Competitive Markets with Informational Asymmetries and Trading Restrictions: Welfare Analysis and Applications to Finance

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Abstract

This thesis consists of three original articles in the field of general equilibrium with incomplete markets and general equilibrium with asymmetric information, and an introduction to the theory, which traces its development and embeds the following chapters in a common framework.

In *Pareto Improving Trade Restrictions in an Incomplete Markets Economy*, we consider a stylised three period one good general equilibrium model with incomplete security markets. We show that the introduction of an indiscriminate marginal constraint on security trades can lead to a Pareto improvement, even though all prices are endogenous and agents are fully rational and have symmetric information.

In *Signaling Credit Quality Independently of Contract Choice: a Non-Transaction Cost Approach to Swaps in Anonymous Markets*, we demonstrate that under two conditions, swaps are non-redundant securities in anonymous financial markets. Firstly, there is asymmetric information over the project payoff which is financed by swaps. And secondly, borrowers are restricted from being investors at the same time. If either of this condition fails, then swaps are redundant assets. Swaps permit a constrained optimal solution to an asymmetric information problem.

Finally, *Anonymous Corporate Bond Markets with Asymmetric Information*, the main article of this thesis, shows that in an anonymous credit market which is characterised by limited liability and asymmetric information between borrowers and lenders, the nominal rate of interest on tradable debt (*the coupon rate*) sorts borrowers by their riskiness and in this way has an *indirect* influence on the price and quantity of bonds traded in equilibrium. This is in contrast to symmetric information models, in which the nominal coupon rate has no function. The paper claims that the adverse sorting effect of the nominal interest rate, as in Stiglitz-Weiss (1981), is maintained in a competitive setting, but that, even though changes in the nominal interest rate result in non-monotonic changes in the deliveries of agents, the orderly functioning of markets is not impaired.

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Contents

Abstract 1

Acknowledgements 2

1 Introduction 6

1.1 Classical General Equilibrium Theory 6

1.2 Developing the Classical Paradigm: changes in the fundamental data of general equilibrium theory 11

1.2.1 Uniformly Restricted Participation: General Equilibrium with Incomplete Markets 12

1.2.2 Restricting Trade Spaces to Types: General Equilibrium with Asymmetric Information 18

1.3 Concluding Remarks 25

2 Pareto Improving Trade Restrictions 27

2.1 Introduction 27

2.2 Intuition for the Effect 30

2.3 The Economy 34

2.3.1 Agents, Endowments and Securities 34

2.3.2 Equilibrium 37

2.4 Trade Restrictions and Utility Change 40

2.5 Conclusion 45

2.6 Appendix 46

3 Non-redundant Swaps **57**

3.1 Introduction to the Problem 57

3.1.1 Related Literature 60

3.2 Structure of the Model 63

3.2.1 Uncertainty 63

3.2.2 Project Finance 67

3.2.3 Utility Functions and Optimisation Problems 69

3.3 A Separating Equilibrium with Swaps 75

3.3.1 Structure of Traded Securities 77

3.4 Extension: Explicit Modeling of the Swap Counterparty 80

3.5 Conclusion 84

3.6 Appendix 85

4 Corporate Bond Markets **97**

4.1 Introduction 97

4.2 The Separation of Nominal Bond Coupon Rates and Market Prices 101

4.3 The Limited Liability Set-up of Stiglitz-Weiss 106

4.4 Modeling Stiglitz-Weiss in General Equilibrium 111

4.4.1 Discrete Uncertainty Structure 112

4.4.2 Security Structure and Optimisation Programs 116

4.4.3 Existence of a General Credit Market Equilibrium . . . 126

4.5 Characterising the Equilibrium 128

4.5.1 Comparative Statics of a Credit Crunch 132

4.5.2 A Comparison with Credit Rationing in Stiglitz-Weiss . 133

4.6 An Extension to the Basic Model: Introducing Collateral . . . 135

4.6.1 Characterisation of a Separating Equilibrium with Collateral 138

4.6.2 Comparative Statics of a Credit Crunch with Collateralised Bonds 138

4.7 Conclusion 139

4.8 Appendix 141

Bibliography **150**

Chapter 1

General Introduction

1.1 Classical General Equilibrium Theory

Traditionally, the focus of general equilibrium theory has been on the analysis of the complete entity of an economic system, described by the behaviour of individuals and firms based on axioms of choice. The data of a general equilibrium economy are preferences, endowments and technology. We will describe them in turn:

