^i < \frac{\ell^i}{\gamma} \mid \Im^i\right) E\left[u\left(y^i - \gamma y^i\right) \mid y^i < \frac{\ell^i}{\gamma}; \Im^i\right]\right\} \end{aligned} \tag{2.1.7}$$

Notice that in equilibrium $\ell^i \leq \gamma y_H$ must hold. Given that $y^i \leq y_H$, no rational lender will buy a bond with a face value higher than γy_H, as this would trigger default with probability one (the only rational interest rate on this loan would be infinite). On the other hand, notice that if ℓ^i lies in the interval $[0, \gamma y_L]$, the loan is effectively riskless and its interest rate is zero because of the free entry condition. As u is assumed to be concave, $\beta < 1$ implies that Borrower i prefers $\ell^i = \gamma y_L$ among this range of riskless contracts.

Thanks to the sequential nature of the model, and to the absence of financial markets after the closure of Market B, the decentralized equilibrium of the model can be found in a straightforward manner using a backward induction argument. Recall that the information set of agents in Market B is given by $\Im^B = \{\emptyset, \ell^A, d^A\}$. At the start of his life, Borrower B might observe one of two possible scenarios: either Borrower A has repaid in full $\ell^A (d^A = 0)$ or he has defaulted on his obligations ($d^A = 1$). The different conditions that these two situations imply for Borrower B's insurance possibilities lie at the heart of the inefficiency mechanism that is proposed in this chapter.

2.1.4.1. Borrower B after $d^A = 0$

Using (2.1.7), the problem of Borrower B under full repayment of ℓ^A by Borrower A can be written as:

$$\begin{aligned}
V^B\left(\ell^A, 0\right) \equiv & \max_{\ell^B \leq \gamma y_H} u\left(\Pr\left(y^B \geq \frac{\ell^B}{\gamma} \mid y^A \geq \frac{\ell^A}{\gamma}\right)\ell^B\right) \\
& + \beta\left\{\Pr\left(y^B \geq \frac{\ell^B}{\gamma} \mid y^A \geq \frac{\ell^A}{\gamma}\right) E\left[u\left(y^B - \ell^B\right) \mid y^B \geq \frac{\ell^B}{\gamma}; y^A \geq \frac{\ell^A}{\gamma}\right]\right\} \\
& + \beta\left\{\Pr\left(y^B < \frac{\ell^B}{\gamma} \mid y^A \geq \frac{\ell^A}{\gamma}\right) E\left[u\left(y^B - \gamma y^B\right) \mid y^B < \frac{\ell^B}{\gamma}; y^A \geq \frac{\ell^A}{\gamma}\right]\right\} \\
= & \max_{\ell^B \leq \gamma y_H} u\left(\left[\frac{1 - F\left(\frac{\ell^A}{\gamma}, y_H\right) - F\left(y_H, \frac{\ell^B}{\gamma}\right) + F\left(\frac{\ell^A}{\gamma}, \frac{\ell^B}{\gamma}\right)}{1 - G\left(\frac{\ell^A}{\gamma}\right)}\right]\ell^B\right) \\
& + \frac{\beta}{1 - G\left(\frac{\ell^A}{\gamma}\right)}\left[\int_{\frac{\ell^B}{\gamma}}^{y_H}\int_{\frac{\ell^A}{\gamma}}^{y_H} u\left(y^B - \ell^B\right) f\left(y^A, y^B\right) dy^A dy^B + \right. \\
& \left. \int_{y_L}^{\frac{\ell^B}{\gamma}}\int_{\frac{\ell^A}{\gamma}}^{y_H} u\left(y^B - \gamma y^B\right) f\left(y^A, y^B\right) dy^A dy^B\right]
\end{aligned} \tag{2.1.8}$$

where the set of state variables $\left(\ell^A, d^A\right)$ has been explicitly introduced as the argument of the indirect utility function V^B. Notice that the problem is explicitly conditioned on $y^A \geq \ell^A/\gamma$, which is the condition of full repayment by Borrower A. This information is useful to calculate the probability of repayment of Borrower B given that y^A and y^B are assumed to be correlated. The assumed conditions on the differentiability of distribution and instantaneous utility functions allow to write the necessary first order condition of this problem as follows:

$$\begin{aligned}\Xi\left(\ell^A,0;\ell^B\right) \equiv\ & u_1\left(\left[\frac{1-F\left(\frac{\ell^A}{\gamma},y_H\right)-F\left(y_H,\frac{\ell^B}{\gamma}\right)+F\left(\frac{\ell^A}{\gamma},\frac{\ell^B}{\gamma}\right)}{1-G\left(\frac{\ell^A}{\gamma}\right)}\right]\ell^B\right) \\ & \times\left[\frac{1-F\left(\frac{\ell^A}{\gamma},y_H\right)-F\left(y_H,\frac{\ell^B}{\gamma}\right)-\frac{\ell^B}{\gamma}F_2\left(y_H,\frac{\ell^B}{\gamma}\right)}{1-G\left(\frac{\ell^A}{\gamma}\right)}\right. \\ & \left.+\frac{F\left(\frac{\ell^A}{\gamma},\frac{\ell^B}{\gamma}\right)+\frac{\ell^B}{\gamma}F_2\left(\frac{\ell^A}{\gamma},\frac{\ell^B}{\gamma}\right)}{1-G\left(\frac{\ell^A}{\gamma}\right)}\right] \\ & -\beta\int_{\frac{\ell^B}{\gamma}}^{y_H}\int_{\frac{\ell^A}{\gamma}}^{y_H}u_1\left(y^B-\ell^B\right)\frac{f\left(y^A,y^B\right)}{1-G\left(\frac{\ell^A}{\gamma}\right)}dy^Ady^B=\mu \qquad (2.1.9)\end{aligned}$$

where x_y represents the derivative of the function x with respect to its y-th argument, and μ is the Lagrange multiplier on the constraint $\ell^B \leq \gamma y_H$. Given conditions on concavity of the utility function, condition (2.1.9) characterizes the solution to the optimization problem of Borrower B. Its solution will be given by a function $\ell^B\left(\ell^A,0\right)$ that maps a fully-repaid level of borrowing from Borrower A into borrowing of Borrower B.

2.1.4.2. Borrower B after $d^A=1$

In this scenario, the problem of Borrower B is very similar to the one in the previous subsection, with the only (but fundamental) change that expectations and probabilities are calculated conditional on $y^A < \ell^A/\gamma$, which is the condition of default by Borrower A:

$$\begin{aligned}
V^B\left(\ell^A,1\right) &\equiv \max_{\ell^B \le \gamma y_H} u\left(\Pr\left(y^B \ge \frac{\ell^B}{\gamma} \mid y^A < \frac{\ell^A}{\gamma}\right)\ell^B\right) \\
&\quad +\beta\left\{\Pr\left(y^B \ge \frac{\ell^B}{\gamma} \mid y^A < \frac{\ell^A}{\gamma}\right) E\left[u\left(y^B-\ell^B\right) \mid y^B \ge \frac{\ell^B}{\gamma}; y^A < \frac{\ell^A}{\gamma}\right]\right\} \\
&\quad +\beta\left\{\Pr\left(y^B < \frac{\ell^B}{\gamma} \mid y^A < \frac{\ell^A}{\gamma}\right) E\left[u\left(y^B-\gamma y^B\right) \mid y^B < \frac{\ell^B}{\gamma}; y^A < \frac{\ell^A}{\gamma}\right]\right\} \\
&= \max_{\ell^B \le \gamma y_H} u\left(\left[\frac{F\left(\frac{\ell^A}{\gamma}, y_H\right) - F\left(\frac{\ell^A}{\gamma}, \frac{\ell^B}{\gamma}\right)}{G\left(\frac{\ell^A}{\gamma}\right)}\right]\ell^B\right) \\
&\quad + \frac{\beta}{G\left(\frac{\ell^A}{\gamma}\right)}\left[\int_{\frac{\ell^B}{\gamma}}^{y_H}\int_{y_L}^{\frac{\ell^A}{\gamma}} u\left(y^B-\ell^B\right) f\left(y^A,y^B\right) dy^A dy^B + \right. \\
&\quad \left. \int_{y_L}^{\frac{\ell^B}{\gamma}}\int_{y_L}^{\frac{\ell^A}{\gamma}} u\left(y^B-\gamma y^B\right) f\left(y^A,y^B\right) dy^A dy^B\right]
\end{aligned} \tag{2.1.10}$$

And the necessary first order condition is given by:

$$\begin{aligned}
\Upsilon\left(\ell^A,1;\ell^B\right) &\equiv u_1\left(\left[\frac{F\left(\frac{\ell^A}{\gamma}, y_H\right) - F\left(\frac{\ell^A}{\gamma}, \frac{\ell^B}{\gamma}\right)}{G\left(\frac{\ell^A}{\gamma}\right)}\right]\ell^B\right) \\
&\quad \times \left[\frac{F\left(\frac{\ell^A}{\gamma}, y_H\right) - F\left(\frac{\ell^A}{\gamma}, \frac{\ell^B}{\gamma}\right) - \frac{\ell^B}{\gamma}F_2\left(\frac{\ell^A}{\gamma}, \frac{\ell^B}{\gamma}\right)}{G\left(\frac{\ell^A}{\gamma}\right)}\right] \\
&\quad -\beta\int_{\frac{\ell^B}{\gamma}}^{y_H}\int_{y_L}^{\frac{\ell^A}{\gamma}} u_1\left(y^B-\ell^B\right)\frac{f\left(y^A,y^B\right)}{G\left(\frac{\ell^A}{\gamma}\right)} dy^A dy^B = \mu
\end{aligned} \tag{2.1.11}$$

The solution of this problem is given by a function $\ell^B\left(\ell^A,1\right)$ that maps a defaulted level of borrowing from Borrower A into borrowing of Borrower B. The functions $\ell^B\left(\ell^A,0\right)$ and $\ell^B\left(\ell^A,1\right)$ characterize the equilibrium behavior of Borrower B under each of the two proposed scenarios. Unlike Borrower B, Borrower A has no observation on which to condition expectations and probabilities, and must therefore rely on unconditional moments only.

2.1.4.3. Borrower A

The problem of Borrower A can be written as follows (see(2.1.7)):

$$
\begin{aligned}
V^A &\equiv \max_{\ell^A} u\left(\left[1-G\left(\frac{\ell^A}{\gamma}\right)\right]\ell^A\right)+\beta\times \\
&\left[\int_{\frac{\ell^A}{\gamma}}^{y_H} u\left(y^A-\ell^A\right)g\left(y^A\right)dy^A+\int_{y_L}^{\frac{\ell^A}{\gamma}} u\left(y^A-\gamma y^A\right)g\left(y^A\right)dy^A\right] \qquad (2.1.12)
\end{aligned}
$$

The necessary first order condition that characterises the solution to this problem is:

$$
\begin{aligned}
\Theta\left(\ell^A\right) &\equiv u_1\left(\left[1-G\left(\frac{\ell^A}{\gamma}\right)\right]\ell^A\right)\left[1-G_1\left(\frac{\ell^A}{\gamma}\right)\frac{\ell^A}{\gamma}-G\left(\frac{\ell^A}{\gamma}\right)\right] \\
&-\beta\int_{\frac{\ell^A}{\gamma}}^{y_H} u_1\left(y^A-\ell^A\right)g\left(y^A\right)dy^A=\mu \qquad (2.1.13)
\end{aligned}
$$

The solution to this problem is a unique optimal value of ℓ^A.

2.1.4.4. Equilibrium

The decentralized equilibrium of the model is defined as follows:

Definition 5. A decentralized equilibrium of the model is a triple $\left\{\ell^{A\star},\ell^{B\star,d^A=0},\ell^{B\star,d^A=1}\right\}$ of face values such that: 1. $\ell^{A\star}$ solves problem (2.1.12), 2. $\ell^{B\star,d^A=0}$ solves problem (2.1.8) given $y^A \geq \ell^{A\star}/\gamma$, 3. $\ell^{B\star,d^A=1}$ solves problem (2.1.10) given $y^A < \ell^{A\star}/\gamma$.

Given the properties of differentiability, continuity and concavity of u, and differentiability and continuity of functions F and G, the decentralized equilibrium of the model is fully characterised by the system of equations (2.1.9), (2.1.11) and (2.1.13). A decentralized allocation is a triple $\left\{\ell^{A\star},\ell^{B\star,d^A=0},\ell^{B\star,d^A=1}\right\}$ that is a solution to this system of equations and $\ell^{B\star,d^A=j}=\ell^B\left(\ell^{A\star},j\right)$ for $j\in\{0,1\}$.

The choice of Borrower A with regard to his own level of borrowing ℓ^A will have several effects. For a start, changes in ℓ^A will change the probability of Borrower A being forced to default, which will entail a consumption cost. Borrower A does take into account this effect of ℓ^A on his own consumption possibilities for the second period of his life. However, ℓ^A will also affect Borrower B in a number of ways that Borrower A does not take into account when in his own optimization problem. First, changes in ℓ^A change the probability of Borrower B being in either of the two situations described in sections (2.1.4.1) and (2.1.4.2). Besides, ℓ^A will also affect, given d^A, the choice set of Borrower B via the pair of functions $\ell^B\left(\ell^A, 0\right)$ and $\ell^B\left(\ell^A, 1\right)$.

2.1.5. The Problem of the Benevolent Policymaker

These externalities imply that, in general, the equilibrium allocation that would be chosen by a benevolent policymaker concerned about the welfare of both Borrowers will be different from the decentralized equilibrium allocation. The reason is that the former will take into account the welfare effects of the choice of ℓ^A on the insurance possibilities of Borrower B. The problem of the policymaker is defined as follows:

$$\max_{\ell^A, \ell^B} \lambda U^A + (1-\lambda) E\left[U^B\right] \tag{2.1.14}$$

where $\lambda \in (0,1)$ is the relative weight attached by the policymaker to Borrower A. It is assumed that the policymaker faces the same informational constraints as the decentralized agents. In other words, it is assumed that the policymaker cannot directly observe income in the same way as Banks themselves cannot do it. It is in this sense that the concept of "efficiency" attached to the solution of the problem of the policymaker corresponds specifically to the notion of "ex-ante constrained efficiency" [14]. Thanks to the sequential nature of the problem (and to the fact that there are not subsequent markets after Market B), it is possible to use again the backward-induction argument to reduce the two-variable problem (2.1.14) into the following single variable program:

$$\max_{\ell^A} \lambda U^A + (1-\lambda)\left\{\left[1-G\left(\frac{\ell^A}{\gamma}\right)\right] V^B\left(\ell^A, 0\right) + G\left(\frac{\ell^A}{\gamma}\right) V^B\left(\ell^A, 1\right)\right\} \tag{2.1.15}$$

The solution to (2.1.15) determines the ex-ante socially optimal level of borrowing by Borrower A. The ex-post socially optimal level of borrowing by Borrower B is determined, by backward induction, by equations (2.1.9) and (2.1.11). Therefore, the only difference between the set of decentralized equilibrium conditions and the solution of the policymaker

[14] For the same reason, this chapter makes reference to the problem of a benevolent policymaker and not of a social planner. The social planner reference would imply the ability of policy to pick equilibrium allocation that are state-contingent and not subject to informational constraints.

will be given by the difference between first order condition (2.1.13) and the first order condition of problem (2.1.15).

Definition 6. Given λ, an ex-ante efficient allocation is a triple $\left\{\ell^{A,P}, \ell^{B,P,d^A=0}, \ell^{B,P,d^A=1}\right\}$ such that[15]: 1. $\ell^{A,P}$ solves problem (2.1.15), 2. $\ell^{B,P,d^A=0}$ solves problem (2.1.8) after $y^A \geq \ell^{A,P}/\gamma$, 3. $\ell^{B,P,d^A=1}$ solves problem (2.1.10) after $y^A < \ell^{A,P}/\gamma$.

The ex-ante efficient allocation is characterised by the system of equations composed by (2.1.9), (2.1.11) and the following first order condition of problem (2.1.15):

$$\Phi\left(\ell^A\right) \equiv \lambda\Theta\left(\ell^A\right) + (1-\lambda)\left\{G_1\left(\frac{\ell^A}{\gamma}\right)\frac{1}{\gamma}\left[V^B\left(\ell^A,1\right) - V^B\left(\ell^A,0\right)\right] + \left[1 - G\left(\frac{\ell^A}{\gamma}\right)\right]V_1^B\left(\ell^A,0\right) + G\left(\frac{\ell^A}{\gamma}\right)V_1^B\left(\ell^A,1\right)\right\} = \mu \quad (2.1.16)$$

The ex-ante efficient allocation is thus a triple $\left\{\ell^{A,P}, \ell^{B,P,d^A=0}, \ell^{B,P,d^A=1}\right\}$ that is a solution to this system of equations and $\ell^{B,P,d^A=j} = \ell^B\left(\ell^{A,P}, j\right)$ for $j \in \{0,1\}$.

The fact that some of the actions of Borrower A have effects on other agents that are not being taken into account in the decentralized equilibrium will imply that there is, in general, a gap between the decentralized allocation and the efficient allocation. The nature and economic intuition behind this gap is discussed in turn.

2.1.6. Inefficiency and Discussion

Definition 7. The decentralized allocation is efficient if there is some $\lambda \in (0,1)$ for which $\ell^{A,P} = \ell^{A\star}$.

Proposition 8. *Suppose the parameter set rules out corner solutions (that is, $\mu = 0$ for all the first order conditions that have been presented in the previous sections). The decentralized allocation is efficient if and only if y^A and y^B are independent. Proof: See Appendix B.1.*

Thus, in the general case of non-independence between the incomes of the two borrowers, the decentralized allocation is inefficient. However, the direction of the inefficiency (that is, whether there is overborrowing or underborrowing ex-ante) is unclear at this point. To be more specific, it is necessary to propose some form of correlation between y^A and

[15] where the superindex P stands for "policymaker".

y^B. The following paragraphs offer a discussion along this lines assuming that y^A and y^B are positively correlated (if the correlation is negative, the discussion follows exactly the opposite line of argumentation).

If y^A and y^B are positively correlated, $d^A = 0$ (conversely, $d^A = 1$) signals to agents in Market B an increased (decreased) probability of observing higher levels of y^B. Thus, for a given level of borrowing ℓ^A, $d^A = 0$ ($d^A = 1$) allows Borrower B to sell a given face value ℓ^B at a higher (lower) price. This fact implies that, for all ℓ^A:

$$V^B\left(\ell^A,1\right) < V^B\left(\ell^A,0\right) \tag{2.1.17}$$

Moreover, given d^A, a higher (conversely, a lower) ℓ^A signals to agents in Market B an increased (decreased) probability of observing higher levels of y^B. Thus, for a given default/repayment choice d^A a higher (lower) ℓ^A allows Borrower B to sell a given face value ℓ^B at a higher (lower) price. This fact implies that, for all ℓ^A:

$$V_1^B\left(\ell^A,0\right) > 0 \qquad V_1^B\left(\ell^A,1\right) > 0 \tag{2.1.18}$$

Both (2.1.17) and (2.1.18) create the ambiguity result. To see this, evaluate the optimality condition for the policymaker (2.1.16) at the decentralized $\ell^{A,\star}$ assuming an interior solution:

$$\begin{aligned}\Phi\left(\ell^{A,\star}\right) \;=\; & (1-\lambda)\left\{G_1\left(\frac{\ell^{A,\star}}{\gamma}\right)\frac{1}{\gamma}\left[V^B\left(\ell^{A,\star},1\right) - V^B\left(\ell^{A,\star},0\right)\right] + \right. \\ & \left.\left[1-G\left(\frac{\ell^{A,\star}}{\gamma}\right)\right]V_1^B\left(\ell^{A,\star},0\right) + G\left(\frac{\ell^{A,\star}}{\gamma}\right)V_1^B\left(\ell^{A,\star},1\right)\right\}\end{aligned} \tag{2.1.19}$$

where $\Theta\left(\ell^{A,\star}\right) = 0$ is used. As $\Phi\left(\ell^{A,P}\right) = 0$, it follows that the decentralized equilibrium features inefficient overborrowing ($\ell^{A,\star} > \ell^{A,P}$) only if:

$$\begin{aligned}& G_1\left(\frac{\ell^{A,\star}}{\gamma}\right)\frac{1}{\gamma}\left[V^B\left(\ell^{A,\star},1\right) - V^B\left(\ell^{A,\star},0\right)\right] + \\ & \left[1-G\left(\frac{\ell^{A,\star}}{\gamma}\right)\right]V_1^B\left(\ell^{A,\star},0\right) + G\left(\frac{\ell^{A,\star}}{\gamma}\right)V_1^B\left(\ell^{A,\star},1\right) < 0\end{aligned} \tag{2.1.20}$$

Of course, if condition (2.1.20) is reversed, the decentralized equilibrium features inefficient underborrowing ($\ell^{A,\star} < \ell^{A,P}$). Proposition 8 implies that the LHS of expression (2.1.20) is generally different from zero, as y^A and y^B are postulated to be dependent. However, (2.1.17) and (2.1.18) imply that the sign of this term is, in principle, ambiguous.

To explain the economic intuition behind this ambiguity, it is useful to decompose the LHS of (2.1.20) in two terms, the relative magnitude of which will determine whether or not overborrowing is observed in the decentralized equilibrium:

1. "Hirshleifer Term"

$$G_1\left(\frac{\ell^{A,\star}}{\gamma}\right)\frac{1}{\gamma}\left[V^B\left(\ell^{A,\star},1\right)-V^B\left(\ell^{A,\star},0\right)\right]<0 \tag{2.1.21}$$

   This term reflects the positive effect of ℓ^A on the probability of a loss of size $V^B\left(\ell^A,1\right)-V^B\left(\ell^A,0\right)$ on Borrower B. The latter term reflects the different insurance possibilities of Borrower B under $d^A=1$ and $d^A=0$ respectively[16]. This term captures the mechanism that has been described before: from an ex-ante perspective, the borrowing decision by Borrower A changes the probability of unleashing a certain impact on the choice set of Borrower B. Notice that this effect occurs through the information flow between financial transactions in the two markets. This mechanism is an example of how, from an ex-ante point of view, the interim partial revelation of information about income creates an externality through its effect on the insurance possibilities of other agents. This term is named after Jack Hirshleifer[44, 1971], who first explored this form of externality.

2. "Cross Reputation Term": This term reflects the positive effect of ℓ^A on the probability of observing high levels of y^B given d^A:

$$\left[1-G\left(\frac{\ell^{A,\star}}{\gamma}\right)\right]V_1^B\left(\ell^{A,\star},0\right)+G\left(\frac{\ell^{A,\star}}{\gamma}\right)V_1^B\left(\ell^{A,\star},1\right)>0 \tag{2.1.22}$$

   This term is the expected derivative of the value function of Borrower B with respect to ℓ^A. As explained before, given d^A, an increased ℓ^A improves the insurance possibilities of Borrower B, by being more informative about a high income in case $d^A=0$ or less informative about a low income in case $d^A=1$. If Borrower A and Borrower B were the same person, this effect could be interpreted as the building of reputation by an individual Borrower, in an analogous way to people in real credit markets sometimes wishing to borrow small quantities with the only purpose of mantaining high credit scores. As this reputation transfers in this context from one Borrower to the next given income correlation, this term has been called the "Cross Reputation Term".

If the "Hirshleifer Term" is greater (smaller) in absolute value than the "Cross Reputation Term", the decentralized equilibrium will feature overborrowing (underborrowing) as the

[16] Recall that both Borrowers are assumed to be risk-averse. This assumption implies that Borrowers wish to smooth income across periods, and use unsecured consumer credit as the main instrument to achieve this goal. Thus, differences in ex-post payoffs due to different values of d^A(a state variable) will reflect different choice sets in the same insurance problem.

concern for the insurance possibilities of Borrower B outweighs (at the efficient allocation) the need to create a reputation for him via an increase in ℓ^A.

2.1.7. Correcting the externality

Regardless of the sign of the inefficiency, it is possible to imagine several mechanisms by which the policymaker could incentivize economic agents to optimally choose the efficient allocation in a decentralized equilibrium. This section explores an increase in the cost of wholesale funds and the introduction of a tax on borrowing in Market A as mecanisms of implementation of the efficient allocation.

