INTERNATIONAL ARBITRAGE PRICING THEORY:

EMPIRICAL EVIDENCE FROM THE UNITED KINGDOM AND THE UNITED STATES

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by

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ABSTRACT

The objective of this thesis was to analyse the empirical applicability of the Arbitrage Pricing Theory to international asset markets (UK stock market and US stock market) and to identify the set of economic variables which correspond most closely with the stock market factors obtained from the traditional factor analysis.

Factor analysis and canonical correlation analysis were used as the principal tools for the empirical testing. Although factor analysis is frequently used, canonical correlation analysis is an new technique in this area and provides a method of linking factors extracted from the two sets of data. Various economic indicators were investigated as systematic influences on stock returns. It was shown that, based on the foundations of the APT and the characteristics of the factor scores from the factor analysis on the security returns and the economic indicators, canonical correlation analysis is an approximate technique to link the stock market and the economic forces.

The results using the UK data imply that there is a good correspondence between factor scores generated by the factor analysis on the UK security returns and on the UK economic indicators. The results using the US data show that there is also a fair correspondence, but lower than that for the UK data, between factor scores generated by the factor analysis on the US security returns and on the US economic indicators. The APT was also investigated in an international setting by considering the UK data and the US data together. The results show that the canonical correlation analysis successfully links the stock returns and economic forces. The conclusion of these empirical findings is that security returns are influenced by a number of systematic economic forces. The validity and applicability of the APT were also empirically evaluated. The regression results show that the explanatory power of the APT model is fairly good. The overall results obtained here appear to suggest that the APT pricing relationship is supported by the testing methodology. In addition, the international correlation structure of financial markets movements between the UK economy and the US economy has been analysed.

On balance, the evidence favours the APT and there is available evidence of inter-market linkage between the UK and the US. Individual sets of economic variables have been identified which correspond most closely with the UK and the US stock market factors by using the canonical correlation analysis. The results, at least partially, contribute to the understanding of security market pricing.

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To

MY GRANDMOTHER

MY PARENTS

GERALD

with

unbounded love, admiration, and gratitude.

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CHAPTER 1

INTRODUCTION

The Arbitrage Pricing Theory (APT) (Ross (1976,1977)) constitutes one of the most important models of security market pricing and has received a great deal of attention in financial economics. The APT assumes that every investor believes that the stochastic properties of capital assets returns are consistent with an unknown factor structure. The APT is an equilibrium model based on individuals arbitraging across multiple factors. By eliminating arbitrage opportunities, arbitragers make the market efficient. Ross argues that, in equilibrium, the expected returns on these capital assets are approximately linearly related to the factor loadings. The beauty of the APT is its generality, for it is actually consistent with a host of other asset pricing theories. The APT is a substitute for the Capital Asset Pricing Model (CAPM), in that both assert a linear relation between assets' expected returns and their covariances with other random variables. The APT requires less restrictive assumptions than the CAPM. In particular, it does not require the existence of the market portfolio, nor any specific utility function, nor the homogeneous expectations of returns. The CAPM assumes either investors' utility functions are quadratic or investors have homogeneous expectations about asset returns which have a joint normal distribution. The APT states that returns on a security are driven by a finite number of factors that reflect basic economic forces. Each of these economic forces represents a fundamental source of nondiversifiable risk in the economy.

1.1 Objectives and Contributions of the Study

Despite the appeal of its generality, the APT does not offer any theoretical or empirical grounds for identifying the economic nature of factors. The greatest weakness of the APT is the high level of ambiguity in its empirical predictions. The APT gives little guidance on the identity of the factors and does not tell us what factors are relevant.

Any test of the APT is a joint test that the factors are correctly identified and that the linear pricing relationship holds. In this study, factor analysis is used to identify the number of stock market and macroeconomic factors and to examine their importance. Factor analysis is a technique of multivariate analysis that attempts to account for the correlation between a large number of variables in terms of a small number of underlying factors. It is an approach that is used to investigate the relationships among variables. The use of independent factors extracted from the macroeconomic and financial variables avoids problems arising from the multicollinearity between such variables. These estimated macroeconomic factors convey the relevant information of the economy in a reduced form of a macro-model.

The thesis addresses two major questions : the applicability of the APT to international asset markets (UK stock market and US stock market); the identification of the set of economic variables which correspond most closely with the stock market factors obtained from the traditional factor analysis. Canonical correlation analysis is applied, for the first time in this area. Canonical correlation analysis provides a method of linking factors extracted from the two sets of data. The technique is in similar descriptive fashion to other related "linear transformation" techniques such as factor analysis. Factor analyses are fine if one wants factors chosen independently of each other. However, canonical correlation analysis is a better procedure for explaining as much as possible between one set of variables (i.e. factor scores of security returns) and another set (i.e. factor scores of economic indicators). It shows that, based on the foundations of the APT and the characteristics of the factor scores from the factor analysis on the security returns and the economic indicators, canonical correlation analysis is an approximate technique to link the economic forces and the stock market. The canonical correlation analysis estimates the factor loadings for two sets of data by examining only the inter-set correlation matrix. If the canonical correlations between the factor scores for corresponding pairs of factors are statistically significant (i.e. the association between the factor scores of the security returns and the factor scores of economic indicators), then they imply the factor comparability of the stock returns and the economic forces. The factor structure is therefore similar. As a result, the APT factors can be identified which are based on the intuition of the APT (i.e. the factors are orthogonal to each other) and hence, we can have a better understanding of the asset pricing. In addition, international correlation structure of financial markets movements between the UK economy and the US economy is analysed.

1.2 Outline of the Thesis

The introductory chapter is followed by eleven chapters. Chapter two covers the theoretical developments and origins of the CAPM and the APT. It also provides a detailed analysis of the similarities and differences between the CAPM and the APT.

In chapter three a literature survey of the empirical research on the APT is presented. Although the APT has attracted the attention of many empirical researchers, almost all of the studies are based on the capital markets of the United States. There are few published studies regarding the validity of the APT in the context of the UK capital markets.

Chapter four covers the description of the techniques of factor analysis and canonical correlation analysis. The chapter also contains the factor extraction techniques and critical aspects of factor analysis. The comparison of factor analysis and principal components is also made.

Chapter five analyses the UK stock market factors and the APT. The UK stock market factors are estimated using principal factor and maximum-likelihood methods of factor analysis. The applicability of the APT to the UK stock market is also empirically evaluated.

Chapter six examines a set of UK economic variables in order to estimate the number and loadings of the factors that represent the UK economy. Factor analysis is used to construct independent economic factors from UK economic indicators. The factors extracted from the macroeconomic and financial variables convey the relevant information of the economy in a reduced form of a macro-model.

The relationships between the UK stock returns and economic forces is discussed in chapter seven. The canonical correlation analysis is a new technique which is used to link the stock market and economic forces.

Chapter eight investigates the US stock market factors and the APT. In estimating the number of factors which affect US security returns, principal factor and maximum-likelihood factor analysis are used. The applicability of the APT to the US stock market is also empirically evaluated.

Chapter nine looks into the US economic factors. It examines a set of US economic indicators in order to estimate the number and loadings of the factors that represent the US economy.

Chapter ten analyses the relationships between the US stock returns and economic indicators. It investigates a set of economic indicators as systematic influences on stock returns using canonical correlation analysis.

Chapter eleven is an attempt to investigate the APT in an international setting and the international correlation structure of financial markets movements between the UK economy and the US economy. The validity and applicability of the APT to the international stock market are also evaluated. The international stock market and economic factors are estimated by factor analysis. Canonical correlation analysis is used to analyse the relationships between the international stock returns and the international economic indicators. The relationships between the UK stock returns and the US stock returns are also investigated. In addition, the canonical correlation analysis is used to analyse the relationship between the UK economic indicators and the US economic indicators.

Finally, chapter twelve presents the conclusions of this study.

CHAPTER 2

MARKET EQUILIBRIUM MODELS

2.1 Introduction

The objective of asset pricing model is to use the concepts of portfolio valuation and market equilibrium in order to determine the market price for risk and the appropriate measure of risk for a single asset. Over time, an equilibrium economic model was developed to determine the expected returns on equity and to specify the relationship among asset yields.

In the late 1960s and early 1970s, the field of financial economics was most closely associated with the CAPM, as evidenced by the large number of articles based on it. Since then, finance theory has expanded and matured, while the concepts behind modern portfolio theory and the CAPM are still being tested and used, and arbitrage pricing theory has assumed increasing importance, both in research and applications. The arbitrage pricing methodology has a very simple objective : to price a set of traded assets using the prices of another set of traded assets. As a theory, the APT has some attractive features : it does not rest on the assumptions that made the CAPM seem so restrictive; it is logical and consistent with activities in the capital markets. The APT offers a testable alternative to the CAPM, and many academics have turned their attention to understanding, testing, and attempting to use this new model.

Section 2.2 of this chapter is an attempt to show the theoretical developments and origins of the CAPM. The restrictions and extensions of the CAPM are discussed in section 2.3. The theoretical and empirical problems of the CAPM are discussed in section 2.4 and 2.5 respectively. Section 2.6 covers the empirical tests of the CAPM. The Roll's critique is discussed in section 2.7. Section 2.8 is the conclusion of the CAPM. The theoretical development and origin of the APT is discussed in section 2.9. Section 2.10 is a comparison of the APT with the CAPM.

2.2 Theoretical Developments and Origins of the CAPM

Over the previous thirty-five years a branch of applied micro-economics has been developed and specialised into what is known as modern finance theory. The financial theorists looked to and applied economic theory to problems of interest in finance.

2.2.1 Mean-variance efficiency criterion

In Tobin's (1958) pioneering application of expected utility maximization to the theory of liquidity preference, he considered the implications of the assumption that an investor's preferences among portfolios is represented in terms of the expected outcome of each portfolio (μ) and its standard deviation (σ). Tobin claimed that the mean-variance analysis is relevant in two cases : if the investor's utility function is quadratic, the expected utility associated with any probability distributions depends only on μ and σ. Alternatively, regardless of the form of the investor's utility function, if the subjective probability distributions of the possible portfolios are all members of a two-parameter family of distributions and normally distributed, preferences can be analysed in terms of μ and σ. The basic conclusions of Tobin's theory of liquidity preference and portfolio choice rest on the properties of the indifference curves that can be obtained from the assumption of either a quadratic utility function or a two-parameter probability function.

Tobin's proof that risk-averters with two-parameter subjective probability distributions have convex indifference curves is summarized as follows:

The expected utility associated with a distribution of R with a two-parameter density function f(R; μ_R,σ_R) is given by:

$$E[U(R)] = \int U(R) f(R;\mu_R,\sigma_R) dR \tag{1}$$

$$\text{Let} \quad Z = \frac{R-\mu_R}{\sigma_R} \tag{2}$$

$$E[U(R)] = E(\mu_R,\sigma_R) = \int U(\mu_R+\sigma_R Z) f(Z;0,1) dZ. \tag{3}$$

Let the investor be indifferent between two distributions f(R;μ_R,σ_R) and f(R;μ_R',σ_R'); i.e. EU(μ_R,σ_R) = EU(μ_R',σ_R') and the two points (μ_R,σ_R) and (μ_R',σ_R') lie on the same indifference curve. Also, diminishing marginal utility implies that for every Z,

$$\frac{1}{2}U(\mu_R+\sigma_R Z) + \frac{1}{2}U(\mu_R'+\sigma_R' Z) < U\left(\frac{\mu_R+\mu_R'}{2} + \frac{\sigma_R'+\sigma_R}{2}Z\right)$$

Consequently, $E\left(\frac{\mu_R+\mu_R'}{2}, \frac{\sigma_R+\sigma_R'}{2}\right)$ is greater than

$E(\mu_R,\sigma_R)$ or $E(\mu_R',\sigma_R')$, and $\left(\frac{\mu_R+\mu_R'}{2}, \frac{\sigma_R+\sigma_R'}{2}\right)$,

which lies on a line between (μ_R,σ_R) and (μ_R',σ_R'), is on a higher locus that those points. Thus, Tobin concluded that a risk-averter's indifference curve is necessarily concave upwards, provided it is derived in this manner from a two-parameter family of probability distributions and declining marginal utility of return.

Thus the twin assumptions of risk aversion and a particular form of the utility function are sufficient to produce decision making solely in terms of mean and standard deviation.

2.2.2 Capital Asset Pricing Model

Over time, innovations and extensions were added to the basic theory. The Capital Asset Pricing Model (CAPM) was developed almost simultaneously by Sharpe (1963, 1964), and Treynor (1961), others who developed it even further were Lintner (1965, 1969), Mossin (1966), and Black (1972). Much work in finance has been devoted to developing equilibrium theories of expected returns on equity.

As in all financial theories, a number of assumptions were made in the development of the S-L CAPM. To derive the S-L CAPM, the following assumptions are made so as to have sufficient conditions that each investor holds a minimum-variance portfolio. The first three assumptions are those that underly the portfolio theory. The last three assumptions are necessary to derive the Sharpe-Lintner (S-L) CAPM.

The six assumptions are as follows:

1. All investors are single-period expected utility of terminal wealth maximizers who choose among alternative portfolios on the basis of means and standard deviations of portfolio returns. Investors have identical time horizons. Under this assumption the potentially optimal portfolios for such investors are therefore those with the greatest expected return for a given level of variance and simultaneously, the smallest variance for a given expected return.
2. Investors are price takers and have homogeneous expectations about asset returns.
3. Asset markets are frictionless and information is freely and simultaneously available to investors.
4. There is a risk-free asset such that investors may borrow or lend unlimited amounts at the risk-free rate.
5. There are no market imperfections such as regulations, restrictions on short

selling, or taxes.

6. The quantities of assets are fixed, and all assets are marketable and perfectly divisible.

The following procedures are used to derive the two-parameter asset pricing theory [taken largely from Roll (1977)] : Any portfolio's mean and variance are given by

$$r_p = X'R, \tag{4}$$

$$\sigma_p^2 = X'VX. \tag{5}$$

where X is a (Nx1) vector of proportions invested in the constituent securities in a portfolio, R is a mean return vector of individual assets, and V is the covariance matrix of individual returns. The efficient set is found by minimizing σ_p^2.

The Lagrangian is

$$L = X'VX - \lambda_1(X'R - r_p) - \lambda_2(X'\underline{1} - 1)$$

where λ_1 and λ_2 are undetermined multipliers.

The first order conditions are the vector

$$VX = \frac{1}{2}(\lambda_1 R + \lambda_2 \underline{1}), \tag{6}$$

plus the constraint of eq.(4) and the sum of the proportions invested in assets equals to unity.

If the joint distribution of individual returns is non-degenerate (i.e. no two distinct linear combinations of assets are perfectly correlated and no asset has zero variance) the covariance matrix is positive definite (and non-singular), and all efficient portfolios satisfy

$$X = \frac{1}{2}V^{-1}(R)\begin{pmatrix}\lambda_1\\ \lambda_2\end{pmatrix}. \tag{7}$$

If no linear combination of assets has zero variance and at least two assets have different mean returns, the investment proportions of a mean-variance efficient portfolio whose mean return is r_p are given by the vector

$$X = V^{-1}(R\underline{1})A^{-1}\begin{pmatrix} r_p \\ 1 \end{pmatrix} \tag{8}$$

where the (2x2) matrix A is defined as

$$A \equiv (R\underline{1})' V^{-1}(R\underline{1}) \tag{9}$$

The matrix A is the "fundamental" matrix of information about the basic data contained in the means and covariances of individual assets. Since A is 2x2 and symmetric, it contains only three distinct constants.

Definition:

$$a = R'V^{-1}R, \quad b = R'V^{-1}\underline{1}, \quad c = \underline{1}'V^{-1}\underline{1}, \tag{10}$$

are the "efficient set constants" contained in the matrix

$$A \equiv \begin{pmatrix} a & b \\ b & c \end{pmatrix}.$$

By using eq.(8), the covariance between any arbitrary pair of efficient portfolios, say between efficient portfolio p and efficient portfolio q is obtained, as

$$\sigma_{qp} = (r_q \;\; 1)A^{-1}\begin{pmatrix} r_p \\ 1 \end{pmatrix}. \tag{11}$$

If p and q are orthogonal, this covariance is zero. Thus, putting q = z gives the equation

$$(r_z\ 1)\begin{pmatrix} c & -b \\ -b & a \end{pmatrix}\begin{pmatrix} r_p \\ 1 \end{pmatrix} = 0,$$

from which eq.(12) follows directly.

$$r_z = (a-br_p)/(b-cr_p). \tag{12}$$

For every efficient portfolio except the global minimum variance portfolio there exists a unique orthogonal efficient portfolio with finite mean. If the first efficient portfolio has mean r_p, its orthogonal portfolio has mean r_z.

Note that the (2x1) vector $A^{-1}\begin{pmatrix} r_p \\ 1 \end{pmatrix}$ can be simplified as

$$\begin{pmatrix} Cr_p - b \\ -br_p + a \end{pmatrix} / (ac-b^2) = \frac{Cr_p-b}{ac-b^2}\begin{pmatrix} 1 \\ -r_z \end{pmatrix} = \frac{\sigma_p^2}{r_p-r_z}\begin{pmatrix} 1 \\ -r_z \end{pmatrix},$$

where z is p's orthogonal portfolio.

Substitution back into eq.(8) gives

$$R = r_z\underline{1} + (r_p-r_z)\beta. \tag{13}$$

where $\beta \equiv VX/\sigma_p^2$ is the vector of simple regression slope coefficients of individual assets on efficient portfolio p (the "betas"). Since the covariances are linear in the mean return, of course the "betas" are too.

The relationship for determining the expected returns from a given asset or portfolio is

$$E(\tilde{R}_i) = R_f + [E(\tilde{R}_m)-R_f]\,\frac{\sigma_{im}}{\sigma_m^2} \tag{14}$$

where

R_i = return from the asset or portfolio;

R_f = return from the risk-free asset;

R_m = return from the market portfolio;

$\sigma_{im}/\sigma_m^2 = \beta_i$ = sensitivity of asset or portfolio relative to market movements.[1]

The above result of the S-L CAPM is developed to analyse the riskiness and the required rates of return on assets when they are held in portfolios. This relationship is also known as the security market line (SML).

Sharpe (1964) noted that the market risk of a given stock can be measured by its tendency to move with the general market. The tendency of a stock to move with the market is reflected in its beta coefficient, which is a measure of the stock's volatility relative to an average stock.[2] An investor evaluates an asset in terms of its marginal contribution to the portfolio. The decision to alter the proportion of the portfolio invested in an asset will depend on whether the cost of doing so in terms of risk is greater or smaller than the benefit in expected return. An investor will be in a personal equilibrium when the marginal rate of transformation between return and risk is equal to his personal marginal rate of substitution between return and risk. Investors must be compensated for bearing risk. The greater the riskiness of a stock, the higher its required return would be. However, investors require compensation for risks that cannot be diversified away. The risk which investors will pay a premium to avoid is covariance risk. This risk is also called systematic, undiversifiable, or market-related risk.[3] For instance, such a risk is caused by socioeconomic and political events that affect the returns of all assets. Market risk stems from such things as inflation, recessions, high interest rates, and war; factors which affect all firms simultaneously. If risk premiums existed for diversifiable risk, well diversified investors would buy these securities and bid up their price, and the final expected returns would reflect only nondiversifiable market risk. That is why stock prices have a tendency to "move together".

Since a stock's beta, β_i, measures its contribution to the riskiness of a portfolio,[4] beta is the appropriate measure of the stock's riskiness. The risk for a well-diversified portfolio depends on the market risk of the stocks included in the portfolio. As the number of assets in a portfolio increases, the risk which an asset contributes to a portfolio reduces to be exclusively the covariance risk. The portion of an asset's risk which is uncorrelated with the economy can be avoided at no cost. The part of the risk of an average stock which can be eliminated is called unsystematic, non-market-related or company-specific risk. Company-specific risk is caused by such things as changes in a company's management, strikes, winning and losing major contracts, lawsuits, successful and unsuccessful marketing programs, and other events that are unique to a particular company. In other words, unsystematic risk stems from the fact that many of the factors that surround an individual company are peculiar to that company and perhaps its immediate competitors. Unsystematic risk is unexpected, unpredictable, and, in prospect, unrewarded. As these events are essentially random, their effects on a portfolio can be eliminated by diversification, bad events in one firm will be offset by good events in another. The company risk can be eliminated by diversification, but not many investors do indeed diversify fully.

Blume and Friend (1975) analysed the major classes of assets (including stock portfolios) and liabilities held by individuals. They found that individuals have remarkably undiversified holdings. Blume and Friend investigated not only share holdings, but home ownership and human capital. It would be interesting to include in an individual's holdings those assets held by their pension funds, however, Blume and Friend did not do so. Generally, there seemed to be greater diversification by older individuals and by those who owned their own businesses. The median number of shareholdings per household with net worth exclusive of homes, associated mortgages, and human wealth in excess of $1 million was only fourteen. The results differed among income groups. Blume and Friend concluded that a large number of households hold poorly diversified portfolios. The investors' heterogeneous expectations and the fact that many investors do not properly assess the risks of the portfolios they hold could cause the CAPM to yield a poor description of investors' behaviour. No rational investor will pay a premium to avoid diversifiable risk. Since these uncertainties can be diversified away, they are not relevant to the investors' forecasts of the future returns. As the number of assets in a portfolio increases, the risk which an asset contributes to a portfolio reduces to be exclusively the covariance risk.

2.3 Restrictions and Extensions of the CAPM

Not all of the CAPM assumptions conform to reality, but this fact is not sufficient to reject the model. A model is judged on the basis of predictions, in which case assumptions are not relevant. Although not all of these assumptions conform to reality, they are simplifications which facilitate the development of the CAPM, which is extremely useful for financial decision making, as it quantifies and prices risk. The theoretical extensions in the literature, attempting to relax the basic CAPM assumptions, have yielded results that are generally consistent with the basic theory. It is reasonably unchanged by the relaxation of many of the unrealistic assumptions which made its derivation simpler.

If markets are frictionless, the borrowing rate equals the lending rate, a linear efficient frontier of the S-L CAPM can then be developed. This is the most crucial assumption for the CAPM: the investor is concerned with return and risk, not with the individual characteristics of each asset. The investor's particular attitude toward risk will determine how much of the risk-free asset and the market portfolio will be held. Risk is increased or decreased by borrowing at the risk-free rate to invest additional funds in the market portfolio or by adding a portion of the risk-free assets.

The assumptions that are used to generate the CAPM provide a concrete foundation on which the theory can be developed. Virtually every one of the assumptions under which the CAPM is derived is violated in the real world. Next, the assumptions are relaxed to determine what can be expected in more realistic circumstances. It will be interesting to see how the basic CAPM can be extended by relaxing the unrealistic assumptions without drastically changing it.

2.4 Theoretical Problems of the CAPM

2.4.1 The absence of risk-free asset and the restrictions on short selling

Some academics have questioned the existence of a truly risk-free asset, and they have developed models which do not depend on the existence of a risk-free asset. Black (1972) suggested a model in which it is not necessary to assume the existence of a riskless rate. Black created an alternative CAPM using short-selling as a proxy for the risk-free asset. Black's replacement for the risk-free asset is a portfolio that has no covariance with the market portfolio, so that its total risk and its unsystematic risk are identical and both have positive quantities. As the relevant risk in the CAPM is systematic risk, a risk-free asset would be one with no volatility relative to the market. Hence, all the returns of portfolios which are uncorrelated with the true market portfolio must have zero covariance with the market portfolio, and they have the same systematic risk (i.e., they have zero beta) and in turn, have the same expected return.

However, the limitation of these two-factor models is that they rely heavily on the assumption that there are no short sales constraints. If the investor can short-sell assets, then any portfolio of risky assets can be balanced by short-sold assets, creating a riskless portfolio in any economic environment. Short-selling is the means that allows market prices to be in equilibrium - that is, to be balanced between buyers and sellers. Profit seeking arbitragers facilitate enforcement of the law of one price by buying the stock in the market where its price is lowest and selling in the market where the stock's price is higher. Arbitragers enforce the law of one price as they pursue their profits. Short sales are not always undertaken in search of a speculative profit. Short sales can be used like insurance to hedge away risks and to arbitrage differential prices into equilibrium. The powerful economic force of arbitrage makes securities prices around the world respond efficiently to new information. Greed motivates arbitragers to do a social good. Shorting selling is used by hedgers and arbitragers in developing the arbitrage pricing theory.

For Black, short-selling is similar to issuing securities at an uncertain rate. Black (1972) assumed that all investors could participate in the short-selling of risky securities, which is not actually true, many large portfolios are restricted from short-selling. Ross (1977) has shown that the linear CAPM is invalid in a world with short sales restrictions and no riskless asset.

The assumption regarding the equality of borrowing and lending rates and the free access to the risk-free asset is a rather inaccurate description of the real world. When there are restrictions on the riskless asset, such as a higher borrowing rate than lending rate or only lending at the risk-free asset (i.e., buy US Treasury securities), but no restrictions on the other assets, then the zero beta version of the CAPM is still valid.

The assumption of no market imperfections has several implications for the CAPM. The assumption of short sales complements the assumption about a risk-free asset. If there was no risk-free asset, the investor could create one by short-selling securities. Roll (1977) shows that there must be either a risk-free asset or a portfolio of short-sold securities for the capital market line to be straight. Ross (1977) has also shown that in a world with short sales restrictions and no riskless asset, the linear CAPM is invalid. On the other hand, the assumption removes the transactions costs and taxes that face the real-world. The CAPM assumes that dividends and capital gains are equivalent and transaction costs are irrelevant. This implies that all returns are equally desirable, as capital gain and dividend income are equally attractive to investors. In reality, different investors may have different taxes and different transaction costs. These differences are important if investors consider these taxes and costs in discriminating between different assets. Such a situation will create diverse expectations and multiple efficient frontiers.

2.4.2 Taxation and transaction costs

The CAPM has been modified to adapt taxes. Brennan (1970) has investigated the effect of differential tax rates on capital gains and dividends. With regard to dividend payout he concluded that for a given level of risk, investors required a higher total return on a security the higher its prospective dividend yield was, because of the higher rate of tax levied on dividends than on capital gains. Although he concluded that beta was the appropriate measure of risk, his model has included a second factor to explain the equilibrium rate of return on securities.

The problem of transaction costs has received some attention (e.g. Constantinides (1986), Garman and Ohlson (1981), Milne and Smith (1980)). The problem has probably been less important than taxes since 1975. For instance the Securities and Exchange Commission deregulated transaction costs in order to let them to attain competitive levels.

Commission rates have declined on large transactions and have risen on small transactions (Harrington, 1987).

2.4.3 The existence of non-marketable assets

The assumption which states that the quantities of assets are fixed, and all assets are marketable and perfectly divisible does suggest that liquidity and new issues of securities can be ignored. However, in reality such an assumption may not be true and hence, the simple CAPM probably cannot capture all that is essential in pricing securities.

Fama and Schwert (1977) found that extending popular two-parameter models of capital market equilibrium to allow for the existence of non-marketable human capital does not provide better empirical descriptions of the expected return-risk relationship for marketable securities than those that come out of the simpler models. Their conclusion derived from the fact that relationships between the return on human capital and the returns on various marketable assets are weak, so that the model which includes human capital leads to estimates of risk for marketable assets which are indistinguishable from those of the simpler models.

The study by Williams (1979) has attempted to examine empirically the effect of non-marketable human capital upon both capital asset pricing and individual portfolio composition. With regard to capital asset pricing, their results appeared to strongly confirm those of Fama and Schwert (1977) that human capital in the aggregate has little to do with capital market pricing as well. Williams found that human capital, both in whole or in part, is weakly related with the financial market - so weakly, in fact, that no meaningful covariation appears to exist overall between changes in labour earnings and the rate of return on financial assets.

Liberman (1980) employed an extension of the S-L CAPM which allows for the existence of non-marketable human capital. His study found that empirically the inclusion of human capital appears to have little meaningful effect upon both general capital asset pricing and individual investor portfolio composition. It has been shown to arise from the fact that relationships between returns on almost all types of human capital and those of marketable financial assets are so weak therefore making these two capital asset groupings effectively separable.

Overall, the above studies should only be viewed as an empirical approximation rather than a theoretical contribution to human capital theory. Human capital lacks complete marketability because of moral hazard and their approaches do not deal with the moral hazard problem.

2.5 Empirical Problems of the CAPM

2.5.1 Theoretical and practical problems with riskless asset

In the CAPM theory, the 90-day Treasury bill rate has been virtually the only proxy used for the risk-free asset. However, there are both theoretical and practical problems with using the treasury bill rate.

If the CAPM is to be accurate, the investors' choices of assets must depend only on expected returns and on their aversion to risk. In turn, the R_f (risk-free rate) proxy must have no variance and no covariance with the returns from the market. The required characteristics for R_f cause some problems when choosing a proxy. First, zero variance can exist only for a single period. In a multi-period world, there would be variance in proxies for R_f from period to period. The second problem is that with variances comes potential covariance. Roll (1970) reported that successive, nonoverlapping, Treasury bill rates are serially correlated, therefore returns and prices do not follow a random walk. He also found that the serial correlation is not perfectly positive, which confirms the existence of some reinvestment risk. If there is covariance between R_f and R_m, the beta for R_f would not equal zero, and the line connecting the R_f and R_m, the capital market line would not be straight, but would be convex. Tobin (1958) suggested that an asset's liquidity is critical to investors. Highly liquid assets e.g. Treasury bills would be available at a premium price. Hence, if Treasury bills are used as R_f proxy, the intercept of the market line would be underestimated and its slope would be overestimated relative to the real relationship. In turn, if the investors were not be able to borrow at the risk-free rate, the expected return from portfolios of above-average risk would be overestimated.

There are other problems with using the Treasury bill. Firstly, short-term Treasury bills may show significant variability over time. The variability could come from either the nominal rate of return or the return to compensate for expected changes in the level of prices. Expected inflation may change over time. Hence, although the dividend of Treasury bills is fixed, the return on Treasury bills is not fixed.

A CAPM which relates risk and return under conditions of changing price levels has been developed by Hagerman and Kim (1976). Their model implies that price-level changes do not affect the expected real returns on individual assets except through their impact on the return of the market portfolio. If real market returns are independent of price-level moments, the model is very much like the standard CAPM expressed in real terms. This version of the CAPM does not, however, resolve all the difficulties associated with changing price levels, since it has been assumed in Hagerman and Kim's study that the nominal default-free rate is determined outside the model and that relative prices do not change. These limitations also apply to all other single-period CAPM. In addition, the model developed by Hagerman and Kim was converted into nominal returns by assuming that price-level changes and the real market returns are uncorrelated.

Another problem in choosing the Treasury bills is that it is not a pure market rate. The rates of the Treasury bills are affected by interest rate control or by the money supply. These rates are determined not just by the investors' required compensation for illiquidity and the expected inflation, but by other factors such as economic growth, employment, the value of the U.S. dollar, and international stability.

Empirically, Black, Jensen, and Scholes (1972) showed that the estimated intercept of the CAPM is different from the risk-free rate (their proxy was the Treasury bill rate). They also concluded that low beta securities earn more than the CAPM would predict and high beta securities earn less. The intercept seems to depend on the beta of any asset; high beta securities have a different intercept than low beta securities.

Fama and MacBeth (1974) found that the intercept exceeds the risk-free proxy. Another study, Fama and MacBeth (1973), calculated the actual risk premium and the predicted intercept from 1935 to 1968 and over a variety of subperiods. Their results showed that the intercept does not equal the risk-free rate in any period.

2.5.2 Empirical distribution of security returns

Fama (1965a) has investigated the empirical distribution of daily returns on New York Stock Exchange securities and found that they are distributed symmetrically, but that the empirical distribution has "fat tails" and no finite variance. Fama (1965b) has shown that as long as the distribution is symmetric and stable, investors can use measures of dispersion other than the variance and the theory of portfolio choice is still valid. Fama (1976) believed that the distribution of returns is close enough to normal so that the assumption of normality was appropriate.

Brealey (1970) concluded that at first glance the distribution of daily rates of return from the British equity market resembles the familiar bell-shaped pattern of the normal distribution. The distribution is highly symmetrical. However, closer examination of the frequency distributions reveals an important difference from the normal pattern. There is an excess of very small changes, a deficiency of medium-sized changes and an excess of very large changes. These results are similar to those observed by Fama (1965a) for individual American stocks.

Cunningham (1973) showed that the individual British stocks exhibit consistent behaviour in relation to the index, and the distribution of returns is approximately normally distributed. Hence, he concluded that the distribution of possible future returns on a portfolio can be assessed.

Ang and Pohlman (1978) have investigated the price behaviour of the stocks of five Far Eastern countries and found that in general, those stocks exhibit greater standard deviation and departure from the normal distribution than the U.S. and European stocks.

2.6 Empirical Tests of the CAPM

The CAPM was the genesis for countless empirical tests (e.g. Black, Jensen and Scholes (1972) and Fama and MacBeth (1973, 1974)). Black, Jensen and Scholes (1972) used a time-series method (using returns for a number of stocks over several time periods). Most studies followed the technique developed by Black, Jensen and Scholes. The general structure of these tests is the combination of the efficient market hypothesis with time series and cross-section econometrics. Some index of the market, such as the value weighted combination of all stocks would be chosen and a sample of firms would be tested to see if their excess returns, $E(R_m)$ - R_f, are explained in cross-section by their betas on the index, i.e. could the SML be rejected. The security market line (SML) depicts the relationship between expected returns and risk for individual stocks under conditions of market equilibrium.

Black, Jensen and Scholes (1972) showed that the empirical market line is linear with a positive trade-off between return and risk. However the intercept term is significantly different from zero (9.79 standard deviations away) and it implies that there might be something "left out" of the CAPM which is captured in the empirically estimated intercept term. The findings led them to a negative conclusion with respect to the S-L CAPM.

Fama and MacBeth (1973) tested the relationship between return and risk for NYSE common stocks. The theoretical basis of the tests is the "two-parameter" portfolio model and models of market equilibrium derived from the two-parameter portfolio model. Fama and MacBeth could not reject the hypothesis of these models that the pricing of common stocks reflects the attempts of risk-averse investors to hold portfolios that are efficient in terms of expected value and dispersion of return.

2.7 The Roll's Critique

Roll (1977) has pointed out that the CAPM is not a good hypothesis to test. "Testing the two-parameter asset pricing theory is difficult (and currently infeasible). Due to a mathematical equivalence between the individual return/'beta' linearity relation and the market portfolio's mean-variance efficiency, any valid test presupposes complete knowledge of the true market portfolio's composition. This implies, inter alia, that every individual asset must be included in a correct test" (Roll (1977)).

Roll's critique has two parts. First, he argues that the tests are of very low power and probably cannot detect departures from mean variance efficiency. His central point shows that tests of the CAPM are tests of the implications of the statement that the entire market portfolio is mean variance efficient, and are not simply tests of the efficiency of some limited index such as can be formed from the stock market. Roll claims that the only way to test the CAPM directly is to see whether or not the true market portfolio is ex post efficient. The CAPM's an expectational model and requires using the full set of assets available to the investor as an index. Roll stresses that the essential point is that the market portfolio is unmeasurable. The market portfolio contains all marketable and non-marketable assets, it is impossible to observe. It is impossible to test the validity of the CAPM and the efficiency of the market portfolio because of the difficulty of measuring the true market portfolio. All tests of the CAPM have been joint hypotheses tests of the model and of the data on which it has been tested. Roll argues that the previous tests of the theory are defective and the theory itself is considerably more difficult to test than had been thought.

2.7.1 Living with the Roll's critique

Stambaugh (1982) has investigated the sensitivity of inference about the linearity to changing the set of individual assets for which the linear relation is tested. Tests are conducted with market portfolios that include returns for bonds, real estate, and consumer durables in addition to common stocks. Even when stocks represent only 10% of the portfolio's value, inferences about the CAPM are virtually identical to those obtained with a stocks-only portfolio. He has found that the addition of just a few assets to the set of assets used to test the linear relation can product changes in inference. The sensitivity of the tests to the number of market model equations is not surprising as this is the nature of statistical inference, and even if the tested market index is inefficient with respect to the set of all the assets included in it, it might still be efficient with respect to some subsets of assets.

Stambaugh (1982) says this sensitivity exists whether or not one can identify the market portfolio. He also has addressed the empirical question, whether alternative market indices produce similar inferences about mean variance efficiency. The tests conducted by him accept linearity and produce identical inferences across all market indices. He concluded that: "The impression ... is that inferences about the CAPM are not sensitive to altering the composition of the market index ... It remains possible that alternative market portfolios can reverse inferences about the model. But the results of this sensitivity analysis almost surely indicate that such an occurrence is less likely than Roll's (1977) arguments suggest".

While the indices used in Stambaugh's tests approximate returns on portfolios of aggregate wealth and include a broad range of assets, it is clear that they are more similar to each other than to the true market portfolio. There are many other assets ("missing assets") whose returns are not perfectly observable every period, and are not included in the construction of these market indices. The question remains whether the lack of sensitivity of Stambaugh's tests to the choice of a particular market index constitutes evidence that these tests really test the theory.

Gibbons (1982) employed maximum-likelihood techniques in a multivariate test of the CAPM. Inference is based on a standard likelihood ratio test (LRT) statistic, in conjunction with its limiting chi-squared distribution. Gibbons claimed that the suggested methodology increases the precision of estimated risk premiums by as much as 76%. Moreover, the approach leads naturally to a likelihood ratio test of the parameter restrictions as a test for a financial model. Using a one-step Gauss-Newton computational method, a strong statistical rejection of the efficiency of the equally-weighted index is obtained. With no additional variable beyond β, the substantive content of the CAPM is rejected for the period 1926-1975 with a significance level less than 0.001.

Kandel (1984) presented an analysis of the testability of the mean variance efficiency of a market index when the returns on some components of the index itself are not perfectly observable. The results are basically not supportive of the notion that mean variance efficiency is testable on a subset of the assets. Bounding the market share of the missing asset and its expected return is not sufficient to produce a valid test. When the variance of the missing asset is bounded, and the amount of wealth that might be missing is small, it is possible, in principle, to reject correctly the mean variance efficiency of a market index.

Shanken (1987) developed a framework in which inferences can be made about the validity of an equilibrium asset pricing relation, even though the market portfolio in this relation is unobservable. A multivariate proxy for the true market portfolio, consisting of an equal-weighted stock index and a long-term government bond index, is employed in an investigation of the Sharpe-Lintner CAPM. The empirical evidence suggests that the joint hypotheses that CAPM is valid and multiple correlation between the true market portfolio and proxy asset exceeds 0.7 can be rejected. The proxies can account for at most two-thirds (rejected at the 0.05 level), or perhaps only one-half (rejected at the 0.10 level), of the variation in the true market return. Hence, it is suggested that the correlation coefficient is sufficiently high to provide a valid test.

Roll's critique is one extreme, the counterarguments are based on the statistical notion of measurement error. First, measurement error is a fact of life in all of economics (and statistical analysis), not just finance. However there are well-developed econometric techniques to confront this situation, usually involving the concept of instrumental variables. The crucial parameter in these techniques is the correlation of the proxy to the unobserved variable, in this case, the market portfolio. If the correlation is high, reliable asymptotic testing procedures are available. If the correlation is low, the tests are less reliable.

Consequently, the counter-argument shifts the focus to a discussion of the size of the correlation of the proxy to the true market and related statistical issues.

2.8 Conclusion

A rejection of the CAPM against an unspecified alternative hypothesis is evidence in favour of an alternative model. If an alternative model is available, the relevant comparison is between the current model and the alternative model. Arbitrage Pricing Theory is one of the most recent alternatives suggested for use in describing investor behaviour.[5]

The tests of the CAPM have shown that it is misspecified and may be inadequate. The rejection of the CAPM is evidence in favour of the APT which is one of the most recent possibilities suggested for use in describing investor behaviour. Yet, the CAPM is still useful since it is an equilibrium model which provides a strong specification of the relationship among asset yields that can be interpreted easily.

2.9 Theoretical Developments and Origins of the APT

2.9.1 Introduction

The arbitrage pricing theory formulated by Ross (1976) claimed to offer a testable alternative to the capital asset pricing model. It is an appropriate alternative because it agrees perfectly with what appears to be the intuition behind the CAPM. The CAPM predicts that security rates of return will be linearly related to a single common factor - the rate of return on the market portfolio. The APT assumes that the rate of return on any security is a linear function of k factors. The APT does not assume that the market is in equilibrium. It depends essentially on the absence of arbitrage possibilities, rather than on the much more restrictive condition that the market be in equilibrium as is required in the mean-variance

CAPM.

2.9.2 Basic Assumptions

The APT takes an approach that is different from the CAPM. One of the arguments favouring the APT over the CAPM is that the APT's greater generality is accomplished by the APT being based on fewer simplifying assumptions. For instance, few assumptions are made about investor preferences.

Of the assumptions made by the CAPM, only two are needed for the APT.

1. The expected return and risk preference assumption : Investors prefer more return to less and are risk averse.
2. The capital markets are perfectly competitive and frictionless. There are no transactions costs, taxes, or restrictions on short selling.

Although the APT has fewer assumptions than the CAPM, it has one that is peculiar to it:

3. The generating process of security return assumption : All investors exhibit homogeneous expectations that the stochastic properties of asset returns are consistent with a linear structure of k factors.

   The actual return on the i-th asset is written as:

$$\begin{aligned} \tilde{R}_{it} &= E(\tilde{R}_{it}) + b_{i1}\tilde{F}_{1t} + \dots + b_{ik}\tilde{F}_{kt} + \tilde{e}_i \\ &= E(R_{it}) + \sum_{k=1}^{K} b_{ik}F_{kt} + \tilde{e}_i \end{aligned} \qquad (15)$$

where $\tilde{R}_{it}$ = the random rate of return on the i-th asset in period t,

$E(\bar{R}_{it})$ = the expected rate of return on the i-th asset in period t,

b_{ik} = the sensitivity of the return on asset i to the fluctuations in factor k,

$\bar{\epsilon}_{it}$ = the "unsystematic" risk component idiosyncratic to the i-th asset. Assumed to be mutually independent over time and negligible for large numbers of assets in period t,

$\bar{F}_{kt}$ = the mean zero k-th factor common to the returns of all assets under consideration in period t.

$$E(\tilde{\epsilon}_i) = E(\tilde{F}_k) = E(\tilde{\epsilon}_i\ \tilde{\epsilon}_j) = E(\tilde{\epsilon}_i\ \tilde{F}_k) = E(\tilde{F}_k\ \tilde{F}_m) = 0$$

The above expression implies that the returns of the assets and the idiosyncratic terms are normally distributed. It is generally assumed that the factors are uncorrelated with mean 0 and variance 1, so the covariance matrix of F is the identity matrix, I. Also the security's ϵ's are independent with any other security's ϵ's and each disturbance has finite variance. The common factors are uncorrelated with one another and with the idiosyncratic terms.

The model of eq.(15), can be rewritten conveniently in vector notation as :

$$R = E + BF + \epsilon \tag{16}$$

In the framework of factor analysis, the B coefficients are referred to as the factor loadings, where the dimension of each of these factor-loading vectors is Kx1. Hence, B is an (NxK) matrix of coefficients or loadings on the K factors for each of the N assets. R_i is a (Nx1) row vector containing the random rate of return for N assets, $E(R_i)$ is a (Nx1) row vector of expected rate of return. F_k is a (Kx1) row vector of common factors, and ϵ_i is a (Nx1) row vector of idiosyncratic terms for each asset. Since the factors are independent and are scaled to have unit variance, E[FF'] = I.

2.9.3 Derivation of APT

A linear additive return generating process like equation (15) underlies the APT. Suppose that asset returns are generated by the k-factor linear model.

Choose a portfolio of N securities, the return on this portfolio is

$$\begin{aligned}\tilde{R}_p &= \sum_i w_i \tilde{R}_i \\ &= \sum_i w_i E(\tilde{R}_i) + \sum_i w_i b_{i1} \tilde{F}_1 + \ldots \\ &+ \sum_i w_i b_{ik} \tilde{F}_k + \sum_i w_i \tilde{e}_i. \end{aligned} \tag{17}$$

Let w_i be the change in the dollar amount invested in the i^{th} asset as a percentage of an individual's total invested wealth.

Ross (1976) indicated that the law of large numbers is the driving force behind the diminishing contribution of the idiosyncratic risks to the overall risks of the arbitrage portfolios. The weak law of large numbers (Connor (1989)) guarantees that if we take a large convex (i.e. linear) combination of uncorrelated random variables and each of the linear coefficients is small, then the randomness approximately disappears from the sum. As a portfolio return is a combination of asset returns, if it consists of weights that are spread evenly across many assets, and asset-specific risks have limited independence, then these risks will disappear from the portfolio return. As the residual risk can be diversified away in a large portfolio, no investor need ever bear this risk. As the number of assets becomes large, the linear approximation improves and most of the assets' mean returns are almost exact linear functions of the covariances of the assets returns with economy-wide common factors. Thus, once again, rational costless diversification eliminates unsystematic risk.

Ross's original derivation assumes that the idiosyncratic risks have zero correlation. This allows the diversification of idiosyncratic risk, but Ross also noted that a weaker condition could suffice. The key requirement for the APT is that non-factor risk can be diversified away in many-asset portfolios. This diversification criterion does not strictly require zero correlation across idiosyncratic returns. It only requires that the correlations be sufficiently weak so that the law of large numbers applies.

Based on this, Chamberlain and Rothschild (1983) and Ingersoll (1984) developed an approximate factor model. In an approximate factor structure the idiosyncratic terms need not be uncorrelated and hence, the idiosyncratic covariance matrix need not be diagonal.

In the strict factor model, random returns can be written in the form :

$$R - E = BF + \epsilon \tag{18}$$

and F_k for every i, j, k, $i \neq j$.

The assumption of an exact factor model is identical to assuming the following form for the return covariance matrix:

$$\sum = BB' + D, \tag{19}$$

where $D = E[\epsilon\epsilon']$ is diagonal.

As the factors are definitionally, market wide, each factor will have a broad-based influence affecting many assets in the economy. This means that each of the columns of B will have many non-zero components, which gives rise to the restriction called the pervasiveness condition. The pervasiveness condition requires that the minimum eigenvalue of BB' approaches infinity as N goes to infinity (where B is the NxK matrix of factor betas).

As the number of cross-sections increases, the proportion of total variation explained by any non-pervasive source of risk will approach zero.

In factor analysis a strict factor structure is assumed (see chapter 4). The return covariance matrix is exactly the same covariance matrix of the factor analysis (see section 4.2).

For the approximate factor model, the assumption that ϵ_i, ϵ_j are uncorrelated is dropped. Asset returns follow an approximate factor model if the sequence of covariance matrices can be written in the form of

$$\sum = BB' + V \tag{20}$$

where $V = E[\epsilon \epsilon']$ need not be diagonal.

The minimum eigenvalue of $B^N B^{N'}$ approaches infinity with N (where B^N is the matrix of (nxk) measures of systematic risk) while the maximum eigenvalue of V^N is bounded for all N (Huberman (1989), Connor (1989)). An asymptotic limit is assumed on the amount of covariance between idiosyncratic returns. This is expressed as a bound on the eigenvalues of the idiosyncratic covariance matrix as the number of cross-sections increases. This limits the amount of cross-sectional correlation in the idiosyncratic returns.

In order to obtain a riskless arbitrage portfolio, it is necessary to eliminate both diversifiable (i.e. unsystematic) and undiversifiable (i.e. systematic) risk. To form an arbitrage portfolio which requires no wealth, the APT no-money-invested assumption presumes that arbitraging short sellers are able to obtain 100% of the proceeds from their short sales to finance the purchase of their long positions. Mathematically, the zero change in wealth is written as :

$$\sum_{i=1}^{n} w_i = 0. \tag{21}$$

If we select the weighted average of the systematic risk components for each factor to be equal to zero ($\Sigma_i w_i b_{ik} = 0$), then the portfolio is riskless; so if arbitrage opportunities are absent, $\Sigma_i w_i E(R_i) = 0$. This eliminates all systematic risk. We have selected an arbitrage portfolio with zero beta in each factor. Consequently, the return on the arbitrage portfolio becomes a constant. Correct choice of the weights has eliminated all uncertainty, so that R_p is not a random variable.

Therefore, eq.(17) becomes

$$R_p = \sum_i w_i E(\tilde{R}_i) . \tag{22}$$

The arbitrage portfolio is constructed so that it has no risk and requires no wealth. In equilibrium, the return on any and all arbitrage portfolios must be zero. In a competitive equilibrium model, the pervasiveness conditions allows that investors can efficiently trade factor risk and idiosyncratic risk by exchanging available securities. The investors can diversify away idiosyncratic risk without restricting their choice of factor risk exposure. Rational investors will take the advantage of these trading opportunities, and, in competitive equilibrium, all investors' portfolios will be free of idiosyncratic risk.

In linear algebra, any vector which is orthogonal to the constant vector, i.e.

$$(\sum_i w_i) \times 1 = 0 \tag{23}$$

and to each of the coefficient vectors, i.e.,

$$\sum_i w_i b_{ik} = 0 \text{ for each } K \tag{24}$$

must also be orthogonal to the vector of expected returns, i.e.,

$$\sum_i w_i E(\tilde{R}_i) = 0. \tag{25}$$

These conditions can be written as :

$$\begin{pmatrix} E(R_1) - R_f & E(R_2) - R_f \ \dots & E(R_N) - R_f \\ b_{11} & b_{12} & b_{1k} \\ & & \\ b_{N1} & b_{N2} & b_{Nk} \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \\ w_N \end{pmatrix} = \begin{pmatrix} 0 \\ \\ \\ 0 \end{pmatrix}.$$

An algebraic consequence of eq.(25) is that the expected return vector is a linear combination of the constant vector and the coefficient vectors. Algebraically, there must exist a set of k+1 coefficients, λ_0, λ_1,..., λ_k such that

$$E(\tilde{R}_i) = \lambda_0 + \lambda_1 + \lambda_1 b_{i1} + \dots + \lambda_k b_{ik} \tag{26}$$

2.9.4 Competitive-equilibrium versions of the APT

Dybvig and Ross (1989) noted that there is no substance in the distinction between the 'equilibrium' derivations of the APT and the 'arbitrage' derivations. One derivation may give a tighter approximation than another (i.e. assuming competitive equilibrium gives a stronger pricing approximation), but all derivations require similar assumptions in one form or another.

If the market is to be in equilibrium, the excess return on the portfolio must be close to zero and there are no arbitrage profits. If the return was positive, investors could earn an arbitrage profit by buying the portfolio. If enough investors take advantage of it the price of the securities of which positive amounts were used in the arbitrage portfolio will rise, thereby forcing their rates of return down and back into equilibrium. Arbitrage profits would thus be eliminated.

In equilibrium, the return on a zero-investment, zero-systematic-risk portfolio is zero, as long as the idiosyncratic effects vanish in a large portfolio.

The expected return on i-th asset is given by eq.(26) :

$$E(R_i) = \lambda_0 + \lambda_1 b_{i1} + \dots + \lambda_k b_{ik}.$$

If there is a riskless asset with a riskless rate of return, R_f, then $b_{0k} = 0$ and $R_f = \lambda_0$. Hence, eq.(26) can be rewritten in "excess returns form" as

$$E(R_i) - R_f = \lambda_1 b_{i1} + \dots + \lambda_k b_{ik} \tag{27}$$

The above equation shows the general form of the APT model. In this competitive-equilibrium version of the APT, there exists a precise linear pricing relation in each asset's factor loadings.

With a positive investment, a portfolio with all B's equal to zero must earn a return equal to the risk-free rate. If the return is less than the risk-free rate, the investor will buy the risk-free security and short the portfolio. Whereas if the return is greater than the risk-free rate, it is possible to earn a profit by buying the portfolio and shorting the risk-free security.

In general, the APT is written as:

$$E(R_i)-R_f = [\bar{\delta}_1-R_f]b_{i1} + \ldots + [\bar{\delta}_k-R_f]b_{ik}. \tag{28}$$

where $\bar{\delta}_k$ is the expected return on a portfolio with unit sensitivity to the k-th factor and zero sensitivity to all other factors. Hence, the risk premium, $\lambda_k = \bar{\delta}_k - R_f$. If Eq.(28) is interpreted as a linear regression equation (it is assumed that the factors have been linearly transformed so that their transformed vectors are orthonormal) then the coefficients, b_{ik}, are defined in exactly the same way as beta in the CAPM, i.e.,

$$b_{ik} = \frac{Cov(\tilde{R}_i, \tilde{\delta}_k)}{Var(\tilde{\delta}_j)}, \tag{29}$$

where

$Cov(R_i, \bar{\delta}_k)$ = the covariance between the i-th asset's returns and the linear transformation of the k-th factor,

$Var(\bar{\delta}_k)$ = the variance of the linear transformation of the k-th factor.

The APT holds that the expected return on a security will be related only to its sensitivities to key factors (e.g. b_{i1} ,..., b_{ik}). The S-L CAPM implies that expected returns are related to the beta values. With the interpretation that a "factor" can be thought of as the return on a portfolio, the S-L CAPM implies that the expected value of each factor should equal its beta, times the expected excess return on the market portfolio.

An exact factor structure implies that there will be arbitrage unless the expected return on each portfolio is equal to a linear combination of the beta coefficients,

$$E(R_i)-R_f = \sum_k \lambda_k \beta_{ik} \tag{30}$$

where λ_k is the risk premium associated with the k-th factor, F_k. This equation is the APT version of the SML in the CAPM. The APT is similar to the CAPM in that it is also an equilibrium asset pricing model. The return on any risky asset is seen to be a linear combination of various factors. The APT requires fewer underlying assumptions and allows more factors to explain the equilibrium return on a risky asset than the CAPM. Therefore, the APT is a more general theory than the CAPM. The two theories are similar because both delineate systematic communalities that form the basis for risk premiums in market prices and returns. The APT appears to be an appropriate alternative because it agrees perfectly with what appears to be the intuition behind the CAPM.

2.10 Comparing the APT with the CAPM

There are two major differences between the APT and the CAPM. First, the APT allows more than just one generating factor, not just "the market". The appeal of the APT is mainly due to its implication that compensation for bearing risk can be comprised of several risk premia, rather than just one risk premium as in the CAPM. The APT does not specify any particular constructions of the factors, and hence they do not have to be linear combinations of all market assets. Second, the APT demonstrates that since any market equilibrium must be consistent with no arbitrage profits, every equilibrium will be characterized by a linear relationship between each assets' expected return and its returns' response loadings on the common factors.

The APT is a multifactor pricing model that describes the source of returns for assets. The model says nothing about market efficiency or inefficiency, equilibrium or disequilibrium. It depends essentially on the absence of arbitrage possibilities rather than on the much more restrictive condition that the market be in equilibrium as is required in the mean variance theory. The APT permits a significant weakening of the assumption that markets are in equilibrium. Consequently, the APT yields a statement about the relative pricing of any subset of assets, hence one need not measure the entire universe of assets in order to test the theory since the APT relation will hold for a subset of asset returns which meets its assumptions even if all asset returns do not, provided that the number of assets actually considered is sufficiently large to permit diversification. At the same time, there is no special role for the market portfolio in the APT, whereas the CAPM requires that the market portfolio be efficient. In other words, it is not essential to find the true market portfolio in the APT. Any fully diversified index can be utilised as a proxy for the market. Hence, the APT can furnish at least a partial answer to the objection that the true market has never been identified.

The greatest weakness of the APT is the large amount of ambiguity in its empirical predictions, particularly when compared to the CAPM. The CAPM is explicitly a one-beta model. The APT only guarantees a k-beta form, with k determined empirically. The CAPM specifies the market portfolio return as its factor. We do not have a perfect proxy for the market portfolio return, but at least we know what we are searching for. The APT gives little guidance on the identity of the factors beyond the restriction that they should obey the pervasiveness condition. In other words, each factor should have a broad-based influence affecting many assets in the economy. The assumption that the market factors are pervasive guarantees that investors can efficiently trade factor risk and idiosyncratic risk by exchanging available securities in the competitive equilibrium model. It allows investors to diversify away idiosyncratic risk without restricting their choice of factor risk exposure.

The APT makes relatively few assumptions, it provides little guidance concerning relationships between expected returns and security attributes (systematic factors), and the identity of the priced factors. The APT is a theoretical construct that says nothing about how the factors are to be identified or measured. The APT makes statements neither about the magnitudes nor even about the signs of the λ's, except λ_0. The values can be positive, negative, or zero. By contrast with the CAPM which prices assets in terms of their relation with a potentially observable and endogenous market aggregate, i.e. wealth for the CAPM, the APT factors are exogenous and unspecified.

The CAPM is explicitly a one-beta model which is mathematically equivalent to the one-factor APT. It is reassuring to find that when only one factor exists in the whole world, that single factor must be the market portfolio, and the single factor APT model turns out to be identical to the CAPM. The CAPM and APT can then be integrated by including the CAPM's market portfolio within an APT model. Hence, the CAPM is seen to be a special case of the APT with the market factor as an aggregate consensus measure of all the underlying factors. This implies that the market factor could incorporate nearly all information that the underlying multiple factors contain.

Variance is a well-known measure of dispersion about the expected. If instead of variance the investor was concerned with standard error, σ, his choice would still lie in the set efficient portfolios.

An average risk stock is defined as one which tends to move up and down in step with the general market as measured by some index such as the Dow Jones or the FT-Actuaries Index.

Although the use of systematic and undiversifiable risk has arisen in the literature as synonymous for covariance risk, they are somewhat misleading. They rely on the existence of costless diversification opportunities and on the existence of a large market portfolio. The definition of covariance risk does not.

The expected rate of return on a portfolio is always a linear function. It is simply a weighted average of the expected returns of the individual securities in the portfolio.

The Arbitrage Pricing Theory (APT) formulated by Ross (1976) claimed to offer a testable alternative to the CAPM.

CHAPTER 3

A LITERATURE SURVEY OF THE EMPIRICAL RESEARCH ON THE ARBITRAGE PRICING THEORY

The Arbitrage Pricing Theory (APT) provides an alternative approach to characterization of expected returns on risky securities to that of CAPM. Although it has attracted the attention of many empirical researchers, almost all of these studies are based on the capital markets of the United States. In spite of the prominence and size of the capital markets of the United Kingdom, there are few published studies regarding the validity of the APT in the context of the UK capital markets.

Section 3.1 covers the early studies that used factor analysis to examine asset returns. The empirical studies of the APT using factor analysis are discussed in sections 3.2, 3.3, 3.4, 3.5 and 3.6 respectively. Section 3.7 covers the previous empirical studies using other approaches in the testing of the APT. The macroeconomic factors model is discussed in section 3.8. The empirical studies of the APT using the measured-macroeconomic factor approach are discussed in sections 3.9 and 3.10. The last section is the conclusion of this chapter.

3.1 Early Studies

As discussed in chapter 2, the idea of multiple factor models that generate returns had been studied before the formulation of the APT. There have been a number of early studies examining the covariance structure of asset returns using factor analysis ((King (1966), Meyers (1973), Farrell (1974), Agmon (1973), Lessard (1974)).

However, most of the early studies, beginning with King's and continuing with others have concentrated on extracting industry factors. This is consistent with the traditional market-industry-firm analysis of securities. In most of these studies a "market factor" is first extracted and then the remaining variance is dissected to extract industry factors. These early studies tend to confirm the notion that at most only a few market wide factors are important. Since the APT was not available to predict the cross-sectional effects of industry factors on expected returns at the time of these studies, no tests were conducted for the presence of such effects. In the empirical research of the APT, the goal is to extract the market wide factors only.

The primary objective of King's analysis (1966) was to determine how much of the cross-sectional interdependence among a set of series of monthly price relatives could be explained by market and industry factors. King used factor analytic procedures to explain industry and market influences on expected returns. He first determined the communalities (the portion of covariance among the variables which could be explained by factors common to more than a single variable) and then used principal component analysis to identify the market factor from the covariance matrix. He next removed from the covariance matrix the portion of variance explained by the market factor before using factor analytic methods to further analyze the residual covariance matrix.

King's factor analysis covered the period 1927-60 period for a sample of 63 stocks classified according to six two-digit SIC industries. Both the cluster results and the correlation among industry factors reported by King indicated that the retail, tobacco, and utility industries and the metals and railroad industries showed sufficient correlation to warrant consideration as two rather than five separate groups. In addition, the predominantly negative correlation between these groups as well as with the oil industry indicated that three separate groups might be formed from the six industries analyzed by King: (1) oil industry,

(2) rail and metals industries, and (3) tobacco, retail and utility industries. This evidence of significant co-movement among these industry groupings implied that another factor, broader than the industry factor and in addition to the market and company factors, was needed to explain the variations in common stock returns. King showed that the variance of stocks over the full 1927-60 study period could be explained in terms of (1) market factor, 50 percent; (2) industry factor, 10 percent; and (3) effects unique to an individual security, 40 percent. An analysis by King over four sub-periods indicated relative stability for the industry effect, but showed a successive decline in the importance of the market effect from 58 percent, to 56 percent, to 41 percent, and finally to 31 percent.

King concluded that one factor explains a large percentage of the variance of stock prices, a factor on which each security tends to weight positively. He interpreted this result to mean that a basic market factor exists which has a major effect on all securities. Although his study has enhanced the understanding of non-market components of asset returns, an equilibrium asset pricing model was not used and major economic variables were not considered.

Meyers (1973) claimed that although the procedures used by King were appropriate in the light of his objectives, more objective results would be obtained from the use of a slightly different method. The two most important differences were the use of true principal component analyses in lieu of the Guttman-Harris and centroid techniques and the omission of the multiple factor analysis of industry factors. In order to avoid the problems associated with estimating communalities (see chapter 4 below), Meyers analyzed the total variance in the variables rather than just the common variance. Less precise factors would be expected by the inclusion of unique and error variance. Once the market factor had been identified, the next step in the Meyers (1973) was to remove from correlation matrix that portion of correlation among the variables which was associated with the market factor. If the market model is valid in practice, with $Cov(\tilde{E}_{it},\tilde{E}_{jt}) = 0$, for $i \neq j$ (where $\tilde{E}_{it}$ represents independent factors unique to asset i); the remaining variance should be unique to each of the separate variables, and no persistently strong factors should result from subsequent analysis of the dependence structure. Finally, the cluster analysis performed in his study used a weighting scheme which was conceptually preferable to the one used by King. King's analysis assigned equal weight to each of two variables forming a cluster regardless of the number of securities in each of the original variables. The cluster analysis technique used in the study of Meyers was almost identical to the technique used by King except that Meyers used a weighting scheme which caused each security in a cluster to have equal weight in determining the correlation of the cluster with other variables in the analysis.

Meyers demonstrated that King's conclusion that industry factors accounted for an average of about 10 percent of the variance in stock price changes overstated the role of industry factors in the market as a whole. In general, Meyers' results tended to confirm that King's observations concerning industry factors were an insufficient basis for denying the independence of the residuals in the market model. For example, the market factor explained 59.9% of the total variance, and the first component after this factor had been removed accounted for an additional 4.8% of the total variance, which translates to 11.9% of the residual variance. The first six components computed from the partial correlation matrix explained a total of 18% of the total variance and 45% of the residual variance. Thus, the results by Meyers provided less than a complete defence of the market model, especially in light of the numerous unexplained components generated by his components analysis of both samples. If these components represent some persistent significant source of interdependence among stock prices, then they, rather than industry factors, represent a limitation of the validity of the market model.

Farrell (1974) considered it appropriate to assign a factor to the explanation of the variance of returns of a common stock additional to market, industry, and company, and based upon a system of classification corresponding to the following categories: growth stocks, stable stocks, cyclical stocks, and oil stocks. His study employed several statistical techniques (i.e. stepwise clustering procedure[1], direct inspection of the correlation matrix of the residuals of the stock returns, index procedure, which is somewhat analogous to the forward selection procedure, etc.) in testing the hypothesis that classification according to (1) growth, (2) stable, and (3) cyclical characteristics represents a factor for grouping stocks. These techniques showed that the residuals obtained by the removal of general market effects from a sample of 100 stocks displayed cross-sectional dependence conforming to four distinct stock categories, including an oil group as well as the three hypothesized groups. In addition, regression analysis results indicated that these stock groupings accounted for an average of 14 percent of the variance in rate of return of stocks in the sample, in comparison to 31 percent represented by general market effects.

Agmon (1973) investigated the significance of country factors for share price co-movements. He showed that although movements of share prices in the equity markets of the U.K., Germany, and Japan were related to price changes in the U.S. market index, there was also another residual factor affecting share-price fluctuations in these three markets. The residual factor could be uniquely associated with the country.

Lessard (1974) recognized the importance of national risk factors. Empirical results were presented, based on a set of sixteen national market indices and thirty international industry indices. These indices could be viewed as portfolios selected in order to maximize the impact of national or industry factors. He found that only a small proportion of the variance of national portfolios is common in an international context, which gives rise to considerable risk reduction (ex post) through international dimension. Further, he found that the industry dimension is much less important than the national dimension in defining groups of securities that share common return elements and, therefore, are a less important part of diversification strategy. Moreover, he also showed that, given the importance of national risk factors and the preponderant position of U.S. securities in the world portfolio, a multi-factor market model is called for and that the world factor should be estimated to minimize the impact of national risk factors. This is only a return generating process with multiple independent variables.

Overall, these early studies strongly suggest that at most only a few market-wide factors are important. In most of these studies a "market factor" is first extracted and then the remaining variance is dissected to extract industry factors.

3.2 Empirical Tests of the APT : Early Studies

The results of the studies mentioned below are summarized in the following table.

Gehr (1975) was the first study to test the APT using US stock price data. Gehr used 24 industry indices and 41 individual stocks. He found that there are at least two and probably three common factors for the stock market which explained a large, but not predominant portion of the variance of the stocks used in this study.

Roll and Ross (1980) were among the first to look specifically for APT factors. R&R used daily returns data for NYSE and AMEX companies listed on the exchanges from 1962 to 1972. R&R employed factor analytic techniques to analyse 1260 NYSE stocks that were divided into 42 groups of 30 stocks. In the first step of their study, R&R estimated factor loadings, for their second step, they ran a separate cross-sectional multiple regression for each

Gehr (1975)	30 years	360	24 industry indices + 41 stocks	M	principal axis extraction, varimax and promax	2 - 3	N.A.	CRSP tapes
Roll and Ross (1980)	3 July 1962 - 31 Dec. 1972	2,619	1,260	D	maximum-likelihood	5	3 to 4	NYSE and AMEX
Reinganum (1981)	1963-1978	3,756	1,457 - 2,500	D		3 - 5		NYSE and AMEX
Beenstock and Chan (1986)	Dec 1961 - Dec 1981		220	M	maximum-likelihood	20		London Stock Exchange
Chen (1983)	1963 - 1978		1,064-1,580	D	maximum-likelihood	5	> 1	NYSE and AMEX
Kryzanowski and To (1983)	Jan. 1948 - Dec. 1977	360 (US) 120 (Canadian)	550 (US) 180 (Canadian)	M	Rao's / Alpha	5 (US) 18-20 (Canadian)		NYSE and AMEX Toronto Stock Exchange
Oldfield and Rogalski (1981)	Jan. 1964 - Dec. 1979	639	1,260	W	maximum-likelihood			NYSE and AMEX
Brown & Weinstein(1983)	3 July 1962 - 31 Dec. 1972	2,619	1,260	D	bilinear paradigm (maximum-likelihood)	3 - 5		NYSE & AMEX
Cho (1984)	3 July 1962 - 31 Dec. 1972	1,719	1,171	D	inter-battery	5 - 6		NYSE & AMEX
Trzcinka (1986)	20 years	1,069	865	W	principal components		1	N/ANYSE & AMEX
Cho, Elton & Gruber (1984)	1 Jan. 1973 - 30 Sept. 1980	1,770	1,740	D	maximum-likelihood	5 - 7	2 - 6	NYSE & AMEX
Dhrymes (1984)	3 July 1962 - 31 Dec. 1972	2,618	1,260	D	maximum-likelihood	5	N.A.	NYSE & AMEX
Dhrymes, Friend and Gultekin (1984)	3 July 1961 - 31 Dec. 1972	2,509 - 2,619	1,260	D	maximum-likelihood	2 - 9	N.A.	NYSE & AMEX
Dhrymes, Friend, Gultekin and Gultekin (1985a)	3 July 1962 - 31 Dec. 1981	2509 - 2619	1,260	D	maximum-likelihood	5	N.A.	NYSE & AMEX
Dhrymes, Friend, Gultekin and Gultekin (1985b)	3 July 1962 - 31 Dec. 1981	4793 - 4892	900	D	maximum-likelihood	7 - 17	1 - 3	NYSE & AMEX
Diacogiannis (1986)	1 Nov. 1956 - 31 Dec. 1981	302	200	M	Rao's	1 - 10		London Stock Exchange
Abeysekera & Mahajan (1987)	Jan 1971 - Dec 1982	144	280	M	maximum-likelihood	6 - 8	0	London Stock Exchange

Cho and Taylor (1987)	2 Jan. 1973 - 30 Dec. 1983		340	M	maximum-likelihood	6 - 7	0	NYSE & AMEX
Gultekin and Gultekin (1987)	3 July 1962 - 31 Dec. 1981	4,793 - 4,893	900	D		7 in 30 security portfolios 17 in 90 security portfolios		NYSE & AMEX
Lehmann and Modest (1988)		1040	750	D		5 - 20		NYSE and AMEX
Conway and Reinganum (1988)	July 1962 - Dec. 1972	1,309	550	D	cross-validation technique	2		NYSE & AMEX
Roll (1988)	Sept. 1982 - Aug. 1987	30	2,030	M		5		NYSE & AMEX
Brown (1989)		80	80	W	principal factor	4	1 - 4	NYSE & AMEX
Shukla & Trzcinka (1990)	July 1962 - Dec. 1982		596	W	maximum-likelihood /principal component	4 - 5		NYSE & AMEX
Shukla and Trzcinka (1991)	Jul 1962-Dec.1983, 1984-1988	1,069	596	W	FA /principal components	1 - 5	1 - 5	NYSE & AMEX

M = monthly
D = daily
W = weekly

of the 42 groups of stocks. The cross-sectional regression coefficient λ_k for the kth factor loading is an empirical estimate of that factor's risk premium. One or more of these regression coefficients should be statistically significantly different from zero if the APT is to be substantiated. R&R found that when the zero-beta or risk-free coefficient, λ_0 is assumed to be 6% per annum during the sample period (1962-72), 88.1% of the groups have at least one significant factor risk premium, 57.1% have two or more significant factors and in one-third of the groups at least three risk premia are significant. When the intercept,[2] λ_0, was estimated, two factors were significant for pricing. Using data for individual securities during the 1962-72 period, R&R found that there are at least three and probably four "priced" factors in the generating process of returns.

