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GAME THEORETIC MODELS OF PRICE DETERMINATION

AND

FINANCIAL INTERMEDIATION

by

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To my Parents

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Abstract

The thesis is concerned with the evolution of prices in market economies.

In the first chapter we analyze price competition among firms with limited capacities in the framework of the classical Bertrand Edgeworth model. For this model it is well known that a Nash equilibrium in pure strategies may not exist. We discuss the nature of this non-existence result. While enlarging the strategy space to include non-linear strategies in general does not suffice for existence in the simultaneous move game, the possibility of reactions to competitors' actions in a dynamic context may restore equilibrium.

In the remaining part of the thesis we analyze intermediation in frictional markets. When market participants are informed only imperfectly about potential trading opportunities, search and negotiations may prove costly. In such markets intermediaries by publicly quoting prices can help to reduce the transactional costs of exchange.

In chapter two we analyze the case, in which intermediaries have access to an information technology which informs the full market. We characterize equilibrium. The inability of market participants to coordinate market participation is reflected in a large variety of subgame perfect Nash equilibria. Using a refinement criterion we find that high valuation traders typically trade with intermediaries, while low valuation traders engage in search. Moreover, price competition among several intermediaries yields Walrasian outcomes. Nevertheless, the market exhibits the features of a natural monopoly.

In chapter three we relax the assumption concerning the information technology. An intermediary's choice of an information network determines the size of his clientele and hence the probability of trading. In this case the size of the information network is viewed as a quality attribute by market participants and imperfect price competition among intermediaries obtains. We characterize the industrial structure of those markets as natural oligopolies. Consequently, there is no convergence to a fragmented industrial structure as the economy grows large. Still, as the largest competitors are of roughly equal size equilibrium allocations tend to be fairly competitive.

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Game Theoretic Models of Price Determination

and

Financial Intermediation

General Introduction

1. Introduction

This dissertation is concerned with the evolution of prices in market economies. Market economies are characterized by the free interaction of buyers and sellers of specific products or services.

It is frequently argued that market economies are adequately described by variants of the classical general equilibrium model as developed by Arrow and Debreu (Arrow, Hahn, 1971, Debreu, 1959). However, market equilibria of such economies do require a central price authority, commonly dubbed as the "invisible hand" or the "Walrasian auctioneer". The auctioneer mediates between buyers and sellers in the various markets. He calculates equilibrium prices for all markets, informs the market participants about the vector of equilibrium prices and finally coordinates the exchange of products. All these "services" in the fiction of the general equilibrium theory are provided costlessly. Therefore, in the Walrasian world it is difficult to incorporate intermediaries that specialize in trade. Ultimately intermediaries like the benevolent auctioneer have to provide their services for free.

While a theory of value for products and services can be based on the Walrasian fiction in order to determine the relative scarcity of products, an explicit analysis of the formation of prices requires a framework that explicitly relates those prices to actions taken by private agents of the market system.

Certainly, we observe a wide variety of mechanisms of price determination. In some markets trade may occur directly between buyers and sellers and prices are determined in bilateral bargaining. In other markets sellers unilaterally quote fixed prices as take-it-or-leave-it offers which the buyer may accept or not. Furthermore there are markets which are organized by specialized private intermediaries as the Stock Exchange, the Metal Exchange or the auction houses.

The thesis focuses on models in which private agents and in particular private intermediaries contribute to the organization of markets and to the determination of prices. In the following section we explain in more detail the informational properties of the Walrasian model. Section 3 discusses our contribution to price competition as presented in chapter 1, while section 4 gives a brief account of our analysis of intermediation in frictional markets.

2. General Equilibrium and the Invisible Hand

Before we discuss the informational aspects of the general equilibrium theory we shall give a short account of the equilibrium model.

Equilibrium theory distinguishes two types of agents, consumers and producers. Consumers are characterized by their preferences for the various products and services (precisely: commodities) and their initial endowments of products including factors of production. Producers are endowed with technologies, which summarize the technological knowledge necessary for the transformation of production factors into intermediate and final products. In deciding about the quantity of factors offered and the quantity of products demanded, consumers are restrained by the value of their initial resources, which restricts their budget. It is assumed that the consumers' choices maximize their utilities for any given price system. Likewise producers maximize profits, defined as the difference between value of the sales and factor costs for a given price system. The economy is said to be in equilibrium, when aggregate demand and aggregate supply are equal for each commodity. Thus, in an equilibrium at the given equilibrium price system consumers and producers will not regret their trades and each of them can realize his plans. If traders therefore are advised to trade at Walrasian prices and if they are told with whom to exchange which quantity, clearly the resulting allocation is stable in the sense that no further improvement through bilateral trade should be possible.

The general equilibrium theory is a theory of value. As such it addresses the question of the "correct price" of the various products or commodities in terms of their relative scarcity value in the economy. In an equilibrium agents planning on the basis of those "*scarcity prices*" will never regret their actions. This entails a remarkable informational efficiency of the market system, emphasized particularly by Friedrich August v.Hayek (1945). Individuals do not need to know the details of sudden and frequently offsetting increases and decreases in supply or demand of specific goods. The scarcity prices summarize all information relevant for individual consumption or investment decisions. Thus, the price system serves as a unique communication device economizing on the transmission of information, sending the essential signals only and coordinating independent individual actions.

This coordination property is summarized in Hayek's own words: *"The whole acts as one market, not because any of its members survey the whole field, but because their limited individual fields of vision sufficiently overlap so that through many intermediaries the relevant information is communicated to all. The mere fact that there is one price for any commodity - or rather that local prices are connected in a manner determined by the cost of transportation, etc.- brings about the solution which (it is just conceptually possible) might have been arrived at by one single mind possessing all the information which is in fact dispersed among all the people involved in the process."* (1945, p.526)

Hayek's argument strongly rests on the "Law of One Price". Given scarcity prices are uniquely defined in a particular equilibrium, there is no doubt about their interpretation in terms of the general scarcity of products. Thus, those scarcity prices can guide individuals' actions to the general benefit. However, Hayek does not give an account of the actual formation of market or *transaction prices.* His reference to local prices suggests that implicitly any arbitrage possibilities are exploited and actual transaction prices therefore coincide with scarcity prices.

This to some extent contradicts an earlier statement of Hayek in the same article in which he explicitly discusses the distinction between technological knowledge and knowledge about trading opportunities. In his words, *"...the estate agent whose whole knowledge is almost exclusively one of temporary opportunities, or the arbitrageur who gains from local differences in commodity prices, are all performing eminently useful functions based on special knowledge of circumstances of the fleeting moment not known to others"* (Hayek, 1945, p.522). This amounts to saying that not all individuals at any point in time enjoy the same information about market prices and hence trading opportunities. Rather private agents are constantly searching for arbitrage opportunities and markets are constantly adapting to new conditions. Markets operate dynamically. Informing the "market", therefore, is an "eminently useful" allocative function. It is useful because it helps to adjust scarcity prices to their "true" values. On the other hand because of the dispersion of knowledge it implies that at any point in time virtually the same physical product at the same location may be traded at different terms between different trading partners.

At this point the question arises what constitutes a market. According to the definition of commodities in the general equilibrium framework the market for a single commodity is defined by the physical quality and the location, time and state of nature of its availability. In such a framework the informational efficiency emphasized by Hayek is quite a strong property. If in addition markets should be differentiated with respect to information that different agents might have at certain points in time and space, including information about trading possibilities, the commodities traded would have to be defined in an exceedingly narrow way. It would be difficult to reconcile the conventional notion of (competitive) markets with such a concept of markets in personalized claims. Certainly, Hayek's view of markets abstracts from informational differences among the various participants.

Based on the strong belief that prices will adjust to their "true" scarcity values, i.e. their equilibrium values in the language of the theory, general equilibrium theory concentrates on a static market framework. Market prices adjust rapidly enough such that individual traders do not incur transaction costs. Therefore, differential knowledge about trading opportunities is not considered an important problem and hence remains unmodelled. The Walrasian auctioneer costlessly provides two important economic roles. First, based on the knowledge of the excess demand functions he can calculate equilibrium prices. Those are the scarcity prices of the various commodities. Note that he does not need to know the individual demand and supply functions; it suffices to know the aggregate quantity. Furthermore the auctioneer has to inform all market participants about the equilibrium prices, because otherwise they cannot condition their behaviour on the correct scarcity signals. Second, once he has determined market clearing prices, the exchange of products has to be coordinated.

The public good character of the auctioneer's price setting function is highlighted in the information paradox discussed in a series of papers by Grossman and Stiglitz [1] . If traders are asymmetrically informed about the returns of a security, uninformed traders may infer information from market clearing prices. High share prices for example may reflect insiders' expectation of high future dividend payments. Under the rational expectations hypothesis uninformed traders by conditioning their behaviour on equilibrium prices can acquire information about the underlying asset. A rational expectations equilibrium may fail to exist. If it exists, however, depending on the source of uncertainty, it may reveal completely the relevant information to the badly informed traders. In this case equilibrium prices are fully revealing and informed traders cannot benefit from superior information. Hence, they have no incentive to acquire the relevant information. If, however, no information is collected the question arises, how this information should be embodied in prices at all. This paradox essentially stems from the auctioneer's role in establishing prices, which allows uninformed traders to deduce information costlessly and therefore precludes well informed traders from taking advantage of superior information. Accordingly, Grossman and Stiglitz conclude that informational efficient markets are impossible.

While general equilibrium focuses on the analysis of scarcity prices the fiction of the Walrasian auctioneer may serve as a good approximation to value theory. If the interest however lies in the discussion of the micro structure of markets and the evolution of prices, clearly, this paradigm cannot be accepted. There are several ways of how one might visualize the emergence of prices in an economy without Walrasian auctioneer. Hayek himself seems to suggest that price competition between sellers for example might generate outcomes compatible with Walrasian outcomes. Alternatively one might explicitly model "frictional" markets in which knowledge about trading opportunities remains incomplete. [2] .

[1] see Grossman, Stiglitz (1980) for example

[2] Since we are interested in the role of "real world auctioneers" we do not discuss "remedies" to the information paradox, which build on auctioneer based models. Such strategies have been pursued by Grossman, Stiglitz (1980) and Hellwig (1980) for example

After the first suggestion is taken up in chapter one the following chapters two and three concentrate on modelling frictional markets. The analysis in the thesis is restricted to a partial market framework, concentrating on the "market" for a single commodity and sidestepping the interaction among "markets" for different commodities.

3. Price Competition and Coordination Failures

Chapter one starts with an analysis of price competition. Price competition is an important element of the strategic interaction among intermediaries. After all, one of the main justifications of intermediation is the provision of an attractive price system. It is well known that price competition among firms with constant marginal costs implements the Walrasian outcome whenever at least two firms are active. This result, however, requires that competitors' capacities are unlimited. With limited capacities actually a Nash equilibrium in prices may fail to exist. The non- existence results from a coordination problem of the competitors in allocating quantities.

This can be illustrated by means of a simple example. Consider a single buyer willing to purchase 3 units of a perishable product at a reservation price of 1 and two firms each disposing of 2 units of that commodity. Competition works as follows. The firms simultaneously and independently offer a price per unit. Having observed these offers the buyer decides how many units to purchase from each seller. There is no further trading opportunity in time. Due to excess supply the Walrasian equilibrium price is zero, however, the game does not possess a Nash equilibrium in pure strategies. The reason is simple. Each competitor enjoys some residual market power, since given his competitor offers zero prices he can sell 1 unit at the monopoly price of 1. Therefore, each of the rivals has an incentive to withhold products at the Walrasian price in order to exert his market power on the remaining unit. On the other side if both rivals demand the monopoly price, one of them will not be able to sell both units. Hence, he has an incentive to slightly lower his prices and undercut his competitor. Actually, undercutting will be profitable for one of the rivals for any positive price. Therefore, an equilibrium in pure strategies does not exist.

To be sure the example is quite special. In particular, if each unit is offered by a different firm equilibrium exists and coincides with the Walrasian equilibrium. On the other hand in the framework of partial equilibrium with perfectly divisible products and increasing marginal costs the non-existence result obtains for any finite number of firms. Thus, the non-existence problem also obtains for large numbers. Price competition lacks the coordinating properties of the Walrasian auctioneer.

Chapter one discusses the robustness of price competition. In particular, the question is posed whether the non-existence phenomenon might result from artificial restrictions of the strategy space.

The motivation for this approach can be illustrated by the given example. Suppose the two firms could commit to sell their products in bundles of 2 units. In this case there is a unique Nash equilibrium in prices with each competitor demanding the price 1 for the whole bundle. Thus, the implied per unit equilibrium price is 1/2, but it cannot be implemented, if the price offers are restricted to per unit prices as seen before. Equilibrium in pure strategies however is possible, when non-linear price schedules are allowed. If the competitors are allowed to offer prices with discounts, in equilibrium they could offer the following schedule. For the first unit they demand the price 1 and all further units are free. This suggests that it might be possible to avoid the non-existence problem by enlarging the strategy set. We shall however find that this intuition is correct only in the case of a single buyer.

The central result of chapter one demonstrates the generality of the non-existence problem. It identifies the failure of coordination in quantities as the main source driving the non-existence result in the context of static competition. It is shown that elements of sequential competition do allow for sufficient coordination as to restore equilibrium. Nevertheless, existence does not imply uniqueness and typically a wide range of equilibrium outcomes leaves additional reason for coordination. In this sense Walrasian equilibria imply a higher degree of coordination than equilibria in games of price competition among private agents. Price competition and Walrasian competition are different equilibrium concepts. [3]

The existence problem for an equilibrium in prices also arises in a differ-

[3] Yanelle (1988) makes a similar point for price competition among intermediaries

ent context. If for example customers are only incompletely informed about the prices on offer, typically there will be customers relatively well informed about competitors' prices and others who may not be aware of any other price. In such cases intermediaries have an incentive to quote high prices to capture rents from the badly informed clients and competitive prices for well informed clients. If it is impossible to discriminate between differentially informed clients a similar coordination problem arises in the price game among intermediaries. It is formally equivalent to the existence problem in case of strictly limited capacities. We shall be concerned with this issue again in chapter 3 in an environment, in which intermediaries can inform their clients about their price quotes only incompletely.

4. Intermediation and Frictional Markets

Chapters two and three analyze frictional markets in which knowledge about trading opportunities is dispersed and incomplete. Hence, we are taking up the theme of the second of Hayek's quotes. In such markets a natural role arises for intermediaries to reduce the impacts of the informational frictions.

In the absence of intermediaries market participants have to *search* for trading partners themselves. This may be time consuming and therefore costly. Intermediaries could help to speed up exchange and even if they charge a fee for their services it might be preferable for traders with an urgent desire for trade to transact via the intermediary rather than search for trading partners.

We think of an intermediary as an institution who informs the market about trading opportunities. He may achieve this by actively advertising widely buying (bid) and selling (ask) prices for certain commodities. Thus, market participants will become aware of the intermediary's identity and the prices at which he is willing to trade, which typically are based partially on the intermediary's views about the fundamental scarcity value and on strategic considerations otherwise. The intermediary's activity offers market participants the possibility to avoid time consuming search for trading partners and to trade directly with him.

In analyzing intermediaries we assume that as any other market participant they do know the distribution of reservation prices and therefore aggregate demand and supply. However, they do not possess any informational advantage about specific trading opportunities. It is their presence in the market and their information generating activity which allows them to attract and match customers. We assume that they have access to an *information technology*, which allows them to communicate their price quotes to a fraction of the market. They are prepared to transact with potential customers at the prices offered. Typically, bid and ask prices differ by a positive amount commonly termed the spread. However, price commitments are limited by the intermediaries' ability to find matching trading partners. If they do not succeed the trading opportunity is lost and we assume no further penalty is imposed on the intermediaries. In other words bankruptcy cannot occur. Nevertheless, intermediaries have a natural incentive to quote prices that allow to transact positive volumes, since they earn revenue in form of the spread for each transaction.

Thus, in purchasing the technology intermediaries generate a degree of *visibility* in the market. Intermediaries informing a large fraction of the market are highly visible, whereas others may be hardly discernible from individual searchers. Because of their visibility in the market they can help to speed up the process of exchange and charge positive spreads.

Market participants facing a list of price quotes and the option of search choose according to the relative attractiveness of the various possibilities. Indirect trade may present the problem that the intermediary himself is not able to fulfil his commitment since he does not find a trading partner on the other market side. In this case costs of delay occur. Furthermore, even if the intermediary should be able to trade his price quote might be quite unattractive relative to the alternatives. Consequently, the tradeoff between the gains from trade and the probability of trading determines market participants' choices.

Now, intermediaries in setting prices not only compete against each other but also against the search market. Thus, we can distinguish between traders who prefer intermediated trade and traders who are active on the search market. Accordingly, we can interpret the set of traders on the intermediated market as constituting the organized market. Price competition among intermediaries is not restricted to the organized market but has to take account also of the unorganized "shadow" market.

It is convenient to separate the analysis into two steps. In chapter two we discuss the case, in which intermediaries either inform the full market or are not active at all, while in chapter three intermediaries can choose to which extent they plan to inform the market.

The results of chapter two can be classified according to the number of active intermediaries in the market. The outcomes in case of a single intermediary substantially differ from the case of several active intermediaries.

If one monopolist intermediary is active the competition between the organized and the unorganized market becomes particularly transparent. In equilibrium the monopolist charges a positive spread. The spread decreases with an increasing matching probability and hence efficiency of the search market. Traders with large gains from trade prefer to deal with the monopolist whereas traders with low gains engage in search. Hence, in equilibrium both the organized and the search market are active.

In addition, however, a wide multiplicity of subgame perfect Nash equilibria illustrates the coordination problems inherent in models of intermediation. For example, there is a sequential equilibrium, in which no trade through the intermediary occurs. If no single trader deals with the intermediary and this is common knowledge, no single trader will waste his time and supply an application to the monopolist, because he knows that the monopolist will not be able to find a matching trade. We shall argue, however, that in some sense the equilibrium singled out above is more appealing.

In the case of competing intermediaries the classical Bertrand result obtains and equilibrium bid and ask prices coincide with the (unique) Walrasian equilibrium prices. Intermediaries' spreads and revenues are zero in equilibrium. Again for the same coordination problems as above a wide variety of subgame perfect Nash equilibria exists. Nevertheless, it may be argued that the zero profit equilibrium is more plausible for the model under consideration and in this sense we have found a rationalization for the view that price competition will entail Walrasian outcomes.

However this result rests on strong assumptions. So far, costs have been totally neglected. Once costs for setting up an information technology have to be incurred this result will no longer hold. Suppose sunk costs have to be incurred for the purchase of the information technology prior to the competition stage. For example intermediaries may have to set up a communications network or some advertising agent. In this case in equilibrium at most one intermediary will enter the intermediation sector, because only as a monopolist he can earn positive revenue and therefore recoup his outlays. Even more, the monopoly result arises for any positive level of sunk costs.

On the other side, as argued before, in equilibrium the monopolist intermediary cannot demand prices at will. He is constrained by competition of the search market. In equilibrium he charges positive spreads which constitute transaction costs of exchange for the ultimate traders. In this sense we have given an endogenous explanation of transaction costs based on trading frictions in the search market and the nature of competition among intermediaries. At this point we should also note that trade typically occurs at different prices in the search market. Trivially, on the organized market there is a single equilibrium price.

In chapter 3 we allow intermediaries to establish their own information network. By constructing a large network intermediaries can inform a large portion of the market. Furthermore, by setting up different networks they can avoid direct contact with rivals or reduce the impact of price competition. The choice of an information network allows them to differentiate themselves from competitors and relax price competition accordingly. Therefore, one might expect several intermediaries to be active in equilibrium.

However, care has to be taken in formulating the model. Especially, if all trading opportunities happen to occur simultaneously and intermediaries cannot target their information signals, existence of a price equilibrium in pure strategies cannot be ascertained. Butters (1977), for example, demonstrates the non-existence of a pure strategy equilibrium in prices, when firms advertise their prices. This is due to the conflict between exploiting market power against badly informed customers and competing for market shares among the relatively well informed clients mentioned at the end of section 3. If the price quotes could be targeted discriminatory pricing would guarantee existence of price equilibrium again and this Bertrand Edgeworth type of problem would not occur.

We choose to model a brokerage market in an "island economy". Market participants are scattered on various islands and their desire for trade is generated by unforeseen liquidity events which for example urge them to sell a large bundle of stock. Typically, several brokers are active on each island and traders select one of them to search for a suitable trading partner. Brokers maintain their presence on various islands connecting the subsidiaries by an *information network*, which allows them to communicate with potential trading partners in the network. The chosen scale of information activity affects the intermediaries' probabilities of finding matching partners for a given client. Intermediaries with larger investments are more likely to satisfy their customers. Therefore, an intermediary's size, as measured by the size of his information network, is viewed as a quality attribute. In other words, traders facing the same price quotes from two intermediaries with different size might prefer to transact with the larger intermediary simply for the reason of a higher probability of trade. Thus, imperfect price competition in the model of chapter 3 corresponds to price competition with *vertically differentiated products* (Shaked, Sutton, 1982).

Industries in which products are vertically differentiated may exert an innate tendency towards concentrated industrial structures. The notion of a concentrated industrial structure is described by the concept of *"natural oligopoly"* developed by Shaked and Sutton (1983). According to their definition an industry is a natural oligopoly, when the number of firms active in this industry is bounded independently of the size of the economy. In particular such an industry does not converge to a fragmented and competitive industrial structure as the economy grows large but remains concentrated.

By arguments similar to those of Shaked and Sutton (1982) we can establish a bound for the number of intermediaries active in equilibrium. In particular we find that the natural industrial structure of our brokerage market consists of three "large" intermediaries and a competitive fringe of "smaller" competitors. This finding contrasts with the monopoly result of chapter two. Nevertheless, the industrial structure is fairly concentrated.

In such an equilibrium intermediaries select different information networks in order to relax price competition. Hence, they also quote different prices. In our model with linear demand and supply the large intermediaries enjoying a relatively high degree of visibility can command larger spreads while maintaining larger market shares at the same time. Typically, all intermediaries earn positive profits.

Consequently, even the organized markets exhibit *price dispersion.* Transaction prices do not coincide with scarcity prices. However, there is a well-defined relation between them. In a symmetric equilibrium the scarcity price simply is the mean of the bid and the ask price. Furthermore, it is identical for all intermediaries. Therefore, the results of chapters two and three should not be taken as contradicting classical general equilibrium theory. Rather, they suggest to distinguish between transaction prices and scarcity values. The model outlined suggests a tool for explaining the evolution of market prices and market institutions. The thrust of the argument is that these institutions arise endogenously in pursuit of private gains by exploiting trading frictions in the market.

Our model combines two different strands of literature. Stahl (1988) and Yanelle (1988) analyze pure price competition among intermediaries. While Stahl considers a fixed market structure that forces buyers and sellers to transact indirectly through the intermediaries, Yanelle also allows for a choice between direct and indirect trade. In her model intermediaries enjoy technological economies of scale. However, they are not explained in terms of market generating activities.

The second connection exists to "middlemen" of Rubinstein and Wolinsky (1985a). They explicitly take into account trading frictions in a bargaining and matching framework based on Rubinstein, Wolinsky (1985b). There, the only advantage intermediaries enjoy is a higher frequency of encounters with trading partners. This is imposed exogenously and not derived from the intermediaries' activities. Since in their model trading opportunities are exclusively determined by the matching technology traders do not have an active choice between several modes of transaction. Thus, their only choice is between trading with the present matching partner or delaying trade, while it is not ascertained that individual traders might ever get a chance to trade with an intermediary. Trading opportunities are exogenously given. Accordingly, in Rubinstein's and Wolinsky's framework intermediaries cannot affect their matching probabilities by increasing their relative attraction to market participants. In our model, by investing in visibility, intermediaries contribute to mediating between potential trading partners and create additional trading opportunities.

In combining these two approaches price setting intermediaries arise endogenously in a search environment. They contribute to the reduction of transaction costs. Therefore, the relation between equilibrium prices on organized and search markets and the industrial structure of the organized market can be analyzed in terms of the primitives of the model that are preferences, endowments and the matching technology.

By way of summarizing, the apparent contradiction between Hayek's quotes does not arise in our model of chapters 2 and 3 and we may conclude in Hayek's words: *"Fundamentally, in a system where the knowledge of the relevant facts is dispersed among many people, prices can act to coördinate the separate actions of different people in the same way as subjective values help the individual to coördinate the parts of his plan"* (Hayek, 1945, p.526).

Chapter 1

Bertrand Edgeworth Competition with Non-Linear Prices

1. Introduction
2. The Model
3. Monopsony
4. Several Buyers: Simultaneous Moves
5. Several Buyers: Sequential Moves

Bertrand Edgeworth Competition with Non- Linear Prices

1. Introduction

a) Motivation

The classical Bertrand game of price competition in homogeneous product markets is often viewed as providing a justification for the model of perfect competition in the absence of a fictitious auctioneer (Allen, Hellwig, 1986). It has the appealing feature that prices are set by the agents who do so in reality. However it is well known that the existence of pure strategy equilibria is problematic as soon as capacity constraints are incorporated (Edgeworth, 1897).

These classical price games restrict the strategy space to constant per unit prices and therefore exclude the possibility of price discounts on large sales or other forms of non-linear pricing. The purpose of this chapter is to extend the analysis to the larger strategy space, which allows competitors to use non-linear price schedules. There are two reasons for doing so. First the non-existence result might be considered as an inconsistency of the price setting model per se. Therefore it is important to understand the underlying forces behind this phenomenon. So it could be argued that non-existence results from an artificial restriction of the strategy space. In fact for a special case we shall demonstrate the validity of this line of reasoning (section 3).

The second motivation stems from considerations concerning the integration of contract theory into a more general framework. Abstracting from the design problems caused by asymmetric information and costly observation the strategic foundation of contract theory is the model of Bertrand competition in non-linear price schedules[1] . Any price quantity contract can be viewed as some discontinuous function of prices in quantities. For this class of games in interdependent markets competitive outcomes are well known to result whenever there are at least two competitors driving any excess profit down to zero in equilibrium. Therefore by imposing a zero profit condition this approach conveniently allows to separate the complexity of strategic interaction from the design problem and help to focus the analysis on the aspects of agency costs implied by the presence of informational asymmetries.

