STRATEGIC TRADE POLICY AND RETALIATION:
Export Subsidies and Countervailing Tariffs

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Abstract

This thesis analyses the effect of retaliation with countervailing tariffs and/or production subsidies on the strategic argument for export subsidies, and also proves the existence and uniqueness of equilibrium in the standard model of international trade under oligopoly. Retaliation will be modelled as a multistage game. At the first stage, the foreign country sets its export subsidy to maximise national welfare. Then, at the second stage, the domestic country sets its trade policy, import tariff and/or production subsidy, to maximise national welfare in response to the foreign export subsidy. The solution concept employed is the subgame perfect equilibrium. When the domestic country uses a tariff in response to a foreign export subsidy, then the optimal domestic response is a partially countervailing tariff, and the foreign country does not usually gain from an export subsidy. There is usually no profit shifting argument for an export subsidy when the foreign country faces retaliation with countervailing tariffs. When the domestic country uses a tariff and a production subsidy in response to a foreign export subsidy, then the surprising result is that an export subsidy may increase foreign welfare. In this case, the foreign export subsidy increases both foreign and domestic welfare. The domestic country will always gain from a foreign export subsidy when it sets its trade policy optimally.
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"Bounties upon exportation are, in Great Britain, frequently petitioned for, and sometimes granted to the produce of particular branches of domestic industry. By means of them our merchants and manufacturers, it is pretended, will be enabled to sell their goods as cheap, or cheaper than their rivals in the foreign market. A greater quantity, it is said, will thus be exported, and the balance of trade consequently turned more in favour of our own country. We cannot give our workmen a monopoly in the foreign as we have in the home market. We cannot force foreigners to buy their goods as we have done to our own countrymen. The next best expedient, it has been thought, therefore, is to pay them for buying. It is in this manner that the mercantile system proposes to enrich the whole country" Adam Smith (1776), An Inquiry into the Nature and Causes of the Wealth of Nations, Book IV, Chapter V, Paragraph 1.

Introduction

This thesis analyses the effect of retaliation with countervailing tariffs and/or production subsidies on the strategic argument for export subsidies, and also proves the existence and uniqueness of equilibrium in the standard model of international trade under oligopoly. This chapter provides an introduction to the problem of subsidies and countervailing duties in international trade.

Since the Second World War, a series of GATT multilateral trade negotiations have achieved dramatic tariff reductions. The average tariff in the United States has been reduced by 92%. And, following the implementation of the Tokyo Round of tariff reductions, the average tariff is 4.9% in the United
States, 6.0% in the European Community, and 5.4% in Japan. However, a number of important sectors such as agriculture, textiles and services have been unaffected by trade liberalisation, although these sectors are now being considered in the Uruguay Round of trade negotiations. As industrial tariffs have been reduced, non-tariff barriers and subsidies have become more significant. Industrial subsidies are widespread in the OECD, and the average subsidy rate is about 1.5% of GDP. Subsidies are highest in declining industries such as steel and shipbuilding, but subsidies are also being used to support new industries such as microelectronics. The effect of subsidies is to distort international trade, and to harm the domestic industry in other countries. Consequently, subsidies and the appropriate response to foreign subsidies are important questions in trade policy.

The General Agreement on Tariffs and Trade (GATT) was drawn up in 1947 to provide a liberal framework for international trade. Article XVI of the GATT prohibits the use of export subsidies, and Article VI allows an importing country to impose countervailing duties on subsidised products. However, these rules were not sufficient to prevent

1 Bhagwati (1988), page 3.
2 OECD (1990), page 12.
3 The relevant GATT rules on subsidies and countervailing duties are detailed in Beseler and Williams (1986).
subsidies and countervailing duties becoming a serious problem in international trade. One problem was the trade distorting effects of domestic subsidies such as subsidies for depressed regions and for research and development. In the Tokyo Round of trade negotiations from 1973 to 1979, subsidies and countervailing duties were a controversial issue, and these negotiations led to the GATT Code on Subsidies and Countervailing Duties. This code confirms that countries shall not grant export subsidies on products other than certain primary products. And, that they will seek to avoid causing injury to the domestic industry in other countries through the use of domestic subsidies.

Under the GATT Code, a country which is adversely effected by a foreign subsidy may take action using Track I and/or Track II. Track I allows an importing country to apply countervailing duties if there is material injury to the domestic industry. Definitive countervailing duties may only be applied following a formal investigation by the importing country. The countervailing duty may not exceed the amount of the subsidy, and should be less than the amount of the subsidy if this is adequate to remove the injury to the domestic industry. A countervailing duty shall not be applied for longer than is necessary to counter the damage caused to the domestic industry by the subsidy. Track II allows a country to apply for authorisation from the GATT to apply countermeasures when a foreign subsidy causes injury
to its domestic industry. These countermeasures may include the withdrawal of GATT concessions. This procedure can be used when a foreign subsidy injures the domestic industry through its effect on a third market. Both the United States and the European Community have enacted countervailing duty laws in line with the GATT Code. 4

During the 1980's, there has been extensive use of countervailing duties in response to foreign subsidies, particularly by the United States. According to Finger and Nogués (1987), 425 countervailing duty cases were initiated by GATT signatories between 1980 and 1985. The United States initiated 252 cases, Chile initiated 135 cases, but the European Community initiated only seven cases. Details of the US countervailing duty cases are contained in Destler (1986), and details of EC cases are contained in Beseler and Williams (1986). A large number of the US countervailing duty cases resulted from complaints by the US steel industry in 1982 about the subsidisation of the EC steel industry. 5

The US Commerce Department determined that the EC industry was being subsidised, and calculated subsidy margins of up to 20-30% for UK, French and Spanish imports. In this case


5 For details of protectionism in the world steel industry, including the countervailing duty cases, see Jones (1986).
rather than face countervailing duties, the European Community agreed to a "voluntary" export restraint (VER) with the United States. There is a widespread belief that the threat of countervailing duties was misused by the United States to bring about a politically expedient VER.

In the United States there has recently been great concern about the "unfair" trade practices used by other countries, and about how the US should respond. It has become the orthodox view in the United States that its industries are suffering from the unfair practices of foreign governments. There is concern about foreign government intervention in key manufacturing industries such as aircraft, steel and semiconductors. In the aircraft industry, Airbus Industrie is alleged to have received large subsidies from European governments, and now challenges the dominance of US producers such as Boeing. In the semiconductor industry, it is argued that the Japanese Government have used industrial targeting and a closed domestic market to give its firms a competitive advantage in the DRAM (dynamic random access memory) chip market at the expense of US firms. It is argued in the United States that foreign governments have used unfair trade practices to take advantage of liberal US trade policies, and there is dissatisfaction about the ability of GATT rules to eliminate these practices. A more aggressive

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6 For details of foreign targeting see Krugman (1984b).
US response to unfair foreign trade practices has been suggested by Goldstein and Krasner (1984). They proposed that the US should adopt a strategy of tit for tat retaliation against unfair trade practices, and that the US should develop its own industrial policy. The Omnibus Trade and Competitiveness Act of 1988 allows the United States to retaliate against unfair foreign trade practices.

This discussion suggests a number of questions about foreign export subsidies and about how the domestic country should respond to such subsidies:

(1) Will a foreign export subsidy harm the domestic country if it pursues a policy of laissez-faire? This question is obviously relevant to the United States where it is widely believed that the US economy has been harmed by foreign subsidies due to its liberal trade policy.

(2) How should the domestic country respond to a foreign export subsidy? This question is related to countervailing duty law and the optimal response to foreign subsidies. Article VI of the GATT allows fully countervailing duties, but is such a response optimal?

(3) What is the optimal foreign export subsidy when the foreign country is faced with retaliation? When the foreign country realises that the domestic country will retaliate,
will it be deterred from using an export subsidy?

Export subsidies and countervailing duties could be analysed in a perfectly competitive model of international trade. A foreign export subsidy will lower the price of the product, and hence reduce the output of the domestic industry. Resources displaced by the contraction of the domestic industry will be employed in other sectors of the economy, where they will produce output of equal value to the country. Therefore, the contraction of the domestic industry will not reduce domestic welfare. The price reduction will reduce the profits of the domestic industry, but it will increase consumer surplus. The gain to consumers will exceed the loss to producers, and the net effect is a terms of trade gain for the domestic country. Therefore, the domestic country should welcome the foreign export subsidy, and the optimal domestic response is to send a note of thanks. If the domestic country applied countervailing duties then it would reduce the welfare gains from the foreign export subsidy, and so a countervailing duty is not optimal. For the foreign country, an export subsidy will worsen its terms of trade, and reduce its welfare. An export subsidy is never optimal for the foreign country. If the domestic country applied fully countervailing tariffs then the net effect of the export subsidy is to transfer income from the foreign to the domestic country, and the foreign country will obviously be worse off. Therefore, in a perfectly competitive model,
Recent, it is difficult to explain the use of export subsidies and countervailing duties.

Recently, trade theorists have realised that many industries are not perfectly competitive, and have started to model trade and trade policy under imperfect competition. An important feature of these models is that price exceeds marginal cost, and firms may earn pure profits. These pure profits earned by a country's firms add to its national income, and a country may gain from profit shifting trade policies. In a Cournot duopoly model, with one foreign and one domestic firm, Brander and Spencer (1985) have shown that the foreign country can use an export subsidy to shift profits from domestic to foreign firms, and hence to increase its welfare. The export subsidy will reduce the profits of the domestic industry, and it may reduce the welfare of the domestic country. Dixit (1988) has shown that the optimal domestic response to a foreign export subsidy may be countervailing duties. Therefore, models of trade policy under imperfect competition may provide a more satisfactory analysis of export subsidies and countervailing duties than perfectly competitive models.

Many trade theorists have argued, without presenting any formal model, that any attempt to use profit shifting trade policies is likely to provoke retaliation by other
countries. For example, if the foreign country uses an export subsidy then the domestic country is likely to retaliate with countervailing duties. And, it is argued that retaliation is likely to leave the country which first uses profit shifting trade policies worse off. However, Brander (1986) has argued that it is naive to believe that the possibility of retaliation undercuts any case for retaliation. Therefore, there is a need for a formal model to assess the effect of retaliation on the profit shifting argument for export subsidies.

This thesis will analyse export subsidies and countervailing duties using game theoretic models of trade policy under Cournot oligopoly. Trade policy will be modelled as a multistage game. At the first stage, the foreign country will set its export subsidy to maximise its national welfare. Then, at the second stage, the domestic country will respond by setting its import tariff and/or production subsidy to maximise its national welfare. Finally, at the third stage, the domestic and foreign firms will set their outputs to maximise profits as under Cournot oligopoly. The relevant solution is the subgame perfect equilibrium. This ensures that any threats of retaliation by the domestic country must be credible, and that the foreign country correctly anticipates the response of the domestic country.

Using a multistage game in which the foreign country has a first mover advantage appears to be the most appropriate method to model export subsidies and countervailing duties. Under GATT rules the domestic country can only apply definitive countervailing duties after a formal investigation has established foreign subsidisation, and hence a country which is considering the use of an export subsidy clearly has a first mover advantage. The alternative is to model trade policy as a simultaneous move game as in Johnson (1953-54), but this does not really capture the essence of retaliation. Since if both governments act simultaneously then the domestic country cannot really react to the foreign export subsidy. Therefore, modelling trade policy as a multistage game in which the foreign country has a first mover advantage appears to be the most satisfactory approach.

Chapter 1 surveys the literature on the new international economics and strategic trade policy. It focuses on trade and trade policy under oligopoly, and in particular on the profit shifting argument for export subsidies. Also, it is shown that Dixit (1988) incorrectly derives the optimal tariff in the case when domestic production is uneconomic, and this error is corrected. Chapter 2 analyses the effect of retaliation on the profit shifting argument for export subsidies in a model with linear demand and product differentiation. The optimal domestic response to a foreign
export subsidy under linear demand and product differentiation has been derived by Dixit (1988), and this chapter extends his analysis by considering the optimal foreign export subsidy when faced with retaliation. Chapter 3 proves the existence and uniqueness of equilibrium in the Dixit (1984) homogeneous product Cournot oligopoly model of international trade. Existence is proved without the usual assumption that profit functions are concave. And, the comparative static results for the effects of trade policy are derived. Chapter 4 uses the basic model of chapter 3 to analyse the effect of retaliation on the profit shifting argument for export subsidies. This extends the analysis of Dixit (1988) and chapter 2 by deriving the optimal domestic response to a foreign export subsidy when demand is non-linear. And, then uses these results to derive the optimal foreign export subsidy when faced with retaliation. Chapter 5 analyses the effect of retaliation in a model where the foreign country can use an export subsidy to deter the entry of domestic firms. The optimal domestic response to a foreign export subsidy is derived, and then the optimal foreign export subsidy when faced with retaliation is obtained. Chapter 6 is the conclusion.
1.1 Introduction

This chapter surveys the literature on the New International Economics, which incorporates imperfect competition into models of international trade, and Strategic Trade Policy, which analyses trade policy under imperfect competition. This literature has provided an explanation for intra-industry trade, identified additional sources of gains from trade, and led to a new rationale for trade policy.

The conventional theory of international trade assumes that firms are perfectly competitive and that there are constant returns to scale. Then, international trade is explained by the principle of comparative advantage. Trade between countries occurs because of differences in technology in the Ricardian model, and because of differences in factor endowments in the Heckscher-Ohlin-Samuelson model. An important result in conventional trade theory is that the countries will undoubtedly gain from trade. Although, trade theorists did realise that economies of scale, product differentiation and imperfect competition were probably important factors in international trade,¹ these factors were not incorporated into formal models until the papers of

¹For example Johnson (1967) realises the need to incorporate monopolistic competition into models of international trade.
Krugman (1979), Dixit and Norman (1980), and Lancaster (1980). They showed how monopolistic competition could explain intra-industry trade between similar countries, which could not be explained by comparative advantage. And, it was shown that economies of scale and product variety provided additional sources of gains from trade. Brander (1981) used a Cournot oligopoly model to explain intra-industry trade in identical products, and showed that the pro-competitive effect of trade provided another source of gains from trade. These models allow trade theory to explain intra-industry trade, and strengthen the view that there are gains from trade.

In conventional trade theory the first-best optimum policy for a small country is free trade, and this is true even if foreign competitors protect or subsidise their industries. Only for a large country, able to influence its terms of trade, is a tariff the first-best optimum policy. Whereas, in models with imperfect competition a country may benefit from strategic trade policy. The explanation is that with imperfect competition there may be pure profits, and a country may gain from profit-shifting trade policy. In a Cournot duopoly model, Brander and Spencer (1985) show that a country can use an export subsidy to give its firm a strategic advantage, and thereby allow it to capture a larger share of industry profits. Strategic trade policy challenges the conclusion of the conventional trade theory
that free trade is the optimal policy. This has led to numerous criticisms of this new strategic rationale for trade policy.


In order to make the task manageable this survey will concentrate on models of international trade under oligopoly. A simple duopoly model will be developed which will be used to illustrate the basic ideas throughout the chapter. Section 1.2 discusses intra-industry trade in identical products and the gains from trade. The profit shifting argument for export subsidies, and the numerous criticisms, are considered in section 1.3. The optimal trade policy for the importing country is discussed, and an error in Dixit (1988) corrected, in section 1.4. The conclusions are in section 1.5.
1.2 Intra-Industry Trade and the Gains from Trade

Intra-industry trade is usually explained by assuming that such trade occurs in slightly differentiated products, if consumers demand variety then there will be two way trade.\(^2\) Brander (1981) showed that intra-industry trade in identical products may occur under Cournot oligopoly. The crucial assumption is that markets are segmented. The usual assumption in international trade is that markets are integrated, arbitrage leads to the same price in each country. Then, each firm views the world as one market, and its strategic variable is the the total quantity to produce for the world market. With segmented markets there is no arbitrage, therefore price differences can occur between countries, and firms view each country as a separate market. The firms' strategic variables are the quantities to be produced for each market.

A simple Cournot duopoly model will demonstrate the possibility of intra-industry trade. There are two identical countries, the domestic and the foreign country, with a single firm in each country. The domestic and foreign firm both have constant marginal cost \(c\), and the cost of transporting the product from one country to another is \(k\).

\(^2\)Intra-industry trade and the gains from trade under monopolistic competition are analysed by Helpman (1981), Krugman (1979, 1980, 1981) and Lancaster (1980).
The domestic firm sells output \( Y \) in the domestic market, and exports output \( Y^* \) to the foreign market. The foreign firm exports output \( X \) to the domestic market, and sells output \( X^* \) in the foreign market. Demand in the domestic market is \( P = P(Q) \), where \( Q = X+Y \), and demand in the foreign market is \( P = P(Q^*) \), where \( Q^* = X^*+Y^* \). Hence, the profits of the domestic and foreign firms are, respectively

\[
\pi_1 = (P(X+Y) - c)Y + (P(X^*+Y^*) - c - k)Y^*
\]

\[
\pi_2 = (P(X+Y) - c - k)X + (P(X^*+Y^*) - c)X^*
\]

(1.1)

The first term is profits from the domestic market, and the second term is profits from the foreign market. With the assumption of segmented markets, the strategic variables for the domestic firm are \( Y \) and \( Y^* \), and for the foreign firm \( X \) and \( X^* \). Assuming each firm's profit function is quasi-concave in its strategic variables, the first order conditions for a Nash equilibrium are

\[
\frac{\partial \pi_1}{\partial Y} = P(X+Y) + YP'(X+Y) - c \leq 0, \quad Y \geq 0, \quad \frac{\partial \pi_1}{\partial Y} Y = 0
\]

(1.2)

\[
\frac{\partial \pi_2}{\partial X} = P(X+Y) + XP'(X+Y) - c - k \leq 0, \quad X \geq 0, \quad \frac{\partial \pi_2}{\partial X} X = 0
\]

\[^3\text{Little attention has been paid to the question of existence and uniqueness of Cournot equilibrium in models of international trade. A proof of existence and uniqueness is presented in chapter 3 of this thesis.}\]
Note that the equilibrium quantities in the domestic market, \( Y \) and \( X \) given by (1.2), are independent of the equilibrium quantities in the foreign market, \( Y^* \) and \( X^* \) given by (1.3). For both the domestic and foreign firm, profits in the domestic market are independent of profits in the foreign market. The two firms are engaged in two independent games, one in the domestic market and one in the foreign market, which can be analysed separately. This is due to the assumption of segmented markets, and the assumption that marginal cost is constant.

Foreign exports to the domestic market will be positive if foreign cost, including the transport cost, is less than the domestic firm’s monopoly price. Then, the market will be supplied by imports and domestic production. With Cournot competition, high cost firms are not driven out of the market by low cost firms. As Cournot (1838) noted the firm with the higher marginal cost will have a smaller market share, its perceived elasticity of demand is higher and hence its price-cost margin is lower. Since the model is symmetric, the foreign market will be supplied by both
foreign and domestic firms. Hence, there is intra-industry trade, the foreign firm exports to the domestic market and the domestic firm exports to the foreign market. Trade occurs despite the fact that it is costly, due to the transport cost, and that neither country has a comparative advantage.

The welfare effects of intra-industry trade have been analysed by Brander and Krugman (1983). They also extend the analysis of intra-industry trade to arbitrary demand functions rather than the linear demand functions considered by Brander (1981). Demand is assumed to be derived from a utility function which is additively separable and linear in a competitive numeraire good, then consumer surplus is a valid welfare measure. Welfare is given by the sum of consumer and producer surplus. Firstly, consider the welfare effects of a multilateral move to free trade, where both the domestic and foreign market are opened up to trade. Since the model is symmetric, consider the domestic market. The effects are shown in figure 1.1. The domestic price is reduced from the monopoly price under autarky, $P^A$, to the duopoly price under free trade, $P^T$, the sales of the domestic firm are reduced from $Y^A$ to $Y^T$, and imports are $X^T$. Trade has a pro-competitive effect, there is an increase in

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4The welfare effects of intra-industry trade in a common market are analysed by Neven and Phlips (1985) in a model with linear demand functions.
consumer surplus due to the lower price which is a welfare gain. There is also a welfare gain from the profits that the domestic firm receives on its exports to the foreign market. By symmetry, this is equal to the foreign firm's profits on its exports to the domestic market. But, there is a welfare loss due to the sales of the domestic firm being displaced by imports which incur a transport cost. With positive transport costs the overall welfare effect is ambiguous, and clearly there could be losses from trade. Obviously, with zero transport costs there is an unambiguous welfare gain.

Now consider the welfare effects of a unilateral move to free trade, the domestic market for the oligopolistic product is opened up but not the foreign market.\(^5\) The effects are shown in figure 1.2. The domestic price is reduced from \(P^A\) to \(P^\ddagger\), domestic output from \(Y^A\) to \(Y^\ddagger\), and imports are \(X^\ddagger\). There is a welfare gain due to the increase in consumer surplus, and a welfare loss due to the reduction in profits of the domestic firm because its output decreases. Whereas, in a multilateral move to free trade the loss of profits from the domestic market was offset by the profits from exports to the foreign market. The overall welfare effect is ambiguous, even when transport costs are zero, and may clearly be negative. Hence, there are more

\(^5\)Since trade must be balanced, the domestic country imports the product of the oligopolistic industry, and exports the competitive numeraire good.
likely to be gains from a multilateral move to free trade than a unilateral move.

With Bertrand competition there is no intra-industry trade. Since, in the domestic market, the domestic firm would undercut the foreign firm by setting a price just below the foreign firm's cost, \( c + k - c \). Then, the domestic firm will capture the entire market. Similarly, the foreign firm will supply the entire foreign market. There is no intra-industry trade, but there are gains from trade. The threat of imports has a pro-competitive effect on the domestic firm. Under autarky the domestic firm sets the monopoly price, but faced with competition it reduces its price to just below the costs of the foreign firm. Thus, there is a welfare gain due to the increase in consumer surplus. There is no welfare loss due to wasteful transport costs, since no trade actually occurs. Hence, the overall welfare effect is positive.

Brander and Krugman (1983) and Venables (1985a) have considered intra-industry trade under Cournot oligopoly with free entry. This leads to a clear result about the gains from trade, even with transport costs. With free entry profits are zero in equilibrium, so domestic welfare is given by consumer surplus. Hence, if trade results in a lower domestic price then it will increase domestic welfare. Consider a multilateral move to free trade, the domestic and
foreign market are opened up to trade, and domestic firms have the opportunity to export to the foreign market. Domestic firms will export to the foreign market if it is profitable. Suppose the domestic price did not fall, then profits from the domestic market are the same as under autarky, zero with free entry, but profits from exports are positive, and so domestic firms make positive profits overall. Hence, with free entry, the domestic price must fall until profits are zero in equilibrium. There are clearly gains from trade. But, note that the gains from trade come from the opportunity of domestic firms to export, and not from the effect of imports on the domestic market.

Hwang (1984) has modelled intra-industry trade in a conjectural variations model of oligopoly. It is claimed that the amount of intra-industry trade increases with the degree of collusion, as measured by the conjectural variation parameter. Obviously, with Cournot oligopoly there is more intra-industry trade than with Bertrand oligopoly, where there is none. But, if firms are colluding to maximise joint profits, they would surely not engage in the costly cross-hauling of products. Hwang (1984) clearly demonstrates the inadequacies of using the conjectural variation approach, which attempts to model dynamic oligopoly in a static setting. A better approach to the modelling of dynamic oligopoly is to use a repeated game model. This method has been used by Pinto (1986) to analyse the Brander
and Krugman (1983) model in a dynamic setting. First, Pinto shows that the joint profit maximising solution involves no trade, and then that it can be supported as a perfect equilibrium in an infinitely repeated game, by the threat of reversion to the one-shot Nash equilibrium, if the discount rate is not too large. With implicit collusion firms refrain from exporting to their competitor’s market and there is no intra-industry trade.

Recently, the assumption of segmented markets has been questioned by Ben-Zvi and Helpman (1988) and Venables (1988). They argue that it is unrealistic to model firms as choosing quantities to be sold in each market. Instead, they suggest a multistage game, an extension of Kreps and Scheinkman (1983), in which worldwide capacity is chosen at the first stage, and then the prices for each market are chosen at the second stage. Ben-Zvi and Helpman consider the case of homogeneous products, and show that in equilibrium there is no intra-industry trade. As transport costs approach zero, the equilibrium approaches the Cournot outcome for integrated markets. Venables (1988) assumes that products are differentiated to avoid the discontinuities which occur in capacity constrained price games with homogeneous products. He shows that the volume of trade lies between that which would occur with integrated markets, and

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6Ben-Zvi and Helpman have a third stage where sales are allocated between markets.
that which would occur with segmented markets. Ben-Zvi and Helpman argue that the results of their model provide a firm basis for the models of trade under oligopoly with integrated markets developed in Helpman and Krugman (1985, chapter 5). Then, a sufficient condition for gains from trade is that, on average, the output of the oligopolistic industries increases.\(^7\)

Shaked and Sutton (1984) consider the welfare effects of trade in a model of natural oligopoly. Competition between firms is modelled as a multistage game. At the first stage firms decide whether or not to enter, then in the second stage firms decide the quality of good to produce, and in the final stage decide the price given the quality choices of other firms. An important property of natural oligopoly models, provided unit variable costs do not rise steeply with quality, is that there is a limit to the number of firms in equilibrium, and firms that enter can earn positive profits even with free entry. Thus, the model provides an explanation for the existence of pure profits. The effect of opening two countries to trade is to lower prices to consumers. In the short run this is due to increased competition, and in the long run due to the exit of some firms, which enhances the economies of scale of the remaining firms. Thus, consumers are obviously better off as

\(^7\)Also, see Markusen (1981) for the gains from trade under Cournot oligopoly with integrated markets.
a result of trade. Whether a country gains from trade will depend also on the effect on profits, and this is not considered by Shaked and Sutton (1984) since it is unknown which firms will exit the industry. If all the firms which exit were located in one country, then that country may not gain from trade due to the loss of profits.

The existence of intra-industry trade in identical products can be explained in a Cournot oligopoly model, but the assumption that markets are segmented is crucial. Without this assumption intra-industry trade can only be explained by product differentiation. Intra-industry trade has a pro-competitive effect which reduces prices, and increases consumer surplus. However, it may have a negative effect on welfare if the output of the oligopolistic industry contracts, and profits are reduced. With free entry there are unambiguous gains form trade.

1.3 Strategic Export Subsidies

The models of intra-industry trade discussed in the previous section have been used to analyse the effects of trade policy. Compared to conventional trade theory, imperfect competition introduces some new aspects into trade policy, the most important is the presence of profits. When a country's firms earn pure profits this adds to the country's real income. Brander and Spencer (1985) have suggested that
a country may gain from profit shifting export subsidies. This possibility is the most significant result in the strategic trade policy literature, and has attracted a great deal of discussion. This section will consider the profit shifting argument for export subsidies, and the numerous criticisms of this new strategic rationale for trade policy.

The basic idea is best illustrated using the Cournot duopoly model of the previous section. Now let the domestic and foreign marginal cost differ. The domestic firm has constant marginal cost $c_1$, and the foreign firm has constant marginal cost $c_2$. The foreign export subsidy is $e$ per unit exported. It is assumed that the foreign government can commit itself to an export subsidy before the firms make their output decisions. Above, it was shown that equilibrium in the domestic market is independent of equilibrium in the foreign market, so the domestic market can be analysed separately.

The profits of the domestic and foreign firm from sales in the domestic market are, respectively

\[ \pi_1 = (P - c_1)Y \]

\[ \pi_2 = (P - c_2 + e)X \]

(1.4)

Assuming that each firm's profit function is quasi-concave in its own output, and that there is an interior equilibrium, the first order conditions for a Cournot
equilibrium are

\[ \frac{\partial \pi_1}{\partial Y} = P(X+Y) + YP'(X+Y) - c_1 = 0 \]

\[ \frac{\partial \pi_2}{\partial X} = P(X+Y) + XP'(X+Y) - c_2 + e = 0 \]

And, the second order conditions for profit maximisation are

\[ \frac{\partial^2 \pi_1}{\partial Y^2} = 2P' + YP'' < 0 \quad \frac{\partial^2 \pi_2}{\partial X^2} = 2P' + XP'' < 0 \]

The first equation in (1.5) defines the domestic reaction function \( Y = Y(X) \), and the second equation defines the foreign reaction function \( X = X(Y) \). Together they define the Cournot equilibrium, which is given by the intersection of the domestic and foreign reaction functions in figure 1.3. The comparative static results for the effects of a foreign export subsidy are obtained by totally differentiating (1.5), which yields

\[
\begin{bmatrix}
2P' + YP'' & P' + YP'' \\
P' + XP'' & 2P' + XP''
\end{bmatrix}
\begin{bmatrix}
dY \\
dX
\end{bmatrix}
= \begin{bmatrix}
0 \\
-d\mu 
\end{bmatrix}
\]

Hence, the effects of an export subsidy are

\[ \frac{\partial X}{\partial \epsilon} = \frac{(2P' + YP'')}{\Delta} > 0 \quad \frac{\partial Y}{\partial \epsilon} = \frac{(P' + YP'')}{\Delta} \quad (1.6) \]
Where $\Delta = P'(3P' + QP'') > 0$, which is positive for a unique Cournot equilibrium. The effect of the export subsidy is to increase foreign exports, but the effect on domestic production is unclear. When domestic output and foreign exports are strategic substitutes for the domestic country, $P' + YP'' < 0$, the domestic reaction function is downward sloping, and a foreign export subsidy reduces domestic production.\(^8\) In figure 1.3 the foreign reaction function is shifted outwards by a foreign export subsidy, and the equilibrium shifts from A to B. This is considered to be the normal case, and is implied by the Hahn (1961-62) stability condition. However, when the domestic reaction function is upward sloping, domestic output and foreign exports are strategic complements, and an export subsidy will increase domestic production. The effect of an export subsidy on price is

$$\frac{\partial P}{\partial e} = P' \frac{\partial X}{\partial e} + P' \frac{\partial Y}{\partial e} = \frac{-(P')^2}{\Delta} < 0 \tag{1.7}$$

As expected, an export subsidy reduces the price in the domestic market.

Consider the effect of a foreign export subsidy on foreign welfare. Since equilibrium in the foreign market is

\(^8\)Strategic substitutes and complements are explained in Bulow et al (1985) and Tirole (1988).
independent of equilibrium in the domestic market, an export subsidy has no effect on the foreign market. Hence, foreign welfare is the profits of the foreign firm from exports net of the export subsidy

$$W_2 = (P - c_2)X \quad (1.8)$$

The effect of an export subsidy on foreign welfare is

$$\frac{\delta W_2}{\delta e} = (P - c_2) \frac{\delta X}{\delta e} + X \frac{\delta P}{\delta e} \quad (1.9)$$

The first term is the profit shifting effect of the export subsidy: An export subsidy increases foreign exports, which is a welfare gain since price exceeds marginal cost. The second term is the terms of trade effect: The export subsidy reduces the price the foreign firm receives for its exports which is a welfare loss. In conventional trade theory, price would equal marginal cost so the profit shifting effect would be zero, and the only effect of an export subsidy would be the negative terms of trade effect.⁹ To evaluate the overall effect of the export subsidy on foreign welfare substitute (1.7) into (1.9) which yields

⁹In the two good model of conventional trade theory, an export subsidy will never increase welfare, but with many goods it is possible that an export subsidy will increase welfare. See Feenstra (1986) and Itoh and Kiyono (1987).
At \( e = 0, \ P + XP' - c^2 = 0 \) from (1.5). Hence, the overall effect of the export subsidy evaluated at \( e = 0 \) is

\[
\frac{\partial W^2}{\partial e} = (P + XP' - c^2) \frac{\partial X}{\partial e} + XP' \frac{\partial Y}{\partial e}
\]

This clearly shows that the welfare effect of the export subsidy comes from the strategic effect it has on the output of the domestic firm. With downward sloping reaction functions, an export subsidy will reduce domestic output, see (1.6), and increase foreign welfare. This is the case considered by Brander and Spencer (1985). An export subsidy will increase foreign welfare in a Cournot duopoly if domestic output and foreign exports are strategic substitutes. However, as Collie and de Meza (1986) have shown, if domestic output and foreign exports are strategic complements then an export subsidy will increase domestic output and reduce foreign welfare. This can occur with quite plausible demand functions.\(^{10}\)

The optimal export subsidy is obtained by setting \( \frac{\partial W^2}{\partial e} = 0 \) and solving for \( e \), which yields

\(^{10}\)For example in a symmetric Cournot duopoly with constant elasticity demand functions, domestic output and foreign exports will be strategic complements if the elasticity of demand is less than unity.
With strategic substitutes (complements) the optimal policy is an export subsidy (tax). With an optimal export subsidy the equilibrium is the same as would occur if the foreign firm acted as a Stackelberg leader, and the domestic firm acted as a Stackelberg follower, in the absence of an export subsidy. This is shown in figure 1.4. Foreign welfare is represented by the iso-profit curves of the zero export subsidy foreign reaction function, since these represent profits net of the export subsidy. Welfare is maximised at B where the iso-profit curve is tangential to the domestic reaction function, which is the familiar Stackelberg equilibrium. The optimal export subsidy commits the foreign firm to produce the Stackelberg leader output, and the optimal response of the domestic firm is to produce the Stackelberg follower output. The strategic effect of the export subsidy comes from the ability of the foreign government to commit itself to a policy before the firms make their output decisions, it has a first mover advantage.

When the foreign country cannot use export subsidies, Spencer and Brander (1983) show that R&D subsidies can have the same strategic effect. A subsidy to R&D commits the foreign firm to increase expenditure on R&D which reduces marginal cost, and this commits the foreign firm to produce

\( e = \frac{-XP'(P' + YP'')}{{2P' + YP''}} \)  

(1.11)
When the domestic and foreign firm both export to a third market, and domestic and foreign exports are strategic substitutes, then both the domestic and foreign government have an incentive to subsidise exports. In a Nash equilibrium in trade policies both governments subsidise exports, and de Meza (1986) has shown that the country with the lowest cost will have the largest subsidy.\textsuperscript{11} Both countries are likely to be worse off than if they both had zero export subsidies, since the strategic effect of one country's export subsidy will be offset by the effect of the other country's subsidy. But, both subsidies will worsen the terms of trade of both countries. The two countries are caught in a Prisoner's Dilemma, they would be better off agreeing not to subsidise exports. With full cooperation they would both tax exports to shift the industry to the monopoly output. According to de Meza (1989) price controls would be superior to export subsidies. And, Cooper and Riezman (1989) show that the use of direct quantity controls would be preferred to subsidies in a world of certainty. The Nash equilibrium would involve each country setting its quantity control at the Cournot output of its firm. Then, welfare would be the same as when neither country

\textsuperscript{11} This only holds for duopoly, and not when there are many domestic and foreign firms. See de Meza (1986) and also Hwang and Mai (1988).
intervenes. With sufficient uncertainty, Cooper and Riezman show that governments would use subsidies since they are a more flexible policy than quantity controls.

For the Cournot duopoly model an export subsidy will generally increase foreign welfare, but even in this case the optimal policy may be an export tax. However, this result is extremely sensitive to any changes in the model. When the number of foreign firms is greater than one, the foreign country has to consider the conventional terms of trade argument for an export tax, as well as the strategic argument for an export subsidy. With a single foreign firm there is no terms of trade argument for an export tax, the single firm realises fully the effect its decision will have on the terms of trade. But with many foreign firms, each firm does not take account of the effect its own output decision will have on the price received by other foreign firms. An export tax allows the foreign country to exploit its monopoly power. In oligopolistic industries, with many foreign firms, there is both a terms of trade effect and a strategic argument for intervention, hence the optimal policy is ambiguous. As the number of foreign firms increases then the terms of trade argument will dominate the strategic argument, and the optimal policy will be an export tax. The optimal export subsidy with many foreign and domestic firms has been derived by Dixit (1984).
Obviously, the profit shifting argument for export subsidies assumes there are pure profits. However, as Dixit (1985) argues, profits in high technology industries may be the result of success in a race to develop a new product. Although one firm may win and earn large profits, there may be many losers that have spent large amounts on R&D. If there was free entry, then firms would enter the race to develop the new product until expected profits were zero.

When domestic and foreign firms compete in a high technology industry, an attempt to use profit shifting export subsidies by the foreign country would lead to the increased entry of foreign firms so that expected profits in the foreign industry were zero. Then, an export subsidy does not increase foreign welfare. In general, an export subsidy may lead to the entry of new firms which will tend to dissipate industry profits. Also, the profit shifting argument for export subsidies assumes that the profits of the foreign firm accrue to foreign citizens. But, with the growing internationalisation of business, this appears to be questionable. The foreign industry could include the subsidiaries of domestic multinational firms, and foreign firms could be partly owned by domestic citizens. And, if foreign citizens held shares in domestic firms, then shifting profits from domestic to foreign firms would be less likely to increase foreign welfare. As business becomes more international, profit shifting export subsidies are less likely to be effective.
The profit shifting argument for an export subsidy assumes that resources for the expansion of the oligopolistic industry are made available by a reduction in the output of the competitively produced numeraire good. Since, price is equal to marginal cost in the competitive sector, a reduction in output has no welfare cost. However, if the economy consists of several oligopolistic industries, then an export subsidy which expands one oligopolistic industry may lead to the contraction of another oligopolistic industry, which will involve a welfare cost. Dixit and Grossman (1986) consider an economy with several identical oligopolistic industries, and all use a factor which is in fixed supply. Then, a profit shifting export subsidy given to one industry will contract the output of the other industries, and reduce profits so that the overall profit shifting effect is zero. Hence, the optimal policy is a zero export subsidy. Although this is an extreme example, it shows that the general equilibrium effects of export subsidies need to be considered.

