Job matching and unemployment:
Applications to the UK labour market
and international comparisons
by
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Abstract

This thesis studies different aspects of job matching, mismatch, and their relationship with aggregate unemployment.

The first part addresses the question of the structural rise in the unemployment rate in OECD countries by looking at the link between sectoral shocks and aggregate performance in an economy with heterogeneous labour. The type of sectoral shock considered in chapter 2 is the introduction of skill-biased technologies, that increase the relative demand for skilled labour at the expenses of the less-skilled. Unless the supply of skills adjusts accordingly to the increased demand, and/or relative wages are perfectly flexible, this shock has permanent effects on the aggregate unemployment rate, as shown in a non-competitive labour market model with skilled and unskilled workers. The calibration of this model predicts that a relevant proportion of the recent rise in British unemployment can be attributed to an unbalanced evolution in the demand and the supply of skills, while in continental Europe skill imbalances do not seem responsible for serious labour market problems. Finally, the impact of skill mismatch on US unemployment was limited in magnitude and almost completely offset by countering forces.

The second part of the work uses a job-search approach to investigate the technical characteristics of the matching process between vacancies and unemployed job-seekers. Chapter 3 reviews the empirical search literature that has estimated hiring functions, concluding that recent work has successfully established the existence of a labour market matching function, in which both vacancies and unemployed workers contribute significantly to job formation.

Chapter 4 considers a plausible alternative to a random meeting technology between employers and job-seekers, based on the existence of cheap information channels that save all traders the effort of locating matching partners. When combined with a proper handling of timing in the matching technology, this set-up provides novel results on the recent performance of the British labour market. In particular, it seems that the claimed deterioration of the search effectiveness of the unemployed cannot be explained by a lack of search effort per se, but by stronger competition that the registered unemployed face by other labour market segments.

Chapter 5 provides an analysis of the matching process at the micro level, using individual duration data obtained from a British sample of unemployment entrants. The determinants of re-employment probabilities are here related to a search model in which transitions into employment depend on the probability of receiving a job offer and that of accepting a job offer. The analysis shows that the hypothesis of constant returns to scale in the matching technology, embodied in most bilateral search models, is not rejected by the data. Individual re-employment probabilities respond in fact to local labour market tightness, and are unaffected by its size.
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To my parents
DECLARATION

1. No part of this thesis has been presented to any University for any degree.

2. Some parts of this thesis draw on joint work with my colleagues at the Centre for Economic Performance of the London School of Economics. Chapter 2, "Skill mismatch and unemployment in OECD countries" and chapter 4, "Non-random matching and the Beveridge curve" were undertaken as joint work with Marco Manacorda and Paul Gregg respectively. For the first of these, I contributed 50% of the work. For the second, I contributed 80% of the work. A statement from my co-authors confirming this is given below.

I confirm the above declaration referring to joint work carried out with Barbara Petrongolo

Marco Manacorda

I confirm the above declaration referring to joint work carried out with Barbara Petrongolo

Paul Gregg
Chapter 1

Introduction

The recent performance of Western labour markets has been characterized by some well known phenomena.

First, following the two oil shocks of the 1970s, most OECD countries experienced remarkable rises in their rates of joblessness. But while in the late 1980s, no more than two decades after the first oil shock, the unemployment rate in the United States had reverted to its pre-shock level, in the countries of the European Union it was still two to three times as high as it was at the beginning of the previous decade. These different trends in the evolution of unemployment across the OECD are extensively documented, and have been the subject of a vast debate (see Bean, 1994 for a survey).

Second, both the North American and European labour markets are characterized by substantial flows of workers and jobs, and consequently high rates of job creation, job destruction and job reallocation. Such massive flows are consistent with the presence of both vacancies and unemployed workers within each segment of the labour market, resulting from the intensity of the job reallocation process and the effectiveness of the labour market in matching slack resources. The fact that in several OECD countries the unemployment rate has substantially increased, in spite of a roughly untrended vacancy rate, seems to point to a strong deterioration in the matching effectiveness of the labour market (see - among others - Jackman, Layard and Pissarides, 1989).
It is now evident that labour market theories that were fashionable in the 1980s cannot provide a convincing explanation of these broad facts. Among these, theories based on institutional rigidities argue that more pervasive labour market regulations have generated the rise in unemployment. However, the rise in unemployment is difficult to reconcile with such institutional changes. Interestingly enough, Manning, Wadsworth and Wilkinson (1996) notice that in the UK, despite the substantial labour market deregulation of the 1980s, there is no time in the last fifteen years when the unemployment rate has been below the highest level experienced in the period 1945-1979. Alternative explanations focus instead on persistence mechanisms of different sources (see Blanchard and Summers, 1986 Lindbeck and Snower, 1988 and Layard, Nickell and Jackman, 1991, ch. 4), that prevented a natural adjustment to various temporary shocks like the oil price rises of the 1970s. However, OECD economies have indeed experienced some positive real shocks in the late 1980s, and notwithstanding this European unemployment is still substantially higher than it was in the late 1960s. What needs to be provided is therefore an explanation for the permanent increase in unemployment, rather than a microfoundation for its incomplete adjustment.

Furthermore, both these theories are specifically addressed to the evolution of stocks in the labour market, and are inherently inadequate to understand the transition of jobs between activity and vacancy and that of workers between employment and unemployment.

Given the weaknesses of existing theories in explaining aggregate labour market performance, the attention of macroeconomists has increasingly shifted towards the study of sectoral shocks and labour market flows as the key elements for the understanding of aggregate evolutions.

On the one hand, this has produced a rich stream of research on the analysis of sectoral shocks and aggregate performance in the labour market, whose basic idea goes back to the seminal work by Lilien (1982). The element that links the different contributions in this literature is the recognition of some mechanism - that may rely on
imperfect labour mobility, rigid wage-setting, and related frictions - that translates more intense sectoral turbulence into poorer aggregate performance.

On the other hand, it has established the so-called "flow approach to labour markets" as the tool that is currently most widely used in shedding light on a number of macro-labour issues. This approach, whose bare-bone structure is described in Blanchard and Diamond (1992), builds on the work on search theory that originated since the early 1970s, with the Phelps' (1970) volume. Its aim is to describe the way in which search decisions at the micro level generate job creation and job destruction flows, and the related worker flows in and out of employment (see Pissarides, 1990 and Mortensen and Pissarides, 1994). One of its central ideas, that relates job creation to the amount of slack resources in the economy, the so-called hiring function, has become an extremely popular tool in assessing the degree of matching effectiveness of labour markets.

The aim of this thesis is to provide a theoretical and empirical advancement in both these approaches, in order to understand different aspects of an important issue such as the recent performance of Western labour markets.

The first part of the work explores the link between sectoral shocks and aggregate unemployment in OECD countries. This is done by investigating the characteristics and the consequences of the implementation of skill-biased technologies that shift labour demand towards more qualified labour and away from the less-skilled. The "sectors" of our analysis are therefore defined over the skill level of workers.

There are several reasons why a potential shift in relative demand against the less-skilled has become an appealing explanation for the labour market problems recently experienced by OECD countries. First, the relative employment conditions of workers at the bottom end of the skill distribution have deteriorated seriously over the past fifteen years in the US and in most European economies. Second, their earnings have dramatically fallen relative to those of skilled workers in the US and the UK, although they have remained pretty stable in continental Europe. This evidence is therefore indicative of a shift in the relative demand for skills, that was not adequately matched by
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a parallel shift in the relative supply, and consequently translated into higher unemployment and/or lower earnings for the unskilled, according to the prevailing institutional setting.

In order to clarify this view, in chapter 2 we develop a simple labour market model with skilled and unskilled labour, in which some non-competitive mechanism (that can be justified by wage bargaining, efficiency-wage considerations and the like) govern the wage-setting process. The framework that we adopt is based on the widely accepted idea, first expressed in Lipsey (1960), that wages are relatively more responsive to unemployment when unemployment is low. The presence of such a convex wage-unemployment relationship implies that a given asymmetric shock, hitting two different types of labour and generating greater unemployment (and wage) dispersion, will also generate higher average unemployment.

With respect to previous related work, this model focuses directly on changes in the demand and the supply of skills, rather than inferring skill mismatch from the dispersion of sectoral relative unemployment rates, as in Layard, Nickell and Jackman (1991, ch. 6). We will argue below that the dispersion of sectoral relative unemployment rates is not a perfect index of mismatch in the sense that it does not distinguish the effect of sectoral shocks to wage pressure from that of pure demand/supply imbalances. Moreover, we claim that our index of mismatch embodies a desirable neutrality property that other indices such as the one used by Nickell and Bell (1995) fail to display.

Estimation of our model with US and European data shows that skill mismatch has been a relevant component in the evolution of unemployment in the Britain, explaining as much as half of its increase between 1974 and 1992. However, no such contribution of mismatch is detected in the US and continental Europe.

An alternative way of thinking about mismatch consists in looking directly at how traders in the labour market meet in order to form new jobs or, in other words, at the "technical" characteristics of the matching process between vacancies and job-seekers. We should in fact conclude that there is mismatch between the demand and the supply
of labour if the flow of job creation does not increase sufficiently when either or both sides of the market expand. In order to investigate this, the flow approach to labour markets suggests the use of the hiring function, that determines the intensity of job formation given the size of the vacancy and the unemployment pools.

These are the premises at the basis of the second part of this work, that takes a systematic look at the characteristics of the matching technology among traders in the labour market. We believe in fact that the study of the matching function is valuable for the reason that it converts optimal search strategies at the individual level into aggregate labour market flows, and therefore may provide some microfoundations for a number of macroeconomic issues such as labour market matching effectiveness, unemployment duration, and persistence.

We start the second part of the work by reviewing in chapter 3 the empirical search literature that estimated alternative specifications of the matching function, from the benchmark aggregate studies of Pissarides (1986) and Blanchard and Diamond (1989) to recent work on disaggregate data. On the whole we find that in recent years empirical labour market studies have successfully established the existence of a hiring function, in which both unemployed workers and vacancies contribute significantly to the process of job formation.

Nevertheless, we identify two main areas in this field that are to date in their infancy. The first concerns alternative characterizations of the interaction between traders in the labour market and of their implications for aggregate job and worker flows. This is explored in chapter 4, that considers some plausible microfoundation for the matching function, introducing a non-random meeting technology between employers and job-seekers, along the lines of Coles and Smith (1997). The non-random nature of the search process derives from the existence of information channels such as employment agencies, that are available at low cost to employers and job-seekers, and therefore have the potential of saving all traders the effort of locating matching partners. We argue that non-random matching has relevant implications for the use of the standard $UV$ curve à
la Jackman, Layard and Pissarides (1989) as a measure of matching effectiveness, and provide novel results concerning the matching performance of the labour market over the past thirty years.

The other area in which new research should be welcome is the characterization of the matching process at the micro level, and the identification of relevant links between micro and macro, black-box types of analyses. We pursue this strategy in chapter 5 where we exploit the link between individual re-employment probabilities and aggregate matching conditions by estimating unemployment hazard functions within self-contained labour markets. We find that the determinants of re-employment probabilities are clearly consistent with a job-search framework, in which transitions from unemployment into jobs are the combination of the probability of receiving a job offer and the probability of accepting the offer. In addition, the effect of local labour market variables on workers' transitions into jobs confirms the existence of constant returns to scale in matching. This finding has key implications for theoretical search models of the labour market, in the sense that it rules out the existence of multiple equilibria along a balanced growth path.

Finally, chapter 6 provides a brief overview of this thesis, with a summary of the main findings of each chapter, and possible directions for future work.
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Part I

Sectoral shocks and aggregate performance
Chapter 2

Skill mismatch and unemployment in OECD countries

2.1 Introduction

Evidence on labour market performance in OECD countries over the last two and a half decades delivers two well known stylized facts.

First, following the two oil shocks of the 1970s, most OECD countries experienced remarkable rises in their unemployment rates. In the late 1980s, no more than two decades after the first oil shock, the unemployment rate in the United States had reverted to its pre-shock level, while in the countries of the European Union it was still two to three times as high as it was at the beginning of the previous decade, and showed a remarkable degree of persistence. These different trends in the evolution of unemployment across the OECD are documented in the existing literature, and have been the subject of a vast debate (see Bean, 1994 for a survey). Secondly, wage inequality - both overall and between a number of dimensions - has been steadily increasing in the US over the last twenty years. This dramatic increase in wage inequality does not appear in most European countries. With the exception of the UK, where wage differentials widened during the 1980s, European countries experienced a pretty stable -
if not declining - dispersion of earnings during the last two and a half decades (see for example OECD, 1993 for summary evidence).¹

It has recently been suggested that these pieces of evidence can be rationalized in terms of the same driving force. Krugman (1994) argues that the rise in European unemployment and the widening wage dispersion in the US might be interpreted as “two sides of the same coin”, namely a pressure towards a rise in the inequality of market wages. The different outcome in terms of unemployment versus wage inequality would then depend on the institutional setting dominating a country’s labour market. In flexible labour markets this pressure would translate into an actual widening of the wage distribution. In highly regulated labour markets, the forces that prevent the widening of earnings’ dispersion would instead translate the rise in the inequality of market wages into higher unemployment at the lower end of the wage distribution. One plausible cause of a tendency towards greater inequality is skill-biased technological progress, increasing the relative demand for skilled labour at the expenses of the less-skilled.²

Any increase in the relative demand for skilled labour would not cause major labour market problems if it were matched by a parallel adjustment of supply. But various factors, such as credit market imperfections and/or indivisibilities in investment in human capital, can prevent the required skill upgrading of the labour force. Along this line, the present work is an attempt to evaluate to what extent any imbalance between the demand and the supply of skills - that we refer to as skill mismatch - can be held

¹The issue of the source of increased wage inequality has generated some debate between those who explain it as being mainly induced by the changing structure of international trade and, more specifically, third world competition in those industries which are less skill intensive (see for example Murphy and Welch, 1992 and Wood, 1994), and those who reckon instead that it was mainly due to skill-biased technological progress across industries (for some evidence in this direction see Katz and Murphy, 1992; Berman et al., 1994; and Machin, 1994). Others stress instead the role played by the declining power of labour market institutions (see Goslin and Machin, 1994; and DiNardo et al., 1996).

²The relevant inequality concept in this framework has a “between-group” nature, where the groups are defined over workers’ skills. Different is the driving force behind the tendency towards greater inequality in Bertola and Ichino (1995), who point at the consequences of increased idiosyncratic uncertainty in the labour demand forcing processes, and again differentiate the outcome in terms of unemployment versus wage dispersion according to the degree of labour market flexibility.
responsible for the secular rise in European unemployment.\(^3\)

Our analysis provides two new contributions to the debate.

The first is an explicit description of how wage inequality and unemployment interact in an economy with heterogeneous labour (skilled and unskilled), in which institutions of varying power govern the wage-setting process. The framework that we adopt is based on the widely accepted idea, first expressed in Lipsey (1960), that wages are relatively more responsive to unemployment when unemployment is low. The presence of such a convex wage-unemployment relationship implies that a given asymmetric shock, hitting two different types of labour and generating greater unemployment (and wage) dispersion, will also generate higher average unemployment. In the same framework we show that the impact of a given shock on unemployment is negatively related to real wage flexibility.

The second element concerns the empirical documentation of the driving force at the basis of the recent major developments in unemployment and wage inequality. By focusing on the evolution of prices and quantities of different educational inputs we try to determine whether a net relative demand shift between different skill groups has in fact occurred, to distinguish its demand and supply components, to assess its magnitude, and finally to discuss its relationship with aggregate unemployment.

Related work by Nickell and Bell (1995) and Manning, Wadsworth and Wilkinson (1996) follows a similar approach to the one presented here. The main difference between their approach and the one put forward in this chapter relies on the specification of the relevant mismatch index adopted. As we argue below, the choice of a particular mismatch indicator has important implications for the definition of a "neutral" demand and supply shock, i.e. a change in the structure of labour demand that is perfectly

\(^3\)It may be argued that the observed pattern of unemployment differentials is also consistent with the occurrence of an adverse aggregate shock, after which the skilled whose jobs are at risk may accept unskilled jobs and further displace the less-skilled. In doing this, the skilled can manage to maintain their wage differential, thanks to their higher average productivity. We argue however that, although this story may fit the European experience, it cannot explain the long-run rise in wage differentials in the UK and the US.
matched by a parallel change in the structure of labour supply. We show in fact that the mismatch index that we propose can be associated with some reasonable concept of neutrality, while other indices fail to display such a property.

The rest of this chapter is organized as follows. Section 2.2 presents some descriptive evidence on the evolution of unemployment and wage differentials by education in a set of eleven OECD countries, characterized by different labour market performances. Section 2.3 introduces the labour demand side of the economy. A simple Cobb-Douglas specification of technology delivers testable predictions in terms of the relationship between relative wages and relative employment, and it is not rejected by our data. This section also provides some estimates for the growth of the demand and the supply of skills in our set of countries over the past two decades. Section 2.4 closes the model by introducing a wage function that relates skill-specific wages to skill-specific unemployment. Here we show that labour demand and supply imbalances hitting those workers with the poorest labour market prospects can in fact worsen the aggregate performance of the economy, by increasing the aggregate unemployment rate. Finally, in the same section we evaluate the impact of increased skill mismatch on aggregate unemployment. Section 2.5 concludes the chapter and states our main findings.

2.2 Unemployment and wage differentials by skill: Some evidence

In this section we introduce some descriptive evidence on the evolution of wages and unemployment by skill in a set of OECD countries for which data are available. The aim of this section is to highlight whether any sign of increasing inequality in wages and/or employment opportunities across skills can be detected and to assess whether this is a generalized phenomenon across the OECD. At this stage we are not able to evaluate to what extent a shift in relative net demand towards the skilled has occurred, and in order to do so in the next section we develop an appropriate framework.
Figure 2-1 plots the standardized unemployment rate for 11 OECD countries over the past two and a half decades. The countries are Australia, Canada, France, Germany, Italy, Netherlands, Norway, Spain, Sweden, United Kingdom\(^4\) and United States.

These countries differ substantially in their unemployment experience. One subset, made up of EU countries, Australia and Canada, shows an overall upward trend in the unemployment rate over the period considered. Here unemployment increases roughly monotonically until it reaches a peak around mid-1980s, then has a local minimum in the late 1980s, followed by a further recession. Note however that the recovery of the second half of the 1980s does not bring unemployment back to the level where it started before the first oil shock. In the US, on the other hand, the unemployment rate experiences pronounced cycles, without any definite trend. Lastly, in the Scandinavian countries, unemployment is stable and very low until the late 1980s, and then peaks during the last recession.

While these aggregate trends are well documented in the literature, less evidence has been provided with regard to the skill composition of employment and the labour force.

The educational attainment of individuals is used as the relevant indicator of skill. This is because education can be assumed to represent an intrinsic characteristic of the individuals, while other classifications, such as occupation, tend to reflect the job's rather than the individual's characteristics. Furthermore, education is a time-invariant personal attribute, provided the individuals under analysis have completed their course of study (or it can be treated as such if individuals have passed the age at which, on average, higher education is achieved).

Cross-country comparisons by education can be quite problematic since educational systems vary widely across countries, and so does the quality of schooling. For this reason, our evidence should be treated with some care, so far as international comparisons are concerned. Despite this, we hope to be able to highlight some basic trends and show

\(^4\)Data for the UK are available for aggregate indicators, while all disaggregated data refer to Britain.
Figure 2-1: The standardised unemployment rate in 11 OECD countries, 1970-1994.
that they are robust to the classification used.

In what follows we adopt a dichotomous classification of skills. We generally define as skilled those individuals who have completed their upper secondary education, (or equivalent vocational qualification) and unskilled all the others (see the Data Appendix for a more detailed definition of skill categories across countries and for data sources). Two exceptions have been made to this taxonomy, for the US and Spain, where skilled individuals are those who have at least some college education or vocational equivalent. For the US this procedure provides a more balanced partition between skill levels. For most countries, in fact, there is a point in the sample period at which the two groups are approximately equally sized. This allows us to keep to a "relative" definition of skills, in which skilled individuals are defined as those who have an education attainment above the median. The exception for Spain is due to the very poor disaggregation between skill levels in the original data, that did not allow the same skill partition obtained for other European countries.

Figure 2-2 plots available time series for the period 1970-1994 of the percentage of skilled people in the population of working age (where available), labour force and employment for each country. The relative size of the skilled group grows monotonically over the whole period in all countries, showing a definite and generalized trend towards higher educational attainment.

To evaluate whether the general tendency towards a skill-upgrading was balanced in its demand and supply components, we look at the evolution of skill-specific unemployment rates. Figure 2-3 plots the evolution of the unemployment rates by education for our set of countries. For ten of the eleven countries considered the unemployment rate of the unskilled is above that of the skilled. The only exception is Italy, where unemployment is more concentrated among highly educated workers, although, as we

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5The age structure of unemployment can probably shed some light on this phenomenon. In Italy high school graduates represent the majority of young people (below 24), since the age at which, on average, university education is completed exceeds 24. And young Italian workers' unemployment rates range from three to four times the average rate over the period considered, while youth unemployment rate in other countries (such as the UK or the US) was always below twice the average rate. Therefore the
Figure 2-2: Shares of skilled individuals: 1970-1994.

will see, the trend in the unemployment differential follows the same pattern as in other countries.

Although we will make this point formally in the next section, it can be shown that, for given relative wages, an increase in the imbalance between the demand and the supply of skills can be identified by looking at the evolution of the difference between the unemployment rates of the two groups.

We can detect two main patterns in the evolution of skill-specific unemployment rates. There is in fact a group of countries where the secular increase in unemployment is mainly concentrated among the unskilled. This is the case for the US and the EU countries: Britain, France, Germany, Italy, the Netherlands and Spain. On the other hand, in Australia, Canada and the Scandinavian countries (with the exception of Norway in the last recession) no remarkable change in the difference between skill-specific unemployment rates has taken place. Overall no clear correlation between the difference in the unemployment rates and aggregate unemployment can be detected in our data.

It is interesting to notice at this stage that the remarkably close behaviour of population shares to labour force shares (see Figure 2-2) implies that non-employment rate differentials move very much in line with unemployment rate differentials. We therefore rule out the possibility that the different patterns of unemployment differentials in the various countries are driven by different patterns of labour force participation across skills.

Turning finally to wage differentials, the recent evolution of wage inequality across a number of dimensions - among which education - is extensively documented in the literature,\(^6\) and has produced global consensus on the recognition of a few stylized facts.

\(^{6}\)Among relevant contributions on different countries, there are the whole February 1992 issue of *Quarterly Journal of Economics*, Davis (1992), Bound and Johnson (1992), Juhn et al. (1993), Blanchflower et al. (1993), Blau and Kahn (1994), Erickson and Ichino (1994), and Gosling et al.
Figure 2-3: Unemployment rates by education: 1970-1994.

Notes. 1=skilled; 2=unskilled. For source and definitions: see Data Appendix.
Below we will simply describe the evolution of wage differentials for the two educational groups already defined for our set of countries.

Figure 2-4 plots the evolution of the skilled to unskilled wage ratio for a subset of countries for which consistent time series for wages are available: Britain, Germany, France, Italy, the Netherlands and the US. In no country except Britain and the US - two countries where the differentials are higher in levels - can any appreciable evidence of widening wage differentials by skill be found. In the remaining countries, wage differentials stay basically unchanged or even fall.

For Australia, Canada, Norway and Sweden, indirect evidence based on OECD (1993, 1994b Table 7.A.1) shows that in none of them (with the exception of Sweden in the late 80s, when differentials increased moderately) can any sign of increasing dispersion be detected.

Given our evidence, we can tentatively conclude that there seems to be a clear

Source and definitions: see Data Appendix.
sign of a net relative demand shift towards skilled labour in Britain, France, Germany, Italy, Spain and the US. Evidence of this shift is represented by changes in the skill distribution of unemployment and/or in wage differentials. It is instead more difficult to detect any sign of this kind in other countries. Australia, Canada, Netherlands, Sweden and Norway seem in fact to have kept the imbalance between the demand and supply of skills to a relatively steady level over the last two decades.

It is worth noting that the only country where both relative wages and unemployment differentials evolved against the less-skilled is the US (from the early 1980s). This seems to point to a peculiar experience of the US labour market as compared to the other countries. We will keep this in mind when we try to assess the magnitude of the shift in net relative demand towards skilled workers in the next section.

2.3 Has there been a shift in net demand?

This section introduces a very simple labour market model that should shed some light on what we mean by a shift in net labour demand and on how we can measure it. Below we estimate this model in order to give a quantitative assessment of such a shift.

2.3.1 Theory: A new measure for mismatch

We consider an economy with heterogeneous labour, defined over 2 skill groups, that produce an homogeneous output $Y$. The technology available to firms is represented by the following Cobb-Douglas production function, involving the 2 labour inputs,$^{7}$

$$Y = AN_1^{\alpha_1}N_2^{\alpha_2},$$

(2.1)

in which $A$ represents the aggregate state of technology, and constant returns to scale are imposed ($\alpha_1 + \alpha_2 = 1$). Under perfect competition in the goods market, this would

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$^{7}$The framework can be easily extended to $n$ groups, as shown in the technical appendix.
give \( w_1 = \alpha_1(Y/N_1) \) as the labour demand equation for input 1 - with \( w_1 \) denoting its real wage and \( \alpha_1 \) denoting its product share - or,

\[
\ln w_1 = \ln \alpha_1 + \ln Y - \ln N_1 \\
= \ln \frac{\alpha_1}{l_1} + \frac{Y}{L} - \ln (1 - u_1),
\]

(2.2)

where \( L \) denotes total labour force, \( l_1 \equiv L_1/L \) denotes group 1 labour force share, and \( u_1 = (L_1 - N_1)/L_1 \) denotes its unemployment rate.

The technology parameter \( \alpha_1 \) represents a relative demand indicator for group 1, and therefore shifts in \( \alpha_1 \) can be thought of as being caused - among other factors - by skill-biased technical change. Similarly, \( l_1 \) represents a relative supply indicator for group 1. The same clearly applies for group 2.

Substituting equation (2.1) into (2.2) gives

\[
\ln w_1 = \ln A + \ln \alpha_1 - \alpha_2 \ln(1 - u_1) + \alpha_2 \ln(1 - u_2) - \alpha_2 \ln \frac{l_1}{l_2}.
\]

(2.3)

Finally, differentiation of (2.3) gives

\[
d \ln \frac{w_1}{w_2} = \left( d \ln \frac{\alpha_1}{\alpha_2} - d \ln \frac{l_1}{l_2} \right) + \left[ d \ln(1 - u_2) - d \ln(1 - u_1) \right],
\]

(2.4)

using the property \( d \ln \alpha = \alpha_2 d \ln[\alpha_1/\alpha_2] \).

Equation (2.4) gives the comparative statics of our economy. The first term in brackets represents the shift in net relative demand towards group 1, that we identify as the skilled. This term refers to a change in the skill composition of labour demand (i.e. a change in relative labour demand) which is not perfectly matched by a parallel change in the skill composition of labour supply (i.e. a change in relative labour supply). We refer to this imbalance as skill mismatch.

Previous concepts of mismatch (see Layard, Nickell and Jackman, 1991, ch. 6) focus instead on the dispersion of relative unemployment rates, rather than on the direct
evolution of sectoral demand and supply of labour. By focusing on the (endogenous) unemployment dispersion, the LNJ index does not distinguish pure demand and supply imbalances from adjustments in relative wages and unemployment rates due to different sources.

The mismatch index adopted here, \( d \ln(\alpha_1/\alpha_2) - d \ln(l_1/l_2) \), displays the property of having the same absolute magnitude and opposite sign for the two groups. If this is the case, sectoral unemployment rates move in opposite directions in the face of a net relative demand shock, as will be shown in the next section. This is one of the main differences between this approach and the one followed in Nickell and Bell (1995) and Manning, Wadsworth and Wilkinson (1996), who focus on an absolute mismatch indicator, \( d \ln(\alpha_1/l_1) \).

Another property of our mismatch index - that derives directly from the Cobb-Douglas specification of the production function - is that it weights equally demand and supply shocks. This would not be the case with a CES production technology, in which supply shocks carry a lower weight than demand shocks, insofar as the elasticity of substitution between labour inputs exceeds one. This is another important difference between the present analysis and Nickell and Bell’s (1995), who assume a CES production function combining skilled and unskilled labour with an elasticity of substitution greater than one. However, our estimates below show that the elasticity of substitution between skills is not significantly different from one, implying that a Cobb-Douglas production function is a satisfactory representation of technology, making both the algebra and the empirical implementation of our framework more easily tractable.

