DYNAMIC MODELLING OF EXPECTATIONS WITH PARTICULAR REFERENCE TO THE U.K. LABOUR MARKET

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ABSTRACT

The thesis deals with the analysis of dynamic econometric models which includes Time-Series methods, Conditional modelling and the General to Specific approach combined with Destructive testing. The modelling strategy used is dependent on the requirements of the modeller, whether he needs to Forecast, to derive policy or to produce results to support or deny a particular theory.

Expectations introduce dynamics into econometric specifications and rational or consistent expectations models in particular have number of representations, which depend on the form of the inter-temporal optimisation problem and the method of solving for the expectations. Here we use the Vector AutoRegressive(VAR) form to estimate predictions of variables which are exogenous, an Errors-in-Variables method to produce initial estimates of structural parameters and a recursive systems approach to estimate the backward-forward representation.

Vector autoregressive models of manufacturing wages, output prices, manufacturing inventory accumulation and vacancies are estimated using a general modelling strategy to derive predictions and one step ahead forecasts. These results are then fed into a structural model of output and employment which is estimated using a recursive estimation technique that solves out the endogenous expectations and then replaces the exogenous ones using the Wiener-Kolmogorov prediction formula.

Finally we discuss generalisations of the first order rational expectations model to produce first order euler conditions which bear a closer correspondence to estimated error correction models with which they are related. The inter-temporal optimisation problem is extended to deal with lags and leads on exogenous variables, non-separabilities and lags on adjustment costs. Local and Global Identification conditions are presented for all of the models in the study.

I dedicate this work to the memory of my Father and my Daughter.

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INTRODUCTION

In this study we are interested in applications of the methodology of rational expectations to the labour market. At the macro level people have looked either at models of prices, generally based on the Phillips curve or at quantities which are either factor demand equations or employment functions. In Phillips (1958) he describes wage changes as being dependent on excess demand in the labour market, the study is not like any of the time series models which followed, because it was estimated on four class intervals for data computed from the period 1861-1913 using a graphical non-linear least squares method. The averaging procedure removed cyclical fluctuations and the results where used by Phillips to determine demand pull inflation over three very different sub-periods of the data 1861-1957. Numerous wage models followed Phillips discovery, notably one by Sargan(1964) which has spawned much research, due to its attention to dynamics and simultaneity. Subsequently wage models of the U.K. have usually been variants of the two relationships cited above; for details of some of the early models see Henry, Sawyer and Smith(1976). The first application of such models to include rational expectations was given by Minford and Brech(1980) and has subsequently been updated in various versions of the Liverpool Macroeconomic model of the U.K. economy. Henry and Ormerod(1979) have also presented alternative rational expectations models based on the NIESR model and Wallis et al(1984) and (1985) present results of varying detail for the main U.K. macro models which all now involve some form of expectations.

In the late 70's Sargent(1978) and Kennan(1979) presented labour demand models that incorporated rational expectations. The models were derive by solving an intertemporal optimisation problem with costs of adjustment in which future values were replaced by rational expectations. Muellbauer and Winter(1980), Nickell(1984) and Muellbauer and Mendis(1983) have estimated similar employment relationships for the UK. The Muellbauer and Winter paper models Employment, Exports and Unemployment using a variant of the errors in variable technique in which the distributed lag on the endogenous expectations is shifted to the left hand side (lhs) of the equation. The employment equation is estimated directly using ordinary least squares(OLS) by imposing plausible coefficients on the past and future values of employment. Nickell(1984) uses the substitution method of Sargent which replaces the actual values of output in the employment equation by the relationship defining them and then the two equations are estimated simultaneously. It is usual to assume a single lag in the dependent variable, but Nickell includes two explaining the second lag using aggregation over two labour markets. Muellbauer and Mendis use two methods to estimate their model of employment: firstly they derive an auxiliary model to derive future output expectations at each period and substitute these values back into the employment equation and secondly they follow the Muellbauer and Winter method augmented by an adjustment for serial correlation which is supposed to take account of the moving average error associated wiyh replacement of the expectations in the euler equation by actual values. The National Institute of Economic and Social Research(NIESR) have spent a considerable amount of time estimating employment equations with rational expectations, they have used a variant of the substitution method in which they curtail the future expectations on the exogenous variables to four or five leads and then they replace them by consistent predictions. The NIESR modelling team and many of the other groups have always found it necessary to include two lags on the dependent variable. The most recent NIESR model uses the Kalman Filter to solve for the future values, while a stochastic trend proxies productivity, see Harvey, Henry, Peters and Wren-Lewis(1985). Engle and Watson(1985) and Burmeister et al(1982) and (1985) have also used the Kalman Filter to estimate simple rational expectations models, but in more general models the State vector can become prohibitively large. The methods described here extend the method described in Sargan(1982), the expectations are replaced by actual values and then an efficient recursive procedure is devised to estimate the model. The Sargan method is extended to take account of some asymmetries, further lags in the endogenous and exogenous variables and different period expectations.

In practical terms estimation of rational expectations is not trivial as it is difficult to know what should be used to replace the expectations and what form the model should or can take. In principle their are two methods that have a number of different variants; those are the substitution method due to Sargent(1978) and the errors in variables method of McCallum(1976) and Wickens(1982). The substitution method replaces the expectations by the Vector Moving Average(VMA), Vector AutoRegressive(VAR) or Vector AutoRegressive Moving Average(VARMA) process which are believed to generate them, this requires the definition of the process and then substitution back into the model. The resultant form is of a VARMA, VMA or VAR form when information is dated at t-1 which requires the rational expectations restrictions to be imposed on the parameters for the method to be efficient. The method suggested by Sargan (1982) is to substitute out the expectations assuming they have a VMA representation and then to estimate the resultant restricted quasi reduced form. The above method is a two stage maximum likelihood method, but there are other variants of these methods which include the instrumental variables techniques of Hayashi (1982) and Hanson and Sargent (1980), the Generalised Method of Moments method of Hanson and Sargent (1982) and the Jordan Canonical decomposition method due to Kollintzas (1985). Chow (1980) and (1983) present the solution to the model in state space form and Whitman (1982) uses the Fourier transform to derive the solution of the rational expectations model. Burmeister et al (1985), Engle and Watson (1985) and Fair and Taylor(1980) describe other techniques.

The errors in variable method does not solve the model, but it directly replaces the expectations in the model by their actual values which produces an error in variables. The resultant model is estimated consistently by replacing the future endogenous variables by fitted values. In Chapter 4 we discuss the relative merits of the two principle methods, but it is fair to say that the substitution method is to be preferred if the modeller truly believes in rational expectations. Sargan(1982) and Nickell(1985) explain the key limitations associated with the errors in variables approach, though Broze et al(1984) and Wickens(1986) improve the technique to account for some of the inefficiencies. The advantage of the errors in variables approach is a bi-product of its' inefficiency, because it does not impose a particular solution it might more naturally be used to detect irregularity. Sargan(1984) covers alternative solutions in a highly generalised framework and he suggests different estimation techniques which may be appropriate for estimating rational expectations models which exhibit both regular(unique saddle point or symmetric backward forward solutions) and irregular solutions(non-symmetric solutions). Irregularity can be detected by a Wald test when the errors-in-variables method is used and by a Lagrange Multiplier test when the regular solution is imposed. Preference for a method on estimation grounds will depend both on the tractability of the solution and the efficiency of the estimation technique while the power of the test will depend on the efficiency of the method and in finite samples on the small sample behaviour of the test statistic.

In Chapter one we will look at the theoretical considerations associated with macro models which include expectations. We will show that we are indebted to Keynes who developed the notion of expectations as the basis of a Macroeconomic theory in which uncertainty about the future was critical. Hicks contribution to the theories of general equilibrium, expectations and macroeconomics are discussed and related to the notion of adaptive expectations and non-market clearing. We then discuss rational expectations which have been used in the context of macro models to justify classical and monetarist perspectives. We show that the classical conclusions of many models depend on the structure of the model rather than the nature of the expectations. A macro model of output and employment is then developed which has Keynesian origins, but it assumes that expectations are rational or consistent. The question of the natural rate is discussed and it is shown that the model developed is a generalisation of the extreme Classical and Keynesian formulations.

Chapter two discusses the general problem of estimating dynamic econometric models; this will include time series models, error correction models and dynamic models based on economic theory. We will look at modelling methodology and the way in which econometric models should be set up and the criterion which it is reasonable for them to meet. The different models will be reviewed to see the extent to which they satisfy such conditions.

In Chapter three we look at different time series representation of rational expectations models and we relate them to different forms of the cointegration model The results in Granger and Engle(1987) are presented as they transform models with cointegration into error correction forms, the Yoo(1986) form is presented as it allows a non-stationary VMA to be transformed into the VAR from often used in estimating rational expectations models. We then look at an alternative factorisation which allows the cointegrating vector to be estimated directly and we relate this to the methods suggested in Breusch and Wickens(1988). Logarithmic models of output prices, wages, vacancies and inventory accumulation are estimated using the VAR approach and the best of these models are used to produce one step ahead forecast errors and future predictions. The predictions are then used in the following chapter to derive estimates of an output employment system.

Chapter four presents results for our output employment model and the derivation of the estimation method first mentioned in Sargan(1982). The method is associated with the solution of an intertemporal problem and the resulting euler conditions are found to be equivalent to the macro econometric model developed in chapter one. The first order case is given a number of different forms and these are related to cointegration and it can also be shown that it is still possible to derive a solution under cointegration. The results in the first section are used to specify an output employment model which can either be treated as a macro relationship that depends on future and past values of the endogenous variables as well as wages, output prices, vacancies and inventory accumulation or as a result of the control model covered in the first section. Estimates of the system are derived by Maximising the Likelihood using a Recursive Technique which considerably reduces the data set. A number of estimates are presented including initial estimates derived by the errors in variables approach and systems estimates with trend variables, innovations and with the exogenous variables replaced by predictions. The solved form of the model nests within it a number of specifications that include the restricted form of the rational expectations model, the partial adjustment model, the static model, a VAR(1) in the exogenous variables and a bivariate random walk. The rational expectations model can then be compared with these restricted models using a likelihood ratio test and that test can also be used to test the restrictions associated with rational expectations. If innovations do appear to be significant in their own right this can be due to quasi rational expectations or innovations appearing directly in either the objective function or the structural form. We can also determine whether the current information set, the lagged information set or a combination of the two is relevant. The models are also tested for higher order serial correlation and predictions, equilibrium values and roots are computed.

In section four we relate rational expectations, error correction and cointegration. The section explains the disparity between initial estimates based on the errors in variables method and the results derived using the substitution method and then deals with the problems in estimating models using the errors in variables approach. The disparity relates to the unit root mentioned in chapter three which depends on whether the variables in the system can be given an error correction, a differenced stationary or a cointegration representation. The solution to a rational expectations system is then related to the Granger representation of the cointegrated system and the advantages of the Sargan approach are explained when the exogenous variables are cointegrated. In the final section we discuss aggregation and derive a number of estimates in which an adjustment has been made for serial correlation. The model due to Sargent(1979) includes an adjustment for serial correlation which is either explained by aggregation or the inclusion of stochastic terms in the loss function (see Nickell(1985)).

The fifth chapter looks at the limitations of the first order rational expectations or first order costs of adjustment model and presents some generalisations of those formulations. The structure of the models seems highly restrictive as they only usually allow current exogenous variables to appear, but in this section we allow future and lagged values to enter into the model specification. The loss function can also be adjusted to allow further lags, so that costs are spread over more than one period, but this complicates matters considerably. We show for quite general symmetric models that recursive solutions can be extended and we reveal two extensions which are in keeping with the structure of the first order model. Finally we discuss Global and Local conditions for Identification in the case of a model with interaction costs, these results are then related to the first order model in chapter four, a first order model with lags and future values of the exogenous variables and for the simple extension of the general model that includes a lagged cost to disequilibrium. The final chapter presents some conclusions and discusses some of the questions raised in the main text.

CHAPTER 1

Macroeconomics and Expectations

In a world in which we do not have certainty or in which actions undertaken today impose a cost or restrict our ability to react in the future, current action will be dependent on our perceptions of the future. Expectations are important in economics when we experience change which is often imperceptible, that is, we are dealing with dynamic economies and the models and econometric practices associated with them. The interest in the role of expectations became stimulated by the situation which beset most countries during the 1930's, when economists began to discern the great disparity between the way in which economies were supposed to respond and the way in which they actually did; in particular the great dislocation of labour experienced during this period. The problem as perceived by Keynes(1936) was due to the lack of coordination in plans that occurs in economies in which people do not have perfect information about the future. Pessimistic future expectation aligned with less than infinite price adjustment in a money economy may prevent a return to full equilibrium of the system. The sticky nature of price adjustment and uncertainty about the future are inter-linked, slow price adjustment is due to uncertainty about the future and the future costs of mistaken actions.

The traditional representations of the Keynesian model elucidates the problems of price stickiness and the associated deflationary process implied by it, but is criticised from both Keynesian and Classical perspectives for its sparse treatment of expectations.

It was Hicks(1937) who first presented the ISLM model much used in standard texts, but his emphasis in developing such a structure related to his own work on General Equilibrium Theory as compared with Marshallian partial equilibrium analysis, so that he saw the role of expectations as secondary to the interrelatedness of markets. It is clear by the considerable room given to expectations in both the General Theory and Keynes (1937) article, that he considered them to be critical in the operation of a decentralised money economy in the aggregate. In fact Keynes fundamental criticism of "Mr Hicks" formulation was the omission of expectations (see Keynes(1974)). The Keynesian insight in relation to expectations is that monetary economies operate differently to barter economies under uncertainty and such uncertainty, aligned with costs in decision making in decentralised economies, negates and even subverts the natural adjustment to equilibrium due to the price mechanism. As Leijonhuvud(1968) explains, the price mechanism needs to provide more than one type of signal, which means that prices do not contain sufficient information to clear the system. Market prices are expected to clear individual markets and to determine the overall price level which then sets the correct level of activity for the economy as a whole. The co-ordination of markets by one set of statistics seems barely credible within the context of an atemporal economy with product uncertainty, but in the context of an economy over time in which there are other forms of uncertainty it seems preposterous. If the whole stream of future prices where known in advance, then continuous market clearing of the Debreu(1960) type would occur, but not in an economy driven by expectations which are never perfectly validated, coordinated or imposed.

1.1 Theories of Expectations:

The notion of expectations clearly exists in the works of Marshall, Wicksell and an appreciation of the problems of uncertainty is clearly understood by Walras, but no author prior to Keynes gave such a central role to such variables. In the General Theory Keynes defines two periods over which expectations are made: the short-run and the long-run. Short-run expectations are not seen as being critical in the context of Keynes short period model.

> "But it will often be safe to omit express reference to short term expectations, in view of the fact that in practice the process of revision ... is a gradual and continuous one carried on largely in the light of realised results"
>
> J.M.Keynes (1936), p50-51.

This explanation clearly bares a strong resemblance to the ideas embodied in both rational and adaptive expectations which has led Begg(1982) to suggest that the short-run expectational theory embodied in Keynes(1936) is the rational one, and although this is possible the importance of such a suggestion is questionable; we will discuss this idea again in the next section. Linked to the distinction between long and short-run expectations there are at least three notions of expectation: exogenous, pessimistic and inelastic. Long-run expectations are seen as being fixed or exogenous for the period of analysis, so that shifts in such expectations are quite often the cause of destabalisation and the reason why the economy is not able to shift out of a depression. Hence, such expectations are usually related to interest rates especially the long-rate and described as pessimistic. It is such expectations which cause investment to be less than what it should be and it is that short fall which does not permit the economy to return to full employment. Inelastic expectations usually relate to prices which are not supposed to react one to one with actual values. The liquidity preference schedule is supposed to epitomise the idea of inelastic expectations and in the view of Hines(1971) all market price responses should be characterised in this way. Keynes clearly sees that there is a problem in an uncertain world in determining a set of equilibria over time and uses expectations with period analysis to determine such dynamic equilibria in a static framework.

To reiterate, short-run expectations are determinate to the point of being ignored. They are crystalised by past decisions and because of that they influence the future directly via the past. Long-run expectations are taken as given, they cannot be easily derived and in essence they encapsulate Knights principle of exogenous uncertainty(1921). Such values are inherently subjective and equally likely to be based on opinion as to rational decision making. The stock market being the market in which long assets are held involves such decisions which leads Keynes to equate the process of long expectations formation with a Beauty contest in which the decision is to select not the most beautiful, but the one which the judges believe the public would select as the most beautiful. We can glean from this the idea that in certain markets expectations are disparate, due to the nature of the market and the nature of the information upon which they are based. Long-term expectations are then 'Animal Spirits' which are more likely to be the result of tastes or gut feelings, than rationality.

Following Keynes development of Macroeconomics and his emphasis on the distinguishing role of expectations, Hicks(1939) elaborated and explained the economy in terms of his extension of the General Equilibrium system devised by Walras. In "Value and Capital" Hicks explains the workings of a dynamic economy using the concepts of equilibrium over time and temporary equilibrium. The issue of uncertainty is side stepped by the assumption of "Definite Expectations" which implies that individuals hold single valued expectations which they act on as if they were certain. This yields an elegant model of the economic behaviour of an economy through time which moves from period to period by the artifice of temporary equilibrium. At each period planned quantities are dependent upon past expectations and current values:

> " It will be past expectations right or wrong which mainly govern current output; the actual current price has a relatively small influence ", Hicks(1939), p.117.

The caveat in relations to expectations is supported by the Marshallian assumption that price movements are small and hence, income effects are also small, otherwise the expectational hypothesis would not lead to equilibrium over time.

Expectations had became embedded in the literature even though they were not initially formalised either theoretically or in an empirically usable way. In macroeconomics the concept of adaptive expectations was introduced by Cagan(1956) to explain hyper inflations, this was closely followed by Nerlove's(1958) paper which includes such behaviour in Hog Cycle models. The Adaptive expectations method is easily applied as it is purely dependent on a distributed lag of actual past values of the expectational variable:

$$(1.1) \quad p^e_t = \delta p^e_{t-1} + (1-\delta)p_t$$

$$(1-\delta L)p^e_t = (1-\delta)p_t \qquad \text{(where L is the lag operator)}$$

$$p^e_t = (1-\delta L)^{-1}(1-\delta)p_t$$

The idea of adaptive expectations is inherent in Hicks view, in the sense that they are point expectations, based on observations of the past which are presumed to be known with certainty. Though interesting in the sense that they yield tractable models the approach has its limitations. Adaptive methods may prove useful as rules of thumb for unsophisticated agents when the world or the markets they operate in are not subject to severe changes, but they imply a backward looking, limited information strategy. Such expectations would not be suggested by enlightened econometric practice or by the existence of such phenomena as on-line databases, insider trading, market research companies and other information retrieval and protection practices. Due to its ease of application and the non-existence of operational alternatives adaptive expectations was the principal method of modelling expectations during the 60's and early 70's. Expectations were used in theory to discredit the simple Phillips curve analysis of inflation and to give support to monetarist theories of policy ineffectiveness. Friedman(1968) and Phelps(1968) use expectations to show how the effects of active demand policies will be limited if they are perceived as being inflationary. Further work by Laidler and Parkin and others of the Manchester School introduce adaptive expectations into Phillips style wage equation models, though their methods were later criticised by Godfrey(1974) and Wallis(1971) (also see Desai(1976) and (1984)).

The notion of rational expectations is due to Muth(1961). In the (1961) paper Muth deals with the problem of Farmers operating in a market in which the dynamic adjustment path is described by a Hog-cycle model. Muth shows that agents who know the structure of the model can take advantage of such information to derive optimal predictions and then use the predictions to move directly to the new equilibrium. The solution based on rational expectations will both be different from and superior to the usual solution in which agents simply react to market conditions. It took some time for the new concept to gain acceptance, partly because Muths' article was an obscure application of a powerfull general principal set in a microeconomic framework and in practice because it set the almost intractible conceptual problem of how to compute the unobserved expectational variables. Further more, Wallis(1980) has shown that the advantages of rational as compared with adaptive expectations are not clearly brought out by the example chosen by Muth, because the rational expectations solution to the Hog-Cycle model can be made to look very similar to an adaptive model.

Lucas(1972) and Sargent and Wallace(1973) used rational expectations to produce macroeconomic supply relationships which when introduced into standard macro models where to produce strong classical results. The Classical supply hypothesis transfers Wicksells notion of a natural price to a quantity such as output, while rational expectations implies that variations from the natural rate are only due to misperceptions in price expectations.

$$(1.2) \quad y^s_t - \bar{y}_t = \alpha(p_t - p^e_t) + \epsilon_t$$

where y^s is aggregate supply, $\bar{y}$ the natural rate and p^e the expected price level and ϵ_t a white noise innovation.

The concept of the natural rate follows from Friedman's inversion of Wicksells notion of a natural price to a natural quantity. It is the structure of the Supply relationship and not the Rational Expectations Hypothesis which produces the strong classical results. The Strong Rational Expectations Hypothesis(SREH) is highly restrictive, as it implies that all agents are prior to the same information which they use in the same way. Agents are not expected to be equally efficient in information retrieval, but such distributional effects are meant to cancel out either across agents or across time. Deterministic elements will be learnt by individual agents or arbitraged away by markets. The SREH assumes that there is a true model which agents use to plan efficiently and New Classical Economists add to that assumption the idea that the true model satisfies the Classical assumptions in the short-run. Agents using false models are presumed to consistently make mistakes and so discover that they are using the wrong model and on the basis of that observation they are supposed to learn the true model. The implication of such a strong expectational hypothesis embedded within a classical model with strong informational assumptions is that the economy operates in a classical manner over time. We can see this by imposing the following equilibrium price relationship:

$$(1.3) \quad p_t = \mu p^*_t + (1 - \mu)z_t$$

where $z_t = x_t - \bar{y}_t - \epsilon_t$ and x is income, then if we assume that

$x_t = \psi x_{t-1} + e_t$ and $\bar{y}_t = a_o + a_1 t$ we can show that deviations from the natural rate are purely random.

$$(1.4) \quad p_t - p_t^e = (1 - \mu)(x_t - \psi x_{t-1}) - (1 - \mu)\epsilon_t$$
$$= (1 - \mu)(e_t - \epsilon_t)$$

Equation(1.3) and (1.4) in combination imply that disequilibria are due to defficiencies in information collection and mistakes. An economy based on such a model may never attain the optimal path, but it will achieve the best attainable alternative path in the best of possible worlds. In the context of the Lucas supply equation, rational expectations implies that the error terms associated with (1.4) are non-deterministic which means that there is no direct role for counter-cyclical government policies. The result above does not depend on the rational expectations assumption, but on the structure of the Lucas supply hypothesis. The limitations of the model are due firstly to the assumption that the natural rate is an attractor from which supply cannot escape and secondly to disequilibria only being dependent on deviations of prices from their expectations. If either of the above assumptions break down then so does the ineffectiveness result. A similar point is made in Begg(1982) and (1982a)

In practice, it is likely that (1.2) would not be well specified which suggests that (1.2) may be the long-run equilibrium to which an econometric model adjusts in the short-run and that means that a far richer explanation of the data is possible. If the more complex dynamic were correct, then the Classical model would only hold in the long-run, so that government policy could be effective in the short-run. The effectiveness of government intervention would then depend on the period of adjustment.

It seems better then to deal with a more general structure which do not impose market clearing or the strong rational expectations hypothesis. One is not necessarily rejecting rational expectations, though one might want to modify the assumption. Nerlove (1972) talks of quasi rational expectations in which the restrictions are not imposed or one can talk of consistent expectations in models which use forecasts to replace actual values. The limitations of rational expectations are discussed in Buiter(1980) and Begg(1982) provides a general discussion of the informational difficulties associated with rationality.

Recently a lot of interest has been generated in the way that expectations are formed, the role of differential information and the use of different expectations. Blume and Easley(1982) show that in a learning environment, with imperfect information agents may only by chance come across the right model. Hence, information problems will differentiate agents and limit convergence to rational expectations. Townsend in the paper entitled " Forecasting the Forecasts of others " (1983) discusses such problems in similar terms to Keynes in his Beauty contest example. Implicit in such discussions is the notion that other agents expectations are then crucial in determining the rational expectations equilibrium. At a simpler level it is possible to proxy learning behaviour by recursive modelling methods, an example of this is presented in Pesaran and Pesaran(1987). It is also possible to include different or alternative types of expectations in such models. Pesaran(1987) looks at many of these issues, especially the informational problems associated with rational expectations and the role of survey data in validating the expectational hypothesis. Expectations are clearly important, but how one models them is the problem. Rational Expectations when the informatioal assumptions are not taken too seriously provides a methodology which it is possible to implement and the methods can be broadened out to deal with some of the problems mentioned above. This does not answer the question of the validity of the "as if" assumption which may not always be good enough. In general, the results of rational expectations models depend on the assumptions about information, costs of adjustment, the distinction between expectations expected and acted upon and the computational capabilities of agents; these all relate to the type of model you embed your expectations in. The solutions of linear rational expectations models is dealt with clearly in Blanchard and Kahn(1980), Begg(1982) and Pesaran(1987) and we cover some of the solutions in chapters 4 and 5.

1.2 The nature of expectations in Keynesian models

Before discussing the type of model within which expectations are placed I would like to discuss the use of rational expectations in the context of a Keynesian model. Begg (1982) and (1982a) shows that rational expectations can be embedded in a Keynesian model and he suggests that Keynes only makes sense in the context of rational expectations. It is clear that Keynes discussion of short-term expectations fits nicely into such a framework, because they can be viewed as modellable point expectations which are subject to constant revision. Rational expectations can be seen as one of a number of rules of thumb used by agents for short-term decision making, but they only explain long-term expectation if Keynes mixed up expectation revision with expectation formation(see Begg (1982)). Begg also states that long-term expectations may be exogenous, a view which seems far more consistent with the tenet of Keynes(1937) and which is supported by Lawson(1981) and Ozga(1965).

Beggs' reason for introducing rational expectations into Keynes(1936) is to makes his consumption model more consistent with the permanent income or life cycle models(see Precious(1987) for similar synthesis of Investment models). If consumption is based on current income with long-term expectations given, then the model can then be related to any type of long-term decision making behaviour. So if we let rational expectations of income determine current consumption then:

$$c_t = a + b\,E(x_t | \Omega_t)$$

(1.5) $$c_t = a + bx_t$$

where a depends on long-term expectations.

Keynes short-run consumption model (1.5) seems to be far closer to the income constrained approach of Clower(1967), than the permanent income hypothesis (PIH) of Friedman(1957). Although, the rational expectations model and the absolute income model are observationally equivalent, there is evidence to show that the consumption function that Keynes had in mind was different from either of the above formulations. Firstly short-term expectations are conditional on long-term expectations ($x(r^e)$), so that:

$$E(x_t | \Omega_t, x(r^e)) = x_t.$$

where $x(r^e)$ may depend on subjective factors or gut feelings and such expectations are not likely to determine a whole stream of future values.

Secondly short-term expectations are different from long-term expectations which depend mainly on long-assets values[1]. Thirdly permanent income proxies wealth which is presumed to have a direct effect on consumption while in the General Theory agents do not spend their wealth directly. In Keynes(1936) wealth enters the consumption function purely through its' effect on income and interest rates. Income depends on the return on wealth and windfall capital gains and interest rates are a proxy for the rate of time preference and they indirectly affect consumption through income. In the short-term theoretical model such factors were either collapsed into the constant term or considered as non-deterministic(see chapter eight and nine of Keynes(1936)). Windfall capital gains are viewed as being irregular and 'changes in expectations of the relation between the present and future level of income' can be seen as innovations. The short period theoretical model has no need of such variables, but many of them can be related to innovations or shocks which could certainly be used to enhance an econometric model. In the longer term demographic factors and tastes enter the model which means that the contradiction found by Kuznets between long-time series

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[1] Keynes borrowed Marshalls period analysis, so that the short-term is a period in which capital and the expectations about long-period assets are fixed. Current capital employed is based on the previous vintages of long-term expectations and current capital expenditure is based on current long-term expectations.

results and cross section results can be explained by shifts in the intercept term. The Keynesian model can then be viewed in terms of the relative income hypothesis(RIH) of Dusenberry(1949). Hines(1980) has favoured the dynamic approach associated with the RIH on sociological and economic grounds and it is one of the few models that is consistent with the general aggregation results of Hildenbrand(1983).

It is often argued that Keynes expectations are adaptive, but although that could be a rule of thumb for short-term expectations, Lawson(1981) shows that the notion does not work in the context of long-term expectations. In fact the forward looking nature of Keynes perspective would suggest that the idea of rational expectations is more appropriate, than adaptive expectations as a short-term rule of thumb. Beggs view that long-term expectations are rational makes sense in terms of jump behaviour or if Keynes had mixed up expectations with innovations. The Turnpike Theorem has fed quite naturally into rational expectations theory, as jumps can occur in markets where some prices can be seen as moving quickly relative to other prices and quantities. Such behaviour is often associated with overshooting and markets in which price movements are not always smooth and as Keynes states:

> " and it is of the nature of long-term expectations that they cannot be checked at short intervals. ..., they are liable to sudden revision. Thus the factor of current long-term expectations cannot even approximately be eliminated or replaced by realised results" J.M.Keynes (1936), p51.

Long-term expectations do look similar to jump variables, but purely because jumps are effectively exogenous in comparison to the usual solution paths of rational expectations models. Pesaran(1987) dismisses Begg as simply assuming that short-term expectations are rational. Finally, it seems likely that at the very least long-term expectations in the General Theory are not single valued or modellable in some deterministic manner which puts considerable doubt on the view that Keynes long-term expectations theory was rational.

Ozga (1965) has a clear explanation of Keynes ideas which he places in the context of Hicks and Shackles theory as well as the subjective nature of expectations. Hicks-Lange expectations are perceived as being sure thing equivalents to which agents give a particular response, so that there is an elasticity of expectation. Expectations are then single valued functions of the stream of future prices, so that:

$$p_t^e = p^e(\{E(p_{t+s}|\Omega_t)\}_{s=1}^{\infty})$$

This is why Hicks assumes that a price rise today induces an equivalent price rise across all future periods so that when the response is elastic future prices change to a greater degree. The higher than expected future prices cause consumption or production to be brought forward and so induces a destabilising response of current prices to market conditions. Prices rising today cause current demand to rise, because an elastic expectational response causes future prices to rise to a greater degree. Hence, inelastic expectations reinforce the usual stable response to excess demands and elastic expectations are neutral. The discussion of stability is circular, because the response to the expectation is predicated by the state of nature.

> "To be able to say that expectations are inelastic we would need to be able to reduce them to their sure-prospect equivalents ; and to reduce them to this form we would have to know whether businessmen behave so as to render the system stable. Even therefore, if we could discover what prospects arise in what circumstances, we would not be able to reduce these prospects to a sure-prospect form if we did not already know the conclusion." S.Ozga (1965), p151.

Keynes cuts through this problem via the short-term long-term distinction which implies that sure thing equivalents are used as if they were the multi-prospect outcomes. Hence, short-term and long-term expectations follow a rule of thumb which accepts the notion that actual values encapsulate the expectations. The distinction between the long and the short-term is used to separate production decisions from investment decisions, as there is little uncertainty in the first instance and an awful lot in the second. If the convention breaks down, then the Keynesian model gives a reason and a policy response, but no treatment of the way in which either short-term or long-term expectations are determined. So that expectations are exogenous in the long-run and equivalenced to actual values in the short-run.

Expectations are not simply single valued, they depend on the present, future and past. In Part this relates to the irreversibility of past decisions which then determines future decision making, so that such factors are encapsulated in long-assets. Rational expectations provide a single valued measure of expectations that can be used in the short-term and as such it approximates some of the ideas in the General Theory and provides a reasonable rule of thumb. Long-term expectations involve the subjective element so that they are much more dependent on fad and fashion, but rational expectations may provide a reasonable proxy for considerable periods of time. As true short-term expectations are determined by long-term values, rational expectations may diverge considerably from them when long-term conditions change. Modelling expectations using consistent methods allows us to include different period expectations which may either be important in their own right or as a sign of the existence of agents with differential information or different subjective viewpoints.

1.3 A Keynesian model of Output and Employment with expectations and Inventories.

Let us use the causal structure of output determination associated with Keynes General Theory, so that output demand determines the level of output in the short-run and then the level of output will determine the level of employment.

$$o^d \rightarrow o^s \rightarrow l^d$$

The joint determination of these variables seems obvious at the macro level, due to the circular flow of income and the quantity adjustment process associated with the multiplier(see Leijonhufud(1968) and Hines(1971)). In fact a one dimensional representation of the Keynesian cross diagram can be formulated in output employment space(see Portes and Muellbauer(1978)). We will assume that all variables are in logarithms and that a simplified model of output and employment can be specified in the following way:

$$x = x(l, w) \qquad \text{(income)}$$

$$q^d = q^d(x) \qquad \text{(output demand)}$$

$$(1.6)\quad o^d = q^d + \Delta i^d \qquad \text{(market demand)}$$

$$o^s - o^d = i_1 \qquad \text{(goods market flow condition)}$$

$$(1.7)\quad o^s = q^s + i \qquad \text{(market supply)}$$

$$q^s = q^s(q^d) \qquad \text{(output supply)}$$

$$l^d = l^d(q^s) \qquad \text{(employment demand)}$$

$$l = l^d - v \qquad \text{(employment equation)}$$

Initially we will assume that prices and wages are determined outside the model, that does not necessarily mean that they are strictly exogenous, but that relationships can be found in which they do not directly depend on output and employment; that is much easier to do when there are inventories in the model. The assumption of exogeneity does not mean that prices are fixed, though it does suggest that the real wage is determined by the level of demand when vacancies and inventories do not exist. In Chapter 20 and 21 of the General Theory, the assumption that wages and prices are fixed is clearly discarded and in Chapter 2 it is suggested that price and wage flexibility may be counter productive. The questions posed relate to the extent to which bargaining is for real rather than money wages, acceptability of real wage changes and the ability of labour to determine their real wage and so equate it with the marginal disutility of labour.

" Since there is imperfect mobility of labour, and wages do not tend to exact equality of net advantage in different occupations any individual or group of individuals who consent a reduction . . . will suffer a relative reduction in real wages which is justification enough for them to resist it " Keynes (1936), p14.

Hahn(1982) explains this problem in terms of an externality associated with peer group pressure in the labour market which implies that wages enter the utility function and which manifests itself in terms of the benefit to be gained from high relative wages and the approbation associated with stepping out of line. Begg(1982) attributes the inability of labour to set a real wage to contractual obligations which then binds part of the labour force into real wages which are too high. The group whose wages are flexible then find that it is sub-optimal for them to shift their wage to the value which will clear the market as a whole. The incentive set by price is not sufficient to induce one group to maximise the net benefit of the other by determining the global market clearing wage.

The two theories presented above still suggest that wage inflexibility is at the root of the problem, but we would prefer to suggest that it is due to the relative speeds of adjustment of prices and quantities under uncertainty. Infinite price adjustment is necessary for trading at false prices to be limited or quantity adjustment needs to be limited during the period within which prices are changing. Infinite price adjustment is limited to a number of asset and exchange rate markets, and only in auctions are quantities limited to adjust after price or allow recontracting to occur. If trading occurs at false prices and recontract is not possible, then we have the possibility of disequilibrium trades. Prices are allowed to change, but usually at a slower rate than quantities. Disequilibrium trading has an income effect which reinforces the initial contraction(see Leijonhufvud(1969)), price movements which may counteract such effects are either too slow or dominated by the quantity changes. Initial quantity movements usually relate to national income changes which are likely to be large relative to price movements in individual markets. In that event it is not surprising that quantity effects dominate price movements, especially as agent effects change market prices rather than the general price level. The situation is compounded by the types of problem mentioned by Hahn(1982) and Begg(1982), and the fact that prices and wages move in line.

If market prices and wages are flexible, then a theoretically neutral assumption would imply that they move in step. Let there be i = 1, . . . I industries and weights k_i which sum to one, then:

$$p_i = k_i w_i \qquad \text{and} \qquad p = 1/I \sum_{i=1}^{I} p_i = \sum_{i=1}^{I} k_i w_i$$

$$p = \sum_{i=1}^{I} k_i w + \sum_{i=1}^{I} k_i (w_i - w)$$

If $\sum_{i=1}^{I} k_i = 1$ and $\sum_{i=1}^{I} k_i (w_i - w) = O(1/I)$ where I is large, then $p \approx w$.

Aggregate wages and prices are simply appropriately weighted sums or integrals of individual market prices and wages, so when individual prices move then so do their aggregates. Individual price movements will adjust to clear micro markets, but they do not then alter the aggregate level of activity in a deterministic manner. Hence, activity in micro markets depends firstly on the overall level of activity and secondly on wages and prices in those markets. Excess demand functions are then dependent on the overall level of activity and individual market prices, given prices in other markets:

(1.8) $\xi_i^g = \xi_i^g(p_i | w_i^e, x)$ (excess demand equation for goods)

(1.9) $\xi_i^l = \xi_i^l(w_i | p_i^e, q)$ (excess demand equation for labour)

We are using the Keynesian convention that contracts are set in money terms, but such values are then conditional on price expectations in the case of (1.9) and wage expectations for (1.8). In addition (1.9) may depend on the user cost of capital, though that is also likely to depend on the level of activity.

Let us assume that it is possible to derive well ordered aggregate excess demand functions by summing (1.8) and (1.9), so that:

$$\xi^g = \sum_{i=1}^{I} \phi_i \xi_i^g(p_i | w_i^e, x) = q^d - q^s$$

$$= \xi^g(\underset{\sim}{p} | \underset{\sim}{w}^e, x)$$

$$= \xi^g(\underset{\sim}{p}, \underset{\sim}{w}^e, x(1, \underset{\sim}{w}))$$

(1.10) $\xi^g = \xi^*(\underset{\sim}{p}, \underset{\sim}{w}^e, 1(q^s))$

and

$$\xi^l = \sum_{i=1}^{I} \theta_i \xi_i^l(w_i | p_i^e, q)$$

$$= \xi^l(\underset{\sim}{w} | \underset{\sim}{p}^e, q^s)$$

$$= \xi^l(\underset{\sim}{w} | \underset{\sim}{p}^e, q^s(1))$$

$$= \xi^l(\underset{\sim}{w} | \underset{\sim}{p}^e, q^s(1^d - v))$$

(1.11) $\xi^{l} = \xi^{*}(\underset{\sim}{w},\underset{\sim}{p}^{e},l^{d},v)$

where $\underset{\sim}{w}' = (w_1,w_2, \ldots,w_I)$ and $\underset{\sim}{p}' = (p_1,p_2, \ldots,p_I)$

We now have a system in which excess demands are not determinate, as the level of goods supply also appears on the right hand side (1.10) and the level of employment also appears on the right hand side of (1.11). If we compare this with the classical case we find that:

(1.12) $\xi^{g} = \xi^{g}(\underset{\sim}{p},\underset{\sim}{w}^{e})$

(1.13) $\xi^{l} = \xi^{l}(\underset{\sim}{w},\underset{\sim}{p}^{e})$

Equations (1.12) and (1.13) are only equivalent to (1.10) and (1.11) when quantities do not change, which either means that quantities are fixed or that price adjustment is infinite relative to quantity adjustment. Hicks and Marshall used a combination of infinite price adjustment and quantity changes being small to take care of false trading. Once no false trading is accepted it is easy to presume that the set of prices which determine equilibrium in micro markets are sufficient to set the level of over all activity. If we can derive an appropriate price and wage index, we have the more usual macroeconomic definition of excess demand which only depends on the real wage or prices in wage units:

(1.14) $\xi^{g} = \xi^{g}(p,w^{e})$

(1.15) $\xi^{l} = \xi^{l}(w,p^{e})$

Excess demands now depend on the level of real wages, so that prices drive the system rather than quantities. Unemployment is due to an excess supply of labour which is due to real wages being too high. As (1.14) and (1.15) are derived from micro excess demand functions we find that price flexibility in individual markets is sufficient to clear all markets. Such a micro foundation allows the wage level and price level to be derived from individual price and wage series and these aggregate series, then determine the over all level of activity. The system works from the bottom upwards, rather than the top down which seems to contradict recent developments in aggregation theory and our observation of reality.

Hildenbrand(1983) shows that very strict conditions on the distribution of income needs to be satisfied for an aggregate demand relationship to mirror the micro relationships. In fact we require the shape of the distribution to remain unchanged over time which suggests that new cohorts of consumers simply replace the previous cohorts in a way which is reminiscent of the relative income hypothesis of Duesenberrys' (1949). Kirman (1989) rejects the notion of micro foundations, as he does not believe that sensible aggregation conditions exist that produce unique excess demand functions.

> "Thus demand and expenditure functions that are to be set against reality must be defined at some reasonably high level of aggregation. The idea that we should start at the level of the isolated individual is one that we may well have to abandon" A.P. Kirman, The Economic Journal p138 (1989).

He seems to be suggesting that market wide and economy wide theories should be constructed at the aggregate level. The aggregation results in themselves do not deny that micro variables are important, but they do imply that the structure of the macro model cannot be directly discerned from micro phenomena. Hence, we may still have macro excess demand functions of the form of equations (1.12) and (1.13), but they are not based on micro foundations and as such they do not automatically satisfy micro principles. Hildenbrand(1983) finds that consistent aggregation may lead to macro demand relationships which are upward sloping in price quantity space. So that even if the excess demand relationships associated with the classical framework are correct, there is no guarantee that they satisfy classical assumptions in the aggregate and then the models no longer have the support of consistent aggregation from micro principles as a justification.

The more general excess demand relationships (1.10) and (1.11), give a role both to prices and the level of demand and at the aggregate level they confirm the econometricians suspicion that aggregate phenomena are equally likely to depend on individual prices as they are on aggregate variables (these issues will be covered in more detail at the end of chapter 4). The Nickell and Layard(1985) model supports the notion that both prices and quantities are important in determining macro variables, and the point seems to be supported in practice by the observation of price insensitivity and contractual arrangements in many markets. Leijonhuvud (1968) seems to suggest that the process by which dynamic adjustment occurs is highly complex and that means that a broader information set than wages and prices is required for the determination of a set of excess demand functions. Prices do not provide sufficient information to clear markets.

"At one extreme of a spectrum of possibilities are traditional full employment models where the whole brunt of adjustment is borne by prices; at the other extreme are the "pure Keynesian" models where prices are essentially given and income moves. In between lies the complications of the real world ...."

A.Leijonhuvud (1968), p58-59

In the light of the discussion above we see that the Lucas supply hypothesis fails, when output supply is not tied to the natural rate and excess demands depend on more than the difference between actual and expected prices. Equations (1.10) and (1.11) imply, that except under special conditions, one set of price information can only determine demand and supply levels in individual markets conditional on the overall level of activity in the economy as a whole. The level of demand as was stated above determines output, demand then determines the price level and the real wage is then the ratio of the two. Once activity levels are determined, the activity levels in different markets are due to individual market prices. The price mechanism does what it is good at which is efficiently allocating a given level of resource by setting prices to clear individual markets, given the level of activity.

Clearly our aggregate demand function (1.10) and (1.11) show that individual prices do have some role to play in determining excess demands and so the overall level of demand, but we have suggested that individual price effects will usually be of second order importance when compared with the income reductions associated with the multiplier. Relative price shifts are likely to be associated with redistributions which then change the production frontier and the structure of aggregate demand. Hence, we will be facing a new full employment equilibrium and a new full employment real wage whenever relative prices change. The process will be further complicated by resistance to relative wage changes that are likely to be as seen as unfair, because they change the prestige and esteem of workers in a way that has more to do with the vagaries of chance, than market efficiency. In the extreme such changes could lead to bankruptcy and the disappearance of certain types of products, as depression and the associated disequilibrium adjustment causes the risk of failure to increase substantially and as Dixit(1977) shows in imperfectly competitive markets with entry barriers market failure is stacked against goods with price inelastic demand curves.

If we can uniquely define aggregate price and wage levels and (1.10) and (1.11) are not significantly affected by relative price movements, then we can derive a macro analogue of these equations to compare with (1.14) and (1.15):

(1.16) $\xi^g = \xi^+(p, w^e, q^s)$

(1.17) $\xi^l = \xi^+(w, p^e, l^d, v)$

The analysis above excludes the possibility of inventories and it suggests that the influence of aggregate price effects, will depend on the level of demand. The real balance, Pigou effect and Keynes effects are seen as alternative adjustment processes through which an equilibrium may be attained, but these Keynesian procedures are not likely to work when unemployment has a classical cause. Real balance effects influence the level of demand rather than supply conditions, so that they are a back door means to alleviate demand deficient unemployment. When unemployment is caused by wages being too high, the real balance effect will only raise demand without reducing real wages which means that unemployment will not fall.

The real balance effect works through the existence of outside money whose value increases with deflation, but such an effect can be attacked from both a Keynesian and New Classical perspective. McCallum(1982) puts forward a Ricardian critique of such effects, as hyper rational agents will balance such increases against the governments increased borrowing cost. Agents discount such increases by the likelihood of higher future tax levels, so that increases in money income do not induce higher levels of expenditure. A more serious criticism relates to the nature of the adjustment process that is being observed. Even in the context of disequilibrium models both prices and quantities change as the multiplier works to attain a new equilibrium. Changes in real balances are then associated with disequilibrium price movements in markets in which demand is collapsing and so by implication the real balance effect is either swamped by such quantity changes or the price changes are endogenised so that a new lower level activity is found after both price and quantity adjustment. As such movements occur and prices fall expenditures may be cut, as agents perceive the benefits of putting off spending decision. So that elastic price expectations(see section two) support Neary and Stiglitz(1981) view that future quantity expectations bring forward the deflationary process. We are observing the effect of the speculative motive on the demand for goods which reflects the view of Hines(1971) that the liquidity preference schedule should be mirrored in all goods. Hence, people become increasingly keen to hold money rather than to spend it which means that some investment and durable consumption decisions are curtailed. It is in these circumstances that we observe the liquidity trap which directly counteracts the effect of real balances and is associated with a willingness to hold money when lower prices are expected. When prices stop falling demand does not bounce back, because long-term expectations have changed, labour and producer incomes have fallen, quantity expectations are pessimistic and bankruptcies have destroyed both physical and financial wealth. In a severe recession the production possibility curve may even shift inwards. As Keynes puts it real balance effects are a slender reed on which to base a recovery, it is equivalent to encouraging the unions to determine monetary policy through moderation wage claims. Finally, price reductions will lead to a higher debt burden for companies which is likely to counteract the short term benefits of lower capital prices, because debt is denoted in nominal terms and interest rates are bounded at zero while price changes are not. When prices fall the real cost of borrowing can remain high, as the cost of servicing a nominal debt rises relative to income.

The liquidity trap and the type of depression dynamics associated with it have not been experienced in recent years as prices have generally risen. Hence, it seems likely that output and employment will be responsive to price, but the order of magnitude of such response is likely to be small. Real balance and distributional effects may be important when adjustments are small or when the level of demand is high which is why we follow Nickell and Layard(1985) in giving a role to quantity as well as price.

$$q^d = q^d(\underset{>0}{x}, \underset{<0}{p})$$

$$l^d = l^d(\underset{>0}{q}, \underset{<0}{w})$$

It is also likely that there may be spill over affects from different markets, although there is a problem with dealing with that directly at the aggregate level. The only thing to do here is to include excess demand variables directly in the models, so that:

(1.17) $q^d = q^d(\underset{>0}{x}, \underset{<0}{p}, \underset{>0}{v}, \underset{<0}{\Delta I})$

(1.18) $l^d = l^d(\underset{>0}{q}, \underset{<0}{w}, \underset{>0}{v}, \underset{?}{\Delta I})$

When there is a stable u-v, then

$u = f(\underset{<0}{v}, \underset{>0}{\Delta I}, \underset{<0}{p}, \underset{>0}{w})$.

If we solve out the above system we can derive temporary equilibrium relationships which take account of vacancies and inventory holding. Especially when we assume that the principle role of stocks is as a buffer and that there is a rationing regime under which inventory demand is satisfied last(the customer always comes first).

$$o^s - o^d = i_1$$

substituting for o^s and o^d using (1.6) and (1.7) above gives:

$$q^d + i_o - q^s - \Delta i^d = i_1$$

$$q^s = q^d + \Delta i^d + \Delta i$$

then substituting out for q^d using (1.17) above implies that:

$$(1.19)\quad q^s = q^d(x\ ,\ p\ ,\ \Delta I\ ,\ v) + \Delta i^d + \Delta i$$

Now inventories are in logarithms and we have made an additional assumption that when demand is high inventory investment takes a second place which means that demand for inventories should reflect excess demand. We will assume that $\Delta i^d + \Delta i = f(\ \Delta I\ ,\ v)$, so that (1.19) becomes:

$$q^s = q^d(x\ ,\ p\ ,\ \Delta I\ ,\ v) - f(\ \Delta I\ ,\ v\ )$$

Substituting out for income using labour income implies that:

$$(1.20)\quad q^s = q^*(\underset{>0}{l}\ ,\ \underset{>0}{w}\ ,\ \underset{<0}{p}\ ,\ \underset{?}{\Delta I}\ ,\ \underset{>0}{v})$$

So that we have a flow relationship in which equilibrium supply is demand determined when we take account of inventories. We now have a relationship in which output depends on employment, prices in wage units, inventory accumulation and vacancies. The same can be done for employment as:

$$l^d = l - v$$

If we use (1.18) to replace l^d in the relationship above, that gives the following equilibrium relationship for employment:

$$l = l^d(q^s\ ,\ w\ ,\ \Delta I,\ v) - v$$

$$(1.21)\quad l = l^d(\underset{>0}{q^s}\ ,\ \underset{<0}{w}\ ,\ \underset{?}{\Delta I},\ \underset{<0}{v})$$

Equations (1.20) and (1.21) represent the temporary equilibrium relationships for output and employment. So far our models do not include expectations and they assume perfect adjustment which means that we need to embed them within a dynamic model. Production goods are clearly storable and they are likely to be durable as well which suggests that the demand model should take account of that. We have a degree of inertia associated with costs of adjustment and partial adjustment which suggests that output will depend on past values of output and employment. also Future quantity and price expectations are also likely to be relevant in determining output and employment. We can place (1.30) and (1.31) into the following structural form which can also be derived from agents minimising or maximising an appropriate objective function:

$$q_t = Q^o_{12} l_t + \beta Q^1_{11} q^e_{t+1} + \beta Q^1_{12} l^e_{t+1} + Q^1_{11} q_{t-1} + Q^1_{12} l_{t-1} +$$
$$h_{11} q^*_t + h_{12} l^*_t$$

where q^*_t is defined by (1.20) and l^*_t by (1.21).

$$l_t = Q^o_{21} q_t + \beta Q^1_{21} q^e_{t+1} + \beta Q^1_{22} l^e_{t+1} + Q^1_{21} q_{t-1} + Q^1_{22} l_{t-1} +$$
$$h_{21} q^*_t + h_{22} l^*_t$$

In an economy over time in which there is forward looking behaviour, adjustment costs and contractual obligation, then current values will depend on the past and on expectations of the future. We can implement the model by solving out for the endogenous variable expectations and we can replace the expectations of current values of wages, prices, inventory accumulation and vacancies by actual values when information is dated at time t, but when this is not the case we will have to use predictions or forecasts. The discussion of consumption in section two suggests that innovations may be important and this could also be the case for employment and income.

1.4 The determination of Excess demands, prices and wages

If we assume that prices depend on excess demands, and that vacancies and inventory accumulation have similar relationships to the market excess demand functions (1.16) and (1.17), then it may be possible to specify these equations in such a way that they do not depend on output or employment. So that:

$$\xi^g = \xi^+(p, w^e, q^s)$$

$$\xi^l = \xi^+(w, p^e, l^d, v)$$

Now if we substitute out for l^d using (1.18) and q^s using (1.20) we have the following relationship in terms of actual variables:

$$(1.22)\quad \xi^g = \xi^*(p, w^e, v, \Delta I, l)$$

$$(1.23)\quad \xi^l = \xi^*(w, p^e, v, \Delta I, q)$$

We will discover in Chapter 3 that we can transform (1.20) and (1.21) into the following backward looking representations:

$$F(L)[q_t, l_t] = B(L)[p_t, w_t, v_t, \Delta i_t]'$$

This bivariate system can be transformed into a reduced form in which output and employment only depend on a lag polynomial in the vector $[p, w, v, \Delta i]'$, so that:

$$q_t = b_1^*(L)[p_t, w_t, v_t, \Delta i_t]'$$

$$l_t = b_2^*(L)[p_t, w_t, v_t, \Delta i_t]'$$

When these values are put back into the excess demand equations (1.22) and (1.23), that produces the following reduced forms:

$$\xi_t^g = \xi^g(L)(p_t, w_t, v_t, \Delta i_t)$$

$$\xi_t^l = \xi^l(L)(w_t, p_t, v_t, \Delta i_t)$$

By analogy with the above results it should be possible to produce similar marginalisations or reparameterisations for prices, wages, inventory accumulation and vacancies to those for the excess demand equations. Therefore:

$$B(L) \begin{bmatrix} \Delta i_t \\ v_t \\ p_t \\ w_t \end{bmatrix} = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \end{bmatrix}$$

where the ε_{it} stand for innovations in the variables chosen as exogenous to the system and B(L) is the associated matrix polynomial.

We estimate VAR models of the exogenous variables in Chapter 3, the models are then tested for misspecification to determine whether the reparameterisations have been successful. We will discover in chapter 2 that a successful reformulation depends on stable parameters and this is partly dependent on whether employment and output determine output prices, wages, vacancies and inventory accumulation. The evidence turns out to be somewhat inconclusive, though their is some suspicion that we cannot substitute perfectly for output and employment. The theory presented here would not totally disagree with the notion that inventories, prices, wages and vacancies do not depend on current output and employment, but it would be difficult to deny all causal links. The problem is then to find any variables without some sort of link that can then be modelled separately or to derive a procedure which would allow the system as a whole to be estimated.

In Chapter 4 we use the results of Chapter 3 to derive models of output and employment which take account of the model structure above. A relatively efficient method is derived which only require one step ahead forecast errors and future predictions of the variables treated as exogenous. The model presented above is also consistent with an alternative theoretical representation, but it is felt that the current explanation may be internally more consistent. It is likely that there are some misspecifications induced by not taking account of openness of the economy and not modelling investment or other factors of production. It seems excessive to rely on Keynes assumption that capital is fixed in the short-term, though other authors such as Sargent(1978), Kennan(1979) and Muellbauer and Winter(1980) have made the same assumption. The models derived in this chapter also form a super set of many of the models estimated in the literature, for example the employment models of Muellbauer and Mendis(1982) only depends on output.

It has been traditional from the inception of macroeconomics to treat the subject as large micro and this conceptual approach has not been effectively extended by any appeal to micro-foundations. The essential dichotomy between agent behaviour in aggregate and the aggregate behaviour of individuals has not really been solved. The heroic macro assumptions of Keynes are subtly elaborated in the General Theory by a detailed analysis of macro responses to differentiated sectoral activity and this may be the best that we can do with aggregate time-series data. Kirman(1989) suggests that the usual equilibrium concepts require group behaviour or some law of large numbers to determine unique equilibria. Hence, micro foundations are only the basis of macro phenomena when strict aggregation conditions are met or when the form of the basic relationship is highly simplistic, otherwise aggregate micro relationships are no better than any other hypothesis at explaining macro behaviour.

CHAPTER 2

Dynamic Econometric Modelling

By its very nature time series data is dynamic, that implies econometric models must either be based on a dynamic economic theory or represent adjustment to some underlying static theory. If we are dealing with static models then dynamic data imply that economic phenomena take time to occur, hence, we are dealing with models which incorporate lags. Economics provides many reasons why models should reveal lagged processes: costly adjustment, durability, expectations, habituation and aggregation over economic agents with different response times. In constructing econometric models we also need to address the problems of time aggregation, the relationship between the timing of the data process and the theoretical one, and the choice of a particular functional form.

The Classical Statistical method associated with regression analysis assumes the axiom of correct specification to qualify the results, that implies either new data to experiment with or the correct specification of the original model. In reality we cannot replicate macro-economic data in a meaningful way which means that we need to derive a procedure for efficiently using the data available without violating the properties of randomness which underlie diagnostic testing . The traditional text book approach to econometrics expounded in Johnstons' 'Econometric Methods ' (McGraw Hill (1984)) assumes knowledge of the true model and suggests by its structure that Econometric technique provides a set of recipes which can be followed if the model does not meet our initial criteria. The approach does not mention analysis of the data which must be a pre-condition of model specification or the difficulties of re-specification and search. It has been traditional for applied modellers to either search the data until they find a model which satisfies their criterion or to simply ignore diagnostics if the results seem to support their prejudice. Unbridled search techniques invalidate the Neyman- Pearson Lemma that underlies statistical testing and ignoring diagnostics can produce nonsense regressions.

In this Chapter we look at Keynes reply to Tinbergen which points out the difficulties with modelling economic time series and provides a taxonomy of potential misspecification, and we discuss three approaches to data search, deal with the important problem of non-observation and look at the extent to which dynamic theory can satisfy the problems of model specification. Keynes(1939) synthesised the concerns of a number of statisticians and economists of the day over the problems involved in applying statistical techniques to economic data. Pesaran and Smith(1985) point out, that since Udny Yule(1923) developed the notion of time series modelling and noticed the potential for spurious correlation the analysis of data in economics became circumspect. Pesaran and Smith in support of Keynes, quote Haavelmo(1943)(see Lawson and Pesaran,p138) who picks out many of the recognised difficulties in analysing time series data. Of particular importance to Keynes was the fact, that Tinbergens method took little account of the conceptual difficulties in linking theoretical models to the actual observed data, the latent variable or non-observability problem. In that light, the results become little more than the products of some children's game, with the data artificially massaged to produce parameters and diagnostics which satisfy the modellers point of view.

Tinbergens work needs to be viewed as a pioneering study which attempts to utilise statistical methods to produce results, but the warnings which reverberate through Keynes can now be seen as justified by much of the applied work which has not satisfied Keynes and Haavelmo's criticisms or answered the conundrum set by Yule. It is hardly surprising that recently Econometrics has undergone a period of soul searching, as many of its leading exponents have worried less about the direction in which the discipline was heading and more about technique. A crisis point was reached following the poor performance of the major macro models after the first Oil Crisis in 1973 and the poor relative performance of many macro-models vis-a-vis simple time series formulations. Lucas(1976) developed a scathing critique of traditional Macro-econometric modelling based on rational expectations which suggested that model parameters would be inherently instable, because the models did not incorporate intelligent agents reaction to policies. The critique does not invalidate model building, but it complicates it as standard parameterisations need to be re-formulated to take account of the deep agent responses. David Hendry and Graham Mizon have approached this problem by looking for dynamic time-series forms which represent the data and satisfy theory in the long-run. The approach has been fairly successful in the case of the consumption function of Davidson et al(1978) and the demand for money study of Hendry and Mizon(1978).

The problem of breakdown has again focused attention on the problem of knowing the true model which has suggested three major methods of data search being adopted. Edward Leamer(1978) produced a novel book which has much affected econometric methodology, though his suggested technique has not been widely taken up in practice. The Search procedure and the Search program emphasise robustness of results through analysis of Extreme Bounds(EBA) and they suggest a Bayesian approach which marries modellers a priori beliefs to the observed results. Cooley and LeRoy(1981) and Leamer(1983) have used such techniques, but they have not fed into the mainstream, because the problem of misspecification has not been addressed and EBA can be more conveniently reformulated into a classical test procedure (see Pagan et al(1985)). The search method is an interesting concept, because the specification of priors emphasises the models prejudice and the formulation can be set up, so that the role of the final model and the data can be clearly determined. We will not deal with the Leamer approach, but we feel that it deserves mention with reservations over the implementation of particular applications and the fact that the approach only distinguishes between models on the basis of robustness, so that no additional methods are suggested to validate results. Selection of a sensible theoretical model and the extension of EBA to deal with the sign of derivatives and other forms of restriction may provide a semi-parametric approach to model selection.

Sims(1980) and Hendry and Richard((1982) and (1983)) have developed more effective modelling strategies to produce econometric specifications which satisfy theoretical principles. The Vector Auto-Regressive(VAR) approach to econometric modelling is a particularisation of standard time series modelling. Time Series models can be viewed as reduced form representations of fully formulated Econometric models and VARS can be derived by substituting out exogenous variable expectations from rational expectations models solved for their future endogenous variables. The data are first differenced to stationarity, then the variables in the system are given a VAR representation to simplify the procedure of identifying the time series structure and to simplify the method of estimation; Vector Moving-Average(VMA) models introduce into the estimation procedure complex non-linearities. The selection of a VAR as the forcing process for the exogenous variables insures that the solved form of the rational expectations model will also produce a VAR in the endogenous variables after substitution. In the next Chapter we use a hybrid of the VAR methodology and the general modelling strategy of Hendry and Mizon(1978) to derive models of the exogenous processes, but the endogenous variables are modelled using the method of Sargan(1982) which allows the deep parameters to be estimated. We will see in this and the next chapter, that there are many parameterisations of the types of general models suggested by Hendry and Mizon(1978) and Hendry and Richard(1983). In Chapters 3 and 4 we will see some of the limitations of the VAR approach and deal with it in relation to cointegration.

Hendry and Richard(1983) synthesises a body of work which commenced with the general modelling and destructive testing approach suggested in Hendry and Mizon(1978) and Davidson et al(1978) and added to that the notion of encompassing developed in Mizon(1984) and Mizon and Richard(1986) and the concepts of endogeneity suggested in Engle, Hendry and Richard(1983). The methodology is aimed at deriving a good model which satisfies the data, dominates other models, has stable parameters within and outside the estimation period and which has a theoretical interpretation in the long-run. The formulation of a general model does not impose strong theoretical restrictions on data in the short-run and in combination with the procedure of downward testing it should satisfy the notion of the true model, because the final representation should be consistent with the general specification and it should satisfy a number of tests which will validate correct specification. A well specified model will be considered to be good if it outperforms competing explanations of the data and yields theoretically consistent parameter estimates; it should also be a parsimonious representation of the data. If all of the criterion are satisfied and the results are invariant to policy changes, then the single equation or sub-system of equations which satisfy them will correctly formulated re-parameterisations of a more general system. Though we accept the importance of formulating general models and agree with the need to validate them, we also feel that econometric models should where possible have a short-run interpretation and that many of the testing procedures should be more closely linked to economic theory.

A valid modelling strategy should eliminate bad models and allow us to select between competing representations of the data. There are a number of models which we would clearly wish to reject: nonsense regressions, estimated models with unbounded variance and models whose results are not coherent. Granger and Newbold(1974) show that it is easy to discover time series which exhibit a strong correspondence amongst the data and this is especially true if the data are close to random walks, but such relationships should be viewed with deep suspicion. Dickey and Fuller(1978) and Sargan and Barghava(1984) show how to test for models with unit roots in the error term and models which do not satisfy such tests should be rejected, because the variance is not bounded which means that the relationship disappears as the sample evolves. The model could then be said not to exist or to be purely spurious, the problem first explained by Yule(1923). Although unit roots are a problem, the literature on Cointegration shows that groups of non-stationary variables may move together to produce a new series which is stationary; static regressions relating such variables together should then reject the unit root tests, even though the univariate time-series are only stationary in differences.

Econometric models are built for a purpose: prediction, policy analysis and the testing of theory. The choice of method and selection of a model may depend on the purpose for which it was built. Ron Smith(1984) suggests, that a models performance will reflect the reason why it was built, so there will be a trade-off between such reasons and how well the model works. That a models construction depends on the modellers requirements does not mean that misspecified models will be acceptable, but suggests that the criterion for selection may be predicated by such modelling decisions.

2.1 The Extreme Keynesian View

Keynes view of econometrics anticipated the lack of clear criterion for the validation of models, but it was his fear of the possible abuses of such methods which made him skeptical of the use of "Multiple Correlation analysis" in relation to economic data.

> "In plain terms, it is evident that if what is really the same factor appearing in several places under various disguises, a free choice of regression coefficients can lead to strange results. It becomes like those puzzles for children where you write down your age, multiply, add this and that and end up with the number of the beast in Revelation." Keynes(1973),p310

Keynes attacks Tinbergens use of least squares for the estimation of investment models, because he considered that the Economic processes driving the variables was prone to change and that expectations of interest rates and profit were not observable. The criticism holds for any method of estimation which does not take account of this problem or of variations in the process driving the data. If taken to its logical conclusion Keynes prognosis is excessively pessimistic and in principal irrefutable, as it implies that the observed data is formulated as part of an errors in variables system whose parameters are changing discretely over time. Hence, we have a large multivariate system, with a joint distribution spanning all time periods. Such a system can be defined as having the following joint density, if the variables are identically distributed.

(2.11) $D(v_1, v_2, \ldots v_T | V_0, \theta) \sim D(\mu, \Omega)$

where $v_t' = (w_t', y_t', x_t')$ and V_0 is the matrix of initial conditions θ is a vector of unknown parameters and μ and Ω are the mean and variances of the multivariate density.

The covariance structure implies intertemporal as well as contemporaneous correlation between variables. This states nothing about the distribution, whose exact form is likely to depend on the variables as well as the individual observations. In general, it will not be possible to identify the parameters we are interested in as the model will be over parameterised and it may not even be possible to estimate it, as the number of parameters may far exceed the data. In order to analyse such observational data we need to impose some structure on the means and covariances of the system. and make assumptions about the distribution of the variables. If we are to estimate any economic relationships we must discover an appropriate way to partition the model so we can at least derive conditional results. If this is possible we will be interested in a subset of variables which will be related to a re-parameterisation $\psi = f(\theta)$ of the original specification. Modelling is then the process by which we choose the conditional form, such an approach may not be futile, but care must be taken in model construction to verify and validate the results.

As Pesaran and Smith explain Keynes article deals with the principle difficulties with modelling time series: omission of variables, Latency, the Non-experimental nature of economic data and the quality of such data, spurious correlation, simultaneity, multicollinearity, linearity, dynamic specification and parameter instability. The methodology of Hendry and Richard pays attention to many of the above issues and we will deal in the next two sections with the problems of model specification, but the problems of latency and the non-experimental nature of the data we use emphasise the nature of model building and the problems which it involves. The first complicates the structure of models and the second invalidates the simple use of standard statistical techniques. Keynes was not against modelling as such, but he was afraid that many economic relationships would be prone to such criticism, so that econometric modelling would be fraught with problems, especially the simplistic use of least squares. Similar skepticism is re-iterated by David Hendry:

> "Econometricians have found their Philosophers' Stone; it is called regression and it is used for transforming data into significant results" David Hendry(1980b)

In practice modellers operate on the basis that such problems are not relevant especially for the variables they are interested in and the existence of observed relationships which are stable questions the extreme case and makes it difficult to refute models. It is very difficult to disprove any relationships validity, as either by chance or through the strong dependence associated with cointegration poor techniques may still reveal true parameter values. Even so, such relationships will not yield appropriate inferences and the chosen model may not have a sensible interpretation in the context of its misspecification(in the case of cointegration the estimated parameters only have a long-run interpretation). In general one would expect poorly specified models to break down, especially if the results are due to chance, or to be dominated by better models which can explain the nature of the misspecification.

It seems likely that there do exist sensible partitions of the data as the view of general instability does not appear to be consistent with the observation of processes which have shown a remarkable degree of stability. Phillips original (1958) article reveals a model estimated over the period 1861 to 1913 which seems to hold good up until 1958 and David Hendry has discovered many dynamic models which appear to have been stable for relatively long periods of time. So that models of change which give a role to adjustment will better approximate reality than simple static forms which are more prone to Keynes criticism. Whether the extreme view is correct depends on the nature of the instability.