On the other hand by sidestepping the strategic analysis the approach deliberately remains partial. The occurrence of rationing on credit markets for example is among the most interesting results of the contract theoretical analysis. In line with the literature on fixed price equilibria (Dreze,1975) one would expect that in a more general treatment through a chain of spillovers the rationing phenomenon might feed back to the lenders or banks effectively constraining their capacity of granting loans. In such a setting the appropriate underlying strategic framework would seem to be a model of capacity constrained competition in contracts,.i.e. in non-linear price functions.

b) Overview: price setting oligopolies in homogeneous product markets

Bertrand's original model of price competition (Bertrand, 1883) views firms as simultaneously quoting prices for a homogeneous product, which subsequently is produced at a constant marginal cost and distributed at the prices arranged. Since there are no constraints to production capacity any of the rivals potentially could serve the whole market. Therefore the competitors will profitably undercut each other until the marginal costs of the second most efficient firm and hence equilibrium is reached. If firms are identical the Walrasian equilibrium with prices equalling marginal costs will obtain.

As capacity constraints are introduced however rival firms might actually

[1] see Hellwig (1988) for a methodological discussion of the contract theoretic approach.

command equilibrium prices above marginal costs. Osborne and Pitchik (1986) demonstrate for a duopoly version of the above game the existence of unique pure strategy equilibria for the case that either capacities are severely restricted or for the case, in which they are sufficiently available. When aggregate capacity is severely constrained relative to demand in a sense to be made precise below, undercutting will become unprofitable at the price at which demand just exhausts total production and equilibria with positive price cost margins emerge. If on the other side the capacity of any individual firm is not strictly necessary, competition will drive margins down to zero and the efficient competitive outcome will result. It is the intermediate region of capacities, in which there does not exist a pure strategy equilibrium if only linear prices are admitted. This point was made already by Edgeworth (1897) and leads us to study a more general strategy space to include non-linear prices.

Consequently we have to model the buyers as discrete players rather than a continuum of negligibly small traders. In the extreme case of only one single buyer we find that there will be a unique Nash equilibrium in pure strategies with non-linear price functions in equilibrium. This finding does not generalize to the case of several buyers. Rather we do find that the Edgeworth non-existence problem generalizes even to the strategy space, which does allow sellers to simultaneously offer price functions. More precisely we develop the concept of an *essential* seller as a capacity constrained seller, who is strictly needed for any efficient allocation. We then show that non-existence of a pure strategy equilibrium occurs exactly, when there are at least two essential sellers. These sellers basically compete in selling off "excess capacities" thus "destabilizing" any equilibrium allocation.

There is a further a assumption implied in the standard Bertrand model of price competition: prices are set simultaneously. Therefore the model is static in character. With a more general strategy space it turns out that mechanisms of sequential competition in contracts may allow for enough quantity commitment to allow the existence of pure strategy equilibria. We illustrate this by means of a game form in which competition takes place in two stages. At stage one the competitors choose maximal quantities for each customer and at stage two, having observed all their rivals quantity allocation they decide about prices. For this game the existence of a continuum of pure strategy equilibria is established. So the "dynamic" model proposed here will have pure strategy equilibria. However, it will exhibit a similar indeterminateness to the one, which F.Y.Edgeworth (1897) might have had in mind, when interpreting such markets as inherently unstable and predicting "cyclical" price movements.

Interpreting these results we conclude that the non-existence problem is closely related to the implied rationing scheme. Note that in case of several buyers any equilibrium implies the use of some rationing scheme, i.e. an assignment of the buyers to the sellers. In the case of simultaneous price competition the conflict between the price game and the coordination on the rationing scheme cannot be solved in general.

Note also that in case of existence, generally the nature of equilibrium will be quite indeterminate. This suggests that the competitors still have to solve a coordination problem. Only in the case of just one buyer can we expect a unique equilibrium. This seems obvious, since in this case there will be no rationing problem.

In a way these results parallel those of Kreps, Scheinkman (1983) and Davidson, Deneckere (1986), who show the existence of pure strategy equilibria in a two stage game, in which firms on stage 1 choose capacities and at stage 2 prices. The equilibria in their models will typically depend on the underlying rationing scheme.

The chapter now proceeds as follows. Section 2 introduces the general model. In section 3 the case of a single buyer is discussed. In sections 4 and 5 the general cases with several buyers are discussed in game forms of simultaneous and sequential competition respectively.

2. The Model

We consider a model with finitely many players. There are B buyers and S sellers. This allows us to model the notion of size, which we need to motivate the use of non-linear quantity dependent price schedules. The agents trade a homogeneous product which is available in a limited amount $X_s > 0$ to any of the sellers $s = 1, ..., S$ at constant marginal cost, which we normalize to zero up to the capacity limit. We interpret this product as an intermediate good, which is used by the buyers as a factor of production and which allows them to generate some profit or surplus $\pi_b(x)$, $b = 1, ..., B$ as a function of the factor input x. In order to avoid complications arising from strategic interactions among the buyers on the final product market and to focus on the strategic interaction in the factor market we assume that the buyers' profit functions are independent. In fact we shall also assume they are equal. The monotonicity assumption implies free disposal of any abundant factors. To avoid technical complications we shall assume without loss to the economic intuition the boundedness of the surplus function. Without the factors no production takes place and hence no surplus can be generated. The economically important condition is the concavity of the surplus function. We shall demonstrate that the case of a convex surplus function is less interesting. Therefore we state this assumption for the sake of clarity already here. Finally to avoid trivialities we assume that there are some real profit possibilities. By means of summarizing we have:

assumption 1: (surplus)

a)$\pi : I\!R_+ \rightarrow I\!R_+$

b)$\pi_b(x) = \pi(x) \quad , \quad b = 1, ..., B$

c)$\lim_{x \to 0} \pi(x) = 0$

d)$\pi(.)$ is bounded

e)$\pi(.)$ is non-decreasing

f)$\pi(.)$ is weakly concave

g)$x^* := \inf \text{argmax } \pi(x) > 0$

The conditions of exchange of the products are determined in contracts. Sellers do write these contracts $c_{sb} = (x_{sb}, p_{sb})$ specifying the quantity x_{sb} sold by seller s to buyer b at a total price of p_{sb}. We shall allow sellers to offer a whole menu of contracts $C_{sb} := [c_{sb} \mid c_{sb} \subset I\!R^2_+] \in I\!R^2_+$ and let the buyers choose among them. These menus might be rather arbitrary and include non-linear price schedules as functions of quantities. Also linear contracts could be offered with the particular offer $C_{sb} = [c_{sb} : p_{sb} = \text{const}\, x_{sb}]$. The only restriction to be imposed is that any contract can be written without recourse to some (convergent) series.

assumption 2: (contracts)

C_{sb} is closed in $I\!R^2_+$, $\quad s = 1, ..., S$, $\quad b = 1, ..., B$

The exact order of moves will vary in the subsequent sections. The general theme however is that contracts are offered by the sellers and that buyers subsequently choose at a later stage. So the sellers' strategies are menus, i.e. sets of contracts, from which the sellers choose at most one. The buyers' strategies therefore can be defined as a vector $d_b = (d_{1b}, ..., d_{sb})$, where $d_{sb} \in C_{sb} \cup (0,0)$ indicates, which contract is chosen from seller s. The zero element allows for rejection of the full menu. After contracts are signed the products are exchanged, production takes place and the players will receive their respective payoffs g_s and h_b. Whereas the seller just earns the sum of the payments from all contracts sold, the payoffs for the buyer are calculated as total returns from the final product market less the factor costs stipulated by the contracts.

$$
\begin{aligned}
g_s &= \sum_{b=1}^{B} proj_2\, d_{sb} = \sum_{b=1}^{B} p_{sb} \quad , \quad \forall s \\
h_b &= \pi\left(\sum_{s=1}^{S} proj_1\, d_{sb}\right) - \sum_{s=1}^{S} proj_2\, d_{sb} \\
&= \pi\left(\sum_{s=1}^{S} x_{sb}\right) - \sum_{s=1}^{S} p_{sb} \quad , \quad \forall b
\end{aligned}
$$

where $proj_1(x,y) := x$ and $proj_2(x,y) := y$

We shall also note that the payoffs are functions of the strategy choices of all players. In the sequel we prefer to suppress the arguments just for brevity of notation, as long as there is no risk of confusion. The proper notation should be:

$$\begin{aligned} g_s &= g_s(C_1, ..., C_S, d_1, ..., d_B) \\ h_b &= h_b(C_1, ..., C_S, d_1, ..., d_B) \end{aligned}$$

$$\text{where } C_s := (C_{s1}, ..., C_{sb})$$

All agents maximize their payoffs in a non-cooperative game, which excludes binding commitments to collusion. Since the sellers by offering the contracts move first, they can influence the outcome in all subsequent subgames and to that extent exert market power. Buyers on the other side react very passively and essentially solve an optimization problem.

We are interested in the *subgame perfect Nash equilibria* in pure strategies. As is known from Dasgupta, Maskin (1986) we might expect the existence of mixed strategy equilibria but in the context of the Bertrand Edgeworth game the interesting phenomenon is exactly the tension between market power and competition, which gives rise to the use of completely mixed strategies.

A *Nash equilibrium* is defined as a strategy combination $(\hat{C}_1, ..., \hat{C}_S, \hat{d}_1, ..., \hat{d}_B)$, where $C_s := (C_{s1}, ..., C_{sb})$, which for any player is the best reply to his opponents' strategy choices, i.e.

$$\begin{aligned} \hat{C}_s &\in \text{argmax}_{C_s} g_s(\hat{C}_1, ..., \hat{C}_S, \hat{d}_1, ..., \hat{d}_B) \\ \hat{d}_b &\in \text{argmax}_{d_b} h_b(\hat{C}_1, ..., \hat{C}_S, \hat{d}_1, ..., \hat{d}_B) \end{aligned}$$

Subgame perfection requires that equilibrium choices of the players constitute a Nash equilibrium in each subgame. The refinement excludes equilibria which are based on players' expectations, which would not support a Nash equilibrium in subsequent subgames. In these subgames players would have a natural incentive to deviate from their equilibrium strategy. In the absence of commitment possibilities such equilibria seem unplausible.

Specifically, subgame perfection excludes Nash equilibria supported by noncredible threats. Threats are credible only if it is in the player's own interest to enforce them once he is called to enact them.

So far we have completed the general description of the game.

3. Monopsony

a) Equilibrium

We shall start with the case of a single buyer, i.e. $B = 1$. This case is particularly interesting, since there is no need to employ some rationing scheme. If the capacity of a single seller is exhausted it does not matter which buyer he will serve. So the sellers offer the contracts at stage 1 and the monopsonist buyer decides about acceptance at stage 2:

stage 1: sellers offer menus of contracts $C_{sb} \in I\!R^2_+$, $s = 1, ..., S$

stage 2: buyer selects a set of contracts $d_b = (d_{1b}, ..., d_{sb})$,
where $d_{sb} \in C_{sb} \cup \{(0,0)\}$
$(0,0) :=$ all offers (of seller s) rejected

For this game form we find Nash equilibria in pure strategies. In equilibrium each seller can earn his 'marginal contribution' to the monopolists' surplus as stated in the following result:

Proposition 1

The generalized Bertrand Edgeworth game with a single buyer has subgame perfect Nash equilibria in pure strategies, whichare payoff-unique:

$$\hat{g}_s = \pi\left(\sum_{i=1}^{S} X_i\right) - \pi\left(\sum_{i \neq s} X_i\right) \quad , \quad \forall s$$

$$\hat{h}_b = \pi\left(\sum_{s=1}^{S} X_s\right) - \sum_{s} \hat{g}_s \quad , \quad b = 1$$

Proof:

1. First note that under assumptions 1 and 2 the proposed equilibrium payoffs are well defined. As long as $\pi(.)$ is concave $h_b > 0$. Therefore the buyer will participate actively in the game.

2. Since the sellers have the right of the first offer they obviously can guarantee the payoffs of at least $\hat{g}_s$ by offering just the contract $\tilde{c}_s = (X_s, \hat{g}_s)$. This contract specifies the maximal price the buyer would be willing to pay for the additional quantity of factors, provided he receives the quantities X_i from the competitors $i \neq s$.

3. On the other side, if all sellers offer to sell the quantities X_i at price $\hat{g}_i$, $i \neq s$, the best price seller s can get is in fact $\hat{g}_s$ as well, because this is exactly the price for which the buyer will be indifferent between acceptance and rejection.

4. If in any potential Nash equilibrium with $\hat{c}_s = (\hat{x}_s, \hat{p}_s)$, $\hat{x}_s = X_s$, $s = 1, ..., S$ the proof is already completed and we have shown that $\hat{p}_s = \hat{g}_s$ in this case.

5. If however there is some seller s with $\hat{x}_s < X_s$, define a new game with capacities $\tilde{X}_i := X_i$, $i \neq s$ and $\tilde{X}_s := \hat{x}_s$. Either the new game satisfies the conditions of step 3 above or there is another seller t with $\hat{x}_t < X_t$. In the latter case redefine a new game with $\tilde{X}_t := \hat{x}_t$. We can now continue this procedure until after finitely many steps all sellers satisfy the conditions of step 3.

Without loss of generality let us assume that seller s is already the last agent, for whom the iteration takes place. Then the new game will satisfy $\hat{x}_i = X_i$, $i = 1, ..., S$ and $(\hat{c}_s)_s$ defines an equilibrium for this game, since by means of restricting capacities only the strategy set of seller s is reduced. Therefore optimal responses will remain optimal for all other players.

Since $\hat{x}_s$ by hypothesis is part of an equilibrium allocation we know from step 1 that the sellers can at least secure their marginal contributions, i.e. $\hat{p}_s \geq \hat{g}_s$, $s = 1, ..., S$.

Furthermore for the constrained game we know from step 4.

$$\hat{p}_s = \hat{g}_s^c = \pi\left(\sum_{s=1}^{S} \tilde{X}_s\right) - \pi\left(\sum_{i \neq s} \tilde{X}_i\right)$$

So we conclude $\hat{g}_s^c \geq \hat{g}_s \quad , \quad s = 1, ..., S$

6. On the other side by construction:

$$\hat{g}_s^c = \pi\left(\hat{x}_s + \sum_{i \neq s} \tilde{X}_i\right) - \pi\left(\sum_{i \neq s} \tilde{X}_i\right)$$

As $\pi(x)$ is non-decreasing in x we get:

$$\hat{g}_s^c - \hat{g}_s = \pi\left(\sum_{s=1}^{S} \tilde{X}_s\right) - \pi\left(X_s + \sum_{i \neq s} \tilde{X}_i\right) \quad \leq \quad 0$$

Therefore in conclusion we have shown $\hat{g}_s^c = \hat{g}_s$ and consequently $\hat{p}_s = \hat{g}_s$ for all $s = 1, ..., S$

Q.E.D.

b) Discussion

In equilibrium we find a theme well known to economists. Each seller receives his 'marginal contribution' to total surplus. Clearly, this contribution will be positive only as long as the factors are necessary to produce the maximal amount of surplus, in a sense made precise in the following corollary.

Corollary 1 (characterization of regimes)

a) When factors are 'abundant', i.e. if $\forall s: \sum_{i \neq s} X_i \geq x^*$ *then* $\hat{g}_s = 0$, $s = 1, ..., S$ *and* $\hat{h}_b = \pi(x^*)$.

b) When factors are 'scarce', i.e. in the remaining cases, $\hat{g}_s \geq 0$ *and* $\hat{h}_b = \pi(x^*)$. *When factors are even 'very scarce', i.e.* $\sum_s X_s \leq x^*$ *then* $\hat{x}_{sb} = X_s$ *and* $\hat{g}_s > 0$.

Proof:

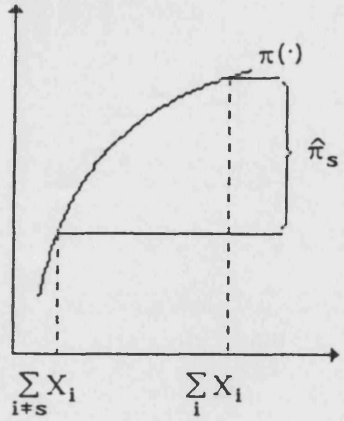
The corollary essentially rewrites proposition 1. Only for the case of very scarce resources we have to show that $\hat{x}_{sb} = X_s$ for any equilibrium contract $(\hat{x}_{sb}, \hat{p}_{sb})$. Now imagine some seller s with $\hat{x}_{sb} < X_s$. So $\sum_s \hat{x}_{sb} < \sum_s X_s$. Since π is strictly increasing in the case considered seller s could profitably deviate from the proposed equilibrium strategy by offering $\tilde{x}_{sb} = X_s$ at a price $\hat{p}_{sb} + \epsilon$ for some positive ϵ, which is small enough to make the buyer accept the offer.

Q.E.D.

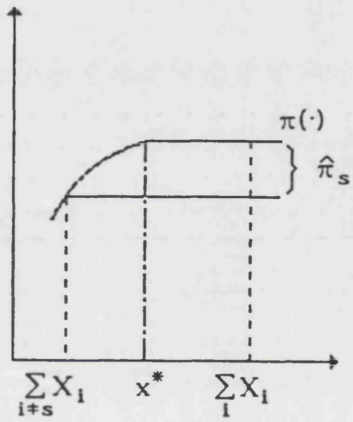
Case a) can be considered the competitive case. Removal of any of the competitors leaves enough capacity to still satisfy the optimal level of production. In this case none of the competitors is strictly necessary for production and hence the equilibrium payoff will not reward any of the sellers and the classical Bertrand result obtains. Clearly, if they could join efforts and collude, the competitive situation might change drastically. For example a grand coalition among all sellers would be able to claim all the surplus. However we have excluded this in the game form considered since we want to focus on the aspects of price competition among capacity constrained sellers.

In case b) any of the sellers is needed for the constrained optimal scale of production and therefore all of them are rewarded positive prices. In fact given all rival sellers claim their 'marginal contribution' for any of them, this also is the highest price they can achieve, because it leaves the monopsonist buyer just indifferent between acceptance and rejection. Any price lower than that would be welcome by the buyer. Figure 1 illustrates the various regimes.

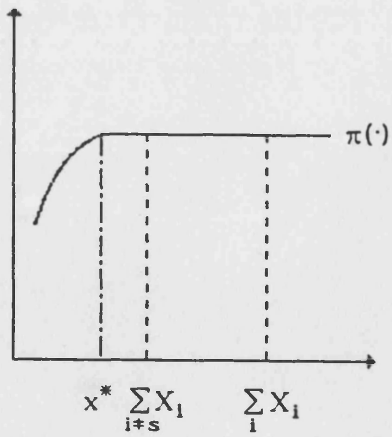
The equilibrium payoff structure exhibits price dispersion although equilibrium is implemented in pure strategies. Sellers with larger capacities can claim larger parts of surplus in the case of scarce endowments of factors. Because of the concavity of the surplus function this is also true for the payoffs per capacity unit as corollary 2 demonstrates. Hence heterogeneity among the sellers causes price dispersion among the implied (per unit) equilibrium prices. As in the literature the observation of price dispersion in the real world is often taken as justification for mixed strategy equilibria. The attraction here is that rational behaviour can also generate this phenomenon in pure strategies. Of course, the distribution of prices is determined by the distribution of factors among the sellers and the shape of the surplus function.



very scarce



scarce



abundant

<u>figure 1</u>

Corollary 2 (price dispersion)

The equilibrium per unit prices $\frac{\hat{p}_{sb}}{\hat{x}_{sb}}$ *are non-decreasing in* $\hat{x}_{sb}$*. They are strictly non-decreasing, when* $\pi(.)$ *is strictly concave.*

Proof:

By virtue of the proof of proposition 1 (step 5) replacing absolute capacities X_s by equilibrium quantities $\hat{x}_{sb}$ we can rewrite $\hat{p}_{sb}$ as

$$\hat{p}_{sb} = \pi\left(\sum_{i=1}^{S} \hat{x}_{ib}\right) - \pi\left(\sum_{i \neq s} \hat{x}_{ib}\right)$$

The rest of the proof is accomplished by establishing the following lemma, which demonstrates a general property for (weakly) concave functions:

Lemma 1

Let $f : \mathbb{R} \to \mathbb{R}$ *be a concave function and* $x > y > 0$*. Then for any* A *the following property holds:*

$$\frac{f(A) - f(A - x)}{x} \geq \frac{f(A) - f(A - y)}{y}$$

Proof:

Given that $A - y = \frac{y}{x}(A - x) + \frac{x-y}{x}A$ the conclusion follows directly from the concavity of f:

$$f(A-y) > \frac{y}{x}f(A-x) + \frac{x-y}{x}f(A)$$

This is equivalent to the statement of the lemma.

Q.E.D.

Now choose $A = \hat{x}_{sb}$, $x = X_s$, $y = \hat{x}_s$ to prove the corollary.

Q.E.D.

Since the larger capacities enable sellers to demand higher per unit prices we can interpret the capacities as a measure of each seller's market power. As long as factors are scarce, market power is limited and competition among the sellers is relaxed. As a seller's capacity and hence his market power increases competition among the sellers intensifies, reducing their equilibrium profits and increasing the monopsonist's payoffs. This is rigorously stated in the next corollary.

Corollary 3 (comparative static properties)

a) If $\sum_s X_s \leq x^*$ then an increase in the endowment of factors for seller s will strictly increase his equilibrium payoffs, while decreasing the returns for all competitors $i \neq s$. Also the buyer will benefit.
b) If $\sum_s X_s > x^*$ then an increase in the endowments of factors for seller s will not change the equilibrium payoff of seller s but it will decrease (weakly) the competitors' payoff, while raising the monopsonist's returns.

Proof: These properties follow immediately from corollary 1.

An immediate consequence of corollary 3 is the convergence to a Walrasian equilibrium, whenever the capacities of at least two sellers are increased towards X^*. Thus a uniform increase in the absolute level of market power will ultimately implement the Walrasian equilibrium. This is a statement of the standard Bertrand justification for competitive outcomes.

However there is also a second theme more intrinsic to Walrasian theory. This views competitive outcomes as the limiting case of vanishing market power. Thus as the relative market power of the competitors shrinks to negligibility, competitive equilibria emerge in markets with a large number of rivals. This is stated in the following corollary.

Corollary 4 (Convergence to competition)

Consider a sequence of economies $(\hat{X}_1, ..., \hat{X}_S)_{S \to \infty}$ *such that* $\max(X_1, ..., X_S) \to 0$ $(S \to \infty)$. *If* $\pi(.)$ *is differentiable then for any subsequence* s_σ *it follows* $lim_{s \to \infty} \frac{\hat{g}_{s_\sigma}}{X_{s_\sigma}} = p^*$ *, where* p^* *is defined as the competitive per unit price:* $p^* = D\pi(\sum_{i=1}^{B} \hat{x}_i)$

Proof:

The theorem establishes:

$$\hat{g}_{s_\sigma} = \pi\left(\sum_{s=1}^{S} X_s\right) - \pi\left(\sum_{s \neq s_\sigma} X_s\right)$$

Hence by definition of differentiability

$$lim_{s_\sigma \to \infty} \frac{\hat{\pi}_{s_\sigma}}{X_{s_\sigma}} = lim \frac{1}{X_{s_\sigma}} \left(\pi(\sum_{s=1}^{S} X_s) - \pi(\sum_{s \neq s_\sigma} X_s)\right) = D\pi\left(\sum_{s=1}^{S} \hat{x}_s\right)$$

Q.E.D.

As in Allen and Hellwig (1986) we find an oligopolistic foundation of perfect competition for the subcase, in which rationing of different buyers cannot occur. In fact we establish the result in pure strategies, whereas theirs is based on equilibrium distributions of mixed strategy equilibria for a given rationing scheme.

The results discussed so far strongly rely on the weak concavity of the surplus function $\pi(.)$. In this case competition intensifies as the endowment constraints are relaxed because the marginal contribution of the factors to total surplus is decreasing. In the case of a convex surplus function however, the marginal contribution of the factors is increasing, easing price competition among the sellers in a drastic way. In this case actually in a subgame perfect Nash equilibrium the sellers will appropriate all the surplus and their only problem is to coordinate the split of profits among themselves.

Proposition 2: (convex surplus)

In the case of a convex surplus function (replacing assumption 1f) there is a continuum of subgame perfect Nash equilibria in pure strategies. In equilibrium $\hat{x}_{sb} = X_s$ *and* $\hat{p}_{sb}$ *such that* $\sum_s \hat{p}_{sb} = \pi\left(\sum_s X_s\right)$.

Proof:

Obviously, for $\hat{x}_{sb} < X_s$ for some s, there will be a profitable deviation for s by offering a higher quantity of factors at a higher price. Therefore $\hat{x}_{sb} = X_s$. Now given any set of equilibrium prices for the rivals $\hat{p}_{ib}$, $i \neq s$ the optimal choice of a contractual price for seller s is

$$\hat{p}_{sb} = \pi\left(\sum_i X_i\right) - \sum_{i \neq s} \hat{p}_{ib}$$

which leaves the buyer just indifferent between acceptance and rejection. So we have established $\hat{c}_{sb} = (\hat{x}_{sb}, \hat{p}_{sb})$ as an equilibrium contract.

Q.E.D.

