On the Theory of Vertical Integration

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Abstract

This thesis explores vertical integration in both competitive and noncompetitive settings. Chapter 2 shows that allocation of ownership matters even in a repeated relationship. The optimal control structure of the static game restricts the gain from deviation to be the lowest but also the punishment will be minimal. The worst ownership structure of the one-shot game is good in the repeated setting because it provides the highest punishment but bad because the gain from deviation is also the highest. We show that two types of equilibria exist: one where partnership and a hostage type solution are optimal and second where the results of the one-shot game apply.

Chapter 3 focuses on vertical oligopolies when both integrated and unintegrated firms coexist. We analyse the integrated firm's strategy in the input market. If the integrated firm is more efficient in transforming the input into final good, it will buy some input to drive up rival's marginal cost. Only if the integrated firm is less efficient will it sell input. If there is no competition in the final good market vertical supply arises because it has no harmful effects on the downstream unit's profits. If competition is very tough overbuying will emerge; by raising rival's costs the integrated firm can achieve a dominant position in a highly competitive market.

Chapter 4 examines integration decisions of successive duopolists. We show that qualitatively the same pattern of integration emerges whether there is Cournot or Bertrand competition in the input market. We find that the degree of integration in the industry is increasing in the size of the downstream market. There is a tendency for partial integration when one upstream firm is relatively efficient compared to its rival.

Chapter 5 takes into account both the firm's internal and external environment. Further, we explicitly model the effect of varying the degree of market competition. We observe a non-monotonic relationship between ownership allocation and competition. We also see greater upstream ownership of assets when the upstream worker is important.

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1 Introduction

This thesis explores vertical integration in both competitive and noncompetitive settings. We apply two approaches to integration. The incomplete contracting approach takes the view that different units of the firm are run by separate managers who are self-interested and cannot be made to act in the best interest of the firm because of incompleteness of contracts.[1] The managers make a firm-specific investment ex ante. Since contracts contingent on investments cannot be written the bargaining over the surplus occurs after the investments are made. The managers foresee that part of the surplus they generate by their investment is expropriated in the bargaining while they pay the full cost of investment. Therefore distortions in investments arise. Ownership matters because it affects the outside options and therefore the outcome of the bargaining game and the incentives to invest.

The second branch of literature assumes that integration leads to profit sharing and removes all the conflicts of interest inside the firm thus giving the main emphasis on the strategic interaction between the firms.[2] The main issue is how integration affects the industry cost structure and the competition in the downstream market. The integration structure is used as a device to change the industry cost structure to the benefit of the firm making the integration decision or of all the industry. Also the possibility of foreclosure of nonintegrated rivals is explored.

[1]Williamson (1975) and (1985), Klein, Crawford and Alchian (1978), Grossman and Hart (1986), Hart and Moore (1990) and Bolton and Whinston (1993).

[2]Vickers (1985), Bonnano and Vickers (1988), Salinger (1988), Hart and Tirole (1990), Ordover, Saloner and Salop (1990) and Gal-Or (1992).

Chapter 2 "Reputation and Allocation of Ownership" adopts the incomplete contracting approach. According to this approach holdup problems arise because of the possibility of opportunistic behaviour. If the agents are in a repeated relationship and care about the future any one-shot gain from opportunistic behaviour should be outweighed by the loss of trust in the future. We examine if there is any scope for allocation of ownership in the dynamic setup. We show that allocation of ownership indeed matters unless agents are very patient. Two types of equilibria can arise: one where partnership and a hostage type solution are optimal and second where the results of the one-shot game apply. The ownership structure is chosen to give the agents best incentives to cooperate. The best ownership structure is such that the gain from deviation is lowest relative to the punishment. The main trade-off is the following: the worst ownership structure of the one-shot game provides maximal punishment but also the gain from deviation will be the highest while the optimal control structure of the static game restricts the gain from deviation to be the smallest but also the punishment will be minimal. The worst ownership structure of the one-shot game is the one that does not give agents any outside options and therefore the noncooperative investments are lowest and the punishment is highest. If an agent cheats in investment the cooperation breaks down immediately; the surplus will be divided noncooperatively. When the agents do not have outside options the bargaining will result in an even split of the surplus; the deviant gets half of the surplus generated by opponent's first-best investment and gains a lot from deviation. While when the agents have an outside option the opponent has a high one since his investment is high and therefore the deviant cannot extract as large share of its value as in the previous case. We show that there are two types of equilibria depending on the parameter values. Partnership and a hostage type solution arise in equilibrium when it is important to maximize punishment. The results of the one-shot game broadly apply when minimizing the gain from deviation is dominant.

Chapters 3, 4 and 5 explore competitive settings. The broad theme in these Chapters is the industry cost structure and the toughness of competition. In Chapter 3 "Vertical Supply, Foreclosure and Overbuying" we examine how these factors affect the integrated firm's strategy in the input market; does it participate in the market as a buyer or seller or does it participate at all. The integrated firm may choose to buy some input in the market although internal supplies would be cheaper. Overbuying drives up the unintegrated rival's marginal costs and the integrated firm is in a better competitive position in the final good market. This comes at a cost; the average (but not the marginal) cost increases. In some other circumstances the integrated firm may sell input to its rival. It understands that this input will ultimately compete with its own final good but the upstream unit makes a profit from input sales. We show that with Cournot competition and homogeneous final goods the integrated firm will overbuy when it is more or equally efficient in transforming the input into the final good than its unintegrated rival. If it is already in a strong position in the downstream market it can afford to raise average costs somewhat in order to gain an even stronger position. Only if the integrated firm is less efficient will it find it optimal to sell input. If the downstream market does not offer much, it may pay to shift attention to input sales and at least make a profit from that. Furthermore, if the integrated firm is very inefficient in the final good production it will drop out from the downstream market and concentrate fully on input production. The second crucial issue for the integrated firm's strategy is the toughness of competition in the downstream market. If there is no competition (the firms operate in different markets) input sales bring revenues to the integrated firm and it has no harmful effects whatsoever on its downstream profit; therefore vertical supply occurs. When competition is tough it becomes important for the integrated firm not to help its rival to compete against itself; vertical supply is not optimal. If the firms are equally efficient the integrated firm can achieve a cost advantage by overbuying input. This gives it a dominant position in a highly competitive market; overbuying emerges in equilibrium.

Whereas Chapter 3 took the industry structure as given, Chapter 4 "Endogenous Industry Structure in Vertical Duopoly" analyses how the industry cost structure and toughness of competition in the input market affect the integration decision and how this can induce further cost changes. We compare two models; one with extreme Bertrand competition in the input market and one with less severe Cournot competition. We show that qualitatively the same pattern of integration emerges in both models. We find that the firms integrate as a result of rising demand. Porter and Livesay (1971) and Chandler (1977) provide empirical support for this prediction. Furthermore, there is a tendency for asymmetric industry structure when one upstream firm is relatively efficient compared to its rival.

In Chapter 5 "Incomplete Contracts, Vertical Integration and Product Market Competition" the industry cost structure arises from a more fundamental source and the strategy in the input market is part of the integration decision. This chapter is a synthesis of the two approaches to integration; we take into account both the firm's internal and external (competitive) environment. Suppose there is an upstream firm, a downstream firm and an integrated firm owned by its downstream manager. Each production unit has a manager who can enhance the value of the firm's product by exerting effort. The integrated firm could use internal input but since that is of low value (the non-owning manager of the upstream unit has low incentives to improve its value) it will choose to buy from the independent supplier. The integrated firm can get a large share of the surplus when negotiating with the supplier since it has an outside option to use internal supplies while the nonintegrated downstream firm is fully dependent on these supplies; therefore the manager of the integrated firm has higher incentives to improve the value of its final good than the manager of the nonintegrated firm. Since there are no capacity constraints an efficient supply arrangement is for the independent upstream firm to supply both downstream units and this is what we would expect in noncompetitive environment. As competition gets tougher in the downstream market the nonintegrated firm suffers since its product is of lower value. It may pay for the nonintegrated downstream firm to buy up the input supplier. Then input will be inferior but the other firm suffers too and the firms are in equal footing. However, if competition gets really tough then the final good market offers little attraction. In which case it may be best to effectively withdraw from the final market and concentrate on input production. This can be achieved by giving the ownership of the upstream unit to its manager. There is therefore a non-monotonic relationship between ownership allocation and competition.

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2 Reputation and Allocation of Ownership

2.1 Introduction

A recent theory of vertical integration relates to situations where contracts are incomplete and parties make specific investments ex ante.[3] Since contracts contingent on investments cannot be written bargaining over the surplus occurs after the investments have been made. The agent foresees that part of her investment can be expropriated in ex post bargaining while she pays the full cost of investment. Therefore distortions in investments typically arise. According to this theory the ownership rights should be allocated to minimize these distortions. When the parties are in a repeated relationship and care about the future, any one-shot gain from opportunistic behaviour should be outweighed by the loss of trust in the future.[4] Can first best be achieved under any ownership structure? Is there any scope for allocation of ownership in the repeated relationship? These are the issues raised in this paper.

We show that allocation of ownership indeed matters even in a repeated relationship (unless agents are very patient). Two types of equilibria exist: one where partnership and a hostage type solution are optimal and second where the results of the one-shot game apply.

The ownership structure is chosen to give the agents best incentives to cooperate. The best ownership structure is such that the gain from deviation is lowest relative to the punishment. One might expect that the optimal ownership structure in

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[3]Williamson (1975) and (1985), Klein, Crawford and Alchian (1978) and more formally Grossman and Hart (1986), Hart and Moore (1990) and Bolton and Whinston (1993).

[4]See Macaulay (1963) for empirical evidence of reputation effects.

the repeated game is the worst structure of the one-shot game (no outside options) because it provides the highest punishment. However, the highest punishment does not imply that cooperation would be most sustainable. It is also true that when the punishment is highest so is the gain from deviation. When an agent deviates in investment the cooperation breaks down immediately: the surplus will be divided noncooperatively. When there are no outside options the bargaining will result in an even split of the surplus; the deviant gets half of the surplus generated by the opponent's first-best investment and gains a lot from deviation. While when the agents have outside options then the deviant cannot extract as much as half of the value of the efficient investment in bargaining. The trade-off present in the repeated game is the following. Ownership structure with no outside options is good because it provides the highest punishment but bad because the gain from deviation is also the highest. The optimal control structure of the static game restricts the gain from deviation to be the lowest but also the punishment will be minimal. We show that two types of equilibria exist: one where ownership is allocated to maximize punishment and another where a control structure that best limits the gain from deviation is chosen.

We show that partnership with a unanimity clause is optimal in a repeated game when the investment costs are very convex. The agents cannot use the assets unless they reach a unanimous agreement; cheating would lead to an outcome with very low surplus in the future and punishment is maximal. Partnership is optimal if compared to the optimal structure of the one shot game the relative increase in the punishment is greater than the relative increase in the gain from deviation. When investment costs are steeply increasing the gain from deviation is high; the first-best investment is expensive and cheating would lead to a big saving in investment costs. In the same time punishment is relatively low; the first-best surplus is not very high when the high investment is expensive. When the gain is high and the punishment is low it is easier to obtain a higher *relative* change in punishment. It is optimal to put all the weight in maximizing punishment although then also the gain from deviation will be the highest; partnership is optimal. Reputation effects thus provide a new explanation for partnerships.[5]

When only one agent has an investment an equally good structure is one where the noninvesting agent owns both assets. This is a hostage type solution to prevent opportunism as discussed in Williamson (1983) and (1985). The investing agent is very vulnerable; she does not have access to her essential asset without the consent of the other agent. Any opportunistic behaviour would lead to a very bad equilibrium. Franchising provides an example of hostages: sometimes the franchisor may require franchisees to rent from them short term the land on which their outlet is located.

When investment costs are almost linear there exists an equilibrium where broadly the results of the one-shot game apply (Hart and Moore (1990)). The optimal control structure gives the agents the highest possible outside options and therefore best restricts the gain from deviation. For not very convex investment costs the gain from deviation is low and the punishment is high and it is easier to induce higher relative change in the gain; thus it is optimal to minimize the gain from deviation. One important determinant for the optimal ownership structure is the degree of complementarity between the assets; when the assets are strictly complementary they should be owned together while if they are economically independent each agent should control her asset. The second result relates to the importance of agent's investment; if only one agent has an investment she should own both assets. Lastly, if an agent is very important as a trading partner so that without his contribution this asset

[5]Radner (1986) shows that partnership can be efficient in a repeated came but he does not provide an explanation for why partnership would be better than some other organizational form.

does not improve the other agent's incentives, then he should own the asset. Interestingly the predictions of the one-shot and the repeated game do not fully coincide in this parameter range: integration is less likely in the repeated game. Klein (1980) and Coase (1988) suggest that reputation and integration are substitutes in dealing with the problem of opportunism. They refer to models where the benefit of integration is reduced holdups and the costs of integration are something else (for example arising from bureaucracy). Clearly then reputation concerns make integration less likely; the benefits are lower and the costs have not changed. In our model both the benefits and costs of integration change and it is not a priori clear which way the reputation effect goes. It turns out that integration is less likely.

Garvey (1991) also analyses the effect of reputation on the optimal allocation of ownership rights in a two-agent two-asset setup. He finds that the basic result of Grossman and Hart (1986) holds in the repeated setting: an agent with a much more important investment should own both assets. Garvey takes ownership as a continuous variable and assumes that the asset returns accrue to the owner whereas Grossman and Hart assume that ownership increases a manager's bargaining power only by raising his outside option.[6] Thus his model is *not* in fact a repeated version of Grossman-Hart. Furthermore, Garvey restricts the division of surplus to be the same under cooperation and noncooperation whereas we take into account that other sharing rules than the outcome of the one-shot bargaining game may be supported under cooperation. In addition we examine the role of outside options.

Friedman and Thisse (1991) have a related paper to ours. In their model the firms choose noncooperatively the location in the Hotelling line and then collude in

[6]Holmstrom and Milgrom (1991) on the other hand make the same assumption about the ownership of asset returns as Garvey.

pricing in the repeated game. They find that the firms will locate in the middle of the Hotelling line to make deviations very costly. Another paper that obtains an inefficient structure from a static point of view as an equilibrium in a dynamic context is Martimort (1993). He shows that multiprincipals charter acts as a commitment device against principal's incentives to renegotiate long term agreements. Both these papers obtain the reverse structure as the only equilibrium. In our model also the outcome of the static game can be an equilibrium in the dynamic game within some parameter range.

The rest of the paper is organized as follows. Section 2.2 introduces our main model where only one agent has an investment. Section 2.3 briefly discusses the results of the one-shot game. The repeated game is analysed in Section 2.4. In Section 2.5 we extend the model to include investment by both agents.

2.2 The Model

Our stage game is a simplified version of Hart and Moore (1990). We analyse a setup where worker 1 uses asset a_1 to supply worker 2 who in turn uses asset a_2 to supply consumers. Ex ante worker 1 makes an investment in human capital which is specific to asset a_1. The investment is denoted by I. The investment makes the worker more productive in using the asset. The worker for example learns to know better the properties of the asset or the environment the firm operates and can therefore generate more surplus. The investment can be either cost reducing or value enhancing. The investment generates a gross surplus equal to $v(I)$. The cost of the investment to worker 1 is $c(I)$. We make the following assumptions about the value and cost of investment:

Assumption 2.1. *$v(0) \geq 0$, $v'(I) > 0$ and $v''(I) < 0$.*

Assumption 2.2. *$c(0) = 0$, $c'(I) > 0$ and $c''(I) > 0$.*

For simplicity we assume that agent 2 does not have an investment. His contribution to the joint surplus is a fixed value, V. Accordingly the joint surplus is equal to:

$$V + v(I) - c(I). \tag{2.1}$$

Investment in human capital is assumed to be too complex to be described adequately in a contract. It is observable to both agents but not verifiable to third parties like the court. Therefore agent 1 chooses the investment noncooperatively. We also assume that it is very difficult to describe the required input characteristics or worker's duties ex ante. As a result the input trade and nonowning worker's wage is also ex ante noncontractible. We also rule out profit-sharing agreements.[7] Ex ante contracts can only be written on the allocation of ownership. The possible ownership structures are *nonintegration* (each asset is owned by its worker), *integration* by agent i (agent i owns both assets), *joint ownership* (the agents jointly own both assets) and *cross ownership* (agent 1 owns asset a_2 and 2 owns a_1).

The assets do not necessarily fully rely on each other but there can be other suppliers/customers available. When each agent owns her asset, worker 1 can produce the input and sell it to an outsider and worker 2 can buy input from an outsider and produce final good from it. The value of this trade to agents 1 and 2 is assumed to be $\mu v(I)$ and μV respectively. The value of μ depends on the relationship between the assets. When the assets are strictly complementary (there are no alternative

[7]See Hart and Moore (1990) for the justification of these assumptions.

suppliers/customers), then $\mu = 0$. The assets are economically independent when agent i can realize the full value without agent j and asset a_j. In this case $\mu = 1$. When the assets are economically independent 1's input is in no way specific for 2 who can obtain equally good input from alternative suppliers. Also 1 has alternative customers who value the input as much as agent 2.

If an agent owns both assets she can work alone with them and sell the final good to the customers. If agent 1 is the owner the value of the trade without agent 2's contribution is $\lambda_2[v(I) + A_2]$ and if 2 is the owner the value is $\lambda_1(V + A_1)$. A_i is related to the value of asset a_i without its worker. The value of λ_i depends on the importance of agent i as a trading partner. If agent i is indispensable to asset a_i so that giving the control of a_i to agent j (who already owns a_j) does not enhance the surplus he can generate on his own by asset a_j, then $\lambda_i = \mu$. If agent i is dispensable so that agent j could replace her by an outsider without loss of value, then $\lambda_i = 1$. μ is the lowerbound for λ_i; an agent cannot do worse when she owns both assets than when she owns only one.

When an agent does not control any asset on her own (nonowning worker of an integrated firm or a partner in joint ownership) she has an outside option to work for another firm. We assume that asset a_1 is essential to worker 1 (or 1's investment is fully specific to a_1) so that the outside wage does not depend on her investment. Without loss of generality we normalize this fixed wage to zero.

Under cross ownership agent i can use asset a_j for outside trade which has value μA_j. This value does not depend on 1's investment because she does not have access to her essential asset.

We summarize the agents' outside options in Assumption 2.3. We denote by $v(i,A)$ the value agent i can generate on her own when she controls a set A of assets.

Assumption 2.3. $v(i,\{\phi\}) = 0,\ \ v(1,\{a_1\}) = \mu v(I),\ \ v(2,\{a_2\}) = \mu V,\ \ v(i,\{a_j\}) = \mu A_j,$ $v(1,\{a_1,a_2\}) = \lambda_2[v(I) + A_2]$ *and* $v(2,\{a_1,a_2\}) = \lambda_1(V + A_1)$ *for* $i,j = 1,2$ *and* $i \neq j$.

Assumption 2.4. $0 \leq \mu \leq \lambda_i \leq 1$ *for* $i = 1,2,\ \ 0 \leq A_1 \leq v(0)$ and $0 \leq A_2 \leq V$.

Assumption 2.4 says that the marginal value of investment is increasing in the number of agents and assets. The assumption furthermore ensures superadditivity: under any ownership structure the joint surplus is at least as great as the sum of the agents' outside options.

Ex post the uncertainty is resolved and the agents negotiate a spot contract on the input trade or the services of non-owning workers. The investment is observable to both agents at the time of bargaining. The incentive of the bargaining parties to reach agreement is driven by the risk of breakdown of negotiation. This will result in the "split-the-difference" rule where each agent gets half of the gains from trade.[8] Finally, production occurs and the final good is sold to the customers. This completes the description of the stage game.

In our dynamic model the stage game described above is always repeated one more period with high probability. At date 0 the agents write a contract on the allocation of ownership to maximize the joint surplus. The contract can give the ownership of an asset to the same agent(s) for all the game or induce changes in

[8]Binmore, Rubinstein and Wolinsky (1986) and Sutton (1986).

ownership. Given our assumptions about contractibility the only event this contract can be contingent on is time. Skills depreciate and the environment changes and further investments can be made in the beginning of each period. We make the extreme assumption that the investment depreciates fully before the next period begins. In the second half of the period the gains from trade are realized and the spot contract on the division of surplus is written. The time line of the game is:

```
0                              1                              2
|------------------------------|------------------------------|------. . .-->
contract on    investment      spot trade     investment      spot trade
ownership
```

2.3 One-Shot Game

In this section we briefly examine the static game. Equation (2.2) gives the joint surplus maximizing investment, I^*:

$$v'(I^*) - c'(I^*) = 0 \qquad (2.2)$$

Since ex ante contracts on input trade or wage cannot be written, the bargaining takes place after the investment is made. Agent 1 foresees that part of the surplus she generates by her investment is expropriated in ex post bargaining while she pays the full cost of investment. Therefore underinvestment (holdup) typically arises. Ownership is allocated to induce the highest investment. Below we give the outcome of the bargaining game and the incentives to invest under each ownership structure.

Under *nonintegration* (NI) the owners negotiate a two-part tariff on the input trade. The unit price is equal to marginal cost and the bargaining is over the fixed fee. The bargaining will result in the following division of surplus:

$$P_1^{NI} = \frac{1}{2}[(1+\mu)v(I) + (1-\mu)V] - c(I) \tag{2.3}$$

$$P_2^{NI} = \frac{1}{2}[(1-\mu)v(I) + (1+\mu)V] \tag{2.4}$$

Accordingly the incentive for investing is:

$$\frac{1}{2}(1+\mu)v'(I) - c'(I) = 0 \tag{2.5}$$

It is easy to see from equation (2.5) that the investment is the greater the less complementary the assets are (the higher is μ). When the assets are economically independent ($\mu = 1$) agent 1 has first-best incentives.

Under *integration by agent 1* (1I) the owner of both assets can unilaterally decide to transfer input at marginal cost but she has to bargain with agent 2 for his services. The payoffs for the agents are:

$$P_1^{1I} = \frac{1}{2}[(1+\lambda_2)v(I) + V + \lambda_2 A_2] - c(I) \tag{2.6}$$

$$P_2^{1I} = \frac{1}{2}[(1-\lambda_2)v(I) + V - \lambda_2 A_2] \tag{2.7}$$

The investment is given by:

$$\frac{1}{2}(1+\lambda_2)v'(I) - c'(I) = 0 \tag{2.8}$$

The investment is the greater the more dispensable the worker is (the higher is λ_2). In the limit when the worker is fully dispensable ($\lambda_2 = 1$), then the owner has first-best incentives. When the worker is indispensable ($\lambda_2 = \mu$), then the investment is equal under nonintegration and agent 1 control. Assumption 2.4 ensures that agent 1's investment is at least as great when she owns both assets than when she owns only one ($\lambda_2 \geq \mu$).

Under *integration by agent 2* (2I), *cross ownership* (CO) and *joint ownership* (JO) agent 1 can realize the value of her investment only by reaching an agreement with agent 2; her investment has no value if she does not have access to her essential asset. Under integration by 2 and cross ownership agent 2 owns her asset and under joint ownership the agents have to reach a unanimous agreement to use the assets. Therefore agent 1 receives half of the value of her investment at the margin and the investment is given by:

(2.9)
$$\frac{1}{2}v'(I) - c'(I) = 0$$

Since any fixed outside options do not affect the incentives the size of the surplus is equal in these three structures - the division of surplus differs in general.

In this setup the ownership decision is very simple. We should allocate ownership to give agent 1 the highest incentives, that is to give her the highest outside option related to investment. Since by assumption $\lambda_2 \geq \mu$ concentrating ownership of both assets in 1's hands gives her the best incentives and generates the highest surplus (see first order conditions (2.5), (2.8) and (2.9)). If assets are economically independent ($\mu = 1$) or agent 2 is indispensable ($\lambda_2 = \mu$), then nonintegration and integration by agent 1 are equally good. Furthermore, joint ownership, cross ownership and integration by agent 2 are strictly dominated for any μ and $\lambda_2 > 0$.[9]

2.4 Repeated Game

When the agents are in a repeated relationship and care about the future, the holdup problems described in the previous section should not be so severe. In this section we analyse when the efficient investment can be supported using the trigger strategy and Nash punishments. Obviously if the agents are very patient (discount factor is close to one) first best can be supported under any ownership structure. We are interested in situations when the agents are not completely patient and our aim is to find an ownership structure that guarantees first best for the greatest range of discount factors.

Agent 1 implicitly agrees to make the efficient investment and both agents implicitly agree to share the surplus according to (P_1^*, P_2^*). (The sharing rule will be determined later.) Deviation from either investment or sharing rule will trigger noncooperative behaviour from the opponent for the rest of the game. In particular, if agent 1 cheats in investment the cooperation breaks down already in the second half of the day: the surplus will be divided noncooperatively. Also if there is no deviation in investment but an agent does not agree to follow the sharing rule (P_1^*, P_2^*), then noncooperative bargaining will take place. The trigger strategy for agent 1 is:

[9]When $\lambda_2 = \mu = 0$ all ownership structures are equally good.

in period 1 choose I^* and follow (P_1^*,P_2^*)

if (P_1^*,P_2^*) in 1,2,...,t-1, then choose I^* and follow (P_1^*,P_2^*) in t

if not (P_1^*,P_2^*) in t-1, then choose I^N and apply (P_1^N,P_2^N) in t,t+1,... where superscript N refers to the Nash equilibrium of the static game

if not (P_1^*,P_2^*) in t, then apply (P_1^N,P_2^N) in t,t+1,... and choose I^N in t+1,t+2,...

and for agent 2:

if $I = I^*$ in 1,2,...,t and (P_1^*,P_2^*) in 1,2,...,t-1, then follow (P_1^*,P_2^*) in t

if either $I \neq I^*$ in t or not (P_1^*,P_2^*) in t-1 or t, then apply (P_1^N,P_2^N) in t,t+1,...

Note that the only relevant information about the previous period when a new period begins is whether there was or was not deviation. Whether the deviation was in investment or sharing rule does not matter. This also means that the extensive form and the outcome of the bargaining game for the static model (as proposed in Sutton (1986)) is appropriate also here for the punishment phase. Whether the agents reach an agreement or fail to do so and have to take the outside option this period does not change the rest of the game. The next period starts from the same node.

It is easy to see that cheating in investment dominates cheating in sharing rule for agent 1. When 1 deviates in investment, she chooses her investment taking into account that the surplus will be divided noncooperatively. (The deviation investment is thus equal to the investment in the one-shot game.) By definition this is more than making the first-best investment and then switching to noncooperative bargaining. Only when agent 1 does not have an incentive to cheat in investment (she has first-best incentives even in the one-shot game) might she choose to deviate in sharing rule. Obviously agent 2 can cheat only in sharing rule since he does not have an investment.

First best will be supported in equilibrium if and only if the discounted payoff stream from cooperation exceeds the payoff stream from the deviation path for both agents.

$$(2.10) \qquad (1 + \delta + \delta^2 + ...)[T - c(I^*)] \geq P_1^d + (\delta + \delta^2 + ...)P_1^p$$

$$(2.11) \qquad (1 + \delta + \delta^2 + ...)[V + v(I^*) - T] \geq P_2^d + (\delta + \delta^2 + ...)P_2^p$$

where δ is the discount factor, T is the transfer agent 1 receives from 2 under cooperation, P_i^d is i's one-shot deviation payoff and P_i^p is i's payoff in the punishment path. If agent 1 deviates in investment, agent 2 observes it already in the same period and he will not pay T to agent 1. Agent 1 saves in investment costs but receives now a share of the surplus that is determined by noncooperative bargaining. Since agent 2 can punish only by sharing rule and the punishment starts in the same period $P_1^d = P_1^p$ and there is in fact no trade-off from gain today versus punishment tomorrow for agent 1. Equation (2.10) simplifies to:

$$(2.12) \qquad T - c(I^*) \geq P_1^p$$

Equation (2.12) is the incentive compatibility constraint for agent 1 and it does not depend on the discount factor - not because future would not matter but because it affects both sides of (2.12) equally. For example under nonintegration agent 1 chooses efficient investment if and only if:

$$(2.13) \qquad T \geq \frac{1}{2}(1+\mu)v(I^{NI}) + \frac{1}{2}(1-\mu)V - c(I^{NI}) + c(I^*)$$

where I^{NI} is the Nash investment under nonintegration.

Agent 2's incentive constraint (2.11) depends on the discount factor. The higher share of the surplus goes to agent 1 under cooperation, the less likely it is that 2 will cooperate. Therefore the best we can do is to choose T^* such that (2.12) is just satisfied. This proves that:

Proposition 2.1. *When only agent 1 has an investment the optimal sharing rule is:*

$$T^* = P_1^p + c(I^*)$$
$$P_1^* = P_1^p$$
$$P_2^* = V + v(I^*) - c(I^*) - P_1^p.$$

Note that this arrangement gives agent 1 the same surplus as in the one-shot game and the non-investing agent 2 gets all the benefits from 1's higher investment.

If agent 2 chooses to cheat in sharing rule, he does not have to pay T^* to agent 1 but the transfer is determined in noncooperative bargaining. His gain from deviation under nonintegration is:

$$\begin{aligned} G^{NI} &= [\tfrac{1}{2}(1+\mu)V + \tfrac{1}{2}(1-\mu)v(I^*)] - [V + v(I^*) - T^*] \\ &= [\tfrac{1}{2}(1+\mu)v(I^{NI}) - c(I^{NI})] - [\tfrac{1}{2}(1+\mu)v(I^*) - c(I^*)] \end{aligned} \tag{2.14}$$

where $G \equiv P_2^d - P_2^*$. This expression is strictly positive for any $I^{NI} < I^*$ since I^{NI} is chosen to maximize the first term is square brackets. The same is true for any ownership structure and therefore agent 2 can gain by cheating; he can extract more of the value of 1's efficient investment in bargaining than by paying T^*. If agent 1 has first-best incentives in the one-shot game ($I^{NI} = I^*$) there is no reason to deviate for agent 2 either since then he cannot extract any value of 1's investment in bargaining (see equation (2.4)).