2.1.7.1. Increase in the cost of wholesale funds

Assume the zero-profit condition of Bank A (2.1.5) is rewritten as:

$$\pi^A \ell^A + \left[1 - \pi^A\right] 0 = q^A \ell^A (1+\tau)$$
$$\frac{\pi^A}{1+\tau} = q^A$$

where τ is an exogenously engineered percentage variation in the cost of funds. In the baseline model, it is assumed that Banks fund loans through deposits from an exogenous source that can be thought of as having an opportunity cost of funds of zero. Therefore, this exercise considers a policy scenario that increases the opportunity cost for outside depositors in some percentage τ. Rewrite the decentralized problem of Borrower A after the introduction of this increase in the cost of funds as:

$$\begin{aligned} V^A \equiv & \max_{\ell^A} u\left(\frac{\left[1-G\left(\frac{\ell^A}{\gamma}\right)\right]}{1+\tau}\ell^A\right) \\ & +\beta\left[\int_{\frac{\ell^A}{\gamma}}^{y_H} u\left(y^A-\ell^A\right) g\left(y^A\right) dy^A + \int_{y_L}^{\frac{\ell^A}{\gamma}} u\left(y^A-\gamma y^A\right) g\left(y^A\right) dy^A\right] \end{aligned}$$

The necessary (and sufficient, given assumed conditions on concavity) first order condition that characterizes the interior solution to this problem is now:

$$u_1\left(\frac{\left[1-G\left(\frac{\ell^A}{\gamma}\right)\right]}{1+\tau}\ell^A\right)\frac{\left[1-G_1\left(\frac{\ell^A}{\gamma}\right)\frac{\ell^A}{\gamma}-G\left(\frac{\ell^A}{\gamma}\right)\right]}{1+\tau}$$
$$-\beta\int_{\frac{\ell^A}{\gamma}}^{y_H}u_1\left(y^A-\ell^A\right)g\left(y^A\right)dy^A=0 \qquad (2.1.23)$$

In order to correct the inefficiency within the framework of the decentralized equilibrium, the policymaker must set τ such that equation (2.1.23) is satisfied when evaluated at $\ell^{A,\star}=\ell^{A,P}$ (the latter being determined by (2.1.16)). Naturally, $\tau>0$ (<0) whenever the decentralized equilibrium features overborrowing (underborrowing).

2.1.7.2. Borrowing Tax

Assume that the consumption of Borrower A is given by:

$$c_1^A=q^A\ell^A\left(1-t\right)$$

where t is an exogenous tax imposed on borrowing by Borrower A, rebated to wither borrower in the form of a lump-sum transfer. Rewrite the problem of Borrower A with this tax as follows:

$$\begin{aligned}V^A \equiv\ & \max_{\ell^A} u\left(\left[1-G\left(\frac{\ell^A}{\gamma}\right)\right]\ell^A\left(1-t\right)\right)+\beta\times \\ & \left[\int_{\frac{\ell^A}{\gamma}}^{y_H}u\left(y^A-\ell^A\right)g\left(y^A\right)dy^A+\int_{y_L}^{\frac{\ell^A}{\gamma}}u\left(y^A-\gamma y^A\right)g\left(y^A\right)dy^A\right]\end{aligned} \qquad (2.1.24)$$

The necessary first order condition that characterizes the solution to this problem is:

$$u_1\left(\left[1-G\left(\frac{\ell^A}{\gamma}\right)\right]\ell^A(1-t)\right)\left[1-G_1\left(\frac{\ell^A}{\gamma}\right)\frac{\ell^A}{\gamma}-G\left(\frac{\ell^A}{\gamma}\right)\right](1-t)$$
$$-\beta\int_{\frac{\ell^A}{\gamma}}^{y_H}u_1\left(y^A-\ell^A\right)g\left(y^A\right)dy^A=0 \tag{2.1.25}$$

In order to correct the inefficiency within the framework of the decentralized equilibrium, the policymaker must set t such that equation (2.1.25) is satisfied when evaluated at $\ell^{A,\star}=\ell^{A,P}$ (the latter being determined by (2.1.16)). Similar to the case of the increase in the cost of funds, $t>0$ (<0) whenever the decentralized equilibrium features overborrowing (underborrowing).

2.2. Simulations

The ambiguity result of the theoretical model suggests the necessity to explore numerically the range of parameters under which either direction of inefficiency will show up in the decentralized equilibrium. The model presented is simple and not designed at this stage to provide quantitative answers to real world policymakers. This section is intended as an exploratory exercise designed to build intuition on the effect of some parameters on the amount of over/underborrowing.

2.2.1. Functional Forms and Parameters

In order to simulate the previous model, it is necessary to specify, together with parameter values, functional forms for the instantaneous utility function u and the joint distribution function F. The utility function is assumed to be:

$$u(c)=\frac{c^{1-\sigma}}{1-\sigma} \tag{2.2.1}$$

where σ is the coefficient of relative risk-aversion. With regard to the joint distribution, it is assumed that $y^A=y^B=y$. This form of extreme correlation between the incomes of the two borrowers implies that the state of nature is fully realized (but not observed by all) in the second period of Market A, and agents in Market B use available information to learn about the realized state. In this context, the only function left to specify is the unconditional distribution of y. It is assumed that:

$$\Pr\left(y \geq x\right) = \begin{cases} \rho + (1-\rho)\left(\frac{y_H - x}{y_H - y_L}\right) & y_L \leq x \leq y_H \\ 0 & x > y_H \end{cases} \tag{2.2.2}$$

That is, there exist a positive mass at $y = y_H$, as $\Pr\left(y \geq y_H\right) = \Pr\left(y = y_H\right) = \rho$, and y is uniformly distributed in the range $[y_L, y_H)$. This distribution function explicitly encompasses both a discrete Bernoulli income distribution ($\rho > 0$ and zero mass after $y = y_L$) and a continuous distribution ($\rho = 0$).

Appendix B.2 presents the optimization problems of Borrower A, Borrower B and the benevolent policymaker under (2.2.1) and (2.2.2). The model is solved computationally with the set of base parameters presented in table 2.1.

Table 2.1.: Base Parameters

Parameter	Value
ρ	0.4
γ	0.1
σ	2.5
β	0.985
y_H	100
y_L	30

Figure 2.2.1 at the end of the chapter presents the effect of changes in ρ (the probability of observing $y = y_H$), γ (the default penalty) and σ (risk aversion) from their baseline values on the Hirshleifer Term, the Cross-Reputation Term and the LHS of (2.1.20) (which corresponds to the sum of the previous two). Clearly, as in (2.1.21) and (2.1.22) these terms are evaluated at $\ell^{A,\star}$ (the optimal decentralized borrowing by Borrower A). To guarantee an interior solution, as assumed throughout the paper, it is necessary to impose $\rho < 0.525$, $\gamma < 0.75$ and $\sigma > 2.5$.

Panels A and B in figure 2.2.1 present respectively the Hirshleifer and Cross Reputation Terms, and the LHS of (2.1.20) for different values of ρ (and keeping the other parameters at their base values). As explained before, the Hirshleifer Term is generally negative, whereas the Cross-Reputation Term is positive (Panel A). For low levels of ρ, underborrowing is prevalent, as the sum of the two terms of interest is positive. As ρ rises, the Hirshleifer Term becomes relatively bigger and, for $\rho > 0.15$, the decentralized equilibrium features overborrowing. The intuition for this result follows closely the description of the two terms of interest. When ρ is relatively high, Borrower A has the incentive to borrow excessively, in the sense that his default sends a very bad signal to Market B with respect to y.

Under this conditions, the policymaker has a strong incentive to protect Borrower B's insurance possibilities. This effect of ρ can be related to the inefficiency of credit booms in the unsecured consumer credit market. In the context of the model, the business cycle can be captured through changes in ρ: a high ρ might characterize an economic boom, whereas a low ρ would define a recession. Under this interpretation, the simulations suggest that there is overborrowing during economic booms (that is, inefficient credit booms) and inefficiently low borrowing during recessions. When ρ is extremely high (in this case, close to $\rho = 0.525$, the value that triggers the corner solution $\ell^{A,\star} = \gamma y_H$), underborrowing emerges again as the decentralized outcome. The intuition in this case is less interesting: when the probability of the highest possible income is very high, the probability of default is so low that the potential costs of default do not outweigh the potential benefit for Borrower B of a likely non-default from Borrower A.

Panels C and D present the same calculations for different values of γ, the default penalty (other parameters kept at their base values). In this case, the spectrum of possible values of γ that imply overborrowing is very large. Underborrowing is only possible when γ is extremely high, and the optimal decentralized borrowing of Borrower A is close to the riskless minimum. Interestingly, as γ increases, the degree of overborrowing falls. The intuition for this is that, as the private cost of default becomes larger, Borrower A's incentive to borrow diminishes, and the private cost of default makes the policymaker less willing to "bet" on a good realization of income.

Finally, Panels D and E study the effect of different values of σ, the coefficient of risk aversion. As is the case with γ, only extremely high levels of risk aversion would induce underborrowing. Similarly, a higher degree of risk aversion reduces the degree of overborrowing, as risk averse Borrower A has less incentives to borrow given the uncertainty of consumption in the second period.

2.3. Conclusions

This chapter has presented a simple theoretical framework in which the effects of the default choice of some agents spread to some other agents in the economy. The existence of this mechanism depends crucially on the dependence between incomes of different groups of agents. This dependence interacts with the flow of information across markets to alter insurance possibilities across the economy. The fact that atomistic agents do not take into account these externalities of individual default choice imply that, under some conditions, the decentralized borrowing will be suboptimal.

This observation suggests an important policy implication given the recent evolution of unsecured consumer credit in several Latin American economies described in the introduction to this chapter. The model developed here captures a number of salient features of the unsecured consumer credit market in those economies. In particular, the lack of broadly used legal mechanisms that allow lenders to seize individuals' assets in the event of default and the big prominence given to aggregate variables (aggregate default) in the credit extension decision from lenders. As the latter is an important mechanism through which information on the aggregate state is obtained, it is plausible to conjecture that individual decisions that alter aggregate outcomes do potentially create benefits and costs that go beyond the private sphere. In this sense, if the rapid growth of consumer credit in Latin America responds (to some extent) to a broad perception of a higher income in the future, the simulations of the model suggest that policymakers may have the incentive to curb the rapid expansion of credit, lest future defaults affect the ability of borrowers in the future to insure and smooth consumption.

Some comments must be made about the abovementioned results that might guide future work along these lines. Firstly, the role of the absence of secured credit should be highlighted. For example, if Bank B had the legal ability to seize assets from Borrower B in case of default, the ex-ante insurance possibilities of the latter might even be the same under $d^A = 0$ that under $d^A = 1$ with only a change in, say, posted collateral. The inability to secure credit in this framework is therefore a crucial element in delivering the inefficiency result.

Second, the inefficiency result obtained in Proposition 8 is symmetric along the business cycle up to changes in the joint distribution function F. As suggested before, only if the business cycle is interpreted -within the context of the model- as changes in F, then it is possible to think of different directions of inefficiency in different stages of the business cycle: for example, underborrowing in recessions (where high incomes are unlikely) and overborrowing in booms (when low incomes are unlikely).

Otherwise, if F is assumed to be invariant across stages of the business cycle, then the model does not feature endogenously inefficient credit booms, in the sense of overborrowing in some states of nature (or stages of the cycle) at the cost of volatility and underborrowing in some other states of nature[17]. Rather, the model would predict permanently inefficient credit, regardless of the state of nature, in a similar way than information asymmetries and other frictions do.

A possible mechanism to introduce asymmetries along the business cycle without changes to F is proposed by Ordonez[59, 2010]. The mechanism is based on the number of transactions

[17] See Lorenzoni[51, 2008]

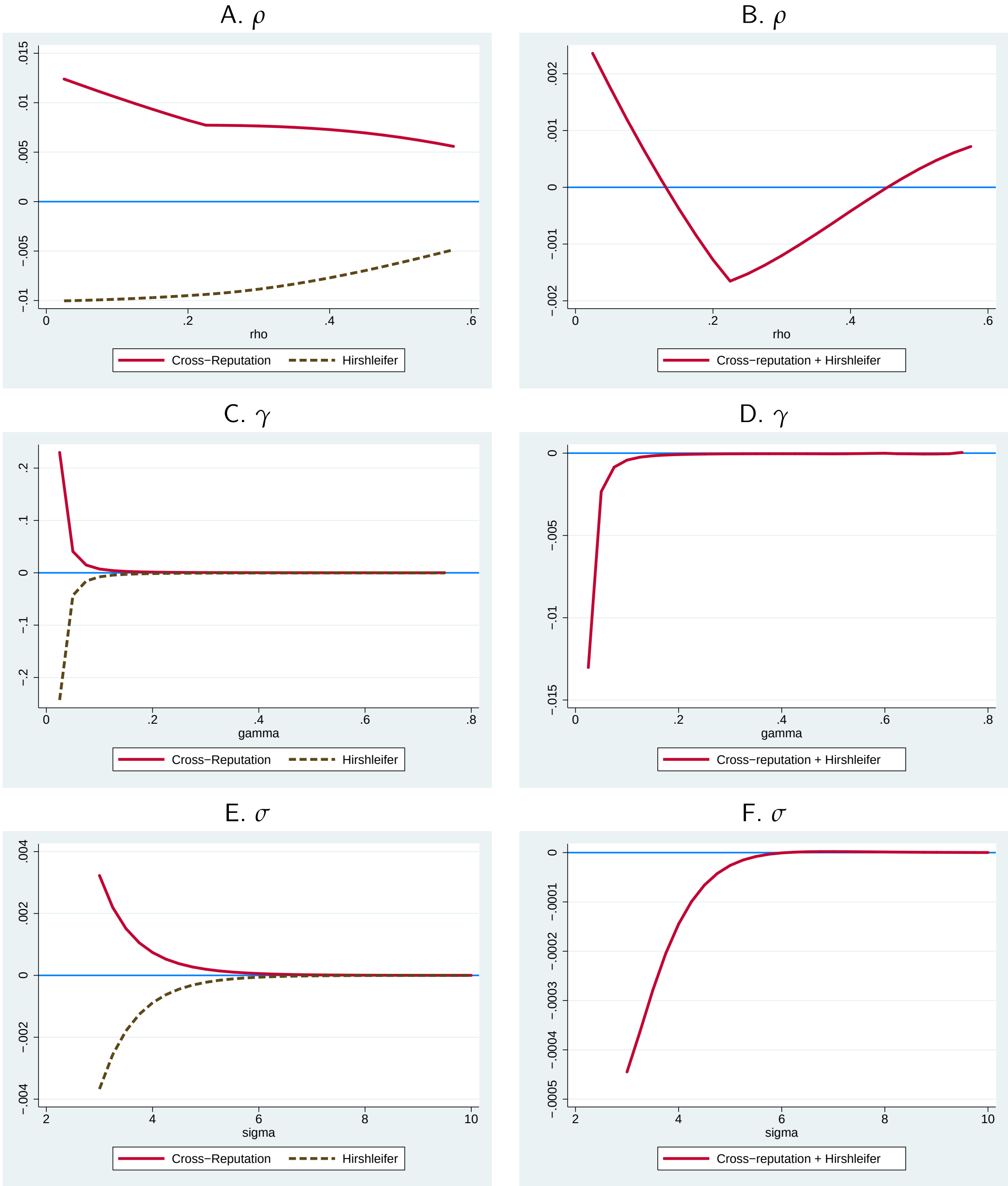
(lending/borrowing) that take place in different stages of the business cycle, that produces a different number of signals allowing lenders to learn at different speeds the changes in the state of nature. This mechanism could be easily implemented in this context after extending the model to several periods and agents.

A final reflection concerns the practical implementation of the solution to the externality. Sections (2.1.7.1) and (2.1.7.2) discussed the theoretical calculation of instruments to implement the efficient allocation in a decentralized equilibrium. However, the pair of solutions discussed are implementable in practice assuming that the policymaker is able to identify Market A from Market B, or more generally, those markets that create externalities with default from those other markets that are affected. In the context of the model, time is the key variable that permits this distinction, but it is clear that in practice this task might be all but impossible at a given point in time. Again, the introduction of asymmetric inefficiency along the business cycle regardless of F might be useful in this case: the realization of the state of nature might serve as an indicator of socially suboptimal trends in borrowing.

Figure 2.2.1.: Overborrowing and Underborrowing

(2.1.20) and (2.1.21)

B: LHS of (2.1.20)



3. Information Sharing and Credit Outcomes: Evidence from a Natural Experiment[1]

In a frictionless capital market, funds will always be available for individuals to smooth their consumption by borrowing against future expected income. In practice, however, borrowers often complain of not being able to borrow enough at reasonable rates. Economic theory suggests that market frictions such as information asymmetries and agency conflicts may explain why capital does not always flow to borrowers with positive expected income flows (see, for example, Stiglitz and Weiss [75, 1981]). From a theoretical point of view, one mechanism lenders could use to overcome these frictions is the exchange of information with other lenders through information brokers, generally known as Credit Bureaus (see Pagano and Jappelli [61, 1993] and Padilla and Pagano [60, 2000])[2].

Consistent with these theories, several empirical studies find that information sharing through credit bureaus is positively correlated to ex-post credit market development, as indicated by higher credit access and lower default rates (see, for example, Djankov et al. [32, 2007], Jappelli and Pagano [47, 2002], Warnock and Warnock [79, 2008] and Galindo and Miller [41, 2001]).

While still potentially increasing the discipline of borrowers, however, certain types of information sharing systems across lenders can also exacerbate "hold-up" problems where lenders extract informational rents from borrowers (see Sharpe [73, 1990], Rajan [66, 1992] and von Thadden [78, 2004]). For instance, if lenders share only negative information (i.e. defaults), or if this negative information is long-lived and carries a stigma of failure,

[1]This chapter is joint work with Juanita González-Uribe.

[2]There are other mechanisms that banks use to overcome these frictions such as requiring potential borrowers to pledge collateral in order to sort observationally equivalent loan applicants through signaling (e.g., Bester [13, 1985] [14, 1987]; Besanko and Thakor [12, 1987a] [11, 1987b]; Chan and Thakor [23, 1987]; Boot, Thakor, and Udell [20, 1991a]) or to induce discipline (e.g., Boot, Thakor, and Udell [19, 1991b]; Boot and Thakor [18, 1994]; Aghion and Bolton [1, 1997]; Holmstrom and Tirole [45, 1997]). In practice, however, not all loans are easily backed by collateral. Individuals may not have enough tangible assets to collateralize, and poor protection of creditor rights may make seizing collateral unfeasible.

inside banks (those with which borrowers already have a lending relationship) are more likely to be able to hold-up customers from receiving competitive financing elsewhere. In this regard, Frisancho [37, 2012] documents that high-quality microfinance borrowers in Peru moved away from their banks when the sharing system was modified to include their positive information (i.e. repayment histories).

This chapter analyzes the relationship between information sharing and credit outcomes in the unsecured consumer credit market, focusing on the potential ex-post disciplinary effect and the ex-ante informational hold-up of long-lived negative information about borrowers. We exploit a natural experiment in Colombia created by Law 1266/2008, whereby detailed information about past defaults that were exogenously "sufficiently old" was erased from Private Credit Bureaus (in what follows, PCB). Specifically, for borrowers in good standing, the law specified that detailed information on all defaults prior to June 2008 was erased with immediate effect in June 2009. Information regarding defaults that occurred between June 2008 and June 2009, as well as positive information of borrowers (e.g., repayment histories), remained unchanged at PCB.

Our empirical strategy compares changes in the credit outcomes for those borrowers whose negative information was erased from PCB (hereafter referred to as the "treatment" group) with changes in credit outcomes for a counterfactual group of borrowers whose information in PCB was not affected by the law ("control" group). In particular, the treatment group corresponds to those borrowers in good standing by June 2009 whose last default episode was recorded before June 2008. As a result of the law, all negative information pertaining the history of these borrowers was destroyed in June 2009. The control group corresponds to borrowers who had not committed a default prior to the enactment of the law, and therefore did not have any negative information to start with. The information about these borrowers in PCB did not change after June 2009. The main identification assumption (to be articulated more precisely below) is that credit outcomes of treated and control borrowers would have evolved in a similar manner in the absence of the change in information sets.

We complement our empirical strategy by comparing credit outcomes for the same borrower across inside and outside banks (the latter being those lenders with which borrowers do not have a pre-existing lending relationship). This comparison is essential to shed light on the effects of the law on the informational hold-up by inside lenders. In Colombia, by legal requirement, banks can only access a borrower's information in PCB after obtaining the explicit written authorization from a prospective customer, something which is done normally as part of the application procedure for a loan. In addition, we focus on unsecured consumer credit, where banks do not have access to reliable sources of information on prospective borrowers for their credit evaluations other than PCB. As a consequence, the change in information induced by the law is only relevant for outside banks, as long as these are unlikely to have private records on prospective borrowers. Outside lenders should therefore be more sensitive to the information change than inside banks.

Using a Differences-in-Differences (DD) specification, we find that *after* old negative information is erased, there is a significant increase in the size and maturity of new loans, no significant changes in interest rates, and an increase in subsequent default rates for the treatment group, relative to the control group. Overall, these results are consistent with both ex-post disciplinary effects, and ex-ante information hold-up from long-lived negative information in PCB. Specifically, consistent with the hold-up theories, we find that most of the increase in the value of loans comes from outside banks. In addition, consistent with the disciplinary role of information sharing, we find evidence of a relative increase in the frequency of default for new loans after negative information is erased.

The main advantage of the choice of treatment and control groups as described above is that both groups committed no default after the announcement of the legislative project. However, one disadvantage is that borrowers who did not commit any default prior to June 2008 may be different from borrowers in the treatment group in ways that we cannot control for even after allowing for differential trends in credit outcomes across treatment and control groups. To deal with this concern, we perform a series of robustness checks that include the use of alternative treatment and control groups in order to deal with potential biases in the estimation that could arise from changes in the expected income of borrowers concurrent to the enactment of the law. Specifically, we consider an alternative treatment group comprised by borrowers whose last default occurred as far in the past as the sample allows us to observe, thus resembling more closely the recent default behavior of the control group. In addition, we consider an alternative control group corresponding to those borrowers in good standing by June 2009 who committed a default between June 2008 and December 2008, and which (in accordance with the law) did not see their prior default history immediately erased from the system in June 2009. The advantage of this alternative control group is that (similar to the treatment group) these borrowers have defaulted in their debt obligations at some point in their recent history. Reassuringly, our main results are robust to the choice of these alternative groups.

This chapter contributes to the literature that explores the impact of information sharing on credit markets (Pagano and Jappelli [61, 1993], Padilla and Pagano ([60, 2000]), Djankov et al. [32, 2007], Jappelli and Pagano [47, 2002], Warnock and Warnock [79, 2008], Galindo and Miller [41, 2001], Galindo and Micco [39, 2007] [40, 2010], Love and Milenko [52, 2003], Brown, Jappeli and Pagano [21, 2009], Frisancho [37, 2012], Hertzberg, Liberty and Paravisini [43, 2011]). Our chapter differs from prior research in that we focus on one specific aspect of information sharing (long-lived negative information) and we exploit exogenous variation in the type of information available in PCB, using detailed loan-level data. In contrast, prior research (with the exceptions of Frisancho[37, 2012] and Hertzberg, Liberty and Paravisini [43, 2011]) uses cross-country differences in the development of PCB to examine the impact of information sharing on credit outcomes. The limitations of this type of approaches are several and well-known.

This chapter also contributes to the literature on hold-up problems (Sharpe [73, 1990], Rajan [66, 1992], Petersen and Rajan [65, 1994], Berger and Udell [9, 1995], Boot [17, 2000], Ongena and Smith [58, 2001], Berger and Udell [10, 2002], Farinha and Santos [35, 2000], Gopalan, Udell, and Yerramilli [42, 2011], Degrysse and Ongena [31, 2005], Schenone [71, 2010] and Ioannidou and Ongena [46, 2010]). Our chapter differs from the ones cited above in that our setting provides an arguably exogenous change in the capability of hold-up by banks. Results from prior research are hard to interpret as long as they may reflect differences between firms that self-select into relationship lending and those that do not. Our main contribution is to propose a new form of hold-up which arises even in the context of information sharing. The key new insight here is that in contexts where stigma of failure is prevalent, sharing systems with long-lived negative information may exacerbate the informational hold-up from inside banks. The results are likely to pertain to the specific characteristics of the Colombian financial system, as there are reasons to believe that it is particularly characterized by a slow, mistake-penalizing nature of information updating by banks. Results may thus generalize to other similar developing economies with shallow financial systems, but are less likely to apply to countries with more developed capital markets where the stigma of failure is, in principle, less prevalent. In this sense, we are limited by the cleanly identified setting to make more general claims in this regard.

This chapter unfolds as follows. Section 3.1 is devoted to a brief exploration of the institutional structure of the Colombian system of credit histories and the main features of the natural experiment that arises from the Law 1266/2008. Section 3.2 presents the empirical strategy employed in the chapter. Section 3.3 describes the dataset and examines the main statistical features of the treatment and control groups created by the experiment. The results are in turn presented in Section 3.4; some concluding comments are discussed in Section 3.5.

3.1. Institutional Background

3.1.1. The Colombian System of Credit Histories

Regulated, formal, financial institutions in Colombia are required by law to frequently record and report information on their loans. This information is collected on behalf of the government by the Public Credit Registry administered by the Financial Superintendence Office

(FSO, Financial Regulation Office). The Registry collects loan-level information covering the formal financial system at quarterly frequency. This information is confidential and is used by the FSO and by the Central Bank with regulatory, policy and research purposes.

Information sharing across formal lenders in the Colombian financial system is done through Private Credit Bureaus (PCB), which started operating in Colombia in 1981[3]. After explicit written authorization from a prospective borrower (something usually included as part of the application process for a loan), a bank can obtain a "credit report" on the borrower from a PCB[4]. Before 2009, these credit reports included detailed information on all current and past loans. Information on the repayment history of utilities (such as electricity bills and telephone services) as well as an internally generated credit score was also provided in the reports[5].

For the case of financial obligations, entries in the credit report included originating bank, loan amount, loan type, value of collateral, repayment and default history, and the value of overdue payments[6].

