WAGES DETERMINATION AND FIRM'S BEHAVIOUR UNDER STRATEGIC MARKET COMPETITION

BY

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Abstract

It is commonplace in wage determination models and, in general, in economic models as a whole, to treat the workers' outside option as given. The main purpose of the present work is to remove, in various ways, this assumption. The work is organized as follows. The first chapter is devoted to introducing the thesis topic and the related literature. The second chapter describes an economy in which the workers hired by a firm acquire without cost a firm-specific skill that enables them to potentially become independent producers. Thus, by modelling explicitly the workers' decision to stay or to leave the firm, a *stable earning profile* for the economy is characterized. Such a stable earning profile can allow for a workers' compensation higher than the basic neo-classical wage and for pay differentials across industries even for initially homogenous workers. The third chapter shows that the existence of a concrete outside option for firms' managers can induce, under specific circumstances, oligopolistic firms to adopt restrictive output practises. In particular, the conditions under which, in a Cournot oligopoly, existing firms behave more collusively than in a standard Cournot model, are carefully defined. The fourth chapter considers the problem of producer co-operatives' (PCs) stability. It shows that PCs' instability argued in the literature can fail to hold in very competitive and low barrier-to-entry markets in which, potentially, dismissed members have a chance to set up new firms. In the fifth and conclusive chapter a new concept of *core-stability* for n-cooperative games is introduced and applied both to the problem of cartel formation under oligopoly and to an economy with a public good. Such a solution concept, denoted ϕ-core, assumes that when a coalition deviates from an agreement, it possesses a first-mover advantage with respect to all other players.

Keywords: *Coalitions, Cooperative Games, Collusion, Core, Managers Compensation, Oligopoly, Public Goods, Self-employment, Wage Determination.*

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Contents

Chapter 1

Introduction

It is commonplace in wage determination models and, in general, in economic models as a whole, to treat the workers' outside option as given. This is the case both in standard bargaining models and in mainstream labour market models in which, usually, the option of every worker is somewhat dichotomous: she or he can either accept a certain pay by a firm or organization whatsoever, or stay inactive, thus obtaining an amount of money that, without loss of generality, can be normalized to zero.

While under certain modelling circumstances this appears as an innocuous and even useful simplifying assumption, in other contexts the same assumption seems, at the best, questionable. In particular, such an implicit presumption of passive behaviour on the part of every worker, either when negotiations with the firm break down or when there is simply an insufficient labour demand, cannot be accepted as a *primitive* of a model as such. In a model without credit constraints and without transaction costs of any sort, why should workers remain idle when facing involuntary unemployment or also when they are voluntarily leaving their workplace?

In a world where either the barriers to entry are not a serious obstacle or in which there is a way in which inactive workforce can become active - through the constitution of self-employed units, partnerships or entrepreneurial firms alike - every worker's outside option cannot be treated as exogenous. It should, indeed, be considered as part of the model variables, i.e., in economic words, be viewed as *endogenous* to the model itself.

The major purpose of the present work (at least, in three of its chapters) is to remove in various ways the above assumption, under the opposite presumption of an active (rather than passive) behaviour on the part of every worker, and, hence, look at the consequence of this point of view. The starting point is that, although the workers that have never been recruited by a firm may not possess any ability to organize their own work, and then their outside option is something given (that can also be normalized to zero), all workers that have been engaged in a firm, own the necessary skill to organize the production process. This dichotomy between skilled and unskilled workers is the key to make *endogenous* the outside option of all skilled workers and hence, derive interesting results.

In most parts of the work, the main modelling tool will be game theory and, in particular, n-person cooperative games. Such a tool, apart from being extremely useful in economic applications, possesses an interest in itself. The last chapter of the thesis will explore a new concept of equilibrium for n-player cooperative games, denoted ϕ-core, that allows every deviating coalition to have a first-mover advantage with respect to the outside players. To present its characteristics, this concept will also be applied to two well known traditional problems of cooperative games: cartel formation under oligopoly and public good provision in a n-player economy.

1.1 Wages determination and labour markets

The second chapter of this thesis presents a model that attempts to analyse the wage determination problem through a completely different approach from that usually adopted by the traditional neoclassical models. It is well known that in the neoclassical models wages are determined competitively by the forces of supply and demand and respond to shortages or surpluses of labour in their various markets. In these models such forces interact to determine a structure of wages and earnings that reflect the willingness of labour to supply itself to the various jobs on offer, and the willingness of employers to pay workers for their marginal products.[1] However, this standard view of

[1] This approach is also in accordance with the seminal Adam Smith's (1776) theory of "compensating wage differentials".

the labour markets has been questioned in many different ways. First, the empirical evidence shows that similar jobs in different firms are often associated with significantly different wage rates.[2] Second, persistent interindustry wage differences for similar jobs are observed in many industrialized countries.[3] Third, the persistence of both wages stickiness and unemployment is inconsistent with the neoclassical competition models.[4]

Although many recent contributions have attempted to tackle these problems, currently a conclusive and convincing explanation does not exist for any of them. To provide just a simple example, efficiency wage theory[5] explanations of interindustry wage differentials, based on the different efficiency wage premiums existing in different industries, has been criticized on empirical and theoretical grounds.[6]

At a completely different level, interindustry wage differentials can be explained, for instance, by the traditional bargaining models, through the observation that workers' bargaining power can vary across different industries even for similar jobs.[7] Nash's (1950) axiomatic bargaining solution probably represents one of the most popular and broadly used frameworks not only in the labour economic literature but in economic analysis as a whole.[8] The extensive use of two-player bargaining models has been also

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[2] The labour market segmentation position can be traced back at least to John Stuart Mill (1871) and Cairnes (1874) who rejected Adam Smith's competitive theory, almost a century later to the American institutionalists (Dunlop (1957), Kerr (1954)), and, more recently to the theories of internal labour market (Doeringer and Piore (1971)) and, among others, to the efficiency wage explanations of wage differentials (see, among the others, Weiss (1980), Krueger and Summer (1988)).

[3] See, for instance Dickens and Katz (1987) and Krueger and Summer (1988), for tentative explanations of such a phenomenon.

[4] Here we do not even attempt to give a detailed list of contributors to this relevant point. An updated survey is contained, for instance, in Bosworth, Dawkins and Stromback (1996).

[5] See, among others, Solow (1989), Salop (1979) and Shapiro and Stiglitz (1984); for accurate surveys, see Weiss (1990) and Akerlof and Yellen (1986).

[6] See, for instance, the criticisms raised by Murphy and Tophel (1990) on Kruger and Summer's (1988) empirical work.

[7] A simple and updated survey of the effect of workers' bargaining power on wage determination both in unionized and non unionized companies, is contained, for instance, in Booth (1995).

[8] For an extensive presentation of cooperative and non-cooperative two-player bargaining games, see Osborne and Rubinstein (1989).

due to the noncooperative foundations recently given to this approach.[9] However, the description of the intra-firm bargaining process through a two-player model (a group of workers on the one hand and a firm or a system of firms on the other), restricts the analysis by imposing in some cases an excessively simple set of institutional and behavioural assumptions. This is especially true when considering the changes which have recently occurred to the structure of negotiations in labour markets. In recent years Western economies have witnessed an increasing degree of decentralization of wage negotiation coupled with an upward trend in self-employment.[10] Associated with the adoption of a wide variety of flexible compensation schemes, such phenomena have somewhat complicated the analysis of industrial relations and labour markets.[11] As underlined at the beginning, another limitation of traditional bargaining models is that reservation wages are usually assumed exogenous. Conversely, it may well be possible that reservation wages vary either across firms (due to different firm-specific skills of workers) or across industries (for the different opportunities existing for the workers hired in different sectors).

The model of the next chapter, although adopting a rather strong assumption, yields results very different from a standard neoclassical model. The model describes an economy in which, initially, there are two different types of individuals: the *entrepreneurs,* endowed with the knowledge of a given production process and the *workers,* endowed with just one unit of labour but without any knowledge on how to produce a commodity. Initially the workers are assumed to be indifferent with respect to the range of jobs offered by different industries. Moreover, once hired by a firm, they are all capable of performing any job, with the training taking the form of a virtually instantaneous on-the-job-training without cost for the firm.[12] The main feature of the

[9] The basic reference of the non-cooperative approach is Rubinstein (1982). For extensive comparisons of Nash and other game-theoretic approaches to the bargaining problem, see Sutton (1986) and Binmore et al. (1986).

[10] See, for instance, Freeman (1995) for a reviews of the decentralization of wage negotiation in the U.S. and U.K., and Blau (1987) and Aronson (1991) for a description of self-employment trends.

[11] When self-employment is interpreted as a pay system typified by highly output-related pay and independence of workers' action (Garen (1996)), it seems to exist a link between the two phenomena indicated above.

[12] This makes the model different from all those models in which workers' training is costly for the firm (see, among others, Lindbeck and Snower's (1988) insider-outsider model).

model is that, once recruited by a firm, workers acquire sufficient skill to *potentially* set up new competitive units. Thus, even if workers are completely substitutable and interchangeable, their potential competing threat may constitute a good reason for existing companies to keep them within the firm through the payment of a wage higher than the usual neoclassical wage. As a result, one of the model features is that workers' reservation wage are not given but rather depend on the specific industry in which the workers are employed. Given such assumptions, the model adopts a cooperative game-theoretic solution concept to determine a *stable earning profile* for the economy. Such an earning profile possesses interesting characteristics. First, workers' equilibrium wage are usually higher than the basic neoclassical (reservation) wage; second, wage differentials across industries for initially homogeneous workers naturally arise in equilibrium; thirdly, the existence of involuntary unemployment can be sustained as a stable equilibrium of the economy.

The impossibility for entrepreneurs to write binding agreements with workers, giving rise to a form of at-will employment contract for the firms, is a feature that the present model shares with the intra-firm multilateral bargaining framework recently introduced by Stole and Zwiebel (1996, 1997). In their model, the firm bargains with each single worker before production starts, under the presumption that a worker can leave at will at any time prior to production. However, there are many differences between the two models. First, in Stole and Zwiebel's model every worker is completely irreplaceable, while we assume, conversely, that workers are completely replaceable. Second, differently from the Cournot type of competition assumed in our model, in their model competition is implicitly introduced (Stole and Zwiebel (1996), p.221). Third, differently from the variable reservation wage raised here, in their model workers' reservation wages are basically given. Finally, in their model, the employees can leave the firm only individually and not as a group, as happens here. As far as their results are concerned, the authors show that the multilateral bargaining model gives rise to overemployment rather than, as in ours, to unemployment.

For the explicit consideration of the possibility of workers forming coalitions and for the use of a n-player cooperative game setting, our model possesses some similarities with the more general approach presented in Ichiishi (1981).[13] Indeed, in Ichiishi's

[13] See also Ichiishi (1997), for a more extended description of his social coalition equilibrium.

work the set of players is not initially partitioned into two different groups and, differently from the multi-sector oligopolistic economy presented here, a general equilibrium framework is adopted; finally, the type of equilibrium the author uses, the social coalitional equilibrium, does not require, as the one presented here, any type of consistency of each coalitional deviation.[14]

1.2 Managers' compensation and firms' collusive behaviour

The idea that the participation in a firm's activity potentially enables employees to organize competing firms by themselves, can very suitably describe the large number of responsibilities that companies often delegate either to their highly skilled workforce or to their company executive officers (CEOs). If such a delegation implies a learning process for the highly qualified employees, it may well be the case that such a knowledge acquisition gives rise to the employees' threat to leave the firm and either set up a competitive venture or just work for another firm, thus revealing relevant information.

Although there exists a well known economic literature[15] claiming that, under oligopoly, the delegation of sales decisions to managers can both be profitable for the firms' owners as well as boost every firm's output with respect to a standard Cournot model, the rise in managers' compensation due to managers' threat of leaving can also be thought to yield opposite results. Whilst in principle the incentive package for managers is the responsibility of the owners, most frequently pay is set by executive compensation committees consisting of non-executive directors or by senior executives themselves.[16] Since managers' compensation is a cost component for the company,

[14] For a more explicit treatment of the cooperative game literature, see section 1.4 and section 5.1.

[15] See, amongst others, Vickers (1985) and Fershtman and Judd (1987).

[16] See, for instance, for detailed and explanatory empirical works, Jensen and Murphy (1990) for the U.S., and Main (1992) for the U.K..

it may well be argued that the "fat cats" phenomenon within the companies reduces competition and output in a given market, when compared to a standard oligopoly model.

The model presented in the third chapter explores this possibility. The model describes a noncooperative game among every company's owner and each hired manager. The latter has to decide whether to accept the offered compensation or leave and setup a new business. As a result, the model shows that, under rather general circumstances, there may be an output restriction with respect to a standard Cournot model. Moreover, equilibrium executives' compensation turns out to be negatively related to the initial number of firms existing in the market and sensitive (although not in a unidirectional way) to existing setup costs. Although the relationship between senior executives' pay and company performance has become a source of controversy in recent years,[17] the direct link found in our model between company's sales and managers' compensation is, usually, empirically confirmed.[18]

As a final remark, it has to be stressed that the model presented here is not meant to describe a world in which firms' managers attempt, whenever possible, to leave their companies to set up new businesses; it rather aims at representing a world in which such a potential threat exists, thus yielding relevant distortions to input allocation and market output.

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[17]See, among others, Gregg, Machin and Szymanski (1993) for a description of this debate.

[18]See, for an empirical work on non-owners managers' compensation and firms' size in the U.K., Watson et al. (1994). Remarkably, also the work by Smith and Szymanski (1995) reverses Jensen and Murphy's result by including a participation constraint in the extimations of companies' performances. In particular, their paper shows that since a "going rate" must be paid to executives to deter them to leave the current firm and sign an incentive contract with another firm, CEOs' outside option will also depend on other firms' performance and hence, the elasticity of top executives to performances will be larger than it would be without this effect.

1.3 The viability of labour-managed organizations

Although by 1981 EC countries had over 14,000 Producer Co-operatives (PCs) with employment in these firms totalling about half a million people, the incidence of PCs remains small relative to conventional organizational forms in Western economies.[19] Theoretical and empirical literature has attempted in various ways to explain both the birth and the viability of PCs in industrialized economies.[20] It is well known that the benchmark model of all theoretical contributions on PCs behaviour is represented by Ward's (1958) seminal model of the "Illyrian" firm, in which the usual profit maximand of the neoclassical firm is replaced by per member value added. The idea of a "life-cycle" for PCs, which can be traced back to Sidney and Beatrice Webb (1890), Tugan-Baranovskii (1921), and more recently to Miyazaki (1984) and Ben-ner (1984), can, in a sense, be derived from Ward's model. The argument is that, whenever PCs are successful, they have a tendency to transform into more conventional profit-maximizing firms. This mainly depends on the rent-seeking behaviour of members who receive a share of value added in excess of the going market wage paid to similar workers. Replacing a member by a hired worker increases the share of value added paid to all remaining workers. As a result, since all members are potentially replaceable by hired workers, over time a PC just becomes a single member profit-maximizing firm.

The aim of the fourth chapter of this work is, once again, to introduce a different presumption on the behaviour of every excluded member. In fact, by explicitly introducing the possibility for every member, when dismissed, to be self-employed or to set up another firm, different results arise with respect to the PCs life-cycle described above. Since every member's outside option is no longer given, a PC can, under certain circumstances, be stable. Two cases are considered: in the first, although the going wage rate is given, a dismissed member can create a new venture; in the second, conversely, the wage rate is treated endogenously through the consideration of

[19] See, for instance, Ben-ner (1988) and Bonin, Jones and Putterman (1993), for empirical surveys on Producer Co-operatives (PCs). These are currently the most spread type of democratic organizations amongst all existing labour-managed forms of organization. We will mostly refer, throughout this section and chapter 4, to PCs.

[20] Surveys are contained, for instance, in Vanek (1970), Ireland and Law (1982), Bonin and Putterman (1983) and Bonin, Jones and Putterman (1993).

stability requirements. Accordingly, two different sets of results follow. The first is that the elasticity of market demand is an important variable in explaining whether or not the process of members' dismissal from a PC is profitable for all remaining members; the second is that, under a stable endogenous wage, newly created PCs can either be stable or unstable depending on the parameters of the model, but it is in general rather unlikely that PCs adopt a member-dismissal strategy. The survival of PCs in many industries, though affected by historical and institutional elements, may represent indirect empirical evidence for some of the results obtained here.[21]

1.4 A new solution concept for transferable utility normal form games

In recent years there has been a increased research into n-person cooperative games. The reasons for this are likely to be found in the growing interest in finding noncooperative foundations of cooperative games and in the endogenous coalition formation models.[22]

A general problem, usually encountered by all economic applications of n-player cooperative games in which the payoff of every agent depends on the strategies of all agents, is certainly represented by the way in which the strategic (or normal) form game is converted into a game with the characteristic function having sensible features.

[21] Case studies describing successful PCs' birth and survival, are contained, for instance, in Jones (1975) and Cornforth et al. (1988) for the U.K., Bradley and Gelb (1987) for Mondragon, Spain, Estrin and Jones (1993) for French PCs, and Gunn (1984) for U.S. Co-operatives.

[22] Indeed, these two topics seem to have proceeded togheter in the literature. About the topic of coalition formation, the major lines of research have proceeded in two main directions: on the one hand, the research for refinements of the concept of stability of coalition structures, as introduced in the seminal works by Thrall and Lucas (1963), Aumann and Drèze (1974), Shenoy (1979) and Hart and Kurz (1983) [see,for instance, Myerson (1991), Derks and Gillies (1995), Ray and Vohra (1997), just as examples of this approach]; on the other hand, the development of non-cooperative models both applied to the coalition formation problem as well as, in general, to n-cooperative games. Relevant contributions of this line of research are, among others, Selten (1981), Chatterje et al. (1993) and Moldovanu (1992). For accurate surveys of both topics, see Greenberg (1994) and Bloch (1997).

Different conversions have been proposed, depending on the assumptions made about the behaviour of the players outside every deviating coalition in the strategic form game. Minmax or Maxmin behaviour assumed on the part of the outsider players give rise to the well known solutions labelled by Aumann (1967) α and β-core. In these conversions it is implicitly assumed that outside players react to a forming coalition either as followers or leaders, respectively. However, in meaningful strategic contexts (as, for instance, in oligopolistic games) there does not seem to be any rationale in assuming that external players adopt such drastic reactions (Minmax or Maxmin) when facing the formation of a coalition. This can explain why, other conversions of strategic form games have been proposed, as the γ and δ conversions, in which outside players react by playing *à la* Nash either sticking or breaking up into singletons, respectively. However, such conversions, although representing an advance in the solution of this problem, do not possess any temporal structures like that included in the Aumann's α and β-core described above. Conversely, it seems natural to assume that, when a coalition forms by playing a strategy in the underlying strategic game, the other players react by playing a strategy that is a reaction to the forming coalition's strategy.

The fifth chapter of the thesis is devoted to reintroducing a natural temporal structure into the strategic form game that underlies a n-cooperative game. This temporal structure is just the usual conjecture of a Stackelberg duopoly model: when a player acts (the leader), it possesses a first-mover advantage with respect to the other player (the follower). Thus, the assumption is that whilst every forming coalition acts as a leader, the outside players react as followers according to their best-reply functions.

The introduction of this new assumption in the traditional γ conversion, yields a new concept of solution, that is denoted here ϕ-core. The application of this solution concept to cartel formation in oligopoly and public good provision, gives rise to new results.

For the cartel formation game under linear Cournot oligopoly, it is shown that while the γ-core yields a very large set of equilibrium allocations, under ϕ-core the equilibrium allocation is unique. Moreover, small perturbation in the linearity of the functions, give rise to either no equilibrium or to multiple equilibria, respectively.

Finally, for the economy with a public good, it is proved that the γ-core allocation recently characterized by Chander and Tulkens (1997) are ϕ-core stable only if agents' preferences are linear. Also here, it is proved that under non linearity of players' utility functions, the set of ϕ-core may well be empty.

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Chapter 2

Earnings, Coalitions and the Stability of the Firm

2.1 Introduction

The idea of considering a firm as an opportunity or 'device' through which a group of people can actively learn how to cooperate for the production of a commodity is certainly not new. Sometimes working together in the same place is just a technical necessity but more often it also improves the skill and the coordination necessary to obtain a more than ordinary result in production. The learning process produces a 'network effect' on what before was just a group of anonymous workers. This process probably requires a certain amount of time, e.g., if the firm starts producing at time 0, after a certain period, at time 1, the group has learned how to cooperate. One of the consequences of this process is that at the time 1 the firm can yield a surplus, equal to the difference between the money value of the firm's production and the market prices of the factors. However, under complete information, the spot market for labour should reflect the value of such a surplus. Indeed, whenever a worker leaves the firm with or without his fellows, he could use his knowledge to set up a new firm in the market.

The aim of this chapter is to explicitly model the workers' decision to stay or to leave the firm in which they are employed, and to observe which firm's *stable earning profile* raises as a result.[1] The main assumption of the model is that at the beginning two different groups of individuals exist in the economy: the *entrepreneurs,* endowed with a specific knowledge of a given industry production process and the *workers,* endowed with just one unit of labour and without any knowledge about how to produce a commodity. However, once an entrepreneur decides to set up a firm in a given industry, he may need to hire a certain number of workers and these workers acquire a firm-specific skill. When this happens, every worker can bargain with the firm his compensation, threatening to leave whenever, given his available outside options, the wage proposed by the entrepreneur is not satisfactory. Since the model assumes oligopolistic competition whithin each industry, in such a strategic environment every entrepreneur is sensitive to the possibility that employees leave their workplace setting up new competing production units. This simple feature of the model, permits to obtain some interesting results. On the one hand, an economy stable earning profile usually comprehends a vector of payments higher than the basic neoclassical (reservation) wage, also giving rise to pay differentials across industries, for initially homogeneous workers; on the other hand, this vector of payments depends on the relative degree of competition of the industry in which every firm operates. Moreover, the framework characterizes a stable earning profile as a particular case of *core of an economy with coalitions of players* behaving *à la* Nash in the product market. The equilibrium earning profile for the economy can in fact be proved to belong to such a solution set.

Among several bargaining models existing in the literature, at least two frameworks include the option for the employees to leave a firm becoming potential competitors. One, by Feinstein and Stein (1988), considers the behaviour of a firm that is aware of the potential danger of its employees' know-how. In this model, the main answer of the firm is to hire more workers, thus yielding a sort of internal employees' redundancy. Hence, the firm is able to lower the workers' outside option and their threat point in the wage negotiation. The reduction of the employees' competing firms value is then used as a device to moderate the employees' wage demand.

[1]Throughout the paper this expression indicates a vector of payments received both by the employees and by the owners of a firm that respects certain stability requirements.

Another model, by Mailath and Postlewaite (1990), shows that when each worker's reservation wage is private information, it might be the case that for leaving employees, even when convenient, finding an agreement on how to distribute their new firm's value can be impossible. The reason for this result is the difficulty, in a multi-agent adverse selection setup, to implement a satisfactory agreement on a collective matter.

In two recent papers Stole and Zwibel (1996, 1997) apply an intra-firm multilateral bargaining framework with non binding agreements between a firm and its worker to yield an equilibrium level of wages and employment. By assuming complete irreplaceability of each single worker, a stable (i.e. non renegotiable) earnings profile for the firm and the workers is characterized and proved to be equal to the Shapley value of the corresponding cooperative game. Two main assumptions seem responsible for their results. The first is the adoption of a decreasing returns of scale production function that modifies the usual split-the-pie bargaining solution by giving the firm an incentive to hire more workers in order to reduce their marginal contractual power. The second is that the behaviour of the firm is substantially parametric when facing each employee's departure. This feature basically implies that workers' reservation wages are unaffected by every action subsequent their departure.