For a Cournot duopoly an export subsidy is generally the optimal policy, but Eaton and Grossman (1986) have shown that an export tax is generally the optimal policy in a Bertrand duopoly.\(^{12}\) Consider a differentiated product

\(^{12}\)Helpman and Krugman (1989) claim that "Whereas export subsidies are sometimes desirable in a Cournot market they
Bertrand duopoly. With segmented markets, the domestic market can be analysed independently of the foreign market. The price of the domestic product is $P_1$, and the price of the foreign product is $P_2$. Demand for the domestic product is $Y = Y(P_1, P_2)$, and demand for the foreign product is $X = X(P_1, P_2)$. Where $Y_1 < 0$ and $Y_2 > (\leq) 0$ for substitutes (complements), and $X_2 < 0$ and $X_1 > (\leq) 0$ for substitutes (complements). Domestic and foreign profits from domestic sales are, respectively

$$\pi_1 = (P_1 - c_1) Y(P_1, P_2)$$

$$\pi_2 = (P_2 - c_2 + e) X(P_1, P_2)$$

Assuming that each firm's profit function is quasi-concave in its own price, and that there is an interior solution, the first order conditions for a Bertrand equilibrium are

$$\frac{\partial \pi_1}{\partial P_1} = (P_1 - c_1) Y_1 + Y = 0$$

$$\frac{\partial \pi_2}{\partial P_2} = (P_2 - c_2 + e) X_2 + X = 0$$

And, the second order conditions for profit maximisation are never desirable in a Bertrand market". But, the conclusion of Eaton and Grossman (1986) is that an export tax is generally the optimal policy for Bertrand oligopoly.
\[ \frac{\partial^2 \pi_1}{\partial P_1^2} = \frac{1}{Y_1} (2Y_1^2 - YY_{11}) < 0 \quad \frac{\partial^2 \pi_2}{\partial P_2^2} = \frac{1}{X_2} (2X_2^2 - XX_{22}) < 0 \]

The comparative static results for the effect of a foreign export subsidy are obtained by totally differentiating the first order conditions (1.13) to obtain

\[
\begin{bmatrix}
2Y_1^2 - YY_{11} & Y_1 Y_2 - YY_{21} \\
X_1 X_2 - XX_{12} & 2X_2^2 - XX_{22}
\end{bmatrix}
\begin{bmatrix}
\frac{dP_1}{dP}
\\
\frac{dP_2}{dP}
\end{bmatrix} =
\begin{bmatrix}
0 \\
-X_2^2 \text{ de}
\end{bmatrix}
\]

(1.14)

The effect of a foreign export subsidy on the domestic and foreign price is

\[
\frac{\partial P_1}{\partial e} = \frac{X_2 (Y_1 Y_2 - YY_{21})}{\Delta'} \quad \frac{\partial P_2}{\partial e} = \frac{-X_2^2 (2Y_1^2 - YY_{11})}{\Delta'} < 0 \quad (1.15)
\]

Where \( \Delta' = (2Y_1^2 - YY_{11})(2X_2^2 - XX_{22}) - (Y_1 Y_2 - YY_{21})(X_1 X_2 - XX_{12}) > 0 \), for a unique equilibrium. A foreign export subsidy will reduce the domestic price if the domestic and foreign price are strategic complements, \( Y_1 Y_2 - YY_{21} < 0 \). For strategic substitutes it will increase the domestic price. A foreign export subsidy reduces the foreign price.

Foreign welfare is producer surplus from exports, \( W_2 = (P_2 - c_2)X \). Hence, the effect of an export subsidy on foreign welfare is
\[
\frac{\partial W}{\partial e} = ((P_2 - c_2)X_2 + X) \frac{\partial P_2}{\partial e} + X_1 \frac{\partial P_1}{\partial e}
\]

At \( e = 0 \), \((P_2 - c_2)X_2 + X = 0 \) from the first order condition (1.13). Thus, the effect of an export subsidy evaluated at \( e = 0 \) is

\[
\frac{\partial W}{\partial e} = X_1 \frac{\partial P_1}{\partial e}
\]  

(1.16)

When the domestic and foreign products are substitutes, \( X_1 > 0 \), then domestic and foreign prices are generally strategic complements, \( Y_{12}Y_{21} < 0 \), so \( \frac{\partial P_1}{\partial e} < 0 \), and the welfare effect of the export subsidy is negative.\(^{13}\) And, when the domestic and foreign products are complements, then domestic and foreign prices are generally strategic substitutes, and the welfare effect of the export subsidy is again negative. However, it is possible that an export subsidy will have a positive welfare effect, for example if the domestic and foreign products are substitutes, and domestic and foreign prices are strategic substitutes. The optimal export subsidy is derived by setting \( \frac{\partial W}{\partial e} = 0 \) and solving for \( e \), yields

\[
e = \frac{-X_1(Y_{12}Y_{21})}{X_2(2Y_{12}Y_{11})}
\]  

(1.17)

\(^{13}\)By the symmetry of the substitution matrix \( X_1 = Y_2 \).
For a Bertrand duopoly, the optimal policy is generally an export tax. The optimal export tax, when the domestic and foreign products are substitutes and domestic and foreign prices are strategic complements, is shown in figure 1.5. This shows that the result of Brander and Spencer (1985) depends on the form of competition between firms.

It has been assumed that the foreign government has a first mover advantage, and can commit itself to an export subsidy before the firms make their output or price decision. A different game structure, based upon the institutional arrangements for export credit subsidies of the Export-Import Bank of the United States, is considered by Carmichael (1987) and Gruenspecht (1988). In this model the domestic and foreign firm export to a third country. At the first stage the two firms set prices, and then in the second stage the two governments set their export subsidies. The two governments maximise producer surplus from exports, profits net of subsidies. Demand for the domestic and foreign product depends on the subsidised prices, the prices set by the firm less the government subsidies. The Nash equilibrium of the second stage is for the two governments to provide subsidies so that the subsidised price of exports maximises producer surplus. These subsidised prices will be the same as the equilibrium prices the two firms would set in the absence of subsidies. At the first stage, the firms
realise how the governments will behave in the second stage.
If a firm increases its price then its government will provide a subsidy so that the subsidised price still maximises producer surplus. The higher the price the firm charges, the larger will be the subsidy it receives from the government and hence the larger will be its profits. In theory, there is no limit to the price the firm could charge, but Carmichael assumes there is a limit to the size of export subsidies. While such models may provide a positive explanation for export subsidies, they are not an argument for the use of export subsidies since firms are basically exploiting the governments. Neary (1989) shows that welfare is always higher if governments have a first mover advantage, and then an export tax is generally the optimal policy.

Brander and Spencer (1985) have shown that a foreign export subsidy can increase foreign welfare by shifting profits from domestic to foreign firms. However, it has been shown that the profit shifting argument is not robust to changes in the assumptions of the model.

1.4 Strategic Import Policy

This section considers the effects of domestic trade policies, such as import tariffs and production subsidies. A tariff can improve the country’s terms of trade, and shift
profits from foreign to domestic firms. Since price exceeds marginal cost in the domestic industry, a production subsidy can be used to correct this distortion. The effect of a foreign export subsidy on domestic welfare, and the optimal domestic response to foreign subsidies are also considered.

Again, consider the Cournot duopoly model of the previous sections. Let the specific, per unit, domestic import tariff be $t$, and the production subsidy $s$. Hence, the profits of the domestic and foreign firm are, respectively

$$\pi_1 = (P - c_1 + s)Y$$

$$\pi_2 = (P - c_2 - t + e)X$$

And, assuming an interior solution, the first order conditions for a Cournot equilibrium are

$$\frac{\partial \pi_1}{\partial Y} = P + YP' - c_1 + s = 0$$  \hspace{1cm} (1.19)

$$\frac{\partial \pi_2}{\partial X} = P + XP' - c_2 - t + e = 0$$

To obtain the comparative static results for the effects of the tariff and production subsidy, totally differentiate the first order conditions, which yields
\[
\begin{bmatrix}
2P' + YP'' & P' + YP'' \\
P' + XP'' & 2P' + XP''
\end{bmatrix}
\begin{bmatrix}
dY \\
dX
\end{bmatrix} = 
\begin{bmatrix}
-ds \\
dt
\end{bmatrix}
\]

By matrix inversion

\[
\begin{bmatrix}
dY \\
dX
\end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix}
2P' + XP'' & -(P' + YP'') \\
-(P' + XP'') & 2P' + YP''
\end{bmatrix}
\begin{bmatrix}
-ds \\
dt
\end{bmatrix}
\] (1.20)

An import tariff reduces foreign exports, and increases (reduces) domestic production if domestic output and foreign exports are strategic substitutes (complements) for the domestic country. The effects of a tariff are the opposite of the effects of an export subsidy. A production subsidy increases domestic production, and reduces (increases) foreign exports if domestic output and foreign exports are strategic substitutes (complements) for the foreign country. The effects on price are

\[
\frac{\partial P}{\partial t} = \frac{(P')^2}{\Delta} > 0 \quad \frac{\partial P}{\partial s} = \frac{-(P')^2}{\Delta} < 0 \] (1.21)

A tariff increases the price and a production subsidy lowers the price.

Since utility is additively separable and linear in the competitive numeraire good, the aggregate indirect utility function is of the form \( V = V(P) + I \), where \( I \) is national income. By Roy's identity \( \frac{\partial V}{\partial P} = -Q \). Demand for \( Q \) is
independent of income so consumer surplus is a valid welfare measure. Domestic welfare is the sum of consumer surplus, producer surplus and government revenue

\[ W_1 = V(P) + (P - c_j)Y + tX \]  

(1.22)

Firstly, consider the optimal domestic tariff and production subsidy, these will be derived using the method employed by Dixit (1984, 1988). Totally differentiating domestic welfare yields

\[ dw_1 = -X(dp - dt) + (P - c_j)dY + tdX \]

The first term is the terms of trade effect, the second term is the domestic distortion effect, and the third term is the tariff revenue effect. From (1.19) \( dp - dt = -P'\, dx + XP''\, dQ \), and \( dY = dQ - dx \). Thus

\[ dw_1 = (P - c_j + X^2\, P')dQ + (c_j - c_2 + e + 2XP')dx \]  

(1.23)

To obtain the optimal policies Dixit (1984, 1988) notes that there are two policies, \( t \) and \( s \), which jointly determine \( X \) and \( Q \). Then, provided the Cournot equilibrium is unique, \( X \) and \( Q \) can be regarded as independent variables with the policies that support them implicitly given by the two first order conditions in (1.19). There are a number of possible solutions depending upon the relative costs of the domestic
and foreign firm. The market may be supplied entirely by domestic production, entirely by imports, or by domestic production and imports.

When \( c_1 \leq c_2 - e \) then the coefficient of \( dX \) is negative for all \( X > 0 \), then it is optimal to set \( X = 0 \), and the market is supplied entirely by domestic production. With \( X = 0 \), the coefficient of \( dQ \) shows that the optimal policy is to set \( P = c_1 \). When the domestic firm has a lower marginal cost than the foreign firm, it is optimal to subsidise domestic production until price equals marginal cost. Then, even with a zero tariff, there will be no imports since price is below foreign marginal cost.

When \( c_1 - c_2 + e + 2XP' > 0 \) for all \( X \leq Q \), then the coefficient of \( dX \) is positive for all \( X \leq Q \) and it is optimal to set \( X = Q \). In this case the foreign firm has such a cost advantage that domestic production is not worthwhile, and the market is entirely supplied by imports. Dixit (1988) incorrectly derives the optimal tariff in this case, what he does is to set the coefficient of \( dQ \) equal to zero ignoring the term in \( dX \), and then uses the foreign firm's first order condition from (1.19) to obtain the optimal tariff.\(^{14}\) However, if the market is entirely supplied by imports then \( Q = X \) so \( dQ = dX \), and the welfare expression (1.23) becomes

\[ dW_1 = (P - c_2 + e + 2XP' + X^2P'')dQ \]  

(1.24)

Setting the coefficient of \( dQ \) equal to zero, and using the foreign firm's first order condition from (1.19) to solve for the optimal tariff yields

\[ t = -X(P' + XP'') \]

An alternative expression for the optimal tariff is

\[ t = -XP'(1 + R) \]  

(1.25)

Where \( R = QP''/P' \) is the relative convexity of demand, which is positive (negative) for concave (convex) demand functions. This is equivalent to the optimal tariff derived by Brander and Spencer (1984a) for a country with no domestic production which imports from a foreign oligopolistic industry. The tariff improves the terms of trade of the domestic country by reducing the price-cost margin of the foreign firm. The use of a tariff to improve the terms of trade by a country which imports from a foreign monopolist was considered by Katrak (1977) and de Meza (1979). The optimal policy will be an import subsidy if \( R \leq -1 \), this occurs if demand is sufficiently convex. For example, with constant elasticity demand functions \( P(Q) = Q^{-1/\eta} \), where \( \eta \) is the elasticity of demand, then
R = -(1 + 1/\eta), and the optimal policy is an import subsidy. When ad valorem tariffs are used instead of specific tariffs, the optimal ad valorem tariff with constant elasticity demand functions is zero. An ad valorem tariff is superior to a specific tariff, when the optimal specific tariff is positive. Conversely, a specific tariff is superior to an ad valorem tariff, when the optimal ad valorem tariff is negative. This has been noted by Hillman and Templeman (1985) and Helpman and Krugman (1989).\textsuperscript{15}

The cost disadvantage required to make domestic production not worthwhile has been analysed by Dixit (1984, 1988) and Venables (1986). For constant elasticity demand functions it can be shown that domestic production is not worthwhile if

\[
\frac{C_1}{C_2 - e} > \frac{\eta^2 + \eta + 1}{\eta^2 - \eta + 1}
\]

(1.26)

The maximum value of the right hand side of this expression is three, and occurs when the elasticity of demand is unity, \(\eta = 1\). In this case, domestic production is worthwhile provided domestic marginal cost is less than three times foreign marginal cost. As the number of foreign firms increases and the foreign industry becomes more competitive, the critical cost ratio tends to one.

\textsuperscript{15}The first best solution cannot be achieved by tariffs, but can be achieved by a price control as de Meza (1979) has noted.
When $0 < c_1 - c_2 + e < -2XP'$, then there will be an interior solution and the market will be supplied by domestic production and imports. To obtain the optimal tariff and production subsidy, set the coefficients of $dQ$ and $dX$ equal to zero in (1.23), and using the first order conditions from (1.19) yields

$$t = -XP'(P' + XP'')$$

(1.27)

$$P - c_1 = -X^2P'' \Rightarrow s = -YP' + X^2P''$$

The optimal tariff is usually positive, but an import subsidy may be optimal if demand is sufficiently convex and the market share of imports is high. When demand is concave (convex) then price will be above (below) marginal cost of the domestic firm, and a production tax may be required if demand is extremely concave.

In the same way that Eaton and Grossman (1986) have shown that the optimal foreign export policy depends upon the nature of competition, Cheng (1988) has shown that the optimal domestic policy also depends on whether there is Bertrand or Cournot competition. In a differentiated product duopoly model with linear demand functions, using the same method as Dixit (1984, 1988), Cheng shows that the optimal tariff and production subsidy are larger for Cournot than
for Bertrand competition. For a homogeneous product Bertrand duopoly, when the domestic firm has a cost advantage, the optimal policy is a production subsidy to bring price equal to marginal cost. When the foreign firm has a cost advantage, the optimal policy is a production subsidy that reduces domestic marginal cost to just above foreign marginal cost, then the foreign firm will set price equal to marginal cost. There is no import tariff.

The optimal tariff for a Cournot oligopoly when the domestic country does not use a production subsidy has been derived by Brander and Spencer (1984b). Then, the method used by Dixit cannot be used, and the usual method is employed. For the Cournot duopoly model, maximising domestic welfare (1.22) with respect to $t$ yields the first order condition

$$\frac{\partial W}{\partial t} = x \left( 1 - \frac{\partial p}{\partial t} \right) + (p - c_i) \frac{\partial y}{\partial t} + t \frac{\partial x}{\partial t} = 0$$

Hence, the optimal tariff is

$$t = - \left[ x \left( 1 - \frac{\partial p}{\partial t} \right) + (p - c_i) \frac{\partial y}{\partial t} \right] / \frac{\partial x}{\partial t}$$ (1.28)

The first term is the terms of trade effect, and the second

\[16\] An alternative policy is an import subsidy to reduce foreign marginal cost to just above domestic marginal cost, then the domestic firm sets price equal to marginal cost. This policy involves no revenue cost.
term is the profit shifting effect. Using the comparative static results from (1.20) and (1.21) yields the optimal tariff

$$t = \frac{-XP'(2P'+QP'') -YP'(P'+YP'')} {2P'+YP''}$$  \hspace{1cm} (1.29)$$

This is generally positive, but may be negative if demand is sufficiently convex. In the absence of a production subsidy, a tariff improves the terms of trade and shifts profits from foreign to domestic firms.

Consider the effect of a foreign export subsidy on domestic welfare when the domestic country pursues a policy of laissez-faire. This has been analysed by Dixit (1984, 1987c) and Mai and Hwang (1987). In the Cournot duopoly model, the effect of a foreign export subsidy on domestic welfare (1.22) is

$$\frac{\partial W_1}{\partial e} = -X \frac{\partial P}{\partial e} + (P - c_1) \frac{\partial Y}{\partial e}$$

The first term is the terms of trade effect: The foreign export subsidy reduces the price of imports which is a welfare gain for the domestic country. The second term is the profit shifting effect: The export subsidy reduces domestic production and reduces the profits of the domestic
firm which is a welfare loss.\(^{17}\) The overall effect can be evaluated using the comparative static results from (1.6) and (1.7), and noting that \(P-c_1 = -YP'\), yields

\[
\frac{\delta W_1}{\delta e} = \frac{P'}{\Delta} \left[ (X-Y)P' - Y^2P'' \right]
\]  

(1.30)

A foreign export subsidy will reduce domestic welfare if domestic output is large relative to imports. For linear demand, domestic welfare will be reduced if the market share of domestic production is greater than a half. When domestic production has a large share of the market, the gain to consumers from lower prices is not sufficient to compensate for the lower profits of the domestic firm. As Dixit (1987c) has noted the domestic country can always gain by imposing a fully countervailing tariff. Then, domestic output, foreign exports and price will not be altered, but the domestic country will gain tariff revenue. But, a fully countervailing tariff is not the optimal response to a foreign export subsidy.

Dixit (1984, 1988) has analysed the optimal domestic response to a foreign export subsidy. When the domestic country uses an import tariff and a production subsidy, and demand is linear, then from (1.27) and using (1.19) the

\(^{17}\)Assuming domestic output and foreign exports are strategic substitutes. If they were strategic complements then domestic production would increase.
optimal policies are

\[ t = \frac{1}{2} (c_1 - c_2 + e) \quad P = c_1 \] (1.31)

The domestic industry is subsidised so that price equals marginal cost, and a tariff is used to extract rent from the foreign firm. The optimal response to a foreign export subsidy is a partially countervailing tariff, \( \frac{dt}{de} = 1/2 \), and to reduce the production subsidy. For constant elasticity demand functions, Dixit (1988) shows that the optimal countervailing tariff is less than a half. When only a tariff is used by the domestic country, the optimal countervailing tariff fraction is one third with linear demand.\(^{18}\)

When marginal cost is decreasing, a tariff may act as a form of export promotion. Then, a tariff expands domestic production which lowers the marginal cost of the domestic firm, and it gains a strategic advantage exporting to the foreign market. This is considered by Krugman (1984a), but he does not consider the welfare effects of such a policy. Trade policy with learning-by-doing is analysed by Dasgupta and Stiglitz (1988).

Trade policy has been analysed by Venables (1985a) in a

\(^{18}\)For a discussion about how a country should respond to other countries trade policies see Dixit (1987a).
homogeneous product Cournot oligopoly with free entry of domestic and foreign firms, and segmented markets. There are assumed to be positive transport costs. The surprising result is that a domestic tariff lowers the price in the domestic market and raises the price in the foreign market. Hence, the domestic country clearly gains from a tariff, even ignoring the tariff revenue. The explanation is that the tariff induces firms to shift from the foreign country to domestic country, where they can produce for the domestic market at lower cost since they no longer incur the transport cost. Similarly, an export subsidy and a production subsidy will also reduce price and increase welfare in the domestic country. The assumption of segmented markets is crucial. Trade policy with integrated markets has been considered by Horstmann and Markusen (1986), with nationally differentiated products, then a domestic tariff raises the price in the domestic market and lowers the price in the foreign market. Markusen and Venables (1988) have compared the effects of trade policy when there is a fixed number of firms or free entry, and when there is segmented or integrated markets. A tariff is more effective when markets are segmented than when they are integrated, and is more effective when there is a fixed number of firms than when there is free entry.

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19 Venables (1987b) analyses trade policy under monopolistic competition with free entry.
The effects of tariffs on cartels has been considered by Davidson (1984) and Fung (1987) in an infinitely repeated game with discounting. Then, implicit collusion can be supported by trigger strategies where if any firm cheats in any period then all firms will revert to the single-shot Nash equilibrium in subsequent periods. A large tariff may then weaken the cartel and make the industry more competitive. Rotemberg and Saloner (1989) consider the effect of a quota in a similar model. Then, a quota set at the free trade level may weaken the cartel and lower the price. Whereas, in a static model Krishna (1989) shows that a quota, at the free trade level, will facilitate collusion and increase price.

The use of protection and subsidies for entry promotion and deterrence has been analysed by Dixit and Kyle (1985). In their model, based upon the Airbus example, an incumbent firm in the US faces the potential entry of a firm in the EC. They model trade policy as a multistage game. The US has the choice of free trade or protection, defined as a complete prohibition of imports. The US has an incentive to use protection to deter the entry of the EC firm, since if entry occurs the profits of the US firm will be reduced. And, the EC can use protection, or subsidies, to make entry profitable for the EC firm. They consider the strategic use of trade policy where one government has a first mover advantage. Then, the equilibrium of the trade policy game
depends on the order of play and the ability of governments to pre-commit to particular policies. Dixit and Kyle provide a useful model with which to start analysing retaliation.

The effects of retaliation are modelled by Gasiorek et al (1989) using a numerical model of international trade under imperfect competition applied to the motor vehicle and computer industries. In their model the world was divided into four countries: the EC, North America, Japan and the Rest of the World. They calculated the multilateral Nash equilibrium in tariffs, and in export subsidies. To model countervailing tariffs, they calculated the Nash equilibrium of a bilateral game where the exporting country sets its export subsidy and the importing country sets its tariff, and compare the outcome with the equilibrium when the export subsidy is set at zero. They interpret the change in the optimal tariff as the countervailing tariff. If the exporting country could commit itself not to subsidise exports then, in the motor vehicle industry, the countervailing tariff deters it from using an export subsidy. But, in the computer industry, the countervailing tariff may not deter the use of an export subsidy. A limitation of this approach is that "it is based on examples,...and examples do not generate general propositions".20

1.5 Conclusions

The recent literature on strategic trade policy has provided a number of surprising results, but the results are often not robust to changes in the model. The profit shifting argument for export subsidies is particularly sensitive in this respect. The effect of retaliation, although often discussed, has not been formally modelled. Although, Dixit (1988) considers the optimal domestic response to foreign export subsidies, he does not consider the optimal policy for the foreign country when faced with such a response. Dixit and Kyle (1985) consider the interaction of domestic and foreign trade policies in a multistage game, and this provides a starting point for the analysis of retaliation in strategic trade policy.
Figure 1.1: Gains From Multilateral Trade
Figure 1.2: Gains From Unilateral Trade
Figure 1.3 A Foreign Export Subsidy
Figure 1.4: The Optimal Export Subsidy
Figure 1.5: Optimal Export Tax
Chapter 2: Strategic Trade Policy and Retaliation

2.1 Introduction

This chapter analyses the effect of retaliation on the profit shifting argument for an export subsidy. Trade policy is modelled as a multistage game, where the foreign country sets its export subsidy in the first stage, and the domestic country responds with an import tariff and/or production subsidy in the second stage. The products of the domestic and foreign industry are nationally differentiated, and demand is linear. Both Cournot oligopoly and Bertrand duopoly are considered. It is shown that when the foreign country faces retaliation with countervailing tariffs there is no profit shifting argument for an export subsidy.

Brander and Spencer (1985) have shown that a foreign export subsidy may increase foreign welfare in oligopolistic industries, by shifting profits from domestic to foreign firms. It has been argued, by Grossman (1986) and Bhagwati (1988), that the use by a government of strategic export subsidies to shift profits, at the expense of trading partners, is likely to lead to retaliation. And, that retaliation would leave the country that first used export subsidies worse off. Whereas, Brander (1986) has argued that it is naive to believe that retaliation would completely undercut the case for strategic trade policy. Until now,
with the exceptions of Dixit and Kyle (1985) and Dixit (1988), there has been no attempt to formally model retaliation. Although, Gasiorek et al (1989) have analysed the effect of retaliation in a numerical model.

In practice, the likely response of the domestic country to a foreign export subsidy, which attempts to shift profits from domestic to foreign firms, would be to apply countervailing tariffs against the subsidised imports. Such a response is allowed under article VI of the General Agreement on Tariffs and Trade, GATT. Dixit (1988) has analysed how the domestic country should respond to a foreign export subsidy in oligopolistic industries. A foreign export subsidy alters the optimal trade policy of the domestic country, and Dixit interprets the change in the level of the optimal tariff as a countervailing duty. Dixit (1988) concludes that the optimal domestic response to a foreign export subsidy is a partially countervailing tariff.

In the classic analysis of tariff retaliation by Johnson (1953-54) each country imposes its optimum tariff on the assumption that the other country's tariff is unchanged. After a reaction process, with successive adjustments of tariffs by countries, a Nash equilibrium in tariffs is

\[1\]For a survey see McMillan (1986). A similar approach is taken by Gros (1987a,b) to analyse tariffs in the Krugman (1979) model of intra-industry trade under monopolistic competition.
reached, and a country may be better or worse off at this Nash equilibrium than at free trade. The dynamic process envisaged by Johnson is not really credible. There is no reason why the countries should not move immediately to the Nash equilibrium. The tariff equilibrium presented by Johnson could be interpreted as a Nash equilibrium in a game where each country independently and simultaneously sets its tariff. Such a model does not permit the analysis of retaliation since neither country can really respond to the tariff of the other country. However, Johnson also suggested a Stackelberg type of analysis, where one country sets its tariff first and then the second country responds.\(^2\) Similarly, in Dixit and Kyle (1985) one country can commit itself to a policy before the other country sets its policy. Dixit and Kyle modelled trade policy as a multistage game, using the concept of subgame perfect equilibrium. Their model, based on the Airbus example, considered the use of protection and subsidies to deter and promote entry.

Spencer (1988a) considers countervailing tariffs in a model where the foreign government gives subsidies to capital, which reduce the marginal cost of foreign firms, giving them a strategic advantage that allows them to capture a larger share of industry profits. In her model the domestic government does not set its trade policy optimally, instead

\(^2\)This alternative model is discussed by Johnson (1953-54) in footnote 5 on page 146.
the countervailing tariff is assumed to be set at the maximum level permitted under the GATT, the so called equal payment tariff. The revenue raised by this tariff must not exceed the foreign subsidy payments. In proposition seven, Spencer shows that, for a Cournot duopoly, a small subsidy to additional capital countervailed by an equal payment tariff, increases foreign welfare. This is not surprising if it is noted that there will be no countervailing tariff in this case, and the result is really just the same as Spencer and Brander (1983). When the foreign country uses export subsidies, a fully countervailing tariff is the maximum permitted under the GATT. It will be shown here that only partially countervailing tariffs are optimal, hence the maximum permitted by the GATT is not relevant.

Following Dixit and Kyle (1985), and the suggestion of Johnson (1953-54), trade policy will be modelled as a multistage game in this chapter. At the first stage, the foreign country sets its export subsidy, and at the second stage the domestic country sets its import tariff and/or production subsidy, as in Dixit (1988). The foreign country is assumed to be able to commit itself to an export subsidy before the domestic country sets its import tariff and production subsidy. Such a commitment could be achieved by a

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3 When the subsidy is close to zero, the countervailing tariff and the effect of a foreign subsidy on the countervailing tariff are also close to zero. See Spencer (1988a) page 60.
constitutional clause, for example, the United States Constitution prohibits export taxes. This game structure seems to be a reasonable description of the institutional arrangements under the GATT. There is a prohibition of export subsidies, but if a country does use an export subsidy then the importing country can respond with countervailing duties under Article VI of the GATT. The prohibition by the GATT of export subsidies could be seen as an attempt to commit countries to a policy of not subsidising exports. The solution concept employed is that of subgame perfect equilibrium. Therefore, any threats of retaliation by the domestic country have to be credible, and when setting its export subsidy the foreign country realises the effect its decision will have on the optimal trade policy of the domestic country.

The effect of a foreign export subsidy on foreign and domestic welfare when the domestic country pursues a policy of laissez-faire is considered. Then, the trade policy games when the domestic country can use an import tariff and/or a production subsidy in response to the foreign export subsidy are analysed. The effect of a foreign export subsidy on domestic welfare when the domestic country pursues an optimal trade policy is considered, and the optimal domestic

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4 This is noted by Carmichael (1987). Article I, section 8, of the Constitution states: "No tax or duty shall be laid on articles exported from any state".
response to a foreign export subsidy is derived. This is used to analyse the effect of a foreign export subsidy on foreign welfare when the foreign country anticipates the response of the domestic country. Then, the optimal foreign export subsidy is derived.

2.2 Cournot Oligopoly

The model used here is basically the same as in section three of Dixit (1988). A number of foreign and domestic firms compete in a differentiated product Cournot oligopoly. The product is nationally differentiated, so the product of the domestic industry is not a perfect substitute for the product of the foreign industry, but all firms in one country produce the same product. In the domestic country there are n identical firms each with constant marginal cost $c_1$, and in the foreign country there are m identical firms each with constant marginal cost $c_2$. The domestic and foreign markets are assumed to be segmented. Therefore, since marginal cost is constant, the domestic and foreign market can be analysed separately. Consider the domestic market, where the domestic industry competes with imports from the foreign industry. Each domestic firm produces output $y$ for domestic consumption, and each foreign firm exports output $x$ to the domestic market. Hence, domestic production is $Y = ny$, and foreign exports, or alternatively domestic imports, are $X = mx$. The price of the domestic
product $Y$ is $P_1$, and the price of the foreign product $X$ is $P_2$. The domestic import tariff is $t$ per unit imported, and the production subsidy is $s$ per unit of domestic production. The foreign export subsidy is $e$ per unit exported.

Domestic consumers are assumed to have preferences, for $X$, $Y$ and a competitive numeraire good, that can be represented by a utility function which is additively separable and linear in the numeraire good. Therefore, the aggregate indirect utility function is of the form $^5$

$$V = V(P_1, P_2) + I$$  \hspace{1cm} (2.1)

Where $I$ is national income. So by Roy’s identity $\frac{\partial V}{\partial P_1} = -Y$ and $\frac{\partial V}{\partial P_2} = -X$. Furthermore, it will be assumed that demand is linear and symmetric, the inverse demand functions are

$$P_1 = \alpha - \beta Y - \gamma X$$

$$P_2 = \alpha - \gamma Y - \beta X$$  \hspace{1cm} (2.2)

If $\beta > \gamma$ then the foreign and domestic products are imperfect substitutes, the degree of product differentiation is given by $\beta - \gamma$. When $\beta = \gamma$ the domestic and foreign products are perfect substitutes and the model reduces to

$^5$For further details of the quasi-linear utility function, and the indirect utility function, see Varian (1984).
the case of homogeneous products.

The profits of domestic and foreign firms from sales in the domestic market are

\[
\pi_1 = (P_1 - c_1 + s)y
\]

\[\pi_2 = (P_2 - c_2 - t + e)x\]

Domestic welfare is given by the sum of consumer surplus, domestic industry profits and government revenue. Demand for X and Y is independent of income therefore consumer surplus is a valid measure of welfare. Distributional considerations will be ignored. Thus, domestic welfare is

\[
W_1 = V(P_1, P_2) + (P_1 - c_1 + s)y + tX - sY
\]

\[= V(P_1, P_2) + (P_1 - c_1)Y + tx\]

(2.4)

Where \((P_1 - c_1)Y\) is domestic producer surplus and \(tx\) is tariff revenue. The production subsidy represents a transfer from the domestic government to domestic firms, so the net income effect of the production subsidy on domestic welfare is zero.

Since the foreign and domestic market are segmented and marginal cost is constant, the relevant measure of foreign
welfare is producer surplus from exports, that is foreign 
profits from exports net of export subsidies

\[ W_2 = (P_2 - c_2 - t)X \]  \hspace{1cm} (2.5)

At a Cournot equilibrium firms set their outputs to maximise 
profits taking the domestic and foreign trade taxes and 
subsidies as given. With linear demand each firm's profit 
function is continuous and concave in its own output so a 
Cournot equilibrium exists by the usual existence proofs for 
concave games. And the equilibrium is unique and symmetric. 
At an interior solution the market is supplied by both 
domestic production \((Y > 0)\) and foreign imports \((X > 0)\). 
Profit shifting arguments for trade policy are only relevant 
if there is an interior solution. Assuming there is an 
interior solution, the first order conditions for a Cournot 
equilibrium are\(^6\)

\[ \frac{\partial \pi_1}{\partial y} = p_1 + y \frac{\partial p_1}{\partial y} - c_1 + s = 0 \]  \hspace{1cm} (2.6)

\[ \frac{\partial \pi_2}{\partial x} = p_2 + x \frac{\partial p_2}{\partial x} - c_2 - t + e = 0 \]

\(^6\)It can be shown that there will be an interior solution if:

\[ c_1 - s < (1 - \frac{y}{2\beta})\alpha + \frac{y}{2\beta} (c_2 + t - e) \]  \hspace{1cm} (Y > 0)

\[ c_2 + t - e < (1 - \frac{y}{2\beta})\alpha + \frac{y}{2\beta} (c_1 - s) \]  \hspace{1cm} (X > 0)
The second order conditions for profit maximisation are

\[ \frac{\partial^2 \pi_1}{\partial y^2} = -2\beta < 0 \]
\[ \frac{\partial^2 \pi_2}{\partial x^2} = -2\beta < 0 \]

The equilibrium of the model is given implicitly by the two first order conditions for profit maximisation, as a function of domestic and foreign trade policies. To obtain the comparative static results for e, t and s totally differentiate (2.6), which yields

\[
\begin{bmatrix}
(n+1)\beta & m\gamma \\
n\gamma & (m+1)\beta
\end{bmatrix}
\begin{bmatrix}
dy \\
dx
\end{bmatrix}
= 
\begin{bmatrix}
ds \\
de - dt
\end{bmatrix}
\]

Inverting this matrix yields

\[
\begin{bmatrix}
dy \\
dx
\end{bmatrix}
= 
\frac{1}{\Delta}
\begin{bmatrix}
(m+1)\beta & -m\gamma \\
-n\gamma & (n+1)\beta
\end{bmatrix}
\begin{bmatrix}
ds \\
de - dt
\end{bmatrix}
\]

(2.7)

Where \( \Delta = (n+m+1)\beta^2 + nm(\beta^2 - \gamma^2) > 0 \).

And the effect on prices is

\[
\begin{bmatrix}
dP_1 \\
dP_2
\end{bmatrix}
= 
\frac{-1}{\Delta}
\begin{bmatrix}
\beta^2 + nm(\beta^2 - \gamma^2) & m\beta\gamma \\
\beta\gamma & \beta^2 + nm(\beta^2 - \gamma^2)
\end{bmatrix}
\begin{bmatrix}
ds \\
de - dt
\end{bmatrix}
\]

(2.8)

The comparative static results all have the expected signs, for instance a tariff reduces imports and increases domestic production, and prices of both the foreign and the domestic product increase. The terms of trade, \( P_2 - t \), is the cost of
imports to the domestic country and also the price the foreign country receives for its exports. The effect of a tariff on the terms of trade is

\[
\frac{\partial P_2}{\partial t} - 1 = \frac{-(m+1)\beta^2}{\Delta} < 0
\]

A tariff improves the domestic country's terms of trade and worsens the foreign country's terms of trade.

2.21 Foreign Export Subsidy

This section considers the effect of a foreign export subsidy on foreign and domestic welfare in the absence of a domestic tariff or production subsidy. Brander and Spencer (1985) have shown that a foreign export subsidy may increase foreign welfare by committing foreign firms to expand output. This leads domestic firms to reduce their output and allows foreign firms to capture a larger share of industry profits. The effect of a foreign export subsidy on foreign welfare (2.5) is

\[
\frac{\delta W}{\delta e} = (P_2 - c_2) \frac{\delta X}{\delta e} + X \frac{\delta P_2}{\delta e} \tag{2.9}
\]

The first term is the profit shifting effect, and the second term is the terms of trade effect. An export subsidy increases foreign exports, and since price exceeds marginal
cost, this increases welfare. Profits are shifted from
domestic to foreign firms. Also, the export subsidy reduces
the price of the foreign product so the foreign country
receives a lower price for its exports. This is the terms of
trade effect which has a negative effect on welfare. The
overall effect can be obtained by using the foreign firms' first order condition from (2.6) together with the
comparative static results from (2.7) and (2.8) to evaluate
at \( e = 0 \), yields

\[
\frac{\partial W_2}{\partial e} = \frac{X}{\Delta} \left[ (n-m+1)\beta^2 - nm(\beta^2 - \gamma^2) \right]
\]  

(2.10)

An export subsidy will increase foreign welfare if the
number of foreign firms is not too large relative to the
number of domestic firms as in Dixit (1984), and the degree
of product differentiation is not too great. For a duopoly,
with one foreign and one domestic firm, an export subsidy
will always increase welfare. The optimal export subsidy is
derived by setting \( \frac{\partial W_2}{\partial e} = 0 \) and solving to obtain

\[
e = \frac{x \left( (n-m+1)\beta^2 - nm(\beta^2 - \gamma^2) \right)}{(n+1)\beta}
\]

(2.11)

The effect of the foreign export subsidy on domestic welfare
(2.4) is given by

\[
\frac{\partial W_1}{\partial e} = -X \frac{\partial p}{\partial e} + (P_1 - c_1) \frac{\partial y}{\partial e}
\]  

(2.12)
The first term is the terms of trade effect and the second term is the profit shifting, or domestic distortion, effect. The export subsidy reduces the price of the foreign product which improves the terms of trade of the domestic country. The export subsidy reduces domestic production, and since price exceeds marginal cost, this reduces domestic industry profits. A foreign export subsidy increases consumer surplus by lowering prices but it reduces the profits of domestic firms. The overall welfare effect is ambiguous. The comparative static results (2.7) and (2.8) together with (2.6) can be used to evaluate the overall welfare effect

$$\frac{\delta W}{\delta e} = \frac{m}{\Delta} \left[ X(\beta^2 + n(\beta^2 - \gamma^2)) - \gamma \beta Y \right]$$ (2.13)

Domestic welfare may be reduced by a foreign export subsidy, particularly if domestic industry output is large relative to imports and the degree of product differentiation is small. Comparing (2.10) and (2.13), a foreign export subsidy which increases foreign welfare is likely to reduce domestic welfare. An export subsidy is basically a beggar-my-neighbour policy.