Before 2009, the activities of PCB in Colombia were, by and large, unregulated. The consequent lack of oversight led to a number of legal conflicts between borrowers and PCB. Borrowers often complained that by reporting default history on all loans, even for those that had been already repaid in full, PCB exacerbated the consequences of default. PCB became known as "black lists", and the perception of the public was that once in the black list, they would not be able to borrow at reasonable rates in the future, if at all. At the time, these issues were at the forefront of the calls for reform of the system[7].

3.1.2. The Reform of 2009: Law 1266/2008

The lack of oversight on PCB did not escape the attention of policymakers. A total of 12 attempts were made prior to 2008 to introduce a regulatory framework on the sector, but all these legislative projects failed to reach approval by either Congress or the Constitutional Court (CC). During the first quarter of 2008, the Colombian government submitted to

[3] For a brief review of the recent history of PCB in the Colombian financial system, see CEMLA [22, 2005]. As of the start of 2013, there were three operating, nation-wide, PCB in the country: Datacrédito, CIFIN, and Procrédito.

[4] This legal requirement implies that it is not possible for any potential lender to get hold at once of the entire information set of PCB unless authorized by each and every single borrower in the economy.

[5] Because credit reports include such detailed information, credit scores have not acquired the relative importance that they have in more developed financial systems.

[6] For the case of unsecured consumer credit operations, apart from the information shared through PCB and the information gathered through a relationship, banks have access to few other sources of information for their credit evaluations.

[7] For a discussion of regulatory flaws and proposals at the time, see Miller and Guadamillas[54, 2006].

Congress the final legislative project that eventually became Law 1266/2008. Fundamentally, this law regulates the market activities of PCB in several dimensions, including -for instance- dispositions about maximum holding time windows for negative items in credit histories and mechanisms to deal with complaints and mistakes in the information.

Law 1266/2008 was enacted by Congress on December 31st, 2008. Importantly, the law allowed a period of adjustment of six months after enactment (that is, from January 2009 to June 2009) during which lenders and borrowers adapted from the old, unregulated system to the new, regulated one. Once finished the transition period, the system became regulated by the body of the law, and no more regulatory changes were scheduled or expected to happen in the future.

Most likely for political reasons (the law was one of the promises of the reelection campaign of incumbent President Alvaro Uribe in 2006), the law included a set of measures whose purpose was to offer *some* borrowers a form of "fresh start" vis-a-vis PCB. This entailed the one-off removal (or destruction) of negative items (i.e. defaults) held in the PCB credit reports of borrowers that satisfied certain conditions. Specifically, histories of prior defaults were erased provided that:

1. Defaults were exogenously "sufficiently old" by June 2009.
2. Borrowers were in good standing with all their debt obligations by the end of the period of adjustment (June 2009).

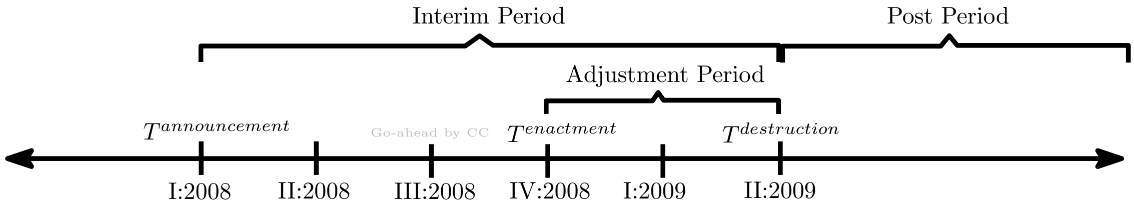
The "fresh start" policy can be thought of as a natural experiment in the set of available information about borrowers in the financial system. To be more specific, for those borrowers in good standing by June 2009, detailed information on all defaults prior to June 2008 was erased. Information on defaults that occurred between June 2008 and June 2009, as well as positive information on borrowers (e.g., repayment histories), remained in principle unchanged. If the borrower continued to be in good standing, information on defaults that occurred between June 2008 and June 2009 was to be erased immediately after these defaults become one year old.

The differential (across borrowers) entitlement to a fresh start implies that the policy did not modify the information shared in PCB homogeneously across borrowers. Instead, the modification depended on the exact timing of prior defaults at the moment in which information was erased. Our identification strategy relies precisely in the differential effect of the law across borrowers.

For the purposes of studying the exact structure of the time effects of the law, Figure 3.1.1 presents the life cycle of the legislative project that culminated in Law 1266/2008. The project was first announced to the public at the beginning of 2008, after it is approved by

Congress and sent to the Constitutional Court (CC) for examination. The first quarter of 2008 is then referred to in what follows as $T^{announcement}$. In April 2008, the CC rejects the project on procedure grounds, mandating Congress to discuss it again without questioning the constitutionality of its body. After an additional round of discussion and approval, the CC finally gave the project the green light of constitutionality in October 2008. As mentioned above, the law was finally enacted on New Year's Eve 2008. In what follows, we refer to the last quarter of 2008 as $T^{enactment}$.

Figure 3.1.1.: Timeline of Law 1266/2008



Because the implementation of the law included a transition period of six months after $T^{enactment}$, the changes in the information of borrowers related to the fresh start policy were not fully implemented until the end of June 2009. After this date, the information sets of PCB were legally required to reflect the changes established by the law. In what follows, we refer to the second semester of 2009 as $T^{destruction}$[8]. In what follows, the period between $T^{announcement}$ and $T^{destruction}$ is referred to as the *interim* period, and the period after $T^{destruction}$ as the *post* (-reform) period. The former comprises the time window in which anticipation effects are likely to be observed. Importantly, during the interim period there was varying uncertainty about the probability of the law being approved and enacted (if not about its characteristics in such a case). Only towards the end of 2008:III and the beginning of 2008:IV, after the final approval by the CC, market agents were certain that the law was to be enacted in the near future.

3.2. Empirical Strategy

An ideal experiment to identify the causal effect of long-lived negative information in PCB on credit outcomes would be as follows. Consider a given borrower with some history of old, past defaults who is in good standing in his/her current financial obligations with all inside lenders. The experiment exogenously makes the borrower's past defaults unobservable to

[8]For the purposes of this investigation, it is assumed that all negative information subject to destruction was deleted at once at $T^{destruction}$. To the extent that information matters at all, the destruction of information previous to $T^{destruction}$ would have the effect of reducing the estimated impact of the reform (for those specifications using $T^{destruction}$ as the limit between pre-reform and post-reform outcomes). Therefore, the assumption that all destruction occurs at once after the six months of adjustment have passed is the most conservative that could be made.

all new, outside lenders. Given that this intervention does not affect either the probability of default of the borrower or the quantity and quality of information at the hands of inside banks, any observed change in his/her credit outcomes post-intervention is to be attributed to the causal effect of the unobservability of negative information for outside lenders[9].

The "fresh start" policy intervention created by Law 1266 resembles closely this ideal experiment. In particular, it created a group of (treated) borrowers who were benefited by an immediate elimination of *all* past negative information about defaults or arrears from the databases of PCB. Given that the law provided for an adjustment period, this group comprises borrowers whose last arrears episode occurred more than one year before $T^{destruction}$ (and were up-to-date in all their obligations at $T^{destruction}$). Thus, borrowers at $T^{destruction}$ whose last default was flagged at PCB *before* June 30th, 2008 are included in this group. As this group emerges from the policy intervention without any stain in their credit histories, it is used as the treatment group throughout most of the empirical specifications below[10].

At the same time, the law kept unchanged the information with regard to another set of (control) borrowers. This group corresponds to borrowers who had not committed a default prior to the enactment of the law (within the sample), and for which information in the credit bureaus does not change as a consequence of the law after $T^{destruction}$[11].

In light of the previous discussion regarding the timing of the approval and implementation of the law, the main advantage of using this group of borrowers as the control group is that (similar to the treatment group) it committed no default between $T^{announcement}$ and $T^{destruction}$. Its main disadvantage (which we mitigate with the robustness checks below) is that borrowers who did not commit a default prior to 2008 may be different in ways that we cannot control for even after allowing for differential trends in credit outcomes across treatment and control groups. The validity of the empirical strategy relies on the assumption that credit outcomes of treated and control borrowers would have evolved in a similar fashion in the absence of the policy change. Thus, the main identification assumption is that the evolution of credit outcomes responds similarly to aggregate shocks

[9] An alternative experiment with a similar spirit would entail handing over detailed information about past defaults of a *given* borrower to a random group of potential lenders. This would create a cross-variation across lenders with regard to the quantity of information about the particular individual. Given that the probability of default is not affected by the random release of information, any change in the credit outcomes of the borrower can be interpreted as a causal effect of the variation in information.

[10] Another group which receives (partial) treatment is comprised by those borrowers that were up-to-date in all their liabilities at $T^{enactment}$ but had at least one arrears episode occurring shortly (at most one year) before. This group is only partially treated as long as only their old arrears episodes are deleted from their credit histories, whereas the younger ones are kept in the credit reports. Provided they keep up-to-date, that negative information would eventually be deleted (once it has been in the PCB set for one year). This group will not be considered in the exercise of this chapter.

[11] Given the restrictions posed by using a sample over time of borrowers, it is impossible to make sure that these borrowers had an unblemished credit history. To the extent that out-of-sample defaults are a concern, they would have the effect of reducing the estimated effect of the policy intervention, and therefore act against the results of this chapter, to be discussed below.

for borrowers whose last default is older than one year, and for borrowers who have never defaulted, at the time when negative information was erased. Evidence at this respect is presented below. Finally, note that a violation of this identification assumption likely biases our results against finding any positive effect in credit outcomes.

These arguments provide the foundations for a baseline Differences-in-Differences (DD) estimation based in the following econometric specification:

$$Y_{i,g,t} = \alpha_i + \lambda_t + \delta_g \times t + \theta \times Treatment_g \times Post_t + \varepsilon_{i,g,t} \tag{3.2.1}$$

where:

$$Treatment_g = \begin{cases} 1 & \text{if firm } i \text{belongs to the Treatment Group} \\ 0 & \text{if firm } i \text{belongs to the Control Group} \end{cases}$$
$$Post_t = \begin{cases} 1 & t > T^{destruction} \\ 0 & t \leq T^{destruction} \end{cases}$$

The dependent variable $Y_{i,g,t}$ is one of a number of potential credit outcomes for borrower i, in group g ($g = \{\text{Treatment, Control}\}$) at time t. These include the natural logarithm of the amount of loans obtained by the borrower, the average interest rate and maturity of these loans, and the frequency of default. Given the large number of financial obligations in the database, we analyze only new loans, thereby ensuring the timeliness of the information or loan terms. The right hand side of specification (3.2.1) includes borrower fixed effects (to control for all time invariant borrower heterogeneity), calendar quarter dummies (to control for economy wide shocks) and group-specific time trends[12]. The interaction (DD) coefficient θ is the coefficient of interest in specification (3.2.1). It represents the conditional difference in credit outcomes across borrowers in the two groups before and after the policy intervention. The specification is designed to measure the effect of the experiment on the credit outcomes of a given borrower[13].

To estimate the effects taking into account the possibility of preemptive behavior by market participants after $T^{announcement}$ (but previous to $T^{destruction}$), we also estimate a more flexible specification where we allow for differences in behavior during the period after the

[12] The inclusion of these trends follow Angrist and Pischke [2, 2009]; it implies that specification (3.2.1) is likely to underestimate any existing permanent effect of the intervention on growth rates of the credit outcome in question.

[13] The database employed in this chapter does not include borrower information that could be used as individual, time-varying controls in specification (3.2.1). Only individual, unobserved, fixed controls and group-level, time-varying controls are taken into account through borrower and time fixed effects.

legislative project was announced:

$$Y_{i,g,t} = \mu_i + \xi_t + \zeta_g \times t + \theta_I \times Treatment_g \times Interim_t + \theta_P \times Treatment_g \times Post_t + \eta_{i,g,t} \quad (3.2.2)$$

where:

$$Interim_t = \begin{cases} 1 & T^{announcement} < t \leq T^{destruction} \\ 0 & \text{otherwise} \end{cases}$$

The coefficient on the first interaction term in specification (3.2.2), θ_I, captures the effect of announcing the characteristics of the law, holding the information set of PCB constant. It measures the difference in credit outcomes of treated and counterfactual borrowers during the interim period relative to any other time window in the sample. The coefficient θ_P represents the difference in credit outcomes of treated and counterfactual borrowers *after* the elimination of negative information from PCB.

In order to explore in more detail whether our results are consistent with the aforementioned theories of hold-up, we complement our empirical strategy by comparing credit outcomes for the same borrower across inside and outside banks. This comparison is useful, as the change in information induced by the law is only relevant for outside banks, which are unlikely to have private records on the borrower[14]. Unlike outside banks, which do not observe any past negative information on treated borrowers after $T^{destruction}$, inside banks do have information on the default history of their borrowers previous to the policy intervention (either because their financial operations with the borrower were directly the subject of default or because they accessed the credit report of the borrower in the past). It is reasonable to assume they kept this information during and after the policy intervention.

If long-lived negative information in PCB carries a persistent stigma in credit markets, it is likely that while this information is shared in the PCB, inside banks are not disciplined by the market to offer the better performing customers competitive rates (a la Sharpe [73, 1990]). In this setting, borrowers would have an incentive to switch banks immediately after $T^{destruction}$, after detailed information on their prior defaults is erased (a la Ioannidou

[14]Recall that, in the case of Colombia, banks can only access a borrower's information in the credit bureau given explicit written authorization from a prospective customer. In the Colombian unsecured consumer credit market banks have no access to loan-level information on borrowers other than the PCB.

and Ongena [46, 2010]). If Colombian banks are indeed using informational holding-up with their clients, we expect new loans to be concentrated in outside banks.

The classification between inside and outside banks is based on the start period of our sample, 2007:IV. For a given borrower, this exercise follows the spirit of the baseline specification and defines as outside banks all lenders that lent for the first time to the borrower after $T^{announcement}$. All other lenders are classified as inside banks. The differential effect of the policy intervention on inside and outside banks across the treatment and control groups can be calculated from the estimation of the following Differences-in-Differences-in-Differences (DDD) specification:

$$Y_{i,g,Outside,t} - Y_{i,g,Inside,t} = \mu_i + \xi_t + +\beta_g \times t + \varphi \times Treatment_g \times Post_t + \nu_{i,g,t} \quad (3.2.3)$$

where $Outside$ and $Inside$ index the outcome for each particular group of banks. The coefficient of interest, φ, reflects the effect of the policy on credit outcome Y for outside banks relative to inside banks for treated borrowers, relative to control borrowers.

3.3. Data: Treatment and Control

3.3.1. The Data: Public Credit Registry on Consumer Credit

Our analysis uses data from the chapter on Consumer Credit of the Colombian Public Credit Registry, administered by the FSO. The database contains detailed loan contract information on a quarterly basis, on all outstanding loans granted by all licensed financial institutions in Colombia. This information is submitted by financial institutions in the form of a quarterly, standardized format (so called "Format 341"). For each loan we have information on the date of origination, maturity date, contract terms, rating at origination, and ex-post performance. For each borrower we have information on their total bank debt, banking relationships, internal bank rating and past repayment history[15].

The key characteristic of this database for our analysis is that, in contrast to PCB, information in the Public Credit Registry was not modified by the law. As explained before, the information managed by the public credit registry is confidential and used only for policy

[15]This rich database is similar to others used in empirical corporate finance and banking papers based on Latin American data (e.g., Ioannidou and Ongena [46, 2010] for the case of Bolivia, and Hertzberg, Liberti and Paravisini [43, 2011] for the case of Argentina, among others).

and regulatory purposes. Hence, we are in a unique position to examine the effect of the law while controlling explicitly for the changes in information.

The confidential character of the information limits the exercise of this chapter to a random sample of borrowers for the period 2007:IV-2011:IV [16]. On average for the period of study, the sample covers 57.72% of the total stock of gross consumer credit in the consolidated balance sheet of the Colombian financial system. This includes non-balanced information for a total of 2,842,726 borrowers and 64 lenders over the length of the 17 quarters of analysis.

3.3.2. The Treatment and Control Groups

3.3.2.1. Construction

Ideally, the treatment and control groups should be constructed using data about credit histories directly extracted from PCB. Unfortunately, this information is confidential and only available under the same conditions imposed on lenders, that is, under the authorization of the borrowers whose information is to be collected. This chapter therefore relies on the construction of individual credit histories from the Public Credit Registry data under the assumption that the latter closely resemble those in PCB databases. In other words, the confidentiality of PCB datasets forces the analysis to make several assumptions with regard to the behavior of PCB using data only from the Public Credit Registry.

It is assumed that, at the borrower (consumer) level, the key variable that PCB use to produce and summarize a credit history is the number of days in arrears for each single outstanding loan. Let d_{ijt} denote this number for borrower i, loan j at a given point in time, t. The key assumption is that a negative information item ("stain") in an individual firm's credit history is triggered (or flagged) whenever $d_{ijt} > \bar{d}$, for some threshold $\bar{d}$ and some loan j. That is, when the number of days in arrears for any loan exceeds some $\bar{d}$. In what follows, the period of time where $d_{ijt} > \bar{d}$ for some j will be interchangeably known as an "arrears episode" or a "default episode" of individual i. The stain is assumed to be a permanent feature of the credit history of borrower i except when altered by the fresh start policy intervention.

Let $\bar{t}_i = \max\left\{t : d_{ijt} > \bar{d} \text{ for some } j \text{ and } t < T^{destruction}\right\}$ denote the date of the last default episode for a given borrower i *before* $T^{destruction}$. Borrower i belongs to the treatment group ($Treatment_g = 1$) whenever he/she satisfies *both* the following conditions:

[16] In addition, the ID codes for borrowers and lenders are anonymized, in the sense that the database accessed by this investigation identifies borrowers and lenders with a code different to their legal ID. Therefore, it is not possible to identify individual borrowers or lenders by name using this database.

1. $\bar{t}_i \in \left[2007:IV, T^{enactment}\right)$. That is, i has at least one default episode in the sample, but i is not flagged in all his/her outstanding obligations at the time of enactment of the law or at the time of destruction of negative information.

2. $\bar{t}_i < T^{destruction} - 1$ year. That is, the last negative flag in the credit history was triggered more than one year previous to the destruction of negative information.

Borrower i belongs to the control group ($Treatment_g = 0$) whenever $d_{ijt} \leq \bar{d}$ for all j and for all $t \in \left[2007:IV, T^{destruction}\right]$. Alternatively, borrower i belongs to the control group whenever $\bar{t}_i$ is not defined. In other words, individuals in this control group do not register any stain during the sample of study.

3.3.2.2. Characterization

The selection of the groups depends naturally on the choice of $\bar{d}$. All the exercises to follow employ two alternative values for $\bar{d}$, 30 days and 0 days. When $\bar{d}$ is set to 30 (0), a negative flag is assumed to appear at the PCB information set whenever there are more than 30 (0) days in arrears for a given borrower in any of his/her obligations. Although setting $\bar{d} = 30$ is closer to the legal and practitioner's definition of a default, $\bar{d} = 0$ is used as a robustness check given that is the most conservative threshold that could be employed (as it is unlikely that lenders and PCB give a lot of informational value to arrears that are settled in a small number of days).

Selecting $\bar{d} = 30$ produces a panel dataset of 29,062,134 observations that includes 2,056,214 borrowers. Of these, 1,900,746 (92.4%) belong to the control group and the remaining 155,648 (7,6%) to the treatment group. Table 3.1 presents a summary of descriptive statistics at the end of the first quarter of 2008 ($T^{announcement}$) for a number of variables that reflect credit outcomes and average contractual characteristics for both groups. The main variables of interest are: 1. Loans, which corresponds to the overall value of all loans obtained by a given borrower in a given quarter (loans are defined as financial obligations whose origination date is recorded to be within the previous 90 days), 2. Average interest rate, which corresponds to the weighted average of contractual (ex-ante) interest rate of loans originated within the previous 90 days, and 3. Average loan maturity, defined as the weighted average of contractual (ex-ante) maturity of loans originated within the previous 90 days. The information in the table reinforces the conjecture that both control and treatment groups are similar in a number of observed dimensions that may help determine future credit outcomes, at the moment in which the policy intervention is announced. Table 3.2 presents the same set of summary statistics when the threshold is set to $\bar{d} = 0$; similar conclusions arise in this case[17].

[17] The maximum interest rates presented in Tables 3.1 and 3.2 are practically the same. This is due to the

Evidence in favour of the "common-trends" identification assumption discussed before is presented in Figure 3.3.1. The figure presents the behavior over time of the average size of (ln) new loans. Panel A is constructed with a threshold of $\bar{d} = 30$ and Panel B with a threshold of $\bar{d} = 0$. The vertical bars in the panels, from left to right, correspond to $T^{announcement}$, $T^{enactment}$ and $T^{destruction}$. Anticipation effects, if at all, are likely to be observed in the window between the first two dates.

The evolution of average (ln) new loans supports the hypothesis of common trends across the two groups before the policy intervention and serves as visual evidence of the effect of the destruction of negative information on the loans obtained after $T^{destruction}$ by treated borrowers. The permanent, positive impact of treatment on the access to credit of borrowers in the treatment group is confirmed below with the estimation of the econometric specifications. Importantly, notice that the average (ln) fresh loans received by the treatment and the control groups starts to diverge right before the actual destruction of negative information, which suggests the absence of strong anticipatory effects. Finally, notice that (controlling for the pre-intervention time trend of loans) the policy intervention did not seem to affect the access to credit of borrowers in the control group.

Figure 3.3.1.: Average (ln) Loans

The figure presents the evolution of the average natural logarithm of new loans across groups. New loans are defined as financial obligations whose origination date is recorded to be within the 90 days previous to report. The vertical bars in the panels, from left to right, correspond to $T^{announcement}$, $T^{enactment}$ and $T^{destruction}$.

Panel A: $\bar{d} = 30$ Panel B: $\bar{d} = 0$

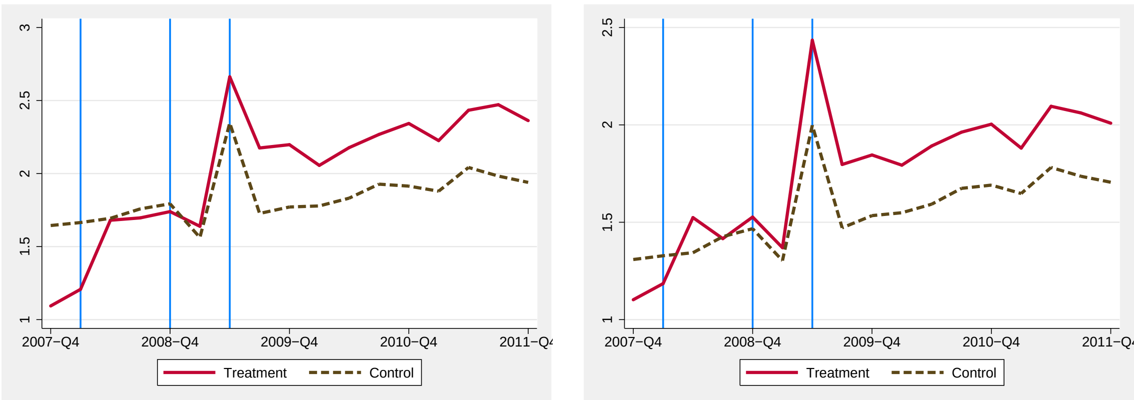


Figure 3.3.2 at the end of the chapter presents the evolution of a number of alternative measures of credit outcomes. These include the interest rate and maturity on loans, and the rating at issuance (an ex-ante measure of risk). Interest rates for both the treatment and the control groups share a downward trend that follows the expansionary stance of monetary policy carried by the Central Bank of Colombia after the first half of 2008. Only

fact that consumer credit lenders in Colombia are subject to a system of ceiling rates. At the other side of the spectrum, zero interest rates are prevalent in consumer loans whose maturity is smaller than 30 days.

in the case of $\bar{d} = 0$ (Panel B) the interest rate charged on loans to the treatment group seemed to have fallen relatively more than that of the control group. The maturity of loans experienced an upward trend for both groups during the sample period. Especially in the case of $\bar{d} = 30$, the maturity on loans taken by the treated borrowers seemed to increase proportionally more (alternatively, the gap between the maturities of the control group loans and the treatment group loans shrank after intervention).

As discussed before, the evolution of the rating at issuance in Figure 3.3.2 deserves some attention[18]. As explained above, by the definition of treatment and control groups, there are quarters in which (some of the) obligations of treated borrowers are in arrears ($d_{ijt} > \bar{d}$). This is true for arrears episodes older than $T^{destruction} - 1$ year, as the exercise imposes no arrears for the treatment group between $T^{destruction} - 1$ year and $T^{destruction}$. Therefore, the rating at issuance of the treatment group falls over time as these borrowers continue to be in good standing over time. As expected, the rating of control borrowers is low throughout the sample window as these borrowers do not have any arrears the period of study. At $T^{destruction}$, the level of the ratings is very similar across groups, and treated borrowers are only different in their negative credit history. Thus, although by construction the rating at issuance of treated and non-treated borrowers does not share a common trend before $T^{enactment}$, its rapid convergence before $T^{destruction}$ yields support to the hypothesis that the behavior of credit outcomes would have evolved in a similar fashion for both groups in the absence of the policy intervention (at least they do not exhibit large differences in terms of their expected future probabilities of default).