The model presented here, albeit through a different framework, takes a first step in the direction of explicitly modelling each worker's outside option and looking at its consequences. The main assumption responsible for the model results, is basically one: after being hired by a firm, the workers dispose (for simplicity, instantaneously) of the necessary know-how to set up a new production unit. As a consequence, although the entrepreneurs can substitute without cost every departing worker with unemployed people, there is an indirect cost to be paid in terms of increased competition in the product market. This simple assumption permits to calculate a stable earning profile, that is, a not improvable payoff vector for all individuals of the economy.

In its basic structure the model can be considered as mainly heuristic. The idea that all employees of a firm can immediately acquire a specific ability to become entrepreneurs or partners of a new firm, without bearing the necessary setup costs, can appear a very extreme assumption. However, once some basic results are obtained, it would not be difficult introducing specific types of transaction costs that, by constraining each individual's behaviour, could make the picture definitively more realistic.

The next section outlines the basic structure of the model. Section 2.3 introduces an application of the model describing the main results of the paper. Section 2.4 is devoted to presenting in greater detail the game-theoretic nature of the solution concept adopted. Section 2.5 discusses some possible extensions of the model. Section 2.6 concludes the paper.

2.2 The structure of the model

2.2.1 A simple oligopolistic multi-sector economy

This section describes a simple economy in which a finite set of individuals $N = \{1, 2, ..., n\}$ is initially distributed among two subsets, such that $N = (\{I_K\} \cup \{I_L\})$, where I_K represents the subset of entrepreneurs, while I_L indicates a subset containing homogenous workers. As anticipated above, a distinctive feature of the economy is that, at the beginning, each *entrepreneur* owns a specific knowledge of a given industry production process, while *workers* just possess a unit of labour to offer.[2] For simplicity, it is assumed that every entrepreneur $i \in I_K$ is allowed to set up just one firm by hiring a certain number of workers from the subset I_L. As a consequence of their lack of know-how, the members of I_L, at least initially, cannot set up a firm without having first been recruited either by one member of I_K, or by someone that has worked before with a member of I_K, and so on. Let us assume m different specific know-how $(l = 1, 2, ..., m)$ and, for each one, k^l entrepreneurs disposing from the beginning of this particular knowledge.[3] Therefore, it turns out that, potentially, the initial number of firms in the economy can be equal to $\underline{\mathbf{v}}^{\mathrm{T}}\underline{\mathbf{k}}^l$, where $\underline{\mathbf{v}}^{\mathrm{T}}$ is the transposed l-dimensional unitary

[2] This assumption is very strong but undoubtedly useful to describe an initial situation under which, it is not so relevant who detains the firms' property rights, but who is entitled to start a business.

[3] This means that each l-th sector can comprehend a different number of entrepreneurs.

vector and $\underline{\mathbf{k}}^l$ is the vector representing the number of entrepreneurs that disposes of the knowledge of the l-th sector production process ($l = 1, 2, ..., m$).

Since not necessarily all workers in I_L will be hired by one amongst the existing entrepreneurs, the potential number of coalitions in every initial coalition structure of the economy comprehends ($\underline{\mathbf{v}}^{\mathrm{T}}\underline{\mathbf{k}}^l + 1$) coalitions (firms) denoted S^{jl}, where $j = 1, 2, ...k^l$. That is, in the economy there are m sectors, each one with a certain number of firms ($j = 1, 2, ...k^l$) devoted to producing a homogeneous commodity y_l. Furthermore, there is in general a coalition (that can also be empty) including all unemployed people that do not belong to any firm S^{jl}. Denoting the set of all firms of industry l as $\sum^l = \bigcup\limits_{j=1,..k^l} S^{jl}$, the set of all unemployed workers can be represented as $U = N \diagdown \bigcup\limits_{l=1,..,m} \sum^l$.

Let production for self-consumption be excluded, by requiring every commodity to be sold in the market. Let also each firm S^{jl} possess a continous and invertible production function specific to the industry $g_l : \mathbf{R}_+ \rightarrow \mathbf{R}_+$ represented by:

$$y_{jl} = g_l(\ell) \tag{2.1}$$

where ℓ indicates the quantity of labour required to produce the commodity. This means that the number of workers hired by every firm S^{jl} will be decided by each entrepreneur according to the inverse function $I^N\left(g^{-1}(y_{jl})\right) = \mid I_L \cap S^{jl} \mid$, where the function $I^N(.) : \mathbf{R}_+ \rightarrow \mathbf{N}_+$ transforms every real number into its closest natural number.[4]

Given the existing production function, every firm is assumed to compete *à la* Cournot in the l-th homogeneous good market, with a payoff function $\pi_{jl} : \mathbf{R}^2_+ \rightarrow \mathbf{R}$ given by:

$$\pi_{jl}(y_{jl}, y_l) = \{p_l(y_l)\, y_{jl}\}_{\substack{\forall j=(1,..k^l) \\ \forall l=(1,..m)}} \tag{2.2}$$

[4] Let us assume that, when a real number is exactly between two integers, the function $I^N(.)$ selects the lowest one.

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inverse demand function of the l-th sector
sure existence and uniqueness of a Cournot
the following assumptions will be considered

sector is a function of its own strategy and
ns in that sector;

m, compact and convex and, in particular,
esent every j-th firm's production boundary

$Y_j \times \Re_+ \rightarrow \Re_+$, is twice continuously differ-

m of the multi-sector oligopolistic economy is
such that, for every $j = 1, ..., k^l$ in that given

$$\pi_j \left(y_j^*, y_{-j}^*\right) \geq \pi_j \left(y_j, y_{-j}^*\right), \; \forall y_j \in Y_j \text{ and } \forall j = 1, ..k^l$$

where y_{-j} is the sum of all firms' output in a given industry minus j.

It is well known that assumptions A.1-A.5 are sufficient to prove the existence of a Cournot-Nash equilibrium. By A.3, every player's payoff function is continuous in the strategy profile $y^l \in \prod_{j=1,..,k^l} Y_j$ and, by A.4 and A.5, strictly concave on y_{jl}. By A.2, firms' strategy sets are non empty, compact and convex, so that existence of a Nash equilibrium follows.

Uniqueness is implied by A.4 and A.5 as follows. Since, for each firm, $p''y_j + p' < 0$ and $p' < 0$, the function $F(y_j, y_l) \equiv p'y_j + p$ is decreasing both in y_j and y_l. In fact, $\frac{\partial F(yj,y)}{\partial y_j} = p' < 0$ and $\frac{\partial F(y_j,y_l)}{\partial y_l} = p''y_j + p' < 0$. Suppose now that in a given sector there exist two Nash equilibria y_l^1 and y_l^2. Suppose also, without loss of generality, that $y_l^1 < y_l^2$. At a Nash equilibrium, $p'y_j + p = 0$, so that, if $\sum_{j=1}^{k^l} y_j^1 < \sum_{j=1}^{k^l} y_j^2$, it follows from A.4 and A.5 that $y_l^1 > y_l^2$ for every $j = 1, .., k^l$, leading to a contradiction.

The uniqueness of y_l^* in each sector implies, in turn, that the multi-sector oligopolistic economy has a unique Nash equilibrium.

In what follows, a further simplification will be made on the compensation system adopted by every firm. The initial reservation wage of unemployed workers, i.e., all $i \in (\{I_L\} \cap \{U\})$, is equal to zero (because they cannot produce without first being recruited by a firm). Moreover, entrepreneurs are assumed to pay every recruited worker a share α_i^l of the firm's surplus (2.2), where $\alpha_i^l \in [0, 1]$ is such that $\sum_{i \in S^{jl}} \alpha_i^l = 1$.[5]

Given such assumptions, it follows that for each coalition S^{jl} the surplus is given by (2.2) and is distributed according to the vector $\underline{\alpha_i}^l$. Moreover, assuming linear preferences for all individuals, the utility of every individual will be given by:

[5] Basically this feature of the model simplifies the problem when compared to the adoption of a fixed wage, without any particular loss of generality. Note also that, since firms are assumed symmetric in every l-th industry, the share to be paid to workers will be the same in each j-th firm of that industry and will be, therefore, indicated as α^l.

$$\begin{cases} u^i = \alpha_i^l \pi_{jl}\left(y_{jl}, y_l\right), \ \forall i \in \left\{I_L \cap S^{jl}\right\}, \\ u^i = \left(1 - \sum\limits_{i \in I_L \cap S^{jl}} \alpha_i^l\right) \pi_{jl}\left(y_{jl}, y_l\right), \forall i \in \left\{I_K \cap S^{jl}\right\}. \end{cases} \tag{2.3}$$

2.2.2 The stability of an earning profile

Given the oligopolistic economy described above, a *stable earning profile* for the economy can be defined as a feasible vector of payments such that, given certain specific constraints for the individuals of the economy, anyone cannot improve upon. In our framework, given Nash equilibrium quantities, a stable earning profile is an income distribution within each firm such that none, individually or in a group, wants to leave the firm to form a new production unit.

In our economy, under the assumptions described above, every firm's equilibrium surplus is given by:

$$\pi_{jl}^* = \left\{\pi_{jl}\left(y_{jl}^*, y_l^*\right)\right\}_{\forall j=1,2,\ldots,k^l} \tag{2.4}$$

where y_l^* represents the Nash equilibrium quantity vector of a l-th industry. This vector also determines the partition of people belonging to I_L into two different groups: one group, denoted by D^*, is the set of all employed workers at a Nash equilibrium, i.e., $D^* = \sum\limits_{l=1}^{m} \sum\limits_{j=1}^{k^l} I^N\left(g^{l^{-1}}\left(y_{jl}^*\right)\right)$; the other, conversely, is made of all unemployed workers $U = I_L \diagdown D^*$. Assuming, for the time being, the existence at the Nash equilibrium of a non empty set U, in the economy there are many different choices available to each individual active in a firm. For every *employee* the choice is between:

(1a) staying in a firm and negotiating with the entrepreneur a share α_i^l of the firm's equilibrium profit (2.4);

(2a) leaving the firm (after having been trained) alone (or with other workers), to become entrepreneur of a new firm in the industry, by recruiting workers either from the same or from another firm (of the same industry), knowing that the entrepreneur in S^{jl} will recruit new workers from the set U according to their best-reply;

(3a) leaving the firm (after having been trained) to become self-employed, producing a certain amount of output (with only one unit of labour), given that existing k^l firms will continue to produce according to their best-reply;

(4a) leaving the firm (after having been trained) with other firm's workers to become member of a partnership, given the best-reply of existing firms;

(5a) entering the unemployed set, obtaining the initial reservation wage of the economy, equal to zero.

For each entrepreneur, apart from the choice to be self-employed and not to hire any worker, choices (1a)-(5a) described above are similarly feasible. Therefore, in equilibrium, the entrepreneur will never earn less than his workers, otherwise he would deviate from the current situation setting up another firm.

Thus, an earning profile of the economy (a vector of remunerations for all individuals of the economy, denoted $\underline{\mathbf{z}}$) can be viewed as stable if, given the partition of N players into $(\underline{\mathbf{v}}^{\mathrm{T}}\underline{\mathbf{k}}^l + 1)$ coalitions (k^l firms S^{jl} in each industry plus the unemployment set U) according to the Nash equilibrium quantity vector of every l-th industry y_l^*, no individual or group of individuals within each firm S^{jl} can improve upon $z^i \in \underline{z}^{S^{jl}}$ by deviating, where $\underline{z}^{S^{jl}}$ represents the vector of payments obtained in equilibrium within every S^{jl}.[6] Note that allowed deviations are as in (1a)-(5a) above. Moreover, the behaviour of every complementary coalition, i.e. the individual in the set $S^{jl}\setminus T$,

[6] By assumption unemployed people cannot autonomously setup firms and, therefore, cannot deviate from $\underline{z}$. Section 2.4 will describe in grater detail the game-theoretic setup on which is based the model.

(where T indicates here a deviating coalition) is supposed to be which to carry on as before the deviation occurred, according to its best-reply. This is possible through the recruitment of a certain number of workers from the unemployment set U.[7] Moreover, note that only consistent deviations are considered, that is, deviations that cannot, in turn, be objected.

It is now time to describe the sufficient conditions for an earning profile $\underline{\mathbf{z}}$ to be stable.

The Nash equilibrium quantity (and consequently the number of I_L employed in each S^{jl}) is, by the symmetry of the equilibrium considered, identical for all existing firms in a given l-th industry. Thus, the set of conditions making each firm's earning profile $z^{S^{jl}}$ stable can be characterized by a share α^l of firm's profits (equal by symmetry for all firms within an industry, so the superscript j can be dropped) paid to *all employees* working for a firm, i.e., $\alpha^l = \sum_{i \in \{I_L \cap S^{jl}\}} \alpha_i^l$. In each l-th sector, the equilibrium share α^l can be determined through the respect of the following constraints:

a) No employee of a firm has to find convenient to become entrepreneur and setting up a *new* firm by hiring an optimal number of workers and paying them a share of the profit sufficient for them to stay. This condition holds when:

$$\frac{\alpha^l\left(k^l\right) \cdot \pi_{jl}^*\left(k^l\right)}{I^N\left(\ell_{jl}^*\left(k^l\right)\right)} \geq \left(1 - \alpha^l\left(k^l + s' + 1\right)\right) \pi_{jl}^*\left(k^l + s' + 1\right) \tag{2.5}$$

[7] This behaviour is justifiable. In fact, suppose as a benchmark a constant returns to scale production function like $y_j = \ell_j$ for each j-th firm, where ℓ_j is the number of workers hired by each entrepreneur of a given sector. In this case, the equilibrium price of each market $p\left(L^*\right)$, where $L^* = \sum_{j=1}^{k^l} \ell_j^*$, is not affected by transferring trained workers from a firm to another. So it easy to see that for a firm there is not any advantage in recruiting a worker that is already active in another firm. This would immediately be substituted by another unemployed worker and the firm's equilibrium payoff would not change. Thus, at least in the case of constant returns to scale, firms' Cournot behaviour after a deviation can be considered reasonable.

where $s' = I^N\left(k^l \cdot \Delta\ell^*_{jl}\left(k^l\right)\right) = I^N\left(k^l\left(\ell^*_{jl}\left(k^l\right) - \ell^*_{jl}\left(k^l+1\right)\right)\right)$ is the number of firms (positive by assumptions A.4 and A.5) created by all workers involuntarily dismissed from any of the k^l firms as a result of the output reduction due to new firm's entry; $\left(1 - \alpha^l\left(k^l + s' + 1\right)\right)$ indicates the share of profit that the leaving worker, now entrepreneur, earns in the new firm; $\pi^*_{jl}\left(k^l + s' + 1\right)$ represents the equilibrium payoff of every firm (now that the market includes $\left(k^l + s' + 1\right)$ firms); $\pi^*_{jl}\left(k^l\right)$ indicates each initial firm's equilibrium payoff and

$I^N\left(\ell^*_{jl}\left(k^l\right)\right) = \left\{I^N\left(g^{l^{-1}}\left(y^*_{jl}\left(k^l\right)\right)\right)\right\}_{\forall j=1,..,k^l}$ is the Nash equilibrium number of employees hired by every firm when the initial number of firms in the market stays as it is. Note that, by symmetry, within each firm, $\left\{\alpha^l_i\left(k\right)\right\}_{i\in I_L\cap S^{jl}} = \frac{\alpha^l\left(k^l\right)}{I^N\left(\ell_{jl}\left(k^l\right)\right)}$. Moreover, note that when condition (2.5) holds, leaving the firm and becoming entrepreneur is certainly not profitable for a group of workers (which obviously need to pay the residual group of recruited unemployed workers their equilibrium share);

b) No employee of a firm has to find convenient to become a member of a newly created partnership with some of the other firm's employees. This condition holds when:

$$\frac{\alpha^l\left(k^l\right) \cdot \pi^*_{jl}\left(k^l\right)}{I^N\left(\ell^*_{jl}\left(k^l\right)\right)} \geq \frac{\pi^*_{jl}\left(k^l + s'' + 1\right)}{I^N\left(\ell^*_{jl}\left(k^l + s'' + 1\right)\right)} \tag{2.6}$$

where RHS represents the equilibrium payoff of each member of a new partnership when a firm of this type (and then a number of induced new entrant firms, denoted s'') enters the market;

c) No employee of a firm has to find convenient to become self-employed:

$$\frac{\alpha^l\left(k^l\right) \cdot \pi^*_{jl}\left(k^l\right)}{I^N\left(\ell^*_{jl}\left(k^l\right)\right)} \geq E^*\left(k^l + s''' + 1\right) \tag{2.7}$$

where E^* is a self-employed's equilibrium payoff (obtained assuming that only 1 unit of labour is used) when a new firm of this type (and a number of induced new entrant firms, denoted s''') enters the market;

d) None of k^l entrepreneurs operating in one of the existing industries has to earn less than each one of his employees. This corresponds to the condition:

$$\left(1-\alpha^l\left(k^l\right)\right)\pi^*_{jl}\left(k^l\right) \geq \frac{\alpha^l\left(k^l\right)\cdot\pi^*_{jl}\left(k^l\right)}{I^N\left(\ell^*_{jl}\left(k^l\right)\right)} \tag{2.8}$$

e) No entrepreneur has to find convenient to be self-employed rather than hiring workers and sharing profit and knowledge with them, given that the other (identical) entrepreneurs of the industry will do the same (thus, giving rise to a *self-employed equilibrium* for that sector):

$$\left(1-\alpha^l\left(k^l\right)\right)\pi^*_{jl}\left(k^l\right) \geq E^*\left(k^l\right); \tag{2.9}$$

f) *internal consistency*:

$$\left(1-\alpha^l\left(k^l\right)\right)\pi^*_{jl}\left(k^l\right) \geq \left(1-\alpha^l\left(k^l+s+1\right)\right)\pi^*_{jl}\left(k^l+s+1\right), \tag{2.10}$$

that expresses the fact that each entrepreneur has to find convenient to pay his employees the equilibrium wage (characterized by the share $\alpha^{k^l,l}\left(k^l\right)$) rather than let one (or more than one) employee(s) leaving in order to pay the newly recruited workers a lower equilibrium wage. However, note that conditions (2.5)-(2.8) always imply (2.10). In fact, when (2.5) is the tightest amongst (2.5)-(2.8), condition (2.8) directly implies (2.10). When, conversely, either (2.6) or (2.7) are the tightest constraint, (2.5) is respected with inequality and, again, (2.8) will imply (2.10). Moreover, due to the recursive nature of condition (2.5), a further assumption is needed. In fact, solving expression (2.5) would be like assuming that, at each round of entry, $t = (1, ..., T)$, where T represents the maximum number of entrants given that the set of unemployed is finite, the most profitable deviation for employees is always that expressed by choice (2a), i.e., to be entrepreneurs in a new firm. In order to avoid that an unprofitable option at a given round of entry becomes profitable in one of following rounds, then making expression (2.5) hard to solve, the following condition will be imposed in what follows.[8]

A.7 *(No-crossing condition) Let α_1, α_2 and α_3 be the share $\alpha^l\left(k^l\right)$ that respect condition (2.5), (2.6) and (2.7), respectively, with equal sign. When, for a given number of existing firms k^l, $\alpha^l\left(k^l\right) = \max\{\alpha_1, \alpha_2, \alpha_3\}$, the same condition holds for every $\left(k^l + t\right)$, for $t = 1, ..., T$.*

Now, when a vector $\alpha^l\left(k^l\right) = (\alpha^1, \alpha^2, ...\alpha^m)$ respects all conditions listed above, the corresponding earning profile of the economy $\underline{\mathbf{z}}\left(\underline{\alpha}^l\left(k^l\right)\right)$ possesses some properties of stability. Using (2.5)-(2.10) and given an arbitrary initial number k^l of firms existing in each industry, the proposition that follows characterizes the vector $\underline{\alpha}^{\,l}$ associated to the stable earning profile $\underline{\mathbf{z}}\left(\underline{\alpha}^l\right)$ for the economy. Let us, for ease of notation, denote $I^N\left(\ell_{jl}\left(k^l\right)\right)$ as $\widehat{\ell}_{jl}\left(k^l\right)$.

Proposition 1 *Under no-crossing condition and for a given number of entrepreneurs (and firms) existing in every industry, a stable earnings profile of the economy is characterized by the following share of profit in every S^{jl} of a given industry:*

$$\alpha^{l*} = \frac{\sum_{t=1}^{T}(-1)^{t+1}\pi^*_{jl}\left(k^l+\left(k^l+t\right)\cdot\sum_{h=1}^{t}\left(\Delta^h\widehat{\ell}^*_{jl}(k+h-1)+t\right)\right)\prod_{h=0}^{t-1}\widehat{\ell}^*_{jl}\left(k^l+\left(k^l+t\right)\cdot\sum_{h=0}^{t-1}\left(\Delta^h_{jl}\widehat{\ell}^*_{jl}(k+h-1)+h-1\right)\right)}{\pi^*_{jl}\left(k^l\right)}$$

$$\text{and } \alpha^{l*} \leq \frac{\pi^*_{jl}\left(k^l\right)-E^*_{j,l}\left(k^l\right)}{\pi^*_{jl}\left(k^l\right)}, \text{ when } \alpha_1 = \max\ \{\alpha_1, \alpha_2, \alpha_3\};$$

$$\min\left\{\frac{\widehat{\ell}^*_{jl}}{\left(1+\widehat{\ell}^*_{jl}\right)}, \frac{\pi^*_{jl}\left(k^l\right)-E^*_{j,l}\left(k^l\right)}{\pi^*_{jl}\left(k^l\right)}\right\} \geq \alpha^{l*} = \frac{\pi^*_{jl}\left(k^l+\Delta\widehat{\ell}^*_{jl}\left(k^l\right)+1\right)\cdot\widehat{\ell}^*_{jl}\left(k^l\right)}{\pi^*_{jl}\left(k^l\right)\cdot\widehat{\ell}^*_{jl}\left(k^l+\Delta\widehat{\ell}^*_{jl}\left(k^l\right)+1\right)}$$

[8] The following section will show how to remove this simplifying assumption.

$$when\ \alpha_2 = \max\ \left\{\alpha_1^{'}, \alpha_2, \alpha_3\right\},$$

$$for\ \alpha_1^{'} = \frac{\pi^*_{jl}\left(k^l+\triangle\widehat{\ell}^*_{jl}\left(k^l\right)+1\right)\cdot\widehat{\ell}^*_{jl}\left(k^l\right)}{\pi^*_{jl}\left(k^l\right)} - \frac{\pi^*_{jl}\left(k^l+\triangle\widehat{\ell}^*_{jl}\left(k^l\right)+\triangle^2\widehat{\ell}^*_{jl}\left(k^l+1\right)+2\right)\widehat{\ell}^*_{jl}\left(k^l\right)\widehat{\ell}^*_{jl}\left(k^l+\triangle\widehat{\ell}^*_{jl}\left(k^l\right)+1\right)}{\pi^*_{jl}\left(k^l\right)\widehat{\ell}^*_{jl}\left(k^l+\triangle\widehat{\ell}^*_{jl}\left(k^l\right)+\triangle^2\widehat{\ell}^*_{jl}\left(k^l+1\right)+2\right)};$$

$$\min\left\{\frac{\widehat{\ell}^*_{jl}\left(k^l\right)}{\left(1+\widehat{\ell}^*_{jl}\right)}, \frac{\pi^*_{jl}\left(k^l\right)-E^*_{j,l}\left(k^l\right)}{\pi^*_{jl}\left(k^l\right)}\right\} \leq \frac{E^{l*}\left(k^l+\triangle\widehat{\ell}^*_{jl}\left(k^l\right)+1\right)\cdot\widehat{\ell}^*_{jl}\left(k^l\right)}{\pi^*_{jl}\left(k^l\right)}$$

$$when\ \alpha_2 = \max\ \left\{\alpha_1^{''}, \alpha_2, \alpha_3\right\}$$

$$for\ \alpha_1^{''} = \frac{\pi^*_{jl}\left(k^l+\triangle\widehat{\ell}^*_{jl}\left(k^l\right)+1\right)\cdot\widehat{\ell}^*_{jl}\left(k^l\right)}{\pi^*_{jl}\left(k^l\right)} - \frac{E^{l*}\left(k^l+\triangle\widehat{\ell}^*_{jl}\left(k^l\right)+\triangle^2\widehat{\ell}^*_{jl}\left(k^l+1\right)+2\right)\widehat{\ell}^*_{jl}\left(k^l\right)\widehat{\ell}^*_{jl}\left(k^l+\triangle\widehat{\ell}^*_{jl}\left(k^l\right)+1\right)}{\pi^*_{jl}\left(k^l\right)}.$$

Proof. When $\alpha_1 = \max\ \{\alpha_1, \alpha_2, \alpha_3\}$, in order for $\underline{\mathbf{z}}\left(\underline{\alpha}^l\right)$ to be stable, α^{l*} must respect condition (2.5) with equality. The solution can be obtained by iteratively solving the differential equation contained in (2.5) for $\alpha^l\left(k^l\right)$, by assuming a number of potential entrants $t = T$. This is the maximum number of available entrants, given that the set of I_L is finite. Note that, for $t = (1, ...T-1)$, we indicate with $\triangle^t\widehat{\ell}^*_{jl}\left(k^l+t+1\right) = \left[\widehat{\ell}^*_{jl}\left(k^l+t\right) - \widehat{\ell}^*_{jl}\left(k^l+t+1\right)\right]$ the number of workers dismissed by every firm at each round of entry as a consequence of the output reduction.