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7 Brander and Spencer (1984b) note that this term can be decomposed into two effects: the effect on industry profits, and the effect on consumer surplus. Domestic industry profit is $$\Pi_1 = (P_1 - c_1)Y$$, therefore $$(P_1 - c_1)dY = d\Pi_1 - YdP_1$$. 

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Mai and Hwang (1987) claim to show conditions under which a foreign export subsidy will not only increase foreign welfare but also the welfare of the domestic country, and hence the world as a whole. They then go on to suggest a relaxation of the GATT prohibition of export subsidies. However, their argument is flawed since they only show the conditions under which a foreign export subsidy will increase domestic welfare. They have implicitly assumed that an export subsidy will necessarily increase foreign welfare but this is not correct. Mai and Hwang consider the case of a symmetric homogeneous product Cournot oligopoly, where \( x = y \), in which case the effect on domestic and foreign welfare is

\[
\frac{\delta W_1}{\delta e} = \frac{-X}{\Delta} (n - m) \beta^2 \\
\frac{\delta W_2}{\delta e} = \frac{X}{\Delta} (n - m + 1) \beta^2
\]

(2.14)

Proposition two of Mai and Hwang (1987) states that if the number of foreign firms exceeds the number of domestic firms, then a foreign export subsidy will increase domestic welfare. This is correct, but in this case the export subsidy will not increase foreign welfare so the foreign country has no incentive to use an export subsidy. And, an export subsidy which increases foreign welfare will not
increase domestic welfare. In this case an export subsidy is always a beggar-my-neighbour policy. Therefore, the conclusion of Mai and Hwang that there should be a relaxation of the GATT prohibition of export subsidies would seem to be unsound.

2.22 Foreign Export Subsidy, Domestic Import Tariff and Production Subsidy

This section considers the situation when the domestic country can respond to the foreign export subsidy with an import tariff and a production subsidy. Trade policy is modelled as a multistage game. At the first stage of the game the foreign government sets its export subsidy to maximise its national welfare. In the second stage the domestic government sets its tariff and production subsidy in response to the foreign export subsidy. In the final stage domestic and foreign firms set their outputs given the various trade taxes and subsidies set by the two governments in previous stages of the game. The equilibrium concept employed is subgame perfection which rules out any non-credible threats. This means that the foreign government will realise the effect its export subsidy will have on the optimal domestic tariff and production subsidy. As usual the game is solved by backward induction.

In the second stage of the game the domestic government sets
its tariff and production subsidy to maximise domestic welfare. The optimal import tariff and production subsidy under oligopoly have been derived by Dixit (1984, 1988) and for duopoly by Cheng (1988). Maximising domestic welfare (2.4) with respect to \( t \) and \( s \), assuming there is an interior solution, yields the first order conditions\(^8\)

\[
\frac{\partial W}{\partial s} = -X \frac{\partial P_2}{\partial s} + (P_1 - c_1) \frac{\partial Y}{\partial s} + t \frac{\partial X}{\partial s} = 0
\]

\[
\frac{\partial W}{\partial t} = -X \left( \frac{\partial P_2}{\partial t} - 1 \right) + (P_1 - c_1) \frac{\partial Y}{\partial t} + t \frac{\partial X}{\partial t} = 0
\]

(2.15)

The first term is the terms of trade effect, the second term is the domestic distortion, or profit shifting, effect and the third term is the tariff revenue effect. Using the comparative static results from (2.7) and (2.8) yields

\[
\begin{bmatrix}
    n(m+1)\gamma \\
    m\gamma
\end{bmatrix}
\begin{bmatrix}
    P_1 - c_1 \\
    t
\end{bmatrix}
= \begin{bmatrix}
    -nmx\beta\gamma \\
    -(n+1)mx\beta^2
\end{bmatrix}
\]

Solving for the optimal policies, and using (2.6) yields

\(^8\)It can be shown that there will be an interior solution if:

\[
c_2 - e < \left( 1 - \frac{\gamma}{\beta} \right) \alpha + \frac{\gamma}{\beta} c_1 \quad (X > 0)
\]

\[
c_1 < \left( 1 - \frac{m\gamma}{(m+2)\beta} \right) \alpha + \frac{m\gamma}{(m+2)\beta} (c_2 - e) \quad (Y > 0)
\]

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\[ P_1 - c_1 = 0 \quad \Rightarrow \quad s = y\beta > 0 \]

\[ t = x\beta = \frac{1}{2} (P_2 - c_2 + e) > 0 \]

These correspond to the optimal policies derived by Dixit (1984, 1988) and Cheng (1988), although they provide explicit solutions for \( t \) and \( s \). The production subsidy is used to eliminate the domestic distortion so that price is equal to marginal cost for the domestic industry. And, the import tariff is used to extract rent from the foreign firms by improving the terms of trade as in Brander and Spencer (1984a). The optimal tariff extracts exactly half the producer surplus from the foreign industry.

In the previous section it was shown that when the domestic country has no tariff or production subsidy a foreign export subsidy may reduce domestic welfare. Now consider the effect of a foreign export subsidy on domestic welfare when the domestic country sets its tariff and production subsidy optimally. The effect of a foreign export subsidy on domestic welfare (2.4) is

\[
\frac{dW_1}{de} = \frac{\partial W_1}{\partial e} + \frac{\partial W_1}{\partial t} \frac{dt}{de} + \frac{\partial W_1}{\partial s} \frac{ds}{de} \quad (2.17)
\]

Since the domestic country is setting the tariff and production subsidy optimally, \( \frac{\partial W_1}{\partial t} = \frac{\partial W_1}{\partial s} = 0 \), so the indirect effect of the export subsidy on domestic welfare

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through its effect on the tariff and production subsidy is zero. This is an example of the envelope theorem. The effect of the foreign export subsidy on domestic welfare is equal to the direct effect evaluated at the optimum. Thus

\[
\frac{dW_1}{de} = \frac{\partial W_1}{\partial e} = -X \frac{\partial P}{\partial e} + (P_1 - c_1) \frac{\partial Y}{\partial e} + t \frac{\partial X}{\partial e}
\]

(2.18)

When the domestic country sets its production subsidy optimally so that price equals marginal cost, the profit shifting effect of the export subsidy is zero. The export subsidy improves the terms of trade and increases tariff revenue, so the overall effect on welfare is positive. Using the comparative static results from (2.7) and (2.8) together with the optimal import tariff and production subsidy from (2.16) to evaluate at the optimum yields

\[
\frac{dW_1}{de} = X > 0
\]

(2.19)

When the domestic country sets its import tariff and production subsidy optimally, a foreign export subsidy will always increase domestic welfare. Obviously, the domestic country would gain if it applied fully countervailing tariffs, \(dt/de = 1\), since then the only effect would be to increase tariff revenue. And, it can do better by pursuing the optimal trade policy. This is analogous to the argument that a country which pursues an optimal trade policy will
not suffer immiserizing growth. Like growth a foreign export subsidy enlarges the domestic country’s opportunity set and so if it pursues an optimal trade policy it will always gain.\textsuperscript{9}

The effect of a foreign export subsidy on the optimal domestic tariff and production subsidy can be obtained by totally differentiating (2.16) which yields

\[
\begin{align*}
\frac{\partial x}{\partial t} &= \beta \frac{\partial x}{\partial t} + \beta \frac{\partial x}{\partial s} + \beta \frac{\partial x}{\partial e} \\
\frac{\partial s}{\partial t} &= \beta \frac{\partial s}{\partial t} + \beta \frac{\partial s}{\partial s} + \beta \frac{\partial s}{\partial e}
\end{align*}
\]

(2.20)

Then using the comparative static results from (2.7) yields

\[
\begin{bmatrix}
\Delta + (n+1)\beta^2 & n\gamma \beta \\
-m\gamma \beta & \Delta - (m+1)\beta^2
\end{bmatrix}
\begin{bmatrix}
\frac{dt}{de} \\
\frac{ds}{de}
\end{bmatrix}
= \begin{bmatrix}
(n+1)\beta^2 \\
-m\gamma \beta
\end{bmatrix}
\]

(2.21)

Hence, the optimal domestic countervailing tariff and production subsidy responses to a foreign export subsidy are

\[
\frac{dt}{de} = \frac{n\beta^2}{2n\beta^2 + nm(\beta^2 - \gamma^2)} > 0
\]

(2.22)

\[
\frac{ds}{de} = \frac{-m\gamma \beta}{2n\beta^2 + nm(\beta^2 - \gamma^2)} < 0
\]

\textsuperscript{9}See Bhagwati and Srinivasan (1983).
The optimal domestic response to a foreign export subsidy is to increase the tariff and decrease the production subsidy, as Dixit (1988) has shown. The optimal countervailing tariff fraction, $\frac{dt}{de}$, is positive but less than a half when domestic and foreign output are imperfect substitutes. For identical products it is a half, which is the well known result of Dixit (1984). An export subsidy increases the foreign rent available for the domestic country to extract with its tariff, see (2.16). And, the export subsidy reduces the price of the domestic product so that a smaller production subsidy is required, to bring price equal to marginal cost, and so correct the domestic distortion. Hence, the optimal domestic response to a foreign export subsidy is to increase the tariff and reduce the production subsidy.

It is now possible to analyse the first stage of the game when the foreign government sets its export subsidy, realising the effect this will have on the optimal tariff and production subsidy set by the domestic government. The effect of an export subsidy on foreign welfare (2.5) is

$$\frac{dW^2}{de} = (P_2 - c - t) \left( \frac{\partial X}{\partial e} + \frac{\partial X}{\partial t} \frac{dt}{de} + \frac{\partial X}{\partial s} \frac{ds}{de} \right)$$

$$+ X \left( \frac{\partial P_2}{\partial e} + \left( \frac{\partial P_2}{\partial t} - 1 \right) \frac{dt}{de} + \frac{\partial P_2}{\partial s} \frac{ds}{de} \right) \quad (2.23)$$
The first term is the profit shifting effect and the second term is the terms of trade effect. The countervailing tariff reduces the profit shifting effect of the export subsidy and worsens the terms of trade. Whereas, the reduction in the domestic production subsidy increases the profit shifting effect and improves the terms of trade. Using the comparative static results from (2.7) and (2.8) together with (2.6), and the optimal countervailing responses from (2.22) to evaluate at \( e = 0 \), yields

\[
\frac{dW_2}{de} = \frac{-nm^2x(\beta^2 - \gamma^2)}{2n\beta^2 + nm(\beta^2 - \gamma^2)} \leq 0 \quad (2.24)
\]

If the domestic and foreign products are imperfect substitutes then the overall effect of a foreign export subsidy on foreign welfare is negative. For identical products the effect is exactly zero. When the foreign country faces retaliation with countervailing tariffs and production subsidies there is no profit shifting argument for an export subsidy. The optimal foreign export subsidy can be obtained by setting \( \frac{dW_2}{de} = 0 \), which yields

\[
e = - \frac{x(\beta^2 - \gamma^2)}{\beta} \leq 0 \quad (2.25)
\]

The optimal foreign policy is to tax exports, and the tax is larger the greater the degree of product differentiation
between the foreign and domestic products. For homogeneous products the optimal export tax is zero.

2.23 Foreign Export Subsidy and Domestic Import Tariff

In the previous section the domestic country used an import tariff and production subsidy to countervail the foreign export subsidy, but in practice governments only use import tariffs. Now consider the situation when the domestic government can only use a tariff. When it could use a tariff and a production subsidy it had two instruments to deal with two distortions, the import tariff was used to improve the terms of trade and the production subsidy to correct the domestic distortion. There is now only one instrument to deal with both distortions. Without a production subsidy to counter the domestic distortion, price will exceed marginal cost in the domestic industry so the tariff is now used to shift profits from foreign to domestic firms, as well as improving the terms of trade. At the second stage of the game the domestic government sets its import tariff to maximise national welfare in response to the foreign export subsidy. Brander and Spencer (1984b) and Dixit (1988) have derived the optimal tariff under oligopoly. Maximising domestic welfare (2.4) with respect to \( t \), assuming there is an interior solution, yields the first order condition\(^{10}\)

\[^{10}\text{It can be shown that there will be an interior solution if:}\]
\[ \frac{\partial W_1}{\partial t} = -X \left( \frac{\partial P}{\partial t} - 1 \right) + (P_1 - c_1) \frac{\partial Y}{\partial t} + t \frac{\partial X}{\partial t} = 0 \quad (2.26) \]

Rearranging yields the optimal tariff

\[ t = X \left( \frac{\partial P}{\partial t} - 1 \right) - (P_1 - c_1) \frac{\partial Y}{\partial t} \bigg/ \frac{\partial X}{\partial t} \quad (2.27) \]

The first term in brackets is the terms of trade effect and the second term is the profit shifting effect. This clearly shows the dual role of the tariff. The tariff improves the terms of trade and expands domestic industry output which shifts profits from foreign to domestic firms. Using the comparative static results from (2.7) and (2.8) together with (2.6) yields the optimal tariff

\[ t = \chi \beta + \frac{n}{n + 1} y \gamma > 0 \quad (2.28) \]

In the previous section the domestic country would always gain from a foreign export subsidy if it set its production subsidy and tariff optimally. The effect of a foreign export subsidy on domestic welfare (2.4) when only the tariff is

\[ c_2 - e < \left( 1 - \frac{n^2 \gamma}{(n+1)^2 \beta} \right) \chi + \frac{n^2 \gamma}{(n+1)^2 \beta} c_1 \quad (X > 0) \]

\[ c_1 < \left( 1 - \frac{m \gamma}{(m+2) \beta} \right) \chi + \frac{m \gamma}{(m+2) \beta} (c_2 - e) \quad (Y > 0) \]
set optimally is

\[
\frac{dW}{de} = \frac{\delta W}{\delta e} + \frac{\delta W}{\delta t} \frac{dt}{de}
\]

Since the tariff is set optimally, \( \frac{\delta W}{\delta t} = 0 \), the overall effect is

\[
\frac{dW}{de} = \frac{\delta W}{\delta e} = -X \frac{\delta P}{\delta e} + (P_1 - c_1) \frac{\delta Y}{\delta e} + t \frac{\delta X}{\delta e} \quad (2.29)
\]

The export subsidy improves the domestic terms of trade and increases tariff revenue, but reduces domestic industry profits. Using the comparative static results from (2.7) and (2.8) together with (2.6), and the optimal tariff from (2.28) to evaluate at the optimum yields

\[
\frac{dW}{de} = X > 0 \quad (2.30)
\]

When the domestic country sets its import tariff optimally a foreign export subsidy will always increase domestic welfare.

The effect of a foreign export subsidy on the optimal domestic tariff can be obtained by totally differentiating (2.28) to obtain
Using the comparative static results from (2.7) yields

\[
\frac{dt}{de} = \frac{(n+1)^2 \beta^2 - nm\gamma^2}{(n+1)\Delta + (n+1)^2 \beta^2 - nm\gamma^2} < 1
\]  

(2.31)

The denominator is positive so the optimal countervailing tariff fraction is clearly less than one, and for a duopoly it is exactly one third as Dixit (1988) has shown. But it is actually always less than a half as can clearly be seen if it is rewritten as

\[
\frac{dt}{de} = \frac{(n+1)^2 \beta^2 - nm\gamma^2}{2((n+1)^2 \beta^2 - nm\gamma^2) + m((2n+1)\beta^2 + n^2(\beta^2 - \gamma^2))} < \frac{1}{2}
\]  

(2.32)

Also, it may be negative if the number of foreign firms is large and the degree of product differentiation is small, if \(m > (n+1)^2 \beta^2/n\gamma^2\). For example, if the domestic and foreign products are perfect substitutes, \(\beta = \gamma\), and there is a single domestic firm, \(n = 1\), then \(dt/de\) is negative if there are more than four foreign firms, \(m > 4\). The foreign export subsidy increases the foreign rent that the domestic country can extract with its tariff, and this will tend to increase the optimal tariff. But the export subsidy reduces domestic price so that price is closer to marginal cost, and this reduces the gains from using a tariff to shift profits to domestic firms, which will tend to reduce the optimal
tariff. Hence, the overall effect of a foreign export subsidy on the optimal tariff is ambiguous.

At the first stage of the game the foreign country sets its export subsidy to maximise its national welfare, realising the effect its decision will have on the optimal domestic tariff. The effect of a foreign export subsidy on foreign welfare (2.5) is

\[
\frac{dW}{de} = (P_2 - c_2 - t) \left( \frac{\delta X}{\delta e} + \frac{\delta X}{\delta t} \frac{dt}{de} \right) + X \left( \frac{\partial P}{\partial e} + \left( \frac{\partial P}{\partial t} - 1 \right) \frac{dt}{de} \right) \tag{2.33}
\]

The first term is the profit shifting effect, and the second term is the terms of trade effect. If the countervailing tariff fraction is positive then it will reduce the profit shifting effect of the export subsidy, and worsen the terms of trade. But it is possible that the countervailing tariff fraction is negative, in which case it will increase the profit shifting effect of the export subsidy and improve the terms of trade. Using the comparative static results from (2.7) and (2.8) together with (2.6), and the optimal tariff fraction from (2.31) to evaluate the welfare effect at \( e = 0 \), yields

\[
\frac{dW_2}{de} = -X \left( \frac{m\beta^2 + (n+2)mn(\beta^2 - \gamma^2)}{(n+1)\Delta + (n+1)^2\beta^2 - nm\gamma^2} \right) < 0 \tag{2.34}
\]
The overall effect of a foreign export subsidy on foreign welfare is negative, even if the optimal countervailing tariff fraction is negative. This is because if the foreign export subsidy will increase foreign welfare in the absence of a countervailing tariff, then the optimal countervailing tariff fraction will be positive, and the overall effect of the export subsidy on welfare is negative. The countervailing tariff fraction will only be negative when an export subsidy reduces foreign welfare in the absence of a countervailing tariff. The optimal export subsidy is obtained by setting $dW^d/e = 0$, which yields

$$e = \frac{-x(m\beta^2 + (n+2)mn(\beta^2 - \gamma^2))}{(n+1)^2 \beta} < 0 \quad (2.35)$$

The optimal foreign policy is an export tax. When the foreign country faces retaliation with a countervailing tariff there is no profit shifting argument for an export subsidy, even when the optimal domestic response to an export subsidy is to reduce its tariff.

2.24 Foreign Export Subsidy and Domestic Production Subsidy

In this section the domestic country is assumed to use only a production subsidy to countervail the foreign export subsidy. Dixit (1988) has derived the optimal production subsidy under oligopoly. Maximising domestic welfare (2.4)
with respect to \( s \), assuming there is an interior solution, yields the first order condition\(^{11}\)

\[
\frac{\partial W_1}{\partial s} = -X \frac{\partial p}{\partial s} + (P_1 - c_1) \frac{\partial Y}{\partial s} = 0 \quad (2.36)
\]

Using the comparative static results (2.7) and (2.8), yields

\[
P_1 - c_1 = \frac{-mx_\gamma}{m + 1} < 0 \quad (2.37)
\]

The optimal policy is to subsidise domestic production so that price is below marginal cost. This is because the production subsidy improves the terms of trade, and so it is worth subsidising domestic production beyond the output where price equals marginal cost. Solving for the optimal production subsidy using (2.6) yields

\[
s = y\beta + \frac{m}{m + 1} x\gamma > 0 \quad (2.38)
\]

The effect of a foreign export subsidy on domestic welfare (2.4), when the domestic country sets its production subsidy

\(^{11}\)It can be shown that there will be an interior solution if:

\[
c_2 - e < \left(1 - \frac{\gamma}{\beta}\right)\alpha + \frac{\gamma}{\beta} c_1 \quad (X > 0)
\]

\[
c_1 < \left(1 - \frac{m^2\gamma}{(m+1)^2\beta}\right)\alpha + \frac{m^2\gamma}{(m+1)^2\beta} (c_2 - e) \quad (Y > 0)
\]
optimally is

\[
\frac{dW}{de} = \frac{\partial W}{\partial e} + \frac{\partial W}{\partial s} \frac{ds}{de}
\]

At an optimum \( \frac{\partial W}{\partial s} = 0 \) so the overall effect is

\[
\frac{dW}{de} = \frac{\partial W}{\partial e} = -X \frac{\partial P^2}{\partial e} + (P - C) \frac{\partial Y}{\partial e} \quad (2.39)
\]

The export subsidy improves the terms of trade and reduces domestic industry output, which increases welfare since price is below marginal cost, so overall it must increase domestic welfare. Using the comparative static results (2.7) and (2.8) together with (2.37) to evaluate at the optimum yields

\[
\frac{dW}{de} = \frac{m}{m + 1} X > 0 \quad (2.40)
\]

A foreign export subsidy always increases domestic welfare if the domestic country sets its production subsidy optimally.

The effect of a foreign export subsidy on the optimal production subsidy is obtained by totally differentiating (2.38), and using the comparative static results from (2.7) yields
\[
\frac{\text{ds}}{\text{de}} = \frac{m}{n} \frac{(n-m)\gamma \beta}{(2m+1)\beta^2 + m^2(\beta^2-\gamma^2)} \tag{2.41}
\]

This is negative if the number of foreign firms exceeds the number of domestic firms. For a duopoly it is zero.

At the first stage of the game the foreign country sets its export subsidy realising the effect its decision will have on the optimal domestic production subsidy. The effect of a foreign export subsidy on foreign welfare (2.5) is

\[
\frac{\delta W}{\delta e} = (P_2 - c_2) \left( \frac{\delta X}{\delta e} + \frac{\delta X}{\delta s} \frac{\text{ds}}{\text{de}} \right) + X \left( \frac{\delta P_2}{\delta e} + \frac{\delta P_2}{\delta s} \frac{\text{ds}}{\text{de}} \right) \tag{2.42}
\]

Using the comparative static results (2.7) and (2.8) together with (2.6) and (2.41) to evaluate at \(e = 0\) yields

\[
\frac{\text{d} W_2}{\text{de}} = \frac{X(\beta^2 - m^2(\beta^2-\gamma^2))}{(2m+1)\beta^2 + m^2(\beta^2-\gamma^2)} \tag{2.43}
\]

An export subsidy will increase foreign welfare provided the degree of product differentiation and the number of foreign firms is not too large. With perfect substitutes the welfare effect is clearly positive. Setting \(\delta W_2/\delta e = 0\) gives the optimal export subsidy

\[
e = \frac{X(\beta^2 - m^2(\beta^2-\gamma^2))}{(m+1)\beta} \tag{2.44}
\]
The optimal foreign policy is an export subsidy if the domestic and foreign products are perfect substitutes. With imperfect substitutes an export tax will be optimal if the number of foreign firms is large.

2.3 Bertrand Duopoly

This section considers the same trade policy games for the case of Bertrand duopoly. The basic model is the same except that there is now only one domestic and one foreign firm, and the firm’s strategic variable is price rather than quantity. From (2.2) the demand facing the domestic and foreign firm as a function of prices is

\[ Y = \frac{1}{\beta^2 - \gamma^2} \left[ \alpha (\beta - \gamma) - \beta P_1 + \gamma P_2 \right] \]

\[ X = \frac{1}{\beta^2 - \gamma^2} \left[ \alpha (\beta - \gamma) + \gamma P_1 - \beta P_2 \right] \]

Assuming there is sufficient product differentiation so that there is an interior solution, the first order conditions for a Bertrand-Nash equilibrium in prices are

\[ c_1 - s < \left( 1 - \frac{\gamma \beta}{2\beta^2 - \gamma^2} \right) \alpha + \frac{\gamma \beta}{2\beta^2 - \gamma^2} (c^*_2 + t - e) \quad (Y > 0) \]

---

12 If there was more than one firm in each country then Bertrand competition would lead to price being equal to marginal cost.

13 It can be shown that there will be an interior solution if:
\[
\frac{\partial \pi_1}{\partial P_1} = (P_1 - c_1 + s) \frac{\partial Y}{\partial P_1} + Y = 0
\]

\[
\frac{\partial \pi_2}{\partial P_2} = (P_2 - c_2 - t + e) \frac{\partial X}{\partial P_2} + X = 0
\]

(2.46)

The comparative static results are obtained by total differentiation of (2.46), which yields

\[
\begin{bmatrix}
2\beta^2-\gamma^2 & \gamma \beta \\
\gamma \beta & 2\beta^2-\gamma^2
\end{bmatrix}
\begin{bmatrix}
\frac{dY}{dP_1} \\
\frac{dX}{dP_1}
\end{bmatrix} =
\begin{bmatrix}
\beta ds \\
\beta de - \beta dt
\end{bmatrix}
\]

By matrix inversion

\[
\begin{bmatrix}
\frac{dY}{dP_1} \\
\frac{dX}{dP_1}
\end{bmatrix} = \frac{\beta}{\Delta'}
\begin{bmatrix}
2\beta^2-\gamma^2 & -\gamma \beta \\
-\gamma \beta & 2\beta^2-\gamma^2
\end{bmatrix}
\begin{bmatrix}
ds \\
de - dt
\end{bmatrix}
\]

(2.47)

Where \(\Delta' = (\beta^2-\gamma^2)(4\beta^2-\gamma^2) > 0\)

And the effect on prices is

\[
\begin{bmatrix}
\frac{dP_1}{dP_1} \\
\frac{dP_2}{dP_2}
\end{bmatrix} = \frac{\beta}{4\beta^2-\gamma^2}
\begin{bmatrix}
-2\beta & -\gamma \\
-\gamma & -2\beta
\end{bmatrix}
\begin{bmatrix}
ds \\
de - dt
\end{bmatrix}
\]

(2.48)

The comparative static results all have the expected signs.

\[
c_2 + t - e < \left(1 - \frac{\gamma \beta}{2\beta^2-\gamma^2}\right)\alpha + \frac{\gamma \beta}{2\beta^2-\gamma^2} (c_1 - s) \quad (X > 0)
\]
2.31 Foreign Export Subsidy

This section considers the effect of a foreign export subsidy on foreign welfare in the absence of any domestic trade policy. Eaton and Grossman (1986) have shown that with Bertrand competition there is no profit shifting argument for an export subsidy. For a Bertrand duopoly the domestic and foreign price are usually strategic complements, then an export subsidy commits the foreign firm to a lower price and the response of the domestic firm is to lower its price to the disadvantage of the foreign country. Whereas, for a Cournot duopoly domestic and foreign output are usually considered to be strategic substitutes, and the domestic firm reduces its output in response to the foreign export subsidy which is to the advantage of the foreign country. The effect of an export subsidy on foreign welfare (2.5) is

\[
\frac{\partial W}{\partial e} = (P_2 - c_2) \frac{\partial X}{\partial e} + X \frac{\partial P}{\partial e}
\]

(2.49)

The first term is the profit shifting effect and the second term is the terms of trade effect. Using the comparative static results from (2.47) and (2.48) together with the foreign firm's first order condition from (2.46) to evaluate at \(e = 0\), yields
\[ \frac{\partial W_2}{\partial e} = \frac{-X \gamma^2}{4\beta^2 - \gamma^2} < 0 \quad (2.50) \]

As Eaton and Grossman (1986) have shown a foreign export subsidy has a negative effect on foreign welfare. Setting \( \frac{\partial W_2}{\partial e} = 0 \) yields the optimal export subsidy

\[ e = \frac{-X(\beta^2 - \gamma^2)\gamma^2}{(2\beta^2 - \gamma^2)\beta} < 0 \quad (2.51) \]

The optimal foreign policy is an export tax. There is no profit shifting argument for an export subsidy under Bertrand duopoly.

The effect of a foreign export subsidy on domestic welfare (2.4) is

\[ \frac{\partial W_1}{\partial e} = -X \frac{\partial \mathcal{P}}{\partial e} + \frac{(P_1 - c_1) \partial Y}{\partial e} \quad (2.52) \]

The first term is the terms of trade effect which is positive and the second term is the profit shifting effect which is negative. Consumers benefit from lower prices but the profits of the domestic firm are reduced. The overall effect is obtained by using the comparative static results from (2.47) and (2.48) together with (2.46), which yields

\[ \frac{\partial W_1}{\partial e} = \frac{\beta(2\beta X - \gamma Y)}{4\beta^2 - \gamma^2} \quad (2.53) \]
The domestic country will gain unless domestic production is much larger than imports, and is more likely to gain under Bertrand than under Cournot competition. But, the optimal foreign policy is to tax exports which is likely to reduce domestic welfare.

2.32 Foreign Export Subsidy, Domestic Import Tariff and Production Subsidy

This section considers the trade policy game when the foreign country sets its export subsidy at the first stage, then at the second stage the domestic country sets its import tariff and production subsidy, and in the final stage the domestic and foreign firm set their prices. The optimal tariff and production subsidy for Bertrand duopoly has been derived by Cheng (1988). At the second stage the domestic country sets its import tariff and production subsidy to maximise domestic welfare (2.4) in response to the foreign export subsidy. Maximising domestic welfare (2.4) with respect to \( t \) and \( s \), assuming there is an interior solution, yields the first order conditions\(^\text{14}\)

\(^{14}\)It can be shown that there will be an interior solution if:

\[
c_2 - e < \left(1 - \frac{\gamma}{\beta}\right) \alpha + \frac{\gamma}{\beta} c_1 \quad (X > 0)
\]

\[
c_1 < \left(1 - \frac{\gamma \beta}{3 \beta^2 - 2 \gamma^2}\right) \alpha + \frac{\gamma \beta}{3 \beta^2 - 2 \gamma^2} (c_2 - e) \quad (Y > 0)
\]
\[
\frac{\partial \Pi}{\partial t} = -X \left( \frac{\partial P}{\partial t} - 1 \right) + (P_1 - c_1) \frac{\partial Y}{\partial t} + t \frac{\partial X}{\partial t} = 0
\]

\[
\frac{\partial \Pi}{\partial s} = -X \frac{\partial P}{\partial s} + (P_1 - c_1) \frac{\partial Y}{\partial s} + t \frac{\partial X}{\partial s} = 0
\]

(2.54)

Using the comparative static results from (2.47) and (2.48) yields

\[
\begin{bmatrix}
\gamma \beta^2 & -(2\beta^2 - \gamma^2) \beta \\
(2\beta^2 - \gamma^2) \beta & -\gamma \beta^2
\end{bmatrix}
\begin{bmatrix}
P_1 - c_1 \\
t
\end{bmatrix}
= \begin{bmatrix}
-(2\beta^2 - \gamma^2)(\beta^2 - \gamma^2) X \\
-\beta \gamma (\beta^2 - \gamma^2) X
\end{bmatrix}
\]

Hence, the optimal policies are

\[
P_1 - c_1 = 0 \quad \Rightarrow \quad s = \frac{\beta^2 - \gamma^2}{\beta} Y
\]

(2.55)

\[
t = \frac{\beta^2 - \gamma^2}{\beta} X = \frac{1}{2} (P_2 - c_2 + e)
\]

As in the case of Cournot oligopoly the production subsidy is used to correct the domestic distortion and the tariff to extract rent from the foreign firm. Cheng (1988) derives explicit solutions for the optimal policies and shows that the optimal tariff and production subsidy are smaller under Bertrand duopoly than under Cournot duopoly.

Again, if the domestic country sets its import tariff and production subsidy optimally it will always gain from a
foreign export subsidy.

The effect of the foreign export subsidy on the optimal domestic tariff and production subsidy is obtained by totally differentiating the optimal policies which yields

\[
\frac{dt}{\beta} = \frac{\beta^2 - \gamma^2}{\beta} \left( \frac{\delta x}{\delta t} dt + \frac{\delta x}{\delta s} ds + \frac{\delta x}{\delta e} de \right)
\]

\[
\frac{ds}{\beta} = \frac{\beta^2 - \gamma^2}{\beta} \left( \frac{\delta y}{\delta t} dt + \frac{\delta y}{\delta s} ds + \frac{\delta y}{\delta e} de \right)
\]

Using the comparative static results from (2.47) gives

\[
\begin{bmatrix}
2(3\beta^2 - \gamma^2) \\
-\gamma \beta
\end{bmatrix}
\begin{bmatrix}
dt/de \\
ds/de
\end{bmatrix}
= 
\begin{bmatrix}
2\beta^2 - \gamma^2 \\
-\gamma \beta
\end{bmatrix}
\]

Hence, the optimal countervailing fractions are

\[
\frac{dt}{de} = \frac{1}{3} > 0 \\
\frac{ds}{de} = \frac{-\gamma}{3\beta} < 0
\]

The optimal domestic response to a foreign export subsidy is to increase the tariff and reduce the production subsidy. The optimal countervailing tariff fraction for Bertrand duopoly, \( dt/de = 1/3 \), is smaller than for Cournot duopoly, \( 1/3 < dt/de < 1/2 \), from equation (2.22).

At the first stage of the game the foreign country sets its
export subsidy to maximise national welfare realising the
effect its decision will have on the optimal domestic tariff
and production subsidy. The effect of a foreign export
subsidy on foreign welfare (2.5) is

\[
\frac{dW^2}{de} = (P_2 - c_e - t) \left( \frac{\partial X}{\partial e} + \frac{\partial X}{\partial t} \frac{dt}{de} + \frac{\partial X}{\partial s} \frac{ds}{de} \right) 
\]

\[
+ X \left( \frac{\partial P_2}{\partial e} + \left( \frac{\partial P_2}{\partial t} - 1 \right) \frac{dt}{de} + \frac{\partial P_2}{\partial s} \frac{ds}{de} \right) (2.57)
\]

The countervailing tariff reduces the profit shifting effect
of the export subsidy and worsens the terms of trade.
Whereas, the reduction in the production subsidy increases
the profit shifting effect and improves the terms of trade.
Using the comparative static results from (2.47) and (2.48)
together with (2.46), and the optimal countervailing
responses from (2.56) to evaluate at \( e = 0 \), yields

\[
\frac{dW^2}{de} = -\frac{X}{3} < 0 
\]

(2.58)

The foreign export subsidy has a negative effect on foreign
welfare. The optimal export subsidy is obtained by setting
\( dW^2/de = 0 \)

\[
e = -\frac{X(\beta^2 - r^2)}{\beta} \leq 0
\]

(2.59)
The optimal foreign policy is an export tax. The expression for the optimal export tax is the same as under Cournot oligopoly, see (2.25). When the foreign country faces retaliation with countervailing tariffs and subsidies, there is no profit shifting argument for an export subsidy under Bertrand or Cournot competition.

2.33 Foreign Export Subsidy and Domestic Import Tariff

In this section the domestic country is assumed to use only an import tariff to countervail the foreign export subsidy. At the second stage of the game the domestic country sets its tariff to maximise national welfare given the foreign export subsidy. Maximising domestic welfare (2.4) with respect to \( t \), assuming an interior solution, yields the first order condition\(^{15}\)

\[
\frac{\partial W}{\partial t} = -X \left( \frac{\partial P}{\partial t} - 1 \right) + (P_1 - C_1) \frac{\partial Y}{\partial t} + t \frac{\partial X}{\partial t} = 0 \quad (2.60)
\]

Rearranging, yields the optimal tariff

\(^{15}\)It can be shown that there will be an interior solution if:

\[
c_2 - e < \left(1 - \frac{\gamma \beta}{2\beta^2 - \gamma^2}\right) \alpha + \frac{\gamma \beta}{2\beta^2 - \gamma^2} C_1 \quad (X > 0)
\]

\[
c_1 < \left(1 - \frac{\gamma \beta}{3\beta^2 - 2\gamma^2}\right) \alpha + \frac{\gamma \beta}{3\beta^2 - 2\gamma^2} (c_2 - e) \quad (Y > 0)
\]
The tariff improves the terms of trade and shifts profits from foreign to domestic firms. Bertrand competition does not alter the profit shifting role of the tariff. Using the comparative static results from (2.47) and (2.48) together with (2.46) yields the optimal tariff

$$t = \left[ x \left( \frac{\partial p^2}{\partial t} - 1 \right) - (p_1 - c_1) \frac{\partial y}{\partial t} \right] \frac{\partial x}{\partial t} \tag{2.61}$$

The optimal tariff is positive. The first term in brackets is the terms of trade effect and the second term is the profit shifting effect.

Again, when the domestic country sets its import tariff optimally it will always gain from a foreign export subsidy.

The effect of a foreign export subsidy on the optimal domestic tariff is obtained by totally differentiating (2.62)

$$dt = \frac{\beta^2 - \gamma^2}{\beta} \left[ \left( \frac{\partial x}{\partial t} + \frac{\gamma \beta}{2\beta^2 - \gamma^2} \frac{\partial y}{\partial t} \right) dt + \left( \frac{\partial x}{\partial e} + \frac{\gamma \beta}{2\beta^2 - \gamma^2} \frac{\partial y}{\partial e} \right) de \right]$$

Using the comparative static results from (2.47) yields the optimal countervailing tariff fraction
\[
\frac{dt}{de} = \frac{\beta^2 - \gamma^2}{\beta^2 + 2(\beta^2 - \gamma^2)} > 0 \quad (2.63)
\]

The optimal countervailing tariff fraction is positive but less than one third, this is smaller than under Cournot duopoly, see equation (2.32), where it is exactly one third.

At the first stage of the game the foreign country sets its export subsidy to maximise its national welfare realising the effect its decision will have on the optimal domestic tariff. The effect of a foreign export subsidy on foreign welfare (2.5) is

\[
\frac{dW}{de} = (P_2 - c^2 - t) \left( \frac{\partial X}{\partial e} + \frac{\partial X}{\partial t} \frac{dt}{de} \right) + X \left( \frac{\partial P}{\partial e} + \left( \frac{\partial P}{\partial t} - 1 \right) \frac{dt}{de} \right) \quad (2.64)
\]

The first term is the profit shifting effect of the export subsidy and the second term is the terms of trade effect. The countervailing tariff reduces the profit shifting effect of the export subsidy and worsens the terms of trade. Using the comparative static results from (2.47) and (2.48) together with (2.46), and the optimal countervailing tariff fraction (2.63) to evaluate at \( e = 0 \), yields

\[
\frac{dW}{de} = \frac{-X\beta^2(4\beta^2 - 3\gamma^2)}{(4\beta^2-\gamma^2)(\beta^2 + 2(\beta^2 - \gamma^2))} < 0 \quad (2.65)
\]
A foreign export subsidy reduces foreign welfare. This is not surprising since an export subsidy reduces welfare even without a countervailing tariff. The optimal export subsidy is obtained by setting \( \frac{dW_2}{de} = 0 \) which yields:

\[
e = \frac{-X\beta(4\beta^2-3\gamma^2)(\beta^2-\gamma^2)}{(2\beta^2-\gamma^2)^2} < 0 \tag{2.66}
\]

The optimal foreign policy is an export tax, as with Cournot oligopoly. So again there is no profit shifting argument for an export subsidy under Bertrand or Cournot competition.