If agent 2 cheats in sharing rule he gains in this period but from the next period on the payoff will be lower because agent 1 chooses Nash investment. Under nonintegration the loss from deviation is equal to:

$$(2.15) \qquad L^{NI} = [V + v(I^*) - T^*] - [\tfrac{1}{2}(1+\mu)V + \tfrac{1}{2}(1-\mu)v(I^{NI})]$$
$$= [v(I^*) - c(I^*)] - [v(I^{NI}) - c(I^{NI})]$$

where $L \equiv P_2^* - P_2^p$. Loss is strictly positive for any $I^{NI} < I^*$ since I^* maximizes the first term in square brackets. L shows how much lower the joint surplus will be in the punishment path. If agent 2 is patient enough the one-shot gain from cheating is outweighed by lower payoff in the future. Agent 2's incentive constraint (2.11) simplifies to:

$$(2.16) \qquad \delta \geq G/(G + L).$$

The main focus of this paper is on equation (2.16). The gain and loss from deviation will differ in general for different ownership structures. Define $\underline{\delta} \equiv G/(G + L)$. In what follows we concentrate on finding the control structure that guarantees first best for the greatest range of discount factors, that is gives the lowest $\underline{\delta}$. The best ownership structure is such that the gain from deviation is lowest relative to the loss. Now it becomes clear that the optimal allocation gives the ownership to the same agent(s) for all the game. For example giving ownership of the assets to agent 1 for the first t periods and then making agent 2 the owner for the rest of the game does not improve the incentives to cooperate in any way (it may do no harm either if $\underline{\delta}$ is equal under both control structures).

It is quite obvious that if agent 1 has first-best incentives even in the one-shot game ($\delta = 0$) under some ownership structure this must be the optimal structure also for the repeated game. This gives our first results on the optimal control structure:

Proposition 2.2.

(i) If assets are economically independent ($\mu = 1$), then nonintegration is (weakly) optimal.

(ii) If the non-investing agent 2 is dispensable ($\lambda_2 = 1$), then integration by agent 1 is (weakly) optimal.

In these cases agent 2 does not have any holdup power and cannot get any share of the value of 1's investment in bargaining; therefore agent 1 has always first-best incentives (see equations (2.5) and (2.8)).

Next we turn to analyse the optimal ownership structure when there is underinvestment problem in the one-shot game, that is μ and $\lambda_2 < 1$. We know that G and L are strictly positive in this case. Then it is appropriate to determine the optimal control structure by minimizing the right-hand-side of (2.16).[10] Furthermore $0 < \underline{\delta} < 1$; if agent 2 is very patient first best can be supported under any ownership structure and if 2 is very impatient underinvestment will occur.

It turns out that as in the one-shot game any fixed values do not affect the incentives. (V cancels out in equations (2.14) and (2.15).) Only outside options related to investments and consequently the level of Nash investment are important. Therefore

---

[10]When $\mu = 1$ then both the numerator and denominator of equation (2.16) are equal to zero under nonintegration and the same is true under integration by agent 1 when $\lambda_2 = 1$.

we can obtain the gain and loss from deviation for cross ownership, joint ownership and agent 2 control from equations (2.14) and (2.15) by setting μ equal to zero and changing the Nash investment level to be appropriate. This proves that:

Lemma 2.1. $\underline{\delta}^{CO} = \underline{\delta}^{JO} = \underline{\delta}^{2I}$.

Not only are cross ownership, joint ownership and agent 2 control equivalent but from the point of view of the static game these are the structures one would not expect to be useful. The common element in these structures is that they do not give control rights to the investing agent 1.

Since only the level of Nash investment affects $\underline{\delta}$ it is clear that:

Lemma 2.2.
(i) $\underline{\delta}^{1I}(\lambda_2) = \underline{\delta}^{NI}(\mu)$ *if* $\lambda_2 = \mu$.
(ii) $\underline{\delta}^{1I}(0) = \underline{\delta}^{NI}(0) = \underline{\delta}^{CO} = \underline{\delta}^{JO} = \underline{\delta}^{2I}$.

When agent 2 is indispensable ($\lambda_2 = \mu$) nonintegration and agent 1 control are equivalent since owning both assets rather than only a_1 does not improve 1's incentives to invest in the punishment path. When the assets are strictly complementary ($\mu = 0$) and agent 2 is indispensable ($\lambda_2 = 0$) neither nonintegration or agent 1 control provides any outside option to agent 1. Agent 2 has the maximal holdup power: agent 1 cannot do anything without agent 2 or asset a_2. Then all the ownership structures are equivalent.

Therefore we are left with the question: is $\underline{\delta}$ minimized by removing agent 1's outside option (joint or cross ownership or agent 2 control) or by giving her an outside option (nonintegration or integration by agent 1)? We can derive the optimal control structure by examining how the lowerbound for the discount factor under nonintegration, $\underline{\delta}^{NI}(\mu)$, and under integration, $\underline{\delta}^{1I}(\lambda_2)$, move with μ and λ_2. Since Lemma 2.2 shows that $\underline{\delta}^{NI}(\mu) = \underline{\delta}^{1I}(\lambda_2)$ when $\mu = \lambda_2$, it is sufficient to concentrate on $\underline{\delta}^{NI}(\mu)$ only. Examining how μ affects $\underline{\delta}^{NI}$ is like comparing different ownership structures.

We start by analysing the gain and loss from deviation.

Proposition 2.3. *Both the gain and loss from deviation are decreasing in μ under nonintegration.*

Proof:

Equation (2.14) gives the gain from deviation under nonintegration. Total differentiation gives:

$$dG^{NI}/d\mu = [\tfrac{1}{2}(1+\mu)v'(I^{NI}) - c'(I^{NI})]\,\partial I^{NI}/\partial\mu + \tfrac{1}{2}[v(I^{NI}) - v(I^{*})] = \tfrac{1}{2}[v(I^{NI}) - v(I^{*})] < 0 \tag{2.17}$$

The investment effect is negligible and therefore we can ignore the first term in (2.17). Accordingly, the gain is decreasing in μ. Equation (2.15) gives the loss from deviation under nonintegration. By total differentiation we obtain:

$$dL^{NI}/d\mu = -\,[v'(I^{NI}) - c'(I^{NI})]\,\partial I^{NI}/\partial\mu = -\,\tfrac{1}{2}(1-\mu)v'(I^{NI})\partial I^{NI}/\partial\mu < 0 \tag{2.18}$$

The first order condition (2.5) helps us to determine the sign of this expression and to simplify it. It is easy to see from (2.5) that $\partial I^{NI}/\partial\mu$ is positive. Therefore (2.18) is unambiguously negative.

Q.E.D.

High loss and low gain from deviation would guarantee good incentives to cooperate. Proposition 2.3 tells that removing agent 1's outside option ($\mu = 0$) provides the highest loss. In the punishment path agent 1 receives only half of the value of her investment at the margin and therefore the Nash investment and joint surplus is the lowest possible. Nonintegration with $\mu = 0$ is like joint ownership, cross ownership and agent 2 control which are the worst structures in the one-shot game. In the repeated game these structures have the advantage that they provide the highest punishment.

However, the highest punishment does not imply that cooperation would be most sustainable.[11] Proposition 2.3 shows that when the punishment is highest so is the gain from deviation. When agent 2 deviates in sharing rule the spot contract will be written with the split-the-difference rule. When there are no outside options the agents simply split the gross surplus 50:50; the deviant gets half of the surplus generated by agent 1's first-best investment and therefore gains a lot from deviation. While when agent 1 has an outside option ($\mu > 0$) agent 2 can extract less than half of the value of the efficient investment and therefore gains less from deviation. (Note that the size of the surplus does not change for small increase in μ; the investment effect is negligible.) The optimal ownership structure of the one-shot game gives the highest possible outside option and consequently the highest share of the surplus under noncooperative bargaining to the investing agent 1 and therefore best restricts the gain from deviation for agent 2. In the same time punishment will be minimal because 1's incentive to invest in the punishment path is maximized. While the worst structure of the static game is good in the repeated game because it provides the highest punishment but bad because the gain from deviation is also the highest.

[11]Note that by cooperation we refer to first-best investment and sharing rule. Of course even under noncooperation the agents get together and make the deal but the investment is lower and the division of surplus is different.

Proposition 2.3 tells that the gain and loss from deviation move to the same direction as we change μ and it is not immediately clear what is the effect on $\underline{\delta}^{NI}$. The change in $\underline{\delta}^{NI}$ is given by:

$$\frac{\partial\underline{\delta}^{NI}}{\partial\mu} \stackrel{s}{=} \frac{(1-\mu)v'(I^{NI})\partial I^{NI}/\partial\mu}{[v(I^*) - c(I^*)] - [v(I^{NI}) - c(I^{NI})]} - \frac{[v(I^*) - v(I^{NI})]}{[(1+\mu)v(I^{NI})/2 - c(I^{NI})] - [(1+\mu)v(I^*)/2 - c(I^*)]} \tag{2.19}$$

where $\stackrel{s}{=}$ denotes that the expressions have the same sign. The sign of (2.19) depends on the difference between the *relative* changes in the gain and loss. If the relative decrease in the gain from deviation is higher than the relative decrease in the punishment then $\underline{\delta}^{NI}$ is decreasing in μ. Analysing such a difference is very subtle. Therefore we introduce explicit functional forms for the value and cost of investment. First in Subsection 2.4.1 we examine a simple example of discrete investments. Then in Subsection 2.4.2 continuous investments with constant cost elasticity are analysed. Subsection 2.4.3 examines several other functional forms.

2.4.1 Discrete Investment

In this Subsection we examine discrete investments. We assume that:

Assumption 2.1'. *I can take three values: 0, v, and 2v.*

Assumption 2.2'. *c(0) = 0, c(v) = c, and c(2v) =* γc *where* $2 < \gamma < 3$.

Assumption 2.2' ensures that $2v$ is the efficient level of investment. We further assume that $v = 2c - \varepsilon$ where ε is very small. This assumption guarantees that zero investment will be chosen in the punishment path when agent 1 does not have an outside option related to investment. While for even a small outside option the medium investment, v, is chosen. This assumption allows us to concentrate on the most interesting part of the parameter space.

Under *joint ownership* zero investment will be chosen in the punishment path. This is ensured by:

(2.20) $$\frac{1}{2}v - c < 0 \quad \text{and}$$

(2.21) $$v - \gamma c < 0$$

Using equations (2.14) and (2.15) the gain and loss from deviation are:

(2.22) $$G^{JO} = \gamma c - v$$

(2.23) $$L^{JO} = 2v - \gamma c$$

Therefore the lowerbound for the discount factor is:

(2.24) $$\underline{\delta}^{JO} = \frac{\gamma c - v}{v}$$

From our earlier analysis we know that cross ownership and agent 2 control are equivalent to joint ownership.

Under *nonintegration* agent 1 will choose v in the punishment path for $\mu \in (0,\overline{\mu})$ where $\overline{\mu} = [2(\gamma\text{-}1)c - v]/v < 1$. Then it is true that:

(2.25) $$\frac{1}{2}(1+\mu)v - c > 0 \quad \text{and}$$

(2.26) $$\frac{1}{2}(1+\mu)v - c > (1+\mu)v - \gamma c$$

The gain and loss from deviation are:

(2.27) $$G^{NI} = (\gamma\text{-}1)c - \frac{1}{2}(1+\mu)v$$

(2.28) $$L^{NI} = v - (\gamma\text{-}1)c$$

And the lowerbound for the discount factor is:

(2.29) $$\underline{\delta}^{NI} = \frac{2(\gamma\text{-}1)c - (1+\mu)v}{(1\text{-}\mu)v} \quad \text{for } \mu \; \varepsilon \; (0,\bar{\mu})$$

Integration by agent 1 is equivalent to nonintegration when $\mu = \lambda_2$. Accordingly:

(2.30) $$\underline{\delta}^{II} = \frac{2(\gamma\text{-}1)c - (1+\lambda_2)v}{(1\text{-}\lambda_2)v} \quad \text{for } \lambda_2 \; \varepsilon \; (0,\bar{\lambda}_2)$$

where $\bar{\lambda}_2 = \bar{\mu}$. In what follows we analyse the case when $\mu \; \varepsilon \; (0,\bar{\mu})$ and $\lambda_2 \; \varepsilon \; (0,\bar{\lambda}_2)$.[12]

It is easy to show that integration by agent 1 (weakly) dominates nonintegration ($\underline{\delta}^{II} \leq \underline{\delta}^{NI}$). The punishment is equal under both structures (Nash investment is equal to v) and the gain is lower under integration since then agent 1's outside option is higher and agent 2 can extract a smaller share of the surplus in noncooperative bargaining.

[12]Since we already know that when $\mu = \lambda_2 = 0$ all ownership structures are equivalent and when either $\mu \geq \bar{\mu}$ or $\lambda_2 \geq \bar{\lambda}_2$ first-best can be obtained even for $\delta = 0$.

Because nonintegration is dominated we can determine the optimal ownership structure by comparing joint ownership and agent 1 control. From (2.24) and (2.30) it follows that:

$$\underline{\delta}^{JO} < \underline{\delta}^{II} \iff \gamma > \frac{2(\lambda_2 v + c)}{c(1+\lambda_2)} \tag{2.31}$$

Therefore for large values of γ joint ownership (and cross ownership and agent 2 control) are optimal and for small values of γ agent 1 control is optimal.[13]

How does γ affect the incentives to cooperate? It is easy to see from equations (2.22) and (2.27) that the gain from deviation is increasing in γ. When the first-best investment becomes more expensive (γ increases) agent 2 has to pay a higher transfer to agent 1 to implement efficient investment. Since the value of the investment has not changed 2's payoff is now lower under cooperation. On the other hand 2's deviation payoff is unchanged because it is not related to the investment costs. Therefore the gain from deviation is higher. Equations (2.23) and (2.28) show that the loss from deviation is decreasing in γ. The drop in surplus after deviation is smaller when the first-best investment is expensive and thus the first-best surplus is not very high.

The equivalent of equation (2.19) in this discrete case is:

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[13]The critical value for γ determined by equation (2.31) has to lie between 2 and 3 for both regions to exist. Denote this critical value by $\tilde{\gamma}$.

(i) $\tilde{\gamma} > 2 \iff v > c$

(ii) $\tilde{\gamma} < 3 \iff 2\lambda_2 v < c(1+3\lambda_2)$

Insert $v = 2c$ in *(ii)* and we have $\lambda_2 < 1$. Clearly this is true.

$$\underline{\delta}^{JO} < \underline{\delta}^{1I} \iff (G^{JO}-G^{1I})/G^{1I} < (L^{JO}-L^{1I})/L^{1I} \tag{2.32}$$

Accordingly joint ownership is optimal if moving from agent 1 control to joint ownership increases the punishment relatively more than the gain from deviation. When γ is high the gain is high and the punishment is low. Therefore it is easier to obtain a higher *relative* change in punishment. (The absolute changes do not in fact depend on γ.) Then joint ownership which maximizes punishment is optimal. When γ is low the opposite is true: the gain is low and the punishment is high. Then the optimal thing is to put all the weight in minimizing the gain since higher relative changes are easier to obtain there.

In the next subsection this result is confirmed for continuous investments.

2.4.2 Continuous Investment: Constant Elasticity

We assume that the value and the cost of investment are:

Assumption 2.1". *$v(I) = I$.*

Assumption 2.2". *$c(I) = I^{\gamma}$ where $\gamma > 1$.*

Lemma 2.3 determines the sign for equation (2.19).

Lemma 2.3. $\underline{\delta}^{NI}$ *is* $\left\{\begin{array}{c} decreasing\ in \\ independent\ of \\ increasing\ in \end{array}\right\}$ μ *if and only if* $\gamma \left\{\begin{array}{c} < \\ = \\ > \end{array}\right\} 2.$

Proof: In Appendix 2.

Lemmas 2.1 to 2.3 help us to construct Figure 2.1. We also include the implication of Proposition 2.2: $\underline{\delta}^{NI}(1) = 0$. The Figure compares the lowerbounds for the discount factor under different ownership structures for various values of the outside option parameters μ and λ_2. This Figure proves to be very useful in examining the optimal ownership structure.

When costs are very elastic the punishment effect dominates[14]; a small increase in μ will lower the punishment more than the gain and cooperation becomes more difficult ($\underline{\delta}^{NI}$ increases). While when costs are quite inelastic the gain effect is more important; a higher μ will lower the gain more than the punishment and cooperation is easier ($\underline{\delta}^{NI}$ decreases). This is in line with the results of the previous subsection. For high values of γ moving from joint ownership ($\mu = 0$) to agent 1 control ($\mu > 0$) weakens the incentives to cooperate while for low values of γ the incentives are improved by this change in the ownership structure.

The result is the same for both discrete and continuous investment but the effects behind the continuous case are more complex. In the discrete case the levels of the first-best and Nash investment are fixed and only the cost of the first-best investment changes in γ. For high values of γ the gain from deviation is high and the punishment is low. Since the absolute changes do not depend on γ moving from joint ownership to agent 1 control will result in a higher relative decrease in the punishment than in the gain; $\underline{\delta}$ increases. In the continuous case the levels of investment are not fixed but adjust to changes in the cost elasticity. When the costs become more elastic the gap between the efficient and the noncooperative investment becomes smaller.

[14]The important parameter is the elasticity of investment costs *relative to* the elasticity of the value of investment. We have assumed, without loss of generality, that the value is unit elastic and therefore our condition depends only on the cost elasticity, γ.

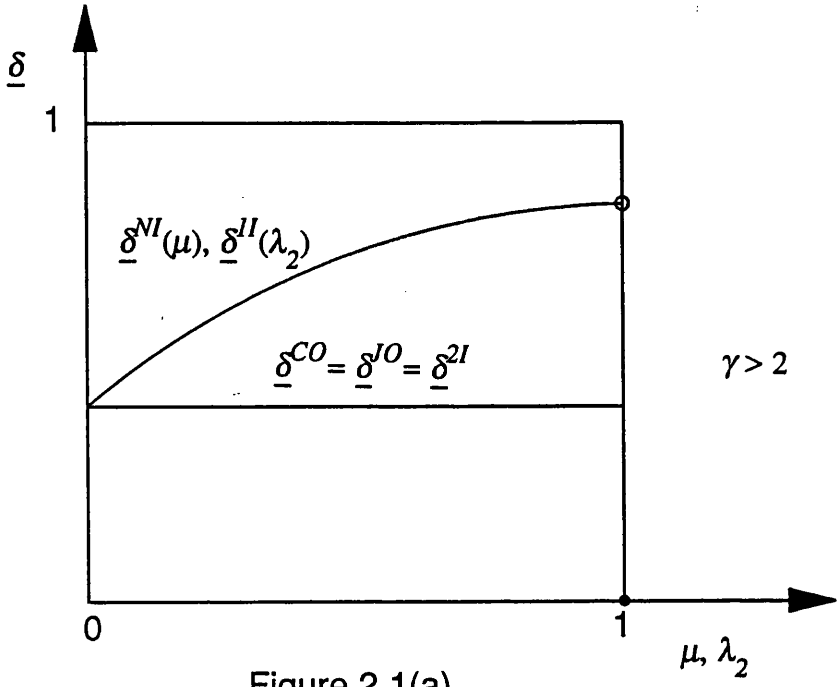

Figure 2.1(a)

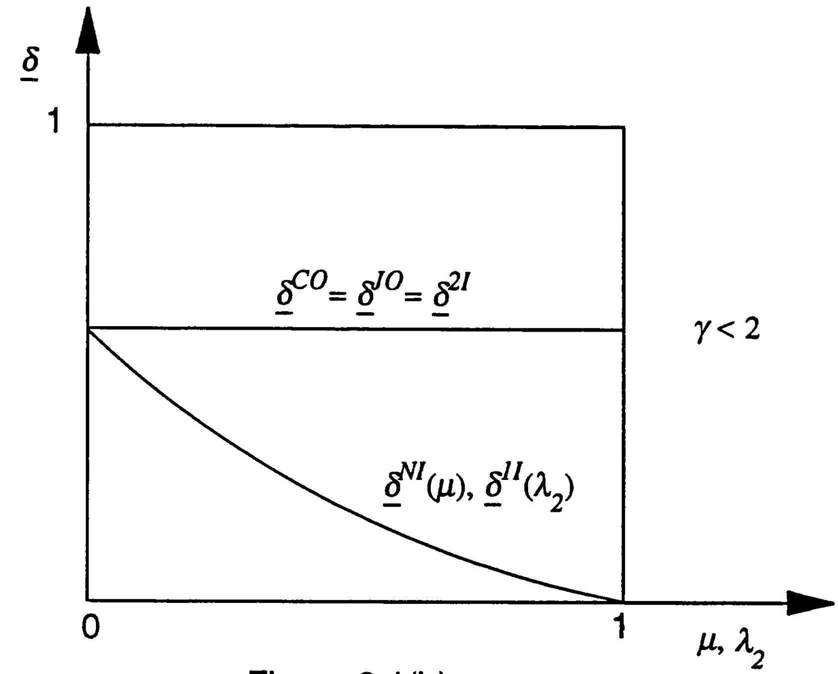

Figure 2.1(b)

Now the important effects are the *absolute* changes in the gain and loss due to higher μ. We rewrite these changes from equations (2.17) and (2.18).

$$\left|\frac{\partial G}{\partial \mu}\right| = \frac{1}{2}(I^{*}-I^{NI}) \tag{2.33}$$

$$\left|\frac{\partial L}{\partial \mu}\right| = \frac{1}{2}(1-\mu)\frac{\partial I^{NI}}{\partial \mu} \tag{2.34}$$

Both these terms describe the change in investment due to higher slope of the value function. In the first one the slope changes from $\frac{1}{2}(1+\mu)$ to 1 and in the second one the slope increases a little from $\frac{1}{2}(1+\mu)$. It is obvious that the faster the slope of the cost function increases the smaller will be the induced changes in the investments due to steeper value function. Furthermore the change in gain is more sensitive to γ than the change in punishment. In (2.33) the change in the slope of the value function (and accordingly the change in the investment) is not marginal and a change in γ will have a greater effect. Because the changes in the gain and punishment are both decreasing in γ and the gain is decreasing faster, for high values of γ the change in punishment is higher and therefore also the *relative* change in punishment is greater. Accordingly, for high γ the punishment effect dominates ($\underline{\delta}^{NI}$ is increasing in μ) and for low γ the gain effect dominates ($\underline{\delta}^{NI}$ is decreasing in μ).[15] This leads to our main results.

Proposition 2.4. *Joint ownership, integration by the noninvesting agent 2 and cross ownership are (weakly) optimal if and only if* $\gamma > 2$, $\mu < 1$ *and* $\lambda_2 < 1$.

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[15]It is also true that now the level of punishment is greater than the level of gain for high values of γ. But the absolute change in punishment is so much greater than in the gain that also the relative change in punishment is greater.

Proof:

It is immediately clear from Figure 2.1(a) that for $\gamma > 2$ $\underline{\delta}^{JO} = \underline{\delta}^{CO} = \underline{\delta}^{2I} \leq \underline{\delta}^{NI}$ and $\underline{\delta}^{JO} = \underline{\delta}^{CO} = \underline{\delta}^{2I} \leq \underline{\delta}^{1I}$ when $\mu < 1$ and $\lambda_2 < 1$.

Q.E.D.

When $\gamma > 2$ it becomes important to ensure that the punishment is maximal. Then joint ownership is optimal. The agents have to reach a unanimous agreement to use the assets. If not they can work for another firm at zero wage. Therefore the joint surplus is the lowest possible in the punishment path and cheating would lead to a very bad equilibrium. Reputation effects thus provide a new explanation for partnerships.

Agent 2 control is equally good in providing maximal punishment. Removing all the control rights from the only investing agent is a hostage type solution to prevent opportunism as discussed in Williamson (1983) and (1985). Under agent 2 control 1's outside option is to work for another firm at zero wage. Therefore the punishment is the highest. Franchising gives an example of hostages: franchisors sometimes control the leases of franchisees or even own the land on which their outlet is located to ensure proper quality standards and after-sales services.[16]

Also cross ownership is equivalent to the above structures in our model. However, if agent 1's investment is not fully specific to asset a_1 but is somewhat useful also for working with asset a_2 cross ownership does not guarantee maximal punishment. Then if 1 owns a_2 she has an outside option related to her investment while joint ownership and agent 2 control remove 1's outside option; cross ownership is not optimal.

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[16]Klein (1980), 359.

Proposition 2.5. *Integration by the only investing agent is (weakly) optimal if $\gamma < 2$.*

Proof:

Now Figure 2.1(*b*) where $\gamma < 2$ is appropriate. Since by assumption $\lambda_2 \geq \mu$, $\underline{\delta}^{II}$ is the lowest as the Figure illustrates.

Q.E.D.

Proposition 2.5 tells that when $\gamma < 2$, the prediction of the one-shot game holds. In this parameter range it is more important to ensure that the gain from deviation is the smallest possible although then also punishment is minimal. This will be guaranteed by giving the ownership of both assets to the investing agent 1.

Proposition 2.6. *Ownership does not matter if (i) $\mu = \lambda_2 = 0$ or (ii) $\gamma = 2$, $\mu < 1$ and $\lambda_2 < 1$.*

Proof:

(*i*) Follows straightforward from Lemma 2.2. (*ii*) If $\gamma = 2$ $\underline{\delta}$ is equal for all ownership structures as Lemmas 2.2 and 2.3 show (if $\mu < 1$ and $\lambda_2 < 1$).

Q.E.D.

Proposition 2.6 gives the only two cases when ownership structure does not matter. First, if all ownership structures are equivalent in the static game (equal Nash investment) they will be equivalent in the dynamic game as well. Second, even when the ownership structures differ in the one-shot game we have the knife-edge result when the punishment and gain effect exactly offset each other and ownership does not matter.

2.4.3 Other Functional Forms

In this Subsection we report the results of experiments with other functional forms. The following alternatives were examined:

(*a*) $v(I) = -e^{-\alpha I}$, $c(I) = (\gamma I - 1)$, $\alpha > 2\gamma$

(*b*) $v(I) = \alpha I$, $c(I) = (e^{\gamma I} - 1)$, $\alpha > 2\gamma$

(*c*) $v(I) = \alpha \ln(I)$, $c(I) = (I^{\gamma} - 1)$, $\alpha > 2\gamma$

(*d*) $v(I) = (\alpha + \beta I - \theta I^2)$, $c(I) = \gamma I$, $\beta > 2\gamma$

(*e*) $v(I) = (\alpha + \beta I - \theta I^2)$, $c(I) = \gamma I^2$

(*f*) $v(I) = (a + I)^2/4b$, $c(I) = cI^2$, $c > 1/4b$

For (*a*) to (*e*) $\underline{\delta}^{NI}$ is increasing in μ and joint ownership, agent 2 control and cross ownership are optimal. While for (*f*) $\underline{\delta}^{NI}$ is decreasing in μ and agent 1 control gives the best incentives to cooperate. These examples show that we should be somewhat cautious about the results of the previous two subsections. It is not only the convexity/elasticity of the cost function that drives the results. For (*a*) to (*d*) $\underline{\delta}^{NI}$ depends only on μ, other parameters cancel out. For (*e*) $\underline{\delta}^{NI}$ depends also on θ and γ but whatever the value of these parameters $\underline{\delta}^{NI}$ is increasing in μ. Likewise for (*f*) $\underline{\delta}^{NI}$ depends on b and c as well. What is common between these results and the constant elasticity ones is that when $\underline{\delta}^{NI}$ is increasing in μ both the level of punishment and the change in punishment due to higher μ are greater than the level of and the change in gain. Whereas when $\underline{\delta}^{NI}$ is decreasing in μ the opposite is true: the level of and the change in gain are greater.

2.5 Two Investments

We chose a very simple structure for our model to make the main trade-off in the repeated game clear. In this Section we analyse the first natural extension: both agents have an investment. We denote agent 1's and 2's investment by I_1 and I_2 respectively and assume that the value of the investment is $v(I_i) = I_i$ and the cost of investment is $c_i(I_i) = I_i^\gamma/\sigma_i$ where $\gamma > 1$ and $\sigma_i > 0$.

In this setup both agents can cheat and punish by investment and the optimal sharing rule is not as simple as in the main model. Proposition 2.7 designs a sharing rule such that both agents have best incentives to cooperate.

Proposition 2.7. *The optimal sharing rule is:*

$$P_1^* = sP_1^d + (1 - s)(S^* - P_2^d)$$

$$P_2^* = (1 - s)P_2^d + s(S^* - P_1^d)$$

where $s = (P_2^d - P_2^p)/(P_1^d + P_2^d - P_1^p - P_2^p)$ *and* $S^* = v(I_1^*)+v(I_2^*)-c_1(I_1^*)-c_2(I_2^*)$.

Proof:

When agent i pays a transfer T to agent j for the input or for the contribution of the worker, the payoffs are:

$$P_i^* = v(I_1^*) + v(I_2^*) - T - c_i(I_i^*) \tag{2.35}$$

$$P_j^* = T - c_j(I_j^*) \tag{2.36}$$

Then agent i will cooperate if and only if:

$$\delta \geq \frac{P_i^d - v(I_1^*) - v(I_2^*) + T + c_i(I_i^*)}{P_i^d - P_i^p} \tag{2.37}$$

Likewise agent j cooperates if and only if:

$$\delta \geq \frac{P^d_j - T + c_j(I^*_j)}{P^d_j - P^p_j} \tag{2.38}$$

Because agent i's incentive to cooperate is decreasing in T while j's incentive is increasing in T, the optimal T^* gives the agents balanced incentives to cooperate. Setting the right-hand-sides of equations (2.37) and (2.38) equal we can solve for T^*:

$$T^* = \frac{(P^d_i-P^p_i)\,[P^d_j+c_j(I^*_j)] \;+\; (P^d_j-P^p_j)\,[v(I^*_1)+v(I^*_2)-P^d_i-c_i(I^*_i)]}{(P^d_1-P^p_1) \;+\; (P^d_2-P^p_2)} \tag{2.39}$$

Inserting T^* in equations (2.35) and (2.36) gives the expressions in the Proposition.

Q.E.D.