2.2 A General Specification of Econometric Models

The generalised errors in variables model (2.11) cannot be estimated without the imposition of more structure, this either entails simplification of the model or the making of a number of auxiliary assumptions. Even if such assumptions or simplifications are not made clear, they are implicit in the model and when they do not hold an arbitrary parameterisation is imposed on the model so that the validity of any restrictions should be tested. A method of model selection should simplify a general model and validate the simplification, such a method is proposed by Hendry and Richard(1983).

The modelling process represented by (2.11) can be made less complicated by using the sequential nature of economic models to justify a similar factorisation:

$$(2.21) \quad D(V_T^1 | V_o, \theta) = \prod_{t=1}^{T} D(v_t | V_{t-1}, \theta)$$

where $V_t' = (W_t', Y_t', Z_t')$, $V_t' = (V_o : V_t^1)'$, $V_t^{1'} = (v_1, \ldots, v_t)$ and the w's are nuisance variables, the y's endogenous variables in the theoretical system and the z's exogenous.

Equation (2.21) is based on the assumption that the economic phenomena and the data operate in a sequential manner. A sensible econometric model may require reformulation of the data, but simple data transformations may not alleviate the latent variables problem. In the case of a switching regression model which is truly dynamic the likelihood will not be conformable with (2.21). We will discuss this more fully in section 2.4, but we note that the assumption is made by most applied and theoretical econometricians. In the sequential form the model is still over-parameterised, to reveal a tractable model we need to reduce the number of variables in the data matrix V and discover a structure for the variance-covariance matrix which will allow us to estimate ψ. The ψ's will then be a function of the original parameters θ; the exact form will depend on the re-parameterisation and conditioning of (2.21). It is more usual for the modeller, conditional on theory to presume that he has the correct information set, but any such formulation is simply a reformulation of the more general structure (2.21). Models should be set up in a general way and then compared with other data and alternative specifications.

The modeller needs to discover an appropriate method to partition the data, to allow him to eliminate variables which may be considered to be of marginal significance or whose effect is not relevant for the purposes of the analysis. Let us describe such nuisance variables as w_t, then the correct marginalisation of (2.21) needed to derive a relationships only in the ys and the zs is given below:

$$(2.22) \qquad D(v_t|V_{t-1},\theta) = D(w_t|y_t,z_t,V_{t-1},\phi_1)\ D(y_t,z_t|V_{t-1},\phi_2)$$

where $v'_t = (w'_t,y'_t,z'_t)$ and w_t are the omitted variables and in general $\phi_1 = \phi_1(\theta)$ and $\phi_2 = \phi_2(\theta)$

If the ws are not to affect the parameters of interest then ψ should depend in a fixed way on ϕ_2 alone and the marginal density of the ys should not depend on current or past values of the ws. The first condition implies that the parameters associated with w_t do not affect those of the marginal density of $s'_t = (y'_t,z'_t)$, so changes in ϕ_1 will not affect the relation between ψ and ϕ_2 and the parameters determining ϕ_2 should not be linked by restrictions on θ to ϕ_1; the re-parameterisation represents a sequential cut, as described by Florens and Mouchart(1980). The second condition means that s_t should not be

Granger caused by W_t (see Granger(1969) and (1980) or Harvey(1981)). The parameters in the marginal density will be constant when both the factorisation and the re-parameterisation are correct. We are then left with a system which can be analysed, given a small enough set of y's and z's. To ease the specification we can condition (2.12) on the exogenous variables:

$$(2.23) \quad D(s_t|S_{t-1}, \phi_2) = D(y_t|z_t, S_{t-1}, \lambda_1)\, D(z_t|S_{t-1}, \lambda_2)$$

where $S_t' = (Y_t', Z_t')$ and $s_t' = (y_t', z_t')$

and λ_1 and λ_2 are vectors of independent parameters

Notice, that for any V matrix there are as many factorisations as there are variables of interest, so that a viable specification will relate different models of the theoretical exogenous variables to the endogenous variables and the nature of such a partition will depend on the system. Different relationships with respect to the same variable are not valid except as evidence of data specifications or in relation to specifically designed separate models. Such formulations will represent different parameterisations of the data which will require alternative methods of estimation. The factorisation and method used will depend on the requirements of the modeller, so a more structured model will be necessary for policy analysis and tests of theories, while prediction models can have less structure.

The method so described is quite general, it implies or suggests that correct marginalisations will be associated with parameter constancy, as the observation of instability of the parameters will be a sign of an inappropriate re-parameterisation. Although parameter constancy is a sensible criterion for a model to satisfy, care should be taken in constructing tests and analysing the results. This is because the simple, sample splitting tests have their limitations and the form of misspecification associated with failure may not be due to the marginalisation or if it is, the effect may be small. Tests are utilised in an environment in which the correct specification is not known, so tests of parameter constancy are an element in the search for a good model, as such they must be correctly applied to give valid inference. The test may be invalidated by repeatedly using the statistic to select a model during data search, by ordering or combining specification tests wrongly or through failure of the distributional assumptions. Leamer(1978) shows that the probability level of the t-statistic is biased towards acceptance when it is used as a model selection criterion in a data search; this will be true of any test used in this manner. Kiviet(1985) observes that specification tests when grouped must be ordered in a particular way and that some tests cannot be used in concert (also see Breusch(1979). All tests are dependent on the validity of the distributional assumptions made, the test may suggest incorrect rejection regions or have low power if the assumptions do not hold or they offer poor approximations. The general model (2.31) is likely to be over-parameterised relative to the data, that means that there are a range of potential starting points for search linked to as many final solutions, so that selection of an economically informative model is likely to be difficult.

> "In practice one may be willing to suffer some loss of efficiency to achieve a tractable model when ϕ_1 and ϕ_2 are not variation free..." Hendry and Richard(1983),p118

Even if the tests of specification are correctly implemented and the model fails at the parameter constancy stage, we may not wish to reject the basic structure, especially if failure is due to truly varying parameters, incorrectly handled endogenous variables or simple structural breaks. Alternatively, we would like to reject models which are poorly specified, suffer from regime shifts or represent none sensible re-parameterisations of the data. In order to distinguish between the forms of misspecification we need to do more than reject models on the basis of a simple Chow test of sample splitting or predictive failure. We should look at the reasons for model break down and adjust the model on the basis of that or re-think the methodology we are using, especially if it is a single equation technique. As far as predictive failure is concerned the criterion is even weaker, as the model may still be appropriate for within period analysis or it may be wrongly rejected outside the period, because of a small number of new data points available for testing.

The standard Chow test(see Johnston(1984)) separates the sample into two sub-periods and compares the parameter estimates to see if they have changed; this is conditional on constant variance. Let us take the standard linear model for the ith endogenous variable:

(2.24) $Y_{it} = Z_t \beta_1 + U_t$ and $U_t \sim N(0, \sigma^2 I)$

so that $\hat{\beta}_1 = (Z_t' Z_t)^{-1} Z_t' Y_{it}$ and $Y_{it}' = (y_{i1}, y_{i2}, \ldots, y_{it})$

and $Y_{is} = Z_s \beta_2 + U_s$ $\beta_j = (\beta_{1j}, \beta_{2j}, \ldots, \beta_{kj})$ and $U_s \sim N(0, \sigma^2 I)$

so that $\hat{\beta}_2 = (Z_s' Z_s)^{-1} Z_s' Y_{is}$ and $Y_{is}' = (y_{it+1}, y_{it+2}, \ldots, y_{it+s})$

where $\hat{\beta}$ is the OLS estimator and the hypothesis to be tested is

$$H_0 : \beta_1 = \beta_2$$

$$H_1 : \beta_1 \neq \beta_2 \qquad \text{s.t.} \quad \sigma_1 \neq \sigma_2$$

The test will be rejected if H_0 is false or the variance is not constant. In the standard text book view of econometrics H_0 will fail when we have time-varying parameters, if the model is inconsistently estimated or if their are regime shifts which alter the structure of the model. The Chow test gives us no idea about the validity of the structure or informs us how the model should be changed. As Newbold suggests in his criticism of H and R(1983) there may be no reason to believe that model parameters are stable, but they may follow a stable process. If the w's cannot be modelled we may not be able to improve on the method used and if the coefficients move in a regular way it may be preferable to use a varying parameters method. The structure of (2.24) has not changed, but the method of estimation will be inappropriate. Hence, we might wish to use the error decomposition to estimate such a model or the Kalman filter(see Maddala(1977).

$$\hat{\beta}_t = \hat{\beta}_{t-1} + \varepsilon^*_t$$

where $\varepsilon^*_t \sim WN(0,\sigma^2)$; a white noise innovation

Variation in the parameters may be caused by inconsistency, as certain variables may have been wrongly assumed exogenous. This relates to strict exogeneity, as defined by Engle et al (1982), so some of the RHS variables in (2.24) are not independent of the equation error. The form of the equation is correct, but it needs to be set within the context of a broader model; we could choose an instrumental variables estimator:

$\tilde{\beta}_1 = (K_t' Z_t)^{-1} K_t' Y_{it}$ where $K_t = (k_1', k_2', \ldots, k_t')$ is the matrix of instruments

The observation of regime shifts causes greater complication, as it implies that variables have been directly omitted from the structure of the model or that the original structure is not valid for the whole period, if at all. At the simplest level this may just require the introduction of dummies to account for institutional changes of which we have no details or it may imply that the model is appropriate for sub-sets of the data and an alternative model should be constructed following the changes. This type of change is exemplified by entry to the Common Market or floating exchange rates or other discrete shifts. In forward looking models the effects of dummies may be small, because of anticipation. It has been shown that a tax dummy in a stock model can change into a simple policy on/off dummy when expectations are rational. In this instance the change in structure may be limited to the adjustments in the constant suggested by dummies or through the development of a different model. If Z_{1s} is the data matrix for the second period which may be in part or wholly different from Z_s the OLS estimator for β_2 is given below:

$$\hat{\beta}_2 = (Z_{1s}' Z_{1s})^{-1} Z_{1s}' Y_{is}$$

This is a simple case of switching regimes where we know when the structural break occurs, but in general this will not be the case. In a disequilibrium framework the economy may appear to shift randomly between models, this may be indistinguishable from an arbitrary movement. Such discrete shifts either have to be handled by a dynamic model or through the discrete choice methods of disequilibrium econometrics. The Kalman Filter can provide a method of estimating models in which the parameters shift in a deterministic way.

$$\hat{\beta}_t = \hat{\beta}_{t-1} + S(z_t^s - z_t^d) + \varepsilon_t \quad \text{and} \quad \varepsilon_t \sim WN(0, \Gamma)$$

Finally such variation in parameters may be due to the conditions of partition not being satisfied, in fact the problem of endogeneity suggested above could be a signal of this more general problem, or parameter shifts may be due to cross equation restrictions which have not been accounted for properly in the original marginalisation of (2.21). This will occur if we have omitted variables which are informative about ψ or violated the conditions of weak exogeneity.

Parameter non-constancy is due to a number of causes, not distinguished by the Chow and predictive failure tests. They are not set up to determine whether the partition is valid, but they are indicators of model misspecification; which in the example above suggest that OLS estimation of (2.36) may not be appropriate. As this simple model shows the form of (2.36) does not always change when the parameters change, though the standard OLS specification has to be augmented to allow the parameters to be correctly estimated. Hence, tests of parameter constancy need to be supported by other information before an equation is rejected. The cases given above suggest that (2.36) should be placed within a more general framework to determine the way in which it should be reformulated. The standard model can be placed in a time varying parameters form to see if the re-specification is valid(see Brown Durbin and Evans(1975)) or made part of a larger system in the case of endogeneity; either of the above ideas could be used in the case of switching regressions to decide whether breakdown is arbitrary or the original form still useful. If we cannot find such a general framework which gives sensible results, then we may wish to start again.

Hendry and Richard(1982 and 1983) see tests of parameter constancy as part of a strategy to eliminate models which are invalid reformulations of the data, they are augmented by tests of serial correlation, heteroscedasticity, and normality, as well as the requirement that the model is theory consistent, data admissable and encompasses other models. This process of weeding out poor models is meant to reveal a Tentatively Acceptable Conditional Data Characterisation (TACDC), but destructive testing may not be the best way to select a good model. The tests presented above provide information about model failure, but they are only informative in particular cases of the way in which the model should be changed. In general, such methods can only illude to the true model, as this really needs to be derived from the detailed comparison of alternative theoretical or well structured specifications. Although we agree with the general form presented by H and R(1983) and believe that it is important to test models, we feel that specifications should yield more structure and tests should be based on a more constructive approach to modelling.

The treatment of (2.23) will depend on the exact factorisation and the purpose for which we wish to use the model. If we wish to analyse the marginal density of the ys taking the zs as weakly exogenous the parameters of interest ψ should depend on λ_1 alone and should not be affected by changes in λ_2; inference will be valid under such circumstances. Valid estimates can be derived with the standard condition that the z's are independent of the stochastic component of the y's, but that does not imply invariance as there may be cross equation restrictions; that is especially true of models which use rational or consistent expectations. Weak exogeneity is the appropriate concept for meaningful estimation and appropriate inference, though the search for an invariant structure must be undertaken carefully with attention paid to the purposes of modelling and the nature of the misspecification.

2.3 Data Determined Dynamics

A variant of the general method can be used to justify the estimation of economic phenomena using single equations, see Hendry and Ericsson(1983). The papers by Davidson et al(1978), Hendry and Mizon(1978) and Hendry and Ericsson(1985) can be qualified in this way, though they also explain particular problems in determining statistically meaningful econometric models. The approach used utilises single equation methods to select by testing a TACDC from a general model. Theory enters this process through selection of the data set to be utilised and by providing the long-run relationship to which the data adjusts; under certain circumstances the adjustment process has an economic rationale. If the model is to be interpreted it must mean that the significant element of the dynamics stem from such adjustment and that the adjustment is uniform across time. The exact lag specification is determined by the data(the Data Generation Process), so simplistic theory based propositions are not wrongly imposed. The chosen model should satisfy the usual classical criterion, have stable parameters and perform well outside the sample. The method has pointed out the need for dynamic models when the data is trended and the limitations of the traditional way of selecting models:

> "Such an approach requires the 'Axiom of Correct Specification' (Leamer, 1978, p.4) that all assumptions of the model are valid and leads to a model-building methodology in which violated assumptions are viewed as 'problems' to be 'corrected'", Hendry and Richard(1983), p117

For Example, Hendry and Mizon in their (1978) paper explained why serial correlation cannot be corrected in the usual way if it is due to more general dynamic misspecification. Even though this work has been informative and has led applied modellers to think more about specification, it can be criticised for spawning models which are difficult to interpret, placing too much emphasis on single equation estimation and allowing destructive testing the major role in deciding the structure of econometric models.

If we start from a general economic model with k endogenous and l exogenous variables, where a * superscript means that we are dealing with a theoretical variable then:

(2.31) $(I-B(\theta^*))y_t^* = A(\theta^*)z_t^*$

when the economic model is dynamic $B(\theta^*)$ and $A(\theta^*)$ will be functions of lags (L) and leads (L^{-1})

It is usual when modelling to analyse a block of equations or one equation from (2.31); we will follow the literature and take a single linear relationship:

(2.32) $y_{it}^* = b_1 y_{1t}^* + b_2 y_{2t}^* + \dots b_k y_{kt}^* + a_1 z_{1t}^* + a_2 z_{2t}^* \dots + a_1 z_{1t}^*$

Equation (2.32) is the long-run solution to a system which is both static and linear and certain of the variables may be omitted from it restricting the long-run parameters a and b. The General econometric model is not assumed to be static, so this short-run model incorporates the dynamic factors. The selection of a lag length J depends on the number of observations, the order of seasonality, other modellers experience with similar data and the nature of the series being modelled. A simple rule of thumb which generally works is $J = J_s+1$ or $J = J_s+2$, where J_s is the order of seasonality. The general model with J lags in all variables which has (3.32) as its long-run solution is given below.

(2.33) $y_{it} = \sum_{j=1}^{J} \beta_j \, y_{t-j} + \sum_{j=o}^{J} \alpha_j z_{t-j} + e_{it}$

where $\beta_j = (\beta_{1j}, \beta_{2j}, \dots, \beta_{kj})$ and $\alpha_j = (\alpha_{1j}, \alpha_{2j}, \dots \alpha_{1j})$

A parsimonious form of (2.33) is derived by testing down using an F statistic or Likelihood ratio criteria to determine whether the restrictions associated with the final specific form are significant. If the specific model is consistent with the general model, then the procedure is validated by testing to see whether the results satisfy the assumptions of the method of estimation. In the case of Ordinary Least Squares(OLS) the errors should be serially independent, the equation variance homoscedastic, there should be no problem with simultaneity and for valid inference we need normality or a large sample. If we estimate (2.33) directly using a single equation method the model can be interpreted directly in terms of the theoretical parameters:

$$(2.34) \quad b_k = \sum_{j=1}^{J} \beta_{jk} \Big/ \Big(1-\sum_{j=1}^{J} \beta_{ji}\Big) \text{ and } a_l = \sum_{j=o}^{J} \alpha_{jl} \Big/ \Big(1-\sum_{j=1}^{J} \beta_{ji}\Big)$$

In addition to being a parsimonious representation of (2.33), the chosen form should be a sensible partition of (2.21) the general errors in variables system. A sensible partition will be an invariant model which has an economic interpretation and which explains other formulations of the data, also Hendry and Richard(1983) assume that the error e_t is a white noise innovation; these are necessary conditions for a TACDC. In terms of the single equation form it must satisfy a range of tests which validate the method of estimation, usually OLS and show that it is consistent with the statistical principals of the general method. In addition to the criterion mentioned above a TACDC should satisfy tests of parameter constancy and predictive failure, also it should be theory consistent, data admissible and encompass other models. A form which satisfies these conditions is considered to be a sensible marginalisation of the original model. If the relationship is invariant the RHS variables should be weakly exogenous with respect to this reduced parameter set, that is necessary for the method of estimation to be valid. If the model is to be used for prediction the RHS variables should be monocausal; they should not depend on Y_{it}.

The reasons for the above tests are obvious in terms of standard text book theory they are direct checks on the relevance of the estimation method. Parameter constancy would normally be expected of an invariant model, so violation of that principal suggests the model may be misspecified and the estimation method inconsistent. In the previous section we were critical of the use of tests of parameter constancy in model selection, because the prediction and Chow tests available are weak tests of misspecification and not direct tests of invariance which would determine the validity of a marginalisation. They do not show that the model is invalid for sub-periods of the data and they give no idea of the way in which it should be reformulated. They are an indicator of either a poor model or an incomplete specification. This distinguishes between the rejection of the model which should be based on more than tests of parameter stability and rejection of the structure. In the single equation framework the problem is more acute, as the models generally have less structure. Hence, it is easier to discover by chance a constant model which is not a true invariant

Discovery of a model which satisfies all of the above criterion would give strong support to the view that a good approximation of the true model had been found, though care must be taken in interpretation of such results in this way. There are additional requirements which economic theory might require of a model, these Hendry and Ericsson(1983) call "Theory Consistency", and

"Encompassing", a model should at worst variance-dominate other models and at best explain their results. Theory consistency suggests a model's parameters should be in line with economic theory and in addition to that certain restrictions implied by theory should also be testable; as Spanos(1981) mentions fitted values should satisfy theoretical identities and the model constructed so that theory based restrictions are testable. The literature on systems of demand equations shows how models can be constructed to allow theory to be tested; flexible functional forms do not impose homogeneity, symmetry and negativity on demand systems(see Deaton and Muellbauer(1980)). In terms of (2.33), theoretical propositions are imposed on the long-run solution, these parameters should have the correct signs and satisfy the constraints of theory.

The error correction form of (2.33) provides a structure which more easily allows the analysis of theory, because the parameters of the long-run model automatically drop out and the short-run dynamics can have a theoretical interpretation. Salmon(1982), Hendry and Spanos(1981) and Nickell(1985) present the adjustment process of the ECM as the solution to an optimal control problem, the dynamic process of a disequilibrium model and the solved form of a rational expectations model, in which the exogenous variables are generated by a first order autoregressive process. The error correction form of (2.33) is:

$$(2.35) \quad \Delta y_{it} = \sum_{j=0}^{J-1} (\beta_j^{+} \Delta y_{t-j} + \alpha_j^{+} \Delta z_{t-j}) + \sum_{j=1}^{J} \tau_j^{+} (y_{t-j} - y_{t-j}^{*})$$

where $\beta_{0j} = 0$ and $\tau_j^{+} = (0 \; 0 \; . \; . \; . \; \tau_{ij} \; 0 \; . \; . \; . \; 0)$

The parameters of (2.33) and (2.35) are related in the following way, where y^*_{it-j} is given by (2.32) the equilibrium model, so that:

$$\beta^+_0 = (0\ 0\ .\ .\ .\ -1\ 0\ .\ .\ .\ 0)$$

$$\beta_j = \beta^+_j - \beta^+_{j-1} + \tau^+_j - \tau_{ij}b$$

$$(2.36)\quad \alpha_0 = \alpha^+_0$$

$$\alpha_j = \alpha^+_j - \alpha^+_{j-1} - \tau_{ij}a \qquad j > 0$$

where $a = (a_1\ a_2 \ldots\ a_l)$ $b = (b_1 b_2 \ldots\ b_k)$

Substituting out β^+, τ^+ and α^+ in (2.35) with some reformulation will give (2.32).

The Error Correction Model(ECM) is preferred, because it has a theoretical basis, but the single equation ECM has been criticised, because it does not allow interaction or spill-over of adjustment and the estimates of the dynamic model are often unstable. In testing down from the general form (2.35) is likely to maintain more structure than (2.33), as the variables have the interpretation of growth or adjustment parameters, while the same exercise on (2.33) may leave an arbitrary form associated with a number of specifications.

The ECM is given in single equation form here, but it also has a systems representation(see Davidson(1985)). Muellbauer(1982) shows in the demand case that the error correction form (2.35) sets all the spill-overs or cross corrections to zero, so that $\tau_{rj} = 0$ for $r \neq i$. In reality there is no reason why this will hold, so that the τ^+_js should be unconstrained vectors of parameters; Anderson and Blundell (1985) estimate such a model.

If cross corrections are important longer adjustment is induced in the single equation form, as (2.35) will only be correctly specified when the cross correction are captured by further lagged variables. If this criticism is correct the structural interpretation given to the estimated parameters using (2.34) will not be valid, as the results are conditional on the omission of the cross market corrections. The long-run static and steady state parameters will be affected, as the cross equation responses are taken to be own adjustments and growth parameters.

The parameters given by (2.34) are the long-run static solution to (2.33) and (2.35), given $y_t = y_{t-1} = y_t^+$ and $z_t = z_{t-1} = z_t^+$; where the time subscripts in the static formulation are superfluous. The static parameters come directly from (2.35), when $\Delta y_t=0$, but the model also has a dynamic representation in which $\Delta y_t=\pi$ the rate of growth when the model is in logarithmic form. In steady state all variables grow at the same rate and (2.35) has the following solution:

$$(2.37) \quad y_{it}^+ = y_{it}^* - (\beta^x + \alpha^x)\pi$$

where $\Delta z = \pi_z = \pi = \pi_y = \Delta y$ in steady state and

$$\beta^x = \Big(\sum_{k=1}^{K} \sum_{j=0}^{J} \beta_{kj} - 1\Big) / \sum_{j=1}^{J} \tau_{ij} \quad \text{and} \quad \alpha^x = \Big(\sum_{l=1}^{L} \sum_{j=0}^{J} \alpha_{lj}\Big) / \sum_{j=1}^{J} \tau_{ij}$$

Currie(1981) has criticised the ECM on the basis of such dynamic results, noting that even when the static solution is stable the dynamic parameters are invariably not, suggesting unrealistically explosive growth paths and suggesting that the period of the data is too short to pick up such long-run effects. In section 4.4 we will show that such instability may be due to the transformation of a model with forward looking expectations into error correction form. In some cases the models will have a theoretical interpretation, but such hypotheses should be tested and compared with other formulations before a model is accepted.

We would expect a model to suggest why an other formulation revealed a particular result, this is encompassing which implies that a model should be so constructed so that the parameters of model can explain the results of less general models (it needs to be possible to make such distinctions). In general such tests may only be feasible over a limited range of alternatives which means that models may be chosen on the basis of their comparative ability to explain incidents in the data, so more general model should fit and perform better.

> "For a single equation estimated by least squares, a necessary condition for encompassing is variance dominance .... . It seems natural that a poorly fitting equation cannot account for why a well fitting equation fits well" Hendry and Ericsson(1983).

Encompassing in this framework is a general criterion which we would like valid models to meet, but in this general sense it is difficult to apply a rigorous test, as nothing is stated about the structure of the alternative. The fit of the single equation model should not be used as a criterion for such a test, as the model specification will depend on the purpose for which it was constructed, if it suggests a valid partition of our unobserved general model then the relative performance on the basis of fit or prediction is irrelevant. In order to properly analyse comparative performance or preference on the grounds of variance dominance, the model should be compared at the level of the sub-system from which they are derived and even so, such preference is still based on too narrow criterion if all we are looking at is fit or prediction. True encompassing tests should deal with the model structure and so analyse omission in terms of the model parameters; such tests will be the same as those for omitted variables in simple cases or when individual parameters cannot be identified. The proper analysis of model specification should be carried out at the level of the intelligible sub-system so that a structure should not be rejected on the basis of the comparative fit of single equations. In addition to that, the preferred model should have a theoretical explanation if it is to be used for policy, otherwise the best fit model will be acceptable.

Hendry and Richard(1983) provides a justification of the single equation method in terms of their methodology, so that the discovery of a correct partition is based on destructive testing Single equation methods are accepted for their simplicity, but such a philosophy has its costs. The data available is limited which means that we cannot be certain that the chosen model is a valid characterisation of the data or that it is the only single equation to be a valid marginalisation of (2.11). This is because destructive testing aims to dispose of statistically poor models, but does not determine what is a good model. The alternative is never specified, so models compete with the idealised data generation process without having a specific model which characterises that. The need to choose models on the basis of tests which are likely to be poorly determined limits the efficiency of the strategy. Models need to be tested, but the methods currently used may be of dubious value, because the tests are prone to problems of biased selection and inappropriate inference by virtue of iteration over selection criterion and limited inferential power, due to the form and number of the tests.

Single equation methods are generally less efficient, so they are likely to be less informative which means they may be difficult to interpret, and prove hard to use for analysis and testing of policy and theory. The relationship between the parameters of (2.33) and (2.32) described in (2.34) only holds good on the premise that the dynamics of (2.33) are due to adjustment over time, if they depend on expectations, durability or aggregation they could be given a different explanation. If the process describing the data is due to such dynamic factors or the static phenomena of omission of variables or non-linearity, then the long and short run variables are not good estimates of the parameters of interest. The model may be a valid representation of the data generation process, but it is not being interpreted in the right way. Reduction of (2.33) to a parsimonious form may reveal a model without a clear short-run theoretical explanation, because of the ad-hoc nature of the lag structure(see Hendry(1980b)); we cannot then validate the short-run form. The ECM is obviously more appealing when using such methods and as a general principal is easier to interpret than reductions from (2.33). Keynes(1973) seems to sum up the problem of lag choice when he discusses Tinbergens' method:

".. he fidgets about until he finds the time lag which does not fit too badly with the theory and the general presuppositions of his method. ... But there is another passage(p.39) where Professor Tinbergen seems to agree that time lags must be given a priori", Keynes(1939,1973), p314.

On the basis of correct inference, utilising and validating the equations by using Hendry and Richard(1983) we may uncover a statistically sensible marginalisation of the likelihood of our general problem (2.11), but there is no guarantee that such a relationship will reveal results which are economically meaningful and it is easy to find strange partitions or parameterisations which are hard to give meaning to. This is not to say that the method always reveals nonsensical results or results which are not useful, but to insist that there are limitations to this method and the purposes to which it can be put. In general it will not be possible to test theories using such methods or to derive more detailed models for economic policy analysis and reveal from that sensible multipliers or elasticities. This is because (2.33) has not been linked to the correct structure, so that the limited single equation form may over parameterise the model or limit the possible detail. Hence we may not be able to analyse what is of interest or determine the validity of theories:

"As regards disproving such a theory, he cannot show that they are not verae causae, and the most he may be able to show is that, if they are verae causae, either the factors are not independent, or the correlations involved are not linear, or the environment is not homogeneous over a period of time

,perhaps because non-statistical factors are relevant"

Keynes(1939,1973), p308.

The inability to disprove a theoretical proposition is not negated by the use of a more general model, as the dynamics may indicate a re-parameterisation which invalidates our explanation of both the short and long run model. The Lucas critique of policy effectiveness would be relevant here, as the parameters of the single equation model would be composed of the theoretical parameters and the policy response. In periods of relative stability in which changes where small or weighted by small parameter values the single equation model may exhibit parameter stability. This will be the case when the estimates are relatively inefficient, allowing larger than expected variation in the parameters, so that the hypothesis of parameter stability is not easily rejected. The model has stable parameters, but in the simple reduced form specification it does not accept the theoretical restrictions, though a final form which specified the reaction function and the theoretical relationship would do.

If general models are constructed in a sensible form, so that theoretical considerations can be tested, this may also yield tests of encompassing as a bi-product, rather than general appeals to superior fit of the data. In specifying encompassing in terms of tests of model parameters, where possible we are negating the possibility of alternative marginalisations explaining different elements of the data or suggesting that such distinctions related to fit will not be important for models which purport to explain the same thing or are very closely related if not involving different parameterisations of the same variables. In addition to the points stressed above we may also find that such tests of structure can be linked to the type of invariance associated with parameter stability. Whether a model is rejected in response to tests depends on the reason for its construction and the cause of the instability. A heavily parameterised model used for policy analysis may not predict too well, as its predictive power will depend on the processes for the exogenous variables and the validity of the conditioning over different periods and states of the world. Econometric specifications do have limitations to their use and applicability, due to the changing nature of the world, hence models are bound to break down over time, that is not a problem, it is why they break down which is important.

2.4 The Latent Variables Approach to Econometrics

The econometric model can be developed in three stages: specification of the economic model, choice of the latent mapping and the formulation of the econometric specification. A well developed model should be based on a theoretical principal derived from the literature or from an aligned discipline, by the observation of institutional reality or the observation of a simple data relationships. Model reformulation should not be based on data search, but the failed econometric specification should be completely re-specified and the modelling process started again. Such an approach yields a scientific way of specifying results and revealing economic information from the data. The method used will depend on the rationale behind the model construct, so that forecast models may not require as strong criterion for analysis as structural representations.

The Economic model will normally be derived from a theoretical proposition which we would like to assess for data acceptability. Equation (2.31) of the previous section represents such a general structure, but it is not automatically amenable to estimation, because we do not know the link between the model and the data or the stochastic structure; without these components we will not be able to decide the appropriate estimation method. The theory model holds exactly, as such relationship are set up to satisfy some equilibrium concept or are arranged so that accounting identities hold exactly. Spanos(1981) suggests that the data will not satisfy most theoretical propositions exactly and the parametric structure of equations will be affected by the imposition of such identities on the data.

Smith and Hunter (1985) show for exchange rate models, that the imposition of arbitrage constraints will alter the specification of cross exchange rate models for all but a limited range of theoretical formulations. We assume that the theoretical model is static, but in the next section we will introduce dynamic economic specifications.

In choosing such a formulation the modeller has to select a functional form, the level of aggregation, the exogenous endogenous split and in some cases the stochastic structure. The literature on systems of demand equations has paid attention to the problem of functional form, because certain utility functions, such as the Linear Expenditure System impose strong restrictions on the model and limit the degree of substitutability (see Deaton and Muellbauer (1980). The difficulty is remedied by choosing flexible functional forms, such as the Almost Ideal Demand System developed by Deaton and Muellbauer(1980a) which allows the restrictions of theory to be tested and provides a local approximation to any demand equation. The aggregate structure of the model, the endogeneity of variables and the stochastic structure all depend on the data and although the theoretical model may be suggestive of them they cannot be imposed, because this may re-parameterise the model in a strange way or invalidate estimation. This implies that there will be a filter between the data and the theoretical model. Therefore:

$$f_z : z_t^* \dashrightarrow z_t \qquad (2.41)$$
$$f_\theta : \theta^* \dashrightarrow \theta$$

is the function which maps the theoretical variables or parameters on to their data equivalents. Usually this process is incorporated into the construction of the data model, so that the conditional model (2.23) represents the transformed version of the general theoretical model (2.31). Similarly, the parameters of (2.32) are mapped via (2.34) onto the econometric model (2.33). Equation (2.41) is not usually specified, but it is inherent in any econometric specification.

Most modellers make the trivial assumption that the data corresponds exactly to the theoretical analogue. Observation of the data would suggest that this is rarely true, because economic data is non-experimental, subject to political and governmental bias in collection and usually does not satisfy directly the requirements of the theory model. In the natural sciences data can be replicated under controlled conditions, in economics such experiments are not possible for all macro and a wide range of the micro problems which interest us, because the sector or unit to be analysed cannot be isolated from the economy or replicated exactly. The majority of economic data is collected by the agencies of government, so it is not value free, because the collection process reflects a statistical or economic perspective and the published series are pre-processed using filters. The data is also subject to updating, change in the base year and redefinition, all of which complicates model building. The trivial assumption would be more acceptable if the modeller collected his own data, by interview or design experiments, this would reveal information not contaminated by the factors described above.

The current approach is modelling rather than experiment intensive which implies the general mapping (2.41), but many modellers choose the trivial mapping:

$$(2.42) \qquad y^{*}_{t} = y_{t} + \epsilon_{t} \qquad \text{s.t. } \epsilon_{t} \sim WN(0,\sigma^{2}_{y})$$

and

$$z^{*}_{t} = z_{t} + u_{t} \qquad \text{s.t. } u_{t} \sim WN(0,\sigma^{2}_{x})$$

In the case of the exogenous variables z it is more usual to assume an exact equivalence, so u would be zero. If the trivial assumption is correct modelling is much simplified, but in many cases we would not wish to make such an assumption; in particular if data series followed random walks or if the theory appears totally uninformative of the data. A mapping may not exist when theory is not useful in constructing the econometric model or when the theoretical parameters are not identified. In the extreme this may be true for all models and then theory would play no part in econometric modelling. Therefore:

(2.43) $\nexists g : y_t^* \to z_t^*$

Proposition (2.43) would be true if the trivial mapping holds and the data are represented by random walks which is the case for time series data when they are smooth and trended, in such circumstances static theoretical models will exhibit strong serial correlation. Granger and Newbold(1974) observed that much empirical work regressed contemporaneous variables on each other in a static economic framework without taking account of strong signs of autocorrelation:

(2.44) $Y_t = Z_t A + U_t$

where $Y_t' = (y_1, y_2, \ldots, y_t)$ and $Z_t' = (z_1, z_2, \ldots, z_t)$
and $U_t' = (u_1, u_2, \ldots, u_t)$ and A is the matrix of parameters

If the Z and Y variables are trended it is likely that (2.42) will provide a reasonable fit of the data with high R^2 and reasonable fit of the parameters, but in many cases such models exhibited strong serial correlation as measured by a low Durbin statistic. Serial correlation is a sign of a moving average error, pure error autocorrelation or of more general dynamic misspecification(the distinction is clearly made in Hendry and Mizon(1978)). All three of the above possibilities shed doubt on inference and in the case in which we have more general misspecification the parameters may not be well specified. As was stated before time-series data are highly smoothed, in the extreme this suggests that Y and X follow random walks:

$$Y_t = Y_{t-1} + E_t$$

(2.45) and

$$Z_t = Z_{t-1} + \epsilon_t$$

where $E'_t = (e_1, e_2, .. , e_t)$ $\epsilon'_t = (\varepsilon_1, \varepsilon_1, .. , \varepsilon_t)$

If this is the true model and the errors are independent, then as Granger and Newbold(1974) show the results observed in (2.45) will be spurious and it will suffer from serial correlation. The error contains everything which is omitted from the model:

$$U_t = Y_{t-1} - Z_t A + E_t$$

lagging (2.45) and using it to substitute out for Y_{t-1} above:

$$U_t = -\Delta Z_t A + U_{t-1} + E_t$$

(2.46) $= U_{t-1} + E_{1t}$

where $E_{1t} = E_t + \epsilon_t A$ and from (2.45) E_t and ϵ_t are innovations in Y_t and Z_t respectively.

It is then obvious from (2.46) that the omission of lagged Y will cause first order serial correlation. Under the trivial correspondence the economic relationship is of no importance to the data generation process; the model is not informative about the data. This is the extreme case which implies for a given economic model, here the static one, that there is no mapping between the data and the latent variables. A test of the random walk hypothesis will be a test of the existence of the postulated economic relationship; independent of the mapping. The results presented above show that (2.44) has an error which follows a random walk which means that the error variance is unbounded. As the sample evolves the standard errors will increase without limit and the parameters becomes less well specified. Asymptotically the parameters of (2.44) will be perfectly consistent with any parameterisation, suggesting that the OLS estimates of A become unidentified(see Sargan (1988)), but If we look at the direction of the bias of the OLS standard errors in finite samples we will find that it is indeterminate so that the estimated parameters may appear to be significant. If the trend component of the two series is similar, then we will also observe a high R^2. This is why Granger and Newbold select high R^2 relative to durbin watson statistic as a sign of spurious correlation.

It is usual to find that a particular model or representation of the data does not to exist, so that:

$$\text{for a given } g : y_t^* \dashrightarrow z_t^*$$

$$\nexists \; f_\theta : \theta^* \dashrightarrow \theta$$

either the mapping does not exist, so that the random walk hypothesis is true or the model parameters are not identified. The first is strong evidence against the model, the second only states that for the present representation the data are not informative of the model, in general the latter proposition will be observed.

Time series will often be close to random walks, but many models exist which out perform such simple models and have an economic interpretation. An example of such a model would be the error correction model, specified by Hendry and Mizon(1978):

$$(2.47) \quad \Delta Y_t = b_1 \Delta Z_t + b_2 (Y - AZ)_{t-1} + E_{1t}$$

As was observed in section 2.3 this form has a number of interpretations associated with a dynamic theoretical models, they imply the trivial latent mapping and a dynamic econometric model. The paper by Hendry and Mizon suggests a dynamic latent mapping linked to a static theory. In the general ECM (2.35) the latent mapping across the parameters is given by the long-run solution (2.34).

If it is correct to represent Z and Y by (2.45), then the relationship (2.44) will be spurious, except when the exogenous and endogenous variables are cointegrated. We will deal with cointegration in more detail in the next chapter, but in this instance it implies that the OLS estimates of A will be consistent when there is one dependence between the Zs and Ys and (2.47) is then an alternative time series representation of the random walk model.

The methodology does not answer the problem of modelling, but it does provide a framework within which the assumptions of the modeller are clearly stated and the models analysed. In most cases it will not be possible to choose between models on these grounds, because different mapping and theory combinations may be observationally equivalent or the same mapping may be associated with a number of models. Therefore:

$$\theta_i^* \varepsilon \theta^*$$

$$\text{s.t } f_\theta^{-1}: \theta \dashrightarrow \theta_i^* \quad \text{for all } i = 1,2, \ldots,n$$

where $n > 1$

Choice of the model will then be based on subjective factors such as the nature of the latent mapping, the signs and dimensions of variables, comparative model performance and the satisfaction of theoretical conditions or correspondences between the data. It may be too strong a condition to expect the data to satisfy theoretical restrictions, but the means of variables or long-run solutions at least should satisfy such conditions. In addition to this the form of the latent mapping may not appear sensible for some models, for example parsimonious forms of (2.33) may not have an interpretation if the trivial mapping is chosen. In conclusion we can re-state the initial proposition of the chapter in terms of latent variables: dynamic data either requires static theory associated with a dynamic latent mapping or a static mapping with dynamic theory.

2.5 Theory Based Econometric Specifications

The methods suggested here are based on the principals of the second section noticing the criticisms linked to the simple theoretical models of the fourth section. Hence, we set up models which following Hendry and Richard(1983) provide a statistically valid partition of the data matrix and, as they are based on theory, should reveal economically meaningful results. The methods used follow the rational or consistent expectations methodology, but do not restrict models to the dynamics of the simplest formulations of the strong rational expectations school. The methods make no assumptions about market clearing or impose strong informational assumptions, where ever possible we test such hypotheses and try to provide a general modelling framework which encompasses associated specifications. The methodology follows current literature in suggesting general models, testing model specification, checking for coherency of models, emphasising dynamics and trying to explain alternative results. It does not suggest single equation methods, except for determining prediction models and it does not place strong reliance on parameter stability and encompassing as model selection criterion. This does not imply that models should perform badly in relation to data based models, but we argue that models specified for analysis, may be allowed to meet weaker statistical criterion, as they are more amenable to direct tests of theoretical principles.

If we start with (2.23) above which partitions the original data into that which is of interest and that which can be omitted:

(2.51) $D(s_t|S_{t-1}\phi_2) = D(y_t|z_t,S_{t-1}\lambda_1)D(z_t|S_{t-1}\lambda_2)$

where $s_t= [y_t,z_t]$ and $S_t= [Y_t,Z_t]$ and z_t, y_t, Z_t, and Y_t are as defined before in sections 2.2 and 2.3

The theoretical model associated with the y's can either be based on the minimisation of a quadratic loss function or the solution of a model with future expectations. If we solve the most general loss function we will derive a model with a number of future exogenous and endogenous variables in it:

$$(2.52) \quad Q(L^{+},L)y_t^e = G^{+}(L^{+},L)z_t^e$$

where $Q(L^{+},L)$ and $G^{+}(L^{+},L)$ are matrix polynomials in the lag operator L and the forward operator L^{+} and s^e is an expectation

In this work we will deal with the first order symmetric case and extend that to include general lags and leads in the exogenous variables, certain forms of non symmetry and strict forms of disaggregation. The first order conditions of the optimisation problem give a rationale to models with future expectations in them, though they do not have to be based on that. If such models are solved for the future values, then the following model will result.

$$(2.53) \quad y_t^e = F\, y_{t-1} + G^{*}(L^{+})z_t^e$$

where L^{+} is the forward operator which does not alter the time subscript associated with the expectations.

The expectations of the exogenous variables are assumed to be generated by the available information which is characterised by st and to maintain the structure of (2.53) we require Y_{t-1} not to Granger cause z_t, so that:

$$(2.54) \quad z_t^e = B_1^{\#}(L)z_{t-1}$$

This model may be related to the data via a mapping between the data and the theoretical model. The trivial mapping will be presumed here, though for this to be a sensible assumption the variables should be manipulated to link them as closely as possible to the theory. In particular the forms of G^{*} and $B^{\#}$ will reflect that. The latent mapping is presented below:

(2.55) $y_t = y_t^e + u_{1t}$ and $z_t = z_t^e + u_{2t}$

where z_t^e and y_t^e are the true expectations of z and y.

We can now construct a general model based on the theoretical propositions implied by the latent mapping (2.55) and the theoretical relationship (2.53) and (2.54). Therefore:

(2.56) $y_t = F\,y_{t-1} + G^*(L^+)B^\#(L)z_{t-1} + u_{1t}$

(2.57) $z_t = B^\#(L)z_{t-1} + u_{2t}$ and $\Omega = \begin{bmatrix} \Omega_{11}\Omega_{12} \\ \Omega_{21}\Omega_{22} \end{bmatrix}$

As Hendry and Richard (1983) point out, it is usual in this literature to conflate the notion of an economic and a mathematical expectation which implies firstly that $z_t^e = z_t^p$ and secondly that the forcing processes of the two series are the same. In general, this will not be the case so that different models for the ys will be observed than those hypothesised by rational expectations. If the two processes are the same and (2.56) includes current values of the zs, then strict exogeneity is not enough for (2.56) to produce efficient estimates. The zs are not weakly exogenous, as λ_1 and λ_2 in (2.51) will depend on the same deep parameters. If the zs are weakly exogenous, then the parameters of (2.56) are invariant to changes in $B^\#(L)$ which means that it can be efficiently estimated on its own. In these circumstances the strict rational expectations hypothesis does not hold. The rational expectations hypothesis is imposed when the optimal predictor is the same as the true expectation, but if that is not the case then the error term will include the difference between the theoretical expectation and the prediction. Therefore:

$$(2.58)\quad y_t = F\, y_{t-1} + G^*(L^+)z_t^p + u_{1t}^*$$

$$\text{and}\quad u_{1t}^* = u_{1t} + G^*(L^+)(B^\#(L)-B(L))z_{t-1}$$

$$\text{where}\quad z_t^p = B(L)z_{t-1}$$

The imposition of rationality may effect the consistency and efficiency of the estimates, as the z^ps may be correlated with the error term.

$$\Longrightarrow \operatorname{plim}((u_{1t}^{*\prime} z_t^p)/N) \neq 0$$

$$\text{as } u_{1t}^* = u_{1t} + G^*(L^+)(z_t^e - z_t^p)$$

This will be a problem if there are considerable differences between the processes driving the expectations and the zs, such as would be the case if the $B^\#$ and B polynomials involved sparse matrices with different zero restrictions. In general one would believe that the parameter differences involved would be small relative to u_{1t}, so that the degree of inconsistency would be small. Alternatively consistency would be satisfied in large samples if the predictors or the expectations tended to rationality:

$$\lim_{t\to\infty} z_t^p \longrightarrow z_t^e$$

$$\text{as}\quad \lim_{t\to\infty} u_{1t}^* \longrightarrow u_{1t}$$

Hence, expectations model with the z_t^e replaced by z_t^p will be consistently estimated using (2.58) when the sample is large. The more information we acquire the better informed we are, so over time we learn the process driving the true expectations. If such an assertion is correct, the process needs to be stable to yield a net gain from new information. The alternative to this view would be that the model of expectations depends on subjective factors, so that we can never capture them perfectly. Hence, the model including predictions will be the best that can be achieved in the short-run:

$$(2.59) \quad y_t = F\, y_{t-1} + G^*(L^+)z_t^p + u_{1t}$$

Equation (2.59) would be the true model, in the sense that it is as close as can be got to the true expectations driven model. In this form estimation would be consistent if the subjective and non-modellable elements of expectations were orthogonal to the exogenous variables. Direct estimation of (2.59) would be fully efficient when it is a legitimate marginalisation of the more general structure (2.22) and the zs are weakly exogenous. If the zs are not weakly exogenous, then we need to take account of the effect of the z process on the parameters in (2.59). We will now outline a method which takes account of such a dependence by replacing the expectations in (2.59) by their actual values; the procedure is illuded to by Sargan(1982) and developed further in chapters four and five.

$$(2.510) \quad y_t = F\, y_{t-1} + G^*(L^+)z_t - D^*u_{2t+1} + u_{1t}$$

where D^* is a complex function of $G^*(.)$ and $B(.)$

This is a hybrid of the other two main methods used in the estimation of rational expectations models, that is the errors in variables method of Wickens(1982) and the substitution method due to Sargent(1978). The errors in variables method estimates (2.52) consistently by initially replacing the expectations with their actual values and then instrumenting these future variables to take account of the endogeneity induced by a forward looking moving average error. The substitution method transforms the solved form (2.53) into an estimable model by replacing the expectations using the process driving the exogenous variables. That is roughly the method described by (2.56), except that we use B(L) rather than $B^{\#}(L)$. The Sargan approach uses the solved form of (2.52) and is in that way linked to the substitution method, but it then replaces the expectations by their actual values which links it to the errors in variables approach. The method should be relatively efficient, as it can take account of any invariance by restricting D^{*} using the exogenous variable parameters.

There are a number of variants of these techniques which have been suggested, in particular the method used by Muellbauer and Winter(1980) and the method of Broze et al(1985). The Muellbauer approach uses the solved model, but eliminates the future expectations in the exogenous variable using a Koyck lead, that can then be estimated by instrumental variables. The Broze technique leaves a relationship similar to the Muellbauer and errors in variables form, but it also takes account of the moving average error term. The Wickens method and the Muellbauer method are convenient in that they use instrumental variables, but they are not efficient, because they do not take account of the moving average error term. The methods of Sargan, Sargent and Broze et al which solve the model are more complex, but they are more efficient. Fully efficient methods either require, simultaneous estimation of the process driving y and z or the weak exogeneity of the zs.

The Sargan form allows comparison amongst a number of different theoretical models and reveals alternatives which remove the strong restrictions implied by the rational expectations approach. In line with the other techniques the underlying model can encompass a range of equilibrium concepts and may be compared with error correction forms. We will show in chapter four that the most parsimonious form of the error correction model may be unstable when the symmetric rational expectations model is the correct model generating the data, as the error correction form may only be correctly specified when the solution to the expectations model is the stable one.

Theory based data generating models firstly have an interpretation which may not be true of data based approaches and secondly allow the data generation process to be tested using theory. A poor model will quite clearly be rejected, as it will either yield poor predictions, meaningless equilibria, or latent roots and parameter estimates which are not consistent with theory. This provides a set of natural criteria to assess the models in addition to testing theoretical restrictions, such as those imposed by rational expectations, homogeneity, durability, habituation or market clearing.

Chapter three

Modelling Expectations in the Labour Market

The reduced form of an econometric model can be given a time series representation and in the case in which series are stationary this time series form can be re-parameterised into a VAR, Vector Moving Average (VMA) and Vector Auto-Regressive Moving Average (VARMA). If the series are stationary in levels the VAR, VMA and VARMA can then be reformulated into an error-correction form which is related to Granger-Engle cointegration when series are non-stationary (this will be discussed in the next section). Series which are not stationary do not have the same correspondence. The VMA exists in terms of the non-stationary series, but the structure is only invertible when the series are cointegrated if the unit roots can be factored out. Yoo(1986) and Engle(1987) explain how this can be done and provide conditions under which simple cointegration structures can be derived from polynomial cointegrated structures. Wickens and Breusch (1988) present an alternative to the cointegration representation which uses the Bewley transformation to directly estimate the long-run parameters of a model. If the series are not stationary this form may still produce consistent estimates of the long- and short-run parameters, but simple attempts to invert the VAR will produce an unstable VMA form.

The quasi reduced form of rational expectations models quite naturally has a VAR form and this result has been used by Sims (1980) to support the multivariate time-series approach to econometric modelling. The problem with the VAR approach is that it does not reveal the structure of the model and because of that it may be inefficient in estimating the parameters of interest. We use a VAR technique augmented by the Hendry and Mizon (1978) general to specific methodology. We attempt to produce models which are congruent in the sense that they satisfy the range of tests consistent with the estimation method and which satisfy the usual parameter stability and prediction criteria. Congruent models should be valid re-parameterisations or marginalisations of the relevant equations of interest solved from the full systems representation of the data generation process. The downward testing approach is supported by the evidence of Granger and Engle (1987) which suggests that in dynamic forms which are close to non-stationarity any reasonable restriction may be preferred to non. The Wickens and Breusch(1988) approach produces a model in levels and differences where the levels parameters are the long-run ones. As the levels term is an order of magnitude different from the parameters on the differences this suggests that a valid downward testing procedure only needs to produce long-run parameters which are not significantly different from those of the general model. If after testing down the long-run parameters are materially different, then the parsimonious representation of the general model has been misspecified and the procedure of eliminating variables should be revised. If the series are stationary correct specification of the dynamics is crucially important, but if they are not then parsimony may be preferred to over-parameterisation. In either case the usual diagnostic tests appear to be valid and the parameters asymptotically normally distributed(see Gourieroux et al(1987) or Wickens and Breusch(1988)).

The technique suggested above is used to produce sub-systems of exogenous variables which are then fed into a model of the endogenous variables estimated using an extension of the method suggested in Sargan (1982). In this chapter we deal with the estimates of the exogenous variable processes and test them to see whether they are correctly formulated. The models are then used to derive parameters of the associated moving average representation, future predictions and one step ahead prediction errors. The models are subjected to a range of tests associated with correct specification which includes tests of serial correlation, heteroscedasticity, Auto-Regressive Conditional Heteroscedasticity (ARCH), functional form, predictive failure, normality and parameter stability. Parameter stability is checked for by Chow-tests and the analysis of the models recursive residuals. We test for Granger-Causality to check whether past output and employment affect the exogenous variable process; the test will indicate whether our marginalisation is correct. The long-run parameters of the specific model are compared with the general model to see if they are consistent with each other and the short-run parameter restrictions are tested using an F-criterion.

We present models of output prices, vacancies, wages and inventory accumulation which are fed into an output employment system. The models are set up as VARs in levels which are then estimated equation by equation using OLS. A general equation is formulated for each variable which is then reduced and further marginalised to produce a parsimonious form which imposes zero and difference restrictions. The final representation is then validated using the tests mentioned above. While the commonly used tests for serial correlation, heteroscedasticity, normality and functional form are generally satisfied at both the 5% and 1% level, the tests of parameter constancy and predictive failure are only satisfied coherently at the 1% level. The predictive failure tests are run over the period 1980q1 to 1980q4, but in 1980q4 there was an enormous de-stocking of inventories which was not repeated in subsequent years. Hence, the predictive failure test is heavily influenced by that event which may explain the poor performance of the inventory model outside the period when compared with the period of estimation. The price and wage models satisfy most of the test, except for the CUSUMSQ test which fails at the 5% level, but recent evidence by Kramer, Plogberger and Alt (1988) suggests that in the case of dynamic regression the CUSUM test has reasonable power and is to be preferred to alternative dynamic tests. A further problem arises when we try to invert the VAR parameters into those for the equivalent VMA representation, as the inversion leads to increasing VMA parameters. The problem of inversion and the fact that predictions grow over time is evidence of non-stationarity, though the predictions do not increase explosively. In estimating the endogenous variables we deal with the non-stationarity by truncating the MA parameters, by not imposing the restriction associated with those parameters and also some efficiency gains could be made by inverting the model into a quasi VARMA which then includes elements in non-inverted auto-regressive difference terms. The cointegration approach is discussed here, as the non-invertibility may indicate that some of the variables grouped in the sub-models may be stationary in combination. The VAR procedure may not impose the correct restrictions or in finite sample there may be sufficient bias to produce estimates which do not invert to a stable VMA. Yoo(1986) and Engle(1987) discuss the decomposition of singular matrices and they show that a model with a cointegration form may be given a VAR representation. The non-explosive nature of the wage and price series is not inconsistent with cointegration, but the method of estimation may not be able to select precisely the appropriate parameters which can be used to invert the VAR into a stationary VMA under cointegration.

At the end of the chapter we discuss the cointegration technique which is well suited to the Sargan procedure, but we feel that we do not have adequate selection criteria to determine whether the series are cointegrated or not and which forms are correct. First step models of the variables used in this chapter seem to satisfy the Sargan Bhargava and Dickey Fuller tests, but the second step estimates do not have significant correction terms and the first step estimates do not seem to satisfy the super-consistency results of Stock(1987). If super-consistency is satisfied, then you would expect the recursive residuals and the recursive parameters of such models to indicate stability. If the series are cointegrated then the OLS residuals of the first step estimates will not be normal, but one would suspect that the absolute values of statistics such as the CUSUM and CUSUMSQ would indicate a high degree of stability when the super-consistency result holds.

3.1 Cointegration and Rational Expectations Systems.

Rational Expectations models have a number of equivalent time-series representation of which the error correction model is one. In this section we will see that it is always possible to specify the system in the form of a vector moving average(VMA), an error correction and a vector autoregressive moving average(VARMA), but a stationary vector autoregression (VAR) can only be derived using the Yoo(1986) procedure to invert polynomials with unit roots. If the system in levels is cointegrated, then the VAR in differences will be misspecified.

We can represent the joint process driving the endogenous and exogenous variables, as a VARMA model:

$$(3.11) \quad A^{+}(L)s_t = D(L)e_t$$

where $s_t'=[y_t' : x_t']$, $e_t'=[u_t' : \varepsilon_t']$ and $e_t \sim WN(0\ ,\ \Sigma\ )$

If the roots of $A^{+}(L)$ lie outside the unit circle, then (3.11) can be inverted to derive a Wold moving average representation, alternatively if the roots of D(L) are outside the unit circle then we can transform the model into a Vector AutoRegressive(VAR) form. Sargent(1978) and Kollintzas(1985) deal with the solution to quasi-symmetric rational expectations models((in the next Chapter we cover the regular solution to the first order condition). The regular solution has a pair of symmetric roots, the stable one is fed back and the unstable one forwards to reveal the forward solution to a rational expectations model:

$$(3.12) \quad F_{11}(L)\ y_t = E(F_{12}(L^{-1})x_t | \Omega_{t-1}) + u_t^{*}$$

$$F_{22}(L)\ x_t = \varepsilon^*_t$$

Where u^*_t and ε^*_t are everything left out of the model

The substitution method solves out for the future expectations using a variant of the Wiener-Kolmogorov prediction formula solved for future predictions: prediction formula:

$$E(x_{t+i}|\Omega_{t-1}) = F_{22}(L)_i x_{t-1}$$

$$\text{and } F_{22}(L)_i = F_{22i}[0] + F_{22i}[1]L + F_{22i}[2]L^2 \ldots$$

Substituting out for the expectations in (3.12) using this formula implies that:

$$F_{11}(L)y_t = F^*_{12}(L)x_{t-1} + u^*_t$$

$$\text{where } F^*_{12}(L) = F^*_{12}[0] + F^*_{12}[1]L + F^*_{12}[2]L^2 + \ldots$$

$$\text{and } F^*_{12}[j] = F_{12}[j]\ F_{22}(L)_j$$

The system can then be represented as a VAR which will be stable if the factorisation holds and if $F_{22}(L)$ has all it's roots inside the unit circle:

$$(3.13)\quad F^*(L)s_t = e^*_t$$

$$\text{where } F^*(L) = \left[\begin{array}{c|c} F_{11}(L) & F^*_{12}(L) \\ \hline 0 & F_{22}(L) \end{array}\right]$$

The VAR technique relates directly back to the original rational expectations solution presented in Muth(1961) where the prediction formula is used to provide an optimal solution to the hog-cycle model. In the literature different solutions have been generated by the different time series representations of the exogenous variables; Sims(1980) emphasises the use of the VAR technique for modelling macro variables with expectations. Estimation of the VAR representation has been favoured, because of the difficulty of statistically identifying multivariate time series models with a moving average component and the ease with which it can be estimated in large samples by the direct application of standard regression techniques. Ordinary least squares is not efficient, because it does not take account of cross equation restrictions or the effect of the lagged endogenous variables on the likelihood, but it is easy to implement and it should produce consistent estimates. If (3.11) is the true model, then a finite VAR form will only be well specified when $F^*(L) = D(L)^{-1}A(L)$ and $F^*(L)$ has a VARMA form.

Identification in the time series context relates to the selection of the structure of the model and the degree of differencing required to make the process stationary (see Granger and Newbold(1986) for an explanation of these issues). Granger and Newbold(1974) have presented evidence that most economic time series are close to random walks which suggests that s_t will be I(1) (integrated of order one), so that the data should be differenced once to produce a stationary formulation of (3.13) above. Therefore:

(3.14) $A^x(L)\Delta s_t = \epsilon_t^+$

where $A^x(L) = (A^x(0) + A^x(1)L + A^x(2)L \ldots)$ has all it's roots outside the unit circle.

Although (3.14) may be statistically valid in economic terms it is rather limited. In their Demand for Money Study, Hendry and Mizon(1978) show that such differenced series do not produce the usual static equilibrium representation of the transactions demand for money and that implies that desired equilibrium may be changed by an infinite amount. Differenced models are problematic, because the growth model suggested by differencing is consistent with a number of theoretical forms and the difference operator may impose restrictions which are not appropriate for the data. Kollintzas(1985) interprets differences in stock variables, as flow movements which may be reasonable in terms of quantities, but does not seem sensible for price series. Certainly the degree of variability in most differenced series seems to be excessive for a pure flow interpretation and models which incorporate prices and quantities will not have a pure static equilibrium solution.

Granger(1983), Granger and Weiss(1983) and Engle and Granger(1987) have suggested an alternative to this approach which satisfies the above criticisms. Cointegration generalises the error correction model of Davidson et al(1978) and Hendry and Mizon(1978) to include higher order correction and different forms of dependency. Cointegration implies that a vector of non-stationary or I(1) variables may become stationary in combination, though each individually would have a univariate moving average and autoregressive representation in first differences. The procedure has the advantage of providing a dynamic model which has a long-run solution related to economic theory. If the data are Cointegrated and (3.12) is the true model, then D(L) will not be directly invertible which means that the VAR in levels will at best be explosive or may not exist and the VAR in differences will have some unit roots in the error term due to over-differencing. Here, we will show that the VMA form is to be preferred for deriving the solution to the system, because the moving average form always exists, but it is not directly invertible when we have cointegration:

(3.15) $\Delta s_t = C(L)\,\epsilon_t$

where $C(z)$ has all its roots on or outside the unit circle, $C(0)=I$, $C(L)= (C(1)+(1-L)C^*(L)$, $rk(C(1))=g-r$ with $0 < r < g$ and ϵ_t is a zero mean white noise innovation.

If we follow Granger and Engle(1987) then (3.15) may be re-parameterised into a VARMA form by inverting the stable part of C(L) which is non-singular. That involves first factoring out the determinant of C(L) which includes r unit roots and then multiplying each side by the adjoint:

(3.16) $A^*(L)s_t = d(L)\epsilon_t$

where $Adj(C(L)) = (1-L)^{r-1}A^*(L)$ and $det(C(L)) = (1-L)^r d(L)$ $d(L)$ is a scaler polynomial and $A^*(0) = I$

The singularity of C(1) is the cointegration assumption, as if that matrix is of full rank then (3.12) can be inverted into a VAR in first differences. We can use the rest of the Granger representation theorem to transform (3.16) into error correction form, given the partition of the polynomial $A^*(L) = A^*(1)L + (1-L)A(L)$.

(3.17) $A(L)\Delta s_t = A^*(1)s_{t-1} + d(L)\epsilon_t$

A vector of cointegrating variables η_t will exist when there are left and right side annihilators of $C(1)\ \delta$ and α' both of rank r and α' produces the vector of cointegrated variables $\eta_t = \alpha' s_t$. If $A^*(1) = \delta\alpha'$ then the error correction can be written in cointegrating form:

$$(3.18) \quad A(L)\Delta s_t = \delta\eta_{t-1} + d(L)\epsilon_t$$

Notice that the s_t series will be I(0) when $A^*(1)$ is of full rank, as the rank of C(1) is zero, so that the unit root can be eliminated. The cointegration form is to be preferred for modelling, because it encompasses both the simple error correction and the VAR. When d(L)=I in equation (3.18) all the roots of C(1) are zero, the data are stationary and both the simple error correction form and the VAR representation are equivalent to the cointegration form. If in addition to the previous condition $A^*(1) = 0$, then C(1) is of full rank and the levels term disappears from the error correction and the VAR in differences is equivalent to equation (3.18). If C(1) and $A^*(1)$ are of less than full rank, then the cointegration form produces a stationary time-series representation of the data and the error correction and VAR forms are misspecified. In those circumstances the error correction model assumes that d(L) = I which means that techniques which do not take account of the moving average error term may be inconsistent as well as inefficient. The VAR in differences will over difference, because it does not take account of the singularity.

When the individual series are non-stationary the Granger-Engle representation theorem either requires equation (3.18) to be estimated with the moving average error or by a consistent method which accounts for the endogeneity introduced by such an error term. The Granger-Engle representation is a final form which is over-parameterised relative to the VMA representation (3.15), as $A^*(L)$ clearly has many more parameters, than $C(L)$.

In the simple case in which r=1 the Engle-Granger two step method can be used, but it is not always valid with r>1. At this point it is worthwhile looking at the solution proposed by Yoo(1986) which entails factoring (3.15) using the Smith-Mcmillan form:

$$(3.19) \quad C(L) = J(L)M(L)V(L)$$

$$M(L) = \begin{bmatrix} I_{g-r} & 0 \\ 0 & \Delta I_r \end{bmatrix}$$ and all the roots of $J(L)$

and $V(L)$ lie outside the unit circle.

It is easy to see that we can invert $V(L)$ and $J(L)$, but $M(L)$ when inverted introduces an infinite lag with unit roots. Rewriting (3.15) using the result above and inverting $J(L)$ we produce the following VARMA:

$$(3.20) \quad J(L)^{-1}\Delta s_t = M(L)V(L)\epsilon_t$$

At this point Yoo explains how a non-invertible VMA can be inverted using the common factor (1-L) on the lhs and rhs of (3.20). Hence (3.20) becomes:

$$(3.21) \quad M^*(L)J(L)^{-1}s_t = V(L)\epsilon_t$$

$$M^*(L) = \begin{bmatrix} \Delta I_{g-r} & 0 \\ 0 & I_r \end{bmatrix}$$

We now have a VAR in levels and differences and when $V(L)$ is block diagonal this will be partitioned in such a way that the first g-r elements are stationary equations in differences and the final r elements are stationary equations in levels. In this form the first equations are VARMA forms in differences and the second group of equations, VARMAs in levels. We can now re-formulate (3.21) into a VAR form, as $V(L)$ can be inverted:

$$(3.22) \qquad B(L)s_t = \epsilon_t$$

$$\text{Where } B(L) = V(L)^{-1} M^*(L) J(L)^{-1}$$

Notice, that (3.22) can only be partitioned into equations in differences and levels when $V(L)$ is block diagonal. We can now derive a stable error correction representations when we set $M^*(L) = \Delta L + M^+(L)$, so that:

$$(3.23) \quad V(L)^{-1}J(L)^{-1}\Delta s_t + V(L)^{-1}M^+(L)J(L)^{-1}s_t = \epsilon_t$$

where

$$M^+(L) = \begin{bmatrix} 0 & 0 \\ 0 & LI_r \end{bmatrix}$$

Partitioning $V(L)^{-1}$ into $[V_1(L)^{-1}: \tau(L)]$ and $J(L)^{-1}$ into $[J_1(L)^{-1}: \alpha(L)']$ appropriately dimensioned submatrices we can produce an error correction representation of (3.22):

$$(3.24) \quad B^*(L)\Delta s_t = -\tau(L)\alpha(L)'s_t + \epsilon_t$$

$$\text{where } B^*(L) = R(L)^{-1}J(L)^{-1}$$

Equation (3.24) produces a model with a polynomial in the error correction term which is only equivalent to the simple error correction form (3.18) when $B(L) = (1-L)(B^*(L) + \tau^*(L)\alpha^*(L)') +$

$\tau(1)\alpha(1)'$. When the Yoo form of polynomial cointegration is correct, the imposition of (3.18) on the data may cause B(L) to have roots inside the unit circle, this will not affect the results when $\tau(1)\alpha(1)'$ is enough to stabilise the model. Engle and Granger(1987) show for a simple bivariate case that the Smith Mcmillan form does produce the usual error correction representation without the scaler polynomial d(L) on the error.