R&R realized that there remains a possibility that other variables are also "priced" even though they are not related to undiversifiable risk (e.g. the total variance of individual returns). For example, the total variance should not affect expected returns if APT is valid, because its diversifiable component would be eliminated by portfolio formation and its non-diversifiable part would depend only upon the factor loadings and factor variances. According to the theory, such variables should not explain expected returns; hence if some were found to be empirically significant the APT would be rejected. To test for added factors, they regressed the expected returns derived using the five factors that they estimated in their factor analysis, against what they called "own" variance or the total variance of individual returns.

However, 45.2% of the groups displayed statistically significant effects from the "own" variance. Roll and Ross found that even though variances and average returns were highly correlated, the variance did not contribute to the explanatory power of an APT model. R&R, after correcting the problem that positive skewness in lognormal returns could create dependence between the sample mean and sample standard deviation, found that the total variance of security returns did not add any explanatory power for estimated expected returns. Thus, Roll and Ross concluded that the theory is supported in that estimated expected returns depend on estimated factor loadings, and variables such as the "own" variance, though highly correlated with estimated returns, do not add any further explanatory power to that of the factor loadings. Therefore, the APT could not be rejected on this basis.

Because the same underlying common factors can be rotated differently in each group, the problem of the non-uniqueness of factor loadings arises. However, there is one parameter, the intercept term, λ_0, which should be identical across groups, whatever the sample rotation of the generating factors. Other factors need not be the same, because the factor loadings are not unique from group to group. R&R tested for the equivalence of the λ_0 terms across 38 groups and found no evidence that the intercept terms were different. Again, the APT could not be rejected. Chen (1983) conducted a series of insightful empirical tests of the APT. He compared the empirical characteristics of the APT and the CAPM using daily stock returns from 1963 to 1978. First, cross-sectional regressions of the average returns from the sampled stocks were related to the APT and the CAPM models. The sensitivity measure on the first factor has the highest statistical significance. The first risk factor somewhat resembles the market portfolio, as the correlation coefficient between the factor loading of the first factor and the market index was found to be high and positive (in excess of 0.9). In addition, the hypothesis that the risk premia of all the factors are insignificantly different from zero was rejected. This suggests that more than one factor should be considered. Chen also found that the APT predicts average returns better than the CAPM. He also employed cross-secional regression to detect unused information about stocks' expected returns that turned up as residue in the random error terms. The tests were based on the idea that if a particular model was valid, its random error term should be white noise, the residuals should contain no additional information. Chen reported that the CAPM appeared to be econometrically misspecified in most cases and that the APT model was able to explain some of the CAPM's unexplained residual returns. In contrast, the CAPM was unable to explain anything about the error terms from the APT model. Furthermore, Chen formulated two additional tests based on empirical anomalies in the CAPM that can be interpreted as evidence against it. The tests were designed to see if the total variance of a stock's returns or the size of the issuing firm were cross-sectionally related to the stock's average return after removing the part of the return that was explained by the APT model. The results indicated that neither the firms' variances nor the firms' sizes had significant explanatory power over the unexplained residual return terms left by the APT. This represents further evidence in support of the APT.

A study by Cho, Elton, and Gruber (1984) showed that the methodology Roll & Ross use (the stocks are grouped in different groups) has a problem of factor comparability. They claimed that very little is known about the properties of the estimates obtained from maximum-likelihood factor analysis or of the sensitivity of the results to the characteristics of the underlying data. The estimated factor loadings are unique only up to an orthogonal transformation and thus if one were to carry out separate factor analyses for each group, it would be necessary to see whether the factors were the same across different groups before making any generalizations over the entire sample. In their study, they examined the results produced by the Roll and Ross procedure when the return generating process was known. They allowed the parameters of the return generating process to change quarterly in two distinct ways. They used both Wilshire Associates' fundamental betas which are estimated using techniques devised by Rosenberg and Marathe (1976), and betas which were estimated quarterly using historical daily return data (1973-80). They grouped the stocks into 58 groups of 30 securities each. It was suggested that extra factors might be identified for two reasons. Firstly, if the betas themselves are related to a set of variables (factors), then the return generating process and the model explaining equilibrium returns may contain several. These extra factors would reflect the factors that influence betas. Ross (1976) has shown that the existence of variables that affect the influences of market returns on securities' returns can lead to a multi-index model. Additional factors might also be identified simply due to random patterns in the data.

Cho, Elton, and Gruber (1984) found that while the Roll and Ross (1980) procedure has a slight tendency to overstate the number of factors at work in the market, this tendency cannot account for the large number of factors Roll and Ross found in their original article. Cho, Elton, and Gruber also concluded that this is true even though the parameters of the two-factor CAPM are linearly related to other variables and changed over time in response to changes in these variables.

3.3 Empirical Tests of the APT : The Dhrymes Critique

In estimating the number of factors, Dhrymes (1984) used a sample similar to that of Roll and Ross (1980), and has concluded that:

> "at the 5% level of significance, with a group of 15 securities, we have at most two 'common risk' factors; with a group of 30 securities we have at most three 'common risk' factors; with a group of 45 securities we have at most four 'common risk' factors; with a group of 60 securities we have at most six 'common risk' factors; and with a group of 90 securities we have at most nine 'common risk' factors" (p.39).

There exists a significantly positive relationship between the number of factors which affect the security returns and the number of securities in the groups to which the factor analytic methods are applied. The number of securities being analyzed has an impact on the number of "common risk" factors being discovered. Such results highlight the fact that the methodology used for testing the APT may not be the appropriate one, and previous tests of the APT are not necessarily tests of the model. Dhrymes, Friend and Gultekin (1984) stressed three points: first, that the method of Roll and Ross (1980) has major pitfalls and is seriously flawed; second, that individual factors should not be tested for their pricing influence; and third, that more than three to five factors can be found by increasing the size of the group analyzed. They commented that the only meaningful tests are those which determine whether any factors are priced, rather than those which test whether some of them are priced and others are not. Actually, R&R (1980) raised the issue of the rotation problem and conducted F-tests of the joint significance of all factor prices. Roll and Ross (1984) disagreed with the critique by DFG, they claimed that despite the rotation problem, tests of individual factor pricing have meaning. As the factors are extracted in the order of their importance in explaining the covariance matrix of returns, it is interesting to ascertain if they each have an influence upon pricing. R&R also argued that there are many reasons why the number of non-priced factors will increase with the group size. There may be as many factors as there are sets of assets, and they could all be detected with a sufficiently powerful test. However, since most of the common factors are diversifiable (e.g. non-pervasive factors), they will not be priced (i.e. they will have no associated risk premia). Hence, those non-priced factors are irrelevant for the APT.

In another study, Dhrymes, Friend, Gultekin, and Gultekin (1985b) used new procedures to test the basic implication of the APT model that only common (factor) risks are priced. The common and unique variance measures are estimated within the sample period, in which they serve as explanatory variables. DFG&G derived the common and unique measures of risk from the daily time-series observations in the first half-period (1962 to 1972) and used them to explain the daily cross-section returns for the second half-period (1972 to 1981). DFG&G were concerned about the question of how the number of factors that are significant (on the first stage) and /or priced (on the second stage) varies with the sizes of securities groups or the length of the time series. They showed that tests results appeared to be extremely sensitive to the number of securities used in two stages of the tests of the APT model. The tests also indicated that unique risk was fully as important as common risk. In another study, Dhrymes, Friend, Gultekin, and Gultekin (1985a) presented a comprehensive set of tests of the implications of the APT. They found that the risk premia was not significant in most groups (at least 36 out of 42), indicating a lack of a linear relationship between the expected rates of return and the measures of risk parameters implied by the APT model. Furthermore, unique variance measures of risk, while generally making only small contributions to the explanation of asset returns, turned out to be as significant as frequently as the covariance measures of risk - which was inconsistent with the APT model. These intercept tests were more mixed, but provided only limited support to the model. One of the important implications of the model is that the intercept terms are, on average, the same in all groups which would be true if the intercepts were either the risk-free or zero-beta rates of return. Such an implication was not rejected by their study; on the other hand, the same evidence suggested that on average the intercept term was insignificantly different from zero for most groups. Moreover, these intercepts were significantly different from the risk-free rate interpreted as the appropriate Treasury Bill rate.

Brown and Weinstein (1983) proposed a new approach to estimating and testing asset pricing models in the context of a bilinear paradigm. It applied to the special case of the arbitrage pricing model where the number of factors was pre-specified. They found that the data appeared to be generally in conflict with a five or seven factor representation of the model used by Roll and Ross (1980). Brown and Weinstein concluded that the three factors that best represent the observed variation in the data do not significantly differ across groups. They suggested that there may be a small number of economy-wide factors that affect security returns.

Cho (1984) tested the APT by estimating the factor loadings that were consistent between two industry groups of securities. Inter-battery factor analysis[3] was employed so that the factor loadings were estimated by constraining the factors to be the same between two different groups. He concluded that there are five or six inter-group common factors that generate daily returns for two groups and that these inter-group common factors do not depend on the size of groups. Also, the APT could not be rejected in the sense that the risk-free rate and the risk premia are the same across groups and that the risk-free rate is different from zero.

Gultekin and Gultekin (1987) used seven factors in 30 security groups and seventeen factors in groups of 90 stocks. They tested the APT on a monthly basis using the same set of factor loadings that were obtained from the maximum-likelihood factor analysis of seven factors during the entire period. They found that these factors are priced for all groups in January and were rarely priced in other months. They concluded that the factor analysis approach would imply that the APT is valid only in January.

Cho and Taylor (1987) indicated that between six and seven factors are usually sufficient for groups of 30 US securities. The number of return-generating factors is rather stable most of the time and for most of the groups. Their results, however, showed that there is a January effect and a small-firm effect on stock returns. They also noted that the APT pricing relationship does not appear to be supported by the standard two-stage process, as the

APT does not hold for the entire period. There is no group that shows any significant statistics. This result is similar to the findings of Dhrymes, Friend, Gultekin and Gultekin (1985b).

3.4 Empirical Tests of the APT : Non-US Studies

A study which used data on Canadian securities was written by Hughes (1982). Hughes used two groups containing 110 securities and a sample size of 120 observations. She found that only three or four factors were priced in the market and the intercept of the APT pricing relation closely predicted the Canadian risk free rate during the test period. However, her tests can be criticised, because she utilized a large group size relative to the number of observations per security. Hughes stated: "The number of factors extracted was increased from five to twelve and the chi-square statistic continued to indicate that many additional factors were needed for adequate factoring" (p.16). However, as the number of factors increases with the group size, the chi-square test Hughes used requires a large number of observations relative to the size of the group. In such a case, therefore, the k-factor generation model could probably be rejected for every possible value of k.

Diacogiannis (1986) utilised time series data from the London Stock Exchange and has concentrated on the empirical verification of the assumption that there exists a security return generating model which remains the same across different security groups and across various time periods. The results indicated that the number of factors change as the group size changes (i.e. as the number of securities increases, the number of factors determined increases) as suggested by Dhrymes, Friend and Gultekin (1984). He also found that the number of factors also changes across various time periods for the same group of securities and for different security groups.

Beenstock and Chan (1986) tested the APT using UK security returns, they concluded that a relatively high proportion of the variance of estimated expected returns for 220 UK securities can be explained in terms of the APT. The mean R^2 were between 0.25 and 0.44. B&C suggested that the number of priced factors in the UK is unlikely to be small, as they argued that the explanation power of a 20 factor APT model was significantly greater than a four factor model. They also noted that the number of factors is proportionate to the sample size.

Abeysekera and Mahajan (1987) empirically tested two hypotheses to evaluate the validity and applicability of the APT to the UK stock market using monthly individual price data. Their study empirically evaluated the validity of the APT by testing two hypotheses. The first hypothesis is that the intercept term (λ_0) of the pricing relation represents the risk-free rate R_f. The second hypothesis is that the APT implies that if k factors are responsible for driving the individual asset returns through time, then there should be a risk premium attached to each of these factors. The monthly returns on a random selection of securities listed continuously in the London Stock Exchange from January 1971 to December 1982 were computed and then seven portfolios were formed, each consisting of 40 randomly selected securities. Each portfolio was then subjected to eight maximum-likelihood factor analyses, pre-specifying between one and eight factors, to determine the factor loadings. Their results supported the first hypothesis that the risk-free rates are equal to the corresponding estimated intercept terms of the models tested. The results also showed that the intercept term was significantly different from zero. However, by utilizing two different procedures (i.e. the chi-square test and the t-test) the results for the second hypothesis that if k factors are responsible for driving the individual asset returns through time, then there should be a risk premium attached to each of these factors, showed that the risk premia are not significantly different from zero. Their latter finding does not support the APT and is in conflict with the results of Roll and Ross (1980) and other studies that are based on US stock market data.

3.5 International APT

Kryzanowski and To (1983) used the factor analysis to examine the factor structures of security returns. The specific purpose of their paper was to empirically test the assumption that security returns are characterized by an explicit underlying factor structure composed of at least one general or common factor. They used the US and the Canadian stock price data to test the APT. Their study concluded that the number of relevant factors is an increasing function of the size of the group being factored. They observed that while five factors were sufficient to represent the US security returns, Canadian securities required eighteen to twenty factors. They suggested that since the first (and maybe the second) is the only factor associated with almost all the securities in each sample, there is even some empirical support for the hypothesis that a very simple one- or two-factor structure may adequately describe the underlying economic structure of security returns. Kryzanowski and To observed that there was a far greater number of relevant factors for the Canadian data (i.e. 18 to 20 factors) as compared to the US data (i.e. 5 factors). The authors claimed it was partly due to the fact that the Canadian samples each consisted of 60 securities, while the U.S. samples each consisted of 50 securities. The number of relevant factors may be an increasing function of the size of the group being factored. Moreover, the study showed that the first factor was relatively less important and was associated with fewer securities for the Canadian data than for the US data. The first US factor accounts for about 70% of the total variance, whereas the first Canadian factor accounts for only 40%. Kryzanowski and To suggested that the difference may be partially due to the greater number of market imperfections in the Canadian capital markets (e.g. market thinness as described by Fowler, Rorke, and Jog (1979)), which may result in the creation of one or more "artificial" factors.

3.6 Empirical Tests of the APT : Non-Equity Studies

Oldfield and Rogalski (1981) analyzed the response of common returns to statistical factors estimated from the weekly returns of a set of U.S. Treasury bills. They assumed that the arbitrage pricing model gives a valid ex post and ex ante return model for both sets of securities. They also assumed common factors were present. A five step procedure was set up. First, they factor analyzed weekly Treasury bill returns and constructed time series of factor scores. Second, the time series of factor scores were used as independent variables in time series regressions with individual stock returns as the dependent variable. This yielded initial estimates of stock response coefficients. In the third step, they set up special stock portfolios, such that a portfolio's returns respond to changes in one factor (or zero factors) only. Oldfield and Rogalski then used intermediate portfolios in which the initial share coefficients from step two were averaged to give portfolio coefficients. From this step they have the actual weekly returns on the special factor portfolios. The fourth step entailed regressing individual share returns on special portfolio returns. This gave a revised estimate of share response coefficients. Finally, in step five, they did cross-sectional regressions on the results from step four. They then used the results in an averaged model to analyze the estimated ex ante arbitrage pricing model. Their results showed that the arbitrage pricing model is a correct specification of ex post and ex ante security returns. In addition, the Treasury bill returns have been shown to provide a source for identifying statistical factors that influence common stock returns.

Gultekin and Rogalski (1985) examined the factor structure of US Treasury security returns and tested the APT in the US Treasury security market. They also compared the empirical performance of the APT with that of the CAPM in the US Treasury security market during the 20-year sample period, 1960-1979. Their study found that mean returns on bond portfolios were linearly related to at least two factor loadings. Furthermore, the multivariate tests were not consistent with one- to seven-factor APT models as descriptive models of the US Treasury securities market. The tests could be viewed as the first empirical attempt to accurately measure interest-rate risk for bonds using factor-generating models. They showed that one-month-ahead forecasts using factor-generating models are somewhat better than corresponding naive predictions or predictions using the "market model" with various market portfolios.

3.7 Other Approaches

Trzcinka (1986) examined the behaviour of the eigenvalues of the sample covariance matrix as the number of securities increased. The purpose of his paper was to test whether sample covariance matrices could be characterized as having k large eigenvalues. Using all available data on the 1983 CRSP tapes, the sample covariance matrices of returns in sequentially larger portfolios of securities were computed. Analyzing their eigenvalues, he found evidence that one eigenvalue dominated the covariance matrix indicating that a one-factor model might describe security pricing. He also found that only the first eigenvalue dominated the matrix. The application in his study indicates that only one factor is required.

Conway and Reinganum (1988) explored the ability of cross-validation procedures[4] to identify the number of stable factors in security returns. In simulations with one-factor and two-factor models, the correct stable factor structures were identified by both the formal likelihood ratio test and the cross-validation method more than 95% of the time. When the cross-validation technique was applied to the actual returns of 11 groups with 50 randomly selected securities, their results showed the presence of one dominant factor and one minor factor. In contrast, formal tests using the likelihood ratio statistic suggested a model with more than five factors. One dominant factor and one relatively minor factor were also identified using cross-validation in groups of both 30 and 60 randomly selected firms. However, the authors admitted that when groups are designed to highlight industry or size effects, the discovery of more than one dominant factor is problematic. Furthermore, Conway and Reinganum claimed that even if there are multiple economic factors generating stock returns, they may be difficult to disentangle if the underlying factors tend to be correlated.

Shukla and Trzcinka (1990,1991) examined the cross-sectional pricing equation of the APT using both the principal components analysis and the maximum-likelihood factor analysis. Their results show that, for data assumed stationary over twenty years, the first eigenvector from principal components analysis is a surprisingly good measure of risk when compared with either a one- or a five-factor model or a five-eigenvector model. Their results indicated that in some cases, the principal components analysis is superior to the factor analysis. They also showed that the APT explains as much as 40% of the variation in mean returns of 865 US companies (weekly data for twenty years). Shukla and Tracinka (1990) showed that the first factor is highly correlated with both the equal and value weighted market betas, and the first eigenvector from the principal components has a much higher correlation with the equal weighted betas than with the value weighted. This supports Brown's (1989) theoretical argument that the first principal component is the equal weighted market index if the idiosyncratic risks are equal across firms.

3.8 Macroeconomic Factors Outside the APT

Fogler, John and Tipton (1981) tried to assign economic meaning to stock market factors and to determine the extent to which these factors were related to the prices of capital in the bond market. The results showed that the returns from stock groups were found to relate to returns in the Government bond market and to corporate bonds with default risk. Moreover, the returns of bond market variables were found to relate to the stock market forces derived from all 100 stocks by principal components analysis, although those bonds with default risk showed a very weak relationship. Fogler, John and Tipton found that the first three sources of variation in 100 stocks were related to the market, the interest rate on US government bonds, and the interest rate on AA utility bonds.

Sharpe (1982) has chosen and used a broader set of factors and examined monthly security returns of 1,325 NYSE stocks over the 1931-79 period. He did not attempt to identify common factors, but drew on previous research and industry practice. He reported finding five "common attributes" and "eight attributes representing 'sectors' of the economy". The five common attributes were

(1) Dividend yield: "prior 12 months' dividends paid to common stockholders divided by the market value at the end of the prior month".

(2) Firm size: "the logarithm (to base 10) of the market value of the firm's equity at the end of the prior month".

(3) Stock beta: the slope coefficient from a regression of "the excess returns on a stock over the prior 60 months on the Standard and Poors' stock index".

(4) Alpha: the intercept from the regression used to calculate the stock beta factor.

(5) Bond beta: the slope coefficient from a regression of "the excess returns on stock over the prior 60 months on the excess returns on long-term government bond returns.

The "eight attributes" representing 'sectors' of the economy were basic industries, capital goods, construction, consumer goods, energy, finance, transportation and utilities.

For each month in the 1931-1979 period, Sharpe ran cross-section regressions of the realized returns against (a) the beta factor, (b) the five common factors, and (c) the five common and the eight sector factors. The mean R^2 over 588 cross-section regressions was 0.037, 0.079 and 0.104 of beta, common factors, and common and sector factors respectively.

Multifactor models of security returns are also available from many investment institutions. For instance, the Salomon Brothers model includes five factors: inflation, real economic growth, oil prices, defence spending, and real interest rates (Estep, Hanson, and Johnson, 1983). One use of these factor models is in performance evaluation, where the focus is on the reasons for the security returns of an investment strategy being what they are (the so-called "performance attribution" stage of performance evaluation).

3.9 Empirical Tests of the APT :

Measured-Macroeconomic Factor Approach

An alternative approach to the use of factor analysis is for that the researcher to use his intuition to choose factors and then to estimate the factor loadings by some sort of regression analysis. These loadings can then be tested to see if they explain the cross-sectional variations in estimated expected returns.

Although more studies take the factor analysis approach, the most influential tests of the multifactor model are those of Chen, Roll, and Ross (1986). The alternative approach in Chen, Roll, and Ross is to look for economic variables that are correlated with stock returns and then to test whether the loadings of returns on these economic factors describe the cross-section of expected returns.

Chen, Roll and Ross (1986) selected a range of business conditions variables that may be related to returns because they are related to shocks to expected future cash flows or to discount rates and tested a set of economic state variables as systematic influences on stock market returns. The study used intertemporal asset pricing theory to choose a set of macroeconomic variables and to construct series of their innovations, and then related this to systematic factors extracted from stock returns by a factor analysis. To ascertain whether the identified economic state variables are related to the underlying factors that explain pricing in the stock market, a version of the Fama-MacBeth (1973) technique was employed. The procedure was as follows: (a) A sample of assets was chosen. (b) The assets' exposure to the economic state variables was estimated by regressing their returns on the unanticipated changes in the economic variables over some estimation period. (c) The resulting estimates of exposure (betas) were used as the independent variables in 12 cross-sectional regressions, one regression for each of the next 12 months, with asset returns for the month being the dependent variable. Each coefficient from a cross-sectional regression provides an estimate of the sum of the risk premium, if any, associated with the state variable and the unanticipated movement in the state variable for that month. (d) Steps b and c were then repeated for each year in the sample, yielding for each macro variable a time series of estimates of its associated risk premium. The time-series means of these estimates were then tested by a t-test for significant difference from zero.

They analyzed data from 1958 to 1984 and found five principal factors existed. As mentioned above, using correlation and regression analysis they analyzed the relationship of the five unknown, but principal factors, to fundamental macroeconomic variables:

1. A change in expected inflation and unexpected inflation.

2. An unexpected change in the term structure of interest rates.
3. The growth rate of, and anticipated and unanticipated changes in industrial production.
4. Unanticipated change in the risk premium.
5. Changes in a stock market index.

The market index was included to capture the effect of any variables that had not been explicitly included.

Chen, Roll and Ross found that several of the economic variables were significant in explaining expected stock returns, most notably, industrial production, changes in the risk premium, twists in the yield curve, and somewhat weakly, measures of unanticipated inflation and changes in expected inflation during periods when these variables were highly volatile. Perhaps the most striking result is that even though a stock market index, such as the value-weighted New York Stock Exchange index, explains a significant portion of the time-series variability of stock returns, it has an insignificant influence on expected returns when compared against the economic state variables. If the market index is important in pricing, even after the other common factors have been accounted for, either the factors have been mismeasured, or one or more factors are missing. Born (1984) claimed that the market portfolio cannot be one of the common factors in the APT's return generating model and finding a statistically significant 'market' factor suggests that additional return generating factors remain to be identified. The identification of the factors that are relevant in pricing assets is still at its inception. Both aggregate consumption and oil price risk have no overall effect on asset pricing.

The economic logic underlying these variables seems to make sense. Common stock prices are the present values of discounted cash flows. Industrial production is obviously related to profitability. The remaining variables are related to the discount rate.

The intuition behind these factors is useful for portfolio management. For example, it has often been stated that common stocks are not a good hedge against inflation. Although it is true if one holds an equally weighted portfolio of all stocks, the logic of factor analysis suggests that there is a well-diversified subset of common stocks that is in fact a good hedge against inflation. Since the factors are mutually orthogonal, one can at least in principle choose a portfolio which is hedged against inflation risk without changing the portfolio sensitivity to any of the other three factors mentioned above.

Kim and Wu (1987) take a different approach by incorporating a multifactor return generating process into the traditional CAPM. This method attempts to remedy the inability of the APT to assign proper economic meanings to return factors. Kim and Wu showed that there are at least three significant factors. The first factor encompasses general economy-wide variables and the second factor is characterisized by interest rate and money supply. The third factor includes the labour market variables.

McElroy and Burmeister (1988) replaced the unknown random factors of factor analysis with observed macroeconomic variables. The set of macroeconomic factors studied by McElroy and Burmeister (1988) is similar to the factors of Chen, Roll and Ross (1986) and is also described in Burmeister and Wall (1986). The economic interpretation of those factors is explored by Berry, Burmeister and McElroy (1988). Five different types of risk factors have been shown to have a significant influence on expected returns : (1) risk of changes in default premiums, (2) risk that the term structure of interest rates may change, (3) risk of unanticipated inflation or deflation, (4) risk that the long-run expected growth rate of profits for the economy will change, and (5) residual market risk, or any remaining risk needed to explain a market index such as the S&P 500. An interesting feature of their work was the inclusion of an additional implicit factor, interpreted as a residual market factor. This factor could be thought of as a proxy for otherwise omitted or incompletely specified factors. Burmeister and McElroy (1988) investigated the APT model in which there are both measured macroeconomic and unoberserved factors. They used both measured and unmeasured factors to estimate the linear factor model, the APT, and the CAPM. Using monthly stock returns and six factors, the January effect could not be rejected. The following are invariant with respect to the inclusion of January effects : the CAPM restrictions on the APT are rejected; the APT restrictions on the linear factor model are not rejected. The result is in contrast to those found by Gultekin and Gultekin (1987) and Cho and Taylor (1987).

3.10 Measured-Macroeconomic Factor Approach : Non-US Studies

Hamao (1989) presented an empirical investigation of the APT in the Japanese equity market using Japanese macroeconomic factors. The variables used were similar to those derived in Chen, Roll and Ross (1986) for the US market. Factors examined included industrial production, inflation, investor confidence, interest rate, foreign exchange, and oil prices. They found that changes in expected inflation, unanticipated changes in the risk premium and unanticipated changes in the slope of the term structure appear to have a significant effect on the Japanese stock market. Weaker evidence of the presence of a risk premium exists in changes in monthly production and changes in the terms of trade. The oil price changes and unanticipated changes in foreign exchange were not priced in the Japanese stock market. The result was surprising, given the importance of international trade in the Japanese economy. In addition, value and equally weighted market indices have neither statistically significant risk premia nor captured extra systematic risks missed by other macroeconomic variables.

Poon and Taylor (1991) reconsidered the results in Chen, Roll and Ross (1986) to see if they are applicable to UK stocks. They carried out a similar set of tests using UK data. Their results showed that variables similar to those of CRR do not affect share prices in the UK in the manner described in CRR. They concluded that it could be other macroeconomic factors at work, or the methodology in CRR is inadequate for detecting such pricing relationships.

3.11 Conclusion

Both the factor analyses approach and the measured-macroeconomic factor approach have their merits. The factor analysis approaches are implemented to conform to the factor structure underlying the APT. The factor analysis approach, suggested by Ross'(1976) where the APT has been used to extract the common factors in returns and then to test whether expected returns are explained by the cross-sections of the loadings of security returns on the factors (Roll and Ross (1980), Chen (1983)). Although we are not primarily concerned with the total number of factors, those factors that are not priced are just as important as those "priced" factors in an investment decision. Even if certain factors are unpriced, it is useful to know the asset loadings on that factor, despite the fact that they do not affect expected returns. For example, in an event study it would be useful to remove the common unpriced component as well as the common priced component of an asset's return to reduce the variation in the residual. By longing and shorting assets, one can form portfolios that have zero factor loadings or mutually uncorrelated. The factor analysis is useful in the exploratory stage as it is a technique that reduces the dimensions of the problem and allows one to focus on the extracted factors and match them against variables that economic theory suggests.

However, the factor analysis approach to tests of the APT leads to unresolvable squabbles about the number of common factors in returns and expected returns (Dhrymes, Friend and Gultekin (1984), Roll and Ross (1984), Dhrymes, Friend, Gultekin and Gultekin (1985b), Trzcinka (1986), Conway and Reinganum (1988)). The difficulty with the factor analysis approach is that the factors cannot be directly associated with macroeconomic variables and hence the factor sensitivities do not have economic interpretations.

While factor analysis may not by itself provide a completely satisfactory solution to the issue of how many factors there are in the stock-return generating process, it is helpful in testing APT against specified alternatives, as well as in linking identifiable economic variables to common stock return fluctuations. The development of the APT is quite separate from the factor analysis. Factor analysis is used only as a statistical tool to uncover the underlying factors in the economy by investigating how asset returns co-vary together. Factor analysis investigates covariance (communality). The goal of factor analysis is to reproduce the correlation matrix with a few orthogonal factors. In the context of APT, we are interested in the theoretical solution uncontaminated by unique and error variability, therefore factor analysis is the choice here. Factor analysis is used to estimate the number of factors and to provide the estimated factor loadings for the APT. Factor analysis can also be used to confirm that there is more than one common factor in returns and expected returns, which is useful.

The measured-macroeconomic factor approach is implemented without regard for the formal factor structure. Its attempt to relate assets expected returns to the covariances of assets' returns with other variables is more in the spirit of Merton's (1973) intertemporal CAPM than in the spirit of the APT. The primary advantages of using measured economic factors are: (1) the factors and their APT prices in principle can be given economic interpretations, while with a factor analysis approach it is unknown what factors are being priced; and (2) rather than using only asset prices to explain asset prices, measured macroeconomic factors introduce additional information, linking asset price behaviour to macroeconomic events.

The Chen, Roll and Ross approach (identifying economic factors that are correlated with returns and testing whether the factor loadings explain the cross-section of expected returns) is a productive way to use multifactor models to improve the understanding of asset-pricing.

However, the CRR approach has some drawbacks (discussed in detail in section 7.2). For example, no satisfactory theory would argue that the relation between financial markets and the macroeconomy is entirely in one direction. Although stock returns are usually considered as responding to external forces, they may also have a feedback on the other variables. In this thesis, the relationships between security returns and economic indicators are analyzed using the canonical correlation analysis. This is the first use of canonical correlation analysis to link the stock market and economic forces. In addition, based on the foundations of the APT and the characteristics of the factor scores from the factor analysis on security returns and economic indicators, the canonical correlation analysis is an appropriate technique to use to link economic forces and the stock market and making it a better alternative method than the CRR approach.

The cluster technique has an objective of separating a large number of variables into a group of subsets or clusters so that the variables within a cluster will be highly intercorrelated, and variables from different clusters, not so highly intercorrelated. King (1967) viewed the routine as a method of exploration properly falling under the heading of "data analysis" rather than "inference", the results of which would be subject to testing and confirmation via other techniques. The primary virtue of this method is its stepwise nature, leading to a simple and rapid computer program in which the steps can be broadly described as follows: (1) search the residual correlation matrix for the two variables with the highest positive correlation coefficient; (2) combine these variables to reduce the matrix by one; and (3) recompute the correlation matrix to include the correlation between the combined variable and the remaining variables. This process continues in an iterative fashion until the last meger is the trivial one in which all of the variables are clustered into one group.

The total variance would not affect expected returns if the APT is valid because its diversifiable component would be eliminated by portfolio formation and its non-diversifiable part would depend only upon the factor loadings and factor variances.

The inter-battery factor analysis is very similar to the canonical correlation analysis in that it estimates the factor loadings for two groups of securities by examining only the inter-group correlation matrix. If two groups had the same set of factors then it should be reflected in the inter-group correlation matrix. The inter-group correlation matrix reflects only those factors that are common to two groups and not those factors that are common for only one group. Thus, this method estimates the factor loadings by constraining the factors to be the same between two groups of securities.

Cross-validation is a general statistical method for checking that estimated models reflect stable features of the underlying process and do not overfit the data. Cross-validation estimates the models using one random sample of data and then uses a second random sample to validate the predictions from the estimated models. Cross-validation can be used to identify stable factor structures by fitting successive models with additional factors and noting when the prediction errors begin to stabilize or increase. The intuition is that when a model is overfit one tends to fit the noise component in a given sample. The noise is incorporated in the out-of-sample forecasts and tends to drive the predictions away from the stable structure. By checking the predictions from an estimated model with a new sample of data, models that are overfit tend to result in greater prediction errors.

CHAPTER 4

FACTOR ANALYSIS

4.1 Introduction

Factor analysis is a method of multivariate analysis that attempts to account for the correlation between a large set of variables in relationship to small number of underlying factors. It is an approach that is used to investigate the relationships between variables. In the factor analytic approach, a matrix of observations of correlated variables is examined to determine whether the data could be generated by a linear model involving a minimum number of unobservable variables (i.e. factors) that are fundamental to the data generating process. These factors and linear combinations of them are used to explain the observed data. In general, factor analysis provides great insight into the patterns of association underlying a set of multivariate data. Empirical estimates of the APT model can be obtained by using factor analysis. Factor analysis is used as a statistical tool to uncover underlying factors in the economy by investigating how asset returns co-vary together.

Section 4.2 contains the mathematical model for the factor structure of the factor analysis. The factor extraction techniques (e.g. maximum-likelihood factor analysis and principal factor analysis) are discussed in section 4.3. The critical aspects of factor analysis are mentioned in section 4.4. The canonical correlation analysis is discussed in section 4.5. In section 4.6, the comparison of factor analysis and principal components analysis is made.

4.2 The Mathematical Model for Factor Structure

A major assumption of factor analysis is that it is not possible to observe the underlying factors directly; the variables depend upon the factors but are also subject to random errors. Some of these factors are assumed to be common to two or more variables and some are assumed to be unique to each variable. The unique factors are then also assumed to be orthogonal to each other. Therefore, by definition the unique factors do not contribute to the correlation between variables. Only the common factors (which are assumed much smaller in number than the number of observed variables) contribute to the correlation among the observed variables.

Factor analysis supposes that the data comes from the well-defined model,

$$x = \beta f + u + \mu$$

where x(px1) is a random vector with mean μ and covariance matrix S, β (pxk) is a matrix of constants and f (kx1) and u(px1) are random vectors. The elements of f are called common factors and the elements of u specific or unique factors; where the underlying factors depend upon the following assumptions:

$$E(f) = 0, \ Var(f) = I, \ (I = \text{identity matrix}),$$

$$E(u) = 0, \ Cov(\mu_i, \mu_j) = 0, \ i \neq j$$

and $$Cov(f,u) = 0 \ ;$$

The covariance matrix of u is denoted by $V(u) = \psi = diag(\psi_{11}, \ldots, \psi_{pp})$. It is generally assumed that the factors are uncorrelated with mean 0 and variance 1, so the covariance matrix of f is the (kxk) identity matrix, I. It is also assumed that f and u and therefore x are normal multivariate distribution. The validity of the multivariate normality assumption provides the distribution required for the accuracy of the maximum-likelihood estimation of the parameters.

As

$$x_i = \sum_{j=1}^{k} b_{ij} f_j + u_i + \mu_i, \quad i = 1,\dots,p,$$

so that

$$\sigma_{ii} = \sum_{j=1}^{k} b_{ij}^2 + \psi_{ii}.$$

Thus, the variance of x is split into two parts. First,

$$\sum_{j=1}^{k} b_{ij}^2$$

is called the communality and represents the variance of x_i which is shared with the other variables via the common factors. In particular, $b_{ij}^2 = C(x_i, f_j)$ represents the extent to which x_i depends on the j^{th} common factor. On the other hand, ψ_{ii} is called the specific or unique variance and is due to the unique factor u_i; it explains the variability in x_i which is not shared with the other variables.

TABLE 4.1

SUMMARY OF THE EXPLORATORY FACTOR MODEL

Matrix	Dimension	Mean	Covariance	Dimension	Description
f	(kx1)	0	ϕ=E(ff')	(kxk)	common factors
x	(px1)	0	R=E(xx')	(pxp)	observed variables
β	(pxk)	-	-	-	loadings of x on f
u	(px1)	0	ψ=E(uu')	(pxp)	unique factors

There are three stages involved in obtaining solutions to factor analysis : (1) the preparation of an appropriate correlation matrix; (2) extraction of initial (orthogonal) factors;

and (3) rotation to a final solution.

4.2.1. Estimation of the factor loadings

When the factors are initially extracted, it is assumed for convenience that the common factors are uncorrelated with each other and have unit variance. It is within the context of these assumptions that common factors explain the correlations among the observed variables. The difference between the correlation ($\bar{R}$) predicted by the factor model and the actual correlation (R) is the residual correlation (i.e. $R_{res} = R - \bar{R}$). The residual correlation will highlight the adequacy of the fitted model. If a model is a good one, correlations in the residual matrix are small, indicating a close fit between observed and reproduced matrices.

The matrix of correlations between variables can often be diagonalized. It is then possible to use on them the matrix algebra of eigenvectors and eigenvalues with factor analysis as the result. When the matrix is diagonalized, it is transformed into a matrix with numbers in the positive diagonal and zeros everywhere else. In this application, the numbers in the positive diagonal of the diagonalized matrix represent variances from the correlation matrix that has been repackaged as follows:

$$\mathbf{L} = \mathbf{V'RV}$$

Diagonalization of R is accompanished by post- and pre-multiplying it by the matrix V and its transpose. The columns in V are called eigenvectors, and the values in the main diagonal of L are called eigenvalues. The first eigenvector corresponds to the first eigenvalue, and so forth. The factor with the largest eigenvalue has the most variance and so on, down to factors with small or negative eigenvalues that are usually omitted from solutions. As the goal of factor analysis is to summarize a pattern of correlations with as few factors as possible, and because each eigenvalue corresponds to a different potential factor, usually only factors with large eigenvalues are retained. These few factors duplicate the correlation matrix as faithfully as possible.

The matrix of eigenvectors pre-multiplied by its transpose produces the identity matrix ($V'V = I$) with ones in the positive diagonal and zeros elsewhere. Calculations for eigenvectors and eigenvalues are extremely laborious and are completed by the computer. The eigenvalues of the (pxp) matrix of correlations between variables (i.e. R) are solutions of the determinantal equation $|R - LI| = 0$. The determinant of a matrix is a mathematical property of a square matrix and as a means of determining the rank (or the number of independent dimensions) of an adjusted correlation matrix. The determinant of a matrix equals the product of its eigenvalues. The determinantal equation has p solutions and therefore R possesses p eigenvalues. Calculations for eigenvalues and eigenvectors require solving p equations in p unknowns.

Once the eigenvalues and eigenvectors are known, the correlation matrix can be considered a product of three matrices - the matrices of eigenvalues and corresponding eigenvectors.

After reorganization, the square root is taken of the matrix of eigenvalues.

$$
\begin{aligned}
R &= V\sqrt{L}\,\sqrt{L}V' \\
&= (V\sqrt{L})(\sqrt{L}V')
\end{aligned}
$$

The correlation matrix can be considered a product of two matrices, each a combination of eigenvectors and the square root of eigenvalues.

If $V\sqrt{L}$ is called β, and $\sqrt{L}V'$ is β', then

$$R = \beta\beta'$$

The (unrotated) factor loading matrix (i.e. the matrix of correlations between factors and variables) is then found by straightforward matrix multiplication as follows :

$$\beta = V\sqrt{L}$$

The validity of the k-factor model can be expressed in terms of a simple condition on R.

$$R = \beta\beta' + \psi.$$

where R is the correlation matrix of the observed variables and ψ is the correlation matrix of the unique factors (which is diagonal because the unique factors are uncorrelated). If R can be broken down into the form above then the k-factor model holds for x.

4.2.2 Factor rotation

The results of factor extraction, unaccompanied by rotation, are likely to be uninterpretable regardless of which extraction technique is used. The objective of rotation is to detect the meaning attached to the common factor axes so as to make them maximally interpretable. Repositioning the axes will change the coordinates of the variable points, (i.e. factors) but not the positions of the points with respect to each other. Thus, rotation makes the solution more interpretable without changing its underlying mathematical properties. Rotation is not and cannot be used to improve the quality of the mathematical fit between the observed and reproduced correlation matrices, because all orthogonally rotated solutions are mathematically equivalent to each another and to the solution before rotation. Since

$$R = \beta\beta'$$

if G is any orthogonal matrix,

$$\begin{aligned} R &= (\beta G)(\beta G)' \\ &= \beta^{*}\beta^{*\prime} \end{aligned}$$

where $\beta^{*} \equiv \beta G$. Thus, regardless of which factor loading estimate is used, it is always possible to rotate β by an orthogonal matrix to yield a new estimate, β^*, that will have the same associated R. Therefore, the estimate of β is not unique; βG is equivalent to β for any orthogonal transformation with GG' = I. Non-uniqueness of the factor loadings is a difficult problem for the testing of the APT which relies upon identifying a particular factor, solely on the basis of the magnitude of the variables' factor loadings, as the factors may be rotated without affecting the validity of the model. One is free to choose such a rotation to make the factors as intuitively meaningful as possible.

There are two principal methods of rotation : orthogonal and oblique, and the difference between them is important. In orthogonal rotation, the factors (i.e. axes) remain orthogonal to each other giving the advantage of ease of description and interpretation of results. If an orthogonal rotation does not produce an interpretable pattern of loadings it may be possible to do so by admitting non-orthogonal (oblique) transformations. An oblique rotation is more general than an orthogonal rotation in that it does not arbitrarily impose the restriction that factors be uncorrelated. However, the loss or orthogonality complicates the interpretation of the parameters and the factors. The oblique rotation has the conceptual advantages that there may be a case when the factors are correlated, and this could not be uncovered if the research is limited to orthogonal factors. Its advantage over orthogonal rotations is that, after making oblique rotations, if the resulting factors are orthogonal, one can be sure that the orthogonality is not an artifact of the method of rotation. It has been argued that employing orthogonal rotation may be preferred over oblique rotation, if for no other reason than that the former is much simpler to understand and interpret. In the present context, because the APT explicitly requires orthogonality of the factors, orthogonal rotation is the choice in this study.

Three orthogonal rotational techniques are used in this study : quartimax, varimax and equamax. Just as the extraction procedures have slightly different statistical goals, the rotational procedures maximize or minimize different statistics.

The goal of varimax is to maximize the variance of the squared loadings for each factor so that loadings which were high after extraction become higher after rotation and loadings that are low become lower. Interpreting a factor is easier because it is more obvious which variables correlate with it.

Quartimax does for variables what varimax does for factors. The objective of the quartimax rotation is to determine the orthgonal transformation which will carry the orginal factor matrix into the rotated factor matrix for which the variance of squared factor loadings for each variable is a maximum. The interpretation of a variable becomes simpler as fewer common factors are involved in it.

Equamax is a hybrid between varimax and quartimax that tries simultaneously to simplify the factors and the variables. Mulaik (1972) reports that equamax tends to behave erratically unless the researcher can specify the number of factors with confidence.

The adequacy of rotation is assessed in several ways. Perhaps the simplest way is to compare the pattern of correlations in the correlation matrix with the factors. If "simple structure" is present, several variables correlate highly with each factor and only one factor correlated highly with each variable. In other words, the columns of the factor loading matrix, which define factors, have several high and many low values while the rows of the factor loading matrix, which define variables vis-a-vis factors, have only one high value. Rows with more than one high correlation correspond to variables that are said to be complex because they reflect the influence of more than one factor.

Tabachnick and Fidell (1989) showed that just as the different methods of extraction tend to give similar results with a good data set, so also do the different methods of rotation tend to give similar results if the pattern of correlations in the data is fairly clear. In other words, a stable solution tends to appear regardless of the method of rotation used. After orthogonal rotation, the importance of the proportion of variance explained by a set of variables can be measured as the sum of squared loadings (SSL) for the factor divided by the number of variables. Although the total variance explained by the set of extracted factors is unaffected, the proportion of variance attributable to individual factors differs before and after rotation because rotation tends to redistribute variance among factors.

An estimate of the internal consistency of the solution, i.e., the certainty with which factor axes are fixed in the variable space, is given by the squared multiple correlations (SMC) of the predicted factor scores and the variables (Tabachnick and Fidell, 1989).

The SMC is also the lower bound for the communality and is an approximation to the communality (Harman, 1976). A high SMC (say, 0.70 or above) means that the factors account for substantial variance for the observed variables (i.e. the security returns). A low SMC means the factors are poorly defined by the observed variables.

After orthogonal rotation, the values in the loading matrix are now correlations between the variables and the rotated factors. The factor loading matrix is the matrix of regression-like weights which is used to estimate the unique contribution of each factor to the variance in a variable. The factor loading matrix is the matrix of regression-like weights which is used to estimate the unique contribution of each factor to the variance in a variable. The greater the correlation between a variable (i.e. returns of a company) and a factor, the more the variable is a pure measure of the factor. Thus, one has to decide which size of correlation (i.e. loading) is meaningful, collect together the variables with loadings in excess of the criterion, and search for a real variable which explains the returns of that group. As a rule of thumb, loadings in excess of 0.30 are eligible for interpretation (Tachbachnick and

Fidell, 1989), whereas lower ones are not, because a factor loading of 0.30 indicates at least a 9% shared variance betweeen the variable and the factor. Comrey (1973) suggests that loadings in excess of 0.71 (50% shared variance) are considered excellent (i.e. such loadings are almost certainly interpretable), 0.63 very good, 0.55 good, 0.45 fair, and 0.32 poor.

4.3 Factor Extraction Techniques

Two most common methods of factor analysis are used in this research and they are discussed here :

(i) the maximum-likelihood factor analysis (MLFA),

(ii) the principal factor analysis (PFA).

4.3.1 Maximum-Likelihood Factor Analysis

The overall objective of the maximum-likelihood factor analysis is to identify the population parameters that have the maximum-likelihood of generating the observed sample distribution. Maximum-likelihood extraction also provides the capability of estimating the number of factors. This is accomplished by specifying an arbitrary number of factors, say k, then solving for the maximum-likelihood conditional on a correlation matrix generated by exactly k factors. In an exploratory factor analysis, one would normally start with the hypotheses of k-common factors and proceed with (k-1) and (k+1) common factors respectively until the best number of parameters to be included in the model (based on the goodness-of-fit criteria) when maximum-likelihood estimation is used.

In maximum-likelihood solutions, unique variance is treated as a nuisance parameter. The general approach to nuisance parameters is to try and eliminate them from the likelihood, and to maximize a modified likelihood. Therefore, the method assigns greater weight to the variables with greater communality (or less unique variance), and this follows the general principle of efficient statistical estimation in which less stable estimates are given less weight[1]. The importance of each of the factors is assessed by the percent of variance it represents.

The maximum-likelihood method also permits an objective determination of the number of factors required to explain the data. Several criteria are available to test the goodness-of-fit for factor analysis. Lawley and Maxwell (1971) propose a likelihood ratio test for factor analysis. By assuming that the factors and errors have independent multivariate normal distributions, the likelihood function of the data from the estimated k-factor model is compared to the unrestricted likelihood. Maximum-likelihood factor analysis makes explicit use of the assumption that the sample is drawn from a multivariate normal distribution. Conditional on the multivariate normality assumption, the MLFA method provides hypothesis-testing opportunities (i.e. the MLFA method is associated with the explicit test for the significance of the assumed number of factors). In general, however, the consequences of violating the assumption of multivariate normality are not clearly understood.

For the factor analysis model, the likelihood ratio statistic is given by

$$\delta = -2 \log_e \lambda$$

$$= n\left[\log_e |\hat{\beta}\hat{\beta}' + \psi| + \mathrm{trace}\{R(\hat{\beta}\hat{\beta}' + \psi)^{-1}\} - \log_e |R| - p\right]$$

$$= n\{\log_e |\hat{\beta}\hat{\beta}' + \psi| - \log_e |R|\}.$$

The likelihood ratio, λ, depends only on sample observations, the sample correlation matrix and the estimate of the population correlation matrix under the hypothesis of k factors. When the hypothesis of k factors is true, the likelihood ratio statistic has an asymptotic chi-squared distribution with $[(p-k)^2-p-k]/2$ degrees of freedom. To improve the chi-squared approximation in moderate-size samples, Barlett (1950) suggested replacing n in the likelihood ratio statistic with the factor [n-(2p+4k+11)/6]. If the hypothesis of k factors is rejected, an alternative hypothesis of some large number of factors may be assumed to explain the observed correlations. However, Conway and Reinganum (1988) show that the conventional chi-squared test for the number of factors tends to fit too many factors.

The usual method is to start with a small value of k, and increase the number of common factors one by one until H_k is not rejected. However, this procedure is open to criticism as the critical values of the test criterion have not been adjusted to allow for the fact that a set of hypotheses is being tested in sequence (Mardia, Kent and Bibby, 1979).

The basic problem is that the more factors that are estimated, the better the fit and the greater the percent of variance in the data "explained" by the factor solution. However, the greater the number of factors included, the less parsimonious the solution. Therefore, one has to include enough factors for an adequate fit, but not so many that parsimony is lost, (analogy with R^2 in regression analysis, the adjusted R^2 can decrease when a new variable is added to the regression model, even though the R^2 will always increase to some extent when new variables are added).

Due to the difficulty with the chi-squared goodness-of-fit test, alternative measures of goodness-of-fit that include a penalty based on the number parameters fitted are used to assist in model selection. The change in the goodness-of-fit statistic must be large enough to justify the more complex model. In an attempt to allow for this effect, two adjusted likelihood ratio statistics (Akaike's information criterion and Schwarz's Bayesian criterion) providing penalties for fitting parameters in the model are used to evaluate the fit of factor analysis models. Cudeck and Browne (1983) suggest that using the two indices will provide information similar to the cross-validation procedure to assess the fit of a model (an obvious disadvantage of the cross-validation is that it reduces the sample size by half), but is computed from a single sample.

The two measures are given by :

Akaike's information criterion (AIC), AIC = L+2q, and

Schwarz's Bayesian criterion (SBC), SBC = L+q ln n^*,

where L is the value of the Bartlett's corrected form of the likelihood ratio statistic, $n^* = [n-(2p+4k+11)/6]$ and $q = [p(k+1)-k(k-1)/2]$ in the k factor model.

As more parameters are added to a model, the decrease in the likelihood ratio statistic must be large enough to warrant the increase in the number of fitted parameters. The two measures effect a trade-off between the bias introduced by fitting the wrong number of factors and the precision with which the parameters are estimated (as the number of factors is increased the bias decreases, but the error increases). The two measures require the number of factors chosen to make the likelihood ratio statistic a minimum.

Akaike's information criterion (Akaike, 1973,1974) as an alternative to the chi-squared goodness-of-fit test is a general criterion for estimating the best number of parameters to include in a model when maximum-likelihood estimation is used. The model based on the number of factors that yields the smallest value of AIC is considered best. The criterion effects a trade-off between the bias introduced by fitting the wrong number of factors and the precision with which the factors are estimated. AIC, like the chi-squared test, tends to include factors that are statistically significant, but inconsequential for practical purposes (Schwarz, 1978, Jobson, 1988).

Another criterion similar to AIC, for determining the best number of factors is Schwarz's Bayesian criterion. The model is based on the number of factors that yields the smallest value of SBC is considered best. SBC appears to be less inclined to include trivial factors than either AIC or the chi-squared test (Schwarz, 1978).

A reliability coefficient developed by Tucker and Lewis (1973) is also designed to meet the objective that the change in the goodness-of-fit statistic must be large enough to justify the more complex model. This reliability coefficient is a ratio of explained covariation to total variation which gives some perspective on the residual variation. The residual variation should be as small as possible without the factor model becoming too cumbersome. This reliability coefficient is based on the residual correlations in the matrix after the effects of final factors are taken out; it is therefore ultimately based on the fit between the observed correlations and correlations based on the factor solution. The reliability coefficient incorporates the adjustment that divides the overall discrepancy by the degrees-of-freedom, thereby adjusting for the potential differences between factor solutions. The coefficient ranges between 0 and 1, the former representing the poorest fit and the latter a complete fit.

4.3.2 Principal Factor Analysis

Usually before applying maximum-likelihood, principal factor analysis is used to get a rough idea of the number of factors. The principal factor method is probably the most widely used technique in factor analysis (Harman, 1976). Principal factor analysis is essentially equivalent to a principal components analysis performed on the reduced correlation matrix (i.e. replacing the observed diagonal elements of the observed correlations with estimated communalities). The principal factoring is the repeated form of principal component analysis. The principal factor method leans heavily on the close resemblance between factor analysis and principal components analysis. The first step is to estimate communalities (squared multiple correlations of each variable with all other variables) which are used to replace the ones in the positive diagonal of the observed correlation matrix producing a reduced correlation matrix (Harman, 1976). The squared multiple correlations are known to be less than (or at most equal to) the communalities (Harman, 1976). The squared multiple correlations are the maximum absolute correlation with any other variable and are used as initital communality estimates. A variable with a low squared multiple correlation (SMC) with all other variables is an outlier among the variables. This is the starting point for the iterative procedure.

Kaiser's measure of sampling adequacy (MSA) (Kaiser, 1970, 1974) provides another approximate idea of whether the data are adequate for factor analysis. Kaiser's measure of sampling adequacy is a summary of how small the partial correlations are in relation to the ordinary correlations. Kaiser's measure of sampling adequacy is a ratio of the sum of squared correlations to the sum of squared correlations plus the sum of squared partial correlations. The value approaches 1 if the partial correlations are small, values of 0.6 and above are required. As MSA approaches unity, the correlation matrix becomes more and more suitable for factor analysis, and Kaiser and Rice (1974) suggested that if MSA < 0.5 the correlation matrix is unacceptable for factor analytic purposes.

$$\mathrm{MSA} = \frac{\sum\sum_{j \neq k} r_{jk}^2}{\sum\sum_{j \neq k} r_{jk}^2 + \sum\sum_{j \neq k} q_{jk}^2}$$

where r_{ij} is an original correlation and q_{ij} is an element of the anti-image correlation matrix, which is given by $Q = SR^{-1}S$, where R is the correlation matrix and $S = (\mathrm{diag}[R^{-1}])^{1/2}$. The index ranges between 0 and 1. In fact, the index only becomes 1 if all the off-diagonal elements of the inverse of the correlation matrix are zero, which in turn implies that every variable can be predicted without error from other variables in the set. Kaiser (1970) claimed that the magnitude of MSA improves as (1) the number of variables increases, (2) the number of common factors decreases, (3) the number of cases increases, and (4) the average magnitude of correlations increases. The guide for interpreting the measure is as follows (Kaiser, 1974) :

in the .90's marvellous

in the .80's meritorious

in the .70's middling

in the .60's mediocre

in the .50's miserable

below .50 unacceptable.

Communality values are used instead of ones to remove the unique and error variance of each observed variable; only the variance a variable shares with the factors is used. The unique factor is an unobservable, hypothetical variable that contributes to the variance of only one of the observed variables. In common factor analysis, the unique factors play the role of residuals, and are defined to be uncorrelated both with each other and with the common factors. In the second stage, principal components analysis is applied to the reduced correlation matrix. The principal factor analysis is the application of principal component analysis to the reduced correlation matrix (i.e. with communalities in place of the ones in the principal diagonal) and the first k components used to provide estimates of the loadings in the k factor model. An initial quick estimate of the number of factors is obtained from the sizes of the eigenvalues reported. The multivariate procedures rely on eigenvalues and their corresponding eigenvectors because they consolidate the variance in a matrix (the eigenvalue) while providing the linear combination of variables (the eigenvector) to do it. The major work of factor analysis is the calculation of eigenvalues and eigenvectors. Once they are known, the (unrotated) factor loading matrix is found by straightforward matrix multiplication

$(V\sqrt{L})$ where the columns in V are eigenvectors; and the values in the main diagonal of L are eigenvalues. The first eigenvector corresponds to the first eigenvalue, and so forth. The variance is accounted for by the eigenvalue. One of the most popular criteria for estimating the number of factors is to retain factors with eigenvalues greater than unity.

As a second estimate of the number of factors, the scree test can also be performed on the graph of the eigenvalues. To perform the test, a graph in which all the potential factors, in descending order, are arranged along an abscissa, with percent of variance (i.e. the eigenvalues) as the ordinate. The test uses the graph of eigenvalues and chooses the number of factors corresponding to the point where the eigenvalues begin to level off, forming an almost horizontal straight line. The straight portion has been named the scree (Cattell, 1966).

The principal factor solution is based on the eigenvalues which serve as the criteria for determining the number of factors to extract, and the measure of variance accounted for. The contributions of the factors to the total variance of the variables decrease with each succeeding factor. Due to sampling variation and estimation effects, the reduced correlation matrix (estimates of communalities rather than unities are inserted in the main diagonal of a correlation matrix) need not be positive semi-definite, and some negative eigenvalues are expected. If a principal factor analysis fails to yield any negative eigenvalues, the previous communality estimates are probably too large. The contributions of the imaginary factors will be negative and will reduce the contributions of the real factors to the actual amount with which the analysis was started. Even to retain all the real eigenvalues would be an overestimation of the number of factors, because the source of the positive eigenvalues is greater than the original sum of communalities (the negative eigenvalues will reduce the sum to the starting value). Since the total communality for the variables is the trace of the reduced correlation matrix, the factorization process should be stopped when the sum of the eigenvalues is equal to the starting value. The cumulative proportion of variance explained by the retained factors should be approximately equal to 1.

The major point in favour of the principal factor analysis is that it does not require any distributional assumptions to be made about the data. An advantage of this fact is that the technique can be applied validly to fairly broad data types, but a restriction is that there is no hypothesis testing associated with it and hence it is predominantly a descriptive technique.

4.4 The Critical Aspects of Factor Analysis

There are a number of critical aspects in the application of factor analysis to a data set, e.g. design and interpretation difficulties, in addition to the methodological and statistical problems which are inherent in the factor analytic methods.

A frustrating problem when using factor analysis to test the APT is that the procedure cannot tell the researchers what the factors are. Hence, there is an interpretation problem of the common factors which determine the security returns. It is difficult to ascertain the nature of the underlying factors which influence the security returns. The factors cannot be directly associated with macroeconomic variables and hence the factor loadings do not have economic interpretations.

Another aspect of the non-uniqueness of the factor loadings is that they are also arbitrary with respect to sign. The sign itself has no intrinsic meaning, and in no way should it be used to assess the direction of the relationship between the variable and the factor. The sign of the loadings on any factor may be reversed without altering the adequacy of the factor solution. Therefore, this too will limit any economic interpretation of such factor loadings as measure of systematic factor risk. However, signs of variables for a given factor have a specific meaning relative to the signs of other variables; the different signs simply mean that the variables are related to that factor in opposite directions.

Another limitation concerning the fundamental data requirements is the effect of missing data on factor methods. Swain, Brynoza and Swain (1979) concluded that the correct parameters cannot be obtained by factor analysis based on correlation coefficients when data are missing.

An improvement in the variables to observations ratio can be obtained in the following methods:

(i) one can reduce the time interval for data collection; or

(ii) extend the time period over which the investigation occurs; or

(iii) limit the sample size; or

(iv) group into portfolios.

However, there are still a number of complications as a result of using the above methods. If method (i) is used by analyzing daily or weekly return data rather than monthly, Scholes and Williams (1977), Dimson (1979) and Roll (1981) showed that the correlation structure of the data is systematically biased due to measurement error associated with infrequent trading if the daily or weekly return data rather than monthly is utilised. If method (ii) is used, then the time series sample of security return data would be subject to shifts in variance (Sinclair,1982). Lastly, if the sample size is limited, then representativeness will become a problem and this is essential to the research design when not much is known about factor structure.

Another major difficulty in using factor analysis to test the validity of the APT is the problem of comparing factors that are estimated from separate factor analyses in different groups. For example, since large security sample sizes are common in finance, large sets of variables are required (in the case of the APT the number of securities is to be large enough to guarantee the application of the law of large numbers). However, the effect of indeterminacy means that factors may differ between studies and between different groups. It is extremely hard to compare the factors in one group of securities with the factors in another group as there is no satisfactory procedure of examining the factor congruency. In turn, it implies that rigorous comparison of the factors between subsamples and between studies is limited.

Brown and Weinstein (1983) made an attempt to compare factors that were obtained in two different groups. They divided a group of 60 securities into two subgroups of 30 securities each, and carried out factor analyses on each of these three groups by forcing the number of factors to be three. Two separate factor analyses on two subgroups of 30 securities did not constraint the factors to be the same, whereas a factor analysis on the group of 60 securities constrained the factors to be the same. They then compared the constrained residual sum of squares to the unconstrained residual sum of squares using F-statistics. The constrained residual sum of squares was obtained by analyzing the entire group of 60 securities, and the unconstrained residual sum of squares was obtained by combining the residual sum of squares from the two subgroups. They concluded that the three factors that best represent the observed variation in the data do not significantly differ across groups. However, Brown and Weinstein did not prohibit the factors from rotating freely by forcing only the number of factors to be the same between two subgroups of securities.

Cho (1984) tried to solve the factor comparability problem by employing inter-battery factor analysis rather than a traditional factor analysis. Inter-battery factor analysis was first introduced by Tucker (1958) and later improved by Browne (1979). He claimed that the advantage of using maximum-likelihood inter-battery factor analysis is that it can be used to estimate factor loadings by constraining the factors to be the same between two different groups. The marketwide factor loadings that are common across all groups could be estimated by extending the above methodology to more than two groups. In estimating inter-group common factor loadings, Tucker used the unweighted least squares methodology that can only be solved iteratively. Traditional factor analyses are based on iterative schemes, which do not guarantee the global optimum. On the other hand, this maximum-likelihood estimate in a closed form solution always yields the global optimum if the assumptions are satisfied. It turns out that maximum-likelihood inter-battery factor analysis is very similar to canonical correlation analysis in that estimates of inter-battery factor loadings may be computed by rescaling correlation coefficients between the original variables and the canonical variables obtained in the canonical correlation analysis. The difference is that inter-battery factor analysis attempts to explain the correlation coefficients among variables using a single set of unobservable factor variables, whereas the canonical correlation analysis attempts to explain the correlation coefficients among variables using two sets of observable linear combinations of variables, i.e. canonical variables. The method is very similar to the canonical correlation analysis in that it estimates the factor loadings for two groups of securities by examining only the inter-group correlation matrix. The inter-group correlation matrix should reflect only those factors that were common to two groups and not those factors that were common for only one group. It has been suggested in the financial economics literature that the residual portion of the correlation matrix is not a diagonal matrix and that the residual factors may represent industry factors. Thus, Cho claimed that such a method could be used to estimate the factor loadings by constraining the factors to be the same between two groups of securities. Furthermore, the author believed that unlike the previous studies which used the same number of factors for the entire sample, the sample could determine how many factors to use without altering the significance levels, hence, there would be no restriction on the number of factors in the cross-sectional analysis.

In conclusion, this section has attempted to highlight the statistical and methodological problems associated with the use of factor techniques as an exploratory research method on security returns. Of course, there is a need to make use of the available factor analytical tools, but to do so with caution in mind.

4.5 Canonical Correlation Analysis

Canonical correlation is a technique for analyzing the relationship between two sets of variables. Many of the problems associated with using canonical correlation are due to jargon. Firstly, there are sets of variables (i.e. the factor scores of security returns and the factor scores of the economic indicators), then there are canonical variates which are linear combinations of variables, one combination from one set (i.e. factor scores of the security returns) and a second combination from the other set (i.e. factor scores of the economic indicators). These two combinations form a pair of canonical variates. Each linear combination is chosen to maximize the correlation between the two canonical variates. The term "canonical correlation" refers to the relationship between a pair of canonical variates of the two sets of variables.

One can view canonical correlation analysis as an extension of multiple regression. In multiple regression analysis, the variables are partitioned into an X-set containing $q \geq 1$ explanatory variables and a Y-set containing $p = 1$ dependent variable. The regression solution involves finding the linear combination B_x' which is most highly correlated with Y.

In canonical correlation analysis, however, there are several variables on both sides (i.e. $p > 1$ and $q > 1$) and there will be several ways to recombine the variables on both sides to relate them to each other. Mathematically, canonical correlation coefficients B_x' and B_y' are obtained so as to maximize the correlation between $B_x'X$ and $B_y'Y$. Formally, canonical correlation analysis involves partitioning the two sets of variables (i.e. an X-set and a Y-set). There is no assumption of causal asymmetry in the mathematics of canonical correlation analysis; X and Y are treated symmetrically.

The object is then to find linear combinations :

$\rho \;=\; B_x'X$ and $\phi \;=\; B_y'Y$

such that ρ and ϕ (note not X and Y or B_x and B_y) have the largest possible correlation (i.e. which maximizes the linear relationship between ρ and ϕ). The correlation between ρ and ϕ is

$$r_{ci}(B_x,B_y) \;=\; \frac{B_x'R_{xy}B_y}{(B_x'R_{xx}B_xB_y'R_{yy}B_y)^{\frac{1}{2}}}$$

where $r_{ci}(B_x,B_y)$ is used to emphasize the fact that the correlation varies with different values of B_x and B_y. In general, $\rho_i = B_{xi}'X$ and $\phi_i = B_{yi}'Y$ are called the i^{th} canonical correlation variates; $r_{ci} = \lambda_i^{1/2}$ is called the i^{th} canonical correlation coefficient. The i^{th} canonical correlation variates for X are uncorrelated and are standardized to have variance 1; similarly for the i^{th} canonical correlation variates for Y.

The first step in a canonical analysis is generation of a canonical matrix, R. The canonical correlation matrix is a product of four correlation matrices. R_{xx} contains the correlations among the variables in the X set, R_{yy} the correlation among the Y set variables, and R_{xy} and R_{yx} the correlations of each of the variables in one set with each of the variables of the other set. By the symmetrical property of a correlation matrix, $R_{xy} = R_{yx}$.

$$R = R_{yy}^{-1}R_{yx}R_{xx}^{-1}R_{xy}$$

Conceptually, the canonical correlation matrix can be thought of as a product of regression coefficients for predicting X's from Y's ($R_{yy}^{-1}R_{yx}$) and regression coefficients for predicting Y's from X's ($R_{xx}^{-1}R_{xy}$).

As discussed in section 4.2.1, correlation matrix R can be diagonalized. Diagonalization of R is accomplished by post- and pre-multiplying it by the matrix V and its transpose (i.e. L = V'RV; where V and L are the eigenvectors and eigenvalues respectively). Canonical analysis proceeds by solving for the eigenvalues and eigenvectors of the canonical correlation matrix R. The eigenvector corresponding to each eigenvalue is transformed into the canonical coefficients which are used to combine the original variables into the canonical variate. Formally, each eigenvalue, λ_i, is equal to the squared canonical correlation, r_{ci}^2, for the i^{th} pair of canonical variates (i.e. ρ_i and ϕ_i) (Tabachnick and Fidell, 1989):

$$\lambda_i = r_{ci}^2$$

The canonical correlation is the square root of the eigenvalue. The canonical correlation, r_{ci}, is interpreted as an ordinary Pearson product-moment correlation coefficient between the pair of canonical variates. When r_{ci} is squared, it represents shared or overlapping variance between two variables, or, in this case, canonical variates. As $r_{ci}^2 = \lambda_i$, the eigenvalues themselves represent overlapping variance between pairs of canonical variates. There will be no more pairs than the number of variables in the smaller set.

Conventional statistical procedures (i.e. likelihood ratio statistic, F test) apply to significance tests for number of reliable canonical variate pairs. Significance tests are used to test whether one or a set of r_c's differs from zero. The number of statistically significant pairs of canonical variates (i.e. i pairs of canonical variates) is often larger than the number of interpretable pairs. Thorndike (1978) noted that analyses using very large samples and relatively few variables may result in small correlations that are statistically significant, but scientifically trivial. He also suggested that it would seem reasonable in most cases to reject as meaningless a relationship in which the squared canonical correlation is less than 0.10. Tabachnick and Fidell (1989) noted that as the canonical correlation values of 0.30 or lower represent, squared, less than a 10% overlap in variance, some researchers do not interpret pairs with a canoncal correlation lower than 0.30 even if significant. However, in some conditions (i.e. in economics) the result is reasonable when the squared canonical correlation is equal to 0.10.

For the application of the canonical correlation analysis in this study, the two sets of canonical correlation coefficients (analogous to regression coefficients) which are required for each canonical correlation, combine, respectively, the factor scores of the security returns and those of the economic indicators.

4.5.1 The Canonical Model

$$B_y = (R_{yy}^{-\frac{1}{2}})'\hat{B}$$

From the equation above, the canonical coefficients for the factor scores of the economic indicators are a product of (the transpose of the inverse square root of[2]) the matrix of correlations between the factor scores of the economic indicators and the matrix of eigenvectors, B, for the factor scores of the economic indicators. If a matrix has been multiplied by itself, there is a parallel in matrix algebra to squaring and taking the square root of a scalar. Once the canonical coefficients for the factor scores of the economic indicators are computed, coefficients for the factor scores of the security returns can be found using the following equation:

$$B_x = LR_{xx}^{-1}R_{xy}B_y .$$

Coefficients for the factor scores of the security returns are a product of four matrices: L, a diagonal matrix of reciprocals of eigenvalues; R_{xx}^{-1}, the inverse of the correlation matrix between the factor scores of the security returns; R_{xy}, the matrix of correlations between the factor scores of the economic indicators and those of the security returns; and B_y, the coefficients for the factor scores of the security returns.

Interpretation of significant pairs of canonical variates is based on the matrices of correlations between the variables and the canonical coefficients, called loading matrices, A_x and A_y. Correlations between variables and canonical variates are found by multiplying the matrix of correlations between variables (R) and the matrix of canonical coefficients (B).

$$A_x = R_{xx}B_x \text{ and } A_y = R_{yy}B_y.$$

4.5.2 Interpretation

A canonical variate is interpreted by considering the pattern of variables highly correlated (loaded) with it.

With respect to separate regression analysis, Kuylen and Verhallen (1981) noted that separate multiple regression analyses of each set of variables would neglect the interrelations of the sets.

Tatsuoka (1973, p.273) notes,

> "The often-heard argument, "I'm more interested in seeing how each variable, in its own right, affects the outcome" overlooks the fact that any variable taken in isolation may affect the criterion differently from the way it will act in the company of other variables. It also overlooks the fact that multivariate analysis - precisely by considering all the variables simultaneously - can throw light on how each one contributes to the relation".

With respect to factor analysis, there are similarities between factor analysis and canonical correlation analysis[3]. Both are variable reduction schemes that use uncorrelated linear combinations. Factor analysis considers interrelationships within a set of variables, the focus of canonical correlation is on the relationship between two groups of variables. The canonical correlation analysis estimates the factor loadings for two groups of securities by examining only the inter-group correlation matrix. The first few pairs of linear combinations of variables (the canonical variates) generally account for most of the between-association. Canonical correlation is viewed as an external factor analysis, in contrast with the internal factor analysis of a single set of variables. Wimmer (1977) noted that independent factor analyses are satisfactory if one wants factors chosen independently of each other. It is not a reliable procedure if one wants to explain as much as possible of one set of variables from the other set.