Let x, y and z denote three consumption bundles, over which individual i has a choice. General equilibrium theory is based on two consistency axioms regarding the choice over these consumption bundles: *Completeness*, that either $x \succeq_i y$, or $y \succeq_i x$, or both, in which case $x \sim_i y$. And *transitivity*, that if $x \succeq_i y$ and $y \succeq_i z$, then $x \succeq_i z$. Individuals whose behaviour complies with these axioms are called 'rational'. With some additions, the main one being that individuals are non-satiated in the consumption of goods, analytically meaningful utility functions $u^i(x, y, z)$ representing these preference axioms can be constructed.

Individuals are endowed with endowments ω. Individuals choose commodity bundles subject to the restriction that the value of the commodity bundles which they choose do not exceed the value of their endowments. If there are L goods, and $p \in I\!R^L$ denotes the vector of prices of a commodity bundle, then the budget constraint is $\mathcal{B}^i(p, \omega^i) = \{p.x^i \leq p.\omega^i\}$.

Technology is referred to as the way in which commodity bundles can be substituted for each other. Smooth substitution implies infinitesimally divisible goods and a smooth transactions technology. Thus continuity and differentiability of the utility function can be interpreted as assumptions on technology.

With the data of the economy specified, demand correspondences can be found, which are the sets of the maximum elements of the problem of maximising the utility of agents subject to the budget constrained over a range of prices, which agents take as given. Denote the demand correspondences by $f^i(p, \omega^i) \in \arg\max \{u^i(x^i)|x^i \in \mathcal{B}^i(p, \omega^i)\}$. By the Maximum Theorem, demand correspondences own certain properties from the construction of utility functions and the budget constraint: boundedness, continuity, homogeneity, budget feasibility and a type of boundary behaviour. The sum of all demand correspondences, the aggregate demand correspondence, or, equivalently, the aggregate excess demand correspondence $Z(p) = \sum_{i=1}^{I}(f^i(p) - \omega^i)$ inherits boundedness, continuity, homogeneity, 'Walras' Law' that $pZ(p) = 0$ for all $p \in I\!R^L$, and the boundary behaviour of the individual demand correspondences by the properties of sequential compactness. A general equilibrium is defined as a tuple (p^*, x^*) of a price vector and an allocation, such that at p^*, all individuals in the economy maximise utility subject to their respective budget constraints and markets clear, ie. the aggregate excess demand function has a zero.

There are three issues which are studied in general equilibrium theory: existence, welfare properties and comparative statics.

Under the conditions on the economy above, invoking a fixed point theorem such as Kakutani's theorem is enough to show existence of a general equilibrium. Existence was first proved by Arrow and Debreu (1954), and independently by McKenzie (1959), who made assumptions on demand correspondences[1]. Existence of a general equilibrium is often associated with the idea of 'the orderly functioning of markets'.

However, the real power of classical general equilibrium theory must be traced back to its welfare properties. The first welfare theorem states that every competitive equilibrium is Pareto efficient. Pareto efficiency means that an equilibrium allocation (p^*, x^*) cannot be improved upon for any one agent, without making at least one other agent worse off. The proof of the first welfare theorem relies on the fact that a superior allocation must lie

[1]There is an earlier claim due to Wald (1936)

in the upper contour set of at least one agent and is therefore not budget feasible, since prices in a particular equilibrium are uniquely determined for all agents.