The disadvantage of the Yoo form is that a simple error correction does not always take account of the non-stationarity, when the roots $B^*(L)$ are all outside the unit circle we may have the less common case of polynomial cointegration. An alternative form can be derived which takes account of the non-stationarity, guarantees the existence of the simple error correction form and represents the long-run parameters directly; this depends on the decomposition of a model similar to (3.20) above. If we assume that $R(L) = J(L)^{-1}$ and $D(L) = M(L)V(L)$ are finite matrix polynomials of degree p and q, where D(L) has r unit roots and D(1) is of rank g-r(though rk(D(1)) will usually be greater than g-r). Then the spectral decomposition of D(L) can be inverted in the following way:

$$D^*(z) = z^q D(1/z)$$

The companion form of $D^*(.)$ can then be written with associated canonical form (see Sargan(1983)):

$$\begin{bmatrix} 0 & I & . & . & . & 0 \\ 0 & 0 & I & . & . & 0 \\ . & . & . & . & . & . \\ D_o^* & D_1^* & . & . & . & D_{q-1}^* \end{bmatrix} P^+ = \begin{bmatrix} I & . & . & . & 0 \\ 0 & I & . & . & 0 \\ . & . & . & . & . \\ 0 & . & . & . & D_q^* \end{bmatrix} P^+\nabla$$

$$\nabla = \left[\begin{array}{c:c} \nabla_1 & 0 \\ \hdashline 0 & I_r \end{array}\right]$$ where all the roots of ∇_1 are within the unit circle

Let us extract a factor from the matrix D(L) which corresponds to the factor with the r unit roots and g-r zero roots. Therefore:

$$(3.25)\quad D_o(z) = H^{-1}\begin{bmatrix} (z-1)I_r & 0 \\ 0 & zI_{g-r} \end{bmatrix} H$$

As was stated above the restriction that D(1) is of rank g-r may be too strong, so that in practice we will be dealing with two possibilities. When $D^*(L)$ has g-r zero roots we will find that $D^*(z) = D_o(z)D_1(z)$ and $D_1(z)$ is of degree q-1. But when we do not have enough zero roots, then we have to add g extra zero roots to the polynomial. We can do this by considering $zD^*(z)$, because $|zD^*(z)| = z^g|D^*(z)|$ and that introduces the g null roots. We now have that $D^*(z) = D_o(z)D_1(z)$ with $D_1(z)$ of degree q and $D_o(z)$ is defined in (3.25) above. We can now use this to factorise D(L):

$$(3.26)\quad R(L)\Delta s_t = D_o^*(L)D_1^*(L)\epsilon_t$$

where $D_1^*(L) = L^{q-1}D_1(L^{-1})$ in the first instance,

in the second $D_1^*(L) = L^qD_1(L^{-1})$ and $D_o^*(L) = LD_o(L^{-1})$

If we now use the Yoo method of transforming non-stationary models (3.26) becomes:

$$D_o^*(L)^{-1}R(L)\Delta s_t = D_1^*(L)\epsilon_t$$

$$(3.27)\quad H^{-1}\begin{bmatrix}(1-L)^{-1}I_r & 0\\ 0 & I_{g-r}\end{bmatrix} H\,R(L)\Delta s_t = D_1(L)\epsilon_t$$

If we use the transformation associated with Yoo, then we get a VARMA representation in differences and levels:

$$(3.28)\quad H^{-1}\begin{bmatrix}I_r & 0\\ 0 & \Delta I_{g-r}\end{bmatrix} H\,R(L)s_t = D_1(L)\epsilon_t$$

$$(3.29)\quad \begin{bmatrix}I_r & 0\\ 0 & \Delta I_{g-r}\end{bmatrix} H\,R(L)s_t = HD_1(L)\epsilon_t$$

The advantage of this model is that the polynomial matrixes are of the same dimension as those in the original representation. Equations (3.28) or (3.29) also can be given an error correction representation which can be transformed to produce the long-run parameters directly. Firstly, we will re-define the matrix polynomial as $H_1R(L) = G(L)$ and then the vector of r cointegrated variables associated with the original r unit roots will be G(1) as $\eta_t = G(1)s_t$. We can now re-write (3.29) in the following way by partitioning $H' = [H_1' : H_2']$, so that we clearly have a model in differences and levels:

$$G(L)s_t = H_1D_1^*(L)\epsilon_t$$

$$H_2R(L)\Delta s_t = H_2D_1^*(L)\epsilon_t$$

Now if we use the usual factorisation and the cointegration form we can represent the first relationship in terms of cointegrated variables:

$$((1-L)G^*(L) + LG(1))s_t = H_1\ D_1^*(L)\ \epsilon_t$$

$$(3.30)\quad G^*(L)\Delta s_t + \eta_{t-1} = H_1\ D_1^*(L)\ \epsilon_t$$

In general we have cointegrated systems in which $r > 1$, which means that the two step method is not valid and that it is unlikely that we will find a version of equation (3.24) for which $\tau(L)\alpha(L)' = \tau\alpha'$. In such circumstances, it is not clear if there is any advantage in using the Yoo approach or any sense in using the two step method of Granger and Engle. On these terms Equation (3.30) has the advantage of producing the long-run parameters in one step and of having the same order lag structure as the VARMA representation. Obviously the method is more complex, due to the VMA error, but that is not likely to be much more difficult than estimating equation (3.16) directly. In terms of the usual time-series data sets it seems likely that cointegration effects, if they exist, are likely to be complex and as a result of this efficient methods are likely to need to take account of the error structure. Finite sample experiments by Hendry et al(1988) seems to suggest that the super consistency result of Stock is not operative on typical sample sizes, which implies that the accuracy of long-run parameter estimates will be significantly improved by efficiently estimating the short-run dynamics.

Equation (3.30) is similar to the Bewley representation of the error correction model in Wickens and Breusch(1988) which is related to the cointegration representation of Engle and Granger(1987) and Yoo(1986). Wickens and Breusch(1988) reformulate the standard error correction form in differences and levels into a model in levels and differences using similar arguments to those which support the notion of cointegration. The advantage of this method which uses the Bewley transformation is that the long-run parameters are computed directly and that the method does not require the super-consistency result to produce efficient estimates. If we take the cointegration model in VAR form then:

(3.31) $B(L)s_t = \epsilon_t$

Where $B(L) = A^*(L)$ when the s_t series are stationary in levels, otherwise (3.31) is the same as (3.22)

In principle, Wickens and Breusch deal with the trivial cointegration case in which the s variables are stationary and $r = n$. Let there exist:

$\Gamma^{\#} = [\gamma_{ij}]$

where $\gamma_{ij} = 1$ for all $i = j$ and γ_{ij} are long-run parameters when $i \neq j$.

Then :

$\Gamma^{\#} = \nabla^{\#}B(1)$

where $B(1) = I + B[1] + B[2] \ldots + B[j]$ and $\nabla^{\#} = (\mathrm{Diag}(B(1))^{-1}$.

Then we can re-write the VAR form in error correction form using the usual factorisation using $B^{+}(L) = [B(1)L + (1 - L)B^{+}(L))$, so that:

$B^{+}(L)\Delta s_t + B(1)s_{t-1} = \epsilon_t$

Pre-multiplying each side by $\nabla^{\#}$ gives:

$\nabla^{\#}B^{+}(L)\Delta s_t + \Gamma^{\#}s_{t-1} + \epsilon^{\#}_t$

$$\Delta s_t + (\nabla^{\#} - I)\Delta s_t + \Gamma^{\#} s_{t-1} + B^{+}[1]\Delta s_{t-1} + B^{+}[2]\Delta s_{t-2} \ldots = \epsilon^{\#}_t$$

$$s_t + (\nabla^{\#} - I)\Delta s_t + \Gamma^{*} s_{t-1} + B^{+}[1]\Delta s_{t-1} + B^{+}[2]\Delta s_{t-2} \ldots = \epsilon^{\#}_t$$

where $\epsilon_t = \nabla^{\#}\epsilon_t$ and $\Gamma^{*} = (\Gamma^{\#} - I)$

The long-run parameters of the model may be directly estimated from Γ^* using instrumental variables. When the data are stationary in levels the model is a re-parameterisation of an error correction model and as the form is linear it allows all of the parameters to be estimated directly in one step. If the data are Cointegrated of order 1, then a problem may arise as the stationary representation can have a VMA error or polynomial terms in the cointegrating variables. The models dealt with so far would suggest, that the Wickens Breusch form may only be efficiently estimated when the data satisfy the super consistency result. Otherwise, the non-trivial cointegration case involves $\tau(L)\alpha(L)' \neq \tau\alpha'$ which firstly suggests that $B^*(L)$ may have roots on or inside the unit circle and secondly that the long-run parameters are likely not to be accurately estimated. The non-stationary form can be estimated consistently and it seems likely to produce Gaussian errors though the estimates may be inefficient, because of the difficulty in determining the lag length. The model is a re-parameterisation of the Yoo form which has a longer and more complex lag structure than an equivalent parameterisation of equation (3.30). Secondly, as the lag structure is not known a priori it is easy to either produce biased estimates by under parameterising it or inefficient estimates by over parameterising it. It is clear that the techniques which do not impose the cointegration constraints are at best likely to be inefficient, while the efficient methods are likely to be difficult to implement. In addition to this it has been the Authors experience that the two step method and the VAR approach are unsatisfactory in the context of quarterly aggregate time-series data. In practice the VAR method has been used as simple alternatives do not exist, but our analysis has been limited by the difficulty of finding appropriate estimates and non-explosive predictions.

Sims Stock and Watson(1986) show that the VAR is asymptotically efficient and we have seen that VARs can be given a number of different representations. The forms that the VAR takes depend on the degree of integration and cointegration of the series, but independent of such considerations the VAR in levels can always be estimated. In finite samples such estimates may be inefficient, as they do not impose the appropriate restrictions and they may tend towards any asymptotic limits more slowly than equivalent formulations derived using FIML or the two step method of Engle and Granger(1987). We have used a general parameterisation to formulate our models, but that has led to models which are not stable in levels. As can be seen from the structure of equation (3.22), that may be due to our inability to impose the restrictions due to cointegration. In terms of estimation there is no problem with estimating such models and Wickens and Breusch(1988) present strong reasons why the usual asymptotic normality result will hold for the parameters which suggests that inference is not affected by the non-stationarity.

3.2 VAR Estimates of models of Inventory accumulation, Vacancies, Output Prices and Manufacturing Wages.

The system has been partitioned into exogenous and endogenous variables. In this section the exogenous variables are modelled using the vector autoregressive representation (3.19) in levels for wages, prices, vacancies and inventory accumulation. Initially a general model is estimated with nine lags on wages, prices and inventories, five lags on vacancies and four seasonals including the constant. The downward testing or general to specific methodology is then followed to produce a parsimonious representation of the data. The models are then validated using the range of tests advocated by Hendry and Richard(1983) and others to determine whether these single equations models are well specified. The problem of endogeneity is addressed by firstly determining whether the partition is acceptable using a variant of the Granger causality test and then we check for invariance by looking at the stability of both the equation variance and the parameters over the sample.

The relationships are re-parameterisations of the original structure which implies that they do not necessarily satisfy theoretical restrictions. If the strong convergence result implicit in the dependence between cointegrated variables is imposed on the model, then the long-run parameters should be theory consistent even if the short-run ones do not satisfy theory. If the series are I(0) or differenced to stationarity, then the long-run parameters may not satisfy theory, due to the re-parameterisation. The evidence in Breusch and Wickens would suggest that the parameters in levels are more influential, because of the difference in the order of magnitude of the series. If the level and difference form is estimated, then the long-run results are computed directly from the levels terms which may or may not satisfy theory automatically.

TABLE 3.1 Ordinary Least Squares Estimates of the Inventory Accumulation Equation using 81 observations for the period from 59Q4 to 79Q4

Regressor	Coefficient	Standard Error	T-Ratio
C	9.4280	.5843	16.1356
S1	.0901	.0261	3.4563
S2	.1164	.0248	4.6907
S3	.0551	.0294	1.8760
$\Delta i(-3)$	-.2290	.0764	-2.9963
$\Delta_2 \Delta\Delta i(-5)$	.1900	.0482	3.9381
$\Delta v_4 p(-1)$	.5136	.0557	9.2277
$\Delta p(-1)$	1.4194	.6085	2.3326
$\Delta\Delta p(-6)$	-3.0529	.9946	-3.0696
$\Delta_2 w(-1)$	-1.4297	.3982	-3.5909
$\Delta_2 \Delta w(-6)$	2.2876	.5124	4.4643

Standard Diagnostics

R-Squared	.7448	F-statistic $F(10, 70)$	20.4293
R-Bar-Squared	.7083	S.E. of Regression	.0712
R.S.S.[1]	.3553	Mean of Δi	7.6935
$\sigma_{\Delta i}$	.1319	Log-likelihood	104.9536
DW-statistic	2.1198		

1 Residual Sum of Squares

The inventory model either has a steady state inventory accumulation or a disequilibrium interpretation[1]. The results reveal an inventory accumulation model in first differences which means that the static equilibrium solution does not exist. If we treat inventory accumulation as a disequilibrium phenomena, then this would imply a long-run state in which the goods market does not clear, otherwise we have a fixed level of inventory accumulation. A long-run inventory demand relationship is not indicated by the results which is not surprising given that the tax and interest rate effects relevant for such a levels relationship are not included. The steady state relationship is derived by setting $\Delta x_t = \pi$ for real variables, $\Delta p_t = (1+\dot{p})$ for nominal variables and if we interpret the dependent variable in terms of excess demand then:

$$(q^s - q^d) = 7.671 + seas - \pi - 1.1716(1+\dot{w})$$

In the main, the non-smooth nature of the data seems to be more consistent with a disequilibrium story, than the inventory accumulation one. In theory in the long-run such disequilibria should disappear, but in the medium term the economy may attain a stable adjustment path or traverse as Hicks(1974) calls it. The actual data may be more strongly related to the traverse path, than the full steady state growth path. In terms of such a medium term interpretation the excess demand relationship is positively affected by growth and positively affected by wage inflation.
If such an interpretation is not valid, then we could say that the relationship explains desired inventory investment which depends negatively on growth and negatively on wage inflation. If the inflation effect is dissected it is found that the demand for

inventories are positively affected by price inflation and negatively affected by wage inflation. The positive price effect can be seen to be related to the capital gain due to stock holding and the negative effect of wages is due to the cost of holding stocks. The cost element seems to dominate or there is a non-homogeneity in terms of the real wage inflation effect which is not surprising. Hendry and Ericsson(1984) explain that the short-run price relationship may not be of the same form as the variable with which you associate it, that means that the imposition of a real effect in the short-run is not valid. The long-run results are consistent with the results of many studies in which the relationship is not real in the short-run.

TABLE 3.2: Additional Diagnostic Tests[2]

Test Statistics	LM Version		F Version	
A:Serial Correlation	CHI-SQ(4)=	2.1336	F(4, 66)=	.4464
:Serial Correlation	CHI-SQ(8)=	4.7344	F(8, 62)=	.4811
B:Functional Form	CHI-SQ(1)=	.5586	F(1, 69)=	.4792
C:Normality	CHI-SQ(2)=	2.6128	Not applicable	
D:Heteroscedasticity	CHI-SQ(1)=	.6754	F(1, 79)=	.6643
E:Predictive Failure	CHI-SQ(5)=	66.8621	F(5, 70)=	13.3724
F:Predictive Failure	CHI-SQ(5)=	82.3233	Not applicable	
G:A.R.C.H.	CHI-SQ(8)=	7.6027	Not applicable	
H:Test of Restrictions	CHI-SQ(24)=	18.3596	F(24, 44)=	.4667

In statistical terms the model performs well within period, it satisfies all of the specification tests at the 5% level except for the predictive failure test which it does not come close to satisfying. The results for the prediction period are strongly affected by the shake out of stocks in the fourth quarter of 1980. In part this may be seen as an aberration, due to factors particular to the time. Alternatively, the results may be particular to the small sub-sample used for the prediction period, the test which is the second one described by Chow(1960) also compares the models using overlapping periods rather than the usual Chow test which splits the sample in two (see Pesaran and Pesaran(1987)). It is possible that 1980q4 is an outlier which cannot be modelled and that idea is consistent with the results of the multiple Chow tests presented later. The general to specific methodology used to produce the parsimonious results presented in table 3.1 is validated by the final test H which determines whether the zero and difference restriction are satisfied. The general model contained four seasonal dummies including the constant, nine lags on inventories, prices and wages and five lags on vacancies. The parsimonious from if we exclude the dummies includes eight parameters which implies twenty four restrictions in testing down to the relationship presented in table 3.1.

Table 3.3 below is based on a variable addition test which determines whether output and employment are relevant to the inventory accumulation model. The test is not significant, but the 4th and 9th lags on income were individually significant. The test is constructed in the spirit of a Granger Causality test if the lags omitted from the stock model are truly insignificant.

TABLE 3.3 Granger Causality Test and Prediction of Inventory Model

Variable deletion test for the omission of output and employment from the inventory accumulation equation.

Lagrange Multiplier test statistic	CHI-SQ(18)= 21.3043
Likelihood Ratio test statistic	CHI-SQ(18)= 24.7203
F statistic	F(18, 52) = 1.0310

Dynamic Forecasts

Observation	Actual	Prediction	Error
80Q1	7.4146	7.5235	-.1089
80Q2	7.6756	7.6541	.0214
80Q3	7.4639	7.4897	-.0258
80Q4	6.7867	7.3658	-.5791
81Q1	7.1470	7.4390	-.2650

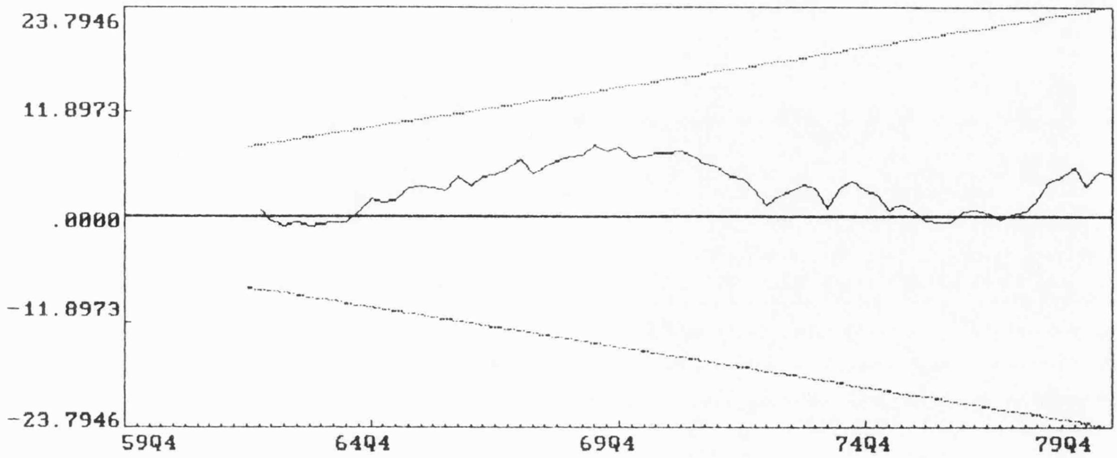
Summary statistics for static forecasts

Based on 4 observations from 80Q1 to 80Q4

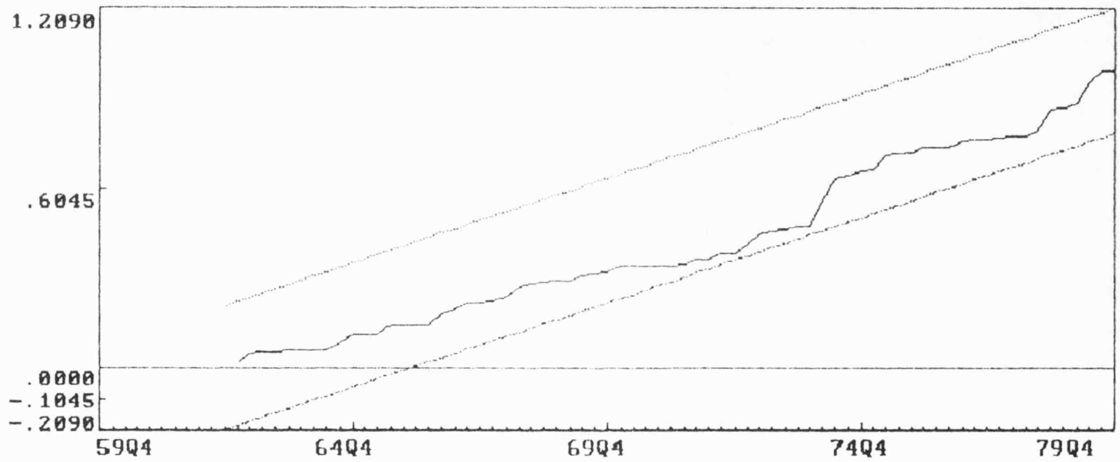
Mean Prediction Errors	-.1915	Mean Sum Abs Pred Errors	.2000
Sum Squares Pred Errors	.0837	Root Mean Sumsq Pred Errors	.0110

Multi-period chow-tests and variances are presented below and CUSUM and CUSUMSQ tests based on recursive residuals are given on the next page:

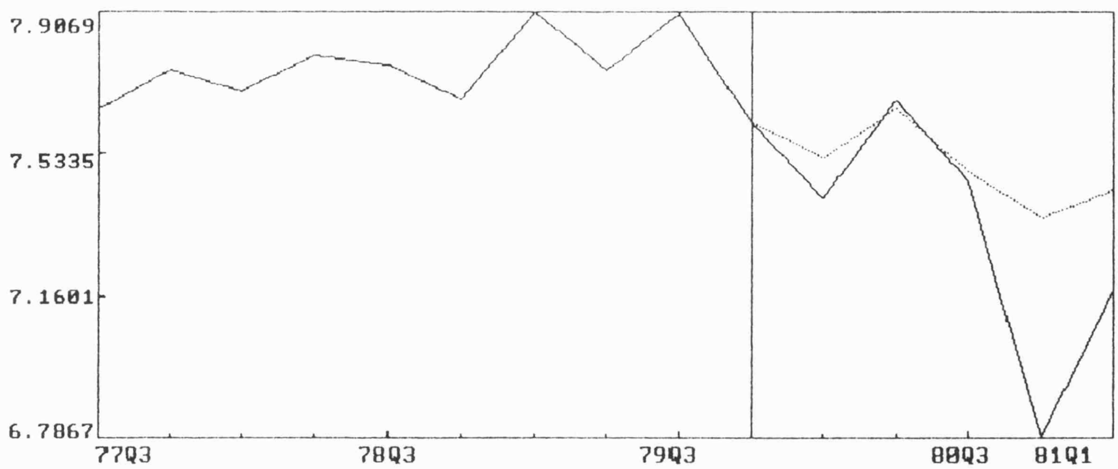
Figure 3.1 Diagnostic Graphs for Inventory Accumulation Equation



CUSUM with 5% confidence bands



CUSUMSQ with 5% confidence bands



Forecasts for five periods ahead

End Period	Chow Test		Standard Error
64Q3	CHI-SQ(11)= 7.2873	F(11,59)= .6625	.0582
67Q2	CHI-SQ(11)= 8.8016	F(11,59)= .8001	.0621
70Q2	CHI-SQ(11)= 5.7552	F(11,59)= .5232	.0615
72Q1	CHI-SQ(11)= 7.0567	F(11,59)= .6415	.0591
73Q3	CHI-SQ(11)=11.1303	F(11,59)=1.0118	.0613
76Q1	CHI-SQ(11)=13.5883	F(11,59)=1.2353	.0693

The cumulative sum of squares and cumulative sum of squares squared test support the regression standard error estimates and the repeated Chow-tests which seem to imply that the regression is invariant. If this is true, then the model represents a sequential cut of the parameter space. This implies that the partition of the parameter space associated with the elimination of all other variables is not invalid and the result supports the pseudo Granger causality test specified above. Outside the estimation period the predictive failure test provides evidence against this proposition, but the shake out in 1980 can be viewed in two ways: either as an effect which no model could predict or as a true indication of a structural break following the change in Government policy.

In moving from a general specification in levels, it has been appropriate to transform the vacancies model into a parsimonious form in differences and levels. The relationship is fairly well specified with levels terms which are significant so that the model has long-run static and dynamic steady state solutions.

TABLE 3.4 Ordinary Least Squares Estimates of the Total Vacancies Equation using 81 observations for the period from 59Q4 to 79Q4

Regressor	Coefficient	Standard Error	T-Ratio
C	3.4102	.8999	3.7894
S1	.0525	.0226	2.3248
S2	.0689	.0257	2.6852
S3	.0120	.0238	.5050
$(1+L^2)\Delta i(-1)$	-.0977	.0485	-2.0155
$\Delta\Delta i(-4)$	-.1304	.0618	-2.1099
$\Delta i(-7)$	-.1687	.0679	-2.4853
$\Delta v(-1)$	.6047	.0947	6.3847
$\Delta v(-2)$	.2276	.1055	2.1582
$(1+L^4)\Delta w(-1)$	-.8007	.2715	-2.9497
$\Delta w(-8)$	-1.0687	.5086	-2.1013
$(v-w+p)(-1)$	-.1064	.0292	-3.6399

Standard Diagnostics

R-Squared	.7218	F-statistic F(12, 68)	16.2433
R-Bar-Squared	.6776	S.E. of Regression	.0599
R.S.S.	.2480	Mean of Δv	.0038242
$\sigma_{\Delta v}$	.1056	Log-likelihood	119.5184
DW-statistic	2.0485		

If we set $x_t = x_{t-1} = x$, $\Delta x_t = 0$ and $\Delta i = (q^s - q^d)$, then the long-run static equilibrium solution is given below:

$$v = 32.051 + seas + 3.422(qd - q^s) + w-p$$

The static relationship states that vacancies or excess demand in the labour market depends on excess demand for goods and real wages. The positive link between vacancies and real wages does not make sense if vacancies are explained in terms of excess demand, but it does seem more reasonable if we are looking at desired vacancies. As firms labour demand grows we observe a higher level of real wages associated with a shift of the demand curve along a relatively fixed long-run supply curve. In the long-run a given real wage will be associated with a rise in vacancies, but in the short-run the model shows that nominal wage growth reduces vacancies. The link between wages and vacancies may be spuriously caused by wages and vacancies rising together, but this seems less likely as the levels effects are truly significant(see Granger and Newbold(1974) and (1986) for discussion of spurious correlation). Engle and Granger(1987) have evidence based on monte carlo simulation, that the power of the t-test for unit roots using an un-restricted regression in differences and levels is low. Hence, significant levels effects may not be enough to reject the hypothesis that a relationship is spurious. It was also found that the term $(v+w-p)^i$ could be included without greatly affecting any of the tests or the predictive power of the model. The results are not presented here, but they do indicate a more conventional negative long-run response of vacancies to wages.

In the steady state real variables grow at a fixed rate π and nominal variables grow at a different rate $(1+\dot{p})$. The steady state solution to the model is given below:

$$v = 32.051 + seas + 3.422(q^d - q^s) + w\text{-}p$$
$$+ 4.4\pi - 25.094(1 + \dot{w})$$

In the steady state vacancies still depend on the level of wages, they are positively affected by excess demand for goods and the growth rate, but negatively affected by wage inflation. The results allow inventory accumulation to be partly explained by investment in stock holding and this places a break on the effect that growth has on the process of vacancies when such an effect is not included the growth effect becomes even stronger.

TABLE 3.5: Additional Diagnostic Tests

Test Statistics	LM Version	F Version
A:Serial Correlation	CHI-SQ(4)= 4.2433	F(4, 64)= .8983
:Serial Correlation	CHI-SQ(8)= 9.2386	F(8, 61)= .9816
B:Functional Form	CHI-SQ(1)= 1.1064	F(1, 67)= .9417
C:Normality	CHI-SQ(2)= 2.7211	Not applicable
D:Heteroscedasticity	CHI-SQ(1)= 1.3060	F(1, 79)= 1.2946
E:Predictive Failure	CHI-SQ(5)= 3.9251	F(5, 69)= .7850
F:Predictive Failure	CHI-SQ(5)= 21.9355	Not applicable
G:A.R.C.H.	CHI-SQ(8)= 3.7538	Not applicable
H:Test of Restrictions	CHI-SQ(25)= 23.2532	F(24,43)= .5501

In statistical terms the vacancies model performs well within period, because it fits the data well and it satisfies all the diagnostic tests at the 5% level. The restrictions associated with the specific model presented in table 3.4 are quite easily satisfied with the F test being close to zero. Outside the estimation period the model performs equally well satisfying the

Chow test, though it does not satisfy the Hendry predictive failure test.

TABLE 3.6 Granger Causality Test and Prediction of the Total Vacancies Model

Variable deletion test for the omission of output and employment from the Total Vacancies equation:

Lagrange Multiplier test statistic	CHI-SQ(18)= 17.8891
Likelihood Ratio test statistic	CHI-SQ(18)= 20.2141
F statistic	F(18, 50) = .8031

Dynamic Forecasts

Observation	Actual	Prediction	Error
80Q1	-.1710	-.1355	-.0356
80Q2	-.2001	-.0885	-.1116
80Q3	-.2835	-.1238	-.1596
80Q4	-.2044	-.0457	-.1587
81Q1	.0204	.1452	-.1248

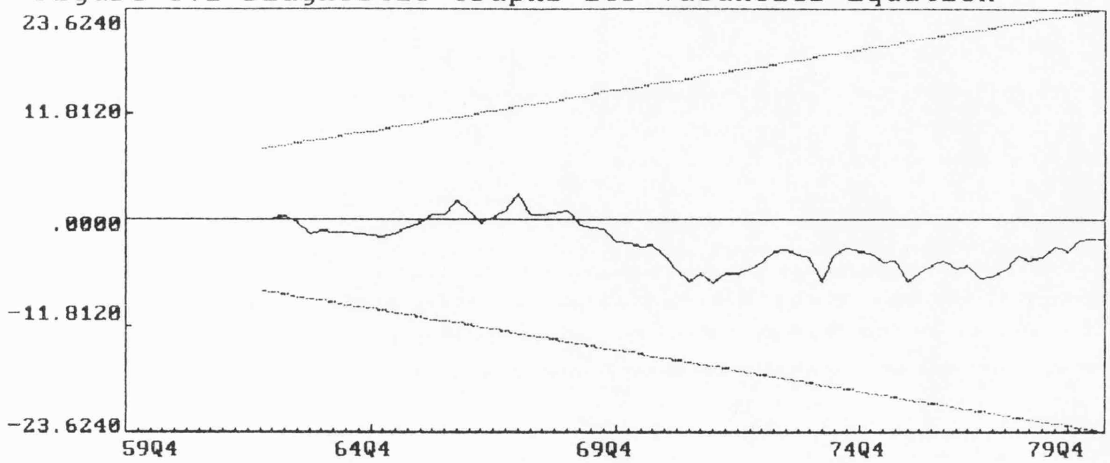
Summary statistics for dynamic forecasts

Based on 5 observations from 80Q1 to 81Q1

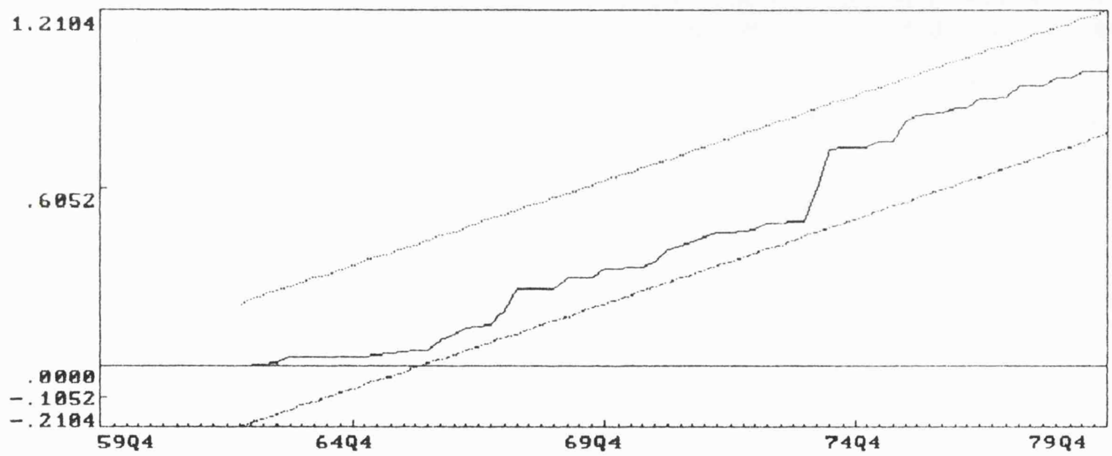
Mean Prediction Errors	-.1181	Mean Sum Abs Pred Errors	.1181
Sum Squares Pred Errors	.0160	√ Mean Sumsq Pred Errors	.1265

A check on parameter Invariance is given by looking at multi-period chow-tests, variances and recursive residuals. CUSUM and CUSUMSQ tests are given on the next two pages.

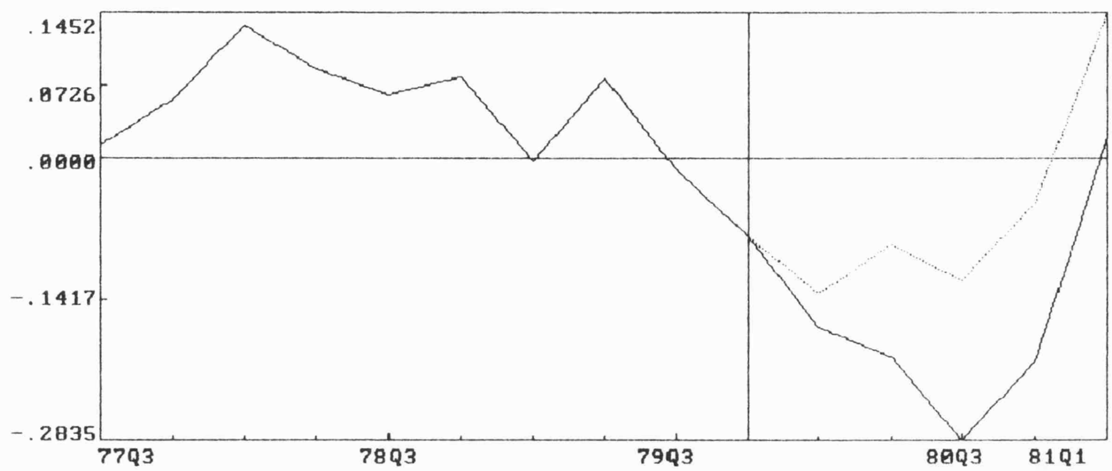
Figure 3.2 Diagnostic Graphs for Vacancies Equation



CUSUM with 5% bands



Cumulative Sum of Squares with 5% confidence bands



Forecasts for five periods ahead

End Period	Chow Test		Standard Deviation
64Q3	CHI-SQ(12)= 11.5668	F(12,57)= .9639	.0312
67Q2	CHI-SQ(12)= 20.9632	F(12,57)=1.7469	.0413
70Q2	CHI-SQ(12)= 16.5490	F(12,57)=1.3791	.0517
72Q1	CHI-SQ(12)= 18.5336	F(12,57)=1.5445	.0544
73Q3	CHI-SQ(12)= 23.3352	F(12,57)=1.9446	.0527
76Q1	CHI-SQ(12)= 6.8157	F(12,57)= .5680	.0624

The tests in table 3.7 give an indication of whether the partition of the parameter space is correct and in particular whether an invariant representation has been found. The inclusion of output and employment does not affect the results with these variables not being significant either jointly or individually. Hence, it seems likely that vacancies are not Granger caused by output and employment. There is some indication that the variance is rising over the period, but whether this is significant given the small initial sample period is questionable. The Lagrange-Multiplier test for Heteroscedasticity gives counter factual evidence against such an hypothesis. The model appears to be relatively robust over the period, except for 1973q4 for which the Chow test is significant at the 5% level($\chi^2(12) = 21.026$), but it is not significant at the 1% level($\chi^2(12) = 26.217$). The CUSUMSQ plot also gives some evidence of structural change, but the shift does not cause the statistic to move outside of the 5% band. The parsimonious form of the output price model is well specified and it seems to fit the data well. The General specification is again simplified to a model in differences and

TABLE 3.8 Ordinary Least Squares Estimates of the Industrial Output Price Equation using 81 observations for the period from 59Q4 to 79Q4

Regressor	Coefficient	Standard Error	T-Ratio
C	-.2138	.0658	-3.2491
S1	.0101	.0022962	4.4060
S2	.0060937	.0026806	2.2733
S3	.0026490	.0026251	1.0091
$\Delta i(-2)$	.0197	.0088738	2.2229
$\Delta\Delta i(-8)$	-.0360	.0065766	-5.4780
$\Delta_4 v(-1)$	-.0202	.0054450	-3.7086
$\Delta v(-2)$	.0576	.0174	3.3129
$\Delta p(-1)$	.7165	.0625	11.4697
$\Delta p(-7)$	-.4535	.1805	-2.5124
$\Delta_3 p(-6)$	.2028	.0647	3.1365
p-w-v(-1)	-.0120	.0033281	-3.6197

Standard Diagnostics

R-Squared	.877	F-statistic $F(10, 70)$	44.7129
R-Bar-Squared	.8574	S.E. of Regression	.0067226
R.S.S.	.0031183	Mean of Δp	.0192
$\sigma_{\Delta p}$	.0178	Log-likelihood	296.7447
DW-statistic	1.8645		

levels where the correction term is well within the usual significance limits. The long-run static solution is derived in the usual way giving the equilibrium model presented below with Δi given an investment interpretation.

$$p = 17.8166 + seas + 1.6417\Delta i + v + w$$

The long-run relationship seems quite reasonable if stock accumulation is due to the desire to hoard and vacancies can either have an excess demand interpretation or a desired vacancy interpretation; the wage coefficient has the correct sign and it implies that we have a wage weighted price model . If the inventory affect was not due to an investment motive, then we could eliminate it in the long-run by setting demand equal to supply.

In the long-run steady state, growth rates are equalised and the inflation rate set at a non-accelerating rate. The resulting model is presented below:

$$p = 17.8166 + seas - 10.71666(1 + \dot{p}) - 3.4285\pi + v + w$$

or

$$p = 17.8166 + seas - 10.71666(1 + p) + 1.6417\Delta i + v + w$$

Growth as would be expected has a negative effect on prices and surprisingly inflation has the same effect, though this can be seen to be an expectational adjustment to an underlying trend. The rest of the relationship stays the same except for the coefficients on inventory accumulation which is accounted for with planned vacancies as a real effect. Inventories can otherwise be viewed as disequilibrium phenomena, though they have the wrong sign or as investment in stock which appropriately adds to demand and raises prices. Vacancies can also be given a disequilibrium interpretation which means that the change in the long-run should not be important.

Table 3.9 Additional Diagnostic Tests [1]

Test Statistics	LM Version	F Version
A:Serial Correlation	CHI-SQ(4)= 1.7351	F(4, 63)= .3557
:Serial Correlation	CHI-SQ(8)= 5.1471	F(8, 59)= .5174
B:Functional Form	CHI-SQ(1)= 6.2152	F(1, 66)= 5.6513
C:Normality	CHI-SQ(2)= .2844	Not applicable
D:Heteroscedasticity	CHI-SQ(1)= 2.8401	F(1, 79)= 2.8706
E:Predictive Failure	CHI-SQ(5)= 14.3151	F(5, 67)= 2.8706
F:Predictive Failure	CHI-SQ(5)= 9.935	Not applicable
G:A.R.C.H.	CHI-SQ(8)= 12.819	Not applicable
H:Test of Restrictions	CHI-SQ(28)= 26.7522	F(28, 41)= .5178

In statistical terms the output price relationship does not perform as well as the others. The functional form test clearly fails at both the 5% level and the Chow predictive failure test is also not satisfied, though it does lie within the 1% band ($\chi^2(5) = 15.086$). The Hendry test contradicts the Chow test, as it is clearly satisfied and the plot of the predictions seem to agree with that. Failure of the Chow test may be due to the size of the sample; the other Chow tests seem to agree with this prognosis, though the evidence is far from clear. The parsimonious model is clearly accepted so that the restrictions imposed in moving from the general to specific do not affect the performance of the model. The variable addition test seems to imply that it is reasonable to exclude output and employment from the output price relationship and that is supported by the fact that individual terms in output and employment are not significant. The Granger Causality test determines whether the dependent variables are important in modelling the exogenous

TABLE 3.10 Granger Causality Test and Prediction of the Industrial Output Prices Model

Variable deletion test for the omission of output and employment from the Industrial Output Price equation.

Lagrange Multiplier test statistic	CHI-SQ(18)= 15.3084
Likelihood Ratio test statistic	CHI-SQ(18)= 16.9677
F statistic	F(18, 49) = .6603

Dynamic Forecasts

Observation	Actual	Prediction	Error
80Q1	.0515	.0448	.0066783
80Q2	.0389	.0351	.0038339
80Q3	.0229	.0270	-.0041168
80Q4	.0122	.0256	-.0134
81Q1	.0296	.0157	.0139

Summary statistics for dynamic forecasts

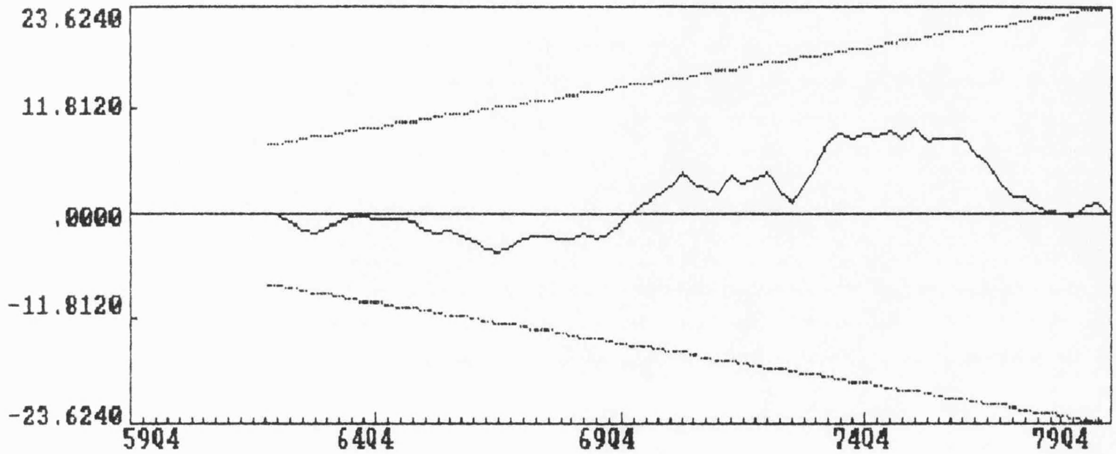
Based on 5 observations from 80Q1 to 81Q1

Mean Prediction Errors .0013812 Mean Sum Abs Pred Errors .00839

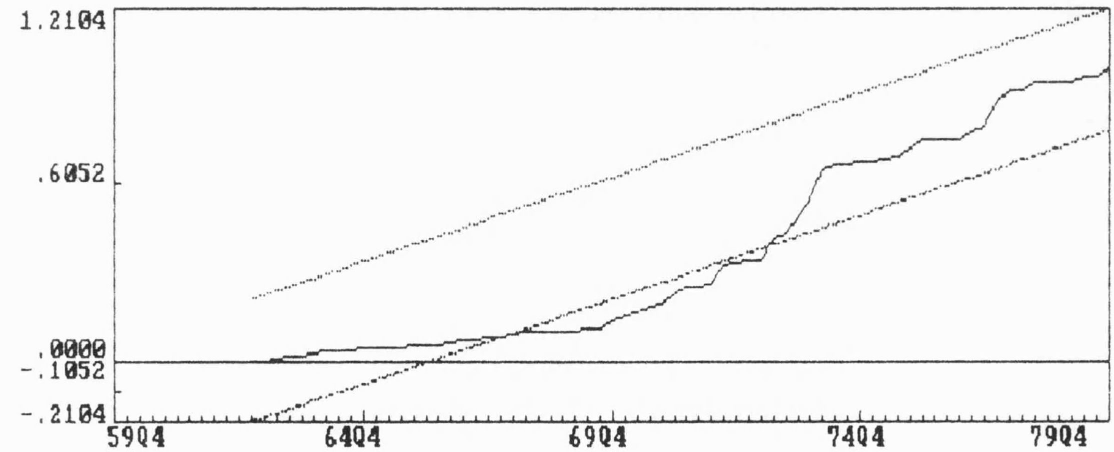
Sum Squares Pred Errors .0000898 √ Mean Sumsq Pred Errors .00948

End Period	Chow Test		Standard Deviation
64Q3	CHI-SQ(12)= 9.0745	F(12,57)= .7562	.00382
67Q2	CHI-SQ(12)=24.4703	F(12,57)=2.0392	.00356
70Q2	CHI-SQ(12)=19.9107	F(12,57)=1.6592	.00411
72Q1	CHI-SQ(12)=14.0917	F(12,57)=1.1743	.00514
73Q3	CHI-SQ(12)=16.5065	F(12,57)=1.3755	.00592
76Q1	CHI-SQ(12)=15.326	F(12,57)=1.2772	.00661

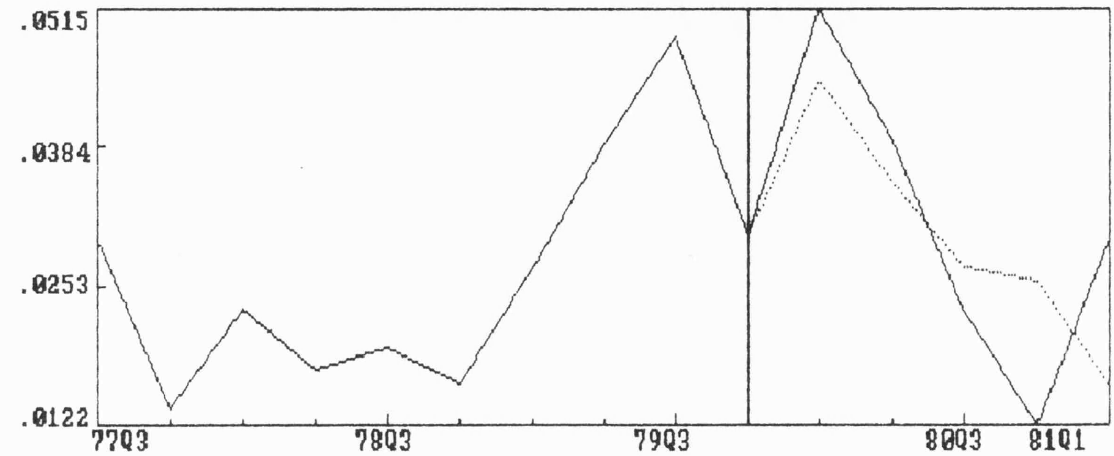
Figure 3.3 Diagnostic Graphs for the Output Price Equation



CUSUM with 5% confidence bands



CUSUMSQ with 5% confidence bands



Forecasts for five periods ahead

variable which is important for the efficiency of the estimates and the predictions. All the forms of this test are clearly satisfied. As we have seen, in the models dealt with so far there is no direct test of Exogeneity.

A check on parameter Invariance is given by looking at the multi-period chow-tests and variances given above, and this information is further supported by the CUSUM and CUSUMSQ tests presented on the previous page.

Weak exogeneity require invariance for the model of the endogenous variable to be stable, but the evidence seems to be indeterminate. The variance terms do vary over the period, though the model does not seem to suffer from heteroscedasticity and the Chow tests do not seem to indicate large variations in the parameters of the model, though the χ^2 version for the sample split in 67q2 is significant at the 5% level($\chi^2(12)$ = 21.026). The predictive failure test presented with the main results provides some evidence to support this, as does the cusumsq test, which suggests a break between 64q4 and 73q3. The model is not completely satisfactory, though the problems are not great enough to totally reject it at this stage; especially given the number of tests and the likelihood that the rejection region has become large relative to the nominal confidence regions.

The model of wages is re-parameterised into a form in differences and levels which has a strong correction term which produces a real wage effect. Again much of the result hangs on the interpretation of the inventories effect without which we get a

TABLE 3.11 Ordinary Least Squares Estimates of the Manufacturing Wage Equation using 81 observations for the period from 59Q4 to 79Q4

Regressor	Coefficient	Standard Error	T-Ratio
C	-.5768	.1511	-3.8185
S1	.0021039	.0040582	0.5184
S2	-.0093918	.0039974	-2.3494
S3	-.0167	.00036286	-4.6046
Time	.0003674	.0000626	5.8673
$\Delta w(-2)+\Delta\Delta_2 w(-4)$	.2261	.0434	5.2130
$\Delta i(-1)$	.0524	.0127	4.1196
$\Delta\Delta i(-3)$	-.0292	.0122	-2.3983
$\Delta i(-7)$	.0309	.0125	2.4782
$\Delta\Delta p(-1)$	.4013	.1697	2.3643
$\Delta_5\Delta p(-3)$	.5969	.0930	6.4180
(p-w-v)(-2)	.0121	.0058197	2.0743

Standard Diagnostics

R-Squared	.7227	F-statistic F(10, 70)	16.3479
R-Bar-Squared	.6785	S.E. of Regression	.0109
R.S.S.	.0081414	Mean of Δw	.0260

$\sigma_{\Delta w}$ .0192 Log-likelihood 257.8785 DW-statistic 2.2090

real wage model which is constant in the long-run or depends on growth factors in the steady state. If we set $x_t = x_{t-1} = x$ and $\Delta x_t = 0$ then the long-run equilibrium solution is given by:

$$w - p = -47.889 + seas + .0304t - 6.874(q^d - qs) - v$$

Real wages are constant in the long-run except for the negative effect of excess demand in the goods market and a negative response of wages to vacancies. If we take the dynamic solution wages are allowed to adjust in response to growth and wage inflation relative to price inflation. In steady state real factors grow at the same rate π and nominal elements grow at the rate $(1+\dot{p})$.

$$w - p = -47.889 + seas + .0304t + 6.874\Delta i + 18.69(1+\dot{w}) + v$$

Real wages in the steady state depend positively on wage inflation, negatively on vacancies and positively on inventory accumulation. If the inventory term relates to the growth rate, then wages would rise with the growth rate.

In statistical terms the model performs well apart from the Chow test for predictive failure which is clearly not satisfied at the 1% level ($\chi^2(4)$ = 13.277). The test of the restrictions associated with the General model are easily satisfied. The

TABLE 3.12 Additional Diagnostic Tests

Test Statistics	LM Version		F Version	
A:Serial Correlation	CHI-SQ(4)=	1.7611	F(4, 65)=	.3612
:Serial Correlation	CHI-SQ(8)=	3.1976	F(8, 61)=	.3875
B:Functional Form	CHI-SQ(1)=	.047400	F(1, 67)=	.039800
C:Normality	CHI-SQ(2)=	1.5549	Not applicable	
D:Heteroscedasticity	CHI-SQ(1)=	.057500	F(1, 79)=	.056100
E:Predictive Failure	CHI-SQ(4)=	16.1756	F(4, 69)=	4.0439
F:Predictive Failure	CHI-SQ(4)=	19.2766	Not applicable	
G:A.R.C.H.	CHI-SQ(8)=	3.2976	Not applicable	
H:Test of Restrictions	CHI-SQ(26)=	14.9204	F(26, 44)=	.3424

Granger causality test is satisfied, excepting the Likelihood ratio form of the test at the 5% level($\chi^2(18)$ =28.869) and the 1st and 2nd lags on output are individually significant at the 5% level, though when they are included on their own they are not significant.

TABLE 3.13 Granger Causality Test and Prediction of the Manufacturing Wage Model

Variable deletion test for the omission of output and employment from the manufacturing wage equation.

Lagrange Multiplier test statistic	CHI-SQ(18)= 26.3867
Likelihood Ratio test statistic	CHI-SQ(18)= 31.9279
F statistic	F(18, 52) = 1.3689

Static Forecasts

Observation	Actual	Prediction	Error
80Q1	.0543	.0495	.0048015
80Q2	.0729	.0560	.0169
80Q3	.0441	.0468	-.0026684
80Q4	.0267	.0710	-.0443

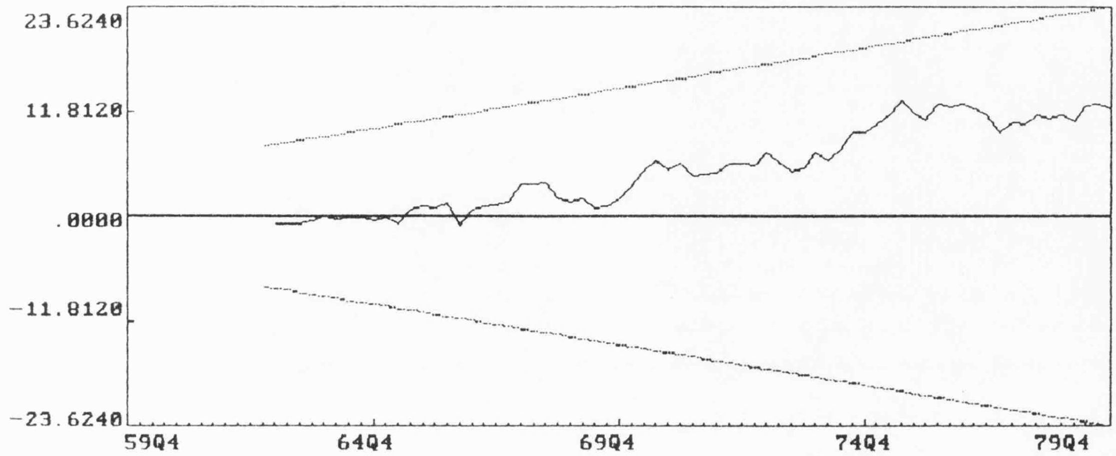
Summary statistics for static forecasts

Based on 4 observations from 80Q1 to 80Q4

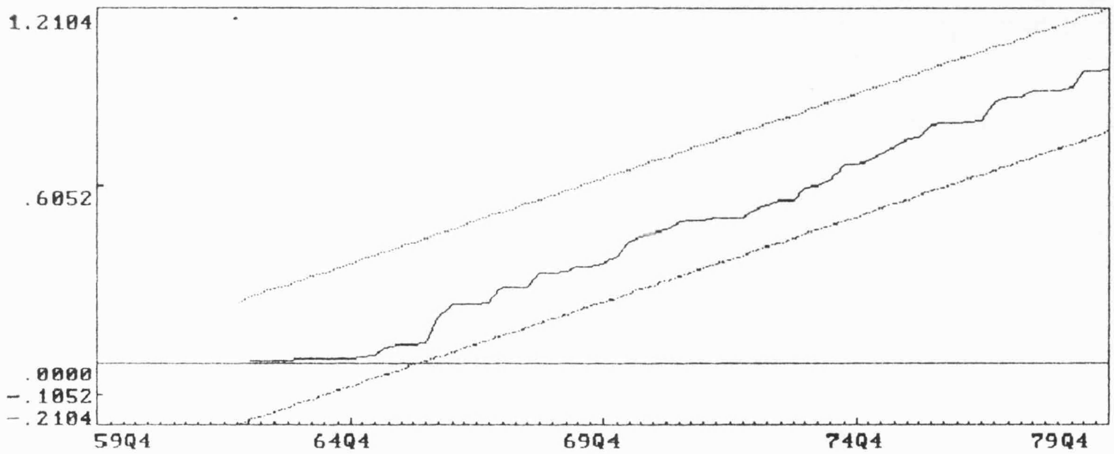
Mean Prediction Errors	.0069191	Mean Sum Abs Pred Errors	.0224
Sum Squares Pred Errors	.0005753	√ Mean Sumsq Pred Errors	.0240

A check on parameter Invariance is given by the multi-period Chow

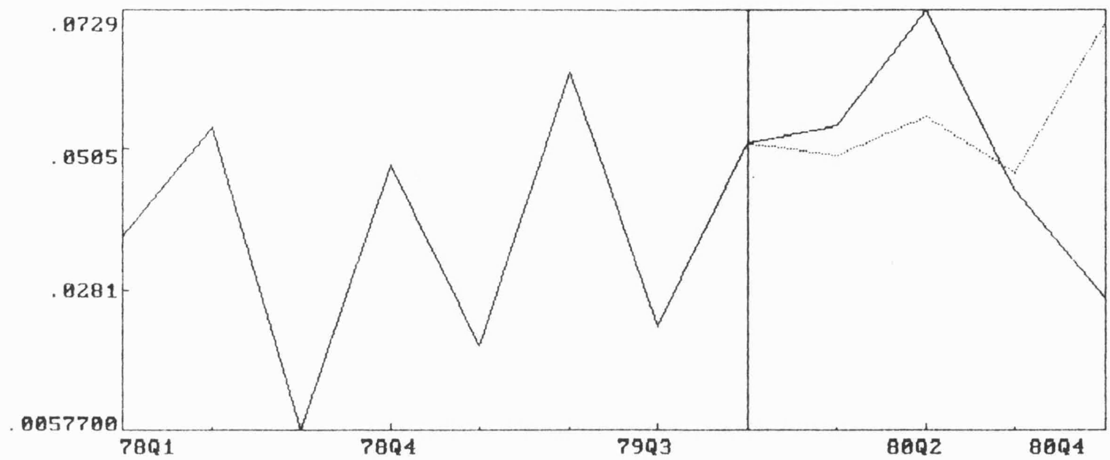
Figure 3.4 Diagnostic Graphs for Manufacturing Wage Equation



CUSUMwith 5% confidence bands



CUSUMSQ with 5% confidence bands



Forecasts for four periods ahead

End Period	Chow Test		Standard Deviation
64Q3	CHI-SQ(12)=13.7250	F(12,57)=1.1437	.0037970
67Q2	CHI-SQ(12)=14.3358	F(12,57)=1.1946	.00936
70Q2	CHI-SQ(12)= 7.7976	F(12,57)= .6498	.0105
72Q1	CHI-SQ(12)=13.0757	F(12,57)=1.1413	.0103
73Q3	CHI-SQ(12)=14.9401	F(12,57)=1.2450	.0102
76Q1	CHI-SQ(12)=12.0862	F(12,57)=1.0072	.0108

-tests, variances and recursive residuals. CUSUM and CUSUMSQ tests are given on the previous page and the other tests above. The models appear to be invariant in terms of the CUSUM, CUSUMSQ statistics and the repeated Chow tests. Apart from the first period estimates the standard deviation is fairly stable around .01.

The models in this section are related to the cointegration theory specified in the first section in the sense that they are models in differences and levels. The models are relatively sensible. though they all suggest long adjustment times and imply long lags on price variables in particular. In period the models perform well in statistical terms and they mainly satisfy the standard tests at the 1% level. In the case of inventories the events of one quarter in particular seem to be important which suggests that failure is due to this one outlier.

The assumption of exogeneity is important when we use these models as forecasts in the models of output and employment in the next section. It is difficult to test for exogeneity, but Davidson et al(1978), Engle et al(1983) and Hendry and Richard(1983) suggest ways in which this may be done. Strict exogeneity implies that variables included in a regression are not correlated with the error, but this definition as we saw in Chapter 2 is not consistent with the reformulations which are undertaken when we derive an econometric model. Weak exogeneity is the appropriate condition, but that requires a model which is parameter invariant. We suggest that the models presented come close to such a definition, given the limits of classical testing procedures in analysing such problems. If the variables are not weakly exogenous, then it seems likely that they are not caused by the endogenous variables under the current parameterisation. Finally the models do not produce stable predictions which is not surprising, but if the series are non-stationary the prediction do not seem to be explosive. The parameters need to be inverted into an MA form, but in this instance that is not possible because of the non-stationarity. In practice we do invert and then truncate the moving average parameters.

3.3 Conclusion

The models presented here perform reasonably well in terms of the standard test procedures suggested in the literature and associated with the correct implementation of ordinary least squares. The wage and price models have long-run solutions which do not reject the notion of real relationships being important in the long-run, though the models are close to rejecting the hypothesis that they have a long-run solution. The standard time series approach adopted by Sims would suggest that we deal with variables in differences rather than levels. The procedure would eliminate the difficulty with inverting the parameters, but as we will see in the next section such a difference model does not seem to be supported by the data. The forecasts of the series seem to increase over time, but in a non-accelerating way which seems to give some credence to the cointegration hypothesis. If the series are cointegrated, then the VAR parameters do suggest a simple way in which we can reverse the Yoo procedure to transform the results into those associated with the VMA form in cointegrating variables. If we had such a technique it would be possible to invert any autoregressive form what ever the order of integration or cointegration into the moving average representation required here.

When the vector series are all stationary in differences or levels it is possible to invert the system into a VMA form, but when the series are cointegrated the process is more complicated. In the first section we showed that it was possible to use the prediction formula to eliminate the expectations in (3.12), but use of the VAR representation of the exogenous variables does not produce the type of rational expectations system suggested at the end of the last chapter. It can be shown that the cointegration representation can always be transformed into a VMA. If we re-write (3.15) in the following way:

$$(3.32) \quad \Delta s_t = (C(1)L + (1-L)C^*(L))\epsilon_t$$

$$\Delta\alpha' s_t = \alpha'(C(1)L + (1-L)C^*(L))\epsilon_t$$

$$\Delta\eta_t = (\alpha' C(1)L + (1-L)\alpha' C^*(L))\epsilon_t$$

$$\Delta\eta_t = (1-L)\alpha' C^*(L))\epsilon_t$$

$$(3.33) \qquad \eta_t = C^+(L)\epsilon_t$$

It is possible to partition the vector of cointegrated variable into those associated with the endogenous variable process n_{1t} and those linked to the exogenous variable process η_{2t}. The partition of the exogenous variables in Equation(3.33) produces the following prediction formula (see Hunter(1988)):

$$(3.34) \qquad E(\eta_{t+i} | \Omega_t) = C^+_{22}(L)_i \varepsilon_t$$

$$\text{where } C^+_{22}(L) = C_{22}[i] + C_{22}[i+1]L + C_{22}[i+2]L^2 \ . \ . \ .$$

Substituting out for the expectations in (3.12) produces a very specific VARMA system and when we also partition the xs out (3.12) becomes:

$$(3.35) \quad F^+(L)s_t = D^+(L)e_t$$

$$F^+(L) = \left[\begin{array}{c|c} F_{11}(L) & 0 \\ \hline 0 & A_{22}(L)(1-L) \end{array}\right]$$

$$D^+(L) = \left[\begin{array}{c|c} I & D^+_{12}(L) \\ \hline 0 & d(L) \end{array}\right]$$

$$\text{where } D^*_{12}(L) = I + D^*_{12}[1]L^* + D^*_{12}[2]L^2 + \ . \ . \ .$$

$$\text{and} \quad D^*_{12}[j] = F_{12}[j]\ C^+_{22}(L)_j$$

In the next section we will use a method which uses such VMA processes and can take account of cointegration in the endogenous variables; that is the recursive method of Hunter(1984) and Sargan(1982). The method requires stable forecasts, one step ahead prediction errors and the moving average parameters of the exogenous processes. The cointegration approach to modelling the exogenous variables naturally feeds into this procedure, because the VMA representation always exists and when the exogenous variables are cointegrated the predictions are highly efficient (see Granger and Engle(1987)). Substitution using the VMA produces a computationally efficient method of estimating the quasi-reduced form which allows one to estimate the deep parameters of the model.

1. In practice the actual data is likely to incorporate the two effects, in the short-run the nature of the data would suggest that the disequilibrium one would dominate while in the long-run disequilibria should disappear or become small and then the investment effect should dominate.

2. A:Lagrange multiplier test of residual serial correlation; see Pesaran and Pesaran(1987) or Harvey(1981)

B:Ramsey's RESET test using the square of the fitted values; see Pesaran and Pesaran(1987) or Harvey(1981)

C:Based on a test of skewness and kurtosis of residuals, due to Jarque and Bera(1981) also see Pesaran and Pesaran(1987)

D:Based on the regression of squared residuals on squared fitted values see Pesaran and Pesaran(1987) or Harvey(1981)

E:A test of adequacy of predictions (Chow's second test) see Pesaran and Pesaran(1987) or Chow(1960)

F:Predictive Failure test in Hendry(1979)

G:Autoregressive Conditional Heteroscedasticity Test of Residuals(ARCH), see Pesaran and Pesaran(1987) or Engle(1982)

H:F and Likelihood Ratio tests of restriction in relation to general model 3.21 see Pesaran and Pesaran(1987) or Harvey(1981)

Chapter four

Dynamic models of output and employment estimated using the solved form of a rational expectations model.

In this chapter we look at first order rational expectations models of output and employment. The methods used here follow the optimal control approach suggested by Chow (1975), (1983) and (1980) which views the economy as minimising the distance of actual values from a target. The target can be seen as an equilibrium for the agent or the economy and this can be related either to a notional relationship or equilibrium point. Here, we extend and use the techniques presented in Sargan (1982) which assume an optimal control framework. Though that paper deals with cost minimisation, the usual duality between cost minimising and the maximisation of an objective function still holds. When the target and the revenue functions are equivalent, then the cost minimising approach produces the same first order conditions as that associated with agents maximising profit subject to costs of adjustment.

In the first section the model presented in Sargan(1982) is used to produce a forward representation of the usual first order rational expectations model in which the expectations are replaced by actual values using the Wiener-Kolmogorov prediction formula. We then deal with the different representations of the rational expectations model and relate them to cointegration.

In the second section we derive the target or equilibrium model associated with cost minimising approach; this is equivalent to selecting a revenue or cost function in the profit or utility maximising case. In section three we specify the results of an output employment model and present the Muellbauer form of the first order condition which is used to produce initial estimates. In section four we discuss the different representations associated with the first order condition and compare them with the full method. In the final section we discuss aggregation and use that to rationalise the model with serially correlated errors. We also produce additional results which introduce the theoretical extensions dealt with in the final chapter.

4.1 Cointegration and First Order Rational Expectations Systems

In the literature it is usual for a rational expectations model to be the solution of a quadratic optimisation problem. Sargan(1982) deals with a control problem in which agents minimise a loss function subject to cost of adjustment matrix K and disequilibrium cost matrix H, where H and K are usually assumed to be positive definite. Sargent(1979) and Sargent and Hansen(1980) have dealt with agents maximising an objective function subject to cost of adjustment; the necessary conditions for a maximum are satisfied if the coefficient matrices H on the stock and K on the flow are negative definite. Kollintzas(1985) derives weaker conditions for a cross product model similar to the one presented in the chapter 5. Here, we will look at a quadratic objective function, though similar results may be derived for more general models:

$$(4.10)\quad \Gamma_t = E\{ \sum_{t=0}^{T} \beta^t (\Delta y_t' K \Delta y_t + (y_t - z_t)' H (y_t - z_t) \}$$

where β is a discount factor y_t an gx1 vector of exogenous variables and z_t an gx1 vector of target variables.

The first order condition related to this first order rational expectations model is well known in the literature, see Sargent(1979), Sargan(1982) and Hunter(1984) and (1985). It can be simply derived by differentiating (4.10) with respect to y_t which gives us:

$$(4.11) \qquad E(Q_o y_t - \beta Q_1' y_{t+1} - Q_1 y_{t-1} = Hz_t | \Omega_t)$$

$$(4.11a) \qquad \lim_{T \to \infty} E(H(y_T - zT) + K(y_T - y_{T-1}) | \Omega_t) \to 0$$

where $Q_o = H + (1+\beta)K$ and $Q_1 = K = Q_1'$ which is the definition in Sargan(1982) and H and K are normally positive-definite when we are dealing with a minimum.

This form of the first order condition will be covered in more detail in section four where we will discuss the problems with estimating the model using such conditions and look at the different representations of (4.11) associated with endogenous variables which are stationary in levels, differences and conjointly. Here we are interested in solving the stochastic difference equation and then using that solution to reveal a variant of the forward solution which allows us to estimate the deep parameters and which only depends on one step ahead forecast errors and a small subset of exogenous variable predictions. Equation (4.11a) is the transversality condition which also has to be satisfied if we wish to derive a full solution to the rational expectations problem.

