INVENTORY INVESTMENT IN THE UK: EXCESS VOLATILITY, FINANCIAL EFFECTS AND THE COST OF CAPITAL

A thesis to be submitted to the University of London
for the degree of doctor of philosophy

by

ALISTAIR KEITH LOVELL MILNE

London School of Economics and Political Science

ABSTRACT

This thesis consists of four self-standing papers (chapter 2 through chapter 5) together with an introduction (chapter 1) and a conclusion (chapter 6).

Chapter 2 examines the data on UK inventory investment. Excess volatility is a minor feature. The cyclical movements of inventory investment - examined using tabulations and graphical techniques - are much more prominent, apply to all categories of inventories, and data encompass the observed excess volatility. A frequency domain analysis (using a simple but novel technique) confirms this finding. The cyclical movements in the frequency domain correspond to slow speed of adjustment in the time domain. These results suggest that the explanation of excess volatility is a degenerate research programme and should be abandoned in favour of a return to explaining the cyclical movements of inventory investment.

Chapter 3 considers the mis-specification testing of the linear quadratic production smoothing model of inventories previously estimated by Blanchard (1983). Estimation results, under instrumental variable estimation, depend on the normalisation of the estimated first order condition. The model is encompassed by, but does not encompass, the alternative stock-adjustment model of Lovell (1961). The West (1986) variance inequality is shown to be equivalent to the setting of some lower bound on residual variance.

Chapter 4 analyses a dynamic model with bankruptcy, under simplifying exogeneity assumptions about financial contracts. When there are constraints on the availability of both debt and equity, then inventory holdings depend on net assets during periods of financial pressure. This implies a link between inventory investment and profitability for firms under financial pressure. Estimation using a panel of UK company accounts provides striking confirmation of this relationship. Aggregation over the panel indicates that the effects of profits explains a large part of the movements in aggregate UK inventory investment.

Chapter 5 provides a detailed analysis of the determinants of the cost of capital for inventory investment paying particular attention to the effects of UK stock relief legislation. The IFS tax model is used to calculate aggregate and sectoral measures of the cost of capital. The tax position of individual companies does not greatly affect the aggregate cost of capital. Stock relief legislation lowers the aggregate cost of capital, by more in the 4th and 1st quarters than in the 2nd and 3rd quarters of each year.

To the memory of my mother, Anita, who always placed the highest value on education.

Acknowledgements

The first three years of the research reported in this thesis were supported by an ESRC post-graduate training award. The first year was spent with the LSE Centre for Labour Economics and final two years with the LSE Financial Markets Group: I have been priviliged to benefit from the excellent research environment provided by both groups. In the subsequent two years my employers, the Centre for Economic Forecasting at the London Business School, have given me a generous amount of time to complete the writing up of the thesis and allowed me to present much of this material at various conferences.

I am grateful to my supervisor, Mark Schankerman, for his detailed and careful reading of earlier drafts and to comments, ideas and advice from James Davidson, Michael Devereaux, Andrew Harvey, Mervyn King, Mark Robson, Sushil Wadhwani and participants at a number of seminars and conferences. Finally the contribution of my wife Margaret, in putting up with my pre-occupation with this work over five years, is warmly appreciated.

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CHAPTER 1

INTRODUCTION

1. Motivation.

The four central chapters of this thesis, chapter 2 through chapter 5, are each offered as self-standing contributions to the applied econometric literature. All use UK data, but there is considerable variety in both analytical approach and in econometric technique. Their common subject matter is the empirical study of what is commonly referred to in United Kingdom as stock investment and in the United States as inventory investment.

The thesis title combines the US terminology with a reference to the use of UK data. This transatlantic syntax is pointed. It acknowledges the now dominant position of the US in academic research on economics. More specifically it is a reference to what is now a decade long flow of US studies of inventory investment, a branch of the applied econometric literature which has prompted little corresponding work in the UK. This thesis is intended as a contribution to this recent literature using UK data and following what may be regarded as more characteristically British traditions of applied econometrics.

Excess volatility

Although the chapters are written as self-standing papers there are three themes, referred to in the subtitle, which link and motivate the various chapters. Almost all the recent work on inventories has related to inventories of manufactured finished goods, discussing the view that these inventories are held so as to smooth production over time. This has been a voluminous literature yet there has been complete failure to reach a consensus (Blinder and Maccini (1990)). The first theme of this thesis is that this failure is a consequence of a false diagnosis: the interpretation of the mis-specification of the production smoothing model as due to the "excess volatility" of production. A careful examination of the UK data and the application of standard tests suggest instead that this mis-specification arises because production smoothing models fail to capture the cyclical movements of inventory investment. The explanation of excess volatility is a degenerate research programme which should be abandoned in favour of a return to explaining the cyclical movements of inventory investment.

This is both a very simple and, to the present writer, a very obvious point. The reason that it has not been emphasised in other studies is the widespread belief that at least one form of production smoothing model (the linear quadratic model of Holt et al (1960) estimated by Blanchard (1983)) is observationally equivalent to the standard stock-adjustment specification of Lovell (1961) (itself a simple partial adjustment specification). The stock-adjustment model captures the cyclical movements of inventory investment fairly well. Hence it is generally thought that production smoothing models can reproduce the cyclical movements of inventory investment and that its mis-specification is therefore due to the excess volatility of production not the cyclicality of inventory investment. In this thesis it is shown that the basic premise of this argument is flawed: the linear quadratic production smoothing model and the stock adjustment model are *not* observationally equivalent. The problem with production smoothing models are after all their failure to reproduce the cyclical movements of inventory investment.

Structural macro-econometric modelling and the LSE econometrics

The persistence of these mistaken views about the mis-specification of the production smoothing model can be traced to a problem of econometric methodology. The excess volatility literature has been pursued within the tradition of structural macro-econometrics stemming from the new-classical macro-economics of Thomas Sargent and Robert Lucas. The arguments in favour of this approach are compelling: only by estimating the underlying structural parameters of taste and technology is it possible to avoid estimating unstable reduced forms. When expectations are forward looking then reduced form parameters alter with any change in the stochastic processes determining variables exogenous to the model. Only structural estimation can be of help in policy analysis.

This is of course the well known Lucas critique and nothing in this thesis challenges the cogency of this argument. The simple point made here is that a structural estimate will only be stable if it corresponds to the true underlying structure as far as that can be known. If the Lucas critique is taken seriously then testing for possible structural mis-specification is an essential part of econometric modelling. The reason this point needs to be made is that, despite the Lucas critique, existing attempts to carry forward the program of structural econometrics are flawed by a failure to apply any systematic procedure for mis-specification testing. The excess volatility literature, with the confusions it has engendered about the mis-specification of the production smoothing model, illustrates this point.

This shortcoming in the literature is all the more glaring because a systematic procedure for mis-specification testing, consistent with the goal of structural macro-econometric modelling, is already offered by the LSE tradition of econometrics. This proceeds by acknowledging the presence of a specification error, which reflects the inevitable discrepancy between the estimated econometric model and the underlying data generating process. Mis-specification testing may then proceed based upon the properties of this induced specification error, and a comparison with the specification error induced by competing models.

The perspective on the excess volatility literature put forward in this thesis have been developed by a careful application of the LSE framework for mis-specification testing. This is a rewarding approach, offering a quite different diagnosis of the mis-specification of the production smoothing model and pointing out a neglected programme of research, on the cyclicality of inventory investment, which may well provide the key to several apparently irresolvable puzzles of this literature. This success suggests that the LSE framework for mis-specification testing may be fruitfully applied in many other areas of applied macro-econometrics.

Financial effects on inventory investment

This emphasis on the cyclicality of inventory investment leads naturally to the second theme of this thesis. This is that financial effects, arising from informational imperfections in capital markets, can explain much of the cyclical movements in inventory investment. A cursory reading of the literature on business cycles reveals that such financial effects are a standard explanation of cyclical fluctuations in both fixed capital investment and in consumer expenditure. But these ideas have, hitherto, not been applied to the study of inventory investment. The theoretical and empirical analysis presented here suggests that much of cyclical movement of inventory investment can indeed be ascribed to this cause.

The theoretical analysis is a first response to a substantial technical challenge: in order to develop a systematic model of cyclical fluctuations in inventory investment it is necessary to go beyond the simple one-period models which characterise the literature on asymmetric information in capital markets to a fully dynamic, infinite-period model. Such models, even in the simplified framework adopted here, are at the very limits of tractability with currently available techniques.

Here the intention is not to provide a complete theoretical analysis but to develop a model which can guide empirical specification of financial effects on inventory investment. In this respect it is very successful. At the expense of a number of strong exogenising assumptions, intuitively appealing results emerge. In periods of normal operation real decisions are unaffected by financial factors: the firm however holds a cash balance as an insurance against the possibility of the triggering of bankruptcy by poor trading conditions in future periods. If poor trading conditions transpire then the firm reduces inventory investment until such time as it can build cash balances up to desired levels. The presence of financial effects varies from company to company depending on its current financial state.

The resulting empirical specification (in which inventory investment depends on profits for the subset of firms under financial pressure) is estimated using a large panel of individual company accounts. This panel is also used to calculate the effects of the estimated financial effects on aggregate UK inventory investment, indicating that they are sufficiently powerful to explain much of the cyclical movements of inventory investment in the UK in the late 1970s and early 1980s.

The cost of capital

The final theme of this thesis is the analysis of the cost of capital for inventory investment. This material is not as closely linked to the main body of the thesis as are the other chapters, in that it does not relate directly to the cyclicality of inventory investment. The only indirect link is that one possible mechanism to explain the cyclicality of aggregate inventory investment is through movements in the cost of capital. This suggestion must address one of the enduring puzzles of the econometric literature on inventory investment: there is almost no econometric evidence that the cost of capital affects inventory holding decisions. One reason for this may be measurement difficulties, associated with the tax status of individual companies and (in the UK) with the availability of stock relief on inventory investment. This is the motivation for the careful and detailed examination of the cost of capital carried out in the last of the main chapters of the thesis.

For the most part this proves to be a standard application of procedures which are already well known in the literature, which results in some modest extensions of existing studies of this topic. The main result of this exercise are quantitative estimates of the cost of capital, by industrial sub-sector, using a second micro-econometric data set (the Institute for Fiscal Studies tax model). This output may in turn be of value in future studies of the cost of capital on UK inventory investment.

The remainder of this introductory chapter

The remainder of this fairly lengthy introductory chapter provides a detailed review of two broad issues raised in the individual chapters. Section 2 discusses the application of the LSE econometrics to the mis-specification testing of econometric models based on explicitly stated dynamic theories. Section 3 surveys the literatures on both inventory investment and on the modelling of business cycles. Traditionally these two literatures have been very closely linked but in recent years they have followed separate paths. The review of business cycle theories suggests a number of possible mechanisms which might explain the cyclicality of inventory investment. Finally section 4 provides a more detailed overview of the contents of the thesis.

2. The LSE approach to econometrics and the mis-specification testing of dynamic econometric models

The LSE approach to econometric modelling.

A particular methodological perspective provides the foundation for the views on the excess volatility of production put forward in chapter 2 and chapter 3. This perspective is what has been referred to as the LSE tradition of econometric modelling. It is associated in with the names of Dennis Sargan and David Hendry, although features of the LSE approach can be traced back to the philosophy of science of Karl Popper and Imre Lakatos, and in particular to the emphasis of Popper on the falsifiability of all scientific propositions which he viewed as necessary for the scientific process of conjecture followed by refutation (Popper (1963)).

Both chapter 2 and chapter 3 appeal to the LSE tradition to provide a framework for the mis-specification testing of econometric models based on explicit dynamic theory. To provide a background to the reading of these two chapters this section reviews the LSE tradition and explains how its application leads to such contrasting conclusions about the modelling of inventory investment than those reached in the recent US literature.

The standard statement of the LSE approach to econometric modelling is Hendry and Richard (1983). Other less technical accounts are given in Hendry (1983) and Gilbert (1990). This tradition has become an accepted standard for the modelling of economic time series in the UK and Europe but is much less widely adopted in the US. The LSE approach stresses that all econometric models are only approximations to an unknown underlying data generation process, and that assessment of econometric models is hampered by lack of experimental data and short sample periods, and hence by the necessity to make strong marginalising and conditioning assumptions before models can be estimated. Chapter 2 and chapter 3 extend the LSE approach by applying it to the mis-specification testing of dynamic models with explicit theoretical foundations.

A characteristic insight of the LSE approach is that econometric models cannot be assessed only in terms of the theoretical perspective from which they are derived. To consider a model only within its own theoretical framework is to make the misleading assumption that there are no alternative models which might approximate the data generating process better than the model under consideration.

In other words to assume what Leamer (1978) has called the axiom of correct specification and hence adopt an inherently uncritical approach to model development.

The axiom of correct specification can be avoided by both looking for evidence that he model fails to approximate the data generating process (mis-specification tests) and by comparing the estimated models performance with other competing models (encompassing tests). An acceptable model is one which both accounts for the main features of the data - without exhibiting structural change over the estimation period and while achieving a satisfactory post-sample forecasting performance - and which can account for the salient features of competing models. The LSE tradition recommends a range of standard mis-specification tests and encompassing tests to aid in this process. It is these procedures which are applied to the linear quadratic production smoothing model in chapter 3, yielding notably different conclusions from those drawn in the US literature on this model.

The reason that these differences arise can be traced to the distinctive view taken by the LSE tradition on the presence of the error term in econometric estimates. This differs from the interpretation put on the presence of the error term in many US econometric studies. This is not however to argue that the LSE approach should replace the techniques often employed by US econometrician. The view put forward here is that these contrasting approaches to econometrics are complementary rather than competitive, and that the LSE tradition offers additional insight into problems of mis-specification which are difficult to deal with from the rather narrower perspective exemplified by much US applied econometrics.

The LSE view is that the presence of an error term reflects the inevitable failure to model all aspects of the data generation process. The true set of relationships generating the data can never be established for several reasons, amongst which may be emphasised the inability to conduct experiments, paucity of data and ongoing structural and policy change. Thus no econometric model can be regarded as correctly specified and for this reason no econometric model can fit the data exactly. All specifications induce a residual error when fitted to available data.

However it is still possible to assess the adequacy of a proposed specification relative to variants of the specification or to other proposed models. The sample distribution of this induced error term then plays a key role in the assessment of estimated models as adequate approximations to the data generating process. In particular serial correlation of the induced error term, or correlation with lagged conditioning variables, is taken as an indication of mis-specification.

Another method of model assessment suggested by the LSE tradition is the encompassing principal. When applied to estimated econometric models, this is the requirement that a model which is an adequate approximation to the unknown data generating process, should be able to replicate (encompass) the findings of other models as if it were the data generating process itself. Mizon and Richard (1986) provide a formal development of this principle, which unifies a wide range of different testing procedures, including the voluminous literature on non-nested hypothesis testing. The encompassing principle is also the appropriate framework for considering a claim of observational equivalence between two competing models.

A particular benefit of applying the encompassing principle is that the consideration of the claims of all competing models guards against the danger of what Lakatos has referred to as a "degenerate" research programme; that is a research programme which devotes effort to producing theories which fail to explain features of the data. One of the main claims of this thesis is that the explanation of excess volatility has been such a degenerate research programme, and that the explanation of the cyclical movements of inventory investment offers a more progressive direction for future research.

The mis-specification testing of dynamic models

Improvements to the technical tools of econometrics associated with the new classical macro-economics now allow the econometric estimation of dynamic models based on explicit theoretical foundations. It is these techniques which have been applied in the recent literature on inventory investment. As usually applied these techniques assume a different interpretation for the presence of the error term than that espoused by the LSE approach to econometrics.

Hansen and Sargent, in their influential presentation of the techniques of linear econometric estimation under the assumption of rational expectations, argue (Hansen and Sargent (1980) page 9) that there are essentially only two sources of error in econometric models:

> "This paper develops two different models of the error terms in behavioral equations. Both models use versions of the assumption that private agents observe and respond to more data than the

econometrician possesses....Together with variants of 'errors in variables models', these models are about the only plausible models of the error processes that we can imagine."

Thus the techniques of Hansen and Sargent embody a quite different view of the error process, and of econometric modelling, than that set out by Hendry and Richard (1983). Hansen and Sargent view equation error as arising *either* because of measurement errors *or* because of information available to the decision making agent but not to the econometrician. They do not consider the possibility that equation error arises through the unavoidable failure to specify a complete model of the data generating process.

The espousal of the LSE approach in this thesis is not be interpreted as a technical criticism of the approach recommended by Hansen and Sargent. They analyse two variants of models in which the econometrician has access to only a subset of the data available to decision making agent. These two models of the error process are:

(i) the econometrician observes all relevant information except "a univariate random process that is observed by private agents but is not observed by the econometrician";

(ii) the econometrician has access to only a subset of the information set used by the private agents in their decision rules.

These two different models of the error process lead to different techniques of estimation. In case (i), where the unobserved random process is AR(q), and the forcing variables are determined by an VAR process, Hansen and Sargent show how to derive a closed form solution of the model which has ARMA(q,q+1) errors. Full information maximum likelihood estimation may be then conducted by imposing cross equation restrictions between the closed form solution of the model and the stochastic process determining the movement of the forcing variables. Technically this is irreproachable. The only criticism that can be made from the LSE perspective, is that the freedom given to the econometrician, to choose an arbitrary AR(q) process, means that any data generating process can be closely fitted by such procedures. To use Popper's terminology such models are difficult to falsify. This makes it difficult to ever reject such a model as mis-specified. Such estimates therefore provide little information about the appropriate underlying micro-economic theory.

In case (ii) full information maximum likelihood estimation is impracticable (without making arbitrary assumptions about the correlation between the information available to the agent and the stochastic shock on the forcing variable). Instead instrumental variable estimation of the first order condition is appropriate, exploiting the orthogonality of equation errors from the information set available to the econometrician. Where it is assumed that either time aggregation or expectational errors induce an MA process in the residuals of the estimated first order condition, then the GMM estimator of Hansen (1982) is asymptotically efficient and yields consistent standard errors. This technique has been applied in much of the applied econometric literature in the US, but as yet little attention has been paid to mis-specification testing of such models. Mis-specification has only been considered by examining whether parameter estimates are consistent with the underlying theory and whether over-identifying restrictions are satisfied.

The estimation of the linear quadratic production smoothing model of inventories by Blanchard (1983) illustrates the Hansen and Sargent approach to case (i). Blanchard applies full information maximum likelihood estimation to a derived closed form with an ARMA error process. His simulation results indicate that, with the incorporation of this serially correlated error process, the linear quadratic production smoothing model fits the data about as well (ie the underlying white noise errors are of about equal variance) as the conventional stock adjustment specification. But what fits the data well is the estimated model plus the associated ARMA(q,q+1) error process. As already noted, with this technique, it is very difficult to ever reject the estimated model as mis-specified and this certainly cannot be done on the basis of a comparison of residual variance.

In fact actual inventory movements do not do a very good job of minimising the objective function assumed by Blanchard. This is the basis of the extended variance comparison of West (1986). He estimates the same model as Blanchard, using the GMM estimator, and shows that the comparison of the value of the objective function under actual behaviour, with its value under the simplest possible alternative decision rule of never allowing inventories to depart from trend, can be expressed in terms of a simple inequality involving variances and co-variances (chapter 3 re-states the derivation of this inequality). The violation of this variance inequality is therefore an indication of model mis-specification.

This finding begs at least two questions. Can standard tests of mis-specification recommended by the LSE tradition be applied to models with explicitly stated linear-quadratic theory for model dynamics (thus greatly enhancing our ability to recognise model mis-specification)? If so how do such tests relate to the West variance comparison based on success of actual behaviour in optimising the assumed objective function? Chapter 3 considers both these questions, and finds that the encompassing tests, as proposed by Mizon and Richard (1986) and Mizon (1984) provide a general procedure for the mis-specification testing of linear-quadratic models, estimated under the assumption (ii) above that the econometrician holds only a subset of the information available to the agent.

Chapter 3 also finds that, at least in principal, tests of residual auto-correlation can be applied under case (ii), but these suffer from the difficulty that a degree of moving average residual auto-correlation can arise from linear-quadratic models, where the estimated first order condition involves expectations over more than one periods.[1] Finally chapter 3 finds that where standard tests of mis-specification are applied, so that the mis-specification error is both residually uncorrelated and uncorrelated with weakly exogenous data, then the West test can be interpreted as the setting of an upper bound on residual variance. This makes intuitive sense. In linear-quadratic models where the estimated error terms are serially uncorrelated and independent of weakly exogenous variables, but have very high variance, an alternative behaviour rule, that predicted by the estimated equation, does much better than actual behaviour in minimising the assumed objective function.

Two extensions of the LSE approach

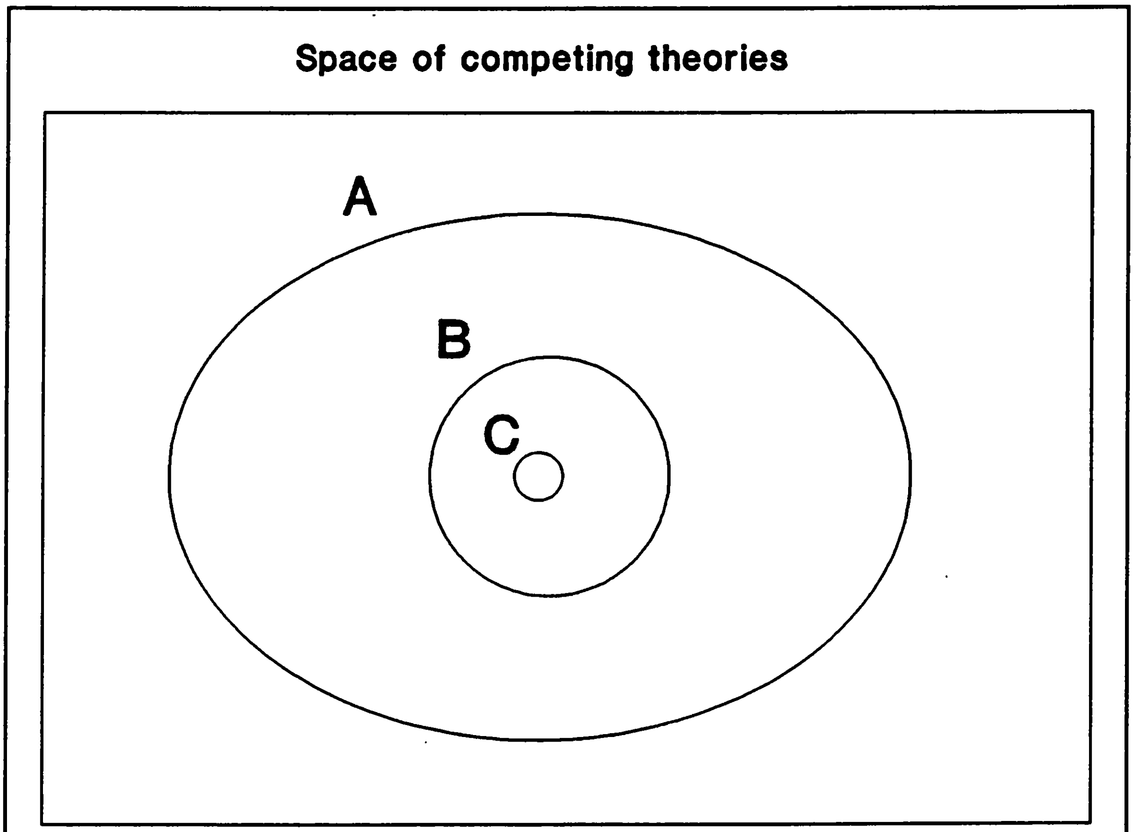
The application made in the present thesis of standard tests of mis-specification to the testing of explicit dynamic theories is an extension of the usual LSE recommendations about dynamic specification. More typical are the views of Hendry, Pagan and Sargan (1986) who start from the proposition (page 1025) "...we consider that as yet economic theory provides relatively little information about lag structures." However they view economic theory as essential for imposing long run relationships. Thus these authors (page 1048-1049) favour, in many contexts, the use of the error correction mechanism which "implements long-run proportionality or homogeneity and ensures that the dynamic equation reproduces in an equilibrium context the associated equilibrium theory." In short, in the standard application of the

LSE approach, theory is used to establish long run homogeneity restrictions and to suggest a set of conditioning variables included in the dynamic specification (and hence implicitly determining the marginalisation of other variables) but is not used to restrict the dynamic specification. The only role of theoretical analysis of dynamics is to provide (Hendry, Pagan and Sargan (1986) section 2.5) "quasi-theoretical bases for dynamic models".

To assess this recommendation it is necessary to be clear about the purpose of the econometric estimation. If is to provide a model for forecasting or policy analysis then the views of Hendry, Pagan and Sargan are clearly quite defensible. If however the purpose of the econometric estimation is to test some theory of the economic dynamics then the approach of Hendry, Pagan and Sargan is no longer appropriate and the model dynamics must be explicitly based on the underlying dynamic theory. This in turn requires that a clear distinction be maintained between the error term resulting from expectations formation and the remaining specification error arising from the inevitable failure of the model to fully describe the data generating process.

A further modest extension of the LSE approach is made in chapter 2. The testing of alternative theories of inventory dynamics depends not only on formal tests of mis-specification but also on informal descriptions of the data generating process (what are sometimes referred to as the "stylised facts"). In the case of inventories there are however at least two competing data descriptions which claim attention. Which of these descriptions should our theoretical models seek to explain? This problem suggests extending the usual LSE methodology by applying the encompassing principle to competing descriptions of the dynamics of inventory investment. The formal analysis of the encompassing principle is not applicable in this context. But the concept of encompassing can still be readily applied as the requirement that any description of the data, if it were correct, should subsume all other available descriptions of the data. Chapter 2 shows that in this sense the "excess volatility" of production is an unsatisfactory description of the process generating observations on inventory investment because it fails to encompass, but is encompassed by the pro-cyclical movements in inventory investment first described by Abramovitz.

The implications of this finding is illustrated in the following figure. Theories generated by the two research programmes (the explanation of excess volatility and of the cyclical movements in inventories) are indicated by A and B. The encompassing relationship between these two descriptions establishes that the set of theories which generate "excess volatility" but not the cyclical movements in inventories is non-empty. Whereas the set of theories which generates the cyclical movements in inventories but not excess volatility is empty. Hence B lies entirely within A. The set of theories consistent with the data generating process (C) can be generated by either research programme but this is done much more efficiently by the pursuit of theories within B, ie by the explanation of cyclical movements in inventories. The "excess volatility" research programme (A) is degenerate because it is dominated by the more progressive research programme (B).



A : generated by "excess volatility" research programme
B : generated by explanation of Abramovitz description
C: consistent with all features of the data

There is one further issue of econometric procedure discussed in this thesis, which emerges from the estimates presented in chapter 3. This is the appropriate choice of dependent and independent variables for the estimation of models based on explicit objective functions, such as the linear quadratic production smoothing model. The difficulty is that the first order conditions, derived from the optimisation of such an objective function, do not yield any insight as to which variable is to be treated as the dependent variable, and which as the independent variables, in the estimation of the first order condition.

This is the normalisation problem, discussed in relation to instrumental variable estimation by Sargan (1958). For any estimation technique, other than full information maximum likelihood, relative parameter estimates are affected in small sample by the chosen normalisation. Alternative estimation results presented in chapter 3 indicate that in the case of the linear quadratic production smoothing model of inventories this makes a considerable difference to estimation results.

Chapter 3 argues that the normalisation of the first order condition, in which the level of inventories is a dependent variable, in to be preferred. This can be argued on two different grounds. The first is that any departure from the data-generating process (the specification error on the estimated first order condition) will be more closely correlated with the level of inventories than with any of the other variables or combinations of variables included in the first order condition. This in turn suggests that superior small sample performance will be obtained by normalising with the level of inventories as the dependent variable. The second grounds for preferring this normalisation is that it is then more convenient to conduct encompassing tests against alternative models of the level of inventories.

3. The literature on business cycles and inventory investment

An overall theme of this thesis, argued specifically in chapter 2 and summarised in the preceding sub-section, is that recent research on inventory investment has been mis-directed towards the explanation of excess volatility. The findings reported in both chapter 2 and chapter 3 suggest instead that the main task of research on inventory investment should be, as it was until the mid 1970s, to explain the pronounced cyclical movements of inventories, providing micro-foundations for the observed cyclical movements in inventory investment and hence for much of observed business cycle movements. This section reviews and compares the separate literatures on the theoretical foundations of business cycles and on the theoretical modelling of inventory investment, suggesting some conclusions of relevance to both literatures and setting the results of chapter 4 and chapter 5 in a broader context.

It is well known, at least since the work of Abramovitz (1950), that cyclical movements in inventory investment account for a major part of the peak to trough and trough to peak movements in expenditure on gross domestic product. Blinder and Holtz-Eakin (1986) report that some 70% of post-war peak to trough movements in expenditure on US GNP is accounted for by falls in inventory investment. Chapter 2 presents a similar calculation using UK data (although these calculations differ from Blinder and Holtz-Eakin in that trend movements in GDP and inventory holdings are removed before the calculation of peak to trough and trough to peak movements, a correction which reduces the share of inventory movements in peak to trough movements in GDP). This suggests that 32% of cyclical fluctuations in expenditure on GDP are accounted for by inventory investment. While there is some, mostly casual, evidence that the cyclical fluctuations in inventory investment are being reduced by new methods of inventory control made possible by information technology, an understanding of cyclical movements in aggregate demand is, to a large extent, an understanding of inventory investment.

This task, the provision of micro-foundations for the understanding of business cycles, has been the research challenge taken up for several years by both equilibrium business cycle and new-keynsian theorists. In the past the study of business cycles and of inventory investment were always closely linked and it is only recently that the two literatures have parted company. A comparative review of the recent developments in the two literatures can therefore clarify both what has already been achieved towards understanding the cyclical movements in inventory investment and what remains to be done. This section provides such a review and suggests some directions for future research, two of which are then taken up in chapters 4 and 5 of the present thesis.

The issue of aggregation is raised at several points in this review of alternative theories. Many micro-economic models of inventory investment, for example the (S,s) inventory model and the analysis of financial effects and of the cost of capital offered in chapter 4 and chapter 5, cannot be applied directly to aggregate data. Does the individual firm behaviour indicated by these models aggregate to the level of the sector or the economy? Aggregation of micro-economic models is always only approximate and usually taken for granted. Nonetheless there are reasons for expecting aggregation to markedly affect the link between inventory investment and output. The discussion offered here suggests both that there is a considerable amount of further research to be conducted on the aggregation of inventory models, and that in many cases, of which the models of chapter 4 and chapter 5 are both examples, only fairly crude numerical solutions are possible.

The analysis of business cycles and inventory movements up to the 1960s.

The provision of micro-foundations for business cycle theories - developing theories of cyclical movements based on a precise statement of the objectives, information and constraints of individual agents - has been adopted as programme of research only in the past twenty years or so. Theories of business cycles and of inventory movements however go back much further and in this earlier literature the link between inventory movements and business cycles is clearly acknowledged.

Awareness of the importance of inventory investment to cyclical fluctuations in output dates back to at least the 1930s. This earlier tradition of business cycle analysis will not be dealt with here but mention should at least be made of the General Theory (Keynes (1936)). In chapter 22, "Notes on the Trade Cycle", Keynes discusses the contribution of inventory investment to fluctuations in aggregate demand. He emphasises the acquisition of surplus inventories at the peak of the cycle, when sales fall short of expectations, and the need for a passage of time to complete the absorbtion of the surplus. He views the cyclical movements in inventory investment as subject to the "uncontrollable and disobedient psychology of the business world" which induces excessively optimistic expectations which increase inventory investment at the peak of the cycle and correspondingly pessimistic expectations which depress investment in inventories at the trough of a slump. This explains the cyclical pattern of inventory investment but begs an explanation of why such expectations should be held by optimising agents.

Formal models of both the business cycle and of fixed capital investment were initiated with the multiplier-accelerator model of business cycles (Samuelson (1939)). Formal models of inventory investment soon followed (Metzler (1941), Nurkse (1952)) which were equally capable of generating cyclical movements in inventory investment and output. However empirical study of the accelerator mechanism suggested that fixed capital investment could not be satisfactorily modelled as a function of the change in output. Early empirical studies of fixed capital investment, such as in the Klein-Goldberger model of the United States economy (Klein and Goldberger (1955)) adopted instead the more general partial adjustment specification, in which the failure to immediately adjust the capital stock to the new current level of output could be loosely justified as reflecting gestation lags in investment and or (Eisner and Strotz (1963)) the costs of introducing new capital equipment.

The pioneering empirical study of inventory investment is that of Abramovitz (1950) who examined the behaviour of inventory investment during the inter-war years, using NBER reference cycle techniques. His main finding, that inventory investment moved pro-cyclically while inventory levels moved cyclically but lagged output by around six quarters, is inconsistent with the simple accelerator models and fits rather better with the less formalised view of inventory investment given by Keynes. Since Abramovitz studies of inventory fluctuations have rarely failed to point out that the contribution of inventory investment to cyclical fluctuations in expenditure is far larger than the long run share of inventory investment in total expenditure.

Early econometric studies found that a satisfactory empirical model was provided by the stock adjustment (partial adjustment) model of Lovell (1961), but that in order to provide a satisfactory fit of the observed fluctuations of inventory investment the stock-adjustment model requires a very slow adjustment towards long-run inventory/output ratios. The stock adjustment specification remains the standard tool for the empirical modelling of inventory investment (this point is confirmed by the review of inventory investment equations in the main macro-economic models of the UK offered by Wallis et al (1987)).

The stock adjustment specification is an empirical success, but raises a number of problems of interpretation. The estimated speeds of adjustment are implausibly slow. In the case of inventory investment it is much more difficult than it is for fixed capital investment to justify a slow adjustment to desired levels. As reported by Blinder (1981), estimation of stock adjustment model on quarterly data typically reveals an adjustment towards target inventory levels of 10 per cent per quarter or less. This implies that in each year firms correct little more than one third of the deviation of inventories from their desired levels.

There is a second fundamental criticism of the stock adjustment model, cogently put by Feldstein and Auerbach (1976). This is that empirical estimation yields coefficient estimates on sales surprises that are inconsistent with the slow adjustment towards target inventory levels. Inventory levels are restored very rapidly following a sales surprise, but very slowly following an change in the anticipated level of sales.

To explain this result Feldstein and Auerbach propose a variant on the stock-adjustment model, which they refer to as the "target adjustment" model, in which it is not the level of inventories which adjusts slowly to the target, but instead it is the target itself which adjusts slowly to changes in expected sales. They cite a number of factors which might lead to slow adjustment of inventory targets: (i) the practice of setting target inventory levels only infrequently; (ii) substantial fixed costs associated with ware-house space which thus changes only slowly; and (iii) small costs associated with holding excess inventory compared with the substantial costs of allowing "stock-outs". Nevertheless an obvious lacuna remains in the literature: there is no formal model which justifies the apparent slow movement of target inventory levels.

Modern theories of inventories.

It is only in past two decades that economic theory has set itself the task of providing the theoretical foundations for an understanding of both inventory holdings and business cycles based on an explicit statement of the objectives, information and constraints facing individual economic agents. The two literatures have followed rather different paths over this period. This review therefore begins by describing the recent literature on inventory investment and then relates these to some well known analyses of the business cycle. These in turn suggest some to possible directions of future research on inventory investment.

It is puzzling that recent research has not stressed the links between inventory movements and business cycles. The recent inventory investment literature has instead been pre-occupied with the production smoothing model of manufacturers finished goods inventories and the explanation of excess volatility, whereas the most important cyclical movements in inventories are for raw materials in manufacturing and distributors inventories (see Blinder (1981) and Chapter 2).

Some attention was in fact paid to the implications for business cycle movements in the early analysis of the production smoothing model offered by Blinder and Fischer (1981). The appeal of this model is that it does embody clearly stated micro-economic foundations for aggregate behaviour. Blinder and Fischer demonstrate that the production smoothing model generates persistent output disturbances in response to transient shocks to demand, thus offering one solution to the problem then troubling the new classical macro-economics as to how temporary disturbances might result in serially correlated deviations in output from trend. However this does not amount to an explanation of the cyclical movements in inventory investment.

The empirical study of the production smoothing model, in the more general linear quadratic formulation of Holt et al (1960) and Blanchard (1983) has been discussed earlier in this introduction and is the subject of chapter 3 of this thesis. This model generalises a linear quadratic version of the model of Blinder and Fischer to allow for quadratic costs of changing the level of production and quadratic costs of departure from a target ratio of inventories to anticipated sales. The estimation and simulation results of Blanchard were initially accepted as very successful. The variance inequality test of West (1986) then showed that the linear quadratic production smoothing model was after all mis-specified but this result provided no guide to an alternative model. The response in the literature has been to produce a series of models (of which the most prominent are Kahn (1987), Ramey (1991), Blinder (1986) and Caplin (1985)) directed at explaining the excess volatility of production, rather than the cyclical movements in inventories.

There is also a widespread perception (Blinder (1981), Fair (1990), Blinder and Maccini (1991)) that the stock-adjustment specification of Lovell (1961) is observationally equivalent to the production smoothing model. The popularity of the stock adjustment model derives from its success in capturing the cyclical variability of inventory investment. This appears to be the reason why the debate over excess volatility has pushed aside the concern of earlier researchers with studying the cyclical movements of inventory investment. There is a general belief (nowhere in the literature is it stated explicitly) that the production smoothing model is mis-specified for reasons unrelated to the cyclicality of investment in inventories of finished goods and hence that this latter phenomena is no longer worthy of study.

Chapters 2 and 3 of this thesis are a direct challenge to this viewpoint. The results of the encompassing tests offered in chapter 3 show that the linear quadratic production-smoothing model is indeed mis-specified but that it is observationally quite distinct from the stock adjustment model. Chapter 2 indicates that the stock adjustment model, with its characteristically slow speeds of adjustment of empirically estimated inventory equations, can generate the observed cyclical movements in inventories. The production smoothing model, in contrast, is dynamically mis-specified and appears incapable of explaining the cyclical movements of inventory investment.

The weakness of the stock adjustment specification is that it is not based on an explicitly stated theory of dynamic optimisation. Eichenbaum (1984) does provide more appropriate micro-economic foundations for the stock adjustment model. These are based not on quadratic costs of production but by incorporating quadratic costs associated with changing the level of inventories. This implies that there is a desired level of inventories which depends on the expected level of sales in all future periods. If in addition sales are AR(1) then the standard stock-adjustment specification emerges. Taken together with the findings of Feldstein and Auerbach which favour their target adjustment model of inventories, this analysis suggests that the appropriate micro-economic foundations might involve substantial costs associated with adjusting the target level of inventories, rather than with the level of inventories themselves; but this only removes the difficulties associated with the target adjustment specification one degree, because we lack a formal underpinning for the presence of such substantial costs of adjusting target levels of inventories.

Non-convex costs of production (Ramey (1991)) offer a way of salvaging the interpretation of inventory holdings as a means of re-allocating production over time and of generating pro-cyclicality of investment in inventories of finished goods. In this case manufacturers optimise by concentrating (batching) production in a few periods, rather than smoothing output over time. There are however both theoretical and empirical difficulties with this analysis: such non-convexities imply a very strong tendency towards the concentration of production which is not evident in the comparison of output and sales made in chapter 2. Moreover this explanation applies only to the least volatile category of inventories, manufacturers finished goods.

A second development has been to consider models which take more explicit account of stock-outs of lines of inventories, an approach which does apply to all categories of inventories. Kahn (1987) analyses the implications of imposing a non-negativity constraint on inventories (at the level of the individual firm) and a penalty cost of stock-out for the relative variance of production and sales. He shows that the variance of production can exceed that of sales if there are linear costs of production and either (i) demand exhibits positive serial correlation or (ii) there is "backlogging" of demand ie demand which is not satisfied in the current period can be met next period. In effect backlogging in this model is an alternative way of generating auto-correlation of effective demand because a shock to demand in the current period spills over into the subsequent period via the sales backlog. A difficulty with the Kahn analysis is that, while it demonstrates that stock-out avoidance can generate excess volatility, it does not show that stock-out avoidance can generate the cyclical movements of inventories. Thus the empirical success of the model remains unestablished.

One difficulty with analysing the empirical behaviour of the stock-out model is aggregation. Given that firms hold inventories so as to reduce the costs of stock-out but that stock-outs still sometimes occur, then what are the implications for aggregate inventory holdings? In this case there it can be plausibly argued that stock-out themselves are rare events and that, at both the level of the individual line of inventory, and in aggregate, inventories are a simple function of anticipated sales or output. This line of argument suggests that the stock-out avoidance model can be viewed as the provision of micro-economic foundations to the accelerator model of Metzler. Indeed the inclusion of target levels of inventories in aggregate model (by Blanchard (1983) and in earlier studies) is conventionally justified by the need to avoid stock-outs. The stock-out avoidance model then suffers from the same difficulties as the original accelerator model: it generates cyclical movements but it fails to re-produce the observed cyclical movements in inventories.

One other model has played a prominent role in the recent literature on inventory investment. This is the (S,s) model of inventory holdings, discussed by

Blinder (1981), Caplin (1985) and Blinder and Maccini (1990). Unlike the remainder of the recent literature on inventory investment, this literature has made explicit reference to the cyclical movements of inventory investment, and in particular to the behaviour of raw material inventories and of inventories held by the distribution sector. Because of these explicit links to the business cycle literature it is more convenient to postpone detailed discussion of the (S,s) model until the following sub-section. However it should be noted that no consensus has yet been reached on the ability of the (S,s) model to explain the prominent characteristics of inventory investment.

To conclude, this review of the recent literature on inventory investment indicates that there are a wide variety of models of inventory investment proposed in the recent literature. None of these as yet offers a convincing explanation of the cyclical movements in inventory investment, or of the slow speeds of adjustment in estimated stock adjustment models.

Business cycle theory and the direction of future research on inventory investment.

Three strands of the modern business cycle literature seem to offer lessons for the study of inventory investment. Real equilibrium business cycle models, of which the interpretation of cyclical fluctuations in employment as reflecting the inter-temporal substitution of labour supply offered by Lucas and Rapping (1969) is an early example, attempt to explain cyclical movements in real variables as the response of optimising agents to stochastic disturbances to productivity.

This view has its parallel in the inventories literature: Blinder (1986) suggests costs shocks as an explanation of why the variance of production exceeds the variance of sales, but as he himself admits this is an unsatisfactory explanation unless the there is some observed proxy to the observed shocks. Studies which incorporate measurable proxies for cost shocks, such as real wages or raw material prices (Maccini and Rossana (1984), Blinder (1986), Miron and Zeldes (1988)) are not successful in explaining the variability of output. Only the unobserved serially correlated cost-shocks assumed in the estimates of Blanchard (1983) and Eichenbaum (1989) offer a means of explaining the data, but as made clear above in the discussion of the LSE approach to econometrics such models are inherently un-testable and cannot therefore be regarded as a satisfactory theoretical explanations of the cyclical movements in inventory investment.

A related equilibrium business cycle analysis of inventory investment is that of Christiano (1988). He analyses inventories as flexible factors of production in an equilibrium business cycle model of consumption and investment. The distinctive role played by inventories is that, unlike fixed capital or employment, they may be altered immediately when new information becomes available. Shocks to productivity and tastes generate fluctuations in output (because consumption is smoothed relative to output) and hence require fluctuations in the stocks of fixed capital, employment and inventories but the fluctuations in inventories are the largest because of their flexibility. Christiano also assumes that there is a signal extraction problem in forecasting (serially correlated) taste and productivity shocks; this increases the relative fluctuation of inventories even more. Christiano estimates this model using US data obtaining parameter estimates which at least partly support the model.

There is however a crucial problem with the Christiano analysis: at no point does he model the decision to hold inventories themselves as an optimising decision, inventories are simply the residual which emerges from the optimising decisions of firms, determining production, capital stock and employment and households, choosing consumption and labour supply. This residual role is perhaps acceptable in a buffer stock model of inventories of finished goods but is unconvincing for the quantitatively more important fluctuations in distributors inventories, raw materials and work in progress.

The approach could be salvaged by modelling the fluctuations in inventory investment as reflecting some optimising decision in which there is a target holding of inventories to sales or output (which could be zero), combined with costs of inventory holdings deviating from target and capital gains on inventories. The desire of consumers to smooth consumption in the face of supply shocks which disturb production, could then lead to cyclical investment in inventories held in the distribution sector through movements in the anticipated rate of capital gain on inventories. The cost of capital, allowing for any speculative capital gains earned by buying low and selling high, is the interest rate less the anticipated capital gain on inventories. The desire by consumers to smooth consumption, by saying consuming less today and more tomorrow, is then signalled by the price system through lower prices today and higher anticipated prices tomorrow. This provides an incentive, through movements in the cost of capital, for investment in inventories which makes production and consumption plans mutually consistent.

This is still problematic because empirical evidence for cost of capital effects on inventory investment is almost entirely lacking. Few studies of inventory investment have found significant interest elasticities of inventory demand. Michael Lovell, who pioneered the estimation of partial adjustment models of inventory demand in the 1960s, was driven to conclude from his considerable experience of estimating models of inventory investment (Lovell (1976) page 400), that:-

> "...the probability of obtaining an interest rate coefficient with a negative sign is 50 percent"

Chapter 5 of this thesis is concerned with this vexing issue. Time series data for the UK in the 1970s and 1980s are a particularly valuable data source for further work on this topic, because of the major changes in tax legislation which introduced, amended and then abolished tax relief (the so called "stock relief" schemes) on inventory holdings. Chapter 5 provides as complete an analysis as is possible of the effect the stock relief schemes had on the cost of capital for inventory investment in the UK.

A key problem is once again aggregation, because the effects of the cost of capital are so heterogenous amongst firms. Chapter 5 deals with this problem by using the Institute for Fiscal Studies tax model to derive sectoral and aggregate measures of the cost of capital adjusted for stock relief. This reveals that the largest shifts in the cost of capital were associated with an incentive to build up inventory holdings at the end of the company accounting year (this temporary build-up is referred to in this chapter as "window dressing") in order to take full advantage of the available stock relief.

A particularly influential real equilibrium business cycle model is that of Kydland and Prescott (1982). Their calibrated model shows that the time taken to build fixed capital equipment, when combined with what is otherwise a classical model of optimising agents and flexible prices, is capable of explaining many of the stylised facts about the cyclical movements in both output and investment. There is no parallel with the inventory investment literature presumably for the reason that, even when allowing for ordering lags, technological delays built into investment in inventories are never long enough to generate cyclical movements of this kind. It is notable that the one prominent stylised fact about cyclical fluctuations not explained by the Kydland and Prescott model are the cyclical fluctuations in inventory investment, their model instead predicting inventory movements that smooth production (because production costs are convex).

One way of rectifying this difficulty may be by an appeal to the (S,s) models of inventory investment in which fixed costs of ordering lead to the triggering of an inventory investment (bringing inventories up to a maximum S level) when inventory levels fall to a minimum (s) level. Cooper and Haltiwanger (1988) reconcile a calibrated real equilibrium business cycle model similar to that of Kydland and Prescott with the observed cyclicality of inventory investment, but this results depends on (S,s) investment by a single economy wide retailing firm. With this assumption a substantial covariance between output and inventory investment is driven by the triggering of inventory investment by the single retailing firm.

It is unclear where this result would survive in a world of many retailers because the aggregation difficulties with the (S,s) model in the context of cyclical fluctuations in aggregate sales, remain unsolved. Results currently available are conflicting. Blinder (1981) and Lovell (1988) both present simulation models with several firms following (S,s) in which production is more variable than final demand. The variance is more marked in the case of Lovell because the (S,s) triggers themselves are altered in the face of shocks to aggregate demand. On the other hand an analysis of the Markov process determining inventory investment in an (S,s) economy by Caplin (1985) shows that in steady state aggregate inventories are proportional to aggregate sales (the accelerator relationship once again) and, intuitively, it seems unclear why (S,s) rules should result in the slow speeds of adjustment to steady state revealed by aggregate empirical equations. In any case none of these studies clearly address the question of the cyclical behaviour of inventories in an aggregated (S,s) world.

A second strand of the business cycle literature which may offer some prospect of explaining the cyclical movements in inventory investment are the models of financial effects on real corporate decisions resulting from informational imperfections in capital markets. This has been a burgeoning literature, following the seminal contributions of Akerlof (1970), Jaffee and Russell (1976) and Stiglitz and Weiss (1981). This literature is reviewed in chapter 4, which then develops a model of financial effects on inventory investment.

Technically this proves rather difficult, since a model of inventory investment must necessarily be a dynamic model. With a number of simplifying assumptions (notably the exogenising of the financial contract and the linearisation of the link between net assets and inventory holdings) a link between current profits and inventory investment emerges for those firms under a degree of financial pressure. Empirical estimation with a large panel data set of UK companies, using a variety of indicators of financial pressure, reveals that there is a powerful effect of profits restricted to the appropriate subset of companies.

A key problem here is again the aggregation of a micro-economic model in which the behaviour of individual firms is so heterogenous. Where the heterogeneity is at the level of the individual company, rather than as in the (S,s) model at the level of the individual line of inventory, then a tractable, numerical solution to the aggregation problem, pursued in chapter 4, it the use of panel data to estimate financial effects at the level of the individual company. Subsequent aggregation across the entire panel suggests that financial effects are sufficiently powerful to explain much of movement in aggregate UK inventory holdings.

A final strand of the business cycle literature, which could help to explain the cyclical movement in inventories but as yet appears not to have been applied to models of inventory investment, are the recent development of a variety of models of market imperfections collectively referred to (Mankiw (1990)) as new keynsian theories. These include analyses of nominal price rigidities arising from the costs associated with changing the prices set by monopolistic firms (Akerlof and Yellen (1985)) and of real wage rigidities arising from efficiency wage models (Stiglitz (1986)). A common feature of these theories is an appeal to second order effects which allow the setting of prices or employment to depart, slightly, from the first order equilibrium.

One argument which supports the idea that small second-order effects could produce large fluctuations in aggregate inventory investment is the finding reported by Feldstein and Auerbach and in chapter 2 below, that fluctuations in inventory holdings over the course of the cycle represent only a few days of individual firm output, but represent a considerably greater proportion of GDP. This is a consequence of the many stages of processing of finished goods, during which they pass through the hands of several different companies. A small fluctuation in terms of gross sales represents, according to the calculations reported in chapter 2, around three times as great a fluctuation in terms of gross output. Thus only a relatively small perturbation of inventory holdings around their equilibrium values requires explanation.

What this literature does not provide is an analysis of second-order costs which produce such perturbation in inventory holdings. Feldstein and Auerbach (1976) suggest a number of factors, described above, which result in costs of altering the target level of inventories. These costs, even if small, could result in target inventory levels being altered only slowly over time, inducing the observed pro-cyclicality of inventory investment. One possible approach might be to allow for informational discrepancies between warehouse managers and the central management of the firm. In such a situation warehouse managers may face a signal extraction problem, of the kind studied by Muth (1960), for which the optimal forecast of expected sales is a exponentially distributed lag on past sales. If the sales process, at the level of the warehouse, is very noisy, then target inventories will indeed adjust very slowly to changes in the underlying sales process, generating the cyclical movements in aggregate inventory investment. This appealing avenue of research is not addressed in this thesis but left for further study.

4. Overview of the thesis

As described in the previous section, the greater part of the US literature on inventory investment over the past decade has consisted of demonstrations, counter-demonstrations and explanations for the presence of "excess volatility" of production, the observation that the variance of manufacturing production exceeds the variance of manufacturing sales. Chapter 2 and chapter 3 offer a critique of this excess volatility literature, written from the perspective of the LSE tradition of econometrics. Chapter 2, using largely descriptive techniques, suggests that the explanation of excess volatility has been, to use the terms of Lakatos (1963), a "degenerate" research programme. It argues that the failure of the literature to reach a consensus view about the determinants of investment in inventories of finished goods in manufacturing is because of a mistaken emphasis on the explanation of excess volatility and a failure to consider other features of the data.