Suppose now \( d \ln(\alpha_1/\alpha_2) - d \ln(l_1/l_2) > 0 \), implying a positive net relative demand shock for the skilled. Equation (2.4) says that this requires either a rise in relative wages for the skilled, or a rise in their relative employment rate \( (d \ln(1-u_2) - d \ln(1-u_1) < 0) \), or both. The way the total impact is split between employment and wage differentials depends on the curvature and the position of a wage-setting schedule, that will be introduced in the next section.
For small enough $u_1$ and $u_2$, we can approximate $d \ln(1 - u_2) - d \ln(1 - u_1)$ as $d(u_1 - u_2)$, implying that a demand shock favouring group 1, with $u_1 < u_2$, will increase the difference between sectoral unemployment rates. In other words, if the evolution of relative demand and supply is perfectly balanced, there is no need for relative wages to change or for the difference between sectoral unemployment rates to change.

2.3.2 Evidence: How to measure mismatch

Having set a broad framework for thought, we proceed by exploring the evolution of the demand and supply of skills in our set of OECD countries. The evolution of labour supply can be easily assessed using labour force figures. As in most of the related literature, we treat labour supply as exogenous, as a limiting case for skill formation when this is not perfectly elastic. If in fact relative labour supply were infinitely elastic to differences in expected income, any mismatch unemployment would be purely transitory.

With regard to the labour demand indicator, below we estimate a more general specification for aggregate technology than equation (2.1), and aim at giving possible measures for the evolution of the relative demand for skills.

To keep things as general as possible, we proceed by estimating a linear homogeneous CES aggregate production function, involving two labour inputs:

$$Y = A \left( \alpha_1 N_1^p + \alpha_2 N_2^p \right)^{\frac{1}{\sigma}}, \quad (2.5)$$

where $\rho = 1 - 1/\sigma$, with $\sigma$ denoting the elasticity of substitution between labour inputs. The $\alpha$'s are, once more, some relative productivity indexes (such that $\alpha_1 + \alpha_2 = 1$), and $A$ represents total factor productivity. Profit maximization yields the following relative

---

8A trans-logarithmic specification for the cost function (see Berndt, 1991, section 9.4) was also estimated for a few countries. This gave quite poor results, probably for lack of suitable data; but confirms the main findings on the net labour demand shift that are obtained using a CES production function.
demand for inputs
\[ \ln \frac{N_1}{N_2} = -\sigma \ln \frac{W_1}{W_2} + \sigma \ln \frac{\alpha_1}{\alpha_2}. \] (2.6)

Due to lack of data on group-specific productivities, we use a linear time trend as a proxy for (log) relative productivities, as in Katz and Murphy (1992). Higher powers of the time trend were included during estimation and found non significant. Moreover, a common elasticity of substitution across countries is imposed between the two labour inputs, to obtain a measure of the "average" elasticity of substitution in OECD countries. The intercept term and the trend coefficient are allowed to differ across countries. Estimation is performed for those six countries on which wages are available (see Figure 2-4).

The regression equation therefore has the form
\[ \ln \frac{N_{c1t}}{N_{c2t}} = a_c + \beta_t t - \sigma \ln \frac{W_{c1t}}{W_{c2t}} + \varepsilon_{ct}, \] (2.7)

where \( c \) and \( t \) index respectively countries and years. In order to improve the precision of our estimates, estimation is performed on a system of seemingly-unrelated equations such as (2.7), with the cross-equation restriction of a common \( \sigma \). The results are reported in Table 2.1. The fit of all equations is close to perfect. The estimate of the elasticity of substitution equals 1.059 (s.e. 0.123), and the one-tail test on \( \sigma \) does not lead to a rejection of the null hypothesis \( \sigma = 1 \) against the alternative \( \sigma > 1 \) at the standard significance levels.

Justified concern for the endogeneity of relative wages would suggest using instrumental variables for \( W_{c1t}/W_{c2t} \). Ignoring dynamics in the relative demand equations, lagged relative wages can serve as proper instruments. IV estimates are however very similar to the ones reported, giving \( \sigma = 1.036 \) (s.e. 0.133).

This implies that - over our set of OECD countries - the production function can be legitimately approximated by a Cobb-Douglas specification. This in turn allows us to exploit the useful properties of Cobb-Douglas production functions, so that we can
### Table 2.1: The labour demand equation in 6 OECD countries

<table>
<thead>
<tr>
<th>Countries</th>
<th>FRA</th>
<th>GER</th>
<th>ITA</th>
<th>NET</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.728</td>
<td>0.769</td>
<td>-1.691</td>
<td>-0.012</td>
<td>-1.591</td>
<td>-0.523</td>
</tr>
<tr>
<td>$t \times 100$</td>
<td>(0.049)</td>
<td>(0.079)</td>
<td>(0.075)</td>
<td>(0.076)</td>
<td>(0.068)</td>
<td>(0.052)</td>
</tr>
<tr>
<td>$\ln (W_1/W_2)$</td>
<td>6.57</td>
<td>5.13</td>
<td>6.51</td>
<td>4.15</td>
<td>7.64</td>
<td>5.25</td>
</tr>
<tr>
<td></td>
<td>(0.20)</td>
<td>(0.52)</td>
<td>(0.32)</td>
<td>(0.28)</td>
<td>(0.23)</td>
<td>(0.12)</td>
</tr>
<tr>
<td></td>
<td>-1.059</td>
<td>-1.059</td>
<td>-1.059</td>
<td>-1.059</td>
<td>-1.059</td>
<td>-1.059</td>
</tr>
<tr>
<td></td>
<td>(0.123)</td>
<td>(0.123)</td>
<td>(0.123)</td>
<td>(0.123)</td>
<td>(0.123)</td>
<td>(0.123)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.987</td>
<td>0.937</td>
<td>0.995</td>
<td>0.990</td>
<td>0.975</td>
<td>0.988</td>
</tr>
<tr>
<td>Sample</td>
<td>84-94</td>
<td>76-89</td>
<td>77-91</td>
<td>79-93</td>
<td>74-92</td>
<td>70-89</td>
</tr>
<tr>
<td>No. obs.</td>
<td>11</td>
<td>7</td>
<td>12</td>
<td>8</td>
<td>19</td>
<td>20</td>
</tr>
</tbody>
</table>


measure the growth rate in relative demand by estimating growth rates in wage bill shares.

Table 2.2 reports estimated annual growth rates (with the respective standard errors) of the following variables: relative labour supply $L_1/L_2$, relative employment $N_1/N_2$, relative employment rates $N_1/L_1 / (N_2/L_2)$, relative demand demand $\alpha_1/\alpha_2$, relative net demand $(\alpha_1/\alpha_2) / (l_1/l_2)$. The estimates for these last two variables are computed only for countries on which wage data are available. Recall finally that, for small enough unemployment rates, the growth rate in relative employment rates provides an approximation for the change in the difference between the groups' unemployment rates.

A few things are worth mentioning. First, all OECD countries experienced a skill upgrading in the structure of both supply and demand (all of the growth rates in columns I, II and IV are significantly positive and of comparable magnitude). Second, this tendency towards skill upgrading meant higher unemployment rates for the unskilled in Germany, Spain and Italy, and, to a lesser extent, in Britain, France, Norway and the US (see column III).

Finally, column V shows that there has been a substantial shift in net relative demand against the unskilled in Britain, France, Germany and the US (during the 1980s.
Table 2.2: Annual growth rates (×100) in supply, employment and demand for skills.

<table>
<thead>
<tr>
<th>Countries</th>
<th>Sample (No. Obs.)</th>
<th>$d\ln \frac{\hat{Y}_1}{L_2}$</th>
<th>$d\ln \frac{N_1}{N_2}$</th>
<th>$d\ln \left( \frac{N_1}{N_2} \right)$</th>
<th>Sample (No. Obs.)</th>
<th>$d\ln \alpha_1$</th>
<th>$d\ln \left( \frac{\alpha_1}{\alpha_2} \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>79 – 93 (15)</td>
<td>5.36 (0.17)</td>
<td>5.43 (0.19)</td>
<td>0.07 (0.05)</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>CAN</td>
<td>79 – 93 (14)</td>
<td>5.49 (0.17)</td>
<td>5.46 (0.17)</td>
<td>0.03 (0.07)</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>FRA</td>
<td>78 – 94 (17)</td>
<td>5.80 (0.15)</td>
<td>6.07 (0.13)</td>
<td>0.27 (0.04)</td>
<td>84 – 94 (11)</td>
<td>6.47 (0.23)</td>
<td>0.36 (0.08)</td>
</tr>
<tr>
<td>GB</td>
<td>74 – 92 (19)</td>
<td>6.82 (0.31)</td>
<td>7.03 (0.32)</td>
<td>0.21 (0.08)</td>
<td>74 – 92 (19)</td>
<td>7.55 (0.27)</td>
<td>0.73 (0.13)</td>
</tr>
<tr>
<td>GER</td>
<td>76 – 89 (7)</td>
<td>4.54 (0.61)</td>
<td>5.29 (0.56)</td>
<td>0.75 (0.12)</td>
<td>76 – 89 (7)</td>
<td>5.11 (0.61)</td>
<td>0.58 (0.11)</td>
</tr>
<tr>
<td>ITA</td>
<td>77 – 92 (16)</td>
<td>6.46 (0.06)</td>
<td>6.86 (0.08)</td>
<td>0.41 (0.02)</td>
<td>77 – 91 (12)</td>
<td>6.52 (0.15)</td>
<td>0.06 (0.14)</td>
</tr>
<tr>
<td>NET</td>
<td>79 – 93 (8)</td>
<td>5.84 (0.34)</td>
<td>5.83 (0.34)</td>
<td>0.00 (0.00)</td>
<td>79 – 93 (8)</td>
<td>4.75 (0.21)</td>
<td>−1.08 (0.20)</td>
</tr>
<tr>
<td>NOR</td>
<td>72 – 93 (22)</td>
<td>6.02 (0.12)</td>
<td>6.23 (0.13)</td>
<td>0.21 (0.03)</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>SPA</td>
<td>77 – 93 (17)</td>
<td>5.05 (0.22)</td>
<td>5.58 (0.24)</td>
<td>0.53 (0.07)</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>SWE</td>
<td>71 – 93 (21)</td>
<td>6.93 (0.10)</td>
<td>6.94 (0.10)</td>
<td>0.01 (0.02)</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>US</td>
<td>70 – 91 (22)</td>
<td>4.59 (0.20)</td>
<td>4.74 (0.22)</td>
<td>0.15 (0.04)</td>
<td>70 – 89 (20)</td>
<td>5.24 (0.13)</td>
<td>0.41 (0.17)</td>
</tr>
<tr>
<td></td>
<td>70 – 79 (10)</td>
<td>6.77 (0.15)</td>
<td>6.94 (0.21)</td>
<td>0.16 (0.09)</td>
<td>70 – 79 (10)</td>
<td>5.67 (0.10)</td>
<td>−1.11 (0.23)</td>
</tr>
<tr>
<td></td>
<td>79 – 91 (13)</td>
<td>3.21 (0.12)</td>
<td>3.25 (0.17)</td>
<td>0.04 (0.01)</td>
<td>79 – 89 (11)</td>
<td>4.73 (0.35)</td>
<td>1.48 (0.24)</td>
</tr>
</tbody>
</table>

Notes. The growth rates of relevant variables are the estimated coefficients on a linear time trend (×100) interpolated through the series of logarithms. Standard errors in brackets. Source and definitions: see Data Appendix.
only). There is in contrast virtually no imbalance in the evolution of the demand and the supply of skills in Italy, while there is a strong net relative demand shift against the skilled in the Netherlands. This is actually consistent with the fact that the unemployment differential did not really change in this country, while wage differentials were falling (see also OECD, 1993, 1994b, Table 7.A.1).

Looking more in depth at the US, the magnitude of the shift during the 1980s appears notably higher, when compared to any other country (no such distinction between decades is made for other countries, because in no other country is such a change found between the 1970s and the 1980s). This result, also found in Katz and Murphy (1992), confirms the findings of section 2.2 regarding the simultaneous shift in relative wages and relative employment rates in the US during the 1980s. A closer look at columns I and IV explains this. A substantial deceleration in the evolution of the supply of skills - rather than an acceleration in demand - seems responsible for the greater gap between the demand and the supply of skills in the US during the 1980s. Declining wage differentials by skill during the 1970s apparently discouraged skill formation for those generations that would enter the labour force in the following decade.

This result might be partly due to the different classification used across the set of countries. In particular, if one is willing to assume concavity in the growth of educational attainment in the population, due to the existence of an upper bound in the level of skills (no more than 100% of the population can achieve skills), this could imply that a country with higher average skill attainment would tend to experience a less rapid growth in the proportion of skilled workers. But this is not the case for the US. The ratio of skilled to unskilled labour force in the US was 0.65 in 1980, and the average across the whole set of countries was just over two thirds. And sorting countries by this ratio, the US occupies the median position.

To check the robustness of this result, we also looked at growth rates in the relative supply of skills using a more restrictive definition of skilled labour across our set of countries, which confirmed previous findings. In conclusion, we can state that the US
experienced a dramatic increase in the gap between the demand and the supply of skills, mainly due to a reduction in the rate of growth of supply. This imbalance is therefore responsible for the peculiar US experience, i.e. widening wage and unemployment differentials during the 1980s.

2.4 How mismatch relates to unemployment

The results of the previous section confirm the occurrence of a net demand shift in many European countries, with high and increasing unemployment, as well as in the US. Despite this, skill mismatch is not a generalized phenomenon, since this does not show up in the data for Australia, Canada, Netherlands, Norway and Sweden, countries that also experienced some rise in unemployment.

The next step is to evaluate whether and to what extent these trends in skill mismatch can be held responsible for the increase in unemployment in those countries where a positive demand shift towards the skilled took place. In order to do so, we close the model by combining the labour demand condition presented in section 2.3 with a widely accepted wage-setting relationship.

2.4.1 Theory

The mechanism at the heart of our model is very simple. It focuses on the idea that wages set by workers and firms are a decreasing convex function of unemployment, being more responsive to unemployment variations when unemployment is low than when it is high. This can be justified on the basis of a bargaining model in which unions and firms negotiate wages at given unemployment, and firms then chose employment at given agreed wages.

Under the assumption that sectoral wages respond solely to sectoral unemployment, such a convex wage function would imply that an asymmetric labour demand shock, hitting some categories of workers and favouring others, would generate some dispersion
in sectoral unemployment rates and therefore increase the average unemployment rate at given average wage.

This can be easily seen from Figure 2-5, where the $WS$ curve represents the wage-setting schedule as a convex function of the employment rate and $LD$ represents labour demand. Assuming for the moment that the curves $WS$ and $LD$ represent labour market conditions for all types of workers, in the initial position $E$ there is no dispersion in real wages or unemployment rates, and $w$ and $u$ therefore indicate both sectoral and average values. If an asymmetric labour demand shock takes place, this shifts up the labour demand schedule for skilled workers and shifts down that for unskilled workers, therefore introducing some unemployment dispersion in the economy. Average unemployment, being some linear combination of $u_1$ and $u_2$, will be situated somewhere on the $SU$ segment. At constant average real wage, determined by average productivity, the aggregate unemployment rate $u'$ would be higher than the level associated with no unemployment dispersion $u$.

Below we generalize this framework, allowing for initial heterogeneity in sectoral wage functions and labour demand functions and for endogenous changes in the average wage, and we derive the conditions under which such an asymmetric shock can have effects on the aggregate unemployment rate.

Sticking to a well established literature\(^9\) we adopt a double-logarithmic wage function for each group $i$, of the form

$$\ln w_i = z_i - \gamma \ln u_i, \quad i = 1, 2$$

(2.8)

where $w_i$ and $u_i$ denote group $i$ real wage and unemployment rate, respectively, $z_i$ represents group-specific wage pressure factors, and $\gamma$ represents (the absolute value of) real wage elasticity with respect to own-group unemployment.

\(^9\)The double-log wage equation is the standard specification adopted by Blanchflower and Oswald (1994). It is also supported by Layard et al. (1991), ch. 6, that note that the log of unemployment dominates the absolute level of unemployment in the wage-unemployment relationship.
Figure 2-5: The effect of an asymmetric labour demand shift on sectoral and aggregate unemployment rates
This last parameter may vary across countries according to how labour market institutions affect the wage-setting process, and may turn out to be a relevant aspect in the comparison of unemployment experiences across the OECD.

In equation (2.8), wage pressure is simply defined as any force that can influence wages at given unemployment. Wage pressure factors are generally identified as union power, the replacement ratio and the proportion of long-term unemployment. In the analysis that follows we do not concentrate on the identification of each of these factors, but just on their overall contribution to sectoral wage evolution.

The double-logarithmic specification adopted can be obtained as a log-linear approximation to a first-order condition for wages derived from a bargaining problem (see Manning, 1993), and it is empirically supported by data from a number of countries (see Jackman and Savouri, 1991 and Blanchflower and Oswald, 1994 for regional wage equations and Gregg and Machin, 1994 and Manacorda and Petrongolo, 1996 for skill-specific wage equations).

If we totally differentiate the demand equation (2.3):

\[ d \ln w_1 = \alpha_2 \left( d \ln \frac{\alpha_1}{\alpha_2} - d \ln \frac{l_1}{l_2} \right) + \alpha_2 [d \ln(1 - u_2) - d \ln(1 - u_1)] + \left( d \ln A + \ln \frac{N_1}{N_2} d \alpha_1 \right) \]

and interact it with the differentiated wage curve (2.8), changes in the unemployment rate for group 1 can be expressed as follows

\[ du_1 = \frac{u_1(1 - u_1)}{\alpha_2 u_1 + \gamma(1 - u_1)} \left( dz_1 - d \ln A - \ln \frac{N_1}{N_2} d \alpha_1 - d \ln \alpha_1 + \alpha_2 d \ln \frac{l_1}{l_2} + \frac{\alpha_2}{1 - u_2} du_2 \right), \]

and similarly for group 2.

Before examining the contribution of any exogenous shock on sectoral unemploymen-
ment, we can see that, the higher real wage flexibility $\gamma$, the lower is the impact of a given shock on unemployment $u_1$, and the higher is the impact on wages $w_1$. Moreover, the effect of exogenous shocks on sectoral unemployment depends on the initial level of $u_1$. In particular, it can be shown that this effect increases until $u_1$ reaches the value $\sqrt{\gamma}/(\sqrt{\alpha_2} + \sqrt{\gamma})$ and decreases thereafter. The ambiguity in the magnitude of the impact of an exogenous shock stems from the fact that both the labour demand (2.3) and the wage curve (2.8) are convex functions in $u_1$. The ambiguity can be easily solved noting that, for values of $\gamma$ above 0.035, a lower bound for the estimated wage elasticity in the empirical literature on skill-specific wage equations, and values of $\alpha_2$ around its average (0.5), the threshold value is above 0.20. Therefore, for most of the observed values of $u_1$, we can expect the term $(1 - u_1)u_1/([\alpha_2 u_1 + \gamma(1 - u_1)]$ to vary positively with $u_1$.

Below we will interpret the various components of $du_1$, with particular attention to the effect of a net relative demand shock on unemployment. In order to clarify the role of demand/supply imbalances in shaping sectoral unemployment rates, we solve equation (2.10) for each skill group. This gives the following closed-form solutions for $du_1$ and $du_2$:

\[
du_1 = \phi \left\{ \frac{\alpha_2 u_1 + \gamma(1 - u_1)}{u_1(1 - u_1)} \left( dz_1 - d\ln A - \ln \frac{N_1}{N_2} d\alpha_1 \right) \right. \\
+ \frac{\alpha_2}{1 - u_2} \left( dz_2 - d\ln A - \ln \frac{N_1}{N_2} d\alpha_1 \right) \\
- \frac{\gamma \alpha_2}{u_2} \left( d\ln \frac{\alpha_1}{\alpha_2} - d\ln \frac{l_1}{l_2} \right) \left\} \right. \\
\]  

\[du_2 = \phi \left\{ \frac{\alpha_1 u_2 + \gamma(1 - u_2)}{u_2(1 - u_2)} \left( dz_2 - d\ln A - \ln \frac{N_1}{N_2} d\alpha_1 \right) \right. \\
+ \frac{\alpha_1}{1 - u_1} \left( dz_1 - d\ln A - \ln \frac{N_1}{N_2} d\alpha_1 \right) \\
- \frac{\gamma \alpha_1}{u_1} \left( d\ln \frac{\alpha_2}{\alpha_1} - d\ln \frac{l_2}{l_1} \right) \left\} \right., 
\]  

(2.11)
where

$$\phi = \frac{u_1 u_2 (1 - u_1)(1 - u_2)}{\gamma [\gamma (1 - u_1)(1 - u_2) + \alpha_1 u_2 + \alpha_2 u_1 - u_1 u_2]} > 0. \quad (2.13)$$

The first two terms in round brackets in equations (2.11) and (2.12) represent the excess of group-specific wage pressure $z_1$ or $z_2$ over the feasible growth in real wages. This is given by the increase in total factor productivity ($d \ln A$) and the growth in output that a given sectoral productivity shock would produce at given employment shares ($\ln (N_1/N_2) d \alpha_1$). This last term, although depending on $d \alpha_1$, cannot be interpreted as a pure imbalance factor and therefore is not included into the net relative demand shock. It represents the output gain (loss) that both groups enjoy (suffer) when sectoral productivities change.

Equations (2.11) and (2.12) show that group-specific unemployment depends both on own excess wage pressure, and on the other group excess wage pressure, with the former having clearly a greater impact than the latter. This second effect derives from the fact that the two factors are co-operant in the production function: higher wage pressure for one type of labour implies lower demand for the type involved and lower employment, which decreases marginal productivity of the other type of labour.

As the last term shows, sectoral unemployment rates are clearly negatively related to net relative demand shocks hitting the group involved. For realistic values of the $u_i$'s, the magnitude of the impact depends positively on the initial level of the unemployment rate of the group. This non-linearity therefore provides a rationale for why aggregate unemployment should rise as a consequence of a net relative demand shock towards skilled labour. Under the plausible assumption that skilled workers have lower unemployment rates than the unskilled, $u_2$ will experience a greater absolute rise than the fall in $u_1$.

Everything else being equal, perfectly balanced changes in sectoral demand and supply - implying a zero mismatch index - are consistent with constant sectoral unemployment rates and real wages. This result naturally defines a neutrality condition of
this model. Neutral changes in relative labour demand and labour supply are such that sectoral unemployment rates and wages are unaffected. This is a stronger condition than the one outlined in section 2.3, where we showed that, along the labour demand schedule, a zero mismatch index implies constant relative wages and unemployment rate differentials between sectors.

Note incidentally that if one used an absolute mismatch index such as $d \ln(\alpha_i/l_i)$, there would be no guarantee that the unemployment rates of both groups stay unchanged, unless the index were zero for both groups, a condition which only holds locally, and if the starting values for $\alpha_1/l_1$ and $\alpha_2/l_2$ are the same. By the same token, for a given change in $\ln(\alpha_1/l_1)$, $\ln(\alpha_2/l_2)$ might increase, decrease or even stay unchanged, the reason being that no constraint hinges on the direction of proportional increases in the two net demand indicators. In the face of a skill-biased change, a model relying on such an index of mismatch would therefore imply no clear correlation between sectoral unemployment rates or between relative wages, which might vary in either the same or opposite directions, depending on the starting values of the demand and supply indexes. We think of this as being an undesirable property of a model trying to explain changes in the relative demand and supply of inputs, since it does not stick to any clear definition of neutrality.

Having divided the labour force into two groups, the aggregate unemployment rate is given by $u = u_1 l_1 + u_2 l_2$. Therefore

$$
du = u_1 dl_1 + u_2 dl_2 + l_1 du_1 + l_2 du_2
$$

$$
= (u_1 - u_2) dl_1 + \phi_1 \left( dz_1 - d \ln A - \ln \frac{N_1}{N_2} d \alpha_1 \right) + \phi_2 \left( dz_2 - d \ln A - \ln \frac{N_1}{N_2} d \alpha_2 \right)
$$

$$
+ \phi_3 \left( d \ln \frac{\alpha_1}{\alpha_2} - d \ln \frac{l_1}{l_2} \right),
$$

where
The first term in equation (2.14) is just a compositional effect, due to the migration of the labour force from one unemployment group to the other. It tends to have a negative impact on unemployment, since the proportion of the labour force in the high unemployment category (i.e. the unskilled group for the vast majority of countries) has shrunk over time.

The other terms illustrate instead the effect of the variation in sectoral unemployment rates. We find once more the excess wage pressure terms and, finally, the net relative demand shock (or mismatch term). It follows that a net relative demand shift towards skilled workers and away from the less-skilled (i.e. \( d \ln(\alpha_1/\alpha_2) - d \ln(l_1/l_2) > 0 \)) increases total unemployment if \( l_2\alpha_1/u_1 > l_1\alpha_2/u_2 \). Given that \( \alpha_1/\alpha_2 = w_1N_1/w_2N_2 \), the relevant condition becomes

\[
\frac{w_1}{w_2} > \frac{u_1/(1-u_1)}{u_2/(1-u_2)}. \tag{2.15}
\]

Condition (2.15) says that, other things equal, a net relative demand shift favouring the group with better labour market prospects (higher wages and/or lower unemployment) will tend to increase total unemployment. This is a natural prediction of a non-linear model such as the one described. Any increase in dispersion, either in wages or unemployment rates, generated by exactly symmetric shocks for the categories involved, is bound to increase unemployment, due to the convexities in the underlying relationships.

The whole discussion above is based on the assumption of homogeneous wage flexibility across skill groups. Some studies find however that the elasticity of pay is higher...
among low-skill workers (see Gregg and Machin, 1994 and Blanchflower and Oswald, 1994), although Nickell and Bell (1995) find the reverse.

When wage flexibility varies across skills, it can be shown that

\[
\frac{\partial u_1}{\partial \ln \left( \frac{\alpha_1}{\alpha_2} \right)} = -\frac{\alpha_2 u_1 (1 - u_1)(1 - u_2)}{\gamma_1 (1 - u_1)(1 - u_2) + \frac{\gamma_2}{\gamma_1} \alpha_1 (1 - u_1) u_2 + \alpha_2 (1 - u_2) u_1} \tag{2.16}
\]

and

\[
\frac{\partial u_2}{\partial \ln \left( \frac{\alpha_1}{\alpha_2} \right)} = \frac{\alpha_1 u_2 (1 - u_1)(1 - u_2)}{\gamma_2 (1 - u_1)(1 - u_2) + \alpha_1 (1 - u_1) u_2 + \frac{\gamma_2}{\gamma_1} \alpha_2 (1 - u_2) u_1}. \tag{2.17}
\]

Therefore the impact of mismatch on \( u_1 \) is greater in absolute value the higher \( \gamma_2 \) and the lower \( \gamma_1 \), and conversely for \( u_2 \). Given that the variation in \( u_1 \) due to mismatch is generally negative \((d \ln (\alpha_1/\alpha_2) - d \ln (l_1/l_2) > 0)\), having \( \gamma_2 < \gamma_1 \) gives a greater fall in \( u_1 \) and a smaller rise in \( u_2 \), therefore producing a smaller rise in aggregate unemployment with respect to the case \( \gamma_2 = \gamma_1 \). If instead \( \gamma_1 > \gamma_2 \), the effect on aggregate unemployment is magnified. One way of reducing the impact of sectoral labour demand and supply shocks on unemployment is therefore to increase wage flexibility for the "losers" and reducing it for the "winners".

### 2.4.2 Evidence

We are now in a position to assess quantitatively the impact of mismatch on sectoral and aggregate unemployment, using equations (2.11), (2.12) and (2.14). In doing so we restrict the analysis to the countries for which we have data on wages and exclude the Netherlands, since clearly no rise in unemployment can be explained there. If anything, the trend in net relative demand for skills is responsible for a decrease in unemployment and some other explanations (that we have generally labelled as wage pressure) must
be put forward to account for the increase in the rate of joblessness.

Our set of countries is fairly representative. It includes three European countries, with high and increasing unemployment and no increase in wage differentials, the US, with no significant increase in unemployment and widening wage differentials, and finally Britain, situated somewhere between these two extremes.

Figure 2-6 plots time series of different mismatch indicators for this set of countries. The diagrams reported in the first column plot the evolution of the unemployment rate $u$, our mismatch index $mm = \ln(\alpha_1/\alpha_2) - \ln(l_1/l_2)$ and the $LNJ$ mismatch index $LNJ = l_1(u_1/u - 1)^2 + l_2(u_2/u - 1)^2$ in each country. For comparison purposes, starting values of all this variables are standardised to 1. In general the $LNJ$ tend to fluctuate more than the unemployment rate, which in turns fluctuate more than the $mm$ index. If anything, the long term pattern of the unemployment rate seems to be better tracked by the $mm$ than the $LNJ$ index. The diagrams of the second column report the Nickell and Bell index for both the skilled ($NBskilled = \ln(\alpha_1/l_1)$) and the unskilled ($NBunskilled = \ln(\alpha_2/l_2)$) respectively. As noted above, according to this index relative demand seems to fall for both groups in the set of countries considered.

