RESEARCH SPILLOVERS, INTERNATIONAL COMPETITION

AND ECONOMIC PERFORMANCE

by

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To my parents

ABSTRACT

This dissertation develops a theoretical framework for the analysis of oligopolistic rivalry through cost-reducing research and development (R&D) expenditures in an environment where firms cannot fully appropriate the results of their R&D investment. This framework is developed in order to examine the incentives of firms to invest in R&D in oligopolistic markets, the implications for the structure and performance of industries subject to such spillovers and the nature of public policy in this context.

Two lines of research are followed: one focuses on *technological* factors affecting incentives and performance, the other on the influence of *strategic behaviour*. With respect to technological factors, it is argued that the impact of research spillovers on market incentives and performance, and on the desirability of a public policy of R&D subsidies, depends crucially on the specific assumptions made about how a firm's production process is affected by its own R&D and that of its rivals. These assumptions are embodied in the knowledge production function and the associated cost function facing each firm.

Issues of substitutability and complementarity between a firm's research and that of its rivals and between various components of a firm's own research are central in determining the impact of spillovers. The level and composition of R&D investment, production and profitability, concentration and monopoly power are all influenced by the impact of rivals' research on the marginal productivity of a firm's own research. Optimal subsidies are similarly influenced. This suggests that the effectiveness of such policies towards R&D investment depends on the nature of technology and on specific appropriability characteristics of industries.

With respect to strategic behaviour, the dissertation extends previous research in the context of a multi-stage model of international competition in R&D to the case where R&D is subject to spillovers. This allows an examination of the importance of different behavioural assumptions (one-stage vs. multi-stage decision-making) on the impact of spillovers. The analysis also questions the results of strategic models obtained with full appropriability of R&D, whereby strategic behaviour results in higher equilibrium levels of R&D and production and lower profits. Strategic behaviour can result in lower R&D and higher profitability if research is difficult to appropriate. The conditions under which this occurs are explored fully.

The characteristics of industrial policy in the context of strategic international competition are then explored. Models that assume fully appropriable R&D suggest the optimality of a policy of positive subsidies to the R&D expenditures of domestic firms. The existence of spillovers may reverse this result. The analysis thus casts doubt on the efficacy of a government policy of R&D subsidies in a strategy of "precommitment". This goes beyond the usual retaliatory arguments against such behaviour and points to the limits of such an interventionist approach in an international environment where R&D has some characteristics typical of public goods.

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CHAPTER ONE

RESEARCH SPILLOVERS:

THEORY, EVIDENCE AND POLICY

I. Introduction

In the context of the economics of technological change, it is widely held that by its nature, innovation has certain characteristics typical of public goods. These characteristics imply that firms which invest in research and development (R&D) activities often find that the fruits of their efforts are being used by other firms or industries without adequate compensation being paid. R&D efforts by firms therefore generate external effects which affect the marginal decisions of other firms. These external effects linked to innovative activities have been labelled "research spillovers" [1].

Research spillovers have an important role to play in determining the incentives of private firms to invest in R&D activities, and in shaping the performance and characteristics of industries that rely heavily on product and process innovation. As a result, they are receiving increasing attention, both in the theoretical literature on innovation, and by policy-makers seeking to formulate policies in science and technology [2].

The increasingly global characteristics of research activities and international trade of R&D-intensive products have added a new dimension to the issues raised by research spillovers. To the extent that trade occurs in oligopolistic markets where firms rely on innovation to gain a competitive advantage, research by one firm can potentially benefit its international competitors. Such international spillovers have implications for the predictions of strategic trade theories and for public policy that attempts to support domestic firms by subsidising R&D.

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[1] One recent attempt at a definition describes research spillovers as "any original, valuable knowledge generated in the research process which becomes publicly accessible, whether it be knowledge fully characterising an innovation, or knowledge of a more intermediate sort" (Cohen and Levinthal, 1989, p. 571).

[2] This dissertation is concerned with the microeconomic aspect of spillovers: their effect on firm incentives and behaviour, and on industry structure and performance. The macroeconomic counterpart of this is the importance of knowledge spillovers for long-term rates of growth in the economy. This is increasingly central in the new growth theories where technology is an endogenous factor in the production process. See for example Romer (1990).

In order to address some of these issues, this dissertation develops models of oligopolistic competition through R&D expenditures in which research spillovers are modelled explicitly. These models allow the examination of the incentives to invest in R&D, the characteristics of industrial structure and performance, as well as of the desirability of a public policy of R&D subsidies in an environment characterised by less than perfect appropriability of research.

a. Characteristics and limitations of the approach

The models that are developed in the main body of the dissertation belong to the literature which takes into account the interaction between competing firms and focuses on the incentives to spend on R&D and on static industry equilibrium. In particular, of all the factors influencing innovative activity (technological opportunity, demand and appropriability conditions), we follow others in focusing on the impact of spillovers. Appropriability is however modelled here in a richer analytical setup and that makes it possible to question and reinterpret some of the existing results, as well as to lay a better claim on accounting for the impact of spillovers on incentives and performance.

In oligopolistic markets where competition occurs through the development of innovations, there are three supply-side channels through which R&D influences a firm's profits: via outputs, unit production costs, and the R&D costs themselves. The explicit modelling of research spillovers in this context implies the introduction of a "borrowed research" variable alongside a firm's own research expenditures. A firm's production of knowledge then depends on the level of its own R&D expenditures and on the size of the knowledge pool that it can draw upon.

In analysing the effect of R&D expenditures in a spillover environment on incentives and performance, a number of factors need to be taken into account. One is direct: both own R&D and the "borrowed" R&D of competitors (via the spillover rate) affect a firm's production costs, and through that its output and earnings. Another effect is indirect: a firm's own R&D expenditures influences the production costs of its competitors, and therefore their output and industry price. It therefore has an indirect effect on own earnings. Finally, own R&D can have an effect on a firm's own production cost by influencing its capacity to take advantage of competitors' R&D expenditures. These effects determine the impact of spillovers; they represent the building blocks for the models that are developed below.

For a particular firm or industry therefore, research spillovers represent outside knowledge capital (Griliches, 1979). Their existence implies that the production of knowledge achieved by the firm or the industry depends not only on its own research efforts but also on outside efforts or more generally on the level and nature of the knowledge pool available to it. Moreover, the productivity of own research be affected by the size and nature of the knowledge pool that it can draw upon.

The main value-added of the approach followed in this dissertation comes from the fact that in analysing the effects of inappropriability, the models developed pay particular attention to different *types* of R&D expenditures, to *technological* factors affecting incentives and performance, and on the influence of *strategic behaviour*. These three factors together determine the impact of inappropriability of research on the incentives of firms to invest in R&D, and on industrial structure and performance.

Specifically, the models in the chapters that follow innovate in a number of ways. First, R&D is not assumed to be homogeneous. Instead, it is broken down into two parts with different appropriability characteristics: a basic research part that is subject to spillovers, and a development part, which is taken to be fully appropriable. This distinction between basic research and development research in terms of appropriability (chapters two and three) is complemented by one in terms of timing in chapter four.

The distinction between the two parts of R&D is meant to capture the fundamental differences between that part of the innovation process where new knowledge is still of the generic type and therefore widely applicable and the part where the R&D output has been focused to a particular range of applications. Both types of R&D may of course in practice be partially or fully non-appropriable and the restriction of spillovers to the basic research part of R&D does carry an implicit assumption that development knowledge cannot be reverse-engineered, either because of patent protection or because the cost of imitation is prohibitive.

Basic research is closer to the idea of innovation as knowledge creation. It is therefore more reasonable to assign to it alone the public good properties of R&D. In effect, the difficulty of patenting basic research has long been identified as a main reason for underinvestment in this type of R&D (Nelson, 1959). Furthermore, it is widely held that social returns from basic research are significant and higher than private returns (Mansfield, 1980; Rosenberg, 1990); this in turn has been used as a justification for substantial public support [3]. The modelling of spillover effects may go some way towards providing a plausible theoretical explanation.

The second distinguishing feature of the models developed below is the focus on *technological* factors and in particular the treatment of "own" and "borrowed" research. Previous theoretical explorations of spillovers have for the most part treated the research of rivals as a perfect substitute for a firm's own research. Empirical evidence suggests that this is often not the case. Consequently closer attention needs to be paid to the way R&D inputs combine to produce cost-reducing innovations.

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[3] In the US, for example, government funds have accounted for the largest fraction of the total funds devoted to basic research. Even though the federal share has declined in recent years, it still represents roughly two-thirds of the total outlays. See *National Science Foundation* (1986) for details.

The impact of research spillovers on incentives for R&D investment, industry profitability and structure, and for optimal R&D subsidies, depend crucially on the specific assumptions made about how a firm's production process is affected by its own R&D and that of its rivals. In the models that follow, these assumptions are embodied in the knowledge production function and the associated cost function facing each firm.

Issues of substitutability and complementarity between a firm's research and that of its rivals and between various components of a firm's own research are shown to be central in determining the impact of spillovers. The level and composition of R&D investment, production and profitability, concentration and monopoly power are all influenced by the impact of rivals' research on the marginal productivity of a firm's own research. Optimal subsidies are similarly influenced. This suggests that the effectiveness of such policies towards R&D investment depends on differences in the nature of technology and on specific appropriability characteristics of industries.

The third characteristic concerns assumptions about behaviour. Chapters two and three develop models where firms act as Cournot oligopolists, choosing the levels of R&D and output in a one-stage game. This is the approach taken in most of the literature on spillovers. Chapter four develops a strategic game, where firms choose basic research expenditures, development research expenditures and output levels in three distinct stages. This allows an analysis of the importance of different behavioural assumptions (simultaneous vs. sequential moves in each firm's decision-making on R&D expenditures and output production) on the impact of spillovers [4]. It also allows us to question the results of strategic models that assume full appropriability of R&D, particularly their prediction that strategic behaviour results in higher equilibrium levels

[4] Firms move *simultaneously* in all models, including the strategic model in chapter four. *Sequential* moves here mean all firms choosing simultaneously R&D levels before choosing output levels, rather than one firm making a move before another.

of R&D and production and lower profits. It is established that strategic behaviour can result in lower R&D and higher profitability if research is difficult to appropriate. The conditions under which this occurs are explored fully.

The framework of the strategic model allows also an extension of multi-stage models of international competition in R&D to the case where R&D is subject to spillovers and an analysis of the characteristics of industrial policy in this context. Models that assume fully appropriable R&D suggest the optimality of a policy of profit-shifting positive subsidies to R&D expenditures of domestic firms. The existence of spillovers suggests that the externality involved may reverse this result. The analysis of chapter five casts doubt on the efficacy of a government policy of R&D subsidies in a strategy of "precommitment". This goes beyond the usual retaliatory arguments against such behaviour and points to the limits of such an interventionist approach in an international environment where R&D has some characteristics typical of public goods.

Other general characteristics of the models that are developed in this dissertation are: the greater importance attached to conditions of appropriability relative to technological opportunity for shaping firms' incentives and industry structure and performance; the focus on process rather than product innovations; and the assumption that firms are integrated ie they operate on the whole range of activities ranging from knowledge creation to output production.

These model attributes seem to be reflecting fairly well the dominant characteristics of R&D-intensive but technologically maturing oligopolistic industries. These would be well represented by the "science-based" industries in Pavitt's (1984) industrial classification or by the high technology sectors in the OECD's classification based on

R&D intensity [5]. In general, these are industries where all firms are expected to commit substantial resources to R&D activities, and which also depend considerably on external sources of generic knowledge (Nelson and Levin, 1986).

Other characteristics of these industries are the small number of participants, all covering the whole range from basic research to production (Gort and Klepper, 1982). Cost-saving advances represent substantial contributions in total effort while the technological environment of these established and well-defined industries is characterised by a body of previously developed knowledge on which firms build. Relatedly, since firms of this type operate within a particular technological trajectory, the relative importance of technological opportunities has declined. Therefore, the conditions of appropriability are very important in an environment where a small group of rivals innovates continuously and incrementally in order to maintain a competitive advantage.

These characteristics aside, the approach followed has certain limiting features. All models are static and deterministic, and as such abstract from some of the most important aspects of the innovation process. Their timeless nature makes them incapable of identifying the *speed of research*, and consequently it is not possible to analyse how this characteristic is related to inappropriability of R&D. No account is taken of uncertainty; neither is there recognition of the risk inherent in every part of the innovation process. This implies that in the firm's calculus decisions related to innovative activities are reduced to the size of R&D expenditures. In practice, firms decide not only on their R&D budget, but also on which projects to undertake, based on their expected probability and timing of success.

[5] Pavitt's (1984) taxonomy organises industrial activities according to (i) the sources from which firms obtain their technology; (ii) the main users of products and the role technology plays in their demand; and (iii) the means available to firms for appropriation of commercial benefits. This allows three broad sectoral categories to emerge: supplier dominated, production-intensive and science-based industries. The science-based group includes aircraft, pharmaceuticals and other chemicals, computers and semiconductors and scientific instruments.

Other limiting features relate to the nature of the equilibria examined. Attention is restricted to *symmetric* equilibria only, so that the Schumpeterian process of "creative destruction" (Schumpeter, 1975, ch. 7) cannot be adequately represented. Strict symmetry implies both identical research outlays and simultaneous innovation by all players; we cannot therefore look at imitation and at the relative size of firms.

A final important limitation relates to the spillover rate itself. Throughout, it is treated as an exogenous parameter in the models. Its size is taken as a datum by the firm and in turn the firm's behaviour has no influence on the extent to which its research becomes available to rivals. In reality, not only is the firm partially in control over how much of its research is proprietary, but also technological change itself has an effect on the underlying conditions that determine the size and productivity of spillovers (Levin and Reiss, 1989). Furthermore, the channels through which spillovers occur are ignored. Different channels (patents, licensing agreements, joint ventures or imitation) may generate different types of spillovers with distinct effects on cost reduction and on the rate of R&D investment (Bernstein, 1988). Endogenising the spillover rate would be an important step in obtaining a more precise relationship between the effect of appropriability and the institutional environment.

It should be evident from these limitations that the models developed in this dissertation are quite constrained in the type of questions they can address. Asking a limited set of questions can however yield some clear answers and useful insights. Thus the representation of spillovers in this simplest of environments, where particular attention is paid to technological and behavioural factors, allows for a more comprehensive examination of their possible effects.

b. An outline of the chapter

In this first chapter of the dissertation we attempt to motivate the theoretical approach taken in the chapters that follow by asking a number of questions: what is meant by "research spillovers"?; do such spillovers exist?; do they matter?; what does economic theory tell us about their impact on incentives, structure and performance?; and what are the weaknesses in current approaches?

The next section of the chapter examines the evidence about the existence of research spillovers and their effects on the incentives of firms to invest in R&D. Rather than attempting a comprehensive survey of the empirical literature on the subject, the aim is to present some apparent contradictions between received theory and empirical research. These contradictions are further explored in the context of the economic theory of innovation and form the motivation for the development of the theory in the subsequent chapters of the dissertation. The section concludes with a discussion of some policy concerns that arise from the existence of spillovers.

Section three examines the main insights revealed by the economic theory of innovation, as these relate to research spillovers. The discussion follows the development of arguments in the area, starting from those that emphasise the static welfare externalities associated with the production of knowledge, and moving on to approaches that stress the dynamic nature of the innovation process and the associated externalities related to the systemic characteristics of technological advance. The main attempts to date to model spillovers in an explicitly oligopolistic context are then briefly reviewed. The section concludes with a discussion of strategic behaviour arguments in international trade, as these impact on the issues at hand when trade occurs through the development of new products and processes. Finally, section four gives an outline of the structure of the main body of the dissertation.

II. Issues of definition, empirical evidence and policy concerns

The diversity of approaches in the empirical investigation of inappropriability of research is best understood in the context of a conceptual distinction between two types of research spillovers. The first type is a question of price measurement: "spillovers" in this case occur because R&D-intensive inputs and outputs are not priced in their fully hedonic value. The second type is a matter of knowledge transmission [6].

The first type of spillovers relates to purchases of R&D-intensive inputs. In the situation where the price at which inputs are purchased do not fully reflect their increased value, in addition to generating an increase in producer surplus in the innovating firm or industry, the innovation lowers the input prices for the purchasing firms or industries and increases their producer surplus (Mohnen, 1989). Through these *embodied* "spillovers" therefore, the R&D conducted in one firm or industry will have welfare side-effects [7].

The second concept of spillovers refers to the transfer of scientific and engineering knowledge. Such transfers have been labelled *disembodied* or *real knowledge* spillovers to emphasize that they do not necessarily come embodied in any piece of

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[6] Griliches (1979) and Schankerman (1979) discuss this conceptual distinction and its implication for empirical work.

[7] There are two possible reasons why we may not have fully "hedonic" (quality-adjusted) prices. The first relates to market conditions: only a perfectly discriminating monopolist with a secure market condition would be able to appropriate all the social returns ("quasi-rents") to the innovation. Failing that, products are sold to using industries at prices that are lower than their true social cost. The second is a technical reason: in the calculation of price indices, adjustment is not made for "costless" quality improvements to new products. New products tend to be "linked" in at their introductory price with the index unchanged. Quality adjustments in the price index therefore reflect only the original private returns of the inventor and the consumer surplus arising from the erosion of the original monopolistic market position, rather than the full social return associated with the invention.

machinery or equipment. It is implicit to most definitions that they refer to cases where one firm or industry can use the knowledge developed elsewhere without providing adequate compensation [8].

There are a number of channels through which the knowledge generated by one firm circulates to other firms. It may be embodied in research personnel which moves between firms or be the result of reverse-engineering on the part of a firm on the products of its rivals. It may be contained in descriptions of new products or processes that are found in publications, catalogues or patent applications or be disseminated in conferences or seminars. Finally, knowledge spillovers may be the by-product of mergers and acquisitions, joint ventures or other forms of inter-firm cooperation [9].

Whatever the channel through which they operate, disembodied or pure knowledge spillovers imply that discoveries made by one firm serve as an inspiration for the research of other firms. They can create new ideas, force research in new directions or make previously unsuccessful research come to successful conclusions. This allows these firms to introduce more efficient processes for production or develop new products. The invention of synthetic fibres for example by the chemical industry found great applications in the textile industry. Similarly, new materials like hard alloy steel could only be exploited when appropriate machinery was developed that allowed its efficient use. Unlike however the case of embodied spillovers, this type of

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[8] This dissertation develops models of oligopolistic rivalry where firms sell homogeneous goods and where the knowledge production function of a firm is augmented to include the R&D expenditures of rivals, in addition to its own R&D expenditures. The channels through which the R&D expenditures of rivals affect a firm's knowledge production function and production costs are not specified. They could be via purchases, at which point we would be looking at embodied R&D spillovers due to non-hedonic prices. Or they could be through the simple transfer of knowledge ie via disembodied (knowledge) spillovers. In principle therefore, the insights of the models developed below are quite general and do not depend on the nature of spillovers assumed.

[9] Levin, Klevorick, Nelson, and Winter (1987) provide some information based on evidence from surveys on the importance of various channels of knowledge transmission.

borrowing is not necessarily related to input purchase flows [10]. The photographing equipment industry and the scientific instruments industry may not be particularly strongly connected in terms of input purchases; they do nevertheless benefit from each other's research (Griliches, 1979). In either case, as knowledge expands for the receiving firms, spillovers contribute to cost-reduction and demand-expansion.

To the extent that firms are rivals in the product market the impact of spillovers has an additional dimension. If one firm can use the results of the research performed by other firms without compensation, the private investment incentive may be too low because the firm spending the R&D does not count the spillover as a benefit. When spillovers benefit competitors, however, the incentive of the potential innovator to invest in R&D can be further lowered relative to the social incentive. Furthermore, the R&D conducted by a firm can increase the effectiveness of the R&D conducted by rivals. In this case therefore, in addition to their cost-reducing aspect, spillovers also affect incentives and the structure of competition in the industry.

This discussion has been couched in terms which suggest that spillovers are domestic, occurring amongst firms or industries located within a single country. The international aspect of spillovers can however be of at least equal importance to the domestic aspect. To the extent that many R&D-intensive oligopolistic industries are heavily internationalised, with competition occurring primarily amongst large firms located in different countries, knowledge will also circulate amongst firms located in different countries, as well as amongst firms located within the same country.

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[10] This implies that in contrast to embodied spillovers, non-embodied spillovers may affect the productivity of the user firm or industry even if prices fully reflect quality improvements. This is because technology developed in one firm may affect the productivity in other firms or industries without the transaction of intermediate and investment goods.

There is furthermore no *a priori* reason to believe that international spillovers occur in any different way than domestic spillovers, nor that their impact on the incentives of firms to invest in R&D or on industry performance differs in the two cases. If the fact that a firm cannot appropriate its R&D expenditures acts as a disincentive for the level of R&D investment by its domestic rivals, it will also reduce R&D expenditures by foreign rivals. If it also reduces the industry-wide cost of achieving a given level of cost reduction, it will do so whether the industry is national or international.

a. Incidence and impact of spillovers

The starting point of empirical studies that have investigated the issue of appropriability of research is a relatively simple observation: once new knowledge has been created, it can often be imitated relatively easily, cheaply and quickly. The studies that have explored the implications of this empirically demonstrated stylized fact fall broadly into two main categories. In the first are attempts to measure the gap between private and social rates of return to R&D without explicitly identifying the interindustry or intraindustry links through which spillovers can operate. Their main conclusion is that the social returns from R&D (and from basic research especially) are significant and higher than private returns [11].

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[11] Mansfield (1985) and Mansfield et al. (1977) for example have shown that only a fraction of the benefits of inventions are captured by the inventor or the innovating firm even when there exist patent rights to protect the newly created knowledge. In the 1977 paper private benefits are measured by the innovator's profits, net of any costs for producing and marketing the innovation, and net of the profits that the innovator would have earned on products displaced by the innovation and with adjustment for unsuccessful R&D. Social benefits are obtained by adding to the private benefits the change in consumer surplus due to lower prices and profits by innovators minus the R&D costs incurred by firms other than the innovating firm. They conclude that the social rate of return greatly exceeds the private rate (a median of 56% against a median private rate of return of 25%), with the private rate too low in many cases to justify undertaking the project. Another method of estimating the social rate of return on R&D involves estimating the marginal product, profitability or cost reduction from R&D by introducing the R&D stock of knowledge as an input into a production, profit or cost function (see Schankerman, 1981, for the interpretation of the resulting estimate). On this basis Mansfield (1980) disaggregated R&D into a basic and a development part and obtained that the return to R&D comes overwhelmingly from basic research.

The second group of studies has explicitly modelled the channels of transmission of spillovers by introducing an outside or borrowed stock of knowledge as an input into the production process alongside a firm's (or industry's) own accumulated R&D stock or expenditures. A number of techniques have been employed, depending on whether the focus is on embodied or on knowledge spillovers and on spillovers between industries or within them [12]. Since our interest is to examine the impact of spillovers on costs, incentives, and profitability we concentrate here on empirical studies that provide information on these matters [13].

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[12] In a survey of work in the area, Mohnen (1990) identifies six approaches. The first approach measures the influence of the R&D spillover econometrically by treating spillovers as an unweighted sum of the R&D of all other firms or industries. Examples are Levin and Reiss (1984, 1988), Levin (1988), Bernstein (1988), and Bernstein and Nadiri (1989). In the second approach, the R&D spillover variable is measured as a weighted sum of all the outside R&D. The papers following this latter approach can be further subdivided into four sub-groups, according to the proximity measure used to construct the weights. Papers in the first sub-group use weights proportional to the flows of intermediate input purchases, as revealed by input-output transactions; this approach is associated with the work of Terleckyj (1974), Griliches and Lichtenberg (1984), Bresnahan (1986), and Goto and Suzuki (1989). The second group uses flows of patents. Scherer, F.M. (1982a, 1982b and 1984), carrying on the lines first suggested by Schmookler (1966), is the best known work using this approach; other papers include Schankerman (1979), Pakes and Schankerman, M. (1984); Englander, Evenson and Hanazaki (1988). The third group focuses on the flows of innovations between firms and industries as in Robson, Townsend and Pavitt (1988), while the fourth uses patent data and the concept of *technological distance* (Jaffe, 1986, following on an idea originally suggested by Griliches). The idea here is to look at the correlation of the position vectors of industries in a technology space, where each element in the technological position vector of each sector is the fraction of the sector's R&D expenditures in a particular technological area. Finally, the last approach adopts a framework whereby each outside R&D is introduced directly and separately. Bernstein (1988) is an example.

[13] In empirical work on spillovers a sharp distinction is made between *intra-* and *inter-*industry spillovers. This distinction is data-driven; for theoretical work on the impact of spillovers, however, it is less useful. Thus, in the context of the oligopolistic models developed in this dissertation, the boundaries of an "industry" are simply defined by the product rivalry and assumption of product homogeneity.

It should also be emphasized that it is not easy to identify separately in empirical work embodied R&D spillovers and knowledge spillovers. Since both types of spillovers can occur between industries and within industries (in the SIC sense of the word "industry"), a distinction between *intra*industry and *inter*industry spillovers does not help. The key here is the particular links used in the analysis. Thus, the use of intermediate input transactions as weights for other sectors' R&D when investigating interindustry spillovers suggests that one is looking at embodied spillovers due to measurement errors; conversely, the position in a technology space, the unweighted sum of others' R&D or a direct estimation approach all are closer to identifying knowledge spillovers. Patent flows are more problematic. When used as carriers of R&D and in order to define "technological closeness", they are closer to identifying links due to knowledge spillovers. When however they are used as proxies for input purchases, they also capture some of the spillover due to incorrect price measurement.

The cost-reducing effect of spillovers is generally confirmed in empirical investigations. Levin and Reiss (1984, 1988) were able to reject the hypothesis that appropriability conditions do not matter. They concluded that the extent of spillovers is higher for processes than for products and that it varies considerably between industries, with electronics industries appearing to have significantly higher spillovers than other industries [14]. Levin (1988), based on survey data, came to similar conclusions and also identified the importance of different channels of spillovers. Bernstein (1988) estimated the effects of intra- and interindustry spillovers in seven Canadian industries (food and beverages, pulp and paper, metal fabricating, non-electrical machinery, aircraft and parts, electrical products, and chemical products). He concluded that both types of spillover affect production costs, with interindustry spillovers exerting greater downward pressure on costs of production than intraindustry spillovers.

These empirical studies address two related questions. The first is whether spillovers affect costs of production. The overwhelming response is that they significantly reduce them. The second question is whether spillovers alter incentives to invest in R&D. Here, the answer is more qualified. Levin and Reiss (1988) find evidence that suggests that variations in the degree of substitution between own and rival R&D may affect private incentives to conduct R&D. In estimating a model that allows possibility that own and rival R&D are imperfect substitutes, they establish that diminished appropriability does not necessarily reduce R&D expenditures.

Bernstein (1988) finds evidence that the response of firms to intraindustry spillovers depends on the nature of the industry they operate in. Firms operating in

[14] Levin and Reiss (1988) estimate a model where the industry R&D pool consists of own R&D expenditures plus a fraction of the sum of all others' R&D expenditures. They are able to estimate separately the productivity of the spillover from the extent of the spillover and to separate the spillover effect due to (cost-reducing) process and (demand-creating) product R&D.

industries with relatively low propensities to invest in R&D (small R&D cost share) tend to substitute their rivals' R&D for their own. In contrast, in industries with relatively higher R&D propensities there is a complementary relationship between firms' own and their rivals' R&D. There is therefore a positive incentive effect of spillovers on the demand for R&D in industries with larger R&D cost shares.

Levin (1988), using a different methodology, arrives to similar conclusions. Based on survey data, he uses cluster analysis to conclude that industries reporting the highest levels of spillovers (primarily electronics-related) are also those with average rates of innovation higher than industries relying upon independent R&D. He suggests that his results support the hypothesis that spillovers are conducive to rapid technical progress but not the hypothesis that they discourage R&D investment.

Finally, Bernstein and Nadiri (1989) provide some evidence that supports the traditional disincentive effects of spillovers. In a sample of four US industries (chemicals, petroleum, machinery and instruments), they observed that both the short- and the long-run demand for R&D capital (and for physical capital) decreased in response to an increase in the intraindustry spillover. They did not find any complementarity effects between intraindustry spillovers and firms' own R&D. Rather, own and borrowed R&D appeared to be substitutes.

Jaffe (1986), using the concept of "technological distance", attempts to establish the importance of spillovers by looking at the effect of other firms' R&D on the productivity of own R&D. His starting point is that since spillovers depend on the technological similarity of the research efforts of different firms, a firm's R&D success is affected by a "potential spillover pool", defined as the weighted sum of other firms' R&D, with weights proportional to the proximity of firms in a technology space.

The framework adopted by Jaffe examines the two distinct effects of spillovers: the purely *technological* effect, whereby R&D spillovers constitute an unambiguously positive externality for receiving firms; and the *economic* effect, due to the effect of others' research on a firm's profits. His estimations lead him to conclude that the productivity of a firm's R&D is increased by the R&D of "technological neighbours" (R&D of other firms increases the knowledge output of the firm directly, and also increases the elasticity with respect to own R&D); neighbours R&D however reduces profits (and market value) of low R&D-intensity firms [15].

While therefore evidence from empirical studies suggests that spillovers exist in practice, it is not conclusive on their impact on R&D incentives and on the productivity of R&D expenditures. In some cases the R&D-incentive impact of spillovers goes in the opposite direction than what theory has until now suggested. In section II below we explore some arguments that try to shed light on this result; a theoretical framework that attempts to understand it more fully is developed in the main body of the dissertation.

b. Policy concerns

A general feeling among policy-makers that resources spent on R&D by private firms acting in isolation are in some sense insufficient (often explicitly because of the existence of research spillovers) has led to the implementation of a number of types of policy aimed at closing the gap between private and public returns to R&D. The more widely used are: direct or indirect subsidies to restore incentives; strengthening appropriability rights; and encouraging cooperation amongst firms at the research stage.

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[15] Jaffe notes that in order to understand the implications of spillovers for industrial structure and public policy it is necessary to focus on variations in the extent of spillovers in different technology areas (he assumes uniform spillover rates) and to examine spillovers in a context where their implications for firms' strategic behaviour are recognized.

Tax policies and direct subsidies are used widely to raise private R&D incentives. In the US for example, 47% of the total spent on private R&D in 1988 came from direct government subsidies [16]. The aim is to lower the costs to individual firms for undertaking research and thus counter some of the disincentives associated with low appropriability of R&D. While however it has been established that a policy of relying on subsidies can counteract effectively the disincentive effect of spillovers (Spence, 1984), such a policy does not address the other side of the coin: the insufficient dissemination of R&D results. Moreover, evidence shows that the social cost of R&D subsidies may be substantial: firms may classify as R&D projects unrelated to research in order to be eligible for subsidies or tax breaks [17].

Another problem with subsidies, addressed directly in the main body of the dissertation, is that they may in fact further distort incentives rather than correct them. In some industries, spillovers may act as a spur to R&D expenditures, rather than as a disincentive. Since subsidies usually do not discriminate between industrial sectors on the basis of their appropriability, technological and behavioural characteristics, they may be too blunt an instrument for encouraging R&D [18].

Subsidies to R&D often also have an explicitly international dimension. Countries deliberately intervene in order to support "national champions" that are engaged in international competition through product and process innovations. By subsidising the R&D expenditures of domestic firms, governments aim to increase their rate of innovation and thereby increase their market share and profits at the expense of rivals.

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[16] Cited in Katz and Ordover (1990) from *Economic Report of the President* (1989, p.226).

[17] In the US, Brown (1984) found discrepancies between the increases in R&D expenditures as reported in tax forms and increases in R&D spending as reported in independent surveys.

[18] Other problems associated with subsidies are related to the fact that they involve resource shifting between sectors (Dixit and Grossman, 1986). When they are targeted, they imply an ability on the part of the government to "pick winners", which is widely debated. Finally, raising the subsidy funds through taxes gives rise to "deadweight losses" (Grossman, 1989).

Such profit-shifting subsidies may however be counter-productive in international environments characterised by low levels of appropriability of research. In circumstances where spillovers are extensive, industrial subsidies in support of domestic firms also benefit foreign rivals. This negative externality can outweigh the increased positive incentives for additional R&D, making such policies counter-productive. The circumstances where this can occur are explored in chapter five below.

Restoring appropriability by creating or extending property rights to knowledge through patents and licensing is another approach that has been followed. If a firm can patent or licence an innovation, it can recapture some of the costs associated with R&D and restore appropriability and incentives. The temporary monopolistic power conferred by a patent over the prospective benefits associated with an innovation constitutes an incentive for investment in the creation of the innovation (Kamien and Schwartz, 1982). As Katz and Ordover (1990) point out, stronger property rights improve appropriability in two ways. First, if the firm chooses not to share its R&D results, it will be shielded from the effects of spillovers. Second, if it does, licensing of the results will be made more profitable. This in turn will raise incentives for R&D outlays and may also increase the dissemination of R&D results.

Patents however have time limits, are sector-specific, and are usually narrowly defined so that they can in many instances be easily evaded. They are therefore an imperfect instrument for restoring appropriability and can never fully offset the large externalities associated with private innovation. They may also, as Arrow (1962) and Spence (1984) separately pointed out, reduce the efficient sharing of R&D. This is because spillovers have the socially beneficial effect of forcing dissemination of research at or near its marginal cost of transmission, which for knowledge can be almost zero [19]. Strengthening intellectual property rights will have the effect of incorrectly pricing the (essentially public) good that R&D has created. Even then where this does not inhibit the diffusion of research findings, research dissemination will occur at inefficient prices and will therefore be at a level that is less than socially optimal.

Encouraging cooperation amongst firms at the R&D stage is a third policy tool which is receiving increasing attention as both the number and variety of such industrial structures are proliferating. The idea is that by sharing research output, research joint ventures can increase the efficiency of R&D efforts and eliminate duplication. Simultaneously, cost sharing agreements restore some of the incentives to conduct R&D so that a technology cooperation agreement may serve as a mechanism to internalise the externalities created by spillovers [20]. The policy is implemented mainly by relaxing the antitrust treatment of joint ventures [21]. The desirability however of public policy to encourage the formation of joint research ventures will depend on a number of factors, including the structural characteristics of the relevant research and product markets, the objectives of the venture and the strategies of the participants [22].

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[19] This argument, together with its pitfalls and limitations, is elaborated in the next section.

[20] Beside problems of low appropriability of the results of R&D, other factors that underlie the formation of technical cooperation agreements between firms include the high and rising costs of innovation, the requirements of a widened scientific and technological base and issues of access to international markets. For a recent review see F. Chesnais (1988). In general, the equilibrium characteristics of industries characterised by firms engaged in research joint ventures as a response to the existence of research spillovers are analysed in a number of recent papers, such as d' Aspremont and Jacquemin (1988), Vonortas (1989) and Henriques (1990). These models are in a sense a counterpart of the types of models developed in the chapters that follow. They investigate the characteristics of *cooperative* equilibria characterised by spillovers, while we look at the characteristics of *non-cooperative* equilibria.

[21] For an analysis of the public policy implications of research joint ventures from the standpoint of reform of antitrust legislation, see Grossman and Shapiro (1984).

[22] Katz (1984) and Katz and Ordover (1990). They point out that *ex ante* cooperation (ie cooperation in research) will not necessarily lead to a greater amount or greater efficiency of R&D investment. An assessment of the effects of ex ante cooperation is only meaningful in the context of analysis that compares R&D levels and efficiency of research consortia both with those in the absence of cooperation and with the case where firms engage in *ex post* cooperation (such as patent licensing).

III. Spillovers and Economic Theory

Innovation policy and empirical studies have one preoccupation in common: the former assume and the latter test for the existence of a tradeoff between incentives to undertake R&D expenditures and the efficiency with which a given level of cost reduction is in fact achieved. In this section we discuss the theory behind this apparent tradeoff, together with some arguments that suggest that it may be overstated. We then examine some attempts to model spillovers in an oligopolistic context that addresses this issue. We conclude with a discussion of strategic behaviour arguments and their implications for the impact of spillovers, especially in the context of international trade.

a. Static welfare externalities

For the efficient operation of the innovation process, markets must provide adequate and appropriate incentives and rewards to inventors and innovators, while ensuring efficient selection and ranking amongst alternatives in terms of resource allocations. Arrow (1962) and Nelson (1959) are credited as being the first to identify a number of reasons why the market might support insufficient scientific effort and allocate this effort in an inefficient manner. Their analysis derives from the characterisation of invention as the production of information or knowledge. They consequently focus on the "public good" aspect of knowledge, the externalities associated with its production and the resulting divergence between private and social costs and benefits.

Knowledge has a number of characteristics that complicate its provision by private agents in a market system. The two fundamental characteristics here are attributes possessed in varying degrees by any economic good: rivalry and excludability. An input is rival if its use by one person precludes it being used by another. A good is excludable if the owner can preclude others from using it. Knowledge is widely considered to be a *partially excludable* and *non-rivalrous* good (Romer, 1990).

Private goods are perfectly excludable; public goods are not. Knowledge possesses the public good attribute of not being excludable in that it is difficult for agents that have devoted resources to generating innovations to appropriate the benefits and prevent others from using the knowledge without adequate compensation. It is however considered to be only imperfectly excludable, in that innovators capture at least part of the social benefits associated with the development of knowledge [23]. Such knowledge "spillovers" imply that in situations where some of the inventive activity is not fully appropriable, there exists an ex ante negative externality operating on reduced incentives to spend on R&D. In such situations, the market may be providing insufficient incentives for the socially optimal provision of knowledge.

Non-rivalry is another characteristic of knowledge and a feature typical of public goods. When a piece of knowledge or information about a particular innovation is non-rival, the number of people using it (consuming the services rendered) can be increased without additional costs, at least up to some capacity constraint [24]. A welfare optimum in this situation requires that the price of the associated product be equal to the marginal variable cost of transmitting the information. Once produced therefore, efficient allocation of R&D requires near-free dissemination and therefore low appropriability. To the extent that because of instruments (such as patents) or restrictive practices arising out of some monopoly situation price diverges from marginal cost, R&D will be under-utilised and the welfare optimum will not be achieved.

[23] Romer (1990) notes that in order to reconcile private provision of innovation with perfect non-excludability of knowledge it is necessary to consider innovation as the unintended consequence of some other activity, such as investment in (traditional) capital or in education. In that situation, it is not possible to account for R&D activities by private firms.

[24] The "lumpiness" of knowledge is important here. To the extent that cost-reducing expenditures are fixed costs of production, the main element of R&D expenditures is the initial outlay. The marginal cost of production is then simply the cost of the transmission of the new information to other firms, which is near zero.

While therefore low appropriability points to an *ex ante* negative externality associated with reduced incentives, the non-rivalrous aspect of knowledge indicates the existence of an *ex post* positive externality of cost reduction. High spillovers can therefore reduce the industry's R&D cost of achieving a given level of product cost reduction and thereby increase dynamic performance. This suggests that a problem exists in reconciling the opposite aims of a framework that must provide innovators with an environment which stimulates innovative activity (by restricting use of the innovation and thereby guaranteeing some gains to the innovator) while at the same time allowing maximum use of its product (by keeping its price low and thereby ensuring imitation, adoption and diffusion) [25].

b. Dynamic externalities and systemic factors

The characterisation of innovation as the production of information and its associated public good characteristics have given rise to the argument that there exists a tradeoff between an ex ante negative externality operating on reduced incentives to conduct R&D and an ex post positive externality operating on cost reduction. A number of issues however related to the dynamic nature of innovation and its systemic characteristics have contributed to a more complete understanding of the forces guiding incentives and performance in markets characterised by innovative activities.

The first concerns the heterogeneity of knowledge. Nelson (1980), among others, has drawn a distinction between two types of technological knowledge: the generic (or basic research) part, consisting of inferences about how things work, the identification of constraints and of possible ways of overcoming them, and heuristics relative to the

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[25] Another potential source of market failure is the (often uninsurable) risk and uncertainty inherent in the research process, and especially at the basic research end of the spectrum. These are not problems unique to research. They are not the focus of this thesis and are therefore only mentioned here. For an analysis that does justice to their implications see Dasgupta (1987) and Dasgupta and Maskin (1987).

problems at hand; and the "operative techniques" (or development) part, consisting of particular ways to make things work which are specific to the task at hand. The generic part is the one possessing the "public good" features to a great extent, since generic knowledge has a wide range of applications, can be communicated without major learning costs, and could considerably limit the capabilities of those denied access. The "techniques" part often possesses the public good properties only to a limited extent; the range of "technique" applicability is narrow, its learning entails high costs because it needs to be applied to the specific needs of the user, and denial of access may not eliminate chances for technological success if the generic part has been assimilated.

In a more general sense, most technological knowledge is "information" that, while being generally applicable and easily reproducible, is specific to particular firms and applications, can be embodied in products and processes, and varies amongst industries in its source and direction (Pavitt, 1984). As a consequence, the concept of a "pool" of knowledge underlying Arrow's analysis misses the firm-specific or process- and product-specific and differentiated nature of most of the expenditures necessary to produce a new technique or product. Specificity and localisation are determining characteristics of most innovations, both in terms of functional applications and in terms of the ability of the innovating firm to appropriate relevant knowledge [26]. This implies that the cost of acquiring technological knowledge in a usable form may be high. Evidence suggests that the resources spent on reverse-engineering or spying are in fact substantial (Levin, 1988), implying that spillovers are not free [27].

[26] The "localised" nature of technological progress is a concept first introduced by Atkinson and Stiglitz (1969). As Stiglitz (1987) points out, it is related to the distinction made between basic knowledge, with wide applicability, and technical knowledge, which is much more specific and localised.

[27] In an early treatment of the subject of spillovers, Nordhaus (1969, p. 37) draws a distinction between the *transmission* costs of information, which are almost certainly low, and the *absorption* costs, which may require the investment of substantial resources.

One interpretation of these attributes of innovation that dilute its public good characteristics is that they should be seen in the context of the current "technological trajectory" and its dominant characteristics [28]. Trajectories have different degrees of generality in different technologies and industrial sectors; the general hypothesis however is that innovation activities are strongly selective, formed in certain directions and that they embody a particular pattern of search for new techniques and a resulting combination of variety and regularities in the outcome of the innovation process.

Current technological trajectories have a number of dominant characteristics. One of them is the cumulativeness of technological change at firm, country or world level (Chesnais, 1986) which points to the importance of learning processes and restricts the "free pool" nature of new technology. It implies that the probability of success in innovating is a function of the level of achieved results. What firms do in the future therefore is strongly conditioned by what they have done in the past. Search processes involved in technological learning are localised around groups of agents and are path-dependant [29].

Interindustry differences in the nature of technology that relate to the cumulativeness of technological change have been suggested as possible explanations for the observed phenomenon of industries characterised both by a low degree of appropriability (high spillovers) and by a high R&D intensity. Levin (1988) has argued that the disincentive effect of appropriability should be expected to prevail only in industries characterised by "discrete" technologies, ie where innovations represent in

[28] "Technological trajectories" are defined as directions of advance within particular "technological paradigms", themselves defined as "..an 'outlook', a set of procedures, a definition of the 'relevant' problems and of the specific knowledge related to their solution." (Dosi, 1982, p. 148).

[29] Stiglitz (1987). He also points out that while localised learning enhances the accumulation of knowledge along a given "trajectory", it creates the possibility that agents (firms, industries or countries) can be effectively "locked-in" to particular trajectories. The welfare implications are not clear, however, because they depend on whether one "locks-in" to the socially efficient trajectory.

effect isolated discoveries. In such an environment, knowledge of one firm's innovation may lower the marginal productivity of the R&D investment of its rivals by making any additional effort duplicative [30]. The chemical and drug industries (before the revolution in genetic engineering) may be considered examples of such industries.

Technical advance can instead be "cumulative", as seems to have been the case in electronics industries, with each advance building on previous technology and incorporating many of the features of the displaced products and processes. Spillovers from the R&D conducted by one firm can in this case raise the marginal product of the R&D conducted by its rivals [31]. In such an environment, spillovers can affect positively the cost of production and at the same time act as a spur to R&D investment.

The idea that technological advance can be of a "cumulative" nature leads naturally to the centrality of the notion of "learning" in the behaviour of firms. Cohen and Levinthal (1989) have in fact suggested that in addition to generating new innovations, research and development expenditures enhance the firm's ability to assimilate and exploit information in the public domain. The recognition of this second role of R&D suggests that the ease of learning within an industry will have a direct effect on the level of R&D expenditures and will indirectly determine the influence that conditions of appropriability have on R&D investment.

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[30] This implies that "own" and "borrowed" research can be assumed to be perfect substitutes, as in Spence (1984) and in chapter two below (for basic research only).

[31] Katz and Ordover (1990) label this the effect of *intermediate* technological spillovers. In the context of their model, an increase in the R&D level of one firm will increase the effectiveness of R&D conducted by rivals when an R&D project consists of a number of stages that must be completed before a patent can be obtained, and one firm can learn how to complete a stage by observing a rival's success. Put another way, the rival's probability of success, conditional on not being pre-empted, rises. They distinguish this from *final* R&D spillovers, whereby even though one firm may have won the patent race, rival firms may benefit form the product or process that the winner has developed. This distinction alludes to substitutability and complementarity effects between the research of different firms and is an alternative interpretation to the nature of technology argument.

This line of argument recognizes a dual role for R&D: in addition to developing a product --new information-- , R&D also develops the capability of firms to utilise more efficiently already existing information, as well as to learn to anticipate and follow future developments. This second aspect has been referred to as "learning to learn" or learning by learning, as distinct from "learning by doing" [32]. R&D helps firms identify, track and potentially take advantage of knowledge initially developed elsewhere: it develops firms' "absorptive" capacity (Cohen and Levinthal, 1989).

A number of authors have drawn attention in the past to the role of learning. Tilton (1971) has asserted that one of the main reasons why firms invest in R&D in the semiconductor industry is in order to "facilitate the assimilation of new technology developed elsewhere" [33]. Rosenberg (1976) and Nelson and Winter (1977) have argued that in order for firms to be able to use freely available knowledge they often have to invest in R&D. Rosenberg (1990) has likened performance in (basic) research as "a ticket of admission to an information network". In all cases, authors argued that these expenditures were necessary because, rather than knowledge being "on the shelf", it often requires a substantial research capability in order to understand and assimilate the knowledge that is in principle in the public domain.

While the recognition of the role of R&D in learning is not new, its implications for the ability of firms to appropriate publicly available knowledge have not been explored until recently [34]. Imitation costs have been interpreted as primarily consisting

[32] Learning by doing refers to a process by which the firm becomes more efficient in what it is already doing. Learning to learn allows a firm to acquire outside knowledge that will help it to do something different (Cohen and Levinthal, 1989). Therefore just as experience in production increases one's productivity in producing (learning by doing), so experience in learning may increase one's productivity in learning (Stiglitz, 1987).

[33] Tilton (1971), p. 71 as cited in Cohen and Levinthal (1989).

[34] The papers of Stiglitz (1987) and Cohen and Levinthal (1989) are two examples of approaches that develop theoretical frameworks in which the capacity to absorb new information is enhanced by one's learning experience.

of the cost of transmission of information, ie as being of a static nature, and small when compared to the cost of generating the innovation. Such costs however may depend crucially on the technological level achieved by a firm (its stock of accumulated R&D). The capacity to imitate and take advantage of technological developments elsewhere may depend crucially on own R&D expenditures.

The question of what determines the costs of assimilating technological knowledge is central to the impact of spillovers. Cohen and Levinthal (1989) argue that these costs are low when the firm has already invested in the development of its absorptive capacity in the relevant field. This suggests that the long-run cost of learning may be substantial and that it is borne by the development of a stock of knowledge. Consequently, in an environment characterised by spillovers and where it is possible to learn from competitors, the incentives for R&D investment should be influenced by the knowledge that they themselves contribute to that learning.

The extent to which R&D is critical to the development of a firm's absorptive capacity (and therefore implicitly to the "cost" of spillovers) depends on the characteristics of outside knowledge. In addition to the degree to which a particular field of knowledge is cumulative, identified also by other authors, Cohen and Levinthal suggest that the complexity of the knowledge to be assimilated, the degree to which the outside knowledge is targeted to the specific needs of the firm, and a field's pace of advance all have a role to play. The more complex outside knowledge is and the more generic and less targeted its nature, the more important are R&D expenditures for identifying and allowing the exploitation of valuable knowledge. Similarly, the faster the pace of advance of the field, the greater is the effort required to keep up with developments.

From the above discussion it should be clear that the simple statement that problems of appropriability reduce the incentives that firms have in R&D needs to be heavily qualified. The costs involved in imitation, learning curve effects, and the nature of technological advance itself point to a more complicated picture where the public good nature of R&D is considerably weakened. This is reinforced by the existence of a whole array of other appropriability devices, apart from patents and commercial secrecy, such as lead times, superior sales and service efforts, or differential technical efficiency related to scale economies. It follows that any welfare criteria that are developed in order to guide policy-making need to take into account the differential importance of the appropriability problem depending on the market structure and characteristics for which the relevant innovation is intended.

c. Modelling research spillovers

Modelling spillovers is relatively recent. While the importance of appropriability for innovative activity has been recognized and modelled since Arrow's (1962) seminal article, it has not been until recently that models appeared in the literature taking explicitly into account the effect of a firm's inventive activity on other firms by formulating "borrowing" functions that allow for an interaction between the size of individual and aggregate R&D effort.

These models have grown out of a literature that attempted to formalise Schumpeter's original insights about market structure and innovation [35] by focusing on firm behaviour with respect to incentives for R&D investments, and on the characteristics of the resulting industrial structure and performance. R&D was treated as fully appropriable (no spillovers) and the degree of innovation was either taken to depend on an exogenous market structure, or alternatively market structure and innovation were

[35] Namely that there is a positive relationship between innovation and monopoly power and that large firms are more innovative than small firms Schumpeter (1975). The second postulate is often attributed to Galbraith (1953). See also the discussion of Dasgupta (1987) about the debate on what Schumpeter *really* meant.

treated as endogenous variables that are determined simultaneously. In this latter case, the characteristics of both industrial structure and innovative performance were traced to more fundamental factors: the structure of demand or extent of the market (Schmookler, 1966); the nature of technological opportunities (Rosenberg, 1976); and the technological and institutional conditions that determine the appropriability of R&D benefits by firms [36].

Published papers that develop a theoretical framework incorporating spillovers in the context of an oligopolistic industry include Levin and Reiss (1983, 1989), Hartwick (1983), Spence (1984), Cohen and Levinthal (1989). Some unpublished work is by Leung (1983) and Vonortas (1987, 1988) [37]. These models take into account the

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[36] Dasgupta and Stiglitz (1980a) is a representative example of the type of models that focus on the incentives that firms have for R&D expenditures and on the resulting industrial structure and performance. Since the model in chapter two of this dissertation is an extension of their model, their main results are reviewed in the beginning of the next chapter. Another type of models in the economics of innovation focus on the *speed* of technological development by formulating R&D competition as a race to innovate. Representative examples are Loury (1979), Dasgupta and Stiglitz (1980b), Lee and Wilde (1980), and Reinganum (1982, 1984, 1985). These models deal with the time-cost tradeoff of research. Firms are assumed to move sequentially in repeated games that are "tournaments" (in the expression of Dasgupta, 1986) ie where firm performance is rewarded on the basis of *rank* within the set of all realized performances. This necessitates asymmetric behaviour on the part of firms. In these models it is possible for firms to duplicate each other's research with the result that the market can generate too much, rather than too little, R&D relative to the social optimum.

Both these types of models are based on the principles of maximisation and equilibrium. An altogether different approach is taken in "evolutionary theory"-type models (Nelson and Winter 1977, 1982). They treat the Schumpetarian hypothesis as a process of evolutionary change and argue that the concepts of equilibrium and maximisation are incompatible with the process of evolution; they rely instead on "routines", "rules of thumb" and stochastic elements to describe firm behaviour. The approach yields useful insights, but the generality adopted has its price: the models are not analytically tractable and are only open to simulation techniques.

[37] There are important precursors to these models in attempts to model spillovers outside an oligopolistic framework. Nordhaus (1969, ch. 3) developed a model of technological change in a competitive market where firms take into account that some of their research "spills out" and benefits other firms. Their productivity depends on the research inputs of all firms via an explicit spillover parameter. Griliches (1979) and Schankerman (1979) have also developed simple models that allow for an interaction between individual and aggregate research effort. While Griliches introduces exogenous (to the firm) knowledge as an additional input into a conventional production function, Schankerman specifies (and estimates) a knowledge production function that includes "borrowed" research. He also develops a simple general equilibrium model in order to examine the question of underinvestment in R&D due to spillovers and the issue of optimal subsidies. His model leads him to conclude that spillovers do not necessarily imply underinvestment in R&D.

interaction between competing firms and focus on the determinants of R&D expenditures at firm and industry level and on the implications for industrial structure.

Hartwick (1984), Leung (1983) and Vonortas (1987) are all simple extensions of the Dasgupta and Stiglitz (1980a) model for the case of spillovers. Hartwick expands the firm's unit production cost function to include the R&D expenditures of all other firms. There is no spillover parameter as such; a firm's own and the total of its rivals' R&D enter into a Cobb-Douglas specification with differential elasticities of cost reduction with respect to R&D [38]. This implies a certain amount of complementarity between own and rival R&D. His simulations lead him to conclude that the socially optimal outcome may involve more than one firm operating [39]. Unlike in the Dasgupta and Stiglitz (1980a) model, the market outcome involves R&D and production levels that are below the social optimum. Subsidies to R&D for each firm in the market can therefore improve welfare.

Leung (1983) and Vonortas (1987) similarly extend the Dasgupta and Stiglitz (1980) framework by formulating a marginal cost function where the part of rivals'

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[38] Dasgupta and Stiglitz (1980a) use a unit production cost function of the form $c_i = c(x_i)$, where x_i is the firm's own R&D expenditures, which for the derivation of explicit results they parameterize as $c_i = \beta x_i^{-\alpha}$, where $1/\beta$ is the scientific level of the industry and α is the elasticity of own reduction with respect to R&D. Hartwick extends this to the form $c_i = c(x_i, X_i)$ where $X_i = \sum_{j \neq i} x_j$. This he parameterizes as $c_i = \beta x_i^{-\alpha} X_i^{-\gamma}$, where γ is the elasticity of cost reduction with respect to rival R&D.

[39] "Letting a thousand flowers bloom", an old idea in R&D economics that can be traced back to Nelson (1961). In the simulations that deliver these results however, the parameters chosen exhibit increasing returns to scale to R&D expenditures; in addition, the elasticity of cost reduction with respect to rivals' R&D is greater than that with respect to own R&D.

R&D that spills out is a perfect substitute for a firm's own R&D. Leung obtains the result that an industry characterised by higher spillovers will be less concentrated, while spillovers will act as a disincentive to R&D when demand for the product is inelastic [40].

Levin and Reiss (1984) generalise the cost formulation by including industry-wide R&D alongside a firm's own R&D expenditures [41]. They distinguish between three dimensions of appropriability. The first is a *technological* dimension. To the extent that unit cost reduction is very elastic with respect to increments in the industry-wide R&D (holding own R&D constant), costless imitation is relatively easy and R&D is (technologically) inappropriable. The second dimension is *structural*: for any given technology of R&D and market size, a firm's appropriable benefits from increasing the common pool of knowledge depend on its market share (which in symmetry is the reverse of the number of firms). Finally, the third dimension is *behavioural*: Levin and Reiss examine the firm's response to the existence of a pool of knowledge it can use (ie be a free rider and cut back on own R&D or not) by a conjectural variation parameter [42].

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[40] Leung's functional specification of the cost function is $c_i = \left(x_i + \theta \sum_{j \neq i} x_j\right)^{-\alpha}$ where θ is the spillover rate. Vonortas (1987) breaks R&D into basic (x) and development research (y) and uses a cost function of the form $c_i = \left(x_i + \delta\theta \sum_{j \neq i} x_j\right)^{-\alpha} y_i^{-\gamma}$ where δ is the firm's absorptive capacity (assumed exogenous) and γ is the elasticity of cost reduction with respect to own development research expenditures. Rather than examining the impact of spillovers however, his primary interest is in comparing the noncooperative equilibrium with the one that results when firms cooperate in (non-appropriable) basic research.

[41] Their cost specification is $c_i = c(x_i, X)$ where X represents the pool of industry-wide R&D. They also use the functional form $c_i = \beta x_i^{-\alpha} X^{-\gamma}$.

[42] All three dimensions of appropriability are reflected in the models developed in the following chapters. The technological dimension is captured through the assumptions about substitutability and complementarity implicit in the use of different functional forms for the knowledge production functions; the structural dimension is captured by endogenising the determination of the equilibrium number of firms. Also both these dimensions are reflected in the spillover rate. Finally the behavioural dimension is captured in the construction of the strategic model and the comparison with the Cournot (one stage) model. Such an approach seems to be preferable to varying exogenously a conjectural variations parameter, as the notion of "reaction" is not very meaningful in a one-stage game.

The model by Spence (1984) is essentially static, but with dynamic connotations: a firm's unit costs depend on its accumulated knowledge. Additions to that knowledge depend on own R&D expenditures and on the part of the R&D expenditures of rivals that spill over. The underlying assumption is that a firm's own R&D capital is a perfect substitute for borrowed R&D capital. He examines the impact of spillovers on incentives and on industry performance. For incentives, his interest is twofold: he analyses the effect of spillovers on the comparison between private and social incentives to invest in R&D and also the sign of their effect on private R&D investment.

Spence's main conclusion is that while spillovers reduce the incentives for cost reduction, they also reduce the costs at the industry level of achieving a given level of cost reduction. Put another way, with appropriability of R&D, the achievable surplus of the market is lower because a high rate of cost reduction can only be achieved with significant R&D outlays. He constructs a simple example that demonstrates that for a given number of firms, the incentives of the market decline with the spillover rate; for a given spillover rate, they decline as the industry becomes less concentrated (except at low spillover rates, where some competition increases the amount of cost reduction) [43]. In the absence of subsidies, the market performs best with moderate spillovers (which reduce industry R&D without excessively removing incentives).

If the government subsidises R&D, Spence shows that both firm incentives and market performance increase with the spillover rate (for a given number of firms); for a given spillover rate, as the number of firms increases, incentives and performance first rise and then fall beyond the point where the redundancy costs overwhelm the

[43] Incentives are reflected in a function that accounts for the present value of the effect of own and of borrowed R&D on a firm's earnings gross of R&D investment. They are expressed as the ratio of these benefits implicitly recognized by the market to the optimal surplus. Market performance is evaluated by calculating total surplus net of the cost of R&D and of any R&D subsidies. Relative market performance is then the ratio of the surplus actually achieved by the market to the first best optimal surplus.

competitive effect on margins. At very high spillover rates however they just rise with the number of firms because of the absence of the redundancy problem. Finally, optimal subsidies are always positive when research is less than fully appropriable and rise with the spillover rate.

In a more recent paper, Levin and Reiss (1989) extend the approach taken in their earlier model by assuming that R&D can be demand-creating, as well as cost-reducing. Technological opportunities and the degree of appropriability differ between (cost-reducing) process R&D and (demand-creating) product R&D; the authors thus examine the effect of spillovers on both the amount and the type of R&D. They also allow for the possibility that own and rival R&D are imperfect substitutes; this permits the consideration of how variations in the degree of substitution between own and rival R&D affect incentives to undertake R&D [44].

When R&D is cost-reducing only, they obtain that concentration and (process) R&D intensity fall with increases in the *extent* of spillovers but rise with increases in the *productivity* of spillovers. The latter result appears because, in a situation where own and rival R&D are imperfect substitutes, own R&D is enhanced by increases in industry knowledge. When R&D is both cost-reducing and demand-creating, it is not possible to sign unambiguously the comparative statics of the complete system. Nevertheless,

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[44] The cost specification in general form in the Levin and Reiss 1989 model is similar to their 1984 model ie $c_i = c(x_i, X)$ where X represents the pool of industry-wide R&D. It is further assumed that $X = x_i + \theta \sum_{j \neq i} x_j$. They parameterize this as $c_i = A x_i^{-\alpha}\left(x_i + \theta \sum_{j \neq i} x_j\right)^{-\gamma}$. While in the specification of industry-wide process R&D own and rival R&D are perfect substitutes, the effect of *im*perfect substitutability is obtained by including separately own R&D and the pool consisting of own and rival R&D in the cost function. They therefore calculate two separate elasticities of unit costs with respect to each type of process R&D. This allows a distinction to be made between the *extent* of spillovers (how much R&D leaks out) and the *productivity* of spillovers (how much borrowed R&D reduces costs).

under certain circumstances [45], increases in the extent of process and product R&D spillovers decrease concentration and R&D intensity, while increases in the productivity of spillovers have the opposite effect. When the extent of spillovers is near zero however, an increase in the extent of spillovers may increase the intensity of R&D.

Cohen and Levinthal (1989) assume that firms invest in R&D for two reasons: in order to produce innovations and in order to learn from competitors and from knowledge sources outside the industry (universities, labs). The *ease of learning* within an industry will therefore affect incentives to spend on R&D; it will also determine the impact of appropriability and of technological opportunity on these incentives. Spillovers in this setting can encourage equilibrium R&D expenditures.