Finally the results also rely on the particular extensive form chosen. If the buyer had the right to offer the contracts and sellers were to respond, he could retain all the surplus generated from production. The interesting result of this section however is that even when all the bargaining power is given to the sellers, competition among them will reduce their claim to surplus, while enabling the buyer to earn a positive fraction as long as the surplus function is strictly concave. It is the strong position of the rivals in the market, which involves them in strong competition to the benefit of the buyer.

c) Relation to the Classical Bertrand Edgeworth Game

The results discussed so far distinguish the model under study sharply from the classical Bertrand Edgeworth model with constant per unit prices. First we observe that the equilibrium contracts of proposition 1 are not generally linear contracts, which allow the buyer to purchase any quantity up to capacity at a constant per unit price. Therefore, the equilibrium allocation cannot generally be implemented with the classical strategy space used by Edgeworth $CL := \{(x,p) \in I\!R^2_+ \mid p = \text{const } x\}$

Corollary 5 (non-linearity of equilibrium contracts)

The equilibrium of proposition 1 can be implemented by linear contracts $\hat{c}_{sb} \in CL$ if and only if $p(x) = const\, x$ or if total factors are abundant, i.e. if $\hat{g}_s > 0$.

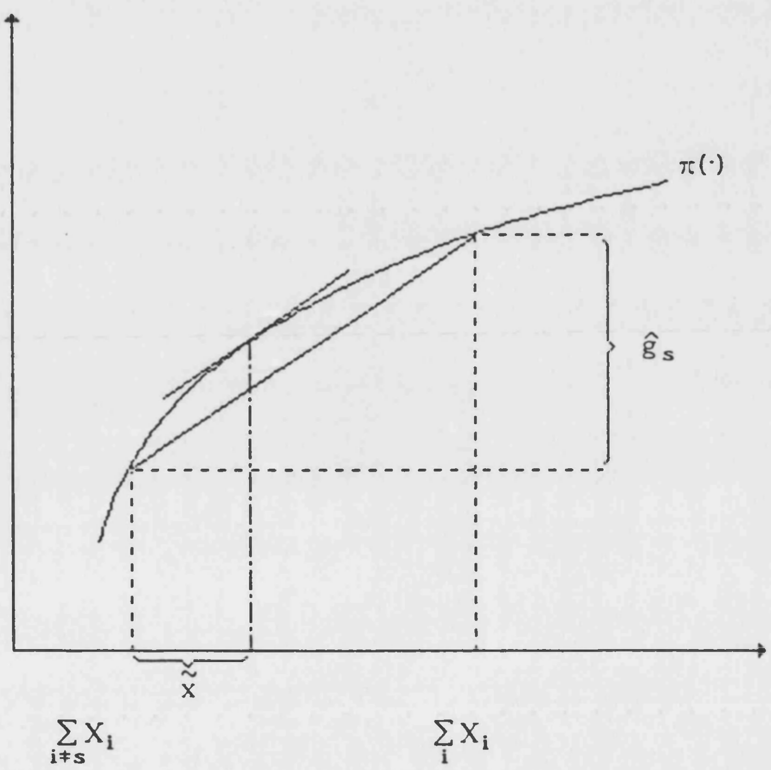
Proof:

The result follows immediately from the concavity of π, which implies that the average contribution exceeds the contribution at the margin, which can be seen as a limiting case of lemma 1. If the buyer was allowed to purchase any quantity at a constant per unit price, he would prefer to purchase $\tilde{X} < \hat{x}_{sb}$, whenever factors are scarce and $\pi(x)$ is concave (see figure 2.)

Q.E.D.

Since it is known that Bertrand Edgeworth equilibria do exist, when resources are sufficiently constrained (Osborne, Pitchik, 1986) and in fact are identical to the respective quantity constrained Cournot equilibria, we might ask how these equilibria compare to those identified here. As it turns out the Bertrand Edgeworth equilibria in linear prices in case of scarce factors do exist only as long as competition among the sellers is weak enough so that sellers in fact prefer to sell all their products despite of the adverse price effect. Therefore the equilibrium price for the classical Bertrand Edgeworth game is given by the slope of the surplus function at the margin, which is the same for all sellers. Since for strictly concave surplus functions this is lower than any average contribution of the sellers also the classical Bertrand Edgeworth equilibria are less profitable to the sellers than the equilibria of the general case. We shall illustrate this in figure 3.

So existence of equilibria in the classical model only occurs for the extreme cases. Either competition among the sellers is so strong that no positive profits are sustainable in equilibrium or it is so weak that sellers would prefer more capacity even at the expense of increased competition. From this we conclude that the more general strategy space is more suitable for the study of price competition under capacity constraints. However so far we have concentrated on a single buyer completely sidestepping the (potential) problems arising from rationing.



<u>figure 2</u>

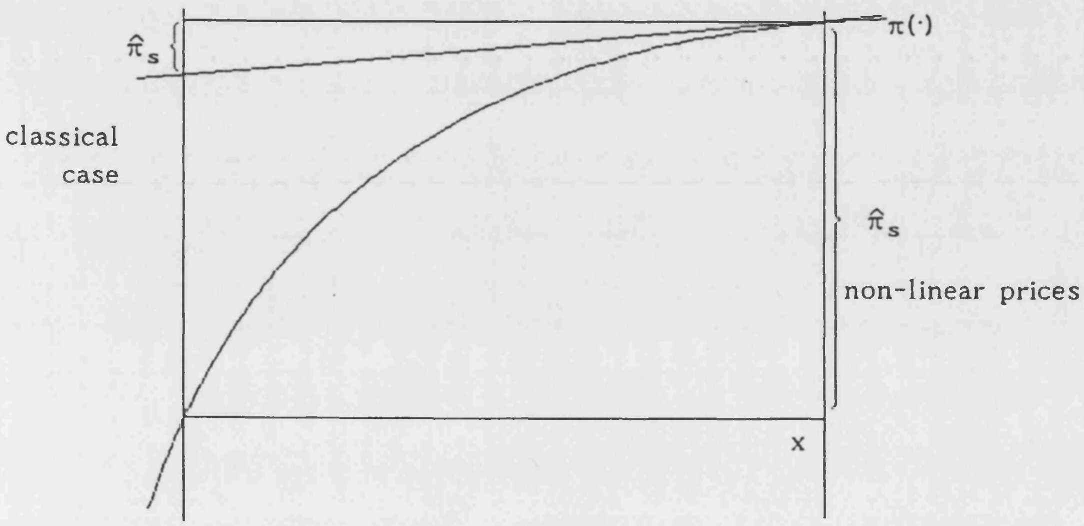


figure 3

4. Several Buyers: Simultaneous Moves

With several buyers additional strategic considerations do arise for the sellers. Not only do they have to worry about the pricing of the contracts but also do they have to decide about whom to offer what quantity of the constrained factor.

In the framework set out in section 2 this situation can be easily modelled since the seller s will offer a specific contract c_{sb} to each individual buyer b. Unequal treatment of the buyers by the sellers such as price discrimination (for identical quantities) is explicitly allowed. For consistency we shall require the total quantity on offer not to exceed the seller's total capacity.

assumption 3

$$C_{sb} \quad = \quad [c_{sb} = (x_{sb}, p_{sb}) \mid \sum_b x_{sb} \leq X_s]$$

Thus sellers are explicitly forced to take their capacity constraints into account when quoting their offers. They have to decide, whom they will supply with what quantity in the case of excess demand. In other words we endogenize the rationing scheme in a very particular way: it is implied in the offers of "personalized" contracts.

This is in marked contrast to the common literature with linear prices, in which sellers only choose prices. The quantity allocation typically is chosen by an exogenously imposed rationing scheme [2] . In this literature all buyers face the same prices and choose sellers according to the best prices. Now it may occur that the seller with the best price cannot serve all demand. He therefore has to decide which buyers to accept and which to reject. The unsuccessful buyers are granted another chance to apply for the next seller, where they could be rationed again.

[2] An exception to this is Davidson, Deneckere, 1986.

This process might continue until all resources are sold. It turns out that mixed strategy equilibria intimately depend on the particular rationing scheme used by the sellers (Davidson, Deneckere, 1986 and Vives, 1986). On the other side pure strategy equilibria do not depend on the rationing schemes, whenever they exist (Osborne, Pitchik, 1986). Since it is our concern to analyze the existence problem in a more general strategic environment, we choose to allow full generality in the choice of contracts. Also we believe that the choice of a rationing mechanism may be an important strategic decision to capacity constrained firms.

Before we state the result let us give a definition. We shall call a seller s *essential*, if his resources are strictly necessary, i.e. if

$$X_s > 0 \quad , \quad \sum_{i \neq s} X_i < \sum_b X^* = BX^*$$

Otherwise he is called *unessential*. So in the language of corollary 1 the factors are scarce if there are essential sellers.

<u>**Proposition 3:**</u>

If $\pi(.)$ is strictly concave and there are at least two essential sellers no subgame perfect Nash equilibrium in pure strategies exists.

If there is at most one essential seller a payoff unique subgame perfect Nash equilibrium exists in pure strategies, in which all inessential sellers earn zero profits.

Before we give the proof it will be convenient to establish a general lemma about concave functions. The proof then is given in several steps. In step 2 the payoffs for a potential equilibrium are determined. Step 3 establishes the efficient use of resources. This implies that essential sellers will earn positive equilibrium payoffs (step 4), whereas unessential sellers will earn zero profits (step 5). Step 6 establishes equilibrium in the relevant cases, whereas step 7 develops profitable deviations for the non-existence case thus concluding the proof.

Lemma 2:

Let $f : I\!R \rightarrow I\!R$ be strictly concave and $A > B$, $z > 0$. Then for any $z < A - B$:

$$f(A - z) + f(B + z) > f(A) + f(B)$$

Proof:

The statement is equivalent to $f(A) - f(A - z) < f(B + z) - f(B)$. Now write : $A = (A - B - z) + (B + z)$. By sub-additivity, a property implied by any concave function $f(A) < f(A - B - z) + f(B + z)$ and likewise $f(A - B - z) < f(A - z) + f(B)$. Therefore

$$\begin{aligned} f(A) - f(A - z) &< f(A - B - z) + f(B + z) - f(A - z) \\ &= f(B + z) + f(A - B - z) - f(A - z) \\ &< f(B + z) - f(B) \end{aligned}$$

Q.E.D.

Proof of Proposition 3:

1. The proof is indirect. So let us consider a hypothetical Nash equilibrium in pure strategies $\hat{c}_{sb} = (\hat{x}_{sb}, \hat{x}_{sb})$

.

2. If we temporarily fix the equilibrium quantities $\hat{x}_{sb}$ as the maximal amounts seller s is willing to sell to buyer b, proposition 1 allows us to calculate the potential equilibrium prices $\hat{p}_{sb}$ as:

$$\hat{p}_{sb} = \hat{g}_{sb} := \pi\Big(\sum_i \hat{x}_{ib}\Big) - \pi\Big(\sum_{i \neq s} \hat{x}_{ib}\Big)$$

Clearly, however, the equilibrium quantities are not fixed and we have to consider alternative quantity allocations.

3. Claim:

In any hypothetical equilibrium resources are used efficiently, because otherwise profitable deviations exist to exploit the inefficiencies. With identical and strictly concave profit functions this implies:

$$\pi\left(\sum_i \hat{x}_{ib}\right) = \pi \quad , \quad b = \{1, ..., B\}$$

Proof of the claim:

If the statement is wrong, then there are buyers b and β with different equilibrium purchases, such that

$$\pi\left(\sum_i \hat{x}_{ib}\right) > \pi\left(\sum_i \hat{x}_{i\beta}\right)$$

Now choose a seller s with $\hat{x}_{sb} > 0$ and consider the strategy $\tilde{c}_{sb} = (\tilde{x}_{sb}, \tilde{p}_{sb})$, in which seller s reduces his sales to b by a sufficiently small amount z, which he uses to increase sales to buyer β. At the same time he reduces the total price for his diminished contribution to b. However because of the concavity of $\pi(.)$ he will be more than compensated by the additional income he will receive from β.

So consider the deviation:

$$\begin{aligned}
\tilde{x}_{sb} &= \hat{x}_{sb} - z \quad , \quad \tilde{p}_{sb} = \pi\left(\sum_i \hat{x}_{ib} - z\right) - \pi\left(\sum_{i \neq s} \hat{x}_{ib}\right) \\
\tilde{x}_{s\beta} &= \hat{x}_{s\beta} + z \quad , \quad \tilde{p}_{s\beta} = \pi\left(\sum_i \hat{x}_{ib} + z\right) - \pi\left(\sum_{i \neq s} \hat{x}_{i\beta}\right) \\
\tilde{x}_{si} &= \hat{x}_{si} \qquad\quad , \quad \tilde{p}_{si} = \hat{p}_{si} \quad , \quad i \neq b, \beta
\end{aligned}$$

Now because of the strict concavity of $\pi(.)$ and lemma 2 we get [3]

[3] Use $A = \sum_i \hat{x}_{ib}$, $B = \sum_i \hat{x}_{i \neq b}$, where $A > B$ because of the monotonicity

$$\tilde{p}_{sb} + \tilde{p}_{s\beta} - \hat{p}_{sb} - \hat{p}_{s\beta} \quad =$$

$$\pi\left(\sum_i \hat{x}_{ib} - z\right) + \pi\left(\sum_i \hat{x}_{i\beta} + z\right) - \pi\left(\sum_i \hat{x}_{ib}\right) - \pi\left(\sum_i \hat{x}_{i\beta}\right)$$

$$> \quad 0$$

Hence the deviation is profitable and we have arrived at a contradiction which proves the claim.

4. From 2. and 3. we conclude that an essential seller s will enjoy positive equilibrium profits, because there is at least one buyer b such that

$$\pi\left(\sum_{i \neq s} \hat{x}_{ib}\right) < \pi\left(\sum_i \hat{x}_{ib}\right)$$

Otherwise $\hat{x}_{sb} = 0$, for all b , $\hat{g}_{sb} = 0$ and

$$\pi\left(\sum_i \hat{x}_{ib}\right) < \pi(X^*)$$

which contradicts the optimality of $\hat{c}_{sb}$, since s could easily improve profits by offering resources $\tilde{x}_{sb} > 0$ at sufficiently low but positive prices.

5. On the other hand we conclude that unessential sellers will earn zero profits in a potential pure strategy equilibrium.

To prove this statement assume to the contrary that the unessential seller σ will earn zero profits in equilibrium, i.e. for some $b \quad : \hat{x}_{\sigma b} > 0 \ , \ \hat{p}_{\sigma b} > 0$.

a. If there is some seller s with excess resources $z < \hat{x}_{\sigma b}$ he can easily use those to undercut σ in competition for buyer b. His profitable deviation is given by means of the following strategy:

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of $\pi(.)$

$$\begin{aligned} \tilde{x}_{sb} &= \hat{x}_{sb} + z \quad , \quad \tilde{p}_{sb} = \hat{p}_{sb} + \epsilon \\ \tilde{x}_{sj} &= \hat{x}_{sj} \quad\quad\;\; , \quad \tilde{p}_{sj} = \tilde{p}_{sj} \quad , \quad j \neq b \end{aligned}$$

with ϵ small enough, such that

$$\pi\left(\sum_{i \neq \sigma} \hat{x}_{ib} + z\right) - \pi\left(\sum_{i \neq \sigma} \hat{x}_{ib}\right) - \epsilon > 0$$

Contract $\tilde{c}_{sb} = (\tilde{x}_{sb}, \tilde{p}_{sb})$ therefore increases buyer b's payoffs by at least ϵ as compared to $\hat{c}_{\sigma b}$ and thus he will accept it. So $\tilde{c}_{sb}$ constitutes a profitable deviation for seller s.

b. If however no seller has excess resources the fact that σ is an unessential seller means that the aggregate equilibrium quantities of some buyer exceed X^*, i.e. there is some buyer β such that

$$\sum \hat{x}_{i\beta} > X^*$$

Now take some seller s with $\hat{x}_{s\beta} > 0$ and consider the strategy, in which s reduces his supply to β by a sufficiently small amount z without affecting his payoff. Simultaneously he increases his sales to b by z at a slight profit:

$$\begin{aligned} \tilde{x}_{sb} &= \hat{x}_{sb} + z \quad , \quad \tilde{p}_{sb} = \hat{p}_{sb} + \epsilon \\ \tilde{x}_{s\beta} &= \hat{x}_{s\beta} - z \quad , \quad \tilde{p}_{s\beta} = \hat{p}_{s\beta} \\ \tilde{x}_{sj} &= \hat{x}_{sj} \quad\quad\;\; , \quad \tilde{p}_{sj} = \hat{p}_{sj} \quad , \quad j \neq b, \beta \end{aligned}$$

such that

$$\begin{gathered} \sum \hat{x}_{i\beta} > X^* \\ \pi\left(\sum_{i \neq \sigma} \hat{x}_{ib} + z\right) - \pi\left(\sum_{i \neq \sigma} \hat{x}_{ib}\right) - \epsilon > 0 \end{gathered}$$

This deviation is profitable as long as $\hat{p}_{\sigma b} > 0$ for the same reason as above thus establishing the contradiction.

In summary we conclude that in any potential pure strategy equilibrium any unessential seller will receive zero profits.

6. This already proves the second assertion of the proposition for the case of zero essential sellers, since obviously contracts at zero prices implement the equilibrium.

But also if there is only one essential seller s, in equilibrium all his rivals will offer zero prices. So he is the dearest source of resources and can only claim any remaining surplus. Hence an equilibrium is given by the following allocation $(\hat{x}_{sb})_{sb}$, in which all unessential sellers sell all their goods at zero prices, whereas the essential seller charges his marginal contribution for any remaining resources[4]

Characterization of equilibrium with one essential seller s:

$$\sum_{\sigma \neq s} \hat{x}_{\sigma b} = X \quad , \quad \hat{p}_{\sigma b} = 0 \quad , \quad \forall b$$

$$\text{such that } \sum_{b} \hat{x}_{\sigma b} = X_\sigma \quad \text{for } \sigma \neq s$$

$$\hat{x}_{sb} = X^* - X \quad , \quad \hat{p}_{sb} = \pi(X^*) - \pi(X)$$

7. In the remaining case with at least two essential sellers 1 and 2 either all essential sellers only deal exclusively with one buyer in equilibrium or there are different buyers $b \neq \beta$, who will both buy from essential sellers.

a. In the first case, if there is one buyer b such that $\hat{x}_{sb} > 0$ and $\hat{x}_{sj} = 0$ for $j \neq b$ all buyers except b receive resources from unessential sellers only.

[4] In fact it is easily seen that this is the unique equilibrium since in any other (asymmetric) quantity allocation for the unessential sellers those could strictly increase their equilibrium payoffs in contradiction to step 5.

In this situation an unessential seller can earn positive profits by undercutting an essential seller in competition for buyer b. This establishes a contradiction to step 5. Hence this case does not constitute an equilibrium.

The case in which there are no inessential sellers is readily seen as not constituting an equilibrium either since each seller could profitably deviate by selling his resources to buyer β at the higher price $\pi(X_j) - \epsilon$ for $j = 1,2$ and ϵ small enough.

b. In the latter case there are buyers b and β such that

$$\hat{x}_{1b} > 0 \qquad \hat{p}_{1b} > 0$$
$$\hat{x}_{2\beta} > 0 \qquad \hat{p}_{2\beta} > 0$$

Without loss of generality choose b such that $\sum_i \hat{x}_{ib} \geq \sum_i \hat{x}_{i\beta}$.

We consider the following profitable deviation for seller 1, which is directed to undercut seller 2 in competition for buyer β.

$$\tilde{x}_{1b} = \hat{x}_{1b} - z \quad , \quad \tilde{p}_{1b} = \pi\Big(\sum_i \hat{x}_{ib} - z\Big) - \pi\Big(\sum_{i\neq 1} \hat{x}_{ib}\Big)$$

$$\tilde{x}_{1\beta} = \hat{x}_{1\beta} + z \quad , \quad \tilde{p}_{1\beta} = \pi\Big(\sum_{i\neq 2} \hat{x}_{i\beta} + z\Big) - \pi\Big(\sum_{i\neq 1,2} \hat{x}_{i\beta}\Big) - \epsilon$$

$$\tilde{x}_{1i} = \hat{x}_{1i} \quad , \quad \tilde{p}_{1i} = \hat{p}_{1i} \quad , \quad i \neq b, \beta$$

where z is small enough, i.e. $z < \frac{1}{2}\min(\hat{x}_{1b}, \hat{x}_{2b})$

This contract reduces the sales to b by an amount of z, that is used to undercut seller 2. Prices are chosen such that seller 2 will not trade with buyer β at all. The increase in seller 1's payoff in this deviation is:

$$\pi\Big(\sum_{i\neq 2} \hat{x}_{i\beta} + z\Big) - \pi\Big(\sum_{i\neq 1.2} \hat{x}_{i\beta}\Big) - \epsilon$$
$$- \pi\Big(\sum_i \hat{x}_{i\beta}\Big) + \pi\Big(\sum_{i\neq 1} \hat{x}_{i\beta}\Big) - \pi\Big(\sum_i \hat{x}_{ib}\Big) + \pi\Big(\sum_i \hat{x}_{ib} - z\Big)$$

Rewriting this expression allows successive application of lemma 2.

$$\pi\left(\sum_{i\neq 2}\hat{x}_{i\beta} + z\right) - \pi\left(\sum_{i\neq 2}\hat{x}_{i\beta}\right) + \pi\left(\sum_{i\neq 2}\hat{x}_{i\beta}\right) - \pi\left(\sum_{i\neq 1,2}\hat{x}_{i\beta}\right) - \epsilon$$
$$- \pi\left(\sum_{i}\hat{x}_{i\beta}\right) + \pi\left(\sum_{i\neq 1}\hat{x}_{i\beta}\right) - \pi\left(\sum_{i}\hat{x}_{ib}\right) + \pi\left(\sum_{i}\hat{x}_{ib} - z\right)$$

Using the concavity of π we can apply lemma 2 and choose $\epsilon > 0$ small enough such that

$$\pi\left(\sum_{i\neq 2}\hat{x}_{i\beta}\right) - \pi\left(\sum_{i\neq 1,2}\hat{x}_{i\beta}\right) - \pi\left(\sum_{i}\hat{x}_{i\beta}\right) + \pi\left(\sum_{i\neq 1}\hat{x}_{i\beta}\right) \geq \epsilon$$

Again using concavity (lemma 2) and the assumption on aggregate capacities we find the remaining partial sum to be positive, i.e. the following inequality holds for small enough ϵ

$$\pi\left(\sum_{i\neq 2}\hat{x}_{i\beta} + z\right) - \pi\left(\sum_{i\neq 2}\hat{x}_{i\beta}\right) - \pi\left(\sum_{i}\hat{x}_{ib}\right) + \pi\left(\sum_{i}\hat{x}_{ib} - z\right) > \epsilon$$

Consequently, both components of the rise in revenues are positive and hence the deviation is profitable for small ϵ.

So far we have only considered a single equilibrium contract of seller 2. The deviation proposed would not be feasible, if there were further contracts in seller 2's offer set that would be preferred by buyers b and β. However, if one of the buyers prefers such a contract to the deviation offered by seller 1 he also prefers it to the original contractual offer, which was assumed to be in equilibrium. This is a contradiction and completes the proof.

Q.E.D.

As the proof demonstrates it is the concavity of the surplus function combined with the lack of commitment to particular buyers, which give rise to profitable deviations from any proposed pure equilibrium strategy. However also with linear profit functions non-existence of equilibrium occurs.

Proposition 4:

If $\pi(.)$ is linear, i.e. if $\pi(x) = \alpha x$, a subgame perfect Nash equilibrium in pure strategies exists and is payoff unique, if

a. either aggregate resources are very scarce, such that $\sum_i X_i < BX^*$

b. or if there is at most one essential seller, in which case all unessential sellers earn zero profits.

Otherwise in the intermediate range with at least two essential sellers, no equilibrium exists in pure strategies.

Proof:

1. As in step 2 of the proof of proposition 3 the hypothetical equilibrium prices can be calculated as $\hat{p}_{sb} = \hat{g}_{sb}$ for any fixed quantity allocation.

2. Now in case a) $\hat{g}_s := \sum_b \hat{g}_{sb} = \alpha X_s$ for all sellers. Undercutting will invariably reduce the defiant's profits , since all available resources are priced implicitly at the monopolistic per unit price α.

3. In b) either there are no essential sellers, in which case there are no effective capacity constraints and the standard Bertrand result obtains, or there is just one single essential seller s. In any equilibrium, in which the essential seller sells all his resources the unessential sellers can earn only zero profits, since they undercut each other in the Bertrand fashion. If he does not sell all his resources the unessential sellers will also earn zero profits, since otherwise s can profitably undercut them by means of the spare resources. Therefore in equilibrium unessential sellers offer their goods at a zero price and the s can earn no

more than his remaining "marginal contribution" to aggregate surplus, which is $B\pi(X^*) - B\pi(\sum_s \frac{X_s}{B}) > 0$.

4. So it is the remaining case, in which aggregate resources are abundant, but in which nevertheless there are at least two essential sellers. As above it can be argued that in a hypothetical equilibrium unessential sellers will earn zero profits charging zero prices. Therefore at least one of the essential sellers either has spare resources or can reduce sales to some buyer b with $\sum_i \hat{x}_{ib} > X^*$ in a revenue neutral way, thus freeing resources for a deviation strategy. These excess goods now can be used in the standard way to undercut any essential rival seller. So equilibrium prices have to be zero for any seller, which however contradicts the fact that essential seller s should earn a positive payoff.

Q.E.D.

In the linear case there is an additional region, in which pure strategy equilibria exist. This occurs, when resources are so scarce that aggregate resources do not suffice to realize all the potential surplus. In this sense proposition 4 resembles the results of Osborne and Pitchik (1986), who show the existence of unique mixed strategy equilibria for a capacity constrained duopoly with linear prices. They show that only in an "intermediate range" completely mixed strategies are employed, whereas in the "Bertrand region" as well as in the "monopoly region" pure strategies suffice.

In summary the existence question is intimately related to the number of essential sellers. If there is at most one of them a unique subgame perfect Nash equilibrium in pure strategies always exists. With two sellers undercutting strategies become individually profitable. Therefore, equilibrium requires that at most one seller earns positive equilibrium profits. This may occur either when the absolute market power of all sellers is large enough, such that no seller earns positive profits, or in monopoly like situations, in which exactly one seller enjoys significantly more relative market power than all of his rivals.