The rest of this sections adopts a "decomposition" approach for the total change in unemployment, based on equations (2.11), (2.12) and (2.14). The results of this exercise are reported in Tables 2.3-2.5, where we estimate the average annual change in skill-specific and aggregate unemployment and the contribution brought about by mismatch. The latter is measured by the last term in (2.11), (2.12) and (2.14), being given by the mismatch index $(d\ln(\alpha_1/\alpha_2) - d\ln(l_1/l_2))$, multiplied by the sample average of the relevant variables $(-\gamma\alpha_2/u_2, \gamma\alpha_1/u_1, \phi_3$ respectively).\textsuperscript{10} Two alternative values of the parameter $\gamma$ are used in turn: 0.1 and 0.035. On the basis of the existing evidence, they can be taken as upper and lower bounds for the actual value of real wage flexibility (see Blanchflower and Oswald, 1994 and Manacorda and Petrongolo, 1996).

\textsuperscript{10}It should be noted that, given the generalised trends in $\alpha_1, \alpha_2, u_1,$ and $u_2$, the use of initial versus average values would predict a higher impact of mismatch (in absolute value) on skilled unemployment and a lower impact on unskilled unemployment.
Figure 2-6: The unemployment rate and various mismatch indexes in 5 OECD countries. Values standardised to 1 in the first year of the sample. Source: see Data Appendix.
Table 2.3: Annual changes in the skilled unemployment rate and impact of mismatch (×100)

<table>
<thead>
<tr>
<th>Countries</th>
<th>Unemployment</th>
<th>Impact of mismatch</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$u_1$</td>
<td>rigid wages</td>
</tr>
<tr>
<td>Britain</td>
<td>0.18</td>
<td>-0.18</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>France</td>
<td>0.24</td>
<td>-0.07</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Germany</td>
<td>0.22</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Italy</td>
<td>0.06</td>
<td>-0.15</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>US</td>
<td>0.00</td>
<td>-0.07</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
</tr>
</tbody>
</table>

Notes. Changes are computed by interpolating a linear trend on the series. Standard errors are reported in brackets. Sample sizes and number of observations are those used in Table 2.1. Rigid wages: $\gamma = 0.035$. Flexible wages: $\gamma = 0.1$.

Table 2.4: Annual changes in the unskilled unemployment rate and impact of mismatch (×100)

<table>
<thead>
<tr>
<th>Countries</th>
<th>Unemployment</th>
<th>Impact of Mismatch</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$u_2$</td>
<td>rigid wages</td>
</tr>
<tr>
<td>Britain</td>
<td>0.39</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>France</td>
<td>0.58</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>Germany</td>
<td>0.88</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>Italy</td>
<td>0.43</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>US</td>
<td>0.14</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.06)</td>
</tr>
</tbody>
</table>

Notes. Changes are computed by interpolating a linear trend on the series. Standard errors are reported in brackets. Sample sizes and number of observations are those used in Table 2.1. Rigid wages: $\gamma = 0.035$. Flexible wages: $\gamma = 0.1$. 

49
Table 2.5: Annual changes in the aggregate unemployment rate, impact of mismatch and compositional effect (×100)

<table>
<thead>
<tr>
<th>Countries</th>
<th>Unemployment $u$</th>
<th>Impact of mismatch</th>
<th>Compositional effect</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>rigid wages</td>
<td>flex wages</td>
<td></td>
</tr>
<tr>
<td>Britain</td>
<td>0.29</td>
<td>0.13</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>France</td>
<td>0.36</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Germany</td>
<td>0.35</td>
<td>0.07</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Italy</td>
<td>0.38</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>US</td>
<td>0.05</td>
<td>0.07</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.03)</td>
<td>(0.01)</td>
</tr>
</tbody>
</table>

Notes. Changes are computed by interpolating a linear trend on the series. Standard errors are reported in brackets. Sample sizes and number of observations are those used in Table 2.1. Rigid wages: $\gamma = 0.035$. Flexible wages: $\gamma = 0.1$.

Tables 2.3 and 2.4 report the annual changes in skill-specific unemployment and the contribution of the increased imbalance between the demand and supply of skills. For example: in France the unemployment rate for the skilled has grown by nearly two and a half percentage points over the last decade.

In all of the five countries there is a tendency for mismatch to reduce skilled unemployment and to increase unskilled unemployment. The estimates provided are somehow sensitive to the value of $\gamma$, implying a greater contribution of mismatch to the change in unemployment when wages are rigid. In no country except the US, is mismatch able to explain all of the rise in the unemployment rate of the low educated, and some increase in wage pressure must be invoked to explain its trend. Analogously, since the increase in mismatch implies a reduction in the skilled unemployment rate, some other factors must be blamed for its modest but generalized increase. Only Britain and the US show a substantial contribution of mismatch to the increase in unskilled unemployment, accounting respectively for more than 50% and about 100% of the total rise.

Moving to aggregate unemployment, Table 2.5 reports the contribution of skill mismatch to the growth in the total rate of joblessness, together with the compositional
effect, i.e. the variation in unemployment brought about by the generalized increase in the educational attainment of the labour force. With the exception of Italy, where the unemployment rate of the skilled is always above the one for the unskilled, the compositional effect implies a decrease in aggregate unemployment of similar magnitude across countries, in the order of 0.05 percentage points a year.

As far as mismatch is concerned, this explains no more than 20% of the increase in unemployment in continental European countries, irrespective of real wage flexibility. Other factors should be hence invoked to explain the bulk of the rise in unemployment. These may be a generalised increase in wage pressure, or an adverse aggregate shock after which the skilled whose jobs are at risk may accept unskilled jobs and further displace the less-skilled.

The relative contribution of mismatch is instead significantly higher in the two remaining countries. Our estimates for Great Britain show that the increased imbalance between the demand and the supply of skills may account for as much as 45% of the total rise in unemployment between 1974 and 1992 if wages are relatively rigid. When we allow for further wage flexibility this reduces to 28%, still leaving some rise in unemployment to be explained by the evolution of wage pressure.\footnote{Along the lines of Nickell and Bell (1995), we also performed an extremely simple supply-side regression for the unemployment rate in Britain. Due to the very short sample period (1974-1992), we only use the real price of oil as a predictor of the change in (log)unemployment (see for example Carruth et al., 1994), together with the mismatch index derived above. We find that including the mismatch index in the regression increases the $R^2$ from 65.8% to 71.3%, and explains a half of the increase in unemployment between 1974 and 1992.}

In the US, there is a sign of increasing mismatch, having the same impact as in Germany and France. US unemployment stays however untrended because its modest increase due to skill mismatch is fully offset by the compositional effect, with no residual rise in unemployment to be explained by wage pressure.

Simulations with $\gamma_1 \neq \gamma_2$ (not reported in the table) give pretty similar results to the ones reported for homogeneous $\gamma$ in France, Germany, Italy and the US, where the change in unemployment explained by mismatch is however fairly small. This change
is instead significant in Britain, and it is therefore worthwhile to report some indicative results for $\gamma_1 \neq \gamma_2$. If $\gamma_1$ is slightly lower than $\gamma_2$, this gives an impact of mismatch that is 31% of the total rise in unemployment when $\gamma_1 = 0.035$ and $\gamma_2 = 0.05$, and 25% when $\gamma_1 = 0.08$ and $\gamma_2 = 0.1$. Conversely, if $\gamma_1$ is slightly higher than $\gamma_2$, the implied contribution of mismatch to the rise in unemployment is 55% when $\gamma_1 = 0.05$ and $\gamma_2 = 0.035$ and 35% when $\gamma_1 = 0.1$ and $\gamma_2 = 0.08$.

To summarize, although some shift in net demand towards the skilled can be detected in the five countries considered, only in Britain and the US - where wage and unemployment differentials did widen - is its magnitude significant if compared to the actual change in unemployment to be explained. In particular, mismatch has a greater relative importance in the US than in Britain, explaining as much as the whole change in unemployment. In absolute terms, however, in the US mismatch has a smaller impact on unemployment than in Britain, accounting for an annual increase of about 0.05 percentage points in unemployment. This can be explained by state dependence in the impact of sectoral disequilibria on aggregate unemployment. Higher sectoral unemployment rates have amplified the effect of demand/supply imbalances in Britain, although their magnitude is smaller than in the US.\(^{12}\)

Overall, Britain stands out as being the only country where the lack of adjustment in the supply of skills severely affected the performance of the labour market.

### 2.5 Concluding remarks

The main concern of this chapter consisted in assessing the role played by the imbalance between the demand and the supply of skills in shaping the evolution of labour market performances across OECD countries over the last two decades. The analysis is guided

\(^{12}\)The results found in Manacorda and Petrongolo (1996) are quantitatively similar to these. There we use the estimates resulting from a British and a US wage equation in turn, to calculate all the terms in (2.11)-(2.14), and therefore reproduce the whole evolution of actual unemployment in the two countries.
by a simple theoretical framework where aggregate technology is characterized by a Cobb-Douglas production function involving two inputs (skilled and unskilled labour), and wage-setting is governed by a double-log wage function. Although rather simplified, this model seems to perform quite satisfactorily on our data, and proves rather enlightening in understanding the effect of skill mismatch on aggregate unemployment.

When the relative demand and supply of skills grow in the same proportion, everything else being equal we expect both relative wages and sectoral unemployment rates to be unaffected. This is therefore the definition of neutrality that stems from our model. When instead demand and supply of skills do not grow in line with each other, we expect that either the wage structure changes or sectoral unemployment rates move in opposite directions, or both.

The first and probably incontrovertible result that stems from our data is that the demand for skills increased steadily in Western countries during the last two decades and probably long before. In many OECD economies this tendency in the evolution of relative demand for skills was not matched by an equal increase in relative supply. This fact, which helps to rationalize the different evolution of unemployment by education and wage differentials in the set of countries considered, was not however a homogeneous phenomenon. It took place with different intensity in different countries, and in some of them, notably Italy and the Netherlands, its trend was negligible or even negative. In contrast, its magnitude was remarkable in Britain and the US during the 1980s, where in fact the difference between skill-specific unemployment rates was rising, and wage differentials widened dramatically.

By estimating our model, we finally assess the quantitative importance of skill mismatch on the evolution of unemployment across OECD economies. Skill mismatch did not cause serious labour market problems in continental Europe, where wage differentials did not widen and the bulk of the rise in unemployment could not be blamed on an unbalanced evolution of the demand and the supply of skills. In Britain instead, skill mismatch explains a substantial part of the nearly 6 percentage points' increase
in unemployment, between 28% and 45%, across different realistic levels of real wage flexibility. Finally, in the US there has been some skill mismatch, but the relatively low starting value of unemployment in this country has kept the impact of mismatch limited in magnitude.

Our results also show that some relevant increase in what we labelled as wage pressure must have taken place in Europe, to explain the residual in the unemployment growth that is not accounted for by mismatch, while there seems to have been no appreciable increase in wage pressure on the other side of the Atlantic. This finding is consistent with the evidence that real wages have grown in Europe by 1.7% per annum during the 1970s and the 1980s, but only by 0.4% in the US (see Bentolila and Dolado, 1994).

It is finally worthwhile to spend a few words on other types of mismatch and their possible impact on unemployment. This work has focused on mismatch along the dimension of education, that can be considered a good proxy for skills. Our index of skill mismatch implicitly accounts for the effect of mismatch along those dimensions that are correlated to education, but it does not include the effect of mismatch along dimensions that are orthogonal to it. On the whole, we believe that there have not been major sectoral shocks along dimensions that are totally uncorrelated to education, so that skill mismatch should reasonably represent recent structural disturbances in the labour market.

2.6 Technical appendix: The \( n \)-groups case

With constant returns to scale Cobb-Douglas technology described as

\[
Y = A \prod_{i=1}^{n} N_i^{\alpha_i}, \quad i = 1, \ldots, n
\]  

(2.18)

labour demand for each input \( i \) is given by \( w_i = \alpha_i(Y/N_i) \), or
\[ \ln w_i = \ln A + \ln \alpha_i + \sum_{i \neq j} \alpha_j \ln N_j - \ln N_i \]
\[ = \ln A + \ln \alpha_i - (1 - \alpha_i) \ln (1 - u_i) + \sum_{i \neq j} \alpha_j \ln (1 - u_j) \]
\[ - (1 - \alpha_i) \ln \frac{l_i}{l_n} + \sum_{i \neq j} \alpha_j \ln \frac{l_j}{l_n}, \] (2.19)

where type \( n \) labour has been chosen as a numeraire. Combining this expression with a wage-setting function of the kind

\[ \ln w_i = z_i - \gamma \ln u_i, \quad i = 1, \ldots, n \]
gives the following expression for the determination of the change in \( u_i \)

\[ du_i = \frac{u_i(1 - u_i)}{(1 - \alpha_i)u_i + \gamma(1 - u_i)} \left( dz_i - d \ln A - \sum_{i \neq j} \ln \frac{N_i}{N_j} d \alpha_i \right) \]
\[ - d \ln \alpha_i + (1 - \alpha_i) d \ln \frac{l_i}{l_j} + \sum_{i \neq j} \frac{1 - \alpha_i}{1 - u_j} du_j \] (2.20)

Given that \( \sum_j d \alpha_j = \sum_j \alpha_j d \ln \alpha_j = 0 \), it follows that

\[ d \ln \alpha_i = d \ln \alpha_i - \sum_j \alpha_j d \ln \alpha_j = \]
\[ = (1 - \alpha_i) d \ln \frac{\alpha_i}{\alpha_n} - \sum_{i \neq j} \alpha_j d \ln \frac{\alpha_j}{\alpha_n}. \] (2.21)

Substituting this into (2.20) gives
The change in the unemployment rate for each group depends on sectoral demand/supply shocks concerning each type of labour, where sectoral shocks are defined with respect to the demand/supply conditions of the group chosen as the numeraire. The change in aggregate unemployment is obtained as

\[ du = \sum_i u_i (1 - u_i) \left( dz_i - d \ln A - \sum_{i \neq j} \frac{N_i}{N_j} d \alpha_i \right) - (1 - \alpha_i) \left( d \ln \frac{\alpha_i}{\alpha_n} - d \ln \frac{l_i}{l_n} \right) + \sum_i \alpha_j \left( d \ln \frac{\alpha_j}{\alpha_n} - d \ln \frac{l_j}{l_n} \right) + \sum_{i \neq j} \frac{\alpha_j}{1 - u_j} du_j \]  

(2.22)

The change in the unemployment rate for each group depends on sectoral demand/supply shocks concerning each type of labour, where sectoral shocks are defined with respect to the demand/supply conditions of the group chosen as the numeraire. The change in aggregate unemployment is obtained as \[ du = \sum_i u_i dl_i + \sum_i l_i du_i, \] where the \( du_i \)'s are evaluated using (2.22).

2.7 Data Appendix

2.7.1 Employment, labour force and unemployment


- **France.** Sample: 1978-1994. Source: *La Population Active d'Apres l'Enquete*
Emploi, INSEE. Selection criteria: males and females, 15 years old and over. Skilled: with baccalauréat general or vocational qualification (CAP or BEP). Unskilled: without either of the above qualifications.


- **Great Britain.** Sample: 1974-1992. Source: General Household Survey individual record files. Selection criteria: males, 16-64 years old; females, 16-60 years old. Skilled: with A-level (or equivalent), including senior vocational qualification. Unskilled: with O-level (or equivalent), including junior vocational qualification.


- **Spain.** Sample: 1977-1993. Source: Encuesta de Poblacion Activa, INE. Selection criteria: males and females, 16 years old and over. Skilled: some college (nivel


2.7.2 Wages

(same skill partition as above)


• **United States** Sample: 1970-1989. Source: Annual demographic files, March Current Population Survey (Outgoing Rotation Group). Selection criteria: wage and salary earners, males and females, 16-69 years old, working at least 40 weeks and earning more than one half the minimum wage on a full time basis. Earning concept: weekly gross wages (annual earnings divided by number of weeks worked). Our thanks to Steve Davis for having provided the data.
Part II

An empirical search approach to the labour market
Chapter 3

Looking into the black box: A survey of empirical matching functions

3.1 Introduction

Since the early work of Marston (1976) and Clark and Summers (1979), the study of labour market flows has been pursued actively in the United States in the late 1980s and early 1990s, generating a highly influential literature on the so called “flow approach to labour markets”. This approach has developed a view of labour markets which builds up from the flow of workers and jobs as those characteristics that are central to the understanding of aggregate evolutions (see Blanchard and Diamond, 1992).

So what are the basic facts underlying the “flow approach”? Blanchard and Diamond (1989, 1990a) note that labour markets in the US are characterized by high worker flows and turnover rates, and compute that nearly seven million of workers move into or out of employment every month. Although these transitions could be consistent with individuals reallocating themselves across a given set of jobs, Davis and Haltiwanger (1990) use longitudinal establishment data to show that labour turnover
is actually associated with substantial gross job creation and job destruction.\footnote{Similar results are also found by Leonard (1987) and Dunne et al. (1989).} They estimate in fact that the average annual rates of job creation and job destruction in manufacturing were respectively 9.2\% and 11.3\% of employment between 1972 and 1986. Among the characteristics of the job reallocation process, they find relevant persistence of establishment-level employment changes and note the significant countercyclical pattern of job reallocation, being the sum of (mildly procyclical) job creation and (strongly countercyclical) job destruction.

Blanchard and Diamond (1990a) confirm most of the basic findings of Davis and Haltiwanger on job flows and investigate the pattern of gross worker flows across three labour market states. Interestingly enough, both flows in and out of unemployment seem to increase in a recession. This is in turn motivated on the basis of a dual labour market model in which there are “primary” workers who do not quit, and always dominate “secondary” workers in firms’ hiring decisions. This idea is extensively developed in Blanchard and Diamond (1994).

According to Burda and Wyplosz (1994), large flows between employment, unemployment and inactivity also characterize European labour markets, and the cyclical behaviour of these movements closely resemble the pattern of US labour market flows.

From a macroeconomic perspective, substantial labour market flows imply the coexistence of unfilled jobs and unemployed workers in each segment of the economy. The coexistence is basically the results of two factors: the intensity of the worker and job reallocation processes, and the effectiveness of the labour market in matching unemployed workers to available vacancies. The equilibrium relationship between the unemployment rate and the vacancy rate in an economy (known as the $UV$ curve) is therefore a useful analytical tool for assessing the degree of labour market matching effectiveness.

The $UV$ curve has a long history in the study of labour markets dynamics. Its existence was noted by Beveridge (1944), after whom the curve is often named.

From a theoretical viewpoint, the curve has its microfoundations in search models
of the labour market in which firms post vacancies and unemployed workers apply for jobs. The main underlying idea is that job search process is imperfect, in the sense that it is costly and time-consuming. Its nature can be represented by a random matching or hiring function (whose basic idea is found in Holt, 1970) that tells how many newly filled jobs an economy creates at each point in time, given the number of unfilled vacancies and unemployed job-seekers existing in the market. A rich development and synthesis of important contributions in this literature is found in Pissarides (1990).

Earlier work by Hansen (1970) derives instead the existence of the $UV$ curve from an aggregation process across distinct labour markets. Given that the short side of the market needs not to be the same in all segments of the economy, local disequilibria are consistent with the coexistence of unemployment and vacancies at the aggregate level. Other studies that follow this approach are Drèze and Bean (1990), Bentolila and Dolado (1991) and Franz (1991). Although this approach does not build on the existence of a matching function, it can be shown that it is consistent with it.

The labour market matching function can be expressed as follows

$$M = \phi m(U, V),$$

with $m(.)$ increasing and concave in both arguments, and $m(U, 0) = m(0, V) = 0$. $M \leq \min(U, V)$ denotes the flow of successful matches generated by the stock of unemployed workers $U$ and vacancies $V$, and $\phi$ is a scale parameter of the matching technology. A number of other variables which do not appear in (3.1) traditionally affect the overall matching effectiveness, such as search effort, heterogeneity of agents and different aspects of mismatch. Their global effect is here simply embodied into $\phi$.

A long-run relationship between the unemployment rate and the vacancy rate derives from (3.1) imposing constant returns to scale in $m(.)$ and noticing that, in steady state, the number of matches $M$ equals the number of job separations $sN$, with $s$ denoting the separation rate and $N$ denoting total employment. This gives
\[ s = \phi m \left( \frac{U}{N}, \frac{V}{N} \right) \] (3.2)

as a representation of the UV curve. Given the separation rate and the characteristics of \( m(.) \), we therefore expect a negative steady-state relationship between the unemployment rate (approximated by \( U/N \)) and the vacancy rate (\( V/N \)) in an economy.

An aggregate UV curve such as equation (4.2) is empirically supported by a number of studies (Jackman and Roper, 1987, Budd, Levine and Smith, 1988 and Jackman, Layard and Pissarides, 1989 for Britain, Abraham, 1987 for the US, Franz, 1987 for Germany, Edin and Holmlund, 1991 for Sweden, and Brunello, 1991 for Japan). They establish the existence of a negative long-run relationship between the vacancy rate and the unemployment rate in most OECD countries. Furthermore, most of them note that the curve is highly unstable in Europe, revealing some deterioration in the matching effectiveness of European labour markets since the early 1970s, when the unemployment rate started its secular rise, despite the absence of any long-run pattern in the vacancy rate.

Estimation of log-linear UV curves, along the lines followed by most of the studies mentioned, may involve some problems, mainly connected with the assumption of constant returns to scale in the matching function, the assumption of flow equilibrium, and the endogeneity of the separation rate. Probably due to these shortcomings, towards the end of the 1980s most of the empirical literature on labour market flows has devoted to the direct estimation of a matching function such as (3.1). Relevant empirical studies provide an incredibly wide span of results, estimating aggregate time-series functions for the whole economy or for some sector (most frequently manufacturing), panel functions for regions or districts, and hazard functions for individual re-employment probabilities.

In this chapter we provide a survey of the empirical search literature that has estimated labour market matching functions. Most frequent econometric problems encountered in estimation are discussed, together with the solutions generally proposed.

The plan of the chapter is as follows. Section 3.2 sketches the methodologies most
frequently adopted in the estimation of the matching function and introduces a few benchmark studies in the field. Section 3.3 deals with measurement issues in the estimation of the matching function, connected with the identification of the relevant pool of job-searchers and the destination of those who leave the pool. Aggregation problems, both across time and space, are discussed in section 3.4 and 3.5 respectively. Section 3.6 addresses instead the issue of the returns to scale in the matching technology. Finally, section 3.7 concludes the chapter.

3.2 Empirical matching functions

Pioneer results in the estimation of matching functions of the labour market are presented in Pissarides (1986), who estimates an aggregate matching function for Britain over the period 1967-1983. The specification adopted uses the monthly outflow rate from male unemployment as the dependent variable, i.e.

\[
\ln \left( \frac{M}{U} \right)_t = \alpha_0 + \alpha_1 \ln \left( \frac{V}{U} \right)_t + \alpha_2 \ln \left( \frac{U}{L} \right)_t + \alpha_3 \ln \left( \frac{V}{L} \right)_t + \alpha_4 t + \alpha_5 t^2 \tag{3.3}
\]

where \( M \) here denotes the number of all those who leave the unemployment register during the measuring interval. Lags of the dependent and the independent variables are included to capture off-steady state dynamics in unemployment and vacancies, and structural variables such as the replacement ratio and the level of mismatch in the economy are proxies for structural shocks to the matching technology. A higher replacement ratio or greater turbulence in the economy should in fact deplete the number of hires at constant unemployment and vacancies. Level variables such as the unemployment \((U/L)\) and the vacancy rate \((V/L)\) should capture any "scale effect" in the matching technology. The fact that they turn out to have a non-significant impact on the unemployment outflow rate suggests the presence of constant returns to scale in the function.
The estimated elasticity of matches with respect to unemployment is found to be around 0.7.

Later work by Blanchard and Diamond (1989, 1990b) estimates a matching function for the US over the period 1968-1981. The estimated equation is a simple log-linear specification in levels such as

\[ \ln M_t = \alpha_0 + \alpha_1 \ln U_t + \alpha_2 \ln V_t + \alpha_3 t, \]  

(3.4)

where the log of monthly national hirings is used as the dependent variable. Estimation is performed using both OLS and IV in turn, but the results are not too sensitive to the estimation method or the specification adopted: the elasticities of matches with respect to vacancies and unemployment are positive and highly significant, and the time trend generally comes in with a negative, significant coefficient, implying a deterioration in the matching effectiveness of the labour market since the late 1960s. Overall, they find clear evidence of the existence of such an aggregate relation, and conclude that “the hiring process is well described by a matching function with constant or mildly increasing returns to scale, unit elasticity of substitution, and relative weights of 0.4 and 0.6 on unemployment and vacancies”.

The substantially lower elasticity of matches with respect to unemployment than in Pissarides (1986) is mainly explained by the different dependent variables used in the two studies. The unemployment outflow, used by Pissarides (1986), is expected to be more highly correlated to the unemployment stock than to the vacancy stock, given that it not only includes flows into employment, but also exits from the labour force, that have little to do with the supply of vacancies. Fairly high elasticities of matches with respect to unemployment are in fact also found by Burda and Wyplosz (1994), who estimate log linear matching functions for France, Germany, Spain and the UK by regressing total exits from unemployment on vacancy and unemployment stocks.

By contrast, the measure used by Blanchard and Diamond (1989, 1990b) consists of all new accessions to employment, including job-to-job moves and flows from inactivity
directly into employment on top of unemployment to employment flows, and is therefore highly correlated to the supply of vacancies, while it displays a much lower correlation with the pool of registered unemployed. Blanchard and Diamond however tackle the issue of the relevant pool of job-searchers in their estimates, by explicitly including out-of-labour force job-seekers into the matching function. These and related results will be described in the next section, where measurement problems of the relevant quantities in the matching function will be discussed.

Blanchard and Diamond (1989, 1990b) also estimate equation (3.4) for the US manufacturing sector alone. The results that they obtain in this case are broadly consistent with the aggregate ones, with the important qualification that the sectoral matching function displays increasing rather than constant returns to scale.

These early results were taken as a benchmark by an incredibly large number of studies in the following years, which further investigated the issue of the returns to scale of the matching function and pointed at several problems connected with its estimation. On the whole, the general tendency that developed since the early 1990s was to switch from time-series, aggregate matching functions to more disaggregate specifications, either in panel form or pure cross-sections.

Anderson and Burgess (1995) estimate a panel of state-industry observations for the US over the period 1978-1984, using a similar specification to Blanchard and Diamond (1989), having included a few demographic and union variables, and distinguishing whether new hires come from employment or non-employment. Their results confirm a sensibly higher elasticity of matches with respect to vacancies than to unemployment, and provide more clear-cut evidence in favour of increasing returns to scale in the matching technology.

In an attempt to apply the matching function analysis to local labour markets, Coles and Smith (1996) and Bennet and Pinto (1994) both provide cross-section estimates of the matching function for self-contained labour markets in Britain. Local labour markets are represented in Coles and Smith (1996) by travel-to-work areas, where people
tend to live and work. Their study shows the importance of the geographic density of unemployment and vacancies in the hiring process, in the sense that more concentrate labour markets have higher matching rates. The analysis of Bennet and Pinto (1994) refers instead to Training and Enterprise Councils, finding that the parameters of the matching technology do not vary sensibly across districts, and therefore denying the existence of major aggregation problems. Both these studies provide a successful empirical microfoundation of the aggregate matching function.

Finally, it is worthwhile mentioning those contributions that studied the determinants of individual re-employment probabilities following a hazard function approach. Re-employment rates have the potential of explaining different stages of the search process, being the combination of two probabilities: the probability of receiving a job offer and the probability of accepting the offer. The first of these depends on the set of characteristics that describe a worker's productivity (such as age, education, experience, etc.) and on labour demand conditions. This latter effect is basically the only one captured by aggregate matching functions. The second probability depends on a worker's reservation wage, and therefore on the expected distribution of wages, the cost of search, unemployment income and the probability of receiving a job offer.

With respect to aggregate matching functions, hazard function specifications have the main advantage of being rather flexible. They allow for a wide spectrum of functional forms for duration distributions, and control for a number of individual characteristics whose importance is only implicit in an aggregate matching function. In particular, they can introduce duration dependence of exit rates from unemployment, which is generally controlled for in aggregate estimates by conditioning job formation on single ad hoc regressors such as the incidence of long term unemployment.