Their model follows Spence (1984) in assuming in the specification of the determination of the firm's stock of knowledge that extra-firm R&D expenditures are a simple addition to a firm's own R&D expenditures; unlike Spence however, the proportion of this knowledge that can actually be absorbed by the firm depends on its absorptive capacity. The latter is an endogenous variable, determined by a firm's own R&D expenditures and by a variable reflecting the characteristics of outside knowledge that make own R&D more or less important to the development of absorptive capacity.

The endogeneity of absorptive capacity implies that contrary to the standard proposition, the effect of spillovers on incentives to invest in R&D is now ambiguous. Thee are two offsetting effects: the loss associated with the diminished appropriability as spillovers increase; and the benefit to the firm of increasing its absorptive capacity. This latter positive absorption effect may therefore offset the negative appropriability

[45] These circumstances relate to the case when the Jacobian of the system is positive. The authors claim that this is indeed the case when it is evaluated using the estimates that they obtain in the empirical part of the paper.

incentive, with the result that spillovers can act as a spur to R&D investment. Furthermore, the more learning becomes dependent on own R&D, the more increasing technological opportunity or spillovers will tend to induce R&D effort.

Despite their differences, these models suggest in any attempt to model spillovers, the type of environment firms are assumed to be operating in is important. It is an environment where there exist substantial opportunities for cost reduction through R&D expenditures, but where the cost of achieving them are substantial, though not prohibitive. Put another way, R&D is neither ineffective in reducing costs, nor very effective and cheap. There is therefore a conflict between allocative efficiency and dynamic technical efficiency (Spence, 1984); in those circumstances, there are potential incentive and performance problems associated with spillovers.

d. Strategic arguments and the international dimension

A number of authors have explicitly or implicitly modelled firm behaviour as a multi-stage process, where the decision on the amount to produce takes place after the expenditures on R&D have already been made. In addition to being a more faithful representation of the timing in actual decision-making, this procedure has interesting implications for firms' strategies with respect to R&D. In particular, in a situation where R&D reduces marginal costs, a multi-stage decision process gives rise to an incentive on the part of firms to shift resources to the "sunk" category of costs in the early (R&D-related) stages of the game so as to gain a strategic advantage in the later (output) stage of the game, where production takes place in an imperfectly competitive market (Brander and Spencer, 1983a). Even therefore in the situation where R&D is cost-reducing only, the elaboration of such a framework allows firms to perceive and act upon more complex strategies that go beyond the use of R&D for straight cost-minimisation [46].

The extension of these models to the case where R&D is subject to spillovers allows an examination of issues related to the interaction of strategic behaviour and inappropriability of research findings [47]. In this context it is possible to perform comparisons along two lines: (i) between the multi-stage equilibrium with spillovers and the multi-stage equilibrium with full appropriability; and (ii) between the strategic (multi-stage) equilibrium and the Cournot (one stage) equilibrium when in both cases R&D is subject to spillovers. These comparisons help to answer two related questions: whether or how spillovers affect the incentives of firms to invest in R&D and the performance of the industry when firms use R&D strategically; and whether the effect of strategic behaviour on R&D expenditures, output and profitability of industries is altered in the presence of spillovers.

Strategic behaviour related to R&D also has wide implications for international trade issues and for government behaviour. Firms often seem to be investing heavily in research and development in the direction of previously unexplored, but potentially profitable, areas with the explicit aim of making it too expensive for rivals to enter an international market. The implication is that the evolution and industrial structure characteristics of certain international oligopolistic markets will depend largely on such

[46] The argument for the strategic use of R&D is similar to "credible threat" arguments in oligopoly theory, such as those developed in Spence (1977, 1979) or Dixit (1980). These models emphasise the role of irreversible investments in establishing market power.

[47] Of the spillover models reviewed in the previous section, only Spence (1984) and Cohen and Levinthal (1989) take partly into account strategic consideration. Their models are implicitly two-stage, in the sense that when deciding on R&D levels, firms take into account the influence of their expenditures on their rivals' cost, production and earnings, and the feedback on their own profits. Spence also compares this setup with a situation where firms "ignore" the effect of their R&D on rivals (the equivalent to a one-stage Cournot equilibrium).

a strategic use of R&D, rather than on considerations of efficient resource allocation. In this second-best world, substantial government support of research and development efforts in particular industrial sectors over extended time periods may be justified [48].

These type of arguments have often provided the theoretical rationale for a number of government measures in technology and trade policy. They are based on the recognition that with trade today seeming to reflect temporary advantages resulting from shifting leads in technological races (Krugman, 1986), the capacity for innovation is critical for international competitiveness and in order to maintain world export shares. The importance of non-price factors in competitiveness (such as quality, reliability, technical flexibility and sophistication) and of imperfectly competitive environments points to a pattern of international competition through process and product innovation within which firms commit substantial resources in order to maintain a competitive advantage and countries intervene in order to support "national champions" [49].

Such arguments implicitly or explicitly recognize that a national government is a player in a multi-stage international game involving both domestic and foreign firms engaged in innovative activities. By virtue of their power, national governments can change the rules of the game, define new strategies, preempt old ones or alter the

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[48] For an elaboration of these arguments see the papers by Krugman and Brander in Krugman (ed., 1986). In the same volume, Grossman provides a powerful critique. For an analysis that deals with these issues in the context of the debate for an industrial strategy in Europe, see Pearce and Sutton (1986).

[49] Note that this rationale for government intervention in international markets does not hinge on assumptions that the process of international competitiveness is assumed to converge to a long run equilibrium. Instead, it is held that at each point in time there exist profitable opportunities for intervention by national governments in international markets in which competition is imperfect and where a number of participants are making positive profits. This is because the division of profits between competitors cannot be determined a priori on grounds of efficiency: it depends largely on their strategic interaction. For an elaboration of this argument see Justman and Teubal (1986), Pearce and Sutton (1986) and Lyons (1987).

payoffs to the other players, thus affecting the outcome of international competition through new products and processes. In this way, they can attempt to reverse the comparative advantage rooted in the cumulative effect of past technologies [50].

The question that arises in the context of the concerns of this dissertation is whether these arguments survive in an environment where research is subject to spillovers. With R&D less than fully appropriable, some of the research of the domestic firm(s) spills over to foreign rivals. This should affect the incentives to invest in R&D and the structure of international production. Since however there is no *a priori* reason to believe that international intrasectoral spillovers are of a different nature from domestic intrasectoral spillovers, firm incentives and industry performance in the international case are affected in the same way as they are in the case of domestic spillovers. If spillovers among domestic rivals acts as a disincentive for R&D, so should spillovers among international rivals.

When considering public policy however, these similarities give way to important differences. To the extent that part of the knowledge generated by an R&D subsidy is

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[50] For a concrete example assume the existence of two firms (or countries) producing two differentiated products making positive profits. Assume also that given their presence a third firm (or country) would make losses if it entered the market, whatever its product specification. Given that the identity of the firms (or countries) is undetermined, the role of a national industrial policy is taken to be to tilt the balance in favour of a domestic potential entrant into the market over one of the foreign rivals. By using subsidies that reduce costs to a domestic firm or procurement policies or tariffs which raise its potential revenues while reducing that of foreign rivals, public policy can in general change the (several) available configurations. Alternatively, a large country by blocking access to its domestic market might exclude an otherwise available situation in which a foreign producer entered. Such policies work by changing the product-development strategies of foreign producers (and only to the extent that they do so). They affect foreign producers' decisions by altering their profitable strategies.

This stylized example illustrates the possibility of using an international competitive strategy in order to enter profitably a new market or forestall entry of a foreign producer. The argument is however equally applicable in the case where a number of firms are sharing an established international market and making profits. By subsidising R&D or exports a national government can decrease the production costs of its domestic firm and thereby increase its market share and profits at the expense of rivals. Relative to export subsidies, however, an R&D subsidy has the advantage of effective subsidisation of an earliest stage of the production process (as opposed to output subsidies) and is as a consequence less likely to invite retaliation, or be illegal on the basis of international codes of acceptable behaviour (such as GATT rules).

available to a domestic firm's foreign competitor, a public policy of subsidising R&D has an important externality attached to it, not previously present when spillovers were absent. In an analogous fashion, when a foreign firm cannot appropriate part of its R&D investment, a foreign government's subsidisation of the research of its national champion cannot be "contained" within that country's national borders. This externality is fundamentally different to the one in the case of domestic competition and domestic R&D subsidies. In that case, the government's objective is to maximise domestic welfare, which involves all firms that are recipients of the subsidy. In the case of international competition, the objective of the subsidy is profit-shifting; in setting subsidy rates the government is only concerned with the interests of the domestic firm.

Attempts to model international competition through R&D in a framework where firms use R&D for strategic purposes have not until now examined the implication of research spillovers for the characteristics of the resulting equilibria and for the appropriate role of public policy. Similarly, spillovers have been exclusively modelled in a domestic framework; the implications of international spillovers for public policy have not been explored. The model developed in chapter five of this dissertation attempts to fill this gap. It addresses the question of whether or how the presence of externalities in R&D in the strategic model alters the incentives on the part of governments to subsidise R&D and how it affects more generally the characteristics of the noncooperative international equilibrium.

IV. Structure of the dissertation

The main body of the thesis consists of four chapters. The structure of chapter two is as follows. After an introductory section, section II develops an framework where *n* firms compete by setting simultaneously output and R&D expenditures. We derive the equilibrium levels of R&D expenditures and output in the presence of spillovers in basic research under the assumption that "own" and "borrowed" basic research are perfect substitutes, and that basic and development research enter as inputs into a knowledge production function of a Cobb-Douglas form. Both the case where there exist barriers to entry and where free entry results in firms earning zero profits in equilibrium are examined. We look at how the key variables at the firm or industry level (R&D levels and composition, cost reduction, profits and industry structure) vary with the degree of appropriability (the extent of spillovers), and we examine whether the main results of similar models that omit spillovers stand in the presence of inappropriability.

In section III of that chapter we turn to the case where the industry is socially managed and derive the optimal levels of output and R&D in the presence of spillovers. These levels serve as a benchmark against which the performance of the market and the incentives it provides for cost reduction through R&D are compared (section IV). In section IV, we also examine how a government policy of subsidising basic research affects the incentives to invest in the partially appropriable part of R&D. Optimal subsidy rates are derived and their determinants are examined. Section V pulls together the main conclusions of the chapter.

Chapter three focuses on the importance of the nature of technology for the impact of spillovers by adopting a knowledge production function of a constant elasticity of substitution form; this allows varying degrees of substitutability and complementarity between three research inputs: a firm's own basic research, the basic research of its rivals and its own development research. Section I discusses the approach taken. Section II solves for the equilibrium levels of R&D expenditures and output in an n-firm oligopoly both when there are barriers to entry for the case of free entry. The effect of spillovers on R&D levels, costs, output and profit under this new parameterization of the knowledge production function are examined. Section III develops the solution for the socially-managed industry and enquires on whether the socially optimal outcome in this case can involve more than one firm operating.

Section IV examines the implications of these results for the incentives of the market to conduct basic research as well as its performance relative to the socially optimal case. A government policy of subsidising basic research is also examined; it is established that its nature and effectiveness depends on the characteristics of the underlying knowledge production function. Section V presents a numerical simulation that illustrates the main results of chapters two and three. Under two alternative functional forms of the knowledge production function, we examine the response of key variables (such as the composition of R&D, R&D intensity, relative costs and market performance, etc.) to variations in the spillover rate and the degree of concentration in the industry. Finally, section VI concludes with a discussion that allows direct comparison of the main conclusions to emerge with those of chapter two.

Chapter four consists of four sections. Section I presents the rationale for developing a strategic model of R&D with spillovers where firms choose the levels of research expenditures and their output levels in a multi-stage game. Section two develops a three-stage model with spillovers in basic research in general form and compares its equilibrium with that of the one-stage (Cournot) model developed in the previous chapters. We examine the effects of different assumptions about basic and development research inappropriability on the comparative statics properties of the model and compare the resulting equilibrium with that of the non-strategic model.

An explicit solution to a two-stage model that does not distinguish between basic and development research is derived in section III on the basis of two different specifications of the knowledge production function: (i) one that assumes perfect substitutability between "own" and "borrowed" R&D; and (ii) a constant elasticity of substitution knowledge production function. These specifications allow us to solve for the case of *n* symmetric firms and compare R&D levels, output and profits in the strategic model with the corresponding levels in the non-strategic model.

Section IV summarises the conclusions of the chapter and relates these to the approach taken in chapters two and three with the help of a simple schema. The schema focuses on the two sets of factors that have been explored throughout these chapters: technology factors, relating to the industry-specific characteristics of technical advance and to the way that a firm's own R&D combines with that of its rivals to reduce costs; and factors relating to the behaviour of firms and to the precise nature of the oligopolistic game being played through R&D. The combination of these factors determine the nature of the impact of spillovers on the incentives that firms have to invest on R&D, on firm profitability, and on industrial structure and performance.

Finally, chapter five is a direct application of the strategic framework with spillovers decveloped in chapter four to international trade. A duopoly model of international competition with spillovers in R&D is developed and its equilibrium characteristics are analysed. The model is then extended to accommodate the possibility of governments subsidising the R&D of their domestic firm in a policy of "precommitment". Under these circumstances, we explore the effect of a domestic subsidy to R&D on R&D expenditures, the nature of optimal subsidies and the rate of return of such government intervention.

CHAPTER TWO

R&D SPILLOVERS AND INDUSTRY PERFORMANCE:

BASIC RESEARCH AS A PUBLIC GOOD

I. The framework

In this chapter we develop a framework that incorporates "spillover effects" of a firm's inventive activity to other firms. "Spillovers" are defined here as that part of the firm's research and development (R&D) expenditures [1] that cannot be appropriated by means of patents, market power or otherwise and which is potentially usable by other firms (perhaps at a cost to them) without any compensation to the firm that originated the research. Given however that the product of R&D --new knowledge-- has some features typical of public goods, the fact that some of the R&D expenditures spill over to other firms does not preclude the full use of the resulting R&D knowledge by the innovating firm as well.

Research spillovers can be *intra*sectoral or *inter*sectoral; some of the R&D may leak to rivals in the same industrial sector or to firms in other sectors with which the innovating firm is technologically linked. Given that the models developed in this and the following chapters are of oligopolistic rivalry, the focus is on spillovers of the first type only. There is no doubt that a comprehensive theoretical treatment of the effects of spillovers would require the inclusion of both types of spillovers; the exclusion of the inter-industry effects may be at the expense of understating the cost-reducing effect of low appropriability.

[1] What "spills out" in effect is the knowledge created as a result of expenditures in research and development; spillovers in other words operate on the *outcome* of R&D expenditures, ie the resulting process or product innovation. Given that the product of R&D --the innovation-- is often intangible, it is usually proxied by, among other things, the patent right acquired by the innovator for the protection of rent accruing from the commercialisation of the innovation. Patents however (their number, cost of acquiring or renewal rate) are at best a measure of the *private*, as opposed to the *social*, value of the innovation (on this, see Schankerman and Pakes (1986)); as such, they are an imperfect tool for the purpose of examining in a theoretical framework the effect of inappropriability of the fruits of research on firm incentives and industry performance. R&D expenditures therefore remain the best proxy available, although the use of an input as a proxy for the output of a certain production process (the generation of knowledge in this case) does abstract from the question of the *productivity* of the use of the input.

Within an industrial sector, two different types of competition between firms are recognized: competition in R&D and competition in output. R&D itself is broken down into two parts/stages: *basic* (or pure) research R, and *development* (or applied) research D. Spillovers are presumed to exist in the basic research stage whereas development research is taken to be fully appropriable [2].

The fraction of the basic research of a firm's rivals that "spills-over" and can be observed constitutes a "pool" of basic research that is available to the firm. This potential basic research spillover pool is transformed into actual "borrowed" research by taking into account the firm's "absorptive capacity". "Own" and "borrowed" basic research, together with development research, are then transformed into cost saving innovations after taking into account the firm's opportunities for cost reduction and the scientific level of the industry.

The basic sources of technological knowledge utilised by the firm therefore are: its own basic research expenditures, the knowledge originating with the spillovers from the basic research expenditures of the other firms in the same industry; and its own development research expenditures.

The structure of the chapter is as follows. In section II, we develop a model of oligopoly where n firms compete by setting simultaneously output and the level of R&D expenditures. We initially derive the equilibrium levels of basic and development research expenditures, as well as of output for the firm and the industry in the presence

[2] Traditionally, R&D is broken down into *basic research*, defined as experimental or theoretical work with the aim of acquiring new knowledge and not focused on any particular application; *applied research*, defined as original work directed primarily towards a particular practical objective; and *development*, that is systematic work drawing on existing knowledge and directed towards new processes and products (for official definitions see OECD (1981)). For our purposes, the distinction between basic research (subject to spillovers) and development research (fully appropriable) will suffice.

of spillovers in basic research. Both the case where there exist barriers to entry and where free entry results in firms earning zero profits in equilibrium are examined. The level of R&D expenditures, cost reduction, profit and industry performance are then compared to the case where spillovers are absent, and we examine whether the main results of similar models that omit spillovers stand in the presence of inappropriability. Furthermore, we look at how the key variables at the firm or industry level vary with the degree of appropriability (the extent of spillovers), as well as how sensitive they are to parameters such as the elasticity of demand for the product, the scientific base of an industry or the opportunities for cost reduction through R&D facing a firm.

In section III we turn to the case where the industry is socially managed and derive the optimal levels of output and R&D in the presence of spillovers. These levels serve as a benchmark against which the performance of the market and the incentives it provides for cost reduction through R&D are compared (section IV). In section IV, we also examine how a government policy of subsidising basic research affects the incentives to invest in the partially appropriable part of R&D. Optimal subsidy rates are derived and their determinants are examined. Section V pulls together the main conclusions.

II. A model of oligopoly with spillovers

This section analyzes an n-firm oligopoly in both product and R&D in the presence of intrasectoral spillovers. Firms are assumed to be integrated, in that they both conduct R&D and market the resulting product, identical for all firms [3]. R&D is cost-reducing only; it is like a fixed cost but its level is a decision variable for the firm [4].

The model takes as its starting point the paper by Dasgupta and Stiglitz (1980a -- henceforth denoted as D&S). In that work, a model of cost-reduction through R&D is developed with firms competing in R&D and output in an oligopolistic market. Both the case of an exogenous (because of barriers to entry) and an endogenous market structure are examined; in the latter case the zero-profit condition determines entry and industrial structure (proxied by the equilibrium number of firms). The model is symmetric, timeless and devoid of uncertainty. The possibility of learning and imitating among firms is ignored; also, R&D is taken to be homogeneous [5].

[3] Dasgupta and Stiglitz (1980a) defend the use of an identical product assumption in an oligopolistic market by supposing that preferences are defined over *characteristics* of commodities. Firms compete to produce different commodities with the same characteristics; commodities are therefore perfect substitutes in consumption and will as a result be subject to the same price.

[4] R&D is of course in principle aimed at both process and product innovations. The two are however in a sense similar as Spence (1984) points out. His argument is along the same lines as the defence of the identical good assumption. Let products deliver services s to consumers with inverted demand $P(s)$. These services are delivered through goods x with a cost function $c(x)$. Let $f(q)$ be the quantity of services per unit of good so that $s=f(q)x$; the cost of producing s is then $c[s/f(q)]$. If $f'(q)>0$ and q is raised by product-development R&D the effect is to reduce the *cost* of services. Formally therefore, this kind of product development is equivalent to cost reduction.

[5] It should also be noted that in the D&S model it is implicitly assumed that competition occurs amongst *n domestic* firms. The model could also be interpreted as one of international competition amongst *n* symmetric firms located in different countries (or a combination of domestic and foreign firms). For this we would require to assume conditions of free trade and no government intervention, leading to one world price for the product. The "industry" over which total R&D expenditures or output are defined would then cover many countries. The same interpretation can also be carried over to the model developed in this chapter, if we recognize that research spillovers amongst firms in an oligopolistic industry can be international, as well as national.

The main predictions of that model are as follows. A positive association between concentration and R&D intensity is established for industries characterised by the same elasticity of demand, but differentiated in terms of their innovative opportunities; no causality is implied however, since both are determined simultaneously. In industries differing only in the size of their output market and their underlying scientific base, firms spend the same amount of R&D relative to their size. Growth in industry demand stimulates R&D activity and industries facing greater innovative opportunities are more concentrated. Finally, the market may be characterised both by excessive expenditure on R&D and too low a rate of technical progress when compared to a socially managed industry [6].

By modifying and extending the D&S framework it is possible to allow for spillovers in the knowledge generated by one part of R&D (basic research), while allowing another part of R&D (development research) to be fully appropriable.. Let R_i represent the pool of basic research knowledge available to each firm i (i=1,...,n). It is determined by the firm's own basic research expenditures x_i and by the proportion of the basic research expenditure of each of all the other firms in the industry x_{-i} ($-i$ denotes all firms other than i), that spills over and that can be absorbed. Let y_i be the development research expenditures of firm i. The unit cost of production c_i is then taken to depend on the pool of basic research R_i and on the development research

[6] The Dasgupta and Stiglitz (1980a) model provides a framework that accounts for a number of empirical observations in the economics of innovation (Dasgupta, 1986): namely that the innovative process does not display any economies of scale with respect to the size of firms; that there is a positive association between the degree of concentration in an industry and its innovative activity, as long as concentration is not too great; that industries facing greater technological opportunities are more concentrated; that growth in demand for the products of industry stimulates R&D activity within it; that research activity is strongest in industries where entry barriers are neither too high nor too low; and that there is a positive relationship between a firm's R&D activity and its profits. The model cannot account for the observation that past R&D successes lead to greater current R&D effort on the part of successful firms and that imitation is a pervasive phenomenon that should influence the incentives to invest in R&D.

expenditure y_i and to be independent of output (constant unit production cost) [7]. R&D expenditures are therefore in this setup like a fixed cost to the firm. This fixed cost however is a choice variable and firms can spend more or less on basic and development research expenditures, with the result being reflected in a lower or higher marginal production cost.

The basic structure of the model is then given by the following equations:

$$(2.1) \qquad P = P(Q) \quad \text{where} \quad Q = \sum_{i=1}^{n} q_i$$

$$(2.2) \qquad c_i = c\,[R(x_i, x_{-i}, \delta_{,i}, \theta), y_i]$$

where $0 \le \theta_i \le 1, \quad 0 \le \delta_{,i} \le 1 \quad$ and $\quad x_{-i} = \sum_{j \neq i} x_j$

and where q_i and Q are firm and industry output respectively, with P the market price of the resulting product.

We assume that the unit cost c_i is a decreasing function of own and competitors' basic research expenditures, ie that $\partial c_i/\partial x_i < 0$ and $\partial c_i/\partial x_j < 0$ or alternatively that the underlying knowledge production function for basic research is an increasing function of own and borrowed basic research, ie that $\partial R_i/\partial x_i > 0$ and $\partial R_i/\partial x_j > 0$. On the other hand, $\partial^2 c_i/\partial x_i^2 > 0$ and $\partial^2 c_i/\partial x_j^2 > 0$ so that there are decreasing returns to both own and borrowed basic research expenditures. Basic research knowledge increases with the

[7] There is nothing in the model to distinguish development research expenditures y_i from any other cost-reducing inputs that are not R&D-related. The reader can interpret y_i as any of a number of other expenditures, that are for example labour or capital related. In such a case, x_i should be understood as representing R&D expenditures in general, with spillovers operating on the total of R&D, and the substitution possibilities between x and y should be reinterpreted as referring to substitution between R&D in general and non R&D-related inputs.

spillover rate ($\partial R_i/\partial \theta_j > 0$) and with the degree of absorptive capacity ($\partial R_i/\partial \delta_{ji} > 0$). Finally, development research expenditures reduce unit production costs ($\partial c_i/\partial y_i < 0$) and exhibit decreasing returns ($\partial^2 c_i/\partial y_i^2 > 0$).

In expression (2.2), the vector of spillover rates from all firms' basic research expenditures is given by $\theta = (\theta_1, \ldots, \theta_n)$. The degree therefore to which the basic research expenditures of any firm become potentially available for use by other firms is given by the parameter θ_j, where $0 \leq \theta_j \leq 1$. A value of zero indicates that there are no spillovers: the results of basic research are exclusively appropriated by the firm that originates it --basic research is in effect a private good. A value of one on the other hand means that spillovers are perfect: the basic research effort of one firm is entirely in the public domain and increases the potentially borrowable research pool available to other firms by the full amount of the original basic research expenditures. For $0 < \theta_j < 1$, spillovers are imperfect.

The spillover parameter may of course vary from one firm to another, depending, among other things, on the firm's capacity to retain proprietary knowledge. We will however assume that it is identical across firms in the same industry (so that $\theta_i = \theta_j = \theta$), reflecting instead exogenous factors such as patent policy as well as the industry-specific nature of the knowledge created. We also assume $\theta_i = 1$ for firm *i*, so that each firm can make full use of its own basic research.

The proportion of the "potentially usable" basic research pool that firm i actually absorbs is given in (2.2) by $\delta_{.i} = (\delta_{1i}, \ldots, \delta_{ni})$. The parameter δ_{ji} in this context indicates the proportion of the "non-appropriable" basic research from firm *j* that firm *i* can actually use. We assume that $0 \leq \delta_{ji} \leq 1$. When $\delta_{ji} = 1$, the firm absorbs all knowledge in the public domain. Alternatively, when $\delta_{ji} = 0$, it absorbs none. This parameter has in other work (Cohen and Levinthal (1989)) been interpreted as indicating imitation costs, by linking the capacity to absorb externally generated knowledge to the R&D effort that is required to maintain that capacity. In the context of this model, we will initially make no specific assumption about the allocation of investment to imitative effort, so that the vector δ_{ji} is taken to be predetermined. This assumption is later relaxed, and a firm's absorptive capacity is assumed to be a function of a firm's own basic research expenditures (ie $\delta = \delta_{.j}(x_i)$) in the sense that higher own basic research expenditures enhance the ability to imitate rivals' basic research. In any case however, we assume $\delta_{ji} = \delta_{ki} = \delta_i$ so that each firm's absorptive capacity, while specific to it, does not vary with the source of outside knowledge, while $\delta_{ii} = 1$, ie a firm can fully absorb its own basic research.

The appropriation of other firms' research is therefore achieved through the interaction of the absorptive capacity parameter δ_i and the spillover parameter θ. The firm can only potentially use the fraction of its competitors' basic research that is not appropriated. Of that fraction, it actually uses only a certain part, the part being determined by its capacity to recognize the spillovers and integrate the results of that public knowledge into its own production mechanism. In this sense, and carrying in the context of the specific model an analogy found elsewhere in economics, $\delta_i\theta \sum_{j \neq i} x_j$ represents the *effective* flow of spillovers, in contrast to the *nominal* flow, given by $\theta \sum_{j \neq i} x_j$ [8]. The firm's basic research knowledge could therefore consist only of the fruits of its own basic research expenditures *either* because other firms can fully appropriate the results of their own basic research expenditures *or* because the firm lacks the capability to use that publicly available knowledge.

[8] The distinction between *nominal* and *effective* spillovers is equivalent to that in Schankerman (1979) between *borrowable* and *borrowed* research.

Firms are assumed to aim to maximize profits net of R&D expenditures. In order to do so, they choose basic research expenditures x_i, development research expenditures y_i and output q_i simultaneously in a one-stage game. We are interested in the properties of the resulting Nash-Cournot equilibrium in output and R&D expenditures (both basic research and development research). The firm's objective function is therefore to maximize

$$(2.3) \qquad \underset{x_i, y_i, q_i}{\Pi_i} = \Pi_i(q_i, Q_{-i}, x_i, x_{-i}, \theta, \delta_{.i}, y_i) = \{P(Q) - c[R(x_i, x_{-i}, \delta_{.i}, \theta), y_i]\}q_i - x_i - y_i$$

The free-entry equilibrium is given by the combination

$$(2.4) \qquad [n^*, (q_1^*, x_1^*, y_1^*), \ldots, (q_i^*, x_i^*, y_i^*), \ldots, (q_n^*, x_n^*, y_n^*)]$$

where for each firm i:

$$(2.5) \qquad \{P(Q^*) - c[R(x_i^*, x_{-i}^*, \delta_{.i}, \theta), y_i^*]\}q_i^* - x_i^* - y_i^* >$$

$$> \left\{P\left[\sum_{j \neq i} q_j^* + q_i\right] - c[R(x_i, x_{-i}^*, \delta_{.i}, \theta), y_i]\right\} q_i - x_i - y_i$$

and

$$(2.6) \qquad \{P(Q^* + q_k) - c_k[(x_k, x_i, \delta_{.i}, \theta), y_i]\}q_k - x_k - y_k \leq 0$$

Condition (2.5) ensures that the firm's behaviour is suboptimal if at equilibrium it chooses $(x, y, q) \neq (x^*, y^*, q^*)$; the condition in other words requires that the firm's profits be lower if, when the rest of the firms in the industry are at their equilibrium output and R&D expenditures, it chooses a different combination. Expression (2.6) is the free-entry condition: when the optimal number of firms, output levels and R&D expenditures have been reached any new entrant makes negative profits at the free-entry equilibrium.

The first-order conditions for the firm's maximizing problem with respect to output, basic research and development research expenditures respectively are:

$$(2.7) \qquad P[1-\varepsilon(Q^*)s_i] = c_i[R_i(x_i^*, x_{-i}^*, \delta_{,i}, \theta), y_i^*]$$

$$(2.8) \qquad -[\partial c_i^* / \partial x_i^*] q_i^* = 1$$

$$(2.9) \qquad -[\partial c_i^* / \partial y_i^*] q_i^* = 1$$

where $\varepsilon(Q)$ is the inverse elasticity of demand and $s_i = q_i/Q$ is the individual firm's share in industry output. For the second-order conditions to hold we need decreasing returns to scale in R&D expenditures ie $\partial^2 c_i/\partial x_i^2 > 0$ and $\partial^2 c_i/\partial y_i^2 > 0$ as well as the own effects on marginal profit to exceed the cross effects ie that $\Pi_{qq}\Pi_{xx} > \Pi_{xq}^2$, $\Pi_{xx}\Pi_{yy} > \Pi_{xy}^2$ and $\Pi_{qq}\Pi_{yy} > \Pi_{qy}^2$. This we assume [9].

Inspection of the first-order conditions (2.7)-(2.9) reveals a number of properties of the equilibrium solution, noted also in Dasgupta and Stiglitz (1980a) and Levin and Reiss (1984) [10]. From (2.7), price deviates from the marginal cost of production so that the market solution is not expected to lead to an optimal allocation of resources. The deviation from optimal pricing depends on the share of each firm in industry output s_i (as the number of firms increases, $s_i \to 0$ and $P \to c_i$) and on the demand elasticity.

[9] The specific conditions for which a symmetric equilibrium ($Q^*/n, x^*, y^*$) with barriers to entry and a free-entry symmetric equilibrium (n^*, q^*, x^*, y^*) exist as a solution to the model described by the first-order conditions (2.7')-(2.10') below are examined in the appendix to this chapter. They hold for the particular parametarizations given below ie where the inverse demand function is given by (2.12) and where the (basic) knowledge production function and the associated cost function are given by (2.13) and (2.14).

[10] In those models however, given that R&D is treated as being homogeneous, the discussion is in terms of output q and an aggregate R&D variable, equivalent here to $x+y$.

Similarly, the left-hand sides of (2.8) and (2.9) indicate that private marginal benefit from basic and development research depends on the firm's own output q_i; social optimality would require industry output Q instead [11].

For the symmetric solution the Nash equilibrium is (n^*, q^*, x^*, y^*). Letting therefore $q_i=q$, $x_i=x$, $y_i=y$, $s_i=1/n^*$ and $\delta_j = \delta$, the first-order conditions now become:

$$(2.7') \qquad P(Q^*)[1-\varepsilon(Q^*)/n^*] = c[R(x^*,(n^*-1)x^*,\delta,\theta),y^*]$$

$$(2.8') \qquad -\{\partial c[R(x^*,(n^*-1)x^*,\delta,\theta),y^*]/\partial x\}(Q^*/n^*) = 1$$

$$(2.9') \qquad -\{\partial c[R(x^*,(n^*-1)x^*,\delta,\theta),y^*]/\partial y\}(Q^*/n^*) = 1$$

With symmetry, the free-entry condition (2.6) becomes:

$$(2.6') \qquad \{P(Q^*+q)-c[R(x,(n-1)x,\delta,\theta),y]\}q - x - y \leq 0$$

which implies that for the representative firm's (q^*, x^*, y^*), industry equilibrium must give non-negative profits ie

$$(2.10) \qquad \{P(Q^*)-c[R(x^*,(n^*-1)x^*,\delta,\theta),y^*]\}Q^* \geq (x^*+y^*)n^*$$

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[11] Note that the marginal return to a firm's basic research here (equation (2.8)) takes account only of the direct effect of own basic research on own profits ie $\Pi^i_{x_i}$, and not of the indirect effects of own basic research on competitors' basic research knowledge $R_j = x_j + \theta\delta_j x_{-j}$ and through that on price and own profit ie of $\theta \sum_{j \neq i} \delta_j \Pi^i_{R_j}$. This is due to the nature of decision-making of the model: firms simultaneously decide on R&D and production levels. Other writers, such as Spence (1984) or Cohen and Levinthal (1989), solving recursively an implicit two-stage game, with the decision to spend on R&D in one stage and the decision on the level of production in another, take both direct and indirect effects into account in deriving the marginal benefit of R&D expenditures. Spence (1984) in particular has interpreted the absence of the indirect effect in the firm's first-order conditions as a situation where firms imperfectly anticipate or totally ignore the effects of their own R&D investments on the costs of other firms and on industry prices; he offers this as an explanation for the empirical observation that in certain industries high spillovers may coincide with a high degree of dynamic efficiency. (This argument is taken up again in chapter three of the thesis, while a multi-stage framework is developed in chapter four).

Under the assumption that free-entry results in firms earning negligible profits at equilibrium, we can replace (2.10) by a zero-profit condition:

$$\text{(2.10')} \qquad \{P(Q^*) - c[R(x^*, (n^*-1)x^*, \delta, \theta), y^*]\}Q^* = (x^* + y^*)n^*$$

Using the first-order condition (2.7') and the zero-profit condition (2.10') we derive the following relationship:

$$\text{(2.11)} \qquad 1/n^* = [1/\varepsilon(Q^*)]\,[R^* + D^*]$$

where $1/n$ is the index of concentration in an industry and $R=nx/PQ$ and $D=ny/PQ$ are respectively industry basic research and development expenditures as a proportion of sales. R^* and D^* can therefore be interpreted as the optimal basic and development research intensities with R^*+D^* the optimal R&D intensity.

Expression (2.11) is similar to the one derived in Dasgupta and Stiglitz (1980a); in their paper R&D is treated as homogeneous so that they derive instead $1/n^* = Z^*/\varepsilon(Q^*)$ where $Z^*=n^*x^*/P(Q^*)Q^*$ stands for optimal R&D expenditures. The expression derived here implies that *without spillovers or for a given spillover rate, in a cross-section of industries with the same elasticity of demand but differing in terms of scientific base or opportunities for cost reduction through R&D, R&D intensity and the degree of concentration in an industry would be positively related*[12]. More concentrated industries would tend to be more R&D intensive and vice versa. Given however that both concentration and R&D are simultaneously determined, no causality is implied. Instead, industrial concentration and R&D intensity are both uniquely determined by a

[12] "For a given spillover rate" is an expression that discounts the effect of partial inappropriability of R&D on the interpretation of equation (2.11). Unlike in Dasgupta and Stiglitz (1980a), in this model under free entry the optimal number of firms is a function of the spillover rate ie $n^* = n^*(\theta)$. Similarly for the optimal R&D intensity so that the degree of inappropriability affects both sides of (2.11).

host of other variables, such as the demand and cost elasticities, the size of the industry, its scientific base, and, more importantly for the present model, by the degree of appropriability (the spillover rate). This result does not depend on any particular functional form of the (constant marginal) cost function.

The same expression further implies that, for given n, industries with small elasticity of demand (ie large ε) would be expected to have high R&D intensity. The reasoning here is that a small demand elasticity allows for high mark-ups so that the equilibrium is maintained by expenditures on R&D rather than by entry (with n fixed). This result reflects the role of R&D as an effective barrier to entry in the manner that Schumpeter argued.

a. Parameterization of the cost function

In order to take the analysis further it is necessary to specify the functional forms of the price and costs functions. This will allow for a sharper characterization of equilibrium and for the derivation of some comparative statics results [13].

The specification of the cost function is crucial to the analysis of equilibrium and of the effects of spillovers on R&D expenditures, given that it determines the relationship between own and "borrowed" basic research, that between basic and development research, and the one between both and cost reduction. It is for this reason that in this and the next chapter we will examine two alternative knowledge production functions and their associated cost functions embodying different assumptions about the manner in which knowledge inputs combine to reduce costs. Throughout, market demand will be assumed to be of the iso-elastic form and given by:

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[13] Rather than exhaustively covering all variables, in the derivation of the comparative statics results we focus particularly on those results that show more clearly the impact of the assumptions made about appropriability conditions.

(2.12) $P(Q) = \sigma Q^{-\varepsilon}$ $\sigma, \varepsilon > 0$

where σ indicates the size of the market.

Initially assume that the basic research knowledge production function is of the form:

(2.13) $R_i = x_i + \sum_{j \neq i} \delta_{ji} \theta_i x_j$

The associated unit cost function is of the form:

(2.14) $c_i = \beta R_i^{-\alpha} y_i^{-\gamma} = \beta\, [x_i + \sum_{j \neq i} \delta_{ji} \theta_i x_j]^{-\alpha} y_i^{-\gamma}$ where $\alpha, \beta, \gamma > 0$

In this expression $1/\beta$ is a measure of the scientific base of the industry (a lower β implies a higher scientific base) and α and γ indicate the responsiveness of cost to basic and development research expenditures. While however γ is the elasticity of cost reduction with respect to a firm's own development research expenditures, α is the elasticity of cost reduction with respect to the total pool of basic research expenditures. The cost elasticity of the firm's own basic research expenditures is then $\alpha/[1 + \delta\theta(n-1)]$ in the symmetric case. This implies that as spillovers decline (appropriability increases), a firm is better able to translate its basic research expenditures into cost-saving innovations.

This cost function exhibits certain characteristics which will be seen to be crucial for the impact of spillovers on incentives to spend on R&D and on industry performance. The underlying (basic) knowledge production function $R_i = x_i + \sum_{j \neq i} \delta_{ji} \theta_i x_j$ assumes that "own" and "borrowed" basic research are perfect substitutes (or equivalently that own and rivals' basic research can be substituted at the rate $1/\delta\theta$). It does introduce a cost to the borrowing firm for using the non-appropriable part of other firms' expenditures through the parameter δ, but that cost is independent of the amount borrowed. It can therefore be interpreted as a "search cost" which forces the borrowing firm to use less than the total basic research spillover pool. Furthermore the specification of the cost function assumes a Cobb-Douglas-type relationship between the basic research of the industry as a whole and the development research of the individual firm, so that the elasticity of substitution between the two is unity [14].

Under the functional forms assumed in expressions (2.12) and (2.14) the first-order conditions for the symmetric solution become:

(2.15) $\sigma(Q^*)^{-\varepsilon}[1-\varepsilon(Q^*)/n^*] = \beta K^{-\alpha}(x^*)^{-\alpha}(y^*)^{-\gamma}$

(2.16) $\alpha\beta K^{-\alpha-1}(x^*)^{-\alpha-1}(y^*)^{-\gamma}(Q^*/n^*) = 1$

(2.17) $\gamma\beta K^{-\alpha}(x^*)^{-\alpha}(y^*)^{-\gamma-1}(Q^*/n^*) = 1$

where $K = [1+\delta\theta(n^*-1)]$.

The variable K has a straightforward interpretation in this context. It is the ratio of the marginal *social* productivity of basic research to its marginal *private* productivity. Marginal private productivity takes into account only the effect of basic research expenditures on own marginal cost and is defined from (2.8) as $-[\partial c_i^*/\partial x_i^*]q_i^*$. Marginal social productivity on the other hand takes also into account the effect of own basic research expenditures on the costs of all other firms via the spillover rate; it is therefore defined as $-\left[(\partial c_i/\partial x_i)+\left(\sum_{j\neq i}\partial c_j/\partial x_i\right)\right]q_i$. Both of these relate to the technological (ie cost-reducing) effects of spillovers, as opposed to their pecuniary effects (ie their effects

[14] Underlying the cost function is a Cobb-Douglas production function of the form $q_i = L_i\left[\left(x_i+\sum_{j\neq i}\theta_i\delta_i x_j\right)^{-\alpha} y_i^{-\gamma}\right]/\beta$ where L_i is the level of all other inputs.

on profits). In symmetry, their ratio is given by $K = [1 + \delta\theta(n^* - 1)]$. Thus, when basic research is fully appropriable ($\theta = 0$), the two are equal. For positive spillovers, the "wedge" between social and private marginal productivity of basic research increases with the spillover rate or with the number of firms in the industry [15].

b. Oligopoly with barriers to entry

We now assume for the moment that there exist barriers to entry so that the number of firms in the industry n is exogenously given. Routine algebraic manipulations enable us then to solve for the equilibrium level of output, and of basic and development research expenditures at firm level (q^*, x^*, y^*):

$$(2.18) \qquad q^* = [\alpha^{\alpha}\, \beta^{-1}\, \gamma^{\gamma}\, \sigma^{1+\rho}\, (1 - \varepsilon/n)^{1+\rho}\, n^{-\varepsilon(1+\rho)}]^{1/[\varepsilon - \rho(1-\varepsilon)]}$$

$$(2.19) \qquad x^* = [\alpha^{\varepsilon - \gamma(1-\varepsilon)}\, \beta^{\varepsilon - 1}\, \gamma^{\gamma(1-\varepsilon)}\, \sigma\, (1 - \varepsilon/n)\, n^{-\varepsilon}]^{1/[\varepsilon - \rho(1-\varepsilon)]} \quad \cdot (1/K)$$

$$(2.20) \qquad y^* = [\alpha^{\alpha(1-\varepsilon)}\, \beta^{\varepsilon - 1}\, \gamma^{\varepsilon - \alpha(1-\varepsilon)}\, \sigma\, (1 - \varepsilon/n)\, n^{-\varepsilon}]^{1/[\varepsilon - \rho(1-\varepsilon)]}$$

where $\rho = \alpha + \gamma$.

Profit for the individual firm in equilibrium is then given by:

$$(2.21) \qquad \Pi^* = [\alpha^{\alpha(1-\varepsilon)}\beta^{\varepsilon - 1}\gamma^{\gamma(1-\varepsilon)}\sigma\, (1 - \varepsilon/n)\, n^{-\varepsilon}]^{1/[\varepsilon - \rho(1-\varepsilon)]} \quad \cdot \{\varepsilon/(n - \varepsilon) - [\gamma + (\alpha/K)]\}$$

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[15] As an illustration, in a duopoly with $\theta = 0.50$, the marginal social productivity of basic research is 1.5 times the marginal private productivity. For 8 firms, the ratio rises to 4.5. With complete spillovers on the other hand ($\theta = 1$), marginal social productivity of basic research is twice the marginal private productivity in a duopoly and eight times in an industry with eight firms.

For the comparative statics results, we assume that $\varepsilon - \rho(1-\varepsilon) > 0$ holds. This condition is satisfied for inelastic demand ($\varepsilon > 1$) or for demand not *too* elastic and is given by the second-order conditions for the existence of equilibrium (see the appendix at the end of the chapter).

A number of observations can now be made. First, and in line with Schmookler's (1966) demand-inducement theories, *R&D and profits are higher in larger markets.* Given that price is a constant mark-up on average cost [16], as output increases ($\partial q^*/\partial\sigma > 0$) with an unchanged cost function, the rise in total revenue increases profits ($\partial\Pi^*/\partial\sigma > 0$). Similarly, as output increases, basic and development research expenditures are higher ($\partial x^*/\partial\sigma > 0$ and $\partial y^*/\partial\sigma > 0$).

Secondly, *an improvement in the scientific level of the industry raises production. It increases (decreases) profits and R&D expenditures when demand is elastic (inelastic).* $\partial q^*/\partial\beta < 0$ while $\partial\Pi^*/\partial\beta > 0$ (< 0) when $\varepsilon > 1$ (< 1) or equivalently when $1/\varepsilon < 1$ (> 1). As the scientific base of the industry improves (β falls), the average cost curve shifts down. With a downward-sloping demand curve output expands. Since price is a constant mark-up on average cost, profit is a fixed fraction of total revenue and the expansion in output increases (decreases) profits when demand is elastic (inelastic). Furthermore, for constant profits (or zero profits in the free-entry solution) to be maintained, the increase (decrease) in total revenue brought about by the improvement in the scientific base when demand is elastic (inelastic) implies an increase in total costs. R&D expenditures therefore increase with $1/\beta$ when demand is elastic and fall when it is inelastic so that $\partial x^*/\partial\beta > 0$ (< 0) as $\varepsilon > 1$ (< 1) (and $\partial y^*/\partial\beta > 0$ (< 0) as $\varepsilon > 1$ (< 1)).

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[16] When the firm maximizes total costs L_i+x_i+y_i (with L_i the level of all non-R&D inputs) subject to a production function of the form given in note 13 above then $MC = AC(1+\rho)$ so that $P/AC = 1/[(1-\varepsilon/n)(1+\rho)]$ is a constant for given n.

The above results mirror those of Dasgupta and Stiglitz (1980a). In the framework of the present model however, the intensity of R&D and the firm's profitability are also influenced by the degree of appropriability of basic research, that is by the spillover rate θ. In the case where there are barriers to entry, greater spillover opportunities reduce basic research expenditures ($\partial x^*/\partial\theta < 0$), while leaving development research expenditures and output unchanged ($\partial y^*/\partial\theta = \partial q^*/\partial\theta = 0$). Because the basic research of rivals that leaks out and that can be absorbed is a perfect substitute for a firm's own, firms cut down on their own basic research expenditures as more of the basic research expenditures by their rivals becomes available to them, so that their direct unit production costs are invariant to the spillover rate ie $\partial c^*/\partial\theta = 0$. Since therefore spillovers in the basic research of rivals affect a firm's basic research expenditures only (ie reduce fixed costs), while leaving the other part of fixed costs, development research expenditures, and marginal cost unchanged, they lead to higher profits ($\partial\Pi^*/\partial\theta > 0$). We summarise these effects of spillovers in the case where there are barriers to entry as Proposition 1:

Proposition 1: **When demand and cost functions are given by (2.12) and (2.14), in the Nash-Cournot equilibrium (q^*, x^*, y^*) with barriers to entry spillovers in basic research:**

(i) reduce basic research expenditures;

(ii) leave unchanged development research expenditures, marginal cost and output;

(iii) increase profits.

It is important to emphasise at this point the precise mechanism by which a higher spillover rate reduces firms' basic research expenditures. The disincentive effect here is *not* due to the indirect effect whereby firms, perceiving that their own basic research expenditures serve to decrease their rivals' costs, respond by cutting down their own basic research expenditures. Such a behavioural sequence necessitates a multi-stage model with R&D decisions preceding output decisions. This type of model is developed in Spence (1984), Cohen and Levinthal (1989) and in chapter four below. In the case at hand, the disincentive effect is more direct and due to two factors: (i) the fact that firms perceive that their own marginal costs depend on their rivals' basic research expenditures via the spillover rate; and (ii) that own and rival basic research expenditures are perfect substitutes in (basic) knowledge production. Due to the simultaneous nature of decision making in this model (decisions on R&D and output are taken by all firms simultaneously in a one-stage game), the additional indirect disincentive effect that exists in multi-stage games is absent. It is as if firms, while cognisant of the fact that their rivals' basic research reduces their own production costs, fail to perceive (and therefore do not take into account) the fact that their own basic research expenditures enter as arguments in their rivals' marginal cost functions. This in effect makes their R&D strategies more aggressive.

While spillovers reduce the incentives at firm and industry level to invest in basic research, they also reduce the costs at the industry level of achieving a particular cost reduction (or basic knowledge generation). This is not immediately obvious from the above given that it was established that $\partial c^*/\partial\theta = 0$, ie that higher spillovers leave marginal costs unaffected. It can be seen however from the following. For a given level of basic knowledge R_i to be generated, basic research expenditures equal to $x_i + \theta\delta x_{-i}$ are needed. In a symmetric equilibrium therefore, basic research expenditures at industry level are $X = nR/K$, where $K = 1 + \delta\theta(n - 1)$. For a given level of basic research knowledge therefore (and, correspondingly, given also development research expenditures, for a given level of cost reduction), higher spillovers reduce the necessary industry-wide basic research expenditures needed to bring it about. We can therefore conclude:

Corollary 1.1: **When demand and cost functions are given by (2.12) and (2.14), spillovers in basic research reduce (ex ante) incentives for cost reduction while also reducing (ex post) the costs at industry level of achieving a given level of cost reduction.**

The impact of a firm's absorptive capacity δ on its incentives to invest in R&D depends crucially on whether it is taken to be exogenous (like the spillover rate) or not. If it is exogenous, its effect on incentives to spend on R&D mirror those of the spillover rate. A higher capacity to incorporate rivals' basic research into one's own production acts as a substitute for own expenditures on basic research ($\partial x^*/\partial\delta < 0$). It does not affect development research, marginal cost or output ($\partial y^*/\partial\delta = \partial c^*/\partial\delta = \partial q^*/\partial\delta = 0$) and therefore increases profits ($\partial\Pi^*/\partial\delta > 0$).

In the case therefore where there exist barriers to entry and the market structure is taken as a datum, high spillovers (or a high absorptive capacity) are associated with low levels of basic research at the firm and industry level. The greater the degree of inappropriability, the higher are the benefits for the individual firm and the lower are the firm and industry incentives to spend on the part of R&D that is not fully appropriable.

c. A digression: endogenous absorptive capacity

We will now digress a little in order to explore briefly whether the results obtained above and in particular the disincentive aspect of spillovers hold in the case where a firm's capacity to absorb the basic knowledge that its rivals cannot appropriate is no longer taken as given, and is instead assumed to depend on the firm's own basic research expenditures. In this case therefore, assimilation of externally available knowledge is no longer passive or costless; it depends instead on the level of own commitment to basic research expenditures.

The motivation behind the assumption that a firm's absorptive capacity should be treated as an endogenous variable in the model, depending on the firm's own basic research expenditures, can be traced to the findings of a number of empirical studies [17], as well as to casual observation of particular industries. A common finding is that while widespread spillovers are thought to exist, they do not seem to be associated with reduced incentives to spend on R&D. High R&D-intensity industries, such as electronics, which exhibit a low degree of appropriability of R&D-related expenditures, also often have a high degree of dynamic efficiency.

One possible explanation rests on the understanding that R&D (or in the context of this model basic research) has a dual role: in addition to directly helping to develop innovations, R&D may also serve as a learning tool [18]. By spending on basic research, firms learn how to better follow developments elsewhere in the industry. This means that they are in a better position to take advantage of the basic research of their rivals that spills over: for given spillovers, their absorptive capacity improves [19].

The logic of this argument leads us to the assumption that absorptive capacity be made to depend on a firm's own basic research expenditures, so that $\delta = \delta(x_i)$ [20]. Higher basic research expenditures improve this capacity ($\delta_x = \partial\delta/\partial x_i > 0$), but at a diminishing rate ($\delta_{xx} = \partial^2\delta/\partial x_i^2 < 0$). It can be shown that under this assumption the effect of spillovers on the incentives to spend on basic research is no longer clear-cut.

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[17] see, for example, Levin (1988) or Levin and Reiss (1988).

[18] see Cohen and Levinthal (1989). The argument was developed in chapter one above.

[19] Endogenising absorptive capacity is only one way of modelling the observation that high spillovers need not necessarily act as a disincentive for R&D expenditures. The idea that firms learn through own R&D so that in the presence of spillovers they might benefit from expenditures on basic research can also be modelled directly through particular assumptions about the nature of the firm's cost function. This approach is followed in chapter three below.

[20] It has been argued (Vonortas (1987)) that in addition to being a function of basic research expenditures, absorptive capacity probably depends on a number of other factors, such as the technological environment that the firm operates in, the type of industry involved, the ease of

Let E_i be the firm's earnings gross of R&D expenditures, ie $E_i \equiv \{P(Q) - c[R(x_i, x_{-i}, \delta_i, \theta), y_i]\}q_i$ and let $E^i_{R_i}$ be the firm's marginal benefit with respect to the knowledge generated by basic research expenditures. In the context of the model with exogenous absorptive capacity, $E^i_{R_i} = E^i_{x_i} = -[\partial c_i^*/\partial x_i^*]q_i^*$ from equation (2.8), or $E^i_{R_i} = E^i_{x_i} = \alpha\beta K^{-\alpha-1}(x^*)^{-\alpha-1}(y^*)^{-\gamma}(Q^*/n) = 1$ for the specific parameterization of the cost function in equation (2.14). It was then shown that $\partial x^*/\partial\theta < 0$, so that higher spillovers acted as a disincentive for expenditures on basic research. In the case where absorptive capacity is endogenous and given as $\delta = \delta(x_i)$, the marginal benefit from the knowledge generated by expenditures on basic research is given by:

$$(2.8') \qquad E^i_{R_i}(1 + \delta_x\theta x_{-i}) \equiv -[\partial c_i^*/\partial x_i^*](1 + \delta_x\theta x_{-i})q_i^*$$

where $\delta_x = \partial\delta/\partial x_i > 0$ and where $x_{-i} = \sum_{j \neq i} x_j$. For the particular parameterization of the cost function given by (2.14), expression (2.8') becomes for the symmetric case:

$$(2.16') \qquad E^i_{R_i}(1 + \delta_x\theta x_{-i}) \equiv \alpha\beta K^{-\alpha-1}(x^*)^{-\alpha-1}(y^*)^{-\gamma}(Q^*/n)K_1 = 1$$

where $\quad K = [1 + \delta(x_i)\theta(n-1)]$ and $\quad K_1 = [1 + \delta_x\theta(n-1)x^*]$

A number of observations can now be made. Comparing (2.8) and (2.8') it is apparent that the marginal benefit of cost reduction due to basic research is, *ceteris paribus*, larger in the case where absorptive capacity is a function of the firm's own basic research expenditures. This is because in addition to directly reducing costs, basic

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learning and communication between firms, the organisation of research within firms themselves, etc. Cohen and Levinthal (1989) have incorporated some of these factors by stipulating that $\delta = \delta(x_i;\beta)$, where β is a variable determined by the characteristics of the underlying knowledge that affects the ease of learning. They assume $\delta_{x\beta} > 0$ and $\delta_\beta < 0$, so that increasing β increases the marginal effect of R&D on absorptive capacity, but diminishes the level of absorptive capacity. In our case, we stick with $\delta = \delta(x_i)$, since we endogenise absorptive capacity only to investigate the effect on incentives to spend on basic research.

research expenditures now also indirectly increase the firm's ability to absorb externally available knowledge (the second term in the parenthesis of expression (2.8')). Given that costs and profits depend also on this external knowledge, the marginal benefit of investing in basic research is higher.

Related to this is the effect of spillovers on the equilibrium basic research expenditures x^* in the case where absorptive capacity is an endogenous variable. The sign of the effect is now given by [21]:

(2.22) $\text{sign } \{\partial x^*/\partial\theta\} = \text{sign } \left\{E^i_{RR}R^i_\theta[1+\delta_x\theta(n-1)x]+E^i_{R_i}\delta_x(n-1)x\right\}$

In this expression, $E^i_{RR}<0$ represents the diminishing returns to basic research, $R^i_\theta > 0$ is the marginal effect of spillovers on the flow of basic research knowledge and $E^i_{R_i}\delta_x(n-1)x$ represents the benefit to the firm from increasing its absorptive capacity.

An inspection of (2.22) reveals that in contrast to the previously derived result with exogenous absorptive capacity that increasing spillovers decreases incentives to invest in basic research, the sign of $\partial x^*/\partial\theta$ is ambiguous. The end result will depend on the relative strength of two countervailing forces: the negative incentive coming from the diminishing returns to basic research (the first term in (2.22)), and the positive adoption incentive (the second term). If diminishing returns to basic research are very strong, they may outweigh the effect due to the endogeneity of absorptive capacity and the disincentive effect of spillovers will remain. If however the extra benefits conferred

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[21] $E^i_{RR}R^i_\theta[1+\delta_x\theta(n-1)x]+E^i_{R_i}\delta_x(n-1)x$ is the result of the differentiation of the expression for the marginal benefit due to basic research when absorptive capacity is endogenous (ie $E^i_{R_i}(1+\delta_x\theta x_{-i})$) with respect to the spillover rate θ. Cohen and Levinthal (1989) show in a similar setting that expression (2.22) will hold as long as output reaction functions are downward sloping. The effect of spillovers on equilibrium basic research expenditures can therefore be deduced from the impact of spillovers on the marginal benefit due to basic research knowledge.

to the firm by the fact that an increased absorptive capacity increases its pool of available basic research knowledge outweigh the effect of diminishing returns, then spillovers will spur further investment in basic research [22].

d. Oligopoly with free entry

Going back to the case where absorptive capacity is determined outside the model and is taken as a datum by firms, when we relax the assumption that positive profits are maintained because of barriers to entry, we can solve for the equilibrium number of firms in the free-entry Nash equilibrium. From (2.21) we have [23]:

(2.23) $n^* = \varepsilon\left(\frac{1+\gamma+(\alpha/K)}{\gamma+(\alpha/K)}\right)$ where $K = [1+\delta\theta(n^*-1)]$.

The first thing to note is that when there are no spillovers (ie $\theta = 0$), K=1 and expression (2.22) becomes $n^* = \varepsilon(1+\rho)/\rho$ where $\rho = \alpha+\gamma$. This is a relationship equivalent to that derived by Dasgupta and Stiglitz for the case where R&D is homogeneous and fully appropriable. The Dasgupta and Stiglitz result therefore falls out as a special case of our model. Furthermore, for n>0 it is the case that

[22] It would be interesting to contrast our approach with the one followed in Cohen and Levinthal (1989). They ignore the effect of diminishing returns and obtain the same result of ambiguity by balancing two offsetting effects: a negative appropriability incentive and a positive absorption incentive. The latter is the same as the one here: it is based on the benefit to the firm of increasing its absorptive capacity. The former however is associated with the diminished appropriabilty of rents as spillovers increase and is based on the marginal cost to the firm of increasing R&D expenditures due to the fact that its own R&D expenditures enter as an argument in competitors' knowledge production functions R_j. This indirect effect feeds back into the firm's own earning function via market output and price, so that $E^i_{R_j} < 0$. Taking it into account implicitly assumes a two-stage procedure, as discussed further above and in chapter four.

[23] While n^* has to be an integer, the right-hand-side of (2.23) will not necessarily be one. For a free-entry equilibrium therefore we define n^* to be the largest integer not exceeding $\varepsilon[1+\gamma+(\alpha/K)]/[\gamma+(\alpha/K)]$. For details, see the appendix to this chapter.

$\varepsilon(1+\rho)/\rho \leq n^* < \varepsilon(1+\gamma)/\gamma$ (see the appendix to this chapter). The equilibrium number of firms therefore derived in (2.23) corresponds to that derived by Dasgupta and Stiglitz only when spillovers are zero. If spillovers are positive, *the equilibrium number of firms that a free-entry oligopoly can sustain in the presence of spillovers is greater than the free-entry equilibrium number of firms when spillovers are absent.*

If therefore freedom of entry results in the number of firms n to be sufficiently large for profits to be zero, the resulting number of firms in equilibrium will be a function of the spillover rate so that $n^* = n^*(\theta)$. Furthermore, $dn^*/d\theta > 0$, so that *greater spillover opportunities increase the number of firms in equilibrium* [24]. This is because higher spillovers reduce basic research expenditures at the level of the firm and lead to positive profits. For the free-entry equilibrium to be maintained, the number of firms in the industry increases through entry. Other things being equal therefore, the degree of industrial concentration should be expected to be higher in industries where the degree of appropriability of basic research is high (ie where θ is low). We summarise this result in the following proposition:

Proposition 2: **When demand and cost functions are given by (2.12) and (2.14), the free-entry Nash-Cournot equilibrium (n^*, q^*, x^*, y^*) with imperfectly appropriable basic research is characterised by a less concentrated industry than the same equilibrium with perfect appropriability. Furthermore, the degree of industrial concentration varies directly with the degree of appropriability (ie indirectly with the spillover rate).**

[24] The derivation of this result requires some explanation. Given that $K = [1+\delta\theta(n^*-1)]$, expression (2.23) for the optimal number of firms n^* can be rewritten as $[\varepsilon-\rho(n^*-\varepsilon)]+\delta\theta(n^*-1)[\varepsilon-\gamma(n^*-\varepsilon)]=0$. By totally differentiating this expression we obtain that $\text{sign}\{dn^*/d\theta\} = \text{sign}\{\delta(n-1)[\varepsilon-\gamma(n^*-\varepsilon)]/[(\rho-\varepsilon\delta\theta)+\gamma\delta\theta[(n^*-1)+(n^*-\varepsilon)]]\}$. Since $[\varepsilon-\gamma(n^*-\varepsilon)]>0$ by the conditions for the existence of the free-entry equilibrium derived in the appendix, $\text{sign}\{dn^*/d\theta\} = \text{sign}\{(\rho-\varepsilon\delta\theta)+\gamma\delta\theta[(n^*-1)+(n^*-\varepsilon)]\}$. For $dn^*/d\theta > 0$, it is then necessary and sufficient that $n^* > (1/2)\{\varepsilon(1+\gamma)/\gamma+1-(\rho/\gamma\delta\theta)\}$. From the condition $n^* > 0$ for the existence of equilibrium however we have $n^* < \varepsilon(1+\gamma)/\gamma$. Simple manipulation then yields that $dn^*/d\theta > 0$ for $\theta > -\{\rho/\delta[\varepsilon-\gamma(1-\varepsilon)]\}$. For a positive spillover rate therefore, $dn^*/d\theta > 0$.