### 2.34 Foreign Export Subsidy and Domestic Production Subsidy

In this section the domestic country is assumed to use only a production subsidy to countervail the foreign export subsidy. Maximising domestic welfare (2.4) with respect to \( s \), assuming an interior solution, yields the first order condition\(^\text{16}\)

\[
c^2 - e < \left(1 - \frac{\gamma}{\beta}\right)\alpha + \frac{\gamma}{\beta} c_1 \quad (X > 0)
\]

\[
c_1 < \left(1 - \frac{\gamma\beta}{2\beta^2-\gamma^2}\right)\alpha + \frac{\gamma\beta}{2\beta^2-\gamma^2} (c^2 - e) \quad (Y > 0)
\]

\(^{16}\)It can be shown that there will be an interior solution if:
Using the comparative static results from (2.47) and (2.48), yields

\[ P_1 - c_1 = -X \gamma \frac{\beta^2 - \gamma^2}{2\beta^2 - \gamma^2} < 0 \]  

(2.68)

The optimal domestic policy is to subsidise domestic production so that price is below marginal cost because the production subsidy improves the terms of trade. Solving for the optimal production subsidy using (2.46) yields

\[ s = \frac{\beta^2 - \gamma^2}{\beta} \left( Y + \frac{\gamma \beta}{2\beta^2 - \gamma^2} X \right) \]  

(2.69)

Again, when the domestic country sets its production subsidy optimally a foreign export subsidy will always increase domestic welfare.

The effect of a foreign export subsidy on the optimal domestic production subsidy is obtained by totally differentiating (2.69), which yields

\[ ds = \frac{\beta^2 - \gamma^2}{\beta} \left[ \left( \frac{\partial Y}{\partial s} + \frac{\gamma \beta}{2\beta^2 - \gamma^2} \frac{\partial X}{\partial s} \right) ds + \left( \frac{\partial Y}{\partial e} + \frac{\gamma \beta}{2\beta^2 - \gamma^2} \frac{\partial X}{\partial e} \right) de \right] \]

Using the comparative static results from (2.47) yields
The optimal domestic response to a foreign export subsidy is zero. The optimal domestic production subsidy is independent of the foreign export subsidy.

The effect of a foreign export subsidy on foreign welfare is

\[
\frac{\partial W_2}{\partial e} = (P_2 - c_2) \left( \frac{\partial X}{\partial e} + \frac{\partial X}{\partial ds} \right) + X \left( \frac{\partial P_2}{\partial e} + \frac{\partial P_2}{\partial ds} \right)
\]

(2.71)

The effect of the export subsidy on foreign welfare is the same as when there was no domestic tariff or production subsidy. Using the comparative static results from (2.47) and (2.48) together with (2.46) and (2.70) to evaluate at \( e = 0 \) yields

\[
\frac{dW_2}{de} = \frac{-X\gamma^2}{4\beta^2 - \gamma^2} < 0
\]

(2.72)

A foreign export subsidy increases foreign welfare. Setting \( \frac{dW_2}{de} = 0 \) yields the optimal export subsidy

\[
e = \frac{-X(\beta^2 - \gamma^2)\gamma^2}{(2\beta^2 - \gamma^2)\beta} < 0
\]

(2.73)

The optimal export subsidy is positive. When the domestic
country uses a production subsidy to countervail the foreign export subsidy, the optimal foreign policy is an export tax under Bertrand duopoly.

2.4 Conclusions

This chapter has modelled retaliation as a multistage game, where the foreign country sets its export subsidy in the first stage, and the domestic country responds with a tariff and/or a production subsidy in the second stage. Demand was assumed to be linear, with nationally differentiated products, and both Cournot oligopoly and Bertrand duopoly were considered.

When the domestic country pursues a policy of laissez-faire, a foreign export subsidy may reduce domestic welfare. A foreign export subsidy is likely to be a beggar-my-neighbour policy, increasing foreign welfare at the expense of domestic welfare. However, when the domestic country pursues an optimal trade policy it will always gain from a foreign export subsidy. The optimal domestic response to a foreign export subsidy, when the domestic country uses a tariff and a production subsidy, is to increase the tariff and reduce the production subsidy. When the domestic country uses only a tariff, the optimal response is usually to increase the tariff, but a reduction may be optimal. The optimal countervailing tariff fractions are larger under Cournot
than Bertrand competition, and are always less than or equal to a half. As Dixit (1988) has argued only partially countervailing tariffs are justified. When the domestic country uses only a production subsidy, the optimal response is ambiguous, but for duopoly it should not alter its subsidy.

The foreign country realises the effect its export subsidy will have on the optimal domestic trade policy, and anticipates such a response when it sets its export subsidy. When the domestic country uses a tariff and a production subsidy, a foreign export subsidy will reduce foreign welfare, and the optimal policy is to tax exports. Similarly, when the domestic country uses only a tariff, a foreign export subsidy will reduce foreign welfare. When the domestic country uses only a production subsidy, the effect of a foreign export subsidy on foreign welfare is ambiguous. Facing retaliation with countervailing tariffs, there is no profit shifting argument for an export subsidy, and the foreign country should commit itself not to subsidise exports.
Chapter 3: International Trade and Cournot Equilibrium: Existence, Uniqueness and Comparative Statics

3.1 Introduction

This chapter will prove the existence and uniqueness of equilibrium in the Dixit (1984) model of international trade under Cournot oligopoly, and will derive the comparative static results for the effects of trade taxes and subsidies. This basic model will be used in chapter four to analyse the effect of retaliation on the strategic argument for export subsidies. The existence of the Cournot equilibrium will be established using a proof, based on McManus (1962, 1964), which does not require the usual assumption that profit functions are concave. A simple proof will be used to establish the uniqueness of the equilibrium. The assumptions required to prove existence and uniqueness will be used to sign the comparative static results for the effects of trade policy. These comparative static results are needed to model trade policy as a multistage game in chapter four.

Models of international trade under imperfect competition have frequently employed the concept of Cournot equilibrium. They have been used to explain intra-industry trade, Brander (1981), Brander and Krugman (1983) etc, and to analyse trade policy, Brander and Spencer (1984a, 1984b, 1985), Dixit (1984, 1988) etc. These models usually have two countries,
the domestic and the foreign country, and in each there are
a number of firms that compete in both markets. It is
assumed that marginal costs are constant and that markets
are segmented, then firms are engaged in two independent
games, one in the domestic market and one in the foreign
market. Hence, the game in one market can be analysed
separately from the game in the other market. Also, the
models are assumed to be symmetric, in the sense that in
each country all firms have identical costs. The question of
the existence and uniqueness of equilibrium in these models
has largely been ignored.

The usual proof of the existence of a Cournot equilibrium,
assumes that each firm's profit function is concave so that
a standard proof for the existence of equilibrium in concave
games can be applied. This approach is adopted by Myles
(1988) in a model of international trade under oligopoly. An
alternative proof by McManus (1962, 1964) does not require
the profit functions to be concave, but assumes that all
firms have identical cost functions. Here, a proof is
developed which adapts the method used by McManus to exploit
the symmetry of these models. The existence of equilibrium
is established under weaker conditions than those required
to obtain concave profit functions. The usual method to
prove the uniqueness of the Cournot equilibrium is to use
the Gale-Nikaidô theorem on univalent mappings. Recently,
Kolstad and Mathiesen (1987) have developed a necessary and
sufficient condition for uniqueness. Here, a much simpler proof will be used to establish uniqueness.

In most models of international trade under Cournot oligopoly the Hahn (1961-62) stability condition is used to sign the comparative static results, despite the irrelevance of stability. This requires the reaction functions to be downward sloping, which rules out the interesting comparative static results which occur with upward sloping reaction functions. The comparative static results derived here are signed without any assumptions, other than those used to establish the existence and uniqueness of the equilibrium. The definitions of strategic substitutes and complements are introduced and used to discuss the comparative static results. Then, the possibility of the price overshifting of a tariff, where price increases by more than the amount of the tariff, and profit overshifting, where profits increase due to a tariff, are discussed.

The Cournot oligopoly model is interpreted as a static, single shot, game of complete information, where firms simultaneously and independently choose their outputs. At a Cournot equilibrium, a Nash equilibrium in quantities, each firm chooses an output which is the optimal response to the output choices of all other firms. The Cournot equilibrium is the only rational solution of this game.\(^1\) Any other

\(^1\)Johansen (1982) argues that if there is a natural solution
output choices could not be an equilibrium, since then at least one firm would not be using its optimal response to the output choices of the other firms, which is not rational. It is often argued that a Cournot equilibrium involves naive or myopic behaviour by firms, they take the output of competitors as fixed and do not realise the effect their action will have on the output of their competitors. But each firm does not know the output of its competitors. It must predict their output, given that they are also maximising profits, so that it can calculate its optimal response. This is a complicated problem since the decision-making of one firm is related to that of all other firms. In effect each firm must determine the Cournot equilibrium. At the Cournot equilibrium all firms have correctly predicted the output of their competitors and their output is the optimal response to the correctly predicted outputs of competitors. Thus, far from assuming naive or myopic behaviour, the Cournot equilibrium involves sophisticated behaviour by firms. For this reason the uniqueness of the equilibrium is an important property. If there are two, or more, Cournot equilibrium then firms cannot predict the output of their competitors, and it is then difficult to argue that the Cournot equilibrium is the rational solution of the game.

concept for non-cooperative games then it must be the Nash equilibrium.
Bertrand's criticism of the Cournot equilibrium, that firms actually choose prices rather than outputs, is a criticism of the model and not the equilibrium concept. In Bertrand oligopoly firms choose prices and the rational solution to the game is the Nash equilibrium in prices. Kreps and Scheinkman (1983) have shown that a two stage game, where firms choose capacities at the first stage and then prices in the second stage, yields the Cournot outcome. Therefore, it can be argued that that the Cournot oligopoly model represents a reduced form of this capacity-price game.2

Oligopoly is often modelled using conjectural variations, these are the conjectures of a firm about how its competitors will react to a change in its output. And, Bresnahan (1981) has argued that conjectures should be consistent, that is a firm's conjecture about how a competitor will react should equal the slope of the competitor's reaction function. Obviously, as the solution to a static oligopoly model such ideas make no sense. In a static game, in which firms independently and simultaneously choose outputs, firms cannot react to the output choices of their competitors. It is often argued that such models represent a reduced form of some dynamic oligopoly model, but there is no basis for this assertion. Another spurious

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2 In international trade, Ben-Zvi and Helpman (1988) and Venables (1988) analyse a two stage game where worldwide capacity is chosen at the first stage, and then prices for each market in the second stage.
concept is the stability of the Cournot equilibrium, which in a static model is obviously meaningless since stability is a dynamic property. But, even in a multi-period model it is a doubtful concept, since it requires firms to be out of equilibrium, and there is no reason for firms to produce any output other than the equilibrium output.

3.2 The Model

There are two countries: the domestic and the foreign country. In the domestic country there are \( n \) identical firms each with constant marginal cost \( c_1 \), and in the foreign country there are \( m \) identical firms each with constant marginal cost \( c_2 \). The domestic and foreign markets are segmented so there is no possibility of arbitrage between them, and so there can be price discrimination. Since marginal cost is constant and markets are segmented, the firms are involved in two independent games, one in the domestic market and one in the foreign market, which can be analysed separately. Consider the domestic market, where the inverse demand function is \( P = P(Q) \). The output of the \( i \)th domestic firm, for domestic consumption, is \( y_i \), the output of all domestic firms but firm \( i \) is \( Y_{-i} \) and total domestic industry output is \( Y \), so \( Y = y_i + Y_{-i} \). Similarly, define the exports, to the domestic market, of the \( i \)th foreign firm as \( x_i \), the exports of all foreign firms but firm \( i \) as \( X_{-i} \) and total foreign exports as \( X \), so \( X = x_i + X_{-i} \). Total domestic
consumption is \( Q = X + Y \), foreign exports plus domestic production. The profits of domestic and foreign firms from sales in the domestic market are

\[
\pi_{1i}(y, x) = (P - c_1)y_i \quad i = 1, \ldots, n
\]

(3.1)

\[
\pi_{2i}(y, x) = (P - c_2)x_i \quad i = 1, \ldots, m
\]

(3.2)

At a Cournot equilibrium, a Nash equilibrium in quantities, each firm's output is an optimal response to the output of all other firms. Therefore a Cournot equilibrium is a vector of outputs \((y^c, x^c) = (y_1^c, \ldots, y_n^c, x_1^c, \ldots, x_m^c)\) such that

\[
\pi_{1i}(y^c, x^c) = \max_{y_i \geq 0} \pi_{1i}(y_1^c, \ldots, y_i^c, \ldots, y_n^c, x_i^c, \ldots, x_m^c) \quad i = 1, \ldots, n
\]

(3.3)

\[
\pi_{2i}(y^c, x^c) = \max_{x_i \geq 0} \pi_{2i}(y_1^c, \ldots, y_i^c, \ldots, y_n^c, x_1^c, \ldots, x_m^c) \quad i = 1, \ldots, m
\]

It will be assumed that the inverse demand function, and hence since marginal cost is constant, profits are twice continuously differentiable. A necessary condition for a Cournot equilibrium is therefore

\[
\frac{\partial \pi_{1i}}{\partial y_i} = P + y_iP' - c_i \leq 0, \quad y_i \geq 0, \quad \frac{\partial \pi_{1i}}{\partial y_i} y_i = 0 \quad i = 1, \ldots, n
\]

(3.3)

\[
\frac{\partial \pi_{2i}}{\partial x_i} = P + x_iP' - c_i \leq 0, \quad x_i \geq 0, \quad \frac{\partial \pi_{2i}}{\partial x_i} x_i = 0 \quad i = 1, \ldots, m
\]
For an interior solution, where all firms produce a positive output, these reduce to the usual first order conditions. Further assumptions will have to be made to ensure the existence and uniqueness of a Cournot equilibrium.

3.3 Existence and Uniqueness

A Cournot equilibrium is a pure strategy Nash equilibrium in quantities, so the existence problem is similar to that for any Nash equilibrium in pure strategies. According to Dasgupta and Maskin (1986) there are two reasons for the possible non-existence of pure strategy Nash equilibrium: if the payoff function is not continuous or if it is not quasi-concave. For Cournot equilibrium the payoff (profit) function is continuous but may not be quasi-concave. Therefore, it is the profit function not being quasi-concave which maybe the cause of non-existence of the Cournot equilibrium. If a Cournot equilibrium does not exist it should not be concluded that the model has no equilibrium. Dasgupta and Maskin have shown that a mixed strategy equilibrium exists for most games, even if payoff functions are discontinuous.\(^3\)

There are three methods to prove the existence of a Cournot equilibrium

\(^3\)If payoff functions are continuous, as in Cournot oligopoly, then a mixed strategy Nash equilibrium exists, see Theorem 3 of Dasgupta and Maskin (1986).
equilibrium each using different assumptions about demand and cost functions. One approach by Frank and Quandt (1963) is to assume that each firm's profit function is concave in its own output, so that a standard existence proof can be applied. Szidarovszky and Yakowitz (1977) assume that the inverse demand function is concave, \( P'' < 0 \), and cost functions are convex, which yields concave profit functions. The assumption that the demand function is concave, rules out many interesting comparative static results which occur with convex demand. But, this assumption is stronger than required to obtain concave profit functions, from (3.1) the second derivatives of the profit functions are

\[
\frac{\partial^2 \pi^{i}}{\partial y_i} = 2P' + y_iP'' \quad i = 1, \ldots, n
\]

\[
\frac{\partial^2 \pi^{i}}{\partial x_i} = 2P' + x_iP'' \quad i = 1, \ldots, m
\]  

(3.4)

Therefore, each firm's profit function will be everywhere concave in its own output if \( 2P' + QP'' < 0 \), so a Cournot equilibrium exists provided demand is not too convex, but this is still a fairly strong assumption. The assumption that profits are concave could be replaced with the assumption that they are quasi-concave, without affecting

---

4For example, Theorem 1 of Dasgupta and Maskin (1986): There exists a pure strategy Nash equilibrium if the strategy set is non-empty, convex and compact, and the payoff function is continuous and quasi-concave.
the proof, but it is not clear what this implies for the shape of demand functions.

The second approach, due to McManus (1962, 1964), does not require the profit functions to be quasi-concave. It assumes that all firms have identical and convex costs, but imposes no restrictions on demand functions, except that it is a non-increasing function and total revenue is bounded. Without the assumption of quasi-concavity of profit functions, the reaction functions need not be continuous so fixed point theorems cannot be applied in the normal way. McManus shows that the cumulative reaction correspondence is non-decreasing, hence any discontinuities must be jumps upwards, and in this way is able to show that a symmetric equilibrium exists.

A more recent approach by Novshek (1985) does not require cost functions to be convex, in this way it is less restrictive than previous approaches, but it assumes that the demand function is such that $P' + QP'' < 0$. This assumption implies that each firm's marginal revenue be everywhere decreasing in the output of other firms, which is the same as the Hahn (1961-62) stability condition. And, this implies that reaction functions are downward sloping for all firms, and rules out the possibility of upward

---

5Roberts and Sonnenschein (1976) have used the same proof, as they acknowledge in Roberts and Sonnenschein (1977).
sloping reaction functions. When all firms have constant marginal costs, this proof is more restrictive than assuming that profit functions are concave.

Proofs that the Cournot equilibrium is unique have generally used the Gale-Nikaidó (1965) theorem for the univalence of mappings. A sufficient condition for uniqueness is that the Jacobian, derived from the first order conditions for profit maximisation, is a P-matrix, all the principal minors are positive. This is restricted to equilibrium in the interior of the strategy space, and does not apply to equilibrium where some firms produce zero output. This condition for uniqueness is equivalent to the Seade (1980a) stability condition. A necessary and sufficient condition for uniqueness has been obtained by Kolstad and Mathiesen (1987) using index analysis.\(^6\) If the Jacobian determinant is positive at all equilibrium then there is a unique Cournot equilibrium, and conversely if the equilibrium is unique then the Jacobian determinant is positive at the equilibrium.

The existence of the Cournot equilibrium will be proved here, without the assumption that profit functions are concave. The proof exploits the symmetry of the model, all firms in each country have identical costs, so although the

\(^6\)See Varian (1984) for a discussion of index analysis, and an application to the uniqueness of equilibrium in the competitive model.
proof by McManus (1962, 1964) is not directly applicable it can be used to show the existence of domestic and foreign industry reaction functions. Despite the fact that reaction functions of individual firms need not be continuous, it can be shown that under certain assumptions the industry reaction functions are continuous. And, since the industry reaction functions are continuous it is possible to show that a Cournot equilibrium exists using a fixed point theorem. A simple proof can then be used to show that the Cournot equilibrium is unique.

To prove the existence and uniqueness of the Cournot equilibrium the following assumptions are required:

(A1) The inverse demand function $P(Q)$ is decreasing, twice continuously differentiable and total revenue, $P(Q).Q$, is bounded.

(A2) The following conditions are satisfied:

$$(n + 1) P'(X+Y) + X P''(X+Y) < 0 \quad \forall X, Y$$

$$(m + 1) P'(X+Y) + Y P''(X+Y) < 0 \quad \forall X, Y$$

(A3) The following condition is satisfied:

$$(n + m + 1) P'(Q) + Q P''(Q) < 0 \quad \forall Q$$

Assumption (A2) ensures that the demand function is not too convex, and it replaces the usual assumption that each
firm's profit function is concave in its own output. For the domestic industry concavity of the profit function requires that \( 2P' + YP'' < 0 \), whereas assumption (A2) requires that \((n+1)P' + YP'' < 0\). When there is a single domestic firm (A2) is equivalent to the concavity of the profit function, but when there is more than one domestic firm it is less restrictive than the concavity assumption. Obviously, to take advantage of the fact that all domestic firms are identical requires that there are at least two firms in the domestic industry. Assumption (A2) can be interpreted as an "aggregate concavity" condition which ensures that the industry reaction functions are continuous, in the same way that concavity ensures that the reaction functions of the firms are continuous. Assumption (A3) is the necessary and sufficient condition for uniqueness of Kolstad and Mathiesen (1987) and also the stability condition of Seade (1980a).

**Theorem:** For a homogeneous product Cournot oligopoly with inverse demand function \( P(Q) \) and \( n \) identical domestic firms each with constant marginal cost \( c_1 > 0 \) and \( m \) identical foreign firms each with constant marginal cost \( c_2 > 0 \). If assumptions (A1), (A2) and (A3) are satisfied, then there exists a unique and symmetric Cournot equilibrium.

Note that (A1) implies that there exists a \( \bar{Q} \) such that price is below marginal cost for all firms if \( Q \geq \bar{Q} \), this follows from the fact that total revenue is bounded and \( P(Q) \) is
decreasing. Let the strategy set of the firms be \([0, \bar{Q}]\), which is non-empty, convex and compact.

Proof. First it is necessary to show that a domestic industry equilibrium exists. That is, for any given \(X\), there exists a \((y_1^*, \ldots, y_n^*)\) such that each domestic firm is setting its output optimally. The proof closely follows McManus (1962, 1964). Define the cumulative reaction correspondence, 
\[ Y = r(Y_{-1}, X), \]
as the values of domestic industry output, \(Y\), when firm \(i\) chooses optimally as a function of the output of all other domestic firms, \(Y_{-1}\), for given foreign exports, \(X\). The cumulative reaction correspondence may not be continuous since profit functions need not be concave, so it is not possible to prove existence in the usual way. However, it can be shown that the cumulative reaction correspondence is non-decreasing, so any discontinuities must be jumps upwards. For \(Y_{-1} = 0\) then \(Y \geq 0\) and for \(Y_{-1} = \bar{Q}\) then \(Y = \bar{Q}\), since the optimal output of firm \(i\) is zero, \(y_i^* = 0\). Hence, it must intersect the line \(Y = \frac{n}{n-1} Y_{-1}\), at say \(Y^*\) in figure 3.1. This yields a symmetric domestic industry equilibrium where each domestic firm produces \(y^* = Y^*/n\). Then \(Y_{-1} = (n-1)y^*\) and the optimal response for each domestic firm is to produce \(y^*\) so that \(Y = ny^*\).

To prove the cumulative reaction correspondence is non-decreasing, in the sense that \(Y_{-1}^B > Y_{-1}^A\) implies \(\min r(Y_{-1}^B, X) \geq \max r(Y_{-1}^A, X)\), define \(Y^A = \max r(Y_{-1}^A, X)\) and
\[ y^A_i = Y^A_i - Y^A_{i-1} \]. Then, \( y^A_i \) is a profit maximising output for firm \( i \) when the rest of the domestic industry produces \( Y^A_{i-1} \), domestic industry output is \( Y^A \), and price is \( P^A = P(Y^A + X) \).

Define \( Y^B = \min \{Y^B_i, X\} \) and \( y^B_i = Y^B_i - Y^B_{i-1} \). Then, \( y^B_i \) is a profit maximising output for firm \( i \) when the rest of the domestic industry produces \( Y^B_{i-1} \), domestic industry output is \( Y^B \), and price is \( P^B = P(Y^B + X) \). If \( y^A_i = 0 \) then \( Y^B > Y^A \) if \( Y^B_{i-1} > Y^A_{i-1} \), and the cumulative reaction correspondence is clearly non-decreasing. For \( y^A_i > 0 \), let \( y^B_i \) be such that \( Y^B_{i-1} < y^B_i < Y^A \). In situation A the \( i \)th firm could produce \( Y^B - Y^A \) instead of output \( y^A \) and the price would be \( P^B \), but since \( y^A \) is a profit maximising output

\[
(P^A - c_1) y^A_i = (P^B - c_1) (Y^B - Y^A) \quad (3.5)
\]

Similarly in situation B the \( i \)th firm could produce \( Y^A - Y^B \), in which case the price would be \( P^A \), and since \( y^B \) is the profit maximising output

\[
(P^B - c_1) y^B_i = (P^A - c_1) (Y^A - Y^B) \quad (3.6)
\]

Adding together (3.5) and (3.6) and then rearranging yields

\[
(P^A - P^B) (Y^B_{i-1} - Y^A_{i-1}) \geq 0 \quad (3.7)
\]

Since, by assumption, \( Y^B_{i-1} > Y^A_{i-1} \) then \( P^A \geq P^B \) therefore \( Y^B \geq Y^A \), which proves that the cumulative reaction
correspondence is non-decreasing, so any discontinuities must be jumps upwards.

Let \( \bar{X} \) be such that \( P(\bar{X}) = c_1 \). Then if \( X > \bar{X} \), price will be below domestic marginal cost for any level of domestic output, so the optimal output for all domestic firms is obviously zero. For \( X = \bar{X} \), a necessary condition for profit maximisation is

\[
\frac{\partial \pi}{\partial y_i} = P + y_i P' - c_i = 0 \quad i = 1, \ldots, n \quad (3.8)
\]

It can be shown that any equilibrium is symmetric, all domestic firms produce the same output. Consider any two firms, say \( i \) and \( j \). Subtract the first order condition for profit maximisation for firm \( j \) from the first order condition for firm \( i \) yields

\[
(y_i - y_j)P' = 0 \quad (3.9)
\]

Since \( P' < 0 \), it follows that \( y_i = y_j \), which holds for any \( i \) and \( j \). Hence, all domestic firms produce the same output in equilibrium.

Summing (3.8) over all domestic firms yields the following necessary condition for a domestic industry equilibrium

\[
F(Y, X) = nP(Y+X) + YP'(Y+X) - nc_1 = 0 \quad (3.10)
\]
Since \( P(Q) \) is twice continuously differentiable, \( F(Y, X) \) is continuously differentiable. For \( X \leq \bar{X} \), \( F(0, X) \geq 0 \) (since \( P(X) \geq c_1 \)), and \( F(\bar{Q}-X, X) < 0 \) (since \( P(\bar{Q}) < c_1 \)). \( F(Y, X) \) is also decreasing in \( Y \) since, by assumption (A2)

\[
\frac{\partial F}{\partial Y} = (n+1)P' + YP'' < 0 \quad (3.11)
\]

Therefore, for any given \( X \leq \bar{X} \), there is a unique \( Y \) which solves \( F(Y, X) = 0 \). Since it has already been shown that, for any \( X \), there exists a symmetric domestic industry equilibrium and (3.10) which is a necessary condition for an equilibrium has a unique solution, then it follows that (3.10) must be a necessary and sufficient condition for an equilibrium. Hence \( F(Y, X) = 0 \) implicitly defines the domestic industry reaction function, \( Y = f(X) \). Since \( F(Y, X) \) is continuously differentiable, and \( \frac{\partial F}{\partial Y} < 0 \), then by the implicit function theorem \( f(X) \) is continuous. Also by the implicit function theorem

\[
f' = \frac{-\frac{\partial F}{\partial X}}{\frac{\partial F}{\partial Y}} = \frac{-(nP'+YP'')}{(n+1)P'+YP''} \quad (3.12)
\]

The right hand side exists and is continuous, since \( P(Q) \) is twice continuously differentiable and the denominator is non-zero by assumption (A2), hence \( f(X) \) is continuously differentiable.
Hence, the domestic industry reaction function is given by

\[
\begin{align*}
f(X) &= \begin{cases} 
  \{ Y \mid F( Y , X ) = 0 \} & X \leq \bar{X} \\
  \emptyset & X > \bar{X}
\end{cases} 
\end{align*}
\tag{3.13}
\]

Which is defined for \( X \in [0, \tilde{Q}] \). The reaction function is shown in figure 3.2.

By similar arguments it can be shown that a foreign industry equilibrium exists and the foreign industry reaction function is

\[
\begin{align*}
g(Y) &= \begin{cases} 
  \{ X \mid G( Y , X ) = 0 \} & Y \leq \bar{Y} \\
  \emptyset & Y > \bar{Y}
\end{cases} 
\end{align*}
\tag{3.14}
\]

Where \( G(X, Y) = mP(Y+X) + XP'(Y+X) - mc^2 \), and \( \bar{Y} \) is defined such that \( P(\bar{Y}) = c^2 \). In equilibrium all foreign firms export the same output, \( x_i = x_j \) for all \( i \) and \( j \). The reaction function \( g(Y) \) is continuous. For \( Y \leq \bar{Y} \), by the implicit function theorem

\[
g' = \frac{-\partial G/\partial Y}{\partial G/\partial X} = \frac{-(mP' + XP'')}{(m+1)P' + XP''}
\tag{3.15}
\]

At a Cournot equilibrium the domestic industry's output \( Y^c \) must be the optimal response to the foreign industry's
exports \( X^c \), which itself must be an optimal response to \( Y^c \).
That is \( Y^c = f(X^c) \) and \( X^c = g(Y^c) \), or equivalently

\[
Y^c = f(g(Y^c)) = f \circ g(Y^c)
\]  

(3.16)

A Cournot equilibrium exists if \( f \circ g(Y) \) has a fixed point. To prove that it does have a fixed point define the function \( h(Y) = f \circ g(Y) - Y \), for \( Y \in [0, \bar{Q}] \). At a Cournot equilibrium \( f \circ g(Y^c) = Y^c \) so \( h(Y^c) = 0 \). The function \( h(Y) \) is obviously continuous since a composite function of two continuous functions, \( f \) and \( g \), is itself continuous. At \( Y = 0 \), \( h(0) \leq 0 \) and at \( Y = \bar{Q} \), \( h(\bar{Q}) < 0 \). Therefore, by the intermediate value theorem, there must exist a \( Y^c \), \( 0 \leq Y^c < \bar{Q} \), such that \( h(Y^c) = 0 \), which proves that a Cournot equilibrium exists.

The Cournot equilibrium will be unique if \( h(Y) \) is decreasing in \( Y \), \( h'(Y) < 0 \), since then \( h(Y) \) is one-to-one. By the chain rule the derivative of \( h(Y) \) is

\[
h'(Y) = f'(g(Y)).g'(Y) - 1
\]  

(3.17)

For \( Y \leq \bar{Y} \) and \( g(Y) \leq \bar{X} \), using (3.12) and (3.15) yields

\[
h'(Y) = \frac{-P'((n+m+1)P' + QP'')}{((n+1)P' + YP'')((m+1)P' + XP'')} < 0
\]  

(3.18)

This is negative by assumptions (A2) and (A3). For \( Y > \bar{Y} \) then \( g' = 0 \) so \( h' = -1 < 0 \), and for \( g(Y) > \bar{X} \) then \( f' = 0 \) so
h' = -1 < 0. Therefore, h(Y) is clearly decreasing in Y, and hence there is a unique Cournot equilibrium. ■

It has been shown that there exists a unique and symmetric Cournot equilibrium.

3.4 Comparative Statics

This section will derive the comparative static results for the effects of various trade policies in the Cournot oligopoly model of the previous section. In chapter two, with linear demand the comparative static results all had the expected signs, but with non-linear demand a number of interesting comparative static results may occur. A general survey of comparative static results under oligopoly is Dixit (1986). The difficult problem with comparative statics under oligopoly is to sign the results and often stability conditions are used. Brander and Spencer (1984a, 1984b, 1985) and Dixit (1984) use the Hahn (1961-62) stability condition, that each firm's marginal revenue is decreasing in the output of all other firms. This requires reaction functions to be downward sloping and rules out the interesting results which occur if they are upward sloping. A less restrictive stability condition is provided by Seade (1980a). However, since stability is a meaningless idea in a

Other useful references on comparative static results under oligopoly are de Meza (1982), Bulow et al (1985), Katz and Rosen (1985), Levin (1985), and Seade (1985).
static Cournot oligopoly model, it would be theoretically more rigorous to use a condition for uniqueness to sign the results. And, fortunately, the Seade (1980a) stability condition is equivalent to the Kolstad and Mathiesen (1987) condition for uniqueness. Here, no assumptions will be made to sign the comparative static results beyond those required to prove the existence and uniqueness of the Cournot equilibrium.

In order to analyse trade policy, now introduce trade taxes and subsidies into the model of the previous section. Let \( t \) be the specific, per unit, domestic import tariff and \( s \) the specific domestic production subsidy. Let, let \( e \) be the specific foreign export subsidy. Trade taxes and subsidies do not affect the proof of existence since \( c_1 \) and \( c_2 \) could be defined to include any taxes or subsidies. Hence, a unique and symmetric equilibrium exists. Since the equilibrium is symmetric let \( y_i = y \), \( \pi_{1i} = \pi_1 \), \( x_i = x \) and \( \pi_{2i} = \pi_2 \) for all \( i \). Then, the profits of domestic and foreign firms from sales in the domestic market become

\[
\pi_1 = (P - c_1 + s)y
\]

(3.19)

\[
\pi_2 = (P - c_2 - t + e)x
\]

Firstly, consider the situation where foreign firms have a sufficient cost advantage, \( c_1 - s > c_2 + t - e + x^0P' \) where \( x^0 \)
is the exports of a foreign firm when there is no domestic production, so that \( X > \bar{X} \) in equilibrium, and hence there is no domestic production. Then, the Cournot equilibrium is given by the first order condition for profit maximisation by foreign firms.

\[
\frac{\partial \pi_2}{\partial x} = p + xP' - c_2 - t + e = 0 \tag{3.20}
\]

The comparative static results are obtained by total differentiation of the first order condition, and noting that \( mx = X = Q \), which yields

\[
((m+1)P' + QP'')
\]

\[dQ = m \, dt - m \, de \]

Hence, the comparative static results are

\[
\frac{\delta Q}{\delta t} = \frac{m}{(m+1)P' + QP''} < 0 \quad \frac{\delta Q}{\delta e} = \frac{-m}{(m+1)P' + QP''} > 0 \tag{3.21}
\]

A tariff reduces imports and increases the price in the domestic market. The effects of an export subsidy are the opposite of the effects of a tariff. The terms of trade for the domestic country is the cost of imports, net of the tariff, \( P - t \). For the foreign country, the terms of trade is the price it receives for its exports, net of the domestic tariff, \( P - t \). A reduction in \( P - t \) will improve the domestic country's terms of trade and worsen the foreign
country's terms of trade. The effect of a tariff on the terms of trade is

\[
\frac{\partial P}{\partial t} - 1 = \frac{-(P' + QP'')}{(m+1)P' + QP''}
\]  

(3.22)

For the domestic country a tariff will usually improve the terms of trade but it will worsen the terms of trade if \( P' + QP'' > 0 \), then the price increases by more than the amount of the tariff. This is what Seade (1985) has called price overshifting,\(^8\) and it occurs if the relative convexity of demand is less than minus one, \( R = QP''/P' < -1 \). A more intriguing possibility is that the tariff may actually increase the profits of foreign firms. The effect of a tariff on the total profits of the foreign industry, \( \Pi_2 = (P - c_2 - t)X \), is

\[
\frac{\partial \Pi_2}{\partial t} = (P - c_2 - t) \frac{\partial X}{\partial t} + X \frac{\partial P^2}{\partial t}
\]

Using the comparative static results (3.21) yields

\[
\frac{\partial \Pi_2}{\partial t} = \frac{-X(2P' + QP'')}{(m+1)P' + QP''}
\]  

(3.23)

Foreign industry profits will increase if \( 2P' + QP'' > 0 \), which will occur if price overshifting is sufficiently

\(^8\)The possibility of price and profit overshifting was first noted by de Meza (1982).
large. Profit overshifting occurs if the relative convexity of demand is less than minus two, $R < -2$. For constant elasticity demand functions, $R = -(1 + 1/\eta)$ where $\eta$ is the elasticity of demand, so there is always price overshifting of a tariff, and there will be profit overshifting if the elasticity of demand is less than unity. With profit overshifting the tariff moves the foreign industry closer to the output they would produce under collusion.

A similar analysis could be used to derive the comparative static results at an equilibrium where there are no foreign imports. Then, the equilibrium would be given by the first order condition for profit maximisation by domestic firms. These results will not be derived since they will not be required in chapter four.

Now consider the interior solution, where both domestic production and foreign exports are positive. There will be an interior solution if $c_1 - s < c_2 + t - e + x^0P' \ (Y > 0)$ and $c_2 + t - e < c_1 - s + y^0P' \ (X > 0)$, where $y^0$ is the output of a domestic firm when there are no foreign imports. The first order conditions for a Cournot equilibrium are

$$\frac{\partial \Pi_1}{\partial y} = P + yP' - c_1 + s = 0$$

(3.24)

---

9 Profit overshifting cannot occur if profit functions are concave, since this implies that $2P' + QP'' < 0$, see (3.4).
\[
\frac{\partial \pi_2}{\partial x} = P + xP' - c_2 - t + e = 0
\]

The second order conditions for profit maximisation are

\[
\frac{\partial^2 \pi_1}{\partial y^2} = 2P' + yP'' < 0
\]

\[
\frac{\partial^2 \pi_2}{\partial x^2} = 2P' + xP'' < 0
\]

To obtain the comparative static results for the effects of the trade taxes and subsidies on the equilibrium outputs of the firms totally differentiate the first order conditions to obtain

\[
\begin{bmatrix}
(n+1)P' + nyP'' & m(P' + yP'') \\
n(P' + xP'') & (m+1)P' + mxP''
\end{bmatrix}
\begin{bmatrix}
dy \\
dx
\end{bmatrix}
= 
\begin{bmatrix}
-ds \\
dt-de
\end{bmatrix}
\]

The solution is obtained by matrix inversion

\[
\begin{bmatrix}
dy \\
dx
\end{bmatrix}
= \frac{1}{\Delta}
\begin{bmatrix}
(m+1)P' + mxP'' & -m(P' + yP'') \\
-n(P' + xP'') & (n+1)P' + nyP''
\end{bmatrix}
\begin{bmatrix}
-ds \\
dt-de
\end{bmatrix}
\]

(3.25)

Where \(\Delta = ((n+m+1)P' + QP'')P' > 0\).