Neither agent would have an incentive to deviate if they could get their deviation payoff even under cooperation. Since this is not feasible the best we can do is to give each agent a certain proportion of her deviation payoff. It is like agent 1 gets her deviation payoff with probability s and agent 2 gets his deviation payoff with probability $(1 - s)$ leaving the rest of the surplus, $(S^* - P^d_2)$, to agent 1. s is chosen to balance the agent's incentives to cooperate. Proposition 2.7 gives $s = (P^d_2-P^p_2)/(P^d_1+P^d_2-P^p_1-P^p_2)$. This weight is related to how close the Nash investment is to the first-best one. For example under nonintegration:

$$(P^d_2-P^p_2) = \tfrac{1}{2}(1-\mu)\,[v(I^*_1) - v(I^{NI}_1)]. \tag{2.40}$$

If agent 1 is better able to punish agent 2 by investment

$$[v(I^*_1) - v(I^{NI}_1)] > [v(I^*_2) - v(I^{NI}_2)] \tag{2.41}$$

then $s > 1/2$ and agent 1 receives a higher proportion of her deviation payoff than agent 2.

Sometimes agent 1 does not have an incentive to deviate in investment: she has first-best incentives even in the one-shot game (she owns both assets and agent 2 is dispensable). Then $P_2^d = P_2^p$ and $s = 0$, that is agent 2 gets his full deviation payoff. This is the same case as we had in our main model; the noninvesting agent could punish only by sharing rule and therefore the investing agent got her full deviation payoff which is equal to her punishment payoff in this case. Here agent 1's Nash investment is equal to its efficient level and therefore reversion to noncooperation provides no punishment in investment.

Inserting the optimal sharing rule of Proposition 2.7 in (2.37) or (2.38) gives us a lowerbound for the discount factor:

(2.42) $$\underline{\delta} = (G_1 + G_2)/(G_1 + G_2 + L_1 + L_2)$$

where G_i is the gain from deviation to agent i and L_i is the loss. Now the best ownership structure is such that the *aggregate* gain from deviation is lowest relative to the *aggregate* punishment. Since we have equalized the incentives it is the aggregate terms that matter.

The same trade-off is present in this version of the model: an ownership structure that provides maximal punishment will also give the highest gain from deviation.[17]

[17]See Proposition A2.1 in Appendix 2.

Now when both agents have an investment obviously agent 2 control is not anymore equivalent to joint ownership and cross ownership; under integration by 2 an investing agent has control rights. The second difference to our main model is that nonintegration and integration by agent i are not equivalent when worker j is indispensable ($\lambda_j = \mu$). Although i's investment is equal under both structures j's investment will differ and therefore $\underline{\delta}$ is not equal. Lemma 2.4 gives some useful properties for the lowerbounds of the discount factor.

Lemma 2.4.

(i) $\partial\underline{\delta}^{NI}/\partial\mu \stackrel{\leq}{=} (\gamma - 2)$.

(ii) $[\underline{\delta}^{NI}(\mu) - \underline{\delta}^{iI}(\lambda_j)] \stackrel{\leq}{=} (\gamma - 2)$ *for* $0 < \mu = \lambda_j < 1$.

(iii) $[\underline{\delta}^{iI}(\lambda_j) - \underline{\delta}^{JO}] \stackrel{\leq}{=} (\gamma - 2)$ *for* $0 < \lambda_j < 1$.

(iv) $\underline{\delta}^{iI}(0) = \underline{\delta}^{iI}(1)$.

Proof: In Appendix 2.

Lemma 2.4 helps us to construct Figure 2.2 which we use to find the optimal ownership structure. The results are in line with our main model.

Proposition 2.8. *When both agents have an investment joint ownership and cross ownership are (weakly) optimal if and only if* $\gamma > 2$ *and* $\mu < 1$.

Proof:

See Figure 2.2(a) where $\gamma > 2$. It is immediately clear from the Figure that $\underline{\delta}^{JO} = \underline{\delta}^{CO} \leq \underline{\delta}^{NI}$ and $\underline{\delta}^{JO} = \underline{\delta}^{CO} \leq \underline{\delta}^{iI}$ when $\mu < 1$.

Q.E.D.

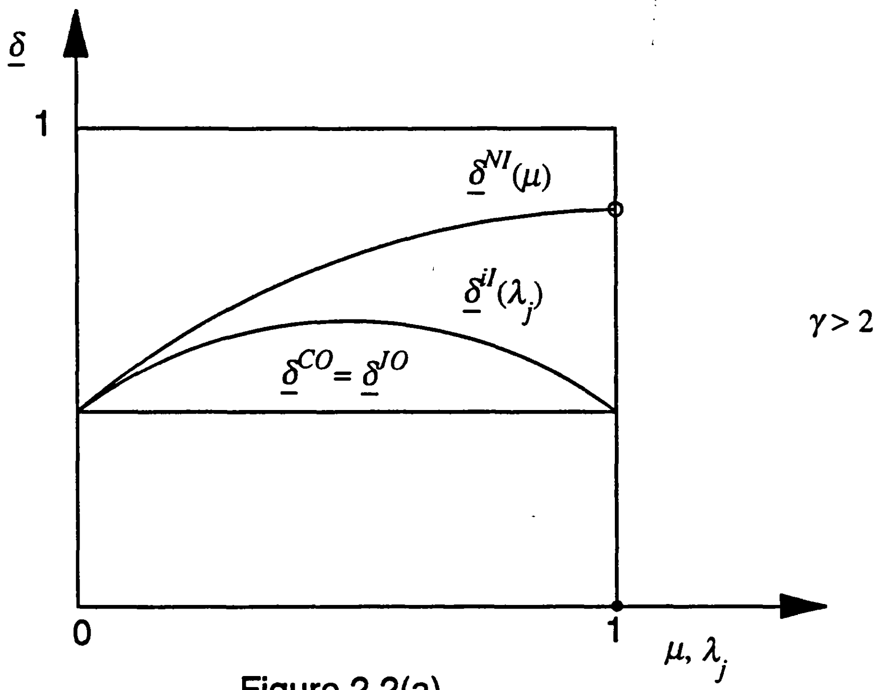

Figure 2.2(a)

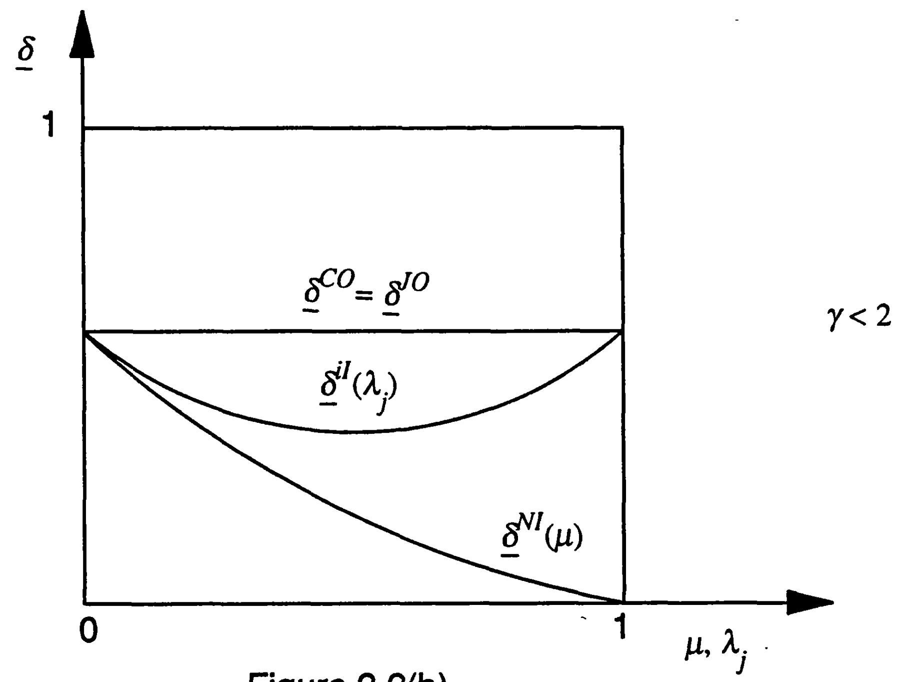

Figure 2.2(b)

As in the main model maximizing punishment becomes important when costs are very elastic and joint ownership and cross ownership are optimal. The same remark given in the previous section applies also here: cross ownership would not be optimal if i's investment is not fully specific to a_i and therefore partnership is the more general prediction.

Proposition 2.9. *The following statements are true if $\gamma < 2$:*

(i) Joint and cross ownership are (weakly) dominated by nonintegration and integration.

(ii) If assets are strictly complementary then nonintegration is (weakly) dominated by integration.

(iii) If agent i is indispensable to asset a_i, then integration by agent j is (weakly) dominated by nonintegration.

Proof:

See Figure 2.2(*b*) where $\gamma < 2$. (*i*) Under joint ownership and cross ownership $\underline{\delta}$ reaches its maximum. Therefore these structures are dominated. (*ii*) When assets are strictly complementary ($\mu = 0$), the value for $\underline{\delta}^{NI}$ is given by the intercept in the vertical axis. Therefore $\underline{\delta}^{NI} \geq \underline{\delta}^{iI}$. (*iii*) When agent j is indispensable to asset a_j, $\lambda_j = \mu$. The Figure shows that then $\underline{\delta}^{NI} \leq \underline{\delta}^{iI}$. Q.E.D.

As in the main model a control structure that minimizes the gain from deviation is best when $\gamma < 2$. Proposition 2.9 gives the same results as Hart and Moore (1990). One important determinant for the optimal ownership structure is the degree of complementarity between the assets; when the assets are strictly complementary they should be owned together. Also, if an agent is very important as a trading partner (indispensable) then he should own his asset. Furthermore, joint and cross ownership are dominated.

In Proposition 2.9 we considered only extreme values of the parameters μ and λ_i. It is interesting to examine if the static and repeated game give exactly the same predictions for all parameter values. For this aim we have constructed Figures 2.3 and 2.4. The Figures are based on numerical simulations of the model. In Figure 2.3 the relative importance of the investment is in the vertical axis (when $\sigma_1/\sigma_2 > 1$ agent 1's investment is more important) and the degree of asset complementarity is in the horizontal axis.[18] The predictions for the limit values of parameter μ are the same: strictly complementary assets should be owned together and for economically independent assets there should be independent control. However, for intermediate values of μ there are some differences and in particular nonintegration is more likely in the repeated game. Figure 2.3 also shows that the more important an agent's investment is the more likely it is that she owns both assets.

In Figure 2.4 we have the importance of an agent as a trading partner in the horizontal axis. Here we assume for simplicity that $\lambda \equiv \lambda_1 = \lambda_2$. λ has a peculiar nonmonotonic effect; although in general in this parameter range it is more important to minimize the gain from deviation the punishment effect starts to dominate under integration for high values of λ. When a worker is almost dispensable if she becomes even more dispensable the punishment decreases more than the gain under integration and it is better to give her the ownership of her asset. Therefore in this setup both a fully indispensable and fully dispensable agent should own her asset while only the former is true in the one-shot game. However, this nonmonotonic effect is not very

[18]The Figure is drawn for given value of λ, where $0 < \lambda < 1$.

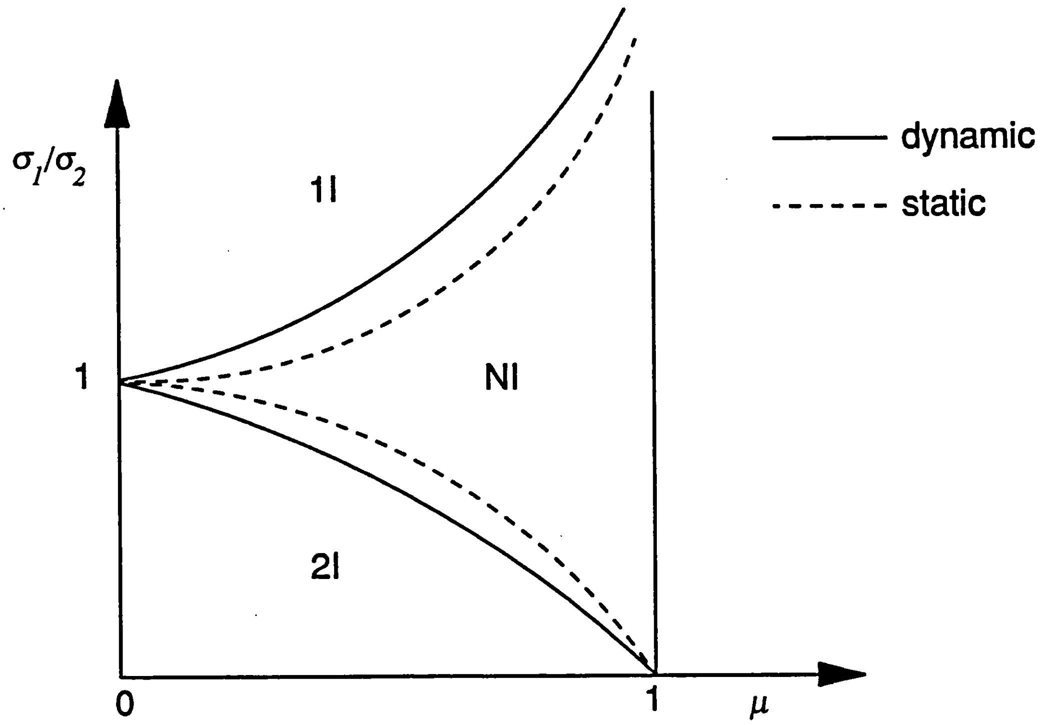

Figure 2.3

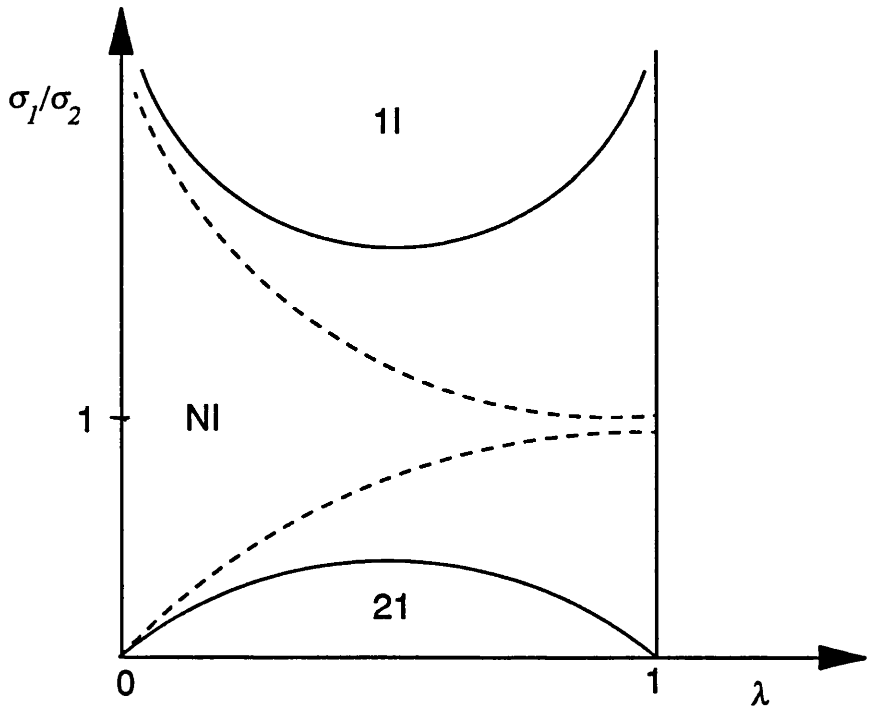

Figure 2.4

robust since it did not occur in our main model. Also Figure 2.4 shows that nonintegration is more likely in the repeated game. This observation can be linked to the discussion about integration and reputation being substitutes (e.g. Klein (1980) and Coase (1988)). This discussion refers to models where the benefit of integration is reduced holdups and costs are something else (like arising from bureaucracy). There clearly one would expect less integration in a repeated setting; the benefits of integration are lower and costs have not changed. In our model both the benefits and costs of integration change in the repeated game and it is not a priori clear which way the reputation effect goes. As Figures 2.3 and 2.4 show nonintegration is more likely when $\gamma < 2$.[19]

[19]Of course when $\gamma > 2$ nonintegration is less likely in the repeated setting. In fact, nonintegration never occurs.

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Appendix 2

Proof of Lemma 2.3:

The joint surplus maximizing investment is:

(A2.1)
$$I^* = \gamma^{\frac{-1}{(\gamma-1)}}$$

and the Nash investment under nonintegration is:

(A2.2)
$$I^{NI} = [(1+\mu)/2\gamma]^{\frac{1}{(\gamma-1)}}$$

Inserting these investments in (2.14) and (2.15) we obtain:

(A2.3)
$$G^{NI} = \gamma^{\frac{-\gamma}{(\gamma-1)}}\left[1 - \left[\tfrac{1+\mu}{2}\right]^{\frac{\gamma}{\gamma-1}}\right] - \tfrac{(1+\mu)}{2}\gamma^{\frac{-1}{(\gamma-1)}}\left[1 - \left[\tfrac{1+\mu}{2}\right]^{\frac{1}{\gamma-1}}\right]$$

(A2.4)
$$L^{NI} = \gamma^{\frac{-1}{(\gamma-1)}}\left[1 - \left[\tfrac{1+\mu}{2}\right]^{\frac{1}{\gamma-1}}\right] - \gamma^{\frac{-\gamma}{(\gamma-1)}}\left[1 - \left[\tfrac{1+\mu}{2}\right]^{\frac{\gamma}{\gamma-1}}\right]$$

Therefore the lowerbound for the discount factor under nonintegration is:

(A2.5)
$$\underline{\delta}^{NI} = \left\{\left[1 - \left[\tfrac{1+\mu}{2}\right]^{\frac{\gamma}{\gamma-1}}\right] - \tfrac{(1+\mu)}{2}\gamma\left[1 - \left[\tfrac{1+\mu}{2}\right]^{\frac{1}{\gamma-1}}\right]\right\} / \tfrac{(1-\mu)}{2}\gamma\left[1 - \left[\tfrac{1+\mu}{2}\right]^{\frac{1}{\gamma-1}}\right]$$

Differentiating (A2.5) with respect to μ we obtain:

(A2.6)
$$\frac{\partial \underline{\delta}^{NI}}{\partial \mu} \overset{s}{=} \left[1 - \left[\tfrac{1+\mu}{2}\right]^{\frac{\gamma}{\gamma-1}}\right]\left\{(\gamma-1) - \left[\tfrac{1+\mu}{2}\right]^{\frac{1}{\gamma-1}}\left[(\gamma-1) - \tfrac{(1-\mu)}{(1+\mu)}\right]\right\} - \gamma\left[1 - \left[\tfrac{1+\mu}{2}\right]^{\frac{1}{\gamma-1}}\right]\left\{(\gamma-1) - \left[\tfrac{1+\mu}{2}\right]^{\frac{1}{\gamma-1}}[(\gamma-1) - (1-\mu)/2]\right\}$$

To simplify notation define $\varepsilon \equiv (\gamma - 2) > -1$ and $\eta \equiv (1+\mu)/2$. Since $0 \leq \mu < 1$, then $1/2 \leq \eta < 1$. Then (A2.6) simplifies to:

(A2.7)
$$F_\eta(\varepsilon) = \left[1 - \eta^{\frac{2+\varepsilon}{1+\varepsilon}}\right]\left\{(1+\varepsilon) - \eta^{\frac{1}{1+\varepsilon}}\left[(1+\varepsilon) - \tfrac{(1-\eta)}{\eta}\right]\right\} - (2+\varepsilon)\left[1 - \eta^{\frac{1}{1+\varepsilon}}\right]\left\{(1+\varepsilon) - \eta^{\frac{1}{1+\varepsilon}}[(1+\varepsilon) - (1-\eta)]\right\}$$

Next define $\nu \equiv \eta^{\frac{1}{1+\varepsilon}}$. Substituting ν in (A2.7) and simplifying we obtain:

(A2.8) $F_\eta(\varepsilon) = (1-\nu\eta)[(1+\varepsilon)(1-\nu) + \nu(1-\eta)/\eta] - (2+\varepsilon)(1-\nu)[(1+\varepsilon)(1-\nu) + \nu(1-\eta)]$

$$= (1-\nu\eta)(\nu-\eta)/\eta - \varepsilon(2+\varepsilon)(1-\nu)^2$$

From ν's definition we have $\varepsilon = \{[\ln(\eta)/\ln(\nu)] - 1\}$. Substituting this in (A2.8) gives:

(A2.9)
$$\begin{aligned} F_\eta(\varepsilon) &= (1-\nu\eta)(\nu-\eta)/\eta - \{[\ln(\eta)/\ln(\nu)]^2 - 1\}(1-\nu)^2 \\ &= \{\nu(1-\eta)^2 - \eta(1-\nu)^2[\ln(\eta)/\ln(\nu)]^2\}/\eta \\ &= \frac{(1-\eta)^2(1-\nu)^2}{\eta[\ln(\nu)]^2}[f(\nu) - f(\eta)] \end{aligned}$$

where $f(\nu) = \dfrac{\nu[\ln(\nu)]^2}{(1-\nu)^2}$. Lemma 2.3 says that $F_\eta(\varepsilon) \lesseqgtr \varepsilon$. It is easy to verify from (A2.9) that $F_\eta(0) = 0$. Furthermore $F_\eta(\varepsilon) \lesseqgtr [f(\nu) - f(\eta)]$. It is straightforward to show that:

$-1 < \varepsilon < 0 \iff 0 < \nu < \eta < 1$

$\varepsilon = 0 \iff 1/2 \leq \nu = \eta < 1$

$\varepsilon > 0 \iff 1/2 \leq \eta < \nu < 1$

Therefore $f'(\nu) > 0$ for $\nu \in (0,1)$ implies that $F_\eta(\varepsilon) \lesseqgtr \varepsilon$.

(A2.10)
$$f'(\nu) = \frac{(1+\nu)[\ln(\nu)]^2}{(1-\nu)^3} + \frac{2\ln(\nu)}{(1-\nu)^2} = g(\nu)h(\nu)$$

where $g(\nu) = [2(1-\nu)/(1+\nu) + \ln(\nu)]$ and $h(\nu) = (1+\nu)\ln(\nu)/(1-\nu)^3$. $h(\nu) < 0$ for $\nu \in (0,1)$. $g(1) = 0$ and $g'(\nu) = (1-\nu)^2/\nu(1+\nu)^2 > 0$ and therefore $g(\nu) < 0$ for $\nu \in (0,1)$. Accordingly $f'(\nu) = h(\nu)g(\nu) > 0$ for $\nu \in (0,1)$.

Q.E.D.

Proposition A2.1. *Both the aggregate gain from deviation, $(P_1^d+P_2^d-S^*)$, and the aggregate loss, $(S^*-P_1^p-P_2^p)$, are decreasing in μ under nonintegration and decreasing in λ_j under integration.*

Proof:

First note that S^* does not depend on μ or λ_j. Agent i's punishment payoff under nonintegration is:

(A2.11) $$P_i^p = \frac{(1+\mu)}{2}v(I_i^{NI}) + \frac{(1-\mu)}{2}v(I_j^{NI}) - c_i(I_i^{NI})$$

Total differentiation gives:

(A2.12) $$dP_i^p/d\mu = \frac{1}{2}[v(I_i^{NI}) - v(I_j^{NI})] + \frac{(1-\mu)}{2}v'(I_j^{NI})\frac{\partial I_j^{NI}}{\partial \mu} \gtrless 0$$

Consequently the aggregate punishment is decreasing in μ:

(A2.13) $$d(S^*-P_1^p-P_2^p)/d\mu = -\frac{(1-\mu)}{2}[v'(I_1^{NI})\frac{\partial I_1^{NI}}{\partial \mu} + v'(I_2)\frac{\partial I_2^{NI}}{\partial \mu}] < 0$$

Agent i's deviation payoff is:

(A2.14) $$P_i^d = \frac{(1+\mu)}{2}v(I_i^{NI}) + \frac{(1-\mu)}{2}v(I_j^*) - c_i(I_i^{NI})$$

By differentiating totally we obtain:

(A2.15) $$dP_i^d/d\mu = \frac{1}{2}[v(I_i^{NI}) - v(I_j^*)] \gtrless 0$$

(A2.16) $$d(P_1^d+P_2^d-S^*)/d\mu = \frac{1}{2}[v(I_1^{NI}) + v(I_2^{NI}) - v(I_1^*) - v(I_2^*)] < 0$$

Repeating the analysis for the integrated structure gives:

(A2.17) $$d(S^*-P_1^p-P_2^p)/d\lambda_j = -\frac{(1-\lambda_j)}{2}v'(I_i^{iI})\frac{\partial I_i^{iI}}{\partial \lambda_j} < 0$$

(A2.18) $$d(P_1^d+P_2^d-S^*)/d\lambda_j = [v(I_i^{iI}) - v(I_i^*)]/2 < 0$$

Q.E.D.

Proof of Lemma 2.4:

(*i*) $\underline{\delta}^{iI}(0) = \underline{\delta}^{iI}(1)$

The first-best and Nash investments are:

(A2.19) $$I_i^* = (\sigma_i/\gamma)^{\frac{1}{(\gamma-1)}}$$

(A2.20) $$I_i^{iI} = [(1+\lambda_j)\sigma_i/2\gamma]^{\frac{1}{(\gamma-1)}}$$

(A2.21) $$I_j^{iI} = (\sigma_i/2\gamma)^{\frac{1}{(\gamma-1)}}$$

Using these equations we obtain the lowerbound for the discount factor under agent i control:

(A2.22) $$\underline{\delta}^{iI}(\lambda_j) = \Big\{\sigma_i^{\frac{1}{(\gamma-1)}}\Big[1 - \Big[\frac{1+\lambda_j}{2}\Big]^{\frac{\gamma}{\gamma-1}}\Big] + \sigma_j^{\frac{1}{(\gamma-1)}}\Big[1 - \Big[\frac{1}{2}\Big]^{\frac{\gamma}{\gamma-1}}\Big]$$
$$- \frac{(1+\lambda_j)}{2}\gamma\sigma_i^{\frac{1}{(\gamma-1)}}\Big[1 - \Big[\frac{1+\lambda_j}{2}\Big]^{\frac{1}{\gamma-1}}\Big] - \frac{1}{2}\gamma\sigma_j^{\frac{1}{(\gamma-1)}}\Big[1 - \Big[\frac{1}{2}\Big]^{\frac{1}{\gamma-1}}\Big]\Big\}/$$
$$\Big\{\frac{(1-\lambda_j)}{2}\gamma\sigma_i^{\frac{1}{(\gamma-1)}}\Big[1 - \Big[\frac{1+\lambda_j}{2}\Big]^{\frac{1}{\gamma-1}}\Big] + \frac{1}{2}\gamma\sigma_j^{\frac{1}{(\gamma-1)}}\Big[1 - \Big[\frac{1}{2}\Big]^{\frac{1}{\gamma-1}}\Big]\Big\}$$

We simplify notation by defining $\tilde{\eta} \equiv (1+\lambda_j)/2$, $c_i \equiv \sigma_i^{\frac{1}{(\gamma-1)}}$ and $\varepsilon \equiv (\gamma - 2)$.

(A2.23) $$\underline{\delta}^{iI}(\lambda_j) = \Big\{c_i\Big[1 - \tilde{\eta}^{\frac{2+\varepsilon}{1+\varepsilon}}\Big] + c_j\Big[1 - \Big[\frac{1}{2}\Big]^{\frac{2+\varepsilon}{1+\varepsilon}}\Big] - \tilde{\eta}c_i(2+\varepsilon)\Big[1 - \tilde{\eta}^{\frac{1}{1+\varepsilon}}\Big] -$$
$$\frac{1}{2}c_j(2+\varepsilon)\Big[1 - \Big[\frac{1}{2}\Big]^{\frac{1}{1+\varepsilon}}\Big]\Big\}/\Big\{(1-\tilde{\eta})(2+\varepsilon)c_i\Big[1 - \tilde{\eta}^{\frac{1}{1+\varepsilon}}\Big] + \frac{1}{2}(2+\varepsilon)c_j\Big[1 - \Big[\frac{1}{2}\Big]^{\frac{1}{1+\varepsilon}}\Big]\Big\}$$

Next define $\tilde{v} \equiv \tilde{\eta}^{1/(1+\varepsilon)}$ and $\varphi \equiv 2^{-1/(1+\varepsilon)}$

(A2.24) $$\underline{\delta}^{iI}(\lambda_j) = [(1-\tilde{\eta}\tilde{v})c_i + (1-\varphi/2)c_j - (2+\varepsilon)(1-\tilde{v})\tilde{\eta}c_i -$$
$$(2+\varepsilon)(1-\varphi)c_j/2]/[(1-\tilde{\eta})(2+\varepsilon)(1-\tilde{v})c_i + (2+\varepsilon)(1-\varphi)c_j/2]$$

For $\lambda_j = 0$: $\tilde{\eta} = 1/2$ and $\tilde{v} = \varphi$ and for $\lambda_j = 1$: $\tilde{\eta} = 1$ and $\tilde{v} = 1$. Substituting these into (A2.24) we obtain:

(A2.25) $$\underline{\delta}^{iI}(0) = [(1-\varphi/2)(c_i+c_j) - (2+\varepsilon)(1-\varphi)(c_i+c_j)/2\ ]/[(2+\varepsilon)(1-\varphi)(c_i+c_j)/2]$$
$$= [(1-\varphi/2) - (2+\varepsilon)(1-\varphi)/2\ ]/[(2+\varepsilon)(1-\varphi)/2]$$

(A2.26) $$\underline{\delta}^{iI}(1) = [(1-\varphi/2)c_j - (2+\varepsilon)(1-\varphi)c_j/2]/[(2+\varepsilon)(1-\varphi)c_j/2]$$
$$= [(1-\varphi/2) - (2+\varepsilon)(1-\varphi)/2]/[(2+\varepsilon)(1-\varphi)/2]$$

Therefore $\underline{\delta}^{iI}(0) = \underline{\delta}^{iI}(1)$.

(ii) $\underline{\delta}^{NI}(\mu) - \underline{\delta}^{iI}(\lambda_j) \overset{\leq}{=} \varepsilon$ **for** $0 < \mu = \lambda_j < 1$ **and** $\underline{\delta}^{iI}(\lambda_j) - \underline{\delta}^{JO} \overset{\leq}{=} \varepsilon$ **for** $0 < \lambda_j < 1$.