Thus, the solution of constraint (2.5) is exactly the first expression shown in proposition 1. It has to be noticed that when $\alpha^{l*} = \alpha_1$, no coalition of workers would have any incentive to become entrepreneur of a new firm. In fact, the share of a new firm's profit should be divided amongst all deviating workers, ensuring, consequently, for each one of them, a level of earning lower than α_i^{l*}.

When the best workers' outside option is characterized by α_2 (corresponding to the option to create a partnership), the whole problem must be modified to take into account that, in the iterative solution of (2.5), the best outside option for the employees working in the new entrepreneurial firm created at the first round of entry is now represented by the share α_2. Provided this, α_1 must include α_2 in the second an last round of entry, thus yielding α_1' as expressed above. Note that α_2 is the solution of constraint (2.6). The same procedure yields α_1'', when the best workers' outside option is which to be self-employed (corresponding to the share α_3). Moreover, as shown above, the difference equation (2.10) is always respected when α^{l*} respects constraints (2.5)-(2.9). Thus, the initial entrepreneurs will always find convenient to pay their employees the equilibrium share of the profit $\alpha^{l*}\left(k^l\right)$ rather than risk their departure and increase the existing market competition. Finally, the LHS expressions in rows fourth and sixth (within the min parentheses) above, represents the solution of constraints (2.8) and (2.9), ensuring that every entrepreneur does not have any incentive to deviate or to give rise to a self-employment equilibrium. When these inequalities hold, provided that constraints (2.5)-(2.7) are satisfied, entrepreneurs will never find convenient either to set up a new firm as entrepreneur, partner or self-employed, or to be self-employed from the beginning. In general then, given the partition of the economy determined by the Nash equilibrium, when the vector $\underline{\alpha}^* = (\alpha^{1*}, \alpha^{2*}, ...\alpha^{m*})$ respects the condition above, the corresponding economy earning profile:

$$\underline{\mathbf{z}}\left(\underline{\alpha}^*\right) = \left[\left\{\left(1-\alpha^{l*}\left(k^l\right)\right)\pi^*_{k^l l}\left(k^l\right)\right\}_{\forall i \in \left(I_K \cap S^{jl}\right)}, \left\{\frac{\alpha^{l*}\left(k^l\right)\pi^*_{k^l,l}\left(k^l\right)}{\widehat{\ell}^*_{k^l,l}\left(k^l\right)}\right\}_{\forall i \in \left(I_L \cap S^{k^l,l}\right)}, \{0\}_{\forall i \in U}\right]$$

for $l = 1,..m$, and $j = 1,..,k^l$, is stable. ■

The main aim of Proposition 1 is which to exactly characterizes a stable earning profile $\underline{z}(\underline{\alpha}^*)$ for the economy. What the proposition shows is that, under simple stability conditions, each firm will find convenient to offer the workers a compensation sufficiently high to keep them inside the firm. This compensation - expressed as a share of every firm's profit - has to be high enough to prevent even the finest firm's subcoalition, represented by a single worker, to have an incentive to setup a new firm in the form of an entrepreneurial, partnership or self-employed type of unit through the recruitment of the available workforce.

2.3 Model applications

2.3.1 A linear example

In order to present the features of a stable earning profile, let us introduce a simple one-industry example. For simplicity, let the production function of every firm be linear in the number of workers, that is, $y^j = \gamma \cdot \ell$, for $\gamma > 0$. In order to make calculations even simpler, let $p(Y) = a - y$ be a linear inverse demand for a homogeneous good, where $y = \sum_{j=1}^{k} y_j$ represents the total quantity of the good sold in the market, with $a > \sum_{j=1}^{k} \overline{y}_j > 0$.

Straightforward calculations yield the results shown in the table below.

VARIABLES	EQUILIBRIUM VALUES
y_j^*, ℓ_j^*	$\frac{a}{(k+1)}, \frac{a}{\gamma(k+1)}$
y^*	$\frac{a \cdot k}{k+1}$
$p(y^*)$	$\frac{a}{k+1}$
$\pi_j^*(k)$	$\frac{a^2}{(k+1)^2}$
$\pi_j^*\left(k^l + s'' + 1\right)$	$\frac{a^2}{\left(k+\frac{ka}{\gamma(k+1)(k+2)}+2\right)^2}$
$\frac{\pi_j^*\left(k^l+s''+1\right)}{\ell_j^*\left(k^l+s''+1\right)}$	$\frac{a\gamma}{\left(k+\frac{ka}{\gamma(k+1)(k+2)}+2\right)}$
$E_j^*(k)$	$(a - \gamma k)$
$E_j^*\left(k^l + s''' + 1\right)$	$\left(\frac{a\gamma-k-\gamma}{(k+1)}\right)$

Tab. 2.1 - Equilibrium values for the linear example

In the picture below, it is presented a numerical simulation.

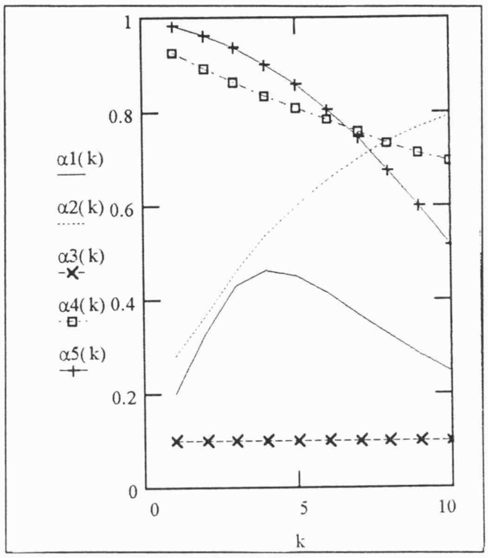

fig.3.1 - Values of $\alpha_1', \alpha_2, \alpha_3, \alpha_4, \alpha_5$
$(\ a = 250,\ \gamma = 10)$ **for** $k = 1, ...10.$

The picture shows the values of α_1', α_2, α_3, for a given number of initial (entrepreneurs) firms existing in the industry $(k = 1, ...10)$. It can be noticed that, for the values of parameters selected and for a given range of k, α_2 is the maximum value among α_1', α_2, α_3, therefore constituting the equilibrium share of profit that every firm has to paid to all its employees. Moreover, as the picture shows, for $(k = 1, ..7)$, α_2 respects the constraint that imposes that each employee has to earn less than his entrepreneur (with a corresponding share denoted α_4) and that no entrepreneur has to prefer to be self-employed (with a corresponding share denoted α_5). However, for $k > 7$, this constraints no longer holds, and (for such numbers of initial firms) a self-employment equilibrium can be expected to raise as a result. This indicates that, under the parameters selected , for $k > 7$, the stable earning profile characterized by proposition 1 is not stable. When, conversely, for a given number of initial entrepreneurs, α^* respects α_5 but not α_4, it is the internal consistency requirement that does not and new firms can be expected to enter the market. Again, this means that for the parameters values, the partition of players into k coalitions characterized by proposition 1, is not stable.

In the figure below we plot the equilibrium earning $z(\alpha^*(k))$ for each employee, according to the equilibrium share represented in figure 3.1. It can be noticed that, in general, apart from the case in which the initial number of firms is very low, $z(\alpha^*(k))$ decreases with the number of firms assumed to exist initially in the industry.

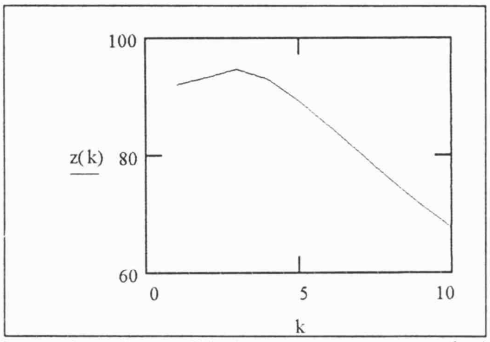

Fig.3.2-Each employee's equilibrium remuneration $z(\alpha^*)$ (k=1,..10).

This is a rather general result of the model that describes the fact that, since workers' outside option varies with any initial partition of the economy (when this constitutes an equilibrium, in our example for $1\leq k \leq 7$), the corresponding equilibrium compensation of each employee changes according to the value of his firm's market share. When either the size of the market (represented by the parameter a), or the number of firms in the economy increase, equilibrium employees' compensation decreases. The reason is that, since workers' training is specific to a firm of a certain industry, each worker can negotiate a share of firm's profit by using the threat of setting up a new firm *only* in that particular industry.

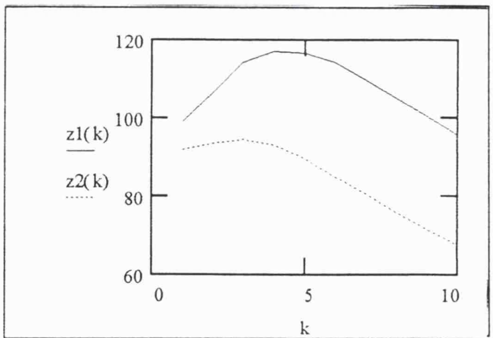

Fig. 3.3 - Employees equilibrium compensation in two different size industries

This makes possible the existence of an equilibrium earning profile characterized by different levels of compensation for individuals that, although homogeneous, work in different industries. The figure above, for instance, represents the equilibrium earnings for employees working in two industries ($l = 1, 2$) characterized by a different market size.

2.3.2 Removing the no-crossing condition

In the previous section, in order to characterize a stable earning profile of the economy, the model imposed, for simplicity, a special condition on the shares solving constraints (2.5)-(2.7), denoted *no-crossing condition* (assumption A.7). This was a simple way to avoid that at a given round of entry t, the value of one share became higher than the others, after having been lower in the previous rounds. Thus, such a condition is just a way to simplify the characterization of a stable earning profile that, due to constraint (2.5) presents an iterative nature. However, numerical simulations show that it may well be possible that the values of the shares representing different outside options intersect during the T rounds of entry considered in the iterative solution of (2.5). This is not a problem as long as the share that at a given round represents the best employees' outside option is inserted into the iterative solution of (2.5) exactly at that stage (that then becomes the last stage). For instance, when at a given round t, α_2 represents employees' best outside option, the expression for $\alpha^{l*} = \alpha_1$ presented in proposition 1, becomes:

$$\alpha^{l*} = \frac{\sum_{i=1}^{T}\left[(-1)^{i+1}\pi^*_{jl}\left(k^l+\left(k^l+i\right)\cdot\sum_{j=1}^{i}\left(\Delta\widehat{\ell}^*_{jl}(k+j-1)+i\right)\right)\right]\prod_{j=0}^{i-1}\widehat{\ell}^*_{jl}\left(k^l+\left(k^l+i\right)\sum_{j=0}^{i-1}\left(\Delta^{j*}_{jl}\widehat{\ell}^*_{jl}(k+j-1)+j-1\right)\right)}{\pi^*_{jl}\left(k^l\right)} +$$

$$+\frac{\alpha_2\cdot\prod_{j=0}^{T-1}\widehat{\ell}^*_{jl}\left(k^l+\left(k^l+i\right)\sum_{j=0}^{T-1}\left(\Delta^{j*}_{jl}\widehat{\ell}^*_{jl}\left(k+j-1\right)+j-1\right)\right)}{\pi^*_{jl}\left(k^l\right)}$$

Similarly, the share α_3 needs to be plugged into expression above when, at a given round t, such a share becomes the maximum among the shares obtained solving constraints (2.5)-(2.7). Let the corresponding values of α^{l*} be denoted as $\alpha_1^{t^2}$ and $\alpha_1^{t^3}$, where t stands for the round in which α^2 or α^3, respectively, becomes higher than α^1.

2.4 The game-theoretic nature of a stable earning profile

This section briefly outlines the game-theoretic nature underlying the model presented above. Here the purpose is mainly to show that the stable earning profile described in the previous section can be characterized through a cooperative game-theoretic setting. In particular, it will be shown that the vector $\left(\underline{y}^l, \underline{\alpha}^l\right)$ belongs to the core of a partition function game obtained starting with a set of players partitioned into entrepreneurs and workers.

Firstly, some basic notions are introduced in order to illustrate the results obtained in this section.

2.4.1 Game and equilibrium concept

Let us introduce the necessary tools to describe the game. In general we can start with a normal form game,

$$\Gamma = \left(\{u_i, X_i\}_{i \in N}, X_S\right) \tag{2.11}$$

where N is the (finite) players set, X_i is the strategy set of player i, and X_S is the strategy set of a coalition of players $S \subseteq N$. Let $\wp(N)$ be the set of all feasible partitions p of the players set N; let X_p denote the set $\prod_{S \in p} X_S$, for any $p \in \wp(N)$. The set $X \equiv \bigcup_{p \in \wp(N)} X_p$ is the set of all possible outcomes (in terms of strategies) of the game Γ. The function $u_i : X \to R_+$ represents players' preferences. We will limit ourselves to the case of transferable utility functions u_i.

We first describe the set of permissible partitions that can be induced in the game.

We start with an initial partition of the players' set N into two different subsets, N_1 and N_2, with identical players within each subset, formed in this way:

$$p^0 = \left(\{i\}_{i \in N_1}, N_2\right) \tag{2.12}$$

such that, for every $i, j \in N_1, \{i\} \cap \{j\} = \emptyset$, and such that $(N_1 \cup N_2) = N$.

Therefore, the initial partition p^0 includes every $i \in N_1$ as singleton and the set of players N_2. From this initial partition, through the choice of a strategy, every player $i \in N_1$ can induce any coalition $S \subseteq N$, such that:

$$S^i = \left(\{i\}_{i \in N_1} \cup \{W \subset N_2\}\right) \tag{2.13}$$

in which a player $i \in N_1$ merges with a subset (that can also be empty) of N_2. Therefore, the induced partition p^1 is:

$$p^1 = \left(\{S^i\}_{i \in N_1}, \left(N_2 \backslash \left(N_2 \cap \{S^i\}_{i \in N_1}\right)\right)\right), \tag{2.14}$$

where every S^i can also be formed by just a single player $i \in N_1$. Again, from p^1 a new partition p^2 can be induced through the strategy of any T^1, such that:

$$T^1 = \left(\left\{\Sigma^1 \subset \{S^i\}_{i \in N_1}\right\} \cup \left\{W^1 \subset \left(N_2 \backslash \{S^i\}_{i \in N_1}\right)\right\}\right) \tag{2.15}$$

i.e., formed by any subset Σ of S^i and a subset (also empty) of excluded players still belonging to N_2. Let every subsequent partition $p^3, p^4, ...p^t$ be induced in the same way, i.e.,

$$T^2 = (\{\Sigma^2 \subset T^1\} \cup W^2), T^3 = (\{\Sigma^3 \subset T^2\} \cup W^3), ..., T^t = \left(\left\{\Sigma^t \subset T^{(t-1)}\right\} \cup \{W^t\}\right),$$

where W^j are all subsets of N_2 (until there are available players).

Let $\Im(N)$ denote the family of all coalitions formed in the way described above, while $\wp_T(N)$ denotes the set of all correspondent partitions (that also includes in each round the players remained in N_2). Let, for every coalition $S \in \Im(N)$ the strategy set be $X_S = \prod\limits_{i \in S} X_i$. Moreover, let the above sequence of partitions be induced by every $S \in \Im(N)$ through the choice of a Nash strategy, given that all other coalitions in the correspondent induced partition $p \in \wp_T(N)$ react noncooperatively according to their best-reply. Let us also assume that a unique vector of Nash equilibrium strategies always exists. Thus, following the conversion of partition function games originally proposed by Ichiishi (1981) and later used by Ray and Vohra (1997), we can uniquely define the worth of a coalition S in every correspondent induced partition $p \in C(N)$, as the aggregate utility of its members in the Nash equilibrium between that coalition (acting as a single player) and the other coalitions, also acting noncooperatively with respect to all other coalitions. That is:

$$v(S,p) = \sum_{i \in S} u_i(x^*) \tag{2.16}$$

where x^* is a vector of strategies such that, for every $S \in p$,

$$\sum_{i \in S} u_i\left(x_S^*, x_{N \backslash S}^*\right) \geq \sum_{i \in S} u_i\left(x_S, x_{N \backslash S}^*\right), \ \forall x_S \in \prod_{i \in S} X_i. \tag{2.17}$$

Definition 2 *The vector of strategy $x^*$for all coalitions belonging to the corresponding partition $p^j \in \wp_T(N)$, $(j = 1, ...t)$ is consistently core-stable if there not exists any coalition $S \in \Im(N)$ such that $v(S, p^{j+1}) > \sum_{i \in S} u_i(x^*)$, where $p^{j+1} \in \wp_T(N)$ is the new partition induced by S from p^j, and the new vector of strategy is not, in turn, objectable in the same way. When such a x^* exists, we define the correspondent $p^* \in \wp_T(N)$ as a core-stable coalition structure.*

Remark 1 *The concept of stability introduced above presents some similarities with the "equilibrium binding agreement" recently introduced by Ray and Vohra (1997). The main differences are that the authors allow all consistent deviations that make the initial partition p^0 finer and, usually, the initial coalition structure is the grand coalition.*

2.4.2 Stable earning profile and core-stability

Having defined the game-theoretic setup required for the analysis, we can now apply it to the definition of a stable earning profile. We restrict our attention to just one industry of the economy described in section 2.2.1, since the analysis can easily be extended to an economy with m sectors. Starting with a set of players $N = (I_k \cup I_L)$ partitioned as in (2.12), we allow every $i \in I_K$ to select a Nash strategy to form a coalition as in (2.13), thus inducing a partition as which expressed in (2.14), given that the all other coalitions act noncooperatively according to their best-reply. From this partition on, every coalition

$$T = \left(\left\{\Sigma \subset \left\{S^i\right\}_{i \in I_K}\right\} \cup \left\{W \subset I_L \backslash \left\{S^i\right\}_{i \in I_L}\right\}\right)$$

can induce new partitions as those described above. The underlying normal form game Γ can thus be defined in the following way. In every partition $p \in \wp_T(N)$, for each player $i \in I_K$ or $i \in (\Sigma \subset S^i)$ or again, for every player $i \in (T^j \subset T^{j-1})$, $(j = 1, 2, ..., t)$, the strategy set is:

$$X_i = \{y_i \in R_+ : y_i \leq \overline{y}_i < \infty\} \equiv Y_i,$$

where y_i represents an output choice; conversely, for every player in N_2 or in any of the subsets $W^j \subset N_2$, $(j = 1, 2, ..., t)$ used to form a new coalition T^j, the strategy set is defined as follows:

$$X_i = \{yes, no\}.$$

This means that under a given partition $p^j \in \wp_T(N)$, every $i \in I_L$ is just allowed to express his agreement to participate in a coalition $S \in \Im(N)$. However, once in S, every $i \in I_L$ becomes able, either as singleton or with other players in S, to deviate and form a new coalition belonging to $\Im(N)$.

Let the preferences of every player in $S \in \Im(N)$ be linear in profit and expressed by the function:

$$u_i = \alpha_i \pi_S(y_S, y_{-S}),$$

where $\alpha_i \in [0,1]$ is such that $\sum\limits_{i \in S} \alpha_i = 1$. For every coalition $S \in \Im(N)$, let the strategy set be:

$$X_S = \left\{ (x_S, \alpha_S) : x_S \in \prod_{i \in S} X_i \text{ and } \alpha_S = (\alpha_1, ..\alpha_S) : \ \sum_{i \in S} \alpha_i = 1 \right\}$$

where α_S is a vector of shares within every S. It follows that, given a partition $p \in \wp_T(N)$, the worth of a coalition S is given by:

$$v(S,p) \equiv \sum_{i \in S} u_i \left(y_S^*, y_{-S}^*\right) = \pi_S \left(y_S^*, y_{-S}^*\right)$$

where y_{-S}^* is the Nash equilibrium vector of quantities of all other coalitions in the induced partition p, minus S. We are now ready to present the main results of this section. These are that, starting from proposition 1 and definition 2, two consistently core-stable equilibria can be characterized, with, associated, two different core-stable coalition structures.

Proposition 2 *(Self-employment equilibrium) When, under the initial partition p^0, the following condition holds*

$$\alpha_5 \leq \max\{\alpha_1, \alpha_2, \alpha_3\}, \tag{2.18}$$

where α_1 must be replaced, depending on the different circumstances described in proposition 1 and section 2.3.2, with $\alpha_1', \alpha_1'', \alpha_1^{t^2}, \alpha_1^{t^3}$, respectively, the vector $x^* = \left(\left(\{y_i^*\}_{\forall i \in I_K}, \{yes\}_{\forall i \in I_L}\right), 1\right)$ is consistently core-stable and the partition $p^0 \in \wp_T(N)$ is the core-stable coalition structure for the economy.