When discussing the comparative static results for the effects of domestic and foreign trade policy the concept of strategic substitutes and complements will be particularly
Domestic output and foreign exports are strategic substitutes (complements) for the domestic country if an increase in foreign exports decreases (increases) the marginal profitability of of the domestic firms, that is $\delta^2 \pi_1 / \delta x \delta y = P' + yP'' < (>) 0$. With strategic substitutes (complements) aggressive action by the foreign country, which increases foreign exports, will reduce (increase) the marginal profitability of domestic firms and hence lead to a reduction (increase) in domestic output. Similarly, domestic output and foreign exports are strategic substitutes (complements) for the foreign country, if an increase in domestic output decreases (increases) the marginal profitability of the foreign firms, that is $\delta^2 \pi_2 / \delta y \delta x = P' + xP'' < (>) 0$. Strategic complements can only occur if demand is sufficiently convex. Domestic output and foreign exports need not be strategic substitutes, or complements, for both countries. It could be that they are strategic substitutes for the domestic country and strategic complements for the foreign country. For this to be the case foreign exports must exceed domestic output, $x > y$, and demand must be sufficiently convex. They will be strategic substitutes for the domestic (foreign) country if the domestic (foreign) industry reaction function is downward sloping, at equilibrium, and strategic complements if it is upward sloping.\textsuperscript{11} Using the Hahn (1961–62) stability

\textsuperscript{10}See Bulow, Geanakoplos and Klemperer (1985) and also Fudenberg and Tirole (1984).

\textsuperscript{11}From the definitions of strategic substitutes, and
condition to sign comparative static results rules out the possibility of strategic complements.

The effect of an import tariff on domestic output and foreign exports is

\[ \frac{\partial Y}{\partial t} = \frac{-mn(P' + yP'')}{\Delta} \]

\[ \frac{\partial X}{\partial t} = \frac{m((n+1)P' + nyP'')}{\Delta} < 0 \] (3.26)

The tariff reduces imports, foreign exports, and will increase (decrease) domestic industry output if domestic output and foreign exports are strategic substitutes (complements). With strategic complements a tariff has the unexpected effect of reducing domestic industry output. The tariff, by reducing imports, shifts the demand curve facing domestic firms outwards, and with convex demand it also makes the demand curve steeper. Domestic firms now face a less elastic demand curve and find it profitable to increase their price-cost margin rather than increase output. This shows that, under some circumstances, domestic firms may use protection with a tariff, to exploit their monopoly power by reducing output and increasing their profit margins. The effect of the tariff on the domestic price is

complements and the slope of the domestic and foreign reaction functions given by (3.12) and (3.15).
As expected the import tariff increases the price in the domestic market. The effect of a tariff on the terms of trade is

\[ \frac{\partial P}{\partial t} = \frac{m(P')^2}{\Delta} > 0 \quad (3.27) \]

A tariff will usually improve the domestic country's terms of trade, but it will worsen the terms of trade if 

\[ ((n+1)P' + QP'') > 0, \]

then the price increases by more than the amount of the tariff. With domestic production there will be price overshifting of a tariff if the relative convexity of demand, \( R < -(n+1) \), whereas with no domestic production it occurred if \( R < -1 \). Hence, with domestic production a tariff is more likely to improve the terms of trade. A tax imposed on both domestic output and imports would be overshifted if \( R < -1 \), hence a tariff is less likely to be overshifted than a general tax. The effect of a tariff on foreign industry profits is

\[ \frac{\partial \Pi^2}{\partial t} = \left( P - c_2 - t \right) \frac{\partial X}{\partial t} + X \left( \frac{\partial P}{\partial t} - 1 \right) \]

Using the comparative static results yields
\[
\frac{\delta \Pi_2}{\delta t} = \frac{-XP'}{\Delta} \left[ ((n+1)P' + nyP'') + ((n+1)P' + QP'') \right] \quad (3.29)
\]

The first term in square brackets is negative but the second term will be positive if there is price overshifting. A tariff will increase the profits of foreign firms if the price overshifting is sufficiently large. Profit overshifting is less likely with a tariff than with a general tax, when it will occur if \(2P' + QP'' > 0\). Both price and profit overshifting of a tariff are less likely when there is domestic production than when there is no domestic production. This is because the domestic firms are untaxed and so are likely to increase rather than reduce output, which will reduce the profits of the foreign industry. Domestic firms will only reduce output as a result of a tariff if domestic and foreign industry output are strategic complements.

The effect of a production subsidy on domestic output and foreign exports is

\[
\frac{\partial Y}{\partial s} = \frac{-n((m+1)P' + mxP'')}{\Delta} > 0 \\
\frac{\partial X}{\partial s} = \frac{nm(P' + xP'')}{\Delta} \quad (3.30)
\]

12 For constant elasticity demand curves, there is always price overshifting of a general tax and there will be profit overshifting if the elasticity of demand is less than unity.
The production subsidy increases domestic output and will decrease (increase) foreign exports if domestic output and foreign exports are strategic substitutes (complements) for the foreign country. The effect of a production subsidy on price in the domestic market is

$$\frac{\partial P}{\partial s} = -\frac{n(P')^2}{\Delta} < 0$$  \hspace{1cm} (3.31)

As expected the production subsidy reduces the price in the domestic market.

The effect of a foreign export subsidy on domestic output and foreign exports is

$$\frac{\partial Y}{\partial e} = \frac{mn(P' + yP'')}{\Delta}$$  \hspace{1cm} (3.32)

$$\frac{\partial X}{\partial e} = \frac{-m((n+1)P' + nyP'')}{\Delta} > 0$$

The foreign export subsidy increases foreign exports and will decrease (increase) domestic industry output if domestic output and foreign exports are strategic substitutes (complements) for the domestic industry. The use by Brander and Spencer (1985) of the Hahn (1961-62) stability condition rules out the possibility of strategic complements, and hence that a foreign export subsidy may
increase domestic industry output. And, the effect of the foreign export subsidy on price in the domestic market is

\[
\frac{\partial P}{\partial e} = \frac{-m(P')^2}{\Delta} < 0 \quad (3.33)
\]

The export subsidy reduces the price in the domestic market. The effects of an export subsidy are the opposite of the effects of an import tariff.

3.5 Conclusions

A proof of the existence of Cournot equilibrium in models of international trade, such as Dixit (1984), has been developed, which does not make the usual assumption that profit functions are concave. And, a simple proof was used to establish the uniqueness of the equilibrium. The comparative static results have been signed using only the assumptions employed to prove the existence and uniqueness of equilibrium. These results will be used in the following chapter to analyse trade policy.
Figure 3.1: Cumulative Reaction Correspondence
Figure 3.2: Industry Reaction Functions
Chapter 4: Export Subsidies and Countervailing Tariffs

4.1 Introduction

This chapter analyses the effect of retaliation on the profit shifting argument for export subsidies. As in chapter two trade policy is modelled as a multistage game, where the foreign country sets its export subsidy in the first stage, and the domestic country responds with an import tariff and/or production subsidy in the second stage. Here, the basic model is the homogeneous product Cournot oligopoly model analysed in chapter three. Surprisingly, it is shown that when the domestic country uses a tariff and production subsidy, a foreign export subsidy will increase both foreign and domestic welfare. When the domestic country uses only a tariff, a foreign export subsidy will generally reduce foreign welfare.

The effect of a foreign export subsidy on foreign and domestic welfare, when the domestic country pursues a policy of laissez-faire, is considered in section 4.2. A trade policy game when there is no domestic production is considered in section 4.3. Then, the trade policy games are analysed when there is domestic production. In section 4.4 the domestic country can use an import tariff and production subsidy, an import tariff only in section 4.4, and a production subsidy only in section 4.5. In these sections,
the effect of a foreign export subsidy on domestic welfare when the domestic country pursues an optimal trade policy is considered, and the optimal domestic response to a foreign export subsidy is derived. This is used to analyse the effect of a foreign export subsidy on foreign welfare, when the foreign country anticipates the response of the domestic country. And, then the optimal foreign export subsidy is derived. The conclusions are in section 4.7.

The basic model used in this chapter is the same as in chapter three, and in Dixit (1984). In chapter three a proof of the existence and uniqueness of equilibrium was presented, and the comparative static results for the effects of a domestic import tariff, production subsidy and a foreign export subsidy were derived. The comparative static results will be used in this chapter to analyse trade policy. In order to derive the optimal trade policies for the domestic and foreign country, measures of welfare are needed. This requires a further assumption about domestic consumer preferences: Domestic consumers are assumed to have preferences, for Q and a competitive numeraire good, that can be represented by a utility function which is additively separable and linear in the numeraire good. Therefore, the aggregate indirect utility function is of the form

\[ V = V(P) + I \]  

(4.1)
Where $I$ is national income. So by Roy's identity $\frac{\partial V}{\partial P} = -Q$.

Domestic welfare is given by the sum of consumer surplus, domestic industry profits and government revenue. Demand for $Q$ is independent of income, so consumer surplus is a valid welfare measure. Distributional considerations will be ignored. Thus, domestic welfare is

$$W_1 = V(P) + (P - c_1 + s)Y + tX - sY$$

$$= V(P) + (P - c_1)Y + tX$$

(4.2)

Where $(P - c_1)Y$ is domestic producer surplus and $tX$ is tariff revenue.

The foreign and domestic market are segmented and marginal cost is constant, hence the only impact exports have on foreign welfare is the profits they generate. Therefore, foreign welfare can be represented by producer surplus from exports, foreign industry profits net of export subsidies, that is

$$W_2 = \Pi_2 - eX$$

$$= (P - c_2 - t)X$$

(4.3)

With these welfare measures it is now possible to analyse
trade policy.

4.2 Foreign Export Subsidy

This section considers the effect of a foreign export subsidy on foreign and domestic welfare, in the absence of any domestic tariff or production subsidy. Firstly, consider the effect of a foreign export subsidy on foreign welfare. Brander and Spencer (1985) have shown that an export subsidy may increase foreign welfare, by committing foreign firms to increase output so that the foreign industry captures a larger share of total industry profits. The effect of a foreign export subsidy on foreign welfare (4.3) is

\[
\frac{\partial \tilde{W}}{\partial \tilde{e}} = (P - c_2) \frac{\partial X}{\partial \tilde{e}} + X \frac{\partial P}{\partial \tilde{e}} \tag{4.4}
\]

The effect of an export subsidy can be divided into two components. The first term is the profit shifting effect, the subsidy increases exports and since price exceeds marginal cost this has a positive effect on welfare. The second term is the terms of trade effect, the subsidy reduces the price foreign firms receive for their exports which has a negative effect on welfare. The overall effect can be evaluated using the comparative static results from chapter three, (3.32) and (3.33), and the first order condition for profit maximisation by foreign firms, from (3.24) \( P - c_2 = -xP' - e \), in equation (4.4). Evaluating at
\( e = 0, \) yields

\[
\frac{\partial \bar{W}_2}{\partial e} = \frac{mxP'}{(n-m+1)P'+nyP''} \quad (4.5)
\]

An export subsidy will increase foreign welfare if the number of foreign firms is not too large relative to the number of domestic firms and demand is not too convex. Brander and Spencer (1985) consider the case of a Cournot duopoly, with one foreign and one domestic firm, they assume the Hahn stability condition is satisfied so that domestic and foreign output are strategic substitutes, in this case an export subsidy always increases foreign welfare. But, if domestic and foreign output are strategic complements then an export subsidy will reduce foreign welfare, this is the case in a symmetric duopoly with constant elasticity demand functions with elasticity less than unity.\(^1\) Eaton and Grossman (1986) have shown that if there is Bertrand competition rather than Cournot competition then an export subsidy will reduce welfare. The optimal export subsidy can be derived by setting \( \frac{\partial \bar{W}_2}{\partial e} = 0 \) and solving this yields

\[
e = \frac{-xP'((n-m+1)P'+nyP'')}{(n+1)P'+nyP''} \quad (4.6)
\]

In the case of a Cournot duopoly, the optimal export subsidy

\(^1\)As Collie and de Meza (1986) have shown.
shifts the industry equilibrium to where it would be if the foreign firm acted as a Stackelberg leader in the absence of the export subsidy.

The effect of the foreign export subsidy on the profits of the domestic industry, $\Pi_1 = (P - c_i)Y$, is obtained by differentiation with respect to $e$, thus

$$\frac{\partial \Pi_1}{\partial e} = (P - c_i) \frac{\partial Y}{\partial e} + Y \frac{\partial P}{\partial e} \quad (4.7)$$

Evaluating this expression using the comparative static results yields

$$\frac{\partial \Pi_1}{\partial e} = -nmyP' \quad (4.8)$$

A foreign export subsidy will always reduce domestic industry profits, so the domestic industry is always going to be concerned about foreign subsidies. However, the effect of a foreign export subsidy on domestic welfare is not so clear. The effect of a foreign export subsidy on domestic welfare has been briefly considered by Dixit (1984) and by Mai and Hwang (1987). The effect of a foreign export subsidy on domestic welfare (4.2) is given by

$$\frac{\partial W_1}{\partial e} = (P - c_i) \frac{\partial Y}{\partial e} - X \frac{\partial P}{\partial e} \quad (4.9)$$
The export subsidy has two effects on domestic welfare. The first term is the profit shifting effect, an export subsidy reduces domestic output, in the case of strategic substitutes, which reduces domestic welfare since price exceeds marginal cost. With strategic complements the profit shifting effect is positive. The second term is the terms of trade effect, an export subsidy reduces the price of imports thus improving the terms of trade. The reduction in the price of domestically produced goods is a gain to consumers but it also represents a loss of profit for domestic firms, and therefore it has no net effect upon welfare. To evaluate the welfare effect use the comparative static results from chapter three and the first order condition for profit maximisation by domestic firms, from (3.24) \( P - c_1 = -yP' \), in the above expression (4.9) yields

\[
\frac{\partial W}{\partial e} = -\frac{mP'}{\Delta} \left[ (ny-mx)P' + ny^2P'' \right] \quad (4.10)
\]

The first term in brackets will be positive if imports exceed domestic production, that is if the import share is over a half. The second term will be positive if demand is convex, \( P'' > 0 \), then an export subsidy shifts the industry to a new equilibrium where the demand curve is less steep and firms act more competitively. For linear demand an

---

2 This is the same as the result derived by Dixit (1984), equation (19), but differs from that derived by Mai and Hwang (1987), equation (11), which is incorrect.
export subsidy will increase domestic welfare if the import share is above a half. Then, imports take a large share of the domestic market so the positive terms of trade effect will dominate the negative profit shifting effect. Comparing the effect of a foreign export subsidy on foreign welfare (4.5) with the effect on domestic welfare (4.10), it is apparent that an export subsidy which increases foreign welfare is likely to reduce domestic welfare. An export subsidy is basically a beggar-my-neighbour-policy.

4.3 Foreign Export Subsidy and Domestic Import Tariff: No Domestic Production

This section considers the interaction of domestic and foreign trade policy when there is no domestic production.\(^3\) The domestic country can use a rent extracting tariff to improve its terms of trade, as in Brander and Spencer (1984). For the foreign country, if there is no domestic production, then there is no profit shifting argument for an export subsidy. In fact, when there is more than one foreign firm the optimal policy is an export tax to exploit the foreign country's market power. However, when the foreign country has a first mover advantage, an export subsidy may be optimal if it brings about a lowering of the domestic

\(^3\)It may be that there is no domestic production because the foreign industry has a large cost advantage, so that domestic production is not worthwhile. See Dixit (1988), Venables (1986) and chapter 1.
tariff.

Trade policy will be analysed as a multistage game. At stage one, the foreign government sets its export subsidy to maximise its national welfare. Then at the second stage, the domestic government sets its import tariff to maximise its national welfare in response to the foreign export subsidy. Finally, in the third stage, of the game firms set their outputs to maximise their profits given the foreign export subsidy and domestic import tariff. In a subgame perfect equilibrium the foreign government realises the effect its export subsidy will have upon the optimal domestic tariff and takes this into account when setting its export subsidy. To obtain a subgame perfect equilibrium the game is solved by backward induction.

The equilibrium of the final stage of the game was derived in chapter three. The comparative static results, for the effects of a domestic tariff and a foreign export subsidy when there is no domestic production are given in (3.21) and (3.22), and these can now be used to solve the second stage of the game.

At the second stage of the game, the domestic country sets its tariff to maximise national welfare given the foreign export subsidy. Maximising domestic welfare (4.2) with respect to \( t \) yields the first order condition
Using the comparative static results above to solve for the optimal tariff yields

\[ t = -x(P' + QP'') \]  \hspace{1cm} (4.12)

This is the same as the optimal tariff derived by Brander and Spencer (1984a). The tariff improves the terms of trade of the domestic country. An import subsidy, a negative tariff, is the optimal policy if the tariff is overshifted, see (3.22), and this occurs if the relative convexity of demand is less than minus one, \( R < -1 \). Then, an import subsidy reduces the price of imports by more than the amount of the subsidy, and so the gain in consumer surplus exceeds the cost of the subsidy.

To obtain the effect of the foreign export subsidy on the optimal import tariff totally differentiate the first order condition for welfare maximisation (4.11), to obtain

\[ \frac{\partial^2 W}{\partial t^2} \frac{dt}{de} + \frac{\partial^2 W}{\partial e \partial t} = 0 \]

Therefore, the effect of the foreign export subsidy on the optimal domestic tariff, the countervailing tariff fraction,
is given by

$$\frac{dt}{de} = - \frac{\delta^2 W}{\partial e \partial t} \Bigg/ \frac{\delta^2 W}{\partial t^2} \quad (4.13)$$

To evaluate this expression the second order partial derivatives have to be derived. Using the comparative static results to evaluate the first derivative of the welfare function (4.11), yields

$$\frac{\partial W_i}{\partial t} = \frac{1}{D} \left[ Q(P' + QP'') + mt \right]$$

Where $D = (m+1)P' + QP' < 0$. To obtain $\frac{\partial^2 W_i}{\partial t^2}$ differentiate $\frac{\partial W_i}{\partial t}$ with respect to $t$, and evaluating at an optimum where the term in square brackets is zero, yields

$$\frac{\partial^2 W_i}{\partial t^2} = \frac{1}{D} \left[ (P' + 3QP'' + Q^2P''') \frac{\delta Q}{\partial t} + m \right]$$

Using the comparative static results from (3.21) yields

$$\frac{\partial^2 W_i}{\partial t^2} = \frac{m}{D^2} \left[ (P' + 3QP'' + Q^2P''') + ((m+1)P' + QP') \right] < 0 \quad (4.14)$$

Which must be negative to satisfy the second order condition for welfare maximisation. To obtain $\frac{\partial^2 W_i}{\partial e \partial t}$ differentiate $\frac{\partial W_i}{\partial t}$ with respect to $e$, and evaluating at an optimum, yields
Using the comparative static results from (3.21) yields

\[
\frac{\partial^2 W}{\partial e \partial t} = \frac{1}{D} \left[ (P' + 3QP'' + Q^2p'') \frac{\partial Q}{\partial e} \right]
\]

Hence, using (4.14) and (4.15) in (4.13) yields the optimal countervailing tariff fraction

\[
\frac{dt}{de} = \frac{(P' + 3QP'' + Q^2p'')}{((m+1)P' + QP'') + (P' + 3QP'' + Q^2p'')} < 1
\]

This is less than one since the denominator is negative from the second order condition for welfare maximisation, and \((m+1)P' + QP'' < 0\) by assumption (A2). Dixit (1988) claimed that a fully countervailing tariff was optimal in this case but this is incorrect since he derived the optimal tariff incorrectly, see chapter 1 and Collie (1990). It is possible that the countervailing tariff fraction is negative if \(P' + 3QP'' + Q^2p'' > 0\). For linear demand the countervailing tariff fraction is positive, \(dt/de = 1/(m+2)\), which is one third when there is a single foreign firm and tends to zero as the number of foreign firms becomes large. For constant elasticity demand functions the countervailing tariff fraction is positive, \(dt/de = 1/(m\eta^2 - \eta + 1)\) where \(\eta\) is the
elasticity of demand, so \( 0 < \frac{dt}{de} < 1 \) since \( \eta > 1 \). And, for demand functions of the form \( P = a - b \ln Q \) it can be shown that \( P' + 3QP'' + Q^2P''' = 0 \) so that the countervailing tariff fraction is zero, and the optimal domestic tariff is independent of the foreign export subsidy. A negative countervailing tariff fraction does not occur with any of the standard functional forms, but it is still possible, in theory.

Now consider the first stage of the game when the foreign government sets its export subsidy to maximise its national welfare realising the effect its decision will have upon the optimal domestic tariff. The effect of an export subsidy on foreign welfare (4.3) is given by

\[
\frac{\partial W_F}{\partial e} = (P + QP' - c_2 - t) \left[ \frac{\partial Q}{\partial e} + \frac{\partial Q}{\partial t} \frac{dt}{de} \right] - Q \frac{dq}{de} \quad (4.17)
\]

The first term is the terms of trade effect it will be zero when there is a single foreign firm, since this firm fully exploits the market power of the foreign country. When there are many foreign firms, they do not fully exploit the foreign country's market power, due to competition among themselves, and in this case an export tax can be used restrict output to the monopoly position. So with many firms

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4 By assumption (A2) \((m+1)+QP' < 0\), which for constant elasticity demand functions implies \( \eta > 1 \).
the terms of trade effect of an export subsidy will be negative. The second term is the countervailing tariff effect which will be negative (positive) if the countervailing tariff fraction is positive (negative). To obtain the optimal export subsidy set $\frac{\partial W_2}{\partial e} = 0$, then solving using the comparative static results (3.21), and the foreign firm's first order condition for profit maximisation (3.20), together with the optimal countervailing tariff fraction (4.16), yields

$$e = x \left[ (m-1)P' + (P' + 3QP'' + Q^2P''') \right]$$  (4.18)

The first term is the terms of trade effect, and the second term is the countervailing tariff effect. When there is a single foreign firm it acts as a monopolist, so the terms of trade effect is zero, and when the number of foreign firms is greater than one the terms of trade effect is negative. The second term is negative (positive) if the countervailing tariff fraction is positive (negative). Therefore, when there is a single foreign firm, the optimal foreign policy is an export subsidy if the countervailing tariff fraction is negative. When the number of foreign firms is greater than one, an export subsidy will be optimal if it reduces the domestic tariff sufficiently to offset the negative terms of trade effect. This is an example of where an export subsidy increases foreign welfare, not by shifting profits from domestic to foreign firms, but by altering the domestic...
tariff to the advantage of the foreign country. However, although a negative countervailing tariff is possible in theory, it seems unlikely in practice.

Competition policy may also be used by the foreign country to alter the domestic import tariff to its advantage. Cowan (1989) shows that an increase in the number of foreign firms will induce a reduction in the domestic tariff if demand is not too convex, $R > -1$, which increases foreign welfare.

4.4 Foreign Export Subsidy, Domestic Import Tariff and Production Subsidy

This section analyses how retaliation effects the profit shifting argument for a foreign export subsidy when there is domestic production, and the domestic country can respond with a countervailing tariff and a production subsidy. In chapter two, with linear demand and national product differentiation, it was shown that with retaliation there was no profit shifting argument for an export subsidy. However, with non-linear demand functions there is an argument for export subsidies despite retaliation.

Trade policy is analysed as a multistage game, for which the appropriate solution is a subgame perfect equilibrium, this excludes the possibility of non-credible threats. At the first stage the foreign government sets its export subsidy
to maximise its national welfare. Then in the second stage, the domestic government responds to the foreign export subsidy by setting its import tariff and production subsidy to maximise its national welfare. In the final stage, firms set their outputs to maximise profits given the trade policies set by the two governments in the previous stages. In a subgame perfect equilibrium the foreign government will set its export subsidy realising the effect this will have on the optimal tariff and production subsidy of the domestic country. The subgame perfect equilibrium is obtained by a process of backward induction. Firstly, the Nash equilibrium of the final stage is obtained then this solution is used to derive the Nash equilibrium of the second stage. The solution to the second stage is then used to derive the Nash equilibrium of the first stage. One thus obtains the subgame perfect equilibrium for the entire game.

The Nash equilibrium of the final stage of the game was obtained in chapter three. The comparative static results for the effects of a domestic tariff, production subsidy and a foreign export subsidy, (3.26) to (3.33), can now be used to solve for the rest of the game. In the second stage of the game, the domestic government sets its import tariff and production subsidy to maximise national welfare given the foreign export subsidy. Assume there is an interior solution, where the market is supplied by both imports and
domestic production. Maximising domestic welfare (4.2) with respect to \( t \) and \( s \) yields the first order conditions

\[
\frac{\partial W}{\partial t} = x\left(1 - \frac{\partial P}{\partial t}\right) + (P - c_1) \frac{\partial Y}{\partial t} + t \frac{\partial X}{\partial t} = 0
\]

\[
\frac{\partial W}{\partial s} = -x \frac{\partial P}{\partial s} + (P - c_1) \frac{\partial Y}{\partial s} + t \frac{\partial X}{\partial s} = 0
\]

(4.19)

The first term is the terms of trade effect, the second term is the domestic distortion, or profit shifting, effect and the third term is the tariff revenue effect. Using the comparative static results from chapter three in (4.19), yields

\[
\begin{bmatrix}
  m((n+1)P' + nyP'') - nm(P' + yP'') \\
  mn(P' + xP'') - n((m+1)P' + mxP'')
\end{bmatrix}
\begin{bmatrix}
  t \\
  P - c_1
\end{bmatrix}

= \begin{bmatrix}
  -mxP'((n+1)P' + QP'') \\
  -mxP' nP'
\end{bmatrix}
\]

Solving for the optimal policies then yields

\[
t = -x(P' + mxP'')
\]

(4.20)

\[5\text{This requires the foreign firms to have a cost advantage, but not so large that domestic production is not worthwhile: } 0 < c_1 - c_2 + e < -2xP'. \text{ See Dixit (1988) and chapter 1.}\]
\[ P - c_1 = -x(mx'P'') \implies s = -yP' + x(mxP'') \]

These correspond to the optimal policies obtained by Dixit (1988) using a different method. The formula for the optimal tariff is a generalisation of the result obtained by Brander and Spencer (1984). It is possible that the optimal policy is an import subsidy, this will be the case if demand is sufficiently convex and import penetration is large. For linear demand domestic production should be subsidised until price equals marginal cost, but when demand is concave (convex) price should be above (below) marginal cost. The explanation is that if demand is concave (convex), then a decrease (increase) in the subsidy shifts the equilibrium up (down) the demand curve making demand less steep, and so reducing the price-cost margin of foreign firms thus improving the terms of trade. So when demand is non-linear the production subsidy has a role in improving the terms of trade as well as correcting the domestic distortion. The overall level of protection for the domestic industry is always positive since \( t + s = -(x+y)P' > 0 \).

Now consider the effect of a foreign export subsidy on domestic welfare, when the domestic government sets its tariff and production subsidy optimally

\footnote{The method used here allows the second order conditions and the comparative static results for the optimal policies to be derived.}
\[ \frac{dW_i}{de} = \frac{\partial W_i}{\partial e} + \frac{\partial W_i}{\partial t} + \frac{\partial W_i}{\partial s} \]

Since the import tariff and production subsidy are set optimally, \( \frac{\partial W_i}{\partial t} = \frac{\partial W_i}{\partial s} = 0 \). Therefore, only the direct effect of a foreign export subsidy on domestic welfare has to be considered, because any induced changes in the tariff and production subsidy will have no effect on welfare since at an optimum their effect on welfare is zero. Hence, the effect of the foreign export subsidy on domestic welfare (4.2) is

\[ \frac{dW_i}{de} = -X \frac{\partial P}{\partial e} + (P - C_t) \frac{\partial Y}{\partial e} + t \frac{\partial X}{\partial e} \]

And using the comparative static results from chapter three and the optimal policies from (4.20) yields

\[ \frac{dW_i}{de} = X > 0 \quad (4.22) \]

A foreign export subsidy always increases domestic welfare if the domestic country pursues an optimal trade and industrial policy. This is not surprising. If the domestic country applied fully countervailing tariffs then the only effect would be to increase tariff revenue, and hence to increase domestic welfare.
To obtain the comparative static results for a change in the foreign export subsidy totally differentiate the first order conditions for welfare maximization (4.19), this yields

\[
\begin{bmatrix}
\frac{\partial^2 W_1}{\partial t^2} & \frac{\partial^2 W_1}{\partial s \partial t} \\
\frac{\partial^2 W_1}{\partial t \partial s} & \frac{\partial^2 W_1}{\partial s^2}
\end{bmatrix}
\begin{bmatrix}
\frac{dt}{de} \\
\frac{ds}{de}
\end{bmatrix}
= 
\begin{bmatrix}
-\frac{\partial^2 W_1}{\partial s \partial t} \\
-\frac{\partial^2 W_1}{\partial s^2}
\end{bmatrix}
\tag{4.23}
\]

The second order conditions for a welfare maximum require that the above Hessian matrix is negative definite. Therefore, the elements in the principal diagonal must be negative and the determinant must be positive.

A detailed derivation of the second order partial derivatives, and the comparative static results is provided in Appendix A. The second order partial derivatives evaluated at the welfare maximum are

\[
\frac{\partial^2 W_1}{\partial t^2} = \frac{mP'}{\Delta^2} \left[ 2((n+1)P' + QP'')^2 + mZ \right]
\]

\[
\frac{\partial^2 W_1}{\partial s^2} = \frac{n^2P'}{\Delta^2} \left[ 2m(P')^2 + Z \right]
\]

\[
\frac{\partial^2 W_1}{\partial t \partial s} = \frac{nmP'}{\Delta^2} \left[ 2P'((n+1)P' + QP'') - Z \right]
\]

\[
\frac{\partial^2 W_1}{\partial s \partial t} = \frac{nmP'}{\Delta^2} \left[ 2P'((n+1)P' + QP'') - Z \right]
\tag{4.24}
\]
\[
\frac{\partial^2 W_1}{\partial t^2} = \frac{-mP'}{\Delta^2} \left[ ((n+1)P' + QP'')((n-m+1)P' + QP'') + mP''((n+m+1)P' + QP'') + mZ \right]
\]

\[
\frac{\partial^2 W_1}{\partial s \partial t} = \frac{-nmP'}{\Delta^2} \left[ P'((n-m+1)P' + QP'') - xP''((n+m+1)P' + QP'') - Z \right]
\]

Where \( Z = [(P')^2 + mx^2(P'P'' - 2(P'')^2)] \)

The determinant of the Hessian matrix can be shown to be

\[
H = \begin{vmatrix}
\frac{\partial^2 W_1}{\partial t^2} & \frac{\partial^2 W_1}{\partial s \partial t} \\
\frac{\partial^2 W_1}{\partial s \partial t} & \frac{\partial^2 W_1}{\partial s^2}
\end{vmatrix} = \frac{2n^2mZ}{\Delta^2} \tag{4.25}
\]

Therefore, the second order conditions for a welfare maximum will be satisfied if \( Z > 0 \). If \( Z \) is positive everywhere, then there is a unique welfare maximum, and unique optimal policies.

Using these results to solve for the optimal countervailing tariff response to a foreign export subsidy yields

\[
dt = \frac{1}{H} \begin{vmatrix}
\frac{\partial^2 W_1}{\partial s \partial t} & \frac{\partial^2 W_1}{\partial s^2} \\
\frac{\partial^2 W_1}{\partial s \partial t} & \frac{\partial^2 W_1}{\partial s^2}
\end{vmatrix}
\]
The optimal response to a foreign export subsidy, when demand is linear, is to increase the import tariff by half the amount of the export subsidy as Dixit (1984) has shown. If demand is convex (concave) then the optimal countervailing fraction is less (greater) than a half. Therefore, it is possible that if demand is sufficiently concave a fully countervailing tariff is optimal. This result generalises Dixit (1988).\(^7\) The effect of a foreign export subsidy on the optimal domestic production subsidy is

\[ \frac{ds}{de} = -\frac{m}{2n} \left[ 1 + \frac{xp''((n+1)p' + Qp'')}{z} \right] \]  

(4.27)

If demand is linear then, \(\frac{ds}{de} = -\frac{m}{2n}\), and a foreign export subsidy reduces the domestic production subsidy. The reduction will be larger the greater the number of foreign firms and the smaller the number of domestic firms. With

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\(^7\) Dixit (1988) shows that the optimal countervailing tariff fraction is a half for linear demand, and less than a half for constant elasticity demand functions.
non-linear demand a foreign export subsidy will usually reduce the optimal domestic production subsidy. It is possible, if demand is sufficiently convex, that a foreign export subsidy will increase the optimal domestic production subsidy. When demand is concave (convex) the increase in the tariff and the reduction in the production subsidy are greater (smaller) than with linear demand. These results can now be used to solve the first stage of the game.

In the first stage of the game the foreign government sets its export subsidy to maximise its national welfare, realising the effect that its decision will have upon the optimal tariff and production subsidy set by the domestic country in the second stage. The effect of an export subsidy on foreign exports is

$$\frac{dX}{de} = \frac{\partial X}{\partial e} + \frac{\partial X}{\partial t} \frac{dt}{de} + \frac{\partial X}{\partial s} \frac{ds}{de}$$

Using the comparative static results from chapter three and the optimal countervailing tariff and production subsidy responses, (4.26) and (4.27), yields

$$\frac{dX}{de} = \frac{-m}{P'} \left[ \frac{1}{2} + \frac{mx^2(P')^2}{Z} \right] > 0$$  \hspace{1cm} (4.28)

An export subsidy increases exports despite the countervailing tariff and production subsidy. The effect of
The export subsidy on foreign welfare (4.3) is

\[
\frac{dW^2}{de} = (P-c_2-t) \left( \frac{\partial X}{\partial e} + \frac{\partial X}{\partial t} \frac{dt}{de} + \frac{\partial X}{\partial s} \frac{ds}{de} \right) \\
+ X \left( \frac{\partial P}{\partial e} + \left( \frac{\partial P}{\partial t} - 1 \right) \frac{dt}{de} + \frac{\partial P}{\partial s} \frac{ds}{de} \right)
\]

(4.29)

The first term is the profit shifting effect, and the second term is the terms of trade effect. The countervailing tariff, if it is positive, will reduce the profit shifting effect of the export subsidy, and worsen the terms of trade unless there is price overshifting of the tariff. Whereas, the countervailing production subsidy, if it is negative, will increase the profit shifting effect of the export subsidy, and improve the terms of trade. Using the comparative static results from chapter three, and the optimal countervailing tariff and production subsidy responses from (4.26) and (4.27), to evaluate at \( e=0 \), yields

\[
\frac{dW^2}{de} = \frac{x(mxP_0^\prime)^2}{Z} \geq 0
\]

(4.30)

An export subsidy increases foreign welfare, if demand is non-linear, despite the countervailing response from the domestic government. The export subsidy will increase foreign welfare when there is retaliation even when it would not increase welfare in the absence of retaliation. This may seem counterintuitive, but it should be remembered that
although the countervailing tariff will tend to reduce foreign welfare, the countervailing production subsidy will tend to increase foreign welfare, so it is not surprising that the overall effect is positive. In chapter two, with linear demand and product differentiation, the overall effect was negative. The foreign export subsidy increases foreign welfare, and it has been shown, see (4.22), that a foreign export subsidy increases domestic welfare. Hence, a foreign export subsidy increases the welfare of both countries. Setting \( \frac{dW_2}{de} = 0 \), and solving yields the optimal foreign export subsidy

\[
e = \frac{-2mx^2P'(P'')^2}{[(P')^2 + mx^2P'P'']} = 0 \tag{4.31}
\]

The optimal foreign export subsidy is positive when demand is non-linear. In chapter two, with linear demand, the optimal foreign policy was an export tax if the domestic and foreign products were imperfect substitutes.

A foreign export subsidy has been shown to increase both domestic and foreign welfare, when the domestic country retaliates with a countervailing tariff and production subsidy.

4.5 Foreign Export Subsidy and Domestic Import Tariff

In the previous section the domestic government was able to
use both an import tariff and a production subsidy to countervail the foreign export subsidy, but in practice governments tend to use only import tariffs. Article VI of the GATT does not mention the possibility of using production subsidies to countervail foreign export subsidies, although this does not prevent a country from using production subsidies. For the EEC, the countervailing duty regulations require that foreign export subsidies are countered with tariffs and there is no role for production subsidies. In this section, the domestic government is assumed to use only an import tariff. Therefore, at the first stage of the game the foreign government sets its export subsidy to maximise its national welfare. Then, in the second stage, the domestic government sets its import tariff to maximise its national welfare given the foreign export subsidy. And, in the final stage, firms set their outputs to maximise profits given the export subsidy and tariff. In a subgame perfect equilibrium, the foreign government realises the effect its export subsidy will have on the optimal domestic tariff and takes this into account when setting the export subsidy. As usual the game is solved by backward induction. The equilibrium of the final stage of the game was derived in the chapter three, and the comparative static results derived there can now be used to solve the second stage of the game.

When the domestic government could use an import tariff and
production subsidy it had two policies to deal with two distortions. The import tariff was used basically to improve the terms of trade whereas the production subsidy was used to correct the domestic distortion, although for non-linear demand the production subsidy had a role in improving the terms of trade. With no production subsidy, price will exceed marginal cost in the domestic industry, and the tariff will be used to shift profits from foreign to domestic firms.