The Nash investments under nonintegration are:

(A2.27) $$I_i^{NI} = [(1+\mu)\sigma_i/2\gamma]^{\frac{1}{(\gamma-1)}}$$

Substituting these into (2.42) we obtain (A2.5); the lowerbound for nonintegration is equal when one or both agents have an investment. Since at $\mu = \lambda_j$ $\eta = \tilde{\eta}$ and $\nu = \tilde{\nu}$ we obtain:

(A2.28)
$$\underline{\delta}^{NI}(\mu) - \underline{\delta}^{iI}(\lambda_j) =$$
$$\frac{(1-\eta\nu) - \eta(2+\varepsilon)(1-\nu)}{(1-\eta)(2+\varepsilon)(1-\nu)} - \frac{(1-\eta\nu)c_i+(1-\varphi/2)c_j-\eta(2+\varepsilon)(1-\nu)c_i-(2+\varepsilon)(1-\varphi)c_j/2}{(1-\eta)(2+\varepsilon)(1-\nu)c_i+(2+\varepsilon)(1-\varphi)c_j/2}$$
$$\stackrel{s}{=} (1-\eta\nu)(1-\varphi) - 2(1-\eta)(1-\nu)(1-\varphi/2) + (1-2\eta)(2+\varepsilon)(1-\nu)(1-\varphi)$$

Note that $\underline{\delta}^{NI}$ is not defined at $\mu = 1$.

Under joint ownership the Nash investments are:

(A2.29)
$$I_i^{JO} = (\sigma_i/2\gamma)^{\frac{1}{(\gamma-1)}}$$

And we have:

(A2.30)
$$\underline{\delta}^{iI}(\lambda_j) - \underline{\delta}^{JO} =$$
$$\frac{(1-\tilde{\eta}\tilde{\nu})c_i+(1-\varphi/2)c_j-\tilde{\eta}(2+\varepsilon)(1-\tilde{\nu})c_i-(2+\varepsilon)(1-\varphi)c_j/2}{(1-\tilde{\eta})(2+\varepsilon)(1-\tilde{\nu})c_i+(2+\varepsilon)(1-\varphi)c_j/2} - \frac{(1-\varphi/2)-(2+\varepsilon)(1-\varphi)/2}{(2+\varepsilon)(1-\varphi)/2} \stackrel{s}{=}$$
$$(1-\tilde{\eta}\tilde{\nu})(1-\varphi) - 2(1-\tilde{\eta})(1-\tilde{\nu})(1-\varphi/2)+ (1-2\tilde{\eta})(2+\varepsilon)(1-\tilde{\nu})(1-\varphi)$$

This proves that $\underline{\delta}^{NI}(\mu) - \underline{\delta}^{iI}(\lambda_j) \stackrel{s}{=} \underline{\delta}^{iI}(\lambda_j) - \underline{\delta}^{JO}$ for $\mu = \lambda_j$. Therefore one of the following has to be true:

(*i*) $\underline{\delta}^{NI}(\mu) < \underline{\delta}^{iI}(\lambda_j) < \underline{\delta}^{JO}$

(*ii*) $\underline{\delta}^{NI}(\mu) = \underline{\delta}^{iI}(\lambda_j) = \underline{\delta}^{JO}$

(*iii*) $\underline{\delta}^{NI}(\mu) > \underline{\delta}^{iI}(\lambda_j) > \underline{\delta}^{JO}$

From Lemmas 2.2 and 2.3 we know that $\underline{\delta}^{NI}(0) = \underline{\delta}^{JO}$ and $\partial\underline{\delta}^{NI}/\partial\mu \stackrel{s}{=} \varepsilon$. Therefore for $\mu = \lambda_j = 0$ and/or $\varepsilon = 0$ $\underline{\delta}^{NI} = \underline{\delta}^{JO}$ and (*ii*) holds. For $\mu = \lambda_j \in (0,1)$ and $\varepsilon < 0$ $\underline{\delta}^{NI} < \underline{\delta}^{JO}$ and therefore (*i*) holds. Respectively for $\mu = \lambda_j \in (0,1)$ and $\varepsilon > 0$ $\underline{\delta}^{NI} > \underline{\delta}^{JO}$ and (*iii*) holds.

Q.E.D.

3 Vertical Supply, Foreclosure and Overbuying

3.1 Introduction

This Chapter focuses on vertical oligopolies when both integrated and unintegrated firms coexist. Our main interest is in an integrated firm's decision about which market to enter, and at what scale to operate. The integrated firm realizes that its actions in the input market affect the competition in the final good market. Although the integrated firm can obtain input at marginal cost from its own upstream unit, it may prefer to buy some in the market aiming to raise input price for the rival; that is, the integrated firm might *overbuy*. In some other circumstances the integrated firm might *vertically supply* its rival. But the input it sells to its rival will ultimately compete with its own final good. Therefore the integrated firm might not be willing to sell (or buy) input: that is, it might choose to *foreclose* in the upstream market. Also, if the integrated firm is very inefficient in the downstream market, it may be optimal to *exit* and operate only in the upstream market. In this Chapter we derive conditions for the integrated firm to overbuy, foreclose, vertically supply its rival or exit from the downstream market. In addition, we look at how the firm's optimal product mix will adjust to changes in costs and demand.

The integrated firm may choose to buy input in the market although internal supplies would be cheaper. Overbuying drives up rival's marginal cost and the integrated firm is in a better competitive position in the final good market. (In our model input is produced with constant returns to scale and therefore the integrated firm cannot completely monopolize the downstream market by buying up the input.) This of course comes at a cost; the average (but not the marginal) cost of the integrated firm increases. If the gain in the downstream market is greater than the increased cost of input, the integrated firm will find it optimal to buy input. Overbuying arises in equilibrium for example when the integrated firm is more or equally efficient in transforming input into the final good than its unintegrated rival. If it is already in a strong position in the downstream market it can afford to raise average costs somewhat in order to gain an even stronger competitive position.

The integrated firm understands that every unit of input it sells to its rival will increase the rival's downstream production and, consequently, decrease the price for its own final good. If this strategic effect on downstream profit is great enough, the integrated firm will not sell input. We show that it is not always optimal for the integrated firm to foreclose, even when, for instance, it has access to the same technology (for producing the final good) as the unintegrated firm to which it is selling. The crucial issue is the integrated firm's relative *overall* efficiency in transforming the input into final good. If the integrated firm is more efficient than the unintegrated firm, it is indeed not optimal to sell input. But if the integrated firm is less efficient, it may be profitable. If the integrated firm is in any case in a weak position in the downstream market, it may well sell some input and at least make a profit from it. In an international trade setting where the unintegrated downstream firm is in the domestic country and the integrated firm is an exporter, vertical supply can be an equilibrium as a result of differences in other factor costs, for example labour costs. Also, if the transportation cost is higher for the final good than for the input, the integrated firm may sell input. Furthermore, the domestic country can induce supply of input by imposing a greater tariff on the final good than on the input.

In our main model foreclosure is a knife-edge. Everywhere else except in one point the integrated firm is active in the input market: it either sells or buys input. In our application to international trade transportation costs paid by the seller introduce a wedge between overbuying and vertical supply regions where foreclosure is an equilibrium.

When the integrated firm chooses its strategy in the input market there is a trade-off between gains in the final market and losses in the input market. The exit decision is simpler: it only depends on the profit margin of the final good. When the integrated firm is very inefficient in transforming input into the final good, it will find it optimal to exit and operate only in the upstream market.

Above we have focused on the cost asymmetries as a driving force behind the integrated firm's strategy in the input market. Also the toughness of competition in the final good market affects the integrated firm's strategy. If there is no competition (the firms operate in different markets) input sales bring revenues to the integrated firm and it has no harmful effects whatsoever on its downstream profit; therefore vertical supply occurs. When competition is tough it becomes important for the integrated firm not to help its rival to compete against itself; vertical supply is not optimal. If the firms are equally efficient the integrated firm can achieve a cost advantage by overbuying input. This gives it a dominant position in a highly competitive market; overbuying emerges in equilibrium.

The integrated firm's decisions can also be affected by purely technological reasons. Vertical supply can arise when there are diminishing returns to scale in final good production (Quirmbach (1986)) while diminishing returns in input production may cause the integrated firm to buy input. Our model assumes constant marginal costs, so these effects are not present. Salinger (1988) examines the impact of a vertical merger on the input and final good price in an oligopoly where integrated and unintegrated firms coexist. He finds that the prices do not necessarily increase after a vertical merger and foreclosure. We show that overbuying would arise in his model, a strategy he did not consider. Therefore there is more tendency for the input and final good prices to increase than his results indicate. Spencer and Jones (1991) and (1992) examine vertical supply and foreclosure decisions in an international trade setting.[20] Their models differ from ours in that the alternative supplier of input is a domestic perfectly competitive industry with high and increasing costs and we have an unintegrated oligopolist. Their main concern is the effect of domestic supply conditions (the absolute quantity of input supplies and the response of these supplies to the input price charged by the integrated firm) on the vertical supply decision. In our model the downstream costs rather than the upstream conditions are the crucial issue for the integrated firm's strategy. Also, Spencer and Jones do not consider the possibility that the integrated firm may buy input from the independent supplier to drive up its rival's costs or even monopolize the downstream market. Ordover, Saloner and Salop (1990) differs from the two previous papers (and from ours) in that they have Bertrand competition in the input market. In their model the integrated firm will foreclose in the upstream market but only because they assume that the integrated firm can commit not to supply rival firm below a certain price. They do not explain why such a commitment is feasible and why the firms have to integrate to make this commitment. In our model foreclosure can arise in equilibrium without incredible commitments. Lastly, Salop and Scheffman (1983) and (1987) analyse cost-raising strategies (including overbuying) of a dominant firm with a competitive fringe. In our model also the "fringe" behaves strategically.

The rest of the Chapter is organized as follows. In Section 3.2 we introduce our

[20]The first draft of this Chapter was completed in December 1990, before the papers by Spencer and Jones came to our knowledge.

model of vertical oligopolies. In Section 3.3 we derive the equilibrium of the two stage game. Sections 3.4 and 3.5 examine the integrated firm's strategies in the input and final good markets. Section 3.6 discusses the comparative statics of the model. In Section 3.7 the role of the toughness of competition in the downstream market is analysed. Section 3.8 applies our model to an international trade setting.

3.2 The Model

We consider a successive Cournot duopoly. There is a final good, y, and an input, x, which are homogeneous. We include in the model a vertically integrated firm I, an unintegrated upstream firm U, and an unintegrated downstream firm D. Our paper focuses on the integrated firm's decision about which market to enter, and at what scale to operate. Firm I *potentially* sells input to D, buys input from U or provides its own input as Figure 3.1 illustrates. We denote by y_D and y_I the final-good production levels of firms D and I, and by x_U and x_I the amount of input firms U and I sell in the upstream market (if x_I is negative I is buying input).

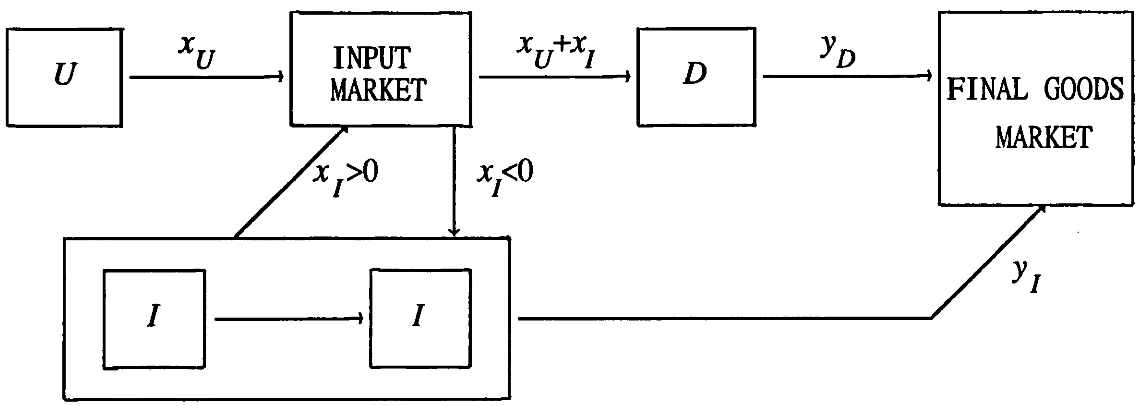

Figure 3.1

In the first stage the upstream firms commit to input sales, and *I* potentially purchases some input. The buyers of input are assumed to be price takers in the upstream market. Price taking in just the extreme case where the upstream firms have all the bargaining power. Our results would not change if the bargaining power were more equally distributed. The crucial assumption is that pricing is linear. Linear prices are justified when the downstream firms could bootleg. In the second stage, *D* and *I* choose the downstream outputs given the input price. (The equilibrium in this downstream market generates the derived demand curve for the input by firm *D* at the first stage.) Firm *I* produces the input for its own use to order. The sequential decision-timing structure makes intuitive sense, because transportation takes time and the input must be available to downstream firms before they can manufacture the final good.

We make the following assumptions about the demand function and technology.

Assumption 3.1. *The demand function for final good is* $p_y = a - b(y_I + y_D)$, *where* p_y *is the price of the final good and a and b are positive constants.*

Assumption 3.2. *Firms have constant returns to scale in the production of input and final good. The final good is produced with a fixed coefficient technology and a unit coefficient for the input x.*

The profit functions of the firms are:

(3.1) $$\pi_U = x_U(p_x - c_x^U);$$

(3.2) $$\pi_D = y_D(p_y - p_x - c_y^D);\text{ and}$$

(3.3) $$\pi_I = x_I(p_x - c_x) + y_I(p_y - c_x - c_y)$$

where p_x is the price of the input, c_x^U is U's marginal cost of input, c_y^D is D's marginal cost of transforming input into final good (that is, the net marginal cost of the final good), c_x is I's marginal cost of input, and c_y is I's net marginal cost of the final good. Note that I's profit function is the same whether it buys or sells input. When it buys input ($x_I < 0$), it produces the remaining input requirements, (y_I+x_I), internally: $\pi_I = x_I p_x - (y_I+x_I)c_x + y_I(p_y - c_y)$, which after rearranging the terms gives equation (3.3).

We further make the following assumption about the size of the downstream market.

Assumption 3.3. *The downstream market is large enough to accommodate all three firms. Specifically, a is sufficiently large that the following two inequalities are satisfied:*

(i) $a > c_y/5 + 2c_x^U + 4c_y^D/5$; *and*

(ii) $a > 13c_y/17 + 7(c_x + c_x^U)/3 + 4c_y^D$.

3.3 Equilibrium

To solve for the subgame perfect equilibrium, we work backwards and solve first for the equilibrium in the downstream market given the input price p_x. It is well known that the equilibrium quantities are:

$$y_D = [a + c_x + c_y - 2(c_y^D + p_x)]/3b; \text{ and} \tag{3.4}$$

$$y_I = [a - 2(c_x + c_y) + c_y^D + p_x]/3b. \tag{3.5}$$

And the equilibrium price is:

$$p_y = (a + c_x + c_y + c_y^D + p_x)/3. \tag{3.6}$$

Substituting the upstream market clearing condition

$$y_D = (x_I + x_U) \tag{3.7}$$

into (3.4) we can solve for the inverse demand for input; this gives the price at which D is willing to buy the input quantity supplied:

$$p_x = [a + c_x + c_y - 2c_y^D - 3b(x_I + x_U)]/2. \tag{3.8}$$

Next we turn to examine the first stage. Inserting p_x from (3.8) into (3.1) we get firm U's profit expressed in terms of x_I and x_U only:

$$\pi_U = x_U\{[a + c_x + c_y - 2c_y^D - 3b(x_I + x_U)]/2 - c_x^U\}. \tag{3.9}$$

We can do the same for firm I. First substitute p_y from (3.6) and y_I from (3.5) into (3.3). Then substitute p_x from (3.8) and we obtain firm I's profit in terms of x_I and x_U only:

$$\begin{aligned} \pi_I = {} & [a - c_x - c_y - b(x_I + x_U)]^2/4b + \\ & x_I\{[a + c_x + c_y - 2c_y^D - 3b(x_I + x_U)]/2 - c_x\}. \end{aligned} \tag{3.10}$$

The first order conditions for firm U's and firm I's equilibrium choice of outputs x_U and x_I are:

(3.11) $$\partial\pi_U/\partial x_U = a + c_x + c_y - 2c_y^D - 2c_x^U - 3bx_I - 6bx_U = 0; \text{ and}$$

(3.12) $$d\pi_I/dx_I = -[a - c_x - c_y - b(x_I + x_U)] + (a - c_x + c_y - 2c_y^D - 3bx_U - 6bx_I) = 0.$$

Firm I is aware that higher input sales or lower input purchases will decrease its downstream profits (the first big term in (3.12), in square brackets), because the final-good price decreases. The equilibrium quantities in the upstream market are thus:

(3.13) $$x_U = (5a + 5c_x - c_y - 10c_x^U - 4c_y^D)/24b; \text{ and}$$

(3.14) $$x_I = (-a - c_x + 5c_y + 2c_x^U - 4c_y^D)/12b.$$

(Notice that, thanks to Assumption 3.3, the right-hand side of (3.13) *is* positive, so that firm U does not drop out of the upstream market.) The equilibrium price is:

(3.15) $$p_x = (5a + 5c_x - c_y + 6c_x^U - 4c_y^D)/16.$$

Finally, we can return back to the downstream market to ascertain the subgame perfect equilibrium in terms of the exogenous parameters only. When we insert (3.15) into (3.4) and (3.5) we get the equilibrium quantities:

(3.16) $$y_D = (a + c_x + 3c_y - 2c_x^U - 4c_y^D)/8b; \text{ and}$$

(3.17) $$y_I = (7a - 9c_x - 11c_y + 2c_x^U + 4c_y^D)/16b.$$

(Notice that, again thanks to Assumption 3.3, the right-hand-side of (3.16) *is* positive, so that firm D will not exit from the downstream market.) By substituting (3.15) into (3.6) we obtain the equilibrium price:

(3.18) $$p_y = (7a + 7c_x + 5c_y + 2c_x^U + 4c_y^D)/16.$$

Lastly, suppose that foreclosure is an equilibrium (the integrated firm neither sells or buys input). It is a simple matter to repeat the earlier analysis with x_I set equal to zero. The subgame perfect equilibrium quantity and price in the upstream market are:

(3.19) $$\hat{x}_U = (a + c_y + c_x - 2c_x^U - 2c_y^D)/6b; \text{ and}$$

(3.20) $$\hat{p}_x = (a + c_y + c_x + 2c_x^U - 2c_y^D)/4$$

where the hats denote the foreclosure equilibrium. The equilibrium quantities in the downstream market are:

(3.21) $$\hat{y}_D = (a + c_y + c_x - 2c_x^U - 2c_y^D)/6b; \text{ and}$$

(3.22) $$\hat{y}_I = (5a - 7c_y - 7c_x + 2c_x^U + 2c_y^D)/12b.$$

And the equilibrium price is:

(3.23) $$\hat{p}_y = (5a + 5c_y + 5c_x + 2c_x^U + 2c_y^D)/12.$$

3.4 Input Market

In this Section we examine firm *I*'s strategy in the input market. *I*'s strategy depends crucially on its downstream net marginal costs. It is useful to define the critical value $\underline{C}$ for c_y to be that which makes the right-hand-side of (3.14) equal to zero:

$$\underline{C} = (a + c_x - 2c_x^U + 4c_y^D)/5. \tag{3.24}$$

Proposition 3.1 follows straightforward from (3.14).

Proposition 3.1. *The integrated firm will*
(i) buy input if and only if $c_y < \underline{C}$,
(ii) foreclose if and only if $c_y = \underline{C}$ *and*
(iii) sell input to its rival if and only if $c_y > \underline{C}$.

The critical value $\underline{C}$ gives the point at which *I* will switch from overbuying to supplying input. It follows from Assumption 3.3 that $\underline{C} > 0$.

When the integrated firm is very efficient in final good production it will overbuy. Because input is produced with constant returns to scale technology *I* cannot completely monopolize the downstream market by buying up the input. However, by buying some input *I* can raise rival's marginal costs (overbuying increases its own average but not marginal costs) and gain a competitive advantage in the final good market.

The condition for overbuying in Proposition 3.1 can be shown to be equivalent to

(3.25) $$(\tilde{p}_y - c_x - c_y) - (\tilde{p}_x - c_x) > 0.$$

where the prices are evaluated at an overbuying equilibrium. Equation (3.25) is thus a necessary condition for overbuying. Accordingly, a sufficient condition for not buying input is:

(3.26) $$(\tilde{p}_y - c_x - c_y) - (\tilde{p}_x - c_x) \leq 0.$$

Corollary 3.1. *The integrated firm will not buy input if at an overbuying equilibrium the price-cost margin of the final good, $(\tilde{p}_y - c_x - c_y)$, is smaller than or equal to the price-cost margin of the input, $(\tilde{p}_x - c_x)$.*

Here the price-cost margin of the input is the *loss* per unit of input when *I* buys it in the market at price $\tilde{p}_x$ rather than uses internal supplies at a lower cost c_x.

The intuition behind Corollary 3.1 is the following. Suppose overbuying is an equilibrium and the equilibrium prices are $\tilde{p}_y$ and $\tilde{p}_x$. Suppose also that overbuying is such a powerful strategy that when *I* buys an amount x of input, *D*'s final good production decreases by x and *I*'s final good production increases by the same amount.[21] (Note that the final good price does not change.) If *I* stopped buying input its profit from final good would decrease by $x(\tilde{p}_y - c_y)$ but it would save $x\tilde{p}_x$. Subtract xc_x from

[21]Clearly overbuying does not improve *I*'s competitive position this much. But the reasoning in the text gives a *sufficient* condition for not buying input.

both terms and we have the condition in Corollary 3.1. Therefore if $(\tilde{p}_y - c_x - c_y)$ is smaller than $(\tilde{p}_x - c_x)$, overbuying cannot be an equilibrium.

Note that in fact this argument is rather general: it does not depend on the linearity of demand. Corollary 3.1, and indeed Corollaries 3.2 and 3.3 below, are true without Assumption 3.1.

When the integrated firm is quite inefficient in the downstream market it will supply input to its rival. Its upstream unit makes a profit from selling input but ultimately this input competes with its own final good and lowers the downstream unit's profit.

In line with Corollary 3.1, we have:

Corollary 3.2. *The integrated firm will not sell input to its rival if at a vertical supply equilibrium the profit margin of the final good, $(p_y - c_x - c_y)$, is greater than or equal to the profit margin of the input, $(p_x - c_x)$.*

We can prove Corollary 3.2 by the same kind of reasoning that was behind Corollary 3.1. Suppose first that firm I sells input to firm D and the equilibrium prices are p_x and p_y. This strategy cannot be an equilibrium if firm I could profitably deviate from it given rivals' strategies. In particular, if firm I shifted all the input it currently sells to firm D to internal final good production[22], I would earn a profit margin $(p_y - c_x - c_y)$ per unit. (Note that the final good price would not change, because firm D's final

[22]This is not necessarily the *best* strategy for firm I to follow; it may choose to produce internally at a different level. But the reasoning in the text gives a *sufficient* condition for foreclosure.

good production would decrease by the same amount as firm I's final good production increased. Firm D's input would be reduced to that supplied by firm U.) Clearly then, if $(p_y - c_x - c_y)$ exceeded $(p_x - c_x)$ it would not be optimal for firm I to supply D.

Foreclosure becomes a knife-edge in our model. Everywhere else except when $c_y = \underline{C}$ the integrated firm is active in the input market; I either sells or buys input. We can show that:

Corollary 3.3. *The integrated firm will sell input to its rival if at the foreclosure equilibrium the profit margin of the final good,* $(\hat{p}_y - c_x - c_y)$*, is smaller than the profit margin of the input,* $(\hat{p}_x - c_x)$*.*[23]

The intuition behind Corollary 3.3 is straightforward. Suppose first that foreclosure is an equilibrium and the equilibrium prices are $\hat{p}_y$ and $\hat{p}_x$. Firm I considers if it can make more profit by selling one unit of input to firm D and reducing final good production by one unit. If the profit margin for its unit of input sold to firm D, $(\hat{p}_x - c_x)$, is greater than that for the unit of input produced internally, $(\hat{p}_y - c_x - c_y)$, then it is clearly not optimal for firm I to foreclose.

Above we have derived intuitive conditions in terms of the difference in the profit margins of final good and input. Now we turn to examining what the relative costs can tell us. First suppose that the downstream firms are equally efficient ($c_y = c_y^D$). Substituting $c_y = c_y^D$ in (3.14) and using Assumption 3.3 shows that the integrated firm will overbuy. This proves that:

[23] Spencer and Jones (1991) obtain an equivalent to this Corollary.

Corollary 3.4. *If the integrated firm is more or equally efficient in transforming the input into the final good than its unintegrated rival, then it will buy input.*

When the integrated firm is more efficient in the final good production, it can afford to raise its average costs in order to have an even better competitive position. While only if the integrated firm is less efficient, it will ever sell input. If it is in any case in weak position in the final good market, it may as well sell some input and make a profit from it.

Note that above we have been comparing the *net* marginal costs of the final good. The *gross* marginal cost of final good for D is ($p_x + c_y^D$) and for I is ($c_x + c_y$). Even if I's gross marginal cost of the final good were lower than D's, it can nevertheless be optimal for I to sell input to D. I's net marginal cost (c_y) can still be higher than D's (c_y^D). The divergence between gross and net cost differentials arises simply because of the fact that the input price is greater than marginal cost ($p_x > c_x$).

If also the upstream firms are equally efficient ($c_x = c_x^U$ and $c_y = c_y^D$) it is still true that the integrated firm overbuys. Salinger (1988) showed that when firms are equally efficient, the integrated firm will not sell input. Our results confirm it. However, we also show that overbuying would arise in his model, a strategy he did not consider.

Equal upstream costs alone ($c_x = c_x^U$) do not fix the integrated firm's strategy. Depending on the relative downstream costs, I either sells or buys input. Spencer and Jones (1991) suggest that identical upstream costs make vertical supply unprofitable. We show that the crucial issue is the relative downstream costs.

3.5 Final Good Market

If I is very inefficient in the downstream market it may exit and operate only in the upstream market. The calculations in Section 3.3 ignored the non-negativity constraint $y_I \geq 0$. In this Section we investigate if this constraint is indeed slack or binding by checking whether the right-hand-side of (3.17) is positive or not. Firm I's decision in which markets to operate depends crucially on its downstream net marginal cost. Again, it is useful to define the critical value $\overline{C}$ for c_y to be that which makes the right-hand-side of (3.17) equal to zero:

$$\overline{C} = (7a - 9c_x + 2c_x^U + 4c_y^D)/11. \tag{3.27}$$

Proposition 3.2 follows directly from (3.17).

Proposition 3.2. *The integrated firm operates in the downstream market if and only if* $c_y < \overline{C}$.

This critical value $\overline{C}$ gives the point at which I will exit from the downstream market.

The right-hand-side of (3.17) can be shown to be equivalent to the profit margin of the final good, $(p_y - c_x - c_y)$.

Corollary 3.5. *The integrated firm will exit from the downstream market if its profit margin of the final good,* $(p_y - c_x - c_y)$, *is non-positive.*

In I's strategy in the input market there was a trade-off between gain in the final market and loss in the input market. Accordingly, the decision depended on the difference in the price-cost margins of the two products. The exit decision is simpler: it depends solely on the profit margin of the final good. When firm I is in the position to choose downstream output, the upstream competition is already past.

3.6 Comparative Statics

Propositions 3.1 and 3.2 are illustrated in Figure 3.2, which shows firm I's final good production as a fraction of its total production, $y_I/(y_I+x_I)$. This fraction describes firm I's *optimal product mix.* (Thanks to fixed coefficient technology with constant returns to scale, and a unit coefficient, we can properly add y_I to x_I.) When I overbuys, x_I is negative and the fraction is greater than one.[24] Under foreclosure x_I is equal to zero and the fraction is equal to one. With vertical supply the fraction is smaller than one. Finally, when I exits y_I and the fraction are equal to zero. It follows from Assumption 3.3 that $0 < \underline{C} < \overline{C}$, that is the regions for overbuying, vertical supply and exit exist.

Below, we will be examining the comparative static properties for firm I's product mix; and also the absolute levels of its input sales or purchases and final good production. Remember in what follows that the optimal x_I (and $\underline{C}$) depend on the difference in the profit margins of the final good and input. And the optimal y_I (and $\overline{C}$) depend on the profit margin of the final good only.

First, however, it is useful to understand how *prices* move with changes in the parameters. One would expect that both input and final good prices would increase in the demand and cost parameters. But in fact there are two important exceptions.

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[24]Assumption 3.3 guarantees that $(y_I+x_I) > 0$.