The issue of multiplicity and determinacy of equilibria is essential for comparative statics. There are several results in this area. Debreu (1970) proved that generically, general equilibria are locally unique. However, three powerful results on the structure of the excess demand functions, by Mantel (1976)-Debreu (1974)-Sonnenschein (1972), show essentially that the existence results are equivalent to the fixed point theorems, implying that if a correspondence satisfies the conditions of a fixed point theorem, then it can be viewed as the aggregate excess demand correspondence of one particular economy. The results imply that very little can be said regarding the structure of the demand correspondence of a particular economy under observation, since the 'real' underlying economy cannot be identified. Without further restrictive assumptions on utility functions, or on the distribution of preferences in the economy (Hildenbrand, (1982)), comparative statics becomes impossible.

In view of these discouraging results on comparative statics, and of solved questions regarding existence and welfare properties, general equilibrium has developed beyond the classical questions, and, in some way, has moved away from the analysis of the entity of an economic system to more specific questions in finance, production, money and policy. In an early contribution, Debreu (1959) and Arrow (1964) show that the underlying logic of the general equilibrium approach can be carried over to the setting of uncertainty and time without any changes, as long as goods can be traded contingent on time and location. Significantly reducing the numbers of markets needed for the equivalence result, Arrow (1964) showed that 'contingent contracts',[2], rational expectations and a system of complete contingent security markets, will suffice to make an economy equivalent to a classical general equilibrium economy again. This result was generalised to the case of a general complete financial structure by Radner (1972). Neither the underlying mathematical techniques, nor the existence question or welfare properties are altered when time and uncertainty are treated in this way. Since the contingent contract construction is extremely helpful and revealing for the analysis of incomplete markets economies, it warrants further study: Let there only be one good in the economy, which can be interpreted as income, and whose price is normalised to one. Let there be two periods, and a finite number of states $s = 1, \cdots, S$ in period $t = 1$. A contingent contract for state $s \in S$ is a promise to deliver one unit of the good in state s and nothing otherwise. Its price, denoted by π_s is payable at $t = 0$. The optimisation problem of agents is transformed into

$$\begin{array}{lcl} max & u^i(x^i) & s.t. \\ x^i & \in & B^i(\pi, \omega^i) = \{\pi.x^i \leq \pi.\omega^i\} \end{array}$$

When writing the first order conditions of this problem, it becomes clear

[2]Contracts that pay off one unit in one particular state and zero otherwise

that the contingent contract prices are equal to the marginal utilities of income in states $s = 0, \cdots, S$, and can therefore be referred to as 'state prices'. The welfare properties of a general equilibrium can be expressed in terms of the collinearity of the marginal utilities of income in the states, hence, the state prices. At a Pareto optimum, the vector of marginal utilities of income point in the same direction for all individuals. This insight carries over to the welfare analysis of more general economies and we will return to it presently.

1.2 Developing the Classical Paradigm: changes in the fundamental data of general equilibrium theory

General equilibrium theory retains its analytical power from the simplicity of its approach. All agents are price takers, prices are linear and contracts are anonymous. Extensions to general equilibrium theory attempt to preserve these methodological underpinnings, while allowing for changes in the fundamental data of preferences, endowments and technology. As we have already stated, research has arguably shifted to answering more specific questions in the fields of policy, finance, macroeconomics. This thesis is written in this spirit. It uses the methodology of anonymity, price taking and non-exclusiveness to analyse very particular phenomena in, from the viewpoint of traditional general equilibrium theory, highly specialised settings.

Several attempts have been made to allow for generalisations of preferences, endowments or technology. *Bounded rationality* can be viewed as a modification of utility functions that allow for non-completeness of preferences. Moral hazard can be interpreted as allowing for non-convex preferences. Both of these modifications are not easily introducible into general equilibrium theory. Moral hazard introduces problems of non-existence (eg. Helpman and Laffont, (1975)), while, more fundamentally, the incompleteness of preferences makes the very construction of utility functions non-obvious at the very least.