The degree of concentration $1/n^*$ is not necessarily the best index of the extent of monopoly power in the industry. Dasgupta and Stiglitz in fact show that in their model the equilibrium can sustain simultaneously a large number of firms and high price margins. The same result can be generated here when $\theta=0$, so that $n^* = \varepsilon(1+\rho)/\rho$ and $P(Q^*)/c[x^*,(n^*-1)x^*,y^*] = 1+\rho$. It would then be possible for an industry to exhibit both low concentration (high n^*) and high price margins if demand was highly inelastic (large ε), leading to large n^*, and if simultaneously firms had significant opportunities for cost reduction through R&D (large ρ), leading to high P/c ratios.

In the case where spillovers are positive however, the ratio of price to production cost is given by $P(Q^*)/c[x^*,(n-1)x^*,y^*] = 1/[1-\varepsilon/n^*(\theta)]$. For a given spillover rate therefore and for given elasticities of cost reduction with respect to R&D, a highly inelastic demand will lead to high n^* and low price margins. Furthermore, since $dn^*/d\theta > 0$ it follows that $d(P/c)/d\theta < 0$. Higher spillovers reduce price-cost margins and thereby promote allocative efficiency [25]. In the context of the free-entry equilibrium derived above (based on the functional forms given by (2.12) and (2.14) and following on Proposition 2, we can therefore state:

Corollary 2.1: **In a cross-section of industries facing the same elasticity of demand and with the same opportunities of cost reduction through R&D, but differing in terms of the degree of appropriability of basic research, one would expect to observe that the industries that exhibit the least appropriability of basic research would be the ones least concentrated and with the weakest monopoly power.**

[25] While the sign of $d(P/c)/d\theta$ is unambiguously negative, the signs of $dP/d\theta$ and $dc/d\theta$ depend on the signs of expressions $\{-\varepsilon+\rho[(n^*-\varepsilon)-\varepsilon]\}$ and $\{n^*-1-\varepsilon\}$ respectively. For $n^*-1-\varepsilon>0$ spillovers may increase or decrease prices, the net effect depending on the resolution of entry on industry output. Marginal production cost increases however, due to the cutbacks in R&D expenditures, and sufficiently so for the reduction in price-cost margins. If on the other hand $n^*-1-\varepsilon<0$, marginal costs decline, while industry output rises and prices fall sufficiently for a reduction in price-cost margins.

In the situation where barriers to entry allow firms to maintain positive profits, we showed that for given n spillovers reduce basic research expenditures at firm and industry level, while leaving development research expenditures, marginal cost and output unchanged. In contrast, when free entry pushes profits to zero, spillovers affect all variables at firm and industry level. Under the assumed functional forms $dx^*/d\theta < 0$ and at the same time $dX^*/d\theta = d(n^*x^*)/d\theta < 0$. Spillovers induce a reduction in firm basic research expenditures at the free-entry equilibrium. Furthermore, despite the fact that with higher spillovers the free-entry equilibrium can sustain a larger number of firms, so that the number of firms undertaking basic research increases, industry-wide basic research expenditures suffer as well [26]. In a free-entry oligopoly therefore *inappropriability of basic research acts as a disincentive for basic research expenditures at both the firm and industry-wide level.*

The effect of inappropriability of basic research on free-entry development research expenditures at firm and industry level is less clear cut. The sign of $dy^*/d\theta$ and $dY^*/d\theta = d(n^*y^*)/d\theta$ depends on the sign of the expressions $1+\varepsilon-n^*$ and $\{\varepsilon-\rho(1-\varepsilon)(n^*-\varepsilon)\}$. For $n^* > 1+\varepsilon$ development research expenditures at firm level decline; industry-wide however, the total effect may be carried by the entry of new firms. Conversely, for $n^* < 1+\varepsilon$ both firm and industry development research expenditures increase in response to a higher spillover rate. Similarly, for total (ie basic and development) R&D expenditures at firm level, the effect of spillovers will be determined by the sign of the expression $\{-n^*[\varepsilon-\rho(1-\varepsilon)]-\varepsilon(n^*-1-\varepsilon)\}$. For

[26] The sign of $d(x^*)/d\theta$ and $d(X^*)/d\theta$ is given by the sign of expressions $\{-n^*[\varepsilon-\rho(1-\varepsilon)]-(n^*-1-\varepsilon)[\varepsilon-\gamma(n^*-\varepsilon)]\}$ and $\{-\gamma n^*(n^*-\varepsilon)[\varepsilon-\rho(1-\varepsilon)]-\varepsilon[(n^*-1)-\rho(1-\varepsilon)][\varepsilon-\gamma(n^*-\varepsilon)]\}$ respectively.

industry-wide R&D, the sign of $d(X^* + Y^*)/d\theta$ is given by the sign of the expression $\{-[(n^* - 1) - \rho(1-\varepsilon)]\}$. For $n^* \geq 1+\varepsilon$ total R&D at firm and industry level decrease [27]. We can therefore summarise these results in the following proposition:

Proposition 3: **When demand and cost functions are given by (2.12) and (2.14), in the free-entry Nash-Cournot equilibrium (n^*, q^*, x^*, y^*) spillovers in basic research:**

(i) reduce firm and industry basic research expenditures;

(ii) reduce firm development research expenditures (for $n^* \geq 1+\varepsilon$);

(iii) reduce firm and industry total R&D expenditures;

For $n^* < 1+\varepsilon$, spillovers increase firm and industry development research expenditures.

A related question is the effect of spillovers on the R&D intensity in a certain industry. We defined R&D intensity as the fraction of an industry's combined basic and development research expenditures in total sales, ie $R^*+D^*=n^*(x^*+y^*)/P(Q^*)Q^*$. In the absence of spillovers, $R^* + D^* = \rho/(1+\rho)$. Given the Cobb-Douglas production function, expenditures in R&D are a constant fraction of total cost. With free entry and zero profits, total costs equal total revenues and R&D expenditures are therefore also a fixed fraction of total revenue. That fraction is determined only by the elasticity of cost reduction with respect to total R&D (which can be interpreted as a firm's technological

[27] In order to be able to sign the results, we have to sign the expression $n^* - 1 - \varepsilon$. In the appendix to this chapter we show that $\varepsilon(1+\rho)/\rho \leq n^* \leq \varepsilon(1+\gamma)/\gamma$. From this result and equation (2.23) for optimal n^* it can be established that for $n^* < 1+\varepsilon$ to hold, we need demand to be inelastic ($\varepsilon > 1$) and in addition $\rho > \varepsilon > \gamma$. This implies that $\varepsilon/\rho < 1$. For $n^* \geq 1+\varepsilon$ on the other hand, we need $\varepsilon \geq \rho > \gamma$. This is satisfied for an elastic or inelastic demand. The question that arises here is what values of ε and ρ are reasonable for the existence of a free-entry symmetric Nash equilibrium where firms earn negligible profits. Since we established $n^* \geq \varepsilon(1+\rho)/\rho$ ($n^* = \varepsilon(1+\rho)/\rho$ for $\theta = 0$), n^* will be an integer at least as large as the smallest integer exceeding $\varepsilon(1+\rho)/\rho$. This implies that for an equilibrium number of firms "large" enough for zero profits to forestall entry, the smallest integer exceeding $\varepsilon(1+\rho)/\rho$ needs to be "large". This will be the case if ε/ρ is "large", which seems to rule out $\varepsilon/\rho < 1$, a condition necessary for $n^* < 1+\varepsilon$ to hold. We therefore interpret $n^* \geq 1+\varepsilon$ to hold as a general case.

opportunity), and is independent of the size of the market and of the scientific base of the industry. When a part of basic research expenditures cannot be appropriated however, R&D intensity also depends on the spillover rate. $R^* + D^* = \varepsilon / n^*(\theta)$ and $d(R^* + D^*)/d\theta = -[\varepsilon / n^2(\theta)]\,(dn^*/d\theta) < 0$ so that when the number of firms is large enough for firms to earn zero profits in the free-entry equilibrium, optimal R&D intensity is negatively related to the spillover rate. For the free-entry solution therefore:

Corollary 3.1: **Inappropriability in basic research is associated with less R&D intensive industries, with the level of R&D intensity varying inversely with the spillover rate.**

Given the spillover rate however, it is still the case that industries with high elasticities of cost reduction with respect to basic and development research expenditures will have higher R&D intensities and be more concentrated. The higher these technological opportunities are the greater will the difference be between overall average and marginal costs, since $MC = AC/(1+\rho)$. This implies a low mark-up of price on total AC and consequently low profits when n is fixed. When n is allowed to vary, this leads to exit and a more concentrated industry.

In the context of this model, spillovers also affect the relative importance of basic versus development research expenditures. From (2.18) and (2.19) we have:

(2.24) $\quad x^*/y^* = (\alpha/\gamma)\,(1/K) \quad$ where $\quad K = [1 + \delta\theta(n^* - 1)]$

In the absence of spillovers, the ratio of basic to development research depends exclusively on the relative elasticities of cost reduction so that for θ=0, K=1 and $x^*/y^* = (\alpha/\gamma)$. If the elasticity of cost reduction with respect to development research is higher than that for basic research (given the nature of the knowledge generated by the latter), a firm would tend to spend more on development research than on basic research. The ratio would also be independent of the number of firms in the industry.

It is not surprising that the introduction of a non-appropriable part of basic research makes the ratio x^*/y^* also a function of the spillover rate. From (2.24) we can see that, ceteris paribus, the ratio is lower in industries where part of R&D is inappropriable. Furthermore, *industries with higher spillover rates will be spending less on basic research as a proportion of total R&D expenditures than industries where spillovers are low.* Spillovers therefore induce a shift away from pure or generic research whose outcome is not fully appropriable towards more product-specific and proprietary development research. This result may go some way in explaining why industries with the same opportunities for cost reduction may have differential ratios of basic to development research expenditures. We can therefore state:

Corollary 3.2: Inappropriability in basic research is associated with a shift from partially appropriable basic research to fully appropriable development research.

In the case where there are barriers to entry into an industry, we established that the proportion of total R&D that is accounted for by basic research is also a function of the number of firms in the industry. Given the elasticities and the spillover rate, as n increases, the ratio of basic to development research expenditures and the fraction of total R&D expenditures accounted for by basic research decline. In the case where free-entry results in firms earning zero profits in equilibrium however, the optimal share of basic research in total R&D is uniquely determined by the elasticities of cost reduction and by the degree of appropriability of basic research.

III. The socially-managed industry

We now turn to the case of the socially-managed industry. The purpose here is to derive the equilibrium levels of output and of resources devoted to R&D in an industry organised by a social planner but where part of basic research is not appropriated by the originating firm. These levels would be optima from the social point of view and would enable us to establish a "benchmark" against which the efficiency of the market could be compared.

We assume that the social planner solves her maximisation problem in two steps. In the first stage the socially optimal number of firms is chosen, and in the second stage the planner chooses the levels of R&D and output that will maximise welfare [28]. The problem is solved recursively, with industrial structure taken as given for the solution of the second stage. Given n firms for the moment therefore, at the second stage the social planner is assumed to organize production so as to maximize net consumer surplus, which is given by:

$$(2.25) \qquad W = U(Q) - \sum_{i=1}^{n} c_i q_i - \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} y_i$$

where $U(Q)$ is the total benefit from the consumption of output Q (the area under the demand curve). The first-order conditions then are:

$$(2.26) \qquad \partial U(Q)/\partial q_i = c\left[\left(x_i, \sum_{j \neq i} x_j\right), y_i\right]$$

[28] This two-step procedure serves two purposes. First, it allows a comparison of the marginal benefits due to basic research in the case of an n-firm oligopoly with that of a socially-optimal industry composed of n firms in the presence of external effects. Secondly, it allows for the examination of the possibility that the socially optimal number of firms exceeds one. While under the functional forms assumed in this chapter it will be established that n_s=1, this procedure allows direct comparison of the results here with those in chapter three, where we define circumstances that give n_s>1.

(2.27) $$-\left[(\partial c_i/\partial x_i)+\left(\sum_{j\neq i}\partial c_j/\partial x_i\right)\right]q_i=1$$

(2.28) $$-[\partial c_i/\partial y_i]q_i=1$$

With symmetry, these conditions become:

(2.26') $$U'(Q)=c\{[x,(n-1)x],y\}$$

(2.27') $$-[(\partial c/\partial x)+(n-1)(\partial c/\partial x)](Q/n)=1$$

(2.28') $$-[\partial c/\partial y](Q/n)=1$$

Expression (2.26') indicates that price equals the marginal cost of production in equilibrium, thus giving us a quasi-competitive solution. In this particular case, marginal and average (variable) production costs are the same. The price therefore does not cover the firm's R&D expenditures and they would need to be fully subsidized.

In the socially-managed industry, the expression for the marginal social benefit from basic research expenditures takes into account the externality generated through spillovers. Comparing therefore (2.27') with the corresponding expression (2.8') in the oligopolistic case we can see that the Nash equilibrium is missing the term $-[(n-1)/n]Q(\partial c/\partial x)$ ie the external effect of the increase in one firm's basic research on the n-1 other firms. Since $\partial c/\partial x<0$, the term is positive and the private marginal benefit of basic research expenditures understates the social one. Ceteris paribus therefore, *each firm in an oligopolistic market tends to underinvest in basic research relative to the social optimum.*

We can now assume explicit forms for the utility and cost functions so as to obtain a sharper characterization of equilibrium. The utility function will be assumed to be of the form:

(2.29') $\quad U(Q) = [1/(1-\varepsilon)]\sigma Q^{1-\varepsilon} \qquad$ so that $\quad U'(Q) = \sigma Q^{-\varepsilon}$

For the cost function, in this and the next chapter we will examine the two specifications that are used in the case of oligopoly. In both cases we will assume that cost and demand elasticities are independent of the market structure [29]. Assume therefore that the cost function is given by (2.14). Under symmetry, in the socially optimal case unit production costs are given by (with the subscript $_s$ denoting the solution in the socially-managed industry):

(2.30') $\quad c = \beta\,([1+\delta\theta(n-1)]x_s)^{-\alpha}(y_s)^{-\gamma}$

and the first-order conditions are then:

(2.31) $\quad \sigma(Q_s)^{-\varepsilon} = \beta\, K^{-\alpha}\,(x_s)^{-\alpha}(y_s)^{-\gamma}$

(2.32) $\quad \alpha\beta K^{-\alpha}(x_s)^{-1-\alpha}(y_s)^{-\gamma}(Q_s/n) = 1$

(2.33) $\quad \gamma\beta K^{-\alpha}(x_s)^{-\alpha}(y_s)^{-1-\gamma}(Q_s/n) = 1$

where $K = 1 + \delta\theta(n-1)$.

For the second-order conditions to hold, we need to assume strict concavity of the welfare function in R&D expenditures. This is satisfied for $\varepsilon - \rho(1-\varepsilon) > 0$. The equilibrium values of basic research expenditures, development research expenditures and output when the social planner takes n as given then are:

[29] The assumption that cost and demand elasticities do not differ in the two market structures facilitates the assessment of the performance of the market vis-a-vis the socially optimal case (where we have to restrict $\varepsilon < 1$ for $W^* > 0$. It can of course be argued that competitive pressure makes the market more efficient and that this should be reflected in a higher elasticity of cost reduction with respect to R&D. If this is indeed the case, the derived performance of the market may understate its true potential.

(2.34) $\quad q_s = [\sigma^{1+\rho}\,\beta^{-1}\,\alpha^{\alpha}\,\gamma^{\gamma}\,K^{\alpha}\,n^{-\varepsilon(1+\rho)}]^{1/[\varepsilon-\rho(1-\varepsilon)]}$

(2.35) $\quad x_s = [\sigma\beta^{\varepsilon-1}\,\alpha^{\varepsilon-\gamma(1-\varepsilon)}\,\gamma^{\gamma(1-\varepsilon)}\,K^{\alpha(1-\varepsilon)}\,n^{-\varepsilon}]^{1/[\varepsilon-\rho(1-\varepsilon)]}$

(2.36) $\quad y_s = [\sigma\beta^{\varepsilon-1}\,\alpha^{\alpha(1-\varepsilon)}\,\gamma^{\varepsilon-\alpha(1-\varepsilon)}\,K^{\alpha(1-\varepsilon)}\,n^{-\varepsilon}]^{1/[\varepsilon-\rho(1-\varepsilon)]}$

From (2.34)-(2.36) it follows immediately that output, as well as basic and development research expenditures are all larger in industries that face larger markets ($\partial q_s/\partial\sigma > 0$, $\partial x_s/\partial\sigma > 0$ and $\partial y_s/\partial\sigma > 0$). Furthermore, output is smaller in industries characterized by a lower scientific base (more costly technology or higher β) so that $\partial q_s/\partial\beta < 0$, while R&D expenditures are less in such circumstances only if demand is elastic ($\partial x_s/\partial\beta < 0$ and $\partial y_s/\partial\beta > 0$ if $\varepsilon < 1$).

The impact of spillovers on the socially optimal levels of R&D expenditures is the following. If n is taken as predetermined for the moment $\partial x_s/\partial\theta > 0$ and $\partial y_s/\partial\theta > 0$ for $\varepsilon < 1$. Higher spillovers increase the basic and development research expenditures undertaken at firm level in equilibrium if demand is elastic ($\varepsilon < 1$). Furthermore $\partial Q^*/\partial\theta > 0$ so that higher spillovers increase output. Finally, with optimum welfare given by $W^* = nx_s[\varepsilon - \rho(1-\varepsilon)]/[\alpha(1-\varepsilon)]$, $\partial W^*/\partial\theta > 0$ so that, if the social planner takes the industry structure as given, *higher spillovers improve welfare.* We can summarise these results in the following proposition:

Proposition 4: **When demand and cost functions are given by (2.12) and (2.14), in the solution (q_s, x_s, y_s) for a socially-managed industry composed of n firms, spillovers in basic research:**
- **(i) increase basic and development research expenditures**
- **(ii) increase output;**
- **(iii) improve welfare.**

With the equilibrium values for R&D and output given in (2.34)-(2.36), the social planner can now determine the number of firms in equilibrium that would maximise net consumer surplus. From $W^* = nx_s[\varepsilon - \rho(1-\varepsilon)]/[\alpha(1-\varepsilon)]$ we can see that for the functional form of the cost function given in (2.14) and for the admissible values of the spillover rate and of absorptive capacity (ie for $0 \leq \theta \leq 1$ and $0 \leq \delta \leq 1$), we have $\partial W^*/\partial n < 0$. This implies that in the socially-managed industry welfare is maximised with at most one firm operating.

This result is not surprising. With own and rival basic research being perfect substitutes, the operation of any number of firms exceeding one is wasteful from the social point of view. The externality associated with spillovers can then be internalised by allowing the operation of one firm only [30]. In that case firm and industry variables coincide and the expressions in (2.34)-(2.36) give the socially optimal levels for (Q_s, X_s, Y_s) for $\theta = 0$, K=1 and n_s=1. These values are in effect those that would have arisen if the social planner had from the start maximised a welfare function of the form $W=U(Q)-cQ-X-Y$, where $P = \sigma Q^{-\varepsilon}$ and $c = \beta X^{-\alpha} Y^{-\gamma}$. We therefore have:

$$(2.34') \qquad Q_s = [\sigma^{1+\rho}\, \beta^{-1}\, \alpha^{\alpha}\, \gamma^{\gamma}]^{1/[\varepsilon-\rho(1-\varepsilon)]}$$

$$(2.35') \qquad X_s = [\sigma\beta^{\varepsilon-1}\, \alpha^{\varepsilon-\gamma(1-\varepsilon)}\, \gamma^{\gamma(1-\varepsilon)}]^{1/[\varepsilon-\rho(1-\varepsilon)]}$$

$$(2.36') \qquad Y_s = [\sigma\beta^{\varepsilon-1}\, \alpha^{\alpha(1-\varepsilon)}\, \gamma^{\varepsilon-\alpha(1-\varepsilon)}]^{1/[\varepsilon-\rho(1-\varepsilon)]}$$

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[30] Hartwick (1984), following a different procedure, derives a socially optimal outcome with external effects where more than one firm is operating. He maximizes net consumer surplus given n firms and attempts to capture the externality in R&D by allowing for differential cost elasticities with respect to "own" and to "borrowed" R&D in his unit production cost function $c(x,X) = \beta X^{-\gamma} x^{-\alpha}$ where X=(n-1)x. His specification therefore assumes a degree of complementarity between a firm's own R&D and that of its rivals. Furthermore, depending on the choice of values for the parameters, the externality generated in R&D can benefit other firms more than the originating firm (in effect, the values chosen for his simulations exhibit increasing returns to scale to R&D expenditures; in addition, the elasticity of cost reduction with respect to rivals' R&D is greater than that with respect to own R&D). In the socially optimal case, where the external effect is accounted for in the maximand, this will result in the optimal number of firms being greater than one. Hartwick's result is reconciled with ours when the cost function is assumed to be of the nested CES form (see the next chapter).

IV. Market performance and government policy

Sections II and III addressed the question of how the equilibrium characteristics of a free-entry oligopoly and of a socially-managed industry are affected by the existence of a non-appropriable part in basic research. We derived results that showed that the impact of spillovers depends to a large extent on the assumptions about how R&D inputs combine to produce cost-saving innovations. In this section, we examine the implications of these results for the incentives of the market to conduct basic research as well as its performance relative to the socially optimal case [31]. We also examine the desirability and implications of a government policy that subsidises basic research.

a. Relative market incentives and performance

The specification of the cost function adopted in this chapter assumed that a firm's own basic research is a perfect substitute for the basic research of its rivals while total basic research and development research are substitutable with an elasticity of one. We want to compare basic research expenditures and total R&D performed in the free-entry oligopolistic market with the socially optimal case under these assumptions. From (2.19) and (2.35') we see that $x^*/X_s = [(n^*)^{-\epsilon}(1-\epsilon/n^*)]^{1/A}(1/K)$. For $\theta = 0$, $n^* = \epsilon(1+\rho)/\rho$ and $x^*/X_s = (n^*)^{-\epsilon}/(1+\rho)$ (since n_s=1, x_s=X_s). It follows that $x^* < X_s$ so

[31] We noted in the beginning of the chapter that the reader could interpret the research spillovers in this model as being international, rather than national, with knowledge generated by one firm benefiting its rivals in other countries. This would make the model one of international oligopolistic rivalry, with firms located in different countries but selling their product at a world price. Under this interpretation, the welfare maximisation problem in section III above that delivers the socially optimal levels of R&D and production has to be understood as referring to *global* welfare, in the sense that it covers all consumers and producers of the product, wherever they are located. Similarly by "the government" subsidising R&D now one has to understand some supra-national agency that attempts to stimulate industry R&D without discriminating between firms. This type of subsidy is of course very different from the profit-shifting subsidies in strategic trade, where governments attempt to assist national firms to capture a greater slice of world profits. A model along those lines is developed in chapter five below.

that *with full appropriability, each firm in the free-entry oligopoly spends* less on basic research than is socially optimal. The result is reinforced when part of basic research is non-appropriable since with the cost function under consideration, higher spillovers act as a disincentive to basic research expenditures at firm level. With $\theta > 0$ therefore, n^* is given by expression (2.23) and $x^* < X_s$ as $[n^{*-\varepsilon}]^{1/A} < [1+\gamma+(\alpha/K)]^{1/A}K$. Similarly, $x^*+y^* < X_s+Y_s$ so that individual firms in the market case underinvest in total R&D as well.

In the market economy however, total basic research expenditures are $X^*=n^*x^*$ and we are primarily interested in X^*/X_s. When basic research is fully appropriable, $X^*/X_s = (n^*)^{\rho(\varepsilon-1)}/(1+\rho)$. It is therefore possible, if ε is large (ie if demand is highly inelastic) for $n^*x^* > x_s$. This is the basis for the conclusion arrived at by Dasgupta and Stiglitz (1980a) that the market economy may be characterized by excessive expenditure on R&D (n^*x^*), while simultaneously having too low a rate of technical progress (x^*). The result that the market may encourage excessive duplication is derived in their model with R&D treated as homogeneous and fully appropriable.

When we allow for the existence of spillovers in basic research, we have that $X^*/X_s = [(n^*)^{\rho(\varepsilon-1)-1}(n^*-\varepsilon)]^{1/A}(1/K)$. It is still the case that we can have $X^*>X_s$ or $X^*<X_s$ but the ratio declines as the degree of appropriability falls (ie as θ rises). Furthermore, $X^*+Y^* > X_s+Y_s$ or $X^*+Y^* < X_s+Y_s$ so that the market in the presence of spillovers may be spending too much on total R&D as well. Excessive duplication of basic research effort and of total R&D in a market economy is therefore still possible in the presence of spillovers, but duplication will be less if the degree of appropriability is low. Put another way, the range of the elasticity of demand for which the market wastes resources on R&D is more restricted if spillovers are widespread. In summary:

Proposition 5: **When demand and cost functions are given by (2.12) and (2.14), in a free-entry oligopoly with spillovers in basic research each firm underinvests in basic research and in total R&D compared with social optimality. The market as a whole however may be spending too much on R&D, although this is less likely if the degree of appropriability of basic research is low.**

A related question concerns the performance of the market relative to the socially optimal case in the presence of spillovers. This can be represented by the ratio TS/W^*. TS is the total surplus achieved in the market, and is obtained as the addition of the consumer surplus $B(Q) - Q\,B'(Q)$ (where $B(Q)$ is the total benefit obtained by the consumption of output Q) and the producer surplus $n\Pi$ so that $TS^* = B(Q^*) - c^*Q^* - n^*x^* - n^*y^*$. Maximum welfare is given by $W^* = U(Q_s) - c_sQ_s - X_s - Y_s$ with one firm operating. We assess first the relative performance of the market when there are barriers to entry so that the number of firms n is exogenously given. From the equilibrium values (q^*, x^*, y^*) and (Q_s, X_s, Y_s) derived in the market and in the socially optimal case respectively, and for the form of the cost function assumed in (2.14):

$$(2.37)\quad TS^*(n)/W^* = (1/A)\,[(1-\varepsilon/n)n^{\rho(\varepsilon-1)}]^{(1/A)} \cdot [\varepsilon(n+1-\varepsilon)/(n-\varepsilon) - (1-\varepsilon)(\gamma + (\alpha/K))]$$

If the industrial structure is taken as a datum for the moment, so that n is exogenously determined, a comparison of the case where basic research is fully appropriable to the case with spillovers shows that $TS^*_{\theta>0}/W^* > TS^*_{\theta=0}/W^*$. Spillovers reduce basic research expenditures and lead to higher profits. When the number of firms is given, this implies a higher producer surplus. Given that the socially optimal outcome in this case involves one firm only, $W^*_{\theta=0} = W^*_{\theta>0}$ and therefore the ratio TS^*/W^* increases. Furthermore, $d(TS^*/W^*)/d\theta > 0$ so that a higher spillover rate improves the performance of the market.

When there are no barriers to entry and the number of firms in equilibrium is large enough for the zero profit condition to hold, the entire surplus generated in the market is consumer surplus. The ratio TS^*/W^* is therefore given by:

$$(2.38) \quad TS^*(n^*)/W^* = (\varepsilon/A)\,[(1-\varepsilon/n^*)^{(1-\varepsilon)(1+\rho)}(n^*)^{\rho(\varepsilon-1)}]^{(1/A)}$$

and it is possible to have $\partial(TS^*/W^*)/\partial\theta > 0$ or $\partial(TS^*/W^*)/\partial\theta < 0$. More specifically, the sign of $\partial(TS^*/W^*)/\partial\theta$ is determined by the sign of the expression $\{\varepsilon(1+\rho)-\rho(n^*-\varepsilon)\}$. Given that the number of firms is a function of the spillover rate and $dn^*/d\theta > 0$, a higher spillover rate is associated with a less concentrated industry. As the spillover rate increases, total R&D expenditures at firm and industry level decline. Unit production costs therefore increase. With entry, industry-wide output may rise or fall. If total output produced falls, total surplus falls and a higher spillover rate can be associated with a lower TS^*/W^* ratio.

Proposition 6: **When demand and cost functions are given by (2.12) and (2.14), in an oligopoly with barriers to entry, spillovers in basic research improve market performance. In a free-entry oligopoly inappropriability improves performance only when spillovers result in higher industry output.**

b. R&D subsidies

The preceding discussion has shown that in the context of the specific model, spillovers act as a disincentive for firms' investments in basic research and in total R&D. An obvious question is whether the government should implement a policy to try and counter this negative effect and what precise form this policy should take. One possible avenue is to attempt to reduce the incidence of spillovers through, for instance, more far-reaching legislation on the right to patent basic research principles. Such a strategy however ignores the positive externality associated with inappropriability of part of R&D: spillovers, by reducing R&D costs for a given level of cost reduction, have the potential to improve the market's performance.

An alternative strategy is for the government to institute a policy which would counter the negative externality associated with spillovers, while preserving the spillover environment. In its simplest form, such a policy would involve a subsidy on basic research. This would have the effect of lowering the cost of basic research for individual firms and therefore to encourage investment in this type of R&D.

With a per-unit subsidy on basic research, denoted s, the firm's profit-maximising problem would now be, instead of (2.3):

$$\text{(2.3')} \qquad \underset{x_i, y_i, q_i}{\Pi_i} = \{P(Q) - c[R(x_i, x_{-i}, \delta_{-i}, \theta), y_i]\}q_i - (1-s)x_i - y_i$$

For the functional forms for demand and cost functions given in (2.12) and (2.14), the corresponding first-order conditions for the symmetric solution are the same as for the situation where firms bear the full cost of basic research (2.15)-(2.17), except for the condition that equalises marginal benefit with marginal cost in basic research, which is replaced by:

$$\text{(2.16')} \qquad \alpha\beta K^{-\alpha-1}(x^*)^{-\alpha-1}(y^*)^{-\gamma}(Q^*/n^*) = 1 - s$$

For the situation where there exist barriers to entry so that the number of firms n can be taken as given, the new system of first-order conditions (2.15), (2.16') and (2.17) yields the following solutions:

$$\text{(2.18')} \qquad \tilde{q} = [\alpha^{\alpha}\beta^{-1}\gamma^{\gamma}\sigma^{1+\rho}(1-\varepsilon/n)^{1+\rho}n^{-\varepsilon(1+\rho)}(1-s)^{-\alpha}]^{1/[\varepsilon-\rho(1-\varepsilon)]}$$

$$\text{(2.19')} \qquad \tilde{x} = [\alpha^{\varepsilon-\gamma(1-\varepsilon)}\beta^{\varepsilon-1}\gamma^{\gamma(1-\varepsilon)}\sigma(1-\varepsilon/n)n^{-\varepsilon}(1-s)^{-\varepsilon+\gamma(1-\varepsilon)}]^{1/[\varepsilon-\rho(1-\varepsilon)]} \cdot (1/K)$$

$$(2.20') \qquad \tilde{y} = [\alpha^{\alpha(1-\varepsilon)}\beta^{\varepsilon-1}\gamma^{\varepsilon-\alpha(1-\varepsilon)}\sigma(1-\varepsilon/n)\,n^{-\varepsilon}(1-s)^{-\alpha(1-\varepsilon)}]^{1/[\varepsilon-\rho(1-\varepsilon)]}$$

A number of observations can now be made. First, for subsidy rates that are positive and less than 100%, basic research expenditures and total R&D are higher in the presence of subsidies and rise with the subsidy rate ie $\partial\tilde{x}/\partial s > 0$ and $\partial(\tilde{x}+\tilde{y})/\partial s > 0$. Development research expenditures increase with the subsidy rate only if demand is inelastic ($\partial\tilde{y}/\partial s > 0$ if $\varepsilon < 1$). Marginal production costs fall ($\partial\tilde{c}/\partial s < 0$), and consequently firm and industry output is higher and prices are lower ($\partial\tilde{Q}/\partial s > 0$, $\partial\tilde{P}/\partial s < 0$). Profit for individual firms rises for elastic demand ($\partial\tilde{\Pi}/\partial s > 0$ for $\varepsilon < 1$).

Secondly, as should be expected, subsidies reduce the disincentive effects of spillovers on basic research. For given n and a subsidy rate s, the marginal disincentive effect of spillovers is given by the negative sign of $\partial\tilde{x}/\partial\theta$. Accordingly, subsidies negatively affect this ie $\partial^2\tilde{x}/(\partial\theta\partial s) < 0$. Their presence therefore improves incentives to invest in basic research in an environment characterised by spillovers.

Finally, subsidies to basic research improve welfare. If we define maximal welfare in the presence of subsidies as $\tilde{W}^* = U(\tilde{Q}_s) - \tilde{c}_s\tilde{Q}_s - (1-s)\tilde{X}_s - \tilde{Y}_s$, we can establish that $\partial\tilde{W}^*/\partial s > 0$. Because of their output-expanding effect, higher subsidy rates to basic research are associated with higher welfare.

In setting a subsidy rate, the government is attempting to maximise the total surplus generated by the market. This surplus is the addition of the producer and consumer surpluses generated by the production of output Q, net of the cost of the subsidy, ie $TS=PS+CS-sX-Y$. For $(\tilde{x}, \tilde{y}, \tilde{Q}/n)$, $\tilde{TS}$ is the maximum surplus generated by the market under the presence of spillovers and subsidies. The optimal subsidy rate can then be calculated as the rate that will maximise this total surplus. Setting $\partial\tilde{TS}/\partial s = 0$ yields the optimum subsidy rate s^*:

(2.39) $$s^* = 1 - \frac{(n-\varepsilon)[\varepsilon-\gamma(1-\varepsilon)]}{\varepsilon+(n-\varepsilon)[\varepsilon-\gamma(1-\varepsilon)]}(1/K)$$

Inspection of (2.40) reveals that the optimal subsidy rate is positive and less than one. It depends on the number of firms in the industry, the spillover rate, the elasticity of demand and the elasticity of cost reduction through development research. With full appropriability, s^* is still positive, reflecting the fact that even in the absence of spillovers, firms in the market underinvest in R&D. As the number of firms increases however, the optimal subsidy declines and as $n \rightarrow \infty, s^* \rightarrow 0$. With positive spillovers $\partial s^*/\partial\theta > 0$, so that the optimal subsidy increases with the spillover rate. The effect of concentration on the optimal subsidy rate is however ambiguous in this case. The reason is that in a spillover environment with many firms, subsidies can significantly reduce industry marginal costs. Finally, *ceteris paribus*, the optimal subsidy to basic research increases with the elasticity of cost reduction through development research.

The table below provides a simple example that helps illustrate these results. It is constructed for an elastic demand ($1/\varepsilon = 1.5$) and for $\gamma = 0.3$. Reading across rows, at any level of concentration (except for a monopolistic industry), optimal subsidies increase with the spillover rate. For four firms, optimal rates vary from 25% with full appropriability to 81% when basic research is a pure public good. Reading down columns, with full appropriability optimal subsidy rates decline sharply as the market becomes less concentrated. With low spillovers, they decline initially; as the number of firms increases however, the effect of a spillover environment on cost reduction overwhelms the cost of subsidies. With moderate or high spillovers, this effect is reinforced, and optimal subsidy rates increase with n. These effects do not change when we vary the elasticity of demand or the elasticity of cost reduction through development research.

Table 1					
Optimal subsidy rates to basic research (using (2.14))					
Firms	Spillover rate				
n	0.00	0.25	0.50	0.75	1.00
1	77.92	77.92	77.92	77.92	77.92
2	46.85	57.50	64.58	69.64	73.43
3	33.51	55.67	66.75	73.40	77.83
4	26.08	57.76	70.43	77.25	81.52
5	21.35	60.67	73.78	80.33	84.27

We can now compare the performance of the market on the basis of the derived optimal subsidy to basic research to that in the absence of any R&D subsidy. This entails the comparison of $TS^*(s^*) = B(Q^*) - c^*Q^* - (1-s^*)nx^* - ny^*$ with $TS^*(s{=}0){=}B(Q^*){-}c^*Q^*{-}nx^*{-}ny^*$. It is easy to establish that for elastic demand ($1/\varepsilon > 1$ -- necessary for both $TS^*(s{=}0){>}0$ and $TS^*(s^*) > 0$), we have that $TS^*(s^*) > TS^*(s = 0)$ [32]. When basic research is subject to spillovers, the market with optimal subsidies in basic research outperforms the market with no R&D subsidies.

We summarise these results in the following proposition:

Proposition 7: **When demand and cost functions are given by (2.12) and (2.14), subsidies in basic research:**
- **(i) reduce the disincentive effect of spillovers;**
- **(ii) improve welfare;**
- **(iii) increase the performance of the market.**

Corollary 7.1: **Optimal subsidies to basic research are positive and increase with the spillover rate.**

The discussion has so far assumed that in setting subsidy rates, the government can identify the part of R&D that represents basic research and which is subject to

[32] $TS^*(s^*)/TS^*(s = 0) = \Gamma/\Delta$ where $\Gamma = (1-s^*)^{-B/A}\{\varepsilon K(n+1-\varepsilon)(1-s^*) - [\alpha+\gamma K(1-s^*)](n-\varepsilon)(1-\varepsilon)\}$, $\Delta = \{\varepsilon K(n+1-\varepsilon) - (\alpha+\gamma K)(n-\varepsilon)(1-\varepsilon)\}$ and where $B = \varepsilon - \gamma(1-\varepsilon)$ and $A = \varepsilon - \rho(1-\varepsilon)$. We can then establish that $TS^*(s^*) > TS^*(s = 0)$.

spillovers. It can therefore subsidise basic research while letting producers bear the total cost of fully appropriable development research. In reality however, there exists a "revelation problem": basic and development research often cannot be easily distinguished from each other, and governments usually tend as a result to subsidise the whole of R&D [33]. The question that arises in this context is whether this practical policy is a second-best. In an environment where basic research only is subject to spillovers, and where it is difficult to identify separately basic and development research expenditures, is there still a case for government subsidising the whole of R&D?

In order to address this question, we derive the firms' equilibrium levels of R&D and output when the total of R&D expenditures is subsidised at a rate $\hat{s}$, but when it is still the case that basic research only is subject to spillovers. Rather than maximising profit as in (2.3') therefore, each firm chooses R&D and output levels so as to maximise:

$$(2.3'') \qquad \underset{x_i, y_i, q_i}{\Pi_i} = \{P(Q) - c[R(x_i, x_{-i}, \delta_{.i}, \theta), y_i]\}q_i - (1-s)(x_i + y_i)$$

The resulting solutions when there are barriers to entry so that the number of firms n can be taken as given are:

$$(2.18'') \qquad \hat{q} = [\alpha^{\alpha}\beta^{-1}\gamma^{\gamma}\sigma^{1+\rho}(1-\varepsilon/n)^{1+\rho}n^{-\varepsilon(1+\rho)}(1-s)^{-\rho}]^{1/[\varepsilon-\rho(1-\varepsilon)]}$$

$$(2.19'') \qquad \hat{x} = [\alpha^{\varepsilon-\gamma(1-\varepsilon)}\beta^{\varepsilon-1}\gamma^{\gamma(1-\varepsilon)}\sigma(1-\varepsilon/n)n^{-\varepsilon}(1-s)^{-\varepsilon}]^{1/[\varepsilon-\rho(1-\varepsilon)]} \cdot (1/K)$$

$$(2.20'') \qquad \hat{y} = [\alpha^{\alpha(1-\varepsilon)}\beta^{\varepsilon-1}\gamma^{\varepsilon-\alpha(1-\varepsilon)}\sigma(1-\varepsilon/n)n^{-\varepsilon}(1-s)^{-\varepsilon}]^{1/[\varepsilon-\rho(1-\varepsilon)]}$$

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[33] This is not the same as saying that often both basic and development research expenditures are subject to spillovers. We continue to maintain the assumption made in this chapter, that is that when "revealed", basic and development research have fundamentally different appropriability characteristics: the former is subject to spillovers, while the latter is not.

It is straightforward to establish that with a subsidy $\hat{s}$ that is positive and less than 100%, basic research, development research and output all increase with the subsidy rate ($\partial\hat{x}/\partial\hat{s} > 0$, $\partial\hat{y}/\partial\hat{s} > 0$ and $\partial\hat{q}/\partial\hat{s} > 0$). Profit for individual firms rises for elastic demand ($\partial\tilde{\Pi}/\partial s > 0$ for $\varepsilon < 1$). Welfare also increases due to the output-expanding effect. If we define maximal welfare in the presence of subsidies to total R&D as $\hat{W}^* = U(\hat{Q}_s) - \hat{c}_s\hat{Q}_s - (1-\hat{s})(\hat{X}_s + \hat{Y}_s)$, we can establish that $\partial\hat{W}^*/\partial\hat{s} > 0$.

The optimal subsidy $\hat{s}^*$ can now be derived by assuming that the government attempts to maximise total surplus, given in this case by $TS=PS+CS-s(X+Y)$. The optimal subsidy per unit of (total) R&D is then given by:

$$(2.39') \qquad \hat{s}^* = 1 - \frac{(n-\varepsilon)\,[\alpha+\gamma K]}{\rho K(n+1-\varepsilon)}(1/K)$$

Comparison of the optimal subsidy rate to basic research (2.39) with the optimal subsidy rate to total R&D (2.39') reveals that for elastic demand ($1/\varepsilon > 0$ -- a necessary condition for W>0), it is the case that irrespective of spillovers, $s^* > \hat{s}^*$. Optimal subsidies are lower when it is the total R&D, rather than the part subject to spillovers, that is being subsidised.

With optimal subsidies $\hat{s}^*$ to total R&D in place, the total surplus achieved is given by $TS^*(\hat{s}^*) = B(\hat{Q}) - \hat{c}\hat{Q} - (1-\hat{s}^*)(n\hat{x} + n\hat{y})$. The question of whether it is better for the government to abstain from any R&D subsidies if there exists a "revelation problem" with basic research can then be addressed by comparing $TS^*(\hat{s}^*)$ to the total surplus that the market can achieve in the absence of R&D subsidies, given by $TS^*(s{=}0)=B(Q^*)-c^*Q^*-nx^*-ny^*$. It can be established that as in the case of optimal subsidies to basic research only, for a given number of firms and spillover rate, $TS^*(\hat{s}^*) > TS^*(s=0)$ [34]. Total surplus in this case exceeds the maximal total surplus in the absence of subsidies.

Of greater interest however is the comparison between $TS^*(s^*)$ and $TS^*(\hat{s}^*)$ [35]. We cannot establish a priori whether the performance of the market under optimal basic research subsidies exceeds or falls short of the market performance with optimal subsidies to total R&D. The ratio depends on the elasticities of demand and of cost reduction with respect to the pool of basic research and to own development research expenditures, and more importantly, on the number of firms in the industry and the degree of appropriability of basic research.

Table 2 below provides an example that illustrates this comparison for different levels of concentration in the industry and spillover rates. For its construction, we have assumed an elastic demand ($1/\varepsilon = 2$) and cost elasticities with respect to the pool of basic research α and to own development research γ of 0.1 and 0.3 respectively. With full appropriability, we see that the performance of the market under optimal basic research subsidies falls short of that under optimal subsidies to total R&D. The same is true for low spillover environments and/or concentrated industries. In high spillover environments and/or less concentrated industries, the reverse is true. Here the

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[34] $TS^*(\hat{s}^*)/TS^*(s=0) = \mathrm{E}/\Delta$ where $\mathrm{E} = (1-\hat{s}^*)^{-\varepsilon/A}\{\varepsilon K(n+1-\varepsilon)(1-\hat{s}^*) - (\alpha+\gamma K)(n-\varepsilon)(1-\varepsilon)\}$, $\Delta = \{\varepsilon K(n+1-\varepsilon) - (\alpha+\gamma K)(n-\varepsilon)(1-\varepsilon)\}$ and where $B = \varepsilon - \gamma(1-\varepsilon)$ and $A = \varepsilon - \rho(1-\varepsilon)$. We can then establish that $TS^*(\hat{s}^*) > TS^*(s=0)$.

[35] This is given by $TS^*(s^*)/TS^*(\hat{s}^*) = \Gamma/\mathrm{E}$ where
$\Gamma = (1-s^*)^{-B/A}\{\varepsilon K(n+1-\varepsilon)(1-s^*) - [\alpha+\gamma K(1-s^*)](n-\varepsilon)(1-\varepsilon)\}$,
$\mathrm{E} = (1-\hat{s}^*)^{-\varepsilon/A}\{\varepsilon K(n+1-\varepsilon)(1-\hat{s}^*) - (\alpha+\gamma K)(n-\varepsilon)(1-\varepsilon)\}$ and where $B = \varepsilon - \gamma(1-\varepsilon)$ and $A = \varepsilon - \rho(1-\varepsilon)$.

revelation problem implies that a policy of subsidising the whole of R&D, rather than just the part subject to spillovers, has a cost, and that cost increases both with the spillover rate and with the number of firms in the industry.

Table 2

Relative market performance under s^* and $\hat{s}^*$ (using (2.14))

Firms	Spillover rate				
n	0.00	0.25	0.50	0.75	1.00
1	0.774	0.774	0.774	0.774	0.774
2	0.931	0.943	0.940	0.948	0.956
3	0.967	0.976	0.993	1.010	1.028
4	0.980	0.998	1.023	1.051	1.075
5	0.987	1.013	1.049	1.083	1.112

The explanation runs along the following lines. With low spillovers/high concentration, the potential cost-reducing benefit of subsidising the total R&D exceeds that of subsidising one part of R&D only. This is especially true since the elasticity of cost reduction with respect to development research is assumed to exceed that of basic research. With high spillovers/low concentration, the potential cost-reducing benefit of subsidising basic research only is very high; it can therefore outweigh the cost of not inducing any increase in the other part of R&D, development research.

V. Conclusions

This chapter set out to investigate the economic characteristics of an environment where a set of interacting firms compete by setting output and R&D levels simultaneously in a one-stage game and where one part of R&D, basic research, is not entirely appropriable by the firm or firms that originate it. In a framework where market structure is in turn taken as a datum or considered as endogenous, we examined the impact of such R&D spillovers on R&D expenditures, costs and output, on industrial concentration and monopoly power, and on the incentives and performance generated by the market relative to a socially optimal case.

The main conclusions are presented below. They rely heavily on the underlying assumptions about the nature of technological advance in an industry as it is revealed in the cost function of the individual firm. This first set of conclusions should be compared with a similar set in the next chapter emanating from an analysis embodying different assumptions about firms' technology. Empirically, the conclusions here seem to be most relevant to industries where technical advance can be thought to have a "discrete" nature, in the sense of innovations in basic research representing isolated advances in knowledge. The basic research of a firm's rivals that leaks out and is absorbed can then be plausibly considered to be a perfect substitute to a firm's own basic research. Furthermore, total basic research and development research are assumed to be substitutable with an elasticity of one, so that the cost function is represented by (2.14). The impact of spillovers is in this case seen to be quite drastic and is summarized below.

(1) If there exist barriers to entry into an industry so that the number of firms can be taken to be exogenously given, inappropriability in basic research will reduce the

incentives at firm level to conduct basic research and improve profitability at the firm level [36]. A higher spillover rate will be associated with a lower rate of basic research expenditures and of total R&D at industry level, with lower fixed costs and consequently with more profitable industries.

(2) In a cross-section of industries facing the same elasticity of demand and with the same opportunities of cost reduction through R&D, but differing in terms of the degree of appropriability of basic research, one would expect to observe that the industries exhibiting the least appropriability of basic research would be the ones least concentrated and with the weakest monopoly power. Spillovers therefore reduce price-cost margins and improve allocative efficiency.

(3) When the number of firms in an industry is large enough for each firm to earn zero profits in the free-entry equilibrium, inappropriability is associated with less R&D-intensive industries.

(4) Despite the fact that higher spillovers sustain a larger number of firms in the free-entry equilibrium, so that the number of firms undertaking basic research increases, industry-wide basic research expenditures suffer. In a free-entry oligopoly therefore inappropriability of basic research acts as a disincentive for basic research expenditures at both the firm and industry-wide level. It may increase industry-wide development research expenditures, but R&D expenditures in total fall.

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[36] All the conclusions are based on the assumption that absorptive capacity is exogenously given. Section II.c demonstrated how with absorptive capacity a function of a firm's basic research expenditures, spillovers need not *necessarily* act as a disincentive to conduct basic research, and will instead encourage it in the case where the positive absorptive incentive is strong enough to outweigh the negative incentive due to diminishing returns. We postpone drawing a full set of conclusions on the basis of this possibility until next chapter, where such a complementarity of own and competitors' basic research is examined more carefully through particular assumptions about the cost function.

(5) Industries with a lower degree of appropriability of basic research will be spending less on basic research as a proportion of total R&D expenditures than industries where spillovers are low. Spillovers therefore induce a shift away from pure or generic research whose outcome is not fully appropriable towards more product-specific and proprietary development research.

(6) Each firm in a free-entry oligopolistic market underinvests in basic research and in total R&D relative to the social optimum. The extent of underinvestment increases with the degree of inappropriability of basic research.

(7) Basic research expenditures in the market, as well as total R&D, will in general be less than socially optimal, whether basic research is appropriable or not; if however demand for the product is highly inelastic total basic research expenditures in an industry with free-entry can exceed the socially optimal level if spillovers are not pervasive. Excessive duplication of basic research and of total R&D in a market economy are therefore possible, even though cost reduction is lower, but duplication will be less if the degree of appropriability of basic research is low.

(8) When there are barriers to entry into an industry, inappropriability of basic research increases the profitability of firms and improves the performance of the market relative to the social optimum. In the free-entry equilibrium however, a higher spillover rate may dampen market performance if the disincentive effect on basic research expenditures increases unit production costs substantially and reduces total industry-wide output.

(9) A government policy of subsidising basic research will stimulate expenditures in the non-appropriable part of R&D and improve welfare. The optimal subsidy rate will be positive, less than one and increase as the degree of appropriability of basic research falls.

VI. APPENDIX TO CHAPTER TWO

We want to locate conditions for which a free-entry symmetric equilibrium (n^*,q^*,x^*,y^*) exists as a solution to the model described by the first-order conditions (2.7')-(2.10'). Assume for the moment that there are barriers to entry so that n is given. The problem reduces to locating conditions for which the choice $(Q^*/n,x^*,y^*)$ yields non-negative profits for the individual firm in equilibrium ie for which:

(A.1) $\Pi(Q^*/n,x^*,y^*) = \{P(Q^*) - c[x^*,(n-1)x^*,y^*]\}Q^*/n - x^* - y^* \geq 0$

We furthermore need the profit function of the individual firm $\{P[(n-1)Q^* + q_i] - c[x_i,(n-1)x^*,y_i]\}q_i - x_i - y_i$ to be concave in the neighbourhood where q_i=Q^*/n, x_i=x^*, y_i=y^*. Before specifying demand and cost functions therefore, for the second-order conditions to hold we need the corresponding Hessian to be negative definite. This implies decreasing returns to scale to R&D expenditures ie $\partial^2 c/\partial x^2 > 0$ and $\partial^2 c/\partial y^2 > 0$. It also implies that own effects (of output and of R&D) on marginal profit must exceed cross effects so that $\Pi_{qq}\Pi_{xx} > \Pi^2_{xq}$, $\Pi_{xx}\Pi_{yy} > \Pi^2_{xy}$ and $\Pi_{qq}\Pi_{yy} > \Pi^2_{qy}$.

If the functional forms of the demand and cost functions are given by (2.12) and (2.14), for non-negative profits we need $n > \varepsilon$ (so that $(Q^*/n,x^*,y^*)$ is real-valued) and:

(A.2) $\varepsilon/(n-\varepsilon) - [\gamma + (\alpha/K)] \geq 0$

Condition (A.2) is satisfied when $\varepsilon \leq n \leq \varepsilon(1+\gamma)/\gamma$ so that:

(A.3) $\Pi(Q^*/n,x^*,y^*) \geq 0 \quad \text{iff} \quad \varepsilon \leq n \leq \varepsilon(1+\gamma)/\gamma$

For zero profits, the condition becomes:

(A.3') $\Pi(Q^*/n,x^*,y^*) = 0 \quad \text{iff} \quad \varepsilon(1+\rho)/\rho \leq n \leq \varepsilon(1+\gamma)/\gamma$

For the profit function to be concave in the neighbourhood of the equilibrium when the inverse demand and cost functions are given by (2.12) and (2.14) the second-order conditions (SOC) are satisfied when:

$$\text{(A.4)} \quad \varepsilon(1+\rho)\left[\frac{(n-\varepsilon)+(n-1)}{n(n-\varepsilon)}\right] > \rho \quad \text{or} \quad \varepsilon(1+\rho)\mu > \rho$$

It can be shown however that $\mu < 1$. [Proof: $(n-\varepsilon)+(n-1) < n(n-\varepsilon) \Rightarrow (n-1) < (n-\varepsilon)(n-1) \Rightarrow n > \varepsilon$ which is given by the FOC. QED]. Condition A.4 can therefore be replaced by $\varepsilon(1+\rho) > \rho$ or $\varepsilon - \rho(1-\varepsilon) > 0$. This is in fact the condition used for the derivation of the comparative statics in the text.

We have therefore proved the following theorem:

Theorem 1: If $P(Q)$ and c_i satisfy (2.12) and (2.14) and if n is a positive integer, then a symmetric Cournot-Nash equilibrium in the presence of spillovers amongst n firms exists if:
(i) $\varepsilon(1+\rho)/\rho \leq n \leq \varepsilon(1+\gamma)/\gamma$ and
(ii) $\varepsilon(1+\rho)\left[\frac{(n-\varepsilon)+(n-1)}{n(n-\varepsilon)}\right] > \rho$ or $\varepsilon(1+\rho)\mu > \rho$.

It can be immediately seen that (ii) is satisfied if $\varepsilon > 1$ ie if demand is inelastic. It can also be satisfied for $\varepsilon < 1$ as long as demand is not *too* elastic. We have therefore shown that if $\varepsilon(1+\rho)/\rho \leq n \leq \varepsilon(1+\gamma)/\gamma$ a Nash equilibrium with spillovers exists even if demand is throughout inelastic. Furthermore, the presence of spillovers allows for a much wider range of ε to satisfy the existence conditions. With $\theta > 0$, the SOC can be satisfied with a more elastic demand than would be necessary in the case where spillovers are absent.

We now want to locate conditions for the existence of a Nash equilibrium when there is free-entry. Suppose there are barriers of entry so that $(Q^*/n, x^*, y^*)$ is the optimal choice for the firm in a symmetric equilibrium with n firms. Following the methodology in Dasgupta and Stiglitz (1980), define by $x' + y'$ the solution of $P(Q^*)= c[x,(n-1)x,y]$. $x' + y'$ is like an entry cost. If there exists an $m>0$ such that $\Pi(Q^*/n, x^*, y^*) < m(x' + y')$, no additional firm will find it profitable to enter the industry and condition (2.6) would then be satisfied.

For the functional forms given in (2.12) and (2.14) using nq^* from (2.20) into $P(Q^*)= c[x^*,(n-1)x^*,y^*]$ we have:

$$\text{(A.5)} \quad x' + y' = [\beta^{\varepsilon-1}\sigma(1-\varepsilon/n)^{\varepsilon(1+\rho)/\varepsilon}\alpha^{\varepsilon}(\gamma/\alpha)^{\gamma(1-\varepsilon)}n^{-\varepsilon}]^{1/[\varepsilon-\rho(1-\varepsilon)]} \cdot [(\gamma/\alpha)+(1/K)]$$

Since

$$\text{(A.6)} \quad \Pi^* = [\sigma(1-\varepsilon/n)\beta^{\varepsilon-1}\alpha^{\alpha(1-\varepsilon)}\gamma^{\gamma(1-\varepsilon)}n^{-\varepsilon}]^{1/[\varepsilon-\rho(1-\varepsilon)]} \cdot [\varepsilon/(n-\varepsilon)-(\gamma+\alpha/K)]$$

from (A.5) and (A.6) it follows that:

$$(A.7) \quad \Pi(Q^*/n, x^*, y^*) \le m(x' + y') \quad \text{iff} \quad [\varepsilon/(n-\varepsilon)-(\gamma+\alpha/K)] \le m(1-\varepsilon/n)^{1/\rho}(\gamma+\alpha/K)$$

For the free-entry Nash equilibrium we will choose n to be the largest positive integer not exceeding $[1+\gamma+(\alpha/K)]/[\gamma+(\alpha/K)]$. We have therefore shown:

Theorem 2: If $P(Q)$ and c_i satisfy (2.12) and (2.14) respectively and in view of Theorem 1, a free-entry Nash equilibrium exists if:

(i) $\varepsilon(1+\rho)\left[\frac{(n-\varepsilon)+(n-1)}{n(n-\varepsilon)}\right] > \rho$ or $\varepsilon(1+\rho)\mu > \rho$

(ii) $n^* = [1+\gamma+(\alpha/K)]/[\gamma+(\alpha/K)]$

(iii) $[\varepsilon/(n-\varepsilon)-(\gamma+\alpha/K)] \le m(1-\varepsilon/n)^{1/\rho}(\gamma+\alpha/K)$.

CHAPTER THREE

COMPLEMENTARITIES IN BASIC RESEARCH

I. Introduction

The results obtained in the previous chapter point to a potentially serious problem associated with the inappropriability of part (or all) of R&D. While spillovers are beneficial from the point of view of *ex post* (production) cost reduction, they may be associated with serious *ex ante* disincentives to invest in the part of R&D that is subject to imitation by rivals. This suggests that industries where spillovers are widespread should be expected to be those characterised by a low rate of technological progress.

A number of empirical studies however seem to contradict this result. They point to evidence suggesting that while spillovers exist there do not seem to be serious disincentive effects associated with them. In a recent paper for example, Levin (1988) reports on the results of an empirical study to ascertain the effects of inappropriability on industrial performance and R&D intensity. He finds that while "involuntary" spillovers do exist, they do not seem to be associated with low R&D intensity. The disincentive effect of spillovers which is predicted in models such as the one by Spence (1984) for the homogeneous R&D case and in the model developed in chapter two above (under the parameterization of the cost function given by (2.14)), is not discernible in the data. Levin's and others' results lend weight to widespread beliefs, based on casual observation, that there exist many high R&D-intensity industries, such as electronics, which exhibit a low degree of appropriability of R&D-related expenditures, but also often have a high degree of dynamic efficiency.

In an attempt to reconcile theory with the stylized facts revealed in empirical work a number of explanations have been offered. The first is *institutional*. It asserts that while spillovers hurt firms' incentives to invest in the part of R&D that is not full appropriable by the originating firm, the fact that we do not observe a diminished rate of effort in R&D can be largely explained by government support of R&D which in the form of subsidies restores incentives. In the model in the previous chapter we showed that the introduction of a subsidy rate to basic research reduces the marginal cost to the firm of investing in that form of R&D expenditures and acts as a counterbalance to the diminished appropriability associated with it. Optimal subsidies are positive for the case where appropriability is imperfect and increase with the spillover rate.

A second explanation runs along *behavioural* lines. In his 1984 paper, Spence conjectures that firms might imperfectly anticipate (or even totally ignore) the effects of their own R&D on the costs of other firms and on industry prices. He interprets this as implying that when making their decisions on the levels of R&D, firms do not take account of the fact that their R&D increases their competitors' knowledge and thereby reduces their costs. This indirect effect reduces a firm's own profits; when it is not therefore taken into account R&D investments are more aggressive (because the perceived benefit is higher) and the disincentive effect due to spillovers is reduced [1]. In the model of the previous chapter however, we showed that even in the absence of this indirect effect, ie in a situation where the marginal benefit of basic research to the firm is defined solely by the reduction in its marginal cost due to its own basic research expenditures, the incentives to invest in basic research in equilibrium decline with the spillover rate. While therefore ignoring spillovers does increase the apparent marginal

[1] Imperfect anticipation of the effects of a firm's own R&D investments on the costs of other firms and on industry prices is reflected in Spence (1984) in the first-order conditions. The marginal return to a firm's R&D expenditures takes account in such a case only of the (positive) direct effect of R&D on own profits ie $\Pi^i_{x_i}$, and not of the indirect effects of R&D on competitors' knowledge level $R_j = x_j + \theta\delta_j x_{-j}$ and through that on price and own profit ie of $\theta \sum_{j \neq i} \delta_j \Pi^i_{R_j}$. Given that this second term is negative, its omission increases the marginal benefit of spending on R&D. Put another way, not taking into account this indirect effect is equivalent to reducing a *two*-stage game (with R&D decisions preceding production decisions) to a *one*-stage game.

benefit to the firm of investing in the partially appropriable part of R&D and thereby makes its investment decisions more aggressive, it does not totally eliminate or reverse the disincentive effect due to spillovers [2].