Proof. By proposition 1, when expression (2.18) is satisfied, the constraint (2.9) does not hold and *no player* $i \in I_K$ in the initial partition p^0 has any incentive to induce the new partition p^1 by merging with a subset of I_L. Therefore, every $i \in I_K$ will select a Nash equilibrium output as singleton. Since all players $i \in I_K$ are identical (because we are treating a one-industry case), they will all do the same, thus receiving a payoff equal to $v\left(\{i\}_{\forall i \in I_K}, p^0\right) = \pi_i\left(y_i^*, y_{-i}^*\right)$, where, for each $i \in I_K$, $\alpha_i = 1$. Hence, the correspondent consistently core-stable vector of strategy will be $x^* = \left(\left(\{y_i^*\}_{i \in I_K}, \{yes\}_{i \in I_L}\right), 1\right)$. Note that in this case the strategy selected by players in I_L, $x_i \in (yes, no)$, is irrelevant, because, no player $i \in I_K$ will be willing to recruit them, irrespective of their choice, and, hence, the equilibrium payoff of every $i \in I_L$ will be equal to zero. Consequently, the corresponding partition $p^0 = \left(\{i\}_{i \in I_K}, I_L\right)$, is the core-stable coalition structure of the economy. ■

Proposition 3 *(Capitalistic economy equilibrium) When, under the initial partition p^0, the following conditions hold:*

$$\min\{\alpha_4, \alpha_5\} \geq \max\{\alpha_1, \alpha_2, \alpha_3\}, \tag{2.19}$$

with α_1 that can also be equal to $\alpha_1', \alpha_1'', \alpha_1^{t2}, \alpha_1^{t3}$, respectively, depending on the different cases considered in proposition 1 and section 2.3.2, the vector of strategies,

$$x^* = \left(\left(\{y^*_{S^i}\}_{\forall i \in I_K}, \{yes\}_{\forall i \in I_L}\right), \left(\{1-\alpha^*_{S^i}\}_{\forall i \in I_K}, \left\{\frac{\alpha^*}{|I_L \cap S^i|}\right\}_{\forall i \in I_L \cap S^i}\right)\right)$$

is consistently core-stable and the partition $p^1 \in \wp_T(N)$, is the economy core-stable coalition structure.

Proof. By proposition 1, when the LHS of condition (2.19) holds, the constraint (2.9) is satisfied and every player in I_K has an incentive to induce from p^0 a new partition through a Nash equilibrium choice of output. This new partition is $p^1 \in \wp_T(N)$, and involves to recruit players from the subset I_L. Since all $i \in I_K$ are identical, they will all take the same choice. From proposition 1 we also know that when LHS of (2.19) holds, also constraint (2.10) is satisfied, and every $i \in I_K$ will prefer to pay the recruited workers $i \in I_L$ a sufficient share rather than let them induce a new partition $p^2 \in \wp_T(N)$. When one of members of RHS of (2.19) constitutes the maximum amongst shares in parenthesis, any allowed subcoalition of S^i is indifferent whether to induce from p^1 a new partition p^2 through the choice of a consistent strategy or just stay where it is. Hence, given the vector of Nash equilibrium quantities, denoting with α^* the share described in proposition 1, the consistently core-stable vector of shares will be, by symmetry, $\alpha_i^* = \frac{\alpha^*}{|I_L \cap S^i|}$ for all $i \in (I_L \cap S^i)$, and $(1-\alpha^*)$ for every $i \in (I_K \cap S^i)$. Moreover, the Nash equilibrium strategy for every player in $I_L \cap \{S^i\}_{i \in I_K}$ will be which to accept to merge with $\{i\}_{i \in S^i}$, i.e., $\left(\{x_i\}_{i \in I_L \cap S^i}\right) = yes$, since the only alternative would be to receive a zero payoff by being unemployed. Again, the strategy of every excluded player $i \in I_L \backslash \{S^i\}_{i \in I_K}$ is irrelevant for the stability of the vector of strategies. Thus, the correspondent partition $p^1 = \left(\{S^i\}_{\forall i \in I_K}, I_L \backslash \left(I_L \cap \{S^i\}_{i \in I_K}\right)\right)$ induced from

p^0 through the strategy choice of every $i \in I_K$, is the economy core-stable coalition structure. ■

The two propositions above describe two among the many *institutional equilibria* that, given the initial partition of players and given all allowed inducible partitions, can arise in the economy.

2.5 Discussion

2.5.1 The case of full (or near full) employment

Section 2.2 has described all possible deviations available to players under the assumption that in the economy a non empty set of unemployed workers exists in equilibrium. When, conversely, this set does not contain enough people to replace every deviating coalition of workers, some further comments are necessary. In particular, for all model shares of profit (described in proposition 1) that do not require an iterative process of firms' entry (cases in which either α^2 or α^3 are the best employees' outside options), the number of equilibrium unemployed required for the correspondent earning profile of the economy to be stable is very low. When, conversely, the best employees' outside option possesses an iterative nature (as in the case of α^1), numerical simulations show that a certain number of unemployed workers may be required for the corresponding share to converge. When, given the initial number of firms, the set of unemployed U is too small for α^1 to converge, the best employees' outside option is usually too high to be sustained as a stable earning profile for the economy. However, since in all cases considered equilibrium workers' compensation is higher than initial reservation wages, the existence of an unemployment set could be a rather plausible situation of the economy. However, what basically the model outlines is that, in a strategic market environment, the unemployment can be characterized as an *equilibrium phenomenon,* that in the model simply depends upon the different initial knowledge of entrepreneurs and workers.

2.5.2 The case of fixed wages

Section 2.2 has assumed a profit-sharing type of compensation for workers. This feature of the model simplifies the analysis, by making the Nash equilibrium vector of output unaffected by the share of profit α assigned to workers. When this assumption is removed and workers receive fixed wages, the equilibrium can be described by a pair $\left(\underline{y}^* \left(\underline{w}^* \left(k^l\right)\right), \underline{w}^* \left(k^l\right)\right)$, representing economy equilibrium quantities and wages, respectively. Since now each firm's output choice depends on the level of wages, a temporal structure has to be introduced in the model to represents every firm's Nash equilibrium output choice conditional on wages. However, simple specifications of the model show that, in general, the qualitative features of the analysis remain very close to those described in the model presented above.[9]

2.5.3 The existence of a setup cost

The model presented assumed absence of setup costs for every active firm in the economy. However, without credit constraints, the existence of a fixed cost borne by every company does not change the nature of the model results more than it does a change in the parameters values. Numerical simulations show that, for symmetric setup costs in every sector, there usually exist a range of k^l for which employees' best outside options are negatively affected by setup costs and a range of k^l for which they are, conversely, positively affected.

2.6 Concluding remarks

This chapter has described an oligopolistic economy in which each worker recruited by a firm obtains a firm-specific skill. This skill enables him, potentially, to leave the

[9]The next chapter will consider a noncooperative version of the model presented here, assuming though that employees are paid with fixed wages.

firm in which is employed to set up a new firm in the same industry. By carefully defining the conditions needed to ensure that, given the initial partition of existing players, no individual in the economy wants to change his situation through a feasible deviation, a stable earning profile for the economy has been characterized. The levels of workers' remunerations associated to this earning profile present two features: they are higher than neoclassical reservation wages (that in the model are equal to zero); they are sensitive to the size of firms' market shares and hence, to the number of firms operating in equilibrium in each industry. Moreover, given the initial players' partition and given a sequence of feasible deviations allowed to each coalition of players and under the assumption of noncooperative behaviour across coalitions, the model has shown that the stable earning profile can be sustained as a consistently core-stable vector of strategies.

Two extensions of the present model (not developed in any of next chapters) would be worthwhile of further exploration. One is to let the typology of the firms initially existing in the economy (entrepreneurial, self-employed, or partnership alike) be completely endogenous. This extension would imply to build a fully developed theory of the firm formation togheter with a theory of the stability of an earning profile as the one presented here A second line of analysis would be which to extend the allowed players' deviations to coalitions of agents belonging to different firms and/or to introduce different constraints meant to describe different bargaining processes and (possibly) more realistic institutional setups of the economy.

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Chapter 3

Managers' Compensation and Collusive Behaviour under Cournot Oligopoly

3.1 Introduction

Usually the outside option of companies executives officers (CEOs) and, in general, of highly trained workers, takes the form of alternative offers made by other (often competitor) firms. This form of at-will employment does not necessarily mean that each company is completely vulnerable to the disclosure of its strategic informations after the CEOs' departure. Trade Secret Acts (like the Uniform TSA in U.S.), corporate policies on trade secrets as well as postemployment restrictive covenants, such as non disclosure and nonsolicitation agreements, are all tools that large companies adopt to avoid a too great temptation for CEOs and other important employees to walk off stealing company's informations.[1]

However, there are well known cases in which CEOs decide to leave their company to set up independent business, mainly as a result of a solid organizational and managerial

[1]See, for instance, the recent case study "When an Executive Defects", *Harvard Business Review*, January-February 1997, pp. 18-34.

experience acquired in the field. In fact, in industries in which a relatively small number of cutthroat competitors control most of the market, a noncompete clause cannot realistically be imposed on executives. It might be either too expensive for the guaranteed contracts that senior officers would demand if asked to accept it, or it might simply be an uncommon practise in the industry. Moreover, when CEOs set up new ventures based on their organizational and market experience, companies do not really have grounds for a good lawsuit. It can be difficult to achieve evidence from which a court can infer that either customer lists, pricing and marketing plans or simply the company organizational style have been stolen.

The relevance of the outside option or "going rate" in affecting executives' pay is empirically recognized [see, for instance, Smith and Szymanski (1995)].[2] CEOs' defection to set up independent businesses can be considered more likely in industries in which fixed costs are not particularly high and the company's experience is easily duplicable. Whether the answer of existing firms' owners should be, on the one hand, that of increasing the existing market competition - to reduce the value of potential entrants - on the other it may simply be that of adopting a collusive output choice as a result. In any case, whenever the company's environment naturally discloses strategic informations to a few firm's insiders, executives' pay should be responsive of the existing outside options and hence, of the features of the market in which the firm operates.

The purpose of this chapter is to show that the existence of a concrete outside option for firms' executives can induce, under specific circumstances, every firm to adopt restrictive output practises. In particular, the chapter characterizes the conditions under which, in a Cournot oligopoly, existing firms behave more collusively than in a standard Cournot model. It is also shown that room may exist for implicit perfect and stable collusive agreements among firms. Other interesting findings are also twofold. Firstly, that the equilibrium executives' pay will usually be dependant upon the number of companies initially disposing of the knowledge required to set up the business. Secondly, that companies' procedures difficult to duplicate can constitute a beneficial form of competition policy, by inducing the firms to behave less collusively in

[2]Related empirical works on managers' pay and firms' performance are, amongst others, Murphy (1985), Jensen and Murphy (1990) and Gregg et al. (1993).

the product market. This is because firms are less worried to lose their informational advantages in favour of potential defecting firm's insiders.

Different setups related to this topic are contained, among the others, in Fenstein and Stein (1988), Mailath and Postlewaite (1990) and Stole and Zwiebel (1997). The results presented in this paper can also be compared to the well known (and opposite) result [Vickers (1985), Fershtman and Judd (1987), Sklivas (1987)] that under Cournot oligopoly the presence of managers' incentives related to sales can induce each company to behave less collusively than simple entrepreneurial firms (i.e., which managed by just the owner).

The chapter is organized as follows. The next section briefly presents the game-theoretic structure of the model. Section 3.3 introduces a simple model specification to show the main paper findings. Section 3.4 analyses the effect of including a fixed cost in the model. Section 3.4 is devoted to extend some of the results to a more general framework. Section 3.5 concludes the chapter.

3.2 The structure of the model

The model describes an oligopolistic industry in which, at the beginning, only n agents (indicated as business *initiators*) possess the knowledge to produce a commodity with a given technology. To be effective, the technology requires both skilled and unskilled workers. Such an exclusive knowledge represents the only barrier to entry for other potential competitors (for instance the skilled workers, henceforth labelled as *managers*) assumed to need a specific on-the-job training to start a new business. Thus, in the industry, the n initiators are assumed to set up n (identical) firms behaving *à la* Cournot and producing a homogenous commodity y. The sequence of strategies described in the model is quite simple. Firstly, every n -th company decides how much commodity to produce (and thus, how many identical managers to hire), according to the usual profit maximization procedure. Secondly, the company has to fix each manager's remuneration, indicated as v. Hence, a manager recruited by the firm can either decide to stay, accepting v, or leave, to set up a competing company in the industry, thus earning a profit of at most $\pi(n+1)$. Every manager that has set up a firm continues the game exactly as before (i.e., first deciding y and then v for her or his recruited managers) and the game goes on in this way, with a potentially infinite number of stages. The solution concept used to solve the game is the subgame perfect Nash equilibrium. The definition that follows describes in detail an equilibrium of the game.

Definition 3 *An equilibrium of the game is a vector of quantities,*
$\left(y_1^*(v^*(n+k), n+k), y_2^*(v^*(n+k), n+k), ..., y_{n+k}^*(v^*(n+k), n+k)\right)$,
where v^* represent every manager's equilibrium compensation and k the number of new firms entered the market in equilibrium, such that, for every $i-th$ firm, for $i = 1, 2, ..., n+k$, whatever the number of entrants ($k = 0, 1, ..., \infty$), it must be that:

$$\pi_i\left[y_i^*(v(n+k)), y_{-i}^*(v(n+k))\right] \geq \pi_i\left[y_i(v(n+k)), y_{-i}^*(v(n+k))\right] \tag{3.1}$$

and,

$$\pi_i\left[y_i^*(v^*(n+k)), y_{-i}^*(v^*(n+k))\right] \geq \pi_i\left[y_i^*(v(n+k)), y_{-i}^*(v^*(n+k))\right] \tag{3.2}$$

while, for each manager recruited by a firm,

$$v^*(n+k) \geq \pi_i\left[y_i(v(n+k+1)), y_{-i}(v(n+k+1))\right] \tag{3.3}$$

where in all expressions above, y_{-i} indicates the vector of quantity selected by all firms different from firm i and, for ease of notation, $y_i(v(n+k))$ stands for $y_i(v(n+k), n+k)$ for any i and any k.

Definition 2 imposes three conditions on an equilibrium of the game. Firstly, that no firm must find profitable to change its selected quantity (condition 3.1); secondly, that, given the quantity chosen, no firm must have an incentive to change managers' equilibrium compensation (condition 3.2); thirdly, that every hired manager must prefers to stay within the firm rather than setup a new business, otherwise the game would continue and an equilibrium would not be reached (condition 3.3). It has to be noticed that the solution concept adopted here focuses on individual players' behaviour and excludes collective deviations from equilibrium.[3] The figure below depicts the game strategy sequence.

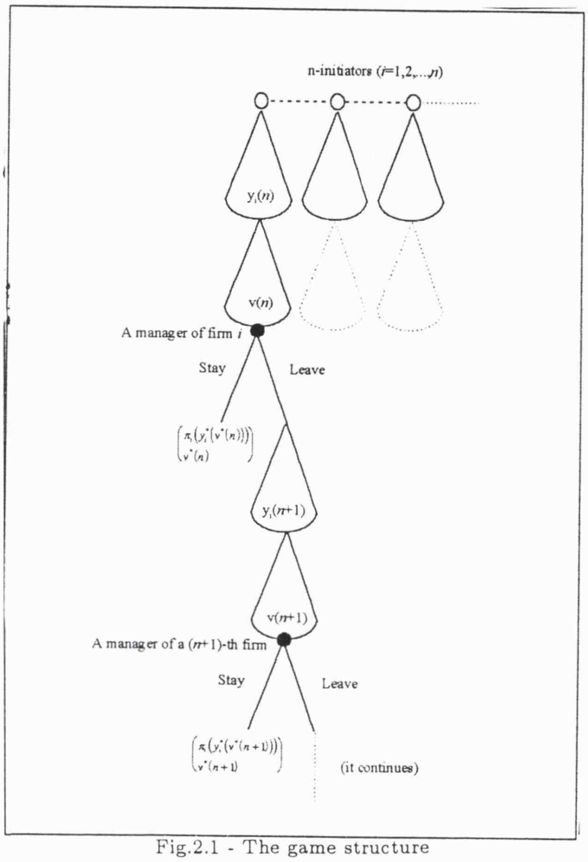


Fig.2.1 - The game structure

Definition 2 imposes three conditions on an equilibrium of the game. Firstly, that no firm must find profitable to change its selected quantity (condition 3.1); secondly, that, given the quantity chosen, no firm must have an incentive to change managers' equilibrium compensation (condition 3.2); thirdly, that every hired manager must prefers to stay within the firm rather than setup a new business, otherwise the game would continue and an equilibrium would not be reached (condition 3.3). It has to be noticed

[3] Moreover, it has to be noticed that, due to the nature of the problem, the type of equilibrium is a *stationary* subgame perfect equilibrium. In fact, at each k-th stage, every firm decides its best strategy assuming that in all following stages new entrant firms will select exactly their equilibrium strategy. Finally, note that all stages included in the game are not meant to represent time, but rather a chain of two-stage strategies happening simultaneously.

that the solution concept adopted here focuses on individual players' behaviour and excludes collective deviations from equilibrium.[4] The figure below depicts the game strategy sequence.

The next section applies the equilibrium definition to a simple model specification, in order to show its main results.

3.3 A simple example

Let us assume that in a certain industry n agents, initially disposing of the knowledge on how to produce a homogenous commodity y, decide to set up n (identical) firms behaving *à la* Cournot. Let also the technology available to them be described by the following Cobb-Douglas constant returns to scale production function:

$$y_i = m^{\theta} \cdot \ell^{(1-\theta)} \tag{3.4}$$

where respectively m is the number of managers (or highly skilled workers) recruited by every i-th firm ($i = 1, ...n$) while ℓ is the number of unskilled workers. Let us assume, for simplicity, that $\theta = 1/2$. Let also every firm's fixed cost be equal to zero. Without loss of generality, the wage paid to unskilled workers is normalized to one, while v denotes each manager's compensation. Moreover, let the market demand be linear and equal to:

$$p(Y) = a - Y \tag{3.5}$$

[4]Moreover, it has to be noticed that, due to the nature of the problem, the type of equilibrium is a *stationary* subgame perfect equilibrium. In fact, at each k-th stage, every firm decides its best strategy assuming that in all following stages new entrant firms will select exactly their equilibrium strategy. Finally, note that all stages included in the game are not meant to represent time, but rather a chain of two-stage strategies happening simultaneously.

where $Y = \sum_{i=1}^{n} y_i$ represents the total quantity of commodity delivered to the market. Deriving by (3.4) every firm's cost function as:

$$C_i\left(y_i\right) = 2\sqrt{v} \cdot y_i \tag{3.6}$$

it is straightforward to get every initial i-th firm's Cournot equilibrium quantity (for any arbitrary managers' compensation) as:

$$y_i^*\left(n\right) = \frac{a - 2\sqrt{v}}{n+1} \tag{3.7}$$

and every i-th firm's equilibrium profit as:

$$\pi_i^*\left(n\right) = \frac{\left(a - 2\sqrt{v}\right)^2}{(n+1)^2} \tag{3.8}$$

As explained above, the basic feature of the model is that, when managers are hired by a firm, they immediately acquire the specific knowledge to become potential competitors of the existing firms. Hence, managers' compensation must be optimally decided by a firm knowing each manager's potential threat of leaving to setup, through the use of unskilled workers and other managers, a new production unit. When a manager leaves the firm, he or she will presumably set up a company of the same type as the one she or he is working for. Thus, if in the previous stages of the game k firms have entered the market, from (3.8), the payoff of a leaving manager is at most equal to:[5]

$$\pi_i\left(n+k+1\right) = \frac{\left(a - 2\sqrt{v\left(n+k+1\right)}\right)^2}{\left(n+k+2\right)^2} \tag{3.9}$$

[5]A simplyfyng assumption made here is that the exit of one manager and the consequent creation of a new firm determines a trascurable reduction in the equilibrium number of managers' hired by all other firms. However, to include this effect and the induced entry of new firms does not change at all the nature of the model results.

where $v(n+k+1)$ represents the wage that the leaving manager will pay to her or his managers. Expression (3.9) it is built under the presumption that every existing firm whose manager(s) has decide to leave, can easily find a substitute. As it will become clear later, such a managers' availability can either be assumed (case 3.3.1), or it is something endogenously generated by the model (case 3.3.2).[6]

Now, we can apply definition 1 to find managers' equilibrium compensation and, hence, an equilibrium of the game. Firstly, we look for the level of compensation the firms have to pay to make a manager indifferent whether to stay or to leave. This can be done by solving expression (3.9). In this way, at any stage of the game a firm knows that, if selected manager's compensation $v(n+k) \geq \pi_i(n+k+1)$, the manager will stay, while, otherwise, she or he will leave. Secondly, it has to be found the number of firms k that enter at the equilibrium, so to fully characterize the final compensation $v^*(n+k)$ and thus $y_i^*(v^*(n+k))$ for every firm active in the market. By using a standard backward induction procedure, we notice that under the specification of the model, two different cases arise. They are both illustrated below.

3.3.1 Infinite number of stages

This describes the case in which there exists a virtual unlimited availability of managers ready to be hired by the firms of the industry and, as a consequence, the corresponding managers' reservation wage is equal to zero. Moreover, denoting as t the number of new entrants that makes every firm's profit (approximately) equal to zero, under the model specification, $\pi_i(n+t)$equal to zero happens for t that tends to infinity, given that t must solve:

$$\pi_i(n+t) = \frac{\left(a - 2\sqrt{v(n+t)}\right)^2}{(n+t+1)^2} = \frac{\left(a - 2\sqrt{0}\right)^2}{(n+t+1)^2} = 0 \tag{3.10}$$

[6] Moreover, as long as there are managers available to work, companies are completely indifferent whether to hire an unemployed manager or just recruit one that is currently working for another firm.

provided that, when $\pi_i(n+t) = 0$, also $v(n+t) = \pi_i(n+t+1) = 0$. Thus, when $(t-1)$ firms have already entered the market, every manager will be indifferent whether to stay in the firm as employee, earning the corresponding wage $v(n+t-1) = \pi_i(n+t) = 0$ or just leave. Firms at the previous stage, knowing this, need to decide whether to pay $v^*(n+t-2) = \pi_i(v^*(n+t-1))$ to their managers (where v^* indicates, at every stage, the wage that respects (3.9)) or pay less, letting at least one manager leave to get a profit of $\pi_i(v^*(n+t-1))$. Thus, since at the last stage a manager will necessarily be paid $v^*(n+t-1) = \pi_i(n+t) = 0$, one stage before firms will find convenient to pay $v^*(n+t-2)$ only if:

$$\pi_i^*(v^*(n+t-2)) \geq \pi_i^*(v^*(n+t-1)) \tag{3.11}$$

The lemma presented below proves that, if every firm's profit is weakly positive and

the next stage firm's optimal strategy is which to fix a compensation that respect (3.9), it will be always convenient for a firm at the previous stage to do the same, paying every manager just enough to keep her or him within the firm.

Lemma 4 *Under the model specification, if* $p(Y^*(n+k)) \geq AC_i(y_i^*(n+k))$, *for any possible* $k = 1, 2, ..., \infty$, *the following inequality holds for every firm* $i = 1, ..., n+k$:

$$\pi_i^*(v^*(n+k)) \geq \pi_i^*(v^*(n+k+1)) \tag{3.12}$$

Proof. (See Appendix).