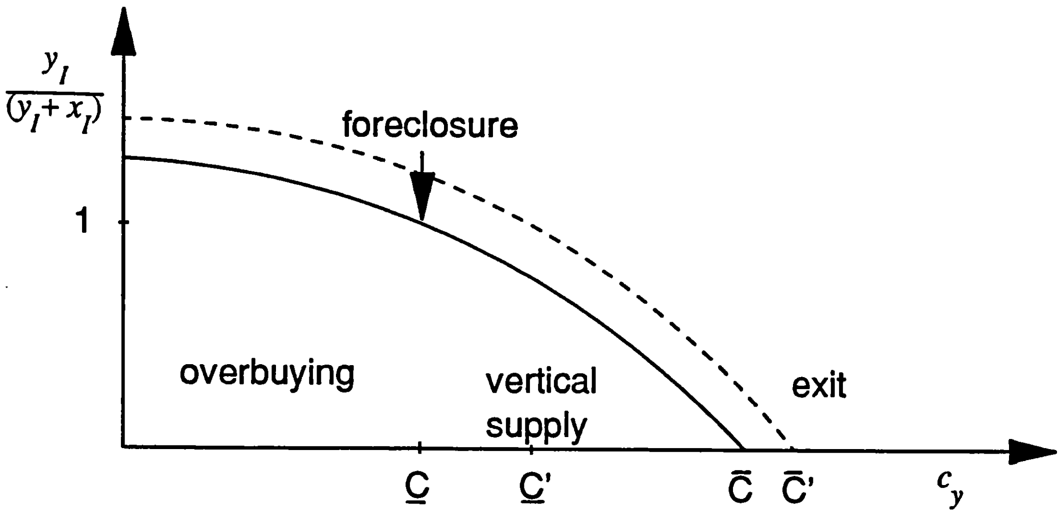

Figure 3.2

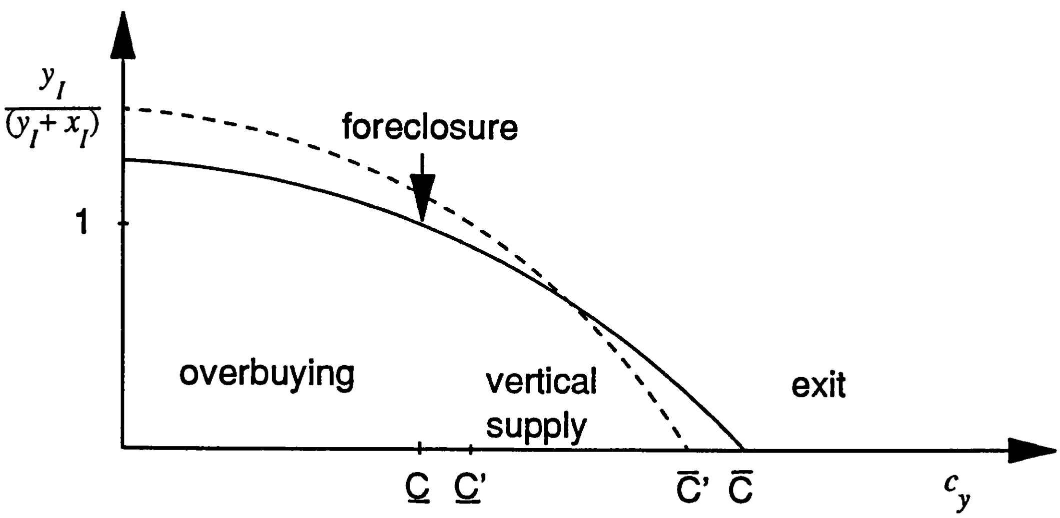

Figure 3.3

(*i*) An increase in c_y^D decreases the input price because the demand for input by firm *D* will shift back. (*ii*) An increase in c_y also decreases the input price. With vertical supply firm *I* will increase its supplies to *D* when selling input becomes relatively more profitable than selling final good. With overbuying the demand for input by *I* will decrease because it is more difficult the gain a competitive advantage in the downstream market.

With these properties in mind, let us now turn to firm *I*'s output decisions, in particular its choice of *quantities* x_I and y_I, and its *product mix* $y_I/(x_I+y_I)$.

An increase in the demand for the final good (a rise in *a*) increases the final good price more than the input price. That is, the difference in the profit margins of the final good and input (and $\underline{C}$), and the profit margin of the final good (and $\overline{C}$), all increase. Accordingly, an increase in demand will increase *I*'s final good production and decrease (increase) the amount of input *I* sells (buys). Overbuying becomes more likely and exit less likely. This effect is illustrated by the broken line in Figure 3.2.[25]

An increase in firm *D*'s downstream marginal cost (c_y^D) increases the final good price but decreases the input price because the demand for input by firm *D* shifts back. The difference in the profit margins (and $\underline{C}$), and the profit margin of the final good (and $\overline{C}$), both increase - and overbuying becomes more likely and exit less likely. Firm *I*'s input sales decrease (purchases increase) and final good production increases; the share of final good in its product mix rises.

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[25]The intercept in the vertical axis does not necessarily increase; the change in product mix is ambiguous.

By contrast, an increase in firm *I*'s upstream marginal cost (c_x) will increase final good price more than the input price and, consequently, will lower the profit margin of the final good less than that of the input. That is, the difference in the profit margins and $\underline{C}$ increase but $\overline{C}$ decreases. The broken line in Figure 3.3 illustrates this comparative static effect; notice that overbuying and exit both become more likely and vertical supply less likely. In absolute terms, both input sales and final good production decrease and input purchases increase. (The shift in the product mix is ambiguous.)

A *decrease* in firm *U*'s upstream marginal cost (c_x^U) will decrease the final good price less than the input price. We have again the case in Figure 3.3 where both overbuying and exit become more likely and vertical supply less likely and firm *I* will sell less of both products.

Finally, an increase in firm *I*'s net downstream marginal cost (c_y) decreases the profit margin of the final good more than that of the input. Accordingly, the difference in the profit margins decreases and the share of the input in firm *I*'s product mix increases. In Figure 3.3 this would mean moving along the solid line to the right. The absolute level of firm *I*'s input sales increases and final good production and input purchases decrease.

Active Integrated Firm in the Input Market

Above we have examined only the signs of the comparative static effects. Their magnitudes are of interest too. For brevity, here we focus only on the magnitudes of the firms' responses to changes in costs. How are these magnitudes affected by the presence of an active integrated firm in the input market?

We rewrite the coefficients from equations (3.13), (3.16), (3.17), (3.19), (3.21) and (3.22) in Table 3.1 to make the comparison of the coefficients clearer.

	$x_U = \tilde{x}_U$	$\hat{x}_U$	$y_D = \tilde{y}_D$	$\hat{y}_D$	$y_I = \tilde{y}_I$	$\hat{y}_I$
c_x^U	-20	-16	-12	-16	6	8
c_x	10	8	6	8	-27	-28
c_y^D	-8	-16	-24	-16	12	8
c_y	-2	8	18	8	-33	-28

Note: All the coefficients should be divided by $48b$. The coefficients for x_I are omitted, since they all equal zero when there is foreclosure.

Table 3.1

One clear and interesting message from the Table is summarized in the following proposition.

Proposition 3.3. *If the integrated firm is active in the input market, firms react more aggressively to changes in costs (either their own, or their rivals') in their own market; but they react less aggressively to changes in costs in the other market.*[26]

Let us take an example. Consider firm *D*'s response to a change in firm *I*'s marginal cost of input, c_x. From the Table we see that the marginal effect on firm *D*'s production, y_D, is 6/48*b* if *I* is active in the input market but 8/48*b* if there is foreclosure. Notice that here we are looking at a change in costs in the "other" market; Proposition 3.3 tells us that firm *D* should respond less aggressively to a change in c_x if *I* is active (6/48*b* < 8/48*b*). Now consider firm *U*. The marginal effect on firm *U*'s production, x_U, of a change in c_x is 10/48*b* if *I* is active, but only 8/48*b* if there is foreclosure. Again, this marries with Proposition 3.3: firm *U* responds more aggressively to a change in costs in its own market if *I* is active in the input market (10/48*b* > 8/48*b*).

In brief, the economics behind these comparisons is the following.

First, consider the case of foreclosure. There is a single chain of effects: a rise in firm *I*'s marginal cost of input, c_x, causes its supply of final good, y_I, to fall. By the usual Cournot logic (strategic substitutes) firm *D*'s supply, y_D, will rise and its demand for input rises concomitantly. Firm *U* (the monopoly supplier in this case of foreclosure) sells more: x_U rises.

Now, if the integrated firm is active in the input market, there is an additional chain of effects. The rise in c_x causes firm *I* to reduce its supply x_I to the input market

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[26]Note that the coefficient for c_y even changes sign in the equation for x_U.

or to increase its demand for input. So the input price, p_x, rises. This rise in input price means that firm D supplies less; y_D falls. Notice that this works in the *opposite* direction than the (dominant) effect given in the previous paragraph. That is, an active integrated firm makes firm D to respond *less* aggressively to a rise in c_x, as per Proposition 3.3. Firm U, on the other hand, will choose to supply more to the input market (x_U rises) as p_x rises. This *reinforces* the effect given in the previous paragraph: vertical supply makes firm U respond *more* aggressively to a rise in c_x - again, as per Proposition 3.3.

From Proposition 3.3 we know that these comparative effects are in fact general, at least to this linear model.

3.7 Toughness of Competition

The previous Sections focused on the cost asymmetries as a driving force behind the integrated firm's strategy in the input market. The second natural candidate affecting the profitability of vertical supply and overbuying is the toughness of competition in the final good market. Now we assume that the firms are equally efficient with c_x the marginal cost of input and c_y the net marginal cost of the final good. We further assume that the demand for firm i's final good takes the following form:

$$p_i = [\alpha(1+2\rho) - (1+\rho)y_i - \rho y_j]/(1+2\rho) \tag{3.28}$$

where $\alpha > 0$ and $0 \leq \rho \leq \bar{\rho}$. This demand function has the property that in the terminology of Shaked and Sutton (1990) the expansion effect is constant and the competition effect is a function of ρ. We can therefore refer to ρ as the degree of competition. The role of ρ is more transparent from the inverse of this demand function: $y_i = \alpha - p_i - \rho(p_i - p_j)$. ρ describes how badly the firm suffers from charging a higher price than its rival. Solving the model gives:

$$x_I = \frac{(a-c_x-c_y)(1+2\rho)(1-\rho^2+\rho)}{(2+3\rho)(2+4\rho+\rho^2)} \tag{3.29}$$

It follows from equation (3.29) that:

Proposition 3.4. *If there is no competition in the final good market the integrated firm will sell input. If competition is very tough the integrated firm will overbuy.*

When the two downstream firms are monopolies in their respective markets ($\rho = 0$) the integrated firm sells input ($x_I > 0$). It makes a profit from vertical supply and that has no harmful effects whatsoever on its profit from the final market. When the degree of competition increases input sales start to have an adverse effect on the downstream profit. When ρ is high enough to make $(1-\rho^2+\rho)$ negative the integrated firm starts buying input. By overbuying the integrated firm can gain a dominant position in a highly competitive market.

3.8 International Trade

One interpretation of our model is that firm *D* is in the domestic country, and *U* and *I* are foreign exporters. Firm *D* sells the final good in the domestic market in competition with *I*'s exports. Furthermore, *D* is dependent for its input on imports from *U* and *I*. Cost asymmetries arise naturally in this international trade setting as a consequence of different endowments or technologies. We further include transportation costs for the integrated firm. Firm I incurres the per unit transportation costs t_x for the input and t_y for the final good. We do not include transportation costs for the unintegrated firms: the marginal cost of the unintegrated firms can be thought as consisting of the marginal cost of production *and* transportation costs.[27] D can be assumed to have zero transportation costs because it is in the domestic country. Furthermore, we could interpret t_x and t_y as tariffs, which are zero for the domestic firm.

Now the integrated firm's profit function is:

(3.3)' $$\pi_I = x_I(p_x - c_x - t_x) + y_I(p_y - c_x - c_y - t_y) \text{ if } x_I > 0$$

(3.3)" $$\pi_I = x_I(p_x - c_x) + y_I(p_y - c_x - c_y - t_y) \text{ if } x_I < 0$$

We need two variants because when I sells input it pays the transportation cost itself but when I buys input the transportation cost is incurred by U.

The sufficient condition for not selling input given by Corollary 3.2 is now:

(3.30) $$(p_y - c_x - c_y - t_y) - (p_x - c_x - t_x) \geq 0$$

We can obtain a (stronger) sufficient condition by comparing the difference in the profit margins with firm D's profit margin (which we know is positive, since y_D is positive). Specifically, subtracting firm D's profit margin, $(p_y - c_y^D - p_x)$, from the difference in firm I's profit margins the price terms drop out, and we are left with a sufficient condition which only involves underlying cost parameters:

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[27]We also assume that the transportation cost U pays for selling to D and I is equal.

(3.31) $$(c_y + t_y) \leq (c_y^D + t_x)$$

Corollary 3.6. *If it is cheaper to produce the final good in the exporting country and transport it to the domestic country (with unit cost* $(c_y + t_y)$*) than it is to transport the input to the domestic country and produce the final good there (with unit cost* $(c_y^D + t_x)$*), then the integrated firm will not sell input to its rival.*

Notice that condition (3.31) depends on the relative efficiency of the downstream firms, $(c_y - c_y^D)$, and on the difference in the transportation costs, $(t_y - t_x)$. Suppose first that the transportation costs are equal $(t_y = t_x)$. This reproduces the result of Corollary 3.4: when the integrated firm is more efficient in the final good production it will not sell input. Whereas if firm *I* is less efficient in final good production than firm *D*, it can be profitable for *I* to sell part of its input production to its rival - even though that will lower the price for its own final good. Note that the difference in net marginal costs of the final good need not be due to different production technologies. It may be that the costs of inputs other than *x* differ between *I* and *D*. For example the labour costs may well differ across countries.

Next, assume instead that that firms *D* and *I* are equally efficient in final good production ($c_y = c_y^D$). Now it can be optimal for firm *I* to sell input to firm *D* if the final good is more expensive to transport than the input. When we interpret the transportation costs as tariffs this comparison implies that the domestic country can induce supply of input by imposing a greater tariff on the final good than on the input.

Transportation cost for the input introduces a wedge between the price-cost margins for input when *I* sells or buys input. This means that foreclosure is not a knife-edge anymore. Between the regions for overbuying and vertical supply there is a region where the integrated firm does not participate in the input market either as a buyer or seller.

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4 Endogenous Industry Structure in Vertical Duopoly

4.1 Introduction

We aim to develop some fairly general properties for the degree of integration in the industry. We first analyse in detail a model where there is Cournot competition in the input market. Then we compare the results to those of a model where nonlinear prices are applied in the input market (Bertrand competition). We show that in both cases qualitatively the same pattern of integration emerges.

Incentives for integration in a vertical duopoly are driven by three externalities. First, we have the vertical externality of double-marginalization. Second, a horizontal externality emerges when downstream firms compete in the product market. Third, there is an excessive supply incentive: every additional unit of input an upstream firm sells to one downstream firm reduces the profit of the other downstream firm by depressing the final good price. An unintegrated upstream firm does not take this marginal effect on the downstream firm's profit into account and therefore sells too much input to the other firm. Double-marginalization effect is proportional to the upstream firm's profit margin since that is the distortion in question. Excessive supply incentive is the greater, the higher is the downstream firm's profit margin because then the loss from increased rival's output is highest. The horizontal effect depends on both margins. High downstream margin tells that there is a large gain from expanding output. While high upstream margin means that the input cost will be much lower after integration.

In our model there are two upstream firms and two downstream firms. The benefit of integration is profit sharing between an upstream firm and a downstream firm: vertical externalities are internalized. Vertical integration does not, of course, internalize the horizontal externality. In fact, integration makes the competition more tough. The cost of integration arises from a loss in efficiency and it is assumed to be fixed. We find that the degree of integration is increasing in the size of the downstream market. The profit margin for both input and final good are higher and therefore all three effects work in the same direction to favour integration. The second result relates to a situation where two upstream firms differ in efficiency. It is intuitively clear that the present model will favour integration by the low-cost firm because it has a higher profit margin and accordingly suffers from greater externalities if unintegrated. Thirdly an industrywide cost increase and higher fixed cost of integration result in a lower degree of integration. All these predictions are robust to the form of competition in the input market.

This analysis also helps us to understand the evolution of the industry structure over time. We predict that in a young market where demand is low and average marginal cost is high (because the learning process is in the beginning) we would see a nonintegrated structure. When the market starts growing and the firms slide down the learning curve the industry becomes more integrated.

Stigler (1951) makes the opposite prediction to us: vertical disintegration is the typical development in growing industries, vertical integration in declining industries. Stigler views production of a final good as a series of distinct functions. Certain functions are subject to decreasing costs. A young market may be too small to support a firm specialized in the function subject to decreasing costs. But when the market expands, the demand for that function becomes sufficient to permit a firm specialized in performing it; the firms spin off the decreasing cost functions and purchase input from the new firm. However, setting up a specialized firm is not the only way to exploit the economies of scale. One integrated firm could produce this input for itself and for the other firms in the industry whatever the size of the market and thus avoid the set-up cost of a new firm. Empirical evidence also suggests that firms frequently integrate as a result of rising, not declining, demand.[28]

Salinger (1988) examines how a vertical merger of successive Cournot oligopolists affects the input and final good price. He finds that a vertical merger does not necessarily increase the prices although a merger leads to foreclosure. The incentives to integrate are the same as in our model. We allow the upstream firms to differ in efficiency and endogenize the integration decision. Ordover, Saloner, and Salop (1990) abstract from double-marginalization by assuming Bertrand competition between equally efficient upstream firms; the input price is driven down to marginal cost. In their model the incentive for integration arises from the assumption that an integrated firm can commit not to supply a rival firm below a certain price. Then the unintegrated upstream firm can raise the input price which will benefit also the integrated firm. Ordover, Saloner, and Salop do not explain why such a commitment is feasible and why the firms have to integrate to be able to commit to such a strategy. In our model foreclosure arises in equilibrium without incredible commitments. In Hart and Tirole (1990) upstream firms set nonlinear prices (essentially Bertrand competition). Low-cost upstream firm has an incentive for integration to restrict competition in the downstream market. Integrated supplier can undercut its high-cost rival slightly, so that the unintegrated firm buys the same total amount as before but now buys from the integrated supplier. This again benefits the integrated firm by

[28]Porter and Livesay (1971), 132, and Chandler (1977), 490.

raising rival's costs. We show that the predictions this model gives for the degree of integration are qualitatively the same as ours.

Several papers focus on integration decisions in a setting where two upstream firms sell exclusively to two respective downstream firms.[29] Double-marginalization and horizontal externality arise in this setting. If integration induces the rival to be less aggressive, the horizontal externality gives another reason for integration. In other words, when final goods are *strategic substitutes* it is good to be a top dog in the downstream market and integrate to have a lower marginal cost.[30] Both vertical and horizontal externality call for integration. However, when final goods are *strategic complements* the profitable strategy is to be a puppy dog in the downstream market. To eliminate the vertical externality the input price should be equal to marginal cost but to relax the horizontal externality the input price should be higher. Relaxing competition proves to be more important. Tougher competition represents the cost of integration.

The rest of the Chapter is organized as follows. In Section 4.2 we introduce our model. Section 4.3 compares the industry structures and Section 4.4 derives the equilibrium industry structure. In Section 4.5 welfare issues are analysed. In Section 4.6 we compare our model to Hart and Tirole's (1990).

[29]E.g. Vickers (1985), Fershtman and Judd (1987), Bonnano and Vickers (1988), Lin (1988), Gal-Or (1991) and (1992).

[30]We use the terminology of Bulow et al. (1985) and Fudenberg and Tirole (1984).

4.2 The Model

There are two upstream firms, *U1* and *U2*, producing a homogeneous input, x_i, and two downstream firms, *D1* and *D2*, producing a homogeneous final good, y_i. Firms have constant returns to scale in the production of input and final good. Furthermore, the final good is produced with a fixed coefficient technology and a unit coefficient for the input x. *U1* is more efficient in producing the input than *U2*; $c_1 \leq c_2$ where c_i is the marginal cost of input for *Ui*. The downstream firms are equally efficient and, without loss of generality, we assume that transforming input into the final good is costless.

Demand function for the final good is linear $p_y = a - b(y_1 + y_2)$, where p_y is the price of the final good and a and b are positive constants. We further make the following assumption about the size of the downstream market.

Assumption 4.1. *The market for the final good is big enough to accommodate both firms. Specifically, a is sufficiently large that the following inequalities are satisfied:*

(i) $a > c_i$ *for all i, and*

(ii) $a > 11c_2 - 10c_1$.

Our focus is on the question when the firms will stay independent and when they will vertically integrate. (Horizontal mergers are ruled out by antitrust statutes.) Four structural configurations can emerge: nonintegration, partial integration by the low-cost firm, partial integration by the high-cost firm, and full integration.

The decision timing structure of our model is illustrated in Figure 4.1.

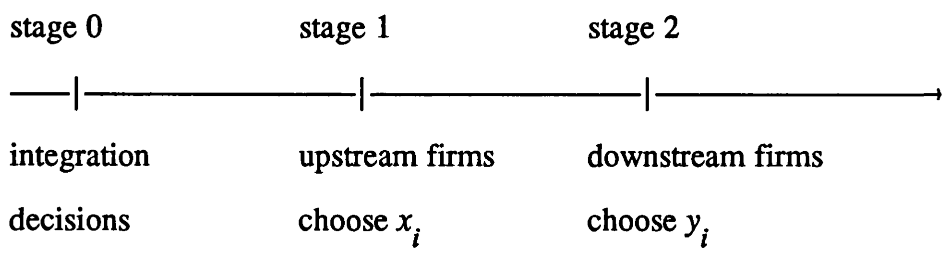

Figure 4.1

At stage 0 the firms decide whether to integrate with a full understanding of the consequences of this decision for the competition in the upstream and downstream markets. We assume that *Ui* can integrate only with *Di.* Because the downstream firms are identical, this is not a restrictive assumption. Integration is irreversible. The benefit of integration is profit sharing between an upstream firm and a downstream firm; the vertical externalities are internalized. The cost of integration arising from a loss in efficiency is assumed to be fixed, *E*. An integrated firm may be less efficient because a non-owning manager has lower incentives to come up with good ideas to reduce production costs or to raise quality because this investment is expropriated by the owner of the firm. Also, there may be a loss in information about the non-owning manager's performance, and therefore less incentive to make improvements. Furthermore, there may be legal costs of the merger.

At stage 1 the upstream firms choose how much input to sell to the downstream firms given the industry structure. In the input market the upstream firms are Cournot duopolists whereas the buyers of input (the downstream firms) take the input price as given. Price taking is just the extreme case where the upstream firms have all the bargaining power. The nature of the vertical externalities would not change if the bargaining power were more equally distributed. The crucial assumption is that pricing is linear. Linear prices apply when the downstream firms could bootleg. When the upstream firms can observe whether or not the downstream firm carries his product, two-part tariffs are optimal contracts (Rey and Tirole (1986)). This alternative assumption about upstream competition is discussed in Section 4.6. Only unintegrated firms are active in the input market; the integrated firms neither sell nor buy input in the market. Chapter 3 showed that when the firms are equally efficient in final good production the integrated firm does not sell input to its rival. However, under partial integration the integrated firm would buy input to raise its unintegrated rival's costs. Including these input purchases would only slightly change the tendencies for integration but would unnecessarily complicate the analysis. Therefore we simply assume that the integrated firm will not buy input. (See Chapter 3 for the overbuying strategy of the integrated firm.) We also assume that exclusive dealing contracts where an upstream firm commits to supplying only one downstream firm are not enforceable.

At stage 2 the downstream firms choose the final good production levels given the input price and the industry structure. The equilibrium in this downstream market generates the derived demand curve for the input at stage 1. Downstream firms behave as Cournot duopolists in the final good market. Cournot competition is justified by our assumptions about the decision timing structure. The downstream market game is played by firms with capacity constraints and the outcome will be Cournot if c_1 and c_2 are high enough.[31]

The profit function of an integrated firm is:

(4.1) $$\pi_i = y_i(p_y - c_i) - E$$

[31] Tirole (1988), 215.

and the profit functions of unintegrated firms are:

(4.2) $\pi_{Ui} = x_i(p_x - c_i)$; and

(4.3) $\pi_{Di} = y_i(p_y - p_x)$

where p_x is the input price.

4.3 Comparison of Industry Structures

We solve the model by backward induction starting from the last stage. First, we solve for the equilibrium in the downstream market given the input price and the industry structure. The equilibrium in the downstream market generates the derived demand function for the input. Second, we insert this demand function in the upstream firms' profit functions and solve for the equilibrium in the upstream market given the industry structure. Third, we return back to the downstream market to ascertain the subgame perfect equilibrium. Substituting the input price we get the equilibrium in the downstream market in terms of exogenous parameters and the industry structure. (See Appendix 4 for details.) This is how we obtain the profit functions relevant for the stage 0 integration decisions. Under nonintegration (NI) the profits for the upstream and downstream firms are:

(4.4) $\pi^{NI}_{Ui} = 2(a - 2c_i + c_j)^2/27b$; and

(4.5) $\pi^{NI}_{Di} = (2a - c_1 - c_2)^2/81b$.

Under partial integration by Ui and Di (PIi) the profits are:

(4.6) $$\pi^{PIi}_{Uj} = (a - 2c_j + c_i)^2/24b;$$

(4.7) $$\pi^{PIi}_{Dj} = (a - 2c_j + c_i)^2/36b; \text{ and}$$

(4.8) $$\pi^{PIi}_{i} = (5a - 7c_i + 2c_j)^2/144b - E.$$

And under full integration (FI) we have:

(4.9) $$\pi^{FI}_{i} = (a - 2c_i + c_j)^2/9b - E.$$

Incentives for integration are driven by three externalities. Consider first a successive monopoly (Spengler (1950)). The vertical externality of *double-marginalization* arises because an unintegrated downstream firm does not take the upstream firm's marginal profit into account when output is increased. Because the downstream firm cares only about its own profit, it tends to make decisions that lead to too low a consumption of input; the industry produces less than the monopoly output. Integration internalizes this externality and enables the industry to earn monopoly profits. The incentive for integration is the greater, the greater is the distortion (p_x - c_i).

Next consider an industry where two upstream firms sell exclusively to two respective downstream firms. Now a *horizontal externality* emerges; the downstream firms destroy profits by competing. Integration does not internalize the horizontal externality since, by assumption, an upstream firm can integrate with only one downstream firm. However, integration has a horizontal effect. High marginal costs (nonintegration) enable the downstream firms to restrict industry output. But given that the rival has high marginal cost the other firm has an incentive to integrate. This will result in lower output by the rival (because reaction functions are downward sloping) which has a positive first order effect on the merged firm's profit. Therefore integration increases the joint profit of the vertical structure not only because double-marginalization is eliminated but also because integration makes the firm a top dog in the downstream market. In the second vertical merger the horizontal effect is the greater, the larger is the change in the merged firm marginal cost, $(p_x^{PIj} - c_i)$, and the higher that firm's profit margin was originally, $(p_y^{PIj} - p_x^{PIj})$.[32] The latter term describes the benefit from expanding output. If the industry output was already very high (low profit margin) there is little gain from expanding output. In the first vertical merger also the input price for the rival changes. Therefore the horizontal effect depends on how much more favourable the cost change is for the merged firm, $(p_x^{NI} - c_i - p_x^{NI} + p_x^{PIi}) = (p_x^{PIi} - c_i)$, and how profitable the expansion of output is, $(p_y^{NI} - p_x^{NI})$.[33]

Lastly suppose that the upstream firms can sell to both downstream firms. Then an *excessive supply incentive* arises. An unintegrated upstream firm *Ui* ignores that every unit of input it sells to *Dj* depresses the final price and reduces *Di*'s profits by $|y_i(\partial p_y/\partial y_j)|$ which is equivalent to *Di*'s profit margin. An unintegrated *Ui* is selling too $\eta|y_i(\partial p_y/\partial y_j)|$ to *Dj* compared with the level that would maximize the joint profit of *Ui-Di*. Excessive supply incentive arises only in nonintegrated industry since under partial integration the integrated firm does not buy input from the unintegrated upstream firm. Table 4.1 summarizes these three effects.

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[32]Taylor series give the following expression for the horizontal effect:

$$(\partial\pi_i^{PIj}/\partial y_j)(\partial y_j/\partial c_i)(c_i - p_x^{PIj}) + (\partial^2\pi_i^{PIj}/\partial y_j\partial c_i)(\partial y_j/\partial c_i)(c_i - p_x^{PIj})^2/2 =$$
$$(p_y^{PIj} - p_x^{PIj})(p_x^{PIj} - c_i)/3b + (p_x^{PIj} - c_i)^2/9b$$

[33]The expression for the horizontal effect of the first vertical merger is very complicated and therefore we omit it here. Also what we say in the text is a simplification but serves well to help the intuition.

	NI	PIj
Excessive supply	$(p_y^{NI} - p_x^{NI})$	0
Double-marginalization	$(p_x^{NI} - c_i)$	$(p_x^{PIj} - c_i)$
Horizontal effect	$(p_y^{NI}-p_x^{NI}),(p_x^{PIi}-c_i)$	$(p_y^{PIj}-p_x^{PIj}),(p_x^{PIj}-c_i)$

Table 4.1

We denote by π_i the vertical structure *Ui-Di*'s joint profit whether it is integrated or not; $\pi_i = \pi_{Ui} + \pi_{Di}$. $\hat{\pi}_i$ stands for the variable profit, that is, profit gross of integration cost. Thus a vertically integrated firm's profit is $\pi_i = \hat{\pi}_i - E$ and an unintegrated firm's profit is $\pi_i = \hat{\pi}_i$. We can easily obtain the following observation about profit levels *ignoring the integration cost.*

Observation 4.1. $\pi_i^{PIj} < \hat{\pi}_i^{FI} < \pi_i^{NI} < \hat{\pi}_i^{PIi}$

Proof: In Appendix 4.

First, integration increases the variable profit of the vertical structure ($\hat{\pi}_i^{PIi} > \pi_i^{NI}$ and $\hat{\pi}_i^{FI} > \pi_i^{PIj}$). The vertical externalities are internalized and in addition integration has a positive horizontal effect as explained earlier. Second, vertical integration imposes a negative externality on the rival ($\pi_i^{PIj} < \pi_i^{NI}$ and $\hat{\pi}_i^{FI} < \hat{\pi}_i^{PIi}$). The merged firm competes more aggressively in the downstream market which makes the rival less aggressive and lowers its profits. Also the upstream unit has lower profits because the demand for input has decreased. Third, each firm is worse off under full integration than under nonintegration ($\hat{\pi}_i^{FI} < \pi_i^{NI}$). Under full integration *both* firms have lower marginal costs and are more aggressive and, consequently, destroy profits by competing. Although the vertical externalities are internalized the negative horizontal effect dominates and joint profits are lower under full integration than under nonintegration. Figure 4.2 helps us to understand this result. The solid line shows the monopoly output (assume for a moment that $c_1 = c_2$). The closer the equilibrium is to the solid line, the higher is the producer surplus ignoring the integration costs. We can see that nonintegration has the advantage of restricting output. In fact, it restricts output too much; the industry produces *less* final good than a monopoly would produce. However, output under nonintegration is closer to the monopoly output than output under full integration.