The first section of chapter 2 provides a careful summary of the main features of post-war inventory investment in the UK. This shows that "excess volatility" is a very minor feature of the data and that inventory investment alters the level of production relative to sales only very slightly. The more prominent feature of the data is the cyclicality of inventory investment, a feature which has been well known at least since the work of Abramovitz (1950).

A variance decomposition is used to show that these cyclical movements encompass excess volatility in that any theory which explains the cyclical movements can generate excess volatility. The converse proposition, that any theory which explains excess volatility can explain the cyclical movements, does not apply. The key implication of this encompassing result is that a "progressive" research programme, one which could ultimately arrive on a consensus about the determinants of inventory investment, is only possible if inventory models are assessed by their ability to explain all aspects of the data generating process. Research should return to the task of understanding the well known pro-cyclical movements in inventory investment, an understanding which must in any case encompass any explanation of excess volatility.

A representation of the cross spectra between inventory usage (output and sales) and inventory holdings is obtained by transferring data-based time domain estimates into the frequency domain, using a simple but novel technique. The resulting cross-spectra confirm the Abramovitz observation for all categories of manufacturing inventories.

A second puzzle about the econometrics of inventory investment is that data based estimation reveals extra-ordinarily slow adjustment to long-run equilibrium. The technique for transferring time domain relationships into the frequency domain also suggests a correspondence between these slow speeds of adjustment and the cyclical movements of inventory investment. It is only by incorporating a very slow speed of adjustment parameter that the simple stock-adjustment specification can generate the observed relationships between inventory usage and inventories at business cycle frequencies. This suggests that a successful theoretical model of the cyclical dynamics of inventory investment promises to provide a solution to this further problem in the econometrics of inventory investment.

Chapter 3 provides a critical assessment of the linear quadratic production smoothing model of inventory investment, estimated in the influential paper of Blanchard (1983) (see section 2 of this introduction). It is a generalisation of the simple production smoothing model to a model in which firms attempt to both smooth both the level and changes in the level of production and minimise deviations from a target inventory to sales ratio. West (1986) demonstrates that, at least with US data, the linear quadratic production smoothing model violates a simple variance inequality implied by the underlying theory. This finding has generally been interpreted as a demonstration of excess volatility and in this way has inspired much of the subsequent literature on finished goods inventory investment. Following Blanchard the linear quadratic production smoothing model has usually been assumed to be observationally equivalent to the stock adjustment model of Lovell.

The main finding of chapter 3 is that the two models are not in fact observationally equivalent. The stock adjustment model encompasses but is not encompassed by the linear quadratic production smoothing model when the two models are estimated by instrumental variable techniques. Hence the mis-specification of the linear quadratic production smoothing model can be attributed to a failure to capture the cyclical dynamics of investment in inventories of finished goods. Thus the detailed assessment of these competing models made in chapter 3 yields a very similar conclusion to that drawn in chapter 2 : that the major puzzle in the theory of inventory holdings is a failure to explain the cyclical movements of inventory investment.

In establishing this finding chapter 3 discusses both the estimation and mis-specification testing of models with explicit theory based dynamics. It finds that encompassing tests are an appropriate basis for mis-specification testing. Indeed in the absence of further identifying restrictions on specification and expectational errors encompassing tests appear to be the only available formal tests of model mis-specification. They can be applied to the comparison of competing models by developing an estimation framework which covers both specifications. This is in turn most easily accomplished by adopting the normalisations of the estimated euler equation derived from the underlying dynamic theory which yields a common dependent variable in the two competing models. The application of the encompassing tests also requires that the euler equation be estimated using instrumental variable techniques under the assumption that expectational errors are orthogonal to the information set available to the econometrician. Encompassing tests are not possible with the alternative approach to the estimation of dynamic models with explicit theoretical foundations where the error term reflects a serially correlated stochastic shock to the agents objective function unobserved by the econometrician.

These results explain why Blanchard is able to find observational equivalence between the two main models of inventory investment while chapter 3 finds that the stock adjustment model encompasses but is not encompassed by the linear quadratic production smoothing model. Blanchard assumes an unobserved AR stochastic disturbance in his estimate of the linear quadratic production smoothing model, which allows the model to fit well to his data on the US automobile industry, but also rules out any meaningful comparison with competing models on the grounds of goodness of fit.

Chapter 3 also discusses a often un-remarked difficulty with the estimation of euler equations using instrumental variable techniques. In small sample relative parameter values depend on the normalisation of the dependent variable. Intuitive arguments are offered, consistent with the LSE approach to econometrics, for favouring one normalisation amongst all others.

The final contribution of chapter 3 is an assessment of the West variance inequality test when compared to the standard tests of model mis-specification recommended by the LSE approach to econometrics. Assuming that the null-hypotheses tested by the other standard tests are accepted then the West test can be interpreted as the setting of an upper bound on the variance of the specification error in the estimated euler equation, consistent with the underlying dynamic theory.

There has been considerable research since the mid-1970s on informational imperfections in capital markets, exploring the mechanisms by which these can generate financial effects on fixed capital investment. The second topic of this thesis pursued in chapter 4 is an extension of this analysis to financial effects on inventory investment. There is a direct link to the findings of the earlier chapters, in that such financial effects are a potential explanation of the pro-cyclical movements in inventory investment. Chapter 4 analyses a dynamic model of these financial effects appropriate to the modelling of inventory investment. This analysis suggests a link between profits and investment for a subset of firms under financial pressure.

Chapter 4 then reports estimates of the relationship between company turnover, profits and inventory investment for a panel of individual UK company accounts. The results are consistent with the analytical model, a significant correlation emerging between inventory investment and current profits but restricted to the sub-set of firms identified as under financial pressure using a number of different indicators. Aggregation across the panel indicates that these financial effects operating on a subset of firms are sufficiently powerful to explain a considerable part of the aggregate movements in UK inventory investment in the late 1970s and early 1980s. The main caveat about these findings is the 13 per cent fall in aggregate panel inventory holdings in 1980/81, a figure which is considerably larger than the corresponding movement in the national accounts measure of inventory holdings and suggests considerable measurement error at least for the aggregate measure of inventories in the panel, perhaps associated with the deflation procedures adopted in the construction of the panel data set.

The final topic of this thesis addressed in chapter 5 is the analysis of the cost of capital for inventory investment. This chapter provides a detailed discussion of the determinants of the cost of capital and how this has been affected by UK tax legislation. The techniques of King and Fullerton (1984) are applied to the case of inventory investment. Allowance is made for stock relief, tax exhaustion, the transition between the two different schemes of stock relief, and the possiblity of corner solutions for the individual company.

Chapter 2 then uses the Institute for Fiscal Studies tax model, itself based on a panel of 390 individual company accounts, to assess the impact of stock relief, in force from 1974 to 1984, on the aggregate cost of capital for inventory investment and to generate quarterly series for the cost of capital for manufacturing sub-sectors and for distribution. The main advantage of using a disaggregated data panel of this kind is that it allows a quantititave assesment to be made of the affect of the tax position of individual companies on the aggregate costs of capital. It also allows a quantification of the effects of seasonality which arise because of the uneven distribution of the end of company accounting years.

CHAPTER 2

EXCESS VOLATILITY AND THE CYCLICAL FLUCTUATIONS OF INVENTORY INVESTMENT

1. Introduction

The explanation of excess production volatility (the observation that the variance of production exceeds the variance of sales for a number of manufacturing sectors) has prompted a substantial US literature on finished goods inventory investment (see Blinder and Maccini (1990) for a review). But this literature has reached no consensus on the sources of excess volatility, which has been variously attributed to cost shocks (Blinder (1986)), stock-out avoidance (Kahn (1987)) and non-convex costs of production (Ramey (1991)). Moreover the very presence of excess production volatility has been questioned (Miron and Zeldes (1989), Fair (1989)). This literature seems far from a satisfactory resolution.

This paper addresses the following issue. Is excess volatility of production, whether or not it is an accurate characterisation of the data, really the key feature of the data that inventory models should explain? There are after all several other ways of describing the movements in inventory investment; these include for example the pro-cyclical movements in inventory investment (recently documented by Blinder and Holtz-Eakin (1986) but well known at least since Abramovitz (1950)) and the characteristically slow dynamics which emerge from unrestricted dynamic estimation of the relationship between sales and inventory holdings (on which see Feldstein and Auerbach (1976) and Blinder (1981)). This paper, using UK data sources, discusses the relationship between these alternative descriptions of the data.

The main finding is that the Abramovitz description of the cyclical movements in inventories, but not excess volatility, is an encompassing data description. In other words the Abramovitz description captures all the features of the data but excess volatility is only a partial description.

This bulk of this paper uses simple descriptive techniques such as graphs and tabulations. A more formal description of the data, using a simple procedure for transforming time-domain estimates into a frequency domain representation of the cross-spectra between inventory usage and inventories, confirms the Abramovitz description. A similar frequency domain transformation suggests that the implausibly slow speeds of adjustment which emerge from data based estimation of inventory equations, also results from the cyclical behaviour of inventories.

The finding that excess volatility is an incomplete data description indicate that the considerable research effort to explain the excess volatility of production has been mis-applied. In the terminology of Lakatos this has been a "degenerate" research programme, which cannot produce adequate models of the data generating process. Instead attention should return to models which explain the cyclical movements of inventory investment. These can also explain excess volatility and the slow speeds of adjustment of estimated inventory equations.

Data descriptions and data encompassing

The relationship between different descriptions of the data can be considered by means of the encompassing principle (Mizon (1984), Mizon and Richard (1986)). It states that an acceptable econometric model (that is one which provides an adequate approximation to the unknown data generating process) will reproduce the findings of other models as if it were the data generating process itself. The value of this criterion of model selection is that encourages (in the terminology of Lakatos) a "progressive" modelling strategy, which aims to capture all features of the data generating process. Moreover the encompassing principle provides a formal unification of the theory of non-nested testing (Mizon and Richard (1986)). This standard application of the encompassing principle is referred to here as "model encompassing".

This paper extends the encompassing principle by applying it not to competing econometric models but to descriptions of the observed data (or stylised facts) which are used to motivate the development of specific models. A data description may be said to encompass a second data description if, whenever the first correction applies the second applies also. This application of the encompassing principle is distinguished by referring to it here as "data encompassing".

The econometric model builder should be concerned with data encompassing because of the role played by formal or informal data descriptions in motivating model specifications. This role can be problematic when, as is the case with inventories of finished goods, there are competing descriptions of the data. To avoid wasting research effort it is advisable to entertain data descriptions which data encompass all others. This reduces the risk of spending great effort on developing theories and models which fail to approximate the actual data.

In the case of model encompassing the Wald principle yields formal encompassing tests of the encompassing of the complete parameter vector (or of elements of that parameter vector) of a competing model (Mizon and Richard (1986)). Data encompassing is however an attribute of a data description, not of an estimated model so a formal test of data encompassing is not available. It can however be examined by assuming that a particular description of the data is correct and examining whether alternative data descriptions must then also apply.

Arrangement of the paper

Different descriptions of the UK inventories data are considered in section 2. Excess volatility of production in manufacturing is observed for the UK, but the variance of production is only slightly greater than the variance of sales. The most marked feature of inventory movements in the UK is the marked cyclical movements in inventory investment. This applies to all categories of inventory investment, not just to finished goods in manufacturing.

Section 3 proposes one particular version of the stylised facts as a data encompassing description. This is the concise description of the cyclical movements in inventory holdings first established on inter-war US data using NBER reference cycle techniques by Abramovitz (1950). This description is found to apply to post-war UK manufacturing. The encompassing relationship between the Abramovitz description and excess volatility is then examined through the identity linking output, inventory investment and sales. This establishes the central finding of the paper that the Abramovitz description encompasses, but is not encompassed by, excess volatility.

Section 4 offers a more formal description of the cyclical relationships between output or sales and inventories. Vector auto-regressions relating inventories and inventory usage are estimated using data on the growth rates (logarithms in first differences). Granger-Sims causality tests indicate that these may be restricted to a univariate relationship from inventory usage onto inventory holdings. This time domain estimate is then transformed into a representation of the cross-spectra in the frequency domain, using a simple but novel technique. The cross-spectra confirm the Abramovitz conclusions for both inventory levels and inventory investment. Section 5 applies the same technique for the transformation from the time-domain into the frequency domain to examine the correspondence between the speed of adjustment in stock-adjustment models and the cyclical behaviour of inventory investment. This indicates that the characteristically slow speeds of adjustment which emerge from time-domain estimation correspond, in the frequency domain, to the observed cyclical behaviour of inventory investment as described by Abramovitz.

2. The major features of inventory investment in the UK

This section considers the stylised facts of UK inventory investment, paying closest attention to the manufacturing sector. The most prominent feature of the data are the cyclical movements which contribute markedly to fluctuations in expenditure on GDP. The "excess" volatility of production relative to sales is a relatively minor feature in that the level of gross manufacturing output differs from sales only to a very small degree.

The basic data used for the manufacturing sub-sectors are quarterly observations on inventory investment in constant 1980 prices and on the index of manufacturing output from 1960q1-1987q4. A gross output series, in 1980 prices, is then derived by grossing up the seasonally un-adjusted manufacturing production index, to obtain an estimate of gross output of the manufacturing sector (including intra-sectoral sales). Gross sales are estimated by adding inventory investment to the gross output series. Note that any measurement error introduced by this indirect method of calculating sales will tend to increase the variance of sales relative to production.

Excess volatility

Excess volatility is revealed by the comparison of the variance of production and the variance of sales shown in table 1. The variance of production exceeds the variance of sales in all sectors. Moreover the variance of the change in production also exceeds the variance of the change in sales for all sectors except chemicals. The ratio of the variance of output to the variance of sales is close to that for US non-durable manufacturing reported by West (1986). "Excess volatility" appears to be equally characteristic of UK manufacturing as of US manufacturing.

TABLE 1 : VOLATILITY OF SALES AND OUTPUT IN UK MANUFACTURING

	Variance of			
	Sales	Output	Δsales	Δoutput
For				
Total UK manufacturing	81.0	92.5	25.6	35.6
Metal manufacturing	100.2	104.9	108.0	121.0
Chemicals	144.2	150.8	39.3	26.5
Engineering and allied products	106.5	119.5	47.9	62.7
Food, drink and tobacco	21.8	24.3	18.6	20.4
Textiles, footwear and clothing	168.7	174.2	32.7	42.6
Other manufacturing	137.8	150.8	24.5	29.5

	Standard Deviations (%) of			
	Sales	Output	Δsales	Δoutput
For				
Total UK manufacturing	9.00	9.62	5.06	5.97
Metal manufacturing	10.01	10.24	10.39	11.00
Chemicals	12.01	12.28	6.27	5.15
Engineering and allied products	10.32	10.93	6.92	7.92
Food, drink and tobacco	4.67	4.93	4.31	4.52
Textiles, footwear and clothing	12.99	13.20	5.72	6.53
Other manufacturing	11.74	12.28	4.95	5.43

Notes: Sales and output measured as % deviations around a deterministic linear time trend 60q1-87q4 (except metal manufacture which is 68q1-87q4). Δsales and Δoutput first difference in deviations of sales and output around trend.

The degree of excess volatility is however very small. This is clear from the comparison of the standard deviations made in the second part of table 1, the units of which (% deviations from trend) are directly interpretable. The standard deviation of output around trend is for no sector more than 0.4% greater than the standard deviation of sales around trend.

This point is reinforced by chart 1 which shows the % deviation of output and sales from trend for total UK manufacturing. These are the same series used for the calculation of the variances in the table. It is almost impossible to distinguish output and sales, so close are their co-movements. Similar charts can be drawn for the manufacturing sub-sectors. It is evident that excess volatility is a very minor feature of the data.

Recent contributions to the US literature (Miron and Zeldes (1988), Fair (1990)) have shown that using different data for production, sales and inventory investment it is possible to reverse the ranking of the variance of output and of sales, suggesting that measurement errors are affecting the outcome of this comparison. The present finding that the variance of output and sales are almost exactly the same, if it also applies to the US, would explains why measurement error should have such an influence on the relative ranking of the variance of output and sales.

It is of interest to decompose the total variance of the previous table into seasonal and non-seasonal components (table 2). The greatest part of the variance of the levels of output and sales is non-seasonal. For all sectors the variance of output remains slightly greater than that of sales when the seasonal component is removed. It is however the seasonal component which makes the greatest contribution to the variance of the change in output and sales. Moreover for two sectors (engineering and other manufacturing) the variance of the seasonal change in output is much greater than the variance of the seasonal change in sales.

TABLE 2 : DECOMPOSITION OF THE VARIANCE OF OUTPUT AND SALES

Non-seasonal component

	Sales	Output	ΔSales	ΔOutput
Total UK manufacturing	74.0	82.1	7.0	7.2
Metal manufacturing	83.0	87.2	61.6	71.1
Chemicals	133.2	144.0	17.1	10.0
Engineering and allied products	92.4	99.0	14.1	12.3
Food, drink and tobacco	18.5	20.9	7.5	8.0
Textiles, footwear and clothing	160.3	162.1	13.6	14.2
Other manufacturing	134.1	145.2	11.8	11.4

Seasonal component

	Sales	Output	ΔSales	ΔOutput
Total UK manufacturing	7.0	10.5	18.6	28.5
Metal manufacturing	17.2	17.6	46.3	49.9
Chemicals	11.1	6.8	22.3	16.5
Engineering and allied products	14.2	20.5	33.7	50.4
Food, drink and tobacco	3.3	3.4	11.1	12.4
Textiles, footwear and clothing	8.5	12.2	19.1	28.4
Other manufacturing	3.7	5.6	12.7	18.1

These results are consistent with the usual finding that the variance of the level of macro-economic aggregates is concentrated at low (non-seasonal) frequencies

((Granger (1966)). An intuition about this point is that the calculations of the variance of output and sales are dominated by the extreme values around trend, corresponding to cyclical peaks and troughs. This suggests that an understanding of cyclical movements is crucial to an explanation of the variance of output and of sales.

Inventory turnover

Table 1 and chart 1 show that investment in finished goods inventories makes only a small difference to the level of output. A related point is that the ratios of inventories of finished goods to final sales are quite small, typically representing less than one month of sales. Table 3 shows inventory holdings and sales, by sector, for the UK. The ratio of finished goods inventory holdings to annual gross sales for UK manufacturing is 5.7%, or less than one month's sales. Total inventory holdings by UK manufacturing are about 18%, or just over two months of gross sales. Given that finished goods inventories represent such a small proportion of total manufacturing output it is hardly surprising that investment in finished goods alters the level of output relative to sales to such a small degree.

TABLE 3 : OUTPUT, SALES AND INVENTORY HOLDINGS

£bn 1980 prices	*OUTPUT AND SALES (1987)* VALUE ADDED	GROSS SALES	*INVENTORY HOLDINGS 1986Q4* FIN-INISHED GOODS	RAW MAT-ERIALS	WORK IN PRO-GRESS	TOTAL	*RATIO* TOTAL/ GROSS SALES
ENERGY AND WATER	24.8	54.0				3.9	0.9
TOTAL MANUFACTURING	58.2	170.9	9.8	9.9	11.4	31.1	2.2
of which:							
METAL MANUFACTURE	5.4	12.4	0.5	0.9	0.8	2.2	2.1
CHEMICALS	6.1	20.8	1.7	1.3	0.5	3.5	2.0
ENGINEERING & ALLIED	25.1	62.4	3.8	3.7	7.7	15.1	2.9
FOOD, DRINK & TOBACCO	7.2	30.6	1.4	1.6	1.2	4.2	1.6
TEXTILES	3.9	11.0	0.9	0.7	0.5	2.1	2.3
OTHER MANUFACTURING	10.4	34.3	1.6	1.6	0.8	4.2	1.5
CONSTRUCTION	13.3	30.6				5.0	2.0
DISTRIBUTIVE TRADES	32.4	54.3				17.9	4.0
OTHER SECTORS*	102.7	35.1				15.1	1.3
WHOLE ECONOMY	290.7	616.3				68.0	1.3

* Excluding motor trades

The contribution of inventory investment to movements in GDP

Despite the limited degree to which the variance of production exceeds that of sales, investment in inventories contributes substantially to cyclical fluctuations in expenditure on GDP, out of all proportion to its long run share in expenditure on GDP. Table 4 documents this familiar point for the UK.[2] Column 6 shows the percentage contribution of inventory investment to the cyclical fluctuations in GDP(E). Over the entire period (1955-1988) this averages 32%, enormously greater than the 0.6% average share of inventory investment in expenditure on GDP over the same period. This large contribution of inventory investment to cyclical movements in GDP is a familiar point documented, for example, by Blinder and Holtz-Eakin (1986) for the United States.[3]

TABLE 4 : CONTRIBUTION OF INVENTORY INVESTMENT (ΔI) TO CYCLICAL MOVEMENTS IN EXPENDITURE ON UK GDP

	LEVELS RELATIVE TO TREND		CHANGES T to P and P to T		CONTRIBUTION
Peaks (P) and Troughs (T)	GDP(E)	ΔI	GDP(E)	ΔI	%
	(1)	(2)	(3)	(4)	(5)
56Q1(P)	861	368			
58Q4(T)	-2183	-196	-3044	-564	19
61Q1(P)	422	820	2605	1016	39
62Q3(T)	-1162	-356	-1583	-1177	74
64Q3(P)	1618	1129	2780	1485	53
70Q4(T)	-561	191	-2179	-938	43
73Q2(P)	4169	1588	4730	1397	30
75Q3(T)	-966	-1602	-5135	-3189	62
79Q1(P)	2334	503	3301	2104	64
82Q2(T)	-6329	-886	-8663	-1389	16
89Q2(P)	6937	834	13266	1720	13
SUM (ABSOLUTE MAGNITUDES)			47286	14978	32

Contribution by inventory category :	
Manufacturers finished goods	1
raw materials	11
work in progress	7
distribution, hotels and repairs	9
other inventory investment	4

The dates of peaks and troughs are established by inspection of the departures of GDP from trend. Trend is estimated by regressing GDP on a deterministic time trend split in 1973q1. Trend inventory investment is 0.53% of trend GDP, its long run share over the period 1955-1989.

The contrast, between the substantial contribution made by inventory investment as a whole to fluctuations in expenditure on GDP and the relatively minor influence which investment in inventories of finished goods has on the level of gross output relative to sales, seems paradoxical. Two observations serve to resolve this paradox. The first is the observation that inventories of finished goods are by no means the most volatile category of inventory investment. Table 4 also decomposes the contribution of inventory investment to cyclical movements in GDP(E) by category of inventory investment. Exactly as reported by Blinder (1981) for the US, it is inventories of raw materials and distributor's inventories which make the greatest contribution to cyclical movements in inventory investment. The contribution of inventories of finished goods is very much smaller.

The second observation which resolves this paradox is the distinction between gross output and value added. Gross output consists of sales and investment in inventories of finished goods aggregated over all firms. Value added is much smaller. As shown by table 3 value added in manufacturing is only one third of gross sales and similar ratios apply to other sectors.[4] Thus an inventory investment which increases gross sales by 1 percent increases aggregate value added (ie GDP) by around 3 percent.

Changes in inventory investment also make a substantial contribution to quarter to quarter movements in expenditure on GDP. This emerges from the following breakdown of the variance of changes in GDP 1955q2-1989q4 (with the variances scaled as a percentage of the total variance of expenditure on GDP).

Decomposition of the variance of changes in expenditure on GDP

Var (Y)	=	Var (Y-ΔI)	+	Var (ΔI)	+	2 Cov (Y-ΔI,ΔI)
100		93		33		-26

Inventory investment contributes about one third of the total variance in quarterly movements in UK GDP but there is a substantial offsetting negative correlation with the other components of expenditure on GDP.

This completes the description of inventory movements in the UK. The behaviour of inventory investment in the UK is very similar to that documented for the US by, amongst others, Blinder (1981), Blanchard (1983), Blinder and Holtz-

Eakin (1986) and West (1986). As measured by the variance of output relative to sales there appears to be a small degree of excess volatility of production in UK manufacturing of a similar extent to that reported for US non-durable manufacturing. But inventories of finished goods represent less than one month of output or of sales and investment in finished goods inventories alters the level of output relative to sales to only a very small degree. Excess volatility is a very minor feature of the data. Much more prominent are the familiar cyclical movements in inventory investment.

3. A data encompassing description of inventory movements.

This section considers the concise description of the cyclical movements in inventory movements first put forward by Abramovitz (1950). Here it is shown to apply to UK manufacturing over the period 1958-1988. The Abramovitz description is then shown to data encompass, but not be data encompassed by, the excess volatility proposition. Thus only the former is of interest.

The Abramovitz observation

Abramovitz (1950), in his study of inter-war US inventories data using NBER reference cycle techniques, found that both inventory levels and inventory investment exhibited pronounced cyclical movements; but, while inventory levels lagged the business cycle by about 6-9 months, inventory investment was coincident with cyclical movements in output and sales, thus making a marked contribution to cyclical movements in expenditure. This Abramovitz description is attractive because it is a concise description of the cyclical movements in both inventory levels and inventory investment, corresponding exactly what the previous section suggests are the main features of inventory investment in the UK. Thus the Abramovitz observation, if it captures all the prominent features of the data generating process, can provide a valuable guide to the development of models of inventory investment.

Although based on a study of inter-war US data the Abramovitz observations hold up equally well on post-war UK data. Cyclical movements are most easily examined using graphical techniques (similar to the NBER reference cycle techniques used by Abramovitz in his original study). Charts 2(a) and 2(b) compare the cyclical movements in manufacturing sales with the level of finished goods inventories and finished goods inventory investment by UK manufacturing. The level of sales and the level of inventories are % deviations from seasonals plus trend (trend estimated here by regression on a linear time trend split in 1973q1, though a very similar picture emerges from using a single deterministic trend or a stochastic trend). The inventory investment series is the change in inventory levels relative to trend, scaled by a constant factor to facilitate comparison. The Abramovitz conclusions are confirmed by inspection of these charts, with inventory investment in phase with the cyclical movements in sales while cyclical movements in the level of inventories lag sales by about three quarters. Also noteworthy is the inventory de-cumulation after 1984, perhaps reflecting the introduction of new methods of inventory control.

Charts 3(a) to 4(b) show that similar conclusions can be drawn for the other two categories of manufacturing inventories. This is of particular interest because it indicates that the Abramovitz description, unlike "excess" volatility of production, generalises to other categories of inventories and not just to inventories of finished goods.

The Abramovitz observation data encompasses excess volatility

The Abramovitz description subsumes the excess volatility proposition. This can be seen from the identity linking output, sales and inventories of finished goods:

$$Q \equiv S + \Delta I \tag{1}$$

Expressing this identity in terms of variances yields:

$$Var(Q) \equiv Var(S) + Var(\Delta I) + 2\,Cov(S,\Delta I) \tag{2}$$

Pro-cyclical movements in inventory investment imply that the covariance is positive and hence that the variance of output exceeds the variance of sales. The Abramovitz observation is sufficient to generate excess volatility.

The converse proposition does not hold. The variance of sales can exceed the variance of output even when the covariance between inventory investment and output is negative, provided only that the variance of inventory investment is large enough. So the excess volatility proposition fails to encompass the Abramovitz observation. A model which explains the Abramovitz observation must also explain excess volatility. In contrast a model which explains excess volatility may not generate pro-cyclical movements in inventory investment and hence can remain mis-specified.

4. Frequency domain analysis

Graphical techniques, such as those used in the previous section or the similar NBER reference cycle techniques used by Abramovitz, are no longer widely applied in the academic literature. The academic community prefers the use of econometric techiques, which for descriptive purposes means the estimation of data based dynamic specifications to capture the main movements in the variables under consideration. This section first examine vector auto-regressions as formal descriptions of the data and then transforms the estimated relationships into the frequency domain, to examine the cyclical relationships between inventories and inventory usage (output or sales) are still apparent. These alternative techniques provide a more formal confirmation of the main conclusions drawn in section 2.

Vector auto-regressions.

Vector auto-regressions were estimated over the period 56q1-90q4, using first differences of the logarithms of the inventories, output and sales data described in section 1, and including seasonal dummies. Table 5 reports the Granger-Sims causality tests calculated from these estimates.

TABLE 5 : GRANGER CAUSALITY TESTS.

	F1(3,104)	F2(3,104)
Finished goods and gross manufacturing sales	3.2	0.7
Work in progress and gross manufacturing sales	3.0	0.8
Raw materials and manufacturing output	8.9	1.0
Distributors inventories and retail sales	1.4	0.5

F1 is the test of the null hypothesis that past observations on inventory usage (sales or output) are of no help in forecasting inventory holdings. F2 is the test of the null hypothesis that the lags on inventory holdings are of no help in predicting inventory usage. Significance levels for F(4,131) are 2.68 (at 5%) and 3.95 (at 1%).

The causality tests indicate that in no case does inventory investment granger cause output or sales. Manufacturing output granger causes investment in raw materials (this is highly significant), while gross manufacturing sales granger causes inventories of both finished goods and of work in progress (significant at the 5 per cent but not 1 per cent level). Retail sales does not granger cause inventories held in the distribution sector.

For completeness table 6, at the end of the paper, shows the estimated vector auto-regressions for the four categories of inventories. For the three categories of manufacturers inventories there are significant positive coefficients on sales from lag one to lag three. For all categories of inventories there are also significant positive coefficients on lagged inventories, although in the case of work in progress this occurs only at the fourth lag, possibly indicating a seasonal non-stationarity in the data rather than a dynamic relationship. For inventories held by the distribution sector only the third and fourth lags on consumer expenditure are significant. Taken together these regressions indicate that not only is there a significant link from lagged inventory usage (as indicated by the Granger causality tests) but that it takes a number of periods for changes in inventory usage to have their full impact on inventory holdings.

Frequency domain representation of the time domain estimates.

A further way of exploring the properties of these descriptive equations is to re-express the relationship between inventory usage and inventory holdings provided by equations 4-6 in the frequency domain. This is done here by calculating the cross-spectrum between the series as the theoretical frequency response functions (cross-spectra) of the estimated vector auto-regressions for inventories. These calculations were carried out for the three categories of manufacturing inventories where the granger causality tests indicate that output or sales are of use in predicting inventory holdings. This is a simple but novel technique, similar to that suggested by Parzen (1967) for the estimation of the spectrum of a single series from an auto-regression estimated in the time domain. Technical details are provided in the annex.

The results are presented in charts 5(a)-7(d) showing gain and phase relationships between inventory usage, inventory holdings and inventory investment. The gain relationships show the degree to which variance in output and sales is carried through onto inventory levels and inventory investment at different frequencies. For inventory levels the highest gain is exhibited at the lowest frequencies (charts 5(a), 6(a) and 7(a)). For inventory investment the gain is highest at fairly high frequencies (charts 5(c), 6(c) and 7(c)).

The phase relationships indicated by the charts support the Abramovitz observation. At the low business cycle frequencies (where the frequency λ is in the range $\pi/10<\lambda<\pi/5$), corresponding to periodicities of between 2 and 5 years, inventory levels are considerably out of phase with output and sales (charts 5(b), 6(b), 7(b)). Finished goods inventory investment and raw material inventory investment are in phase with sales and output at the low business cycle frequencies (charts 6(d) and 8(d)). Again this is consistent with the Abramovitz observation. Investment in work in progress is however out of phase with sales (chart 7(d)), a conflict with the Abramovitz observation.

5. Speeds of adjustment of inventory equations and the frequency domain.

As noted by Blinder (1981) it is characteristic of estimated inventory equations that they have coefficients close to unity on lagged inventory levels and hence exhibit extremely slow speeds of adjustment. Estimated stock adjustment specification usually suggest that less than 10 per cent of any discrepancy between inventories and the target level of inventories is corrected in each quarter. These slow speeds of adjustment are difficult to reconcile with any known model of inventory behaviour: they cannot for example be explained by costs of adjustment because they require implausibly high costs associated with inventory purchase.

It may be conjectured that the reason that such slow speeds of adjustment emerge is that they reflect the cyclical movements of inventory investment. This conjecture is illustrated by transforming the simple stock adjustment model into the frequency domain, using the same techniques applied in the previous section. The stock adjustment model may be written:

$$I_t = \lambda I_{t-1} + (1-\lambda) S_t \qquad (3)$$

Again this may be interpreted as a linear filter from sales onto inventory holdings, and from sales onto inventory investment. The techniques described in the annex may be used to transform the time domain filter into the frequency domain.

Chart 8(a)-8(d) show the gain and phase relationship between sales and inventory investment that emerges for values of $1-\lambda$ (the speed of adjustment) ranging from 0.8 (rapid adjustment) down to 0.1. The comments on these charts relate to the range of business cycle frequencies from $\pi/5$ - $\pi/10$ radians. The gain charts (8(a)) and 8(b)) indicate that a large part of the variance of sales, at business cycle frequencies is transferred to the level of inventories, when the speed of adjustment is high, but that the gain falls as the speed of adjustment is reduced.

The phase charts (8(b) and 8(d) indicate a correspondence between the Abramovitz description and slow speeds of adjustment. At business cycle frequencies sales and inventory levels are in phase when adjustment is rapid, but inventory levels lag sales by nearly half a cycle ($\pi/2$) when speeds of adjustment are very slow (0.1). For inventory investment the opposite picture arises: with rapid speeds of adjustment inventory investment leads sales at business cycle frequencies, but with a speed of adjustment as slow as 0.1, then inventory investment is in phase with sales. Thus the stock adjustment model reproduces the Abramovitz observation, that inventory investment is in phase with the business cycle and inventory levels lag the business cycle, if and only if the speed of adjustment parameter is very low, around 0.1 on quarterly data.

6. Conclusions

This paper argues that the stylised fact about inventory investment which has been the centre of recent research on inventory investment - the so-called "excess" volatility of production - is in fact a misleading description of the data. Excess volatility does apply to the UK but it is at best a minor feature of the data generating process. The cyclical movements in inventory investment are of much greater quantitative importance.

The Abramovitz description of the cyclical movements in inventories says that both inventory investment and inventory holdings exhibit cyclical movements, but that whereas inventory investment moves in phase with cyclical movements in GDP, the cyclical movements in inventory holdings lag GDP by 6-9 months. Abramovitz (1950) derived this description from inter-war data for the US. The present paper shows that it is equally applicable to post-war data for the UK.

Not only is the Abramovitz description applicable to UK inventories but it data encompasses excess volatility in the sense that pro-cyclical movements in inventory investment imply that the variance of production exceeds the variance of sales. The converse however does not apply. This is the central data encompassing result of this paper. Furthermore the Abramovitz description, unlike excess volatility, applies to all categories of inventories.

These results are confirmed by the frequency domain analysis of section 4, which indicates pro-cyclical movements of inventory investment using more formal descriptive techniques. This technique also illustrates (section 5) that the apparently slow speeds to adjustment of estimated inventory equations can be seen as reflecting the cyclical movements of inventory investment.

These findings have the following implications for the direction of future research on inventory investment:

(i) Models proposed in the literature to explain "excess volatility of production" remain unsatisfactory if they do not also explain the pro-cyclical movements in inventory investment. For example stock-out avoidance models such as Kahn (1987) are of little interest since they do not generate procyclical movements of inventory investment.[5] On the other hand models which generate pro-cyclical movements in inventory investment must, an implication of the data encompassing finding of this paper, also explain "excess volatility". Hence only the latter programme of research is of interest. This programme of research can also explain the puzzle about the characteristically slow speeds of adjustment in estimated inventory equations.

(ii) Pro-cyclical movements in manufacturing inventory investment arise for raw material inventories as well as for finished goods. Excess volatility on the other hand is a description applicable only to finished goods. Therefore a more promising path for future research is on a model applicable to both categories of inventory investment. This casts doubt on two explanations that have been suggested for excess volatility, namely non-convex costs of production (Ramey (1987)) or cost shocks (suggested by Blinder (1986)), since these are incapable of explaining pro-cyclical movements in holdings of manufacturer's raw materials.

(iii) The literature on business cycle fluctuations is likely to be a fruitful source of hypotheses which may explain the behaviour of inventories.

These considerations suggest particular avenues of further research on models which can potentially explain the cyclical movements of all categories of inventories. One is an equilibrium business cycle model of inventory investment. Despite the major role of inventories in cyclical movements in GDP no existing equilibrium business cycle models, known to this author, offers an explanation of inventory movements. The challenge is to specify inventory investment decisions as the outcome of an optimisation decision in such a way that the observed correlations with output emerge. A second possible explanation of the cyclical movements in all categories of inventory investment are (S,s) models (see Blinder and Maccini (1990)) although the aggregation problems associated with this model remain unsolved. A further possible explanation, which has not received much attention in the literature, is an appeal to capital market imperfections. These have frequently been cited as a source of cyclical fluctuations in fixed capital investment and in personal consumption: their application to a model of inventories has not yet been made in the literature.

Table 6 : Vector Auto-Regressions for UK inventories

Estimated 1956q1 - 1990q4.

Dependent Variable	Finished Goods in Manufacturing $\Delta\ln(I^f)$		Work in Progress in Manufacturing $\Delta\ln(I^w)$		Raw Materials in Manufacturing $\Delta\ln(I^r)$		Inventories in Distribution $\Delta\ln(I^d)$	
Coefficients on lagged inventories (t-stats)	0.200 (2.12)	$\Delta\ln(I^f_{-1})$	0.035 (0.39)	$\Delta\ln(I^w_{-1})$	0.147 (1.63)	$\Delta\ln(I^r_{-1})$	0.162 (1.87)	$\Delta\ln(I^d_{-1})$
	-0.108 (1.17)	$\Delta\ln(I^f_{-2})$	0.065 (0.75)	$\Delta\ln(I^w_{-2})$	0.217 (2.37)	$\Delta\ln(I^r_{-2})$	0.065 (0.76)	$\Delta\ln(I^d_{-2})$
	0.148 (1.62)	$\Delta\ln(I^f_{-3})$	0.058 (0.68)	$\Delta\ln(I^w_{-3})$	-0.131 (1.49)	$\Delta\ln(I^r_{-3})$	-0.173 (2.01)	$\Delta\ln(I^d_{-3})$
	0.152 (1.71)	$\Delta\ln(I^f_{-4})$	0.271 (3.16)	$\Delta\ln(I^w_{-4})$	0.151 (1.88)	$\Delta\ln(I^r_{-4})$	0.260 (3.02)	$\Delta\ln(I^d_{-4})$
Coefficients on lagged inventory usage (t-stats)	0.156 (1.43)	$\Delta\ln(S_{-1})$	0.128 (2.14)	$\Delta\ln(S_{-1})$	0.293 (5.36)	$\Delta\ln(Y_{-1})$	0.013 (0.22)	$\Delta\ln(C_{-1})$
	0.191 (1.76)	$\Delta\ln(S_{-2})$	0.117 (1.98)	$\Delta\ln(S_{-2})$	0.155 (2.62)	$\Delta\ln(Y_{-2})$	0.074 (1.23)	$\Delta\ln(C_{-2})$
	0.248 (2.28)	$\Delta\ln(S_{-3})$	0.108 (1.79)	$\Delta\ln(S_{-3})$	0.086 (1.43)	$\Delta\ln(Y_{-3})$	0.110 (1.83)	$\Delta\ln(C_{-3})$
	0.027 (0.24)	$\Delta\ln(S_{-4})$	0.004 (0.07)	$\Delta\ln(S_{-4})$	0.139 (2.39)	$\Delta\ln(Y_{-4})$	0.111 (1.93)	$\Delta\ln(C_{-4})$
Standard Error	2.29 %		1.29 %		1.15 %		1.11 %	
Measure of Inventory Usage	Gross Manufacturing Sales (S)		Gross Manufacturing Sales (S)		Manufacturing Output (Y)		Consumer Expenditure (C)	

Annex Spectral analysis of data based inventory/inventory usage relationships.

This annex describes the manner in which the cross-spectrum between inventory usage and inventory holdings has been derived from the time-domain estimates of section 4. Further details on the relationship between the time and frequency domains are available in standard textbooks such as Sargent (1979) chapter XI and Harvey (1981) chapter 3.

The vector auto-regressions of section 4 can be interpreted as yielding the log of inventory holdings as a linear filter of the log of inventory usage. This linear filter can be written, with an independent stationary and un-correlated error term, as:

$$\Gamma^1(L)y_t = \Gamma^2(L)x_t + \epsilon_t \qquad \text{(A1)}$$

The relationship between the spectrum of x_t and y_t at frequency λ ($0 \leq \lambda \leq \pi$) which results from the application of this linear filter may be described by the frequency response function $W(\lambda)$ given (where the γ_j^1 and γ_k^2 are the coefficients of the lag polynomials $\Gamma^1(L)$ and $\Gamma^2(L)$) by:

$$W(\lambda) = \frac{\sum_{k=0}^{\infty} \gamma_k^2 e^{-i\lambda k}}{\sum_{j=0}^{\infty} \gamma_j^1 e^{-i\lambda j}} \qquad \text{(A2)}$$

The numerator and denominator of this expression can be decomposed into real and imaginary components (using the identity $e^{-i\lambda} = \cos\lambda + i \sin\lambda$):

$$W(\lambda) = \frac{\sum_{k=0}^{\infty} \gamma_k^2 [Cos(\lambda k) + i Sin(\lambda k)]}{\sum_{j=0}^{\infty} \gamma_j^1 [Cos(\lambda k) + i Sin(\lambda k)]} = \frac{\bar{\gamma}_a^1 + i\bar{\gamma}_b^1}{\bar{\gamma}_a^2 + i\bar{\gamma}_b^2} \tag{A3}$$

Then the gain and phase relationship between x_t and y_t at frequency λ can be expressed in terms of the real and imaginary components. The gain $G(\lambda)$ is given by:

$$G(\lambda) = \sqrt{\frac{\left\{\bar{\gamma}_a^2(\lambda)\right\}^2 + \left\{\bar{\gamma}_b^2(\lambda)\right\}^2}{\left\{\bar{\gamma}_a^1(\lambda)\right\}^2 + \left\{\bar{\gamma}_b^1(\lambda)\right\}^2}} \tag{A4}$$

while the phase is given by:

$$Ph(\lambda) = \tan^{-1}\left(-\frac{\bar{\gamma}_b^2}{\bar{\gamma}_a^2}\right) - \tan^{-1}\left(-\frac{\bar{\gamma}_b^1}{\bar{\gamma}_a^1}\right) \tag{A5}$$

CHART 1 - UK MANUFACTURING
GROSS OUTPUT AND SALES 1957-1989

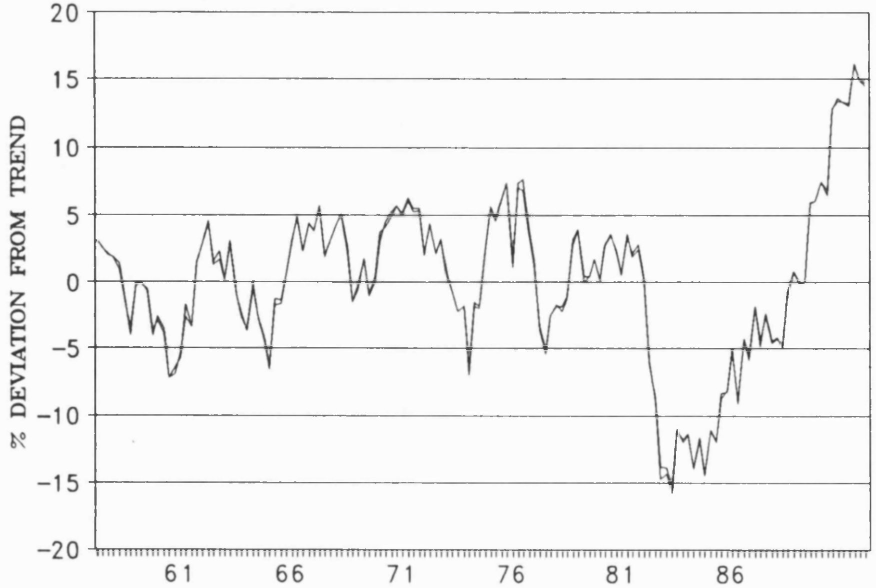


Chart 2(a) UK gross manufacturing sales & finished goods inventory levels

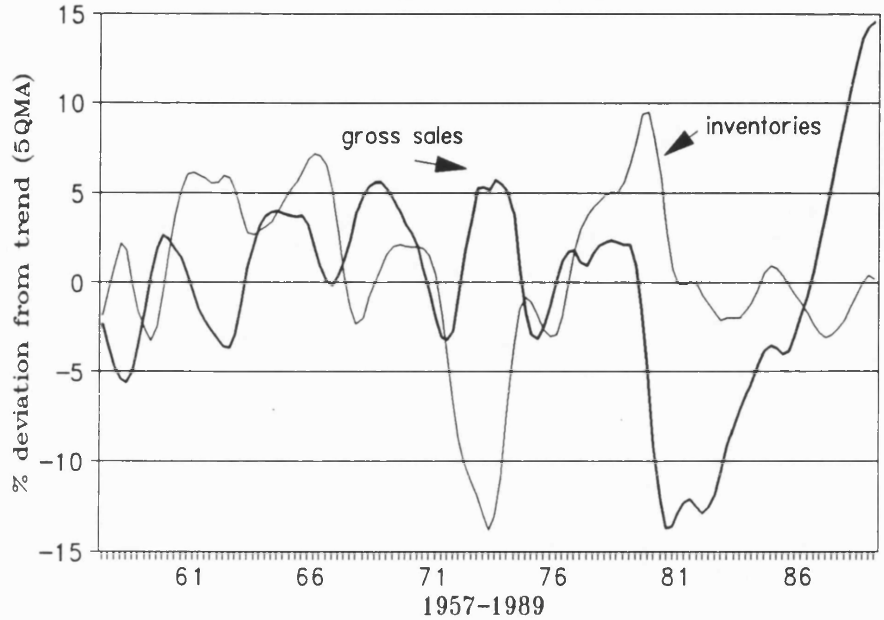
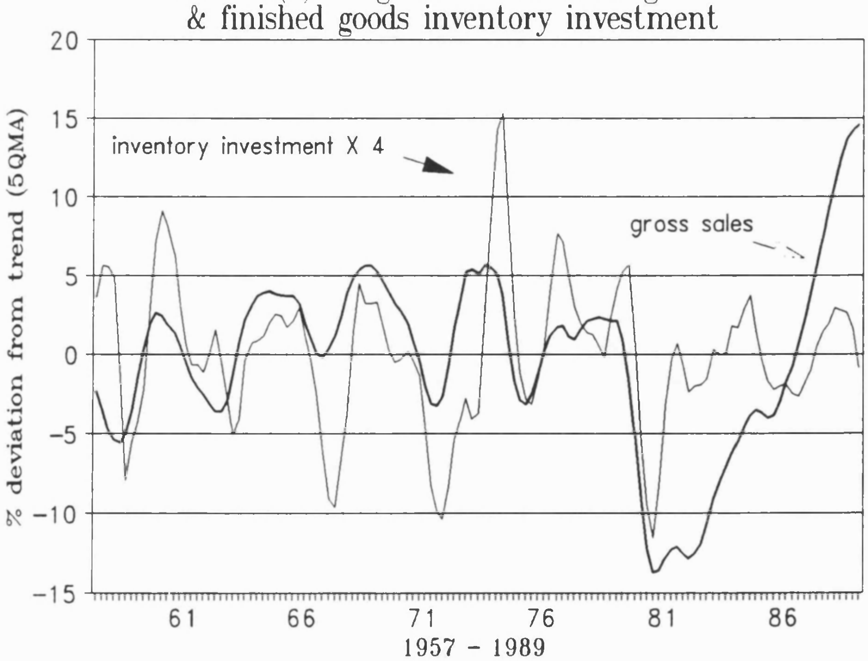


Chart 2(b) UK gross manufacturing sales & finished goods inventory investment



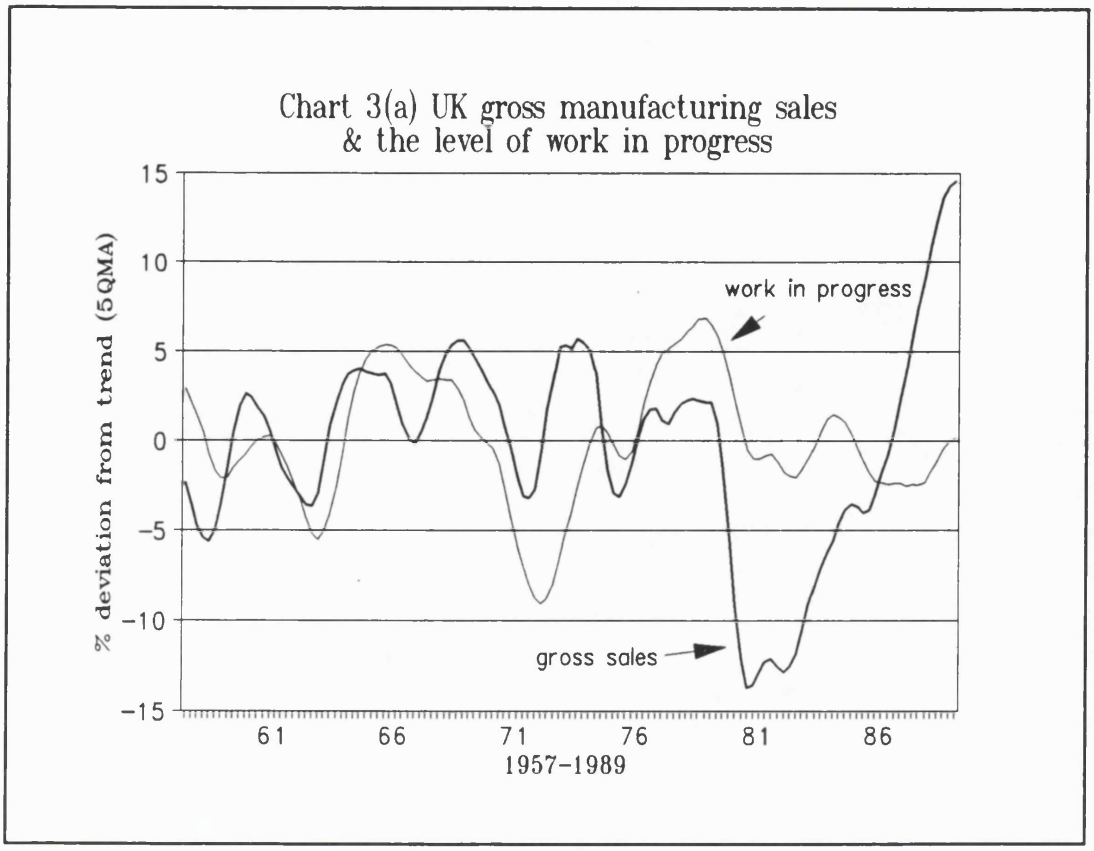


Chart 3(a) UK gross manufacturing sales
& the level of work in progress
% deviation from trend (5QMA)
15
10
5
0
-5
-10
-15
work in progress
gross sales
61
66
71
76
81
86
1957-1989