Since lemma 1 holds at any stage of the game, every firm's optimal strategy will be which of paying the minimal remuneration sufficient to induce its managers to stay within the firm, i.e., a wage that respects (3.9). By backward induction, at the first stage of the game, when the n initiators decide whether to let their managers stay (paying them accordingly) or leave (thus increasing the existing number of firms), they will certainly set $v = v^*(n)$ and the equilibrium number of entrants will be $k = 0$. The equilibrium remuneration $v^*(n)$ is thus obtainable by solving the following equation:

$$v^*(n) = \pi_i^*(n+1) = \frac{\left(a - 2\sqrt{v^*(n+1)}\right)^2}{(n+2)^2} \tag{3.13}$$

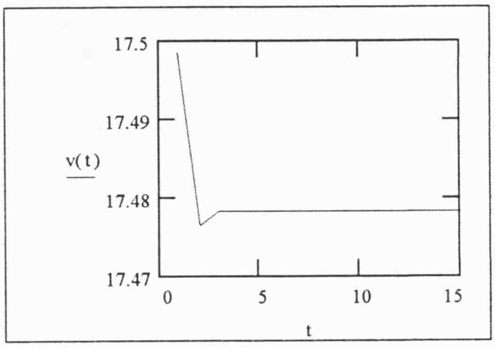
Expression (3.13) is a non linear difference equation that can be solved by iteration on the potential number of entrants k. A straightforward substitution process yields:

$$v^*(n) = \frac{\left(a(n+3)(n+4)\cdot\cdot(n+k) - 2a(n+4)\cdot\cdot(n+k) + 4a(n+5)\cdot\cdot(n+k) - 8a\cdot\cdot + (-1)^{k+1}2^{k+1}\sqrt{v^*(n+k-1)}\right)^2}{(n+2)^2(n+3)^2\cdot\cdot(n+k)^2} \tag{3.14}$$

Now, from (3.10) we know that, for $k = t \to \infty$, $v^*(n+t-1) = \pi_i^*(n+t) = 0$. Hence, putting $v^*(n+t-1) = 0$ into expression (3.14) and rearranging, we get:

$$v^*(n) = \lim_{t\to\infty} \frac{\left\{\sum\limits_{i=0}^{t-3}\left[(-1)^i(2^i a)\right]\prod\limits_{j=t-3}^{i}(n+3+j) + (-1)^{t-2}(2^{t-2}a)\right\}^2}{\prod\limits_{i=0}^{t-2}(n+2+i)^2} \tag{3.15}$$

Interesting properties of expression (3.15) are that it is unique for every set of parameters values and that takes finite values, even for a finite and very low number of stage t. The picture below shows that the value of $v^*(n)$ already converges for $t = 4$.



Tab. 3.1 - Values of $\mathbf{v}^*(\mathbf{n})$, for $\mathbf{t} = \mathbf{0},...\mathbf{15}$ (**a=1000, n=20**)

Moreover, the value of $v^*(n)$ is monotonically decreasing in the number n of initiators assumed to exists at the beginning of the game.

Now, by embedding expression (3.15) into (3.7), it ensues that, according to definition 1, the unique equilibrium of the game is $(y_1^*(v^*(n), n), y_2^*(v^*(n), n), ..., y_n^*(v^*(n), n))$.

3.3.2 Finite number of stages

This is the case in which a market for managers exists in the economy and no manager is available to be hired for less than the market clearing wage, denoted as $\overline{v}$. When this is the case, the reasoning above just needs a few changes. Since an equilibrium compensation must be sufficiently high to keep a manager within the firm (by condition 3.3 of equilibrium), it needs to be:

$$v^*(n+k) = \max\{\overline{v}, \pi_i(v^*(n+k+1))\} \tag{3.16}$$

and then,

$$v^*(n+k) \geq \overline{v} \tag{3.17}$$

By expression (3.8) and (3.17) it follows that:

$$\pi_i(\overline{v}, n+k) \geq \pi_i(v^*(n+k), n+k) \tag{3.18}$$

It is clear from (3.18) that there exists a finite $t \geq k$, for which $\pi_i(\overline{v}, n+t) = \overline{v}$, with a negligible approximation, due to the integer problem. Thus, $v^*(n+t-1) = \overline{v}$ for t that solves:

$$\pi_i(\overline{v}, n+k) = \frac{\left(a - 2\sqrt{\overline{v}}\right)^2}{(n+t+1)^2} = \overline{v} \tag{3.19}$$

whose only positive solution is:

$$t = \frac{a - (n+3)\sqrt{\overline{v}}}{\sqrt{\overline{v}}} \tag{3.20}$$

Hence, at stage $(n+t-2)$ every firm will pay a manager a compensation equal to $v^*(n+t-2)$ only if:

$$\pi_i(v^*(n+t-2)) \geq \pi_i(v^*(n+t-1)) \tag{3.21}$$

That lemma that follows proves that the above inequality always holds, at any stage of the game.

Lemma 5 *Under the model specification and for $v^* \geq \overline{v}$, at any stage of the game $(k = 1, 2, ..., t)$, and for every firm $(i = 1, ..., n+k)$, the following inequality holds:*

$$\pi_i^*(v^*(n+k)) \geq \pi_i^*(v^*(n+k+1)) \tag{3.22}$$

Proof. (See Appendix).

Once again, expression (3.22) ensures that the n initiators will decide in equilibrium to pay $v = v^*(n)$ and the equilibrium number of entrants will be $k = 0$. Hence, the equilibrium managers' remuneration $v^*(n)$ is obtainable by iteration, through the choice of the appropriate number of entrants $t(\overline{v})$ for which $v^*(n+t-1) = \pi_i^*(n+t) = \overline{v}$, as:

$$v^*(n) = \frac{\left\{ \sum_{i=0}^{t-3} \left[(-1)^i (2^i a) \right] \prod_{j=t-3}^{i} (n+3+j) + (-1)^{t-2} 2^{t-2} \left(a - 2\sqrt{\overline{v}} \right) \right\}^2}{\prod_{i=0}^{t-2} (n+2+i)^2} \tag{3.23}$$

Also in this case $v^*(n)$ is unique for every set of parameters and takes a finite value even for a very low t.

In figure below, $v^*(n)$ is plotted against different number of business initiators and compared to a given market clearing wage $\overline{v}$. Managers' compensation is decreasing with the number of firms assumed to exist in the market. Notice also that, since for a range of n equilibrium managers' compensation is higher than the market clearing level, there will always be, within this range, managers available to be hired by the firms. This endogenous availability of managers allows for their substitution when they decide to leave the firm and gives consistency to the game described above.

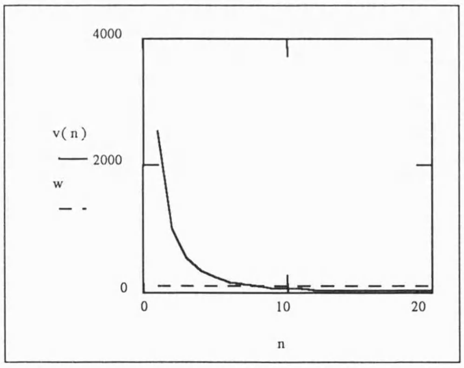


Fig 3.2-Values of $v^*(n)$ and $w = \overline{v}$ for $a=350$ and $n=1,..20$.
In this example $v^*(n)$ is equal to $\overline{v}=10$ for $\underline{n}=8$.

The analysis that follows is conducted under the case, definitively more realistic, of a finite number of stages. This gives rise to a few results presented below. The first one is concerned with every firm's equilibrium choice of managers, and it is expressed in the next proposition.

Proposition 6 *Under the model assumptions, when the number of initiators is no greater than* $\underline{n}$*, the equilibrium number of managers* m^* *selected by every firm is less than it would be under standard neoclassical assumptions in the market for managers.*

Proof. Since from (3.4) an efficient choice of managers implies $m^* = \frac{y^{i*}}{\sqrt{v^*}}$, and, from (3.7) Cournot equilibrium output y_i^* is monotonically decreasing in v, it ensues that $m^*(v^*(n)) < m^*(\overline{v})$ whenever $v^*(n) > \overline{v}$,where $\overline{v}$ indicates the neoclassical market clearing wage. Since $v^*(n) = \pi_i(v^*(n+1))$, and, by lemma 2, the second member is monotonically decreasing in n, there will certainly be a value of $n = \underline{n}$ for which $v^*(\underline{n}) = \overline{v}$. Thus, for $n < \underline{n}$, $m^*(v^*(n)) < m^*(\overline{v})$ and the result follows. ■

It can be noticed that equilibrium quantity $y_i^*(v^*(n), n)$ is, for every firm, dependent on equilibrium managers' compensation. Thus, since $v^*(n)$ is higher than $\overline{v}$ for $n < \underline{n}$, it turns out that, when $n < \underline{n}$, $y_i^*(v^*(n), n) < y_i^*(\overline{v}, n)$. This means that each firm is, within a given range of n, more collusive in terms of output than under usual market clearing conditions. There also exists an initial number of firms for which $y_i^*(v^*(n), n)$ exactly coincides with the perfectly collusive output choice under market clearing wage, i.e., that obtained when all firms cooperatively maximize their joint profit. These results are described in the next proposition.

Proposition 7 *The output selected by every firm under Cournot equilibrium and managers' threat to leave is more collusive than under Cournot equilibrium and managers' competitive market for* $n < \underline{n}$*, that is,* $y_i^*(v^*(n), n) < y_i^*(\overline{v}, n)$ *for* $n < \underline{n}$*. Moreover, there exists a level of* $n = n^*$ *for which,* $y_i^*(v^*(n^*), n^*) = y_i^{c*}(\overline{v}, n^*)$*, where* y_i^{c*} *is the output resulting by cooperative agreement among firms.*

Proof. By proposition 1, for $n < \underline{n}$, $v^*(n)$ is greater than $\overline{v}$. Since firm's equilibrium output is monotonically decreasing in v, it follows that, for $n < \underline{n}$, $y_i^*(v^*(n), n) <$

$y_i^*(\overline{v}, n)$. Moreover, straightforward manipulation of expression (3.23) show that it is equal to:

$$v^*(n) = \frac{a^2\left(n^5 + 23n^4 + 205n^3 + 881n^2 + 1818n + 1424\right)^2}{(n+2)^2(n+3)^2(n+4)^2(n+5)^2(n+6)^2(n+7)^2} \tag{3.24}$$

Substituting expression (3.24) for $v^*(n)$ into y_i^* yields:

$$y_i^*(v^*(n), n) = \frac{a - 2\sqrt{v^*(n)}}{n+1} \tag{3.25}$$

while, the collusive quantity under market clearing managers' compensation $\overline{v}$ is:

$$y_i^{c*}(\overline{v}, n) = \frac{a - 2\sqrt{\overline{v}}}{2n} \tag{3.26}$$

Thus, expression (3.25) is equal to (3.26) , for $n = n^*$, where n^* is the only positive solution of an equation that, for ease of brevity, is not presented here. It can be noticed that the higher the managers' market clearing wage and the lower will be the number of firms for which $y^{i*}(v^*(n), n) = y_i^{c*}(\overline{v}, n)$. ■

A particular example of the result above is presented in figure 3.3. Given $\overline{v}$, for $n = 220$, the collusive equilibrium quantity $y_i^{c*}(\overline{v}, n)$ coincides with $y^{i*}(v^*(n), n)$. In the example, for $n < \underline{n}$, $y^{i*}(v^*(n), n)$ turns out to be even more collusive than $y_i^{c*}(\overline{v}, n)$. Moreover, since every firm's quantity $y^{i*}(v^*(n), n)$ is also a Nash equilibrium quantity, it will be stable against each firm's temptation to deviate from the equilibrium choice of output, differently to what normally happens under collusive agreement. It can be noticed that such a particular example of *non-cooperative collusive* solution can take place either through mergers among firms (when the number of initiators is greater than n^*) or through controlled departure of managers induced by a firm (when n is lower than n^*). The latter can specially be the case when the initiators maintain a share of the new companies' control, a relatively widespread practise in high-tech industries.[7]

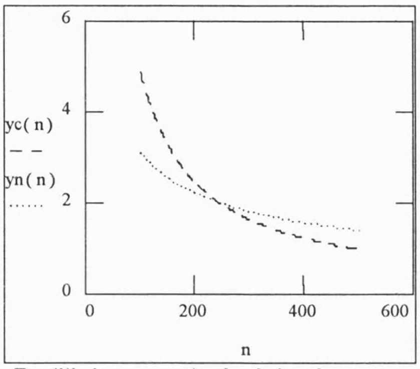


Fig. 3.3 - Equilibrium quantity both for the managerial firm $yn(n)$ and for the perfectly collusive firm ($yc(n)$) (a=1000, $\overline{v}$=10,n=0,..600).

Anyway, the basic result of the model is that, whether the behaviour of existing firms in a market is less or more collusive than in usual Cournot models depends upon the number of business initiators that dispose of the basic know-how to set up a firm.

3.4 The effect of a fixed cost

The analysis of previous section was conducted under the extreme assumption that both business initiators and potential entrants do not bear any fixed cost to set up a firm. This section is devoted to explore which consequences arise when a positive fixed cost is needed to set up a business. In the model a fixed cost affects both the outside options of potential competitors as well as the profit of business initiators. It must be noticed that, including a fixed cost - under absence of whatsoever firm's credit constraint - does not change the qualitative nature of equilibrium. Lemma 2

[7] For a description of the so called *Spin-off* practises in high-tech industries, see, among the others, Gordon (1992), and Seward and Walsh (1996), .

still holds (although now the number t of stages of the game changes) and expression (3.13) becomes:

$$v^*(n) = \pi_i^*(n+1) = \frac{\left(a - 2\sqrt{v^*(n+1)}\right)^2}{(n+2)^2} - F \tag{3.27}$$

where F indicates the fixed cost. Again, expression (3.27) can be solved for $v^*(n)$, yielding:

$$v^*(n) = \frac{\left\{\sum_{i=0}^{t-3}\left[(-1)^i\left(2^i a + 2^{i+1}\sqrt{F}(n+2+i)\right)\prod_{j=t-3}^{i}(n+3+j)\right] + (-1)^{t-2}\left[2^{t-2}\left(a - 2\sqrt{\overline{v}} - 2\sqrt{F}\right)\right]\right\}^2}{\prod_{i=0}^{t-2}(n+2+i)^2} - F \tag{3.28}$$

Once again, expression (3.28) is monotonically decreasing in the number of initiators n. However, the effect of the fixed cost is not unidirectional. For relatively low levels of F, every leaving manager's outside option increases and the initiators behaviour will be in equilibrium even more collusive than before. Conversely, for high levels of F, manager's outside option tends to decrease and so does the equilibrium degree of collusion put in place by initiators. The next proposition explains this result.

Proposition 8 *If the fixed cost needed to set up a firm is lower than* $\underline{F}$*, every firm's equilibrium output will be more collusive than under entry threat and total absence of fixed cost. For* $F > \underline{F}$ *, the opposite result holds.*

Proof. Since expression $v^*(n)$ converges to a finite value for a very low t, it can be twice derived with respect to F. Second derivative turns out to be negative, and function $v^*(n, F)$ concave with respect to F. It thus straightforward to find a finite value $\underline{F}$ for which $v^*(n, \underline{F}) = v^*(n, 0)$. It follows that, for $F \in [0, \underline{F}]$, $v^*(n, F) \geq v^*(n, 0)$; conversely, for $F \in (\underline{F}, \overline{F}]$, $v^*(n, F) < v^*(n, 0)$, where $\overline{F}$ indicates the level for which $v^*(n, F) = 0$. ■

3.5 Some generalizations of the model

This section is devoted to give some generality to the results obtained above as well as to discuss which basic assumptions are strictly required to achieve the main model findings.

3.5.1 Assumptions

We list below a number of assumptions required to obtain the collusive result in a Cournot model. These assumptions are that:

A.5.1 the payoff of each firm is a function of its own strategy and of the sum of strategies of all existing firms (usually defined as *aggregation axiom*; see, for instance, Dubey, Mas-Colell and Shubik (1980));

A.5.2 strategy sets Y_i are, for every firm, compact and convex;

A.5.3 every firm's payoff function, $\pi_i : Y_i \times \Re_+ \rightarrow \Re_+$, is twice continuously differentiable;

A.5.4 $\frac{\partial^2 p(Y)}{\partial Y^2} y_i + \frac{\partial p(Y)}{\partial Y} < 0$;

A.5.5 $\frac{\partial p(Y)}{\partial Y} - \frac{\partial^2 C_i(y_i)}{\partial y_i^2} < 0$,

(A.5.4 and A.5.5 are standard assumptions for second order conditions to hold, see, for instance, Friedman (1977));

A.5.6 The output of every firm is strictly decreasing in manager's compensation v;

3.5.2 Basic results and discussion

Under assumptions A.5.1-A.5.6, it can be proved that:

i) A Cournot-Nash equilibrium $(y_1^*, y_2^*, ..., y_n^*)$ always exists and is unique (see, for instance, Corchon (1996), p.15);

ii) Condition $\pi_i^*(n) \geq \pi_i^*(n+k)$ for $(k = 1, 2, ..., t)$ always hold under firm's positive profitability (see lemma 3, in appendix).

From i) and ii), the following proposition can be derived:

Proposition 9 *Under Cournot oligopoly and managers' threat to leave, when assumptions A.5.1-A.5.6 hold, there always exists a number of initiators below which equilibrium managers' compensation is higher than market clearing neoclassical wage. Moreover, within this range of n, equilibrium output is more collusive than in a standard Cournot model.*

Proof. Result ii) implies that, for $(k = 0, 1, ..., t)$, $v^*(n)$ is monotonically decreasing in n. Hence, it is always possible to find a $n = \underline{n}$ such that $v^*(\underline{n}) = \overline{v}$ and then for $n < \underline{n}$, $v^*(\underline{n}) > \overline{v}$. Furthermore, from A.5.5 it ensues that, for $n < \underline{n}$, $y_i^*(v^*(n)) < y_i^*(\overline{v})$. ■

It can be interesting to spend a few words in discussing the meaning of A.5.5. Coupled with A.5.4, it ensures that every firm's payoff is concave, from which, second orders conditions for a maximum are satisfied. This in turn requires either a "not too convex" demand function or a "not too concave" cost function with respect to output. By assuming (without loss of generality) a linear technology, it is easy to see that:

$$p' - C_i'' < 0 \Rightarrow p' - \frac{\partial^2 v^*(n)}{\partial y_i^2} y_i^* - 2\frac{\partial v^*(n)}{\partial y_i} < 0 \qquad (3.29)$$

Note in general that the model assumptions always ensure that both $\frac{\partial \pi^*(n+1)}{\partial y_i} < 0$ and thus, $\frac{\partial v^*(n)}{\partial y_i} < 0$ and that $\frac{\partial^2 \pi^*(n+1)}{\partial y_i^2} < 0$ and then $\frac{\partial^2 v^*(n)}{\partial y_i^2} < 0$. Hence, every firm's cost function will be concave. Given a regular demand function, what is thus required for the collusive effect to take place is "a not too concave" cost function. Condition (3.29) shows that for every firm the choice of y_i yields, beside the usual negative effect on the demand function, an indirect effect on managers' compensation. This effect has two components: the first is which to reduce the pay of every manager through the fall of her or his outside option, i.e., the threat to set up a new firm. The second is which to reduce the other subsequent potential k leaving managers' outside option and, hence, increasing which of the first leaving manager. By taking the second derivative of $v^*(n)$ with respect to y_i and applying the model specification used in section 3.3, it turns out that:

$$\frac{\partial^2 v^*(n)}{\partial y_i^2} = \lim_{k \to t} \sum_{j=1}^{k} (-1)^{(j+1)} \cdot p'' \cdot \prod_{j=1}^{k} y_{n+j}^* \tag{3.30}$$

Condition (3.30) may certainly hold when expression (3.29) is not "too negative". This implies to impose that the chain effect on every k-th entrants has sufficient strength to almost offset the direct negative effect of an output increase on the first leaving manager's outside option. However, the same result would ensue by assuming that every manager's learning process requires a certain period of time to be completed. In this case, it would always exist a discount factor $\delta \in (0,1)$ sufficiently low for firms' payoff to fall with the number of entrants k and hence, for the collusive effect to take place.

3.6 Concluding remarks

We have described, through an extremely simple model, that companies owners' need to fix a level of compensation high enough to keep managers within the firm can give rise to a collusive choice of output stable against individual firm's deviations. The result holds when the depressive effect of leaving managers on firms' profit prevails on the positive effect due to a reduction of their compensation, and the existing number of firms is not very high. Furthermore, the model generates the empirically appealing property (see, for instance, Watson et al.(1994)) that managers' compensation is decreasing with the number of firms existing in the market and, consequently, with their size. The nature of every company's knowledge may also play a role as long as firms' (organizational or technological) procedures difficult to duplicate decrease managers' outside option hence increasing the number of hired managers in the industry. On the other hand, easily replicable companies' procedures would be coupled with a reduced number of recruited managers (due to the very high managers' outside option) and companies' output restrictions. In this respect, given the initial number of firms operating in an industry, complex and heterogeneous company's procedures could be beneficial in terms of level of output and competition generated in the market.

3.7 Appendix

Proof of Lemma 1. *As long as,* $p\left(Y^*\left(n+k\right)\right) \geq AC_i\left(y_i^*\left(n+k\right)\right)$, *the following inequality holds for every* $i = 1, ..., n+k$

$$\pi_i^*\left(v^*\left(n+k\right), n+k\right) \geq \pi_i^*\left(v^*\left(n+k+1\right), n+k+1\right) \tag{3.31}$$

Proof. The meaning of expression (3.31) is that, under positive profitability of existing firms, every company finds convenient to pay each manager the equilibrium wage $v^*\left(n+k\right)$ rather than let her or him go, starting a new negotiation with another manager. Let us prove the lemma by contradiction.

Suppose inequality (3.31) does not hold, that is:

$$\left[p\left(Y^*\left(n+k\right)\right) - 2\sqrt{v^*\left(n+k\right)}\right] y_i^*\left(n+k\right) <$$

$$< \left[p\left(Y^*\left(n+k+1\right)\right) - 2\sqrt{v^*\left(n+k+1\right)}\right] y_i^*\left(n+k+1\right)$$

This expression can be solved by iteration and, for each firm under the potential market entry of t firms, the following result ensues:

$$v^*(n+k) > \frac{p(Y^*(n+k))^2}{4} - \frac{[p(Y^*(n+t))\,y_i^*(n+t) - v^*(n+t)\,4y_i^*(n+t)]^2}{4(y_i^*(n+t))^2}$$

where t indicates the number of firms that can enter before every firm's profit is equal

to zero and the game ends. Since in this case there are no entry costs, room potentially exists for an infinite number of entrants. Hence, taking the limit of expression above for t that tends to infinite, we get:

$$v^*(n+k) > \frac{p(Y^*(n+k))^2}{4} - \lim_{t\to\infty} \frac{[p(Y^*(n+t))\,y_i^*(n+t) - v^*(n+t)\,4y_i^*(n+t)]^2}{4(y_i^*(n+k))^2}$$

that can be rewritten as:

$$2\sqrt{v^*(n+k)} \cdot y_i^*(n+k) > p(Y^*(n+k)) \cdot y_i^*(n+k) - \lim_{t\to\infty} \pi_i(v^*(n+t))$$

and then,

$$2\sqrt{v^*(n+k)} \cdot y_i^*(n+k) > p(Y^*(n+k)) \cdot y_i^*(n+k) - 0$$

from which:

$$2\sqrt{v^*(n+k)} = AC(y_i^*(n+k)) > p(Y^*(n))$$

that contradicts the assumption of every firm's weak positive profitability. This concludes the proof. ■

Proof of Lemma 2. *Under the existence of a given market clearing wage for managers equal to* $\overline{v}$, *the following inequality holds for every* $i = 1, ..., n + k$:

$$\pi_i^* \left(v^* \left(n+k\right), n+k\right) \geq \pi_i^* \left(v^* \left(n+k+1\right), n+k+1\right) \tag{3.32}$$

Proof.