The solution and the derivation of tractable identification conditions is often assumed to hang on the auxiliary assumption that the xs and ys are at least weakly stationary. Time series data are then transformed into first differences or de-trended before the model is specified, but as we shall see in section four that is not always appropriate. If the endogenous variables are cointegrated, then the g levels terms in the loss function will exhibit a dependence relationship $n_{1t} = \alpha' y_t$ where n_{1t} is a sub-vector of r variables which are linear combinations of the original ys. H is then rank deficient and that singularity forces us to transform the model to remove any zero roots, but the transformed model can then be reformulated to produce the usual first order condition (4.11) (see Appendix A1 for details). When the ys are cointegrated we just have a special case of the standard result in which the parameters have to satisfy additional restrictions, but all of the results for the usual case in which r=g also go through under cointegration.

The standard solution to the difference equation (4.11) is covered in Appendix A2, so that we have what Sargan(1984) calls regular or saddle point solution in which there are equal numbers of stable and unstable roots. Hence, equation (4.11) can be replaced by equation (x) in Appendix A2 when H is positive-semi-definite and K positive definite (Kollintzas(1985) proves this result for the cross product model dealt with in chapter 5):

$$E((\beta F + F^{-1})y_t - \beta y_{t+1} - y_{t-1} = B_o z_t \mid \Omega_t)$$

where $B_o = (\beta F + F^{-1} - (1+\beta)I)$ and $F = PMP^{-1}$.

We can confirm Kollintzas result as the singularity associated with cointegration implies that H either has a g-r zero roots or r positive ones, but as we will see below this does not affect the form of the forward solution. We can pre-multiply the equation above by F to reveal a form of the first order condition which can then be easily transformed into a forward looking model:

$$E((\beta F^2 + I)y_t - \beta F y_{t+1} - F y_{t-1} = FB_o z_t \mid \Omega_t)$$

Where $B_o = (\beta F + F^{-1} - (1+\beta)I)$

Re-writing this in a more appropriate from we have:

(4.12) $E((I - \beta FL^{-1})(I - FL)y_t = FB_o z_t \mid \Omega_t)$

In equation (4.12) F may have a number of unit roots as the effect of cointegration is to remove g-r levels terms from the system, but that does not preclude us from inverting $(I - \beta FL^{-1})$ so that a forward representation exists even with cointegration:

(4.13) $$y_t - F\, y_{t-1} = \sum_{s=0}^{\infty} (\beta F)^s F B_o E(z_{t+s} \mid \Omega_t) + u_t$$

Equation (4.13) is exactly the same open loop solution, as that presented in Nickell(1987) as the forward solution associated with a system of factor demand equations derived by maximising an intertemporal profits function. In that instance z_t depends on the structure of the revenue function, so that the solved form is no different when the target is the same as the revenue function associated with no costs of adjustment. A unit root in the forward convolution does not affect the forward solution, as the unstable roots in F are in the null space of B_o. If we look at canonical form of the forward convolution we have that:

$$(\beta F)^s F B_o = \beta^s P\, M^{s+1} P^{-1} (\beta F + F^{-1} - (1+\beta)I)$$

where $F = PMP^{-1}$, $PP^{-1} = I$ and $M = \mathrm{diag}(\mu_1, \mu_2 \ldots \mu_g)$ and $\mu_i = 1$ for $i = g-r, \ldots, g$ then:

$$(\beta F)^s FB_o = \beta^s P\, M^{s+1} P^{-1} P(\beta M + M^{-1} - (1+\beta)I)P^{-1}$$

$$= \beta^s P\, M^{s+1}\left(\begin{bmatrix} \beta M_1 & 0 \\ 0 & \beta I \end{bmatrix} + \begin{bmatrix} M_1^{-1} & 0 \\ 0 & I \end{bmatrix} - (1+\beta)\begin{bmatrix} I_r & 0 \\ 0 & I \end{bmatrix}\right)P^{-1}$$

$$= \beta^s P\, M^{s+1}\left(\begin{bmatrix} (\beta M_1 + M_1^{-1} - (1+\beta)I_r) & 0 \\ 0 & 0 \end{bmatrix}\right)P^{-1}$$

$$= \beta^s P \begin{bmatrix} M_1^{s+1}(\beta M_1 + M_1^{-1} - (1+\beta)I_r) & 0 \\ 0 & 0 \end{bmatrix} P^{-1}$$

When there is cointegration the forward solution only depends on the r stable roots of the system. Usually we would use the substitution method to replace the forward expectations using a finite order Vector Auto-Regressive(VAR) representation. Hence, when $z_t = Ax_t$ (A may or may not be of full rank) we may have the stable VAR representation of the exogenous variables given below:

(4.14) $\quad x_t = \sum_{i=1}^{p} B_i x_{t-i} + \varepsilon_t \quad$ and $\quad E(x_{t+s} | \Omega_t) = \sum_{i=1}^{p} B_i E(x_{t+s-i} | \Omega_t)$

when the xs are also cointegrated $B(1) = \tau_1 \alpha_1'$ and some of the exogenous variables then may appear in differences and others in levels (we do not deal with this here).

Given that it is always possible either to derive a VMA representations in difference or in levels we can re-write (4.13) as a generalised Wold moving average form:

$$x_t = \sum_{s=o}^{\infty} C_r\, \varepsilon_{t+s} \quad \text{and} \quad E(x_{t+s} | \Omega_t) = \sum_{r=s}^{\infty} C_r \varepsilon_{t+s-r}$$

where ε_t is a Martingale difference or white noise innovation.

The Wold Moving Average form is convenient to use, as cross period expectational differences simply depend on the innovation or news. If we compare the t period expectation with the t+1 period expectation then we have the following result which can be used to substitute for future expectations of the exogenous variables. Therefore:

(4.15a) $E(x_{t+s}|\Omega_t) - E(x_{t+s}|\Omega_{t+1}) = -C_{s-1}\varepsilon_{t+1}$

(4.15b) $E(x_{t+s}|\Omega_{t-1}) - E(x_{t+s}|\Omega_t) = -C_s\varepsilon_t$

If we pre-multiply the left and right hand side of equation (4.13) by $(I - G_1L^{-1})$ where $G_1 = \beta F$ and $z_t = Ax_t$, then:

(4.16) $$(I - G_1L^{-1})(y_t - Fy_{t-1} - u_t) = \sum_{s=0}^{\infty}(\beta F)^s FB_oAE(x_{t+s}|\Omega_t) - \beta F \sum_{s=0}^{\infty}(\beta F)^s FB_oAE(x_{t+s}|\Omega_{t+1})$$

$$= FBAx_t + \sum_{s=1}^{\infty}(\beta F)^s FB_oA\{E(x_{t+s}|\Omega_t) - E(x_{t+s}|\Omega_{t+1})\}$$

If information is truly dated at time t, then we can use (4.15a) to replace the differential in expectations in (4.16) above, so that we replace the future expectations by a term in the one step ahead forecast error:

(4.17) $$(I - G_1L^{-1})(y_t - F\, y_{t-1} - u\,) = FB_oAx_t + FB_o\sum_{s=1}^{\infty}(G_1)^s AC_{s-1}\varepsilon_{t+1}$$

We can now reverse the transformation to produce a model which involves the one-step ahead forecast error and actual values of the xs.

$$y_t - Fy_{t-1} - u_t = (I - G_1L^{-1})^{-1}\{G_o^* x_t - G_3\ \varepsilon_{t+1}\}$$

where $G_o^* = FB_oA$, $G_3 = FB_oD$ and $D = \sum_{j=1}^{\infty} G_1^j\ AC_{j-1}$

This obviously has a representation which is equivalent to

(4.13), but we can re-write this in a recursive form which clearly indicates the benefits of this approach:

(4.18) $y_t - Fy_{t-1} - u_t = h_t$

(4.19) $h_t = G_o^* x_t - G_3 \varepsilon_{t+1} + G_1 h_{t+1}$

Equation (4.19) represents the infinite lead which we can truncate by arbitrarily selecting a point T such that $h_{T+1} = 0$ and then we can replace h_t in (4.18) using the series derived by the repeated application of (4.19). For a large enough post sample period the terminal conditions do not seem to matter, as the influence of the future convolution seems to decay quite quickly. This technique should be relatively efficient and in comparison with (4.13) it involves a considerable saving as one step forecast errors are required rather than T-N future expectations at each period. In section three we will use this representation to estimate a model of output and employment, but first we deal with the other solutions and discuss the impact of cointegration.

The backward solution which is attributed to Sargent(1978) can be derived by repeatedly replacing the exogenous variable expectations in (4.13) using the formula for x in (4.14).

(4.110) $F(L)y_t = \sum_{s=o}^{p-1} G_s^+ x_{t-s} + u_t$

where F(L) = I - FL has all of its roots on or outside the unit circle.

A stable backward solution can be derived as long as the xs grow at a rate less than or equal too 1/β(see Sargent 1978). Hence, the derivation of a solution which imposes the rational expectations restrictions does not necessitate stable processes for the exogenous variables. Obviously, it would be sufficient for these series to be jointly stationarity as occurs with cointegration, but that is not necessary.

It is usual to give rational expectations and cointegration models different time series representations and the case in which the endogenous variables are cointegrated is not an exception. When we have cointegration amongst the endogenous variables there are g-r unit roots in the polynomial associated with the endogenous variables. The unit roots imply that the autoregressive parameters cannot be directly inverted to produce a stable Vector Moving-Average with eXogenous variables(VMAX) representation in levels, but we have a result due to Yoo(1986) which allows us to invert such polynomials. If we let F(L) = V(L) M*(L) U(L), then we can re-write (4.110) above in the following way:

$$(4.111)\quad V(L)M^*(L)U(L)y_t = \sum_{s=o}^{p} G_s^+ x_{t-s} + u_t$$

where V(L) = P(L) and U(L) = P^{-1} are non-singular matrixes

$$\text{and } M^*(L) = \begin{bmatrix} I_r & 0 \\ 0 & \Delta\, I_{g-r} \end{bmatrix} \quad P(L) = P\begin{bmatrix} I - M_1 L & 0 \\ 0 & I_{g-r} \end{bmatrix}$$

We can now invert this form of the model to produce the VMAX or VMA error form of the rational expectations model, taking care to introduce a first difference at the appropriate point:

$$(4.112)\quad \Delta y_t = G^*(L)x_{t-s} + C(L)u_t$$

where $G^*[i]=U(L)^{-1}M^+(L)V(L)^{-1}G_i$ and $C(L) = U(L)^{-1}M^+(L)V(L)^{-1}$

$$\text{and} \qquad M^+(L) = \begin{bmatrix} \Delta I_r & 0 \\ 0 & I_{g-r} \end{bmatrix}$$

The Smith-Mcmillan form can also be used to give the autoregressive parameters an error-correction representation when we have a dependence between the exogenous variables . Here, the correction term associated with cointegration of the endogenous variables is separate from the correction term in the loss function which relates to a cost to disequilibrium. We can directly reformulate F(L) using the following factorisation $F(L) = ((I-FL) - (1-L)F^*(L))$ and $F^*(L) = I$. Therefore:

$$\Delta y_t + (I - F)y_{t-1} = G^*(L)x_t + u_t$$

When we have non-trivial cointegration $I - F = \tau_1\alpha_1'$ and $(I - F)$ has g-r zero roots so that $rk(I - F) = rk(\tau_1) = rk(\alpha_1') = r$.

We can also factor $G^*(L)$ in a similar way using $G\ (L) = (G^*(1)L + (1-L)G^+(L))$ and if $G^*(1) = (I - F)A$ then we have a stable error correction representation which produces the same long-run parameters as the rational expectations system.

$$(4.113)\quad \Delta y_t = (I - F)(Ax_{t-1} - y_{t-1}) + G^+(L)\Delta x_t + u_t$$

where $(I - F)$ is singular.

Due to the disequilibrium structure of the model the endogenous and exogenous variables are either jointly cointegrated or error correcting between the target and the actual value. When the endogenous variables are cointegrated it may be possible to derive the long-run parameters of the rational expectations model from an error correction representation like (4.113), but such a representation is likely to be inefficient unless the rational expectations restrictions can be imposed. We will also find in section four that a more natural error correction from exists which allows the parameters of the system to be estimated more efficiently.

The error correction model is a specialisation of the backward solution which has often been used to estimate rational expectations models, for example see the employment model in Nickell and Layard(1985). In general, such estimates of the backward solution do not usually reveal the deep parameters, the method due to Kollintzas(1985) is a notable exception. In this light, equation (4.18) and (4.19) provide an approach to the problem which is efficient and which allows all of the parameters of interest to be estimated. In the next section we will use such results to rationalise a model of output and employment and that model will then be estimated in section three.

3.2 Definition of the equilibrium model of output and employment

The cost minimising approach presented in the previous section can be seen as one of three possible ways to derive the first order condition equation (4.11) for an output employment system. The other two methods involve either maximising an intertemporal profits function or the use of the simple artefact of assuming a dynamic macro relationship which is the same as the other two solutions.

Given acceptance of the loss function in the previous section or the model from chapter 1 we will start from the first order condition (4.11) which in the case of the output employment equation becomes:

$$(4.21) \qquad E(Q_o \begin{bmatrix} q_t \\ l_t \end{bmatrix} - \beta Q_1' \begin{bmatrix} q_{t+1} \\ l_{t+1} \end{bmatrix} - Q_1 \begin{bmatrix} q_{t-1} \\ l_{t-1} \end{bmatrix} = H \begin{bmatrix} q_t^* \\ l_t^* \end{bmatrix} \mid \Omega \;)$$

where q_t and l_t stand for the log of actual output and employment, the * denotes target or equilibrium values and the matrices Q_i and H are two by two.

Equation (4.21) is in logs which means that it does not have a direct analogue based on profit maximisation, but it does have two possible interpretations. The first assumes that (4.21) is a structural form of unknown origin which corresponds to the solution of the loss function and that is the model derived in Chapter 1 the second relates to the cost minimising model of the previous section. The problem with that method is that we require a model for the target variables, as our cost matrices relate to adjustment cost and disequilibrium cost or cost of false trading to mirror the results of Chapter 1.

The models in the previous section give some credence to the ideas that wages and prices are jointly determined with inventories and vacancies. In turn output and employment decisions are then conditional on such values. Output is chosen in the context of quarterly data as being a variable over which plans are made because firstly it is costly to adjust and secondly it is a relatively free variable when price is set and inventories are used as a buffer stock. Inventory investment decisions are then seen to involve a longer time profile, while in the short-run inventory adjustment is assumed to take up the slack. Employment has long been considered as a slow adjusting variable, the only question is whether employment decisions are independent of other factor demand decisions. Partialing out other factor demands either implies that they adjust instantaneously or that they are separable from other such decision. Separability may be relevant for the majority of investment decisions, as labour costs at the inception of large projects are of second order importance and when the project has been implemented the investment cost has been met so that then there may be no substitutability between labour and capital. Decisions are of the putty clay type which means that labour costs may only impinge on the decision making for a small period of time. Even when a project is running short-term investment decisions may be more dependent on the initial capital decision than on labour costs.

So far in this section we have dealt with standard relationship in which agents make choices conditional on a vector of exogenous variables which are given. In the second instance it is assumed that agents attempt to minimise a loss function subject to costs of adjustment and costs of being away from equilibrium. Implicit in this is the notion of a two stage problem in which agents know the true equilibrium relationship or their notional response function, but due to costs of adjustment or price stickiness they are not able to attain that equilibrium immediately. The observed model represents the adjustment process to equilibrium which is based on the feedback responses generated by solution of the control problem. The equilibrium which is being chased is dependent on the concept of equilibrium we wish to select, it is obviously an attainable point in price quantity space, but what determines that point depends on the period at hand, the speed of adjustment and the perspective of the agent. A standard neo-classical framework would suggest that the agent is attempting to attain the full equilibrium of the system, but that seems a strong assumption to make and it would appear to be inconsistent with the idea that the agent only knows his own notional reaction function or such relationships in the market in which he is operating. If we do not attain full equilibrium we may still observe the agent attaining points on his function which are consistent with free unconstrained action, but once full market clearing is denied as a useful equilibrium concept we need to assume that the agent needs to take account of the fact that it may not be possible to attain a point on a notional curve. If the data used are of a relatively short period relative to the known period of adjustment of the market or a particular contract holds for a given market then it is quite feasible that the min condition is the appropriate condition to choose.

If there is inertia in the system, so that prices do not adjust to clear markets, then the non-forced trading condition holds and trades occur at the minimum of supply and demand curves. Malinvaud(1977) and Barro and Grossman(1974) deal with such disequilibrium models in which agents are constrained in the short-period by the order of price and quantity adjustment being reversed. If quantities adjust to clear markets first, then trading occurs at false prices and if agents cannot be forced to purchase more than they wish the short side of the market dominates. The multiple contractions produced by such quantity adjustment lead to sequence of rationed equilibria which are generally not consistent with full equilibrium of the system. In addition to this, the effect of non attainment of equilibrium in other markets will spill over into the market of interest. If the min condition is to be of use then we must concede either price or quantity adjustment in the market concerned leads to at least one notional curve being attained. Otherwise we are left with any point in the wedge to the left of the min condition being a valid equilibrium point. In principle that is not a problem as spillovers or fixed prices could lead to that being the case, but in practice any point being an equilibrium seems only to be consistent with total inertia. The min condition is then accepted as the long-run or medium-run or temporary equilibrium to which agents adjust.

Maddala(1983) revues the methods used to estimate models of disequilibrium, which includes switching regressions, continuous switching and the latent variables approach of Hendry and Spanos(1980). The min condition implies a discrete switching model which complicates our analysis considerably, because in our case the error term which helps to identify the appropriate curve is a theoretical error related to the equilibrium condition and not the equation error which is estimated. The min condition is also not appropriate, because the data to be used are aggregate time-series data which relate not just to a single market, but to a sequence of markets in different states of equilibrium. In the aggregate, if their is orderly trading, then Muellbauer(1979) has shown that the discontinuous function defined by the min condition is replaced by a continuous non-linear function bounded to the right by the min condition(see fig 3.1 below).

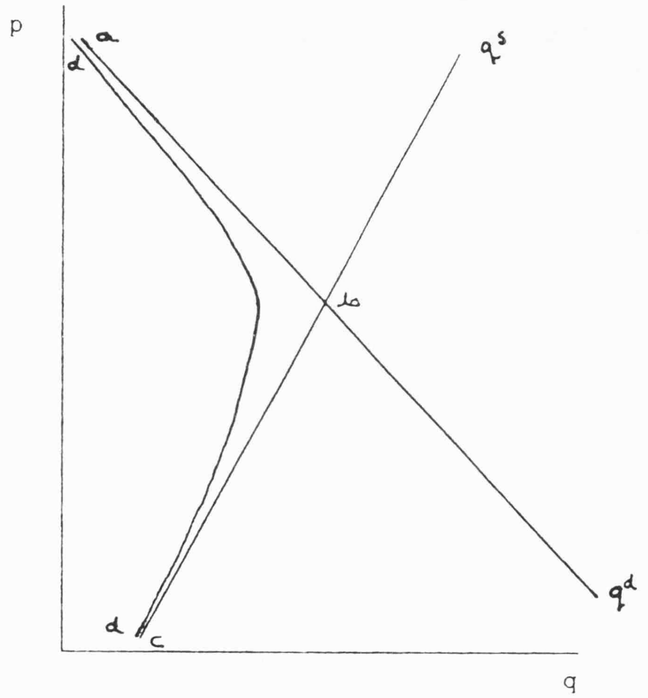


FIG 4.1 A Model of Continuous Switching

The min condition is defined by the v shaped curve abc which is the minimum of supply and demand with respect to quantity and the equilibrium relationship defined by continuous switching is given by the dd curve. The continuous switching model has recently been applied to models of the labour market by Muellbauer and Winter(1980) and Nickell and Andrews(1983). Muellbauer assumes a distribution of firms and households experiencing different degrees of rationing which in the aggregate produces a continuous shift between markets in excess demand and those in excess supply. Hence we observe the continuous line dd rather than a discrete shift in the aggregate. The model is also consistent with the notion of a natural quantity for a good or natural rate of unemployment in the labour market, as we never attain a full equilibrium in which all markets clear.

The simplest employment model is derived by assuming a uniform distribution of firms, otherwise an excess demand term appears as a function of the density and distribution function selected. Nickell and Andrews deal with the case in which the firms are distributed normally, but here we follow Muellbauer(1979) and assume a uniform distribution. Muellbauer and Winter show that the following relationship exists between employment(l), labour demand(l^d) and vacancies(v):

$$(4.21) \qquad l = l^d - v$$

For a given labour demand function we can derive the actual employment relationship. Therefore:

$$l^d = Z_2^d A_2 + \epsilon_1$$

Where ϵ_1 describes the unexplained element of demand.

Taking all variables to be in logarithms labour is chosen initially to depend on wages(w), prices(p), vacancies and the change in inventories(Δi). Vacancies are included to take account of spillovers from other labour markets and inventories to take account of similar factors from the goods market. Substituting out for the demand curve in (4.21) gives the static employment equation associated with the model.

$$(4.22) \qquad l = a_{20} + \underset{(-)}{a_{21}}\Delta i + (\underset{(+)}{a_{22}} - 1)v + \underset{(-)}{a_{23}}p + \underset{(+)}{a_{24}}w + \epsilon_1$$

and in a real employment equation $a_{23} = -a_{24}$

The demand relationship is similar to that described in Muellbauer and Portes(1978) (which is reproduced in Branson(1979)) in which employment depends on real wages initial endowments and a number of expectational variables related to the state of markets.

The goods market can be treated in a similar way if we assume that stocks act as a buffer and if the extent of the market is limited in the sense of Arrow(1959). Hence, we observe the demand function and firms have downward sloping demand curves. In the short-run prices do not adjust, because any slack is taken up by stock holding. Such behaviour is quite consistent with the observation that most firms do not continuously change prices and the possibility that firms target the market to satisfy demand. Such a story is quite compatible with firms facing costs in adjusting output and a reputation cost when they do not meet demand. It may be possible for customers not to be able to find a particular brand, but it is unusual for them to be completely rationed in a good, because of the existance of substitutes. Individuals may make second best decisions or delay purchase, but that will rarely effect employment or produce a strong spillover into other markets, because the consumer will either buy another brand or order the good they wish to buy. Consumption is usually delayed rather than not undertaken. If speculation in goods is small or inventory accumulation fairly constant then the non-constant/non-trend component of the inventory series mainly takes account of disequilibria. If o^d and o^s are the demand and supply of goods and q^s and q^d are the demand and supply of output then:

$$o^s = q^s + i_0$$

$$o^d = q^d$$

$$o^s - o^d = q^s + i_0 - q^d \quad = i_1$$

$$q^s - q^d = \Delta i$$

The analysis assumes that hording is either expensive, constant or that inventory investment can be modelled by a trend, but in reality such assumptions are likely to be affected by the governments tax position and interest rates. In this section we will assume that such factors are not important, but later on we will deal with them. Using the disequilibrium argument for stock building we can derive a similar expression for output to that derived for the employment equation:

$$(4.23) \qquad q = q^d + \Delta i$$

Given a particular output demand equation we have assumed in the goods market that the analogue of dd is the notional demand function. It is assumed that demand is either met out of output or stocks which implies an aggregate relationship slightly to the left of the true demand curve.

$$q^d = z_1^d A_1 + \epsilon_q$$

where ϵ_q is the unexplained component of output demand

Taking all variables in logarithms output demand is assumed to depend on real wages, inventory accumulation and vacancies. Where vacancies determine spillovers from the labour market and inventory accumulation spillovers from other goods markets. Otherwise they can be thought of as taking account of the state of these markets. Substitution of a logarithmic demand function produces the following model of output:

$$q = a_{10} + \underset{(-)}{(a_{11}} + 1)\Delta i^{1} + \underset{(+)}{a_{12}}v + \underset{(-)}{a_{13}}p + \underset{(+)}{a_{14}}w + \epsilon_{q}$$

In selecting targets of this type we do not presume market clearing, but in the same way as the min condition presumes that we are either on the demand or the supply curve the equilibrium or notional relationships assume that the observations are consistent with that equilibrium concept. Hence, consumers are not forced to consume more of a good than they desire or firms to take more labour than they would wish. In the case of manufacturing output this seems to be reasonable, as consumers usualy have no control over merit goods or goods centrally provided. Hendry and Spanos(1980) deal with this problem, as they use the notion suggested by Frisch which does not limit our observed model to lie on the wedge given by the min condition or the continuous line given by dd. The Hendry and Spanos approach treats the disequilibrium phenomena as a latent variable, the resultant model is an error correction model. The approach presented here is similar, as the first order form of the rational expectations model has an error correction representation and the short-run model does not constrain the results to satisfy the min condition. Hence, actual output may lie anywhere in output-price space, but in the long-run agents are constrained to attempt to hit the demand curve for output. The short-run model of output and employment is based on the

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[1]In practice $(a_{13}+1)$ will be a composite which partly depends on the fact that the actual series used is not the logarithmic difference, but the log of the difference in levels.

solved rational expectations form (4.18) and (4.19) which then embeds the long-run relationship within it:

$$(4.24)\quad \begin{bmatrix} q_t \\ l_t \end{bmatrix} - F \begin{bmatrix} q_{t-1} \\ l_{t-1} \end{bmatrix} - \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} = \begin{bmatrix} h_{1t} \\ h_{2t} \end{bmatrix}$$

$$(4.25)\quad \begin{bmatrix} h_{1t} \\ h_{2t} \end{bmatrix} = F\, B_o \begin{bmatrix} a_{10} & a_{11}\{+1\} & a_{12} & a_{13} & a_{14} \\ a_{20} & a_{21} & a_{22}\{-1\} & a_{23} & a_{24} \end{bmatrix} \begin{bmatrix} 1 \\ \Delta i_t \\ v_t \\ p_t \\ w_t \end{bmatrix} -$$

$$G_3 \begin{bmatrix} \varepsilon_{1t-1} \\ \varepsilon_{2t-1} \end{bmatrix} + \beta F \begin{bmatrix} h_{1t-1} \\ h_{2t-1} \end{bmatrix}$$

Next we will deal with the estimation of the model associated with equations (4.24) and (4.25)

4.3 A Rational Expectations model of output and Employment.

In this section we deal with the Muellbauer form of the first order condition which we use to produce initial estimates of our output employment model by instrumental variables, then the system is estimated efficiently by maximising the concentrated Likelihood function associated with (4.17) and (4.18) above. The results are then analysed, a test for serial correlation presented and the models re-estimated using an approximate adjustment for serial correlation.

Initial estimates are derived using a transform of the first-order condition by the errors in variables method due to

Wickens(1982) which is the approach used by Muellbauer and Winter(1980). If we work the expectations operator through (4.17) in section one, then:

$$(y_t - F\, y_{t-1} - u_t) - G_1(y_{t+1} - Fy_t - u_{t+1}) = FB_oAx_t - FB_oD\varepsilon_{t+1}$$

If we then pre-multiply each side by F^{-1} and re-arrange the terms in y:

$$(\beta F + F^{-1})y_t - (y_{t-1} + \beta y_{t+1}) = B_oAx_t - B_oD\varepsilon_{t+1} + F^{-1}u_t - \beta u_{t+1}$$

then by subtracting $(1+\beta)y_t$ from the first and second term on the rhs of the equation above we have that:

$$(\beta F + F^{-1} - (1+\beta)I)y_t - (y_{t-1} + \beta y_{t+1} - (1+\beta)y_t) =$$

$$B_oAx_t - B_oD\varepsilon_{t+1} + F^{-1}u_t - \beta u_{t+1}$$

$$B_oy_t - (\beta\Delta y_{t+1} - \Delta y_t) = B_oAx_t - B_oD\varepsilon_{t+1} + F^{-1}u_t - \beta u_{t+1}$$

where $B_o = (\beta F + F^{-1} - (1+\beta)I)$

$$y_t - B_o^{-1}(\beta\Delta y_{t+1} - \Delta y_t) = Ax_t - D\varepsilon_{t+1} + F^{-1}B_o^{-1}u_t - \beta B_o^{-1}u_{t+1}$$

(4.31) $y_t = B_o^{-1}(\beta\Delta y_{t+1} - \Delta y_t) + Ax_t + \xi_t$

where $\xi_t = - D\varepsilon_{t+1} + F^{-1}B_o^{-1}u_t - \beta B_o^{-1}u_{t+1}$

In the context of the output employment equation, (4.31) is in the errors in variables form associated with Wickens (1982). The original expectations have been replaced by actual values which creates an error in variables, but the structural parameters can be estimated consistently by applying the generalised instrumental variables estimator to (4.31)(see Harvey(1981) or Sargan(1988) for an explanation of GIVE). The method can be much simplified by setting $\beta = 1$, so that the first term on the lhs of (4.31) becomes an acceleration in y. This is similar to the method used by Muellbauer and Winter(1980), except that they impose the coefficients on the acceleration term in an employment equation using cross section results. The instrument set we use involves six lags on output and employment and the current values plus four lags on wages, prices, vacancies and inventory accumulation.

The GIVE results of our output employment model are presented in the first column of table 1. The deep parameters seem to accord reasonably well with theory, except for a positive coefficient on price in the employment equation. The B_0 matrix which is directly related to F has negative roots which is incorrect when the model is based on agent optimisation and the model is not well specified. We will see in the next section that their are two possible reasons for such a result: firstly unit roots and secondly benefits associated with bringing forward production or consumption (see Kollintzas(1985) for an explanation). The two equations suffer from serial correlation which is predicted by the exclusion of any adjustment for the VMA error in (4.31), although the forward looking nature of the error structure should guarantee consistency when we estimate the model using instrumental variables. There is no guarantee that the VMA error is the sole cause of the serial correlation and additional tests would suggest that the model is not well specified. In particular, the output model fails the Sargan(1964) test for instrument validity at the 1% level($\chi(26) = 52.34$) and the employment equation at the 5%($\chi(26)=41.23$). In the light of serial correlation this is not surprising, as the test is a general test of misspecification. Hence, rejection of the instrument set is only valid when the other specification tests are satisfied.

The errors in variables approach should consistent, but inefficient parameter estimates when the model is correctly specified. To produce estimates which are fully efficient we need to take account of the moving average error term implicit in (4.31), but there are two problems with using such methods. Firstly the moving average term has roots inside the unit circle which can only be dealt with when an exact maximum likelihood method is used (see Pesaran (1978)). Secondly a difficult non-linear procedure is further complicated by imposition of the rational expectations restriction which require F and B_o to be inverted at each step. At this point we would suggest that equally efficient estimates can be derived by iterating over equations (4.24) and (4.25) above. If the errors are normal or tend asymptotically to normality, then we have the following likelihood function:

$$(4.32) \quad \text{Log}(L) = -NG\ \text{Log}(2\pi) - \tfrac{1}{2}N\ \text{Log}|\Omega| - \tfrac{1}{2}\ \text{tr}(\Omega^{-1}\sum_{t=1}^{N} u_t u_t')$$

where $u_t = y_t - Fy_{t-1} - h_t$ and $h_t = G_o^* x_t - G_3\varepsilon_{t+1} + G_1 h_{t+1}$

We can simplify the problem by concentrating out Ω and then replacing it by a consistent estimator or we can use a consistent estimator to produce a quasi likelihood function which should be optimal in large samples(see White(1982) or Heijmans and Magnus(1986)).

$$\text{Log}(L_c) = C - \text{Log}|\Omega|$$

Hence the objective function that we are going to minimise using the Quasi Newton Method suggested by Gill Murray and Pitfield(see Wolf(1978) or Sargan(1988)) is minus the concentrated likelihood divided by an adjustment factor @ which scales the problem to lie between zero and one(these are limits which are optimal for the Nag routine E04JBF).

$$\text{Log } L^{*} = \tfrac{1}{2}N\,(\log|\hat{S}|)/@$$

$$\text{where } \hat{S} = 1/N\left(\sum_{t=1}^{N} u_t u_t'\right) \quad \text{is a consistent estimator of } \Sigma$$

The method chosen has the advantage of not requiring first derivatives which are difficult to compute given the infinite lead and the complexity of the non-linearities. Quasi-Newton methods use the steepest descent approach and when we have a quadratic objective function the method selects an optimal conjugate direction (they are H conjugate, so that the search directions form a basis of the parameter space). Variable metric methods are a special case of the steepest descent approach which adjust for non-singularities of the Hessian or second derivative matrix. The method of Gill Murray Pitfield method uses a rank one update to compute a new estimate of the inverse of the Hessian based on the Cholesky decomposition. The update approximates the inverse by selecting the diagonal factors in such a way that it is guaranteed to be non-singular and it converges to the true matrix in a neighbourhood of the optimum. The inverse of the Hessian can be used to produce an estimate of the variance-covariance matrix of the parameters.

$$\underset{N \to \infty}{\text{Hes} \longrightarrow \text{IA}} \qquad \text{s.t.} \qquad \text{Hes} = \left.\frac{\delta^2 \text{Log}(L^{*})}{\delta\theta'\delta\theta}\right|_{\theta = \hat{\theta}}$$

then:

$$(Hes)^{-1} = V(\hat{\theta}) \qquad s.t. \qquad Hes^{*} = Hes/@$$

$$\sqrt{N}(\theta - \theta) \sim N(0, V(\theta)) \quad \text{and} \quad t = \frac{\hat{\theta}_i - \theta_o}{\sqrt{avar(\theta_i)}}$$

The estimates of the variance are computed directly and indirectly using the Hessian and an estimate of the Hessian computed using a local approximation to the second derivative matrix in a neighbourhood of the optimum. The standard errors can only be expected to be approximate, due to the bias associated with the generated expectational variables and the associated parameters of the moving average processes of the exogenous variables(see Pagan(1984) for discussion of this problem). Sargan and Marwaha(1986) have produced some simulation evidence for the single equation case which shows that the bias may be small when either the roots of the exogenous variable processes or those of the rational expectations system are close to the unit circle, but the evidence is not strong enough to suggest that the problem can be ignored. Unfortunately the second derivative matrix cannot be computed directly and estimation of the appropriate second derivatives is somewhat cumbersome as it also requires alternative estimates of the innovations. I believe that the standard errors are likely to be under estimates, although the effect on the likelihood of dropping certain variables would suggest that they are not too far from the truth.

In table 1 below the column 2 and 3 are estimates of equation (4.24) and (4.25), that have been derived recursively using a zero terminal condition and 51 future predictions. In terms of

Table 4.1 Output and Employment Models

MODEL	1	2	3	4	5
MANUFACTURING					
OUTPUT					
F_{11}		0.69384 (0.04066)	0.67837 (0.07006)	0.41628 (0.07370)	0.70097 (0.06460)
F_{12}		ZERO	-0.17758 (0.16557)	-0.28047 (0.13886)	-0.21779 (0.06779)
B_{o11}	-0.1685 (0.1538)	0.28394	0.27333	0.92612	0.16832
B_{o12}	-1.6546 (1.3531)	0.00000	0.20126	0.55754	0.15781
D_{11}		0.05438	-0.06221	-0.07471	-0.31898 (0.23396)
D_{12}		0.06613	0.05159	-0.00831	-0.43366 (0.43751)
D_{13}		0.40608	0.44822	0.08451	-2.86588 (2.98395)
D_{14}		0.63728	0.67737	0.18038	1.56476 (1.72885)
a_{11}	0.0942 (0.0286)	0.36652 (0.07464)	0.12668 (0.13184)	-0.03334 (0.05669)	-0.15150 (0.21619)
a_{12}	0.0542 (0.0135)	ZERO	0.02608 (0.02265)	0.03259 (0.00931)	-0.05181 (0.05279)
a_{13}	-0.6337 (0.0385)	-0.66542 (0.05292)	-0.73457 (0.06699)	-0.38821 (0.02632)	-0.74501 (0.06118)
a_{14}	0.6542 (0.0316)	0.66542 (0.05292)	0.73687 (0.05756)	0.16712 (0.03270)	0.75353 (0.05204)
SEASONALS	3.6538 (0.1992)	1.18157 (0.11982)	1.55309 (0.24026)	1.25334 (0.08084)	2.14862 (0.38673)
	0.0401 (0.0261)	-0.61439 (0.12002)	-0.43460 (0.13245)	-0.12165 0.05117	-0.22657 (0.34692)
	0.0185 (0.0254)	0.20450 (0.05266)	0.03424 (0.09858)	-0.07015 (0.04919)	-0.33856 (0.23002)
	-0.0225 (0.0402)	-0.12755 (0.04153)	-0.07370 (0.04806)	0.00265 (0.01705)	0.00542 (0.13094)
TREND				1.04665 (0.06907)	

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F_{21}		0.05909 (0.00794)	0.06092 (0.01278)	0.07984 (0.01726)	0.05763 (0.01125)
F_{22}		0.87168 (0.02647)	0.86433 (0.03218)	0.88205 (0.03057)	0.93407 (0.00145)
B_{o21}	0.0282 (0.0714)	-0.06734	-0.06904	-0.15872	-0.04176
B_{o22}	-2.9240 (0.6293)	0.08127	0.06257	0.00025	-0.00056
D_{21}		0.18214	0.11912	0.13605	-0.47911 (0.27702)
D_{22}		0.06764	0.10025	0.06985	1.12376 (0.40736)
D_{23}		0.09376	0.20897	-0.01244	8.01957 (3.25270)
D_{24}		0.07389	0.12993	-0.19486	-3.90276 (2.21590)
a_{21}	-0.0216 (0.0133)	0.40957 (0.13444)	0.26715 (0.11934)	0.24843 (0.11015)	0.62823 (0.20030)
a_{22}	0.0212 (0.0063)	0.03009 (0.01209)	0.05125 (0.01997)	0.05658 (0.02034)	0.13846 (0.05070)
a_{23}	-0.0556 (0.0179)	0.01308 (0.04906)	-0.03339 (0.05079)	0.04444 (0.05739)	-0.03512 (0.09880)
a_{24}	-0.0593 (0.0147)	-0.14031 (0.04680)	-0.09260 (0.04380)	-0.20552 (0.07918)	-0.09572 (0.08357)
SEASONALS	8.6911 (0.0926)	0.21595 (0.21499)	0.44072 (0.21726)	0.35539 (0.20118)	-0.10157 (0.41160)
	0.0367 (0.0121)	-0.35894 (0.13927)	-0.28708 (0.13662)	-0.21317 (0.11044)	-1.08948 (0.34043)
	0.0448 (0.0118)	0.31469 (0.09922)	0.22410 (0.08706)	0.22892 (0.08744)	0.37968 (0.17001)
	0.0170 (0.0187)	-0.11189 (0.04979)	-0.09476 (0.05075)	-0.07444 (0.03946)	-0.39942 (0.13785)
TREND				0.17660 (0.14971)	
β	1.0000	0.51406 (0.07826)	0.54126 (0.07729)	0.57907 (0.06834)	0.77397 (0.05059)

LOG-LIKELIHOOD		408.04865	408.79339	424.33395	417.77141
VARIANCE-COVARIANCES	0.00054	0.00027	0.00027	0.00020	0.00027
$x10^{-4}$	0.00000	0.17507	0.16789	0.20652	0.19503
$x10^{-4}$	1.176	0.07259	0.07114	0.07225	0.06020
LM(1) =		14.67957	13.45434	11.69369	8.26012
LM(2) =		17.97791	16.33301	15.08973	11.31238
LM(4)[2] =	37.4882	29.19508	21.76417	23.88619	20.79480
LM(4)[2] =	21.2257				
LM(5) =		32.05677	23.52046	26.41067	22.12342

regression variance there do seem to be benefits from using the system method and such gains may be further advanced by the inclusion of extra exogenous variables. The results do not appear to be sensitive to the terminal conditions, as reasonable perturbations in them do not greatly affect either the estimates or the likelihood function. This may in part be due to the large value of the discount factor in these models. The maximum interval estimate of the discount factor in column 3 is .696 which implies an quarterly discount rate of 47%. There are three possible explanations of this: firstly a huge risk premium, secondly that the future or estimates of the future are highly discounted and thirdly that the discount rate is overcoming some non-stationarity. All three may be relevant, especially non-stationarity which can either relate to unit roots in the system

[2] Separate tests of serial correlation are presented for each equation in the IV case.

or to non-stationarity in the exogenous variable processes. We already know that their is some non-stationarity in the exogenous variable processes which is compounded by our inability to invert the AR parameters and in the next section we will see that cointegration in the endogenous variables may also be a problem.

In terms of the output equation the results in column (2) and (3) seem to be theory consistent when they are compared with the models in both chapter 1 and this chapter, but such conclusions must treated with care as the model suffers from serial correlation. The smoothing procedure associated with the forward lead in h_t seems to remove the higher order serial correlation, but it does not get rid of first order effects (the test for first order serial correlation is $\chi^2(4)$ so that the statistic at the 5%/1% level should be compared with 9.49/13.3); the test is explained in appendix B of this chapter. Unfortunately serial correlation causes the parameter estimates to be inconsistent, because there is a lagged dependent variable. Consistency is a minimal requirement for any model which means that we cannot be confident about the results in column (2) and (3), though they should not be ignored as it is always possible for a poorly formulated estimator to produce useful results. This possibility would be supported by the cointegration results presented in the previous chapter and the view expressed recently by Sir Karl Popper that we are observing long-run propensities which may not depend on classical statistical foundations. The results presented may have some validity for comparison or when the degree of the inconsistency is small. For this reason, it is difficult to accept the conclusion that employment does not affect output, especially when we compare these results with those in column (4) and (5). The price homogeneity restriction on the output equation is satisfied and this result does not change when we introduce the ε_{t+1}s without restriction.

In the case of the employment equation some of the parameters are not theory consistent, as the price and inventory coefficients have the wrong sign. The price coefficient should mirror the coefficient on wages when we have a real model of labour demand in the long-run, otherwise agents suffer from money illusion. In this instance the price coefficient is insignificant and such non-heterogeneities are compatible with certain strong Keynesian theories, but more realistically these results may be due to excluded cost or relative price variables. While homogeneity is critical in the context of a factor demand model, the influence of inventories is less clear. The positive sign on inventory accumulation may be due to investment in inventories or speculation. Such investment or speculation would lead to a higher demand for output and so higher levels of employment.

A further check on model performance is given by looking at the equilibrium values and the roots of the system which need to be both real and positive. In general, the equilibrium values for the output and employment equations are quite reasonable, they suggest in the output case that demand has always been met during the estimation period. The roots to the system associated with Column(3) are μ_i = .771345 ± .04298i so that they are almost the same for output and employment, except for a small imaginary term which either suggests that the cost matrixes are not positive definite or that the equation needs to be transformed to take account of further asymmetries.

The deep parameters presented in columns (2), (3) and to a lesser extent (4) and (5) are consistent with the target real demand equation suggested in section two, as it depends positively on inventories and vacancies and negatively on prices in terms of wage units. The introduction of a time trend causes a large jump in the likelihood which is clearly significant. The trend appears to be especially important for the output equation as its inclusion reduces the test statistic for first order serial correlation so that it is below the 1% critical value and the discount rate falls slightly to 40%. The roots of the system are $\mu_i = .649165 \pm .178445$ which is consistent with the optimisation story, as they are both real and less than unity. The inclusion of the time trend assists in stabilising the model and it reduces serial correlation slightly, but the output equation does not satisfy the homogeneity constraint and the coefficient on inventories becomes negative which indicates that the buffer stock effect is dominated by spillovers. The coefficients of the employment equation appear slightly more plausible as homogeneity would be satisfied if the standard errors on wages could be used to determine the test. In terms of fit the model with the time trend is to be preferred, but the test for serial correlation is not convincing.

We can reformulate our original model to test the proposition that the appropriate information set is being used. In its most general form the test is a test of specification, as it determines whether including the contemporaneous predictions has an effect on the model structure. Take the usual model:

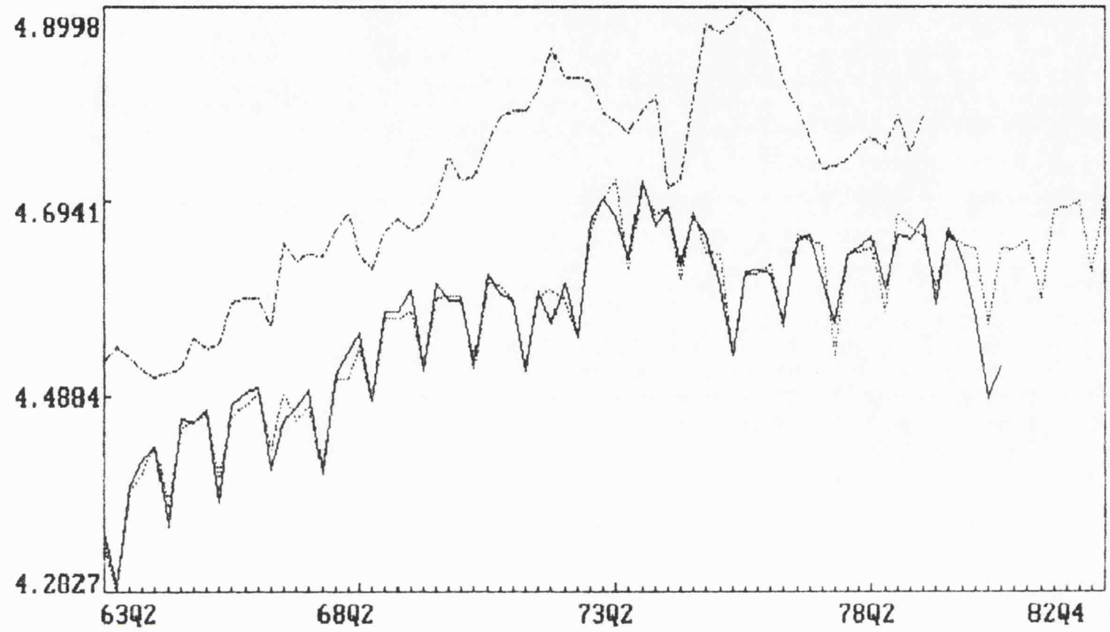
$$y_t = Fy_{t-1} + h_t + u_t$$

then h_t determines the nature of the test. If we need a general test of misspecification, then we can compute the following relationship:

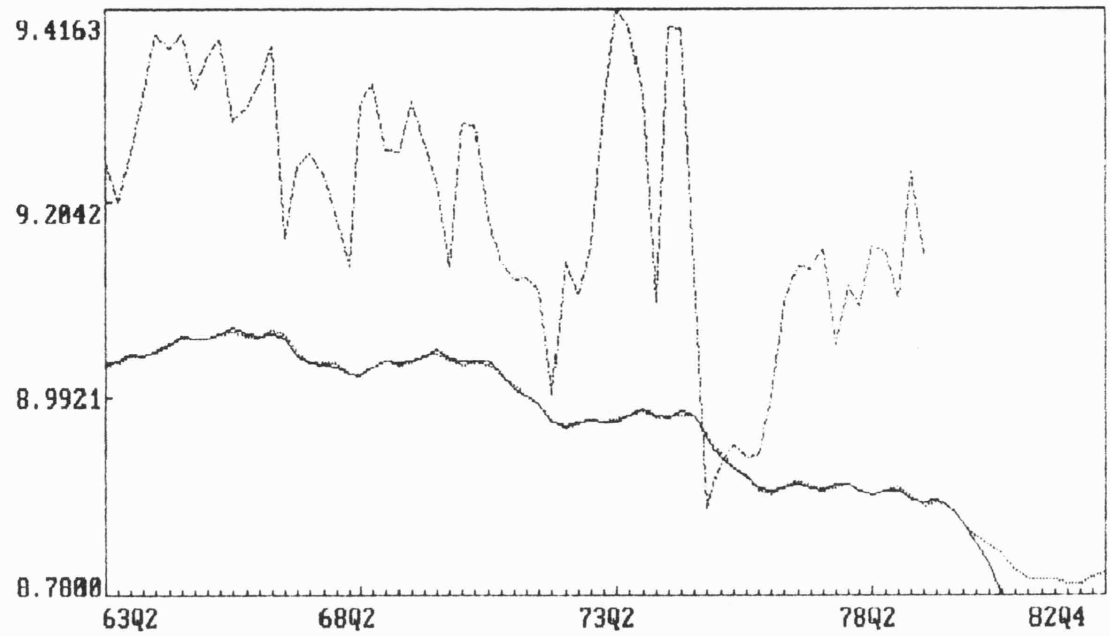
$$(4.33)\quad h_t = G_o x_t - FBD\varepsilon_{t+1} - FBD_1\varepsilon_t/\beta + G_1 h_{t+1}$$

If D and D_1 are estimated unconstrainedly in (4.33), then we would have a general test of misspecification which depends on the validity of the rational expectations restrictions, the appropriateness of the conditioning and the period over which the expectations are taken. When $D_1 = 0$ we are dealing with the model presented in column 5, so that a Likelihood ratio test between 3 and 5 determines whether the restrictions are relevant or not. The test statistic is 17.1 which fails when the test is set at the 5% level, but is easily satisfied when compared with a $\chi^2(8)$ value at the 1% level(20.09). The unrestricted model is better formulated, as it satisfies all the tests for serial correlation at the 5% level and the discount factor is more reasonable with an upper interval estimate of around .87 which implies a rate of return of 15%. In the long-run the Output equation is compatible with a demand equation which has an own price elasticity of .75 which is similar to the model in column (3) and price homogeneity is also satisfied. The inventory and vacancies terms have negative coefficients which is contrary to the theoretical assumptions, as it suggests that inventory investment is competing with and vacancies reducing output demand. The vacancies and inventory accumulation coefficients are not

FIGURE 4.2 Plots of Equilibrium, Fitted and Actual values for Output and Employment models in column 5 of Table 4.1 that include innovation parameters estimated unrestrictedly



actual output ——— fitted .... equilibrium -.-.-



actual employment ——— fitted .... equilibrium -.-.-

significant and when this is combined with the large standard errors on the innovations it suggests that it may be better to impose the rational expectations restrictions on this model. The long-run employment equation seems to gives an important role to unexpected events, the innovations appear to be significant and they suggest the unanticipated increases in the real wage and inventories reduce employment demand and unanticipated increases in vacancies increase employment demand. Anticipated inventory accumulation which is due to investment, then raises demand the vacancies term takes account of supply and spillover effects and anticipated wage and price effects are not significant. We can see from Figure 4.2, that the models produce a reasonable fit and in this instance the equilibrium value appears to make some sense. In the case of the output equation demand always exceeds the level of output which is what we would expect when stocks are taking up the slack. The employment equation is less believable as it suggests excess demand for labour through out most of the period, but the innovations have not been included in the computation of the equilibrium model.

If we look at the roots of the system, then we find that μ_i = .83189 ± .169377 so that we have one stable root and one unit root. Unit roots cause problems, because they imply a zero root in the B_o matrix, but in this case that cancels out the non-stationary path, so that the exogenous variable parameters are identified. It must be noted, that an alternative maximum was discovered with a likelihood value of 416 and a strong correspondence, suggestive of cointegration between the long-run coefficients of the two equations. The B_o matrix coefficients suggest that the unit root is associated with the employment equation and the negative employment feedback in the output equation also makes sense in the context of cointegration.

Table 4.2 Alternative dynamics for Employment models

Sargent (1978)	$\mu_1 = .957$ $\mu_2 = .409$	Un-adjusted data for U.S. employees on private agricultural payrolls
Meese (1980) (static expectations)	$\mu_1 = .967$ $\mu_2 = 0$	Seasonally adjusted data for U.S. production workers on private non-agricultural payrolls.
Mendis and Muellbauer (1982)	$\mu_1 = .819$ $\mu_2 = .786$	Un-adjusted data for British Manufacturing employment (in logs)
Nickell (1984)	$\mu_i = .85(\cos(\theta) \pm i\sin(\theta))$, $\theta = 23.5°$	Un-adjusted data for U.K. Manufacturing employment
Column(3)	$\mu_{11} = 0.772554(\cos(\theta) \pm i\sin(\theta))$ $\theta = 3.19°$ $\mu_{12} = 0.0$	
Column(4)	$\mu_{11} = 0.82761$ $\mu_{11} = 0.0$	Model with time trend
Column(5)	$\mu_{11} = 1.00126$ $\mu_{12} = 0.0$	Model with innovations in the exogenous variables

we use the same data as Nickell(1984)

For column(3) the solution implies a stable model with highly damped cycles, in the trend model both equations are stable and in the innovation case we have the unit root. As Nickell (1987) states, the dynamic results in Table 4.3 seem to accord with theory, though the effects are somewhat large in the case of Sargent (1978) and Mendis and Muellbauer(1982) and the model in column(5). The B_0 matrix above indicates for the models here, that the larger roots are associated with the employment equations, so that adjustment is slower and in the innovation model infinite. Such large roots may be a sign of cointegration which suggests that the system should be re-specified in terms of the output employment ratio and the difference in employment. Large roots may indicate model reformulation rather than the negation of the theory; we will deal with this in the next section. Nickell suggests that large roots are due to aggregation which would explain the stability of Meese results which are based on industry data and which only require one lag on employment. The models due to Sargent(1978), Mendis and Muellbauer(1982) and Nickell(1984) include a second order lag in employment, so that the regular solution requires two stable roots. Nickell justifies this on the grounds of aggregation, Sargent and Muellbauer and Mendis on omitted serially correlated effects and we present similar results in section 5.

It is only possible to use the coefficient restrictions on the D matrix as a test of the rational expectations restrictions when the objective function or structural equations do not depend directly on the innovations. In reality the test of the rational expectations hypothesis is far more complex than is suggested by the restrictions on D, as it involves a number of assumptions about model conditioning, the period of expectation, aggregation across agents and the regularity of the model. The structure assumes a regular solution and endogeneity is bound up with the period of the expectation. When expectations are determined at t-1 we should either replace current xs by their predictions or replace ε_{t+1} with ε_t. The first suggestion produces a model which is not nested, the latter presumes that t-1 is the period of the expectation as D is set to zero. When we estimate (4.33) with D restricted and D_1 freely estimated, then we find that the coefficients on the contemporaneous innovations are individually insignificant and that the likelihood only increases to 411. The test can be seen as Hausman style test of strict exogeneity, while the test of the rational expectations restrictions can be thought of as a test of weak exogeneity when the models parameters are stable. The test statistic for endogeneity, conditional on the rational expectation hypothesis is 4.8 which is clearly not significant when compared with the usual critical values for a $\chi^2(8)$ random variable. This in combination with the insignificance of individual innovations seems to indicate at least strict exogeneity and if the model in column (5) is preferred we may even have weak exogeneity; the invariance of the paramters in the y equation to changes in the parameters of the x process. The two conditions together can imply strong exogeneity which can be tested by estimating the model with D and D_1 in (4.33) unconstrained. For strong exogeneity we also require Granger non-causality of xs by the ys. The endogeneity test formulated above is not a powerful test of the period of expectations which suggests an alternative test under the rationality assumption. If we impose the same restriction on D and D_1, then (4.33) becomes:

$$(4.33)\quad h_t = G_o x_t - \varnothing FBD\varepsilon_{t+1} - (1-\varnothing)FBD\varepsilon_t/\beta + G_1 h_{t+1}$$

where $\varnothing = 0$ implies that expectations are formed at t-1 and $\varnothing = 1$ implies that they are set at period t.

If we estimate this model we find that $\varnothing = .56$ and the likelihood ratio test is highly significant at 5.88 which suggests either that the period of the expectation lies between t and t-1 or that there is differential information. When innovations are included separately as the model in column (5) shows, the results are only altered marginaly and the individual parameters are not significant.

The trend model certainly produces a better fit, but it still suffers from serial correlation which leads to the question of whether their may not be an alternative specification which performs better. The information set is a super set of some output and employment models, but it is too narrow to produce well formulated demand relationships and the results are also not fully consistent with the Keynesian macro model of the first chapter. In addition to extending the infromation set and possibly determining better models of the exogenous variables, there are three possible extensions which may improve the results: firstly to transform the model into a true cointegration form, secondly to adjust for serial correlation and thirdly to extend the loss function model to include more lags on endogenous and exogenous variables. The first issue is discussed in some detail in the next section, results for the autoregressive model are presented in the final section and the both the loss function and the equilibrium condition are extended in chapter 5.

4.4 Rational Expectations models in Error Correction Form

The first order condition for the Sargan(1982) model is given in a similar form to that presented in Kollintzas(1985). Therefore:

(4.41) $E(Q_o y_t - \beta Q_1' y_{t+1} - Q_1 y_{t-1} | \Omega_t) = E(Hz_t | \Omega_t)$

where $Q_o = H + (1+\beta)K$ and $Q_1 = K = Q_1'$ and K must be positive definite and H positive semi definite for a minimum(see appendix A4.1)

(4.42) $\lim_{T \to \infty} \beta^{\frac{1}{2}T} |E(y_T | \Omega_t)| \to 0$

(4.43) $\lim_{T \to \infty} E(H(y_T - z_T) + K(y_T - y_{T-1}) | \Omega_t) \to 0$

where (4.43) is the usual transversality condition which is satisfied if (4.42) holds.

If the information set is dated at time t with respect to the exogenous variables and we do not know u_t, then equation(4.41) will have a moving average error when we solve out for expectations. Therefore:

(4.44) $Q_o y_t - \beta Q_1' y_{t+1} - Q_1 y_{t-1} - Q_o u_t + \beta Q_1' u_{t+1} + \beta Q_1' D\varepsilon_{t+1} = Hz_t$

Reformulating (4.44) and lagging it provides the error correction form, where the equilibrium or target condition is given by $z_t = Ax_t$ and the form of D depends on the process driving the exogenous variables.

$$(4.45)\quad \beta Q_1\Delta y_{t+1} = Q_1\Delta y_t - H(y_t - z_t) + \beta Q_1' u_{t+1} - Q_o u_t - \beta Q_1' D\epsilon_{t+1}$$

Lagging (4.45) and multiplying through by $1/\beta Q_1^{-1}$ gives us the more usual error correction model which also has a moving average error. Therefore:

$$(4.46)\quad \Delta y_t = 1/\beta\Delta y_{t-1} - H^*(y_{t-1} - z_{t-1}) + u_t - 1/\beta(H^* + (1+\beta)I)u_{t-1} - D\epsilon_t$$

where $H^* = 1/\beta Q_1^{-1}H = (F + 1/\beta F^{-1} - (1 + 1/\beta)I)$ and $F = PMP^{-1}$. M is a matrix of stable roots associated with the solution to the second order difference equation(i) and P the associated eigen vectors(see Appendix A4.1)

In section 1 we saw that the symmetric form of the loss function reveals a saddle point solution which produces the usual backward forward rational expectations model; Sargan(1984) calls such symmetric results regular solutions. Sargent(1978) explains that the forward solution to the problem which satisfies both the transversality condition and the euler condition can be derived by feeding the unstable roots forward and the stable ones backward. The roots associated with that solution are linked to the roots of the matrix of coefficients on the correction term and the moving average error. Therefore:

$$H^* = (F + 1/\beta F^{-1} - (1+1/\beta)I)$$

$$= (PMP^{-1} + 1/\beta PM^{-1}P^{-1} - (1 + 1/\beta)PP^{-1})$$

for the appropriate choice of P

Diagonalising H^* using P^{-1} and P reveals the roots ϕ_i of H^* which are related to the roots of the second order difference equation(4.41). Therefore:

$$(4.47)\quad P^{-1}H^*P = \Phi = M + 1/\beta\, M^{-1} - (1+1/\beta)I$$

If we look at a single root ϕ_i from H* we can derive conditions under which the error correction system is related to the rational expectations model and the vector of endogenous variables is cointegrated.

(4.48) $\phi_i = (\mu_i - 1) + 1/\beta(1/\mu_i - 1)$

where $\phi_i \to \infty$ as $\mu_i \to 0$ and $\mu_i \to \infty$

and $\phi_i = 0$ if $\mu_i = 1$

The error correction term disappears when all the roots of the second order difference equation(4.41) are unity which leaves a vector ARMA(1,1) model:

(4.49) $\Delta y_t = 1/\beta \Delta y_{t-1} + u^*_t - (\beta/(\beta + 1))u^*_{t-1} - D\varepsilon_{t+1}$

where we have used the observational equivalence of an MA(1) error with roots inside the unit circle to a model with roots outside the unit circle.

Equation (4.49) is not consistent with the usual agent optimisation problem in which all costs are positive, as the regular solution negates the possibility of unit roots and with g unit roots the levels term in the objective function disappears. If (4.49) is poorly specified, then by differencing the ys to stationarity, the rational expectations objective function can be reformulated, but if it is well specified, then (4.49) is not based on an objective function in differences and second differences. In differenced form the first order condition would be the same as (4.41) and any number of unit roots could be handled in this way without affecting the nature of the solution to the rational expectations model. If we partition y into a g_1

vector of variables y_1 which are stationary in levels and a g_2 vector of variables y_2 which are stationary in first differences, then the loss function could be redefined in terms of the new variables and the error correction form of the first order condition would become:

$$(4.410)\quad \Delta y^*_t = 1/\beta \Delta y^*_{t-1} - H^*(y^*_{t-1} - z^*_{t-1}) + u^*_t - 1/\beta(H^* + (1+\beta)I)u^*_{t-1}$$
$$- D^* w_{t+1}$$

where $y^{*\prime}_t = [y'_{1t} : y^{*\prime}_{2t}]$, $y^*_{2t} = \Delta y_{2t}$, u^*_t, u^*_{t+1} are innovations in $y\overset{*}{s}$ and $z^{*\prime}_t = [z'_{1t} : z^{*\prime}_{2t}]$, $z^*_{2t} = \Delta z_{2t}$, and w^*_{t+1} innovations in the $z\overset{*}{s}$.

Equation (4.410) can be thought of as a model in terms of either flow effects which do not depend on the stock or growth and levels variables which can be controlled independently by the agent.

Reducing the order of H^* would eliminate the endogenous variables associated with a unit root, but that would only be reasonable if the system was triangular or the omitted series independent of the other endogenous variables. If the loss function involves cointegrated variables, then the associated singularity implies that the $rk(H^*) = r$ where $r \leq g-1$ and $\Delta\Delta y_t$ terms do not appear in the loss function. Equation(4.41) is then the appropriate first order condition(see appendix A4.2), otherwise equation (4.49) is the first order condition and H^* has full rank.

The roots of the system can be used to determine the structure or test whether the model is consistent with rational expectations theory. Unit roots are one indication of model misspecification and imaginary roots are not consistent with H^* being of full rank. The assumption that we are dealing with a cost minimising problem or a maximising one relates to all the roots of the system μ_i being greater than zero; otherwise there are benefits to bringing actions forward. The ability to detect such differences in the structure or consistency in formulation will depend on the efficiency and appropriateness of the method of estimation. Here, we will show that there are a number of limitations associated with estimating the first order condition.

The errors in variables method of Wickens(1982) has been used to estimate the first order condition, but that method does not take account of the moving average error and the innovation in the exogenous variables. A number of adjustments have been made to the errors-in-variables method to take account of these deficiencies, but the technique is limited in this context, because it does not satisfy all of the conditions associated with the optimisation problem. Nickell(1985) points out that (4.41) does not always satisfy the objective function, as the technique does not automatically select the roots associated with the optimal plan. The Lagrange-Euler first order condition is necessary, but not sufficient, as we also require the transversality condition (4.42) to be met.

If we look at stable roots $\mu_i < 1$, then H^* is guaranteed to be positive definite as the symmetric solution usually keeps the model away from that singularity, but the first order condition method does not impose such restrictions. If the roots μ_i lie in the interval $[1,1/\beta]$ for $0 < \beta < 1$, then they are not consistent with cost minimisation, as this implies that the roots of H^* are negative. Hence, some of the roots of H^* may appear to be negative when the reverse is true and this problem is further compounded when the method is inefficient and the roots are near the unit circle.

In practice such problems are likely to occur, because as Pagan(1984) explains both standard errors and t-statistics will be biased when innovations and expectations are introduced into single equation regressions or limited information models. Power and Ullah(1987) using Monte Carlo experiments show that such bias can be quite considerable and as a result of that, parameter estimates of the first order conditions are more likely to be linked with changes in sign(the parameters differ noticeably from their true values for a range of constructed models). In the previous section we found that the roots of the bivariate rational expectations model of output and employment where negative when the errors-in-variables method was used and positive when the system was estimated by a full solution method.

The above arguments relate to estimates of the rational expectations model, but they follow through with error correction models. Estimates of the simple error correction system will be inefficient when the data are cointegrated (see section 1 of chapter 3 and section 1 of this chapter for discussion of such issues) and the rational expectations solution needs to be considered when we interpret the coefficients of the correction term. Traditionally error correction models are presented as disequilibrium adjustment to a long-run equilibrium or steady state growth model; the consumption function of Davidson et al(1978) and the money demand model of Hendry and Mizon(1978) are interpreted in this way. In the long-run $s_t = s_{t-1} = s^*$ where the * denotes an equilibrium value and in the static solution $\Delta s_t=0$ so the long-run static equilibrium of equation(4.46) will be:

$$y^* = z^* = Ax^*$$

The long-run static solution to the rational expectations model is the same as this. In the dynamic steady state $\Delta s_t = \pi$, so that (iv) becomes:

$$(4.411) \quad y^* = Ax^* + (1 + 1/\beta)(H^*)^{-1}\pi$$

where π is a gx1 vector of growth rates.

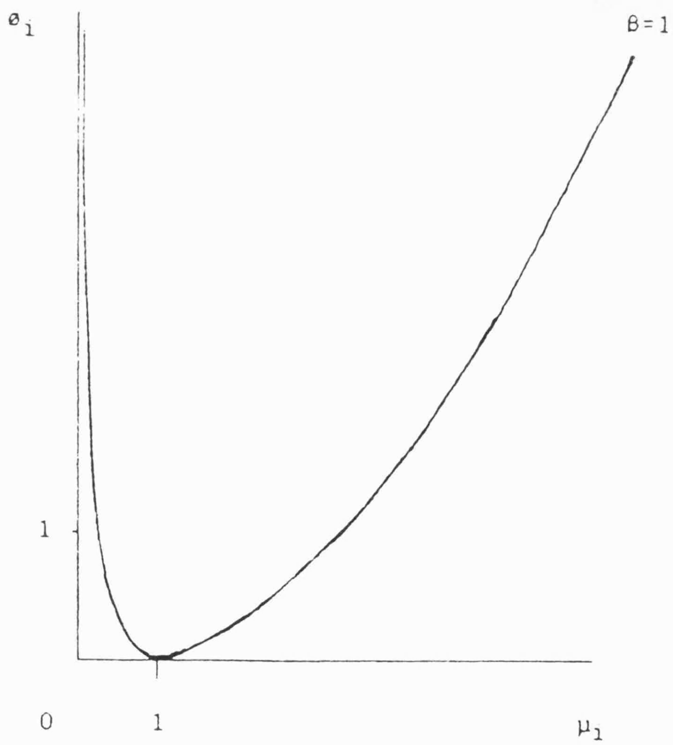
As Currie(1981) explains the static equilibrium is not effected by the stability of the process driving the model, but the dynamic solution will be. Currie notes that the dynamic solution of the original Davidson and Hendry consumption function is not stable and he specifies restrictions on the lag process which will impose stability. Drobny and Hall(1987) use such procedures to restrict their cointegrated model of wages. The Currie thesis may be valid, but it may not be relevant in the case of a correction form estimated from the first order conditions of a rational expectations model. If we assume that H^* is positive definite which is consistent with a cost minimising model then we require the coefficient on the growth term in (4.411) to be stable. The condition needed for dynamic stability depends on the roots of the matrix of coefficients $(1 + 1/\beta)(H^*)^{-1}$. Therefore:

$1/\phi_i < \beta/(1 + \beta)$ or $\phi_i \geq (\beta + 1)/\beta$ for all $i = 1, \ldots, g$

where β is a discount factor which should lie in the range [0,1] and the lower bound of ϕ_i is 2.