Two or more essential sellers will compete to sell the "spare resources" at a slight profit. This is like the problem of the "hot potato", in which each player just wants to get rid of it and passes it on to some of his competitors. Similarly here no essential seller wants to keep spare resources. Since there are no commitment possibilities in the game even cooperative agreements such as proportionally disposing of the spare resources cannot occur in a pure strategy equilibrium.

This "hot potato" problem ceases, if aggregate resources are scarce and $\pi(.)$ is linear. With concave surplus functions however an additional motive for undercutting is given by the decreasing returns property. If a competitor were driven out, in principle higher payoffs could be realized by the others. This gives rise to deviations from any pure equilibrium strategy and there is no way the sellers could commit not to undercut each other.

In the last section we want to give two examples of different game forms, which implicitly allow for some commitment in situations of several successive decisions.

5. Several Buyers: Sequential Moves

We want to conclude this chapter by giving an example, in which the sequential nature of the decision process allows enough implicit commitment to restore equilibrium in pure strategies.

This example is motivated by the proof of the non-existence result (proposition 3). Non- existence basically occurs because the sellers cannot agree on a rationing scheme. If however there is a preceding stage, in which they have to decide and commit about the maximal amount of resources X_{sb} they are going to sell to each buyer b, the strategic responses are limited and equilibrium obtains. Specifically we have in mind the following extensive form (B):

stage 0: Sellers s decide about maximal quantities X_{sb}, they want to offer to buyer b, such that $\sum_b X_{sb} = X_s$

stage 1: sellers offer menus of contracts $C_{sb} \subset I\!R^2_+$, $s = 1, ..., S$

stage 2: buyer b selects a set of contracts $d_b = (d_{1b}, ..., d_{sb})$,
where $d_{sb} \in C_{sb} \cup \{(0,0)\}$
$(0,0) :=$ all offers rejected (of seller s)

This game form may be interpreted as a model with a hierarchical decision process. In the credit market we could think of the process of granting loans for example. While at a prior stage managers decide about the industry portfolio the specific loan to a particular client is arranged at a lower level under the constraints passed down.

For this game existence of equilibrium is readily shown.

Proposition 5

The game described by game form (B) has subgame perfect Nash equilibria in pure strategies. They are characterized by $\hat{c}_{sb} = (\hat{x}_{sb}, \hat{p}_{sb})$*, such that:*

$$\pi\left(\sum_i \hat{X}_{ib}\right) = \pi \quad , \qquad \forall b \text{ and}$$

$$\hat{p}_{sb} = \pi\left(\sum_i \hat{X}_{ib}\right) - \pi\left(\sum_{i \neq s} \hat{X}_{ib}\right) \quad , \quad \forall s$$

Proof:

The proof follows immediately from step 2 of the proof of proposition 3. For any allocation $(X_{sb})_{s,b}$ we can use proposition 1 to establish equilibrium for the corresponding subgame.

Now at stage 0 sellers effectively have to calculate the split of the surplus. Given the strategy choices of his rivals the best response of seller s is to offer his resources efficiently, i.e. to the buyers with the largest "marginal returns". If he has spare resources, by means of corollary 3 (comparative statics) he cannot increase his profits by increasing his commitments. This would only lower the rivals' equilibrium payoffs. Therefore, any efficient allocation $\hat{x}_{sb}$ will implement an equilibrium.

Q.E.D.

It is worth emphasizing that unessential sellers earning positive profits is compatible with equilibrium. So not surprisingly the commitment possibility strongly affects the nature of price competition.

Since any efficient allocation of the selling constraints $\hat{X}_{sb}$ will implement an equilibrium typically the payoffs will be different in alternative equilibria. For example in equilibria, in which all sellers treat their buyers identically and sell the same amount to each of them, competition among sellers is rather fierce. In this case aggregate payoffs for the sellers are lowest and maximal for the buyers. By building up exclusive relationships with their clients sellers can relax competition and claim larger parts of the surplus. This is spelled out rigorously in the next corollary.

Corollary 6

a. multiple equilibria:

There is a large multiplicity of subgame perfect Nash equilibria in pure strategies for the game defined by the game form (B). In general they exhibit different payoffs.

b. competitive equilibria:

The lowest payoffs for the sellers are associated with "competitive equilibria", in which $\hat{x}_{s1} = ... = \hat{x}_{sB}$ for all sellers.

c. relationship equilibria:

Let $B = nS$, $n \in I\!N$. Then there are equilibria, in which each seller exclusively deals with n buyers. In this case sellers earn all the available profits:

$$\hat{g}_s = n\hat{g}_{sb} \quad , \quad \hat{g}_{sb} = \pi\left(\frac{1}{n}X_s\right) \quad , \quad \forall s$$

$$\hat{h}_b = 0 \quad , \quad \forall b$$

Proof:

The proof follows immediately from corollaries 2 and 3.

Corollary 6 again highlights the tension between the monopolistic and competitive elements in capacity constrained price games. Although quantity commitments guarantee the existence of equilibrium now the choice of equilibrium is indeterminate and any allocation "between the two extremes" can be implemented as an equilibrium.

Note that the commitment implies that the quantity decisions at stage 0 are publicly observed by all rivals. Incomplete information about the quantity decisions leads back to the case of section 4 and non-existence of equilibrium. So the precise nature of repeated interaction is crucial for existence of equilibrium.

Thus, while dynamic interaction may restore pure strategy equilibria the only equilibrium property of efficiency in the sale of resources is fairly weak and leaves the choice of equilibrium largely indeterminate. So one could view the actual choice of equilibrium by the players as a coordination game in beliefs. In this context Edgeworth cycles may reappear although in pure strategies.

6. Conclusion

This chapter addressed the question of equilibrium in capacity constrained price games. F.Y. Edgeworth was the first in 1897 to point out that the existence of pure strategy equilibria may be problematic. While his analysis is cast in a special model here the question has been studied in a more general framework allowing for non-linear prices and discriminatory pricing. Nevertheless, the results in important aspects accord with Edgeworth's analysis. Existence of price equilibrium in simultaneous move games is problematic.

While Edgeworth suggests to resolve the non-existence in a dynamic interpretation we find existence may be ascertained in a dynamic framework in which competitors can react to each others actions. On the other hand for the game form we present, a large variety of equilibria highlights the coordination problems among capacity constrained competitors. So Edgeworth cycles reappear in beliefs. While a "dynamic equilibrium" may exist in general it is not unique.

The implications of these findings for the proposed embedding of contract theory into a more general framework suggest that such a theory should be truly dynamic.

Chapter 2

Intermediation in Search Markets

1. Introduction
2. The Model
3. A Monopolistic Intermediary
4. Price Competition among Intermediaries
5. Conclusion

"Not the least of the torments which plague our existence is the constant pressure of time, which never lets us so much as draw breath but pursues us all like a taskmaster with a whip."

(Arthur Schopenhauer, On the Suffering of the World, 1851)

Intermediation in Search Markets

1. Introduction

Mechanisms of price determination do vary considerably across different markets. There are personal markets, where agents meet more or less randomly and bargain about the price of the products they want to exchange such as bazaars or street markets for fruit, textiles or even transistor radios or watches. On the other side there are well organized markets, in which prices are determined according to well specified rules, as on the Stock Exchange, the Metal Exchange or the auction houses such as Christie's or Sotheby's. Also it is quite common to encounter list prices which are set by the manufacturers of durable goods. In those cases the dealers often have little discretion in granting discounts and little room for bargaining remains. Typically, the same would apply to supermarkets as well, where bargaining in general is not possible.

Accordingly, there are situations in which firms fix the prices of their products independently of the particular client they may face at the time. In a bazaar sellers would show far more flexibility and take into account the relative need of the buyer as well as his bargaining tactics. But again any trade will be conducted directly between buyers and sellers.

Auction markets, on the contrary, are intermediated markets, in which the products are either sold directly to or at least commissioned to an intermediary. Trade between buyers and sellers therefore is conducted indirectly. Since generally intermediaries' services are costly to the traders, the question arises, why after all intermediaries are necessary.

Several aspects are discussed in the literature. One line of thought argues that intermediation may help to complete the market system in situations, in which asymmetric information causes market failure otherwise (Garella, 1989). Another line stresses economies of scale in the provision of incentive schemes (Diamond, 1984, Yanelle, 1988). Intermediaries might save on transaction (monitoring) costs to the benefit of society and could be considered as part of an efficient mechanism minimizing the impact of informational costs.

In their paper about "middlemen" (1986) Rubinstein and Wolinsky pursue a totally different route. They incorporate intermediaries in a bargaining and matching framework. Intermediaries enjoy the advantage that their probabilities of meeting customers is higher than for customers meeting customers. In equilibrium all agents may interact with each other. A buyer might happen to bargain either with an intermediary or a seller. In this sense both direct and indirect trade take place in equilibrium. However, neither buyers nor sellers can affect their matching probabilities by deliberately dealing with the intermediary. So the model is silent about explaining the advantages of intermediation.

In this paper we analyze markets in which intermediaries provide *immediacy*. By their very presence in the market they can be contacted by buyers and sellers at any time. They quote prices, which can be observed by all market participants. Therefore searching for intermediaries is costless for all agents. However, intermediaries typically quote different buying (bid) and selling (ask) prices. Generally it does not pay to sell a product to the intermediary and repurchase it thereupon, since the price differential between ask and bid price is lost in such a round trip transaction. The implied bid-ask spread imposes costs of transacting for market participants. Therefore, they might find it profitable to search for a trading partner themselves and not deal with the intermediary at all. In this case they will incur search costs, which are endogenously determined in our model. So market participants have an active choice between transacting indirectly via intermediaries or directly on the search markets. We develop a market model, in which both markets are active in equilibrium.

The role of immediacy is particularly important in financial markets. There prices react continuously upon arrival of new information, which may affect the value of the underlying security. Since this information generally is revealed only gradually to market participants, either via direct communication or indirectly through the movement of prices, financial markets may face serious problems of asymmetric information. This is manifested in recurring debates about insider dealing as well as in the growing literature on optimal market micro structures (Milgrom, Glosten (1985), Kyle (1985), Roell (1987), Dennert (1989)). This literature concentrates on the design of optimal bid-ask-spreads for market makers in situations of "informed dealing". To prevent a market break down, additional trading motives are necessary. Typically, these are modelled as "noise traders", who may be considered as traders dealing for reasons unexplained by the model. Undoubtedly, asymmetric information will affect the design of the intermediaries' price schedules. Yet, there has to be a prior motive for trade, which keeps the "noise" or "liquidity" traders in the market. It is this motive we concentrate on, sidestepping the impact of insider information.

In his seminal article "The Cost of Transacting", Harold Demsetz (1968) develops the idea of immediacy to explain actually observed transaction costs at the New York Stock Exchange (NYSE). In his view, the exchange's specialists provide the service of immediate order execution by maintaining inventories of shares. In markets with large price fluctuations such services might be of great value to customers. Intermediaries can recoup any opportunity costs and positioning risks they incur, by charging positive bid-ask spreads. He develops a model, in which transaction costs are entirely determined by the costs of providing the service of immediacy.

Although the "specialists" for a particular stock at the NYSE are regulated monopolists, Demsetz argues verbally that there should be enough competitive pressure from related and rival markets so as to avoid excessive spreads. "Competition of several types will keep the observed spread close to cost. The main types of competition emanate from (1) rivalry for the specialist's job, (2) competing markets, (3) outsiders who submit limit orders rather than market orders, (4) floor traders who may bypass the specialist by crossing buy and sell orders themselves, (5) and other specialists" (Demsetz, 1968, p.43). Thus, competitive pressure should guarantee fairly competitive pricing of the specialist. Observed positive bid-ask spreads therefore are reliable measures of the cost of intermediation. So, while leaving the confines of his model, Demsetz implicitly relies on the beneficial effects of price competition.

The recent literature on two sided price competition however contests such a view. It demonstrates the sensitivity of equilibria of price setting games on the exact nature of competition. Stahl (1988) and Yanelle (1988) offer examples of two sided price competition, in which non-Walrasian equilibria with positive bid- ask spreads emerge, even when the intermediation technology is costless. These examples hinge on the impossibility of short sales. In such a non-Walrasian equilibrium intermediaries offer attractive bid prices and hence virtually attract all sellers. This gives them a monopoly position towards buyers. The examples given by Stahl and Yanelle are given in a static context, but it is not clear, how dynamic considerations should annihilate these phenomena. Furthermore, even the existence of equilibrium may be problematic in intermediated markets. (Yanelle, 1988).

This calls for a rigorous treatment of the intuition developed above. In such a framework then also the question can be addressed, to what extent an intermediary's liberty to set prices will be constrained by the various sources of competition. In particular, how will bid-ask spreads and the nature of competition among intermediaries depend on frictions in the market for direct transactions between buyers and sellers?

Our model formalizes Demsetz's central insight: organized markets reduce the impact of trading frictions.Therefore, these frictions which give rise to the intermediated market are modelled explicitly in the direct exchange market. It takes time to find a suitable trading partner and to negotiate the price. Intermediaries, by offering fixed prices, stand ready to speed up the process of exchange. Their price quotes are widely visible and therefore allow traders to trade without delay, provided the intermediary can satisfy the order at all. To provide this service, intermediaries are guided by the principle of gain in charging bid and ask prices. However, they are bound by competition from rival intermediaries as well as from the search market. In fact, we shall see that the presence of this search market qualitatively changes the nature of competition among the intermediaries. In addition, they have to compete against the search market and cannot exploit a monopoly position as in the examples discussed above. Competition among several intermediaries is shown to result in Walrasian outcomes. It depresses bid-ask spreads down to zero. Consequently, buyers and sellers can transact at Walrasian prices and zero transaction costs via the intermediaries. In the absence of any costs for the intermediation services the Walrasian auctioneer can be replaced by competing intermediaries.

If, however, there are fixed costs of entry into the intermediation business, the natural industrial structure is that of a natural monopoly in the sense of Shaked and Sutton (1983). In their definition an industry is a natural oligopoly, when the number of firms entering the industry is bounded independently of the size of the economy. Thus, in a natural oligopoly the convergence to a fragmented industrial structure is explicitly prevented, as the economy grows large. Here, in equilibrium only one intermediary enters. He charges positive spreads. In the choice of prices, however, he is constrained by competition against the search market. In equilibrium, typically, the search market is active. Buyers and sellers with large gains from trade prefer intermediated trade, whereas traders with lower gains engage in search.

The paper is organized as follows: Section 2 introduces the general model. Section 3 analyzes the case of a monopolistic intermediary, competing against the search market. Section 4 also allows for competition among intermediaries. Finally, section 5 concludes by discussing the implications for the industrial structure of the intermediation business.

2. The Model

Let us consider a market for a perishable homogeneous product. Three types of market participants will be active on this market, buyers, sellers and intermediaries.

We assume a continuum of ultimate traders, i.e. of buyers and sellers. They want to trade at most one unit of an indivisible product. Their preferences are described by reservation prices, which for both are assumed to be uniformly distributed along the unit interval, $r \in [0,1]$, thus generating linear market supply and demand functions, which give rise to a unique Walrasian equilibrium.

preferences:

buyer $\quad U_{0k}(r) \quad = \quad r - p_t$

seller $\quad U_{0v}(s) \quad = \quad p_t - s$

r, s := reservation prices

p_t := price of transaction at period t

supply and demand functions:

$$
\begin{aligned}
S(p) &= p \\
D(p) &= 1 - p \quad , \qquad \text{where} \quad p \in [0,1]
\end{aligned}
$$

Furthermore, we assume the reservation prices of all market participants to be private information. Only the aggregate distributions of types, $G_0(.)$ and $F_0(.)$, and therefore supply and demand function are common knowledge. So in principle all traders can calculate Walrasian equilibrium prices and the corresponding allocations. However since there is no auctioneer quoting market clearing prices and coordinating the trading activities, the agents are forced to establish the equilibrium allocation by their own actions, which does imply costly search for trading partners and some sort of price bargaining.

Alternatively, they may trade with intermediaries, who in contrast to the auctioneer, will quote prices with the purpose of generating profits. Typically these prices will not be market clearing in the Walrasian sense and inefficient allocations do result.

Search Market

When traders choose to enter the search market we shall assume that buyers and sellers are matched at random by some matching technology. The number of agents on the two sides of the market may differ. The technology will be such that each market participant on the short side of the market will be matched with some probability $\lambda \in [0,1]$ with an agent of the opposite type. The matching probabilities of agents on the long side consequently are adjusted by the relative numbers and therefore less than λ. The probability of being matched to a particular subset of trading partners will be the same for all subsets of the same size (i.e with the same Lebesgue measure) on the same market side. In the symmetric case of equally many sellers and buyers in the search market λ is simply the probability of being matched for any participant. This is the continuous analog of assuming that the probability of being matched to a particular partner will be constant for all possible partners. Such a technology can easily be shown to exist for the discrete case.

Once a match is established the traders will engage in negotiations about the price of the product. Since they do not know their counterpart's reservation price, sequential bargaining may become quite complex (Rubinstein, 1985). Therefore we choose to model the bargaining process in a highly simplified version. Nature selects one of the partners at random to announce a take-it-or-leave-it offer, which the counterpart may accept or reject. Upon acceptance trade is accomplished. After rejection however, the trading opportunity is lost and the traders leave the market all together.

For technical reasons we shall also assume that only traders, who expect positive utility from trading will enter the market. This will prevent the market from being "overcrowded" by agents unwilling or incapable of trading, i.e. with an expected value from trade of zero. Since these superfluous traders could affect the matching probabilities, we prefer to exclude them by means of this assumption.

We could also have introduced a small entry cost for the search market. Since this however would complicate equilibrium calculations considerably, we also discard this possibility and prefer to think of the limit of vanishing entry costs.

So essentially the implicit cost of search for traders consists of the probability of disagreement in a particular match, which urges them to delay trade by one period.

Intermediaries

Alternatively traders may choose to deal immediately with the intermediaries at the quoted prices. We shall assume that the intermediaries $i = 1, ..., I$ quote fixed ask- and bid prices (a_i, b_i), which are visible for the whole market. For the period in question they are committed to these prices for any transaction they engage in. As long as they face the same number of buyers and sellers at the prices quoted they can easily match their clients and service all deals. In case of a mismatch however they cannot serve all customers on one side of the market. For this contingency we assume that they randomly ration the side of the market they cannot serve fully. In fact the intermediaries price commitments are contingent on their (personal) ability to match the two market sides. They will never experience bankruptcy. We shall neglect the possibility that the intermediary enters the search market to satisfy remaining customers. So their profits are $q(a - b)$, where q is the number of traders on the short side of the market.

Consequently, the attraction of dealing with the intermediary for the traders depends critically on the probability of being served at the quoted prices. Typically in equilibrium this probability will be very high since intermediaries profits are proportional to the trading volume with the spread as proportionality factor. So market participants essentially have to trade off a relatively firm quote of the intermediary against its price.

If on the other side all market participants expect the intermediary to be unable to generate any positive transaction volume, based on such a belief it is preferable not to deal with him.

Let buyer r's and seller s's choice of market be defined by $d_k(r) \in \{0, 1, ..., I\} \cup \emptyset$ and $d_v(s) \in \{0, 1, ..., I\} \cup \emptyset$, where 0 denotes the search market, i the respective intermediary and $\emptyset$ the choice not to enter at all.

Information

Only intermediaries are assumed to have access to an *information technology* which allows to inform all potential traders uniformly. This technology may be thought of a broadcasting or a computer network or simply a newspaper. If access was not restricted each trader could quote prices himself. However, we have in mind that access to the technology is costly and in consequence only few agents will acquire it.

Intermediaries' price quotes, therefore, are *public information.* Also the distributions of buyers and sellers $F_0(r)$ and $G_0(s)$ are public information which incorporates an element of rational expectations about the commodity's scarcity value.

Trader's reservation prices, however, are *private information.* It is this informational friction, which makes trade costly. Even in equilibrium these private valuations will not be revealed completely even though the choice of transaction of a particular trader is informative to some extent. If a buyer prefers intermediated trade his valuation should exceed the ask price. The precise valuation however may never be discovered by other market participants. Also the identity of their clients is assumed to be private information of the intermediaries. So the choice at stage 2 is (partially) observed only by intermediaries.

Game Form

Let us summarize the model and define the exact pattern of timing in the decisions of the particular game form Γ to base our further analysis upon. It is convenient to model the different "decision stages" as a multistage game.

Extensive form Γ

stage 1:
The intermediaries choose ask and bid prices (a_i, b_i), $\quad i = 1, ..., I$, to which they will be committed for the period under consideration.

stage 2:
Buyers and sellers choose the market they want to enter, if at all
$d_k(r) \in \{0, 1, ..., I\} \cup \emptyset$ and $d_v(s) \in \{0, 1, ..., I\} \cup \emptyset$

$0 :=$ search market

$i :=$ intermediary, $i \in \{1, ..., I\}$

$\emptyset :=$ no activity

stage 3:
Nature matches randomly the players in the search market and determines who will make the first and final offer.
Furthermore, nature rations randomly the long sides of any mismatched positions of the intermediaries.

stage 4:
Market participants as determined by the random process supply a take-it-or-leave-it offer.

stage 5:
Their matching partners accept or reject the offer. Acceptance completes the trade, whereas rejection leaves the players unsatisfied.

The specific extensive form chosen does imply some form of commitment of the intermediaries to their price quotes. In particular, once the traders have chosen their market we do not allow them to renegotiate the price with the intermediaries at stage 3. Given that also the intermediaries do not know their customers' identity, i.e. reservation price, this assumption does not seem too restrictive however. Although the intermediary might be willing to grant concessions to low valuation traders, he would rather prefer to extract more surplus from high valuation traders.

Equilibrium Concept

We are interested in the *subform perfect Nash equilibria* of the game described so far. Before we can formally define an equilibrium the players' payoffs have to be determined.

Buyers' and sellers' payoffs consist of the surplus which can be generated in a particular transaction and of the probability of trade actually taking place at the given transaction. It will be important to distinguish between two components of the probability of trade. Trade may fail to take place because the intermediary has to ration one side of the market. In this case the short side of the market determines the intermediary's transaction volume, which in general will be positive.

On the other hand it is readily evident that an intermediary will be unable to generate any positive transaction volume when everybody expects him not to be active in business. In this case potential clients will rationally expect the intermediary not to be able to fulfil his price offers. Therefore they abstain from trade with him at any price quote. Obviously, this phenomenon stems from common conjectures of market participants.

By $\mu_{ki}(r)$ and $\mu_{vi}(s)$ we shall denote the probability buyer r or seller s assign to the event of intermediary i actually being able to trade at the prices quoted. So for any choice $d_k(r) \in \{1, ..., I\}$ and $d_v(s) \in \{1, ..., I\}$ buyer r's and seller s's payoffs are $\pi_k(r) = \mu_{k.d_k(r)}(r)(r - a_{d_k(r)})$ and $\pi_v(s) = \mu_{v.d_v(s)}(s)(b_{d_v(s)} - s)$ respectively. If traders prefer to remain inactive their payoffs are zero and if they select the search market they receive the expected utility from search, which will be defined more rigorously in the next section.

Intermediary i's payoff, $i \in \{1, ..., I\}$, is determined by $\pi_i = q_i(a_i - b_i)$, where $q_i := \min \{\nu(\{r \mid d_k(r) = i\}), \nu(\{s \mid \delta_v(s) = i\})\}$ with the Lebesgue measure ν.

A *Nash equilibrium* is a constellation $((\hat{a}_i, \hat{b}_i)_{i=1}^I, (\hat{d}_k(r)), (\hat{d}_v(s)))$ such that

$$\begin{aligned} (\hat{a}_i, \hat{b}_i) &\in \text{argmax } \pi_i \quad \forall i \\ \hat{d}_k(r) &\in \text{argmax } \pi_k(r) \quad \forall r \\ \hat{d}_v(s) &\in \text{argmax } \pi_v(s) \quad \forall s \end{aligned}$$

The Nash equilibrium is *subform perfect* if it induces a Nash equilibrium in each subform .

3. A Monopolistic Intermediary

a) Characterization of Equilibria

Before we get into the intricacies of competition among the intermediaries in section 4, let us consider the case of a monopolistic intermediary.

In order to derive his optimal pricing strategy the monopolist has to know the market reaction to his price quotes. So we start analyzing the choice problem for the market participants at temporarily given prices and later determine the optimal spread. Let $a \in [0,1]$ denote the monopolist's ask price and $b \in [0,1]$ his bid price. (Since we consider a single intermediary in this section, we suppress the subscript i.)

As a first trivial observation we note that for any positive bid-ask-spread the search market will be active. This is true as sellers with reservation prices above the intermediary's bid price b and buyers with a valuation below the ask price a can obtain a gain from trade only on the search market.

Observation 1:
For any positive bid ask spread, $a - b > 0$, there will be some traders actively trading in the search market.

Now let $G(s)$ and $F(r)$ be the conditional equilibrium distribution functions of sellers and buyers active in the search market. Buyer r's utility $U_k(r)$ when entering the search market consists of three components. If he is lucky, he finds a trading partner and receives the right to supply a take-it-or-leave-it bid. Otherwise he may still find a matching partner and respond passively accepting or rejecting a bid and finally in the worst case he might not even find a partner on the search market. His utility therefore consists of the value of the bidder's game plus the value of the respondent's game weighted by the appropriate probabilities. The same applies to the seller's utility $U_v(s)$.

In order to determine the value of the search game for a bidder we have to define his optimal bid for given conditional distributions of sellers $G(s)$ and buyers $F(r)$ active in the search market. Let $x(r)$ and $y(s)$ denote the buyer's and the seller's bid. In case buyer r's bid is accepted, he receives utility $r - x(r)$. So he has an incentive to bid a low price. On the other side by lowering the bid he also reduces the probability of the bid being accepted because only sellers with reservation prices $s \leq x(r)$ might accept. This trade-off between maximal surplus and a high probability of trade determines the actual offer.

A passive matching partner will accept a bid only as long as his utility gain is positive. So buyer r's utility is $\max\{0, r - y(s)\}$ in case seller s has the right to bid. Likewise seller s's utility is $\max\{0, x(r) - s\}$ when it is buyer r to bid.