Such a reversal in the role of spillovers, ie a situation where spillovers become an incentive to invest in R&D, is possible in the context of a third explanation that focuses on the *technology* itself, and to inter-industry differences in the nature of technical advance. Take for example the case where own and rival basic research are perfect substitutes, as in (2.14). This would seem to best characterize industries where technical advance has a "discrete", as opposed to a "cumulative" nature, so that innovation represents isolated advances in knowledge. In such a situation, spillovers from rivals lower the marginal productivity of a firm's basic research by making further such outlays duplicative, and, in the context of this model, also shift some of the firm's resources to another type of R&D, development research.

Technical advance may however instead be "cumulative". Rather than representing radical departures in previously unexplored directions, many of today's innovations are often closely related to existing knowledge; the technology embodied in new processes is then founded on the technology it replaces. If findings in a field build on past findings, the capacity to assimilate new products and processes depends to a large extent on the firm's own past R&D expenditures [3]. In addition therefore to generating innovations, R&D in this case also develops the ability of firms to utilise technological knowledge from its competitors in the same industry or to assimilate

[2] The effect of behavioural assumptions on the impact of spillovers on R&D incentives and firm profitability are explored in detail in chapter four below.

[3] Levin (1988) offers this idea of industries dominated by "isolated increases in knowledge" as opposed to industries dominated by "cumulative advance" as a plausible explanation for the contradiction between previous theoretical predictions on the effects of spillovers and the results of empirical work.

innovations developed in other industrial sectors. In such a situation, the marginal productivity of a rival's basic research can enhance the productivity of the firm's own basic research. Consequently, a high spillover rate may also stimulate expenditures in basic research and thereby raise R&D intensity.

Another aspect of outside knowledge that may make a firm's own R&D expenditures critical to its capacity to assimilate externally generated knowledge is the complexity of knowledge and the degree to which outside knowledge is targeted to the needs of the firm [4]. When extra-firm knowledge is particularly complex, or when it is of a generic nature and less directly targeted to the capabilities already developed by a firm, own R&D expenditures become particularly important in allowing the firm to track the evolution of this knowledge and assimilate it. This suggests that the effect of spillovers on incentives and performance can be most sharply visualised if, as has been the case in the models developed here, it is assumed that spillovers exist in basic research, whereas development research is taken to be fully appropriable.

There are at least two ways of attempting to model and examine the implications of the main elements of this third "technological" explanation to the apparent paradox. The first has been followed by Cohen and Levinthal (1989) and was also illustrated as a digression (section II.c) in the previous chapter. It consists of assuming that a firm's capacity to absorb external knowledge depends on its own R&D expenditures. Higher expenditures mean that firms are in a better position to take advantage of the research of their rivals that spills over: for given spillovers, their absorptive capacity improves. Cohen and Levinthal show that with absorptive capacity an endogenous variable in the model, spillovers will in certain circumstances spur, rather than discourage,

[4] Cohen and Levinthal (1989) and Pavitt (1987) discuss some of the characteristics of knowledge and technology mentioned here.

expenditures in R&D. In the previous chapter, we illustrated this result for the case where spillovers are in basic research only, and where basic and development research enter the firm's cost function with a unitary elasticity.

An alternative way of capturing the idea that firms invest in basic research (or in R&D in general) in order to be better able to utilise information available externally is to focus directly on the idea that competitors' basic research (or R&D in general) may be *complementary* to a firm's own expenditures, rather than simply substituting for it. This implies allowing in the formulation of the model for the possibility that spillovers from rivals can increase, as well as decrease, the marginal productivity of a firm's own basic research. In the context of a model where cost reduction depends on expenditures in (partially appropriable) basic research and in (fully appropriable) development research, this boils down to a particular specification of the cost function that is more flexible with respect to the possibilities of substitutability and complementarity between three research inputs: own basic research, rivals' basic research, and own development research [5]. This chapter develops such a model, analyses its implications for incentives at the firm level and industry performance, and compares the results with those of the previous chapter with the use of a simple example.

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[5] The specification adopted in this chapter is a more general one than that of Cohen and Levinthal (1989). They focus on the idea of *learning*, and on the implications of an endogenous absorption parameter on the incentives that firms have for R&D expenditures. We focus on the technological interaction between firms. Thus with our approach it would in principle be possible to distinguish between two industries which, while having similar absorptive capacities, also have different technological characteristics and thus respond differently to spillovers. This is reflected in the specification chosen. We assume that for each firm, the marginal cost function is given in general form by $c_i = \phi(D_i, R_i, R_j)$, where D_i is the firm's development research knowledge, R_i its basic research knowledge and R_j the basic research knowledge of its competitors. For heterogeneous R&D, the Cohen and Levinthal specification would be of a weakly separable form $c_i = \phi(D_i, \delta(R_i)R_j) = \phi[D_i, \psi(R_i, R_j)]$, where δ is the firm's absorptive capacity.

II. Oligopoly with *n* firms

The results of the last chapter were derived on the basis of the functional form of the cost function in (2.14). This formulation embodies certain assumptions about the manner in which a firm's own and its borrowed basic research combine with development research to reduce production costs. Own basic research is assumed to be a perfect substitute for that part of the basic research of rivals that spills over and that can be absorbed. Total basic research on the other hand, both own and borrowed, is taken to be substitutable with development research with an elasticity of one. These assumptions are fairly stringent and it may be that they exert an unduly strong influence on the effects of spillovers.

a. Complementarities in basic research

In order to explore whether it is true that the disincentive effect of spillovers rests critically on the underlying assumptions about the firm's technology, we examine a different parameterization of the cost function. Rather than using a cost function whose underlying knowledge production function is of the Cobb-Douglas type, we now assume a cost function based on a two-level (nested) constant elasticity of substitution (CES) knowledge production function. For a particular firm *i*, the (basic) knowledge production function is given by the form:

$$(3.1) \qquad R_i = \left[x_i^{-\rho_1} + \left(\sum_{j \neq i} \delta_{ji} \theta_j x_j \right)^{-\rho_1} \right]^{-1/\rho_1}$$

with a corresponding cost function:

$$(3.2) \qquad c_i = \beta \left\{ \alpha R_i^{-\rho_2} + \gamma y_i^{-\rho_2} \right\}^{\lambda/\rho_2} = \beta \left\{ \alpha \left[x_i^{-\rho_1} + \left(\sum_{j \neq i} \delta_{ji} \theta_j x_j \right)^{-\rho_1} \right]^{\rho_2/\rho_1} + \gamma y_i^{-\rho_2} \right\}^{\lambda/\rho_2}$$

$$\text{where} \quad -1 < \rho_1 < \infty, \quad -1 < \rho_2 < \infty, \quad \rho_1, \rho_2 \neq 0, \quad \alpha, \gamma > 0, \quad \alpha + \gamma = 1$$

The firm's own basic research expenditures together with that part of the basic research expenditures of its rivals that spills over and that can be absorbed are nested in a subaggregate R_i that represents the total basic research pool. Own and borrowed basic research however are no longer perfect substitutes, and their direct elasticity of substitution is given by $s_{xx} = 1/(1+\rho_1)$. The subaggregate input R_i on the other hand enters into the cost function together with development research expenditures and with a direct elasticity of substitution between the two of $s_{Ry} = 1/(1+\rho_2)$ [6]. This is in contrast to the previous specification of the cost function where the elasticity of substitution between basic research and development research was equal to one.

A nested CES cost function allows for two different types of elasticity of substitution: (i) that between the inputs within the nest ie between "own" and "borrowed" basic research; and (ii) that between the nest as a whole and the input outside it ie between the total pool of basic research and (own) development research. In our case however, we are primarily interested in comparing the ease of substitution between "own" and "borrowed" basic research on the one hand with that between "borrowed" basic research and own development research on the other, rather than with that between the "nest" as a whole and development research. Put another way, we want to explore the implications of a situation where the public part of rivals' basic research is more complementary with a firm's own basic research than with its development research. To this end it is more appropriate to use the concept of

[6] In effect, the subaggregate input R_i, together with development research expenditures y_i, enter into a nested CES knowledge production function of the form $f_i = \left\{ \alpha R_i^{-\rho_2} + \gamma y_i^{-\rho_2} \right\}^{-1/\rho_2}$. This knowledge, generated through own basic research, borrowed basic research, and development research expenditures, is in turn transformed into cost reduction through a function $c_i = \beta f_i^{-\lambda}$ where λ is the elasticity of cost reduction with respect to knowledge generated by R&D.

"elasticity of complementarity" [7]. Let c_{xx} be the elasticity of complementarity between "own" and "borrowed" basic research and c_{xy} that between "borrowed" basic research and own development research [8]. For $c_{xx} > c_{xy}$ we require that $\partial \log f_{x_i}/\partial x_j > \partial \log f_{y_i}/\partial x_j$ ie that borrowed basic research raises the marginal productivity of own basic research by more that it raises the marginal productivity of development research. It can be easily verified that $c_{xx} > c_{xy}$ ($c_{xx} < c_{xy}$) when $\rho_1 > \rho_2$ ($\rho_1 < \rho_2$) [9].

The new specification of the cost function therefore allows us to account for the following possibility: that the firm may be faced with a situation where the basic research of its rivals that "spills over" is more easily substitutable with the firm's own

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[7] Hicks (1970) introduces the concept of "elasticity of complementarity" c as a dual concept to the (Allen partial) elasticity of substitution σ. For two inputs i and j it is defined as $c_{ij} = f\, f_{ij}/f_i\, f_j$, $i \neq j$, with $q=f(a_1,\ldots,a_n)$ the firm's production function where q is output and a_i the ith input. Similarly the partial elasticity of substitution is defined as $\sigma_{ij} = g\, g_{ij}/g_i g_j$ where $C_{ij} = g(p_1,\ldots,p_n,q)$ is the cost function dual to the production function. The two definitions are equivalent under a two-factor constant returns to scale technology. Using duality theory, Sato and Koizumi (1973) show that the partial elasticity of complementarity measures the inverse of the cross elasticity of derived demand with marginal cost held constant, while the partial elasticity of substitution measures the same with output held constant. Furthermore, in the same way that for more than two factors the Allen partial elasticity of substitution is used in order to characterise inputs as substitutes and complements (with respect to changes in *prices*), the partial elasticity of complementarity can be used for the same purpose (with respect to changes in *quantities*). For $c_{ij}>0$ therefore, the two inputs are defined to be quantity-*complements*, while for $c_{ij}<0$ they are quantity-*substitutes*.

[8] $c_{x_i x_{-i}} = c_{xx} = (1+\rho_2) + (\rho_1 - \rho_2) f_i^{-\rho_2} / \left(\alpha R_i^{-\rho_2}\right)$ and $c_{x_i y_i} = c_{x_{-i} y_i} = c_{xy} = 1/s_{Ry} = 1 + \rho_2$. Following textbook notation, we use c for the elasticity of complementarity, s for the direct elasticity of substitution and σ for the Allen elasticity of substitution. These symbols are used in this section only and should not be confused with, respectively, the marginal cost function c, the subsidy rate s and the extent of the market σ, used throughout the dissertation.

[9] A brief proof is as follows. Define a nested CES knowledge production function $f_i^{-\rho_2} = \alpha R_i^{-\rho_2} + \gamma y_i^{-\rho_2}$ where R_i is as defined in (3.1). Then $f_{x_{-i}} = \alpha (f_i/R_i)^{1+\rho_2} (R_i/\sum \delta\theta x_{-i})^{1+\rho_1}$ and $\partial \log f_{x_{-i}}/\partial x_i = (1+\rho_2)\partial \log f_i/\partial x_i + (\rho_1 - \rho_2)\partial \log R_i/\partial x_i$. Similarly, $f_{y_i} = \gamma (f_i/y_i)^{1+\rho_2}$ and $\partial \log f_{y_i}/\partial x_i = (1+\rho_2)\partial \log f_i/\partial x_i$. Therefore $c_{xx} > c_{xy}$ or $\partial \log f_{x_{-i}}/\partial x_i > \partial \log f_{y_i}/\partial x_i$ when $\rho_1 > \rho_2$. It is also the case that pairs of factors that are more complementary than others in terms of the elasticity of complementarity c will tend to be more complementary in terms of the Allen elasticity of substitution σ as well, so that $\rho_1 > \rho_2$ ($\rho_1 < \rho_2$) implies $c_{xx} > c_{xy}$ ($c_{xx} < c_{xy}$) and $\sigma_{xx} < \sigma_{xy}$ ($\sigma_{xx} > \sigma_{xy}$).

development research than it is with its own basic research [10]. In such a case, a higher spillover rate may not automatically induce the firm to cut back on its own basic research. In the light of the (relative) complementarity between "own" and "borrowed" basic research, a firm may instead increase its own basic research expenditures.

As in the previous chapter, firms are assumed to aim to maximize profits net of R&D expenditures. In this respect, they choose basic research expenditures x_i, development research expenditures y_i and output q_i simultaneously in a one-stage Cournot game. The firm's objective function is then to maximize:

$$(3.3) \qquad \underset{x_i, y_i, q_i}{\Pi_i} = \Pi_i(q_i, Q_{-i}, x_i, x_{-i}, \theta, \delta_{,i}, y_i) = \{P(Q) - c[R(x_i, x_{-i}, \delta_{,i}, \theta), y_i]\}q_i - x_i - y_i$$

where the cost function is now given by the (nested) CES form of (3.2) and where demand takes again an iso-elastic form ie:

$$(3.4) \qquad P(Q) = \sigma Q^{-\epsilon} \qquad\qquad \sigma, \varepsilon > 0$$

where σ indicates the size of the market.

b. Equilibrium with barriers to entry

Under the cost and demand functions assumed in (3.2) and (3.4), the first-order conditions for the symmetric solution in the case where there exist barriers to entry are:

$$(3.5) \qquad \sigma(Q^*)^{-\epsilon}[1 - \varepsilon(Q^*)/n] = \beta\left[\alpha K^{\rho_2/\rho_1}(x^*)^{-\rho_2} + \gamma(y^*)^{-\rho_2}\right]^{\lambda/\rho 2}$$

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[10] As was noted also in the previous chapter, nothing in the model distinguishes development research expenditures y_i from any other cost-reducing inputs that are not R&D-related. In the situation where y_i is interpreted as any of a number of other such expenditures, x_i should be understood as representing R&D expenditures in general, with spillovers operating on the total of R&D. The concepts of "substitutability" and "complementarity" between x and y should then be interpreted as referring to R&D in general in relation to non R&D-related inputs.

$$(3.6) \qquad \lambda\alpha\beta\left[\alpha K^{\rho_2/\rho_1}(x^*)^{-\rho_2}+\gamma(y^*)^{-\rho_2}\right]^{(\lambda-\rho_2)/\rho_2} K^{(\rho_2-\rho_1)/\rho_1}(x^*)^{-1-\rho_2} q = 1$$

$$(3.7) \qquad \lambda\gamma\beta\left[\alpha K^{\rho_2/\rho_1}(x^*)^{-\rho_2}+\gamma(y^*)^{-\rho_2}\right]^{(\lambda-\rho_2)/\rho_2} (y^*)^{-1-\rho_2} q = 1$$

where $K = 1 + (n-1)\,[\delta\theta]^{-\rho_1}$

For the second-order conditions to hold, it is necessary that $\varepsilon - \lambda(1-\varepsilon) > 0$. We assume that this is indeed the case. The explicit solutions for $(Q^*/n, x^*, y^*)$ when the industrial structure is taken as a datum are then given by:

$$(3.8) \qquad q^* = \left[\sigma^{1+\lambda}(1-\varepsilon/n)^{1+\lambda}\,\beta^{-1}\,\lambda^{\lambda}\,\alpha^{-\lambda/\rho_2}\,K^{-\lambda(1+\rho_1)/\rho_1}\,(1+\psi K^{\phi-1})^{-\lambda(1+\rho_2)/\rho_2}\,n^{-\varepsilon(1+\lambda)}\right]^{1/[\varepsilon-\lambda(1-\varepsilon)]}$$

$$(3.9) \qquad x^* = \left[\sigma(1-\varepsilon/n)\,\beta^{\varepsilon-1}\,\lambda^{\varepsilon}\,\alpha^{\lambda(\varepsilon-1)/\rho_2}\,K^{\lambda(\varepsilon-1)/\rho_1-\varepsilon}\,(1+\psi K^{\phi-1})^{\lambda(\varepsilon-1)/\rho_2-\varepsilon}\,n^{-\varepsilon}\right]^{1/[\varepsilon-\lambda(1-\varepsilon)]}$$

$$(3.10) \qquad y^* = x^*\,\psi\,K^{\phi}$$

where $\psi = (\gamma/\alpha)^{1/(1+\rho_2)}$ and $\phi = (\rho_1-\rho_2)/[\rho_1(1+\rho_2)]$

We can now examine the impact of spillovers under the CES specification of the cost function, starting with the case where there exist barriers to entry so that n can be taken as given [11]. In the specification of the unit cost function used in chapter two above, "own" and "borrowed" basic research were perfectly substitutable. This meant that when the number of firms was given, an increase in the spillover rate induced a corresponding decrease in the basic research expenditures of the representative firm

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[11] In the comparative statics based on the CES cost function we concentrate on the cases where the specification of the cost function gives qualitatively, as opposed to quantitatively different results to the case where the cost function is as in (2.14) of chapter two. We therefore ignore in the comparative statics the impact of changes in the scientific base β and of the market size σ on the equilibrium values of (q^*, x^*, y^*), given that the sign of the effects is independent of the specification of the cost function.

equal to the increase in its effective borrowed basic research, while leaving development research and direct production costs unchanged. Total costs inclusive of R&D costs fell of course, since basic research outlays had fallen. Profits therefore rose leading to entry in the absence of barriers.

In contrast, with the nested CES cost function, the comparative statics results of the impact of spillovers on R&D, costs, output and profits, are no longer as straight-forward. They are discussed each in turn and summarised in Table 1 below.

Starting with the impact of spillovers on basic research expenditures in equilibrium, the sign of $\partial x^*/\partial\theta$ is ambiguous; a firm's basic research expenditures may rise or fall as a result of a higher degree of inappropriability [12]. The effect will depend on three factors: first, on the *relative strength of complementarity* between own and borrowed basic research as opposed to that between own basic research and development research (ie the magnitude of c_{xx} as opposed to that of c_{xy}); second, on the *ease of substitutability* between "own" basic research, "borrowed" basic research and development research (ie the absolute values of s_{xx} and s_{Ry}); and lastly, on the elasticities of demand and of cost reduction with respect to R&D-related knowledge.

Take for example the case where $\rho_1 > \rho_2 > 0$, so that borrowed basic research is more complementary with "own" basic research than it is with development research ($c_{xx} > c_{xy}$), while both own with borrowed basic research and total basic with

[12] Differentiation of (3.9) yields that the sign of $\partial x^*/\partial\theta$ is determined by the sign of the expression $\{[\rho_1\varepsilon+\lambda(1-\varepsilon)](1+\rho_2)+(\rho_1-\rho_2)\psi K^{\phi-1}[\varepsilon-\lambda(1-\varepsilon)]$ where $\phi=(\rho_1-\rho_2)/[\rho_1(1+\rho_2)]$ and $\psi=(\gamma/\alpha)^{1/(1+\rho_2)}$. Note also that it is not just the sign of $\partial x^*/\partial\theta$ that is ambiguous; unlike in the Cobb-Douglas case, in the CES case we cannot even say that, compared with industries where basic research is fully appropriable, firms in industries where spillovers exist will be spending less in basic research. In effect $x^*|_{\theta>0}=x^*|_{\theta=0}K^{\lambda(\varepsilon-1)/\rho_1-\varepsilon}(1+\psi K^{\phi-1})^{\lambda(\varepsilon-1)/\rho_2-\varepsilon}$ so that for $0>\rho_1>\rho_2$, $\varepsilon<1$ and $\lambda>\varepsilon\rho_1/(\varepsilon-1)$, we have $x^*|_{\theta>0}>x^*|_{\theta=0}$.

Table 1: Comparative statics results using (3.2)

			$\partial x/\partial\theta$	$\partial y/\partial\theta$	$\partial c/\partial\theta$	$\partial q/\partial\theta$	$\partial\Pi/\partial\theta$
$\rho_1>\rho_2>0$	$\varepsilon>1$	$\lambda<\rho_2\varepsilon/(\varepsilon-1)$	+	+	-	+	-
	$\varepsilon>1$	$\rho_2\varepsilon/(\varepsilon-1)<\lambda<\rho_1\varepsilon/(\varepsilon-1)$	+	-	-	+	+/-
	$\varepsilon>1$	$\varepsilon/(1-\varepsilon)>\lambda>\rho_1\varepsilon/(\varepsilon-1)$	+/-	-	-	+	+/-
	$\varepsilon<1$	$\lambda<\varepsilon/(1-\varepsilon)$	+	+	-	+	+/-
$\rho_1=\rho_2>0$	$\varepsilon>1$	$\lambda<\rho_1\varepsilon/(\varepsilon-1)$	+	+	-	+	-
	$\varepsilon>1$	$\varepsilon/(1-\varepsilon)>\lambda>\rho_1\varepsilon/(\varepsilon-1)$	-	-	-	+	+/-
	$\varepsilon<1$	$\lambda<\varepsilon/(1-\varepsilon)$	+	+	-	+	+/-
$\rho_2>\rho_1>0$	$\varepsilon>1$	$\lambda<\rho_1\varepsilon/(\varepsilon-1)$	+/-	+	-	+	-
	$\varepsilon>1$	$\lambda>\rho_2\varepsilon/(\varepsilon-1)$	-	-	-	+	+/-
	$\varepsilon>1$	$\rho_2\varepsilon/(\varepsilon-1)>\lambda>\rho_1\varepsilon/(\varepsilon-1)$	-	+	-	+	+/-
	$\varepsilon<1$	$\lambda<\varepsilon/(1-\varepsilon)$	+/-	+	-	+	+/-
$0>\rho_1>\rho_2$	$\varepsilon>1$		+/-	-	-	+	-
	$\varepsilon<1$	$\varepsilon/(1-\varepsilon)>\lambda>\rho_2\varepsilon/(\varepsilon-1)$	+	+	-	+	+/-
	$\varepsilon<1$	$\rho_1\varepsilon/(\varepsilon-1)<\lambda<\rho_2\varepsilon/(\varepsilon-1)$	+	-	-	+	+/-
	$\varepsilon<1$	$\lambda<\rho_1\varepsilon/(\varepsilon-1)$	+/-	-	-	+	+
$0>\rho_1=\rho_2$	$\varepsilon>1$		-	-	-	+	+/-
	$\varepsilon<1$	$\varepsilon/(1-\varepsilon)>\lambda>\rho_1\varepsilon/(\varepsilon-1)$	+	+	-	+	+/-
	$\varepsilon<1$	$\lambda<\rho_1\varepsilon/(\varepsilon-1)$	-	-	-	+	+
$0>\rho_2>\rho_1$	$\varepsilon>1$		-	-	-	+	+/-
	$\varepsilon<1$	$\varepsilon/(1-\varepsilon)>\lambda>\rho_2\varepsilon/(\varepsilon-1)$	+/-	+	-	+	+/-
	$\varepsilon<1$	$\lambda<\rho_1\varepsilon/(\varepsilon-1)$	-	-	-	+	+
	$\varepsilon<1$	$\rho_2\varepsilon/(\varepsilon-1)>\lambda>\rho_1\varepsilon/(\varepsilon-1)$	+/-	-	-	+	+/-
$\rho_1>0>\rho_2$	$\varepsilon>1$	$\lambda<\rho_1\varepsilon/(\varepsilon-1)$	+	-	-	+	+/-
	$\varepsilon>1$	$\lambda>\rho_1\varepsilon/(\varepsilon-1)$	+/-	-	-	+	+/-
	$\varepsilon<1$	$\varepsilon/(1-\varepsilon)>\lambda>\rho_2\varepsilon/(\varepsilon-1)$	+	+	-	+	+/-
	$\varepsilon<1$	$\lambda<\rho_2\varepsilon/(\varepsilon-1)$	+	-	-	+	+/-
$\rho_2>0>\rho_1$	$\varepsilon>1$	$\lambda<\rho_2\varepsilon/(\varepsilon-1)$	-	+	-	+	+/-
	$\varepsilon>1$	$\lambda>\rho_2\varepsilon/(\varepsilon-1)$	-	-	-	+	+/-
	$\varepsilon<1$	$\varepsilon/(1-\varepsilon)>\lambda>\rho_1\varepsilon/(\varepsilon-1)$	+/-	+	-	+	+/-
	$\varepsilon<1$	$\lambda<\rho_1\varepsilon/(\varepsilon-1)$	-	+	-	+	+/-

development research are not easily substitutable with each other ($s_{xx} < 1, s_{R_y} < 1$). If in addition demand is elastic ($1/\varepsilon > 1$), then $\partial x^*/\partial\theta > 0$ [13]. In such a case spillovers, rather than acting as a disincentive on a firm's basic research, will instead act as a stimulus. In fact even if $\rho_1 = \rho_2$ (ie $c_{xx} = c_{xy}$), higher spillovers increase basic research, as long as $\rho_1\varepsilon + \lambda(1-\varepsilon) > 0$. This last requirement is satisfied either for inelastic demand, or for elastic demand provided $\varepsilon\rho_1/(\varepsilon-1) > \lambda > \varepsilon/(1-\varepsilon)$.

For a wide range of parameter values therefore, spillovers act as an incentive to expenditures in basic research [14]. Under the assumed functional forms, we can still have $\partial x^*/\partial\theta < 0$ if $\rho_2 > \rho_1$ and $\rho_1 < 0$ (with demand inelastic or elastic provided that $\varepsilon\rho_1/(\varepsilon-1) > \lambda > \varepsilon/(1-\varepsilon)$ holds), or if $\rho_2 \gg \rho_1$ by a significant amount, with $\rho_1>0$, but then only with inelastic demand and assuming in addition that $\lambda > \varepsilon\rho_1/(\varepsilon-1)$. To obtain therefore the disincentive effect of spillovers on basic research effort in this case, one needs to assume $c_{xx}<c_{xy}$ and $s_{xx}>1$ or $c_{xx}<<c_{xy}$ and in addition impose certain restrictions on the parameters. Individual basic research effort is therefore certain to fall only if borrowed research is more complementary with development research than it is with "own" basic research *and* if in addition substitutability between "own" and "borrowed" basic research is high.

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[13] If instead demand is inelastic ($\varepsilon > 1$), we can still have $\partial x^*/\partial\theta > 0$, as long as $\rho_1\varepsilon + \lambda(1-\varepsilon) > 0$ (which necessitates that $\varepsilon\rho_1/(\varepsilon-1) > \lambda > \varepsilon/(1-\varepsilon)$) or as long as $\psi K^{\phi-1}(\rho_1-\rho_2)[\varepsilon-\lambda(1-\varepsilon)] > |(1+\rho_2)[\rho_1\varepsilon+\lambda(1-\varepsilon)]|$.

[14] It should be noted that this reversal of the traditional result that spillovers act as a disincentive to basic research, which we obtained in the CES case, does not hinge on the breakdown of R&D into (partially appropriable) basic research and (non-appropriable) development research and on the resulting possibilities for relative complementarity and substitutability between the three pairs of factors. Suppose instead that R&D were homogeneous, so that a firm's cost reduction depended on *x* only, where *x* is subject to spillovers. Rather than (3.2), suppose a firm was faced with a cost function such as $c_i = \beta\left\{\alpha x_i^{-\rho} + \gamma\left(\sum_{j\neq i}\delta_{ji}\theta_j x_j\right)^{-\rho}\right\}^{\lambda/\rho}$. The sign of $\partial x^*/\partial\theta$ would still be ambiguous and $\partial x^*/\partial\theta > 0$ for $\varepsilon\rho + \lambda(1-\varepsilon) > 0$.

Turning now to development research, differentiation of (3.9) with respect to the spillover rate reveals that for $\varepsilon\rho_2 + \lambda(1-\varepsilon) > 0$, $\partial y^*/\partial\theta > 0$. The effect on development research expenditures therefore depends on the elasticity of demand and on the elasticity of substitution between basic and development research $s_{R_y} = 1/(1+\rho_2)$. For spillovers to affect positively development research expenditures, it is sufficient for $\rho_2 > \lambda - \lambda/\varepsilon$. For an elastic demand, this condition is satisfied if development research is not easily substitutable with basic research ($\rho_2 > 0$ or $s_{R_y}<1$). If demand is inelastic, it is also necessary that in addition the elasticity of cost reduction with respect to knowledge generated by R&D (λ) be low. If on the other hand basic and development research are good substitutes for each other ($\rho_2 < 0$), the positive incentive effect of spillovers can only survive if demand is elastic and if in addition $\lambda > \varepsilon\rho_2/(\varepsilon - 1)$. In any other case spillovers have a disincentive effect.

Irrespective of their (incentive or disincentive) impact on basic and development research expenditures, spillovers have a beneficial effect on marginal production costs in equilibrium. Substitution of the equilibrium values of x^* and y^* into (3.1) and differentiation with respect to the spillover rate yields that $\partial c^*/\partial\theta < 0$. Put another way, whether expenditures in R&D decline or not, the production of knowledge generated by R&D in equilibrium increases with the spillover rate. Accordingly, as the marginal cost function shifts down due to a higher spillover rate, with a downward demand schedule output expands: $\partial q^*/\partial\theta > 0$. Given n therefore, higher spillover rates reduce unit production costs and increase output.

Lastly, when n is fixed, higher spillovers may increase or decrease individual firm profits ie $\partial\Pi^*/\partial\theta > 0$ or <0 [15]. The overall effect on profits is the combination of the separate effects on R&D expenditures, production costs and revenues. Given that spillovers reduce equilibrium production costs and thereby increase output, if demand is inelastic ($\varepsilon > 1$) profits gross of R&D expenditures (ie Pq-cq) will fall. Whether net profits fall as well will depend on the strength of the effect of spillovers on basic and development research expenditures. If for example $\rho_1 > \rho_2 > 0$, and if in addition $\varepsilon/(1-\varepsilon) < \lambda < \rho_1\varepsilon/(1-\varepsilon)$, so that spillovers stimulate expenditures in both basic and development expenditures, net profits fall. They also fall however for $\rho_2 > \rho_1 > 0$, and $\varepsilon/(1-\varepsilon) < \lambda < \rho_1\varepsilon/(1-\varepsilon)$. In that situation, higher spillovers increase output and development research while reducing production costs. With an inelastic demand total revenue falls and so do net profits, even in the case where weak complementarity between "own" and "borrowed" basic research induces the firm to spend less on its own basic research when appropriability declines (thereby reducing one element of fixed costs).

In the situation where instead we assume that demand is elastic and where $0 > \rho_1 > \rho_2$ and $\varepsilon/(1-\varepsilon) < \lambda < \rho_1\varepsilon/(1-\varepsilon)$, then $\partial\Pi^*/\partial\theta > 0$. Here the elastic demand leads to a fall in profits gross of R&D expenditures, while the high substitutability of (borrowed) basic research with development research causes the latter to decrease. Even if basic research rises, net profits fall so that equilibrium profits are higher as the appropriability of basic research declines.

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[15] Differentiation of $\Pi^* = \Pi^*(x^*, (n-1)\delta\theta x^*, y^*, q^*)$ with respect to the spillover rate shows that the sign of $\partial\Pi^*/\partial\theta$ is determined by the sign of the expression $\{[\varepsilon/(n-\varepsilon)]K(1+\psi K^{\phi-1})(1+\rho_1)(1+\rho_2)(1-\varepsilon)+A\psi K^{\phi-1}(\rho_1-\rho_2)(K-1)-(1+\psi K^{\phi})(1+\rho_2)B\}$ where $A = [\varepsilon-\lambda(1-\varepsilon)]$ and $B = [\rho_1\varepsilon+\lambda(1-\varepsilon)]$.

Summarising the main results of this subsection we develop proposition 1 (to be compared with proposition 1 in chapter two above):

Proposition 1: **When cost and demand functions are given by (3.2) and (3.4), in the Nash-Cournot equilibrium (q^*, x^*, y^*) with barriers to entry spillovers in basic research:**

- **(i) will *increase* basic research expenditures when the basic research of rivals is at least as complementary with a firm's own basic research as with its own development research $c_{xx} \geq c_{xy}$ and when in addition $\varepsilon\rho_1 + \lambda(1-\varepsilon) > 0$;**
- **(ii) will increase development research expenditures when $\varepsilon\rho_2 + \lambda(1-\varepsilon) > 0$, and decrease them when $\varepsilon\rho_2 + \lambda(1-\varepsilon) < 0$;**
- **(iii) will increase the total knowledge generated due to R&D expenditures, and thereby reduce marginal production cost and expand output;**

Corollary 1.1: **Spillovers in basic research can increase or decrease (ex ante) incentives for cost reduction while reducing (ex post) the costs at industry level of achieving a given level of cost reduction.**

c. Free-entry equilibrium

For the case where there are no barriers to entry, and where n^* is large enough so that firms are earning zero profits in equilibrium, the expression equivalent to (2.23) in the last chapter, indicating the number of firms in the free-entry equilibrium is given by:

$$(3.11) \qquad n^* = (\varepsilon/\lambda)\left[\lambda + \frac{K + \psi K^{\phi}}{1 + \psi K^{\phi}}\right] \quad \text{or} \quad n^* = \frac{\varepsilon(1+\lambda)}{\lambda} + (\varepsilon/\lambda)\frac{[\delta\theta(n^*-1)]^{-\rho_1}}{1+\psi K^{\phi}}$$

where $\psi = (\gamma/\alpha)^{1/(1+\rho_2)}$ and $\phi = (\rho_1 - \rho_2)/\rho_1(1+\rho_2)$

It can be readily checked that for $\theta = 0$ (no spillovers), expression (3.11) reduces to $n^* = \varepsilon(1+\lambda)/\lambda$ so that the equilibrium number of firms depends then only on the elasticity of demand and on the responsiveness of cost reduction to the knowledge generated by R&D expenditures. With positive spillovers however, n^* is also a function of the spillover rate θ and the free-entry equilibrium with spillovers sustains a larger number of firms than the corresponding equilibrium with full appropriability. Even in the case where own and borrowed research are not perfect substitutes therefore, inappropriability is associated with less concentrated industries.

By total differentiation of (3.11) with respect to the spillover rate and the number of firms we obtain that while $n^* |_{\theta>0} > n^* |_{\theta=0}$, we can have $dn^*/d\theta > 0$ or <0 [16]. It is not necessarily the case that a higher spillover rate implies a larger number of firms in the free-entry equilibrium. As was indicated above for the equilibrium where barriers to entry prevent profits going to zero, in the case for example where demand is inelastic, and where $\rho_2 > \rho_1 > 0$ and $\lambda < \rho_1\varepsilon/(1-\varepsilon)$, higher spillovers will reduce profits [17]. The free-entry solution with zero profits and a higher spillover rate will then be associated with a more concentrated industry.

The explanation for this apparent contradiction runs along the following lines. In the situation when spillovers are zero, the firm's cost function is of a normal

[16] The sign of $\partial n^*/\partial\theta$ is determined by the sign of the expression $\mu\nu(n-1)/(1-\theta\mu\nu)$, where $\mu = (\varepsilon/\lambda)(K-1)/[(1+\psi K^{\phi})^2\theta(n-1)(1+\rho_2]$ and $\nu = -\rho_1(1+\rho_2)(1+\psi K^{\phi}) + (K-1)\psi k^{\phi-1}(\rho_1-\rho_2)$. Since $\mu > 0$, when $\nu < 0$ we will have $\partial n^*/\partial\theta < 0$. This will occur when ρ_1 and ρ_2 are both positive, or alternatively when $s_{xx} = 1/(1+\rho_1) < 1$ and $s_{Ry} = 1/(1+\rho_2) < 1$, and irrespective of whether we have $\rho_1 > \rho_2$ ($c_{xx} > c_{xy}$) or not.

[17] This result we can obtain analytically from the expression $\partial\Pi^*/\partial\theta$. In general however, because of the functional form it is not possible to derive analytically the complete range of parameter values for which we have $dn^*/d\theta > 0$ or <0. The elasticity of substitution parameters for which higher spillover rates reduce the number of firms in equilibrium can only be established with simple simulations of (3.11). As an example, consider the situation where we have ρ_1=10 and ρ_2=2, giving s_{xx}=0.09 and s_{Ry}=0.33 respectively. Assume also that demand is slightly elastic ($1/\varepsilon$=1.25) and that the elasticity of cost reduction with respect to R&D is λ=0.4. With these parameters, in the situation with no spillovers zero profits would obtain with a duopoly (the equilibrium number of firms being derived as the largest integer not exceeding the value of n^* that satisfies (3.11)). With θ=0.25, zero profits obtain with four firms. For θ=0.50, n^*=3 while for complete spillovers (θ=1), the free-entry equilibrium number of firms is two.

(non-nested) CES type with the elasticity of substitution between basic and development research given by $s_{xy} = 1/(1+\rho_2)$. Compared to this situation, even a small positive spillover rate will decrease production costs, raise profits and lead to higher n^* in a zero-profit equilibrium. Once some inappropriability has been established however, the effect of a higher spillover rate involves taking account of the elasticities of substitution between research inputs and of the differential impact of a higher effective borrowed research on the marginal productivities of "own" basic research and of development research respectively. Once the firm adjusts its R&D spending in response to that, its maximal profits under the higher spillover rate may be lower, leading to a lower number of firms at the zero-profit equilibrium. We can therefore summarise in the following proposition:

Proposition 2: **When cost and demand functions are given by (3.2) and (3.4), the free-entry Nash-Cournot equilibrium (n^*, q^*, x^*, y^*) with imperfectly appropriable basic research is characterised by a less concentrated industry than the same equilibrium with perfect appropriability. However, the degree of industrial concentration can vary either directly or indirectly with the degree of appropriability (ie with the spillover rate).**

The impact of spillovers on monopoly power in the industry is similarly ambiguous under the assumed two-level (nested) CES cost function. $P(Q^*)/c^* = 1/(1-\varepsilon/n^*)$ and therefore it is possible that the expression $\partial(P(Q^*)/c^*)/\partial\theta = -[\varepsilon/(n^*-\varepsilon)^2](\partial n^*/\partial\theta)$ is positive or negative. Price-cost margins can in fact increase if higher spillovers lead to a smaller equilibrium number of firms. When comparing however the case with spillovers to the case where spillovers are absent it is still true that the *existence* of spillovers (as opposed to higher rates of spillovers) reduces monopoly power in an industry. In this respect therefore the qualitative impact of the existence of spillovers does not hinge on the specification of the cost function.

Taking now into account the values of the elasticities between the various inputs needed to determine whether spillovers lead to a more or less concentrated industry, we can develop the following:

Corollary 2.1: **In a cross-section of industries facing the same elasticity of demand and with the same opportunities of cost reduction through R&D, but differing in terms of the degree of appropriability of basic research, one would expect to observe that the industries that exhibit the least appropriability of basic research would be the ones *most* concentrated and with the *strongest* monopoly power when the elasticities of substitution between own and "borrowed" basic research and between total (both own and "borrowed") effective basic research and development research are low (s_{xx}<1 and s_{Ry}<1).**

Turning next to the effect of inappropriability on the total industry-wide basic research when the cost function is of the nested CES type and when zero profits are guarantied by free-entry, we see that no clear pattern emerges. Under the parameterization of the cost function in chapter two we obtained that $n^*x^* |_{\theta=0} > n^*x^* |_{\theta>0}$ and that $\partial(n^*x^*)/\partial\theta < 0$. The existence of inappropriability and an increase in the spillover rate both resulted in lower basic research expenditures for the industry as a whole in the free-entry equilibrium. In the present case however, with the cost function of the nested CES type, $n^*x^* |_{\theta=0} > n^*x^* |_{\theta>0}$ or $n^*x^* |_{\theta=0} < n^*x^* |_{\theta>0}$ and $\partial(n^*x^*)/\partial\theta > 0$ or $\partial(n^*x^*)/\partial\theta < 0$. In the free-entry case neither the existence of inappropriability nor a higher spillover rate necessarily imply lower total industry-wide basic research. Industries where part of basic research is non-appropriable may, ceteris paribus, spend more on basic research than similar industries where spillovers are absent.

An example of a case where a greater degree of inappropriability is associated with higher basic research expenditures at firm level and with a greater number of firms in the free-entry equilibrium is the following. Assume that $0 > \rho_1 > \rho_2$ (ie c_{xx}>c_{xy} and

s_{xx}<s_{Ry}<1) and that in addition $(1+\rho_2)\,[\varepsilon\rho_1+\lambda(1-\varepsilon)]+\phi(\rho_1-\rho_2)A>0$ so that the strong complementarity between "own" and "borrowed" basic research induces each firm to spend more on basic research as a result of a higher spillover rate. With elastic demand ($\varepsilon<1$), profits also rise leading to a higher number of firms in the zero-profit equilibrium. In this case therefore, spillovers lead to higher basic research expenditures at firm and industry level.

Proposition 3: **When demand and cost functions are given by (3.2) and (3.4), in the free-entry Nash-Cournot equilibrium (n^*, q^*, x^*, y^*) spillovers in basic research *increase* firm and industry basic and development research expenditures if demand is elastic, "borrowed" basic research is more complementary with a firm's own research than with its development research (c_{xx}>c_{xy}) and when the elasticities of substitution between own and "borrowed" basic research and between total (both own and "borrowed") effective basic research and development research are low (s_{xx}<1 and s_{Ry}<1).**

A related question is the effect of spillovers on the <u>R&D intensity</u> in a certain industry. R&D intensity is defined as the fraction of an industry's combined basic and development research expenditures in total sales, ie $R^*+D^*=n^*(x^*+y^*)/P(Q^*)Q^*$. When basic research can be fully appropriated, $R^*+D^*=\varepsilon/n^*=\lambda/(1+\lambda)$. With a CES knowledge production function, expenditures in R&D are a constant fraction of total cost. With free entry and zero profits, total costs equal total revenues and R&D expenditures are therefore also a fixed fraction of total revenue. That fraction is solely determined by the elasticity of cost reduction with respect to the knowledge generated by R&D expenditures. When a part of basic research expenditures cannot be appropriated however, R&D intensity also depends on the spillover rate. $R^*+D^*=\varepsilon/\,n^*(\theta)$ and $d(R^*+D^*)/d\theta=-[\varepsilon/\,n^2(\theta)]\,(dn^*/d\theta)$. Given that in the present context, spillovers may lead to a more or to a less concentrated industry, the effect on

R&D intensity is similarly ambiguous. It will increase with the spillover rate when a lower degree of appropriability is associated with more concentrated industries. We can therefore conclude that for the free-entry solution:

Corollary 3.1: **Inappropriability in basic research is associated with less R&D intensive industries, but the level of R&D intensity varies positively with the spillover rate when the elasticities of substitution between own and "borrowed" basic research and between total (both own and "borrowed") effective basic research and development research are low (ie when $s_{xx} = 1/(1+\rho_1) < 1$ and $s_{Ry} = 1/(1+\rho_2) < 1$).**

Turning finally to the relative importance of basic versus development research in a firm's total R&D outlays, under the assumed specification of the cost function, the expression equivalent to (2.24) in chapter two is:

$$(3.12) \qquad x^*/y^* = (\alpha/\gamma)^{1/(1+\rho_2)} K^{(\rho_2-\rho_1)/\rho_1(1+\rho_2)}$$

When basic research is fully appropriable, $\theta = 0$, K=1 and the ratio of basic to development research depends only on the relative distribution of their shares in production costs and on the elasticity of substitution between basic and development research. With the cost function of the Cobb-Douglas type, the latter was equal to one so that the ratio depended only on the relative shares.

When spillovers are positive, taking initially the case where the numbers of firms is exogenously given (reverting, in other words, to the model with barriers to entry), we can see that $\partial(x^*/y^*)/\partial\theta > 0$ (<0) as $\rho_1 > \rho_2$ ($\rho_1 < \rho_2$). If $\rho_1 < \rho_2$ so that an increase in effective borrowed research (due to a higher spillover rate) raises the marginal productivity of own basic research less than that of development research, the ratio of basic to development research at the firm level declines. If $\rho_1 = \rho_2$, so that c_{xx}=c_{xy}, the spillover rate does not affect the ratio. It raises the proportion of basic research in total R&D

only in the case where the marginal productivity of development research is more sensitive than the marginal productivity of "own" basic research to increases in "borrowed" basic research.

In the case where the optimal number of firms is determined by free-entry, the sign of $\partial(x^*/y^*)/\partial\theta$ is given by the sign of $[(\rho_1 - \rho_2)/(1+\rho_2)]\,[(n-1)+\theta(\partial n^*/\partial\theta)]$. The proportion of basic research in total R&D is then raised if borrowed basic research is more complementary with a firm's own basic research than with its development research and if in addition spillovers lead to positive profits and consequently to entry of firms.

Corollary 3.2: **Inappropriability in basic research will be associated with a shift from fully appropriable development research to partially appropriable basic research when borrowed basic research is more complementary with a firm's own basic research than with its development research and if in addition research inputs are easily substitutable with each other.**

III. Socially-managed Industry

We now turn to the case of the socially-managed industry. As in the previous chapter, the purpose here is to derive the equilibrium levels of output and of resources devoted to R&D in an industry organised by a social planner but where part of basic research is not appropriated by the originating firm. These levels are taken to be optima from the social point of view and enable us to establish a "benchmark" against which the efficiency of the market can be judged.

The social planner's maximisation problem is solved in two steps. In the first stage the socially optimal number of firms is chosen, and in the second stage the levels of R&D and output that will maximise welfare are chosen. This two-step procedure allows for the examination of the possibility that the socially optimal number of firms exceeds one. The problem is solved recursively, with industrial structure taken as given for the solution of the second stage. Given n firms for the moment therefore, at the second stage the social planner is assumed to organize production so as to maximize net consumer surplus, which is given by (an expression equivalent to (2.25) in chapter two and repeated here for the sake of convenience):

$$(3.13) \qquad W = U(Q) - \sum_{i=1}^{n} c_i q_i - \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} y_i$$

where $U(Q)$ is the total benefit from the consumption of output Q (the area under the demand curve). The first-order conditions then are:

$$(3.14) \qquad \partial U(Q^*)/\partial q_i^* = c\left[\left(x_i^*, \sum_{j\neq i} x_j^*\right), y_i^*\right]$$

$$(3.15) \qquad -\left[(\partial c_i^*/\partial x_i^*) + \left(\sum_{j\neq i} \partial c_j^*/\partial x_i^*\right)\right] q_i^* = 1$$

$$(3.16) \qquad -[\partial c_i^*/\partial y_i^*] q_i^* = 1$$

With symmetry, these conditions become:

(3.14') $U'(Q) = c\{[x^*, (n-1)x^*], y^*\}$

(3.15') $-[(\partial c^*/\partial x^*) + (n-1)(\partial c^*/\partial x^*)](Q^*/n) = 1$

(3.16') $-[\partial c^*/\partial y^*](Q^*/n) = 1$

In order to solve (3.14')-(3.16') explicitly and obtain a sharper characterization of equilibrium, the utility function will be assumed to be of the form:

(3.17) $U(Q) = [1/(1-\varepsilon)]\sigma Q^{1-\varepsilon}$ so that $U'(Q) = \sigma Q^{-\varepsilon}$

In this social planner's problem, the cost function is assumed to be given by (3.2). Under symmetry, unit production costs therefore are:

(3.18) $c = \beta\left\{\alpha\left(1 + [\delta\theta(n-1)]^{-\rho_1}\right)^{\rho_2/\rho_1} x^{-\rho_2} + \gamma y^{-\rho_2}\right\}^{\lambda/\rho_2}$

If the social planner takes the number of firms in the industry as given, the first-order conditions to the maximisation problem are:

(3.19) $\sigma(Q_s)^{-\varepsilon} = \beta\left[\alpha K^{\rho_2/\rho_1}(x_s)^{-\rho_2} + \gamma(y_s)^{-\rho_2}\right]^{\lambda/\rho_2}$

(3.20) $\lambda\alpha\beta\left[\alpha K^{\rho_2/\rho_1}(x_s)^{-\rho_2} + \gamma(y_s)^{-\rho_2}\right]^{(\lambda-\rho_2)/\rho_2} K^{\rho_2/\rho_1}(x_s)^{-1-\rho_2} q_s = 1$

(3.21) $\lambda\gamma\beta\left[\alpha K^{\rho_2/\rho_1}(x_s)^{-\rho_2} + \gamma(y_s)^{-\rho_2}\right]^{(\lambda-\rho_2)/\rho_2} (y_s)^{-1-\rho_2} q_s = 1$

where $K = 1 + [\delta\theta(n-1)]^{-\rho_1}$ and the subscript $_s$ denotes the socially-optimal solution.

The equilibrium values for (Q_s, x_s, y_s) when the second-order conditions hold are then given by:

$$(3.22)\qquad q_s = \left[\sigma^{1+\lambda}\,\beta^{-1}\,\alpha^{-\lambda/\rho_2}\,\lambda^{\lambda}\,n^{-\varepsilon(1-\lambda)}\left(1+\psi K^{\phi_1-1}\right)^{-\lambda(1+\rho_2)/\rho_2} K^{-\lambda/\rho_1}\right]^{1/A}$$

$$(3.23)\qquad x_s = \left[\sigma\,\beta^{\varepsilon-1}\,\alpha^{(A-\varepsilon)/\rho_2}\,\lambda^{\varepsilon}\,n^{-\varepsilon}\left(1+\psi K^{\phi_1-1}\right)^{(A-\varepsilon)/\rho_2-\varepsilon} K^{(A-\varepsilon)/\rho_1}\right]^{1/A}$$

$$(3.24)\qquad y_s = \psi K^{\phi_1} x_s$$

where $\psi = (\gamma/\alpha)^{1/(1+\rho_2)}$, $\phi_1 = -\rho_2/\rho_1(1+\rho_2)$ and $A = \varepsilon - \lambda(1-\varepsilon)$

With n given, the effect of spillovers on the socially optimal levels of basic and of development research expenditures (the sign of $\partial x_s/\partial\theta$ and $\partial y_s/\partial\theta$) depends on the elasticity of substitution between basic and development research. For $\rho_2 < 0$ (so that $s_{R_y} = 1/(1+\rho_2) > 1$), basic research in the socially-managed industry increases as a result of a higher spillover rate [18]. Taking account of the positive externality generated by basic research in the socially optimal case implies that each firm spends more on basic research as a result of a higher spillover rate, as long as the substitutability between basic and development research is high. The proportion of total R&D accounted for by basic research then rises.

The effect on development research depends on the sign of the expression $\varepsilon\rho_2 + \lambda(1-\varepsilon)$. Spillovers increase the socially optimal level of the fully appropriable part of R&D for $\rho_2 > \lambda(\varepsilon-1)/\varepsilon$, ie for all but very high levels of substitutability between basic and development research. Even however in the case where high substitutability between appropriable and non-appropriable research inputs leads to a decline of

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[18] The sign of $\partial x_s^*/\partial\theta$ is determined by the sign of the expression $\lambda(1-\varepsilon)(1+\rho_2) - \rho_2\psi K^{\phi_1}A$. Given that the functional form of the utility function is assumed to be given by $U(Q) = [1/(1-\varepsilon)]\sigma Q^{1-\varepsilon}$, demand is taken to be elastic for positive levels of utility. We therefore have $\partial x_s^*/\partial\theta > 0$ for $\rho_2 < 0$.

development research as spillovers increase, total R&D expenditures increase. Furthermore, $\partial q^*/\partial\theta > 0$ and $\partial W^*/\partial\theta > 0$. Higher spillovers increase output and social welfare. We can summarise these results in the following proposition:

Proposition 4: **When demand and cost functions are given by (3.2) and (3.4), in the solution (q_s^*, x_s^*, y_s^*) for the socially-managed industry, spillovers in basic research:**

(i) increase basic research expenditures when the substitutability of basic with development research is high (s_{Ry}>1);

(ii) increase development research expenditures, *except* where the substitutability of basic with development research is very high (ie except when $\rho_2 > \lambda(\varepsilon - 1)/\varepsilon$);

(iii) increase total R&D expenditures;

(iv) increase output;

(v) improve welfare.

a. Optimal number of firms in the industry

In order to determine the socially optimal number of firms in this case, the planner solves $W^*(q_s, x_s, y_s)$ for n_s. The equilibrium number of firms is then given by the expression:

$$(3.25) \qquad n_s = 1 + \frac{[\delta\theta(n_s^* - 1)]^{-\rho_1}}{1 + \psi K^{\phi_1 + 1}}$$

From expression (3.25) we can observe that, for the range of values of θ and δ postulated in the model (ie $0 \leq \theta \leq 1$, $0 \leq \delta \leq 1$), it is possible to have n_s>1. In the situation where the technology is described by a nested CES cost function as in (3.2), the socially optimal outcome may involve more than one firm operating. This is in contrast to the case in chapter two where "own" and "borrowed" basic research were perfect substitutes and where the socially optimal outcome involved at most one firm operating.

Consider as an example the situation where research inputs are not easily substitutable with each other and where in addition the non-appropriable basic research of rivals enhances the marginal productivity of a firm's own basic research by more than it does the marginal productivity of development research (so that "own" and "borrowed" basic research are highly complementary). Assuming $s_{xx} = 1/(1+\rho_1) = 0.09$ and $s_{Ry} = 1/(1+\rho_2) = 0.2$, for a spillover rate of 0.10, the socially optimal outcome will involve three firms operating. For a spillover rate of 0.50, it will involve two firms while for complete spillovers (θ=1), welfare will be maximised with only one firm operating, just as it would be in the situation with no spillovers [19].

The reasoning for this result runs along the following lines. In the case where research is subject to spillovers, the social planner takes into account the externality generated by basic research when maximizing net consumer surplus. In the structure of the previous chapter, with own and borrowed R&D perfect substitutes, the research paths of different firms were perfect substitutes. Additional R&D investment of rivals that becomes public generates a cost reduction for a firm equivalent to investing the same amount on R&D. This implies that in the socially managed case, allowing more than one firms to operate generates needless duplication. The social planner therefore would only ever have one firm undertaking R&D [20].

[19] The example is calculated for ρ_1=10, ρ_2=4, giving elasticities of substitution between own and borrowed basic research and between the pool of basic research and own development research of $s_{xx} = 1/(1+\rho_1) = 0.09$ and $s_{Ry} = 1/(1+\rho_2) = 0.2$ respectively. We also assume $\alpha = 0.3$ and $\gamma = 0.7$. The socially optimal number of firms is then calculated as the largest integer not exceeding the value of n that satisfies expression (3.25).

[20] Production would then be undertaken by the firm investing in R&D, with regulation enforcing a price equal to marginal cost; alternatively, the planner could costlessly disseminate the technology to a large number of firms and allow competition to determine prices. The point is made in Katsoulakos and Ulph (1990).

In the case where research inputs are subject to spillovers and poorly substitutable, welfare is increased by allowing more than one firm to operate. With research paths no longer perfect substitutes, spillovers lead to lower production costs and higher output, thus generating higher welfare. In effect, in this case the planner has to reconcile two separate effects: introducing more than one firm will allow different research paths to emerge and society will benefit from the positive externalities associated with spillovers; with a segmented market however each firm will be producing less, and the incentive to invest in R&D will be reduced.

For given values of the elasticities of substitution, the relative strength of the two effects is determined by the spillover rate. As the example above illustrates, with high or moderate appropriability (low or moderate spillovers), the positive external effect dominates and welfare is maximised with more than one firm operating. With high or complete spillovers, on the other hand, costs are reduced significantly; together with the segmentation of the market, this reduces incentives to invest in R&D. The socially optimal outcome then once more involves at most one firm operating [21].

In the situation therefore where the social planner takes the industrial structure as given, spillovers increase output and improve welfare, irrespective of the specification of the cost function and the relationship between "own" and "borrowed" basic research

[21] In a recent paper, Katsoulakos and Ulph (1990) obtain certain results that parallel our results in this chapter. They extend the Dasgupta and Stiglitz (1980a) model to examine the case of product differentiation. In their model, products are imperfect substitutes and R&D is product-specific. The social planner then faces a conflict between (i) proliferating products to satisfy consumers' taste for variety, but with each firm producing little output and therefore undertaking little cost-reducing R&D; and (ii) having a small number of products, each with a large output and consequently a lot of cost-reducing R&D. They obtain that when the degree of substitutability between products is low relative to the productivity of R&D, the socially optimal outcome may involve more firms operating than the equilibrium number of firms in the market. Thus the assumption of product differentiation has an analogous impact to the assumption of externalities in R&D with poor substitutability between a firm's own and its rivals' research inputs.

with development research. The existence of inappropriability or higher spillover rates increase total resources devoted to R&D in the socially-managed industry, while the separate effect on basic and development research depends on the ease of substitutability between the two inputs. Furthermore, if the social planner can adjust the number of firms to achieve the maximum net consumer surplus, the socially optimal outcome may involve more than one firm operating in the case where "own" and "borrowed" basic research are not perfect substitutes and where the technology is described by a nested CES cost function. This last result can be formulated into a proposition:

Proposition 5: **When demand and cost functions are given by (3.2) and (3.4), the solution (n_s^*, q_s^*, x_s^*, y_s^*) for the socially-managed industry may involve more than one firm operating if research inputs are not easily substitutable with each other (s_{xx}<1, s_{Ry}<1).**

IV. Market performance and government policy

Sections II and III of this and the previous chapter addressed the question of how the equilibrium characteristics of an n-firm oligopoly and of a socially-managed industry are affected by the existence of a non-appropriable part in basic research. We derived results that showed that the impact of spillovers depends to a large extent on the assumptions about how R&D inputs combine to produce cost-saving innovations, as this is reflected in the cost function with which firms operate. In this section, we first examine the implications of the results of this chapter (ie under the assumption of a nested CES cost function) for the incentives of the market to conduct basic research as well as its performance relative to the socially optimal case. Secondly, we briefly examine a government policy that provides a subsidy to expenditures in the non-appropriable part of R&D, basic research, and show how the nature and effectiveness of such a policy is heavily dependant on the characteristics of the underlying knowledge production function that firms in a particular industry are faced with.

a. Relative market incentives and performance

Under the assumption that a firm's technology is described by a two-level (nested) CES cost function, as in (3.2), we examine the situation where in the market we have free-entry so that the (optimal) number of firms n^* is determined by the zero-profit condition while in the socially-managed industry the planner can choose the number of operating firms n_s so as to maximise social welfare. We start with the comparison of basic research expenditures by a firm in an oligopolistic market structure with that in the socially-managed industry. If basic research is fully appropriable, $n^* = \varepsilon(1+\lambda)/\lambda$ for the oligopoly while n_s=1. It follows that $x^* < x_s$ as $(n^*)^{-\varepsilon} < (1+\lambda)$. For $\theta > 0$ on the other hand, the ratio x^*/x_s is given by:

$$(3.26) \qquad \frac{x^*}{x_s} = \left[(1-\varepsilon/n^*) \left(\frac{K}{K_s} \right)^{\lambda(\varepsilon-1)/\rho_1 - \varepsilon} K_s^{-\varepsilon} \left(\frac{n^*}{n_s} \right)^{-\varepsilon} \left(\frac{1+\psi K^{\phi-1}}{1+\psi K_s^{\phi_1 - 1}} \right)^{\lambda(\varepsilon-1)/\rho_2 - \varepsilon} \right]^{1/A}$$

where $K_s = 1 + [\delta\theta(n_s - 1)]^{-\rho_1}$.

In order to determine whether the ratio x^*/x_s is less than one or not, we need to determine whether the market equilibrium with free entry sustains a greater number of firms than the number of firms maximises social welfare, where n^* is given by (3.11) and n_s by (3.25). We assume that $n^* \geq n_s$ [22]. Inspection of expression (3.26) then shows that $x^* < x_s^*$: in the presence of inappropriability each firm in the market underinvests in basic research compared to the socially optimum level. For different spillover rates however we have that $\partial(x^*/x_s)/\partial\theta > 0$ or < 0, with the sign depending on the assumptions about substitutability amongst inputs, the elasticity of cost reduction with respect to the knowledge generated from R&D expenditures and the elasticity of demand for the product. The possibility that higher spillover rates improve the basic research expenditures of firms in the market relative to the optimal level is a reflection of the fact that under the current specificaton of the cost function, spillovers can act as a spur to basic research expenditures of oligopolistic firms.

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[22] This is not as arbitrary an assumption as it seems. In the case of the oligopoly, from (3.11) we have $n^*|_{\theta>0} = n^*|_{\theta=0} + (\varepsilon/\lambda)[\delta\theta(n^* - 1)]^{-\rho_1}/(1+\psi K^{\phi})$ while in the case of the socially-managed industry from (3.25) we have $n_s|_{\theta>0} = n_s|_{\theta=0} + [\delta\theta(n_s^* - 1)]^{-\rho_1}/\left(1+\psi K_s^{\phi_1+1}\right)$. Given that $n^*|_{\theta=0} = \varepsilon(1+\lambda)/\lambda$ while $n_s|_{\theta=0} = 1$ we can guarantee that $n^*|_{\theta>0} > n_s|_{\theta>0}$ as long as $\varepsilon(1+\lambda)/\lambda \gg 1$. If $\varepsilon(1+\lambda)/\lambda$ does not greatly exceed one, it is possible for $n^*|_{\theta>0} < n_s|_{\theta>0}$ marginally; given however that the optimal number of firms is defined to be the largest integer not exceeding n^* in (3.11) and n_s in (3.25) respectively, we take it that in the least $n^*|_{\theta>0} = n_s|_{\theta>0}$ will hold.