We will now proceed to discuss the comparative statics for the incentives for integration. We can show that:

Proposition 4.1. *(i) The more efficient firm has a greater incentive to integrate.*
The incentive to integrate is
(ii) decreasing in the degree of integration,
(iii) increasing in the size of the downstream market,
(iv) decreasing in firm's own marginal costs,
(v) increasing in rival's marginal costs, and
(vi) decreasing in the average marginal cost of the industry.

Proof: In Appendix 4.

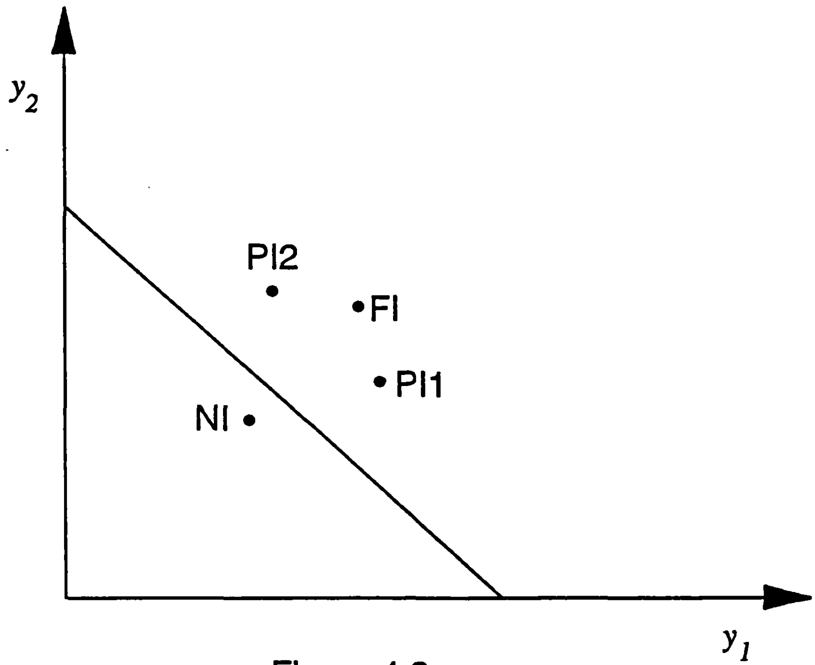

Figure 4.2

The proof of Proposition 4.1 is done with the profit functions (4.4)-(4.9) but here we offer intuitive discussion in terms of the profit margins that describe the three effects of a vertical merger (see Table 4.1).

The low-cost upstream firm has a greater incentive to integrate because its profit margin is greater than that of the high-cost firm; both double-marginalization and horizontal effect are greater for the low-cost firm.

The incentive to integrate is greater under nonintegration than under partial integration. Suppose first that the input price is the same under both structures and the upstream firms do not cross supply (that is *Ui* sells input to *Di* only). Even in this setting there are diminishing returns to integration. Expansion of output is less profitable if the rival has already expanded because of the horizontal effect. Now take into account that the input price actually decreases after a vertical merger. (We explain the reason for the lower input price in the next paragraph.) Therefore the double-marginalization is smaller under partial integration and this further reduces the incentive for integration. Lastly consider cross supply. Under partial integration there is no excessive supply incentive which gives another reason for lower incentive for integration under partial integration.

A vertical merger results in a lower input price because of two effects on the input market. First, the market power of the remaining supplier increases (in fact, it has now monopoly). Second, the merged firm produces more final good which shifts the residual demand curve of the unintegrated downstream firm back. Accordingly, the demand for the input decreases. These two effects on the input price go in the opposite direction. Salinger (1988) showed that the second effect dominates (and the input price decreases) if and only if less than half of the upstream firms are integrated before the merger. This is why we get this rather surprising effect in duopoly: a vertical merger decreases input price although the integrated firm forecloses the upstream market.

The incentive to integrate is the greater, the greater is the market for final good. All three effects work in the same direction to favour integration. The input price (and the upstream firm's profit margin) is increasing in the size of the market and the final price increases more than the input price and therefore the downstream firm's profit margin increases as well.

The incentive is decreasing in the firm's own marginal cost because a per cent increase in marginal cost increases input price by less than a per cent, which results in a lower profit margin for the upstream firm. Downstream firm's profit margin decreases as well because final price increases less than the input price.

An increase in rival's cost increases both double-marginalization and horizontal effect under partial integration. Under nonintegration double-marginalization is higher, excessive supply incentive is lower and the change in horizontal effect is ambiguous since the upstream firm's profit margin increases and the downstream firm's profit margin decreases. The first effect is dominant and the incentive to integrate is higher. An increase in rival's marginal cost increases input price. It is obvious that input price is increasing in rival's marginal cost when it is selling input. When the rival is integrated (and does not sell input) an increase in its marginal cost decreases its final good production which increases the unintegrated firm's final good production, its demand for input and, accordingly, the input price. An increase in rival's marginal cost increases the downstream firm's profit margin under partial integration but decreases it under nonintegration. Under nonintegration it is not only rival's cost that increases because both downstream firms buy input from both upstream firms. We can also show that an industrywide cost increase lowers the incentive to integrate; the firm's own cost effect is dominant.

4.4 Equilibrium Industry Structure

Proposition 4.1 allows us to construct Figure 4.3 which shows the equilibrium industry structures for various pairs of integration costs (E) and efficiency differences (c_2 - c_1). The figure is drawn for given values of market size and average marginal cost of the industry. The efficiency difference cannot be too big, otherwise $U2$ would not have positive output (which Assumption 4.1 ensures). The two middle locuses can cross when the market for final good is quite small. In the diagram we take the simplest case but Proposition 4.2 takes into account the possibility that these locuses may cross. We restrict ourselves to pure strategy equilibria.

Proposition 4.2. *The equilibrium industry structure is*
(i) full integration, if and only if $\hat{\pi}_2^{FI} - \pi_2^{PI1} > E$,
(ii) partial integration by the low-cost firm, if $\hat{\pi}_2^{FI} - \pi_2^{PI1} < E$ *and* $\hat{\pi}_1^{FI} - \pi_1^{PI2} > E$, *or* $\hat{\pi}_2^{PI2} - \pi_2^{NI} < E$ *and* $\hat{\pi}_1^{PI1} - \pi_1^{NI} > E$,
(iii) partial integration by either firm (i.e. there are two equilibria), if and only if $\hat{\pi}_1^{FI} - \pi_1^{PI2} < E$ *and* $\hat{\pi}_2^{PI2} - \pi_2^{NI} > E$
(iv) nonintegration, if and only if $\hat{\pi}_1^{PI1} - \pi_1^{NI} < E$.

To interpret Figure 4.3, consider the effect of greater integration cost and fix the values of other variables. When the integration cost is high neither firm has an incentive to integrate; nonintegration emerges. When we lower the integration cost it becomes profitable for the low-cost firm to integrate; we have partial integration by the low-cost firm. For even lower integration cost also the high-cost firm can bear the

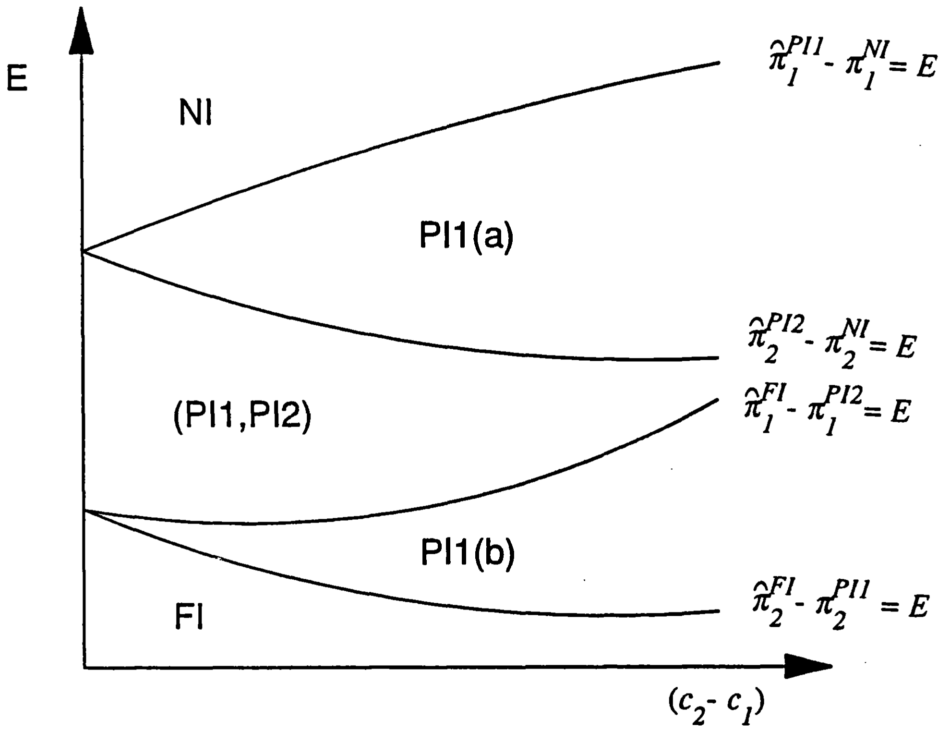

Figure 4.3

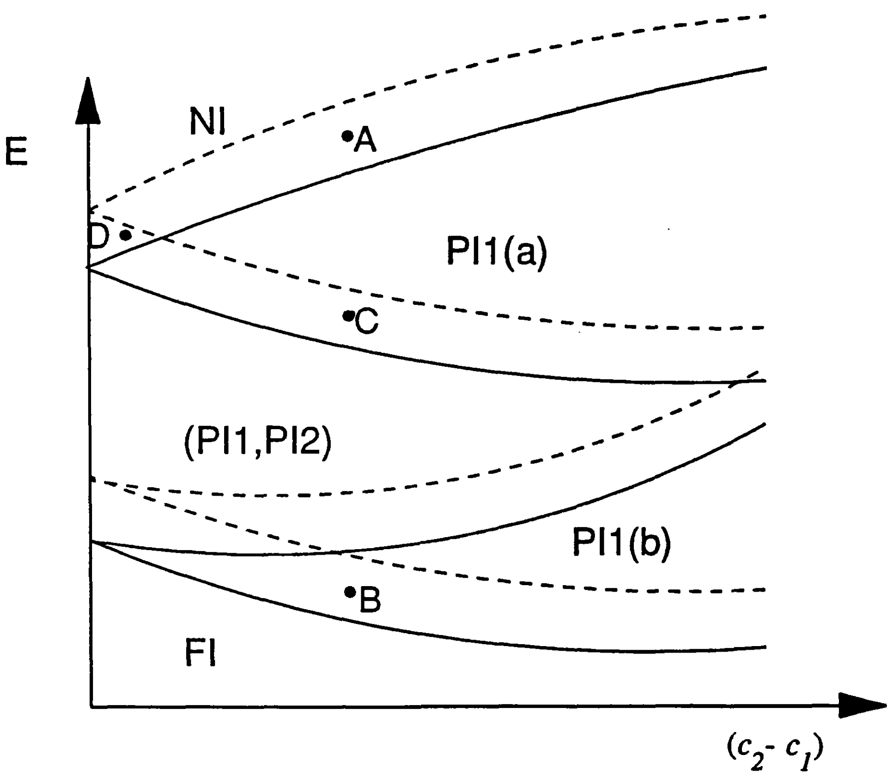

Figure 4.4

integration cost if the other firm does not bandwagon. Also the low-cost firm integrates if the other firm does not. We have two equilibria: partial integration by either firm. For still lower integration cost the low-cost firm integrates whatever the high-cost firm does and, accordingly, the high-cost firm does not integrate; we have again partial integration by the low-cost firm. When integration cost is very low even the high-cost firm can bear it whatever the rival does and full integration occurs. Note that we have two completely different regions where partial integration by the low-cost firm is the unique equilibrium. In region PI1(a) only the low-cost firm can bear the integration cost. And in region PI1(b) the high-cost firm can bear the integration cost only if the rival does not bandwagon. But because the low-cost firm integrates whatever the high-cost firm does, the high-cost firm will not integrate.

Next consider the effect of the cost difference keeping integration cost constant (that is, choose a point from the vertical axis). When the firms are equally efficient ($c_1 = c_2$) we are in the vertical axis. For the intermediate values of E only one firm can integrate. Because the firms are identical either firm can integrate in all of this range; there are two equilibria PI1 and PI2. Now increase the efficiency difference. The greater is the efficiency difference, the more likely it is that we end up in a region where only the low-cost firm integrates (PI1). However, starting from *any* point in the vertical axis and increasing the cost difference does not necessarily lead to PI1. For very low values of E we have always full integration, for "very intermediate" values there are always two equilibria, and for very high values of E nonintegration always occurs. The region where PI1 is an equilibrium increases in the efficiency difference and, consequently, the likelihood of PI1 to be an equilibrium is greater.

Then consider an increase in the size of the market for final good. The firms' incentives to integrate are increasing in the size of the market and, accordingly, all the critical locuses in Figure 4.3 shift upward (see Figure 4.4). The basic effect is that the degree of integration increases; point *A* which used to be in the region of nonintegration is now in the region of partial integration and at point *B* where partial integration occurred the industry becomes fully integrated. By this way we can also select which equilibrium of the multiple ones will emerge in the process of growing market for the final good. Point *C* which was in the region of partial integration of the low-cost firm is now in the region of two equilibria. Because *U1-D1* was already integrated the equilibrium that will be selected is partial integration by the low-cost firm. However, in our model also partial integration by the high-cost firm can be an equilibrium. Consider point *D*; the integration cost is fairly high and the firms are almost equally efficient. Originally the industry was nonintegrated. Now, when the size of the market increases we come to the region of two equilibria. Because neither firm was originally integrated it is now possible to have an industry structure where only the high-cost firm integrates.

An increase in the average marginal cost of the industry decreases the firms' incentives to integrate. All the boundaries will shift downwards and have the same effects as lowering the size of the market.

We sum up the comparative static results for the industry structure in the following propositions.

Proposition 4.3. *The degree of integration is*
(i) increasing in the size of the downstream market,
(ii) decreasing in the integration cost, and
(iii) decreasing in the average marginal cost of the industry.

Proposition 4.4. *The greater is the efficiency difference between the upstream firms, the more likely is an asymmetric industry structure where only the more efficient firm integrates.*

4.5 Welfare

In this Section we derive the welfare maximizing industry structure. We use the sum of consumer surplus and producer surplus as a welfare notion (W). There are three sources of deadweight losses in this model: (*i*) Harberger triangle (final good price is greater than the marginal cost), (*ii*) production inefficiency (also the high-cost upstream firm has positive output), and (*iii*) the fixed costs of integration.

In our model a vertical merger always decreases the final good price. The newly integrated firm obtains input at marginal cost which is lower than its Cournot price. This *ceteris paribus* decreases the final good price. However, if there remains an unintegrated firm in the market we have to take into account the effect of the merger on the input price. As was explained earlier a vertical merger lowers the input price which further decreases the final good price. Consequently, the social gain from integration is the lower final good price which reduces the Harberger triangle.

The production inefficiency is equal to $(c_2-c_1)x_2$ under NI and PI1 and $(c_2-c_1)y_2$ under PI2 and FI. The merger by the low-cost firm makes production allocation more efficient and the merger by the high-cost firm makes the production allocation less efficient. We can, however, show that even the merger by the high-cost firm increases variable welfare; the positive effect of the lower final good price outweighs the negative effect of less efficient production allocation. We can show that:

Observation 4.2. $W^{NI} < \hat{W}^{PI2} < \hat{W}^{PI1} < \hat{W}^{FI}$ *where hats denote the variable (gross of E) values.*

Proof: In Appendix 4.

Partial integration by the low-cost firm dominates partial integration by the high-cost firm ($W^{PI2} < W^{PI1}$). The fixed costs of integration are equal under both industry structures but both the final good price and the high-cost firm's output are lower under partial integration by the low-cost firm. Consequently, PI2 is never a social optimum. Variable welfare is the greater, the greater is the degree of integration ($W^{NI} < \hat{W}^{PI1} < \hat{W}^{FI}$), but so is the sum of integration costs. Accordingly, for different parameter values either full integration, partial integration by the low-cost firm or nonintegration can be the social optimum:

Proposition 4.5. *The welfare maximizing industry structure is*
(i) full integration if and only if $\hat{W}^{FI} - \hat{W}^{PI1} > E$,
(ii) partial integration by the low-cost firm if and only if $\hat{W}^{FI} - \hat{W}^{PI1} < E$ *and* $\hat{W}^{PI1} - \hat{W}^{NI} > E$, *and*
(iii) nonintegration if and only if $\hat{W}^{PI1} - \hat{W}^{NI} < E$.

Proof: In Appendix 4.

Figure 4.5 illustrates the critical locuses for both social optimum (solid line) and for Nash equilibrium (broken line). We show in each region the socially optimal structure and the Nash equilibrium structure in brackets if it is different from the socially optimal one. It is straightforward from Figure 4.5 that:

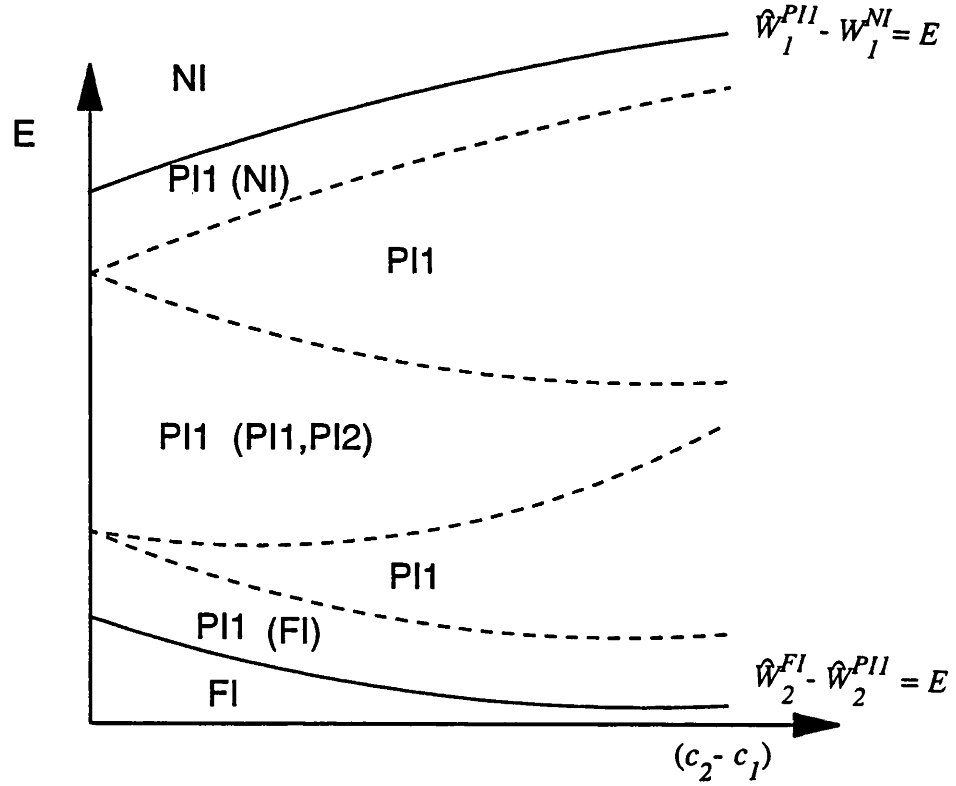

Figure 4.5

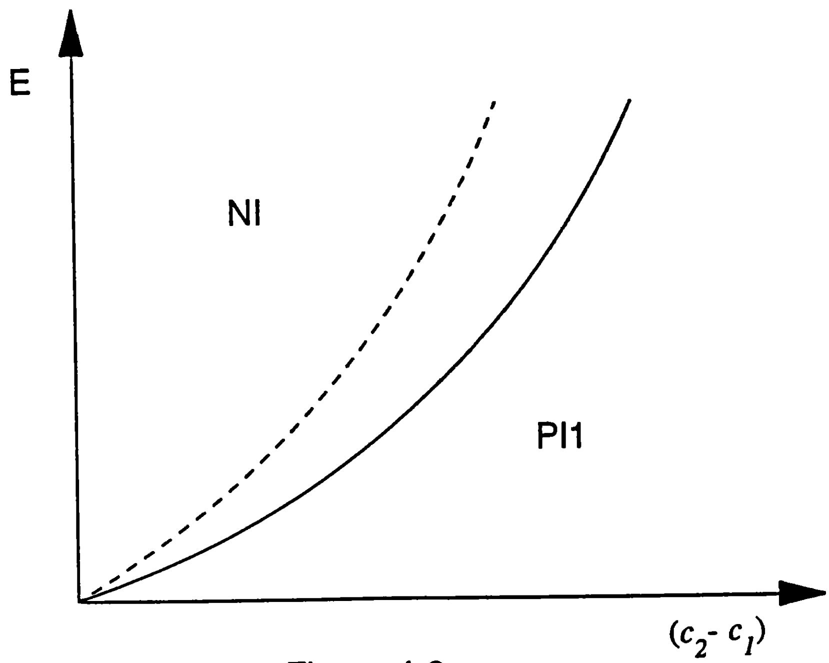

Figure 4.6

Proposition 4.6. *Partial integration by the low-cost upstream firm is less likely than what is welfare maximizing.*

Proof: In Appendix 4.

When making its integration decision the firm ignores the negative externality for rival and the positive externality for consumers. It turns out that the first merger is very beneficial for the consumers and is not very harmful for the rival; the first merger occurs too late (in terms of growing market). The second merger does not offer a much lower price for the consumers but harms the rival a lot; the second merger emerges too early.

4.6 Nonlinear Prices

To conclude, we compare our results to those of Hart and Tirole's (1990) Model 1 (hereafter H-T) which has nonlinear prices in the input market (essentially Bertrand competition). Both models have Cournot competition in the final good market. In both models the benefit of integration is profit sharing and there is a fixed cost of integration. We follow Sutton (1991) in using Cournot and Bertrand models as special examples within a general class of models which differ in toughness of price competition. Bertrand has the most severe price competition where only the most efficient firm can survive. Cournot corresponds to more relaxed price competition where also the less efficient firm has a positive market share. Our aim was to have predictions for the degree of integration that are robust to the toughness of price competition in the input market.

H-T do not provide comparative static analysis for the industry structure but it is a simple matter to do that for linear demand. Figure 4.6 illustrates the equilibrium industry structures in their model. Competition in the input market is so severe that the high-cost firm cannot sell any input. Even if it integrated with a downstream firm, its downstream unit would buy all its input from the low-cost firm. Accordingly, high-cost firm does not gain anything from integration. Its incentive to integrate would be on the horizontal axis in Figure 4.6. The upward sloping critical locus gives the low-cost firm's incentive to integrate. When the firms have equal marginal costs neither firm has an incentive to integrate. The incentive is increasing in the efficiency difference. Low-cost upstream firm has an incentive for integration to restrict competition in the downstream market. If the low-cost supplier is unintegrated it cannot commit to supply one downstream firm only; excessive supply incentive arises. Integrated supplier internalizes this externality and can undercut its high-cost rival slightly, so that the unintegrated firm buys the same total amount as before but now buys from the integrated supplier. This increases rival's costs which has a positive horizontal effect on the integrated firm's profits. The equilibrium industry structure is either nonintegration or partial integration by the low-cost firm.

A greater market size or a lower average marginal cost will bend the critical locus upwards (broken line). We find that the comparative static results for the degree of integration are the same as in our model. The degree of integration is the greater (*i*) the greater the size of the downstream market, (*ii*) the lower the integration cost, and (*iii*) the lower the average marginal cost of the industry. Also, the greater is the efficiency difference, the more likely is an asymmetric industry structure where only the low-cost firm is integrated. The main point of our paper is that qualitatively the same pattern of integration emerges whether there is Cournot or Bertrand competition in the input market.

Welfare results differ. In H-T nonintegration is always the social optimum because integration restricts output and increases fixed costs of integration. In our model also an industry structure with vertical integration can be welfare maximizing because integration increases output although it also increases the fixed costs. H-T find excessive integration in Nash equilibrium and we find a less asymmetric industry structure than what is welfare maximizing.

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Appendix 4

We solve the nonintegrated case as an example. Full integration and partial integration follow in a similar manner. The profit functions of the firms are:

(A4.1) $$\pi_{Di} = y_i(a - by_i - by_j - p_x) \qquad i,j = 1,2 \quad i \neq j$$

(A4.2) $$\pi_{Ui} = x_i(p_x - c_i) \qquad i = 1,2$$

To solve for the subgame perfect equilibrium, we work backwards and solve first for the equilibrium in the downstream market given the input price p_x. The equilibrium quantities are:

(A4.3) $$y_i = (a - p_x)/3b$$

Substituting the upstream market clearing condition

(A4.4) $$y_1 + y_2 = x_1 + x_2$$

into (A4.3) we can solve for the inverse demand for input; this gives the price at which the unintegrated downstream firms are willing to buy the input quantity supplied by the unintegrated upstream firms:

(A4.5) $$p_x = [2a - 3b(x_1+x_2)]/2.$$

Next we turn to examine the first stage. Inserting p_x from (A4.5) to (A4.2) we get the unintegrated upstream firm's profit expressed in terms of x_i's only:

(A4.6) $$\pi_{Ui} = x_i\{[2a - 3b(x_1+x_2)]/2 - c_i\}$$

The equilibrium quantities in the upstream market are thus:

(A4.7) $$x_i = 2(a - 2c_i + c_j)/9b.$$

The equilibrium price is:

(A4.8) $$p_x = (a + c_1 + c_2)/3.$$

Finally, we can return back to the downstream market to ascertain the subgame perfect equilibrium in terms of the exogenous parameters only. When we insert (A4.8) into (A4.3) we get the equilibrium quantities:

(A4.9) $$y_i = (2a - c_1 - c_2)/9b$$

The equilibrium price is:

(A4.10) $$p_y = [5a + 2(c_1 + c_2)]/9$$

And the equilibrium profits of the firms are:

(A4.11) $$\pi_{Di} = (2a - c_1 - c_2)^2/81b$$

(A4.12) $$\pi_{Ui} = 2(a - 2c_i + c_j)^2/27b$$

Under partial integration by Ui and Di we can solve for the equilibrium in a similar manner. The equilibrium quantities, prices and profits are:

(A4.13) $$x_j = y_j = (a - 2c_j + c_i)/6b$$

(A4.14) $$y_i = (5a - 7c_i + 2c_j)/12b$$

(A4.15) $$p_x = (a + 2c_j + c_i)/4$$

(A4.16) $$p_y = (5a + 2c_j + 5c_i)/12$$

(A4.17) $$\pi_{Uj} = (a - 2c_j + c_i)^2/24b$$

(A4.18) $$\pi_{Dj} = (a - 2c_j + c_i)^2/36b$$

(A4.19) $$\pi_i = (5a - 7c_i + 2c_j)^2/144b - E$$

And under full integration:

(A4.20) $$y_i = (a - 2c_i + c_j)/3b$$

(A4.21) $$p_y = (a + c_i + c_j)/3$$

(A4.22) $$\pi_i = (a - 2c_i + c_j)^2/9b - E$$

Proof of Observation 4.1:

Step 1: $\pi_i^{PIj} < \hat{\pi}_i^{FI}$

(A4.23) $$(a - 2c_i + c_j)^2/24b + (a - 2c_i + c_j)^2/36b < (a - 2c_i + c_j)^2/9b$$

Simplifying

(A4.24) $$3(a - 2c_i + c_j)^2/72b > 0$$

Step 2: $\hat{\pi}_i^{FI} < \pi_i^{NI}$

(A4.25) $$(a - 2c_i + c_j)^2/9b < 2(a - 2c_i + c_j)^2/27b + (2a - c_1 - c_2)^2/81b$$

Simplifying

(A4.26) $\quad [(2a - c_1 - c_2) - \sqrt{3}(a - 2c_i + c_j)]/9\sqrt{b} > 0$

(A4.27) $\quad \Leftrightarrow a > [(2\sqrt{3} - 1)c_j - (\sqrt{3} + 1)c_i]/(2 - \sqrt{3})$

Step 3: $\pi_i^{NI} < \hat{\pi}_i^{PIi}$

(A4.28) $\quad 2(a - 2c_i + c_j)^2/27b + (2a - c_1 - c_2)^2/81b < (5a - 7c_i + 2c_j)^2/144b$

Simplifying

(A4.29) $\quad [(a-c_j)(65a + 117c_j - 182c_i) + 41(c_i-c_j)^2]/1296b > 0$

(A4.30) $\quad \Leftarrow\ a > (182/65)c_i - (117/65)c_j$

Assumption 4.1 guarantees that (A4.24), (A4.27) and (A4.30) are satisfied.

Q.E.D.

Proof of Proposition 4.1:

The incentives to integrate are:

(A4.31) $\quad \hat{\pi}_i^{PIi} - \pi_i^{NI} = [(a-c_j)(65a - 182c_i + 117c_j) + 41(c_j-c_i)^2]/1296b$

(A4.32) $\quad \hat{\pi}_i^{FI} - \pi_i^{PIj} = (a - 2c_i + c_j)^2/24b$

(*ii*) the incentive is decreasing in the degree of integration: $\hat{\pi}_i^{PIi} - \pi_i^{NI} > \hat{\pi}_i^{FI} - \pi_i^{PIj}$

(A4.33) $\quad (5a - 7c_i + 2c_j)^2/144b - 2(a - 2c_i + c_j)^2/27b - (2a - c_1 + c_2)^2/81b >$
$(a - 2c_i + c_j)^2/24b$

Simplifying

(A4.34) $\quad [(a-c_i)(11a+45c_i-56c_j) - 130(c_j-c_i)^2]/1296b > 0$

(A4.34) holds if the following conditions are satisfied:

(A4.35) $\quad 11a+45c_1-56c_2 > 0 \Leftrightarrow a > (56/11)c_2 - (45/11)c_1$

(A4.36) $\quad 11a+45c_1-56c_2 > 65(c_2-c_1) \Leftrightarrow a > (121/11)c_2 - (110/11)c_1$

(A4.37) $\quad a-c_1 > 2(c_2-c_1) \Leftrightarrow a > 2c_2 - c_1$

Assumption 4.1 guarantees that these conditions are satisfied.