Define the expected utility from search as the sum of the expected value from bidding and the value of the subgame, in which the matching partner offers a bid. We shall do this for the special case, in which the total numbers of sellers and buyers in the search market are equal since the general case is readily established by adjusting the respective utilities by the relative measure of traders on the "long" side of the market. Buyer r expects the following utility from search:

$$U_k(r) \quad = \quad \frac{\lambda}{2}\int_{s \leq x(r)} (r - x(r))\; dG(s) \quad + \quad \frac{\lambda}{2}\int_{y(s) \leq r} (r - y(s))\; dG(s)$$

Likewise the sellers' utility attainable from search can be written as:

$$U_v(s) \quad = \quad \frac{\lambda}{2}\int_{r \geq y(s)} (y(s) - s)\; dF(r) \quad + \quad \frac{\lambda}{2}\int_{x(r) \geq s} (x(r) - s)\; dF(r)$$

Besides search market participants may as well choose intermediated trade. In this case the value of an intermediated transaction will depend on the surplus, which can be attained at a given price quote of the intermediary and the probability of trade actually taking place. If the intermediary can match the application of a particular client, this value is simply the difference between the client's reservation price and the intermediary' s quote. In case the monopolist cannot match all clients, there is either a chance $\alpha_k \leq 1$ or $\alpha_v \leq 1$ that a buyer or a seller has to be rationed. Then the value of intermediation for buyer r and seller s , $W_k(r)$ and $W_v(s)$, can be given by:

$$\begin{aligned} W_k(r) &= \alpha_k(r-a) \\ W_v(s) &= \alpha_v(b-s) \end{aligned}$$

In equilibrium, clearly, at most one market side will be rationed, i.e. $\max\{\alpha_k, \alpha_v\} = 1$.

Market participants compare the utility from search with the value of a transaction via the intermediary. They will deal with the intermediary only if

$$\begin{aligned} &W_k(r) \geq U_k(r) \qquad \text{or} \\ &W_v(s) \geq U_v(s) \end{aligned}$$

Alternatively, if neither intermediated trade nor search yields market participants a chance to engage in profitable trade they might decide to remain inactive at all.

In order to classify traders according to their preferred mode of transaction in equilibrium a monotonicity property is quite useful. If in equilibrium buyer r chooses to remain inactive, then also all buyers $r' < r$ remain inactive. Clearly, should buyer r' strictly prefer search or even intermediated trade buyer r could as well imitate the strategy of r' and attain at least the same level of utility. Symmetrically if seller s remains inactive in equilibrium all sellers $s' > s$ will choose to remain inactive. Observe that seller 1 and buyer 0 never can achieve positive gains from trade and therefore abstain from trade. Denote the set of buyers remaining inactive by O_k and the set of inactive seller by O_v. Accordingly, in equilibrium these sets are convex and bounded.

Observation 2

In equilibrium the sets of inactive buyers O_k and inactive sellers O_v are closed and convex sets such that $0 \in O_k$ and $1 \in O_v$.

For the search market a similar monotonicity property can be established. In particular in the next observation we demonstrate that if in equilibrium buyer r chooses search then there is no buyer $r' < r$ who prefers intermediated trade. So r' either enters the search market as well or remains inactive. Together with the previous observation this implies that the set of buyers who strictly prefer search to intermediated trade is a convex set. A symmetric statement holds for sellers, where S_v denotes the set of sellers who strictly prefer search to any other transactional form. If more buyers than sellers enter the search market denote the degree of mismatch by β_k. In other words $\beta_k = \max\left(1, \frac{\nu(S_v)}{\nu(S_k)}\right)$, where $\nu(S)$ denotes the measure of set S. Analogously $\beta_v = \max\left(1, \frac{\nu(S_k)}{\nu(S_v)}\right)$. So the sets S_k and S_v are simply $S_k = \{r \mid \beta_k U_k(r) > W_k(r)\}$ and $S_v = \{s \mid \beta_v U_v(s) > W_v(s)\}$.

Observation 3

In equilibrium the sets S_k of buyers and S_v of sellers who strictly prefer search to intermediated trade are convex sets.

Proof:

1) In order to prove this claim we establish the monotonicity property. Let $r_0 \in S_k$ and $s_0 \in S_v$ be buyers and sellers who strictly prefer search in a given equilibrium. Then we establish that any buyer $r < r_0$ and any seller $s > s_0$ in equilibrium will not deal with the intermediary, i.e. $r \notin I_k$ and $s \notin I_v$. Since $\{r \mid r < r_0\}$

and O_k are convex subsets of the real line also S_k as the complement of O_k in $\{r \mid r < r_0\}$ is a convex set. Likewise S_v is convex.

2) In equilibrium at most one side of the market will be rationed. So either $\alpha_k = 1$ and $\alpha_v \leq 1$ or $\alpha_k < 1$ and $\alpha_v = 1$. In step 2 we shall establish the monotonicity property for the first case while for the latter case it will be proved in the next step.

As $\alpha_k = 1$ there may be fewer buyers on the search market than sellers and therefore $\beta_k \leq 1$.

Now let us establish the monotonicity property for buyers. So consider buyer $r < r_0$. By assumption $W_k(r_0) < \beta_k U_k(r_0)$.

Recall

$$U_k(r) = \frac{\lambda}{2} \int_{s \leq x(r)} (r - x(r))\, dG(s) + \frac{\lambda}{2} \int_{y(s) \leq r} (r - y(s))\, dG(s)$$

$$W_k(r) = \alpha_k (r - a)$$

Imitating the bidding strategy $x(r_0)$ of buyer r_0 in general is not optimal for buyer r and his utility from bidding his best bid $x(r)$ will certainly not be lower. So we get the following list of inequalities, which ultimately demonstrate $\beta_k U_k(r) > W_k(r)$. This is a contradiction to our maintained hypothesis.

$$
\begin{aligned}
U_k(r) &\geq \frac{\lambda}{2} \int_{s \leq x(r_0)} (r - x(r_0))\, dG(s) + \frac{\lambda}{2} \int_{y(s) \leq r} (r - y(s))\, dG(s) \\
&= \frac{\lambda}{2} \Bigg(\int_{s \leq x(r_0)} (r_0 - x(r_0))\, dG(s) + \int_{y(s) \leq r} (r_0 - y(s))\, dG(s) \\
&\qquad - \int_{s \leq x(r_0)} (r_0 - r)\, dG(s) - \int_{y(s) \leq r} (r_0 - r)\, dG(s) \Bigg) \\
&= \frac{\lambda}{2} \Bigg(\int_{s \leq x(r_0)} (r_0 - x(r_0))\, dG(s) + \int_{y(s) \leq r_0} (r_0 - y(s))\, dG(s) \\
&\qquad - \int_{s \leq x(r_0)} (r_0 - r)\, dG(s) - \int_{y(s) \leq r} (r_0 - r)\, dG(s) \\
&\qquad - \int_{r \leq y(s) \leq r_0} (r_0 - y(s))\, dG(s) \Bigg)
\end{aligned}
$$

Thus we find the first two integrals to equal $U_k(r_0)$. Since by assumption $\alpha_k = 1$ it follows $\beta_k U_k(r_0) > r_0 - a$ and we find the following sequence of implications:

$$
\begin{aligned}
\beta_k U_k(r) &\geq \beta_k U_k(r_0) - \frac{\lambda}{2} \Bigg(\int_{s \leq x(r_0)} (r_0 - r)\, dG(s) + \int_{y(s) \leq r_0} (r_0 - r)\, dG(s) \Bigg) \\
&\geq \beta_k U_k(r_0) - \lambda (r_0 - r) \nu(S_v) \\
&\geq \beta_k U_k(r_0) - (r_0 - r) \\
&> r - a \\
&= W_k(r)
\end{aligned}
$$

Consequently $r \notin I_k$.

3) If $\alpha_k < 1$ necessarily $a_v = 1$. By symmetry a result analogous to step 2 holds for sellers. In this case for $s_0 \in S_v$ implies $s \notin I_v$ for all $s > s_0$. Hence, buyers

with valuations $r \in I_v$ cannot expect any profitable trade and remain inactive. Accordingly, $\nu(I_v + S_v) = \nu(I_k + S_k)$, which implies $\beta_k = 1$ in this subcase.

As in the previous step we consider an imitation strategy, in which some buyer r attempts to imitate the search strategy of some buyer r_0. Accordingly the following relations hold:

$$\begin{aligned}
U_k(r) &\geq \frac{\lambda}{2}\int_{s\leq x(r_0)} (r - x(r_0))\, dG(s) + \frac{\lambda}{2}\int_{y(s)\leq r_0} (r - y(s))\, dG(s) \\
&= \frac{\lambda}{2}\left(\int_{s\leq x(r_0)} (r_0 - x(r_0))\, dG(s) + \int_{y(s)\leq r_0} (r_0 - y(s))\right) dG(s) \\
&\qquad + \frac{\lambda}{2}(r - r_0)\left(\int_{s\leq x(r_0)} dG(s) + \int_{y(s)\leq r_0} dG(s)\right) \\
&= U_k(r_0) + \frac{\lambda}{2}(r - r_0)\left(\int_{s\leq x(r_0)} dG(s) + \int_{y(s)\leq r_0} dG(s)\right)
\end{aligned}$$

Depending on the sign of $\frac{\lambda}{2}\left(\int_{s\leq x(r_0)} dG(s) + \int_{y(s)\leq r_0} dG(s)\right)$ two cases have to be considered.

i) $\frac{\lambda}{2}\left(\int_{s\leq x(r_0)} dG(s) + \int_{y(s)\leq r_0} dG(s)\right) \geq \alpha_k$

In this case choose $r > r_0$. Using the imitation strategy for buyer r we get the inequality:

$$\begin{aligned}
U_k(r) &\geq U_k(r_0) + \alpha_k(r - r_0) \\
&= U_k(r_0) - \alpha_k(r_0 - a) + a_k(r - a)
\end{aligned}$$

Now $r_0 \in S_k$ implies $r \in S_k$ for any $r > r_0$. In equilibrium $W_k(a) = 0$ and $U_k(a) > 0$. This statement clearly holds for $a > b$ since then buyer a will be able to generate positive surplus with a positive probability. Also if $a = b$ seller b will have positive search utility $U_v(b) > 0$ as long as $\nu(I_k) > 0$. Accordingly,

S_v includes a positive measure of sellers with reservation prices less than b. This implies $U_k(a) > 0$. Accordingly with the above result $\nu(I_k) = 0$ which contradicts the assumption $\alpha_k < 1$. Therefore this case cannot occur in equilibrium.

ii) $\frac{\lambda}{2}\left(\int_{s \leq x(r_0)} dG(s) + \int_{y(s) \leq r_0} dG(s)\right) < \alpha_k$

Here consider some buyer $r < r_0$. For him the following relation holds:

$$\begin{aligned} U_k(r) &\geq U_k(r_0) - \alpha_k(r_0 - r) \\ &= U_k(r_0) - \alpha_k(r_0 - a) + a_k(r - a) \end{aligned}$$

Accordingly, $r_0 \in S_k$ implies $r \notin I_k$ for any $r < r_0$, the monotonicity property.

4) Finally by a completely symmetric argument the monotonicity property also holds for sellers.

Q.E.D.

In conclusion it follows readily that the sets I_k and I_v of buyers and sellers choosing intermediated trade are convex sets. If they are not empty they contain the traders with the largest potential gains from trade.

<u>Observation 4</u>

In equilibrium I_k and I_v are convex sets such that $0 \in I_v$ if $\nu(I_v) > 0$ and $1 \in I_k$ if $\nu(I_k) > 0$.

By way of summarizing in equilibrium we can decompose the set of buyers into disjoint set I_k, S_k and O_k such that $I_k \cup S_k \cup O_k = [0,1]$ and $0 \in O_k$ and $1 \in I_k$. Likewise $I_v \cup S_v \cup O_v = [0,1]$ represents the partition of the set of sellers, where $0 \in I_v$ and $1 \in O_v$.

Next existence of equilibrium will be analyzed. Indeed we find an equilibrium in which all the subsets of the decomposition have positive measure. This is stated in the first result of this section.

Result 1

a) The intermediation game has a subform perfect Nash equilibrium, which is characterized as the unique solution of

$$\begin{aligned} W_k(1-c) &= U_k(1-c) \\ W_v(c) &= U_v(c) \end{aligned}$$

at the equilibrium prices of

$$\hat{a}(\lambda) = \frac{3}{4} - \frac{\lambda}{8}$$

$$\hat{b}(\lambda) = \frac{1}{4} + \frac{\lambda}{8}$$

b) There is a critical seller $\hat{c}(\lambda) = \frac{1}{4}$ *such that sellers* $s \in [0, \hat{c}]$ *and buyers* $r \in [1-\hat{c}, 1]$ *choose to transact with the intermediary. Buyers and sellers from* $[\hat{c}, 1-\hat{c}]$ *will enter the search market, whereas remaining agents cannot profitably trade in any market.*

Proof:

By virtue of the observations above if in equilibrium $a > b$ then the search market will be active and the sets of buyers and sellers trading with the intermediary can be written as intervals $I_v = [0, c_v]$ and $I_k = [c_k, 1]$ unless $I_v = \emptyset$ or $I_k = \emptyset$. This immediately implies $O_k = I_v = [0, c_v]$. Buyer $r < c_v$ will find no trading partner on the search market since all potential sellers with lower valuations prefer intermediated trade. In equilibrium r cannot deal profitably with the intermediary either and therefore remains inactive. Likewise $O_v = I_k = [c_k, 1]$. Consequently, the sets of agents active in search are identical $S_v = S_k =]c_v, c_k[$. So in equilibrium $\beta_k = \beta_v = 1$. Moreover, in equilibrium $c_v < c_k$ since otherwise the monopolist quotes prices that generate losses. We shall see that indeed in equilibrium $a > b$. So the intermediary can earn positive revenues.

This characterization of the search market allows a simple representation of the optimal bid schedule for any trader considering entry into the search market. In step 1 the optimal bid schedule is derived, while in step 2 the utility from search is determined. This allows to explicitly calculate the critical valuations c_k and c_v as functions of the monopolist's price quotes in step 3. In step 4 we establish that in equilibrium the intermediary quotes symmetric prices $a = 1 - b$ and finally in the last step 5 the optimal spread is determined.

step 1:

For buyers $r \in S_k$ the optimal bid can be calculated as follows:

$$
\begin{aligned}
x(r) \quad := \quad & \operatorname{argmax} \int_{s \leq x(r)} (r - x(r))\, dG(s) \\
= \quad & \operatorname{argmax} \int_{c_v}^{x(r)} \frac{1}{c_k - c_v} (r - x(r))\, ds \\
= \quad & \operatorname{argmax} \big(r - x(r)\big)\big(x(r) - c_v\big) \\
= \quad & \frac{r + c_v}{2}
\end{aligned}
$$

Likewise for $s \in S_v$:

$$\begin{aligned}
y(s) \quad &:= \quad \operatorname{argmax} \int_{y(s)\leq r} (y(s) - s)\, dF(r) \\
&= \quad \operatorname{argmax} \int_{y(s)}^{c_k} \frac{1}{c_k - c_v} (y(s) - s)\, dr \\
&= \quad \operatorname{argmax} \big(c_k - y(s)\big)\, \big(y(s) - s\big) \\
&= \quad \frac{c_k + s}{2}
\end{aligned}$$

step 2:

Using the optimal bid schedule in equilibrium the utility of participation in the search market can be calculated for $r \in S_k$ and $s \in S_v$

$$\begin{aligned}
U_k(r) &= \frac{\lambda}{2} \int_{s \leq x(r)} (r - x(r))\, dG(s) \quad + \quad \frac{\lambda}{2} \int_{y(s)\leq r} (r - y(s))\, dG(s) \\
&= \frac{\lambda}{2} \frac{1}{c_k - c_v} \int_{c_v}^{x(r)} \big(r - x(r)\big) ds + \frac{\lambda}{2} \frac{1}{c_k - c_v} \int_{c_v}^{2r - c_k} \big(r - y(s)\big) ds \\
&= \frac{\lambda}{2} \frac{1}{c_k - c_v} \int_{c_v}^{x(r)} \frac{r - c_v}{2} ds + \frac{\lambda}{2} \frac{1}{c_k - c_v} \int_{c_v}^{2r - c_k} \frac{2r - c_k - s}{2} ds \\
&= \frac{\lambda}{2} \frac{1}{c_k - c_v} \Bigg(\frac{(r - c_v)^2}{4} + \frac{1}{2} (2r - c_k)(2r - c_k - c_v) \\
&\qquad\qquad - \frac{1}{4}(2r - c_k)^2 + \frac{1}{4} c_v^2 \Bigg) \\
&= \frac{\lambda}{2} \frac{1}{c_k - c_v} \left(\frac{(r - c_v)^2}{4} + \frac{1}{4} (2r - c_k - c_v)^2 \right) \\
&= \frac{\lambda}{8} \frac{1}{c_k - c_v} \left((r - c_v)^2 + (2r - c_k - c_v)^2 \right)
\end{aligned}$$

Analogously

$$
\begin{aligned}
U_v(s) &= \frac{\lambda}{2}\int_{r\geq y(s)} \big(y(s)-s\big)\, dF(r) \quad + \quad \frac{\lambda}{2}\int_{x(r)\geq s} \big(x(r)-s\big)\, dF(r) \\
&= \frac{\lambda}{2}\frac{1}{c_k-c_v}\int_{y(s)}^{c_k} \big(y(s)-s\big)dr + \frac{\lambda}{2}\frac{1}{c_k-c_v}\int_{2s-c_v}^{c_k} \big(x(r)-s\big)dr \\
&= \frac{\lambda}{2}\frac{1}{c_k-c_v}\int_{y(s)}^{c_k} \frac{c_k-s}{2}dr + \frac{\lambda}{2}\frac{1}{c_k-c_v}\int_{2s-c_v}^{c_k} \frac{r+c_v-2s}{2}dr \\
&= \frac{\lambda}{2}\frac{1}{c_k-c_v}\Bigg(\frac{(s-c_k)^2}{4} + \frac{1}{2}(2s-c_v)(2s-c_k-c_v) \\
&\qquad\qquad\qquad\qquad - \frac{1}{4}(2s-c_v)^2 + \frac{1}{4}c_k^2\Bigg) \\
&= \frac{\lambda}{2}\frac{1}{c_k-c_v}\left(\frac{(s-c_k)^2}{4} + \frac{1}{4}(2s-c_k-c_v)^2\right) \\
&= \frac{\lambda}{8}\frac{1}{c_k-c_v}\left((s-c_k)^2 + (2s-c_k-c_v)^2\right)
\end{aligned}
$$

In particular the utility levels of the critical traders c_k and c_v are identical and can be determined as

$$
U_k(c_k) = U_v(c_v) = \frac{\lambda}{4}(c_k - c_v)
$$

Buyers with larger valuations $r \in I_k$ and sellers with lower valuations $s \in I_v$ according to observation 3 expect at least $U_k(c_k)$ or $U_v(c_v)$ respectively since they could always immitate the bidding behaviour of the critical agents. Any remaining traders cannot profitably trade.

step 3:

On the other hand at given prices (a, b) the value of intermediated trade can be easily determined for the critical buyers and sellers c_k and c_v by $W_k(c_k)$ and $W_v(c_v)$. Since these critical agents are defined by indifference between search and intermediated trade and since W_j and U_j , $j = k, v$ are continuous functions their valuations have to satisfy the following equation system:

$$\begin{aligned} U_k(c_k) &= W_k(c_k) \\ U_v(c_v) &= W_v(c_v) \end{aligned}$$

Since at most one side of the intermediated market has to be rationed at most one of α_k and α_v is less than 1. Without loss of generality let us assume $0 < 1 - c_k \leq c_v$ or in other words $\alpha_v \leq 1$ and $\alpha_k = 1$. Now, using the result of step 2, the equation system reads:

$$\begin{aligned} \frac{\lambda}{4}(c_k - c_v) &= c_k - a \\ \frac{\lambda}{4}(c_k - c_v) &= \frac{1 - c_k}{c_v}(b - c_v) \end{aligned}$$

or equivalently

$$\begin{aligned} \frac{\lambda}{4}(c_k - c_v) &= c_k - a \\ c_k - a &= \frac{1 - c_k}{c_v}(b - c_v) \end{aligned}$$

The second equation implies $b\,(1 - c_k) = (1 - a)\,c_v$. Given $c_v \geq 1 - c_k$ this implies further $b \geq 1 - a$ or equivalently $a \geq 1 - b$. In equilibrium sellers can only be rationed if the ask price a is more distant from the hypothetical Walrasian equilibrium price of $\frac{1}{2}$ than the bid price b.

Straightforward algebraic transformation of the second equation yields:

$$1 - c_k \quad = \quad \frac{1 - a}{b}\, c_v$$

This can be inserted into the first equation to yield after some elementary manipulation

$$c_v \quad = \quad (1 - a)\, \frac{4(1 - a) - \lambda}{(4 - \lambda)(1 - a) - \lambda b}\, b$$

Finally the trading volume can be determined for the case $\alpha_k = 1$ at prices $a \geq 1 - b$ as

$$1 - c_k \quad = \quad (1 - a)\, \frac{4(1 - a) - \lambda}{(4 - \lambda)(1 - a) - \lambda b}$$

Symmetrically, for prices $a < 1 - b$ in equilibrium buyers will be rationed and $\alpha_v = 1$. Analogously, one finds

$$c_v \quad = \quad b\, \frac{4b - \lambda}{(4 - \lambda)b - \lambda(1 - a)}$$

step 4:

Next we demonstrate equilibrium prices to be symmetric relative to the hypothetical Walrasian equilibrium price, i.e. $\hat{a} = 1 - \hat{b}$.

In order to prove this claim assume to the contrary $\hat{a} > 1-\hat{b}$, which according to step 3 implies $\hat{c}_v > 1 - \hat{c}_k$. We shall establish that the following deviation $(\tilde{a}, \tilde{b})$ is profitable for the monopolist for small enough ϵ. This contradicts the maintained assumption. By symmetry also the case $\hat{a} < 1 - \hat{b}$ can be ruled out in equilibrium and the claim will be established.

$$\begin{aligned} \tilde{a} &= \hat{a} - \epsilon \\ \tilde{b} &= \hat{b} - \epsilon \end{aligned}$$

The deviation does not affect the spread, $\tilde{a} - \tilde{b} = \hat{a} - \hat{b}$. We shall find however that the deviation increases trading volume. In order to establish $1 - \tilde{c}_k > 1 - \hat{c}_k$ the following inequality has to be satisfied:

$$(1-\tilde{a})\,\frac{4(1-\tilde{a})-\lambda}{(4-\lambda)(1-\tilde{a})-\tilde{b}\lambda} \quad > \quad (1-\hat{a})\,\frac{4(1-\hat{a})-\lambda}{(4-\lambda)(1-\hat{a})-\hat{b}\lambda}$$

$\tilde{a} < \hat{a}$ implies directly $1 - \tilde{a} > 1 - \hat{a}$. For the remaining factors we claim

$$\frac{4(1-\tilde{a})-\lambda}{(4-\lambda)(1-\tilde{a})-\tilde{b}\lambda} \quad > \quad \frac{4(1-\hat{a})-\lambda}{(4-\lambda)(1-\hat{a})-\hat{b}\lambda}$$

This claim is established by going backwards the following sequence of equivalent relations

$$
\begin{aligned}
4\big((1-\tilde{a})-\lambda\big)\ \big((4-\lambda)(1-\hat{a})-\hat{b}\lambda\big) \quad &> \\
&\big(4(1-\hat{a})-\lambda\big)\ \big((4-\lambda)(1-\tilde{a})-\tilde{b}\lambda\big)
\end{aligned}
$$

$$
\begin{aligned}
4\big((1-\hat{a}+\epsilon)-\lambda\big)\ \big((4-\lambda)(1-\hat{a})-\hat{b}\lambda\big) \quad &> \\
&\big(4(1-\hat{a})-\lambda\big)\ \big((4-\lambda)(1-\hat{a}+\epsilon)-(\hat{b}\lambda-\epsilon)\big)
\end{aligned}
$$

$$
\begin{aligned}
4(4-\lambda)(1-\hat{a}+\epsilon)(1-\hat{a})-(4-\lambda)\lambda(1-\hat{a})-4\hat{b}\lambda(1-\hat{a}+\epsilon)+\hat{b}\lambda^2 \quad &> \\
&4(4-\lambda)(1-\hat{a}+\epsilon)(1-\hat{a})-(4-\lambda)\lambda(1-\hat{a}+\epsilon) \\
&-4(\hat{b}\lambda-\epsilon)\lambda(1-\hat{a})+(\hat{b}-\epsilon)\lambda^2
\end{aligned}
$$

$$
-4\hat{b}\lambda\epsilon \quad > \quad -(4-\lambda)\lambda\epsilon+4\lambda\epsilon(1-\hat{a})-\lambda^2\epsilon
$$

$$
4\hat{b}\lambda\epsilon \quad < \quad 4\hat{a}\lambda\epsilon
$$

$$
\hat{b} \quad < \quad \hat{a}
$$

In summary the deviation $(\tilde{a},\tilde{b})$ is profitable for the intermediary. This establishes a contradiction to the maintained assumption. Accordingly, in equilibrium symmetric pricing is optimal for the intermediary.

step 5:

Finally, the optimal choice of prices has to be determined. Given optimal prices are symmetric $\hat{a} = 1 - \hat{b}$ trading volume $\hat{c}_v = 1 - \hat{c}_k$ is given by $\hat{c}_v = \frac{4(1-\hat{a})-\lambda}{2(2-\lambda)}$. In equilibrium the monopolist's profits are $\hat{c}_v(\hat{a} - \hat{b}) = \hat{c}_v(2\hat{a} - 1)$. The optimal choice of $\hat{a}$ can be determined as

$$\begin{aligned}
\hat{a} &= \text{argmax } \hat{c}_v \ (2\hat{a} - 1) \\
&= \text{argmax } \frac{4(1-\hat{a}) - \lambda}{2(2-\lambda)} \ (2\hat{a} - 1) \\
&= \frac{3}{4} - \frac{\lambda}{8}
\end{aligned}$$

Consequently

$$\hat{b} = 1 - \hat{a} = \frac{1}{4} + \frac{\lambda}{8}$$

and

$$\hat{c}_v = 1 - \hat{c}_k = \frac{1}{4}$$

This implies that the monopolist can earn positive revenues for any λ. By quoting prices $a \leq b$ he can earn non-positive revenues only. Therefore, the described allocation is an equilibrium.