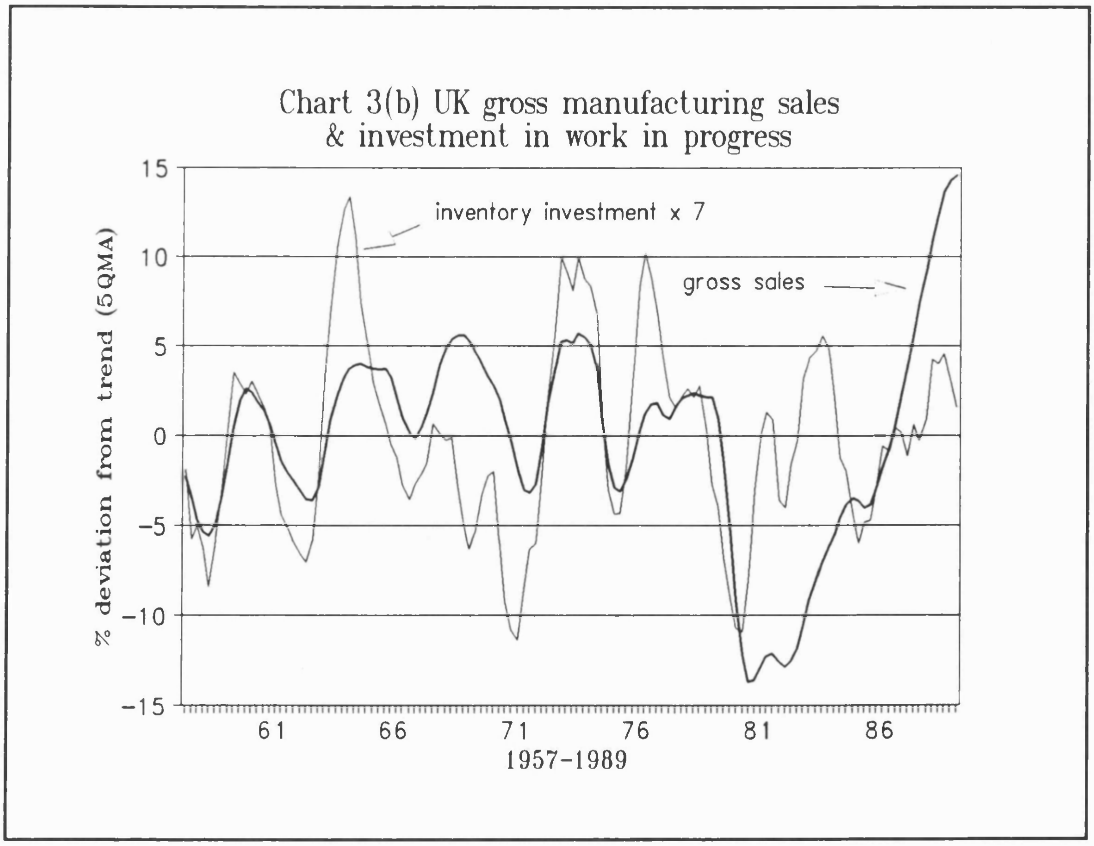


Chart 3(b) UK gross manufacturing sales
& investment in work in progress
% deviation from trend (5QMA)
15
10
5
0
-5
-10
-15
inventory investment x 7
gross sales
61
66
71
76
81
86
1957-1989

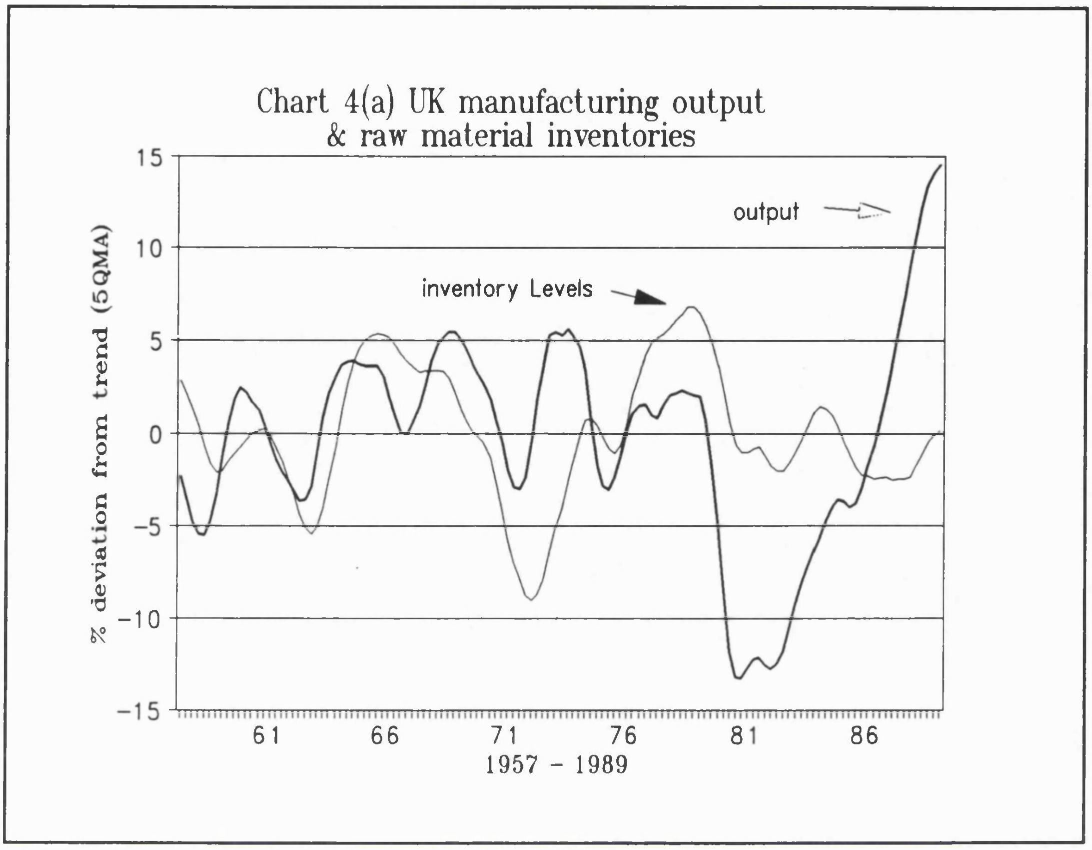


Chart 4(a) UK manufacturing output
& raw material inventories
output
inventory Levels
% deviation from trend (5QMA)
15
10
5
0
-5
-10
-15
61
66
71
76
81
86
1957 - 1989

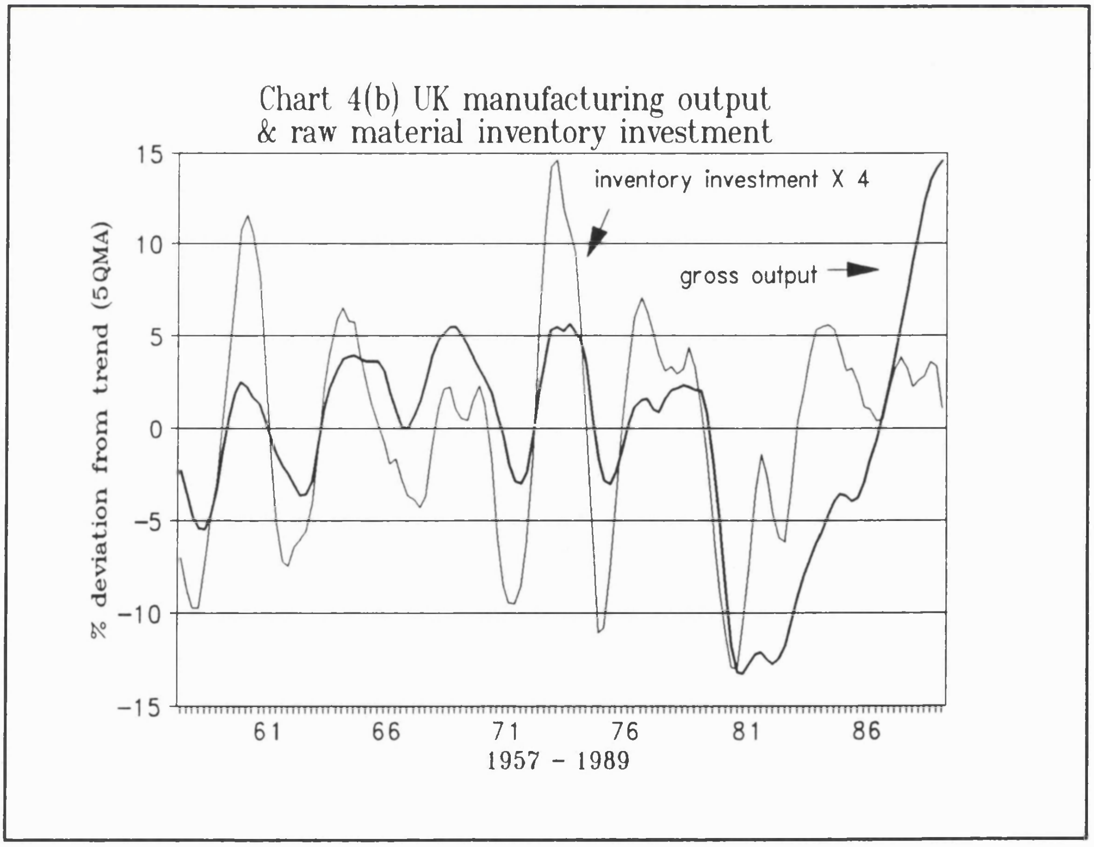


Chart 4(b) UK manufacturing output
& raw material inventory investment
inventory investment X 4
gross output
% deviation from trend (5QMA)
15
10
5
0
-5
-10
-15
61
66
71
76
81
86
1957 - 1989

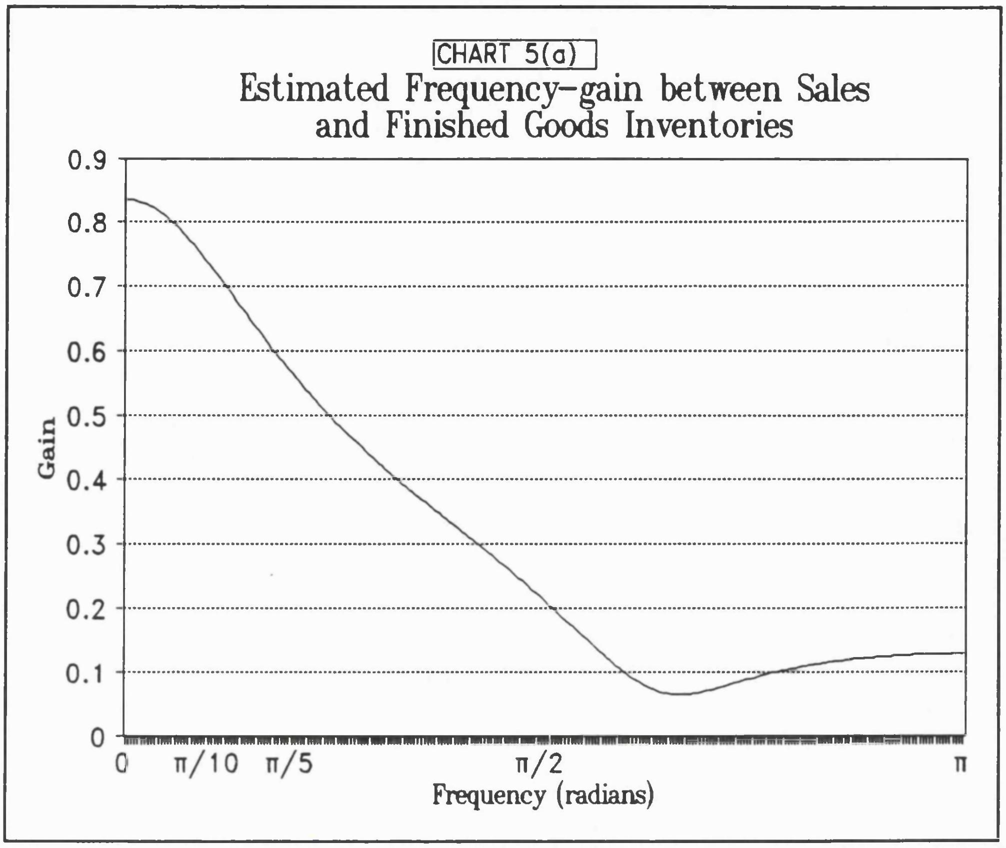


CHART 5(a)
Estimated Frequency-gain between Sales and Finished Goods Inventories
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Gain
0
π/10
π/5
π/2
π
Frequency (radians)

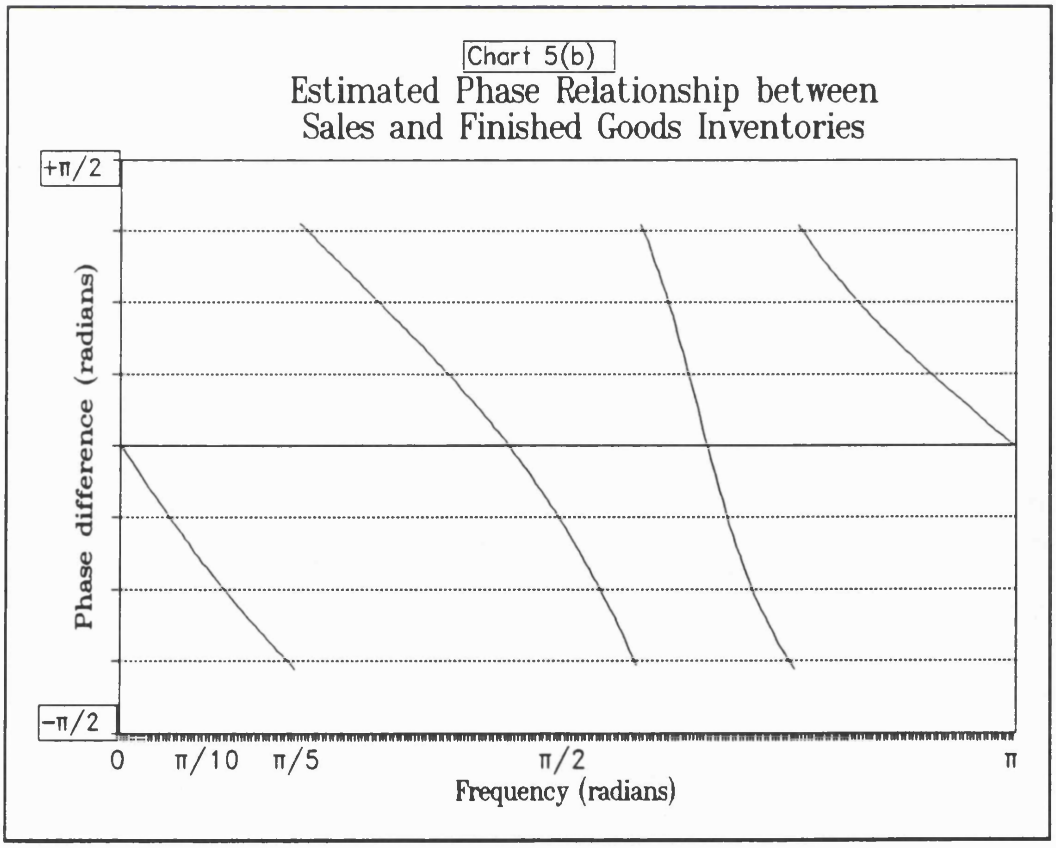


Chart 5(b)
Estimated Phase Relationship between Sales and Finished Goods Inventories
+π/2
-π/2
Phase difference (radians)
0
π/10
π/5
π/2
π
Frequency (radians)

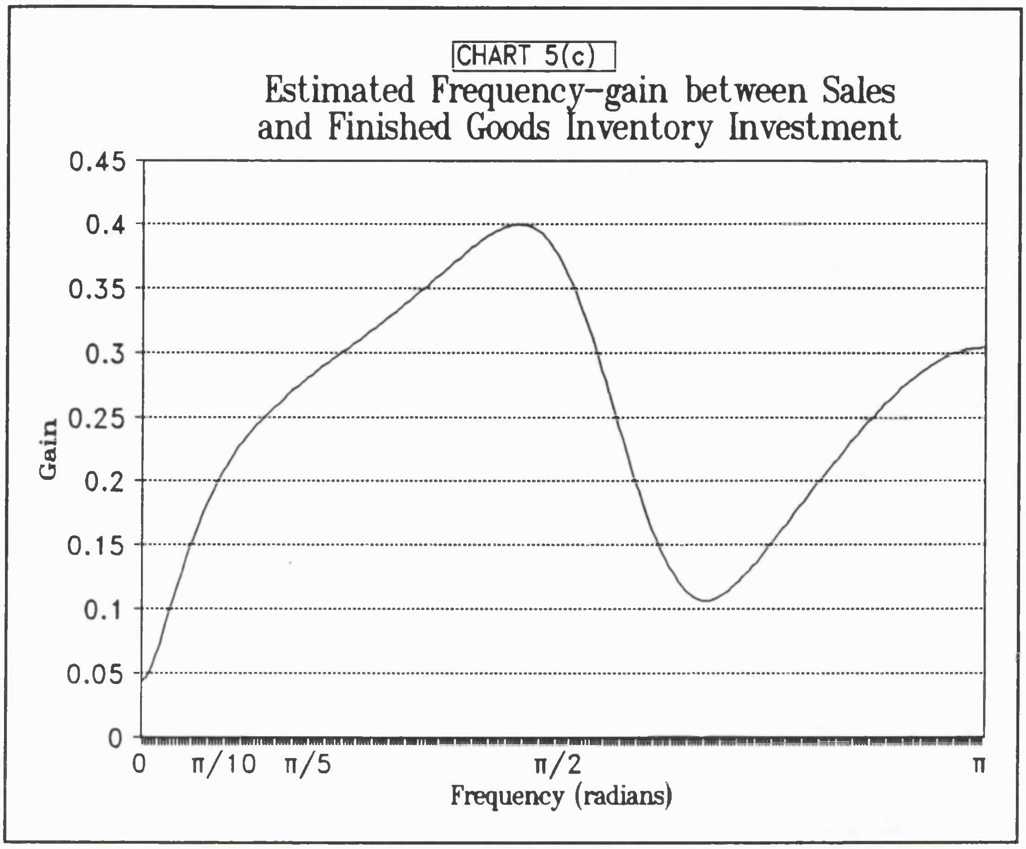


CHART 5(c)
Estimated Frequency-gain between Sales and Finished Goods Inventory Investment
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Gain
0
π/10
π/5
π/2
π
Frequency (radians)

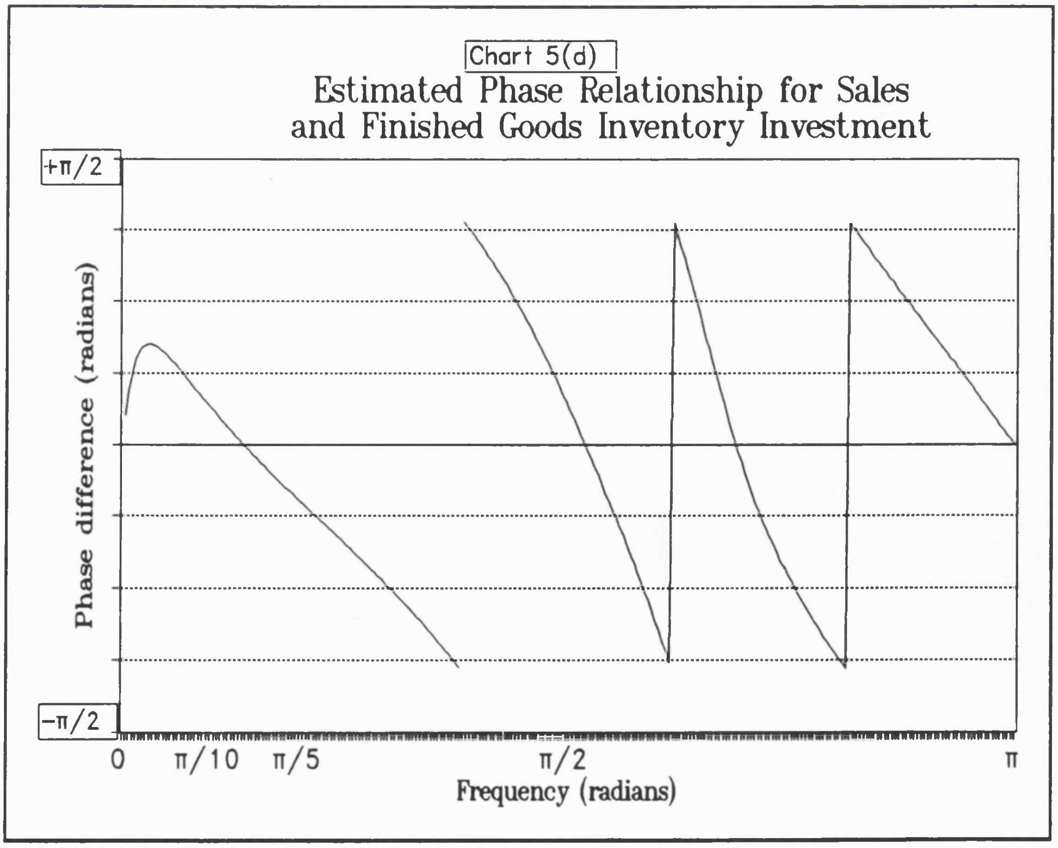


Chart 5(d)
Estimated Phase Relationship for Sales and Finished Goods Inventory Investment
+π/2
-π/2
Phase difference (radians)
0
π/10
π/5
π/2
π
Frequency (radians)

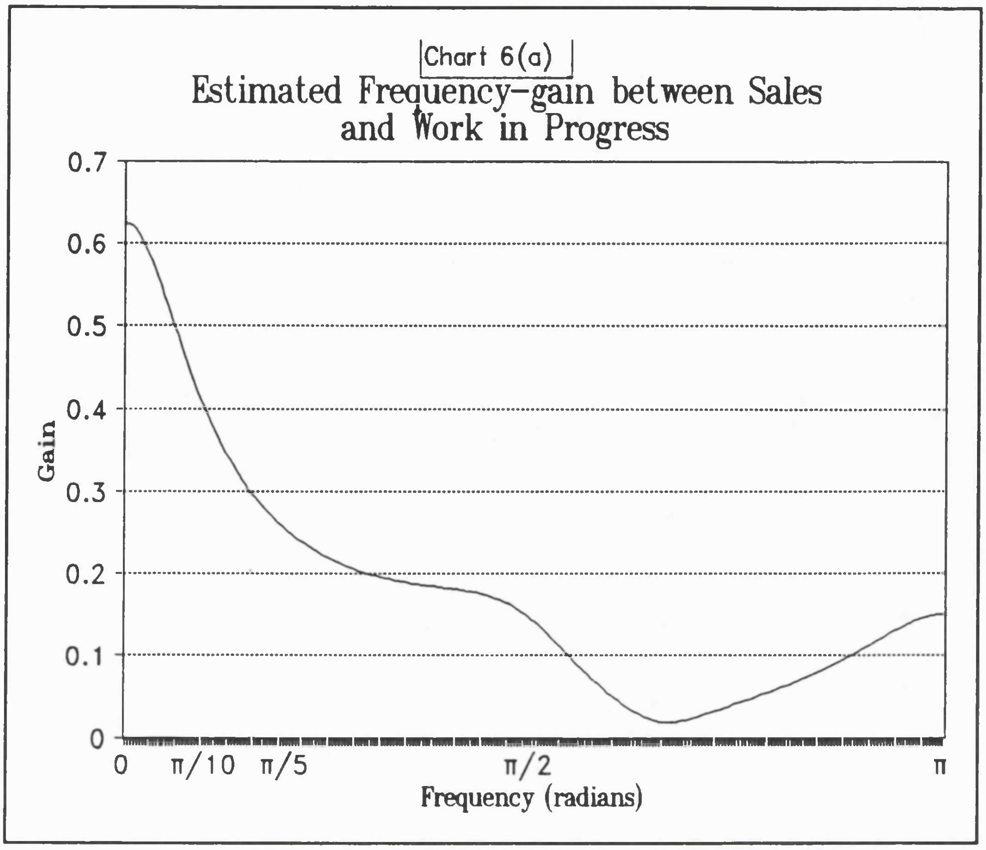


Chart 6(a)
Estimated Frequency–gain between Sales and Work in Progress
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Gain
0
π/10
π/5
π/2
π
Frequency (radians)

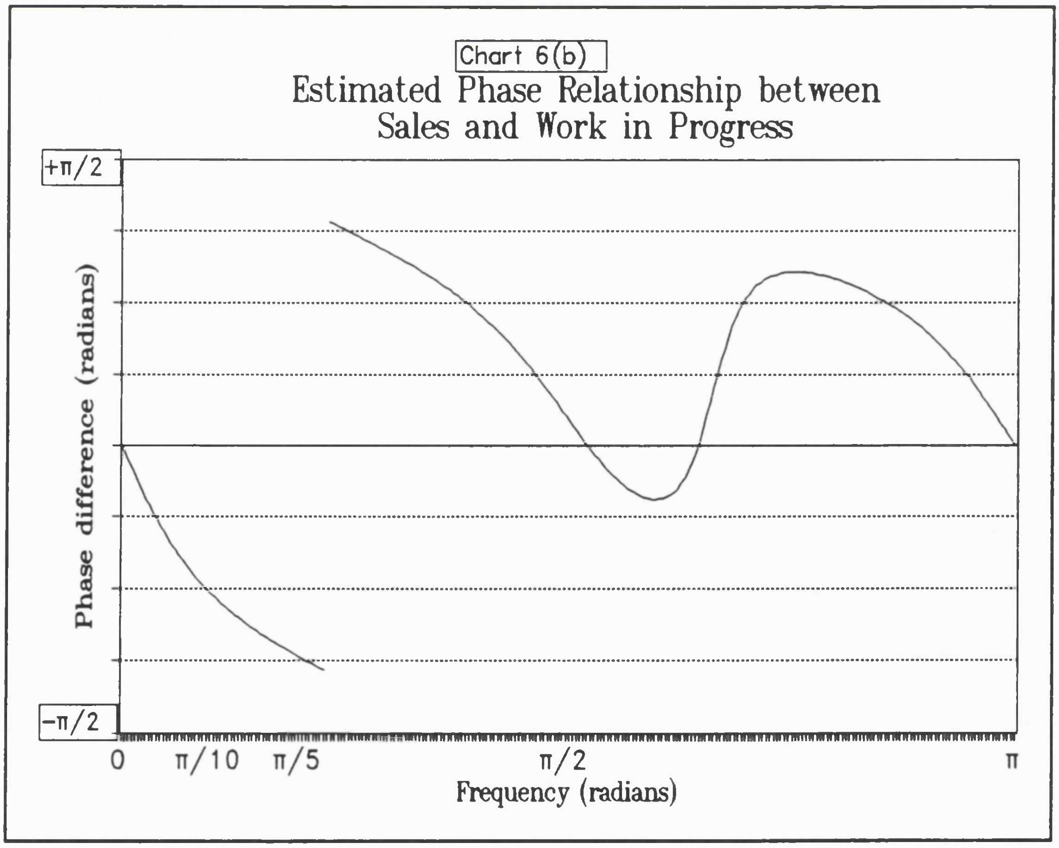


Chart 6(b)
Estimated Phase Relationship between Sales and Work in Progress
+π/2
−π/2
Phase difference (radians)
0
π/10
π/5
π/2
π
Frequency (radians)

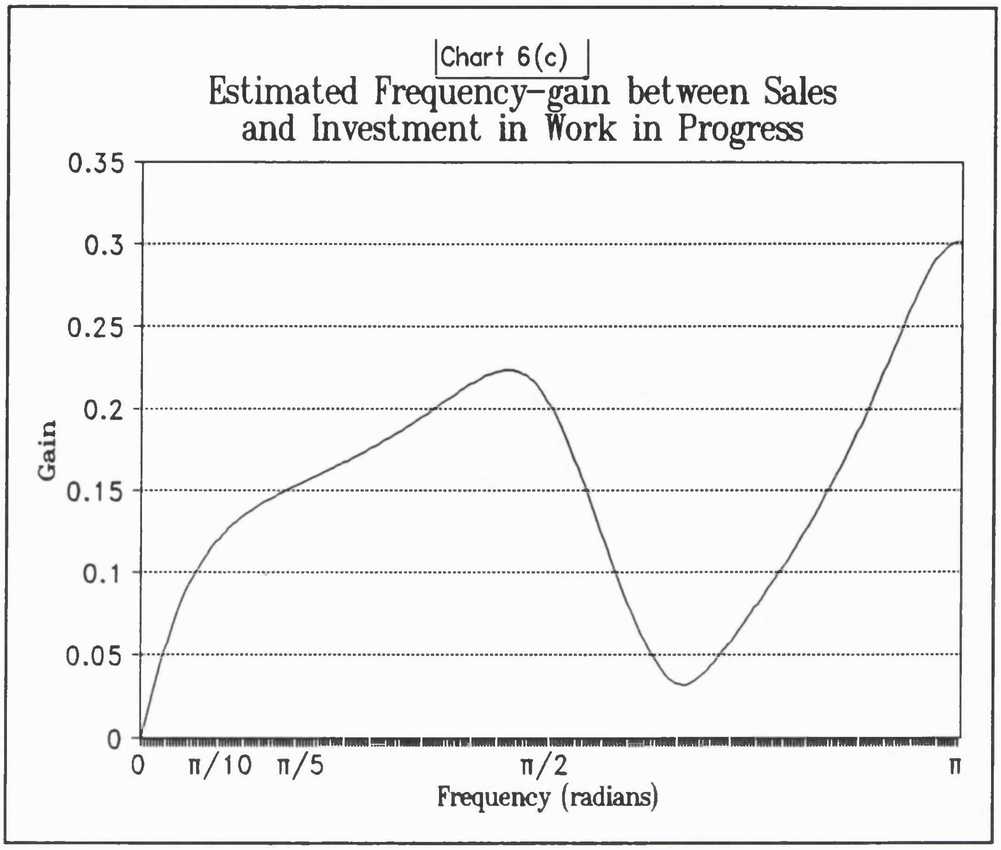


Chart 6(c)
Estimated Frequency-gain between Sales and Investment in Work in Progress
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Gain
0
π/10
π/5
π/2
π
Frequency (radians)

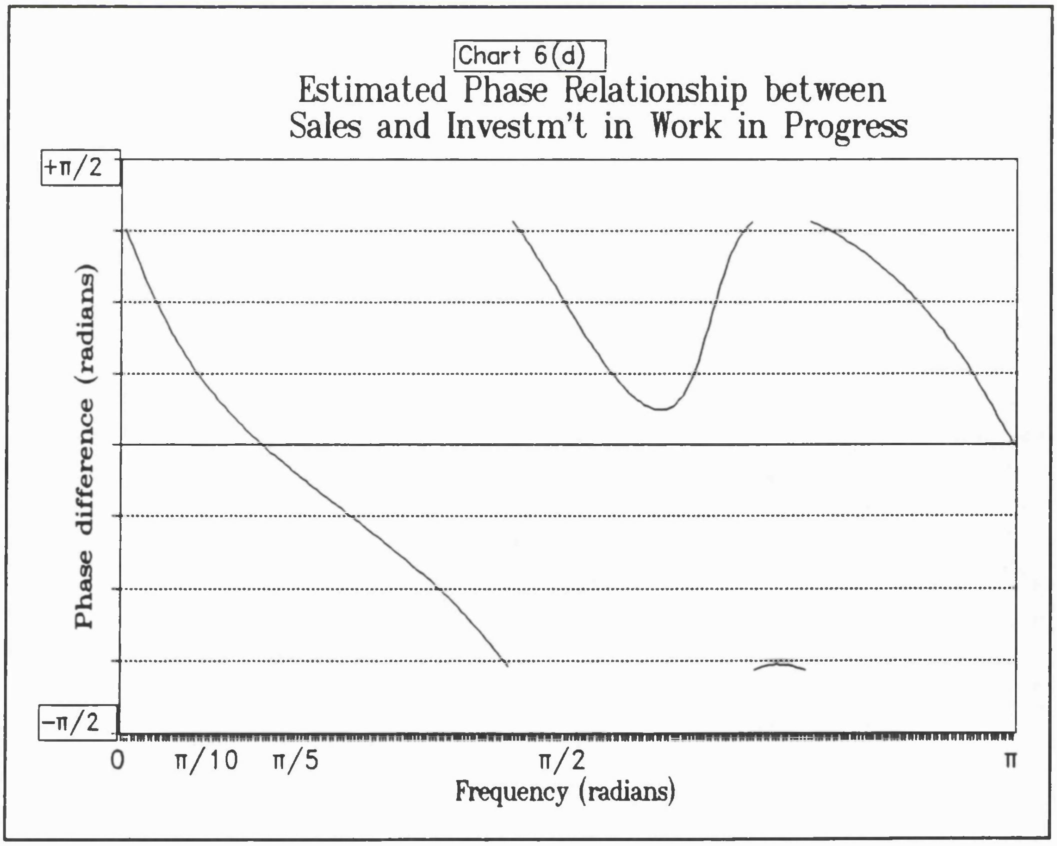


Chart 6(d)
Estimated Phase Relationship between Sales and Investm't in Work in Progress
+π/2
−π/2
Phase difference (radians)
0
π/10
π/5
π/2
π
Frequency (radians)

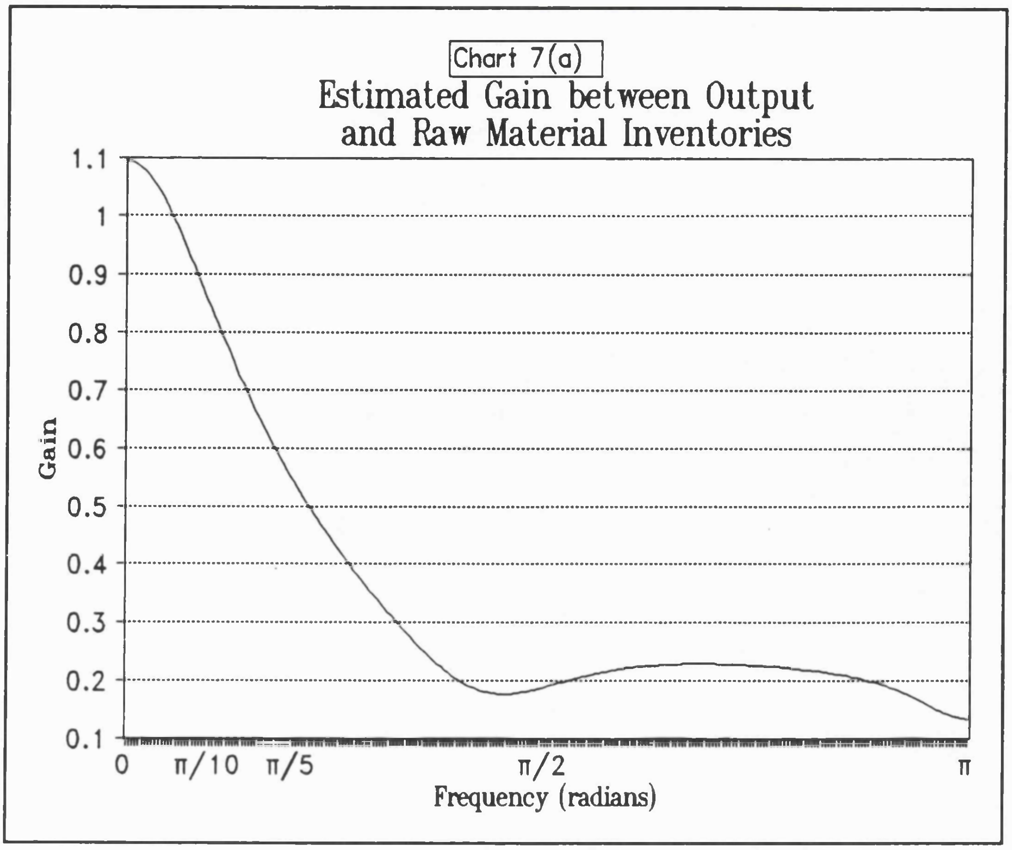


Chart 7(a)
Estimated Gain between Output and Raw Material Inventories
Gain
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
π/10
π/5
π/2
π
Frequency (radians)

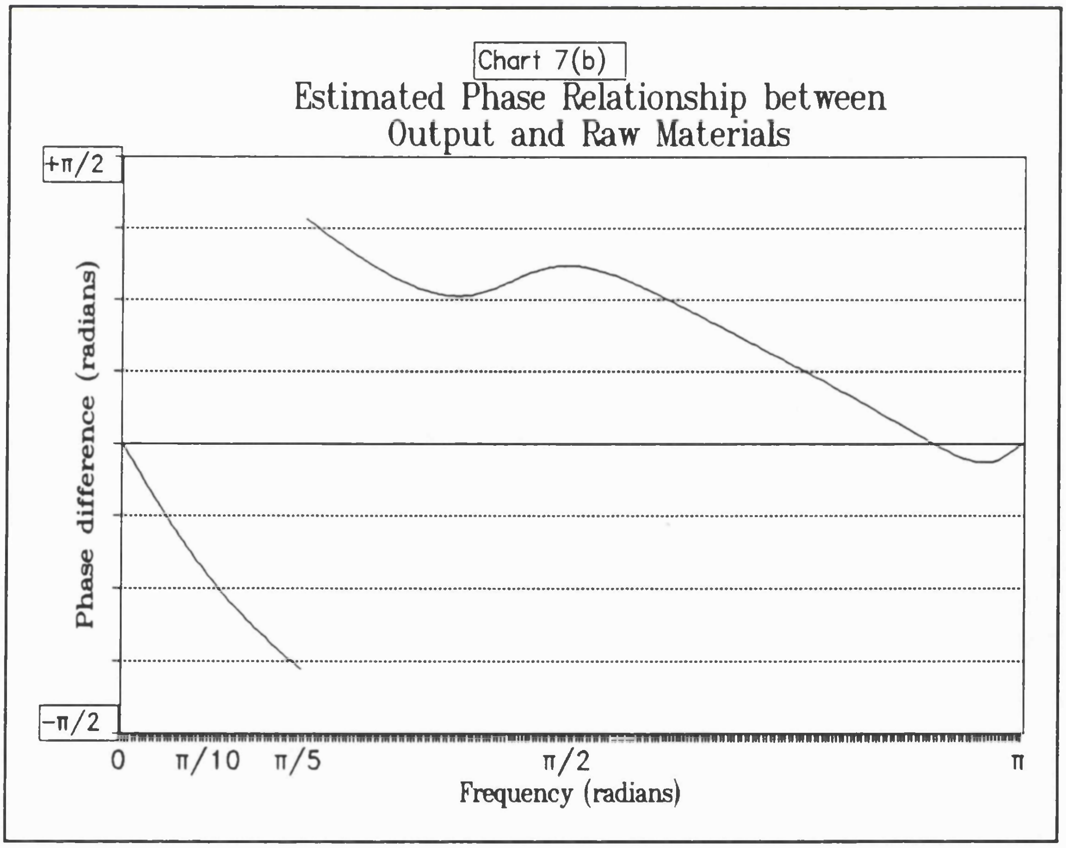


Chart 7(b)
Estimated Phase Relationship between Output and Raw Materials
Phase difference (radians)
+π/2
-π/2
0
π/10
π/5
π/2
π
Frequency (radians)

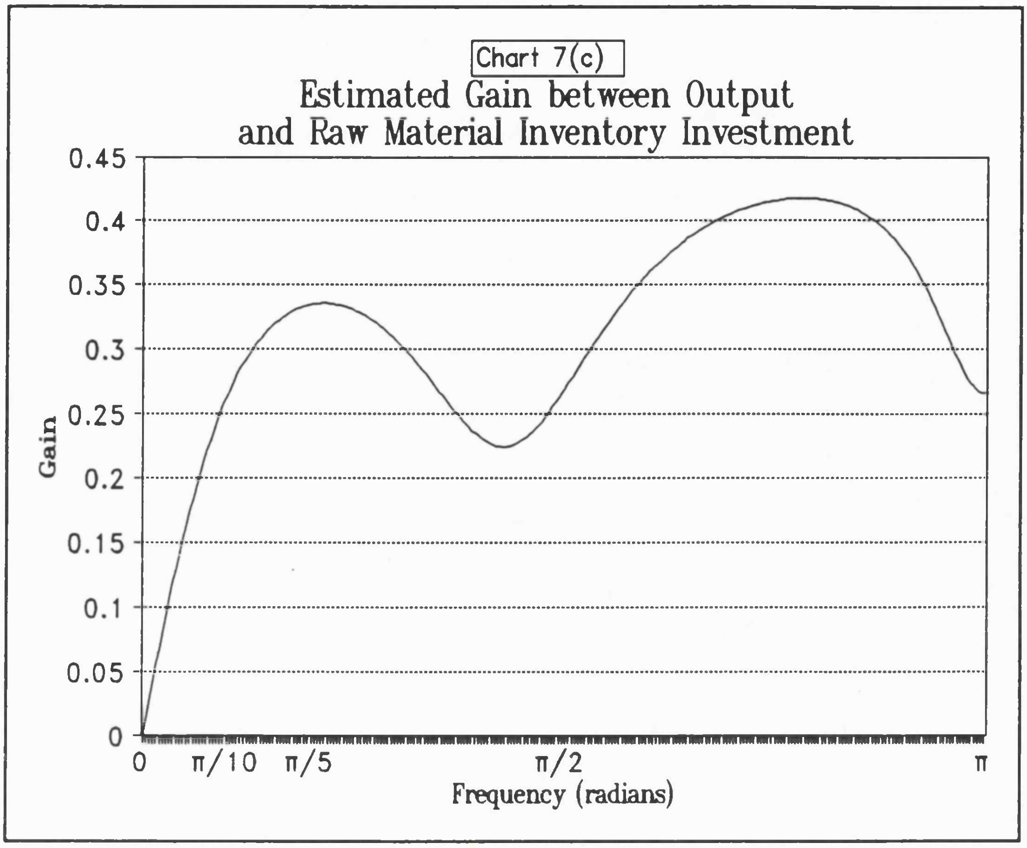


Chart 7(c)
Estimated Gain between Output
and Raw Material Inventory Investment
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Gain
0
π/10
π/5
π/2
π
Frequency (radians)

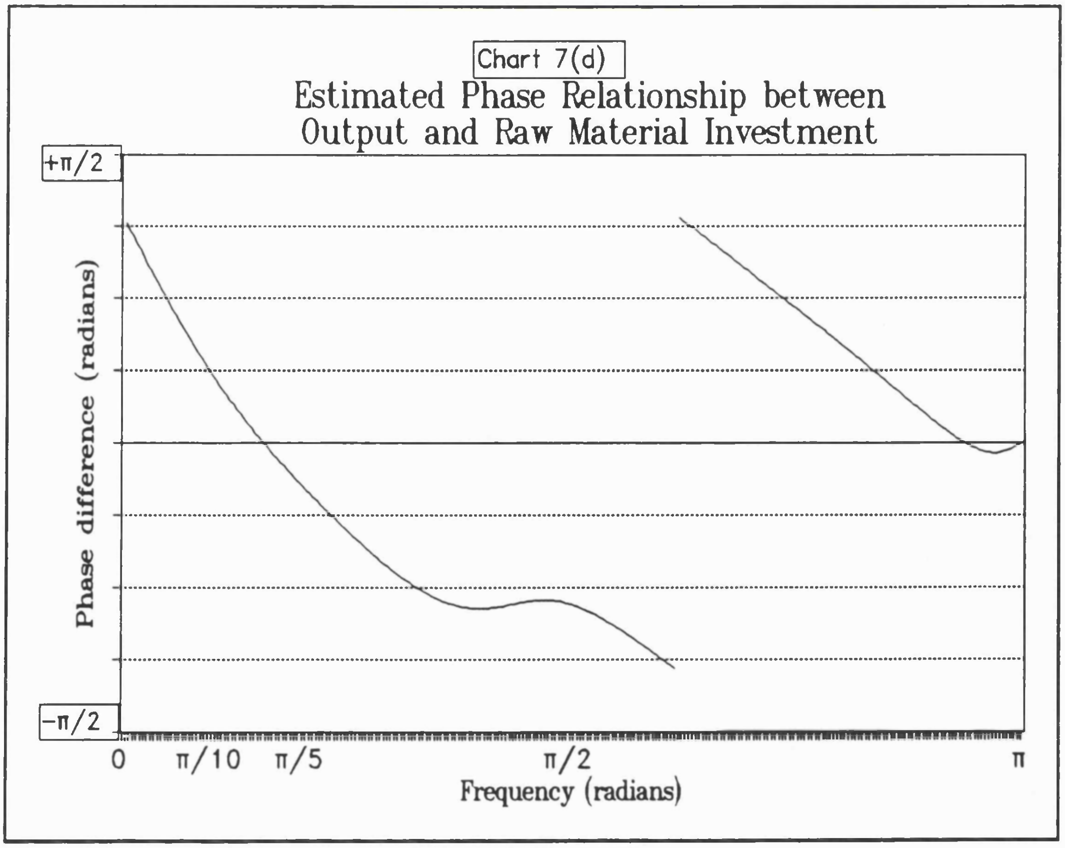


Chart 7(d)
Estimated Phase Relationship between
Output and Raw Material Investment
+π/2
−π/2
Phase difference (radians)
0
π/10
π/5
π/2
π
Frequency (radians)

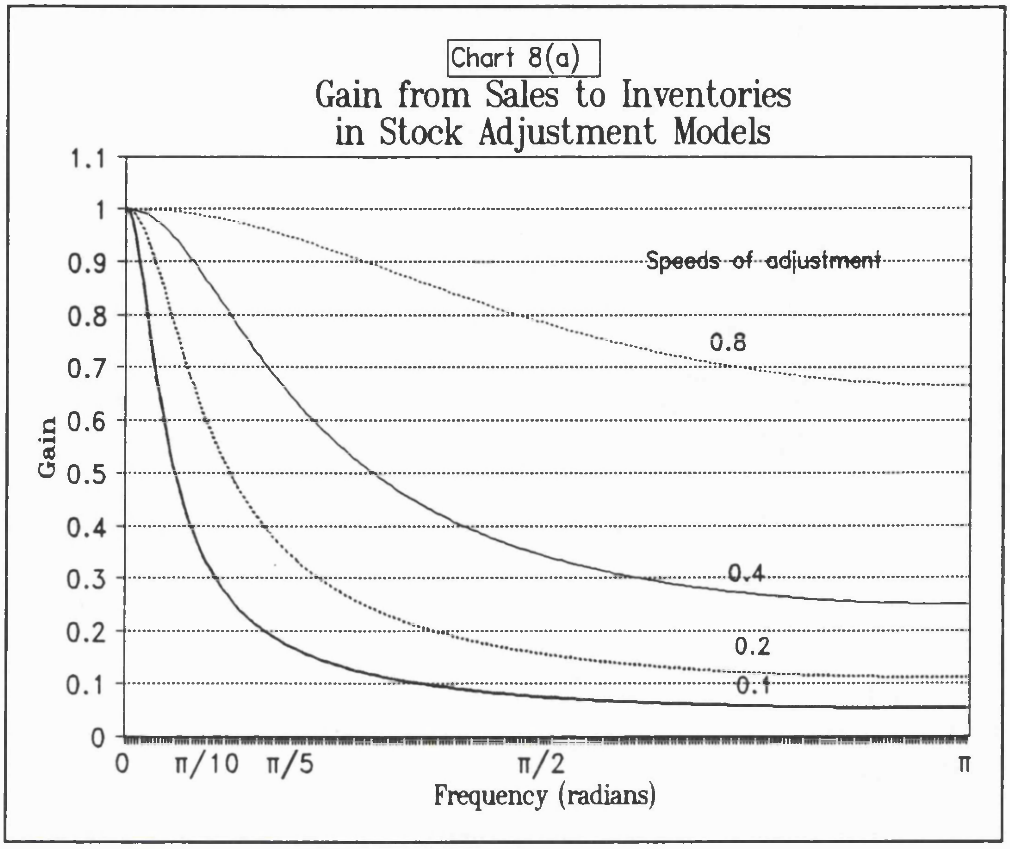


Chart 8(a)
Gain from Sales to Inventories
in Stock Adjustment Models
Gain
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Speeds of adjustment
0.8
0.4
0.2
0.1
0
π/10
π/5
π/2
π
Frequency (radians)

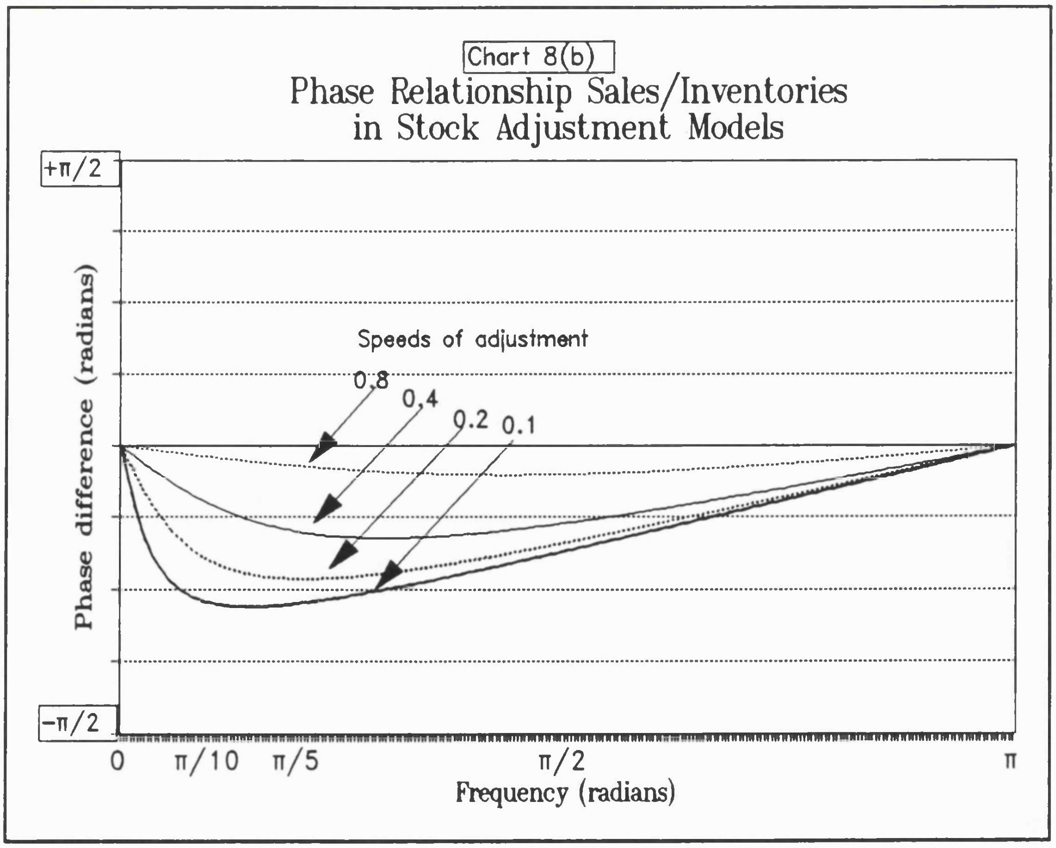


Chart 8(b)
Phase Relationship Sales/Inventories
in Stock Adjustment Models
+π/2
−π/2
Phase difference (radians)
Speeds of adjustment
0.8
0.4
0.2
0.1
0
π/10
π/5
π/2
π
Frequency (radians)

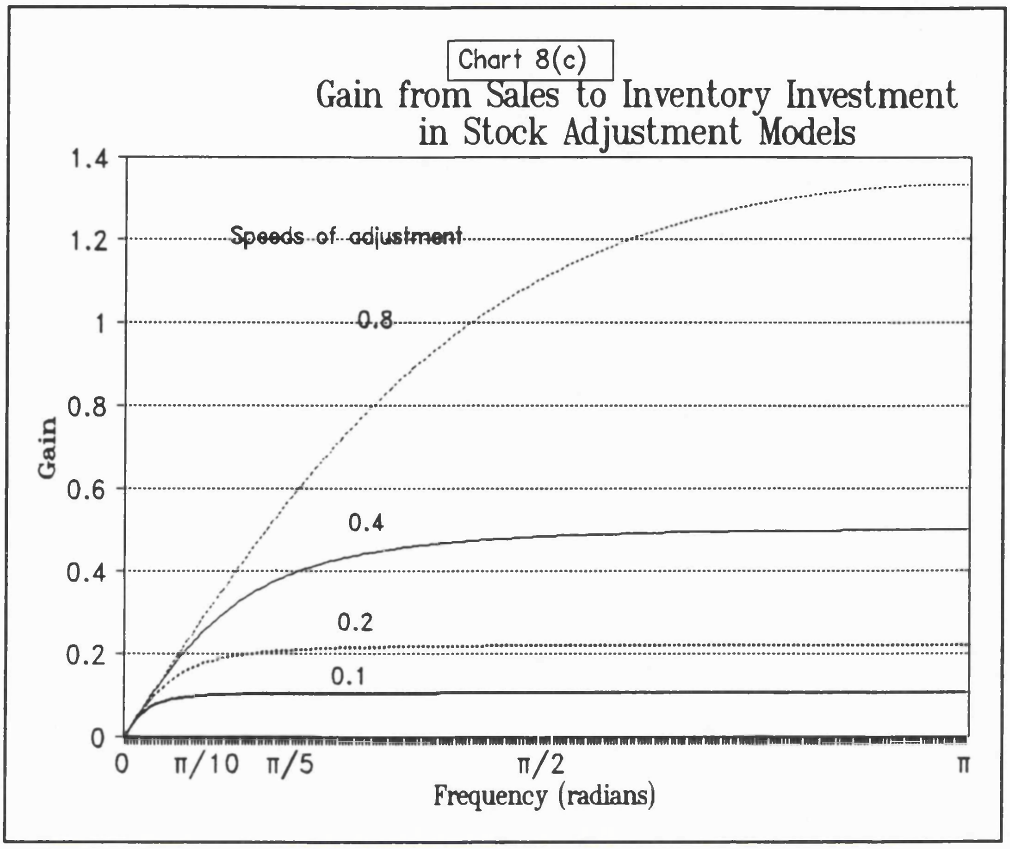


Chart 8(c)
Gain from Sales to Inventory Investment
in Stock Adjustment Models
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Gain
Speeds of adjustment
0.8
0.4
0.2
0.1
0
π/10
π/5
π/2
π
Frequency (radians)

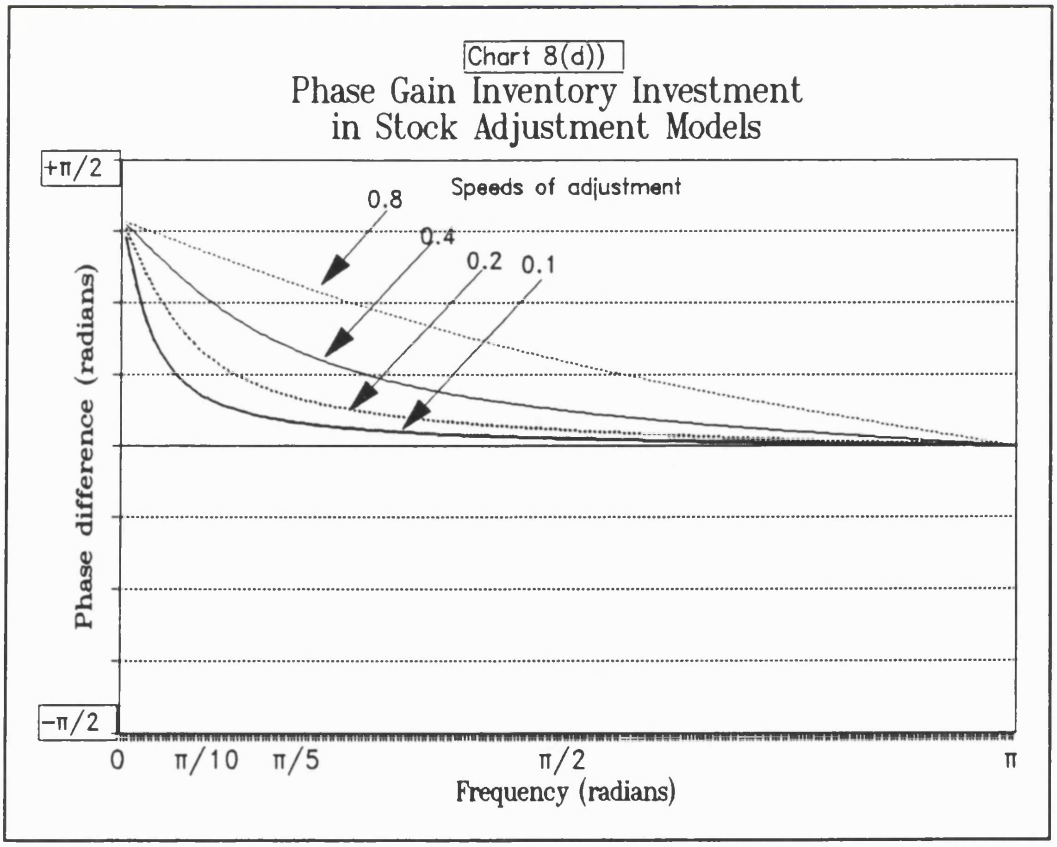


Chart 8(d))
Phase Gain Inventory Investment
in Stock Adjustment Models
+π/2
−π/2
Phase difference (radians)
Speeds of adjustment
0.8
0.4
0.2
0.1
0
π/10
π/5
π/2
π
Frequency (radians)

CHAPTER 3

THE PRODUCTION SMOOTHING MODEL OF INVENTORIES: A METHODOLOGICAL PERSPECTIVE

1. Introduction and outline

Over the past decade a substantial number of articles have appeared in the US which discuss manufacturers investment in inventories of finished goods, but no consensus on the determinants of this category of inventory investment has been reached. Production smoothing models have attracted a great deal of attention but are regarded as empirically unsatisfactory. No alternative model has gained general acceptance in their place. The interest in production smoothing models has eclipsed the much older programme of research into the cyclical movements of inventory investment.