3.4. Results

This section presents the results of the estimation of the aforementioned specifications together with additional robustness checks. All estimations below are performed on a 17-quarter panel that includes 2 quarters pre-announcement, 5 interim quarters (that is, 7 pre-intervention quarters) and 10 post-intervention quarters. Robust standard errors are clustered at the two-way bank-time level[19][20]. Results are shown separately with and without the inclusion of group-specific time trends (coefficients δ_g in the specifications above) and for two different values of the threshold $\bar{d}$ (30 and 0). Only results that are statistically

[18] In the original reporting structure of Format 341, the variable "Rating" is coded in the scale A-E, where "A" correspond to the highest rating. The rating has been recodified here in the scale 1-5, where the equivalence is given by: "A"=1, ..., "E"=5.

[19] In order to construct the panel this exercise selects the "main" bank for each borrower at each quarter. The "main" bank is defined as the counterparty with which the borrower holds his/her largest financial obligation at a given moment in time.

[20] Standard errors were also clustered at the one-way borrower level. These errors were without exception smaller than those under the two-way clustering level, and therefore are not reported.

significant regardless of the choice of $\bar{d}$ and of the inclusion/exclusion of group-specific time trends are considered robust.

3.4.1. Baseline Results

Tables 3.3-3.5 present the results of the estimation of specifications (3.2.1) and (3.2.2). The main result of this chapter is presented in Table 3.3: the increase in loans received by the treated borrowers *after* the destruction of negative information is statistically significant and robust to the choice of $\bar{d}$, to the inclusion of group-specific time trends and to the choice of control group. When choosing $\bar{d} = 30$ as the threshold for a negative flag (Panel A), the estimates in column 3 of table 3.3 indicate that treated borrowers experience an increase of 54.7% in loans after their negative information is deleted compared to borrowers in the control group. This point estimate falls to 29.7% when a threshold of $\bar{d} = 0$ is chosen instead (Panel B), and is in both cases relatively unaffected by the inclusion of group-specific time trends.

As expected, the point estimates fall with the lower threshold, as a relatively small number of days in arrears is less informative than a large one. Thus, the choice of $\bar{d} = 0$ is the most conservative scenario that could be considered and the results in this case indicate a lower bound for the effects of the policy intervention. The estimates of the interim effects, although statistically significant when a large threshold is chosen, are not robust to picking $\bar{d} = 0$. The absence of interim effects on the amount of loans can in principle be interpreted as the result of uncertainty regarding the characteristics of the law. However, the interim effects will be manifest in alternative credit outcomes, as will be seen below.

There is no robust, statistically significant effect of the policy intervention or its announcement on the interest rate on loans across groups (Table 3.4). Although almost always the point estimate suggest a reduction in the interest rate for treated borrowers, this results is not statistically significant when including group-specific time trends or reducing the threshold for $\bar{d}$. The absence of strong effects on the interest rate is expected in a credit market characterized by the prevalent use of ceiling rates on consumer credit.

Interim effects are observed when considering the effect of the policy intervention on the maturity of loans (Table 3.5). Choosing $\bar{d} = 30$, the maturity of loans received by treated borrowers increased on average 30.5% during the interim period and 51.1% after the intervention (Column 3), compared to the control group. That is, even though during the interim period treated borrowers do not receive a larger amount of loans compared to the control group, they are able to obtain larger maturities on average. The direction of these results is robust to picking $\bar{d} = 0$ as threshold. Point estimates are not significantly affected by the inclusion of group-specific time trends.

The results presented thus far indicate that the destruction of negative information caused by Law 1266 allowed affected borrowers to obtain larger loans at longer maturities. In the case of loans, the effect of the policy intervention is only manifest after the destruction of information; although the policy was announced in advance, anticipation effects are not found to be robust or statistically significant.

3.4.2. Robustness Checks

In addition to the inclusion of group-specific time trends and different choices of $\bar{d}$ (as performed above), this section implements two additional robustness checks on the estimated effect of the policy intervention on loans and average maturity.

3.4.2.1. Alternative Treatment and Control Groups

A concern with respect to the identification strategy is related to the fact that, by construction, borrowers in either treatment and control group may be different in their repayment ability as of time $T^{destruction}$ simply because the treatment group does have default episodes during the period of study. To mitigate this concern, we employ alternative treatment and control groups as follows.

First, we pick an alternative treatment group according to $\bar{t}_i$, that is, according to the last date in which the borrowers register a default episode. An alternative sub-treatment group comprised only by borrowers that have the minimum possible $\bar{t}_i$ (oldest default in-sample before $T^{destruction}$) is the least likely to be affected by the concern, as long as the predictive power of old defaults on repayment ability at $T^{destruction}$ is expected to decay with the age of the default episode.

Tables 3.6 and 3.7 present the results of the estimation of specifications (3.2.1) and (3.2.2), following the structure of the previous subsection, when the treatment group is comprised only by borrowers whose last default is $\bar{t}_i = 2007 : IV$ (the first period of the sample). The control group in each case is the same employed in the baseline specifications. The choice of this groups provides the most conservative configuration of treatment and control groups that is feasible to construct with the sample.

At a significance level of 10%, the result that treated borrowers obtain more loans after the policy intervention relative to controlled borrowers is robust to picking the new (sub)treatment group. In this case, loans to borrowers in the treatment group increase 51.4% (Column 3) after the destruction of information relative to the control group. This result is robust to the inclusion of group-specific time trends and to the choice of threshold[21].

From the results presented in table 3.7, it is clear that borrowers in the treatment group are also able to obtain longer maturities after the destruction of information. Similar to the baseline scenario, these borrowers get an average of 50.7% (Column 3) higher maturities *after* intervention compared to the control group. This result is robust to the choice of $\bar{d}$ and the inclusion of group-specific trends. However, the result that maturities increase *during the interim period* found above (see table 3.5) is not robust to the change in the treatment group: although the estimate of θ_I is significant when $\bar{d} = 30$ and time trends are excluded, it is not when the threshold is changed and/or group-specific trends are included. From this exercise it is possible to conclude that, as is the case for loans, the positive effect of the policy intervention on maturities of treated borrowers is restricted to the period after the destruction of negative information, whereas anticipation effects are relatively weak (or non-existent).

Finally, we also submit our main result to the choice of an alternative control group, corresponding to borrowers who committed a default between June 2008 and December 2008, and which therefore did not see their prior default history immediately erased from the system at $T^{destruction}$.The main advantage of this alternative control group is that (similar to the treatment group) these borrowers have also defaulted at some point in the past on some of their financial obligations. The main disadvantage of this group, however, is that these borrowers necessarily defaulted after the legislative project was announced. This behavior suggests that these control borrowers may be unsophisticated vis-a-vis the treatment group, something which can potentially bias our estimates. Thus, the main identification assumption in using this alternative control group is that borrowers who defaulted after the announcement of the legislative project are not fundamentally different to the treatment group, such that aggregate shocks concurrent to the policy change have a similar direct effect on their expected income (we refer to this assumption as common trends assumption in what follows). To be consistent with previous notation, borrower j belongs to the alternative control group whenever $\bar{t}_i \in \left[T^{destruction} - 1 \text{ year}, T^{enactment}\right)$.

Table 3.8 presents the results of the estimation of specifications (3.2.1) and (3.2.2), following the structure of the previous subsection when the alternative control group is employed. Reassuringly, the main result of this chapter, the increase in the average loan size to borrowers whose negative information is deleted from PCB after the policy intervention, is robust to this choice.

[21] The only exception in this regard is the lack of significance after including group-specific time trends under $\bar{d} = 0$. Together with the choice of the subtreatment group, this is the most conservative scenario that could be studied from the sample. The robustness of the main result to all other specifications and to additional checks below is, however, reassuring with regard to its statistical and economic significance.

3.4.2.2. Placebo Test

An additional concern is that the sample selection procedure somehow produces the results presented above in a mechanical way. To verify that this is not the case, this subsection performs a "placebo" policy intervention using the sub-sample of the data that starts at $T^{enactment}$. The exercise assumes that the policy intervention is announced 2 quarters after (and is implemented 7 quarters after) the start of this "placebo sample". The placebo announcement occurs therefore at $T^{enactment} + 2$ and the implementation of the placebo intervention occurs at $T^{enactment} + 7$. Importantly, treatment and control groups are re-constructed with the new placebo sample. In this sense, the time structure imposed on the placebo sample resembles the one of the original dataset. Naturally, if the results presented above are not driven by the sample selection procedure, it is expected that the estimates of the coefficients of interest are either not significant or not robust to small changes in the specification.

Reassuringly, this concern on sample selection procedure is rejected by the results in table 3.9, which presents estimates of the baseline specifications (3.2.1) and (3.2.2) with the new "placebo" sample[22]. The estimate of coefficient θ_P is significant when $\bar{d} = 30$ is chosen and linear time trends are excluded. However, this result is not robust to either the inclusion of trends or the choice of $\bar{d}$ (in all other configurations the estimate is not statistically different from zero).

This check confirms that the baseline results discussed above are a consequence of the economic and financial effects of the policy intervention, and not of any sample/group procedure selection.

3.4.3. On the Ex-post Cost of Information Destruction

The baseline results indicate that the policy intervention potentially increases the utility of borrowers in the treatment groupby allowing them to increase the size and maturity of their loans. However, this does not necessarily imply that policies towards the elimination of public negative information are welfare-improving. As discussed above, there is a well established literature on the social benefits of information sharing created by the reduction of adverse selection and moral hazard problems in credit markets. As a consequence, one of the potential social costs of destroying public negative information is precisely the ex-post quality of outstanding loans and the severity of asymmetric information problems.

[22] For the sake of brevity, only results using the natural logarithm of loans as the dependent variable are reported.

Evidence on the potential social costs of the policy intervention is provided in Figure 3.4.1 at the end of this chapter. The figure presents the evolution of the average default status (borrower i is in a state of default if $d_{ijt} > \bar{d}$ for some j) across groups. The figure focuses only on the post-intervention period, as no borrower in the treatment or control group is in arrears during the interim period by contruction. The figure suggest that after negative information is deleted both groups experienced an increase in the incidence of defaults, but this increase was proportionately larger for the treatment group. This indicates that the additional borrowing treated borrowers are able to obtain after the policy intervention is on average more risky ex-post.

Following Hertzberg, Liberty and Paravisini [43, 2011], the effect of the destruction of public negative information on the hazard rate of default episodes across the treatment and control groups is estimated using two specifications analogous to (3.2.1), where the dependent variable is a measure of the frequency of arrears episodes:

$$
\begin{aligned}
1\left\{d_{ijt} > \bar{d} | d_{ijt-1} > \bar{d} \text{ for some } j\right\}_{i,g,t} &= \alpha_i + \lambda_t + \delta_g \times t + \\
&\quad \theta \times Treatment_g \times Post_t + \varepsilon_{i,g,t}
\end{aligned}
\tag{3.4.1}
$$

The dependent variable is an indicator that switches to the value of 1 whenever borrower i registers a new episode of default after being in good standing in the previous period. The dependent variable takes the value of zero everywhere else. This indicator is a measure of the frequency with which borrower i in group g *falls* into arrears (or default). The variables at the right-hand side of (3.4.1) follow the same structure of baseline specification (3.2.1). The results presented below also include the estimation of the following specification

$$
\begin{aligned}
1\left\{d_{ijt} > \bar{d} \text{ for some } j\right\}_{i,g,t} &= \alpha_i + \lambda_t + \delta_g \times t + \\
&\quad \theta \times Treatment_g \times Post_t + \varepsilon_{i,g,t}
\end{aligned}
\tag{3.4.2}
$$

where the dependent variable takes the value of 1 whenever borrower i registers a (not necessarily new) default epiode. Unlike the indicator variable in the main text, this dependent variable measures the frequency with which borrower i in group g *is* in a state of arrears (or default). As will be seen, the estimated direction of the effects is robust to the use of this alternative specification. Given that the treatment and the control group are constructed assuming they do not have arrears episodes during the interim period, specification (3.2.2) cannot be employed in this context.

Table 3.10 reports the estimation results. Consistent with the evidence presented in figure

3.4.1, there is a statistically significant and robust increase in the frequency of defaults of treated borrowers post-intervention, compared to the control group. Taking $\bar{d} = 30$ as threshold, Column 1 (3) of table 3.10 indicates that the hazard rate of new (existing) defaults on any debt increases on average 3.4% (6.3%). Similar to previous results, the introduction of group-specific time trends do not change the direction of these effects.

Overall, the evidence thus far indicates that borrowers whose negative information was destroyed, while increasing their loan sizes at better conditions as a consequence of the law, turn out to experience a higher frequency of episodes of arrears after the policy intervention. This result is consistent with the "discipline effect" discussed before of information sharing. The identification assumption posed in this chapter implies that, although the quality of the pool of borrowers composed by the treatment and the control group is not affected by the policy intervention, the latter has the joint effect of increasing proportionately more the borrowing of one group in a way that has a proportionately larger effect on the ex-post risk of their loans. The effect of the policy intervention on ex-post defaults is therefore a clear downside of policy interventions of the type studied in this chapter.

In future versions of the chapter, we plan to quantify the overall ex-post disciplinary effect of sharing information on old defaults and the associated costs of this practice due to informational hold-up, to make more general statements about the welfare effects of the fresh-start policy.

3.4.4. On the Evidence of Ex-ante Informational Hold-up

The results of the estimation of specification (3.2.3) for loans size and the maturity of loans are presented respectively in tables 3.11 and 3.12. The estimations follow the baseline exercise in including separately different thresholds and group-specific linear time trends. The first column of Table 3.11 indicates that, on average after the destruction of information, borrowers in the treatment group (under $\bar{d} = 30$) obtain 20.9% more loans from outside banks than from inside banks, compared to the control group. Although the point estimates vary mildly, the direction of this result is robust to the inclusion of time trends and a change in the threshold $\bar{d}$.

These results indicate that the increase in loans obtained by treated borrowers was attained proportionately more by establishing new relationships than by increasing their borrowing from existing lenders at the time of the announcement of the law. This result is consistent with the idea that borrowers whose negative information is destroyed have a strong incentive to substitute away from lenders who can access information about their past default behavior. This evidence is consistent with inside banks using informational holding-up with their clients prior to the fresh start policy intervention. The hold-up is partially broken by the destruction of negative information, allowing treated borrowers to engage in fresh relationships with outside lenders.

The results in table 3.12 suggest that despite obtaining more loans from outside banks, treated borrowers do not experience a statistically significant difference between inside and outside banks in terms of the maturity of loans after the policy intervention, compared to the control group. The increase in maturities post-intervention presented in tables 3.5 and 3.7 seems therefore to have been shared proportionately by both types of banks. Admittedly, this result is less consistent with theories of hold-up. In future versions of the chapter where certain data restrictions have been overcome, we plan to explore in more depth the differences in contract conditions for new loans across old and new banks in order to shed light on the existence of information hold-up on the contractual conditions of new loans.

3.5. Conclusions

We analyze the relationship between information sharing and credit outcomes in the unsecured consumer credit market, focusing on the potential ex-post disciplinary effect, and ex-ante informational hold-up, of long-lived negative information about borrowers. We exploit a natural experiment in Colombia where detailed information about past defaults that were exogenously "sufficiently old" by June 2009 was erased from the private credit bureaus by law.

We find a significant relative increase in the size and maturity of new loans, no significant changes in interest rates, and an increase in subsequent default rates, for treated borrowers whose old negative information is erased, relative to control borrowers whose information does not change. Overall, results are consistent with both ex-post disciplinary effects, and ex-ante information hold-up from long-lived negative information in credit bureaus. Specifically, consistent with the hold-up theories, we find that most of the increase in the value of loans comes from outside banks. For a given borrower, we find an increase in the value and maturity of new loans by outside relative to inside banks. Finally, consistent with the disciplinary role of information sharing we find an increase in the probability of default for loans obtained after the destruction of information.

In future versions of the chapter we plan to examine more carefully the variation of a given borrower across inside and outside banks with respect to loan size, maturity and price of new loans, as well as quantify the aggregate consequences of this policy, using information about the universe of borrowers in Colombia. Access to the universe of borrowers would also allow us to perform calculations with regard to the welfare implications of changes in publicly available negative information, as it would allow us to compare the aggregate benefit of breaking informational hold-ups against the aggregate cost of a less efficient discipline device.

Table 3.1.: March 2008 Cross-Section of Borrowers. Descriptive Statistics **($\bar{d} = 30$)**

This table presents a summary of descriptive statistics for a number of credit outcomes using a snapshot of the treatment and control groups at the end of the first quarter of 2008 ($T^{announcement}$). The threshold $\bar{d}$ is set to 30 days. Loans are defined as financial obligations whose origination date is recorded to be within the 90 days previous to the end of the quarter. The variable "Rating" is coded in Format 341 in the scale A-E, where "A" correspond to the highest rating. For the purposes of constructing the table, the rating has been recodified in the scale 1-5, where the equivalence is given by: "A"=1, "B"=2, "C"=3, "D"=4, "E"=5.

	Sample				
	Mean	S.D.	Median	Max.	Min.
Loans (mill. COP)	8.12	16.3	3.50	1860	0.00
Total Debt (mill. COP)	8.16	20.0	2.84	8590	0.00
Nr. of Lenders	1.79	1.21	1	17	1
Average Interest Rate (%)	22.59	10.34	26.82	33.08	0.00
Average Maturity (years)	2.89	2.38	3.00	24.26	0.00
Average Rating	1.04	0.29	1.00	5.00	1.00

	Treatment (125,522 individuals)				
	Mean	S.D.	Median	Max.	Min.
Loans (mill. COP)	7.42	18.9	2.16	841	0.00
Total Debt (mill. COP)	8.18	19.7	2.32	1500	0.00
Nr. of Lenders	1.99	1.36	1	16	1
Average Interest Rate (%)	23.35	10.26	27.27	32.75	0.00
Average Maturity (years)	2.24	2.54	1.33	23.54	0.003
Average Rating	1.54	0.96	1.00	5.00	1.00

	Control (1,754,571 individuals)				
	Mean	S.D.	Median	Max.	Min.
Loans (mill. COP)	8.15	16.2	3.55	1860	0.00
Total Debt (mill. COP)	8.16	20.0	2.87	8590	0.00
Nr. of Lenders	1.77	1.20	1	17	1
Average Interest Rate (%)	22.55	10.34	26.81	33.08	0.00
Average Maturity (years)	2.92	2.36	3.01	24.26	0.003
Average Rating	1.01	0.15	1.00	5.00	1.00

Table 3.2.: March 2008 Cross-Section of Borrowers . Descriptive Statistics ($\bar{d} = 0$)

This table presents a summary of descriptive statistics for a number of credit outcomes using a snapshot of the treatment and control groups at the end of the first quarter of 2008 ($T^{announcement}$). The threshold $\bar{d}$ is set to 0 days. Loans are defined as financial obligations whose origination date is recorded to be within the 90 days previous to the end of the quarter. The variable "Rating" is coded in Format 341 in the scale A-E, where "A" correspond to the highest rating. For the purposes of constructing the table, the rating has been recodified in the scale 1-5, where the equivalence is given by: "A"=1, "B"=2, "C"=3, "D"=4, "E"=5.

	Sample				
	Mean	S.D.	Median	Max.	Min.
Loans (mill. COP)	7.41	11.7	3.80	700	0.00
Total Debt (mill. COP)	5.89	14.4	2.28	8590	0.00
Nr. of Lenders	1.47	0.86	1	13	1
Average Interest Rate (%)	22.30	10.35	26.81	33.08	0.00
Average Maturity (years)	3.15	2.26	3.04	23.54	0.003
Average Rating	1.03	0.27	1.00	5.00	1.00

	Treatment (184,766 individuals)				
	Mean	S.D.	Median	Max.	Min.
Loans (mill. COP)	6.62	14.2	2.15	700	0.00
Total Debt (mill. COP)	6.31	14.1	1.93	852	0.00
Nr. of Lenders	1.72	1.07	1	13	1
Average Interest Rate (%)	23.21	10.49	27.27	33.08	0.00
Average Maturity (years)	2.51	2.46	3.00	23.54	0.003
Average Rating	1.16	0.61	1.00	5.00	1.00

	Control (967,856 individuals)				
	Mean	S.D.	Median	Max.	Min.
Loans (mill. COP)	7.56	11.2	4.00	600	0.00
Total Debt (mill. COP)	5.81	14.4	2.35	8590	0.00
Nr. of Lenders	1.43	0.80	1	13.00	1
Average Interest Rate (%)	22.14	10.32	26.81	33.08	0.00
Average Maturity (years)	3.26	2.21	3.05	22.90	0.003
Average Rating	1.01	0.13	1.00	5.00	1.00

Figure 3.3.2.: Alternative Credit Outcomes

The figure presents the evolution of alternative measures of credit outcomes during the sample period. The variable "Rating" is coded in Format 341 in the scale A-E, where "A" correspond to the highest rating. The rating has been recodified here in the scale 1-5, where the equivalence is given by: "A"=1, "B"=2, "C"=3, "D"=4, "E"=5. The vertical bars in the panels, from left to right, correspond to $T^{announcement}$, $T^{enactment}$ and $T^{destruction}$.

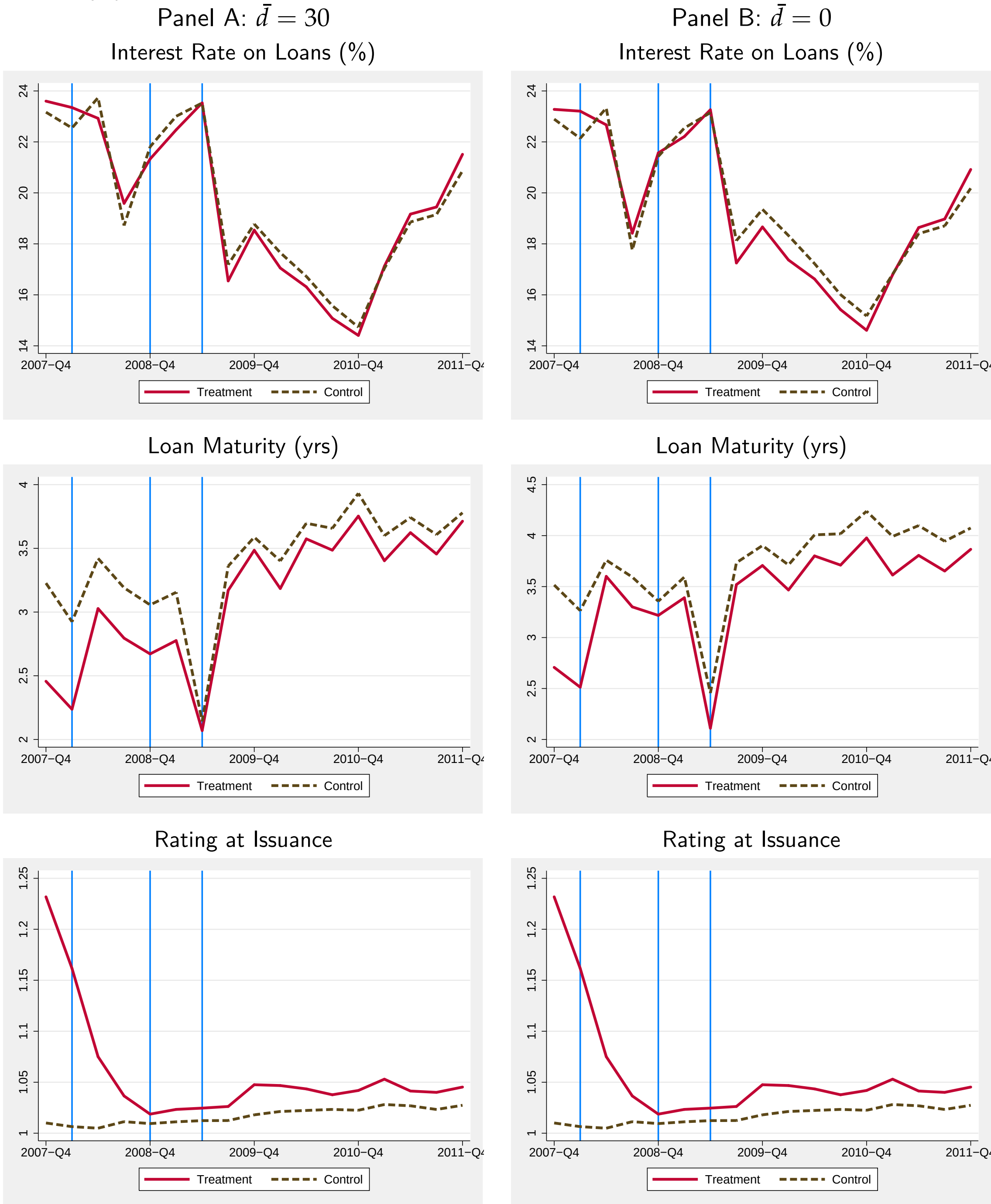


Figure 3.4.1.: Average Default Status

The figure presents the evolution of the average default status (borrower i is in a state of default if $d_{ijt} > \bar{d}$ for some j) across groups. Average default in the interim period is zero by contruction for both the treatment and the control groups, thus the figure focuses only on the post-intervention period.