General equilibrium theory has been extended with a lot more success to allow for more restrictive transactions technologies, as in the case of general equilibrium with incomplete markets (GEI). The next section will elaborate on this point. A very recent development has been the introduction of adverse selection, which could be viewed as allowing for type-specific endowments, and whose integration necessitates non-type specific transactions technologies. The three papers that make up this thesis are based on these two extensions, and we will now briefly turn to discuss the issues raised when extending general equilibrium along these lines.

1.2.1 Uniformly Restricted Participation: General Equilibrium with Incomplete Markets

Market incompleteness of financial markets can be viewed as a restriction on the trading technology of individuals. Specifically, individuals are restricted to trade bundles of goods and are not able to disentangle the bundles. In the mathematical construction, it implies that individuals face multiple budget constraints, which can not be trivially reduced to a single budget constraint as in the case of a 'contingent market equilibrium'. Denote the vector of security prices by q and the matrix of security payoffs as V, where $V = (V)_{s=1\cdots S}^{j=1\cdots J}$. Then a financial markets budget constraint is

$$\mathcal{B}^i(q, \omega^i, V) = \left\{ (x^i; z^i) \in I\!R_+^{S+1} \;\middle|\; \begin{array}{ll} x_0^i - \omega_0^i = qz^i & z^i \in I\!R^J \\ x_s^i - \omega_s^i = Vz^i & s = 1, \cdots, S \end{array} \right\}$$

In contrast to the original budget constraint, the financial market budget constraint consists of a set of $S+1$ constraints. The market is said to be complete if $S = J$, where j are the linearly independent securities available in the market. Since the matrix V becomes square when markets are complete, by a basic theorem of linear algebra, there are portfolios z^i that generate every possible income stream $y \in I\!R^S$, since $z^i = yV^{-1}$ always has a solution for every y. This allows to rewrite the complete market financial budget set as a single budget equation. If markets are non-trivially incomplete, not every income stream y lies in the span of $\langle V \rangle$. However, the no-arbitrage principle of contingent market pricing, that there is no arbitrage if and only if there exists a vector of positive state prices (or prices of Arrow securities), carries over to incomplete markets. Thus the fundamental pricing relation that the price of a security equals its discounted value of payoffs under some probability measure, $q = \pi V$, carries over straight from the contingent market construction[3], and every income stream in the span of $\langle V \rangle$ can be uniquely

[3]To see this, normalise the marginal utility of income at period $t = 0$ to one. Then denoting by λ_s^i the Lagrange multipliers of agent i in state s, the first order conditions

priced: $c_q(y) = qz = \pi V z = [\pi, y]$ determines a unique cost c of the income stream y. Note, however, that now the vector of state prices π ceases to be unique across agents.

These basic observations of incomplete markets theory suffice to give an intuition for the results on existence and welfare, and, perhaps more fundamentally reveal why the utility function construction is not endangered by the introduction of restrictions on technology: since agents are forced, in equilibrium, to agree on the value of the bundles of goods which they can trade, even though they disagree about the valuation of individual income streams, the value of 'everything which is traded' is uniquely determined and an equilibrium always exists in the case of one good and two periods, and generically (for an open dense subset of the parameter space) for multigood or multiperiod economies, since then the payoff matrix is not independent of prices anymore (Hart (1975)). Existence was proved by Werner (1985) for nominal assets, by Geanakoplos-Polemarchakis (1986) for a numéraire commodity and by Duffie and Shafer (1985, 1986) for general asset structures.