Q.E.D.

By construction, we have shown the existence of the equilibrium of result 1. Moreover, we have shown that it is the unique equilibrium candidate satisfying the equation system of step 3. Note that in the equilibrium described $\mu_k(r) = \mu_v(s) = 1$. Rationing does not occur.

The equilibrium characterized above however is not the unique subform perfect Nash equilibrium of the intermediation game. For example also a no trade result constitutes a subform perfect Nash equilibrium, supported by the beliefs that all traders refrain from dealing with the intermediary at whatever attractive prices he offers. Since in this case the intermediary cannot find a matching partner for any single client, nobody can profit from trade with him and therefore nobody will trade with him, thus reinforcing the original beliefs.

Result 2

The intermediation game exhibits a continuum of subform perfect Nash equilibria.

Proof:

By the same logic as given above any price pair $(\tilde{a}, \tilde{b})$ with $\tilde{b} = 1 - \tilde{a}$ and $\tilde{a} \in [\frac{1}{2}, 1]$ can be supported as a perfect Nash equilibrium.

Just let the strategy of each trader be to enter the intermediated market if and only if the intermediary quotes the prices $(\tilde{a}, \tilde{b})$ and those are advantageous for the particular trader as compared to the search market. Supported by the conjecture that everybody else is behaving the same way, it is indeed an equilibrium strategy, since for deals with the intermediary at any other price he expects utility of zero. In this case the optimal strategy for each trader off the equilibrium path is to resort to search thus reinforcing the initial conjecture.

Such an equilibrium can be described conveniently by the probabilities of trade which read as:

$$\forall r\ \forall s: \quad \hat{\mu}_k(r) = \hat{\mu}_v(s) = \begin{cases} 1 & \text{if } (a,b) = (\tilde{a},\tilde{b}); \\ 0 & \text{otherwise.} \end{cases}$$

So the monopolist has no alternative but to offer the prices $(\hat{a},\hat{b}) = (\tilde{a},\tilde{b})$. As the proof of result 1 reveals for most admissible prices $\tilde{a}$ the intermediary can generate a positive volume of trade and hence positive revenues.

Q.E.D.

b) An alternative Game Form

The multiplicity of equilibria stems from the fact that the participation decision among the various market participants is uncoordinated. As long as $\mu_k(r) = \mu_v(s) = 1$ all potential gains from trade are exhausted for any trader. In any other equilibrium efficiency costs arise. If the intermediary's price quotes are symmetric these efficiency costs obtain from a lack of coordination. The source of the multiplicity is the possibility that buyers and sellers assign trading probabilities $\mu_k(r) = \mu_v(s) = 0$. Since in general the monopolist has no incentive to quote prices, which validate such assignments, the cause of the multiplicity is seen to originate in the traders inability to coordinate market participation. In this section we shall illustrate this point by means of a slightly modified game form.

If agents were allowed to enter the search market after having been rationed at stage 3, supplying an offer to the preferred intermediary is a weakly dominant strategy to all traders. Thus any particular buyer or seller will enter the search market directly if he prefers to do so, or if according to his beliefs the preferred intermediary will have no business. Nevertheless since applications are costless, he may as well do so before search actually starts. In this section we shall discuss this intuition.

So consider the alternative game form $\tilde{\Gamma}$.

Extensive form $\tilde{\Gamma}$

stage 1:
The monopolist intermediary chooses ask and bid prices (a, b) , to which he will be committed for the period under consideration.

stage 2:
Buyers and sellers choose the market they want to enter, if at all
$d_k(r) \in \{0,1\} \cup \emptyset$ and $d_v(s) \in \{0,1\} \cup \emptyset$

$0 :=$ search market

$1 :=$ monopolist intermediary

$\emptyset :=$ no activity

stage 3:
Nature randomly rations the long sides of any mismatched positions of the intermediaries. Rationed agents will enter the search market.

stage 4:
Nature randomly matches buyers and sellers on the search market and determines the bidder in each match

stage 5:
Market participants as determined by the random process supply a take-it-or-leave-it offer.

stage 6:
Their matching partners accept or reject the offer. Acceptance completes the trade, whereas rejection leaves the players unsatisfied.

The game described by this extensive form also exhibits a large variety of subform perfect Nash equilibria. However, in most of these equilibria at least some market participants use weakly dominated strategies. These players forego the chance to accept the intermediary's offer because they conjecture that he would not be able to find a matching trading partner, even though supplying an unsuccessful application is costless for them under the given constellation. In the previous game defined by game form Γ an unsuccessful application implies the loss of the trading possibility.

Neglecting equilibria with (weakly) dominated strategies we find a unique subform perfect Nash equilibrium.

Result 3

The game $\tilde{\Gamma}$ has a unique subform perfect Nash equilibrium in which no player uses a weakly dominated strategy. This equilibrium coincides with the equilibrium of game form Γ described in result 1.

Proof:

a) Clearly the subform perfect Nash equilibrium of result 1 is also an equilibrium of $\tilde{\Gamma}$. This follows from the proof of result 1 and the observation that sending an application to the intermediary is a weakly dominant strategy for buyers and sellers. In equilibrium, therefore, $\hat{\mu}_k(r) = \hat{\mu}_v(s) = \hat{\mu} = 1$

b) Since accepting the monopolist's offer is always a weakly dominant strategy for buyers $r \in I_k$ and sellers $s \in I_v$ they will apply. In contrast the strategy of not applying at all at stage 2 is weakly dominated. The monopolist, therefore, can expect to generate a positive volume of trade. So as long as he offers the equilibrium prices stated in result 1 he can implement that Nash equilibrium, which we have shown to be unique in the proof of result 1.

Q.E.D.

Game form $\tilde{\Gamma}$ clearly demonstrates the nature of the multiplicity of equilibria in the present set up. The indeterminacy of equilibrium derives from the problem of market participants to coordinate their entry decision. Intermediated trade is viable only if sufficiently many traders believe so and participate in intermediated trade.

On the other hand game form $\tilde{\Gamma}$ gives market participants an advantage in the number of moves. If they are unsuccessful at stage 2 they simply participate in the search market. This destroys the simultaneity of decisions game form Γ attempts to capture. Therefore, in the sequel we shall prefer to concentrate on game form Γ. The purpose of this subsection is merely to highlight the coordination issue.

c) Discussion and Interpretation

Interpreting result 1 and 3 we have developed a model, in which the amount of intermediation is endogenously determined under the presence of trade frictions (search costs) on the unintermediated market. It takes time to find a good matching partner and as long as time is valued by the agents they may be willing to pay for intermediation services. Intermediaries provide immediacy and thus help to economize on search costs. On the other hand as long as they charge a positive spread they impose another transaction cost, which not all market participants are willing to pay. So the tradeoffs of the gains from immediacy against the transactional costs will determine the amount of trading activity via the intermediaries and the importance of the search or "shadow" market.

In equilibrium only traders with large gains from trade will prefer to trade with the intermediary. For them the chances to meet inadequate matching partners on the search market is particularly high. Hence search may prove relatively more expensive for them.

But also in equilibrium there is an active search market. Since match specific prices may be established there, we do observe a distribution of prices, at which trade takes place. This is in contrast to a unique market clearing price in the Walrasian theory or Cournot's monopoly theory.

The total number of active market participants in both markets exceeds the equilibrium number of traders in the Walrasian equilibrium. Clearly, in some matches on the search market no trade may take place. However some traders may engage in profitable trade, who in Walrasian theory cannot participate in the markets. This is illustrated in figure 1.

In the case of the monopolist the result is particularly interesting. In the absence of a search market ($\lambda = 0$) a "classical" monopolist would trade with the same types of buyers and sellers at a higher margin however. Since the intermediary's economic role is purely to reduce the impact of trade frictions in the market, in choosing prices he is therefore bound by the size of these frictions. We may interpret λ as a partial measure of search market efficiency. An efficient search market, which matches the short side with certainty (i.e. $\lambda = 1$), will restrain the intermediary's choice of prices most severely. In the model under consideration it will not render him totally redundant because the search market cannot achieve full efficiency. As the efficiency of the search market however vanishes the monopolist can afford to set prices, which correspond to his monopoly prices. So the introduction of a market, which allows agents to circumvent the monopolist will weaken his market power and depress his margins. This is restated in the following corollary.

Corollary

$$\hat{a}(0) = \frac{3}{4} \qquad \hat{b}(0) = \frac{1}{4}$$
$$\hat{a}(1) = \frac{5}{8} \qquad \hat{b}(1) = \frac{3}{8}$$

The parameter λ can also be interpreted as a measure of the competitive pressure on the intermediary from a competing market. Even protected monopolists may have to face the competition of shadow markets, in which trade takes place on an unobservable individual level. Especially in financial markets such a dual structure of operation is found. So stocks are commonly traded on organized exchanges. However quite frequently there are also well established search markets, in which prices are set by bilateral agreement. Often such search markets themselves are organized by financial intermediaries, who thus compete with the organized exchanges.

Returning to Demsetz's example at the New York Stock Exchange NYSE the question of competition off-market dealing is vital for the exchange, which is organized as a specialist system. The exchange grants exclusive rights to specialists to make the market in a specific stock. This gives the specialist the exclusive right to quote the prices for this stock. On the other hand the specialist has to comply with the rules of the exchange, which somewhat restrain his freedom in pricing and more importantly commit him to deal any normal quantity of stock at the price quoted. Only for large imbalances in the specialist's books due to large orders this commitment may be suspended. Therefore particularly for large deals an "upstairs dealer market" has developed, in which large trades, typically block trades, are matched. The participants in this upstairs market are a few large intermediary houses, which due to their large customer base can accommodate large deals better than the exchange. Obviously as the efficiency of the upstairs market increases and small transactions are collected and bundled into larger blocks, there is concern about the viability of the specialist exchange.

Finally note that we cannot Pareto rank the equilibria for alternative measures of search market efficiency. Increasing λ will reduce equilibrium spreads and the participation in the search market as is seen in figure 1. Increasing efficiency of search may leave out market participants with rather low gains from trade.

Pagano (1986) also cites the Italian stock market as an extreme example of the dual market structure. According to him about $\frac{4}{5}$ of the trade in Italian stocks takes place off the exchanges. His explanation of this phenomenon however employs a quite different argument. In an environment of aggregate price risk he argues that search markets are deeper and hence more liquid. Especially for large transactions the price risks associated with the organized market are larger. A search fee, which could be thought of as a brokerage fee to acquire access to the upstairs market, prevents all traders from concentrating on the search market and particularly traders with small transaction volumes will concentrate on the official stock exchanges. Thus he assumes that organized exchanges costlessly provide the price setting mechanism. In contrast, our model tries to explain the emergence of prices on organized markets. Therefore, the price mechanism is costly and traders will trade with the intermediary only, if the search costs exceed the bid ask spread. Otherwise the traders with low gain from trade will resort to less costly search. Search in our context means direct search of the market participants, whereas the upstairs market can be thought of constituting already some kind of organized and possibly efficient search.

In establishing fragmentation as an equilibrium phenomenon, Pagano strongly relies on the multiplicity of "conjectural" equilibria. Of course, full concentration of trade on the organized exchange provides another equilibrium of his model.

So there is a similarity to our model concerning the coordination issue. Result 2 emphasizes the sensitivity of intermediated markets with respect to beliefs about potential market participation. In order for intermediation to be viable market participants have to believe in its value and such a conviction implies some degree of coordination of the entry decisions. Otherwise chances of profitable intermediation may be foregone.

One could argue that the no trade phenomenon and the many conjectural equilibria sustained by appropriate beliefs in result 2 would never really happen in the game under consideration, since at given prices (a, b) the agents with valuations from I_k and I_v should simply choose intermediated transactions. Thus,in the subgame of stage 2, they could enforce Nash equilibria with higher payoffs for all agents from I_k and I_v. These equilibria are not payoff dominant for all traders however, since the expected quality of matches will decrease for the participants in the search market. Nevertheless the coordination required among the traders in I_v and I_k to play the game with a positive level of intermediation seems to be minimal.

On the other side in a more complex and realistic environment with several related markets, which give rise to profitable intermediation, such a simple selection argument would probably fail and the coordination issue would become far more serious.

4. Price Competition among Intermediaries:

a) Characterization of Equilibria

Introducing another intermediary allows us to discuss the aspects of competition among intermediaries. We shall see that we can concentrate on the case of just two intermediaries. So, now traders will have the choice of dealing either with one of the two intermediaries or to engage in active search themselves.

As the participation decisions of the traders are concerned, the same phenomena as in the monopolist case arise. This implies that intermediation may not occur even though several intermediaries are in the market offering attractive prices. Again the pure attraction of prices is irrelevant, if the intermediaries cannot transact at those prices. This again will support consistent beliefs about the intermediaries' inability of generating enough trading volume.

An additional phenomenon will occur now, since traders beliefs may discriminate among the intermediaries, thus impairing effective competition among them. So for example if the general belief is upheld that intermediary 1 alone can generate enough volume to render his services profitable, this fortunate intermediary will enjoy a monopoly position. Competition will not be effective, since it is counteracted by the traders' psychology, which in this example does not give competitors any chance. This example already indicates that competition among intermediaries very sensitively depends on the general belief structure in the economy. This is summarized in the next result.

Result 4

The intermediation game Γ with competing intermediaries $I \geq 2$ exhibits a continuum of subform perfect Nash equilibria.

Proof:

By the argument given above any price quote of any intermediary can be supported as a subform perfect Nash equilibrium. If traders simply collectively refute to deal with any intermediary, other than i, none of the competitors will engage in business. Now result 2 can be applied for intermediary i.

Q.E.D.

All of these equilibria, except the one we are going to describe below, are sustained by particular belief structures, which rely on the fact that profitable trading opportunities are foregone because of a coordination problem among exactly those agents, who could profit. In other words at stage 2 in the game tree subsets of traders cannot agree to play the equilibrium, which yields the higher payoff for the entire subsets. The inability to coordinate the participation decision creates the anticompetitive character of these equilibria. Since we do not model the emergence of belief structures we cannot discriminate among them. However we can analyze the particular case of a "competitive climate", in which traders exploit any profitable opportunity in the mutual understanding that everybody else behaves the same way. In this case the intermediaries are exposed to the full force of competition.

In order to formalize this notion define the set of buyers $I_k^i(p)$ and sellers $I_v^i(p)$, who prefer to trade with intermediary $i = 1,2$ at the given set of prices $p = ((a_1, b_1), (a_2, b_2))$, provided the intermediary generates positive trading volume. If at stage 2, after the intermediaries' choice of prices, both of these sets for intermediary i are not empty, there are two subgame perfect Nash equilibria of the subsequent subgame. In one equilibrium agents $I_k^i(p)$ and $I_v^i(p)$ actually do trade with intermediary i, whereas in the remaining one agents from both sets do not trade with him. The latter case is payoff inferior for these agents. We can now analyze the set of equilibria, which is supported by the selection of the dominant equilibria for players in non empty sets $I_k^i(p)$ and $I_v^i(p)$. Note that typically these equilibria are not payoff dominant in the sense of Harsanyi, Selten (1989), since the quality of the search market is impaired as high valuation traders leave the market.

The dominant equilibria in this sense do not allow any pair of buyers and sellers to improve their equilibrium payoffs if they were allowed to coordinate their participation decision. So in this sense all potential gains from trade are exploited in such an equilibrium.

<u>**Result 5:**</u>

There is a payoff unique subform perfect Nash equilibrium for the game form Γ, *in which no pair of buyers and sellers could improve by bilateral coordination of the participation decision.*

Furthermore, this equilibrium corresponds to the competitive equilibrium outcome, with the intermediaries choosing the Walrasian price $\hat{a}_i = \hat{b}_i = \frac{1}{2}$, *for* $i = 1, 2$. *In this equilibrium the amount of search activity and the volume of intermediated trade is indeterminate.*

Proof:

Suppose intermediary 1 offers uniformly better spreads, i.e. $a_2 > a_1 > b_1 > b_2$. Then under the chosen selection criterion intermediary 2 will not be able to generate any positive level of activity, since any potential customer is better off dealing with his competitor 1. The volume of trade of intermediary 1 corresponds to the volume a monopolist could achieve at the same price quote and is therefore determined as in step 3 of the proof of result 2.

Therefore in any relevant equilibrium the profits of the intermediaries have to be equal, i.e. $\hat{\pi}_1 = \hat{\pi}_2$. Otherwise, if $\hat{\pi}_2 > \hat{\pi}_1$ intermediary 1 can undercut his competitor by choosing prices $\tilde{a}_1 = \hat{a}_2 - \epsilon$ and $\tilde{b}_1 = \hat{b}_2 + \epsilon$,where ϵ is small enough, such that $\hat{\pi}_2 > \tilde{\pi}_1 > \hat{\pi}_2$. The existence of such an ϵ follows from the continuity of the volume of trade in prices (see step 3 in the proof of result 2).

If however profits are equal, they must be equally zero for any relevant equilibrium, because otherwise by the same argument a slight reduction in the spread causes a discontinuous increase in profits for any deviant. So necessarily the equilibrium spread is zero and it remains to show that the unique equilibrium price implied by this class of equilibria is the Walrasian price indeed.

Assume $\hat{a}_2 = \hat{b}_2 = p_2 > \frac{1}{2}$. What allocation would this price imply in the absence of intermediary 1 and what would his optimal response therefore be?

Suppose there was an active search market, then clearly intermediary 1 could take advantage of it by offering prices, which are good enough to attract customers from both sides of the market, in close similarity to the monopoly case. Most importantly an active search market would give him an opportunity to earn positive profits, given his competitor's prices. This contradicts our finding however that in any relevant equilibrium profits are zero.

Alternatively, if there was no active search market, sellers will be rationed with $\alpha_v = \frac{1-p}{p}$, since all sellers with $s \leq p$ and all buyers $r \geq p$ will trade with intermediary 2. By lowering his ask price $\tilde{a}_1 = p - \epsilon$, intermediary 1 can attract at least the buyers with reservation values in $[p - \epsilon, p]$. Now intermediary 1 can afford to offer a positive spread, i.e. $\tilde{b}_1 = \tilde{a}_1 - \bar{\epsilon}$, $\bar{\epsilon} > 0$ (but small enough). Faced with the choice of price p with probability $\frac{1-p}{p}$ and price $p - \epsilon$ with probability 1, sellers with reservation values close enough to zero will prefer to trade with intermediary 1. Thus again he could earn positive profits in contradiction to our earlier finding.

Finally the allocation, in which both intermediaries quote Walrasian ask and bid prices is a subform perfect Nash equilibrium, when trading volumes of all intermediaries are equal for example.

Q.E.D.

b) Discussion and Interpretation

Result 5 reconfirms the sensitivity of intermediated markets with respect to widely held beliefs and participation decisions of potential clients. In addition to the monopoly case, also the degree of competition among intermediaries may be affected by the general belief structure.

In a "competitive environment", however, Bertrand like undercutting is effective. In this sense we have provided a model explaining the emergence of equilibrium prices without resorting to the coordinating function of an auctioneer, but exclusively relying on the rational choice of prices by the market participants.

This is in contrast to recent work on intermediation. Stahl (1988) and Yanelle (1988) offer models of two sided price competition, in which non-Walrasian equilibria emerge. In their models there is no active search market, which corresponds to $\lambda = 0$ in our model. Hence the only way buyers and sellers can transact is via the intermediaries. Both authors rely on a sequential notion of intermediation. Intermediaries have to buy the products first, before they can resell them again. Short sales are not allowed. An intermediary succeeding in purchasing all the products by offering the most attractive bid price can afford to set revenue maximizing ask prices, which in general will exceed the Walrasian equilibrium price.

With an active search market ($\lambda > 0$) such a phenomenon could not occur. The reason is that intermediaries also face competition from the search market. Therefore they cannot maintain both, a monopoly position in both markets and a positive spread. Whenever their equilibrium spread were positive, $\hat{a}_i > \hat{b}_i$, buyers and sellers from $[\hat{b}_i, \hat{a}_i]$ would resort to the search market. This would create an opportunity for competitors to earn positive profits. In equilibrium however profits have to be zero and thus $\hat{a}_i = \hat{b}_i$.

Our model predicts the Walrasian price also for the extreme case of $\lambda = 0$. This is a consequence of the particular rationing scheme we employ. Matching the short side of the market implies that intermediary i's total volume of trade is bounded by $\min\{1-a_i, b_i\}$. Therefore sellers will typically be rationed for $b_i > \frac{1}{2}$. In contrast the model sketched above requires that any bid $\tilde{b}$ is a firm price, at which intermediary i will purchase any quantity supplied. Thus he obtains the monopoly position which allows him to sell the products at a positive margin $\tilde{a}_i > \tilde{b}_i$. So the latter rationing rule explicitly introduces an asymmetry in favour of the sellers, which causes the non-Walrasian result.

Note that our argument for the general case $\lambda > 0$ is independent of the particular rationing scheme employed. It is therefore the existence of a parallel market, which substantially alters the nature of price competition. In particular it prevents the occurrence of non-Walrasian equilibria. So equilibria of the model of two sided price competition resemble those of one sided price competition as in the classical Bertrand game, essentially because the search market helps to preserve symmetry.

5. Conclusion

Costs of intermediation have not been mentioned so far in our model. Implicitly we have assumed that only intermediaries did have free access to an information technology, which intermediaries could use to inform each trader instantaneously about their price quotes. Clearly, if each trader has free access to such a technology efficient search will result and nothing remains to be explained. We shall now allow equal access to such a technology for each trader, however at a price.

Assume that the information technology is generally available at a common fixed cost k, payable at stage 0 before the intermediation game starts.

Again multiple equilibria due to coordination issues arise. In a "competitive environment" , defined as in result 4, however just a single intermediary will be active because only a single intermediary can earn positive revenues in the ensuing price game. So the monopolistic equilibrium of result 1 is the unique equilibrium of the entry game. This is an artifact of the Bertrand nature of competition in the intermediation game. As soon as two intermediaries are active they drive down each other's equilibrium profits to zero, which does not allow to recover the sunk cost paid for access to the information technology at stage 0.

This argument is valid for any positive level of costs $k > 0$. So the market we describe is a natural monopoly in the sense of Shaked and Sutton (1983). As long as costs are positive only one intermediary can "survive" in the market. He will provide the service of immediacy at a positive spread. However in choosing the spread he is bound by the degree of efficiency of the search market λ. So Demsetz's intuition about the beneficial effects of competition across markets carries over to our model. However in contrast to his view there is no reason to expect full efficiency, as long as rival markets are not frictionless. In our model the spread actually is bounded from below (see corollary). So the bid ask spread is a biased measure of the real costs of transacting.

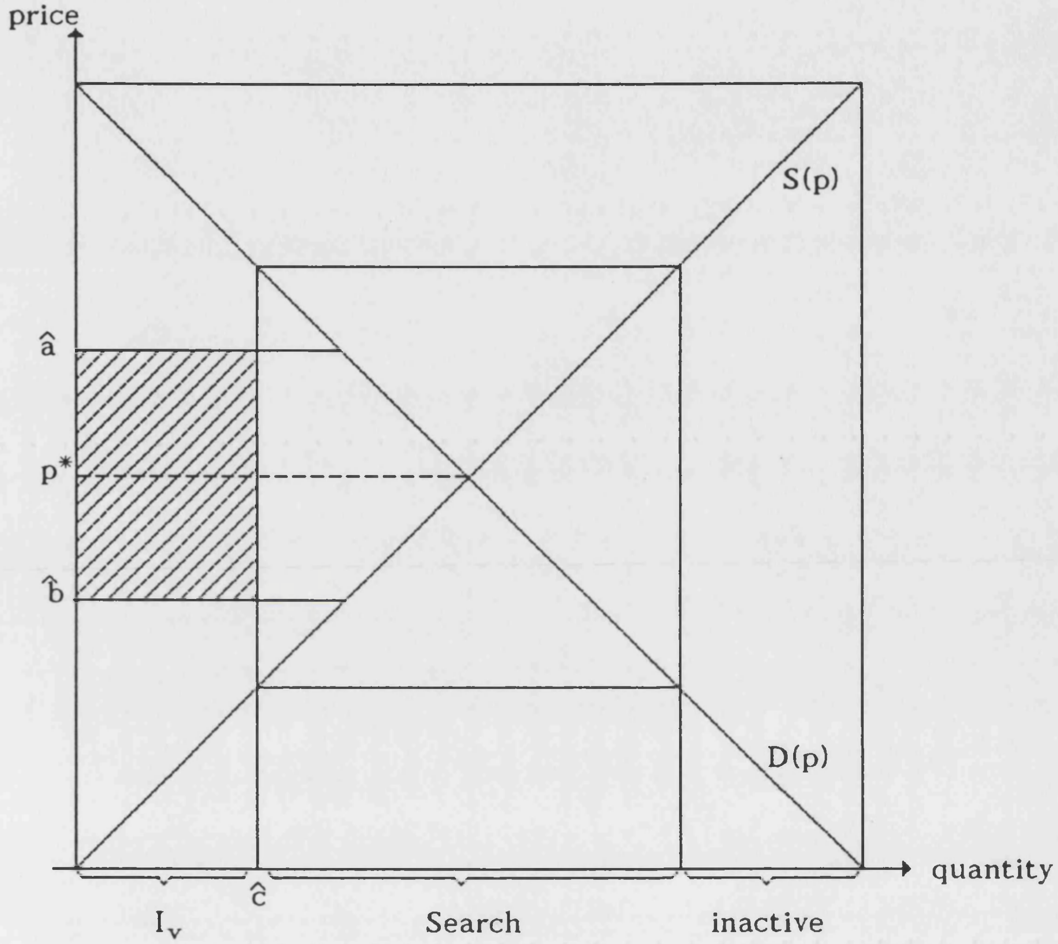

Figure 1

Chapter 3

Natural Oligopoly in Intermediated Markets

1. Introduction
2. Outline of the Argument
3. The Model
4. Price Competition
5. Industrial Structure
6. Relation to the Literature
7. Conclusion

"The way to make money is to get, if you can, a monopoly for yourself."