Panel A: $\bar{d} = 30$ Panel B: $\bar{d} = 0$

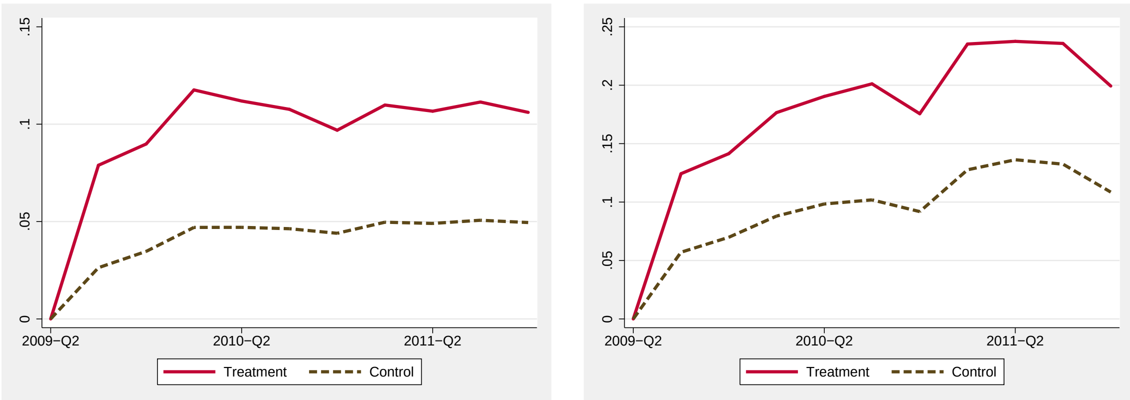


Table 3.3.: Effects of the Policy Intervention. Loans

This table presents the estimations of specifications (3.2.1) and (3.2.2) using as the dependent variable the natural logarithm of Loans:

$$\begin{aligned} (3.2.1) \qquad \ln\left(Loans_{i,g,t}\right) &= \alpha_i + \lambda_t + \delta_g \times t + \theta \times Treatment_g \times Post_t + \varepsilon_{i,g,t} \\ (3.2.2) \qquad \ln\left(Loans_{i,g,t}\right) &= \alpha_i + \lambda_t + \delta_g \times t + \theta_I \times Treatment_g \times Interim_t \\ &\quad + \theta_P \times Treatment_g \times Post_t + \eta_{i,g,t} \end{aligned}$$

Robust standard errors clustered at the two-way main bank-time level are presented in parentheses below the coefficient estimates. ** indicates estimates are statistically different from zero at the 5% level. Panel A (B) uses a threshold for negative flag of $\bar{d} = 30$ ($\bar{d} = 0$) days.

	Panel A: $\bar{d} = 30$			
	(3.2.1)	(3.2.1)	(3.2.2)	(3.2.2)
θ	0.386**	0.276**		
	(0.077)	(0.138)		
θ_I			0.236**	0.232**
			(0.087)	(0.092)
θ_P			0.547**	0.536**
			(0.080)	(0.147)
Borrower FE and Quarter FE	Yes	Yes	Yes	Yes
Group Time Trends	No	Yes	No	Yes
Nr. of Observations	29,062,134			
Level of Clustering	Bank × Time	Bank × Time	Bank × Time	Bank × Time
Nr. of Clusters	61×17	61×17	61×17	61×17

	Panel B: $\bar{d} = 0$			
	(3.2.1)	(3.2.1)	(3.2.2)	(3.2.2)
θ	0.187**	0.112		
	(0.070)	(0.129)		
θ_I			0.161	0.158
			(0.086)	(0.088)
θ_P			0.297**	0.288**
			(0.066)	(0.129)
Borrower FE and Quarter FE	Yes	Yes	Yes	Yes
Group Time Trends	No	Yes	No	Yes
Nr. of Observations	17,416,323			
Level of Clustering	Bank × Time	Bank × Time	Bank × Time	Bank × Time
Nr. of Clusters	61×17	61×17	61×17	61×17

Table 3.4.: Effects of the Policy Intervention. Interest Rate on Loans

This table presents the estimations of specifications (3.2.1) and (3.2.2) using as the dependent variable the interest rate on Loans:

$$(3.2.1) \quad Interest_{i,g,t} = \alpha_i + \lambda_t + \delta_g \times t + \theta \times Treatment_g \times Post_t + \varepsilon_{i,g,t}$$

$$(3.2.2) \quad Interest_{i,g,t} = \alpha_i + \lambda_t + \delta_g \times t + \theta_I \times Treatment_g \times Interim_t + \theta_P \times Treatment_g \times Post_t + \eta_{i,g,t}$$

Robust standard errors clustered at the two-way main bank-time level are presented in parentheses below the coefficient estimates. ** indicates estimates are statistically different from zero at the 5% level. Panel A (B) uses a threshold for negative flag of $\bar{d} = 30$ ($\bar{d} = 0$) days.

	Panel A: $\bar{d} = 30$			
	(3.2.1)	(3.2.1)	(3.2.2)	(3.2.2)
θ	0.033	-0.582**		
	(0.201)	(0.236)		
θ_I			-0.515**	-0.941**
			(0.168)	(0.182)
θ_P			-0.338	-1.611
			(0.263)	(1.630)
Borrower FE and Quarter FE	Yes	Yes	Yes	Yes
Group Time Trends	No	Yes	No	Yes
Nr. of Observations	3,311,359			
Level of Clustering	Bank × Time	Bank × Time	Bank × Time	Bank × Time
Nr. of Clusters	56×17	56×17	56×17	56×17

	Panel B: $\bar{d} = 0$			
	(3.2.1)	(3.2.1)	(3.2.2)	(3.2.2)
θ	-0.212	-1.175**		
	(0.286)	(0.403)		
θ_I			-0.638	-1.256**
			(0.323)	(0.377)
θ_P			-0.658	-2.520**
			(0.474)	(0.719)
Borrower FE and Quarter FE	Yes	Yes	Yes	Yes
Group Time Trends	No	Yes	No	Yes
Nr. of Observations	1,677,661			
Level of Clustering	Bank × Time	Bank × Time	Bank × Time	Bank × Time
Nr. of Clusters	55×17	55×17	55×17	55×17

Table 3.5.: Effects of the Policy Intervention. Maturity of Loans

This table presents the estimations of specifications (3.2.1) and (3.2.2) using as the dependent variable the natural logarithm of the maturity of Loans:

$$(3.2.1) \quad \ln\left(Maturity_{i,g,t}\right) = \alpha_i + \lambda_t + \delta_g \times t + \theta \times Treatment_g \times Post_t + \varepsilon_{i,g,t}$$

$$(3.2.2) \quad \ln\left(Maturity_{i,g,t}\right) = \alpha_i + \lambda_t + \delta_g \times t + \theta_I \times Treatment_g \times Interim_t + \theta_P \times Treatment_g \times Post_t + \eta_{i,g,t}$$

Robust standard errors clustered at the two-way main bank-time level are presented in parentheses below the coefficient estimates. ** indicates estimates are statistically different from zero at the 5% level. Panel A (B) uses a threshold for negative flag of $\bar{d} = 30$ ($\bar{d} = 0$) days.

	Panel A: $\bar{d} = 30$			
	(3.2.1)	(3.2.1)	(3.2.2)	(3.2.2)
θ	0.289**	0.088		
	(0.076)	(0.068)		
θ_I			0.305**	0.257**
			(0.043)	(0.028)
θ_P			0.511**	0.367**
			(0.076)	(0.056)
Borrower FE and Quarter FE	Yes	Yes	Yes	Yes
Group Time Trends	No	Yes	No	Yes
Nr. of Observations	3,572,443			
Level of Clustering	Bank × Time	Bank × Time	Bank × Time	Bank × Time
Nr. of Clusters	56×17	56×17	56×17	56×17

	Panel B: $\bar{d} = 0$			
	(3.2.1)	(3.2.1)	(3.2.2)	(3.2.2)
θ	0.262**	0.149		
	(0.095)	(0.135)		
θ_I			0.516**	0.554**
			(0.209)	(0.193)
θ_P			0.628**	0.742**
			(0.156)	(0.162)
Borrower FE and Quarter FE	Yes	Yes	Yes	Yes
Group Time Trends	No	Yes	No	Yes
Nr. of Observations	1,818,966			
Level of Clustering	Bank × Time	Bank × Time	Bank × Time	Bank × Time
Nr. of Clusters	55×17	55×17	55×17	55×17

Table 3.6.: Robustness Check: Subtreatment Group with Oldest Default. Loans

This table presents the estimations of specifications (3.2.1) and (3.2.2) using as the dependent variable the natural logarithm of Loans:

$$(3.2.1) \quad \ln\left(Loans_{i,g,t}\right) = \alpha_i + \lambda_t + \delta_g \times t + \theta \times Subtreatment_g \times Post_t + \varepsilon_{i,g,t}$$

$$(3.2.2) \quad \ln\left(Loans_{i,g,t}\right) = \alpha_i + \lambda_t + \delta_g \times t + \theta_I \times Subtreatment_g \times Interim_t + \theta_P \times Treatment_g \times Post_t + \eta_{i,g,t}$$

where $Subtreatment_g$ is a dummy that takes a value of 1 if borrower i's only default episode in-sample (before $T^{destruction}$) occurred in 2007:IV, and zero if the borrower does not experience any default episode in-sample (before $T^{destruction}$). Robust standard errors clustered at the two-way main bank-time level are presented in parentheses below the coefficient estimates. **(*) indicates estimates are statistically different from zero at the 5% (10%) level. Panel A (B) uses a threshold for negative flag of $\bar{d} = 30$ ($\bar{d} = 0$) days.

	Panel A: $\bar{d} = 30$			
	(3.2.1)	(3.2.1)	(3.2.2)	(3.2.2)
θ	0.374**	0.273*		
	(0.115)	(0.158)		
θ_I			0.204	0.197
			(0.224)	(0.224)
θ_P			0.514**	0.493*
			(0.233)	(0.269)
Borrower FE and Quarter FE	Yes	Yes	Yes	Yes
Group Time Trends	No	Yes	No	Yes
Nr. of Observations	28,163,670			
Level of Clustering	Bank × Time	Bank × Time	Bank × Time	Bank × Time
Nr. of Clusters	61×17	61×17	61×17	61×17

	Panel B: $\bar{d} = 0$			
	(3.2.1)	(3.2.1)	(3.2.2)	(3.2.2)
θ	0.137*	0.063		
	(0.083)	(0.139)		
θ_I			0.159	0.156
			(0.114)	(0.111)
θ_P			0.246**	0.237
			(0.112)	(0.151)
Borrower FE and Quarter FE	Yes	Yes	Yes	Yes
Group Time Trends	No	Yes	No	Yes
Nr. of Observations	16,099,313			
Level of Clustering	Bank × Time	Bank × Time	Bank × Time	Bank × Time
Nr. of Clusters	61×17	61×17	61×17	61×17

Table 3.7.: Robustness Check: Subtreatment Group with Oldest Default. Maturity of Loans

This table presents the estimations of specifications (3.2.1) and (3.2.2) using as the dependent variable the natural logarithm of Loans:

$$(3.2.1) \quad \ln\left(Maturity_{i,g,t}\right) = \alpha_i + \lambda_t + \delta_g \times t + \theta \times Subtreatment_g \times Post_t + \varepsilon_{i,g,t}$$

$$(3.2.2) \quad \ln\left(Maturity_{i,g,t}\right) = \alpha_i + \lambda_t + \delta_g \times t + \theta_I \times Subtreatment_g \times Interim_t + \theta_P \times Treatment_g \times Post_t + \eta_{i,g,t}$$

where $Subtreatment_g$ is a dummy that takes a value of 1 if borrower i's only default episode in-sample (before $T^{destruction}$) occurred in 2007:IV, and zero if the borrower does not experience any default episode in-sample (before $T^{destruction}$). Robust standard errors clustered at the two-way main bank-time level are presented in parentheses below the coefficient estimates. **(*) indicates estimates are statistically different from zero at the 5% (10%) level. Panel A (B) uses a threshold for negative flag of $\bar{d} = 30$ ($\bar{d} = 0$) days.

	Panel A: $\bar{d} = 30$			
	(3.2.1)	(3.2.1)	(3.2.2)	(3.2.2)
θ	0.259**	0.026		
	(0.095)	(0.099)		
θ_I			0.342*	0.282
			(0.189)	(0.183)
θ_P			0.507**	0.330*
			(0.201)	(0.191)
Borrower FE and Quarter FE	Yes	Yes	Yes	Yes
Group Time Trends	No	Yes	No	Yes
Nr. of Observations	3,454,204			
Level of Clustering	Bank × Time	Bank × Time	Bank × Time	Bank × Time
Nr. of Clusters	56×17	56×17	56×17	56×17

	Panel B: $\bar{d} = 0$			
	(3.2.1)	(3.2.1)	(3.2.2)	(3.2.2)
θ	0.263**	0.180		
	(0.126)	(0.165)		
θ_I			0.355	0.379
			(0.332)	(0.322)
θ_P			0.514*	0.586*
			(0.308)	(0.318)
Borrower FE and Quarter FE	Yes	Yes	Yes	Yes
Group Time Trends	No	Yes	No	Yes
Nr. of Observations	1,665,413			
Level of Clustering	Bank × Time	Bank × Time	Bank × Time	Bank × Time
Nr. of Clusters	55×17	55×17	55×17	55×17

Table 3.8.: Robustness Check: Alternative Control Group. Loans

This table presents the estimations of specifications (3.2.1) and (3.2.2) using as the dependent variable the natural logarithm of Loans:

$$
\begin{aligned}
(3.2.1) \qquad \ln\left(Loans_{i,g,t}\right) &= \alpha_i + \lambda_t + \delta_g \times t + \theta \times Treatment_g \times Post_t + \varepsilon_{i,g,t} \\
(3.2.2) \qquad \ln\left(Loans_{i,g,t}\right) &= \alpha_i + \lambda_t + \delta_g \times t + \theta_I \times Treatment_g \times Interim_t \\
&\quad + \theta_P \times Treatment_g \times Post_t + \eta_{i,g,t}
\end{aligned}
$$

where $Treatment_g$ takes a value of 0 if borrower i's last default episode in-sample (before $T^{destruction}$) occurred in $\bar{t}_i \in \left[T^{destruction} - 1 \text{ year}, T^{enactment}\right)$, and 1 as in the baseline specification of table 3.3. Robust standard errors clustered at the two-way main bank-time level are presented in parentheses below the coefficient estimates. **(*) indicates estimates are statistically different from zero at the 5% (10%) level. Panel A (B) uses a threshold for negative flag of $\bar{d} = 30$ ($\bar{d} = 0$) days.

	Panel A: $\bar{d} = 30$			
	(3.2.1)	(3.2.1)	(3.2.2)	(3.2.2)
θ	0.209	-0.204		
	(0.116)	(0.101)		
θ_I			0.462**	0.421**
			(0.104)	(0.171)
θ_P			0.532**	0.445*
			(0.095)	(0.253)
Borrower FE and Quarter FE	Yes	Yes	Yes	Yes
Group Time Trends	No	Yes	No	Yes
Level of Clustering	Bank × Time	Bank × Time	Bank × Time	Bank × Time
Nr. of Clusters	61×17	61×17	61×17	61×17

	Panel B: $\bar{d} = 0$			
	(3.2.1)	(3.2.1)	(3.2.2)	(3.2.2)
θ	0.083	-0.226**		
	(0.081)	(0.066)		
θ_I			0.274**	0.123
			(0.080)	(0.154)
θ_P			0.275**	-0.037
			(0.060)	(0.226)
Borrower FE and Quarter FE	Yes	Yes	Yes	Yes
Group Time Trends	No	Yes	No	Yes
Level of Clustering	Bank × Time	Bank × Time	Bank × Time	Bank × Time
Nr. of Clusters	61×17	61×17	61×17	61×17

Table 3.9.: Placebo Test. Loans

This table presents the estimations of specifications (3.2.1) and (3.2.2) using as the dependent variable the natural logarithm of Loans:

$$
\begin{aligned}
(3.2.1) \qquad \ln\left(Loans_{i,g,t}\right) &= \alpha_i + \lambda_t + \delta_g \times t + \theta \times Treatment_g \times Post_t + \varepsilon_{i,g,t} \\
(3.2.2) \qquad \ln\left(Loans_{i,g,t}\right) &= \alpha_i + \lambda_t + \delta_g \times t + \theta_I \times Treatment_g \times Interim_t \\
&\quad + \theta_P \times Treatment_g \times Post_t + \eta_{i,g,t}
\end{aligned}
$$

for a placebo sample that starts at $T^{destruction}$. The placebo policy intervention takes place 7 quarters after the start of the placebo sample and is announced 2 quarters after the start of the placebo sample. Robust standard errors clustered at the two-way main bank-time level are presented in parentheses below the coefficient estimates. ** indicates estimates are statistically different from zero at the 5% level. Panel A (B) uses a threshold for negative flag of $\bar{d} = 30$ ($\bar{d} = 0$) days.

	Panel A: $\bar{d} = 30$			
	(3.2.1)	(3.2.1)	(3.2.2)	(3.2.2)
θ	0.307**	0.120		
	(0.057)	(0.078)		
θ_I			0.121	0.008
			(0.068)	(0.124)
θ_P			0.392**	0.133
			(0.028)	(0.169)
Borrower FE and Quarter FE	Yes	Yes	Yes	Yes
Group Time Trends	No	Yes	No	Yes
Nr. of Observations	29,062,134			
Level of Clustering	Bank × Time	Bank × Time	Bank × Time	Bank × Time
Nr. of Clusters	61×17	61×17	61×17	61×17

	Panel B: $\bar{d} = 0$			
	(3.2.1)	(3.2.1)	(3.2.2)	(3.2.2)
θ	0.159**	0.131**		
	(0.030)	(0.056)		
θ_I			0.000	-0.036
			(0.052)	(0.085)
θ_P			0.159**	0.074
			(0.043)	(0.120)
Borrower FE and Quarter FE	Yes	Yes	Yes	Yes
Group Time Trends	No	Yes	No	Yes
Nr. of Observations	17,416,323			
Level of Clustering	Bank × Time	Bank × Time	Bank × Time	Bank × Time
Nr. of Clusters	61×17	61×17	61×17	61×17

Table 3.10.: Effects of the Policy Intervention: Default/Arrears

This table presents the estimations of specifications (3.4.1) and (3.4.2):

$$\text{(3.4.1)} \quad 1\left\{d_{ijt} > \bar{d} | d_{ijt-1} > \bar{d} \text{ for some } j\right\}_{i,g,t} = \alpha_i + \lambda_t + \delta_g \times t + \theta \times Treatment_g \times Post_t + \varepsilon_{i,g,t}$$

$$\text{(3.4.2)} \quad 1\left\{d_{ijt} > \bar{d} \text{ for some } j\right\}_{i,g,t} = \alpha_i + \lambda_t + \delta_g \times t + \theta \times Treatment_g \times Post_t + \varepsilon_{i,g,t}$$

Robust standard errors clustered at the two-way main bank-time level are presented in parentheses below the coefficient estimates. ** indicates estimates are statistically different from zero at the 5% level. Panel A (B) uses a threshold for negative flag of $\bar{d} = 30$ ($\bar{d} = 0$) days.

	Panel A: $\bar{d} = 30$			
	(3.4.1)	(3.4.1)	(3.4.2)	(3.4.2)
θ	0.034**	0.043**		
	(0.003)	(0.006)		
θ			0.063**	0.052**
			(0.002)	(0.005)
Borrower FE and Quarter FE	Yes	Yes	Yes	Yes
Group Time Trends	No	Yes	No	Yes
Nr. of Observations	25,252,962			
Level of Clustering	Bank × Time	Bank × Time	Bank × Time	Bank × Time
Nr. of Clusters	62×15	62×15	62×15	62×15

	Panel B: $\bar{d} = 0$			
	(3.4.1)	(3.4.1)	(3.4.2)	(3.4.2)
θ	0.043**	0.056**		
	(0.003)	(0.008)		
θ			0.086**	0.062**
			(0.004)	(0.005)
Borrower FE and Quarter FE	Yes	Yes	Yes	Yes
Group Time Trends	No	Yes	No	Yes
Nr. of Observations	15,058,194			
Level of Clustering	Bank × Time	Bank × Time	Bank × Time	Bank × Time
Nr. of Clusters	61×15	61×15	61×15	61×15

Table 3.11.: Inside Banks vs. Outside Banks. Loans

This table presents the estimation of specification (3.2.3) using as the dependent variable the natural logarithm of Loans:

$$\ln\left(Loans_{i,g,Outside,t}\right) - \ln\left(Loans_{i,g,Inside,t}\right) = \mu_i + \xi_t + +\beta_g \times t + \varphi \times Treatment_g \times Post_t + \nu_{i,g,t} \quad (3.2.3)$$

where $Outside$ and $Inside$ index the credit outcome for outside and inside banks starting at $T^{announcement}$. Robust standard errors clustered at the two-way main bank-time level are presented in parentheses below the coefficient estimates. ** indicates estimates are statistically different from zero at the 5% level. Panel A (B) uses a threshold for negative flag of $\bar{d} = 30$ ($\bar{d} = 0$) days.

	Panel A: $\bar{d} = 30$	
	(3.2.3)	(3.2.3)
φ	0.209**	0.401**
	(0.074)	(0.058)
Borrower FE and Quarter FE	Yes	Yes
Group Time Trends	No	Yes
Nr. of Observations	1,752,894	
Level of Clustering	Bank × Time	Bank × Time
Nr. of Clusters	53×14	53×14

	Panel B: $\bar{d} = 0$	
	(3.2.3)	(3.2.3)
φ	0.352**	0.332**
	(0.137)	(0.081)
Borrower FE and Quarter FE	Yes	Yes
Group Time Trends	No	Yes
Nr. of Observations	983,966	
Level of Clustering	Bank × Time	Bank × Time
Nr. of Clusters	53×15	53×15

Table 3.12.: Inside Banks vs. Outside Banks. Maturity

This table presents the estimation of specification (3.2.3) using as the dependent variable the natural logarithm of Maturity on Loans:

$$\ln\left(Maturity_{i,g,Outside,t}\right) - \ln\left(Maturity_{i,g,Inside,t}\right) = \mu_i + \xi_t + +\beta_g \times t + \varphi \times Treatment_g \times Post_t + \nu_{i,g,t} \tag{3.2.3}$$

where $Outside$ and $Inside$ index the credit outcome for outside and inside banks starting at $T^{announcement}$. Robust standard errors clustered at the two-way main bank-time level are presented in parentheses below the coefficient estimates. ** indicates estimates are statistically different from zero at the 5% level. Panel A (B) uses a threshold for negative flag of $\bar{d} = 30$ ($\bar{d} = 0$) days.

	Panel A: $\bar{d} = 30$	
	(3.2.3)	(3.2.3)
φ	-0.166	0.057
	(0.090)	(0.063)
Borrower FE and Quarter FE	Yes	Yes
Group Time Trends	No	Yes
Nr. of Observations	1,752,894	
Level of Clustering	Bank × Time	Bank × Time
Nr. of Clusters	53×14	53×14

	Panel B: $\bar{d} = 0$	
	(3.2.3)	(3.2.3)
φ	-0.026	-0.087
	(0.127)	(0.186)
Borrower FE and Quarter FE	Yes	Yes
Group Time Trends	No	Yes
Nr. of Observations	983,966	
Level of Clustering	Bank × Time	Bank × Time
Nr. of Clusters	53×15	53×15

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A. Appendices to Chapter 1

A.1. Proof of Proposition 1

The system of equations of the equilibrium is given by:

$$\beta E_t \left[\frac{R_{t+1}}{G_{t+1}} \left(\frac{c_{y,t}}{c_{m,t+1}} \right) \right] = 1 \tag{A.1.1}$$

$$\beta E_t \left[\frac{R_{t+1}}{G_{t+1}} \left(\frac{c_{m,t}}{c_{o,t+1}} \right) \right] = 1 \tag{A.1.2}$$

$$\beta E_t \left[\frac{I_t}{\Pi_{t+1} G_{t+1}} \left(\frac{c_{y,t}}{c_{m,t+1}} \right) \right] = 1 \tag{A.1.3}$$

$$c_{y,t} + l_t = 1 - \beta\gamma \tag{A.1.4}$$

$$c_{m,t} - l_t = 1 + (1+\beta)\gamma + d_t \tag{A.1.5}$$

$$c_{o,t} = 1 - \gamma - d_t \tag{A.1.6}$$

$$\left[a_H S_t^{\phi-1} + (1-a_H) \right]^{\frac{\phi}{\phi-1}} \Delta_t = S_t^{1-\phi} \tag{A.1.7}$$

$$E_{t-1} \left\{ \left(\check{p}_t^H \right)^{-\epsilon} Y_t^H \left(\check{p}_t^H - \left[\frac{1}{1-\kappa} - \frac{\kappa}{1-\kappa} \left(\check{p}_t^H \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \right) \right\} = 0 \tag{A.1.8}$$

$$\bigstar_t^{-\eta} (A_t)^{-\eta-1} Y_t^{(1+\eta)} c_{y,t}^{\frac{\alpha_y}{3}} c_{m,t}^{\frac{\alpha_m}{3}} c_{o,t}^{\frac{\alpha_o}{3}} \times \tag{A.1.9}$$

$$\left[a_H + (1-a_H) S_t^{1-\phi} \right]^{\frac{1+\eta}{1-\phi}} = \left[\frac{1}{1-\kappa} - \frac{\kappa}{1-\kappa} \left(\check{p}_t^H \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}$$

$$\left\{(1-\kappa)\left[\frac{1}{1-\kappa}-\frac{\kappa}{1-\kappa}\left(\check{p}_t^H\right)^{1-\epsilon}\right]^{\frac{-\epsilon}{1-\epsilon}}+\kappa\left(\check{p}_t^H\right)^{-\epsilon}\right\}^{-1} = \bigstar_t \qquad \text{(A.1.10)}$$

$$R_t = \frac{\bar{I}^* \xi_t}{\Pi_t} \qquad \text{(A.1.11)}$$

The non-stochastic steady state of the Home economy is characterized by a situation where $A_t = A_{t-1} = \bar{A} = 1$. Given that utility is logarithmic, Sheedy[74, 2013] has proved that there exists a unique steady state to the subsystem of equations composed by (A.1.1), (A.1.2) and (A.1.4)-(A.1.6) in which $\bar{G}_t = \frac{\bar{Y}_t}{\bar{Y}_{t-1}} = 1$ and $\bar{c}_y = 1$, $\bar{c}_m = 1$, $\bar{c}_o = 1$, $\bar{l} = -\beta\gamma$, $\bar{d} = -\gamma$ and $\beta\bar{R} = 1$. Given this, the remaining of the proof amounts to check that the remaining set of equations imply a steady state in the values described in the proposition that is consistent with $\bar{G}_t = 1$. Given that there is no uncertainty, all firms pick the same price in the steady state and thus $\bar{\check{p}}^H = \bar{\bigstar} = 1$. From the steady state of the Foreign economy, $\beta\bar{I}^* = 1$. From (A.1.11), $1 = \frac{\bar{\xi}_t}{\bar{\Pi}_t}$. By definition, $\frac{S_t}{S_{t-1}} = \frac{\xi_t}{\Pi_t}$. Thus, in steady state, S_t and Y_t are constant and the system of equations reduces to (A.1.7) and (A.1.9) with $\bar{S}$ and $\bar{Y}$ unknowns. A solution of this system is $\bar{S} = 1$ and $\bar{Y} = 1$.