(*iii*) the incentive is increasing in the size of the downstream market

(A4.38) $\partial(\hat{\pi}_i^{PIi} - \pi_i^{NI})/\partial a = (130a - 182c_i + 52c_j)/1296b > 0$

(A4.39) $\partial(\hat{\pi}_i^{FI} - \pi_i^{PIj})/\partial a = 2(a - 2c_i + c_j)/24b > 0$

(*iv*) the incentive is decreasing in firm's own marginal cost

(A4.40) $\partial(\hat{\pi}_i^{PIi} - \pi_i^{NI})/\partial c_i = -(182a - 82c_i - 100c_j)/1296b < 0$

(A4.41) $\partial(\hat{\pi}_i^{FI} - \pi_i^{PIj})/\partial c_i = -4(a - 2c_i + c_j)/24b < 0$

(*v*) the incentive is increasing in rival's marginal cost

(A4.42) $\partial(\hat{\pi}_i^{PIi} - \pi_i^{NI})/\partial c_j = (52a - 100c_i - 152c_j)/1296b > 0$

(A4.43) $\partial(\hat{\pi}_i^{FI} - \pi_i^{PIj})/\partial c_j = 2(a - 2c_i + c_j)/24b > 0$

(*vi*) the incentive is decreasing in the average marginal cost of the industry

(A4.44) $\sum_{j=1}^{2}\partial(\hat{\pi}_i^{PIi} - \pi_i^{NI})/\partial c_j = -(130a + 18c_i + 52c_j)/1296b < 0$

(A4.45) $\sum_{j=1}^{2}\partial(\hat{\pi}_i^{FI} - \pi_i^{PIj})/\partial c_j = -2(a - 2c_i + c_j)/24b < 0$

(*i*) the more efficient firm has a greater incentive to integrate

When the firms have equal marginal costs the incentives to integrate are equal (equations (A4.31) and (A4.32)). Now increase c_2. (A4.40)-(A4.43) show that *U1-D1*'s incentive increases and *U2-D2*'s incentive decreases. Therefore *U1-D1*'s incentive is greater than *U2-D2*'s when $c_1 < c_2$.

Q.E.D.

Proof of Observation 4.2:

Step 1: $W^{NI} < \hat{W}^{PI2}$

(A4.46) $$2(2a - c_1 - c_2)^2/81b + 2(a - 2c_1 + c_2)^2/27b + 2(a - 2c_2 + c_1)^2/27b + 2(2a-c_1-c_2)^2/81b < (7a - 2c_1 - 5c_2)^2/288b + 5(a - 2c_1 + c_2)^2/72b + (5a - 7c_2 + 2c_1)^2/144b$$

Simplifying

(A4.47) $$[(a-c_1)(175a+459c_1-634c_2) + 199(c_2-c_1)^2]/2592b > 0$$

(A4.48) $$<= a > (634/175)c_2 -(459/175)c_1$$

Step 2: $\hat{W}^{PI2} < \hat{W}^{PI1}$

Step 2(*a*): $CS^{PI1} > CS^{PI2}$ $<=>$ $p_y^{PI1} < p_y^{PI2}$

where *CS* is consumer surplus

(A4.49) $$(5a + 2c_2 + 5c_1)/12 < (5a + 2c_1 + 5c_2)/12$$

(A4.50) $$<=> c_1 < c_2$$

Step 2(*b*): $\pi_1^{PI1} + \pi_2^{PI1} > \pi_2^{PI2} + \pi_1^{PI2}$

(A4.51) $$(5a - 7c_1 + 2c_2)^2/144b + 5(a - 2c_2 + c_1)^2/72b > (5a - 7c_2 + 2c_1)^2/144b + 5(a - 2c_1 + c_2)^2/72b$$

(A4.52) $$<=> (a-c_1) > (a-c_2)$$

Step 3: $\hat{W}^{PI1} < \hat{W}^{FI}$

(A4.53) $$(7a - 5c_1 - 2c_2)^2/288b + (5a - 7c_1 + 2c_2)^2/144b + 5(a - 2c_2 + c_1)^2/72b < (2a - c_1 - c_2)^2/18b + (a - 2c_1 + c_2)^2/9b + (a - 2c_2 + c_1)^2/9b$$

Simplifying

(A4.54) $$[(a-c_1)(9a+51c_1-60c_2) + 84(c_2-c_1)^2]/2592b > 0$$

(A4.55) $$<= a > (60/9)c_2 - (51/9)c_1$$

Assumption 4.1 ensures that (A4.48), (A4.50), (A4.52) and (A4.55) are satisfied.

Q.E.D.

Proof of Proposition 4.5:

$\hat{W}^{FI} - \hat{W}^{PI1} < \hat{W}^{PI1} - \hat{W}^{NI}$

(A4.56) $\quad \Leftrightarrow CS^{FI} + \pi_1^{FI} + \pi_2^{FI} + CS^{NI} + \pi_1^{NI} + \pi_2^{NI} < 2CS^{PI1} + 2\pi_1^{PI1} + 2\pi_2^{PI1}$

(A4.57) $\quad \Leftrightarrow (2a - c_1 - c_2)^2/18b + (a - 2c_1 + c_2)^2/9b + (a - 2c_2 + c_1)^2/9b +$
$2(2a - c_1 - c_2)^2/81b + 2(a - 2c_1 + c_2)^2/27b + 2(a - 2c_2 + c_1)^2/27b +$
$2(2a-c_1-c_2)^2/81b < 2(7a - 5c_1 - 2c_2)^2/288b + 2(5a - 7c_1 + 2c_2)^2/144b +$
$10(a - 2c_2 + c_1)^2/72b$

Simplifying

(A4.58) $\quad [47(a - c_1)^2 + 316(c_2 - c_1)(a - c_2) + 96(c_2 - c_1)(a - 2c_2 + c_1)]/2592b > 0$

Q.E.D

Proof of Proposition 4.6:

Step 1: $\hat{W}^{FI} - \hat{W}^{PI1} < \hat{\pi}_2^{FI} - \pi_2^{PI1}$

(A4.59) $\quad \Leftrightarrow CS^{FI} + \hat{\pi}_1^{FI} + \hat{\pi}_2^{FI} - CS^{PI1} - \hat{\pi}_1^{PI1} - \pi_2^{PI1} < \hat{\pi}_2^{FI} - \pi_2^{PI1}$

(A4.60) $\quad \Leftrightarrow CS^{FI} + \hat{\pi}_1^{FI} < CS^{PI1} + \hat{\pi}_1^{PI1}$

(A4.61) $\quad \Leftrightarrow (2a - c_1 - c_2)^2/81b + (a - 2c_1 + c_2)^2/9b$
$< (7a - 5c_1 - 2c_2)^2/288b + (5a - 7c_1 + 2c_2)^2/144b$

Simplifying

(A4.62) $\quad [(a-c_1)(475a-135c_1-340c_2) - 212(c_2-c_1)^2]/2592b > 0$

True if the following conditions are satisfied:

(A4.63) $\quad 475a-135c_1-340c_2 > 0 \Leftrightarrow a > (135/475)c_1 + (340/475)c_2$

(A4.64) $\quad 475a-135c_1-340c_2 > 212(c_2-c_1) \Leftrightarrow a > (552/475)c_2 - (77/475)c_1$

(A4.65) $\quad a-c_1 > c_2-c_1 \Leftrightarrow a > c_2$

Assumption 4.1 guarantees that these conditions are satisfied.

Step 2: $\hat{W}^{PI1} - W^{NI} > \hat{\pi}_1^{PI1} - \pi_1^{NI}$

(A4.66) $\iff CS^{PI1} + \hat{\pi}_1^{PI1} + \pi_2^{PI1} - CS^{NI} - \pi_1^{NI} - \pi_2^{NI} > \hat{\pi}_1^{PI1} - \pi_1^{NI}$

(A4.67) $\iff CS^{PI1} + \pi_2^{PI1} > CS^{NI} + \pi_2^{NI}$

(A4.68) $\iff (7a - 5c_1 - 2c_2)^2/288b + 5(a + c_1 - 2c_2)^2/72b >$

$2(2a - c_1 - c_2)^2/81b + 2(a + c_1 - 2c_2)^2/27b + (2a - c_1 - c_2)^2/81b$

(A4.69) $\iff [45(a-c_1)^2 + 108(c_2-c_1)(a-c_2) + 72(c_2-c_1)(a-c_1)]/2592b > 0$

Q.E.D.

5 Incomplete Contracts, Vertical Integration and Product Market Competition

5.1 Introduction

In recent years there has been renewed interest in the possible anti-competitive effects of vertical mergers. In particular attention has focused on the possibility that integration will result in the foreclosure of nonintegrated rivals i.e the restriction of buyers' access to suppliers and/or suppliers' access to buyers. Clearly in order to analyse such questions the precise effects of integration decisions, and the competitive environment within which firms operate must be considered in detail. In this paper we will adopt an incomplete contracts/optimal control rights approach to integration. Further, we will explicitly model the effect of varying the degree of market competition. This approach enables us to consider the nature of and motives for foreclosure. We will also consider the decisions firms make with regard to specialisation i.e. will a group of assets be assembled to concentrate on selling the final product, to specialise in input production or to combine both tasks.

Following Grossman and Hart (1986) we will assume that ownership confers residual control rights. If an agent owns an asset then he/she makes the decisions regarding the use of that asset, except where such decision powers are granted to others through contractual agreements. In a complex world where it may be difficult to verify or even write the clauses of a complete contract the allocation of ownership rights will therefore be important.

As an example suppose that an agent can work on a machine, and by doing so improve its performance. If a contract rewarding the agent for his/her effort or for the performance improvement could be written then we might expect such an agreement to be drawn up between the agent and the factory owner. The agent would put in the effort and the owner would benefit from an improved machine. However, if agreements cannot be written then once the agent has overhauled the machine he can be sacked, without compensation. As a result the worker will not exert any effort. An obvious solution to this problem would be to give the worker the ownership rights to the machine. However then the factory owner must negotiate with the agent for use of the machine. In turn this may dull the owner's incentives to find new markets for the product, since the machine-owning worker can capture some of the benefits. The various ownership structures thus have costs and benefits. Which structure dominates depends on the relative sizes of these two effects. If the maintenance worker is pretty ineffective we might expect the factory owner to control the machine. On the other hand if maintenance is crucial the worker will own the machine.

Now suppose that another firm, selling in a separate market desires the machine's product. This will tend to further encourage worker ownership of the machine. The dulled incentives of the factory owner are now offset by extra benefits to a high performance machine, through external sales.

But what if the other firm competes against the factory in the downstream market? Then selling the machine output to the rival allows it to concentrate on competing in the downstream market. (Assume the rival has other less efficient supply sources available, and so does not fully rely on the machine operator.) In contrast, the factory owner's incentives are dulled and it will suffer in a competitive market. If competition on the downstream market is weak the gains from intermediate output trade outweigh the adverse effects on the final good market. However, as competition gets tougher it may pay to transfer the machine to the factory owner. Machine output is of course inferior, but the external buyer suffers too. In addition the factory owner has improved incentives to fight on the downstream market, since he/she no longer has to negotiate for access to the machine. If competition on the final good market gets really tough then it offers little attraction. In which case it may be best to effectively withdraw from the final market and concentrate on intermediate good production. This can be achieved by giving the agent ownership of the machine. Industry profits are then shared between the input producer and the remaining downstream firm. There is therefore a non-monotonic relationship between ownership allocation and competition.

The above example provides a flavour of our results. Broadly we see greater upstream ownership of assets (nonintegration) when the upstream worker is important/effective. We also observe a non-monotonic relationship between ownership structure and the toughness of competition.

There are two lines of literature on vertical integration that we intend to bring together. The first one takes the view that different units of the firm are run by separate managers who are self interested and cannot be made to act in the best interest of the firm because of incompleteness of contracts.[34] The importance of a manager's investment is a major determinant for the optimal allocation of ownership. In these papers the issue of product market competition is not raised. The second branch of literature concentrates on the effect of integration on product market competition and explores the possibility of foreclosure.[35] These papers assume that integration leads to

[34]Grossman and Hart (1986), Hart and Moore (1990) and Bolton and Whinston (1993).

[35]Vickers (1985), Bonnano and Vickers (1988), Salinger (1988), Hart and Tirole (1990), Ordover, Saloner and Salop (1990) and Gal-Or (1992).

profit sharing and removes all the conflicts of interest inside the firm.[36] Integration structure is used as a device to gain competitive advantage in the downstream market by changing industry cost structure. Our paper has elements from both strands of literature. We provide an explicit treatment of the internal organisation of the firm in the strategic context of a competitive environment. The key determinants for the optimal allocation of ownership rights in our paper are the relative importance of managers' efforts and the toughness of competition in the product market.

Holmström and Tirole (1991) and Helper and Levine (1992) are also related to our paper. Holmström and Tirole examine transfer pricing problem in the context of organizational choice. In a decentralized firm the units are allowed to trade externally and bargain over the transfer price. The price will then reflect the quality and thus give the managers good incentives for quality enhancement. On the other hand the external trade may draw excess attention and deter relationship-specific investments. The optimal degree of decentralization is determined by this trade-off. In our model the vertical supply decision is part of the organizational choice. Helper and Levine examine the effect of product market competition in transaction cost model. They show that downstream firms with oligopoly rents may prefer inefficient arm's-length supplier relations to long-term contracts, if that reduces the ability of the supplier to bargain for a share of the oligopoly rents. In our paper a downstream manager may buy up an upstream unit and so make its input inefficient to increase his share of oligopoly rent relative to the other downstream firm. Helper and Levine assume that oligopoly rent is fixed while in our model it depends on the toughness of competition and on the managers' investments.

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[36]Bolton and Whinston (1991) compare these two approaches and in particular the papers by Hart and Tirole (1990) and Bolton and Whinston (1993).

The rest of the Chapter is organized as follows. Section 5.2 introduces the model. In Section 5.3 we present some useful preliminary results. This prepares us for Section 5.4 where payoff functions and investment incentives are examined. Finally industry ownership structure is analysed in Section 5.5.

5.2 The Model

We consider two upstream assets, *U1* and *U2*, and two downstream assets, *D1* and *D2*. The upstream assets produce input which is used by the downstream units to produce the final good. All technologies are constant returns to scale and one unit of input makes one unit of final good. Both input and final good production is costless.

Each unit is operated by a manager. Before production takes place the manager of a unit can increase the value of its output by exerting effort. If the manager of *Di* chooses effort level J_i then the unit value added in final good production is $v(J_i)$. Similarly if the manager of *Ui* chooses effort level I_i then the unit value of input is $v(I_i)/k$, where k is a measure of the relative effectiveness of upstream effort.

Assumption 5.1. $v(I_i)/k = I_i/k$ *and* $v(J_i) = J_i$.

The cost of investment to the manager is:

Assumption 5.2. $c(I_i) = I_i^\beta$ *and* $c(J_i) = J_i^\beta$ *where* $\beta >> 0$.

Once a manager has expended effort in value-enhancing investment he/she can be replaced costlessly for the remaining stages of production, that is the owner can hire an equally capable manager to take care of the production.

In line with the previous literature on incomplete contracts we assume that the effort requirements are too complex to be described adequately in a contract. As a result effort levels are chosen non-cooperatively. It is for this reason that inefficiencies can arise and the allocation of ownership rights matters in our model. Contractual incompleteness means that any compensation a manager receives for his effort must occur after the effort has been undertaken. We also assume that it is very difficult to describe the required input characteristics ex ante. Therefore the input trade too is ex ante noncontractible. We also rule out profit-sharing agreements.

Ex ante contracts can however be written on the allocation of ownership rights. Amongst the rights ownership confers is the power to hire and fire managers. The ownership of the assets *Ui* and *Di* is allocated between the initial managers of the upstream and downstream units. The managers of *Ui* and *Di* allocate ownership rights to maximize their joint profits given the ownership structure of *Uj-Dj*. As a considerable simplification we will consider the following industry configurations:
(*i*) partial integration (PI); one set of assets (*U2,D2*) is owned by a downstream manager and the second set of assets (*U1,D1*) is owned separately, and
(*ii*) integration (I); both sets of assets are owned by a downstream manager.
Industry structure is therefore determined by *U1* and *D1*'s integration decision.

Assumption 5.3. $k \geq 1$.

This assumption ensures that a given level of effort is more effective in increasing value when applied during final good production as opposed to input production. It is consistent with our focus on downstream control.

Ex post the uncertainty about the required characteristics of input is resolved and the owners of upstream and downstream assets negotiate a spot contract on the procurement of input. The contract will take the form of a two part tariff. Input will be exchanged at price zero, while the fixed fee negotiated will depend on the bargaining power of the parties. Also the bargaining on the non-owning manager's compensation for effort takes place ex post.

Assumption 5.4. *Internally sourced input can be utilised for internal production only.*

Assumption 5.4 is made for convenience. In a more complete model that includes firms' technology choices (specific or general) we could derive this assumption as a result.

Ultimately, production of final good occurs and the firms compete in the downstream market. We assume that total demand is 1, and this is allocated between the two downstream firms according to the values of their products, and the degree of market competition. The profits of the downstream firms are given by:

$$\pi_{Di} = \pi_i(v_i, v_j; \rho) \quad \text{for } i,j = 1,2 \;\; i \neq j \tag{5.1}$$

where the product values are v_i and v_j and ρ is the degree of competition, $0 \leq \rho \leq \bar{\rho}$. When $\rho = 0$ there is no competition; each firm is a monopoly in its half of the market.

Assumption 5.5. $\pi_i(v_i,v_j;0) = v_i/2$

We further assume:

Assumption 5.6. *For* $\rho > 0$

(i) $\partial\pi_i/\partial v_i > 0$ *and* $\partial^2\pi_i/\partial v_i^2 < 0$

(ii) $\partial\pi_i/\partial v_j < 0$

(iii) $\partial^2\pi_i/\partial v_i\partial v_j < 0$

(iv) $\partial^2\pi_i/\partial v_i^2 - \partial^2\pi_i/\partial v_i\partial v_j < 0.$

The assumption that the marginal value of investment is decreasing in the value of rival's product does not imply anything about the strategies in the final good market. This assumption can hold both with price competition (strategic complements) and quantity competition (strategic substitutes).

Assumption 5.7.

(i) *For* $v_i < v_j$ $\partial\pi_i/\partial\rho < 0$, $\partial\pi_i/\partial\rho < \partial\pi_j/\partial\rho$, $\partial^2\pi_j/\partial v_j\partial\rho > 0$ *and* $\partial^2\pi_i/\partial v_i\partial\rho <$ $\partial^2\pi_j/\partial v_j\partial\rho$.

(ii) *For* $v_i = v_j$ $\partial\pi_i/\partial\rho < 0$ *and* $\partial^2\pi_j/\partial v_j\partial\rho > 0$.

Assumption 5.7 says that the low-value firm suffers from an increase in competition while the high-value firm may gain (its market share is higher but unit margin is lower). Also, the marginal value of the investment is increasing in the degree of competition for the high-value firm. Competition lowers the profit but enhances the marginal incentives to invest for firms whose product values are equal.

As described above the firm's profit is its market share times the unit margin (since the size of the market is one). In general it is sufficient to give structure to this total profit term but we shall make one assumption regarding the unit margin, μ_i:

Assumption 5.8. *At $\rho = 0$ $\partial\mu_i/\partial\rho = -\gamma$ where $\gamma > 0$.*

Introducing a little competition to monopoly markets results in an equal drop in unit margins for both firms.

The time line of the game is:

Ex ante		Ex post	
contract on ownership	managers choose efforts	spot contract on input trade and effort compensation production of input	production of final good competition in the downstream market

Note that all the actions are observable. When the managers choose effort levels they know the ownership structure of the competing vertical structure. Bargaining over the procurement of the input and effort compensation occurs under symmetric information: effort levels are observable (but not verifiable). Finally, the downstream managers know rival's input purchases when they make their pricing decisions.

5.3 Preliminary Results

In this Section we derive a useful set of results that form a basis for the analysis of the optimal ownership structure.

Proposition 5.1.

A non-owning manager will receive no compensation for his effort and as a result will undertake no effort.

Since contracts contingent on effort cannot be written ex ante the manager must be rewarded after effort has been made. However once effort is made its value is sunk in the asset. The manager can therefore be sacked by the owner at no cost. As a result the manager will not be rewarded for the initial effort (unless he/she also owns the asset). Foreseeing no reward a non-owning manager will therefore make no investment in effort. Since $I_2 = 0$ we can simplify notation by defining $I \equiv I_1$.

If a downstream firm *Di* can only be supplied by an independent upstream firm *U1* the bargaining will result in an even split of the surplus:

$$(5.2) \qquad R_{di} = R_{u1} = \pi_i[v(J_i)+v(I),v_j;\rho]/2 \qquad \text{for } i,j = 1,2 \quad i \neq j$$

where *di* is *Di*'s owner, *u1* is *U1*'s owner and *R* is the revenue.

If a downstream firm *Di* is supplied internally by an upstream unit it owns then the owner keeps all the surplus:

$$(5.3) \qquad R_{di} = \pi_i[v(J_i),v_j;\rho]$$

Proposition 5.1 tells that the non-owning manager makes no effort and therefore the value of the final good manufactured from internal input is the value of the downstream investment only (since $v(0) = 0$).

Finally if a downstream firm *Di* can obtain input either internally or from an independent supplier *U1* then the profit shares are given by:

(5.4) $$R_{di} = \{\pi_i[v(J_i)+v(I),v_j;\rho] + \pi_i[v(J_i),v_j;\rho]\}/2$$

(5.5) $$R_{u1} = \{\pi_i[v(J_i)+v(I),v_j;\rho] - \pi_i[v(J_i),v_j;\rho]\}/2$$

The gains from external over internal sourcing are shared by the downstream firm and the independent upstream supplier.

Since there are no capacity constraints on firms' output and exclusive dealing contracts are not enforceable an independent upstream firm cannot commit to serve one downstream firm only. Having dealt with one downstream firm on an exclusive basis it has every incentive to trade with its rival - given that input is traded in spot exchanges before final good sales occur. During bargaining each downstream firm will foresee that its rival will also have access to the high-quality input and will negotiate the upstream firm's fee accordingly.

Proposition 5.2.

Under partial integration the independent upstream firm (U1) will supply both downstream firms. As a result it will receive revenue:

$$R_{u1} = \pi_1(v_1,v_2)/2 + [\pi_2(v_2,v_1)-\pi_2(\bar{v}_2,v_1)]/2$$

where $v_1 = v(J_1) + v(I)$, $v_2 = v(J_2) + v(I)$ *and* $\bar{v}_2 = v(J_2)$.

Assumption 5.4 ensures that under integration both firms will source input internally.

5.4 Manager Payoffs and Effort Incentives

5.4.1 Payoff Functions

We are now in a position to calculate the payoffs for the managers under the two possible ownership regimes.

Partial Integration (PI):

With a partially integrated industry structure there are three independent firms - the separate upstream and downstream firms *U1* and *D1*, and the integrated downstream firm *D2*. The payoffs for the owners are given by:

(5.6) $$\pi_{u1} = \pi_1(v_1,v_2;\rho)/2 + [\pi_2(v_1,v_2;\rho) - \pi_2(v_1,\bar{v}_2;\rho)]/2 - c(I)$$

(5.7) $$\pi_{d1} = \pi_1(v_1,v_2;\rho)/2 - c(J_1)$$

(5.8) $$\pi_{d2} = [\pi_2(v_1,v_2;\rho) + \pi_2(v_1,\bar{v}_2;\rho)]/2 - c(J_2)$$

where $v_1 = v(J_1) + v(I)$, $v_2 = v(J_2) + v(I)$ and $\bar{v}_2 = v(J_2)$.

Under the partial integration regime the independent upstream firm, *U1*, receives half of *D1*'s profits. Since *D1* has no alternative source of input bargaining results in this 50:50 split. *U1* also receives half of its contribution to the profits of the integrated downstream firm, *D2*. If *D2* fails to come to an agreement with *U1* it can source input from its own (inefficient) upstream plant. *U1* and *D2* therefore bargain over the incremental contribution of the superior input and again share the gains 50:50. *D2* receives the average of the revenues generated with and without *U1*'s superior input. Since contracts contingent on effort cannot be written, bargaining for input takes place after effort has been expended. As a result each party bears the full cost of its own investment.

Integration (I):

In the integrated setting there are only two firms. Each downstream manager also controls an upstream plant. Payoffs for the owners are given by:

(5.9) $$\pi_{di} = \pi_i(v_i, v_j; \rho) - c(J_i) \quad i = 1,2$$

where $v_i = v(J_i)$.

Since there are no independent upstream producers each downstream firm must source its input needs internally. Given that non-owning upstream managers do not exert effort these inputs are of basic, unenhanced quality. However the downstream owner-managers have free access to input and as a result retain all the revenues generated by their sales.

For the remainder of the analysis we will drop the arguments of the profit functions and let $\pi_i = \pi_i(v_i, v_j; \rho)$ and $\overline{\pi}_i = \pi_i(\overline{v}_i, v_j; \rho)$.

5.4.2 Incentives to Invest

Each party's incentives to invest in effort are determined by the above payoff functions. Where a manager receives all revenue increments generated by his or her effort incentives are maximised, while if the revenues are shared incentives to invest are diluted too. Each manager will choose an effort level such that the marginal cost of extra effort (borne solely by the manager) is equated with the marginal returns of extra effort to that manager. Below we list these relations for each ownership structure.

Partial Integration (PI):

$$(5.10) \qquad u1: \left\{ \left[\frac{\partial \pi_1}{\partial v_1} + \frac{\partial \pi_1}{\partial v_2} \right] + \left[\frac{\partial \pi_2}{\partial v_1} + \frac{\partial \pi_2}{\partial v_2} \right] - \frac{\partial \bar{\pi}_2}{\partial v_1} \right\} \frac{v'(I)}{2k} = c'(I)$$

An increase in effort by $u1$ raises the value of the input used by $D1$ and $D2$, and hence the value of the final products. Since $u1$ must negotiate with a buyer its contribution to profits will be shared 50:50 with the downstream firm. With greater effort by the upstream manager the value of $D1$'s product rises which will tend to increase its revenues. However the improved input is also used by $D1$'s rival on the downstream market, $D2$, yielding a counteracting negative effect on $D1$'s profits. Clearly the scale of this negative effect will depend on the degree of competition in the downstream market. If competition is weak then increases in the value of $D2$'s product have very little effect on $D1$'s profits, and vice versa. If competition is strong the negative effects will be quite considerable. Similar forces are at work in the impact of increased upstream investment on $D2$'s profits. The first two big terms inside the curly brackets show that $u1$ takes into account the effect on half the industry profit. However if $D2$ were to utilise its internal input source it would not benefit from greater $U1$ effort. Indeed profits would be reduced by the improvements in competing $D1$'s product. Thus greater effort by $u1$ will weaken $D2$'s internal sourcing option thereby increasing $u1$'s share of the benefits of trade (the last term inside curly brackets).

(5.11) $$d1:\ \frac{\partial \pi_1}{\partial v_1}\frac{v'(J_1)}{2} = c'(J_1)$$

When *d1* exerts greater effort it increases its profits. Given the bargaining for input with *U1*, *d1* receives half of its marginal contribution which it equates with the marginal cost of extra investment in effort.

(5.12) $$d2:\ \left[\frac{\partial \pi_2}{\partial v_2} + \frac{\partial \bar{\pi}_2}{\partial v_2}\right]\frac{v'(J_2)}{2} = c'(J_2)$$

By investing in more effort *d2* not only increases its revenues, but it also increases its share of these revenues (by raising the value of internal sourcing). Both these effects have a half weighting in determining *d2*'s effort.

Let the efforts of *u1*, *d1* and *d2* in the partial integration case be denoted by I^{PI}, J_1^{PI} and J_2^{PI} respectively.

Integration (I):

(5.13) $$di:\ \frac{\partial \pi_i}{\partial v_i} v'(J_i) = c'(J_i) \qquad i = 1,2$$

Under the integrated regime each downstream firm must source input internally. However the absence of external deals and the resultant bargaining means that each downstream manager keeps the full incremental value of an extra unit of effort.

We will denote *d1* and *d2*'s investments in the integrated case by J_1^I and J_2^I respectively.

Nash equilibrium in investments is determined by the first order conditions (5.10), (5.11) and (5.12) under partial integration and by (5.13) under integration. Our assumptions ensure that a unique equilibrium in investments exists for both structures.

From these effort equations we can already observe some of the trade-offs that the different ownership structures present. Under the integrated regime downstream managers keep all of the gains from extra effort, but lose out on the benefits of upstream investment. With a partially integrated industry structure firms benefit from upstream investment increasing the value of the products, but the returns from extra effort for the downstream managers are diluted. Of course the precise nature of these trade offs will depend on the relative value of upstream investment, and on the influence of competition on the downstream market. These factors will be considered further below. First it will prove useful to examine investment incentives when there is no competition i.e. $\rho = 0$.

5.4.3 No Competition Case

When there is no competition on the product market the profits of each downstream firm are unaffected by the value of the rivals output. Market share for each firm is set at 1/2 and the unit margin depends only on own value. As a result the effort equations have a particularly simple form.

Partial Integration (PI):

(5.14) $$u1:\ \frac{v'(I^{PI})}{2k} = c'(I^{PI})$$

(5.15) $$d1:\ \frac{v'(J_1^{PI})}{4} = c'(J_1^{PI})$$

(5.16) $$d2:\ \frac{v'(J_2^{PI})}{2} = c'(J_2^{PI})$$

U1 serves the whole market (=1) and the contribution of its effort increment to the unit margins ($v'(I^{PI})/k$) is shared 50:50 with the relevant downstream firm. *D1* has a market of 1/2 and receives half the increment to unit value. *D2* too has a market of 1/2 but keeps the full revenue increment of its extra effort ($v'(J_2^{PI})$) since it can realise this by sourcing internally.