This paper argues that the unsatisfactory state of this literature is, in large part, due to a weakness of econometric methodology. The careful application of tests of mis-specification, offered by the LSE econometrics, clarifies the reasons for the empirical failure of the production smoothing model. This indicates that the appropriate direction of future research is a return to the now unfashionable topic of explaining the cyclical behaviour of inventories.

The production smoothing literature

The production smoothing model of inventories, elaborated by Blinder and Fischer (1981), is the standard theoretical analysis of inventories of finished goods. It views production decisions as an inter-temporal optimisation and inventory holding decisions as the means by which production in one period may be used to meet sales in another period. With the usual assumption of convex costs of production this implies that inventories are used to smooth production relative to sales. Despite its analytical attractions a great deal of evidence, in particular the observation that the variance of production generally exceeds the variance of sales (the so called "excess volatility" of production), suggests that this simple model is inconsistent with observed inventory investment decisions.

Empirical studies (Blanchard(1983), West(1986), Miron and Zeldes (1988)) have mostly considered versions of the production smoothing model which combine production smoothing with a desire to maintain inventories as a target proportion of sales. Following Blanchard this model is referred to in this paper as the linear quadratic production smoothing model of inventories. The original formulation is due to Holt et al (1960). It is a model which combines quadratic costs of production, with costs of changing production and costs of allowing inventories to depart from a target ratio of sales.[6]

The linear quadratic production smoothing model has attracted attention because it is a potential explanation of the excess volatility of production, which maintains a production smoothing motive for holding inventories. It can do so because of the inclusion of a target level of inventories proportional to anticipated sales. This target can generate a variance of production which exceeds the variance of sales, provided only that the sales process is auto-correlated.[7]

Blanchard estimates the linear-quadratic production smoothing model, and shows that (in the version he estimates) it captures the movements of the data as well as a conventional partial adjustment specification (the stock adjustment model of Lovell (1959). This finding has led subsequent researchers to assume that the linear-quadratic production smoothing model is observationally equivalent to the stock adjustment model.

Subsequent contributions by West (1986) and Miron and Zeldes (1988) were much less favourable to the linear quadratic production smoothing model in doubt. They found that actual inventory behaviour worsens the estimated objective function relative to the simple alternative policy of maintaining inventories at trend level. As shown by West, this test can be captured by a simple variance inequality. Both West and Miron and Zeldes demonstrate the rejection of this inequality on US 2-digit SIC data. This failure is generally attributed to "excess volatility" and has prompted the development of a number of models in which the volatility of production exceeds the volatility of sales.[8] None of these alternatives has however commanded general acceptance.

The West variance inequality is the only test of model mis-specification which has been applied to the linear-quadratic production smoothing model. This has been a shortcoming because other standard tests of model mis-specification, such as the encompassing tests applied in this paper, may prove a more reliable guide than the West test to the construction of alternative models.

Most researchers in this field regard the linear quadratic production smoothing model as observationally equivalent to the conventional stock adjustment model (a partial adjustment specification) first estimated by Lovell (1961). This view appears to rest on the demonstration by Blanchard (1983) that his version of the linear quadratic production smoothing model yields a residual variance similar to that of the stock adjustment specification. A formal demonstration of observational equivalence has not however ever been made. The encompassing tests reported below indicate that not only are these models not observationally equivalent, but also that on UK data the linear quadratic production smoothing model is dominated by the stock-adjustment specification.[9]

The mis-specification testing of dynamic econometric models

The failure to apply a full range of mis-specification tests to the linear quadratic production smoothing model can be related to the contrast between two influential traditions of applied econometrics of the past twenty years. The new classical approach to econometrics associated with Christopher Sims, Tom Sargent and Lars Hansen, which has dominated applied research in the US, emphasises the goal of extracting the deep structural parameters which describe the tastes and technology governing the optimisation decisions of underlying agents, and the need to take account of forward looking expectations based on full use of available information. Estimation of dynamic optimisation models has typically been undertaken in this tradition. The LSE econometric methodology associated with Dennis Sargan and David Hendry emphasises instead the inadequacies of time series econometric modelling, given our short data samples and the lack of experimental evidence. The LSE approach indicates that all models must be assessed in terms of how well they approximate the unknown data generating process.

This paper demonstrates that mis-specification testing of empirical models of dynamic optimisation, of the kind proposed by the new-classical school, can be carried out by applying the procedures of the LSE approach to econometrics. This is a departure from the recommendations of, for example, Hendry, Pagan and Sargan (1984), who suggest that the dynamics of empirical models should be data determined and that role of theory should be the determination of long run relationships. While many proponents of the LSE econometric tradition eschew the use of theory for the determination of short run dynamics this is not a necessary feature of the LSE approach. The logic of the LSE approach can still be applied to models of dynamic optimisation.

The tests of mis-specification offered by the LSE approach may be considered as falling into two broad groups, tests based on residual auto-correlation and tests based on the encompassing principle. If the induced specification error (the discrepancy between the model and the unknown data generating process) exhibits serial correlation then the model is failing to capture some of the dynamics of the data. It is for this reason that residual auto-correlation is regarded as a indication of model mis-specification. This inference is however hazardous when applied to empirically estimated models of dynamic optimisation, because the model itself may induce a moving average error resulting from expectational errors of variables over a time horizon of greater than one period. Because expectational errors and the specification error are not separately identified standard formal tests of residual auto-correlation are not applicable (although an informal judgment about the degree of acceptable residual auto-correlation can still be made).

The encompassing principle emphasises that any successful model will be able to explain (encompass) the parameter estimates that emerge from the estimation of competing models. Mizon and Richard (1986) discuss the encompassing principle and show how it incorporates all forms of non-nested testing. For linear models, sharing a common dependent variable, the general encompassing tests simplifies to F-tests of restrictions within a general model nesting both of the competing models. This is the test applied below in section 5.

Note that the encompassing principle, as with all non-nested testing, do not always yield clear-cut results. A common result of applying the principle is that a model both encompasses and is encompassed by some competitor: such mutual encompassing does not indicate model mis-specification. It only indicates that the two competing models cannot be distinguished using the data set concerned. Failure to encompass a competing model, on the other hand, is an indicator of model mis-specification. If both models fail to encompass each other then neither can be considered a satisfactory approximation to the data generating process.

Even if the model under consideration has explicit theoretical foundations it is *not* necessary for the application of encompassing tests that the competing model has a similar theoretical pedigree. Adequate testing for model mis-specification requires that a proposed model be compared with all competing models, whatever their origins. Finally it should be noted that the encompassing framework is the appropriate test of a claim of observational equivalence between two models: if indeed they are exactly observationally equivalent then they will be mutually encompassing on all data sets, which will be revealed by a breakdown in the calculation of the encompassing tests.

Failure of a dynamic optimisation model to encompass a competing models indicates that the residual is not orthogonal to the information set used by the econometrician, violating the basic rational expectations assumption of such models. This property is conventionally examined by applying standard tests of over-identification (Sargan (1959), Hansen (1982)). The advantage of applying encompassing tests is that they gain improved power by utilising out of sample information about the specification of alternative models.

Outline of the paper

The paper begins, in section 2, by setting out an estimation framework which incorporates all the major competing models of inventory investment, allowing a comparison between recent models of inventories and the older tradition of stock adjustment models. This estimation framework includes not only the linear quadratic production smoothing model, and the stock adjustment model, but also the target-adjustment model of Feldstein and Auerbach (1976). The specification of the linear quadratic production smoothing model includes two error terms: one reflecting expectational discrepancies between the agent and the econometrician, the other being the specification error emphasised by the LSE tradition.

Sections 3 and 4 then discuss the methodology of estimating models, such as the linear quadratic production smoothing model, where short run dynamics are the outcome of an intertemporal optimisation with quadratic objective functions and forward looking expectations of future variables. A conventional procedure is to estimate first order conditions derived from such models using instrumental variable techniques. Section 3 shows that the results of this standard procedure are sensitive, in small sample, to the normalisation of the first order condition. While a full analysis of this issue is beyond the scope of this paper there are intuitive grounds for estimating the first order condition in the form of a decision rule, based on past variables and expectations of current and future variables, for the control variable of the underlying optimisation. This normalisation, which has not been applied by all other investigators of the hybrid production smoothing model, is the normalisation used for the empirical results of section 5.

Section 4 considers the mis-specification testing of such dynamic optimisation models. The LSE approach to econometrics offers a range of mis-specification tests based on an acknowledgement of the presence of a specification error in addition to other possible sources of stochastic error. This specification error reflects the inevitable failure of any empirical model to completely capture the data generating process and is why all econometric models must exhibit some residual error. The usual discussions of the estimation of models with forward looking expectations, including for example Hansen and Sargent (1980), do not recognise the possibility of specification error, effectively ruling out any systematic treatment of tests of mis-specification.

Section 4 also considers the status of the West variance inequality, in the context of other tests of model mis-specification offered by the LSE approach to econometrics. It is shown that the West inequality can be interpreted as the setting of some lower bound on the variance of the residuals from the estimated first order-condition.

Section 5 presents estimates of the linear quadratic production smoothing model using UK data and then considers standard tests of mis-specification. Parameter estimates are generally not very satisfactory. There is substantial residual auto-correlation in the hybrid production smoothing model. Encompassing tests are applicable even when both specification error and informational discrepancies apply. These indicate that the linear quadratic production smoothing model is encompassed by, but does not encompass, the stock adjustment model.

2. An estimation framework

This section reviews the linear quadratic production smoothing model, which features prominently in the literature since the influential paper by Blanchard (1983), and establishes a common estimation framework for comparison with the familiar stock adjustment model of inventory investment. This framework restricts attention to models of inventory investment which are linear in inventories, output and sales.

A standard view of investment in inventories of finished goods is that it is undertaken in order to smooth production relative to sales. This model has been elegantly elaborated by Blinder and Fischer as an explanation of the persistence of cyclical movements in output. This simple model is however inadequate because it fails to explain the fact that the variance of output exceeds the variance of sales for most manufacturing sectors.[10] This is inconsistent with the pure production smoothing model, in which inventories of finished goods are held solely to smooth the level of output.

This difficulty has focused attention on a linear quadratic production smoothing model in which inventories are held not only to smooth output but also because companies desire to maintain inventories near some target proportion of sales. This linear quadratic production smoothing model was developed originally by Holt et al (1960) for application to production and inventory control problems in operations research. It assumes that firms minimise the expected value of the discounted sum over all periods of the following one-period objective function:

$$\frac{1}{2}\left\{a_0 Q^2 + a_1(\Delta Q)^2 + a_2(I-a_3 S_{+1})^2\right\} \tag{2.1}$$

The first term represents quadratic costs of producing output above or below trend levels. The second term represents costs of changing the level of output, such as the costs of hiring or firing labour, or of setting up or removing capital equipment. The third term reflects a desire to maintain inventories as a target proportion of next period sales. It is a linear quadratic production smoothing model because it combines both convex costs of production and the cost of deviating from a target level of inventories.

The linear quadratic production smoothing model includes as a special case, when $a_1 = a_2 = 0$, the simple model in which inventories are held in proportion to expected sales. This simple model is of interest because is capable of generating "excess volatility" with the variance of production exceeding the variance of sales, provided only that sales exhibits positive first order auto-correlation. This suggests that the linear quadratic production smoothing model is more promising than a simple production smoothing model because it is can potentially explain the observation that the variance of production exceeds the variance of sales.

Estimation of the linear quadratic production smoothing model proceeds as follows. Given the objective function (1.1) an Euler condition relating the expected costs of an increase in inventories of finished goods in current and subsequent periods can be derived. With a discount rate ß the expected present value of the objective function is given by:

$$\mathscr{E}\left\{ \sum_{j=0}^{\infty} \frac{1}{2}\beta^{j}\left(a_0 Q^2 + a_1(\Delta Q)^2 + a_2(I - a_3 S_{+1})^2\right) \mid \Omega_t \right\} \qquad (2.2)$$

Differentiation with respect to the current level of inventories (assuming sales are constant) yields the following first order condition:[11]

$$\mathscr{E}\left\{ a_0(Q_t - \beta Q_t) + a_1\left(\Delta Q_t - 2\beta\Delta Q_{t+1} + \beta^2\Delta Q_{t+2}\right) + a_2(I_t - a_3 S_{t+1}) \mid \Omega_t \right\} = 0 \qquad (2.3)$$

This first order condition is satisfied provided that sales are unaffected by the amount of inventory investment. This will be the case for standard models of the firm, whether the firm is a price taker or faces a downward sloping demand curve.

Inventory investment decisions are an inter-temporal optimisation and hence the first order condition may be interpreted as an Euler equation. It indicates, for given expected sales, that the marginal cost of increasing production by a small amount in the current period, less the marginal benefit of increasing end period inventories, equals the marginal cost of production in the following period. Thus the firm is indifferent between scheduling marginal production in the current or the following period. This first order condition may be estimated using instrumental variable procedures, using instruments drawn from the information set available to the econometrician. This standard estimation procedure, first applied to this model by West (1986), is followed in this paper.

Only relative parameter estimates affect behaviour and therefore only relative parameter values can be recovered from estimation. Thus in order to conduct estimation some normalisation of the parameters is required. When any estimation technique other than FIML is adopted this normalisation affects the estimation results. The difficulties are discussed in section 2. The conclusion reached there is that in order to achieve better small sample properties and in order to allow comparison with other models of inventory investment it is best to express the linear quadratic production smoothing model using the decision variable normalisation rule in which inventories are determined by terms in sales and in past and future inventory holdings. This is derived (from (1.3) making use of the identity $Q = S + I - I_{-1}$) as:

$$I_t = \mathscr{E}\left\{\frac{C + D}{e} \; | \Omega_t\right\} + v_t$$

where: -

$$C = - a_0(S_t - \beta S_{t+1}) - a_1(\Delta S - 2\beta \Delta S_{t+1} + \beta^2 \Delta S_{t+2}) + a_2 a_3 S_{t+1} \tag{2.4}$$

$$D = -a_1(I_{t-2} + \beta^2 I_{t+2}) + [a_0 + 2a_1(1+\beta)](I_{t-1} + \beta I_{t+1})$$

$$e = a_0(1+\beta) + a_1(1+4\beta+\beta^2) + a_2$$

The final step is to adopt the normalisation by which the weighted parameter sum e, (the denominator on the right hand side of the decision rule) equal to unity, and to re-arrange terms in a_0 and a_1, yielding a linear regression for I_t:

$$I_t = \mathscr{E}\{a_0 F_t + a_1 G_t + a_2 a_3 S_t \mid \Omega_t\} + v_t \tag{2.5}$$

where: -

$$F_t = + I_{t-1} + \beta I_{t+1} - (S_t - \beta S_{t+1})$$

$$G_t = - I_{t-2} - \beta^2 I_{t+2} + 2(1+\beta)(I_{t-1} + \beta I_{t+1}) - (\Delta S_t - 2\beta \Delta S_{t+1} + \beta^2 \Delta S_{t+2})$$

v_t is a specification error, included because the model can only be an approximation to the unknown data generating process. The inclusion of this specification error reflects the adoption of the LSE approach to mis-specification testing, and forms the basis for the encompassing tests applied in the final section of the paper.

The estimation framework is now extended to cover the stock adjustment model of inventories. This simple partial adjustment model was first applied to inventories by Lovell (1961). A linear version of his specification is as follows:

$$I_t = (1-\theta)I_{t-1} + \theta b \mathscr{E}\{S_{t+1} | \Omega_t\} \tag{2.6}$$

where $0<\theta<1$ is the speed of adjustment parameter, and b is the desired ratio of inventories to next period sales.

The usual specification of the stock adjustment model is in terms of current, rather than expected future, sales. The reason for departing from the usual specification in the present paper is that an additional independent variable is then shared between the two specifications, increasing the power of the encompassing tests. A major flaw with the stock adjustment specification was pointed out by Feldstein and Auerbach (1973). This appears when the model is extended to include terms in sales surprises:

$$I_t = (1-\theta)I_{t-1} + \theta b \mathscr{E}\{S_{t+1} | \Omega_t\} + (1-\gamma)(S_t - \mathscr{E}\{S_t | \Omega_{t-1}\}) \tag{2.7}$$

The difficulty noted by Feldstein and Auerbach is an evident inconsistency between the estimates of the speed of adjustment parameter θ and the parameter γ. Estimated values of θ are typically less than 0.1, when estimated on quarterly data, suggesting a very slow speed of adjustment, whereas the parameter γ, which measures the speed of adjustment to a sales surprise, is typically greater than 0.9, suggesting that inventory holdings react very quickly to correct the draw-down of inventories arising through sales surprises.

Feldstein and Auerbach suggest what they refer to as the target adjustment model. Here inventory levels adjust instantaneously to their target levels, but the target itself adjusts slowly:

$$
\begin{aligned}
I_t &= I_t^* + (1-\gamma)\left(S_t - \mathscr{E}\{S_t|\Omega_{t-1}\}\right) \\
I_t^* &= (1-\mu)I_{t-1}^* + \mu b\mathscr{E}\{S_{t+1}|\Omega_t\} + \epsilon_t
\end{aligned}
\tag{2.8}
$$

An observable relationship can be derived by substitution, (where $w_t = S_t - \mathscr{E}\{S_t|\Omega_{t-1}\}$):

$$
I_t = (1-\mu)I_{t-1} + \mu b\mathscr{E}\{S_{t+1}|\Omega_t\} + (1-\gamma)(w_t - (1-\mu)w_{t-1}) + \epsilon_t \tag{2.9}
$$

The subsequent estimation makes no attempt to measure contemporaneous sales surprises. Estimation is by instrumental variables using as instruments the observable subset of the information set Ω_{t-2}. This is the simplest possible estimation procedure. While general method of moments estimation would deliver more efficient estimation, under the null of correct specification, this procedure is still consistent and is all that is required for the encompassing tests conducted at the end of the paper. A consequence of applying this method is that the sales surprises are indistinguishable from the current error terms and, in the estimation framework of this paper, the target adjustment model is distinguished from the standard stock adjustment model only by the presence of negative first order residual auto-correlation. The estimation framework covers both models.

There is a case to be made for a logarithmic instead of a linear functional form for the stock adjustment model.[12] A log linear specification of the stock adjustment model is usually be preferred to a linear specification because of data admissibility; a log linear specification automatically ensures that the non-negativity constraint on inventories holds, and it is in this form that the stock adjustment model has usually been estimated. A linear specification is preferred here because a linear specification of both the linear quadratic production smoothing and the stock adjustment model increases the power of the encompassing tests.

3. Alternative normalisations of the estimated euler equation.

Parameter normalisation and instrumental variable estimation

This discussion of the estimation framework has left unresolved two issues of econometric methodology. The first arises because in estimating a model with a formally specified objective function, such as the linear quadratic production smoothing model, only relative parameter estimates can be derived. This implies some normalisation of the parameters in order to undertake estimation, or equivalently some choice of dependent and independent variables. But this choice of dependent and independent variables affects the relative parameter estimates (for any estimation technique other than FIML) when the first order condition is over-identified.

Note that the linear regression derived, in the previous section, for the estimation of the linear quadratic production smoothing model, is not that estimated by West (1986). He instead estimates a linear regression with $\beta Q_{t+1} - Q_t$ as the dependent variable, corresponding to a different normalisation of the first order condition than that applied in section 1 of this paper. The present section demonstrates that this normalisation affects the estimation results to a non-trivial extent, and argues that the normalisation adopted in this paper, which yields a linear regression in the form of a decision rule for inventories, is to be preferred.

Beginning with any quadratic objective function the derived first order condition (which in the context of an intertemporal optimisation can be interpreted as an Euler equation) can be written as:

$$\mathscr{E}\{\ \omega'\beta\,|\,\Omega\ \} = 0 \tag{3.1}$$

β is the k element parameter vector in the objective function. ω is the k element vector of variables which appear in the objective function. Ω is the information set available to the optimising agent.

The estimated equation is a sample analogue to this first order condition:

$$\begin{gathered} W'\beta = u \\ \underset{n\to\infty}{Plim}\left\{ \frac{1}{n}Z'u=0 \right\} \end{gathered} \tag{3.2}$$

W is the n by k matrix of observations of ω. u is the n element vector of residuals. u is the sum of both expectational errors and the specification error emphasised by the LSE methodology. Estimation is by instrumental variables, using the subset (Z) of the full information set (Ω) available to the econometrician as instruments.

Some linear constraint or normalisation must be imposed on ß and only relative parameter estimates are obtainable. A common normalisation is to set the value of one of the parameters equal to 1. More generally it is possible to assume some linear constraint amongst the parameters. parameters from the first order condition.

In either case the normalisation corresponds to a choice of dependent and independent variables for the subsequent instrumental variables estimation. Thus the normalisation $\beta_i=-1$ corresponds to the choice of W_i as the dependent variable (y), keeping all other columns of W as independent variables. Formally W is partitioned into {y:X}. IV regression of y (W_i) on X (W_j j=1,...i-1,i+1,...k) yields a consistent estimate of the parameter vector ß. However any linear combination of the columns of W can also be chosen as the dependent variable.

This correspondence, between the choice of parameter normalisation and the selection of dependent and independent variables, is not one-to-one. This is because each parameter normalisation can be estimated with a range of possible choices of independent variables.

In all cases, after normalisation, the sample analogue to the first order condition can be re-expressed as:

$$\begin{gathered} y = Xb + u \\ \underset{n\to\infty}{Plim}\left\{ \frac{1}{n}Z'u=0 \right\} \end{gathered} \tag{3.3}$$

When the equation is exactly identified, then the IV estimator is:

$$\hat{b} = (Z'X)^{-1}Z'y \tag{3.4}$$

If, on the other hand, the equation is over-identified, with q > k-1 instruments, then the instrumental variable estimator may be generalised as:

$$\hat{b} = (MZ'X)^{-1}MZ'y \tag{3.5}$$

where M is a k-1 x q weighting matrix. Note that M can depend on X and Z. Sargan (1958) establishes that the choice:

$$M = X'Z(Z'Z)^{-1} \tag{3.6}$$

is asymptotically efficient in the sense that the choice of any other weighting matrix results in an asymptotic variance-covariance matrix which exceeds that obtained from (2.5) by a positive semi-definite matrix.[13] This choice is the generalised instrumental variable estimator (GIVE). Moreover the asymptotic variance-covariance matrix does not depend on the normalisation chosen. Thus asymptotically the normalisation does not matter.

However in practice, for finite sample reasons, the normalisation does not matter. It alters the (relative) parameter estimates whenever there are more than k-1 instruments. One way of expressing this point is to view generalised instrumental variable estimation as a method of moments estimator. The set of instruments is chosen so that the residual vector and the instrument set are uncorrelated (this is usually ensured by the assumption of rational expectations formation). The method of moments interpretation of instrumental variable estimation is that the parameter vector is chosen so as to set the sample correlations between the induced residuals and the set of instruments as close as possible to zero. If the equation is exactly identified (with the number of instruments - the members of the econometrician's information set - equal to k-1) then the k-1 sample correlations can all be set exactly equal to zero by the appropriate choice of relative parameter values. The same relative parameter estimates and the same induced residuals emerge from all possible normalisations. Thus the implied decision rule and the relative parameter estimates must be the same for all normalisations.

If however the equation is over-identified (with the number of instruments exceeding k-1) then the parameter vector cannot be chosen so that all sample correlations between the induced residuals and the instruments are zero. It is only possible to choose the k-1 x 1 parameter vector so that the induced residuals exhibit zero sample correlations with a set of k-1 linear combinations of the instruments. Some weighting of the instruments must be made (or equivalently some metric must be chosen for the measurement of the correlation between the induced residuals and the instruments). This is the role of the matrix M in the formal statement of the generalised IV estimator (2.5). It is in these circumstances, when the equation is over-identified, that the estimation results depend on the chosen normalisation.

A useful interpretation may be given to the weighting matrix M in terms of canonical correlations, which brings out this last point. The matrix M is such that it chooses the k-1 linear combinations of the q instruments Z which are the best linear predictors of the k-1 independent variables. It is in this sense that the weighting of instruments is made so as to yield the maximum correlation with the independent variables of the estimated first order condition. But the set of independent variables and the choice of dependent variable depend on the normalisation of the first-order condition. Hence the weighting of the instruments and thus the relative parameter estimates are sensitive to the chosen normalisation. The problem does not arise with full-information maximum likelihood estimation. Consider the statement of the complete system in the usual matrix notation:

$$BY + \Gamma X = U \tag{3.7}$$

From this it is clear that the likelihood function is unaffected by any re-scaling of the parameter matrices B and Γ ; hence the results of FIML estimation are unaffected by the chosen normalisation.[14]

Asymptotically the dependence of the estimated parameter vector on the choice of dependent and independent variables will not matter if the over-identifying restrictions for the first-order conditions are satisfied. In this case the asymptotic distribution of the resulting parameter estimates is not sensitive to the normalisation chosen. The different normalisations are all consistent estimates of the relative parameter vectors if the model is not mis-specified (Sargan (1958)). The Sargan analysis also yields the standard test of the over-identifying restrictions which is distributed chi-squared (q-k+1) when the model is correctly specified. Thus from an asymptotic point of view it does not matter which normalisation is used, provided the Sargan test of over-identifying restrictions is satisfied.

Alternative normalisations of the production smoothing model

The effect of choosing different normalisations is illustrated by presenting different estimates of the linear quadratic production smoothing model (the data are as described in section 4.) Alternative normalisations applied to the production smoothing model are those of West (1986) and Blanchard (1983) (though as noted above Blanchard uses FIML estimation, so that his estimates are not affected by the implicit normalisation). West (1986) uses the normalisation:

$$a_0 + a_1(1+\beta) = 1 \tag{3.8}$$

He applies this to yield the following linear relationship for estimation (where $q_t = \beta Q_{t+1} - Q_t$):

$$q_t = a_1(\beta q_{t+1} + q_{t-1}) + a_2 I_t - a_2 a_3 S_{t+1} + u_t \tag{3.9}$$

As an illustration that the parameter normalisation, and choice of dependent variables, are not in one to one correspondence, note that the same parameter normalisation can be used instead to substitute out a_1 using $(1+\beta)\, a_1 = 1 - a_0$. This yields the estimated equation:

Another possible normalisation is that used by Blanchard (1983) $a_1=1$. He applies maximum likelihood estimation so his estimation results are not affected by

$$r_t = a_0(r_t - (1+\beta)q_t) + a_2(1+\beta)I_t - a_2a_3(1+\beta)S_{t+1}$$
$$\text{where:} \quad r_t = -\Delta Q_t + 2\beta\,\Delta Q_{t+1} - \beta^2\Delta Q_{t+2} \tag{3.10}$$

the normalisation, but if a limited information estimation technique is instead followed then that normalisation yields the estimable equation:

$$r_t = -a_0q_t + a_2I_t - a_2a_3S_{t+1} + u_t \tag{3.11}$$

Table 1 presents IV estimation results for these three alternative estimable equations. The data, described in section 4, is for total UK manufacturing. The main message of this table is the extent to which these results differ both from each other. The standard errors of the equations (relative to the mean of the level of inventories) range from 4.4% to 8.4%. The relative parameter values and the precision of the parameter estimates vary greatly from one normalisation to another. Yet for only one of these estimations does the Sargan statistic indicate violation of the over-identifying restrictions at the 95% significance level (15.51) (although in this case it is highly significant with a value of 31.9). These contradictory results illustrate how over-identification can lead to different results, depending on the normalisation chosen, even when formal tests of the over-identifying restrictions are passed. It is noteworthy how similar regression (i) is to the same regression reported by West (1986) using monthly US data. As in his paper none of the directly estimated parameters are significant, but the derived estimate for a_0 is highly significant with a plausible value.

Grounds for preferring a decision variable normalisation

There are grounds for preferring one amongst all the possible normalisations. Under the null of correct specification then asymptotically the choice of normalisation does not matter. The normalisation may still matter if it affects either (i) the asymptotic power of tests of mis-specification or (ii) estimation results in small samples. A full discussion of this issue is beyond the scope of this paper. Nevertheless the adoption of the LSE approach suggests that the preferred normalisation is that in which the estimated equation corresponds to the decision rule used by the optimising agent. This may be referred to as the decision variable normalisation. The following arguments support the choice of this normalisation.

Improved small sample performance. The decision rule normalisation can improve small sample performance by minimising the correlation between the instruments and the error term. Although the instruments, if valid, are chosen to be independent of the error on the equation, it is still the case that, due to sampling variation, some of the instruments are correlated with the error term in finite sample. GIVE weighting maximises the canonical correlation between the instruments and the independent variables, and hence results in finite sample correlation between the weighted instrument set and the error term. The decision rule normalisation is to be preferred on the grounds that it minimises this finite sample correlation.

The reason it does so is simply that, if the model is correctly specified, the decision rule normalisation reflects the optimisation decisions by which other, weakly exogenous, variables determine the decision variable; in the present context this is the determination of inventories. Hence both expectational errors (reflecting the informational advantage of the optimising agent) w_t and specification error v_t are correlated more highly with the decision variable than with any of the independent variables in the decision rule. The presence of the specification error on the right-hand side of the decision rule implies that this normalisation delivers better small sample performance.

Substantial measurement errors on one of the independent variables of the decision rule relative to measurement errors on the dependent variable can alter this conclusion. In this case it is possible that the error on the estimated equation is most highly correlated with this variable than with the dependent variable of the decision rule. In the case of inventories, for which the data is of poor quality, the worst measurement errors are likely to be on the dependent variable and this objection is unlikely to apply.

Zero parameter values. The second reason for adopting the decision rule normalisation is the possibility of zero parameter values, even when the model is correctly specified. Suppose that the normalisation is made around a parameter β_i, which in fact is zero. This implies that the dependent variable will in fact be unrelated to the independent variables, even when the model is correctly specified. Normalising the first order condition as a decision rule avoids this problem, because the normalisation must be a linear combination of all the parameters in the model.

Encompassing. The final reason for adopting the decision variable normalisation is that this normalisation facilitates the formal testing of encompassing relationships between the model with an explicitly stated objective function and other competing models. The reason for comparing the performance of competing models is that they all claim to be good approximations to the data generating process determining the decision variable. It is therefore appropriate to estimate all these models with this as the dependent variable. This consideration again suggests choosing the decision rule normalisation.

4. Mis-specification tests of models based on dynamic optimisation theory

The methodology of mis-specification testing.

The main grounds for rejection of the linear quadratic production smoothing model in the US literature has been the failure of the West variance bounds test (West (1986)) but the model has not been subjected to other tests of mis-specification. This section discusses the mis-specification testing of econometric models derived from dynamic optimisation theory. It then examines the West variance inequality test and its relationship to other standard tests of model mis-specification.

The LSE econometric methodology emphasises that any model is only an approximation to the unknown data generating process. No estimated model can fit the data exactly. All models induce some residual error which will be referred to here as specification error. Mis-specification testing of econometric models derived from quadratic objective functions is only possible once this source of stochastic error is recognised. The LSE approach emphasises that all models are subject to this induced specification error; and the properties of this induced error (induced that is by the model specification) are crucial to assessing whether the model is in fact mis-specified. If the specification error is correlated, either with its own past, or with weakly exogenous variables, this is taken as evidence of mis-specification.

Specification error is not the only possible source of stochastic error. Two others are measurement error and an informational advantage over the investigator possessed by the optimising agent. Each of these provide a further reason why the first order condition estimated using observed data is not exactly satisfied. Hansen and Sargent (1980), while not recognising the possibility of specification error, offer two alternative interpretations of this informational advantage. The first is that it reflects some underlying disturbance to the objective function, which they assume to take the form of an AR(q) process. Hansen and Sargent derive the maximum likelihood estimator of the underlying parameters under this first assumption, taking account of cross-equation restrictions with the process determining the forcing variables. In so doing they show that the estimated decision rule has an ARMA(q,q-1) error term. It is this technique that was applied by Blanchard in his estimate of the linear quadratic production smoothing model. Blinder (1986) discusses whether such an unobserved disturbance (which he refers to as a "cost shock") can be said to explain the excess volatility of production.

The difficulty with this unobserved disturbance interpretation of the stochastic disturbance is that, unless there is some arbitrary restriction on the order of the AR process, the specification error can no longer be identified. All models provide an adequate approximation to the data generating process, mis-specification testing is no longer possible and there remain no grounds for choosing amongst competing models. Thus this interpretation of the stochastic error is fundamentally inconsistent with the LSE methodology applied by this paper.

Hansen and Sargent (1980) also discuss an alternative interpretation of the error term as reflecting information, unavailable to the econometrician, on which the optimising agents condition their expectations of future variables. Remaining expectational error must be orthogonal to the econometrician's information set so instrumental variable estimation is appropriate, in the manner of McCallum (1976). Where expectational discrepancies between the agent and the econometrician are not resolved within a single period there is the possibility that the expectational errors have an MA component, and in this case the standard errors of the instrumental variable estimates are inconsistent, suggesting the use of the Cumby, Huizinga and Obstfeld (1983) correction of the standard errors, or the application of the GMM estimator of Hansen (1982).

Mis-specification testing can still be conducted when expectational error orthogonal to the econometrician's information are combined with specification error. However there are difficulties in interpreting tests of residual auto-correlation, since these can reflect an MA component in the expectational error as well as auto-correlation of the specification error. The usual tests of mis-specification based on residual auto-correlation can only be applied when there is no-moving average error generated by errors in the expectations of variables more than one period ahead. This will be the case for the present model only if (i)the agent shares the same information set as the econometrician or (ii) the agent has perfect foresight and the model is estimated by ordinary least squares.[15]

A key aspect of testing for model mis-specification is to assess model performance relative to other competing models, and this is most appropriately conducted using the encompassing tests of Mizon (1983) and Mizon and Richard (1983). These tests, whose results depend on the correlation between the residual and weakly exogenous variables suggested by the competing model, are applicable even when there is auto-correlation of the residuals, so in applying these tests there is no need to make the simplifying assumptions required to conduct tests of residual auto-correlation. When comparing two linear models with a common dependent variable, as in the estimation framework set out in section 2, the encompassing test simplifies to an F-test of the restriction within a general specification which nests both competing models.

The advantage of applying the encompassing principle to estimated dynamic optimisation models, stems from their improved power in tests of the orthogonality of the stochastic residual. Standard tests of over-identification (Sargan (1959), Hansen (1982)) suffer from lack of power when the instrument set is even moderately large, because the size of the test must allow for chance correlation between the residual and the instruments. Encompassing tests obtain improved power by utilising out of sample information drawn from research on other data relating to different countries and different time periods. Thus the stock adjustment specification emerges from the work of Lovell (1959) on US post-war data, not from data-based specification search over the sample used for the encompassing tests. By entertaining this specific alternative model the dimensionality of the test is greatly reduced (from the 9 over-identifying restrictions examined by the Sargan test to the 1 degrees of freedom of the test that the linear quadratic production smoothing model encompasses the stock-adjustment model) and hence yields a considerable increase in power.

The other major advantage of the encompassing principle is, that by emphasising comparison with competing models, it encourages a progressive programme of research on models which can explain all competing formulations and hence explain all aspects of the data generating process.

The West variance inequality as a test of mis-specification

The literature has applied only one test of mis-specification to the linear quadratic production smoothing model. This is the West test which applies a variance inequality derived by comparing the expected value of the objective function of the linear quadratic production smoothing model over the sample period (which result from the inventory management policies actually followed) with the expected value that would have been obtained if inventories had never been allowed to depart from their trend values. If the minimised objective function is correctly specified, then the expected value under the alternative policy should be no lower than the expected value which results from the policy actually followed. Otherwise the estimated objective function is not being minimised and so the model is mis-specified. This is a general test principle which can be applied to any econometric model derived from the optimisation of a quadratic objective function.

Formally the restriction embodied in the West test is expressed through the following inequality:

$$\begin{aligned} &\mathscr{E}\left[\sum_{j=0}^{\infty}\left\{a_0Q_{t+j}^2 + a_1(\Delta Q_{t+j})^2 + a_2(I_{t+j}-a_3S_{t+j+1})^2\right\} \mid \Omega_t\right] \\ \leq\ &\mathscr{E}\left[\sum_{j=0}^{\infty}\left\{a_0S_{t+j}^2 + a_1(\Delta S_{t+j})^2 + a_2(a_3S_{t+j+1})^2\right\} \mid \Omega_t\right] \end{aligned} \qquad 4.1$$

The left hand side of the inequality is the expected value of the discounted objective function, given the firm's actual decision rule. The right hand side is the expected value of the discounted objective function, given the alternative decision rule $I_t = 0$.

Inequality (4.1) is a conditional on the information set at time t, Ω_t. Re-expressing the inequality in terms of unconditional expectations, followed by some simple manipulation, yields the West variance inequality:[16]

$$a_0\,\{Var(Q)-Var(S)\} + a_1\,\{Var(\Delta Q)-Var(\Delta S)\} + a_3\,\{Var(I)-2Cov(I,S_{+1})\}\leq 0 \qquad 4.2$$

The interpretation of this inequality as a mis-specification test is persuasive mainly because the alternative rule, maintaining inventories at their trend values, is so simple. If this very simple alternative does indeed result in an improvement in the assumed objective function, relative to actual behaviour, then the model must be mis-specified. West (1986) evaluates this inequality for a number of US manufacturing sectors using his estimates of the underlying parameters. He finds that it is violated for all sectors, although the violations are mostly of marginal statistical significance. Note that the simple production smoothing model is the special case where $a_1=a_2=0$ and the inequality then states that the variance of production is less than that of sales. Thus the rejection of the linear quadratic production smoothing model by inequality

(4.2) is a generalisation of the rejection of the simplest form of production smoothing model on the grounds that the variance of production exceeds the variance of sales.

The following table shows calculations of the left hand side of the West inequality, using the coefficient estimates of the linear quadratic production smoothing model estimated in section 5. If the there is no mis-specification then the calculated value, the left hand side of the inequality, should be negative. The inequality is violated by all sectors and for total manufacturing, although the violation is significant for only four of the sectors and not for total manufacturing.[17]

The West variance inequality.

	Inequality	Standard Error
Total Manufacturing	739000	(398000)
Metal Manufacturing	1915	(432)
Chemical Manufacturing	1313	(24084)
Engineering and Allied	125000	(43000)
Food, drink and tobacco	46090	(17900)
Textiles, footwear and clothing	4880	(1555)
Other Manufacturing	6703	(19581)

A stronger version of the West variance inequality.

The remainder of this section develops a stronger version of the West test, appropriate for testing the restriction that the specification error on the estimated equation is zero. It then considers the relationship between the West test and the standard tests of mis-specification recommended by the LSE approach to econometrics and, finally, the interpretation of failure of the West test.

This discussion assumes that the parameters of the linear quadratic production smoothing model have been estimated and uses the following notation:

I	Inventory levels
$\hat{I}$	Predicted inventory levels from the estimated model
O(I)	The unconditional expectation of the objective function given the estimated values of the parameters.
u	The residual, $u = I - \hat{I}$, on the estimated first order condition
v	Specification error
w	Expectational discrepancies between the agent and the econometrician.

v and w are the decomposition of u into its component parts: u = v + w. The stronger version of the West test which will now be developed must take account of possible covariance between the w component of the current residual and current sales (S), next period sales (S_{+1}) and next period but one sales (S_{+2}); denote these three covariances by σ_0, σ_1 and σ_2 respectively. Such covariance will occur because w reflects the informational advantage of the optimising agent in predicting current and future levels of sales. If the model is properly specified, v is uncorrelated both with current and past disturbances, expectational errors, with past values of all weakly exogenous variables, and with past and future values of strongly exogenous variables.

The same manipulations that were used to derive the West variance inequality establish that:

$$(1-\beta)O(I) = a_0 Var(S + \Delta I) + a_1 Var(\Delta[S + \Delta I]) + a_2 Var(I - a_3 S_{+1}) \qquad 4.3$$

$\hat{I}$ achieves the minimum value of O(I) for all possible rules based on current information. The strictest version of the West test can then be derived on the assumption that there is no specification error (v_t= 0). In this case any departure of actual inventory behaviour (I) from predicted behaviour ($\hat{I}$) must be such as to improve the objective O(I) and therefore (since the objective is minimised) O(I) ≤ O($\hat{I}$). Substituting (4.3) into this inequality, and evaluating the appropriate covariances, yields the further inequality:

$$\sigma_u \leq - (2a_0+6a_1+a_2)^{-1} \left[(a_0+a_1)\sigma_0 + (a_0-2a_1-a_2a_3)\sigma_1 + a_1\sigma_2\right] \qquad 4.4$$

The left hand side of this inequality reflects the increase in O(I) relative to O($\hat{I}$) which results from the variance of u = I-$\hat{I}$. The right-hand side of the inequality reflects the reduction in O(I) which results from co-variance between u and the current and predicted values of sales. If this reduction is outweighed by the increase due to the variance of u, then the inequality is violated and error cannot only reflect an informational advantage used to better predict current and future values of sales. This is a stricter version of the West inequality, applicable to testing the null hypothesis that there is no specification error in the equation.

This analysis also provides an insight into (4.2), the version of the inequality applied by West, and clarifies its relationship to other standard tests of mis-specification. Suppose now that a specification error (v) is now present. If the model is not mis-specified, the specification error is uncorrelated with all past sales and with past specification error. Suppose also that the specification error is uncorrelated with current and future sales (a strong exogeneity assumption) then O(I) may be written:

$$(1-\beta)^{-1}\, O(I) = (2a_0+6a_1+a_2)\,(\,\sigma_v + \sigma_w\,) - (a_0+a_1)\sigma_0 \qquad 4.5$$
$$- (a_0-2a_1-a_2a_3)\sigma_1 - a_1\sigma_2$$

The West inequality (3.2) is failed when O(I) exceeds the value of O(0) associated with the simple alternative rule I=0. It is apparent from (3.5) that O(I) will exceed this level, and the West inequality be violated, if σ_v exceeds some value that depends on O(0) and the variances and covariances of w. Thus failure of the West test, when other tests of mis-specification are passed, can be interpreted as a test of residual variance. Relaxation of the strong exogeneity assumption about sales rules out this simple interpretation of the West test, but it still suggests that the West test can be regarded as an setting an upper bound on some combination of the variances and co-variances of the specification error v. Inventory movements ("excess volatility") unexplained by the linear quadratic production smoothing model (the specification error) are too substantial for it to be a satisfactory model of inventory movements.

The West test may also be failed because the specification error, v, is correlated either with weakly exogenous variables or with its own past. This is no longer an "excess volatility" explanation of the failure of the West test: rather this source of failure indicates that there is some other alternative model which can generate a more adequate approximation to the data generating process. Thus failure of the West test can be an indicator of general dynamic mis-specification, rather than of excess volatility.

5. Estimation results and encompassing tests.

This section presents the results of estimating both the linear quadratic production smoothing model and the stock adjustment model, using quarterly data on gross sales (S_t) and inventories of finished goods (I_t). Results are presented for total UK manufacturing and for a breakdown into six manufacturing sectors at approximately the SIC 2-digit level. Data definitions and sources are as follows. The inventories data, in constant 1980 prices, are from 1960q1 to 1987q4. Inventory investment in finished goods by UK manufacturing, published in the quarterly UK national accounts, is available from the Central Statistical Office data tape. The sectoral data on inventory investment is an unpublished breakdown of the published data for aggregate manufacturing.[18] Data on the level of inventories is calculated by cumulating the investment data from bench-mark values for 1986q4, obtained from the national accounts blue-book. Output indices, published by the Central Statistical Office and released in their data tape, are available from the 1950s onwards, except for metal manufacturing for which output indices are only available from 1968q1. Output indices are grossed up to yield measures of gross output in constant prices. The bench-mark 1980 figure for gross output is derived by applying the gross/net output ratio (taken from the 1979 input/output tables and allowing for intra-sectoral sales) to 1980 value added. Gross sales are derived from the identity linking output, sales and inventories.

A distinction is commonly made between manufactures produced for stock and manufactures produced to order (Belsley (1967)). The linear quadratic production-smoothing model is applicable only to sectors which produce to stock. In the case where manufacturers produce to order production is smoothed by increased backlogs on orders during periods of peak demand (West (1987)), with inventories of finished goods held to meet forthcoming deliveries, not to smooth production. Regrettably it is not possible, using this UK data, to distinguish sectors which produce to order. Four sectors - chemicals, FDT (food, drink and tobacco), TFC (textiles, footwear and clothing) and other manufacturing - produce largely for stock. But the output of metal manufacturing and engineering and allied industries includes both production to order and production to stock.

The instruments used are dated t-2 or earlier. Instruments dated t-1 are excluded because of possible MA(1) correlation of the residuals arising because of time aggregation, the presence of informational discrepancies about S_{t+1} or (in the case of the target adjustment specification) through sales surprises. The instruments are sales, inventories, the margin of output prices over input prices for each sector and a measure of the real exchange rate (relative producer output prices) lagged back to t-4.

Table 2 reports the results of estimating the linear quadratic production smoothing model in the decision rule form described in the section 1. The results are similar for all sectors. The parameter estimates for a_1 are (with the exception of metals for which there are many fewer observations) significant and sensibly signed. The parameter estimates for a_0 and $a_2.a_3$ are however insignificant (there is one exception which is a significantly negative estimate of a_0 for food, drink and tobacco). The implied parameter estimates for a_2 are always insignificant.

These parameter estimates provide some support for the linear quadratic production smoothing model, but there is evidence of mis-specification. There is substantial first order residual auto-correlation indicated by the LM1 statistic. As noted above, under the assumption that the agent and the econometrician share the same information set, this can be formally interpreted as a mis-specification test. For chemicals and for textiles, footwear and clothing, the Sargan test rejects the over-identifying restrictions.

The most convincing evidence that the linear quadratic production smoothing model is an unsatisfactory approximation to the data generating process comes not from these mis-specification tests but from the comparison with the simple stock-adjustment model. Table 3 reports the results of estimating the stock adjustment model, together with the F-tests of encompassing. The stock adjustment model has deliberately been specified using S_{t+1} as a measure of inventory usage since it is in this form (rather than the more usual inclusion of current sales S_t) that the encompassing tests have greatest power. The test F1 indicates that for total manufacturing and for four out of the six manufacturing sectors the stock adjustment model provides a complete parametric encompassing of the linear quadratic production smoothing model. For the remaining two sectors - food, drink and tobacco and textiles footwear and clothing - the linear quadratic production smoothing model parameters are significant in the general equation, but only at the 95% level.

In contrast the linear quadratic production smoothing model is, with the sole exception of other manufacturing, unable to provide a complete parametric encompassing of the stock adjustment model. Moreover for three sectors and for total manufacturing the F-test (a test of the significance of the lagged dependent variable in the nesting model) is significant at well over the 99% level.

A further indication of the relative performance of the two models is the comparison of the residual standard errors, expressed as a percentage of the mean of the dependent variable. For all but two sectors (chemicals and engineering) the standard error is considerably lower for the stock adjustment model than for the linear quadratic production smoothing model. This again indicates that the linear quadratic production smoothing model is not a satisfactory measure of the unknown data generating process.