We require the correction term to have explosive roots greatly in excess of 1 if we are to have a stable steady state solution to equation (4.46).The problems presented above can be graphically illustrated by mapping out equation(4.48) for a particular value of β.

Figure 4.3 Relationship between ϕ and μ



If we look at the diagram above which selects β=1, then μ=.382 is associated with a unit root in ϕ which means that values of μ less than that are linked with explosive roots in the correction term. Imposing the restriction that the steady state solution to the error correction model is stable implies that the pair of symmetric roots associated with the regular solution are bounded from above in the case of $\mu_i < .268$ and from below in the case of its reciprocal $1/\mu_i > 3.639$. Obviously the upper and lower bounds change with β, as does the relationship in the roots. The stable steady state solution to (4.411) restricts the upper bound of the roots μ_i to lie in the range [.5,.268] for β in the open interval (0,1).

Discovery of a correction term with roots outside the unit circle may not be a sign of an unstable model, but an indication that it may be appropriate to interpret the error correction form as a reparameterisation of a symmetric rational expectations model. The observation of an unstable long-run steady state solution may also be an indication of forward looking behaviour and not a sign of instability.

Aggregation and Distributed lags in the equilibrium condition

In the light of the aggregation problem there are four possible strategies which one might follow, firstly to ignore it, secondly to include a set of extra exogenous variables which characterise the industry level information, thirdly to use general functional form or conditions that allow for perfect aggregation or finally to assume a functional form to account for the misspecification associated with it.

In section three of this chapter we assumed the problem away or rather we suggested that the problem was not relevant for macroeconomic analysis. Ignoring the aggregation problem either pre-supposes a method for perfect aggregation, such as the AIDS of Deaton and Muellbauer (1980a) or suggests that micro effects cancel out. We would like to presume that the later is correct and that would be consistent with Keynes (1936) discussion of the influence of individual income and wealth effects on the consumption function. In fact much of the discussion of aggregation in demand studies has depended heavily on distributional assumptions. In his study of the Demand for food Tobin (1950) discussed this issue and he suggested that perfect aggregation for a standard log linear model of food demand depends on constant income and population distributions. Hildenbrand (1983) has used a more sophisticated approach to analyse the problem, though his conclusions imply that a distributional assumption akin to the relative income hypothesis is required for the aggregation of demand systems. If we are to aggregate perfectly we require the income distribution to stay the same shape which means that income cohorts should maintain the same relative position or new cohorts or agents entering a new cohort should replace the dying or misplaced agents. The condition is less stringent than the constancy assumption of Tobin.

The integral conditions associated with the distributional assumptions imply particular weights when series are summed, as does the more traditional approach, due to Theil(1954). Our discussion of such issues will use certain conditions on the parameters of the cross product matrixes which will lead to approximate results. The aggregate model is squeezed into the same structure as in section 1, except for a VAR(1) error. An alternative explanation of such models is given by Nickell(1984) who suggests that the structure may involve a more complex stochastic process. If we take the micro system, so that industry or agent specific variables are defined thus:

$$y_t = [y_{it}], \quad h_t = [h_{it}] \quad \text{and} \quad u_t = [u_{it}]$$

then our system can be defined by (4.18) above, where we are now looking at a micro relationship:

$$y_t = F\, y_{t-1} + h_t + u_t$$

If we now have a set of aggregate variables y^* and their residual y^+, then:

$$y_t^* = \nabla_1 y_t \quad \text{and} \quad \nabla = \begin{bmatrix} \nabla_1 \\ \nabla_2 \end{bmatrix} \text{ is a square non-singular matrix}$$

then:

$$(4.51) \quad \begin{bmatrix} y_t^* \\ y_t^+ \end{bmatrix} = \nabla F \nabla^{-1} \begin{bmatrix} y_{t-1}^* \\ y_{t-1}^+ \end{bmatrix} + \nabla\, h_t + \nabla\, u_t$$

$$\text{where } \nabla F \nabla^{-1} = \Gamma \text{ and } \Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix}$$

The system (4.51) determines two equations one in terms of the aggregate variables and the other in terms of the residual, they can be solved to produce a relationship purely in terms of the aggregate variables and the equation errors which hopefully can be treated as a determinate random process.

$$(4.52) \quad y_t^* = \Gamma_{11} L y_t^* + \Gamma_{12} L y_t^+ + \nabla_1 (h_t + u_t)$$

$$(4.53) \quad y_t^+ = \Gamma_{21} L y_t^* + \Gamma_{22} L y_t^+ + \nabla_2 (h_t + u_t)$$

Hence we can use (4.53) to derive a relationship for y^+ and then replace that in (4.52) which leaves with the following model purely in terms of the y*, h and u.

$$(4.54)\quad (I - \Gamma_{11}L)y_t^* = \Gamma_{12}(I - \Gamma_{22})^{-1}(\Gamma_{21}Ly_t^* + \nabla_2(h_t + u_t)) + \nabla_1 (h_t + u_t)$$

If we look at one series, it may be possible that the parameters in individual markets are not too different so that $\Gamma = \gamma I + \Delta\Gamma$ which suggests that:

$$\begin{aligned}(I - \Gamma \quad L)^{-1} &= [(1 - \gamma L)I - \Delta\Gamma \quad L]^{-1}\\ &= (1 - \gamma L)^{-1}I + (1 - \gamma L)^{-2}\Delta\Gamma_{22}L\\ &\quad + (1 - \gamma L)^{-3}(\Delta\Gamma_{22}^2)\end{aligned}$$

If we substitute out for $(I - \Gamma \quad L)^{-1}$ in (5.44) above,then:

$$\begin{aligned}(4.55)\quad (I - \Gamma_{11}L)y_t^* &= \Gamma_{12}((1 - \gamma L)^{-1}I + (1 - \gamma L)^{-2}\Delta\Gamma_{22}L\\ &\quad + (1 - \gamma L)^{-2}(\Delta\Gamma_{22}^2))(\Gamma_{21}Ly_t^* + \nabla_2(h_t + u_t))\\ &\quad + \nabla_1 (h_t + u_t)\end{aligned}$$

We can considerably simplify the above relationship, by firstly eliminating terms of order less than $o(\Delta\Gamma^2)$ and then multiplying through by $(1-\gamma L)$, so that:

$$\begin{aligned}(1 -\gamma L)(I - \Gamma_{11}L)y_t^* &= \Gamma_{12}\Gamma_{21}Ly_t^*\\ &\quad + ((1 - \gamma L)\nabla_1 + (\Gamma_{12}\nabla_2 + \Gamma_{12}\Delta\Gamma_{22}\nabla_2)(h_t + u_t)\end{aligned}$$

We now have a moving average error term linked to a more complex dynamic than the model in section 1, but this can be simplified when the $o(\Gamma_{12}\Delta\Gamma_{22}\nabla_2) < o(\Delta\Gamma^2)$ and $\Gamma_{12}\Gamma_{21}$ and $\Gamma_{12}\Delta\Gamma_{22}$ are relatively small. Therefore our model becomes:

$$(4.56)\quad (I - \Gamma_{11}L)y_t^* = (\nabla_1 + (1 - \gamma L)^{-1}\Gamma_{12}\nabla_2)(h_t + u_t)$$

This relationship has an autoregressive error associated with the infinite order moving average term $(1 - \gamma L)^{-1}u_t$. We can show that such equations exist for all g aggregate variable when the system can be diagonalised, so that (4.51) becomes:

$$\begin{bmatrix} y_t^x \\ y_t^{\#} \end{bmatrix} = \Gamma^{\#} \begin{bmatrix} y_{t-1}^x \\ y_{t-1}^{\#} \end{bmatrix} + \nabla^{\#}(h_t + u_t)$$

where $P\Gamma P^{-1} = \Gamma^{\#}$ and $\Gamma^{\#} = \begin{bmatrix} P_1\Gamma_{11}P_1^{-1} & P_1\Gamma_{12}P_2^{-1} \\ P_1\Gamma_{11}P_1^{-1} & P_1\Gamma_{12}P_2^{-1} \end{bmatrix}$

If we take a system in which each y_{it}^x is stacked on top of the the $y^{\#}$s, then we will end up with g equations of the form (4.56):

$$(I - \Gamma_{11}L)y_{it}^x = (\nabla_{1i}^{\#} + (1 - \gamma_i L)^{-1}\Gamma_{12}^{\#}[i]\nabla_{2i}^{\#})(h_t + u_t)$$

If we now stack such equations we can produce a system in terms of the diagonalised aggregate variables:

$$(4.56) \quad (I - \Gamma_{11}^{\#}L)y_t^x = (\nabla_1^{\#} + (1 - R^{\#}L)^{-1}\Gamma_{12}^{\#}\nabla_2^{\#})(h_t + u_t)$$

Inverting the diagonalisation we now have the multivariate analogue of (4.56):

$$(4.56) \quad (I - \Gamma_{11}L)y_t^{*} = (\nabla_1 + (I - \Gamma_o L)^{-1}\Gamma_{12}\nabla_2)(h_t + u_t)$$

If we let $u_t^{+} = \Gamma_{12}\nabla_2 u_t$, then:

$$(I - \Gamma_o L)^{-1}u_t^{+} = e_t$$

so that $e_t = \Gamma_o e_t + u_t^{+}$ is an autoregressive error where:

$$e_t = (I - \Gamma_{11}L)y_t^{*} - (\nabla_1 + (I - \Gamma_o L)^{-1}\Gamma_{12}\nabla_2)h_t + \nabla_1 u_t$$

Serial correlation can be eliminated using the quasi-differenced form of the Generalised Least Squares Estimator (see Harvey(1981)):

$$u_t^+ = (I - \Gamma_o)(I - \Gamma_{11}L)(y_t^* - \nabla_1(h_t + u_t)) + \Gamma_{12}\nabla_2 h_t$$

It is likely that we can eliminate $\Gamma_{12}\nabla_2 h_t$ when the roots of the system are not far from the unit circle and cross product terms are relatively small, because the elements of h_t are all multiplied by $\Gamma_{12}\nabla_2 FB_o$, where FB_o = $(I-\beta F)(I-F)$. If we then bring together terms in the error we see that the transformation has introduced a moving average error, so that:

$$(\Gamma_{12}\nabla_2 + \nabla_1)u_t + \Gamma_o\nabla_1 u_{t-1} = (I - \Gamma_o^*)(I - \Gamma_{11}L)(y_t^* - \nabla_1 h_t)$$

The problem associated with this error structure may be ameliorated when $(\Gamma_{12}\nabla_2 + \nabla_2)$ is large relative to $\Gamma_o\nabla_1$ or when $\Gamma_o\nabla_1$ is small relative to the other parameters. We may then be able to re-write our relationship as a straight forward autoregressive model:

$$u_t^{\#} = (I - \Gamma_o)(I - \Gamma_{11}L)(y_t^* - h_t^*)$$

Alternatively may find that the u_t errors are considerably smaller than e_t which means that the moving average term may turn out to be negligible in terms of the data, but even so when the aggregation story is believed it is important to test such a model for first order serial correlation. The same result can also be derived by solving out for the moving average in (4.56) and then assuming that terms $o(\Gamma_o^2\Gamma_{12}\nabla_2)$ are negligible. We then have a VMA(1) model which may be approximated by a VAR(1) error when Γ_o is relatively small.

Now we need to look at the effect on the structure of aggregation over the exogenous variables. We have from section one that:

$$(4.57) \quad h_t = G_o^* x_t - G_3 \varepsilon_{t+1} + G_1 h_{t+1}$$

where $G_o^* = FB_oA$ $G_3 = FB_oD$, $G_1 = \beta F$ and $D = \sum_{s=0}^{\infty} (G_1)^s AC_{s-1}$

We also know from chapter 3 that x_t has a Wold autoregressive form, which can be formulated as a system in terms of both micro and macro exogenous variables, so that:

$$B(L)x_t = \varepsilon_t$$

and

$$B^{\#}(L)\begin{bmatrix} x_t^* \\ x_t^+ \end{bmatrix} = \begin{bmatrix} \varepsilon_t^* \\ \varepsilon_t^+ \end{bmatrix} \quad \text{and} \quad B^{\#}(L) = \xi B(L)\xi^{-1}$$

$$\text{where } \xi = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} \qquad B^{\#}(L) = \begin{bmatrix} B_{11}^{\#}(L) & B_{12}^{\#}(L) \\ B_{21}^{\#}(L) & B_{22}^{\#}(L) \end{bmatrix}$$

We can now eliminate x^+ from the relationship for x^* using the following equation:

$$(4.58) \quad x_t^+ = -B_{22}^{\#}(L)^{-1}(B_{21}^{\#}(L)x_t^* - \xi_2\varepsilon_t)$$

Hence, we have a relationship which takes account of the effects of exogenous variable aggregation by extending the lag structure of the model by using the relationship above to eliminate the x^+s from the equation for the x^*s:

$$B_{11}^{\#}(L)x_t^* = B_{12}^{\#}(L)B_{22}^{\#}(L)^{-1}(B_{21}^{\#}(L)x_t^* - \xi_2\varepsilon_t) + \xi_1\varepsilon_t$$

$$(B_{11}^{\#}(L) - B_{12}^{\#}(L)B_{22}^{\#}(L)^{-1}B_{21}^{\#}(L))x_t^* = (\xi_1 - B_{12}^{\#}(L)B_{22}^{\#}(L)^{-1}\xi_2)\varepsilon_t$$

If we invert the terms on the right hand side we can derive the VMA form for the aggregate variables:

$$x_t^* = (B_{11}^{\#}(L) - B_{12}^{\#}(L)B_{22}^{\#}(L)^{-1}B_{21}^{\#}(L))^{-1}(\xi_1 - B_{12}^{\#}(L)B_{22}^{\#}(L)^{-1}\xi_2)\varepsilon_t$$

(4.59) $x_t^* = D_1^{\#}(L)\ \varepsilon_t$

and by analogy a similar form exists for x^+:

(4.510) $x_t^+ = D_2^{\#}(L)\ \varepsilon_t$

We can now use these results to derive the aggregate form of (4.57) above:

(4.511) $h_t = G_{o1}^* x_t^* + G_{o2}^* x_t^+ - G_{31}^* \varepsilon_{t+1} - G_{32}^* \varepsilon_{t+1} + G_1 h_{t+1}$

where $G_{oi}^* = FB_o A \xi_i^-$, $G_{3i} = FB_o D_i$, $G_1 = \beta F$ and
$D_i = \sum_{s=0}^{\infty} (G_1)^s A \xi_i^- D_{is-1}^{\#}$

Now if we replace x^+ in (4.511) above using (4.58) we produce the recursive relationship below.

$$h_t = G_{o1}^* x_t^* - G_{o2}^* B_{22}^{\#}(L)^{-1}(B_{21}^{\#}(L) x_t^* - \xi_2 \varepsilon_t)$$

$$- G_{31}^* \varepsilon_{t+1} - G_{32}^* \varepsilon_{t+1} + G_1 h_{t+1}$$

(4.512) $h_t = G_{o1}^* x_t^* - G_{o2}^*((\theta_1 + \theta_2 + \ \ldots) x_t^* - B_{22}^{\#}(L)^{-1} \xi_2 \varepsilon_t)$

$$- G_{31}^* \varepsilon_{t+1} - G_{32}^* \varepsilon_{t+1} + G_1 h_{t+1}$$

It seems likely that higher order terms in $FB_o A \xi_1^- B_{22}^{\#}(L)^{-1}$ are likely to be small, so that it might be possible to ignore them and by analogy we might be able to discard the extra error terms which should also be small. If we also can let $\xi_1^- D_{1s-1}^{\#} + \xi_2^- D_{2s-1}^{\#} \approx (\xi_1^- + \xi_2^- \theta_1) D_{1s-1}^{\#}$, where the $D_i^{\#}$ terms comes from a VMA representation of x^*, then we can give (4.512) the more familiar form:

(4.513) $h_t = (G_{o1}^* - G_{o2}^* \theta_1) x_t^* - G_3^{\#} \varepsilon_{t+1}^{\#} + G_1 h_{t+1}$

where $G_{oi}^* = FB_o A \xi_i^-$, $G_3^{\#} = FB_o D$, $G_1 = \beta F$ and
$D = \sum_{s=0}^{\infty} (G_1)^s A (\xi_1^- + \xi_2^- \theta_1) D_{is-1}^{+}$

Clearly there are a number of assumptions here which may or may not do justice to the truth, but when the model is based on the aggregation procedure suggested above it does seem important to determine whether innovations of the disaggregated series are significant. If we find no role for such information, then we can be more certain of the validity of the aggregate model. The problem would be simplified by assuming that industry specific models only depend on aggregate variables and such an assumption would produce a relationship observationally equivalent to (4.513). The assumption above seems to be too strong and testing the proposition is complicated by the need to build sectoral models which leaves us with the approach we have here.

Having taken care of the aggregation effects of the endogenous variables we can see how endogenous variable aggregation affects the recursive form of the forward convolution:

$$\begin{bmatrix} h_t^* \\ h_t^+ \end{bmatrix} = \Gamma B_o^{\#}(\nabla A(\xi_1^- - \xi_2^-\theta_1)x_t^* - \sum_{s=0}^{\infty}(\Gamma\ )^s\nabla A(\xi_1^- - \xi_2^-\theta_1)D_{1s-1}\varepsilon_{t+1}^{\#})$$

$$+ \beta\Gamma \begin{bmatrix} h_{t+1}^* \\ h_{t+1}^+ \end{bmatrix}$$

$$\Gamma B_o^{\#} = \begin{bmatrix} \beta(\Gamma_{11}^2 + \Gamma_{12}\Gamma_{21}) + I - (1+\beta)\Gamma_{11} & \beta(\Gamma_{11}\Gamma_{12} + \Gamma_{12}\Gamma_{22}) - (1+\beta)\Gamma_{12} \\ \beta(\Gamma_{21}\Gamma_{11} + \Gamma_{22}\Gamma_{21}) - (1+\beta)\Gamma_{21} & \beta(\Gamma_{22}^2 + \Gamma_{21}\Gamma_{12}) + I - (1+\beta)\Gamma_{22} \end{bmatrix}$$

Let us look at the relationship for h^*, so that:

$$h_t^* = (\beta(\Gamma_{11}^2 + \Gamma_{12}\Gamma_{21}) + I - (1+\beta)\Gamma_{11})(\nabla_1 A^{\#}x_t^* - \nabla_1 D\ \varepsilon_{t+1}) +$$

$$(\beta(\Gamma_{11}\Gamma_{12} + \Gamma_{12}\Gamma_{22}) - (1+\beta)\Gamma_{12})(\nabla_2 A^{\#}x_t^* - \nabla_2 D\ \varepsilon_{t+1}) +$$

$$\beta\Gamma_{11}h_t^* + \beta\Gamma_{12}\nabla_2 h_{t+1}$$

where $A^{\#} = A(\xi_1^- - \xi_2^-\theta_1)$

We have already assumed that $\Gamma_{12}\Gamma_{21}$ and $\Gamma_{12}\Delta\Gamma_{22}$ are relatively small and it was suggested in deriving the autoregressive form of the model that $\Gamma_{12}\nabla_2 h_t$ would be small when the roots of the system are close to unity, because all the terms in h_t are then multiplied by $\Gamma_{12}\nabla_2(I-\beta F)(I-F)$. Now if we combine the conditions above with the possibility that $(\beta(\Gamma_{11}\Gamma_{12}+ \Gamma_{12}\Gamma_{22}) -(1+\beta)\Gamma_{12})$ is small, because it is an off diagonal term in the dissaggregated variables which is then mulitplied by ∇_2, then we end up with:

$$(4.514)\quad h^*_t = (\beta(\Gamma^2_{11}+ I +(1+\beta)\Gamma_{11})(\nabla A_1 x^*_t - D_1\epsilon^{\#}_{t+1}) + \beta\Gamma_{11}h^*_{t+1} \qquad (\text{where } D_1 = \nabla_1 D)$$

As (4.514) stands we would not expect to impose the usual restrictions on D_1 unless additional condition could be imposed. As cross product terms are small or appear to be relatively small, then it may be possible that the following approximation holds:

$$\Gamma^s = (\Delta\Gamma^s) + \begin{bmatrix} \Gamma^s_{11} & 0 \\ 0 & \Gamma^s_{22} \end{bmatrix}$$

If we let $A_i = \nabla_i A^{\#}$ and use the result above, then:

$$D_1 = \sum_{s=1}^{\infty} [\nabla_{11}(\beta\Gamma_{11})^s A_1 + \nabla_{12}(\beta\Gamma_{22})^s A_2] D_{1s-1} + \nabla_1 \sum_{s=1}^{\infty} (\beta\Delta\Gamma)^s \nabla A^{\#} D_{is-1}$$

$$(4.515)\quad D \approx \sum_{s=1}^{\infty} (\beta\Gamma_{11})^s A_1 D_{is-1}$$

If we combine (4.514) with our autoregressive form for y^* we have a system that is exactly the same as that derived in section 1 except for a VAR(1) error which has a rationale based on aggregation, this is a slightly more general model to the one used by Nickell(1983):

$$(4.516)\quad (I - \Gamma_o)(I - \Gamma_{11}L)(y^*_t - h^*_t) = u^*_t$$

We will use the same recursive maximum likelihood procedure explained in section 3, except that the error is defined by (4.516) above, the standard errors are based on the estimate of the Hessian produced by the Gill Murrey Pitfield procedure. The lagrange multiplier test for serial correlation should be treated with some caution, because it is more like a multivariate Box-Pierce statistic, as the program was not adjusted to take account of the implicit inclusion of second and first order lags on y^* introduced by the autoregressive error. The small size of the absolute value of the test suggests that serial correlation is not a problem and if their is some bias this is likely to be small given the size of the parameters.

The output employment systems presented in table 4.2 are estimated for the period 1962q4 to 1979q4 and the program generates predictions for the period 1980q1 to 1992q3. Column (1) in the table below reports the results not adjusted for serial correlation, column (2) the results with adjustment and Column (3) the model with VAR(1) error and time trend. Column (4) presents similar results for a model which includes past values of the exogenous variables, but does not have the autoregressive error, the method of estimation for this model is dealt with in more detail in the next chapter. The final model is used as a link with the next chapter, but it also provides a check on the validity of the GLS adjustment. If the model including lagged exogenous variables significantly outperforms the autoregressive model, then the common factor restriction cannot hold (a test for a common factor restriction was considered to be outside the scope of the current work). A quick check on the validity of the

TABLE 4.3 Autoregressive Models of Output and Employment

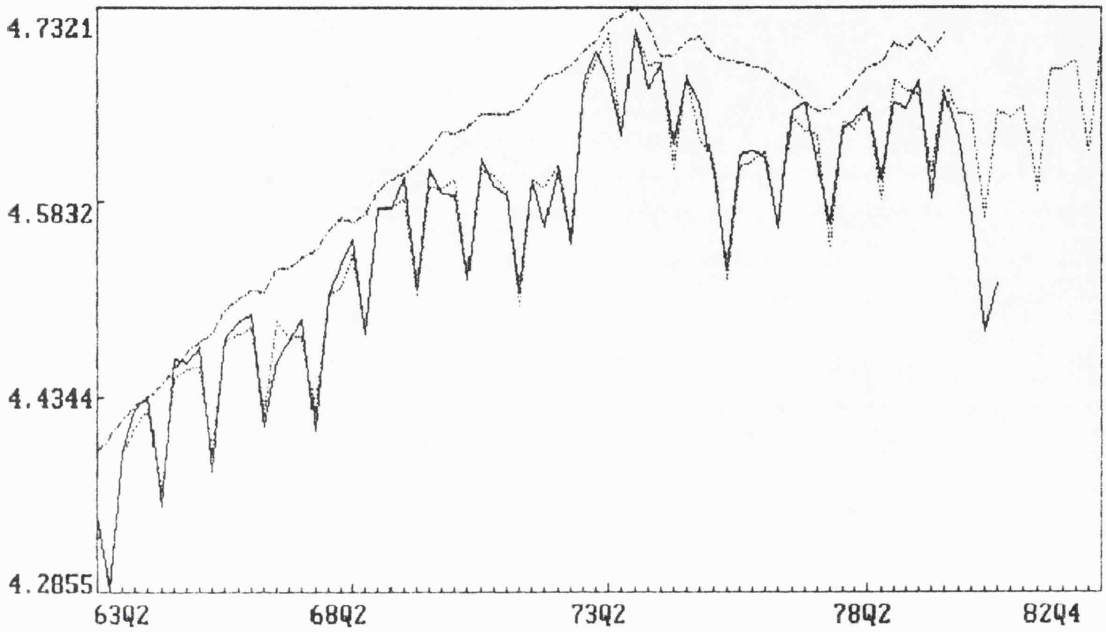
	1	2	3	4	4 continued
Manufacturing Output					
F_{11}	0.67383	0.76620	0.52583	0.68353	
	(0.05799)	(0.06605)	(0.08904)	(0.08438)	
F_{12}	-0.19262	-0.34526	-0.36251	0.28691	
	(0.16868)	(0.15132)	(0.10438)	(0.10345)	
B_{o11}	0.27905	0.15659	0.46705	0.42980	
B_{o12}	0.22083	0.40726	0.52570	-0.39878	
D_{11}	-0.06644	-0.04545	-0.09179	0.36087	
$D1_{2}$	0.04509	0.03208	-0.03985	2.48207	
D_{13}	0.42374	0.36147	0.07360	3.83789	
D_{14}	0.64973	0.63303	0.20467	-2.75992	
					A_{11}
a_{11}	0.11045	0.13604	-0.03097	0.73827	0.14252
	(0.13482)	(0.08628)	(0.04146)	(0.30716)	(0.30716)
a_{12}	0.02197	0.00559	0.02478	2.48207	-2.25515
	(0.02202)	(0.02457)	(0.01001)	(0.26385)	(0.29819)
a_{13}	-0.71390	-0.71694	-0.39675	3.83789	-3.94297
	(0.06495)	(0.06365)	(0.03411)	(0.64088)	(0.69435)
a_{14}	0.71846	0.71797	0.17668	-2.75992	2.77408
	(0.05745)	(0.05415)	(0.05164)	(0.19480)	(0.22959)
SEASONALS	1.16229	1.60195	1.26615	-1.30350	
	(0.32369)	(0.16665)	(0.08731)	(0.30716)	
	0.44790	-0.59481	-0.11904	2.25515	
	(0.19920)	(0.18256)	(0.08225)	(0.29819)	
	0.34931	0.09196	-0.08408	0.55286	
	(0.08216)	(0.07912)	(0.04594)	(0.98262)	
	0.41812	-0.10890	0.00834	0.98262	
	(0.11637)	(0.06365)	(0.02584)	(0.13436)	
TREND			1.03402		
			(0.09714)		

Employment Equation					A_{12}
F_{21}	0.06061 (0.00925)	0.05216 (0.01037)	0.09728 (0.03033)	0.05022 (0.00901)	
F_{22}	0.86319 (0.03250)	0.76789 (0.06174)	0.85003 (0.04306)	0.93464 (0.01132)	
B_{o21}	-0.06061	-0.06153	-0.14107	-0.06980	
B_{o22}	0.06196	0.15459	-0.00310	0.08079	
D_{21}	0.11410	0.02475	0.08534	0.37645	
D_{22}	0.09125	0.04661	0.10690	0.43980	
D_{23}	0.18464	0.13065	0.20184	-0.24364	
D_{24}	0.10473	0.06814	-0.03846	-0.83717	
a_{21}	0.25596 (0.11669)	0.09159 (0.04276)	0.16928 (0.08412)	0.72583 (0.10713)	0.16160 (0.34144)
$a_{22}+1$	0.04840 (0.01975)	0.04447 (0.01470)	0.05093 (0.01825)	2.33369 (0.17488)	-2.11819 (0.20524)
a_{23}	-0.02275 (0.04823)	-0.06414 (0.04397)	-0.02221 (0.04479)	4.55343 (1.18231)	-4.01380 (1.16126)
a_{24}	-0.10208 (0.04311)	-0.06098 (0.03784)	-0.12429 (0.06742)	-2.99614 (0.27645)	2.23457 (0.29454)
SEASONALS	0.18522 (0.27492)	0.74011 (0.08178)	0.57977 (0.14279)	-1.84511 (0.35035)	
	0.49405 (0.15190)	-0.15896 (0.07670)	-0.28643 (0.16745)	1.93701 (0.29150)	
	0.18379 (0.05419)	0.11221 (0.03503)	0.17724 (0.07063)	0.29150 (0.13760)	
	0.27426 (0.08181)	-0.05190 (0.02841)	-0.09930 (0.05847)	0.70247 (0.12008)	
TREND			0.09784 (0.13717)		
β	0.53900 (0.07560)	0.46979 (0.13542)	0.62370 (0.08029)	0.21153 (0.07433)	

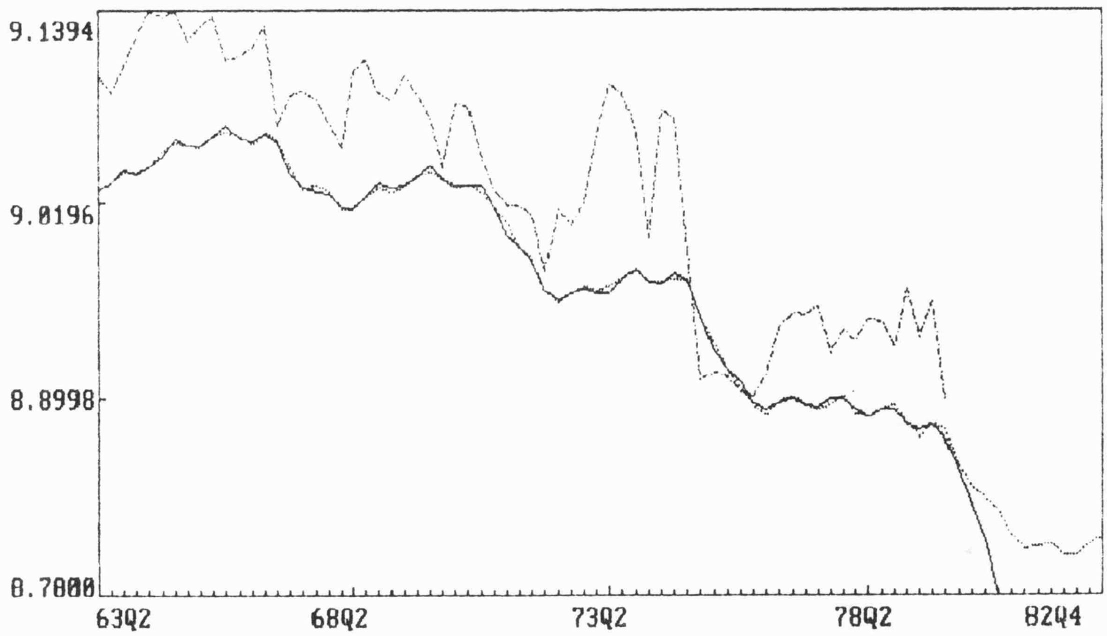
AR(1)MATRIX				
Γ_{o11}		-0.32377 (0.12944)	-0.42020 (0.13045)	
Γ_{o12}		0.99696 (0.81122)	2.21388 (0.17882)	
Γ_{o21}		-0.03136 (0.02292)	-0.07760 (0.03892)	
Γ_{o22}		0.65825 (0.16067)	0.55730 (0.11187)	
LIKELIHOOD	403.0279	413.3694	425.6767	408.506
VARIANCE	0.00026	0.00024	0.00018	0.00024
$x10^{-4}$	0.16808	0.14801	0.15225	0.13111
$x10^{-4}$	0.07228	0.05809	0.05881	0.06522
LM(1)	15.85196	1.93617	1.25152	11.77907
LM(2)	19.04442	2.40925	4.69223	15.40356
LM(4)	24.95076	10.64854	15.33008	29.13100
LM(5)	26.52296	14.83810	20.48444	33.65383
N	68	68	68	68

adjustment can be given by comparing the estimates in column(1) and column(2), when the common factor restriction holds then their should be little difference between the two sets of parameters which in this instance is the case. The four restrictions associated with setting $\Gamma_o = 0$ are clearly not satisfied which implies that it is necessary to adjust for serial correlation. If such an adjustment is due to either aggregation or a more complex error process associated with the endogenous variables, then the roots of the system are stable, but not real, as $\mu_i = .767 \pm .13426i$. The roots here suggest dampened oscillations linked to a process which converges after one and a half to two years. The rate of convergence is similar to that of

Figure 4.4 Plots of Equilibrium, Fitted and Actual values of Output and Employment for the VAR(1) error model with trend from column 3 of Table 4.4



actual output ——— fitted .... equilibrium -.-.-



actual employment ——— fitted .... equilibrium -.-.-

the Nickell(1984), though these results are slightly more stable. Roots which are not real suggest that one of the cost matrices is not positive definite or that the system has an alternative structure which is less symmetric (such models are dealt with in the next chapter). If we solve out for the autoregressive error, then we introduce second order lags which would be related to the solution of a higher order loss function. The solution to the roots of the system in column(3) are μ_1 = .3045, μ_2 = -.1926 and μ_3, μ_4 = .57894(cos(θ) ± isin(θ)) where θ = 30.8°, but the negative root is incompatible with the dynamic being caused by a higher order system as it implies benefits to either adjustment or disequilibrium. If such a root is associated with Γ_0, then we a stable first order system in which the extra dynamic can be attributed to aggregation.

In terms of fit the model in column(3) seems to dominate, the other models, but the model without produces an output demand equation in the long-run which satisfies price homogeneity. The results for the trend models have price coefficients that have the right sign and the employment equation is closer to satisfying homogeneity at the cost of it not being satisfied by the output equation. The discount factor is higher at .624(upper interval estimate .784), but not as large as in the innovation model and when the restriction that β = .9 is imposed it is rejected(Log-Likelihood of 421.145 versus 425.676). Figure 4.3 suggests that the fit for the output and employment equations is good and the equilibrium values for the output equation seem to be reasonable. The equilibrium values for the employment equation relate to the demand for labour, so that we may be suspicious of their accuracy given the failure of homogeneity and question the imposition of that restriction. The difference between the actual values and demand is a measure of excess supply of goods in the case of output and vacancies in the case of the employment which suggests in the latter case that the actual value and the equilibrium value should be roughly in line after 1974.

The final model does not dominate the VAR(1) error model and the restrictions do seem to be warranted, when column (1) and (4) are compared. The model also has a very small discount factor and the long-run parameters of the employment and the output equation are not totally independent, they are roughly proportional to each other. Again we have an indication of a relationship between employment and output of the type suggested by cointegration, though one of the roots given by $\mu_i = .809085 \pm .1737026$ is close to the unit circle it is not as close as that in column(5) of table 4.1. Alternative reasons for the link between parameters would be a lack of identification, as there are signs of ill-conditioning and the coefficients of FB_0 are roughly proportional. When $\beta=0$ the future values of the Xs are not important and the model collapses to a partial adjustment form. Here, the size of the A coefficients seems to contradict this as it appears that the estimates are being compensated for the effect of small β on future xs and the small B_0 coefficients on the target variables in the employment equation. The employment coefficients, then dominate the smaller effects in the output equation. The small value of β seems to emphasise this so that the output and employment coefficients in the long-run are similar and this is linked to cointegration under which one might expect that $z_t = \alpha' A x_t$. Although a long-run model in differences and levels does produce price effects compatible with long-run demand models of output and employment, only the wage coefficient in the employment equation looks to be significant. This wage term is the only thing which differentiates between the employment and output equations in the long-run. The model is not appealing, because of the large long-run parameters, the poorly determined equilibrium relationship and the suggestion that first order serial correlation is still a problem.

Conclusion

In this chapter we have discussed what are called first order rational expectations models in chapter 5. An efficient method of estimation has been developed whose form can be attributed to Sargan (1982). The approach uses the Muellbauer and Winter(1980) transformation to remove future expectations and when that is reversed the forward convolution in the exogenous variables and one step ahead forecast errors has the recursive representation first derived in Hunter(1984).

In section one we derive the estimator and relate it to the backward looking representation of the model which is shown to have an error correction and cointegration form. It is shown that the forward representation still exists when we have unit roots and it is postulated that it may even be possible to derive estimates of the deep parameters, because the unit root is in the null-space of B_0. In section two we specify the long-run form of the particular output employment model that is to be estimated and this is related to the optimising theory of section 1 and specific models of the long-run or deep parameters. The model suggests that we should observe demand equations for output and employment in the long-run and this turns out to be roughly consistent with the results. The long-run models can either be thought of as targets for a cost minimising control approach or as being derived from appropriate revenue or utility functions in the choice theoretic or profit maximising framework. Under such an optimising framework we would normally expect the coefficients on the difference and levels terms to satisfy conditions associated with the underlying criterion. In this case we would expect cost matrices to be positive definite or at the least positive-semi-definite, such conditions are related to the existence of unit roots/cointegration and as we shall see in the next chapter conditions for identification.

In section 3 we specify the Muellbauer form of the model which is used to derive initial estimates, these models suffer from serial correlation, but under the rational expectations assumption the instrumental variables method of Wickens(1982) produces consistent estimates. The IV estimates of the deep parameters are roughly consistent with theory, though the estimates of B_0 are not. The problem may be attributed to cointegration, as unit roots can lead to negative estimates of the cost parameters, this idea is partially supported by the discovery of a unit root and the possibility that output and employment are cointegrated. The Maximum Likelihood method appears to produce more efficient estimates, than the IV approach and the estimates of F are quite consistent with some form of optimisation story. Unfortunately this method does not seem to be able to remove first order serial correlation and the model suggests a discount factor which is unrealistic. The deep parameters are mainly consistent with theory, excepting that employment depends on nominal wages and anticipated inventory changes appear to represent investment or speculative rather than disequilibrium effects. In terms of explicability, the model in which the innovations are included directly seems to be preferred, and though it still suggests a nominal employment model, it does not suffer from serial correlation. The model prduces a more reasonable discount factor and it suggests that unanticipated factors do influence employment in the way expected by theory. The model with a trend has deep parameters with signs which can be given a theoretical justification, but neither of the long-run demand equations satisfies homogeneity and the test for first order serial correlation is marginal at the 1% level. The model produces deep parameters which can be used to derive equilibrium values and on inspection such value are not totally unreasonable, obviously better formulated long-run models would produce more accurate equilibria.

Further experimentation with the first order model shows that the question of period of expectation is not straight forward and their is evidence, that either the model combines agent expectation for period t and t-1 or that the true period lies between these two values. The question of the period of expectation is complicated, because it is bound up with the test of the rational expectations restrictions, the nature of exogeneity and the role of innovations in the model. It appears from the estimates derived that the xs are strictly exogenous when a Hausman(1978) test is acceptable, but whether we have weak exogeneity as well depends on the invariance of the parameters and the test of restrictions on the one step ahead forecast errors.

The discovery of a unit root, suggests cointegration and the errors-in-variables approach which estimates the first-order condition has an error correction form in which the endogenous variables are cointegrated when the matrix H^* on the correction term is singular. The cointegration form in this guise is unstable which suggests that error correction models with unstable dynamic effects or explosive coefficients on the correction matrix are reparameterisations of the first order rational expectations model. The results of section 4 suggest that it is more appropriate to estimate the model using a method which imposes the rational expectations restrictions, than the un-restricted first order condition. This is especially true when we have cointegration, as the form of the first order condition may produce estimates which are not compatible with agent optimisation of a loss or profit function.

We have assumed that the aggregate models are representative or that simplistic transformations are sufficient to eliminate the problem. Nickell(1984) deals with aggregation by assuming dual labour markets which produces a model with two lags on the dependent variable. We use an autoregressive model that imposes the rational expectations restrictions, the results for this system are similar to those of section 3 which suggests that the transformation is appropriate. As to whether the serial correlation is due to aggregation or to the omission of variables is more difficult to ascertain. To answer such a question it is necessary to have comparable disaggregate data and sufficient observations to estimate the most general specification or to be able to generate individual industry innovations and to include individual market expectations and their lags in models.

The adjustment for serial correlation eliminates first order effects and produces a model which is compatible with the pure error autocorrelation. If there is a common factor, then the parameter with and without the common factor restriction should be the same. The trend model seems to be preferred in this case, even though the output equation dose not satisfy homogeneity the system has a higher likelihood and the discount rate is more reasonable. The undeniable conclusion that has to be made is that their are variables missing and when individual errors are regressed on other variables it has been found that hours, the exchange rate and the retail price index have a significant effect. It is clearly the case that a properly formulated factor demand system would require additional price terms and the capital stock as well as hours as endogenous variables. In the context of a macro model it is likely that other factors may be relevant: an alternative structural form or the inclusion of additional variables to capture excess demands, the openness of the economy or the effect of the monetary and financial sectors on output and employment.

Appendix 4A.1 Solutions to rational expectations models with cointegrated endogenous variables.

If we take the Sargan Loss function:

$$\text{(i)}\ \Gamma_t = E\{\sum_{t=0}^{T} \beta^t (\Delta y_t' K\, \Delta y_t + (y_t - z_t)' H (y_t - z_t) \mid \Omega_t\}$$

where $rk(H) = r$ and $0 < r < g$, then the endogenous variables are cointegrated. If we factor $H = E'E$, then the $rk(E) = r$ and we can so define N that $[E' : N']$ is a non-singular square matrix.

Then we can redefine (i) above in terms of new variables:

$$\text{(ii)}\ \Gamma_t = E\{\sum_{t=0}^{T} \beta^t (\Delta y_t^{*\prime} K^* \,\Delta y_t^* + (y_t^* - z_t^*)' H (y_t^* - z_t^*) \mid \Omega_t\}$$

$$\text{where } y_t^* = \begin{bmatrix} y_{1t}^* \\ --- \\ y_{2t}^* \end{bmatrix} = \begin{bmatrix} E \\ --- \\ N \end{bmatrix} y_t \quad \text{and } K^* = [\,E' : N'\,]^{-1} K \begin{bmatrix} E \\ --- \\ N \end{bmatrix}^{-1}$$

with z_t^* defined in the same way as y_t^*.

$$\text{(iii)}\ \Gamma_t = E\{\sum_{t=0}^{T} \beta^t (\Delta y_{1t}^{*\prime} K_{11}^* \Delta y_{1t}^* + (y_{1t}^* - z_{1t}^*)' (y_{1t}^* - z_{1t}^*)$$

$$+ 2\Delta y_{1t}^{*\prime} K_{12}^* \Delta y_{2t}^* + \Delta y_{2t}^{*\prime} K_{22}^* \Delta y_{2t}^* \mid \Omega_t\}$$

If we reformulate (iii) above in terms of new variables which are stationary then $y_{1t}^+ = y_{1t}^*$, $y_{2t}^+ = \Delta y_{2t}^*$ and it is assumed that z_{2t}^+ and $\Delta\Delta y_{2t}$ are null vectors and that a similar transformation holds for the zs where appropriate.

$$\text{(iv)}\ \Gamma_t = E\{\sum_{t=0}^{T} \beta^t (\Delta y_{1t}^{+\prime} K_{11}^* \Delta y_{1t}^+ + (y_{1t}^+ - z_{1t}^+)' (y_{1t}^+ - z_{1t}^+)$$

$$+ 2\Delta y_{1t}^{+\prime} K_{12}^* y_{2t}^+ + y_{2t}^{+\prime} K_{22}^* y_{2t}^+ \mid \Omega_t\}$$

Differentiating (iv) with respect to y_{1t}^+ we get:

(v) $E\{ \beta^t (K^*_{11}\Delta y^+_{1t} - \beta K^*_{11}\Delta y^+_{1t+1} + (y^+_{1t} - z^+_{1t}) +$

$K^*_{12}(y^+_{2t} - \beta\, y^+_{2t+1}) | \Omega_t \} = 0$

cancelling out 2 and with respect to y^+_{2t}:

(vi) $E\{\beta^t (K^*_{21}\Delta y^+_{1t} + K^*_{22} y^+_{2t} | \Omega_t\} = 0$

Equation (vi) does not immediately look like the usual form of the first order condition, but we can easily re-write (vi) by subtracting it from itself lead forward one period. In terms of * variable (vi) becomes:

(vii) $E\{ K^*_{21}(\Delta y^*_{1t} - \beta\Delta y^*_{1t+1}) + K^*_{22}(\Delta y_{2t} - \beta\Delta y_{2t+1} | \Omega_t\} = 0$

We can now return to a more familiar form of the first order condition if we put (v) and (vii) together with (v) in terms of (*) variables:

$$E\{ K^*(\Delta y^*_t - \beta\Delta y^*_{t+1}) + \left[\begin{array}{c:c} I & 0 \\ \hdashline 0 & 0 \end{array}\right] (y^*_t - z^*_t) \mid \Omega_t) = 0$$

The first order condition is simply equation (4.11) in section 4.1 when we reverse the original transformation:

$$E\{ K\,(\Delta y_t - \beta\Delta y_{t+1}) + H\,(y_t - z_t) \mid \Omega_t\} = 0$$

The formula is the same, as that in the case in which the variables are not cointegrated.

Appendix 4A.2 Symmetric Solution to Quadratic Difference Equations

1 The first order condition to the standard multivariate costs of adjustment model is given by

$$E(Q_o\, y_t - \beta Q_1 y_{t+1} - Q_1 y_{t-1} = Hz_t | \Omega_t)$$

if we let $y_t^* = \beta^{\frac{1}{2}t} y_t$ and $Q_i^* = \beta^{\frac{1}{2}(i-1)} Q_i$ for $i = 0, 1$ where $Q_o = ((1 + \beta)K + H)$, $Q_1 = K$ and $z_t^* = \beta^{\frac{1}{2}t} z_t$, then

(viii) $E(Q_o^*\, y_t^* - Q_1^* y_{t+1}^* - Q_1^* y_{t-1} = \beta^{-\frac{1}{2}}\, Hz_t^* | \Omega_t)$

$$E((Q_1^*)^{-1} Q_o^* y_t^* - y_{t+1}^* - y_{t-1}^* = B_o^* z_t^* | \Omega_t)$$

where $B_o^* = (Q_1^*)^{-1} \beta^{-\frac{1}{2}} H$

This is a difference equation which has a standard solution for the homogeneous case given by $y_t^* = \mu^t p_t$, the associated characteristic function is given below.

(ix) $((Q_1^*)^{-1} Q_o^* \mu - \mu^2 - 1) = 0$

$$|(Q_1^*)^{-1} Q_o^*\, \mu - \mu^2 - 1| = 0$$

$$|\ H^+ - \emptyset I\ | = 0$$

Where $H^+ = (Q_1^*)^{-1} Q_o^*$ and $\emptyset = (\mu + 1/\mu)$, hence the characteristic equation has a solution which is composed of pairs of roots associated with each root to the system v. If we select the stable solution we can stack the characteristic equations given by (ix) with their associated eigen values to derive the factorisation associated with the saddle point solution.

Therefore:

$$(H\, PM^+ - PM^2 - P) = 0$$

where $M = \begin{bmatrix} \mu_1 & o & . & . & . & o \\ o & \mu_2 & & . & . & . \\ . & & . & & & . \\ . & . & & & . & . \\ o & . & . & o & \mu_G & \end{bmatrix}$, so post multiplying by

P^{-1} and setting $PMP^{-1} = G$ with $G^2 = PM^2P^{-1}$ and $PP^{-1} = I$ gives us the form that has been factored, G also satisfies the characteristic equation. Therefore:

$$H^{+}G - G^{2} - I = 0$$

where $H^{+} = G + G^{-1}$,

Alternatively the quadratic above has the following factorisation:

$$(G'\mu - I)W(G - \mu I) = (G'WG + W)\mu - G'W\mu^{2} - WG$$

so that:

$$Q_o^{*} = G'WG + W \text{ , } Q_1^{*} = G'W \text{ and } Q_1^{*} = WG$$

$$Q_o^{*} = Q_1^{*}G + W \qquad \text{and} \qquad Q_o^{*}G = Q_1^{*}G^{2} + WG = Q_1^{*}(G^{2} + I)$$

$$Q_o^{*} = Q_1^{*}(G + G^{-1}) \text{ and } \quad H^{+} = (Q_1^{*})^{-1}Q_o^{*} = G + G^{-1}$$

Which is the same formula for H^{+} as that derived above, so that when we replace y_t^{*} by y_t and H^{+} by H^{*} we get:

$$H^{*} = \beta F + F^{-1} \qquad \text{where } F = \beta^{-1/2}G$$

using H^{*} to replace $(Q_1^{*})^{-1}Q_o^{*}$ in (viii) above gives:

(x) $E((\beta F + F^{-1})y_t - \beta y_{t+1} - y_{t-1} = B_o z_t \mid \Omega_t)$

where $B_o = \beta^{-1/2}B_o^{*}$

Appendix 4B A Lagrange Multiplier Test for Serial Correlation

Let us specify a general non-linear system of which the model in section 1 is a special case:

$$y_t = P[\theta_1]\, x_t^* + v_t$$

where $\theta' = [\theta_1 : \theta_2]$ and x_t^* the vector of all the pre-determined

Then the concentrated likelihood function, associated with V_t being white-noise (the null hypothesis for which $E(\frac{V'V}{N}) = \Omega_v$ and $E(\underset{\sim}{v}) = 0$) is given as

$$L_v = -\frac{N}{2} \log \left|\frac{V'V}{N}\right| \qquad \text{and} \quad V = Y - X^*P[\theta_1]'$$

The alternative hypothesis which assumes that we have first order serial correlation, then $v_t = Rv_{t-1} + e_t$, where e_t is a white noise error and $(E\,[e_t e_t'] = \Omega)$ and the concentrated likelihood function associated with the alternative is:

$$\text{(ib)} \quad L_e = -\frac{N}{2} \log\left|\frac{E'E}{N}\right| \qquad \text{and} \quad E = V - V_1R'$$

We can now determine a multivariate representation of Lagrange-Multiplier (LM) test for serial correlation associated with Godfrey (1978). The test imposes the restriction that $\theta_2 = \text{vec}(R) = 0$, so that

$$H_o : \theta_2 = 0 \qquad \text{is the null}$$

$$H_1 : \theta_2 = 0$$

Harvey (1981) derives the LM test and Sargan (1988) presents the multivariate analogue which is the score test:

$$LM = \frac{1}{N} \left[\frac{\delta L}{\delta \theta_2}\right] V_{22} \left[\frac{\delta L}{\delta \theta_2}\right]$$

where V_{22} = plim $\begin{bmatrix} \frac{-1}{n} & \frac{\delta^2 L}{\delta\theta_2\, \delta\theta_2} \end{bmatrix}$ $\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}$ is the score

and it can be shown using Cramers' linear transformation theorem, that:

$LM \sim \chi^2_{g*}$

where θ_2 is a g* x 1 vector of constrained parameters.

Hence, to derive the test we need to produce first and second order derivatives of (ib) with respect to θ_i and vec(R) under the null (R = 0). If we totally differentiate (ib) (see Magnus and Neudecher [1988]) we find that:

$dL_c = -tr\,(\dot{\Omega}^{-1} E' dE)$

where $\dot{\Omega}^{-1} = (\frac{E'E}{n}^{-1})$ the total derivatives of E w.r.t. θ are:

$dE = -V_1\ dR'$

$dE' = dV'$ (under the null) and

$dV' = -dPX^{*'}$

Then taking first derivatives w.r.t. θ_1 and R we have that:

$$\frac{\delta Lc}{\delta R} = [\dot{\Omega}_e^{-1}\ (E'V_1)]$$

(iib) $$= \begin{bmatrix} \frac{(V'V)^{-1}}{n} & \frac{(V'V_1)}{n} \end{bmatrix}$$ (under the null)

this is a multivariate form of the Durbin-Watson statistic:

(iiib) $$\frac{\delta Le}{\delta\theta_{1i}} = tr\left[\hat{\Omega}^{-1} E'Z \frac{\delta p(\theta_1)'}{\delta\theta_{1i}}\right]$$

$$\frac{\delta Le}{\delta\theta_{1i}} = tr\left[\frac{(V\ V)}{N}^{-1}(V\ X^*) \frac{\delta P(\theta_1)}{\delta\theta_1}\right] \quad \text{(under the null)}$$

Now let us derive the second derivative by taking the total differential again, so that:

$$\frac{1}{N} d\left[\frac{\delta L}{\delta R}\right] = -\Omega\ (d\Omega)\Omega^{-1}\left[\frac{E'V_1}{N}\right] + \frac{1}{N}\Omega^{-1} dE'V_1$$

when R = 0, then $plim\left[\frac{E'V_1}{N}\right] = plim\left[\frac{V'V_1}{N}\right] = 0$

$$\Rightarrow \quad \frac{1}{N} d\left[\frac{\delta L}{dR}\right] = \frac{1}{N}\ \Omega^{-1}\ dE'V_1$$

$$= \frac{1}{N}\ \Omega^{-1}\ dRV_1'V_1$$

when we are considering changes in R above and we are looking at changes in θ_{1i} then we obtain:

$$\frac{1}{N} d\left[\frac{dL}{dR}\right] = -\Omega^{-1} dP(\theta_1)\frac{(X^{*\prime}V_1)}{N}$$

(ivb) $$= -\Omega^{-1}\left[\frac{\delta P(\theta_1)}{\delta\theta_{1i}}\right] \frac{[X^{*\prime}V_1]}{N} d\theta_{1i}$$

The $plim(X^{*\prime}V_1/N)$ is not zero, as we have a lagged endogenous variable. Now if we take the total derivative of (iiib) (see Magnus and Neudecher [1988]).

$$\frac{1}{N} d\left[\frac{\delta L_e}{\delta\theta_{1i}}\right] = -tr\left[\Omega^{-1} d\Omega\ \Omega^{-1}\left(\frac{E'X^*}{N}\right)\left[\frac{\delta P(\theta_1)}{\delta\theta_{1i}}\right]\right]$$

$$+ \operatorname{tr}\left[\Omega^{-1}\frac{(E'X^{*})}{n}\, d\left[\frac{\delta P(\theta_1)}{\delta\theta_{1i}}\right]'\right]$$

$$+ \frac{1}{n}\operatorname{tr}\left[\Omega^{-1} dE'Z\, \frac{\delta P(\theta_1)}{\delta\theta_{1i}}\right]$$

Under the null plim$(E'X^{\bullet}/n) = 0$ which means that the first two terms are likely to be negligible, that leaves us with:

$$\text{(vb)} \quad \frac{1}{n}\, d\left[\frac{\delta L}{\delta\theta_{1i}}\right] \approx -\operatorname{tr}\left[\Omega^{-1}\, \frac{\delta P(\theta_1)}{\delta\theta_{1i}} \frac{(X^{\bullet\prime}X^{\bullet})}{n}\, \frac{\delta P(\theta_1)'}{\delta\theta_{1i}}\right] \delta\theta_{1i}$$

as $dE' = -dP(\theta_1)X^{\bullet\prime} = -\left[\frac{\delta P(\theta_1)X^{\bullet\prime}\delta\theta_{1i}}{\delta\theta_{1i}}\right] \approx o$

so that when we stack the θ_{1i}s into a vector we have the following matrix form:

$$\text{(vib)} \quad -(\text{Hes})_{11} = -\left[\frac{\delta \operatorname{vec}P(\theta_1)'}{\delta\theta_1}\left(\Omega^{-1}\otimes\frac{(X^{\bullet\prime}X^{\bullet})}{n}\right)\frac{\delta \operatorname{vec} P}{\delta\theta}\right]$$

and (ivb) becomes

$$-(\text{Hes})_{12} = \left[\Omega^{-1}\otimes \frac{V_1'Z}{n}\right] \frac{\delta \operatorname{vec}(P(\theta_1))}{\delta\theta_1}$$

$$(\text{Hes})_{22} = \Omega^{-1}\otimes \left(\frac{V'V}{n}\right)$$

so from the formula for the partitioned inverse (see Dhrymes [1984]) we have that:

$$n\, V_{22} = [(\text{Hes})_{22} - (\text{Hes})_{21}\, (\text{Hes})_{11}^{-1}\, (\text{Hes})_{12}]^{-1}$$

$$= (\text{Hes}_{22})^{-1} + (\text{Hes}_{22})^{-1}(\text{Hes})_{21}V_{11}(\text{Hes})_{12}(\text{Hes}_{22})^{-1}$$

where $V_{11} = [(\text{Hes})_{11} - (\text{Hes})_{12}(\text{Hes}_{22})^{-1}(\text{Hes})_{21}]^{-1}$

Now we have that:

$$\left(\frac{\delta L}{\delta \theta_2}\right) H_{22}^{-1} \left[\frac{\delta L}{\delta \theta_2}\right]' = N^2 tr\left[\frac{(V'V)}{N}^{-1} \frac{V'V_1}{N} (V_1'V_1/N)^{-1} \frac{V_1'V}{N} \right]$$

which is asymptotically equivalent to:

$$N^2 tr \left(\Omega_v^{-1} \frac{V'V_1}{N} \Omega_v^{-1} \frac{V_1'V}{N}\right)$$

which is a multivariate Durbin-Watson statistic, and

$$\left[\left(\frac{\delta L}{\delta \theta_2}\right)' (Hes\ )_{22}^{-1} (Hes)_{21}\right] = N\left[vec\left(\frac{V'V_1}{N}\right)\right]' \left(\Omega^{-1} \otimes \left(\frac{V_1'V_1}{N}\right) \frac{V_1'X^*}{N}\right) \frac{\delta vec[P(\theta_1)]}{\delta \theta_1}$$

so that the test statistic is:

$$\text{(viib)} \quad LM_1(g^2) = N\ tr\left(\Omega^{-1} \frac{V'V_1}{N} \Omega^{-1} \frac{V_1'V}{N}\right) +$$

$$\left[vec\left(\frac{V'V_1}{N}\right)\right]' \left(\Omega^{-1} \otimes \left(\frac{V_1'V_1}{N}\right) \frac{V_1'X^*}{N}\right) \frac{\delta vec[P(\theta_1)]}{\delta \theta_1} V_{11} \left[\frac{\delta vec[P(\theta_1)]}{\delta \theta_1}\right]'$$

$$\left[\left(\Omega^{-1} \otimes \left(\frac{X^{*\prime}V_1}{N}\right) \frac{V_1'V_1}{N}\right) vec\left(\frac{V'V_1}{N}\right)\right] \sim \chi^2_{g^2}$$

The above test can be generalised to take account of any order of serial correlation by replacing V_1 with V_i and when such tests are independent we can derive a portmanteau test:

$$LM_i(ig^2) = \sum_{j=1}^{i} LM_j(g^2) \sim \chi^2_{(ig^2)}$$

We can simplify (viib) when we believe that the lagged endogenous variables are the cause of bias in the Box-Pierce or multivariate Durbin Watson test, because they enter the model linearly. Let us partition P to separate out the lagged endogenous variables:

$$y_t = P_1 x^*_{1t} + P_2(\theta_o)\, x^*_{2t} + v_t$$

where P_1 are linear in parameters and x^*_{1t} are lagged endogenous variables, then taking the last term we have:

$$\text{(viiib)}\quad N\left[\frac{\mathrm{vec}(V'V_1)}{N}\right]'\left(\Omega^{-1}\otimes\frac{(V_1'V_1)}{N}\frac{V_1'X^*}{N}\right)\frac{\delta\mathrm{vec}[P(\theta_1)]}{\delta\theta_1}\; V_{11}\left[\frac{\delta\mathrm{vec}[P(\theta_1)]}{\delta\theta_1}\right]'$$

$$\left[\left(\Omega^{-1}\otimes\frac{(X^{*\prime}V_1)}{N}\frac{V_1'V_1)}{N}\right)\mathrm{vec}\frac{(V'V_1)}{N}\right]$$

now we have $\theta_1' = [\mathrm{vec}(P_1)' : \theta_o']$ and $\delta\mathrm{vec}(P_1)/\delta\theta_1 = [\ I : 0\ ]$ and when we partition $V_{11} = \begin{bmatrix} V_{33} & V_{34} \\ V_{43} & V_{44} \end{bmatrix}$ we can re-write (viiib) as

$$N\left[\frac{\mathrm{vec}(V'V_1)}{N}\right]'\left(\Omega^{-1}\otimes\frac{(V_1'V_1)}{N}\frac{V_1'X_1^*)}{N}\right) V_{33}\left[\left(\Omega^{-1}\otimes\frac{(X_1^{*\prime}V_1)}{N}\frac{V_1'V_1)}{N}\right)\mathrm{vec}\frac{(V'V_1)}{N}\right]$$

and when we only have y_{t-1} then we can replace X_1^* by Y_1 a g x N matrix of observations on the first order lags on y. Godfrey and Wickens (1982) show that the first order test given above does not distinguish between a VAR(1) error and a VMA(1), because of the approximations used to derive the score test.

Chapter Five

Dynamic Extensions to Models of Adjustment Based on Quadratic Loss Functions

In the previous Chapter we saw that econometric models which incorporate rational expectations can be presented as the solution to an optimal control problem. The usual quadratic optimisation problem restricts the parameters of the rational expectations model and it confines the resulting dynamic model. There are two primary methods of estimating such models: the first is based on the spectral decomposition of the lag polynomial produced by minimising the loss function in the control problem, the second uses the state space form to derive an estimable model based on iterating over the matrix ricatti equations; Chow (1983) explains the relationship between these methods. Here, we follow Sargent (1978) and Sargent and Hansen (1981) in using the explicit solution method, because it gives a more explicit treatment of the economic problem. Kollintzas(1985) has derived an equivalent method which diagonalises the problem to simplify the algebra of the solution, but it only works for the symmetric case. We extend the Sargan approach of the previous chapter, because we feel that it has computational advantages over the other methods. In particular, the state space method quickly produces large state space vectors and fixed parameter matrices even for quite small models.

A number of the results presented here are similar to those of Kollintzas (1985) and Kollintzas and Geerts (1984), though we deal with a forward solution rather than a backward one and we show that the recursive approach can be generalised to any order of quadratic loss function. The method uses the actual future values and allows the restrictions implied by rational expectations to be imposed.

The Chapter is split in to four sections: the first deals with the introduction of lags and leads in the equilibrium condition and the second extends the loss function to allow adjustment costs to be spread over a number of periods and the final sections cover global and local identification of the non-symmetric model. The purpose of this exercise is to derive econometric specifications which are more closely related to observed dynamic models. Wallis (1980) suggests, that rational expectations models should be compared with unrestricted distributed lag models as a tests of their validity. If such comparisons are to be successful, then the lag structure implied by the rational expectations model should have some relation to the data generation process. If we take the money and consumption models of Hendry and Mizon (1978) and Davidson et al (1978) as examples of modelling time series data, then clearly the simple first order rational expectations model is not easily related to such models.

In the previous chapter we saw that the Lagrange-Euler condition for a rational expectations model is similar to a simple error correction, except that the correction term can be explosive if the rational expectations model has a regular saddle point solution. Error correction models which have been observed as representing the data are very different to the type of euler condition produced by the solution of first order objective functions. Comparison of single equations is problematic, especially in the light of cointegration, because it does not capture the inter-action between dependent variables. It is only be relevant in the multivariate case, when the series are independent. However, it provides some measure of the possible distortions which may occur when too a simple dynamic is imposed on the data.

The solution offered in this chapter is to derive more general models based on the principal of rational expectations or consistent prediction which nest the simple models within them. The strong rational expectations hypothesis aligned with a restrictive dynamic is not then imposed on the data.

5.1 Dynamic extensions to the Econometric Specification via the equilibrium condition.

Sargent and Hansen (1981) and Sargan (1982) specify methods of estimation and solutions to the multivariate costs of adjustment rational expectations models derived from the optimisation of quadratic objective functions. Sargent and Hansen deal with agents maximising a profit or utility function subject to symmetric costs of adjustment. The econometric model is derived by finding the optimal solution to the agent problem, eliminating the endogenous variable expectations using the forward solution method and substituting out for the exogenous variable expectations by the Wiener-Kolmogorov prediction formula. The approach is limited, because it assumes that the observed data correspond exactly to the model derived from the solution of the agent problem and it does not reveal the deep parameters of the model. Here, we follow Sargan (1982) in assuming that the agent problem needs to account for a cost of disequilibrium, and in specifying a method which solves the model forward to reveal the deep parameters. The Sargan approach is explained in detail in the context of an econometric specifications which allows for lagged variables and which does not impose the usual strong rational expectations restrictions on the forward evolution of the exogenous variables.

The agent or social planner determines his optimal plans conditional on the current information set Ω_t which is a superset of the available data. The plans are derived by minimising a quadratic loss function with two elements: an adjustment cost K and a cost of being away from the target or equilibrium H. The expected loss is given by:

$$(5.1)\quad E(C_t \mid \Omega_t) = E\left(\sum_{t=o}^{T} \beta^t (\Delta y_t' K \, \Delta y_t + (y_t - z_t)' H (y_t - z_t) \mid \Omega_t\right)$$

where $E(x_{j+s} \mid \Omega_j)$ is the expectation of x_{j+s} conditional on the information available at period j and K and H are positive definite matrixes of costs; this final condition is necessary for a unique local minimum and it implies that the cost function is convex.

The conditions on the cost matrixes of the Hansen and Sargent (1981) model are the mirror image of those presented here, because that method involves the maximisation of an objective function which means that the cost matrixes need to be negative definite. Such models would also require symmetry to be imposed, because of the theoretical model underlying the maximisation problem. If (5.1) represents the maximisation of utility subject to costs of adjustment, then the zs would be prices or demand shocks and the condition on the Hessian of cross price derivatives would require symmetric cost matrixes to satisfy the axioms of choice. As Deaton (1975) explains, symmetry in the cross Hessian of compensated price coefficients is necessary so that agents make consistent choices, otherwise they would be confused by the monetary evaluations due to a change in the good of account. In our case such theoretical restrictions do not affect the parameters of the cost function, but they may influence the coefficients of the static equilibrium condition below:

(5.2) $z_t = Ax_t + \varepsilon_t$

where A is a matrix of fixed parameters, x_t a vector of observables and ε_t the unobservable or stochastic part of z_t.

The loss function attenuates adjustment to a target value z_t which can be viewed as an equilibrium or optimal point on an agent or economy specific response function. The target is a notional point to which the economy adjusts in the short-run. The implication of this approach is that agents have desires which they would like to achieve and which are based on the solution to an idealised problem, but the desires are notional in the sense that they cannot be achieved automatically. This leads to the subsidiary problem of selecting a course of action with respect to actual control variables to meet the target or desired values. Minimisation of the loss function reveals a set of plans or contingencies which limit the expense of adjusting to a moving target.

The target of an agent response function is static, but we have no reason to believe that this is the case. If we look at the Consumer Maximising utility, his target is the optimal point on a demand curve. If the consumer problem is an intertemporal one or the commodities are addictive or habit forming, then the demand curve will be a function of current and future prices, in the first case or past prices in the last. In general our target relationship will be dynamic or, in the direct optimisation approach of Hansen and Sargent, the utility of profit function will depend on past and future values of exogenous variables:

(5.3) $z_t = A(L)\, x_t + A\_(L^{-1})\, x_{t+1} + \xi_t$

where $A(L) = I + A_1L + A_2L^2 + \ldots . A_pL^p$

and $A\_(L) = I + A_{-1}L^{-1} + A_{-2}L^{-2} + \ldots A_{-r}L^{-r}$

with the $p < s$ where s is the order of the process driving the exogenous variable and L is the lag operator and L^{-1} the lead operator

The short-run model depends on the solution to the rational expectations model and the stochastic process forcing the exogenous variables. If the endogenous variables can be described as a vector autoregressive model which is weakly stationary, then that can be inverted to reveal an infinite order moving average model:

(5.4) $x_t = \sum_{s=o}^{\infty} C_s \, \varepsilon_{t+s}$

where w_t is a Martingale difference or white noise innovation.