McLaughlin and Otto (1981) noted that in general, it can be said that canonical correlation requires the same set of assumptions as employed in the more commonly utilized general linear model techniques, such as multiple correlation, regression, and factor analysis, but also shares the robustness of these techniques with regard to violations of those assumptions. Therefore, although there is no requirement that the variables be normally distributed when canonical correlation is used descriptively, inference regarding the number of significant canonical variate pairs does require the assumption of multivariate normality.

Linearity is related to canonical correlation analysis in at least two ways. The first is that the analysis is performed on correlation of variance-covariance matrices that are sensitive to linear, but not higher order relationships. If the relationship between two variables is curvilinear, it is not "captured" by these statistics and the canonical result misses the nonlinear part of the relationship unless the variables are transformed. The second is that canonical correlation maximizes the linear relationship between a variate from one set of variables and a variate from the other set. If the relationship between variates is not linear, canonical correlation analysis misses it.

Similar to multiple regression analysis, canonical correlation analysis is best when relationships among pairs of variables are homoscedastic, that is, when the variance of one variable is the same at all levels of the other variable (Tabachnick and Fidell, 1989).

4.6 Factor Analysis and Principal Components Analysis

Principal components analysis, like factor analysis, is an attempt to explain a set of data in a smaller number of dimensions than one starts with, but the procedures used in the two methods to achieve the goal are essentially quite different. The goal of PCA is to extract maximum variance from a data set with a few orthogonal components. The goal of FA is to reproduce the correlation matrix with a few orthogonal factors. Factor analysis, unlike principal components analysis, begins with a hypothesis about the correlational structure of the variables. The hypothesis is that a set of k factors exists and these are adequate to account for the interrelationships of the variables. Principal components analysis, on the other hand, is a method of orthogonal transformation of any set of variables into a set of new variables which are uncorrelated with each other. Since principal component analysis is merely a transformation of the data, no assumptions are made about the form of the correlation matrix derived from the sample data.

One of the most important decisions when testing the APT is the choice between principal components analysis and factor analysis. Factor analysis is a model-based technique which has as a primary aim the explanation of the associations among the variables, by contrast, principal component analysis aims to explain the variances and has no underlying model as a basis. Mathematically, the difference involves the contents of the positive diagonal in the correlation matrix (the diagonal that contains the correlation between a variable and itself). In either PCA or FA, the variance that is analyzed is the sum of the values in the positive diagonal. In PCA ones are in the diagonal and there is the same amount of variance to be analyzed as there are observed variables; each variable contributes a unit of variance by contributing a 1 to the positive diagonal of the correlation matrix. All the variance is distributed to components, including error and unique variance for each observed variable. Therefore if all components are retained, PCA duplicates exactly the observed correlation matrix and the standard scores of the observed variables.

In FA, only the variance that each observed variable shares with other observed variables is available for analysis. Exclusion of error and unique variance from FA is based on the belief that such variance only confuses the picture of underlying processes. Shared variance is estimated by communalities, values between 0 and 1 that are inserted in the positive diagonal of the correlation matrix. Maximum-likelihood extraction manipulates off-diagonal elements rather than values in the diagonal. The solution in FA concentrates on variables with high communality values. The sum of the communalities (sum of the SSLs) is the variance that is distributed among factors and is less than the total variance in the set of observed variables. As a result of unique and error variances being omitted, a linear combination of factors approximates, but does not duplicate, the observed correlation matrix and scores on observed variables.

In the context of APT, we are interested in the theoretical solution uncontaminated by unique and error variability, therefore FA is the choice for this study.

1. The usual method for computing variance accounted for by a factor, is to take the sum of squares of the corresponding column of the factor pattern (loading), yielding an unweighted result. If the square of each loading is multiplied by the weight of the variable before the sum is taken, the result is the weighted variance explained, which is equal to the corresponding eigenvalue. Sum of squares are equivalent to eigenvalues in the unrotated solution and this value divided by the number of variables gives the proportion of variance explained by that factor.

2. If one has a matrix, M, had M been multiplied by itself, $MM = R_{yy}$, then $R_{yy}^{1/2} = M$. That is, there is a parallel in matrix algebra to squaring and taking the square root of a scalar, but it is a complicated business because of the complexity of matrix multiplication. If, however, one has a matrix R_{yy} from which a square root is desired (as in canonical correlation), one searches for a matrix, M, which, when multiplied by itself, produces R_{yy}.

3. Cooley and Lohnes (1971) noted that "the factor model selects linear functions of tests that have maximum variances, subject to the restriction of orthogonality. The canonical model selects linear functions that have maximum covariances between domains, subject to restrictions or orthogonality".
Tatsuoka (1971) claimed that the technique may therefore be loosely characterized as a set of variables that are most highly related (linearly) to the components of the other set of variables.

CHAPTER 5

STOCK MARKET FACTORS AND APT: THE UK EVIDENCE

5.1 Introduction

Although the Arbitrage Pricing Theory (APT) of Ross (1976) has been intensively investigated in the United States, there are relatively few empirical investigations into the application of APT to the pricing of UK stocks.

This chapter contains the results of a "traditional" test of the APT. The first objective of the chapter is to estimate the number of factors which determine UK stock returns and the correlations between stock returns and factors. A maximum-likelihood factor analysis of the empirical variance-correlation matrix of returns is used to provide the estimated factor loadings for the APT. The size of these loadings reflects the relationship between each stock's return and the factors. The second objective is to use the individual security factor loadings (the APT analogues of multiple betas) to explain the cross-sectional variation of individual expected returns. The use of a standard methodology in this chapter not only captures behaviour of the UK stock market, but also provides results which can be compared with those obtained for the US stock market. After this, the object of chapters 6 and 7 is to make a partial identification of the factors by comparing this collection of factor scores with those of the real economy.

This chapter differs from other UK studies (e.g. Diacogiannis (1986), and Abeysekera and Mahajan (1987)) in that a longer time period was used (i.e. 1965-1988); both the maximum-likelihood factor analytic method and principal factor analysis was used to give a rough idea of the number of factors before proceeding to the maximum-likelihood factor analysis. Then, the factor scores for the securities are correlated with those of the economic variables through canonical correlation analysis. This is the main theme of the thesis.

Section 5.2 contains the background of this chapter. The data description of the UK security returns is discussed in section 5.3. The method used in the study is considered in section 5.4. The results of the principal factor analysis and the maximum-likelihood factor analysis are discussed in sections 5.5 and 5.6 respectively. In section 5.7, the individual-security factor loading estimates are used to explain the cross-sectional variation of individual estimated expected returns. In section 5.8, the results are discussed and the problem of non-stationarity is considered in section 5.9. The last section is the summary of the findings.

5.2 Background

Most empirical studies attempting to test the APT have used US stock price data, (as discussed in chapter 3). There are very few empirical studies related to the APT which have used UK stock price data.

Diacogiannis (1986) utilized time series data from the London Stock Exchange and has concentrated upon the empirical verification of the assumption that there exists a security return generating model which remains the same across different security groups and across various time periods. The results indicated that the number of factors increases as the group size increases as suggested by Dhrymes, Friend and Gultekin (1984). The number of factors also changes across various time periods for the same group of securities and for different security groups.

Abeysekera and Mahajan (1987) empirically tested two hypotheses to evaluate the validity and applicability of the APT to the UK stock market using monthly individual price data. Their study empirically evaluated the validity of the APT by testing the following two hypotheses. The first hypothesis is that the intercept term (λ_0) of the pricing relation represents the risk free rate R_f. The second hypothesis is that the APT implies that if k factors are responsible for driving the individual asset returns through time, then there should be a risk premium attached to each of these factors. The monthly returns on random selections of securities listed continuously in the London Stock Exchange from January 1971 to December 1982 were computed and then seven portfolios were formed, each consisting of 40 randomly selected securities. Each portfolio was then subjected to eight maximum-likelihood factor analyses, prespecifying between one and eight factors, to determine the factor loadings. Their results supported the first hypothesis that the risk free rates are equal to the corresponding estimated intercept terms of the models tested. The results also showed that the intercept term was significantly different from zero. However, the results for the second hypotheses that if k factors are responsible for driving the individual asset returns through time, then there should be a risk premium attached to each of these factors, showed by utilizing two different procedures (i.e., the χ^2 test and the t-test) that the risk premia are not significantly different from zero. Their latter finding does not support the APT and is in conflict with the results of Roll and Ross (1980) and others that are based on US stock market data.

Interestingly, determining the number of factors underlying security returns and testing the validity and applicability of the APT to the UK stock market have shown to be an elusive and difficult endeavor. The estimated number of factors varies widely across different studies. More importantly, my study attempts to investigate the applicability of the APT and to interpret the factors by relating them to other aspects of the UK economy.

5.3 Data Description

The data source is the London Share Price Database of the London Business School (monthly-returns file) which contains share returns after adjustment for all capital changes and dividends. The sample period is January 1965-December 1988 inclusive, giving a maximum of

288 monthly security returns. The choice of this period is based on the availability of the data on the economic variables which are used in the analysis contained in chapter 6 below. Some of these macroeconomic series are not available before January 1965, and the period investigated for the security returns should correspond to that of the economic variables. One month is the shortest interval over which data is available from the London Share Price Database.

The use of factor analysis imposes two requirements on the sample selection process. Firstly, in factor analysis, observations with missing values for any variable in the analysis should be omitted from the computations, because calculation of correlations requires simultaneous observations. Therefore, only securities with no missing observations between January 1965 and December 1988 are included. Altogether, 234 securities have continuous data and were frequently traded for the entire sample period.

This selection criterion may introduce a bias in favour of survival in that only firms which were in existence for the entire sample period are included. This survival bias will exclude failed firms, takeover and merger victims, and newly listed companies, therefore those risk factors peculiar to an individual or all of these types of firm will not be represented in the sample. Furthermore, over time, a company can change its basic character through acquisitions and purposeful strategic choices as well as by changes in the markets in which it operates. These changes will result in changes in its exposure to the underlying economic factors. In order to maintain sufficient degrees of freedom, the number of companies cannot exceed the time series dimension. This survival bias will increase with the length of the sample period and will, therefore, be common to all tests which require long data series. The sample in similarity to other studies is therefore biased towards long-lasting firms.

Secondly, the returns of the securities are required by factor analysis to have a multivariate normal distribution. If factor analysis is used descriptively as a convenient way to summarize the relationships in a large set of observed variables, assumptions regarding the distributions of variables are not required; although, if variables are normally distributed, the accuracy of the factor analytic solution will increase (Tabachnick and Fidell, 1989). The maximum-likelihood factor analysis is applicable when the data is assumed to be normally distributed, and enables significance tests to be made about the validity of the k-factor model. In general, the consequences of violating the multivariate normal distribution assumption are not clearly understood (Kim and Mueller, 1978).

The assumption of multivariate normality is not readily tested because it is impractical to test an infinite number of linear combinations of variables for normality. However, univariate normality is necessary for multivariate normality. Although normality of all linear combinations of variables is not testable, the Kolmogorov-Smirnov test[1] can be used to test whether a set of observations are from a completely specified continuous distribution (i.e. univariate normality). 173 securities have been excluded by the normality criterion. Sixty-one securities fulfill the requirements that the returns do not have any missing observations and are normally distributed. The normality requirement potentially causes bias as it results in the inclusion of only "well-behaved" firms (i.e. it excludes those firms that have extreme rates of returns in one part of the sample period). However, if the model does not apply to this group of "well-behaved" firms then it is unlikely to apply to the full market.

The sixty-one sample securities were classified and grouped by the classification used by the Stock Exchange and the Institute of Actuaries. Table 5.1 suggests that the distribution of the securities in the sample appears to be an accurate representation of the distribution of the total number of securities listed in the Stock Exchange Year Book in each industry group, although the sample appears to contain a relatively higher proportion of securities in the non-durable consumer goods group and a relatively lower proportion of securities in the financial group than

TABLE 5.1

DISTRIBUTION OF SAMPLE SECURITIES IN EACH INDUSTRY GROUP

Industry Classification	Number of securities sample	Percentage of securities sample	Number of securities in the Stock Exchange Year Book	Percentage of securities the Stock Exchange Year Book
Capital Goods	14	22.95	508	19.29
Commodity Goods	3	4.92	185	7.03
Consumer Goods (Durable)	4	6.56	243	9.23
Consumer Goods (Non Durable)	20	32.79	503	19.10
Financial	10	16.39	695	26.40
Other	10	16.39	499	18.95
	61	100.00	2633	100.00

that in the stock market in general. This might be explained by the relatively large number of long lived "traditional" manufacturing companies in the UK and the relatively recent growth of the financial sector.

5.4 Method

One of the difficulties of empirically studying the APT is that it does not offer any theoretical or empirical grounds for identifying the economic nature of the factors. The APT gives little guidance on the identity of the factors beyond the restriction that they should obey the pervasiveness condition (i.e., that a set of economic factors systematically influences the returns on all stocks).

In estimating the number of factors which affect UK security returns, two factor extraction techniques were used:

(i) Principal factor analysis (PFA) to get an approximation of the number of factors before proceeding to a maximum-likelihood factor analysis.

(ii) Maximum-likelihood factor analysis (MLFA) is used to identify more precisely the number of factors, their factor loadings and factor scores.

The maximum-likelihood method not only provides a firm theoretical basis for the estimation process, but is also one of the few methods that permits a statistical test of the number of factors (i.e. likelihood ratio test of goodness-of-fit) required to explain the data (Krzanowski, 1990).

5.5 Principal Factor Analysis

Before turning to maximum-likelihood, the monthly returns of the sixty-one securities were subjected to principal factor analysis to determine the number of factors which account for a meaningful percentage of common variance. The communalities (squared multiple correlations of each variable with all other variables) are shown in Table 5.2 and reveal that, using the squared multiple correlations for the communality estimates, the average communality value is 0.64. The communalities are used to replace the ones in the positive diagonal of the observed correlation matrix to produce the reduced correlation matrix. This mean communality is acceptable and indicates that the variables are correlated with each other, as it is expected since the security returns should somehow correlate with each other, therefore the data are acceptable

TABLE 5.2

PRIOR COMMUNALITY ESTIMATES: Squared Multiple Correlations

CO#1	CO#2	CO#3	CO#4	CO#5	CO#6
0.633613	0.574026	0.638279	0.460627	0.614310	0.741980
CO#7	CO#8	CO#9	CO#10	CO#11	CO#12
0.389467	0.621736	0.693815	0.649808	0.669774	0.721235
CO#13	CO#14	CO#15	CO#16	CO#17	CO#18
0.606242	0.653111	0.551533	0.513748	0.754200	0.667721
CO#19	CO#20	CO#21	CO#22	CO#23	CO#24
0.620214	0.634056	0.664507	0.454803	0.574956	0.454224
CO#25	CO#26	CO#27	CO#28	CO#29	CO#30
0.839364	0.698260	0.519409	0.652742	0.610817	0.769178
CO#31	CO#32	CO#33	CO#34	CO#35	CO#36
0.723729	0.719536	0.481835	0.710168	0.448908	0.673490
CO#37	CO#38	CO#39	CO#40	CO#41	CO#42
0.805464	0.722946	0.443500	0.624366	0.684715	0.606789
CO#43	CO#44	CO#45	CO#46	CO#47	CO#48
0.652620	0.836951	0.650607	0.582854	0.504005	0.514791
CO#49	CO#50	CO#51	CO#52	CO#53	CO#54
0.846473	0.658413	0.738677	0.671517	0.622981	0.827370
CO#55	CO#56	CO#57	CO#58	CO#59	CO#60
0.606443	0.657651	0.627094	0.565196	0.666904	0.644793
CO#61	Mean SMC 0.64		Min SMC 0.39		
0.592423	Max SMC 0.85				

CO# denotes company number.

for factor analysis. Table 5.3 shows that the mean of Kaiser's measure of sampling adequacy is 0.97, which implies that the data are well suited for factor analysis. The ones in the positive diagonal of the correlation matrix are replaced by the communality estimates (i.e. squared multiple correlations) in preparation for factor extraction.

Moving on to the factor extraction stage, an initial estimate of the number of factors is obtained from the sizes of the eigenvalues (refer to section 4.3.2). One of the most popular criteria (refer to section 4.3.2) for estimating the number of factors is to retain factors with eigenvalues greater than 1. The results in Table 5.4 indicate that three factors have eigenvalues greater than 1, and these three factors account for 81.63% of total explained variance. The first factor explains nearly 74% of the total variation in stock market returns, the second factor explains only 4.1% of the total variance, and the third factor 3.9%. The second and third factors are unusually low when compared with the first factor and this implies that these factors are much less important than the first factor. Thirty-seven of the eigenvalues are positive while twenty-four are negative; which is to be expected when estimates of communalities, rather than unities are inserted in the main diagonal of a correlation matrix[2].

As a second estimate of the number of factors, the scree test was also performed on the graph of the eigenvalues. If the eigenvalues are plotted against their rank order they will lie on a descending curve. One then looks for an "elbow" in the curve, as this would indicate the point at which the further addition of factors shows diminishing returns in terms of variation explained. The rule is to examine the curve and to stop factoring at the point where the eigenvalues begin to level off forming a straight line with an almost horizontal slope. The straight portion has been named the scree (Cattell, 1966). Applying the scree test, it would appear that at least four factors should be extracted[3].

TABLE 5.3

KAISER'S MEASURE OF SAMPLING ADEQUACY

CO#1	CO#2	CO#3	CO#4	CO#5	CO#6
0.972051	0.971718	0.971311	0.979782	0.957968	0.968370
CO#7	CO#8	CO#9	CO#10	CO#11	CO#12
0.968078	0.977127	0.973955	0.938735	0.926115	0.974470
CO#13	CO#14	CO#15	CO#16	CO#17	CO#18
0.858487	0.970306	0.978038	0.969363	0.969567	0.974165
CO#19	CO#20	CO#21	CO#22	CO#23	CO#24
0.975387	0.970692	0.983639	0.948038	0.973910	0.973019
CO#25	CO#26	CO#27	CO#28	CO#29	CO#30
0.952505	0.972093	0.946081	0.966290	0.974358	0.966452
CO#31	CO#32	CO#33	CO#34	CO#35	CO#36
0.963759	0.975353	0.961857	0.969187	0.963058	0.964536
CO#37	CO#38	CO#39	CO#40	CO#41	CO#42
0.968568	0.965965	0.940974	0.971211	0.974864	0.972296
CO#43	CO#44	CO#45	CO#46	CO#47	CO#48
0.965597	0.969634	0.973667	0.979430	0.970052	0.970177
CO#49	CO#50	CO#51	CO#52	CO#53	CO#54
0.943259	0.978067	0.951564	0.978687	0.978008	0.968255
CO#55	CO#56	CO#57	CO#58	CO#59	CO#60
0.973157	0.971101	0.978962	0.953860	0.984149	0.983953
CO#61	Mean MSA 0.97	Min MSA 0.93			
0.977040	Max MSA 0.98				

CO# denotes company number.

TABLE 5.4

EIGENVALUES OF THE REDUCED CORRELATION MATRIX

	Eigenvalue	Difference	Proportion	Cumulative
1	28.5451	-	0.7364	0.7364
2	1.5942	26.9508	0.0411	0.7776
3	1.5025	0.0917	0.0388	0.8163
4	0.9249	0.5775	0.0239	0.8617
5	0.8343	0.0906	0.0215	0.8617
6	0.6676	0.1667	0.0172	0.8790
7	0.6602	0.0074	0.0170	0.8960
8	0.5851	0.0750	0.0151	0.9111
9	0.5550	0.0300	0.0143	0.9254
10	0.5062	0.0488	0.0131	0.9385
11	0.4576	0.0485	0.0118	0.9503
12	0.4084	0.0491	0.0105	0.9603
13	0.3793	0.0291	0.0098	0.9706
14	0.3706	0.0086	0.0096	0.9802
15	0.3646	0.0060	0.0094	0.9896
16	0.3388	0.0257	0.0087	0.9983
17	0.3135	0.0252	0.0081	1.0064
18	0.2873	0.0262	0.0074	1.0138
19	0.2724	0.0148	0.0070	1.0208
20	0.2598	0.0125	0.0067	1.0275
21	0.2366	0.0232	0.0061	1.0336
22	0.2187	0.0178	0.0056	1.0393
23	0.1907	0.0279	0.0049	1.0442
24	0.1695	0.0212	0.0044	1.0486
25	0.1489	0.0206	0.0038	1.0524
26	0.1354	0.0134	0.0035	1.0559
27	0.1249	0.0104	0.0032	1.0591
28	0.1091	0.0158	0.0028	1.0620
29	0.0892	0.0198	0.0023	1.0643
30	0.0710	0.0181	0.0018	1.0661
31	0.0621	0.0089	0.0016	1.0677
32	0.0515	0.0106	0.0013	1.0690
33	0.0472	0.0042	0.0012	1.0702
34	0.0239	0.0222	0.0006	1.0709
35	0.0223	0.0025	0.0006	1.0715
36	0.0098	0.0124	0.0003	1.0717
37	0.0025	0.0073	0.0001	1.0718

TABLE 5.4 (continued)

	Eigenvalue	Difference	Proportion	Cumulative
38	-0.0073	0.0098	-0.0002	1.0716
39	-0.0105	0.0031	-0.0003	1.0713
40	-0.0297	0.0192	-0.0008	1.0706
41	-0.0360	0.0063	-0.0009	1.0696
42	-0.0473	0.0113	-0.0012	1.0684
43	-0.0617	0.0143	-0.0016	1.0668
44	-0.0682	0.0064	-0.0018	1.0651
45	-0.0718	0.0036	-0.0019	1.0632
46	-0.0808	0.0089	-0.0021	1.0611
47	-0.0817	0.0009	-0.0021	1.0590
48	-0.1010	0.0193	-0.0026	1.0564
49	-0.1135	0.0124	-0.0029	1.0535
50	-0.1242	0.0107	-0.0032	1.0503
51	-0.1281	0.0038	-0.0033	1.0470
52	-0.1387	0.0105	-0.0036	1.0434
53	-0.1466	0.0078	-0.0038	1.0396
54	-0.1688	0.0221	-0.0044	1.0352
55	-0.1708	0.0020	-0.0044	1.0308
56	-0.1733	0.0024	-0.0045	1.0264
57	-0.1843	0.0110	-0.0048	1.0216
58	-0.1899	0.0056	-0.0049	1.0167
59	-0.1981	0.0082	-0.0051	1.0116
60	-0.2187	0.0205	-0.0056	1.0059
61	-0.2301	0.0114	-0.0059	1.0000

This section used principal factor analysis to reveal the probable number and size of the UK stock market factors. The results suggest that at most four factors should be extracted from security returns. The next stage of the analysis is to use a more powerful technique (maximum-likelihood factor analysis) to extract the factors and their factor loadings. The estimated factor loadings are then used to explain the cross-sectional variation of individual estimated expected returns, and to measure the size and statistical significance of the estimated risk premium associated with each factor.

5.6 Maximum-Likelihood Factor Analysis

Maximum-likelihood factor analysis not only provides a firm theoretical basis for the factor estimation process, but also, unlike principal factor analysis, permits a statistical test of the number of factors required to explain the data.

Based on the analysis of the previous section (in which at most, four factors were found) the monthly returns of the sixty-one securities were subjected to maximum-likelihood factor analysis to determine the number of, and factor loadings of the common factors; the results are summarized below:

TABLE 5.5

DIFFERENT CRITERIA FOR ESTIMATING THE BEST NUMBER OF PARAMETERS TO INCLUDE IN A MODEL

Number of factors	Schwarz's Bayesian criterion (SBC)	Akaike's information criterion (AIC)	Tucker and Lewis's reliability coefficient (T&L)
1	2,094.32	3,742.35	0.86
2	2,081.18	3,495.70	0.89
3	2,086.00	3,289.23	0.91

When the number of factors is equal to 4, some of the communality estimates are greater than 1. Since communalities are squared correlations, they must lie between 0 and 1. If the communality is equal to unity, the situation is referred to as a Heywood case, and if a communality exceeds unity, it is an ultra-Heywood case. An ultra-Heywood case implies that a factor has negative variance, a clear indication that something is wrong. The possible cause of the anomaly is the extraction of too many factors which renders a factor solution invalid. With fewer than four factors all the communality estimates are less than 1, therefore, the Table 5.5 shows only the results with fewer than four factors. The results show that the SBC measure is at a minimum if returns are modelled as being explained by two factors[4]. We therefore regard the two factor models as dominant and analyze the results for this case. Although the value of the AIC measure for three factors is at a minimum, the choice should be based on the SBC measure as it appears to be less inclined to include trivial factors than the AIC measure (Schwarz, 1978). The AIC measure tends to include factors that are statistically significant, but inconsequential for practical purposes (Schwarz, 1978, Jobson, 1988). The Tucker & Lewis (1973) reliability coefficient is a ratio of explained covariation to total variation, which gives some perspective on the residual variation. The residual variation should be as small as possible without the factor model becoming too cumbersome. The Tucker & Lewis reliability coefficients for the two factor model and three factor model are 0.89 and 0.91 respectively. They both indicate that there is a good fit between observed and reproduced matrices.

5.6.1 Factor Patterns

The values of the factor patterns reflect the extent of relationship between each stock and each factor. A factor is interpreted from the stocks that have high loadings on it. Stocks for which factor loadings are large are thus more closely linked with the factor than those for which it is not. Usually, a factor is most interpretable when a few stocks are highly correlated with it and the rest are not.

Table 5.6 contains the factor pattern for the two significant factors and shows that the highest factor loading is 0.8399 and the lowest factor loading is 0.3575 for the first factor. All the factor loadings have the same sign (i.e. unipolar factor) and are statistically significant (i.e. same factor accounts for all stocks), such a factor is known as a general factor. The interpretation is that there is a market factor operating which has a major effect on all stocks. For the second factor, 23% of the stocks have negative loadings, while 77% have positive loadings. As a rule of thumb, only variables with loadings of 0.30 (in absolute terms) and above are interpreted (Tabachnick and Fidell, 1989). Only four stocks have loadings in excess of 0.30 (in absolute terms). It is interesting to note that those four stocks belong to the financial industry. This implies that the second factor is either trivial or nontrivial, but not general (i.e. important only for specific stocks or specific time periods). The second factor is common only to a group of stocks (i.e. it is a statistically insignificant factor for most of the stocks).

5.6.2 Rotation of Factors

The next step in factor analysis involves finding simpler and more easily interpretable factors through rotation, while keeping the number of factors and communalities of each factor fixed.

As the APT explicitly assumes that the factors are uncorrelated, orthogonal rotation is used here. Recall that three orthogonal rotational techniques are used: quartimax, varimax, equamax (see section 4.2.2). The variances explained by factor 1 and factor 2 with and without weights are shown in Table 5.7. The aim of rotation is to lead to a simpler and more easily interpretable factor, the achievement of simple structure would mean essentially that the observed variables fall into mutually exclusive groups whose loadings are high on single factors, perhaps moderate to low on a few factors and of negligible size on the remaining factors. The quartimax

TABLE 5.6

UNROTATED FACTOR PATTERN

	FACTOR 1	FACTOR 2		FACTOR 1	FACTOR 2
CO#44	0.83990	-0.18343	CO#54	0.83456	-0.27921
CO#37	0.81721	-0.15725	CO#49	0.80821	-0.32368
CO#25	0.80076	-0.40515	CO#17	0.78506	0.10360
CO#30	0.77445	-0.33315	CO#6	0.77355	-0.02686
CO#59	0.76499	0.02136	CO#21	0.76281	0.05510
CO#60	0.76002	0.05442	CO#32	0.75902	0.18023
CO#9	0.75404	0.15041	CO#38	0.74949	-0.13367
CO#12	0.74483	-0.32379	CO#26	0.74139	-0.01848
CO#34	0.74119	0.09756	CO#41	0.73767	-0.11541
CO#52	0.73227	0.07838	CO#31	0.72688	0.17908
CO#57	0.72496	0.05571	CO#18	0.72359	0.20442
CO#45	0.72054	0.17181	CO#50	0.71761	0.06976
CO#56	0.71375	0.15403	CO#28	0.70885	0.03483
CO#20	0.70057	0.10723	CO#29	0.69402	-0.01601
CO#53	0.69330	0.18787	CO#1	0.69239	-0.05121
CO#55	0.69046	0.06572	CO#8	0.68942	0.00242
CO#43	0.68880	0.10363	CO#19	0.68591	0.14437
CO#46	0.68326	0.01558	CO#36	0.68291	0.26427
CO#14	0.68260	0.21910	CO#3	0.67612	-0.04433
CO#40	0.67567	0.13642	CO#61	0.67096	0.00398
CO#42	0.66628	0.22198	CO#5	0.65756	0.21870
CO#23	0.65690	0.05529	CO#51	0.65132	0.14634
CO#15	0.63195	0.19626	CO#2	0.63142	0.14628
CO#16	0.59880	0.04154	CO#47	0.58549	0.16796
CO#4	0.57926	0.07770	CO#48	0.57658	0.21062
CO#58	0.55367	0.03278	CO#27	0.55309	0.15810
CO#10	0.54628	0.14519	CO#24	0.54368	0.10873
CO#11	0.53288	0.05774	CO#33	0.52937	0.21630
CO#35	0.51143	0.13907	CO#22	0.50200	0.05781
CO#7	0.46234	0.20173	CO#39	0.42930	0.05875
CO#13	0.35754	0.08860			

CO# denotes company number.

TABLE 5.7

VARIANCE EXPLAINED BY FACTORS ON DIFFERENT ROTATIONAL TECHNIQUES

Rotational technique		Variance explained by each factor	
		Factor 1	Factor 2
Unrotated	(weighted)	65.52	4.31
	(unweighted)	28.29	1.55
Quartimax	(weighted)	64.65	5.17
	(unweighted)	28.32	1.52
Varimax	(weighted)	35.30	34.52
	(unweighted)	16.81	13.03
Equamax	(weighted)	35.30	34.52
	(unweighted)	16.81	13.03

method maximizes the variance of the factor contributions (i.e., the squared factor loadings). Since the total variance must remain constant, a consequence of the quartimax method would be to increase the number of zero or near-zero loadings as well as the size of the larger loadings. The quartimax rotation is the rotation of choice here, because it aims to make the variables as simple as possible by maximizing the variance of the loadings on each factor in order to achieve the simple structure. The variances are explained by factor 1 (64.65 with weights, 28.32 without weights) and factor 2 (5.14 with weights, 1.52 without weights). The results shows that the first factor is still the dominant factor through the quartimax rotation. The squared multiple correlations (SMCs) are the estimates of communality between variables and the factors. The

SMCs represent the proportion of variance in variables that are predictable from the factors underlying them. The squared multiple correlations of the variables with factor 1, and factor 2 are 0.98 and 0.81 respectively which implies that the two factors are internally consistent and well defined by the stocks.

The results in Table 5.8 show that the highest factor loading is 0.8121 and the lowest factor loading is 0.3655 for the first factor. Although the loadings of the first factor appear to be statistically significant, there are differences of up to 55% in the magnitude of the highest and the lowest factor loadings. However, the coefficients of the first factor are all large and positive, indicating an important general factor among the stocks. A general factor has impacts on all security returns. The second factor has loadings of opposite signs, that is 49% of the stocks have positive loadings on factor 2 while 51% of the stocks have negative loadings. The second factor retains the mixture of signs in the loadings of the stocks, indicating that the stocks have different reactions to the second factor. However, as only five stocks (all belong to the financial industry) have loadings in excess of 0.30 (in absolute terms), the second factor is minor (i.e. important only for those five stocks).

A summary of the results appears in Table 5.9. All of the companies in the financial group, and 60% of the companies in the consumer goods (non-durable) group are positively related to the second factor; while 70% of the companies belonging to other groups, 67% of the companies of commodity groups, 75% of the companies of consumer goods (durables), and 79% of the companies of capital goods are negatively related to the second factor. Given King's (1966) finding that secondary factors which can be interpreted as industry related, accounted for an average of about 10% of the variance in stock price changes, it is interesting to see how

TABLE 5.8

ROTATED FACTOR PATTERN (QUARTIMAX)

	FACTOR 1	FACTOR 2		FACTOR 1	FACTOR 2
CO#44	0.81211	0.28205	CO#55	0.69624	0.01689
CO#54	0.79542	0.37651	CO#42	0.68796	-0.14114
CO#37	0.79270	0.25335	CO#29	0.68718	0.09846
CO#17	0.79181	-0.00947	CO#40	0.68710	-0.05507
CO#32	0.77508	-0.08866	CO#8	0.68481	0.07961
CO#9	0.76658	-0.05964	CO#1	0.68138	0.13322
CO#6	0.76486	0.11869	CO#46	0.68026	0.06582
CO#49	0.76396	0.41753	CO#5	0.67891	-0.13892
CO#21	0.76395	0.03604	CO#61	0.66667	0.07587
CO#59	0.76210	0.06980	CO#3	0.66604	0.12445
CO#60	0.76109	0.03638	CO#51	0.66410	-0.06782
CO#34	0.74753	-0.00869	CO#23	0.65881	0.02325
CO#34	0.74688	0.49753	CO#15	0.65081	-0.11968
CO#31	0.74303	-0.09134	CO#2	0.64434	-0.07012
CO#18	0.74277	-0.11689	CO#47	0.60132	-0.09712
CO#52	0.73639	0.00929	CO#16	0.59949	0.02999
CO#45	0.73586	-0.08487	CO#48	0.59754	-0.14053
CO#26	0.73392	0.10655	CO#4	0.58439	-0.00824
CO#30	0.72932	0.42292	CO#27	0.56797	-0.09118
CO#38	0.72827	0.22188	CO#10	0.55968	-0.07917
CO#56	0.72700	-0.06802	CO#58	0.55364	0.03332
CO#57	0.72644	0.03094	CO#24	0.55276	-0.04328
CO#50	0.72082	0.01611	CO#33	0.55134	-0.15179
CO#41	0.71870	0.20235	CO#11	0.53596	0.00606
CO#53	0.71073	-0.10405	CO#35	0.52435	-0.07724
CO#36	0.70950	-0.18116	CO#22	0.50532	0.00233
CO#20	0.70835	-0.02312	CO#7	0.48305	-0.14529
CO#28	0.70796	0.04975	CO#39	0.43324	-0.00726
CO#14	0.70382	-0.13634	CO#13	0.36554	-0.04543
CO#12	0.70102	0.41010			
CO#19	0.69822	-0.06175			
CO#43	0.69624	-0.02095			

CO# denotes company number.

different industrial groupings correspond to the second factor. Keep in mind that while a company may be squarely situated in a particular industry grouping, companies in the same

TABLE 5.9

DISTRIBUTION OF LOADINGS ON FACTOR 2

Industry Classification	Positive Number	Positive Percentage	Negative Number	Negative Percentage
Capital Goods	3	21	11	79
Commodity Goods	1	33	2	67
Consumer Goods (Durable)	1	25	3	75
Consumer Goods (Non Durable)	12	60	8	40
Financial	10	100	0	0
Others	3	30	7	70
	30	49%	31	51%

grouping can behave quite differently. Even within the same industry two companies can have quite different patterns of sensitivities. Individual companies vary widely in their sensitivities to the economic factors.

5.7 Risk Measures and Average Returns

The validity and applicability of the APT to the UK stock market is empirically evaluated. The APT implies that there is a linear relationship between the risk measures embodied in the factor loadings and the expected returns. An important implication of the APT is that the intercept term (λ_0) should be significantly different from zero. The APT further implies that if k factors are responsible for driving the individual asset returns through time, then there should be a risk premium attached to each of these factors (i.e. they must have non-zero prices).

In this section, the individual-security factor loading estimates are used to explain the cross-sectional variation of individual estimated expected returns. The APT will be supported if the actual returns depend on estimated factor loadings (i.e., factor beta coefficients of the security returns generating model)[5].

The general approach developed for pricing tests is straightforward (e.g. Roll and Ross (1980)). The factor loadings (beta coefficients) are used as independent variables to explain the cross-sectional variation in the mean returns of all the securities which comprise the sample. The mean returns are used as the proxy for the expected returns.

$$\bar{R}_i = \hat{\lambda}_0 + \hat{\lambda}_1 \hat{b}_{i1} + \hat{\lambda}_2 \hat{b}_{i2}$$

where λ_i is the risk premium on factor i,

b_{ik} is the factor loading of security i on the k^{th} factor,

$\bar{R}_i$ is the expected returns on the i^{th} security.

The regression results are shown in Tables 5.10 and 5.11. The regression results show that the APT explains 11% (in terms of adjusted R^2) of the variation in mean returns of the sample. This suggests that the explanatory power of the model is quite modest. The F value is used to test the null hypothesis that parameters λ_1 and λ_2 are simultaneously zero. The calculated F statistic is greater than the theoretical F value at the five per cent level, indicating that the null hypothesis can be rejected. The explanatory power of the model will be the same whether the rotated or unrotated factor patterns are used as independent variables in the regression analysis. Rotation cannot be used to improve the fit between the observed and

TABLE 5.10

REGRESSION RESULTS USING UNROTATED FACTOR PATTERNS AS INDEPENDENT VARIABLES

Variable	Parameter Estimate	Standard Error	T for H_0: parameter=0	Prob>|T|
λ_0	0.02133	0.00280	7.607	0.0001
λ_1	-0.01215	0.00403	-3.018	0.0038
λ_2	-0.00498	0.00271	-1.840	0.0708
R^2	0.1387	F-value	4.669	
Adj R^2	0.1090	Prob>F	0.0132	

TABLE 5.11

REGRESSION RESULTS USING ROTATED FACTOR PATTERNS AS INDEPENDENT VARIABLES

Variable	Parameter Estimate	Standard Error	T for H_0: parameter=0	Prob > |T|
λ_0	0.02133	0.00280	7.607	0.0001
λ_1	-0.01266	0.00416	-3.044	0.0035
λ_2	0.00350	0.00250	1.401	0.1667
R^2	0.1387	F-value	4.669	
Adj R^2	0.1090	Prob>F	0.0132	

reproduced correlation matrices because all orthogonally rotated solutions are mathematically equivalent to each another and to the solution before rotation.

During the sample period, January 1965 to December 1988, the risk-free coefficient, λ_0, was equivalent to 28.82% annually, or 2.13% monthly, as shown in Table 5.10. The intercept term is always the same for unrotated or rotated factor patterns. The intercept term is significantly greater than zero at the 5% level of significance. The positive intercept term is consistent with the APT model, as one testable implication of the APT is that the intercept term should be positive. Although it is often argued that the intercept term should equal the risk-free rate, the APT does not, in fact, require that the zero-beta rate equal the observed return on 30-day Treasury bill rates. Ingersoll (1984) argues that the intercept in the APT could be a zero beta asset even though a risk-free asset exists[6].

Measurement error biases the ordinary least-squares estimate of parameters. Pindyck and Rubinfeld (1981) noted that the ordinary least-squares estimates of the regression parameters will be biased due to the measurement error. Recently, Shukla and Trzcinka (1990) commented that the intercept of the model (as well as the other parameters) will be affected by measurement error, so that deviation of the intercept from the zero-beta rate may be interpreted as pricing and measurement errors[7]. The resultant large empirical pricing errors will result in large intercepts, and large estimates of idiosyncratic risk and parameters λ_i. A better measure of systematic risk should result in a better fit of the pricing equation. The better measure should have lower pricing errors (APT assumes that the pricing errors are negligible). This can be tested by examining the magnitudes of intercepts across the models. The magnitude of deviation of the intercept from the zero-beta rate will be higher for a poor measure of systematic risk. A small deviation of the intercept from the zero-beta rate implying lower pricing error plus measurement error.

The risk premium of the first rotated factor, λ_1, is -14.17% annually, or -1.27% monthly, during January 1965 to December 1988 as shown in Table 5.11. The price associated with an APT factor can be negative if investors want, perhaps for hedging purposes, to hold stocks whose returns increase when there is an unanticipated negative realization of that factor (and whose returns decrease when there is an unanticipated positive realization). This negative price reflects an attribute that investors find desirable.

The results of this replication of the standard testing approach show that there are two factors in the UK stock market, but that only one factor and the risk-free coefficient (λ_0) are important for pricing.

5.8 Discussion

The above sections estimate the number of the UK stock market factors using principal factor and maximum-likelihood methods of factor analysis. The results show that there are two stock market factors in the UK. It has been shown by principal factor analysis that the first factor accounts for nearly 74% of the proportion of the total variation in stock market returns, the second factor explains only 4.1%. By using maximum-likelihood factor analysis, the results confirmed the earlier findings by principal factor analysis that the first factor is an important general factor among the stocks. The coefficients of the first factor are all positive and statistically significant. The relatively small size of the second factor is unusually low (i.e. only four stocks have loadings in excess of 0.30 (in absolute terms)), and it implies that the second factor is a minor one and is much less important than the first factor.

The validity and applicability of the APT to the UK stock market are empirically evaluated. The APT implies that there is a linear relationship between the risk measures embodied in the factor loadings and the expected returns. An important implication of the APT is that the intercept term (λ_0) should be significantly different from zero. The APT further implies that if k factors are responsible for driving the individual asset returns through time, then there should be a risk premium attached to each of these factors(i.e. they must have non-zero prices). Most researchers are not primarily concerned with the total number of factors; instead, they are interested in the number of 'priced' factors, which are those with non-zero means. Chen (1983), however, claimed that those factors that are not priced are just as important as those that are priced in an individual's investment decision. They are irrelevant only in predicting expected returns.

The individual-security factor loading estimates were then used as independent variables to explain the cross-sectional variation in the mean returns of the securities that comprise the sample. The mean returns (as the proxy for the expected returns) for securities were regressed against the factor loadings. Only the first factor and the risk-free coefficients are priced. It is clear from the cross-sectional regression results that the APT has some empirical power (in terms of adjusted R^2). The APT explains 11% of the variation in the twenty-four years average returns. The validity of the APT in pricing UK stocks is supported in that the intercept term is significantly different from zero and the risk premium of the first factor is also significantly different from zero. Note that the findings are consistent with the CAPM. The CAPM is seen to be a special case of the APT. Although statistically significant, the explanatory power of the APT in pricing UK stocks is not high. For example, Roll (1988) showed that regressions of individual monthly stock returns on the multiple factors produced explanatory power, as measured by the average adjusted R^2, of 24.4% for 2030 US companies (monthly data for five years). Shukla and Trzcinka (1990) showed that the APT explains as much as 40% of the variation in mean returns of 865 US companies (weekly data for twenty years). This difference in results is not due to the statistical approach used. The US results in chapter 8 show that the APT explains up to 30% of the variation in the twenty-four years average returns which is comparable to the R^2 of Shukla and Trzcinka.

The findings of this chapter are in conflict with the results of Abeysekera and Mahajan (1987) with regard to the UK stock market. Their results showed that the risk premia are not statistically different from zero and their finding did not support the APT (see chapter 3). The results from this chapter are also quite different to the findings of the studies that were based on the US stock market. The possible explanations of conflicting evidence regarding the number of factors could be due to different time period and different groupings of stocks. The fact that a different number of factors is significant in a different time span is not surprising. The relevant factors (i.e. those that had a significant impact on the factor structure of security returns) may not be the same from stock grouping to stock grouping for the same time period, and from time period to time period for the same grouping of stocks.

For comparison, several studies which evaluated the validity and applicability of the CAPM to the UK stock market are reported here. Theobald (1980) analyzed the beta factors of the market model. He used the ordinary least squares technique and showed that the market model explains 19.9% of the sample of 201 UK companies for his whole sample period (1963 to 1972) and explains 20.2% and 21.3% for the two equal, non-overlapping sub-periods (i.e. 1963 to 1967 and 1968 to 1972) respectively.

Corhay, Hawawin and Michel (1988) also tested the validity of the CAPM on the London Stock Exchange using a methodology similar to that of Fama and MacBeth (1973). They reported evidence of seasonality in the estimated coefficients of the relationship between average returns and risk. The relationship between average monthly returns and systematic risk over the entire twenty-seven years period from January 1957 to December 1983 is not statistically significant.

Overall, the results obtained in this chapter show that the APT pricing relationship is supported by the testing methodology. Although statistically significant, the modest explanatory power of the model may be due to the following reasons, namely, the non-stationarity of the risk and expected returns during the twenty-four years period; the APT pricing relationship holds only in some months of the year[8]; the possibility of non-linear pricing relationships; and the market environment associated with the London Stock Exchange[9]. However, the results exclusively use criteria that measure how closely the factor analysis model fits and predicts observed correlations of returns. If instead the objective is to predict mean returns, higher-order factor models would provide more accurate predictions of mean returns. Minor factors relatively unimportant in explaining return covariances, may turn out to be important in predicting mean returns. If the estimated factor loadings on the higher-order factors will contain more noise than information as the standard errors are likely to be as large as the coefficients, this would lead to unstable predictions of mean returns and the price of risk. The results of this chapter suggest that for the purpose of predicting observed correlations, a simpler, more parsimonious model is preferable.

5.9 Non Stationarity

In testing the APT, the return distribution is assumed to be stationary over time so that measures of systematic risk can be estimated from a correlation matrix based on, in this case, twenty-four years of data. There are a number of studies which demonstrate that security returns are not stationary[10].

Non-stationarity is a major econometric difficulty of every asset pricing study. Empirical studies that involve long time-series must assume that the underlying economic parameters being estimated remain constant over the period examined. In the real world it is possible that some of the factors found to affect the security returns in one period are unimportant in the following period. In such a case, the number of factors determining security returns changes over time. Over time, a company may change its basic character through acquisitions and purposeful strategic choices as well as by changes in the markets in which it operates. These changes will result in changes in its exposure to the underlying economic factors. If different return generating models were found across various time periods for different security groups, it can be accepted that there is a violation of the assumption about the uniqueness of the security returns generating model across various time periods for the same group of securities. In addition, the instability of the number of factors through time also shows the violation of the major assumption required to transform the model into a testable relationship.

In the present chapter, it has been simply assumed that the non-stationarity problem does not exist. Thus, risk and expected returns were assumed not to have changed during the twenty-four years period. By taking no measures to mitigate any problems arising from this, these tests are biased toward finding that the risk measures are not significant. However, Shukla and Trzcinka (1990, 1991) commented that the factor analysis measures are better able to handle the problem of non-stationarity than principal components analysis. If idiosyncratic risks of securities vary, factor analysis is less constrained than the components analysis, because factor analysis estimates idiosyncratic risks simultaneously with factor loadings, while idiosyncratic risks are ignored when components are estimated. In practice, this means that any non-stationarity and measurement errors will affect the estimation of components more than the factors because factor loadings are always estimated with more degrees of freedom than components.

5.10 Summary

This chapter estimates the number of UK stock market factors using principal factor and maximum-likelihood methods of factor analysis. The results show that there are two stock market factors in the UK. It has been shown that when the intercept is estimated, only the first factor and the intercept emerged as significant for pricing. Hence, it appears that there is only one "priced" factor in the UK stock market. The "priced" factor is the dominant factor as it accounts for nearly 74% of the proportion of the total explained variation in stock market returns. The first factor is an important general factor.

The validity of the APT in pricing UK stocks is supported by the fact that the intercept term is significantly different from zero and the risk premium of the first factor is also significantly different from zero. It is clear from the cross-sectional regression results that the APT has some empirical power (in terms of adjusted R^2). The APT explains 11% of the variation in the twenty-four years average returns, this result is quite encouraging. In this study, it has been simply assumed that the non-stationarity problem does not exist. Thus, risk and expected returns were assumed not to have changed during the twenty-four years period. By taking no measures to mitigate any problems arising from this, these tests are biased toward finding that risk measures are not significant.

Having examined the APT as a statistical construct, the next step is to interpret the factors and relate them to other aspects of the economy.

1. The Kolmogorov-Smirnov test is used as a test statistic for goodness of fit. The test assumed that the null hypothesis was simple, that is, the null hypothesis completely specified the distribution of the population (i.e. that the input data values are a random sample from a normal distribution). When the data are tested against a normal distribution with mean and variance equal to the sample mean and variance, the usual Kolmogorov statistic is computed. Lilliefors (1967) reports that the standard tables used for the Kolmogorov-Smirnov test are valid when testing whether a set of observations are from a completing specified continuous distribution. If one or more parameters must be estimated from the sample then the tables are no longer valid. A table is given for use with the Kolmogorov-Smirnov statistic for testing whether a set of observations is from a normal population when the mean and variance are not specified, but must be estimated from the sample. The table is obtained from a Monte Carlo calculation.

Level of Significance for D (0.01)

Sample size (N)	Kolmogorov-Smirnov test	Monte Carlo calculation
Over 35	$\frac{1.63}{\sqrt{N}}$	$\frac{1.031}{\sqrt{N}}$

2. Since multiplication by the square root of the eigenvalue is involved in getting the factor weights, eleven of the principal factors are imaginary. For practical interpretation this means that the number of relevant factors necessary to describe the total communality (as estimated) certainly must be less than or equal to thirty-seven by using the number of positive eigenvalues as the criterion for choice of the number of factors to include in the model. As a result of sampling variation and estimation effects, the reduced correlation matrix need not be positive semi-definite, and some negative eigenvalues are expected. If the analysis is made in terms of all thirty-seven real factors, the communality resulting from this solution will exceed the starting communality. This follows from the mathematical property that the contributions of the twenty-four imaginary factors will be negative and will reduce the contributions of the thirty-seven real factors to the actual amount with which the analysis was started except for round-off errors. The first three eigenvalues account for most of the total starting communality, and, therefore, only these factors have any practical significance.

3. Unfortunately, the scree test is not exact; it involves judgement of where the discontinuity in eigenvalues occurs. As Gorsuch (1983) reports, results of the scree test are more obvious and reliable when sample size is large, communality values are high, and each factor has several variables with high loadings. Under less than optimal conditions, the scree test is still usually accurate to within one or two factors.

4. As discussed in section 4.3.1, the model based on the number of factors that yields the smallest value of SBC is considered best.

5. Roll and Ross (1980) and others, evaluated the hypothesis implied by the APT that there is a linear relationship between the risk measure embodied in the factor loadings and the expected returns for US security pricing using the individual t-statistics, which were computed by utilizing the results of regression analysis similar to that used in section 5.7. Such a procedure, however, has two major drawbacks. First, the t-statistics generated are not independent across factors. This dependence arises as a result of augmenting the loadings matrix to estimate λ_0 where the augmented matrix is not diagonal. Derivation of the APT assumes the loadings matrix to be diagonal (Roll and Ross, 1980; Abeysekera and Mahajan, 1987). Second, the factors from different regressions may have different interpretations. This is due to the non-uniqueness of the factor scores, i.e., the first factor from one group may or may not correspond to the first factor obtained from another group. One way out of this dilemma, as suggested by Dhrymes, Friend and Gultekin (1984), is to conduct a joint test of the complete vector of risk premia, rather than the significance test of the individual risk premia. In my study here, the t-statistics and F-test are both used in the regression analysis.

6. Ingersoll commented that as with the CAPM there are two versions of APT. One corresponds in form to the Black or zero-beta version of the asset model; the other is in form similar to the Sharpe-Lintner version of the model. While the forms are similar, the interpretations are not identical. The zero-beta CAPM arises when there is no riskless asset. The "zero-beta" APT arises even though a separate riskless asset is available in the economy. If it is possible to construct a well-diversified portfolio that is also free of factor risk, the Sharpe-Lintner version is appropriate. If this construction is not possible, then the zero-beta version is correct. APT is not based on the equilibrium between the risky and riskless assets as in the CAPM. It is based on the absence of arbitrage opportunities. If there is no way to create a portfolio of risky assets which is free of risk, no arbitrage comparisons can be made between the risky and riskless assets.

7. Ross (1976) assumes that the APT equation follows a strict factor structure, i.e. the diagonal covariance matrix of the unique factors. It is based on the intuition that the firm specific return represents diversifiable risk which should have a zero price in an economy with no arbitrage opportunities. The firm specific risks will be diversified out of large portfolios. Ross shows that the sum of squared approximation errors (the pricing errors) is finite as the number of securities in the economy approaches infinity. The maximum-likelihood estimation has asymptotic properties (i.e. statistical approximations which are valid as the number of cross-section observations grows large). To test the APT, the return distribution is assumed to be stationary over time. If the stationarity assumptions are violated, the estimates of the systematic risk will be subject to measurement error problems. A better measure of systematic risk should result in a better fit of the pricing equation. The regression coefficients will likely reflect measurement error. The measurement error biases the ordinary least squares estimate of the intercept. In practice, the pricing errors and the measurement errors will be inseparable. The pricing errors plus the measurement errors are interpreted as the empirical pricing error.

8. Tinic and West (1984) and Gultekin and Gultekin (1987) reported that the APT explains the return-risk relationship only in January.

9. Theobald and Price (1984) found evidence indicating inefficiency in the UK equity market.

10. The mean returns may not be a good proxy for the expected returns. Ferson, Kandel and Stambaugh (1987) showed that expected risk premia and asset betas vary over time. Fama and French (1989) also showed that expected returns on common stocks and long-term bonds contain a term or maturity premium that has a clear business-cycle pattern (low near peaks, high near troughs). Expected returns also contain a risk premium that is related to longer-term aspects of business conditions. The variation through time in this premium is stronger for low-grade bonds than for high-grade bonds and stronger for stocks than for bonds. The general message is that expected returns are lower where economic conditions are strong and higher when conditions are weak. Harvey (1991) showed that the variation in expected returns is common across international markets.

CHAPTER 6

THE FACTOR STRUCTURE OF THE UK ECONOMY

6.1 Introduction

The APT has one major shortcoming in that the factors determining asset returns are not associated with specific economic variables by the model. The APT offers no theoretical or empirical grounds for identifying the economic nature of the factors. This chapter uses a new approach to the identification of the sets of economic variables associated with security returns. This approach is based on an explicit recognition of the complex multicollinearity which exists between economic variables. The procedure involves two stages: first, the use of factor analysis on a range of economic variables, to extract the independent factors; and second, to use canonical correlation analysis to compare these economic factors with those already extracted from the sample of UK stock returns.

The objective of this chapter is to examine a set of UK economic variables in order to estimate the number and loadings of the factors that represent the UK economy. The sizes of the factor loadings reflect the extent of the relationship between each economic variable and each factor. The comparison of stock market and economic factors is contained in chapter 7.

Section 6.2 contains the background of this chapter. The data description of the economic variables is discussed in section 6.3. The method used in the study is mentioned in section 6.4. In sections 6.5 and 6.6, the results of the principal factor analysis and the maximum-likelihood factor analysis are discussed respectively. In section 6.7, the results are discussed and the last section presents the conclusions drawn from these results.

6.2 Background

Although most studies of the APT take the factor analysis approach, the most influential tests of the multifactor model are those of Chen, Roll and Ross (1986), CRR. CRR examined a range of business condition variables that may be related to stock returns and attempted to identify economic factors that are correlated with stock returns, testing whether the factor loadings explain the cross-section of expected returns. Hamao (1989) performed a parallel analysis in Japanese markets (using Japanese macroeconomic variables) as a test of the robustness of the CRR results. Poon and Taylor (1991) reconsidered the results in CRR to see if they are applicable to UK stocks.

Chen, Roll and Ross (1986), have tested the Arbitrage Pricing Model with data drawn from the US securities markets and found exogenous economic factors in the multivariate Arbitrage Pricing Model. CRR (1986) identify a set of five factors as affecting expected returns in their data set: industrial production, changes in the risk premium, twists in the yield curve, and, more weakly, measures of anticipated inflation, and changes in expected inflation during periods when these variables are highly volatile. These macroeconomic variables are assumed to have influenced either future cash flows or the risk-adjusted discount rate, two key variables when stocks are priced by the expectation of the present value of future cash flows.

Although CRR do not provide a formal model, their results indicate that the common factors are related to the fundamental economic aggregates. By design, the variables in the CRR study are chosen outside of the equity market to model stock returns as functions of macroeconomic variables and nonequity asset returns. They are looking for exogenous macrovariables that affect the future cash flows or the risk-adjusted discount rate of a company.

To measure the risk of these economic factors, Chen, Roll and Ross use the "innovations" in rather than the "levels" of these variables. For example, in measuring the term structure risk, they use the return difference between long- and short-term government debt (which is the change in the yield spread, properly normalized) rather than the yield spread (the difference in the implied internal rates of return which is more appropriate for predicting expected returns and premiums). It is not so much the absolute change in an indicator that is important, but how it compares to market expectations.

The two-stage regression technique used in CRR was adapted from Fama and MacBeth (1973). Using the two-stage regression technique they carried out analyses on each individual macroeconomic factor. However, Poon and Taylor (1991) reported that the two-stage regression technique used in Chen, Roll and Ross (1986) is very sensitive to the number of independent variables included in the regression. A particular factor may appear to be significant in one multivariate analysis, but not when other independent variables have been changed, or when analysed alone in an univariate model, and vice versa. Separate multiple regression analyses of each set of variables would neglect the interrelations of the sets.

The correlations between macroeconomic variables could produce a collinearity problem. In this study, factor analysis is used to construct independent economic factors from UK economic variables. The factors extracted from the macroeconomic and financial variables eliminate multicollinearity among independent variables. These estimated economic factors convey the relevant information of the economy in a reduced form of a macro-model. In chapter 7, the factor scores from the factor analysis on security returns and economic indicators will be used to investigate the link between the stock market and economic forces.

6.3 Data Description

Monthly data were obtained from Datastream. The study period is from January 1965 through to December 1988 inclusive, which corresponds to that of the security returns used in chapter 5. The major categories of macroeconomic variables considered in the analysis are those representing the stock market, money supply, industrial production, and labour market, as well as international trade. The variables are measured by widely used indicators which cover a wide spread of economic processes and sectors of the economy. In addition, these macroeconomic variables are assumed to influence either future cash flows or the risk-adusted discount rate, two key variables when stocks are priced by the expectation of the present value of future cash flows. The selection of these variables is also based on the availability of the data.

Unemployment Rate: Wholly Unemployed Rate in G.B. (Old Base) (seasonally adjusted), this is referred to below as (ECON1).

Government Securities: Average Gross Redemption Yield on 20 Year Government Securities (ECON2).

Interest Rate: Interest Rate on 3-Month Bank Bills (Cyclical Indicator Series) (seasonally adjusted) (ECON3).

Coincident Indicator: A "roughly coincident" index, showing current movements in production, composed of Gross Domestic Product; retail sales volume; output in manufacturing industry; and CBI quarterly surveys of capital utilization and actual changes in stocks (ECON4).

Having found groups of indicators with similar timing relationships to the reference cycle, it is of interest to see whether the indicators in a group can be combined into one synthetic indicator. Such a combination provides a convenient summary of the group, and it is less affected by irregular variations than the individual indicators.

A complete index is formed by combining together in some way the actual values of the indicators at corresponding times. The usual procedure is to scale each indicator by dividing the values by a measure of the amplitude of the cyclical variations, and then add a constant so that the series takes the value 100 at a selected date. This process, which is known as amplitude standardisation, turns each indicator into a form of index number showing cyclical variations around 100 with a common amplitude.

Market Index: FT30 Share Price Index (ECON5); FT Actuaries 500 Share Price Index (ECON6); FT Actuaries Capital Goods Share Price Index - monthly average (ECON7); FT Actuaries Financial Group Share Price Index - monthly average (ECON8); FT Actuaries Industrial Share Price Index - monthly average (ECON9).
The market indices should reflect both the real information in the industrial production series and the nominal influence of the inflation variables.
An advantage of the FT-Actuaries series as a whole is that it allows investors to track the performance of particular sectors. Among the more important component indices are the Industrial Group Index, the 500-Share Index, which is the same as the Industrial Group Index but including oils, and the Financial Group Index.

Government Securities: FT government Securities Price Index (End Period) (ECON10).

Lagging Indicator: A "lagged index", indicating the pattern of production about a year after it has happened, composed of unemployment; unfilled vacancies; investment in manufacturing plant and machinery; orders in engineering; and the level of manufacturers' stocks and work in progress (ECON11).
When many series of economic indicators showing cyclical behaviour, are considered together, it is often found that there are systematic timing relationships between their corresponding turning points. Those variables which regularly turn latest as lagging variables.

Longer Leading Indicator: Central Statistical Office's Longer Leading Indicator - a longer leading index indicating trends about a year in advance, composed of the rate of interest on three-month prime bank bills; the financial balance of industrial and commercial companies; housing starts; the Financial Times - Actuaries 500 Share index; and the quarterly survey of business confidence conducted by the Confederation of British Industry (ECON12).

Industrial Production - Total: (Volume - seasonally adjusted) (ECON13).

Inflation: Retail Prices Index - All Items (not seasonally adjusted) (ECON14).

Shorter Leading Indicator: Shorter Leading Indicator - a "shorter leading index", indicating trends about six months ahead, composed of new car registrations; CBI quarterly surveys on the expected change in new orders and in stocks; credit granted; and gross trading profits of companies excluding stock appreciation and oil and gas extraction (ECON15).

Exchange Rate: Average Exchange Rate - US \$ to £1 (ECON16).
The average exchange rate was the midpoint between the spot buying and selling rates recorded by the Bank of England on the last working day of each month.

Fuel & Oil Prices: Wholesale Prices, Manufacturing Input - Fuel (not seasonally adjusted) (ECON17); Consumer Prices - Gasoline and Oil (not seasonally adjusted) (ECON18).

Consumer Expenditure: Consumers Expenditure on Durable Goods (constant prices - seasonally adjusted) (ECON19).

GDP: Gross Domestic Production, average estimate (ECON20).

Money Supply: Money Supply M1 End Quarter Level (current prices - seasonally adjusted) (ECON21).

Many of these series are themselves indices which will be sensitive to a broader range of forces than those explicitly included here. Also, different market indices which have differences in coverage will cause slight differences among them. Factor analysis will eliminate the collinearity problem.

All the economic variables examined are measured by rates of change rather than absolute values. There are three reasons for differencing : (a) for comparison with stock returns which are themselves differenced. It is the rate of change rather than the level itself that is significant. (b) First differencing is applied to render the series stationary ((Nelson and Plosser, 1982), (Wasserfallen, 1989), (Eun and Shim, 1989)). (c) It is not so much the absolute change in an indicator that is important, but how it compares to market expectations. Economically, if macroeconomic variables are random walks, the first differences are equivalent to unexpected values which are the unanticipated innovations in the economic variables.

6.4 Method

In estimating the number of factors representing the economic activities of the UK economy, two factor extraction techniques are used (as discussed in chapter 5):

(i) Principal factor analysis (PFA) is used to reveal the probable number and size of the UK economic factors before proceeding to a maximum-likelihood factor analysis;

(ii) Maximum-likelihood factor analysis (MLFA) is used to identify precisely the number of UK economic factors and their factor loadings.

The factors extracted from the macroeconomic and financial variables eliminate multicollinearity among independent variables since factor analysis extracts independent factors from the range of economic variables.

6.5 Principal Factor Analysis

As discussed in chapter 5, we use PFA to get an approximate idea of the number of factors. The results of applying PFA to this set of returns on the economic indicators show that the overall Kaiser's measure of sampling adequacy (MSA) is 0.79 (Table 6.1) and the squared multiple correlations (SMC) of all the variables are 0.53 (Table 6.2) on average, therefore the results imply that the data are quite adequate for factor analysis. In chapter 5, it was shown that for the sample of security returns, the overall MSA and SMC are 0.97 and 0.64 respectively. The lower values of the MSA and SMC for the economic variables imply that the economic variables are less strongly correlated with each other than are the security returns. This is not unexpected because the set of macroeconomic data includes a much more

TABLE 6.1

KAISER'S MEASURE OF SAMPLING ADEQUACY

ECON1	ECON2	ECON3	ECON4	ECON5	ECON6	
0.661625	0.745157	0.725893	0.636730	0.913850	0.873261	
ECON7	ECON8	ECON9	ECON10	ECON11	ECON12	
0.884405	0.948959	0.808137	0.688158	0.688652	0.831123	
ECON13	ECON14	ECON15	ECON16	ECON17	ECON18	
0.709755	0.598899	0.700692	0.721029	0.589629	0.547367	
ECON19	ECON20	ECON21	Mean SMC	0.79	Min SMC	0.55
0.751406	0.748081	0.812990			Max SMC	0.95

TABLE 6.2

PRIOR COMMUNALITY ESTIMATES: SMC

ECON1	ECON2	ECON3	ECON4	ECON5	ECON6	
0.315770	0.690338	0.449430	0.676428	0.506380	0.894804	
ECON7	ECON8	ECON9	ECON10	ECON11	ECON12	
0.911788	0.850708	0.954754	0.572502	0.530565	0.684097	
ECON13	ECON14	ECON15	ECON16	ECON17	ECON18	
0.232984	0.342543	0.622558	0.200795	0.213500	0.354607	
ECON19	ECON20	ECON21	Mean SMC	0.53	Min SMC	0.20
0.279265	0.541285	0.242951			Max SMC	0.95

diverse set of variables. Table 6.3 shows the eigenvalues of the reduced correlation matrix. Based on the eigenvalue 1 criterion, four factors are retained, and, those four factors account for 92.22% of the common variance. The first factor accounts for nearly 49% of the proportion of total variation, the second factor accounts for over 22% of the proportion of total variation, the third factor accounts for 11.75%, whereas the fourth accounts for 9.40%. Eleven of the eigenvalues are positive while ten are negative; which is to be expected. The scree test based on the graph of eigenvalues also shows that no more than four factors should be extracted.

TABLE 6.3

EIGENVALUES OF THE REDUCED CORRELATION MATRIX

	EIGENVALUE	DIFFERENCE	PROPORTION	CUMULATIVE
1	5.413547		0.4891	0.4891
2	2.452335	2.961212	0.2216	0.7107
3	1.300948	1.151387	0.1175	0.8282
4	1.039921	0.261028	0.0940	0.9222
5	0.880913	0.159008	0.0796	1.0018
6	0.520631	0.360282	0.0470	1.0488
7	0.246758	0.273872	0.0223	1.0711
8	0.232736	0.014022	0.0210	1.0921
9	0.086730	0.146006	0.0078	1.1000
10	0.074838	0.011892	0.0068	1.1067
11	0.017032	0.057805	0.0015	1.1083
12	-0.008972	0.026004	-0.0008	1.1075
13	-0.019792	0.010821	-0.0018	1.1057
14	-0.049178	0.029386	-0.0044	1.1012
15	-0.070956	0.021778	-0.0064	1.0948
16	-0.080131	0.009175	-0.0072	1.0876
17	-0.127063	0.046932	-0.0115	1.0761
18	-0.154138	0.027074	-0.0139	1.0622
19	-0.198339	0.044201	-0.0179	1.0443
20	-0.227730	0.029391	-0.0206	1.0237
21	-0.262039	0.034309	-0.0237	1.0000

It is interesting to note that the second factor accounts for nearly 45% of the proportion of total variation explained by the first factor. In chapter 5, it was shown that the second UK stock market factor can only explain 5.5% of the proportion of total variation of the security returns as explained by the first UK stock market factor. The results reflect the importance of the market factor in the UK security returns; while, in the wider UK economy, several factors have an important part in representing the economy.

6.6 Maximum-Likelihood Factor Analysis

The monthly returns of the economic and financial variables were subjected to maximum-likelihood factor analysis to determine the number and factor loadings of the common factors. The goodness of fit results for the UK economic factors are summarized in Table 6.4.

When the number of factors is equal to 4, some of the communality estimates are greater than 1. If the communality exceeds unity, it is an ultra-Heywood case. An ultra-Heywood case implies that a factor has negative variance, a clear indication that something is wrong. The possible cause of the anomaly is the extraction of too many common factors which renders a factor solution invalid. With fewer than four factors all the communality estimates are less than 1. Therefore, the Table 6.4 shows only the results with fewer than four factors.

The results in Table 6.4 show that Akaike's information criterion (AIC) and Schwarz's Bayesian criterion (SBC) for three factors are lower than that those for two factors. As discussed above, the Heywood case occurs when the number of factors is equal to four. Tucker and Lewis's (T&L) reliability coefficient is the ratio of explained covariation to total variation which provides some perspective on the residual variation. The residual variation

TABLE 6.4

DIFFERENT CRITERIA FOR ESTIMATING THE BEST NUMBER OF PARAMETERS TO INCLUDE IN A MODEL

Number of factors	Akaike's information criterion	Schwarz's Bayesian criterion	Tucker & Lewis's reliability coefficient
2	1175.60	701.35	0.69
3	883.84	590.27	0.77

should be as small as possible without having the factor model becoming too cumbersome. The T&L coefficient for the three factor model is 0.77 which implies that there is a good fit between observed and reproduced matrices. Therefore, three factors are considered for further investigation. In chapter 5, it was shown that the Tucker and Lewis's reliability coefficient for the two factor model for the UK security returns is 0.89. In comparison, it appears that there is a slightly better fit between observed and reproduced matrices of the UK stock market factors model than that of the UK economic factors model.

Table 6.5 shows the factor pattern for the three extracted factors. The highest absolute loading on factor 1 is 0.9955 and the lowest is 0.0192. For all three factors, there is a mixture of positive and negative loadings. Some variables have negative loadings on all factors (e.g. wholesale prices, manufacturing input - fuel (ECON17)), some are positive for all factors (e.g. gross domestic production (ECON20)), while most show a mixture of positive and negative loadings. The signs for variables for a given factor have a specific meaning relative to the signs for other variables; the different signs simply mean that the variables are related to that factor in opposite directions.