At the second stage the domestic government sets its import tariff to maximise its national welfare in response to the foreign export subsidy. The optimal tariff when there is domestic production has been derived by Brander and Spencer (1984b). Assume there is an interior solution. Maximising domestic welfare (4.2) with respect to \( t \) yields the first order condition

\[
\frac{\partial W_1}{\partial t} = x \left( 1 - \frac{\partial P}{\partial t} \right) + (P - c_1) \frac{\partial Y}{\partial t} + t \frac{\partial X}{\partial t} = 0 \quad (4.32)
\]

Solving for the optimal tariff yields

---

\(^8\) Venables (1986), for the case of constant elasticity demand functions, shows that there may be two local welfare maximum. One with a negative tariff and no domestic production, and one with a positive tariff and domestic production.
The dual role of the tariff in this case is clear. The first term in square brackets is the terms of trade effect, and the second is the profit shifting effect. Substitute the comparative static results from chapter three into the above, and using the profit maximisation condition for domestic firms, \( P - c_1 = -yP' \), then yields the optimal tariff

\[
t = -\left[ x\left(1 - \frac{\partial P}{\partial t}\right) + (P - c_1) \frac{\partial Y}{\partial t}\right] / \frac{\partial X}{\partial t}
\]

The sign of the optimal tariff is ambiguous, for linear and concave demand functions it is clearly positive, but for convex demand it may be negative. If demand is sufficiently convex there will be price over-shifting of the tariff, this will occur if the relative convexity of demand, \( R < -(n+1) \). When this occurs the tariff increases the price by more than the amount of the tariff which worsens the terms of trade. In this case the terms of trade effect is negative. Also, with convex demand a tariff may actually reduce domestic production, this occurs when domestic and foreign output are strategic complements \( nP' + nyP'' < 0 \), and then the profit shifting effect of the tariff is negative. So for sufficiently convex demand functions it is possible that both the terms of trade effect and the profit shifting effect may be negative in which case the optimal tariff will
be negative. If the optimal policy is an import subsidy then the level of protection for domestic firms will be negative whereas when both a production subsidy and a tariff were used the overall level of protection was always positive.

An interesting point to note is that the optimal tariff may exceed the maximum revenue tariff, in contrast to conventional trade theory where the optimum tariff is always less than the maximum revenue tariff.\(^9\) Since the conventional optimum tariff maximises welfare, which is the sum of consumer surplus and tariff revenue, then an increase in the tariff beyond the maximum revenue level will reduce both tariff revenue and, since it increases the price, consumer surplus. Therefore, the optimum tariff cannot exceed the maximum revenue tariff. But, with oligopolistic industries there are pure profits which have to be included in welfare, and it is possible that an increase in the tariff beyond the maximum revenue level will increase profits by more than tariff revenue and consumer surplus are reduced. So the optimal tariff may exceed the maximum revenue tariff. The condition for this to occur can be seen from

\[
\frac{\partial W}{\partial t} = \left( X + t \frac{\partial X}{\partial t} \right) - X \frac{\partial p}{\partial t} + (P - c_1) \frac{\partial y}{\partial t}
\]

\(^9\)See Johnson (1951-52). He notes that this result will not apply if a tariff causes the price of imports to fall, but here a tariff increases the price.
At the maximum revenue tariff, the first term in brackets is zero so the optimal tariff will exceed the maximum revenue tariff if \( \frac{\partial W_I}{\partial t} \) is positive at this point. The second term is the effect on consumer surplus of the tariff which is negative, and the third term is the profit shifting effect of the tariff which is positive if domestic and foreign output are strategic substitutes. The condition for the optimal tariff to exceed the maximum revenue tariff can be seen by using the comparative static results in the above which yields

\[
\frac{\partial W_I}{\partial t} = \frac{mP'}{\Delta} \left[ (ny-mx)P' + ny^2P'' \right] > 0 \tag{4.35}
\]

If this is positive, when evaluated at the maximum revenue tariff, then the optimal tariff exceeds the maximum revenue tariff. For linear demand this requires that domestic production exceeds imports.\(^{10}\)

In section 4.2 it was shown that, in the absence of domestic intervention, domestic welfare may be reduced by a foreign export subsidy. However, in the previous section it was shown that when the domestic government sets its import

\(^{10}\)For a Cournot duopoly where the domestic and foreign firm have identical costs, domestic production will exceed imports since the maximum revenue tariff is obviously positive.
tariff and production subsidy optimally then a foreign export subsidy will always increase domestic welfare. The effect of a foreign export subsidy on domestic welfare when the domestic country sets its import tariff optimally is given by

\[
\frac{dW_1}{d\epsilon} = \frac{\partial W_1}{\partial \epsilon} + \frac{\partial W_1}{\partial t} \frac{dt}{d\epsilon}
\]

Since the import tariff is set optimally, \( \frac{\partial W_1}{\partial t} = 0 \). Therefore, only the direct effect of the foreign export subsidy on domestic welfare has to be considered since any induced changes in the tariff will have no effect on welfare, because at an optimum its effect on welfare is zero. Hence, the effect of the foreign export subsidy on domestic welfare (4.2) is

\[
\frac{dW_1}{d\epsilon} = \frac{\partial W_1}{\partial \epsilon} = -X \frac{\partial P}{\partial \epsilon} + (P - c) \frac{\partial Y}{\partial \epsilon} + t \frac{\partial X}{\partial \epsilon}
\]  (4.36)

The first term, the terms of trade effect, is positive since an export subsidy reduces the domestic price. The second term, the profit shifting effect, is negative if domestic and foreign output are strategic substitutes. Then, an export subsidy reduces domestic production and hence since price exceeds marginal cost reduces domestic profits. The third term, the tariff revenue effect, is positive if there is a positive tariff since the export subsidy increases imports and hence tariff revenue.
To determine the overall effect note that the effect of an export subsidy is the negative of the effect of a tariff so

\[
\frac{dW}{de} = -\left[ -x \frac{\partial P}{\partial t} + (P - c_1) \frac{\partial Y}{\partial t} + t \frac{\partial X}{\partial t} \right]
\]

From the first order condition for the optimal tariff (4.32), this reduces to

\[
\frac{dW}{de} = x > 0 \tag{4.37}
\]

Whenever the domestic government sets its import tariff optimally it will gain from a foreign export subsidy. An increase in the foreign export subsidy is equivalent to a reduction in the tariff for the domestic country except that there is no tariff revenue effect. So if the domestic tariff is set optimally then a reduction in the tariff has no effect on welfare, but an increase in the export subsidy will increase welfare since it does not involve the loss of tariff revenue that would occur with a tariff reduction. Thus, the welfare gain is equal to this tariff revenue effect.

To obtain the effect of a foreign export subsidy on the optimal domestic tariff totally differentiate the first order condition (4.32) which yields
Therefore, the effect of a foreign export subsidy on the optimal import tariff, the optimal countervailing fraction, is given by

\[ \frac{\partial^2 W}{\partial t^2} \frac{dt}{de} + \frac{\partial^2 W}{\partial e \partial t} = 0 \]

From appendix B the second order partial derivatives are

\[ \frac{\partial^2 W_1}{\partial t^2} = \frac{mN}{\Delta} + \frac{mP'}{\Delta^2 N} \left[ ((n-m+1)P' + nyP'')N^2 + B \right] < 0 \]

Which must be negative from the second order condition for welfare maximisation.

\[ \frac{\partial^2 W_1}{\partial e \partial t} = \frac{-mP'}{\Delta^2 N} \left[ ((n-m+1)P' + nyP'')N^2 + B \right] \]

where Z, N and B are defined as

\[ Z = [(P')^2 + mx^2(P'P'' - 2(P')^2)] > 0 \]
\[ N = (n+1)P' + nyP'' < 0 \]
\[ B = m(n+1)P'Z + (mxP'')^2(nP' + 2N) + 3m^2P''N^2 + mny^2P' (P'P'' - 2(P'')^2) \]

The sign of \( \frac{dt}{de} \) will be the same as the sign of \( \frac{\partial^2 W_1}{\partial e \partial t} \)

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which has the opposite sign to the term in square brackets. This cannot be signed from the second order conditions. The first term in square brackets will be negative, if a foreign export subsidy increases foreign welfare in the absence of any countervailing response by the domestic country see equation (4.5). This will tend to make $\frac{dt}{de}$ positive. So the larger the potential for profit shifting export subsidies the larger will be the countervailing tariffs that the domestic country applies to foreign exports. The second term in square brackets, $B$, has an ambiguous sign. Looking at the definition of $B$, the first two terms are clearly negative, the third term is negative (positive) for concave (convex) demand functions and the fourth term is ambiguous. For linear demand functions $B$ will be negative, but for constant elasticity demand functions both the third and fourth terms will be positive so the overall sign is unclear. Although there is some ambiguity about the sign of $B$ it seems most likely that it is negative. If $B$ is negative this will tend to make $\frac{dt}{de}$ positive. The overall sign of $\frac{dt}{de}$ is unclear even for linear demand functions and will be negative if the number of foreign firms is large relative to the number of domestic firms. However, if the export subsidy would increase foreign welfare, when there are no countervailing tariffs $(n-m+1)P'_r + nyP'' < 0$ from (4.5), and $B$ is negative then the domestic country will respond with countervailing tariffs, $\frac{dt}{de}$ is positive. It is possible to show that fully countervailing tariffs are never justified.
from above

\[ \frac{dt}{de} = \frac{mN}{\Delta \Omega} > 0 \]  \hspace{1cm} (4.39)

Where \( \Omega = \frac{\partial^2 W_1}{\partial t^2} < 0 \). Therefore, it follows that \( \frac{dt}{de} < 1 \).

When the domestic government uses only an import tariff then the optimal policy is a less than fully countervailing tariff. This supports the conclusion of Dixit (1988) that only partially countervailing tariffs are justified. And, it is even possible that the optimal response to a foreign export subsidy is to reduce the tariff.

Now consider the first stage of the game, when the foreign government sets its export subsidy to maximise its national welfare realising the effect this will have upon the optimal tariff set by the domestic government in the next stage of the game. The effect of an export subsidy on foreign welfare (4.3), taking into account the effect it has on the optimal domestic tariff, is given by

\[ \frac{dW}{de} = (P-C-\frac{\partial P}{\partial t} - 1) \left( \frac{\partial X}{\partial e} + \frac{\partial X}{\partial t} \frac{dt}{de} \right) + X \left( \frac{\partial P}{\partial e} + \left( \frac{\partial P}{\partial t} - 1 \right) \frac{dt}{de} \right) \]  \hspace{1cm} (4.40)

The first term is the profit shifting effect, and the second term is the terms of trade effect. The countervailing tariff will, if it is positive, reduce the profit shifting effect of the export subsidy and worsen the terms of trade, unless
there is price overshifting of the tariff. To evaluate the overall effect, use the comparative static results from chapter three, and the optimal countervailing tariff fraction from above, in (4.40). Hence, the overall effect, evaluated at $e = 0$, is

$$\frac{dW_2}{de} = \frac{-m^2 xP'B}{\Delta^2 \Omega N}$$  (4.41)

The sign of the welfare effect is the same as the sign of $B$, which although it ambiguous is likely to be negative. For linear demand $B$ is certainly negative, but for constant elasticity demand functions it is ambiguous. Therefore, the effect of a foreign export subsidy on foreign welfare is likely to be negative when the domestic government responds with countervailing tariffs. This confirms the assertion of Bhagwati (1988) that a country which uses profit shifting policies is likely to lose if retaliation occurs.

The optimal export subsidy can be derived by setting $dW_2/de = 0$ and solving for the optimal $e$, which yields

$$e = \frac{xP'B}{N^3}$$  (4.42)

The optimal policy will be an export tax if $B$ is negative. As in chapter two, the foreign country should tax its exports. There is no profit shifting argument for export
subsidies when the domestic country retaliates with countervailing tariffs.

In the numerical model of Gasiorek et al (1989), a Japanese export subsidy to the computer industry will increase Japanese welfare despite retaliation with a countervailing tariff by the EC. Whereas, a Japanese export subsidy to the vehicle industry will reduce Japanese welfare when there is retaliation. The result for the computer industry contrasts with the above proposition that a foreign export subsidy will usually reduce foreign welfare when there is retaliation with a countervailing tariff. The explanation is that in the numerical model of Gasiorek et al marginal cost is decreasing, and a Japanese export subsidy will have an impact on the Japanese market. An export subsidy will reduce the price in the Japanese market and increase consumer surplus. The export subsidy is operating like a production subsidy, and correcting the distortion in the Japanese market, as well as shifting profits to Japanese firms. Since, economies of scale are assumed to be larger in the computer industry than in the vehicle industry, the domestic distortion effect will be more significant in the case of the computer industry. If Japan could use a production subsidy and an export subsidy, then the production subsidy would be used to correct the domestic distortion and the export subsidy to shift profits. Then, the optimal Japanese policy when faced with retaliation would probably be a
production subsidy and an export tax. There will be no profit shifting argument for an export subsidy when there is retaliation with a countervailing tariff.

4.6 Foreign Export Subsidy and Domestic Production Subsidy

In the previous section the domestic government used an import tariff to improve its terms of trade and to shift profits to the domestic industry. An alternative policy is to use a production subsidy, and this section will consider the same trade policy game when the domestic government uses only a production subsidy. At the first stage of the game, the foreign government sets its export subsidy to maximise its national welfare. Then, in the second stage, the domestic government sets its production subsidy to maximise its national welfare given the foreign export subsidy. And, in the final stage, the firms set their outputs given the foreign export subsidy and domestic production subsidy. Again, a subgame perfect equilibrium is required so that any threats of retaliation have to be credible. The foreign government realises the effect its export subsidy will have upon the the optimal production subsidy set by the domestic country when it makes its decision. As usual the game is solved by backward induction. The equilibrium of the final stage of the game was derived in chapter three and the comparative static results derived there can now be used to solve the second stage of the game.
At the second stage of the game the domestic government sets its production subsidy to maximise its national welfare given the foreign export subsidy. So the effect of a domestic production subsidy on welfare (4.2) is

\[
\frac{\partial W_1}{\partial s} = -X \frac{\partial P}{\partial s} + (P - c_1) \frac{\partial Y}{\partial s}
\]  

(4.43)

The first term is the terms of trade effect, the production subsidy lowers the domestic price and so reduces the cost of imports to the domestic economy. The second term is the profit shifting effect, a production subsidy expands the output of the domestic industry which increases welfare if price exceeds marginal cost. To obtain the optimal policy set \( \frac{\partial W_1}{\partial s} = 0 \) and solve for \( P - c_1 \), yields

\[
P - c_1 = X \frac{\partial P}{\partial s} \frac{\partial Y}{\partial s}
\]

Using the comparative static results from chapter three in the above leads to

\[
P - c_1 = \frac{mx(P')^2}{(m+1)P' + mxP''} < 0
\]  

(4.44)

The optimal policy is to subsidise domestic production so that price is actually below marginal cost. If price was above marginal cost then both the profit shifting effect and
the terms of trade effect are positive and so the overall effect on welfare is positive. Therefore, at the optimum price is below marginal cost so that, at the margin, the loss of producer surplus is just equal to the terms of trade gain. Since the production subsidy improves the terms of trade, it pays to subsidise production beyond the point required to correct the domestic distortion, where price equals marginal cost.

In the previous section it was shown that when the domestic government sets its import tariff optimally it will always gain from a foreign export subsidy. Now consider the effect of a foreign export subsidy when the domestic government sets its production subsidy optimally. The effect of a foreign export subsidy on domestic welfare (4.2) is given by

\[ \frac{dW_1}{de} = \frac{\partial W_1}{\partial e} + \frac{\partial W_1}{\partial s} \frac{ds}{de} \]

Since at an optimum the effect of a change in the production subsidy on domestic welfare is zero, \( \partial W_1/\partial s = 0 \), only the direct effect of the export subsidy on domestic welfare, evaluated at the optimum, is relevant. Thus

\[ \frac{\partial W_1}{\partial e} = -X \frac{\partial P}{\partial e} + (P-C_1) \frac{\partial Y}{\partial e} \]  

(4.45)

The first term, the terms of trade effect, will be positive
since the export subsidy reduces the domestic price and so lowers the cost of imports. The second term, the profit shifting effect, will be positive (negative) if domestic and foreign output are strategic substitutes (complements) since price is below marginal cost at the optimum. Hence, a reduction in domestic output will increase domestic welfare. The overall effect can be evaluated by using the comparative static results in chapter three and the optimal $\rho - c_1$ from above, yields

$$\frac{dW_1}{de} = \frac{mXP'}{(m+1)P'+mxP''} > 0 \quad (4.46)$$

If the domestic government sets its production subsidy optimally it will always gain from a foreign export subsidy. In the previous sections the gain was equal to $X$, but here the gain will be larger (smaller) than this if $P'+mxP''$ is positive (negative).

To obtain the effect of a foreign export subsidy on the optimal domestic production subsidy totally differentiate the first order condition (4.43), which yields

$$\frac{\partial^2 W_1}{\partial s^2} \frac{ds}{de} + \frac{\partial^2 W_1}{\partial e \partial s} = 0$$

Therefore, the effect of a foreign export subsidy on the optimal production subsidy, the countervailing subsidy
fraction, is given by

\[
\frac{ds}{de} = - \frac{\partial^2 W_1}{\partial e \partial s} \bigg/ \frac{\partial^2 W_1}{\partial s^2}
\]  

(4.47)

From appendix C the second order partial derivatives evaluated at the welfare maximum are

\[
\frac{\partial^2 W_1}{\partial s^2} = \frac{n^2 (P')^2}{\Delta^2 M} \left[ M^2 + m^2 (P')^2 + mZ \right] < 0
\]

Which must be negative to satisfy the second order conditions for a welfare maximum.

\[
\frac{\partial^2 W_1}{\partial e \partial s} = \frac{nm (P')^2}{\Delta^2 M} \left[ M^2 + m^2 (P')^2 + mZ - (m+1)\Delta \right]
\]

Where

\[ M = (m+1)P' + mxP'' < 0 \]
\[ Z = [(P')^2 + mx^2 (P'' - 2(P')^2)] > 0 \]

The countervailing subsidy fraction is

\[
\frac{ds}{de} = - \frac{m}{n} \left[ 1 - \frac{(m+1)\Delta}{M^2 + m^2 (P')^2 + mZ} \right] 
\]  

(4.48)

The sign of this is ambiguous, but for linear demand functions it reduces to
In the case of duopoly, with one foreign and one domestic firm, the countervailing subsidy fraction is zero so the optimal domestic production subsidy is independent of the foreign export subsidy. The countervailing subsidy fraction will be positive (negative) if the number of domestic firms is greater (smaller) than the number of foreign firms. It was shown in section 4.2 that that a foreign export subsidy would increase foreign welfare, in the absence of retaliation, if \( n-m+1 > 0 \). Therefore, if the countervailing subsidy fraction is positive then the foreign export subsidy would increase foreign welfare in the absence of retaliation. And if the export subsidy would reduce foreign welfare, in the absence of retaliation, then the countervailing subsidy fraction will be negative.

Now consider the final stage of the game when the foreign government sets its export subsidy to maximise its national welfare, realising the effect its decision will have upon the optimal production subsidy set by the domestic government in the second stage. The effect of an export subsidy on foreign welfare (4.3), taking into account the effect it has on the domestic production subsidy is given by

\[
\frac{dW}{de} = (P-c_2) \left( \frac{\partial X}{\partial e} + \frac{\partial X}{\partial s} \frac{ds}{de} \right) + X \left( \frac{\partial P}{\partial e} + \frac{\partial P}{\partial s} \frac{ds}{de} \right) \quad (4.50)
\]
The first term is the profit shifting effect, and the second term is the terms of trade effect. If the countervailing subsidy fraction is positive (negative) then this will weaken (strengthen) the welfare effect of the foreign export subsidy. Using the comparative static results from chapter three, and the countervailing subsidy fraction, to evaluate the welfare effect at \( e = 0 \), yields

\[
\frac{dW}{de} = X \left[ \frac{(P' + mxP'')^2 + mZ + (m-1)mxP'P''}{M^2 + m^2(P')^2 + mZ} \right] \tag{4.51}
\]

When demand is concave or there is only one foreign firm, the effect of the export subsidy on foreign welfare is positive. It may be negative if demand is convex and the number of foreign firms is greater than one. To obtain the optimal export subsidy set \( \frac{dW}{de} = 0 \) and use \( P - c_2 = -xP' - e \), yields

\[
e = -xP' \left[ \frac{(P' + mxP'')^2 + mZ + (m-1)mxP'P''}{(P' + mxP'')^2 + mZ^2 + (m-1)P'M + (m+1)(P')^2} \right] \tag{4.52}
\]

The optimal policy will be an export subsidy if demand is concave or if there is only one foreign firm. But if demand is convex and there is more than one foreign firm then the optimal policy may be an export tax.
4.7 Conclusions

The domestic country may suffer a welfare loss as a result of a foreign export subsidy if it pursues a policy of laissez-faire. It was shown that export subsidies are basically a beggar-my-neighbour policy, they increase foreign welfare at the expense of the domestic country. However, when the domestic country pursues an optimal trade and industrial policy, using an import tariff, production subsidy or both, then it will always gain from a foreign export subsidy. The optimal domestic response to a foreign export subsidy, when the domestic country uses a tariff and a production subsidy is generally to increase the tariff and reduce the production subsidy. The optimal countervailing tariff fraction is greater (less) than a half when demand is concave (convex). When the domestic country uses a tariff, its optimal response to a foreign export subsidy is usually to increase the tariff, but a reduction may be optimal. The optimal countervailing tariff is always less than one, so a fully countervailing tariff is never justified. The optimal domestic response to a foreign export subsidy when the domestic country uses a production subsidy is ambiguous.

When the domestic country uses an import tariff and production subsidy to countervail foreign export subsidies then the foreign country gains from an export subsidy, unless demand is linear when the effect is zero, despite the
retaliation. The optimal foreign policy is to subsidise exports. The domestic and foreign country both gain from a foreign export subsidy. This is a surprising result. When the domestic country uses only a tariff a foreign export subsidy will generally reduce foreign welfare. There is usually no profit shifting argument for an export subsidy when the domestic country retaliates with countervailing tariffs. When the domestic country uses only a production subsidy, the effect of a foreign export subsidy on foreign welfare is ambiguous.
Chapter 5: Export Subsidies, Entry Deterrence and Countervailing Tariffs

5.1 Introduction

This chapter analyses the effect of retaliation, when the foreign country uses an export subsidy to deter the entry of domestic firms. Although there is now a large literature on strategic trade policy little attention has been paid to the formal modelling of trade wars and retaliation. The exceptions are Dixit and Kyle (1985) and Dixit (1988). Despite this lack of any formal models, Bhagwati (1988) and Grossman (1986) have asserted that if retaliation occurs then both countries will be left worse off. Hence, it is argued that retaliation negates the strategic rationale for trade policies such as export subsidies. It seems plausible that a foreign export subsidy is likely to result in the domestic country retaliating with countervailing tariffs which will leave the foreign country worse off. This chapter follows Dixit and Kyle (1985) and models trade policy as a multistage game. The results are that when the domestic country retaliates with countervailing tariffs the optimal foreign policy is a zero export subsidy. However, if the domestic country retaliates with countervailing tariffs and production subsidies then the optimal foreign policy may be an export subsidy.
Dixit and Kyle (1985) analysed the use of protection and subsidies for entry promotion and deterrence. In their model, based upon the Airbus example, an incumbent firm in the US faces the potential entry of a firm in the EC. They model trade policy as a multistage game in which the two countries have the choice of free trade or protection: a complete prohibition of imports. The US has an incentive to use protection to deter the entry of the EC firm since if entry occurs the US firm will earn duopoly profits rather than monopoly profits. The EC can use protection to make entry profitable for the EC firm. The equilibrium of the trade policy game depends on the order of play and the ability of governments to pre-commit to particular policies. One possibility is that the US may gain by committing itself to free trade which makes it unnecessary for the EC to use protection to promote entry. Dixit (1988) analyses the optimal trade policy in an oligopoly, and derives the domestic country’s optimal countervailing tariff and subsidy response to a foreign export subsidy. He shows that only partially countervailing tariffs are justified. Chapter four extended this analysis to derive the optimal export subsidy for the foreign country when the domestic country retaliates with countervailing tariffs and/or production subsidies.

In this model a number of incumbent foreign firms face the potential entry of domestic firms in a homogeneous product Cournot oligopoly. The foreign country can use an export
subsidy to deter the entry of domestic firms. When there is no domestic tariff the optimal foreign policy is to subsidise exports so that no domestic firms enter the industry. An example of subsidies being used to deter entry is the semiconductor industry. The US semiconductor industry has accused MITI of protecting the Japanese market and of giving subsidies to favoured Japanese firms. This industrial targeting encourages investment by the favoured Japanese firms and discourages investment by US firms. Such subsidies may have deterred US firms from entering the market for the latest generation of memory chips.¹

As in Dixit and Kyle (1985) trade policy will be modelled as a multistage game for which the appropriate solution is the subgame perfect equilibrium which ensures that any threats of retaliation must be credible. When the foreign country uses an export subsidy and the domestic country uses an import tariff, the Nash equilibrium in trade policies has a foreign export subsidy and a fully countervailing domestic tariff. In a Stackelberg equilibrium the foreign country has a first mover advantage so it can commit itself to an export subsidy before the domestic country sets its tariff. This seems the appropriate method to model retaliation and countervailing tariffs, since it allows the domestic country

¹For details see the Semiconductor Industry Association (1983), "The Effects of Government Targeting on World Semiconductor Competition: A case history of Japanese industrial strategy and its costs for America".
to respond to the foreign export subsidy. At a Stackelberg equilibrium the foreign country realises that the domestic country will retaliate with a countervailing tariff and its optimal policy is to set a zero export subsidy. With retaliation there is no strategic rationale for an export subsidy.

When the domestic country can use a production subsidy as well as a tariff the Nash equilibrium in trade policies has a foreign export subsidy which is fully countervailed by a domestic tariff and a production subsidy or tax. The optimal policy is a production subsidy (tax) if demand is convex (concave), this shifts domestic firms down their average cost curve. At the Stackelberg equilibrium the foreign country realises the effect its export subsidy will have on the optimal domestic tariff and production subsidy. If foreign firms have a lower marginal cost than domestic firms, then an export subsidy is optimal despite the countervailing tariff and production subsidy. This is a case where retaliation does not remove the strategic rationale for an export subsidy.

The effect of product differentiation can be studied using linear demand functions. In a Nash equilibrium in trade policies there is a foreign export subsidy or tax and a domestic tariff. The effect of competition policy on the Nash equilibrium of the trade policy game can be studied as
in Cowan (1989). An increase in the number of foreign firms results in a reduction in the domestic tariff and the foreign export subsidy. The effect of the lower tariff is to increase foreign welfare. Competition policy can shift the equilibrium of the trade policy game in favour of the foreign country. At a Stackelberg equilibrium in trade policies there is a foreign export tax and a domestic tariff. Again, with retaliation there is no strategic rationale for an export subsidy.

5.2 Basic Model

Consider a homogeneous product Cournot oligopoly in which a number of foreign firms have already entered the industry incurring a sunk fixed cost. They are faced with the entry of domestic firms who will enter if they can earn positive profits so if there is free entry the profits of domestic firms will be zero in equilibrium. It is assumed that the domestic and foreign markets are segmented, that domestic firms sell only in the domestic market, and that marginal cost is constant. These assumptions allow the analysis of the domestic market to be undertaken independently of the foreign market thus making the analysis much simpler. The foreign government is assumed to maximise its national welfare which in this case is producer surplus from exports. It can use an export subsidy to deter the entry of domestic firms. The subsidy is assumed to be given only to incumbent
firms, and not to new entrants, hence the number of foreign firms can be regarded as fixed. If subsidies were given to new entrants as well as incumbent firms then free entry of foreign firms would lead to zero profits and no role for export subsidies. In Japan MITI appears able to pursue a policy of selective subsidies to "favoured" firms in a way which does not seem to occur in the U.S.A. and Western Europe. The domestic government is assumed to maximise its national welfare which is given by the sum of consumer surplus, producer surplus and government revenue, but with free entry producer surplus is zero in this model. The domestic government can use an import tariff and a production subsidy to extract rent from foreign firms, and to encourage the entry of domestic firms.

The assumptions made here may seem stronger than in other models of trade policy, but it should be remembered that the equilibrium of the trade policy game is being analysed. To do this the optimal policies and the effect of a change in one country's trade policy on the other country's optimal policy is required. Many other models of trade policy only provide the welfare effects of small policy changes, often evaluated at the free trade position. The assumptions made here make a difficult problem tractable, which is the reason for making any assumption.

There are $m$ foreign firms that have already incurred a sunk
fixed cost $F_2$ with constant marginal cost $c_2$ each producing output $x$ for export to the domestic market. Domestic firms must incur a sunk fixed cost $F_1$ before entering the industry, the number of domestic firms that enter the market is $n$, they have constant marginal cost $c_1$ and produce an output $y$. Demand for the homogeneous product in the domestic market is given by the inverse demand function $P = P(Q)$ where $Q$ is total consumption, imports are $X = mx$ and domestic production is $Y = ny$ so $Q = X + Y$. The specific, per unit, domestic import tariff is $t$, the domestic production subsidy is $s$, and the foreign export subsidy is $e$. Domestic consumers are assumed to have preferences, for $Q$ and a competitive numeraire good, that can be represented by a utility function which is additively separable and linear in the numeraire good. Therefore, the aggregate indirect utility function is of the form: $V = V(P) + I$, where $I$ is national income. By Roy’s identity $\frac{\partial V}{\partial P} = -Q$. Domestic welfare is given by the sum of consumer surplus, domestic industry profits and government revenue. Demand for $Q$ is independent of income, hence consumer surplus is a valid measure of welfare. Distributional considerations will be ignored. Thus, domestic welfare is

$$ W_1 = V(P) + n\pi_1 + tX - sY $$

$$ = V(P) + tX - sY $$

(5.1)
With free entry, domestic industry profits are zero, \( \pi_1 = 0 \).
Unlike chapters two and four, subsidy payments have a direct income effect on domestic welfare. Foreign welfare is producer surplus from exports, that is profits of the foreign industry net of export subsidies

\[
W_2 = (P - c_2 - t)X \quad (5.2)
\]

The following assumptions will be made to ensure the existence and uniqueness of the Cournot equilibrium:

(A1) The inverse demand function \( P(Q) \) is decreasing, twice continuously differentiable and total revenue, \( P(Q)Q \), is bounded.

(A2) The following conditions are satisfied:

\[
(n + 1)P' + nyP'' < 0 \\
(m + 1)P' + mxP'' < 0
\]

(A3) The following condition is satisfied:

\[
(n + m + 1)P' + QP'' < 0
\]

Then, there exists a unique and symmetric Cournot equilibrium, for a proof see chapter three.

At a Cournot equilibrium firms maximise profits with quantity as their strategic variable, they take the number of domestic firms that have entered the market, and domestic
and foreign trade policies as given. The profits of domestic and foreign firms are

\[ \pi_1 = (P - c_1 + s)y - F_1 \]  

(5.3)

\[ \pi_2 = (P - c_2 - t + e)x - F_2 \]

The first order conditions for a Cournot equilibrium are

\[ \frac{\partial \pi_1}{\partial y} = P + yP' - c_1 + s = 0 \]  

(5.4)

\[ \frac{\partial \pi_2}{\partial x} = P + xP' - c_2 - t + e = 0 \]

And, the second order conditions for profit maximisation are

\[ \frac{\partial^2 \pi_1}{\partial y^2} = 2P' + yP'' < 0 \]  

\[ \frac{\partial^2 \pi_2}{\partial x^2} = 2P' + xP'' < 0 \]

Domestic firms will enter the industry if they can earn positive profits and so with free entry profits will be driven to zero in equilibrium. This will determine the number of domestic firms that enter the industry.\(^2\) The zero profit condition is

\[\]

\(^2\)The number of domestic firms \( n \) is assumed to be a continuous variable. This ignores the integer problem, but see Seade (1980b) for a justification of this approach.
\[
\pi_1 = (P - c_1 + s)y - F_1 = 0
\] 

(5.5)

The two first order conditions (5.4) together with the zero profit condition for domestic firms (5.5) define the equilibrium of the model given the domestic and foreign trade policies. To obtain the comparative static results totally differentiate the first order conditions and the zero profit condition, which yields

\[
\begin{pmatrix}
(m+1)P' + mxP'' & n(P' + xP'') & y(P' + xP'') \\
m(P' + yP'') & (n+1)P' + nyP'' & y(P' + yP'') \\
myP' & (n-1)YP' & y^2P'
\end{pmatrix}
\begin{pmatrix}
dx \\
dy \\
dn
\end{pmatrix}
= \begin{pmatrix}
1 & -1 & 0 \\
0 & 0 & -1 \\
0 & 0 & -y
\end{pmatrix}
\begin{pmatrix}
dt \\
de \\
ds
\end{pmatrix}
\] 

(5.6)

The minors of the matrix above are

\[
\begin{align*}
D_{11} &= y^2P' (2P' + yP'') > 0 \\
D_{12} &= 0 \\
D_{13} &= -yP' (2P' + yP'') < 0 \\
D_{21} &= y^2P' (P' + xP'') \\
D_{22} &= y^2(P')^2 > 0 \\
D_{23} &= yP' [(n-1)P' - m(P' + xP'')] \\
D_{31} &= -yP' (P' + xP'') \\
D_{32} &= yP' (P' + yP'') \\
D_{33} &= P' ((n+m+1)P' + QP'') > 0
\end{align*}
\]
And the determinant is

\[ D = y^2 (P')^2 (2P' + yP'') < 0 \]

Hence, using Cramer's rule the comparative static results can now be obtained. The effects of an export subsidy or tariff are

\[
\frac{\partial x}{\partial e} = - \frac{\partial x}{\partial t} = \frac{-D_{11}}{D} = \frac{-1}{P'} > 0 \quad \frac{\partial y}{\partial e} = - \frac{\partial y}{\partial t} = \frac{-D_{12}}{D} = 0
\]

\[
\frac{\partial n}{\partial e} = - \frac{\partial n}{\partial t} = \frac{-D_{13}}{D} = \frac{m}{yP'} < 0 \quad (5.7)
\]

\[
\frac{\partial X}{\partial e} = - \frac{\partial X}{\partial t} = \frac{-m}{P'} > 0 \quad \frac{\partial Y}{\partial e} = - \frac{\partial Y}{\partial t} = \frac{m}{P'} < 0
\]

\[
\frac{\partial P}{\partial e} = - \frac{\partial P}{\partial t} = 0 \quad \frac{\partial Q}{\partial e} = - \frac{\partial Q}{\partial t} = 0
\]

An export subsidy increases the exports of foreign firms but does not alter the scale of domestic firms that enter the industry, however there is a reduction in the number of domestic firms that enter. The exports of the foreign industry increases while the output of domestic industry contracts by the same amount to leave the price unaltered.\(^3\)

The effects of a tariff are the opposite of the effects of

\(^3\)In Nakao (1989) a foreign export subsidy does not affect the price in the domestic market, when there is free entry of domestic firms. In this model price is set by the domestic oligopoly and, foreign firms act as price takers.
an export subsidy, note that a tariff has no effect on price.\textsuperscript{4} The effects of a production subsidy are

$$\frac{\partial x}{\partial s} = \frac{D_{21} - yD_{31}}{D} = \frac{2(P' + xP'')}{\Delta}$$

$$\frac{\partial y}{\partial s} = \frac{-D_{22} + yD_{32}}{D} = \frac{yP''}{\Delta}$$

$$\frac{\partial n}{\partial s} = \frac{D_{23} - yD_{33}}{D} = \frac{-[2m(P' + xP'') + (2P' + nyP'')]}{\Delta y}$$

$$\frac{\partial X}{\partial s} = \frac{2m(P' + xP'')}{\Delta}$$

$$\frac{\partial Y}{\partial s} = \frac{-2((m+1)P' + mxP'')}{\Delta} > 0$$

$$\frac{\partial Q}{\partial s} = \frac{-2P'}{\Delta} > 0$$

$$\frac{\partial P}{\partial s} = \frac{-2(P')^2}{\Delta} < 0$$

Where $\Delta = P'(2P' + yP'') > 0$. A domestic production subsidy reduces (increases) the exports of the foreign industry if domestic output and foreign exports are strategic substitutes (complements). Domestic output and foreign exports are strategic substitutes (complements) for the foreign country if an increase in domestic output reduces (increases) the marginal profitability of foreign firms.

\textsuperscript{4}The result that a tariff does not increase price is similar to the result obtained by Brander and Spencer (1981) where a foreign monopolist is faced with the potential entry of a domestic firm and produces the entry deterring "limit" output. A tariff then does not increase price if the monopolist continues to produce the limit output to deter the entry of the domestic firm.
\[
\frac{\partial^2 \pi}{\partial y \partial x} = P' + xP'' < (>) 0.5
\]

The scale of production of domestic firms will increase (decrease) as demand is convex (concave). The effect on the number of domestic firms is ambiguous, it is possible that if there is a large increase in the scale of domestic firms then the number of domestic firms may actually fall. The production subsidy increases domestic industry output and lowers the price.

Before examining the trade policy game involving the two governments consider the effect of a foreign export subsidy in the absence of any domestic trade policy, \( t = s = 0 \). The effect of an export subsidy on foreign welfare, from (5.2) is

\[
\frac{\partial W}{\partial e} = (P - c_2) \frac{\partial X}{\partial e} + X \frac{\partial P}{\partial e}
\]  

(5.9)

The first term is the profit shifting effect: exports are increased and since price exceeds marginal cost this increases foreign welfare. The second term is the terms of trade effect: an export subsidy usually results in a fall in the price of exports which reduces foreign welfare. In this model an export subsidy does not affect the price so the terms of trade effect is zero, therefore only the positive profit shifting effect is present. Using the comparative

\[\text{Strategic substitutes and complements are explained in Bulow et al (1985) and chapter three.}\]
static results the overall effect is

\[
\frac{\partial W}{\partial e} = (P - c_2)^{-m} \frac{m}{p'} > 0 \quad (5.10)
\]

An export subsidy is unambiguously welfare improving for the foreign country. The export subsidy commits foreign firms to produce a larger output which reduces the number of domestic firms that can profitably enter the industry. This reduces domestic industry output by an amount equal to the increase in foreign industry exports so that price does not fall. The optimal policy is to subsidise exports so that no domestic firms will enter the industry and foreign firms capture the entire market. This argument for an export subsidy is stronger than the profit shifting argument of Brander and Spencer (1985), discussed in chapters two and four.