Regarding welfare, note that the indeterminacy of state prices allows the non-collinear alignment of marginal rates of substitution for different agents at the same equilibrium prices. Consequently, a central planner who reallocated payoffs in such a way as to make state prices/marginal rates of substitution collinear, would be able to Pareto improve upon the equilbrium allo-

with respect to the portfolio choice z^i are $-\lambda_0^i q + \sum_{s=1}^{S} \lambda_s^i V_s = 0$. Since $\nabla u^i(x^i) = \lambda^i$, and by the normalisation, $q = \pi V$.

cation. However, a stronger result is true. Geanakoplos and Polemarchakis (1986) establish the generic constrained suboptimality of GEI equilibria with many goods (or many periods). A GEI equilibrium is called 'constrained optimal' if a budget feasible reallocation of portfolios in the trading period with the existing assets cannot improve upon the equilibrium allocation. A constrained optimal *re*allocation must respect budget feasibility *and* the security structure available. At first sight, considering the maximum properties of individual choices, it seems surprising that individuals, when left to their own devices, cannot find the choice which truly maximises their utility. However, the intuition is the following: a portfolio reallocation changes relative income levels of agents in all states of nature, and in doing so, affects relative prices. The relative price change, however, cannot be decomposed into portfolio reallocations. In other words, faced with the relative prices of the Pareto improving allocation, individuals would not trade portfolios in such a way that at those prices sport markets and asset markets cleared. The proofs of constrained suboptimality rely heavily on tools of differential topology, and all the analysis is local. It is an open question to construct a mechanism which finds the constrained optimum, and there is debate as to the definition of constrained suboptimality employed in Geanakoplos and Polememarchakis (1986) (see Kajii (1995)).

Pareto Improving Trade Restrictions in an Incomplete Markets Economy

The contribution of this thesis to the theory of incomplete markets is contained in chapter two. In this chapter, we provide another interpretation of the constrained suboptimality result of multiperiod incomplete market economies. We conduct the analysis in a highly specialised setting, and our main result does not hold for every arbitrary economy satisfying the standard assumptions.

The result is the following: in a two agent replica economy with three periods and uncertainty only between $t = 0$ and $t = 1$, with general preferences and short-lived securities, imposing a marginal indiscrimate borrowing constraint on agents can induce a Pareto improving reallocation, even though all prices are endogenous.

The mechanism used to generate the result is that a marginal borrowing constraint induces one of the agents to save more in the previous period. As a consequence of his saving, the individual arrives at the period in which borrowing is constrained with a greater level of wealth. The changed distribution of wealth induces a price change. The price change may make both agents better off, and compensate for an adverse price change in the period prior to the constraint period. Hence the allocation may be improved with respect to the GEI equilibrium.

Again, the result seems surprising at first sight. It suggests that individuals borrow too much in equilibrium, and with rational expectations and utility maximisation, one does not expect this to happen. However, the intuition is the following: when individuals are confronted with the prices that are the prices of the Pareto improved allocation, they choose portfolios that would yield an ever greater utility level for them, than the borrowing constraint allocation does. However, these portfolio choices are not market clearing, and hence are not an equilibrium. Another way to look at the same problem, taking into account that utility functions are additively separable, would be to see that the multiplicity of budget constraints 'disconnects' the portfolio choice problems from periods $t = 0$ to $t = 1$ and from $t = 1$ to $t = 2$. The portfolio choice problem from $t = 1$ to $t = 2$ determines the prices of the securities traded in these periods. These prices depend on the endowments in both periods, wheighted by the preferences of the individuals. However, agents do not take into account the fact that the previous choice problem from $t = 0$ to $t = 1$ changes the distribution of wealth in period $t = 1$ and therefore influences the choice problem in subsequent periods.

Methodologically, the contribution of the article is that the borrowing constraint is 'non-discriminating'. In contrast to the interventions suggested by Geanakoplos and Polemarchakis (1986, 1990), or by Herings and Polemarchakis (1997), the knowledge requirement is very low. However, the result is, even in the highly special economy, not true all values of the parameters. Economies can be found for which the borrowing constraint induces an adverse price effect in the previous period which destroys the Pareto improvement of the portfolio reallocation.