As noted above the performance of the linear quadratic production smoothing model should really be assessed only for those sectors which are known to produce largely to stock. Slight evidence in favour of the model is that in two of the four production to stock sectors the stock adjustment model fails to encompass the linear quadratic production smoothing model. However in both cases the linear quadratic production smoothing model remains mis-specified on the basis of tests of residual auto-correlation and is also clearly unable to encompass the stock adjustment specification.

The estimates of the stock adjustment model in table 3 are characterised by well determined coefficients on the lagged dependent variable in the range 0.7-0.95. As noted by Blinder (1981) this is characteristic of the stock adjustment model for inventories not only for finished goods but for all other categories of inventories. In the context of the stock adjustment model it implies very slow, indeed implausibly slow, adjustment towards long run equilibrium. These dynamics are the central, long-standing yet still unresolved puzzle in the econometric study of inventory investment.

The verdict on the linear quadratic production smoothing model delivered by the estimates in this paper, with the possible exception of other manufacturing for which no clear encompassing results emerge, is that the linear quadratic production smoothing model is unable to capture these inventory dynamics. Because of this failure and other evidence of mis-specification the linear quadratic production smoothing model must be held to be an unsatisfactory model of the underlying data generating process.

6. Conclusions

This paper has considered the mis-specification testing of the linear quadratic production smoothing model of inventory investment from the methodological perspective of the LSE approach to econometrics. It argues that the unsettled state of the literature on investment in inventories of finished goods has arisen because of the failure to apply a systematic procedure for examining model mis-specification. The application of standard tests of model mis-specification in this paper indicates that when estimated on UK data the production smoothing model is indeed mis-specified, but goes beyond the current literature by offering a more specific agenda for future research. Five general conclusions may be drawn.

(i) The first of these, the main methodological point argued throughout this paper, is that the LSE approach to econometrics of Sargan and Hendry should be used to supplement the new classical econometric procedures of Lucas, Hansen and Sargent. While accepting the new classical emphasis on the desirability of estimating underlying structural parameters, as a means of avoiding the instabilities associated with the Lucas critique and developing adequate dynamic theory, only the LSE approach to econometrics offers a systematic framework for mis-specification testing.

This has implications for estimation technique. The LSE econometrics rules out the inclusion of an auto-correlated error, unobserved by the econometrician, in the agents objective function. This is because mis-specification tests can then no longer be applied. By suitable choice of auto-correlation process the assumed model can be made to fit any data whatsoever and is no longer subject to scientific testing. This is the technique used by Blanchard (1983) in his estimation of the linear quadratic production smoothing model. The inclusion of an auto-correlated error explains why he finds that this model fits the data as well as the standard stock-adjustment specification.

The more common estimation procedure pursued in the new classical tradition is to derive an euler equation from the assumed theory and then estimate by instrumental variable techniques assuming orthogonality to the information set. The application of the LSE approach requires that the presence of a specification error is also acknowledged; this specification error reflects the inevitable failure of any model to capture all aspects of the data-generating process.

(ii) A related issue discussed in section 3 is the appropriate normalisation of the estimated euler equation. When conducting instrumental variables estimation the parameter estimates are, in small sample, sensitive to the chosen normalisation. The presence of a specification error suggests that a "decision variable normalisation" is to be preferred, in which the dependent variable to which the specification error is attached is the decision variable of the agent, and in which the decision variable has zero weighting as an independent variable. The normalisation does not matter if the specification is known with certainty to be correctly specified. In such a case maximum likelihood estimation is appropriate, and the parameter estimates are then unaffected by the normalisation.

(iii) The third general conclusion is that the encompassing principle offers a standard procedure for the mis-specification testing of dynamic econometric models against alternative specifications. This requires that a single estimation framework be set out incorporating both the euler equation derived from the assumed theory and a competing empirical model. As set out in section 2 the encompassing framework is linear and the competing models share a common dependent variable, so the encompassing tests consist of F-tests against the general model nesting the two competing models.

Failure to encompass a competing model indicates that the equation residuals from the estimated euler equation are in fact correlated with the econometrician's information set, violating the joint assumption of rational expectations and correct model specification. This testing procedure is more powerful than the common procedure of examining only tests of the over-identifying restrictions (the Sargan or Hansen statistics) because it utilises out of sample information about the choice of competing model.

The West test is the other main test of mis-specification which has been applied to the linear quadratic production smoothing model. As shown in section 4 failure of the West test can arise *either* because the induced equation specification residuals are correlated with their own past or correlated with elements of the econometrician's information set *or* because the variance of the equation residuals exceeds an upper bound. The LSE approach already provides a framework for testing whether the specification error is a residual, and encompassing tests will reveal if there is any competing model which results in a smaller residual variance. Since the

West test is rather inconvenient to implement it is unclear that it adds much to the standard range of mis-specification tests.

(iv) The fourth main conclusion, which results from the encompassing tests applied in section 5, is that the linear quadratic production smoothing model is encompassed by but fails to encompass a simple stock adjustment specification. Not only does this reveal model mis-specification but it also shows that the linear quadratic production smoothing model and the stock adjustment specification are *not*, as is commonly assumed, observationally equivalent. The stock adjustment model provides a much more satisfactory approximation to inventory dynamics.

(v) The final conclusion, suggested by the results of the encompassing tests, is that research on finished goods inventories should concentrate on theories of inventory investment which can explain the empirical success of the stock adjustment specification. In particular, given that stock-adjustment specifications can successfully reproduce the cyclical movements of inventories, research should return to the task of developing theoretical models of the cyclical movements of inventories which are the most prominent feature of the time series dynamics of UK finished goods inventories.

Table 1 Estimation results : Linear quadratic Production Smoothing Model Alternative normalisations. Total Manufacturing

Normalisation: $a_0 + (1+\beta)\, a_1 = 1$

(i) Substituting out a_0 60q4-87q2 LM1=17.7 LM4=22.7 SARGAN=11.6

	a_1	a_2	$a_2.a_3$	
$q_t =$	$0.060\ (\beta q_{t+1}+q_{t-1})$	$-\ 0.583\ I_t$	$+\ 0.083\ (-S_{t+1})$	$+\ u_t$
	(0.156)	(0.379)	(0.130)	(8.4%)

Implied value $a_0 = 0.880$ (SE=0.310)

(ii) Substituting out a_1 60q4-87q2 LM1=43.1 LM4=49.1 SARGAN=13.9

	a_0	a_2	$a_2.a_3$	
$r_t =$	$0.327\ ((1+\beta)q_t - r_t)$	$-\ 0.325\ I_t/(1+\beta)$	$-\ 0.378\ (-S_{t+1}/(1+\beta))$	$+\ u_t$
	(0.260)	(0.277)	(0.206)	(8.1%)

Implied value $a_1 = 0.338$ (SE=0.131)

Normalisation $a_1 = 1$ 60q4-87q2 LM1= 6.7 LM4=52.3 SARGAN=31.9

	a_0	a_2	$a_2.a_3$	
$r_t =$	$-\ 1.959\ (\ -q_t\ )$	$+\ 1.526\ I_t$	$-0.372\ (-S_{t+1})$	$+\ u_t$
	(0.156)	(0.498)	(0.154)	(4.4%)

Normalisation $a_0(1+\beta)+a_1(1+4\beta+\beta^2)+a_2 = 1$

Substituting out a_2 60q4-87q2 LM1=57.9 LM4=64.3 SARGAN=14.1

$I_t =$	$-\ 0.045\ F_t$	$+\ 0.149\ G_t$	$-\ 0.021\ S_{t+1}$	$+\ u_t$
	(0.150)	(0.040)	(0.049)	(4.7%)

Implied value $a_2 = 0.203$ (SE=0.309)

To aid comparison the standard errors of the equations are expressed as percentages of the mean of I_t.[19]

Table 2 Estimation results : Linear Quadratic Production Smoothing Model

	a_0	a_1	$a_2.a_3$		
Total manufacturing 60q4-87q2 LM1=57.9 LM4=64.3 SARGAN=14.1					
$I_t =$	$- 0.045\ F_t$	$+ 0.149\ G_t$	$- 0.021\ S_{t+1}$	$+ u_t$	$a_2 = 0.203$
	(0.150)	(0.040)	(0.049)	(4.7%)	(0.309)
Metals 69Q1-87q2 LM1=34.8 LM4=38.5 SARGAN= 5.0					
$I_t =$	$+ 0.015\ F_t$	$+ 0.107\ G_t$	$- 0.065\ S_{t+1}$	$+ u_t$	$a_2 = 0.334$
	(0.169)	(0.074)	(0.074)	(17.3%)	(0.450)
Chemicals 60q4-87q2 LM1=60.8 LM4=60.7 SARGAN=24.5					
$I_t =$	$+ 0.024\ F_t$	$+ 0.141\ G_t$	$+ 0.045\ S_{t+1}$	$+ u_t$	$a_2 = 0.112$
	(0.166)	(0.044)	(0.052)	(5.6%)	(0.333)
Engineering and allied 60q4-87q2 LM1=48.0 LM4=54.5 SARGAN= 6.6					
$I_t =$	$- 0.032\ F_t$	$+ 0.138\ G_t$	$- 0.052\ S_{t+1}$	$+ u_t$	$a_2 = 0.243$
	(0.187)	(0.059)	(0.061)	(7.2%)	(0.405)
Food, drink and tobacco 60q4-87q2 LM1=41.8 LM4=48.6 SARGAN= 8.9					
$I_t =$	$- 0.432\ F_t$	$+ 0.305\ G_t$	$- 0.165\ S_{t+1}$	$+ u_t$	$a_2 = 0.049$
	(0.213)	(0.071)	(0.092)	(6.1%)	(0.463)
Textiles, footwear and clothing 60q4-87q2 LM1=38.2 LM4=46.9 SARGAN=21.1					
$I_t =$	$+ 0.034\ F_t$	$+ 0.123\ G_t$	$- 0.040\ S_{t+1}$	$+ u_t$	$a_2 = 0.204$
	(0.110)	(0.036)	(0.040)	(4.7%)	(0.244)
Other manufacturing 60q4-87q2 LM1=52.5 LM4=73.8 SARGAN= 1.42					
$I_t =$	$- 0.003\ F_t$	$+ 0.173\ G_t$	$- 0.017\ S_{t+1}$	$+ u_t$	$a_2 =-0.021$
	(0.126)	(0.054)	(0.049)	(8.3%)	(0.353)

- F_t and G_t are as defined in the text. β is the quarterly discount rate assumed equal to 0.99.
- GIVE estimation using instruments dated t-2 to t-4. Standard errors in brackets. Standard error of equation expressed as a % of the mean of the dependent variable. Deterministic time trends and quarterly dummies not reported.
- LM1 and LM4 are lagrange multiplier tests of auto-correlation, of first order and of order up to four respectively. Sargan tests validity of the nine over-identifying restrictions. Significance levels:

95% $\chi^2(1)$ 3.84 $\chi^2(4)$ 9.49 $\chi^2(9)$ 16.92
99% $\chi^2(1)$ 6.63 $\chi^2(4)$ 13.28 $\chi^2(9)$ 21.67

Table 3 Stock adjustment model and encompassing tests.

Total manufacturing 60q4-87q2 LM1=10.5 LM4=14.5 SARGAN=7.6

$$I_t = \underset{(0.054)}{+\,0.920\, I_{t-1}} \quad \underset{(0.022)}{+\,0.078\, S_{t+1}} \quad \underset{(2.2\%)}{+\,u_t} \qquad \begin{array}{l} F1(2,96)=\ 2.2 \\ F2(1,96)=58.8 \end{array}$$

Metals 69Q1-87q2 LM1= 1.4 LM4=6.68 SARGAN=12.4

$$I_t = \underset{(0.071)}{+\,0.851\, I_{t-1}} \quad \underset{(0.021)}{-\,0.024\, S_{t+1}} \quad \underset{(5.0\%)}{+\,u_t} \qquad \begin{array}{l} F1(2,63)=\ 1.0 \\ F2(1,63)=26.6 \end{array}$$

Chemicals 60q4-87q2 LM1=33.3 LM4=42.7 SARGAN=22.5

$$I_t = \underset{(0.087)}{+\,0.897\, I_{t-1}} \quad \underset{(0.048)}{+\,0.116\, S_{t+1}} \quad \underset{(5.3\%)}{+\,u_t} \qquad \begin{array}{l} F1(2,96)=\ 0.3 \\ F2(1,96)=\ 4.8 \end{array}$$

Engineering and allied 60q4-87q2 LM1=13.3 LM4=21.3 SARGAN= 5.0

$$I_t = \underset{(0.054)}{+\,0.935\, I_{t-1}} \quad \underset{(0.022)}{+\,0.061\, S_{t+1}} \quad \underset{(6.8\%)}{+\,u_t} \qquad \begin{array}{l} F1(2,96)=\ 1.3 \\ F2(1,96)=\ 5.1 \end{array}$$

Food, drink and tobacco 60q4-87q2 LM1= 0.4 LM4=21.1 SARGAN=24.2

$$I_t = \underset{(0.051)}{+\,0.921\, I_{t-1}} \quad \underset{(0.045)}{+\,0.041\, S_{t+1}} \quad \underset{(3.7\%)}{+\,u_t} \qquad \begin{array}{l} F1(2,96)=\ 4.7 \\ F2(1,96)=\ 9.0 \end{array}$$

Textiles, footwear and clothing 60q4-87q2 LM1= 9.5 LM4= 9.5 SARGAN=16.9

$$I_t = \underset{(0.058)}{+\,0.897\, I_{t-1}} \quad \underset{(0.025)}{+\,0.057\, S_{t+1}} \quad \underset{(3.3\%)}{+\,u_t} \qquad \begin{array}{l} F1(2,96)=\ 4.2 \\ F2(1,96)=53.9 \end{array}$$

Other manufacturing 60q4-87q2 LM1= 2.8 LM4= 7.6 SARGAN=14.2

$$I_t = \underset{(0.140)}{+\,0.732\, I_{t-1}} \quad \underset{(0.032)}{+\,0.017\, S_{t+1}} \quad \underset{(5.6\%)}{+\,u_t} \qquad \begin{array}{l} F1(2,96)=\ 2.7 \\ F2(1,96)=\ 9.6 \end{array}$$

- Estimation techniques and sample periods exactly as in table 2. LM1, LM4 are as in table 1. Sargan tests for 10 over-identifying restrictions. Significance levels: 95% - 18.31 99% - 23.21
- The F-tests are as follows. F1 tests the restriction of the general nesting model to the stock adjustment model. Acceptance indicates that the stock adjustment model encompasses the linear quadratic productions smoothing model. F2 tests the restriction of the general model to the linear quadratic production smoothing model. Acceptance indicates that the linear quadratic production smoothing model encompasses the stock adjustment model.

Significance levels:

95%	F(1,96)=3.96	F(2,96)=3.11	F(1,63)=4.00	F(2,63)=3.15
99%	F(1,96)=6.96	F(2,96)=4.88	F(1,63)=7.07	F(2,63)=4.97

CHAPTER 4

FINANCIAL EFFECTS ON INVENTORY INVESTMENT

1. Introduction

Informational imperfections in capital markets invalidate the Modigliani-Miller theorem. The financial costs of investment can no longer be identified with real rates of interest on tradeable securities, and other financial variables have effects on real decisions. Much recent research has discussed the consequences of informational imperfections for the efficiency of investment decisions and for movements in aggregate fixed capital investment. This paper considers the implications for inventory investment and in particular whether these considerations provide an explanation of the pronounced cyclical fluctuations in inventory investment in the UK in the late 1970s and early 1980s.

A model of the dynamics of inventory investment is proposed. This extends the existing literature on financial effects on corporate investment to a dynamic setting. The predictions of this model are then investigated using a panel of individual UK company accounts data. The principal objectives in conducting this estimation are to examine whether profits affect inventory investment in the manner suggested by the model and to what extent movements in profitability provide an explanation of the aggregate movements in inventory investment.

The technical solution of the model is of interest and should be applicable to other aspects of company decision making. The presence of constraints make the model intractable using standard methods of guess and verification. Thus a *regime analysis* is applied to yield a solution of the firm's dynamic programming problem. This regime analysis consists of a qualitative analysis of the value function and the firm's decision rules under the variety of different regimes that emerge as the set of binding constraints is altered.

The literature on informational imperfections in capital markets and their effect on investment decisions is reviewed in section 2. These ideas have been applied in the study of fixed capital investment and to explain the depth and severity of the 1930s US depression but have not hitherto been applied to the study of inventory investment. A difficulty in extending these ideas to the study of inventory investment is that the formal models of this literature are all one period models. While this may be an acceptable way of modelling a fixed capital investment decision which can be regarded as a once only opportunity, such models are less convincing when applied to decisions (such as inventory holding decisions) which are repeated over time.

Section 3 develops a dynamic model of financial effects on inventory holdings, utilising the sequential analysis of an infinite period dynamic programme. This formalisation has two advantages over one period models. It clarifies the role of bankruptcy in the link between financial imperfections and real decisions, showing how the possibility of bankruptcy can affect real expenditure decisions even if immediate bankruptcy is very unlikely and the direct costs of bankruptcy are small. It also suggests that links between financial measures, such as net assets or cash flow, and real decisions can arise for any firm which comes under financial pressure. In this model it is the recent history of stochastic shocks faced by the firm, rather than fixed firm characteristics, which induce these financial effects on inventory investment.

These insights are purchased at the price of failure to endogenise either the rate of interest or the size of debt constraint and by modelling corporate decision making in a very simplified fashion, with inventories used as the only factor of production. The generalisation of the model, to allow for the endogeneity of interest rates and the financial contract, and to other decisions by an imperfectly competitive firm employing several factors of production are left for further work. An annex provides the guess and verification solution of the model with a constraint on debt but not on equity, and the regime analysis of the model with both an equity and a debt constraint.

Section 4 presents empirical results using a panel of individual company accounts, collected by the UK Department of Trade and Industry, with a particularly large representation of small firms.[20] Section 4 compares the movements in inventory investment and turnover obtained by aggregation over this data with the corresponding national accounts aggregates and examines the distribution of a number of indicators of financial pressure within the panel. It then presents some simple estimates of the short run determinants of inventory investment. Finally it examines whether the link between profitability and investment revealed in this data set can explain the pronounced aggregate movements in UK inventory investment over the late 1970s and early 1980s.

2. Models of financial effects on real expenditure decisions.

Recent literature demonstrates how, as a result of informational imperfections, the cost of finance to individual firms and households can be affected by balance sheet measures, cash flow, firm or household characteristics and the aggregate supply of credit, as well as market rates of interest.[21] These results provide theoretical support for the much longer standing empirical literature on the links between cash flows, profitability or balance sheet measures, such as liquidity or net worth, and real decisions by both households and firms. This section reviews this literature.

Allowing for the presence of informational imperfections in financial markets constitutes a major challenge to the standard Arrow-Debreu tradition and has prompted a re-assessment of the conventional view that freely operating capital markets ensure an efficient allocation of savings. The other major implication, and the one of greater relevance to this paper, is that the cost and availability of finance vary considerably over the business cycle and between firms, to a much greater extent than market rates of interest, and hence provide a mechanism by which financial factors affect real expenditures and output and can induce procyclical fluctuations in investment.

Early views on financial factors and real expenditures

The view that financial factors play a central role in the propagation of the business cycle is not of course new. A well known example is the debt-deflation argument of Fisher (1933). According to Fisher price deflation in the 1930s led to an increase in the real value of debt, and a cut in business and household expenditures. But this argument does not justify the underlying assertion that market rates of interest do not fully capture the impact of financial factors on the real economy. If planned expenditures are desired at existing (real) rates of interest and at current income levels, why should the burden of debt or the insolvency of banks have additional real effects?

Such financial explanations of cyclical movements in real expenditures assume (without rigorous justification) that financial factors influence the cost of finance other than through interest rates alone. There is of course substantial casual evidence in favour of this view. For example banking practitioners commonly state that their approval of lines of credit is conditional on the satisfactory behaviour of a number of key accounting ratios. Lines of credit can be sharply curtailed when these balance sheet measures deteriorate, with an immediate impact on holdings of inventories and other assets.

This casual evidence is supported by the work of Meyer and Kuh (1957) who emphasise the limited access to bank finance of small firms. However this line of research was eclipsed by later contributions which explored the links between centralised capital markets and the cost of investment finance. The Modigliani and Miller (1958) proof of the irrelevance of financial structure, even when investment returns are uncertain, is derived under assumptions which ensure that investment decisions are based on a single, economy-wide cost of finance. Similarly the influential work of Jorgenson on the user cost of capital assumes that financing costs are completely captured by market rates of interest. Links between net worth or cash flows and investment were still supported by some empirical contributions (see for example Eisner (1978) and also Mishkin (1978) on the effect of household net worth on consumer expenditure in the 1930s) but the theoretical case for such links remained weak.

Theoretical models of informational assymetries in capital markets

It is only in the last two decades that academic research has produced convincing theoretical arguments for anticipating a relationship between measures of financial worth or retained earnings and the costs of financing investment. These arguments all rest on capital market imperfections arising because of informational asymmetries. These asymmetries result in a "lemons" problem which leads to an increased cost of finance and possibly to credit rationing. The original lemons article (Akerlof (1970)) discussed credit markets in developing countries. Jaffee and Russell (1976) show how imperfect information can lead to restrictions on the size of loan which banks are prepared to advance. Stiglitz and Weiss (1981) present striking and simple examples of how adverse project selection or moral hazard can generate credit rationing. This suggests either a complete breakdown of bank credit, or an upper limit on interest rates, with more firms applying for loans than the bank can profitably supply (assuming an upward sloping supply of funds) and with loan applicants being turned away on arbitrary grounds. In either case interest rates, even adjusted for risk and taxation, no longer reflect the cost of debt finance.

Similar informational arguments can be applied to equity finance, although they are perhaps not quite so convincing. For example Greenwald, Stiglitz and Weiss (1984) propose a model based on bankruptcy costs. Banks observe the internal cash flow of the firm, but this is hidden from the equity market. Firms with good internal cash flow can use this as security for debt finance, avoiding the costs of bankruptcy. Firms with poor cash flow are forced, in part, to make use of relatively expensive equity finance rather than accept the costs of bankruptcy. The additional expense of equity arises because its issue signals that the firm has poor internal cash flow.

There are difficulties with this argument, especially if the decision to issue equity or retain earnings signals good investment opportunities rather than liquidity difficulties.[22] More successful is the model of Myers and Majluf (1984) in which new equity issues must be subscribed to by new investors. This dilutes the interests of existing shareholders and thus, if management acts in the interests of existing shareholders, raises the costs of new issue finance. The effects of informational imperfections on the costs of equity finance thus remain an area of ongoing research. The discussion in this paper considers only the choice between retained earnings and debt finance and the resulting effects of cash flow and balance sheet measures on inventory investment. New equity finance, which is rarely used for investment in current assets, is assumed to be unavailable.[23]

A further branch of this literature endogenises the form of the financial contract. In some circumstances this can eliminate the lemons problem (De Meza and Webb (1987)), but generally the lemons problem remains. For example "costly state verification" (firm actions can be observed but there is a cost to doing so) together with firm objectives which diverge from those of shareholders (agency costs) make debt the optimal financial instrument with verification being made whenever the firm is unable to pay interest costs.[24] Debt provides an incentive compatible form of finance in that it ensures the firm operates in the interests of the providers of finance, but the costs of verification increase the cost of debt. If state verification is sufficiently costly it can also lead to credit rationing (Williamson (1987)).

More recent contributions to the literature on informational imperfections and bank lending extend the analysis to a general equilibrium framework incorporating savers and borrowers. This is done in order to demonstrate, in an internally consistent fashion, how informational imperfections can lead to cyclical fluctuations in real corporate expenditures. The general argument is that there are close links between borrower net worth or other balance sheet measures and the cost of debt finance. Higher net worth means that the borrower has more collateral for obtaining outside funds and for using directly for project finance. This reduces the lemons premium on debt finance and leads to higher levels of investment.

The cyclical relationship emerges because periods of above average demand improve balance sheet measures and increase cash flow and profitability hence encouraging increased investment. Conversely periods of below average output and demand result in deteriorating balance sheet measures and reduced investment. Potentially this can be a very marked effect. Mankiw (1986) demonstrates the possibility of "credit collapse" with a fall in borrower collateral leading to the unavailability of credit to all borrowers.

Empirical evidence

These later contributions suggest an important empirical prediction obtained from these financial models of real expenditure fluctuations. This is most clearly expressed by Calomiris and Hubbard (1987a) in their distinction between "information intensive" and "full information" borrowers. The latter, typically large and mature firms with established credit-worthiness, can issue debt or borrow from banks at close to market rates of interest. The former, typically small and newly established, only have limited access to bank debt and must provide security for their borrowing. It is these "information intensive" firms who are expected to exhibit pronounced real effects in response to changes in financial magnitudes.

Bernanke (1983) considers these points in the context of explaining the severity of the great depression in the US. He notes how the fall in output and employment was concentrated amongst small and medium sized businesses. He argues that the substantial number of bank failures broke long-standing credit relationships and forced remaining banks to adopt very risk-averse lending policies. He finds further empirical support for these ideas in that the volume of credit advanced is a significant determinant of output when added to an estimated Barro-Lucas supply relationship between money surprises and output.

A variety of other empirical evidence is consistent with these models. Calomiris and Hubbard (1987b) find evidence of links between credit availability and the volume of economic activity in the period before the establishment of the US Federal Reserve, while Friedman (1982) provides similar evidence for the more recent past.

The most striking evidence is probably that of Fazzari et al (1988) and Srinivasan (1986) obtained from analysis of US individual company data. Fazzari et al analyse a large panel of company data covering the period 1970-1984. They distinguish "information intensive" and "full information" firms on the basis of dividend pay-out ratios. Using a variety of different specifications for the investment equation, they find that investment responds to cash flow and other balance sheet measures. Moreover it does so to a much greater extent for those with the lowest dividend pay-out ratios (the more "information intensive" firms.) Srinivasan finds that small and medium sized companies, over the period 1960 to 1980, make little use of new equity issues and that large companies have disproportionately greater access to debt finance during cyclical downturns.[25] Small companies also exhibit greater volatility of dividends and earnings and their investments and sales are more procyclical than large firms.

Together these contributions suggest a strong case for financial effects on firm's real expenditure decisions, but all the models described here are simple one-period financing models. While the development of multi-period models is an ongoing area of research these have not yet resulted in internally consistent general equilibrium models of financial effects on real expenditures. One drawback of considering only one-period models is that long period credit relationships are one way of overcoming informational imperfections and may substantially mitigate the conclusions to be drawn from these models. This issue, which requires the endogenising of the financial contract, is not pursued in this paper.

Contribution of this paper

This paper addresses a distinct problem with the use of one period models. One-period analyses lead automatically to the views (such a those expressed by Calomiris and Hubbard on the distinction between information intensive and full information firms) which suggest that it is firm specific characteristics which lead to a firm being amongst the subset of firms which experience financial effects on real expenditure decisions. The dynamic model set out in the following section suggests instead that it is the recent history of stochastic shocks at firm level which, by putting the firm under financial pressure, lead to financial effects on real decision making. This implies that period specific indicators of financial pressure at the level of the individual firm will be more successful at predicting financial effects on real decisions than firm specific characteristics which do not change over time.

3. A dynamic programming model of inventory investment subject to financial constraints.

Structure and assumptions

This section presents a simple formal model in which debt issue and inventory investment are the control variables of an infinite period dynamic programme. This is a "bankruptcy avoidance" model with some of the features of the one-period model of Wadhwani (1986). This model supports the claim that financial factors may affect inventory investment, but suggests that financial effects operate when the firm is under financial pressure not during normal periods of operation.

This development of a fully dynamic model has been pursued by adopting highly simplifying assumptions about the nature of the financial constraints. Unlike much of the literature reviewed in the previous section this model does not endogenise the financial contract, or even interest rates and the constraint on debt finance, within a model of informational imperfections. Firms are assumed to be able to borrow up to some pre-established limit of credit-worthiness. This assumption comes close to how bankers themselves describe their lending practices. Nonetheless it would be desirable to extend the present model to allow for the endogeneity of interest rates and the debt constraint subject to informational imperfections. This avenue of research is left for further work.

The structure of the model is as follows. Revenues in each period, net of production costs, are assumed to be given by:

$$R_t = \theta_t \Psi(Y_t) - \Xi(Y_t , I_{t-1}) \tag{3.1}$$

The first term, representing gross revenues, applies to firms in both perfect and imperfect competition. For the perfectly competitive firm θ_t is the relative price of perfectly competitive output and $\Psi(Y_t) = Y_t$. For the firm facing a downward sloping demand curve $\theta_t \frac{\partial \Psi(Y_t)}{\partial Y_t}$ is the marginal revenue of the firm, with shifts in demand indexed by θ_t.

Uncertainty about future profits arises because θ_t is stochastic and distributed around a fixed mean:

$$\theta_t = \bar{\theta} + \epsilon_t \qquad \mathcal{E}(\epsilon_t) = 0 \tag{3.2}$$

It is assumed that ϵ_t is stationary and serially independent with a continuous probability density function $f(\epsilon)$ defined over the interval $(-\infty,+\infty)$. This rules out both discrete and mixed discrete and continuous distributions for ϵ_t. The further assumption that $f(\epsilon)$ is single peaked will also be made in order to establish certain features of the model.

The second term in (3.1) is the cost of producing a given level of output, Y_t, given the amount of inventories held over from the previous period. These are most easily thought of as raw material inventories or distributor's inventories, but the same specification can apply also to inventories of work in progress or of finished goods. There is no stochastic disturbance to costs of production.

It is assumed throughout the paper that the expected value of this objective is such that the expected value of future dividends if the firm continues in operation is greater than zero, and hence that the firm never chooses voluntarily to cease operations.

The cost and revenue functions have standard properties. There are decreasing returns to net revenue with respect to output implying, in the case of perfect competition, decreasing returns to scale in production. An increase in the level of inventories reduces the cost of production, but at a diminishing rate. Some inventories are required to produce at all. These properties may be formally stated:

$$\begin{gathered} 1 \geq \frac{\partial \Psi}{\partial Y} > 0\,, \quad \frac{\partial \Xi}{\partial Y_t} > 0\,, \quad \bar{\theta}\frac{\partial^2 \Psi}{\partial Y_t^2} - \frac{\partial^2 \Xi}{\partial Y_t^2} < 0 \\ \Xi(Y_t{>}0,0) = +\infty\,, \quad \frac{\partial \Xi}{\partial I_{t-1}} < 0\,, \quad \frac{\partial^2 \Xi}{\partial I_{t-1}^2} > 0 \end{gathered} \tag{3.3}$$

The sequence of decision making is as follows. At the end of each period managers of the firm determine three control variables, namely inventory holdings (I_t), debt carried over to the next period (D_t) and next period's output (Y_{t+1}). D_t may be negative, in which case the firm holds a cash balance. Then, at the beginning of the following period, the uncertainty about θ_{t+1} is resolved: production costs are then incurred, revenues received and interest payments are made. At the end of each period the firm inherits, as a consequence of its past decisions, net assets H_t where:

$$H_t = R_t - (1+r)D_{t-1} \tag{3.4}$$

Inventories are purchased at a price γ. Following the usual historical cost convention profits are defined as:

$$\pi_t = R_t - \gamma I_{t-1} - rD_{t-1} \tag{3.5}$$

The firm's objective is the maximisation of the discounted sum of dividend payments using the shareholder's discount rate δ:

$$\sum_{j=0}^{\infty} \delta^j d_{t+j} \tag{3.6}$$

The dividend payments depend on net assets, inventory holdings and the amount of debt carried forward:

$$d_t \equiv H_t - \gamma I_t + D_t \tag{3.7}$$

Debt finance takes the form of bank lending available at exogenously determined fixed rates of interest with an exogenous limit S on the amount of debt that the company is allowed to carry. There is a discrepancy between the interest rates on debt and on cash holdings. The interest paid on the outstanding debt is assumed to be higher when the firm owes money than when it holds a cash balance, and the spread of rates is assumed to include the rate of interest implied by the shareholder's rate of time discount. These interest rates are denoted by r^+ (applied on the firm's borrowing when D is positive) and r^- (paid on the firm's cash holdings which are held when D is negative) respectively. The rate of interest on outstanding debt will also be referred to as r, which equals either r^+ or r^- depending on the sign of D_t. r^+ and r^- are subject to the inequalities:-

$$1 + r^+ > \frac{1}{\delta} > 1 + r^- \tag{3.8}$$

The right hand inequality in (3.8) ensures that dividend payments are made. Otherwise, if the interest rate on cash holdings $r^- = \delta^{-1} - 1$ and there is risk of bankruptcy, the firm will accumulate cash indefinitely paying only a single terminal dividend. The other inequality arises because the bank's shareholders require some compensation for the risk of bankruptcy.

Solution

Bellman's equation for this dynamic programme, in which the state variable is the inherited amount of net assets H_t and the control variables are I_t, D_t and Y_{t+1}, may be written as:

$$V(H_t) = \underset{I_t,D_t,Y_{t+1}}{Max} \left[d_t + \delta \mathscr{E} V(H_{t+1}) \right] \tag{3.9}$$

The annex discusses the solution of this problem. When there is no non-negativity constraint on dividends (so that firms can seek additional equity finance in each decision period) then the solution takes a very simple form and the debt constraint does not affect the firm's decisions. Debt is not issued, nor are cash balances held. Inventory holdings are determined by the same first order conditions which emerge from maximisation of the one period objective Ω:

$$\Omega = \delta_t \mathscr{E} \underset{Y_{t+1},I_t}{Max} R(\theta_{t+1}, Y_{t+1}, I_t) - \gamma I_t \tag{3.10}$$

Thus in the absence of an equity financing constraint inventory investment is determined by expectations of future prices and by the rate of discount of the firms shareholders. The firm's investment in end of period inventories is dynamically efficient and unaffected by any financial variables other than the shareholder's discount rate δ.

Financial effects on inventory holdings emerge only when there is a constraint on the provision of additional equity finance. At least two considerations suggest that such a constraint should operate. There are substantial costs associated with equity issue making this an impractical source of finance for incremental amounts of investment. More fundamentally, and as discussed in section 2, there are informational assymetries in equity markets as well as debt markets, which lead to constraints on the issue of new equity.

The second solution of the model assumes that new equity is not available for investment in current assets, resulting in a non-negativity constraint on dividend payments:

$$d_t \geq 0 \tag{3.11}$$

A solution for the amended dynamic programme exists with a value function $V(H_t)$ which represents the expected value of all future dividend payments. It is further assumed that the expectation $\mathcal{E}_t[V(H_t)]$ always exists for the optimal setting of the control variables at time t. The value function then satisfies the Bellman equation and there are a set of first order conditions (emerging from the maximisation of the right hand side of the Bellman equation) which determine inventory holdings and debt issue in each period. An analytical expression for the value function in terms of the state variable H_t is not obtainable. It is however possible to apply a regime analysis which yields insight into the effects of financial factors on the dynamic behaviour of inventory holdings. The following propositions may be established (proofs given in the annex):

(a) When $H_t \leq -S$ then $V(H)=0$

(b) When $-S < H_t$ then the value function is a continuous and differentiable function of H_t, and the derivative of the value function satisfies:

$$\frac{\partial V(H_t)}{\partial H_t} > 1 \tag{3.12}$$

(c) Suppose that the distribution of the stochastic shock ϵ is such that (i) $f(\epsilon)$ is single peaked; (ii) bankruptcy is unlikely when a dividend is paid; and (iii) the probability of a dividend payment in the subsequent period is high when a dividend payment is made in the current period. There then exists a finite, non-negative critical value of H_t, $H^* \geq 0$, such that for $H_t > H^*$ the value function may be written:

$$V(H_t) = V(H_t^*) + H_t - H_t^* \tag{3.13}$$

and a dividend is paid if, and only if, H_t exceeds H^* according to the rule:

$$d_t = H_t - H_t^* \tag{3.14}$$

(d) If $H_t \geq H^*$ then D_t, I_t and next period's output Y_{t+1} are set independently of the state variable H_t. Thus we can write $D_t = D^* < 0$, $I_t = I^*$ and $Y_{t+1} = Y^*$. Y^* is less than planned output when there are no constraints on equity finance. I^* may be either above or below inventory holdings when there are no constraints on equity finance.

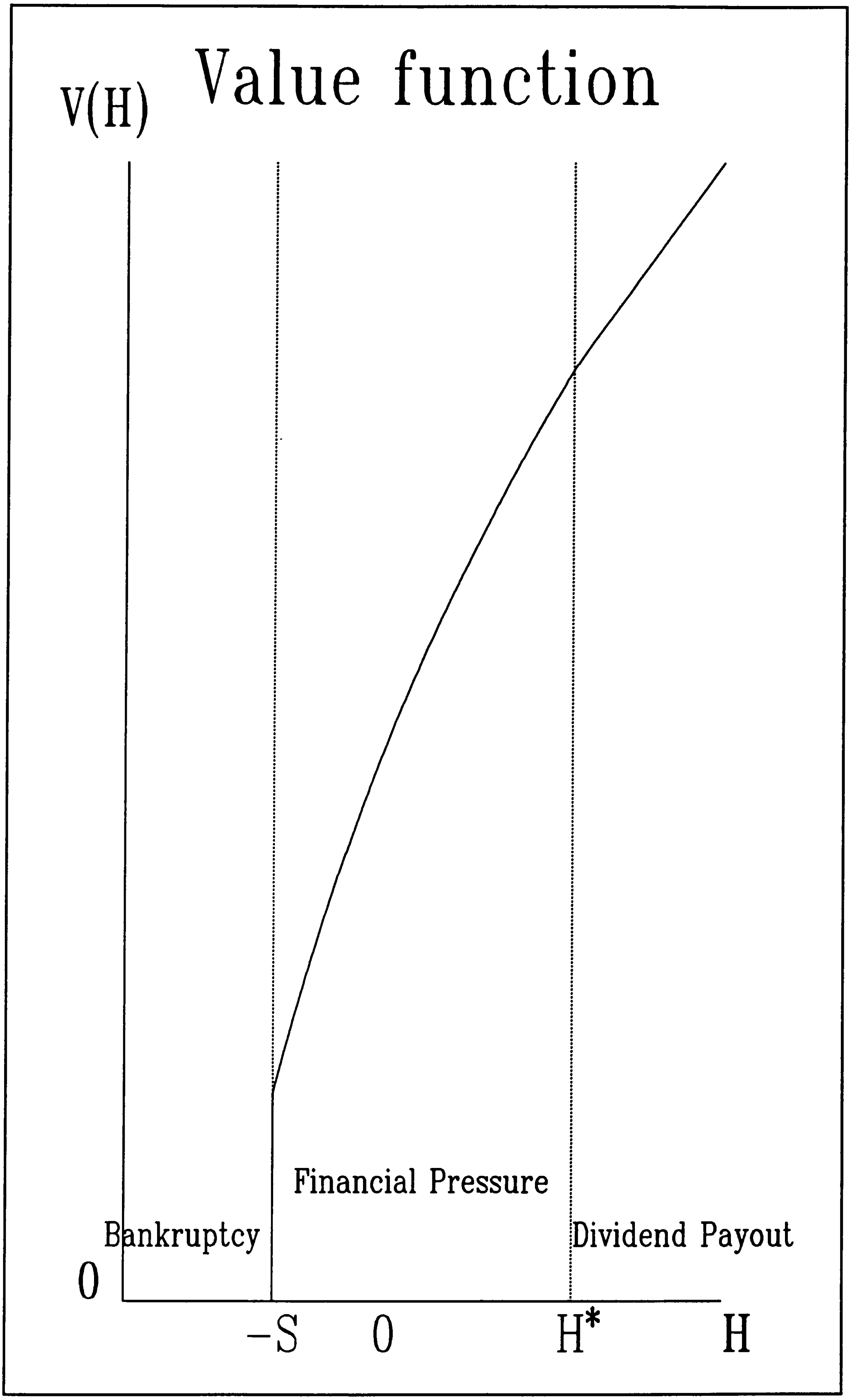
(e) When $-S < H_t < H^*$ then the inequality in (3.12) is strict. Inventory holdings, debt issue and planned output may then be written as:

$$
\begin{aligned}
D_t &= D^* + \phi_D(H_t) \\
I_t &= I^* + \phi_I(H_t) \\
Y_{t+1} &= Y^* + \phi_Y(H_t) \\
&\text{where} \\
\phi_D(H^*) &= 0 \;\;, \;\; \phi_D(-S) = -D^* + S \\
\phi_I(H^*) &= 0 \;\;, \;\; \phi_I(-S) = -I^* \\
\phi_Y(H^*) &= 0 \;\;, \;\; \phi_I(-S) = -Y^* \\
\gamma\phi_I - \phi_D &= H - H^*
\end{aligned}
\tag{3.15}
$$

Little can be said about the form of the decision rules in (3.15), although as indicated the extreme values, when $H=H^*$ and $H=-S$, are known. Between these values, and especially where H is close to -S, there is the possibility both of discontinuities and of reversals of sign of the first derivatives with respect to H. For the empirical estimates presented in the following section it has been necessary to approximate these decision rules by linear functions of H_t.

These conclusions about the value function and the relationship between the state variable H_t on the control variables D_t, I_t and are summarised in the accompanying diagram. The x-axis shows the value of net assets H_t. Three regions may be distinguished: bankruptcy ($H_t<-S$), financial pressure ($-S<H_t<H^*$), and dividend pay-out ($H_t>H^*$). When net assets exceed the threshold value H^* then the firm is in the dividend pay-out region, it carries a cash balance forward to the next period, and inventory investment is independent of net assets H_t.

When net assets are less than the threshold value H^* then the firm is in the region of financial pressure. No dividends are paid and inventory holdings are reduced by an amount which depends on the extent to which net assets H_t fall below H^*. Finally if net assets fall below -S then the firm is bankrupt. A relationship between inventory holdings and net assets emerges when the firm enters the regime of financial pressure, characterised by non-payment of dividends. Otherwise output is reduced to below and inventory holdings may be above or below, what they would be were there no capital market imperfections, but are unaffected by the amount of


Value function
V(H)
Financial Pressure
Bankruptcy
Dividend Payout
0
−S
0
H*
H

net assets.

The intuition behind these results is that the firm carries a cash balance instead of paying out a dividend, to insure it against the risk of a poor outcome (or series of poor outcomes) for future prices, leading to bankruptcy. Bankruptcy is costly to the firm's shareholders because the entire future stream of returns from the firm is then lost. The marginal return on the holding of cash thus exceeds the interest rate r^- because it reduces the probability of future bankruptcy. The firm holds cash up to the point at which the marginal reduction in the expectation of lost future streams of profits equals the wedge between the shareholders discount rate (δ) and the interest rate on cash balances (r^-).

In periods of normal operation the firm's planned output, Y^*, is less than in the case where there is no constraint on the provision of equity finance. This reduction in planned output increases H_{t+1} for low values of ϵ_{t+1} and reduces H_{t+1} for high values of ϵ_{t+1}, providing further insurance against a poor outturn.

4. Panel data estimation of the bankruptcy avoidance model.

The log-linearisation of the inventory equation

The empirical specification used in this section is based on a log-linear version of the model analysed in section 3. This specification is obtained by assuming a linear version of (3.5) applicable to the individual firm and then by deriving a log-linearisation applicable to all firms.

Linearising equation (3.5) when the firm is under financial pressure inventory:

$$I_t = I^* - \phi_I H^* + \phi_I H_t \tag{4.1}$$

Following common panel data practice the estimation is conducted in first differences, removing fixed effects on the level of inventory holdings, requiring a relationship for the change, rather than the level of inventories. When the firm is under financial pressure (in both the current and previous accounting years) this is:

$$\Delta I_t = \phi_I \Delta H_t \tag{4.2}$$

Difficulties with finding an accurate measure of the firm's net assets H_t suggest a further step in the analysis: rather than taking first differences of data on firms net capital (which corresponds most closely to net assets H in the theory but is subject to large measurement error) it is possible instead to use current profits (net of interest, taxes and dividend payments) as a measure of the change in net assets. That this is simply an alternative way of measuring the change in net assets may be seen by substituting the identity constraining dividend payments (3.6) and the definition of net assets (3.4) into the definition of profits (3.5) yielding:

$$
\begin{aligned}
\pi_t - d_t &= R_t - \gamma I_{t-1} - r D_{t-1} - d_t \\
&= R_t - (H_{t-1} - d_{t-1}) - (1+r)D_{t-1} - d_t \\
&= (H_t - d_t) - (H_{t-1} - d_{t-1})
\end{aligned}
\tag{4.3}
$$

and noting that when the firm is under financial pressure in both the current and previous periods dividend payments $d_t = d_{t-1} = 0$. Hence the investment in inventories may be expressed as:

$$\Delta I_t = \phi_I \pi_t \tag{4.4}$$

This re-expression of the link between net assets and inventories as a relationship between profits and inventory investment reveals a parallel with the literature on liquidity constraints and household consumption. The effect of profits on inventory investment for the firm under financial pressure corresponds to the result from the theory of the consumer that the consumption of liquidity constrained households exhibits greater sensitivity to current income than the consumption of unconstrained households.

The reason for estimating a log-linear, rather than a linear, version of (4.4) is that the coefficient ϕ_I will differ from firm to firm. If it is assumed that the limit on indebtedness S is a fixed proportion of H^*, $S=sH^*$, then an equivalent log-linear specification invariant to the value of ϕ_I, in which profits are scaled by the capital of the firm, is available:[26]

$$\Delta \ln(I) = \frac{1}{1+s}\frac{\pi}{K} \tag{4.5}$$

The estimation also allows for a correlation between inventory holdings (at the end of the accounting year) and turnover (in the same year) both for all firms and for the subset of firms under financial pressure. Such a correlation is not formally included in the analysis of section 3, but can be viewed as arising from serial correlation in the stochastic process driving firm revenues. Suppose that the stochastic component of output prices is determined by an AR(1) process:

$$\epsilon_t = \psi \epsilon_{t-1} + v_t \,, \; |\psi| < 1 \,, \; \mathscr{E}(v_t) = 0 \tag{4.6}$$

The state variable H_t must then be augmented by a second state variable ϵ_t and the setting of both the control variables and the threshold value of net assets, H^*, depends upon ϵ_t. A high value of ϵ_t increases the marginal return to both inventories and planned output. As a result, for a given level of H_t, both inventory holdings and planned output increase with ϵ_t. Treating current period turnover ($(\overline{\theta}_t + \epsilon_t)\ \Psi(Y_t)$) as a measure of ϵ_t then a correlation between inventory holdings and current period turnover will arise. Note that when the firm is under financial pressure, Y_t is affected by net asset holdings, implying that current period turnover will vary by more than ϵ_t, and raising the possibility of a reduced correlation between inventory holdings and current turnover during periods of financial pressure.

This relationship between inventory holdings and turnover is almost more appropriately estimated as a log-linear specification. The ratio of inventories to turnover, and hence the coefficient on the change in turnover in a linear specification, will differ from firm to firm. In a log-linear specification the ratio of inventories to turnover is a fixed firm effect, removed by estimation in first differences. The elasticity of a change in inventory holdings with respect to a change in turnover also emerges directly from a log-linear specification.

Alternative indicators of financial pressure

The predictions for dividend behaviour emerging from this model do not accord entirely with the empirical literature on the payment of dividends. Studies of dividend payments by publicly quoted companies suggest that dividend payments when they are made, are a fairly constant fraction of earnings, whereas the model implies that as earnings rise the dividend pay-out ratio should increase. Also, perhaps because dividends play a signalling role, publicly quoted firms often continue to pay dividends even when under financial pressure, and are reluctant to cut dividends as a ratio of earnings below some lower bound. This second consideration suggests that non-payment of dividends may not be an entirely satisfactory indicator of financial pressure.

Because of this difficulty a number of alternative indicators of financial pressure were also used. A ratio of dividend payments to post-tax, post-interest earnings of less than 8.5 per cent (approximately the median dividend pay-out ratio in the sample) was used as a second dividend based indicator of financial pressure. The size of the firm (as indicated by its capital assets in constant 1980 prices) was a further indicator. The remaining indicators of financial pressure were accounting ratios, commonly used by practitioners in assessing company credit-worthiness and liquidity. These were the interest cover ratio (the ratio of pre-tax and pre-interest earnings to total long and short term interest payments) and the return on net assets (pre-tax and pre-interest earnings as a percentage of the book value of net assets) as indicators of credit-worthiness, and two measures of liquidity: the current ratio (the ratio of current assets to current liabilities) and the quick ratio (the ratio of current assets other than inventories to current liabilities).

Interviews with banking practitioners suggested the following threshold values, representing the levels at which bankers might begin to feel concern about the ability of a company to pay back its debts, which were then used to distinguish companies under financial pressure:[27]

	Manufacturing	Retailing
Current ratio	1.5	1
Quick ratio	1	0.5
Interest cover	5	10
Return on assets	25%	25%

There is a problem in applying equation (4.5) to the case where the firm is under financial pressure in the previous accounting year but has now emerged from financial pressure in the current accounting year. This arises because the linear relationship between net assets and inventory holdings only holds within the regime of financial pressure and no longer applies once the firm is free of financial pressure. The model predicts that profits should only be related to inventory investment up to the point at which $I_t = I^*$. Thereafter further increases in profits should be paid out as dividends (or increased dividends if the firm pays out some minimum level of dividends in period t-1), and should have no effect on inventory investment. A relationship which applies when the firm has just emerged from a period of financial pressure must therefore correct profits for the payment of dividends in the second period, when the firm is no longer under financial pressure. When indicators of financial pressure are used, other than non-payment of dividends, this correction must also allow for the level of dividend payment made in the previous period when the firm is under financial pressure. Therefore, for all observations for which it is indicated that the firm was under financial pressure in t-1 but not in t, the following alternative version of (4.5) was used:

$$\Delta \ln(I_t) = \frac{1}{1+s} \frac{\pi_t - d_t + d_{t-1}}{K_t} \tag{4.7}$$

A final consideration is that the historical cost profits reported in the company accounts forming this panel data set over-state the change in the net assets of the firm. Zero historical cost profits, for the firm under financial pressure, will result in a fall in net assets and inventory decumulation. For the estimation it is assumed that a fixed level of positive historical cost profits, as a proportion of firm capital, is required to prevent inventory decumulation. This level of profits is estimated as an additional coefficient with an expected positive sign.

The estimated specification

In the log-linear specification which incorporates all these features, the investment in inventories ($\Delta\ln(I)$) is driven by the growth of turnover ($\Delta\ln(T)$) and by profits (π) expressed as a proportion of the total capital of the firm (K). Let z be the (1,0) dummy variable for the presence of financial pressure in the current accounting year and z_{-1} be the corresponding (1,0) dummy variable for financial pressure in the previous accounting year. The values of z and z_{-1} depend upon the various indicators of financial pressure discussed above. The estimated equation may then be written:

$$\Delta\ln(I) = a_0\Delta\ln(T) + b_0\frac{\pi}{C} + c_0 + z(a_1\Delta\ln(T) + b_1\frac{\pi}{K} + c_1)$$
$$+ (z_{-1}-zz_{-1})(a_1\Delta\ln(T) + b_1\frac{(\pi-d+d_{-1})}{K} + c_1) \qquad (4.8)$$

The coefficients a_0 and b_0 are on turnover and profits for all firms (whether or not they experience financial pressure). a_1 and b_1 are additional coefficients on turnover and profits for the subset of firms experiencing financial pressure. c_0 and c_1 (c_0,c_1<0) are the estimates of the level of historical cost profits at which no inventory decumulation occurs. The dummy variable z picks out firms which are currently under financial distress. The term (z_{-1}–zz_{-1}) picks out observations where the firm has just emerged from financial distress. The predictions of the model are (i) that $a_0 \approx 0.5$ (ii) that $b_0 = 0$ and $c_0=0$; (iii) that $b_1 > 0$ and $c_1<0$; and (iv) that $a_1 \leq 0$ (with strict inequality if as discussed above financial pressure reduces the correlation between turnover in the current accounting year and inventory holdings).