Industry-wide total basic research expenditures in the market for the case where the zero profit condition holds are $n^* x^*$ whereas total basic research expenditures in the socially-managed industry are $n_s x_s$. For $\theta = 0$, the ratio of the two is: $n^* x^* / x_s = [(1 - \varepsilon/n^*)(n^*)^{\lambda(\varepsilon - 1)}]^{(1/A)}$. It follows that $n^* x^* > x_s\,(= n_s x_s)$ as $(n^*)^{\lambda(\varepsilon - 1)} > (1 + \lambda)$ (or equivalently $n^* x^* < x_s\,(= n_s x_s)$ as $(n^*)^{\lambda(\varepsilon - 1)} < (1 + \lambda)$). Where spillovers are absent therefore, the market as a whole may be spending too much on basic research, while each firm is underinvesting compared to the social optimum. This result was derived in Dasgupta and Stiglitz for the case where R&D is homogeneous, in addition to being fully appropriable [23].

With positive spillovers, both $n^* x^*$ and $n_s x_s$ can be higher, depending on the cost and demand elasticities, so that the market as a whole may in this case also be wasting resources on basic research. Furthermore, the sign of $\partial(n^* x^* / n_s x_s)/\partial\theta$ is ambiguous: a higher spillover rate may increase or decrease the ratio. Under the assumed cost function, the reasoning for this ambiguous result is no longer that spillovers necessarily tend to discourage basic research in the market while leaving the socially optimal level unchanged. Inappropriability may instead stimulate basic research in the market case, as discussed above. In the situation where it does however, the external effect generated increases also the socially optimal level of basic research, and may result in more than one firm operating in the socially-managed industry. The ratio $n^* x^* / n_s x_s$ with positive spillovers will therefore depend on the strength of the two effects. In summary:

[23] It should be noted however that the result that the market may be encouraging too much duplication rests on demand being inelastic ($\varepsilon > 1$). A positive social benefit out of the consumption of Q however (and an associated positive social welfare), is only defined in the functional form adopted by Dasgupta and Stiglitz (1980a) --and used also here as expression (3.16)-- if demand is *elastic*.

Proposition 6: **When cost and demand functions are given by (3.2) and (3.4), in a free-entry oligopoly with spillovers in basic research the industry as a whole may be spending too much on basic research compared to the socially optimal levels.**

The performance of the market relative to the maximum surplus generated in the socially managed industry can be represented by the ratio TS^*/W^*. TS is the total surplus achievable in the market, and is obtained as the addition of the consumer surplus $B(Q) - Q\,B'(Q)$ (where $B(Q)$ is the total benefit obtained by the consumption of output Q) and the producer surplus $n\Pi$ so that $TS^* = B(Q^*) - c^*Q^* - n^*x^* - n^*y^*$. Maximum welfare is given by W^*, so that $W^* = U(Q_s^*) - c_s^*Q_s^* - X_s^* - Y_s^*$. In the situation where the firm's technology is described by (3.2) the ratio TS^*/W^* is:

$$(3.28) \qquad TS^*/W^* = (\varepsilon/A)\left[(1-\varepsilon/n^*)\left(\frac{n^*}{n_s^*}\right)^{\lambda(\varepsilon-1)}\left(\frac{K}{K_s}\right)^{\lambda(\varepsilon-1)/\rho_1} K^{\lambda(\varepsilon-1)}\left(\frac{1+\psi K^{\phi-1}}{1+\psi K_s^{\phi_1}}\right)^{\lambda(\varepsilon-1)(1+\rho_2)/\rho_2}\right]^{1/A}$$

Expression (3.28) is derived for the case where the zero-profit condition holds in the market, so that n^* is the endogenously determined number of firms. The surplus generated is therefore entirely consumer surplus. In the socially-managed industry, the number of firms n_s is chosen to maximize net consumer surplus.

In chapter two, under the assumption that "own" and "borrowed" basic research are perfect substitutes we obtained the result that, while a higher spillover rate does not necessarily improve the performance of the market, the existence of inappropriability implies that the total surplus generated in the market as a percentage of the optimal surplus will always exceed the surplus in the case where spillovers are absent. In the case where the firm's technology is described by a nested CES cost function, this is no longer the case. When spillovers stimulate, rather than inhibit, basic research expenditures, market performance in the presence of inappropriability may fall short of the performance in the case where spillovers are absent. In addition therefore to the indeterminacy of the effect of a higher spillover rate on performance (ie of the sign of $\partial(TS^*/W^*)$), in this case we can also have either $TS^*/W^*|_{\theta=0} > TS^*/W^*|_{\theta>0}$ or $TS^*/W^*|_{\theta=0} < TS^*/W^*|_{\theta>0}$.

Proposition 6: When cost and demand functions are given by (3.2) and (3.4), in a free-entry oligopoly, neither the existence of inappropriability nor higher spillover rates in basic research necessarily improve the relative performance of the market.

b. Subsidies to basic research

We now turn briefly to examine the equilibrium characteristics of a market environment where the government subsidises a part of basic research expenditures of each firm. This is an exercise already developed in the last chapter where we assumed that a firm's technology is given by a Cobb-Douglas knowledge production function, and a firm's basic research is a perfect substitute with the part of the basic research of its rivals that leaks out and that can be absorbed. In such a situation, a government policy that subsidises basic research will restore incentives to spend in the partially appropriable part of R&D and thereby improve welfare. It followed that optimal subsidy rates were positive and increasing with the degree of inappropriability.

The aim in this section is to examine whether these results hold when the firm's technology is described by a nested CES knowledge production function and an associated cost function as in (3.2). Such a cost function gives greater flexibility in substitutability between inputs and allows for the possibility that the basic research expenditures of rivals that leak out and that can be absorbed increase the marginal productivity of a firm's basic research by more than that of its development research.

We showed that in such circumstances spillovers can act as an incentive, rather than as a disincentive, for investment in basic research, thus questioning the desirability of a policy that subsidises non-appropriable R&D.

In order to examine the implications of the assumptions made in this chapter about technology on government policy, we assume that firms can take advantage of a per-unit subsidy on basic research, denoted s. The firm's profit-maximising problem would now be, instead of (3.3):

$$(3.3') \qquad \Pi_{\substack{i \\ x_i, y_i, q_i}} = \{P(Q) - c[R(x_i, x_{-i}, \delta_{.i}, \theta), y_i]\}q_i - (1-s)x_i - y_i$$

For the functional forms for cost and demand functions given in (3.2) and (3.4), the corresponding first-order conditions for the symmetric solution are the same as for the situation where firms bear the full cost of basic research (3.5)-(3.7), except for the condition that equalises marginal benefit with marginal cost in basic research, which is replaced by:

$$(3.6') \qquad \lambda\alpha\beta\left[\alpha K^{\rho_2/\rho_1}(x^*)^{-\rho_2} + \gamma(y^*)^{-\rho_2}\right]^{(\lambda-\rho_2)/\rho_2} K^{(\rho_2-\rho_1)/\rho_1}(x^*)^{-1-\rho_2} q = 1 - s$$

For the situation where there exist barriers to entry so that the number of firms n can be taken as given, the new system of first-order conditions (3.5), (3.6') and (3.7) yields the following solutions:

$$(3.8') \quad \tilde{q} = \left[(\sigma(1-\varepsilon/n))^{1+\lambda} \beta^{-1} \lambda^{\lambda} \alpha^{-\lambda/\rho_2} K^{-\lambda(1+\rho_1)/\rho_1} \left(1 + \psi K^{\phi-1}(1-s)^{-\rho_2/(1+\rho_2)}\right)^{-\lambda(1+\rho_2)/\rho_2} n^{-\varepsilon(1+\lambda)}(1-s)^{-\lambda}\right]^{1/A}$$

$$(3.9') \quad \tilde{x} = \left[\sigma(1-\varepsilon/n)\, \beta^{\varepsilon-1} \lambda^{\varepsilon} \alpha^{\lambda(\varepsilon-1)/\rho_2} K^{\lambda(\varepsilon-1)/\rho_1-\varepsilon} \left(1 + \psi K^{\phi-1}(1-s)^{-\rho_2/(1+\rho_2)}\right)^{\lambda(\varepsilon-1)/\rho_2-\varepsilon} n^{-\varepsilon}(1-s)^{-\varepsilon}\right]^{1/A}$$

$$(3.10) \quad \tilde{y} = \tilde{x}\, \psi K^{\phi}(1-s)^{1/(1+\rho_2)}$$

where $A = [\varepsilon - \lambda(1-\varepsilon)]$, $\psi = (\gamma/\alpha)^{1/(1+\rho_2)}$ and $\phi = (\rho_1 - \rho_2)/[\rho_1(1+\rho_2)]$

The per unit subsidy of basic research effectively reduces the cost to each firm of such expenditures. Accordingly, basic research expenditures rise with the subsidy rate, as long as that rate is positive and less than 100%, so that $\partial\bar{x}/\partial s > 0$. The impact of subsidies on development research expenditures on the other hand depends on the sign of the expression $\varepsilon\rho_2 + \lambda(1-\varepsilon)$. For elastic demand, they rise if $\rho_2 > 0$ or if $\rho_2 < 0$ and $\lambda > \varepsilon\rho_2/(\varepsilon - 1)$. For inelastic demand, they will only rise if $\rho_2 > 0$ and in addition $\lambda < \varepsilon\rho_2/(\varepsilon - 1)$. Furthermore, $\partial\bar{c}/\partial s < 0$ and $\partial\bar{q}/\partial s > 0$: subsidies reduce marginal production costs and thereby increase output. For elastic demand profits gross of R&D expenditures increase and so do net profits, as long as the increase in fixed costs is not too great.

The interesting question in this setting however is how basic research subsidies affect the impact of spillovers on the incentives to conduct basic research. Since the latter effect is given by the sign of $\partial\bar{x}/\partial\theta$, the impact of subsidies is given by $\partial^2\bar{x}/\partial\theta\partial s$. In chapter two we saw that $\partial\bar{x}/\partial\theta < 0$ and $\partial^2\bar{x}/\partial\theta\partial s < 0$. Spillovers acted as a disincentive to basic research and a subsidy to basic research reduced that disincentive.

In this chapter, with technology described by a cost function of the form given in expression (3.2), it was shown that spillovers could instead act as an incentive to invest in that part of R&D that is only partially appropriable. Take for example the case where demand is elastic, and where $\rho_1 > 0 > \rho_2$ and $\lambda < \varepsilon\rho_2/(\varepsilon - 1)$. This is a situation where borrowed basic research is more complementary with a firm's own basic research than with its development research, and where basic and development research are easily substitutable with each other while own and borrowed basic research inputs are not. Spillovers here act as a spur to further basic research expenditures ie $\partial\bar{x}/\partial\theta > 0$.

Simultaneously however, for the assumed range of parameter values, subsidies that are positive and less than 100% ($0 < s < 1$) encourage further that (positive) incentive ie $\partial^2 \tilde{x} / \partial\theta\partial s > 0$. In the context of the model in this chapter therefore, the effect of subsidies on the incentives to invest in basic research are no longer straightforward.

In setting a subsidy rate, the government is attempting to maximise the total surplus generated by the market. This surplus is the sum of the producer and consumer surpluses generated by the production of output Q, minus the cost of the subsidy, ie $TS=PS+CS-sX$. For $(\tilde{x}, \tilde{y}, \tilde{Q}/n)$, the maximum surplus generated by the market under the presence of spillovers and subsidies is $\tilde{TS}$. The optimal subsidy rate can then be calculated as the rate that will maximise this total surplus. Setting $\partial \tilde{TS} / \partial s = 0$ yields the optimum subsidy rate s^*:

$$(3.29) \qquad s^* = 1 - \frac{(n-\varepsilon)(A + [\varepsilon\rho_2 + \lambda(1-\varepsilon)]\omega)}{\varepsilon(n+1-\varepsilon)(1+\rho_2)K}$$

$$\text{where } \omega = \frac{1 + \psi K^{\phi}(1-s^*)^{1/(1+\rho_2)}}{1 + \psi K^{\phi-1}(1-s^*)^{-\rho_2/(1+\rho_2)}}$$

Despite the fact that in (3.29) s^* is not expressed in reduced form, a number of observations can be made. The optimal subsidy rate depends on a host of variables: the number of firms in the industry, the spillover rate, the elasticity/substitutability conditions amongst research inputs, and on the elasticities of demand and of cost reduction with respect to the knowledge generated through R&D expenditures. As a result, higher spillover rates in this case do not necessarily imply higher optimal subsidy rates. They do so only when spillovers act as a disincentive to basic research. If instead spillovers spur expenditures in the part of R&D that is only partially appropriable, then the optimal subsidy rate falls as the spillover rate rises. We therefore have $\partial s^*/\partial\theta > 0$ as $\partial\bar{x}/\partial\theta < 0$ and $\partial s^*/\partial\theta < 0$ as $\partial\bar{x}/\partial\theta > 0$ [24].

The importance of technology assumptions on the sign and size of basic research subsidy rates can be demonstrated by a simple example. Tables 2 and 3 below show how optimal subsidy rates vary with respect to the number of firms in the industry and the spillover rate under two different assumptions about elasticities of substitution between research inputs. In table 2, we assume that research inputs are easily substitutable with each other ($0 > \rho_1 > \rho_2$ or $s_{xx}>1$, $s_{Ry}>1$), while a firm's basic research is more complementary with the basic research of its rivals than with its own development research ($c_{xx}>c_{xy}$) [25]. With the exception of the case of the monopolist, for any given number of firms optimal subsidy rates increase with the spillover rate. For fully appropriable basic research or for a given spillover rate, they fall with the number of firms.

This last result is in contrast to the case in chapter two, where with own and borrowed basic research perfect substitutes, for a given positive spillover rate subsidy rates initially fall and then rose as the market became more fragmented. The rise was due to the effect of the spillover environment on cost reduction overwhelming the cost

[24] The sign of $\partial\bar{x}/\partial\theta$ (and therefore of $\partial s^*/\partial\theta$) is determined by the sign of the expression $D\{(1+\rho_2)[\varepsilon\rho_1+\lambda(1-\varepsilon)]+A(\rho_1-\rho_2)E\}$ where A, D and E are all positive. Thus for $\rho_1 \geq \rho_2$ and $\varepsilon\rho_1+\lambda(1-\varepsilon)>0$ we have $\partial\bar{x}/\partial\theta > 0$ and $\partial s^*/\partial\theta < 0$.

[25] The optimal subsidy rates in table 2 have been derived for ρ_1=-0.1 and ρ_2=-0.5 (s_{xx}=1.1, s_{Ry}=2); those of table 3 have been derived for ρ_1=2 and ρ_2=0.5 (s_{xx}=0.33, s_{Ry}=0.66). In both tables, we have assumed elasticities of demand and of cost reduction with respect to R&D of 0.8 and 0.2 respectively.

of subsidies. Here, because research inputs are not perfect substitutes, the cost of subsidies as the number of firms increases outweighs the effect of spillovers on cost reduction so that optimal subsidy rates decline.

Table 2

Optimal subsidy rates to basic research using (3.2): $0 > \rho_1 > \rho_2$

Firms	Spillover rate				
n	0.00	0.25	0.50	0.75	1.00
1	0.88	0.88	0.88	0.88	0.88
2	0.62	0.81	0.81	0.82	0.82
3	0.52	0.76	0.77	0.78	0.78
4	0.47	0.74	0.75	0.76	0.76
5	0.44	0.73	0.74	0.75	0.75

Table 3

Optimal subsidy rates to basic research using (3.2): $\rho_1 > \rho_2 > 0$

Firms	Spillover rate				
n	0.00	0.25	0.50	0.75	1.00
1	0.88	0.88	0.88	0.88	0.88
2	0.23	0.95	0.84	0.71	0.61
3	-0.27	0.75	0.37	0.12	0.00
4	-0.62	0.43	-0.11	-0.35	-0.45
5	-0.88	0.00	-0.48	-0.68	-0.76

Table 3 has been derived under the assumption that it is difficult to substitute between research inputs ($\rho_1 > \rho_2 > 0$ or $s_{xx}<1$, $s_{Ry}<1$), and where again a firm's basic research is assumed to be more complementary with the basic research of its rivals than with its own development research ($c_{xx}>c_{xy}$). The pattern of optimal subsidies is now altered. As before, optimal subsidy rates fall with the number of firms. In this case, they may also turn negative as the market becomes more fragmented. Higher spillover rates however do not necessarily imply higher optimal subsidy rates. Because of the low elasticities of substitution between research inputs and of the relative complementarity between own and borrowed basic research, spillovers act as an incentive to basic research expenditures. While therefore compared with a situation of full appropriability, spillovers imply higher optimal subsidies, a higher spillover rate leading to higher expenditures to basic research will also be associated with a lower optimal subsidy rate. In the example in question, with high spillovers and a fragmented market, the nature of the technology implies that the optimal policy is to tax basic research [26].

We summarise these results in the following proposition:

Proposition 7: **When cost and demand functions are given by (3.2) and (3.4), and for a given market structure and degree of appropriability, optimal subsidies in basic research are uniquely determined by ρ_1, ρ_1, and by the elasticities of demand and of cost reduction with respect to R&D expenditures. Optimal subsidy rates rise with the spillover rate (for given *n*) when spillovers act as a disincentive to basic research; they fall (and can turn negative) when spillovers spur such investments.**

[26] It should be noted that what determines whether $\partial s^*/\partial\theta$ is positive or negative is the absolute elasticity of substitution between own and borrowed basic research and between total basic research and development research ie whether $\rho_1, \rho_2 > 0$ or <0 (s_{xx}, s_{Ry} >0 or <0), rather than whether own basic research is more complementary with borrowed research than with own development research ie whether $c_{xx}>c_{xy}$ or $c_{xx}<c_{xy}$ (both tables were derived under the assumption that $c_{xx}>c_{xy}$).

V. An extended example

We will now illustrate the main findings of this and of the last chapter by developing a simple example aimed at bringing out the implications of spillovers for incentives and performance. Using the equilibrium values for (n, q, x, y) derived for the market case and for the socially-managed industry under the Cobb-Douglas cost function (2.14) of chapter two and the nested CES form of (3.2) given here, we simulate the effects of spillovers in different economic environments. The parameters for these simulations are chosen in order to obtain a more quantitative picture and explore some of the possibilities that arise in environments where part of basic research is inappropriable. The actual values of the parameters or of the variables derived lay no claim to reality in the sense of being empirically observed.

Assume for the moment that market structure is exogenous so that the number of firms n is given. Letting the spillover rate θ vary between 0 and 1 in increments of 0.25, tables 4 and 5 below demonstrate how basic research expenditures at firm level vary with the degree of appropriability and with the number of firms in the industry for the two alternative specifications of the cost function. Table 4 is derived for the case where "own" and "borrowed" basic research are perfect substitutes so that the cost function is given by (2.14). We have assumed $\alpha = 0.3$, $\gamma = 0.7$ and $\varepsilon = 0.8$. Table 5 is derived on the basis of the cost function given by (3.2). We have assumed $\alpha = 0.3, \gamma = 0.7, \varepsilon = 0.8$, and $\rho_1 = -0.1, \rho_2 = -0.2$, so that $0 > \rho_1 > \rho_2$. In this case therefore "borrowed" basic research is assumed to be more complementary with "own" basic research than with development research while the elasticity of substitution between basic research as a whole and development research is 1.25 and that between "own" and "borrowed" basic research 1.11. For all tables, and in order to simplify matters and concentrate on the main issues at hand, we have chosen $\beta = \sigma = \delta = 1$.

Table 4					
Basic research expenditures at firm level (using (2.14))					
Firms	Spillover rate				
n	0.00	0.25	0.50	0.75	1.00
1	0.017	0.017	0.017	0.017	0.017
2	0.041	0.033	0.028	0.024	0.021
3	0.034	0.022	0.017	0.013	0.011
4	0.027	0.015	0.011	0.008	0.007
5	0.021	0.011	0.007	0.005	0.004
6	0.014	0.008	0.005	0.004	0.003

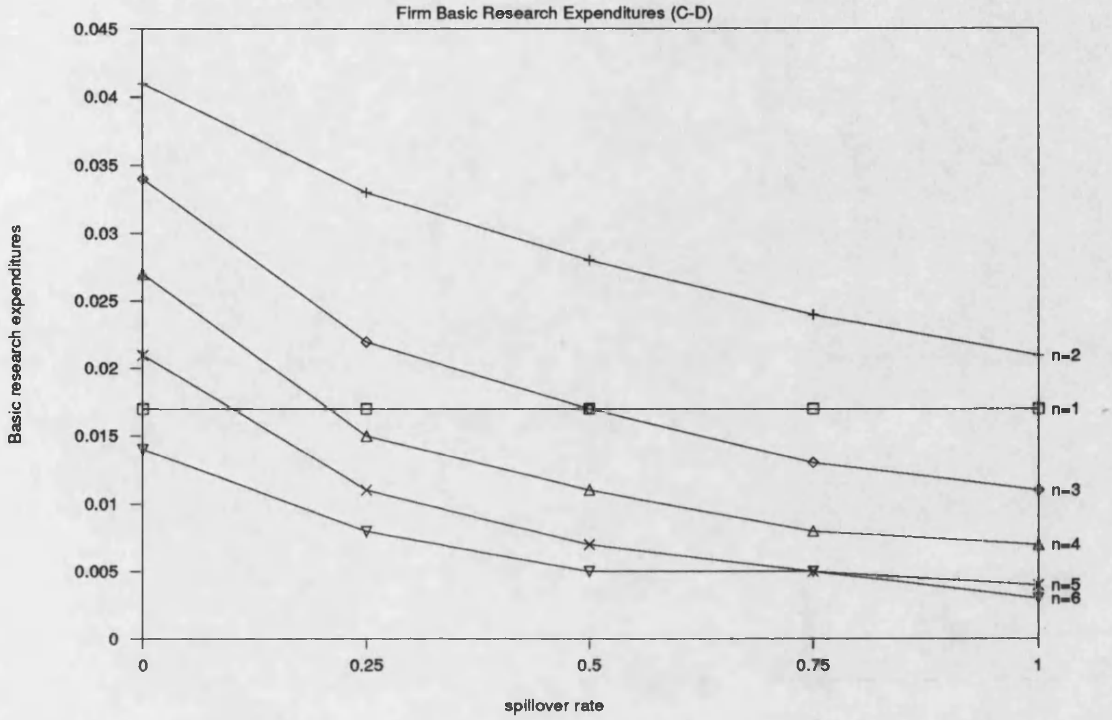
Table 5					
Basic research expenditures at firm level (using (3.2))					
Firms	Spillover rate				
n	0.00	0.25	0.50	0.75	1.00
1	0.014	0.014	0.014	0.014	0.014
2	0.032	0.070	0.072	0.074	0.076
3	0.026	0.060	0.062	0.064	0.065
4	0.021	0.049	0.051	0.052	0.053
5	0.017	0.041	0.043	0.044	0.045
6	0.014	0.035	0.036	0.037	0.038

The impact of different degrees of inappropriability can be seen by reading across rows, while for each spillover rate the effect of the fragmentation of the market is seen by reading down columns. In table 4, the disincentive effect of inappropriability on firm outlays on basic research is apparent for every $n>1$. When $n=6$ for example, expenditures when $\theta = 1$ are 16.6% of what they would be with full appropriability. Furthermore, while two firms each spend more than a monopolist, the fragmentation of the market implies that thereafter basic research outlays by each firm decline. In contrast, table 5 illustrates a case where spillovers act as a stimulus to basic research. A higher spillover rate here induces each firm to spend more on basic research. With n=6 for example, expenditures when $\theta = 1$ are 2.7 times the level with full appropriability. Furthermore, the higher the spillover rate, the slower is in this case the reduction in expenditures per firm as a result of the fragmentation of the market.

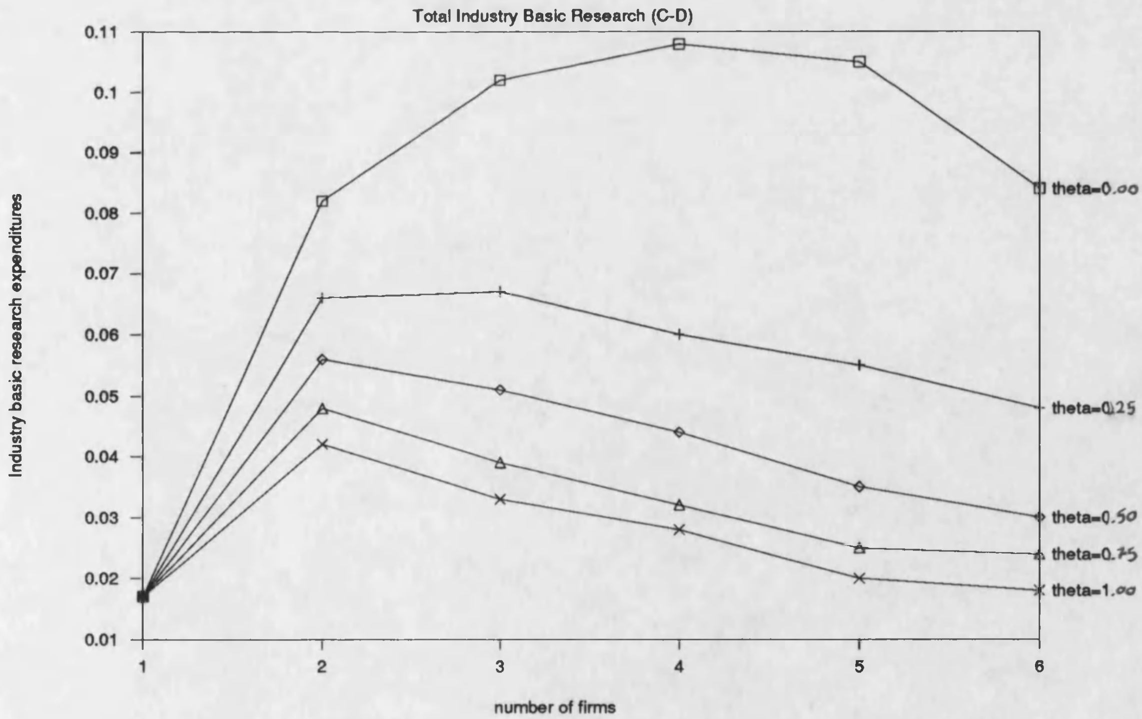
These results can be best visualised with the help of graphs. Graphs 1 and 2 below are constructed on the basis of the cost function given in (2.14) and graphs 3 and 4 on the basis of the nested CES cost function given in (3.2). Graph 1 gives the relationship between the spillover rate and a firm's basic research expenditures for different levels of concentration in the industry. With the exception of the case of the monopolist, where spillovers are by definition irrelevant, lower appropriability is associated with reduced effort in basic research whatever the number of firms in the market. In contrast, when the cost function allows for a higher complementarity between "borrowed" basic research and a firm's "own" basic research than between "borrowed" research and its development research (Graph 3), spillovers encourage basic research expenditures, although as the spillover rate approaches one, the incentive effect --although still positive-- diminishes, especially in the cases where the market is fragmented.

The effect of inappropriability and market fragmentation on *industry-wide* expenditures on basic research is depicted in graphs 2 and 4. Graph 2 represents the case where "own" basic research is a perfect substitute for "borrowed" basic research and where basic and development research enter the cost function with a unitary elasticity of substitution. With full appropriability, the total resources devoted to R&D increase with the number of firms up to a point where the fragmentation of the market and falling profit margins induce a fall in total expenditures in basic research. When spillovers reduce the ability of firms to appropriate rents, the decline in total industry-wide basic research effort occurs at a significantly higher degree of concentration for the industry. In most cases in effect, total basic research is at a maximum when the industry is represented by a duopoly.

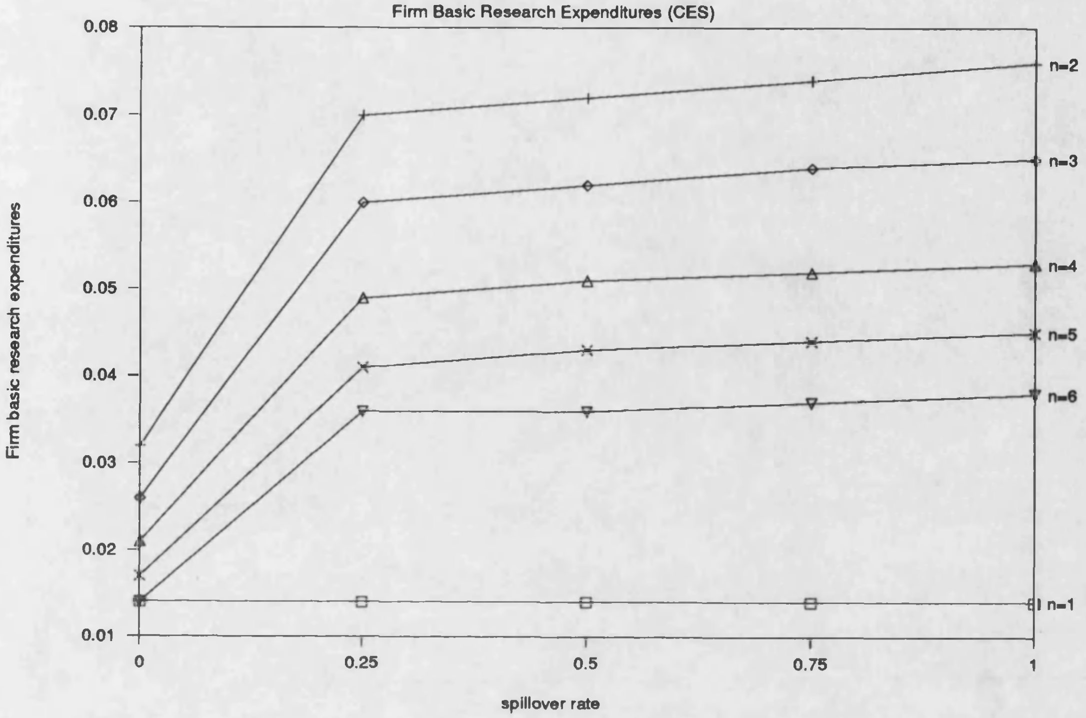
Graph 1



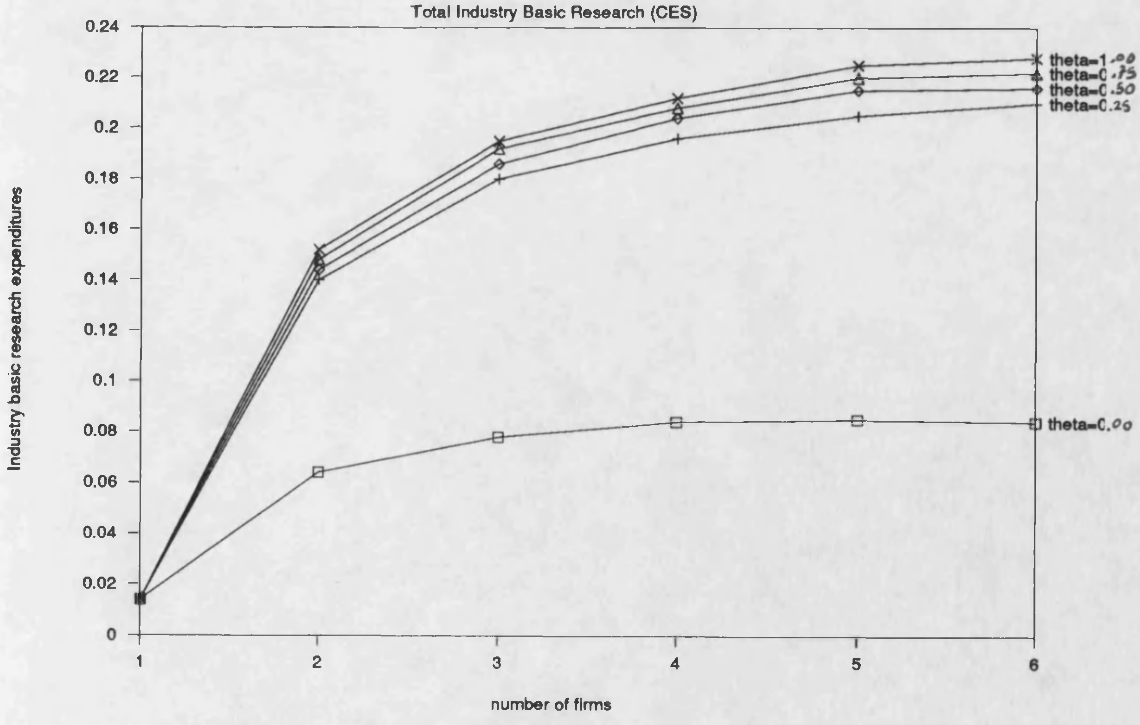
Graph 2



Graph 3



Graph 4



In the case of the nested CES cost function, the result is reversed. While with full appropriability, the fragmentation of the market induces a fall in total effort in basic research, the high relative complementarity between a firm's own basic research and that of its rivals reflected in the parameters chosen imply that the maximum level of total expenditures on basic research is reached at a level of market fragmentation greater than in the situation with full appropriability. Furthermore, industry-wide basic research expenditures increase with the spillover rate, although the rate of increase diminishes as n increases.

We next assume that market structure is endogenous so that the number of firms in the market is determined by the zero-profit condition (2.10') given in chapter two. Tables 6-11 examine the effect of differential degrees of appropriability on a number of variables in the market case and compare the market with the socially-managed industry. Table 6 has been derived on the basis of the Cobb-Douglas cost function represented in expression (2.13) while tables 7-11 have been derived using the nested CES cost function given in (3.2).

In all tables, column 1 gives the spillover rate. Column 2 shows the zero-profit equilibrium number of firms n^* in the market for different spillover rates. Column 3 is the industry-wide basic research expenditures in the market n^*x^* while column 4 is the optimal ratio of basic to development research expenditures x^*/y^*. Columns 5 and 6 give the total output Q^* and the price $P(Q^*)$ prevailing in the market, while column 7 shows the mark-up of price over marginal cost P/c, and is therefore a proxy for monopoly power. Column 8 represents R&D expenditures as a fraction of sales (R^*+D^*) and can be interpreted as a measure of the optimal R&D intensity.

Columns 9-11 compare the market with the socially managed industry. Column 9 shows total basic research in the market as a fraction of optimal basic research n^*x^*/n_sx_s

and column 10 is a ratio of unit production costs c_m/c_s. Finally, column 11 shows the performance of the market relative to the socially-managed industry (the ratio of the maximum surplus achievable in the market to the welfare optimum ie TS^*/W^*).

The parameters used in the derivation of the tables are chosen in order to illustrate some contrasting effects of inappropriability. In all tables, demand is assumed to be slightly elastic ($\varepsilon = 0.8$ or $1/\varepsilon = 1.25$), and $\beta = \sigma = \delta = 1$. In table 6, we let $\alpha = 0.05, \gamma = 0.1$. In tables 7-11, $\alpha = 0.3, \gamma = 0.7$ and we assume a cost elasticity with respect to total R&D (λ) of 0.2; we then vary the parameters that determine the substitutability and complementarity among R&D inputs. In tables 7-9 we assume that the elasticities of "own" with "borrowed" basic research and that of total basic research with development research are low (ie $\rho_1, \rho_2 > 0$ or $s_{xx} = 1/(1+\rho_1) < 1, s_{Ry} = 1/(1+\rho_2) < 1$); in tables 10 and 11, the reverse assumption is made ($\rho_1, \rho_2 < 0$ or $s_{xx} > 1, s_{Ry} > 1$.

We also examine the effect of alternative assumptions about the relative magnitudes of the elasticity of complementarity between "borrowed" and "own" basic research on the one hand and "borrowed" research with development research on the other. In table 7 we assume that ρ_1=4 and ρ_2=2 ($s_{xx} = 1/(1+\rho_1) = 0.2$, $s_{Ry} = 1/(1+\rho_2) = 0.33$) so that "borrowed" basic research enhances the marginal productivity of "own" basic research more than that of development research (c_{xx}>c_{xy}). In table 8 we assume $\rho_1 = \rho_2$=4 so that c_{xx}=c_{xy} and in table 9 ρ_1=0.5 and ρ_2=4 so that c_{xx}<c_{xy}. In tables 10 and 11 we reverse the assumption about ease of substitutability. With $\rho_1, \rho_2 < 0$ we have $s_{Ry}, s_{xx} > 1$ so that it is relatively easy to substitute one knowledge input for another. Table 10 is derived with $\rho_1 = -0.1$ and $\rho_2 = -0.5$ so that c_{xx}>c_{xy} while in table 11 $\rho_1 = -0.5$ and $\rho_2 = -0.1$ so that c_{xx}<c_{xy}.

Table 6: Free entry simulations with (2.14)										
1	2	3	4	5	6	7	8	9	10	11
θ	n^*	n^*x^*	x^*/y^*	Q^*	$P(Q^*)$	P/c	R^*+D^*	$\frac{n^*x^*}{x_s}$	c_m/c_s	TS/W^*
0.00	5	0.030	0.500	0.297	2.339	1.162	0.14	0.711	1.340	0.884
0.25	6	0.014	0.222	0.299	2.328	1.132	0.11	0.325	1.371	0.885
0.50	7	0.008	0.125	0.298	2.333	1.111	0.10	0.186	1.399	0.884
0.75	7	0.006	0.091	0.298	2.333	1.111	0.10	0.135	1.399	0.884
1.00	7	0.004	0.071	0.298	2.333	1.111	0.10	0.106	1.399	0.884

Table 7 : Free entry simulations with (3.2) and $\rho_1 > \rho_2 > 0$										
1	2	3	4	5	6	7	8	9	10	11
θ	n^*	n^*x^*	x^*/y^*	Q^*	$P(Q^*)$	P/c	R^*+D^*	$\frac{n^*x^*}{n_s x_s}$	c_m/c_s	TS/W^*
0.00	4	0.052	0.75	0.268	2.86	1.25	0.20	0.692	1.422	0.912
0.25	5	0.036	0.67	0.243	3.09	1.19	0.16	0.471	1.612	0.903
0.50	4	0.047	0.73	0.261	2.92	1.25	0.20	0.623	1.451	0.919
0.75	4	0.051	0.74	0.266	2.87	1.25	0.20	0.676	1.426	0.92
1.00	4	0.052	0.75	0.267	2.86	1.25	0.20	0.686	1.421	0.921

Table 8: Free entry simulations with (3.2) and $\rho_1 = \rho_2 > 0$										
1	2	3	4	5	6	7	8	9	10	11
θ	n^*	n^*x^*	x^*/y^*	Q^*	$P(Q^*)$	P/c	R^*+D^*	$\frac{n^*x^*}{n_s x_s}$	c_m/c_s	TS/W^*
0.00	4	0.056	0.84	0.267	2.871	1.25	0.20	0.696	1.419	0.911
0.25	5	0.039	0.84	0.24	3.124	1.19	0.16	0.487	1.622	0.902
0.50	4	0.051	0.84	0.26	2.937	1.25	0.20	0.632	1.452	0.918
0.75	4	0.055	0.84	0.266	2.884	1.25	0.20	0.681	1.426	0.92
1.00	4	0.056	0.84	0.267	2.875	1.25	0.20	0.691	1.421	0.921

Table 9: Free entry simulations with (3.2) and $\rho_2 > \rho_1 > 0$										
1	2	3	4	5	6	7	8	9	10	11
θ	n^*	n^*x^*	x^*/y^*	Q^*	$P(Q^*)$	P/c	R^*+D^*	$\frac{n^*x^*}{n_s x_s}$	c_m/c_s	TS/W^*
0.00	4	0.056	0.84	0.267	2.871	1.25	0.20	0.696	1.419	0.912
0.25	6	0.055	2.06	0.2	3.615	1.15	0.13	0.688	1.936	0.865
0.50	5	0.057	1.75	0.213	3.446	1.19	0.16	0.703	1.788	0.879
0.75	5	0.059	1.6	0.222	3.326	1.19	0.16	0.728	1.726	0.886
1.00	5	0.06	1.49	0.228	3.256	1.19	0.16	0.734	1.69	0.889

Table 10: Free entry simulations with (3.2) and $0 > \rho_1 > \rho_2$										
1	2	3	4	5	6	7	8	9	10	11
θ	n^*	n^*x^*	x^*/y^*	Q^*	$P(Q^*)$	P/c	R^*+D^*	$\frac{n^*x^*}{n_s x_s}$	c_m/c_s	TS/W^*
0.00	4	0.02	0.184	0.277	2.793	1.25	0.20	0.74	1.42	0.912
0.25	12	0.06	71.20	0.876	1.112	1.07	0.06	0.28	1.44	0.944
0.50	12	0.06	95.72	0.952	1.04	1.07	0.06	0.27	1.444	0.943
0.75	13	0.065	118.8	0.999	1.00	1.065	0.066	0.29	1.45	0.943
1.00	13	0.065	135.1	1.036	0.972	1.065	0.066	0.29	1.456	0.943

Table 11: Free entry simulations for (3.2) and $0 > \rho_2 > \rho_1$										
1	2	3	4	5	6	7	8	9	10	11
θ	n^*	n^*x^*	x^*/y^*	Q^*	$P(Q^*)$	P/c	R^*+D^*	$\frac{n^*x^*}{n_s x_s}$	c_m/c_s	TS/W^*
0.00	4	0.036	0.39	0.273	2.828	1.25	0.20	0.72	1.419	0.912
0.25	4	0.024	0.22	0.305	2.583	1.25	0.20	0.48	1.297	0.932
0.50	4	0.024	0.19	0.315	2.518	1.25	0.20	0.48	1.264	0.939
0.75	4	0.024	0.17	0.322	2.475	1.25	0.20	0.48	2.242	0.942
1.00	4	0.02	0.16	0.328	2.442	1.25	0.20	0.40	2.226	0.946

In Table 6, with "own" and "borrowed" basic research perfect substitutes, industry-wide R&D expenditures fall as the degree of inappropriability rises despite the fact that at higher spillover rates the number of firms spending on basic research has increased. The ratio of basic to development research in each firm's R&D budget falls from 50% to 7%. Price-cost margins and R&D intensity decline, to the extent that the number of firms in the industry has risen. Prices with imperfect appropriability are lower than with full appropriability of basic research, but rise with the spillover rate to reflect the higher unit production costs and lower output. The market underperforms the socially-managed industry, but the degree of inappropriability does not affect the ratio substantially. The lower fixed costs, due to the disincentive to spend on R&D as a result of the higher spillover rate, and the higher unit production costs, together with the higher prices at high spillover rates counterbalance each other so that the total effect of spillovers on performance is minimal.

Tables 7-11 illustrate how the impact of spillovers rests crucially on the assumptions about the technology. In contrast to Table 6, tables 7-9 show that a higher spillover rate can stimulate basic research and total R&D in the industry, even though the level of basic research expenditures in industries with no spillovers will tend to exceed the levels in industries where part of R&D leaks out. A higher spillover rate acts as a disincentive to basic research only in the case where "borrowed" basic research is highly complementary with development research and in addition it is relatively easy to substitute one input for another (table 11). Compared to a situation of full appropri-ability however, spillovers increase total basic research expenditures only in the case where "borrowed" basic research is highly complementary with a firm's own basic research and in addition it is relatively easy to substitute (Table 10). In that situation, the fact that the effective basic research available to the firm increases when some of the basic research of its rivals leaks out induces the firm to intensify its efforts in basic research and can even reverse the shares of basic and development research in total R&D.

Industrial concentration, R&D intensity and monopoly power all respond in a variety of ways to inappropriability, reflecting the assumptions underlying the cost function. In table 7 for example, where "borrowed" basic research enhances more the marginal productivity of "own" basic research than that of development research, but where it is relatively difficult to substitute among inputs, the number of firms that a free-entry equilibrium can sustain in the presence of spillovers exceeds that with full appropriability of basic research, but a higher spillover rate leads to a decline in the number of firms in equilibrium. Price mark-ups and R&D intensity initially fall but soon recover with a higher spillover rate. The initial impact of the inappropriability in basic research is to reduce total R&D expenditures at firm and industry level. If there were barriers to entry into the industry, this would have led to higher profits. In the free-entry equilibrium, it leads to a higher number of firms. As the spillover rate increases however, the complementarity that exists among R&D inputs causes each firm to spend more on both basic and development research. Unit production costs fall, but fixed costs and total costs rise for each firm in the industry, so that at the higher spillover rate, the equilibrium number of firms falls.

In the comparison of total resources devoted to basic research expenditures in the market with those in the socially-optimal case (ie n^*x^* vs. n_sx_s), tables 7-11 reveal that the impact of spillovers rests on the ease of substitutability among R&D inputs. Compared with a situation where basic research is fully appropriable, the extent of market underinvestment in basic research tends to be greater when this type of R&D is subject to spillovers. The ratio of market basic research expenditures to the socially optimal levels may however increase or decrease with the spillover rate. The ratio rises with the spillover rate in the cases where higher spillovers cause higher total basic research expenditures in the market due to entry and/or due to spillovers acting as a spur to basic research investment. This is the case for example in tables 7, 8 and 9 where research inputs are not easily substitutable with each other.

In contrast, tables 10 and 11, where elasticities of substitution exceed one, illustrate the possibility of higher spillover rates being associated with an increase in the underinvestment of the market in total basic research expenditures. In table 11 this is a reflection of spillovers acting as a disincentive for basic research outlays by existing forms in the market (there is no entry or exit as we vary the spillover rate). In table 10, where research inputs are easily substitutable with each other and where in addition "borrowed" basic research is more complementary with a firm's own basic research than with its development research, the ratio falls and then rises with the spillover rate. The fall is due to the socially-managed industry increasing its spending on basic research and on total R&D at higher spillover rates, so that the ratio of R&D expenditures in the two market structures declines. The rise in the ratio as the spillover rate approaches one is due to a higher equilibrium number of firms and of total market expenditures on basic research.

Finally, compared with the situation where basic research is fully appropriable, inappropriability increases the performance of the market relative to the welfare optimum when the substitutability amongst research inputs exceeds unity (Tables 10 and 11). Given that in a free-entry equilibrium the entire surplus generated in the market is consumer surplus, the higher total output and lower product price when appropriability is imperfect imply an increase in the relative performance of the market. A higher spillover rate may however reduce the ratio of the total market surplus to maximum welfare (table 10). This is despite the further increase in consumer surplus and is due to the fact that with the socially optimal number of firms exceeding one, maximum welfare increases with the spillover rate, so that the ratio TS/W^* may fall. In contrast, when it is difficult to substitute between R&D inputs, the relative performance of the market with spillovers can exceed or fall short to the case with full appropriability, while a higher spillover rate will increase the ratio TS^*/W^* (Tables 7-9). Output with spillovers is lower and price higher than with full appropriability, so that consumer surplus and market performance suffer, but a higher spillover rate may reduce marginal cost, increase output and thereby improve performance.

VI. Conclusions

If there is one central conclusion to be drawn from the analysis, it is that the impact of spillovers is not as straightforward as previously thought. Accordingly, most theoretical papers that have attempted to capture the inappropriability of some of R&D in models of strategic interaction amongst firms are in a sense misleading. By treating R&D as homogeneous and by ignoring possibilities of complementarity between the R&D performed by a particular firm and that of its rivals, as well as between different types of R&D (basic and applied), they have on the main arrived at the conclusion that spillovers act as a disincentive to R&D while enhancing profitability and market performance. The analysis of this chapter has shown that these results apply only for particular assumptions about the firm's technology, as revealed in its cost function, and may in fact be reversed under alternative assumptions about the manner in which R&D inputs combine to reduce costs.

The group of results presented here is consistent with technical advance in an industry being "cumulative" rather than "discrete" in the sense of innovations building on and enhancing existing knowledge, rather than replacing it. Since the model is entirely formulated in terms of expenditures per period, the essence of "cumulative" technical advance is captured by the supposition that the basic research of rivals that spills over and is absorbed can increase the marginal productivity of a firm's own basic research and of its development research at different rates. Furthermore, the elasticity of substitution between basic research and development research is allowed to vary in the manner of a two-level (nested) CES cost function as in (3.2). Under these assumptions, the effects of inappropriability depend critically on two factors: first, on the *relative complementarity* between "borrowed" and "own" basic research on the one hand and that between "borrowed" basic research and development research on the other; and secondly, the impact of spillovers depends on the (absolute) *ease of substitutability* between research inputs.

The main conclusions of this chapter are presented below. They are presented in a form that makes them directly comparable with the set of conclusions derived in chapter two of the thesis so as to bring out the effect of the alternative assumptions about the cost function on the impact of spillovers on incentives, market structure and performance.

(1') If the number of firms in an industry is taken to be exogenously determined, higher spillover rates will be associated with higher levels of basic and development research expenditures at firm level if R&D inputs are not easily substitutable with each other, if demand is elastic (or not too inelastic), and (for basic research only) if in addition "borrowed" basic research is at least as complementary with a firm's development research as it is with its own basic research. Higher spillover rates will always be associated with lower marginal production costs and higher output.

(2') In a cross-section of industries facing the same elasticity of demand and with the same opportunities of cost reduction through R&D, but differing in terms of the degree of appropriability of basic research, one would expect to observe that the industries that exhibit the least appropriability of basic research would be the ones most concentrated and with the strongest monopoly power when the elasticities of substitution between own and "borrowed" basic research and between total (both own and "borrowed") effective basic research and development research are low.

(3') In the free-entry equilibrium, R&D intensity in oligopolistic industries where basic research is fully appropriable will be at least as great as in industries where part of basic research is inappropriable. Industries facing a low degree of appropriability (a high spillover rate) will however be more R&D-intensive than industries where the degree of appropriability is high when the elasticities of substitution between own and "borrowed" basic research and between total (both own and "borrowed") effective basic research and development research are low.

(4') In the comparison of two free-entry oligopolistic industries where basic research is partially appropriable, the industry with the lowest degree of appropriability in basic research (the highest spillover rate) will be spending more on basic and development research expenditures if demand is elastic, if "borrowed" basic research is more complementary with a firm's own research than with its development research and if the elasticities of substitution between own and "borrowed" basic research and between total (both own and "borrowed") effective basic research and development research are low.

(5') As long as "borrowed" basic research is more (less) complementary with a firm's own basic research than with its development research and if in addition research inputs are easily substitutable with each other, a higher spillover rate will increase (decrease) the share of basic research expenditures in total R&D. Compared to the case with full appropriability however, industries with spillovers can be associated with a higher ratio of basic to development research expenditures even when "borrowed" basic research is more complementary with a firm's own basic research than with its development research, as long as it is difficult to substitute between R&D inputs.

(6') Each firm in a free-entry oligopolistic market where basic research is partially appropriable underinvests in basic research relative to the social optimum. The ratio of market to socially optimal levels of basic research may however increase or decrease with the extent of spillovers in basic research.

(7') Total basic research expenditures in the free-entry oligopolistic market, as well as total R&D, can be greater than is socially optimal, when basic research is only partially appropriable. Furthermore, while with full appropriability excessive duplication of basic research and of total R&D in a market economy are possible only when demand is highly inelastic, with spillovers they are possible even if demand is throughout elastic.

(8') When spillovers stimulate, rather than inhibit, basic research expenditures, market performance relative to the social optimum in the presence of inappropriability may fall short of the performance in the case where spillovers are absent. Furthermore, a higher spillover rate can reduce the relative performance of the market. Higher spillovers reduce unit production costs and expand output in the market, thus generating a higher producer surplus; relative performance can still suffer however if the cost reduction and output expansion in the socially-managed industry due to the less appropriable basic research is greater than that in the case of the market.

(9') A government policy of subsidising basic research will stimulate expenditures in the non-appropriable part of R&D. For a given market structure and degree of appropriability, optimal subsidy rates are determined by the ease of substitutability and the relative complementarity between research inputs, and by the elasticities of demand and of cost reduction with respect to R&D expenditures. Optimal rates increase with the degree of inappropriability only when spillovers act as a disincentive to investments in basic research; they decrease (and can turn negative) when instead spillovers spur such investments.

The comparison of the set of conclusions presented here with the one previously derived in chapter two indicate that even within the context of a simple model, it is not easy to generalise on the impact of spillovers in an oligopolistic industry. In a static model, devoid of risk and uncertainty, where product and R&D decisions on the part of firms are represented in a one-stage game, the importance of the nature of the technology facing individual firms was shown to be crucial in determining both firm behaviour and industry characteristics when part of R&D is not fully appropriable by the firm that originates it.

CHAPTER FOUR

RESEARCH SPILLOVERS AND STRATEGIC BEHAVIOUR

I. Introduction

In the preceding two chapters we developed a model of cost reduction through R&D. The model was aimed at examining the incentives of oligopolistic firms to invest in research and development and the resulting static and dynamic performance of industries where part of R&D is not entirely appropriable by the originating firm(s) and is instead subject to spillovers. We showed that the impact of spillovers depends crucially on the specific assumptions made about the firm's knowledge and technology and how these relate to the knowledge produced by its competitors. These assumptions and relationships were taken to be codified in the cost functions facing each firm.

One particular characteristic of the model developed in the second and third chapter of the thesis concerns the assumptions made about the behaviour of firms. We assumed that firms competed by deciding on R&D and output levels simultaneously in a one-stage game, taking the decisions of other firms as given. This produced a Nash equilibrium as a result of a Cournot game in quantities (quantities being represented in our case by the level of output and that of basic and development research expenditures). With the exception of some brief references in the footnotes, we did not examine the case where the firm's decision making is a multi-stage process, with decisions on R&D being determined before decisions on production.

Alternatively, firm decision-making can be modelled as a multi-stage process, where decisions on the amount to produce take place after decisions on R&D investment. This procedure gives rise to incentives on the part of firms to shift resources to the "sunk category" of costs in early stages so as to gain a strategic advantage in later stages. By perceiving such strategic considerations, firms depart from simple cost-minimisation. This chapter develops such a model where firms act strategically by competing in an oligopolistic environment and where basic research, development research and output levels are chosen successively in three stages. The framework therefore allows us to look closely at issues related to the interaction of strategic behaviour and inappropriability of research [1].

The model developed below follows closely the work of Brander and Spencer (1983a). They develop a two-stage model of competition in (fully appropriable) R&D and output for the case of a duopoly. The resulting strategic equilibrium is compared to the non-strategic equilibrium that arises from a Cournot game in R&D and output (ie a situation where firms decide on R&D levels and output simultaneously in a one-period game). They reach a number of conclusions. First, strategic behaviour induces each firm to spend more on R&D than is necessary for cost-minimisation so that the equilibrium with strategic behaviour involves some inefficiency. Secondly, the strategic equilibrium involves higher total R&D expenditures than the corresponding Cournot equilibrium. With perfect symmetry [2], this implies that each firm spends more on R&D in the strategic case than in the Cournot model. Third, at the equilibrium resulting from the two-stage (strategic) game output is higher, prices are lower, and each firm earns less profit than at the corresponding equilibrium of the Cournot (one stage) game.

We want to extend this model to the case where R&D is subject to externalities and where as a result part of R&D expenditures are not appropriable by the originating

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[1] An alternative approach to capturing interaction effects among firms and linking these to the degree of appropriability of research has been to develop "conjectural variations" models (see for example Levin and Reiss (1984)). In such models firms are assumed to anticipate and take into account their rivals' instantaneous responses to their own unobservable actions. This approach has been widely criticized in the literature for attempting to capture the notion of "reaction" (a concept of a dynamic nature) in an inappropriate one period static framework.

[2] Perfect symmetry in this context implies that in equilibrium, demand and cost functions, as well as expenditures on R&D, are identical for all firms in the market.

firm(s). Our interest is in making comparisons along two lines: (i) between the multi-stage equilibrium with spillovers and the multi-stage equilibrium with full appropriability; and (ii) between the strategic (multi-stage) equilibrium and the Cournot (one stage) equilibrium when in both cases R&D is subject to spillovers. The objective is to try and answer a number of questions that suggest themselves in the two comparisons. In the examination of the strategic case with and without spillovers, the question is whether or how spillovers affect the incentives of firms to invest in R&D and the performance of the industry when firms use R&D for strategies that are more complex than simple cost minimisation. In the comparison between the strategic and the Cournot equilibrium on the other hand, we want to address the question of whether the presence of spillovers alters the effect of strategic behaviour on R&D expenditures, output and profitability of industries. We want for example to examine whether, compared to the Cournot case, the strategic use of R&D increases the total amount of R&D undertaken, increases total output, lowers prices and reduces industry profit.

The structure of this chapter is as follows. In the next section we develop a three stage model of R&D with spillovers in general form. We examine the effects of differential assumptions about basic and development research inappropriability on its comparative statics properties and compare the resulting equilibrium with that of the non-strategic model. In section three the three-stage model is collapsed to two stages by assuming that research is homogeneous and that spillovers operate on the whole of R&D, rather than just on basic research. By specifying two different functional forms for the knowledge production function, we can then solve explicitly for the case of n symmetric firms and compare the strategic and Cournot equilibria, with and without spillovers. Section four has the concluding remarks.

II. A three-stage model of R&D with spillovers

In order to model the strategic use of R&D it is necessary to formulate a multi-period oligopoly model of R&D and production [3]. We choose to develop a three-period model in general form, although for the illustration of particular points and for the derivation of explicit solutions it will be collapsed into a two-stage model, by abandoning the distinction between basic and development research expenditures. The structure of the general model is as follows. In the first period/stage, each firm chooses its level of expenditures in basic research, which is assumed to be only partially appropriable. In the second period it decides on the level of expenditures on development research, with the basic research levels of all firms already determined and known to each other. Development research is taken to be fully appropriable by the firm that originates it. Finally, in the third period the representative firm chooses an output level, given the choices on basic and on development research that itself and its competitors have made in the previous two steps [4].

The model therefore involves a three-step equilibrium, where the equilibria in the first and second stages of the game are assumed to be the outcome of noncooperative Nash games in R&D levels and the final third step is resolved as a Cournot game [5]. In deciding on basic research levels in the first step firms form expectations regarding the Nash game in development research in the second step and the quantity game in output in the third step. Similarly, in deciding on their investment on development research in

[3] The model is an example of a multi-period model without time dependence (Friedman, 1986).

[4] The choice of competition in quantities rather than in price in the last stage of the model can be explained on a number of levels. A practical argument is that this allows a direct comparison with the results obtained in the one-stage models in chapters two and three. A conceptual argument is that it would be hard to see the advantages of introducing Bertrand competition when products are homogeneous. Finally, a technical argument is that "quantity precommitment and Bertrand competition yield Cournot outcomes" (Kreps and Sceinkman, 1983).

[5] For a discussion of the concept of the Nash noncooperative equilibrium, see Johansen (1982).

the second stage of the game, in addition to the knowledge of the basic research levels chosen by all firms in the first stage, firms form expectations on the quantity game in output to follow. We assume that in equilibrium their expectations are confirmed so that the outcome is an example of a "subgame perfect equilibrium" [6].

a. The basic model

Let the market be represented by rivalry between two firms, denoted *1* and *2*. (In some cases we use *i* and *j* instead; it is then understood that when *i* refers to firm *1*, *j* refers to firm *2*). Each firm *i* produces output q^i, has revenue E^i and costs C^i. R&D consists of basic research x_i and development research y_i. Basic research is subject to spillovers at the rate θ while development research is taken to be fully appropriable. For $\theta = 0$, appropriability of R&D is perfect. For $\theta = 1$, basic research is a pure public good and each firm can make full use of the basic research expenditures of its rival. For $0 < \theta < 1$, appropriability is imperfect, with each firm *i* benefiting from a proportion θx_j of the basic research expenditures of its rival [7]. Profit for firm *i* can then be written as:

$$\Pi^i(q^1, q^2; y_i, x_1, x_2) = E^i(q^1, q^2) - C^i(q^i, y_i, x_1, x_2) - y_i - x_i \tag{4.1}$$

The firms produce a homogeneous good and we assume that increasing the output of good *j* decreases the total and marginal revenue of firm *i* [8]. We therefore have that $\partial E^i/\partial q^j = E^i_j < 0$ and $\partial^2 E^i/\partial q^i \partial q^j = E^i_{ij} < 0$.

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[6] For a definition of a "subgame perfect equilibrium", see Friedman (1983).

[7] The emphasis of this chapter is on the difference that an alternative behavioural assumption --*strategic* behaviour-- has on the impact of inappropraiability. We will therefore assume that a firm can fully absorb the basic research knowledge of its rival that spills out. In the context of the models in chapters two and three, this is equivalent to assuming that the parameter of absorptive capacity, δ_i is set equal to one. *Nominal* and *effective* spillovers are therefore here one and the same and are represented by the parameter θ.

[8] The output of the two firms are in other words assumed to be *strategic substitutes*, in the sense used by Bulow, Genakopoulos and Klemperer (1985). This condition satisfies the requirements for stability in a simple Cournot game.