The next step is to rotate the factors in order to find more easily interpretable results,

TABLE 6.5

UNROTATED FACTOR PATTERN

	FACTOR 1	FACTOR 2	FACTOR 3
ECON9	0.9956	-0.0075	0.0270
ECON7	0.9522	0.0139	-0.0062
ECON6	0.9344	0.0040	-0.0337
ECON8	0.9156	-0.0354	0.0396
ECON5	0.6427	-0.0134	-0.0906
ECON10	0.3237	-0.0077	-0.2473
ECON17	-0.1185	-0.1173	-0.0466
ECON2	-0.4752	-0.0116	0.2750
ECON4	0.0192	0.8717	0.1848
ECON15	0.1808	0.7458	-0.2615
ECON20	0.0541	0.7312	0.0488
ECON13	0.0586	0.4160	0.1421
ECON19	0.1111	0.3781	-0.1238
ECON18	-0.0278	-0.1457	0.0104
ECON14	0.1069	-0.2351	0.0047
ECON11	-0.2888	0.2127	0.5529
ECON3	-0.2556	0.0693	0.4797
ECON16	0.0743	-0.0641	-0.1430
ECON1	0.1372	-0.2372	-0.2811
ECON21	0.2190	0.0882	-0.3765
ECON12	0.5725	0.1553	-0.6819

while keeping the number of factors and the communalities of each variable fixed. Recall that three orthogonal rotational techniques are often used: quartimax, varimax and equamax. The variances explained by the three factors, with and without weights, are shown in Table 6.6. Although the three rotation techniques give similar results, as before, the quartimax rotation is preferred because it aims to make the variables as simple as possible by maximizing the variance of the loadings on each variable. The results in Table 6.6 are consistent with the earlier results that the first factor is the dominant factor through different rotations. The squared multiple correlations (SMCs) are the estimates of communality

TABLE 6.6

VARIANCE EXPLAINED BY FACTORS USING DIFFERENT ROTATIONAL TECHNIQUES

Rotational technique		Variance explained by each factor		
		Factor 1	Factor 2	Factor 3
Unrotated	Weighted	148.50	7.28	4.42
	Unweighted	4.98	2.40	1.53
Quartimax	(weighted)	135.82	16.61	7.76
	(unweighted)	4.05	2.44	2.43
Varimax	(weighted)	138.11	7.67	14.41
	(unweighted)	4.16	2.40	2.33
Equamax	(weighted)	138.58	7.63	13.98
	(unweighted)	4.18	2.42	2.31

between variables and the factors. The SMCs represent the proportion of variance in variables that are predictable from the factors underlying them. The squared multiple correlations of the variables with factor 1, factor 2, and factor 3 are 0.9778, 0.8309, and 0.8794 respectively, which implies that the three factors are internally consistent and well defined by the variables.

The results in Table 6.7 show the pattern of factor loadings after the quartimax rotation. All three factors retain the mixture of signs in the loadings of the economic variables, indicating that the economic variables have different reactions to the factors.

Table 6.8 identifies the economic variables grouped by the statistically significant

TABLE 6.7

ROTATED FACTOR PATTERN (QUARTIMAX)

	FACTOR 1	FACTOR 2	FACTOR 3
ECON9	0.9590	0.2643	0.0498
ECON7	0.9070	0.2818	0.0703
ECON8	0.8875	0.2310	0.0165
ECON6	0.8822	0.3035	0.0610
ECON5	0.5876	0.2743	0.0297
ECON17	-0.1222	0.0183	-0.1213
ECON12	0.3401	0.8061	0.2269
ECON21	0.0950	0.4167	0.1220
ECON10	0.2367	0.3306	0.0254
ECON1	0.0578	0.3249	-0.2125
ECON16	0.0314	0.1626	-0.0515
ECON2	-0.3725	-0.3997	-0.0551
ECON3	-0.1058	-0.5369	0.0271
ECON11	-0.1218	-0.6267	0.1639
ECON4	0.0379	-0.2326	0.8595
ECON15	0.0660	0.2491	0.7687
ECON20	0.0368	-0.0828	0.7292
ECON13	0.0811	-0.1481	0.4101
ECON19	0.0546	0.1236	0.3903
ECON18	-0.0177	-0.0076	-0.1474
ECON14	0.1128	0.0433	-0.2283

factor loadings of the three factors. A factor is most affected by the economic variables that have high loadings on it. Tabachnick and Fidell (1989) suggested that variables which have loadings in excess of 0.30 (in absolute terms) are considered "statistically significant".

The results of this section suggest that there are three major factors underlying the UK economy. The first factor encompasses general market-wide variables and is composed of various market indices. The second factor includes longer leading indicator, lagging indicator, money supply, interest rate, gross redemption yield on gilts, market indices and unemployment rate. The third factor represents variables such as the coincident indicator, GDP, shorter leading indicator, industrial production, and consumers expenditure on durable

TABLE 6.8

IDENTIFICATION OF THE ECONOMIC VARIABLES GROUPED BY THE FACTOR LOADINGS

Factor 1:	FT Actuaries Industrial Share Price Index - monthly average	(ECON9)	0.9590
	FT Actuaries Capital Goods Share Price Index - monthly average	(ECON7)	0.9070
	FT Actuaries Financial Group Share Price Index - monthly average	(ECON8)	0.8875
	Financial Times Actuaries 500 Share Price Index	(ECON6)	0.8822
	FT 30 Share Price Index (End Period)	(ECON5)	0.5876
	Central Statistical Office's Longer Leading Indicator	(ECON12)	0.3401
	Average Gross Redemption Yield on 20 year Government Securities	(ECON2)	-0.3725
Factor 2:	Central Statistical Office's Longer Leading Indicator	(ECON12)	0.8061
	Money Supply (M1) End Quarter Level	(ECON21)	0.4167
	UK FT Government Securities Price Index	(ECON10)	0.3306
	Wholly Unemployed Rate in Great Britain	(ECON1)	0.3249
	Lagging Indicator	(ECON11)	-0.6267
	UK Interest Rate on 3 Month Bank Bills	(ECON3)	-0.5369
	UK Gross Redemption Yield on 20 Year Gilts	(ECON2)	-0.3997
Factor 3:	Coincident Indicator	(ECON4)	0.8595
	Gross Domestic Production, Average	(ECON20)	0.7687
	Shorter Leading Indicator Estimate	(ECON15)	0.7292
	Industrial Production - Total (volume)	(ECON13)	0.4101
	Consumers Expenditure on Durable Goods	(ECON19)	0.3903

goods. The reduced form of the macroeconomic model demonstrates that there is residual macro variability which is not predicted by the market portfolio.

The relationships between these economic variables in each of the three factors seem to follow the logic of economic activity. According to the discounted cash flow (DCF) valuation formula, stock prices are the expected discounted dividends. It follows from this valuation formula that changes in stock prices occur because of changes in either expected cash flows or in the risk adjusted discount rate. For the first factor, the positive factor loadings of the market indices and the longer leading indicator reflect the fact that the stock price indices are a component of the index of leading economic indicator. As for the yield of government securities, whose factor loading is negative, the story is reversed; the stock market returns are inversely related to changes in the yields of government securities. The government securities represent alternative investment opportunities; whenever they rise, investors tend to switch out of stock, causing stock prices to fall. For the second factor, the signs of the factor loadings are consistent with economic reasonings. Money supply is inversely related to the interest rate. An increase in money supply stimulates the economy and increases spending on goods and services. As a result, the increased economic activity tends to increase employment. As it is expected, the longer leading and lagging indicators are inversely related to each other, because the longer leading indicator shows the trends approximately a year in advance while the lagging indicator shows the pattern of production approximately a year after it has occurred. A major share price index is positively related to the money supply. The negative relationship of interest rates and stock price index is expected, as higher interest rates increase the attractiveness of alternative investments. In addition, the money demand theory implies a negative relation between the inflation rate and the growth rate of economic activity. Because stock returns predict economic activity, a negative correlation is induced between stock returns and inflation. The unemployment rate is positively related to the stock price index as an increase in the unemployment rate appears to lead to an expansionary monetary policy. For the third factor, the coincident indicator which indicates current movements in production is positively related to the GDP, industrial production, the shorter leading indicator and the expenditure on durable goods. The variables in the third factor reflect the economic activity. A higher economic activity should result in an increase in share prices because expected profits of firms in the future will increase.

6.7 Discussion

By the maximum-likelihood method of factor analysis, it has been shown that there are three UK economic factors. Although the results here are fairly similar to the findings of Kim and Wu (1987) who extracted factors from US economic indicators, the market return measure (i.e. the first factor encompasses the market indices) appears to be the most important factor in explaining the overall economic activities in the UK, whereas the broad market return did not appear to be the most significant factor in their study. The market indices may capture unexpected shocks to the economy more rapidly than smoothed series of economic variables, because of the smoothing of the economic time series in short holding periods, such as a single month, these series cannot be expected to capture all the information available to the market in the same period. On the other hand, stock prices respond very quickly to information. The market returns are of interest in their own right as proxies whose efficiency is relevant to CAPM tests. In view of the traditional CAPM where the market return plays a major role, this is an interesting result. This is consistent with the idea that views the market factor as an aggregate consensus measure of all the underlying factors. In this study, the reduced form of the macroeconomic model demonstrates that there is residual macro-variability which is not predicted by the market portfolio. The cumulative proportion of the three economic factors accounts for almost 83% of the variations in UK economic activities. Thus, it can be assumed that the three factors are good representations of economic activities.

6.8 Conclusions

This chapter suggests that there were three major factors underlying the UK economy during the study period (1965-1988). Several macroeconomic factors have been extracted from the security returns. The first factor encompasses general market-wide variables and is composed of various market indices. The second factor includes a longer leading indicator, a lagging indicator, money supply, interest rate, gross redemption yield on gilts, and unemployment rate. The third factor represents variables such as the coincident indicator, GDP, shorter leading indicator, industrial production, and consumers expenditure on durable goods.

The analysis shows that three factors form a good representation of the economic activities which describe the economy; in total the three factors account for almost 83% of the variations in all economic variables. The market return measures appear to be the most important factor as the market indices account for a significant proportion of variation.

Based on the foundations of the APT and the characteristics of the factor scores from the factor analysis on security returns and economic indicators, the canonical correlation analysis will be used in the next chapter as an alternative and more reliable technique than that used by Chen, Roll and Ross (1986).

CHAPTER 7

STOCK RETURNS AND ECONOMIC FORCES: THE UK EXPERIENCE

7.1 Introduction

The objective of this chapter is to analyze the relationships between UK security returns and economic indicators for the UK.

No satisfactory theory would argue that the relation between financial markets and the macroeconomy is entirely in one direction. Although stock returns are usually considered to respond to external forces, they may also have a feedback effect on the macroeconomy. The APT gives little guidance as to the identity of the factors beyond the restriction that they should obey the pervasiveness condition (see chapter 2).

Based on the foundations of the APT and the characteristics of the factor scores from the factor analysis on security returns and economic indicators, the canonical correlation analysis is a new technique which is used to link economic forces and the stock market in this chapter. The canonical correlations are the association between the factor scores of the security returns and the factor scores of economic indicators. The technique is similar in descriptive fashion to other related "linear transformation" techniques such as factor analysis. If the correlations between the factor scores for corresponding pairs of factors are statistically significant, then they imply the factor comparability of the stock returns and the economic forces. To determine whether the same factors influence the security returns and the economic indicators, it is not sufficient to just examine the factor loadings. An intuitive way to view canonical correlation is to think of two new variables (canonical variates) being created, each of these being a linear combination of the original sets of variables (i.e. factor scores of security returns and factor scores of economic variables). Each linear combination will be such that it maximizes the correlation between the two canonical variates. The technique of canonical correlation analysis is discussed in greater detail in section 4.5.

The next section contains the background of this chapter. Section 7.3 investigates the nature of the links and patterns of interdependency between stock returns and economic forces and the number of (statistically significant) links between them. The interpretation of canonical variates is discussed in section 7.4. The results are discussed in section 7.5 and the last section contains the summary of the results.

7.2 Background

Although most studies employ the factor analytic approach, the most influential tests of the multifactor model are those of Chen, Roll and Ross (CRR)(1986). Their approach is to look for economic variables which are correlated with stock returns. This approach thus attempts to address the economic interpretation of factors, left unsatisfied in the factor analytic approach. However, CRR used a version of the Fama-MacBeth (1973) technique which consists of a two-stage regression. The first set of regressions estimates the portfolios' exposures to pricing factors (betas). The second set of regressions estimates the market prices for the beta values obtained from the first set of regressions. The result of this two-stage regression methodology is to generate time series of estimated premia for each risk factor. The time series of risk premia estimates are then tested to see if they are significantly different from zero. The multiple regression analysis that CRR used is very sensitive to the number of independent variables included in the regression. A particular factor may appear to be significant in one multivariate analysis, but not when other independent variables have been changed or when analysed alone in an univariate model, and vice versa. In addition, separate multiple regression analyses of each set of variables would neglect the interrelations of the sets. The multicollinearity among economic variables presents another drawback of the CRR approach. However, despite the drawbacks, the CRR approach is a first step in using multifactor models to improve the understanding of asset pricing.

In this chapter, the canonical correlation analysis is used to analyse the linkage between the factor scores of the security returns and those of the economic indicators. In chapter 6, factor analysis is used to construct independent economic factors which are orthogonal to each other. The factors extracted from the macroeconomic and financial variables eliminate multicollinearity among independent variables. These estimated economic factors convey the relevant information of the economy in a reduced form of a macro-model. Factor analyses are satisfactory if one wants factors chosen independently of each other, however, canonical correlation analysis is a more reliable procedure used to explain as much as possible between one set of variables (i.e. factor scores of security returns) and another set (i.e. factor scores of economic indicators). Canonical correlation is viewed as an external factor analysis, in contrast with the internal factor analysis of a set of variables. As a result, APT factors can be identified which are based on the intuition of the APT (i.e. the factors are orthogonal to each other) and hence, we can have a better understanding of the asset pricing.

7.3 Empirical Results Using the Canonical Correlation Analysis Approach

The factor scores of the factors extracted from the security returns and from the economic indicators in chapter 5 and chapter 6 are subject to canonical correlation analysis in order to find the relationship between the security returns and the economic indicators.

The simple univariate statistic shows that the seven variables (i.e. factor scores of the factors extracted from the security returns and economic indicators), namely, FSEC 1, FSEC 2 and FECON 1, FECON 2 and FECON 3 have a mean which is approximately equal to zero, and a standard deviation is equal to the multiple correlation of the factor with the variables (i.e. security returns, economic indicators). Since the computed factor scores are only estimates of the true factor scores, the estimated factor scores may have small non-zero correlations. There are often correlations among scores for factors even if factors are orthogonal and factor scores sometimes correlate with other factors in addition to the one they are estimating (Tabachnick and Fidell, 1989).

TABLE 7.1

SIMPLE UNIVARIATE STATISTIC

VARIABLE	ST DEV
FSEC1	0.9912
FSEC2	0.9022
FECON1	0.9888
FECON2	0.9115
FECON3	0.9377

The first step in the canonical analysis is generation of a correlation matrix, R (Table 7.2). The correlation matrix is subdivided into four parts: the correlations among the factor scores of the security returns (R_{xx}), the correlations among the factor scores of the economic indicators (R_{yy}), and the two matrices of correlations between the factor scores of the security returns and those of the economic indicators ($R_{xy} = R'_{yx}$).

The correlations between the factor scores of the security returns and those of the

TABLE 7.2

CORRELATIONS AMONG THE SECURITY RETURNS, ECONOMIC INDICATORS AND BETWEEN THE SECURITY RETURNS AND ECONOMIC INDICATORS

CORRELATIONS AMONG THE SECURITY RETURNS (R_{xx})

	FSEC1	FSEC2
FSEC1	1.0000	0.0229
FSEC2	0.0229	1.0000

CORRELATIONS AMONG THE ECONOMIC INDICATORS (R_{yy})

	FECON1	FECON2	FECON3
FECON1	1.0000	0.0550	-0.0081
FECON2	0.0550	1.0000	-0.0017
FECON3	-0.0081	-0.0017	1.0000

CORRELATIONS BETWEEN THE SECURITY RETURNS AND THE ECONOMIC INDICATORS (R_{xy})

	FECON1	FECON2	FECON3
FSEC1	0.6641	0.2795	0.0356
FSEC2	-0.1422	0.0171	-0.1678

economic indicators are fairly high, the largest being 0.6641 between FSEC 1 and FECON 1. This correlation between the factor scores of the first stock market factor and those of the first economic factor is rather high. However, significance cannot yet be assessed.

As shown in Table 7.3, the first canonical correlation is 0.7243, representing 52.46% overlapping variance between the first pair of canonical variates (i.e. linear combination of the factor scores of the security returns and that of the economic indicators), which appears to be larger than any of the direct between-set correlations. This implies that the first pair of canonical variates are highly related to one another. The second canonical correlation is

TABLE 7.3

CANONICAL CORRELATION ANALYSIS

	1	2
CANONICAL CORRELATION (r_c)	0.7243	0.1726
SQUARED CANONICAL CORRELATION (r_c^2)	0.5246	0.0298

TESTS OF H_0: THE CANONICAL CORRELATION IN THE CURRENT COLUMN AND ALL THAT FOLLOW ARE ZERO

	1	2
Likelihood Ratio	0.46123428	0.97020713
F-test	44.5674	4.3605
Pr > F	0.0001	0.0136

0.1726, representing 2.97% overlapping variance for the second pair of canonical variates. Therefore, there are two statistically significant pairs of canonical variates. The first canonical correlation represents a substantial relationship between the first pair of canonical variates. Interpretation of the second canonical correlation and its corresponding pair of canonical variates is especially marginal, because the canonical correlation value of the second pair of 0.1726 represent, squared, less than a 3% overlap in variance. Though the second pair is a statistically significant link, it accounts for a trivial amount of common variance.

The last panel of Table 7.3 shows the probability level for the null hypothesis that all the canonical correlations are zero in the population is 0.0136, hence both pairs of canonical variates reach significance ($\alpha = 0.05$) and they account for the significant relationships between the two sets of variables. The first F test is for all pairs taken together, the second test is for all pairs of canonical variates with the first and the most important pair of canonical variates removed. All pairs produced after the first one are constrained to be uncorrelated with all the preceding combinations. There will never be more pairs than the number of variables in the smaller set. Bentler and Bonett (1980) noted that a major difficulty with the chi-square goodness-of-fit test in covariance structure models has been that in small samples many competing models are found to be equally acceptable, whereas in large samples virtually any model tends to be rejected as inadequate, large samples often yield models that attempt to explain residuals which are negligible for practical purposes. The number of statistically significant pairs of canonical variates is often larger than the number of interpretable pairs if the number of observations is at all sizable. The sample here is also a large one and therefore an overemphasis on probability values is particularly dangerous.

As shown in Table 7.4, the first canonical correlation vectors are

$$\rho_1 \;=\; 0.9811 \text{ FSEC1} - 0.2174 \text{ FSEC2},$$

and

$$\phi_1 \;=\; 0.9238 \text{ FECON1} + 0.3228 \text{ FECON2} + 0.0917 \text{ FECON3},$$

with $r_c = 0.7243$.

7.4 Interpretation of Canonical Variates

After the canonical correlation creates the canonical variates, the matrix of correlations of the original variables (i.e. factor scores of the stock market factors) with the canonical variates (ρ and ϕ) is a factor loading matrix. It contains the correlations of the original variables with the canonical coefficients. The content of the canonical variates is interpreted via the factor loading matrix. Interpretation of reliable pairs of canonical variates is based on the factor loading matrices, A_x and A_y. Usually correlations between original variables

TABLE 7.4

CANONICAL CORRELATION ANALYSIS: STANDARDIZED CANONICAL COEFFICIENTS FOR THE SECURITY RETURNS AND THE ECONOMIC INDICATORS

STANDARDIZED CANONICAL COEFFICIENTS (B_x) FOR THE SECURITY RETURNS

	SEC1	SEC2
FSEC1	0.9811	0.1950
FSEC2	-0.2174	0.9763

STANDARDIZED CANONICAL COEFFICIENTS (B_y) FOR THE ECONOMIC INDICATORS

	ECON1	ECON2
FECON1	0.9238	-0.0694
FECON2	0.3228	0.4148
FECON3	0.0917	-0.9077

and canonical coefficients in excess of 0.3 are interpreted (Tabachnick and Fidell, 1989). As shown in Table 7.5, the first pair of canonical variates has high loading on FSEC 1 (0.9761) of the factor scores of the security returns and on FECON 1 (0.9423) of the factor scores of the economic indicators. Thus, the first canonical variates are primarily FSEC 1 for the security returns and FECON 1 for the economic variables. The results in chapter 6 show that the first UK economic factor is composed of market indices and encompasses general market-wide variables hence it somewhat resembles the market portfolio.

The second pair of canonical variates has high loading on FSEC 2 (0.9808) of the factor scores of the security returns and on FECON 2 (0.4126), FECON 3 (-0.9090) of the factor scores of the economic indicators. Hence, the second canonical variates are primarily FSEC 2 for the security returns and FECON 2, FECON 3 for the economic variables. From

TABLE 7.5

CANONICAL STRUCTURE

CORRELATIONS BETWEEN THE SECURITY RETURNS AND THEIR CANONICAL COEFFICIENTS, (A_x)

	SEC1	SEC2
FSEC1	0.9761	-0.2174
FSEC2	-0.1950	0.9808

CORRELATIONS BETWEEN THE ECONOMIC INDICATORS AND THEIR CANONICAL COEFFICIENTS, (A_y)

	ECON1	ECON2
FECON1	0.9423	-0.0539
FECON2	0.0951	0.4126
FECON3	-0.2062	-0.9090

CORRELATIONS BETWEEN THE SECURITY RETURNS AND THE CANONICAL COEFFICIENTS OF THE ECONOMIC INDICATORS, ($R_{xx}B_y$)

	ECON1	ECON2
FSEC1	0.7070	-0.0375
FSEC2	-0.1412	-0.0193

CORRELATIONS BETWEEN THE ECONOMIC INDICATORS AND THE CANONICAL COEFFICIENTS OF THE SECURITY RETURNS, ($R_{yy}B_x$)

	SEC1	SEC2
FECON1	0.6825	-0.0093
FECON2	0.2705	0.0712
FECON3	-0.0715	-0.1569

the results in chapter 6, the second and third economic factors represent variables such as industrial production, coincident indicator, GDP, consumer expenditure on durable goods, shorter leading indicator and is also composed of longer leading indicator, lagging indicator, money supply, interest rate, gross redemption yield on gilts, and unemployment rate.

The utilization of canonical correlation analysis not only provides information about the nature of the number of (statistically significant) links between the sets, but also shows the extent to which the variance in one set is conditional upon or redundant given the other set.

As we have seen, canonical correlation analysis involves finding the canonical variates from the factor scores of security returns that are maximally correlated with the canonical variates from the factor scores of economic indicators. The canonical correlation does not refer to the relationship between the factor scores of security returns and that of economic indicators themselves. In canonical correlation, a squared canonical correlation tells us the amount of variance that the two canonical variates share, and does not necessarily indicate significant correlation between the two sets of variables (i.e. factor scores of security returns and that of economic indicators). In multiple regression, however, the squared multiple correlation represents the proportion of criterion variance accounted for by the optimal linear combination of the predictors. Hence, there is a danger of obtaining highly correlated, but unimportant factors in a canonical correlation analysis which may be present when there are two variables (i.e. factor scores of security returns and that of economic indicators), one in each set of variables, which are not characteristic for the whole set, but yet highly correlated with each other. In this case one may find a factor pair of essentially unique factors as the first canonical factors[1].

It is therefore interesting to know how much variance the canonical variates from the security returns extract from the economic indicators, and vice versa. In canonical analysis, this variance is called redundancy (Stewart and Love, 1968; Miller and Farr, 1971):

$$r_d = (pv)(r_c^2) .$$

The redundancy in a canonical variate is the percent of variance (pv) it extracts from its own set of variables times the canonical correlation squared for the pair of canonical variates[2].

As shown in Table 7.6, canonical redundancy analysis illustrates that the first pair of canonical variates is a fairly good overall predictor of the opposite set of variables, the proportions of variance explained being 0.2599 and 0.1813. Although the second pair of canonical variates is statistically significant, it is not economically meaningful, the proportions of variance being 0.0150 and 0.0099. The first canonical variate of the security returns extracts 49.54% of the variance of the security returns, the second canonical variate extracts 50.46% of the variance. In summing for the two variates, 100% of the variance in the security returns is extracted by the two canonical variates (because there are only two pairs). For the economic indicators, the first canonical variate extracts 34.57% of variance of the economic variables, while the second canonical variate extracts 33.31% of variance. Together the two canonical variates extract 67.88% of the variance in the economic indicators.

The squared multiple correlations in Table 7.7 indicate that the first canonical variate of the economic indicators has fairly good predictive power for FSEC 1, but has virtually no predicting power for FSEC 2. The first canonical variate of the security returns is a fairly good predictor of FECON 1, but again is almost worthless for predicting FECON 2 and FECON 3.

The squared multiple correlations in Table 7.7 show that both the second canonical variate of the economic indicators and the second canonical variate of the security returns are ineffectual in predicting FSEC 2, and FECON 2 and FECON 3 respectively.

TABLE 7.6

CANONICAL REDUNDANCY ANALYSIS

STANDARDIZED VARIANCE OF THE SECURITY RETURNS EXPLAINED BY

	THEIR OWN CANONICAL VARIATES			THE OPPOSITE CANONICAL VARIATES	
	PROPORTION	CUMULATIVE PROPORTION	CANONICAL R-SQUARED	PROPORTION	CUMULATIVE PROPORTION
1	0.4954	0.4954	0.5246	0.2599	0.2599
2	0.5046	1.0000	0.0298	0.0150	0.2749

STANDARDIZED VARIANCE OF THE ECONOMIC INDICATORS EXPLAINED BY

	THEIR OWN CANONICAL VARIATES			THE OPPOSITE CANONICAL VARIATES	
	PROPORTION	CUMULATIVE PROPORTION	CANONICAL R-SQUARED	PROPORTION	CUMULATIVE PROPORTION
1	0.3457	0.3457	0.5246	0.1813	0.1813
2	0.3331	0.6788	0.0298	0.0099	0.1913

7.5 Discussion

In this chapter, the relationships between security returns and economic indicators are analysed by linking and comparing the two sets of factors extracted from the security returns and the economic indicators. The results from section 7.3 imply that the canonical correlation between the first canonical variate of the security returns and that of the economic indicators is 0.7243. This is the highest correlation between any linear combination of the security returns and the economic indicators. The first canonical variate formed from the security returns is the most successful linear combination of the security returns to predict the

TABLE 7.7

SQUARED MULTIPLE CORRELATIONS

SQUARED MULTIPLE CORRELATIONS BETWEEN THE SECURITY RETURNS AND THE FIRST 'M' CANONICAL VARIATES OF THE ECONOMIC INDICATORS

M	1	2
FSEC1	0.4998	0.5012
FSEC2	0.0199	0.0486

SQUARED MULTIPLE CORRELATIONS BETWEEN THE ECONOMIC INDICATORS AND THE FIRST 'M' CANONICAL VARIATES OF THE SECURITY RETURNS

M	1	2
FECON1	0.4658	0.4658
FECON2	0.0731	0.0782
FECON3	0.0051	0.0297

first canonical variate formed from the economic indicators. Likewise the first canonical variate formed from the economic indicators is the best linear combination of the economic indicators for predicting the first canonical variate formed from the security returns. It is interesting to note that the signs of the correlations between the security returns and their canonical coefficients and those between the economic indicators and their canonical coefficients in Table 7.5 are consistent with macroeconomic reasoning. For example, the results indicate that there is a positive correlation (0.6825) between the first canonical coefficients of the security returns and the first economic factor. It implies that the security returns, the market indices and the longer leading indicator are positively related.

Chen, Roll and Ross (1986) commented that because of the smoothing and averaging characteristics of most macroeconomic time series in short holding periods, such as a single month, these series cannot be expected to capture all the information available to the market in the same period. Stock prices, on the other hand, respond very quickly to public information. The effect of this is to guarantee that market returns will be, at best, weakly related and very noisy relative to innovations in macroeconomic factors. This could bias the results in favour of finding a stronger linkage between the time-series returns on market indices and the stock returns than between the stock returns and innovations in the macro variables. This is because stock returns are relatively more closely related to the market indices.

Table 7.5 shows a positive correlation (0.2705) and a negative correlation (-0.0715) between the first canonical coefficients of the security returns and the second and third economic factors respectively. There is also a positive correlation (0.0712) and a negative correlation (-0.1569) between the second canonical coefficients of the security returns and the second and third economic factors respectively. However, the canonical correlation of the second pair of canonical variates of only 0.1726 represents, squared, less than a 3% overlap in variance. As the canonical correlation is lower than 0.30, the pair of canonical variates will not be economically significant even if statistically significant. Overall, these results imply that the security returns are positively related to the longer leading indicator, money supply, government securities price index, and unemployment rate respectively. There is also a very small negative correlation between the security returns, the lagging indicator and interest rate.

The relationship between stock prices and the economy is interpreted as follows. The leading indicators are supposed to be able to forecast the direction of the real economy and to anticipate movements in the economy. The stock price indexes are one of the components of the index of leading economic indicators. On average, the index of leading indicators grows at about the same rate as the real economy. The lagging indicator shows the pattern of production about a year after it has occurred. Hence, as is expected, the security returns are positively and negatively related to the leading and lagging indicators respectively. The coincident indicator which indicates current movements in production is also negatively related to the security returns as expected.

With regard to money supply, stock prices are likely to be rising and interest rates falling if there is an excess supply of money. The reverse occurs if the real supply of money is below the needs of individuals. The increased money supply has a positive liquidity effect on stock prices. The results indicate that stock prices appear to react negatively to rising interest rates (and vice versa for falling interest rates). The negative relationship of interest rates and stock prices would be expected as higher interest rates increase the attractiveness of alternative investments. Whenever interest rates rise, investors tend to switch out of stock, causing stock prices to fall. The opposite would occur with falling rates. With lower interest rates, the attractions of borrowing are increased. If interest rates decline, there will be a wider range of assets when the return exceeds the costs of interest. A cut in interest rates is often followed in the first instance more by a surge in asset values than by an upturn in output growth. Lower interest rates encourage and stimulate capital investment. Firms' sales will thus increase, boosting their own earnings. Any decrease in the interest rate will have a positive effect on expected future earnings and so raise stock prices. In addition, lower interest rates will decrease the attractions of saving and hence increase the attractions of investing in the stock market. On the other hand, the leveraged firms will reduce the burden of interest payments due to a decline in interest rates and thus will have positive effect on earnings.

The unemployment rate works as a leading indicator because jobs become hard to find as actual production decreases. The unemployment rate and related statistics provide the first comprehensive pieces of information about the general state of the economy for any given month. Interestingly, there is a positive relationship between the stock returns and the unemployment rate. The reasons may be due to the fact that employment is expected to increase only in the later stages of a boom period at a point when declining earnings are expected for most companies. Also, an increase in the unemployment rate appears to lead to expansionary monetary policy which initially is positive for security prices.

The results above show that the canonical correlation analysis successfully links the stock market and the economic forces. Based on the foundation of the APT and the characteristic of the factor scores from the factor analysis, the canonical correlation analysis is a better technique than the multiple regression analysis used in the Chen, Roll and Ross (1986). The drawbacks of the CRR are discussed in section 7.2. The approach here is superior to that of CRR. Canonical correlation analysis examines the relationship between the security returns and the economic indicators by creating a linear combination of the factor scores of the security returns and those of the economic indicators. The canonical correlation technique considers all the variables (factor scores of security returns and those of economic indicators) simultaneously rather than considering the possibility that a particular variable may be significant in one multiple regression but not when other independent variables have been changed or when analysed alone in an univariate model. Separate multiple regression analyses of each set of variables would neglect the interrelations of the two sets.

7.6 Conclusion

This chapter investigates a set of economic indicators as systematic influences on stock returns using the canonical correlation analysis. Based on the foundations of the APT and the characteristics of the factor scores from the factor analysis on security returns and economic indicators, canonical correlation analysis is an appropriate technique to link the economic forces and the stock market and is a better method than the Chen, Roll and Ross (1986) approach (identifying economic forces that are correlated with returns and testing whether the factor loadings explain the cross-section of expected returns).

The first pair of canonical variates is composed of the factor scores of the first factor of the security returns and those of the first factor of the economic indicators. As it is shown in the previous chapter, the first economic factor is encompassed mainly of market indices.

Although the second pair of canonical variates is statistically significant, there is only a very weak correlation between them. The second pair of canonical variates is composed of the factor scores of the second factor of the security returns and those of the second and third factors of the economic indicators. As shown in the previous chapter, the second economic factor represents primarily the longer leading indicator, lagging indicator, money supply, interest rate, gross redemption yield on gilts, and unemployment. Whereas the third factor encompasses variables such as industrial production, coincident indicator, GDP, consumer expenditures on durable goods, and shorter leading indicator.

The conclusion of these empirical findings is that security returns are influenced by a number of systematic economic forces. However, the market return is the most important factor linking to the security returns. It implies that the market return plays a major role in the APT in the UK security market. The market return explains a significant portion of the time series variability of stock returns. In view of the traditional CAPM where the market return plays a major role, this is an interesting result. The result is consistent with the view that the market factor is an aggregate consensus measure of all the underlying factors. This implies that the market factor alone could incorporate nearly all information that the underlying multiple factors contain. One may view CAPM as a one-factor APT model, with the market model being the return generation process.

1. The proportion of variance extracted from the set of factor scores of the security returns by the canonical variates of the security returns is the sum of the squared correlations between the factor scores of the security returns and their canonical variates divided by the number of factor scores of security returns (i.e. 2) in the set.

$$pv_{xc} = \sum_{i=1}^{K_x} \frac{a_{ixc}^2}{k_x}$$

and

$$pv_{yc} = \sum_{i=1}^{K_y} \frac{a_{iyc}^2}{k_y},$$

which is extracted from the set of factor scores of the economic indicators by the canonical variates of the economic indicators is the sum of the squared correlations between the factor scores of the economic indicators and their canonical variables divided by the number of factor scores of economic indicators (i.e. 3) in the set.

2. The correlations between the variables in a set and a canonical variates of the set are the canonical variate loadings. Each of these loadings is a bivariate correlation, the square of which can be interpreted as the amount of variance of the variable that is accounted for by the canonical variate. By taking the sum of squared canonical variate loadings for a particular canonical variate, one gets the amount of variance of the set that is accounted for by the canonical variate. The proportion of variance extracted from a set of variables by a canonical variate of the set is the sum of the squared canonical variate loadings divided by the number of variables in the set.

CHAPTER 8

STOCK MARKET FACTORS AND APT: THE US EVIDENCE

8.1 Introduction

This chapter contains the results of a test of the APT. The first objective is to estimate the number of factors which determine US stock returns and the correlations between observed variables and factors. The second objective is to use the individual security factor loadings to explain the cross-sectional variation of individual expected returns. The results can be compared with those obtained earlier for the UK stock market. After this, the object of chapters 9 and 10 is to make an identification of the factors by comparing this collection of factor scores with those of the real economy. Section 8.2 contains the data description of the US security returns. The method used in the study is considered in section 8.3. The results of the principal factor analysis and the maximum-likelihood factor analysis are discussed in sections 8.4 and 8.5 respectively. In section 8.6, the individual-security factor loading estimates are used to explain the cross-sectional variation of individual estimated expected returns. In section 8.7, the results are discussed and the last section is the summary of the results.

8.2 Data Description

The data source is the Centre for Research in Security Prices, Graduate School of Business, University of Chicago (monthly-returns file) which contains share returns after adjustment for all capital changes and dividends. The sample period is January 1965 - December 1988 inclusive, giving a maximum of 288 monthly security returns. The choice of this period is based on the availability of the data on the economic variables which are used in the analysis contained in chapter 9 below. Some of these macroeconomic series are not available before January 1965, and the period investigated for the security returns should correspond to that of the economic variables. As discussed in chapter 5, the use of factor analysis imposes two requirements on the sample selection process, namely, the observations with missing values for any variable in the analysis should be omitted from the computations, because calculation of correlations requires simultaneous observations; and the returns of the securities are required to be multivariate normally distributed. As a result, 217 securities fulfill the requirements that the returns do not have missing observations and are normally distributed. The 217 companies sample securities were classified and grouped by the classification used by the Enterprise standard industrial classification. Table 8.1 suggests that compared to the distribution of the number of securities listed in the Standard & Poor's 500 in each industry group, our sample seems to contain a relatively high proportion of securities in the transportation and public utilities groups and a relatively lower proportion of securities in the financial group than that in the stock market in general. This might be explained by the relatively large number of long lived transportation and public utilities companies in the US and the relatively recent growth in the financial sector.

8.3 Method

In estimating the number of factors which affect US security returns, two factor extraction techniques were used:

(i) Principal factor analysis (PFA) to get an approximate idea of the number of factors before proceeding to a maximum-likelihood analysis.

(ii) Maximum-likelihood factor analysis (MLFA) is used to identify more precisely the number of factors, their factor loadings and factor scores.

TABLE 8.1

DISTRIBUTION OF SAMPLE SECURITIES IN EACH GROUP

Industry Classification	Number of securities in the sample	Percentage of securities in the sample	Industry Classification*	Number of securities in the S&P 500	Percentage of securities in the S&P 500
Finance, Insurance & Real Estate	15	6.91	Financial	53	10.60
Manufacturing	137	63.13	Industrials	388	77.60
Mining	9	4.15	Transportation	18	3.60
Retail Trade	5	2.30	Utilities	41	8.20
Services	7	3.23			
Transportation & Public Utilities	43	19.82			
Wholesale Trade	1	0.46			
	217	100.00		500	100.00

* As of June 30, 1989, there were a total of 84 industry groups that made up the S&P 500. The industry categories are, in turn, grouped into four major sectors. The major sectors are industrials, utilities, financials, and transportation.

8.4 Principal Factor Analysis

Before turning to maximum-likelihood factor analysis, the monthly returns of the 217 securities were subjected to principal factor analysis to determine the number of factors which account for a meaningful percentage of common variance. The communalities (squared multiple correlations) are shown in Table 8.2 and reveal that the average communality value is 0.90. This mean communality is acceptable and indicates that the variables are correlated with each other, therefore the data are acceptable for factor analysis. Table 8.3 shows that the mean Kaisers' measure of sampling adequacy is 0.90, which implies that the data are well suited for factor analysis. The ones in the positive diagonal of correlation martrix are replaced by the communality estimates in preparation for factor extraction.

Moving on to the factor extraction stage, a first quick estimate of the number of factors is obtained from the sizes of the eigenvalues. One of the most popular criteria for estimating the number of factors is to retain factors with eigenvalues greater than 1. The results in Table 8.4 indicate that thirty-five factors have eigenvalues greater than 1, and these thirty-five factors account for 55.30% of total explained variance. The first factor explains nearly 39% of the total variation in stock market returns, the second explains only 8% of the total variance, the third 3.4 %, the fourth factor 3% and the fifth factor only 2.1%. Those factors which account for less than 2% of total variance are ignored as they are insignificant (e.g. the sixth factor accounts for only 1.6% of variation and the 35th factor only 0.5%). The size of the second and the other factors is rather low and it implies that these factors are much less important than the first factor.

As a second estimate of the number of factors, the scree test was also performed to examine the graph of eigenvalues. Applying the scree test, it would seem that at most only

TABLE 8.2

PRIOR COMMUNALITY ESTIMATES: SQUARED MULTIPLE CORRELATIONS

Mean SMC 0.89

CO#1	CO#3	CO#6	CO#7	CO#9	CO#10
0.910682	0.870783	0.869838	0.914027	0.925835	0.926819
CO#11	CO#12	CO#13	CO#14	CO#15	CO#16
0.899654	0.903081	0.885682	0.861304	0.944578	0.912169
CO#18	CO#19	CO#20	CO#21	CO#23	CO#26
0.900740	0.909370	0.911223	0.819398	0.903962	0.827384
CO#27	CO#28	CO#30	CO#32	CO#33	CO#34
0.900826	0.904204	0.937979	0.950878	0.806415	0.878675
CO#35	CO#36	CO#39	CO#40	CO#41	CO#42
0.853808	0.905209	0.890300	0.903556	0.881611	0.882671
CO#43	CO#44	CO#46	CO#47	CO#49	CO#51
0.895037	0.892850	0.824140	0.873041	0.906278	0.841419
CO#52	CO#53	CO#56	CO#57	CO#58	CO#60
0.835351	0.938846	0.850979	0.854138	0.908904	0.937033
CO#62	CO#67	CO#70	CO#72	CO#73	CO#74
0.884332	0.925602	0.919008	0.847094	0.874064	0.924571
CO#75	CO#77	CO#78	CO#79	CO#80	CO#82
0.834834	0.876268	0.887394	0.848383	0.896807	0.868162

TABLE 8.2 continued

CO#85	CO#87	CO#90	CO#91	CO#92	CO#93
0.848943	0.904169	0.876103	0.875504	0.856580	0.920918
CO#94	CO#95	CO#99	CO#100	CO#101	CO#104
0.883280	0.898105	0.878824	0.916229	0.863322	0.907913
CO#107	CO#109	CO#110	CO#113	CO#114	CO#117
0.923629	0.938433	0.857311	0.851959	0.878113	0.926932
CO#120	CO#121	CO#122	CO#124	CO#125	CO#127
0.882833	0.905323	0.853204	0.925812	0.902335	0.937879
CO#129	CO#131	CO#132	CO#135	CO#137	CO#140
0.898804	0.920832	0.838440	0.868816	0.882713	0.854281
CO#142	CO#143	CO#145	CO#146	CO#147	CO#148
0.828020	0.900368	0.903143	0.912766	0.888151	0.869616
CO#150	CO#151	CO#152	CO#153	CO#154	CO#155
0.887355	0.899838	0.919070	0.907100	0.908355	0.883840
CO#157	CO#158	CO#159	CO#161	CO#162	CO#164
0.927938	0.917525	0.932746	0.899912	0.905820	0.907176
CO#167	CO#169	CO#171	CO#172	CO#173	CO#174
0.929297	0.862905	0.876066	0.874163	0.886565	0.923764
CO#177	CO#179	CO#180	CO#182	CO#184	CO#185
0.919051	0.868987	0.875881	0.893927	0.925210	0.906866

TABLE 8.2 (continued)

CO#186	CO#187	CO#190	CO#192	CO#196	CO#197
0.891168	0.848010	0.848604	0.859333	0.931259	0.873590
CO#202	CO#203	CO#204	CO#205	CO#207	CO#209
0.890693	0.914720	0.886317	0.898385	0.885962	0.915823
CO#212	CO#214	CO#215	CO#216	CO#217	CO#219
0.869768	0.888795	0.888453	0.897180	0.876665	0.889409
CO#221	CO#224	CO#225	CO#226	CO#228	CO#230
0.876380	0.937814	0.915479	0.900991	0.861725	0.887979
CO#231	CO#234	CO#236	CO#241	CO#243	CO#244
0.865586	0.915885	0.877569	0.892633	0.827409	0.900328
CO#246	CO#247	CO#248	CO#252	CO#253	CO#255
0.880031	0.913359	0.904924	0.924991	0.931530	0.906793
CO#258	CO#259	CO#261	CO#262	CO#263	CO#265
0.929179	0.907528	0.909810	0.859522	0.873485	0.895806
CO#266	CO#268	CO#269	CO#274	CO#275	CO#276
0.905307	0.903868	0.877539	0.896448	0.905531	0.952030
CO#278	CO#279	CO#280	CO#282	CO#283	CO#284
0.857454	0.884325	0.883832	0.859554	0.873621	0.940978
CO#287	CO#288	CO#289	CO#290	CO#291	CO#295
0.913135	0.917021	0.901833	0.924391	0.888712	0.847531

TABLE 8.2 continued

CO#296	CO#297	CO#298	CO#299	CO#302	CO#304
0.936635	0.896118	0.897769	0.893285	0.899263	0.906287
CO#306	CO#307	CO#308	CO#309	CO#310	CO#311
0.911508	0.924993	0.897038	0.872600	0.930893	0.943053
CO#312	CO#313	CO#314	CO#315	CO#316	CO#317
0.895066	0.918222	0.911062	0.920817	0.930506	0.909885
CO#320	CO#322	CO#324	CO#325	CO#326	CO#327
0.843992	0.918723	0.879302	0.855540	0.913561	0.893845
CO#328	CO#329	CO#330	CO#332	CO#333	CO#334
0.899405	0.862900	0.878403	0.896621	0.891809	0.889853
CO#335	CO#337	CO#339	CO#341	CO#343	CO#345
0.900307	0.886074	0.888172	0.919712	0.888330	0.928938
CO#346	CO#347	CO#349	CO#350	CO#351	
0.916502	0.876648	0.832900	0.909655	0.854203	

CO# denotes the individual US company.

the first five factors should be extracted.

This section used principal factor analysis to reveal the probable number and size of the US stock market factors. The results show that not more than five factors should be extracted from security returns. The next stage of the analysis is to use a more powerful technique (maximum-likelihood factor analysis) to extract the factors and their factor

TABLE 8.3

KAISER'S MEASURE OF SAMPLING ADEQUACY

Overall MSA = 0.90

CO#1	CO#3	CO#6	CO#7	CO#9	CO#10
0.872329	0.930107	0.921033	0.889813	0.893082	0.885853
CO#11	CO#12	CO#13	CO#14	CO#15	CO#16
0.901884	0.830951	0.912552	0.916895	0.901857	0.914000
CO#18	CO#19	CO#20	CO#21	CO#23	CO#26
0.902464	0.936070	0.857228	0.837787	0.914411	0.923666
CO#27	CO#28	CO#30	CO#32	CO#33	CO#34
0.878400	0.923668	0.904020	0.847631	0.905623	0.911851
CO#35	CO#36	CO#39	CO#40	CO#41	CO#42
0.936377	0.905730	0.942580	0.905002	0.903285	0.907473
CO#43	CO#44	CO#46	CO#47	CO#49	CO#51
0.883175	0.930772	0.934507	0.907851	0.880674	0.901561
CO#52	CO#53	CO#56	CO#57	CO#58	CO#60
0.922251	0.886581	0.891945	0.853950	0.908016	0.887930
CO#62	CO#67	CO#70	CO#72	CO#73	CO#74
0.868829	0.853670	0.921952	0.836098	0.914497	0.899010
CO#75	CO#77	CO#78	CO#79	CO#80	CO#82
0.910225	0.882077	0.899557	0.944392	0.860945	0.919081
CO#85	CO#87	CO#90	CO#91	CO#92	CO#93
0.903291	0.878568	0.847046	0.910508	0.938843	0.939382

TABLE 8.3 (continued)

CO#94	CO#95	CO#99	CO#100	CO#101	CO#104
0.927893	0.932973	0.929917	0.932838	0.869414	0.897330
CO#107	CO#109	CO#110	CO#113	CO#114	CO#117
0.878666	0.906592	0.886770	0.896882	0.925387	0.902284
CO#120	CO#121	CO#122	CO#124	CO#125	CO#127
0.918795	0.939948	0.908726	0.922332	0.906979	0.911233
CO#129	CO#131	CO#132	CO#135	CO#137	CO#140
0.917327	0.932613	0.827005	0.934522	0.914803	0.890649
CO#142	CO#143	CO#145	CO#146	CO#147	CO#148
0.846202	0.901737	0.906382	0.917123	0.892122	0.894661
CO#150	CO#151	CO#152	CO#153	CO#154	CO#155
0.868893	0.849902	0.886177	0.900080	0.866806	0.929484
CO#157	CO#158	CO#159	CO#161	CO#162	CO#164
0.877682	0.916531	0.885103	0.939262	0.884970	0.878115
CO#167	CO#169	CO#171	CO#172	CO#173	CO#174
0.885854	0.901275	0.937491	0.920012	0.781717	0.950028
CO#177	CO#179	CO#190	CO#182	CO#184	CO#185
0.891083	0.901457	0.919057	0.881600	0.878154	0.835990
CO#186	CO#187	CO#190	CO#192	CO#196	CO#197
0.888413	0.877363	0.926610	0.930114	0.819521	0.897794

TABLE 8.3 (continued)

CO#202	CO#203	CO#204	CO#205	CO#207	CO#209
0.915639	0.888469	0.911910	0.895366	0.888499	0.926571
CO#212	CO#214	CO#215	CO#216	CO#217	CO#219
0.931409	0.886568	0.945742	0.882895	0.933721	0.905064
CO#221	CO#224	CO#225	CO#226	CO#228	CO#230
0.923893	0.886606	0.903609	0.824536	0.910263	0.852288
CO#231	CO#234	CO#236	CO#241	CO#243	CO#244
0.910264	0.903217	0.917855	0.926736	0.924393	0.933300
CO#246	CO#247	CO#248	CO#252	CO#253	CO#255
0.885781	0.914660	0.917378	0.904782	0.891078	0.918108
CO#258	CO#259	CO#261	CO#262	CO#263	CO#265
0.843072	0.847120	0.875018	0.902365	0.874640	0.911583
CO#266	CO#268	CO#269	CO#274	CO#275	CO#276
0.877527	0.891200	0.889933	0.837907	0.882553	0.861604
CO#278	CO#279	CO#280	CO#282	CO#283	CO#284
0.825254	0.894858	0.924018	0.852711	0.950747	0.864028
CO#287	CO#288	CO#289	CO#290	CO#291	CO#295
0.857969	0.934303	0.844114	0.874123	0.894590	0.874680
CO#296	CO#297	CO#298	CO#299	CO#302	CO#304
0.894787	0.879665	0.882880	0.919559	0.920147	0.909350

TABLE 8.3 (continued)

CO#306	CO#307	CO#308	CO#309	CO#310	CO#311
0.866381	0.886102	0.889402	0.902410	0.866668	0.870855
CO#312	CO#313	CO#314	CO#315	CO#316	CO#317
0.925376	0.926584	0.890116	0.918968	0.938833	0.924095
CO#329	CO#322	CO#324	CO#325	CO#326	CO#327
0.874591	0.934906	0.916192	0.906838	0.901988	0.915675
CO#328	CO#329	CO#330	CO#332	CO#333	CO#334
0.834910	0.843013	0.916552	0.891839	0.914722	0.881405
CO#335	CO#337	CO#339	CO#341	CO#343	CO#344
0.915601	0.884857	0.904256	0.919860	0.938604	0.880238
CO#345	CO#346	CO#347	CO#349	CO#350	CO#351
0.832737	0.852053	0.907575	0.839689	0.902039	0.944517

CO# denotes the individual company.

loadings. The estimated factor loadings are then used to explain the cross-sectional variation of individual estimated expected returns, and to measure the size and statistical significance of the estimated risk premium associated with each factor.

TABLE 8.4

EIGENVALUES OF THE REDUCED CORRELATION MATRIX

	Eigenvalue	Difference	Proportion	Cumulative
1	75.233794		0.3884	0.3884
2	15.418737	59.815057	0.0796	0.4680
3	6.516334	8.902403	0.0336	0.5016
4	5.797964	0.718370	0.0300	0.5316
5	4.149978	1.647987	0.0214	0.5530
6	3.111782	1.038196	0.0161	0.5690
7	2.300336	0.811446	0.0119	0.5809
8	2.212119	0.088217	0.0114	0.5923
9	1.961089	0.251030	0.0101	0.6024
10	1.900186	0.060903	0.0098	0.6123
11	1.799680	0.100506	0.0093	0.6215
12	1.715561	0.084119	0.0089	0.6304
13	1.615405	0.100156	0.0083	0.6387
14	1.569184	0.046221	0.0081	0.6483
15	1.552767	0.016417	0.0080	0.6549
16	1.509558	0.043210	0.0078	0.6627
17	1.490064	0.019494	0.0077	0.6703
18	1.480229	0.009836	0.0076	0.6780
19	1.391536	0.088692	0.0072	0.6852
20	1.370284	0.021252	0.0071	0.6922
21	1.364027	0.006257	0.0070	0.6995
22	1.312916	0.051111	0.0068	0.7061
23	1.293091	0.019825	0.0067	0.7127
24	1.267913	0.025178	0.0065	0.7193
25	1.231939	0.035974	0.0064	0.7256
26	1.201312	0.030627	0.0062	0.7318
27	1.165444	0.035868	0.0060	0.7379
28	1.150470	0.014974	0.0059	0.7438
29	1.123239	0.027231	0.0058	0.7496
30	1.092449	0.030791	0.0056	0.7552
31	1.082972	0.009477	0.0056	0.7608
32	1.072665	0.010307	0.0055	0.7664
33	1.047302	0.025363	0.0054	0.7718
34	1.035216	0.012086	0.0053	0.7771
35	1.016144	0.019072	0.0052	0.7824
36	0.991082	0.025062	0.0051	0.7875
37	0.972842	0.018240	0.0050	0.7925
38	0.952197	0.020645	0.0049	0.7974

TABLE 8.4 (continued)

	Eigenvalue	Difference	Proportion	Cumulative
39	0.926816	0.025381	0.0048	0.8022
40	0.910199	0.016616	0.0047	0.8069
41	0.894664	0.015535	0.0046	0.8115
42	0.886801	0.007863	0.0046	0.8161
43	0.870345	0.016456	0.0045	0.8206
44	0.845111	0.025234	0.0044	0.8249
45	0.841306	0.003806	0.0043	0.8293
46	0.832258	0.009048	0.0043	0.8336
47	0.808469	0.023789	0.0042	0.8378
48	0.796029	0.012439	0.0041	0.8419
49	0.784534	0.011495	0.0040	0.8459
50	0.753073	0.031461	0.0039	0.8498
51	0.739185	0.013888	0.0038	0.8536
52	0.726613	0.012573	0.0038	0.8574
53	0.723240	0.003373	0.0037	0.8611
54	0.708337	0.014903	0.0037	0.8648
55	0.694315	0.014022	0.0036	0.8683
56	0.677439	0.016876	0.0035	0.8718
57	0.670334	0.007105	0.0035	0.8753
58	0.661900	0.008434	0.0034	0.8787
59	0.641744	0.020156	0.0033	0.8820
60	0.638307	0.003437	0.0033	0.8853
61	0.615809	0.022498	0.0032	0.8885
62	0.609194	0.006615	0.0031	0.8917
63	0.602176	0.007018	0.0031	0.8948
64	0.594891	0.007285	0.0031	0.8978
65	0.578942	0.015949	0.0030	0.9008
66	0.565149	0.013793	0.0029	0.9037
67	0.551833	0.013317	0.0028	0.9066
68	0.541050	0.010782	0.0028	0.9094
69	0.535639	0.005411	0.0028	0.9121
70	0.518114	0.017525	0.0027	0.9148
71	0.516826	0.001288	0.0027	0.9175
72	0.506182	0.010644	0.0026	0.9201
73	0.499752	0.006430	0.0026	0.9227
74	0.486409	0.013343	0.0025	0.9252
75	0.463726	0.022683	0.0024	0.9276
76	0.458684	0.005042	0.0024	0.9300
77	0.453086	0.005598	0.0023	0.9323
78	0.451220	0.001866	0.0023	0.9346

TABLE 8.4 (continued)

	Eigenvalue	Difference	Proportion	Cumulative
79	0.426544	0.024676	0.0022	0.9368
80	0.425054	0.001490	0.0022	0.9290
81	0.420439	0.004616	0.0022	0.9412
82	0.410876	0.009562	0.0021	0.9433
83	0.403137	0.007739	0.0021	0.9454
84	0.389133	0.014005	0.0020	0.9474
85	0.386230	0.002902	0.0020	0.9494
86	0.381234	0.004996	0.0020	0.9514
87	0.364810	0.016424	0.0019	0.9532
88	0.360329	0.004482	0.0019	0.9551
89	0.358323	0.002006	0.0018	0.9570
90	0.353464	0.004858	0.0018	0.9588
91	0.342226	0.011238	0.0018	0.9605
92	0.333555	0.008670	0.0017	0.9623
93	0.327083	0.006472	0.0017	0.9640
94	0.321400	0.005683	0.0017	0.9656
95	0.319558	0.001842	0.9673	0.9673
96	0.311608	0.007950	0.0016	0.9689
97	0.306423	0.005185	0.0016	0.9705
98	0.297924	0.008499	0.0015	0.9720
99	0.296722	0.001202	0.0015	0.9735
100	0.288890	0.007833	0.0015	0.9750
101	0.282435	0.006454	0.0015	0.9765
102	0.268305	0.014131	0.0014	0.9779
103	0.259221	0.009084	0.0013	0.9792
104	0.253366	0.005855	0.0013	0.9805
105	0.251005	0.002361	0.0013	0.9818
106	0.243493	0.007512	0.0013	0.9831
107	0.236999	0.006494	0.0012	0.9843
108	0.232676	0.004323	0.0012	0.9855
109	0.227665	0.005011	0.0012	0.9867
110	0.219547	0.008118	0.0011	0.9878
111	0.215086	0.004461	0.0011	0.9889
112	0.209412	0.005673	0.0011	0.9900
113	0.208321	0.001092	0.0011	0.9911
114	0.195541	0.012780	0.0010	0.9921
115	0.194102	0.001440	0.0010	0.9931
116	0.186879	0.007222	0.0010	0.9940
117	0.184252	0.002627	0.0010	0.9950
118	0.175307	0.008945	0.0009	0.9959

TABLE 8.4 (continued)

	Eigenvalue	Difference	Proportion	Cumulative
119	0.173288	0.002019	0.0009	0.9968
120	0.163965	0.009324	0.0008	0.9976
121	0.161720	0.002245	0.0008	0.9985
122	0.154622	0.007098	0.0008	0.9993
123	0.151780	0.002842	0.0008	1.0001
124	0.147150	0.004630	0.0008	1.0008
125	0.141050	0.006100	0.0007	1.0015
126	0.138756	0.002294	0.0007	1.0023
127	0.134588	0.004168	0.0007	1.0029
128	0.130385	0.004203	0.0007	1.0036
129	0.126500	0.003885	0.0007	1.0043
130	0.123072	0.003428	0.0006	1.0049
131	0.117125	0.005947	0.0006	1.0055
132	0.114046	0.003079	0.0006	1.0061
133	0.107197	0.006850	0.0006	1.0067
134	0.105612	0.001585	0.0005	1.0072
135	0.097050	0.008562	0.0005	1.0077
136	0.093225	0.003825	0.0005	1.0082
137	0.087767	0.005458	0.0005	1.0086
138	0.083844	0.003923	0.0004	1.0091
139	0.082186	0.001658	0.0004	1.0095
140	0.080992	0.001195	0.0004	1.0099
141	0.077010	0.003982	0.0004	1.0103
142	0.073329	0.003681	0.0004	1.0107
143	0.072954	0.000376	0.0004	1.0111
144	0.066771	0.006182	0.0003	1.0114
145	0.063769	0.003002	0.0003	1.0117
146	0.060982	0.002787	0.0003	1.0121
147	0.054770	0.006213	0.0003	1.0123
148	0.051678	0.003092	0.0003	1.0126
149	0.047829	0.003849	0.0002	1.0129
150	0.044472	0.003357	0.0002	1.0131
151	0.041545	0.002927	0.0002	1.0133
152	0.039737	0.001808	0.0002	1.0135
153	0.037936	0.001801	0.0002	1.0137
154	0.035345	0.002592	0.0002	1.0139
155	0.032165	0.003179	0.0002	1.0140
156	0.030284	0.001881	0.0002	1.0142
157	0.024440	0.005844	0.0001	1.0143
158	0.022568	0.001872	0.0001	1.0144

TABLE 8.4 (continued)

	Eigenvalue	Difference	Proportion	Cumulative
159	0.019932	0.002637	0.0001	1.0145
160	0.015662	0.004270	0.0001	1.0146
161	0.013568	0.002094	0.0001	1.0147
162	0.008425	0.005143	0.0000	1.0147
163	0.004597	0.003829	0.0000	1.0148
164	0.003763	0.000833	0.0000	1.0148
165	0.002925	0.000838	0.0000	1.0148
166	-0.002359	0.005284	-0.0000	1.0148
167	-0.003730	0.001371	-0.0000	1.0148
168	-0.006264	0.002533	-0.0000	1.0147
169	-0.008629	0.002366	-0.0000	1.0147
170	-0.010224	0.001595	-0.0001	1.0146
171	-0.014535	0.004310	-0.0001	1.0146
172	-0.019504	0.003969	-0.0001	1.0145
173	-0.021307	0.001803	-0.0001	1.0144
174	-0.023111	0.001805	-0.0001	1.0142
175	-0.024509	0.001397	-0.0001	1.0141
176	-0.026315	0.001806	-0.0001	1.0140
177	-0.028275	0.001960	-0.0001	1.0138
178	-0.031284	0.003009	-0.0002	1.0137
179	-0.033149	0.001864	-0.0002	1.0135
180	-0.034608	0.001459	-0.0002	1.0133
181	-0.038238	0.003631	-0.0002	1.0131
182	-0.041689	0.003451	-0.0002	1.0129
183	-0.043456	0.001766	-0.0002	1.0127
184	-0.044715	0.001259	-0.0002	1.0124
185	-0.046425	0.001710	-0.0002	1.0122
186	-0.046802	0.000378	-0.0002	1.0120
187	-0.050017	0.003214	-0.0003	1.0117
188	-0.053152	0.003135	-0.0003	1.0114
189	-0.055763	0.002611	-0.0003	1.0111
190	-0.058489	0.002726	-0.0003	1.0108
191	-0.059843	0.001353	-0.0003	1.0105
192	-0.061908	0.002066	-0.0003	1.0102
193	-0.062389	0.000480	-0.0003	1.0099
194	-0.063713	0.001324	-0.0003	1.0096
195	-0.066515	0.002802	-0.0003	1.0092
196	-0.067269	0.000754	-0.0003	1.0089
197	-0.068317	0.001047	-0.0004	1.0085
198	-0.068723	0.000406	-0.0004	1.0082

TABLE 8.4 (continued)

199	-0.070114	0.001391	-0.0004	1.0078
200	-0.072717	0.002603	-0.0004	1.0074
201	-0.072941	0.000224	-0.0004	1.0071
202	-0.074992	0.002051	-0.0004	1.0067
203	-0.076326	0.001334	-0.0004	1.0063
204	-0.077211	0.000885	-0.0004	1.0059
205	-0.079227	0.002016	-0.0004	1.0055
206	-0.081411	0.002184	-0.0004	1.0050
207	-0.082586	0.001175	-0.0004	1.0046
208	-0.082972	0.000386	-0.0004	1.0042
209	-0.083934	0.000962	-0.0004	1.0038
210	-0.084924	0.000991	-0.0004	1.0033
211	-0.087368	0.002444	-0.0005	1.0029
212	-0.087852	0.000484	-0.0005	1.0024
213	-0.088677	0.000825	-0.0005	1.0020
214	-0.091583	0.002906	-0.0005	1.0015
215	-0.093670	0.002087	-0.0005	1.0010
216	-0.095187	0.001517	-0.0005	1.0005
217	-0.097884	0.002697	-0.0005	1.0000

8.5 Maximum-Likelihood Factor Analysis

Based on the analysis of the previous section (in which at most five factors were found) the monthly returns of the 217 securities were subjected to maximum-likelihood factor analysis to determine the number and factor loadings of the common factors; the results are summarized below:

The value of the SBC measure for five factors is at a minimum, which is consistent with the results of the principal factor analysis that not more than five factors should determine the US security returns. Therefore, we regard the five factor models as dominant and analyze the results for this case. Although the value of the AIC measure for six factors is at a minimum, the choice should be based on the SBC measure as it seems to be less inclined to

TABLE 8.5

DIFFERENT CRITERIA FOR ESTIMATING THE BEST NUMBER OF PARAMETERS TO INCLUDE IN A MODEL

Number of factors	Schwarz's Bayesian criterion (SBC)	Akaike's information criterion (AIC)	Tucker and Lewis's reliability coefficient (T&L)
2	25,105.61	47,830.29	0.67
3	24,617.19	46,065.92	0.71
4	24,109.66	44,266.99	0.75
5	23,980.21	43,227.87	0.78
6	24,072.66	42,636.24	0.80

include trivial factors than the AIC measure (Schwarz,1978). The Tucker & Lewis's reliability coefficient which represents the ratio of explained covariation to total variation for the five factor model and six factor model are 0.78 and 0.80 respectively. They both indicate that there is a good fit between observed and reproduced matrices.

8.5.1 Factor Patterns

Table 8.6 contains the factor pattern for the five significant factors and shows that the highest factor loading is 0.7691 and the lowest factor loading is 0.4119 for the first factor. For the second factor, 29% of the stocks have negative loadings, while 71% have positive loadings. For the third factor, 56% of the stocks have negative loadings, while 44% of the stocks have positive loadings. For the fourth factor, 42% of the stocks have negative loadings, while 58% of the stocks have positive loadings, and for the fifth factor, 89% of the stocks have negative loadings, while 11% of the stocks have positive loadings.