Suppose inequality (3.32) does not hold, that is:

$$\left[p\left(Y^* \left(n+k\right)\right) - 2\sqrt{v^* \left(n+k\right)}\right] y_i^* \left(n+k\right) <$$

$$< \left[p\left(Y^* \left(n+k+1\right)\right) - 2\sqrt{v^* \left(n+k+1\right)}\right] y_i^* \left(n+k+1\right)$$

This expression can be solved by iteration and, for every firm under the potential market entry of t firms, the following result ensues:

$$v^* \left(n+k\right) > \tfrac{p(Y^*(n+k))^2}{4} - \lim_{s \to t} \pi_i \left(v^* \left(n+t\right)\right) = \tfrac{p(Y^*(n+k))^2}{4} - \overline{v}$$

from which:

$$\pi_i \left(v^* \left(n+k\right)\right) = v^* \left(n+k-1\right) < \overline{v}$$

that contradicts the fact that every manager's compensation must necessarily be greater than market clearing wage. This concludes the proof. ■

Proof of Lemma 3. *Under the following standard assumptions:*

$$\frac{\partial^2 p\left(Y\right)}{\partial Y^2} y_i + \frac{\partial p\left(Y\right)}{\partial Y} < 0 \tag{A.5.4}$$

and,

$$\frac{\partial p\left(Y\right)}{\partial Y}-\frac{\partial^{2}C_{i}\left(y_{i}\right)}{\partial y_{i}^{2}}<0 \tag{A.5.5}$$

the following inequality holds:

$$\pi_{i}^{*}\left(v^{*}\left(n\right),n\right)\geq\pi_{i}^{*}\left(v^{*}\left(n+1\right),n+1\right) \tag{3.33}$$

Proof. (Standard) Assumptions (A.5.4) and (A.5.5) always imply that $Y^{*}\left(n\right)<Y^{*}\left(n+1\right)$ and $y_{i}^{*}\left(n\right)>y_{i}^{*}\left(n+1\right)$. Provided this, and using first order conditions, it follows that:

$$\frac{d\pi_{i}^{*}(n)}{dn}=\frac{dy_{i}^{*}}{dn}p+y_{i}^{*}\frac{dp^{*}}{dY^{*}}\frac{dY^{*}}{dn}-\frac{dC}{dy_{i}^{*}}\frac{dy_{i}^{*}}{dn}=$$

$$=\left(p-C'\right)\frac{dy_{i}^{*}}{dn}+y_{i}^{*}p'\frac{dY^{*}}{dn}=y_{i}^{*}p'\left(\frac{dY^{*}}{dn}-\frac{dy_{i}^{*}}{dn}\right)<0. \blacksquare$$

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Chapter 4

Stable Producer Co-operatives in Competitive Market

4.1 Introduction

Modern capitalist economies as well as economies in transition are often characterized by a puzzling mixture of large private and public corporations, small-size companies, producer co-operatives and self-employment type of ventures, sometimes coexisting in the same industries. There are, however, sectors in which certain types of firms seem to be particularly at ease. In particular, different forms of producer co-operatives (henceforth PCs) and partnerships can usually be found - although not exclusively - in construction, printing, glass-making, woodworking and service industries (for instance in Italy, France and Sweden), in clothing and footwear (in U.K.) and in plywood, reforestation and taxi cab industries (in U.S.). In other cases, these forms of organization appear concentrated in relatively restricted areas, with a diversified range of activities (as, for instance, in Mondragon, Spain).[1] However, in all these cases some common features of the industries in which PCs are clustered exist and seem to be, among the others, low barriers to entry, small size of firm, a rather specific and trained workforce and a relatively high degree of competition. Do these features have something to do with the arguments usually adopted to explain the PCs' (in)stability?

[1] See Bonin, Jones and Putterman (1993) for a general overview on Producer Co-operatives.

There is an extensive body of literature addressing the reason behind the birth of labour-managed types of organizations.[2] In particular, one element of this of literature concerns the feasibility and stability of democratic forms of enterprise in capitalist markets.[3] The bulk of the instability argument is that, in profitable industries, PCs' members can find convenient to dismiss part of the firm's membership to recruit less expensive fixed-wage workers. However, since the industries in which PCs usually operate appear to be (according to the above mentioned features) vulnerable to competition from new entrants into the market, it seems natural to look at what happens if, when dismissed, every PC's member has the possibility to set up a new firm. Furthermore, as said above, PCs generally occur in an environment in which companies' setup costs are low and employees often possess highly specific skills. These features allow the workers recruited both by PCs and capitalist firms (henceforth CF) alike to possess a concrete outside option during the wage bargaining process. Therefore, the effect of existing workers' outside options should be considered in the determination of every industry equilibrium wage. In which case, dropping the usual exogenous. wage assumption could help better explain the mechanism of PC's instability. Since PC's members are usually assumed to compare their remuneration with a given equilibrium wage, ultimately the stability of a PC's membership should be tested against a wage endogenously determined.

The aim of this chapter is to include some of the described features in a standard labour-managed firms setup. Two main insights are provided. Firstly, that in an environment in which dismissed members can set up new firms, the PC's instability does not hold if not under specific conditions, mainly concerning the market demand elasticity. Secondly, that there are particular situations in which workers have an incentive to set up PCs while, under other circumstances, such incentives do not exist. Finally, if a co-operative is profitably formed in such environment, it can be shown to be stable within a range of market parameters.

This chapter uses a simple short-run perfectly competitive model with n firms to

[2]For accurate surveys see, for instance, Ireland and Law (1984), Bonin and Putterman (.), Bonin, Jones and Putterman (1993) and Dow and Putterman (1995).

[3]See, among the others, Webb and Webb (1920), Tugan-Baranovskii (1921), Jones (1975,1979), Ben-ner (1984,1988), Myizaki (1988), Estrin and Jones (1992).

present its main results. Extensions to imperfectly competitive market and heterogeneous workers setups are possible but, for ease of simplicity, are not presented here. The paper is organized as follows. Section 4.2 extends the traditional PC's instability result and introduces the basic idea of the paper. Section 4.3 describes a simple model of endogenous wage determination from which the main results of the paper are obtained. Section 4.4 concludes.

4.2 The PC's instability argument

The traditional Ward's (1958) and Vanek's (1970) per capita value-added objective function assumed for the producer co-operatives is the feature usually thought to make these forms of enterprise unstable under positive profitability. Since a successful PC can profitably acquire new workers in the spot labour market at a given market wage to replace its members, this feature is supposed to yield an iterative process of substitution leading eventually a PC to be owned and managed by just one member and become a capitalist firm.[4]

Ben-ner (1984), for instance, establishes this result in a standard perfect competition and partial equilibrium framework either with fixed or with variable use of capital, also admitting the possibility of an individual members' productivity higher than that of workers hired in the spot labour market.[5]

In a model with uncertainty Miyazaki (1984) generalizes the PC's instability result, showing that, whenever a CF's expected profit is positive, the twin PC tends to

[4]In some countries PCs are obliged by law to respect a given member-employee ratio according to the "open door" principle. However, this principle is usually not respected by employee-owned firms without a co-operative *status*. Moreover, in several countries legislative reforms have recently weakened the above mentioned constraint. For a survey of the main legislative changes occured in Europe and a theoretical analysis of their consequences, see, for instance, Marini and Zevi (1995, 1996) and Monzon C. & al. (1996).

[5]In Ben-ner' s framework the only possible exception to this result occurs when there is a "network externality" among PC's members. This makes unprofitable to break the network of members to exchange inexpensive wage-labourers with expensive PC's members. Obviously, such an exception specifically depends upon the form assumed by the network externality.

degenerate into a CF. As a consequence, the only *raison d'être* of a PC can be an environment in which CFs are expected to become insolvent.[6]

The PC's instability result can easily be described as follows. Suppose a perfectly competitive PC that has optimally recruited a certain number of members m and of fixed-wage workers ℓ to maximize its per member value added:

$$V(m^*, \ell^*) = \frac{p \cdot y_i (m^*, \ell^*) - w\ell^* - F}{m^*} \tag{4.1}$$

where p indicates the market price for a certain commodity, y_i represents the commodity itself produced by every i-th firm $(i = 1, ..., n)$ with the use of members and fixed-wage workers, w is a given market wage and F a fixed cost. Now, by dismissing one member and hiring one fixed-wage worker (assumed equally productive) to replace him, each remaining member gets the following:

$$V(m^* - 1, \ell^* + 1) = \frac{p \cdot y_i (m^* - 1, \ell^* + 1) - w(\ell^* + 1) - F}{(m^* - 1)} \tag{4.2}$$

By straightforward manipulations of (4.2) the following result ensues:

$$V(m^* - 1, \ell^* + 1) = V(m^*, \ell^*) + \frac{V(m^*, \ell^*) - w}{(m^* - 1)} \tag{4.3}$$

It is obvious from expression (4.3) that the member-reducing strategy is convenient as long as $V(m^*, \ell^*) - w > 0$, that is, the value added of each member is higher than the current market wage w. Furthermore, since $V(m^*, \ell^*) = w + \frac{\pi}{m^*}$, where π represents every CF's profit, it turns out that as long as every CF is profitable, $V > w$ and every PC is unstable against the above mentioned process of members' substitution.

[6]This is contained in Miyazaki (1984), proposition 1, p.917. The author also extends his analysis to include the possibility of both an incomplete insurance and an imperfect capital market.

However, usually this argument does not take into account the behaviour of a member after his dismissal. In fact, in an environment in which the people associated with a PC have developed an entrepreneurial skill and barriers to entry are relatively low, a dismissed member may have the opportunity to set up a new firm either as entrepreneur, self-employed or member of a newly created PC, rather than be unemployed. In which case, every PC's payoff should include such a new entrant effect, even when just one member has been dismissed.

Two comparative statics exercises can be performed as a consequence of this observation. In the first it can be assumed that, although the dismissed member sets up a new firm in the same industry, the subsequent market price change does not affect the levels of m^* and ℓ^* optimally decided by the PC before the new firm's entry. Hence, the substitution of one member with one fixed-wage worker affects every remaining member' s payoff as follows:

$$V(m^* - 1, \ell^* + 1) = \frac{p(Y^*(n+1))\, y_i(m^* - 1, \ell^* + 1) - w \cdot (\ell^* + 1) - F}{(m^* - 1)} \tag{4.4}$$

where $p(Y^*(n+1))$ is the market clearing price when a new firm has entered the market. Expression (4.4) can easily be rewritten as:

$$V(m^* - 1, \ell^* + 1) = V(m^*, \ell^*) + \frac{V(m^*, \ell^*) + \Delta R^*(n) / \Delta n - w}{(m^* - 1)} \tag{4.5}$$

where $\Delta R^*(n) / \Delta n$ represents the change in the firm's equilibrium revenue as due to the new firm's entry in the industry.[7]

In this case, even under positive profitability of a twin CF, the usual PC's instability does not necessarily arise. In general, in a market with n identical (PC) firms, a change

[7] A market price reduction has traditionally the effect on existing PCs to decrease ℓ^* and increase m^*. However, expression (2.5) implies an out-of equilibrium reduction of 1 unit for m^* and a rise of 1 unit for ℓ^*,with obviuosly no effect on $y_i(m^*, \ell^*)$.

in the firm's equilibrium revenue $R^*(n) = p(Y^*(n))\, y_i^*$ as due to the entry of new firms can be expressed as:

$$\frac{dR^*(n)}{dn} = \frac{dP(Y^*(n))}{dY^*} \cdot \frac{dY^*}{dn} \cdot y_i^* + P(Y^*(n)) \frac{dy_i^*}{dn} = \frac{dY^*}{dn} \cdot \frac{P(Y^*(n))}{n} \left(1 - \tfrac{1}{\eta}\right) \quad (4.6)$$

where η indicates the industry demand elasticity. Expression (4.6) shows that if the industry demand elasticity is (in absolute value) less than 1, new firms' entry, by decreasing the price, reduce as well the revenue of every PC in the market. Furthermore, the lower the number of firms in the market, the stronger will be the negative entry effect of a dismissed member on every PC's revenue and consequently more unlikely will be the phenomenon of PC members' substitution. More precisely, for a member's dismissal to be surely profitable, it must be that, for $\eta < 1$:

$$V(m^*, \ell^*) + \Delta R^*(n) / \Delta n - w > 0 \quad (4.7)$$

It is easy to see that, for $\Delta R^*(n) / \Delta n < 0$, the condition above may fail to hold also for $V > w$.

A second comparative statics exercise can be performed by taking into account the whole change caused by the replacement of one member with one fixed worker. After this happens, every PC's payoff can be expressed as:

$$V(m^*(n+1), \ell^*(n+1)) = \frac{p(Y^*(n+1))\, y_i(m^*(n+1), \ell^*(n+1)) - w\ell^*(n+1) - F}{(m^*(n+1))} \quad (4.8)$$

where respectively $m^*(n+1)$ and $\ell^*(n+1)$ represent the new equilibrium number of members and workers after the new firm's entry. The following example shows that expression (4.8) can be lower than the original payoff $V(m^*, \ell^*)$ even for $\eta \geq 1$.

Example. *Let* $P(Q) = a - Y$ be *the inverse market demand function. Let every LMF's production function be equal to* $y_i = \min\left\{\frac{\sqrt[3]{\ell}}{\sqrt[3]{\alpha}}, \frac{\sqrt[3]{\ell}}{\sqrt[3]{\beta}}\right\}$, *where* α *and* β *are parameters belonging to the interval* $(0,1)$. *It ensues that:*

$$V(m^*(n), \ell^*(n)) = \frac{\left(\sqrt{a^2 - 6Fn}\right)(2a^2 - 3Fn) - 3aFn + 2a^2 - 27wF^2\beta}{27F^2\alpha}$$

The expression above is monotonically decreasing in n, *and the equilibrium demand elasticity* η *is greater than 1. Therefore, in this case, also for* $\eta > 1$, *existing LMFs do not have any incentive to substitute members with fixed-wage workers.*

The next proposition generalizes the instability result under the entry threat of every dismissed member.

Proposition 10 *Under a given market wage for workers, sufficient conditions for every PC to conveniently dismiss a member and hire a fixed wage-earner are, for* $\eta \leq 1$ *that:*

$$[R^*(n+1) - w\ell^*(n+1) - F]\, m^*(n) > [R^*(n) - w\ell^*(n) - F]\, m^*(n+1)$$

and, for $\eta > 1$ *that:*

$$R(n+1)\, m^*(n) > R(n)\, m^*(n+1)$$

Proof. This simply follows by straightforward manipulations of expressions (4.1) and (4.8). ■

The proposition shows that, under the threat of a new firm's entry, the instability of a PC is not granted anymore. The argument does not apply to the case of members' retirement, in which usually there is not formation of new firms. Similarly, high barriers to entry in the market and high individual members' risk aversion, by reducing the chance of the creation of new companies, also weaken the relevance of the argument. Perhaps thus, it is not so casual that usually PCs populate industries characterized by significantly high skilled workforce (with an easily transferable human capital) and low entry barriers.

4.3 Endogenous wage and firm stability

It has been stressed that "the different choices of labour, and therefore output in the short run (between a PC and a CF), are attributable to the different decision-making problems used to represent the two organizational forms, i.e., the wage is exogenous. to the CF but endogenous and higher in the PC." (Bonin, Jones and Putterman (1993), p.1298). The analysis presented in this section seeks to remove the above asymmetric treatment adopted for CF workers and PC members' compensation, so as to make both endogenous. If a PC has to be judged for its stability, why should not the same criteria apply to a CF? In a highly competitive industry there might be reasons why very low wages are not sustainable by the existing CFs. Were the current wage lower than every worker's outside option, employees would start leaving their workplace and set up new firms either as entrepreneurs, self-employed or members of new PCs. By taking this point of view, an endogenous equilibrium wage for a given industry (a wage for which a CF is stable) can be described. As a result, it is under this equilibrium wage that PC's birth and stability has to be tested against alternative forms of organization.

This section models a standard n-profit maximizing capitalistic firms' market in which every identical firm produces a homogeneous good y with the use of labour ℓ and a given setup cost F.[8] The basic feature of the model that drives its particular

[8] It may well be possible to start with a mixed market in which a certain number of CFs and PCs coexist togheter. However, a logical objection would be: why have PCs been created if they are not stable? Since the main paper concern is precisely the PCs' stability, it seems more reasonable to start with a given number of CFs in the market in order to check if room potentially exists for the subsequent formation and survival of PCs.

results is given by the nature of workers' entrepreneurial skill and by the firm-worker relationship. What is assumed is basically that, once recruited as employees by a firm, workers acquire a specific skill (for instance a certain knowledge on how to organize the production) that *potentially* enables them to set up new production units through different organizational forms. The common knowledge of this possibility gives rise to a *concrete* outside option for every firm's employee. This outside option is taken into account by every firm's owner when fixing the wage. The employees' exit and the consequent creation of new firms can in fact lower existing firms' profitability. Under these circumstances, a PC can be set up by a group of workers only when the newly created PC's expected value added is at least equal to the equilibrium workers' remuneration, endogenously determined within the industry. Moreover, for a new PC to be a stable form of organization, its payoff has to be stable against a process of further members' dismissal and creation of new firms in the market.

4.3.1 The equilibrium wage in a capitalistic industry

Let us assume n profitable capitalist firms existing in a given product market. After bearing a given fixed cost let every firm need a certain quantity of labour to produce just one commodity. Given this simple framework, a level of wage having certain features of stability can be introduced. The conditions required to characterize an equilibrium wage can be described as follows:

$$w(n) \geq p(Y^*(n+1))\, y^*_{CF}(n+1) - w(n+1)\, \ell^*_{CF}(n+1) - F \tag{4.9}$$

$$w(n) \geq v + \frac{p(Y^*(n+1))\, y^*_{PC}(n+1) - F}{\ell^*_{PC}(n+1)} \tag{4.10}$$

$$w(n) \geq p(Y^*(n+1))\, y_{SE} - F \tag{4.11}$$

$$p\left(Y^*\left(n\right)\right)y^*_{CF}\left(n\right) - w\left(n\right)\ell^*_{CF}\left(n\right) - F \geq w\left(n\right) \tag{4.12}$$

where respectively $y^*_{CF}\left(n+1\right)$ and $\ell^*_{CF}\left(n+1\right)$ represent the quantity of good and the number of workers optimally selected by a newly set up capitalist firm, $y^*_{PC}\left(n+1\right)$ and $\ell^*_{PC}\left(n+1\right)$ the quantity and the workers chosen by a new entrant producer co-operative and $p\left(Y^*\left(n+1\right)\right)$ is the correspondent market price when one more firm enters the market. As long as the wage respects the above conditions, none of the hired workers will find convenient to leave the firm to set up new firms as entrepreneurs (4.9), PC's members (4.10) or self-employed (4.11). Notice that the third condition concerns the self-employed option, in which one worker produces alone a given output y_{SE}. The assumption, in each of conditions above, is that when employees leave from a firm, the latter is assumed to recruit one or a group of unemployed to replace them. This implies that a pool of unemployed workers exist in equilibrium, even if, when not hired by a firm, their outside option is just equal to v.[9] Moreover, condition (4.12) ensures that none of n entrepreneurs earn less than his hired workers, otherwise it would become convenient to let workers go and set up a new firm under a lower wage. Denoting the wage obtained by solving expressions (4.9), (4.10) and (4.11) respectively as $w_{I,}w_{II}$ and $w_{III,}$ the next definition simply illustrates the fact that under the above framework the equilibrium wage needs to respect a few basic conditions.

Definition 4 *For each number n of CFs initially existing in the market, the equilibrium wage paid by every firm must respect the following conditions:*

$$\frac{p(Y^*(n))y^*_{CF}(n)-F}{\left(1+\ell^*_{CF}(n)\right)} \geq w^*\left(n\right) = \max\left\{w_{I,}w_{II}, w_{III}\right\} \geq v \tag{4.13}$$

[9]The existence of a group of unemployed workers can be made endogenous whenever $w^*\left(n\right) > v$, where v is the given reservation wage of the economy. Furthermore, every entrepreneur is assumed to behave *à la* Nash, in the sense that any other' s strategy (for instance which of the other recruited workers) is taken as given during the negotiation with every employee.

Explanation. Given the existing number of firms, the equilibrium wage must respect the RHS of expression (4.13), otherwise every employee, given the behaviour of other employees, would prefer to leave the firm. The LHS of (4.13) is the solution of (4.12) and simply ensures that every CF's entrepreneur will never prefers to set up a new firm. This corresponds to the condition, $\pi(w^*(n)) \geq w^*(n)$. When this expression holds, since by (9), $w^*(n) \geq w_I$, it immediately follows that

$$\pi_i^*(w^*(n), n) \geq \pi_i^*(w^*(n+1), n+1),$$

where $\pi_i^*(w^*(n+1), n+1)$ is every firm's profit after one more firm has entered the market (also affecting the equilibrium wage). Thus, given n firms, this condition makes the equilibrium consistent: every CF will always try to keep its employees inside the firm through the payment of the equilibrium wage. Finally, for the workers to participate in production, the equilibrium wage must never be lower than the reservation wage v payable in the economy.

4.3.2 Birth and stability of a PC

A direct consequence of the simple definition introduced above is that, as long as the equilibrium industry wage respects all the stability conditions, PCs can be formed only under particular circumstances. These are expressed in the next proposition.

Proposition 11 *Under the industry equilibrium wage $w^*(n)$ all existing active workers are indifferent between being members of a new PC and being employees of an existing CF only if the following condition holds:*

$$w^*(n) = w_{II} \geq \max\{w_I, w_{III}\} \tag{4.14}$$

When, conversely, $w^*(n) = w_I > w_{II}$ or $w^*(n) = w_{III} > w_{II}$, every worker will prefer to be employee, CF entrepreneur or self-employed rather than form a new PC.

Proof. (Self-evident) In all subcases included in expression (4.14) workers are indifferent between being employed in a CF or being members of a newly created PC. Thus, their choice will mainly depend upon their personal preferences. When, conversely, $w^*(n) = w_I > w_{II}$, workers will be indifferent between being entrepreneurs in a new CF (using unemployed workers paid their current equilibrium wage $w^*(n+1)$) and being employees for an existing CF. However, since $w^*(n) > w_{II}$ any worker or group of workers will have incentives to set up a PC. The same situation arises (with the self-employment option) when $w^*(n) = w_{III} > w_{II}$. Finally, since unemployed workers do not possess the specific knowledge required to set up a firm, their only possibility is to be inactive, obtaining the reservation wage v (represented, for instance, by a public subsidy). ■

The above proposition describes a market in which employees' threat to leave their firm and become direct competitors affects their final equilibrium wage. This framework can suitably describe only those industries in which existing firms require both highly trained workers and relatively low setup costs to produce a commodity. The next example constitutes a specific application of the results of proposition 11.

Example 1 *Suppose, as before, an inverse market demand function isoelastic and equal to* $P(Y(n)) = AY^{-\eta}$, *with* $\eta = 1$, *where* $Y = \sum_{i=1}^{n} y_i$, *and* y_i *is the quantity produced by every identical CF in the market. Furthermore, let each CF's production function be* $y_i = \sqrt{\ell}$ *and let* F *be the fixed cost required to start the production. By condition (3.6) included in definition 1, the equilibrium wage must respect the following conditions:*

$$w_{IV} \geq w^*(n) = Max\{w_I, w_{II}, w_{III}\} \geq v$$

where $w_{IV} = \frac{p(Y^*(n))y^*_{CF}(n)-F}{\left(1+\ell^*_{CF}(n)\right)}$ *. By straightforward calculations, the following expressions ensue:*

$$w_I = \frac{A}{2(n+1)} - F \tag{4.15}$$

$$w_{II} = \frac{v}{2AFn - \frac{1}{n} - 1} \tag{4.16}$$

$$w_{III} = \sqrt{\frac{A - F\left(n+1\right) + \sqrt{A^2 - 2AF(n+1) + F^2}}{(2+n)}} \tag{4.17}$$

$$w_{IV} = \frac{A}{2n} - F. \tag{4.18}$$

In figure below, these values are plotted against different number of CFs assumed to exist in the market. It can be noticed that, for $n = 16$, $w_{IV} > w_{II} \geq w_{III} > w_I$. Hence, given this initial number of firms, by definition 4 it follows that $w^*(n) = w_{II}$ and, by proposition 9, that existing employees have incentives to form PCs whenever, for this level of earnings, they prefer to be PC's members than be employees for a CF.