The benchmark study in this literature is probably the one by Lancaster (1979), that uses a sample of British unskilled male workers to show that exit rates from unemployment are negatively affected by the local unemployment rate and the duration of search. Following this pioneer application of duration models to re-employment prob-
abilities, a large number of papers have studied the determinants of exits rates from unemployment, looking for example at the effect of unemployment income (see Nickell, 1979 and Narendranathan, Nickell and Stern, 1985), of training (see Ridder, 1988), or concentrating on youth employment probabilities (see Lynch, 1985, 1989 and Imbens and Lynch, 1993). In conditioning on the state of the local labour market, only two of them (Nickell, 1979 and Atkinson, Gomulka, Micklewright and Rau, 1984) actually take into consideration the labour demand side and employers' search, by controlling for the local vacancy-unemployment ratio. In what follows we will not review hazard studies individually, and address the interested reader to the excellent survey by Devine and Kiefer (1991). We will instead concentrate on the more recent time-series or panel studies that estimate reduced-form types of matching functions for mainly two reasons. First, this literature is pretty young, and lacks for the time being a comprehensive review. Second, despite the numerous shortcomings involved by aggregate estimates, we believe that it is worthwhile to document to which extent the predictions of search theory for individual behaviour can be reflected in aggregate labour market flows.

We will therefore start in next session with the discussion of an important estimation issue with aggregate data: the measurement of the relevant pool of job-searchers and of the number of successful matches.

### 3.3 Measurement problems and on-the-job search

Following Pissarides (1990), we start by assuming that search and production in the economy are two completely separate activities. This means that each worker is either employed and producing or unemployed and searching and, similarly, each job is either filled and producing or vacant and searching. Under this assumption, total job creation

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2 Despite a rich literature on the study of unemployment duration, little work has been done so far, mainly because of data limitation problems, on vacancy durations, with the notable exceptions of van Ours and Ridder (1992, 1993).
in the economy is adequately represented by a matching function such as (3.1), in which
the dependent variable $M$ denotes the number of newly formed pairs between $U$ and $V$. Due to limitations in the provision of detailed data on labour market flows, very few studies adopt exactly this measure. This is the case of Burda and Profit (1996) and Boeri and Burda (1996), who measure $M$ as the outflow from registered unemployment due to job finds. In most works, however, different proxies for $M$ are used.

One of the proxies most frequently used is the total number of exits from unemployment, as in Pissarides (1986) and Burda and Wyplosz (1994). As mentioned above, Pissarides (1986) estimates an outflow rate equation for Britain, suggesting an elasticity of matches with respect to unemployment around 0.7. Burda and Wyplosz estimate Cobb-Douglas matching functions for France (1971-1993), Germany (1968-1991), Spain (1977-1992) and the UK (1985-1993), usually finding an elasticity of matches with respect to unemployment in the range 0.5-0.7, and a much lower elasticity with respect to vacancies. The main disadvantage of this measure is that some of those leaving the unemployment registry flow into inactivity rather than into employment. When a log-linear version of the matching function is estimated, the use of this proxy remains valid, however, if flows from unemployment into inactivity are a constant fraction of total exits from unemployment over the business cycle. Alternatively, under the belief that most transitions out of unemployment for men are into jobs, male unemployment outflows can be used instead, as in Attfield and Burgess (1995).

The total number of hires is the other commonly used measure for $M$, as well as the outflow from the stock of unfilled vacancies, which differs from the number of hires insofar it may include vacancies that are withdrawn by the employer without being filled. Total hires however obviously include job-to-job flows and flows from inactivity into employment. Once more, there are problems connected with the cyclical pattern of these extra components of total hires, especially of job-to-job flows and with their effects on the job-finding probabilities of the unemployed.

This opens the problem of the identification of the relevant pool of searching agents,
which, whenever it is not proportional to the unemployment stock, generates a bias in estimation of (3.1). The relevant question is therefore how important are employment inflows that do not originate out of registered unemployment, and what are their cyclical properties.

Evidence on worker flows in fact strongly suggests dropping the assumption made at the beginning of this section, that only the registered unemployed search, allowing for employed job search and search by those that, although not working, are not registered as unemployed (classified as inactive or out of the labour force).

Blanchard and Diamond (1989) construct a job-to-job flow series for the US assuming that these flows account for 40% of all job quits,\(^3\) and that the quit rate for the economy as a whole is the same as the quit rate in manufacturing. This procedure leads them to conclude that job-to-job movements account for 15% of total hires in the period 1968-1981. The remainder 85% is accounted for by hires from unemployment (45%) and hires from out of the labour force (40%).

Similar information for the UK can be derived from the Employment Audit, that uses the quarterly Labour Force Survey data. Job-to-job moves in 1992 represented as much as 51% of total hires, while flows from unemployment and inactivity represented 21% and 27% respectively. Due to the three months gap between one observation and the next, these data tend to overstate the importance of job-to-job moves (and understate those of the other two flows), given that some individuals may flow into unemployment (or inactivity) and back again into employment before the end of the quarter. Even allowing for some correction, Pissarides (1994) suggests a lower bound for job-to-job moves of 40% of total hires. Elsewhere in Europe, job-switches seem to have lower relative importance than in the UK. Burda and Wyplosz (1994) estimate that in Germany in 1987 job-to-job flows represented 16% of employment inflows, with the rest being shared in equal proportions by unemployment and inactivity outflows. The whole picture for German worker flows does not look too different from the US one.

\(^3\)This is the proportion estimated by Akerlof et al. (1988).
In France, 67% of the employment inflow was represented by unemployment outflows, with job-to-job flows accounting for a mere 10%, and flows out of inactivity for 23%.

While evidence on inactivity inflows and outflows is somehow mixed, a rich body of evidence confirms that job-to-job flows are procyclical, being closely linked to the quit rate.

Burgess (1993) provides a model of competition between employed and unemployed job-searchers, and explains the procyclicality job switches by modelling employed job search on the basis of a reservation wage rule. Employed workers whose wages fall below the (endogenous) reservation wage start searching for a new job. The reservation wage is a decreasing function of the attractiveness of the job market, represented by the likelihood of receiving a job offer. This means that, when the arrival rate of job offers rises in a boom, the employed have a stronger incentive to engage in search, partially crowding the unemployed out of new jobs. On top of this kind of congestion effect, Pissarides (1994) considers the possibility that during a boom employers tend to open vacancies that are particularly attractive to the employed, given that the proportion of these in the pool of job-searchers rises in booms, and tend instead to destroy jobs that employed workers quit, since they would only be acceptable by unemployed job-seekers. This is a further element that enhances the procyclicality of job-to-job flows. More recently, Boeri (1995) combines endogenous employed job search with the possibility that unemployed job search has different intensities at different durations.

Empirical work that explicitly takes into account employed job search and/or out-of-labour-force job search include Blanchard and Diamond (1989), Attfield and Burgess (1995), Boeri (1995) and Mumford and Smith (1997). Blanchard and Diamond (1989) use alternative definitions of the relevant pool of searchers, allowing different types of workers to be perfect substitutes up to a scalar level. They assume a relation of the form

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4Blanchard and Diamond (1989, 1990a) note that the flow of hires from out of the labour force is procyclical in the US, while Burda and Wyplosz (1994) conclude that flows in and out of the labour force do not exhibit any particular cyclical pattern in Europe.
\[ \ln M_t = \alpha_0 + \alpha_1 \ln (X_{1t} + \alpha X_{2t}) + (1 - \alpha_1) \ln V_t, \]  

(3.5)

where \( X_{1t} \) and \( X_{2t} \) denote two possible components of the pool. They estimate (3.5) for the US, using unemployed and inactive people for \( X_{1t} \) and \( X_{2t} \) respectively, and find a non significant coefficient on \( X_{2t} \). Following a similar procedure, Boeri (1995) finds that the unemployment outflow in Germany, Spain and the UK is not affected by employed job search \textit{tout court} but only by search of those on temporary jobs or jobs that are at risk.

For the UK, Burgess (1993) and Attfield and Burgess (1995) find evidence of endogenous job competition, obtaining an elasticity of the unemployment outflow with respect to total hires below 1. The standard matching function in \( U \) and \( V \) is then reinterpreted as a reduced-form relation for the unemployment outflow arising from the simultaneous determination of matching and job competition between employed and unemployed job-seekers, with on-the-job search being expressed as a function of the unemployment level. As a consequence, the estimated properties of the reduced-form equation do not necessarily reflect the properties of the underlying matching function. Similarly, Mumford and Smith (1997) use Australian data to extend the job search competition to those that are out of labour force, and find evidence of inactive people ranking after the unemployed, who in turn rank after the employed in the process of filling vacancies.

### 3.4 Temporal aggregation

The matching function describes a process of job creation that takes place in continuous time within an integrated marketplace. The use of discrete time data to estimate aggregate matching functions induces both temporal and spatial aggregation problems. Time aggregation problems arise when flow variables are estimated as functions of stock conditioning variables. This happens in the empirical production literature,
where a production function is used to describe the flow of output from the stocks of factors of production. Similarly, the empirical search literature generally estimates a matching function that describes the flow of job creation as a function of the stocks of unemployment and vacancies.

In order to analyse the problems involved by this common procedure, we consider for convenience an explicit version of (3.1), imposing a unit elasticity of substitution between $U$ and $V$, as in equation (3.4), and including a well behaved disturbance term $\varepsilon_t$

$$\ln M_t = \alpha_0 + \alpha_1 \ln V_t + \alpha_2 \ln U_t + \varepsilon_t. \quad (3.6)$$

As Blanchard and Diamond (1989) remark, there is no clean way to handle timing in equation (3.6). If $M_t$ is measured as a flow over a time period, and $U_t$ and $V_t$ as stocks at some point during the period, stocks $U_t$ and $V_t$ are depleted by matches $M_t$, and this generates a downward bias in the estimated coefficients $\alpha_1$ and $\alpha_2$. This problem is often dealt with using beginning-of-period stocks $U_{t-1}$ and $V_{t-1}$ as conditioning variables or as instruments for $U_t$ and $V_t$. If there is no serial correlation in the error term, lagged stocks $U_{t-1}$ and $V_{t-1}$ are uncorrelated with $\varepsilon_t$ and therefore perform as proper instruments.

Even so, whatever stock variable is used on the right hand side of the equation, the dependent variable is mismeasured, being the aggregated flow over a time interval. In particular, the measured outflow over some time interval does not only include the outflow from the initial stock, but also the outflow from the inflow over the same interval. This can generate a situation in which the total outflow during the interval exceeds the initial stock. This is actually the case for vacancies, whose average completed duration is in most cases under a month. So, whenever the data frequency is monthly or lower, an equation such as (3.6) would make little sense.

Here we try to describe this problem more formally using an exponential probability distribution of duration, characterized by constant hazard with respect to duration.

With hazard rate $\lambda$, the survivor probability of an agent is given by $R(t) = \exp(-\lambda t)$,
with \( t \) denoting the elapsed duration of search. The probability of being matched (or outflow rate) over a time period of length \( t \) is therefore \( F(t) = 1 - \exp(-\lambda t) \). The total number of matches from a given stock of unemployed and vacancies can be then computed as either \( V[1 - \exp(-\lambda V t)] \) or \( U[1 - \exp(-\lambda U t)] \).

Let's consider a period of time whose length is set equal to 1. Supposing that we have an initial stock of searching agents \( S \) (that can either represent unemployed workers or vacancies), and a subsequent inflow \( I(t) \), total matches are given by

\[
M = (1 - e^{-\lambda}) S + \int_0^1 [1 - e^{-\lambda(1-t)}] I(t) dt
\]  

(3.7)

where the first term denotes the outflow from the initial stock and the second denotes the outflow from the inflow. Estimating (3.6) on discrete data using beginning-of-period stocks as conditioning variables therefore leaves out the number of matches represented by the second term in (3.7).

Under the simplifying assumption of uniform inflow \( I \) during the whole period, we have

\[
M = (1 - e^{-\lambda}) S + \left[ 1 - \frac{1}{\lambda} (1 - e^{-\lambda}) \right] I.
\]  

(3.8)

It can be noted that the term in square brackets is bounded between zero and one, and therefore describes a plausible outflow rate from the inflow. Also note that the outflow rate from the inflow is lower than the one from the stock, for the reason that the inflow has, on average, less time available for a successful match.

So, how can estimation take into account the matches generated by the inflow \( I \)? Right hand side variables in (3.4) should include the whole set of matching agents during the period over which the number of matches is measured. This will be the beginning-of-period stock, plus some proportion of the inflow. This proportion should control for different outflow rates of stocks and flows, due to the different search time available to the two categories of agents. Given that each agent in \( S \) has a matching probability
which is \[ [1 - \frac{1}{\lambda} (1 - e^{-\lambda})] (1 - e^{-\lambda})^{-1} \] times the matching probability of each agent in \( I \), the pool of matching agents between time 0 and time 1 can be expressed in homogeneous "matching units" as

\[ A = S + \left( (1 - e^{-\lambda})^{-1} - \frac{1}{\lambda} \right) I. \] (3.9)

In order to compute \( A \), the hazard rate \( \lambda \) can be obtained by estimating equation (3.8) on stocks and flows. Alternatively, for small enough \( \lambda \), the term in square brackets in (3.9) can be approximated by \( 1/2 \), using a second order Taylor expansion of \( \exp(-\lambda) \) around \( \lambda = 0 \).

This is the procedure proposed by Gregg and Petrongolo (1996) and used in the next chapter in order to deal with the time aggregation problem in the estimation of an aggregate matching function for the UK for the years 1967-1995. The results in terms of matching effectiveness of vacancies change substantially from the case in which the conditioning variables are measured as pure stocks. Berman (1994) uses instead the sum of beginning-of-period stock and flows over the period to form a proper instrument for \( U_t \) and \( V_t \) in estimating a referral function for Israel over the period 1978-1990.

Burdett, Coles and van Ours (1994) show that the use of beginning-of-period stocks as conditioning variables generates a bias in the resulting elasticities of matches with respect to \( U_{t-1} \) and \( V_{t-1} \), that depends on the time series properties of the two stocks. Suppose that both \( U_{t-1} \) and \( V_{t-1} \) are mean-reverting series, an assumption which is somehow implicit in a matching function where the number of matches is a positive function of \( U_{t-1} \) and \( V_{t-1} \). In this case the average size of a stock over a time period tends to be negatively correlated with the size at the beginning of the period. This implies that, when unemployment (or the number of vacancies) is above the mean, the average size of the stock during the following period will be relatively small, depleting the number of aggregate matches during the period. On the other hand, when the initial stock is below the mean, its size will tend to increase over the following period, generating a higher number of intra-period matches. This mechanism generates a downward
bias in the estimated elasticities of $M_t$ with respect to $U_{t-1}$ and $V_{t-1}$.

It is shown however that, for a small enough measuring interval, the size of the bias can be approximated as a linear function of its length. Thus the size of the bias can be estimated by doubling the length of the measuring interval and comparing the obtained coefficients with those estimated using the original data frequency. This procedure suggests that the bias is not too important whenever the data frequency is monthly or higher and the cycle frequency is yearly or lower.

### 3.5 Spatial aggregation

The other issue that links aggregate production and matching technologies is naturally aggregation across space. Similarly to the empirical production literature, most search literature simply aggregates the number of unemployed workers and open vacancies across space to predict the flow of job creation in the same area.

What spatial aggregation does is to consider an aggregate economy as a single labour market, while being instead a collection of spatially distinct labour markets among which there might or might not be interaction. So the relevant issue is whether aggregating local labour market data might bias the resulting estimates.

Coles and Smith (1996) show that spatial aggregation might bias the results towards constant returns to scale in the matching function, while the matching process could display increasing returns. They argue that, replicating for a thousand times a marketplace, characterized by a given geographical size and endowed with a given number of unemployed workers and vacancies, the total number of matches is multiplied by a thousand if there is no interaction among the identical marketplaces created. However, if the number of unemployed and vacancies is multiplied by a thousand within the original market of given size, the number of matches should be multiplied by more than a thousand.

As a corollary of this, increasing returns to matching in aggregate functions may
only result if there are interactions between single markets that compose the aggregate economy. Therefore aggregating across markets where there is no interaction may bias the estimated results downwards. Moreover, matching is more effective if the market is concentrated, implying that it is the density and not the absolute number of agents which matter.

In order to test whether regional aggregation has biased empirical results on the matching function, Coles and Smith (1996) estimate the function using a cross section of spatially distinct labour markets in England and Wales, defined within travel-to-work areas. They find no substantial difference with respect to the estimates on aggregate matching functions of Blanchard and Diamond (1989) and cannot reject the hypothesis of constant returns to scale, even within narrowly defined markets. When the geographic size of the market is used to control for a density effect, it turns out to have a negative impact on matches, confirming that concentrated markets are more effective.

Constant returns to scale are also not rejected in a similar study by Bennet and Pinto (1994), who estimate separate local matching functions over the period 1985-1991 for 104 areas of Training and Enterprise Councils that cover Britain. They find that most of the estimates for the returns to scale range between 0.7 and 1.15.

A further issue concerns the interaction between local matching conditions and regional migration or commuting behaviour.

The importance of job search considerations in worker migration is explicitly recognized in Jackman and Savouri (1992). They note that the direction of gross migration flows in Britain seems consistent with a job search approach, in which migration is seen as a special case of job matching, and that the magnitude of flows is best explained in time series regressions by the evolution of the total number of job-worker engagements. Regional migration stylized facts are instead difficult to reconcile with the human capital theory predictions, mainly on the ground that high wage regions do not attract significant in-migration.

The effects of regional migration and commuting on local matching conditions are
analysed by Burda and Profit (1996). They represent an aggregate economy as a two-dimensional space divided into a number of districts. Workers in each district engage in search and firms are passive. Workers’ decisions determine search intensity in all districts: how many jobs to apply for - if any - in each district. This extension of the matching function to the spatial dimension relates job creation in a district to economic conditions everywhere in the economy, inducing a network of complex spillover effects between neighbour districts. Burda and Profit estimate a matching function that embodies this kind of spillovers for 76 Czech labour market districts, and find significant effects of neighbour unemployment on local matching. Even when regional spillovers are considered, the hypothesis of constant returns to scale in the matching function cannot be rejected.

Similarly, Petrongolo and Wasmer (1998) estimate a matching function for Britain (1986-1995) and France (1983-1994), using a regional panel for each country. Cross-regional spillovers are considered allowing each worker to search in her own or other regions with different search intensities. It is shown that search intensity is positive and significant in regions that are adjacent to the one where the worker resides, although around 10% of the level of search intensity within the region of residence. Constant returns to scale in the matching function are not rejected by both British and French data.

To conclude, although the problem of spatial aggregation has only been recently perceived in the estimation of labour market matching functions, the findings of those who explicitly embody a spatial dimension into the estimation do not seem to invalidate earlier results on aggregate matching functions.

3.6 Constant or increasing returns to scale?

When search is coordinated by a matching process, the nature of interactions between traders and the externalities that their search decisions generate determine the rela-
relationship between matching efficiency and the size of the economy, as measured by the number of agents that engage in search. In other words, they determine the degree of homogeneity of the matching technology.

Widely used models of bilateral search, such as Pissarides (1990), consider two different kinds of externalities that each agent involved in search generates. Suppose that each agent of one type can only trade with any agent of the other type. An agent’s decision to involve in search generates a positive (thin market) externality, while enhancing the probability of finding a trading partner among agents of the opposite type, and therefore decreasing the cost of search to the other side of the market. At the same time she generates a negative (congestion) externality on agents of her same type, by increasing the number of competitors for potential trading partners, and therefore increasing the cost of search on her same side. It can be argued that the net effect of positive and negative externalities from trade leaves the matching efficiency of a marketplace independent of its size, so that constant returns in the matching technology is a plausible assumption for search models. First glance evidence supporting constant returns is the fact that economies of diverse size such as the UK and US have closely comparable unemployment rates.

Diamond (1982) however argues that, if greater search effort on one side not only decreases the cost of search on the other side but also leads the other side to increase its own search effort, then matching may display increasing returns.

More recent studies by Coles and Smith (1996, 1997) and Coles (1994) take into consideration possible alternatives to a random, space-independent technology, and argue that it is theoretically plausible that the matching technology exhibit increasing returns. Coles and Smith (1996) infer increasing returns to scale from a replication argument, in which replicated marketplaces can interact with one another. However, it is not clear why interactions between spatially distinct markets should not generate both positive and negative trading externalities, whose net result in terms of matching efficiency is ambiguous. Coles (1994) and Coles and Smith (1997) consider instead a
non random matching process, in which the stock of unmatched traders on one side of
the market can only match with the flow of traders on the other side. This derives from
a plausible sampling procedure in which the stock of traders on one side of the market
can only match with the inflow of traders on the other side. In this case, although
a conventional matching function defined over stocks of traders may display constant
or even decreasing returns to scale, the correct matching function in stocks and flows
should exhibit increasing returns. We will come back to this idea in more detail in the
next chapter.

Despite a few contributions on the theoretical possibility that the matching tech­
nology exhibits increasing returns, most search models rest on the assumption that
the matching function is linear homogeneous. The assumption of constant returns to
scale in the matching function is in fact a crucial one, being the element that ensures
a constant unemployment rate along a balanced-growth path, see Pissarides (1990),
Mortensen and Pissarides (1994, 1995) and Aghion and Howitt (1994). Multiple (rank­
able) equilibria arise instead when the matching function exhibits increasing returns to
scale, see Diamond (1982, 1984).

Determining whether there are constant or increasing returns in the matching func­
tion is ultimately an empirical matter. Empirical studies on the matching function
cannot reject in most cases the CRS hypothesis or find, in few cases, evidence of weakly
increasing returns. However, possible misspecification problems (such as those arising
from temporal or geographical aggregation), inducing a downward bias in the resulting
estimates, would still leave the question quite open.

The benchmark study of Blanchard and Diamond (1989) finds evidence of constant
or weakly increasing returns estimating a Cobb-Douglas matching function for the US
aggregate economy. They find more clear-cut evidence in favour of increasing returns
when they restrict the estimation to the US manufacturing sector and use a set of
instruments for unemployment and vacancies. On the whole, they tend to dismiss the
result of strongly increasing returns to scale on the ground of possible misspecification.
Their analysis was updated and modified in several ways by later work. As far as aggregation bias are concerned, Burdett, Coles and van Ours (1994) show that temporal aggregation problems are nearly irrelevant in the work of Blanchard and Diamond (1989), given the sufficiently high (monthly) frequency of the data series that they use and the relatively low frequency of cycles in the conditioning variables. According to Coles and Smith (1996), geographical aggregation should also play no role in practice, despite the claimed validity of the IRS hypothesis on theoretical grounds. They estimate in fact a cross section of matching functions for England and Wales, and cannot reject the CRS hypothesis in matching, even within perfectly integrated labour markets such as travel-to-work areas. Similar results are obtained by Bennet and Pinto (1994) and Burda and Profit (1996).

Increasing returns are obtained instead by Anderson and Burgess (1995). They estimate a matching function using panel data on state-industry level matches in four US states, using therefore a lower level of aggregation than Blanchard and Diamond. They also estimate separate matching functions, using hires from employment and hires from non-employment in turn as dependent variables. Constant returns cannot be rejected in the first case, while they are strongly rejected in the second.

The studies mentioned estimate Cobb-Douglas matching functions in unemployment and vacancies. On the one hand, this specification is globally well-behaved, in the sense that it adequately embodies the property that no jobs are created when one of the argument of the function is zero. On the other hand, it is rather restrictive, and it is surprisingly low the number of studies who attempt alternative specifications. Blanchard and Diamond (1989) also estimate a CES function, and fail to reject the hypothesis that the elasticity of substitution between unemployment and vacancies is one. However, in doing this they impose constant returns.

Warren (1996) explicitly addresses the issue of increasing versus constant returns to scale using a flexible (translog) specification of the matching technology, on the ground that this gives the least biased estimate (compared with the generalized-Leontief and
extended generalized Cobb-Douglas forms) of the degree of returns to scale of a known technology.\(^5\) This reads

\[
\ln M_t = \beta_0 + \beta_1 \ln U_t + \beta_2 \ln V_t + \frac{1}{2} \beta_{11} (\ln U_t)^2 + \frac{1}{2} \beta_{22} (\ln V_t)^2 + \beta_{12} \ln (U_t + V_t). \tag{3.10}
\]

The equation is estimated using monthly data for the US manufacturing sector over the period 1969-1973, including a linear trend. The Cobb-Douglas specification is retrieved imposing the restriction \(\beta_{11} = \beta_{22} = \beta_{12} = 0\). The scale elasticity is computed as

\[
\eta = \eta_U + \eta_V,
\]

where \(\eta_U = \beta_1 + \beta_{11} \ln U_t + \beta_{12} \ln V_t\) and \(\eta_V = \beta_2 + \beta_{22} \ln V_t + \beta_{12} \ln U_t\). Two different measures for the number of matches are used in turn: new hires (that exclude recalled employees) and total accessions (that include recalled employees). The CRS hypothesis is rejected in both cases in favour of increasing returns. Another important finding is that the coefficients \(\beta_{11}, \beta_{22}\) and \(\beta_{12}\) are significantly different from zero at the 5% significance level, providing evidence in favour of a more sophisticated specification than the Cobb-Douglas.

On the whole, those who estimate log linear matching functions on highly aggregated data tend to find evidence in favour of constant returns to scale. Using a lower level of aggregation and/or estimating a more flexible functional form sometimes leads rejecting the constant returns hypothesis in favour of increasing returns.

An alternative way of assessing the relationship between market size and matching efficiency consists in estimating hazard functions for unemployment or vacancy durations on micro data. If the underlying matching technology displays constant returns to scale, the hazard rate for an unemployed worker should only depend on the degree of labour market tightness \(\theta = V/U\), and not on the absolute size of the pool of searching agents \(U\). If instead matching displays increasing returns, the hazard rate should depend positively on the level of \(U\), after controlling for labour market tightness \(\theta\).

Despite the importance of micro duration analysis for the understanding of ag-

\(^5\)See Guilkey et al. (1983).
aggregate matching performance, macro and micro approaches have so far mainly been used in the empirical search literature for answering different questions. On the one hand, aggregate matching function studies have mainly addressed the issue of aggregate matching effectiveness and of the returns to scale in the matching technology. On the other hand, hazard functions were mainly used to investigate the determinants of exits from unemployment, without concern for the returns to scale in the underlying matching technology. An exception to this is the work by Lindeboom, van Ours and Renes (1994), who explicitly exploit the link between the aggregate matching function and hazard rate specifications for evaluating the relative effectiveness of alternative search channels.

Chapter 5 of this thesis adopts a hazard function specification of unemployment duration in order to consider explicitly the issue of the returns to scale in the matching function. Re-employment probabilities are conditioned on the number of unemployed workers and vacancies within the travel-to-work area of each worker. The fact the coefficients on \( \ln U_t \) and \( \ln V_t \) are not significantly different from each other across a number of different specifications implies that re-employment probabilities are only affected by local labour market tightness and not by the absolute size of the pool of searchers, which confirms the existence of constant returns to scale in matching.

This result however does not preclude the possibility that the matching process displays increasing returns as far as the quality, as opposed to the number, of matches are concerned. As a first glance evidence of this phenomenon, we can easily notice that the maximum wage paid in a market such as London exceeds by far and large the maximum wage paid in any other town in Britain. It is in fact plausible that thick markets enhance the quality of newly-formed jobs simply by displaying larger tails in the distribution of the quality of job opportunities, as measured by the productivity or the wage paid by the job. If the reservation quality of a new job is independent of the size of the market, the larger left tail of the distribution does not affect the minimum quality acceptable, while the larger right tail does improve the quality of the
best job accepted, thereby increasing the average job quality in the market. This field of research, although unexplored at the moment, would possibly shed some light on more disaggregated aspects of the matching process.

3.7 Summary and conclusions

The study of the labour market matching function has become increasingly popular during the last fifteen years. The rich theoretical and empirical literature that this stream of research has generated can be interpreted as the response to two main objectives.

First, from a theoretical viewpoint, the analysis of the mechanisms underlying job formation has provided some microfoundations to widely used search models of the labour market. Second, establishing the existence of a matching function on empirical grounds has explained the observed behaviour of job and worker flows, and could shed some light on the performance of the labour market across space and time.

This chapter has focused on the second of these points, by reviewing the empirical search literature that has estimated matching functions of the labour market. Different econometric specifications were discussed, together with the problems most frequently encountered in estimation.

A few broad issues are frequently addressed by estimation. First, the need to identify correctly the relevant pool of searching agents that may contribute to job creation and the state where they flow into has led to consider possible alternatives to standard outflow equations in unemployment and vacancies only. Second, aggregation problems both across time and space has expanded the analysis of the matching function at the micro level - until recently fairly unexplored - to assess the extent to which existing time series studies were affected by an aggregation bias. Thirdly, the characterization of the equilibrium properties of widely used search models of the labour market has generated deep empirical investigation of the degree of homogeneity of the underlying matching technology.
On the whole, it can be concluded that the empirical search literature has successfully established the existence of a labour market matching function, in which both unemployed workers and vacancies contribute significantly to the process of job formation. Interestingly enough, available disaggregate studies were consistent with most of the previous aggregate findings. This has confirmed the importance of the matching function as a powerful black-box tool of analysis insofar as it reflects optimal search decisions at the micro level into aggregate labour market flows.