TABLE 8.6

UNROTATED FACTOR PATTERN

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5
CO#313	0.76913	0.08955	-0.10614	0.02666	0.00569
CO#316	0.76622	-0.04307	0.05388	0.09341	0.00656
CO#124	0.73672	0.15247	-0.07211	-0.08351	-0.11012
CO#161	0.73353	0.03565	-0.05286	0.08807	0.17294
CO#315	0.72553	-0.09877	-0.00192	0.19152	0.06585
CO#131	0.71875	0.04954	-0.08219	0.06790	-0.27515
CO#121	0.70600	-0.26162	-0.00668	0.08783	0.03481
CO#100	0.69857	0.07103	-0.03505	0.27761	-0.12041
CO#174	0.69798	0.21289	0.10225	0.10511	-0.36081
CO#93	0.69023	0.24730	-0.01842	-0.01246	-0.31938
CO#312	0.68744	0.17087	-0.03303	0.08260	0.09226
CO#104	0.68659	0.13657	0.02157	0.12633	0.03769
CO#158	0.68274	-0.15428	-0.01651	0.01511	0.00058
CO#30	0.68246	0.22773	0.02643	0.23296	0.24762
CO#341	0.68016	0.15299	-0.01140	0.03647	-0.30080
CO#209	0.67919	0.15698	-0.18784	-0.26710	-0.09367
CO#241	0.67758	0.07874	-0.03141	0.15846	-0.19492
CO#146	0.67670	0.20974	-0.02520	0.12381	0.12640
CO#343	0.67625	0.06399	-0.19620	0.00381	-0.17466
CO#92	0.67040	0.01602	-0.09563	0.08715	0.05061
CO#39	0.66996	0.23780	-0.14466	-0.16006	-0.00697
CO#19	0.66487	0.30609	-0.15895	-0.11858	0.00477
CO#253	0.66358	0.30477	-0.06631	-0.01914	0.15863
CO#129	0.66202	0.19999	0.02038	0.11875	-0.01814
CO#192	0.66185	0.10624	-0.02246	0.01347	0.13507
CO#317	0.66065	0.06195	-0.01390	0.27908	-0.09088
CO#127	0.66027	0.06220	-0.11494	0.24904	-0.16669
CO#322	0.66003	0.19906	-0.01568	0.14730	-0.31106
CO#304	0.65694	0.20792	-0.02808	0.01038	0.13651
CO#221	0.65659	0.08146	-0.06557	0.01155	-0.03018
CO#135	0.65508	0.13256	0.06099	0.16751	-0.17309
CO#247	0.65374	0.00542	-0.33890	-0.01501	-0.07632
CO#335	0.65126	0.03090	-0.17013	-0.36293	0.02251
CO#333	0.65107	0.07474	-0.15436	0.11081	0.02940
CO#339	0.65099	0.14446	-0.00878	0.08397	0.05818
CO#95	0.64783	0.12666	0.05507	0.09993	-0.33019
CO#44	0.64749	0.09163	-0.21875	-0.36887	0.03200
CO#280	0.64745	0.11611	-0.05338	0.07871	-0.17841

TABLE 8.6 (continued)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5
CO#302	0.64569	0.15406	0.10952	0.10148	0.20050
CO#155	0.64446	0.16558	-0.14658	0.17874	0.16397
CO#350	0.64396	0.22690	-0.20068	-0.02888	0.07329
CO#299	0.64392	0.08966	-0.00676	0.11983	-0.19409
CO#23	0.64224	0.09133	-0.11174	0.00845	-0.15406
CO#79	0.64057	0.21013	-0.17085	0.06722	0.04327
CO#157	0.64033	0.18024	-0.13226	0.20464	0.14576
CO#70	0.64017	0.01435	-0.31702	-0.27759	-0.04183
CO#203	0.63999	0.08551	-0.16534	0.08807	0.13499
CO#58	0.63998	0.25707	-0.02018	0.00984	-0.19011
CO#120	0.63827	0.06469	0.14151	-0.01785	-0.15012
CO#212	0.63768	0.14312	0.01441	0.09619	-0.28944
CO#347	0.63709	0.17036	0.04790	0.17079	-0.06596
CO#283	0.63516	0.13316	-0.12677	0.06356	0.01430
CO#307	0.63424	0.10651	0.39658	-0.09995	0.03792
CO#47	0.63338	0.11118	-0.17243	0.06027	0.09433
CO#203	0.63318	-0.00883	-0.01542	0.07677	0.03315
CO#167	0.63302	0.29354	0.12400	-0.06887	-0.23428
CO#6	0.63300	0.24335	-0.07790	0.09395	-0.00819
CO#236	0.63060	0.16038	0.06931	0.17154	-0.09144
CO#252	0.62953	0.00815	-0.23002	-0.30583	-0.11601
CO#14	0.62939	0.16051	0.05074	-0.04441	-0.06313
CO#327	0.62884	-0.28654	0.01408	-0.00728	0.09825
CO#18	0.62853	0.22909	0.07995	0.15671	0.17397
CO#205	0.62817	0.16902	0.02060	0.18021	-0.16036
CO#114	0.62720	-0.22104	0.03990	0.05290	-0.05232
CO#153	0.62673	0.35797	-0.13290	-0.06139	0.09251
CO#351	0.62636	0.23229	-0.10122	0.20768	0.05540
CO#13	0.61855	-0.12892	-0.10562	-0.02342	-0.08487
CO#137	0.61849	0.15827	0.20292	0.06672	-0.11545
CO#35	0.61643	0.07563	-0.12780	-0.19558	0.14053
CO#326	0.61620	0.24532	0.00762	0.10093	0.21580
CO#314	0.61614	0.23072	0.10438	0.20525	-0.08025
CO#169	0.61522	0.01940	-0.03840	0.07291	0.02271
CO#180	0.61475	0.06130	-0.27142	-0.12274	-0.01159
CO#171	0.61396	0.24910	-0.14075	-0.13818	0.07041
CO#152	0.61182	0.17868	0.00912	-0.04134	-0.29388
CO#217	0.60878	0.27466	-0.16654	0.15901	0.08133
CO#91	0.60856	0.10586	-0.23228	-0.06885	0.00962
CO#43	0.60851	0.14225	-0.06545	-0.05171	-0.14369

TABLE 8.6 (continued)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5
CO#261	0.60837	0.10919	-0.03549	0.08572	0.07202
CO#263	0.60682	0.07518	-0.16486	-0.12628	0.08859
CO#154	0.60666	0.10050	-0.03906	0.15634	0.14643
CO#143	0.60641	0.18965	-0.01290	0.11005	0.21547
CO#15	0.60610	-0.59752	0.04832	0.06682	0.00073
CO#36	0.60570	-0.21505	0.09415	0.05523	-0.00079
CO#219	0.60507	0.11539	0.35026	0.09191	0.04794
CO#172	0.60408	0.08897	-0.22352	-0.28606	0.06819
CO#34	0.60396	0.13312	-0.22299	-0.33498	0.05651
CO#49	0.60300	-0.05881	0.01705	-0.07359	-0.12980
CO#288	0.60296	-0.54103	0.02472	-0.06896	0.02593
CO#332	0.60208	0.24474	-0.03371	0.11378	0.22272
CO#287	0.60193	-0.10962	-0.12697	-0.10029	0.02264
CO#78	0.60019	0.25447	-0.08227	0.08886	0.25283
CO#276	0.59869	0.23499	0.06661	0.30402	-0.21351
CO#28	0.59850	-0.49653	-0.01218	0.07931	-0.06422
CO#41	0.59818	-0.12895	-0.02084	0.03105	-0.10495
CO#62	0.59714	0.14579	0.03441	0.08372	-0.04110
CO#265	0.59708	-0.03627	-0.22583	-0.31158	-0.11078
CO#46	0.59647	0.18346	-0.12103	0.14161	0.08337
CO#215	0.59382	0.36519	-0.11827	0.06543	0.05333
CO#296	0.59361	-0.55849	0.02468	-0.01599	-0.00215
CO#150	0.59321	0.03246	-0.09286	-0.23456	-0.10935
CO#99	0.59174	0.17108	-0.31834	-0.18792	-0.12004
CO#3	0.59154	0.07177	-0.18313	-0.34636	-0.05260
CO#117	0.59121	0.04154	-0.11640	0.17616	-0.16821
CO#125	0.59038	0.31877	-0.07008	0.17465	0.28455
CO#87	0.59019	0.17543	-0.17721	0.01811	0.03373
CO#1	0.58946	0.12944	-0.16225	0.16911	0.03781
CO#231	0.58935	0.22277	0.15384	0.18949	-0.07347
CO#74	0.58875	-0.51906	0.04161	-0.12914	-0.07066
CO#255	0.58670	0.09645	-0.06815	-0.49613	0.03492
CO#228	0.58593	0.16719	0.09012	0.09206	0.06784
CO#291	0.58587	0.15030	-0.03376	0.02485	-0.20515
CO#16	0.58500	0.08079	-0.20817	-0.45913	0.05595
CO#325	0.58034	0.08362	-0.01676	0.10359	0.00057
CO#73	0.57995	0.17760	0.09903	0.00071	0.07358
CO#75	0.57986	0.07364	0.01384	-0.03108	0.18614
CO#177	0.57844	-0.47035	-0.00343	-0.00641	-0.01258
CO#214	0.57834	0.12794	0.03613	0.01209	0.13184

TABLE 8.6 (continued)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5
CO#186	0.57682	0.02419	-0.04171	-0.03778	-0.28493
CO#225	0.57599	-0.50573	-0.08091	0.05459	0.01267
CO#202	0.57356	0.13771	-0.05443	0.07147	0.25240
CO#82	0.57349	0.23530	-0.06853	0.25999	-0.02700
CO#122	0.57235	-0.00019	-0.00607	0.18974	0.02331
CO#40	0.57203	0.17726	-0.04097	0.07257	0.30565
CO#279	0.57203	0.18410	0.05458	0.10393	0.11352
CO#52	0.57085	0.23884	0.13057	0.00231	-0.01716
CO#310	0.56929	0.40509	-0.11145	-0.01686	0.00738
CO#275	0.56806	0.29126	-0.03126	0.03919	0.31573
CO#113	0.56797	0.11535	0.07545	0.23536	-0.12460
CO#224	0.56687	-0.54358	-0.03255	0.03474	0.05638
CO#268	0.56587	-0.43835	-0.06458	0.14108	-0.01660
CO#258	0.56571	0.05979	-0.11480	-0.35793	-0.14850
CO#179	0.56562	0.13092	-0.20172	-0.38823	0.04510
CO#274	0.56460	-0.03230	-0.11412	-0.15101	0.00573
CO#182	0.56448	-0.44008	0.01634	0.05776	0.01175
CO#94	0.56148	0.30408	0.36915	-0.09823	-0.00119
CO#140	0.56137	-0.14646	0.02216	-0.08321	0.00685
CO#56	0.56042	0.02102	-0.07476	-0.02300	0.06983
CO#80	0.55964	0.23973	0.08914	0.22813	0.02584
CO#109	0.55943	-0.53362	-0.03350	-0.04623	0.12893
CO#306	0.55770	0.31822	-0.15876	0.06522	0.23946
CO#147	0.55769	0.14995	0.04467	0.19017	-0.03084
CO#197	0.55715	0.18478	-0.02032	0.03968	0.13031
CO#67	0.55629	-0.54117	-0.05131	0.08188	-0.03208
CO#190	0.55507	0.09006	-0.04186	0.32509	0.27968
CO#324	0.55488	0.33818	-0.16112	-0.12112	0.08756
CO#107	0.55306	-0.04312	0.43295	-0.19358	-0.10413
CO#262	0.55240	0.16562	0.18231	0.02569	0.05001
CO#26	0.55053	0.02210	0.23839	0.09884	0.05345
CO#159	0.54880	-0.48637	0.07218	-0.16939	0.02908
CO#148	0.54688	0.07918	-0.11859	0.12920	0.09636
CO#85	0.54479	0.23298	0.09939	-0.10883	-0.13283
CO#320	0.53966	0.00217	0.00770	-0.00353	-0.08619
CO#330	0.53963	-0.47570	0.03630	-0.04184	-0.03012
CO#334	0.53811	0.13770	-0.04467	0.13884	0.27359
CO#243	0.53779	-0.36976	0.07219	0.10316	0.04198
CO#207	0.53757	0.15372	-0.24182	-0.46820	-0.01542
CO#234	0.53728	-0.51300	0.04440	0.07204	-0.04630

TABLE 8.6 (continued)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5
CO#269	0.53298	-0.12071	-0.13125	-0.13544	0.02818
CO#246	0.53176	-0.08771	0.37809	-0.14365	-0.01971
CO#90	0.53058	0.05532	-0.06069	0.03768	0.11102
CO#10	0.52741	0.24887	0.09869	0.28893	-0.28128
CO#266	0.52713	-0.45558	0.15134	-0.00869	0.01248
CO#289	0.52335	0.16744	-0.24600	-0.38679	0.02018
CO#298	0.52291	-0.51003	0.04513	-0.07905	0.05319
CO#145	0.51944	0.29762	0.39140	-0.28311	0.01061
CO#297	0.51893	-0.46269	-0.04511	0.09681	0.05154
CO#337	0.51537	-0.39239	0.00149	0.06750	-0.03416
CO#162	0.51442	-0.49406	0.04665	0.00309	-0.10902
CO#344	0.51188	-0.15121	-0.05924	0.03573	-0.01786
CO#216	0.51129	0.14194	0.47156	-0.12586	0.17743
CO#51	0.51006	-0.15021	-0.09319	-0.06434	-0.07136
CO#282	0.50849	0.10300	0.01982	-0.06126	0.13119
CO#11	0.50749	0.26252	0.23857	0.12835	-0.14638
CO#226	0.50747	0.22786	0.24165	0.10648	-0.13243
CO#77	0.50657	-0.22273	0.26447	0.08660	0.06354
CO#101	0.50612	0.13851	-0.01531	-0.07375	-0.07220
CO#7	0.50430	0.30061	0.17068	0.16066	-0.24945
CO#311	0.50380	-0.50379	0.05526	-0.23623	0.08207
CO#57	0.50337	0.14440	0.15123	-0.00021	-0.01967
CO#309	0.50047	0.02387	0.41136	-0.17648	0.02690
CO#72	0.49964	-0.11568	0.18616	-0.01145	0.02014
CO#187	0.49651	-0.08941	0.00065	0.06591	0.01194
CO#230	0.49517	0.12637	0.12856	-0.04775	0.28243
CO#21	0.49047	0.07849	0.16239	-0.08749	-0.00895
CO#308	0.48925	0.02931	0.48844	0.00337	0.03416
CO#295	0.48576	0.05637	0.32565	-0.05212	0.11791
CO#151	0.48507	0.23549	0.42238	-0.16710	0.16484
CO#33	0.47714	0.12868	0.01790	0.03741	0.21582
CO#185	0.46945	0.20941	0.34464	-0.35946	0.07971
CO#349	0.46896	0.04940	-0.12966	-0.03035	-0.00244
CO#290	0.46587	0.31787	0.35390	-0.36731	0.03249
CO#110	0.46386	0.22761	-0.02066	0.12362	0.36320
CO#259	0.46266	0.19584	0.45426	-0.33361	-0.05411
CO#132	0.44375	-0.05092	0.15578	-0.01191	0.00969
CO#278	0.43072	-0.16574	0.03018	0.00475	0.11300
CO#142	0.42611	0.13015	-0.01978	0.09310	0.18140
CO#329	0.42513	0.20141	-0.11438	-0.34295	-0.01269

TABLE 8.6 (continued)

CO#173	0.41801	0.25036	0.17507	-0.06588	0.01668
CO#42	0.50008	-0.50700	0.02124	0.14844	0.04106
CO#345	0.45641	-0.52100	0.08200	-0.20445	-0.08911
CO#164	0.52344	-0.52684	0.05017	-0.03799	0.00711
CO#346	0.48284	-0.53600	0.02058	0.03197	-0.00016
CO#115	0.53772	-0.53956	0.00344	0.00097	0.13424
CO#184	0.53513	-0.54753	-0.03596	0.15369	-0.02637
CO#248	0.53727	-0.54918	-0.01838	0.02690	-0.03539
CO#284	0.56024	-0.56228	0.09630	-0.07176	0.09301
CO#60	0.53861	-0.56359	-0.02803	0.12871	-0.08124
CO#244	0.51914	-0.56704	0.06695	-0.09535	-0.01374
CO#53	0.51135	-0.57085	-0.01252	0.10358	0.17661
CO#196	0.49600	-0.57385	0.04026	0.02791	-0.00653
CO#9	0.55524	-0.59429	0.02315	-0.04936	0.01357
CO#32	0.50981	-0.60406	-0.02903	-0.00899	0.04182
CO#20	0.41186	0.09810	0.59313	-0.19942	-0.02029
CO#328	0.43199	0.22116	0.56471	-0.14567	0.08507
CO#27	0.42999	0.13507	0.55708	-0.23195	0.01940
CO#12	0.44715	0.11261	0146852	0.00309	0.15323

CO# denotes the individual company.

A criterion is used for meaningful correlation (usually 0.30 or larger), the stocks with loadings in excess of the criterion are collected (refer to chapter 5).

8.5.2 Rotation of Factors

The next step in factor analysis involves finding simpler and more easily interpretable factors through rotation, while keeping the number of factors and communalities of each stock fixed.

As the APT explicitly assumes that the factors are uncorrelated, orthogonal rotation is used here. The variances explained by the five factors with and without weights are shown in Table 8.7. The quartimax rotation is probably the rotation of choice because it aims to

TABLE 8.7

VARIANCE EXPLAINED BY FACTORS ON DIFFERENT ROTATIONAL TECHNIQUES

		Factor 1	Factor 2	Factor 3	Factor 4	Factor 5
Unrotated	(Weighted)	152.63	35.22	12.79	11.69	7.73
	(Unweighted)	74.72	15.13	6.12	5.43	3.75
Quartimax	(Weighted)	142.53	44.14	13.29	12.28	7.81
	(Unweighted)	70.88	18.48	6.30	5.72	3.77
Varimax	(Weighted)	65.69	55.96	35.61	35.94	26.85
	(Unweighted)	28.66	28.63	17.49	17.26	13.11
Equamax	(Weighted)	60.26	41.53	39.84	42.00	36.43
	(Unweighted)	26.03	20.48	20.44	20.27	17.92

make the variables as simple as possible by maximizing the variance of the loadings on each factor in order to achieve the simple structure. The quartimax rotation shows that the first factor is still the dominant factor. The squared multiple correlations of the stocks with factor 1, factor 2, factor 3, factor 4, and factor 5 are 0.99, 0.97, 0.93, 0.92 and 0.89 respectively, which implies that the five factors are internally consistent and well defined by the stocks.

The results in Table 8.8 show that the highest factor loading is 0.7686 and the lowest factor loading is 0.2724 for the first factor. The coefficients of the first factor are positive and relatively large, indicating an important general factor among the stocks. The first factor

TABLE 8.8

ROTATED FACTOR PATTERN (QUARTIMAX)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5
CO#313	0.76859	0.12022	-0.05194	0.06084	0.00229
CO#124	0.74173	0.05486	0.01792	0.14881	0.12106
CO#30	0.73687	-0.03128	0.01106	-0.18700	-0.23418
CO#316	0.72672	0.25476	0.05590	-0.06604	0.01102
CO#161	0.72465	0.16510	-0.02297	-0.02159	-0.16257
CO#174	0.72370	-0.00969	0.08836	-0.09117	0.37977
CO#93	0.72282	-0.04903	0.03000	0.06403	0.33176
CO#253	0.72216	-0.11051	0.02511	0.08197	-0.14857
CO#146	0.71829	-0.01706	-0.00330	-0.06540	-0.11548
CO#19	0.71815	-0.11588	-0.03053	0.21214	-0.00004
CO#312	0.71446	0.02345	0.00069	-0.02403	-0.08129
CO#100	0.71147	0.11557	-0.10034	-0.19953	0.12811
CO#131	0.70752	0.14284	-0.07211	0.01415	0.28239
CO#104	0.70423	0.05794	0.02646	-0.08558	-0.02383
CO#153	0.70042	-0.17530	-0.02108	0.14683	-0.08719
CO#39	0.70003	-0.04720	-0.00766	0.24391	0.01314
CO#129	0.69651	-0.00998	0.02857	-0.07787	0.03152
CO#322	0.69440	-0.01490	-0.03575	-0.08570	0.32083
CO#341	0.69173	0.03795	0.00961	0.01436	0.31265
CO#304	0.68960	-0.01807	0.03979	0.03833	-0.12466
CO#157	0.68847	-0.00735	-0.13604	-0.09933	-0.14390
CO#350	0.68753	-0.05005	-0.10727	0.14351	-0.07299
CO#155	0.68732	0.00776	-0.13868	-0.07014	-0.16264
CO#351	0.68642	-0.05997	-0.11146	-0.11329	-0.05187
CO#79	0.68579	-0.03580	-0.12159	0.04367	-0.04240
CO#315	0.68489	0.29144	-0.03675	-0.13738	-0.05413
CO#6	0.68394	-0.06497	-0.04852	-0.01711	0.01479
CO#241	0.68210	0.10496	-0.05593	-0.09194	0.20386
CO#58	0.68137	-0.07196	0.02627	0.03994	0.20143
CO#217	0.68095	-0.10808	-0.14744	-0.04344	-0.08177
CO#215	0.68022	-0.19391	-0.06165	0.02378	-0.04943
CO#209	0.67897	0.03214	-0.01735	0.34969	0.09811
CO#18	0.67515	-0.03291	0.08155	-0.14044	-0.15716
CO#125	0.67436	-0.14828	-0.04910	-0.09941	-0.27841
CO#127	0.67369	0.10882	-0.16817	-0.14309	0.16846
CO#343	0.67309	0.11255	-0.14524	0.11476	0.17447
CO#317	0.67212	0.11525	-0.08275	-0.21190	0.09930
CO#135	0.67117	0.05279	0.02948	-0.13785	0.18789

TABLE 8.8 (continued)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5
CO#339	0.67069	0.03985	0.01538	-0.03683	-0.04645
CO#167	0.66930	-0.09833	0.18825	0.05522	0.25623
CO#326	0.66763	-0.06463	0.04111	-0.06157	-0.20325
CO#347	0.66680	0.01153	0.02580	-0.13757	0.07983
CO#192	0.66665	0.08101	0.03321	0.03216	-0.12301
CO#302	0.66534	0.03717	0.13650	-0.10214	-0.18072
CO#314	0.66272	-0.04961	0.06671	-0.19175	0.09717
CO#333	0.66169	0.09680	-0.13580	-0.00389	-0.02812
CO#310	0.65955	-0.23685	-0.02427	0.09622	-0.00245
CO#92	0.65936	0.16283	-0.07550	-0.00570	-0.04473
CO#78	0.69525	-0.08280	-0.03367	-0.01594	-0.24641
CO#205	0.65879	0.00768	-0.00989	-0.13451	0.17183
CO#280	0.65810	0.06154	-0.04206	-0.01154	0.18644
CO#332	0.65742	-0.07081	-0.00203	-0.05773	-0.21340
CO#283	0.65657	0.03916	-0.08903	0.02849	-0.01075
CO#236	0.65639	0.02040	0.04199	-0.14693	0.10654
CO#276	0.65562	-0.06439	-0.01632	-0.26613	0.22589
CO#95	0.65492	0.05651	0.03893	-0.07194	0.34492
CO#221	0.65464	0.09937	-0.01909	0.05277	0.03876
CO#203	0.65425	0.08413	-0.12939	0.01939	-0.13408
CO#47	0.65232	0.05763	-0.12587	0.04821	-0.09371
CO#171	0.65228	-0.07324	-0.00984	0.21837	-0.06516
CO#212	0.65219	0.03582	0.00698	-0.05319	0.30141
CO#299	0.64824	0.08764	-0.01935	-0.06810	0.20455
CO#143	0.64505	-0.01549	0.01336	-0.06303	-0.20475
CO#23	0.64401	0.08219	-0.06842	0.07508	0.15897
CO#247	0.64336	0.15487	-0.26757	0.18551	0.06640
CO#46	0.64049	-0.02082	-0.10772	-0.04760	-0.08095
CO#14	0.63846	0.02439	0.11180	0.05766	0.07987
CO#82	0.63805	-0.07745	-0.11085	-0.17589	0.03079
CO#306	0.63787	-0.15993	-0.09200	0.03494	-0.23864
CO#275	0.63242	-0.12209	0.03786	0.00706	-0.30560
CO#231	0.63098	-0.04592	0.11609	-0.19870	0.09341
CO#137	0.62768	0.02974	0.20231	-0.10463	0.14067
CO#44	0.62587	0.08760	-0.00455	0.46081	-0.02876
CO#152	0.62569	-0.00236	0.05741	0.07394	0.30685
CO#154	0.62544	0.06672	-0.04106	-0.09523	-0.13946
CO#87	0.62421	-0.01565	-0.11484	0.08807	-0.03374
CO#1	0.62304	0.02542	-0.16432	-0.05683	-0.03892
CO#324	0.52270	-0.17654	-0.02977	0.20865	-0.08484

TABLE 8.8 (continued)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5
CO#261	0.62215	0.06025	-0.01437	-0.03077	-0.06303
CO#91	0.61824	0.05469	-0.13743	0.19000	-0.01207
CO#43	0.61808	0.02847	0.00047	0.11041	0.15199
CO#120	0.61694	0.12295	0.17028	-0.00246	0.17239
CO#62	0.61565	0.02553	0.04485	-0.05535	0.05442
CO#80	0.61544	-0.07458	0.04843	-0.21102	-0.01108
CO#158	0.61381	0.33576	0.00735	0.02773	0.01134
CO#121	0.61307	0.44416	-0.01786	-0.04343	-0.02287
CO#99	0.61256	-0.01587	-0.17324	0.33507	0.11282
CO#204	0.61145	0.18140	-0.00401	-0.03030	-0.02254
CO#335	0.61070	0.14963	0.03168	0.43549	-0.01615
CO#228	0.60999	0.00601	0.10095	-0.08702	-0.05075
CO#70	0.60991	0.15072	-0.14216	0.41602	0.03670
CO#180	0.60936	0.09787	-0.15686	0.25495	0.00718
CO#40	0.60850	-0.01323	0.00551	-0.02082	-0.29681
CO#307	0.60629	0.10104	0.45114	-0.02127	0.00320
CO#279	0.60491	-0.01618	0.06766	-0.08487	-0.09911
CO#52	0.60392	-0.06278	0.17256	-0.01967	0.03782
CO#35	0.60307	0.09649	0.01433	0.26254	-0.13380
CO#169	0.60270	0.14796	-0.02277	-0.01806	-0.01393
CO#291	0.60241	0.01405	-0.00564	0.02743	0.21400
CO#263	0.60088	0.08983	-0.05083	0.21408	-0.08561
CO#219	0.59969	0.07688	0.33324	-0.19000	-0.01302
CO#202	0.59909	0.02387	-0.01331	-0.01416	-0.24466
CO#73	0.59897	-0.00276	0.14589	-0.00723	-0.05476
CO#34	0.59892	0.03485	-0.01909	0.42935	-0.05472
CO#117	0.59590	0.11118	-0.14699	-0.07982	0.16942
CO#214	0.59320	0.03622	0.01787	0.03468	-0.12231
CO#172	0.59140	0.07605	-0.04184	0.38402	-0.06715
CO#94	0.59131	-0.11054	0.43557	-0.02262	0.03933
CO#252	0.58978	0.15920	-0.05809	0.40753	0.11673
CO#197	0.58978	-0.02385	0.02498	0.00277	-0.12037
CO#190	0.58962	0.05910	-0.10494	-0.25333	-0.27484
CO#113	0.58880	0.04473	0.01249	-0.21135	0.13783
CO#325	0.58782	0.07714	-0.01316	-0.05547	0.00866
CO#147	0.58761	0.00867	0.01101	-0.15910	0.04270
CO#10	0.58712	-0.09563	0.01104	-0.26786	0.29446
CO#75	0.57435	0.09328	0.07942	0.05285	-0.17261
CO#334	0.57063	0.01316	-0.03162	-0.08194	-0.26673
CO#85	0.56799	-0.06447	0.17781	0.09458	0.15202

TABLE 8.8 (continued)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5
CO#262	0.56655	0.00543	0.20758	-0.06463	-0.02651
CO#3	0.56563	0.09210	0.00810	0.42386	0.05658
CO#7	0.56472	-0.14373	0.13229	-0.18015	0.26895
CO#122	0.56446	0.15394	-0.04351	-0.14047	-0.01478
CO#148	0.56375	0.06594	-0.11185	-0.04151	-0.09479
CO#150	0.56021	0.13258	0.03919	0.28588	0.11776
CO#13	0.55960	0.28844	-0.06624	0.09656	0.08983
CO#179	0.55579	0.02889	0.01788	0.46765	-0.04196
CO#16	0.55527	0.08376	0.03742	0.53533	-0.05200
CO#186	0.55350	0.13284	-0.00528	0.08681	0.29340
CO#11	0.55252	-0.10053	0.21075	-0.17943	0.17115
CO#49	0.55210	0.22540	0.06729	0.09430	0.14346
CO#255	0.55002	0.07865	0.17953	0.51396	-0.02099
CO#265	0.54604	0.19322	-0.05751	0.40869	0.11113
CO#56	0.54528	0.13175	-0.01818	0.08062	-0.06321
CO#287	0.54520	0.26677	-0.04745	0.17330	-0.01812
CO#114	0.54185	0.38648	0.03105	-0.03255	0.06617
CO#226	0.54129	-0.06647	0.22008	-0.16131	0.15765
CO#41	0.53976	0.28654	-0.01304	0.01207	0.11447
CO#145	0.53264	-0.10994	0.52616	0.13518	0.03067
CO#258	0.53150	0.10017	0.06570	0.40696	0.15646
CO#90	0.53100	0.09027	-0.02523	0.01775	-0.10466
CO#26	0.52899	0.14414	0.21753	-0.15605	-0.02747
CO#289	0.52742	-0.02081	-0.02412	0.48253	-0.02088
CO#274	0.52521	0.18423	-0.01252	0.21386	-0.00029
CO#327	0.52422	0.45080	0.03587	0.02994	-0.08508
CO#110	0.52268	-0.09104	0.00567	-0.08162	-0.35555
CO#36	0.52088	0.37847	0.08228	-0.05812	0.01800
CO#101	0.51552	0.00787	0.05311	0.10398	0.08233
CO#320	0.51510	0.14742	0.03568	0.03069	0.09728
CO#57	0.51234	0.01035	0.17984	-0.03053	0.04020
CO#282	0.51030	0.04604	0.09116	0.07538	-0.11835
CO#230	0.50132	0.02752	0.19647	0.01717	-0.26223
CO#33	0.49696	0.01082	0.05676	-0.01669	-0.20476
CO#140	0.48942	0.29950	0.07489	0.09572	0.00661
CO#21	0.47429	0.07289	0.21849	0.04316	0.03101
CO#269	0.47355	0.25894	-0.04234	0.20321	-0.02486
CO#349	0.46675	0.07539	-0.07336	0.10566	0.00350
CO#295	0.45970	0.10300	0.35977	-0.05662	-0.08508

TABLE 8.8 (continued)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5
CO#187	0.45667	0.22184	-0.00193	-0.03500	-0.00305
CO#344	0.45491	0.28215	-0.05111	0.01690	0.02318
CO#142	0.45479	-0.00872	-0.00478	-0.05491	-0.17473
CO#173	0.45306	-0.11148	0.23386	0.01639	0.00497
CO#246	0.45199	0.25828	0.42432	0.00867	0.05754
CO#51	0.44605	0.28066	-0.04640	0.12268	0.07553
CO#329	0.43862	-0.07382	0.07324	0.38552	0.01836
CO#72	0.43596	0.26134	0.19590	-0.03847	0.00235
CO#77	0.41819	0.36860	0.22275	-0.16112	-0.03704
CO#132	0.40150	0.18182	0.16992	-0.02794	0.00975
CO#278	0.36771	0.28090	0.04523	0.00319	-0.10252
CO#15	0.42104	0.74206	0.00277	-0.05567	0.01238
CO#9	0.36499	0.72630	0.02354	0.05780	-0.00160
CO#32	0.32556	0.71906	-0.04154	0.03867	-0.03476
CO#284	0.37374	0.70271	0.10751	0.04875	-0.07541
CO#296	0.41393	0.70176	0.01622	0.02944	0.01458
CO#53	0.38493	0.69803	-0.05607	-0.07010	-0.16868
CO#244	0.33113	0.69381	0.07981	0.08122	0.02855
CO#196	0.31913	0.68926	0.00555	-0.02240	0.01745
CO#288	0.42355	0.68916	0.04109	0.07827	-0.01250
CO#60	0.37385	0.68392	-0.09787	-0.08394	0.08689
CO#224	0.40067	0.67573	-0.05217	0.00377	-0.04883
CO#248	0.36802	0.67323	-0.04476	0.00488	0.04316
CO#115	0.37019	0.66747	-0.00222	0.01773	-0.12418
CO#67	0.39490	0.66745	-0.09427	-0.03121	0.03729
CO#184	0.37824	0.66688	-0.10996	-0.10438	0.03125
CO#109	0.39054	0.66665	-0.01584	0.07699	-0.12031
CO#74	0.40846	0.66594	0.07454	0.12747	0.08556
CO#164	0.35224	0.65417	0.04718	0.03590	0.00604
CO#346	0.31832	0.64807	-0.01104	-0.01856	0.00953
CO#234	0.37806	0.64122	-0.00290	-0.06082	0.05778
CO#298	0.35378	0.63899	0.06331	0.07506	-0.03969
CO#28	0.44457	0.63859	-0.05335	-0.04112	0.07297
CO#225	0.42335	0.63814	-0.10315	0.00650	-0.00851
CO#345	0.27242	0.63534	0.13140	0.17333	0.10505
CO#311	0.32377	0.63248	0.13565	0.21353	-0.06609
CO#159	0.37550	0.62721	0.12545	0.14902	-0.01205
CO#42	0.35311	0.62252	-0.04980	-0.12466	-0.03266
CO#162	0.35443	0.61810	0.02219	0.00127	0.12099
CO#177	0.42551	0.61102	-0.00711	0.03241	0.02285

TABLE 8.8 (continued)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5
CO#330	0.38148	0.60859	0.03896	0.04705	0.04272
CO#266	0.37210	0.59246	0.13462	-0.03015	0.00738
CO#297	0.38260	0.58255	-0.08421	-0.04947	-0.04645
CO#182	0.42506	0.57796	-0.01086	-0.03489	-0.00113
CO#268	0.43792	0.56939	-0.11901	-0.07840	0.02060
CO#337	0.39195	0.51711	-0.03021	-0.03927	0.04266
CO#243	0.41981	0.50572	0.02873	-0.09962	-0.02847
CO#20	0.37111	0.06161	0.65093	-0.03000	0.07145
CO#27	0.39831	0.03010	0.63793	0.01509	0.03029
CO#328	0.43143	-0.05302	0.62309	-0.06654	-0.03560
CO#259	0.44202	-0.02349	0.58648	0.15276	0.09878
CO#216	0.49400	0.03937	0.53473	-0.04558	-0.13289
CO#290	0.48214	-0.14424	0.52507	0.22426	0.00640
CO#151	0.49329	-0.05967	0.51183	0.01180	-0.12343
CO#185	0.45802	-0.03969	0.50754	0.21883	-0.04161
CO#107	0.47599	0.22541	0.49333	0.03543	0.14664
CO#309	0.44840	0.14589	0.47853	0.02463	0.01336
CO#309	0.45021	0.13792	0.47836	-0.17134	0.00875
CO#12	0.43535	0.04612	0.47317	-0.16594	-0.11200
CO#207	0.52967	-0.00172	0.00892	0.55646	0.01624

has impacts on all security returns. For the other four factors, some of the stocks have negative loadings on these factors, while some of the stocks have positive loadings. Those four factors retain the mixture of signs in the loadings of the stocks, indicating that the stocks have different reactions to those factors. The absolute factor loadings on the other factors are smaller than that of the first factor. For example, only forty-one stocks have loadings in excess of 0.30 (in absolute terms), the second factor is minor (i.e. important only for those 41 stocks).

It is interesting to see how different industrial groupings correspond to the other four factors. A summary of the results appears in Table 8.9. For example, the company in the wholesale trade group, 100% of the retail trade, 95% of the transportation and public

TABLE 8.9

DISTRIBUTION OF FACTOR LOADINGS

Distribution of Loadings on Factor 2

Industry Classification	Positive Number	Positive Percentage	Negative Number	Negative Percentage
Mining	2	22	7	78
Manufacturing	86	63	51	37
Transportation & Public Utilities	41	95	2	5
Wholesale Trade	1	100	0	0
Retail Trade	5	100	0	0
Finance, Insurance & Real Estate	13	87	2	13
Services	6	86	1	14
	154	71	63	29

Distribution of Loadings on Factor 3

Industry Classification	Positive Number	Positive Percentage	Negative Number	Negative Percentage
Mining	9	100	0	9
Manufacturing	70	51	67	49
Transportation & Public Utilities	23	53	20	47
Wholesale Trade	0	0	1	100
Retail Trade	0	0	5	100
Finance, Insurance & Real Estate	12	80	3	86
Services	1	14	6	86
	115	53	102	47

TABLE 8.9 (continued)

Distribution of Loadings on Factor 4

Industry Classification	Positive Number	Positive Percentage	Negative Number	Negative Percentage
Mining	3	33	6	67
Manufacturing	72	53	65	47
Transportation & Public Utilities	24	56	19	44
Wholesale Trade	0	0	1	100
Retail Trade	3	60	2	40
Finance, Insurance & Real Estate	4	27	11	73
Services	4	57	3	43
	110	51	107	49

Distribution of Loadings on Factor 5

Industry Classification	Positive Number	Positive Percentage	Negative Number	Negative Percentage
Mining	5	56	4	44
Manufacturing	72	53	65	47
Transportation & Public Utilities	22	51	21	49
Wholesale Trade	0	0	1	100
Retail Trade	3	60	2	40
Finance, Insurance & Real Estate	7	47	8	53
Services	1	14	6	86
	110	51	107	49

utilities, 87% of the finance, insurance, and real estate group, 85% of the services, 63% of the manufacturing are positively related to the second factor. While 78% of the mining group are negatively related to the second factor. The results show that individual companies vary widely in their sensitivities to the economic factors. Even within the same industry, different companies have quite different patterns of sensitivities.

8.6 Risk Measures and Average Returns

In this section, the individual-security factor loadings are used to explain the cross-sectional variation of individual estimated expected returns. The APT will be supported if the actual returns depend on estimated factor loadings (i.e. factor beta coefficients of the security returns generating model).

The general approach developed for pricing tests is straightforward (e.g. Roll and Ross(1980)). The factor loadings (beta coefficients) are used as independent variables to explain the cross-sectional variation in the mean returns of all the securities which comprise the sample. The mean returns are used as the proxy for the expected returns.

$$\bar{R}_i = \hat{\lambda}_0 + \hat{\lambda}_1 b_{i1} + \hat{\lambda}_2 b_{i2} + \hat{\lambda}_3 b_{i3} + \hat{\lambda}_4 b_{i4} + \hat{\lambda}_5 b_{i5} \text{ } (\lambda_0 \text{ estimated})$$

The regression results are shown in Tables 8.10 and 8.11. The regression results show that the APT explains 30% (in terms of adjusted R^2) of the variation in mean returns of the sample. This suggests that the explanatory power of the model is fairly good. The F value is used to test the null hypothesis that all parameters (i.e. $\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5$) are simultaneously zero. The calculated F statistic is greater than the theoretical F value at the five per cent level, indicating that the null hypothesis can be rejected. The explanatory power of the model will be the same whether the rotated or unrotated factor patterns are used as the independent variables in the regression analysis. Rotation cannot be used to improve the fit between the observed and reproduced correlation matrices because all orthogonally rotated solutions are mathematically equivalent to one another and to the solution before rotation.

During the sample period, January 1965 to December 1988, the risk-free coefficient,

TABLE 8.10

REGRESSION RESULTS USING UNROTATED FACTOR PATTERNS AS INDEPENDENT VARIABLES

Variable	Parameter Estimate	Standard Error	T for H_0: parameter=0	Prob > |T|
λ_0	0.01501	0.00209	7.192	0.0001
λ_1	-0.00539	0.00355	-1.516	0.1309
λ_2	0.00637	0.00078	8.214	0.0001
λ_3	-0.00215	0.00136	-1.586	0.1142
λ_4	-0.00088	0.00131	-0.673	0.5016
λ_5	0.00696	0.00156	4.466	0.0001
R^2	0.3162	F-value	19.512	
Adj R^2	0.3000	Prob>F	0.0001	

λ_0, was equivalent to 19.58% annually or 1.50% monthly as shown in Table 8.10. The intercept term is significantly greater than zero at the 5% level of significance. The positive intercept term is consistent with the APT model, as one testable implication of the APT is that the intercept term should be positive. Although it is often argued that this should equal the risk-free rate, the APT does not, in fact, require that the zero-beta equal the observed return on 30-day Treasury bill rates (refer to chapter 5).

The risk premium of the second rotated factor, λ_2, is -8.80% annually or -0.77% monthly during January 1965 to December 1988 as shown in Table 8.11. While the risk premium of the fifth rotated factor, λ_5, is -8.27% annually or -0.72% monthly. The price associated with an APT factor can be negative for hedging purposes. The negative price reflects that investors want to hold stocks whose returns increase when there is an

TABLE 8.11

REGRESSION RESULTS USING ROTATED FACTOR PATTERNS AS INDEPENDENT VARIABLES

Variable	Parameter Estimate	Standard Error	T for H_0: parameter=0	Prob > |T|
λ_0	0.01501	0.00209	7.192	0.0001
λ_1	-0.00328	0.00329	-0.998	0.3195
λ_2	-0.00765	0.00143	-5.345	0.0001
λ_3	-0.00091	0.00155	-1.588	0.5571
λ_4	0.00136	0.00126	1.079	0.2816
λ_5	-0.00717	0.00153	-4.691	0.0001
R^2	0.3162	F-value	19.512	
Adj R^2	0.3000	Prob>F	0.0001	

unanticipated negative realization of that factor (and whose returns decrease when there is an unanticipated positive realization).

The results of this standard testing approach show that there are five factors in the US stock market, but that only two factors and the risk-free coefficient (λ_0) are important for pricing.

8.7 Discussion

The above sections estimate the number of the US stock market factors using principal and maximum-likelihood methods of factor analysis. The results show that there are five stock market factors in the US. It has been shown by principal factor analysis that the first factor accounts for nearly 39% of the proportion of the total explained variation while the second factor accounts for nearly 8%. By maximum-likelihood factor analysis, the results confirmed the earlier findings by principal factor analysis that the first factor is an important factor among the stocks. The coefficients of the first factor are all positive and statistically significant. The absolute factor loadings on the other factors are smaller than that of the first factor. For example, only forty-one stocks have loadings in excess of 0.30 (in absolute terms), therefore the second factor is minor (i.e. important only for those 41 stocks).

It is interesting to note that the UK results in chapter 5 which show that there are only two UK stock market factors. The first UK stock market factor is also a dominant one and it accounts for nearly 74% of proportion of the total variation in the UK stock market returns. By comparison, the first US stock market factor is less important than the first UK stock market factor in determining the security returns in the domestic market.

The validity and applicability of the APT to the US stock market are also evaluated. One of the important implications of the APT is that the intercept term (λ_0) should be significantly different from zero. The APT further implies that if k factors are responsible for driving the individual asset returns through time, then there should be a risk premium attached to each of these factors.

The individual-security factor loading estimates were then used as independent variables to explain the cross-sectional variation in the mean returns of the securities that comprise the sample. The mean returns (as the proxy for the expected returns) for securities were regressed against the factor loadings. The second and the fifth rotated factors and the risk-free coefficents are priced. It is clear from the cross-sectional regression results that the APT has some empiricial power (in terms of adjusted R^2). The APT explains 30% of the variation in the twenty-four years average returns as compared with only 11% of that of the UK results in chapter 5. Kim and Wu (1987) showed that the APT explains from 26% to

29% of the variation in returns of 464 US stocks (monthly data) for 1973-1979 and 1980-1985 respectively. Shukla and Trzcinka (1990) showed that the APT explains 40% of the variation in mean returns of 865 US companies (weekly data for twenty years). The results of the two studies are comparable to the results of this chapter. The result of this chapter is quite encouraging as modelling twenty-four years returns is a difficult task because there is a high variation in the measures of risk and return when long time periods are used. In testing the APT, the return distribution is assumed to be stationary over time so that measures of systematic risk can be estimated from a correlation matrix based on, in this case, twenty-four years of data. In this chapter, it has been assumed that the non-stationarity problem does not exist. Thus, risk and expected returns were assumed not to have changed during the twenty-four years period. By taking no measures to mitigate the non-stationarity problem, these tests are biased toward finding that the risk measures are not significant.

Overall, the results obtained in this chapter show that the APT pricing relationship is supported by the testing methodology.

8.8 Summary

This chapter estimates the number of the US stock market factors using principal factor and maximum-likelihood methods of factor analysis. The results show that there are five stock market factors in the US. It has been shown that when the intercept is estimated, the second and the fifth factors and the intercept emerged as significant for pricing. Hence, it seems that there are two "priced" factors in the US stock market.

The validity of the APT in pricing US stocks is supported by the fact that the intercept term is significantly different from zero and the risk premia of the second and the fifth factors are also significantly different from zero. It is clear from the cross-sectional regression results that the APT explains 30% of the variation in the twenty-four years average returns. The result is quite surprising and encouraging. In this study, it has been simply assumed that the non-stationarity problem does not exist. By taking no measures to mitigate any problems arising from this, these tests are biased toward finding that risk measures are not significant.

The next step is to interpret the factors extracted from the US security returns and to relate them to other aspects of the economy.

CHAPTER 9

THE FACTOR STRUCTURE OF THE US ECONOMY

9.1 Introduction

The objective of this chapter is to examine a set of US economic variables in order to estimate the number and loadings of the factors that represent the US economy. The sizes of the factor loadings reflect the extent of the relationship between each economic variable and each factor. The comparison of security and economic factors is contained in chapter 10.

The data description of the economic variables is discussed in section 9.2. The method used in the study is mentioned in section 9.3. In sections 9.4 and 9.5, the results of the principal factor analysis and the maximum-likelihood factor analysis are discussed respectively. In section 9.6, the results are discussed and the last section contains the conclusions.

9.2 Data Description

Monthly data were obtained from Datastream. The study period is from January 1965 through December 1988 inclusive, which corresponds to that of the security returns used in chapter 8. The major categories of macroeconomic variables considered in the analysis are those representing the stock market, money supply, industrial production, and labour market, as well as international trade. The variables are measured by widely used indicators which cover a wide spread of economic processes and sectors of the economy.

The major economic and financial variables selected in this study can be classified into the following categories:

Balance of Payments:	Imports CIF (ECON1);ExportsFOB (ECON36).
Capital Formation:	Construction - Value of Contracts: Total (ECON18); Construction - Work Put in Place: Residential, Private Sector (ECON19).
Coincident Indicator:	Bcd Coincident Composite Index (ECON12);
Consumer Expenditure:	Consumer Credit Outstanding - Financial Institutions (ECON34); Loans (Commercial Banks) (ECON33).
Demand Deposits:	(ECON30).
Fuel & Oil Prices:	Wholesale Prices - Gas Fuels (ECON24); Producer Prices - Refined Petroleum Products (ECON26); Output of Crude Petroleum (ECON10);
Government Securities:	Yield of Long-Term Government Bonds (End Period) (ECON38).
Gross National Product:	Gross National Product (at Annual Rates) (ECON37); Personal Income - Total (at Annual Rates) (ECON4);
Industrial Production:	Industrial Production - Total (ECON5); Industrial Production - Durable Goods (ECON6); Industrial Production - Non-Durable Goods (ECON7); Industrial Production - Investment Goods (ECON8); Industrial Production - Consumer Goods (ECON9); Manufacturing Deliveries - Durable Goods (ECON14). Manufacturing Deliveries - Nondurable Goods (ECON15); Manufacturing Net New Orders - Total (ECON16); Manufacturing Net New Orders - Nondurable Goods (ECON17);
Inflation:	Consumer Price Index (All Urban Consumers); Consumer Prices - All Items (ECON27); Producer Prices - Total (ECON25);
Interest Rate:	Interest Rate on 3-Mth (Top Rated) Bankers' acceptances (Discount) (ECON2); Interest Rate on 3 Month US$ Deposits in London (End Period) (ECON3);
Lagging Indicator:	Bcd Lagging Composite Index (ECON13);
Leading Indicator:	Bcd Leading Composite Index (ECON11);
Market Index:	Share Prices - Industrials (Standard & Poor) (ECON35);

Money Supply:	Money Supply M1 (ECON28); Money Supply M2 (ECON29);
New Capital Issues by Corporations: (ECON32).	
Sales:	Retail Sales: Value (ECON21); Wholesale Sales: Value (ECON20);
Total Reserves:	(ECON31).
Unemployment:	Unemployment Total (ECON23); Employment in Manufacturing Industry (ECON22).

All the economic variables examined are measured by rates of change rather than absolute values. The economic variables selected in this section are similar to the UK economic variables in chapter 6. The macroeconomic variables are assumed to influence either future cash flows or the risk-adjusted discount rate, two key variables when stocks are priced by the expectation of the present value of future cash flows. However, the number of the UK economic variables is smaller than the US economic variables, there are 38 US economic variables selected in the analysis of the factor structure of the US economy as compared with only 21 UK economic variables in the analysis of that of the UK economy. The selection of these variables is based on the availability of the data. In factor analysis, observations with missing values for any variable in the analysis should be omitted from the computations because calculation of correlations requires simultaneous observations. Therefore, only variables with no missing observations between January 1965 and December 1988 are included.

9.3 Method

In estimating the number of factors representing the economic activities of the US economy, two factor extraction techniques are used (as discussed in chapter 5) :

(i) Principal factor analysis (PFA) is used to reveal the probable number and size of the US economic factors before proceeding to a maximum-likelihood analysis;

(ii) Maximum-likelihood factor analysis (MLFA) is used to identify precisely the number of US economic factors and their factor loadings.

The factors extracted from the macroeconomic and financial variables eliminate multicollinearity among independent variables since factor analysis extracts independent factors from the range of US economic variables.

9.4 Principal Factor Analysis

Before turning to maximum-likelihood, the principal factor analysis is used to get an approximate idea of the number of factors. The results show that the overall Kaiser's measure of sampling adequacy (MSA) is 0.74 (Table 9.1) and the squared multiple correlations (SMC) of all the variables are 0.57 (Table 9.2) on average, therefore the results imply that the data are quite adequate for factor analysis. In chapter 8, it was shown that for the sample of security returns, the overall MSA and SMC are 0.90 and 0.89 respectively. This is expected as the set of macroeconomic data includes a much more diverse group of variables.

Table 9.3 shows the eigenvalues of the reduced correlation matrix. Based on the eigenvalue 1 criterion, six factors are retained, and, those six factors account for 84.83% of common variance. The first factor accounts for nearly 30% of the total variation, the second factor accounts for over 15% of variance, the third factor accounts for nearly 14%, the fourth factor accounts for 10%, whereas the fifth and sixth factors account for 9.12% and 6.53% respectively. The scree test based on the graph of eigenvalues shows that there are no more than six factors which should be extracted.

TABLE 9.1

KAISER'S MEASURE OF SAMPLING ADEQUACY

ECON1	ECON2	ECON3	ECON4	ECON5	ECON6
0.701514	0.740800	0.766186	0.654625	0.824262	0.787561
ECON7	ECON8	ECON9	ECON10	ECON11	ECON12
0.674348	0.793558	0.698974	0.655756	0.777826	0.728487
ECON13	ECON14	ECON15	ECON16	ECON17	ECON18
0.621258	0.810662	0.721610	0.797410	0.710198	0.844084
ECON19	ECON20	ECON21	ECON22	ECON23	ECON24
0.833545	0.784321	0.630868	0.796428	0.923204	0.752803
ECON25	ECON26	ECON27	ECON28	ECON29	ECON30
0.718100	0.745109	0.710466	0.589868	0.714056	0.679493
ECON31	ECON32	ECON33	ECON34	ECON35	ECON36
0.640579	0.636977	0.726214	0.606024	0.664858	0.616153
ECON37	ECON38	Mean MSA	0.74	Min MSA	0.59
0.856015	0.779241			Max MSA	0.92

It is interesting to note that the second US macroeconomic factor accounts for over 50% of total variation explained by the first factor. In chapter 6, it was shown that the second UK macroeconomic factor accounts for nearly 45% of total variation explained by the first factor. It was also shown in chapter 6 that the four factors that were retained based on the "eigenvalue 1" criterion accounted for 92.22%. The results reflect the similarity of the number of factors underlying these two economies.

TABLE 9.2

PRIOR COMMUNALITY ESTIMATES: SMC

ECON1	ECON2	ECON3	ECON4	ECON5	ECON6
0.494954	0.636548	0.511005	0.494806	0.826136	0.912274
ECON7	ECON8	ECON9	ECON10	ECON11	ECON12
0.912060	0.839645	0.935058	0.542173	0.729555	0.926277
ECON13	ECON14	ECON15	ECON16	ECON17	ECON18
0.640830	0.660793	0.941561	0.486875	0.943495	0.471884
ECON19	ECON20	ECON21	ECON22	ECON23	ECON24
0.421034	0.328228	0.396162	0.784219	0.352169	0.203180
ECON25	ECON26	ECON27	ECON28	ECON29	ECON30
0.601385	0.428462	0.535597	0.438844	0.469896	0.415514
ECON31	ECON32	ECON33	ECON34	ECON35	ECON36
0.254262	0.370477	0.477511	0.489425	0.379053	0.575383
ECON37	ECON38	Mean SMC	0.57	Min SMC	0.20
0.491500	0.499013			Max SMC	0.94

It has been noted that in chapter 8, the second US stock market factor explains only 20.5% of total variation of the security returns as explained by the first US stock market factor. The results reflect the importance of the first stock market factor in the US security returns; while, in the wider US economy, several factors have an important role in representing the US economy. After all, the economy is a superset of the stock market.

TABLE 9.3

EIGENVALUES OF THE REDUCED CORRELATION MATRIX

	EIGENVALUE	DIFFERENCE	PROPORTION	CUMULATIVE
1	6.380357		0.2924	0.2924
2	3.325740	3.054617	0.1524	0.4449
3	3.028419	0.297321	0.1382	0.5837
4	2.358480	0.669939	0.1081	0.6918
5	1.988726	0.369754	0.0912	0.7829
6	1.425174	0.563552	0.0653	0.8483
7	0.946303	0.478871	0.0434	0.8916
8	0.796518	0.149785	0.0365	0.9282
9	0.708242	0.088276	0.0325	0.9606
10	0.475927	0.232315	0.0218	0.9824
11	0.459913	0.016013	0.0211	1.0035
12	0.369921	0.089993	0.0170	1.0205
13	0.332475	0.037446	0.0152	1.0357
14	0.272135	0.060340	0.0125	1.0482
15	0.208039	0.064095	0.0095	1.0577
16	0.190415	0.017625	0.0087	1.0664
17	0.153492	0.036922	0.0070	1.0735
18	0.131945	0.021547	0.0060	1.0795
19	0.084323	0.047622	0.0039	1.0834
20	0.050546	0.033777	0.0023	1.0857
21	0.015373	0.035173	0.0007	1.0864
22	0.007823	0.007549	0.0004	1.0868
23	-0.003282	0.011106	-0.0002	1.0866
24	-0.022100	0.018818	-0.0010	1.0856
25	-0.036640	0.014540	-0.0017	1.0839
26	-0.043461	0.006821	-0.0020	1.0819
27	-0.050699	0.007237	-0.0023	1.0796
28	-0.062742	0.012043	-0.0029	1.0767
29	-0.079070	0.016328	-0.0036	1.0731
30	-0.107735	0.028665	-0.0049	1.0682
31	-0.115873	0.008138	-0.0058	1.0629
32	-0.131228	0.015355	-0.0060	1.0568
33	-0.159234	0.028006	-0.0073	1.0495
34	-0.175596	0.016362	-0.0080	1.0415
35	-0.201109	0.025513	-0.0092	1.0323
36	-0.224729	0.023620	-0.0103	1.0220
37	-0.228717	0.003989	-0.0105	1.0115
38	-0.250822	0.022105	-0.0115	1.0000

9.5 Maximum-Likelihood Factor Analysis

The monthly returns of the economic and financial variables were subjected to maximum-likelihood factor analysis to determine the number and factor loadings of the common factors. The goodness of fit results for the US economic factors are summarized in Table 9.4.

When the number of factors is equal to 7, several communality estimates are greater than 1 (since communalities are squared correlations, they must lie between 0 and 1). The possible cause of the ultra-Heywood case is the extraction of too many factors which renders a factor solution invalid. With fewer than seven factors, all the communality estimates are less than 1. Therefore, the Table 9.4 shows only the result with fewer than seven factors.

TABLE 9.4

DIFFERENT CRITERIA FOR ESTIMATING THE BEST NUMBER OF PARAMETERS TO INCLUDE IN A MODEL

Number of factors	AIC	SBC	T&L
2	4188.03	2300.97	0.40
3	3272.34	1909.06	0.55
4	2769.79	1721.89	0.63
5	2447.80	1623.17	0.69
6	2135.31	1527.36	0.74

The results above show that Akaike's information criterion (AIC) and Schwarz's Bayesian criterion (SBC) both have the smallest values at six factors. As discussed above, the Heywood case occurs when the number of factors is equal to seven. The Tucker and Lewis's reliability coefficient for the six factor model is 0.74 which implies that there is a good fit between observed and reproduced matrices. Therefore, six factors are considered for further investigation. In chapter 6, it was shown that the Tucker and Lewis's reliability coefficient for the six factor model for the UK macroeconomic variables is 0.77. By comparison, it appears that there is a slightly better fit between observed and reproduced matrices of the UK macroeconomic factors model than that of the US.

Table 9.5 shows the factor pattern for the six extracted factors. The highest factor loading is 0.9006 and the lowest factor loading is 0.0315 (in absolute terms) for the first factor. For all these factors, there is a mixture of positive and negative loadings for the economic variables. Some variables are positive for all factors (e.g interest rate on 3th month deposits in London with US$ (ECON3)), some variables are negative for all factors (e.g. total unempolyment (ECON23)), while most show a mixture of positive and negative loadings. The sign itself may have no intrinsic meaning. However, signs for variables for a given factor have a specific meaning relative to the signs for other variables; the different signs simply mean that the variables are related to that factor in opposite directions.

The next step is to rotate the factors in order to find more easily interpretable results, while keeping the number of factors and the communalities of each variable fixed. Recall that three orthogonal rotational techniques are used: quartimax, varimax, equamax. The variances explained by those six factors with and without weights are shown in Table 9.6. The quartimax rotation is the rotation of choice as the aim of quartimax rotation is to make the variables as simple as possible by maximizing the variance of the loadings on each variable. The squared multiple correlations of the variables with factor 1, factor 2, factor 3, factor 4, factor 5, and factor 6 are 0.9711, 0.9772, 0.9621, 0.8345, 0.7980 and 0.8187 respectively which implies that the six factors are internally consistent and well defined by the economic variables.

Table 9.7 shows the pattern of factor loadings after the quartimax rotation. All six

TABLE 9.5

UNROTATED FACTOR PATTERN

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5	FACTOR6
ECON17	0.90056	-0.31796	-0.24669	-0.02510	0.00487	-0.01473
ECON15	0.89147	-0.32887	-0.23171	0.00827	-0.00365	-0.00707
ECON14	0.56207	-0.12687	0.37089	-0.08348	-0.05642	0.07244
ECON37	0.39675	-0.12096	0.33553	-0.02245	0.17651	0.02623
ECON20	0.39582	-0.14879	0.12320	0.01948	-0.06703	0.06220
ECON16	0.38204	-0.06277	0.25530	-0.13285	0.01659	0.19291
ECON4	0.34182	-0.12188	0.28803	0.03404	-0.00781	-0.00331
ECON21	0.29223	0.00411	0.21548	-0.13169	0.07324	0.00552
ECON9	0.42542	0.87871	-0.12883	-0.01779	0.02384	-0.01012
ECON7	0.40986	0.92398	-0.13969	0.02515	0.01531	0.01744
ECON6	0.48123	0.76390	0.09701	0.04747	-0.13270	0.03920
ECON8	0.43963	0.67880	0.10458	0.13804	-0.14865	-0.00809
ECON28	-0.07274	-0.32879	0.12125	-0.11772	0.07398	0.06251
ECON12	0.63257	-0.05264	0.73846	-0.00178	-0.04865	-0.07229
ECON5	0.57943	0.09521	0.63971	0.06745	0.10121	-0.05044
ECON22	0.51450	-0.04052	0.59703	0.28577	0.00173	-0.09473
ECON34	0.14588	0.01166	0.31347	0.27905	0.00683	0.02545
ECON24	0.07609	-0.13551	-0.17507	0.04742	0.02471	0.10728
ECON26	0.25518	-0.13371	-0.31606	0.27736	0.07526	0.05874
ECON25	0.31234	-0.22198	-0.33735	0.28743	0.11564	0.12755
ECON23	-0.31282	-0.00719	-0.37800	-0.07464	-0.12033	-0.06634
ECON27	0.13925	-0.02832	-0.38428	0.34927	-0.01456	0.08050
ECON13	-0.17996	0.03327	0.04363	0.65542	-0.14635	-0.17920
ECON2	0.21999	-0.07778	0.20556	0.57671	0.31057	0.37187
ECON38	0.24232	-0.02322	-0.02574	0.42772	0.31733	0.38542
ECON3	0.17375	0.05801	0.13496	0.42501	0.34099	0.41052
ECON33	0.22020	-0.05796	0.23407	0.38540	0.07397	0.02710
ECON29	0.08121	0.04400	0.09374	-0.45236	0.28363	0.27731
ECON35	0.06000	0.02370	0.02333	-0.51494	0.05263	0.09840
ECON11	0.34462	0.06767	0.43676	-0.54596	0.26709	0.28692
ECON31	-0.06709	-0.00802	0.10750	-0.04838	0.19209	0.14830
ECON32	0.03150	0.05212	-0.04773	-0.14307	-0.51970	0.07492
ECON1	0.19266	-0.03841	-0.01795	0.01578	-0.58247	0.36244
ECON36	0.19124	-0.01761	0.06651	0.06743	-0.64121	0.37931
ECON18	0.14282	0.11269	0.03411	-0.07357	-0.35804	0.44798
ECON10	-0.09777	-0.33834	0.12129	-0.01014	-0.36245	0.37133
ECON19	0.22202	-0.00819	0.34003	-0.19519	0.05445	0.35722
ECON30	0.03603	-0.00758	0.15877	-0.18188	0.21368	0.23830

TABLE 9.6

VARIANCE EXPLAINED BY FACTORS ON DIFFERENT ROTATIONAL TECHNIQUES

Rotational technique:	Quartimax		Varimax		Equamax		Unrotated	
	(w)	(u.w)	(w)	(u.w)	(w)	(u.w)	(w)	(u.w.)
Factor 1	48.30	5.45	46.64	5.15	42.48	4.52	73.62	5.18
Factor 2	46.89	3.55	47.29	3.59	47.95	3.66	43.95	3.14
Factor 3	41.63	2.44	42.98	2.45	7.58	2.52	22.68	3.08
Factor 4	6.13	2.32	6.51	2.38	44.62	2.50	5.96	2.68
Factor 5	4.23	2.01	6.06	2.07	6.27	2.37	4.28	2.00
Factor 6	6.72	1.91	4.42	2.03	5.01	2.10	3.40	1.60

factors retain the mixture of signs in the loadings of the economic variables, indicating that the economic variables have different reactions to the factors.

Table 9.8 identifies the economic variables grouped by the statistically significant factor loadings of the six factors. Tabachnick and Fidell (1989) suggested that variables which have loadings in excess of 0.30 (in absolute terms) are considered "statistically significant". The greater the loading, the more the variable is a pure measure of the factor. However, choice of the cutoff for size of loading to be interpreted is a matter of researcher preference. Sometimes, there is a gap in loadings across the factors, and, if the cutoff is in the gap, it is easy to specify which variables load and which do not. Other times, the cutoff is selected because one can interpret factors with that cutoff but not with a lower cutoff.

It can be concluded from Table 9.8 that the six factors here are representations of economic activities. The first factor is composed of general economy-wide variables, interest rate, GNP, employment, and encompasses coincident and leading composite indicators. The second factor represents mainly variables such as industrial production, money supply (M1).

TABLE 9.7

ROTATED FACTOR PATTERN (QUARTIMAX)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5	FACTOR6
ECON12	0.96385	0.05983	-0.13501	0.02162	0.05510	-0.03886
ECON5	0.84182	0.18917	-0.11863	0.03247	-0.06309	0.08839
ECON22	0.80247	0.06826	-0.05633	-0.21320	-0.01666	0.12612
ECON14	0.65818	0.01774	0.10534	0.13723	0.13638	-0.05486
ECON37	0.52925	-0.03471	0.06296	0.13510	-0.08965	0.07350
ECON4	0.45940	-0.03462	0.04947	0.00187	0.03445	0.00210
ECON16	0.43193	0.01970	0.07139	0.24121	0.13553	0.04014
ECON20	0.38127	-0.01339	0.20311	0.02743	0.12246	-0.02624
ECON33	0.34816	0.00520	0.06901	-0.24580	-0.03145	0.27197
ECON21	0.34606	0.06117	0.00610	0.16872	-0.02927	-0.04158
ECON34	0.33127	0.02538	-0.08065	-0.19094	0.02025	0.21094
ECON23	-0.48086	-0.05151	0.04486	-0.05195	0.02985	-0.16051
ECON9	0.07366	0.97594	0.02168	0.10810	-0.02582	0.00281
ECON7	0.06430	0.92386	0.04969	0.07875	-0.00348	0.03575
ECON6	0.28244	0.86146	-0.06017	0.02211	0.13993	0.01880
ECON8	0.28039	0.77566	-0.05065	-0.08657	0.12233	0.03070
ECON28	0.07337	-0.36001	-0.01359	0.12649	-0.01170	0.00442
ECON17	0.51469	0.07359	0.80215	0.09958	0.03994	-0.22035
ECON15	0.52156	0.05983	0.79641	0.07173	0.05070	-0.19775
ECON25	0.03476	-0.02729	0.57363	-0.10608	-0.01020	0.17474
ECON26	-0.00129	0.03343	0.47932	-0.14640	-0.02337	0.12683
ECON27	-0.14553	0.10681	0.42647	-0.23536	0.04852	0.15600
ECON24	-0.04949	-0.06995	0.23753	0.02123	0.04587	0.05318
ECON11	0.48308	0.05526	-0.15578	0.71011	0.00083	0.02999
ECON29	0.07309	0.00210	-0.04755	0.60496	-0.04491	0.06081
ECON35	0.01680	0.00202	-0.06383	0.48426	0.03227	-0.20592
ECON19	0.35376	-0.01484	-0.08622	0.36516	0.19648	0.16999
ECON30	0.11181	-0.05139	-0.06442	0.33761	-0.01801	0.16880
ECON13	-0.05198	0.01164	-0.06785	-0.67859	-0.01498	0.22439
ECON36	0.14406	0.05637	0.05811	-0.09713	0.75063	-0.02643
ECON1	0.08988	0.04730	0.11972	-0.04296	0.69292	-0.05629
ECON18	0.06035	0.13986	0.02103	0.15252	0.56126	0.08144
ECON10	0.03083	-0.36610	-0.01698	0.01672	0.51760	0.06816
ECON32	-0.04546	0.07771	-0.04289	-0.03771	0.46312	-0.27624
ECON2	0.32380	-0.02233	0.19933	-0.15503	-0.02068	0.70320
ECON3	0.21009	0.08501	0.15352	-0.00367	-0.02647	0.66450
ECON38	0.16243	0.06482	0.32763	-0.02294	-0.02133	0.59431
ECON31	0.01843	-0.06733	-0.07872	0.16998	-0.06485	0.18217

TABLE 9.8

IDENTIFICATION OF THE ECONOMIC VARIABLES GROUPED BY THE FACTOR LOADINGS

Factor 1:	Coincident composite index	(ECON12)	0.9639
	Industrial Production - total	(ECON5)	0.8418
	Employment in manufacturing industry	(ECON22)	0.8025
	Manufacturing deliveries - durable goods	(ECON14)	0.6582
	GNP	(ECON37)	0.5293
	Personal income	(ECON4)	0.4594
	Manufacturing net new order - total	(ECON16)	0.4319
	Wholesale sales: value	(ECON20)	0.3813
	Loans (commercial banks)	(ECON33)	0.3482
	Retail sales: value	(ECON21)	0.3461
	Consumer credit outstanding - financial institutions	(ECON34)	0.3313
	Manufacturing net new orders - non-durable goods	(ECON17)	0.5147
	Manufacturing deliveries - non-durable goods	(ECON15)	0.5216
	Leading composite index	(ECON11)	0.4831
	Construction - work put in place: residential (private sector)	(ECON19)	0.3538
	Interest rate on 3 mth	(ECON2)	0.3238
	Unemployment: total	(ECON23)	-0.4809
	Industrial production: durable goods	(ECON6)	0.2824
	Industrial production: investment goods	(ECON8)	0.2804
Factor 2:	Industrial production: consumer goods	(ECON9)	0.9759
	Industrial production: non-durable goods	(ECON7)	0.9239
	Industrial production: durable goods	(ECON6)	0.8615
	Industrial production: investment goods	(ECON8)	0.7757

TABLE 9.8 (continued)

	Money supply (M1)	(ECON28)	-0.3600
	Output of crude petroleum	(ECON10)	-0.3661
Factor 3:	Manufacturing net new orders - non-durable goods	(ECON17)	0.8022
	Manufacturing deliveries - non-durable goods	(ECON15)	0.7964
	Producer prices: total	(ECON25)	0.5736
	Producer prices - refined petroleum products	(ECON26)	0.4793
	Consumer prices - all items	(ECON27)	0.4265
	Yield of long-term government bonds	(ECON38)	0.3276
Factor 4:	Leading composite index	(ECON11)	0.7101
	Money supply (M2)	(ECON29)	0.6050
	Share prices - industrials (S&P)	(ECON35)	0.4843
	Construction - work put in place: residential (private sector)	(ECON19)	0.3652
	Demand deposits	(ECON30)	0.3376
	Lagging composite index	(ECON13)	-0.6786
Factor 5:	Exports FOB	(ECON36)	0.7506
	Imports CIF	(ECON1)	0.6929
	Construction - value of contracts: total	(ECON18)	0.5613
	Output of crude petroleum	(ECON10)	0.5176
	New capital issues by corporations	(ECON32)	0.4631
Factor 6:	Interest rate on 3 mth	(ECON2)	0.7032
	Interest rate on 3 mth with US$ deposits in London	(ECON3)	0.6645
	Yield of long-term government bonds	(ECON38)	0.5943
	Loans (commercial banks)	(ECON33)	0.2720
	New capital issues by corporations	(ECON32)	-0.2762

The third factor is composed of manufacturing net new orders and deliveries, producer prices index, consumer prices index, wholesale prices on gas fuels, and yield on long-term government bonds. The fourth factor encompasses leading composite index, money supply (M2), share prices - industrials, construction of residential property (private sector), demand deposits level and lagging composite index. The fifth factor represents balance of payments (e.g. exports FOB and imports CIF), the total value of the contracts of construction, the output of crude petroleum, and the amount of new capital issues by corporations. The final factor is composed primarily of interest rate, yield of long-term government bonds, the amount of loans of commercial banks, and lagging composite index.

The relationships among these US economic variables in each of the six US economic factors appear to follow the logic of economic activity. For the first factor, the signs of the factor loadings are consistent with economic reasonings. The general economy-wide indicators (i.e. industrial production, GNP, retail sales, consumer credit, coincident and leading indicators, employment, etc.) have positive loadings and those indicators more or less reflect the general economic activities. The unemployment level is inversely related to the other economy-wide indicators as expected. A lower level of economic activities (i.e. actual production is low) mean layoffs, and a high unemployment rate. For the second factor, industrial production is negatively related to the money supply and the output of crude petroleum. For the third factor, an increase in the manufacturing orders and deliveries indicates an increase in aggregate demand which causes an acceleration of of inflation (i.e. producer and consumer prices levels). For the fourth factor, as expected, the leading and lagging indicators are inversely related to each other because the lagging indicator shows the pattern of production about a year after it has occurred while the leading indicator shows the trends about a year in advance. The positive relationship of money supply (M2) and industrial share prices is as would be expected, as an increase in the money supply not only reduces the interest rate, but also causes an increase in income. The negative relationship of industrial share prices and interest rates is expected as investment and production are stimulated by the lower interest rates. For the fifth factor, it mainly reflects the balance of payments and the new capital formation of the economy. For the final factor, the interest rates and the yield of government bonds are positively related. The negative relationship of new capital issues and interest rates is as would be expected, as higher interest rates increase the attractiveness of alternative investments to investing in stocks. Hence, an increase in interest rates decreases the amount of new issues.

9.6 Discussion

By the maximum-likelihood method of factor analysis, it has been shown that there are six US economic factors. The results here are fairly similar to the findings of Kim and Wu (1987) who extracted factors from the US economic indicators. The market return measure also does not appear to be the most important factor for the US results here. This is probably due to the fact that the market return does not add explanatory power to the other macroeconomic factors. The cumulative proportion of the six US economic factors accounts for almost 85% of the variations in US economic activities. Hence, it can be assumed that the six economic factors are good representations of US economic activities.

As compared with the UK results, it has been shown that the cumulative proportion of the three UK economic factors accounts for almost 83% of the variations in UK economic activities. Similar categories of macroeconomic factors are extracted from the UK economic variables. In chapter 6, it has been shown that the first factor encompasses general market-wide variables and is composed of various market indices. The second factor includes longer leading indicator, lagging indicator, money supply, interest rate, gross redemption yield on gilts, and unemployment rate. The third factor represents variables such as the coincident indicator, GDP, shorter leading indicator, industrial production, and consumers expenditure on durable goods.

9.7 Conclusions

This chapter suggests that there were six major factors underlying the US economy during the study period (1965-1988). The first factor encompasses general market-wide variables, industrial production, GNP, employment, consumer credit, and concident and leading composite indicators. The second factor represents variables such as industrial production, money supply (M1). The third factor is composed of manufacturing net new orders and deliveries, producer prices index, consumer prices index, wholesale prices on gas fuels, and yield on long-term government bonds. The fourth factor encompasses leading composite index, money supply (M2), share prices - industrials, construction of residential property (private sector), demand deposits level and lagging composite index. Whereas the fifth factor represents balance of payments (e.g. exports FOB and imports CIF), the total value of the contracts of construction, the output of crude petroleum, and the amount of new capital issues by corporations. The final factor is composed primarily of interest rate, yield of long-term government bonds, the amount of loans of commercial banks, and lagging composite index.

The analysis shows that these six factors form a good representation of the economic activities which describe the economy; in total the six factors account for almost 85% of the variation in all US economic variables.

Based on the foundations of the APT and the characteristics of the factor scores from the analysis on security returns and economic indicators, the canonical correlation analysis will be used in the next chapter to analyse the relationships between the US security returns and the US economic indicators.

CHAPTER 10

STOCK RETURNS AND ECONOMIC FORCES : THE US EXPERIENCE

10.1 Introduction

The objective of this chapter is to analyse the relationships between security returns and economic indicators. This chapter investigates the association between the set of economic indicators examined in chapter 9 and the sample of stock returns discussed in chapter 8 using canonical correlation analysis.

The next section investigates the nature of the links and patterns of interdependency that relate the stock returns and the economic forces; the number of (statistically significant) links between them; and the extent to which stock returns are conditional upon or redundant given the economic forces and vice versa by using canonical correlation analysis and canonical redundancy analysis. The interpretation of the canonical variates is discussed in section 10.3. Section 10.4 discusses the results and the last section is the summary of the results.

10.2 Empirical Results Using the Canonical Correlation Analysis Approach

The factor scores of the factors extracted from the security returns in chapter 8 and from the economic indicators in chapter 9 are subject to canonical correlation analysis in order to analyse the relationship between the security returns and the economic indicators. The simple univariate statistics show that the eleven variables (i.e. factor scores of the factors extracted from the security returns and economic indicators), namely FSEC 1, FSEC 2, FSEC 3, FSEC 4, and FSEC 5 and FECON 1, FECON 2, FECON 3, FECON 4, FECON 5, and FECON 6 have means which are approximately equal to zero, and standard deviations equal to the multiple correlation of the factor with the variables (i.e. security returns, economic indicators). Since the computed factor scores are only estimates of the true factor scores, the estimated factor scores may have small non-zero correlations.

TABLE 10.1

SIMPLE UNIVARIATE STATISTIC

VARIABLE	ST DEV
FSEC 1	0.9955
FSEC 2	0.9868
FSEC 3	0.9627
FSEC 4	0.9604
FSEC 5	0.9411
FECON 1	0.9854
FECON 2	0.9885
FECON 3	0.9808
FECON 4	0.9135
FECON 5	0.8932
FECON 6	0.9048

The first step in the canonical analysis is the generation of a correlation matrix, R (Table 10.2). The correlation matrix is subdivided into four parts: the correlations among the factor scores of the security returns (R_{xx}), the correlations among the factor scores of the economic indicators (R_{yy}), and the two matrices of correlations between the factor scores of the security returns and that of the economic indicators ($R_{xy} = R'_{yx}$).