The panel data

The company accounts panel data set used provided a total of 9,143 observations on 2,269 companies for the financial years 1977/78-1985/86. The source of the data was the Department of Trade and Industry company accounts tape (obtained from the ESRC data archive). These tapes contain company accounts of over 3,000 UK registered companies, and a total of 18,854 separate company accounts over the financial years 1976/77-1985/86. A variety of filters applied to the data eliminated many companies and reduced the number of observations used for estimation. Only firms operating wholly or mainly in the United Kingdom and classified by the Department of Trade and Industry as operating either in manufacturing or in distribution were included in the panel (reducing the number of observations to 17,290). A large number of these observations had to be dropped because of missing data, 5051 because accounts failed to report either inventory holdings and a further 656 observations where the ratio of profits to current earnings was not available. A further 155 observations were dropped because the ratio of inventory to turnover was less than 0.1 months of turnover or more than 50 months of turnover (in manufacturing) or less than 0.1 months of turnover or more than 100 months of turnover (in distribution).

The turnover and inventories data were deflated into 1980 prices using producer price indices for the appropriate manufacturing sector and the consumer expenditure deflator for distribution.[28] The price indices were those appropriate to the period covered by each company account, with end-account indices for inventories and account average indices for turnover. The inventories deflator used for manufacturing was the arithmetic mean of the producer price output and producer price input indices. The rate of inventory turnover was then calculated and used to correct the inventories data for the use of the first in first out (FIFO) valuation convention at historic cost prices.[29] The taking of first differences resulted in the dropping of a further 2269 observations and a final 6 observations were dropped because inventories or turnover rose by more than 1000% or fell by more than 95% or because the inventory turnover ratio rose by more than 400% or fell by more than 80%.

Main features of the data

This subsection describes the main features of the panel data, with the help of a number of descriptive tabulations (tables 1-4). Not all companies are represented in the panel in each year. On average there are around 4 observations for each company (with observations after a break treated as observations on a new company), although there are many more observations on the largest companies. The distribution of firms by number of observations is as follows:

Number of observations	1	2	3	4	5	6	7	8	9
Number of firms	276	240	839	155	154	141	81	380	3

This highly skewed distribution reflects the procedures used by the department of trade and industry in building up the panel. Around 500 of the largest UK companies are included in the sample in every financial year. The sample of smaller companies rotates over time, with medium sized companies typically providing between 4 and 7 observations and the smallest companies providing 3 or less observations.

The Department of Trade and Industry sampling procedure means that the number of smaller firms included in the panel falls gradually from 1978/79 to 1982/83, jumps by a large amount in 1983/84 (as the result of a major re-sampling exercise was undertaken) and then falls off again until 1986/87. Since the variation in coverage largely relates to smaller firms this does not greatly affect aggregate inventory holdings and turnover in the panel.

Table 1 compares aggregate inventory holdings and turnover in the panel with the corresponding national accounts aggregates. Aggregate turnover and inventories for company accounts ending in the financial year are compared with corresponding national accounts statistics for the corresponding calendar year. Using the industrial classification provided on the Department of Trade and Industry tape this indicates that the panel, even after removing a large number of observations and the varying number of companies covered, represents over 70% of turnover in the UK distribution sector and over 75% of turnover in the UK manufacturing sector throughout the period of the panel.

Combining manufacturing and distribution the movement of panel aggregate turnover compares fairly well with the equivalent national accounts aggregates. However there is some discrepancy between the panel and the national accounts for the two individual sectors. The fall in manufacturing output in 1980/81 is not fully reflected in the panel data while the fall in turnover in distribution recorded for the panel is rather greater than the corresponding fall in retail sales. It is likely that this discrepancy reflects a difference between the national accounts industrial classification and the industrial classification provided on the Department of Trade and Industry tape, with firms classified as in distribution on the tape actually partly engaged in manufacturing.

There are more marked discrepancies between the movements in the panel measure of aggregate inventory holdings and the national accounts measure of inventory holdings. There is a sharp increase in aggregate panel inventories for company accounts ending during the financial year 1979/80, but this is not reflected in the national accounts data for 1979. The subsequent fall in the panel aggregate during 1980/81 is much greater even than the fall in the national accounts measure of inventories. Moreover the aggregate panel inventories in manufacturing exhibit very erratic movements in 1981/82 (a rise of 4 per cent) and in 1982/83 (a fall of 9 per cent) which are not reflected in the national accounts.

These discrepancies must be regarded as a serious shortcoming of this data set, especially when it is used to draw conclusions about the determinants of aggregate inventory movements. The discrepancies probably reflect the difficulties of adjusting company accounts data for the historical cost convention and the predominant practice amongst UK firms of first in first out accounting of inventory holdings, especially during periods of high inflation.

There is considerable variation in the data at the level of the individual company. The following tabulation shows the quartiles of the distribution of the growth rates of inventories and turnover for all observations. While median per annum growth rates of turnover and inventories are respectively 1 and 0 percent, the corresponding quartiles are around 8 percent (for turnover) and 13 percent (for inventories) either side of these medians.

Quartiles of Growth Rates in the Panel Data

	Distribution		Manufacturing	
%	Turnover	Inventories	Turnover	Inventories
Q1	-8.2	-14.6	-7.7	-13.4
Median	1.3	0.2	1.0	-0.4
Q3	10.1	13.0	9.2	11.4

Table 2 shows the distribution of the various indicators of financial pressure by financial year. All the indicators (with the exception of company size) show a similar pattern, with the proportion classified as under financial pressure rising from 1978/79 to 1980/81 or 1981/82, and then falling in remaining years. For a high proportion of observations no dividend payments are made, but this proportion falls sharply towards the end of the sample.

Table 3 shows the distribution of the various indicators of financial pressure, both unweighted and weighted by reported capitalisation (in £mn 1980 prices). Based on the number of observations, the incidence of financial pressure ranges from 22 percent of the sample (the current ratio) and 28 percent (interest cover) to 50 percent of the sample (dividend pay-out ratio less than 8.5 percent). Weighted by reported capitalisation the incidence of financial pressure ranges from 17 percent (non-payment of dividends) to 37 percent (the return on assets) and 45 percent (the quick ratio).

The weighting makes a substantial difference to the proportion of the sample classified as under financial pressure by dividend behaviour because non-payment of dividends is much more common amongst smaller companies. For 40 per cent of observations there are no dividend payments but these fall to only 17 percent when the observations are weighted by capitalisation. The weighting makes an even greater difference for distribution companies (which have a smaller average capitalisation than manufacturing companies), the proportion not paying dividends falling from 48 per cent to 9 per cent after weighting. The weighting by capitalisation results in a similar reduction in the incidence of financial pressure, as indicated by a dividend pay-out ratio of less than 8.5 percent.

When financial pressure is indicated by interest cover, or by the return on assets, then the weighting results in some fall in the proportion of the sample under financial pressure, but only from 28 percent to 21 percent (interest cover) and from 43 to 37 percent (return on assets). For the quick and current ratio the weighting makes little difference.

Table 2 and table 3 also show that large numbers of individual firms either come under financial pressure or escape from financial pressure in each period. Depending on the indicator of financial pressure adopted, 5-12 percent of companies move into and a further 4-11 percent of companies move out of financial pressure over the sample (weighting reduces both these ranges to 4-9 percent). Company size, which does not change from one period to another, is again an exception.

Table 4 examines this point in a different way by showing the proportion of companies which are respectively classified as never, partly and always under financial pressure. As the interpretation of these ratios is sensitive to the number of observations available on each firm, these proportions are tabulated by the number of observations. For large companies, where observations are available over most of the panel, there still are a substantial proportion which are never under financial pressure (this proportion varying from 50 per cent where dividend behaviour or interest cover is used as an indicator down to 25 per cent where return on assets is used as an indicator). A much lower proportion of large companies (varying from 4-12 percent according to indicator) are always under financial pressure. While around half of large companies are under financial pressure during part of the sample (or around two thirds where return on assets is the indicator of financial pressure).

For smaller companies, for which there are typically only around 3 observations, both the proportion of companies which are always under financial distress and the proportion of companies which are never under financial distress are much higher. However even with only three observations between one-quarter and one-third (depending on the indicator of distress) of all companies are under financial distress part but not all of the time. Thus table 4 also indicates the frequency with which the classification of the individual company alters within the sample.

Estimation results

The results of estimating equation (4.8) for the change in inventory holdings, without including time dummies, are reported in table 5. Almost identical results are obtained when time dummies are included (table 6). The first three columns of the tables show coefficients on turnover and profits and the constant term for all firms in the sample (a_0, b_0 and c_0). The final three columns of the tables show the additional coefficients on those firms identified as under financial pressure (a_1, b_1 and c_1).

The first row in each of these tables shows the results of estimating a model in which firms under financial pressure are not separately distinguished. There is a highly significant link between increases in turnover and inventory investment. The elasticity of inventory investment on the change in turnover is around 0.55, remarkably close to the coefficient of 0.5 that would be predicted by a square root law of inventory investment. The impact of profits is also powerful and statistically highly significant. However the presence of this profit term could be due to the effects of some omitted real variable, perhaps proxying expectations of future growth of output and sales. On its own this finding provides little evidence for financial effects on inventory investment.

Strong evidence in favour of financial effects on inventory investment is revealed in the remaining rows of tables 5 and 6, where the impact of profits on inventory investment is confined to the subset of firms under financial pressure as indicated by dividend behaviour, the interest cover ratio or the return on net assets. This finding is exactly as predicted by the model. When other indicators of financial pressure were used, namely firm size, the quick ratio or the current ratio, there is no difference in the correlation between profits and inventory investment for firms under financial pressure.

There is some evidence, on the borderline of statistical significance, that the elasticity of inventory investment on the change in turnover is smaller for those firms where financial pressure is revealed by dividend behaviour or the interest cover ratio falling below the threshold value. Again this is consistent with the model. Of greater statistical significance is the difference between the turnover elasticities for larger and smaller firms; smaller firms exhibit a significantly lower elasticity on turnover, a finding for which there is no immediately apparent explanation.

A number of estimates were conducted combining two indicators of financial pressure (with separately estimated coefficients for the two categories of financial pressure). These all suggested that the interest cover ratio was the most successful indicator, in the sense that other indicators, when combined with the interest cover ratio, identified a set of companies with insignificant coefficients. In one case reported in the final rows of tables 5 and 6, where both non-payment of dividends and interest cover were used as indicators of financial pressure, coefficients resulted that were significant for both groups of companies identified as under financial pressure.

The financial effects on inventory investment revealed by these estimates are unrelated to firm size (except that the prevalence of dividend non-payment is much greater amongst smaller companies). In tables 5 and 6, smaller firms, with a capitalisation of less than £10mn in 1980 prices, do not exhibit a greater correlation between profits and inventory investment. It is also possible that smaller firms, when they are under financial pressure, have less access to outside finance and are therefore forced to a greater degree than larger firms to alter inventory investment in response to current profits. This hypothesis is examined using the results reported in table 7.

In the reported results turnover coefficients are restricted to be the same for those firms under financial pressure as for all firms. This restriction makes no difference to the conclusions. There is in fact no significant difference between the profit coefficients for large, medium or small firms under financial pressure. The alternative hypothesis that financial effects are unrelated to firm size is once again accepted.

Table 8 shows the results that emerge from a re-estimation of (4.8) weighting observations by the capitalisation of the company in 1980 prices. The results are similar to those reported in tables 5 and 6. With firms under financial pressure not separately distinguished profitability has a powerful and highly significant effect on inventory investment. The effects of profitability can be restricted to firms under financial pressure, when this is indicated by dividend behaviour, interest cover or return on assets. A somewhat more surprising result is that the elasticity on turnover, for those companies in financial distress as indicated by the interest cover ratio, is around 0.4 smaller than for other companies. The interest cover ratio is now always the dominating indicator of financial pressure, in the sense that other indicators combined with the interest cover ratio, always result in insignificant coefficients on companies under financial pressure (see the final row of table 8).

These results are entirely consistent with the predictions of the version of the model set out at the beginning of this section. There is a highly significant link between profitability and inventory investment, restricted to subsets of companies identified as under financial pressure. The most successful indicator is the interest cover ratio, although dividend behaviour and return on assets also work well. These findings are consistent with the earlier empirical work of Fazzari et al (1988) who use firm level data on a panel of US manufacturing companies to demonstrate that internal cash flow affects expenditure on fixed capital investment.

Fixed versus varying classification of financial pressure

These results reported here contrast with Fazzari et al in one key respect; they use a varying rather than a fixed classification of firms. Fazzari et al argue that mature companies, whose prospects are well understood, are appropriately modelled by the standard model in which external capital is a good substitute for internal funds. In contrast informational imperfections result in external finance being more costly than internal finance (a "financing hierarchy") for a subset of "information intensive" companies whose prospects for future earnings are not well understood by the providers of external finance. This analysis leads Fazzari et al to use dividend pay-out behaviour to classify companies in groups which remain fixed over the entire sample.

Fazzari et al divide their sample (422 publicly quoted firms over a 15 year period) into three groups, according to dividend pay-out ratios over the full 15 years. The first group consists of 49 firms who made a dividend pay-out of less than 10 per cent in 10 of the 15 years. The second group consists of a further 39 of the remaining firms who made a dividend pay-out of less than 20 per cent in 10 of the 15 years. The remaining 334 companies were placed in a third class of mature firms. Using this fixed classification, and estimating a variety of different models of fixed capital investment, they find effects of cash flow on investment for all firms, but with the effects of cash flow being more powerful for the first two classes of firms with lower dividend pay-out ratios and most powerful for the first class of firms with the lowest dividend pay-out ratios.

Is a fixed classification (of the kind adopted by Fazzari et al) or a varying classification (as suggested by the model of section 3 and used in this paper) a more successful predictor of the effects of profitability on inventory investment? Table 9 shows the results of estimates which combine both fixed and varying classifications. The fixed classifications adopted here use the same threshold values as the varying classifications used for the estimation reported in the earlier tables, but a company is classified as under financial pressure if the indicator falls below the threshold for 50 per cent or more of the available observations for that company. Adopting this procedure the number of companies classified as under financial pressure is approximately the same as with the varying classifications, but the classification of each company does not change over time.

Table 9 compares fixed and varying classifications for three different indicators of financial pressure, namely non-payment of dividends, interest cover and return on assets. Turnover is omitted for firms under financial pressure but this makes little difference to the results. In all cases, when combining both a fixed and a varying classification, it is only the profitability of companies which are treated as under financial pressure using the varying classification that is statistically significant. The restriction to a fixed classification alone is very strongly rejected, and when this is done the coefficient on profits for all companies (b_0) once again becomes significant. It is noteworthy that the constant term for companies under financial pressure has a significant positive sign in the combined equation, perhaps indicating that companies which regularly experience financial pressure maintain tighter control over inventory investment. Because of the significance of this constant term the restriction to the model using a varying classification alone is also rejected, but at much lower levels of statistical significance than the restriction to a model using only a fixed classification.

Aggregation of the financial effects on inventory investment

The remaining question to be considered using this panel of company accounts data is whether the estimated financial effects, operating through the profitability of firms under financial pressure, provide a potential explanation of the pronounced cyclical movements in inventory investment over the period of the panel. This particular panel, which covers a large percentage of total corporate inventory holdings in the UK, should be well suited to this task. There is however the obvious difficulty with these comparisons, revealed in table 1 and discussed earlier, that aggregate inventory holdings in the panel and in the national accounts do not tally at all well.

Table 10 reports an analysis of the aggregate movements in panel inventory holdings, comparing predicted aggregate inventory investment by the companies in the panel, using the estimated equation presented in the final row of table 5, with the actual panel aggregate and the corresponding national accounts aggregate. The table indicates that the fitted movements in inventories do a fairly good job of explaining the national accounts aggregates, but are much poorer at explaining the movements in the actual panel aggregates. The inventory movements induced by financial pressure (column (4)) are quantitatively very large, and sufficiently powerful to provide a possible explanation of the substantial cyclical fluctuations in aggregate inventory investment. The residuals in column (5) indicate that fitted inventory investment under-predicts the national accounts aggregates in 1978/79, and 1979/80; coincides with the aggregate movements in inventory investment during the two years of most rapid inventory decumulation (1980/81 and 1981/82); under-predicts slightly in 1982/83 and 1983/84; and over-predicts somewhat in 1984/85 and 1985/86.

The fitted movements in aggregate inventory investment correspond much less well to the actual panel aggregates. Column (6) indicates substantial residuals, of the order of ± 5 percent, which alternate in sign from one financial year to the next. This pattern is consistent with the conjecture that the discrepancy between the panel aggregates and the national accounts aggregates are due to measurement error in the panel aggregates.

The aggregation of these estimation results suggests that the impact of financial effects, of the kind analysed in this paper, on aggregate inventory investment are very powerful indeed. Combined with changes in turnover they capture the movement of inventory investment, especially the dramatic inventory decumulation of 1980/81, fairly well. It seems that financial effects can explain the long-standing puzzle of the pronounced pro-cyclicality of inventory investment.

There are however some remaining doubts about the quality of the inventories data in this particular data set which urge a degree of caution in accepting these conclusions. The obvious way to resolve these doubts would be further work on the determinants of inventory investment, along the lines reported here, with company level data for other time periods and from other countries.

5. Conclusions

This paper has presented an infinite-period model of inventory dynamics in the form of a dynamic programme for a firm subject to an exogenous limit on debt finance and bankruptcy when debt exceeds this level. The model is solved, using standard methods, when equity finance is unconstrained and in this case the debt finance constraint has no effect on the behaviour of the firm. With an additional exogenous restriction on equity finance then financial effects on inventory investment emerge.

This latter finding rests on a qualitative solution of the model subject to both equity and debt constraints. This is examined (in the annex) by analysing the different regimes which result from each possible set of binding constraints on the firm's actions. This technique of *regime analysis* has potential application in other dynamic models of firm behaviour where constraints limit the setting of control variables.

The version of the model with constraints on both debt and equity finance can be understood as a dynamic generalisation of the one period "bankruptcy avoidance" model of Wadhwani (1986), but unlike that model it has not been necessary to include an aversion to bankruptcy in the objective function of the firm. Instead the firm desires to hold a certain level of liquid assets, and thus reduce the probability of bankruptcy, because bankruptcy results in the loss of an otherwise valuable stream of future earnings.

The firm's behaviour is driven by the net assets it inherits from its current operations and financial structure. When its net assets exceed a threshold value then the firm holds its desired level of liquid assets and inventories and pays any remaining assets out in the form of dividend payments. If however net assets fall below the threshold value then the firm is unable to hold the desired level of liquid assets. Such firms, described here as being under financial pressure, pay no dividends but instead use revenue to build up inventories and reduce indebtedness.

The analysis indicates the possibility of highly non-linear relationships between net assets and inventory holdings and between net assets and indebtedness. If however linearity is assumed, then inventory investment is driven by current profits during periods of financial pressure. This parallels the finding, from the literature on the life cycle model of consumption, that when liquidity constraints bind, consumption depends on current income rather than lifetime wealth. It is this prediction about the relationship between profits and inventory investment which is examined in the empirical section of the paper.

The most obvious theoretical weakness of this model is the exogeneity of the firm's debt and equity contracts. This suggests that the model should be further developed by endogenising the constraints on both debt and equity finance. Such constraints can only be justified by the presence of informational imperfections in both debt and equity markets. The analysis would thus require a more explicit statement of the information available to both banks and shareholders. The set up of the model would have to allow for a distribution of unobserved parameters amongst firms so that some firms, unknown to their creditors, yield negative expected returns. It remains to be seen whether the analysis of this paper would remain tractable in this more sophisticated environment.

Inventory investment in the panel of individual company accounts data examined here is remarkably consistent with the theoretical analysis. A highly significant link between profits and inventory investment emerges for the subset of firms identified as under financial pressure, at least as indicated by non-payment of dividends or, and with an even more significant coefficient, as indicated by the values of certain key accounting ratios (interest cover and the return on assets). For other firms there is no link between profits and inventory investment. In this regard the theoretical model is a striking empirical success, although its predictions about dividend payments seem to accord less closely to firm behaviour.

Aggregation across the panel (which covers 70-75 per cent of inventories in UK manufacturing and distribution) shows that the magnitude of the estimated financial effects on inventory investment are very large and can explain the pronounced cyclical decumulation of inventories in the UK in the early 1980s. However they manage less well at explaining the recorded movements in aggregate inventory investment in the panel itself (as opposed to aggregate inventory investment in the national accounts). This last finding may reflect errors in the panel inventory data. Despite this last shortcoming the empirical results suggest that not only are financial effects a significant determinant of inventory investment, but they can explain much of the well known but little understood cyclical fluctuations of inventory investment.

Annex : Solution of the dynamic programme.

Preliminaries

This annex presents the solution of the dynamic programme of section 3. The solution when there is no constraint on equity finance emerges from the standard technique of guess and verification. The solution when there is a non-negativity constraint on the payment of dividends is obtained by a regime analysis applied across different sets of binding constraints. In both cases it is helpful to distinguish positive and negative debt as two distinct control variables, D^+ and D^-, with the first subject to a non-negativity constraint while the second is subject to a non-positivity constraint:

$$D^+ \geq 0 \,, \quad D^- \leq 0 \tag{A1}$$

The Bellman equation, which is maximised subject to the constraints on D^+ and D^-, together with the constraint determining d_t and the equation of motion for the state variable H_t, are then written:

$$\begin{aligned} V(H_t) &= \max_{I_t, Y_{t+1}, D_t^+, D_t^-} \left[d_t + \delta \mathscr{E}\, V(H_{t+1}) \right] \\ d_t &= H_t - \gamma I_t + D_t^+ + D_t^- \\ H_{t+1} &= R(\epsilon_{t+1}, I_t, Y_{t+1}) - (1+r^+)D_t^+ - (1+r^-)D_t^- \end{aligned} \tag{A2}$$

Three further variables, v, w and z, are used repeatedly in the regime analysis of the dynamic programme with a non-negativity constraint on dividend payments. These are the expected present discounted value of a marginal increment to output in the next period t+1, to inventories at end of the current period t, and to the expected value of net assets in the period t+1. Formally these may be defined as:

$$
\begin{aligned}
v &= \delta \int_{-\infty}^{+\infty} \frac{\partial V(H_{t+1})}{\partial Y_{t+1}} f(\epsilon)\, d\epsilon \\
w &= \delta \int_{-\infty}^{+\infty} \frac{\partial V(H_{t+1})}{\partial I_t} f(\epsilon)\, d\epsilon \qquad \text{(A3)} \\
z &= \delta \int_{-\infty}^{+\infty} \frac{\partial V(H_{t+1})}{\partial \bar{H}_{t+1}} f(\epsilon)\, d\epsilon
\end{aligned}
$$

where $\bar{H}_{t+1}$ is the expected value of $H_{t+1} = R(\epsilon_{t+1}, I_t, Y_{t+1}) - (1+r_t)D_t$.

These marginal present discounted values all appear in the first order conditions for the maximisation of the right hand side of the Bellman equation. Since I_t affects $V(H_{t+1})$ only through the revenue function R, and $\frac{\partial^2 R}{\partial \epsilon \partial I} = 0$, it follows that $w = z \frac{\partial R}{\partial I}$. Note also that the interaction between I_t and Y_{t+1} in R() means that an increase in end period inventories increases the marginal value of an increment to output in the next period ($\frac{\partial v}{\partial I_t} > 0$) while an increase in planned output increases the marginal value of an increment to end period inventories ($\frac{\partial w}{\partial Y_{t+1}} > 0$).

Measurability of the expectation $\mathscr{E}V(H_{t+1})$

The regime analysis of the dynamic programme with a constraint on equity finance assumes that there exists a value function (V(H)) which is a solution to the Bellman equation. Then by the principle of optimality (for a proof see Stokey and Lucas (1989), theorem 9.2, pp 246-247) this function V(H) also attains the supremum of the firm's objective:

$$
\mathscr{E} \sum_{j=0}^{\infty} \delta^j d_{t+j} \qquad \text{(A4)}
$$

and may be interpreted as the expected present discounted value of the future stream of dividends.

The assumption that there exists a value function which satisfies the Bellman equation raises difficult technical issues relating to the measurability of $\mathscr{E}V(H_{t+1})$. The general statement of stochastic dynamic programmes allows for the possibility that the optimal plan does not satisfy the Bellman equation. This will occur if the optimal plan is such that the expectation of the value function in period t+1 is non-measurable on the space of the stochastic shocks in period t+1 implying that the right hand side of the Bellman equation cannot be evaluated. See Stokey and Lucas (1989) chapter 9 for an extended discussion.

Such non-measurability requires that the agent behave according to decision rules which defy intuitive interpretation in the context of economic models. It rules out, for example, decision rules which are almost everywhere continuous functions of the state variables of the dynamic programme. It may be conjectured that additional restrictions on the structure of the dynamic programme, restrictions which would be weak as far as the construction of economic models is concerned, would be sufficient to ensure that the optimal plan always satisfies the Bellman equation. Such a result is not available in the literature but it would seem that the assumption of a solution to the Bellman equation is, in the present context, only technically restrictive and is not critical to the economic analysis of the model.

Solution with no constraints on equity finance

Applying the method of guess and verification yields the solution when there is no equity financing constraint. The guess is that the firm holds neither cash, nor debt; that inventory holding and employment decisions are made so as to maximise operating profits less the cost of inventory investment, with an appropriate discounting between the two periods using the discount rate δ; and that the value of the firm to its shareholders is given in terms of net assets H_t as:-

$$V(H_t) = H_t + K \tag{A5}$$

The form of this guess at the value function is prompted by the intuition that the value of the shareholding in the current period should reflect current net assets plus a constant K which corresponds to the value to shareholders of the continued operation of the firm.

The maximisation on the right hand side of the Bellman equation yields four first order necessary conditions. The first two, derived with respect to D^+_t and D^-_t, are subject to complementary slackness. The others are derived with respect to the end period level of inventories I_t and the planned level of output Y_{t+1}:

$$\begin{aligned} z(1 + r^+) \geq 1 \quad D^+ \geq 0 \\ z(1 + r^-) \leq 1 \quad D^- \leq 0 \\ w = \gamma \\ v = 0 \end{aligned} \tag{A6}$$

The first two conditions are only satisfied by $D^+_t = D^-_t = 0$ (since, with $r^+ > \delta^{-1} - 1 > r^-$, the first inequality in each constraint is always strict). Let $\bar{I}_t$ and $\bar{Y}_{t+1}$ be the levels of inventory investment and planned output

which maximise the present value of the following one period maximand Φ:

$$\Phi_t = \delta \mathscr{E}\left[R(\epsilon_{t+1}, I_t, Y_{t+1})\right] - \gamma I_t \tag{A7}$$

Evaluating v and w, when V(H) = H + K, shows that $\bar{I}_t$ and $\bar{Y}_{t+1}$ also satisfy the final two first order conditions in (A6). With $I = \bar{I}_t$ the current dividend is given by $H - \upsilon \bar{I}_t$ and the verification is completed by substituting $\bar{I}_t$ and $\bar{Y}_{t+1}$ and $D^+_t = D^-_t = 0$ in the Bellman equation yielding:

$$V(H) = H_t - \gamma \bar{I}_t + \delta \mathscr{E}\left[H_{t+1} + K \mid \bar{I}_t\right] \tag{A8}$$

From this it follows that the Bellman equation is satisfied by the initial guess when K is given by:

$$K = -\gamma \bar{I}_t + \delta \{ \mathscr{E}\left[R(e_{t+1}, \bar{I}_t, \bar{Y}_{t+1})\right] + K\}$$
$$= \{\Phi(\bar{I}_t, \bar{Y}_{t+1}) + \delta K\} = \frac{1}{1 - \delta} \Phi(\bar{I}_t, \bar{Y}_{t+1}) \tag{A9}$$

This completes the verification. This analysis omits one point. Suppose that the value function becomes negative because H < -K. In this case, if the shareholders are subject to limited liability, it is in their interests for the firm to switch over to pure debt finance, rather than to issue new equity. This possibility can be ruled out by assuming *either* that the shareholders are subject to unlimited liability (so that it is always in the shareholder's interests for the firm to issue new equity rather than switch to debt finance) *or* that the distribution of e_t is such that $H_t \geq -K$. If neither of these assumptions are made then there is effectively a dividend constraint of the form $d_t \geq -K$ and the analysis of the following sub-section then applies.

Solution with a non-negativity constraint on dividend payments.

Suppose that informational imperfections mean that there is an upper bound on equity issue, in the form of a non-negativity constraint on dividends, $d_t \geq 0$. This constraint may be written:-

$$H_t + D_t^+ + D_t^- - \gamma I_t \geq 0 \tag{A10}$$

The solution of this version of the model proceeds by a regime analysis of the properties of the value function, and the associated decision rules, under each of the possible sets of binding constraints. This makes the assumption (discussed above) that the expectation of next period's value function is well defined.

The proofs of the propositions (a) to (e) in section 3 are then as follows:

Proposition (a) This follows because if H<S, even if the firm pays no dividend and makes no investment in inventory, the debt finance constraint is violated and the firm is bankrupt. Since by assumption nothing can be raised from re-flotation of the firm the discounted expected value of current and future payments to shareholders V(H)=0.

Proposition (b) The intuition behind this proof is that since ϵ has a continuous probability distribution it has the effect of smoothing out any discontinuities inside the expectations operator on the right hand side of the Bellman equation. Small changes to I_t, D_t or Y_{t+1} only result in small changes to the expectation of $V(H_{t+1})$. As a result, and provided the firm is not bankrupt ($H \geq -S$), the value function (the left hand side of the Bellman equation) is both continuous and differentiable.

Three lemmas will be used in the formal proof of this proposition.

Lemma 1 : $\mathrm{Max}(0,H) \leq V(H) \leq H + K$. The value function is always greater than, or equal to, the greater of zero or H but is less than or equal to the corresponding value function that applies when there is no constraint on equity finance.

Proof: With a non-negativity constraint on dividend payments, the present value of expected future dividend payments is always non-negative implying that $0 \leq V(H)$. The firm also always has the option of declaring voluntary bankruptcy distributing all net assets to shareholders. Hence $H \leq V(H)$. Finally consider the contingent plan associated with the value function V(H). This plan, together with voluntary bankruptcy in the event of $H \leq -S$, is a feasible plan for the case where there is no constraint on the issue of equity. But this cannot yield a higher present discounted value of future dividend payments than the optimal plan for the case where there is no constraint on new equity finance. Hence $V(H) \leq H + K$.

Lemma 2. Any increase in the value function $\Delta V(H)$ resulting from an increase of net assets of ΔH, is subject to the inequality $\Delta V(H) \geq \Delta H$. Hence any discontinuities in V(H) must be positive.

Proof: The increment of net assets can always be used to finance a dividend payment, increasing shareholder wealth by ΔH. Hence whatever policy is chosen by the firm the value function must increase by at least ΔH. A negative discontinuity in V(H) would violate this inequality.

Lemma 3. By assumption ϵ has a continuous (not a discrete or mixed discrete/continuous) distribution. Let f^m be the maximum value of the probability density function for ϵ, corresponding to a value of ϵ^m. Then w,z (defined above in section (i) of this appendix) satisfy the following inequalities:

$$\begin{array}{rcccl} \delta & < & z & \leq & \delta[1 + Kf^m] \\ \delta\,\Xi_I & < & w & \leq & \delta\,\Xi_I[1 + Kf^m] \end{array} \qquad \text{(A11)}$$

Proof: Given lemma 1 the minima of z,w arise if the entire distribution of ϵ lies in a region where the value function is linear with a slope of +1. The maxima of z,w arise if the entire distribution of ϵ, except for ϵ^m, lies in a region where the value function is linear with a slope of +1, while at ϵ^m there is a positive discontinuity in V(H) of +K (the maximum possible discontinuity consistent with lemma 1 and lemma 2).

Proposition (b) is now proven as follows. Consider a small positive increment to H_t, ΔH (when $H_t > -S_t$). This can be applied in one of four ways:

(1) An increase in end period dividends $\Delta d_t = \Delta H_t$;

(2) An increase in end period inventories ($\Delta I_t = \gamma^{-1}\Delta H_t$);

(3) A reduction of end period debt ($\Delta D^+_t = -\Delta H_t$); or

(4) An increase in end period cash holdings ($\Delta D^-_t = -\Delta H_t$).

Differentiability will be established by evaluating the limit as $\Delta H \rightarrow 0$ of $\frac{\Delta V(H)}{\Delta H}$ and showing that this limit exists under all four alternative policies:

Under (1) $\Delta V(H) = \Delta H$ and $\frac{\Delta V(H)}{\Delta H} = 1$.

Under (2) the direct effect on the value function is $w\Delta I = w\gamma^{-1}\Delta H$. There is also an indirect effect, arising through the re-optimisation of output, of $v\Delta Y$, but since $v=0$ this is of second order and the limit as $\Delta H \rightarrow 0$ of $\frac{\Delta V(H)}{\Delta H} = \gamma^{-1}w$

Under (3) (which is only feasible when $D^+>0$) then the limit as $\Delta H \rightarrow 0$ of $\frac{\Delta V(H)}{\Delta H} = (1+r^+)z$.

Under (4) (which is only feasible when $D^+=0$) then the limit as $\Delta H \rightarrow 0$ of $\frac{\Delta V(H)}{\Delta H} = (1+r^-)z$.

Hence, under the optimal plan:

$$\frac{\partial V(H)}{\partial H} = \lim_{\Delta H \to 0} \frac{V(H+\Delta H)-V(\Delta H)}{\Delta H} \qquad \text{(A12)}$$
$$= \mathrm{Max}(1, \gamma^{-}1w, (1+r)z)$$

where r corresponds to r^+ when $D^+>0$, r^- otherwise.

By lemma 3, the right hand side of (A12) is finite so that the partial derivative of V(H) exists and is finite and V(H) is therefore continuous and differentiable. The inequality in $\frac{\Delta V(H)}{\Delta H}$ then follows immediately from (A12).

Proposition (c).

Proposition (c) is then established in three steps. The first is to show that if H ≤ 0, then no dividend is paid. Suppose instead that $H \leq 0$ and a dividend is paid. $I_t>0$ (since otherwise expected revenues in t+1 are $-\infty$) so that $D^+_t> 0$ and $D^-_t=0$. Now consider the effect on $V(H_t)$ of a small cut in the dividend paid of Δd accompanied by a corresponding reduction in D^+_t. This increases the value function by $\Delta d(\delta z(1+r^+)-1) > 0$ (the inequality follows from A11). This in turn implies that when $H \leq 0$ it is always in the shareholder's interests for a positive dividend to be reduced and, therefore, when H<0 no dividend is paid.

The second part of the proof of proposition (c) is considerably more difficult and requires stronger assumptions about the distribution of the stochastic shockϵ than are needed for the proof of propositions (a) and (b). The reason for requiring stronger conditions on the distribution of ϵ is that there remains the possibility that, as H increases, the firm might switch back from dividend payments to retention of net assets. These stronger conditions are needed to rule out the possibility of multiple regimes and ensure that, once H is sufficiently large to trigger a dividend payment, any increment to net assets will always be paid out as an increase in dividends, rather than being used to increase inventory holdings or cash holdings.

What is required are conditions to ensure that the retention of net assets ΔH always increases the value function by less than ΔH when dividends are paid (so that the firm always pays out the increment as dividends). A fairly weak set of sufficient conditions is that $f(\epsilon)$ be single peaked; that when a dividend payment is made in the current period the probability of bankruptcy in the following period is low (so that bankruptcy is triggered on the left-hand tail of the probability density function); and that when a dividend is made in the current period the probability of a dividend payment in the following period is high enough to include all events in the right hand tail of $f(\epsilon_{t+1})$. These conditions must apply to both finite horizon and infinite-horizon solutions of the dynamic programme.

The sufficiency of these conditions may then be established through the following contraction mapping argument. Consider the class of functions ς, such that for all $V(H) \in \varsigma$ (i) there exists a fixed value of H, H^*, associated with each V(H) for which $V(H) = V(H^*) + H - H^*$, for all $H > H^*$; (ii) $\frac{dV(H)}{dH} \geq 1$ for all H in the range $-S \leq H < H^*$; and (iii) $V(H) = 0$ for $H < -S$ (iv) H

Let V_0 be a value function on the right hand side of the Bellman equation and V_1 be the value function that then results on the left hand side of the Bellman equation from maximisation of the Bellman maximisation. Assume that the additional conditions on ϵ apply to the maximisation of V_1. The contraction mapping argument then proceeds by assuming that $V_0 \in \varsigma$ and showing that this implies that $V_1 \in \varsigma$. Hence by iteration on the Bellman equation (and assuming that the additional conditions on ϵ apply at each stage of the iteration) $V(H) \in \varsigma$.

The following lemma will be used:

Lemma 4. Consider the maximisation of the Bellman equation when $V_0 \in \varsigma$. Let the expectation $\mathscr{E}\epsilon = \bar{\epsilon}$. Suppose that bankruptcy under V_0 ($H<-S$) is triggered by events on the extreme left-hand tail of the distribution of $f(\epsilon)$, and that $H>H^*_0$ over the entire right hand tail of $f(\epsilon)$ then:

$$\text{letting:}\quad X = \int_{\epsilon_b}^{+\infty} V_0(H) f(\epsilon)\, d\epsilon$$

$$\frac{d\frac{dX}{d\bar{\epsilon}}}{d\bar{\epsilon}} < 0 \qquad \text{(A13)}$$

Proof: Suppose that bankruptcy is triggered by a value of ϵ of ϵ_b, associated with a probability density of f_b. This implies that:

$$\frac{dX}{d\bar{\epsilon}} = f_b V(-S) + \int_{\epsilon_b}^{+\infty} \frac{dV}{dH} f(\epsilon)\, d\epsilon \qquad \text{(A14)}$$

Since bankruptcy is triggered by events on the left hand side of $f(\epsilon)$ the increase in $\bar{\epsilon}$ reduces the probability of bankruptcy. It also reduces the weight given to values of $H<H^*$ in the evaluation of $\frac{dx}{dH}$ (because $V < H^*$ only occurs when ϵ is in the left hand tail of $f(\epsilon)$). Since both effects operate in the same direction $\frac{dx}{dH}$ falls as $\bar{\epsilon}$ increases establishing the lemma.

With this lemma the contraction mapping argument may be completed. Let H^*_1 be the minimum value of H for which the maximisation of the right hand side of the Bellman equation with V_0 on the right hand side involves a dividend payment. Now consider an increment (of any possible size) to H at $H=H^*_1$ of ΔH. The proof proceeds by showing that the right-hand side of the Bellman equation is then maximised by paying out ΔH as a dividend, no matter how large ΔH and hence that $V_1 \in \varsigma$.

The right hand side of the Bellman equation is maximised for $H=H^*_1$. Any part of the increment to net assets of ΔH not paid out as a dividend may be used either to increase cash holdings or to increase inventories. The increase cash balances is equivalent to a rightward shift in ϵ of $(1+r^-)\Delta H$. Since, by lemma 4, $\frac{dx}{d\epsilon}$ now falls the marginal benefit of holding additional cash balances is reduced and the right hand side of the Bellman equation can no longer be maximised.

If instead it is used to increase inventory holdings there is a fall in the cost of production (increasing $\overline{\epsilon}$) but the marginal reductions in the costs of production fall as I is increased (this follows from the assumption of the convexity of Ξ). Once again, whatever the size of ΔH, the first order conditions on the right hand side of the Bellman equation can no longer be satisfied.

This establishes that in the optimisation underlying $V_1(H)$ the retention of the additional net assets ΔH, when $H=H^*_1$, results in a violation of the first order conditions for the maximisation of the Bellman equation. Hence all additional assets when $H=H^*_1$ are paid out as dividends and $V_1(H) \in \varsigma$. Thus membership of ς is preserved through iterations on the Bellman equation provided that the additional conditions on ϵ are satisfied.

V(H) is the unique fixed point from repeated iteration on the Bellman equation, beginning with any arbitrary value function. Since membership of the set ς is preserved in this iteration (on the assumption that the additional conditions on ϵ apply at each iteration), it follows that $V(H) \in \varsigma$ and the second part of (c) is established.

To complete the proof of (c) the existence of a non-negative finite H^* will be proved by contradiction. Assume that there is no finite H^* at which a dividend is paid. The proof proceeds by showing that in this case the value function is zero at all values of H, a contradiction with (b).

Note that V(H), and the associated decision rules, are the limit, as $T \to \infty$, of the value function and decision rules derived from the finite period optimisation (which will be referred to here as the T-period optimisation) in which the activity of the firm ceases after T periods, with all net assets at time T paid out as a dividend (this is a consequence of the contraction mapping arguments outlined in Stokey and Lucas (1989)).

Consider also the closely related optimisation (which will be referred to here as the terminal dividend T-period optimisation) in which the firm operates for T periods, but in which the firm is constrained to pay no dividends until time T, and then pays out all net assets as a single terminal dividend. If no dividend is paid for any value of H under the optimal contingent plan, then the value function V(H) and the associated decision rules must be the limit, as $T \to \infty$, of the terminal dividend T-period optimisation.

Let $V_T(H)$ be the expected present value of the terminal dividend paid after T periods under the terminal dividend T-period optimisation. Let ϕ^- be maximum of $[(1+r^-)^{-1}\mathscr{E}R(Y,I)-\gamma I]$. The following lemma will now be established:

Lemma 5. $$V_T(H) < \delta^T(1+r^-)^T\left[H+\frac{1-(1+r^-)^{-T}}{1-(1+r^-)^{-1}}\phi^-\right]$$

Proof. Consider the optimal contingent plan for solving the terminal dividend T-period optimisation. Consider now the following relaxations of the constraints facing the firm: suppose that the interest rate on debt is reduced from r^+ to r^-; and that the limit of S on indebtedness is removed. Let the new value of the firm be $V_T(H)^\circ$. Both these relaxations must increase the value of the firm so $V_T(H)<V_T(H)^\circ$.

Since the firm is still constrained to pay a single terminal dividend it will aim to maximise the terminal dividend, which is achieved by holding in each period inventory I to maximise $[(1+r^-)^{-1}\mathscr{E}R(,I)-\gamma I]$. The expected value of the terminal dividend $\mathscr{E}d_T(H)^\circ$ then satisfies the first order difference equation: $\mathscr{E}d_T(H)^\circ = [\mathscr{E}d_{T-1}(H)^\circ+\phi^-](1+r^-)$, with the initial value $\mathscr{E}d_0(H)^\circ=H$. The solution to this difference equation is:

$$\mathscr{E}d_T(H)^\circ = \left[H+\phi\frac{1+r^-}{r^-}\right](1+r^-)^T-\phi\frac{1+r^-}{r^-} \qquad \text{(A15)}$$

and hence, since $V_T(H)<V_T(H)^\circ=\delta^T\mathscr{E}d_T(H)^\circ$, $V_T(H)$ satisfies lemma 4.

Now from lemma 4, in the limit as $T\to\infty$, $V(H)=V_T(H)<0$ (because $\delta^{-1} > (1+r^-)$). But we also know that $V(H)\geq 0$. Hence $V(H)=0$ for all H, conflicting with proposition (b) above. This contradicts the initial assumption, that there is no level of H at which a dividend is paid, and hence there must be some minimum, non-negative value of H, H^* at which a dividend is paid.

Propositions (d) and (e).

Inventory holdings, debt and output are determined by the first order conditions which emerge from the maximisation of the right hand side of the Bellman equation. Let λ be the lagrange multiplier from the constraint that net assets and debt are used to finance dividend payments and inventory investment (λ may be interpreted as the present value of an increment to current assets). The first order conditions may then be written:

$$
\begin{aligned}
&\frac{z}{\lambda}(1 + r^{+}) \geq 1 \quad D^{+} \geq 0 \\
&\frac{z}{\lambda}(1 + r^{-}) \leq 1 \quad D^{-} \leq 0 \\
&\frac{w}{\lambda} = \gamma \\
&\nu = 0 \\
&\lambda - 1 \geq 0 \quad d \geq 0
\end{aligned}
\tag{A16}
$$

The three pairs of first order conditions, for D_t^+, D_t^- and d_t hold with complementary slackness. When a dividend is paid the present value of an increment to net assets, $\lambda = 1$. Otherwise $\lambda \geq 1$.

The determination of inventory holdings I_t, debt holdings D_t and next period's output Y_{t+1} are now considered under the two regimes of dividend payment. Suppose first that $d_t>0$, and hence that $\lambda=1$. From proposition (c), $D^+=0$ while $D^-<0$. Hence:

$$z = (1+r^{-})^{-1} \tag{A17}$$

z is the marginal value of an increment to expected assets in the next period. The second-order condition that $\frac{\partial z}{\partial D^-}>0$ (which follows from lemma 4) indicates the uniqueness and stability of the choice of D^-. Inventory holdings are then determined by the condition that $w = z \frac{\partial \Xi}{\partial I}$. The assumptions made in the main text ensure that the second order conditions are satisfied and there is a unique and stable choice of I. Finally planned output is determined by the penultimate first order condition (with second order conditions again satisfied from the assumptions made in the main text). D^-, I and Y are all independent of the level of net assets H and may be expressed as in proposition (d).

Proposition (e) is established by considering the regime where $H<H^*$ and hence no dividend is paid. A difficulty arises here from the possibility that $\frac{\partial z}{\partial D^-} \leq 0$. This would be the case where a small increment to expected net assets in the following period does little to avert the probability of bankruptcy and is therefore of little value to the firm. It is likely, as net assets decline to the point where $\frac{\partial z}{\partial D^-} \leq 0$, that the firm then jumps, from a solution where it minimises borrowing, to one where it maximises borrowing and increases inventory investment and planned output as far as possible. Such a strategy is a gamble on a favourable outturn for ϵ in order to have some chance of escaping from bankruptcy. The non-linearities in the value-function effectively make the firm risk-loving in the region of probable bankruptcy. Whether this possibility transpires depends on the distribution of $f(\epsilon)$: no further analysis will be made here.

If it is assumed that unique plans for indebtedness, inventory holdings and planned output emerge from the decision rules when $H<H^*$, then these decision rules can be expressed as functions of the deviation of net assets (H) from the level at which dividends are paid (H^*). Formally this can be set out by noting that:

(A18)

and writing I and D as functions of the lagrange multiplier λ:

$$\begin{aligned} I-I^* &= \eta_I(\lambda-1) \qquad \eta_I > 0 \\ D-D^* &= \eta_D(\lambda-1) \qquad \eta_D < 0 \end{aligned} \tag{A19}$$

It is then possible to express $H - H^*$ as a function of λ, and invert this function to obtain λ as a function of $H-H^*$:

$$
\begin{aligned}
&H - H^* = \gamma\,\eta_I(\lambda-1) - \eta_D(\lambda-1) = \eta\,(\lambda-1) \quad \eta' > 0 \\
&\rightarrow \lambda - 1 = \phi(H - H^*) \quad \phi' > 0
\end{aligned} \tag{A20}
$$

Finally substitution into (A19) yields the results set out in proposition (e).

Table 1 : Comparison of Panel and Economy Wide Aggregates

	Distribution							
	Turnover				Inventories			
	Panel		Retail Sales		Panel		National Accounts	
	Level (£bn 1980 prices)	Annual % growth	Level (£bn 1980 prices)	Annual % growth	Level (£bn 1980 prices)	Annual % growth	Level (£bn 1980 prices)	Annual % growth
1978	33	3.9	56	5.4	4.1	8.5	9.6	5.3
1979	33	3.0	59	4.3	4.3	4.4	10.3	7.7
1980	38	-5.2	58	-0.5	5.0	-11.8	10.0	-3.7
1981	36	-2.9	58	0.1	4.3	-3.4	10.0	0.8
1982	35	5.0	60	2.0	4.1	-1.8	10.3	2.4
1983	49	5.1	63	5.1	5.7	3.8	10.0	-2.4
1984	45	7.7	65	3.7	5.6	11.4	10.1	1.1
1985	44	8.6	68	4.5	5.5	-2.9	10.6	4.4
	Manufacturing							
	Turnover				Inventories			
	Panel		National Accounts*		Panel		National Accounts	
	Level (£bn 1980 prices)	Annual % growth	Level (£bn 1980 prices)	Annual % growth	Level (£bn 1980 prices)	Annual % growth	Level (£bn 1980 prices)	Annual % growth
1978	121.6	0.5	171	0.9	31.2	1.4	36.2	1.3
1979	119.0	0.7	170	-0.4	31.2	7.3	36.4	0.8
1980	116.7	-6.1	156	-8.2	32.6	-12.3	34.1	-6.4
1981	99.6	0.7	147	-5.9	26.2	3.9	32.6	-4.5
1982	107.2	0.5	147	0.0	33.2	-8.9	31.5	-3.4
1983	121.0	2.9	151	2.8	30.2	-1.3	31.4	-0.4
1984	119.7	6.9	157	3.8	28.9	1.1	31.9	1.7
1985	117.6	2.2	162	3.2	26.9	-3.8	31.5	-1.3

* Calculated from the index of manufacturing output, grossed up using the 1984 input/output tables, with a correction for investment in inventories of finished goods.

Table 2 : Proportions of sample under financial pressure
By financial year

Indicator of financial pressure		77/78	78/79	79/80	80/81	81/82	82/83	83/84	84/85	85/86
dividend pay-out ratio =0	%	53	43	44	51	52	48	33	27	25
	Entry	0	3	5	10	8	6	6	6	5
	Exit	7	11	4	3	5	7	9	7	5
dividend pay-out ratio <0.085	%	64	52	53	59	59	56	47	42	38
	Entry	5	4	5	10	8	7	6	6	5
	Exit	10	18	5	4	6	8	17	7	8
interest cover	%	14	15	22	38	37	36	29	25	25
	Entry	3	6	11	17	8	9	5	6	7
	Exit	5	4	4	2	7	8	13	8	6
capital <£10mn	%	62	45	45	46	46	44	44	36	34
	Entry	0	0	0	0	0	0	0	0	0
	Exit	0	0	0	0	0	0	0	0	0
return on assets	%	21	33	34	50	57	53	43	40	36
	Entry	10	13	12	24	16	11	6	9	8
	Exit	9	8	11	6	8	13	15	11	11
quick ratio	%	48	39	47	47	45	45	41	42	42
	Entry	7	7	12	7	6	7	6	7	5
	Exit	7	6	3	8	7	7	7	6	6
current ratio	%	22	16	17	20	23	26	23	24	24
	Entry	2	2	5	7	6	7	5	6	5
	Exit	3	5	4	3	2	3	4	4	4
Number of observations		58	1093	1049	1047	971	962	1511	1304	1148

Table 3 : Proportions of sample under financial pressure
By industry, unweighted and weighted.