These being the main achievements of the existing literature, it can be noted that two areas of research are to date still in their infancy. The first concerns alternative characterizations of the interaction between traders in the labour market and of the implications that they have for aggregate job and worker flows. The second concerns further investigation at the micro-level, so as to identify as many of those aspects of job formation that are only implicit in aggregate matching functions.

The second part of this thesis is an attempt to address both these issues. Chapter 4 considers a plausible microfoundation for the matching function, introducing a non-random meeting technology between employers and job-seekers and deriving some implications for the use of the $UV$ curve as a measure of matching effectiveness. Finally, chapter 5 estimates a duration model of unemployment on micro data, in order to derive predictions for the returns to scale of the matching process within self-contained labour markets.
Chapter 4

Non-random matching and the Beveridge curve

4.1 Introduction

The Beveridge curve is a powerful analytical tool largely used because of its simplicity and easy observability. The progressive outward shift of the curve in Britain since the early 1970s (see Figure 4-1) appears to make a clear statement about the performance of the labour market. In their seminal paper on the analysis of vacancies, Jackman, Layard and Pissarides (1989) conclude that this outward shift reflects a deterioration in the efficiency of the labour market in matching job-seekers to vacancies. Arguing against increased mismatch by regions, skill or occupation groupings, they suggest that search effectiveness by the unemployed had declined both in terms of their own search effort and their attractiveness to employers.

This chapter starts by focusing on the Beveridge curve analysis as an indicator of labour market effectiveness. Our approach rests on the theoretical work on job search undertaken since the early 1970s, which shows that a steady state relationship between the unemployment rate and the vacancy rate can be derived from a matching function between vacancies and unemployed job-seekers, having imposed a few simplifying
restrictions.

The first of these is to assume a constant returns to scale matching technology between unemployed workers and vacancies. Secondly, embedded into the Beveridge curve derivation is the assumption that the economy is in steady state, with the number of separations being equal to the number of job matches. These two restrictions are easily removed whenever an aggregate matching function is estimated directly.

Additional restrictions apply to most of the empirical literature on either Beveridge curve or matching function estimation. Here we concentrate on two of them. First, when discrete-time data are used to estimate a continuous-time matching processes, a time aggregation problem arises. As shown in Burdett, Coles and van Ours (1994) and discussed in the previous chapter, this generates a bias in the resulting estimates, whose magnitude depends on the time-series properties of the conditioning variables and is inversely related to the frequency of the data available. We try to deal with time aggregation problem by controlling for the total number of agents that can generate new matches within some time interval, i.e. the beginning-of-period stock plus the new inflow.

Second, the Beveridge curve rests on the assumption of random matching, with trade in the labour market taking place when agents randomly meet one another, implying a constant likelihood of a match of any unfilled vacancy with any uncompleted spell of unemployment. We contrast this assumption with one possible model of non-random matching, as proposed by Coles (1994). They present a model in which “the stock of unmatched traders on one side of the market is trying to match with the flow of new traders on the other side”. In other words, a newly posted vacancy is considered by the whole stock of unemployed workers, but, if the match is rejected by one side or the other, there is a zero probability of a match between that individual and that vacancy in the future. This implies that the stock of unfilled vacancies, sampled but yet unmatched, can only match with the flow of new job-seekers. A symmetric argument applies to the screening of unemployed workers. This mechanism produces an initial
high probability of a match for traders in the labour market and a much lower one in subsequent periods.

Building on this idea of stock/flow matching, in this chapter we test its main predictions in terms of trading probabilities and illustrate its implications for the measurement of matching effectiveness.

In particular, stock/flow matching has relevant implications for the use of the Beveridge curve as a measure of labour market effectiveness. The two sides of the matching process, unfilled vacancies and uncompleted unemployment spells, do not match with each other but with the corresponding flows. Thus the two stocks can move independently of each other without necessarily implying a deterioration in labour market effectiveness. This view suggests that ignoring flows in matching is not an appropriate simplifying assumption and may produce a seriously misleading view of matching effectiveness over time.

We go on to consider effectiveness in this alternative specification of a matching function and the results suggest there was no aggregate decline in the matching effectiveness of vacancies between the 1960s and 1990s. Nevertheless, there remains a deterioration in the matching effectiveness of the claimant unemployed since the early 1970s, however slightly less severe than that implied using the standard assumption of random matching.

This chapter maps out as follows. Section 4.2 considers the standard Beveridge curve as laid out by Jackman, Layard and Pissarides (1989) and updates the analysis until 1996. It suggests a deteriorating job search performance over the past forty years, affecting both short-term and long-term unemployed. Section 4.3 outlines the main predictions of a stock/flow matching model, which are tested in section 4.4. Evidence on completed and uncompleted duration of search spells and on matching probabilities of “new” agents supports a non-random matching technology. The implications of non-random matching for measuring the effectiveness of the labour market are illustrated in Section 4.5. Including explicitly new flows into the matching function, both for the
treatment of time aggregation and to control for the non-random nature of the matching process, implies a revision of some widespread conclusions on labour market efficiency. Section 4.6 concludes the chapter.

4.2 The Beveridge curve in Britain: 1967-1996

There has been a rich literature establishing that the degree of the matching effectiveness of the labour market can be assessed on the basis of the performance of the aggregate equilibrium relationship between the unemployed rate and the vacancy rate in an economy, known as the UV or Beveridge curve. Relevant contributions include Jackman and Roper (1987), Jackman, Layard and Pissarides (1989), Blanchard and Diamond (1989) and Layard, Nickell and Jackman (1991). Figure 4-1 plots the standard UV curve for Britain from 1953 to 1996 and shows its outwards shift through the 1960s, 1970s and 1980s, and the more recent modest shift inwards.

Broadly speaking, shifts in the curve are generated by reallocation shocks, that cause the unemployment and the vacancy rate to move in the same direction. Movement of unemployment and vacancies along an upward-sloping locus are the stylized fact generally observed to draw conclusions on matching effectiveness. The curve also displays anti-clockwise loops, whose size has increased across decades. Loops are generated by the different speed of adjustment of unemployment and vacancies to changes in the level of aggregate activity. Although the effects of the two kinds of shock could not be so easily separated, below we will stick to the common practice of interpreting shifts in the curve as the outcome of shocks to the matching technology.

The aggregate UV curve has its microfoundations in search models of the labour market in which firms post vacancies and unemployed workers apply for jobs. In section 3.1 we argued that the UV curve can be derived from a hiring function such as

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1See Pissarides (1985).

\[ M = \phi m(U, V), \]

with \( m(\cdot) \) increasing and concave in both arguments, and \( m(U, 0) = m(0, V) = 0 \). \( M \leq \min(U, V) \) denotes the flow of matches generated by the stock of unemployed workers \( U \) and open vacancies \( V \), and \( \phi \) is a scale parameter of the matching technology.

A long-run relationship between the unemployment rate \( U/L \) and the vacancy rate \( V/N \) derives from (4.1) imposing constant returns to scale in \( m(\cdot) \) and noticing that, in steady state, the number of matches \( M \) equals the number of job separations \( sN \), with \( s \) denoting the separation rate. This gives

\[ s \approx \phi m \left( \frac{U}{L}, \frac{V}{N} \right) \]

as a representation of the Beveridge curve. Given \( s \) and \( \phi \), equation (4.2) describes a negative relationship between the vacancy rate and the unemployment rate. Equation
(4.2) is generally estimated expressing \( m(\cdot) \) as a constant returns to scale Cobb-Douglas function in unemployment and vacancies. The regression equation has generally the form

\[
\ln \left( \frac{U}{L} \right)_t = \alpha_0 + \alpha_1 \ln s_t + \alpha_2 \ln \left( \frac{U}{L} \right)_{t-1} + \alpha_3 \ln \left( \frac{V}{N} \right)_t, 
\]

where the lag of the dependent variable captures off-steady state dynamics, allowing for the different speed of adjustment in the behaviour of unemployment and vacancies.

The contributions mentioned in section 3.1 generally agree in documenting a deterioration of the relationship in (4.3) over the post-war period in most OECD countries, with a roughly untrended vacancy rate being consistent with a progressively higher unemployment rate than in the past. Börsch-Supan (1991) actually concludes with a rather pessimistic view about the use of the \( UV \) curve as a tool of macroeconomic analysis, remarking that its instability over time is not eliminated once different potential sources of matching deterioration are controlled for.

Here we resume and update this kind of analysis. The equation is estimated on aggregate quarterly data on unemployment, vacancies, employment and labour force in Britain, using instrumental variables.\(^2\) The sample period is 1967:3-1996:3. The data are seasonally adjusted.

Stability of equation (4.3) over time is assessed by looking at the behaviour of the function across four sub-periods, defined between subsequent peaks in the number of open vacancies: 1967:3-1973:4, 1974:1-1979:3, 1979:4-1987:4 and 1988:1-1996:3, represented by four dummy variables \( d_1, \ldots, d_4 \). Given that the claimed deterioration in (4.3) is often blamed on a higher incidence of long-term unemployment, characterized by lower search effectiveness, we also perform the stability analysis leaving out all the long-term unemployed, defined as those with duration longer than six months. The

\(^2\)The lagged vacancy rate is used as an instrument for the current vacancy rate. No substantial difference is found in the estimates when the GDP per employee (current and/or lagged) is used as an additional instrument.
Table 4.1: Estimates for a $UV$ curve for Britain, 1967:3-1996:3.

<table>
<thead>
<tr>
<th></th>
<th>Total unempl.</th>
<th>Short term unempl.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>$\ln s_t$</td>
<td>0.185 (0.029)</td>
<td>0.200 (0.056)</td>
</tr>
<tr>
<td>$\ln (U/L)_{t-1}$</td>
<td>0.907 (0.012)</td>
<td>0.890 (0.011)</td>
</tr>
<tr>
<td>$\ln (V/N)_t$</td>
<td>-0.346 (0.079)</td>
<td>-0.137 (0.010)</td>
</tr>
<tr>
<td>$d_2$</td>
<td>0.553 (0.071)</td>
<td>0.080 (0.011)</td>
</tr>
<tr>
<td>$d_3$</td>
<td>1.290 (0.064)</td>
<td>0.121 (0.015)</td>
</tr>
<tr>
<td>$d_4$</td>
<td>1.090 (0.063)</td>
<td>0.077 (0.016)</td>
</tr>
<tr>
<td>const.</td>
<td>-5.355 (0.388)</td>
<td>-1.004 (0.070)</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>0.83</td>
<td>0.99</td>
</tr>
<tr>
<td>No. obs.</td>
<td>113</td>
<td>113</td>
</tr>
</tbody>
</table>


Results are presented in Table 4.1. Columns I-III contain the results obtained using total unemployment, while columns IV-VI refer to short term unemployment only.

Both sets of regressions, i.e. with total and short term unemployment, show a deterioration of matching effectiveness in periods 2 and 3. When we control for adjustment lags in the unemployment rate, including the lagged dependent variable in the regression, the long-run coefficients on $d_2 - d_4$ are of the same order of magnitude of those obtained in the static relationship for total unemployment, and slightly higher for short-term unemployment.

However the results of the stability analysis do not change substantially whether the pool of job-searchers is represented by all unemployed or just the short term unemployed. This should remove the concern that the deterioration in matching effectiveness
of the labour market is simply an outcome of increased unemployment duration.

Matching performance in period 4 shows a modest improvement in the first two columns of each specification. Yet, when the separation rate is included, the estimated coefficients on \( d_3 \) and \( d_4 \) are no longer significantly different from each other in column III, and in column VI they are only different from each other at the 10% significance level. Including the separation rate controls for some of the cyclical fluctuations in activity and, given the exaggerated cycle over the last period (see Figure 4-1), this result suggests that the improvement is mainly a cyclical phenomenon.

4.3 The matching process

The previous section replicated and updated the existing work on the UV curve analysis and none of the results are unusual or controversial. In this section we modify the standard matching function analysis in order to deal with two specific issues. First, we treat matching in continuous time and provide a way of dealing with time aggregation problems that arise when discrete time data are used to describe a continuous-time matching process. Second, we consider a non-random matching technology in which the current stock of unmatched traders on one side of the market can only match with the flow of new traders on the other side, as proposed in Coles (1994) and Coles and Smith (1997). We start however by briefly reviewing the main predictions of a random matching model of the labour market.

4.3.1 Random matching: a few predictions

A simple random matching framework can be exemplified in the following way. Unmatched identical workers and vacancies\(^3\) are scattered across the market, and need to locate possible trading partners before creating a match. The matching technology that

\(^3\)The predictions of the random matching model do not differ substantially if heterogeneous agents are considered instead, see Pissarides (1990, ch. 5).
expresses job formation per unit of time as a function of the number of trading agents existing in the market is given by

\[ M = \phi m(U, V), \]  

(4.4)
displaying the properties stated above. Equation (4.4) implies that, during the small interval \( dt \), each vacancy is contacted on average by \( [\phi m(U, V)/V] \) unemployed workers. The probability of a given worker being selected for that vacancy is \([\phi m(U, V)/(UV)]\) \( dt \). The survival probability for an unemployed worker during \( dt \) can be therefore computed as

\[ R_U(dt) = \left(1 - \frac{\phi m(U, V)}{UV} dt\right)^V \approx \exp\left(-\frac{\phi m(U, V)}{U} dt\right) \]  

(4.5)
for large enough \( U \) and \( V \). Consequently, the probability for a given worker of being matched (or exit probability during \( dt \)) is

\[ F_U(dt) = 1 - \exp\left(-\frac{\phi m(U, V)}{U} dt\right) \]  

(4.6)

The transition from unemployment to employment is therefore a process with hazard rate \( \lambda_U = \phi m(U, V)/U \). Similarly, the transition of jobs from vacancy to activity is a process with hazard rate \( \lambda_V = \phi m(U, V)/V \). The total number of matches during \( dt \) can be then computed as either \( U[1 - \exp(-\lambda_U dt)] \) or \( V[1 - \exp(-\lambda_V dt)] \).

The matching function describes a continuous time process. If we are using a discrete interval as our time unit, we deal with time aggregation by taking into account both the matches generated by the beginning-of-period stock of unemployed \( U \) (or vacancies \( V \)) and those generated by the inflow \( u \) (or \( v \)) during the period. Let's consider a period of time whose length is set equal to 1. Suppose that we have an initial stock of

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\(^4\)It would be more appropriate to refer simply to \textit{outflow} of agents from the existing pool, rather than to \textit{matches} between unemployment and vacancies, which is a more restrictive concept. In this section we will assume for simplicity that they coincide. When it comes to estimation, \( M \) will be measured using the appropriate outflow.
searching agents $S_i$ (that can either represent unemployed workers or vacancies), and a subsequent inflow $I_i(t)$. Total matches are given by\textsuperscript{5}

$$M = \left(1 - e^{-\lambda}\right) S + \int_0^1 \left[1 - e^{-\lambda(1-t)}\right] I(t) dt,$$

where the first term denotes the outflow from the initial stock and the second denotes the outflow from the inflow. Under the simplifying assumption of uniform inflow $I$ during the period, we have

$$M = \left(1 - e^{-\lambda}\right) S + \left[1 - \frac{1}{\lambda} \left(1 - e^{-\lambda}\right)\right] I. \tag{4.7}$$

It can be noted that the term in square brackets is bounded between zero and one, and therefore describes a plausible outflow rate from the inflow. Also note that the outflow rate from the inflow is lower than the one from the stock, for the reason that the inflow has, on average, less time available for a successful match. For small enough $\lambda$, the outflow rate from the inflow can be approximated as half the outflow rate from the stock, using a second order Taylor expansion of both outflow rates around $\lambda = 0$.

It is also useful to consider the predictions of matching models for completed and uncompleted durations of search spells.

We start by looking at the observed average duration of uncompleted search spells at some point in time $t_i$. At time $t_0 < t_1$ there is a stock of agents $S$ with average uncompleted duration $d_0$. The surviving stock at $t_1$ is $\exp[-\lambda(t_1 - t_0)] S$, among whom the average uncompleted duration is $d_0 + (t_1 - t_0)$. At time $t_1$, there is also some unmatched inflow in the economy, $\int_{t_0}^{t_1} \exp[-\lambda(1-t)] I(t) dt$, with accumulated uncompleted duration of $\int_{t_0}^{t_1} (1-t) \exp[-\lambda(1-t)] I(t) dt$. Setting $t_0 = 0$ and $t_1 = 1$ for simplicity, average uncompleted duration among the pool of unmatched agents at time $t_1$ is given by

\textsuperscript{5}Dropping for simplicity all the subscripts, given that they refer to the same type of agents.
\[ d_{1, \text{unc}} = \frac{(d_0 + 1)e^{-\lambda}S + \int_0^1 (1-t)e^{-\lambda(1-t)}I(t)dt}{e^{-\lambda}S + \int_0^1 e^{-\lambda(1-t)}I(t)dt}. \]

Using once more the assumption of uniform inflow between 0 and 1, this gives

\[ d_{1, \text{unc}} = \frac{(d_0 + 1)e^{-\lambda}S + \frac{1}{\lambda} \left[ 1 - (1 + \lambda)e^{-\lambda} \right] I}{e^{-\lambda}S + \frac{1}{\lambda} (1 - e^{-\lambda}) I}, \] (4.8)

with \( \partial d_{1, \text{unc}}/\partial \lambda < 0. \)

In a similar vein we compute the mean completed duration of search for agents that are matched between 0 and 1. Each agent in \( S \) has a probability \( \lambda \exp(-\lambda \tau) \) of getting matched at some time \( \tau \) between 0 and 1, therefore experiencing on average completed search duration \( \tau + d_0 \). Each agent in \( I(t) \) is flowing into the pool at some time \( t \) and has a probability \( \lambda \exp[-\lambda(\tau - t)] \) of being matched at \( \tau > t \), having completed search duration \( \tau - t \). Mean completed search duration at \( t_1 \) is therefore

\[ d_{1, \text{com.}} = \frac{\lambda S \int_0^1 (\tau + d_0)e^{-\lambda \tau}d\tau + \lambda \int_0^1 \left[ \int_0^1 (\tau - t)e^{-\lambda(\tau-t)}d\tau \right] I(t)dt}{(1 - e^{-\lambda}) S + \int_0^1 [1 - e^{-\lambda(1-t)}] I(t)dt} \]

or

\[ d_{1, \text{com.}} = \frac{\left[ \frac{1}{\lambda} + d_0 - (1 + \frac{1}{\lambda} + d_0) e^{-\lambda} \right] S + \frac{1}{\lambda} \left[ 1 - \frac{1}{\lambda} (2 - (2 + \lambda) e^{-\lambda}) \right] I}{(1 - e^{-\lambda}) S + \left[ 1 - \frac{1}{\lambda} (1 - e^{-\lambda}) \right] I}, \] (4.9)

with \( \partial d_{1, \text{com.}}/\partial \lambda < 0. \)

It is easy to check that, for observed values of \( d_0 \), the ratio \( S/I \) and \( \lambda \), the average uncompleted duration among the existing stock of unmatched agents, \( d_{1, \text{unc}} \), falls short of the average completed duration among a cohort of agents, \( d_{1, \text{com.}} \), for both vacancies and unemployment. This is very much at odds with evidence on vacancy and unemployment durations, that tells that completed durations are generally half the uncompleted...
4.3.2 Non-random matching

The introduction of non-random matching into this framework has been pioneered by Coles (1994), who explicitly recognizes the role played by information channels such as employment agencies in an environment of heterogeneous agents. If the information provided by these channels is accessible to all searchers, they do not need to spend time and resources in order to locate one another. In the limit we can think that every trader can sample the whole pool of possible matching partners in each period. This triggers a matching process in which unemployed job-seekers will search over the whole existing stock of open vacancies only in the first period they are on the market. If they find no suitable partner among the existing stock, in the following period they will just search among the newly opened vacancies, given that the existing stock has already been sampled and rejected. The same argument applies to employers with open vacancies, that first screen the whole unemployment stock and, after that, only the new inflow.

This implies a step-wise relationship between matching probabilities and duration: exit rates are higher at zero duration and lower (but constant) after the period it takes to sample the existing pool. This prediction is very much in line with evidence on vacancy hazard rates. Coles and Smith (1997) compute that one quarter of all vacancies in Britain are filled on the first day they are opened and, after that, their exit probability declines abruptly. The jump in exit rates is however much less pronounced for the unemployment stock. This may be due to the fact that the search process is highly asymmetric, with vacancies posted at the Jobcentre and unemployed workers sampling them at their preferred pace. Moreover, for each type of traders, the size of the step in the exit rate depends negatively on the flow/stock ratio of traders of the other type. As we will see below, vacancy inflows relative to vacancy stocks are fairly large, at least compared to the correspondent ratio for unemployment. This implies
that unemployment exit rates need not fall as sharply as vacancies exit rates when an agent switches from the flow to the stock status.

There are issues about how long the initial search period is. It is driven by how long it takes for a newly posted vacancy to be sampled by all the existing unemployed - and vice versa. As mentioned above, it is possible that the intensity of sampling by the unemployed is also time dependent. Such that, say, on the first day half the unemployed sample a new vacancy at the Jobcentre, three quarters in the first week and all in a month. For reasons of simplicity, below we will keep any "smooth" duration dependence, both within the initial search period and during the subsequent search life, out of the scene.

In order to describe this matching mechanism more formally, we can consider a time interval \((t - 1, t)\) and the four categories of agents \(U_{t-1}, V_{t-1}, u_t\) and \(v_t\) that represent respectively the beginning-of-period stocks and the subsequent flows of unemployment and vacancies. Firms and workers are ex-ante homogeneous and the quality (or surplus) of each job-worker pair is only revealed after agents have met. Assuming for simplicity that the value of the match has a Bernoulli distribution, agents either obtain a constant positive surplus from the match, with ex-ante probability \(\pi\), in which case they decide to match and start production, or obtain a zero surplus, with ex-ante probability \(1 - \pi\), in which case they keep searching. If agents have positive discount rates, there is no reason to delay matching when this yields a positive surplus. Therefore a specific job-worker match that is once rejected cannot be accepted in the future.\(^6\)

Given the role of information channels, all agents in \(u_t\) can sample \(V_{t-1} + v_t\) vacancies as soon as they enter the market. The probability that a newly unemployed worker matches with an old vacancy depends on how many old vacancies are available and how

---

\(^6\)Search in this model implies that an agent's reservation wage is not higher than the positive value of the Bernoulli distribution. The Bernoulli distribution for the surplus of a match is easily tractable but by no means necessary to obtain our results. If we consider instead a continuous distribution, all we need to rule out recalls of rejected offers is the stationarity of the reservation surplus (or reservation wage). Although in practice the reservation wage of unemployed workers may fall with duration of search (see Kiefer and Neumann, 1979), the extremely short vacancy duration should rule out the opportunity of an effective recall on practical grounds.
many competitors there are for them. Each old vacancy is contacted by $u_t$ unemployed workers, each of whom has a probability $1/u_t$ of being selected. In order to match with a new vacancy, a new unemployed worker faces more competition than in the match with an old vacancy, because both $U_{t-1}$ and $u_t$ are valid candidates for $v_t$. The probability of such a match is therefore $1/(U_{t-1} + u_t)$. Given that new vacancies do not distinguish between new and old unemployed, this is also the probability that an old unemployed matches with a new vacancy.

By a symmetry argument, the probabilities that a new vacancy is filled by a new and an old unemployed are $1/v_t$ and $1/(V_{t-1} + v_t)$ respectively, the latter also denoting the probability that an old vacancy is filled by a new unemployed.

Matching probabilities for the four categories of agents in the unit time interval are therefore given by the following expressions

$$F_U(t) = 1 - \exp \left( -\frac{v_t}{U_{t-1} + u_t} \right), \quad (4.10)$$

$$F_u(t) = 1 - \exp \left( -\frac{v_t}{U_{t-1} + u_t} \right) \exp \left( -\frac{V_{t-1}}{u_t} \right) > F_U(t), \quad (4.11)$$

$$F_V(t) = 1 - \exp \left( -\frac{u_t}{V_{t-1} + v_t} \right), \quad (4.12)$$

$$F_v(t) = 1 - \exp \left( -\frac{u_t}{V_{t-1} + v_t} \right) \exp \left( -\frac{U_{t-1}}{v_t} \right) > F_V(t). \quad (4.13)$$

Equations (4.10)-(4.13) show that inflows have a greater probability of finding a suitable match than stocks, represented by the possibility of finding a partner in the existing stock when they first enter the market, with no need to wait for the new inflow of possible matching partners.

Note incidentally that all exit rates only depend on some ratio of relevant traders, therefore implying that the matching technology displays constant returns to scale. This
is an important difference from the matching process outlined in Coles and Smith (1997), that predicts instead increasing returns. The different prediction of this framework derives from the fact that congestion externalities are explicitly taken into account when the whole number of competitors for each vacancy is considered (or - conversely - for each unmatched worker). No such externality is instead considered by Coles and Smith, because they assume that traders flow into the market individually.

In the empirical analysis that follows, higher matching probabilities of inflows per unit of time will be treated in a slightly different way from specifications (4.11) and (4.13). They will be simply represented by allowing for a positive instantaneous probability \( p \) that agents are matched as soon as they enter the pool. Clearly, according to (4.11) and (4.13), \( p_u \) is an increasing function of \( V_{t-1}/u_t \) and \( p_v \) is an increasing function of \( U_{t-1}/v_t \). With probability \( 1 - p \), the inflow will instead match later with new traders, with hazard rate \( \lambda \). Using (4.10) and (4.12), \( \lambda_U \) is an increasing function of \( v_t/(U_{t-1} + u_t) \) and \( \lambda_V \) is an increasing function of \( u_t/(V_{t-1} + u_t) \). Note therefore that \( p \) represents a probability of instantaneous matching as soon as an agent enters the market, while \( \lambda \) represents the hazard rate of those surviving after entrance.

The specification in terms of \( p \) and \( \lambda \) implies that equations (4.7), (4.8) and (4.9) can be rewritten as

\[
M = (1 - e^{-\lambda}) S + p I + (1 - p) \left[ 1 - \frac{1}{\lambda} (1 - e^{-\lambda}) \right] I \\
M = (1 - e^{-\lambda}) S + \left[ 1 - (1 - p) \frac{1}{\lambda} (1 - e^{-\lambda}) \right] I \\
(4.14)
\]

\[
d_{1,unc.} = \frac{(d_0 + 1) e^{-\lambda} S + (1 - p) \frac{1}{\lambda^2} \left[ 1 - (1 + \lambda) e^{-\lambda} \right] I}{e^{-\lambda} S + (1 - p) \frac{1}{\lambda} (1 - e^{-\lambda}) I} \\
(4.15)
\]

and
\[ d_{1,\text{com.}} = \frac{\left[ \frac{1}{\lambda} + d_0 - (1 + \frac{1}{\lambda} + d_0) e^{-\lambda} \right] S + (1 - p) \frac{1}{\lambda} \left[ 1 - \frac{1}{\lambda} (2 - (2 + \lambda) e^{-\lambda}) \right] I}{(1 - e^{-\lambda}) S + \left[ 1 - (1 - p) \frac{1}{\lambda} (1 - e^{-\lambda}) \right] I}. \] (4.16)

The total number of matches increases with respect to the case \( p = 0 \). It can be shown that \( \partial d_{1,\text{unc.}} / \partial p > 0 \), provided that the mean uncompleted duration among the surviving inflow (i.e. among \((1 - p)I\)) is shorter than mean uncompleted duration among the stock. This last condition is assumed to be satisfied, given that the stock and the surviving inflow have the same hazard rate, and that the stock has a longer duration to start with. The positive effect of \( p \) in determining uncompleted duration consists in removing from the whole set of unmatched agents a proportion \( p \) of those that have lower average duration. We have instead \( \partial d_{1,\text{com.}} / \partial p < 0 \), given that a proportion \( p \) of the inflow has zero completed duration.

We noted that, under random matching (with \( p = 0 \)), \( d_{1,\text{unc.}} \) was too small compared to \( d_{1,\text{com.}} \) for both vacancies and unemployment. Given \( \partial d_{1,\text{unc.}} / \partial p > 0 \) and \( \partial d_{1,\text{com.}} / \partial p < 0 \), we can therefore think of a positive value of \( p \) that brings predictions in terms of completed and uncompleted durations of search spells closer to what is observed in reality.