The correlations between the factor scores of the security returns and that of the economic indicators are moderate, the large ones are 0.3329 between FSEC1 and FECON4, -0.3085 between FSEC1 and FECON6, and -0.3770 between FSEC2 and FECON6. However, significance cannot yet be assumed.

As shown in Table 10.3, the first canonical correlation is 0.5505, representing 30.31%

TABLE 10.2

CORRELATIONS AMONG THE SECURITY RETURNS, ECONOMIC INDICATORS, AND BETWEEN THE SECURITY RETURNS AND THE ECONOMIC INDICATORS

Correlations Among The Security Returns (R_{xx})

	FSEC 1	FSEC 2	FSEC 3	FSEC 4	FSEC 5
FSEC 1	1.0000	0.0060	0.0053	0.0043	0.0020
FSEC 2	0.0060	1.0000	-0.0029	0.0003	0.0006
FSEC 3	0.0053	-0.0029	1.0000	-0.0021	0.0032
FSEC 4	0.0043	0.0003	-0.0021	1.0000	-0.0006
FSEC 5	0.0020	0.0006	0.0032	-0.0006	1.0000

Correlations Among the Economic Indicators (R_{yy})

	FECON1	FECON2	FECON3	FECON4	FECON5	FECON6
FECON1	1.0000	0.0037	0.0136	0.0134	0.0139	0.0005
FECON2	0.0037	1.0000	0.0002	0.0144	0.0001	0.0058
FECON3	0.0136	0.0002	1.0000	-0.0006	0.0016	-0.0402
FECON4	0.0134	0.0144	-0.0006	1.0000	-0.0104	-0.0407
FECON5	0.0139	0.0001	0.0016	-0.0104	1.0000	-0.0201
FECON6	0.0005	0.0058	-0.0402	-0.0407	-0.0201	1.0000

Correlations Between the Security Returns and the Economic Indicators (R_{xy})

	FECON1	FECON2	FECON3	FECON4	FECON5	FECON6
FSEC1	-0.0092	0.0093	-0.0717	0.3329	0.0353	-0.3085
FSEC2	-0.1338	-0.0207	-0.1402	-0.0947	-0.0036	-0.3770
FSEC3	0.1132	-0.0408	0.0790	-0.0005	0.0029	0.0332
FSEC4	-0.0971	-0.0576	-0.0548	-0.1193	0.1088	0.0029
FSEC5	-0.0261	-0.0735	0.0538	-0.0183	0.0023	-0.0771

of overlapping variance between the first pair of canonical variates (i.e. linear combination of the factor scores of the security returns and that of the economic indicators), which appears to be larger than any of the direct between-set correlations. The high correlation between the first pair of canonical variates is expected as each linear combination maximizes the correlation between the pair of canonical variates (refer to section 4.5). The second canonical correlation is 0.3653; representing 13.35% of overlapping variance for the second pair of canonical variates. The second pair accounts for nearly 45% of the common variance of that of the first pair.

The last panel of Table 10.3 shows that the probability level for the null hypotheses that the first two canonical correlations are zero in the population is only 0.0001, hence both pairs of canonical variates reach significance ($\alpha = 0.05$) and they account for the significant relationships between the two sets of variables. The other canonical correlations are not significantly different from zero. All pairs produced after the first one are constrained to be uncorrelated with all the preceding combinations. The process of constructing canonical variates continues until the number of pairs of canonical variates equals the number of variables in the smaller set (i.e. there are five variables in the set of factor scores of US security returns).

As shown in Table 10.4, the first canonical correlation vectors are

$$\rho_1 = 0.7005\ \mathrm{FSEC1} + 0.6884\ \mathrm{FSEC2} - 0.1432\ \mathrm{FSEC3} + 0.0105\ \mathrm{FSEC4} + 0.0957\ \mathrm{FSEC5}$$

and

$$\phi_1 = -0.2142\ \mathrm{FECON1} - 0.0152\ \mathrm{FECON2} - 0.3114\ \mathrm{FECON3} + 0.2670\ \mathrm{FECON4} + 0.0305\ \mathrm{FECON5} - 0.8867\ \mathrm{FECON6},$$

with $r_c(\rho_1,\phi_1) = 0.5505$.

10.3 Interpretation of Canonical Variates

After the canonical correlation creates the canonical variates, the factor loading matrix

TABLE 10.3

CANONICAL CORRELATION ANALYSIS

	1	2	3	4	5
Canonical correlation (r_c)	0.5505	0.3653	0.1487	0.1280	0.0622
Squared canonical correlation (r_c^2)	0.3031	0.1335	0.0221	0.0164	0.0039

Tests of H_0: The canonical correlation in the current column and all that follow are zero.

	1	2	3	4	5
Likelihood	0.57862199	0.83026407	0.95814448	0.97980338	0.99613148
F-test	5.4232	2.6622	1.0026	0.9570	0.5456
Pr > F	0.0001	0.0001	0.4445	0.4537	0.5801

contains the correlations of the original variables (i.e. factor scores of the security returns) with the canonical coefficients. Usually correlations between original variables and canonical coefficients in excess of 0.3 are interpreted (Tabachnick and Fidell, 1989). The content of the reliable pairs of canonical variates is interpreted via the factor loading matrix. As shown in Table 10.5, the first pair of canonical variates has high loading on FSEC1 (0.7041) of the factor scores of the security returns, on FSEC2 (0.6931) of the factor scores of the security returns and on FECON6 (-0.8859) of the factor scores of the economic indicators. Thus, the first canonical variates are primarily FSEC1, FSEC2 for the security returns and FECON6 for the economic variables. That is, the first pair of variates is primarily variables such as interest rate, yield of long-term government bonds, the amount of loans of commercial banks, the amount of new capital issues by corporations and lagging indicators.

TABLE 10.4

CANONICAL CORRELATION ANALYSIS: STANDARDIZED CANONICAL COEFFICIENTS FOR THE SECURITY RETURNS AND THE ECONOMIC INDICATORS

Standardized Canonical Coefficients (B_x) for the Security Returns

	SEC1	SEC2	SEC3	SEC4	SEC5
FSEC1	0.7005	0.6415	-0.2094	0.2245	0.0604
FSEC2	0.6884	-0.5989	0.2909	-0.2363	0.1643
FSEC3	-0.1432	0.1472	0.6163	0.4040	0.6441
FSEC4	0.0105	-0.4517	-0.5619	0.6506	0.2383
FSEC5	0.0957	-0.0978	0.4184	0.5522	-0.7081

Standardized Canonical Coefficients (B_y) for the Economic Indicators

	ECON1	ECON2	ECON3	ECON4	ECON5
FECON1	-0.2142	0.3617	0.5190	-0.0302	0.7371
FECON2	-0.0152	0.1097	-0.2073	-0.6853	0.1475
FECON3	-0.3114	0.1907	0.4905	0.3347	-0.4581
FECON4	0.2670	0.8909	-0.2813	0.0881	-0.1922
FECON5	0.0305	-0.0593	-0.4684	0.6408	0.4330
FECON6	-0.8867	0.1488	-0.3937	-0.0243	-0.0820

The second pair of canonical variates has high loadings on FSEC1 (0.6365), on FSEC2 (-0.5957), on FSEC4 (-0.4494) of the factor scores of the security returns, on FECON4 (0.8918) and on FECON1 (0.3758) of the factor scores of the economic indicators. Hence, the second pair of variates represents the leading indicator, money supply (M2), share prices (industrials), construction of private sector residential property, demand deposits level and lagging indicator; general economy-wide variables (i.e. industrial production, GNP, consumer credit, commercial bank loans, unemployment, and coincident and leading

TABLE 10.5

CANONICAL STRUCTURE

Correlations Between the Security Returns and their Canonical Coefficients, (A_x)

	SEC1	SEC2	SEC3	SEC4	SEC5
FSEC1	0.7041	0.6365	-0.2059	0.2292	0.0643
FSEC2	0.6931	-0.5957	0.2880	-0.2356	0.1624
FSEC3	-0.1412	0.1529	0.6169	0.4063	0.6412
FSEC4	0.0139	-0.4494	-0.5643	0.6503	0.2377
FSEC5	0.0971	-0.0961	0.4205	0.5534	-0.7059

Correlations Between the Economic Indicators and their Canonical Coefficients, (A_y)

	ECON1	ECON2	ECON3	ECON4	ECON5
FECON1	-0.2149	0.3758	0.5144	-0.0181	0.7349
FECON2	-0.0173	0.1247	-0.2116	-0.6842	0.1469
FECON3	-0.2788	0.1891	0.5127	0.3361	-0.4440
FECON4	0.2999	0.8918	-0.2567	0.0720	-0.1811
FECON5	0.0421	-0.0662	-0.4496	0.6404	0.4462
FECON6	-0.8859	0.1069	-0.3935	-0.0583	-0.0632

Correlations Between the Security Returns and the Canonical Coefficients of the Economic Indicators, $(R_{xx}B_y)$

	ECON1	ECON2	ECON3	ECON4	ECON5
FSEC1	0.3876	0.2325	-0.0306	0.0293	0.0040
FSEC2	0.3815	-0.2176	0.0428	-0.0302	0.0101
FSEC3	-0.0777	0.0559	0.0917	0.0520	0.0399
FSEC4	0.0077	-0.1642	-0.0839	0.0833	0.0148
FSEC5	0.0535	-0.0351	0.0625	0.0709	-0.0439

Correlations Between the Economic Indicators and the Canonical Coefficients of the Security Returns, $(R_{yy}B_x)$

	SEC1	SEC2	SEC3	SEC4	SEC5
FECON1	-0.1183	0.1373	0.0765	-0.0023	0.0457
FECON2	-0.0095	0.0456	-0.0315	-0.0876	0.0091
FECON3	-0.1535	0.0691	0.0762	0.0430	-0.0276
FECON4	0.1651	0.3258	-0.0382	0.0092	-0.0113
FECON5	0.0232	-0.0242	-0.0668	0.0820	0.0278
FECON6	-0.4877	0.0391	-0.0585	-0.0075	-0.0039

indicators).

It is interesting to know how much variance the canonical variates from the security returns extract from the economic indicators, and vice versa. As shown in Table 10.6, the first pair of canonical variates is only a moderate overall predictor of the opposite set of variables, the proportions of variance explained being 0.0610 and 0.0505. Although the second pair of canonical variates is statistically significant, the proportions of variance are only being 0.0266 and 0.0223. The first canonical variate of the security returns extracts 20.11% of the variance of the security returns, the second canonical variate extracts 19.89% of the variance. In summing for the two variates, 40.01% of the variance in the security returns is extracted by the two canonical variates (because there are five canonical pairs, but only two pairs are statistically significant). For the economic indicators, together the two canonical variates extract only 33.41% of variance.

The squared multiple correlations in Table 10.7 indicate that the first canonical variate of the economic indicators has moderate predictive power for FSEC1 and FSEC2, but is almost useless for predicting FSEC3, FSEC4, and FSEC5. The first canonical variate of the security returns is a fairly good predictor of FECON6, but has almost no predictive power for FECON1, FECON2, FECON3, FECON4, and FECON5.

The squared multiple correlations in Table 10.7 show that the second canonical variate of the economic indicators was nearly useless for prediciing FSEC1 and FSEC2 and useless for predicting FSEC3, FSEC4 and FSEC5. The second canonical variate of the security returns has only moderate predictive power for FECON4, but almost none for FECON1, FECON2, FECON3, FECON5 and FECON6.

TABLE 10.6

CANONICAL REDUNDANCY ANALYSIS

Standardized Variance of the Security Returns Explained By

	Their own Canonical Variates			The opposite Canonical Variates	
	Proportion	Cumulative Proportion	Canonical R-Squared	Proportion	Cumulative Proportion
1	0.2011	0.2011	0.3031	0.0610	0.0610
2	0.1989	0.4001	0.1335	0.0266	0.0875
3	0.2002	0.6003	0.0221	0.0044	0.0919
4	0.2005	0.8007	0.0164	0.0033	0.0952
5	0.1993	1.0000	0.0039	0.0008	0.0960

Standardized Variance of the Economic Indicators Explained By

	Their own Canonical Variates			The opposite Canonical Variates	
	Proportion	Cumulative Proportion	Canonical R-Squared	Proportion	Cumulative Proportion
1	0.1668	0.1668	0.3031	0.0505	0.0505
2	0.1673	0.3341	0.1335	0.0223	0.0729
3	0.1658	0.4999	0.0221	0.0037	0.0765
4	0.1667	0.6666	0.0164	0.0027	0.0793
5	0.1658	0.8324	0.0039	0.0006	0.0799

10.4 Discussion

In this chapter, the two sets of factors extracted from the US security returns and the US economic indicators are used to analyse the links between them. The results show that the canonical correlation between the first canonical variate of the security returns and that of the economic indicators is 0.5505. This is the highest correlation between any linear

TABLE 10.7

SQUARED MULTIPLE CORRELATIONS

Squared Multiple Correlations Between the Security Returns and the First 'M' Canonical Variates of the Economic Indicators

M	1	2	3	4	5
FSEC1	0.1503	0.2043	0.2053	0.2061	0.2061
FSEC2	0.1456	0.1929	0.1948	0.1957	0.1958
FSEC3	0.0060	0.0092	0.0176	0.0203	0.0219
FSEC4	0.0001	0.0270	0.0341	0.0410	0.0412
FSEC5	0.0029	0.0041	0.0080	0.0130	0.0149

Squared Multiple Correlations Between the Economic Indicators and the First 'M' Canonical Variates of the Security Returns

M	1	2	3	4	5
FECON1	0.0140	0.0328	0.0387	0.0387	0.0408
FECON2	0.0001	0.0022	0.0032	0.0108	0.0109
FECON3	0.0236	0.0283	0.0341	0.0360	0.0368
FECON4	0.0273	0.1334	0.1348	0.1349	0.1351
FECON5	0.0005	0.0011	0.0056	0.0123	0.0131
FECON6	0.2379	0.2394	0.2428	0.2429	0.2429

combination of the security returns and the economic indicators. The first canonical variate formed from the economic indicators is the best linear combination of the economic indicators for predicting the first canonical variate formed from the security returns.

It is interesting to note that the signs of the correlations between the security returns and their canonical coefficients and those between the economic indicators and their canonical coefficients in Table 10.5 are consistent with macroeconomic reasoning. Table 10.5 shows a negative correlation (-0.4877) between the first canonical coefficients of the security returns and the sixth economic factor. It implies the first canonical coefficients of the security returns and the sixth economic factor are negatively related.

For the first pair of canonical variates, the effect of the economic indicators on the economy can be interpreted as follows. With lower interest rates, the attractions of borrowing are increased. If interest rates decline, there will be a wider range of assets where the return exceeds the costs of capital. Lower interest rates encourage and stimulate capital investment. Firms' sales will thus increase, boosting their own earnings. Any decrease in the interest rate would have a positive effect on expected future earnings and so raise the stock prices. The yield of long-term government bonds is found to be positively related to the interest rate as is expected.

The second pair of canonical variates has high loading on FSEC1 (0.6365), on FSEC2 (-0.5957), on FSEC4 (-0.4494) of the factor scores of the security returns, on FECON4 (0.8918) and on FECON1 (0.3758) of the factor scores of the economic indicators. Hence, the second pair of variates represents variables such as leading indicators, money supply (M2), share prices - industrials, construction of residential (private sector), demand deposits level and lagging indicator; and general economy-wide variables, industrial production, GNP, unemployment, and encompasses coincident and leading composite indicators.

Table 10.5 shows a positive correlation (0.3258) and a negative correlation (0.1373) between the second canonical coefficients of the security returns and the fourth and first economic factors respectively. The correlation of the second pair of canonical variates is 0.3653 and it represents, squared, 13.35% overlap in variance. Hence, the second pair of canonical variates is also economically significant.

For the second pair of variates, the economic indicators have the following impact on the economy. The leading indicator is expected to be able to forecast the direction of the real economy and to anticipate movements in the economy. The lagging indicator, on the other hand, shows the pattern of production about a year after it has happened. Hence, as is expected, the security returns are positively and negatively related to the leading and lagging indicators respectively.

With regard to interest rates and money supply, stock prices are likely to be rising and interest rates falling when there is an excess money supply. In recessions and the early stages of recoveries, wealth should be lower than that in boom periods when actual income is high. The demand deposits level shows the procyclical fluctuation. Whenever individuals have more wealth in boom periods, they are likely to use the extra wealth to purchase stocks and other assets. Any increase in the demand for stocks drives up their prices.

The results also show that the stock prices are positively related to private residential construction. Private residential construction is an important indicator of future economic activity because most turnarounds in the US economy have been precipitated by changes in household spending habits. A sustained decline in residential construction suggests that the economy is slowing down and could dip into recession at some point. A rise in residential construction would suggest that the economy is expanding or is about to expand.

The positive relationship between the first economic factor and security returns implies that stock prices are positively related to the general economy-wide variables (e.g. coincident and leading indicators, industrial production, GNP, total employment, consumer credit, retail sales, interest rate, commercial bank loans, etc). The general economy-wide variables reflect a major portion of economic activity. The results suggest that US security returns are responsive to a large number of macroeconomic variables in the US economy. Since financial assets (i.e. stocks) are claims against the output of the economy, changes in the macroeconomy are reflected in the macroeconomic indicators. Higher economic activity should result in an increase in share prices as the expected profits of firms in the future will increase.

10.5 Conclusion

This chapter investigates a set of economic indicators as systematic influences on stock returns using canonical correlation analysis. The results from this chapter imply that the canonical correlation analysis successfully links the US stock market and the US economic forces.

The first pair of canonical variates is composed of the factor scores of the first and second factors of the security returns and those of the sixth factor of the economic indicators. As it is shown in the previous chapter, the sixth economic factor encompasses the interest rate, the yield of long-term government bonds, the commercial bank loans, the amount of new capital issues by corporations, and lagging indicators.

The second pair of canonical variates is composed of the factor scores of the first, second, and fourth factors of the security returns and those of the fourth and the first factor of the economic indicators. As shown in the previous chapter, the fourth economic factor represents primarily the leading indicators, money supply (M2), share prices - industrials, construction of residential and private sector and demand deposits level; while the first economic factor is composed of general economy-wide variables (i.e. industrial production, GNP, unemployment rate, consumer credit, coincident and leading composite indicators).

When the UK and the US results are compared, there is a better correspondence between factor scores generated by the factor analysis on security returns and that on economic indicators of the UK results than that of the US results.

The APT proposes that the expected returns can be explained by the sensitivities of stock returns to innovations in the macroeconomic variables. The conclusion of the empirical findings in this chapter is that the US security returns are influenced by a number of systematic economic forces.

CHAPTER 11

INTERNATIONAL ARBITRAGE PRICING THEORY

11.1 Introduction

In this chapter, the Arbitrage Pricing Theory is investigated in an international setting. Most empirical studies of asset pricing models use securities from a single country (see chapter 3). The benefits that can be obtained from multinational diversification are typically enhanced by barriers to investment (e.g. lack of information about local accounting conventions). When such barriers exist, they cause international market segmentation, and diversification tends to be more beneficial. The intercountry correlations between securities markets are high because different countries and their economic prospects are closely tied, and vice versa. In view of the trend toward stock market integration, one might expect that increasing global diversification would lead to a greater role for international factors in asset pricing.

Section 11.2 is the data description. The empirical results and interpretation of the relationship between the UK security returns and that of the US are discussed in section 11.3. The method of investigation of the international stock markets factors is discussed in section 11.4. The empirical results of international stock market factors and economic factors are discussed in sections 11.5 and 11.6 respectively. The relationships between international security returns and economic indicators are discussed in section 11.7. Section 11.8 discusses the canonical correlation analysis between the UK economic indicators and the US economic indicators. And the last section presents the conclusions of this chapter.

11.2 Data Description

The data sources are from the London Share Price Database of the London Business School (monthly-returns file) and the Centre for Research in Security Prices, Graduate School of Business, University of Chicago (monthly-returns file). The sample period is January 1965 to December 1988 inclusive, giving a maximum sample size per security of 288 monthly returns. Only securities with no missing observations during the entire period were included. Altogether the sample consists of 278 stocks representing two different countries: (i) United Kingdom [London Stock Exchange (61 stocks)]; (ii) United States [New York Stock Exchange (217 stocks)]. The stocks are the same as those that were used in the domestic sections.

11.3 Canonical Correlation Analysis between the UK and the US Security Returns

The factor scores of the factors extracted from the UK and US security returns in chapters 5 and 8 are subject to canonical correlation analysis. There are two UK and five US stock market factors that were extracted in the domestic sections.

The first step in the canonical analysis is the generation of a correlation matrix (Table 11.1). The correlations among the factor scores of the UK security returns and that of the US are moderate, the largest one is 0.4909 between FSECA1 (UK security returns) and FSECC1 (US security returns). This correlation between the factor scores of the first UK stock market factor and those of the first US stock market factor is rather high. However, significance cannot yet be assumed.

As shown in Table 11.2, the first canonical correlation is 0.5041, representing 25.41% of overlapping variance between the first pair of canonical variates, which appears to be larger than any of the direct between-set correlations. The second canonical correlation is

TABLE 11.1

CORRELATIONS BETWEEN THE UK SECURITY RETURNS AND THE US SECURITY RETURNS.

	FSECC1	FSECC2	FSECC3	FSECC4	FSECC5
FSECA1	0.4909	0.0675	-0.0279	-0.0356	0.0647
FSECA2	-0.0537	0.1020	-0.0558	0.0613	0.0248

(FSECA = Factor scores of the UK security returns)
(FSECC = Factor scores of the US security returns)

0.1386, representing 1.92% of overlapping variance for the second pair of canonical variates. Therefore, there are two statistically significant pairs of canonical variates. The first canonical correlation represents a substantial relationship between the first pair of canonical variates. Interpretation of the second canonical correlation and its corresponding pair of canonical variates is marginal, because the canonical correlation value of the second pair of 0.1386 represents, squared, less than a 2% overlap in variance. Though the second pair is a statistically significant link, it accounts for a trivial amount of common variance.

The last panel of Table 11.2 shows that the probability level for the null hypothesis that all the canonical correlations are zero in the population is only 0.0001, hence the first pair of canonical variates reach significance (α = 0.05). The first pair of canonical variates account for the significant relationships between the two sets of variables (i.e. the factor scores of the UK security returns and the US security returns).

As shown in Table 11.3, the first canonical correlation vectors are

$$\rho_1 = 0.9969 \, FSECA1 - 0.1051 \, FSECA2$$

and

TABLE 11.2

CANONICAL CORRELATION ANALYSIS

	1	2
Canonical Correlation (r_c)	0.5041	0.1386
Approx. Standard Error	0.0440	0.0578
Squared Canonical Correlation (r_c^2)	0.2541	0.0192

Tests of H_0: The canonical correlation in the current column and all that follows are zero.

	1	2
Likelihood Ratio	0.73155781	0.98079868
F-test	9.5071	1.3802
PR > F	0.0001	0.2409

$$\phi_1 = 0.9816\ FSECC1 + 0.1063\ FSECC2 - 0.0490\ FSECC3 - 0.0874\ FSECC4 + 0.1208\ FSECC5$$

with $r_c(\rho_1, \phi_1) = 0.5041$.

11.3.1 Interpretation of canonical variates

After the canonical correlation creates the canonical variates, the matrix of correlations of the original variables (i.e. factor scores of the UK and US security returns) with the canonical coefficients is a factor loading matrix.

As shown in Table 11.4, the first pair of canonical variates has high loading on

TABLE 11.3

CANONICAL CORRELATION ANALYSIS: STANDARDIZED CANONICAL COEFFICIENTS FOR THE SECURITY RETURNS AND THE ECONOMIC INDICATORS

Standardized Canonical Coefficients for the UK Security Returns

	Standardized canonical coefficients 1	Standardized canonical coefficients 2
FSECA1	0.9969	0.0823
FSECA2	-0.1051	0.9947

(FSECA = Factor scores of the UK security returns)

Standardized Canonical Coefficients for the US Security Returns

	Standardized canonical coefficients 1	Standardized canonical coefficients 2
FSECC1	0.9816	-0.0983
FSECC2	0.1063	0.7713
FSECC3	-0.0490	-0.4146
FSECC4	-0.0874	0.4183
FSECC5	0.1208	0.2175

(FSECC = Factor scores of the US security returns)

FSECA1 (0.9945) of the factor scores of the UK security returns and on FSECC1 (0.9819) of the factor scores of the US security returns. Thus, the first canonical variates are primarily FSECA1 for the UK security returns and FSECC1 for the US security returns.

The canonical correlation analysis involves finding the canonical variates from the factor scores of the UK security returns that are maximally correlated with the canonical variates from the factor scores of the US security returns. It is interesting to know how much

TABLE 11.4

CANONICAL STRUCTURE

Correlations between the UK Security Returns and their Canonical Coefficients

	SECA1	SECA2
FSECA1	0.9945	0.1051
FSECA2	-0.0823	0.9966

Correlations between the US Security Returns and their Canonical Coefficients

	SECC1	SECC2
FSECC1	0.9819	-0.0937
FSECC2	0.1123	0.7721
FSECC3	-0.0435	-0.4175
FSECC4	-0.0832	0.4188
FSECC5	0.1227	0.2162

Correlations between the UK Security Returns and the Canonical Coefficients of the US Security Returns

	SECC1	SECC2
FSECA1	0.5013	0.0146
FSECA2	-0.0415	0.1381

Correlations between the US Security Returns and the Canonical Coefficients of the UK Security Returns

	SECA1	SECA2
FSECC1	0.4950	-0.0130
FSECC2	0.0566	0.1070
FSECC3	-0.0219	-0.0578
FSECC4	-0.0419	0.0580
FSECC5	0.0619	0.0300

(FSECA = Factor scores of the UK security returns)
(FSECC = Factor scores of the US security returns)

variance the canonical variates from the UK security returns extract from the US security returns, and vice versa.

As shown in Table 11.5, canonical redundancy analysis illustrates that the first pair of canonical variates is a moderate overall predictor of the opposite set of variables, the proportions of variance explained being 0.1265 and 0.0509.

TABLE 11.5

CANONICAL REDUNDANCY ANALYSIS

Standardized Variance of the UK Security Returns Explained by:

	Their Own Canonical Variates			Their Opposite Canonical Variates	
	Proportion	Cumulative Proportion	Canonical R-Squared	Proportion	Cumulative Proportion
1	0.4979	0.4979	0.2541	0.1265	0.1265
2	0.5021	1.0000	0.0192	0.0096	0.1362

Standardized Variance of the US Security Returns Explained by:

	Their Own Canonical Variates			Their Opposite Canonical Variates	
	Proportion	Cumulative Proportion	Canonical R-Squared	Proportion	Cumulative Proportion
1	0.2001	0.2001	0.2541	0.0509	0.0509
2	0.2003	0.4004	0.0192	0.0038	0.0547

The squared multiple correlations in Table 11.6 indicate that the first canonical variate of the US security returns has fairly good predictive power for the first factor scores of the UK stock market factor. The first canonical variate of the UK security returns is also

TABLE 11.6

SQUARED MULTIPLE CORRELATIONS

Squared Multiple Correlations between the UK Security Returns and the First 'M' Canonical Variates of the US Security Returns

M	1	2
FSECA1	0.2513	0.2515
FSECA2	0.0017	0.0208

Squared Multiple Correlations between the US Security Returns and the First 'M' Canonical Variates of the UK Security Returns

M	1	2
FSECC1	0.2450	0.2452
FSECC2	0.0032	0.0147
FSECC3	0.0005	0.0038
FSECC4	0.0018	0.0051
FSECC5	0.0038	0.0047

a fairly good predictor of the first factor scores of the US stock market factor.

11.3.2 Summary

In the last section, the relationships between the UK and the US security returns are analysed by linking and comparing the two sets of factors extracted from the UK and the US security returns respectively. In checking to determine the same factors have appeared in both cases it is not sufficient to just examine the factor loadings. One needs to obtain the correlations between the factor scores for corresponding pairs of factors. If these correlations are high, then one may have confidence of factor stability. The results imply that the canonical correlation between the first canonical variate of the UK security returns and that of the US security returns is 0.5041. This is the highest possible correlation between any linear combination of the UK and the US security returns. The first canonical variate formed from the UK security returns is the most successful linear combination of the UK security returns to predict the first canonical variate formed from the US security returns. Also the first canonical variate formed from the US security returns is the best linear combination of the US security returns for predicting the first canonical variate formed from the UK security returns. The results imply that there is a fair correspondence between the UK and the US security returns.

11.4 International APT

The main purpose of this section is to investigate the APT in an international setting. The approach used is to :

(i) Extract the number of international stock market factors common to the UK and US security returns;

(ii) Use the individual-asset factor loading estimates from factor analysis to explain the cross-sectional variation of individual estimated expected returns;

(iii) Estimate from the cross-sectional model and use it to measure the size and statistical significance of risk premia associated with the estimated factors;

(iv) Construct international macroeconomic factors from various macroeconomic variables.

(v) The canonical correlation analysis is used to analyse the relationships between international security returns and macroeconomic indicators.

11.5 International Stock Market Factors

In estimating the number of factors which affect the UK and the US security returns, two factor extraction techniques were used :

(i) Principal factor analysis (PFA) to get an approximate idea of the number of factors before proceeding to a maximum-likelihood analysis.

(ii) Maximum-likelihood factor analysis (MLFA) is used to identify more precisely the number of factors, their factor loadings and factor scores. After this, the collection of factor scores is used to compare with the common factors (factor scores) of the macroeconomic factors. This is the subject of section 11.7.

11.5.1 Principal factor analysis

Before turning to maximum-likelihood analysis, the monthly returns of the securities were subjected to principal factor analysis to determine the number of factors which account for a meaningful percentage of common variance. The communalities (squared multiple correlations) are shown in Table 11.7 and reveal that the average communality value is 0.99, which implies that the data are well suited for factor analysis. This mean communality is very good and indicates that the securities are correlated with each other, therefore the data are well suited for factor analysis. Table 11.8 shows that the mean Kaisers' measure of sampling adequacy is 0.52, which implies that the data are fairly acceptable for factor analysis. The ones in the positive diagonal of correlation matrix are replaced by the communality estimates in preparation for factor extraction.

Moving on to the factor extraction stage, an initial quick estimate of the number of factors is obtained from the sizes of the eigenvalues. One of the most popular criteria for estimating the number of factors is to retain factors with eigenvalues greater than 1.

TABLE 11.7

PRIOR COMMUNALITY ESTIMATES: SMC

Mean SMC 0.99

(UK)CO#1	(UK)CO#2	(UK)CO#3	(UK)CO#4	(UK)CO#5	(UK)CO#6
0.982905	0.993613	0.990673	0.987512	0.982231	0.988668
(UK)CO#7	(UK)CO#8	(UK)CO#9	(UK)CO10	(UK)CO11	(UK)CO12
0.986834	0.990891	0.990336	0.989881	0.994484	0.988681
(UK)CO13	(UK)CO14	(UK)CO15	(UK)CO16	(UK)CO17	(UK)CO18
0.988902	0.990737	0.981009	0.992948	0.995949	0.996071
(UK)CO19	(UK)CO20	(UK)CO21	(UK)CO22	(UK)CO23	(UK)CO24
0.979476	0.991873	0.986408	0.995374	0.983745	0.978743
(UK)CO25	(UK)CO26	(UK)CO27	(UK)CO28	(UK)CO29	(UK)CO30
0.997318	0.981102	0.984746	0.995889	0.992041	0.995023
(UK)CO31	(UK)CO32	(UK)CO33	(UK)CO34	(UK)CO35	(UK)CO36
0.993920	0.986626	0.979750	0.991349	0.992767	0.992739
(UK)CO37	(UK)CO38	(UK)CO39	(UK)CO40	(UK)CO41	(UK)CO42
0.993029	0.980915	0.990256	0.986864	0.988853	0.991955
(UK)CO43	(UK)CO44	(UK)CO45	(UK)CO46	(UK)CO47	(UK)CO48
0.991253	0.994273	0.993258	0.990980	0.969372	0.989572
(UK)CO49	(UK)CO50	(UK)CO51	(UK)CO52	(UK)CO53	(UK)CO54
0.996508	0.990146	0.986725	0.992789	0.984229	0.997255
(UK)CO55	(UK)CO56	(UK)CO57	(UK)CO58	(UK)CO59	(UK)CO60
0.988062	0.987289	0.989774	0.970734	0.991869	0.994320
(UK)CO61	(US)CO#1	(US)CO#3	(US)CO#6	(US)CO#7	(US)CO#9
0.981767	0.987464	0.989929	0.990807	0.983996	0.995022
(US)CO10	(US)CO11	(US)CO12	(US)CO13	(US)CO14	(US)CO15
0.986471	0.975777	0.994910	0.984681	0.978668	0.997066
(US)CO16	(US)CO18	(US)CO19	(US)CO20	(US)CO21	(US)CO23
0.993671	0.988946	0.994842	0.990918	0.982346	0.984370

TABLE 11.7 (Continued)

(US)CO26	(US)CO27	(US)CO28	(US)CO30	(US)CO32	(US)CO33
0.980649	0.993602	0.989519	0.993921	0.997458	0.980983
(US)CO34	(US)CO35	(US)CO36	(US)CO39	(US)CO40	(US)CO41
0.983668	0.993901	0.982910	0.985864	0.991514	0.975816
(US)CO42	(US)CO43	(US)CO44	(US)CO46	(US)CO47	(US)CO49
0.966686	0.987543	0.993350	0.986888	0.995453	0.995434
(US)CO51	(US)CO52	(US)CO53	(US)CO56	(US)CO57	(US)CO58
0.991824	0.979766	0.994088	0.975164	0.985192	0.987777
(US)CO60	(US)CO62	(US)CO67	(US)CO70	(US)CO72	(US)CO73
0.994278	0.987847	0.993552	0.993683	0.988786	0.987579
(US)CO74	(US)CO75	(US)CO77	(US)CO78	(US)CO79	(US)CO80
0.991900	0.984569	0.991254	0.989582	0.979667	0.986883
(US)CO82	(US)CO85	(US)CO87	(US)CO90	(US)CO91	(US)CO92
0.987688	0.986428	0.990593	0.987626	0.985585	0.972379
(US)CO93	(US)CO94	(US)CO95	(US)CO99	(US)CO100	(US)CO101
0.996238	0.987748	0.990769	0.987162	0.997034	0.984935
(US)CO104	(US)CO107	(US)CO109	(US)CO110	(US)CO113	(US)CO114
0.979735	0.993345	0.990214	0.987599	0.982526	0.978522
(US)CO115	(US)CO117	(US)CO120	(US)CO121	(US)CO122	(US)CO124
0.993044	0.995327	0.988863	0.993495	0.980876	0.992843
(US)CO125	(US)CO127	(US)CO129	(US)CO131	(US)CO132	(US)CO135
0.990193	0.991125	0.987391	0.994309	0.985698	0.985229
(US)CO137	(US)CO140	(US)CO142	(US)CO143	(US)CO145	(US)CO146
0.987906	0.980143	0.982307	0.991567	0.989636	0.991298
(US)CO147	(US)CO148	(US)CO150	(US)CO151	(US)CO152	(US)CO153
0.988174	0.984482	0.987883	0.992603	0.982289	0.987249
(US)CO154	(US)CO155	(US)CO157	(US)CO158	(US)CO159	(US)CO161
0.988379	0.987774	0.992303	0.994008	0.995361	0.995284

TABLE 11.7 (Continued)

(US)CO162 0.995376	(US)CO164 0.991308	(US)C0167 0.992810	(US)CO169 0.985388	(US)CO171 0.989737	(US)CO172 0.987776
(US)CO173 0.982900	(US)CO174 0.984731	(US)CO177 0.983768	(US)CO179 0.994227	(US)CO180 0.985472	(US)CO182 0.984371
(US)CO184 0.991406	(US)CO185 0.989080	(US)CO186 0.983668	(US)CO187 0.982568	(US)CO190 0.985402	(US)CO192 0.986401
(US)CO196 0.993826	(US)CO197 0.984050	(US)CO202 0.980825	(US)CO203 0.988724	(US)CO204 0.985985	(US)CO205 0.995887
(US)CO207 0.974809	(US)CO209 0.994333	(US)CO212 0.993845	(US)CO214 0.988618	(US)CO215 0.994826	(US)CO216 0.990057
(US)CO217 0.976707	(US)CO219 0.987502	(US)CO221 0.995343	(US)CO224 0.996234	(US)CO225 0.993061	(US)CO226 0.990849
(US)CO228 0.983746	(US)CO230 0.985103	(US)CO231 0.983469	(US)CO234 0.991294	(US)CO236 0.991361	(US)CO241 0.987564
(US)CO243 0.973900	(US)CO244 0.985305	(US)CO246 0.987705	(US)CO247 0.994543	(US)CO248 0.993878	(US)CO252 0.993764
(US)CO253 0.989305	(US)CO255 0.995158	(US)CO258 0.990353	(US)CO259 0.993258	(US)CO261 0.991335	(US)CO262 0.988316
(US)CO263 0.988274	(US)CO265 0.981839	(US)CO266 0.971454	(US)CO268 0.980539	(US)CO269 0.981258	(US)CO274 0.989974
(US)CO275 0.994693	(US)CO276 0.990642	(US)CO278 0.974012	(US)CO279 0.987338	(US)CO280 0.985244	(US)CO282 0.989352
(US)CO283 0.993699	(US)CO284 0.994792	(US)CO287 0.990522	(US)CO288 0.991342	(US)CO289 0.988765	(US)CO290 0.987235
(US)CO291 0.989805	(US)CO295 0.969213	(US)CO296 0.996054	(US)CO297 0.984777	(US)CO298 0.978922	(US)CO299 0.992232
(US)CO302 0.994943	(US)CO304 0.986749	(US)CO206 0.993286	(US)CO207 0.995181	(US)CO308 0.990949	(US)CO309 0.989685

TABLE 11.7 (Continued)

(US)CO310	(US)CO311	(US)CO312	(US)CO313	(US)CO314	(US)CO315
0.994098	0.996332	0.991701	0.990929	0.989838	0.993615
(US)CO316	(US)CO317	(US)CO320	(US)CO322	(US)CO324	(US)CO325
0.996901	0.984370	0.977199	0.987788	0.993017	0.982483
(US)CO326	(US)CO327	(US)CO328	(US)CO329	(US)CO330	(US)CO332
0.992242	0.991925	0.990846	0.990995	0.984322	0.983866
(US)CO333	(US)CO334	(US)CO335	(US)CO337	(US)CO339	(US)CO341
0.987920	0.976648	0.984204	0.988067	0.992834	0.994452
(US)CO343	(US)CO344	(US)CO345	(US)CO346	(US)CO347	(US)CO349
0.982014	0.992983	0.994572	0.984529	0.984919	0.988541
(US)CO350	(US)CO351				
0.986187	0.972842				

(UK)CO denotes the individual UK company;
(US)CO denotes the individual US company.

The results in Table 11.9 indicate that there are fifty-four factors which have eigenvalues greater than 1, the first six factors accounting for 50.26% of total explained variance. The first factor accounts for nearly 31% of the variation, the second factor with 7.4%, the third factor with only 5.7%, and the fourth, fifth, and sixth factor with only 2.6%, 2.2% and 1.6% respectively. The relatively small size of the second and the other factors are rather low and it implies that these factors are much less important than the first factor.

As a second estimate of the number of factors, the scree test was also performed. Applying the scree test, it would appear that no more than the first few factors should be extracted.

This section used principal factor analysis to reveal the probable number and size of

TABLE 11.8

KAISER'S MEASURE OF SAMPLING ADEQUACY:
MEAN MSA = 0.52

(UK)CO#1	(UK)CO#2	(UK)CO#3	(UK)CO#4	(UK)CO#5	(UK)CO#6
0.472051	0.287235	0.401361	0.321708	0.453771	0.682619
(UK)CO#7	(UK)CO#8	(UK)CO#9	(UK)CO10	(UK)CO11	(UK)CO12
0.226646	0.509980	0.438669	0.420736	0.295734	0.586193
(UK)CO13	(UK)CO14	(UK)CO15	(UK)CO16	(UK)CO17	(UK)CO18
0.204399	0.420033	0.577848	0.319553	0.414603	0.397724
(UK)CO19	(UK)CO20	(UK)CO21	(UK)CO22	(UK)CO23	(UK)CO24
0.555786	0.425543	0.471106	0.313047	0.518548	0.525143
(UK)CO25	(UK)CO26	(UK)CO27	(UK)CO28	(UK)CO29	(UK)CO30
0.440265	0.679707	0.354859	0.371370	0.401740	0.485235
(UK)CO31	(UK)CO32	(UK)CO33	(UK)CO34	(UK)CO35	(UK)CO36
0.398036	0.618069	0.472154	0.654636	0.251422	0.466298
(UK)CO37	(UK)CO38	(UK)CO39	(UK)CO40	(UK)CO41	(UK)CO42
0.523635	0.615837	0.214231	0.503749	0.470560	0.380121
(UK)CO43	(UK)CO44	(UK)CO45	(UK)CO46	(UK)CO47	(UK)CO48
0.520861	0.513625	0.518576	0.339670	0.531275	0.351723
(UK)CO49	(UK)CO50	(UK)CO51	(UK)CO52	(UK)CO53	(UK)CO54
0.427944	0.459084	0.643599	0.430887	0.605669	0.481040
(UK)CO55	(UK)CO56	(UK)CO57	(UK)CO58	(UK)CO59	(UK)CO60
0.654734	0.505722	0.419555	0.516363	0.486958	0.386900
(UK)CO61	(US)CO#1	(US)CO#3	(US)CO#6	(US)CO#7	(US)CO#9
0.458388	0.682544	0.500008	0.591666	0.609675	0.525803
(US)CO10	(US)CO11	(US)CO12	(US)CO13	(US)CO14	(US)CO15
0.604864	0.557543	0.326634	0.716623	0.659030	0.495104
(US)CO16	(US)CO18	(US)CO19	(US)CO20	(US)CO21	(US)CO23
0.524247	0.504783	0.505865	0.404851	0.468832	0.684244

TABLE 11.8 (continued)

(US)CO26	(US)CO27	(US)CO28	(US)CO30	(US)CO32	(US)CO33
0.519724	0.335925	0.587896	0.555776	0.440173	0.414171
(US)CO34	(US)CO35	(US)CO36	(US)CO39	(US)CO40	(US)CO41
0.563190	0.429268	0.672417	0.695330	0.504177	0.764352
(US)CO42	(US)CO43	(US)CO44	(US)CO46	(US)CO47	(US)CO49
0.730186	0.569718	0.463784	0.510601	0.443184	0.521489
(US)CO51	(US)CO52	(US)CO53	(US)CO56	(US)CO57	(US)CO58
0.365202	0.666467	0.537657	0.516416	0.474510	0.527754
(US)CO60	(US)CO62	(US)CO67	(US)CO70	(US)CO72	(US)CO73
0.530019	0.449508	0.525011	0.479131	0.415816	0.464686
(US)CO74	(US)CO75	(US)CO77	(US)CO78	(US)CO79	(US)CO80
0.592216	0.451975	0.404050	0.488390	0.724412	0.496511
(US)CO82	(US)CO85	(US)CO87	(US)CO90	(US)CO91	(US)CO92
0.524754	0.445373	0.610533	0.441799	0.591941	0.679276
(US)CO93	(US)CO94	(US)CO95	(US)CO99	(US)CO100	(US)CO101
0.529318	0.518419	0.565954	0.667336	0.503059	0.474914
(US)CO104	(US)CO107	(US)CO109	(US)CO110	(US)CO113	(US)CO114
0.687952	0.491139	0.545585	0.404325	0.628549	0.689510
(US)CO115	(US)CO117	(US)CO120	(US)CO121	(US)CO122	(US)CO124
0.432913	0.477450	0.550761	0.538824	0.508825	0.565237
(US)CO125	(US)CO127	(US)CO129	(US)CO131	(US)CO132	(US)CO135
0.575896	0.660406	0.668628	0.574024	0.538546	0.624185
(US)CO137	(US)CO140	(US)CO142	(US)CO143	(US)CO145	(US)CO146
0.481118	0.601175	0.346867	0.545550	0.450321	0.605545
(US)CO147	(US)CO148	(US)CO150	(US)CO151	(US)CO152	(US)CO153
0.510083	0.476346	0.625448	0.337308	0.756023	0.643862
(US)CO154	(US)CO155	(US)CO157	(US)CO158	(US)CO159	(US)CO161
0.609575	0.622050	0.627259	0.594138	0.428022	0.567612

TABLE 11.8 (continued)

(US)CO162 0.406001	(US)CO164 0.520882	(US)CO167 0.677399	(US)CO169 0.489361	(US)CO171 0.522836	(US)CO172 0.654210
(US)CO173 0.500235	(US)CO174 0.661031	(US)CO177 0.701337	(US)CO179 0.396591	(US)CO180 0.556723	(US)CO182 0.644503
(US)CO184 0.650151	(US)CO185 0.571594	(US)CO186 0.595299	(US)CO187 0.518695	(US)CO190 0.476905	(US)CO192 0.555074
(US)CO196 0.415172	(US)CO197 0.547086	(US)CO202 0.614343	(US)CO203 0.661584	(US)CO204 0.532405	(US)CO205 0.423155
(US)CO207 0.673636	(US)CO209 0.557944	(US)CO212 0.496183	(US)CO214 0.506079	(US)CO215 0.432132	(US)CO216 0.490186
(US)CO217 0.712056	(US)CO219 0.622225	(US)CO221 0.445537	(US)CO224 0.490599	(US)CO225 0.440119	(US)CO226 0.396795
(US)CO228 0.559914	(US)CO230 0.489524	(US)CO231 0.649102	(US)CO234 0.491750	(US)CO236 0.546527	(US)CO241 0.672856
(US)CO243 0.566548	(US)CO244 0.533347	(US)CO246 0.435099	(US)CO247 0.570885	(US)CO248 0.478910	(US)CO252 0.580764
(US)CO253 0.626103	(US)CO255 0.425239	(US)CO258 0.633913	(US)CO259 0.375344	(US)CO261 0.619484	(US)CO262 0.468392
(US)CO263 0.499589	(US)CO265 0.687711	(US)CO266 0.708648	(US)CO268 0.580678	(US)CO269 0.415096	(US)CO274 0.477355
(US)CO275 0.513008	(US)CO276 0.702334	(US)CO278 0.444308	(US)CO279 0.445348	(US)CO280 0.608243	(US)CO282 0.448305
(US)CO283 0.434630	(US)CO284 0.477458	(US)CO287 0.541945	(US)CO288 0.584577	(US)CO289 0.430398	(US)CO290 0.507926
(US)CO291 0.583153	(US)CO295 0.594714	(US)CO296 0.509307	(US)CO297 0.450361	(US)CO298 0.628525	(US)CO299 0.479593
(US)CO302 0.472757	(US)CO304 0.661686	(US)CO306 0.497956	(US)CO307 0.502370	(US)CO308 0.473121	(US)CO309 0.410331

TABLE 11.8 (continued)

(US)CO310	(US)CO311	(US)CO312	(US)CO313	(US)CO314	(US)CO315
0.539878	0.419226	0.510245	0.593953	0.563966	0.578030
(US)CO316	(US)CO317	(US)CO320	(US)CO322	(US)CO324	(US)CO325
0.531439	0.808202	0.512865	0.735766	0.489917	0.466484
(US)CO326	(US)CO327	(US)CO328	(US)CO329	(US)CO330	(US)CO332
0.514101	0.491923	0.387792	0.301983	0.592026	0.533467
(US)CO333	(US)CO334	(US)CO335	(US)CO337	(US)CO339	(US)CO341
0.564139	0.708353	0.753448	0.467919	0.454729	0.557090
(US)CO343	(US)CO344	(US)CO345	(US)CO346	(US)CO347	(US)CO349
0.692935	0.368628	0.393777	0.490182	0.589067	0.382397
(US)CO350	(US)CO351	(UK)CO	denotes the individual UK company;		
0.754628	0.742365	(US)CO	denotes the individual US company.		

the international stock market factors. The results show that not more than six factors should be extracted from security returns. The next stage of the analysis is to use a more powerful technique (maximum-likelihood factor analysis) to extract the factors and their factor loadings. The estimated factor loadings are then used to explain the cross-sectional variation of individual estimated expected returns, and to measure the size and statistical significance of the estimated risk premium associated with each factor.

11.5.2 Maximum-likelihood factor analysis

The monthly returns of the 278 (UK and US) securities were subjected to maximum-likelihood factor analysis to determine the number and factor loadings of the common factors; the results are summarized below:

TABLE 11.9

EIGENVALUES OF THE REDUCED CORRELATION MATRIX

	Eigenvalue	Difference	Proportion	Cumulative
1	84.771669	64.358238	0.3086	0.3086
2	20.413431	4.896281	0.0743	0.3829
3	15.517150	8.514321	0.0565	0.4393
4	7.002829	0.996986	0.0255	0.4648
5	6.005843	1.629066	0.0219	0.4867
6	4.376777	1.046780	0.0159	0.5026
7	3.329997	0.489117	0.0121	0.5147
8	2.840880	0.202990	0.0103	0.5251
9	2.637890	0.299008	0.0096	0.5347
10	2.338882	0.108616	0.0085	0.5432
11	2.230266	0.020107	0.0081	0.5513
12	2.210159	0.110766	0.0080	0.5593
13	2.099393	0.127091	0.0076	0.5670
14	1.972301	0.054829	0.0072	0.5742
15	1.917473	0.008287	0.0070	0.5811
16	1.909186	0.026258	0.0069	0.5881
17	1.882928	0.062965	0.0069	0.5949
18	1.819963	0.027895	0.0066	0.6016
19	1.792068	0.047108	0.0065	0.6081
20	1.744960	0.029705	0.0064	0.6144
21	1.715254	0.036385	0.0062	0.6207
22	1.678870	0.008549	0.0061	0.6268
23	1.670321	0.055542	0.0061	0.6329
24	1.614779	0.044727	0.0059	0.6388
25	1.570052	0.006675	0.0057	0.6445
26	1.563377	0.032441	0.0057	0.6502
27	1.530937	0.006604	0.0056	0.6557
28	1.524333	0.053894	0.0055	0.6613
29	1.470438	0.007618	0.0054	0.6666
30	1.462820	0.016131	0.0053	0.6720
31	1.446689	0.030582	0.0053	0.6772
32	1.416108	0.030952	0.0052	0.6824
33	1.385155	0.013302	0.0050	0.6874
34	1.371854	0.015797	0.0050	0.6924
35	1.356057	0.017671	0.0049	0.6973
36	1.338386	0.040328	0.0049	0.7022
37	1.298058	0.015099	0.0047	0.7069
38	1.282959	0.013870	0.0047	0.7116

TABLE 11.9 (continued)

	Eigenvalue	Difference	Proportion	Cumulative
39	1.269090	0.028553	0.0046	0.7162
40	1.240537	0.007239	0.0045	0.7207
41	1.233298	0.011673	0.0045	0.7252
42	1.221625	0.017449	0.0044	0.7297
43	1.204176	0.023962	0.0044	0.7341
44	1.180214	0.024571	0.0043	0.7384
45	1.155643	0.007480	0.0042	0.7426
46	1.148163	0.014407	0.0042	0.7467
47	1.133756	0.019777	0.0041	0.7509
48	1.113979	0.020058	0.0041	0.7549
49	1.093921	0.022573	0.0040	0.7589
50	1.071348	0.025039	0.0039	0.7628
51	1.046309	0.018694	0.0038	0.7666
52	1.027615	0.004263	0.0037	0.7704
53	1.023352	0.005865	0.0037	0.7741
54	1.017487	0.020677	0.0037	0.7778
55	1.996811	0.006145	0.0036	0.7814
56	0.990666	0.009931	0.0036	0.7850
57	0.980734	0.020171	0.0036	0.7886
58	0.960563	0.009233	0.0035	0.7921
59	0.952330	0.019179	0.0035	0.7956
60	0.933152	0.011909	0.0034	0.7990
61	0.921243	0.018664	0.0034	0.8023
62	0.902579	0.006758	0.0033	0.8056
63	0.895821	0.015330	0.0033	0.8088
64	0.880490	0.009850	0.0032	0.8121
65	0.870640	0.007870	0.0032	0.8152
66	0.862770	0.018208	0.0031	0.8184
67	0.844563	0.011827	0.0031	0.8214
68	0.832736	0.018418	0.0030	0.8245
69	0.814318	0.007951	0.0030	0.8274
70	0.806367	0.012882	0.0029	0.8304
71	0.793485	0.007017	0.0029	0.8333
72	0.786468	0.009786	0.0029	0.8361
73	0.776682	0.009990	0.0028	0.8389
74	0.766691	0.009988	0.0028	0.8417
75	0.756704	0.010024	0.0028	0.8445
76	0.746680	0.006847	0.0027	0.8472
77	0.739833	0.026006	0.0027	0.8499
78	0.713828	0.004344	0.0026	0.8525

TABLE 11.9 (continued)

	Eigenvalue	Difference	Proportion	Cumulative
79	0.709483	0.013291	0.0026	0.8551
80	0.696192	0.016386	0.0025	0.8576
81	0.679807	0.004019	0.0025	0.8601
82	0.675788	0.008054	0.0025	0.8625
83	0.667734	0.004049	0.0024	0.8650
84	0.663684	0.012973	0.0024	0.8674
85	0.650712	0.015112	0.0024	0.8698
86	0.635600	0.007962	0.0023	0.8721
87	0.627638	0.002422	0.0023	0.8744
88	0.625216	0.014384	0.0023	0.8766
89	0.610832	0.009109	0.0022	0.8789
90	0.601724	0.014138	0.0022	0.8811
91	0.587586	0.002317	0.0021	0.8832
92	0.585269	0.007628	0.0021	0.8853
93	0.577641	0.011298	0.0021	0.8874
94	0.566343	0.006052	0.0021	0.8895
95	0.560291	0.006837	0.0020	0.8915
96	0.553453	0.007703	0.0020	0.8935
97	0.545750	0.011305	0.0020	0.8955
98	0.534446	0.003239	0.0019	0.8975
99	0.531207	0.009380	0.0019	0.8994
100	0.521827	0.005073	0.0019	0.9013
101	0.516754	0.004996	0.0019	0.9032
102	0.511758	0.008401	0.0019	0.9050
103	0.503357	0.012752	0.0018	0.9069
104	0.490605	0.003380	0.0018	0.9087
105	0.487225	0.004856	0.0018	0.9104
106	0.482369	0.006248	0.0018	0.9122
107	0.476121	0.006387	0.0017	0.9139
108	0.469735	0.004690	0.0017	0.9156
109	0.465044	0.008630	0.0017	0.9173
110	0.456414	0.010277	0.0017	0.9190
111	0.446137	0.009040	0.0016	0.9206
112	0.437097	0.005565	0.0016	0.9222
113	0.431532	0.002897	0.0016	0.9238
114	0.428635	0.010937	0.0016	0.9253
115	0.417698	0.006600	0.0015	0.9269
116	0.411098	0.002088	0.0015	0.9284
117	0.409010	0.005099	0.0015	0.9298
118	0.403911	0.008075	0.0015	0.9315

TABLE 11.9 (continued)

	Eigenvalue	Difference	Proportion	Cumulative
119	0.395836	0.005315	0.0014	0.9328
120	0.390521	0.005969	0.0014	0.9342
121	0.384552	0.006602	0.0014	0.9356
122	0.377951	0.003804	0.0014	0.9369
123	0.374146	0.007691	0.0014	0.9385
124	0.366456	0.003382	0.0013	0.9396
125	0.363074	0.010727	0.0013	0.9410
126	0.352347	0.007291	0.0013	0.9422
127	0.345055	0.007671	0.0013	0.9435
128	0.337384	0.001759	0.0012	0.9447
129	0.335625	0.007077	0.0012	0.9460
130	0.328548	0.002506	0.0012	0.9471
131	0.326041	0.006523	0.0012	0.9483
132	0.319518	0.002249	0.0012	0.9495
133	0.317269	0.004629	0.0012	0.9507
134	0.312641	0.003989	0.0011	0.9518
135	0.308651	0.006859	0.0011	0.9529
136	0.301792	0.003576	0.0011	0.9540
137	0.298216	0.004245	0.0011	0.9551
138	0.293971	0.004938	0.0011	0.9562
139	0.289033	0.008273	0.0011	0.9572
140	0.280760	0.001395	0.0010	0.9582
141	0.279365	0.005252	0.0010	0.9593
142	0.274114	0.002103	0.0010	0.9603
143	0.272011	0.005439	0.0010	0.9612
144	0.266571	0.001502	0.0010	0.9622
145	0.265070	0.007660	0.0010	0.9632
146	0.257410	0.003877	0.0009	0.9641
147	0.253533	0.006830	0.0009	0.9650
148	0.246703	0.002606	0.0009	0.9659
149	0.244098	0.001671	0.0009	0.9668
150	0.242427	0.005137	0.0009	0.9677
151	0.237290	0.003631	0.0009	0.9686
152	0.233659	0.006163	0.0009	0.9694
153	0.227496	0.006219	0.0008	0.9703
154	0.221277	0.005003	0.0008	0.9711
155	0.216274	0.005441	0.0008	0.9718
156	0.210834	0.001056	0.0008	0.9726
157	0.209778	0.004082	0.0008	0.9734
158	0.205696	0.002158	0.0007	0.9741

TABLE 11.9 (continued)

	Eigenvalue	Difference	Proportion	Cumulative
159	0.203537	0.003689	0.0007	0.9749
160	0.199849	0.003844	0.0007	0.9756
161	0.196005	0.001857	0.0007	0.9763
162	0.194148	0.006539	0.0007	0.9770
163	0.187609	0.004907	0.0007	0.9777
164	0.182702	0.004157	0.0007	0.9784
165	0.178544	0.003877	0.0006	0.9790
166	0.174667	0.000957	0.0006	0.9796
167	0.173710	0.003016	0.0006	0.9803
168	0.170694	0.003600	0.0006	0.9809
169	0.167094	0.005015	0.0006	0.9815
170	0.162079	0.000706	0.0006	0.9821
171	0.161373	0.007336	0.0006	0.9827
172	0.154037	0.000777	0.0006	0.9832
173	0.153260	0.003955	0.0006	0.9838
174	0.149306	0.003720	0.0005	0.9843
175	0.145586	0.002315	0.0005	0.9849
176	0.143271	0.003517	0.0005	0.9854
177	0.139754	0.003575	0.0005	0.9859
178	0.136180	0.003731	0.0005	0.9864
179	0.132449	0.002307	0.0005	0.9869
180	0.130142	0.002969	0.0005	0.9874
181	0.127173	0.002395	0.0005	0.9878
182	0.124778	0.002450	0.0005	0.9883
183	0.122328	0.001656	0.0004	0.9887
184	0.120672	0.005197	0.0004	0.9892
185	0.115475	0.003086	0.0004	0.9896
186	0.112389	0.003950	0.0004	0.9900
187	0.108439	0.002335	0.0004	0.9904
188	0.106103	0.003142	0.0004	0.9908
189	0.102962	0.001312	0.0004	0.9911
190	0.101650	0.000805	0.0004	0.9915
191	0.100845	0.003122	0.0004	0.9919
192	0.097724	0.003344	0.0004	0.9922
193	0.094380	0.003026	0.0003	0.9926
194	0.091354	0.001882	0.0003	0.9929
195	0.089472	0.001264	0.0003	0.9932
196	0.088208	0.002279	0.0003	0.9936
197	0.085929	0.001457	0.0003	0.9939
198	0.084472	0.005853	0.0003	0.9942

TABLE 11.9 (continued)

	Eigenvalue	Difference	Proportion	Cumulative
199	0.078620	0.001986	0.0003	0.9945
200	0.076634	0.002188	0.0003	0.9947
201	0.074446	0.000883	0.0003	0.9950
202	0.073563	0.002241	0.0003	0.9953
203	0.071322	0.003280	0.0003	0.9955
204	0.068042	0.002338	0.0002	0.9958
205	0.065704	0.003322	0.0002	0.9960
206	0.062382	0.002123	0.0002	0.9963
207	0.060259	0.002611	0.0002	0.9965
208	0.057648	0.001309	0.0002	0.9967
209	0.056339	0.002235	0.0002	0.9969
210	0.054104	0.000676	0.0002	0.9971
211	0.053427	0.002695	0.0002	0.9973
212	0.050732	0.001721	0.0002	0.9975
213	0.049011	0.001766	0.0002	0.9976
214	0.047245	0.001044	0.0002	0.9978
215	0.046201	0.001498	0.0002	0.9980
216	0.044703	0.000514	0.0002	0.9981
217	0.044189	0.001025	0.0002	0.9983
218	0.043164	0.001789	0.0002	0.9985
219	0.041375	0.000696	0.0002	0.9986
220	0.040680	0.001966	0.0001	0.9988
221	0.038714	0.002158	0.0001	0.9989
222	0.036556	0.002763	0.0001	0.9990
223	0.033793	0.001862	0.0001	0.9992
224	0.031931	0.001158	0.0001	0.9993
225	0.030773	0.000464	0.0001	0.9994
226	0.030309	0.001476	0.0001	0.9995
227	0.028833	0.003718	0.0001	0.9996
228	0.025115	0.001682	0.0001	0.9997
229	0.023434	0.000717	0.0001	0.9998
230	0.022717	0.000848	0.0001	0.9999
231	0.021869	0.001633	0.0001	0.9999
232	0.020236	0.001041	0.0001	1.0000
233	0.019195	0.000946	0.0001	1.0001
234	0.018248	0.002004	0.0001	1.0002
235	0.016245	0.001035	0.0001	1.0002
236	0.015210	0.000957	0.0001	1.0003
237	0.014253	0.001664	0.0001	1.0003
238	0.012589	0.000187	0.0000	1.0004

TABLE 11.9 (Continued)

	Eigenvalue	Difference	Proportion	Cumulative
239	0.012402	0.001977	0.0000	1.0004
240	0.010425	0.000966	0.0000	1.0005
241	0.009459	0.000450	0.0000	1.0005
242	0.009009	0.002109	0.0000	1.0005
243	0.006900	0.000361	0.0000	1.0005
244	0.006539	0.001594	0.0000	1.0006
245	0.004946	0.000441	0.0000	1.0006
246	0.004505	0.000347	0.0000	1.0006
247	0.004159	0.000750	0.0000	1.0006
248	0.003409	0.000957	0.0000	1.0006
249	0.002452	0.001279	0.0000	1.0006
250	0.001173	0.000150	0.0000	1.0006
251	0.001022	0.000604	0.0000	1.0006
252	0.000418	0.001556	0.0000	1.0006
253	-0.001138	0.000539	-0.0000	1.0006
254	-0.001677	0.000672	-0.0000	1.0006
255	-0.002349	0.000626	-0.0000	1.0006
256	-0.002975	0.000670	-0.0000	1.0006
257	-0.003645	0.000515	-0.0000	1.0006
258	-0.004161	0.000453	-0.0000	1.0006
259	-0.004614	0.000606	-0.0000	1.0006
260	-0.005220	0.000344	-0.0000	1.0006
261	-0.005565	0.000585	-0.0000	1.0005
262	-0.006150	0.000274	-0.0000	1.0005
263	-0.006424	0.000519	-0.0000	1.0005
264	-0.006942	0.000483	-0.0000	1.0005
265	-0.007426	0.000211	-0.0000	1.0004
266	-0.007637	0.000354	-0.0000	1.0004
267	-0.007990	0.000105	-0.0000	1.0004
268	-0.008096	0.000272	-0.0000	1.0003
269	-0.008367	0.000311	-0.0000	1.0003
270	-0.008679	0.000143	-0.0000	1.0003
271	-0.008822	0.000228	-0.0000	1.0003
272	-0.009050	0.000212	-0.0000	1.0002
273	-0.009262	0.000201	-0.0000	1.0002
274	-0.009463	0.000221	-0.0000	1.0002
275	-0.009684	0.000659	-0.0000	1.0001
276	-0.010343	0.000504	-0.0000	1.0001
277	-0.010847	0.000450	-0.0000	1.0000
278	-0.011296		-0.0000	1.0000

Table 11.10 shows that the value of the SBC measure for six factors is at a minimum, which is consistent with the results of the principal factor analysis that only the first few factors are important to determine the security returns. Therefore, we regard the six factor models as dominant and analyse the results for this case. Although the value of

TABLE 11.10

DIFFERENT CRITERIA FOR ESTIMATING THE BEST NUMBER OF PARAMETERS TO INCLUDE IN A MODEL

Number of factors	Schwarz's Bayesian Criterion	Akaike's Information Criterion	Tucker & Lewis Reliability Coefficient
2	49,458.23	95,865.22	0.49
3	46,909.60	89,756.98	0.57
4	46,489.01	87,908.47	0.60
5	46,119.33	86,165.48	0.63
6	46,117.64	85,162.10	0.65
7	46,347.38	84,625.25	0.66

the AIC measure for seven factors is at a minimum, the choice should be based on the SBC measure as it seems to be less inclined to include trivial factors than the AIC measure (Schwarz, 1978). The Tucker and Lewis reliability coefficient for the six-factor model and seven-factor model are 0.65 and 0.66 respectively. They both indicate that there is a good fit between observed and reproduced matrices.