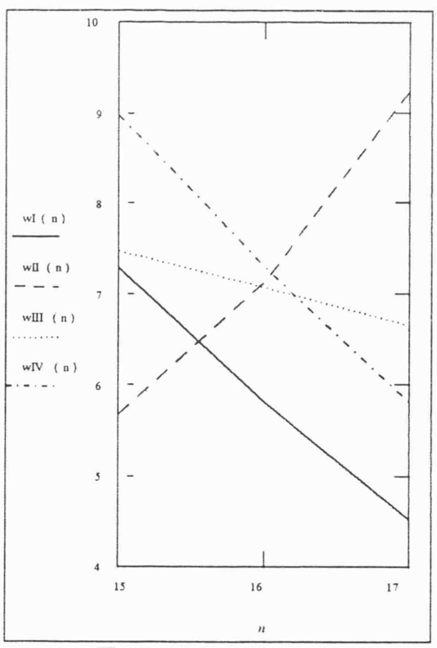

Fig. 4.1 - Levels of the different workers' outside options for n=15,..17 (A=800, v=1, F=17.5).

Figure 4.1 shows that w_{II} (the PC's type of outside option) is the only outside option increasing with the number of CFs supposed to exist in the market. The reason is that PCs react to price reductions (due to the higher number of firms) with an increase in the number of members and hence with a rise in the output. In the above example, with unitary elasticity, this effect always rises every PC's payoff.

We can now turn to the stability of every newly formed PC. In a framework like the one described above, when a PC is set up it can also be proved to be stable against the subsequent process of substitution of members with spot labour market workers in the sense of Ben-ner (1988) and Myizaki (1988). In fact, although at a first glance it may look profitable to exchange existing members with unemployed workers, once trained, the newly recruited workers have to be paid the current equilibrium wage. That a newly formed PC can be stable, however, needs to be proved. This is done in the next proposition.

Proposition 12 *When new PCs are set up by trained workers leaving an existing CF, they are always stable against the temptation to reduce their membership through the recruitment of available unemployed workers as long as the market elasticity $\eta \leq 1$. When, conversely, $\eta > 1$, PC's stability is guaranteed only under specific conditions.*

Proof. Proposition 9 showed that new PCs can only be formed under condition (4.14). Thus, if an unemployed worker is hired by a newly created PC to substitute a member, this worker will become trained and will threaten to leave unless paid the new equilibrium wage. By the results of the model, this wage will be greater or equal to $w_{II}(n+1)$. Thus, to decide whether or not a PC is stable against the process of members' dismissal it has to be checked that:

$$\frac{P(Y^*(n+2))\, y_{PC}(n+2) - w_{II}(n+1) - F}{m^*(n+2)} \leq \frac{P(Y^*(n+1))\, y_{PC}(n+1) - F}{m^*(n+1)} \tag{4.19}$$

When $\eta \leq 1$ it follows that, in terms of revenues, $R_{PC}(n+2) \leq R_{PC}(n+1)$. Moreover, in general, $m^*(n+2) > m^*(n+1)$. Hence, for $\eta \leq 1$, expression (4.19) holds with strict inequality even for $w_{II}(n+1) = 0$, so that PC's stability is always ensured as a result. For $\eta > 1$,in order to undermine PC's stability the rise in revenue due to every member substitution must prevail over a sum of negative effects. Specifically, straightforward manipulations show that PC's stability is preserved as long as:

$$[R_{PC}(n+2) - F - w_{II}(n+1)]\, m^{*}(n+1) \leq [R_{PC}(n+1) - F]\, m^{*}(n+2) \qquad (4.20)$$

Thus, under the existing equilibrium wage, if new recruited workers are not more productive than existing members it turns out that only under specific conditions (that represented in expression (4.20)) PCs do not find profitable to substitute members with fixed-wage workers. ■

In example 2, since $\eta = 1$, it can be checked that,

$$P(Y^{*}(n+2))\, y_{PC}(n+2) = P(Y^{*}(n+1))\, y_{PC}(n+1)$$

and, since, in general, $p(Y^{*}(n+2)) < P(Y^{*}(n+1))$, it ensues that:

$$\begin{aligned} m^{*}(n+2) &= (y^{*}_{PC}(n+2))^2 = \left(\tfrac{2F}{p(Y^{*}(n+2))}\right)^2 > \\ m^{*}(n+1) &= (y^{*}_{PC}(n+1))^2 = \left(\tfrac{2F}{p(Y^{*}(n+1))}\right)^2 \end{aligned}$$

Hence, condition (4.20) is always respected with strict inequality.

In general, proposition 10 helps to see that, in a given industry, the presence of an endogenous equilibrium wage makes PCs more robust against the internal instability process.

4.3.3 Other forms of instability: an example

The model presented above has described a situation in which, once workers become trained and ready to set up new companies, a certain equilibrium wage can endogenously be obtained. However, when a change takes place in the existing market conditions, it may well be possible that other forms of PCs' instability enter the scene. The next example describes a situation in which an exogenous shock in the market demand gives rise to a switch in the equilibrium wage $w^*(n)$ from w_{II} to w_{III}. When this happens, the newly created PC's members can suddenly find convenient either to become employees for existing CFs (with the further advantage for CFs to get rid of some of existing PCs and hence reduce the market competition), or, alternatively, to become self-employed. This is just one example of instability usually not considered in the traditional models of labour-managed firms.

Example 2 *Adopting the same model specification introduced above, let A, the demand size parameter, be subjected to an exogenous shock. The figure below depicts the effects of an increase in A, from $A = 860$ to $A' = 920$, for a given number of existing firms* $(n = 15)$ *and for certain values of parameters* $(F = 17.5,\ v = 2.5)$.

The figure 4.2 shows that, as long as $860 \leq A \leq 865$, the equilibrium wage $w^*(n)$ is equal to w_{II} and the entry of new PCs in the market has then been viable. However, a small change in A (such to make $A > 865$) makes incumbent PCs potentially unstable. Under the new value of A, the equilibrium wage $w^*(n)$ becomes either equal to w_{III} or w_I and hence PCs' members find now attractive either to be hired as employee by existing CFs or to set up new units of production respectively as self-employees or entrepreneurs.

That represented is just one of the possible forms of instability that a PC can suffer during its life-cycle. Similar examples can certainly be found, showing that, even in a very simple setup, the economics of companies institutional changes is indeed a very complex matter.

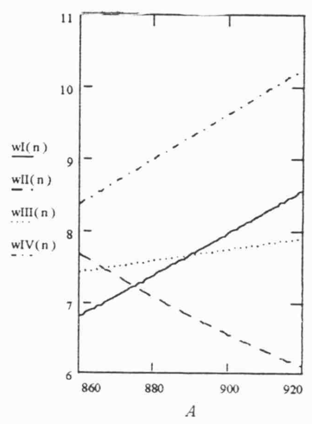

Fig.4.2 - Levesl of workers' outside options for A=860..920, (n=15, v=2.5, F=18.5).

4.4 Concluding Remarks

This chapter has shown that the usual argument for Producer Co-operatives' instability may not hold if an active (rather than passive) behaviour is assumed on the part of every member after their dismissal. Whether this is a reasonable description of a member's behaviour is probably dependant on the specific context in which a firm is assumed to sell its product. In markets characterized by low entry barriers and a highly skilled workforce, dismissed members are probably able to set up new production units rather than be inactive. If this is the case, the traditional conclusions on Producer Co-operatives's instability have to be corrected and further analysis is needed to draw conclusive results on the general robustness of this form of enterprise. Moreover, since labour-managed forms of organization are often observed in markets characterized by a highly skilled workforce, low entry barriers and high potential competition (e.g., in fisheries, potteries or services industries), assuming an endogenously determined workers' compensation does not seem too unrealistic for these markets. The model has shown that under such circumstances, Producer Co-operatives' instability is even less likely to arise. However, other sources of instability then become possible. Particular instances of instability can occur as a consequence of exogenous shocks that, by changing the current equilibrium wage, may undermine the internal stability of newly formed Producer Co-operatives.

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Chapter 5

The Core of Economies with Stackelberg Leaders

5.1 Introduction

The traditional representation of cooperative games with transferable utility is based on a "characteristic" function, specifying for each coalition the amount of utility that its members can ensure themselves in the underlying normal form game. This formulation is meant to isolate coalitional decisions, abstracting from the strategic complexity of the cooperation process. However, unless the payoffs of the members of a coalition and of its complement are independent (orthogonal games) or opposite (constant sum games), the characteristic function fails to be well defined[1]. Indeed, this is the case of many meaningful strategic situations, in which the payoff of each player may generally depend on the strategies of all players in the game. In such cases, the characteristic function can still be well defined by introducing some assumptions on the strategies of players in the complementary coalitions (the "outside players").

One way to deal with this problem, first proposed by von Neumann-Morgenstern (1944) and considered by Aumann (1967), is to assume that outside players coordinate

[1]In Shubik (1982) terminology, the game is not a c-game.

their strategies to minimize the aggregate payoff of the forming coalition. A temporal structure is implicitly introduced in the players' choice of strategies. In the so called α-core, the forming coalition acts as a leader, and chooses its best strategies, given the minimizing behaviour of outside players; in the β-core, conversely, it behaves as a follower, and maximizes its payoff given the coordinated strategies of outside players. Since in both cases deviations are very costly, α and β-core are usually very large. Moreover, still fulfilling a rationality requirement in constant sum games, α and β-assumptions do not seem justifiable in most economic settings[2].

An alternative approach proposed by Aumann (1959) extends Nash Equilibrium "-passive" expectations to the cooperative framework. The concept of strong equilibrium defined by the author assumes that deviating coalitions take as given the strategies of outside players. Being immune from the deviations of any coalition, thus including the grand coalition and every individual player, strong equilibria are both Nash equilibria and efficient strategies. However, since in games with positive externalities the efficient strategies of excluded players make coalitional deviations "too" profitable, strong equilibria do not exist for many economic problems.

In the contest of some recent economic applications, a different approach has proved useful in ensuring a non-empty core without making use of extreme assumptions on the behaviour of outside players such as the α and β conjectures. This approach, named γ-approach by Chander-Tulkens (1997), assumes that outside players neither jointly minimize the payoff of a deviating coalition (as in the α and β-core), nor keep their strategies fixed (as in the Strong Nash Equilibrium), but they rather maximize their own utility as singletons. Here, the behaviour of deviating players and which of outside players is implicitly assumed to develop in two stages. In the first stage, similarly to the Γ game by Hart and Kurtz (1983),[3] a coalition forms and the excluded players split

[2]Indeed, in costant sum games, the α-core coincides with the *modified characteristic function* proposed by Harsanyi (1959), assigning to each coalition the solution of the variable threats Nash bargaining problem with the respective complementary coalition.

[3]The Γ game is indeed a strategic coalition formation game with fixed payoff division, in which the strategies consist of the choice of a coalition. Despite the different nature of the two games, there is an analogy concerning the coalition structure induced by a deviation from the grand coalition. In the Γ game, any deviation from the the grand coalition's strategy profile induces a coalition structure in which the deviating coalition stay together and the outside players split up.

up as singletons; in the second stage, members of the deviating coalition and excluded players simultaneously choose their strategies in the underlying normal form game, *given* the specific coalition structure originated in the first stage. Consequently, the strategy profile induced by the deviation of a coalition $S \subset N$ is the Nash equilibrium among S and each individual player in $N \backslash S$.

In this chapter we modify the γ-assumption by removing this two stage structure and reintroducing the temporal sequence in the choice of players' strategies *in the underlying normal form game,* typical of the α and β-core. We assume that the formation of a coalition and the choice of a coordinated strategy by its members in the underlying game are two simultaneous events, that can be thought of as a unique action. When a set of players form a coalition, at the same time they choose a coordinated strategy, taking as given the (non-cooperative) reaction of the excluded players as singletons. In this respect, deviating coalitions possess a first mover advantage with respect to the outside players. We thus associate with the deviation of every coalition S the Stackelberg equilibrium in which S acts as leader and players in $N \backslash S$ play (individually) as followers.

According to this assumption, we define a modified version of the γ-core, denoted ϕ-core. We then show how some recent applications of the γ-core to oligopolistic markets and public goods production problems are affected by our assumption. For the linear oligopoly case, we prove that, although the γ-core is very large, the only allocation in the ϕ-core is the equal split allocation. For the linear-quadratic oligopoly, conversely, we show that, differently from the γ-core, the ϕ-core is empty. For the case of public goods production, we consider a simple economy with one public and one private good, and we discuss the validity of Chander and Tulkens (1997) result of non-emptiness of the γ-core. We consider the case of symmetric agents, and show that if preferences are linear in the public good, then the allocation the authors propose belongs to the ϕ-core. However, if preferences are strictly concave, the ϕ-core is shown to be empty for the specific case of quadratic utility and quadratic cost.

5.2 The general set-up

Let $\Gamma = \left(\{X_i, u_i\}_{i \in N}, \{X_S\}_{S \subseteq N}\right)$ be a strategic form game, where N is the (finite) players set, X_i is the strategy set of player i, and X_S is the strategy set of a coalition of players S.[4] Let $P(N)$ be the set of all possible partitions π of the players set N; let X_π denote the set $\prod\limits_{T \in \pi} X_T$, for any $\pi \in P(N)$. The set $X \equiv \bigcup\limits_{\pi \in P(N)} X_\pi$ is the set of all possible outcomes (in terms of strategies) of the game Γ. The function $u_i : X \rightarrow R_+$ represents players' preferences. We restrict our attention to transferable utility functions u_i.

Definition 5 *A Nash Equilibrium of* Γ *is a strategy profile* $\overline{x}$ *such that, for all* $i \in N$, $\overline{x}_i \in X_i$ *and, for all* $x_i \in X_i$, $u_i(\overline{x}) \geq u_i(x_i, \overline{x}_{-i})$.

5.2.1 The value function under the γ-assumption

The γ-assumption postulates that the worth of a coalition is the aggregate utility of its members in the Nash equilibrium between that coalition (acting as a single player) and the outside players (acting as singletons). The value function $v_\gamma(S)$ is defined for all $S \subseteq N$ by:

$$v_\gamma(S) = \sum_{i \in S} u_i \left(\hat{x}_S, \{\hat{x}_j\}_{j \in N \setminus S}\right) \tag{5.1}$$

where,

$$\hat{x}_S = \arg\max_{x_S \in X_S} \sum_{i \in S} u_i \left(x_S, \{\hat{x}_j\}_{j \in N \setminus S}\right) \tag{5.2}$$

and, $\forall j \in N \setminus S$,

$$\hat{x}_j = \arg\max_{x_j \in X_j} u_j \left(\hat{x}_S, \{\hat{x}_k\}_{k \in (N \setminus S) \setminus \{j\}}, x_j\right). \tag{5.3}$$

[4] Note that, in general, X_S may not coincide with the set $\prod\limits_{i \in S} X_j$.

Definition 6 *The joint strategy $\widehat{x} \in X_N$ is in the γ-core, if there exists no coalition S such that $v_\gamma(S) > \sum_{i \in S} u_i(\widehat{x})$.*

5.2.2 The value function under the ϕ-assumption

The new value function we introduce is based on the assumption that deviating coalitions exploit a first-mover advantage. As under the γ-assumption, when a coalition S forms, players in $N \backslash S$ split up as singletons. Differently from the γ case, the members of S choose a coordinated strategy as leaders, thus anticipating the reaction of the players in $N \backslash S$, who simultaneously choose their best response as singletons. The strategy profile associated to the deviation of a coalition S is the Stackelberg equilibrium of the game in which S is the leader and the players in $N \backslash S$ are, individually, the followers. We denote this strategy profile as a *partial equilibrium* with respect to S. Formally, this is the strategy profile $\tilde{x}(S) = (\tilde{x}_S, x_j(\tilde{x}_S))$ such that

$$\tilde{x}_S = \underset{x_S \in X_S}{\arg\max} \sum_{i \in S} u_i \left(x_S, \{x_j(x_S)\}_{j \in N \backslash S} \right) \tag{5.4}$$

and, $\forall j \in N \backslash S$,

$$x_j(x_S) = \underset{x_j \in X_j}{\arg\max}\, u_j \left(x_S, \{x_k(x_S)\}_{k \in (N \backslash S) \backslash \{j\}}, x_j \right). \tag{5.5}$$

We first establish sufficient condition for the existence of $\tilde{x}(S)$.

For every coalition $S \subset N$ and strategy profile $x_S \in X_S$, we define the restriction $\Gamma(N \backslash S, x_S)$ of the game Γ to the set of players $N \backslash S$, given the fixed profile x_S.

Proposition 13 *Let Γ be a strategic form game. For every $S \subset N$ and $x_S \in X_S$, let the game $\Gamma(N \backslash S, x_S)$ possess a unique Nash Equilibrium. For every $S \subset N$, let X_S be compact. Let each player's payoff be continuous in every other player's strategy. Then,* *for every $S \subset N$, there exists a partial equilibrium of Γ with respect to S. Moreover, if payoffs are strictly concave in each players strategy, such a partial equilibrium is unique.*

Proof. By condition (5.5), the strategy profile $\{x_j(x_S)\}_{j\in N\backslash S}$ is the unique Nash equilibrium of $\Gamma(N\backslash S, x_S)$. By the closedness of the Nash equilibrium correspondence (see, for instance, Fudenberg and Tirole (1991), pag.30), members of S maximize a continuous function over a compact set (condition (5.4)); thus, by Weiestrass Theorem, a maximum exists. Uniqueness comes as a straightforward consequence of the strict concavity of the leader's maximization problem. ■

We can thus define the value function $v_\phi(S)$ as follows:

$$v_\phi(S) = \sum_{i\in S} u_i\left(\widetilde{x}_S, \{x_j(\widetilde{x}_S)\}_{j\in N\backslash S}\right). \tag{5.6}$$

Definition 7 *The joint strategy $\widetilde{x} \in X_N$ is in the ϕ-core, if there exists no coalition S such that $v_\phi(S) > \sum_{i\in S} u_i(\widetilde{x})$.*

In the next to sections we apply the concept of ϕ-core to two widely studied economic problems: cartel formation in oligopolies and resource allocation in economies with public goods.

5.3 Cartel formation in oligopoly

In recent years there has been a renewed interest in the application of cooperative solution concepts to the problem of cartel formation under oligopoly [see, for a survey, Bloch (1997)]. A specific use of the γ-core is contained, for instance, in Rajan (1989). The author shows that in a symmetric Cournot oligopoly with linear demand and quadratic costs, for a number of firms $n \geq 3$, firms never chose to stay separate (i.e., giving rise to the coalition structure $\{1, 1, ..., 1\}$); moreover, it is proved that, for $n \leq 4$, the γ-core is non empty.

In what follows, after a short description of the Cournot setting, we first show that, in a symmetric oligopoly with *linear* demand and *linear* costs, the γ-core strictly includes the equal split allocation for any number of firms. For the same model specification we then prove that the equal split allocation is the *unique* allocation contained in the ϕ-core. Finally, we show that, when costs are *quadratic,* the ϕ-core can be empty.

5.3.1 The Cournot setting

Let $\pi_i(y, y_i) = p(y)\, y_i - C_i(y_i)$ be the profit function of every firm $i \in N = \{1, 2, ..., n\}$, where y_i is the output of a firm, $y = \sum_{i=1}^{n} y_i$ the total output, $p(y)$ the usual inverse demand function and $C_i(y_i)$ the cost function of every firm. Let also $C_i(.) = C_j(.)$, for every i, j in N.

We introduce the following standard assumptions:

A.1 The function $\pi_i(.)$ is twice continuously differentiable;

A.2 For every firm i, the capacity constraint $\overline{y}_i < \infty$ determines the maximum production level;

A.3 $p''(.)\, y_i + p'(.) < 0$ and $p'(.) - C_i'' < 0$.

Consistently with Section 2, we now define the normal form game, denoted as Γ_1, associated to our problem. Each player (firm) strategy set is:

$$X_i = \{y_i \in R_+ : y_i \leq \overline{y}_i\} \equiv Y_i. \tag{5.7}$$

Players' preferences are linear in profit and, for every coalition S, the strategy set is represented by:

$$X_S \equiv \left\{ (y_S, t_S) : y_S \in \prod_{i \in S} Y_i, \text{ and } t_S = (t_1, ..., t_s), \text{ such that } \sum_{i \in S} t_i = 0 \right\} \tag{5.8}$$

where t_S is a vector of transfers.

Proposition 14 *There exists a unique Nash equilibrium of the game* Γ_1.

Proof. By A.1, every player's payoff functions is continuous in the strategy profile $y \in Y_N$ and, by A.3, strictly concave on y_i. By A.2, strategy sets are non empty, compact and convex, so that existence of a Nash equilibrium follows. Uniqueness is implied by A.3 as follows. Since, for each firm, $p''y_i + p' < 0$ and $p' - C_i'' < 0$, the function $F(y_i, y) \equiv p'y_i + p - C'$ is decreasing both in y_i and y. In fact, $\frac{\partial F(y_i,y)}{\partial y_i} = p' - C_i'' < 0$ and $\frac{\partial F(y_i,y)}{\partial y} = p''y_i + p' < 0$. Suppose now that there exist two Nash Equilibria y^1 and y^2 of Γ_1. Suppose also, without loss of generality, that $y^1 > y^2$. At a Nash Equilibrium, $p'y_i + p - C_i' = 0$, so that, if $\sum_{i=1}^{n} y_i^1 > \sum_{i=1}^{n} y_i^2$, it follows from A.3 that $y_i^1 < y_i^2$ for every $i = 1, .., n$, leading to a contradiction. ■

5.3.2 The γ-core

By applying the definition of $v_\gamma(S)$ to the Cournot setting introduced above, we obtain the following expression:

$$v_\gamma(S) = \sum_{i \in S} \left[p(\hat{y}_S, \hat{y}_{-S}) \hat{y}_i - C_i(\hat{y}_i) + \hat{t}_i \right] \tag{5.9}$$

where

$$\hat{y}_S = \arg\max_{y_S \in Y_S} \sum_{i \in S} \left[p(y_S, \hat{y}_{-S}) y_i - C_i(y_i) + \hat{t}_i \right] \tag{5.10}$$

and where $\widehat{t_i}$ is the equilibrium lump-sum transfer for every $i \in S$, and

$$\hat{y}_j = \underset{y_j \in Y_j}{\arg\max}\, p\left(y_j, \hat{y}_S, \underset{\substack{k \neq j \\ k \notin S}}{\hat{y}_k} \right) y_j - C_i\left(y_j\right), \quad \forall j \in N \backslash S. \tag{5.11}$$

By A.1, we can differentiate $v_\gamma\left(S\right)$ and, by symmetry of players, the strategy profile $\hat{y} \in Y_N$ characterizing $v_\gamma\left(S\right)$ is such that, for every $i \in S$, $\hat{y}_i$ respects:

$$p\left(\hat{y}\right) + p'\left(\hat{y}\right) s\hat{y}_i = C_i'\left(\hat{y}_i\right), \tag{5.12}$$

where $s = |S|$, while, for every $j \in N \backslash S$, $\hat{y}_j$ respects:

$$p\left(\hat{y}\right) + p'\left(\hat{y}\right) \hat{y}_j = C_j'\left(\hat{y}_j\right). \tag{5.13}$$

5.3.3 The ϕ-core

We now apply our equilibrium concept to the oligopolistic setting described above. According to the general setup, the function $v_\phi(S)$ is as follows:

$$v_\phi(S) = \sum_{i \in S} \left[p\left(\tilde{y}_S, \{y_j(\tilde{y}_S)\}_{j \in N \backslash S}\right) \tilde{y}_i - C_i(\tilde{y}_S) + \tilde{t}_i \right] \tag{5.14}$$

where

$$\tilde{y}_S = \underset{y_S \in Y^S}{\arg\max} \sum_{i \in S} \left[p\left(y_S, \{y_j(y_S)\}_{j \in N \backslash S}\right) y_i - C_i(y_i) + \tilde{t}_i \right] \tag{5.15}$$

and $\forall j \in N \backslash S$,

$$y_j(y_S) = \underset{y_j \in Y^j}{\arg\max}\, p\left(y_S, \{y_k(y_S)\}_{k \in (N \backslash S) \backslash \{j\}}, y_j\right) y_j - C_j(y_j). \tag{5.16}$$

Note first that, as $\sum_{i \in S} \tilde{t}_i = 0$, the function $v_\phi(S)$ is fully defined by the choice of a vector $\tilde{y}_S$ by the members of S.