A.2. Parameters

The combinations of structural parameters that have been mentioned throughout the chapter are:

$$\begin{aligned}
\theta &\equiv \frac{2\left(1+\beta\right)+\beta\gamma+\sqrt{\left(1+2\beta\right)^{2}+3\left[1-\left(\beta\gamma\right)^{2}\right]}}{2\left(1+\gamma\right)} \\
\lambda &\equiv \frac{2\left(\beta\gamma-1\right)}{2\left(1+\beta\right)-\beta\gamma+\sqrt{\left(1+2\beta\right)^{2}+3\left[1-\left(\beta\gamma\right)^{2}\right]}} \\
\chi &\equiv \frac{1}{3}\left[\left(\gamma\beta\right)^{2}+\gamma\left(\theta-\beta\right)^{2}+\left(-\gamma\theta\right)^{2}\right] \\
&\quad +\frac{1}{1-\eta}\frac{1}{3}\left[\alpha_{y}\left(\gamma\beta-\zeta\right)^{2}+\alpha_{m}\left(\gamma\left(\theta-\beta\right)-\zeta\right)^{2}+\alpha_{o}\left(-\gamma\theta-\zeta\right)^{2}\right] \\
\Theta_{A} &\equiv \frac{\varrho}{\left(1+\eta\right)\left(1-\psi+\psi a_{H}\right)} \\
&\quad -\frac{\left(1+a_{H}\psi\right)\zeta\varrho}{\left(1+\eta\right)\left(1-\psi+\psi a_{H}\right)\left[\theta\left(1+\eta\right)\left(1-\psi+\psi a_{H}\right)+\left(1+a_{H}\psi\right)\zeta\right]\left(1-\lambda\beta\Gamma\right)} \\
\Theta_{d} &\equiv \frac{\delta\zeta}{\left[\left(1+\eta\right)\left(1-\psi+\psi a_{H}\right)\right]^{2}} \\
&\quad -\frac{\left(1+a_{H}\psi\right)\left\{\chi\left[\left(1+\eta\right)\left(1-\psi+\psi a_{H}\right)\right]^{2}+\delta\zeta^{2}\right\}}{\left[\left(1+\eta\right)\left(1-\psi+\psi a_{H}\right)\right]^{2}\left[\theta\left(1+\eta\right)\left(1-\psi+\psi a_{H}\right)+\left(1+a_{H}\psi\right)\zeta\right]\left(1-\lambda^{2}\beta\right)} \\
\Theta_{\pi} &\equiv \epsilon+\frac{\delta\alpha}{\left(1-\alpha\right)\left(1+\eta\right)\left(1-\psi+\psi a_{H}\right)}\left[\frac{\theta}{\theta\left(1+\eta\right)\left(1-\psi+\psi a_{H}\right)+\left(1+a_{H}\psi\right)\zeta}\right]
\end{aligned}$$

where:

$$\begin{aligned}
\delta &\equiv \left(1+\eta\right)-2\left(1+\eta\right)\left(1-a_{H}\right)\psi+\left(1-a_{H}\right)\left[\left(\phi+\eta\right)\left(1-a_{H}\right)+1-\phi\right]\psi^{2} \\
\varrho &\equiv \left(1-a_{H}\right)a_{H}\left(1-\phi\right)\psi
\end{aligned}$$

A.3. The Loss Function

The social planner of the home economy maximizes the following social welfare function:

$$\begin{aligned} W_o &= E_0\left[\frac{1}{3}\sum_{t=-2}^{\infty}\beta^t U_t\right] \\ &= E_0\left\{\sum_{t=0}^{\infty}\beta^t\left[\frac{1}{3}\left(\ln C_{y,t}+\ln C_{m,t}+\ln C_{o,t}\right)\right.\right. \\ &\quad \left.\left. -\frac{1}{1+\eta}\frac{1}{3}\left({\alpha_y}^{-\eta}H_{y,t}^{1+\eta}+{\alpha_m}^{-\eta}H_{m,t}^{1+\eta}+{\alpha_o}^{-\eta}H_{o,t}^{1+\eta}\right)\right]\right\} \\ &\quad +tip \end{aligned}$$

where tip stands for "terms independent of monetary policy". We will proceed step by step.

A.3.1. The term $\left({\alpha_y}^{-\eta}H_{y,t}^{1+\eta}+{\alpha_m}^{-\eta}H_{m,t}^{1+\eta}+{\alpha_o}^{-\eta}H_{o,t}^{1+\eta}\right)$

From the intratemporal first order condition of the consumers in the Home economy:

$$\alpha_i^{-\eta}=\frac{H_{i,t}^{-\eta}w_{i,t}}{C_{i,t}}$$

Plugging this into our term of interest:

$$\begin{aligned} \left({\alpha_y}^{-\eta}H_{y,t}^{1+\eta}+{\alpha_m}^{-\eta}H_{m,t}^{1+\eta}+{\alpha_o}^{-\eta}H_{o,t}^{1+\eta}\right) &= \frac{H_{y,t}w_{y,t}}{C_{y,t}}+\frac{H_{m,t}w_{m,t}}{C_{m,t}}+\frac{H_{o,t}w_{o,t}}{C_{o,t}} \\ &= \frac{1}{Y_t}\left(\frac{H_{y,t}w_{y,t}}{c_{y,t}}+\frac{H_{m,t}w_{m,t}}{c_{m,t}}+\frac{H_{o,t}w_{o,t}}{c_{o,t}}\right) \end{aligned}$$

From the allocation of labour demands by firms, the equilibrium in the labour market at Home and taking into account $x_t A_t = w_t$:

$$\left({\alpha_y}^{-\eta}H_{y,t}^{1+\eta}+{\alpha_m}^{-\eta}H_{m,t}^{1+\eta}+{\alpha_o}^{-\eta}H_{o,t}^{1+\eta}\right)=\frac{x_t A_t N_t}{Y_t}\left(\frac{\alpha_y}{c_{y,t}}+\frac{\alpha_m}{c_{m,t}}+\frac{\alpha_o}{c_{o,t}}\right)$$

From the aggregate production function, $\frac{Y_t}{\bigstar_t}\left(\frac{P_t}{P_t^H}\right)=A_t N_t$, thus:

$$\left({\alpha_y}^{-\eta}H_{y,t}^{1+\eta}+{\alpha_m}^{-\eta}H_{m,t}^{1+\eta}+{\alpha_o}^{-\eta}H_{o,t}^{1+\eta}\right) = \frac{x_t}{\bigstar_t}\left(\frac{P_t}{P_t^H}\right)\left(\frac{\alpha_y}{c_{y,t}}+\frac{\alpha_m}{c_{m,t}}+\frac{\alpha_o}{c_{o,t}}\right)$$

From the expression for x_t - recall $\hat{Y}_t \equiv \frac{Y_t}{(A_t)^{\frac{1}{1-(1-a_H)\psi}}}$:

$$\begin{aligned} \left(\alpha_y^{-\eta} H_{y,t}^{1+\eta} + \alpha_m^{-\eta} H_{m,t}^{1+\eta} + \alpha_o^{-\eta} H_{o,t}^{1+\eta}\right) &= c_{y,t}^{\frac{\alpha_y}{3}} c_{m,t}^{\frac{\alpha_m}{3}} c_{o,t}^{\frac{\alpha_o}{3}} \hat{Y}_t^{1+\eta} \times \\ & \left(\frac{P_t}{P_t^H}\right)^{1+\eta} \bigstar_t^{-\eta-1} \times \\ & (A_t)^{\frac{(1+\eta)(1-a_H)\psi}{1-(1-a_H)\psi}} \left(\frac{\alpha_y}{c_{y,t}} + \frac{\alpha_m}{c_{m,t}} + \frac{\alpha_o}{c_{o,t}}\right) \end{aligned} \tag{A.3.1}$$

Employing the definition of the natural terms of trade $S_t^n = A_t^{-\frac{\psi}{1-(1-a_H)\psi}}$, we can rewrite this as:

$$\begin{aligned} \left(\alpha_y^{-\eta} H_{y,t}^{1+\eta} + \alpha_m^{-\eta} H_{m,t}^{1+\eta} + \alpha_o^{-\eta} H_{o,t}^{1+\eta}\right) &= c_{y,t}^{\frac{\alpha_y}{3}} c_{m,t}^{\frac{\alpha_m}{3}} c_{o,t}^{\frac{\alpha_o}{3}} \hat{Y}_t^{1+\eta} \times \\ & \left(\frac{P_t}{P_t^H}\right)^{1+\eta} \bigstar_t^{-\eta-1} \times \\ & (S_t^n)^{-(1+\eta)(1-a_H)} \left(\frac{\alpha_y}{c_{y,t}} + \frac{\alpha_m}{c_{m,t}} + \frac{\alpha_o}{c_{o,t}}\right) \end{aligned} \tag{A.3.2}$$

A.3.2. The term $\frac{1}{3}\left(\ln C_{y,t} + \ln C_{m,t} + \ln C_{o,t}\right)$

$$\begin{aligned} \frac{1}{3}\left(\ln C_{y,t} + \ln C_{m,t} + \ln C_{o,t}\right) &= \frac{1}{3}\left(\ln c_{y,t} + \ln c_{m,t} + \ln c_{o,t} + 3\ln Y_t\right) \\ &= \frac{1}{3}\left(\ln c_{y,t} + \ln c_{m,t} + \ln c_{o,t}\right) + \ln Y_t \\ &= \frac{1}{3}\left(\ln c_{y,t} + \ln c_{m,t} + \ln c_{o,t}\right) + \ln \hat{Y}_t + \\ & \frac{1}{1-(1-a_H)\psi} \ln A_t \\ &= \frac{1}{3}\left(\ln c_{y,t} + \ln c_{m,t} + \ln c_{o,t}\right) + \ln \hat{Y}_t + tip \\ &= \frac{1}{3}\left(\tilde{c}_{y,t} + \tilde{c}_{m,t} + \tilde{c}_{o,t}\right) + \hat{Y}_t + tip \end{aligned} \tag{A.3.3}$$

where the last equality uses the fact that the steady state values for those variables are equal to one.

A.3.3. Second-order approximations

The approximation of the loss function will employ the following second-order approximations

1.

$$\begin{aligned}\left(\bigstar_t\right)^{-\eta-1} &= e^{\tilde{\bigstar}_t(-\eta-1)} \\ &= 1+(-\eta-1)\,\tilde{\bigstar}_t+\frac{(-\eta-1)^2}{2}\left(\tilde{\bigstar}_t\right)^2+O(3)\end{aligned}$$

2.

$$\begin{aligned}\left(\hat{Y}_t\right)^{1+\eta} &= e^{\hat{Y}_t(1+\eta)} \\ &= 1+(1+\eta)\,\hat{Y}_t+\frac{(1+\eta)^2}{2}\left(\hat{Y}_t\right)^2+O(3)\end{aligned}$$

3.

$$\frac{1}{3}\left(\tilde{c}_{y,t}+\tilde{c}_{m,t}+\tilde{c}_{o,t}\right)=-\frac{1}{2}\frac{1}{3}\left(\tilde{c}_{y,t}^2+\tilde{c}_{m,t}^2+\tilde{c}_{o,t}^2\right)+O(3)$$

4.

$$\begin{aligned}c_{y,t}^{\frac{\alpha_y}{3}}c_{m,t}^{\frac{\alpha_m}{3}}c_{o,t}^{\frac{\alpha_o}{3}} &= e^{\tilde{c}_{y,t}\frac{\alpha_y}{3}+\tilde{c}_{m,t}\frac{\alpha_m}{3}+\tilde{c}_{o,t}\frac{\alpha_o}{3}} \\ &= 1+\left(\tilde{c}_{y,t}\frac{\alpha_y}{3}+\tilde{c}_{m,t}\frac{\alpha_m}{3}+\tilde{c}_{o,t}\frac{\alpha_o}{3}\right)+ \\ &\quad \frac{1}{2}\left(\tilde{c}_{y,t}\frac{\alpha_y}{3}+\tilde{c}_{m,t}\frac{\alpha_m}{3}+\tilde{c}_{o,t}\frac{\alpha_o}{3}\right)^2+O(3)\end{aligned}$$

5.

$$\begin{aligned}\frac{\alpha_y}{c_{y,t}}+\frac{\alpha_m}{c_{m,t}}+\frac{\alpha_o}{c_{o,t}} &= \alpha_y e^{-\tilde{c}_{y,t}}+\alpha_m e^{-\tilde{c}_{m,t}}+\alpha_o e^{-\tilde{c}_{o,t}} \\ &= 3-\left(\alpha_y\tilde{c}_{y,t}+\alpha_m\tilde{c}_{m,t}+\alpha_o\tilde{c}_{o,t}\right)+ \\ &\quad \frac{1}{2}\left[\alpha_y\tilde{c}_{y,t}^2+\alpha_m\tilde{c}_{m,t}^2+\alpha_o\tilde{c}_{o,t}^2\right]+O(3)\end{aligned}$$

6.

$$\begin{aligned}
\bigstar_t &= \left\{(1-\kappa)\left[\frac{1}{1-\kappa}-\frac{\kappa}{1-\kappa}e^{\tilde{p}_t^H(1-\epsilon)}\right]^{\frac{-\epsilon}{1-\epsilon}}+\kappa e^{-\epsilon\tilde{p}_t^H}\right\}^{-1} \\
\tilde{\bigstar}_t &= -\frac{\kappa\epsilon}{2(1-\kappa)}\left(\tilde{p}_t^H\right)^2+O(3) \\
&= -\frac{\kappa\epsilon}{2(1-\kappa)}\left(\tilde{\Pi}_t^H-E_{t-1}\tilde{\Pi}_t^H\right)^2+O(3)
\end{aligned}$$

where the last equality uses the fact that $E_{t-1}\tilde{p}_t^H = 0 + O(2)$.

7.

$$\begin{aligned}
\left(\frac{P_t}{P_t^H}\right)^{1+\eta} &= \left[a_H+(1-a_H)S_t^{1-\phi}\right]^{\frac{1+\eta}{1-\phi}} \\
&= \left[a_H+(1-a_H)e^{\tilde{S}_t(1-\phi)}\right]^{\frac{1+\eta}{1-\phi}} \\
&= 1+(1+\eta)(1-a_H)\tilde{S}_t+ \\
&\quad \frac{1}{2}(1+\eta)(1-a_H)\left[(\phi+\eta)(1-a_H)+(1-\phi)\right]\tilde{S}_t^2
\end{aligned}$$

8.

$$\begin{aligned}
(A_t)^{\frac{(1+\eta)(1-a_H)\psi}{1-(1-a_H)\psi}} &= e^{\tilde{A}_t\left[\frac{(1+\eta)(1-a_H)\psi}{1-(1-a_H)\psi}\right]} \\
&= 1+\tilde{A}_t\left[\frac{(1+\eta)(1-a_H)\psi}{1-(1-a_H)\psi}\right]+ \\
&\quad \frac{1}{2}\left(\tilde{A}_t\right)^2\left[\frac{(1+\eta)(1-a_H)\psi}{1-(1-a_H)\psi}\right]^2+O(3)
\end{aligned}$$

9.

$$\begin{aligned}
(S_t^n)^{-(1+\eta)(1-a_H)} &= e^{-\tilde{S}_t^n(1+\eta)(1-a_H)} \\
&= 1-\tilde{S}_t^n(1+\eta)(1-a_H) \\
&\quad +\frac{1}{2}(1+\eta)^2(1-a_H)^2\left(\tilde{S}_t^n\right)^2+O(3)
\end{aligned}$$

A.3.4. The Second-Order Approximation to the Welfare Function Approx II: Terms of Trade Gap

The objective of the social planner is thus to maximize:

$$
\begin{aligned}
W_o &= E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{3} \left(\tilde{c}_{y,t} + \tilde{c}_{m,t} + \tilde{c}_{o,t} \right) + \hat{Y}_t - \frac{1}{1+\eta} c_{y,t}^{\frac{\alpha_y}{3}} c_{m,t}^{\frac{\alpha_m}{3}} c_{o,t}^{\frac{\alpha_o}{3}} \hat{Y}_t^{1+\eta} \left(\frac{P_t}{P_t^H} \right)^{1-\eta} \bigstar_t^{-\eta-1} \right. \right. \\
&\quad \left. \left. \times \left(S_t^n \right)^{-(1+\eta)(1-a_H)} \left(\frac{1}{3} \frac{\alpha_y}{c_{y,t}} + \frac{1}{3} \frac{\alpha_m}{c_{m,t}} + \frac{1}{3} \frac{\alpha_o}{c_{o,t}} \right) \right] \right\} + tip
\end{aligned}
$$

Now we plug all our results from before, and eliminating all elements of order greater than 2:

$$
\begin{aligned}
W_o &= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -\frac{1}{2}\frac{1}{3} \left(\tilde{c}_{y,t}^2 + \tilde{c}_{m,t}^2 + \tilde{c}_{o,t}^2 \right) - \frac{(1+\eta)}{2} \left(\hat{Y}_t \right)^2 + \tilde{\bigstar}_t \right. \\
&\quad - \frac{1}{1+\eta} \frac{1}{2} \left[\left(\frac{\alpha_y}{3} \tilde{c}_{y,t}^2 + \frac{\alpha_m}{3} \tilde{c}_{m,t}^2 + \frac{\alpha_o}{3} \tilde{c}_{o,t}^2 \right) - \left(\tilde{c}_{y,t} \frac{\alpha_y}{3} + \tilde{c}_{m,t} \frac{\alpha_m}{3} + \tilde{c}_{o,t} \frac{\alpha_o}{3} \right)^2 \right] \\
&\quad - (1-a_H) \tilde{S}_t - (1+\eta)(1-a_H) \tilde{S}_t \hat{Y}_t - \frac{1}{2} (1-a_H) \left[(\phi+\eta)(1-a_H) + (1-\phi) \right] \tilde{S}_t^2 \\
&\quad \left. + (1-a_H) \tilde{S}_t^n + (1+\eta)(1-a_H) \tilde{S}_t^n \hat{Y}_t + (1+\eta)(1-a_H)^2 \tilde{S}_t \tilde{S}_t^n \right\} \\
&\quad + O(3) + tip \\
&= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -\frac{1}{2}\frac{1}{3} \left(\tilde{c}_{y,t}^2 + \tilde{c}_{m,t}^2 + \tilde{c}_{o,t}^2 \right) - \frac{(1+\eta)}{2} \left(\hat{Y}_t \right)^2 + \tilde{\bigstar}_t \right. \\
&\quad - \frac{1}{1+\eta} \frac{1}{2} \left[\left(\frac{\alpha_y}{3} \tilde{c}_{y,t}^2 + \frac{\alpha_m}{3} \tilde{c}_{m,t}^2 + \frac{\alpha_o}{3} \tilde{c}_{o,t}^2 \right) - \left(\tilde{c}_{y,t} \frac{\alpha_y}{3} + \tilde{c}_{m,t} \frac{\alpha_m}{3} + \tilde{c}_{o,t} \frac{\alpha_o}{3} \right)^2 \right] \\
&\quad - (1-a_H) \hat{S}_t - (1+\eta)(1-a_H) \hat{Y}_t \hat{S}_t - \frac{1}{2} (1-a_H) \left[(\phi+\eta)(1-a_H) + (1-\phi) \right] \hat{S}_t^2 \\
&\quad \left. - (1-a_H) a_H (1-\phi) \hat{S}_t \tilde{S}_t^n \right\} \\
&\quad + O(3) + tip
\end{aligned}
$$

where the last equalities use the definition of the terms of trade gap, $\hat{S}_t + \tilde{S}_t^n = \tilde{S}_t$. From the equilibrium conditions:

$$
\begin{aligned}
\left(\frac{\alpha_y}{3} \tilde{c}_{y,t}^2 + \frac{\alpha_m}{3} \tilde{c}_{m,t}^2 + \frac{\alpha_o}{3} \tilde{c}_{o,t}^2 \right) &= \frac{\tilde{d}_t^2}{\theta^2} \left[\frac{\alpha_y}{3} (\gamma\beta - \zeta)^2 \right. \\
- \left(\tilde{c}_{y,t} \frac{\alpha_y}{3} + \tilde{c}_{m,t} \frac{\alpha_m}{3} + \tilde{c}_{o,t} \frac{\alpha_o}{3} \right)^2 &\quad \left. + \frac{\alpha_m}{3} \left(\gamma (\theta - \beta) - \zeta \right)^2 + \frac{\alpha_o}{3} (-\gamma\theta - \zeta)^2 \right]
\end{aligned}
$$

and:

$$
\frac{1}{3} \left(\tilde{c}_{y,t}^2 + \tilde{c}_{m,t}^2 + \tilde{c}_{o,t}^2 \right) = \frac{1}{3} \left[(\gamma\beta)^2 + \gamma (\theta - \beta)^2 + (-\gamma\theta)^2 \right] \frac{\tilde{d}_t^2}{\theta^2}
$$

to rewrite the welfare function as:

$$
\begin{aligned}
W_o \;=\; & E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -\frac{1}{2}\frac{1}{3}\left[(\gamma\beta)^2 + \gamma(\theta-\beta)^2 + (-\gamma\theta)^2\right]\frac{\tilde{d}_t^2}{\theta^2} - \frac{(1+\eta)}{2}\hat{Y}_t^2 \right. \\
& -\frac{\kappa\epsilon}{2(1-\kappa)}\left(\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H\right)^2 \\
& -\frac{1}{1+\eta}\frac{1}{2}\left[\frac{\alpha_y}{3}(\gamma\beta-\zeta)^2 + \frac{\alpha_m}{3}(\gamma(\theta-\beta)-\zeta)^2 + \frac{\alpha_o}{3}(-\gamma\theta-\zeta)^2\right]\frac{\tilde{d}_t^2}{\theta^2} \\
& -(1-a_H)\hat{S}_t - (1+\eta)(1-a_H)\hat{Y}_t\hat{S}_t - \\
& \frac{1}{2}(1-a_H)\left[(\phi+\eta)(1-a_H)+(1-\phi)\right]\hat{S}_t^2 \\
& \left. -(1-a_H)a_H(1-\phi)\hat{S}_t\tilde{S}_t^n \right\} \\
& +O(3) + tip
\end{aligned}
$$

or:

$$
\begin{aligned}
W_o \;=\; & E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -\frac{1}{2}\frac{\chi}{\theta^2}\tilde{d}_t^2 - \frac{(1-\eta)}{2}\hat{Y}_t^2 - \frac{\kappa\epsilon}{2(1-\kappa)}\left(\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H\right)^2 \right. \\
& -(1-a_H)\hat{S}_t - (1+\eta)(1-a_H)\hat{Y}_t\hat{S}_t - \frac{1}{2}(1-a_H)\left[(\phi+\eta)(1-a_H)+(1-\phi)\right]\hat{S}_t^2 \\
& \left. -(1-a_H)a_H(1-\phi)\hat{S}_t\tilde{S}_t^n \right\} \\
& +O(3) + tip
\end{aligned}
$$

with:

$$
\begin{aligned}
\chi \equiv & \frac{1}{3}\left[(\gamma\beta)^2 + \gamma(\theta-\beta)^2 + (-\gamma\theta)^2\right] \\
& + \frac{1}{1+\eta}\frac{1}{3}\left[\alpha_y(\gamma\beta-\zeta)^2 + \alpha_m(\gamma(\theta-\beta)-\zeta)^2 + \alpha_o(-\gamma\theta-\zeta)^2\right]
\end{aligned}
$$

The loss function $\mathcal{L}_0$ is defined as $-W_0$ excluding terms of order higher than 2 and terms independent of monetary policy::

$$
\begin{aligned}
\mathcal{L}_0 \;=\; & E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2}\frac{\chi}{\theta^2}\tilde{d}_t^2 + \frac{(1+\eta)}{2}\hat{Y}_t^2 + \frac{\epsilon}{2}\left(\frac{\kappa}{1-\kappa}\right)\left(\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H\right)^2 \right. \\
& +(1-a_H)\hat{S}_t + (1+\eta)(1-a_H)\hat{Y}_t\hat{S}_t + \frac{1}{2}(1-a_H)\left[(\phi+\eta)(1-a_H)+(1-\phi)\right]\hat{S}_t^2 \\
& \left. +(1-a_H)a_H(1-\phi)\hat{S}_t\tilde{S}_t^n \right\}
\end{aligned}
$$

The loss function in the main text just replaces the definition of $\tilde{S}_t^n$.