11.5.3 Factor patterns

Table 11.11 contains the factor pattern for the six significant factors and shows that the highest factor loading is 0.7584 and the lowest factor loading is 0.2405 for the first

TABLE 11.11

UNROTATED FACTOR PATTERN

	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6
US/CO313	0.75842	0.09629	0.11648	-0.10826	0.02358	0.01160
US/CO316	0.73837	0.21633	0.02185	0.03648	0.11039	-0.00095
US/CO161	0.72921	0.08022	0.05552	-0.05976	0.08875	0.17225
US/CO124	0.72415	0.08491	0.18177	-0.07451	-0.08630	-0.10288
US/CO131	0.71111	0.08602	0.07101	-0.08890	0.05890	-0.27212
US/CO315	0.69381	0.24041	-0.02924	-0.02084	0.20398	0.05703
US/CO93	0.68948	-0.00013	0.24999	-0.02784	-0.02010	-0.32866
US/CO100	0.67964	0.12867	0.11158	-0.05572	0.28132	-0.13543
US/CO174	0.67746	0.09863	0.25348	0.08316	0.11274	-0.36924
US/CO312	0.67739	0.06575	0.19503	-0.05049	0.08407	0.07635
US/CO121	0.67680	0.27480	-0.18839	-0.01349	0.09745	0.03125
US/CO39	0.67611	-0.02457	0.23166	-0.13726	-0.17353	0.00002
US/CO343	0.67525	0.04813	0.07258	-0.19866	-0.01322	-0.16590
US/CO30	0.67449	0.03966	0.24472	0.00167	0.24165	0.23665
US/CO241	0.67113	0.06898	0.09823	-0.04437	0.15791	-0.18757
US/CO221	0.66723	-0.00662	0.07252	-0.05484	-0.00005	-0.01285
US/CO341	0.66582	0.08676	0.18423	-0.01940	0.03385	-0.29752
US/CO209	0.66336	0.09701	0.19250	-0.17649	-0.27105	-0.08898
US/CO146	0.66237	0.07403	0.24035	-0.03697	0.13077	0.11958
US/CO104	0.66197	0.14073	0.18756	-0.00149	0.14395	0.02292
US/CO92	0.65955	0.10822	0.04485	-0.10681	0.08534	0.04953
US/CO127	0.65929	0.04371	0.06922	-0.12790	0.24518	-0.15637
US/CO158	0.65876	0.22020	-0.09317	-0.01862	0.02412	-0.00496
US/CO317	0.65821	0.05512	0.07571	-0.02306	0.27940	-0.08199
US/CO129	0.65624	0.03335	0.21372	0.00649	0.12077	-0.03000
US/CO192	0.65608	0.06461	0.12557	-0.03214	0.01507	0.12361
US/CO135	0.65511	0.02637	0.14097	0.04435	0.16630	-0.17988
US/CO322	0.65364	0.03260	0.21133	-0.03250	0.13861	-0.30860
US/CO95	0.65359	-0.00410	0.12120	0.05281	0.09192	-0.32103
US/CO19	0.65302	0.03525	0.32799	-0.15590	-0.12210	0.00523
US/CO339	0.65299	0.01248	0.14862	-0.01675	0.08389	0.05281
US/CO299	0.65202	-0.00324	0.08281	-0.00980	0.10473	-0.19531
US/CO350	0.64925	-0.02485	0.21780	-0.19776	-0.04029	0.08400
US/CO280	0.64921	0.01993	0.11972	-0.04754	0.06756	-0.16918
US/CO58	0.64778	-0.04698	0.24528	-0.03019	-0.00004	-0.19358
US/CO304	0.64729	0.05263	0.23125	-0.03078	0.01313	0.13696
US/CO79	0.64635	-0.02418	0.20169	-0.16451	0.05577	0.05377

TABLE 11.11 (continued)

	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6
US/CO302	0.64611	0.02218	0.16098	0.10212	0.10558	0.20371
US/CO155	0.64589	0.00722	0.16499	-0.14914	0.17109	0.16934
US/CO247	0.64314	0.10383	0.03018	-0.33778	-0.02631	-0.05969
US/CO253	0.64303	0.07734	0.34246	-0.07121	-0.00863	0.15548
US/CO333	0.64148	0.08047	0.09538	-0.16558	0.10837	0.02235
US/CO157	0.63541	0.03112	0.19231	-0.13418	0.20229	0.15283
US/CO70	0.63508	0.08116	0.03174	-0.30036	-0.29638	-0.02225
US/CO347	0.63375	0.02917	0.18103	0.03293	0.17080	-0.07498
US/CO212	0.63299	0.03919	0.15435	0.00547	0.09273	-0.28113
US/CO44	0.63185	0.11438	0.13052	-0.19766	-0.37648	0.04376
US/CO335	0.63136	0.15282	0.08100	-0.15430	-0.36519	0.02649
US/CO203	0.63090	0.08080	0.10995	-0.16847	0.08615	0.13746
US/CO18	0.62719	0.00437	0.23474	0.06636	0.16462	0.17127
US/CO205	0.62651	0.01691	0.17311	0.00174	0.17837	-0.16646
US/CO6	0.62631	0.02512	0.25762	-0.08518	0.09320	-0.01130
US/CO351	0.62435	-0.00018	0.23287	-0.10968	0.20355	0.05691
US/CO23	0.62425	0.11573	0.12866	-0.12293	0.01021	-0.15376
US/CO167	0.62407	0.02286	0.31278	0.10550	-0.06517	-0.25435
US/CO120	0.62379	0.11256	0.10250	0.13148	-0.01410	-0.16186
US/CO236	0.62357	0.04642	0.17799	0.05721	0.17362	-0.10225
US/CO47	0.62333	0.07339	0.13461	-0.17346	0.05322	0.09795
US/CO204	0.62147	0.11729	0.02432	-0.02038	0.07716	0.02527
US/CO252	0.62060	0.09871	0.03180	-0.21882	-0.32114	-0.10867
US/CO14	0.61982	0.06372	0.18656	0.05127	-0.04113	-0.06386
US/CO283	0.61831	0.09779	0.16531	-0.13387	0.06193	0.01627
US/CO171	0.61784	-0.02165	0.24765	-0.13057	-0.14500	0.08841
US/CO91	0.61550	-0.00310	0.10050	-0.22076	-0.08664	0.01950
US/CO153	0.61509	0.01716	0.37671	-0.13446	-0.05603	0.09838
US/CO13	0.61343	0.11754	-0.10599	-0.10354	-0.03101	-0.07672
US/CO154	0.61331	0.00051	0.09403	-0.04862	0.14896	0.14566
US/CO169	0.61226	0.06704	0.03428	-0.04259	0.07214	0.02820
US/CO217	0.60971	-0.02442	0.27171	-0.17170	0.15079	0.09089
US/CO137	0.60853	0.06350	0.18437	0.19646	0.07347	-0.12324
US/CO78	0.60773	-0.04547	0.24248	-0.07602	0.08233	0.26569
US/CO314	0.60687	0.03508	0.24756	0.07872	0.21792	-0.09278
US/CO114	0.60521	0.22154	-0.16433	0.03373	0.06005	-0.05402
US/CO219	0.60390	0.04068	0.13026	0.34542	0.10398	0.03988
US/CO261	0.60282	0.05308	0.12796	-0.03925	0.08581	0.07244
US/CO150	0.60239	0.01101	0.02610	-0.08520	-0.24938	-0.10017

TABLE 11.11 (continued)

	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6
US/CO99	0.59979	-0.02800	0.15785	-0.30359	-0.21115	-0.09809
US/CO152	0.59901	0.06485	0.20262	-0.00032	-0.04337	-0.30037
US/CO287	0.59895	0.10479	-0.08565	-0.10833	-0.10940	0.04209
US/CO180	0.59889	0.11481	0.09755	-0.26357	-0.13209	-0.00330
US/CO49	0.59787	0.09935	-0.03368	0.02170	-0.07628	-0.12954
US/CO41	0.59770	0.09453	-0.11393	-0.02448	0.02026	-0.10407
US/CO307	0.59665	0.21661	0.19034	0.38625	-0.05801	0.01999
US/CO326	0.59636	0.08616	0.28327	-0.00857	0.11450	0.19921
US/CO288	0.59518	0.24912	-0.49407	0.03591	-0.08004	0.03296
US/CO263	0.59508	0.09218	0.10219	-0.16060	-0.13486	0.08715
US/CO1	0.59458	-0.00380	0.12389	-0.16990	0.15503	0.04134
US/CO172	0.59195	0.09120	0.11603	-0.21492	-0.29377	0.07732
US/CO276	0.59029	0.02450	0.24870	0.03903	0.30689	-0.23570
US/CO43	0.58902	0.10863	0.18146	-0.07618	-0.04949	-0.14463
US/CO281	0.58789	0.00309	0.15004	-0.04227	0.01578	-0.20318
US/CO327	0.58626	0.33911	-0.18744	-0.00130	0.01283	0.08061
US/CO35	0.58581	0.18575	0.14215	-0.12219	-0.18222	0.13620
US/CO15	0.58573	0.31927	-0.52892	0.05136	0.06838	0.00032
US/CO28	0.58446	0.25901	-0.44385	-0.00859	0.07357	-0.06486
US/CO325	0.58203	0.02782	0.08862	-0.01820	0.10253	0.00529
US/CO143	0.57984	0.12777	0.23916	-0.03855	0.12667	0.18575
US/CO228	0.57926	0.04491	0.18748	0.08809	0.09996	0.07218
US/CO73	0.57880	0.01480	0.18375	0.09061	-0.00122	0.06464
US/CO34	0.57853	0.14285	0.18501	-0.21568	-0.33517	0.05642
US/CO231	0.57796	0.04738	0.24722	0.13545	0.20400	-0.08314
US/CO117	0.57767	0.10214	0.07023	-0.13160	0.17717	-0.16266
US/CO125	0.57744	0.02836	0.33892	-0.07623	0.18267	0.28048
US/CO36	0.57680	0.25208	-0.14506	0.08100	0.06897	-0.00953
US/CO62	0.57619	0.11342	0.18928	0.02463	0.09882	-0.04686
US/CO3	0.57612	0.11084	0.10829	-0.16492	-0.35200	-0.04102
US/CO215	0.57444	0.04408	0.39499	-0.12471	0.07176	0.04693
US/CO52	0.57434	-0.02204	0.23851	0.12878	0.00748	-0.01785
US/CO332	0.57422	0.12085	0.29446	-0.05236	0.13109	0.20533
US/CO296	0.57208	0.31487	-0.48823	0.03571	-0.01246	-0.00374
US/CO265	0.57183	0.18359	0.01757	-0.21300	-0.31686	-0.09195
US/CO46	0.56937	0.12728	0.23369	-0.13809	0.15261	0.06541
US/CO87	0.56836	0.10793	0.21619	-0.18952	0.02222	0.02416
US/CO113	0.56830	0.02097	0.12251	0.06812	0.23754	-0.11876
US/CO214	0.56628	0.07761	0.15600	-0.04411	0.01603	0.12448

TABLE 11.11 (continued)

	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6
US/CO147	0.56521	-0.02220	0.14296	0.03594	0.18564	-0.03911
US/CO107	0.56519	0.02514	-0.04215	0.45691	-0.18667	-0.07573
US/CO82	0.56461	0.02745	0.24874	-0.09608	0.26433	-0.03846
US/CO255	0.56355	0.14968	0.15259	-0.05934	-0.48505	0.02959
US/CO202	0.56273	0.06820	0.16030	-0.05825	0.07548	0.25306
US/CO197	0.56233	-0.01906	0.17862	-0.03152	0.03806	0.12450
US/CO186	0.56118	0.11853	0.06077	-0.05259	-0.03576	-0.28586
US/CO74	0.56025	0.33871	-0.43778	0.04171	-0.12030	-0.07777
US/CO165	0.55946	-0.47477	-0.07115	0.02540	-0.03807	0.04756
US/CO275	0.55932	0.01738	0.30649	-0.03717	0.04654	0.30826
US/CO182	0.55827	0.20587	-0.40173	0.02184	0.04887	0.01657
US/CO310	0.55698	-0.00130	0.42107	-0.12005	-0.01356	0.00760
US/CO67	0.55625	0.20216	-0.51331	-0.04136	0.06343	-0.01907
US/CO268	0.55398	0.22884	-0.39383	-0.07156	0.13418	-0.02464
US/CO279	0.55335	0.09206	0.22093	0.04238	0.12181	0.10233
US/CO16	0.55241	0.19253	0.15161	-0.19075	-0.45390	0.05357
US/CO75	0.55241	0.16777	0.13283	0.01150	-0.01252	0.17593
US/CO224	0.55207	0.27256	-0.48879	-0.03148	0.02679	0.05584
US/CO225	0.54950	0.31483	-0.43223	-0.07816	0.04814	0.00579
US/CO190	0.54919	0.05186	0.10272	-0.05672	0.33464	0.27643
US/CO80	0.54895	0.03457	0.26050	0.06536	0.23903	0.01243
US/CO122	0.54863	0.15910	0.04999	-0.01529	0.20063	0.00971
US/CO274	0.54829	0.14385	0.01066	-0.11345	-0.15192	0.00340
US/CO177	0.54827	0.32533	-0.39040	-0.00621	-0.00034	-0.02537
US/CO109	0.54779	0.26021	-0.48164	-0.02543	-0.04860	0.13747
US/CO179	0.54738	0.10928	0.17148	-0.18586	-0.38637	0.05199
US/CO306	0.54584	0.02026	0.33414	-0.16861	0.07305	0.24071
US/CO56	0.54482	0.12202	0.05818	-0.08575	-0.02136	0.05883
US/CO40	0.54452	0.13433	0.23089	-0.05211	0.08835	0.29025
UK/CO34	0.54414	-0.48548	-0.17434	-0.00910	0.05005	0.06834
US/CO26	0.54341	0.08656	0.04820	0.23548	0.11278	0.04652
US/CO258	0.54279	0.14620	0.10913	-0.10924	-0.35609	-0.14536
US/CO94	0.54164	0.07124	0.34466	0.35235	-0.06936	-0.02369
US/CO324	0.54091	0.02650	0.36156	-0.16359	-0.12021	0.09013
US/CO60	0.53951	0.20081	-0.53882	-0.01902	0.10955	-0.06767
US/CO9	0.53728	0.30366	-0.53085	0.02600	-0.04985	0.00904
US/CO262	0.53705	0.08292	0.19967	0.17490	0.04178	0.03081
US/CO53	0.53649	0.28233	-0.51302	-0.00663	0.10350	0.17728
US/CO140	0.53566	0.21625	-0.08402	0.00973	-0.07151	-0.00003

TABLE 11.11 (continued)

	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6
US/CO148	0.53403	0.08645	0.10586	-0.12803	0.12682	0.08989
UK/CO55	0.53318	-0.44506	-0.12673	-0.04089	0.10435	-0.06842
US/CO334	0.53082	0.04897	0.15655	-0.04385	0.14290	0.27123
US/CO248	0.52888	0.24148	-0.50549	-0.01605	0.01720	-0.03225
UK/CO8	0.52745	-0.43025	-0.15871	-0.02929	0.01086	0.06914
US/CO10	0.42493	-0.01812	0.24860	0.07807	0.29065	-0.29443
US/CO184	0.52476	0.24735	-0.50223	-0.03262	0.14529	-0.02180
US/CO115	0.52469	0.26479	-0.48547	0.01305	-0.00140	0.13952
US/CO85	0.52438	0.08582	0.27218	0.07260	-0.09050	-0.15570
US/CO320	0.52437	0.11854	0.03526	-0.00852	-0.00178	-0.08936
UK/CO51	0.52298	-0.41318	-0.08221	0.31544	-0.00198	0.11419
US/CO344	0.52135	0.04769	-0.15509	-0.04971	0.02426	0.00017
US/CO243	0.52069	0.23475	-0.31557	0.06850	0.10586	0.03526
US/CO234	0.51872	0.27924	-0.45178	0.04941	0.07694	-0.04666
US/CO90	0.51747	0.09789	0.08641	-0.06487	0.04412	0.11599
US/CO284	0.51702	0.41382	-0.45349	0.08885	-0.05150	0.07808
US/CO207	0.51680	0.11110	0.19841	-0.22699	-0.46857	-0.00539
US/CO216	0.51403	0.01518	0.15553	0.48864	-0.10637	0.18243
US/CO159	0.51379	0.35687	-0.39081	0.07558	-0.15783	0.01676
US/CO269	0.51268	0.18073	-0.07023	-0.11970	-0.12974	0.03585
US/CO51	0.51149	0.08653	-0.13812	-0.08430	-0.07673	-0.06384
US/CO289	0.51117	0.06433	0.19629	-0.22934	-0.39144	0.01976
UK/CO28	0.50730	-0.49537	-0.07720	-0.00651	-0.07799	0.07140
US/CO11	0.50669	-0.01607	0.26686	0.22909	0.13903	-0.16279
US/CO101	0.50660	0.01239	0.14351	-0.01942	-0.07714	-0.07617
US/CO7	0.50602	-0.04389	0.29606	0.14955	0.16390	-0.26442
US/CO226	0.50517	-0.00071	0.23723	0.22504	0.11472	-0.14705
US/CO282	0.50316	0.05066	0.12158	0.01280	-0.05600	0.12513
US/CO246	0.50199	0.22375	-0.01277	0.37767	-0.11334	-0.02643
US/CO298	0.50165	0.29419	-0.44164	0.05242	-0.07991	0.05234
US/CO266	0.50083	0.30195	-0.38354	0.14620	0.00899	0.00583
US/CO77	0.50044	0.14458	-0.18944	0.25749	0.10149	0.06237
US/CO145	0.49727	0.08436	0.34644	0.38795	-0.25238	0.00179
US/CO164	0.49677	0.32239	-0.45135	0.04802	-0.02883	0.00540
US/CO187	0.49676	0.07270	-0.07698	0.00547	0.05772	0.01945
US/CO162	0.49587	0.27313	-0.43444	0.04872	0.00333	-0.11337
US/CO330	0.49571	0.38752	-0.37023	0.03855	-0.02705	-0.04315
US/CO297	0.49421	0.29232	-0.39112	-0.04398	0.10010	0.04813
US/CO337	0.39103	0.26868	-0.32631	-0.00172	0.07410	-0.03642

TABLE 11.11 (continued)

	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6
US/CO72	0.48842	0.13605	-0.07808	0.17776	0.00126	0.01604
US/CO42	0.48448	0.26044	-0.45334	0.01082	0.15173	0.03485
US/CO21	0.48118	0.07452	0.10543	0.16617	-0.07952	-0.00991
US/CO57	0.47947	0.12178	0.19449	0.13540	0.01632	-0.04661
UK/CO15	0.47914	-0.41633	-0.11854	-0.01652	0.01007	-0.00055
US/CO309	0.47770	0.15935	0.08420	0.41441	-0.14794	0.02038
US/CO151	0.47662	0.03526	0.26265	0.42806	-0.14289	0.16007
UK/CO19	0.47427	-0.47425	-0.16706	0.01126	0.03194	-0.00381
US/CO349	0.47407	0.00560	0.04272	-0.12880	-0.04321	0.00578
US/CO244	0.47280	0.42407	-0.45400	0.05762	-0.07568	-0.03024
US/CO295	0.47235	0.10302	0.09663	0.32343	-0.02648	0.11003
UK/CO23	0.47044	-0.42979	-0.17373	-0.04083	-0.03304	0.12220
UK/CO20	0.47006	-0.46558	-0.26860	-0.02110	0.02498	0.01222
US/CO230	0.46941	0.13936	0.18046	0.11831	-0.02003	0.27167
US/CO33	0.46825	0.05499	0.14771	0.00524	0.04237	0.20638
US/CO311	0.46509	0.37470	-0.40268	0.05490	-0.21737	0.07041
US/CO110	0.46328	-0.01411	0.23025	-0.02974	0.12826	0.35437
US/CO185	0.46193	0.03673	0.23446	0.35018	-0.33925	0.07478
US/CO132	0.45593	0.00799	-0.05872	0.15679	-0.01149	0.01053
US/CO290	0.44884	0.05280	0.35685	0.36038	-0.33754	0.03266
UK/CO22	0.42694	-0.27377	-0.13677	0.03523	0.08618	0.04843
UK/CO10	0.42662	-0.35145	-0.08481	0.32230	-0.00943	0.10708
US/CO278	0.41921	0.14380	-0.12827	0.02856	0.00760	0.10774
US/CO329	0.41557	0.03592	0.22218	-0.11199	-0.34208	-0.01338
US/CO142	0.41101	0.07668	0.15899	-0.03581	0.10098	0.16672
UK/CO58	0.40718	-0.39474	-0.00903	0.20982	-0.12978	0.08121
US/CO173	0.40169	0.05244	0.28264	0.15655	-0.05002	-0.00748
UK/CO11	0.39850	-0.37238	-0.00831	0.19575	-0.08730	-0.01271
UK/CO24	0.38369	-0.37500	-0.11336	0.07229	-0.01135	-0.01773
UK/CO24	0.37369	-0.35593	-0.06041	-0.05069	0.00147	0.01199
UK/CO35	0.24053	-0.30216	0.09661	0.20341	-0.14533	-0.04543
UK/CO13	0.30669	-0.34020	-0.10800	-0.06676	0.02722	-0.03731
UK/CO7	0.24438	-0.37850	0.00150	0.07449	0.05641	0.03082
UK/CO39	0.38213	-0.40042	-0.20782	-0.02699	0.02787	0.03262
UK/CO4	0.38511	-0.41637	-0.12371	-0.08277	-0.00432	0.05789
UK/CO48	0.35520	-0.44126	-0.19317	0.04044	0.00995	0.08838
UK/CO16	0.37326	-0.44654	-0.12903	0.02850	0.05752	-0.01776
UK/CO47	0.31493	-0.46182	0.01853	0.03900	0.03068	0.00623
UK/CO33	0.31808	-0.46552	-0.06676	-0.09736	0.03247	-0.01423

TABLE 11.11 (Continued)

	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6
UK/CO46	0.45154	-0.47404	-0.19836	0.02812	-0.00615	-0.04722
UK/CO#5	0.41763	-0.47829	-0.19108	0.02389	-0.08124	0.10737
UK/CO42	0.44224	-0.48950	-0.12349	0.02830	-0.09460	-0.03756
UK/CO25	0.48421	-0.49669	-0.10869	0.01926	0.08225	-0.00526
UK/CO#2	0.37557	-0.49681	-0.13180	0.12570	-0.01988	-0.01505
UK/CO#3	0.42536	-0.49725	-0.18208	0.00343	-0.01055	0.02942
UK/CO36	0.45882	-0.50068	-0.12593	0.02087	0.01973	-0.04695
UK/CO43	0.45222	-0.50525	-0.13060	-0.01961	-0.10400	-0.01097
UK/CO#1	0.46267	-0.50557	-0.10575	-0.03912	-0.01888	0.05908
UK/CO40	0.45104	-0.50750	-0.05451	-0.00513	-0.04831	-0.00292
UK/CO61	0.41826	-0.51044	-0.13769	0.02535	-0.00604	-0.07313
UK/CO50	0.46682	-0.51469	-0.19505	0.02920	-0.02505	-0.01152
UK/CO57	0.48312	-0.51732	-0.16676	-0.01387	0.00299	0.00039
UK/CO52	0.47863	-0.52136	-0.20129	-0.01489	-0.06721	-0.05363
UK/CO14	0.42876	-0.52438	-0.13270	-0.02747	0.01357	-0.01275
UK/CO30	0.46778	-0.53302	-0.29892	0.03068	-0.04356	-0.00868
UK/CO29	0.42617	-0.53563	-0.13437	0.08519	0.04799	-0.03067
UK/CO18	0.45435	-0.53981	-0.17857	-0.01042	0.05001	-0.02925
UK/CO31	0.46694	-0.54161	-0.14404	-0.04453	-0.03787	0.03089
UK/CO59	0.52404	-0.54212	-0.14792	-0.02367	0.00494	0.13370
UK/CO38	0.46749	-0.54498	-0.19068	-0.03340	-0.08117	0.02350
UK/CO26	0.48277	-0.54597	-0.14530	0.00743	0.03491	0.08016
UK/CO56	0.43053	-0.54691	-0.18570	-0.07074	0.03230	0.07338
UK/CO12	0.41779	-0.55890	-0.22109	-0.05419	-0.07769	-0.01085
UK/CO32	0.49536	-0.56350	-0.14044	0.05361	-0.00821	0.03620
UK/CO21	0.49841	-0.56510	-0.12321	0.04497	-0.03946	-0.03471
UK/CO#9	0.46422	-0.56515	-0.19402	0.05983	-0.06453	-0.03040
UK/CO41	0.44497	-0.56913	-0.14586	0.01374	-0.01944	0.06784
UK/CO54	0.53225	-0.57365	-0.26814	-0.03482	-0.04376	-0.01705
UK/CO25	0.47191	-0.57672	-0.22980	-0.00425	-0.06468	-0.01039
UK/CO60	0.47913	-0.57742	-0.14945	-0.05378	-0.00748	-0.09942
UK/CO49	0.50486	-0.58073	-0.19115	-0.07121	-0.03096	-0.02234
UK/CO#6	0.45965	-0.58980	-0.21526	-0.04771	-0.03610	0.00901
UK/CO44	0.53891	-0.59534	-0.21657	0.02655	-0.05227	-0.01087
UK/CO37	0.50024	-0.60533	-0.21179	-0.00674	-0.04887	-0.03636
UK/CO17	0.47390	-0.62038	-0.13128	-0.01482	-0.02497	-0.09458
US/CO345	0.42696	0.32963	-0.43793	0.08419	-0.19571	-0.10035
US/CO346	0.45952	0.30061	-0.46821	0.01436	0.03657	-0.01010
US/CO196	0.47801	0.29128	-0.51171	0.03961	0.02535	-0.01210

TABLE 11.11 (Continued)

	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5	FACTOR 6
US/CO32	0.48674	0.32201	-0.53267	-0.02604	-0.00962	0.03762
US/CO20	0.39067	0.12601	0.15187	0.59835	-0.15991	-0.02156
US/CO27	0.41714	0.08172	0.17547	0.57533	-0.19930	0.02391
US/CO328	0.42130	0.04790	0.25211	0.57068	-0.11126	0.08337
US/CO308	0.49843	0.01009	0.03360	0.50226	0.02132	0.04469
US/CO12	0.45700	-0.02004	0.10950	0.48708	0.02251	0.16725
US/CO259	0.45551	0.04221	0.22483	0.47017	-0.30880	-0.05277

US/CO denotes individual US company;
UK/CO denotes individual UK company.

factor. The first factor is the general factor as all factor loadings have the same sign and are of comparable magnitude. For the second factor, 31% of the stocks have negative loadings, while 69% have positive loadings. For the third factor, 41% of the stocks have negative loadings, 59% of the stocks have positive loadings. For the fourth factor, 57% of the stocks have negative loadings and 43% of the stocks have positive loadings. For the fifth and the sixth factors, 46% of the stocks have negative loadings and 54% of the stocks have positive loadings respectively.

11.5.4 Rotation of factors

The next step in factor analysis involves finding simpler and more easily interpretable factors through rotation, while keeping the number of factors and communalities of each stock fixed.

As the APT explicitly assumes that the factors are uncorrelated, orthogonal rotation is used here. The variances explained by the factors with and without weights are shown in

Table 11.12. The quartimax rotation is the rotation chosen because it aims to make the variables as simple as possible by maximizing the variance of the loadings on each factor in order to achieve the simple structure. The quartimax rotation shows that the first factor is still the dominant factor. The squared multiple correlations of the stocks with factor 1, factor 2, factor 3, factor 4, factor 5 and factor 6 are 0.99, 0.98, 0.97, 0.93, 0.92 and 0.89 respectively which implies that the six factors are internally consistent and well defined by the stocks.

TABLE 11.12

VARIANCE EXPLAINED BY FACTORS ON DIFFERENT ROTATIONAL TECHNIQUES

Rotational technique	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6
Unrotated						
(weighted)	173.47	43.26	34.95	13.67	11.88	8.07
(unweighted)	84.17	19.81	15.25	6.51	5.54	3.88
Quartimax						
(weighted)	150.03	55.18	45.10	14.29	12.54	8.16
(unweighted)	74.11	25.75	18.79	6.76	5.84	3.91
Varimax						
(weighted)	88.92	66.53	67.03	25.80	23.96	13.07
(unweighted)	44.84	31.29	29.08	12.19	11.55	6.21
Equemax						
(weighted)	60.54	61.78	41.02	41.94	42.76	37.26
(unweighted)	28.36	26.55	20.93	20.57	20.56	18.19

The results in Table 11.13 show that the highest factor loading is 0.7599 and the lowest factor loading is 0.1534 for the first factor. The coefficients of the first factor are all

TABLE 11.13

ROTATED FACTOR PATTERN (QUARTIMAX)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5	FACTOR6
US/CO313	0.75990	0.11045	0.12265	-0.04695	0.05942	-0.00406
US/CO124	0.73505	0.09274	0.05711	0.01882	0.15023	0.11463
US/CO30	0.73257	0.08585	-0.03003	0.00509	-0.18571	-0.22660
US/CO316	0.72835	0.03017	0.25885	0.04812	-0.07106	0.01484
US/CO253	0.72577	0.01821	-0.10650	0.02734	0.07673	-0.14493
US/CO174	0.72354	0.03487	-0.00500	0.08422	-0.09000	0.38594
US/CO146	0.71738	0.05605	-0.01532	-0.00163	-0.06766	-0.11029
US/CO18	0.71577	0.06490	-0.11323	-0.02555	0.21089	0.00197
US/CO161	0.71371	0.13589	0.16376	-0.02186	-0.02259	-0.16346
US/CO93	0.71126	0.13038	-0.04567	0.02697	0.06919	0.34144
US/CO100	0.71068	0.04700	0.11842	-0.10081	-0.20197	0.13876
US/CO312	0.70989	0.08484	0.02417	-0.00590	-0.02022	-0.06707
US/CO104	0.70844	0.01441	0.06057	0.01564	-0.09102	-0.01235
US/CO153	0.70054	0.05085	-0.17235	-0.01864	0.14041	-0.09131
US/CO131	0.69735	0.11651	0.14555	-0.06955	0.01608	0.27820
US/CO129	0.68900	0.09962	-0.00820	0.02692	-0.07613	0.04177
US/CO315	0.68810	0.00651	0.29619	-0.04068	-0.14106	-0.05013
US/CO341	0.68776	0.06630	0.04175	0.01205	0.01640	0.30864
US/CO322	0.68661	0.09587	-0.01107	-0.03566	-0.07889	0.31658
US/CO39	0.68608	0.16060	-0.04691	-0.00245	0.24809	0.00942
US/CO215	0.68534	0.00196	-0.18927	-0.05635	0.01749	-0.04288
US/CO304	0.68531	0.07909	-0.01723	0.04292	0.03811	-0.12493
US/CO157	0.68237	0.09581	-0.00704	-0.12213	-0.10495	-0.15226
US/CO#6	0.67980	0.08127	-0.06309	-0.04438	-0.01872	0.01740
US/CO351	0.67768	0.10616	-0.05684	-0.10350	-0.11521	-0.05486
US/CO155	0.67642	0.13063	0.00880	-0.12738	-0.07192	-0.16898
US/CO209	0.67602	0.06481	0.03457	-0.01392	0.35645	0.09695
US/CO125	0.67548	0.03412	-0.14494	-0.03919	-0.10565	-0.27535
US/CO350	0.67465	0.14978	-0.04875	-0.10118	0.14304	-0.08192
US/CO241	0.67412	0.10716	0.10584	-0.05632	-0.09392	0.20408
US/CO217	0.67338	0.11070	-0.10759	-0.13772	-0.04481	-0.09165
US/CO79	0.67266	0.15003	-0.03509	-0.10721	0.04113	-0.05185
US/CO326	0.67177	0.01227	-0.06137	0.03733	-0.06389	-0.18845
US/CO332	0.66672	-0.03124	-0.06535	-0.00761	-0.06328	-0.19825
US/CO18	0.66672	0.11049	-0.04219	0.08182	-0.14038	-0.15629
US/CO58	0.66670	0.16042	-0.07219	0.02331	0.04696	0.20538
US/CO167	0.66468	0.07491	-0.09615	0.17537	0.06415	0.27633
US/CO127	0.66334	0.12939	0.10957	-0.16536	-0.14871	0.15513

TABLE 11.13 (continued)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5	FACTOR6
US/CO317	0.66319	0.11898	0.11538	-0.07342	-0.21692	0.08716
US/CO310	0.66129	0.03153	-0.23343	-0.02503	0.09363	-0.00152
US/CO135	0.66098	0.12779	0.05282	0.02723	-0.13419	0.19224
US/CO343	0.66085	0.13911	0.11324	-0.14224	0.11848	0.16597
US/CO314	0.65903	0.06821	-0.04645	0.05741	-0.19333	0.10633
US/CO192	0.65897	0.10677	0.08147	0.02873	0.03458	-0.11235
US/CO339	0.65889	0.14080	0.03794	0.01557	-0.03665	-0.04203
US/CO347	0.65852	0.10538	0.01331	0.02601	-0.13426	0.08662
US/CO333	0.65528	0.08912	0.09891	-0.13803	-0.00653	-0.02283
US/CO283	0.65491	0.04588	0.04308	-0.08606	0.02745	-0.01361
US/CO302	0.65396	0.12973	0.03797	0.13054	-0.09837	-0.18517
US/CO92	0.65325	0.09043	0.16376	-0.07769	-0.00514	-0.04560
US/CO143	0.65225	-0.01642	-0.01206	-0.00052	-0.06506	-0.17801
US/CO276	0.65172	0.06541	-0.06129	-0.02106	-0.26347	0.24398
US/CO203	0.65036	0.08250	0.08439	-0.12387	0.01500	-0.13743
US/CO236	0.64952	0.08830	0.02229	0.04515	-0.14520	0.11513
US/CO46	0.64888	-0.02268	-0.01812	-0.11158	-0.05487	-0.06542
US/CO205	0.64872	0.11454	0.01051	-0.01371	-0.13134	0.17546
US/CO47	0.64744	0.07935	0.05823	-0.11792	0.04763	-0.09758
US/CO78	0.64639	0.14614	-0.08292	-0.01718	-0.01656	-0.25832
US/CO280	0.64469	0.14070	0.06274	-0.02662	-0.01096	0.17739
US/CO23	0.64314	0.04555	0.08537	-0.07379	0.07215	0.15764
US/CO212	0.64253	0.10631	0.03968	0.00958	-0.05180	0.29191
US/CO171	0.64235	0.13386	-0.07409	-0.00117	0.21679	-0.07993
US/CO306	0.63954	0.03487	-0.15642	-0.09306	0.02719	-0.24002
US/CO95	0.63774	0.16345	0.05845	0.04717	-0.06914	0.33511
US/CO247	0.63748	0.09136	0.15632	-0.26458	0.17888	0.05004
US/CO82	0.63729	0.05426	-0.07456	-0.11933	-0.17777	0.03872
US/CO221	0.63604	0.19010	0.09834	-0.00430	0.05281	0.02255
US/CO14	0.63319	0.07900	0.02570	0.11678	0.05818	0.08114
US/CO299	0.63051	0.17378	0.08739	-0.01082	-0.06081	0.20498
US/CO275	0.63048	0.05680	-0.12022	0.04032	0.00544	-0.29800
US/CO231	0.62951	0.05076	-0.04386	0.11408	-0.20110	0.10021
US/CO87	0.62939	0.00417	-0.01230	-0.12044	0.08361	-0.02492
US/CO324	0.62543	0.02632	-0.17458	-0.03027	0.20700	-0.08515
US/CO44	0.62242	0.06584	0.08894	0.00329	0.45951	-0.03561
US/CO137	0.62141	0.07493	0.03221	0.20733	-0.10026	0.14725
US/CO152	0.62056	0.06251	0.00275	0.05344	0.07829	0.31335
US/CO43	0.61918	0.02711	0.03239	-0.00561	0.11110	0.15268
US/CO62	0.61877	0.01320	0.02855	0.04468	-0.06309	0.05840

TABLE 11.13 (continued)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5	FACTOR6
US/CO40	0.61715	-0.02832	-0.01081	0.00455	-0.02633	-0.28304
US/CO307	0.61669	-0.05326	0.10579	0.44264	-0.03814	0.01940
US/CO261	0.61558	0.09543	0.05889	-0.00879	-0.03186	-0.06434
US/CO80	0.61491	0.04553	-0.07356	0.04338	-0.21091	-0.00098
US/CO121	0.61292	0.02948	0.44722	-0.01778	-0.04820	-0.02312
US/CO158	0.61250	0.04282	0.33813	0.00826	0.02180	0.01472
US/CO120	0.61207	0.06580	0.12506	0.16692	0.00473	0.18298
US/CO#1	0.61172	0.13712	0.02534	-0.15835	-0.05356	-0.04360
US/CO35	0.61146	-0.02169	0.09889	0.01432	0.25267	-0.12802
US/CO154	0.61113	0.15415	0.06676	-0.03850	-0.09040	-0.13931
US/CO335	0.60956	0.04906	0.15081	0.03378	0.43392	-0.01619
US/CO180	0.60799	0.05251	0.09921	-0.15021	0.25193	0.00067
US/CO279	0.60773	0.01493	-0.01252	0.06567	-0.09207	-0.08989
US/CO204	0.60569	0.08012	0.18204	-0.00077	-0.02979	-0.01638
US/CO228	0.60524	0.07827	0.00659	0.10979	-0.08940	-0.05594
US/CO91	0.60373	0.16046	0.05285	-0.12622	0.19032	-0.01963
US/CO34	0.60333	0.00304	0.03826	-0.02270	0.42753	-0.05103
US/CO70	0.59905	0.12229	0.15204	-0.13609	0.41663	0.02106
US/CO99	0.59766	0.16039	-0.01481	-0.16408	0.33574	0.09505
US/CO202	0.59578	0.05991	0.02660	-0.00827	-0.01614	-0.24609
US/CO263	0.59531	0.07378	0.09222	-0.04756	0.21727	-0.08262
US/CO94	0.59406	0.00494	-0.10578	0.42247	-0.01976	0.06132
US/CO117	0.59395	0.05458	0.11419	-0.14918	-0.08524	0.16097
US/CO52	0.59334	0.12343	-0.06264	0.17639	-0.01871	0.03901
US/CO169	0.59265	0.11789	0.14831	-0.02023	-0.01960	-0.02066
US/CO214	0.59106	0.05624	0.03716	0.01591	0.03507	-0.11558
US/CO291	0.59055	0.12826	0.01530	-0.00709	0.03289	0.21187
US/CO73	0.58825	0.11072	-0.00069	0.14478	0.00281	-0.04600
US/CO219	0.58821	0.11646	0.07857	0.33971	-0.18404	-0.00665
US/CO172	0.58612	0.07460	0.07867	-0.04316	0.38496	-0.07287
US/CO190	0.58477	0.07783	0.06061	-0.10133	-0.26178	-0.27566
US/CO75	0.58098	-0.01664	0.09674	0.07989	0.04463	-0.16319
US/CO113	0.58011	0.10937	0.04495	0.02200	-0.21404	0.12949
US/CO10	0.58000	0.08428	-0.09255	0.01156	-0.26675	0.30436
US/CO252	0.57975	0.10504	0.16184	-0.05852	0.41136	0.11292
US/CO197	0.57854	0.13223	-0.02408	0.01983	0.00648	-0.11495
US/CO325	0.57745	0.12381	0.07794	-0.00618	-0.05797	0.00303
US/CO147	0.57494	0.14179	0.00724	0.01648	-0.15579	0.04924
US/CO85	0.57267	0.00250	-0.06080	0.15166	0.09592	0.17398
US/CO334	0.56729	0.06560	0.01324	-0.01941	-0.08662	-0.26535

TABLE 11.13 (continued)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5	FACTOR6
US/CO122	0.56688	0.00577	0.15680	-0.03942	-0.14778	-0.00456
US/CO262	0.56574	0.03216	0.00884	0.20718	-0.06547	-0.00844
US/CO16	0.56419	-0.03164	0.08609	0.03665	0.52954	-0.04476
US/CO#3	0.56204	0.05804	0.09421	0.01336	0.42185	0.04936
US/CO148	0.56191	0.04724	0.06778	-0.10962	-0.04156	-0.09010
US/CO179	0.55701	0.03073	0.03111	0.01780	0.46159	-0.04413
US/CO#7	0.55693	0.09304	-0.14168	0.12680	-0.17411	0.28193
US/CO255	0.55394	0.01467	0.07975	0.16712	0.51134	-0.01111
US/CO186	0.55148	0.04925	0.13512	-0.01371	0.08574	0.29338
US/CO13	0.54788	0.12306	0.28926	-0.06383	0.09630	0.08111
US/CO265	0.54625	0.01948	0.19831	-0.05557	0.40616	0.09529
US/CO11	0.54568	0.08292	-0.09794	0.21378	-0.17860	0.18621
US/CO56	0.54448	0.04317	0.13308	-0.02599	0.08265	-0.05325
US/CO150	0.54196	0.17961	0.13284	0.03755	0.29274	0.11151
US/CO49	0.54137	0.11521	0.22465	0.07097	0.09540	0.14338
US/CO114	0.53813	0.04831	0.38965	0.02981	-0.03463	0.06479
US/CO145	0.53733	-0.01152	-0.10628	0.51763	0.13715	0.04145
US/CO226	0.53510	0.08001	-0.06583	0.21634	-0.15481	0.17050
US/CO327	0.53447	-0.05160	0.45379	0.02241	0.02493	-0.07154
US/CO287	0.53394	0.12739	0.26432	-0.03197	0.17037	-0.03607
US/CO258	0.53272	0.01692	0.10382	0.05809	0.40626	0.15650
US/CO90	0.53018	0.04443	0.09168	-0.02386	0.01260	-0.11059
US/CO289	0.52659	0.04933	-0.02165	-0.02364	0.47874	-0.01518
US/CO41	0.52517	0.14092	0.28678	-0.01234	0.01879	0.11221
US/CO274	0.52434	0.04575	0.18516	-0.01584	0.21405	0.00275
US/CO36	0.52251	0.00720	0.38205	0.07519	-0.05908	0.02289
US/CO26	0.52201	0.08086	0.14600	0.22339	-0.15709	-0.02261
US/CO110	0.51830	0.07699	-0.09283	0.00738	-0.08171	-0.34763
US/CO57	0.51818	-0.01932	0.01288	0.17073	-0.02895	0.06549
US/CO320	0.51152	0.04770	0.15090	0.02452	0.03573	0.09869
US/CO230	0.50931	-0.02940	0.03050	0.18822	0.01031	-0.25261
US/CO101	0.50719	0.10267	0.00849	0.04953	0.10779	0.08713
US/CO282	0.50571	0.07687	0.04527	0.08491	0.07723	-0.11226
US/CO33	0.49451	0.04910	0.01322	0.05132	-0.01285	-0.19646
US/CO140	0.49180	0.00970	0.30168	0.06119	0.09543	0.01151
US/CO269	0.47357	0.02830	0.26112	-0.03728	0.19295	-0.03186
US/CO21	0.46935	0.05884	0.07483	0.22313	0.04513	0.03254
US/CO295	0.45850	0.03714	0.10261	0.35882	-0.05920	-0.07809
US/CO142	0.45822	0.00374	-0.00606	-0.01115	-0.05456	-0.16200
US/CO173	0.45780	-0.00828	-0.10944	0.21943	0.02152	0.02862

TABLE 11.13 (continued)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5	FACTOR6
US/CO246	0.45609	-0.01884	0.26149	0.42146	0.00638	0.06306
US/CO349	0.45350	0.12833	0.07652	-0.07140	0.10949	-0.00395
US/CO187	0.44482	0.11612	0.22174	0.00921	-0.03151	-0.01146
US/CO329	0.43875	0.03652	-0.07310	0.06267	0.38659	0.02297
US/CO344	0.43616	0.17214	0.28179	-0.04020	0.01689	0.00481
US/CO51	0.43255	0.13201	0.28070	-0.04004	0.12513	0.06844
US/CO72	0.43065	0.06561	0.26089	0.18921	-0.03692	0.00434
US/CO77	0.40863	0.09767	0.36759	0.22050	-0.16094	-0.03937
US/CO132	0.38433	0.16201	0.18036	0.17052	-0.02405	0.00851
US/CO278	0.36425	0.04987	0.27986	0.04548	0.00486	-0.09888
UK/CO44	0.29892	0.77319	0.07349	0.03232	0.03340	0.02384
UK/CO54	0.28417	0.76705	0.12118	-0.03371	0.04605	0.02520
UK/CO37	0.26591	0.76683	0.05287	-0.00356	0.03982	0.04629
UK/CO17	0.26913	0.74276	-0.03118	-0.01605	0.02094	0.10339
UK/CO#6	0.23534	0.74002	0.04922	-0.04506	0.04082	-0.00281
UK/CO25	0.23780	0.73970	0.07410	0.00303	0.05300	0.02017
UK/CO49	0.28841	0.73687	0.04166	-0.06517	0.04756	0.02781
UK/CO30	0.21757	0.72320	0.15351	0.02138	0.02063	0.01965
UK/CO#9	0.24043	0.71669	0.05054	0.06500	0.03612	0.04446
UK/CO60	0.27997	0.71051	0.00098	-0.05912	0.01980	0.10512
UK/CO32	0.29427	0.70471	0.01115	0.05284	-0.01576	-0.02263
UK/CO12	0.19722	0.70300	0.05726	-0.04062	0.08112	0.01658
UK/CO21	0.29956	0.70183	-0.00382	0.05253	0.01716	0.04839
UK/CO38	0.25426	0.69716	0.05097	-0.01089	0.08088	-0.01508
UK/CO59	0.32990	0.69526	0.02847	-0.01501	0.00094	-0.12485
UK/CO41	0.24769	0.69521	-0.00169	0.01817	0.00586	-0.05759
UK/CO26	0.29231	0.68385	0.01556	-0.00240	-0.03991	-0.07082
UK/CO52	0.26407	0.68262	0.07377	-0.00388	0.06268	0.06285
UK/CO56	0.23588	0.67958	0.03154	-0.08230	-0.01378	-0.07049
UK/CO18	0.25731	0.67852	0.03794	-0.03725	-0.04965	0.03598
UK/CO31	0.27552	0.67596	0.01013	-0.03001	0.04583	-0.02383
UK/CO50	0.25795	0.67062	0.07053	0.02588	0.00769	0.02266
UK/CO#5	0.28805	0.66602	0.04538	-0.01865	-0.00191	0.00782
UK/CO34	0.35184	0.65833	0.08031	-0.02181	-0.04362	-0.05962
UK/CO29	0.24054	0.65395	0.00111	0.05562	-0.08113	0.04356
UK/CO20	0.25310	0.64944	0.15134	-0.04309	-0.02139	-0.00603
UK/CO14	0.25083	0.64258	-0.00205	-0.03530	-0.00886	0.01900
UK/CO43	0.26498	0.63805	0.01332	0.01432	0.10016	0.02077
UK/CO#3	0.23164	0.63662	0.05424	-0.00053	0.00228	-0.02094
UK/CO61	0.23564	0.63061	0.00954	0.01528	-0.00903	0.08290

TABLE 11.13 (continued)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5	FACTOR6
UK/CO46	0.25178	0.62974	0.08621	0.01572	-0.00863	0.05737
UK/CO36	0.28157	0.62970	0.01288	0.00823	-0.02888	0.05676
UK/CO#1	0.29332	0.62930	-0.00878	-0.02432	0.02862	-0.05188
UK/CO28	0.33806	0.62852	-0.01432	0.03315	0.07577	-0.05966
UK/CO53	0.31791	0.62576	0.00466	-0.00800	-0.08488	0.01430
UK/CO#5	0.21789	0.62501	0.07138	0.04712	0.06116	-0.09602
UK/CO19	0.28866	0.62461	0.06282	-0.00487	-0.03636	0.01278
UK/CO45	0.39261	0.62361	0.00362	0.05181	0.03086	-0.03364
UK/CO42	0.25830	0.61934	0.01443	0.05435	0.07503	0.05009
UK/CO40	0.29413	0.61185	-0.05406	0.01799	0.04518	0.01300
UK/CO#2	0.19341	0.60726	0.00704	0.11582	-0.03265	0.03114
UK/CO#8	0.35012	0.60001	0.08546	-0.02484	0.00207	-0.06121
UK/CO55	0.37150	0.59825	0.04962	-0.07365	-0.08036	0.07351
UK/CO23	0.29045	0.58883	0.08464	-0.02188	0.04389	-0.11514
UK/CO16	0.17415	0.56937	0.07230	0.02715	-0.03123	-0.07929
UK/CO51	0.34992	0.56901	0.05005	0.31171	-0.10409	-0.08252
UK/CO15	0.32070	0.55868	0.04512	-0.01553	-0.00207	0.00848
UK/CO47	0.21586	0.55458	0.01682	0.00059	-0.06875	0.02554
UK/CO#4	0.20706	0.54334	0.10379	-0.04450	-0.02165	-0.02823
UK/CO27	0.19056	0.53028	-0.06740	-0.10405	-0.00279	0.01364
UK/CO48	0.23742	0.52997	0.02173	-0.07409	0.02990	-0.05573
UK/CO33	0.20812	0.50154	-0.13081	0.03431	-0.04642	0.00287
UK/CO58	0.26857	0.49384	-0.03941	0.25696	0.04981	-0.05557
UK/CO24	0.23443	0.49377	0.04085	0.06906	-0.01579	0.03011
UK/CO10	0.27132	0.48536	0.05490	0.31581	-0.10156	-0.07682
UK/CO11	0.26884	0.46788	-0.03493	0.22291	0.01578	0.03601
UK/CO35	0.25779	0.45178	-0.00833	-0.04130	0.01722	-0.00784
UK/CO#7	0.18605	0.43072	0.02088	-0.07750	-0.00504	0.03802
UK/CO22	0.29711	0.42070	0.11043	0.00877	-0.08834	-0.04005
UK/CO39	0.15343	0.41141	-0.09641	0.05638	-0.08325	-0.02152
UK/CO13	0.16786	0.32305	-0.13405	0.25212	0.06596	0.06800
US/CO15	0.41207	0.07963	0.74340	0.00679	-0.05631	0.00839
US/CO9	0.35530	0.08358	0.72710	0.02065	0.05997	-0.00029
US/CO32	0.31991	0.04883	0.71928	-0.04201	0.03892	-0.03417
US/CO284	0.38021	-0.04388	0.70642	0.09517	0.04537	-0.06500
US/CO296	0.40651	0.06868	0.70351	0.02326	0.02506	0.01322
US/CO244	0.34027	-0.06749	0.69810	0.06430	0.07740	0.04060
US/CO53	0.37565	0.08952	0.69720	-0.04846	-0.07249	-0.17348
US/CO196	0.31136	0.06610	0.68959	0.00452	-0.01776	0.01906
US/CO288	0.40752	0.13957	0.68808	0.04798	0.08618	-0.02133

TABLE 11.13 (continued)

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5	FACTOR6
US/CO60	0.35269	0.16860	0.68398	-0.08450	-0.07748	0.06990
US/CO224	0.38957	0.09666	0.67556	-0.05061	0.00911	-0.05184
US/CO248	0.35373	0.12265	0.67204	-0.04344	0.00987	0.03676
US/CO74	0.40621	0.03132	0.66953	0.06605	0.12573	0.08992
US/CO184	0.36547	0.10871	0.66671	-0.10142	-0.10380	0.02234
US/CO115	0.35951	0.09714	0.66650	0.00484	0.01816	-0.13220
US/CO67	0.37502	0.16592	0.66624	-0.08210	-0.02504	0.02147
US/CO109	0.37943	0.10819	0.66538	-0.01327	0.07674	-0.13118
US/CO164	0.34974	0.02700	0.65705	0.04093	0.03386	0.00409
US/CO346	0.31425	0.03604	0.64917	-0.01843	-0.01858	0.01507
US/CO234	0.37070	0.06695	0.54224	0.00254	-0.06612	0.05428
US/CO225	0.41945	0.03649	0.63994	-0.09728	0.00897	-0.00522
US/CO298	0.34785	0.05311	0.63947	0.06639	0.07927	-0.04109
US/CO28	0.43333	0.10116	0.63925	-0.04720	-0.04011	0.07001
US/CO345	0.27255	0.00193	0.63686	0.12144	0.17459	0.11439
US/CO311	0.33019	-0.03626	0.63396	0.12133	0.20958	-0.05650
US/CO159	0.37876	-0.01173	0.62835	0.12016	0.14873	-0.00187
US/CO42	0.34630	0.06938	0.62285	-0.05565	-0.12504	-0.03186
US/CO162	0.34709	0.06209	0.62025	0.02270	0.00215	0.12209
US/CO177	0.42461	0.01681	0.61389	-0.01118	0.03069	0.03196
US/CO330	0.38923	-0.05919	0.61329	0.03911	0.04072	0.05226
US/CO266	0.36954	0.02464	0.59673	0.12668	-0.03454	0.00995
US/CO297	0.38204	0.02404	0.58223	-0.07938	-0.05397	-0.04705
US/CO182	0.41075	0.12842	0.57652	-0.00289	-0.03008	-0.00873
US/CO268	0.42808	0.09695	0.56975	-0.12029	-0.07594	0.02420
US/CO337	0.39066	0.02379	0.51854	-0.03070	-0.04358	0.04111
US/CO243	0.41388	0.06080	0.50675	0.03082	-0.09769	-0.02570
US/CP20	0.37341	-0.01503	0.06627	0.65223	-0.03199	0.07314
US/CO27	0.39597	0.02676	0.03270	0.65058	0.01284	0.02790
US/CO328	0.42941	0.02837	-0.04911	0.62865	-0.06680	-0.03295
US/CO259	0.43728	0.05836	-0.02242	0.59167	0.15371	0.10099
US/CO216	0.48207	0.11740	0.03840	0.55137	-0.04263	-0.13614
US/CO290	0.48543	0.00098	-0.14087	0.52073	0.22296	0.00993
US/CO151	0.48962	0.05126	-0.05885	0.51665	0.01502	-0.11705
US/CO107	0.45348	0.19213	0.22558	0.51142	0.04087	0.12054
US/CO185	0.45302	0.06086	-0.03890	0.50111	0.22437	-0.03332
US/CO308	0.43373	0.15154	0.13572	0.49561	-0.17101	-0.00185
US/CO12	0.41952	0.14000	0.04599	0.49532	-0.16799	-0.12531
US/CO309	0.45038	0.00079	0.14749	0.47866	0.02474	0.01994
US/CO207	0.53343	0.01243	-0.00041	0.00487	0.55230	0.01178

positive and relatively large, indicating an important general factor among the factors. For the other five factors, some of the stocks have positive loadings on these factors, while some of the stocks have negative loadings. Those five factors retain the mixture of signs in the loadings of the stocks, indicating that the stocks have different reactions to those factors. The absolute factor loadings on the other factors are smaller than that of the first factor. For example, only sixty-one stocks have loadings in excess of 0.30 (in absolute terms). It implies that the second factor is important only for those sixty-one stocks.

11.5.5 Risk measures and average returns

In this section, the individual-security factor loadings are used to explain the cross-sectional variation of individual estimated expected returns. The APT will be supported if the actual returns depend on estimated factor loadings (i.e. factor beta coefficients of the security returns generating model). It will also be interesting to see whether the international version of the APT has greater explanatory power than the domestic version (i.e. the results in chapters 5 and 8) of the APT.

The general approach of these pricing tests is rather straightforward. The factor loadings (beta coefficients) are used as independent variables to explain the cross-sectional variation in the mean returns of all the securities which comprise the sample. The mean returns are used as the proxy for the expected returns.

$$\overline{R_i} = \hat{\lambda}_0 + \hat{\lambda}_1\hat{b}_{i1} + \hat{\lambda}_2\hat{b}_{i2} + \hat{\lambda}_3\hat{b}_{i3} + \hat{\lambda}_4\hat{b}_{i4}$$

$$+ \hat{\lambda}_5\hat{b}_{i5} + \hat{\lambda}_6\hat{b}_{i6} \quad (\lambda_0 \text{ estimated})$$

where λ is the risk premium,

b_{ik} is the factor loading of security i on the k^{th} factor,

$\bar{R}_i$ is the expected returns on the i^{th} security.

The regression results are shown in Tables 11.14 and 11.15. The regression results show that the APT explains 26% (in terms of adjusted R^2) of the variation in mean returns of the sample. This suggests that the explanatory power of the model is fairly good. The result is quite encouraging. On the basis of the adjusted R^2, it appears that the explanatory power of the international version here (i.e. 26%), marginally underperforms the domestic US version of the APT (i.e. 30%) in chapter 8, but outperforms the domestic UK version of the APT (i.e. 10%). The F value is used to test the null hypothesis that all parameters (i.e. λ_1, λ_2, λ_3, λ_4, λ_5, λ_6) are zero except for the intercept (λ_0). The calculated F statistic is greater than the theoretical F value at the five per cent level, indicating that the null hypothesis can be rejected. The explanatory power of the model will be the same whether the rotated or unrotated factor patterns are used as independent variables in the regression analysis. Rotation cannot be used to improve the fit between the observed and reproduced correlation matrices, because all orthogonally rotated solutions are mathematically equivalent to one another and to the solution before rotation.

During the sample period, January 1965 through December 1988, the risk-free coefficient, λ_0 has been equivalent to 19.94% annually (λ_0 is estimated) as shown in Table 11.15. The intercept term is significantly greater than zero at the 5% of significance. The positive intercept term is consistent with the APT model, as one testable implication of the APT is that the intercept term should be positive. While the risk premia of the third and sixth rotated factors are -8.84% and -8.32% during the same period.

It is concluded that there are two factors which are important for pricing. The price associated with an APT factor can be negative if investors want, perhaps for hedging

TABLE 11.14

REGRESSION RESULTS USING UNROTATED FACTOR PATTERNS AS INDEPENDENT VARIABLES

Variable		Parameter Estimate	Standard Error	T for H_0: parameter=0	Prob > |T|
λ_0		0.01527	0.00158	9.676	0.0001
λ_1		-0.00587	0.00286	-2.052	0.0411
λ_2		-0.00201	0.00078	-2.579	0.0104
λ_3		0.00606	0.00082	7.364	0.0001
λ_4		-0.00185	0.00128	-1.441	0.1507
λ_5		-0.00063	0.00129	-0.488	0.6259
λ_6		0.00705	0.00152	4.629	0.0001
R^2	0.2752		F-value	17.144	
Adj R^2	0.2591		Prob>F	0.0001	

purposes, to hold stocks whose returns increase when there is an unanticipated negative realization of that factor (and whose returns decrease when there is an unanticipated positive realization). This negative price reflects an attribute that investors find desirable.

The results of this standard testing approach show that there are six factors in the international stock market, but that only two factors and the risk-free coefficient are important for pricing.

11.5.6 Discussion

The above sections estimate the number of the international stock market factors using principal factor and maximum-likelihood methods of factor analysis. The results show that

TABLE 11.15

REGRESSION RESULTS USING ROTATED FACTOR PATTERNS AS INDEPENDENT VARIABLES

Variable	Parameter Estimate	Standard Error	T for H_0: parameter=0	Prob > |T|
λ_0	0.01527	0.00158	9.676	0.0001
λ_1	-0.00345	0.00236	-1.463	0.1446
λ_2	-0.00209	0.00156	-1.341	0.1810
λ_3	-0.00768	0.00115	-6.697	0.0001
λ_4	-0.00070	0.00141	-0.499	0.6182
λ_5	0.00104	0.00125	0.837	0.4036
λ_6	-0.00721	0.00150	-4.804	0.0001
R^2	0.2751	F-value	17.144	
Adj R^2	0.2591	Prob>F	0.0001	

there are six international stock market factors. It has been shown by principal factor analysis that the first factor accounts for nearly 31% of the variation while the second factor accounts for nearly 8%. By maximum-likelihood factor analysis, the results confirmed the earlier findings by principal factor analysis that the first factor is an important factor among the stocks. The coefficients of the first factor are all positive and most of them are statistically significant (i.e. loadings in excess of 0.30 in absolute terms).

The validity and applicability of the APT to the international stock market are also evaluated. One of the important implications of the APT is that the intercept term (λ_0) should be significantly different from zero. The APT also implies that if k factors are responsible for driving the individual asset returns through time, then there should be a risk premium attached to each of these factors.

The individual-security factor loading estimates were then used as independent variables to explain the cross-sectional variation in the mean returns of the securities that comprise the sample. The mean returns (as the proxy for the expected returns) for securities were regressed against the factor loadings. The third and the sixth rotated factors and the intercept term are priced. It is clear from the cross-sectional regression results that the APT has some empirical power (in terms of adjusted R^2). The APT explains nearly 26% of the variation in the twenty-four years average returns as compared with only 11% of that of the UK results in chapter 5. The results here are comparable to the US results in chapter 8 (i.e. the APT explains 30% of the variation of the US stocks). The results here are quite encouraging as modelling twenty-four years returns is not an easy task, because there is high variation in the measures of risk and return when long time periods are used. In this chapter as in others, it has been assumed that the non-stationarity problem does not exist. Thus, risk and expected returns were assumed not to have changed during the twenty-four years period. Therefore by taking no measures to mitigate the non-stationarity problem, these tests are biased toward finding that the risk measures are not significant. The overall results obtained here seem to suggest that the APT pricing relationship is supported by the testing methodology.

11.6 International Economic Factors

In estimating the number of macroeconomic factors affecting the UK and US economies, two factor extraction techniques were used :

(i) Principal factor analysis to get an approximate idea of the number of factors before proceeding to a maximum-likelihood factor analysis.

(ii) Maximum-likelihood factor analysis is used to acquire more precisely the number of factors, their factor loadings and factor scores. After this, the macroeconomic factor scores are compared with the factor scores of the common factors of the security returns. This is the subject of section 11.7.

11.6.1 Principal factor analysis

As discussed in section 11.5.1, we use PFA to get an approximate idea of the number of factors. The major economic variables used here were those that have been used in chapters 6 and 9. Before actually extracting any factor, it is useful to assess the suitability of the data for analysis.

The results of applying PFA to the set of returns on the UK and US economic variables show that the overall Kaiser's measure of sampling adequacy is 0.74 (Table 11.16) and the average communality value (SMC) is 0.63 which implies that the data are suitable for factor analysis. The ones in the positive diagonal of the correlation matrix are replaced by the communality estimates in preparation for factor extraction. The communalities are shown in Table 11.17 and reveal that, the average communality value is 0.63. This mean communality is acceptable and indicates that the variables are correlated with each other, so that the data are acceptable for factor analysis.

Table 11.18 shows the eigenvalues of the reduced correlation matrix. Based on the eigenvalue 1 criterion, eleven factors are retained, and, those eleven factors account for 86.77% of common variance. The first two factors account for nearly 37% of the total variance, the third factor accounts for 9.57%, whereas the eleventh factor accounts for 2.8%. The scree test based on the graph of eigenvalues also shows that not more than nine factors should be extracted.

It is interesting to note that the second factor accounts for nearly 96.5% of the total

TABLE 11.16

KAISER'S MEASURE OF SAMPLING ADEQUACY

USECON1	USECON2	USECON3	USECON4	USECON5	USECON6
0.675788	0.708955	0.735103	0.646256	0.827642	0.742028
USECON7	USECON8	USECON9	USECON10	USECON11	USECON12
0.685348	0.748608	0.695460	0.622497	0.818706	0.746290
USECON13	USECON14	USECON15	USECON16	USECON17	USECON18
0.730884	0.794748	0.702704	0.772318	0.689871	0.615103
USECON19	USECON20	USECON21	USECON22	USECON23	USECON24
0.797966	0.779544	0.636803	0.787415	0.887855	0.745581
USECON25	USECON26	USECON27	USECON28	USECON29	USECON30
0.772064	0.762032	0.707941	0.489132	0.643122	0.589510
USECON31	USECON32	USECON33	USECON34	USECON35	USECON36
0.622324	0.620537	0.737519	0.694136	0.827042	0.619950
USECON37	USECON38	UKECON1	UKECON2	UKECON3	UKECON4
0.807660	0.705237	0.607502	0.721682	0.657935	0.673682
UKECON5	UKECON6	UKECON7	UKECON8	UKECON9	UKECON10
0.845881	0.836271	0.880544	0.930699	0.784865	0.635623
UKECON11	UKECON12	UKECON13	UKECON14	UKECON15	UKECON16
0.727720	0.823259	0.636300	0.587024	0.697780	0.424047
UKECON17	UKECON18	UKECON19	UKECON20	UKECON21	
0.727813	0.733416	0.647249	0.752048	0.753043	
Mean MSA	0.74	Min MSA	0.42	Max MSA	0.93

UKECON denotes the UK economic indicators;
USECON denotes the US economic indicators.

TABLE 11.17

PRIOR COMMUNALITY ESTIMATES : SMC

USECON1	USECON2	USECON3	USECON4	USECON5	USECON6
0.538073	0.691121	0.566338	0.519758	0.837508	0.926504
USECON7	USECON8	USECON9	USECON10	USECON11	USECON12
0.917184	0.867638	0.938991	0.581786	0.753077	0.930471
USECON13	USECON14	USECON15	USECON16	USECON17	USECON18
0.672000	0.692281	0.946608	0.515182	0.948747	0.591002
USECON19	USECON20	USECON21	USECON22	USECON23	USECON24
0.479008	0.359234	0.441300	0.808977	0.402673	0.251414
USECON25	USECON26	USECON27	USECON28	USECON29	USECON30
0.631192	0.540846	0.626484	0.543959	0.578826	0.502141
USECON31	USECON32	USECON33	USECON34	USECON35	USECON36
0.329664	0.410039	0.560108	0.552624	0.533894	0.599169
USECON37	USECON38	UKECON1	UKECON2	UKECON3	UKECON4
0.559566	0.598761	0.463261	0.762963	0.576421	0.710041
UKECON5	UKECON6	UKECON7	UKECON8	UKECON9	UKECON10
0.612851	0.922544	0.923305	0.873677	0.966947	0.677210
UKECON11	UKECON12	UKECON13	UKECON14	UKECON15	UKECON16
0.636030	0.734495	0.336952	0.614061	0.682913	0.399037
UKECON17	UKECON18	UKECON19	UKECON20	UKECON21	
0.424409	0.507116	0.386300	0.605587	0.357201	
Mean SMC	0.63	Min SMC	0.25	Max SMC	0.97

UKECON denotes the UK economic indicators;
USECON denotes the US economic indicators.

TABLE 11.18

EIGENVALUES OF THE REDUCED CORRELATION MATRIX

	Eigenvalue	Difference	Proportion	Cumulative
1	6.826141	-	0.1849	0.1849
2	6.582897	0.243243	0.1783	0.3632
3	3.532312	3.050585	0.0957	0.4589
4	3.442898	0.089414	0.0933	0.5522
5	2.359549	1.083350	0.0639	0.6161
6	2.263895	0.095654	0.0613	0.6774
7	1.692632	0.571263	0.0458	0.7232
8	1.646888	0.045745	0.0446	0.7679
9	1.372694	0.274194	0.0372	0.8050
10	1.252600	0.120094	0.0339	0.8390
11	1.059050	0.193550	0.0287	0.8677
12	0.827793	0.231257	0.0224	0.8901
13	0.798106	0.029687	0.0216	0.9117
14	0.691640	0.106466	0.0187	0.9304
15	0.625149	0.066491	0.0169	0.9474
16	0.597316	0.027833	0.0162	0.9635
17	0.570562	0.026754	0.0155	0.9790
18	0.449936	0.120626	0.0122	0.9912
19	0.407821	0.042115	0.0110	1.0022
20	0.390550	0.017271	0.0106	1.0128
21	0.328673	0.061878	0.0089	1.0217
22	0.270756	0.057917	0.0073	1.0290
23	0.251507	0.019248	0.0068	1.0359
24	0.236578	0.014929	0.0064	1.0423
25	0.224893	0.011685	0.0061	1.0484
26	0.193017	0.031876	0.0052	1.0536
27	0.160679	0.032338	0.0044	1.0579
28	0.145287	0.015391	0.0039	1.0619
29	0.127438	0.017850	0.0035	1.0653
30	0.104885	0.022552	0.0028	1.0682
31	0.069291	0.035594	0.0019	1.0700
32	0.064820	0.004471	0.0018	1.0718
33	0.046667	0.018152	0.0013	1.0731
34	0.031553	0.015114	0.0009	1.0739
35	0.002110	0.029443	0.0001	1.0740
36	-0.004054	0.006164	-0.0001	1.0739
37	-0.011426	0.007372	-0.0003	1.0736
38	-0.019340	0.007914	-0.0005	1.0730

TABLE 11.18 (continued)

	Eigenvalue	Difference	Proportion	Cumulative
39	-0.025878	0.006538	-0.0007	1.0723
40	-0.027741	0.001862	-0.0008	1.0716
41	-0.036801	0.009061	-0.0010	1.0706
42	-0.039583	0.002782	-0.0011	1.0695
43	-0.059077	0.019494	-0.0016	1.0679
44	-0.066171	0.007094	-0.0018	1.0661
45	-0.078534	0.012363	-0.0021	1,0640
46	-0.092841	0.014307	-0.0025	1.0615
47	-0.104849	0.012008	-0.0028	1.0586
48	-0.111971	0.007121	-0.0030	1.0556
49	-0.126234	0.014263	-0.0034	1.0522
50	-0.128886	0.002653	-0.0035	1.0487
51	-0.144048	0.015162	-0.0039	1.0448
52	-0.164143	0.020095	-0.0044	1.0403
53	-0.168240	0.004096	-0.0046	1.0358
54	-0.176152	0.007912	-0.0048	1.0310
55	-0.194024	0.017872	-0.0053	1.0258
56	-0.211808	0.017784	-0.0057	1.0200
57	-0.225482	0.013673	-0.0061	1.0139
58	-0.245504	0.020022	-0.0067	1.0073
59	-0.268330	0.022826	-0.0073	1.0000

variation explained by the first factor. In section 11.5.1, it was shown that the second international stock market can only explain 24% of the total variation of the international security returns as explained by the first international stock market factor. The results reflect the equal importance of the first two international economic factors in representing the UK and US economy.

11.6.2 Maximum-likelihood factor analysis

The monthly returns of the economic and financial variables were subjected to maximum-likelihood factor analysis to determine the number and factor loadings of the common factors. The goodness of fit results for the economic factors are summarized in Table 11.19.

TABLE 11.19

DIFFERENT CRITERIA FOR ESTIMATING THE BEST NUMBER OF PARAMETERS TO INCLUDE IN A MODEL

Number of factors	SBC	AIC	T&L
2	4,474.13	8,303.57	0.35
3	3,960.69	7,067.91	0.47
4	3,587.79	6,116.99	0.56
5	3,426.86	5,593.67	0.61
6	3,345.94	5,234.03	0.65
7	3,246.48	4,840.96	0.69
8	3,196.60	4,550.74	0.72
9	3,187.25	4,345.22	0.75

When the number of factors is equal to 10, some of the communality estimates are greater than 1. If the communality exceeds unity, it is an ultra-Heywood case. An ultra-Heywood case implies that a factor has negative variance, a clear indication that something is wrong. The possible cause of the anomaly is the extraction of too many common factors which renders a factor solution invalid. With fewer than ten factors all the communality estimates are less than 1. Therefore, the Table 11.19 shows the results with only nine factors.

The results in Table 11.19 show that Akaike's information criterion (AIC) and Schwarz's Bayesian criterion (SBC) are at a minimum with nine factors. The Tucker and Lewis's reliability coefficient for the nine factors model is 0.75 which implies that there is a good fit between observed and reproduced matrices. Therefore, nine factors are considered for further investigation. In section 11.5.2, it was shown that the Tucker and Lewis's reliability coefficient for the international stock market factors model is 0.65. For comparison, it seems that there is a slightly better fit between observed and reproduced matrices of the international economic factors model than that of the international stock market factors model.

11.6.3 Factor patterns

Table 11.20 shows that the highest factor loading is 0.9965 and the lowest factor loading is 0.0035 (in absolute terms) for the first factor. Out of fifty-nine variables, twenty-five variables have negative loadings. The other eight factors have both positive and negative loadings. The absolute factor loadings of the remaining eight factors are smaller than that of the first factor, this is a feature of factor analysis.

11.6.4 Rotation of factors

The next step in factor analysis involves finding simpler structure through rotations, while keeping the number of factors and communalities of each variable fixed. The quartimax rotation is the chosen rotation as the goal of quartimax rotation is to make the variables as simple as possible by maximizing the variance of the loadings on each variable. The importance of a factor is best evaluated by the proportion of variance explained by the factor after rotation. The variances explained by the nine factors, with and without weights are shown in Table 11.21.