### 4.4 Evidence on non-random matching

The introduction of non-random matching is a major departure from conventional matching analysis, so it is worthwhile looking at whether there is supporting evidence for it. In the empirical analysis that follows we test the main predictions of the non-random matching model by estimating matching functions such as equation (4.14) and comparing completed and uncompleted durations of search spells.

We use British data on unemployment and vacancies. They are aggregate, quarterly time series, covering the period 1967:1-1996:3.

Unemployment data prior to October 1982 consist of the registrant count, includ-
ing all workers that registered themselves as unemployed at the Ministry of Labour's Employment Exchanges (to become Jobcentres), at Branch Employment Offices, or at Youth Employment Bureaux (to become Youth Employment Service Career Offices). Registration for unemployment became voluntary in October 1982, and the administrative measure of unemployment was changed to the count of workers claiming unemployment-related benefits at Unemployment Benefit Offices. An accurate description of British unemployment data can be found in *Labour Market Trends*, January 1996. Vacancy data are collected at Employment Service Jobcentres, and include job opportunities for self-employed workers as well as part-time jobs, on top of standard full-time vacancies. Vacancy data used here and their limitations are fully described in *Labour Market Trends*, November 1995.

The data used come from two sources: all series previous to 1985:3 come from the Employment Gazette and are seasonally adjusted, while those for the later period come from the NOMIS databank and are not seasonally adjusted. We therefore proceed by removing seasonality from all series after 1985:3.

Figures 4-2 and 4-3 plot time series of these variables. Quarterly inflows and outflows of vacancies track each other very closely, and are on average 3.7 times the beginning-of-quarter stock of unfilled vacancies. This ratio is strongly countercyclical but does not display any definite trend over the sample period. The ratio between unemployment flows and stock is 0.7 on average, starting off nearly at 2 and falling at 0.3 in the early 1990s.

Data on vacancy and unemployment duration are available for a shorter period, from 1986:2 to 1995:4. They come in the following form: (i) average completed duration of vacancies that are filled within a quarter; (ii) average uncompleted duration among the stock of unfilled vacancies at the end of the quarter; (iii) workers leaving unemployment by duration classes; (iv) unemployment stock at the end of the quarter by duration classes. From this set of information we derive completed and uncompleted durations of all spells as represented by $d_{1,\text{com}}$ and $d_{1,\text{unc}}$. 

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Figure 4-2: Vacancy stock, inflow and outflow in Britain (thousands): 1967:1-1996:3. Data seasonally adjusted. Source: Employment Gazette (various issues) and NOMIS.

Figure 4-3: Unemployment stock, inflow and outflow in Britain (thousands): 1967:1-1996:3. Data seasonally adjusted. Source: Employment Gazette (various issues) and NOMIS.
Figure 4-4 plots the average duration of filled and unfilled vacancies against time. The average duration of filled vacancies is around 3 weeks and that of unfilled vacancies is 5 to 9 weeks. In all periods the completed duration of employers’ search falls short of the uncompleted one.

For unemployment, completed and uncompleted average durations are computed by assigning at each duration class the mid-value of the class (having closed the last open class with duration>260 weeks at 312 weeks). Figure 4-5 plots average completed and uncompleted durations for the unemployed. Observed durations are one order of magnitude larger than the corresponding ones for vacancies. Similarly as for vacancies, the ratio between uncompleted and completed durations is in the range 2/2.5.

These ratios are at odds with the predictions of equations (4.15) and (4.16). Random matching is therefore strongly rejected by empirical evidence on duration of search spells. As argued in the previous section, a positive initial matching probability $p$ may reconcile the predictions of a non-random matching model with the observed duration time series.

Below we will let the data decide the values of $\lambda$ and $p$ by estimating outflow equations (4.14) over the period 1967:1-1996:3. These are estimated both imposing a fixed hazard rate $\lambda_i$, $i = V,U$, and estimating a proportional hazard in which the regressors are relevant labour market variables.

In case of non-random matching, $\lambda_i$ denotes the hazard rate of old agents. Assuming that the matching technology has constant returns to scale and that stocks and flows are in general substitutes up to a parameter $\beta_1$ (see (4.10)-(4.13)), we can express $\lambda_i$ as

$$\lambda_i = \exp(\beta_0) \left( \frac{I_j + \beta_1 S_j}{S_i} \right)^{\beta_2} \quad \text{for} \quad i,j = V,U \quad \text{and} \quad i \neq j.$$ (4.17)

An estimated value $\beta_1 = 0$ would predict that stocks of unmatched agents do not trade
Figure 4-4: Average duration of filled and unfilled vacancies in Britain (weeks), 1986:2-1995:4. Data not seasonally adjusted. Source: NOMIS.

Figure 4-5: Average completed and uncompleted unemployment duration in Britain (weeks), 1986:2-1995:4. Data not seasonally adjusted. Source: NOMIS.
with each other.

The outflow equation becomes

\[
M = \left[ 1 - \exp \left( -\exp(\beta_0) \left( \frac{I_j + \beta_1 S_j}{S_i} \right)^{\beta_2} \right) \right] S_i \\
+ \left[ 1 - \frac{(1 - p_i) \left[ 1 - \exp \left( -\exp(\beta_0) \left( \frac{I_j + \beta_2 S_j}{S_i} \right)^{\beta_2} \right) \right]}{\exp(\beta_0) \left( \frac{I_j + \beta_2 S_j}{S_i} \right)^{\beta_2}} \right] I_i.
\]

Equation (4.18) (or its analogue with fixed \( \lambda_i \)) is estimated using the data set described. In the vacancy outflow equation, \( S_i \) denotes the beginning-of-quarter stock of unfilled vacancies, \( I_i \) and \( I_j \) denote the inflow of vacancies and unemployed respectively during the quarter and \( M \) is represented by the vacancy outflow.

For unemployment, \( S_i \) is measured by the beginning-of-quarter unemployment stock, \( I_i \) by the unemployment inflow, \( I_j \) by the vacancy outflow, and \( M \) by the unemployment outflow.

The results are reported in Table 4.2.

From estimated values of \( \beta_0 \), \( \beta_1 \) and \( \beta_2 \) we can retrieve point estimates for \( \lambda_V \) and \( \lambda_U \) for each quarter. Their average values are reported in square brackets. The third last row reports the expected completed duration of search, computed as \((1 - p_i) / \lambda_i\). This is then multiplied by 13 to convert durations from quarters to weeks.

When \( \lambda_i \) is set constant (columns (1) and (5)), virtually the whole outflow from either the vacancy or the unemployment stock seems captured by \( p_i \). Predicted vacancy duration is just above four weeks; unemployment duration is clearly overestimated at 76 weeks.

When \( \lambda_i \) is estimated as a function of its determinants, we find the positive effect coming through the inflow of matching partners, represented by \( \beta_2 > 0 \), and an insignif-

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\(^7\)The equation is estimated using non-linear least squares. Convergence to a global minimum was ensured using several starting values for the parameters, leading to equivalent estimates.

<table>
<thead>
<tr>
<th>Dep. var.:</th>
<th>vacancy outflow</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>unempl. outflow</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>0.126</td>
<td>[0.872]</td>
<td>[0.898]</td>
<td>[0.915]</td>
<td>0.010</td>
<td>[0.345]</td>
<td>[0.344]</td>
<td>[0.248]</td>
<td></td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>$-1.034$</td>
<td>(0.298)</td>
<td>$-1.019$</td>
<td>(0.286)</td>
<td>$-0.832$</td>
<td>(0.225)</td>
<td>$-0.419$</td>
<td>(0.178)</td>
<td>$-0.422$</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.503</td>
<td>(0.042)</td>
<td>0.501</td>
<td>(0.041)</td>
<td>0.410</td>
<td>(0.061)</td>
<td>0.846</td>
<td>(0.054)</td>
<td>0.845</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>$-0.019$</td>
<td>(0.075)</td>
<td>$-0.009$</td>
<td>(0.378)</td>
<td>$-0.768$</td>
<td>(0.046)</td>
<td>0.493</td>
<td>(0.057)</td>
<td>0.493</td>
</tr>
<tr>
<td>$p_i$</td>
<td>0.960</td>
<td>(0.015)</td>
<td>0.765</td>
<td>(0.077)</td>
<td>0.759</td>
<td>(0.072)</td>
<td>[0.768]</td>
<td>(0.046)</td>
<td>0.937</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>$-0.305$</td>
<td>(0.082)</td>
<td>$-0.768$</td>
<td>(0.046)</td>
<td>$-0.019$</td>
<td>(0.019)</td>
<td>$-0.276$</td>
<td>(0.238)</td>
<td>$-0.115$</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.042</td>
<td>(0.019)</td>
<td>0.115</td>
<td>(0.062)</td>
<td>0.042</td>
<td>(0.019)</td>
<td>0.115</td>
<td>(0.062)</td>
<td>0.115</td>
</tr>
<tr>
<td>$(1 - p_i)/\lambda_i$</td>
<td>4.12</td>
<td>3.65</td>
<td>3.62</td>
<td>3.42</td>
<td>76.59</td>
<td>25.44</td>
<td>25.43</td>
<td>25.60</td>
<td></td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>0.95</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>0.46</td>
<td>0.78</td>
<td>0.78</td>
<td>0.78</td>
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</tr>
<tr>
<td>No. obs.</td>
<td>115</td>
<td>115</td>
<td>115</td>
<td>115</td>
<td>115</td>
<td>115</td>
<td>115</td>
<td>115</td>
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</tr>
</tbody>
</table>

Data seasonally adjusted. Estimation method: non linear least squares. The fitting process is iterative (modified Gauss-Newton). Convergence to a unique maximum likelihood solution was ensured using alternative starting values for the parameters to be estimated. Standard errors in brackets. Source: Employment Gazette (various issues) and NOMIS.
icant effect coming through the stock of matching partners ($\beta_1$ is in fact insignificantly different from zero both in column (2) and in column (6)). Both $p_V$ and $p_U$ are positive and significant, confirming that new agents flowing into the market have a higher probability of matching than the existing stock. Furthermore, we find $p_V > p_U$, revealing that new vacancies have on average a higher initial probability of match than “new” unemployed workers. In columns (3) and (7) we remove the non significant regressor $S_j$ and the results are virtually unchanged from columns (2) and (6) respectively.

In columns (4) and (8) we let $p_V$ and $p_U$ vary with labour market conditions in a similar way as for $\lambda_V$ and $\lambda_U$. We argued that $p_V$ and $p_U$ measure instantaneous matching probabilities of agents flowing into the market. Therefore we replace $p_i$ with

$$
p_i = \exp(\delta_0) \left( \frac{S_j}{I_i} \right)^{\delta_1}
\quad i, j = V, U \quad \text{and} \quad i \neq j.
\tag{4.19}
$$

Note that, for identification problems, $p_i$ can be estimated as from equation (4.19) only once we establish that $\lambda_i$ is independent of $S_j$. Time averages of $p_i$ are reported in square brackets.

In column (4) we find that $\delta_1$ is positive and significant for new vacancies, confirming that they match with the stock of unemployed workers. Similarly, columns (8) gives a significant $\delta_1$ for new unemployed workers, implying that they can match with the stock of unfilled vacancies.

The estimates of columns (2)-(4) predict an average vacancy duration around three and a half weeks. However, a failure to match initially implies a much longer duration between 14 and 15 weeks. Regressions (6)-(8) predict unemployment duration around 25 weeks, but a failure to match at the first round of interviews implies a predicted duration between 38 and 52 weeks. The picture painted is therefore that three quarters of vacancies and between half and 60% of the unemployed match almost immediately, but those that are not successful at this juncture face low matching probabilities and
fairly long expected remaining duration. The relatively long uncompleted durations observed at the beginning of this section therefore refer to the tail of traders that are unlucky at their first round of samplings.

Quantitatively speaking, the results reported in Table (4.2) show that new agents flowing into the market have higher matching probabilities than those that have already been searching without success. Furthermore, from a qualitative viewpoint, they imply that stocks are actually matching with flows. In other terms, although the present analysis does not rule out the presence of unobserved heterogeneity and ranking among unemployed workers and vacancies, it shows that these are not such to lower down to zero the matching probabilities of stocks, which can still match with flows of potential partners.

In order to control more precisely for heterogeneity and ranking in the matching process, we estimate separate outflow equations for different classes of completed durations, and test the responsiveness of outflow rates to stocks and flows of potential matching partners.

We compute Kaplan-Meyer exit rates for two classes of unemployment completed durations - less or equal to a quarter, more than a quarter - using data on unemployment durations described above. The risk set for the exit rate at short duration is given by the unemployment inflow during each quarter, plus that part of the beginning-of-period stock that had duration shorter than one quarter, while the one for the exit rate at long duration is given by the whole beginning-of-period stock.\(^8\)

According to stock/flow matching, exit rates for unemployed with high durations should only depend on the inflow of new vacancies, while those for unemployed with low durations should equally depend on the inflow of new vacancies and the stock of old vacancies. When ranking matters, however, we would expect the importance of the inflow of new vacancies to dominate in both unemployment outflow equations, for

\(^8\)Unfortunately we cannot repeat the same experiment for vacancy outflow, because the outflow of vacancies is not disaggregated by duration classes.
Table 4.3: The determinants of unemployment outflow rates in Britain for two duration classes, 1967:3-1995:4.

<table>
<thead>
<tr>
<th></th>
<th>Short duration</th>
<th>Long duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>dep. var.</td>
<td>ln $O_{S_t}$</td>
<td>ln $O_{L_t}$</td>
</tr>
<tr>
<td>$\ln v_t$</td>
<td>0.289</td>
<td>0.463</td>
</tr>
<tr>
<td></td>
<td>(0.180)</td>
<td>(0.117)</td>
</tr>
<tr>
<td>$\ln V_{t-1}$</td>
<td>0.307</td>
<td>0.085</td>
</tr>
<tr>
<td></td>
<td>(0.097)</td>
<td>(0.074)</td>
</tr>
<tr>
<td>const.</td>
<td>-3.728</td>
<td>-4.951</td>
</tr>
<tr>
<td></td>
<td>(0.893)</td>
<td>(0.686)</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td>No. obs.</td>
<td>38</td>
<td>38</td>
</tr>
</tbody>
</table>

Notes. S=duration less or equal to 1 quarter. L=duration greater than 1 quarter. Data not seasonally adjusted (3 quarterly dummies are included). Estimation method: OLS. Standard errors in brackets. Source: NOMIS.

the reason that new vacancies are on average of better quality than the old stock, and therefore they tend to be preferred to old vacancies by unemployed workers at all durations.

To investigate this we estimate the following unemployment outflow equations:

\[
\ln O_{S_t} = a_0 + c_S \ln v_t + d_S \ln V_{t-1} \quad (4.20)
\]

\[
\ln O_{L_t} = b_0 + c_L \ln v_t + d_L \ln V_{t-1} \quad (4.21)
\]

where $O_{S_t}$ and $O_{L_t}$ denote Kaplan-Meyer unemployment outflow rates at short ($\leq 1$ quarter) and long ($> 1$ quarter) duration respectively.

When both stock/flow matching and ranking matter, we expect $c_S > d_S$ and $c_L > d_L$. However, while the difference $c_L - d_L$ depends on both effects, the difference $c_S - d_S$ should only be explained by ranking. One way to check for the importance of pure stock/flow matching is therefore to control for the amount $c_S - d_S$ in the difference $c_L - d_L$. Estimation results are reported in Table 4.3.

The effect of new vacancies is positive and significant in the equation for short-
duration outflow rates, however it is not significantly different from the one of old vacancies. The difference between the two would explain the presence of ranking in matching, but the results show that the ranking of vacancies is not relevant to unemployment outflow. As we would expect, unemployment outflow rates at long durations are instead unaffected by the supply of old vacancies, and only respond to the inflow of new vacancies.

Having confirmed some basic predictions of a stock/flow matching model, we should turn to testing whether embodying these predictions into an aggregate matching function affects the standard conclusions on the evolution of matching effectiveness over time. We devote to this in the next section.

4.5 The performance of the matching function

In this section we turn back to matching effectiveness issues avoiding the restrictions generally imposed in the estimation of a $UV$ curve. We also try to deal with the two further problems connected with empirical matching functions, concerning time aggregation issues and the non-random nature of the matching process. Thus we can see if this re-formulation of the matching technology should alter our perception of changing effectiveness in the labour market as implied by the $UV$ curve analysis.

We consider a Cobb-Douglas version of an aggregate matching function such as (4.1)

$$\ln M_t = \gamma_0 + \gamma_1 \ln V_t + \gamma_2 \ln U_t + \epsilon_t$$

(4.22)

and estimate alternative specifications of it.

In section 3.4 we discussed in length the problems involved in handling timing in equation (4.22).

Here we deal with the simultaneity problem by using beginning-of-period stocks as regressors in (4.22). As far as the temporal aggregation problem is concerned, if the time-aggregated number of matches is to be used as the dependent variable, the relevant
pool of traders that can produce them is made up of the total number of vacancies and unemployed workers (old stock plus inflow), independently of non-random matching considerations, as in equation (4.7).

One further problem concerns finding an appropriate measure for the flow of matches between unemployed job-seekers and open vacancies during a time interval. Given that no such indicator is readily available, two possible candidates can be used as proxies: the outflow from the vacancy stock or the outflow from the unemployment stock. Neither of them is perfectly satisfactory as a direct measure of matches, for the reasons explained in section 3.3. However, due to the lack of better measures for matches, we provide two sets of estimates for (4.22), using either the unemployment outflow or the vacancy outflow as the dependent variable. We interpret these estimates as coming from two outflow equations, without claiming to estimate the structural parameters of an aggregate matching function. Hence we obtain two sources of information on matching effectiveness over time, one coming from vacancies, and the other from the claimant unemployed.

Moving away from the traditional stock-based analysis of matching involves two major steps. The first deals with time aggregation issues but does not negate the standard assumption of random matching. The second allows new flows to have higher exit rates than stocks.

Estimation is performed on the same data set used in section 4.4: vacancy stocks and flows and unemployment stocks and flows in Britain for the period 1967:1-1996:3.

For each outflow equation four specifications are adopted. The first one, which is the most directly comparable to the UV curve of section 4.2, is obtained using lagged stocks in the place of current stocks in order to avoid the simultaneity problem, and including time dummies defined on the same sub-periods as in section 4.2.

The second specification handles the time aggregation problem by including into right hand side variables in (4.22) the whole set of matching traders during the period over which \( M_t \) is measured. This is made up by the beginning-of-period stock, plus
some proportion of the inflow. This proportion should control for different outflow rates of stocks and flows, due to the different search time available to the two categories of traders. For small enough $\lambda$, this proportion can be approximated by $\frac{1}{2}$. As shown in section 3.4, it is actually possible to do slightly better than that. Equation (4.7) tells each agent in $S$ has a matching probability which is $\left[1 - \frac{1}{\lambda} (1 - e^{-\lambda})\right] (1 - e^{-\lambda})^{-1}$ times the matching probability of each agent in $I$. Hence the pool of vacancies and unemployed between time $t-1$ and time $t$ can be expressed in homogeneous “matching units” as

$$V_{t-1,t} = V_{t-1} + \left[\left(1 - e^{-\lambda_V}\right)^{-1} - \frac{1}{\lambda_V}\right] v_t$$

(4.23)

and

$$U_{t-1,t} = U_{t-1} + \left[\left(1 - e^{-\lambda_U}\right)^{-1} - \frac{1}{\lambda_U}\right] u_t$$

(4.24)

respectively. $\lambda_V$ and $\lambda_U$ are sample average values obtained under random matching estimation$^9$, giving $V_{t-1,t} = V_{t-1} + 0.74v_t$ and $U_{t-1,t} = U_{t-1} + 0.56u_t$. Note that we are keeping the assumption of constant hazard rates across stocks and flows, and that flow variables come into the regression with a constrained coefficient.

The third specification introduces non-random matching, including flow variables in the regression, but does not correct for the time aggregation bias. Finally, the fourth specification allows for non-random matching, on top of dealing with the time aggregation problem in the same fashion as the second specification. Unemployment and vacancy flows therefore appear in the regression both with a constrained coefficient (in $\ln V_{t-1,t}$ and $\ln U_{t-1,t}$) and as independent regressors. Given the correlation between $\ln V_{t-1,t}$ and $\ln v_t$ and between $\ln U_{t-1,t}$ and $\ln u_t$, the estimated coefficients are bound to be biased upwards in the “intermediate specifications”.

Tables (4.4) and (4.5) report estimates of equation (4.22) in the form of vacancy

---

$^9$See equation (4.7), results not reported.
Table 4.4: Estimates of vacancy outflow equations for Britain, 1967:1-1996:3

<table>
<thead>
<tr>
<th>dep. var: ln(vacancy outflow)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln V_{t-1}</td>
<td>0.382</td>
<td>0.093</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.017)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln U_{t-1}</td>
<td>0.147</td>
<td>0.010</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.013)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln V_{t-1,t}</td>
<td></td>
<td>0.804</td>
<td>0.298</td>
<td>0.325</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.036)</td>
<td>(0.056)</td>
<td>(0.035)</td>
<td></td>
</tr>
<tr>
<td>ln U_{t-1,t}</td>
<td></td>
<td>0.099</td>
<td>0.011</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.033)</td>
<td>(0.020)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln v_t</td>
<td></td>
<td>0.746</td>
<td>0.550</td>
<td>0.522</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.041)</td>
<td>(0.076)</td>
<td>(0.046)</td>
<td></td>
</tr>
<tr>
<td>ln u_t</td>
<td></td>
<td>-0.044</td>
<td>-0.038</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.032)</td>
<td>(0.036)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d_2</td>
<td>-0.013</td>
<td>-0.064</td>
<td>0.017</td>
<td>0.014</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.022)</td>
<td>(0.011)</td>
<td>(0.011)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>d_3</td>
<td>-0.119</td>
<td>-0.075</td>
<td>0.039</td>
<td>0.038</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.031)</td>
<td>(0.017)</td>
<td>(0.018)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>d_4</td>
<td>-0.064</td>
<td>-0.060</td>
<td>0.045</td>
<td>0.044</td>
<td>0.036</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.033)</td>
<td>(0.016)</td>
<td>(0.017)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>const.</td>
<td>3.362</td>
<td>1.430</td>
<td>1.457</td>
<td>1.266</td>
<td>1.049</td>
</tr>
<tr>
<td></td>
<td>(0.177)</td>
<td>(0.156)</td>
<td>(0.321)</td>
<td>(0.322)</td>
<td>(0.150)</td>
</tr>
<tr>
<td>adj. R^2</td>
<td>0.84</td>
<td>0.95</td>
<td>0.98</td>
<td>0.98</td>
<td>0.97</td>
</tr>
<tr>
<td>DW</td>
<td>0.88</td>
<td>0.90</td>
<td>1.40</td>
<td>1.34</td>
<td>1.23</td>
</tr>
<tr>
<td>No.obs.</td>
<td>115</td>
<td>115</td>
<td>115</td>
<td>115</td>
<td>115</td>
</tr>
</tbody>
</table>

Notes. V_{t-1,t} = V_{t-1} + 0.74v_t; U_{t-1,t} = U_{t-1} + 0.56u_t. d_1=1967:1-1973:4 (dropped); d_2=1974:1-1979:3; d_3=1979:4-1987:4; d_4=1988:1-1996:3. Data seasonally adjusted. Estimation method: OLS. Standard errors in brackets. Augmented Dickey-Fuller test (with four lags) rejected the hypothesis of a unit root in the residuals in all regressions. Source: Employment Gazette (various issues) and NOMIS.

From the standard specification in regressions (1) and (6) we find a qualitatively similar result to that implied by the estimates for a UV curve in section 4.2. However the deterioration in matching effectiveness over time looks much more severe for unemployed workers than for vacancies, and the slight recovery in matching effectiveness witnessed by the UV curve in the last period is not evident in this case. The coefficient on d_3 is not significantly different from the one on d_4 in column (1) and significantly higher in absolute value in column (6). Both equations give strongly decreasing returns to scale.

<table>
<thead>
<tr>
<th>dep. var: ln(unempl. outflow)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln $V_{t-1}$</td>
<td>0.088</td>
<td>0.001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>(0.042)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln $U_{t-1}$</td>
<td>0.239</td>
<td>0.123</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
<td>(0.033)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln $V_{t-1,t}$</td>
<td>0.139</td>
<td>0.121</td>
<td>0.067</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.042)</td>
<td>(0.130)</td>
<td>(0.123)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln $U_{t-1,t}$</td>
<td>0.362</td>
<td>0.226</td>
<td>0.367</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.038)</td>
<td>(0.044)</td>
<td>(0.053)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln $v_t$</td>
<td>0.412</td>
<td>0.225</td>
<td>0.420</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.108)</td>
<td>(0.138)</td>
<td>(0.180)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln $u_t$</td>
<td>0.831</td>
<td>0.699</td>
<td>0.661</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.076)</td>
<td>(0.082)</td>
<td>(0.080)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln $(LTU/U)_t$</td>
<td></td>
<td></td>
<td></td>
<td>-0.314</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.074)</td>
<td></td>
</tr>
<tr>
<td>$d_2$</td>
<td>-0.233</td>
<td>-0.227</td>
<td>-0.136</td>
<td>-0.147</td>
<td>-0.145</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.026)</td>
<td>(0.024)</td>
<td>(0.022)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>$d_3$</td>
<td>-0.289</td>
<td>-0.327</td>
<td>-0.232</td>
<td>-0.267</td>
<td>-0.230</td>
</tr>
<tr>
<td></td>
<td>(0.050)</td>
<td>(0.043)</td>
<td>(0.040)</td>
<td>(0.038)</td>
<td>(0.036)</td>
</tr>
<tr>
<td>$d_4$</td>
<td>-0.343</td>
<td>-0.367</td>
<td>-0.219</td>
<td>-0.253</td>
<td>-0.191</td>
</tr>
<tr>
<td></td>
<td>(0.046)</td>
<td>(0.038)</td>
<td>(0.038)</td>
<td>(0.035)</td>
<td>(0.036)</td>
</tr>
<tr>
<td>const.</td>
<td>4.854</td>
<td>3.408</td>
<td>-2.240</td>
<td>-1.732</td>
<td>-3.757</td>
</tr>
<tr>
<td></td>
<td>(2.61)</td>
<td>(0.391)</td>
<td>(0.790)</td>
<td>(0.727)</td>
<td>(0.795)</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>0.44</td>
<td>0.55</td>
<td>0.76</td>
<td>0.78</td>
<td>0.82</td>
</tr>
<tr>
<td>DW</td>
<td>0.73</td>
<td>0.84</td>
<td>1.10</td>
<td>1.21</td>
<td>1.58</td>
</tr>
<tr>
<td>No.obs.</td>
<td>115</td>
<td>115</td>
<td>115</td>
<td>115</td>
<td>110</td>
</tr>
</tbody>
</table>

Notes. See Table 4.4
When the time aggregation problem is dealt with in regression (2), the estimates for a vacancy outflow equation produce a slight deterioration in the second period and stability thereafter. Moreover the hypothesis of constant returns in the matching technology cannot be rejected, implying that the time aggregation bias in quarterly time series is rather serious. The unemployment outflow in column (7) shows the more familiar deterioration across the four periods considered.

Allowing for stock/flow matching (columns (3)-(5) and (8)-(10)) shows a greater importance of flows in the matching process, both in the vacancy and in the unemployment equation.

Unemployment variables are no longer significant in the vacancy outflow equations (3)-(5), which deliver, if anything, a very slight improvement in matching effectiveness. In the unemployment outflow equations (8) and (9), matching effectiveness exhibits an unchanged deterioration pattern in periods two and three, but remains stable in period four.

Finally, in regression (10) we include the proportion of long-term unemployment in total unemployment as an extra regressor in the unemployment outflow equation. This should control for the traditional concept of duration dependence that mostly happens at high unemployment durations, due to skill obsolescence and/or to the stigma attached to long-term unemployed. The $\frac{LTU}{U}$ ratio should reduce the explanatory power of flow variables in the case that higher matching probability of flows were a mere consequence of early stigma or loss of skills. We find instead that flow variables remain highly significant and that controlling for the incidence of long-term unemployment simply produces a slight improvement in matching effectiveness in the last sub-period.