Proposition 15 *There exists a unique value $v_\phi(S)$ for every $S \subseteq N$.*

Proof. We apply Proposition 13. By Proposition 14, there exists a unique Nash equilibrium for every restricted game $\Gamma_1(N \backslash S, y_S)$. Continuity of payoffs follows from A.1 and compactness of every strategy set from A.2. Moreover, by A.3 payoffs are strictly concave, so that the value $v_\phi(S)$ is unique. ■

According to the above result, under A.1 and symmetry, the FOCs characterizing $\tilde{y} \in Y_N$ are, for every $i \in S$:

$$p(\tilde{y}) + p'(\tilde{y})\, s\tilde{y}_i = C_i'(\tilde{y}_i) \tag{5.17}$$

and, $\forall j \in N \backslash S$,

$$p(\tilde{y}) + p'(\tilde{y})\, y_j(\tilde{y}_S) = C_j'(y_j(\tilde{y}_S)). \tag{5.18}$$

5.3.4 The linear case

Having defined the γ and ϕ-core for the Cournot setting, we now study the linear case, i.e. the case in which $p(y) = a - by$, and, for every $i \in N$, $C_i(y_i) = cy_i$, with $a > c \geq 0$ and $b > 0$.

Proposition 16 *Under linearity and symmetry, the γ-core of the game Γ_1 is non empty and strictly includes the equal split allocation.*

Proof. Condition (5.12) implies that:

$$v_\gamma(N) = \frac{(a-c)^2}{(2b)^2}$$

and

$$v_\gamma(S) = \frac{(a-c)^2}{b^2(n-s+2)^2}$$

where $s = |S|$ and $n = |N|$. Without loss of generality let us normalize $\frac{(a-c)^2}{b^2} = 1$, so that the equal split allocation gives to each player in N a payoff of $\frac{v_\gamma(N)}{|N|} = \frac{1}{4n}$ and $v_\gamma(S) = \frac{1}{(n-s+2)^2}$.

Consider now the equal split allocation for a coalition S, $\frac{v_\gamma(S)}{|S|} = \frac{1}{s(n-s+2)^2}$. Whatever distribution of the worth $v_\gamma(S)$ may be chosen by S, at least one player in S must get a payoff not greater than $\frac{1}{s(n-s+2)^2}$. This implies that coalition S improves upon the equal split allocation for N if and only if

$$\frac{1}{s(n-s+2)^2} > \frac{1}{4n}.$$

Straightforward calculations show that the above inequality is satisfied respectively for:

$$\begin{aligned} s &> n \\ s &< 2 + \frac{n - \sqrt{n^2+8n}}{2} < 1 \\ s &> 2 + \frac{n + \sqrt{n^2+8n}}{2} > n \end{aligned}$$

and hence, it is never satisfied for $1 < s \leq n$. It follows that the equal split allocation for N, characterized by the strategy vectors $(\widehat{y}, \widehat{t})$, where $\widehat{y}$ respects (5.12) and $\widehat{y} = (0, 0, ..., 0)$, belongs to the γ-core. To see that this allocation is strictly included in the γ-core, note that, since individual deviations assign to a player just $v_\gamma(\{i\}) = \frac{1}{(n+1)^2} <$

$\frac{v_\gamma(N)}{|N|} = \frac{1}{4n}$, different and unequal allocations belong as well to the γ-core. In particular, any allocation giving to a player i his worth $v_\gamma(\{i\})$, and $\frac{v_\gamma(N\backslash\{i\})}{|N-1|} = \frac{v_\gamma(N)-v_\gamma(\{i\})}{|N-1|}$ to any remaining player, is not objectable. ■

We now characterize the ϕ-core of the game Γ_1 under linearity and symmetry. The next proposition shows that, once deviating coalitions are allowed to exploit a first mover advantage, all allocations but the equal split one are blocked.

Proposition 17 *In a linear symmetric oligopoly the equal-split allocation is the unique allocation belonging to the ϕ-core.*

Proof. As in the proof of Proposition 16, under normalization, we get:

$$v_\phi(N) = \frac{1}{4}$$

and, from condition (5.17),

$$v_\phi(S) = \frac{1}{4(n-s+1)}.$$

Hence, straightforward calculations show that, for every $S \subset N$, $\frac{v_\phi(S)}{|S|}$ is less than $\frac{v_\phi(N)}{|N|}$ for $1 < s < n$, and equal to $\frac{v_\phi(N)}{|N|}$ either for $s = n$ or $s = 1$. It follows that, since in any deviating coalition $S \subset N$ at least one player gets a payoff less than or equal to $\frac{v_\phi(S)}{|S|}$, no coalition $S \subset N$ can make all its member better off than in the equal split allocation $\frac{v_\phi(N)}{|N|}$, which is then in the ϕ-core. To see that the equal-split is the unique allocation in the ϕ-core, note that any other allocation would require to give to at least one player less than $\frac{v_\phi(N)}{|N|}$. However, such a player could always improve his payoff by deviating and, from the result above, getting a worth equal to $v_\phi(\{i\}) = \frac{1}{4n}$. ■

5.3.5 The linear-quadratic case

We now consider the case of linear demand function $p(y) = a - y$ and quadratic cost function $C_i(y_i) = \frac{y_i^2}{2}$. As indicated above, we know from Rajan (1989) that, for $n = 2$, $n = 3$ and $n = 4$, the γ-core is non empty. We now show that this result does not hold under the ϕ-core assumption.

By conditions (5.17) and (5.18), the following result can be proved.[5]

Proposition 18 *Under linear demand and quadratic costs for every firm, the ϕ-core can be empty.*

Proof. From first order conditions, it is obtained that:

$$v_\phi(N) = \frac{a^2 n^2}{(1 + 2n)^2}$$

and

$$v_\phi(\{i\}) = \frac{a^2(a^2 + 5n - 1)}{(n+1)(n+5)^2}.$$

Simple calculations show that, for every $i \in N$, and for $n \geq 2$, $v_\phi(\{i\}) > \frac{v_\phi(N)}{|N|}$. By efficiency of the equal split solution, in any other efficient allocation at least one player would receive a lower utility. This fact together with the above result that any player can improve upon the equal split allocation by deviating as singleton, imply that any efficient allocation can be objected by the deviation of a single player. This, in turn, implies that the ϕ-core is empty. ■

[5] Which presented here is just an example that removes the assumption of functions linearity and gives rise to emptiness of the ϕ-core. It is plausible to claim that other non linear examples can be found in which the solution set (in terms of allocations), is either empty or possesses more than one allocation, respectively, as a result of the specific functions selected.

5.4 The core of a public good economy

In this section we study the ϕ-core of an economy with one private and one public good. We mostly refer to the work on γ-core by Chander and Tulkens (1997) (C-T hereafter), and show that their results carry over to the ϕ-core if and only if preferences are linear in the public good.[6]

5.4.1 The economy

We consider an economy with one public good q and one private good y. The set of agents is $N = \{1, ..., n\}$; each agent i is endowed with ω_i units of the private good, and produces the public good out of the private good with convex cost $C_i(q_i)$. For every $S \subset N$, we denote by q_S the vector $(q_i)_{i \in S}$, and by Q_S the term $\sum_{i \in S} q_i$; for simplicity, we write q instead of q_N and Q instead of Q_N. Preferences are represented by a quasilinear utility function $u_i(q, y_i) \equiv v_i(Q) + y_i$. We denote by $\pi_i(Q) \equiv \frac{\partial v_i(Q)}{\partial Q}$ the marginal rate of substitution between public and private good for player i, and for all coalitions $S \subseteq N$, we let $\pi_S(Q)$ denote the term $\sum_{i \in S} \pi_i(Q)$.

We make the following assumptions.

A.4: $v_i(Q)$ concave, twice differentiable and such that $\pi_i(Q) > 0$ for all q such that $\sum_{i \in N} C^i(q_i) \leq \sum_{i \in N} \omega_i$.

A.5: $C_i(q_i)$ strictly concave, twice differentiable and such that $C_i'(q_i) \geq 0$ for all $q_i \geq 0$ and $C_i'(q_i) = 0$ for $q_i = 0$.

We associate to this economy the normal form game denoted Γ_2, where strategy sets and preferences are as follows:

[6] Although C-T's results are obtained for an economy with pollution, they generalize to public goods economies under the assumptions made in this paper.

$$
\begin{aligned}
X_i &= \left\{(q_i, y_i) \in R_+^2 : C(q_i) + y_i \leq \omega_i\right\};\\
X_S &= \left\{(q_S, y_S) \in R_+^{2\#S} : \sum_{i \in S} C_i(q_i) \leq \sum_{i \in S} \omega_i - \sum_{i \in S} y_i\right\};\\
u_i(x) &= v_i(Q) + y_i.
\end{aligned}
$$

Proposition 19 ***(Chander-Tulkens):*** *There exists a unique Nash Equilibrium of the game* Γ_2*.*

The Nash Equilibrium $(\bar{q}, \bar{y}) = (\bar{q}_1, ..., \bar{q}_n, \bar{y}_1, .., \bar{y}_n)$ of Γ_2 is characterized by the following FOC's:

$$\pi_i\left(\bar{Q}\right) = C_i'(\bar{q}_i), \quad \forall i \in N. \tag{5.19}$$

5.4.2 The γ-core

Chander an Tulkens propose a specific allocation (q^*, y^*), bearing for an equilibrium interpretation of the economy E, and show by construction that it belongs to the γ-core of the game Γ_2. We report their result in the following proposition.

Proposition 20 ***(Chander-Tulkens):*** *The joint strategy* (q^*, y^*) *where:*

$$q^* \textit{ is such that } \pi_N(Q^*) = C_i'(q_i^*), \textit{ for all } i \in N;$$

$$y_i^* = \omega_i - C_i(\bar{q}_i) - \frac{\pi_i(Q^*)}{\pi_N(Q^*)}\left[\sum_{i \in N}\left(C_i(q_i^*) - C_i(\bar{q}_i)\right)\right]$$

is in the γ*-core.*

In what follows we will refer to (q^*, y^*) as the C-T allocation.

5.4.3 The ϕ-core

In this section we analyze the symmetric case (identical players) and we show that under linear preferences, Proposition 20 carries over to the case of ϕ-core. However, we also show that, if preferences are strictly concave, the ϕ-core may be empty.

The function v_ϕ

By definition, any partial equilibrium $[(\tilde{q}_S, \tilde{y}_S), (q_j, y_j)(\tilde{q}_S, \tilde{y}_S)]$ of Γ_2 with respect to S is such that

$$\begin{aligned} \tilde{q}_S &\in \arg\max_{q_S, y_S} \sum_{i\in S} v_i \left(Q_S + \sum_{j\in N\backslash S} q_j(q_S) \right) + \sum_{i\in S} y_i \\ \text{s.t.} \quad \sum_{i\in S} \omega_i &\geq \sum_{i\in S} [C_i(q_i) + y_i] \end{aligned}$$

and, $\forall j \in N\backslash S$

$$\begin{aligned} q_j(q_S) &= \arg\max_{q_j, y_j} v_j \left(Q_S + \sum_{k\in (N\backslash S)\backslash\{j\}} q_j(q_S) + q_j \right) + y_j \\ \text{s.t.} \quad \omega_j &\geq C_j(q_j) + y_j \end{aligned}$$

Proposition 21 *For every $S \subseteq N$, there exists a partial equilibrium of Γ_2 with respect to S. Moreover, all partial equilibria with respect to S are characterized by the same vector $\tilde{q}$.*

Proof. By Proposition 19, the Nash equilibrium of $\Gamma_2(N\backslash S, q_S)$ exists and is unique for all S and q_S. By continuity of v_i, (A.4), and of $C_i(q_i)$, (A.5), Proposition 13 can be applied here. Moreover, as the maximization problem of S can be written as a function of just q_S, by concavity of v_i and strict convexity of $C_i(q_i)$, Proposition 13 can again be applied to show uniqueness. ■

Some characterization of the partial equilibria of Γ_2

We now analyze in greater detail the partial equilibria of Γ_2.

We first consider the first order condition for every player $j \in N \backslash S$: by symmetry, we can write

$$\pi_j\left(q_j + (n-s-1)q_j + Q_S\right) - C'\left(q_j\right) = 0. \tag{5.20}$$

By Assumptions A.4 and a.5 and applying the implicit function theorem to the mapping $f\left(q_j, q_S\right) \equiv \pi_j\left((n-s)q_j + Q_S\right) - C'\left(q_j\right)$, we conclude that the function $q_j\left(q_S\right)$ is differentiable. Thus, totally differentiating the FOC above, we obtain, in equilibrium, the condition

$$\frac{\partial \pi_j}{\partial q}\left[1 + (n-s)\frac{\partial q_j}{\partial Q_S}\right] - C''\left(q_j\right)\frac{\partial q_j}{\partial Q_S} = 0$$

yielding the reaction function

$$\frac{\partial q_j}{\partial Q_S} = \frac{\frac{\partial \pi_j}{\partial Q}}{C''\left(q_j\right) - (n-s)\frac{\partial \pi_j}{\partial Q_S}} < 0.$$

The term $\frac{\partial q_j}{\partial Q_S}$ gives us the reaction of player j to changes in the vector q_S as determined by the changes in j's Nash equilibrium strategy in the game $\Gamma_2\left(N \backslash S, q_S\right)$.

Given the reaction function of each outside player j, the maximization problem of coalition S yields the following FOCs:

$$\pi_S\left(\tilde{Q}\right)\left(1 + (n-s)\frac{\partial q_j}{\partial Q_S}\right) = C'\left(\tilde{q}_i\right), \quad \forall i \in S. \tag{5.21}$$

By plugging the expression for $\frac{\partial q_j}{\partial Q_S}$ into (5.21), we obtain

$$\pi_S\left(Q_S + (n-s)\cdot q_j\left(q_S\right)\right)(1-k) = C_i'\left(q_i\right) \tag{5.22}$$

where

$$0 < (1-k) = \left((n-s) \frac{\frac{\partial \pi_j}{\partial Q}}{C_i''(q_j) - (n-s)\frac{\partial \pi_j}{\partial Q_S}} + 1 \right) \leq 1. \tag{5.23}$$

Indeed, the presence of the term $(1-k)$ is the only difference between our optimality conditions and the ones obtained by C-T. Comparing the conditions characterizing v_γ and v_ϕ, it can be easily checked that the aggregate amount of public good induced by the deviation of a coalition S under the γ-assumption is greater than or equal to that induced under the ϕ-assumption.

In order to prepare the analysis of the next section, we establish here some properties of partial equilibria. We will refer to the original concept of partial equilibrium introduced by C-T as to the partial equilibria under the γ-assumption.

Lemma 22 *The aggregate amount of public good produced in the partial equilibrium with respect to S is not greater under the ϕ-assumption than under the γ-assumption.*

Proof. Let $Q^\phi(S)$ and $Q^\gamma(S)$ be the aggregate levels of public goods in the partial equilibrium w.r.t. S under ϕ and γ-assumption, respectively. Suppose that $Q^\phi(S) > Q^\gamma(S)$; then, by FOC (5.20), for each player $j \in N \backslash S$, $q_j^\phi(S) \leq q_j^\gamma(S)$. Moreover, as $(1-k) \leq 1$, by FOC (5.22) for every player $i \in S$, $q_i^\phi(S) \leq q_i^\gamma(S)$. The two inequalities imply a contradiction. ■

Lemma (22) and Proposition 5 in Chander-Tulkens (1997) imply that the aggregate amount of public good produced in the partial equilibrium w.r.t. S under the ϕ assumption is not greater than the efficient one.

Lemma 23 *If preferences are linear in the public good, then:*

i) $q_i^\phi(S) \leq q_i^*$, $\forall i \in N$;

ii) $\bar{q}_i \leq q_i^\phi(S)$, $\forall i \in N$;

iii) $\bar{q}_j = q_j^\phi(S)$, $\forall j \in N \backslash S$.

Proof. *i):* By definition of the term $(1-k)$ in condition (5.23), if preferences are linear then $(1-k)=1$. By condition (5.22) this implies the following implications for all $i \in S$:

$$C_i'\left(q_i^{\phi}(S)\right)=\pi_S<\pi_N=C_i'(q_i^*).$$

Similarly, for all $j \in N\backslash S$, condition (5.20) implies:

$$C_j'\left(q_j^{\phi}(S)\right)=\pi_j<\pi_N=C_j'(q_j^*).$$

The two implications, together with strict convexity of $C_i(.)$ for every $i \in N$, imply the result.

ii) and iii): By conditions (5.22) and (5.19), for all $i \in S$:

$$C_i'(\bar{q}_i)=\pi_i<\pi_S=C_i'\left(q_i^{\phi}(S)\right).$$

By conditions (5.20) and (5.19), for all $j \in N\backslash S$:

$$C_j'(\bar{q}_j)=\pi_j=C_j'\left(q_j^{\phi}(S)\right).$$

Again by convexity of cost functions, the results follow. ■

The robustness of Chander-Tulkens result under linear preferences

We are now able to show that under linear preferences for the public good, Proposition 20 by C-T generalizes to the ϕ-core.

Proposition 24 *If preferences are linear, then the C-T allocation (q^*, y^*) belongs to the ϕ-core.*

Proof. The proof of Proposition 2 in Chander-Tulkens (1997) can be directly applied using Lemma (23). Indeed, Lemma (23) establishes all the properties that are needed in the proof of that proposition. ∎

The ϕ-instability of Chander-Tulkens allocation under non-linear preferences

Under non linear preferences, C-T's result requires an additional assumption (Assumption 1" in their paper) concerning the marginal rate of substitution characterizing respectively a Nash and an efficient allocation. Under this assumption, and using a few properties both of Nash and partial equilibrium allocations under the γ-assumption, the authors prove Proposition 20 also for the non linear case. Using the notation introduced in the previous sections, such properties are that $q_i^\gamma(S) \geq \bar{q}_i$, for all $i \in S$, and that $q_j^\gamma(S) \leq \bar{q}_j$, for all $j \in N \backslash S$.
It is easy to check that the first property does not longer hold under the ϕ-assumption: indeed, in C-T's paper this property is proved through the following chain of implications:

$$C_i'(q_i^\gamma(S)) = \pi_S(Q^\gamma(S)) \geq \pi_S(Q^*) \geq \pi_j(\bar{Q}) = C_i'(\bar{q}_i),$$

where the inequality $\pi_S(Q^*) \geq \pi_j(\bar{Q})$ is indeed Assumption 1".

Under ϕ-assumption, the above chain of implications would write

$$C_i'\left(q_i^\phi(S)\right) = \pi_S(Q^\phi(S))(1-k) \geq \pi_S(Q^*) \geq \pi_j(\bar{Q}) = C_i'(\bar{q}_i)$$

which, as $(1-k) < 1$ by non-linearity of preferences, may well not be true. Actually, as Example 1 below shows, linearity turns out to be a necessary condition for C-T result to carry over under ϕ-assumption. Indeed, as it is proved in Proposition (25), in Example 1 the ϕ-core is empty.

Example 3. Let preferences be described by the utility function

$$u_i(q, x_i) = \left(Q - \alpha Q^2\right) + y_i$$

and let costs be described by the function

$$C(q) = \frac{q^2}{2}.$$

It can be easily checked that Assumption 1" in Chander-Tulkens (1997) is satisfied if $\alpha \geq \frac{1}{2}$.

We consider the deviation of a single player i, producing a zero amount of public good. By showing that, given the reactions of the other players, this strategy represents for him an improvement upon the allocation proposed by C-T, we show that he can improve upon it under the ϕ-assumption, as zero production is always a feasible strategy for him. The reaction of the other $(n-1)$ players to the "no production" strategy of i is obtained by the FOC

$$1 - 2\alpha q_j (n-1) = q_j$$

yielding

$$q_j = \frac{1}{1 + 2\alpha(n-1)}$$

and

$$Q = \frac{n-1}{1 + 2\alpha(n-1)}.$$

By using Samuelson's efficiency condition,

$$n(1 - 2\alpha Q^*) = \frac{Q^*}{n}$$

we obtain the efficient level of public good

$$Q^* = \frac{n^2}{1+2n^2\alpha}.$$

We are then able to compare the utility (u_i^*) received by i in the C-T allocation with the utility u_i^0 that i receives through a (zero production) deviation:

$$\begin{aligned} u_i^* &= \frac{n^2}{1+2n^2\alpha} - \alpha\left[\frac{n^2}{1+2n^2\alpha}\right]^2 - \frac{1}{2}\left(\frac{n}{1+2n^2\alpha}\right)^2; \\ u_i^0 &= \frac{n-1}{1+2\alpha(n-1)} - \alpha\left[\frac{n-1}{1+2\alpha(n-1)}\right]^2. \end{aligned}$$

By straightforward calculations, it turns out that, for $n \geq 2$ and $\alpha \geq 0.5$, $(u_i^0 - u_i^*)$ is always positive; hence, every player can individually improve upon the C-T allocation, which, therefore, is not in the ϕ-core. We report in the table below a few numerical values for $(u_i^0 - u_i^*)$.

$$\begin{array}{ll} n = 2,\ \alpha = 0.5 & (u_i^0 - u_i^*) = 0.224 \\ n = 10,\ \alpha = 0.5 & (u_i^0 - u_i^*) = 0.8 \\ n = 50,\ \alpha = 0.5 & (u_i^0 - u_i^*) = 0.96 \\ n = 100,\ \alpha = 0.5 & (u_i^0 - u_i^*) = 0.98 \end{array}.$$

Proposition 25 *Let costs and preference be as in Example 1. Then the ϕ-core of the associated cooperative game is empty.*

Proof. It is shown in Example 1 that any player could improve upon C-T's solution by exploiting a first mover advantage. By efficiency of that solution, for any other efficient solution (q, y), at least one player i would receive a lower utility than in (q^*, y^*). But as any player can improve upon (q^*, y^*) by deviating as singleton, than player i can improve upon (q, y) in the same way. ■

5.5 Concluding remarks

This paper has presented a new solution concept for cooperative games. Our concept modifies the γ-core by introducing a temporal structure in the choices of strategies in the underlying normal form game which is similar to the one adopted in the α-core. At the same time, it is maintained the γ-assumption that outside players react to a forming coalition by splitting up into singletons. This approach is meant to account for those cases in which coalitions can break an agreement and, in so doing, force the outside players to react to their new strategy.

In this paper we have focused our attention on two applications: Cournot oligopolies and public good provision. Our results on cartel formation show that, in a linear symmetric oligopoly, considering the ϕ-core restricts the set of core outcomes to the equal split allocation. Moreover, differently from the γ-core, under quadratic costs the ϕ-core may be empty. In the second application, Chander and Tulkens (1997) results are shown to be robust against the temporal structure assumed in the ϕ-core if and only if preferences are linear in the public good. In the case of non linear preferences, conversely, whenever a coalition can exploit a first mover advantage, the γ-assumption on coalition formation is no longer sufficient to yield a non-empty core.

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