A.4. The First Order Condition of the Central Planner

In the decentralized equilibrium of the model, the following holds (see Sheedy[74, 2013]):

$$\tilde{l}_t = -\frac{\tilde{d}_t}{\theta}$$

The loss function can then be rewritten as:

$$\begin{aligned}
\mathcal{L}_0 \quad = \quad & E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\chi}{2}\tilde{l}_t^2 + \frac{(1-\eta)}{2}\hat{Y}_t^2 + \frac{\epsilon}{2}\left(\frac{\kappa}{1-\kappa}\right)\left(\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H\right)^2 \right. \\
& + (1-a_H)\,\hat{S}_t + (1+\eta)(1-a_H)\,\hat{Y}_t\hat{S}_t + \frac{1}{2}(1-a_H)\left[(\phi+\eta)(1-a_H) + (1-\phi)\right]\hat{S}_t^2 \\
& \left. + (1-a_H)\,a_H(1-\phi)\,\hat{S}_t\tilde{S}_t^n \right\} \qquad \text{(A.4.1)}
\end{aligned}$$

The constraints of this minimization problem are given by the equilibrium conditions as follows:

$$\begin{aligned}
E_t\tilde{l}_{t+1} &= \lambda\tilde{l}_t && \text{(A.4.2)} \\
(1+a_H\psi)\left(\hat{Y}_t - \hat{Y}_{t-1}\right) &= \theta\tilde{l}_t + \tilde{l}_{t-1} + (1+a_H)(1-\phi)\left(\tilde{S}_t^n - \tilde{S}_{t-1}^n\right) && \text{(A.4.3)} \\
-\frac{\kappa}{1-\kappa}\left(E_{t-1}\tilde{\Pi}_t^H - \tilde{\Pi}_t^H\right) &= (1+\eta)\,\hat{Y}_t + \zeta\tilde{l}_t + (1+\eta)(1-a_H)\,\hat{S}_t && \text{(A.4.4)} \\
\hat{S}_t &= -\psi\hat{Y}_t && \text{(A.4.5)}
\end{aligned}$$

The endogenous variables are $\tilde{l}_t$, $\hat{Y}_t$, $\hat{S}_t$, and $\tilde{\Pi}_t^H$. After plugging the definition of the terms of trade, the Lagrangian is given by:

$$
\begin{aligned}
L &= E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\chi}{2} \tilde{l}_t^2 + \frac{\epsilon}{2} \left(\frac{\kappa}{1-\kappa} \right) \left(\tilde{\Pi}_t^H - E_{t-1} \tilde{\Pi}_t^H \right)^2 \right. \\
& \left. + \frac{\delta}{2} \hat{Y}_t^2 - \left[\psi \left(1 - a_H\right) + \varrho \tilde{S}_t^n \right] \hat{Y}_t \right\} \\
& + \sum_{t=0}^{\infty} \beta^t E_0 \left\{ \gimel_t \left[\left(1+\eta\right)\left(1 - \psi + \psi a_H\right) \hat{Y}_t + \zeta \tilde{l}_t - \frac{\kappa}{1-\kappa} \left(\tilde{\Pi}_t^H - E_{t-1} \tilde{\Pi}_t^H \right) \right] \right\} \\
& + \sum_{t=0}^{\infty} \beta^t E_0 \left\{ \daleth_t \left[\lambda \tilde{l}_t - \tilde{l}_{t+1} \right] \right\} \\
& + \sum_{t=0}^{\infty} \beta^t E_0 \left\{ \beth_t \left[\theta \tilde{l}_t + \tilde{l}_{t-1} + \left(1 + a_H\right)\left(1 - \phi\right) \left(\tilde{S}_t^n - \tilde{S}_{t-1}^n \right) - \left(1 + a_H \psi\right) \left(\hat{Y}_t - \hat{Y}_{t-1} \right) \right] \right\}
\end{aligned} \tag{A.4.6}
$$

with

$$
\begin{aligned}
\delta &= (1+\eta) - 2(1+\eta)(1-a_H)\psi + (1-a_H)\left[(\phi+\eta)(1-a_H) + 1 - \phi\right]\psi^2 \\
\varrho &= (1-a_H)\, a_H (1-\phi)\, \psi
\end{aligned}
$$

The FOC with respect to $\tilde{\Pi}_t^H$, $\tilde{l}_t$ and $\hat{Y}_t$:

$$
\epsilon \left[\tilde{\Pi}_t^H - E_{t-1} \tilde{\Pi}_t^H \right] = \gimel_t - E_{t-1} \gimel_t \tag{A.4.7}
$$

$$
\chi \tilde{l}_t + \gimel_t \zeta + \daleth_t \lambda - \beta^{-1} \daleth_{t-1} + \beth_t \theta + \beta E_t \beth_{t+1} = 0 \tag{A.4.8}
$$

$$
\begin{aligned}
& \delta \hat{Y}_t - \left[\psi \left(1 - a_H\right) + \varrho \tilde{S}_t^n \right] \\
& + \gimel_t (1+\eta)(1 - \psi + \psi a_H) \\
& - \beth_t (1 + a_H \psi) + \beta (1 + a_H \psi) E_t \beth_{t+1} = 0
\end{aligned} \tag{A.4.9}
$$

Conjecture $E_t \beth_{t+1} = 0$ (to be verified later). Then, the system is:

$$
\epsilon \left[\tilde{\Pi}_t^H - E_{t-1} \tilde{\Pi}_t^H \right] = \gimel_t - E_{t-1} \gimel_t \tag{A.4.10}
$$

$$
\chi \tilde{l}_t + \gimel_t \zeta + \daleth_t \lambda - \beta^{-1} \daleth_{t-1} + \beth_t \theta = 0 \tag{A.4.11}
$$

$$
\begin{aligned}
& \delta \hat{Y}_t - \left[\psi \left(1 - a_H\right) + \varrho \tilde{S}_t^n \right] \\
& + \gimel_t (1+\eta)(1 - \psi + \psi a_H) - \beth_t (1 + a_H \psi) = 0
\end{aligned} \tag{A.4.12}
$$

It's evident that the key to solving the system is to find $\gimel_t - E_{t-1}\gimel_t$. To find this term, we start from (A.4.12):

$$\gimel_t = \beth_t \frac{(1+a_H\psi)}{(1+\eta)(1-\psi+\psi a_H)} + \frac{\psi(1-a_H)+\varrho\tilde{S}_t^n}{(1+\eta)(1-\psi+\psi a_H)} - \frac{\delta}{(1+\eta)(1-\psi+\psi a_H)}\hat{Y}_t$$

From the Phillips Curve:

$$\hat{Y}_t = \frac{\kappa}{1-\kappa}\frac{1}{(1+\eta)(1-\psi+\psi a_H)}\left(\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H\right) - \frac{\zeta}{(1+\eta)(1-\psi+\psi a_H)}\tilde{l}_t$$

Plugging in the previous equation:

$$\begin{aligned}
\gimel_t \;=\; & \beth_t \frac{(1+a_H\psi)}{(1+\eta)(1-\psi+\psi a_H)} - \frac{\kappa}{1-\kappa}\frac{\delta}{[(1+\eta)(1-\psi+\psi a_H)]^2}\left(\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H\right) \\
& + \frac{\delta\zeta}{[(1+\eta)(1-\psi+\psi a_H)]^2}\tilde{l}_t + \frac{\psi(1-a_H)}{(1+\eta)(1-\psi+\psi a_H)} \\
& + \frac{\varrho}{(1+\eta)(1-\psi+\psi a_H)}\tilde{S}_t^n
\end{aligned}$$

Taking conditional expectations and employing the guess:

$$\begin{aligned}
E_t\gimel_{t+1} \;=\; & \frac{\delta\zeta}{[(1+\eta)(1-\psi+\psi a_H)]^2}E_t\tilde{l}_{t+1} + \frac{\psi(1-a_H)}{(1+\eta)(1-\psi+\psi a_H)} \\
& + \frac{\varrho}{(1+\eta)(1-\psi+\psi a_H)}E_t\tilde{S}_{t+1}^n
\end{aligned}$$

Combining the last two:

$$\begin{aligned}
\gimel_t - E_{t-1}\gimel_t \;=\; & \beth_t \frac{(1+a_H\psi)}{(1+\eta)(1-\psi+\psi a_H)} && \text{(A.4.13)} \\
& - \frac{\kappa}{1-\kappa}\frac{\delta}{[(1+\eta)(1-\psi+\psi a_H)]^2}\left(\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H\right) \\
& + \frac{\delta\zeta}{[(1+\eta)(1-\psi+\psi a_H)]^2}\left[\tilde{l}_t - E_{t-1}\tilde{l}_t\right] \\
& + \frac{\varrho}{(1+\eta)(1-\psi+\psi a_H)}\left(\tilde{S}_t^n - E_{t-1}\tilde{S}_t^n\right) && \text{(A.4.14)}
\end{aligned}$$

Now it is necessary to solve for $\beth_t$ (which will confirm the guess). It turns out it is first necessary to solve for $\daleth_t$. Taking conditional expectation of (A.4.11) at $t+1$:

$$\daleth_t = \chi\beta E_t \tilde{l}_{t+1} + \zeta\beta E_t \gimel_{t+1} + \lambda\beta E_t \daleth_{t+1}$$

From previous equations, we can replace $E_t \gimel_{t+1}$ in the last expression:

$$\begin{aligned}
\daleth_t &= \left[\chi\beta + \frac{\zeta^2\beta\delta}{\left[(1+\eta)\left(1-\psi+\psi a_H\right)\right]^2}\right] E_t \tilde{l}_{t+1} + \frac{\zeta\beta\psi\left(1-a_H\right)}{(1+\eta)\left(1-\psi+\psi a_H\right)} \\
&\quad + \frac{\zeta\beta\varrho}{(1+\eta)\left(1-\psi+\psi a_H\right)} E_t \tilde{S}^n_{t+1} + \lambda\beta E_t \daleth_{t+1}
\end{aligned}$$

We can employ forward operators to solve for $\daleth_t$ ($F^l X_t = E_t X_{t+l}$):

$$\begin{aligned}
\daleth_t \left(1-\lambda\beta F\right) &= \left[\chi\beta + \frac{\zeta^2\beta\delta}{\left[(1+\eta)\left(1-\psi+\psi a_H\right)\right]^2}\right] F\tilde{l}_t + \frac{\zeta\beta\psi\left(1-a_H\right)}{(1+\eta)\left(1-\psi+\psi a_H\right)} \\
&\quad + \frac{\zeta\beta\varrho}{(1+\eta)\left(1-\psi+\psi a_H\right)} F\tilde{S}^n_t \\
\daleth_t &= \left[\chi\beta + \frac{\zeta^2\beta\delta}{\left[(1+\eta)\left(1-\psi+\psi a_H\right)\right]^2}\right] \frac{F}{1-\lambda\beta F}\tilde{l}_t \\
&\quad + \frac{\zeta\beta\psi\left(1-a_H\right)}{(1+\eta)\left(1-\psi+\psi a_H\right)} \frac{1}{1-\lambda\beta F} \\
&\quad + \frac{\zeta\beta\varrho}{(1+\eta)\left(1-\psi+\psi a_H\right)} \frac{F}{1-\lambda\beta F}\tilde{S}^n_t
\end{aligned}$$

Recall $F\tilde{l}_t = E_t\tilde{l}_{t+1} = \lambda\tilde{l}_t$ and assume $F\tilde{S}^n_t = E_t\tilde{S}^n_{t+1} = \Gamma\tilde{S}^n_t$. Then:

$$\begin{aligned}
\daleth_t &= \left[\chi + \frac{\zeta^2\delta}{\left[(1+\eta)\left(1-\psi+\psi a_H\right)\right]^2}\right] \frac{\beta\lambda}{1-\lambda^2\beta}\tilde{l}_t + \frac{\zeta\psi\left(1-a_H\right)}{(1+\eta)\left(1-\psi+\psi a_H\right)} \frac{\beta}{1-\lambda\beta} \\
&\quad + \frac{\zeta\varrho}{(1+\eta)\left(1-\psi+\psi a_H\right)} \frac{\beta\Gamma}{1-\lambda\beta\Gamma}\tilde{S}^n_t
\end{aligned}$$

Now we plug this result back in (A.4.11) together with our solution for $\daleth_t$ above:

$$\begin{aligned} \daleth_t &= \frac{\kappa}{1-\kappa} \frac{\delta\zeta\left(\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H\right)}{(1+\eta)\left(1-\psi+\psi a_H\right)\left[\zeta\left(1+a_H\psi\right)+\theta\left(1+\eta\right)\left(1-\psi+\psi a_H\right)\right]} \\ &\quad - \frac{\zeta\varrho\left(\tilde{S}_t^n - E_{t-1}\tilde{S}_t^n\right)}{\left[\zeta\left(1+a_H\psi\right)+\theta\left(1+\eta\right)\left(1-\psi+\psi a_H\right)\right]} \frac{1}{1-\lambda\beta\Gamma} \\ &\quad - \frac{\chi\left[(1+\eta)\left(1-\psi+\psi a_H\right)\right]^2+\zeta^2\delta}{(1+\eta)\left(1-\psi+\psi a_H\right)\left[\zeta\left(1+a_H\psi\right)+\theta\left(1+\eta\right)\left(1-\psi+\psi a_H\right)\right]} \frac{\left(\tilde{l}_t - E_{t-1}\tilde{l}_t\right)}{1-\lambda^2\beta} \end{aligned}$$

which confirms the guess and where the definitions $\lambda\tilde{l}_{t-1} = E_{t-1}\tilde{l}_t$ and $\Gamma\tilde{S}_{t-1}^n = E_{t-1}\tilde{S}_t^n$ were employed. Finally, we plug the solution for $\daleth_t$ and (A.4.13) in (A.4.10):

$$\begin{gathered} \left(\epsilon + \tfrac{\alpha}{1-\alpha}\tfrac{\delta\theta}{(1+\eta)(1-\psi+\psi a_H)[\zeta(1+a_H\psi)+\theta(1+\eta)(1-\psi+\psi a_H)]}\right)\left(\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H\right) \\ = \\ \left(\tfrac{\delta\zeta}{[(1+\eta)(1-\psi+\psi a_H)]^2} - \tfrac{\chi(1+a_H\psi)[(1+\eta)(1-\psi+\psi a_H)]^2+\zeta^2\delta(1+a_H\psi)}{[(1+\eta)(1-\psi+\psi a_H)]^2[\zeta(1+a_H\psi)+\theta(1+\eta)(1-\psi+\psi a_H)](1-\lambda^2\beta)}\right)\left(\tilde{l}_t - E_{t-1}\tilde{l}_t\right) \\ + \left(\tfrac{\varrho}{(1+\eta)(1-\psi+\psi a_H)} - \tfrac{(1+a_H\psi)}{(1+\eta)(1-\psi+\psi a_H)}\tfrac{\zeta\varrho}{[\zeta(1+a_H\psi)+\theta(1+\eta)(1-\psi+\psi a_H)](1-\lambda\beta\Gamma)}\right)\left(\tilde{S}_t^n - E_{t-1}\tilde{S}_t^n\right) \end{gathered}$$

Defining Θ_A, Θ_d and Θ_π as in Appendix A.2, the first order condition is rewritten as:

$$\Theta_\pi\left(\tilde{\Pi}_t^H - E_{t-1}\tilde{\Pi}_t^H\right) = \Theta_d\left(\tilde{l}_t - E_{t-1}\tilde{l}_t\right) + \Theta_A\left(\tilde{S}_t^n - E_{t-1}\tilde{S}_t^n\right) \tag{A.4.15}$$

The first order condition of the main text replaces the equilibrium condition for $\tilde{l}_t$ and the definition of $\tilde{S}_t^n$. Writing the Phillips curve and the definition of the real return in the form of unanticipated responses, the system of equations of the main text is complete.

B. Appendices to Chapter 2

B.1. Proof of Proposition 8

1. ($\Leftarrow$): If y^A and y^B are independent, it is possible to write $f\left(y^A, y^B\right) = g\left(y^A\right) h\left(y^B\right)$ and $F\left(y^A, y^B\right) = G\left(y^A\right) H\left(y^B\right)$. Imposing this in (2.1.9) and (2.1.11), and using $\mu = 0$, it is straightforward to show that both of them collapse to:

$$
\begin{aligned}
u_1\left(\left[1 - H\left(\frac{\ell^B}{\gamma}\right)\right]\ell^B\right)\left[1 - H_1\left(\frac{\ell^B}{\gamma}\right)\frac{\ell^B}{\gamma} - H\left(\frac{\ell^B}{\gamma}\right)\right] & \\
-\beta \int_{\frac{\ell^B}{\gamma}}^{y_H} u_1\left(y^B - \ell^B\right) h\left(y^B\right) dy^B &= 0
\end{aligned}
$$

which does not depend on ℓ^A. Thus, $V^B\left(\ell^A, 1\right) = V^B\left(\ell^A, 0\right)$ and $V_1^B\left(\ell^A, 0\right) = V_1^B\left(\ell^A, 1\right) = 0$. Imposing these on (2.1.16), this condition is reduced to $\Theta\left(\ell^{A,P}\right) = 0$ as $\lambda > 0$, which is the same optimality condition of the decentralized equilibrium assuming interior solutions.

2. ($\Rightarrow$): Suppose that the decentralized allocation is ex-ante efficient. Then, there exists some λ for which $\Theta\left(\ell^{A,P}\right) = 0$. For the same value of λ, from (2.1.16), it follows that, for interior solutions:

$$
\begin{aligned}
G'\left(\frac{\ell^{A,SP}}{\gamma}\right)\frac{1}{\gamma}\left[V^B\left(\ell^{A,SP}, 1\right) - V^B\left(\ell^{A,SP}, 0\right)\right] + & \\
\left[1 - G\left(\frac{\ell^{A,SP}}{\gamma}\right)\right] V_1^B\left(\ell^{A,SP}, 0\right) + G\left(\frac{\ell^{A,SP}}{\gamma}\right) V_1^B\left(\ell^{A,SP}, 1\right) &= 0
\end{aligned}
$$

This condition is not true in general as long as it involves both preferences and the distribution of income, which can be chosen arbitrarily and separately from one another. As for interior solutions $G(\cdot) > 0$ and increasing, efficiency requires:

$$\begin{aligned} V^B\left(\ell^A,1\right) - V^B\left(\ell^A,0\right) &= 0 \\ V_1^B\left(\ell^{A,SP},0\right) &= 0 \\ V_1^B\left(\ell^{A,SP},1\right) &= 0 \end{aligned}$$

which are jointly true only if $\Xi\left(\ell^A,0;\ell^B\right)$ and $\Upsilon\left(\ell^A,0;\ell^B\right)$ do not depend on ℓ^A. As $u_1 > 0$ (Inada condition) and $\ell^B < \gamma y_H$, this holds only if $f\left(y^A,y^B\right) = g\left(y^A\right)h\left(y^B\right)$and $F\left(y^A,y^B\right) = G\left(y^A\right)H\left(y^B\right)$.□

B.2. Solution of the simulated model

B.2.1. Basics

The probability density function for y is:

$$f(y) = \begin{cases} \frac{1-\rho}{y_H-y_L} & y_L \leq y < y_H \\ \rho & y = y_H \\ 0 & o.w. \end{cases}$$

Using this, we can calculate the following moments for the problem of Borrower A:

$$\begin{aligned}
\Pr(y \geq t) &= \begin{cases} \rho + (1-\rho)\left(\frac{y_H - t}{y_H - y_L}\right) & y_L \leq t \leq y_H \\ 0 & t > y_H \end{cases} \\
\Pr(y < t) &= \begin{cases} (1-\rho)\left(\frac{t - y_L}{y_H - y_L}\right) & y_L \leq t \leq y_H \\ 1 & t > y_H \end{cases} \\
E\left[u(y) \mid y \geq t\right] &= \int_t^{y_H} u(y) \frac{g(y)}{\Pr(y \geq t)} dy + \frac{\rho}{\Pr(y \geq t)} u(y_H) \\
&= \int_t^{y_H} u(y) \frac{\frac{1-\rho}{y_H - y_L}}{\rho + (1-\rho)\left(\frac{y_H - t}{y_H - y_L}\right)} dy + \frac{\rho}{\rho + (1-\rho)\left(\frac{y_H - t}{y_H - y_L}\right)} u(y_H) \\
&= \frac{\frac{1-\rho}{y_H - y_L}}{\rho + (1-\rho)\left(\frac{y_H - t}{y_H - y_L}\right)} \int_t^{y_H} u(y)\, dy + \frac{\rho}{\rho + (1-\rho)\left(\frac{y_H - t}{y_H - y_L}\right)} u(y_H) \\
E\left[u(y) \mid y < t\right] &= \int_{y_L}^{t} u(y) \frac{g(y)}{\Pr(y < t)} dy \\
&= \int_{y_L}^{t} u(y) \frac{\frac{1-\rho}{(y_H - y_L)}}{(1-\rho)\left(\frac{t - y_L}{y_H - y_L}\right)} dy \\
&= \frac{1}{(t - y_L)} \int_{y_L}^{t} u(y)\, dy
\end{aligned}$$

Assuming that Borrower A chose to repay on $\ell^A = x < \gamma y_H$, his action reveals $y \geq \frac{x}{\gamma}$ and we can calculate the following moments for the problem of Borrower B:

$$\Pr\left(y \geq t \mid y \geq \frac{x}{\gamma}\right) = \frac{\rho}{\rho + (1-\rho)\left(\frac{y_H - \frac{x}{\gamma}}{y_H - y_L}\right)} + \int_t^{y_H} \frac{\frac{1-\rho}{y_H - y_L}}{\rho + (1-\rho)\left(\frac{y_H - \frac{x}{\gamma}}{y_H - y_L}\right)} dy$$

$$= \frac{\rho + (1-\rho)\left(\frac{y_H - t}{y_H - y_L}\right)}{\rho + (1-\rho)\left(\frac{y_H - \frac{x}{\gamma}}{y_H - y_L}\right)}$$

$$\Pr\left(y < t \mid y \geq \frac{x}{\gamma}\right) = 1 - \frac{\rho + (1-\rho)\left(\frac{y_H - t}{y_H - y_L}\right)}{\rho + (1-\rho)\left(\frac{y_H - \frac{x}{\gamma}}{y_H - y_L}\right)}$$

$$= \frac{(1-\rho)\left(\frac{t - \frac{x}{\gamma}}{y_H - y_L}\right)}{\rho + (1-\rho)\left(\frac{y_H - \frac{x}{\gamma}}{y_H - y_L}\right)}$$

$$E\left[u(y) \mid y \geq t, y \geq \frac{x}{\gamma}\right] = \int_t^{y_H} u(y) \frac{\frac{\frac{1-\rho}{y_H - y_L}}{\rho + (1-\rho)\left(\frac{y_H - \frac{x}{\gamma}}{y_H - y_L}\right)}}{\frac{\rho + (1-\rho)\left(\frac{y_H - t}{y_H - y_L}\right)}{\rho + (1-\rho)\left(\frac{y_H - \frac{x}{\gamma}}{y_H - y_L}\right)}} dy$$

$$+ \frac{\frac{\rho}{\rho + (1-\rho)\left(\frac{y_H - \frac{x}{\gamma}}{y_H - y_L}\right)}}{\frac{\rho + (1-\rho)\left(\frac{y_H - t}{y_H - y_L}\right)}{\rho + (1-\rho)\left(\frac{y_H - \frac{x}{\gamma}}{y_H - y_L}\right)}} u(y_H)$$

$$= \frac{\frac{1-\rho}{y_H - y_L}}{\rho + (1-\rho)\left(\frac{y_H - t}{y_H - y_L}\right)} \int_t^{y_H} u(y)\, dy$$

$$+ \frac{\rho}{\rho + (1-\rho)\left(\frac{y_H - t}{y_H - y_L}\right)} u(y_H)$$

$$E\left[u(y) \mid y < t, y \geq \frac{x}{\gamma}\right] = \frac{\frac{1-\rho}{y_H - y_L}}{(1-\rho)\left(\frac{t - \frac{x}{\gamma}}{y_H - y_L}\right)} \int_{\frac{x}{\gamma}}^{t} u(y)\, dy$$