(Aristoteles, 384-322 B.C., The Politics)

Natural Oligopoly in Intermediated Markets

1. Introduction

It is a characteristic feature of intermediated markets that the economic benefits derived from intermediary services can be fully exploited only, when those services are offered by few firms. In the literature the function of intermediaries is generally associated with the presence of some sort of economies of scale [1] or the specialization of intermediaries in reducing transaction costs like search costs for example [2] . In either case a single intermediary would minimize the costs of the intermediary activity. On the other hand a monopolist intermediary has strong incentives to exert his market power thus preventing an overall efficient allocation. Consequently, the intermediation industry exhibits an innate conflict between cost efficiency and competitiveness.

It is not surprising, therefore, that much of the discussion about the optimal industrial structure for intermediation services centers around the regulation of monopoly [3] . If one views intermediated markets as natural monopolies one might like to curb the market power of the monopolists by regulatory action. The New York Stock Exchange (NYSE) provides an example of a market, where the market making activities for a particular stock are delegated to single specialists subject to the rules of the exchange. In addition, as argued by Demsetz (1968) and shown in the preceding chapter, the monopolist's conduct is restrained by the competition from the search market and other exchanges.

The view of intermediated markets as natural monopolies, however, has to be contrasted against the strategic analysis of intermediation by Yanelle (1988)

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[1] see Diamond (1984) and Yanelle (1988) for example

[2] see chapter 2

[3] as an early example see Demsetz (1968)

and the results of the preceding chapter. In both studies in general there is no unique equilibrium outcome. While monopoly in both cases is compatible with equilibrium also equilibria with several active intermediaries are possible. Due to coordination problems intrinsic to intermediated markets in both cases a multiplicity of equilibria is sustainable. In general an intermediary can be active in the market only, when he expects a positive transaction volume. If traders perceive that a particular intermediary might not be able to find a matching trade or that he incurs relatively high risks of bankruptcy they may be unwilling to engage in business with him at any price offer. Based on such perceptions price competition among intermediaries will be impaired and a variety of equilibria with several active intermediaries can be sustained. Nevertheless, as in the previous chapter, additional refinement concepts may single out the natural monopoly as a unique industrial structure.

Except for regulatory reasons, few intermediated markets are truly monopolistic. Many intermediated markets may seem concentrated with few dominant firms of roughly equal size. Large block transactions in American stocks, for example, are typically not traded at the floor of the NYSE. Instead they are brokered in the so-called "upstairs market" by few investment banks like Merill Lynch, Goldman Sachs or Salomon Brothers. These investment houses rely on their network of contacts to institutional clients in order to manage directly block transactions, which because of their size could easily exceed any specialist's capacity to absorb risks.

Also the banking structure in many European countries is characterized by few dominant banks and a wide class of small banks. So in Germany and Switzerland, for example, three banks of similar size dominate the industry. These dominant banks also appear to possess the largest networks of branches and foreign affiliations. Since the banking industry probably belongs to the most heavily regulated industries care has to be taken with such observations, however. Nevertheless, regulatory action seems to be more concerned about entry into the industry than about non-fraudulent exit.

The purpose of the present chapter is to present a model of *oligopolistic competition* among intermediaries.

As in chapter 2 we take the view that intermediaries help to reduce trading frictions. By publicly quoting prices they inform the market about trading opportunities. In contrast to the previous chapter, however, we allow intermediaries to decide about the scale of their advertising activities. By setting up a large information network intermediaries can quote prices to a large potential clientele. Smaller networks result in less market participants being aware about the smaller intermediary.

If all intermediaries choose to inform the full market all potential clients are fully informed and consequently will deal at the best price quotes available. In this case price competition drives spreads down to zero and no intermediary can earn positive revenues necessary to recoup the fixed costs for setting up the information network. This is the case of the preceding chapter, where intermediaries have access to a single information technology only, which informs the whole market. If on the other hand each intermediary specializes on a different market segment each of them can exploit a local monopoly position, since the potential clients are informed of at most one price quote.

In general, however, intermediaries compete for each other's market shares and consequently the potential clienteles of various intermediaries will overlap. So there are market participants who receive price offers of various intermediaries. Which offer should they accept? Since intermediaries might not be able to find a matching trading partner and hence trade at the prices quoted, market participants have to weight the price on offer against the probability of trade actually taking place. Consequently, they might not choose to trade at the cheapest price if that deal seems highly risky [4] . A trader facing identical quotes of intermediaries with information networks of different size will prefer to deal with the larger intermediary because of a higher probability of trade. The larger intermediary in

[4] In Yanelle (1988) the multiplicity of equilibria is sustained by a similar consideration. In her model the probability of trade derives from the participation decision of the market participants and not from a deliberate strategic choice

the perception of the customers offers "better" liquidity services. He provides the service of immediate exchange at the prices quoted at a higher probability than rivals with smaller outlays. Accordingly, the services on offer exhibit the feature of *vertically differentiated products* [5] .

Markets of vertically differentiated products exhibit an innate tendency towards *natural oligopolies.* The concept of a natural oligopoly was introduced by Shaked and Sutton (1983) and refers to markets, in which for any market size the number of active firms is limited by an upper bound, which is independent of the size of the economy (or the market). Natural oligopolies describe concentrated markets, in which no convergence to a fragmented structure obtains as the economy grows large.

The bound on active firms decisively depends on the structure of preferences and income distributions. Natural oligopolies occur in differentiated product markets, where all consumers unanimously agree on the ranking of the various products priced at marginal costs. Price competition among the higher ranked products tends to reduce prices so far that only a small number of firms may be able to earn revenues in excess of their costs of entry. Moreover, competition among high quality products depresses prices such that certain lower quality products may seem unattractive to customers even if offered at marginal cost prices. In this case even the poorest consumer may purchase products of relatively high quality. Thus low quality products may never gain positive market shares. Consequently, only few high quality products can remain in the market.

If there are different rankings across customers, in close analogy to markets with horizontally differentiated products, necessarily a fragmented structure results [6] . With horizontally differentiated products there is always room between two rivals for another competitor to enter. The entrant by charging low enough prices can always attract sufficient customers to earn positive revenues. So as the economy grows he can cover his fixed expenses. This mechanism allows to introduce an unbounded number of firms into the market. Hence, the element of

[5] Shaked, Sutton, 1982

[6] see Shaked, Sutton, 1983

vertical product differentiation is central to the notion of a natural oligopoly.

As a standard example of a market, which may be described as a natural oligopoly one could think of a technological product market such as the market for personal computers. In this case the processing time may be viewed as the vertical quality characteristic. If the technology is such that product improvement only adds in "small" amounts to fixed costs competing firms in the market will try to differentiate themselves from their rivals in order to escape the competitive forces of price competition. Different firms offer different qualities and it can be shown that price competition limits the number of firms which can earn positive revenues and thus recover fixed cost outlays (Shaked, Sutton,1982).[7]

In this vein we argue that services offered by intermediaries also exhibit an important quality element, which may support a *natural oligopoly* as the natural industrial structure. In particular, we analyze a brokerage market, in which intermediaries can select the size of their potential clientele.

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[7] If the only difference between computers were their colour and preferences were uniformly distributed along the spectrum arbitrarily many firms could operate in large markets, each offering products of the same quality in different colours, however.

2. Outline of the Argument

We choose a model in which market participants are assumed to be scattered on various isolated islands, which, for example, may be thought of as different stock exchanges [8] . Agents on different islands can communicate only via communication channels such as telephone lines. To set up an information link is costly. Therefore, only intermediaries will choose to operate various channels which they connect to communication networks. At the investment stage they decide, which islands to include in their information network. Once such a network is established they can quote prices on each island included. Also they receive all the local information of islands in their network. Consequently, intermediaries with larger networks can address more islands and clients. This in particular allows them to search more efficiently for trading partners.

The market we consider is a brokerage market in which intermediaries do not actually take possession of the good to be traded. They merely provide the service of searching for a trading partner and are compensated by a commission fee only in case of success.

The trading opportunities arise over time. We may view them as "liquidity events", which occur at times and levels of urgency, which are unforeseen by the individual traders. Each liquidity event is characterized by the fact that there are only few sellers and buyers in the market who would like to exchange a commodity or a block of securities. The traders problem is that they do not know about the identity of their trading partners. Building up a communication network for a single trading event could be quite expensive. So they may prefer to delegate the search to intermediaries rather than search for their own. In order to simplify the exposition we abstract from the possibility of private search. Since the choice between direct search and delegated search has been analyzed in detail already in the previous chapter we can concentrate on the choice of market participants among the various intermediaries here.

[8] Some German stocks are traded on eight national exchanges and on some international exchanges, for example.

Competition among intermediaries is modelled as a two stage game. Initially, at stage one, intermediaries decide about their long run strategic variable in designing their information network. At stage two, in the short run, they compete in prices and offer commission charges.

The success of an intermediaries' search obviously depends on the size of his network. The larger the clientele the higher is the probability of generating a trade. If the cost for setting up the information system is largely sunk, there is limited entry into the industry with competitors choosing different information networks. Since intermediaries with identical networks cannot earn positive revenues they try to differentiate themselves from their rivals by choosing different networks. This strategy may not always be available. It fails in particular when one intermediary serves all islands. Nevertheless, in this case competitors can select a smaller and less expensive network. In order to attract customers they have to demand cheaper prices than their larger rivals as in contrast to the globally active intermediary they cannot guarantee to find a matching partner with certainty. If market participants valuation of the deal is high they will prefer to deal with the less risky intermediary even at the expense of a higher commission charge. Otherwise, they may prefer the cheaper offer at the risk of not concluding a trade at all.

The probability of trading introduces an element of vertical product differentiation. Customers prefer to trade with intermediaries whose probability of trading is high and may pay a premium for better chances. Intermediaries typically try to differentiate themselves from competitors by establishing different information networks which affect their trading probabilities. This allows them to relax price competition.

Arguments familiar from the literature on vertical product differentiation help to characterize the market as a natural oligopoly. As in Shaked and Sutton (1982) it can be shown that in equilibrium different intermediaries will enjoy different market shares. Moreover, we demonstrate the validity of another version of the *finiteness property* (Shaked, Sutton, 1983) in our model. In particular, we shall find that as the economy grows large exactly three intermediaries will generate positive transaction volume on almost all islands. The largest intermediary covers all islands, while his closest rivals are active on exactly one island less. Since the closest rivals are of equal size competition among them drives their commission charges to the competitive level on all islands, on which both are active. Consequently, each of them has exactly one niche island, which allows him to earn positive revenues. The niche islands give room for a competitive fringe of many small intermediaries all trying to participate in the rents available on those particular markets. Thus the natural industrial structure in our model is that of three large and globally active intermediaries and a competitive fringe of small intermediaries competing in small niche markets.

Although, our model predicts a fairly concentrated industrial structure, as the number of islands increases the differences in size among the top three firms vanishes and price margins tend to their competitive values. In this sense in our brokerage market concentration ratios are not informative about the competitiveness of the industry.

3. The Model

Islands

Let us consider the market for a perishable homogeneous product. Three types of market participants will be active in this market, buyers, sellers and intermediaries.

Moreover, let the market be subdivided into M isolated and structurally identical islands $m \in \{1, ..., M\}$. Traders cannot move across islands and the only links between the islands are *communication channels* such as telephone lines. These channels between islands can be connected to larger or even global communication networks. Since access to these communication channels, however, is costly only few agents will actually use them.

We prefer to think of these islands as independent geographically separated markets with a large number of economic agents active on each island. In an extreme case one could also view these islands as the basic economic agents themselves. [9]

Liquidity Events

The desire for urgent trade is originated by *liquidity events*, which do occur sequentially over time and are unpredictable by market participants. In order to keep the model manageable we employ a highly stylized version of those events. Each liquidity event consists of two components: on some island, due to liquidity considerations, one particular client suddenly would like to sell a block of stock, for example, and on another island another client would like to buy the same amount of the same stock. Aggregate excess demand across all islands remains constant and the equilibrium price of the security should not be affected. Both

[9] Such an interpretation is particularly relevant to describe the market for large block transactions in securities among large institutional investors.

traders attach symmetric valuations to the particular trade, in the sense that their reservation prices add up to one. When a buyer's urgency to purchase is given by a valuation of r the corresponding seller's valuation is assumed to be $1 - r$. Also the valuations across liquidity events may differ. Hence, a buyer's valuation of a purchase at price p is $r - p$, while a seller's valuation of a sale at the same price is $(1 - r) - p$. Furthermore, we assume that the reservation value r is drawn from a uniform distribution on the interval $r \in [\frac{1}{2}, 1]$.

The trading opportunity is short term and does exist only for that period. In order to exploit the potential gains from trade buyer and seller have to transact in the same period correspondingly. We take the view that over time there is a succession of many independent liquidity events with the same structure. Since, however, any transactions have to be completed within one period, we can omit the subscripts for time. Rather we introduce a scale parameter $A \in I\!R_{>0}$. It serves as a measure for the total number of transactions and hence as a measure of the size of the whole market.

Liquidity events are uniformly distributed across islands and across economic agents on each island. For each island m we can define an expected demand function $D(p, m)$ and an expected supply function $S(p, m)$ as:

$$\begin{aligned} S(p,m) &= A\,p \\ D(p,m) &= A(1-p) \quad , \\ &\qquad \text{where } p \in [0, \frac{1}{2}] \quad \text{and } A \in I\!R_{>0} \end{aligned}$$

The market participants' problem is to find the suitable trading partner in the same period. Knowing the structure of the game they know about his existence and about his reservation value. However, in order to conclude a transaction in addition they need to know his identity and location. In principle, they could search by randomly calling various telephone numbers. However, this is unlikely to be a cheap strategy and it might be preferable for them to contact a broker who professionally maintains an information and communication network, which allows him readily to evaluate the various needs of his clientele. Since in the previous chapter we have already discussed the endogenous choice between private search and intermediated search we concentrate on delegated search here. So, having incurred a liquidity event market participants are only allowed to contact an intermediary active on their own island. The possibility of private search will not be considered in this chapter.

In order to emphasize the brokerage function of intermediaries we decompose the liquidity event into two subperiods with either the buying event or the selling event occurring in the first subperiod and the offsetting counterevent happening in the second subperiod. Without loss of generality we may assume that the desire to purchase occurs first in the first subperiod. Because of the decomposition of the time period buyers can contact intermediaries in the first subperiod to search for the trading partner in the second subperiod.

Intermediaries

Intermediaries offer the service of searching for a trading partner.

Specifically, they offer contracts, which specify a commission fee $P_i(m)$ for successful search. Intermediaries can communicate their price offers to potential traders in island m only by means of an *information technology.* We could think of this information technology as a computer network, which informs clients about the intermediaries' current price quotes instantaneously. Such technologies are provided for example by Reuters or Datastream in the foreign exchange or in bond and stock markets. Access to this information technology requires a fixed payment of $k > 0$ per island.

The local communication networks can be connected to larger information networks. Any agent with access to such an information network may engage in intermediary activities. Since the market size, as measured by A, is limited and $k > 0$ only finitely many agents can profitably purchase access to those information media. Without loss of generality we can restrict the set of potential intermediaries[10] i to a countable set, i.e. $i \in \mathbb{N}$.

Each intermediary i selects a set of islands $M_i \subseteq \{1, ..., M\}$, on which he plans to offer his brokerage services. For an application to $\#M_i$ islands a cost of $k\#M_i$ has to be borne. By means of his information network an intermediary can advertise his commission fees $P_i(m)$ on each island included in the network $m \in M_i$. He may choose to quote different prices across islands.

The access decision has to be undertaken before any liquidity event takes place and before prices can be quoted. Implicitly, we assume that by choosing M_i the intermediaries define their long run positions. The process of installation and granting access permission to the information network is lengthy relative to the urgency of trade. However, liquidity events may occur quite frequently and in principle intermediaries could change prices after each transaction. We view the investment decision as the long run strategic decision, while prices are taken as short run strategic variables readily to be adjusted to changes in the market environment if necessary.

Once a client applies the intermediary attempts to find the counterpart by searching in his potential clientele. If he is successful the two traders are matched and the intermediary earns the advertised commission fee. If the search, however, remains unsuccessful no trade can take place and the trading opportunity is lost. No further obligations for the intermediary do arise. The only commitment he undertakes is to fix the price quote for the period under consideration. This excludes the possibility of renegotiation after successful search.

The process of search should be viewed as "direct search" in the sense that intermediaries directly contact their potential customers and inform them about the possibility of a trading opportunity without disclosing the identity of their client before concluding the match. Intermediaries can only search on islands included in their network. Implicitly we assume that successful search requires

[10] In the model set out below any agent having purchased the information technology has a strict incentive to engage in the intermediation business. In particular, there is no risk of bankruptcy.

some "intimate" knowledge of the local market, which can be acquired only when being present on the local market.

Furthermore we assume an efficient search mechanism. So whenever the trading partner wanted happens to live on one of the islands $m \in M_i$ we assume intermediary i will find him with certainty. Thus intermediary i's probability of concluding a trade is given by $\frac{\# M_i}{M}$.

In the first subperiod market participants observing the range of prices of the intermediaries active on their island select an intermediary to search for the matching partner in the second subperiod. If the matching partner happens to be on an island served by the chosen intermediary he will be aware of the intermediary's search and apply. Accordingly, the match takes place. The intermediary is rewarded his commission fee and the matching partners bargain about the terms of trade. We assume that they select the Nash bargaining solution and split the surplus after deduction of the commission fee. This implies that they also split the brokerage fee $P_i(m)$. Hence, each trader pays $p_i(m) = \frac{1}{2}P_i(m)$. If no match is generated the trading opportunity is lost.

Market participants have the choice between several intermediaries. Prices are not absolutely certain since they depend on the intermediary's ability to match both sides. A given buyer before selecting an intermediary has to consider the probability that the intermediary might also attract the seller in order to conclude the deal. We assume that market participants are informed about the identity and in particular about the network M_i of each intermediary active on their island. They do not observe their price quotes on other islands however. For a buyer r residing on island m the expected utility from trading with intermediary i is determined as the product of the probability of trade $\frac{\# M_i}{M}$ and the surplus the trader can achieve from trade with intermediary i at the price $p_i(m)$ given the expected outcome of negotiations with his trading partner. It reads $W_i(r,m) = \frac{\# M_i}{M}(r - \frac{1}{2} - p_i(m))$. Since the buyer cannot travel across islands he cannot take advantage of possibly better prices at another island.

Finally, buyer r on island m selects the value maximizing offer $d(r,m)$. Using the convention $W_0(r,m) = 0$ and interpreting $i = 0$ as "no trade" we can write

$$d(r,m) \quad := \quad \text{argmax}_{\,i}\{W_i(r,m)\}$$

In case of indifference between several intermediaries he chooses randomly between those alternatives.

Note that all market participants agree on the choice among the various intermediaries, if all price quotes are identical and the $\frac{\# M_i}{M}$ can be ranked in a strict order. All agents prefer to trade with the intermediary offering the highest probability of trade. Thus, the intermediaries' services of immediate exchange exhibit the feature of ***vertically differentiated products*** as defined by Shaked and Sutton (1982). Intermediaries with larger networks offer a higher probability of trade and consequently a better product. Thus, they gain market power and may command higher prices than their less reliable rivals.

Intermediaries' expected revenues R_i consist of the sum of expected revenues on each island $R_i = \sum_{m \in M_i} R_i(m)$. Those again can be calculated as the product of market size A, the expected trading volume per period $q_i(m)$ on island m and the price advertised, $R_i(m) = Aq_i(m)P_i(m)$. By $q_i(m)$ and $R_i(m)$ we mean explicitly the expected volume of trade and the revenue intermediary i expects to originate on island m. So in our framework it refers to the number of buyers, intermediary i expects to attract on island m.

Game Form

By way of summarizing we set out the extensive form of this game:

stage 1:
Intermediaries $i \in I\!N$ establish a network M_i of islands
$M_i \subseteq \{1, ..., M\}$

stage 2.1.t:
Intermediaries choose prices $P_i(m)$, $\quad i \in M_i$, to which they will be committed for the period under consideration.

stage 2.2.t:
Nature selects a realization of the liquidity event
$(r, m) \in [\frac{1}{2}, 1] \times \{1, ..., M\}$

stage 2.3.t
Buyer r chooses the intermediary to whom he delegates
the search $d(r, m) \in \{i \mid m \in M_i\} \cup \{0\}$

$0 :=$ no trade

$i :=$ intermediary

stage 2.4.t:
At this stage no strategic action takes place. The second subperiod is realized. If the seller is in reach of the network of intermediary $d(r, m)$ trade is concluded. Intermediary $d(r, m)$ is paid the commission fee $p_{d(r,m)}$ and the traders split the remaining surplus to get $\frac{1-p_{d(r,m)}}{2}$.
Otherwise no trade takes place and each participant receives a zero payoff.

The index $t = 1, ..., A$ denotes the succession of independent liquidity events. Since we concentrate on stationarity and serial independence of the trading events it suffices to analyze one single such event. The scale parameter A is simply the number of liquidity events.

Furthermore, observe that no external uncertainty enters the model. The only uncertainty is endogenous and derives from the limited knowledge of individual traders about their trading partners. Intermediaries do not possess any particular advantage in knowledge. However, they may invest in superior communication technologies.

We are interested in the *subgame perfect Nash equilibria* of this game.

4. Price Competition

In our model intermediaries undertake two strategic decisions. Initially, they decide about the scale of their operations and establish a communication network across the islands they plan to engage in business on. Having established their presence in the market they compete for market shares by quoting prices. To solve for the subgame perfect Nash equilibria we solve the game backwards. In this section we focus on the last subgame and analyze the nature of short run competition in prices for given investment decisions. Long run competition and the choice of the network is the topic of the next section.

Note that at a given price market participants prefer to deal with the intermediary who enjoys the higher probability of trade. In equilibrium intermediaries with smaller networks have to compensate a lower likelihood of concluding a successful trade by offering more attractive prices. Accordingly, in equilibrium $M_j \subseteq M_i$ implies $P_j(m) \leq P_i(m)$ for $m \in M_j$.

Competition on say island m between two intermediaries i and j with networks of identical size, $\#M_i = \#M_j$, drives their spreads down to zero and the classical Bertrand type result obtains on island m for all intermediaries with networks of the same or smaller sizes. From the viewpoint of the market participants intermediaries i and j are identical competitors. In this situation they strictly prefer the cheaper offer. Hence, each intermediary has an incentive to undercut any positive price offer of his rival and the competitive allocation with zero spreads obtains. We summarize these observations in the first result.

Result 1

In equilibrium

$M_j \subset M_i$ *implies* $P_i(m) \geq P_j(m)$, $m \in M_j$ *and* $R_i(m) \geq R_j(m)$

$M_j = M_i$ *implies* $P_i(m) = P_j(m) = 0$, $m \in M_i = M_j$ *and* $R_i(m) = R_j(m) = 0.$

Proof:

Since intermediary i enjoys a wider network he can always imitate j's choice of prices and drive him out of business, as i's trading probability is higher than j's. Consequently, j's prices and revenues in equilibrium do not exceed those of i. When both possess the same network of islands Bertrand price competition drives the spreads down to zero in each submarket m.

Q.E.D.

The result implies that in equilibrium at most finitely many intermediaries can earn positive prices on each island because the number of islands is finite.

Before establishing the existence of equilibrium in prices we have to study the choice of intermediary by the market participants (here: buyers). Consider island m and relabel intermediaries active on island m such that they are ranked in decreasing size $\#M_1 \geq \#M_2 \geq ... \geq \#M_l \geq 1$. For the moment we allow l different intermediaries to trade on island m. Observe that the individual value functions $W_i(r,m) = \frac{\#M_i}{M}\left(r - \frac{1}{2} - p_i(m)\right)$, $i \in \{j \mid m \in M_j\}$ are linear in r. Hence, following Shaked and Sutton (1983) we can define critical buyers $t_i(m)$ who are just indifferent between intermediary i and $i+1$. Buyers with larger valuations $r - \frac{1}{2} > t_i(m)$ prefer trade with the larger intermediary and buyers with lower valuations will trade with the smaller intermediary. In accordance with result 1 in equilibrium the solution obviously requires $P_i(m) > P_{i+1}(m)$. Buyers with valuations less than $P_l(m)$ cannot gain from intermediated trade. Since we do not allow them to engage in private search they remain inactive.

The critical buyers are defined as the solution to the system of indifference relations at given price quotes:

$$\begin{aligned} W_i(t_i(m),m) &= W_{i+1}(t_i(m),m) \quad , \quad i < l \\ W_l(t_l(m),m) &= p_l \end{aligned}$$

By employing the definition for $W_i(r,m)$ this equation system can be rewritten.

$$\begin{aligned}
\frac{\#M_i}{M}\left(t_i(m)-p_i(m)\right) &= \frac{\#M_{i+1}}{M}\left(t_i(m)-p_{i+1}(m)\right) \quad , \quad i<l \\
\frac{\#M_l}{M}\left(t_l(m)-p_l(m)\right) &= p_l(m)
\end{aligned}$$

Expressing the $t_i(m)$ in terms of the strategic variables we find

$$\begin{aligned}
t_i(m) &= \frac{1}{\#M_i-\#M_{i+1}}\left(\#M_i p_i(m)-\#M_{i+1}p_{i+1}(m)\right) \\
t_l(m) &= p_l(m)
\end{aligned}$$

Given the choice of market participants the market shares of the intermediaries can be determined. On island m intermediary i expects a share of $q_1(m)=1-t_1(m)$ and $q_i(m)=t_{i-1}(m)-t_i(m)$ for $i\geq 2$. His expected revenue on that island is $R_i(m)=q_i(m)P_i(m)$.