The marginal cost function is taken to be independent of output so that $C^i = c^i q^i$ with $\partial C^i/\partial q^i = c^i$ and $\partial c^i/\partial q^i = 0$. It includes all costs except for R&D, which is a fixed cost. Variable total and marginal costs decline with a firm's expenditures on own basic research and on development research ie $\partial C^i/\partial x_i = C^i_{x_i} < 0$ and $\partial C^i/\partial y_i = C^i_{y_i} < 0$ ($\partial c^i/\partial x_i = c^i_{x_i} < 0$ and $\partial c^i/\partial y_i = c^i_{y_i} < 0$), but at a declining rate ie $\partial^2 C^i/\partial x_i^2 = C^i_{x_i x_i} > 0$ and $\partial^2 C^i/\partial y_i^2 = C^i_{y_i y_i} > 0$ ($\partial^2 c^i/\partial x_i^2 = c^i_{x_i x_i} > 0$ and $\partial^2 c^i/\partial y_i^2 = c^i_{y_i y_i} > 0$). Because of the existence of spillovers in basic research however, they also decline with competitors' basic research ie $\partial C^i/\partial x_j = C^i_{x_j} < 0$ and $\partial^2 C^i/\partial x_j^2 = C^i_{x_j x_j} > 0$ ($\partial c^i/\partial x_j = c^i_{x_j} < 0$ and $\partial^2 c^i/\partial x_j^2 = c^i_{x_j x_j} > 0$). The issue of whether cost reduction due to own basic research exceeds that due to the rivals' basic research is left open at this stage and is taken up below in the context of particular functional forms.

Finally we assume that the effect of own development research on a firm's marginal cost reduction due to own basic research expenditures is positive, so that diminishing returns apply, ie that $\partial^2 C^i/\partial x_i \partial y_i = C^i_{x_i y_i} > 0$ (or $\partial^2 c^i/\partial x_i \partial y_i = c^i_{x_i y_i} > 0$). Similarly for the effect of rivals' basic research on a firm's marginal cost reduction due to own development research expenditures ie $\partial^2 C^i/\partial y_i x_j = C^i_{y_i x_j} > 0$ (or $\partial^2 c^i/\partial y_i x_j = c^i_{y_i x_j} > 0$. We leave however open the possibility that the effect of borrowed basic research on a firm's marginal cost reduction due to own basic research expenditures be positive or negative so that the sign of $\partial^2 C^i/\partial x_i \partial x_j = C^i_{x_i x_j}$ (and similarly of $\partial^2 c^i/\partial x_i \partial x_j = c^i_{x_i x_j}$) is not predetermined. This issue will be explored further below in the context of particular cost functions.

Following the usual procedure in these type of models, we solve recursively. At the *last stage*, and with basic and development research expenditures already sunk, the firm maximises its profit function with respect to its output. The first-order conditions for a maximum are then given by:

(4.2) $\Pi^i_i \equiv \partial\Pi^i/\partial q^i \equiv E^i_i(q^1,q^2) - c^i(y_i;x_1,x_2) = 0$

where $E^i_i = \partial E^i/\partial q^i$. The second-order conditions are satisfied for:

(4.3) $\Pi^i_{ii} \equiv \partial^2\Pi^i/\partial q^i \equiv E^i_{ii} < 0$

and for

(4.4) $\mathrm{A} \equiv \Pi^1_{11}\Pi^2_{22} - \Pi^1_{12}\Pi^2_{21} = E^1_{11}E^2_{22} - E^1_{12}E^2_{21} > 0$

Expression (4.4) holds if own output effects on profits exceed cross effects. In our case, with marginal cost constant, it will hold if own revenue effects exceed cross revenue effects. Together with expression (4.3), they guarantee that the profit function is strictly concave and relate to the stability of the reaction functions. At a symmetric equilibrium, it is also the case that $\Pi^j_{jj}(=E^j_{jj}) < \Pi^j_{ji}(=E^j_{ji})$.

The slope of the output reaction function can be derived by totally differentiating (4.2) to obtain $\Pi^i_{ii}dq^i + \Pi^i_{ij}dq^j = 0$, from which we have:

(4.5) $dq^i/dq^j = -\Pi^i_{ij}/\Pi^i_{ii} = -E^i_{ij}/E^i_{ii} < 0$

In its reduced form, the solution of the first-order conditions for the two firms depends on each firm's own basic and development research expenditures, as well as on the basic research expenditures of its rival through the respective marginal cost functions ie:

(4.6) $q^1 = q^1[c^1(y_1;x_1,x_2), c^2(y_2;x_1,x_2)]; \qquad q^2 = q^2[c^1(y_1;x_1,x_2), c^2(y_2;x_1,x_2)]$

The impact of inappropriability of basic research in the strategic model can be illustrated simply by the comparison of the effect of an increase in (fully appropriable) development research as opposed to that of an increase in (partially appropriable) basic research on a firm's output. By totally differentiating (4.2) with respect to q^1, q^2 and x_1, y_1 we obtain the system:

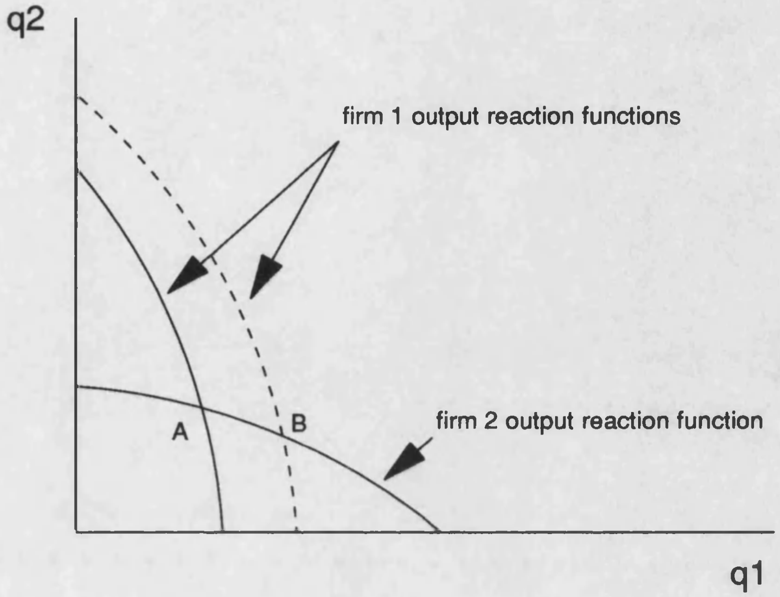
$$(4.7)\qquad \begin{aligned} E_{11}^1 dq^1 + E_{12}^1 dq^2 &= c_{x_1}^1 dx_1 + c_{y_1}^1 dy_1 \\ E_{21}^2 dq^1 + E_{22}^2 dq^2 &= c_{x_1}^2 dx_1 \end{aligned}$$

Holding the basic research of both firms and the development research of firm *2* constant, the effect on the output of firm *1* due to an increase in its development research is given by Cramer's rule as $q_{y_1}^1 = \partial q^1/\partial y_1 = c_{y_1}^1 E_{22}^2/\mathrm{A}$ where A>0 and given by (4.3). Since $E^2{}_{22}$<0 and $c_{y_1}^1 < 0$, then $q_{y_1}^1 > 0$. Similarly, $q_{y_1}^2 = -c_{y_1}^1 E_{21}^2/\mathrm{A}<0$. Because of symmetry, we can then conclude that in general terms:

$$(4.8)\qquad q_{y_i}^i > 0; \quad q_{y_i}^j < 0$$

An increase in development research by a particular firm will lower its marginal cost, shift out its reaction function and increase its output, for constant levels of development research by the other firm and for given levels of basic research. In the process it will decrease the output of its rival. The overall change is given by $q_{y_i}^i + q_{y_i}^j = c_{y_i}^i (E_{jj}^j - E_{ji}^j)/\mathrm{A}$. Since $E_{jj}^j < E_{ji}^j < 0$ and given that $c_{y_i}^i < 0$, it is the case that $q_{y_i}^i + q_{y_i}^j > 0$, so that total output increases as a result of higher development research expenditures. These results are illustrated in Figure 1 below. Starting with an initial equilibrium at point *A*, the increase in firm *1*'s development research expenditures shifts out its reaction function. The new equilibrium at *B* involves higher output for firm *1*, lower for firm *2*, and higher total output in the symmetric case.

Figure 1: Output effects of an increase in development research



Contrast now this result with the comparative statics of an increase in a firm's basic research expenditures. From the system (4.7), keeping the basic research of firm j, as well as development research by either firm constant, the effect of an increase in the basic research of firm i on its output is given by:

(4.9) $\quad q^i_{x_i} = \partial q^i / \partial x_i = \left(c^i_{x_i} E^j_{jj} - c^j_{x_i} E^i_{ij} \right) / \mathrm{A}$

The net marginal impact on output now consists of two separate effects going in opposite directions: the first is a positive effect due to the reduction in the firm's marginal cost which shifts its reaction function outwards; and the second is a negative effect due to the reduction in the marginal cost of firm j as a result of the higher basic research of firm i. This second effect is due to spillovers, it is due in other words to the fact that the basic research of firm i enters as an argument in the marginal cost function of its rival. It shifts the output reaction function of firm j outwards, so that the new equilibrium point lies at the intersection of the two new output reaction functions.

If, instead of being subject to spillovers, basic research was perfectly appropriable, the marginal effect on output would be positive, its sign being determined by $c^i_{x_i}E^j_{jj}$. Inappropriability of basic research however makes the sign of $q^i_{x_i}$ ambiguous in principle. In effect, in general terms, since A>0, we will have $q^i_{x_i} > 0$ as long as $c^i_{x_i}E^j_{jj} > c^j_{x_i}E^i_{ij}$. Since we have however established that in symmetry $E^j_{jj} < E^j_{ji}$, then it follows that it is sufficient for $c^i_{x_i} < c^j_{x_i}$ for $q^i_{x_i}$ to be positive. This latter requirement implies that a particular firm's basic research reduces its marginal cost by more than it reduces the marginal cost of its rival [9].

In order to explore the conditions for which a firm's basic research expenditures reduce the cost of its rival more than its own ie for which we have $c^i_{x_i} > c^j_{x_i}$, we will examine briefly the two parameterizations of the cost function used in the previous two chapters. In chapter two, the firm's marginal cost function involved a linear basic knowledge production function where own and rival basic research where substitutable at the rate 1/θ. We reproduce it here for firm i in the duopoly case [10]:

$$(4.10) \qquad c^i = \beta R_i^{-\alpha} y_i^{-\gamma} = \beta (x_i + \theta x_j)^{-\alpha} y_i^{-\gamma} \qquad \text{where} \quad \alpha, \beta, \gamma > 0$$

The marginal cost reduction of each firm due to the basic research expenditures of firm i is given by differentiation of (4.10) with respect to x_i and by differentiation of c^j with respect to x_i; they are $c^i_{x_i}$ and $c^j_{x_i}$ respectively. A comparison of the two shows that $c^i_{x_i} < c^j_{x_i}$ for $\theta < 1$: own basic research reduces a firm's marginal production cost by more

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[9] Note that this condition is sufficient, though not necessary. It is also possible to have $q^i_{x_i} > 0$, even if $c^i_{x_i} > c^j_{x_i}$, as long as it is still the case that $c^i_{x_i}E^j_{jj} > c^j_{x_i}E^i_{ij}$.

[10] Apart from setting n=2, the only other difference between (4.10) and (2.13) of chapter two is that here we have also set $\delta = 1$. As mentioned above, for the purposes of this chapter, we assume that firms can absorb all of the basic research of rivals that leaks out. There is therefore no distinction between *nominal* and *effective* spillovers.

than it does that of its rival as long as the spillover rate is smaller than its limit value of one [11]. The condition is therefore automatically satisfied in the case of this and any other cost function where basic research knowledge is of the form $R^i = f(x_i + \theta x_j)$.

The situation is different in the case where the cost function has a nested CES form, as explained in chapter three above. That particular specification becomes in the duopoly case for firm i:

$$(4.11) \qquad c^i = \beta \left\{ \alpha \left[x_i^{-\rho_1} + (\theta x_j)^{-\rho_1} \right]^{\rho_2/\rho_1} + \gamma\, y_i^{-\rho_2} \right\}^{\lambda/\rho_2}$$

Differentiation of (4.11) and of the marginal cost function of firm j with respect to x_i yields respectively $c^i_{x_i}$ and $c^j_{x_i}$. In this situation however, for $c^i_{x_i} < c^j_{x_i}$, the condition is $\theta^{-\rho_1} < 1$ [12]. This is satisfied for $\theta < 1$, but only as long as $\rho_1 < 0$. Since $s_{xx} = 1/(1+\rho_1)$ represents the direct elasticity of substitution parameter between own and "borrowed" basic research within the "nest" of the CES, and given that $-1 < \rho_1 < \infty$, it is implied that the condition is fulfilled only when $s_{xx} > 1$, ie only when the two are easily substitutable with each other. If instead $\rho_1 > 0$, that is if the basic research of a firm's rival is not easily substitutable with its own, then $s_{xx} < 1$ and the condition is violated resulting in $c^i_{x_i} > c^j_{x_i}$. If in addition $c^i_{x_i} E^j_{jj} < c^j_{x_i} E^i_{ij}$, then $q^i_{x_i}$ can be negative.

In the presence of spillovers therefore, the marginal effect of basic research on output in the strategic model is not necessarily positive. The different possibilities that are open are illustrated in Figure 2. Starting from an initial equilibrium at A, an increase in basic research by firm i reduces its marginal cost and shifts out its reaction function.

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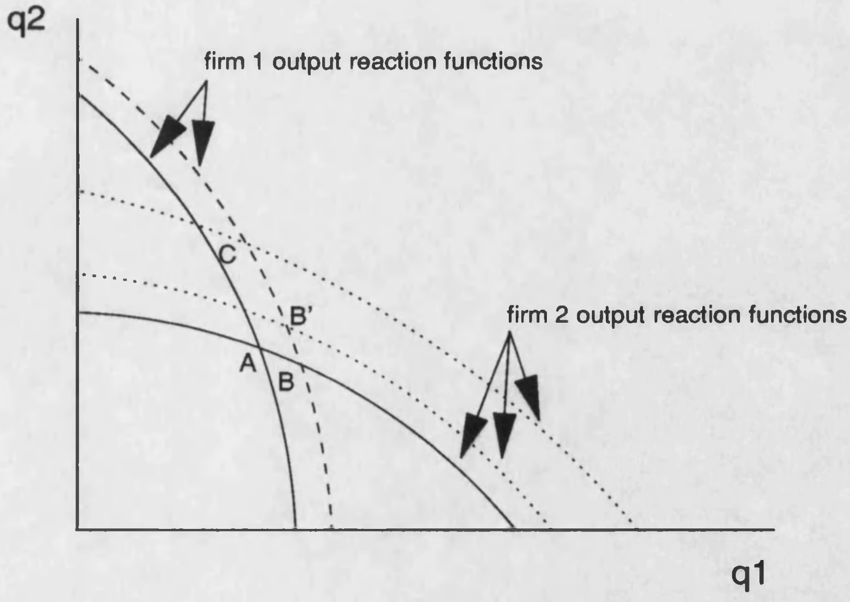
[11] For complete spillovers, ie for $\theta = 1$, $c^i_{x_i} = c^j_{x_i}$. Since however $E^j_{jj} < E^j_{ji}$, we still have $q^i_{x_i} > 0$.

[12] Let $c^i = \beta \tilde{f}_i^{-\lambda}$ where $\tilde{f}_i = \left[\alpha R_i^{-\rho_2} + \gamma y_i^{-\rho_2}\right]^{-1/\rho_2}$ and $R_i = \left[x_i^{-\rho_1} + (\theta x_j)^{-\rho_1}\right]^{-1/\rho_1}$. We then have $c^i_{x_i} = -\lambda\beta\alpha \tilde{f}_i^{\rho_2-\lambda} R_i^{\rho_1-\rho_2} x_i^{-\rho_1-1}$ and $c^j_{x_i} = -\lambda\beta\alpha \tilde{f}_j^{\rho_2-\lambda} R_j^{\rho_1-\rho_2} x_j^{-\rho_1-1} \theta^{-\rho_1}$ so that $c^i_{x_i} < c^j_{x_i}$ for $\theta^{-\rho_1} < 1$.

In the absence of spillovers in basic research, the new reaction function would give an equilibrium at *B* with higher output for firm *i*. If however basic research is subject to spillovers, an increase in basic research expenditures reduces the marginal cost of firm *j* and results in an outward shift of its output reaction function. The resulting output equilibrium will lie at the intersection of the two new reaction functions, both exhibiting lower production costs, with the new levels of output for the two firms depending on the relative shifts of the two curves. In the situation where the effect of the reduction in own costs is stronger, the new equilibrium point *B'* will lie between *A* and *B* and the output of firm *i* will expand. If instead $c^i_{x_i} > c^j_{x_i}$ and in addition $c^i_{x_i}E^j_{jj} < c^j_{x_i}E^i_{ij}$, the effect of the reduction in the marginal cost of its rival brought about by the firm's spillovers (the negative term in (4.8)) will be stronger. This will result in a new equilibrium point such as *C* and a fall in output for firm *i*.

Figure 2: Output effects of an increase in basic research



We can similarly examine the impact on a firm's output of an increase in the basic research expenditures of its rival (or equivalently the impact on a rival's output of an increase in a firm's basic research expenditures). Application of Cramer's rule yields $q^2_{x_1} = \partial q^2/\partial x_1 = \left(c^2_{x_1}E^1_{11} - c^1_{x_1}E^2_{21}\right)/\mathrm{A}$. In general terms therefore, $q^j_{x_i} < 0$ as long as $c^j_{x_i}E^i_{ii} < c^i_{x_i}E^j_{ji}$. Since by symmetry we have $E^i_{ii} < E^j_{ji}$, this implies that a necessary condition is $c^j_{x_i} > c^i_{x_i}$. It is not however a sufficient condition as it is possible for $c^j_{x_i} > c^i_{x_i}$ and still for $c^j_{x_i}E^i_{ii} > c^i_{x_i}E^j_{ji}$, giving $q^j_{x_i} > 0$. An increase in basic research expenditures by firm i therefore can in principle increase the output of firm j even in the situation where cost reduction through basic research benefits the originating firm more than it does its rival. This would be the case with point B' in Figure 2. They will certainly lead to a higher level of output for firm j if $c^j_{x_i} > c^i_{x_i}$ ie in the situation where, because of spillovers, basic research expenditures reduce the marginal costs of a firm's rival by more than they reduce the costs of the firm originating the expenditures.

The total effect of the two changes is given by:

$$(4.12) \qquad q^i_{x_i} + q^j_{x_i} = \left(c^i_{x_i}E^j_{jj} - c^j_{x_i}E^i_{ij} + c^j_{x_i}E^i_{ii} - c^i_{x_i}E^j_{ji}\right)/\mathrm{A}$$

$$= (c^i_{x_i}[E^j_{jj} - E^j_{ji}] + c^j_{x_i}[E^i_{ii} - E^i_{ij}])/\mathrm{A} > 0$$

From the second-order conditions above we established that in a symmetric equilibrium $\Pi^j_{jj}(=E^j_{jj}) < \Pi^j_{ji}(=E^j_{ji})$ (and similarly that $\Pi^i_{ii}(=E^i_{ii}) < \Pi^i_{ij}(E^i_{ij})$). It therefore follows that $E^j_{jj} - E^j_{ji} < 0$ and $E^i_{ii} - E^i_{ij} < 0$. Expression (4.9) is therefore positive ie industry-wide output increases as a result of higher basic research expenditures by firm i irrespective of spillovers (ie whether the disincentive effect dominates or not).

In the *second stage* of the game, and given the dependence of output on basic and on development research expenditures as in (4.6), profit can be written directly as a function of R&D. If we let this function be represented by f^i, it is given by:

$$
\begin{aligned}
(4.13) \qquad f^i \equiv\ & \Pi^i\{q^1[c^1(y_1;x_1,x_2),c^2(y_2;x_1,x_2)],q^2[c^1(y_1;x_1,x_2),c^2(y_2;x_1,x_2)]; \\
& C^i(q^i[c^1(y_1;x_1,x_2),c^2(y_2;x_1,x_2)];y_i;x_1,x_2);y_i,x_i\} \\
=\ & E^i(q^1[c^1(y_1;x_1,x_2),c^2(y_2;x_1,x_2)],q^2[c^1(y_1;x_1,x_2),(c^2(y_2;x_1,x_2)] \\
& -C^i(q^i[c^1(y_1;x_1,x_2),c^2(y_2;x_1,x_2)];y_i;x_1,x_2)-y_i-x_i
\end{aligned}
$$

Each firm maximises this function with respect to its development research expenditures, taking the development research expenditures of its rival as given. From (4.1), (4.2) and (4.13), the first-order conditions of the resulting Nash equilibrium are:

$$
\begin{aligned}
(4.14) \qquad f_i^i \equiv df^i/dy_i \equiv\ & \Pi_i^i q_{y_i}^i + \Pi_j^i q_{y_i}^j + \Pi_{c_i}^i C_{y_i}^i + \Pi_{y_i}^i = 0 \Rightarrow \\
=\ & E_j^i q_{y_i}^j - c_{y_i}^i q^i - 1 = 0
\end{aligned}
$$

since $\Pi_i^i = 0$ and $\Pi_j^i = E_j^i$. The second-order conditions are satisfied for:

$$(4.15) \qquad f_{ii}^i \equiv \partial f_i^i/\partial y_i \equiv E_j^i q_{y_i y_i}^j + q_{y_i}^j(\partial E_j^i/\partial y_i) - c_{y_i y_i}^i q^i - c_{y_i}^i q_{y_i}^i < 0$$

and for $A_1 \equiv f_{ii}^i f_{jj}^j - f_{ij}^i f_{ji}^j > 0$. In symmetry, this implies [13]:

$$(4.16) \qquad |f_{ii}^i| > |f_{ij}^i|$$

where $f_{ij}^i \equiv q_{y_i}^j(\partial E_j^i/\partial y_j) - c_{y_i}^i q_{y_j}^i$.

---

[13] Condition (4.16) states that own effects of development research on profit exceed cross effects. It implies reaction function stability in the second stage of the game and, together with (4.15), are necessary for the existence and uniqueness of equilibrium. As Brander and Spencer (1983) note however, these second-order conditions will not necessarily hold. Particularly, at least one term in (4.15), $q_{y_i}^j(\partial E_j^i/\partial y_i) = q_{y_i}^j\left(E_{ji}^i q_{y_i}^i + E_{jj}^i q_{y_i}^j\right)$, may be positive if $E_{jj}^i \geq 0$. Existence and uniqueness of equilibrium may therefore be a problem in multi-stage models. The conditions will however hold in both the second and first stages of the game if the marginal cost reducing of development and of basic research expenditures is strongly diminishing so that $c_{y_i y_i}^i$ and $c_{x_i x_i}^i$ respectively are large and positive.

The solution to the first-order condition (4.13) depends on the levels of basic research expenditures of each firm. We can therefore write:

$$(4.17) \qquad y^1 = y^1(x_1,x_2); \qquad y^2 = y^2(x_1,x_2)$$

The comparative statics of an increase in the basic research expenditures of one firm on its own development research expenditures and on that of its rival can be found by totally differentiating (4.14) with respect to q_1, y_1 and x_1. This yields the system:

$$(4.18) \qquad \begin{aligned} f_{11}^1 dy^1 + f_{12}^1 dy^2 &= (\partial f_1^1/\partial x_1)dx_1 \\ f_{21}^2 dy^1 + f_{22}^2 dy^2 &= (\partial f_2^2/\partial x_1)dx_1 \end{aligned}$$

From the system (4.18) by a simple application of Cramer's rule we have that $y_{x_1}^1 = \partial y_1/\partial x_1 = [(\partial f_1^1/\partial x_1)f_{22}^2 - (\partial f_2^2/\partial x_1)f_{12}^1]/A_1$. Since $A_1>0$, the sign of $y_{x_1}^1$ depends on the sign of $(\partial f_1^1/\partial x_1)f_{22}^2 - (\partial f_2^2/\partial x_1)f_{12}^1$. From the second-order conditions in symmetry $f_{22}^2 < f_{12}^1$ holds, and in addition $f_{22}^2 < 0$. In general terms therefore if $\partial f_i^i/\partial x_i > 0$, for $y_{x_i}^i < 0$ it is sufficient that $\partial f_i^i/\partial x_i > \partial f_j^j/\partial x_i$. If instead $\partial f_i^i/\partial x_i < 0$, then $\partial f_i^i/\partial x_i > \partial f_j^j/\partial x_i$ is a necessary (but not sufficient) condition for $y_{x_i}^i < 0$. Alternatively, for $y_{x_i}^i > 0$ a sufficient condition is $\partial f_i^i/\partial x_i < \partial f_j^j/\partial x_i$ if $\partial f_i^i/\partial x_i < 0$; it is a necessary (but not sufficient) condition if instead $\partial f_i^i/\partial x_i < 0$. Substituting from expression (4.14) however for each firm and manipulation of the resulting equations reveals that the satisfaction of these conditions depends largely on the sign of $c_{y_i x_i}^i - c_{y_j x_i}^j$ and $c_{x_i}^i - c_{x_i}^j$ [14].

[14] From (4.14) we have $\partial f_i^i/\partial x_i = E_j^i q_{y_j x_i}^j + (\partial E_j^i/\partial x_i)q_{y_i}^j - c_{y_i x_i}^i q^i - c_{y_i}^i q_{x_i}^i$ and $\partial f_j^j/\partial x_i = E_i^j q_{y_j x_i}^i + (\partial E_i^j/\partial x_i)q_{y_j}^i - c_{y_j x_i}^j q^j - c_{y_j}^j q_{x_i}^j$. In symmetry $E_j^i = E_i^j$, $E_{ij}^i = E_{ji}^j$ and $q_{y_i}^j = q_{y_j}^i$. Furthermore, given that $E^i = Pq^i$, we have that $\partial E_j^i/\partial x_i = P' q_{x_i}^i$ and $\partial E_i^j/\partial x_i = P' q_{x_i}^j$. We can then use the derived values of $q_{y_i}^j = -c_{y_i}^i E_{ji}^j/A$, $q_{x_i}^i = \left(c_{x_i}^i E_{jj}^j - c_{x_i}^j E_{ij}^i\right)/A$ and $q_{x_i}^j = \left(c_{x_i}^j E_{ii}^i - c_{x_i}^i E_{ji}^j\right)/A$ and further derive $q_{y_j x_i}^i = -c_{y_j x_i}^j E_{ij}^i/A$ and $q_{y_i x_i}^j = -c_{y_i x_i}^i E_{ji}^j/A$ (ignoring third-order effects on earnings). Substitution then

Put another way, these conditions imply that an increase in a firm's basic research expenditures will induce it to also increase its development expenditures in the following two cases. First, if an increase in its basic research *reduces* the marginal profitability of its development research and does so at a higher rate than the marginal profitability of its rival's development research (which may in fact increase). Second, if its additional basic research *increases* the marginal profitability of its development research but at a lower rate than that of its rival. In this second case, in order for the inducement for higher (marginal) development research expenditures to be present, the negative effect of the rival's increase in output on its marginal product must be low.

Similarly, the effect of an increase in the basic research expenditures of one firm on the equilibrium development research expenditures of its rival are given by $y^j_{x_i} = \partial y_j/\partial x_i = [(\partial f^j_j/\partial x_i)f^i_{ii} - (\partial f^i_i/\partial x_i)f^j_{ji}]/A_1$. In an analogous manner therefore, for $y^j_{x_i} < 0$, if $\partial f^j_j/\partial x_i > 0$ it is sufficient that $\partial f^i_i/\partial x_i < \partial f^j_j/\partial x_i$; if instead $\partial f^j_j/\partial x_i < 0$, $\partial f^i_i/\partial x_i < \partial f^j_j/\partial x_i$ is a necessary (though not sufficient) condition. For $y^j_{x_i} > 0$ on the other hand, $\partial f^i_i/\partial x_i > \partial f^j_j/\partial x_i$ is a necessary condition when $\partial f^j_j/\partial x_i > 0$ and a sufficient condition when $\partial f^j_j/\partial x_i < 0$.

The impact of a marginal increase in a firm's basic research expenditures on its own and on its rival's equilibrium development research expenditures rests therefore with the specification of the cost function. We noted earlier that for a cost function as in (4.10) where own and rival basic research are perfect substitutes, the condition

---

gives $\partial f^i_i/\partial x_i = -c^i_{y_i x_i}[q^i + E^i_j(E^j_{ji}/A)] - c^i_{y_i} q^i_{x_i}[1 + P'(E^j_{ji}/A)]$ and $\partial f^j_j/\partial x_i = -c^j_{y_j x_i}[q^j + E^j_i(E^i_{ij}/A)] - c^j_{y_j} q^j_{x_i}[1 + P'(E^i_{ij}/A)]$. Assuming that the sign of $\partial f^i_i/\partial x_i$ is determined by the sign of $q^i_{x_i}$, for $q^i_{x_i} > 0$, we have that $\partial f^i_i/\partial x_i > 0$. A sufficient condition however for $q^i_{x_i} > 0$ is that $c^i_{x_i} < c^j_{x_i}$. It therefore follows that a sufficient condition for $y^i_{x_i} < 0$ is that $c^i_{x_i} < c^j_{x_i}$ and in addition that $\partial f^i_i/\partial x_i > \partial f^j_j/\partial x_i$. This latter condition is satisfied when $q^j_{x_i} < 0$.

$c^i_{x_i} < c^j_{x_i}$ is satisfied for $\theta < 1$. The same limit value of the spillover rate also satisfies the condition $c^i_{y_i x_i} > c^j_{y_j x_i}$. Under these assumptions about the technology therefore, an increase in one firm's basic research will decrease its development research and increase that of its rival.

In the situation where the cost function has a nested CES form as in (4.11), for both $c^i_{x_i} < c^j_{x_i}$ and $c^i_{y_i x_i} > c^j_{y_j x_i}$ the condition is $\theta^{-\rho_1} < 1$. This is satisfied for $\theta < 1$ as long as $\rho_1 < 0$. If instead $\rho_1 > 0$ ie if the basic research of a firm's rival is not easily substitutable with its own, then both conditions are violated and an increase in one firm's basic research can increase its development research and reduce that of its rival.

In the light of (4.9) and of (4.17), profit can be written as a function of basic research expenditures only. Letting this function be represented by g_i and using the reduced form solutions obtained in the other stages:

$$
\begin{aligned}
(4.19)\quad g^i \equiv E^i\{ & q^1(c^1(y_1(x_1,x_2);x_1,x_2),(c^2(y_2(x_1,x_2);x_1,x_2)),\\
& q^2(c^1(y_1(x_1,x_2);x_1,x_2),(c^2(y_2(x_1,x_2);x_1,x_2)\}\\
& - C^i(q^i(c^1(y_1(x_1,x_2);x_1,x_2),(c^2(y_2(x_1,x_2);x_1,x_2));y_i(x_1,x_2),x_1,x_2))\\
& - y_i(x_1,x_2) - x_i
\end{aligned}
$$

In the *first stage*, the Nash equilibrium of the strategic game occurs where each firm is maximising its profit with respect to its basic research expenditures, given the level chosen by its rival. From (4.1), (4.2), (4.10) and (4.14), the first-order condition for a maximum for firm i is:

$$
\begin{aligned}
(4.20)\qquad g^i_i \equiv dg^i/dx_i & \equiv E^i_i\left(q^i_{y_i} y^i_{x_i} + q^i_{x_i}\right) + E^i_j\left(q^j_{y_i} y^i_{x_i} + q^j_{x_i}\right) - \left(C^i_{y_i} y^i_{x_i} + C^i_{x_i}\right) - y^i_{x_i} - 1 = 0\\
& \Rightarrow y^i_{x_i}\left(E^i_j q^j_{y_i} - c^i_{y_i} q^i - 1\right) + E^i_j q^j_{x_i} - c^i_{x_i} q^i - 1 = 0\\
& \Rightarrow E^i_j q^j_{x_i} - c^i_{x_i} q^i - 1 = 0
\end{aligned}
$$

since $E_i^i = \Pi_i^i = 0$ from (4.2) and $E_j^i q_{y_i}^j - c_{y_i}^i - 1 = f_i^i = 0$ from (4.11). The second-order conditions are given by:

(4.21) $$g_{ii}^i \equiv E_j^i q_{x_i x_i}^j + q_{x_i}^j (\partial E_j^i / \partial x_i) - c_{x_i x_i}^i q^i - c_{x_i}^i q_{x_i}^i < 0$$

and (in symmetry) by:

(4.22) $$|g_{ii}^i| > |g_{ij}^i|$$

where $g_{ij}^i \equiv E_j^i q_{x_i x_j}^j + q_{x_i}^j (\partial E_j^i / \partial x_j) - c_{x_i x_j}^i q^i - c_{x_i}^i q_{x_j}^i$.

b. Strategic vs. non-strategic behaviour

We initially compare basic and development research expenditures at firm and industry level in the three-stage model with those of the corresponding one-stage Cournot case. In the one-stage Cournot game firms set simultaneously R&D (both basic and development) and output in order to maximise profits:

(4.23) $$\underset{q_i, y_i, x_i}{\Pi^i} (q^1, q^2; y_i, x_1, x_2) = E^i(q^1, q^2) - C^i(q^i, y_i, x_1, x_2) - y_i - x_i$$

The first-order conditions are given by:

(4.24) $$\Pi_i^i \equiv \partial \Pi^i / \partial q^i \equiv E_i^i(q^1, q^2) - c^i(y_i; x_1, x_2) = 0$$

(4.25) $$\Pi_{y_i}^i \equiv \partial \Pi^i / \partial y^i \equiv -C_{y_i}^i - 1 \equiv -c_{y_i}^i q^i - 1 = 0$$

(4.26) $$\Pi_{x_i}^i \equiv \partial \Pi^i / \partial x^i \equiv -C_{x_i}^i - 1 \equiv -c_{x_i}^i q^i - 1 \equiv 0$$

The comparison of the two sets of first-order conditions ie of (4.2), (4.14) and (4.20) with (4.24)-(4.26) reveals that while (4.2) is identical to (4.24), the marginal benefits to conducting R&D are different in each case. In the strategic model the marginal benefit to spending on basic or on development research expenditures includes an extra term, reflecting the interaction of the two terms in the step-by-step decision-making. Higher R&D expenditures in the early stages of the game reduce a firm's marginal cost relative to that of its rival in the production stage. In the case of development research, this implies an increase in the output of the firm originating the development research expenditures, a fall in the output of its rival, and an increase in the first firm's profits [15]. This indirect effect is given by the extra term $E^i_j q^j_{y_i}$; it is positive given that $E^i_j < 0$ and $q^j_{y_i} < 0$. We can therefore conclude:

Proposition 1: **In the presence of spillovers in basic research, the marginal benefit of *development* research in the strategic game exceeds that of the Cournot game. Strategic behaviour therefore induces each firm and the total industry to spend *more* on development research than is required to minimise costs.**

Proof: The total cost of producing output q^i is $C^i+y_i+x_i$. With respect to development research expenditures, this cost is minimised for given q^i when $C^i_{y_i} + 1 = 0$, with a second-order condition $C^i_{y_i y_i} > 0$. In the strategic case however, we have from (4.14) that $c^i_{y_i} q^i + 1 = E^i_j q^j_{y_i} > 0$, since $E^i_j < 0$ and $q^j_{y_i} < 0$ by (4.8). Since $C^i_{y_i y_i} > 0$, this implies that development research expenditures exceed the level required to minimise costs. QED.

[15] A model with a similar structure and preoccupations to the one in this chapter is Anderson and Fischer (1989). They develop a two-stage model of a multi-market oligopoly with production taking place before sales and compare the resulting ("strategic") equilibrium with the equilibrium in a game where production and sales decisions are taken simultaneously. In the comparison between the two equilibria, they identify two conditions that are necessary for the two alternative behavioural assumptions to result in different equilibria. First, firms must have the *incentive* to try to alter the strategy (and thereby the production) of rivals (ie there must exist profitable opportunities in doing so); second, they must have the *ability* to do so. Both incentive and ability are reflected in the interaction terms. In the context of this model, the incentive aspect is reflected in $E^i_j < 0$ of (4.14) and (4.20); the ability is reflected in the term $q^j_{y_i} < 0$ of (4.14) for development research, and in the term $q^j_{x_i} < 0$ of (4.20) for basic research. If either $E^i_j = 0$ or both $q^j_{y_i}$ and $q^j_{x_i}$ are zero, the strategic equilibrium is identical to the Cournot equilibrium.

Turning to the partially appropriable basic research, in the Cournot case, the marginal benefit of investing in basic research is equal to the cost reduction achieved due to it. In the strategic case however there is an additional indirect effect and its impact is more complicated when there exist spillovers. In the absence of spillovers, this indirect effect would consist of the (negative) impact on the output of the firm's rival caused by the firm's own basic research expenditures due to the strategic interaction. This tends to increase the firm's own profits and thereby increase the marginal benefit of basic research expenditures.

In the presence of spillovers, the indirect effect due to the step-by-step decision-making is mitigated by the fact that own basic research expenditures enter the rival's cost function through the spillover rate and decrease those costs, thereby potentially reducing the profits of the firm undertaking the basic research in the first instance. These two opposing forces are reflected in the term $E^i_j q^j_{x_i}$. We have established that $E^i_j < 0$; in this case however the sign of $q^j_{x_i}$ is ambiguous and depends on the exact specification of the cost function. More specifically, we saw earlier that $q^j_{x_i}$ can be positive (ie an increase in own basic research expenditures leading to an expansion of production by the rival) in the situation where the marginal cost function is given by (4.11) and where in addition own and rival's basic research are poor substitutes [16]. In that situation, the term $E^i_j q^j_{x_i}$ is negative, so that the marginal benefit of investment in basic research in the strategic case can fall short of that in the Cournot case. We can therefore conclude:

---

[16] We can also have $q^j_{x_i} > 0$ when the marginal cost function is given by (4.10) (ie with $c^i_{x_i} < c^j_{x_i}$), as long as $c^j_{x_i} E^i_{ii} > c^i_{x_i} E^j_{ji}$.

Proposition 2: In the presence of spillovers in basic research, the marginal benefit of *basic* research in the strategic game may exceed or fall short of that in the Cournot game. In the situation where a firm's basic research and that of its rival are poor substitutes, strategic behaviour can induce each firm and the total industry to spend *less* on basic research than is required to minimise costs.

Proof: The total cost of producing output q^i is C^i+y_i+x_i. With respect to basic research expenditures, this cost is minimised for given q^i when $C^i_{x_i} + 1 = 0$, with a second-order condition $C^i_{x_i x_i} > 0$. In the strategic case however, we have from (4.20) that $c^i_{x_i} q^i + 1 = E^i_j q^j_{x_i}$. While $E^i_j < 0$, the sign of $q^j_{x_i}$ is ambiguous. When $q^j_{x_i} > 0$, we have that $c^i_{x_i} q^i + 1 = E^i_j q^j_{x_i} < 0$. Since $C^i_{x_i x_i} > 0$, this implies that development research expenditures fall short of the level required to minimise costs. QED.

These results require some elaboration. They are discussed below, by examining in turn the incentives to conduct R&D in the absence and in the presence of spillovers for both the Cournot and the strategic case. In this discussion, in order to simplify matters and to focus attention on the forces in operation, no distinction is made between basic and development research expenditures. The whole of R&D (denoted x) is assumed to be either fully appropriable by the originating firm or subject to spillovers.

In the Cournot case each firm attempts to maximise profits by setting simultaneously the levels of R&D expenditures and of production. Assume initially that there are no spillovers in research. Since R&D expenditures reduce production costs, each firm has an incentive to invest in such cost reduction, the incentive increasing with the elasticity of cost reduction with respect to R&D. This marginal benefit to the firm due to research expenditures is given by $-C^i_{x_i}$ and is positive.

Assume now that the firm perceives strategic considerations beyond simple cost minimisation. These strategic considerations arise from the fact that decisions on the level of expenditures in R&D precede decisions on output levels. Given that R&D costs are fixed costs, by investing in R&D firms can reduce their marginal costs in the later (production) stage of the game and thereby gain an advantage. Compared to the simple cost minimisation of the Cournot case therefore, the cost-reducing incentive to spend in research is enhanced in this case and in addition to $-C^i_{x_i}$, the marginal benefit to a firm of R&D investment is increased by $E^i_j q^j_{x_i}$. Given that both E^i_j and $q^j_{x_i}$ are negative (a decrease in a rival's output increases a firm's earnings; an increase in own R&D shifts out a firm's reaction function and reduces the output of the rival), the total marginal benefit due to R&D investment is positive and exceeds that in the Cournot case. Each firm ends up spending more on R&D than they would if they were cost-minimisers.

It should be noted here that given the assumption of perfect symmetry and simultaneous moves, in equilibrium no firm actually enjoys the advantage that motivated the strategic behaviour in the first place: we have a classic Prisoner's dilemma [17]. The strategic equilibrium is a Nash equilibrium that leaves each firm worse off relative to the simple Cournot rule, though still better off than if it had chosen to invest on R&D so as to minimise costs, while its rival was acting in a strategic manner.

Looking now again at the Cournot case, assume that research is not entirely appropriable by the firm that generates it. Furthermore, assume initially that the research of each firm is a perfect substitute to the research of its rival that comes into the public domain. In a symmetric equilibrium, the original benefit of R&D investment is now less than in the case where basic research was fully appropriable (ie $-C^{i,\theta=0}_{x_i} > -C^{i,\theta>0}_{x_i}$). This is because the firm knows that by benefiting from its rival's

[17] In recognizing the Prisoner's dilemma nature of the strategic equilibrium, Brander and Spencer also note that the firm's suboptimal behaviour is reinforced by the "once and for all" aspect of rivalry through R&D. In a repeated game, an incentive to collude and move to the joint optimum might develop in time.

research through the existence of spillovers it can minimise production costs by spending less on R&D than before. In equilibrium therefore, each firm ends up spending less on R&D than they would have done if that were perfectly appropriable.

If, rather than being perfect substitutes, the research of each firm was a poor substitute (or a "complement") to its rival's research, the effects of inappropriability can be reversed. Here, the knowledge that a rivals' effort in R&D increases a firm's knowledge pool (and thereby decreases its production costs) can intensify the firm's own efforts in research. The marginal benefit of R&D expenditures in the presence of spillovers then exceeds that with full appropriability of R&D (ie $-C_{x_i}^{i,\theta=0} < -C_{x_i}^{i,\theta>0}$). Rather than acting as a disincentive, the existence of inappropriability in R&D in this situation can therefore enhance the incentive to invest in this type of cost reduction.

How do these incentives and disincentives due to spillovers in the Cournot environment translate in the strategic case? In the strategic case with no spillovers, R&D affected profits in two ways: in a direct way, since R&D reduces marginal production costs and therefore increases profits (by E_i^i); and in an indirect way, since R&D reduces a rival's production in a later stage and therefore increases again the firm's profits (by E_j^i). Now consider spillovers. In the strategic case, they also have two ways of affecting profits. The first is direct and is common to both the Cournot and strategic setups. Spillovers reduce a firm's costs because they make available to it part of the R&D expenditures of its rival.

There is however also an indirect route as well, present only when strategic considerations guide firm behaviour. In the strategic case, a firm perceives (and can act upon the fact that) its own R&D expenditures enter as arguments in the marginal cost function of its rival. Given the spillovers, higher R&D expenditures on its part increase its rival's knowledge pool, reduce its marginal production cost and expand its output.

This results in lower profits for the firm originating the R&D. This indirect effect of spillovers runs counter to the indirect effect due to the strategic nature of the environment. The strategic effect says that higher R&D shifts out a firm's reaction function, reduces the rival's output and increases the firm's profits; the spillover effect says that higher R&D increases the rival's output (by reducing its production costs and shifting out its reaction function) and reduces the firm's profit. The question of which effect dominates depends in large part on the assumptions about the technology.

III. An explicit solution to a two-stage model with *n* symmetric firms

In order to make the comparisons between the strategic and the Cournot cases with and without spillovers more concrete, we solve in the strategic case explicitly for the levels of R&D and output in a two-stage model. The three-stage framework developed above in general form is collapsed here into two stages by assuming that R&D is homogeneous, so that there is no distinction between basic and development research expenditures. In the first stage each firm chooses its total R&D investment x_i ;in the second stage, with R&D levels already determined and known, each firm chooses its production level q^i. We assume that spillovers operate on the entire R&D outlays. This simplifying assumption is at the expense of being able to look at changes in the composition of R&D relative to changes in appropriability conditions; it does however make the model more tractable and allows explicit solutions to be derived for the case where the industry is composed of *n* firms, rather than looking only at the duopoly case.

Solving recursively, at the second stage each firm chooses its production level q_i so as to maximise its profit $\Pi^i = \left[P\left(q^i + \sum_{j \neq i} q^j\right) - c^i\right]q^i - x^i$. We assume that the demand function is of the iso-elastic form $P = \sigma Q^{-\varepsilon}$. In symmetry, the resulting equilibrium output is a function of the firm's and its rivals' cost functions and is given by:

$$(4.27) \qquad q^i = [\sigma(n-\varepsilon)]^{1/\varepsilon} \frac{\left[\sum_i c^i - c^i(n-\varepsilon)\right]}{\varepsilon\left(\sum_i c^i\right)^{(1+\varepsilon)/\varepsilon}}$$

a. Perfect substitution in the knowledge production function

In the first stage, in order to get explicit solutions, we need to assume a particular functional form for the cost function. As before, we will derive solutions based on two such parameterizations. In the first, we assume that the part of the research of a firm's rivals that spills out is a perfect substitute for the firm's own research. The resulting knowledge production function R_i that is specific to the firm enters into its production cost function with an iso-elastic form, the elasticity given by α. We therefore have:

$$(4.28) \qquad R_i = x_i + \theta \sum_{j \neq i} x_j \quad \text{and} \qquad c^i = \beta R_i^{-\alpha}$$

Under this parameterization of the cost and knowledge production functions, we can explicitly solve for the equilibrium level of R&D in the strategic case with positive spillovers. Let this be represented by $x_{ST}^{\theta>0}$. It is given by:

$$(4.29) \qquad x_{ST}^{\theta>0} = [\beta^{\varepsilon-1}\sigma(\alpha/n)^{\varepsilon}(1-\varepsilon/n)]^{1/A}\left[\frac{2n(n-\varepsilon-K)+(1+\varepsilon)K}{n(n-\varepsilon)}\right]^{\varepsilon/A}(1/K)$$

where $A = \varepsilon - \alpha(1-\varepsilon)$ and $K = 1 + \theta(n-1)$.

We can now compare the equilibrium level of firm R&D expenditures in the strategic case under the presence of spillovers with that in the absence of spillovers ($x_{ST}^{\theta=0}$), as well as with the same equilibria in the Cournot model (denoted $x_C^{\theta>0}$ and $x_C^{\theta=0}$ respectively). It is first easy to establish that $x_C^{\theta>0} < x_C^{\theta=0}$. In the Cournot equilibrium each firm spends less on R&D under the presence of spillovers than with full appropriability [18]. It is also easy to establish that in the situation where spillovers are absent, $x_{ST}^{\theta=0} > x_C^{\theta=0}$. The strategic equilibrium will involve higher R&D investment than the

[18] The equilibrium characteristics when firms are Cournot oligopolists were examined extensively in chapters two and three of the thesis in the context of a model that distinguished between (fully appropriable) development research and (partially appropriable) basic research. In the context of the model in this chapter, where R&D is homogeneous, the Cournot model consists of firms choosing simultaneously R&D and production levels so as to maximise profits. The resulting equilibrium level of firm R&D expenditures when some of R&D is not appropriable is given by $x_C^{\theta>0} = [\beta^{\varepsilon-1}\sigma(\alpha/n)^{\varepsilon}(1-\varepsilon/n)]^{1/A}(1/K)$. For $\theta = 0$, $K=1$ and the solution is identical to that derived in Dasgupta and Stiglitz (1980a) for the case with no spillovers.

Cournot equilibrium. This is an extension to the n-firm oligopoly of the duopoly result by Brander and Spencer (1983a) for the case where the knowledge production function and the corresponding cost function are given by (4.28) [19].

We turn next to the impact of spillovers on the equilibrium level of R&D in the strategic case. This involves the comparison of $x_{ST}^{\theta>0}$ with $x_{ST}^{\theta=0}$. From (4.29) we have:

$$(4.30) \qquad x_{ST}^{\theta>0} = x_{ST}^{\theta=0}\left[\frac{2n(n-\varepsilon-K)+(1+\varepsilon)K}{2n(n-\varepsilon)-2n+1+\varepsilon}\right]^{\varepsilon/A}(1/K)$$

Since $(1/K)<1$ and $A>0$ (by the second-order conditions for a maximum), it is sufficient for the term in brackets in (4.30) to be less than one for spillovers to lead to lower R&D in equilibrium in the strategic case. It can be checked that this is the case for $n > (1+\varepsilon)/2$. Since $n > \varepsilon$, this latter condition is satisfied, so that $x_{ST}^{\theta>0} < x_{ST}^{\theta=0}$ [20].

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[19] Setting $\theta=0$, we can see from (4.28) and from the footnote above that $x_{ST}^{\theta=0} = x_{C}^{\theta=0}\left[\frac{2n(n-\varepsilon-1)+(1+\varepsilon)}{n(n-\varepsilon)}\right]^{\varepsilon/A}$. For $x_{ST}^{\theta=0} > x_{C}^{\theta=0}$ it is necessary and sufficient for the term in brackets to be less than one. Upon manipulation, this reduces to the requirement that $n > 1+\varepsilon$. While however from first-order conditions $n > \varepsilon$ is satisfied for any positive level of output, the condition $n > 1+\varepsilon$ is more restrictive. In particular, when n=2, it implies that $x_{ST}^{\theta=0} > x_{C}^{\theta=0}$ only when demand is elastic ($1/\varepsilon > 1$ or $\varepsilon < 1$). As the number of firms increases, the condition is less restrictive; for n=4, for example, the condition is $\varepsilon < 3$, ie that the demand elasticity exceed 0.33. The comparison therefore of the equilibrium level of R&D expenditures in the strategic model with that in the Cournot model, when cost and demand functions have isoelastic forms, is sensitive to the assumed demand elasticity. This is problematic in the sense that it has no clear intuitive explanation; furthermore, it limits the generality of the results claimed by Brander and Spencer (1983a and 1983b). This criticism is also made elsewhere in the literature. In a recent paper Collie and de Meza (1986) have demonstrated "some unappealing implications of strategic models" in the context of a discussion of the strategic rationale for export subsidies; namely that for reasonable functional forms the results of such models (in their paper the case for export subsidies) rests on the elasticity of demand being greater than one. The purpose of this chapter however is to re-examine the implications of strategic models when R&D is not fully appropriable. We will therefore confine ourselves to the cases that satisfy the condition $n > 1+\varepsilon$, so that when research is fully appropriable, we have $x_{ST}^{\theta=0} > x_{C}^{\theta=0}$.

[20] Since $(1/K)<1$, $x_{ST}^{\theta>0} < x_{ST}^{\theta=0}$ if $\left[\frac{2n(n-\varepsilon-K)+(1+\varepsilon)K}{2n(n-\varepsilon)-2n+1+\varepsilon}\right] < 1$. Algebraic manipulation however reduces this to $n > (1+\varepsilon)/2$. For n>2 this latter condition is satisfied since for positive levels of output $n > \varepsilon$ by the first-order conditions of the last stage of the game. QED.

The comparison of the equilibrium levels of R&D in the strategic and the Cournot models when part of R&D is subject to spillovers concerns the comparison of $x_{ST}^{\theta>0}$ with $x_C^{\theta>0}$. With R&D fully appropriable, we obtained $x_{ST}^{\theta=0} > x_C^{\theta=0}$. Strategic considerations led to higher equilibrium levels in R&D than when firms were simply concerned with cost minimisation. In this case, with externalities in R&D:

$$x_{ST}^{\theta>0} = x_C^{\theta>0}\left[\frac{2n(n-\varepsilon-K)+(1+\varepsilon)K}{n(n-\varepsilon)}\right]^{\varepsilon/A} \tag{4.31}$$

Once more the result rests on whether the term in brackets is greater or less than one. This time however, the answer depends on the extent of inappropriability assumed. Simple algebraic manipulation yields that $x_{ST}^{\theta>0} > x_C^{\theta>0}$ for $\theta < (n-\varepsilon-1)/(2n-\varepsilon-1)$ and $x_{ST}^{\theta>0} < x_C^{\theta>0}$ for $\theta > (n-\varepsilon-1)/(2n-\varepsilon-1)$. With low spillovers, the strategic effect whereby high R&D expenditures in the first stage benefit the firm by placing it in a comparative advantage in the second (production) stage of the game outweighs the disincentive effect of inappropriability (due to the fact that the same level of cost reduction can be achieved with lower R&D than before). At low levels of appropriability however (high spillover rates), a given level of cost reduction can be achieved with a relatively much lower level of R&D expenditures, while the strategic aspect loses its power in that aggressive R&D investments now have a large (beneficial) impact on rivals. R&D externalities therefore reverse the result obtained in the situation where research is fully appropriable [21].

Finally, in order to be able to rank the R&D levels in the four possible equilibria (Cournot and strategic, with and without spillovers), we need to compare $x_{ST}^{\theta>0}$ with

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[21] As an example, for slightly elastic demand ($\varepsilon = 0.8$), $x_{ST}^{\theta>0} > x_C^{\theta>0}$ as long as $\theta < 0.10$ (for a duopoly) or $\theta < 0.43$ for an industry composed of 8 firms. The thresholds vary little as we change the elasticity of demand. For appropriability levels that exceed 50% therefore, basic research expenditures in the strategic case fall short of those in the Cournot case.

$x_C^{\theta=0}$. From the above it can be established that in general $x_{ST}^{\theta>0} < x_C^{\theta=0}$, except at very low spillover rates, where the reverse is possible [22]. We can therefore conclude that at very low spillover rates, $x_{ST}^{\theta=0} > x_{ST}^{\theta>0} > x_C^{\theta=0} > x_C^{\theta>0}$. At low spillover rates, $x_{ST}^{\theta=0} > x_C^{\theta=0} > x_{ST}^{\theta>0} > x_C^{\theta>0}$, while at moderate and high spillover rates $x_{ST}^{\theta=0} > x_C^{\theta=0} > x_C^{\theta>0} > x_{ST}^{\theta>0}$ will hold. The threshold levels of appropriability that determine the ranking depend on the concentration in the industry, the price elasticity of demand and on the elasticity of cost reduction with respect to R&D.

In a similar fashion, we can compare the levels of production at the four equilibria, as well as the profitability of the industry. On the basis of (4.29), we can derive from (4.27) the industry-wide equilibrium level of production in the strategic model when R&D is subject to spillovers, ie we can compute $Q_{ST}^{\theta>0}$ as:

(4.32) $$Q_{ST}^{\theta>0} = (x_{ST}^{\theta>0})^{1+\alpha}(n/\alpha\beta)\left[\frac{n(n-\varepsilon)}{2n(n-\varepsilon-K)+(1+\varepsilon)K}\right]K^{1+\alpha}$$

The equilibrium level of production in the strategic case when spillovers are absent ($Q_{ST}^{\theta=0}$) can be deduced from the above expression by setting K=1. Similarly the level of production in the Cournot model when spillovers are positive is given by $Q_C^{\theta>0} = (x_C^{\theta>0})^{1+\alpha}(n/\alpha\beta)K^{1+\alpha}$, while for fully appropriable R&D it is given by $Q_C^{\theta=0} = (x_C^{\theta=0})^{1+\alpha}(n/\alpha\beta)$ (which is the level of industry production in the Dasgupta and Stiglitz 1980a model).

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[22] $x_{ST}^{\theta>0} = x_C^{\theta=0}\left[\frac{2n(n-\varepsilon-K)+(1+\varepsilon)K}{n(n-\varepsilon)}\right]^{\varepsilon/A}(1/K)$. We saw that $\left[\frac{2n(n-\varepsilon-K)+(1+\varepsilon)K}{n(n-\varepsilon)}\right] < 1$ where $\theta > (n-\varepsilon-1)/(2n-\varepsilon-1)$. We can therefore have $x_{ST}^{\theta>0} > x_C^{\theta=0}$ only if $\theta < (n-\varepsilon-1)/(2n-\varepsilon-1)$ and if in addition $\left[\frac{2n(n-\varepsilon-K)+(1+\varepsilon)K}{n(n-\varepsilon)}\right]^{\varepsilon/A}(1/K) > 1$. As an example, for $\varepsilon = 0.8$, $\alpha = 0.1$ and n=4, the threshold of inappropriability beyond which we have $x_{ST}^{\theta>0} > x_C^{\theta=0}$ is a spillover rate around $\theta = 0.10$. For rates above 10%, in this specific example, $x_{ST}^{\theta>0} < x_C^{\theta=0}$ holds.

We can now proceed to obtain a ranking between these four equilibrium production levels. Within each model, the impact of spillovers is different. It can be immediately established that $Q_C^{\theta>0} = Q_C^{\theta=0}$ [23]. As was explained at length in chapter two, in the simple Cournot model, and with a cost function such as the one given by (4.28), the reduction in R&D induced by spillovers exactly matches the R&D that becomes available to the firm from its competitors. This implies that marginal production costs do not change as a result of the inappropriability of R&D ($c_C^{\theta>0} = c_C^{\theta=0}$) and neither do production levels. In contrast, in the strategic case we have that $Q_{ST}^{\theta>0} < Q_{ST}^{\theta=0}$ [24]. The reduction in R&D expenditures due to spillovers leaves firm marginal production costs higher than in the case where R&D is fully appropriable (ie $c_{ST}^{\theta>0} > c_{ST}^{\theta=0}$); with a downward sloping demand curve, production suffers as a result.

Comparing across the two models, in the situation where R&D is fully appropriable, we have that $Q_{ST}^{\theta=0} > Q_C^{\theta=0}$ [25]. This is the result of Brander and Spencer. Compared with the Cournot case, in the strategic case firms have an additional incentive to spend on R&D. As a result their marginal production costs are lower and industry output is higher. When spillovers exist, the result mirrors that for R&D expenditures obtained above. For low spillovers (ie $\theta < (n-\varepsilon-1)/(2n-\varepsilon-1)$) we have that $Q_{ST}^{\theta>0} > Q_C^{\theta>0}$ while for high spillovers (ie $\theta > (n-\varepsilon-1)/(2n-\varepsilon-1)$) we have that

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[23] $Q_C^{\theta>0} = (x_C^{\theta>0})^{1+\alpha}(n/\alpha\beta)K^{1+\alpha} = (x_C^{\theta=0})^{1+\alpha}(n/\alpha\beta) = Q_C^{\theta=0}$.

[24] $Q_{ST}^{\theta>0} = (x_{ST}^{\theta>0})^{1+\alpha}\frac{n}{\alpha\beta}\left[\frac{n(n-\varepsilon)}{2n(n-\varepsilon-K)+(1+\varepsilon)K}\right]K^{1+\alpha} = (x_{ST}^{\theta=0})^{1+\alpha}\left[\frac{2n(n-\varepsilon-K)+(1+\varepsilon)K}{2n(n-\varepsilon)-2n+1+\varepsilon}\right]^{\frac{\varepsilon(1+\alpha)}{A}}\frac{n}{\alpha\beta}\left[\frac{n(n-\varepsilon)}{2n(n-\varepsilon-K)+(1+\varepsilon)K}\right] =$

$= Q_{ST}^{\theta=0}\left[\frac{2n(n-\varepsilon-K)+(1+\varepsilon)K}{2n(n-\varepsilon)-2n+1+\varepsilon}\right]^{\alpha/A}$. Since the term in parentheses is less than one, we have $Q_{ST}^{\theta>0} < Q_{ST}^{\theta=0}$.

[25] $Q_{ST}^{\theta=0} = (x_{ST}^{\theta=0})^{1+\alpha}(n/\alpha\beta)\left[\frac{n(n-\varepsilon)}{2n(n-\varepsilon-1)+(1+\varepsilon)}\right] = (x_C^{\theta=0})^{1+\alpha}\left[\frac{2n(n-\varepsilon-1)+(1+\varepsilon)}{n(n-\varepsilon)}\right]^{\varepsilon(1+\alpha)/A}(n/\alpha\beta)\left[\frac{n(n-\varepsilon)}{2n(n-\varepsilon-1)+(1+\varepsilon)}\right] =$

$= Q_C^{\theta=0}\left[\frac{2n(n-\varepsilon-1)+(1+\varepsilon)}{n(n-\varepsilon)}\right]^{\alpha/A}$. Since the term in parentheses is greater than one, we have $Q_{ST}^{\theta=0} > Q_C^{\theta=0}$.

$Q_{ST}^{\theta>0} < Q_C^{\theta>0}$ [26]. It was established earlier that with low spillovers, $x_{ST}^{\theta>0} > x_C^{\theta>0}$: the strategic effect outweighs the disincentive effect of inappropriability. This leads to a lower level of production costs in the strategic case ($c_{ST}^{\theta>0} < c_C^{\theta>0}$) and to higher industry production. At low levels of appropriability however (high spillover rates), we established that $x_{ST}^{\theta>0} < x_C^{\theta>0}$. R&D externalities reversed the result obtained in the situation where research is fully appropriable. This implies that marginal costs are now higher in the strategic case ($c_{ST}^{\theta>0} > c_C^{\theta>0}$) and as a result industry output is lower.