Brander and Spencer (1985) consider a Cournot duopoly consisting of a foreign firm and a domestic firm. A foreign export subsidy commits the foreign firm to produce a larger output then, if domestic and foreign output are strategic substitutes, the domestic firm will reduce output. The result is to shift profits from the domestic to the foreign firm. It has been shown by Eaton and Grossman (1986) that if there is Bertrand rather than Cournot competition then the

---

\[ \text{6This is similar to firms deterring entry by using investment to commit themselves to a larger output, as in Dixit (1980).} \]
optimal policy is generally an export tax. This argument does not apply to the use of export subsidies to deter entry. Consider a single foreign firm faced with the potential entry of a domestic firm. The product is homogeneous and there is Bertrand competition. The foreign firm has a constant marginal cost $c_2$ and the domestic firm has a constant marginal cost $c_1$ where $c_1 < c_2$, so the domestic firm has a cost advantage. To enter the industry the domestic firm must incur a sunk cost $F_1 > 0$. With no foreign export subsidy the domestic firm will enter the industry, if $F_1$ is not too large, and take the entire market with a price $c_2 - c$. The domestic firm will make positive profits and the foreign firm will make zero profits. If the foreign government gives an export subsidy $e$ such that $c_2 - e < c_1$ then it will not be profitable for the domestic firm to enter since it would be undercut by the foreign firm on price. Therefore, the foreign firm will have a monopoly and earn positive profits. The foreign country will gain if profit net of the export subsidy is positive as in figure 5.1.

In the absence of any domestic trade policy, the optimal foreign policy is to subsidise exports so that no domestic firms will enter the industry, and the foreign firms capture the entire market. There is a strong strategic argument for an export subsidy when there is no retaliation.
5.3 Export Subsidies and Countervailing Tariffs

This section considers the interaction of domestic and foreign trade policies when the domestic country uses an import tariff and the foreign country uses an export subsidy. The model is analysed as a multistage game. At the first stage the domestic and foreign government set their trade policies to maximise national welfare. Then, in the second stage domestic firms decide whether to enter the industry given the foreign and domestic trade policies. In the final stage the foreign firms and any domestic firms which enter set outputs to maximise profits given the foreign and domestic trade policies. A subgame perfect equilibrium is required which rules out any non-credible threats. As usual the equilibrium is obtained by backward induction. First, the Nash equilibrium of the final stage of the game is obtained and then this is used to solve the penultimate stage and so on until one obtains the subgame perfect equilibrium for the entire game. The Nash equilibrium of the final two stages of the game was derived in the previous section and the comparative static results obtained there can now be used to solve the first stage of the game. At a Nash equilibrium in trade policies both countries independently and simultaneously choose their trade policies to maximise their national welfare. The first order conditions for a Nash equilibrium in trade policies are
\[
\frac{\partial W_1}{\partial t} = -Q \frac{\partial P}{\partial t} + t \frac{\partial X}{\partial t} + X = 0
\]

\[
\frac{\partial W_2}{\partial e} = (P - c_2 - t) \frac{\partial X}{\partial e} + \frac{\partial P}{\partial e} = 0
\]

(5.11)

Using the comparative static results from (5.7), and noting that an export subsidy or tariff has no effect on price, together with the first order conditions for profit maximisation from (5.4) yields the optimal trade policies

\[
e^N = t^N = -XP' = (P - c_2) > 0
\]

(5.12)

At the Nash equilibrium in trade policies there is a foreign export subsidy and a fully countervailing domestic tariff. The domestic country extracts all the rent from the foreign country since \( P - c_2 - t = 0 \) so all the gains from trade are captured by the domestic country.\(^7\) If the Nash equilibrium is viewed as the outcome of a trade war then the foreign country has obviously lost.

The situation is shown in figure 5.2. The foreign reaction function can be derived from equation (5.11). If the domestic tariff is set such that \( P - c_2 - t > 0 \), then the optimal foreign policy is to subsidise exports so that no

\(^7\)In Brander and Spencer (1981) the domestic country may extract all the rent from the foreign monopolist.
domestic firms enter the industry. An increase in the domestic tariff will increase the export subsidy, required to deter the entry of all domestic firms, by the same amount so \( \frac{de}{dt} = 1 \). When \( P - c_2 - t = 0 \) then the foreign country is indifferent about any export subsidy between zero and the level required to deter the entry of all domestic firms, since its payoff is zero regardless. And when \( P - c_2 - t < 0 \) then obviously a zero export subsidy is optimal, in fact, any export subsidy or tax is optimal provided it results in zero exports. The slope of the domestic reaction function can be derived by totally differentiating the formula for the optimal tariff, \( t = -xP' \), which yields \( \frac{dt}{de} = 1/2 \). And the optimal tariff is positive when the foreign export subsidy is zero. The Nash equilibrium (NE) is given by the intersection of the foreign and domestic reaction curves.

Now consider how a parameter change alters the equilibrium of the trade policy game. By totally differentiating the optimal policies in (5.12) it can easily be shown that

\[
\frac{\partial e}{\partial c_2} = \frac{\partial t}{\partial c_2} = -1
\]

\[
\frac{\partial e}{\partial c_1} = \frac{\partial t}{\partial c_1} = \frac{2P'}{2P' + yP''} > 0
\]

\[
\frac{\partial e}{\partial F_1} = \frac{\partial t}{\partial F_1} = \frac{P'}{y(2P' + yP'')} > 0
\]
The higher is foreign marginal cost then the lower is the optimal export subsidy and tariff, and the higher is domestic marginal and fixed cost the larger is the optimal export subsidy and tariff in the Nash equilibrium. For the Cournot duopoly model of Brander and Spencer (1985) it has been shown by de Meza (1986) that the country with the lowest cost will have the largest export subsidy. Here, the optimal foreign export subsidy will be larger the greater the relative cost advantage of foreign firms.

To model retaliation the structure of the game will be altered so that the foreign country has a first mover advantage. At the first stage of the new game, call it the Stackelberg game, the foreign country sets its export subsidy to maximise its national welfare. Then, in the second stage the domestic country sets its tariff, in response to the foreign export subsidy, to maximise its national welfare. In the third stage the domestic firms make their entry decision and in the final stage domestic and foreign firms set their outputs to maximise profits. The equilibrium of the final two stages of the game was derived in section 5.2 and the comparative static results derived there can now be used to solve the second stage.

At the second stage the domestic country sets its tariff given the foreign export subsidy. The first order condition
Using the comparative static results from (5.7) together with (5.4) yields the optimal tariff

\[ t = \frac{1}{2}(P - c_2 + e) > 0 \]  

(5.14)

The effect of a change in the foreign export subsidy on the optimal tariff is obvious from above, if it is noted that an export subsidy or tariff have no effect on price, hence

\[ \frac{dt}{de} = \frac{1}{2} \]  

(5.15)

The optimal response to a foreign export subsidy is to increase the tariff by half the amount of the subsidy. A foreign export subsidy should only be partially countervailed. Dixit (1988) shows that for a Cournot duopoly with one domestic and one foreign firm and linear demand the optimal countervailing tariff fraction is one third.

At the first stage of the game the foreign country sets its export subsidy to maximise national welfare realising the effect its decision will have on the optimal domestic tariff. The first order condition for welfare maximisation
is

\[
\frac{\text{d}W^2}{\text{d}e} = (P - c_2 - t) \left( \frac{\partial X}{\partial e} + \frac{\partial X}{\partial t} \frac{\text{d}t}{\text{d}e} \right) + X \left( \frac{\partial P}{\partial e} + \left( \frac{\partial P}{\partial t} - 1 \right) \frac{\text{d}t}{\text{d}e} \right) = 0 \quad (5.16)
\]

The first term is the profit shifting effect, and the second term is the terms of trade effect. The profit shifting effect of the export subsidy is reduced by the effect of the countervailing tariff on foreign exports. The terms of trade effect is negative, since both an export subsidy and a tariff have no effect on the domestic price, but foreign firms are faced with a higher tariff. The countervailing tariff reduces the profit shifting effect and worsens the terms of trade. The optimal foreign export subsidy is derived by using the comparative static results from (5.7) together with the optimal countervailing tariff from (5.15) in (5.16), and noting from (5.4) that \( P - c_2 - t = -xP' - e \). The optimal domestic tariff is obtained by substituting the optimal export subsidy into (5.14). Hence, the optimal trade policies at a Stackelberg equilibrium are

\[
e^* = 0 \quad \quad t^* = \frac{1}{2}(P - c_2) > 0 \quad (5.17)
\]

When the foreign country is faced with retaliation by the domestic country with a countervailing tariff its optimal policy is a zero export subsidy. The foreign country gains by committing itself to a zero export subsidy, since then producer surplus is positive \( P - c_2 - t = \frac{1}{2}(P - c_2) \). Half of
the producer surplus accrues to the foreign country, and half is captured as tariff revenue by the domestic country. The foreign country is better off and the domestic country is worse off at the Stackelberg equilibrium compared to the Nash equilibrium. In figure 5.2, the foreign iso-welfare locus is tangential to the domestic reaction function at the Stackelberg equilibrium (SE).

The Stackelberg equilibrium yields the same outcome as the consistent conjecture equilibrium.\(^8\) The foreign country's consistent conjecture is for the domestic country to increase its tariff by half the amount of any increase in the foreign export subsidy. And, the domestic country's consistent conjecture is that a change in its tariff will have no effect on the export subsidy set by the foreign country. But, consistent conjecture equilibrium are of questionable value.

If the domestic country could commit itself to a tariff before the foreign country sets its export subsidy, then its optimal policy is to set a tariff just below the Nash equilibrium tariff. Then, the foreign country will subsidise exports until foreign firms capture the entire market since profits are positive. And both domestic and foreign welfare

\[^8\text{Turnovsky (1986) considered the consistent conjecture equilibrium of the Johnson (1953-54) tariff game. Tanaka (1989) considers the consistent conjecture equilibrium of the Brander and Spencer (1985) export subsidy game.}\]
are higher than at the Nash equilibrium. Domestic welfare is higher since imports have increased so tariff revenue is higher. Foreign welfare is higher since it now has positive producer surplus.

When the domestic country can retaliate with a countervailing tariff, the optimal foreign export subsidy is zero, and there is no strategic argument for an export subsidy.

5.4 Export Subsidies, Production Subsidies and Tariffs

In the previous section the domestic government only used an import tariff, in this section the domestic government can use a production subsidy and an import tariff. The domestic production subsidy is paid to all domestic firms that enter the industry.\(^9\) Firstly, consider the Nash equilibrium in trade policies. The first order conditions for a Nash equilibrium are

\[
\frac{\partial W}{\partial t} = -Q \frac{\partial P}{\partial t} + t \frac{\partial X}{\partial t} + X - s \frac{\partial Y}{\partial t} = 0
\]

\[
\frac{\partial W}{\partial s} = -Q \frac{\partial P}{\partial s} + t \frac{\partial X}{\partial s} - s \frac{\partial Y}{\partial s} - Y = 0
\]  

\(9\)The first best policy would be to pay the subsidy to one firm, and prevent other firms from entering the industry. Then the model would be as in chapter four.
\[ \frac{\partial W}{\partial e} = (P - c_2 - t) \frac{\partial X}{\partial e} + X \frac{\partial P}{\partial e} = 0 \]

Using the comparative static results from (5.7) and (5.8) together with the first order conditions from (5.4) to solve for the optimal policies yields

\[ t^N + s^N = -xP' > 0 \]

\[ s^N = (mx^2 + \frac{1}{2}ny^2)P'' \quad (5.19) \]

\[ e^N = -xP' > 0 \]

In the Nash equilibrium in trade policies there is an export subsidy which is fully countervailed by a domestic tariff and a production subsidy (tax). The optimal policy will be a production subsidy (tax) when demand is convex (concave), this increases domestic welfare by shifting firms down their average cost curve.\(^{10}\) If demand is sufficiently convex then an import subsidy may be optimal. As in the previous section the domestic country extracts all the rent from the foreign country since \( P - c_2 - t = 0 \). It is often argued that a production subsidy is preferable to a tariff to protect the

\(^{10}\) In a closed economy de Meza (1982) shows that a production subsidy or tax can shift firms down their average cost curve and hence improve welfare. See also Horstmann and Markusen (1986).
domestic industry, but with concave demand the optimal policy is a tariff and a production tax. The explanation is that with free entry and fixed costs inefficient entry will occur. Also, the tariff is not being used to protect the domestic industry but to extract rent from foreign firms.

Now consider the Stackelberg game where the foreign government sets its export subsidy at the first stage. Then at the second stage the domestic government sets its import tariff and production subsidy to maximise national welfare taking the export subsidy set by the foreign country at the first stage as given. The first order conditions for the optimal policies are

\[
\frac{\partial W}{\partial t} = -Q \frac{\partial P}{\partial t} + t \frac{\partial X}{\partial t} + X - s \frac{\partial Y}{\partial t} = 0
\]

\[
\frac{\partial W}{\partial s} = -Q \frac{\partial P}{\partial s} + t \frac{\partial X}{\partial s} - s \frac{\partial Y}{\partial s} - Y = 0
\]

Using the comparative static results (5.7) and (5.8) to solve for the optimal policies yields

\[
t + s = -XP' > 0
\]

\[
s = (mx^2 + \frac{1}{2}ny^2)P''
\]

The effect of a foreign export subsidy on the optimal
domestic tariff and production subsidy can be obtained by totally differentiating the first order conditions for welfare maximisation (5.20) which yields

$$
\begin{bmatrix}
\frac{\partial^2 W_1}{\partial t^2} & \frac{\partial^2 W_1}{\partial s \partial t} \\
\frac{\partial^2 W_1}{\partial t \partial s} & \frac{\partial^2 W_1}{\partial s^2}
\end{bmatrix}
\begin{bmatrix}
dt \\
ds \\
de
\end{bmatrix}
= 
\begin{bmatrix}
-\frac{\partial^2 W_1}{\partial e \partial t} \\
-\frac{\partial^2 W_1}{\partial e \partial s}
\end{bmatrix}
$$

(5.22)

Where all the partial derivatives are evaluated at the welfare optimum. The second order conditions for welfare maximisation require that the above Hessian matrix is negative definite, so

$$
\frac{\partial^2 W_1}{\partial t^2} < 0 \quad \frac{\partial^2 W_1}{\partial s^2} < 0
$$

$$
H =
\begin{vmatrix}
\frac{\partial^2 W_1}{\partial t^2} & \frac{\partial^2 W_1}{\partial s \partial t} \\
\frac{\partial^2 W_1}{\partial t \partial s} & \frac{\partial^2 W_1}{\partial s^2}
\end{vmatrix}
> 0
$$

(5.23)

To obtain the second order partial derivatives use the comparative static results from (5.7) in (5.20), this yields

$$
\frac{\partial W_1}{\partial t} = \frac{1}{p'} (XP' + mt + ms)
$$

(5.24)

Differentiate $\frac{\partial W_1}{\partial t}$ with respect to $t$, and evaluating at
the welfare maximum, yields using (5.7)

\[
\frac{\partial^2 W_1}{\partial t^2} = \frac{1}{P'} \left( P' \frac{\partial X}{\partial t} + m \right) = \frac{2m}{P'} < 0
\] (5.25)

Differentiate \( \partial W_1 / \partial t \) with respect to \( s \), and evaluating at the welfare maximum, yields using (5.8)

\[
\frac{\partial^2 W_1}{\partial s \partial t} = \frac{1}{P'} \left( XP'' + P' \frac{\partial X}{\partial s} + m \right) = \frac{m}{P'} \left( \frac{4P' + yP''}{2P' + yP''} \right) < 0
\] (5.26)

Differentiate \( \partial W_1 / \partial t \) with respect to \( e \), and evaluating at the welfare maximum, yields using (5.7)

\[
\frac{\partial^2 W_1}{\partial e \partial t} = \frac{1}{P'} \left( P' \frac{\partial X}{\partial e} \right) = -\frac{m}{P'} < 0
\] (5.27)

Using the comparative static results from (5.8) in \( \partial W_1 / \partial s \) from (5.20) yields

\[
\frac{\partial W_1}{\partial s} = \frac{1}{\Delta} \left[ 2Q(P')^2 + 2tm(P' + xP'') + 2s((m+1)P' + mxP'') - YP'(2P' + yP'') \right]
\] (5.28)

Differentiate \( \partial W_1 / \partial s \) with respect to \( t \), and evaluating at the welfare maximum, yields

\[
\frac{\partial^2 W_1}{\partial t \partial s} = \frac{1}{\Delta} \left[ 2tP'' \frac{\partial X}{\partial t} + 2sP'' \frac{\partial X}{\partial t} + 2m(P' + xP'') - P'(2P' + yP'') \frac{\partial Y}{\partial t} \right]
\]
Using the comparative static results from (5.7) and the optimal policies from (5.21) yields

\[ \frac{\partial^2 W_1}{\partial t \partial s} = \frac{m}{P'} \left( \frac{4P' + yP''}{2P' + yP''} \right) < 0 \]  

(5.29)

Differentiate \( \frac{\partial W_1}{\partial s} \) with respect to \( e \), and evaluating at the welfare maximum, yields

\[ \frac{\partial^2 W_1}{\partial e \partial s} = \frac{1}{\Delta} \left[ 2tP'' \frac{\partial X}{\partial e} + 2sP'' \frac{\partial X}{\partial e} - P'(2P' + yP'') \frac{\partial Y}{\partial e} \right] \]

Using the comparative static results from (5.7) and the optimal policies from (5.21) yields

\[ \frac{\partial^2 W_1}{\partial e \partial s} = \frac{m}{P'} \left( \frac{2P' + (y - 2x)P''}{2P' + yP''} \right) \]  

(5.30)

The second order partial derivative \( \frac{\partial^2 W_1}{\partial s^2} \) is not required to evaluate the comparative static results, and so need not be derived.

These can then be used to evaluate the comparative static results. The effect of a foreign export subsidy on the optimal domestic tariff is by Cramer’s rule.
\[
\frac{dt}{de} = \frac{1}{H} \begin{vmatrix}
\frac{\partial^2 w_1}{\partial \delta t} & \frac{\partial^2 w_1}{\partial s \delta t} \\
\frac{\partial^2 w_1}{\partial \delta s} & \frac{\partial^2 w_1}{\partial s^2}
\end{vmatrix}
\]

To evaluate this note that from (5.25), (5.27), (5.29) and (5.30)

\[
- \frac{\partial^2 w_1}{\partial \delta t} = \frac{1}{2} \frac{\partial^2 w_1}{\partial t^2} \quad \quad \quad - \frac{\partial^2 w_1}{\partial \delta s} = \frac{1}{2} \frac{\partial^2 w_1}{\partial t \delta s} \quad \quad \quad - \frac{1}{2} \frac{m(4x-y)P''}{\Delta}
\]

Using these in (5.31) yields

\[
\frac{dt}{de} = \frac{1}{2 H} \begin{vmatrix}
\frac{\partial^2 w_1}{\partial t^2} & \frac{\partial^2 w_1}{\partial s \delta t} & 1 \\
\frac{\partial^2 w_1}{\partial \delta s} & \frac{\partial^2 w_1}{\partial s^2} & \frac{-m(4x-y)P''}{\Delta} \\
& & \frac{\partial^2 w_1}{\partial s^2}
\end{vmatrix}
\]

Using (5.26) it can be shown that

\[
\frac{dt}{de} = \frac{1}{2} + \frac{m^2(4P' + yP'')(4x-y)P''}{2 \Delta^2 H}
\]

It will be assumed that \((4x-y) > 0\), this will hold unless the domestic firms have a significant cost advantage. Hence if demand is convex (concave), then the optimal countervailing tariff fraction will be smaller (greater) than a half. For linear demand the countervailing tariff fraction is a half. Dixit (1988) has shown that, for a
homogeneous product Cournot oligopoly with a fixed number of domestic and foreign firms, the optimal countervailing tariff fraction is a half for linear demand and less than a half for constant elasticity demand functions. In chapter four it was shown that the optimal countervailing tariff fraction is less (greater) than a half if demand is convex (concave), which is similar to the result obtained here. A fully countervailing tariff could be optimal if demand was sufficiently concave but, in general, only partially countervailing tariffs are justified.

From (5.26), the effect of a foreign export subsidy on the optimal domestic production subsidy is by Cramer's rule

\[
\frac{ds}{de} = \frac{1}{H} \begin{vmatrix} \frac{\delta^2 W_1}{\delta t^2} & - \frac{\delta^2 W_1}{\delta e \delta t} \\ \frac{\delta^2 W_1}{\delta t \delta s} & \frac{\delta^2 W_1}{\delta e \delta s} \end{vmatrix}
\]

Using (5.25), (5.27), (5.29) and (5.30) in (5.33) yields

\[
\frac{ds}{de} = \frac{1}{H} \begin{vmatrix} \frac{2m}{P'} & m \\ \frac{m}{P'} & \frac{m}{P'} \end{vmatrix} = \frac{1}{H} \begin{vmatrix} \frac{m (4P' + yP'')}{P'(2P' + yP'')} & \frac{m 2P' + (y-2x)P''}{P'(2P' + yP'')} \\ \frac{m}{P'} & \frac{m}{P'} \end{vmatrix}
\]

Therefore, it can be shown that
When demand is convex (concave) an increase in the foreign export subsidy will increase (decrease) the optimal domestic production subsidy. This differs from the result of chapter four where a foreign export subsidy would usually reduce the optimal domestic production subsidy.

It is now possible to analyse the first stage of the game when the foreign country sets its export subsidy to maximise national welfare realising the effect its decision will have on the optimal domestic tariff and production subsidy. The effect of an export subsidy on foreign welfare is

\[
\frac{dW}{de} = (P - c - t) \left( \frac{\partial X}{\partial e} + \frac{\partial X}{\partial t} + \frac{\partial X}{\partial s} \frac{ds}{de} \right) + X \left( \frac{\partial P}{\partial e} + \left( \frac{\partial P}{\partial t} - 1 \right) \frac{dt}{de} + \frac{\partial P}{\partial s} \frac{ds}{de} \right) \tag{5.35}
\]

Note that the export subsidy and tariff have no effect on price. Using the comparative static results to evaluate this expression when \( e = 0 \) so that \( P - c - t = -xP' \) yields

\[
\frac{dW}{de} = \frac{-m^2(4x-y)}{\Delta} \frac{dP}{de} \tag{5.36}
\]

The effect of an export subsidy on foreign welfare will be
zero if demand is linear but with non-linear demand it will be positive if \((2x-y) > 0\). The conditions under which this will occur can be seen by subtracting the domestic firm’s first order condition for profit maximisation from that of the foreign firm (5.4) and noting that \(t + s = -xP'\) from (5.21) yields

\[ c_1 - c_2 + e + (2x-y)P' = 0 \]  \hspace{1cm} (5.37)

Therefore at \(e = 0\), \((2x-y) > 0\) when foreign marginal cost is lower than domestic marginal cost. So an export subsidy will increase foreign welfare if foreign firms have a lower marginal cost than domestic firms. In this case the export subsidy increases foreign welfare, despite retaliation from the domestic country with countervailing tariffs and subsidies. The optimal export subsidy can be derived by setting \(dW_2/de = 0\) and using (5.37) yields

\[ e = \frac{2m^2x(4x-y)(c_1-c_2)(P'')^2}{\Delta^2H + m^2(4x-y)(2x-y)(P'')^2} \]  \hspace{1cm} (5.38)

With linear demand the optimal export subsidy is zero. When foreign firms have a cost advantage and demand is non-linear the optimal policy is an export subsidy despite the countervailing tariffs and subsidies. This shows that, in theory, an export subsidy can increase foreign welfare even when there is retaliation.

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5.5 Product Differentiation

In the previous sections the domestic and foreign outputs were perfect substitutes in this section it will be assumed that the domestic and foreign output are not perfect substitutes, also it will be assumed that demand is linear. The assumption of national product differentiation, where domestic and foreign output are not perfect substitutes but the output of a firm is a perfect substitute for the output of another firm in the same country, may not seem realistic but the main impact that product differentiation has on the model comes from the fact that domestic and foreign output are not perfect substitutes. Assuming that all firms produce a differentiated product would make the arithmetic more tedious but would offer no additional insight.

The prices that domestic and foreign firms face are now different, let \( P_1 \) be the price for domestic output and \( P_2 \) the price for foreign output. Consumer preferences are given by the indirect utility function \( V = V(P_1, P_2) + I \), so by Roy's identity \( \frac{\delta V}{\delta P_1} = -Y \) and \( \frac{\delta V}{\delta P_2} = -X \). Domestic welfare is given by \( W_1 = V(P_1, P_2) + tX - sY \). Foreign welfare is producer surplus from exports \( W_2 = (P_2 - c_2 - t)X \). Demand is assumed to be linear and symmetric. The domestic and foreign inverse demand functions are

\[
P_1 = \alpha - \beta Y - \gamma X
\]
The degree of product differentiation is measured by $\beta - \gamma$, if $\beta = \gamma$ then domestic and foreign output are perfect substitutes. Domestic and foreign profits are given by

$$\pi_1 = (P_1 - c_1 + s)y - F_1$$

(5.40)

$$\pi_2 = (P_2 - c_2 - t + e)x - F_2$$

The first order conditions for a Cournot equilibrium given the number of domestic firms that have entered and given the domestic and foreign trade policies are

$$\frac{\partial \pi_1}{\partial y} = P_1 + y \frac{\partial P_1}{\partial y} - c_1 + s = 0$$

(5.41)

$$\frac{\partial \pi_2}{\partial x} = P_2 + y \frac{\partial P_2}{\partial x} - c_2 - t + e = 0$$

Domestic firms will enter if they can earn positive profits so free entry will lead to zero profits in equilibrium

$$\pi_1 = (P_1 - c_1 + s)y - F_1 = 0$$

(5.42)

The two first order conditions and the zero profit condition determine the equilibrium as a function of the domestic
tariff and production subsidy and the foreign export subsidy. The comparative static results for the effect of the trade policies, and the effect of an increase in the number of foreign firms, are obtained by totally differentiating the two first order conditions (5.41) and the zero profit condition (5.42), to obtain

\[
\begin{pmatrix}
(m+1)\beta & n\gamma & y\gamma \\
m\gamma & (n+1)\beta & y\beta \\
my\gamma & (n-1)y\beta & y^2\beta
\end{pmatrix}
\begin{pmatrix}
dx \\
dy \\
dn
\end{pmatrix}
= \begin{pmatrix}
1 & -1 & -x\beta \\
0 & 0 & -x\gamma \\
0 & 0 & -xy\gamma
\end{pmatrix}
\begin{pmatrix}
dt \\
de \\
dm
\end{pmatrix}
(5.43)
\]

The minors of the above matrix are

- \(D_{11} = 2y^2\beta > 0\)
- \(D_{12} = 0\)
- \(D_{13} = -2my\beta\gamma < 0\)
- \(D_{21} = -y^2\beta\gamma < 0\)
- \(D_{22} = y^2(\beta^2 + m(\beta^2-\gamma^2)) > 0\)
- \(D_{23} = (m - n + 1)y\beta^2 - mn\gamma(\beta^2-\gamma^2)\)
- \(D_{31} = -y\beta\gamma < 0\)
- \(D_{32} = -y(\beta^2 + m(\beta^2-\gamma^2)) < 0\)
- \(D_{33} = (n + m + 1)\beta^2 + nm(\beta^2-\gamma^2) > 0\)

And the determinant of the matrix is

- \(D = 2y^2\beta(\beta^2 + m(\beta^2-\gamma^2)) > 0\)

The comparative static results for the effect of a foreign export subsidy or a domestic import tariff are
\[
\frac{dx}{de} = - \frac{\partial x}{\partial t} = \frac{\beta}{\beta^2 + m(\beta^2 - \gamma^2)} > 0
\]

\[
\frac{dy}{de} = - \frac{\partial y}{\partial t} = 0 \quad \text{(5.44)}
\]

\[
\frac{dn}{de} = - \frac{\partial n}{\partial t} = \frac{-m\gamma}{\beta^2 + m(\beta^2 - \gamma^2)} < 0
\]

An export subsidy increases the output of foreign firms but, as in the case of homogeneous products, it has no effect on the scale of domestic firms. The number of domestic firms which enter the industry decreases. The effect of an export subsidy on domestic and foreign output is

\[
\frac{\partial Y}{\partial e} = n \frac{\partial y}{\partial e} + y \frac{\partial n}{\partial e} = \frac{-m\gamma}{\beta^2 + m(\beta^2 - \gamma^2)} < 0
\]

\[
\frac{\partial X}{\partial e} = \frac{m\beta}{\beta^2 + m(\beta^2 - \gamma^2)} > 0 \quad \text{(5.45)}
\]

Foreign industry output increases as a result of the export subsidy and domestic industry output decreases, but the decrease in domestic output is less than the increase in foreign output. The effect on prices is

\[
\frac{\partial P}{\partial e} = - \beta \frac{\partial y}{\partial e} - \gamma \frac{\partial x}{\partial e} = 0
\]

\[
\text{(5.46)}
\]
\[ \frac{\partial P_2}{\partial e} = -\frac{\partial Y}{\partial e} - \beta \frac{\partial X}{\partial e} = \frac{-m(\beta^2 - \gamma^2)}{\beta^2 + m(\beta^2 - \gamma^2)} < 0 \]

The export subsidy has no effect on the price of the domestic product but reduces the price of the foreign product.

At the Nash equilibrium in trade policies both countries independently and simultaneously choose their trade policies, tariff and export subsidy, to maximise their national welfare. The first order conditions for a Nash equilibrium in trade policies are

\[ \frac{\partial W_1}{\partial t} = -Y \frac{\partial P_1}{\partial t} - X \left( \frac{\partial P_2}{\partial t} - 1 \right) + t \frac{\partial X}{\partial t} = 0 \]

\[ \frac{\partial W_2}{\partial e} = (P_2 - c_2 - t) \frac{\partial X}{\partial e} + X \frac{\partial P_2}{\partial e} = 0 \]

Using the comparative static results to solve for the optimal policies yields

\[ t^N = x\beta > 0 \quad (5.47) \]

\[ \epsilon^N = \frac{x (\beta^2 - m(\beta^2 - \gamma^2))}{\beta} \]

The optimal tariff is positive but the export subsidy may be positive or negative. When the products were perfect
substitutes the Nash equilibrium had a foreign export subsidy which was fully countervailed by a domestic tariff, and the domestic country extracted all the rent from the foreign country. Here the foreign export subsidy is smaller than the domestic tariff and the foreign country earns positive producer surplus on exports since \( P_2 - c_2 - t = m(\beta^2 - \gamma^2)/\beta > 0 \). If there is only a single foreign firm then

\[
t^N = x\beta > 0
\]

(5.48)

\[
e^N = \frac{x\gamma^2}{\beta} > 0
\]

Then the optimal foreign export subsidy and domestic tariff are both positive.

Cowan (1989) analysed how competition policy could alter the equilibrium of the trade policy game. In his model an extension of Brander and Spencer (1984), there is no domestic production and the domestic country uses a tariff to extract rent from the foreign firms while the foreign country uses an export tax to improve the terms of trade. At the first stage of the game the foreign country sets the number of foreign firms, its competition policy, then at the second stage the foreign country sets its export tax and the domestic country sets its import tariff. Cowan shows that if demand is not too convex then an increase in the number of
foreign firms will result in a lowering of the domestic tariff which increases foreign welfare. The foreign country can use competition policy to alter the equilibrium of the game in its favour.

From (5.43), the comparative static results for the effects of foreign competition, an increase in the number of foreign firms \( m \), are

\[
\frac{\partial x}{\partial m} = \frac{-x(\beta^2 - \gamma^2)}{\beta^2 + m(\beta^2 - \gamma^2)} < 0
\]

\[
\frac{\partial y}{\partial m} = 0 \quad \text{(5.49)}
\]

\[
\frac{\partial n}{\partial m} = -\frac{x \gamma \beta}{\gamma \beta^2 + m(\beta^2 - \gamma^2)} < 0
\]

An increase in the number of foreign firms reduces the exports of foreign firms, but has no effect on the scale of domestic firms. And, there is a reduction in the number of domestic firms that enter the industry. The effect on domestic output and foreign exports is

\[
\frac{\partial y}{\partial m} = \frac{x \beta^2}{\beta^2 + m(\beta^2 - \gamma^2)} > 0 \quad \text{(5.50)}
\]

\[
\frac{\partial x}{\partial m} = \frac{-x \gamma \beta}{\beta^2 + m(\beta^2 - \gamma^2)} > 0
\]
An increase in the number of foreign firms increases foreign exports and reduces the output of the domestic industry. The effect on the prices is

$$\begin{align*}
\frac{\partial P_1}{\partial m} &= -\beta \frac{\partial Y}{\partial m} - \gamma \frac{\partial X}{\partial m} = 0 \\
\frac{\partial P_2}{\partial m} &= -\gamma \frac{\partial Y}{\partial m} - \beta \frac{\partial X}{\partial m} = -x\beta (\beta^2 - \gamma^2) < 0
\end{align*}$$

An increase in the number of foreign firms reduces the price of the foreign product, but has no effect on domestic price.

Now consider how foreign competition policy alters the Nash equilibrium in trade policies in this model. The Nash equilibrium policies are given by (5.47). To obtain the effect of foreign competition policy on the foreign export subsidy and domestic tariff totally differentiate (5.47) and using the comparative static results yields

$$\begin{align*}
\begin{bmatrix}
2\beta^2 + m(\beta^2 - \gamma^2) & -\beta^2 \\
\beta^2 - m(\beta^2 - \gamma^2) & 2m(\beta^2 - \gamma^2)
\end{bmatrix}
\begin{bmatrix}
\frac{dt}{dm} \\
\frac{de}{dm}
\end{bmatrix} =
\begin{bmatrix}
-x\beta (\beta^2 - \gamma^2) \\
-2x\beta (\beta^2 - \gamma^2)
\end{bmatrix}
\end{align*}$$

This yields

$$\begin{align*}
\frac{dt}{dm} &= \frac{-2x\beta (\beta^2 - \gamma^2)}{\beta^2 + 2m(\beta^2 - \gamma^2)} < 0 \\
\frac{de}{dm} &= \frac{-3x\beta (\beta^2 - \gamma^2)}{\beta^2 + 2m(\beta^2 - \gamma^2)} < 0
\end{align*}$$

(5.52)
An increase in the number of foreign firms results in a reduction of the foreign export subsidy and the domestic tariff. The reduction in the foreign export subsidy is larger than the reduction in the domestic tariff. The effect of competition policy on foreign welfare is

\[
\frac{dW}{dm} = \frac{\partial W}{\partial m} + \frac{\partial W}{\partial t} \frac{dt}{dm} + \frac{\partial W}{\partial e} \frac{de}{dm} \tag{5.53}
\]

But since the foreign country is setting its export subsidy optimally at the Nash equilibrium \( \frac{\partial W}{\partial e} = 0 \), so the change in the export subsidy has no effect on welfare. Hence the overall welfare effect is

\[
\frac{dW}{dm} = (P_2 - c_2 - t) \left( \frac{\partial X}{\partial m} + \frac{\partial X}{\partial t} \frac{dt}{dm} \right) + X \left( \frac{\partial P_2}{\partial m} + \left( \frac{\partial P_2}{\partial t} - 1 \right) \frac{dt}{dm} \right) = 0 \tag{5.54}
\]

Using the comparative static results it can be shown that the overall effect is just the direct tariff revenue effect

\[
\frac{dW}{dm} = -X \frac{dt}{dm} = \frac{2mx^2\beta(\beta^2 - \gamma^2)}{\beta^2 + 2m(\beta^2 - \gamma^2)} > 0 \tag{5.55}
\]

As in Cowan (1989) an increase in the number of foreign firms can, by reducing the domestic tariff, increase foreign
welfare. This ignores the sunk costs of entry. Competition policy will increase foreign welfare if the gain from the reduction in the domestic tariff exceeds the sunk costs of entry.

Now consider the Stackelberg equilibrium of the game where the foreign country has a first mover advantage and sets its export subsidy before the domestic country sets its tariff. At the second stage the domestic country sets its tariff taking the foreign export subsidy as given. The first order condition for the optimal tariff is

\[
\frac{\partial W}{\partial t} = -Y \frac{\partial P}{\partial t} - X \left( \frac{\partial P^2}{\partial t} - 1 \right) + \frac{\partial X}{\partial t} = 0 \quad (5.56)
\]

Hence using the comparative static results yields the optimal domestic tariff

\[
t = x\beta > 0 \quad (5.57)
\]

To obtain the effect of the foreign export subsidy on the optimal domestic tariff, totally differentiate the above and using the comparative static results yields

\[
\frac{dt}{de} = \frac{\beta^2}{2\beta^2 + m (\beta^2 - \gamma^2)} > 0 \quad (5.58)
\]

The optimal countervailing tariff fraction is positive but

\[
231
\]
less than a half.

In the first stage of the game the foreign country sets its export subsidy realising the effect its decision will have upon the optimal domestic tariff. The first order condition for welfare maximisation for the foreign country is

\[
\frac{dW}{de} = (P_2 - c_2 - t) \left( \frac{\partial X}{\partial e} + \frac{\partial X}{\partial t} \frac{dt}{de} \right) + X \left( \frac{\partial P_2}{\partial e} + \left( \frac{\partial P_2}{\partial t} - 1 \right) \frac{dt}{de} \right) = 0 \quad (5.59)
\]

Using the comparative static results, and noting that \( P_2 - c_2 - t = -e + x\beta, \) yields the optimal export subsidy

\[ e = \frac{-mx(\beta^2 - \gamma^2)}{\beta} < 0 \quad (5.60) \]

When the foreign country can commit itself to an export subsidy, before the domestic country sets its tariff, the optimal policy is an export tax. The foreign country realises that an export subsidy will result in a countervailing tariff so it taxes exports to bring about a lower tariff. The export tax is larger the greater the degree of product differentiation. This result is similar to those obtained in chapter two.

The situation where the domestic country uses an import
tariff and a production subsidy will not be considered since with linear demand the optimal production subsidy is zero, and the results would be the same as when the domestic country only uses a tariff.

5.6 Conclusions

This chapter has considered retaliation by the domestic country when the foreign country uses an export subsidy to deter the entry of domestic firms. For a homogeneous product Cournot oligopoly, in the absence of any domestic trade policy, an export subsidy unambiguously increases foreign welfare. The optimal foreign policy is to subsidise exports so that no domestic firms enter, and the foreign firms capture the entire market. This is a more robust argument for an export subsidy than the profit shifting argument of Brander and Spencer (1985).

However, retaliation with countervailing tariffs negates the argument for an export subsidy. At a Nash equilibrium in trade policies, when the domestic country uses an import tariff, there is a foreign export subsidy and a fully countervailing tariff. The domestic country extracts all the rent from the foreign country. And, the foreign country is better off committing itself not to subsidise exports. At a Stackelberg equilibrium, where the foreign country has a first mover advantage, the domestic country responds to a
foreign export subsidy with a partially countervailing tariff, and the optimal foreign export subsidy is zero. When the foreign country faces retaliation with countervailing tariffs it should not subsidise exports.