Going back to vacancy outflow equations, it is somehow surprising that unemployment variables are in many cases not significant in vacancy outflow equations. Evidence would suggest that this is due to the substantially higher unemployment rates of the 1980s and the 1990s, and to the break-down of the usual negative relationship between

---

10With duration higher than six months.
Table 4.6: Estimates of vacancy outflow equations in Britain: 1967:1-1979:3

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln $V_{t-1}$</td>
<td>0.440</td>
<td>0.492</td>
<td>0.379</td>
<td>0.359</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.031)</td>
<td>(0.020)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>ln $U_{t-1}$</td>
<td>0.297</td>
<td>0.315</td>
<td>0.225</td>
<td>0.211</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>(0.037)</td>
<td>(0.019)</td>
<td>(0.047)</td>
</tr>
<tr>
<td>ln $V_{t-1,t}$</td>
<td>0.733</td>
<td>0.620</td>
<td>0.733</td>
<td>0.734</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(0.081)</td>
<td>(0.018)</td>
<td>(0.047)</td>
</tr>
<tr>
<td>ln $U_{t-1,t}$</td>
<td>0.225</td>
<td>0.211</td>
<td>0.225</td>
<td>0.211</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.047)</td>
<td>(0.019)</td>
<td>(0.047)</td>
</tr>
<tr>
<td>ln $v_t$</td>
<td>0.670</td>
<td>0.210</td>
<td>0.670</td>
<td>0.210</td>
</tr>
<tr>
<td></td>
<td>(0.060)</td>
<td>(0.093)</td>
<td>(0.060)</td>
<td>(0.093)</td>
</tr>
<tr>
<td>ln $u_t$</td>
<td>0.302</td>
<td>0.182</td>
<td>0.302</td>
<td>0.182</td>
</tr>
<tr>
<td></td>
<td>(0.137)</td>
<td>(0.098)</td>
<td>(0.137)</td>
<td>(0.098)</td>
</tr>
<tr>
<td>$d_2$</td>
<td>-0.098</td>
<td>-0.041</td>
<td>-0.029</td>
<td>-0.031</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.069)</td>
<td>(0.015)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>const.</td>
<td>2.089</td>
<td>0.032</td>
<td>-1.871</td>
<td>-1.726</td>
</tr>
<tr>
<td></td>
<td>(0.268)</td>
<td>(0.197)</td>
<td>(1.324)</td>
<td>(1.100)</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>0.89</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>DW</td>
<td>1.15</td>
<td>1.51</td>
<td>1.49</td>
<td>1.47</td>
</tr>
<tr>
<td>No.obs.</td>
<td>47</td>
<td>47</td>
<td>47</td>
<td>47</td>
</tr>
</tbody>
</table>

Notes. See Table 4.4.

the vacancy rate and the unemployment rate (see figure 4-1). To illustrate this, in Table 4.6 we provide estimates for vacancy outflow equations over the period 1967:3-1979:3, that coincides with the first and the second sub-periods of the sample. Both the unemployment stock and the inflow are now significant. Moreover, controlling for non-random matching still washes out most of the shift in the matching technology between the first and the second period.

Two important results emerge from the estimates of Tables (4.4)-(4.6). First, controlling for time aggregation reduces the downward bias in the elasticity of matches with respect to unemployment and vacancies stocks. Second, allowing for non-random matching implies nearly complete stability in the matching effectiveness of vacancies, and reduces the deterioration in the matching effectiveness of the unemployed. Among the concluding remarks of the next section, we suggest a plausible interpretation for this asymmetric behaviour of worker and employer search effectiveness.
4.6 Concluding remarks

The UV curve is a simple and powerful tool for analysing labour market effectiveness in matching vacancies and unemployed job-seekers. The standard UV curve analysis has been interpreted as providing clear evidence of a deteriorating labour market effectiveness up until 1988, which has been halted or very slightly reversed since. But appearances can be deceptive.

In this chapter we dealt with two major problems concerned with empirical matching functions. One is the time aggregation problem that arises when discrete time data are used to describe a continuous time process. We dealt with it by considering the whole pool of searching traders during the interval between two subsequent observations.

The second problem derives from the view of the matching process implied by random matching of unemployed and vacancies. Non-random matching is supported instead by empirical evidence on duration of search spells and by estimates of outflow equations which allow for a higher probability of matching for new inflows.

We explained higher initial matching probabilities using a stock/flow matching model, in which the stock of unmatched traders on one side of the market is matching with the inflow of traders on the other side. This interpretation is reinforced by the fact that ranking of heterogeneous agents does not seem to play too a strong role in raising initial matching probabilities, at least for vacancies.

Evidence provided suggests that most new vacancies match very quickly with the stock of the existing unemployed and the newly unemployed. However, a minority do not match with the stock and have to rely on new unemployed workers to match. These unfilled vacancies thus have a very low matching probability and make up the bulk of the stock of unfilled vacancies at any point in time. Symmetric considerations hold for the pool of unemployed workers. The implications are clearly that the stock of unfilled vacancies and uncompleted unemployment spells do not match with each other and thus can move independently over time without implying much about the effectiveness of the matching process.
When combined with a proper treatment of time aggregation in the matching function, this framework suggests that there has been no deterioration in the matching effectiveness of vacancies but the standard deterioration results still apply (although with smaller magnitude) to the matching effectiveness of the unemployed.

These different conclusions are probably explained by the fact that not all vacancy outflows involve a match with a claimant unemployed - including flows from out-of-labour force directly into jobs and job-to-job moves - and not all unemployment outflows represent moves into jobs. Interestingly enough, Gregg and Wadsworth (1996) show that the job entry probabilities of out-of-work women with a working partner have been rising in Britain over the past two decades, while those for women and men with no partner or a non-working one were falling. This is consistent with the evidence on matching functions provided in this chapter. Those with a working partner will have a low propensity to be unemployed claimants. As they are increasing their share of job matches, outflows of claimants will deteriorate with no change in the matching effectiveness of vacancies. Concerning job switches, Fuentes (1997) documents an increase in on-the-job search since the mid-1970s, that may have altered the position of the Beveridge curve. The implication is that overall matching effectiveness in the labour market has not deteriorated since the late 1960s, but that claimants of unemployment-related benefits are facing a stronger competition from other labour market segments and therefore are filling a lower proportion of the available vacancies.
Chapter 5

Re-employment probabilities and returns to matching

5.1 Introduction

Search models of the labour market impinge on the existence of a matching function that describes the technology of the job formation process by relating hirings to unemployment and vacancies. The equilibrium properties of such models crucially depend on the characteristics of the matching technology. In particular, the assumption of constant returns to scale in the matching function is the element that ensures a constant unemployment rate along a balanced-growth path, as shown in Pissarides (1990), Mortensen and Pissarides (1995) and Aghion and Howitt (1994). Multiple (rankable) equilibria arise instead when the matching function exhibits increasing returns to scale, as in Diamond (1982, 1984), thus raising a number of policy questions.

The aim of this chapter is to test the empirical relevance of this assumption, by estimating individual hazard functions on a sample of unemployment entrants.

The hazard rate denotes the probability of a transition out of unemployment within some small time interval, conditional on the worker being still unemployed when the interval started. If the underlying matching technology displays constant returns to
scale, the hazard rate for an unemployed worker should only depend on the degree of labour market tightness, measured by the vacancy-unemployment ratio, and not on the absolute size of the pool of searching agents as well. If instead matching displays increasing returns, the hazard rate should depend positively on the size of the market, after controlling for labour market tightness.

We argued above that re-employment probabilities have the potential of explaining different stages of the search process, being the combination of two probabilities: the probability of receiving a job offer and the probability of accepting the offer. The first of these depends on the set of characteristics that describe a worker's productivity (such as age, education, experience, etc.) and on labour demand conditions. The second probability depends on a worker's reservation wage, and therefore on the expected distribution of wages, family needs, the cost of search, unemployment income and labour market conditions. The effect of labour market conditions on re-employment probabilities is basically the only one captured by aggregate matching functions.

Moreover, with respect to aggregate matching functions, hazard function specifications have the main advantage of being rather flexible. They allow for a wide spectrum of functional forms for duration distributions, and control for a number of individual characteristics whose importance is only implicit in an aggregate matching function. In particular, they can introduce duration dependence of exit rates from unemployment, which is generally controlled for in aggregate estimates by conditioning job formation on single *ad hoc* regressors such as the incidence of long term unemployment.

Despite the importance of micro duration analysis for the understanding of aggregate matching performance, macro and micro approaches have so far mainly been used in the empirical search literature for answering different questions. On the one hand, aggregate matching function studies have mainly addressed the issue of aggregate matching effectiveness and of the returns to scale in the matching technology. Section 3.6 has reviewed a large number of studies that investigated the issue of constant versus increasing returns to scale in the matching function by using time series or panel data.
On the other hand, hazard functions were mainly used to investigate the determinants of exits from unemployment, without concern for the returns to scale in the underlying matching technology. An exception to this is the work by Lindeboom, van Ours and Renes (1994), who exploits the link between aggregate matching function and hazard rate specifications for evaluating the relative effectiveness of alternative search channels. The empirical analysis of this chapter however differs from theirs mainly on the grounds of the specification of the hazard. In particular, this is the first attempt to use individual data to estimate the returns to scale in the matching technology.

The data used in this chapter comes from the Survey of Incomes in and Out of Work, which examines labour market transitions of a sample of British workers who became unemployed in March/April 1987. In order to avoid a geographical aggregation bias, exit rates from unemployment are conditioned on local labour market variables, measured within travel-to-work areas, that are the closest approximation to self-contained labour markets. The constant returns hypothesis is tested by checking whether re-employment probabilities only depend on local labour market tightness, or whether they are also affected by the absolute number of traders in the market. In this second case the hypothesis of constant returns to scale would not be viable.

The rest of the chapter is organized as follows. Section 5.2 specifies the alternative econometric models to be estimated: a fully parametric hazard model with Weibull duration dependence, and a semi-parametric Cox proportional hazard model. Section 5.3 describes the data set used and section 5.4 provides the estimation results. Finally, section 5.5 concludes the work.

1 Devine and Kiefer (1991) review a number of hazard function studies, and among them only Nickell (1979) and Atkinson et al. (1984) include the labour market tightness as a determinant of the exit probability from unemployment, but they do not test for constant returns to scale in the matching technology by controlling for the labour market size.
5.2 The model

In order to study the determinants of the exit from unemployment, we will apply hazard models to data on the duration of unemployment spells.\(^2\)

The probability distribution of durations can be specified by the cumulative distribution function \( F(t) = \Pr(T < t) \), that provides the probability that a continuous random variable \( T \) denoting duration is less than some value \( t \). The corresponding density function is \( f(t) = dF(t)/dt \). The joint probability distribution of a sample of \( n \) observations \( t_i \) can be represented by the log-likelihood function

\[
L = \sum_{i=1}^{n} \ln f(t_i). \tag{5.1}
\]

In the case that some of the \( n \) observations are right censored, and hence represent uncompleted spells, the contribution to likelihood of each censored observation is the survivor function \( S(t) = 1 - F(t) \), which gives the probability that the duration is longer than \( t \). Let us introduce the censoring indicator \( c_i \), such that \( c_i = 1 \) if the \( i \)th observation is uncensored, and \( c_i = 0 \) otherwise. The likelihood function is given by

\[
L = \sum_{i=1}^{n} c_i \ln f(t_i) + \sum_{i=1}^{n} (1 - c_i) \ln S(t_i). \tag{5.2}
\]

It is convenient to express (5.2) in terms of the hazard rate \( \lambda(t) \), which denotes the probability of completing duration in the short interval of length \( dt \) after \( t \), conditional on duration being still uncompleted at time \( t \). The hazard rate is given by \( \lambda(t) = f(t)/S(t) = -d\ln S(t)/dt \).

Substituting (5.2) becomes

\[
L = \sum_{i=1}^{n} c_i \ln \lambda(t_i) - \sum_{i=1}^{n} \ln S(t_i). \tag{5.3}
\]

\(^2\)Econometric applications of hazard models are extensively described in Kiefer (1988) and Lancaster (1979, 1990).
with \( S(t) = \exp \left( - \int_0^t \lambda(s)ds \right) \).

Besides duration \( t \), a set of explanatory variables can affect the hazard function \( \lambda(t) \). Below we will consider the general case in which at least some of the regressors are time-varying, i.e. they assume more than one value during individuals’ unemployment spells. In particular, this is to condition re-employment probabilities on the whole evolution of local labour market variables during job search.

We consider a proportional hazard model, i.e.

\[
\lambda(t, x(t)) = \phi_1(t)\phi_2(x(t)),
\]

(5.4)

with the survivor function being given by

\[
S(t, x(t)) = \exp \left\{ - \int_0^t \phi_1(s)\phi_2(x(s))ds \right\}.
\]

(5.5)

The baseline hazard \( \phi_1(\cdot) \) is a functional form for the dependence of \( \lambda \) on duration. The second component \( \phi_2(\cdot) \) describes the way in which \( \lambda \) shifts, at given duration \( t \), between individuals endowed with different \( x \)'s.

If one wants to assess the impact of duration on re-employment probabilities, the baseline hazard needs to be represented by an explicit function of duration, e.g. the Weibull

\[
\phi_1(t) = \alpha t^{\alpha - 1},
\]

(5.6)

where \( \alpha \geq 1 \) denotes positive, zero or negative duration dependence, respectively.

The term \( \phi_2(\cdot) \) is conveniently specified as

\[
\phi_2(x(t)) = \exp(x(t)\beta),
\]

(5.7)

in order to ensure a non-negative \( \lambda \) without constraining the parameter space for \( \beta \).

The model outlined specifies the determinant of a single risk: that of leaving the unemployment register. Unemployment duration can terminate with job finding or
alternative states. Given that we are clearly interested in the first type of transition, we need to estimate a competing risk model, that distinguishes exit into employment from exit into alternative states.

Suppose that there are $J$ alternative states: then the contribution of the $i$th individual with destination $k$ to the log-likelihood is

$$
\ln L_i = c_{ik} \ln \lambda_k(t_i) - \sum_{j=1}^{J} \ln S_j(t_i)
$$

$$
= c_{ik} \ln \lambda_k(t_i) - \ln S_k(t_i) - \sum_{j \neq k} \ln S_j(t_i).
$$

The full log-likelihood is

$$
\ln L = \sum_i \ln L_i = \sum_j \ln L_j,
$$

with

$$
\ln L_j = \sum_{i=1}^{n} c_{ij} \ln \lambda_j(t_i) - \sum_{i=1}^{n} \ln S_j(t_i).
$$

Equation (5.9) shows that the parameters of a given cause-specific hazard can be estimated by treating durations finishing for other reasons as censored at time of exit (see Narendranathan and Stewart, 1993). We therefore treat all durations that end in non-employment as censored at the time the worker left the unemployment register. Having said this, the proportional hazard specification (5.4) used for the single-risk model can be applied for the job-finding hazard.

Explanatory variables $x(t)$ to be included into job-finding hazard are determined by a simple labour market matching model. For this purpose we consider the standard matching function in unemployment and vacancies, augmented with a search-effectiveness parameter:

$$
M_t = m(eU_t, V_t).
$$

This relates the amount of job creation to efficiency units of unemployment and the
number of vacancies. \( \bar{e} \) represents the average search effectiveness faced by employers. Ignoring for the moment duration dependence, the re-employment probability for an unemployed worker, \( \lambda(x_i(t)) \), is given by

\[
\lambda(x_i(t)) = e_i \frac{M_t}{\bar{e}U_t},
\]

(5.11)

where \( e_i \) denotes individual search effectiveness. Using a Cobb-Douglas specification for the function \( m(.) \), with elasticities \( a \) and \( b \) respectively, equation (5.11) becomes

\[
\lambda(x_i(t)) = \exp (\ln e_i - (1 - a) \ln \bar{e} - (1 - a) \ln U_t + b \ln V_t),
\]

(5.12)

so that \( x_i(t)'\beta = \ln e_i - (1 - a) \ln \bar{e} - (1 - a) \ln U_t + b \ln V_t \). Personal characteristics are used as proxies for individual search effectiveness \( e_i \). Average search effectiveness \( \bar{e} \) is captured by the constant term in \( x(t) \). Finally, \( U_t \) and \( V_t \) are measured in the local labour market where the \( i \)th individual lives and supposedly looks for a job.

If the matching function (5.10) displays constant returns to scale, \( a + b = 1 \), so that the hazard rate (5.12) only depends on the labour market tightness \( \theta_t = V_t/U_t \). If instead matching displays increasing returns, we expect a lower absolute coefficient on \( \ln U_t \) than on \( \ln V_t \).

The effect of possibly omitted regressors in the exit from unemployment will be controlled for by conditioning the hazard rate on an individual’s unobserved characteristics, summarised into the variable \( v \). The hazard rate is therefore rewritten as

\[
\lambda(v, t, x(t)) = v\phi_1(t)\phi_2(x(t)),
\]

with the survivor function being given by \( S(v, t, x(t)) = \exp \left\{-v \int_0^t \phi_1(s)\phi_2(x(s)) ds \right\} \). Following Lancaster (1979), we assume that \( v \) is distributed as a Gamma variate of unit mean and variance \( \sigma^2 \), taking the form

\[
f(v) \propto v^{\sigma^2-2} \exp \left(-\sigma^2v\right).
\]

(5.13)

Equation (5.13) assumes that \( v \) is independent of \( t \) and \( x(t) \). This assumption, combined with the proportional hazard model specification \( \lambda(v, t, x(t)) = \nu\phi_1(t)\phi_2(x(t)) \), is
sufficient to identify the three sources of variation among individual hazard rates. These are: the duration of search \((t)\), the observable differences among individuals \((x(t))\) and the unobservable ones \((v)\).

The hazard function and the survivor function, conditional on included regressors only, are computed as \(\int_0^\infty \lambda(v,t,x(t))f(v)dv\) and \(\int_0^\infty S(v,t,x(t))f(v)dv\), which give

\[
\lambda(t,x(t)) = \frac{\phi_1(t)\phi_2(x(t))}{1 + \sigma^2 \int_0^t \phi_1(s)\phi_2(x(s))ds} \quad (5.14)
\]

\[
S(t,x(t)) = \left\{1 + \sigma^2 \int_0^t \phi_1(s)\phi_2(x(s))ds\right\}^{-\sigma^{-2}} \quad (5.15)
\]

The discussion so far concerned a fully parametric specification of the hazard. However, for identifying the impact of explanatory variables \(x(t)\) on the hazard rate \(\lambda\), there is no need to specify the functional form of the baseline hazard \(\phi_1(.)\), in which case estimation is semi-parametric, as in the Cox (1972) proportional hazard model. This model exploits the ranking of observed durations: \(t_1 < t_2 < ... < t_i < ... t_n\). The conditional probability that some observation \(i\) could have completed a spell at duration \(t_i\), given that all those observations with longer duration could have completed a spell at the same duration, is \(\lambda(t_i, x_i(t))/\sum_{j=1}^n \lambda(t_j, x_j(t))\), which reduces to \(\phi_2(x_i(t))/\sum_{j=1}^n \phi_2(x_j(t))\) for the proportional hazard model (5.4). The resulting partial log likelihood is therefore

\[
L = \sum_{i=1}^n \left\{\ln \phi_2(x_i(t)) - \ln \left(\sum_{j=1}^n \phi_2(x_j(t))\right)\right\} \quad (5.16)
\]

Having described the likelihood functions that are the objective of the present analysis (see the Appendix for more details), we now turn to the description of all variables included and the data set used.

\[\text{See Lancaster (1979).}\]
5.3 The data

The data used comes from the Survey of Incomes In and Out of Work. The Survey collects individual information on a British representative sample of men and women who started a spell of unemployment and registered at any of the 88 Unemployment Benefit Offices (UBO) selected, in the four weeks starting March 16, 1987.

By focusing on unemployment entrants, the use of these data does not involve a stock sample bias, and allows to adopt semi-parametric methods such as the Cox proportional hazard model that do not condition on the length of unemployment at the first interview date.

Data was collected from two personal interviews. The first interview was carried out shortly after unemployment began - between April and July 1987 - and a total of 3003 interviews was completed with the selected respondents. The second interview was held about nine months later, in January 1988, on respondents who had been interviewed in 1987 and had consented to a second interview. A total of 2146 interviews was completed at this second stage.  

The first interview focused on individuals' personal details and their employment history since one year before the interview, including employment and unemployment income, type of job(s) held and job search activities while unemployed. The follow-up interview covered individuals' employment history since the first interview.

Given the competing-risk framework described, the duration of unemployment - treated in continuous time - is measured as the number of days between the date the worker signed at the UBO and the date she re-entered employment, provided she did not leave the unemployment register before that. In the case that the worker left the register before finding a job, the unemployment spell is censored and is measured as the duration of registered unemployment. Similarly, in the case that by the time of second

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4There is clearly some attrition in the data collected, with 28% of the observations being lost by the time of the second interview. Although we will use available information also on those who only had one interview, we nevertheless need to assume that attrition is random.
interview (or the first interview for those who only had one interview) she is not back yet into employment and she has not left the unemployment register, the unemployment spell is censored and is measured as the number of days between the day of signing at the UBO and the interview.

As said above, unemployment duration or, conversely, re-employment probabilities, depend on the probability of receiving a job offer and the probability of accepting the offer. The first of these depends on local labour market conditions, personal human capital variables such as education and the past (un)employment history, and characteristics such as sex, age, race and health status. The second probability is clearly influenced by the opportunity cost of being employed, measured by the replacement ratio, and the family composition of the unemployed.

As far as the characterisation of local labour markets is concerned, for confidentiality reasons the Survey does not attach explicit geographic identifiers to interviewees. The only geographical information that can be used is the code of the UBO at which the worker was registered. The first two digits of the UBO code denote the region where the UBO is located. Therefore the mapping between British regions and UBOs is non controversial.

However, in order to characterise more precisely local labour market conditions, it is advisable to switch to a much narrower definition of a local market such as the travel-to-work-area. TTWAs are approximations to self-contained labour markets, i.e. areas in which people live, work or look for jobs. According to the most recent definition, TTWAs meet the following criteria: a minimum of working population of 3,500; 75% of those living in the area work there; 75% of those working in the area live there.

The mapping between TTWAs and UBOs is more problematic. Using information from the NOMIS databank it is possible to associate a name to each UBO code. The mapping is then constructed using the TTWA classification provided by NOMIS in order to obtain the closest match between TTWAs and Jobcentres. UBOs which had the same name as a TTWA (e.g. Leeds) were easily located within the corresponding
TTWA. This allowed to locate 49 of the 88 UBOs selected in the Survey. Further progress is made using some implicit geographical information contained in the Survey. Attached to each worker is in fact the unemployment rate of the TTWA in which her UBO is situated. This permitted to locate 26 more UBOs making cross-section comparisons between the unemployment rates attached. Finally, 9 further UBOs were located using unambiguous associations between the name of the UBO and that of the TTWA (e.g. Stockport-Manchester). 4 remaining UBOs could not be located precisely and the corresponding 220 observations were dropped.

Once the mapping is done, the unemployment and vacancy data for the 61 resulting TTWAs are obtained from NOMIS. They refer to the unemployment claimant count and the number of vacancies advertised at Jobcentres.

Tables 5.1 and 5.2 reports some descriptive statistics concerning the sample used.

Before proceeding with estimation, we have a preliminary look at the time pattern of exit rates by computing Kaplan-Meier estimates for the survivor function and the hazard rate. This should give a preliminary idea of the shape of the baseline hazard. Figure 5-1 reports the Kaplan-Meier estimates for the survivor function, which shows that, by the end of the survey period, roughly 35% of the sample was still unemployed. If the Weibull baseline hazard is an adequate representation of reality we would expect the log-log of the survivor function to be a linear function of ln(t) (given that $S(t) = \exp(-t^\alpha)$), and the hazard rate to be a monotonic function of t. These two series are reported in Figures 5-2 and 5-3 respectively. By simple visual inspection, the Weibull assumption for duration dependence does not seem to be badly rejected by our raw data, in the sense that it is not possible to perceive systematic non-monotonicities in the hazard function. The results provided in next section will compare estimates obtained imposing a Weibull baseline hazard with those obtained with a non-parametric baseline hazard. As we would expect, they do not differ significantly.
Table 5.1: Sample characteristics of the unemployment inflow

<table>
<thead>
<tr>
<th>Variables</th>
<th>Males</th>
<th>Females</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>St. dev.</td>
</tr>
<tr>
<td>% exit in employment</td>
<td>51.5</td>
<td>1652</td>
</tr>
<tr>
<td>(% exit in employment) (c_t=1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>% exit in non-employment</td>
<td>10.2</td>
<td>1652</td>
</tr>
<tr>
<td>(% exit in non-employment) (c_t=0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>% stay unemployed</td>
<td>38.3</td>
<td>1652</td>
</tr>
<tr>
<td>(% stay unemployed) (c_t=0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>uncensored duration</td>
<td>11.9</td>
<td>850</td>
</tr>
<tr>
<td>censored duration</td>
<td>25.4</td>
<td>852</td>
</tr>
<tr>
<td>% not white</td>
<td>8.0</td>
<td>1652</td>
</tr>
<tr>
<td>% with health problems</td>
<td>35.9</td>
<td>1652</td>
</tr>
<tr>
<td>% with high education</td>
<td>43.9</td>
<td>1652</td>
</tr>
<tr>
<td>% married</td>
<td>76.3</td>
<td>1652</td>
</tr>
<tr>
<td>children</td>
<td>1.1</td>
<td>1.4</td>
</tr>
<tr>
<td>% home owners</td>
<td>49.3</td>
<td>1652</td>
</tr>
<tr>
<td>% lost full-time job</td>
<td>82.6</td>
<td>1652</td>
</tr>
<tr>
<td>% union members</td>
<td>30.5</td>
<td>1652</td>
</tr>
<tr>
<td>past year unemployment</td>
<td>1.6</td>
<td>5.5</td>
</tr>
<tr>
<td>age</td>
<td>37.7</td>
<td>12.0</td>
</tr>
<tr>
<td>replacement ratio</td>
<td>0.46</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Notes. The proportion of workers with high education includes all those who attended school or vocational training courses until the age of 18, plus those with higher education. The “children” variables denotes the total number of dependent children for males, and the presence of children under the age of six in the household for females. The replacement ratio is computed as the ratio between the total amount of benefits received by the worker (general + supplementary + housing benefits) and the take-home pay in the last job before registering to the UBO. Source: SIIOW

Table 5.2: Local labour markets in Britain

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>St. dev.</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>V/U - April 1987</td>
<td>0.085</td>
<td>0.057</td>
<td>61</td>
</tr>
<tr>
<td>V/U - July 1987</td>
<td>0.103</td>
<td>0.070</td>
<td>61</td>
</tr>
<tr>
<td>V/U - October 1987</td>
<td>0.121</td>
<td>0.084</td>
<td>61</td>
</tr>
<tr>
<td>V/U - January 1988</td>
<td>0.100</td>
<td>0.081</td>
<td>61</td>
</tr>
<tr>
<td>geog. size of TTWA (acres)</td>
<td>71669</td>
<td>80566</td>
<td>61</td>
</tr>
</tbody>
</table>

Source: NOMIS
Figure 5-1: The Kaplan-Meier estimate of the survivor function. Source: SIOW.

Figure 5-2: The Kaplan-Meier estimate of the log(-log(survivor function)). Source: SIOW.

Figure 5-3: The Kaplan-Meier estimate of the hazard function. Source: SIOW.
5.4 Empirical implementation

In order to estimate the models outlined in section 5.2, we let local labour market variables embodied into $x(t)$ to vary monthly, because this is the highest frequency available for unemployment and vacancy data, so that re-employment probabilities are conditioned on the series of monthly $U_t$ and $V_t$ during the individual's unemployment spell. For comparison purposes, the likelihood functions are also estimated using time-invariant $U$ and $V$. In this case the values used for $U$ and $V$ are those recorded in April 1987, when most workers in the sample started their unemployment spell.

One further local labour market variable that is included in the hazard is the geographical size of the TTWA where the worker lives. This should control for a density effect in the matching process.

There is a potential aggregation problem in using “grouped” data for the state of the labour market, as pointed out by Moulton (1986). This might lead to an overestimation of the significance level of aggregate regressors such as $U_t$, $V_t$, and the geographical size of the TTWA, due to the presence of common-group effects. In this specific case it is unclear what the solution may be. Controlling for common-group effects by including TTWAs dummies would in fact saturate the model when labour market regressors are time invariant, and would anyway overload the estimation when they are time-varying.

It can be said, however, that since we are interested in whether the coefficients on $\ln U_t$ and $\ln V_t$ are significantly different from each other, all that is required is that any potential overestimation of the $t$-statistics affect equally the two coefficients.

The model is estimated separately for both men and women, given that not only re-employment probabilities differ across genders, but they also respond differently to some of the controls used (see also Lynch, 1989). In particular, when controlling for family status, males re-employment probabilities are conditioned on the total number of dependent children, while female ones are conditioned on the presence of children under the age of six in the household.
Table 5.3 provides the estimation results using time-invariant regressors.\(^5\)

Column I reports the estimates of re-employment probabilities for men, not controlling for unobserved heterogeneity. The results overall look fairly consistent with the predictions of a simple search model and with previous empirical findings (see also the results collected in Devine and Kiefer, 1991). Personal characteristics that lower the re-employment probabilities of men include age, the replacement ratio, the time spent as unemployed during the year previous to the survey, belonging to ethnic minorities, having health problems and having been union member during the last job held. Higher education increases instead the probability of finding a job, and so does home ownership and being married, while the total number of dependent children does not.