FACTOR PATTERN

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5	FACTOR6	FACTOR7	FACTOR8	FACTOR9
UKECON9	0.99649	-0.01086	-0.02222	-0.00430	0.02982	0.00062	-0.00575	0.00347	-0.00471
UKECON7	0.95122	0.00423	-0.01655	-0.00807	-0.00845	0.03799	-0.04539	0.00303	0.02934
UKECON6	0.93031	-0.01534	-0.07913	0.04683	0.05155	-0.03533	0.04187	0.00545	-0.00179
UKECON8	0.91304	-0.05237	-0.08680	0.02946	-0.02367	-0.02008	-0.04783	-0.01617	-0.04208
UKECON5	0.64138	-0.00972	-0.05253	-0.02770	-0.10485	-0.02648	0.05561	0.03297	-0.13825
UKECON12	0.56242	-0.12523	-0.03368	0.00635	-0.24542	0.00727	0.28449	-0.03938	0.13112
USECON35	0.51028	0.03558	-0.04526	-0.00259	-0.02570	-0.12281	0.29978	0.12603	-0.02983
UKECON21	0.21647	-0.06660	0.01571	0.04144	-0.17108	0.00019	0.18796	-0.07705	0.09656
USECON17	0.00351	0.88530	-0.30033	-0.30666	-0.01251	0.01331	0.03169	-0.00461	-0.01739
USECON15	-0.01235	0.88100	-0.31109	-0.29376	-0.02164	0.03751	-0.00223	-0.01063	-0.00357
USECON5	-0.00884	0.62152	0.10592	0.59627	0.03950	-0.06470	0.00587	-0.11832	0.02778
USECON14	0.01124	0.58632	-0.12254	0.31684	-0.10641	-0.13862	-0.01954	0.06994	0.06725
USECON37	0.02429	0.41861	-0.12189	0.29314	0.11064	-0.09990	0.08277	-0.10186	0.05038
USECON20	-0.04910	0.40912	-0.13567	0.09896	0.00918	0.00378	-0.01229	0.09338	0.03199
USECON16	0.03260	0.39496	-0.06271	0.21512	-0.04325	-0.09008	0.06810	0.07808	0.13606
USECON4	-0.00355	0.36343	-0.11818	0.25622	-0.03648	-0.12635	-0.09986	-0.00467	0.03583
USECON21	0.07443	0.30430	-0.00240	0.19806	0.03091	-0.06342	0.12761	-0.00508	-0.03021
USECON9	0.13461	0.39684	0.87638	-0.15389	0.00105	-0.01842	0.02282	-0.01949	-0.00722
USECON7	0.11679	0.38293	0.82676	-0.15885	0.00540	0.04246	0.01209	-0.01069	0.03863
USECON6	0.10142	0.47083	0.76725	0.06494	-0.02388	0.00969	-0.06988	0.10274	-0.00283
USECON8	0.05996	0.43458	0.68522	0.07961	0.05828	0.02783	-0.11756	0.10345	-0.07039
UKECON17	-0.13093	-0.09935	-0.24299	-0.06006	0.20068	-0.17939	0.11110	-0.15686	-0.17869
USECON28	-0.07746	-0.05892	-0.32673	0.11922	-0.05125	-0.03550	0.14024	-0.02379	0.05107
USECON12	0.04004	0.68258	-0.04798	0.69479	-0.04793	-0.07094	-0.03966	0.00982	-0.05141
USECON22	-0.10552	0.56163	-0.01651	0.56338	0.08833	0.07398	-0.19959	-0.08300	-0.04713
USECON34	-0.13848	0.17780	0.03352	0.32462	0.21652	0.18649	-0.11459	0.00511	-0.03466
USECON24	-0.04854	0.06910	-0.13062	-0.18647	0.09048	-0.02604	-0.00216	0.03649	0.07122
UKECON18	-0.03314	-0.16293	-0.08040	-0.26151	0.03807	-0.00337	-0.12151	0.06805	-0.09501
USECON26	-0.15021	0.24166	-0.11090	-0.33957	0.18488	0.00400	-0.17101	-0.0693	0.06242
UKECON14	0.10481	0.01465	-0.14278	-0.34240	0.01657	-0.11570	-0.06299	-0.01269	-0.02146
USECON25	-0.17634	0.30207	-0.19362	-0.35800	0.23005	0.06513	-0.14944	-0.05462	0.11836
USECON23	-0.05399	-0.33621	-0.00761	-0.36071	-0.05977	-0.13000	-0.00578	0.10734	-0.09713
USECON27	-0.14031	0.12285	-0.00759	-0.39842	0.19532	0.02966	-0.25687	0.01284	0.08741
UKECON2	-0.48732	0.08011	-0.02802	0.02912	0.75811	-0.12200	0.17113	0.14983	-0.10689
USECON38	-0.03940	0.24631	-0.01440	-0.04544	0.46508	0.05649	-0.19211	-0.14215	0.43300
UKECON3	-0.25493	0.10291	-0.00672	0.00781	0.45437	0.07918	0.04515	0.14381	-0.02657
UKECON16	0.07408	-0.06029	-0.06153	-0.10377	-0.28601	0.06194	-0.16463	-0.06240	0.01565
UKECON10	0.33171	-0.06395	0.03898	-0.04733	-0.62282	0.07626	-0.12562	-0.08850	-0.02680
UKECON4	0.01918	0.08321	0.04296	0.29770	0.06331	0.76192	0.24475	0.00246	-0.05773
UKECON20	0.05530	0.03354	0.07809	0.33284	-0.01945	0.59150	0.28937	-0.00077	-0.06776
UKECON15	0.17726	0.06870	0.03126	0.25921	-0.05134	0.51796	0.46278	-0.07079	0.08228
UKECON11	-0.28629	0.04059	0.00429	0.05016	0.20293	0.38824	-0.26611	0.01448	-0.24770
USECON33	-0.06031	0.24327	-0.04217	0.23766	0.20381	0.33657	-0.23736	-0.11800	-0.01145
UKECON13	0.05801	0.09217	-0.00688	0.21493	0.09367	0.33200	0.14235	-0.01333	-0.08372
UKECON1	0.13212	-0.10652	-0.07691	-0.10915	-0.08069	-0.27750	0.02908	0.01987	0.14199
USECON11	0.32017	0.35147	0.02742	0.38234	-0.05378	-0.22936	0.42725	-0.00164	0.25729
USECON29	0.23265	0.07039	0.01065	0.06610	-0.04921	-0.14729	0.41638	-0.06674	0.26456
UKECON19	0.11037	-0.07389	0.03033	0.08086	-0.09325	0.28923	0.29309	0.01927	0.03528
USECON13	-0.23553	-0.15929	0.06637	0.08485	0.25075	0.36931	-0.49571	-0.04148	-0.22338
USECON36	-0.02992	0.19858	-0.00646	0.05541	-0.09456	0.10210	-0.18821	0.65519	0.15901
USECON1	-0.08433	0.19481	-0.02173	-0.02853	-0.12402	0.10183	-0.11129	0.63229	0.14641
USECON32	0.01969	0.02491	0.05197	-0.04735	-0.16218	0.04003	-0.01384	0.52116	-0.14325
USECON18	0.13891	0.13750	0.10315	0.01950	-0.15853	0.14461	-0.08905	0.47817	0.35729
USECON10	-0.03124	-0.08222	-0.33737	0.12758	-0.11345	0.11835	-0.09650	0.44662	0.26991
USECON3	-0.07440	0.18658	0.07354	0.12319	0.34405	0.10612	-0.20408	-0.20974	0.49826
USECON2	-0.14589	0.24444	-0.05271	0.19328	0.36735	0.16839	-0.30996	-0.21971	0.47612
USECON19	0.03754	0.23997	-0.01020	0.32029	0.00014	-0.01940	0.28036	0.13015	0.32801
USECON30	0.08912	0.04003	-0.02128	0.14375	0.01370	-0.07554	0.15377	-0.06730	0.21743
USECON31	0.06731	-0.06302	-0.02013	0.11120	0.11668	0.01094	0.13934	-0.09152	0.15808

TABLE 11.21

VARIANCE EXPLAINED BY EACH FACTOR

	Weighted	Unwighted
Variance explained by each factor:		
Factor 1	212.17	5.69
Factor 2	73.57	5.49
Factor 3	41.66	3.22
Factor 4	26.06	3.40
Factor 5	9.37	2.38
Factor 6	5.97	2.28
Factor 7	4.86	2.13
Factor 8	3.95	1.90
Factor 9	3.44	1.60

Rotational technique	Quartimax w	Quartimax unw	Varimax w	Varimax unw	Equimax w	Equimax unw
Factor 1	52.95	5.66	52.24	5.44	196.67	4.68
Factor 2	199.96	4.82	199.14	4.78	47.90	4.65
Factor 3	46.09	3.65	46.44	3.68	47.37	3.79
Factor 4	10.39	2.77	10.05	2.74	8.07	2.74
Factor 5	42.43	2.58	7.10	2.61	8.85	2.68
Factor 6	6.82	2.55	42.53	2.58	42.83	2.61
Factor 7	4.42	2.12	4.53	2.14	6.25	2.44
Factor 8	5.51	2.00	5.35	2.10	17.90	2.27
Factor 9	12.48	1.93	13.68	2.01	5.22	2.22

w = weighted; unw = unweighted

The squared multiple correlations (SMCs) are the estimates of communality between variables and the factors. The SMCs represent the proportion of variance in variables that are predictable from the underlying factors. The squared multiple correlations of the variables with factor 1, factor 2, factor 3, factor 4, factor 5, factor 6, factor 7, factor 8 and factor 9 are 0.9739, 0.9879, 0.9766, 0.8270, 0.9680, 0.8541, 0.8018, 0.8103 and 0.8841 respectively, which implies that the nine factors are internally consistent and well defined by the variables.

The results in Table 11.22 show the pattern of factor loadings after the quartimax rotation. The highest factor loading on factor 1 is 0.9663 and the lowest is 0.0010 (in absolute terms). All nine factors retain the mixture of signs in the loadings of the economic variables, indicating that the economic variables have different reactions to the factors.

Table 11.23 identifies the economic variables grouped by the statistically significant factor loadings of the nine factors. Tabachnick and Fidell (1989) suggested that variables which have loadings in excess of 0.30 (in absolute terms) are considered "statistically significant".

The results of this section suggest that there are nine factors underlying the UK and US economy. The first factor is composed of the US general economy-wide variables, US interest rate, US GNP, US unemployment and encompass US coincident and US leading indicators. The first factor is basically similar to the first US economic factor in the analysis of US economy in chapter 9. The second factor represents primarily the UK and US market indices, UK longer leading indicator, and UK gross redemption yield on 20 year gilts. The second factor is more or less identical to the first UK economic factor in chapter 6. The third factor is composed of variables such as the US industrial production, US money supply (M1) and US output of crude petroleum (similar to the second US economic factor in chapter 9). The fourth factor represents variables such as the US and UK leading indicators, US money supply (M2), US share prices - industrials, US construction of residential and private sector, US loans (commercial banks) and US lagging indicator (similar to the fourth US economic

ROTATED FACTOR PATTERN

	FACTOR1	FACTOR2	FACTOR3	FACTOR4	FACTOR5	FACTOR6	FACTOR7	FACTOR8	FACTOR9
USECON12	0.96629	0.02763	0.05215	0.05953	-0.12431	0.06893	0.05233	-0.02254	-0.00167
USECON5	0.83883	-0.04238	0.18184	-0.00091	-0.11411	0.08282	-0.06761	0.09488	0.04153
USECON22	0.77042	-0.08671	0.06354	0.26620	-0.06126	0.09095	-0.01257	0.13329	0.02314
USECON14	0.67641	-0.00879	0.01443	-0.07134	0.10950	-0.06209	0.12916	-0.03493	-0.05464
USECON37	0.52923	0.02596	-0.03641	-0.08201	0.07405	0.02059	-0.08868	0.07073	0.10492
USECON4	0.47148	-0.00189	-0.03808	0.02001	0.03908	-0.10278	0.04139	0.02786	-0.03941
USECON16	0.44894	0.00473	0.02352	-0.14995	0.07367	-0.00399	0.12618	0.02186	0.00660
USECON20	0.38295	-0.03787	-0.01462	0.01521	0.20571	0.01671	0.12533	-0.01573	0.02938
USECON19	0.36998	-0.02437	-0.01376	-0.35945	-0.08110	0.16141	0.19550	0.13290	0.09905
USECON21	0.35969	0.06348	0.06039	-0.09268	0.02060	0.05595	-0.02945	-0.05776	0.07857
USECON34	0.29821	-0.09814	0.02694	0.25896	-0.09074	0.16767	0.02252	0.15479	0.16175
UKECON18	-0.27572	0.01301	-0.06906	0.13623	0.12068	-0.11557	0.03003	-0.06165	0.01537
USECON23	-0.46136	-0.03205	-0.05114	-0.01769	0.04029	-0.19454	0.02588	-0.17494	0.00556
UKECON9	0.00096	0.97385	0.08070	-0.14653	-0.03436	0.03755	0.00196	0.03966	-0.11866
UKECON7	0.00529	0.92167	0.08766	-0.12364	-0.02150	0.04760	0.03275	0.06828	-0.16936
UKECON6	0.03988	0.91573	0.00765	-0.17411	-0.05287	0.03331	-0.01095	0.02863	-0.06707
UKECON8	0.00551	0.89965	-0.00499	-0.09798	-0.06786	0.00131	-0.01544	0.00760	-0.16169
UKECON5	-0.00590	0.62660	0.02018	-0.10343	-0.02146	0.02661	-0.00982	-0.16565	-0.13233
USECON35	0.04234	0.48359	0.00735	-0.32839	-0.00912	0.05060	0.04067	-0.18499	0.06336
UKECON12	-0.07855	0.47376	-0.03957	-0.41251	-0.10533	0.15474	-0.00239	-0.05794	-0.22887
USECON9	0.08811	0.02000	0.97506	-0.08859	0.02552	0.01606	-0.02976	-0.00817	-0.00256
USECON7	0.07170	0.00484	0.92416	-0.07671	0.05012	0.05732	0.00466	0.03780	-0.01600
USECON6	0.28694	0.00649	0.86050	0.00872	-0.05864	0.02647	0.11778	-0.00049	-0.00600
USECON8	0.27566	0.00112	0.77311	0.11366	-0.05111	0.01977	0.09345	-0.00089	0.06028
USECON28	0.07838	-0.06271	-0.36090	-0.11829	-0.00931	0.03995	-0.01817	-0.02874	0.00286
USECON13	-0.12657	-0.13306	0.01332	0.70352	-0.09797	0.09848	-0.01868	0.19415	0.05362
UKECON11	0.00187	-0.19743	0.01122	0.55521	0.05735	0.21734	0.00695	0.04354	0.10290
USECON33	0.29231	-0.01747	0.00944	0.36745	0.05643	0.23193	-0.02770	0.25915	0.02280
USECON30	0.13272	0.04699	-0.04259	-0.25303	-0.06803	0.03204	-0.02238	0.14092	0.01827
UKECON21	-0.01980	0.15236	-0.01038	-0.25654	-0.09253	0.10243	-0.04309	-0.01934	-0.14935
USECON29	0.11088	0.14740	0.00801	-0.53655	-0.02411	0.08596	-0.05292	0.04361	0.02929
USECON11	0.52657	0.22526	0.06295	-0.56378	-0.11974	0.08748	0.01205	0.02079	0.06953
USECON17	0.51748	0.01887	0.06878	-0.03010	0.82168	-0.00590	0.04312	-0.12766	-0.05047
USECON15	0.52131	0.00373	0.05490	-0.00223	0.81715	-0.00022	0.05393	-0.10218	-0.07427
USECON25	0.01924	-0.12385	-0.02206	0.12625	0.54981	-0.08187	0.00319	0.18746	0.09060
USECON26	-0.01423	-0.10586	0.03475	0.13271	0.45589	-0.13688	-0.03117	0.13782	0.06130
USECON27	-0.16108	-0.09603	0.10865	0.18657	0.39713	-0.17132	0.05560	0.17580	0.05960
UKECON14	-0.16242	0.12966	-0.04823	-0.01887	0.28571	-0.18793	-0.03107	-0.04841	-0.03074
USECON24	-0.04747	-0.02480	-0.06798	-0.02538	0.23145	-0.06469	0.04078	0.04199	0.07018
UKECON4	0.12398	0.00634	0.01404	0.16147	-0.07422	0.83044	0.06238	0.06479	0.02516
UKECON15	0.12967	0.10677	-0.00147	-0.21156	-0.09050	0.71980	-0.01493	0.03024	-0.02773
UKECON20	0.13050	0.02362	0.02103	0.05006	-0.17489	0.71231	0.03191	-0.01089	-0.00113
UKECON19	-0.05534	0.06404	-0.01396	-0.15617	-0.08396	0.40095	0.03714	-0.04641	-0.04326
UKECON13	0.15133	0.06610	-0.00912	0.08767	-0.04969	0.40046	-0.01651	0.00893	0.07953
UKECON1	-0.09584	0.10952	-0.08308	-0.23810	-0.00003	-0.25208	0.02306	0.00461	-0.05179
USECON36	0.14255	-0.01765	0.05825	0.09496	0.05091	-0.02686	0.71702	-0.04087	0.02523
USECON1	0.08971	-0.07784	0.04877	0.04225	0.11441	-0.00709	0.68200	-0.09347	0.02102
USECON18	0.06245	0.08291	0.14792	-0.11566	0.01714	0.05916	0.61865	0.12192	-0.10994
USECON10	0.02720	-0.01025	-0.36241	-0.01217	-0.02715	0.02907	0.55117	0.08141	-0.04482
USECON32	-0.04393	0.03533	0.07764	0.06852	-0.02144	0.00065	0.45342	-0.32479	0.02704
UKECON17	-0.05386	-0.05922	-0.25849	0.01324	0.07712	-0.11263	-0.28307	-0.09443	0.22342
USECON2	0.28954	-0.13428	-0.01512	0.15006	0.12921	0.02754	0.00119	0.71250	0.07603
USECON3	0.19254	-0.09137	0.09493	0.01395	0.09153	0.01483	-0.01509	0.67285	0.09100
USECON38	0.14533	-0.01761	0.07309	0.03419	0.27767	-0.05168	-0.00442	0.61179	0.21578
USECON31	0.02036	0.05093	-0.06530	-0.15670	-0.07926	0.09354	-0.07174	0.16647	0.08983
UKECON2	0.06021	-0.34448	-0.04744	0.13793	0.12490	-0.03738	-0.07485	0.07573	0.85230
UKECON3	0.04664	-0.17069	0.00916	0.15410	0.12705	0.07914	0.05339	0.10161	0.47023
UKECON16	-0.09389	0.04339	-0.04536	0.05234	0.03242	-0.04237	0.03241	-0.02385	-0.34727
UKECON10	-0.06345	0.22999	0.05502	-0.06386	-0.09898	0.01363	0.04657	-0.17361	-0.65545

TABLE 11.23

IDENTIFICATION OF THE ECONOMIC VARIABLES GROUPED BY THE FACTOR LOADINGS

			Rotated Factor Pattern
Factor 1:	Coincident composite index	(USECON12)	0.9663
	Industrial Production - total	(USECON5)	0.8388
	Employment in manufacturing industry	(USECON22)	0.7704
	Manufacturing deliveries - durable goods	(USECON14)	0.6764
	GNP	(USECON37)	0.5292
	Leading composite index	(USECON11)	0.5266
	Manufacturing deliveries: non-durable goods	(USECON15)	0.5213
	Manufacturing net new orders: non-durable goods	(USECON17)	0.5175
	Personal income	(USECON14)	0.4615
	Manufacturing net new orders - total	(USECON16)	0.4489
	Wholesale sales: value	(USECON20)	0.3830
	Construction-work put in place: residential (private sector)	(USECON19)	0.3700
	Retail sales: value	(USECON21)	0.3597
	Consumer credit outstanding - financial institutions	(USECON34)	0.2982
	Loans (commercial banks)	(USECON33)	0.2923
	Interest rate on 3 mth	(USECON2)	0.2895
	Industrial production: durable goods	(USECON6)	0.2869
	Unemployment: total	(USECON23)	0.4614
Factor 2:	UK FT Actuaries Industrial Share Price Index - monthly average	(UKECON9)	0.9738
	UK FT Actuaries Capital Goods Share Price Index - monthly average	(UKECON7)	0.9217
	UK FT Actuaries 500 Share Price Index	(UKECON6)	0.9157

TABLE 11.23 (continued)

	UK FT Actuaries Financial Group Share Price Index - monthly average	(UKECON8)	0.8997
	UK FT 30 Share Price Index (End Period)	(UKECON5)	0.6266
	Share Prices - industrials (S&P)	(USECON35)	0.4836
	UK Longer Leading Indicator	(UKECON12)	0.4738
	UK Gross Redemption Yield on 20 year Gilts	(UKECON2)	-0.3445
Factor 3:	Industrial production: consumer goods	(USECON9)	0.9751
	Industrial production: non-durable goods	(USECON7)	0.9242
	Industrial production: durable goods	(USECON6)	0.8605
	Industrial production: investment goods	(USECON8)	0.7731
	Output of crude petroleum	(USECON10)	-0.3624
	Money Supply (M1)	(USECON28)	-0.3609
Factor 4:	Lagging composite index	(USECON13)	0.7035
	Loans (commercial banks)	(USECON33)	0.5552
	Leading composite index	(USECON11)	-0.5638
	Money Supply (M2)	(USECON29)	-0.5366
	UK Longer Leading Indicator	(UKECON12)	-0.4125
	Construction - work put in place: residential (private sector)	(USECON19)	-0.3595
	Share prices - industrials (S&P)	(USECON35)	-0.3284
Factor 5:	Manufacturing net new orders: non-durable goods	(USECON17)	0.8217
	Manufacturing deliveries: non-durable goods	(USECON15)	0.8172
	Producer prices: total	(USECON25)	0.5498
	Producer prices: refined petroleum products	(USECON26)	0.4559
	Consumer prices: all items	(USECON27)	0.3971
	UK Retail Prices Index	(UKECON14)	0.2857
	Yield of long-term government bonds	(USECON38)	0.2777

TABLE 11.23 (continued)

Factor 6:	UK Coincident Indicator	(UKECON4)	0.8304
	UK Shorter Leading Indicator	(UKECON15)	0.7198
	UK GDP	(UKECON20)	0.7123
	UK Consumers Expenditure on Durable Goods	(UKECON19)	0.4010
	UK Industrial Production	(UKECON13)	0.4005
Factor 7:	Exports FOB	(USECON36)	0.7170
	Imports CIF	(USECON1)	0.6820
	Construction - value of Contracts: total	(USECON18)	0.6187
	Output of crude petroleum	(USECON10)	0.5512
	New capital issues by corporations	(USECON32)	0.4534
	UK Wholesale Prices, Manufacturing Input - Fuel	(UKECON10)	-0.2831
Factor 8:	Interest rate on 3 mth	(USECON2)	0.7125
	Interest rate on 3 mth US $ deposits in London	(USECON3)	0.6729
	Yield of long-term government bonds	(USECON38)	0.6118
	New capital issues by corporations	(USECON32)	-0.3248
Factor 9:	UK Gross Redemption Yield on 20 Year Gilts	(UKECON2)	0.8523
	UK Interest Rate on 3 mth Bank Bills	(UKECON3)	0.4702
	UK FT Government Securities Price Index	(UKECON10)	-0.6555
	UK Average Exchange Rate - US Dollars to £1	(UKECON16)	-0.3473

factor in chapter 9). It is interesting to note that the UK leading indicator and the US lagging indicator are also inversely related to each other as in the case of the US leading indicator. The fifth factor is composed of the US manufacturing net new orders and deliveries, US producer prices index, US consumer prices index, US producer prices on refined petroleum products, UK retail prices index and US yield on long-term government bonds (similar to the third US economic factor in chapter 9). As expected, the UK retail prices index is related to the US prices indices. The sixth factor is composed of the UK coincident indicator, UK shorter leading indicator, UK GDP, UK consumer expenditures on durable goods and UK industrial production (similar to the third UK economic factor in chapter 6). The seventh factor is composed primarily of the US balance of payments (e.g. exports FOB and imports CIF), US total value of the contracts of construction, US output of crude petroleum and US new capital issues by corporations (similar to the fifth US economic factor in chapter 9). It is interesting to note that the UK wholesale prices of the fuel are negatively related to the output of crude petroleum. The eighth factor represents primarily the US interest rate, US yield of long-term government bonds and US new capital issues by corporations (similar to the sixth US economic factor in chapter 9). The final factor is composed of UK gross redemption yield on 20 year gilts, UK interest rate, UK FT government securities price index and UK exchange rate (US $ to £) (similar to the second UK economic factor in chapter 6).

11.6.5 Discussion

By the maximum-likelihood method of factor analysis, it has been shown that there are nine international economic factors. The cumulative proportion of the nine economic factors accounts for almost 81% of the variations in the UK and US economic activities. The nine international economic factors are basically the same UK economic factors (i.e. three UK factors) and the same US economic factors (six US factors) that have been extracted in chapters 6 & 9).

11.7 Stock Returns and the Economic Forces : International Evidence

The objective of this section is to analyse the relationships between the international security returns and international economic indicators.

11.7.1 Empirical results using the canonical correlation analysis approach

The factor scores of the international stock market factors and international economic factors extracted in sections 11.5 and 11.6 respectively are subject to canonical correlation analysis in order to find the relationship between the security returns and the economic indicators.

The simple univariate statistics show that the fourteen variables (i.e. factor scores of the factors extracted from the security returns and economic indicators), namely, FSEC1, FSEC2, FSEC3, FSEC4, FSEC5, FSEC6, FECON1, FECON2, FECON3, FECON4, FECON5, FECON6, FECON7, FECON8 and FECON9 have mean which is approximately equal to zero, and standard deviation is equal to the multiple correlation of the factor with the variables (i.e. security returns, economic indicators).

The first step in the canonical analysis is the generation of a correlation matrix (Table 11.25). The correlation matrix is subdivided into four parts: the correlations between the factor scores of the security returns, the correlations between the factor scores of the economic indicators, and the two matrices of correlations between the factor scores of the security returns and of the economic indicators.

The correlations between the factor scores of the security returns and that of the economic indicators are fairly high, the largest being 0.6229 between FSEC2 and FECON2. This correlation between the factor scores of the second international stock market factor and those of the second international economic factor is rather high. However, significance cannot yet be assumed.

The first canonical correlation is 0.7340, representing 53.87% overlapping variance

TABLE 11.24

SIMPLE UNIVARIATE STATISTIC

VARIABLE	ST DEV
FSEC1	0.9950
FSEC2	0.9891
FSEC3	0.9870
FSEC4	0.9650
FSEC5	0.9611
FSEC6	0.9434
FECON1	0.9868
FECON2	0.9939
FECON3	0.9882
FECON4	0.9094
FECON5	0.9838
FECON6	0.9241
FECON7	0.8954
FECON8	0.9001
FECON9	0.9402

FSEC = Factor scores of security returns
FECON = Factor scores of economic indicators.

between the first pair of canonical variates (i.e. linear combination of the factor scores of the international security returns and of the international economic indicators) which appears to be larger than any of the direct between-set correlations. This implies that the first pair is highly related to each other. The second canonical correlation is 0.5086, representing 25.87% of overlapping variance for the second pair of canonical variates. The third canonical correlation is 0.2842, representing 8.08% of overlapping variance for the third pair of canonical variates. The fourth canonical correlation is 0.2564, representing 6.07% of overlapping variance for the fourth pair of canonical variates.

The last panel of Table 11.26 shows that the probability level for the null hypothesis that all the canonical correlations are zero in the population is only 0.0147 based on the

TABLE 11.25

CORRELATIONS AMONG THE SECURITY RETURNS, ECONOMIC INDICATORS AND BETWEEN THE SECURITY RETURNS AND ECONOMIC INDICATORS

Correlations Among the Security Returns

	FSEC1	FSEC2	FSEC3	FSEC4	FSEC5	FSEC6
FSEC1	1.0000	0.0050	0.0059	0.0057	0.0045	0.0018
FSEC2	0.0059	1.0000	-0.0004	-0.0018	-0.0017	0.0008
FSEC3	0.0059	-0.0004	1.0000	-0.0028	0.0003	0.0001
FSEC4	0.0057	-0.0018	-0.0028	1.0000	-0.0028	0.0033
FSEC5	0.0045	-0.0017	0.0003	-0.0028	1.0000	-0.0002
FSEC6	0.0018	0.0008	0.0001	0.0033	-0.0002	1.0000

Correlations Among the Economic Indicators

	FECON1	FECON2	FECON3	FECON4	FECON5
FECON1	1.0000	0.0005	0.0044	0.0039	0.0130
FECON2	0.0005	1.0000	-0.0000	-0.0292	-0.0002
FECON3	0.0044	-0.0000	1.0000	-0.0119	0.0001
FECON4	0.0039	-0.0292	-0.0119	1.0000	0.0006
FECON5	0.0130	-0.0002	0.0001	0.0006	1.0000
FECON6	0.0162	0.0062	0.0028	0.0094	-0.0140
FECON7	0.0121	-0.0003	-0.0011	0.0095	0.0043
FECON8	-0.0005	0.0163	0.0032	0.0367	-0.0197
FECON9	0.0020	-0.0165	-0.0002	0.0142	-0.0006

	FECON6	FECON7	FECON8	FECON9
FECON1	0.0162	0.0121	-0.0005	0.0020
FECON2	0.0062	-0.0003	0.0163	-0.0165
FECON3	0.0028	-0.0011	0.0032	-0.0002
FECON4	0.0094	0.0095	0.0367	0.0142
FECON5	-0.0140	0.0043	-0.0197	-0.0006
FECON6	1.0000	0.0081	0.0101	-0.0034
FECON7	0.0081	1.0000	-0.0123	-0.0210
FECON8	0.0101	-0.0123	1.0000	0.0401
FECON9	-0.0034	-0.0210	0.0401	1.0000

TABLE 11.25 (continued)

Correlations Between the Security Returns and the Economic Indicators

	FSEC1	FSEC2	FSEC3	FSEC4	FSEC5	FSEC6
FECON1	0.0122	0.0005	-0.1264	0.1115	-0.1050	-0.0330
FECON2	0.2801	0.6229	-0.0437	0.0004	0.0794	-0.1055
FECON3	0.0096	0.0136	-0.0289	-0.0385	-0.0665	-0.0811
FECON4	-0.2311	-0.0401	0.0608	0.0144	0.1030	0.0399
FECON5	-0.0262	0.0004	-0.1131	0.0792	-0.0446	0.0707
FECON6	0.0062	0.0225	-0.0355	-0.0673	0.0615	0.1305
FECON7	0.0335	-0.0190	-0.0161	-0.0059	0.1156	0.0034
FECON8	-0.3012	-0.0376	-0.4044	0.0192	0.0023	-0.0769
FECON9	0.0267	-0.1653	-0.0314	0.1085	0.1354	-0.0522

F-test, hence there are four pairs of canonical variates which reach significance ($\alpha = 0.05$) and they account for the significant relationships between the two sets of variables. As shown in Table 11.27, the first canonical correlation vectors are

$$\rho_1 = 0.5233FSEC1 + 0.8304FSEC2 + 0.1230FSEC3 - 0.0434FSEC4 + 0.0568FSEC5 - 0.1044FSEC6$$

and

$$\phi_1 = -0.0213FECON1 + 0.9167FECON2 + 0.0250FECON3 - 0.1578FECON4 - 0.0611FECON5 + 0.0121FECON6 + 0.0040FECON7 - 0.3204FECON8 - 0.1314FECON9$$

and $r_c = 0.7340$.

11.7.2 Interpretation of canonical variates

After the canonical correlation analysis creates the canonical variates, the matrix of correlations of the original variables (i.e. factor scores of the security returns) with the

TABLE 11.26

CANONICAL CORRELATION ANALYSIS

	1	2	3	4
Canonical Correlation (r_c)	0.7340	0.5086	0.2842	0.2464
Squared Canonical Correlation (r_c^2)	0.5387	0.2587	0.0808	0.0607

	5	6
Canonical Correlation (r_c)	0.1847	0.1494
Squared Canonical Correlation (r_c^2)	0.0341	0.0223

Tests of H_0: The canonical correlation in the current column and all that follow are zero.

	1	2	3	4
Likelihood Ratio	0.27882341	0.60441166	0.81531261	0.88697704
F-test	7.3617	3.6646	2.0662	1.8797
PR > F	0.0001	0.0001	0.0010	0.0147

	5	6
Likelihood Ratio	0.94431551	0.97767379
F-test	1.6100	1.5871
PR > F	0.1001	0.1779

TABLE 11.27

CANONICAL CORRELATION ANALYSIS: STANDARDIZED CANONICAL COEFFICIENTS FOR THE SECURITY RETURNS AND THE ECONOMIC INDICATORS

Standardized Canonical Coefficients (B_x) for the Security Returns

	SEC1	SEC2	SEC3	SEC4	SEC5	SEC6
FSEC1	0.5233	-0.3611	-0.5498	0.3530	0.3217	-0.2560
FSEC2	0.8304	0.3409	0.2858	-0.2535	-0.0492	0.2142
FSEC3	0.1230	-0.8376	0.2222	-0.1982	-0.3689	0.2421
FSEC4	-0.0434	0.0990	-0.2648	0.4232	-0.0714	0.8568
FSEC5	0.0568	-0.0536	0.6641	0.7284	0.0600	-0.1371
FSEC6	-0.1044	-0.1911	0.2456	-0.2476	0.8664	0.2833

Standardized Canonical Coefficients (B_y) for the Economic Indicators

	ECON1	ECON2	ECON3	ECON4	ECON5	ECON6
FECON1	-0.0213	0.2427	-0.5065	0.0278	0.0246	0.4479
FECON2	0.9167	0.3058	0.1605	0.1497	-0.0738	0.0928
FECON3	0.0250	0.0758	-0.2075	-0.1581	-0.3217	-0.3525
FECON4	-0.1578	-0.0084	0.7124	-0.0640	-0.3100	0.5099
FECON5	-0.0611	0.2131	-0.1424	-0.0157	0.4753	0.4782
FECON6	0.0121	-0.0114	0.2983	-0.0519	0.7437	-0.2306
FECON7	0.0040	-0.0189	0.1792	0.4282	0.1432	-0.2569
FECON8	-0.3204	0.8896	0.1211	0.0200	-0.0538	-0.2442
FECON9	-0.1314	-0.0828	-0.0789	0.8840	-0.0785	0.0621

canonical variates is the factor loading matrix. The content of the canonical variates is interpreted via the factor loading matrix. As shown in Table 11.28, the first pair of canonical variates has high loading on FSEC2 (0.8333) of the factor scores of the security returns and on FSEC1 (0.5287) of the factor scores of the security returns and on FECON2 (0.9183) and

TABLE 11.28

CANONICAL STRUCTURE

Correlations Between the Security Returns and their Canonical Coefficients, (A_x)

	SEC1	SEC2	SEC3	SEC4	SEC5	SEC6
FSEC1	0.5287	-0.3641	-0.5449	0.3556	0.3207	-0.2485
FSEC2	0.8333	0.3389	0.2820	-0.2535	-0.0465	0.2116
FSEC3	0.1260	-0.8401	0.2198	-0.1970	-0.3667	0.2382
FSEC4	-0.0427	0.0982	-0.2702	0.4233	-0.0657	0.8556
FSEC5	0.0579	-0.0563	0.6619	0.7292	0.0614	-0.1410
FSEC6	-0.1029	-0.1913	0.2438	-0.2460	0.8667	0.2859

Correlations Between the Economic Indicators and their Canonical Coefficients, (A_y)

	ECON1	ECON2	ECON3	ECON4	ECON5	ECON6
FECON1	-0.0220	0.2449	-0.4996	0.0329	0.0417	0.4480
FECON2	0.9183	0.3219	0.1445	0.1370	-0.0598	0.0717
FECON3	0.0258	0.0798	-0.2172	-0.1579	-0.3160	-0.3577
FECON4	-0.1985	0.0140	0.7160	-0.0496	-0.2984	0.5008
FECON5	-0.0554	0.1987	-0.1543	-0.0138	0.4667	0.4911
FECON6	0.0141	0.0006	0.3022	-0.0507	0.7340	-0.2304
FECON7	0.0085	-0.0246	0.1820	0.4088	0.1513	-0.2444
FECON8	-0.3151	0.8870	0.1499	0.0495	-0.0742	-0.2314
FECON9	-0.1617	-0.0516	-0.0723	0.8727	-0.0896	0.0648

Correlations Between the Security Returns and the Canonical Coefficients of the Economic Indicators, ($R_{xx}B_y$)

	ECON1	ECON2	ECON3	ECON4	ECON5	ECON6
FSEC1	0.3880	-0.1852	-0.1549	0.0876	0.0592	-0.0371
FSEC2	0.6116	0.1723	0.0802	-0.0625	-0.0086	0.0316
FSEC3	0.0924	-0.4273	0.0625	-0.0485	-0.0677	0.0356
FSEC4	-0.0314	0.0499	-0.0768	0.1043	-0.0121	0.1278
FSEC5	0.0425	-0.0286	0.1881	0.1797	0.0113	-0.0211
FSEC6	-0.0755	-0.0973	0.0693	-0.0606	0.1601	0.0427

TABLE 11.28 continued

Correlations Between the Economic Indicators and the Canonical Coefficients of the Security Returns, ($R_{yy}B_x$)

	SEC1	SEC2	SEC3	SEC4	SEC5	SEC6
FECON1	-0.0161	0.1246	-0.1420	0.0081	0.0077	0.0669
FECON2	0.6740	0.1637	0.0411	0.0338	-0.0111	0.0107
FECON3	0.0189	0.0406	-0.0617	-0.0389	-0.0584	-0.0535
FECON4	-0.1457	0.0071	0.2035	-0.0122	-0.0551	0.0748
FECON5	-0.0407	0.1011	-0.0439	-0.0034	0.0862	0.0734
FECON6	0.0104	0.0003	0.0859	-0.0125	0.1356	-0.0344
FECON7	0.0062	-0.0125	0.0517	0.1007	0.0280	-0.0365
FECON8	-0.2313	0.4511	0.0426	0.0122	-0.0137	-0.0346
FECON9	-0.1187	-0.0262	-0.0205	0.2151	-0.0165	0.0097

FECON8 (-0.3151) of the factor scores of the economic indicators. Thus, the first canonical variates are primarily FSEC1, FSEC2 for the security returns and FECON2, FECON8 for the economic variables. The results in the last section show that the second and the eighth economic factors represent variables such as the UK and US market indices, UK longer leading indicator, UK gross redemption yield on 20 year gilts; and the US interest rate, US yield of long-term government bonds and US new capital issues by corporations.

The second pair of canonical variates has high loading on FSEC3 (-0.8401), FSEC1 (-0.3641) and on FSEC2 (0.3389) of the factor scores of the security returns and on FECON8 (0.8870) and FECON2 (0.3219) of the factor scores of the economic indicators. Hence, from the results of the previous section, the second pair of variates represents variables such as the US interest rate, US yield of long-term government bonds and US new capital issues by corporations; and the UK and US market indices, UK longer leading indicator, and UK gross redemption yield on 20 year gilts.

The third pair of canonical variates has high loading on FSEC5 (0.6619) and on FSEC1 (-0.5449) of the factor scores of the security returns and on FECON4 (0.7160), FECON1 (-0.4996) and FECON6 (0.3022) of the factor scores of the economic indicators. The third pair of variates consist primarily of the US and UK leading indicators, US money supply (M2), US share prices - industrials, US construction of residential and private sector, US loans (commercial banks), US and UK lagging indicator; the US general economy-wide variables, US interest rate, US GNP, US employment, US coincident and US leading indicators; and the UK coincident indicator, UK shorter leading indicator, UK GDP, UK consumer expenditures on durable goods, and UK industrial production.

The fourth pair of canonical variates has high loading on FSEC5 (0.7292), FSEC4 (0.4233) and on FSEC1 (0.3556) of the factor scores of the security returns and on FECON9 (0.8727) and FECON7 (0.4088) of the factor scores of the economic indicators. Hence, the fourth pair of variates are primarily the UK gross redemption yield on 20 year gilts, UK interest rate, UK FT government securities price index, UK exchange rate (US $ to £); and the US balance of payments, US total value of the contracts of construction, US output of crude petroleum, US new capital issues by corporations, and UK wholesale prices (manufacturing input-fuel).

The utilization of canonical correlation analysis not only provides information about the nature of (statistically significant) links between the sets, but also shows the extent to which the variance in one set is conditional upon or redundant given the other set.

As shown in Table 11.29, canonical redundancy analysis illustrates that the first pair of canonical variates is only a moderate overall predictor of the opposite set of variables, the proportions of variance explained being 0.0903 and 0.0606. The second pair of canonical

TABLE 11.29

CANONICAL REDUNDANCY ANALYSIS

Standardized Variance of the Security Returns Explained By

	Their Own Canonical Variates			The Opposite Canonical Variates	
	Proportion	Cumulative Proportion	Canonical R-Squared	Proportion	Cumulative Proportion
1	0.1676	0.1676	0.5387	0.0903	0.0903
2	0.1671	0.3347	0.2587	0.0432	0.1335
3	0.1659	0.5006	0.0808	0.0134	0.1469
4	0.1668	0.6674	0.0607	0.0101	0.1570
5	0.1664	0.8338	0.0341	0.0057	0.1627
6	0.1662	1.0000	0.0223	0.0037	0.1664

Standardized Variance of the Economic Indicators Explained by

	Their Own Canonical Variates			The Opposite Canonical Variates	
	Proportion	Cumulative Proportion	Canonical R-Squared	Proportion	Cumulative Proportion
1	0.1125	0.1125	0.5387	0.0606	0.0606
2	0.1111	0.2236	0.2587	0.0287	0.0893
3	0.1118	0.3354	0.0808	0.0090	0.0984
4	0.1090	0.4444	0.0607	0.0066	0.1050
5	0.1097	0.5541	0.0341	0.0037	0.1087
6	0.1107	0.6648	0.0223	0.0025	0.1112

variates is only moderately related, the proportions of variance being 0.0432 and 0.0287. The third pair of canonical variates is only slightly related, the proportions of variance being 0.0134 and 0.0090. The fourth pair of canonical variates is also only slightly related, the proportions of variance being 0.0101 and 0.0066.

The squared multiple correlations in Table 11.30 indicate that the first canonical variate of the economic indicators has fairly good predictive power for FSEC2 and moderate predictive power for FSEC1, but almost none for predicting FSEC3, FSEC4, FSEC5, and FSEC6. The first canonical variate of the security returns is a fairly good predictor of FECON2, but has almost no predictive power for FECON8 and is useless for predicting FECON1, FECON3, FECON4, FECON5, FECON6, FECON7 and FECON9.

TABLE 11.30

SQUARED MULTIPLE CORRELATIONS

Squared Multiple Correlations Between the Security Returns and the First 'M' Canonical Variates of the Economic Indicators

M	1	2	3	4	5	6
FSEC1	0.1506	0.1849	0.2088	0.2165	0.2200	0.2214
FSEC2	0.3741	0.4038	0.4102	0.4141	0.4142	0.4152
FSEC3	0.0085	0.1911	0.1950	0.1874	0.2020	0.2032
FSEC4	0.0010	0.0035	0.0094	0.0203	0.0204	0.0367
FSEC5	0.0018	0.0026	0.0380	0.0703	0.0704	0.0709
FSEC6	0.0057	0.0152	0.0200	0.0236	0.0493	0.0511

Squared Multiple Correlations Between the Economic Indicators and the First 'M' Canonical Variates of the Security Returns

M	1	2	3	4	5	6
FECON1	0.0003	0.0158	0.0359	0.0360	0.0361	0.0406
FECON2	0.4542	0.4810	0.4827	0.4839	0.4840	0.4841
FECON3	0.0004	0.0020	0.0058	0.0073	0.0107	0.0136
FECON4	0.0212	0.0213	0.0627	0.0628	0.0659	0.0715
FECON5	0.0017	0.0119	0.0138	0.0138	0.0212	0.0266
FECON6	0.0001	0.0001	0.0075	0.0076	0.0260	0.0272
FECON7	0.0000	0.0002	0.0029	0.0130	0.0138	0.0151
FECON8	0.0535	0.2570	0.2588	0.2590	0.2592	0.2604
FECON9	0.0141	0.0148	0.0152	0.0614	0.0617	0.0618

The squared multiple correlations show that the second canonical variate of the economic indicators has a moderate predictive power for FSEC3, but is almost useless for predicting the others. The second canonical variate of the security returns has moderate predictive power for FECON7, FECON2 and FECON1, but is almost useless for predicting the others.

The squared multiple correlations indicate that the third canonical variate of the economic indicators only has moderate predictive power for FSEC5 and FSEC1, but is almost useless for predicting the others. The third canonical variate of the security returns also has only moderate predictive power for FECON4, FECON1 and FECON6, but almost none for the others.

Whereas the squared multiple correlations in Table 11.30 show that the fourth canonical variate of the economic indicators has only small predictive power for FSEC5 and is almost useless for predicting the others. The fourth canonical variate of the security returns also has very small predictive power for FECON9, but almost none for the others.

11.7.3 Discussion

In section 11.7, the relationships between international security returns and economic indicators are analysed by linking and comparing the two sets of factors extracted from the security returns and the economic indicators. The results from section 11.7.1 imply that the canonical correlation between the first canonical variate of the security returns and that of the economic indicators is 0.7340. This is the highest correlation between any linear combination of the security returns and the economic indicators. The results imply that there is a good correspondence between the security returns and the economic indicators. The results in section 11.7.2 indicate that there is a strong linkage between the time-series returns on market indices and the UK and the US security returns. In general, the US interest rates are also related to the security returns. The UK and US security returns seem to be influenced by the US lagging and leading indices, US money supply, US umemployment rate, residential construction; the coincident composite index, GNP, industrial production, the performance of US manufacturing sectors, the oil prices and the consumers expenditure on durable goods. To a lesser extent, the security returns are also determined by the UK interest rates and the exchange rate of US$ to sterling.

11.8 Canonical Correlation Analysis between the UK Economic Indicators and the US Economic Indicators

The objective of this section is to investigate the relationships between the UK and the US economic indicators. The results will show the correlation structure of the two economies.

11.8.1 Empirical results using the canonical correlation analysis approach

The factor scores of the factors extracted from the UK and the US economic indicators in chapter 6 and chapter 9 were subject to canonical correlation analysis in order to find the relationship between the two economies. There are three UK economic factors and six US economic factors, they are the same ones as those that were extracted in the domestic sections in chapters 6 & 9.

As shown in Table 11.31, the correlations among the factor scores of the UK economic indicators and those of the US are moderate, the largest one is 0.3351 between FECONA2 (UK) and FECONB4 (US). This correlation is between the factor scores of the second UK economic factor and the fourth US economic factor. However, significance cannot yet be assumed.

TABLE 11.31

CORRELATIONS BETWEEN THE FACTOR SCORES OF THE UK ECONOMIC INDICATORS AND THAT OF THE US

	FECONA1	FECONA2	FECONA3
FECONB1	0.0251	-0.1457	0.2050
FECONB2	0.0886	-0.0492	0.0228
FECONB3	-0.0449	-0.1063	-0.1780
FECONB4	0.3059	0.3351	0.0163
FECONB5	-0.0099	-0.0056	0.0329
FECONB6	-0.0626	-0.1928	0.0678

(FECONA = Factor scores of UK economic indicators)
(FECONB = Factor scores of US economic indicators)

As shown in Table 11.32, the first canonical correlation is 0.4973, representing 24.73% of overlapping variance between the first pair of canonical variates (i.e. linear combination of the factor scores of the security returns and that of the economic indicators), which also appears to be larger than any of the direct between-set correlations. This implies that the first pair of canonical variates is related to one another. The second canonical correlation is 0.2989, representing 8.93% of overlapping variance for the second pair of canonical variates. Therefore, there are two statistically significant relationships between the first pair of canonical variates.

It is shown in Table 11.32 that the first and second pairs of canonical variates reach significance ($\alpha = 0.05$) and they account for the significant relationships between the two sets of variables (i.e. the factor scores of the UK economic indicators and US economic indicators).

As shown in Table 11.33, the first canonical correlation vectors are

$$\rho_1 = 0.5432 \text{ FECONA1} + 0.8093 \text{ FECONA2} - 0.0446 \text{ FECONA3}$$

TABLE 11.32

CANONICAL CORRELATION ANALYSIS

	1	2	3
Canonical Correlation	0.4973	0.2989	0.1416
Squared Canonical Correlation	0.2473	0.0893	0.0200

Tests of H_0: The canonical correlation in the current column and all that follow are zero

	1	2	3
Likelihood Ratio	0.67175413	0.89241721	0.97995216
F-test	6.6258	3.2794	1.4372
PR > F	0.0001	0.0004	0.2218

and

$$\phi_1 = -0.2363 \text{ FECONB1} + 0.0052 \text{ FECONB2} - 0.2168 \text{ FECONB3}$$
$$+ 0.8611 \text{ FECONB4} - 0.0174 \text{ FECONB5} - 0.3619 \text{ FECONB6}$$

with $r_c = 0.4973$.

11.8.2 Interpretation of canonical variates

After the canonical correlation creates the canonical variates, the factor loading matrix contains the correlations of the original variables (i.e. factor scores of the UK and US economic factors) with the canonical coefficients. As shown in Table 11.34, the first pair of canonical variates has high loading on FECONA2 (0.8392) and FECONA1 (0.5873) of the factor scores of the UK economic indicators and on FECONB4 (0.8781) and FECONB6 (-0.3882) of the factor scores of the US economic indicators. Thus, the first canonical

TABLE 11.33

STANDARDIZED CANONICAL COEFFICIENTS FOR THE SECURITY RETURNS AND THE ECONOMIC INDICATORS

Standardized Canonical Coefficients (B_x) for the Factor Scores of the UK Economic Indicators

	ECONA1	ECONA2	ECONA3
FECONA1	0.5432	0.3328	0.7728
FECONA2	0.8093	-0.1871	-0.5596
FECONA3	-0.0446	0.9249	-0.3776

Standardized Canonical Coefficients (B_y) for the Factor Scores of the US Economic Indicators

	ECONB1	ECONB2	ECONB3
FECONB1	-0.2363	0.7562	0.1526
FECONB2	0.0052	0.1934	0.6106
FECONB3	-0.2168	-0.5349	0.6587
FECONB4	0.8661	0.1793	0.3014
FECONB5	-0.0174	0.0913	-0.1143
FECONB6	-0.3619	0.2470	0.2722

(FECONA = Factor scores of UK economic indicators)
(FECONB = Factor scores of US economic indicators)

variates are primarily FECONA2 and FECONA1 for the UK economic indicators and FECONB4 and FECONB6 for the US economic indicators.

The second pair of canonical variates has high loading on FECONA3 (0.9280) of the factor scores of the UK economic indicator and on FECONB1 (0.7535) and FECONB3 (-0.5345) of the factor scores of the US economic indicators. Hence, the second canonical variates are primarily FECONA3 for the UK economic indicators and FECONB1 and FECONB3 for the US economic indicators.

As shown in Table 11.35, canonical redundancy analysis illustrates that the first pair

TABLE 11.34

CANONICAL STRUCTURE

Correlations Between the Factor Scores of the UK Economic Indicators and their Canonical Coefficients, (A_x)

	ECONA1	ECONA2	ECONA3
FECONA1	0.5873	0.3300	0.7390
FECONA2	0.8392	-0.1703	-0.5165
FECONA3	-0.0416	0.9280	-0.3704

Correlations Between the Factor Scores of the US Economic Indicators and their Canonical Coefficients, (A_y)

	ECONB1	ECONB2	ECONB3
FECONB1	-0.2281	0.7535	0.1663
FECONB2	0.0147	0.2002	0.6172
FECONB3	-0.2060	-0.5345	0.6496
FECONB4	0.8781	0.1814	0.3019
FECONB5	-0.0228	0.0941	-0.1198
FECONB6	-0.3882	0.2608	0.2394

Correlations Between the Factor Scores of the UK Economic Indicators and the Canonical Coefficients of the Factor Scores of the US Economic Indicators, ($R_{xx}B_y$)

	ECONB1	ECONB2	ECONB3
FECONA1	0.2920	0.0986	0.1046
FECONA2	0.4173	-0.0509	-0.0731
FECONA3	-0.0207	0.2773	-0.0524

Correlations Between the Factor Scores of the US Economic Indicators and the Canonical Coefficients of the Factor Scores of the UK Economic Indicators, ($R_{yy}B_x$)

	ECONA1	ECONA2	ECONA3
FECONB1	-0.1134	0.2252	0.0236
FECONB2	0.0073	0.0598	0.0874
FECONB3	-0.1024	-0.1598	0.0920
FECONB4	0.4366	0.0542	0.0427
FECONB5	-0.0113	0.0281	-0.0170
FECONB6	-0.1930	0.0780	0.0339

(FECONA = Factor scores of UK economic indicators)
(FECONB = Factor scores of US economic indicators)

TABLE 11.35

CANONICAL REDUNDANCY ANALYSIS

Standardized Variance of the UK Economic Indicators Explained by:

	1	2	3
Their Own Canonical Variates:			
Proportion	0.3503	0.3330	0.3167
Cumulative Proportion	0.3503	0.6833	1.0000
Canonical R-Squared	0.2473	0.0893	0.0200
The Opposite Canonical Variates:			
Proportion	0.0866	0.0297	0.0063
Cumulative Proportion	0.0866	0.1164	0.1227

Standardized Variance of the US Economic Indicators Explained by:

	1	2	3
The Own Canonical Variates:			
Proportion	0.1695	0.1672	0.1656
Cumulative Proportion	0.1695	0.3367	0.5023
Canonical R-Squared	0.2473	0.0893	0.0200
The Opposite Canonical Variates:			
Proportion	0.0419	0.0149	0.0033
Cumulative Proportion	0.0419	0.0568	0.0602

of canonical variables is a moderate overall predictor of the opposite set of variables, the proportions of variance explained being 0.0866 and 0.0419. Although the second pair of canonical variates is statistically significant, it is not economically meaningful, the proportions of variance explained being 0.0297 and 0.0149.

The squared multiple correlations in Table 11.36 indicate that the first canonical variate of the US economic indicators has moderate predictive power for the second factor scores of the UK economic indicators and has some predictive power for the first factor scores of the US economic indicators. The first canonical variate of the UK economic indicators is also a moderate predictor for the fourth factor scores of the US economic indicators. Whereas, the second canonical variate of the US economic indicators only has some predictive power for the third factor scores of the UK economic indicators. On the other hand, the second canonical variate of the UK economic indicators has only little predictive power for the first factor scores of the US economic indicators.

11.8.3 Discussion

In section 11.8, the relationships between the UK economic indicators and the US economic indicators are analysed by canonical correlation analysis. The results from section 11.8.1 shows that the canonical correlation between the first canonical variate of the UK economic indicators and that of the US economic indicators is 0.4973. This is the highest correlation between any linear combination of the UK and the US economic indicators. The first and second UK economic factors are related to the fourth and sixth US economic factors. In other words, the major economic variables of the UK (market indices, longer leading indicator, money supply, interest rate, lagging indicator, unemployment rate and gross redemption yield on gilts) correspond with the economic variables of the US (leading composite index, money supply (M2), share prices - industrials, residential construction (private sector), demand deposits level, lagging composite index; interest rate, yield of long-term government bonds, commercial bank loans, and lagging composite index). The above UK and US economic factors are also the major economic factors that correspond with the

TABLE 11.36

SQUARED MULTIPLE CORRELATIONS

Canonical Redundancy Analysis

Squared Multiple Correlations Between the UK Economic Indicators and the First'M' Canonical Variates of the US Economic Indicators

M	1	2	3
FECONA1	0.0853	0.0950	0.1060
FECONA2	0.1741	0.1767	0.1821
FECONA3	0.0004	0.0773	0.0801

Squared Multiple Correlations Between the US Economic Indicators and the First 'M' Canonical Variates of the UK Economic Indicators

M	1	2	3
FECONB1	0.0129	0.0636	0.0641
FECONB2	0.0001	0.0036	0.0113
FECONB3	0.0105	0.0360	0.0445
FECONB4	0.1906	0.1936	0.1954
FECONB5	0.0001	0.0009	0.0012
FECONB6	0.0373	0.0433	0.0445

(FECONA = Factor scores of UK economic indicators)
(FECONB = Factor scores of US economic indicators)

UK and US security returns in the domestic country respectively (i.e. the results of chapters 7 and 10).

To a lesser extent, the third UK economic factor is positively related to the first US economic factor. It implies that the UK economic variables (industrial production, coincident indicator, GDP, consumer expenditures on durable goods, and shorter leading indicator) correspond with the US variables (general market-wide variables, interest rate, GNP, the coincident and leading composite indices). And to a lesser extent, the third UK economic factor is negatively related to the third US economic factor. It implies that the UK variables (industrial production, coincident indicator, GDP, consumer expenditures on durable goods, and shorter leading indicator) are negatively related to the US variables (manufacturing net new orders and deliveries, producer and consumer prices indices, wholesale prices on gas fuels, and yield on long-term government bonds).

11.9 Conclusion

This chapter investigates the APT in an international setting, namely, the UK and the US. Canonical correlation analysis is used to investigate a set of economic indicators as systematic influences on stock returns.

The results from this chapter indicate that there is good correspondence between factor scores generated by the factor analysis on security returns and that on economic indicators.

The IAPT has been investigated using two separate methods. In section 11.3, the canonical correlation analysis is used to analyse the correlation between the factor scores of the factors extracted from the UK security returns and that of the US security returns. The results show that there is one significant pair of canonical variates and it is composed of the first factor of the UK security returns and that of the US security returns. As it has been shown in the previous chapters (i.e. chapters 7 & 10), the first factor of the UK security returns encompasses the UK market indices, longer leading indicator, and average gross redemption yield on 20 year government securities. While the first factor of the US security returns consists of the US economic indicators such as the interest rate, yield of long-term government bonds, the commercial banks loans, the amount of new capital issues by corporations, and lagging indicators.

In addition, canonical redundancy analysis has shown that 12.65% of the standardized variance of the UK security returns can be explained by the canonical variate of the US security returns whereas only 5.09% of the standardized variance of the US security returns is explained by the canonical variate of the UK security returns. It can be concluded that the US stock market is a more influential market than the UK stock market, as the US stock market has a higher capability of accounting for the variances of the UK stock market.

The second method is discussed in section 11.4, the factor scores of the factors extracted from the UK and the US security returns and those from the UK and the US economic indicators are subject to canonical correlation analysis. The results show that there are four significant pairs of canonical variates.

The first pair of canonical variates is composed of (UK and US) market indices, (UK) leading indicators, general economy-wide variables; (US and UK) interest rate, (US and UK) yield of long-term government bonds, the (US) amount of loans of commercial banks.

The second pair of variates represents variables such as (US and UK) interest rate, (US and UK) yield of long-term government bonds, the (US) amount of loans of commercial banks; and (US) general economy-wide variables, (US) interest rate, (US) GNP, (US) employment, (US) coincident and leading indicators.

The third pair of variates are primarily (US and UK) leading indicators, (US) money supply (M2), (US) share prices - industrials, (US) construction of residential and private sector, (US) demand deposits level, (UK) lagging indicator; (US) general economy-wide variables, (US) interest rate, (US) GNP, (US) employment, (US) coincident and leading indicators; (UK) industrial production, (UK) coincident indicator, (UK) GNP, (UK) consumer expenditure on durable goods, and (UK) shorter leading indicator.

The fourth pair of variates represents variables such as (UK) gross redemption yield on 20 year gilts, (UK) interest rate, (UK) FT government securities price index, (UK) exchange rate (US$ to sterling); and the (US) balance of payments, (US) total value of the contracts of construction, (US) output of crude petroleum, (US) new capital issues by corporations, and (UK) wholesale prices (manufacturing input - fuel).

These four pairs of canonical variates represent a combination of the UK and the US economic indicators. The results conclude that the international security returns are influenced by the combination of the UK and US economic forces. Global diversification leads to the important role of international factors in asset pricing. The evidence is consistent with non-trivial international influences in asset pricing.

Section 11.8 analyses the relationships between UK economic indicators and US economic indicators. The factor scores of the UK economic factors and those of the US economic factors are subject to canonical correlation analysis. The correlation pattern reflects the degree of economic integration between the two countries. This is so because the more integrated two economies are, the more strongly the stock market movements in one country would be correlated to those in another country. The results show that there are two significant pairs of canonical variates and they consist of the first two factors of the UK economic indicators and the fourth factor of the US economic indicators; and the third factor of the UK economic indicators and the first factor of the US economic indicators. As it is shown in the previous chapters, the first two factors of the UK economic indicators encompass the market indices, general market-wide variables, and the longer leading indicator, lagging indicator, money supply, interest rate, gross redemption yield on gilts and unemployment rate. While the fourth factor of the US economic indicators is composed of leading indicators, money supply (M2), share prices - industrials, construction of residential and private sector, and demand deposits level.

The second pair of canonical variates consists of the third factor of the UK economic indicators and the first and the third factors of the US economic indicators. The third factor of the UK economic indicators encompasses variables such as industrial production, coincident indicator, GDP, consumer expenditures on durable goods, and shorter leading indicator. While the first factor of the US economic indicators consists of general economy-wide variables, interest rate, GNP, employment, and encompasses coincident and leading indicators, the third factor of the US economic indicators represents variables such as manufacturing net new orders and deliveries, producer and consumer prices indices, wholesale prices on gas fuels, and the yield on long-term government bonds.

The results above reflect a high degree of economic and financial integration between the UK and the US economies. The number of pairs of canonical variates is interpreted as reflecting the complexity of the economic relationship between the economies of the two countries. If two countries are integrated through many levels of economic activity (i.e. high economic integration), then more significant pairs are expected to be found. However, if two countries are integrated only through limited levels of economic activity (i.e. low economic integration), then fewer significant pairs are expected.

In addition, canonical redundancy analysis has shown that 8.66% of the standardized variance of the UK economic indicators can be explained by the first canonical variate of the US economic indicators whereas only 4.19% of the standardized variance of the US economic indicators is explained by the first canonical variate of the UK economic indicators. However, it has been shown that 2.97% of the standardized variance of the UK economic indicators can be explained by the second canonical variate of the US economic indicators and only 1.49% of the standardized variance of the US economic indicators is explained by the second canonical variate of the UK economic indicators. Therefore, it can be concluded that the US economy is more influential than the UK economy as the US economic indicators have a higher capability of accounting for the variances of the UK economy. The results also imply that there is a fair correspondence between factor scores generated by the factor analysis on the UK economic indicators and that on the US economic indicators. As expected, there is a certain level of economic integration between the two economies.

CHAPTER 12

CONCLUSIONS

12.1 Introduction

The objective of this study was to analyse the empirical applicability of the APT to international asset markets (UK stock market and US stock market) and to identify the set of economic variables which correspond most closely with the stock market factors obtained from the traditional factor analysis.

Factor analysis and canonical correlation analysis were used as the principal tools for the empirical testing. Although factor analysis is frequently used, canonical correlation analysis is a new technique in this area and provides a method of linking factors extracted from the two sets of data. Various economic indicators were investigated as systematic influences on stock returns. It was shown that, based on the foundations of the APT and the characteristics of the factor scores from the factor analysis on the security returns and the economic indicators, canonical correlation analysis is an appropriate technique to link the economic forces and the stock market.

12.2 Stock Returns and Economic Forces : UK Results

The results using the UK data imply that there is good correspondence between factor scores generated by the factor analysis on the UK security returns and on the UK economic indicators.

The first pair of canonical variates is composed of the factor scores of the first factor of the UK security returns and those of the first factor of the UK economic indicators. The first economic factor encompasses market indices. The second pair of canonical variates is composed of the factor scores of the second factor of the security returns and those of the second and third factors of the economic indicators. The second economic factor represents primarily longer leading indicator, lagging indicator, money supply, interest rate, gross redemption yield on gilts, and unemployment rate. Whereas the third factor encompasses variables such as industrial production, coincident indicator, GDP, consumer expenditures on durable goods, and shorter leading indicator.

The results show that the canonical correlation analysis successfully links the stock returns and economic forces. The conclusion of these empirical findings is that security returns are influenced by a number of systematic economic forces.

The interesting result obtained is that the UK market return plays a major role in the APT in the UK security market. The market return explains a significant portion of the time series variability of stock returns. The result is consistent with the view that the market factor is an aggregate consensus measure of all the underlying factors. One may view CAPM as a one-factor APT model, with the market model being the return generation process. The CAPM is a special case of the APT.

The validity and applicability of the APT to the UK stock market were also empirically evaluated. The APT implies that there is a linear relationship between the risk measures embodied in the factor loadings and the expected returns. The regression results show that the APT explains 11% of the variation in the twenty-four years average returns. Although statistically significant, the explanatory power of the APT in pricing UK stocks is not high. The validity of the APT in pricing UK stocks is supported in that the intercept term and the risk premium of the first stock market factor are significantly different from zero. The positive intercept term is consistent with the APT model, as one testable implication of the APT is that the intercept term should be positive. It is clear from the cross-sectional regression results that the APT has some empirical power (in terms of adjusted R^2). The result is quite encouraging. In this study, it has been assumed that the non-stationarity problem does not exist. By taking no measures to mitigate any problems arising from this, these tests are biased toward finding that the risk measures are not significant. In testing the APT, the return distribution is assumed to be stationary over time so that measures of systematic risk can be estimated from a correlation matrix based on, in this case, twenty-four years of data. It is a special case when the risk premia are assumed to be constant through time, although the theory does not require this.

12.3 Stock Returns and Economic Forces : US Results

The results using the US data show that there is also a fair correspondence, but lower than that for the UK data, between factor scores generated by the factor analysis on the US security returns and on the US economic indicators.

The first pair of canonical variates is composed of the factor scores of the first and second factors of the US security returns and those of the sixth factor of the US economic indicators. The sixth economic factor encompasses the interest rate, yield of long-term government bonds, the amount of new capital issues by corporations and the commercial banks loans.

The second pair of canonical variates is composed of the factor scores of the first, second, and fourth factors of the security returns and those of the fourth factor of the US economic indicators. The fourth economic factor represents primarily the leading indicators, money supply (M2), share prices - industrials, construction of residential and private sector and demand deposits level.

The signs of the correlations between the security returns and the canonical coefficients of the economic indicators, and between the economic indicators and canonical coefficients of the security returns are consistent with macroeconomic reasonings.

There is a better correspondence between factor scores generated by the factor analysis on security returns and on economic indicators of the UK than that of the US results.

The validity of the APT in pricing US stocks is supported by the fact that the intercept term and the risk premia of the second and the fifth stock market factors are also significantly different from zero. The positive intercept term is consistent with the APT model, as one testable implication of the APT is that the intercept term should be positive. The cross-sectional regression results show that the APT explains 30% of the variation in the twenty-four years average returns. The result is quite surprising and very encouraging as modelling twenty-four years returns is a difficult task because there is a high variation in the measures of risk and return when long time periods are used. In this study, it has been assumed that the non-stationarity problem does not exist.

12.4 International Arbitrage Pricing Theory

The APT was also investigated in an international setting by considering the UK data and the US data together. The tests of international APT require the uses of the APT to analyse asset returns across two or more countries. Not many empirical studies have used data from two or more countries. The total market capitalisation of the New York Stock Exchange and London Stock Exchange represents over 40% of the total market capitalisation of the world's major equity markets. The International Arbitrage Pricing Theory (IAPT) was investigated by two separate methods. Firstly, canonical correlation analysis was used to analyse the correlation between the factor scores of the factors extracted from the UK security returns and those of the US security returns. The results show that there is one common factor between the UK security returns and the US security returns. From the results of chapters 7 & 10, it has been shown that the first UK stock market factor is correlated with the UK market indices. While the first US stock market factor is correlated with the US economic indicators such as the interest rate, yield of long-term government bonds, the amount of loans of commercial banks, the amount of new capital issues by corporations and lagging indicators. It has also been concluded that the US security returns are more influential than the UK security returns as the US security returns have a higher capability of accounting for the variances of the UK security returns.

For the second method, the factor scores of the factors extracted from the UK and the US security returns and those from the UK and the US economic indicators are subject to canonical correlation analysis. The international (i.e. UK and US) economic factors are also the major economic factors that correspond with the UK and US security returns in the domestic country respectively. There are in total four significant pairs of canonical variates.

The first pair of canonical variates is composed of (UK and US) market indices, (UK) leading indicators, general economy-wide variables; (US and UK) interest rate, (US and UK) yield of long-term government bonds and the (US) amount of loans of commercial banks.

The second pair of variates represents variables such as (US and UK) interest rate, (US and UK) yield of long-term government bonds, the (US) amount of loans of commercial banks; (US) general economy-wide variables, (US) interest rate, (US) GNP, (US) employment and (US) coincident and leading indicators.

The third pair of variates are primarily (US and UK) leading indicators, (US) money supply (M2), (US) share prices - industrials, (US) construction of residential and private sector, (US) demand deposits level, (UK) lagging indicator; (US) general economy-wide variables, (US) interest rate, (US) GNP, (US) employment, (US) coincident and leading indicators; (UK) industrial production, (UK) coincident indicator, (UK) GNP, (UK) consumer expenditure on durable goods and (UK) shorter leading indicator.

The fourth pair of variates represents variables such as (UK) gross redemption yield on 20 year gilts, (UK) interest rate, (UK) FT government securities price index, (UK) exchange rate (US$ to sterling), the (US) balance of payments, (US) total values of the contracts of construction, (US) output of crude petroleum, (US) new capital issues by corporations and (UK) wholesale prices of manufacturing input (fuel).

The number of pairs of canonical variates is interpreted as reflecting the complexity of the economic relationship between the two countries. The results reflect a high degree of economic and financial integration of the two countries.

The validity and applicability of the APT to the international stock market were also evaluated. The regression results show that the APT explains 26% of the variation in mean returns of the sample from January 1965 through December 1988. This suggests that the explanatory power of the model is fairly good. The results show that the third and sixth rotated factors and the intercept term are priced and the positive intercept term is also consistent with the APT model, as one testable implication of the APT is that the intercept term should be positive. The overall results obtained here appear to suggest that the APT pricing relationship is supported by the testing methodology.

12.5 Linkages between the UK and the US Economies

The canonical correlation analysis was also used to analyse the correlation between the factor scores of the factors extracted from the UK economic indicators and those from the US economic indicators. There are two statistically significant pairs of canonical variates and they consist of the first two UK economic factors and the fourth US economic factor; and the UK third economic factor and the first and the third US economic factors.

The first two UK economic factors encompass the market indices, market-wide variables, the longer leading indicator, lagging indicator, money supply, interest rate, gross redemption yield on gilts and unemployment rate. While the fourth US economic factor is composed of leading indicators, money supply (M2), share prices -industrials, construction of residential and private sector and demand deposits level.

The second pair of canonical variates consists of the third UK economic factor and the first and the third US economic factors. The third UK economic factor represents variables such as industrial production, coincident indicator, GDP, consumer expenditures on durable good and shorter leading indicator. The first US economic factor consists of general economy-wide variables, interest rate, GNP, employment, and encompasses coincident and leading indicators. While the third US economic factor represents variables such as the amount of manufacturing net new orders and deliveries, producer prices index, consumer prices index, wholesale prices on gas fuels and yield on long-term government bonds.

It has also been shown that the US economy is more influential than the UK economy on the international transmission of financial market movements, as the US economic indicators have a higher capability of accounting for the variances of the UK economy.

12.6 Contributions of the Study

The APT is based on a simple and intuitive insight. Despite the appeal of its generality, the APT does not offer any theoretical or empirical grounds for identifying the economic nature of factors. The APT gives little guidance on the identity of the factors and does not tell us what factors are relevant.

In this study, factor analysis was used to identify the number of stock market and macroeconomic factors and to examine their importance. The correlations between macroeconomic variables could produce a collinearity problem. Factor analysis was used to construct independent economic factors. The independent macroeconomic factors extracted from the macroeconomic and financial variables eliminate multicollinearity among independent variables. These estimated economic factors convey the relevant information of the economy in a reduced form of a macro-model. However, factor analysis on the security returns was merely concerned with statistical correlations and was blind to aggregate economic considerations. In investigating whether the same factors have appeared in both in the set of the stock market factors and that of the economic factors, it is not sufficient to just examine the factor loadings. Based on the foundations of the APT and the characteristics of the factor scores from the factor analysis on security returns and economic indicators. Canonical correlation analysis is a better procedure for explaining as much as possible between one set of variables (i.e. factor scores of security returns) and another set (i.e. factor scores of economic indicators). If the canonical correlations between the factor scores for corresponding pairs of factors are statistically significant, then they imply the factor comparability of the stock returns and the economic forces. The factor structure is therefore similar. As a result, the APT factors are identified which are based on the intuition of the APT (i.e. the factors are orthogonal to each other) and hence, we have a better APT model which we could successfully relate the factors more closely to identifiable sources of economic risk. The approach here of using canonical correlation analysis is superior to that of Chen, Roll and Ross (1986). Canonical correlation analysis examines the relationship between the security returns and the economic indicators by creating a linear combination of the factor scores of the security returns and those of the economic indicators. The canonical correlation technique considers all the variables (factor scores of security returns and those of economic indicators) simultaneously rather than considering the possibility that a particular variable may be significant in one multiple regression but not when other independent variables have been changed or when analysed alone in an univariate model. Separate multiple regression analyses of each set of variables would neglect the interrelations of the two sets.

Overall, the results from this study suggests that the APT is a better model as it improves our understanding of security returns. We have a better understanding of the relationship between return factors and economic forces through the work in asset pricing theory, macroeconomics, econometrics and statistical techniques.

12.7 Summary

In this thesis, several major issues of the applicability of the APT to the London Stock Exchange and the New York Stock Exchange have been addressed. Individual sets of economic variables have been identified which correspond most closely with the UK and the US stock market factors by using the canonical correlation analysis. Such a method appears to represent an innovation for empirical research on the APT. In addition, the international perspective of the APT has been investigated and the international correlation structure of financial markets movements between the UK economy and the US economy has been analysed. On balance, the evidence favours the APT and there is available evidence of inter-market linkage between the UK and the US. The results, at least partially, contribute to the understanding of security market pricing.

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