A full ranking of the four industry output equilibria therefore is as follows. For low spillover rates (ie for $\theta < (n-\varepsilon-1)/(2n-\varepsilon-1)$) we have $Q_{ST}^{\theta=0} > Q_{ST}^{\theta>0} > Q_C^{\theta>0} = Q_C^{\theta=0}$. For moderate and high spillover rates (ie for $\theta > (n-\varepsilon-1)/(2n-\varepsilon-1)$), the ranking becomes instead $Q_{ST}^{\theta=0} > Q_C^{\theta>0} = Q_C^{\theta=0} > Q_{ST}^{\theta>0}$.

We now turn to the examination of firm profitability in the two models in the situation where R&D is fully appropriable and where it is instead characterised by spillovers. In each case, we can express individual firm profit as a function of R&D expenditures. In the strategic case with spillovers, profit in equilibrium is given by [27]:

$$\text{(4.33)} \qquad \Pi_{ST}^{\theta>0} = x_{ST}^{\theta>0}\left[\frac{\delta K\varepsilon - \alpha(n-\varepsilon)}{\alpha(n-\varepsilon)}\right] \qquad \text{where} \qquad \delta = \frac{n(n-\varepsilon)}{2n(n-\varepsilon-K)+(1+\varepsilon)K}$$

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[26] $Q_{ST}^{\theta>0} = (x_{ST}^{\theta>0})^{1+\alpha}\frac{n}{\alpha\beta}\left[\frac{n(n-\varepsilon)}{2n(n-\varepsilon-K)+(1+\varepsilon)K}\right]K^{1+\alpha} = (x_C^{\theta>0})^{1+\alpha}\left[\frac{2n(n-\varepsilon-K)+(1+\varepsilon)K}{n(n-\varepsilon)}\right]^{\frac{\varepsilon(1+\alpha)}{A}}\frac{n}{\alpha\beta}\left[\frac{n(n-\varepsilon)}{2n(n-\varepsilon-K)+(1+\varepsilon)K}\right] =$

$= Q_C^{\theta>0}\left[\frac{2n(n-\varepsilon-K)+(1+\varepsilon)K}{n(n-\varepsilon)}\right]^{\alpha/A}$. The term in parentheses is greater than one for $\theta < (n-\varepsilon-1)/(2n-\varepsilon-1)$; we therefore have $Q_{ST}^{\theta>0} > Q_C^{\theta>0}$ for $\theta < (n-\varepsilon-1)/(2n-\varepsilon-1)$ and $Q_{ST}^{\theta>0} < Q_C^{\theta>0}$ for $\theta < (n-\varepsilon-1)/(2n-\varepsilon-1)$.

[27] From the preceding discussion we have that $\delta = \frac{n(n-\varepsilon)}{2n(n-\varepsilon-K)+(1+\varepsilon)K} > 1$ when $\theta > (n-\varepsilon-1)/(2n-\varepsilon-1)$ and $\delta < 1$ when $\theta < (n-\varepsilon-1)/(2n-\varepsilon-1)$. Similarly, the equilibrium profit in the strategic case when spillovers are absent is given by $\Pi_{ST}^{\theta=0} = x_{ST}^{\theta=0}\left[\frac{\delta\varepsilon - \alpha(n-\varepsilon)}{\alpha(n-\varepsilon)}\right]$ where $\delta = \frac{n(n-\varepsilon)}{2n(n-\varepsilon-1)+(1+\varepsilon)} < 1$.

In the Cournot case with spillovers, the equilibrium profit for the individual firm is given by $\Pi_C^{\theta>0} = x_C^{\theta>0}[K\varepsilon - \alpha(n-\varepsilon)]/[\alpha(n-\varepsilon)]$. Since spillovers cause a reduction in R&D expenditures while leaving production costs, prices and output unchanged, profit with spillovers will exceed profit with full appropriability [28]: $\Pi_C^{\theta>0} > \Pi_C^{\theta=0}$. This result was discussed in detail in chapter two for the case where R&D is heterogeneous and where spillovers operate only on part of R&D (basic research).

In the strategic model, compared with the situation of full appropriability of R&D, spillovers lead to lower R&D expenditures, higher production costs, and thereby lower output and higher prices. With an inelastic demand, net revenues gross of R&D expenditures (Pq-cq) rise and so do profits relative to the case with full appropriability. Profits are also higher even if demand is elastic: the fall in R&D expenditures outweighs the fall in net operating revenues and $\Pi_{ST}^{\theta>0} > \Pi_{ST}^{\theta=0}$ [29].

In the comparison of profitability across models, when R&D is fully appropriable, firms in an industry of Cournot oligopolists earn higher profits than firms that operate in industries where strategic considerations guide behaviour ($\Pi_C^{\theta=0} > \Pi_{ST}^{\theta=0}$) [30]. When R&D is characterised by spillovers however, strategic behaviour can result in higher profits per firm than simple cost minimisation if appropriability is sufficiently low ie if

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[28] $\Pi_C^{\theta>0} = x_C^{\theta>0}\left[\frac{K\varepsilon-\alpha(n-\varepsilon)}{\alpha(n-\varepsilon)}\right] = x_C^{\theta=0}(1/K)\left[\frac{K\varepsilon-\alpha(n-\varepsilon)}{\alpha(n-\varepsilon)}\right] = \Pi_C^{\theta=0}\left[\frac{K\varepsilon-\alpha(n-\varepsilon)}{K\varepsilon-\alpha K(n-\varepsilon)}\right]$. Since K>1, the term in parentheses is greater than one, so that $\Pi_C^{\theta>0} > \Pi_C^{\theta=0}$.

[29] $\Pi_{ST}^{\theta>0} = x_{ST}^{\theta>0}\left[\frac{\delta K\varepsilon-\alpha(n-\varepsilon)}{\alpha(n-\varepsilon)}\right] = \Pi_{ST}^{\theta=0}\left[\frac{\delta K\varepsilon-\alpha(n-\varepsilon)}{\delta\varepsilon-\alpha(n-\varepsilon)}\right](\delta/\tilde{\delta})^{\varepsilon/A}(1/K)$ where δ and $\tilde{\delta}$ are as defined above. While $(\delta/\tilde{\delta})^{\varepsilon/A}(1/K) < 1$, the first term in brackets as well as the entire term multiplying $\Pi_{ST}^{\theta=0}$ exceeds one, so that $\Pi_{ST}^{\theta>0} > \Pi_{ST}^{\theta=0}$.

[30] $\Pi_{ST}^{\theta=0} = x_{ST}^{\theta=0}\left[\frac{\delta\varepsilon-\alpha(n-\varepsilon)}{\alpha(n-\varepsilon)}\right] = \Pi_C^{\theta=0}\left[\frac{\delta\varepsilon-\alpha(n-\varepsilon)}{\varepsilon-\alpha(n-\varepsilon)}\right](1/\delta)^{\varepsilon/A}$ where δ is as defined above. We have however that $[\delta\varepsilon-\alpha(n-\varepsilon)]/[\varepsilon-\alpha(n-\varepsilon)] < \delta^{\varepsilon/A}$, so that $\Pi_{ST}^{\theta=0} < \Pi_C^{\theta=0}$.

spillovers are pervasive ($\theta > (n - \varepsilon - 1)/(2n - \varepsilon - 1)$). In that case, we will have $\Pi_{ST}^{\theta>0} > \Pi_{C}^{\theta>0}$. If spillovers are limited ($\theta < (n - \varepsilon - 1)/(2n - \varepsilon - 1)$), profits per firm will be higher in the Cournot case ie $\Pi_{C}^{\theta>0} > \Pi_{ST}^{\theta>0}$ [31].

Finally, and in order to be able to rank the four profit equilibria, we compare the profits that each firm earns in equilibrium in the strategic case when there exist spillovers ($\Pi_{ST}^{\theta>0}$) with the equilibrium level of profits in the Cournot case when R&D is fully appropriable ($\Pi_{C}^{\theta=0}$). We obtained earlier that for high spillovers, $\Pi_{ST}^{\theta>0} > \Pi_{C}^{\theta>0}$. Since also $\Pi_{C}^{\theta>0} > \Pi_{C}^{\theta=0}$, this implies that with low appropriability of R&D, $\Pi_{ST}^{\theta>0} > \Pi_{C}^{\theta=0}$. In the case where R&D is highly appropriable (low spillovers), we established that $\Pi_{C}^{\theta>0} > \Pi_{ST}^{\theta>0}$. For $\Pi_{C}^{\theta=0} > \Pi_{ST}^{\theta=0}$, the threshold of appropriability is higher (ie the spillover rate much lower) [32]. We can therefore rank the four equilibria in the following manner: for high or moderate R&D spillovers, we have $\Pi_{ST}^{\theta>0} > \Pi_{C}^{\theta>0} > \Pi_{C}^{\theta=0} > \Pi_{ST}^{\theta=0}$; for low spillovers $\Pi_{C}^{\theta>0} > \Pi_{ST}^{\theta>0} > \Pi_{C}^{\theta=0} > \Pi_{ST}^{\theta=0}$; and for a very high degree of appropriability of R&D (very low spillover rate) we have that $\Pi_{C}^{\theta>0} > \Pi_{C}^{\theta=0} > \Pi_{ST}^{\theta>0} > \Pi_{ST}^{\theta=0}$.

We summarise the main results from the comparison of the strategic with the Cournot model when R&D is subject to spillovers and when the R&D production function and the associated marginal cost function are given by (4.28) -- ie where own and borrowed R&D are perfect substitutes -- in the following proposition:

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[31] $\Pi_{ST}^{\theta>0} = x_{ST}^{\theta>0}\left[\frac{\delta K\varepsilon - \alpha(n-\varepsilon)}{\alpha(n-\varepsilon)}\right] = \Pi_{C}^{\theta>0}\left[\frac{\delta K\varepsilon - \alpha(n-\varepsilon)}{K\varepsilon - \alpha(n-\varepsilon)}\right](1/\delta)^{\varepsilon/A}$. $\delta < 1$ when $\theta < (n - \varepsilon - 1)/(2n - \varepsilon - 1)$. In that case $[\delta K\varepsilon - \alpha(n - \varepsilon)]/[K\varepsilon - \alpha(n - \varepsilon)] < \delta^{\varepsilon/A}$, so that $\Pi_{ST}^{\theta>0} < \Pi_{C}^{\theta>0}$.

[32] $\Pi_{ST}^{\theta>0} = x_{ST}^{\theta>0}\left[\frac{\delta K\varepsilon - \alpha(n-\varepsilon)}{\alpha(n-\varepsilon)}\right] = \Pi_{C}^{\theta=0}\left[\frac{\delta K\varepsilon - \alpha(n-\varepsilon)}{\varepsilon - \alpha(n-\varepsilon)}\right](1/K)(1/\delta)^{\varepsilon/A}$. For $\Pi_{ST}^{\theta>0} > \Pi_{C}^{\theta=0}$, we need that $\delta K > 1$. Since $K>1$, it is sufficient for $\delta > 1$ which is true when $\theta > (n - \varepsilon - 1)/(2n - \varepsilon - 1)$. Even however for $\theta < (n - \varepsilon - 1)/(2n - \varepsilon - 1)$ ($\delta < 1$) it is possible for $\delta K > 1$ as long as spillovers are not too low or the industry too concentrated. Similarly, for $\Pi_{ST}^{\theta>0} < \Pi_{C}^{\theta=0}$ we need that $\delta K < 1$, which requires as a necessary condition that $\delta \ll 1$ (ie a very low spillover rate in a fragmented industry).

Proposition 3: In a situation where the R&D production function and the associated marginal cost function are given by (4.28), under perfect symmetry, at the strategic equilibrium with spillovers in R&D:

(i) each firm spends less on R&D
(ii) prices are higher
(iii) each firm earns more profit

than at the corresponding Cournot equilibrium with spillovers, *provided* the spillover rate is moderate or high ie for $\theta > (n - \varepsilon - 1)/(2n - \varepsilon - 1)$.

b. CES knowledge production function

These results rest on a particular specification of the cost function, where the part of the research of a firm's rivals that spills out is a perfect substitute for the firm's own research. This assumption is unduly restrictive and may be responsible for the ranking obtained above. Instead we could allow the elasticity of substitution between a firm's R&D and that of its rivals to vary, reflecting, among other things, the nature of the technology in question, the degree to which a technological field is cumulative or the ease of learning. In that situation we could have a knowledge production function of a CES form and an associated cost function such as:

$$(4.28') \qquad R_i = \left[x_i^{-\rho} + \left(\theta \sum_{j \neq i} x_j\right)^{-\rho}\right]^{-1/\rho} \quad \text{and} \quad c^i = \beta R_i^{-\alpha}$$

Under this new parameterization of the cost and knowledge production functions, we can once again explicitly solve for the equilibrium level of R&D in the strategic case with positive spillovers $x_{ST}^{\theta>0}$. It is given by:

$$(4.29') \qquad x_{ST}^{\theta>0} = [\beta^{\varepsilon-1}\sigma(\alpha/n)^{\varepsilon}(1-\varepsilon/n)]^{1/A}\left[\frac{2n(n-\varepsilon-K_1)+(1+\varepsilon)K_1}{n(n-\varepsilon)}\right]^{\varepsilon/A} K_1^{-[\varepsilon\rho+\alpha(1-\varepsilon)]/\rho A}$$

where $A = \varepsilon - \alpha(1-\varepsilon)$ and $K_1 = 1 + [\theta(n-1)]^{-\rho}$.

The same comparisons performed before can now be made with the CES knowledge production function. In the Cournot model, equilibrium R&D expenditures when R&D is fully appropriable is $x_C^{\theta=0}$; with positive spillovers, it is $x_C^{\theta>0}$ [33]. Unlike in the previous parameterization of the cost function, where $x_C^{\theta=0} > x_C^{\theta>0}$, in this case this is true only if $-[\varepsilon\rho + \alpha(1-\varepsilon)]/\rho A < 0$. If the reverse holds, spillovers act as a spur to investment in R&D in the Cournot case [34]. The relationship $x_{ST}^{\theta=0} > x_C^{\theta=0}$ however for the case of fully appropriable R&D still holds. When spillovers are absent, the two parameterizations of the cost function collapse to the same form [35].

Turning next to the impact of spillovers on the equilibrium level of R&D in the strategic case, we compare $x_{ST}^{\theta>0}$ and $x_{ST}^{\theta=0}$. From (4.29') we have that:

$$\text{(4.30')} \qquad x_{ST}^{\theta>0} = x_{ST}^{\theta=0}\left[\frac{2n(n-\varepsilon-K_1)+(1+\varepsilon)K_1}{2n(n-\varepsilon)-2n+1+\varepsilon}\right]^{\varepsilon/A} K_1^{-[\varepsilon\rho+\alpha(1-\varepsilon)]/\rho A}$$

It was established earlier that the term in brackets in (4.30') (similar to the same term in (4.30)) is less than one. For spillovers to be associated with lower R&D in the strategic equilibrium therefore, a sufficient condition is for $K_1^{-[\varepsilon\rho+\alpha(1-\varepsilon)]/\rho A} < 1$. Since $K_1>1$ and $A>0$, this requirement reduces to $-[\varepsilon\rho + \alpha(1-\varepsilon)] < 0$. If instead

[33] The equilibrium level of firm R&D expenditures in the Cournot model when some of R&D is not appropriable and when the knowledge production function is as in (4.28') is given by $x_C^{\theta>0} = [\beta^{\varepsilon-1}\sigma(\alpha/n)^{\varepsilon}(1-\varepsilon/n)]^{1/A} K_1^{-[\varepsilon\rho+\lambda(1-\varepsilon)]/\rho A}$. For $\theta = 0$, K_1=1 and the solution again reduces to that derived in Dasgupta and Stiglitz (1980a) for the case with no spillovers.

[34] We will not dwell on the conditions for which spillovers can spur R&D investment, as these have been covered exhaustively in chapter three of the thesis for the case where R&D is composed of basic and of development research, with spillovers operating on the former only. In brief, $x_C^{\theta=0} < x_C^{\theta>0}$ here in two cases: first, if own and borrowed R&D are not easily substitutable with each other ($\rho > 0$ or $s_{xx} = 1/(1+\rho) < 1$), as long as $\varepsilon\rho + \lambda(1-\varepsilon) < 0$. This implies an inelastic demand ($\varepsilon > 1$) and $\alpha > -\varepsilon\rho/(1-\varepsilon)$ or equivalently that $0 < \rho < \alpha(\varepsilon - 1)/\varepsilon$. Secondly, if own and borrowed R&D are easily substitutable with each other ($\rho < 0$ or $s_{xx} = 1/(1+\rho) > 1$), as long as $\varepsilon\rho + \lambda(1-\varepsilon) > 0$. This implies an elastic demand ($\varepsilon < 1$) and is true for $0 > \rho > \alpha(\varepsilon - 1)/\varepsilon$.

[35] Once more, this is under the assumption that the condition $n > 1 + \varepsilon$ holds.

$-[\varepsilon\rho + \alpha(1-\varepsilon)] > 0$ and in addition the spillover rate is high so that $K_1^{-[\varepsilon\rho+\alpha(1-\varepsilon)]/\rho A} \gg 1$, it is possible for inappropriability in R&D to act as an incentive to equilibrium R&D expenditures in the strategic model, just as it could in the Cournot case. The range of parameters however for which this is possible is more restricted in the situation where firms use R&D for strategic purposes than when it is used for cost-minimisation only.

Under the parameterization given in (4.28'), we can also compare the equilibrium levels of R&D in the strategic and the Cournot models when part of R&D is subject to spillovers ie $x_{ST}^{\theta>0}$ and $x_C^{\theta>0}$. With R&D fully appropriable, we obtained under both parameterizations of the R&D production function that $x_{ST}^{\theta=0} > x_C^{\theta=0}$. Strategic considerations lead to higher equilibrium levels in R&D compared with the case where firms are simply concerned with cost minimisation. With externalities in R&D, and an R&D production function given by (4.38'):

$$(4.31') \qquad x_{ST}^{\theta>0} = x_C^{\theta>0}\left[\frac{2n(n-\varepsilon-K_1)+(1+\varepsilon)K_1}{n(n-\varepsilon)}\right]^{\varepsilon/A}$$

Once more the result rests on whether the term in brackets is greater or less than one. This time however, the answer depends on the combination of two factors: the extent of inappropriability and the ease of substitutability between a firm's R&D and that of its rivals. Simple algebraic manipulation yields that for the term in brackets to exceed unity (giving $x_{ST}^{\theta>0} > x_C^{\theta>0}$), the relationship $\theta^{-\rho} < (n-1)^{1+\rho}(n-\varepsilon-1)/(2n-\varepsilon-1)$ needs to hold [36]. Conversely, for $\theta^{-\rho} > (n-1)^{1+\rho}(n-\varepsilon-1)/(2n-\varepsilon-1)$, we have that $x_{ST}^{\theta>0} < x_C^{\theta>0}$.

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[36] For $\theta^{-\rho} < (n-1)^{1+\rho}(n-\varepsilon-1)/(2n-\varepsilon-1)$, we distinguish two cases. When $\rho > 0$, the requirement is for the spillover rate to exceed $(n-1)^{-(1+\rho)/\rho}[(n-\varepsilon-1)/(2n-\varepsilon-1)]^{-1/\rho}$. When $\rho < 0$, the requirement is that $\theta < (n-1)^{-(1+\rho)/\rho}[(n-\varepsilon-1)/(2n-\varepsilon-1)]^{-1/\rho}$. The reverse holds for $\theta^{-\rho} > (n-1)^{1+\rho}(n-\varepsilon-1)/(2n-\varepsilon-1)$.

Graphs 1 and 2 below illustrate these relationships. They show the threshold spillover rates that determine whether firms spend more on R&D at the strategic or the Cournot equilibrium. These thresholds depend on three variables: the elasticity of substitution between a firm's own R&D and that part of its rivals' R&D that becomes public s_{xx}; the number of firms in the industry n; and the elasticity of demand $1/\varepsilon$. In the graphs the threshold spillover rates are plotted on the y-axis, and the number of firms in the industry on the x-axis. The relationship between n and θ is then drawn for different values of the elasticity of substitution between research inputs, while assuming throughout that the demand for the product is slightly elastic ($\varepsilon = 0.8$).

The first graph is drawn under the assumption that own and rivals' R&D are easily substitutable with each other ($\rho < 0$ or $s_{xx} = 1/(1+\rho) > 1$). The four lines represent four values of ρ: -0.1, -0.5, -0.7 and -0.9, giving elasticities of substitution of 1.11, 2, 3.33 and 10 respectively. The area to the right and below each line then shows the combinations of n and θ for which $x_{ST}^{\theta>0} > x_C^{\theta>0}$. For areas to the left and above the lines we have $x_{ST}^{\theta>0} < x_C^{\theta>0}$. For given n, as spillovers increase we tend to pass from a situation where $x_{ST}^{\theta>0} > x_C^{\theta>0}$ to one where $x_{ST}^{\theta>0} < x_C^{\theta>0}$. Alternatively, for a given spillover rate, as the market becomes more fragmented, we pass from a situation where $x_{ST}^{\theta>0} < x_C^{\theta>0}$ to one where $x_{ST}^{\theta>0} > x_C^{\theta>0}$. Graph 2 is drawn for the case where own and rivals' R&D are *not* easily substitutable with each other ($\rho > 0$ or $s_{xx} = 1/(1+\rho) < 1$), and in particular for ρ equal to 0.1, 0.9, 3 and 10, giving elasticities of substitution of 0.9, 0.52, 0.25 and 0.09 respectively. As n increases (for given θ) -- and as θ increases (for given n) -- we move from a situation where $x_{ST}^{\theta>0} < x_C^{\theta>0}$ to one where $x_{ST}^{\theta>0} > x_C^{\theta>0}$ [37].

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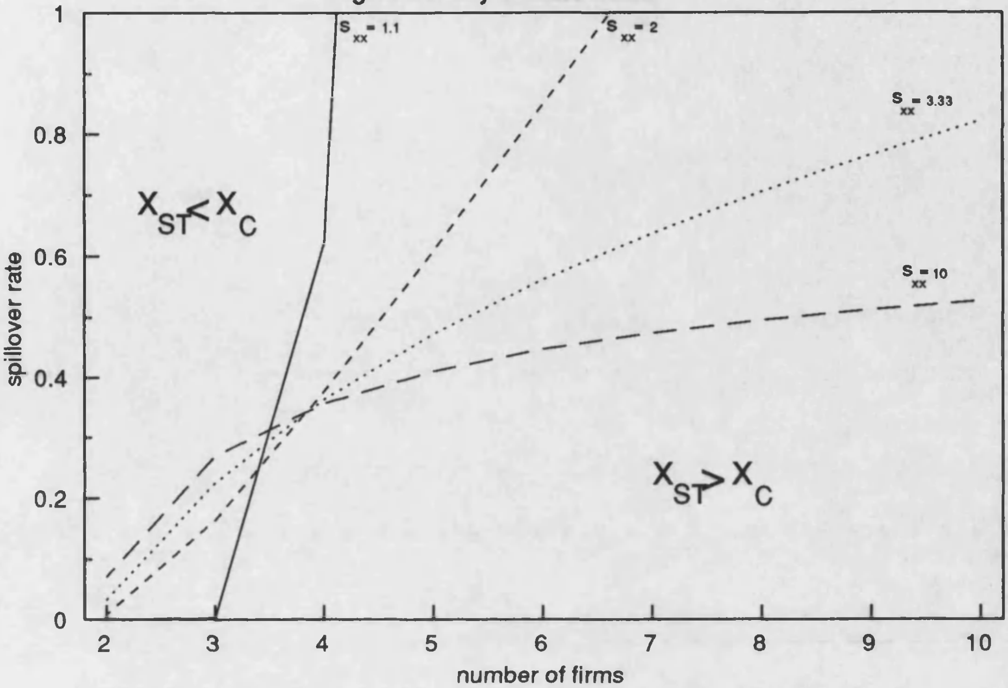
[37] For $\rho = -0.9$ (ie own and rival R&D almost perfect substitutes), $x_{ST}^{\theta>0} < x_C^{\theta>0}$ for any θ>0.07 in a duopoly and θ>0.35 in an industry with four firms. For $\rho = 0.1$, $x_{ST}^{\theta>0} < x_C^{\theta>0}$ for any θ<1 in a duopoly; in contrast when n=4, $x_{ST}^{\theta>0} > x_C^{\theta>0}$ for any θ>0.17. Finally, at the other end of the spectrum, for θ=10, when n=2 $x_{ST}^{\theta>0} < x_C^{\theta>0}$ for any $\theta < 1$, while for n=4, this is only true for θ<0.37.

Graph 1

Spillover rate thresholds

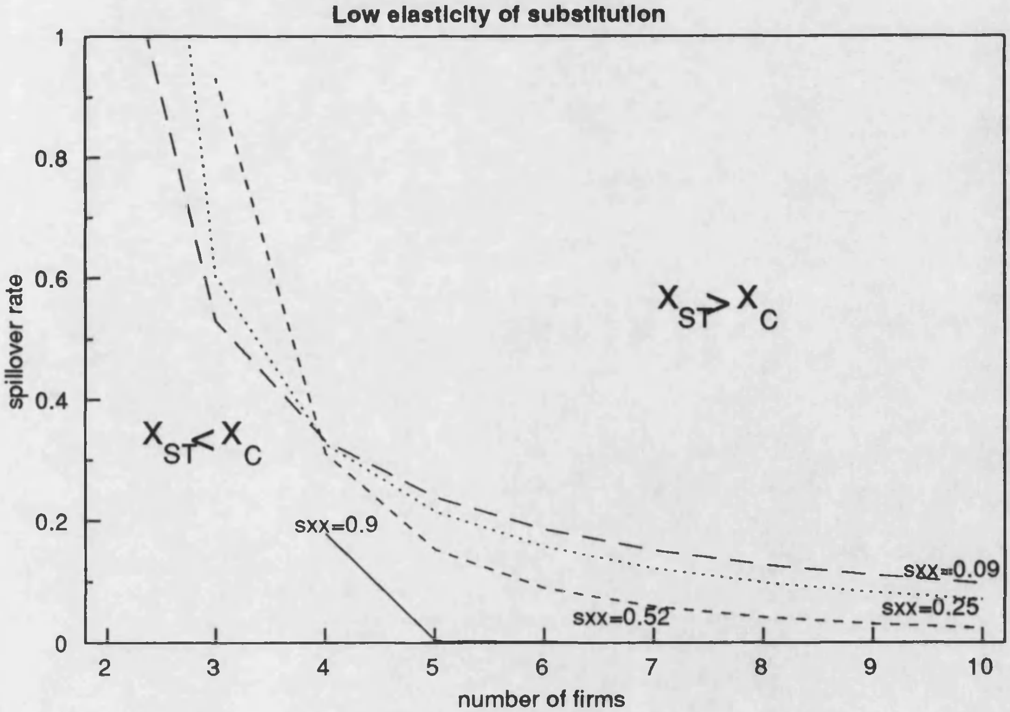
High elasticity of substitution



Graph 2

Spillover rate thresholds

Low elasticity of substitution



The explanation of these patterns runs along the following lines. In the situation where it is relatively easy to substitute between one's own R&D and the part of rivals' R&D that has become public, as spillover rates increase, the strategic effect loses its power. The strategic effect implies that high R&D expenditures in the first stage benefit the firm by placing it in a comparative advantage in the second (production) stage of the game. With high spillovers however, this effect is swamped by the disincentive effect of inappropriability, due to the fact that firms now realise that rivals benefit greatly from their own R&D expenditures and that they can achieve the same comparative level of cost reduction with lower R&D than before. Furthermore, the more concentrated the industry is, the stronger is the logic of this argument and the lower the threshold level of the spillover rate beyond which the strategic firm will cut back its R&D outlays to levels below those dictated by cost-minimising behaviour. This is in fact true irrespective of the value of the elasticity of substitution variable.

In the opposite case, the elasticity of substitution of own R&D with that of rivals' is low; put differently, the public part of a firm's own R&D raises significantly the marginal productivity of the research of its rivals. The situation is now more complicated. When spillovers are limited, the strategic equilibrium involves firms spending less on R&D than in the Cournot equilibrium. As spillovers become more widespread, however, a given level of R&D expenditures by one firm induces its rivals to also increase their R&D outlays. The strategic aspect then regains its power, in the sense that the firm is now obliged to spend more on R&D in order to counter the increased spending by rivals. The relative value of "commitment", in the sense of committing resources to investment in own R&D, increases with the spillover rate, so that it pays again to act strategically by spending more on R&D than would be necessary for simple cost-minimisation.

In addition to the comparison of R&D expenditures, when the R&D production function is given by (4.28') we can also compare production and profitability levels at the four equilibria. Using (4.29'), we derive from (4.27) a new industry-wide equilibrium level of production in the strategic model when R&D is subject to spillovers, ie we compute $Q_{ST}^{\theta>0}$ as:

$$\text{(4.32')} \qquad Q_{ST}^{\theta>0} = (x_{ST}^{\theta>0})^{1+\alpha}(n/\alpha\beta)\left[\frac{n(n-\varepsilon)}{2n(n-\varepsilon-K_1)+(1+\varepsilon)K_1}\right]K_1^{1-\alpha/\rho}$$

When spillovers are absent, the level of industry production in the strategic equilibrium ($Q_{ST}^{\theta=0}$) can be deduced from the above expression by setting K_1=1. Similarly industry production in the Cournot equilibrium with spillovers is given by $Q_C^{\theta>0} = (x_C^{\theta>0})^{1+\alpha}(n/\alpha\beta)K_1^{1-\alpha/\rho}$, while for fully appropriable R&D it is given by $Q_C^{\theta=0} = (x_C^{\theta=0})^{1+\alpha}(n/\alpha\beta)$.

Under the present parameterization of the knowledge production function, in the Cournot model, equilibrium industry production when part of R&D is not appropriable exceeds the equilibrium level under fully appropriable R&D when own and rivals' R&D are easily substitutable with each other ($\rho < 0$). When it is not easy to substitute ($\rho > 0$), we have instead that $Q_C^{\theta>0} < Q_C^{\theta=0}$[38].

The explanation of this result can be traced to the effect of spillovers on the cost-reducing knowledge generated by R&D expenditures that is available to each firm in equilibrium when the knowledge production function is given by (4.28'). In this case, the total *effective* knowledge related to R&D expenditures that the firm has access

[38] $Q_C^{\theta>0} = Q_C^{\theta=0}K_1^{-\alpha(1+\rho)/\rho A}$. Since A>0 and K_1>1, for $\rho < 0$ (ie $s_{xx} = 1/(1+\rho) > 1$) we have that $Q_C^{\theta>0} > Q_C^{\theta=0}$. For $\rho > 0$ (ie $s_{xx} = 1/(1+\rho) < 1$) we have instead that $Q_C^{\theta>0} < Q_C^{\theta=0}$.

to depends on its own R&D expenditures, and on that part its rivals' R&D expenditures that is in the public domain. When however the R&D of a firm's rivals cannot be easily substituted for its own, in the symmetric equilibrium total research knowledge when R&D is subject to spillovers falls short of that same knowledge when spillovers are absent ie $R_C^{\theta>0} < R_C^{\theta=0}$ [39]. Marginal production cost is in that case higher, resulting (with a downward-sloping demand curve) in a lower level of industry output in equilibrium than in the situation where R&D is fully appropriable.

In the strategic case, the comparison of equilibrium levels of industry production with and without spillovers is less straightforward. In addition to the effect due to the conditions of substitutability between own and rivals' R&D, the difference between the two equilibrium production levels will also depend on the interaction effect, due to the strategic behaviour of firms. Thus, for a low degree of substitutability of R&D ($\rho > 0$), the equilibrium production level with spillovers falls short of the same level when spillovers are absent. If however own and rivals' R&D are easily substitutable ($\rho < 0$), the result is not necessarily reversed as in the Cournot case [40]. For a given (high) degree of substitutability however, the R&D-related knowledge available to a firm in a symmetric strategic equilibrium with spillovers is more likely to exceed the level of knowledge associated with full R&D appropriability if the industry is fragmented and/or if the spillover rate is high. In such a situation, marginal production costs are lower and industry production is higher than in the case with no spillovers.

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[39] $R_C^{\theta>0} = R_C^{\theta=0} K_1^{-\epsilon(1+\rho)/\rho A}$. For $\rho < 0$ (ie $s_{xx} = 1/(1+\rho) > 1$) we have that $R_C^{\theta>0} > R_C^{\theta=0}$. For $\rho > 0$ (ie $s_{xx} = 1/(1+\rho) < 1$) we have instead that $R_C^{\theta>0} < R_C^{\theta=0}$.

[40] $Q_{ST}^{\theta>0} = Q_{ST}^{\theta=0} K_1^{-\alpha(1+\rho)/\rho A} (\delta/\tilde{\delta})^{\alpha/A}$. Since $\delta < \tilde{\delta}$, for $\rho > 0$ (ie $s_{xx} = 1/(1+\rho) < 1$) we have that $Q_{ST}^{\theta>0} < Q_{ST}^{\theta=0}$. For $\rho < 0$ however (ie $s_{xx} = 1/(1+\rho) > 1$) we only have that $Q_C^{\theta>0} > Q_C^{\theta=0}$ if $K_1^{-\alpha(1+\rho)/\rho A} (\delta/\tilde{\delta})^{\alpha/A} > 1$. For a given $\rho < 0$, this is more likely for high n and/or high θ.

Comparing across models, the relationship between industry production in the symmetric strategic equilibrium with spillovers ($Q_{ST}^{\theta>0}$) and that in the symmetric Cournot equilibrium with spillovers ($Q_{ST}^{\theta>0}$) is along the same lines as the comparison of R&D expenditures. In order therefore to avoid unnecessary duplication, we simply note that $Q_{ST}^{\theta>0} > Q_{C}^{\theta>0}$ for $\theta^{-\rho} < (n-1)^{1+\rho}(n-\varepsilon-1)/(2n-\varepsilon-1)$ and $Q_{ST}^{\theta>0} < Q_{C}^{\theta>0}$ for $\theta^{-\rho} > (n-1)^{1+\rho}(n-\varepsilon-1)/(2n-\varepsilon-1)$. These conditions are satisfied for particular combinations of the spillover rate, the number of firms in the industry, the elasticity of demand for the product and the elasticity of substitution between a firm's own R&D and that of its rivals [41].

We turn finally to the examination of firm profitability in the two models in the situation where the knowledge production function is given by (4.28'). Individual firm profit can be expressed as a function of R&D expenditures. In the strategic case with spillovers, profit in equilibrium is given by:

$$\text{(4.33')} \qquad \Pi_{ST}^{\theta>0} = x_{ST}^{\theta>0}\left[\frac{\delta K_1\varepsilon - \alpha(n-\varepsilon)}{\alpha(n-\varepsilon)}\right] \qquad \text{where} \qquad \tilde{\delta} = \frac{n(n-\varepsilon)}{2n(n-\varepsilon-K_1)+(1+\varepsilon)K_1}$$

In the Cournot case with spillovers, the equilibrium profit for the individual firm is given by $\Pi_{C}^{\theta>0} = x_{C}^{\theta>0}[\tilde{\delta}K_1\varepsilon - \alpha(n-\varepsilon)]/[\alpha(n-\varepsilon)]$. In the case where own and rival R&D were prefect substitutes, spillovers led to higher profits since they caused a reduction in R&D expenditures while leaving production costs, prices and output unchanged. In this case however, we have seen that spillovers may lead to higher or lower R&D expenditures and production and consequently to higher or lower profits than in the case

[41] $Q_{ST}^{\theta>0} = Q_{C}^{\theta>0}(1/\tilde{\delta})^{\alpha/A}$. We therefore have that $Q_{ST}^{\theta>0} > Q_{C}^{\theta>0}$ for $\theta^{-\rho} < (n-1)^{1+\rho}(n-\varepsilon-1)/(2n-\varepsilon-1)$ and $Q_{ST}^{\theta>0} < Q_{C}^{\theta>0}$ for $\theta^{-\rho} > (n-1)^{1+\rho}(n-\varepsilon-1)/(2n-\varepsilon-1)$. See footnotes 36 and 37 above.

where R&D was fully appropriable [42]. Profit with spillovers can be lower than profit with full appropriability (ie $\Pi_C^{\theta>0} < \Pi_C^{\theta=0}$) if for example demand is elastic ($\varepsilon < 1$) and it is relatively difficult to substitute one's own R&D with the R&D of rivals ($\rho > 0$). In that situation, in an environment characterised with spillovers, revenue *P(Q)q* and R&D expenditures are less while production costs *c(R)q* are more than in the case where spillovers are absent. Despite the lower R&D expenditures, spillovers can in this situation lead to lower profits compared with a situation of full appropriability. This result was discussed in detail in chapter three for the case where R&D is heterogeneous and where spillovers operate only on part of R&D (basic research).

Similar conditions apply in the strategic model. Here, the comparison of profitability in the case where R&D is subject to spillovers with the case where it is fully appropriable is more complicated because of the existence of the interaction effect due to strategic behaviour. We have already seen that compared with the situation of full appropriability of R&D, spillovers can in this case lead to lower or higher R&D expenditures and production costs, and thereby lower or higher output and prices [43].

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[42] $\Pi_C^{\theta>0} = x_C^{\theta>0}\left[\frac{\delta K_1\varepsilon-\alpha(n-\varepsilon)}{\alpha(n-\varepsilon)}\right] = x_C^{\theta=0}K_1^{-(\varepsilon\rho+\alpha(1-\varepsilon))/\rho A}\left[\frac{K_1\varepsilon-\alpha(n-\varepsilon)}{\alpha(n-\varepsilon)}\right] = \Pi_C^{\theta=0}\left[\frac{K_1\varepsilon-\alpha(n-\varepsilon)}{K_1\varepsilon-\alpha K_1(n-\varepsilon)}\right]K_1^{-\alpha(1-\varepsilon)(1+\rho)/\rho A}$. Since $K_1>1$, the term in parentheses is greater than one. For $\Pi_C^{\theta>0} > \Pi_C^{\theta=0}$ therefore, it is sufficient for $K_1^{-\alpha(1-\varepsilon)(1+\rho)/\rho A} > 1$. This is the case when $\rho > 0$ and $\varepsilon > 1$ and when $\rho < 0$ and $\varepsilon < 1$. For $\Pi_C^{\theta>0} < \Pi_C^{\theta=0}$, on the other hand, it is necessary that $\rho < 0$ and $\varepsilon > 1$ or $\rho > 0$ and $\varepsilon < 1$ (giving $K_1^{-\alpha(1-\varepsilon)(1+\rho)/\rho A} < 1$) and in addition that $\left[\frac{K_1\varepsilon-\alpha(n-\varepsilon)}{K_1\varepsilon-\alpha K_1(n-\varepsilon)}\right]K_1^{-\alpha(1-\varepsilon)(1+\rho)/\rho A} < 1$.

[43] $\Pi_{ST}^{\theta>0} = x_{ST}^{\theta>0}\left[\frac{\tilde{\delta}K_1\varepsilon-\alpha(n-\varepsilon)}{\alpha(n-\varepsilon)}\right] = \Pi_{ST}^{\theta=0}\left[\frac{\tilde{\delta}K_1\varepsilon-\alpha(n-\varepsilon)}{\delta\varepsilon-\alpha(n-\varepsilon)}\right](\delta/\tilde{\delta}K_1)^{\varepsilon/A}K_1^{-\alpha(1-\varepsilon)/\rho A}$ where δ and $\tilde{\delta}$ are as defined above. Of the terms in the right-hand-side, the first term in parentheses is greater than one, while $(\delta/\tilde{\delta}K_1)^{\varepsilon/A} < 1$. $K_1^{-\alpha(1-\varepsilon)/\rho A} > 1$ for $\rho > 0$ and $\varepsilon > 1$ or for $\rho < 0$ and $\varepsilon < 1$ and $K_1^{-\alpha(1-\varepsilon)/\rho A} < 1$ for $\rho > 0$ and $\varepsilon < 1$ or for $\rho < 0$ and $\varepsilon > 1$. A necessary (but not sufficient) condition therefore for $\Pi_{ST}^{\theta>0} < \Pi_{ST}^{\theta=0}$ is that $\rho > 0$ and $\varepsilon < 1$ or that $\rho < 0$ and $\varepsilon > 1$.

The effect on profits then depends not only on the direction, but also on the magnitude of the effect of spillovers on revenues and on marginal and fixed costs. These latter effects, meanwhile, are made more complex by the strategies that firms are pursuing.

The effect of spillovers in the strategic and in the Cournot models is therefore along similar lines. Under certain assumptions about elasticities, spillovers can lead to lower profits compared with a situation of full R&D appropriability. The difference of the two modes of behaviour comes out in the range of parameters that allow this result, and in particular the range of the spillover rate and the assumption about the number of firms operating in the industry. In order therefore to focus on the importance of behavioural assumptions on the impact of spillovers when the knowledge production function for each firm is given by (4.28'), we compare profitability across models when R&D is characterised by spillovers.

Comparing profitability across models, we have seen that when R&D is fully appropriable, firms in an industry of Cournot oligopolists earn higher profits than firms that operate in industries where behaviour is guided by strategic considerations (ie $\Pi_C^{\theta=0} > \Pi_{ST}^{\theta=0}$). When R&D is characterised by spillovers however, strategic behaviour can result in higher or lower profits per firm compared to simple cost minimisation in an environment of partially appropriable R&D [44]. The conditions for which profitability is higher in the strategic or the Cournot case are analogous to the conditions in the case of

[44] $\Pi_{ST}^{\theta>0} = x_{ST}^{\theta>0}\left[\frac{\delta K_1\varepsilon - \alpha(n-\varepsilon)}{\alpha(n-\varepsilon)}\right] = \Pi_C^{\theta>0}\left[\frac{\delta K_1\varepsilon - \alpha(n-\varepsilon)}{K_1\varepsilon - \alpha(n-\varepsilon)}\right](1/\tilde{\delta})^{\varepsilon/A}$. When $\theta^{-\rho} < (n-1)^{1+\rho}(n-\varepsilon-1)/(2n-\varepsilon-1)$ we have that $\tilde{\delta} < 1$ and consequently that $\left[\frac{\delta K_1\varepsilon - \alpha(n-\varepsilon)}{K_1\varepsilon - \alpha(n-\varepsilon)}\right] < \tilde{\delta}^{\varepsilon/A}$ giving $\Pi_{ST}^{\theta>0} < \Pi_C^{\theta>0}$. Alternatively when $\theta^{-\rho} > (n-1)^{1+\rho}(n-\varepsilon-1)/(2n-\varepsilon-1)$ we have that $\tilde{\delta} > 1$ and consequently that $\left[\frac{\delta K_1\varepsilon - \alpha(n-\varepsilon)}{K_1\varepsilon - \alpha(n-\varepsilon)}\right] > \tilde{\delta}^{\varepsilon/A}$ giving $\Pi_{ST}^{\theta>0} > \Pi_C^{\theta>0}$. Footnotes 36 and 37 above discuss the ranges of θ and n that satisfy the two conditions.

R&D expenditures and of production which have been analysed above.

In short, for all three variables (R&D expenditures, production an profitability) the comparison of the Cournot and strategic outcomes with spillovers rests on whether $\theta^{-\rho}$ exceeds or fall shorts of $(n-1)^{1+\rho}(n-\varepsilon-1)/(2n-\varepsilon-1)$. If instead $\theta^{-\rho} < (n-1)^{1+\rho}(n-\varepsilon-1)/(2n-\varepsilon-1)$, then in the symmetric strategic equilibrium with spillovers, equilibrium R&D expenditures and production are higher, while profits are lower than at the symmetric Cournot equilibrium with spillovers. Conversely, if $\theta^{-\rho} > (n-1)^{1+\rho}(n-\varepsilon-1)/(2n-\varepsilon-1)$, the Cournot outcome will exhibit higher equilibrium R&D and production, and lower profits than the strategic case.

The main results from the comparison of the strategic with the Cournot model when R&D is subject to spillovers and when the R&D production function and the associated marginal cost function are given by (4.28') -- ie where the knowledge production function is given by a CES form -- are summarised in the following proposition:

Proposition 4: **In a situation where the R&D production function and the associated marginal cost function are given by (4.28'), under perfect symmetry, at the strategic equilibrium with spillovers in R&D:**

(i) each firm spends less on R&D
(ii) prices are higher
(iii) each firm earns more profit

than at the corresponding Cournot equilibrium with spillovers, *provided* the spillover rate satisfies the condition $\theta^{-\rho} > (n-1)^{1+\rho}(n-\varepsilon-1)/(2n-\varepsilon-1)$.

IV. Conclusions

This chapter developed a framework in order to examine the implications of inappropriability of R&D in an oligopolistic environment where firms engage in strategic behaviour. This issue was approached from different angles, corresponding to different sets of questions. The first concerns the issue of how spillovers affect the incentives of firms to invest in R&D and the industry characteristics when firms act strategically in setting R&D and production levels; it therefore involves the examination of the equilibrium characteristics of a strategic model of competition through R&D when the degree of appropriability varies. The second set of questions concerns the issue of how strategic behaviour affects incentives and performance in an environment where R&D is not fully appropriable; this involves the comparison of two different equilibria: the Cournot equilibrium with spillovers and the strategic equilibrium with spillovers.

The simple schema below illustrates how the models developed in this and the two previous chapters of the dissertation fit into a framework that addresses the issues of inappropriability of R&D and strategic behaviour. In the first quadrant are situated models that examine the equilibrium and welfare characteristics of industries where R&D is fully appropriable and where firms are Cournot oligopolists. Dasgupta and Stiglitz (1980) is a prime example of this type of model. In the second quadrant are models that while still assuming R&D to be fully appropriable, interpret oligopolistic rivalry through R&D as being characterised by strategic considerations; in addition they compare the equilibrium characteristics of such strategic models with those of Cournot models ie they compare quadrants 1 and 2. Brander and Spencer's (1983a and 1983b) multi-stage models fall into this category.

	Cournot	**Strategic**
$\theta = 0$	1	2
$\theta > 0$	3a	4a
	3b	4b

The models developed in chapters two and three of this thesis are situated in the third quadrant of the schema. They examine the equilibrium and welfare characteristics of industries where R&D is not fully appropriable and where firms are Cournot oligopolists. Each of the two chapters makes different assumptions about the way in which a firm's own R&D combines with the public component of its rivals' R&D: the resulting equilibria (sections a and b in quadrant 3 of the schema) have different characteristics. These equilibria are then compared to the Cournot equilibrium with no spillovers (ie to quadrant 1).

Chapter four develops a multi-stage (strategic) model of oligopolistic rivalry through non-appropriable R&D (quadrant four). Once more, the resulting equilibrium characteristics differ according to the assumptions made about the technology (quadrants 4a and 4b). These equilibria are compared in two dimensions: with the equilibrium in the strategic model with no spillovers (quadrant 2); and with the corresponding equilibria in the Cournot model with spillovers (ie sections 3a and 3b in quadrant 3).

The main conclusions of this chapter are summarised below. They can all be traced to comparisons between quadrants in this simple schema.

The conclusions relate to the level of research and development investment, levels of output and profitability in oligopolistic industries composed of firms acting strategically when the appropriability of R&D expenditures is imperfect. With perfect appropriability of R&D, firms perceiving strategic considerations beyond cost-minimisation invest more heavily in research than cost-minimising firms. The incentive for this deviation from cost-minimising (Cournot) behaviour lies in the nature of the game being played: by precommitting to a higher level of R&D in early stages, firms aim at reducing their marginal production costs and increasing their market share in the later stages.

With spillovers in R&D, these strategic considerations need to take into account the fact that a part of a firm's R&D outlays indirectly benefits its competitors. This negative externality may outweigh the advantage that the firm seeks to achieve by precommitting to higher levels of R&D than would be necessary in order to minimise costs. The marginal benefit of conducting non-appropriable R&D can in that case exceed or fall short of the level implied by cost-minimising behaviour. The level of R&D investment in such industries composed of strategic firms will then depend on the degree of appropriability of R&D and on the assumptions about technology. It will depend in other words on the spillover rate and on how a firm's their own R&D combines with that of their rivals to reduce production costs.

The implications of two alternative assumptions about technology were examined. Under the first, a firm's R&D was taken to be a perfect substitute to that part of its rivals' R&D that becomes public through the spillover rate. In that situation, the strategic model was found to exhibit some of the same characteristics as the Cournot model: under the presence of spillovers, firms invest less in R&D than when research is fully appropriable. Unlike the Cournot model however, where marginal costs, output and prices with and without spillovers are the same, when firms have strategic considerations, lower R&D implies higher marginal costs, lower output and higher prices in the presence of spillovers than when research is fully appropriable. Under both Cournot and strategic behaviour however, profits are higher when spillovers are positive than when they are absent.

Comparing across models, strategic firms were found to invest more in research than Cournot firms only when the degree of appropriability is high (ie when spillovers are low). For moderate or high spillovers, the disincentive effect due to the negative externality associated with spillovers outweighs the strategic considerations: strategic firms are then characterised by lower levels of R&D expenditures than Cournot firms. The threshold spillover rate depends on the number of firms in the industry and on the elasticity of demand for the product and of cost reduction with respect to R&D.

Furthermore, under the assumption of perfect substitution about own and rival R&D, with low spillovers the strategic equilibrium will be characterised by higher output, lower prices and lower profitability for each firm than the Cournot equilibrium with spillovers. With moderate or high spillovers, the reverse is true: strategic firms will be producing less, facing higher prices and earning more profit than Cournot firms.

The other alternative assumption about the technology made was to postulate that a firm's knowledge production function linking its own research expenditures with that part of its rivals' research that becomes public is of a constant elasticity of substitution form. The substitutability of "own" and "borrowed" research can therefore in this case vary, reflecting, among other things, the nature of the technology in question, the degree to which a technological field is cumulative or the ease of learning.

In the comparison of R&D outlays with and without spillovers within each model, it was established that both under Cournot and under strategic behaviour, spillovers could act as an incentive, rather than a disincentive, for R&D investments, unlike under the assumption of perfect substitutability of research across firms. In a situation for example where "own" and "borrowed" research are not easily substitutable with each other, both Cournot firms and strategic firms could end up spending more on R&D when its appropriability is low than when it is high.

While the reversal of the traditional disincentive result of spillovers is possible under both Cournot and strategic behaviour, a direct comparison between R&D outlays in the two models when research is not fully appropriable revealed important differences. As before, it was established that there is a threshold spillover rate which determines the comparison of equilibrium R&D outlays in the strategic model with those in the Cournot model. In this case however, in addition to depending on the number of firms in the industry, the elasticity of demand for the product and that of cost reduction with respect to R&D, the threshold of the spillover rate depended also on the ease of substitutability of research inputs among rival firms.

This additional variable produced a more complicated and richer analytical setup, with significant possibilities for firms behaving strategically of underinvesting relative to firms operating under Cournot rules both when substitutability of research inputs was easy and when it was difficult. It was established however that strategic firms had a tendency to move from a situation of higher R&D expenditures relative to the Cournot equilibrium to one of lower when substitutability of research inputs is easy and vice versa when it is difficult. In all cases, ceteris paribus, more concentrated industries tended to be characterised with lower R&D expenditures under strategic assumptions than under Cournot assumptions.

The comparison of costs, production, prices and profit across the two models when research is subject to spillovers is along the same lines as the comparison of R&D outlays. The same values for the elasticity and industry structure variables that determine the threshold spillover rate and consequently the comparison between R&D outlays in the Cournot and strategic equilibria determine also output and profitability comparisons.

In this and the previous two chapters we have attempted to explore some of the issues relating to firms' incentives, industry structure and public policy in oligopolistic structures where firms compete in R&D and output and where research cannot be fully appropriated by the firm(s) that originate it. The central conclusion that emerges is assumptions about the behaviour of firms and about the nature of their technology, as these are reflected respectively in the nature of the oligopolistic game being played and in the cost functions facing each firm, are crucial elements that determine the effect of inappropriability. This general conclusion is in line with a number of empirical investigations of the impact of R&D spillovers that cast doubt on the simple results of many theoretical models that attempt to capture the effects of inappropriability.

CHAPTER FIVE

INTERNATIONAL R&D SPILLOVERS

AND PUBLIC POLICY

I. Introduction

Some of the most interesting applications of the type of framework presented in chapter four above are in the field of international trade and in particular with respect to public policy. Strategic behaviour arguments of the type discussed here are in effect often invoked for the justification of an actively interventionist approach to international trade in the form of R&D subsidies.

The framework of a multi-stage game of cost-reducing R&D and production can be readily adapted to look at the issue of public policy in international trade, as Brander and Spencer (1983b) have shown. They applied the results of their first model of strategic use of R&D (1983a) to the analysis of government intervention in oligopolistic international markets through export or R&D subsidies. By modelling government behaviour as the first-move in a multi-stage game (thus affecting the subsequent strategies of firms), their model examines the optimality and welfare implications of a number of different forms of government intervention in support of the R&D performed or of the exports of "national champions". R&D is assumed to be homogeneous and perfectly appropriable; the game that the domestic and the foreign firm play then consists of choosing R&D levels before output levels.

The introduction of public policy in the form of government taxes or subsidies alters the structure of the game. The government of each country is now a player who can make a move one period before firms choose their R&D levels. Assume first that the government of the domestic country, wishing to maximise the net rent accruing to the country (ie the profit to the domestic firm minus the cost of the public policy), makes a prior commitment to subsidise R&D. Brander and Spencer show that the introduction of such a third stage in the game, prior to the other two stages, affects the strategies and payoffs of the domestic and the foreign firm. A domestic R&D subsidy would in such a case increase domestic R&D while decreasing or increasing the R&D of the foreign firm (their Proposition 1). The optimal rate of such a subsidy would be positive (Proposition 2). Furthermore the resulting outcome is equivalent to that which would have come about in the game with no government intervention if the domestic firm was in the position of a leader in a leader-follower equilibrium at the stage where R&D levels are chosen.

The natural question that arises in this context is why the domestic firm cannot move to the leader-follower equilibrium by itself. Two possible interpretations are offered. The first is that in the absence of subsidies, the firm's choice of R&D level is the one that maximises its profit within the context of the two-stage Nash equilibrium. Any different strategy entails the risk of lower profits. The subsidy on the other hand alters the perceived cost structure and thereby changes the set of actions that are consistent with the two-stage Nash equilibrium. Given that the firm cannot credibly subsidise itself, "the government does for the firm what the firm cannot do for itself" [1]. The case of European governments' involvement in aerospace (Concorde, Airbus) is an often cited example in this respect. The second (less plausible) interpretation is that governments have a better knowledge of firm behaviour (ie that firms are naive) or alternatively that they has a different, and possibly incorrect, view of industrial structure to firms. Unlike in the previous interpretation, the stress here is on information asymmetries.

If, instead of only one government subsidising R&D, both simultaneously set R&D subsidies, the resulting noncooperative equilibrium will involve positive subsidies. Both countries are worse off since they earn less rent that they would if they could coordinate so as not to subsidise R&D, although the third --consuming-- country

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[1] Brander and Spencer (1983b), p. 714.

gains from the lower prices that result from higher production. The jointly optimal policy would therefore be to tax R&D, so as to offset the negative effect of own R&D on the other firm's profit and revert to the Cournot equilibrium. This underlines the beggar-thy-neighbour nature of most such policy based on strategic arguments whereby each country gains at the expense of the other [2].

The nature of public policy in this setting is fundamentally different to that explored in chapters two and three. In those chapters it was implicit in the formulation of public policy that competition took place exclusively amongst domestic firms. Similarly, R&D spillovers benefited only the domestic rivals of any one domestic firm undertaking R&D. R&D subsidies in this context applied to all firms in the industry. The rationale was that in the absence of subsidies, firms would be spending less on R&D than socially optimal. Subsidies led to higher R&D, lower production costs, and thus higher production and consumer surplus. With spillovers, under certain assumptions about the cost functions, this effect was accentuated, suggesting that optimal subsidies rise as the degree of appropriability of R&D falls.

In the context of a model of strategic trade, the role of public policy is instead one of supporting the domestic firm(s) against its foreign rivals. The government's objective is still to maximise domestic welfare. In this case, however, under the

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[2] A final result of the Brander and Spencer (1983b) paper is that, in the case where countries are allowed to set both R&D and export subsidies, the noncooperative international equilibrium involves both countries imposing export subsidies and R&D taxes. Export subsidies increase exports while the R&D tax forces firms to operate at levels of R&D that minimise costs, thus achieving overall production efficiency. Once again however, noncooperative behaviour reduces the total rent over which the two countries compete relative to the collusion case. Given that the focus of this thesis is on the effects of inappropriability of research, we do not discuss export subsidies in this chapter. For a criticism however of the results by Brander and Spencer that focuses on implications of assumptions on parameters such as the elasticity of demand on the results, see Collie and de Meza (1986). Another line of criticism focuses on the strategic variable assumed. In the Brander and Spencer international duopoly, output is the strategic variable. Eaton and Grossman (1986) have demonstrated that in the case where instead the strategic variable is price, an export tax, rather than an export subsidy is called for; de Meza (1989) has shown price controls to be at least as attractive as export subsidies in maximising domestic benefits.

assumption that all domestic output is exported, domestic welfare consist of producer's surplus only; consumer's surplus is absent. The objective of the subsidy is thus one of profit-shifting only; in setting subsidy rates the government is concerned exclusively with the interests of the domestic firm. Policy is thus guided by strategic considerations of profit-shifting, not efficiency considerations [3].

This introduction to the results obtained by Brander and Spencer serves as a backdrop for the examination of government policy in a framework of international competition through cost-reducing R&D where R&D is subject to spillovers. In this chapter we extend the Brander and Spencer framework to take account of the possibility that some of R&D is not appropriable by the originating firm.

The extension to the case with spillovers is a natural one in the following sense. In the "new trade" literature there are two types of arguments which are invoked for the justification of departures from principles of free trade. There are first the arguments that are grouped under the "strategic trade" heading. The main point made here is that in imperfectly competitive international environments characterised by persistent excess profits intervention allows governments to "shift" some of these profits to their domestic firm(s): this is a "beggar-thy-neighbour" struggle over rents; put another way, a zero-sum game.

The second type of argument is a more traditional one and is based on externalities: the inability of firms to fully appropriate the results of innovations. Intervention

[3] In the Brander and Spencer models, "profit-shifting" is interpreted as the use of public policy in order to shift profits from foreign to domestic firms. The interpretation is dictated by the structure of their model, particularly the assumption that all the output of the domestic firm and of its foreign rival is exported to a third -- non-producing -- country. This is not however the only incentive that governments have for intervention in strategic trade models. As Norman (1989) notes, through active policies that impact on production and trade, governments can attempt to shift profits from foreign firms to domestic consumers or to the home government, or they can attempt to improve the terms of trade by policies that make domestic exporters compete less aggressively with each other or make foreign suppliers move down their marginal-cost curves.

here is a positive-sum game: the support of R&D can have beneficial effects beyond the borders of the intervening country. The argument is therefore a reformulation in the context of international trade of the basis for R&D subsidies developed in chapters two and three above.

These two arguments tend to be sharply separated in attempts at modelling trade in imperfectly competitive environments and are often confused in policy debates [4]. This chapter develops a framework that allows the treatment of both and the examination of possible conflicts in the nature of public policy based on the different starting points and nature of the two arguments [5].

The starting point of the theory developed here is that the existence of spillovers implies that national innovation policies which subsidise the research expenditures of "national champions" can no longer be considered in isolation. To the extent that part of the knowledge generated by an R&D subsidy is available to a domestic firm's foreign competitor, such a policy of subsidising R&D has an important externality attached to it, not previously present when research was fully appropriable. In an analogous fashion, when a foreign firm cannot appropriate part of its R&D investment, a foreign government's subsidisation of the research of its national champion cannot be

[4] Spence (1984a) has referred to the fact that in environments where governments invest in R&D, the benefits will spill across national borders as "a second level of the free-rider problem at the country level" (p. 360). He suggests that this complicates the evaluation of strategic policies and that differences across countries in dealing with these externalities will lead directly to shifts in relative competitive positions. The objective of policy in such an environment would be to "internalise" R&D externalities internationally, implying that certain forms of cooperative or quasi-cooperative behaviour among firms and countries are required in order to achieve dynamic efficiency.

[5] Another reason why the extension for the case of spillovers is a potentially useful one is that with full R&D appropriability there is nothing to distinguish R&D from any other type of investment. By extending the model for the case of spillovers in R&D, we come closer to justifying such a distinction, based on the unique public good features of R&D. This also allows to distinguish more credibly R&D subsidies from any other type of subsidies.

"contained" within that country's national borders. This has important implications (through the spillover rate) for the level of production costs of the domestic firm, its strategic response and for the appropriate policy of the domestic government.

The chapter is organised as follows. In the next section we develop a three-stage model of international competition with spillovers in general form. This framework will allow us to address the question of whether the same incentives for subsidising R&D carry over in the situation where part of R&D is not appropriable by the originating firm. We first examine the effects of a domestic R&D subsidy on the levels of R&D investment of the domestic and foreign firms and. This leads to a discussion of the determinants of the optimal R&D subsidy rate and the conditions under which R&D should be taxed. We then examine the determinants of the rate of return to government intervention, and how this is affected by conditions of appropriability and the nature of technology, in the sense of the ease of substitutability between the R&D of the domestic firm and that of its foreign rival. The characteristics of a jointly optimal policy are then briefly reviewed. Concluding remarks are in section III.