The forward looking solution to the rational expectations model is found by minimising (5.1) with respect to y_t, that reveals the first order or lagrange Euler conditions; Sargent and Hansen (1981) and Sargan (1982) present this result for the multivariate costs of adjustment model. Therefore:

(5.5) $E(Q_o^* \, y_t - \beta K \, y_{t+1} - Ky_{t-1} = Hz_t | \Omega_t)$

where $Q_o^* = ((1 + \beta)K + H)$

(5.6) $\lim_{T \to \infty} \quad \beta^{\frac{1}{2}T} E(y_T | \Omega_t) \quad \to \quad 0$

Condition (5.5) equates the marginal cost of adjustment with the discounted benefit of achieving the target and condition (5.6) is the necessary condition for stability. If (5.6) does not hold then we would not get a finite solution to our problem. Equation (5.5) will be valid when we either have two stage decision making process, so that the target values do not depend on the solution to the cost minimisation problem or a target based on a global solution which is not affected by the actions of individual agents. The two problems need to be separable which implies a general problem that is linear, so that the solution to the cost function and the utility function or profit function determines the z_ts. The approach of Sargent and Hansen do not allow for disequilibria, but they do allow for exogenous variables in the structure of the revenue or expenditure functions. Nickell(1987) shows that the two problems are identical.

In (5.5) we have a second order difference equation for which there is a standard solution. Sargent (1979) covers the univariate case in detail and Sargent and Hansen (1981) and Sargan (1982) deal with the multivariate rational expectations model. The problem can be factorised into a backward-forward solution given an equal number of stable and unstable roots, the well known saddle point result for rational expectations models.

(5.7) $E(y_t - Fy_{t-1} = (I - G_1L^{-1})^{-1}FB_oz_t \mid \Omega_t)$

where $G_1 = \beta F$, $B_o = K^{-1}H = (\beta F + F^{-1} - (1 + \beta)I)$, $Q_o = \beta^{\frac{1}{2}}Q_o^*$ and $F = \beta^{-\frac{1}{2}}PMP^{-1}$ where M is composed of the stable roots of the system where $|H^*\mu - (1 + \mu^2)I| = 0$ and μ, $1/\mu$ and $\beta\mu$ will be real roots given that $H^* = K^{-1}Q_o$ is positive definite (see appendix 4.A1 for details).

Solving forward and working through the expectations operator in (5.7) implies that:

(5.8) $y_t - Fy_{t-1} = \sum_{s=0}^{\infty}(G_1)^s G_2 \; E(z_t \mid \Omega_t) + u_t$

where u_t is the error associated with elements of u_t which have either been left our or cannot be modelled; u_t will be white noise if the model is correctly specified.

Transforming (5.8) above using a first order Koyck lead eliminates the future values of the target variables; Muellbauer and Winter(1980) have dealt with this in the univariate case and Sargan (1982) and Hunter (1984) for the multivariate case. Here, the problem is complicated by the lag and lead terms, but future expectations in the exogenous variables may still be removed by applying the Wiener-Kolmogorov prediction formula (see Appendix 4.A1 for details). If (5.3) involves future values then some of those expectations will not be removed by this process:

$$(5.9)\quad (I-G_1L^{-1})(y_t-Fy_{t-1}-u_t) = \sum_{i=-p}^{r}(G^*\ E(x_{t+i}|\Omega_t)-(G_1)^{-i}D_{-i}\varepsilon_{t+1})$$

where $G^*_{-i} = FB_oA_{-i}$ and $D_{-i} = \sum_{j=1}^{\infty}G_1^jFB_oA_{-i}C_{j-1}$ and $1 = i+1$ for all $i \geq 0$ and $i = 1$ otherwise.

At this stage the model could be estimated using the errors in variables approach of Wickens (1982), but as was explained in the previous chapter, that would not take account of the cross equation restrictions or the moving average error term (u_t - G_1u_{t+1}). One alternative to the approach we adopt here would be to explicitly model the error process, but that would introduce further complications. The more usual formulation of the VMA error would require exact maximum likelihood estimates of the parameters, because the roots of the matrix of coefficients lie outside the unit circle when the rational expectations costs of adjustment model is correct. An alternative formulation could be derived with the moving average parameters inside the unit circle; Pesaran(1987) calls this a forward filter method. A further non-linearity would be introduced into such exact methods, because the likelihood would have to be conditioned on the current, future and lagged ys. Here, we use the approach of the previous chapter which reverses the Koyck lead to reveal a forward looking solution in terms of the actual x_ts:

$$(5.10)\quad y_t-Fy_{t-1}-u_t= (I-G_1L^{-1})^{-1}(\sum_{i=-p}^{r}(G^*\ E(x_{t+i}|\Omega_t)-(G_1)^{-i}D_{-i}\varepsilon_{t+1})$$

If we let the RHS of (5.10) be equal to h_t then we have a model which can be estimated recursively using maximum likelihood techniques (see Appendix A2 for details). Therefore:

$$(5.11)\quad y_t - Fy_{t-1} - u_t = h_t$$

$$(5.12)\quad h_t = \sum_{i=-p}^{r} (G^* E(x_{t+i}|\Omega_t) - (G_1)^{-i} D_{-i}\varepsilon_{t+1}) + G_1 h_{t+1}$$

and $E(x_{t+i}|\Omega_t) = x_{t+i}$ for all $i \le 0$

If the parameters of the exogenous process C_i are known and outside the sample we use predictions to compute h_{t+1}, then for a sample and prediction period T + N, where N is sufficiently large, the truncation of the backward iteration should be $o(T^{-½})$, given that (5.6) holds. The dynamics specified appear an unnecessary complication, but they are both consistent with the solution of rational expectations models and with a general modelling strategy. Setting up models in this way is more likely to generate correctly specified models, because it allows the restrictions to be tested rather than imposed. A number of dynamic models are nested within (5.11) and (5.12), but we will limit ourselves to the restrictions which differentiate this model from the static equilibrium model, present a method of testing the backward forward solution and use a demand system to explain the relevance of our results.

The standard costs of adjustment model presented by Sargan (1982) and Hunter (1984) is a special case of this model, in which the target is specified by (5.2) so that $A_{-i} = 0$ for all $i \neq 0$. If the target relationship is the relevant model of the dependent variable, then we would expect the future convolution not to be important and the distributed lag on the exogenous variables to depend on stable roots not related to F. The structure of the quadratic loss function imposes the symmetric backward-forward solution, but we can test the assumption in this case by imposing the correct restrictions on the lagged coefficients. When the first order cost of adjustment model from chapter four is appropriate, then we would expect A_{-i} for all $i < 0$ to be equal to $F^{-i}A_0$ and $\beta=0$. If $\beta = 0$ was the only restriction that held, then that could be caused by either the future expectations being heavily discounted or the exogenous variable models not being very good.

We can show that (5.11) and (5.12) allow a range of models to be tested within one structure and in the utility maximising framework we can formulate three hypotheses which would produce dynamic models: habit persistence, durable goods and non-separable intertemporal choice. The framework presented above allows us to compare some of these dynamics with those due to rational expectations. If we take the stock-flow model of Stone and Rowe (1957) and Nerlove (1956), as presented in Deaton and Muellbauer (1980), and generalise it to g goods:

(5.13) $d_t = R_o(S_t^* - S_{t-1}) + \delta S_{t-1}$

$$d_t = \begin{bmatrix} d_{1t} \\ \cdot \\ \cdot \\ \cdot \\ d_{gt} \end{bmatrix}, \quad S_t^* = \begin{bmatrix} s_{1t}^* \\ \cdot \\ \cdot \\ \cdot \\ s_{gt}^* \end{bmatrix} \text{ and } S_t = \begin{bmatrix} s_{1t} \\ \cdot \\ \cdot \\ \cdot \\ s_{gt} \end{bmatrix}$$

Hence d_t is the demand for the vector of goods, based on a stock adjustment principal, given a relationship for the desired stock last period and the stock held:

$$S_t^* = R_1 x_t$$

where $x_t = [1, y_t^d/P_t]$ in Deaton and Muellbauer, but in general it depends on many more variables.

Application of a Koyck lead on (5.13) allows us to eliminate the stock S^* which is not observed from the demand relationship:

$$d_t - (1-\delta)d_{t-1} = R_o R_1 (x_t - (1-\delta)x_{t-1})$$

$$+ (\delta I - R_o)[S_{t-1} - (1-\delta)S_{t-2}]$$

$$= R_o R_1 (x_t - (1-\delta)x_{t-1}) + (\delta I - R_o)d_{t-1}$$

$$(I - (I - R_o)L)d_t = R_o R_1 (x_t - (1-\delta)x_{t-1})$$

Inverting the Koyck lead gives the demand model purely in terms of the observed exogenous variables x_t.

$$d_t = \sum_{i=0}^{\infty} (I - R_o)^i R_o R_1 [x_t - (1-\delta)x_{t-1}]$$

$$= A_o x_t + A_1 x_{t-1} + A_2 x_{t-2} + \ldots\ldots$$

where $A_o = R_o R_1$, $A_j = [\delta I - R_o][I - R_o]^{j-1} R_o R_1$
for all $j \geq 1$

Using (5.11) and (5.12) to reveal the model in terms of actual demands solved for future expectations, we have:

$$d^a = F d^a_{t-1} + h_t + u_t$$

$$h_t = \sum_{i=-p}^{0} (G^* \; E(x_{t+i} \mid \Omega_t) - (G_1)^{-i} D_{-i} \varepsilon_{t+1}) + G_1 h_{t+1}$$

where d^a_t is the vector of actual real expenditure on g goods and $G^*_{-i} = F \; B_o^{-1} A_{-i}$, A_i is defined above, $D_{-i} = \sum_{j=1}^{\infty} G_1^j F B_o A_{-i} C_{j-1}$
$G_1 = \beta F$ and C_{j-1} are from the moving average form for the xs.

An appropriate criterion function would be minimised with respect to β, δ, R_o, R_1 and F, given the restrictions on B_o and the C_{j-1}s.

5.2 General Quadratic Objective Functions

In this section we extend the first order cost of adjustment form of the previous section to allow for interaction between the stock and flow and to introduce higher order costs of adjustment. Kollintzas (1985) extended the model due to Hansen and Sargent (1981) to include such terms for the case in which the costs of adjustment matrixes are symmetric and he presents the multivariate result, as the solution to a number of univariate problems. Here, we analyse a similar model in the same manner as in the previous section, but we do not impose the symmetry which implies that the transformation to a univariate formulation is not possible. The more general framework allows the restrictions implied by the simpler formulations to be tested and provides models which do not impose such a strong variant of the theory, but allow the econometric specification to correspond more closely to the data generation process.

The inter-action term takes account of the additional effect that distance from the target may have on adjustment. This is similar to an acceleration cost, because shifts to a point far from an agent optimum or made far away from such an optimum becomes increasingly more expensive. The cost function to be minimised is the same as in the previous section except that it includes a term bilinear in y_t and $(y_t - z_t)$. Therefore:

$$(5.14)\quad E(C_t \mid \Omega_t) = E\Big(\sum_{t=o}^{T} \beta^t (\Delta y_t' K\, \Delta y_t + (y_t - z_t)' H (y_t - z_t) + 2\Delta y_t' J\, (y_t - z_t) \mid \Omega_t\Big)$$

where y_t and z_t are as specified for equation (5.1) , $E(x_{j+s} \mid \Omega_j)$ is the expectation of x_{j+s} conditional on the information available at period j and $\begin{bmatrix} K & J \\ J' & H \end{bmatrix}$ is the positive definite cost matrix; a necessary condition for a a minimum and a cost function that is convex[1].

The target z_t is taken to be static in this case, so that its relationship is given by (5.2) in the previous section. The set of optimal plans is devised by minimising (5.14) with respect to y_t.

Kollintzas' approach is defined in terms of maximising an objective function subject to symmetric cost matrixes. The full matrix of costs needs to be negative semi-definite for the solution to reveal a maximum, the method of deriving the solution is not materially different, but the result is. Obviously the two underlying agent problems suggest different coefficient values implying that in most cases only one of these models is likely to be validated by the data. In special cases we may find two models which are observationally equivalent, but it seems unlikely that both formulations will be both correctly specified and theory consistent. Symmetry is a sensible condition for a model to satisfy when it is based on utility or profit maximisation, but it should be criterion to be tested rather than imposed. The example presented in Kollintzas relates to the interrelated factor demand model which should satisfy symmetry restrictions, but as Deaton and Muellbauer(1980) show in the case of consumer theory that such restrictions are an important check on the validity of the model. Kollintzas' approach simplifies the solution of rational expectations models at the cost of

[1] J = J' implies that we can use Kollintzas method

imposing symmetry which makes this approach questionable, because inappropriate restrictions are likely to misspecify the model.

Here, we do not impose symmetry and the stochastic environment for the exogenous variables is given by (5.3) above. The Lagrange-Euler first order conditions are given by minimising (5.1) subject to y_t. Therefore:

$$(5.15)\quad E(Q_o^* y_t - \beta Q_1^* y_{t+1} - Q_1^{*\prime} y_{t-1} = (J + H)z_t - \beta J z_{t+1} | \Omega_t)$$

where $Q_o^* = ((1 + \beta)K + J + J' + H)$ and $Q_1^* = J + K$

$$(5.16)\quad \lim_{T \to \infty} \beta^{\frac{1}{2}T} E(y_T | \Omega_t) \to 0$$

The first order condition is similar to the separable cost model presented by Sargan (1982), except for the asymmetry caused by $J \neq J'$ the non-symmetric interaction cost and the introduction of a first-order lead in the target. The standard solution to the multivariate costs of adjustment problem can be augmented to take account of the asymmetry (see Appendix B2 for detail). The lead in the target does not affect the forward solution to the dynamic problem presented in (5.15) and (5.16). Therefore

$$(5.17)\quad E(y_t - Fy_{t-1} = (I - G_1 L^{-1})^{-1}(F(Q_1^+ + (Q_1^*)^{-1}(H - K))z_t + \beta((Q_1^{*\prime})^{-1}K - Q_1^*)z_{t+1} | \Omega_t)$$

where $G_1 = \beta F Q_1^+$, $Q_1^+ = (Q_1^{*\prime})^{-1} Q_1^*$ and $F = \beta^{-\frac{1}{2}} PMP^{-1}$ and M is composed of the stable roots of the characteristic equation $|Q_o^* \mu - Q_1^* \mu^2 - Q_1^{*\prime}| = 0$ and μ, $1/\mu$ and $\beta\mu$ are real roots of the system if Q_o^* is positive definite and J is a symmetric matrix, symmetry of Q_o^* is sufficient for a solution to exist, but that will not guarantee real roots.

It is possible from (5.17) to derive the same type of forward looking econometric specification by working through the expectations operator and replacing the unobserved target variables using the static agent response or equilibrium condition (5.2), so that:

$$(5.18)\quad y_t - Fy_{t-1} = \sum_{s=0}^{\infty}(G_1)^s FE(B_o z_t - B_1 z_{t+1} | \Omega_t) + u_t$$

where u_t is a white noise error term if (5.18) is well specified and $B_o = (Q_1^+ + (Q_1^{*}{}')^{-1}(H - K))$ and $B_1 = (Q_1^+ - (Q_1^{*}{}')^{-1}K)$ and $B_o = \beta Q_1^+(F - I) + F^{-1} - I + B_1$

By analogy with the previous section we can perform the same operations to transform (5.18) into an iterative model from which the future expectations have been removed. The first stage utilises the Koyck lead and the Wiener-Kolmogorov prediction formula to eliminate the predictions of the exogenous variables, but that reveals the model with the moving average error (see Appendix 4.A1). The second stage gets rid of the moving average error term by reversing the Koyck lead and this converts our original forward model in expectations into a forward looking model in actual exogenous variables which can be represented in iterative form (see Appendix A2 of this chapter):

$$(5.19)\quad y_t - Fy_{t-1} - u_t = h_t$$

$$(5.20)\quad h_t = G_2 x_t - G_3 x_{t+1} - G^{*}\varepsilon_{t+1} + G_1 h_{t+1}$$

where $G_1 = \beta F Q_1^+ \quad G_2 + FB_oA \quad G_3 = FB_1A \quad G^{*} = FD$

$D = \sum_{s=1}^{\infty}(G_1)^{s-1}(G_1B_o - B_1)AC_{s-1}$ and

$B_o = \beta Q_1^+ (F - I) + F^{-1} - I + B_1$

The model can be solved recursively by iteration over h_t, given that the terminal condition $h_{t+n} = 0$ and the stability condition

(5.16) are satisfied. The appropriate criterion function will be optimised with respect to the discount parameter β the deep parameters A the factor matrix F and the matrixes B_1 and Q_1^+ which are composed of the original cost matrixes H, K and J; B_o can be derived from $B_o = \beta Q_1^+(F-I)+F^{-1} - I + B_1$, given βF, B_1 and Q_1^+

The system specified by (5.19) and (5.20) nests a number of models within it: the static model in which $y_t = z_t$, a vector AR(1) model, a partial adjustment model and the separable cost of adjustment model with static target. The restrictions associated with different econometric specifications similarly apply to the separable cost model.

(5.21) $y_t = Ax_t + u_t$

when F = 0, so that we have a simple static model

(5.22) $y_t = Fy_{t-1} + u_t$

when A = 0, which is a VAR(1) time series model. Certain exchange rate models and the Hall consumption function have been justified in this way.

(5.23) $y_t = Fy_{t-1} + (I - F)Ax_t + u_t$

when $\beta = 0$, so that the future convolution is not important we are left with a partial adjustment model. The long-run solution to (5.23) is given by the target condition, so that $y^* = Ax^*$.

The other restrictions transform the model into costs of adjustment rational expectations models with specific restrictions on the loss function. If $B_o = 0$, then the target cost and inter-action cost cancel each other out, so that $J = -H$ and the term in x_t is left out of (5.20). It is then possible to estimate all the parameters of this model directly, because B_1 is computed using $-(\beta Q_1^+(I - F) + I - F^{-1})$. If $B_1 = 0$ we are left with the separable cost model from chapter four, because the restriction implies that $J = 0$ and $Q_1^+ = I$. We could also include lags and leads in the target relationship of the separable cost model, but some parameters would not have to exceed the lag length of the exogenous process and in an unrestricted model the coefficients on the first lead term would not all be identified if the target condition (5.3) contained the same lead.

The separable cost of adjustment model has a solution with real roots when Q_o^* and Q_1^* are positive definite, while no such condition holds in the non-separable case. The solution requires Q_o^* to be symmetric in the separable case, but that does not ensure positive definiteness and positive definiteness does not ensure real roots. Kollintzas confirms this assertion in the case in which Q_o^* and Q_1^* are symmetric by showing that the roots may be imaginary for this special case. If the cost matrixes are positive definite, then the backward forward solution for all the models presented in chapter four and five cannot have roots on the unit circle (see Appendix C for proof of this result in the most general case).

Econometric specifications are usually reductions or marginalisations of a more general formulation, such models can be represented as multivariate autoregressive moving average models with exogenous variables. Therefore:

$$(5.24)\quad \theta(L)s_t = \Phi(L)\,\epsilon_t$$

$$\text{where}\quad s_t = \begin{bmatrix} y_t \\ x_t \end{bmatrix} \quad \theta(L) = \begin{bmatrix} \theta_{11}(L) & \theta_{12}(L) \\ \theta_{21}(L) & \theta_{22}(L) \end{bmatrix}$$

$$\Phi(L) = \begin{bmatrix} \Phi_{11}(L) & \Phi_{12}(L) \\ \Phi_{21}(L) & \Phi_{22}(L) \end{bmatrix}$$

$$\epsilon_t = \begin{bmatrix} u_t \\ \varepsilon_t \end{bmatrix} \sim WN(0, \Gamma)$$

The models presented so far restrict $\theta_{21}(L)$ and $\Phi_{21}(L)$ to be zero which implies that the x's are strictly exogenous, so that there is no feedback into the process driving the y's. The backward and forward solutions impose specific restrictions on the other parameters in the model, in the forward case: $\theta_{11}(L) = (I\text{-}FL)$, $\theta_{12}(L) = (I\text{-}G_1L^{-1})^{-1} B^*(L)$, $\Phi_{11}(L) = I$, $\Phi_{12}(L) = D^*(L^{-1})$ and $\Phi_{22}(L) = C(L)$ where B^* and D^* depend on the specific form of the loss function and the equilibrium condition. The principals of general modelling suggest that such restrictions should be tested by starting with a more general formulation and moving to a more specific one, even if the specific form appears to be well specified (Davidson et al 1978 and Hendry and Mizon (1978) explain such methods for single equations). Such a procedure would suggest that we either start with a model which is an order higher than we believe is correct or select a form which is consistent with the existing evidence. Hendry, Pagan and Sargan (1984) point out that the general qth order cost of adjustment model is the only form which will be compatible with (5.24), so that:

$$(5.25)\quad E(C_t \mid \Omega_t) = E\left(\sum_{t=o}^{T} \beta^t (\Delta y_t^{+\prime} K^+ \Delta y_t^+ + \tilde{y}_t' H \tilde{y}_t) \mid \Omega_t\right)$$

$$\text{where} \quad y^+ = \begin{bmatrix} y_t \\ y_{t-1} \\ \vdots \\ y_{t-q} \end{bmatrix}, \quad \tilde{y} = y_t^+ - z_t^+ \quad \text{and} \quad z_t^+ = \begin{bmatrix} z_t \\ z_{t-1} \\ \vdots \\ z_{t-q} \end{bmatrix}$$

and K^+ is a positive definite matrix with typical elements matrix K_{ij} and H^+ is positive definite with element matrix H_{ij} where $i = 0, \ldots q$ and $j = 0, 1, \ldots q$.

The loss function could be thought of directly in state space terms with (5.25) being minimised subject to y_t^+ which would yield a first order representation of the model, but this does not apply in this case, because the solution with respect to y_t^+ depends on past values of y. Hence, the first order analogue of the usual model does not take account of the restrictions implied by minimising (5.25) separately with respect to all of the y_ts, because it assumes that plans do not overlap or such restrictions are not important. The first order form in y_t^+ will only be valid for rational expectations models derived from the solution of (5.25) when K^+ or H^+ are diagonal or can be diagonalised simultaneously. It is only possible to diagonalise the system if H^+ and K^+ are symmetric and $H^+ + (1 + \beta) K^+$ is positive definite. The only other case in which a simple first order model holds is when there are no adjustment lags and one lag in the target cost term. If $K_t^+ = K$ $\;y_t^+ = y_t$ and $\tilde{y}_t' = [(y_t - z_t)'(y_{t-1} - z_{t-1})']$ then (5.25) becomes:

(5.26) $E(C_t \mid \Omega_t) = E(\sum_{t=o}^{T} \beta^t (\Delta y_t' K \, \Delta y_t + \tilde{y}_t' H^+ \tilde{y}_t \mid \Omega_t)$

where K and $H^+ = \begin{bmatrix} H_{oo} & H_{o1} \\ H_{1o} & H_{11} \end{bmatrix}$ are positive definite and z_t is the static target given by condition (5.2).

The solution to (5.26) is given by minimising the loss function with respect to y_t which gives us a similar first order condition to (5.14). Therefore:

(5.27) $E(Q_o^* y_t - \beta Q_1^* y_{t+1} - Q_1^{*\prime} y_{t-1} = (H_{oo} + \beta H_{11}) z_t + H_{o1} z_{t-1} + \beta H_{1o} z_{t+1} \mid \Omega_t)$

where $Q_o^* = ((1+\beta)K + H_{oo} + \beta H_{11}) = (Q_1^{*\prime} + \beta Q_1^* + H_{oo} + \beta H_{11} - \beta H_{10} - H_{01})$

and $Q_1^* = H_{10} + K$, $Q_1^{*\prime} = H_{01} + K$

(5.28) $\lim_{T \to \infty} \beta^{\frac{1}{2}T} E(y_T \mid \Omega_t) \to 0$

The Lagrange Euler condition (5.27) is the same as (5.15) except for the coefficients on the target variables z_t, z_{t-1} and z_{t+1} which means that the forward looking solution may be derived by analogy with the results for the inter-action model; equation (5.28) is the usual stability condition:

(5.29) $E(y_t - F y_{t-1} = (I - G_1 L^{-1})^{-1} (F(B_o z_t + B_{+1} z_{t-1} + \beta B_{-1} z_{t+1} \mid \Omega_t)$

where $G_1 = \beta Q_1^+ F$, $Q_1^+ = (Q_1^{*\prime}) Q_1^*$, $B_o = (Q_1^{*\prime})(H_{oo} + \beta H_{11})$,

$B_{+1} = (Q_1^{*\prime})^{-1} H_{01}$ and $B_{-1} = (Q_1^{*\prime})^{-1} H_{10}$.

$B_o = \beta Q_1^+ (F - I) + F^{-1} - I + \beta B_{-1} + B_{+1}$.

The forward solution is arrived at by straight forward application of the same techniques applied to (5.17) which reveal a similar type of recursive model, except for the addition of a lag term in the exogenous variable(see Appendix B2 for details).

(5.30) $y_t - F y_{t-1} - u_t = h_t$

(5.31) $h_t = G_{+1}x_{t-1} + G_o x_t + \beta G_{-1}x_{t+1} - G^x \epsilon_{t+1} + G_1 h_{t+1}$

where $G_1 = \beta F Q_1^+$, $G_i = FB_iA$ for $i=-1,0,+1$ $\quad G^x = \sum_{i=-1}^{1} (G_1)^{1-i} D_{-i}$

and $D_i = \sum_{s=1}^{\infty} (G_1)^{s-1} G_i C_{s-1}$

The model can be solved recursively subject to a criterion function for the composite cost parameters Q_1^+, B_1' and B_1; the matrix related to the stable solution to the system F; the deep parameters A and the discount factor β. The coefficients on ϵ_{t+1} are imposed using the parameters from the process determining the exogenous variables, and B_o by the restriction on the cost matrix determined by the solution.

Equations (5.30) and (5.31) provide a general model within which we can nest (5.21) when F=0 and (5.22) when A = 0. When $\beta = 0$ we get a pure partial adjustment model when $B_{-1} = 0$ and a more usual dynamic econometric model otherwise:

(5.32) $y_t - Fy_{t-1} - u_t = FB_1' Ax_{t-1} + FB_o Ax_t$

which has the long-run solution $y^* = Ax^*$, given the restriction $B_o = (F - I - B_1')$.

We get the first order model from chapter four when $B'_1 = 0$ and $B_1 = 0$, but as this is observationally equivalent to a model with a diagonal target cost matrix it is not direct evidence that longer lagged effects do not exist. If $B_{+1} = 0$ we have a model that is observationally equivalent to the inter-action cost model and when $B_{-1} = 0$ we have a triangular cost model which cannot be confused with inter-action costs, but must be seen as a pure asymmetric cost of disequilibrium in which the effect only feeds backwards.

The general solution to the intertemporal problem presented in (5.25) does not usually lead to first order models, but to more complex results based on the solution to higher order difference equations. Differentiation of (5.25) with respect to y_t reveals the following Lagrange-Euler first order condition which becomes in compact form:

$$(5.33) \qquad dE\,(C_t \mid \Omega_t) = E(K^{*}\Delta\, y^{+}_{t+1} + K^{*}\Delta y^{+}_{t} + H^{*}\,\Delta\tilde{y}_t \mid \Omega_t)$$

$$K^{*} = I^{*}K^{+} = \begin{bmatrix} K^{*}_{00}+K^{*}_{01}\cdots+K^{*}_{0q} \\ K^{*}_{10}+K^{*}_{11}\cdots+K^{*}_{1q} \\ \vdots \qquad\qquad \vdots \\ K^{*}_{q0}+\;\cdots\;K^{*}_{qq} \end{bmatrix} \quad \text{and} \quad H^{*} = I^{*}H^{+} = \begin{bmatrix} H^{*}_{00}+H^{*}_{01}\cdots+H^{*}_{0q} \\ H^{*}_{10}+H^{*}_{11}\cdots+H^{*}_{1q} \\ \vdots \qquad\qquad \vdots \\ H^{*}_{q0}+\cdots\cdots+H^{*}_{qq} \end{bmatrix}$$

where $K^{*}_{ij} = K_{ij}(L/\beta)^{-i}$ and $H^{*}_{ij} = H_{ij}(L/\beta)^{-i}$

The result may be easily validated by reference to the first order condition (5.27) where $K^{*} = K$, $y^{+}_{t} = y_t$,

$I^{*} = [I : (L/\beta)^{-1}I]$ and $H^{*} = I^{*}\begin{bmatrix} H_{oo} & H_{o1} \\ H_{1o} & H_{11} \end{bmatrix}$

The general model can be factorised in a similar way to the relationships already presented and that factorisation is dealt with in Appendix B1. When the cost matrixes are positive definite the factorisation eliminates the possibility of unit roots and it suggests that the stable roots are all in modulus less than $\beta^{1/2}$; the first proposition is proved in Appendix C. If the saddle point solution is correct the forward solution to the model reveals similar recursive results to those already obtained (Appendix A2 deals with the general result) which is:

$$(5.34) \quad G^{+}_{2}(L)y_t - u_t = h_t$$

(5.35) $$h_t = (G_1^+(L^{-1}))^{-1} g_t$$

where $g_t = E(\sum_{s=-q}^{g} \beta^s \sum_{i=o}^{q-s} \beta^i H_{i(s+1)} z_{t+s} \mid \Omega_t)$ and H_{ij} is an element of the cost matrix H^+, $G_1^+(L^{-1}) = G'(L^{-1}\beta^{½})$, $G_2^+(L) = WG(\beta^{-½}L)$ and W, G(.) and G'(.) come from the factorisation of the polynomial in y from the less compact version of the Lagrange Euler equation (5.33).

If we re-write (5.35) to bring out the recursive nature of the solution, then:

(5.36) $$h_t = g_t + F_1' h_{t+1} + F_2' h_{t+2} + \ldots F_q' h_{t+q}$$

$$\text{as } G_1^+(L^{-1}) = (I - F_1' L^{-1} - F_2' L^{-2} \ldots . F_q' L^{-q})$$

The model can be solved backwards, given a terminal condition h_t and a method of replacing the expectations in g_t. In the first order models it was sufficient to combine a Koyck lead of the form $G_1^+(L^{-1})$ and the Wiener-Kolmogorov prediction formula to eliminate the infinite distributed lead in exogenous expectations implied by the forward solution, but that is not possible here, because the order of the polynomial is greatly increased (the second order costs of adjustment model can be quite easily solved). As in all of these types of model it may only be possible to compute the costs of adjustment terms to a factor. It is difficult to know whether it is easier to estimate the model using (5.34) and (5.36) or to use the first order condition.

5.3 Identification of the cross product model of adjustment

In this section we look at the identification of the cross product model, but these results can be related to most of the other models presented here and in chapter 4. It is difficult to derive both necessary and sufficient conditions for identification in non-linear models, both Rothenberg (1971) and Sargan (1975,1983b) do this for quite general structures. Rothenberg (1971) produces local conditions for the identification of non-linear models, but global conditions usually depend on the form of the non-linearity. In this section we use the Lagrange-Euler first order condition to produce necessary conditions for global identification and then these can sometimes be augmented by sufficient conditions. In the next section we derive local conditions which do not depend on the global ones. A direct analogy can be made between the results presented here and the identification of models with autoregressive errors (see Sargan (1983a)). Such results usually rely on knowledge of the lag structure and such information is useful in identifying rational expectations models. Pesaran (1987) presents a number of conditions drawn from the literature which suggest that the difficult task of identifying rational expectations models may be simplified by our knowledge of the structure.

Initially we use the first order condition which incorporates the same restrictions as the quasi-reduced form in the previous section and the likelihood of this quasi structural form is also equivalent to that of the reduced form (see Pesaran (1987) for a related result). It can be shown that asymptotic identification and consistency are intimately related (see Sargan (1975) for a proof of this). If we can then relate consistent estimates of the parameters of the model to the quasi-reduced form, it can be shown that the parameters of that model will be identified. We start with the first order condition to simplify this procedure, as the non-linearities are less complex in that case. We then relate the first order condition to its reduced form, because those parameters can be identified under very weak conditions. When the reduced form is identified, then these less restricted parameters can be relate back to the structure. Hence, the Identification conditions are derived sequentially by firstly showing that the reduced form parameters are identified and then showing that a unique relationship exists between those parameters and the parameters of the structural form. Rothenberg(1971) shows that this is a sufficient condition for identification.

The cross product model can be presented in the following first order form which can be consistently estimated using either instrumental variables or an exact maximum likelihood method that estimates the moving average term encapsulated in the composite error u_t^+. The equivalent conditions for identification are much easier to derive in the instrumental variables case.

$$(5.37) \quad Q_o^* y_t - \beta Q_1^* y_{t+1} - Q_1^{*\prime} y_{t-1} - HAx_t - JA(x_t - \beta x_{t+1}) = u_t^+$$

where $Q_o^* = (1 + \beta)K + J + J' + H$ and $Q_1^* = K + J$

Clearly we can derive an order condition which is necessary for identification in this instance by comparing the parameters in (5.37) with those in the freely estimated reduced form:

$$(5.38) \quad y_t = P_1 \hat{y}_{t+1} + P_2 y_{t-1} + P_3 \hat{x}_t + P_4 \hat{x}_{t+1} + v_t^*$$

where $\beta Q_1^* = Q_o^* P_1$, $Q_1^{*\prime} = Q_o^* P_2$,

$$\beta A_2 = Q_o^* P_4 \quad \text{and} \quad A_1 + A_2 = Q_o^* P_3$$

Then, if we can identify P_1, P_2, P_3 and P_4 it may be possible to solve out for the parameters β, K, J, H and A. Such a solution will be derived in two stages the first relating (5.38) to a slightly more restricted form which is then related back to (5.37). We can simplify (5.37) in the following way:

$$(5.39) \qquad Q_o^* y_t - \beta Q_1^* \hat{y}_{t+1} - Q_1^{*'} \hat{y}_{t-1} - A_1 x_t - A_2(x_t - \beta \hat{x}_{t+1}) = v_t$$

Let us deal with the identification of the parameters in (5.38) first. If there is a matrix of optimal instruments $[\hat{y}_{t+1} : y_{t-1} : x_t ; \hat{x}_{t+1}]$, then a necessary condition identification is that this matrix is of full rank, so that its cross product is non-singular. This condition is somewhat complicated by the fact that we have to estimate (5.38) by instrumental variables which means that the condition for identification depends on the whole set of instruments.

If we have the process for x_t approximated by an sth order vector autoregressive model we find that the unrestricted backward solution for y_t is given by the following polynomial distributed lag model:

$$(5.40) \qquad y_t - F\, y_{t-1} = \sum_{i=0}^{s-1} \Gamma_i x_{t-i} + u_t$$

where the Γ depends on the parameters of the autoregressive process and those for the loss function, β,H,K,J and A.

It is clear that we need enough additional information to estimate the parameters in the reduced form (5.38) which means that $s > 2$ is required, as $\hat{x}_{t+1}$ depends on x_{t-k} for $k = 0$, 1..(s-1) and y_{t+1} depends on the same exogenous variables plus y_{t-1}. We can see when we lead equation (5.40) one period and then take expectations $\hat{y}_{t+1}$ will depend on $\hat{x}_{t+1}$, y_t and x_{t-k} for $k = 0, \ldots,(s-2)$. As y_t is clearly correlated with the error we need to instrument it which from (5.40) means that the instrument set is y_{t-1}, x_{t-k} for $k=0, \ldots, (s-1)$. Identification of all the parameters in (5.38), requires there to be information in addition to x_t and y_{t-1} so that the x_t process should on average have more than two lags on each variable. The first order model estimated in chapter 4 has the same backward solution (5.40) and it requires $s > 1$ to identify the reduced form coefficients, the model in the first section of this chapter requires $s > r + p + 1$ where r is the number of leads and p the number of lags on the exogenous variables. Equation (5.38) sets $r = 1$ and $p = 0$, so that we need at least a VAR(3) or exogenous variable processes with third order lags on g of the variables.

The reduced form (5.38) is linear in parameters which means that a necessary and sufficient condition for identification is the independence of $\hat{x}_{t+1}$, $\hat{y}_{t+1}$, y_{t-1} and x_t, and that in turn depends on the number of lags in the autoregressive model forcing x_t. If equation (5.38) satisfies the condition of independence, then the more stringent condition $s > r + p + 1$ should also be satisfied. At this stage we can use the fact that when the less restricted parameters associated with (5.38) are identified, then any consistent method of estimation which imposes additional restrictions should also be identified. Hence, we can derive an order condition associated with the parameters in (5.37) which will be necessary for global identification and a step-wise procedure which produces side conditions which are sufficient firstly to identify the parameters in (5.39) and finally those in (5.37). With $2kg + 2k^2$ parameters in the reduced from (5.38), the order condition for the identification of β,K,H,J and A will be:

$$2gk + 2g^2 \geq 3g^2 + gk + 1$$

$$gk \geq g^2 + 1 \qquad \text{or} \quad k > g$$

As K and H are assumed to be positive definite, though we only need K to be positive definite we can introduce the additional restriction that H and K are symmetric. The symmetry restrictions can be tested by comparing the likelihood of the model which imposes them with that which does not, then if it is valid we have the weaker condition that:

$$2gk + 2g^2 \geq g(g + 1) + g^2 + gk + 1$$

$$gk \geq g + 1 \quad k > 1$$

The order conditions presented above are intuitively appealing, but they are not easily augmented by additional conditions of a similar degree of simplicity. Let the parameters of the reduced form (5.38) be denoted by the vector ψ, the less restricted form (5.39) by the vector ξ and the parameters of the quasi structural form (5.37) by the vector θ, then we can derive a sufficient conditions for identification by applying sequentially the global conditions presented in Rothenberg(1971). If the conditions for the identification of the reduced form parameters are met, so that $\psi=\psi^*$, then the parameters of equation (5.39) are identified when a unique solution $\xi=\xi^*$ exists to the following equation:

$\psi^* = P(\xi)$

where $P(\xi)$ is a vector function of the parameters of equation (5.39).

Conditional on the existence of such a solution the sufficient condition for the identification of the quasi structural form is the existence of a unique solution $\theta = \theta^*$ to the following relationship:

$\xi^* = Q(\theta)$

where $Q(\theta)$ is a vector function of the parameters of equation (5.37).

Firstly we will deal with the identification of the parameters of (5.39), this is a linearisation of the original model and then we use that to relate the parameters Q_o^*, Q_1^*, A_1, A_2 and β to the P_is. Equation (5.39) is linear except for β which means that it should be relatively easy to derive sufficient conditions for those parameters. We have four equations relating the parameters in (5.39) to those in (5.38):

$$\beta Q_1^* = Q_o^* P_1 , \qquad Q_1^{*\prime} = Q_o^* P_2,$$

$$\beta A_2 = Q_o^* P_4 \quad \text{and} \quad A_1 + A_2 = Q_o^* P_3$$

The existence of the reduced form representation in terms of the structural from parameters ξ depends on Q_o^* being non-singular which in the context of the our cost minimisation problem means at least one positive definite cost matrix with the others being non-negative definite. It seems reasonable to assume that their are positive costs of adjustment which implies that K is positive definite. If we take the first term above and relate it to the transpose of the second then:

$$Q_o^* P_1 = \beta P_2' Q_o^{*\prime}$$

$$Q_o^* P_1 - \beta P_2' Q_o^* = 0$$

Vectorising this relationship (see Dhrymes (1984) chapter 4) gives:

(5.41) $\quad \mathrm{Vec}\,(Q_o^* P_1 - \beta P_2' Q_o) = ((P_1' \otimes I) - \beta\,(I \otimes P_2'))\ \mathrm{vec}\,(Q_o^*)$

assuming that Q_o^* and hence H and K are symmetric positive definite matrixes.

We now have an homogeneous system of linear equations for which vec (Q_o^*) is an eigen vector and given the conditions mentioned above P_2' should have an inverse. Therefore multiplying (5.41) by $(P_2')^{-1}$ gives us:

$$((P_1' \otimes (P_2')^{-1}) - \beta\, I)\ \mathrm{vec}\,(Q_o^*) = 0$$

It can be shown that the roots of the two matrixes P_1 and P_2 are related. If λ and μ are the latent roots of (P'_1) and (P'_2) respectively, then $(P_1' \otimes (P_2')^{-1})$ has roots λ/μ which are in theory equal to β from (5.42). Hence, we have g^2 estimates of $\beta = \lambda_i/\mu_j$ for all $i,j = 1, .. g$, but we have no way of choosing the roots to guarentee that the restriction holds. As far as the identification of β is concerned this is not a problem as we only need to find one such root. We can derive an estimate of β by looking at the following symmetric family of functions of the roots:

$$\beta = \mathrm{tr}(P_1^i)/\mathrm{tr}(P_2^i)$$

As estimation of the reduced form produces g^2 estimates, then β will usually be over-identified and i above will usually be 1, but as any choice of i will do we only need to find one for identification. Hence, the condition for non-identification of β is the non-existance of any i for which the above condition holds and this only occurs when P_1 has finite roots and P_2 has no roots for which the sum of the i^{th} powers is non zero. Conditional on P_1 having finite roots, we will fail to identify the model when the all of the roots of P_2 are zero or equivalently $P_2 = 0$. An appropriate i will always be found and β identified when P_2 is not nilpotent. If Q_0^* is positive definite, then nilpotency implies that all of the roots of P_2 are zero which means that $Q_1^* = 0$. Now $Q_1^* = K + J$, so that the nilpotency condition means that $K = -J$ and non-identification of β occurs when that is true. When K is positive definite we only require non-negative definite J to generate non-nilpotency and that is sufficient to identify β. In the case of the model in chapter 4, $J = 0$ and β is identified when K is non-zero or positive semi-definite with minimum rank one.

Given that β is identified we can use equation (5.41) to provide us with a rank condition for the identification of Q_0^* and from this follows identification of the other parameters in the system. In deriving (5.38) in terms of the structural form parameters we require Q_0^* to be positive definite, this is guaranteed by the structure of the model as K needs to be positive definite for a solution to the rational expectations problem to exist and J to be non-negative definite for the identification of β. If Q_0^* is positive definite, then without loss of generality we can always transform it into a symmetric form, so that we can re-write (5.41) in terms of the vector of non-similar elements q and the elimination matrix of supra diagonal elements due to Magnus and Neudecker (1980).

(5.43) $((P_1' \otimes I) - \beta\ (I \otimes P_2')L_o q = 0$

where L_o is the elimination matrix and $vec(Q_o^*) = L_o q$

and $P^+ = ((P_1' \otimes I) - \beta\ (I \otimes P_2')$

We have a linear system of equations for which q is now an eigen vector and β is an eigen value. As q has $\frac{1}{2}g(g + 1)$ terms in it and the dimension of L_o is $g^2 x \frac{1}{2} g(g+1)$, then for a unique solution to exist we require:

(5.44) $rk[((P_1' \otimes I) - \beta\ (I \otimes P_2'))L_o] = \frac{1}{2}(g+1)g - 1$

(5.44b) $rk[((P_1' \otimes I) - \beta\ (I \otimes P_2')] \leq g(g-1)$.

The choice of β in (5.44b) produces a multiplicity of g similar roots, as the roots of P_1 and P_2 are proportional when the appropriate restrictions are imposed. Hence, we have g roots μ_i of P_2 and g roots of P_1 for which $\lambda_i = \beta\mu_i$ for all i = 1 ... g which means that g columns of the matrix in (5.44b) cancel, so that the rank of (5.44b) is less than or equal to g(g-1). The rank of (5.44), then has to be less than g(g-1).

When the rank condition (5.44) is satisfied we will be able to identify q to a scaler multiple and so Q_o^*. Notice, that when the rank condition is equal to 1/2g(g+1) the system is satisfied by q = 0, but that is inconsistent with our method. When the rank is 1/2g(g + 1)-i, then we have multiplicity of solutions the number of which depends on i the nullity of $(P^+ L_o)$, so for i > 1 the system cannot be identified without imposing further restrictions. If we do apply further restrictions to Q_1^* and Q_o^*, then we can identify those parameters by looking at the family of solutions associated with the linear form (5.43):

$$q = q_o + \varnothing_1 q_1 + \varnothing_2 q_2 + \varnothing_3 q_3 + \cdots$$

where the $\varnothing_i$ are arbitrary scalers.

We will assume that (5.44) is satisfied, so that Q_o^* or the dissimilar elements will be identified. As we can see it is possible to identify firstly β and then Q_o^* from the reduced form (5.38) and once this has been done it is easy enough to derive estimates of Q_1^*, A_1 and A_2 using the following relationships.

$$Q_1^* = (1/\beta)Q_o^* P_1 , \qquad Q_1^{*'} = Q_o^* P_2,$$

$$A_2 = (1/\beta)Q_o^* P_4 \quad \text{and} \quad A_1 = Q_o^* P_3 - A_2$$

It is then possible to derive the original structural parameters from the linearised parameters using:

$$Q_1^* A = KA + A_1 \qquad A_1 = JA \qquad A_2 = HA$$

$$Q_o^* A = (1 + \beta)KA + J'A + A_2 + A_1 \qquad Q_1^{*'} A = KA + J'A$$

We can initially solve for A using the above formulae and then use those parameters to derive solutions for H,J and K. Therefore:

$$Q_o^* A - \beta Q_1^* A - Q_1^{*'} A = A_2 + A_1 - \beta A_1$$

$$(Q_o^* - \beta Q_1^* - Q_1^{*'})A = A_2 + (1 - \beta)A_1$$

$$A = (Q_o^* - \beta Q_1^* - Q_1^{*'})^{-1} A_2 + (1 - \beta)A_1$$

As long as $(Q_o^* - \beta Q_1^* - Q_1^{*'}))$ is non-singular we will be able to solve for A and as this term is $(H + (1 - \beta)J)$, positive cost matrixes will guarentee non-singularity in the context of our cost minimising approach. Matrices H and J only need to be positive semi-definite for theory consistency and positive definiteness is too strong, as the sufficient condition is $rk(H + (1 - \beta)J) = g$. In a thoery consistent context, positive definiteness of H and J is more than we need as we only require any negative element in $(1 - \beta)J$ to be dominated by the positive elements in H or positive roots in H and J to compensate for negative and zero roots. If we limit ourselves to cases in which $\beta < 1$ we can have a non-singular matrix when J is positive definite, but we cannot have any unit roots as the conditions in appendix C1 are not satisfied. If there is rank deficiency in both H and J we will either need to transform the model to take account of it or to impose additional restrictions on A.

Once we can solve for A, then we should be able to identify J and H. When rk(A) = g one or more partitions of A exist, so that we can compute a number of sub matrixes $A^{(i)}$ from which we can produce different estimates of the parameters H and J. If (5.39) is used to estimate the model, then H and J will be over-identified when the $k > g$ and exactly identified when $k = g$. If $A_1^{(1)}$ is the partition associated with $A^{(1)}$ then:

$$H^{(1)} = A_1^{(1)}(A^{(1)})^{-1}$$

and correspondingly for A and A_2.

$$J^{(1)} = A_2^{(1)}(A^{(1)})^{-1}$$

and

$K = Q_1^* - J$ and then H can also be derived from Q_o^*

The non-singularity and the rk(A) = g, provide sufficient conditions for the identification of A, J, H and K when we estimate the model using (5.39), so that identification will only be strengthened when (5.37) is estimated as it imposes all of the restrictions associated with the interaction cost model. The order conditions which supports our side conditions depends on either positive definiteness or symmetry of H and K. We cannot guarantee that H and K will be symmetric, but we can provide symmetric estimates:

$$\tilde{H} = (\hat{H} + \hat{H}')/2 \qquad \text{and} \qquad \tilde{K} = (\hat{K} + \hat{K}')/2$$

We could then test for symmetry by comparing the estimates of the off diagonal elements or by estimating the models parameters with and without the restrictions and then comparing the likelihoods to see whether symmetry was valid.

The order conditions presented in this section are not very stringent, but they are global and they can be augmented by additional conditions which are sufficient for global identification. A direct relationship exists between the sufficient conditions and the structure of the model as theory consistency requires some positive-negative definite cost matrixes. The derivation of the backward forward solution needs Q_1^* to be non-singular and we require Q_o^* to be non--singular for the reduced form parameters in (5.38) to have a structural interpretation. If the two conditions presented above are to be met then we need either K or J to be positive definite. The sufficient conditions presented in this section are also based on the Identification of the reduced from parameters P_1, P_2, P_3 and P_4 which in turn depends on the instruments being independent. A necessary condition for the independence of the instruments is given by the order condition $s > p + r + 1$, where s is the number of lags in the forcing equation for the xs and r the number of future expectations and p the number of lags in the first order condition. Conditional on the above conditions being met the following conditions are both necessary and sufficient for the identification of the parameters β, H, J, K and A.

1) $k > 1$ (order condition which is necessary)

2) $Q_1^* \neq 0$ (Q_1^* non-nilpotent or $rk(K + J) > 0$)

3) $rk[((P_1' \otimes I) - \beta (I \otimes P_2')L_o] = ½(g+1)g - 1$

4) $rk(Q_o^* - \beta Q_1^* - Q_1^{*\prime}) = g$ (H or J non-singular)

5) $rk(A) = g$

The order condition (1) depends on the symmetry of Q_o^*, so when that is not valid we need to fall back on the more stringent condition $k > g$. Conditions (2)-(4) will hold as long as J is non-singular, though that could be replaced by H and K non-singular and for theory consistency in the context of the cost minimising case we would require positive definiteness. If the rank conditions (3) and (4) are not met, then it still may be possible to identify all of the parameters by imposing additional restrictions on Q_o^* and A respectively. When the other conditions hold, but Q_o^* is not symmetric, then we can usually identify all of the parameters by imposing g normalisation restrictions on the parameter matrix of the endogenous variables. The conditions presented here can be extended to deal with the other models in the chapter and the model in chapter 4 can be dealt with as a special case in which $J = 0$. Under cointegration H and J cannot be positive definite as positive cost matrixes do not allow unit roots and when this does not hold we may need to impose further restrictions to identify the A matrix.

5.4 Local Conditions for Identification

A necessary and sufficient condition for local identification can be derived using the Jacobian matrix and the moment matrix of the data. Rothenberg(1971) deals with conditions for local identification of quite general models and we can extended those conditions when there are enough appropriate instruments for the endogenous variables and the future exogenous variables. Sargan(1983b) shows that the first order condition for identification based on the rank of the jacobian matrix is only necessary for non-identification and that it can be augmented by extra conditions on the second moment matrix of the data and the variance-covariance matrix. In particular this confirms the need to have more lags in the processes driving the exogenous variables as there are also lags of exogenous variables in the model for the endogenous variables. If we take the case in which there are no constraints on A, then we require the data matrix to have at least rank $2g^2 + 2gk$ and the parameter matrix to be of full rank. We know that the following condition should hold in the Generalised Instrumental Variables framework, given the orthogonality of instruments and the data matrix.

Theorem 1: In a neighbourhood of the true parameter values θ^*

(5.45) $V(\theta) \operatorname{plim} \dfrac{(X^{*\prime}Z^*)}{N} = 0$

where $X^* = [Y \; Y_{+1} \; Y_{-1} \; X \; X_{+1}]$ and $Z^* = [\hat{Y} \; \hat{Y}_{+1} \; Y_{-1} \; X \; \hat{X}_{+1}]$

and $V(\theta) = [Q_o^* : -\beta Q_1^* : -Q_1^{*\prime} : -(H + J)A : \beta JA]$

$= [(1 + \beta)K + J + J' + H : -\beta(K + J) : -K + J' :$

$-(H + J)A : \beta JA]$

We can make this condition opperational by replacing $\hat{Y}_{+1}$, $\hat{Y}$ and $\hat{X}_{+1}$ in Z^* and then using $Z^+ = (Y_{-1}, X, X_{-1}, X_{-2}, \ldots X_{-s})$.

If we vectorise (5.41) and let $p \lim \dfrac{(X^{*\prime}Z^+)}{N} = M^{*\prime}$ then:

$$\text{Vec}\left(V(\theta)\ \text{p lim} \frac{(X^{*\prime}Z^{+})}{N}\right) = (M^{*}\otimes I)\ \text{vec}(V) = 0$$

Sargan(1983b) shows for a generalised instrumental variables system that a necessary and sufficient condition for local identification is given by looking at the first derivative of the probability limit specified above. Hence, the necessary and sufficient depends on the both the second moment matrix having rank $2g^2 + 2gk$ and the Jacobian being of full rank.

$$\text{rk}\left[(M^{*}\otimes I)\ \frac{d\text{vec}(V)}{d\theta}\right] = m^{*} = gk + g^{2} + g(g+1)$$

where $\text{vec}(V)' = [((1+\beta)\text{vec}(K) + \text{vec}(J+J') + \text{vec}(H)':$
$(-\beta\text{vec}(K+J)'(-\text{vec}(K + J')': (-(A'\otimes I)\text{vec}(J) - (A'\otimes I)\text{vec}(H)':$
$\beta(A'\otimes I)\text{vec}(J)']$

When K and H are symmetric we know that:

$\text{vec}(K) = L_o k_o$ and $\text{vec}(H) = L_o h_o$

and it is also useful to remember that:

$$-(A'\otimes I)(\text{vec}(J) + \text{vec}(H)) = -((I\otimes H) + (I\otimes J))\ \text{vec}(A)$$

$$(A'\otimes I)\ \text{vec}(J) = (I\otimes J)\ \text{vec}(A) = \text{vec}(JA)$$

The moment matrix of the data can be written as

$$M^{*} = \text{p lim}\ \frac{1}{N}\left[Z^{+\prime}Y\ :\ Z^{+\prime}\ Y_{+1}\ :\ Z^{+\prime}Y_{-1}:\ Z^{+\prime}X_{1}:\ Z^{+\prime}X_{+1}\right]$$

$$= [M_o : M_1 : M_2 : M_3 : M_4]$$

If we now let $\theta' = [\beta : \text{vec}(A)'\ \text{vec}(J)' : h_o' : k_o']$ then:

$$\frac{\delta \text{ vec } (V(\theta))}{\delta\theta} =$$

$$\begin{bmatrix} \text{vec}(K) & 0 & I + \Delta_o & L_o & (1+\beta)L_o \\ \text{vec}(K+J) & 0 & -\beta I & 0 & -\beta L_o \\ 0 & 0 & -\Delta_o & 0 & -L_o \\ 0 & -(I \otimes (H+J)) & -(A' \otimes I) & -(A' \otimes I)L_o & 0 \\ \text{vec}(JA) & \beta(I \otimes J) & \beta(A' \otimes I) & 0 & 0 \end{bmatrix}$$

where Δ_o is the transposition matrix for the cases in which we differentiate vec(J') with respect to vec(J).

Now we can re-write (5.45) using $(M^* \otimes I) = [M_o \otimes I : \ldots \quad | M_4 \otimes I]$

$$\therefore \quad (M^* \otimes I) \frac{\delta \log V(\theta)]'}{\delta\theta} = [(M_o \otimes I) \text{ vec}(K) - (M_1 \otimes I) \text{ vec}(K + J)$$

$$+ (M_4 \otimes I)\text{vec}(JA) : \beta(M_4 \otimes J) - (M_3 \otimes (H + J)) : (M_o \otimes I)(I + \Delta_o)$$

$$- \beta(M_1 \otimes I) - (M_2 \otimes I)\Delta_o - (M_3 \otimes I)(A' \otimes I) + \beta(M_4 \otimes I)(A' \otimes I)$$

$$: (M_o \otimes I)L_o - (M_3 \otimes I)(A' \otimes I)L_o : ((1 + \beta)M_o - \beta M_1 - M_2) \otimes I)L_o]$$

$$= [((M_o - M_1) \otimes I)\text{vec}(K) + (M_4 \otimes I)\text{vec}(JA) - (M_1 \otimes I)\text{vec}(J):$$

$$\beta(M_4 \otimes J) - (M_3 \otimes (H + J)((M_o - \beta M_1) \otimes I) + ((M_o - M_2) \otimes I) \Delta_o$$

$$+ ((\beta M_4 - M_3) \otimes I)(A' \otimes I) : ((M_o \otimes I) - (M_3 \otimes I)(A' \otimes I)) L_o$$

$$: [((1 + \beta)M_o - \beta M_1 - M_2) \otimes I) L_o]$$

$$= [\text{vec}(K(M_o - M_1)' + JAM_4' - M_1'J) : \beta(M_4 \otimes J) - (M_3 \otimes (H + J) :$$

$$((M_o - \beta M_1) \otimes I) + ((M_o - M_2) \otimes I)\Delta_o + ((\beta M_4 - M_3)A' \otimes I) :$$

$$((M_o - M_3A') \otimes I)L_o : (M_7 \otimes I)L_o]$$

$$= [\mathrm{vec}(K(M_o - M_1) + J(AM_4' - M_1')) : \beta(M_4 \otimes J) - (M_3 \otimes (H + J)) :$$

$$((M_o - \beta M_1 + (\beta M_4 - M_3)A') \otimes I) + ((M_o - M_2) \otimes I)\Delta_o :$$

$$((M_o - M_3 A')L_o \; : \quad (M_7 \otimes I)L_o \;]$$

$$= [\mathrm{vec}(K(M_o - M_1)' - JM_6') : \beta(M_4 \otimes J) - (M_3 \otimes (H + J)) :$$

$$((M_o - M_2) \otimes I)L_o + ((M_5 - \beta M_6) \otimes I) : (M_5 \otimes I)L_o : (M_7 \otimes I)L_o]$$

where $\quad M_5 = M_o - M_3 A' \quad M_6 = (M_1 - M_4 A')$ and

$M_7 = ((1 + \beta)M_o - \beta M_1 + M_2)$

It is the rank of this matrix which determines local identification.

$$\mathrm{rk}\begin{bmatrix} \mathrm{vec}(K\,(M_o - M_1)' - JM_6') \\ \beta(M_4 \otimes J) - (M_3 \otimes (H + J)) \\ ((M_o - M_2) \otimes I)\Delta_o + ((M_5 - \beta M_6) \otimes I) \\ (M_5 \otimes I)L_o \\ (M_7 \otimes I)L_o \end{bmatrix} = m^*$$

If there are restrictions on the A matrix or any other of parameters then this condition will change. In the case of A, if we let $\mathrm{vec}(A) = Ra + a_o$, then the rank condition becomes:

$$
\text{rk}\begin{bmatrix} \text{vec}(K\,(M_o - M_1)' - JM_6') \\ [\beta(M_4 \otimes J) - (M_3 \otimes (H + J))]R \\ ((M_o - M_2)\otimes I)\Delta_o + ((M_5 - \beta M_6)\otimes I) \\ (M_5 \otimes I)L_o \\ (M_7 \otimes I)L_o \end{bmatrix} = m^* - r
$$

As Pesaran(1983) notes such local conditions are not easy to interpret or to use in setting up direct tests of non-identifiability, though an equivalent condition can be derived by looking at the condition of the Hessian which is an approximation of the information matrix. In the case of non-linear models non-singularity of the Hessian of second derivatives is necessary for local identifiability, but given our ability to invert almost singular matrices we must be skeptical of relying on such conditions alone. As Sargan(1983b) states it seems more reasonable to deal with probabilities of unidentifiability which suggests satisfaction of the conditions presented in this section should be interpreted as meaning that the chance of identification was high, but not certain. It is for this reason that it is important to augment some of the conditions presented here by the global conditions presented in the previous section. In addition the use of the condition of non-singularity of the Hessian combined with global conditions which mainly depend on the structure of the underlying model has a natural appeal.

5.5 Conclusions

In the framework of rational expectations models or models which utilise the prediction decomposition there are a broad category of results associated with the regular or saddle point solutions. The models differ from the standard first order multivariate costs of adjustment model, because they incorporate richer dynamics. In section 5.1 the results are similar to the types of model covered in the literature, except for the type of problem which is being optimised, because we allow disequilibrium and adjustment to a dynamic target. In this sense rational expectations models are the product of a two stage optimisation procedure: first, agents derive optimal behavioural relationships and then they decide the best way of reaching a point on that relationship. The two stage procedure implies we are a hostage to our desires which we then try to attain at the least cost. The modelling implication is that the strong rational expectations restrictions on the forward solution are broken which leaves a model with a number of freely estimated future exogenous variables. If such a method is not believed on theoretical grounds, then it is still valid in general modelling terms, because it provides a framework within which the strong first order rational expectations model may be tested. In this light, the future expectations are an indication that the original model is not well specified which either suggests that the regular solution is not correct or that the underlying optimisation problem includes the wrong variables.

In section 5.2 we dealt with the same cost of adjustment model, as Kollintzas(1985), but we do not restrict the adjustment matrixes in the optimisation problem to be symmetric. A similar model comes from allowing a lag in the target cost term and it is noted that those two models may be observationally equivalent in operational terms. The final result of this section of the paper covers the general solution to saddle point models of up to qth order. The details of the full solution to the model would require decomposition of the future expectations terms and then their replacement by actual values.

In the last two sections we looked at identification of the non-symmetric cost model. It is shown that the parameters can all be identified using a combination of local and global techniques. Approximate conditions for global identification have been derived in association with a rank condition which is both necessary and sufficient for local identification. Similar conditions can be derived for some of the models presented here, though the cost parameters for the first order cost of adjustment model can only be identified when the symmetry restriction is imposed, otherwise they are only determinate as a ratio of the original cost elements of the loss function.

The essence of this chapter is to show, that it is possible to derive general models within a structured method of modelling which will allow theory to be tested or model types to be compared. We emphasise general models, as do Hendry and Mizon (1978) and Davidson et al (1978), because we believe that a particular implementation of a theoretical model should be tested rather than imposed on the data. We also feel that the models presented here will allow any modeller to set up his model in framework which will make it possible to compare a number of diverse alternatives.

The method of estimation which we would prescribe is maximum likelihood, because it is more efficient than many of the alternatives. We prefer the likelihood approach to method moments estimation, because that approach imposes orthogonality conditions which really should be tested. If we deal with the most general form (5.34) and (5.36), then that could be solved recursively using the Quasi Newton method suggested by Gill, Murrey, Pitfield. The criterion function would be the concentrated likelihood solved for the variance covariance matrix Σ. Therefore:

$$\text{Log } L^{*} = \log|\hat{S}|$$

where $\hat{S} = 1/N(\sum_{t=1}^{N} u_t u_t')$ is a consistent estimator of Σ

and $u_t = G_2^{+}(L)y_t - h_t$

The criterion would be minimised with respect to F_i, i = 1, ...q, A (given that (5.2) in section 5.1 is the target condition) and the Hij's for all i = 1, ...q and j = 1, ...q. Equations (5.11) and (5.12), (5.18) and (5.19), and (5.30) and (5.31) are all special cases of this which implies that these models may be estimated by restricting the parameters of the more general model.

The extent to which different parameters may be estimated will be affected by our ability to identify them, but usually it is enough for the order of the moving average or vector autoregressive representations of the exogenous variables to exceed either q the order of the cost of adjustment in the loss function or p the lag in the target condition in the first order models in section 5.2. In practice such conditions need to be effective, so that the parameters in the exogenous processes need to be significant if they are to identify the model; the result also presumes that the lag length is known. In addition we have the usual requirement that the conditions associated with a maximum are satisfied and that the underlying form of the model is correctly specified. Hence, non-identification will either be associated with the non-existence of a solution or with estimated parameters which are not consistent with cost minimisation.

Appendix 5.A Replacement of Expectations using the Generalised Wiener-Kolmogorov Prediction Formula in models with Past and Future Exogenous Variables

1 Given, that x_t has a moving average representation of the form of (5.4) in section (5.1) then:

$$E(x_{t+s}|\Omega_t) = \sum_{r=s}^{\infty} C_s \varepsilon_{t+s-r}$$

which implies that the difference between the expectations of period t and t + 1 is the innovation ε_{t+1}:

(ai) $$E(x_{t+s}|\Omega_t) - E(x_{t+s}|\Omega_{t+1}) = -C_{s-1}\varepsilon_{t+1}$$

z_t the target or equilibrium relationship is replaced by the static equilibrium form to produce the results in section 5.2:

$$z_t = Ax_t + \xi_t$$

and the dynamic form in section 5.1.

$$z_t = A(L)\, x_t + A_-(L^{-1})x_{t+1} + v_t.$$

If we make the substitution in the case of the multivariate costs of adjustment model from section 5.1, this becomes after taking expectations:

(aii) $$y_t - Fy_{t-1} - u_t = (I - G_1L^{-1})^{-1}F\, B_oE(A(L)x_t + A_-(L^{-1})x_{t+1}|\Omega_t)$$

where $G_1 = \beta F$

The difference in the expectation is used in conjunction with a Koyck lead to remove the infinite lead in the future expectations. Imposing a Koyck lead on (aii) above gives:

$$(I - G_1L^{-1})(y_t - Fy_{t-1} - u_t) = \sum_{s=o}^{\infty} G_1^s FB_o E(A(L)x_{t+s} +$$

$$A\_(L^{-1})x_{t+s+1}|\Omega_t) - G_1 \sum_{s=1}^{\infty} G_1^s FB_o E(A(L)x_{t+s+1} + A\_(L^{-1})x_{t+s+2}|\Omega_{t+1})$$

$$= FB_o A(L)x_t + FB\ E(A\_(L^{-1})x_{t+1}|\Omega_t)$$

$$+ \sum_{s=1}^{\infty} G_1^s FB_o E(A(L)x_{t+s} + A\_(L^{-1})x_{t+s+1}|\Omega_t)$$

$$- E(A(L)x_{t+s} - A\_(L^{-1})x_{t+s+1}|\Omega_{t+1}) \qquad \text{(aiii)}$$

The difference in the expectations, given $A(L)x_t = \sum_{i=o}^{-p} A_i x_{t-i}$ and $A\_(L^{-1})x_{t+1} = \sum_{i=0}^{r} A_{-i} x_{t+i+1}$ is defined below as:

$$D(L)x_{t+1} = \sum_{s=1}^{\infty} (G_1)^s FB_o (E(A(L)x_{t+s} + A\_(L^{-1})x_{t+s+1}|\Omega_t)$$

$$- E(A(L)x_{t+s} + A\_(L^{-1})x_{t+s+1}|\Omega \quad)$$

$$= \sum_{s=1}^{\infty} (G_1^s) FB_o \left(\sum_{i=-p}^{r} A_{-i}(x_{t+s-i}|\Omega_t) - \sum_{i=-p}^{r} A_{-i}E(x_{t+s+i}|\Omega_{t+1})\right)$$

(aiv) $$= \sum_{s=1}^{\infty} (G_1^s) FB_o \left(\sum_{i=-p}^{r} A_{-i}(E(x_{t+s-i}|\Omega_t) - E(x_{t+s-i}|\Omega_{t+1}))\right)$$

If we use (ai) above to replace the difference in the expectation then (aiii) becomes:

$$D^*(L)\ \varepsilon_{t+1} = -\sum_{s=1}^{\infty} (G^s)\ FB \sum_{i=-p}^{r} A_{-i} C_{s-i-1} \varepsilon_{t+1}$$

reversing the summation signs

$$D^*(L) = -(FB_o \sum_{i=-p}^{r} \sum_{s=1}^{\infty} (G_1^s) A_{-i} C_{s-i-1})$$

$$= (-FB_o \sum_{i=-p}^{r} G_1^{-i} \sum_{j=i+1}^{\infty} G_1^j\ A_{-i} C_{j-1})$$

If $j < 0$, then $j + i < 1$ and $C_{j-1} = 0$. Splitting the first summation sign about $i = 0$ means that:

$$-D^*(L) = FB_o \left(\sum_{i=-p}^{-1} G_1^{-i} \sum_{j=i+1}^{\infty} G_1^j A_{-i} C_{j-1} + \sum_{i=0}^{r} G_1^{-i} \sum_{j=i}^{\infty} G_1^j A_{-i} C_{j-1}\right)$$

$$= FB_o \left(\sum_{i=-p}^{-1} G_1^{-i} \sum_{j=1}^{\infty} G_1^j A_{-i} C_{j-1} + \sum_{i=0}^{r} G_1^{-i} \sum_{j=i+1}^{\infty} G_1^j A_{-i} C_{j-1}\right)$$

$$= \sum_{i=-p}^{-1} G_1^{-1} \sum_{j=1}^{\infty} G_1^j FB_o A_{-i} C_{j-1} + \sum_{i=0}^{r} G_1^{-1} \sum_{j=i+1}^{\infty} G_1^j A_{-i} C_{j-1}$$

$$= \sum_{i=-p}^{-1} G_1^{-i} D_{-i} + \sum_{i=0}^{r} G_1^{-i} D_{-i}$$

where $D_{-i} = \sum_{j=l}^{\infty} G_1^j FB_o A_{-i} C_{j-1}$ and $l = 1$ for $i<0$ and $l = i+1$ for $i \geq 0$. Hence we can re-write (aii) above in terms of $D(L)\, x_{t+1}$:

$$(I - G_1 L^{-1})(y_t - Fy_t - u_t) = FB_o A(L)x_t + FB_o E(A\_(L^{-1})x_{t+1} | \Omega_t)$$

$$+ D(L)x_{t+1}$$

$$= FB_o A(L)x_t + FB_o E(A\_(L^{-1})x_{t+1} | \Omega_t)$$

$$- D^*(L)\varepsilon_{t+1}$$

Where $D^*(L)$ is defined above and we can rewrite the relationship thus:

$$(I - G_1 L^{-1})(y_t - Fy_{t-1} - u_t) = \sum_{i=-p}^{r} (G^*_{-i} E(x_{t+i} | \Omega_t) - \bar{G}_1^{i} D_{-i}\varepsilon_{t+1})$$

where $G^*_{-i} = FB_o A_{-i}$ and $D_{-i} = \sum_{j=1}^{\infty} G_1^j G^*_{-i} C_{j-1}$ and $l=1$ when $i<1$ and $l = i+1$ when $i \geq 0$.