		Unweighted			Weighted by capital of firm		
Indicator of financial pressure		Distrib-ution	Manufac-turing	All	Distrib-ution	Manufa-cturing	All
dividend pay-out ratio =0	%	48	36	40	9	19	17
	Entry	5	7	6	2	5	4
	Exit	5	7	7	3	6	5
dividend pay-out ratio <0.085	%	60	46	50	15	23	21
	Entry	5	7	6	3	5	5
	Exit	8	10	10	5	7	6
interest cover	%	46	21	28	41	16	21
	Entry	9	8	8	6	5	5
	Exit	7	7	7	6	3	4
capital <£10mn	%	58	36	42	3	1	1
	Entry	0	0	0	0	0	0
	Exit	0	0	0	0	0	0
return on assets	%	42	43	43	31	39	37
	Entry	12	12	12	7	10	9
	Exit	11	11	11	8	9	9
quick ratio	%	25	51	43	19	51	45
	Entry	5	8	7	4	8	7
	Exit	4	7	6	3	6	6
current ratio	%	12	26	22	16	24	22
	Entry	3	6	5	6	7	7
	Exit	2	4	4	2	5	4
Number of observations		2676	6467	9143	2676	6467	9143

Table 4 : Proportion of firms classified as never, partly and always under financial pressure, by number of observations per firm (%)

Indicator of financial pressure		1	2	3	4	5	6	7	8	9
dividend pay-out ratio =0	Never	37	38	57	26	13	28	41	51	100
	Partly	-	13	18	23	32	33	37	41	-
	Always	63	49	25	50	55	40	22	8	-
dividend pay-out ratio <0.085	Never	29	29	41	18	9	16	36	43	33
	Partly	-	13	19	21	29	37	40	45	33
	Always	71	56	40	61	62	48	25	12	33
interest cover	Never	68	56	58	40	42	35	44	49	100
	Partly	-	19	25	47	48	56	53	47	-
	Always	32	25	18	13	10	9	2	4	-
capital <£10mn	Never	29	35	39	26	19	28	75	90	100
	Partly	-	-	-	-	-	-	-	-	-
	Always	71	61	53	66	79	68	17	3	0
return on assets	Never	60	47	39	26	18	16	21	25	67
	Partly	-	26	36	55	72	72	65	68	33
	Always	40	27	26	19	10	13	14	7	-
quick ratio	Never	54	45	49	43	40	41	30	31	-
	Partly	-	15	20	26	40	45	49	48	100
	Always	46	40	31	31	21	14	21	21	-
current ratio	Never	65	65	69	67	62	62	62	60	33
	Partly	-	12	16	22	31	33	37	35	67
	Always	35	23	15	11	7	4	1	6	-
Number of firms		276	240	839	155	154	141	81	380	3

Table 5 : Estimation results (no time dummies)

	All firms (9143)			Additional coefficients for firms under financial pressure			
Indicator of financial pressure:	ΔT	π/K	Const	Number of Observations	ΔT	π/K	Const
No indicator	0.537 (38.5)	0.419 (11.9)	-0.042 (11.9)				
No dividend payment	0.560 (26.0)	0.080 (1.0)	-0.009 (1.1)	3619	-0.044 (1.6)	0.414 (4.6)	-0.042 (4.6)
Dividend pay-out less than 8.5% of earnings	0.571 (24.0)	0.021 (0.2)	-0.003 (0.3)	4579	-0.053 (1.8)	0.451 (4.3)	-0.046 (4.4)
Interest cover less than threshold	0.552 (32.4)	0.038 (0.7)	0.005 (0.8)	2497	-0.055 (1.9)	0.524 (6.6)	-0.072 (9.2)
Quick Ratio less than threshold	0.567 (30.2)	0.360 (6.5)	-0.041 (7.1)	3918	-0.068 (2.5)	0.115 (1.6)	-0.001 (0.2)
Current ratio less than threshold	0.526 (31.8)	0.400 (8.5)	-0.043 (9.4)	1901	0.032 (1.1)	0.069 (1.0)	0.006 (0.8)
Return on assets less than threshold	0.534 (28.2)	-0.034 (0.5)	0.021 (1.7)	3844	-0.015 (0.6)	0.495 (5.6)	-0.077 (7.7)
Capital less than £10mn	0.614 (29.3)	0.394 (7.7)	-0.043 (8.4)	3862	-0.137 (4.9)	0.064 (0.9)	-0.002 (0.2)
No dividend payment and/or interest cover less than threshold	0.564 (25.8)	-0.123 (1.5)	0.019 (2.2)	3619 (no dividend)	-0.028 (0.9)	0.262 (2.7)	-0.018 (1.9)
				2497 (int cover)	-0.045 (1.4)	0.434 (5.0)	-0.069 (8.4)

Panel covers the financial years 1977/78 to 1985/86. Total number of observations 9143. Method of estimation ordinary least squares. The final three columns show the additional coefficients for those firms identified as under financial pressure. The total coefficients for these firms are the sum of the coefficients in the first three and final three columns.

Table 6 : Estimation results (with time dummies)

	All firms (9143 observations)			Additional coefficients for firms under financial pressure			
Indicator of financial pressure:	ΔT	π/K	Const	Number of observations	ΔT	π/K	Const
No indicator	0.515 (36.7)	0.373 (10.6)					
No dividend payment	0.538 (24.9)	0.058 (0.7)		3619	-0.044 (1.6)	0.380 (4.3)	-0.041 (4.6)
Dividend pay-out less than 8.5% of earnings	0.551 (23.0)	-0.010 (0.1)		4579	-0.055 (1.9)	0.430 (4.1)	-0.047 (4.5)
Interest cover less than threshold	0.531 (31.2)	0.033 (0.6)		2497	-0.056 (2.0)	0.496 (6.3)	-0.063 (8.0)
Quick ratio less than threshold	0.539 (28.6)	0.328 (5.9)		3918	-0.057 (2.1)	0.095 (1.3)	-0.002 (0.3)
Current ratio less than threshold	0.500 (30.1)	0.352 (7.6)		1901	0.044 (1.4)	0.068 (1.0)	0.008 (1.1)
Return on assets less than threshold	0.518 (27.5)	-0.028 (0.4)		3844	-0.024 (0.9)	0.450 (5.2)	-0.068 (6.9)
Capital less than £10mn	0.553 (31.7)	0.380 (8.7)		3862	-0.104 (3.6)	0.013 (0.2)	0.017 (2.1)
No dividend payment and/or interest cover less than threshold	0.543 (24.8)	-0.110 (1.4)		3619 (no dividend)	-0.027 (0.9)	0.231 (2.4)	-0.020 (2.0)
				2497 (int cover)	-0.047 (1.5)	0.413 (4.8)	-0.059 (7.2)

Notes as for table 5

Table 7 : Estimation results (no time dummies, weighted by capital of firm)

	All firms (9143)			Additional coefficients for firms under financial pressure			
Indicator of financial pressure:	ΔT	π/K	Const	Number of Observations	ΔT	π/K	Const
No indicator	0.593 (43.3)	0.379 (11.8)	-0.040 (12.6)				
No dividend payment	0.617 (39.6)	0.084 (1.5)	-0.010 (1.8)	3619	-0.106 (3.3)	0.426 (5.9)	-0.046 (6.5)
Dividend pay-out less than 8.5% of earnings	0.617 (38.6)	0.089 (1.5)	-0.009 (1.6)	4579	-0.091 (3.0)	0.386 (5.3)	-0.046 (6.4)
Interest cover less than threshold	0.708 (43.6)	-0.063 (1.2)	0.004 (0.7)	2497	-0.404 (13.8)	0.767 (10.0)	-0.070 (9.8)
Quick Ratio less than threshold	0.494 (25.3)	0.243 (4.1)	-0.021 (3.4)	3918	0.192 (7.0)	0.136 (1.9)	-0.025 (3.4)
Current ratio less than threshold	0.538 (30.8)	0.381 (8.1)	-0.039 (8.4)	1901	0.150 (5.3)	-0.023 (0.4)	-0.006 (0.9)
Return on assets less than threshold	0.574 (35.2)	-0.163 (2.5)	0.027 (3.6)	3844	0.020 (0.7)	0.671 (8.4)	-0.080 (9.6)
Capital less than £10mn	0.596 (42.9)	0.376 (11.5)	-0.039 (12.3)	3862	-0.133 (1.5)	0.149 (0.7)	-0.018 (0.8)
No dividend payment and/or interest cover less than threshold	0.700 (41.8)	-0.064 (1.1)	0.005 (0.8)	3619 (no dividend)	0.076 (2.2)	0.021 (0.2)	-0.004 (0.4)
				2497 (int cover)	-0.433 (13.5)	0.721 (6.6)	-0.067 (7.6)

Notes as for table 5.

Table 8 : Estimation results distinguishing size of firm

	All firms			Those identified as under financial pressure					
				Cap < £2.5mn		£2.5mn < Cap < £25mn		£25mn < Cap	
Indicator of financial pressure:	ΔT	π/K	Const	π/K	Const	π/K	Const	π/K	Const
No dividend payment	0.537 (38.5)	0.093 (1.2)	-0.009 (1.2)	0.297 (3.1)	-0.011 (1.1)	0.468 (4.3)	-0.067 (6.3)	0.374 (3.4)	-0.497 (4.3)
Interest cover less than threshold	0.535 (38.4)	0.049 (0.9)	0.005 (0.7)	0.398 (3.7)	-0.051 (4.6)	0.525 (5.1)	-0.082 (8.8)	0.568 (4.8)	-0.069 (6.7)
Return on assets less than threshold	0.528 (37.7)	-0.032 (0.4)	0.021 (2.3)	0.364 (3.5)	-0.065 (5.5)	0.593 (5.5)	-0.087 (7.9)	0.551 (4.9)	-0.076 (6.6)

Notes as for table 5.

Table 9 : Estimation results comparing fixed and varying classifications

	All firms			Those identified as under financial pressure				
				Fixed classification		Varying classification		F-test of restrictions (p-values)
Indicator of financial pressure:	ΔT	π/K	Const	π/K	Const	π/K	Const	
No dividend payment	0.534 (38.3)	0.113 (1.4)	-0.012 (1.5)	-0.184 (1.8)	0.034 (3.3)	0.524 (4.3)	-0.066 (5.6)	
	0.537 (38.5)	0.374 (6.1)	-0.040 (6.5)	0.074 (1.0)	-0.002 (0.2)			16.0 (0.0000)
	0.534 (38.3)	0.095 (1.2)	-0.009 (1.2)			0.386 (4.4)	-0.041 (4.5)	5.6 (0.0038)
Interest cover less than threshold	0.533 (38.4)	0.054 (1.0)	0.002 (0.4)	0.063 (0.6)	0.028 (3.0)	0.461 (4.5)	-0.087 (8.8)	
	0.536 (38.5)	0.271 (5.6)	-0.027 (5.2)	0.328 (4.4)	-0.023 (3.2)			38.9 (0.0000)
	0.533 (38.4)	0.050 (0.9)	0.005 (0.7)			0.486 (6.3)	-0.070 (9.0)	7.5 (0.0006)
Return on assets less than threshold	0.528 (37.8)	-0.015 (0.2)	0.017 (1.9)	-0.139 (1.4)	0.028 (2.9)	0.599 (5.3)	-0.095 (8.0)	
	0.535 (38.3)	0.334 (5.8)	-0.032 (4.8)	0.120 (1.6)	-0.014 (1.8)			32.8 (0.0000)
	0.527 (37.7)	-0.032 (0.4)	0.021 (2.3)			0.488 (5.6)	-0.076 (7.7)	4.6 (0.0103)

Notes as for table 5. No time dummies included in estimation.

Table 10 : Analysis of aggregate inventory investment

	National Accounts	Panel Actual	Explained by:		CSO Residual	Panel Residual
	(1)	(2)	Turnover (3)	Financial Pressure (4)	= (1) - (3) - (4)	= (2) - (3) - (4)
1978	2.1	2.2	0.7	-0.9	2.3	2.4
1979	2.0	7.0	0.7	-1.4	2.7	7.7
1980	-5.6	-12.2	-2.7	-2.8	-0.1	-6.7
1981	-3.4	2.9	-0.1	-2.6	-0.7	5.6
1982	-1.9	-8.1	0.9	-3.8	1.0	-5.2
1983	-1.0	-0.5	2.0	-3.3	0.3	0.8
1984	1.4	2.8	4.1	-1.3	-1.4	-0.0
1985	0.2	-3.7	2.2	-1.0	-1.0	-4.9

Columns (2) and (3) are calculated by applying the coefficients reported in the final row of table 5.

CHAPTER 5

THE COST OF CAPITAL FOR INVENTORY INVESTMENT IN THE UK

1. Introduction

Scope of the paper

This paper has two purposes. The first is analysis of the factors affecting the cost of capital for inventory investment. A number of authors have treated this subject but none offers a complete analysis of all the factors which affect the cost of capital for inventory investment. The first three sections of this paper are intended to fill this gap.

The second purpose is to develop measures of the cost of capital for aggregate UK inventory investment in the UK taking account of legislative changes and the tax position of individual companies.[30] These measures make use of the individual company accounts data available on the IFS tax model. There are three principle reasons why the calculations presented here make use of this model. Firstly they allow for the fact that, under the first of the two stock relief schemes applicable in the UK, stock relief was available to some companies and not to others depending on whether the increase in the book value of inventories exceeded some threshold value. Such an allowance is only possible with individual company accounts data. Second by using the IFS tax model it is possible to take account of the effects of tax exhaustion on the cost of capital for inventory investment. Finally the use of individual company accounts data allows the quantification of a seasonal element of stock relief first noted by Sumner (1984).

In contrast to fixed capital investment, inventory investment does not have to be planned well in advance and once undertaken can relatively easily be reversed. Thus inventory investment should respond quickly to changes in the cost of capital. It is therefore surprising that very few econometric studies have found significant cost of capital effects on inventory holdings.[31] One possible explanation is that the cost of capital for inventory investment has not been measured accurately because of changes in tax laws and the tax status of individual companies. The measures presented here go some way towards remedying deficiencies with the UK data.

This paper extends two earlier studies which consider the cost of capital for inventory investment in the UK. Sumner (1984) is the standard analysis of the effects of the UK stock relief schemes on the cost of capital. Sumner derives expressions which are equivalent to those obtained in section 4 below. The analytical sections of this paper goes beyond his results in two respects. First the Sumner analysis takes no account of the source of finance. His results are presented in terms the cost of finance (r in his notation) without discussing how the cost of finance relates to the nominal interest rate. Here we argue that the appropriate cost of finance is that for debt finance. This allows the calculation of quantitative estimates of the cost of capital presented in section 5.

The second extension of Sumners results is to take account of the tax status of individual companies. His results apply only to those companies which always qualify for relief and who pay corporation tax; but under the first stock relief scheme, which applied until November 1980, a high proportion of companies were unable to obtain relief in some or all years, because investment in inventories fell below the threshold for relief. Moreover a high proportion of companies, especially in heavy manufacturing, were tax exhausted in the 1970's and early 1980's. All these cases are formally analysed below.

The results of section 5, which uses the IFS corporation tax model to derive measures of the aggregate cost of capital, build on the work of Devereaux (1988), who makes calculations of the cost of capital for aggregate investment. He also provides calculations of the tax wedge for individual categories of investment including inventory investment. His results assume a mix of finance. They take account of eligibility for tax relief under the various stock relief schemes but only consider the effects of tax exhaustion on investment in plant and machinery.

This paper provides more detailed results than Devereaux but relating only to inventory investment. For the most part debt finance is assumed, although one set of calculations based on a mix of finance is also presented. The calculations of the cost of capital given here, unlike those of Devereaux, take explicit account of the rate of inflation of inventory prices. An estimate of the cost of capital after allowing for tax exhaustion is made. The analysis improves on Devereaux (1988) in one further respect by taking account of the transition arrangements which governed the change-over from the 1975 to the 1981 stock relief scheme.

Outline

The paper is arranged as follows. The analysis of the cost of capital for inventory investment is conducted in sections 2, 3 and 4. Section 2 discusses the various factors that influence the cost of capital, paying particular attention to accounting conventions and the effects of taxation. This section also describes the methodology of King and Fullerton (1984), which is then applied in section 4 to derive a number of formal expressions for the cost of capital.

The cost of capital for inventory investment in the UK has been affected by major changes in the tax treatment of inventories. These are the so-called "stock relief" schemes which offered tax relief on inventory holdings from the mid 1970s to 1984. The operations of these schemes, especially the earlier 1975 scheme, were complex and most descriptions, which are written for accountants or tax lawyers, deal largely with administrative problems. Section 3 of this paper provides a detailed summary of the operation of these schemes focussing on incentives to invest in inventories.[32]

The formal analysis presented in section 4 allows both for the two UK stock relief schemes and for the presence of tax exhaustion. The results are consistent with the earlier work of Sumner (1984), but extend his results by allowing for companies which are tax exhausted or which move into or out of eligibility for stock relief.

Finally section 5 presents the quantitative estimates of the cost of capital, on an annual and quarterly basis, using the Institute for Fiscal Studies corporate tax model. The estimates are weighted averages of the cost of capital for individual companies, using the book value of inventory holdings as weights. They take account of the tax position of the underlying individual companies, including tax exhaustion where this arises. Results are presented for aggregate inventory holdings and for manufacturing, distribution and other sectors. Section 6 concludes the paper.

2. Taxation and the cost of capital for inventory investment.

The cost of capital in the absence of taxation

Three factors determine the cost of capital for inventory investment. These are the nominal rate of interest, inflationary gains on holding inventories and the tax system. Neglecting (for the moment) the tax system, the cost of capital p is given by:

$$p = i - \pi \tag{1.1}$$

The cost of borrowed money is the nominal interest rate i. This is offset by the anticipated rate of inflation π associated with items held in inventory. Note that this second term captures the "speculative" motive for holding inventories based on anticipated price increases.[33] The optimal level of inventories is that at which the marginal return to increasing the level of inventories (net of storage costs) equals the cost of capital. If the Modigliani-Miller theorem applies (the irrelevance of corporate financial structure in the absence of taxation) then this is the appropriate expression whether the source of finance for inventory investment is debt, retained earnings or new equity issues.[34]

Taxation and the cost of finance

How does taxation affect the cost of capital? First it affects the cost of finance. This is a standard analysis and will not be discussed in detail here.[35] The most important consequence is that the source of finance will now matter. In the UK, as under most corporate tax systems, interest payments are tax deductable and so the cost of finance from borrowing is correspondingly reduced. Thus, for tax deductable debt with a rate of corporation tax τ, the cost of finance (c) is given by:

$$c = i*(1 - \tau) \tag{2.2}$$

Retained earnings finance and new issue finance are more complicated matters. If the shareholders are not tax payers (eg pension funds) then the cost of retained earnings finance is the nominal interest rate. If on the other hand the shareholders are tax payers then the cost of retained earnings finance (c) depends on both the rate of personal income tax (m) and the effective rate of capital gains tax (z) in addition to the nominal interest rate. In these circumstances the cost of finance is given by:

$$c = i * \frac{1-m}{1-z} \tag{2.3}$$

Typically m is fairly high for a tax paying shareholder, whereas the effective rate of capital gains tax is low (because of allowances and the ease with which gains may be deferred). Hence with tax paying share-holders the cost of retained finance is less than the nominal rate of interest. However a substantial proportion of shareholders are tax exempt institutions, for whom m and z are 0, so this pushes the cost of finance much closer to the nominal rate of interest.

The cost of new issue finance is different again. Under an imputation system of corporation tax (such as in the UK after 1972) the cost of new issue finance is less than the nominal interest rate, regardless of the tax status of the share purchaser. In this case the cost of capital (c) is given by:

$$c = \frac{i}{\theta} \tag{2.4}$$

θ is the additional dividend shareholders receive when one unit of earnings is distributed. Under an imputation system of corporation tax θ is greater than unity and the cost of new issue finance is less than the nominal rate of interest.

How are the different sources of finance to be taken into account in estimating the cost of capital? One approach (that of King and Fullerton (1984) and of Devereaux (1988)) is to assume that the marginal source of finance is a mix of the three main sources, provided in the same proportions for all investment projects. This does not seem appropriate in the present context. Here we assume that marginal investment in inventories is largely debt financed. The justification for this assumption is that inventory investment fluctuates considerably in the short run and only debt finance provides the necessary flexibility. Moreover the inventories themselves provide security for loans and are therefore more likely to be debt financed. As a check on the quantitative importance of this assumption a calculation of the cost of capital assuming a mix of all three sources of finance is also presented in section 5.

Calculating the effects of taxation on the cost of capital

Taxation affects the cost of capital not only by altering the cost of finance but also because of tax relief offered on the specific investment project and because the revenues of the project are subject to corporation tax. The methodology of King and Fullerton (1984) ensures that all these factors are properly taken into account in calculations of the cost of capital. The basis of the King and Fullerton methodology is that the cost of capital is the rate of return before tax (but net of any depreciation) of a marginally viable project.[36] Thus investment takes place up to the point at which the marginal rate of return equals the cost of capital.

The King-Fullerton approach is implemented by considering a hypothetical investment. The after-tax net present value of the investment is expressed in terms of the pre-tax marginal rate of return on the project (MRR), any project specific allowances and the cost of finance of the investment. For a given tax system this expression depends on the source of finance and the specific investment project undertaken. The cost of capital (p) is then obtained by inverting the expression to obtain the marginal rate of return on an investment project of zero net present value.

To see how this applies in the case of inventory investment consider a debt-financed investment in inventory with a nominal rate of interest i and a rate of inflation of inventory prices π. The present value of a marginal increase in inventories is given by:

$$V = MRR(1-\tau) - i(1-\tau) + \pi(1-\tau) \tag{2.5}$$

The first term on the RHS is the marginal rate of return on the investment net of tax. The second term is the cost of finance. The third term is the inflationary gain on the inventory holding, again net of corporation tax. Inverting this expression to obtain the cost of capital p (ie MRR when V=0) yields:

$$p = i - \pi \tag{2.6}$$

Thus in this example the cost of capital is unaffected by the tax system because the tax relief on the debt finance offsets the tax both on marginal revenues and on the inflationary gains.[37] This result assumes the FIFO accounting convention and no tax exhaustion. Both of these assumptions are discussed below.

Over what period is the hypothetical marginal investment in inventory maintained? As we have described the calculation of the cost of capital this does not matter. The marginal rate of return, the interest rate and the rate of inflation of inventory prices are those which are expected to obtain over the period of the investment. King and Fullerton (1984) assume that the investment is not resold and they therefore calculate the marginal returns and the cost of finance as discounted sums over all future periods. For inventory investment a shorter holding period is more appropriate. This is for two reasons. The assumption that an inventory investment is made in the current period and unwound in the subsequent period is analytically convenient, as it avoids the necessity to consider expectations of the cost of finance over all future periods. The same advantage does not apply in the case of fixed assets because the assumption of a finite holding period means that it is then necessary to consider the resale market for the same assets. Also it is for a hypothetical holding period of one year (or less) that year to year changes in the tax status of an individual company can most easily be allowed for. Thus this paper assumes, in the formal analysis of the cost of capital in section 4 and for the annual cost of capital series presented in section 5, an inventory investment which is unwound after one year.

In practice inventory holding decisions must be considered more frequently than once a year. This matters particularly when, as is the case in the UK between 1975 and 1984, stock relief is offered which is based on end year inventory holdings. If the decision period over which inventory investment decisions are made is less than one year then the availability of tax relief gives rise to an incentive to build up end accounting year inventory levels, relative to intra-accounting year inventory levels, in order to claim maximum relief. This complication is allowed for in the calculations of the quarterly cost of capital presented in section 5.

Accounting conventions and the cost of capital

Inventory is valued in UK company accounts at the lower of cost and "net realisable value" (ie the value that could be obtained by selling or using the inventory). Except where operating losses arise the lower of these will be at cost so in most circumstances this is the relevant basis of inventory valuation. In principle accurate historic cost accounting would require the true historic cost of each item taken out of inventory. In practice such detailed records are not kept and some convention about the cost of items taken from inventory must be adopted. The taxation of inflationary gains on inventories depends on the cost convention used for measuring inventory. If this is the "first in first out" (FIFO) convention, as is common in the UK, then any inflationary gain on inventories is immediately counted as part of operating profit and is subject to corporation tax.[38]

If on the other hand the convention is "last in first out" (LIFO) then the inflationary gains are not included in operating profits and are not immediately subject to corporation tax. The inflationary gains do not dissappear under LIFO conventions, but they are hidden as a systematic undervaluation of the level of inventory. The inflationary profits may be declared through an inventory revaluation as revaluation profits, but this is likely to happen if at all only at a much later date. Hence the effective taxation of inflationary gains is much lower with the LIFO convention. Subject to certain criteria being satisfied LIFO is accepted for tax purposes in the US, but is not acceptable in the UK and is therefore not considered further in this paper.[39]

The other common inventory valuation convention, accepted for tax purposes in the UK, is average cost valuation. This estimates the historic costs of items drawn from inventory by assuming that the age structure of these items corresponds to the age structure of the entire inventory. Items withdrawn from inventory are a representative sample of the entire inventory. Hence the appropriate measure of cost is a weighted average of the historic cost of items currently thought to be in inventory. The age structure of items remaining in inventory is re-calculated at the same time as the historic cost of the items withdrawn from inventory is derived. With average cost valuation all inflationary gains on inventory holdings appear in operating profit, but will be divided between the current and future accounting periods. In practice, with typical inventory turnover of one to two months, only a small part of the inflationary gains will carry over to subsequent accounting periods. For this reason the cost of capital under the average cost convention is very similar to that obtaining under the FIFO convention and the average cost convention is not treated seperately in this paper.

3. The effect of UK stock relief schemes on the cost of capital.

The operation of stock relief schemes in the UK

The rise in inflation in the 1970's and the consequent rise in nominal interest rates resulted in severe liquidity problems for companies throughout the industrialised world.[40] In the United States a common response to these difficulties was a switch to a LIFO accounting convention for the valuation of inventories. The consequence of this change was that inflationary gains on inventory holdings were not immediately reflected in company profits and liquidity difficulties were eased.

LIFO inventory valuation has never been acceptable for tax purposes in the UK, so the growing liquidity problems of the UK corporate sector in 1973 and 1974 led to powerful lobbying for some form of tax relief on inventory appreciation. This resulted in the first system of stock relief. This was replaced by a new stock relief scheme, which operated on entirely different principles, in 1980/1981. Preparatory to the formal analysis of the cost of capital under these two schemes offered in section 3 this section summarises how these schemes operated.

The first scheme was an emergency measure announced in the autumn of 1974 and legislated in the 1975 finance act (it applied retrospectively to accounts ending in the financial years 1973/74 and 1974/75). This was subsequently put on a permanent footing by the 1976 finance act. This scheme treated all increases in the book value of inventory in excess of a threshold level as a tax allowance. The threshold was originally set at 10% of taxable income before deduction of capital allowances. The 1976 finance act amended this threshold to 15% of taxable income after deduction of capital allowances. Under the 1975 scheme physical as well as inflationary increases in inventory attract tax relief. An increase in inventory in the current year attracts, at the margin, tax relief equal to the full investment. In this respect, as well as offering relief on the inflationary gains on inventory, the scheme operated somewhat like the 100% capital allowances on fixed capital investment.

The 1975 scheme provides a very substantial subsidy to investment in inventories when no relief is anticipated in the following year. The incentive is greatly reduced when the firm expects to claim stock relief for the year subsequent to the current one; this is because an increase in the end year book value of inventories increases stock relief in the current accounting year but reduces it by the same amount in the next year.

In the event of a fall in the book value of inventories clawback of previous relief applied up to the total of relief previously granted. This clawback increases the incentive to invest in inventories in exactly the same as when stock relief itself can be claimed. Under the 1975 scheme a rise in the book value of inventory at the end of the accounting year can reduce the tax assesment for that accounting year either in the form of an increase in stock relief or a fall in clawback.

The 1979 and 1980 finance acts introduced two further amendments to the scheme. The 1979 act restricted clawback to stock relief granted in the six previous financial years. This does not affect the analysis of the cost of capital though it does alter the number of companies subject to clawback. The 1980 finance act introduced deferral of clawback; clawback in excess of 5% of the opening value of stocks could be deferred by one financial year. This applied only to temporary dips in the value of inventories. When the value of inventories had already fallen in the previous accounting year deferral was not allowed (and thus deferral could not take place for more than one year).

Deferral of clawback from the current year increases the cost of capital because the marginal investment in inventory no longer reduces the tax liability for the current year but for the subsequent accounting year. When stock relief is available in the subsequent year the effects of the marginal investment on stock relief exactly offset each other and the cost of capital is exactly as if no stock relief is available.

A second system of stock relief replaced the 1975 scheme in 1981. Under this system stock relief was available only on the inflationary increase in inventory. This was calculated by applying an official index of inventory prices (the all-stocks index) to the opening value of inventory (less £2000). This second scheme was itself abolished, with immediate effect, by the 1984 finance act.

The transition between the two stock relief schemes

The switch between the 1975 and the 1981 stock relief schemes introduced a potentially powerful temporary incentive to invest in inventories. This arose (i) because a company which elected to be assessed under the old scheme during the transition period obtains relief on end period inventories twice over (in the current accounting year under the old scheme and in the subsequent accounting year under the new scheme) and (ii) because the tax benefit in the last year of the 1975 scheme is not offset by a corresponding tax penalty in the subsequent accounting year.

In practice the tax incentive is not as clear cut as this. Transitional arrangements for the changeover between the two schemes of stock relief were framed so as to discourage companies from claiming large amounts of stock relief in the last year under the old scheme. These transitional arrangements worked in the following way. The new stock relief scheme was first announced in a consultative paper published on the 14th November 1980. The new scheme automatically applied to all accounting years ending on or after 14th November 1981. For accounting years ending prior to 14th November 1980 the old scheme applied. For accounting years which ended between 14th November 1980 and 13th November 1981 (these were known as the straddling year accounts) companies were allowed to opt for assessment in one of the three following ways:

(i) they could opt for stock relief under the new scheme;
(ii) they could claim stock relief under the old scheme up to the level of clawback deferred from the previous accounting year; or
(iii) they could claim the full level of stock relief under the old scheme. This was subject to the restriction that the end year book value of inventories be no greater than the the book value of inventories on the 14th November 1980. Otherwise the relief was based on the 14th November value of inventories (and the threshold level of profits was reduced according to the proportion of the accounting year which fell before 14th November.)

Of these options (i) and (iii) were described in the consultative document of 14th November 1980. Option (ii) was not made public until the 1981 budget. Thus any ex-ante incentive to invest in inventories arising from the exercise of option (ii) only applied from April 1981 onwards.

The restriction on option (iii) was intended to prevent companies taking advantage of the substantial ex-ante incentive to invest in inventories created by the changeover to the 1981 scheme. In practice some ex-ante incentive may have remained. This is because the book value of inventory on 14th November was an estimate agreed with the Inland Revenue and not the result of a stock taking exercise. The common basis for the agreed value of inventory was a linear interpolation between the beginning of year and end of year inventory valuations in the company accounts. Thus an ex ante incentive to invest in inventories remained because an increase in end year inventories resulted in an increase of the level of inventories agreed for the 14th November. The magnitude of the ex-ante incentive depends on the proportion of the accounting year which falls before November 14th 1980. The higher this proportion the greater the tax relief attracted by an increase in inventories at the end of the accounting year.

Nonetheless the transitional arrangements were probably fairly succesful at restricting the incentive to invest in inventory immediately after the publication of the consutltative document. This is because the detailed implementation of option (iii) was not made clear until the end of April 1981. Before this time the basis on which the 14th November inventory valuation would be agreed was uncertain. Thus it seems most appropriate to assume that initially there was no ex-ante incentive arising from the transition between the two schemes, even when option (iii) was eventually exercised, and that such an incentive emerged only from May 1981 onwards when the working of the transitional arrangements became clear.

4. Formal analysis of the cost of capital.

This section applies the King-Fullerton methodology to derive formal statements of the cost of capital for inventory investment taking account of both stock relief schemes and tax exhaustion.

Notation

The following notation is used:

p	-	the cost of capital for inventory investment
V	-	the present value of a marginal investment in inventories
MRR	-	the pre-tax marginal rate of return to an additional investment in inventories. When inventory investment is optimal V=0 and MRR=p.
i	-	the nominal rate of interest at which companies can borrow and deposit. This is both the cost of finance and also the companies nominal discount rate.
τ_c	-	the rate of corporation tax
π	-	the rate of inflation of inventory prices
π_s	-	the rate of inflation of the all-stocks price index over the course of the next accounting year
σ	-	the proportion of the accounting year "straddling" 14th November 1980 which falls before the 14th November. This measures the ex-ante incentive to increase inventories arising from the transitional arrangements between the two stock relief schemes from May 1981. For accounting years ending on or before April 1981 σ is taken to be 0.

The cost of capital with no tax exhaustion

This sub-section considers a debt financed increase in the level of inventories at the end of the current accounting period which is unwound before the end of the subsequent accounting year. Six cases are distinguished. Debt finance is assumed throughout.

These expressions should be altered to allow for the timing of tax payments. Usually companies are required to pay mainstream corporation tax nine months after the end of the relevant company accounting year. However, due to a quirk of company law, companies which are registered before 1962 are only required to make their mainstream corporation tax payments twenty one months after the end of the accounting period. In the calculations of the cost of capital, presented in section 5 of this paper, a twelve months delay on tax payments and stock relief is assumed. This is achieved by appropriately discounting the rate of corporation tax which appears in the following expressions.

Case 1 There is no stock relief available. This case applies before the introduction of stock relief (first announced in November 1974) and after its abolition in 1984. The same expression for the cost of capital arises during the operation of the 1975 scheme <u>either</u> when the threshold provisions result in no relief being granted for the current and the subsequent accounting year <u>or</u> when there is deferral of clawback from the current accounting year. Net interest payments are i $(1-\tau_c)$. The additional tax arising because of inflation of inventories is $\pi\tau_c$. Capital gain on the inventory holding is π. The post tax rate of return is $(1-\tau_c)$ times MRR. The value of the marginal investment V is given by:

$$V = MRR(1-\tau_c) - i(1-\tau_c) - \pi\tau_c + \pi \tag{4.1}$$

so the cost of capital p, which is the MRR when V=0, is given by:

$$p = i - \pi \tag{4.2}$$

Case 2 The 1975 stock relief scheme. Stock relief (or clawback) under the 1975 stock relief scheme is expected to apply in both years. Net interest payments and wealth tax are as in case 1. The new factor is the stock relief. This is $+\tau_c$ in the current accounting year and $-\tau_c$ in the subsequent accounting year. (The effects of this relief are like the first year tax allowances granted on fixed capital investment. There is an allowance of τ_c per unit of investment when it is made and an offsetting allowance of τ_c when it is unwound.) The amount of capital which has to borrowed for a unit investment falls from 1 to $(1-\tau_c)$. Thus:

$$V = MRR(1-\tau_c) - i(1-\tau_c)^2 - \pi\tau_c + \pi \tag{4.3}$$

so the cost of capital is:

$$p = i(1-\tau_c) - \pi \tag{4.4}$$

Case 3 The 1981 stock relief scheme. This case also applies during the transition from the old stock relief scheme to the new stock relief scheme when there is no ex-ante expectation of stock relief under the 1975 scheme which we assume to be the case before May 1981.

Stock relief is based on the increase in the all stocks index of stock prices π_s in the subsequent accounting year. The additional stock relief is $\pi_s\tau_c$, which must be discounted using the post-tax nominal discount rate since the relief is offered only in the subsequent year. Thus the cost of capital falls to:

$$p = i - \pi - \pi_s \frac{\tau_c}{1-\tau_c}(1+i(1-\tau_c))^{-1} \tag{4.5}$$

When the all-stocks index accurately captures inventory appreciation ($\pi_s = \pi$) this expression simplifies to:

$$p = i - \frac{\pi}{1-\tau_c}\frac{1+i}{1+i(1-\tau_c)} \tag{4.6}$$

The analysis of cases 1 and 2 yields results equivalent to those of Sumner (1984). They differ only because they assume that the source of finance is tax deductable debt and because they are expressed in terms of the nominal interest rate

i. Sumner's results are expressed in terms of the cost of finance to the firm (the firm's nominal discount rate) which Sumner calls r. The formal equivalence can be seen by making the substitution $r=i(1-\tau_c)$ in Sumner's expressions. The results given here for case 3, the 1981 stock relief scheme, differs more significantly from the expression given by Sumner. This is because inflation of the all stocks price index π_s is distinguished from the current rate of inventory inflation π, and because of discounting of the relief offered under the 1981 scheme.

Changes of regime

Cases 1 to 3 are the main analytical expressions for the cost of capital. Other expressions are obtained when changes of regime are allowed for, either from one stock relief scheme to another, or when a company moves into or out of eligibility for stock relief. It is therefore useful to derive further analytical expressions. The most important of these is that relating to the replacement of the 1975 stock relief scheme by the 1981 scheme since this affects the cost of capital for inventory investment by all companies. Other examples of regime change are when the firm can claim stock relief under the 1975 scheme in either of the current or subsequent years but not in both.

Case 4 Transition to the 1981 scheme when the company elects for assesment under the 1975 scheme, and the accounting year ends in May 1981 or later. Section 3 outlined the working of the transition arrangements between the 1975 and 1981 stock relief schemes. A subsidy to marginal investment in inventories under the 1981 scheme applies to all the straddling year accounts. This is because relief under the 1981 scheme on a marginal increase in inventories is offered in the subsequent financial year, by which time the 1981 scheme is fully operational. If the firm elects to claim relief under the 1981 scheme then no additional subsidy is offered on the marginal inventory investment (relief depending on inventory holdings in the previous year) and case 3 is applicable. An additional subsidy to marginal investment in inventories, offering double relief on a marginal inventory investment, arises if the company elects to claim relief under the 1975 scheme based on the straddling year accounts. As noted in section 3, the opportunity to take advantage of this double relief was only available from April 1981 onwards when the operation of the transitional arrangements became clear. After this date the subsidy to marginal investment under the 1975 scheme can be captured by the coefficient σ, the proportion of the accounting year which falls before November 14th 1980. This proportion is relevant because, as a result of the technique of linear interpolation used by the inland revenue, a unit increase in end year inventory increases the agreed book value of inventory on 14th November by σ. Thus there is relief of $\sigma\tau_c$ for the current accounting year in addition to the relief of $\pi_s\tau_c$ in the subsequent accounting year. The financial cost of capital then becomes:

$$p = i - \pi - (\sigma + \pi_s(1 + i(1 - \tau_c))^{-1})\frac{\tau_c}{1 - \tau_c} \qquad (4.7)$$

Case 5 The firm can claim stock relief under the 1975 scheme for the current financial year but not for the subsequent financial year. This results in a big incentive for inventory investment. Stock relief is only available under the 1975 scheme but it is expected in full. The financial cost of capital becomes:

$$p = i - \pi - \frac{\tau_c}{1 - \tau_c} \qquad (4.8)$$

The last term is around 100% during the operation of the 1975 scheme induces a very sharp fall in the cost of capital.

Case 6 The firm can claim stock relief under the 1975 scheme for the second accounting year but not for the current accounting year. Here there is a substantial disincentive to invest in inventory. The financial cost of capital becomes:[41]

$$p = i(1 - \tau_c) - \pi + \frac{\tau_c}{1 - \tau_c} \qquad (4.9)$$

Only the expression derived for case 4 is used in the calculations of the cost of capital presented in section 4. The reason for not using the expressions derived for cases 5 and 6 is that these lead to very big changes in the cost of capital for the individual firm. Such is the impact on the cost of capital that it is questionable whether these expressions are accurate measures of the incentive to invest in inventory. The reason for scepticism is that if the company is aware of the tax incentive to increase (or decrease) inventory levels then it is likely to do so to such an extent that a corner solution is obtained and the change in regime no longer applies.[42]

An example may clarify this point. Suppose the company is eligible for stock relief in the current year, but not in the subsequent year. As shown above (case 5) there is now a very substantial subsidy to marginal investment in inventory. Thus the company is likely to increase end year inventory levels to the point at which it expects clawback of relief to be applied in the subsequent year and the marginal subsidy falls to normal levels. For this reason the preferred measure of the cost of capital, reported in section 4, is calculated on the basis of expressions 1 to 4 alone. Expressions 5 and 6 are used only for an alternative measure of the cost of capital to examine the potential effect of these corner colutions on the aggregate cost of capital.

Tax exhaustion

The cost of capital for inventory investment is affected by mainstream corporation tax exhaustion (henceforth MCT exhaustion).[43] This arises when companies make a tax loss or are unable to fully set their tax allowances against their current taxable income. This can occur even when companies are reporting profits in their accounts because of accelerated depreciation, first year investment allowances and stock relief. MCT exhaustion frequently arises for heavy manufacturing companies because they received substantial allowances of these kinds. Tax exhaustion has become much less common in the mid 1980's both because of the withdrawal of the various allowances in the 1984 budget and the sharp increase in corporate profits. MCT exhaustion means that there can be no additional tax liabilities in the year of tax exhaustion arising from a marginal investment in inventories. This affects the cost of capital because tax liabilities and stock relief arising from a marginal investment in inventories can no longer be claimed for the current accounting year.

In the most extreme case of MCT exhaustion - where the company expects never to be able to set interest expenditures and stock relief against tax and never to be taxed on the marginal revenue of the inventory investment - all terms in the rate of corporation tax vanish from the expression for the cost of capital. In all cases the cost of capital becomes:

$$p = i - \pi \tag{4.10}$$

In practice this case of extreme MCT exhaustion rarely arises. The company can usually expect to set interest payments and stock relief against taxable income at some date and if this is the case tax exhaustion does not have the simple consequences of extreme MCT exhaustion. This makes allowance for tax exhaustion more difficult. Two sets of considerations determine the effect of tax exhaustion on the cost of capital. The first of these is whether, and how far, tax losses are carried forward or back. The second is in which year the marginal benefits of holding inventory accrue. To begin with assume that the marginal benefits of holding inventory all accrue in the current year and concentrate on the carry forward and carry back of tax losses.

The effects of tax losses carried forward are greatest under the 1975 stock relief scheme. Consider case 2, where stock relief is claimed under this scheme in both the current and the subsequent year, and assume that tax losses are carried forward for one year only. Now the benefit of stock relief is not received until the subsequent year and the effective 100% capital allowance for inventory investment vanishes. Thus under the 1975 scheme a single year of tax exhaustion has the same effect on the cost of capital as permanent tax exhaustion.

Under the 1981 scheme tax exhaustion, with losses carried forward, has less marked effects and depends on the duration of tax exhaustion. The tax relief under this scheme accrues in the subsequent tax year. If the company is MCT exhausted for more than a single year then the relief is delayed and must be discounted by an additional amount. Suppose that tax losses are carried forward by n years. Then the cost of capital in case 3 becomes:[44]

$$p = i - \pi - \pi_s(1+i(1-\tau_c))^{-n}\frac{\tau_c}{1-\tau_c} \tag{4.11}$$

A similar discounting of the present value of stock relief arises for case 4, 5 and 6. Notice that in case 1, where no stock relief is available, the carry forward of tax losses has no effect on the cost of capital since all tax terms in the expression for the cost of capital cancel out.

A further complication arises with the carry back of tax losses. The UK tax system allows losses to be carried back and set against taxable profits in the previous year up to the level of the previous years profits. What matters for the cost of capital is whether, at the margin, additional taxable income is carried back, instead of being carried forward. This is the case if tax losses are carried back and are not fully absorbed by previous taxable income. Very different conclusions about the cost of capital then emerge, since a nominal interest credit is added to the tax repayment and the value of tax allowances are increased by tax exhaustion. This reduces the cost of capital. This is in complete contrast to carry forward which reduces the value of tax allowances and increases the cost of capital.

How do the provisions for carrying back tax losses operate? If the losses arise from capital allowances (abolished in the 1984 budget) then they can be carried back up to three years. Otherwise carry back is only allowed for a single year. Nominal interest payments are credited on the tax repayments for the number of years of carry back. A company is allowed to re-arrange its declaration carry back of tax allowances in all previous years in order to obtain maximum tax repayment. The calculations reported in section 4 assume that tax is carried back to the first previous year in which the IFS calculations indicate that it was not MCT exhausted, upto a maximum of three years.[45]

The most dramatic effect of carry back on the cost of capital again arises with the 1975 stock relief scheme. If losses are carried back (but not forward so that marginal earnings are taxed at a higher than normal rate) then the value of stock relief in the current year under the 1975 scheme is increased. Thus the cost of capital in case 2 becomes (with carry back of one year, no carry forward and tax exhaustion ending in the next tax year):

$$p = i(1-\tau_c-\frac{\tau_c}{1-\tau_c}) - \pi \tag{4.12}$$

This is a sharp reduction in the cost of capital. With a corporate tax rate of 52% (that which applied throughout the period of the 1975 scheme) the third term within the brackets is about -1 and the cost of capital is a negative function of the interest rate. This raises a similar problem to that noted above for changes of regime within the 1975 stock relief scheme. The cost of capital falls by so much that, if the above expression is an accurate measure of the incentive to invest in inventories, the inventory holding is likely to be driven to a corner solution where marginal tax losses are once again carried forward. For this reason this expression is not used for the preferred measure of the cost of capital in section 4.

Other than the 1975 scheme the effects of carry-back are more limited. If no stock relief is available then the cost of capital is unaffected by the carry back of marginal tax losses. With the 1981 scheme (and again assuming that tax exhaustion ends in the following year) the cost of capital becomes:

$$p = i - \pi - \pi_s(1+i(1-\tau_c))^{-1}\frac{\tau_c}{1-\tau_c(1+i)} \tag{4.13}$$

Note that since the relief under the 1981 scheme is received only in the subsequent year, the carry back does not increase the current value of the relief; instead it must be discounted according to n, the number of years for which tax exhaustion is expected to continue. However this some increase in the value of relief because the denominator of the third term is reduced by the interest credit on tax payments in the current period. At the level of nominal interest rates recorded for the UK in the early 1980's this results in a fairly small fall in the cost of capital, in comparison to the case of losses carried forward.

A final consideration arising from tax exhaustion is an additional effect on the cost of capital when the benefits of holding inventory (the marginal rate of return MRR in the preceding analysis) arise in future years. When there is no tax exhaustion and future tax rates are expected to be the same as today the dating of the MRR does not affect the cost of capital as the tax rate on MRR is the same no matter when it arises. If however future tax rates are expected to change, or if the company is temporarily tax exhausted, then the tax on future MRR is different than the tax on present MRR and the cost of capital is changed. Permanent tax exhaustion yields the same cost of capital whatever the date of MRR as there is no effective tax on MRR at any date.

To make this point formally suppose that the MRR accrues m years in the future. Let the cost of capital with MRR in the current period (under any of the cases discussed above) be given by p_0. And suppose that tax rates are expected to change from τ^m_cin the current period to τ^m_cm periods hence. Then the cost of capital with MRR m years in the future is given by:[46]

$$p^m = \frac{1-\tau^0_c}{1-\tau^m_c}p^0 \tag{4.14}$$

If tax losses are expected to be carried forward for n years then (3.14) still applies with expected effective tax rates at date 0 and date m are (in terms of the rate of corporation tax τ_c which is not expected to change):

$$\begin{aligned} \tau^0_c &= \tau_c(1+i(1-\tau_c))^{-n} & \\ \tau^m_c &= \tau_c(1+i(1-\tau_c))^{m-n} & m \le n \\ \tau^m_c &= \tau_c(1+i(1-\tau_c)) & m \ge n \end{aligned} \tag{4.15}$$

With tax losses carried forward the effective rate of tax on current earnings is more heavily discounted, and thus lower, than the effective rate of tax on future earnings. This is because the carry forward to the next period of tax payment is greater. Since the current period effective tax rate is lower than the future effective tax rate, it follows from (3.15) that $p_m > p_0$. To summarise when the MRR arises in future periods the increase in the cost of capital, resulting from tax exhaustion, is greater than the rise when MRR is immediate. The effects of tax exhaustion analysed above are re-inforced by the delay in MRR.

If losses are carried back, but not forward, tax exhaustion reduces the cost of capital, under the two stock relief schemes. In this case the delay in MRR again re-inforces the effects of tax exhaustion, this time by reducing the cost of capital further. This is because the effective rate of tax on current MRR is now greater than the effective rate of tax on future MRR, as a result of the interest rate credit on tax repayment and thus $p_m < p_0$. When the MRR arises in future periods, the fall in the cost of capital resulting from the carry back of marginal earnings is greater than the fall when the MRR is immediate. This fall in the cost of capital when marginal earnings are carried back, and MRR is delayed instead of immediate, is not restricted to the two stock relief schemes. It also arises when no relief is available.

Allowing for marginal returns which accrue in future periods will affect the cost of capital even when there is no tax exhaustion, if the company expects a change in the rate of corporation tax. Typically changes in the corporation tax are announced in the budget at the beginning of the financial year, so any change to future rates of corporation tax come as a surprise. The exception in the UK is the 1984 budget which announced lower rates of corporation tax in both 1985/86 and 1986/87. Thus an allowance for marginal returns in future periods reduces the cost of capital in 1984/85 and 1985/86 for all companies, tax exhausted or not.

Over what period do the marginal return to inventory investment accrue, so that we can appropriately adjust the cost of capital? There is no direct evidence on this point. The calculations of the cost of capital in the section 4 make two alternative assumptions. The first is that all returns acccrue in the current accounting year ($m=0$). The second is that returns are spread evenly over the current year and the subsequent two years. The results suggest that in practice the cost of capital for inventory investment is not greatly affected by the profile of marginal returns to inventory investment.

5. Calculations of the cost of capital in the UK

This section presents the calculations of the cost of capital for inventory investment in the UK from 1968 to 1987. These use the Institute for Fiscal Studies corporate tax model to allow for the tax status of individual firms, and (in calculating a quarterly cost of capital series) for the distribution of the end of accounting years.[47] The results are weighted averages of the cost of the capital calculated for the 397 companies included in the IFS model. The weights are the book values of inventory reported in the company accounts. The results are presented in tables 1 to 4 and in a chart.

The IFS corporate tax model

A full description of the IFS model is given by Devereaux (1986). This detailed model of the UK corporation tax system has been developed over several years. It uses publicly available company accounts data, taken from Datastream for recent years and from the Whittington-Meeks version of the DTI company accounts data base for earlier years, to model the tax position of individual companies. It applies the rules of the corporate tax system in an attempt to mirror the computations of the companies themselves.

The model covers 397 companies operating wholly or mainly in the UK, which together account for about 40% of the total non-oil non-financial UK corporate sector. These companies, which are amongst the largest registered in the UK, are those companies for which continous data is available from 1968 to 1984 but excluding companies which earn more than half their profits overseas. In addition a further 11 companies are excluded where independent evidence on tax liabilites suggests that the model performs particularly poorly.

Devereaux (1986) discusses how representative this group of companies is of the entire non-oil corporate sector. Comparison with the larger DTI sample of company accounts data, grossed up by the DTI to reperesent the entire non-oil corporate sector, reveals that the 397 companies are fairly representative (over the period 1977 to 1982) for most important accounting measures. Gross profits, inventory holdings, depreciation, retained income and investment are all close to 40% of the corresponding aggregate figures. Dividend payments, capital employed and holdings of net current assets (excluding inventories) are a somewhat higher proportion. Year to year movements in these variables for companies included in the model are fairly similar to those reported by the DTI.

Use made of the IFS model

The aggregate measures of the cost of capital presented here are weighted averages of the cost of capital for the individual companies in the IFS model. The weights used for aggregation are the book value of inventories reported by each company. The cost of capital for individual companies use information on company tax status taken from the model together with aggregate data on interest rates and the inflation of inventory prices.

The IFS model provides sufficient information to determine, for each company in each financial year, which of cases 1 to 6 above is the relevant expression for the cost of capital. In particular it identifies whether the company obtains stock relief, or faces clawback of relief, under the 1975 stock relief scheme. The IFS model was used also as a source for accounts data on the book value of inventories; for the month and year in which each account ends; and for the all stocks price index at the end of the accounting year used in the calculation of relief under the 1981 scheme.

Finally the IFS model was used to identify MCT exhaustion and whether marginal losses are carried forward or back, and if carried back by how many years. This information, together with the cost of finance in each financial year, is enough to determine the appropriate tax variables for including in the expressions for the cost of capital. The tax variables are all calculated assuming that payment of tax does not occur until 12 months after the end of the accounting year and all tax payments are discounted by one period.

A seperate sectoral breakdown of corporate sales from Datastream was used to obtain an aprroximate sectoral breakdown of individual company sales in the IFS model. These are used as weights in the construction of sector specific measures of the cost of capital.

Other data used in the calculation of the cost of capital

The calculations of the cost of capital require both the cost of finance and the rate of inflation of current inventory prices. The rate of inventory price inflation is calculated from the producer price index (available from 1974 onwards), the wholesale price index (for years prior to 1974 when it was replaced by the producer price index) and the retail price index. The rate of inflation for all inventories is a weighted average of the rate of inflation of manufacturing output and input prices and retail prices, the weights being 1985 inventory holdings by the manufacturing sector and distribution as recorded in the National Income and Expenditure and Accounts. The price index for manufacturers work in progress is assumed to be the arithmetic mean of output and input prices. There is no price data available for inventories held by other sectors, so here the rate of inflation is assumed to be the same as this weighted average rate of inflation for manufacturing and distribution. Sectoral rates of inflation are based on the same indices. The inflation rates used for the calculation of the cost of capital in each financial year are annual averages over the period from July to the following June.