II. A duopoly model of international competition with spillovers

In order to address the question of the implications of R&D spillovers for the nature of public policy in the context of international trade, we extend the framework first developed by Brander and Spencer to the case where R&D is only partially appropriable by the firm that originates it. We examine the impact of an R&D subsidy and the optimal subsidy rate when the government of one country attempts to help its "national champion" capture a bigger slice of the world market and to increase profits and rent to the domestic country, net of the cost of the subsidy. We then examine the rate of return of such intervention and investigate the characteristics of the equilibrium that prevails when both governments attempt to jointly maximise rents.

a. The basic model

Rather than thinking of a duopoly amongst firms *i* and *j* in the domestic market as in chapter four, firm *i* now represents the *domestic* firm and firm *j* the *foreign* firm [6]. In the good that they produce, each is the sole supplier of the country it represents (a "national champion") and both sell to a third market [7]. Furthermore, the structure of the game is different. Rather than firm rivalry being represented by a three-stage game, where firms successively choose basic research expenditures, development research expenditures and production levels, we assume instead that R&D is homogeneous. Firms therefore choose R&D levels in one stage that preceeds the choice of output

[6] Katz and Shapiro (1990) note the difficulties that often surface in attempting to distinguish between "domestic" and "foreign" firms. These difficulties are related to institutional arrangements such as extensive linkages among firms, location of production and shareholding in many countries etc. and complicate the formulation of social welfare functions that serve as a basis for policy. In what follows we assume that the identity of firms is well-defined so that these problems do not arise.

[7] For simplicity we assume that all of the output that each firm produces is exported to the third market where the two foreign firms compete. This third market has no domestic production of its own in the good in question. The possibility of domestic consumption would generally increase incentives for subsidisation (since it would lead to lower prices and higher quantities).

levels. Before R&D levels are chosen however, governments set R&D subsidy rates that are communicated to both firms. Subsequent decisions on the levels of R&D and output therefore reflect the level of public support of R&D.

In the last (output) stage of the game, with R&D expenditures sunk and known to each other, and with a subsidy s per unit of its R&D, the domestic firm chooses the level of output q^1 (ie its exports to the third country) that maximises:

$$\Pi^1(q^1, q^2; x_1, x_2) = E^1(q^1, q^2) - C^1(q^1, x_1, x_2) - (1-s)x_1 \tag{5.1}$$

As before, we assume that the good produced is homogeneous and that increasing the output of good j decreases the total and marginal revenue of firm i. We therefore have that $\partial E^i/\partial q^j = E^i_j < 0$ and $\partial^2 E^i/\partial q^i \partial q^j = E^i_{ij} < 0$. Marginal cost is taken to be independent of output so that $C^i = c^i q^i$ with $\partial C^i/\partial q^i = c^i$ and $\partial c^i/\partial q^i = 0$. Variable total and marginal costs decline with a firm's expenditures on R&D ie $\partial C^i/\partial x_i = C^i_{x_i} < 0$ ($\partial c^i/\partial x_i = c^i_{x_i} < 0$), but at a declining rate ie $\partial^2 C^i/\partial x_i^2 = C^i_{x_i x_i} > 0$ ($\partial^2 c^i/\partial x_i^2 = c^i_{x_i x_i} > 0$). Because of the existence of spillovers however, they also decline with the R&D of its rival ie $\partial C^i/\partial x_j = C^i_{x_j} < 0$ and $\partial^2 C^i/\partial x_j^2 = C^i_{x_j x_j} > 0$ ($\partial c^i/\partial x_j = c^i_{x_j} < 0$ and $\partial^2 c^i/\partial x_j^2 = c^i_{x_j x_j} > 0$). Finally we leave open the possibility that the effect of borrowed basic research on a firm's marginal cost reduction due to own basic research expenditures be positive or negative so that the sign of $\partial^2 C^i/\partial x_i \partial x_j = C^i_{x_i x_j}$ (and similarly of $\partial^2 c^i/\partial x_i \partial x_j = c^i_{x_i x_j}$) is not predetermined.

The first-order conditions for a maximum are then given by:

$$\Pi^1_1 \equiv \partial \Pi^1/\partial q^1 \equiv E^1_1(q^1, q^2) - c^1(x_1, x_2; s) = 0 \tag{5.2}$$

where $E_1^1 = \partial E^1/\partial q^1$. The second-order conditions are satisfied for

$\Pi_{11}^1 \equiv \partial^2\Pi^1/\partial q^1 \equiv E_{11}^1 < 0$ and for $A \equiv \Pi_{11}^1\Pi_{22}^2 - \Pi_{12}^1\Pi_{21}^2 = E_{11}^1E_{22}^2 - E_{12}^1E_{21}^2 > 0$. In symmetry, this implies $\Pi_{jj}^j(=E_{jj}^j) < \Pi_{ji}^j(=E_{ji}^j)$.

The subsidy does not affect directly the resolution of the output game when R&D levels are given, although it does indirectly affect production through its effect on R&D levels committed by each firm. Given therefore that from the last (output) stage we have that $q^1 = q^1[c^1(x_1,x_2), c^2(x_1,x_2)]$ and $q^2 = q^2[c^1(x_1,x_2), c^2(x_1,x_2)]$, profit for the domestic firm is given by:

$$(5.3)\quad g^1(x_1,x_2;s) \equiv \Pi^1\{q^1(x_1,x_2;s), q^2(x_1,x_2;s)\} \equiv$$
$$\equiv E^1\{q^1(c^1(x_1,x_2),(c^2(x_1,x_2)), q^2(c^1(x_1,x_2),(c^2(x_1,x_2))\} -$$
$$- C^1(q^i(c^1(x_1,x_2),(c^2(x_1,x_2));x_1,x_2)) - (1-s)x_1$$

At the *second stage*, each firm chooses the level of its R&D expenditures. In choosing its x_1, the domestic firm balances two considerations: on the one hand, higher R&D expenditures reduce production costs and raise its market share; on the other, they increase its fixed costs and result in a lower industry price. Given the R&D expenditures of its rival therefore, and with diminishing returns to R&D, each firm chooses an optimal x_1 in order to maximise profits. The Nash equilibrium of the strategic game then occurs where each firm is maximising its profit with respect to its R&D expenditures, given the level chosen by its rival. For the domestic firm, therefore, benefiting from an R&D subsidy, the first-order condition for a maximum is:

$$(5.4)\quad g_1^1 \equiv \partial g^1/\partial x_1 \equiv \Pi_1^1 q_{x_1}^1 + \Pi_2^1 q_{x_1}^2 + \Pi_{C_1}^1 C_{x_1}^1 + \Pi_{x_1}^1 = 0$$
$$\Rightarrow E_1^1 q_{x_1}^1 + E_2^1 q_{x_1}^2 - C_{x_1}^1 - (1-s) = 0$$
$$\Rightarrow E_2^1 q_{x_1}^2 - c_{x_1}^1 q^1 = 1 - s$$

since $\Pi_1^1 = E_1^1 = 0$. The second-order conditions are given by:

$$(5.5) \qquad g_{11}^1 \equiv \partial g_1^1/\partial x_1 \equiv E_2^1 q_{x_1x_1}^2 + q_{x_1}^2(\partial E_2^1/\partial x_1) - c_{x_1x_1}^1 q^1 - c_{x_1}^1 q_{x_1}^1 < 0$$

and by $D = g_{11}^1 g_{22}^2 - g_{12}^1 g_{21}^2 > 0$.

The levels of R&D expenditures of the domestic and the foreign firm in equilibrium are determined at the intersection of the R&D reaction functions for each firm. These reaction functions describe for each firm its best response in terms of expenditures in R&D to different levels of R&D by its rival. Taking the case of the domestic firm as an example, the slope of the reaction function will be determined by the domestic firm's reactions to different levels of R&D expenditures by the foreign firm. Starting with a particular combination of domestic and foreign R&D expenditures, the question is whether, as a response to higher foreign R&D, the profits of the domestic firm are maximised with a higher or with a lower R&D budget.

In the situation where R&D is fully appropriable by the firm that originates it, an increase in R&D expenditures by the foreign firm implies that its marginal costs are lower than before. It will therefore tend to produce more as a result. This implies that the expected market size for the domestic firm is now lower than before. Since this reduces the marginal returns to research, it is reasonable to suppose that the domestic firm's best reply is a lower level of R&D expenditures, implying a negatively sloped R&D reaction function. In a symmetric situation, the same will be true for the reaction function of the foreign firm.

When however R&D is no longer fully appropriable by the originating firm and is instead subject to spillovers, this assumption is no longer clear-cut. The R&D externality implies that an increase in the foreign firm's R&D expenditures, while still lowering its own costs, will also have the external effect of lowering the costs of the domestic firm. While therefore the increase in the foreign firm's production implies a fall in the expected market size for the domestic firm, tending to reduce the marginal return of research, the externality effect associated with R&D spillovers tends to increase it. In the situation where this latter effect is strong, the net marginal return of the domestic firm's R&D will increase. It will then be reasonable to suppose that the domestic firm's best reply to a higher level of R&D expenditures by its foreign rival is to increase its own R&D, giving (in a situation of symmetry) upward-sloping R&D reaction functions [8].

The slope of the R&D reaction functions can be derived by the total differentiation of $g_1^1 = 0$ and $g_2^2 = 0$ with respect to x_1 and x_2 to obtain:

(5.6) $$dx^1/dx^2 = -g_{12}^1/g_{11}^1 \qquad \text{and} \qquad dx^2/dx^1 = -g_{21}^2/g_{22}^2$$

From the second-order conditions for the existence and uniqueness of equilibrium we require that $g_{11}^1 < 0$ [9]. The sign of dx^1/dx^2 is therefore determined by the sign of g_{12}^1. If an increase in the foreign firm's R&D reduces the effect of the domestic firm's own R&D on its own profit ($g_{12}^1 < 0$), then the R&D reaction function is negatively sloped. If

[8] d' Aspremont and Jacquemin (1988) have developed a duopoly model with spillovers, parameterized with linear forms, in order to compare the R&D and production levels of the noncooperative equilibrium with those of two equilibria that result from different forms of cooperation (in R&D only, or in both R&D and output) and with the socially optimal levels. Henricques (1990), in a short note, undertakes stability analysis of the d' Aspremont and Jacquemin model and comes to conclusions regarding the shape of output reaction functions that are similar to ours. She concludes that for "large spillovers" (greater than 50% in her case) R&D reaction functions cross correctly (thus ensuring stability) but are upward-sloping.

[9] As was noted earlier in this chapter, an inspection of (5.5) shows that at least one of the terms of g_{11}^1 can be positive ($E_2^1 q_{x_1x_1}^2$) even in the absence of spillovers. With spillovers in R&D a second term can also be positive ($q_{x_1}^2(\partial E_2^1/\partial x_1)$ since we can have $q_{x_1}^2 > 0$), making it all the more possible that the second order conditions will not hold. While this is not a trivial problem, we restrict our attention to the cases where there are sharply diminishing returns to own R&D (ie where $c_{x_1x_1}^1$ is positive and relatively large), so that this effect is dominant and ensures well-behaved second-order conditions.

instead an increase in the rival's R&D increases also the effect of own R&D on own profit ($g_{12}^1 > 0$), the R&D reaction function is positively sloped. In a symmetric situation, this is true for the R&D reaction functions of both firms.

The question that arises here is under what conditions g^i_{ij} might be positive ie when would an increase in the R&D of a firm's rival also raise the effect of its own R&D on profits. Take first the case where R&D is fully appropriable, so that a firm's production costs are a function solely of its R&D investment. Brander and Spencer acknowledge that at least one of the terms of g^i_{ij} is positive [10]. Since however in the case where spillovers are absent we have that $c^1_{x_1x_2} = 0$ and also that $q^1_{x_2} < 0$, it is reasonable to conclude that the effect of the negative term is outweighed, giving $g^1_{12} < 0$.

In the situation where R&D expenditures are subject to spillovers, differentiation of the expression g_1^1 with respect to x_2 gives $g^1_{12} = E^1_2 q^2_{x_1x_2} + q^2_{x_1}(\partial E^1_2/\partial x_2) - c^1_{x_1x_2}q^1 - c^1_{x_1}q^1_{x_2}$. A number of terms in this expression are now of an ambiguous sign. As was discussed in chapter four above in the context of a three-stage model in general form (with R&D broken down into basic and development research), with spillovers it is possible to have $q^2_{x_1} = \left(c^2_{x_1}E^1_{11} - c^1_{x_1}E^2_{21}\right)/\mathrm{A} > 0$ (and similarly for $q^1_{x_2} > 0$). This can be because cost reduction through R&D benefits a firm's rival more than the firm itself ($c^2_{x_1} < c^1_{x_1}$); it can also occur however when R&D reduces the costs of the originating firm more than that

[10] When spillovers are absent, $g^1_{12} = E^1_2 q^2_{x_1x_2} + q^2_{x_1}(\partial E^1_2/\partial x_2) - c^1_{x_1}q^1_{x_2}$. The first term of this expression can be positive, since $E^1_2 < 0$ and we can have $q^2_{x_1x_2} < 0$. In the second term, $\partial E^1_2/\partial x_2 = E^1_{21}q^1_{x_2} + E^1_{22}q^2_{x_2}$ is positive since $E^1_{21} < 0$, $q^1_{x_2} = -c^2_{x_2}E^1_{12}/A < 0$, $q^2_{x_2} = c^2_{x_2}E^1_{11}/A > 0$ and $E^1_{22} \geq 0$ (ie an increase in q^2 reducing E^1 but at a diminishing rate). Together with $q^2_{x_1} < 0$ this means that the second term is negative, as is the third term ($q^1_{x_2} < 0$ and $c^1_{x_1} < 0$). We will then have $g^1_{12} < 0$ as long as $E^1_2 q^2_{x_1x_2}$ is not too positive.

of its rival ($c^2_{x_1} > c^1_{x_1}$) as long as $c^2_{x_1}E^1_{11} > c^1_{x_1}E^2_{21}$ [11]. Similarly $c^1_{x_1x_2}$ can be negative [12], with the implication that it is possible for all terms in g^1_{12} to be positive, giving (in symmetry) unambiguously upward sloping R&D reaction functions for both firms.

b. The effect of a domestic R&D subsidy

We now want to examine the effect of the R&D subsidy given to the domestic firm on R&D expenditures in an environment where R&D is characterised by spillovers. The comparative statics of an increase in the R&D subsidy to the domestic firm on the R&D expenditures of both the domestic and foreign firms can be determined by totally differentiating $g^1_1(x_1,x_2;s)=0$ from (4.37) and $g^2_2(x_1,x_2)=0$ with respect to x_1, x_2 and s. This yields the system:

$$g^1_{11}dx^1 + g^1_{12}dy^2 + ds = 0$$

(5.7)

$$g^2_{21}dx^1 + g^2_{22}dx^2 = 0$$

From this system by a simple application of Cramer's rule we conclude that the effect of the subsidy on the firm's own level of R&D is given by $x^1_s = \partial x^1/\partial s = -g^2_{22}/D$, while the effect on the rival's R&D is given by $x^2_s = \partial x^2/\partial s = -g^2_{21}/D$. Since $g^2_{22} < 0$ by

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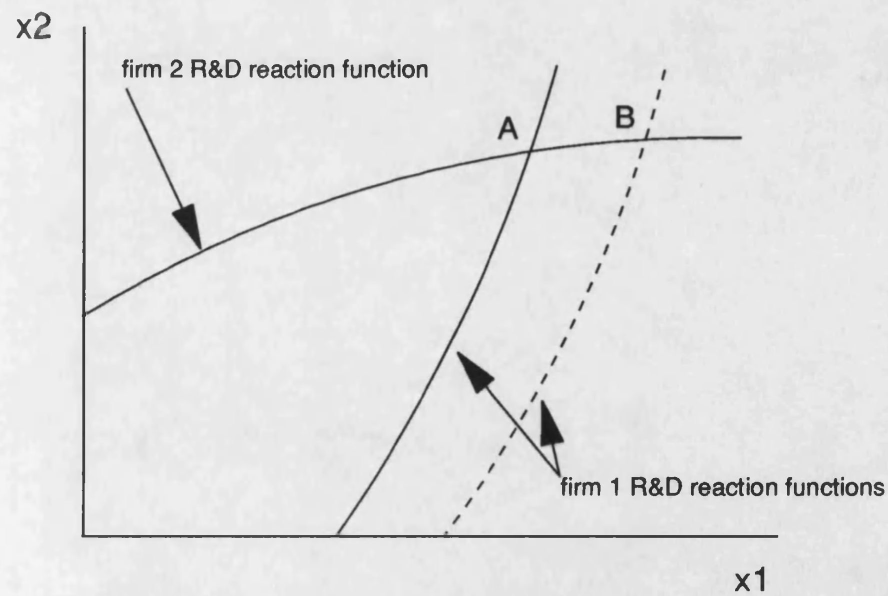
[11] Note that the conditions for which we can have $q^i_{x_j} > 0$ are more general than the conditions for which $q^i_{x_i}$ reverses sign and becomes negative. In the latter case for an increase in a firm's R&D to lead to a decline in its output it is a *necessary* condition that its R&D expenditures reduce the cost of its rival by more than its own ie $c^j_{x_i} < c^i_{x_i}$. In the former case, for an increase in a firm's R&D to lead to an increase, rather than to a decline, in its rival's R&D it is *sufficient* that $c^j_{x_i} < c^i_{x_i}$. This aggressive response from rivals can also come about however in the "normal" case where R&D expenditures reduce the (marginal) cost of the initiating firm by more than that of its rival (ie where $c^j_{x_i} > c^i_{x_i}$).

[12] For a functional form of the R&D production function and of the corresponding (marginal) cost function as that given in expression (4.11) of chapter four, the R&D of a firm's rival increases the rate of marginal cost reduction due to a firm's own R&D ($c^i_{x_ix_j} < 0$) when $\alpha < \rho$, that is when the elasticity of cost reduction with respect to knowledge is greater than the elasticity of substitution between a firm's own R&D and that of its rivals.

the second-order conditions, the domestic subsidy expands the R&D expenditures of the domestic firm [13]. The effect on the rival's R&D effort depends on the sign of g_{21}^2. If $g_{21}^2 < 0$, the subsidy reduces the R&D outlays of the rival firm. In the case on the other hand where due to spillovers $g_{21}^2 > 0$, the subsidy to the domestic firm induces an aggressive response and higher R&D expenditures on the part of its rival.

This outcome is illustrated in Figure 1. With upward sloping R&D reaction functions (ie under the assumption that due to spillovers $g_{ji}^j > 0$) and an initial equilibrium at point A, the subsidy to the domestic firm shifts out its reaction function. The new equilibrium at point B involves higher R&D levels by both firms.

Figure 1: Effects on R&D expenditures of an R&D subsidy



[13] Note that the relationship between the effect of the subsidy on the R&D of the domestic firm and on that of its rival is given by $x_s^1 = -x_s^2(g_{21}^2/g_{22}^2) = -x_s^2(dx_2/dx_1)$.

Proposition 1: **With spillovers in R&D, a domestic R&D subsidy increases domestic R&D and, provided $g^2_{21}>0$, increases foreign R&D as well. If $g^2_{21}<0$, foreign R&D falls.**

c. Optimal R&D subsidies

By "precommitting" to a subsidy of the R&D expenditures of its firm, the domestic government attempts to put the firm in a better competitive position. The subsidy aims at achieving lower marginal production costs, higher market share and profit than the foreign competitor. With spillovers however, we have seen that a domestic subsidy, while inducing higher R&D levels domestically, can also induce higher R&D levels abroad. The natural question that arises in this context is whether the optimal subsidy rate is positive or negative ie whether the incentives for the government to subsidise R&D remain in place or are reversed.

The optimal subsidy s^* is the rate that maximises welfare in the domestic country net of the cost of the subsidy; this is simply equal to the profit of the domestic firm minus the cost of the R&D subsidy [14]. The government therefore chooses s so as to maximise:

(5.8) $$W^1(s) = g^1(x_1, x_2; s) - sx_1$$

From (5.8), the first-order condition for the welfare maximising subsidy is:

(5.9) $$dW^1/ds = g_1^1 x_s^1 + g_2^1 x_s^2 + g_s^1 - x_1 - s x_s^1 = 0$$

$$\Rightarrow x_s^1(g_2^1(dx^2/dx^1) - s) = 0$$

[14] In chapters two and three we derived optimal subsidies in the one-stage game with n domestic firms under the assumption that the government attempted to subsidise total domestic surplus $TS = CS + n\Pi - sX$ (consumer surplus plus producer surplus minus the cost of the subsidy). The difference between that maximand and (5.8) is that here consumer surplus is zero since all output is exported and that producer surplus is the profit of the (unique) domestic firm.

since $g_1^1 = 0$, $x_s^1 = -x_s^2(g_{21}^2/g_{22}^2) = -x_s^2(dx_2/dx_1)$ and $g_s^1 = x_1$. The optimal subsidy rate is:

$$(5.10) \qquad s^* = g_2^1(dx^2/dx^1) = -g_2^1(g_{21}^2/g_{22}^2)$$

From expression (5.10) it can be seen that the optimal subsidy rate is equal to the change in the domestic firm's own profit brought about by the effect of an increase in the domestic firm's R&D on the foreign rival's R&D. The sign of the subsidy rate therefore will depend on two effects: the effect of an increase in own R&D on the R&D of the rival firm ie dx^2/dx^1; and the effect of the change in the R&D of the rival on a firm's profitability ie g_2^1. Since $dx^2/dx^1 = -g_{21}^2/g_{22}^2$ and $g_{22}^2 < 0$ by the second-order conditions for a maximum in the R&D game, the foreign rival's response to higher domestic R&D levels will depend on the impact of domestic R&D on the marginal profitability of foreign R&D. We can label this first effect a (marginal) *R&D productivity* effect; g_2^1 is then an *R&D profitability* effect [15]. Their respective signs, and their interaction, determine whether optimal subsidies are positive, or whether instead R&D should be taxed.

In order to establish the conditions that determine the sign of the optimal subsidy rate, it is necessary to look more closely at the conditions that determine the sign of g_2^1 and of dx^2/dx^1. From (4.36), we have that $g_2^1 = \Pi_1^1 q_{x_2}^1 + \Pi_2^1 q_{x_2}^2$. Since $\Pi_1^1 = 0$ from (4.35), this reduces to $g_2^1 = E_2^1 q_{x_2}^2$. Since $\Pi_2^1 = E_2^1 < 0$, we will have $g_2^1 < 0$ when $q_{x_2}^2 > 0$ and $g_2^1 > 0$ when $q_{x_2}^2 < 0$. The conditions for which we can have this latter possibility were explored at length in section II.a of chapter four above. It was established that for $q_{x_2}^2 < 0$ a necessary (though not sufficient) condition is that $c_{x_2}^2 > c_{x_2}^1$. In order therefore for an increase in the foreign firm's R&D to have the potential to lead to higher profits

[15] Jaffe (1986) makes a similar distinction in an empirical investigation of the effects of spillovers. In his paper, the productivity of a firm's R&D is increased by the R&D of "technological neighbours"; the R&D of rivals however also lowers the profits and market value of firms with low R&D intensity.

for the domestic firm, the foreign firm's R&D expenditures must, because of spillovers and substitutability conditions, decrease its production costs at the margin by less than they decrease the production costs of the domestic firm. For a spillover rate less than unity, this can occur with a knowledge production function of a CES form such as the one in (4.28') of chapter four above, and where in addition the R&D of the foreign firm is not easily substitutable with that of the domestic firm [16].

The effect of the domestic firm's own R&D on the marginal profitability of the foreign rival's R&D (g^2_{21}) determines the slope of the reaction functions and therefore the response of the foreign firm to a higher level of domestic R&D. The conditions under which $g^2_{21} > 0$, giving upward-sloping reaction functions, are less straight-forward and were explored earlier in this chapter (section III.a). It was established that for this to happen it is sufficient for $q^2_{x_1} = \left(c^2_{x_1}E^1_{11} - c^1_{x_1}E^2_{21}\right)/\mathrm{A} > 0$, $q^1_{x_2} > 0$ and $c^1_{x_1x_2} < 0$.

From the variables that determine the signs of g^1_2 and g^2_{21}, it can be seen that the R&D productivity effect and the R&D profitability effect are related to the conditions for appropriability of R&D, as well as to conditions relating to the ease of substitutability between the R&D of the domestic firm and that of its foreign rival. Consequently, the interaction of these conditions determine whether the optimal subsidy rate is positive or negative. On this basis, it is possible to distinguish between three cases, corresponding to different conditions relating to appropriability and to substitutability of research inputs. This taxonomy can be interpreted as corresponding to three types of international oligopolistic industries.

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[16] With spillovers in R&D, $q^2_{x_2} = \left(c^2_{x_2}E^1_{11} - c^1_{x_2}E^2_{21}\right)/D$. Since $D>0$, in order to have $q^2_{x_2} < 0$ a necessary and sufficient condition is that $c^2_{x_2}E^1_{11} < c^1_{x_2}E^2_{21}$which, given that $E^1_{11} < E^2_{21}$, implies $c^2_{x_2} > c^1_{x_2}$ as a necessary condition. With $\theta < 1$, this condition is satisfied under an R&D production function of a CES form as in $R_i = \left[x_i^{-\rho} + \left(\theta \sum_{j \neq i} x_j\right)^{-\rho}\right]^{-1/\rho}$and when in addition $\theta^{-\rho} > 1$ ie when $\rho > 0$ giving an elasticity of substitution between own and borrowed R&D that is less than one.

In the first case (first type of industry), the increase in the domestic firm's R&D due to subsidies reduces the R&D outlays of the foreign rival ($g^2_{21} < 0$, leading to $dx^2/dx^1 < 0$). With a larger expected market, own profits increase as a result ($g^1_2 < 0$) and the optimal subsidy is positive. This is the result obtained by Brander and Spencer (1983a) in an environment where R&D was fully appropriated by the originating firm(s). The result may however also stand in the situation where appropriability of R&D is not perfect, as long as the spillover rate is not high. With moderate spillovers, an increase in the R&D expenditures of the domestic firm may still reduce the marginal profitability of the foreign rival's R&D, despite the fact that some of the domestic R&D expenditures cannot be fully appropriated. The foreign rival's best response to a higher level of domestic R&D would in that case still be to reduce its own R&D outlays, leading to higher domestic profits. In terms of the specifications of the knowledge production function presented in section II.a of chapter four above, this type of situation can arise both with domestic and foreign R&D being perfect substitutes as in (4.28) and in the case where we have a CES form as in (4.28'), as long as domestic and foreign R&D are easily substitutable with each other.

The other two cases both relate to an environment where R&D is subject to spillovers. In the first, higher domestic R&D expenditures increase the marginal profitability of the foreign rival's R&D expenditures ($g^2_{21} > 0$). This gives upward-sloping R&D reaction functions, with the result that higher R&D levels by the domestic firm are matched by higher R&D outlays by the foreign firm ($dx^2/dx^1 > 0$). If a higher level of rival R&D reduces a firm's own profits, the effect of this aggressive response from the foreign rival will be to reduce the profits for the domestic firm ($g^1_2 < 0$). Put another way, the (negative) effect due to spillovers dominates in this case the cost reduction achieved by R&D. In that event, the optimal subsidy rate is negative: R&D should be taxed.

The same assumptions about technology and spillovers that resulted in a positive subsidy rate in the first case examined above can also deliver a negative optimal subsidy rate. Both a parameterization of the knowledge production function where domestic and foreign R&D are perfect substitutes as in (4.28) and one with a CES form as in (4.28') -- as long as domestic and foreign R&D are easily substitutable with each other -- can result in $g^2_{21} < 0$ or $g^2_{21} > 0$, leading (with $g^1_2 > 0$ in both cases) to a positive or a negative subsidy rate respectively. The difference between the two rests on the degree of appropriability, with higher spillover rates reversing the benefit for the domestic firm and making the case of taxing R&D. This suggests that a successful government policy needs to be sensitive to both the technological characteristics of industries (that determine the ease of substitutability of own with borrowed R&D) and to the degree of inappropriability involved.

The final case relates to a situation where $g^2_{21}>0$, leading to $dx^2/dx^1>0$, and where in addition $g^1_2>0$ [17]. The increase in the domestic firm's R&D outlays increases, because of spillovers, the marginal profitability of the foreign firm's R&D. Here however, the increase in the rival's R&D expenditures induced by higher domestic R&D, leads to higher, rather than lower, profits for the domestic firm. The key to the explanation is this: for $g^1_2>0$, we need $q^2_{x_2} < 0$; a necessary condition for this is that $c^2_{x_2} > c^1_{x_2}$, or that the foreign firm's R&D expenditures reduce its production costs at the margin by less than they reduce those of the domestic firm. Because of this, the higher foreign R&D (as a response to the higher level of domestic R&D) leads to lower foreign production and consequently to higher domestic profits. This can occur with a

[17] A fourth case where $g^2_{21}<0$, leading to $dx^2/dx^1<0$, and where in addition $g^1_2>0$ can be ruled out because of inconsistency. This is because $g^1_2>0$ when $q^2_{x_2} < 0$; a necessary condition for this is that $c^2_{x_2} > c^1_{x_2}$. This however implies $q^1_{x_2} > 0$ (and equivalently $q^2_{x_1} > 0$) and leads to $g^2_{21}>0$.

knowledge production function of a CES form as in (4.28'), where domestic and foreign R&D are not easily substitutable with each other. In this situation therefore, the subsidy rate is again positive [18].

We can summarise these points in the following proposition:

Proposition 2: With spillovers in R&D, the optimal domestic subsidy rate will be:

> **(i) *positive*, when higher domestic R&D reduces foreign R&D which in turn increases domestic profits ($g^2_{21}<0$ and $g^1_2<0$) or when higher domestic R&D increases foreign R&D but still increases domestic profits ($g^2_{21}>0$ and $g^1_2>0$);**
>
> **(ii) *negative*, when higher domestic R&D increases foreign R&D which in turn reduces domestic profits ($g^2_{21}>0$ and $g^1_2<0$).**

d. Rates of return to intervention

One question that is often raised in the context of the evaluation of "profit-shifting" strategic policies is whether the rate of return to such intervention is anything near the "normal" rates of return that governments could obtain in capital markets. Assume that a government policy that subsidises the R&D investment of its domestic firm can actually achieve higher profits for it and that it does not invite

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[18] In a recent paper, Beath, Katsoulakos and Ulph (1989) develop a model of an R&D race that has many similarities to ours. Their model accounts for two effects the determine R&D expenditures: the *profit incentive* and the *competitive threat*. A major determinant of the relative magnitude of the two effects is the ease of imitation. If imitation is impossible or difficult, the incentive to do R&D in order to prevent the rival from winning exceeds the profit incentive to invest in R&D. In such a case, if the rival increases R&D spending, the competitive response for the firm is to increase its own R&D effort, giving an upward-sloping reaction function. If instead imitation is easy and the rival's R&D is a very good substitute of one's own, the competitive threat virtually disappears. The optimal response to an increase in the R&D of the rival is then to spend less on R&D, giving a downward-sloping reaction function. These two effects determine also the nature of R&D subsidies. The authors conclude that R&D subsidies are only beneficial when the two countries *differ* in the relative importance of the forces that determine their R&D. If instead in *both* countries *either* competitive threats dominate profit incentives *or* if profit incentives dominate competitive threats, then R&D should be taxed. These results mirror ours: we conclude that an R&D subsidy is appropriate when the R&D productivity effect and the R&D profitability effect go in the same direction; in contrast, it should be taxed when the two effects go in opposite directions.

retaliation by the foreign government. Based on a cost-benefit calculation, and taking into account the existence of alternative investments to R&D subsidies, is it still worthwile asking whether such R&D subsidies are the best policy alternative.

The framework that we have developed in this chapter allows us to address this question directly. By specifying functional forms for cost and demand functions, we can first calculate the optimal subsidy rate s^* that will maximise domestic welfare $W^1(s) = g^1(x_1, x_2; s) - sx_1$, as defined in (5.8). On the basis of this optimal subsidy rate, we can then calculate the rate of return to the government of its policy of subsidising R&D. These calculations can be performed under the assumption the R&D is perfectly appropriable, so that the spillover rate θ is zero, and for the case where R&D is subject to spillovers at the rate $0 \leq \theta \leq 1$. In that latter case, we will use the two functional forms of the cost function developed in the chapters above in order to investigate the effects of alternative assumptions about technology on the rate of return of a policy of R&D subsidies.

We start by assuming that before any R&D subsidy, the domestic firm spends x_1 on R&D and earns profits $\Pi^1 = P(Q)q^1 - c^1q^1 - x_1$. The government now introduces a subsidy s^* to R&D. As a result, the domestic firm will now be spending $\hat{x}_1$ on R&D expenditures. The cost of the R&D to the firm is however only $(1 - s^*)\hat{x}_1$, giving profits equal to $\hat{\Pi}^1 = P(\hat{q}^1, q^2)\hat{q}^1 - \hat{c}^1\hat{q}^1 - (1 - s^*)\hat{x}_1$. The change in the profit of the domestic firm due to the subsidy is thus equal to $\Delta\Pi^1 = \hat{\Pi}^1 - \Pi^1 = (\hat{E}^1 - E^1) - (\hat{x}_1 - x_1) + s^*\hat{x}_1$; this is the total real gross return from the policy. The subsidy costs the government an amount $s^*\hat{x}_1$ so that the total real *net* return is $\Delta E^1 - \Delta x_1$. The *per-unit net* return then is $(\Delta E^1 - \Delta x_1)/s^*\hat{x}_1$; we call this R.

Assuming that the government can raise the amount necessary for the subsidy $(s^*\hat{x}_1)$ without any welfare costs, it is faced with two options. The first is to invest $s^*\hat{x}_1$

in the capital markets, earning a net payoff of $rs^*\hat{x}_1$, with r being the rate of return. The second option is to invest in its domestic firm by subsidising its R&D outlays and recoup the investment from the additional profits that this intervention will generate at the expense of the profits of the firm's foreign rival. From the above, it follows that this second option should be followed if its net excess return is positive ie if $R>r$.

Examining first the situation where R&D is fully appropriable, the introduction of the optimal subsidy s^* induces the domestic firm to increase its R&D expenditures by $\Delta x_1 = \hat{x}_1 - x_1$. This implies that the expected market share for the foreign rival is now lower, and consequently so are the potential returns to R&D expenditures. With downward-sloping R&D reaction functions therefore, the foreign rival will decrease its R&D outlays by dx^2/dx^1. This in turn will cause the profits of the domestic firm to increase by g_2^1. The total *net* gain to the domestic firm/government is thus $g_2^1(dx^2/dx^1)(\hat{x}_1 - x_1)$ [19]. From (5.10) however we have that $s^* = g_2^1(dx^2/dx^1)$ so that the total net return can be rewritten as $s^*(\hat{x}_1 - x_1)$. The *per-unit net* return then is $R = s^*(\hat{x}_1 - x_1)/s^*\hat{x}_1 = 1 - x_1/\hat{x}_1$.

For iso-elastic demand and cost functions of the form assumed in chapter four above for the case where spillovers are absent ie $P(Q) = \sigma Q^{-\epsilon}$ and $c^i = \beta x_i^{-\alpha}$ (where σ and $1/\beta$ are the size of the market and the scientific level of the industry redspectively), and from (5.10) we can explicitly derive s^* and R. With complete R&D appropriability, both depend exclusively on the elasticities of demand and of cost reduction with respect

[19] Calculating the net gain due to the R&D subsidy is equivalent to calculating the gain to the domestic firm of moving from the original point on its reaction function to what would have been the Stackelberg leader-follower point in the absence of subsidies. The subsidy in effect shifts the R&D reaction function of the domestic firm so that it intersects the reaction function of the foreign firm at the Stackelberg point.

to R&D, $1/\varepsilon$ and α respectively. For a given $1/\varepsilon$, both s^* and R increase with the elasticity of cost reduction with respect to R&D expenditures. Conversely, for a given α, both rise as demand becomes more elastic [20].

Empirical research suggests that the elasticity of cost reduction with respect to R&D is in the order of 0.1 to 0.2 [21]. In our model, for α=0.1 we obtain s^*=0.06 when demand is inelastic ($1/\varepsilon$=.8) and s^*=0.05 when demand is slightly elastic ($1/\varepsilon$=1.25). For α=0.2 we obtain s^*=0.11 when $1/\varepsilon$=.8 and s^*=0.12 when $1/\varepsilon$=1.25. On the basis of these optimal subsidies, we can calculate the net per unit rate of return $R = 1 - x_1/\hat{x}_1$. For the functional forms assumed here, this simplifies to $R = 1 - (1 - s^*)^{-\varepsilon/A}$ [22]. For α=0.1, this gives R=0.06, a figure that is not sensitive (to the third decimal point) as we vary the elasticity of demand from $1/\varepsilon$=.8 to $1/\varepsilon$=1.25. For α=0.2 we obtain R=0.11 when $1/\varepsilon$=.8 and R=0.12 when $1/\varepsilon$=1.25.

A number of conclusions can be drawn for the case where spillovers are absent. The first is that optimal subsidies rates are relatively low. This is due to the absence of a consumer surplus in the maximand of the government and in the strategic nature of the environment, with firms investing heavily in R&D to begin with. Secondly, net rates of

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[20] Before we derive s^* and R, some remarks on second-order conditions are in order. For the functional forms assumed and for θ=0, $q^2_{x_2} = c^2_{x_2}E^1_{11}/A > 0$ for ε<3 and $q^1_{x_2} = -c^2_{x_2}E^1_{12}/A < 0$ for ε<1. Collie and de Meza (1986) have already noted that in this type of strategic model, results depend on demand being elastic. Furthermore, from (5.10) $s^* = g^1_2(dx^2/dx^1)$. We have first that $g^1_2 = -(1-s^*)(3-\varepsilon)/(5-3\varepsilon) < 0$ for $\varepsilon < 5/3$. Calculating g^2_{21} and g^2_{22} we then get $s^*|_{\theta=0} = \alpha\omega(3-\varepsilon)/[(5-3\varepsilon)+\alpha\omega(3-\varepsilon)]$ where $\omega = \mu/\nu$ and where $\mu = [-2(1-\varepsilon)+(1-2\varepsilon)(1+\varepsilon)/4]$ and $\nu = \{\alpha[(5-3\varepsilon)(3-\varepsilon)/4+(1+\varepsilon)(2-\varepsilon)/4] - \varepsilon(1+\alpha)(5-3\varepsilon)/2\}$. For the second-order conditions to hold (g^2_{22}<0) we need to restrict $\nu < 0$ and for positive profits we need to restrict $\alpha < 2\varepsilon/(5-3\varepsilon)$.

[21] Stoneman (1987) provides a summary of these studies.

[22] For $P(Q) = \sigma Q^{-\varepsilon}$ and $c^i = \beta x_i^{-\alpha}$, for the case with no subsidies we obtain $x_1^{\theta=0} = [\sigma(\alpha/2)^{\varepsilon}\beta^{\varepsilon-1}(1-\varepsilon/2)]^{(1/A)}\{(5-3\varepsilon)/[2(2-\varepsilon)]\}^{(\varepsilon/A)}$; when R&D is subsidised at the optimal rate s^* we have $\hat{x}_1^{\theta=0} = [\sigma(\alpha/2)^{\varepsilon}\beta^{\varepsilon-1}(1-\varepsilon/2)]^{(1/A)}\{(5-3\varepsilon)/[2(2-\varepsilon)(1-s^*)]\}^{(\varepsilon/A)}$. This implies that $R = 1 - x_1/\hat{x}_1$ simplifies to $R = 1-(1-s^*)^{-\varepsilon/A}$ where $A = \varepsilon - \alpha(1-\varepsilon) > 0$.

return to public intervention through R&D subsidies in this setup are much more sensitive to the ease by which costs respond to R&D expenditures than to variations in the demand elasticity. Finally, and on the basis of the criterion that we must have $R>r$ in order to justify intervention, in an environment of complete R&D appropriability the decision seems to be a marginal one that will rest critically on how easy it is to achieve cost reduction through R&D.

Turning now to the case where R&D is subject to spillovers, we first assume that the R&D of the domestic firm is a perfect substitute for the R&D of its foreign rival, so that the cost function is given by $c^i = \beta(x_i + \theta x_j)^{-\alpha}$. We keep the assumption of the iso-elastic demand function. On this basis, we can derive the optimal subsidy rate s^* which now depends on the appropriability conditions, in addition to the elasticities of demand and of cost reduction with respect to R&D [23].

We saw earlier that with positive spillovers, R&D reaction functions could be positively sloped and subsidies could then turn negative in the case where the domestic firm's R&D increases the marginal profitability of the rival's R&D ($g^2_{21}>0$) and this in turn decreases the domestic firm's profits ($g^1_2<0$). In the case in question, this can occur with an inelastic demand and as the spillover rate increases. For example with $\alpha=0.1$ and $1/\varepsilon=0.9$ we obtain $s^*=0.03$ for $\theta=0.1$ and $s^*=-0.02$ for $\theta=0.3$; with $\alpha=0.2$ we obtain

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[23] With $c^i = \beta(x_i + \theta x_j)^{-\alpha}$ we have $g^1_2 = -(1-s^*)[2(\theta-1+\varepsilon)-(1+\varepsilon)(1+\theta)]/(5-3\varepsilon-3\theta+\varepsilon\theta)$. For $x^{\theta>0}>0$ we need $5-3\varepsilon-3\theta+\varepsilon\theta>0$; it follows that $g^1_2<0$ for all $0<\theta\leq 1$. For the second-order conditions to hold ($g^2_{22}<0$) we need $\alpha\mu<\varepsilon(1+\alpha)\nu$ where $\nu=(1+\varepsilon)(1+\theta^2)/2-2(\theta^2-1+\varepsilon)$ and $\mu=2(\theta-1+\varepsilon)^2-2(\theta-1+\varepsilon)(1+\varepsilon)(1+\theta)+(1+\varepsilon)(1+\theta)^2/4$. The sign of $s^*=g^1_2(dx^2/dx^1)$ is then determined by the sign of g^2_{21} which in turn depends on the sign of the expression $\alpha\psi-\varepsilon(1+\alpha)\theta(1-\varepsilon)$ where $\psi=[1-\theta(1-\varepsilon)][2(\theta-1+\varepsilon)-(1+\varepsilon)(1+\theta)]-(\theta-1+\varepsilon)(1+\varepsilon)(1+\theta)+(1+\varepsilon)(1+2\varepsilon)(1+\theta)^2/4$. For positive profits, we also need to restrict $\delta\varepsilon(1+\theta)/[\alpha(2-\varepsilon)]>1$ where $\delta=2(2-\varepsilon)/[4(1-\varepsilon-\theta)+(1+\theta)(1+\varepsilon)]$. On this basis and from (5.10) we can calculate $s^*=-\omega\tau/(1-\omega\tau)$ where $\tau=[2(\theta-1+\varepsilon)-(1+\varepsilon)(1+\theta)]/(5-3\varepsilon-3\theta+\varepsilon\theta)$ and $\omega=[\alpha\psi-\varepsilon(1+\alpha)\theta(1-\varepsilon)]/[\alpha\mu-\varepsilon(1+\alpha)\nu]$.

s^*=0.08 for θ=0.1 and s^*=-0.04 for θ=0.4. In general with inelastic demand, optimal subsidies fall as the spillover rate increases and turn negative at moderate or high spillover rates. They remain positive for a larger range of θ when the elasticity of cost reduction with respect to R&D is larger. Similarly, rates of return to intervention decline as appropriability falls and are negative when R&D is taxed.

For an elastic demand, both s^* and R increase with the spillover rate, in addition to increasing with the ease of cost reduction with respect to R&D. For a slightly elastic demand ($1/\varepsilon$=1.25) and α=0.1, we obtain s^*=0.08 and R=0.09 for θ=0.25 and s^*=0.33, R=0.34 for θ=0.75; with α=0.2 we obtain s^*=0.12, R=0.13 for θ=0.25 and s^*=0.35, R=0.37 for θ=0.75. Thus in environments where spillovers are pervasive, and as long as demand is elastic, a good case can be made for a policy of R&D subsidies of the type discussed here on the basis that its net rate of return largely exceeds the "normal" rate of return in capital markets.

We turn finally to the case where R&D is subject to spillovers but where "own" and "borrowed" R&D are imperfect substitutes. We assume that the knowledge production function is of a constant elasticity of substitution form so that the corresponding cost function is $c^i = \beta[x_i^{-\rho} + (\theta x_j)^{-\rho}]^{\alpha/\rho}$. The elasticity of substitution between the R&D of the domestic firm and that of its rival is then given by $1/(1+\rho)$. The optimal subsidy rate s^* and the rate of return R now depend on this ease of substitutability, in addition to depending on appropriability conditions and on the elasticities of demand and of cost reduction with respect to R&D [24].

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[24] With $c^i = \beta[x_i^{-\rho} + (\theta x_j)^{-\rho}]^{\alpha/\rho}$ we have $g_2^1 = -(1-s^*)[2(\theta^{-\rho}-1+\varepsilon)-(1+\varepsilon)(1+\theta^{-\rho})]/(5-3\varepsilon-3\theta^{-\rho}+\varepsilon\theta^{-\rho})$. For $\mathcal{X}^{\theta>0}>0$ we need to restrict $5-3\varepsilon-3\theta^{-\rho}+\varepsilon\theta^{-\rho}>0$ or $\theta^{-\rho}<(5-3\varepsilon)/(3-\varepsilon)$; it follows that $g_2^1>0$ for $\theta^{-\rho}>(3-\varepsilon)/(1-\varepsilon)$, which is only possible when $\rho>0$. For the second-order conditions to hold (g_{22}^2<0) we need $\alpha\mu<\varepsilon[(\alpha-\rho)+(1+\rho)(1+\theta^{-\rho})]\nu$ where $\nu=\theta^{-2\rho}(1+\varepsilon)/2-2(\theta^{-2\rho}-1+\varepsilon)$ and $\mu=2(\theta^{-\rho}-1+\varepsilon)^2-2(\theta^{-\rho}-1+\varepsilon)(1+\varepsilon)(1+\theta^{-\rho})+(1+\varepsilon)(1+\theta^{-\rho})^2/4$. The sign of $s^*=g_2^1(dx^2/dx^1)$ is then determined by the sign of g_{21}^2 which in turn depends on the sign of the expression $\alpha\psi-\varepsilon(\alpha-\rho)\theta^{-\rho}(1-\varepsilon)$ where

For an elastic demand, both s^* and R increase with the spillover rate, as before. Ceteris paribus, they decrease as the elasticity of substitution between the firm's R&D and the R&D of its rival declines. Thus when "own" and "borrowed" R&D are almost perfect substitutes ($\rho = -0.9$) and for $1/\varepsilon$=1.25 and α=0.1, we obtain s^*=0.08 and R=0.09 for θ=0.25 and s^*=0.33, R=0.34 for θ=0.75; with α=0.2 we obtain s^*=0.12, R=0.13 for θ=0.25 and s^*=0.34, R=0.35 for θ=0.75. In the situation however where the elasticity between own and borrowed R&D is lower ($\rho = -0.1$, giving an elasticity of 1.11) we obtain s^*=0.06 and R=0.07 for θ=0.25 and s^*=0.13, R=0.14 for θ=0.75 (with α=0.1); with α=0.2 we obtain s^*=0.05, R=0.06 for θ=0.25 and s^*=0.09, R=0.10 for θ=0.75 [25].

These results suggest that with research inputs imperfect substitutes, rates of return to intervention may be low, even in environments characterised by widespread spillovers. Whereas under the perfect substitutability assumption for R&D high spillovers implied that the rate of return to intervention was significantly higher than any "normal" rate of return, the introduction of imperfect substitutability generates rates

$\psi = 2(\theta^{-\rho} - 1 + \varepsilon)[1 - \theta^{-\rho}(1-\varepsilon)] - (\theta^{-\rho} - 1 + \varepsilon)(1+\varepsilon)(1+\theta^{-\rho}) - [1 - \theta^{-\rho}(1-\varepsilon)](1+\varepsilon)(1+\theta^{-\rho}) + (1+\varepsilon)(1+2\varepsilon)(1+\theta^{-\rho})^2/4$. For positive profits, we also need to restrict $\delta\varepsilon(1+\theta^{-\rho})/[\alpha(2-\varepsilon)] > 1$ where $\delta = 2(2-\varepsilon)/[4(1-\varepsilon-\theta^{-\rho}) + (1+\theta^{-\rho})(1+\varepsilon)]$. On this basis and from (5.10) we can calculate $s^* = -\omega\tau/(1-\omega\tau)$ where $\tau = [2(\theta^{-\rho} - 1 + \varepsilon) - (1+\varepsilon)(1+\theta^{-\rho})]/(5 - 3\varepsilon - 3\theta^{-\rho} + \varepsilon\theta^{-\rho})$ and $\omega = [\alpha\psi - \varepsilon(\alpha-\rho)\theta^{-\rho}(1-\varepsilon)]/\{\alpha\mu - \varepsilon[(\alpha-\rho) + (1+\rho)(1+\theta^{-\rho})]\nu\}$.

[25] The case with inelastic demand and $\rho < 0$ gives the same pattern as the case with perfect substitutability. Rates of return fall as the spillover rate increases and can turn negative when it becomes optimal for the government to tax R&D. Beyond a certain spillover rate, R&D taxes become in this case appropriate because an increase in the domestic firm's R&D increases the marginal productivity of its rival's R&D (g^2_{21}>0) and thus reduces the domestic firm's profits (g^1_2<0). The case for $\rho > 0$ (inelastic R&D inputs) is more complicated because under most spillover rates and demand/cost elasticities, either the second-order conditions or the conditions for a positive level of output are not satisfied. It is however possible to generate appropriability-/demand conditions where an increase in the domestic firm's R&D increases the marginal productivity of its rival's R&D (g^2_{21}>0) but still increases the domestic firm's profits (g^1_2>0). In that case, R can again be large and positive and increasing with the spillover rate.

of return much closer to "normal" levels. This implies that decisions to intervene are once more of a marginal nature, and similar to the case with full appropriability of R&D.

This analysis and the numerical results that are based on it are subject to certain caveats. First, it was assumed that the government can raise the funds necessary for the R&D subsidy without incurring any welfare cost. To the extent that this is not true, ignoring this deadweight loss implies that the calculated net rates of return R are overestimates. Secondly, all output was assumed to be for export. This means that there is no domestic consumer surplus so that all gains from lower prices benefit consumers of third countries. If part of domestic production is in fact consumed within the country's borders, this will tend to increase the incentives that the government has to subsidise the R&D of its domestic firm. The net rates of return that we calculated then underestimate the true rates of return.

Finally, there is the "asset" aspect. The gains due to intervention may be cumulative and may be manifested in a number of ways: higher productivity of own future R&D, lower learning costs in the "next round", etc . If this cumulative aspect is important enough so that it outweighs the decay of existing knowledge due to its commercialisation, the calculated rates of return will once more be underestimates [26]. The net effect of all these factors is thus hard to ascertain a priori. They are bound however to affect the calculations and the policy decisions where these appear to be marginal.

[26] In effect it is this "asset" aspect that provides the intellectual backing and underlies the active government intervention in areas such as semiconductors where there are repeated rounds of competition, as over 16K and 256K memory chips. I am grateful to Mark Schankerman for making this point.

e. Jointly optimal policy

A final issue concerns the characteristics of a jointly optimal policy, whereby governments act together in maximising total welfare $W=W^1+W^2$. and the nature of R&D subsidies in this equilibrium. Total welfare can be written as:

$$(5.11) \qquad W(s^1,s^2)=W^1(s^1)+W^2(s^2)=g^1(x_1,x_2;s^1)-s^1x_1+g^2(x_1,x_2;s^2)-s^2x_2$$

The first-order conditions for a maximum can be obtained by differentiating (5.11) with respect to s^1 and s^2. Since $g_1^1=g_2^2=0$, $dx^j/ds^i=(dx_j/dx_i)dx^i/ds^i$ and $dg^i/ds^i=x_i$ we have:

$$(5.12) \qquad \begin{aligned} \partial W^1/\partial s^1 &= [g_2^1(dx_2/dx_1)-s^1+g_1^2-s^2(dx_2/dx_1)]\,(dx^1/ds^1)=0 \\ \partial W^2/\partial s^2 &= [g_2^1-s^1(dx_1/dx_1)+g_1^2(dx_1/dx_2)-s^2]\,(dx^2/ds^2)=0 \end{aligned}$$

Rearranging terms, we obtain:

$$(5.13) \qquad \begin{aligned} s^1+s^2(dx_2/dx_1) &= g_2^1(dx_2/dx_1)+g_1^2 \\ s^1(dx_1/dx_1)+s^2 &= g_2^1+g_1^2(dx_1/dx_2) \end{aligned}$$

Solving the system (5.13) simultaneously for s^1 and s^2 we obtain:

$$(5.14) \qquad s^1=g_1^2 \qquad \text{and} \qquad s^2=g_2^1$$

Brander and Spencer have shown that in an environment with fully appropriable R&D, we will have $g_2^1<0$ and $g_1^2<0$; a jointly optimal policy will therefore involve both governments taxing R&D so as to offset the negative effect of each firm's R&D on the other frim's profit. When R&D is subject to spillovers, we demonstrated earlier in this chapter that there are circumstances where $g_j^i>0$. In particular, since $g_j^i=E_j^iq_{x_j}^j$ and

$E^i_j < 0$, we will have $g^i_j > 0$ when $q^j_{x_j} < 0$. Given that $q^j_{x_j} = \left(c^j_{x_j} E^i_{ii} - c^i_{x_j} E^j_{ji}\right)/D$

$E^i_j < 0$, we will have $g^i_j > 0$ when $q^j_{x_j} < 0$. Given that $q^j_{x_j} = \left(c^j_{x_j} E^i_{ii} - c^i_{x_j} E^j_{ji}\right)/D$ and with D>0, a necessary and sufficient condition for $q^j_{x_j} < 0$ is that $c^j_{x_j} E^i_{ii} < c^i_{x_j} E^j_{ji}$. Since in symmetry $E^i_{ii} < E^j_{ji}$, this implies $c^j_{x_j} > c^i_{x_j}$ as a necessary condition.

Of the two functional forms that we have examined, and for a spillover rate $0 \leq \theta \leq 1$, it is possible to obtain $c^j_{x_j} > c^i_{x_j}$ only with the CES cost function that allows for the possibility that the R&D of the domestic frim is not easily substitutable with the R&D of its rival. We will then have $c^j_{x_j} > c^i_{x_j}$ if $\theta^{-\rho} > 1$. Since the limit value of the spillover rate is one, this can occur only in the situation where ρ>0, so that the elasticity of substitution between own and borrowed R&D $1/(1+\rho)$ is less than one. If in addition $c^j_{x_j} E^i_{ii} < c^i_{x_j} E^j_{ji}$, then $q^j_{x_j} < 0$ leading to $g^i_j > 0$. In that situation, the jointly optimal policy will involve positive subsidies to R&D expenditures. Thus:

Proposition 3: With spillovers in R&D, the jointly optimal policy will involve positive domestic subsidies when $c^j_{x_j} > c^i_{x_j}$ and where in addition $c^j_{x_j} E^i_{ii} < c^i_{x_j} E^j_{ji}$. This can only occur in an environment where "own" and "borrowed" R&D are not easily substitutable with each other.

III. Conclusions

The issue explored in this chapter concerned the characteristics of public policy in support of "national champions" through R&D subsidies. The multi-period model of oligopolistic rivalry of chapter four was recast in a setting where competition occurs amongst firms located in different countries. Public policy then consists of governments subsidising the R&D expenditures of their domestic firm in an profit-shifting attempt that operates by altering firms' strategies. The question that was addressed was whether the incentives for such a policy survived in an environment characterised by research spillovers.

The first step in answering this question consists of determining the effect of a domestic R&D subsidy on the R&D levels of the domestic and of the foreign firm. It was established that while a domestic R&D subsidy increases the R&D expenditures of the recipient firm, it may decrease or increase the R&D outlays of its foreign rival. The effect on the rival's R&D will depend on two factors: the extent of spillovers and the substitutability between own and rival R&D.

Under conditions of fully appropriable R&D, it has been established that the international noncooperative equilibrium involves positive subsidies. This was shown not to hold in general when R&D is subject to spillovers. In particular, it was shown that government behaviour with respect to subsidising R&D should take into account two effects: that of the impact of domestic R&D on the marginal profitability of foreign R&D (labelled an R&D productivity effect) and that of the impact of domestic R&D on the profits of the foreign rival (a profitability effect). Positive subsidies are appropriate when domestic R&D reduces foreign R&D which in turn increases domestic profits or when domestic R&D increases foreign R&D but still increases domestic profits. In the situation however where, because of spillovers and of the nature of technology and strategic interaction, domestic R&D induces higher foreign R&D which in turn reduces domestic profits, the appropriate policy involves taxing R&D.

From the framework that was developed in this chapter and from the analysis of R&D subsidies in chapters two and three above follows an important implication for public policy. Positive R&D subsidies can only be justified in the context of the particular cost, demand and appropriability conditions facing individual industries. They cannot be generalised to apply to all cases where R&D is less than fully appropriable. This is especially true in the case of subsidies to domestic firms that are competing in an international oligopolistic environment. In that case, in the absence of the indirect benefit to domestic consumers from lower prices, present only when subsidising firms that are competing domestically, a policy of subsidising R&D has an important externality attached to it which may thwart its aim of putting the domestic firm in a better competitive position.

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ERRATA

p.6 last line: replace OECD with OECD (1981)

p.14 fn. 12: replace Levin and Reiss (1988) with: Levin and Reiss (1989) -- same in p.15, first para. and p.64, fn.17; replace Pakes and Schankerman (1984) with: Pakes and Schankerman (1984b); end of footnote: replace Bernstein (1988) with: Bernstein (1989)

p.18 fn. 18: replace Grossman (1989) with: Grossman (1990)

p.20 fn. 20: replace Vonortas (1989) with Vonortas (1988)

p.23 fn. 25: replace Dasgupta and Maskin (1987) with: Link and Tassey (1987)

p.30 last paragraph: replace Levin and Reiss (1983, 1989) with: Levin and Reiss (1984, 1989); replace Hartwick (1983) with: Hartwick (1984); replace Vonortas (1987, 1988) with: Vonortas (1986, 1988) --same in p.31, last para. and p.64, fn.20

p.30 fn. 36: delete Lee and Wilde (1980); replace: ...and Reinganum (1982, 1984, 1985) with: Reinganum (1980, 1989) and Dixit (1988); replace Nelson and Winter (1977, 1982) with Nelson and Winter (1977, 1978, 1982)

p.31 last para.: replace Dasgupta and Stiglitz (1980) with Dasgupta and Stiglitz (1980a) --also in p.200, second para.

p.60 fn. 16: replace maximizes with: minimizes; replace ..in note 13 above with:..in note 14 above

p.77 end of first para.: replace ...and $\partial y_s/\partial\beta > 0$ if $\varepsilon < 1$ with ...and $\partial y_s/\partial\beta < 0$ if $\varepsilon < 1$

p.84 first para., 4th line: replace ...if demand is inelastic.. with: ...if demand is elastic

p.85 second para., 4th line: replace 25% with: 26%

p.104 fn. 8: after ..Following textbook notation.. add: (see Layard and Walters, 1978)

p.106 third line: replace $K = 1 + (n-1)[\delta\theta]^{-\rho_1}$ with: $K = 1 + [\delta\theta(n-1)]^{-\rho_1}$

p.159 at the end of fn. 8 add: More generally, for a review of comparative statics issues in models of oligopoly, see Dixit (1986).

p.185 end of fn. 26: replace: ...and $Q_{ST}^{\theta>0} < Q_C^{\theta>0}$ for $\theta < (n-\varepsilon-1)/(2n-\varepsilon-1)$ with: ...and $Q_{ST}^{\theta>0} < Q_C^{\theta>0}$ for $\theta > (n-\varepsilon-1)/(2n-\varepsilon-1)$

p.189 fn. 33: in the expression for $x_C^{\theta>0}$ replace λ by α; also in fn.34 all expressions $\varepsilon\rho + \lambda(1-\varepsilon)$ should be replaced by: $\varepsilon\rho + \alpha(1-\varepsilon)$

p.189 fn. 34: after sentence ending ...on the former only., next sentence should start by: In brief, since $K_1 > 1$, the *sufficient* condition for $x_C^{\theta=0} < x_C^{\theta>0}$ is $-[\varepsilon\rho + \alpha(1-\varepsilon)]/(\rho A) > 0$. This is satisfied in two cases:...

p.189 last line of text: replace $-[\varepsilon\rho + \alpha(1-\varepsilon)] < 0$ with $-[\varepsilon\rho + \alpha(1-\varepsilon)]/\rho < 0$

p.190 first line of text: replace $-[\varepsilon\rho + \alpha(1-\varepsilon)] > 0$ with $-[\varepsilon\rho + \alpha(1-\varepsilon)]/\rho > 0$

p.212 footnote 6: replace Katz and Shapiro (1990) with Katz and Ordover (1990)

p.216 footnote 8: replace d' Aspremont and Jacquemin (1988) with d' Aspremont and Jacquemin (1988, 1990)

p.242 delete references to Mc Fadden (1963) and Mowery (1983)

p.243 delete reference to OECD (1987); also delete reference to OECD (1990) in p.244

p.245 add SCHANKERMAN, M. (1981) "The Effects of 'Double Counting' and Expensing on the Measured Returns to R&D", *Review of Economics and Statistics*, August.