The result is much simplified when we use the static equilibrium condition from the second section, because A_i in the definition of D_{-i} will always be the same. The results above hold except that $A_i = A$.

2 General Recursive Solutions for Rational Expectations Models

If we have a model of the form:

$$G_2^+(L)y_t - u_t = (G_1^+(L^{-1}))^{-1}g_t$$

Then all of the formulations of Section 5.1 and 5.2 can be related to it by placing specific restrictions on $G_2^+(L)$, $G_1^+(L^{-1})$ and g_t. If $G_1^+(L^{-1})$ is invertible we can derive the following recursive solution. Let:

$$h_t = (G_1^+(L^{-1}))^{-1}g_t$$

then:

$$G_1^+(L^{-1})h_t = g_t$$

$$h_t = g_t + G_1'h_{t+1} + G_2'h_{t+2} + \;\; \ldots .$$

In Section 5.1 $g_t = FB_oA(L)x_t + FB_oE(A\;(L^{-1})x_{t+1}|\Omega_t) - D^*(L)\varepsilon_{t+1}$, $G_2^+(L) = (I - FL)$, $G_1^+(L^{-1}) = (I - \beta FL^{-1})$ and $D^*(L)$ is defined in section 5.A1 and B_o is defined in section 5.1.

In section 5.2 the cross product model derived by minimising (5.14) uses the following definitions: $g_t = FB_oAx_t - FB_1Ax_{t+1} - FD\varepsilon_{t+1}$ and $G_2^+(L) = (I - FL)$ and $G_1^+(L^{-1}) = (I - BFQ_1^+)$; B_o, B_1 and Q_1^+ are defined at the beginning of section 5.1.

The model which is derived from (5.26) uses the same definitions for $G_1^+(L^{-1})$ and $G_2^+(L)$, but $g_t = FB_1Ax_{t-1} + FB_oAx_t + \beta FB_{-1}Ax_{t+1} - G^*\varepsilon_{t+1}$ where B_o, B_1, B_{-1} and Q_1^+ are as defined in the second part of section 5.2.

Appendix 5.B1 First Order conditions and Symmetric Factorisations for General Systems.

$$\text{(bi)}\quad E(C_t \mid \Omega_t) = \left(\sum_{t=0}^{N} \beta^t \Delta y_t^{+} K^{+} \Delta y_t^{+} + \tilde{y}_t H^{+} \tilde{y}_t\right) \mid \Omega_t)$$

$$\text{where}\quad y^{+} = \begin{bmatrix} y_t \\ y_{t-1} \\ \vdots \\ y_{t-q} \end{bmatrix}, \quad \tilde{y} = y_t^{+} - z_t^{+} \quad \text{and}\quad z_t^{+} = \begin{bmatrix} z_t \\ z_{t-1} \\ \vdots \\ z_{t-q} \end{bmatrix}$$

$$K^{+} = \begin{bmatrix} K_{00} & K_{01} \cdots & K_{0q} \\ K_{10} & K_{11} \cdots & K_{1q} \\ \vdots & & \vdots \\ K_{q0} & & K_{qq} \end{bmatrix} \text{and } H^{+} = \begin{bmatrix} H_{00} & H_{01} \cdots & H_{0q} \\ H_{10} & H_{11} \cdots & H_{1q} \\ \vdots & & \vdots \\ H_{q0} & & H_{qq} \end{bmatrix}$$

Differentiation of (bi) w.r.t. y_t reveals a minimum if K^{+} and H^{+} are positive definite and a maximum if K^{+} and H^{+} are negative definite. Positive definiteness suggests the cost minimising approach of Sargan (1982) and negative definiteness a model with benefits to adjustment or disequilibrium. Equation (5.33) presents the first order condition associated with (bi). Therefore:

$$\text{(bii)}\quad dE\,(C_t \mid \Omega_t) = E(K^{*}\Delta\, y_{t+1}^{+} + K^{*}\Delta y_t^{+} + H^{*}\, \Delta\tilde{y}_t \mid \Omega_t)$$

$$K^{*} = I^{*}K^{+} = \begin{bmatrix} K_{00}^{*} + K_{01}^{*} \cdots + K_{0q}^{*} \\ K_{10}^{*} + K_{11}^{*} \cdots + K_{1q}^{*} \\ \vdots \qquad\qquad \vdots \\ K_{q0}^{*} + \;.\;.\;.\; K_{qq}^{*} \end{bmatrix} \text{ and }\quad H^{*} = I^{*}H^{+} = \begin{bmatrix} H_{00}^{*} + H_{01}^{*} \cdots + H_{0q}^{*} \\ H_{10}^{*} + H_{11}^{*} \cdots + H_{1q}^{*} \\ \vdots \qquad\qquad \vdots \\ H_{q0}^{*} + \cdots\cdots + H_{qq}^{*} \end{bmatrix}$$

$$\text{where}\quad K_{ij}^{*} = K_{ij}^{+}(\,L/\beta)^{-i} \quad\text{and}\quad H_{ij}^{*} = H_{ij}^{+}(\,L/\beta)^{-i}$$

To bring out the symmetric nature of the solution we can re-write (bii) in less compact form.

(biii) $$\frac{dE(C_t|\Omega_t)}{dy_t} = E\Big(2\sum_{s=1}^{q} [K^{(s)}(\Delta y_{t-s} - \beta\Delta y_{t+1-s}) + \beta^s K^{(s)\prime}(\Delta y_{t+s} - \beta\Delta y_{t+s+1})] + K^{(0)}(\Delta y_t - \beta\Delta y_{t+1}) + \sum_{s=1}^{q} [H^{(s)}\tilde{y}_{t-s} + \beta^s H^{(s)}\tilde{y}_{t+s}] + H^{(0)}\tilde{y}_t | \Omega_t\Big)$$

where $K^{(s)} = \sum_{i=0}^{q-s} \beta^i K_{i(s+i)}$ and $H^{(s)} = \sum_{i=0}^{q-s} \beta^i H_{i(s+i)}$

For an optimum $dE(C_t|\Omega_t)/dy_t = 0$, which means that we can re-formulate (biii) into a $2(q+1)^{th}$ order difference equation:

(biv) $$E\Big(\sum_{s=-(q+1)}^{q+1} Q_s \beta^{\frac{1}{2}s} y_{t+s} = \sum_{s=-q}^{q} H_s^* \beta^{\frac{1}{2}s} z_{t+s} | \Omega_t\Big)$$

where $H_s^* = \beta^{\frac{1}{2}s} H^{(s)\prime}$ for $s \geq 0$ and $H_{(-s)}^* = H_{(s)}^{*\prime}$ for $s > 0$,

$Q = \beta^{\frac{1}{2}(s+1)}[(\beta^{\frac{1}{2}} + \beta^{-\frac{1}{2}})K^{(s)} - \beta^{\frac{1}{2}}K^{(s+1)} - \beta^{-\frac{1}{2}}K^{(s+1)} + \beta^{-\frac{1}{2}(s)}H_s]$,

$Q_{(-s)} = Q_s'$ for $s>0$ and

$Q_o = \beta^{\frac{1}{2}}[(\beta^{\frac{1}{2}} + \beta^{-\frac{1}{2}})K^{(0)} - \beta^{\frac{1}{2}}K^{(1)} - \beta^{\frac{1}{2}}K^{(1)\prime} + \beta^{-\frac{1}{2}}H^{(s)}]$

To derive a solution to the rational expectations problem we need to factorise (biv). Notice that the form is symmetric about Q_o, as $Q_o = Q_o'$ and $Q_s = Q_{-s}'$. The spectral decomposition of (biv)

is $$Q(x) = \sum_{s=-(q+1)}^{q+1} Q_s x^s$$

The roots are usually split into two sets stable and unstable, given the symmetric structure of Q(x) this suggests the following factorisation:

$$Q(x) = G^*(x)WG(1/x)$$

where W is chosen so that $G_o^* = G_o = I$, $G^*(x) = \sum_{s=0}^{q+1} G_s^* x^s$ and $G(x) = \sum_{s=0}^{q+1} G_s x^s$. As the reciprocal of x also satisfies the characteristic equation we will have an equal number of stable and unstable roots.

$$Q(1/x) = G^*(1/x)WG(x)$$

while the transpose of Q(x) is given by

$$Q'(x) = G'(1/x)W'G^{*\prime}(x)$$

which suggests that:

(bv) $Q'(x) = Q(1/x)$

The separation of the roots into two equal sets implies, that we have a unique solution (see Sargan (1983) for the standardisation conditions). Condition (bv) above implies that:

$$Q'(x) = G^*(1/x)\ W\ G\ (x)$$

transposition of Q'(x) implies:

$$Q(x) = G'(x)\ W'\ G^{*\prime}\ (1/x)$$

which given uniqueness means $W = W'$ and $G^*(x) = G'(x)$

$$Q(x) = G'(x)WG(1/x)$$

If we let:

$$G_1^+(x) = G'(x/\beta^{1/2}),\quad G_2^+ = W\ G\ (\beta^{1/2}x) \quad \text{and} \quad g_t = E\ (\sum_{s=-q}^{q} \beta^{1/2}H^*z_{t+s}|\Omega_t)$$

then the solution to our general problem is given by:

$$E\ (G_1^+\ (L^{-1})\ G_2^+\ (L)\ y_t\ |\Omega_t) = g_t$$

$$G_2^+\ (L)\ y_t = [G_1^+\ (L^{-1})]^{-1}\ g_t + u_t$$

The recursive solution to this type of problem can be found in Appendix 5A

5.B2 Particular Solutions for the Interaction Cost Model and a Model with Second Order Cost of Disequilibrium

The solution to all of the models in section 5.1 and 5.2 can be derived from the results presented for the general model derived in Appendix 5.B1. Here we will deal with the solution to (5.14) and (5.26). The first order conditions in both cases are very similar and they can be nested within the following framework:

(bvi) $E(Q_o^* y_t - Q_1^* y_{t+1} - Q_1^{*} y_{t-1} | \Omega_t) = E(H_1\ z^*_{t-1} + H_o z^*_t + \beta H_{-1} z^*_{t+1} | \Omega_t)$

Where $Q^*_o\ y_t$ and Q^*_1 are defined appropriately for the different models presented in section 5.2 and in the case of the costs of adjustment model (5.14) $H^*_1 = 0$, $H^*_0 = (J + H)$ and $H^*_{-1} = J$. In the lagged target case $H^*_1 = H_{01}$, $H^*_0 = H_{00} + \beta H_{11}$ and $H^*_{-1} = H_{10}$.
We can, then derive a symmetric form from (bvi) above by letting $y^*_t = (½\beta)^t y_t$ and $Q_i = \beta^{-½ i} Q^*_i$ for i = 0, 1, and setting the r.h.s. to zero reveals an homogeneous difference equation which has the following characteristic equation if $y^*_t = \mu^t p_t$:

(bvii) $(Q_o\mu - Q_1\mu^2 - Q_1') g = 0$

$$|Q_o\mu - Q_1\mu^2 - Q_1'| = 0$$

If $Q(x) = Q_o\ x - Q_1\ x^2 - Q_1'$ then this polynomial has two sets of distinct roots μ and $1/\mu$. If we utilise the factorisation theorem presented above the solution is of the form:

$$Q(x) = G'(x)WG(1/x)$$

If we let $x = \mu$, then:

$$Q_o\mu - Q_1\mu^2 - Q_1' = (G'\mu - I)W(G - \mu I)$$

$$= (W + G'WG)\mu - G'W\mu \ - WG^2$$

which implies, that:

(bviii) $Q_o = W + G'WG$, $Q_1 = G'W$ and $Q_1' = WG$

From (bviii) we have

$$Q_o = W + Q_1 G$$

and

$$W = (G')^{-1} Q_1 \qquad \text{and} \quad W = Q_1'G^{-1} \text{ which implies:}$$

$$Q_o = Q'G_1^{-1} + Q\, G_1$$

If we revert back to our original form in terms of y_t and we let $F = \beta^{-\frac{1}{2}}G$, then we can re-write the first order condition in the following way using the factorisation presented in (bviii) above:

$$E([\beta Q_1 F + Q_1'F^{-1}]y_t - \beta Q_1 y_{t+1} - Q_1'y_{t-1} | \Omega_t) = E(H^*_{+1}\ z_{t-1} + H^*_o z_t$$
$$+ \beta H^*_{-1} z_{t+1}\ | \Omega_t)$$

Multiplying through by $(Q_1'\ F^{-1})$ and then setting $Q_1^+ = (Q_1^*)^{-1}\ Q_1^*$ and $G_1 = \beta F Q_1^+$ implies that:

$$E((I - \beta F Q_1^+ L^{-1})(I - FL)y_t | \Omega_t) = E(F(B_o z_t + B_1 z_{t-1}$$
$$+ \beta B_{-1} z_{t+1} | \Omega_t)$$

where L^{-1} is the forward lead operator and L the lag operator, and B_o, B_{-1} and B_{+1} are appropriately defined in the relevant parts of section 5.2 of chapter five for the cross product and lagged target cost models.

If we multiply through by $(I - G_1L^{-1})^{-1}$ we get the forward looking solution:

$$E(y_t - Fy_{t-1} | \Omega_t) = E((I - G_1L^{-1})^{-1} (F(B_o z_t + B_1 z_{t-1} + \beta B_{-1} z_{t+1})) | \Omega_t)$$

Appendix 5.C1 Proof of the non-existence of unit roots when the Cost Matrices are Positive Definite

We can show for any of the models presented here that the latent roots cannot lie on the unit circle. To prove this proposition we will use the most general model specified at the end of Section 5.2 and explained in detail in Appendix 5.B1. After elimination of the discount factor the loss function can be written in the following form (see (bi) in Appendix 5.B1).

$$\text{(ci)} \quad E(C_t | \Omega_t) = E(\sum_{t=1}^{N} (y_t^* - \beta^{½} y_{t-1}^*)' K^x (y_t^* - \beta^{½} y_{t-1}^*) + \tilde{y}_t^{*\prime} H^x \tilde{y}_t^*) | \Omega_t)$$

where

$$y_t^* = \begin{bmatrix} y_{1(t-q)} \\ y_{1(t-q+1)} \\ . \\ . \\ y_{1t} \end{bmatrix} \qquad \tilde{y}^* = \begin{bmatrix} \tilde{y}_{1(t-q)} \\ . \\ . \\ . \\ \tilde{y}_{1t} \end{bmatrix}$$

$$\text{and } y_{1t}^* = \beta^{½t} y_t \quad \text{and } \tilde{y}_{1t}^* = (y_{1t}^* - \beta^{½t} z_t)$$

$$H^* = \begin{bmatrix} H_q^* \\ H_{q-1}^* \\ . \\ . \\ H_0^* \end{bmatrix} = \begin{bmatrix} \beta^q & H_{qq} & . & . & . \\ & . & . & . & . \\ & & & . \quad . & . \\ & . & . & \beta H_{11} & \beta^{½} H_{10} \\ & . & . & \beta\, H_{01} & H_{00} \end{bmatrix}$$

$$K^* = \begin{bmatrix} K^*_q \\ K^*_{q-1} \\ \vdots \\ K^*_0 \end{bmatrix} = \begin{bmatrix} K^*_{qq} & K^*_{qq-1} & \cdot & \cdot & K^*_{qo} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & K^*_{11} & K^*_{10} \\ K^*_{oq} & \cdot & \cdot & \cdot & K^*_{00} \end{bmatrix}$$

where $K^*_{rs} = \beta^{\frac{1}{2}(r+s)} K_{rs}$ and $H^*_{rs} = \beta^{\frac{1}{2}(r+s)} H_{rs}$

Then the first order condition is :

$$\text{(cii)} \quad E(\sum_{s=0}^{q} (K^*_s \, (y^*_{t+s} - \beta^{\frac{1}{2}} \; y^*_{t+s-1} \;) - \; \beta^{\frac{1}{2}}{}_s K^x (y^*_{t+s+1} - \beta \; y^*_{t+s})$$

$$+ \tilde{y}^{*\prime}_t H^x \tilde{y}^*_t) \mid \Omega_t) = 0$$

If we take a latent root μ and corresponding latent vector p_o then $p_s = p_o \mu^s$ and $p_{sk} = \mu^{s-k} \; p_o$, so that P^*_s is an extended latent vector defined below:

$$P^{*\prime}_s = [\mu^{s-p} p^\prime_o, \; \mu^{s-p+1} p^\prime_o, \; \dots, \; \mu^{s-1} p^\prime_o, \; \mu^s p^\prime_o]$$

then substitution for y_{1t+s} in (cii) above gives an euler condition in terms of the latent roots.

$$\sum_{s=0}^{q} [K^*_s \; (\beta^{\frac{1}{2}} + \beta^{-\frac{1}{2}} - (\mu + 1/\mu)) \; P^*_s + H^*_s \; P^*_s] = 0$$

$$\text{since} \quad y^*_{t+s} = \begin{bmatrix} y_{1t-q} \\ y_{1t-q+1} \\ \cdot \\ \cdot \end{bmatrix} = \begin{bmatrix} \mu^{s-q} \; p_o \\ \mu^{s-q+1} p_o \\ \cdot \\ \cdot \end{bmatrix} = P^*_s$$

and as $P^*_s = \mu^s P^*_o$ we have

$$\text{(c.iv)} \quad \sum_{s=0}^{q} \; \mu^s [K^x_s \; (\beta^{\frac{1}{2}} + \beta^{\frac{1}{2}} - (\mu + 1/\mu)) + H^x_s \;] \; P^*_o = 0$$

<u>Lemma 1</u>

The latent roots of the system (ciii) cannot lie on the unit circle if the system is to be consistent with an optimising model with positive cost matrices.

Proof

Let $\mu = e^{iw}$ where w is real and p_o^+ be the complex conjugate of p_o

then $\beta^{½} + \beta^{-½} - (\mu + 1/\mu) = \beta^{½} + \beta^{-½} - 2\cos w \geq 0$

since $\beta^{½} + \beta^{-½} \geq 0$ and $\cos w \leq 1$

pre-multiplying (civ) by p_o^+ implies

(c.v) $\sum_{s=0}^{q} \mu^s \; p_o^+ \; (K_s^x \; (\beta^{½} + \beta^{-½} - 2\cos w) + H_s^x \;) \; P_o^*$

Let $M^x = K^x \; (\beta^{½} + \beta^{-½} - 2\cos w) + H^x$

where $M^x = \begin{bmatrix} M_{qq}^x & M_{q(q-1)}^x & \cdots\cdots & M_{qo}^x \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ M_{oq}^x & M_{o(q-1)} & & M_{oo}^x \end{bmatrix}$

then we can re-write (cv) in matrix form:

$$\begin{bmatrix} \mu^q \; p_o^{+'} \vdots \; \mu^{q-1} \; p_o^{+'} \; \vdots \; \ldots \; p_o^{+'} \end{bmatrix} \quad M^x \begin{bmatrix} \mu^{-q} \; p_o^* \\ \mu^{1-q} p_o^* \\ \cdot \\ \cdot \\ p_o^* \end{bmatrix} = 0$$

as $\mu = e^{iw}$, then $\mu^s \; p_o^+ = e^{isw} \; p_o^+$ is the complex conjugate of $\mu^{-s} p_o^+$

Hence, when P_o^x is the complex conjugate of P_o^*

then $P_o^x \; M^x \; P_o^* = 0$

But this contradicts the standard result with respect to positive definite matrixes:

$d^+ Ad > 0$

where A is positive definite and d^{+} is the complex conjugate of d.

5.C2 Conditions on the size of the roots

Given the form of (civ) it seems likely that $|\mu| < \beta^{1/2}$ for β in the neighbourhood of 1.

If $\max |\mu| = \mu_m(\beta)$ where $|\mu| \le 1$ then $\mu_m(\beta) \le 1$ and μ_m is a continuous function of β given the quadratic loss function. If $\beta = 1$ we know from Lemma 1, that $\mu_m(\beta) < 1$, because unit roots are not usually valid given the structure of the loss function.

Then:

$$\left.\begin{array}{l} \mu_m(\beta) < \beta^{1/2} \\ \text{and } \mu_m(\beta) - \beta^{1/2} < 0 \end{array}\right\} \quad \text{when } \beta = 1$$

and in a neighbourhood of β

$$\left.\begin{array}{l} \mu_m(\beta) \le \beta^{1/2} \\ \text{and } |\mu| \le \beta^{1/2} \end{array}\right\} \quad \text{when } 1 \ge \beta \ge \beta_o$$

where μ is a stable root of the system.

Conclusion

Sargent(1978) assumes that aggregation can be dealt with by summing across companies which have identical production surfaces, but such analysis is ingenuous, it produces a trivial correspondence between micro and macro phenomena, as it does not take account of the inherent differences between productive units or the associated distributional problems. Deaton and Muellbauer (1980a) and (1980) produce a far more elegant statement of the representative agent theory in relation to consumer behaviour. The literature on systems of demand equations has many examples of attempts to derive aggregate relationships which are capable of being determined by individual agents maximising utility. Even so, this approach has been criticised by Hildenbrand (1983) who presents conditions on the distribution of income and household characteristics that enable us to be able aggregate perfectly individual agent phenomena for general functions. Kirman(1989) summarises the recent state of debate and the tenet of the article suggests that the usual equilibrium concepts require group behaviour or some law of large numbers for uniqueness. Hence, micro foundations are only the basis of macro phenomena when strict aggregation conditions are met or when the form of the basic relationship is highly simplistic, otherwise aggregate micro relationships are no better than any other hypothesis at explaining macro behaviour. This view is consistent with the old fashioned idea that the macro phenomena behave in a distinctly different way to micro ones. In this light, notions of natural prices and unique equilibria cannot be seen to be derived from any micro foundations and this then separates such ideas from the Wallrasian principles used to justify them. Once this knot is cut the associated optimality of such phenomena is put into doubt and the likelihood that they are the observed state of nature is diminished.

In theoretical terms the non-existence of natural prices or unique equilibria supports the notion of disequilibrium as the natural state of markets which have a high degree of inertia and then macroeconomics is a gestalt of Keynesian demand side theory associated with aggregate expectational behaviour and large micro phenomena on the supply side. Markets which adjust quickly are then seen to exhibit jump behaviour and overshooting unless institutional arrangement exist or large operators dominate and stabilise trade. In practice there are a number of questions which need to be addressed if we wish to transport such ideas into the domain of econometrics. Aggregation is still a key issue when analysing data as it affects the behaviour of the data and our ability to find a stable constant parameterisation. If one reads Keynes General Theory there is a clear appreciation of the way in which macro phenomena mesh with individual market responses to produce the behaviour of an economy in the aggregate. An elegant story is spun from the movement of the whole which is itself woven consistently from the fragmented activity of individual firms and households.

The Keynesian model, suggests that at any moment of time the whole process should be coalescing to a state of a balance constructed from the entropy associated with individual actions.

The observation of cohesive aggregate behaviour depends on the random behaviour of the atoms relative to markets as a whole; micro phenomena are then brownian motion in relation to the overall movement of the economy. Individual activity is not necessarily irrational or non-deterministic within a micro context, but it appears so at the macro level. If such a view is not true or we do not have all agents being the same or the exact aggregation conditions required for the representative consumer theory or the distributional conditions associated with Hildenbrand (1983), then representations of macro phenomena using time-series data will exhibit non-constancy of the parameters. The omission of such variables will then cause highly restrictive models to be biased and general models either to be non-constant or verbose and difficult to interpret.

The results presented here do not seem to be able to reject a Keynesian explanation of the data, as vacancies seem to have a disequilibrium role, price homogeneity fails and output employment relationships seem to dominate price effects. The more classical explanation of the output employment model is better specified in the autoregressive from, though the factor demand interpretation of the employment model depends on money wages. Models with the time trend seems to dominate, but then neither output nor employment equation satisfy homogeneity. It seems likely that there are better models, but the ones estimated do seem to move somewhat closer to producing acceptable models with future expectations. Price Homogeneity is a restriction to be tested, rather than imposed and in the context of most standard models it should be satisfied unless relative price effects are important. The question of aggregation are only partially answered here and they may only be addressed properly when we have sufficient panel data to augment aggregate models with such information.

Main Findings

We have attempted to construct models which both produce a statistically valid partition of the data matrix and reveal results with an economically meaningful interpretation. We have used rational or consistent expectations, but without always restricting the dynamics to the simplest form of the first order models which impose strong rational expectations restrictions. The methods make no assumptions about market clearing and they do not impose strong informational assumptions. Where ever possible we have tried to test hypotheses and construct general models which encompass associated specifications. The models determining the exogenous variables are estimated by OLS and recursive least squares and the models are then validated. We have attempted to eliminate models that perform badly, but we would like to feel that models specified for theoretical analysis need to satisfy additional criterion based on Economic theory.

We assume that we can partition the original data by eliminating variables which are not of interest and this leaves a conditional model in which y depends on z where $z_t = Ax_t$

(6.1) $D(s_t | S_{t-1}, \emptyset_2) = D(y_t | z_t, S_{t-1}, \lambda_1) D(z_t | S_{t-1}, \lambda_2)$

where $s_t = [y_t, z_t]$ and $S_t = [Y_t, Z_t]$

The partition above assumes that S_t is sufficient information to determine the parameters of interest and when we construct the rational expectations model we also assume that the xs or zs are not Granger caused by the ys. The xs are then assumed to be strictly exogenous and the data seems to confirm this, though there are some signs of the xs being Granger Caused by specific lags on the ys. The xs feed into the y process which means that they are not weakly exogenous and when we do not have weak exogeneity, efficiency and valid inference depends on modelling the two processes. Estimation of the joint system will not necessarily lead to correct standard errors as we need to take account of parameter variation both directly and through the generated variables. We have constructed separate autoregressive models of the exogenous variables to derive efficient predictors which are assumed to be replace expectations generated by the available information.

$$(6.2) \qquad z_t^e = B^{\#}(L)z_{t-1}$$

We try to produce well specified marginal models of prices, wages, inventory accumulation and vacancies by a restricted VAR which should be equivalent to a cointegration form of the model:

$$B(L)\begin{bmatrix}\Delta i_t \\ v_t \\ p_t \\ w_t\end{bmatrix} = \begin{bmatrix}\varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t}\end{bmatrix}$$

where the ε_{it} stand for innovations in the variables chosen as exogenous to the system and B(L) is the associated matrix polynomial.

In Chapter 3 we estimated such models of the exogenous variables by OLS. The models were validated using the methods explained in chapter 2 and such methods are used to try to determine whether the marginalisation is correct. It is very important that the parameters are stable and this is partly dependent on whether employment and output or other variables determine output prices, wages, vacancies and inventory accumulation. The evidence was not conclusive, though there is some suspicion that some instability exists and that individual output and employment terms are important. Chow and CUSUM tests seem to imply that the models are stable in period, but the models do not always predict well which may be due to the difficulty in predicting the stock shake out and fall in output and employment associated with the Thatcher experiment. The theory presented would not disagree with the notion that inventories, prices, wages and vacancies do not depend on current output and employment, but it would be difficult to deny all causal links. A further cause of breakdown may be due to the omission of other variables and there is much evidence in favour of such an hypothesis, though stable prediction models could not be derived when the set of exogenous variables was extended. It is difficult to find variables without some sort of link or to derive a procedure which would allow the system as a whole to be estimated.

Cointegration or unit root tests of the models above show that wages and prices might not be stationary in first differences, but such tests are problematic, because they lack power and they do not directly test for cointegration. Dolado et al (1989) show that the loss of power may depend on the form of the Dickey

Fuller tests used. It seems likely from the Dolado, Ericsson and Kremers results, that tests should be treated with skepticism and that the simple t-tests on the error correction may be preferred. We would discard the notion that wages and prices are I(2) on this basis and suggest that all variables are I(1) excluding inventory accumulation which is assumed to be I(0).

The models presented here perform reasonably well in terms of the standard test procedures suggested in the literature which suggests that ordinary least squares is both consistent and relatively efficient. The wage and price models have long-run solutions which do not reject the notion of real relationships being important in the long-run. The forecasts of the series seem to increase over time, but in a non-accelerating way which seems to give some credence to the cointegration hypothesis. If the series are cointegrated, then the VAR parameters do suggest that a way exists by which we can reverse the Smith-Mcmillan-Yoo form to transform these results into the VMA form in cointegrating variables. If we had such a technique it would be possible to invert any autoregressive form what ever the order of integration or cointegration into the moving average representation required here. The partition of the system into exogenous and endogenous variables (see Hunter(1988)) produces the following prediction formula:

$$(6.3) \qquad E(\eta_{t+i}|\Omega_t) = C^{+}_{22}(L)_i\varepsilon_t$$

$$\text{where } C^{+}_{22}(L) = C_{22}[i] + C_{22}[i+1]L + C_{22}[i+2]L^2 \ldots$$

Substituting out for the expectations produces a very specific VARMA system which is similar to the open form presented in Davidson and Hall(1979):

$$(6.4)\qquad F^{+}(L)s_t = D^{+}(L)e_t$$

$$F^{+}(L) = \left[\begin{array}{c|c} F_{11}(L) & 0 \\ \hline 0 & A_{22}(L)(1-L) \end{array}\right]$$

$$D^{+}(L) = \left[\begin{array}{c|c} I & D^{+}_{12}(L) \\ \hline 0 & d(L) \end{array}\right]$$

where $D^{*}_{12}(L) = I + D^{*}_{12}[1]L^{*} + D^{*}_{12}[2]L^{2} + \ldots$

and $D^{*}_{12}[j] = F_{12}[j]\; C^{+}_{22}(L)_{j}$

The cointegration approach to modelling the exogenous variables naturally feeds into this procedure, because the VMA representation always exists and when the exogenous variables are cointegrated the predictions are highly efficient (see Granger and Engle(1987)). Substitution using the VMA produces a computationaly efficient method of estimating the quasi-reduced form which allows one to estimate the deep parameters of the model.

In Chapter 4 we used these results to derive models of output and employment which take account of the model structure above. The method of estimation derived only requires one step ahead forecast errors and future predictions of the variables treated as exogenous. The first order conditions of the optimisation problem gives a rationale to models with future expectations in them, though they do not have to be based on that. If such models are solved for the future values, then the following model will result:

$$(6.5) \quad y^e_t = F\, y_{t-1} + G^*(L^+) z^e_t$$

where L^+ is the forward operator which does not alter the time subscript associated with the expectations.

We derive the estimator and relate it to the backward looking representation of the model which is shown to have an error correction and cointegration form. A reduced rank solution to the forward representation still exists when we have unit roots and it may even be possible to derive estimates of the deep parameters as the unit root is in the null-space of B_0. The long-run form of the output employment model is related to the optimising theory and specific models of the long-run or deep parameters. The model suggests that we should observe demand equations for output and employment in the long-run and this turns out to be roughly consistent with the results. The long-run models can either be thought of as targets for a cost minimising control approach or as being derived from appropriate revenue or utility functions in the choice theoretic or profit maximising framework. Under such an optimising framework we would normally expect the coefficients on the difference and levels terms to satisfy conditions associated with the underlying criterion. In this case we would expect cost matrices to be positive definite or at the least positive-semi-definite, such conditions are related to the existence of unit roots or cointegration and to identification. The Forward form in actual xs is given below:

$$(6.6) \quad y_t = F\, y_{t-1} + G^*(L^+) z_t - D^* u_{2t+1} + u_{1t}$$

where D^* is a complex function of $G^*(.)$ and $B(.)$

We use the Muellbauer form of the model to derive initial estimates, these models suffer from serial correlation, but under the rational expectations assumption the instrumental variables method of Wickens(1982) should produce consistent estimates. The IV estimates of the deep parameters are roughly consistent with theory, though those of B_0 are not. The problem may be attributed to cointegration, as unit roots can cause us to observe negative estimates of the cost parameters, this idea is partially supported by the discovery of a unit root and the possibility that output and employment are cointegrated. The Maximum Likelihood method appears to produce more efficient estimates, than the IV approach and the estimates of F are quite consistent with some form of optimisation story. Unfortunately in its simplest form this method does not seem to be able to remove first order serial correlation and the model suggests a discount factor which is unrealistic. The deep parameters are mainly consistent with theory, excepting that employment depends on nominal wages and anticipated inventory changes appear to represent investment or speculative rather than disequilibrium effects. In terms of explicability the model in which the innovations are included separately seems to be preferred and although it still suggests a nominal employment model, it does not suffer from serial correlation and it is compatible with a more reasonable discount factor. The model suggests that unanticipated factors do influence employment in the way expected by theory. The model with a trend has deep parameters with signs which can be given a theoretical justification, but neither of the long-run demand equations satisfies homogeneity and the test for first order serial correlation is marginal at the 1% level.

The models estimated produce deep parameters which can be used to derive equilibrium values and on inspection such value are not totally unreasonable, better formulated long-run models would produce more accurate equilibria.

Generalised estimates of the first order model show that the period of expectation may either lie between t and t-1 or that there should be agent expectations for the two periods. The question is complicated, because it is bound up with the test of the rational expectations restrictions, the nature of exogeneity and the role of innovations in the model. It appears from the estimates derived that the xs are strictly exogenous when a Hausman test is acceptable, but weak exogeneity also depends on the invariance of the parameters and the test of restrictions on the one step ahead forecast errors. The invarience condition may not be satisfied, as there is some indication of parameter change when different periods are selected.

The discovery of a unit root, suggests cointegration and then the first-order condition has an error correction form in which the the matrix on the correction term is singular. The cointegration form in this guise is unstable which suggests that error correction models with unstable dynamic effects or explosive coefficients on the correction matrix are reparameterisations of the first order rational expectations model. It then seems more appropriate to estimate the model using a method which imposes the rational expectations restrictions, than the un-restricted first order condition.

We have used aggregate models which may be representative or that allow a simple transform to eliminate the problem, this is similar to Nickell(1984). We present a more general model with an autoregressive error, the adjustment for serial correlation eliminates first order effects and produces a model which is compatible with pure error autocorrelation. The trend model seems to be preferred in this case, even though the output equation dose not satisfy homogeneity the system has a higher likelihood and the discount rate is more reasonable. Serial correlation may either be due to aggregation or to the omission of unobservable effects, but to answer that question we would either need to estimate the most general specification or to be able to include individual market effects.

It seems excessive to rely on Keynes Marshallian assumption that capital is fixed in the short-term, though other authors such as Sargent(1978), Kennan(1979) and Muellbauer and Winter(1980) have made such an assumption. It may be that the model should be extended to include hours or the capital stock as an endogenous variable and the exchange rate and other prices as exogenous variables. Alternatively we may need to have more general models which better approximate the actual data generation process. We have started to look at more general models by including autoregressive errors and lagged exogenous variables, but these simple extensions only go part way to generating the type of models which are theoretically possible.

The backward forward model has the following form when there are a number of future exogenous and endogenous variables:

(6.7) $Q(L^+,L)y_t^e = G^+(L^+,L)z_t^e$

where $Q(L^+,L)$ and $G^+(L^+,L)$ are matrix polynomials in the lag operator L and the forward operator L^+ and s^e is an expectation

In the framework of rational expectations models or models which utilise the prediction decomposition there are a broad category of results associated with the regular or saddle point solution. The models differ from the standard first order multivariate costs of adjustment model, because they incorporate richer dynamics. We allow disequilibrium and adjustment to a dynamic target and because of that rational expectations models may be the product of a two stage optimisation procedure. The modelling implication is that the strong rational expectations restrictions on the forward solution are broken which leaves a model with a number of freely estimated future exogenous variables. If such a method is not believed on theoretical grounds, then it is still valid as it provides a framework within which the strong first order rational expectations model may be tested.

We have dealt with the same cost of adjustment model, as Kollintzas(1985), but we do not restrict the adjustment matrixes in the optimisation problem to be symmetric. A similar model comes from allowing a lag in the target cost term and it is noted that those two models may be observationally equivalent. The final result of chapter 5 covers the general solution to saddle point models of up to qth order. The details of the full solution to the model would require decomposition of the future expectations terms and then their replacement by actual values. The solutions are possible to derive, though quite complex in the case of general models; the second order model gives a standard symmetric solution.

Identification of the non-symmetric cost model depends on a combination of local and global techniques. Quasi sufficient global identification conditions have been derived in association with a rank condition which is both necessary and sufficient for local identification. Similar conditions can be derived for some of the models presented here, though the cost parameters for the first order cost of adjustment model can only be identified when the symmetry restriction is imposed, otherwise they are only determinate as a ratio of the original cost elements of the loss function. In the case of symmetric models, it seems likely that these conditions will generalise as it is then possible to derive a canonical representation of the qth order system.

It is possible to derive general models within a structured method of modelling which will allow theory to be tested or model types to be compared. We emphasise general models, as do Hendry and Mizon (1978) and Davidson et al (1978), because we believe that a particular implementation of a theoretical model should be tested rather than imposed on the data. We also feel that the models presented here will allow any modeler to set up his model in framework which will make it possible to compare a number of diverse alternatives.

The extent to which different parameters may be estimated will be affected by our ability to identify them, but usually it is enough for the order of the moving average or vector autoregressive representations of the exogenous variables to exceed either q the order of the cost of adjustment in the loss function or p the lag in the target condition. In addition we have the usual requirement that the conditions associated with a maximum are satisfied and that the underlying form of the model is correctly specified. Hence, non-identification will either be associated with the non-existence of a solution or with estimated parameters which are not consistent with cost minimisation. If the system is not identified, we may have the reduced rank solution associated with cointegration, but that model may still be directly estimated subject to restrictions on the coefficients of the deep parameters.

Suggestions for future research

The results seem to imply that the systems method is to be preferred to methods which produce unrestricted estimates of the first order condition and there seem to be benefits to the techniques used and developed when we compare them with unrestricted VARS. In the VAR case systems estimation may be preferred and cointegration theory would strengthen this idea. It seems likely that the exogenous processes should be jointly modelled and that the appropriate form of the vector time series or cointegration system will depend on its original parsimonious parameterisation. In such a systems context it seems likely that the issues of unit roots, feedback versus feedforward and the role of differential information will be increasingly important.

As a result of this study I feel that it is important to derive new methods to estimate the exogenous processes and it seems important that the techniques suggested in Chapter 3 should be developed for this purpose. There also seems to be some merit in producing efficient estimates of the first order condition, as that would more easily allow the questions of cointegration and irregularity to be addressed. In addition to this the first order condition has an error correction form which may allow the feedforward versus feedback debate to be more clearly dealt with.

There are also interesting questions of information, differential expectations and the very nature of expectations which have not been fully addressed. The results specified here are predicated on the actual expectational processes and the mathematical expectation being the same which implies that expectations are not a characteristic for the future path of all prices. Strict exogeneity is not enough for (6.5) to produce efficient estimates, as λ_1 and λ_2 in (6.1) will depend on the same deep parameters. If the zs are weakly exogenous, then the parameters of (6.5) are invariant to changes in $B^{\#}(L)$ which means that the endogenous variable relationship can be efficiently estimated on its own. In these circumstances the strict rational expectations hypothesis does not hold. The rational expectations hypothesis is imposed when the optimal predictor is the same as the true expectation, but if that is not the case then the error term will include the difference between the theoretical expectation and the prediction. Therefore:

(6.8) $y_t = F\, y_{t-1} + G^*(L^+)z_t^p + u_{1t}^*$

and $u_{1t}^* = u_{1t} + G(L^+)(B^{\#}(L)-B(L))z_{t-1}$

where $z_t^p = B(L)z_{t-1}$

The imposition of rationality may affect the consistency and efficiency of the estimates, as the zPs may be correlated with the error term. This will be a problem if there are considerable differences between the processes driving the expectations and the zs. In general one would believe that the parameter differences involved would be small relative to u_{1t}, so that the degree of inconsistency would be small. Alternatively consistency would be satisfied in large samples if the predictors or the expectations tended to rationality. The evidence sighted by Blume and Easley(1982) would be counterfactual to this, as would the notion that people took up ideologically different views of the world based on belief. In practice, it may only be possible to replace expectations by predictions, as subjective factors may never be captured perfectly. Hence, the model including predictions will be the best that can be achieved in the short-run and estimation would be consistent if the subjective and non-modelable elements of expectations are orthogonal to the exogenous variables.

Appendix 6.A Program REXP

The program uses the Numerical Algorithms Library(NAG) routines, E04JBF, E04HBF, F04AEF, F03ABF and F01ABF. The first is the Gill Murray Pitfield algorithm which is used to solve the models, the second provides initial estimates of the Hessian, F04AEF produces the accurate solution of a set of linear equations and the last two the determinant and inverse of a symmetric matrix using the Cholesky decomposition. In practice only E04HBF and E04JBF are actually used as we know the inverse of a 2x2 matrix.

```
      PROGRAM REXP
      IMPLICIT REAL (A-H,O-Z)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
C    PROGRAM TO ESTIMATE A MODEL WITH EXOGENOUS RATIONAL EXPECTATIONS
C
C    DATA MATRICESE
C
C    Y[120:6]  = ENDOGENOUS VARIABLES
C    X[120:15] = EXOGENOUS VARIABLES
C    WC[120:15]= FORCAST ERRORS OF EXOGENOUS VARIABLES
C    U[120:6]  = EQUATION ERRORS
C    XPR[30:15]=PREDICTIONS ON EXOGENOUS VARIABLES
C
C    GENERAL MATRICESE OF PARAMETERS
C
C    P[140]
C
C    D[6:15] =  PARAMETERS OF FORCAST ERRORS
C
C
C    PARAMETERS OF THE MODEL
C
C    NEQ     = NUMBER OF EQUATIONS
C    NEX     = NUMBER OF EXOGENOUS VARIABLES
C    NOBS    = NUMBER OF OBSERVATIONS
C    NPR     = NUMBER OF PREDICTION PERIODS
C    NSEAS   = NUMBER OF SEASONALS : 1 CONSTANT : 2 + TREND : + SEASONALS
C    : 4 + SEASONALS : 5 + SEASONALS AND TREND
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      REAL Y,X,XPR,WC,P,ETA,CST,HT,UT,S,RLOGL,YSTAR(160,6),
     1YFIT(160,6),ADFC
      INTEGER IRSTN(6,15),IRSTNF(6,6),IRSTNT(6),IRSTNR(6,6)
```

```
      CHARACTER NAMEX(15)*5,NAMEY(6)*5,FORM(10)*8
      DIMENSION Y(120,6),X(120,15),XPR(60,15),WC(120,15),P(100),
     1          CST(160,5),HT(160,6),U(120,6),S(6,6),BO(15,15,10)
      COMMON /CHAR/ NAMEX,NAMEY,FORM
      COMMON /DATA/ Y,X,XPR,WC,CST,HT,U,S,BO
      COMMON /EXTRA/ ADFC
      COMMON /FORM/ ITAPE,ITVAR,ITAPE1,NFORM,IFORM
      COMMON /MODEL/ NEQ,NEX,NOBS,NPR,NLPAR,NSEAS,NLOC,NTYPE,NDTS
      COMMON /REST/ RRLMDA,IRSTN,IREST,NREST,IRSTNF,IRSTNT,NFRES,
     1NTRES,IRSTNR,NRES
       DATA ZERO/0.0/,HALF/0.5/,ONE/1.0/,TWO/2.0/
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
C     INPUT SECTION
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
 1000 FORMAT(20X,40(1H*),/,2(20X,1H*,38X,1H*,/),20X,1H*,7X,'PROGRAM
   1 TO ESTIMATE',7X,1H*,/,20X,1H*,5X,'RATIONAL EXPECTATIONS MODELS',
     25X,1H*,/,20X,1H*,38X,1H*,/,20X,1H*,8X,'PROGRAM BY JOHN HUNTER',
     38X,1H*,/,20X,1H*,10X,'COPYRIGHT MAY 1983',10X,1H*,/,2(20X,1H*,
     438X,1H*,/),20X,40(1H*),///)
 1001 FORMAT(5X,'MODEL SPECIFICATION ',/,5X,19(1H-),//,5X,'NEQ=',I4,2X,
     1'NEX=',I4,2X,'NOBS=',I4,2X,'NPR=',I4,2X,'NSEAS=',I4,2X)
 1002
FORMAT(5X,'NLPAR=',I4,2X,'NTYPE=',I4,2X,'NDTS=',I4,2X,'IBO=',I4,2X,
     1'IREST=',I4/,5X,'NTRES=',I4,2X,'NFRES=',I4//)
 1003 FORMAT(5X,'ADFC=',F6.3,2X,'ETA=',F5.4,2X,
     1'EP=',F13.7,2X,'IERR=',I2,/)
      READ(1,*)NEQ,NEX,NOBS,NPR,NSEAS,ITAPE,ITVAR,IFORM,NLPAR,
     1ITAPE1,NLOC,NTYPE,NDTS,IBO,IREST,NTRES,NFRES
      WRITE(2,1000)
      WRITE(2,1001)NEQ,NEX,NOBS,NPR,NSEAS
      WRITE(2,1002)NLPAR,NTYPE,NDTS,IBO,IREST,NTRES,NFRES
      IF(IFORM.EQ.1) THEN
      READ(2,'(8A10)')FORM
      READ(2,'(I3)')NFORM
      ELSE
      NFORM=0
      ENDIF
      READ(1,*)ADFC,ETA,EP,IERR
      WRITE(2,1003)ADFC,ETA,EP,IERR
      READ(1,'(6A5)')(NAMEY(I),I=1,NEQ)
      READ(1,'(15A5)')(NAMEX(I),I=1,NEX)
      N=NEQ*(2*NEQ+NEX+NSEAS)+1
      IF(NTYPE.EQ.2) N=NEQ*(NEQ+NEX+NSEAS)+1
      IF(NTYPE.EQ.3) N=NEQ*(NEQ+NEX+NSEAS+1)+1
      IF(NTYPE.EQ.4) N=NEQ*(NEQ+NEX+NSEAS+NEQ)+1
      IF(NTYPE.EQ.5) N=NEQ*(NEQ+NEX+NSEAS+NEX)+1
      IF(NDTS.GT.1) N=N+NEQ*NEX
      IF(NDTS.EQ.3) N=N+NEQ*NEX
      IF(IREST.EQ.1.OR.IREST.EQ.3) THEN
      N=N-1
      READ(1,*)RRLMDA
      ENDIF
      IF(IREST.GT.1) THEN
      READ(1,*)NREST
      N=N-NREST
      READ(1,*)((IRSTN(I,J),J=1,NEX),I=1,NEQ)
```

```
      ENDIF
      IF(NFRES.GT.0)THEN
      N=N-NFRES
      READ(1,*)((IRSTNF(I,J),J=1,NEQ),I=1,NEQ)
      ENDIF
      IF(NTRES.GT.0)THEN
      N=N-NTRES
      READ(1,*)(IRSTNT(I),I=1,NEQ)
      ENDIF
      IF(NTYPE.EQ.4) THEN
      READ(1,*)NRES
      IF(NRES.NE.0) THEN
      N=N-NRES
      READ(1,*)((IRSTNR(I,J),J=1,NEQ),I=1,NEQ)
      ENDIF
      ENDIF
      CALL DATAIN(P,NEQ,NEX,NOBS,NPR,N,NLPAR)
      NTN=NOBS+NPR
      IF(NSEAS.GT.0) THEN
      CALL SETZER(CST,5,160,ZERO)
      DO 1 I=1,NTN
    1 CST(I,1)=ONE
      IF(NSEAS.EQ.4.OR.NSEAS.EQ.5) THEN
      DO 2 I=1,NTN,4
      J=I+1
      K=I+2
      CST(I,2)=ONE
      CST(J,3)=ONE
    2 CST(K,4)=ONE
      ENDIF
      IF(NSEAS.EQ.2.OR.NSEAS.EQ.5) THEN
      DO 3 I=1,NTN
    3 CST(I,NSEAS)=FLOAT(I)/FLOAT(NTN)
      ENDIF
      ENDIF
      CALL EST(NTN,N,RLOGL,P,ETA,IBO,EP,IERR,YSTAR,YFIT)
      STOP
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C       ESTIMATION OF MODEL WITH RESTRICTED D MATRIX
C       Y[T]=FY[T-1] + SUM[1:T][RLMDA*F**I*BINV*A(X[I]-D*WC[I+1]]
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE EST(NTN,N,RLOGL,P,ETA,IBO,EP,IERR,YSTAR,YFIT)
      IMPLICIT REAL (A-H,O-Z)
      INTEGER IW(2),ISTATE(100)
      REAL RLOGL,DELTA(50),P(50),HESD(50),HESL(2450),G(50),
     1YFIT(NTN,NEQ),
     2VCOV(50,50),C(50,50),CTD(50,50),W(900),BL(50),BU(50),
     3YSTAR(NOBS,NEQ),A1(6,15)
      LOGICAL LOSCH
      COMMON /MODEL/ NEQ,NEX,NOBS,NPR,NLPAR,NSEAS,NLOC,NTYPE,NDTS
      EXTERNAL MONIT,FUNCT,E04JBQ
       DATA ZERO/0.0/,HALF/0.5/,ONE/1.0/,TWO/2.0/
      DATA DOTWO/0.2/,TEN/10.0/,HUNTH/100000.0/
      MAXCAL=80*N*N
      IF(NLOC.EQ.1) THEN
      LOSCH=.TRUE.
```

```
      ELSE
      LOSCH=.FALSE.
      ENDIF
      LH=N*(N-1)/2
      IF(N.EQ.1)LH=1
      LW=2*N
      IFAIL=0
      CALL E04HBF(N,FUNCT,P,NF,DELTA,HESL,LH,HESD,RLOGL,G,IW,1,W,LW,
     1IFAIL)
      FEST=DOTWO*RLOGL
      LW=9*N
      IFAIL=1
      IF(IBO.NE.1) THEN
      DO 1 I=1,N
      BU(I)=TEN**(6)
    1 BL(I)=-TEN**(6)
      BL(N)=ZERO
      ENDIF
      CALL E04JBF(N,FUNCT,MONIT,10,LOSCH,0,E04JBQ,MAXCAL,ETA,ZERO,
     1HUNTH,FEST,DELTA,IBO,BL,BU,P,HESL,LH,HESD,ISTATE,RLOGL,G
     2,IW,2,W,LW,IFAIL)
      WRITE(2,'(I4)')IFAIL
      CALL TEST(VCOV,STLOG,HESD,HESL,N,LH,RLOGL,C,CTD,P,EP,IERR)
      CALL PRNT(YSTAR,YFIT,NTN,N,CTD,C,VCOV,P,A1)
      RETURN
      END
      SUBROUTINE FUNCT(IFLAG,N,XC,FC,GC,IW,LIW,W,LW)
      IMPLICIT REAL (A-H,O-Z)
      INTEGER IFLAG,N,IW,LIW,LW
      REAL XC,FC,GC,W,TEMP(6,6),TEMP1(6,15),TEMP2(15,15),TEMP3(6,15),
     1G1,G2,G3,F,BINV,A,Q,D
      DIMENSION XC(N),GC(N),W(LW),IW(LIW),G1(6,6),G2(6,15),G3(6,15),
     1F(6,6),BINV(6,6),A(6,15),Q(6,5),D(6,15),R(6,6),A1(6,15),
     2G4(6,15),GS(6,5),APR(6,15),D1(6,15)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     INITIALISING MATRICESE F BINV A FROM P
C     FOR FUNCTION EVALUATION
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      IFLAG=0
      CALL EQUAT(FC,XC,G1,G2,G3,TEMP,TEMP1,TEMP2,TEMP3,F,A,BINV,Q,D,N,
     1R,A1,G4,GS,APR,D1)
      RETURN
      END
      SUBROUTINE BIN(F,BINV,NEQ,G1,RLMDA,T1)
      IMPLICIT REAL (A-H,O-Z)
      REAL F(NEQ,NEQ),BINV(NEQ,NEQ),G1(NEQ,NEQ),TEMP(7,7),Z(6),RLMDA,
     1TEMP1(7,7),T1(NEQ,NEQ)
       DATA ZERO/0.0/,HALF/0.5/,ONE/1.0/,TWO/2.0/
      CALL SETZER(T1,NEQ,NEQ,ONE)
      IFAIL=0
      CALL F04AEF(F,NEQ,T1,NEQ,NEQ,NEQ,BINV,NEQ,Z,TEMP,7,TEMP1,7,IFAIL)
      IF(IFAIL.NE.0) WRITE(2,1000)IFAIL
      DO 2 I=1,NEQ
      DO 2 K=1,NEQ
      TMP=BINV(I,K)
      IF(I.EQ.K)BINV(I,K)=G1(I,K)+TMP-(RLMDA+1)
      IF(I.NE.K)BINV(I,K)=G1(I,K)+TMP
```

```
   2 CONTINUE
     RETURN
1000 FORMAT(2X,'INVERSION ROUTINE FAIL WITH IFAIL= ',I2,/,2X,
    1'1 MEANS NOT PDF AND 2 MEANS ILL CONDITIONED MATRIX')
     END
     SUBROUTINE EQUAT(FC,XC,G1,G2,G3,TEMP,TEMP1,TEMP2,TEMP3,F,A,BINV,
    1Q,D,N,R,A1,G4,GS,APR,D1)
     IMPLICIT REAL (A-H,O-Z)
     REAL FC,XC(N),G1(NEQ,NEQ),G2(NEQ,NEX),G3(NEQ,NEX),TEMP(NEQ,NEQ),
    1TEMP1(NEQ,NEX),TEMP2(NEX,NEX),TEMP3(NEQ,NEX),F(NEQ,NEQ),
    2BINV(NEQ,NEQ),A(NEQ,NEX),Q(NEQ,NSEAS),D(NEQ,NEX),H1,Y1,SE,DET,
    3WKSPC(6),RLMDA,R(NEQ,NEQ),A1(NEQ,NEX),G4(NEQ,NEX),GS(NEQ,NSEAS),
    4APR(NEQ,NEX),D1(NEQ,NEX)
     COMMON /DATA/ Y(120,6),X(120,15),XPR(60,15),WC(120,15),
    1CST(160,5),HT(160,6),U(120,6),S(6,6),BO(15,15,10)
     COMMON /EXTRA/ ADFC
     COMMON /MODEL/ NEQ,NEX,NOBS,NPR,NLPAR,NSEAS,NLOC,NTYPE,NDTS
     COMMON /BANDD/ VECBIN(36),VECD(90)
     COMMON /REST/ RRLMDA,IRSTN(6,15),IREST,NREST,IRSTNF(6,6),
    1IRSTNT(6),NFRES,NTRES,IRSTNR(6,6),NRES
      DATA ZERO/0.0/,HALF/0.5/,ONE/1.0/,TWO/2.0/
     DATA DOTWO/2.0/,TEN/10.0/,HUNTH/100000.0/
     DATA TWENTY/20.0/
     NK=0
     IF(NFRES.GT.0)THEN
     CALL TRANSA(F,NEQ,NEQ,NK,XC,N,IRSTNF,6)
     NK=NK+NEQ*NEQ-NFRES
     ELSE
     CALL TRANS(F,NEQ,NEQ,NK,XC,N)
     ENDIF
     IF(NTYPE.EQ.1)CALL TRANS(BINV,NEQ,NEQ,NK,XC,N)
     IF(NDTS.GT.1) THEN
     IF(NDTS.LE.3) CALL TRANS(D,NEQ,NEX,NK,XC,N)
     IF(NDTS.GT.3) CALL TRANS(D1,NEQ,NEX,NK,XC,N)
     ENDIF
     IF(IREST.GT.1) THEN
     CALL TRANSA(A,NEQ,NEX,NK,XC,N,IRSTN,15)
     NK=NK+NEQ*NEX-NREST
     ELSE
     CALL TRANS(A,NEQ,NEX,NK,XC,N)
     ENDIF
     IF(NSEAS.NE.0)THEN
     IF(NTRES.GT.0) THEN
     DO 2 I=1,NEQ
     DO 2 J=1,NSEAS
     IF(J.EQ.NSEAS.AND.IRSTNT(I).EQ.0) THEN
     Q(I,J)=ZERO
     ELSE
     NK=NK+1
     Q(I,J)=XC(NK)
     ENDIF
   2 CONTINUE
     ELSE
     CALL TRANS(Q,NEQ,NSEAS,NK,XC,N)
     ENDIF
     ENDIF
     IF(NTYPE.EQ.4) THEN
     IF(NRES.EQ.0) THEN
     CALL TRANS(R,NEQ,NEQ,NK,XC,N)
```

```
      ELSE
      CALL TRANSA(R,NEQ,NEQ,NK,XC,N,IRSTNR,6)
      NK=NK+NEQ*NEQ-NRES
      ENDIF
      ENDIF
      IF(NTYPE.EQ.5) CALL TRANS(A1,NEQ,NEX,NK,XC,N)
      RLMDA=XC(N)
      IF(IREST.EQ.1.OR.IREST.EQ.3) RLMDA=RRLMDA
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     SET UP OF MATRICESE TO BE MODELLED G1 G2 G3
C
C     G1=RLMDA*F
C     G2=F*BINV*A
C     G3=F*BINV*D
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 5 J=1,NEQ
      DO 5 I=1,NEQ
    5 G1(I,J)=RLMDA*F(I,J)
      IF(NTYPE.NE.1) THEN
      BINV(1,1)=G1(1,1)+F(1,1)**(-1)-(RLMDA+1)
      IF(NEQ.GT.1) CALL BIN(F,BINV,NEQ,G1,RLMDA,TEMP)
      ENDIF
      INO=0
      CALL VEC(INO,NEQ,NEQ,VECBIN,BINV)
      IF(NDTS.LE.1.OR.NDTS.EQ.4) THEN
      IF(NDTS.EQ.0)CALL SETZER(D,NEQ,NEX,ZERO)
      IF(NDTS.EQ.1) THEN
      IF(NTYPE.EQ.5) THEN
      DO 6 I=1,NEQ
      DO 6 J=1,NEX
    6 APR(I,J)=A(I,J)
      CALL MATMLT(G1,A1,APR,NEQ,NEQ,NEX,1)
      CALL DMA(D,NEQ,NEX,BO,NLPAR,G1,APR,TEMP1,TEMP3,TEMP2)
      ELSE
      CALL DMA(D,NEQ,NEX,BO,NLPAR,G1,A,TEMP1,TEMP3,
     1TEMP2)
      ENDIF
      INO=0
      CALL VEC(INO,NEQ,NEX,VECD,D)
      ENDIF
      ENDIF
      CALL MATMLT(F,BINV,TEMP,NEQ,NEQ,NEQ,0)
      CALL MATMLT(TEMP,A,G2,NEQ,NEQ,NEX,0)
      CALL MATMLT(TEMP,D,G3,NEQ,NEQ,NEX,0)
      IF(NTYPE.EQ.5) CALL MATMLT(TEMP,A1,G4,NEQ,NEQ,NEX,0)
      CALL MATMLT(TEMP,Q,GS,NEQ,NEQ,NSEAS,0)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     CREATION OF HT MATRICES  AND EVALUATION OF FC
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      CALL SETZER(S,6,6,ZERO)
      NT=NOBS+NPR
      NT1=NT+1
      NTL=NT-1
      NE=NEQ+1
      DO 10 K=1,NEQ
```

```
      IF(NTYPE.EQ.3) THEN
      NK=NK+1
      HT(NT1,K)=XC(NK)
      ELSE
      HT(NT1,K)=ZERO
      ENDIF
   10 CONTINUE
      NDF=1
      IF(NTYPE.EQ.4) NDF=2
      NBS1=NOBS+1
      DO 24 I=1,NTL
      I1=NT1-I+1
      I2=I1-1
      I3=I1-NOBS-1
      I4=I2-1
      I5=I3-1
      DO 19 K=1,NEQ
      Y1=ZERO
      H1=ZERO
      DO 14 J=1,NEQ
      IF(I2.GT.NOBS) THEN
      H1=H1+G1(K,J)*HT(I1,J)+G2(K,J)*XPR(I3,J)
      IF(NTYPE.EQ.5) THEN
      IF(I2.EQ.NBS1) THEN
      H1=H1+G4(K,J)*X(NOBS,J)
      ELSE
      H1=H1+G4(K,J)*XPR(I5,J)
      ENDIF
      ENDIF
      ELSE
      H1=H1+G1(K,J)*HT(I1,J)+G2(K,J)*X(I2,J)-G3(K,J)*WC(I2,J)
      Y1=Y1+F(K,J)*Y(I4,J)
      IF(NDTS.GT.3) H1=H1+D1(K,J)*WC(I4,J)
      IF(NTYPE.EQ.5) H1=H1+G4(K,J)*X(I4,J)
      ENDIF
   14 CONTINUE
      IF(NEX.GT.NEQ) THEN
      DO 16 J=NE,NEX
      IF(I2.GT.NOBS) THEN
      H1=H1+G2(K,J)*XPR(I3,J)
      IF(NTYPE.EQ.5) THEN
      IF(I2.EQ.NBS1) THEN
      H1=H1+G4(K,J)*X(NOBS,J)
      ELSE
      H1=H1+G4(K,J)*XPR(I5,J)
      ENDIF
      ENDIF
      ELSE
      H1=H1+G2(K,J)*X(I2,J)-G3(K,J)*WC(I2,J)
      IF(NDTS.GT.3) H1=H1+D1(K,J)*WC(I4,J)
      IF(NTYPE.EQ.5) H1=H1+G4(K,J)*X(I4,J)
      ENDIF
   16 CONTINUE
      ENDIF
      SE=ZERO
      IF(NSEAS.NE.0) THEN
      DO 17 J=1,NSEAS
   17 SE=SE+GS(K,J)*CST(I2,J)
      ENDIF
```

```
      HT(I2,K)=H1+SE
      IF(I2.LE.NOBS) THEN
      U(I2,K)=Y(I2,K)-Y1-HT(I2,K)
      ENDIF
   19 CONTINUE
      IF(I2.LT.NOBS) THEN
      IF(NTYPE.EQ.4) THEN
      DO 21 K=1,NEQ
      E1=ZERO
      DO 20 J=1,NEQ
   20 E1=E1+R(K,J)*U(I2,J)
      U1=U(I1,K)
   21 U(I1,K)=U1-E1
      ENDIF
      DO 22 IJ=1,NEQ
      II=IJ
      DO 22 K=II,NEQ
      IF(NTYPE.EQ.4) THEN
      TT=S(II,K)+(U(I1,K)*U(I1,II)/(FLOAT(NOBS-NDF)))
      ELSE
      TT=S(II,K)+(U(I2,K)*U(I2,II)/(FLOAT(NOBS-NDF)))
      ENDIF
   22 S(II,K)=TT
      ENDIF
   24 CONTINUE
      DO 26 I=1,NEQ
      DO 25 K=1,NEQ
      IF(I.GT.K) S(I,K)=S(K,I)
   25 TEMP(I,K)=S(I,K)
   26 CONTINUE
      IF(NTYPE.NE.4) THEN
      DO 30 I=1,NEQ
      DO 30 J=1,NEQ
      TT=S(I,J)+(U(NOBS,I)*U(NOBS,J)/(FLOAT(NOBS-NDF)))
   30 S(I,J)=TT
      ENDIF
      IF(NEQ.EQ.1) THEN
      FC=LOG(S(1,1))/ADFC
      ELSE
      IF(NEQ.EQ.2) THEN
      DET=S(1,1)*S(2,2)-S(1,2)**2
      ELSE
      IFAIL=0
      CALL F03ABF(TEMP,NEQ,NEQ,DET,WKSPC,IFAIL)
      WRITE(2,1010)IFAIL
      IFAIL=0
      ENDIF
      IF(DET.LE.ZERO) THEN
      FC=TEN**20
      WRITE(2,1000)RLMDA,((S(I,K),I=1,NEQ),K=1,NEQ)
      ELSE
      FC=FLOAT(NOBS-NDF)*LOG(DET)/TWO/ADFC
      ENDIF
      ENDIF
      RETURN
 1000 FORMAT(5X,' UNSTABLE VALUES OF EITHER RLMDA ',F13.7,' OR F',/,
     15X,'HAVE GENERATED LARGE EQUATION VARIANCES ',/,
     26(5X,6E13.7,/))
 1010 FORMAT(5X,' IFAIL FOR DETERMINANT SOLUTION ',I2,/)
```

```
      END
      SUBROUTINE DATAIN(P,NEQ,NEX,NOBS,NPR,N,NLPAR)
      IMPLICIT REAL (A-H,O-Z)
      CHARACTER NAMEX(15)*5,NAMEY(6)*5,FORM(10)*8
      REAL P(N),A1(120,15),A2(120,15),A3(120,15)
      COMMON /CHAR/ NAMEX,NAMEY,FORM
      COMMON /FORM/ ITAPE,ITVAR,ITAPE1,NFORM,IFORM
      COMMON /DATA/ Y(120,6),X(120,15),XPR(60,15),WC(120,15),
     1             CST(160,5),HT(160,6),U(120,6),S(6,6),BO(15,15,10)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     READ DATA SECTION
C       IN  ORDER :  ENDOGENOUS ,  EXOGENOUS ,  PREDICTIONS , PREDICTION
ERRORS
C     THEN      : P MATRIX OF INITIAL VALUES , BO MATRIX OF REDUCED FORM
C     COEFFICIENTS
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      IF(ITAPE.EQ.0) ITAPE1=1
      IF(ITAPE.LT.3) ITAPE=1
      IF(ITVAR.EQ.1) THEN
      READ(ITAPE,*)((Y(I,J),I=1,NOBS),J=1,NEQ)
      READ(ITAPE,*)((A1(I,J),I=1,NOBS),J=1,NEX)
      READ(ITAPE,*)((A2(I,J),I=1,NPR),J=1,NEX)
      READ(ITAPE,*)((A3(I,J),I=1,NOBS),J=1,NEX)
      ELSE
      IF(IFORM.EQ.0) THEN
      READ(ITAPE,*)((Y(I,J),J=1,NEQ),I=1,NOBS)
      READ(ITAPE,*)((A1(I,J),J=1,NEX),I=1,NOBS)
      READ(ITAPE,*)((A2(I,J),J=1,NEX),I=1,NPR)
      READ(ITAPE,*)((A3(I,J),J=1,NEX),I=1,NOBS)
      ELSE
      READ(ITAPE,FORM)((Y(I,J),J=1,NEQ),I=1,NOBS)
      CALL INFORM(A1,NFORM,NEQ,NOBS)
      CALL INFORM(A2,NFORM,NEQ,NPR)
      CALL INFORM(A3,NFORM,NEQ,NOBS)
      ENDIF
      ENDIF
      IF(ITAPE1.LT.3) ITAPE1=1
      READ(ITAPE1,*)(P(I),I=1,N)
      READ(ITAPE1,*)(((BO(I,J,K),K=1,NLPAR),J=1,NEX),I=1,NEX)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     WRITE DATA INPUT TO OUTPUT
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      WRITE(2,1000)(NAMEY(I),I=1,NEQ)
      DO 1 I=1,NOBS
    1 WRITE(2,1001)(Y(I,J),J=1,NEQ)
      WRITE(2,1002)
      CALL DATOUT(A1,NEX,NOBS)
      WRITE(2,1003)
      CALL DATOUT(A2,NEX,NPR)
      WRITE(2,1004)
      CALL DATOUT(A3,NEX,NOBS)
      DO 2 J=1,NEX
      DO 2 I=1,NOBS
      X(I,J)=A1(I,J)
      IF(I.LE.NPR) XPR(I,J)=A2(I,J)
```

```
    2 WC(I,J)=A3(I,J)
      WRITE(2,1005)
      RETURN
 1000
FORMAT(/,10X,'ENDOGENOUSVARIABLES',/,10X,19(1H-),/,1X,6(4X,A5,4X),/,
     1/,1X,10(4X,5(1H-),4X,/))
 1001 FORMAT(6F13.5)
 1002 FORMAT(/,10X,'EXOGENOUS VARIABLES',/,10X,19(1(1H-)),/)
 1003 FORMAT(/,10X,'PREDICTIONS OF EXOGENOUSVARIABLES',/,10X,34(1H-),/)
 1004 FORMAT(/,10X,'ONE STEP AHEAD FORCAST ERRORS',/,10X,29(1H-),/)
 1005 FORMAT(/,10X,'END OF DATA SECTION',/,10X,17(1H-),/)
      END
      SUBROUTINE DMA(D,N1,N2,BO,NLPAR,G1,A,TEMP,TEMP1,TEMP2)
      IMPLICIT REAL (A-H,O-Z)
      REAL D,BO,G1,TEMP,TEMP1,TEMP2
      DIMENSION D(N1,N2),TEMP(N1,N2),TEMP1(N1,N2),TEMP2(N2,N2)
      DIMENSION G1(N1,N1),BO(15,15,10),A(N1,N2)
      CALL MATMLT(G1,A,D,N1,N1,N2,0)
      DO 1 I=1,N1
      DO 1 J=1,N2
    1 TEMP(I,J)=D(I,J)
      DO 4 J=1,NLPAR
      CALL MATMLT(G1,TEMP,TEMP1,N1,N1,N2,0)
      DO 2 I=1,N1
      DO 2 K=1,N2
    2 TEMP(I,K)=TEMP1(I,K)
      DO 3 K=1,N2
      DO 3 I=1,N2
    3 TEMP2(I,K)=BO(I,K,J)
    4 CALL MATMLT(TEMP1,TEMP2,D,N1,N2,N2,1)
      RETURN
      END
      SUBROUTINE DATOUT(XS,NEX,N5)
      IMPLICIT REAL (A-H,O-Z)
      CHARACTER NAMEX(15)*5,NAMEY(6)*5,FORM(10)*8
      REAL XS
      DIMENSION XS(120,15)
      COMMON /CHAR/ NAMEX,NAMEY,FORM
      N1=1
      N2=10
    1 CONTINUE
      IF(NEX-N2.LT.0) N2=NEX
      WRITE(2,1000)(NAMEX(K),K=N1,N2)
      DO 2 I=1,N5
    2 WRITE(2,1001)(XS(I,J),J=N1,N2)
      IF(N2.EQ.NEX) RETURN
      N1=N2+1
      N2=N2+10
      GOTO 1
 1000 FORMAT(//,1X,10(4X,A5,4X),/,10(4X,5(1H-),4X),/)
 1001 FORMAT(10F13.5)
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C         GHESS CALCULATES NUMERICAL SECOND DERIVATIVES
C         WHICH ARE USED TO DERIVE ALTERNATIVE ESTIMATES
C         OF THE VARIANCE-COVARIANCE MATRIX
C         CTD CONTAINS THE HESSIAN
C
```