Interest rates are those obtaining over the financial year. It is argued above that debt is the appropriate source of finance for inventory investment. In this case the cost of capital depends only on the nominal interest rate and the rate of corporation tax and on stock relief.[48] The nominal interest rate is the clearing bank base rate plus a 3% mark-up.

A further estimate of the cost of capital is based on a mix of finance. For this measure of the cost of capital, finance is assumed to be used in the fixed proportions 19.3% debt, 76.3% retentions and 4.4% new issues.[49] As noted above the cost of new issue finance depends on θ, the value of one unit of retained earnings in terms of gross dividends foregone. Data on θ is taken from King (1977) and King and Fullerton (1984) and updated to 1987/88 using the relevant UK imputation rate (the basic rate of personal taxation.)

The mix of finance calculations also require the cost of finance from retained earnings. This depends on the effective rate of capital gains tax z and the rate at which interest income is taxed m, for the three main groups of shareholders; households, tax exempt institutions and insurance companies. King and Fullerton (1984) ch 3 give values for m and z in 1980 for both households and insurance companies. (m and z are 0 for tax exempt institutions). The only large change in these variables over the period 1967-1987 is that resulting from the cut in personal tax rates for the year 1979/80. King and Fullerton estimate that this reduced m for households by some 12% points. The mixed finance calculation of this paper assume that prior to 1979 m was constant at the higher value, and after 1979 constant at the lower value. This calculation also assumes a gradual increase in the proportion of company shares held by tax exempt institutions and the corresponding reduction in the proportion held by households. These proportions are assumed to be linear interpolations of the benchmark figures given in King (1977) and King and Fullerton (1984). Finally data on UK tax rates is taken from King (1977), King and Fullerton (1984) and Board of Inland Revenue (1987).

Foresight assumptions.

The calculations of the cost of capital are affected by degree of foresight which each company is assumed to posess. In the extreme case of myopia the company anticipates no changes in either its tax status or in the tax regime. The calculations of this paper assume that the company is unable to anticipate changes in tax regime, before they are announced, but that it has perfect foresight about its own tax position. In particular it knows for exactly how many years it will remain tax exhausted. Expectations of future interest rates are assumed to equal current interest rates. Movements in the all-stocks price index (which generates relief under the 1981 stock relief scheme) are also assumed to be perfectly anticipated.

There is a deviation from this assumption in one respect. Perfect foresight about the duration of tax exhaustion is unconvincing for expectations, formed prior to the 1984 budget, about tax exhaustion in the period 1984-1988. These expectations of the duration of tax exhaustion were probably systematicly greater than the outturn, since the abolition of capital allowances and stock relief in the 1984 budget was responsible for much of the fall in the number of MCT exhausted companies after 1984. Two alternative treatments of expectations formed before the 1984 budget about the period after the 1984 budgetare possible. One assumes perfect foresight about future tax exhaustion. The other assumes myopia so that a company tax exhausted in 1983/84 is expected to remain permanently tax exhausted. The preferred measure of the cost of capital assumes expected future tax rates which are an arithmetic mean of these perfect foresight and perfect myopia procedures.

Results.

Table 1 documents the extent to which the tax status of individual companies varies within the IFS model. Column 1 shows the percentage of companies which are MCT exhausted. This rises rapidly in the early 1970's reaching a peak of 40% of the sample in 1974/75. The proportion of tax exhausted companies remains over 30%

until the reforms of corporate taxation in the 1984 budget. Thereafter the proportion of tax exhausted companies drops fairly rapidly. Note that individual companies move into and out of tax exhaustion frequently; this is reflected in the average number of years of tax exhaustion remaining for each company. Even in the mid-70's this never rises above five years.

The remaining columns of table 1 summarises the tax position of the firms in the IFS model under the 1975 stock relief scheme. Case 2 - stock relief or clawback applicable in both the current and subsequent year - applies to around 90% of all firms (weighted by the book value of inventories). Cases 5 and 6 are those where relief or clawback applies in only one of the two years. This applies to between 5 and 13% of all companies in the IFS tax model, during the period of the 1975 stock relief sheme. As discussed in section 3 these cases lead to extreme values for the cost of capital and the company is likely to be driven to a corner solution, where inventory holdings are relatively insensitive to the cost of capital. Finally the number of companies who carry back marginal tax losses, and are eligible for relief under the 1975 scheme is about 30% in 74/75 and remains around 10% thereafter. As discussed in section 3 this combination leads to a larger fall in the cost of capital than arises from the stock relief scheme alone.

Tables 2 builds up the preferred measure of the cost of capital for inventory investment from 1968/69 to 1987/88. Columns 1 and 2 are the rates of inventory price inflation and the assumed rate of interest for corporate borrowing. The rate of inventory price inflation is calculated over the twelve months ending in the June following that financial year. Interest rates are those over the financial year. Column 3 -the difference between columns 1 and 2 - is the simple measure of the cost of capital which takes no account of stock relief or of tax exhaustion. The commodity price inflation of 1973 and 1974 leads to a sharp fall in this simple measure of the cost of capital reaching a trough of -20% on an annual basis. This measure then rises through the rest of the 1970's reaching a peak of +10% in 1981/82 and remaining high throughout the 1980's. Column 4 shows the cost of capital allowing for stock relief, but not tax exhaustion. The effects of stock relief, under both the 1975 and the 1981 scheme, is to reduce the cost of capital for inventory investment by between 4 and 8 percentage points. The cost of capital now remains negative throughout the 1970's, and only rises to around 2% in the early 1980's. After the abolition of stock relief in the 1984 budget the cost of capital rises to over 10%.

Column 5, the preferred measure of the cost of capital, allows for the carry forward and carry back of tax losses. In the absence of stock relief tax exhaustion does not affect the cost of capital. Under the 1981 stock relief scheme tax exhaustion makes very little difference to the cost of capital. It is only under the 1975 stock relief scheme that tax exhaustion has an important effect on the cost of capital, offsetting the reduction in the cost of capital arising from tax relief. During the operation of the 1975 scheme tax exhaustion increases the cost of capital by between 1 and 3 percentage points.

Table 3 compares the preferred measure of the cost of capital (column 1) with measures derived on slightly different assumptions. Column 2 assumes a mix of finance, instead of pure debt finance, increasing the cost of finance because full advantage is no longer taken of the tax deductability of debt finance. The increase is most pronounced (around 5 percentage points) when nominal interest rates are high in the 1980's. All but one of the remaining measures of the cost of capital are very close to the preferred measure. Allowing for perfect foresight of the companies tax position after the 1984 budget (column 3), restricting carry back of tax losses to one year (column 4), or allowing for a delay in the MRR on inventory holdings (column 5) makes very little difference to the cost of capital.

The final column includes, rather than excludes, potential corner solutions (cases 5 and 6) in the cost of capital. This markedly reduces the cost of capital under the 1975 scheme, particularly in 1974/75 and 1979/80. A comparison with table 1 suggests that the substantial fall in the cost of capital is mostly due to the high proportion of companies carrying back tax losses, and also claiming stock relief, in 1974/75. Carry back of losses increases considerably in 1979/80, and here again this measure of the cost of capital falls sharply. These results suggest that many companies have been driven to corner solutions, where the calculated cost of capital no longer reflects the incentive to invest in inventories. The preferred measure of the cost of capital must under-estimate the incentive to invest in inventories, while the calculation including potential corner solutions must over-estimate the incentive to invest in inventories. The degree of under and over estimation is unknown.

Table 4 shows the preferred measure of the cost of capital for distribution, manufacturing (distinguishing raw materials, finished goods and work in progress) and other industries. At the foot of the table there is a comparison between national accounts measures of the book value of inventories and aggregate inventory holdings by the companies included within the IFS model. The IFS model covers only 6% of inventory holdings in retailing and distribution; this is because much of the output of this sector is by small companies or non-corporate enterprises.[50] Almost all manufacturing takes place within the corporate sector so the IFS model is more representative, covering about 40% of total manufacturing inventories. The model covers around 30% of the inventory holdings of other industries - mostly construction, agriculture, transport and communications.

The cost of capital for inventory investment in manufacturing, differs from the aggregate measure of table 2 largely because of the effects of inventory inflation. There is a particularly sharp fall in the cost of capital in 1973 and 1974, a renewed fall in the cost of capital in 1975/76 and a rise in the cost of capital to 16% in 1985/86. Although the IFS model does not represent the tax position of retail businesses at all well, the cost of capital in this sector is evidently much smoother than in manufacturing or other industries, again reflecting contrasting movements in inventory price inflation.

Chart 1 shows the quarterly profile of the simple measure of the cost of capital, before allowing for stock relief and tax exhaustion, and the preferred measure of the cost of capital, aggregated over all sectors. These measures use the interest rate in that quarter, but the six monthly rate of inflation from the previous to the subsequent quarter. Both measures fall to particularly low values in 1974 due to the rapid rate of inflation of inventory prices.

Chart 2 shows the difference made by allowing for stock relief and tax exhaustion to the quarterly profile of the cost of capital. This adjustment for stock relief and tax exhaustion exhibits a pronounced seasonal pattern during the period of stock relief. This arises because stock relief reduces the cost of capital only at the end of company accounting years. Stock relief has an impact on the quarterly cost of capital four times that indicated by the expressions given in section 3 (because a given amount of tax relief is four times as large relative to the net benefits of holding inventory) but only in the quarter in which the company account ends. The intra-account cost of capital is unaffected by the availability of stock relief. As company accounts end more frequently in the 4th and 1st quarters this effect carries through onto the aggregate measure of the cost of capital. Stock relief reduces the cost of capital by more in the 4th and 1st quarters than in the 2nd and 3rd quarters.[51]

6. Conclusions

This paper has provided a detailed analysis of the cost of capital for inventory investment and derived measures of the aggregate cost of capital in the UK, allowing for the tax status of individual companies. Section 2 discussed the various factors - the cost of finance, accounting conventions, inflation of inventory prices and the tax system - which affect the cost of capital. When FIFO accounting is used for tax purposes, and assuming debt finance - these assumptions are appropriate for the UK - the cost of capital for inventory investment is simply the nominal interest rate less the rate of inflation of inventory prices. This expression for the cost of capital allows for the cost of finance, the tax system and any anticipated holding gains on inventories but takes no account of stock relief or of tax exhaustion.

Section 3 summarises the operation of the two UK stock relief schemes and the transition arrangements which governed the change over between the two schemes. The 1975 stock relief scheme acted somewhat like the 100% investment allowance for fixed capital investment. Relief was granted on any increase, and relief withdrawn for any fall, in the book value of inventories. Thus the tax system provided a share of the finance of any increase in inventory holdings.

The 1981 stock relief scheme was a simpler indexation system. Here relief took the form, not of a share in the financing of inventory investment, but was offered through the tax deductability of the inflationary profits on inventory holdings. The transition arrangements, governing the introduction of the 1981 scheme, were fairly effective at preventing a potentially substantial incentive to invest in inventory.

Formal expressions for the cost of capital, derived in section 4, are consistent with the earlier results of Sumner (1984). Section 4 extends the Sumner results to allow for the effects of tax exhaustion. Tax exhaustion has its greatest effect on the cost of capital during the operation of the 1975 stock relief scheme. When losses are carried forward the tax incentive to invest in inventories arising from stock relief vanishes. When losses are carried back the tax incentive to invest in inventories is increased. Tax exhaustion has a much lesser effect on the cost of capital under the 1981 scheme, and - under the debt finance assumption made in this paper - has no effect on the cost of capital in the absence of stock relief.

Section 4 also demonstrates that companies which move into or out of stock relief, under the 1975 scheme, are subject to particularly large incentives to invest or dis-invest from inventories, and are therefore likely to be driven to corner solutions.

This is the main caveat to the preferred measure of the cost of capital presented in section 5. This probably under-estimates the incentive to invest in inventory during the 1975 stock relief scheme, but the degree of under-estimation is unkown. In principle it should make a difference whether marginal returns to inventory investment accrue in the current or in future accounting years, but the use of the IFS model suggests that this factor does not make a great difference to aggregate measures of the cost of capital.

Section 5 presents the estimates of the cost of capital for aggregate UK inventory investment derived using the IFS tax model. These supports two conclusions concerning the effect of the tax status of individual companies on the aggregate cost of capital:

(i) Ineligibility of companies for stock relief under the 1975 stock relief scheme applied to only around 10% of companies and so did not affect the aggregate cost of capital to any great extent.
(ii) The effects of tax exhaustion on the cost of capital, which are modelled in some detail here, are only quantitatively important during the operation of the 1975 stock relief scheme. Here tax exhaustion increases the cost of capital, by between 1 and 3 percentage points. Otherwise the effects of tax exhaustion on the aggregate cost of capital are very small.

It is useful to take account of the tax status of individual firms in calculating the aggregate cost of capital. Nonetheless the most important influences on the cost of capital are aggregate factors: the level of nominal interest rates, the rate of inventory price inflation, and the presence of stock relief. It is these which cause the cost of capital to fall sharply during the 1973 commodity price inflation, and to reach record levels from 1984/85 when interest rates are high and stock relief is abolished. Stock relief, in operation between 1974 and 1984, reduces the cost of capital for inventory investment by between 6 and 8 percentage points.

The calculations reported in section 5 also reveal a pronounced seasonality in the aggregate cost of capital induced by the availability of stock relief. This occurs because relief is paid on inventories held at the end of the company accounting year and, since the end of company accounting years falls much more commonly in December or March than in other months, there is a considerably greater reduction in the aggregate cost of capital for the holding of inventories at the end of the fourth and first quarters. This seasonality may prove a particularly useful source of variation in the cost of capital for in the estimation of aggregate time series equations: it remains to be seen whether estimates with this data will yield significant and correctly signed coefficients on the cost of capital for inventory investment.

Table 1: the Tax Position of Individual Companies in the IFS Model

FINANCIAL YEAR	% OF SAMPLE MCT EXHAUSTED	NUMBER OF YEARS TAX EXHAUSTION REMAINING	AVERAGE % OF SAMPLE ELIGIBLE FOR RELIEF UNDER THE 1975 STOCK RELIEF SCHEME IN:			
			THIS YEAR AND NEXT	THIS YEAR ONLY	NEXT YEAR ONLY	THIS YEAR WITH TAX CARRY BACK
68/69	2	3.8				
69/70	1	5.0				
70/71	3	5.9				
71/72	8	5.3				
72/73	10	6.2				
73/74	32	4.2				
74/75	45	3.3	66	2	3	31
75/76	26	4.7	92	4	3	9
76/77	32	4.6	89	8	3	7
77/78	30	4.7	86	7	5	10
78/79	29	4.7	87	4	7	9
79/80	40	4.2	94	0	5	14
80/81	39	4.2	25	0	1	3
81/82	38	4.0				
82/83	38	3.4				
83/84	30	3.2				
84/85	28	2.9				
85/86	24	2.5				
86/87	19	2.0				
87/88	15	1.4				

Table 2 : Cost of Capital for Aggregate UK Inventory Investment

FINANCIAL YEAR	AGGREGATE DATA		COST OF CAPITAL		
	NOMINAL INTEREST RATE (1)	INVENTORY PRICE INFLATION (2)	BEFORE TAX =(1)-(2) (3)	AFTER TAX, CALCULATED WITH THE IFS TAX MODEL, WITH ALLOWANCE MADE FOR:	
				STOCK RELIEF ONLY	STOCK RELIEF PLUS TAX EXHAUSTION
68/69	10.4	4.9	5.5	5.5	5.5
69/70	10.5	5.3	5.2	5.2	5.2
70/71	9.6	7.3	2.2	2.2	2.2
71/72	7.7	4.7	3.1	3.1	3.1
72/73	10.6	25.4	-14.8	-14.8	-14.8
73/74	15.0	36.1	-21.1	-21.1	-21.1
74/75	13.5	20.6	-7.1	-11.6	-8.1
75/76	13.4	18.0	-4.6	-10.9	-8.6
76/77	14.0	15.4	-1.5	-8.0	-5.0
77/78	9.8	7.6	2.2	-2.3	-1.1
78/79	14.7	14.0	0.7	-5.7	-4.8
79/80	18.9	12.2	6.7	-1.3	2.9
80/81	16.1	10.3	5.8	-2.4	-0.4
81/82	17.2	6.7	10.5	2.1	3.4
82/83	13.0	6.0	7	1.8	2.6
83/84	12.0	5.9	6.1	1.2	1.7
84/85	14.2	3.7	10.5	10.5	10.5
85/86	13.8	0.0	13.8	13.8	13.8
86/87	12.8	5.1	7.7	7.7	7.7
87/88	11.8	5.3	6.5	6.5	6.5

Table 3 : Alternative calculations of the cost of capital

FINANCIAL YEAR	PREFERRED MEASURE	MIX OF FINANCE	TAX LOSS CARRY BACK RESTRICTED TO ONE YEAR	PERFECT FORESIGHT OF TAX EXHAUSTION	ALLOWING FOR DELAY IN MRR	WITH EXTREME VALUES FOR COST OF CAPITAL
	(1)	(2)	(3)	(4)	(5)	(6)
68/69	5.5	6.3	5.5	5.5	5.5	5.5
69/70	5.2	5.8	5.2	5.2	5.2	5.2
70/71	2.2	2.6	2.2	2.2	2.2	2.2
71/72	3.1	3.5	3.1	3.1	3.1	3.1
72/73	-14.8	-14.3	-14.8	-14.8	-14.9	-14.8
73/74	-21.1	-17.7	-21.1	-21.1	-21.1	-21.1
74/75	-8.1	-5.1	-8.1	-8.1	-7.8	-14.6
75/76	-8.6	-6.8	-8.6	-8.4	-8.4	-12.5
76/77	-5.0	-3.1	-5.0	-5.0	-5.0	-11.1
77/78	-1.1	0.6	-1.1	-0.8	-0.9	-4.5
78/79	-4.8	-2.2	-4.8	-3.6	-4.0	-5.2
79/80	2.9	7.9	2.9	3.0	3.2	-1.8
80/81	-0.4	5.2	-0.4	-0.9	-0.4	-1.2
81/82	3.4	9.9	3.4	2.9	3.5	3.4
82/83	2.6	8.2	2.7	2.2	2.8	2.6
83/84	1.7	6.6	1.7	1.5	1.9	1.7
84/85	10.5	15.8	10.5	10.5	10.0	10.5
85/86	13.8	18.0	13.8	13.8	13.3	13.8
86/87	7.7	10.9	7.7	7.7	7.8	7.7
87/88	6.5	9.7	6.5	6.5	6.5	6.5

Table 4 : Cost of Capital for inventory investment by sector

FINANCIAL YEAR	MANUFACTURING TOTAL	MANUFACTURING RAW MATERIALS	MANUFACTURING FINISHED GOODS	MANUFACTURING WORK IN PROGRESS	RETAIL AND WHOLESALE	OTHER
68/69	5.8	5.8	5.8	5.8	5.0	5.5
69/70	5.6	5.6	5.6	5.6	4.7	5.2
70/71	3.7	3.7	3.7	3.7	0.2	2.2
71/72	4.2	4.2	4.2	4.2	1.4	3.1
72/73	-27.2	-27.2	-27.2	-27.2	2.9	-14.8
73/74	-33.5	-26.5	-40.4	-33.5	-3.5	-21.1
74/75	-4.7	0.6	-9.8	-4.9	-13.3	-8.5
75/76	-11.6	-17.8	-5.5	-11.5	-2.0	-8.7
76/77	-5.0	-2.6	-7.6	-4.9	-5.7	-4.2
77/78	-0.5	1.9	-3.7	0.1	-0.3	0.1
78/79	-3.1	-3.4	-2.4	-3.5	-2.6	-11.8
79/80	4.2	7.9	0.3	4.5	2.0	3.7
80/81	0.1	-2.2	2.0	0.5	-1.5	0.5
81/82	3.8	4.7	2.5	4.2	0.0	3.8
82/83	1.8	0.3	3.1	2.0	3.2	2.9
83/84	0.9	0.0	1.7	1.0	2.5	1.7
84/85	11.6	15.0	8.2	11.6	9.0	10.5
85/86	16.4	23.1	9.7	16.4	10.2	13.8
86/87	6.6	4.4	8.8	6.6	9.4	7.7
87/88	6.6	6.5	6.8	6.6	6.2	6.5
BOOK VALUE OF INVENTORIES 85/86 £bn						
IFS MODEL: (1)	17				2	3
N. ACCOUNTS: (2)	42				28	12
(1) AS % OF (2)	39				6	24

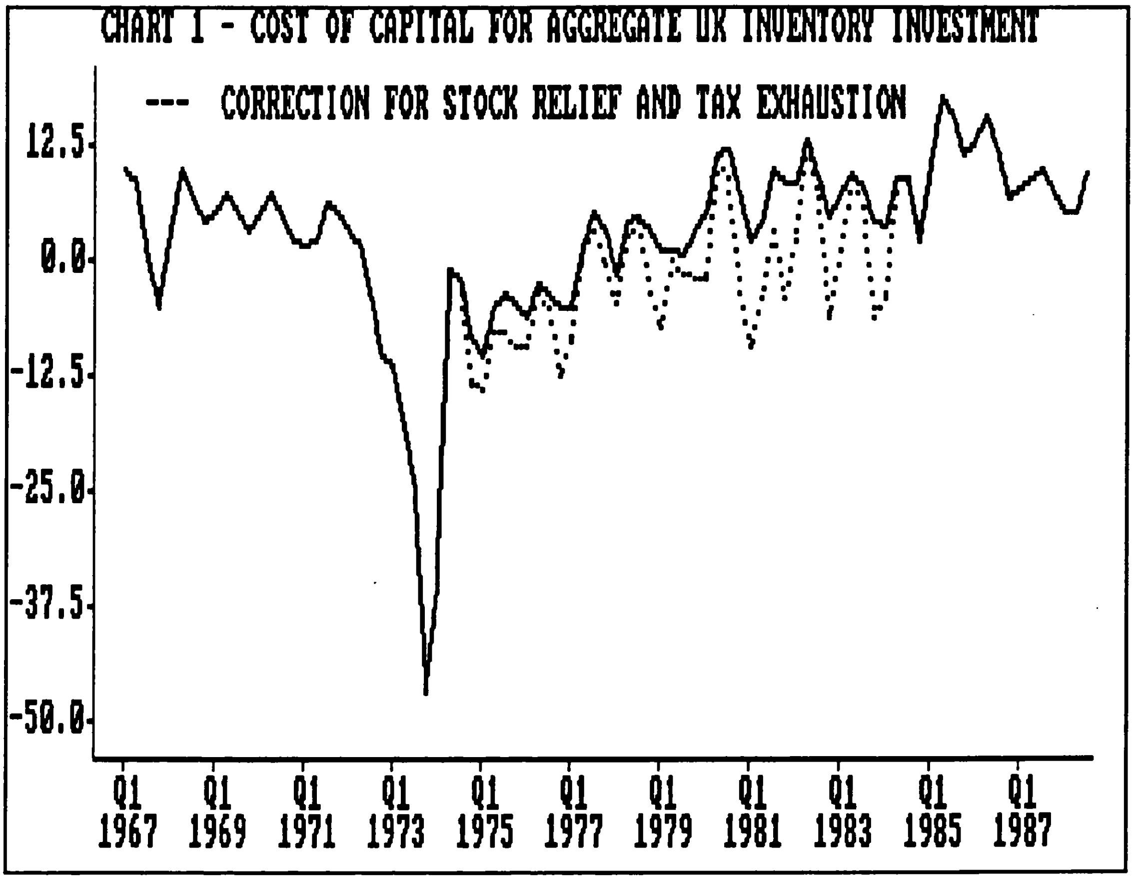


CHART 1 - COST OF CAPITAL FOR AGGREGATE UK INVENTORY INVESTMENT
--- CORRECTION FOR STOCK RELIEF AND TAX EXHAUSTION
12.5
0.0
-12.5
-25.0
-37.5
-50.0
Q1 1967
Q1 1969
Q1 1971
Q1 1973
Q1 1975
Q1 1977
Q1 1979
Q1 1981
Q1 1983
Q1 1985
Q1 1987

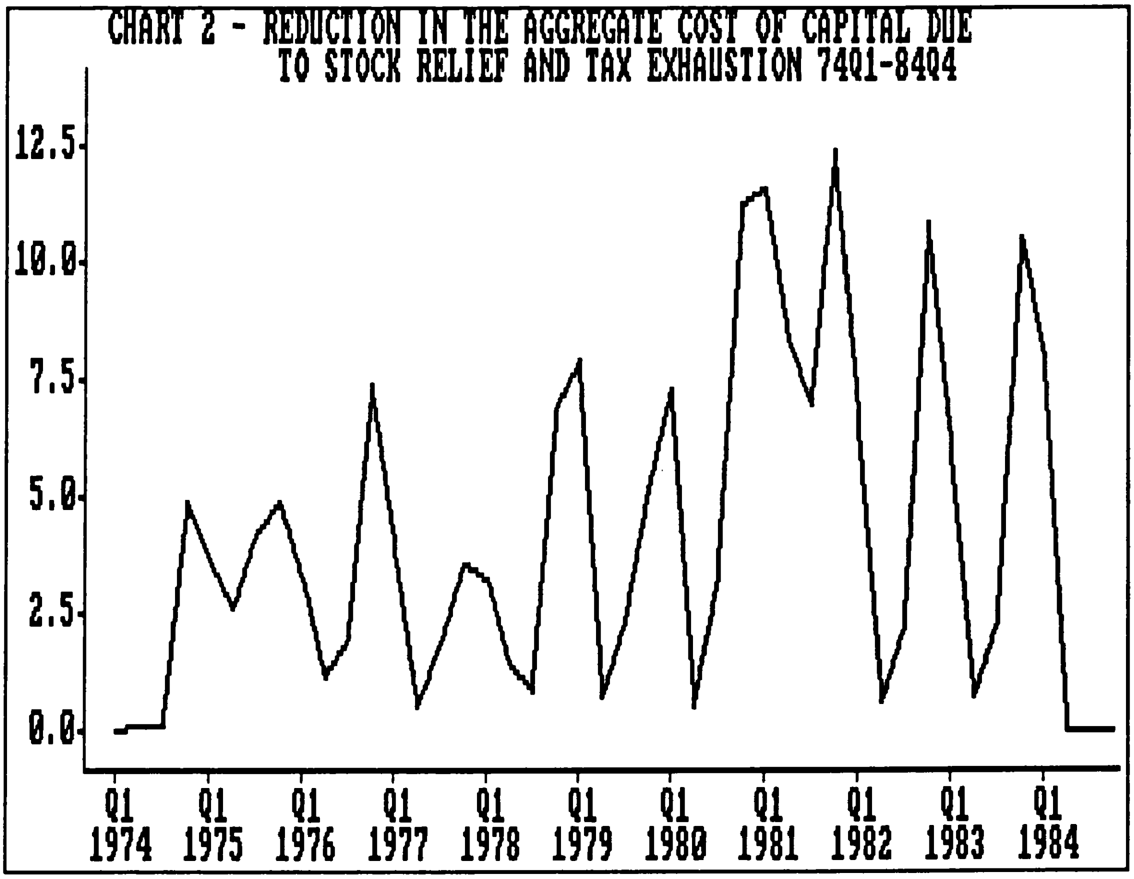


CHART 2 - REDUCTION IN THE AGGREGATE COST OF CAPITAL DUE TO STOCK RELIEF AND TAX EXHAUSTION 74Q1-84Q4
12.5
10.0
7.5
5.0
2.5
0.0
Q1 1974
Q1 1975
Q1 1976
Q1 1977
Q1 1978
Q1 1979
Q1 1980
Q1 1981
Q1 1982
Q1 1983
Q1 1984

CHAPTER 6

CONCLUSIONS AND FURTHER WORK

Excess volatility

a. The degeneracy of the excess volatility research programme

The explanation of excess volatility has been the dominant research programme in the econometric study of inventory investment over the past decade. The literature has proposed a number of models of finished goods inventory investment in which the variance of production exceeds the variance of sales. Chapter 2 examines UK time series data for inventory investment from the perspective of the LSE approach to econometrics. This suggests a number of conclusions relating to this research programme:

(i) Excess volatility is a very minor feature of the UK data and (judging by the variance comparisons reported in the US literature) also a very minor feature of the US data. This point is illustrated by chart 1 of chapter 2. Production and sales are almost indistinguishable for aggregate UK manufacturing. This can explain why there are conflicting findings on the presence of excess volatility: only a small degree of measurement error can reverse the ranking of the variance of output and sales.

(ii) The most pronounced feature of UK inventory investment over the post-war period has been the cyclical fluctuations in inventory investment which account for some [33]% of the cyclical movements in expenditure on GDP. Graphical analysis (similar to the NBER reference cycle techniques) indicate that these fluctuations exhibit precisely the same features as were reported by Abramovitz (1950) for inventory investment in the US during the inter-war period. Inventory investment moves pro-cyclically coincident with the cycle movements in output and sales. Inventory levels also move cyclically but lag sales and output by about 4-6 quarters.

(iii) Any model which explains the cyclical movements of inventory investment also provides and explanation of excess volatility. The converse does not apply: a model which explains excess volatility may or may not provide and explanation of the cyclical movements in inventory investment. This conclusion is reached by the application in chapter 2 of the encompassing principle to informal data descriptions. This application is an extension of the encompassing principle, as proposed by Mizon (1984) and Mizon and Richard (1986) to informal data descriptions existing procedures of the LSE approach to econometrics. Consideration of the variance decomposition of the identity linking output, sales and inventory investment shows that the cyclical movements in inventory investment encompass, but are not encompassed by, the observation of "excess volatility".

(iv) The explanation of "excess volatility" is a degenerate research programme. A degenerate research programme is one which accepts theories which explain only some features of the data. Such theories may be inconsistent with other features of the data which have not been taken into account in assessing the theory. They must then be extended by some supplementary theory (which the degenerate research programme does not provide) or abandoned in favour of more successful theories which explain all features of the data.

(v) The research programme which seeks to explain the cyclical movements in inventory investment is more progressive than that which seeks an explanation of "excess volatility", because any theory consistent with the data which is generated by the latter research programme can also be generated by the former, whereas many theories inconsistent with the data which might be accepted under the latter research programme are ruled out by the former.

(vi) The cyclical movements in inventory investment are common to all categories of inventories whereas excess volatility is a feature only of finished goods inventories. This suggests that in the search for theoretical models of the cyclical movements in inventory investment priority should be accorded to models which apply to all categories of inventories. Only if no such models emerge is it appropriate to consider distinct theories for each category.

(vii) The need to provide an explanation of the cyclical movements in inventory investment suggests that there is scope for further research exploring the links between the theory of business cycles and of inventory investment. Chapter 1, anticipating this conclusion, reviews these two literatures in tandem. This indicates a number of promising areas for future research on inventory investment including:

- Models of financial effects on inventory investment

- Models in which costs shocks are transmitted to all the different categories of inventories by movements in the cost of capital for inventory investment

- Models in the new-keynsian tradition in which small second order disturbances have large aggregate effects

- (S,s) models

Chapters 4 and 5 of this thesis relate to the first two of these areas for further research.

(vi) A theoretical model which explains the cyclical movements in inventory investment is likely also to explain the well known puzzle about the slow speeds of adjustment in empirically estimated models of inventory investment. The support for this conclusion rests on the frequency domain analysis of the data based estimation presented in chapter 2. This shows that simple data based log-linear models can capture the cyclical movements in inventory investment first summarised by Abramovitz. These data based estimates also exhibit the characteristically slow speeds of adjustment associated with the conventional stock-adjustment model of inventory investment. Moreover increasing the speed of adjustment alters the cyclical characteristics of the bivariate relationship determining inventory movements. Thus any theoretical model which supports an log-linear empirical formulation which captures the cyclical movements in inventory investment also provides an explanation of slow adjustment speeds.

b. The mis-specification testing of the production-smoothing model.

Chapter 3 of this thesis re-examines the econometric mis-specification of what is referred to in this thesis as the linear quadratic production smoothing model of inventories. This model was introduced into the recent literature by Blanchard (1983), but its evident mis-specification, revealed by West (1983), was a major stimulus to the subsequent literature on "excess-volatility". Chapter 3 considers the mis-specification of this model from the perspective provided by the LSE approach to econometrics. This necessitates a fairly wide ranging discussion of the econometric estimation of models with explicit theoretical foundations based on quadratic objective functions, of which the linear quadratic production-smoothing model is but one example. This discussion supports a number of conclusions:

(i) Such models are often estimated using the standard techniques of Hansen and Sargent (1980). However these techniques are then usually applied with no allowance for specification error (the unavoidable discrepancy between the estimated model and the unknown data-generating process) and hence cloud any discussion of the mis-specification of such models. A combination of the LSE approach to econometrics with the techniques of Hansen and Sargent provides much clearer insight into the mis-specification testing of such models.

(ii) There is however a degree of conflict between these two traditions of econometrics and criticism of the techniques introduced by Hansen and Sargent can be made from the perspective of the LSE approach to econometrics. The freedom to introduce a serially correlated unobserved disturbance to the underlying objective can give the econometrician unwonted freedom to fit theory to data, making the theory impossible to reject and thus effectively making all theories whatsoever observationally equivalent and making scientific progress impossible.

(iii) There is a often overlooked small-sample difficulty associated with the normalisation of the estimated first-order condition implied by the choice of dependent variable. A full analysis is not provided in this chapter but a number of intuitive grounds for choosing one particular normalisation are offered. There appears to be scope for a further monte-carlo study of this issue.

(iv) The encompassing tests of Mizon (1984) and Mizon and Richard (1986) can be applied to models based on explicit theory with quadratic objective functions and estimated using the instrumental variables approach of the GMM estimator. Encompassing tests comparing the linear quadratic production smoothing model with the standard stock-adjustment specification show that the production smoothing model is clearly rejected. This rejection should however be interpreted as due to general dynamic mis-specification, not to "excess volatility", since it reflects a failure of the specification error associated with the linear quadratic production smoothing model to be uncorrelated with past observations on production and sales.

(v) Standard tests of residual auto-correlation can only be applied as tests of mis-specification to models based on quadratic objective functions under restrictive assumptions about the information set available to the econometrician, assuming either perfect foresight or that the econometrician and the agent share the same information set. There may be scope for further research to examine procedures for separately identifying expectational and specification errors; if this can be done then amended tests of residual auto-correlation can be applied to these standard models under more general assumptions about expectations formation.

Financial effects on inventory investment

Chapter 4 discusses financial effects on inventory investment resulting from informational asymmetries in capital markets and considers, using a panel of individual company accounts, whether these can explain the cyclical movements in inventory investment. Conclusions and scope for further work are as follows:

(i) By assuming an exogenous financial contract it is possible to analyse a stochastic dynamic programming model of inventory investment in which, with constraints on both debt and equity finance, financial effects on inventory investment emerge. An explicit solution is not possible but, under certain assumptions about the distribution of the stochastic shock, a qualitative solution can be derived. This indicates a distinction between periods of normal operation, when dividends are paid but the firm retains a cash balance, and periods of financial pressure, when the firm pays no dividends in order to build up a cash balance and inventory holdings depend on the firms net assets and are less than in periods of normal operation.

(ii) This can be viewed as a bankruptcy avoidance model in which firms desire to hold a cash balance because of the risk of poor future outcomes for the stochastic shock driving the firm into bankruptcy, which results in the loss to shareholders of the entire stream of expected future dividends.

(iii) With a further assumption of linearity in the relationship between net assets and inventory holdings then there is a relationship between current profits and inventory investment for firms under financial pressure. This parallels the finding of a relationship between current income and consumption for liquidity constrained households.

(iv) Estimation of the link between current profits, change in turnover and inventory investment using a panel of UK company accounts provides is strikingly consistent with this model: only those firms under financial pressure, as revealed by a variety of different indicators, exhibit a link between profits and inventory investment and this link is statistically highly significant.

(v) Aggregation over the panel reveals that, while the link between profits and inventory investment does affect the holding of inventories in the entire panel, financial effects are of insufficient magnitude to explain the major cyclical collapse in aggregate UK inventory investment of 1981. Moreover financial effects reduce inventory investment to a greater degree during 1982 and 1983, when more firms were under financial pressure, rather than in 1980. Both these findings suggest that financial effects, while apparently important at the level of the individual firm, are not the explanation of the cyclical movements in inventory investment.

(vi) Several avenues for further research still emerge from this paper. Further investigation of the assumptions required to support the proposed solution of the dynamic programming model is warranted. This model could be applied to other aspects of firm behaviour such as investment and employment. Finally there is a clear interest in extending this model to the more general case of an endogenous financial contracts.

The cost of capital

An alternative explanation of cyclical movements in inventory investment, implicitly appealed to by the work of Christiano (1988), is that inventory holdings are held by optimising households and firms as a means of ensuring consistency between inherently volatile movements in production (due to shocks to technology) with the households desire to smooth consumption over time. Such an optimising view of inventory investment movements requires that inventory investment at the level of the individual firm or household are driven by movements in the cost of capital. A fall in the price of output today (when the technology shock is favourable) relative to the expected price of output tomorrow) induces firms, wholesalers and households to increase inventories.

The main empirical difficulty with this kind of theory is that there is remarkably little evidence of any sensitivity of inventory holdings to the cost of capital: estimated are elasticities of inventory holdings on the rate of interest less the expected rate of output price inflation are rarely significant and often wrongly (positively) signed.

Chapter 6 is a step towards a better understanding of the effects of the cost of capital on inventory investment in two ways. Firstly it offers a detailed summary of the various factors influencing the cost of capital for inventory investment, using the same method as have been applied to calculating the cost of capital for fixed capital investment by King and Fullerton (1984), and taking account of the source of finance, the tax regime and accounting conventions. Second it investigates the marked effects on the cost of capital for inventory investment by individual firms arising from the tax relief on inventory holdings (the so called stock relief legislation) using the IFS tax model. The main conclusions are as follows:

(i) With debt finance (which is favoured by the tax system relative to equity finance), no stock relief and where tax assessment is based on the FIFO accounting convention (the dominant method in the UK) the cost of capital for inventory investment is given by rate of interest less the expected rate of inflation in the price of inventory.

(ii) The two schemes of stock relief which operated in the UK each lowered the cost of capital and simple formulas (consistent with those of Sumner (1983) are stated). Care must however be taken with the transition between the two schemes. An automatic application of the proposed formulas suggests a very sharp fall in the cost of capital in 1981 but this did not in fact occur due to the way in which the tax authorities handled the transition period.

(iii) The first scheme of stock relief operating from 1974 to 1981 offered very marked incentives for inventory investment by some individual firms. These extreme incentives apply to those firms obtaining relief in one year but not in the next or those firms tax exhausted and carrying back or expecting to carry forward tax payments. Such firms may well alter there inventory holdings to the extent that they are at a corner solution.

(iv) These extreme incentives are quantitatively very important at the level of the individual firm, but when aggregated over all the firms in the panel which makes up the IFS tax model, they make only a small difference to the aggregate cost of capital. Thus on aggregate annual data the simple formulas for the effects on the cost of capital are valid.

(v) For quarterly data on the cost of capital there is a further complication. It is then necessary to take account of the uneven distribution of the end dates of company accounts over the financial year. The reason that this matters is that the benefits of the stock relief legislation are all based on end year inventory holdings, offering an incentive to companies to build up inventory levels, for the purpose of attracting stock relief, at the end of the accounting year. Since accounting years fall more often in December and March this is an incentive, in aggregate, for inventories to be increased in the fourth and first quarters. The calculations of the aggregate cost of capital in chapter 6, using the IFS tax model, suggests that the induced seasonal variation effect on the cost of capital is in fact quantitatively even more important than the overall reduction in the annual cost of capital for inventory investment.

(vi) The substantial seasonal variation in the cost of capital over the period 1974-1984 offers an immediate opportunity for further research into the impact of the cost of capital on inventory dynamics.

NOTES

1. Time-aggregation is another reason for anticipating a degree of moving average residual auto-correlation.

2. Trend GDP is estimated by regressing GDP on a deterministic time trend. Trend inventory investment is taken to be 0.6% of trend GDP (the share of inventory investment in the expenditure measure of GDP is over the period 1955-1988)

3. Blinder and Holtz-Eakin also discuss a related point. This is that inventory investment accounts for a large proportion of the quarter to quarter changes in GDP(E). This presumably reflects the large amount of "noise" in inventories data. This particular observation will not play a role in the argument of this chapter.

4. All intra-sectoral sales are included in the measure of gross output; if these are excluded then the gross-net ratio for UK manufacturing is closer to 2.

5. In their simplest form stock out avoidance models result in inventory holdings which are proportional to next periods sales. Such models result in pro-cyclicality of the level of inventories rather than of inventory investment.

6. One confusion in the literature is over the application of the term production smoothing. Blinder (1981) refers to the stock adjustment specification as the "production-smoothing buffer-stock model" although this model is not based on an explicit dynamic theory. Fair (1990) also estimates a partial adjustment specification and refers to it as a production smoothing model.

7. This may be shown as follows. Suppose that sales are AR(1), and inventories are held as a fixed proportion of expected next period sales:

$$
\begin{aligned}
& S_t = \lambda S_{t-1} + u_t \\
\Rightarrow\quad & \mathrm{Cov}(S_t, S_{t-1}) = \lambda \mathrm{Var}(S_t) && (1) \\
& I_t = \alpha \mathcal{E}_t[S_{t+1}] = \alpha\lambda S_t \\
& Y_t = S_t + \Delta I_t = (1+\alpha\lambda)S_t - \alpha\lambda S_{t-1} \\
\Rightarrow\quad & \mathrm{Var}(Y_t) = [(1+\alpha\lambda)^2 + (\alpha\lambda)^2]\, \mathrm{Var}(S_t) - 2(1+\alpha\lambda)\alpha\lambda\, \mathrm{Cov}(S_t, S_{t-1}) && (2) \\
(1)+(2)\Rightarrow\quad & \mathrm{Var}(Y_t) = [\, 1 + 2\alpha\lambda\, (1+\alpha\lambda)\, (1-\lambda)\,]\, \mathrm{Var}(S_t) > \mathrm{Var}(S_t)
\end{aligned}
$$

8. These include non-convex costs of production (Ramey (1991)), technology shocks (Blinder (1986)) and stock-out avoidance (Kahn(1987)).

9. Application of the same tests using US data is being written up by the author in a forthcoming paper.

10. Blanchard (1983) and West (1986) document this finding for the US. There is however an ongoing debate as to whether the excess volatility of production in the US is due to measurement error. On this see Fair (1990).

11. The derivation of this first order condition makes use of the following marginal derivatives with respect to the level of inventories:

$$\partial Q/\partial I = +1,\ \partial Q_{+1}/\partial I = -1,\ \partial \Delta Q/\partial I = +1,\ \partial \Delta Q_{+1}/\partial I = -2,\ \partial \Delta Q_{+2}/\partial I = 1$$

These marginal derivatives are all derived from the identity linking output, inventories and sales.

12. A log-linear version of the hybrid production smoothing model is also possible but this has the disadvantage that the linear identity linking production, sales and inventories of finished goods can no longer be used to obtain a linear decision rule for estimation.

13. The Sargan result applies only when there is a spherical error structure:-

$$Plim \frac{1}{n} u'u = \sigma^2 I$$

Hansen (1982) establishes that in the non-spherical case where:-

$$Plim \frac{1}{n} u'u = \Sigma$$

asymptotic efficiency is obtained by using the weighting matrix:-

$$M = Z'(Z'\Sigma Z)^{-1} Z$$

14. Limited information maximum likelihood estimates are also unaffected by the parameter normalisation.

15. Time aggregation can also induce a moving average error which will bias conventional tests of mis-specification based on residual auto-correlation.

16. (4.2) is derived by taking unconditional expectations of (4.1), applying the law of iterated expectation, taking the expectations operator inside the summation sign and then applying the standard formula for geometric summation.

17. Standard errors for the inequality are based on the variance-covariance matrix for the coefficient estimates alone and take no account of the imprecision in the estimation of the various variances and co-variances.

18. This breakdown was provided by the Central Statistical Office. It is a quarterly version of the series published in table (10.4) of the 1988 UK National Accounts.

19. To make these comparable a re-scaling is required. This is necessary because when these estimates are re-arranged (using the first normalisation $a_0+(1+\beta)a_1$) the decision rule for inventories emerges as:-

$$(1+\beta+2\beta a_1+a_2)\ I_t = a_0\ F_t + a_1\ G_t + a_2 a_3\ S_{t+1} + u_t$$

and the standard error must be divided by the coefficient on I_t to make them comparable with the results in tables 1 and 2.

In the same way using the second normalisation $a_1=1$ results in the decision rule for inventories:-

$$(a_0(1+\beta)+1+2\beta+\beta^2+a_2)\ I_t = a_0\ F_t + G_t + a_2a_3\ S_{t+1} + u_t$$

and the standard errors must again be divided by the coefficient on I_t.

20. This data was obtained from the ESRC data archive at the University of Essex.

21. The survey by Gertler (1988) has been a particularly illuminating source.

22. Specified in a slightly different way (with a continuous range for internal cash flow instead of two discrete values) the signalling equilibrium becomes fully revealing and all bankruptcy costs are avoided through use of equity finance. Furthermore bankruptcy costs need to be justified rather than assumed, and it is difficult to believe that they of sufficient magnitude to induce major distortions to the cost of finance. Finally equity issue may signal that the firm possesses particularly good opportunities for investment, thus lowering the cost of equity relative to other sources of finance.

23. An informal argument for considering only models of debt and retained earnings finance for inventory investment is that issuing new equity to sustain investment in current assets, rather than in new investment projects, is a signal of liquidity difficulties and therefore likely to greatly depress the share price. This in turn is likely to rule out new issue finance for inventory investment.

24. Gale and Hellwig (1985)

25. Srinivasan is unpublished but described in Calomiris and Hubbard (1988) and by Fazzari et al.

26. When $H=-S=-sH^*$, $I=0$. Substitution in (4.1) yields:

$$\phi_I = \frac{1}{1+s}\frac{I^*}{H^*} \approx \frac{1}{1+s}\frac{I}{K}$$

Substitution in (4.4) then yields (4.5).

27. I am especially grateful to Mr Gerald Threadgold of Kleinwort Benson Ltd for advice on appropriate levels for these indicators.

28. The industrial classification available with the data set does not correspond exactly to the 1980 SIC classification of the producer price indices. The producer price indices used were therefore weighted combinations of the published SIC indices.

29. The formula used for this correction was $I' = \dfrac{I}{1 - \pi z}$, where z is inventory turnover (in years) and π is the rate of inflation of inventory prices.

30. For consistency with the US literature this paper uses the terms inventories and inventory investment in place of stocks and stock investment.

31. Blinder (1986) makes this point when reviewing US studies of inventory investment.

32. I am particularly indebted to Mark Robson, of the Inland Revenue and the Financial Markets Group, for advice on this part of the paper.

33. But it takes no account of the uncertainty of changes in the prices of inventories. If companies are risk averse then the expected variance of π reduces the desire to hold inventories.

34. This paper takes no account of informational imperfections which may invalidate the Modigliani-Miller theorem.

35. King and Fullerton (1984) pgs 18-24 offer a fuller discussion.

36. The cost of capital differs from the Jorgenson "user cost of capital" because the latter is gross of depreciation. In the case of inventories depreciation is not relevant and the two concepts are identical.

37. This conclusion appears to be directly opposite to that reached by Sumner (1984) who concludes that the tax system, in the absence of stock relief, "...imposed a substantial penalty on carrying stocks forward to the next accounting year by treating the two components of the real interest rate differently". (Sumner (1984) pg 170). In fact the results of this paper are consistent with Sumner's analysis, the absence of any affect of the tax system on the cost of capital reflecting the present assumption that debt (which attracts tax relief) is the marginal source of finance.

38. The reason that FIFO leads to inflationary inventory gains being included in operating profits is that the cost of sale is based on costs of production or costs of purchase relating to the period when the items concerned enter inventory. Hence any inflationary gain over this period is counted as part of operating profits.

39. Until the 1970's FIFO was the dominant convention in both the US and the UK. Following the high rates of inflation of the middle 1970's the US accounting profession accepted the widespread use of the LIFO conventions, reducing the tax liabilities of US corporations. It is an interesting reflection of contrasting atitudes and institutions in the UK and US that this has not been possible in the UK. In the UK LIFO conventions are not accepted for tax purposes except in the unusual circumstances that they reflect the actual procedure for withdrawing items from inventory.

40. In the previous section it is argued that debt finance is the appropriate source of finance for marginal increases in inventory. If this is correct how can liquidity difficulties arise since the nominal interest costs of debt attract tax relief? The answer here is to distinguish marginal and total finance. It is likely that marginal increases in inventory are fully debt financed whereas only a proportion of total inventory holdings are debt financed.

41. Note that because the stock relief falls in the year subsequent to the current accounting year it has a discounted value slightly less than the stock relief in case 4(b). This is reflected by the replacement of i by i(1-τ_c). When stock relief applies in both years the two additional terms in 4(b) and 4(c) offset each other, only the i(1-τ_c) term remains and we return to case 2.

42. A further example of regime change arises during the transition period. This is when the company opts to claim relief on end year inventory levels up to the amount of deferred clawback. Since this further option was only announced in the 1981 budget it alters the ex-ante incentive to invest in inventories from March 1981 onwards. Where it is exercised for accounts ending from March 1981 onwards, and the relief claimed is less than the amount of deferred clawback then once again there is a very substantial subdsidy to marginal investment in inventories. We assume that where this incentive is recognised the company increases end year inventories upto the point at which the regime change no longer applies. Hence whenever the company opts for this form of assesment the cost of capital is determined by case 3.

43. There is a second kind of tax exhaustion known as advanced corporation tax exhaustion. This arises when a company is unable to offset all of its advanced corporation tax against mainstream corporation tax becasue of the limitation that the amount of ACT set off be no greater than τ_p times MCT. This typically arises for companies incorporated in the UK but with substantial overseas operations. Thus their dividend payments are large compared to their UK taxable income. This does not affect the incentive to invest in inventories because MCT is still payable on marginal increases in income.

44. Note that a discounting of the relief due under the 1981 scheme is required even when there is no tax exhaustion, because the relief is due in the subsequent year. Thus the expression for the cost of capital is that given here with n=1. For simplicity of exposition this discounting is not included in the derivation of the cost of capital for case 3 made above.

45. This procedure is not completely accurate because tax law allows UK companies to re-arrange their tax payments in all previous years so as to obtain the maximum benefit from tax carry back, but such re-arrangement is not reflected in the company accounts on which the IFS model is based. Robson (1985) develops an algorithm to deal with this point, but this approach has not been used here.

46. This can be derived by substitution of the King-Fullerton expression for p_0 into the expression for p_m. All terms other than the tax rate on the MRR cancel out.

47. I am very grateful to Michael Devereaux for providing the output from the Institute for Fiscal Studies model which allowed the calculations of this section to be made. The use made of the IFS model and the resulting calculations are my own, and were not conducted on behalf of the Institute for Fiscal Studies.

48. UK rates of corporation tax are from Board of Inland Revenue (1987).

49. These proportions are those used by King and Fullerton (1984) for the UK.

50. This observation, that the IFS model is not very representative of retailing and distribution, also calls into question whether the measures in tables 2 and 3 are representative aggregate measures of the cost of capital. The inflation component of these measures of the cost of capital is not subject to this criticism, since the inflation measure used for tables 2 and 3 is an aggregate measure weighted using national accounts, not IFS, weights. The more serious difficulty is that these measures do not accurately capture the effects of tax exhaustion and stock relief, because the IFS model does not accurately represent the tax position of individual enterprises in retail and distribution. The resulting errors are of ambigous sign. Stock relief was not available to non-corporate enterprises (so the fall in the aggregate cost of capital during the periods of stock relief may be overstated) and that non-manufacturing companies are much less likely to be tax-exhausted because they can claim fewer capital allowances (this would mean that the fall in the cost of capital under the 1975 stock relief scheme is under-stated).

51. Sumner (1984) discussed this seasonal effect on the cost of capital, but does not offer any quantitative estimate of its impact.

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