Finally, assuming that Borrower A chose to default on $\ell^A = x < \gamma y_H$, his action reveals $y < \frac{x}{\gamma}$ and we can calculate the following moments for the problem of Borrower B:

$$\Pr\left(y \geq t \mid y < \frac{x}{\gamma}\right) = \int_t^{\frac{x}{\gamma}} \frac{1}{\frac{x}{\gamma} - y_L} dy$$

$$= \frac{\frac{x}{\gamma} - t}{\frac{x}{\gamma} - y_L}$$

$$\Pr\left(y < t \mid y < \frac{x}{\gamma}\right) = 1 - \frac{\frac{x}{\gamma} - t}{\frac{x}{\gamma} - y_L}$$

$$= \frac{t - y_L}{\frac{x}{\gamma} - y_L}$$

$$E\left[u(y) \mid y \geq t, y < \frac{x}{\gamma}\right] = \int_t^{\frac{x}{\gamma}} u(y) \frac{\frac{1}{\frac{x}{\gamma} - y_L}}{\Pr(y \geq t)} dy$$

$$= \frac{1}{\frac{x}{\gamma} - t} \int_t^{\frac{x}{\gamma}} u(y)\, dy$$

$$E\left[u(y) \mid y < t, y < \frac{x}{\gamma}\right] = \frac{1}{t - y_L} \int_{y_L}^{t} u(y)\, dy$$

B.2.2. Decentralized Equilibrium

B.2.2.1. Problem of Borrower A

Using the results above, the problem of Borrower A can be expressed as:

$$
\begin{aligned}
& \max_{\ell^A \in [\gamma y_L, \gamma y_H]} \quad u\left(\Pr\left(y \geq \frac{\ell^A}{\gamma}\right)\ell^A\right) + \beta\left\{\Pr\left(y \geq \frac{\ell^A}{\gamma}\right) E\left[u\left(y-\ell^A\right) \mid y \geq \frac{\ell^A}{\gamma}\right]\right. \\
& \qquad\qquad \left. + \left[\Pr\left(y < \frac{\ell^A}{\gamma}\right)\right] E\left[u\left(y-\gamma y\right) \mid y < \frac{\ell^A}{\gamma}\right]\right\} \\
= & \max_{\ell^A \in [\gamma y_L, \gamma y_H]} \quad u\left(\left[\rho + (1-\rho)\left(\frac{y_H - \frac{\ell^A}{\gamma}}{y_H - y_L}\right)\right]\ell^A\right) \\
& \qquad\qquad + \beta\left[\rho + (1-\rho)\left(\frac{y_H - \frac{\ell^A}{\gamma}}{y_H - y_L}\right)\right] \times \\
& \qquad\qquad \left[\frac{\frac{1-\rho}{y_H - y_L}}{\rho + (1-\rho)\left(\frac{y_H - \frac{\ell^A}{\gamma}}{y_H - y_L}\right)} \int_{\frac{\ell^A}{\gamma}}^{y_H} u\left(y - \ell^A\right) dy \right. \\
& \qquad\qquad \left. + \frac{\rho}{\rho + (1-\rho)\left(\frac{y_H - \frac{\ell^A}{\gamma}}{y_H - y_L}\right)} u\left(y_H - \ell^A\right)\right] \\
& \qquad\qquad + \beta\left[(1-\rho)\left(\frac{\frac{\ell^A}{\gamma} - y_L}{y_H - y_L}\right)\right]\left[\frac{1}{\left(\frac{\ell^A}{\gamma} - y_L\right)} \int_{y_L}^{\frac{\ell^A}{\gamma}} u\left(y\left(1-\gamma\right)\right) dy\right]
\end{aligned}
$$

Using (2.2.1):

$$\begin{aligned}
& \max_{\ell^A \in [\gamma y_L, \gamma y_H]} \frac{\left(\left[\rho + (1-\rho)\left(\frac{y_H - \frac{\ell^A}{\gamma}}{y_H - y_L}\right)\right]\ell^A\right)^{1-\sigma}}{1-\sigma} \\
& +\beta\left[\rho + (1-\rho)\left(\frac{y_H - \frac{\ell^A}{\gamma}}{y_H - y_L}\right)\right] \times \\
& \left[\frac{\frac{1-\rho}{y_H - y_L}}{\rho + (1-\rho)\left(\frac{y_H - \frac{\ell^A}{\gamma}}{y_H - y_L}\right)} \int_{\frac{\ell^A}{\gamma}}^{y_H} \frac{\left(y - \ell^A\right)^{1-\sigma}}{1-\sigma} dy \right. \\
& \left. + \frac{\rho}{\rho + (1-\rho)\left(\frac{y_H - \frac{\ell^A}{\gamma}}{y_H - y_L}\right)} \frac{\left(y_H - \ell^A\right)^{1-\sigma}}{1-\sigma}\right] \\
& +\beta\left[(1-\rho)\left(\frac{\frac{\ell^A}{\gamma} - y_L}{y_H - y_L}\right)\right]\left[\frac{1}{\left(\frac{\ell^A}{\gamma} - y_L\right)} \int_{y_L}^{\frac{\ell^A}{\gamma}} \frac{\left[y\left(1-\gamma\right)\right]^{1-\sigma}}{1-\sigma} dy\right] \\
= \; & \max_{\ell^A \in [\gamma y_L, \gamma y_H]} \frac{\left[\rho + (1-\rho)\left(\frac{y_H - \frac{\ell^A}{\gamma}}{y_H - y_L}\right)\right]^{1-\sigma} \left(\ell^A\right)^{1-\sigma}}{1-\sigma} \\
& +\beta\left[\rho + (1-\rho)\left(\frac{y_H - \frac{\ell^A}{\gamma}}{y_H - y_L}\right)\right] \times \\
& \left\{\frac{\frac{1-\rho}{y_H - y_L}\left[\left(y_H - \ell^A\right)^{2-\sigma} - \left(\ell^A\left(\frac{1-\gamma}{\gamma}\right)\right)^{2-\sigma}\right]}{\left[\rho + (1-\rho)\left(\frac{y_H - \frac{\ell^A}{\gamma}}{y_H - y_L}\right)\right](1-\sigma)(2-\sigma)} \right. \\
& \left. + \frac{\rho\left[\left(y_H - \ell^A\right)^{1-\sigma}\right]}{\left[\rho + (1-\rho)\left(\frac{y_H - \frac{\ell^A}{\gamma}}{y_H - y_L}\right)\right](1-\sigma)}\right\} \\
& +\beta\left[(1-\rho)\left(\frac{\frac{\ell^A}{\gamma} - y_L}{y_H - y_L}\right)\right] \frac{(1-\gamma)^{1-\sigma}\left[\left(\frac{\ell^A}{\gamma}\right)^{2-\sigma} - (y_L)^{2-\sigma}\right]}{\left(\frac{\ell^A}{\gamma} - y_L\right)(1-\sigma)(2-\sigma)}
\end{aligned}$$

$$
\begin{aligned}
&= \max_{\ell^A \in [\gamma y_L, \gamma y_H]} \frac{\left[\rho + \left(\frac{1-\rho}{y_H - y_L}\right)\left(y_H - \frac{\ell^A}{\gamma}\right)\right]^{1-\sigma} \left(\ell^A\right)^{1-\sigma}}{1-\sigma} \\
&+ \frac{\beta}{(1-\sigma)(2-\sigma)} \left[\frac{1-\rho}{y_H - y_L}\left(y_H - \ell^A\right)^{2-\sigma}\right. \\
&\left. - \frac{1-\rho}{y_H - y_L}\left(\ell^A\right)^{2-\sigma}\left(\frac{1-\gamma}{\gamma}\right)^{2-\sigma} + \rho(2-\sigma)\left(y_H - \ell^A\right)^{1-\sigma}\right] \\
&+ \frac{\beta}{(1-\sigma)(2-\sigma)} \left[\left(\frac{1-\rho}{y_H - y_L}\right)\right](1-\gamma)^{1-\sigma}\left[\left(\frac{\ell^A}{\gamma}\right)^{2-\sigma} - (y_L)^{2-\sigma}\right]
\end{aligned}
$$

B.2.2.2. Optimization by Borrower A: conditions on interior solution

The aforementioned maximization problem has two constraints: $\ell^A \geq \gamma y_L$and $\ell^A \leq \gamma y_H$. This constraints can be rewritten as:

$$
\begin{aligned}
\gamma y_L - \ell^A &\leq 0 \\
\ell^A - \gamma y_H &\leq 0
\end{aligned}
$$

Let μ and θ be the respective Lagrange multipliers on this two constraints. The first order condition of the problem of Borrower A is:

$$
\begin{aligned}
\nabla\left(\ell^A\right) &= \left[\rho\ell^A + y_H\left(\frac{1-\rho}{y_H-y_L}\right)\ell^A - \frac{1}{\gamma}\left(\frac{1-\rho}{y_H-y_L}\right)\left(\ell^A\right)^2\right]^{-\sigma} \times \\
&\quad \left\{\rho + y_H\left(\frac{1-\rho}{y_H-y_L}\right) - \frac{2}{\gamma}\left(\frac{1-\rho}{y_H-y_L}\right)\ell^A\right\} \\
&\quad + \frac{\beta}{(1-\sigma)(2-\sigma)}\left[-(2-\sigma)\frac{1-\rho}{y_H-y_L}\left(y_H-\ell^A\right)^{1-\sigma}\right. \\
&\quad \left. -(2-\sigma)\frac{1-\rho}{y_H-y_L}\left(\ell^A\right)^{1-\sigma}\left(\frac{1-\gamma}{\gamma}\right)^{2-\sigma} - \rho(2-\sigma)(1-\sigma)\left(y_H-\ell^A\right)^{-\sigma}\right] \\
&\quad + \frac{\beta}{(1-\sigma)(2-\sigma)}\left[\left(\frac{1-\rho}{y_H-y_L}\right)\right](1-\gamma)^{1-\sigma}\left[(2-\sigma)\left(\frac{\ell^A}{\gamma}\right)^{1-\sigma}\left(\frac{1}{\gamma}\right)\right] \\
&\quad = -\mu + \theta \\
&= \left[\rho\ell^A + \frac{y_H(1-\rho)}{y_H-y_L}\ell^A - \frac{1-\rho}{\gamma(y_H-y_L)}\left(\ell^A\right)^2\right]^{-\sigma} \times \\
&\quad \left[\rho + \frac{y_H(1-\rho)}{y_H-y_L} - \frac{2(1-\rho)}{\gamma(y_H-y_L)}\ell^A\right] \\
&\quad + \frac{\beta}{(1-\sigma)}\frac{1-\rho}{y_H-y_L}\left[\left(\frac{1-\gamma}{\gamma}\right)^{1-\sigma}\left(\ell^A\right)^{1-\sigma} - \left(y_H-\ell^A\right)^{1-\sigma}\right] \\
&\quad - \beta\rho\left(y_H-\ell^A\right)^{-\sigma} = -\mu + \theta
\end{aligned}
$$

In the following expression I explore the conditions under which the first constraint is slack but the second binds with equality: $\ell^A = \gamma y_H$. That is, conditions under which $\mu = 0$ and $\theta > 0$ at $\ell^A = \gamma y_H$. Imposing these, the condition collapses to:

$$
\rho^{-\sigma}\left[1 - \left(\frac{1-\rho}{\rho}\right)\frac{y_H}{y_H-y_L}\right] \;>\; \beta\left(\frac{1}{\gamma}-1\right)^{-\sigma} \tag{B.2.1}
$$

B.2.2.3. Problem of Borrower B

Assume $\ell^A = x < \gamma y_H$. As in the main text, the optimization problem of Borrower B depends on the default choice of Borrower A given x.

1. If Borrower A repays, then this action will reveal $\frac{x}{\gamma} \leq y$, and nothing more than that.

The problem of Borrower B is:

$$
\begin{aligned}
& \max_{\ell^B \in [x, \gamma y_H]} && u\left(\Pr\left(y \geq \frac{\ell^B}{\gamma}\right)\ell^B\right) \\
& && +\beta\left\{\Pr\left(y \geq \frac{\ell^B}{\gamma}\right) E\left[u\left(y-\ell^B\right) \mid y \geq \frac{\ell^B}{\gamma}, \Im^B\right] + \right. \\
& && \left.\left[\Pr\left(y < \frac{\ell^B}{\gamma}\right)\right] E\left[u\left(y-\gamma y\right) \mid y < \frac{\ell^B}{\gamma}, \Im^B\right]\right\} \\
= & \max_{\ell^B \in [x, \gamma y_H]} && u\left(\frac{\rho+(1-\rho)\left(\frac{y_H-\frac{\ell^B}{\gamma}}{y_H-y_L}\right)}{\rho+(1-\rho)\left(\frac{y_H-\frac{x}{\gamma}}{y_H-y_L}\right)}\ell^B\right) + \beta\frac{\rho+(1-\rho)\left(\frac{y_H-\frac{\ell^B}{\gamma}}{y_H-y_L}\right)}{\rho+(1-\rho)\left(\frac{y_H-\frac{x}{\gamma}}{y_H-y_L}\right)} \times \\
& && \left[\frac{\frac{1-\rho}{y_H-y_L}}{\rho+(1-\rho)\left(\frac{y_H-\frac{\ell^B}{\gamma}}{y_H-y_L}\right)}\int_{\frac{\ell^B}{\gamma}}^{y_H} u\left(y-\ell^B\right) dy\right. \\
& && \left.+\frac{\rho}{\rho+(1-\rho)\left(\frac{y_H-\frac{\ell^B}{\gamma}}{y_H-y_L}\right)} u\left(y_H-\ell^B\right)\right] \\
& && +\beta\left[\frac{(1-\rho)\left(\frac{\frac{\ell^B}{\gamma}-\frac{x}{\gamma}}{y_H-y_L}\right)}{\rho+(1-\rho)\left(\frac{y_H-\frac{x}{\gamma}}{y_H-y_L}\right)}\right]\frac{1}{\frac{\ell^B}{\gamma}-\frac{x}{\gamma}}\int_{\frac{x}{\gamma}}^{\frac{\ell^B}{\gamma}} u\left(y\left(1-\gamma\right)\right) dy \\
= & \max_{\ell^B \in [x, \gamma y_H]} && u\left(\frac{\rho+(1-\rho)\left(\frac{y_H-\frac{\ell^B}{\gamma}}{y_H-y_L}\right)}{\rho+(1-\rho)\left(\frac{y_H-\frac{x}{\gamma}}{y_H-y_L}\right)}\ell^B\right) + \frac{\beta}{\rho+(1-\rho)\left(\frac{y_H-\frac{x}{\gamma}}{y_H-y_L}\right)} \times \\
& && \left[\frac{1-\rho}{y_H-y_L}\int_{\frac{\ell^B}{\gamma}}^{y_H} u\left(y-\ell^B\right) dy + \rho u\left(y_H-\ell^B\right)\right. \\
& && \left.+\left(\frac{1-\rho}{y_H-y_L}\right)\int_{\frac{x}{\gamma}}^{\frac{\ell^B}{\gamma}} u\left(y\left(1-\gamma\right)\right) dy\right]
\end{aligned}
$$

using (2.2.1):

$$
\begin{aligned}
& \max_{\ell^B \in [x, \gamma y_H]} \quad \frac{\left(\frac{\rho + (1-\rho)\left(\frac{y_H - \frac{\ell^B}{\gamma}}{y_H - y_L}\right)}{\rho + (1-\rho)\left(\frac{y_H - \frac{x}{\gamma}}{y_H - y_L}\right)} \ell^B \right)^{1-\sigma}}{1-\sigma} + \frac{\beta}{\rho + (1-\rho)\left(\frac{y_H - \frac{x}{\gamma}}{y_H - y_L}\right)} \times \\
& \left[\frac{1-\rho}{y_H - y_L} \int_{\frac{\ell^B}{\gamma}}^{y_H} \frac{\left(y - \ell^B\right)^{1-\sigma}}{1-\sigma} dy + \rho \frac{\left(y_H - \ell^B\right)^{1-\sigma}}{1-\sigma} \right. \\
& \left. + \frac{1-\rho}{y_H - y_L} \int_{\frac{x}{\gamma}}^{\frac{\ell^B}{\gamma}} \frac{\left(y\left(1-\gamma\right)\right)^{1-\sigma}}{1-\sigma} dy \right] \\
= \quad & \max_{\ell^B \in [x, \gamma y_H]} \quad \frac{1}{\left[\rho + (1-\rho)\left(\frac{y_H - \frac{x}{\gamma}}{y_H - y_L}\right)\right]^{1-\sigma}} \times \\
& \frac{\left[\rho + \left(\frac{1-\rho}{y_H - y_L}\right)\left(y_H - \frac{\ell^B}{\gamma}\right)\right]^{1-\sigma} \left(\ell^B\right)^{1-\sigma}}{1-\sigma} \\
& + \frac{1}{\rho + (1-\rho)\left(\frac{y_H - \frac{x}{\gamma}}{y_H - y_L}\right)} \times \frac{\beta}{(1-\sigma)(2-\sigma)} \\
& \left[\left(\frac{1-\rho}{y_H - y_L}\right) \left(\left(y_H - \ell^B\right)^{2-\sigma} - \left(\frac{1-\gamma}{\gamma}\right)^{2-\sigma} \left(\ell^B\right)^{2-\sigma} \right) + \right. \\
& \rho\left(2-\sigma\right)\left(y_H - \ell^B\right)^{1-\sigma} \\
& \left. + \left(\frac{1-\rho}{y_H - y_L}\right)\left(1-\gamma\right)^{1-\sigma} \left(\frac{\ell^B}{\gamma}^{2-\sigma} - \frac{x}{\gamma}^{2-\sigma} \right) \right]
\end{aligned}
$$

The aforementioned maximization problem has two constraints: $\ell^B \geq x$ and $\ell^B \leq \gamma y_H$. This constraints can be rewritten as:

$$
\begin{aligned}
x - \ell^B &\leq 0 \\
\ell^B - \gamma y_H &\leq 0
\end{aligned}
$$

Let φ and υ be the respective Lagrange multipliers on this two constraints. The first order condition of the problem of Borrower B is:

$$
\begin{aligned}
\nabla\left(\ell^{B,R,d^A=0}\right) &= \frac{1}{\left[\rho+(1-\rho)\left(\frac{y_H-\frac{x}{\gamma}}{y_H-y_L}\right)\right]^{1-\sigma}}\times \\
&\left[\rho\ell^B+\ell^B\left(\frac{1-\rho}{y_H-y_L}\right)y_H-\left(\ell^B\right)^2\left(\frac{1-\rho}{y_H-y_L}\right)\frac{1}{\gamma}\right]^{-\sigma}\times \\
&\left[\rho+\left(\frac{1-\rho}{y_H-y_L}\right)y_H-\ell^B\left(\frac{1-\rho}{y_H-y_L}\right)\frac{2}{\gamma}\right] \\
&+\frac{1}{\rho+(1-\rho)\left(\frac{y_H-\frac{x}{\gamma}}{y_H-y_L}\right)}\times \\
&\left\{\frac{\beta}{(1-\sigma)(2-\sigma)}\times\right. \\
&\left[-(2-\sigma)\frac{1-\rho}{y_H-y_L}\left(y_H-\ell^B\right)^{1-\sigma}-\right. \\
&\left.\left(\frac{1-\gamma}{\gamma}\right)^{2-\sigma}(2-\sigma)\frac{1-\rho}{y_H-y_L}\left(\ell^B\right)^{1-\sigma}\right] \\
&-\rho\beta\left(y_H-\ell^B\right)^{-\sigma} \\
&+\frac{\beta}{(1-\sigma)(2-\sigma)}\left(\frac{1-\rho}{y_H-y_L}\right)(1-\gamma)^{1-\sigma}\times \\
&\left.(2-\sigma)\left(\frac{\ell^B}{\gamma}\right)^{1-\sigma}\left(\frac{1}{\gamma}\right)\right\} \\
&= -\varphi+\nu \\
&= \frac{1}{\left[\rho+(1-\rho)\left(\frac{y_H-\frac{x}{\gamma}}{y_H-y_L}\right)\right]^{1-\sigma}}\times \\
&\left[\rho\ell^B+\ell^B\left(\frac{1-\rho}{y_H-y_L}\right)y_H-\left(\ell^B\right)^2\left(\frac{1-\rho}{y_H-y_L}\right)\frac{1}{\gamma}\right]^{-\sigma}\times \\
&\left[\rho+\left(\frac{1-\rho}{y_H-y_L}\right)y_H-\ell^B\left(\frac{1-\rho}{y_H-y_L}\right)\frac{2}{\gamma}\right] \\
&+\frac{1}{\rho+(1-\rho)\left(\frac{y_H-\frac{x}{\gamma}}{y_H-y_L}\right)}\times \\
&\left\{\frac{\beta}{(1-\sigma)}\frac{1-\rho}{y_H-y_L}\left[\left(\frac{1-\gamma}{\gamma}\right)^{1-\sigma}\left(\ell^B\right)^{1-\sigma}-\left(y_H-\ell^B\right)^{1-\sigma}\right]\right. \\
&\left.-\beta\rho\left(y_H-\ell^B\right)^{-\sigma}\right\} \\
&= -\varphi+\nu
\end{aligned}
$$

In the following expression I explore the conditions under which the first constraint is slack but the second binds with equality: $\ell^B = \gamma y_H$. That is, conditions under which $\varphi = 0$ and $\nu > 0$ at $\ell^B = \gamma y_H$. Imposing these, the condition collapses to

$$\left[\rho + (1-\rho)\left(\frac{y_H - \frac{x}{\gamma}}{y_H - y_L}\right)\right]^{\sigma} \times \tag{B.2.2}$$
$$\rho^{-\sigma}\left[1 - \left(\frac{1-\rho}{\rho}\right)\left(\frac{y_H}{y_H - y_L}\right)\right] \;>\; \beta\left(\frac{1}{\gamma} - 1\right)^{-\sigma}$$

2. Conversely, if Borrower A defaults, then his action will reveal $y < \frac{x}{\gamma}$. The problem of Borrower B is:

$$\begin{aligned}
&\max_{\ell^B \in [\gamma y_L, x)} \; u\left(\Pr\left(y \geq \frac{\ell^B}{\gamma}\right)\ell^B\right) \\
&\quad +\beta\left\{\Pr\left(y \geq \frac{\ell^B}{\gamma}\right) E\left[u\left(y - \ell^B\right) \mid y \geq \frac{\ell^B}{\gamma}, \Im^B\right] + \right. \\
&\quad \left.\left[\Pr\left(y < \frac{\ell^B}{\gamma}\right)\right] E\left[u\left(y - \gamma y\right) \mid y < \frac{\ell^B}{\gamma}, \Im^B\right]\right\} \\
= &\max_{\ell^B \in [\gamma y_L, x)} \; u\left(\frac{\frac{x}{\gamma} - \frac{\ell^B}{\gamma}}{\frac{x}{\gamma} - y_L}\ell^B\right) + \\
&\quad \frac{\beta}{\frac{x}{\gamma} - y_L}\left\{\int_{\frac{\ell^B}{\gamma}}^{\frac{x}{\gamma}} u\left(y - \ell^B\right) dy + \int_{y_L}^{\frac{\ell^B}{\gamma}} u\left(y\left(1-\gamma\right)\right) dy\right\}
\end{aligned}$$

Using (2.2.1):

$$\begin{aligned}
= &\max_{\ell^B \in [\gamma y_L, x)} \; \frac{1}{1-\sigma}\left(\frac{\ell^B \frac{x}{\gamma} - \frac{1}{\gamma}\left(\ell^B\right)^2}{\frac{x}{\gamma} - y_L}\right)^{1-\sigma} \\
&\quad + \frac{\beta}{\left(\frac{x}{\gamma} - y_L\right)(1-\sigma)(2-\sigma)}\left(\frac{x}{\gamma} - \ell^B\right)^{2-\sigma} \\
&\quad - \frac{\beta}{\left(\frac{x}{\gamma} - y_L\right)(1-\sigma)(2-\sigma)}\left(\frac{1-\gamma}{\gamma}\right)^{2-\sigma}\left(\ell^B\right)^{2-\sigma} \\
&\quad + \frac{\beta\left(1-\gamma\right)^{1-\sigma}}{\left(\frac{x}{\gamma} - y_L\right)(1-\sigma)(2-\sigma)}\left(\frac{\ell^B}{\gamma}\right)^{2-\sigma} \\
&\quad - \frac{\beta\left(1-\gamma\right)^{1-\sigma}}{\left(\frac{x}{\gamma} - y_L\right)(1-\sigma)(2-\sigma)} y_L^{2-\sigma}
\end{aligned}$$

and the first order condition is:

$$\begin{aligned}
\nabla\left(\ell^{B,R,d^A=1}\right) &= \left[\frac{\ell^B\frac{x}{\gamma}-\frac{1}{\gamma}\left(\ell^B\right)^2}{\frac{x}{\gamma}-y_L}\right]^{-\sigma}\left(\frac{x}{\gamma}-\frac{2}{\gamma}\ell^B\right) \\
&\quad -\frac{\beta}{(1-\sigma)}\left(\frac{x}{\gamma}-\ell^B\right)^{1-\sigma} \\
&\quad +\frac{\beta}{(1-\sigma)}\left(\frac{1-\gamma}{\gamma}\right)^{1-\sigma}\left(\ell^B\right)^{1-\sigma}=0.
\end{aligned}$$