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE GHESS(N,CTD,C,XC,RLOGL,EP)
      IMPLICIT REAL (A-H,O-Z)
      REAL CTD(N,N),C(N,N),XC(N),RLOGL,EP,DIV,ZAP
      DATA ZERO/0.0/,HALF/0.5/,ONE/1.0/,TWO/2.0/
      DATA DOTWO/2.0/,TEN/10.0/,HUNTH/100000.0/
      IF(ABS(RLOGL).LT.EP) THEN
      DIV=ONE
      ELSE
      DIV=RLOGL
      ENDIF
      IVIEW=0
      DO 3 I=1,N
      C(I,1)=EP
      ZAP=XC(I)
    2 XC(I)=XC(I)+C(I,1)
      CALL FUNCT(IFLAG,N,XC,C(I,2),GC,IW,1,W,1)
      IDEF=1
      IF((ABS((C(I,2)-RLOGL)/DIV).LT.EP).AND.(C(I,1).LT.ZAP)
     1.AND.(IDEF.LT.1)) THEN
      C(I,1)=C(I,1)*TEN
      GOTO 2
      ELSE
      ENDIF
      XC(I)=XC(I)-TWO*C(I,1)
      CALL FUNCT(IFLAG,N,XC,C(I,3),GC,IW,1,W,1)
      XC(I)=XC(I)+C(I,1)
      II=I
      DO 3 J=1,II
      XC(I)=XC(I)+C(I,1)
      XC(J)=XC(J)+C(J,1)
      CALL FUNCT(IFLAG,N,XC,D,GC,IW,1,W,1)
      CTD(I,J)=(D+TWO*RLOGL-C(I,2)-C(J,2)-C(I,3)-C(J,3))
      XC(I)=XC(I)-TWO*C(I,1)
      XC(J)=XC(J)-TWO*C(J,1)
      CALL FUNCT(IFLAG,N,XC,D,GC,IW,1,W,1)
      CTD(I,J)=(CTD(I,J)+D)/(TWO*C(I,1)*C(J,1))
      XC(I)=XC(I)+C(I,1)
    3 XC(J)=XC(J)+C(J,1)
      N1=N-1
      DO 4 I=1,N1
      II=I+1
      DO 4 J=II,N
    4 CTD(I,J)=CTD(J,I)
      IF(IVIEW.GT.0) THEN
      DO 5 I=1,N
      II=I
    5 WRITE(2,'(6(G17.9))')(CTD(I,J),J=1,II)
      ENDIF
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     GAUSS-JORDAN SWEEP OPERATOR TO INVERT A SYMMETRIC
C     POSITIVE MATRIX
C     USED TO INVERT THE HESSIAN
C     SYM CONTAINS THE HESSIAN
C     AND VCOV ITS INVERSE
C
```

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE GJS(SYM,VCOV,N)
      IMPLICIT REAL (A-H,O-Z)
      REAL SYM(N,N),VCOV(N,N)
      INTEGER I,J,K,L1,L2
       REAL ZERO,HALF,ONE
       DATA ZERO/0.0/,HALF/0.5/,ONE/1.0/,TWO/2.0/
        DO 99 L1=1,N
        DO 98 L2=L1,N
                     VCOV(L2,L1)=SYM(L2,L1)
   98   CONTINUE
   99   CONTINUE
C
C
      DO 1 K=1,N
      TEMP=(-ONE)/VCOV(K,K)
           VCOV(K,K)=TEMP
      DO 3 I=1,N
      IF(I.NE.K)THEN
      IF(I.LT.K)THEN
C
C CHANGE ROW K
C
               VCOV(K,I)=VCOV(K,I)*TEMP
               T2=VCOV(K,I)
      ELSE
C
C CHANGE COL K
C
              VCOV(I,K)=VCOV(I,K)*TEMP
               T2=VCOV(I,K)
      ENDIF
C
C CHANGE THE REST OF THE LOWER TRIANGLE
C
      DO 4 J=1,I
          IF(J.NE.K)THEN
             IF(J.LT.K)THEN
                           T4=VCOV(K,J)
               ELSE IF (I.GT.K)THEN
                                   T4=VCOV(J,K)
             ENDIF
        VCOV(I,J)=VCOV(I,J)+T2*T4/TEMP
      ENDIF
    4 CONTINUE
      ENDIF
    3 CONTINUE
    1 CONTINUE
      DO 5 I=1,N
      VCOV(I,I)=-VCOV(I,I)
      DO 5 J=I+1,N
      VCOV(I,J)=-VCOV(J,I)
      VCOV(J,I)=-VCOV(J,I)
    5 CONTINUE
      RETURN
      END
      SUBROUTINE INFORM(XS,N4,N5)
      IMPLICIT REAL (A-H,O-Z)
      CHARACTER NAMEX(15)*5,NAMEY(6)*5,FORM(10)*8
```

```
      REAL XS
      DIMENSION XS(120,15)
      COMMON /CHAR/ NAMEX,NAMEY,FORM
      COMMON /FORM/ ITAPE,ITVAR,ITAPE1,NFORM,IFORM
      N1=1
      N2=NFORM
    1 CONTINUE
      IF(N2-N4)2,4,3
    2 READ(ITAPE,FORM)((XS(I,J),J=N1,N2),I=1,N5)
      N1=N2+1
      N2=N2+NFORM
      GOTO 1
    3 N2=N4
    4 READ(ITAPE,FORM)((XS(I,J),J=N1,N2),I=1,N5)
      RETURN
      END
      SUBROUTINE MATMLT(XX,YY,NEW,N1,N2,N3,NS2)
      IMPLICIT REAL (A-H,O-Z)
      REAL XX,YY,NEW,F
      DIMENSION XX(N1,N2),YY(N2,N3),NEW(N1,N3)
       DATA ZERO/0.0/,HALF/0.5/,ONE/1.0/,TWO/2.0/
      DO 2 K=1,N3
      DO 2 I=1,N1
      F=ZERO
      DO 1 J=1,N2
    1 F=F+XX(I,J)*YY(J,K)
      IF(NS2.EQ.0)THEN
      NEW(I,K)=F
      ELSE
      NEW(I,K)=NEW(I,K)+F
      ENDIF
    2 CONTINUE
      RETURN
      END
      SUBROUTINE MONIT(N,XC,FC,GC,ISTATE,GPJNRM,COND,POSDEF,NITER,
     1                  NF,IW,LIW,W,LW)
      IMPLICIT REAL (A-H,O-Z)
      COMMON /MODEL/ NEQ,NEX,NOBS,NPR,NLPAR,NSEAS,NLOC,NTYPE,NDTS
      COMMON /EXTRA/ ADFC
      DATA ZERO/0.0/,HALF/0.5/,ONE/1.0/,TWO/2.0/,TWENTY/20.0/
      LOGICAL POSDEF
      INTEGER N,ISTATE,NITER,NF,IW,LIW,LW
      REAL XC(N),FC,GC(N),GPJNRM,COND,W,RLOGL
      NDF=NOBS-1
      IF(NTYPE.EQ.4) NDF=NOBS-2
      RLOGL=-HALF*(NDF)*(NEQ*2*LOG(2*3.1743)+ONE)-(FC*ADFC)
      WRITE(2,1000)
      WRITE(2,1001)NITER,FC,RLOGL
      WRITE(2,1010)(XC(I),I=1,N)
      WRITE(2,1012)
      WRITE(2,1010)(GC(I),I=1,N)
      WRITE(2,1020)GPJNRM,COND
      RETURN
 1000 FORMAT('  NO ITERATIONS   FUNCTION VALUE ',/)
 1001 FORMAT(8X,I5,8X,F10.5,8X,F10.5,/,5X,'PARAMETER VALUES',/)
 1010 FORMAT(7F11.6)
 1012 FORMAT(5X,'GRADIENT OF FUNCTION AT XC',/)
 1020 FORMAT(6X,'EUCLIDEAN NORM   CONDITION OF HESSIAN',/,7X,F10.5,7X,
     1F13.5)
```

```
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
C     MAXI CALCULATES THE MAX AND MIN OF A SERIES Y
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE MAXI(Y,RMX,RMN,I,N,N2,N3,N4)
      IMPLICIT REAL(A-H,O,Z)
      DIMENSION Y(N,N2)
      DO 1 K=N3,N4
      IF(Y(K,I).GT.RMX) RMX=Y(K,I)
      IF(Y(K,I).LT.RMN) RMN=Y(K,I)
    1 CONTINUE
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C     PRINTY PRINTS OUT THE EXOGENOUS VARIABLES Y FOR THE ITH
C     EQUATION, THEIR FITTED VALUES YFIT AND THE CALCULATED
C     EQUILIBRIUM VALUES YSTAR FOLLOWING THIS ARE PREDICTIONS
C     STORED IN YFIT
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE PRNTY(YSTAR,YFIT,I,NTN,NSTRT)
      IMPLICIT REAL(A-H,O-Z)
      CHARACTER NAMEX(15)*5,NAMEY(6)*5,FORM(10)*8,LINE(61)*1,
     1LINE1(9)*1
      DIMENSION YSTAR(NOBS,NEQ),YFIT(NTN,NEQ)
      COMMON /CHAR/ NAMEX,NAMEY,FORM
      COMMON /DATA/ Y(120,6),X(120,15),XPR(60,15),WC(120,15),
     1CST(160,5),HT(160,6),U(120,6),S(6,6),BO(15,15,10)
      COMMON /MODEL/ NEQ,NEX,NOBS,NPR,NLPAR,NSEAS,NLOC,NTYPE,NDTS
      DATA TWO/2.0/,NINE/9.0/,SXTY1/61.0/,AETY1/81.0/,ZERO/0.0/,
     1HALF/0.5/,FOUR5/4.5/
      RMX=Y(1,I)
      RMN=Y(1,I)
      CALL MAXI(Y,RMX,RMN,I,120,6,NSTRT,NOBS)
      CALL MAXI(YSTAR,RMX,RMN,I,NOBS,NEQ,NSTRT,NOBS)
      CALL MAXI(YFIT,RMX,RMN,I,NTN,NEQ,NSTRT,NTN)
      R=RMX-RMN
      SS=R/SXTY1
      H=(RMX+RMN)/TWO
      SE=S(I,I)**HALF
      R2=SE*FOUR5
      WRITE(2,1005)RMN,H,RMX
      DO 5 K=NSTRT,NOBS
      DO 1 J=1,80
    1 LINE(J)=' '
      DO 2 J=1,9
    2 LINE1(J)=' '
      J=MAXO(1,IFIX((Y(K,I)-RMN)/SS))
      LINE(J)='+'
      J=MAXO(1,IFIX((YSTAR(K,I)-RMN)/SS))
      LINE(J)='*'
      J=MAXO(1,IFIX((YFIT(K,I)-RMN)/SS))
      LINE(J)='x'
      IF(ABS(U(K,I)).LT.R2)THEN
```

```
      J=MAXO(1,IFIX((U(K,I)+FOUR5*SE)/SE))
      LINE1(J)='x'
      ELSE
      IF(U(K,I).GT.ZERO)LINE1(9)='*'
      IF(U(K,I).LT.ZERO)LINE1(1)='*'
      ENDIF
    5 WRITE(2,1010)Y(K,I),YFIT(K,I),YSTAR(K,I),LINE,
     1U(K,I),LINE1
      NO1=NOBS+1
      DO 10 K=NO1,NTN
      DO 7 J=1,80
    7 LINE(J)=' '
      J=MAXO(1,IFIX((YFIT(K,I)-RMN)/SS))
      LINE(J)='x'
   10 WRITE(2,1015)YFIT(K,I),LINE
      RETURN
 1005 FORMAT(7X,'Y',12X,'FIT',12X,'EQ',6X,G13.5,11X,G13.5,11X,
     1G13.5,5X,'ERROR',9X,'0',/,7X,'-',12X,'---',12X,'--',6X,30(1H-),
     21H+,30(1H-),5X,5(1H-),5X,4(1H-),1H+,4(1H-),/)
 1010 FORMAT(1X,3(G13.5,1X),61A1,1X,G13.5,1X,9A1)
 1015 FORMAT(15X,G13.5,15X,80A1)
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      SUBROUTINE PRNT CALCULATES THE FITTED VALUES YFIT AND
C      PREDICTIONS FOR NPR FUTURE PERIODS STORED IN THE LAST
C      NPR SPACES OF YFIT
C      IT ALSO CALCULATES EQUILIBRIUM VALUES YSTAR USING
C
C               YSTAR=A*X
C
C      THESE ARE PASSED INTO PRINTY FOR PRINTING THE ITH
C      THESE ITH EQUATION VALUES
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE PRNT(YSTAR,YFIT,NTN,N,F,A,Q,P,A1)
      IMPLICIT REAL(A-H,O-Z)
      DIMENSION YSTAR(NOBS,NEQ),YFIT(NTN,NEQ),F(NEQ,NEQ),A(NEQ,NEX),
     1Q(NEQ,NSEAS),P(N),A1(NEQ,NEX)
      COMMON /MODEL/ NEQ,NEX,NOBS,NPR,NLPAR,NSEAS,NLOC,NTYPE,NDTS
      COMMON /DATA/ Y(120,6),X(120,15),XPR(60,15),WC(120,15),
     1CST(160,5),HT(160,6),U(120,6),S(6,6),BO(15,15,10)
      COMMON /REST/ RRLMDA,IRSTN(6,15),IREST,NREST,IRSTNF(6,6),
     1IRSTNT(6),NFRES,NTRES,IRSTNR(6,6),NRES
       DATA ZERO/0.0/,HALF/0.5/,ONE/1.0/,TWO/2.0/
      DATA DOTWO/0.2/,TEN/10.0/,HUNTH/100000.0/
      NK=0
      IF(NFRES.GT.0) THEN
      CALL TRANSA(F,NEQ,NEQ,NK,P,N,IRSTNF,6)
      NK=NK+NEQ*NEQ-NFRES
      ELSE
      CALL TRANS(F,NEQ,NEQ,NK,P,N)
      ENDIF
      IF(NDTS.GT.1)NK=NK+NEQ*NEX
      IF(IREST.GT.1) THEN
      CALL TRANSA(A,NEQ,NEX,NK,P,N,IRSTN,15)
      NK=NK+NEQ*NEX-NREST
      ELSE
      CALL TRANS(A,NEQ,NEX,NK,P,N)
```

```
      ENDIF
      DO 3 I=1,NEQ
      DO 3 J=1,NSEAS
      IF(J.EQ.NSEAS.AND.IRSTNT(I).EQ.ZERO.AND.NTRES.GT.0) THEN
      Q(I,J)=ZERO
      ELSE
      NK=NK+1
      Q(I,J)=P(NK)
      ENDIF
    3 CONTINUE
      IF(NTYPE.EQ.5) THEN
      CALL TRANS(A1,NEQ,NEX,NK,P,N)
      ENDIF
      NSTRT=2
      IF(NTYPE.EQ.4) NSTRT=3
      DO 7 I=1,NEQ
      DO 7 J=NSTRT,NOBS
      J1=J-1
      YFIT(J,I)=Y(J,I)-U(J,I)
      YO=ZERO
      DO 5 K=1,NEX
      YO=YO+A(I,K)*X(J,K)
      IF(NTYPE.EQ.5) YO=YO+A1(I,K)*X(J1,K)
    5 CONTINUE
      IF(NSEAS.GT.0)THEN
      DO 6 K=1,NSEAS
      IF(K.EQ.1.OR.K.EQ.5) YO=YO+Q(I,K)*CST(J,K)
    6 CONTINUE
      ENDIF
    7 YSTAR(J,I)=YO
      DO 10 I=1,NEQ
      YO=ZERO
      DO 9 K=1,NEQ
    9 YO=YO+F(I,K)*Y(NOBS,K)
   10 YFIT(NOBS+1,I)=YO+HT(NOBS+1,I)
      NO2=NOBS+2
      DO 14 J=NO2,NTN
      DO 14 I=1,NEQ
      YO=ZERO
      DO 11 K=1,NEQ
   11 YO=YO+F(I,K)*YFIT(J-1,K)
   14 YFIT(J,I)=YO+HT(J,I)
      DO 16 I=1,NEQ
      WRITE(2,1005)I
      CALL PRNTY(YSTAR,YFIT,I,NTN,NSTRT)
   16 CONTINUE
      RETURN
 1005 FORMAT(5X,'PLOT OF FITTED VALUES AND RESIDUALS FOR EQUATION:',
     1I2,//)
      END
      SUBROUTINE SETZER(XX,N1,N2,AN)
      IMPLICIT REAL (A-H,O-Z)
      REAL XX,AN
      DIMENSION XX(N1,N2)
       DATA ZERO/0.0/,HALF/0.5/,ONE/1.0/,TWO/2.0/
      DO 1 J=1,N2
      DO 1 I=1,N1
      IF(I.EQ.J) THEN
      XX(I,J)=AN
```

```
      ELSE
      XX(I,J)=ZERO
      ENDIF
    1 CONTINUE
      RETURN
      END
      SUBROUTINE TEST(VCOV,STLOGL,HESD,HESL,N,LH,RLOGL,C,CTD,XC,EP,
     1IERR)
      IMPLICIT REAL (A-H,O-Z)
      CHARACTER NAMEX(15)*5,NAMEY(6)*5,FORM(10)*8
      REAL VCOV(N,N),STLOGL,HESD(N),HESL(LH),RLOGL,CTD(N,N),C(N,N),
     1XC(N),SE(120),T(120),RRLOGL,T1(120)
      COMMON /BANDD/ VECBIN(36),VECD(90)
      COMMON /DATA/ Y(120,6),X(120,15),XPR(60,15),WC(120,15),
     1CST(160,5),HT(160,6),U(120,6),S(6,6),BO(15,15,10)
      COMMON /CHAR/ NAMEX,NAMEY,FORM
      COMMON /EXTRA/ ADFC
      COMMON /MODEL/ NEQ,NEX,NOBS,NPR,NLPAR,NSEAS,NLOC,NTYPE,NDTS
      COMMON /REST/ RRLMDA,IRSTN(6,15),IREST,NREST,IRSTNF(6,6),
     1IRSTNT(6),NFRES,NTRES,IRSTNR(6,6),NRES
       DATA ZERO/0.0/,HALF/0.5/,ONE/1.0/,TWO/2.0/,TWENTY/20.0/
      DATA TEN/10.0/
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C       CREATION OF ASSYMPTOTIC VARIANCE COVARIANCE MATRIX
C       H = L D L'
C       VCOV = H(INV) = L'(INV) D(INV) L(INV)
C       L IS REPRESENTED BY THE MATRIX C WHICH IS INVERTED BY BACK
C       SUBSTITUTION
C
C       HESD CONTAINS THE DIAGONAL ELEMENTS OF D AND HESL CONTAINS
C       THE N(N-)/2 LOWER TRIANGULAR ELEMENTS OF L EXCLUDING THE
C       LEADING DIAGONAL WHICH IS MADE UP OF 1'S . THIS FED INTO
C       HESL BY ROWS
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      WRITE(2,'(7F13.5)')(HESD(I),I=1,N)
      WRITE(2,299)
  299 FORMAT(/,5X,'HESD AND HESL',/)
      NN=N*(N-1)/2
      WRITE(2,'(7F13.5)')(HESL(I),I=1,NN)
      I1=0
      DO 2 J=1,N
      I1=I1+J
      I2=I1-1
      DO 2 I=1,N
      IF(I.LT.J)C(I,J)=ZERO
      IF(I.EQ.J)C(I,J)=ONE
      IF(I.GT.J) THEN
      TEM=ZERO
      II=I-1
      DO 1 NN=J,II
      I2=I2+1
    1 TEM=TEM-HESL(I2)*C(NN,J)
      C(I,J)=TEM
      I2=I2+J-1
      ENDIF
    2 CONTINUE
      DO 3 I=1,N
```

```
      DO 3 J=1,N
      IF(I.LT.J) CTD(J,I)=ZERO
      IF(I.EQ.J) CTD(J,I)=1/HESD(J)
      IF(I.GT.J) CTD(J,I)=C(I,J)/HESD(I)
    3 CONTINUE
      CALL MATMLT(CTD,C,VCOV,N,N,N,0)
      NN=N*(N-1)/2
C     CALL TESVCO(VCOV,HESL,HESD,CTD,C,N,LH,0)
      IF(IERR.GT.0) THEN
      CALL GHESS(N,CTD,C,XC,RLOGL,EP)
      CALL GJS(CTD,C,N)
C     CALL TESVCO(VCOV,HESL,HESD,CTD,C,N,LH,1)
      ENDIF
      DO 4 I=1,N
      SE(I)=(VCOV(I,I)/ADFC)**HALF
      HESD(I)=(C(I,I)/ADFC)**HALF
      T1(I)=(XC(I)/HESD(I))
    4 T(I)=(XC(I)/SE(I))
      RRLOGL=RLOGL
      NDF=NOBS-1
      IF(NTYPE.EQ.4) NDF=NOBS-2
      RLOGL=-HALF*(NDF)*(NEQ*2*LOG(2*3.1743)+ONE)
     1-(RRLOGL*ADFC)
      WRITE(2,290)RLOGL
      WRITE(2,295)((S(I,J),I=1,NEQ),J=1,NEQ)
      WRITE(2,300)
      WRITE(2,301)
      IF(IREST.EQ.0.OR.IREST.EQ.2) THEN
      WRITE(2,302)XC(N),SE(N),HESD(N),T(N),T1(N)
      ELSE
      WRITE(2,'(5X,'' RLMDA  '',F13.5)')RRLMDA
      ENDIF
      NO=0
      NO1=0
      DO 9 I=1,NEQ
      NCN=0
      WRITE(2,303)I,NAMEY(I)
      WRITE(2,304)
      IF(NFRES.GT.0)THEN
      CALL WRITR(6,6,NCN,N,I,NEQ,XC,SE,HESD,T,T1,IRSTNF)
      NCN=NCN+NEQ*NEQ-NFRES
      ELSE
      CALL WRIT(NEQ,N,NCN,NEQ,I,N4,XC,SE,HESD,T,T1)
      ENDIF
      WRITE(2,305)
      IF(NTYPE.EQ.1) THEN
      CALL WRIT(NEQ,N,NCN,NEQ,I,N4,XC,SE,HESD,T,T1)
      ELSE
      DO 5 J=1,NEQ
      NO=NO+1
    5 WRITE(2,405)VECBIN(NO)
      ENDIF
      WRITE(2,306)
      IF(NDTS.GT.1) THEN
      CALL WRIT(NEQ,N,NCN,NEX,I,N4,XC,SE,HESD,T,T1)
      ELSE
      DO 6 J=1,NEX
      NO1=NO1+1
    6 WRITE(2,405)VECD(NO1)
```

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      ENDIF
      WRITE(2,307)
      IF(IREST.GT.1) THEN
      CALL WRITR(6,15,NCN,N,I,NEX,XC,SE,HESD,T,T1,IRSTN)
      NCN=NCN+NEQ*NEX-NREST
      ELSE
      CALL WRIT(NEQ,N,NCN,NEX,I,N4,XC,SE,HESD,T,T1)
      ENDIF
      IF(NSEAS.NE.0) THEN
      WRITE(2,308)
      NS=NSEAS-1
      IF(NTRES.EQ.0)NS=NSEAS
      CALL WRIT(NEQ,N,NCN,NS,I,N4,XC,SE,HESD,T,T1)
      IF(NTRES.GT.0)THEN
      IF(IRSTNT(I).EQ.0)THEN
      WRITE(2,'(5X,''ZERO COEFF'')')
      ELSE
      WRITE(2,309)XC(NCN),SE(NCN),HESD(NCN),T(NCN),T1(NCN)
      ENDIF
      ENDIF
      ENDIF
      IF(NTYPE.EQ.3) THEN
      WRITE(2,310)
      NCN=NCN+1
      WRITE(2,309) XC(NCN),SE(NCN),HESD(NCN),T(NCN),T1(NCN)
      ENDIF
      IF(NTYPE.EQ.4) THEN
      WRITE(2,311)
      IF(NRES.EQ.0) THEN
      CALL WRIT(NEQ,N,NCN,NEQ,I,N4,XC,SE,HESD,T,T1)
      ELSE
      CALL WRITR(6,6,NCN,N,I,NEQ,XC,HESD,T,T1,IRSTNR)
      NCN=NCN+NEQ*NEQ-NRES
      ENDIF
      ENDIF
      IF(NTYPE.EQ.5) THEN
      WRITE(2,312)
      CALL WRIT(NEQ,N,NCN,NEX,I,N4,XC,SE,HESD,T,T1)
      ENDIF
    9 CONTINUE
      DO 14 I=1,N
      DO 12 J=1,N
      CNEW=C(I,J)/ADFC
      VNEW=VCOV(I,J)/ADFC
      C(I,J)=CNEW
   12 VCOV(I,J)=VNEW
   14 CONTINUE
      NLM=1
      IF(NTYPE.EQ.4) NLM=2
      CALL TESTLM(S,VCOV,NLM,N,NEQ,NOBS)
      IF(IERR.GT.0) CALL TESTLM(S,C,NLM,N,NEQ,NOBS)
      RETURN
  290 FORMAT(5X,'LOG LIKELIHOOD =',F13.5,/,5X,16(1H-),/)
  295 FORMAT(5X,'VARIANCE COVARIANCE MATRIX OF THE EQUATIONS',/,5X,
     143(1H-),/,6(5X,6F13.9,/))
  300 FORMAT(5X,'PARAMETER ESTIMATES BY EQUATION ',/,5X,31(1H-)//)
  301 FORMAT(16X,'COEFFICIENT   STANDARD ERROR  STANDARD ERROR ',
     1' T-STATISTICS  T1-STATISTICS',/,16X,71(1H-)/)
  302 FORMAT(5X,'LMDA',5(2X,F13.5))
```

```
303 FORMAT(5X,'EQUATION ',I2,' DEPENDENT VARIABLE NAME ',A5,/,5X,
   111(1H-))
304 FORMAT(5X,'F MATRIX')
305 FORMAT(5X,'B-1 MATRIX')
306 FORMAT(5X,'D MATRIX')
307 FORMAT(5X,'A MATRIX')
308 FORMAT(5X,'SEASONALS')
309 FORMAT(9X,5(2X,F13.5))
310 FORMAT(5X,'TERMINAL CONDITIONS')
311 FORMAT(5X,'AR(1) MATRIX')
312 FORMAT(5X,'A1 MATRIX')
405 FORMAT(11X,F13.5)
    END
    SUBROUTINE SCALC(N,N1,N2,N4,V1,V2,SS)
    IMPLICIT REAL (A-H,O-Z)
    REAL SS(NEQ,NEQ),V1(120,6),S,V2(120,6)
    COMMON /MODEL/ NEQ,NEX,NOBS,NPR,NLPAR,NSEAS,NLOC,NTYPE,NDTS
     DATA ZERO/0.0/,HALF/0.5/,ONE/1.0/,TWO/2.0/
    N3=NOBS-1
    IF(NTYPE.EQ.4) N3=NOBS-2
    DO 2 I=1,NEQ
    DO 2 J=1,NEQ
    IF(J.GE.I.OR.N.LT.1) THEN
    S=ZERO
    DO 1 K=N1,N2
    N5=K+N4
  1 S=S+(V1(K,I)*V2(N5,J)/(FLOAT(N3)))
    SS(I,J)=S
    ELSE
    SS(I,J)=SS(J,I)
    ENDIF
  2 CONTINUE
    RETURN
    END
    SUBROUTINE TESTLM(S,VCOV,NLM,N,NEQ,NOBS)
    IMPLICIT REAL (A-H,O-Z)
    REAL VCOV(N,N),SS(7,6),S(6,6),SINV(6,6)
    IA=NEQ+1
    CALL LM(SS,VCOV,SINV,NEQ,NLM,NOBS,N,IA,S)
    RETURN
    END
    SUBROUTINE LM(SS,VCOV,SINV,NEQ,NLM,NOBS,N,IA,S)
    IMPLICIT REAL (A-H,O-Z)
    REAL S(6,6),VCOV(N,N),TEMP(6,6),TEMP1(6,6),SINV(NEQ,NEQ),Z(7),
   1S2(6,6),SS(IA,NEQ),TLM3,TLM1
    IFAIL=0
    DO 3 I=1,NEQ
    DO 3 J=1,NEQ
  3 SS(I,J)=S(I,J)
    CALL F01ABF(SS,IA,NEQ,SINV,NEQ,Z,IFAIL)
    IF(IFAIL.GT.0) WRITE(2,100)IFAIL
    DO 5 J=2,NEQ
    II=J-1
    DO 5 I=1,II
  5 SINV(I,J)=SINV(J,I)
    CALL LM1(SINV,S2,VCOV,TEMP1,SS,TEMP,TLM3,1,1,N,IA)
    WRITE(2,104)TLM3
    TLM1=ZERO
    CALL LM1(SINV,S2,VCOV,TEMP1,SS,TEMP,TLM1,2,0,N,IA)
```

```
      TLM3=TLM3+TLM1
      WRITE(2,106)TLM3
      DO 7 I=3,4
      TLM1=ZERO
      CALL LM1(SINV,S2,VCOV,TEMP1,SS,TEMP,TLM1,I,0,N,IA)
    7 TLM3=TLM3+TLM1
      WRITE(2,108)TLM3
      TLM1=ZERO
      CALL LM1(SINV,S2,VCOV,TEMP1,SS,TEMP,TLM1,5,0,N,IA)
      TLM3=TLM3+TLM1
      WRITE(2,110)TLM3
      RETURN
  100 FORMAT(2X,'INVERSION ROUTINE FAIL WITH IFAIL= ',I2,/,2X,
     1'1 MEANS NOT PDF AND 2 MEANS ILL CONDITIONED MATRIX')
  104 FORMAT(5X,'LM TEST FOR 1ST ORDER SERIAL CORRELATION =',F8.5,/)
  106 FORMAT(5X,'LM TEST FOR 2ND ORDER SERIAL CORRELATION =',F8.5,/)
  108 FORMAT(5X,'LM TEST FOR 4TH ORDER SERIAL CORRELATION =',F8.5,/)
  110 FORMAT(5X,'LM TEST FOR 5TH ORDER SERIAL CORRELATION =',F8.5,/)
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C          SUBROUTINE LM1 CALCULATES THE MULTIPLE EQUATION
C          VERSION OF THE BOX PIERCE STATISTIC BY SETTING
C          NLM > 0 IT TAKES ACCOUNT OF LAGGED DEPENDENT
C          VARIABLES IN THE MODEL IN CALCULATING THE RELEVANT
C          LM TEST
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE LM1(SINV,S2,VCOV,TEMP1,SS,TEMP,TLM,NLM,LDEP,N,IA)
      IMPLICIT REAL (A-H,O-Z)
      REAL S2(NEQ,NEQ),S3(6,6),SINV(NEQ,NEQ),VCOV(N,N),
     1TEMP1(NEQ,NEQ),TEMP(NEQ,NEQ),SS(IA,NEQ),VEC(36),VEC1(36),TLM,TLM1
      COMMON /MODEL/ NEQ,NEX,NOBS,NPR,NLPAR,NSEAS,NLOC,NTYPE,NDTS
      COMMON /DATA/ Y(120,6),X(120,15),XPR(60,15),WC(120,15),
     1CST(160,5),HT(160,6),U(120,6),S(6,6),BO(15,15,10)
       DATA ZERO/0.0/,HALF/0.5/,ONE/1.0/,TWO/2.0/
      NDF=1
      IF(NTYPE.EQ.4) THEN
      NDF=2
      NLM=NLM+1
      ENDIF
      NLM1=NLM+2
      N4=2-NLM1
      CALL SCALC(-1,NLM1,NOBS,N4,U,U,S2)
      CALL MATMLT(SINV,S2,TEMP,NEQ,NEQ,NEQ,0)
      CALL MATMLT(TEMP,SINV,TEMP1,NEQ,NEQ,NEQ,0)
      TLM=ZERO
      DO 6 I=1,NEQ
      TLM1=ZERO
      DO 5 J=1,NEQ
    5 TLM1=TLM1+TEMP1(I,J)*S2(I,J)
    6 TLM=TLM+TLM1
      TLM=FLOAT((NOBS-NDF))*TLM
      TLM1=ZERO
      IF(LDEP.EQ.1) THEN
      NEQ2=NEQ**2
      CALL LMLAG(SINV,TEMP1,S2,S3,SS,VCOV,TEMP,VEC,VEC1,IA,NEQ2,TLM1,
     1NLM,N,NDF)
      TLM=TLM+TLM1
```

```
      ENDIF
      RETURN
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C        SUBROUTINE LMLAG ADJUSTS THE LM TEST FOR THE EXISTANCE
C        OF LAGGED DEPENDENT VARIABLES
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE LMLAG(SINV,SY,S2,S3,SS,VCOV,TEMP,VEC,VEC1,IA,NEQ2,TLM1,
     1NLM,N,NDF)
      IMPLICIT REAL (A-H,O-Z)
      REAL S2(NEQ,NEQ),S3(NEQ,NEQ),SY(NEQ,NEQ),SINV(NEQ,NEQ),
     1TEMP(NEQ,NEQ),SS(IA,NEQ),VEC(NEQ2),VEC1(NEQ2),Z(7),TLM2,TLM,TLM1,
     1VCOV(N,N)
      COMMON /DATA/ Y(120,6),X(120,15),XPR(60,15),WC(120,15),
     1CST(160,5),HT(160,6),U(120,6),S(6,6),BO(15,15,10)
      COMMON /MODEL/ NEQ,NEX,NOBS
       DATA ZERO/0.0/,HALF/0.5/,ONE/1.0/,TWO/2.0/
      N1=2
      IF(NDF.EQ.2) THEN
      NLM=NLM+1
      N1=N1+1
      ENDIF
      N2=NLM+1
      CALL SCALC(1,N1,N2,0,U,U,TEMP)
      DO 2 I=1,NEQ
      DO 2 J=1,NEQ
    2 SS(I,J)=S(I,J)-TEMP(I,J)
      IFAIL=0
      CALL F01ABF(SS,IA,NEQ,S3,NEQ,Z,IFAIL)
      IF(IFAIL.GT.0) WRITE(2,100)IFAIL
      DO 4 J=2,NEQ
      II=J-1
      DO 4 I=1,II
    4 S3(I,J)=S3(J,I)
      NLM1=NLM+2
      CALL SCALC(0,NLM1,NOBS,0,U,Y,SY)
      CALL MATMLT(S3,SY,TEMP,NEQ,NEQ,NEQ,0)
      II=0
      DO 8 L=1,NEQ
      DO 8 I=1,NEQ
      TLM1=ZERO
      DO 7 J=1,NEQ
      TLM2=ZERO
      DO 6 K=1,NEQ
    6 TLM2=TLM2+S2(J,K)*TEMP(K,I)
    7 TLM1=TLM1+SINV(J,L)*TLM2
      II=II+1
    8 VEC(II)=TLM1
      DO 11 J=1,NEQ2
      TLM2=ZERO
      DO 10 K=1,NEQ2
   10 TLM2=TLM2+VEC(K)*VCOV(K,J)
   11 VEC1(J)=TLM2
      TLM1=ZERO
      DO 12 I=1,NEQ2
   12 TLM1=TLM1+VEC(I)*VEC1(I)
      TLM1=FLOAT(NOBS-NDF)*TLM1
```

```
      RETURN
  100 FORMAT(2X,'INVERSION ROUTINE FAIL WITH IFAIL= ',I2,/,2X,
     1'1 MEANS NOT PDF AND 2 MEANS ILL CONDITIONED MATRIX')
      END
      SUBROUTINE TRANS(XX,N1,N2,K,NC,N)
      IMPLICIT REAL (A-H,O-Z)
      REAL XX,NC
      DIMENSION XX(N1,N2),NC(N)
      DO 1 I=1,N1
      DO 1 J=1,N2
      K=K+1
    1 XX(I,J)=NC(K)
      RETURN
      END
      SUBROUTINE TRANSA(XX,N1,N2,K,XC,N,IRE,N3)
      IMPLICIT REAL (A-H,O-Z)
      REAL XX(N1,N2),XC(N)
      INTEGER IRE(6,N3)
       DATA ZERO/0.0/,HALF/0.5/,ONE/1.0/,TWO/2.0/
      DO 1 I=1,N1
      DO 1 J=1,N2
      IF(IRE(I,J).LT.0) THEN
      IR=K-IRE(I,J)
      XX(I,J)=-XC(IR)
      ENDIF
      IF(IRE(I,J).EQ.0) XX(I,J)=ZERO
      IF(IRE(I,J).GT.0) THEN
      IF(IRE(I,J).EQ.99) THEN
      XX(I,J)=ONE
      ELSE
      IR=IRE(I,J)+K
      XX(I,J)=XC(IR)
      ENDIF
      ENDIF
    1 CONTINUE
      RETURN
      END
      SUBROUTINE VEC(INO,NEQ,NEX,VECTOR,MATRIX)
      IMPLICIT REAL (A-H,O-Z)
      REAL VECTOR(*);MATRIX(NEQ,NEX)
      DO 1 I=1,NEQ
      DO 1 J=1,NEX
      INO=INO+1
    1 VECTOR(INO)=MATRIX(I,J)
      RETURN
      END
      SUBROUTINE WRIT(NEQ,N,N1,N2,N3,N4,XC,SE,HESD,T,T1)
      IMPLICIT REAL (A-H,O-Z)
      REAL HESD(N),XC(N),SE(N),T(N),T1(N)
      N1=N2*(N3-1)+1+N1
      N4=N1+N2-1
      DO 1 I=N1,N4
    1 WRITE(2,1000)XC(I),SE(I),HESD(I),T(I),T1(I)
      N1=N4+N2*(NEQ-N3)
      RETURN
 1000 FORMAT(9X,5(2X,F13.5))
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
```

```
C     SUBROUTINE WRITR PRINTS OUT THE PARAMETERS WHICH ARE
C     RESTRICTED. THESE INCLUDE F A AND THE LAST COLUMN OF
C     Q IF THEIR IS A TIME TREND
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE WRITR(N1,N2,NCN,N,I,N4,XC,SE,HESD,T,T1,IRSTN)
      IMPLICIT REAL(A-H,O-Z)
      DIMENSION XC(N),SE(N),T(N),T1(N),HESD(N),IRSTN(N1,N2)
      DO 7 J=1,N4
      IF(IRSTN(I,J).LT.0) THEN
      IR=NCN-IRSTN(I,J)
      TEM=-XC(IR)
      WRITE(2,309)TEM,SE(IR),HESD(IR),T(IR),T1(IR)
      ENDIF
      IF(IRSTN(I,J).EQ.0) WRITE(2,'(5X,'' COEFF ZERO '')')
      IF(IRSTN(I,J).EQ.99) WRITE(2,'(5X,''COEFF ONE '')')
      IF(IRSTN(I,J).GT.0) THEN
      IR=NCN+IRSTN(I,J)
      WRITE(2,309)XC(IR),SE(IR),HESD(IR),T(IR),T1(IR)
      ENDIF
    7 CONTINUE
      RETURN
  309 FORMAT(9X,5(2X,F13.5))
      END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      SUBROUTINE TESVCOV CHECKS TO SEE IF BOTH ESTIMATES OF
C      OF THE VARIANCE MATRIX GIVE IDENTITY WHEN MULTIPLIED
C      BY THE HESSIAN
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      SUBROUTINE TESVCO(VCOV,HESL,HESD,CTD,C,N,LH,IMET)
      IMPLICIT REAL (A-H,O-Z)
      REAL CTD(N,N),C(N,N),VCOV(N,N),HESD(N),HESL(LH)
       DATA ZERO/0.0/,HALF/0.5/,ONE/1.0/,TWO/2.0/
      IF(IMET.LT.1) THEN
      I2=0
      DO 3 I=1,N
      II=I
      DO 1 J=1,II
      I2=I2+1
    1 C(I,J)=HESL(I2)
      I2=I2-1
      DO 2 J=II,N
    2 C(I,J)=ZERO
    3 C(I,I)=ONE
      CALL MATMLT(VCOV,C,CTD,N,N,N,0)
      DO 6 I=1,N
      II=I
      DO 4 J=II,N
    4 C(I,J)=C(J,I)*HESD(I)
      DO 5 J=1,II
    5 C(I,J)=ZERO
    6 C(I,I)=HESD(I)
      I2=0
      ENDIF
      DO 8 I=1,N
      HESD(I)=VCOV(I,I)
      II=I
```

```
      DO 7 J=1,II
      I2=I2+1
    7 HESL(I2)=VCOV(I,J)
    8 I2=I2-1
      CALL MATMLT(CTD,C,VCOV,N,N,N,0)
      DO 10 I=1,N
   10 WRITE(2,100)(VCOV(I,J),J=1,N)
  100 FORMAT(5X,(6G14.8,/))
      I2=0
      DO 12 I=1,N
      II=I
      DO 11 J=1,II
      I2=I2+1
      VCOV(I,J)=HESL(I2)
   11 VCOV(J,I)=VCOV(I,J)
      I2=I2-1
   12 VCOV(I,I)=HESD(I)
      RETURN
      END
```

Appendix 6.B Data Definitions

The un-generated data come from three sources: C.S.O. Economic Trends, International Financial Statistics and a series generated by the Centre for Labour Economics. The predictions and one step ahead forecast errors were generated using unrestricted vector AR processes and the initial estimates using an instrumental variables technique. The series for output, employment, stocks and vacancies came from the C.S.O.'s Economic Trends. The output series was the index for industrial production for British manufacturing industry. The series for employment statistics and vacancies were taken as deviations from their logarithmic means. The employment series was employment in British manufacturing, vacancies were the series for total vacancies notified and remaining unfilled, stocks were the value of physical increase of stocks; 2000 was added to this series so that it remained positive. The price series was the series for industrial output prices taken from International Financial Statistics. The wage series was an index generated by the Centre for Labour Economics; this series took account of hours worked, assuming a 45-hour week for more detail see S Wadhwani (1982) and J S Symons (1981).

The prediction series were generated using unrestricted VARs, over the period 1959(IV) to 1979(IV). General models were estimated and tested downwards to derive parsimonious final forms. The models were estimated recursively for the period from 1962(IV) to 1980(I) to generate the one step ahead forecast errors for the exogenous variables; wages, prices, Δstocks and vacancies. The parameters from the 1979(IV) models were used to derive the C_r parameters used in the D matrix in section 3 of Chapter 4. These are the data used to estimate the models in Chapter 4.

Data for the output and employment models in logarithms

o	l
1.24400	1.04400
1.31500	1.04300
1.29600	1.03200
1.34500	1.02800
1.28700	1.03200
1.39400	1.04000
1.42300	1.03800
1.43600	1.04200
1.35200	1.04900
1.46400	1.05900
1.46000	1.05500
1.47300	1.05500
1.37900	1.06100
1.47900	1.06700
1.49300	1.06000
1.49800	1.05800
1.41200	1.06200
1.46300	1.05600
1.47700	1.03800
1.49500	1.03000
1.40900	1.02700
1.51100	1.02600
1.53600	1.01700
1.55500	1.01600
1.48200	1.02400
1.57900	1.03200

1.57800	1.02900
1.60200	1.03100
1.51600	1.03700
1.60900	1.04300
1.59100	1.03500
1.58900	1.03100
1.52400	1.03100
1.61800	1.03100
1.59600	1.01800
1.58900	1.00000
1.51500	0.99300
1.60000	0.98300
1.56600	0.96600
1.60900	0.96000
1.55100	0.96400
1.67500	0.96700
1.69800	0.96400
1.67900	0.96500
1.63400	0.97200
1.71600	0.97900
1.67000	0.97100
1.68800	0.97000
1.62800	0.97700
1.67800	0.97200
1.65900	0.95100
1.60500	0.92900
1.53200	0.91700
1.62000	0.91000
1.62200	0.89600
1.61700	0.89300

1.56600	0.89600
1.65300	0.90000
1.65900	0.89500
1.61200	0.89400
1.56700	0.90000
1.63900	0.90000
1.64600	0.89300
1.65700	0.88900
1.60100	0.89300
1.65900	0.89200
1.65500	0.88300
1.67600	0.88000
1.58600	0.88300
1.66700	0.87500

Exogenous variables

Δi	v	p	w
1.65728	0.94876	-0.89160	-1.32583
1.53796	0.85203	-0.88916	-1.31635
1.61431	0.84419	-0.88673	-1.30769
1.70210	0.91998	-0.88673	-1.29909
1.62364	0.96284	-0.88431	-1.28696
1.67415	1.14166	-0.87707	-1.27487
1.73762	1.29832	-0.86988	-1.25998
1.83716	1.36129	-0.85567	-1.23125
1.79811	1.43372	-0.84863	-1.22242
1.81440	1.49717	-0.84397	-1.20223
1.71423	1.52146	-0.83241	-1.16693
1.75577	1.60212	-0.82098	-1.15686

1.79606	1.60212	-0.81193	-1.13323
1.65634	1.59842	-0.80968	-1.10242
1.67786	1.62040	-0.80073	-1.08176
1.72091	1.63121	-0.79186	-1.06160
1.79811	1.55683	-0.78526	-1.08594
1.52078	1.32301	-0.78746	-1.04398
1.67322	1.20949	-0.78526	-1.04294
1.71512	1.11799	-0.78526	-1.03198
1.68202	1.11199	-0.77871	-1.03357
1.59438	1.18739	-0.76787	-1.00085
1.51425	1.19850	-0.75502	-0.97653
1.79893	1.19850	-0.74234	-0.96761
1.82445	1.23644	-0.73814	-0.97555
1.69803	1.30827	-0.72981	-0.93700
1.70120	1.30330	-0.71539	-0.90342
1.79194	1.30330	-0.70725	-0.89292
1.72356	1.29330	-0.69917	-0.89016
1.66200	1.28320	-0.68717	-0.85649
1.52510	1.26786	-0.66553	-0.80521
1.78945	1.24175	-0.64817	-0.76688
1.79647	1.22036	-0.63111	-0.74735
1.65017	1.15906	-0.61065	-0.71086
1.62071	1.01064	-0.57448	-0.65843
1.63095	0.84419	-0.55165	-0.63500
1.65302	0.78749	-0.53614	-0.61927
1.63530	0.77068	-0.53273	-0.59267
1.45876	0.82028	-0.50088	-0.53514
1.67369	0.89035	-0.48776	-0.51422
1.59890	0.99043	-0.47000	-0.50365
1.64922	1.17048	-0.44785	-0.46225

1.83281	1.44242	-0.43541	-0.44014
1.95893	1.68358	-0.43078	-0.40741
1.90286	1.80814	-0.39750	-0.39393
1.78114	1.87774	-0.35810	-0.34284
1.47250	1.66643	-0.28502	-0.32539
1.94520	1.74939	-0.21691	-0.28760
1.95928	1.71703	-0.17793	-0.23514
1.57968	1.57973	-0.12783	-0.15698
1.23201	1.38450	-0.06507	-0.07747
1.39142	1.07517	-0.01511	-0.02934
1.47364	0.87520	0.02078	0.02512
1.49443	0.72739	0.05543	0.07512
1.50934	0.71850	0.09803	0.10769
1.60738	0.73620	0.13540	0.14195
1.75620	0.82028	0.17563	0.14817
1.81763	0.87520	0.22474	0.17992
1.79523	0.97673	0.28518	0.19597
1.82764	1.01728	0.33361	0.21104
1.65444	1.03044	0.36325	0.20857
1.75534	1.08760	0.37707	0.25833
1.69803	1.22036	0.40012	0.29497
1.79194	1.30827	0.41739	0.34918
1.76726	1.37064	0.43696	0.35495
1.67740	1.45104	0.45298	0.40298
1.90692	1.44674	0.47995	0.42228
1.75620	1.52545	0.51879	0.48526
1.90175	1.51343	0.56758	0.50749
1.61481	1.43372	0.59774	0.55912

Predictions for 51 future periods from 80q1 to 92q2

Δi	v	p	w
1.5233345	1.2980938	.6420898	.6003236
1.6808429	1.2047858	.6761237	.6504613
1.5889025	1.0778918	.7046930	.6857339
1.4994731	.9774995	.7344052	.7437041
1.4972076	.9339538	.7606076	.7788034
1.6261125	.8849554	.7876015	.8100430
1.7033944	.9109278	.8124157	.8178712
1.6802158	.9714270	.8384063	.8506738
1.7154078	1.0413594	.8760573	.8746650
1.7948208	1.1355982	.9085183	.8937582
1.8223944	1.2175565	.9400029	.8981110
1.6970630	1.2876554	.9718125	.9310237
1.7122431	1.3496499	1.0099611	.9674275
1.7804184	1.3990688	1.0426002	1.0004580
1.7560005	1.4248190	1.0726681	1.0220343
1.6587582	1.4269371	1.1000851	1.0650420
1.6350203	1.4038224	1.1368110	1.1045111
1.7443080	1.3776374	1.1720649	1.1426789
1.7588916	1.3415847	1.2050183	1.1608872
1.6378655	1.2915955	1.2391529	1.2019992
1.6131792	1.2303243	1.2815765	1.2372288
1.7047944	1.1801200	1.3197771	1.2686878
1.7003984	1.1296945	1.3555369	1.2862258
1.6143684	1.1018348	1.3882594	1.3253570
1.5878897	1.0803237	1.4272461	1.3599460
1.6897659	1.0811367	1.4609348	1.3902653
1.7230287	1.0987072	1.4904929	1.4034327

1.6595030	1.1304283	1.5185632	1.4389184
1.6658554	1.1685920	1.5553303	1.4691217
1.7767706	1.2175436	1.5880128	1.4949948
1.7851315	1.2522454	1.6193467	1.5045505
1.7078247	1.2821493	1.6493584	1.5380656
1.6853480	1.2938175	1.6879213	1.5683324
1.7675719	1.3011680	1.7224203	1.5978889
1.7486973	1.2895255	1.7537076	1.6121244
1.6500735	1.2684603	1.7821819	1.6505926
1.6206317	1.2359781	1.8181703	1.6850663
1.7195010	1.2142262	1.8490120	1.7173384
1.7140756	1.1879020	1.8773116	1.7325262
1.6348510	1.1685987	1.9033778	1.7703251
1.6236839	1.1494555	1.9375749	1.8015971
1.7336245	1.1473908	1.9675426	1.8301115
1.7381406	1.1464348	1.9956019	1.8415473
1.6605520	1.1530671	2.0214434	1.8765010
1.6407795	1.1559234	2.0554211	1.9061935
1.7469535	1.1722674	2.0841913	1.9341393
1.7431493	1.1833663	2.1104410	1.9457207
1.6627440	1.1971278	2.1341255	1.9819714
1.6425872	1.2034674	2.1657391	2.0128019
1.7487402	1.2182217	2.1924050	2.0419197
1.7456684	1.2241049	2.2171090	2.0539756

One step ahead forecast errors for 63q1 to 80q1

$\varepsilon_{\Delta i}$	ε_v	ε_p	ε_w
-.1074748	.0200527	-.0016097	-.0080406
-.0406501	.0038699	-.0043580	.0001839
.0283296	-.0437200	-.0052444	-.0006037
-.0271608	-.0699275	-.0031523	.0030652
.0209148	.0206738	.0044225	.0056514
.0079970	-.0064300	.0045683	-.0022844
-.0041508	-.0081375	.0041279	.0015609
.0983910	-.0055759	.0003846	-.0011400
.0930972	-.0077625	-.0030084	-.0034120
-.0261221	-.0121544	.0012317	.0040415
.0113274	.0199692	-.0013761	-.0072713
.1030089	.0397769	-.0021798	.0169455
.0183241	.0352650	-.0061837	.0041606
-.0196740	.0469371	-.0002975	-.0026979
-.0141638	.0085945	.0007647	.0070550
.1116294	.0878741	-.0029845	-.0274702
-.0794642	-.0678324	-.0054199	.0180654
.0824694	-.0742514	-.0052443	.0046234
.0131454	.0324350	-.0018546	.0022081
.0508117	.0478489	.0023481	.0033456
.0736289	.1073453	.0065766	.0208505
-.1116240	-.1346229	.0033970	.0013951
.0456909	.0020130	.0000338	.0009424
.0485348	.0062310	-.0022672	-.0196935
.0385409	.0182635	-.0000924	-.0040098
.0172539	-.0962926	.0037767	.0054248

.0788855	-.0200663	-.0037917	-.0113475
-.0513600	-.0086186	.0042226	.0006555
.0296569	-.0883730	.0082297	.0103186
-.0847945	-.0130064	.0080196	.0145347
.0174594	-.0205135	.0060445	.0199747
.0210127	.0104070	.0065249	.0110691
-.0008435	-.0690384	.0062176	-.0106544
.0190030	-.1026225	.0104220	.0074709
-.0465984	-.0661487	-.0081976	-.0158911
-.0665347	.0508398	-.0030560	.0030452
-.0138625	-.0623448	-.0042571	.0016197
-.0876647	.0595873	.0140852	.0108354
-.0027955	.0045886	-.0065939	.0005138
-.1107861	.0306863	.0039980	-.0016612
-.1057978	.0461563	.0043950	.0144781
.0661261	.0710177	-.0146018	-.0109096
.0435789	.0143333	-.0077016	-.0111660
.0542179	-.0350571	.0136703	.0041366
-.0286741	-.0197681	.0132678	.0180188
-.1703661	-.1631977	.0181247	-.0083458
.1669667	.1841233	.0069491	.0128958
.0565331	.0385755	-.0042748	.0204161
-.0748233	-.0206987	.0039773	.0004192
-.0318076	-.0063498	-.0020832	.0101262
-.1324047	-.0643090	.0043704	.0123871
.0420566	.0039977	-.0065814	.0157709
-.0471160	-.1349717	.0083330	-.0142080
-.0788013	.0555945	-.0096952	-.0084460
-.0118588	.0463356	.0015788	.0187110
.0055709	.0394845	-.0002619	-.0027995

.0898412	-.0590099	-.0003119	.0034395
.0097025	.0181037	-.0095343	-.0063467
-.0318881	-.0845901	-.0084016	-.0065692
-.0492864	.0186935	-.0154615	-.0224353
.0389393	.0348603	-.0111075	.0126170
.0256613	.0964117	.0015517	-.0028035
.0856710	-.0235906	-.0089092	.0125487
.1544580	.0132063	-.0015911	-.0033465
.0412267	.0740345	-.0028205	.0032009
.0797465	-.0244758	-.0015999	-.0087730
-.1563542	.0692327	.0072512	.0208705
.1130612	.0161270	.0018719	.0002630
-.0208115	-.0007135	-.0089524	-.0055350
-.1045748	-.0331254	.0063227	.0043489

The VMA Inverse of the AR parameters estimated in Chapter 3 computed for 15 lags back.

MA parameters of the Inventory equation.

	0.0000000000000E+000	-1.2514980000000E-001	-3.1765101342000E-001
	-3.2646452682350E-002	1.0053285475554E-001	-1.8625840971845E-001
Δi	-2.3976516055017E-001	-1.0824541141211E-001	-2.2069477792570E-003
	2.2231332743825E-001	9.4339491013653E-002	1.3231999222392E-001
	6.9041119789058E-002	1.7640793001392E-001	-1.3215072234279E-002

	5.1400000000000E-001	2.4448220000000E-001	2.0419718358000E-001
	4.7406884478336E-001	1.9033950499763E-001	6.0497918792057E-002
v	-3.3730172674992E-001	-6.3450125012490E-001	-4.3228210848726E-001
	-5.1648281182742E-001	-3.6146511471744E-001	-2.5495904791473E-001
	-1.2375046870575E-001	1.2073158183619E-001	3.4401705278002E-001

	1.42000000000000	3.7184140000000E-001	-1.4700681504000E-001
	-9.0402642657385E-001	-1.72442142693320	-4.54143309073680
p	3.3436803451184E-001	-6.1346697713902E-001	1.43632473107330
	1.35368278251790	6.8495374018799E-001	9.1244309189129E-001
	1.39258438329700	-3.5917198637737E-001	9.4932367541358E-002

	-1.43000000000000	-1.76983020000000	-2.2060808066000E-001
	2.3043091693053E-001	-4.8717021934422E-002	1.15749449018990
w	-1.0689292929222E-001	-7.6682552156615E-001	-1.5067834105956E-001
	3.9082473478804E-001	9.2877675590360E-001	-1.16994683526090
	-9.1538952455004E-001	1.9051788766211E-001	1.12075396092610

MA parameters for the Vacancies equation

	-9.7700000000000E-002	-1.8276523000000E-001	-3.1914020426500E-001
	-4.7392729836707E-001	-4.0922845296995E-001	-3.8027364922477E-001
Δi	-4.4641933403057E-001	-4.3894339106107E-001	-4.0659666612778E-001
	-2.2850233751810E-001	-8.7077800704250E-002	1.3791920673694E-002
	9.8825580461375E-002	1.5421569050559E-001	1.8908502267012E-001
	1.49830000000000	1.81845757000000	1.89675161189300
	1.72772265179820	1.32791762235970	8.6196873706096E-001
v	3.0855151945645E-001	-3.1989746254630E-001	-7.5894808962767E-001
	-1.09979349885140	-1.25946543611990	-1.23481999640430
	-1.13626503070700	-8.6676791040784E-001	-5.0543996628760E-001
	-1.0640000000000E-001	-7.5813451000000E-001	-1.29085566648000
	-2.40840094213420	-3.48770853116060	-4.50833792903650
p	-4.58218331301940	-5.05938728367990	-4.82432722713430
	-4.40332322078560	-4.87283020731310	-4.77211606441480
	-4.14430362691100	-3.74783014275530	-2.74095592620630
	-6.9430000000000E-001	-7.9543549000000E-001	-7.5462530607100E-001
	-3.2757371466812E-001	-5.6026467091979E-001	-4.9019342308045E-001
w	-5.9599316111245E-001	-1.14062880794230	-1.09316443957410
	-1.37079097246100	-8.4685735770871E-001	-4.8034965088913E-001
	-3.8000137349206E-001	8.8637805670867E-002	8.2013414053780E-001

MA parameters for the Output Price equation

	0.0000000000000E+000	2.1129940000000E-002	3.2515937616000E-002
	3.5985763408269E-002	2.8385713203580E-002	1.5273734848366E-002
Δi	1.2069811151307E-002	-2.5551440063643E-002	-3.2605600870051E-002
	-3.6248451299001E-002	-3.4533953980145E-002	-3.6024166132161E-002
	-3.8635980084262E-002	-3.9385282846486E-002	-2.9285647869892E-002
	-8.2000000000000E-003	3.1337040000000E-002	8.3344327886000E-002
	1.2761620169335E-001	1.7238993275455E-001	2.2179566086094E-001
v	2.6389151318858E-001	3.1197244040444E-001	3.2324010338656E-001
	3.2666282515369E-001	3.2904250208076E-001	3.1293859686547E-001
	3.0140220926118E-001	2.9441868538869E-001	2.8716333889621E-001
	1.70450000000000	2.19450833000000	2.55158874766700
	2.76881845988880	2.89273273857710	3.11457697178570
p	3.07764625503620	3.02814975634250	3.01397303948680
	3.00826752532400	3.03062238266740	3.10465176628470
	3.04038686113820	3.06457360504710	2.87516128116420
	1.2000000000000E-002	3.8147260000000E-002	8.5104988880000E-003
	-3.4692312214126E-002	-4.8671092390624E-002	-1.9717659359955E-002
w	-6.4158091464568E-003	3.6655753205810E-002	1.1640320286375E-001
	1.5243196705628E-001	1.3633194707881E-001	1.0523177988310E-001
	1.2500260979537E-001	9.5837296883110E-002	9.7485142569482E-002

MA parameters for the wage equation

	5.2400000000000E-002	5.2400000000000E-002	3.7517365402000E-002
	4.7697367623093E-002	7.0902049888077E-002	1.0337971678592E-001
Δi	1.1012779822012E-001	9.0867064815238E-002	8.4547937964914E-002
	8.7481983782055E-002	7.0252492871607E-002	5.8097501510809E-002
	3.5520212978476E-002	4.2383571311621E-002	4.4842242348602E-002
	0.0000000000000E+000	1.1542940000000E-002	2.5397651432000E-002
	1.6026928112438E-003	3.4517113056900E-002	5.2961520570049E-002
v	7.2890180139729E-002	9.0973737776476E-002	1.0154055984832E-001
	1.0366305754362E-001	1.1074604156467E-001	8.0525254758717E-002
	5.2038150656641E-002	5.1502430249560E-003	-4.2472943771564E-002
	4.0130000000000E-001	3.5512385000000E-001	9.6918947218900E-001
	1.27724137867270	1.72520365975460	1.98378159502990
p	2.09137017458770	1.76092235874900	1.62022777594770
	1.15861804446400	9.0279317359622E-001	4.1741756023198E-001
	6.5498028385177E-002	-1.5529780611283E-001	-1.8997240557726E-001
	1.00000000000000	1.14388360000000	1.05309882295800
	1.32241119716820	1.55651707404700	1.35811989797070
w	1.22392288048020	1.12017382794970	1.07603929000270
	1.04972285686790	9.2342088493451E-001	9.4081823390516E-001
	9.3530476275553E-001	8.9787133523443E-001	8.9001434085094E-001

If we assume that the following VAR is used to generate the data, then:

$$B(L)s_t = \epsilon_t \qquad \text{and} \qquad s_t = C(L)\epsilon_t$$

Then when the VAR parameters are invertible into a VMA:

$$C(L) = (B(L))^{-1} \qquad \text{so that} \qquad B(L)C(L) = I$$

We can solve for C_i using the following recursive formula:

$$C_i = \sum_{s=1}^{k} B_s C_{i-s} \qquad \text{where} \quad C_o = I \quad \text{and } C_j = 0 \quad \text{for } j < 0$$

When there are unit roots or roots within the unit circle, then we need to take account of that. The Smith-Mcmillan-Yoo transformation can be used to invert a VAR with unit roots, but in practice this is slightly more difficult. We have either truncated the VMA parameters, estimated D without restriction or observed that many of the coefficients do decay after a long decline in the lag structure or cycle.

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