1.2.2 Restricting Trade Spaces to Types: General Equilibrium with Asymmetric Information

Following the fundamental contributions by Akerlof (1970), Mirrlees (1974), Spence (1974) and Rothschild and Stiglitz (1976), problems of asymmetric information have taken centre stage in economic theory. Several attempts have been made to analyse the implications of informational asymmetries in a general equilibrium setting. Helpman and Laffont (1975) provide an example of non-existence of a general equilibrium with moral hazard. Prescott and Townsend (1984) establish a framework to analyse existence and welfare properties. Gale (1992, 1996), has developed a framework from a different, more contractual approach. The problems of general equilibrium with asymmetric information (GEAA) have become clearer with a series of articles by Geanakoplos (1990), Gottardi and Bisin (1997), Polemarchakis and Minelli (1993) and a joint effort by the above authors. The issues are that agents have market power if they possess information that other individuals in the economy do not have. Even though they remain price takers, their private knowledge over the payoff of a contract individualises the contract for them. Hence, a generic model of asymmetric information could be written in an Arrow-Debreu style fashion by making the payoff matrix V dependent on the individuals i. When doing so, two problems naturally arise. Equilibria with asymmetric information may not exist, since agents are given additional arbitrage opportunities and because new feasibility problems are introduced. Both complications can be seen with relative ease, and suggest two restrictions which are needed to make them compatible with the anonymous market set-up. One, that agents are small, and two, that trading restrictions are introduced which prevent individuals from excessively exploiting their private information. In a way, asymmetric information can be introduced as long as it is confined to small trades and as long as only the informational advantage cannot be exploited for speculation beyond its immediate allocational advantage. There are several ways in which 'speculation' can be prevented. They all aim at restricting individuals from holding unlimited long *and* short positions in the same asymmetric information security. Constructions achieving this are: imposing an upper bound on short sales, separating the long and short sides of the market, either by decree or by introducing bid-ask spreads, and the construction of pool securities. Since we make extensive use of the last, we will present a brief formal statement of the pool security construction.

Let security purchases be denoted by ϕ^i, and sales by θ^i. For simplicity, let there be only one standard security, whose payoff depends on the characteristics of the individuals in the economy, and only one type of agent, but infinitely many agents of the same type. For example, one may think of a mortgage as a security which individuals sell to a bank, and which is almost completely standardised, yet the sellers of the security may have superior information regarding the likelihood of repayment. Then the payoff V^i of the security depends on the individual who sells it. On the other side of the market, pool all the individual securities, and denote the average delivery on one unit of the pooled security by V^p. To emphasise the distinction between portfolios of pool securities and of individual securities, denote purchases by $\phi^{i,p}$. If certain assumptions are made regarding the distribution of payoffs of the individual security, namely that there are infinitely many securities of the same type, whose payoffs are iid., then, in the limit for the number of sellers of the security approaching infinity, the law of large numbers can be invoked to make the payoff of the security constant across all social states of nature. Recall that the law of large numbers states that under iid. assumption on the distribution of random variables, their partial sum tends to the average for every sequence of outcomes. In contrast to individual states, which we call s, we denote the partial sums of the random variable (the aggregate states) by σ. Then the law of large numbers states that $V^p(\sigma)$, the payoff of a share in the pool, tends to the average simple average payoff of the individual securities for the number of projects becoming infinitely large. The deliveries into the pool security are endogenous, whereas for the individual security, it is the price which is the equilibrating variable. This leaves the price for the pool security undetermined. For every price, in equilibrium the deliveries will adjust such that the 'effective' return fulfills the requirements of the market. Denote the price of the security by q. A convenient normalisation for the price of the pooled security is that $q = q_p$. Considering that individuals of a certain type will make the same choices, their optimisation becomes:

$$\begin{array}{ccc} max & u^i(x^i) & s.t. \\ (x^i, \phi^i, \theta^i) & \in & B^i(q, V^i, V^p, \omega^i) \end{array}$$

where the budget set is:

$$B^i(q, V^i, V^p, \omega^i) = \left\{ \begin{array}{l} x_0^i = \omega_0^i - q\left(\phi^{i,p}(\sigma) - \theta^i(s)\right) \\ x_s^i = \omega_s^i + V^p(\sigma)\phi^{i,p}(\sigma) - V^i(s)\theta^i(s) \\ \forall s, \forall \sigma \end{array} \right\}$$