In the absence of strategic effects we clearly see that *D2*'s internal sourcing option ensures its manager exerts more effort than *D1*. *u1*'s effort level depends on *k*, the measure of the relative value of upstream investment. When *k* is large an investment in upstream effort has little value. Consequently there is little return on such investments, and upstream effort will be low. When *k* is low upstream effort is highly productive and large rewards ensure high effort levels.

Integration (I):

(5.17) $$d1:\ \frac{v'(J_1^{I})}{2} = c'(J_1^{I})$$

(5.18) $$d2:\ \frac{v'(J_2^{I})}{2} = c'(J_2^{I})$$

With an integrated structure the downstream firms (with market share = 1/2) each receive the full benefit to their revenues of increased effort, thus maximising their incentives. Symmetry ensures that the effort levels of both downstream firms are identical. Note that for $\rho = 0$ *d2*'s investment is the same under both industry ownership configurations.

Proposition 5.3.

When there is no competition between the downstream firms ($\rho = 0$) then investment by the owner of the integrated downstream firm (D2) is identical under integrated and partially integrated structures. In addition it is identical to the effort investment made by D1 in the integrated regime i.e.

$$J_2^{PI} = J_2^I = J_1^I.$$

We are now in a position to consider the equilibrium industry ownership structure.

5.5 Industry Ownership Structure

In the context of our simple model the equilibrium industry ownership structure is determined by the integration decision of *U1* and *D1*. If assets *U1* and *D1* are jointly controlled the industry will be fully integrated, while if *U1* and *D1* are independently controlled the industry will be partially integrated (since by assumption assets *U2* and *D2* are always jointly owned). The initial owners of *U1* and *D1* allocate control of the upstream and downstream assets to maximise their joint profits. The joint profits under each ownership regime are given below:

$$\pi^{I}_{u1\text{-}d1} = \pi^{I}_{1} - c(J^{I}_{1}) \tag{5.19}$$

$$\pi^{PI}_{u1\text{-}d1} = \pi^{PI}_{1} + [\pi^{PI}_{2} - \bar{\pi}^{PI}_{2}]/2 - c(J^{PI}_{1}) - c(I^{PI}) \tag{5.20}$$

We are particularly interested in the dependence of industry structure on two key parameters - the effectiveness of upstream effort (k) and the degree of product market competition (ρ). The locus of points separating the Integrated and Partially Integrated regions of our parameter space is therefore the set of (k,ρ) combinations where $u1$-$d1$ profits under the two ownership structures are equal. In plotting this critical locus it will prove useful to begin at the point where $\rho = 0$ i.e. when there is no competition between the downstream firms.

Proposition 5.4.

Suppose that $\rho = 0$ i.e. there is no competition between the downstream firms. Then there exists a $\tilde{k}$ such that for $k \geq \tilde{k}$ the industry will be integrated, while for $k < \tilde{k}$ the industry will be partially integrated.

Proof:

The difference between $u1$ and $d1$'s joint profits under the integrated and partially integrated regimes is given by the following term:

$$\pi^{I}_{u1\text{-}d1} - \pi^{PI}_{u1\text{-}d1} = [\tfrac{1}{2}v(J^{I}_{1}) - c(J^{I}_{1})] - [\tfrac{1}{2}v(J^{PI}_{1}) + \tfrac{3}{4k}v(I^{PI}) - c(J^{PI}_{1}) - c(I^{PI})] \tag{5.21}$$

From the effort incentive equations of the previous section

$$\tfrac{1}{2}v(J^{I}_{1}) - c(J^{I}_{1}) > \tfrac{1}{2}v(J^{PI}_{1}) - c(J^{PI}_{1}) > 0 \tag{5.22}$$

Also if $k = 1$

$$\tfrac{1}{2}v(I^{PI}) - c(I^{PI}) = \tfrac{1}{2}v(J^{PI}_{2}) - c(J^{PI}_{2}) = \tfrac{1}{2}v(J^{I}_{1}) - c(J^{I}_{1}) \tag{5.23}$$

and as $k \to \infty$

$$\tfrac{3}{4k}v(I^{PI}) - c(I^{PI}) \to 0 \tag{5.24}$$

Hence for $k = 1$ $\pi^{J}_{u1\text{-}d1} - \pi^{PI}_{u1\text{-}d1} < 0$ and for k large $\pi^{J}_{u1\text{-}d1} - \pi^{PI}_{u1\text{-}d1} > 0$. By continuity there is a $k = \tilde{k}$ such that $\pi^{J}_{u1\text{-}d1} - \pi^{PI}_{u1\text{-}d1} = 0$.

Q.E.D.

The economic intuition for this result is quite simple. The value of allocating ownership and hence control of the upstream asset to its manager is the enhanced incentive that manager then has to exert effort. This raises the value of *D1*'s product and also results in input sales to *D2*, each of which generates additional profits. The disadvantage of separation of control of upstream and downstream assets is due to the dilution of manager *d1*'s effort incentives. The benefits of his/her increased effort must then be shared with an input supplier. When the value of upstream investment is high (k low) the benefits of *u1* effort through higher product value and external sales outweigh the weakening of *d1*'s incentives. However when k is high the value of *u1*'s investment will be low, leading to a small contribution to product value and low revenue from external sales. In this situation control of both upstream and downstream assets should be concentrated in the hands of the downstream manager, maximising his/her incentives to exert greater effort.

As we increase the degree of competition between the downstream firms this general pattern will continue with the integrated structure being preferred by *u1* and *d1* when k is high and the non-integrated structure dominating when k is low. However the value of k where the agents are indifferent between integration and non-integration will in general vary as the degree of competition is increased. It is to this relationship that we now turn.

We have already determined that when $\rho = 0$ *u1* and *d1* are indifferent between integrated and non-integrated structures when $k = \tilde{k}$. Which structure dominates however when we increase ρ i.e. we introduce product market competition?

There are several factors that we might expect to influence this. Firstly if *D1* has the higher value product then we might expect its market share to increase as competition increases. On the other hand if *D2* has the better product then we would expect *D1*'s market share to be squeezed. In addition an increase in competition is likely to squeeze margins. Of course even if *D1*'s market share does contract we would expect an offsetting benefit from *U1*'s increased sales to *D2*.

It will be useful to begin by considering the relative values of *D2* and *D1*'s products at $\rho = 0$ and $k = \tilde{k}$.

Proposition 5.5.

At $\rho = 0$ and $k = \tilde{k}$ the following relationships between product values hold:

(i) $v_2^{PI} > \bar{v}_2^{PI} > v_1^{PI}$ and

(ii) $v_2^I = v_1^I$.

Proof:

(*i*) At $\rho = 0$ and $k = \tilde{k}$, $\pi^I_{u1\text{-}d1} = \pi^{PI}_{u1\text{-}d1}$ implies that

(5.25) $$\tfrac{1}{2}v(J_1^I) - c(J_1^I) = \tfrac{1}{2}v(J_1^{PI}) + \frac{3}{4\tilde{k}}v(I^{PI}) - c(J_1^{PI}) - c(I^{PI})$$

But from Proposition 5.3: $J_1^I = J_2^I = J_2^{PI}$.

Hence $\tfrac{1}{2}v(J_2^{PI}) - c(J_2^{PI}) = \tfrac{1}{2}v(J_1^{PI}) + \frac{3}{4\tilde{k}}v(I^{PI}) - c(J_1^{PI}) - c(I^{PI})$.

Re-arranging:

(5.26) $$v(J_2^{PI}) - v(J_1^{PI}) - v(I^{PI})/\tilde{k} = 4\{[\tfrac{1}{4}v(J_1^{PI}) - c(J_1^{PI})] - [\tfrac{1}{4}v(J_2^{PI}) - c(J_2^{PI})]\} + 4[\tfrac{1}{2}v(I^{PI}) - c(I^{PI})]$$

Now from the profit maximising effort conditions:

(5.27) $\frac{1}{4}v(J_1^{PI}) - c(J_1^{PI}) > \frac{1}{4}v(J_2^{PI}) - c(J_2^{PI})$; and

(5.28) $\frac{1}{2}v(I^{PI}) - c(I^{PI}) > 0$

Therefore $v(J_2^{PI}) > v(J_1^{PI}) + v(I^{PI})/\tilde{k}$

(*ii*) From Proposition 5.3: $J_1^I = J_2^I$. It follows immediately that $v(J_1^I) = v(J_2^I)$.

Q.E.D.

Consider the set of parameter values such that there is no competition ($\rho = 0$) and *u1* and *d1* are indifferent between integration and separation ($k = \tilde{k}$) i.e. we are at the point in parameter space where the critical boundary between Integrated and Partially Integrated regimes cuts the k axis (see Figure 5.1). Then at this point the values of the integrated firms' products are identical (as they are everywhere) while the value of *D2*'s product (even when using internally sourced input) under the partially integrated regime exceeds the value of *D1*'s product (using the efficient independent supplier).

From the above result, when *U1* and *D1* are non-integrated we would expect *D2* to increasingly dominate the market as competition increases. On the other hand when *U1* and *D1* are integrated the values of the two downstream products are the same and hence both firms will maintain a market of 1/2 (though increasing competition will reduce margins).

This leads to our main result.

Proposition 5.6.

The relationship between $\tilde{k}$ and ρ along the critical boundary between the Integrated (I) and Partially Integrated (PI) regions of parameter space is non-monotonic. Initially as ρ increases from 0 $\tilde{k}$ falls, while for high values of ρ $\tilde{k}$ is increasing in ρ.

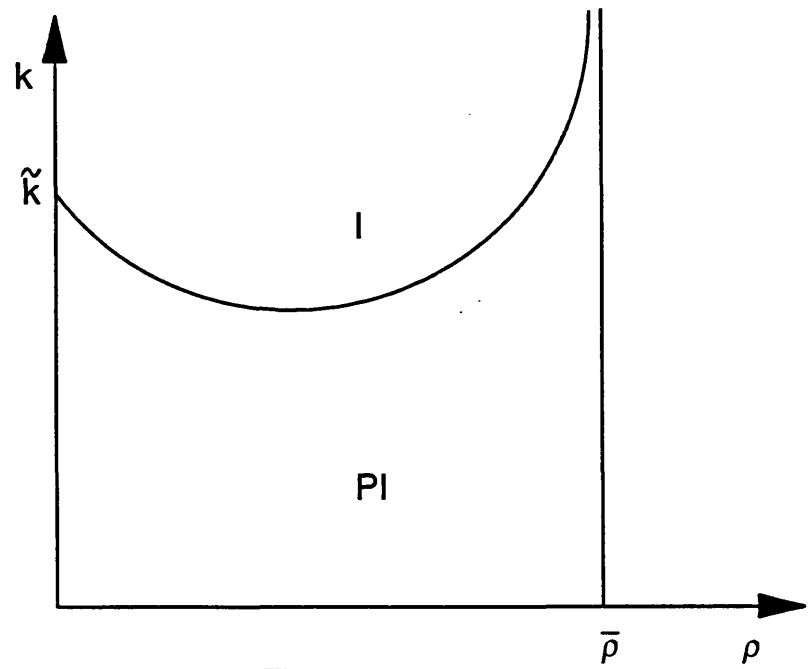

Figure 5.1

Proof: In Appendix 5.

Again the economic intuition is simple. Suppose we begin from the point $\rho = 0$ and $k = \tilde{k}$. At this point *u1* and *d1* are indifferent between integration and non-integration. When non-integrated the value of *D1*'s product is lower than that of *D2*, due to its weaker incentives to perform value-enhancing effort. As competition increases therefore the less efficient upstream-downstream combination (*U1-D1*) is squeezed by *U1-D2*. In addition since k is relatively high *U1*'s contribution to *D2*'s profits is low, and so its share of the gains is small. When integrated *U1* and *D1* lose these small revenues from input sales to *D2*, but *D1* is put on an equal footing with *D2* on the downstream market and hence is squeezed less by increased competition. Therefore with $k = \tilde{k}$ and ρ small the integrated structure dominates. At even stronger levels of competition however the asymmetric structure re-emerges. Tough competition results in considerable dissipation of profits under the symmetric structure, with both downstream firms integrated with input suppliers. Both downstream managers make high investments but since they are equal market share stays at 1/2. At $\rho = \bar{\rho}$ all the profits are dissipated under the symmetric structure. In contrast if *D1* and *U1* separate *U1* can always make a positive profit by supplying a downstream firm. A likely scenario is the one where separation effectively eliminates *D1* on the final good market and gives *D2* a dominant position, but this is more than compensated for by the (admittedly small) supplier profits accruing to *U1*. Thus for strong levels of competition the non-integrated structure dominates for *U1* and *D1*. Rather than being grouped towards head to head competition in the downstream market assets are more advantageously organised to emphasise the buyer-supplier relationship.

This result is illustrated in Figure 5.1. The critical level of k that separates the Integrated and Partially Integrated regions of our parameter space initially falls as ρ is increased from 0 (since an integrated structure is increasingly attractive for *U1* and *D1*), but as ρ rises further the separated structure becomes more and more attractive leading to the dominance of the Partially Integrated industry structure and a rise in the critical value $\tilde{k}$.

Briefly we can compare the above results with the structures that maximise producer surplus and overall welfare.

Proposition 5.7.

When there is no competition between downstream firms ($\rho = 0$) there exists a range of values of k such that the equilibrium industry ownership structure does not maximise producer (and hence total) surplus. Within this parameter range integration by U1 and D1 results in inefficient foreclosure of D2's input source.

Proof:

When $\rho = 0$ both downstream firms are monopolists so consumer surplus = 0. Hence overall welfare is maximised when producer surplus is maximised.

By definition, at $\rho = 0$ and $k = \tilde{k}$: $\pi^{PI}_{u1\text{-}d1} = \pi^{I}_{u1\text{-}d1}$.

From Proposition 5.3: $J^{PI}_2 = J^{I}_2$.

Therefore: $\pi^{PI}_{d2} = v(J^{PI}_2) + \frac{1}{2}v(I^{PI}) - c(J^{PI}_2) > v(J^{I}_2) - c(J^{I}_2) = \pi^{I}_{d2}$

Define $\hat{k}$ such that when $\rho = 0$ and $k = \hat{k}$: $\pi^{PI}_{u1\text{-}d1} + \pi^{PI}_{d2} = \pi^{I}_{u1\text{-}d1} + \pi^{I}_{d2}$

By continuity $\hat{k} > \tilde{k}$.

Q.E.D.

The intuition for this result is again straightforward. When *u1* and *d1* choose an integrated structure they consider only their own joint profits. However when integration occurs *D2* loses an efficient input supplier, and its profits fall. Since *U1* only receives half of the extra revenues generated for *D2* by the superior input *u1* and *d1* fail to consider the other 50% of the benefits (accruing to *d2*), when making their integration decision. Integration thus imposes an externality on *d2* through the foreclosure of its most attractive input source. Since producers capture all the surplus in the no competition setting, an inefficient ownership structure from a producer view is also inefficient in terms of overall welfare maximisation. (Note that with our demand structure there are no inefficient demand effects generated by monopoly).

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Appendix 5

Proposition A5.1

At $\rho = 0$ the changes in ul and di's efforts induced by increased competition will be small i.e.

$$\partial I^{PI}/\partial\rho,\ \partial J_i^{PI}/\partial\rho < \varepsilon$$

where ε is very small.

Proof:

At $\rho = 0$ the changes in investment are simply:

(A5.1) $$\frac{\partial I^{PI}}{\partial\rho} = \left[\frac{\partial^2\pi_1}{\partial v_1\partial\rho} + \frac{\partial^2\pi_1}{\partial v_2\partial\rho} + \frac{\partial^2\pi_2}{\partial v_1\partial\rho} + \frac{\partial^2\pi_2}{\partial v_2\partial\rho} - \frac{\partial^2\bar{\pi}_2}{\partial v_1\partial\rho}\right]/\beta(\beta\text{-}1)I^{\beta-2}$$

(A5.2) $$\frac{\partial J_i^{PI}}{\partial\rho} = \left[\frac{\partial^2\pi_i}{\partial v_i\partial\rho}\right]/\beta(\beta\text{-}1)J_i^{\beta-2}$$

$\beta >> 0$ implies that these terms are very small.

Q.E.D.

Proof of Proposition 5.6:

We shall proceed in 2 stages. First we show that for ρ large and $k = \tilde{k}$ $\pi^{J}_{ul\text{-}dl} < \pi^{PI}_{ul\text{-}dl}$. Second we prove that $d(\pi^{J}_{ul\text{-}dl} - \pi^{PI}_{ul\text{-}dl})/d\rho > 0$ at $\rho = 0$ and $k = \tilde{k}$.

Step 1: for ρ large and $k = \tilde{k}$ $\pi^{I}_{ul\text{-}dl} < \pi^{PI}_{ul\text{-}dl}$

(1*a*) Define $\bar{\rho}$ such that $\pi^{I}_{ul\text{-}dl} = 0$. $\bar{\rho}$ exists since:

$$\text{(A5.3)} \qquad \frac{\partial J^{I}_{i}}{\partial \rho} = \frac{\pi_{i\rho}(\pi_{ij}-\pi_{jj})}{\pi_{11}\pi_{22}-\pi_{12}\pi_{21}} > 0$$

$$\text{(A5.4)} \qquad \frac{d\pi^{I}_{ul\text{-}dl}}{d\rho} = \frac{\partial \pi_{1}}{\partial \rho} + \frac{\partial \pi_{1}}{\partial v_{2}}v'(J^{I}_{2})\frac{\partial J^{I}_{2}}{\partial \rho} < 0$$

where $\pi_{i\rho} = \frac{\partial^2 \pi_i}{\partial v_i \partial \rho}v'(J_i)$, $\pi_{ii} = \frac{\partial^2 \pi_i}{\partial v_i^2}[v'(J_i)]^2 - c''(J_i)$ and $\pi_{ij} = \frac{\partial^2 \pi_i}{\partial v_i \partial v_j}[v'(J_i)]^2$. $\bar{\rho}$ is the upperbound for ρ in our analysis. This ensures that payoff functions are continuous. For $\rho > \bar{\rho}$ only one manager can invest and make nonnegative profit.

(1*b*) $\pi^{PI}_{ul\text{-}dl} > 0$ for $\rho = \bar{\rho}$ and $k = \tilde{k}$

There are four possible scenarios for downstream investments and we analyse each of them in turn.

(*i*) Suppose $J^{PI}_{1} = J^{PI}_{2} = 0$ is an equilibrium. If $I^{PI} = 0$ then the products have no value and $\pi^{PI}_{ul} = 0$. If $0 < I^{PI} < J^{I}_{i}$ then $\pi^{PI}_{ul} = \pi_i[v(I^{PI}),v(I^{PI});\bar{\rho}] - c(I^{PI}) > \pi_i[v(J^{I}_{i}),v(J^{I}_{i});\bar{\rho}] - c(J^{I}_{i}) = 0$. Therefore $I^{PI} = 0$ cannot be an equilibrium.

(*ii*) Suppose $J^{PI}_{1} > 0$ and $J^{PI}_{2} = 0$ is an equilibrium. This implies that $\pi^{PI}_{dl} = \pi_1(v_1,v_2;\bar{\rho})/2 - c(J^{PI}_{1}) \geq 0$ and $\pi_1(v_1,v_2;\bar{\rho}) > 0$. For very small I (at negligible cost since $c'(0) = 0$) $\pi^{PI}_{ul} \geq \pi_1(v_1,v_2;\bar{\rho})/2 > 0$. Therefore $I^{PI} = 0$ cannot be an equilibrium.

(*iii*) Suppose $J^{PI}_{1} = 0$ and $J^{PI}_{2} > 0$ is an equilibrium. This implies that $\pi_2(v_2,v_1;\bar{\rho}) > 0$. For very small I (at negligible cost) $\pi^{PI}_{ul} \geq \frac{1}{2k}\frac{\partial \pi_2}{\partial v_2}v'(I) > 0$. Therefore $I^{PI} = 0$ cannot be an equilibrium.

(*iv*) Suppose $J^{PI}_{1} > 0$ and $J^{PI}_{2} > 0$ is an equilibrium. This implies that $\pi_i(v_i,v_j;\bar{\rho}) > 0$ for

$i = 1,2$. For very small I (at negligible cost) $\pi^{PI}_{u1} \geq \pi_1(v_1,v_2;\bar{\rho})/2 > 0$. Therefore $I^{PI} = 0$ cannot be an equilibrium.

Step 2: *At $\rho = 0$, $k = \bar{k}$ $d(\pi^{I}_{u1\text{-}d1} - \pi^{PI}_{u1\text{-}d1})/d\rho > 0$.*

Under integration introducing competition has the following effect:

$$\text{(A5.5)} \qquad \frac{d\pi^{I}_{u1\text{-}d1}}{d\rho} = \frac{\partial\pi_1}{\partial\rho} + \frac{\partial\pi_1}{\partial v_2}v'(J_2)\frac{\partial J_2}{\partial\rho} + \left[\frac{\partial\pi_1}{\partial v_1}v'(J_1) - c'(J_1)\right]\frac{\partial J_1}{\partial\rho}$$

Given the first order condition (5.13) and the assumption that $\partial\pi_1/\partial v_2 = 0$ at $\rho = 0$ we have:

$$\text{(A5.6)} \qquad \frac{d\pi^{I}_{u1\text{-}d1}}{d\rho} = \frac{\partial\pi_1(v^I_1, v^I_2;0)}{\partial\rho} < 0$$

Under partial integration the changes in payoffs are:

$$\text{(A5.7)} \qquad \frac{d\pi^{PI}_{u1}}{d\rho} = \frac{1}{2}\frac{\partial\pi_1}{\partial\rho} + \frac{1}{2}\frac{\partial\pi_2}{\partial\rho} - \frac{1}{2}\frac{\partial\bar{\pi}_2}{\partial\rho} + \frac{1}{2}\left[\frac{\partial\pi_1}{\partial v_1} + \frac{\partial\pi_2}{\partial v_1} - \frac{\partial\bar{\pi}_2}{\partial v_1}\right]v'(J_1)\frac{\partial J_1}{\partial\rho} +$$
$$\frac{1}{2}\left[\frac{\partial\pi_1}{\partial v_2} + \frac{\partial\pi_2}{\partial v_2} - \frac{\partial\bar{\pi}_2}{\partial v_2}\right]v'(J_2)\frac{\partial J_2}{\partial\rho} + \frac{\partial\pi^{PI}_{u1}}{\partial I}\frac{\partial I}{\partial\rho}$$

$$\text{(A5.8)} \qquad \frac{d\pi^{PI}_{d1}}{d\rho} = \frac{1}{2}\frac{\partial\pi_1}{\partial\rho} + \frac{1}{2}\left[\frac{\partial\pi_1}{\partial v_1} + \frac{\partial\pi_1}{\partial v_2}\right]v'(I)\frac{\partial I}{\partial\rho} + \frac{1}{2}\frac{\partial\pi_1}{\partial v_2}v'(J_2)\frac{\partial J_2}{\partial\rho} + \frac{\partial\pi^{PI}_{d1}}{\partial J_1}\frac{\partial J_1}{\partial\rho}$$

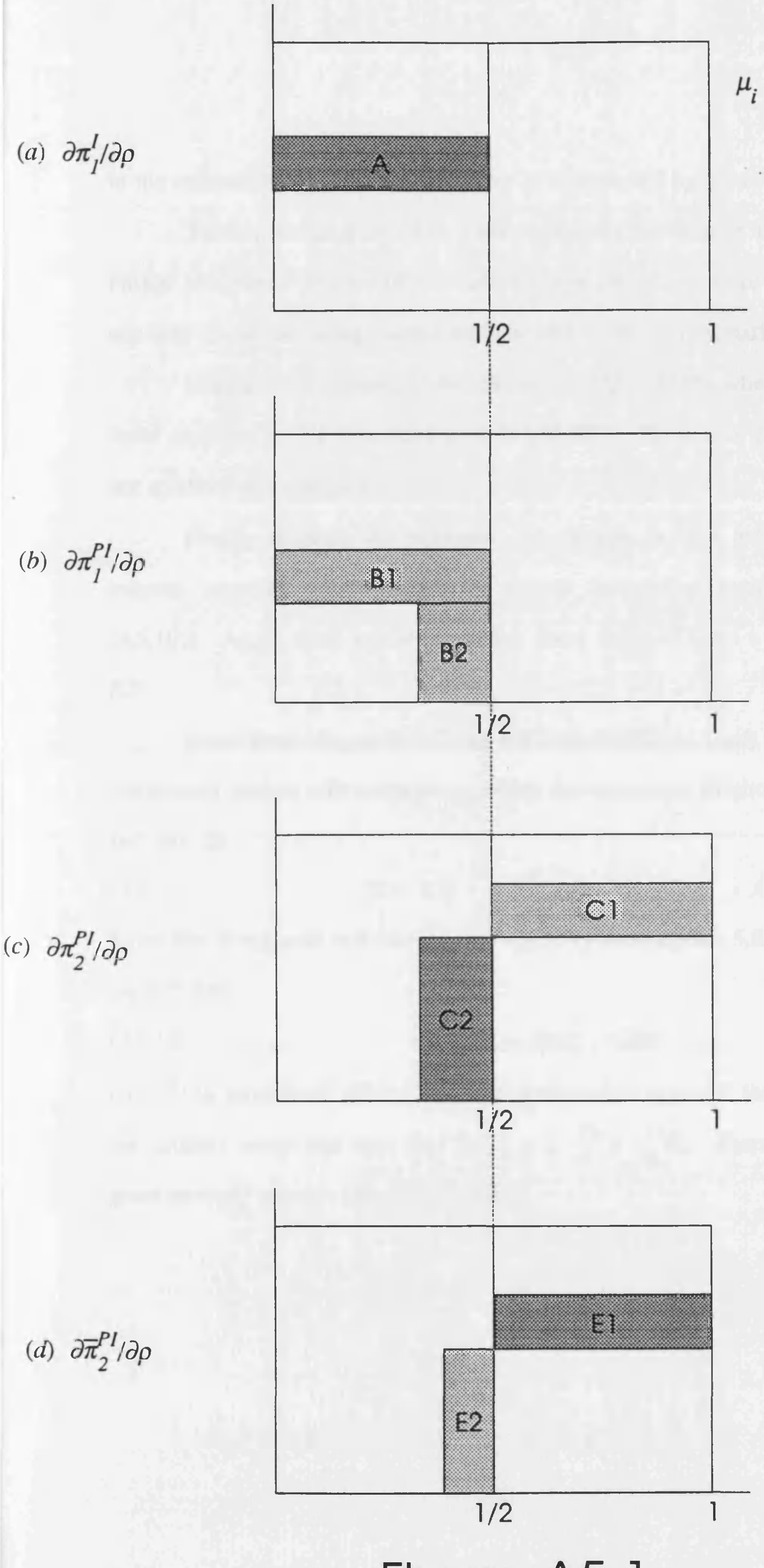
Given the first order condition (5.10), (5.11) and Proposition A5.1 we are left with:

$$\text{(A5.9)} \qquad \frac{d\pi^{PI}_{u1\text{-}d1}}{d\rho} = \frac{\partial\pi_1(v^{PI}_1, v^{PI}_2;0)}{\partial\rho} + \frac{1}{2}\frac{\partial\pi_2(v^{PI}_2, v^{PI}_1;0)}{\partial\rho} - \frac{1}{2}\frac{\partial\bar{\pi}_2(\bar{v}^{PI}_2, v^{PI}_1;0)}{\partial\rho}$$

Therefore

$$\text{(A5.10)} \qquad \frac{d(\pi^{I}_{u1\text{-}d1} - \pi^{PI}_{u1\text{-}d1})}{d\rho} = \frac{\partial\pi_1(v^I_1, v^I_2;0)}{\partial\rho} - \frac{\partial\pi_1(v^{PI}_1, v^{PI}_2;0)}{\partial\rho} - \frac{1}{2}\frac{\partial\pi_2(v^{PI}_2, v^{PI}_1;0)}{\partial\rho} +$$
$$\frac{1}{2}\frac{\partial\bar{\pi}_2(\bar{v}^{PI}_2, v^{PI}_1;0)}{\partial\rho}$$

It is easiest to determine the sign of (A5.10) diagrammatically. Diagram (*a*) in Figure A5.1 represents the change in *D1*'s profits in the integrated case (the first term in equation (A5.10)). Since *d1* and *d2*'s efforts are identical there is no change in market share (=1/2) when competition increases. There is however a fall in profits due



μ_i
(a) $\partial\pi_1^I/\partial\rho$
A
1/2
1
(b) $\partial\pi_1^{PI}/\partial\rho$
B1
B2
1/2
1
(c) $\partial\pi_2^{PI}/\partial\rho$
C1
C2
1/2
1
(d) $\partial\bar{\pi}_2^{PI}/\partial\rho$
E1
E2
1/2
1

Figure A5.1

to the reduced margin *D1* enjoys. This is represented by area A.

Turning to diagram (*b*). This represents the change in *D1*'s profits under the Partial Integration regime (the second term in (A5.10)). Here the fall in profits is due not only to the shrinking margin but also due to the fall in market share ($v_1^{PI} < v_2^{PI}$).

Diagram (*c*) represents the change in *D2*'s profits when it utilises the efficient input supplied by *U1* (the third term in (A5.10)). There is a gain in market share (*C2*) but a fall in unit margins (*C1*).

Finally diagram (*d*) represents the change in *D2*'s profits if it were to use its internal sourcing option under the Partial Integration regime (the fourth term in (A5.10)). Again there is rise in market share (area *E2*) and a fall in unit margin (area *E1*).

From these diagrams we can see that the fall in *u1-d1* profits under the Partial Integration regime will exceed that under the Integrated Regime ((A5.10) is positive) if and only if:

(A5.11) $$(B1+B2) + (C1-C2)/2 - (E1-E2)/2 > A$$

Since the changes is unit margins are equal by Assumption 5.8 ($A = B1 = C1 = E1$) we are left with:

(A5.12) $$B2 + E2/2 > C2/2$$

(A5.12) is satisfied if $B2 \geq C2/2$. Remember that at $\rho = 0$ the unit margin is equal to the product value and note that for $\beta \geq 2$ $v_1^{PI} \geq v_2^{PI}/2$. Therefore Assumption 5.2 (β great enough) ensures that (A5.12) holds.

Q.E.D.