When the domestic country uses an import tariff and production subsidy, the Nash equilibrium in trade policies has a foreign export subsidy which is fully countervailed by a domestic tariff and production subsidy. And, the domestic country extracts all the rent from the foreign country. A production subsidy (tax) is used to shift domestic firms down their average cost curve as demand is convex (concave). At a Stackelberg equilibrium, the domestic country responds to a foreign export subsidy with a countervailing tariff and production subsidy, and the optimal foreign policy is an export subsidy if foreign firms have a cost advantage. This shows that, in theory, a country may gain from an export subsidy despite retaliation. But, in practice, governments only use tariffs to countervail foreign export subsidies.

With product differentiation the results are less dramatic. In the absence of any domestic trade policy, an export subsidy will increase foreign welfare if the number of foreign firms is not too large. At a Nash equilibrium in trade policies, when the domestic country uses a tariff, there is a foreign export subsidy or tax and a partially countervailing tariff. The domestic country cannot extract
all the rent from the foreign country in this case. At a Stackelberg equilibrium the optimal foreign policy is an export tax.
Figure 5.1: Entry Deterring Export Subsidy under Bertrand Duopoly
Figure 5.2: Trade Policy Reaction Functions
Chapter 6: Conclusions

This thesis has analysed the effect of retaliation with countervailing tariffs and/or production subsidies on the strategic argument for export subsidies, and also proved the existence and uniqueness of equilibrium in the standard model of international trade under oligopoly. This chapter will briefly summarise the main results of this thesis, and then draw out any general conclusions.

Chapter 1 surveyed the literature on the new international economics and strategic trade policy, and in particular on trade and trade policy under oligopoly. Firstly, chapter 1 considered the literature on intra-industry trade and the gains from trade. It was shown that intra-industry trade in identical products may occur under oligopoly, and that countries may gain from such trade due to its pro-competitive effect. Secondly, chapter 1 considered the strategic argument for profit shifting export subsidies advanced by Brander and Spencer (1985), and the many criticisms of the profit shifting argument were discussed. Finally, chapter 1 considered the use of strategic import policies such as import tariffs and production subsidies. These policies can be used to shift profits from foreign to domestic firms, to improve the terms of trade, and to correct domestic distortions. It was shown that Dixit (1988) incorrectly derived the optimal tariff for the case when
domestic production is uneconomic, and this error was corrected.

Chapter 2 analysed the effect of retaliation on the profit shifting argument for export subsidies in a Cournot oligopoly model with linear demand and nationally differentiated products. Trade policy was modelled as a multistage game in which the foreign country set its export subsidy in the first stage and the domestic country responded with an import tariff and/or production subsidy in the second stage. When the domestic country pursues a policy of laissez-faire, it was shown that a foreign export subsidy may reduce domestic welfare. However, when the domestic country pursues an optimal trade policy, a foreign export subsidy will always increase domestic welfare. The optimal domestic response to a foreign export subsidy under linear demand and product differentiation has been derived by Dixit (1988), and in chapter 2 this analysis was extended by considering the optimal export subsidy for the foreign country when faced with such a domestic response. When the domestic country uses an import tariff and a production subsidy, the optimal domestic response is to increase the tariff and reduce the production subsidy. Only a partially countervailing tariff is justified, the optimal countervailing tariff fraction is less than or equal to a half. When the foreign country anticipates the response of the domestic country, the optimal foreign policy is an
export tax. Retaliation negates the profit shifting argument for an export subsidy. When the domestic country uses only an import tariff, the optimal domestic response is a less than fully countervailing tariff. The optimal countervailing tariff fraction is less than a half. When the foreign country anticipates the response by the domestic country, the optimal foreign policy is an export tax. Again, retaliation negates the profit shifting argument for an export subsidy. The results when the domestic country uses only a production subsidy are ambiguous. Chapter 2 also analysed the effect of retaliation in a Bertrand duopoly model with linear demand and product differentiation. It was shown that the optimal countervailing tariff fraction is smaller under Bertrand than under Cournot duopoly. And, it was shown that when the domestic country responds with a countervailing tariff, or a countervailing tariff and production subsidy, the optimal foreign policy is an export tax.

Chapter 3 proved the existence and uniqueness of the Cournot equilibrium in the Dixit (1984) model of international trade under oligopoly, and derived the comparative static results for the effects of trade policy. The question of the existence of equilibrium in models of international trade under oligopoly has largely been ignored. The proof of existence used in chapter 3 did not make the usual assumption that profit functions are concave. Instead, the
proof adapted the method employed by McManus (1962, 1964) and used a weaker "aggregate concavity" assumption. A simple proof was used to establish the uniqueness of the equilibrium. Then, the assumptions required to prove the existence and uniqueness of equilibrium were used to sign the comparative static results for the effects of trade policy. It was shown that these assumptions allow domestic output and foreign exports to be strategic substitutes or complements. The comparative static results derived in chapter 3 were used in chapter 4 to analyse the effects of retaliation.

Chapter 4 analysed the effect of retaliation on the profit shifting argument for export subsidies using the homogeneous product Cournot oligopoly model of chapter 3. As in chapter 2 trade policy was modelled as a multistage game. Chapter 4 extended the results of Dixit (1988) and chapter 2 by considering general demand functions rather than just linear demands. When the domestic country pursues a policy of laissez-faire, it was shown that a foreign export subsidy may reduce domestic welfare. However, when the domestic country pursues an optimal trade policy, a foreign export subsidy will always increase domestic welfare. When the domestic country uses an import tariff and a production subsidy, the optimal domestic response to a foreign export subsidy is generally to increase the tariff and reduce the production subsidy. The optimal countervailing tariff
fraction is less (greater) than a half if demand is convex (concave). When the foreign country anticipates the response of the domestic country, the optimal foreign policy is an export subsidy if demand is non-linear. There is a profit shifting argument for an export subsidy despite retaliation. When the domestic country uses only an import tariff, the optimal domestic response is a less than fully countervailing tariff. The countervailing tariff fraction is less than one. When the foreign country anticipates the response of the domestic country, the optimal foreign policy is usually an export tax. Retaliation with a countervailing tariff will usually negate the profit shifting argument for an export subsidy. The results when the domestic country uses only a production subsidy are ambiguous.

Chapter 5 analysed the effect of retaliation in a homogeneous product Cournot oligopoly model where the foreign country can use an export subsidy to deter the entry of domestic firms. This chapter considered the Nash equilibrium in trade policies, where the domestic and foreign countries set their trade policies independently and simultaneously, and the Stackelberg equilibrium, where the foreign country has a first mover advantage. In the absence of any domestic trade policy, the optimal foreign policy is to subsidise exports so that no domestic firms will enter the industry and the foreign firms capture the entire market. This is a strong argument for a strategic export
subsidy, but it ignores the effect of retaliation by the domestic country. At the Nash equilibrium in trade policies, when the domestic country uses only a tariff, there is a foreign export subsidy which is fully countervailed by a domestic tariff and the domestic country extracts all the producer surplus from the foreign country. At the Stackelberg equilibrium, the optimal domestic response to a foreign export subsidy is a partially countervailing tariff. The optimal countervailing tariff fraction is a half. When the foreign country anticipates the response of the domestic country, it will commit itself not to subsidise exports. The optimal export subsidy is zero when faced with retaliation. At the Nash equilibrium, when the domestic country can use an import tariff and a production subsidy, there is a foreign export subsidy which is fully countervailed by a domestic tariff and a production subsidy (tax) and the domestic country extracts all the producer surplus from the foreign country. There is a production subsidy (tax) if demand is convex (concave). At the Stackelberg equilibrium, the optimal domestic response to a foreign export subsidy is generally to increase the tariff and to increase (reduce) the production subsidy if demand is convex (concave). The optimal countervailing tariff fraction is less (greater) than a half if demand is convex (concave). When the foreign country anticipates the response of the domestic country, it will subsidise exports if foreign firms have a cost advantage. An export subsidy can increase welfare despite
retaliation. Chapter 5 also considered the effect of introducing product differentiation into the analysis using a model with linear demand and nationally differentiated products. Then, the optimal foreign policy when faced with retaliation is an export tax.

Modelling trade policy as a multistage game seems to provide a reasonable analysis of export subsidies and countervailing tariffs. Firstly, the structure of the game with the foreign country having a first mover advantage reflects the GATT rules on countervailing duties. The domestic country can only impose definitive countervailing duties after a formal, and usually lengthy, investigation has established foreign subsidisation, and therefore the foreign country does have a first mover advantage. Secondly, the alternative would be to model trade policy as a simultaneous move game as in chapter 5, but this ignores the essence of retaliation which is that the domestic country responds to the foreign export subsidy. A multistage game gives the model some dynamics, and allows the domestic country to respond to the foreign export subsidy. Finally, the multistage game modelling of trade policy yields plausible results.

The modelling of trade policy as a multistage game, where the foreign country has a first mover advantage, yields a number of general conclusions:
Firstly, if the domestic country pursues an optimal trade policy then a foreign export subsidy will always increase domestic welfare. A country will not be harmed by foreign subsidies if its trade policy is set optimally. It is often argued that countervailing duties are intended to deter foreign subsidies, but if a country pursues an optimal trade policy it will not want to deter foreign subsidies.

Secondly, the optimal domestic response to a foreign export subsidy is generally a partially countervailing tariff. When the domestic country uses only a tariff, its optimal response is always a less than fully countervailing tariff. This is obviously relevant to policy making since countries generally impose fully countervailing duties. The results of this thesis suggest that if governments are maximising economic welfare then they will never use fully countervailing duties.

Finally, a foreign export subsidy will usually reduce foreign welfare if the domestic country retaliates with a countervailing tariff. It is possible that the foreign country may gain from an export subsidy despite retaliation, when the domestic country responds with a countervailing tariff and production subsidy. However, in practice, countries only use countervailing tariffs in response to foreign export subsidies, and then an export subsidy will usually reduce foreign welfare. Therefore, retaliation with
a countervailing tariff negates the profit shifting argument for an export subsidy. This is yet another argument against the use of profit shifting export subsidies.
Mathematical Appendix A

In this appendix the comparative static results used in chapter four, section 4.4, for the effect of the foreign export subsidy on the optimal domestic import tariff and production subsidy will be derived. From equation (4.19) the first order conditions for a welfare maximum are

\[
\frac{\partial W_1}{\partial t} = X \left( 1 - \frac{\partial P}{\partial t} \right) + (P - c_1) \frac{\partial Y}{\partial t} + t \frac{\partial X}{\partial t} = 0 \quad (A1)
\]

\[
\frac{\partial W_1}{\partial s} = -X \frac{\partial P}{\partial s} + (P - c_1) \frac{\partial Y}{\partial s} + t \frac{\partial X}{\partial s} = 0 \quad (A2)
\]

The second order conditions for a welfare maximum are

\[
\frac{\partial^2 W_1}{\partial t^2} < 0 \quad \frac{\partial^2 W_1}{\partial s^2} < 0
\]

\[
H = \begin{vmatrix}
\frac{\partial^2 W_1}{\partial t^2} & \frac{\partial^2 W_1}{\partial s \partial t} \\
\frac{\partial^2 W_1}{\partial t \partial s} & \frac{\partial^2 W_1}{\partial s^2}
\end{vmatrix} > 0 \quad (A3)
\]

The comparative static results are obtained by totally differentiating the first order conditions, (A1) and (A2), which yields
To obtain the second order partial derivatives substitute the comparative static results from chapter three into equation (A1) then yields

\[
\begin{pmatrix}
\frac{\partial^2 W_1}{\partial t^2} & \frac{\partial^2 W_1}{\partial s \partial t} \\
\frac{\partial^2 W_1}{\partial t \partial s} & \frac{\partial^2 W_1}{\partial s^2}
\end{pmatrix}
\begin{pmatrix}
dt \\
\frac{\partial}{\partial e dt}
\end{pmatrix}
= 
\begin{pmatrix}
-\frac{\partial^2 W_1}{\partial e \partial t} \\
-\frac{\partial^2 W_1}{\partial e \partial s}
\end{pmatrix}
\tag{A4}
\]

To obtain \( \frac{\partial W_1}{\partial t} \) differentiate \( \frac{\partial W_1}{\partial t} \) with respect to \( t \), and evaluating at a welfare maximum, yields

\[
\frac{\partial W_1}{\partial t} = \frac{1}{\Delta} \left[ tm((n+1)P'+nyP'') - nm(P'+yP'')(P-c_1) \\
+ mxP'((n+1)P'+QP'') \right] \tag{A5}
\]

To obtain \( \frac{\partial^2 W_1}{\partial t^2} \) differentiate \( \frac{\partial W_1}{\partial t} \) with respect to \( t \), and evaluating at a welfare maximum, yields

\[
\frac{\partial^2 W_1}{\partial t^2} = \frac{1}{\Delta} \left[ m((n+1)P'+nyP'') + tm((n+1)P''+nyP''') \frac{\partial Q}{\partial t} + tmP'' \frac{\partial Y}{\partial t} \\
- nm(P'+yP'') P' \frac{\partial Q}{\partial t} - nm(P''+yP''')(P-c_1) \frac{\partial Q}{\partial t} - (P-c_1)mp'' \frac{\partial Y}{\partial t} \\
+ mxP'((n+2)P''+QP'''') \frac{\partial Q}{\partial t} + P'((n+1)P'+QP'') \frac{\partial Q}{\partial t} \right]
\]

Using the comparative static results from chapter three
together with the optimal import tariff and production subsidy from (4.20) and after some simplification, yields

\[
\frac{\partial^2 W_1}{\partial t^2} = \frac{mP'}{\Delta^2} \left[ ((n+1)P'+nyP')((n+m+1)P'+QP') - nmP'(P''+yP'') \right.
\]

\[
+ mxP''(P'+mxP') + nmXP''(P'+yP') - nmP'(P'+yP')
\]

\[
+ mxP'((n+2)P''+QP''') + ((n+1)P'+nyP')((n+1)P'+QP'')
\]

\[
+ mxP'((n+1)P'+QP'') \right]
\]

Multiplying out the terms then yields

\[
\frac{\partial^2 W_1}{\partial t^2} = \frac{mP'}{\Delta^2} \left[ (2n^2+4n+2)(P')^2 + 4(n+1)QP'P'' + 2Q^2(P'')^2 \right.
\]

\[
+ m(P')^2 + m^2x^2(P'P'' - 2(P'')^2) \right]
\]

Factorising then yields

\[
\frac{\partial^2 W_1}{\partial t^2} = \frac{mP'}{\Delta^2} \left[ 2((n+1)P'+QP'')^2 + m[(P')^2 + mx^2(P'P'' - 2(P'')^2)] \right]
\]

Which is equivalent to the form given in the text

\[
\frac{\partial^2 W_1}{\partial t^2} = \frac{mP'}{\Delta^2} \left[ 2((n+1)P'+QP'')^2 + mZ \right] \quad (A6)
\]
Where $Z = (P')^2 + mx^2(P''P'' - 2(P')^2) > 0$

To obtain $\frac{\partial^2 W_1}{\partial s \partial t}$ differentiate $\frac{\partial W_1}{\partial t}$ with respect to $s$, and evaluating at a welfare maximum, yields

$$\frac{\partial^2 W_1}{\partial s \partial t} = \frac{1}{\Delta} \left[ tm((n+1)P''+nyP''') - \frac{\partial Q}{\partial s} + tmP'' \frac{\partial Y}{\partial s} - nm(P' + yP'')P' \frac{\partial Q}{\partial s} \\
- n(P''+yP''')(P-c_1) \frac{\partial Q}{\partial s} - (P-c_1) mP'' \frac{\partial Y}{\partial s} + P'(n+1)P' + QP'' \frac{\partial X}{\partial s} \\
+ mxP'(n+2)P'' + QP'' \frac{\partial Q}{\partial s} + mxP'(n+1)P' + QP'' \frac{\partial Q}{\partial s} \right]$$

Using the comparative static results from chapter three together with the optimal import tariff and production subsidy from (4.20), and after some simplification yields

$$\frac{\partial^2 W_1}{\partial s \partial t} = \frac{nmP'}{\Delta^2} \left[ -nxP'(P''+yP''') - xP''(P'+mxP'') \\
+ xP''((n+1)P'+mxP'') - nP'(P'+yP'') + xP'(n+2)P''+QP'' \\
+ ((n+1)P'+QP')(P'+xP'') + xP'(n+1)P' + QP'' \right]$$

Which simplifies to

$$\frac{\partial^2 W_1}{\partial s \partial t} = \frac{nmP'}{\Delta^2} \left[ (2n+1)(P')^2 + 2QP'P'' - mx^2(P'P''-2(P')^2) \right]$$
Which is equivalent to the form given in the text

\[ \frac{\partial^2 W_1}{\partial e \partial t} = \frac{nmP'}{\Delta^2} \left[ 2P'((n+1)P'+QP'') - Z \right] \]  
(A7)

To obtain \( \frac{\partial^2 W_1}{\partial e \partial t} \) differentiate \( \frac{\partial W}{\partial t} \) with respect to \( e \), and evaluating at a welfare maximum, yields

\[
\frac{\partial^2 W_1}{\partial e \partial t} = \frac{1}{\Delta^2} \left[ tm((n+1)P''+nP'') \frac{\partial Q}{\partial e} + tmP'' \frac{\partial Y}{\partial e} - nm(P'+yP'')P' \frac{\partial Q}{\partial e} \\
- nm(P''+yP'')(P-c_1) \frac{\partial Q}{\partial e} - (P-c_1)mP'' \frac{\partial Y}{\partial e} + P'((n+1)P'+QP'') \frac{\partial X}{\partial e} \\
+ mX'((n+2)P''+QP') \frac{\partial Q}{\partial e} + mX''((n+1)P'+QP') \frac{\partial Q}{\partial e} \right]
\]

Using the comparative static results from chapter three together with the optimal import tariff and production subsidy from (4.20) and after some simplification, yields

\[
\frac{\partial^2 W_1}{\partial e \partial t} = \frac{-mP'}{\Delta^2} \left[ - nmP'(P''+yP'') - mxP''(P'+mxP') + nmP''(P'+yP') \\
- nm(P'+yP'') + mxP'((n+2)P''+QP') + mxP''((n+1)P'+QP') \right]
\]

Which simplifies to

\[
\frac{\partial^2 W_1}{\partial e \partial t} = \frac{-mP'}{\Delta^2} \left[ ((n+1)P'+QP'')(n-m+1)P'+QP'') \right]
\]

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\[ + mxP''((n+m+1)P'+QP) + m[(P')^2 + mx^2(P''-2(P')^2)] \]

Which is equivalent to the form given in the text

\[
\frac{\partial^2 W_1}{\partial s \partial t} = \frac{-mP'}{\Delta^2} \left[ ((n+1)P'+QP'')((n-m+1)P'+QP) + mxP''((n+m+1)P'+QP) + mZ \right] \tag{A8}
\]

Substitute the comparative static results from chapter three into (A2) yields

\[
\frac{\partial W_1}{\partial s} = \frac{1}{\Delta} \left[ nm(P')^2 + tnm(P'+xP'') - n((m+1)P'+mxP'')(P_c_1') \right] \tag{A9}
\]

To obtain \( \frac{\partial^2 W_1}{\partial t \partial s} \) differentiate \( \frac{\partial W_1}{\partial s} \) with respect to \( t \), and evaluating at a welfare maximum, yields

\[
\frac{\partial^2 W_1}{\partial t \partial s} = \frac{1}{\Delta} \left[ 2nmxP'' \frac{\partial Q}{\partial t} + n(P')^2 \frac{\partial X}{\partial t} + tnm(P''+xP''') \frac{\partial Q}{\partial t} + \frac{\partial X}{\partial t} - \frac{\partial (P-c_1)P''}{\partial t} - n((m+1)P'+mxP'')(P'P''') \frac{\partial Q}{\partial t} \right]
\]

Using the comparative static results from chapter three together with the optimal import tariff and production
subsidy from (4.20) and after some simplification yields

$$\frac{\delta^2 W_1}{\delta t \delta s} = \frac{nmP'}{\Delta^2} \left[ 2mxP'P'' + ((n+1)P' + nyP'')(P' - xP'') - mxP'(P'' + xP'') + mx^2(P'')^2 - ((m+1)P' + mxP'')P' \right]$$

Which simplifies to

$$\frac{\delta^2 W_1}{\delta t \delta s} = \frac{nmP'}{\Delta^2} \left[ 2P'((n+1)P' + QP'') - [(P')^2 + mx^2(P'')^2 - 2(P'')^2] \right]$$

Which is equivalent to the form given in the text

$$\frac{\delta^2 W_1}{\delta t \delta s} = \frac{nmP'}{\Delta^2} \left[ 2P'((n+1)P' + QP'') - Z \right] \quad (A10)$$

To obtain $\delta^2 W_1/\delta s^2$ differentiate $\delta W_1/\delta s$ with respect to $s$, and evaluating at a welfare maximum, yields

$$\frac{\delta^2 W_1}{\delta s^2} = \frac{1}{\Delta} \left[ 2nxP'' \frac{\partial Q}{\partial s} + n(P')^2 \frac{\partial X}{\partial s} + tnm(P'' + xP'') \frac{\partial Q}{\partial s} \right]$$

$$+ tnP'' \frac{\partial X}{\partial s} - n(P-c_1)((m+1)P'' + mxP'') \frac{\partial Q}{\partial s}$$

$$- n(P-c_1)P'' \frac{\partial X}{\partial s} - n((m+1)P' + mxP'')P' \frac{\partial Q}{\partial s} \right]$$

Using the comparative static results from chapter three
together with the optimal import tariff and production subsidy from (4.20), and after some simplification yields

\[ \frac{\delta^2 W_1}{\delta s^2} = \frac{n^2 P'}{\Delta^2} \left[ 2mP'P'' + mP'(P'^x + P'') - m(P' + mP'')(P'' + xP'') \right. \]

\[ \left. - mxP''(P'+xP'') + mx^2P''((m+1)P'^x + mP'') - ((m+1)P'+mP'')(P') \right] \]

This simplifies to

\[ \frac{\delta^2 W_1}{\delta s^2} = \frac{n^2 P'}{\Delta^2} \left[ (2m+1)(P')^2 + mx^2(P'' + 2(P')^2) \right] \]

Which is equivalent to the form given in the text

\[ \frac{\delta^2 W_1}{\delta s^2} = \frac{n^2 P'}{\Delta^2} \left[ 2m(P')^2 + Z \right] \quad (All) \]

To obtain \( \frac{\delta^2 W_1}{\delta \delta s} \), differentiate \( \frac{\delta W_1}{\delta s} \) with respect to \( e \), and evaluating at a welfare maximum, yields

\[ \frac{\delta^2 W_1}{\delta \delta s} = \frac{1}{\Delta} \left[ 2mnP'P'' \frac{\partial Q}{\partial e} + n(P')^2 \frac{\partial X}{\partial e} + tnm(P'' + xP'') \frac{\partial Q}{\partial e} \right. \]

\[ \left. + tnP'' \frac{\partial X}{\partial e} - n(P-C_1)((m+1)P'' + mP'') \frac{\partial Q}{\partial e} \right] \]

\[ - n(P-C_1)P'' \frac{\partial X}{\partial e} - n((m+1)P' + mP'')P' \frac{\partial Q}{\partial e} \]
Using the comparative static results from chapter three together with the optimal import tariff and production subsidy from (4.20), and after some simplification yields

\[
\frac{\partial^2 W}{\partial e \partial s} = \frac{-nmP'}{\Delta^2} \left[ -2mxP'P'' + P'((n+1)P'+nYP'') \\
+ mx(P'+mxP'')(P''+xP''') - xP''((n+1)P'+nYP'') \\
- mx^2P''((m+1)P''+mxP''') + ((m+1)P'+mxP'')P' \right]
\]

This simplifies to

\[
\frac{\partial^2 W}{\partial e \partial s} = \frac{-nmP'}{\Delta^2} \left[ P'((n-m+1)P'+QP'') - xP''((n+m+1)P'+QP'') \\
- [(P')^2 + mx^2(P'P'' - 2(P'')^2)] \right]
\]

Which is equivalent to the form given in the text

\[
\frac{\partial^2 W}{\partial e \partial s} = \frac{-nmP'}{\Delta^2} \left[ P'((n-m+1)P'+QP'') - xP''((n+m+1)P'+QP'') - Z \right] (A12)
\]

These results can now be used to evaluate the determinants required to derive the comparative static results. Firstly, the Hessian determinant
\[ H = \begin{vmatrix}
\frac{\partial^2 W_1}{\partial t^2} & \frac{\partial^2 W_1}{\partial s \partial t} \\
\frac{\partial^2 W_1}{\partial s \partial t} & \frac{\partial^2 W_1}{\partial s^2}
\end{vmatrix} = \frac{\partial^2 W_1}{\partial t^2} \frac{\partial^2 W_1}{\partial s^2} - \frac{\partial^2 W_1}{\partial t \partial s} \frac{\partial^2 W_1}{\partial s \partial t} \]

Substitute the second order partial derivatives into the above yields

\[ H = \frac{n^2 m (P')^2}{\Delta^4} \left[ \left( 2 ((n+1) P' + Q P'')^2 + m Z \right) \left( 2 m (P')^2 + Z \right) \right. \]
\[ \left. - m \left( 2 P' ((n+1) P' + Q P'') - Z \right)^2 \right] \]
\[ = \frac{2n^2 m Z (P')^2}{\Delta^4} \left[ (n+1) P' + Q P'' \right]^2 + 2 m P' ((n+1) P' + Q P'') + (m P')^2 \]
\[ = \frac{2n^2 m Z (P')^2}{\Delta^4} \left[ ((n+m+1) P' + Q P'')^2 \right] \]

\[ H = \frac{2n^2 m Z}{\Delta^2} \quad \text{(A13)} \]

Similarly

\[ H_{t} = \begin{vmatrix}
\frac{\partial^2 W_1}{\partial t \partial t} & \frac{\partial^2 W_1}{\partial s \partial t} \\
\frac{\partial^2 W_1}{\partial s \partial t} & \frac{\partial^2 W_1}{\partial s^2}
\end{vmatrix} = - \frac{\partial^2 W_1}{\partial t \partial t} \frac{\partial^2 W_1}{\partial s^2} + \frac{\partial^2 W_1}{\partial s \partial t} \frac{\partial^2 W_1}{\partial s \partial t} \]

Substitute the second order partial derivatives into the
above yields

\[
H_t = \frac{n^2m(P')^2}{\Delta^4} \left[ ((n+1)P' + QP'')(n-m+1)P' + QP'') \right]
+ mXP''((n+1)P' + QP'') + mZ \left( 2m(P')^2 + Z \right) - m \left[ P'((n-m+1)P' + QP'') \right.
- xP''((n+1)P' + QP'') - Z \left( 2P'((n+1)P' + QP'') - Z \right] \]

\[
= \frac{n^2m(P')^2}{\Delta^4} \left[ ((n+1)P' + QP'')^2(2mxP'' + Z) \right]
\]

\[
H_t = \frac{n^2m}{\Delta^2} \left( 2mxP'' + Z \right)
\]

(A14)

Similarly

\[
H_s = \begin{vmatrix}
\frac{\partial^2 W_1}{\partial t^2} & -\frac{\partial^2 W_1}{\partial \sigma \tau} \\
\frac{\partial^2 W_1}{\partial \sigma \tau} & -\frac{\partial^2 W_1}{\partial \sigma \partial \tau} \\
\frac{\partial^2 W_1}{\partial \sigma \tau} & \frac{\partial^2 W_1}{\partial \sigma \partial \tau}
\end{vmatrix}
\]

\[
= -\frac{\partial^2 W_1}{\partial t^2} \frac{\partial^2 W_1}{\partial \sigma \partial \tau} + \frac{\partial^2 W_1}{\partial \sigma \tau} \frac{\partial^2 W_1}{\partial \sigma \partial \tau}
\]

Substitute the second order partial derivatives into the above yields

\[
H_s = \frac{n^2m(P')^2}{\Delta^4} \left[ 2((n+1)P' + QP'')^2 + mZ \right] \left( P'((n-m+1)P' + QP'') \right)
\]
It is now possible to obtain the comparative static results. Using Cramer's rule the effect of a change in the foreign export subsidy on the optimal import tariff is from (A4)

\[
\frac{\partial^2 W_1}{\partial \delta \partial t} \quad \frac{\partial^2 W_1}{\partial s \partial t} \\
\frac{\partial^2 W_1}{\partial \delta \partial s} \quad - \frac{\partial^2 W_1}{\partial s^2}
\]

Using the determinants in (A13) and (A14) yields

\[
\frac{dt}{de} = \frac{1}{H} \left[ -m x P'(n+1) P' + Q P'' + m Z \right]
\]

Similarly, using Cramer's rule the effect of a change in the foreign export subsidy on the optimal import tariff is from (A4)
\[
\frac{\text{ds}}{\text{de}} = \frac{1}{\text{H}} \left| \begin{array}{cc}
\frac{\partial^2 W_1}{\partial t^2} & - \frac{\partial^2 W_1}{\partial \delta t} \\
\frac{\partial^2 W_1}{\partial \delta s} & - \frac{\partial^2 W_1}{\partial \delta s}
\end{array} \right|
\]

Using the determinants in (A13) and (A14) yields

\[
\frac{\text{ds}}{\text{de}} = \frac{-m}{n} \left[ \frac{1}{2} + \frac{xP''((n+1)P' + QP'')}{Z} \right]
\]

(A17)
Mathematical Appendix B

In this appendix the second order partial derivatives used in chapter four, section 4.5, will be derived. The first derivative of the domestic welfare function with respect to \( t \) is

\[
\frac{\partial W}{\partial t} = (P-C_1) \frac{\partial Y}{\partial t} + X \left( 1 - \frac{\partial P}{\partial t} \right) + t \frac{\partial X}{\partial t} \quad \text{(B1)}
\]

Using the comparative static results in chapter three yields

\[
\frac{\partial W}{\partial t} = \frac{m}{\Delta} \left[ yP' (nP' + nyP'') + xP' ((n+1)P' +QP'') + t((n+1)P' +nyP'') \right] \quad \text{(B2)}
\]

To obtain \( \frac{\partial^2 W}{\partial t^2} \) differentiate \( \frac{\partial W}{\partial t} \) with respect to \( t \), evaluating at a welfare maximum, where the above term in square brackets is zero, yields

\[
\frac{\partial^2 W}{\partial t^2} = \frac{m}{\Delta} \left[ yP' (nP''+nyP''') + yP' P'' \frac{\partial Y}{\partial t} + (nP'+nyP'') P' \frac{\partial Y}{\partial t} \right.
\]

\[
+ (nP'+nyP'') yP'' \frac{\partial Q}{\partial t} + xP' ((n+2)P'' +QP''') \frac{\partial Q}{\partial t}
\]

\[
+ ((n+1)P'+QP'') P' \frac{\partial X}{\partial t} + ((n+1)P'+QP'') xP'' \frac{\partial Q}{\partial t}
\]

\[
+ t((n+1)P''+nyP''') \frac{\partial Q}{\partial t} + tP'' + ((n+1)P'+nyP'') \right]
\]

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Using the comparative static results from chapter three together with the optimal tariff from (4.34) yields

\[
\frac{\partial^2 W_1}{\partial t^2} = \frac{mP'}{\Delta^2} \left[ -mnyP'(P''+yP''')N + mnyP'(P'+yP'')(n+1)P''+nyP'''ight]
\]

\[
+ mn(P'+yP'')^2N - ((n+1)P'+QP'')N^2
\]

\[
- mxP''((n+1)P'+QP'')(nP'+nyP'') - mxP''((n+1)P'+QP'')N
\]

\[
- mn^2yP''(P'+yP'')^2 + mxP'((n+1)P'+QP'')(n+1)P''+nyP'''
\]

\[
- mxP'((n+1)P'+nyP'')(n+2)P''+QP''') + ((n+m+1)P'+QP''')N^2 \]

Further simplification then yields

\[
\frac{\partial^2 W_1}{\partial t^2} = \frac{mN}{\Delta} + \frac{mP'}{\Delta^2 N} \left[ ((n-m+1)P'+nyP'')N^2 + (n+1)mP'Z +
\right.
\]

\[
\left. (mxP'')^2(nP'+2N) + 3mxP''N^2 + ny^2mP'(P''-2(P'')^2) \right]
\]

Which reduces to the expression in the text

\[
\frac{\partial^2 W_1}{\partial t^2} = \frac{mN}{\Delta} + \frac{mP'}{\Delta^2 N} \left[ ((n-m+1)P'+nyP'')N^2 + B \right]
\]

Where Z, N and B are defined as,
\[ Z = [(P')^2 + mx^2(P'P'' - 2(P')^2)] > 0 \]

\[ N = (n+1)P' + nyP'' < 0 \]

\[ B = m(n+1)P'Z + (mxP'')^2(nP'+2N) + 3mxP''N^2 + mny^2P'(P'P'' - 2(P')^2) \]

To obtain \( \delta^2 W_1 / \delta \delta t \) differentiate \( \delta W_1 / \delta t \) with respect to \( \epsilon \), evaluating at a welfare maximum, yields

\[
\frac{\delta^2 W_1}{\delta \delta t} = \frac{m}{\Delta} \left[ yP'(nP''+nyP'') \frac{\partial Q}{\partial \epsilon} + yP'P'' \frac{\partial Y}{\partial \epsilon} + (nP''+nyP'')P' \frac{\partial Y}{\partial \epsilon} \right. \\
+ (nP''+nyP'')yP'' \frac{\partial Q}{\partial \epsilon} + xP'((n+2)P''+QP'') \frac{\partial Q}{\partial \epsilon} \\
+ ((n+1)P'+QP'')P' \frac{\partial Q}{\partial \epsilon} + ((n+1)P'+QP'')xP'' \frac{\partial Q}{\partial \epsilon} \\
+ t((n+1)P''+nyP'') \frac{\partial Q}{\partial \epsilon} + tP'' \frac{\partial Y}{\partial \epsilon} \right]
\]

Using the comparative static results from chapter three together with the optimal tariff from (4.34) yields

\[
\frac{\delta^2 W_1}{\delta \delta t} = \frac{mP'}{\Delta^2 N} \left[ -mnyP'(P''+yP'')N + mnyP'(P''+yP'')((n+1)P''+nyP'') \right. \\
+ mn(P''+yP'')^2N - ((n+1)P'+QP'')N^2 \\
- mxP''((n+1)P'+QP'')(nP''+nyP'') - mxP''((n+1)P'+QP'')N \right]
\]
\[-mn^2 yP''(P' + yP'')^2 + mxP' ((n+1) P' + QP'') ((n+1) P'' + nyP''')\]

\[-mxP' ((n+1) P' + nyP'') ((n+2) P'' + QP'')\]

Simplification then yields

\[
\frac{\partial^2 W}{\partial \theta \partial t} = \frac{-mP'}{\Delta^2 N} \left[ ((n-m+1) P' + nyP'') N^2 + (n+1) mP' Z + (mxP'')^2 (nP' + 2N) + 3mxP'' N^2 + ny^2 mP' (P' P''' - 2(P'')^2) \right]
\]

Which reduces to the expression given in the text,

\[
\frac{\partial^2 W}{\partial \theta \partial t} = \frac{-mP'}{\Delta^2 N} \left[ ((n-m+1) P' + nyP'') N^2 + B \right]
\]
Mathematical Appendix C

This appendix derives the second order partial derivatives used in chapter four, section 4.6. Using the comparative static results from chapter three in (4.43), the first derivative of the domestic welfare function is

\[
\frac{\partial W_1}{\partial s} = \frac{n}{\Delta} \left[ X(P')^2 - (P-c_1)((m+1)P'+mxP'') \right] \tag{C1}
\]

The second partial derivative \( \frac{\partial^2 W_1}{\partial s^2} \) is obtained by differentiating \( \frac{\partial W_1}{\partial s} \) with respect to \( s \), and evaluating at a welfare maximum, yields

\[
\frac{\partial^2 W_1}{\partial s^2} = \frac{n}{\Delta} \left[ (P')^2 \frac{\partial X}{\partial s} + 2XP' \frac{\partial Q}{\partial s} - ((m+1)P'+mxP'') \frac{\partial Q}{\partial s} \right.
\]

\[
\left. - (P-c_1)((m+1)P''+mxP''') \frac{\partial Q}{\partial s} - (P-c_1)P'' \frac{\partial X}{\partial s} \right]
\]

Substitute the comparative static results and the optimal \( P-c_1 \) from (4.44) into the above and rearranging gives

\[
\frac{\partial^2 W_1}{\partial s^2} = \frac{n^2 (P')^2}{\Delta^2 M} \left[ (mP'+mxP'')(m+1)P'+mxP'' - 2mxP''((m+1)P'+mxP'') \right.
\]

\[
+ ((m+1)P'+mxP'')^2 + mP'((m+1)P''+mxP''') - mxP''(mP'+mxP'') \right]
\]

After some simplification this yields the expression given
Where,

\[ M = (m+1)P' + mX'' < 0 \]
\[ Z = (P')^2 + mx^2(P'' - 2P')^2 > 0 \]

To obtain \( \frac{\partial^2 W}{\partial \delta s^2} \), differentiate \( \frac{\partial W}{\partial s} \) with respect to \( s \), and evaluating at a welfare maximum, yields

\[
\frac{\partial^2 W}{\partial \delta s^2} = \frac{n^2 (P')^2}{\Delta^2 M} \left[ M^2 + m^2(P')^2 + mZ \right]
\]  

(C2)

Substitute the comparative static results and the optimal \( P - c \) from (4.44) into the above and rearranging gives

\[
\frac{\partial^2 W}{\partial \delta s^2} = \frac{n^2 (P')^2}{\Delta^2 M} \left[ -(n+1)X' + nyP'' + (m+1)P' + mX'' + 2mxP'' \left( (m+1)P' + mX'' \right) + mxP'' \left( (m+1)P' + mX'' \right) \right]
\]

After some simplification this yields the expression given in the text

\[
\frac{\partial^2 W}{\partial \delta s^2} = \frac{n^2 (P')^2}{\Delta^2 M} \left[ M^2 + m^2(P')^2 + mZ - (m+1)\Delta \right]
\]  

(C3)
Bibliography


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