Contrary to expectations, having lost a full-time job in the past does not enhance significantly the probability of finding a new job. This is possibly explained by the negative correlation (for both men and women) between the replacement ratio and the full-time control. Workers that lost a full-time job have lower replacement ratios, and it is difficult to distinguish the two effects on re-employment probabilities. Estimation was also performed dropping the replacement ratio, delivering a positive and highly significant effect on the full-time status.

Local labour market variables have the expected impact on the hazard. Moreover, the coefficient on \(\ln U\) is not significantly different from the one on \(\ln V\), providing evidence in favour of constant returns to scale in the matching function. The geographical size of the local labour market has instead no significant impact on individual hazards, providing therefore no evidence in favour of a density effect in matching. Concerning duration dependence, the estimated Weibull coefficient \(\alpha\) is significantly lower than 1, implying that the hazard is slightly declining with duration.

Turning to re-employment probabilities for women in column III, they seem to be affected positively by the educational level, and negatively by the replacement ratio,

\(^5\)The estimates presented were obtained using a \textit{quasi-Newton} method (the Broyden, Fletcher, Goldfarb, Shannon method), with the covariance matrix computed as the inverse of the Hessian. Alternative methods used provided equivalent results.
Table 5.3: Maximum likelihood estimates of re-employment probabilities with time-invariant regressors

<table>
<thead>
<tr>
<th>Baseline hazard</th>
<th>Weibull</th>
<th>Non parametric</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Males I</td>
<td>Males II</td>
</tr>
<tr>
<td>constant</td>
<td>2.271</td>
<td>5.894</td>
</tr>
<tr>
<td></td>
<td>(0.778)</td>
<td>(1.956)</td>
</tr>
<tr>
<td>ln(age)</td>
<td>-1.066</td>
<td>-1.693</td>
</tr>
<tr>
<td></td>
<td>(0.196)</td>
<td>(0.284)</td>
</tr>
<tr>
<td>not white</td>
<td>-0.267</td>
<td>-0.343</td>
</tr>
<tr>
<td></td>
<td>(0.168)</td>
<td>(0.277)</td>
</tr>
<tr>
<td>health problems</td>
<td>-0.173</td>
<td>-0.316</td>
</tr>
<tr>
<td></td>
<td>(0.086)</td>
<td>(0.149)</td>
</tr>
<tr>
<td>high education</td>
<td>0.532</td>
<td>0.628</td>
</tr>
<tr>
<td></td>
<td>(0.081)</td>
<td>(0.138)</td>
</tr>
<tr>
<td>married</td>
<td>0.511</td>
<td>0.584</td>
</tr>
<tr>
<td></td>
<td>(0.217)</td>
<td>(0.238)</td>
</tr>
<tr>
<td>children</td>
<td>0.037</td>
<td>0.096</td>
</tr>
<tr>
<td></td>
<td>(.074)</td>
<td>(.058)</td>
</tr>
<tr>
<td>home owner</td>
<td>0.145</td>
<td>0.240</td>
</tr>
<tr>
<td></td>
<td>(0.089)</td>
<td>(0.149)</td>
</tr>
<tr>
<td>had FT job</td>
<td>0.061</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>(0.253)</td>
<td>(0.098)</td>
</tr>
<tr>
<td>union member</td>
<td>-0.265</td>
<td>-0.442</td>
</tr>
<tr>
<td></td>
<td>(0.094)</td>
<td>(0.150)</td>
</tr>
<tr>
<td>past unemp.</td>
<td>-0.028</td>
<td>-0.034</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>ln(repl. ratio)</td>
<td>-0.392</td>
<td>-0.689</td>
</tr>
<tr>
<td></td>
<td>(.043)</td>
<td>(.099)</td>
</tr>
<tr>
<td>ln(U)</td>
<td>-0.205</td>
<td>-0.356</td>
</tr>
<tr>
<td></td>
<td>(0.072)</td>
<td>(0.124)</td>
</tr>
<tr>
<td>ln(V)</td>
<td>0.171</td>
<td>0.294</td>
</tr>
<tr>
<td></td>
<td>(0.082)</td>
<td>(0.136)</td>
</tr>
<tr>
<td>ln(area)</td>
<td>-0.011</td>
<td>-0.074</td>
</tr>
<tr>
<td></td>
<td>(0.057)</td>
<td>(0.143)</td>
</tr>
<tr>
<td>α</td>
<td>0.910</td>
<td>1.406</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.102)</td>
</tr>
<tr>
<td>σ²</td>
<td>-</td>
<td>1.292</td>
</tr>
<tr>
<td></td>
<td>(0.133)</td>
<td>(0.249)</td>
</tr>
<tr>
<td>No. Obs.</td>
<td>1239</td>
<td>1239</td>
</tr>
</tbody>
</table>

Notes. Standard errors in brackets. Source: SIOW and NOMIS.
the presence of young children in the household, union membership and past full-time status. Similarly as for men, the full-time variable had instead a positive significant impact when the replacement ratio was dropped.

Unemployment and vacancy variables have an opposite sign to what one would have expected, although neither coefficient is significantly different from zero. This can be at least partly explained considering that the controls used - the number of registered unemployed and the number of vacancies advertised at job-centres - typically reflect males’ labour market variables. The design of the British unemployment insurance system is in fact such that out-of-work women are less likely to be registered unemployed (see Gregg, 1994), so that the related figures are much closer to the male rather than the female unemployment rate. Furthermore, the information given by the S1IOW shows that the proportion of unemployed women who find a job through a Jobcentre is lower than that for unemployed men, so that vacancies advertised there may only affect weakly the probability of a woman going back into work. Interestingly enough, there seems to be a moderate density effect in female re-employment probabilities, given that coefficient on the geographical size of the local labour market is significantly lower than zero (at the 10% significance level).

Similarly as for men, unemployment duration negatively affects female re-employment probabilities.

However, before concluding that there is negative duration dependence in the transition probabilities from unemployment to employment, we should consider the possibility that the estimates obtained in column I are biased due to the omission of unobserved variables. As Lancaster (1979) recognises, the estimate for \( \alpha \) is in fact at least in part an index of the misspecification of the model, measuring the extent of unrecognised heterogeneity within the sample. With the present sample, this is found in column II and IV, where the control for gamma-distributed unobserved heterogeneity delivers a value of \( \alpha \) well above 1 for both males and females. If anything, the presence of unobserved heterogeneity seems more relevant in the female sample, as shown by higher values of \( \alpha \)
and $\sigma^2$ for women than for men. Once controlling for higher $\alpha$, the magnitude of most coefficients in columns II and IV is closely comparable to that of coefficients in columns I and III.$^6$

Even so, it cannot be concluded at this stage that re-employment probabilities are genuinely increasing with duration. The sample is in fact constructed in such a way that it is not possible to distinguish between genuine duration dependence and calendar time dependence, given that all individuals have started an unemployment spell within the same four weeks. As it will be shown by the results that follow, this is a serious problem in the estimates provided, given that the British economy experienced some recovery during 1987 (see also the $\theta$ ratios reported in Table 5.1). This may in fact have improved the re-employment prospects for all those who have been jobless long enough to benefit from the recovery, thus introducing some spurious positive duration dependence in hazard functions.

It can be argued that the dependence of re-employment probabilities on the state of the labour market is a combination of two factors: a purely aggregate factor, represented by business cycle and seasonal fluctuations that affect equally all workers in the sample, irrespective of the area where they live, and local deviations from these aggregate trends, represented by the time pattern of local labour market characteristics. While in the sample used the first component cannot be distinguished from the genuine duration dependence, an attempt to control for local labour market trends can be made by conditioning re-employment probabilities on the time pattern of local labour demand during the whole unemployment spell.

The estimation results using time-varying regressors are reported in Table 5.4.

The sign and the significance of most explanatory variables in columns I-IV have hardly changed, for both males and females, with respect to the case in which all regressors are time-invariant. In particular, local labour market variables have the

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$^6$The coefficient on marital status and on home ownership status switch sign in the regression for females, when unobserved heterogeneity is controlled for. However in no case they are significantly different from zero.
Table 5.4: Maximum likelihood estimates of re-employment probabilities with time-varying regressors

<table>
<thead>
<tr>
<th>Baseline hazard</th>
<th>Weibull</th>
<th>Non parametric</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Males I</td>
<td>Males II</td>
</tr>
<tr>
<td>constant</td>
<td>3.131</td>
<td>3.137</td>
</tr>
<tr>
<td>(1.168)</td>
<td>(0.999)</td>
<td>(1.426)</td>
</tr>
<tr>
<td>ln(age)</td>
<td>-1.038</td>
<td>-1.037</td>
</tr>
<tr>
<td>(0.153)</td>
<td>(0.174)</td>
<td>(0.183)</td>
</tr>
<tr>
<td>not white</td>
<td>-0.272</td>
<td>-0.272</td>
</tr>
<tr>
<td>(0.165)</td>
<td>(0.157)</td>
<td>(0.313)</td>
</tr>
<tr>
<td>health problems</td>
<td>-0.180</td>
<td>-0.180</td>
</tr>
<tr>
<td>(0.088)</td>
<td>(0.087)</td>
<td>(0.114)</td>
</tr>
<tr>
<td>high education</td>
<td>0.501</td>
<td>0.500</td>
</tr>
<tr>
<td>(0.084)</td>
<td>(0.085)</td>
<td>(0.115)</td>
</tr>
<tr>
<td>married</td>
<td>0.478</td>
<td>0.478</td>
</tr>
<tr>
<td>(0.187)</td>
<td>(0.141)</td>
<td>(0.128)</td>
</tr>
<tr>
<td>children</td>
<td>0.040</td>
<td>0.040</td>
</tr>
<tr>
<td>(0.047)</td>
<td>(0.035)</td>
<td>(0.203)</td>
</tr>
<tr>
<td>home owner</td>
<td>0.124</td>
<td>0.124</td>
</tr>
<tr>
<td>(0.102)</td>
<td>(0.088)</td>
<td>(0.139)</td>
</tr>
<tr>
<td>had FT job</td>
<td>0.008</td>
<td>0.008</td>
</tr>
<tr>
<td>(0.190)</td>
<td>(0.170)</td>
<td>(0.125)</td>
</tr>
<tr>
<td>union member</td>
<td>-0.177</td>
<td>-0.176</td>
</tr>
<tr>
<td>(0.088)</td>
<td>(0.090)</td>
<td>(0.148)</td>
</tr>
<tr>
<td>past unemp.</td>
<td>-0.030</td>
<td>-0.030</td>
</tr>
<tr>
<td>(0.012)</td>
<td>(0.011)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>ln(repl. ratio)</td>
<td>-0.368</td>
<td>-0.367</td>
</tr>
<tr>
<td>(0.043)</td>
<td>(0.049)</td>
<td>(0.057)</td>
</tr>
<tr>
<td>ln(U_t)</td>
<td>-0.413</td>
<td>-0.414</td>
</tr>
<tr>
<td>(0.072)</td>
<td>(0.082)</td>
<td>(0.114)</td>
</tr>
<tr>
<td>ln(V_t)</td>
<td>0.412</td>
<td>0.416</td>
</tr>
<tr>
<td>(0.077)</td>
<td>(0.076)</td>
<td>(0.118)</td>
</tr>
<tr>
<td>ln(area)</td>
<td>-0.062</td>
<td>-0.063</td>
</tr>
<tr>
<td>(0.096)</td>
<td>(0.069)</td>
<td>(0.113)</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.844</td>
<td>0.850</td>
</tr>
<tr>
<td>(0.029)</td>
<td>(0.061)</td>
<td>(0.041)</td>
</tr>
<tr>
<td>(\sigma^2)</td>
<td>-</td>
<td>0.015</td>
</tr>
<tr>
<td>(2.233)</td>
<td>(3.357)</td>
<td></td>
</tr>
<tr>
<td>No. Obs.</td>
<td>1239</td>
<td>1239</td>
</tr>
</tbody>
</table>

Notes. Standard errors in brackets. Source: SIIOW and NOMIS.
expected sign on the re-employment probabilities for men. Absolute coefficients on \( \ln U_t \) and \( \ln V_t \) are substantially higher than those on time-invariant regressors \( \ln U \) and \( \ln V \) (see Table 5.3) and strikingly close to each other, confirming the presence of constant returns to scale in matching. Local labour market conditions instead do not affect significantly the re-employment probabilities of women, but their coefficients now have the expected sign.\(^7\)

What changes significantly from Table 5.3 is the relative importance of state dependence versus unobserved heterogeneity. Apparently, when re-employment probabilities are conditioned on the whole evolution of the state of local labour markets over time, there is evidence of negative duration dependence of hazard rates, as shown in columns I-IV of Table 5.4. The inclusion of time-varying regressors captures in fact the rise in re-employment probabilities due to the improving prospects of the British economy though the second half of 1987. Interestingly enough, there is no residual unobserved heterogeneity to be accounted for in individual hazard functions, as shown by the non significant value of \( \sigma^2 \) for both males and females.

Concerning the robustness of the CRS result, it may be argued that the fully parametric approach adopted, where the functional form for duration dependence is specified as a Weibull distribution, has imposed some unnecessary restrictions on the shape of re-employment probabilities. In order to obtain some more general results, a Cox (1972) proportional hazard model is also estimated. This model is semiparametric in the sense that it does not specify any functional form for duration dependence, and therefore does not predict whether the hazard is upward or downward sloping with duration. The results obtained are reported in the last two columns of Tables 5.3 and 5.4, using time-invariant and time-varying regressors in turn.

\(^7\)Female re-employment probabilities were also estimated for a subsample of women, with supposedly stronger average attachment to the labour market than the whole sample. The subsample was constructed selecting women that lost a full time job and have not been unemployed in the year before the first interview, for a total of 374 women. Coefficients on \( \ln U_t \) and \( \ln V_t \) for this subsample were -0.311 (s.e. 0.143) and 0.310 (s.e. 0.147) respectively, in line with the constant returns to scale hypothesis. Similarly, the estimates with Gamma-distributed heterogeneity were -0.393 (s.e. 0.178) and 0.387 (s.e. 0.181) respectively.
Column V of Table 5.3 shows a vector of estimated coefficients for men that is virtually unchanged from the one obtained using a fully parametric model with Weibull duration dependence. The Weibull baseline hazard therefore seems to be a reasonable characterisation of the duration distribution of males’ unemployment spells. Things change slightly for women: the coefficient on the age variable becomes positive and significant, and the relevant family variable is now the marital status, as opposed to the presence of children under six in the household. In other words, the negative effect of the presence of young children on female transitions into jobs is now captured by the fact that re-employment probabilities increase as women get over the age of child care.

For both women and men, the effect of local labour market variables replicate pretty closely the results of columns I and II. Very similar considerations hold for estimates that use time-varying regressors, reported in columns V and IV of Table 5.4: the impact of local labour market variables does not lead to reject the assumption of CRS in matching for males, while is not significantly different from zero for females.

Finally, we test whether the effect of local unemployment and vacancies on hazard rates is also time-varying, along the lines of the results reported in chapter 2. We would in fact expect the vacancy stock to have a significant effect on re-employment probabilities at the beginning of the unemployment spell, and an non significant one thereafter. Conversely, we expect the congestion effect deriving from local unemployment to start biting as the unemployment spells persists, given that the unemployment inflow should not experience substantial competition for jobs from the old unemployed.

To investigate this we provide estimates of hazard functions with Weibull baseline hazard and no unobserved heterogeneity, in which the effect of \( \ln(U) \) and \( \ln(V) \) on re-employment probabilities is allowed to be duration dependent in two alternative ways.\(^8\) The first consists in interacting \( \ln(U) \) and \( \ln(V) \) with \( \log(\text{duration}) \), and the second consists in interacting the two regressors with a dummy variable that represents

---

\(^8\) Similar results to the ones reported were obtained using time-varying regressors and/or a non-parametric baseline hazard. Also, given that the effect of local labour market conditions is not clearly interpretable for female transitions into employment, below we focus on male data.
Table 5.5: The joint effect of unemployment duration and labour market variables on re-employment probabilities for males.

<table>
<thead>
<tr>
<th>Baseline hazard</th>
<th>Weibull</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Males</td>
</tr>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>ln(U)</td>
<td>-0.294</td>
</tr>
<tr>
<td>(0.105)</td>
<td>(0.101)</td>
</tr>
<tr>
<td>ln(V)</td>
<td>0.186</td>
</tr>
<tr>
<td>(0.101)</td>
<td>(0.116)</td>
</tr>
<tr>
<td>ln(U) x ln(duration)</td>
<td>-0.044</td>
</tr>
<tr>
<td>(0.042)</td>
<td>(0.046)</td>
</tr>
<tr>
<td>ln(V) x ln(duration)</td>
<td>-0.058</td>
</tr>
<tr>
<td>(0.046)</td>
<td></td>
</tr>
<tr>
<td>ln(U) x Q</td>
<td>-0.251</td>
</tr>
<tr>
<td>(0.287)</td>
<td></td>
</tr>
<tr>
<td>ln(V) x Q</td>
<td>-0.313</td>
</tr>
<tr>
<td>(0.290)</td>
<td></td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.890</td>
</tr>
<tr>
<td>(0.029)</td>
<td>(0.102)</td>
</tr>
<tr>
<td>average log-likel.</td>
<td>-2.372</td>
</tr>
<tr>
<td>No. Obs.</td>
<td>1239</td>
</tr>
</tbody>
</table>

Notes. Regressors included: see Table \ref{invar}. \( Q = 1 \) if duration is longer than a quarter. Standard errors in brackets. Source: SIIOW and NOMIS.

durations longer than one quarter. The likelihood functions estimated are reported in the Appendix.

The results are reported in Table 5.5. In both columns, the effect of the interaction between labour market variables and duration has the expected sign, although in neither case such an effect is statistically significant.

### 5.5 Conclusions

The assumption of constant returns in the matching function is a property embodied in most bilateral search models, being necessary to ensure the uniqueness of the unemployment rate along a steady state growth path. This chapter has investigated whether this is a plausible assumption, by estimating re-employment probabilities on a British sample of entrants into unemployment.
The analysis was led in the context of two alternative duration models: a fully parametric hazard model, with Weibull type of duration dependence and Gamma-distributed unobserved heterogeneity, and a more general Cox proportional hazard model, that does not impose any specific functional form for the baseline hazard. The two specifications delivered pretty consistent estimates.

The results obtained broadly confirm previous findings on the determinants of re-employment probabilities for men and women (see Devine and Kiefer, 1991, for a survey), and are generally consistent with the predictions of a job search framework. The probability of receiving a job offer should be related to the individual's educational attainment and to the state of local labour demand, which in fact affect positively re-employment probabilities for men. No effect of local labour demand is instead detected in re-employment probabilities for women, although this may be at least in part a consequence of how local labour market conditions are measured. The probability of accepting a job offer should instead depend on the replacement ratio and on the family composition of the individual, which in fact have the expected effect on re-employment probabilities of both men and women.

Concerning the shape of the baseline hazard, clear evidence is found of negative duration dependence in hazard rates for both males and females, when re-employment probabilities are conditioned on the whole evolution of local labour demand during the unemployment spell. This result is also robust to the introduction of gamma distributed unobserved heterogeneity.

In no specification does the absolute coefficient on local unemployment differ from that on the number of vacancies, implying that the absolute size of the searching pool does not affect matching conditions. This therefore allows not to reject the CRS hypothesis in the matching technology between unemployment and vacancies. This finding in turn implies that results of several aggregate studies à la Blanchard and Diamond (1989) where not too seriously biased by geographical aggregation problems.
5.6 Appendix: Some likelihood functions

According to (5.12), the $x(t)$ vector includes some variables that are time-invariant, that we represent by $y$, and some that are time-varying, represented by $z(t)$, so that

$$\phi_2(x(t)) = \exp(y'\gamma + z(t)'\delta).$$

Therefore

$$\ln S(t, x(t)) = -\exp(y'\gamma) \int_0^t \alpha s^{\alpha-1} \exp(z(s)'\delta) ds.$$  (5.17)

Suppose now that variables in $z(s)$ assume a finite number of values between time 0 and time $t$, say 2 for simplicity, such that $z(s) = z_1$ for $0 < s < u$, and $z(s) = z_2$ for $u < s < t$, implying

$$\ln S(t, x(t)) = -\exp(y'\gamma) \left[ \exp(z_1'\delta_1) u^\alpha + \exp(z_2'\delta_2) (t^\alpha - u^\alpha) \right].$$  (5.18)

Equation (5.3) can hence be rewritten as

$$L = \sum_{i=1}^{n} c_i \{\ln \alpha + (\alpha - 1) \ln t_i + x_i(t)'\beta\}$$

$$- \sum_{i=1}^{n} \exp(y_i'\gamma) \left[ \exp(z_1'\delta_1) u_i^\alpha + \exp(z_2'\delta_2) (t_i^\alpha - u_i^\alpha) \right].$$  (5.19)

When Gamma-distributed unobserved heterogeneity is included in the form $\lambda(v, t, x(t)) = v \phi_1(t) \phi_2(x(t))$, with $f(v) \propto v^{\alpha-2} \exp(-\sigma^{-2}v)$, the log-likelihood becomes

$$L = \sum_{i=1}^{n} c_i \{\ln \alpha + (\alpha - 1) \ln t_i + x_i(t)'\beta\}$$  (5.20)
\[-\sum_{i=1}^{n} c_i \ln \left\{ 1 + \sigma^2 \exp(y_i') \left[ \exp(z_i'_{i1} \delta_1) u_i^\alpha + \exp(z_i'_{i2} \delta_2) (t_i^\alpha - u_i^\alpha) \right] \right\} \]

\[-\sigma^{-2} \sum_{i=1}^{n} \ln \left\{ 1 + \sigma^2 \exp(y_i') \left[ \exp(z_i'_{i1} \delta_1) u_i^\alpha + \exp(z_i'_{i2} \delta_2) (t_i^\alpha - u_i^\alpha) \right] \right\}, \]

which tends to the log-likelihood in (5.19) as \( \sigma^2 \to 0. \)

Finally, when the effect of labour market variables is allowed to be duration-dependent, two alternative specifications for the hazard are fitted:

\[\lambda(t, x(t)) = \alpha t^{\alpha-1} \exp (\beta_1 \ln U + \beta_2 \ln V + \beta_3 \ln U \ln t + \beta_4 \ln V \ln t + y'/\gamma) \quad (5.21)\]

and

\[\lambda(t, x(t)) = \alpha t^{\alpha-1} \exp (\beta_1 \ln U + \beta_2 \ln V + \beta_3 q \ln U + \beta_4 q \ln V + y'/\gamma), \quad (5.22)\]

where \( y \) is a vector of time-invariant personal controls and \( q = 1 \) if duration \( t \) is longer than one quarter. Relevant likelihood functions are

\[L = \sum_{i=1}^{n} c_i \left\{ \ln \alpha + (\alpha - 1 + \beta_3 \ln U + \beta_4 \ln V) \ln t_i + \beta_1 \ln U + \beta_2 \ln V + y_i' \gamma \right\} \]

\[-\sum_{i=1}^{n} \exp (\beta_1 \ln U + \beta_2 \ln V + y_i' \gamma) \frac{\alpha}{\alpha + \beta_3 \ln U + \beta_4 \ln V} t_i^{\alpha + \beta_3 \ln U + \beta_4 \ln V}. \quad (5.23)\]

and

\[L = \sum_{i=1}^{n} c_i \left\{ \ln \alpha + (\alpha - 1) \ln t_i + \beta_1 \ln U + \beta_2 \ln V + \beta_3 q_i \ln U + \beta_4 q_i \ln V + y_i' \gamma \right\} \]

145
\[- \sum_{i=1}^{n} (1 - q_i) \exp (\beta_1 \ln U + \beta_2 \ln V + \gamma_i \gamma) t_i^o \]

\[- \sum_{i=1}^{n} q_i \exp (\beta_1 \ln U + \beta_2 \ln V + \gamma_i \gamma) [Q_i^o + (t_i^o - Q_i^o) \exp (\beta_3 \ln U + \beta_4 \ln (\beta \tau))] \]

respectively, where $Q$ denotes the length of a quarter.
Chapter 6

Conclusions

Since the early 1970s, most OECD countries have experienced dramatic rises in their unemployment stocks, a substantial proportion of which displayed the features of a permanent rather than transitory increase. While in fact stable inflation in the 1960s would suggest that the NAIRU in the countries of the European Union was below 3%, the rising inflation in the late 1980s, associated with a 9% unemployment rate, would imply that the NAIRU had in the meanwhile risen above 9%.

In addition, parallel evidence on the evolution of flows of workers and jobs seemed to point to a strong deterioration in the effectiveness of the labour market in matching unemployed workers to available vacancies, with the outflow rate from unemployment falling in many countries despite a roughly untrended supply of new vacancies.

This broad evidence is hard to reconcile with persistence-based labour market theories that focus on the sluggish adjustment of the unemployment level in the aftermath of a temporary recession, and more generally with stock-based theories that do not explicitly take into account the behaviour of worker and job flows.

Given these weaknesses, and the existing evidence on the structure of unemployment, the first part of this thesis addresses the question of the permanent rise in the unemployment rate in a number of OECD countries by looking at the link between sectoral shocks and aggregate performance in an economy with heterogeneous labour.
The type of sectoral shock that we consider in chapter 2 is the introduction of skill-biased technologies, that increase the relative demand for skilled labour at the expenses of the less-skilled. Unless the supply of skills adjusts accordingly to the increased demand, and/or relative wages are perfectly flexible, this type of shock is bound to have permanent effects on the aggregate unemployment rate, as we show in a simple non-competitive labour market model with skilled and unskilled workers. When confronted with data from a number of OECD economies, this models predicts that a relevant proportion of the recent rise in British unemployment can be attributed to an unbalanced evolution in the demand and the supply of skills, while in continental Europe skill imbalances do not seem responsible for serious labour market problems. Finally, the impact of skill mismatch on US unemployment was limited in magnitude and almost completely offset by a fall in wage pressure. However, why the evolution of wage pressure in the US could effectively pin down the unemployment rate around 6%, while instead generating in Europe extra unemployment growth, is still an open question.

The second half of this thesis uses a search approach to take a systematic look at the characteristics of the matching process between traders in the labour market. An alternative way of thinking about the imperfect match between the demand and the supply of labour consists in fact in exploring the “technical” characteristics of the hiring function, which expresses the number of newly formed jobs as a function of the number of agents that search on either side of the labour market.

We begin this part of the thesis in chapter 3, with a survey of the empirical search literature that has estimated labour market matching functions. Although still quite young, this stream of research has successfully established the existence of a labour market matching function that translates optimal search decisions at the micro level into aggregate worker and job flows. This survey chapter reveals the importance of the hiring function as a powerful tool for assessing labour market performance and identifies a few aspects that should require further investigation.

One of these aspects is addressed in chapter 4, that tackles the issue of match-
ing effectiveness by removing some of the restrictions generally imposed in previous theoretical and empirical work on the matching function. We consider a plausible alternative to the random meeting technology between employers and job-seekers, based on the existence of cheap information channels that save all traders the effort of locating matching partners. When combined with a proper handling of timing in the estimation of an aggregate matching function, this set-up provides novel results on the matching effectiveness of the British labour market over the past thirty years. In particular, it seems that the claimed recent deterioration of the search effectiveness of the unemployed cannot be explained by a lack of search effort per se, but by stronger competition that registered unemployed face by other labour market segments. This preliminary conclusion naturally welcomes future research on the evolution and the effects of the search behaviour of all those that are not registered as unemployed.

Finally, chapter 5 provides an analysis of the matching process at the micro level, based on individual duration data obtained from a British sample of unemployment entrants. The determinants of re-employment probabilities are here related to a search model in which the transition into employment depends on the probability of receiving a job offer and that of accepting a job offer. The main result provided by this chapter shows that the hypothesis of constant returns to scale in the matching technology, embodied in most bilateral search models, is not rejected by our data. Individual re-employment probabilities respond in fact to local labour market tightness, as measured by the vacancy-unemployment ratio, and are unaffected by its size. A natural extension of this work consists in assessing whether the matching process may instead display increasing returns as far as the quality - as opposed to the number - of matches are concerned. This idea rests on the premise that thick markets should provide better matching opportunities to highly specialized labour.

Further microeconomic work on the matching function is bound to follow. In recent years, micro duration studies and aggregate studies on the hiring function have mainly addressed different issues, but we argue that this needs not to be the case. More
generally, we believe that providing sound microfoundations for the matching process - both theoretical and empirical - is one of the most valuable inputs for advancing search models of the labour market.
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