The next result establishes the existence of an equilibrium in prices for any given constellation of information networks and subsidiaries. Each constellation defines an *industrial structure* $(M_i)_{i\in I\!N}$.

<u>**Result 2**</u>

For each industrial structure $(M_i)_{i\in I\!N}$, $M_i\subseteq\{1,...,M\}$ *there is a Nash equilibrium in prices.*

Proof:

1) While the network of islands determines the trading probability for each intermediary price competition remains localized on each island. Intermediaries' total revenues are additive in the local revenues $R_i=\sum_{m\in M_i}R_i(m)$. There are

no spill overs of prices across islands. On each island each intermediary competes for potential buyers in a different competitive environment. Once he has been successful in attracting a buyer his payoff depends on whether he can find the seller on one of the islands included in his communication network. Hence, the analysis can be reduced to price competition for buyers on each single island. If the existence of price equilibrium is established for an arbitrary island it carries over to the whole network.

2) As already remarked earlier on each island only finitely many intermediaries can be active. We rank intermediaries on island m in decreasing size of their network. After relabelling we get $\#M_1 \geq \#M_2 \geq ... \geq \#M_l \geq 1$

According to result 1 for $\#M_i = \#M_{i+1}$ it follows that $P_j(m) = 0$, $\forall j \geq i$. Therefore, without loss of generality the analysis can be reduced to the case of

$\#M_1 > \#M_2 > ... > \#M_l \geq 1$

3) In this situation we can follow Shaked and Sutton (1983, pp.1475-76) to establish existence of a price equilibrium.

Observe that $t_i(m)$ formally are globally linear functions of the prices $p_i(m)$ and $p_{i+1}(m)$. Hence the revenue function $R_i(m)$ is a quadratic function for the range of prices $p_i(m)$ in which $t_{i-1}(m) - t_i(m) \geq 0$.

Now fix all prices $p_j(m)$, $j \neq i$. We plan to demonstrate that $R_i(m)$ is a single peaked function of $p_i(m)$ and hence quasi-concave.

While it is clear that for large $p_i(m)$ the critical buyer $t_i(m) \leq 0$, intermediary i earns zero revenues. By lowering the price he can generate a positive volume of trade and positive revenues, provided

$$p_i(m) < \frac{\#M_{i-1}}{\#M_i} \frac{\#M_i - \#M_{i+1}}{\#M_{i-1} - \#M_{i+1}} p_{i-1}(m) + \frac{\#M_{i+1}}{\#M_i} \frac{\#M_{i-1} - \#M_{i+1}}{\#M_{i-1} - \#M_i} p_{i+1}(m)$$

By lowering $p_i(m)$ further it will ultimately happen that intermediary i

pushes intermediary $i+1$ out of the market. This occurs for $t_i = t_{i+1}$ or equivalently for

$$\bar{p}_i(m) = \frac{\#M_i - \#M_{i+2}}{\#M_{i+1} - \#M_{i+2}} p_{i+1}(m) - \frac{\#M_{i+2}}{\#M_i} \frac{\#M_i - \#M_{i+1}}{\#M_{i+1} - \#M_{i+2}} p_{i+2}(m)$$

At slightly lower prices than $\bar{p}_i(m)$ intermediary i's lower neighbour is $i+1$. As this process of reducing prices continues successively i will drive all his smaller competitors out of business.

We shall concentrate however on the shape of the revenue function $R_i(m)$ at the critical price $\bar{p}_i(m)$. For slightly lower prices it coincides with the function $\bar{R}_i(m)$ defined by

$$\bar{R}_i(m) = \left(t_{i-1}(m) - t_{i+1}(m)\right) P_i(m)$$

Since $t_{i-1}(m)$ and $t_{i+1}(m)$ are linear in $p_i(m)$ the function $\bar{R}_i(m)$ is globally defined and a single peaked quadratic. Of course, only a segment $\bar{R}_i(m)$ will be part of $R_i(m)$ for prices slightly lower than $\bar{p}_i(m)$. We shall establish that the derivative of $\bar{R}_i(m)$ at the point of intersection $\bar{p}_i(m)$ is larger than the derivative of $R_i(m)$. Since this property holds for any further 'kink point' of $R_i(m)$ it implies that $R_i(m)$ is a single peaked function. To show this final claim evaluate the derivatives

$$D_{p_i} R_i = 2t_{i-1} - 2t_i - \frac{\#M_i}{\#M_{i-1} - \#M_i} 2p_i - \frac{\#M_i}{\#M_i - \#M_{i+1}} 2p_i$$

$$D_{p_i} \bar{R}_i = 2t_{i-1} - 2t_{i+1} - \frac{\#M_i}{\#M_{i-1} - \#M_i} 2p_i$$

Given $\bar{p}_i(m)$ is defined by the condition $t_i = t_{i+1}$ the claim follows immediately.

$$D_{p_i}\bar{R}_i \mid_{\bar{p}_i} \quad \geq \quad D_{p_i}R_i \mid_{\bar{p}_i}$$

Now the process of price reductions can be continued until intermediary $i+2$ is driven out of business. At his critical price another kink point with the same property as $\bar{p}_i(m)$ arises. Continuing this procedure $R_i(m)$ is shown to be single peaked.

4) Therefore, the revenue functions $R_i(m)$ are quasi-concave functions of $p_i(m)$. Moreover they are bounded. The strategy set for each intermediary is the convex interval $[0,1]$ and hence a compact set. This allows to apply a standard fixed point argument to demonstrate the existence of equilibrium [11] .

Q.E.D.

[11] see Friedman, 1977, pp.152-154,for example

5. Industrial Structure

Based on their expectations about the ensuing price games intermediaries decide about their network investment. Since it is costly to establish a communication and information network, only investments will be undertaken which allow to recoup the outlays. Given positive fixed costs only a finite number of competitors may be active in a market of finite size. However, the number of competitors may increase as market size increases relative to fixed costs and in the limit in large markets competitive equilibria may emerge. While this intuition is borne out in the Cournot model of imperfect competition it is not true in general in models of price competition with vertically differentiated products [12] . Likewise in our model the number of competitors is limited as the market grows in size. In fact, the equilibrium number of active intermediaries is limited as we shall see.

At a given market size A and given fixed costs k per communication technology and per island only a limited number of intermediaries will enter the industry at the investment stage 0. As long as the market is relatively small multiple industrial structures may be compatible with equilibrium. For example suppose that each island may support only a single intermediary in the sense that the monopolistic rents on a particular island barely exceed the fixed expenses and assume further that only independent non-overlapping networks of monopolistic intermediaries would be profitable. In this situation a global monopoly as well as two independent monopolists, i and j, with networks of similar or identical size, $\#M_i = \#M_j = \frac{M}{2}$, could be equilibrium industrial structures. But also in larger markets which allow several competitors to earn positive revenues in general no unique industrial structure can be expected as long as the markets are not too large relative to costs.

Multiple industrial structures are possible in small markets because the incentives to expand a given network are weakened by the force of price competition. The transaction volume achievable at low margins in a competitive environ-

[12] see for example Shaked, Sutton, 1982

ment on a given island may not compensate for the costs of entry into the island concerned. As the market grows, however, the role of costs is reduced. The larger transaction volume may generate the revenue necessary to render entry into a particular island profitable. The monopolistic structures of the preceding example do not obtain in large markets. A global monopoly will always be challenged by small intermediaries, who need to acquire small market shares only to justify entry. Hence, in large markets the industrial structure is truly oligopolistic.

The next result presents an industrial structure which is the unique equilibrium structure for any large enough A relative to k. We shall view this constellation as the *natural industrial structure.*

In case of a single island, as in chapter two, the natural industrial structure obviously is a monopoly with a single intermediary investing in the communication technology. But also for any number of islands in such a natural industrial structure the largest intermediary maintains his presence on all islands. However, with several islands there is room for profitable entry of further intermediaries. By choosing smaller networks they can differentiate themselves from their larger competitors and thus relax price competition. So they can generate positive revenues which cover their fixed outlays in large enough markets.

In the natural industrial structure of our brokerage market, as we shall see, there are exactly two intermediaries quoting prices on $M-1$ islands if $M \geq 4$. They choose different networks leaving a joint of their networks consisting of $M-2$ islands. For convenience we label those islands $\{2, ..., M-1\}$. On these islands competition among the two intermediaries is quite intense. It drives margins down to zero since both intermediaries offer the same trading probabilities of $\frac{M-1}{M}$. Only on the remaining "niche islands", 1 and M, price competition is relaxed leaving space for further entrants.

Let us now state the result.

Result 3

a) As $k \to 0$ and/or $A \to \infty$, a unique[13] *industrial structure emerges :*

$$
\begin{array}{lcll}
M = 1 & : & M_1 = \{1\} & \\
 & & M_j = \emptyset \quad , & j \geq 2 \\
M = 2 & : & M_1 = \{1,2\} & \\
 & & M_2 = \{1\} & \\
 & & M_3 = \{2\} & \\
 & & M_j = \emptyset \quad , & j \geq 4 \\
M = 3 & : & M_1 = \{1,2,3\} & \\
 & & M_2 = \{1,2\} & \\
 & & M_3 = \{2,3\} & \\
 & & M_4 = \{1\} & \\
 & & M_5 = \{3\} & \\
 & & M_j = \emptyset \quad , & j \geq 6
\end{array}
$$

$$
\begin{array}{lcll}
M \geq 4 & : & M_1 = \{1,...,M\} & \\
 & & M_2 = \{1,...,M-1\} & \\
 & & M_3 = \{2,...,M\} & \\
 & & \cdot \quad \cdot \quad \cdot & \\
 & & M_l = \{1,...,M-l+1,M\} \quad , & 4 \leq l \leq M \\
 & & \cdot \quad \cdot \quad \cdot & \\
 & & M_{M+1} = \{1\} & \\
 & & M_{M+2} = \{M\} & \\
 & & M_j = \emptyset \quad , & j \geq M+3
\end{array}
$$

[13] unique up to labels

b) Equilibria are characterized by

$$M = 1 \quad : \quad P_1 = \frac{1}{2}$$

$$R_1 = \frac{A}{16}$$

$$M = 2 \quad : \quad P_1(m) > P_i(m) \quad , \quad m \in \{1,2\} \quad , \quad i = 2,3$$

$$P_2(1) = P_3(2) > 0$$

$$R_1(1) = R_1(2) > R_2(1) = R_3(2) \geq 0$$

$$M = 3 \quad : \quad P_1(m) > P_i(m) \quad , \quad m \in \{1,2,3\} \quad , \quad i = 2,3,4,5$$

$$P_2(2) = P_3(2) = 0$$

$$P_2(1) = P_3(3) > P_4(1) = P_5(3) > 0$$

$$R_1(1) = R_1(3) > R_2(1) = R_3(3) > R_4 = R_5 > 0$$

$$R_1(2) > R_2(2) = R_3(2) = 0$$

$$M \geq 4 \quad : \quad P_1(m) > P_i(m) \quad , \quad m \in M_1 \cap M_i \quad , \quad i \geq 2$$

$$P_2(m) = P_3(m) = 0 \quad , \quad m \in \{2, ..., M-1\}$$

$$P_2(1) = P_3(M) > P_j(\mu) \quad , \quad \mu \in \{1, M\} \quad , \quad j \geq 4$$

$$P_4(\mu) > P_5(\mu) > ... > P_{M+1}(1) = P_{M+2}(M) > 0$$

$$R_i(m) \geq R_j(m) \quad , \quad m \in M_i \cap M_j \quad , \quad j > i$$

$$R_1(m) > R_2(m) = R_3(m) = 0 \quad , \quad m \in \{2, ..., M-1\}$$

$$R_1 > R_2 = R_3 > R_4 > ... > R_{M+1} = R_{M+2} > 0$$

Furthermore in the case $M \geq 4$ the expected transaction volume generated on islands $\{2, ..., M-1\}$ is positive for intermediaries 1,2 and 3 only. However, all active intermediaries attract positive market shares on the two niche islands 1 and M.

For $m \in \{2, ..., M-1\}$

$$q_i(m) \begin{cases} > 0 & \text{for } i \leq 3 \\ = 0 & \text{for } i \geq 4 \end{cases}$$

for $m \in \{1, M\}$

$$q_i(m) \quad > \quad 0$$

Proof:

1) In the case $M = 1$ no possibility of differentiation exists for intermediaries. Hence, monopoly is the unique and natural industrial structure. The characterization of equilibrium follows immediately from $t_1 = p_1$, $P_1 = 2p_1$ and $p_1 = \text{argmax}_p(\frac{1}{2} - p)2p = \frac{1}{4}$.

2) In the remaining cases as the economy grows the largest intermediary has an incentive to include all islands in his network. Since he is the largest firm in each island according to result 1 he generates the largest revenues. In particular, these revenues are positive and allow to recover any fixed costs as those decline relative to the size of the market. Therefore, a natural industrial structure has exactly one omnipresent intermediary.

3) The only chance for further intermediaries to engage in profitable trading is by differentiating themselves from the largest intermediary and hence setting up

smaller networks. How many islands will the second largest intermediary include in his network? Obviously, he needs to include $M-1$ islands. If he selected less islands a rival could choose the same network and add one island. So the rival would be larger and could enjoy a higher probability of trade. According to result 1 he would earn larger revenues on each island in the subsequent price game. As $\frac{A}{k}$ is sufficiently large such a strategy would be profitable. Therefore, the second largest intermediary has to be active on $M-1$ islands in large markets.

4) Now there is scope for two intermediaries to engage in profitable trade on $M-1$ islands. If they choose the same network, of course, both will earn zero revenue in total. However, they can operate different networks. In this case the networks overlap on $M-2$ islands $\{2,...,M-1\}$. Since there they offer the same probability of trade, in equilibrium they cannot command any positive margin and equilibrium prices are zero. However, on the niche islands 1 and M only one of them is present. Therefore, on the niche islands price competition is imperfect and allows them to earn positive revenues, which allow to recover the fixed expenses for the whole network as k is sufficiently small relative to A.

In order to validate this statement we have to establish that in equilibrium $\hat{p}_2(1) > 0$ and likewise $\hat{p}_2(M) > 0$. According to result 2 we are ascertained the existence of an equilibrium. If the claim were wrong at least for one of the niche islands the equilibrium price of intermediary 2 has to equal zero. So assume $\hat{p}_2(1) = 0$, which according to result 1 implies $\hat{p}_i(1) = 0 \quad , \quad i > 2$. We shall demonstrate that this is inconsistent with equilibrium.

If under the maintained assumption $\hat{t}_1 > 0$ because of the continuity of the function defining the critical agent t_1 with respect to p_2 there is a profitable deviation $\tilde{p}_2(1) = \epsilon$, which yields $\tilde{R}_2(1) > 0$ for small enough ϵ. This establishes the contradiction. On the other side $\hat{t}_1 = 0$, by definition of t_1, implies $\frac{1}{M-(M-1)}(M\hat{p}_1(1) - (M-1)\hat{p}_2(1)) = 0$. Therefore, $\hat{p}_1(1) = 0$ and $\hat{R}_1(1) = 0$. Now the same type of deviation is profitable for intermediary 1, i.e. $\tilde{p}_1(1) = \epsilon$ yields $\tilde{R}_1(1) > 0$ if ϵ is small enough contradicting equilibrium.

So by establishing $\hat{p}_2(1) > 0$ we have demonstrated the profitability of the niche markets for intermediary 2 and 3. Observe that the same argument now can be applied for any further entrant of smaller size. Since none of them will quote zero prices in equilibrium there is always space for another intermediary with smaller trading probabilities.

5) In fact, in a natural industrial structure exactly two intermediaries possess networks of size $M-1$. This follows from the fact that only finitely many intermediaries can earn positive revenues. Any further potential intermediary prefers to remain inactive.

Suppose intermediary 1 operates M_1 and only intermediary 2 chooses a network of size $M-1$. Any further intermediary selects a smaller network with correspondingly smaller trading probabilities. In this case one of the inactive intermediaries could choose network M_3 and gain the same probability of trade as intermediary 2. In this case the "inactive" intermediary would earn positive revenue on island M. When the market is large enough this contradicts equilibrium. Therefore, in large markets exactly two intermediaries will operate networks of the size $M-1$.

6) If $M \geq 4$ the niche islands 1 and M provide space for further intermediaries operating in both niche islands. On these islands they can generate positive revenues by demanding positive equilibrium prices as argued in step 4.

They cannot profitably quote prices on the remaining islands $\{2, ..., M-1\}$. Nevertheless, they can maintain a presence on these islands in order to search for buyers. In fact, they have an incentive to establish a large network for search in order to increase their attraction for sellers on the niche islands and thus boost revenues. Accordingly, they will purchase access to the communication system not only on the niche islands but also on islands where they cannot originate any trade. In order to differentiate themselves from their competitors they have to select networks of different size. Provided all of them offer different trading probabilities, all of them can earn possibly "small" but positive revenues in trade generated on the niche islands.

Because the number of islands limits the possibilities of differentiation, for $M \geq 4$ there are exactly $M-3$ intermediaries generating positive trading volume on the two niche islands only. Finally, in equilibrium there is still space for two further intermediaries of size 1 concentrating exclusively on trade on one of the niche islands.

7) The industrial structures in the cases $2 \leq M \leq 3$ are readily verified as equilibrium structures by direct application of result 1.

8) Finally, in equilibrium only the networks of intermediaries 2 and 3 do overlap, where both intermediaries earn zero revenues on the overlapping islands. Therefore, the characterization of equilibrium follows immediately from result 1.

Q.E.D.

Accordingly, in a natural industrial structure exactly $M+2$ intermediaries are active. However, most of them are rather small generating positive transaction volumes only on the two niche islands 1 and M. Exactly three intermediaries, 1,2 and 3, generate positive transaction volumes on at least $M-1$ islands. They also enjoy the largest market shares. Jointly they share more than $\frac{M-2}{M}$ of the market while the remaining "niche players" attract a market share of less than $\frac{2}{M}$.

In this sense the natural industrial structure is fairly concentrated with three large intermediaries and a competitive fringe of niche players. Also no convergence to a fragmented structure obtains on islands $\{2, ..., M-1\}$. Rather, as M grows the relative importance of the smaller intermediaries diminishes while the degree of differentiation, as measured by the difference in trading probabilities, among the three large intermediaries vanishes. As they become increasingly similar price competition among the "big three" tightens and equilibrium prices converge to Walrasian prices.

Result 4

Equilibrium prices for the industrial structure described in result 3 converge to Walrasian prices as the number of islands increases. I.e. $P_i(m) \to 0$, $m \in M_i$ *and* $R_i \to 0$, $\forall i$ *as* $M \to \infty$.

Proof:

Given the industrial structure of result 3 on each island intermediary 1 has a lower neighbour with trading probability of $\frac{M-1}{M}$. As M increase this probability tends to 1 and competitions tightens. To see this we write intermediary 1's revenue function:

$$R_1(m) = \left(1 - t_1(m)\right) P_1(m)$$

where

$$t_1(m) = \frac{1}{M - (M-1)} \left(MP_1(m) - (M-1)P_i(m)\right) \quad , \quad i \in \{2,3\}$$

The first order condition for profit maximization on island $m \in \{1, ..., M\}$ yields

$$1 - t_1(m) - MP_1(m) = 0$$

Accordingly

$$P_1(m) = \frac{1 - t_1(m)}{M} \to 0 \quad (M \to \infty)$$

Since $P_1(m) > P_i(m)$ for $m \in M_i$ we have established $\lim_{M \to \infty} P_i(m) = 0$.

Q.E.D.

Accordingly, the convergence of prices does not result from increased entry of intermediaries in the niche markets but from increased competition among the big intermediaries.

In summary, the number of active intermediaries in large markets, as measured by A, is bounded. While this bound in general depends on M the number of "large" intermediaries is three for any $M \geq 2$. Equilibrium payoffs decisively depend on the number of islands. As their number increases the degree of differentiation among the large intermediaries is reduced. Price competition strengthens. It is worth noting that albeit the strong incentive to monopolization of the information generating activities, as evident in the case $M = 1$, there is enough scope for effective competition if $M \geq 2$. The possibility of creating niche markets allows competitors to generate the revenue necessary for recovering their fixed expenses. Thus, Bertrand price competition on heterogenous markets yields distinctly different outcomes as compared to price competition on a homogenous market.

The model predicts that larger intermediaries can earn extra margins for the better quality of their products or services. Indeed this seems to correspond to observed behaviour in the West German and Swiss banking market. A common explanation given by bankers of the larger institutions often is the superior quality of their products. Our model validates such an explanation at most for brokerage services. However, care has to be taken since both in Switzerland and in West Germany universal banking is common and banks' pricing policies traditionally are based on mixed accounting (Krümmel, 1964). In such an industry it is difficult to analyze single product lines in isolation. Nevertheless, it is evident that the three largest banks in both countries also dispose of the most wide spread network of international subsidiaries and affiliations. This gives them a competitive edge in the business of trade finance for example.

Interestingly, the competitiveness of our brokerage industry cannot be judged by concentration ratios or by counting the number of competitors alone. In fact these measures may be misleading. As M increases both the concentration ratio CR_3, defined as the market share of the three largest firms relative to the whole market, and the number of active firms rises, while equilibrium spreads converge to their competitive level. Rather the existence and the profitability of niche markets is decisive in determining the degree of competitiveness in the industry.

The empirical question concerning the "number of islands" and the possibility of creating niche markets is likely to be a difficult one. According to our model, for example, where islands essentially are defined by a lack of communication and information links between subgroups of potential traders, the task of counting islands would amount to analyze the micro structure of the communication and information system between the potential market participants.

Finally, we conclude this section with a remark on the convergence process. Obviously, in our model the market can grow in at least two different ways. Either the scale parameter A may increase or the number of islands M or even a combination of the two. In our analysis implicitly we looked at processes where A grows sufficiently fast relative to M. This procedure is justified in markets in which the number of independent and separated islands does not increase rapidly while market volume increases steadily.

International stock markets provide an example of markets in which transaction volumes have been rising significantly over years and where innovations in communication and information technologies have reduced the informational differences across national stock exchanges. In the metaphor of our model we could describe this process of communicational integration of the stock exchanges by a reduction of islands. For this case the model predicts increasing margins once the natural industrial structure is reached. The degree of concentration remains unaffected but as the number of islands is reduced niche markets become more relevant for differentiating the competitors and price competition is relaxed. In this sense advances in communication technologies may reduce the competitiveness of the brokerage markets. In the limit the natural monopoly of the previous chapter obtains.

While this prediction may seem quite strong and practitioners may claim that advances in information technologies have increased the possibilities of creating niche markets, we hasten to add that the market under consideration is a market for a single product or security, such as an ordinary IBM-share certificate. We do not address ourselves to the interesting questions of multiproduct competition and financial innovation.

6. Relations to the Literature

There is an important difference between our model of network competition and Shaked and Sutton's (1982) model of vertical product differentiation. In our model the 'qualities' of the top firms are 'very close' as $M \to \infty$ and equilibrium prices converge to marginal costs. No result of this kind occurs in Shaked, Sutton, though it should be made clear that a precise analogy between the models is not possible, as there is no analog in Shaked, Sutton to our 'number of islands' M.

That said it is still of interest to ask why the present convergence result holds. We start by providing a short account of Shaked and Sutton's basic model (1982). They analyze competition in a market with vertically differentiated products as the subgame perfect Nash equilibrium of a three stage game. In the first stage firms decide about entry. Entry is costly. In the second stage they select a product quality u from an intervall of technologically available qualities $[\underline{u}, \bar{u}]$. Finally at stage three competition in prices takes place.

As important differences Shaked and Sutton use a continuum of qualities and the gameform of a three stage game. If u is to be chosen from a discrete set $\{u_1, ..., u_n\}$ such that $u_1 > ... > u_n$ their results are not seriously impaired. This is readily seen, since their proof of existence of a Nash equilibrium in qualities only requires u to be selected from a compact set and does not depend on the fact that u is drawn from a continuum (1982, pp.9,10).

Now an important difference between Shaked, Sutton and the present model lies in our use of a two stage game. The three stage game allows competitors to react to changes in the number of entrants in their choice of quality. In the two stage game used here a potential entrant cannot take as given the quality of incumbents. This has the effect of forcing the 'second highest' quality higher, since if this quality is 'far below' the top quality, then any such configuration can be 'broken' by the entry of a new firm offering a higher quality. It is this feature which in our model leads the second largest intermediary to select a network of size $M-1$. Hence, we get 'convergence in qualities' as $M \to \infty$.

Moreover, in contrast to Shaked and Sutton (1982), the model of network competition exhibits an industrial structure with very large and very small firms. In general there are no moderately sized intermediaries. This effect is a particularity of the network model, which allows small intermediaries to enter niches left by large intermediaries in their attempt to differentiate themselves from their large rivals. For real world comparisons we should bear in mind the restriction of the analysis to a single product market. Most real world intermediaries offer a variety of products and services.

7. Conclusion

We have analyzed a brokerage market in which intermediaries can affect their trading probabilities by establishing an information and communication network. As communication possibilities across potential traders are imperfect we found that in general several firms will offer intermediary services. Nevertheless, the industrial structure turns out to be fairly concentrated with few large firms competing across most submarkets.

However, we also find that the concentration ratio may not be a good guide for measuring the degree of competitiveness of the industry. Since the few active firms turn out to be of roughly similar size price competition is fairly intense and quite competitive allocations result.

Although we have not explicitly modelled the possibility of private search following the lead of chapter two it can be easily accommodated in the present framework. In this case both the phenomena of chapters two and three will arise. Intermediaries compete against the search market in addition and only those who can increase trading probabilities sufficiently will be able to earn positive margins. In equilibrium high valuation traders prefer trade with large intermediaries, while only the lowest valuation traders may have to resort to private search [14] . Multiple equilibria derive from the inability of market participants to coordinate market participation. The industrial structure in any of these equilibria, however, is at least as concentrated as in the present analysis.

[14] In the equilibrium of result 3 only on the niche islands 1 and M some market participants with valuations $r < p_{M+2}$ prefer direct search.

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