The trading restrictions we must impose are that $\phi^{i,p}, \theta^i \geq 0$. Under the conditions which we have listed above, a general equilibrium with asymmetric information exists. The mechanism can be thought of in the following way: prices for individual securities are announced; these prices induce a certain supply; depending on which individuals supply the securities there is a quantity of deliveries into the pool; given that the price of the pooled contract is set equal to the price of the individual contract, the deliveries induce an effective rate of return. There is a demand for pool securities at this return. If demand and supply coincide, markets clear and there is an equilibrium.

Since there are restrictions on unbounded arbitrage sales, equilibria in this way are not unconstrained efficient. However, no notion of 'constrained efficiency' has been devised.

In this thesis, the GEAA setting is exploited to analyse specific phenomena of financial markets.

A Non-Transactions Cost Approach to Swaps in Anonymous Markets with Asymmetric Information

There are two different views one could take of a swap market: one which emphasises the bilateral relationship between the counterparties, the other which views swaps as anonymous exchanges of payment profiles across time, with bilateral payments taking place at every period.

Taking the first viewpoint, it is almost trivial to see that with symmetric information and complete markets, swaps, which are exchanges of payment streams which were traded prior to the swap trade, are redundant assets. Likewise, since interest rate swaps are usually exchanges of coupon streams on a notional principal, swaps trivially make an incomplete market more complete in the same way that 'asset strips' do.

It is then clear that some type of complication or inefficiency must be introduced into a financial market to give swaps a role, over and above the savings on transactions costs which they provide by implicitly allowing retrade of bonds. The paper provides a model in which swaps are non-redundant, and are used to signal good future credit quality to a myopic market. There are two firms who seek finance for the same type of project, but who have private information over the different probabilities of success of the projects. The projects pay off after two periods, however the financial market can only distinguish between the two projects one period ahead. Consequently the good risk firm would like to signal its good credit quality to the market. If there are short sale constraints, and for a particular preference structure, (all firms prefer smooth repayments), swaps will be employed by the good firm in its financing decision. The good risk firm will issue short term one period debt and swap it for long term debt, while the bad firm will issue long term bonds from the start. The result holds true if there are restrictions on short sales and there is asymmetric information in the market. As a restriction on short sales is required for the existence of equilibrium in a GEAA model, its use comes natural in our application.

The model is completely written from a unilateral viewpoint, which means that the swap dealer and the investors are the same individuals and the swap is like a bond contract with a future-type add-on. In an extension to the model, we show that preferences for a counterparty can be found which transform the set-up into a more traditional bilateral treatment of swap contracts.

Anonymous Corporate Bond Markets with Asymmetric Information

The central question of the last paper is whether the nominal coupon rate on bonds, the 'coupon rate', has a role to play in competitive markets with asymmetric information. If information is complete and symmetric, then the nominal rate plays no role as the price of bonds adjusts to equilibrate the supply and demand of credit. With asymmetric information *and* limited liability, however, we show that the nominal coupon rate sorts borrowers by their riskiness, and pooling equilibria exist in which the good risk borrowers subsidise the bad risk borrowers.

The uncertainty construction expands upon a seminal article by Stiglitz and Weiss (1981) (SW) on credit rationing in markets with asymmetric information. The SW model is game theoretic. There is a monopolistic bank which is a price setter in its credit market and a quantity setter in its deposit market. SW show that it *may* be the case that, if borrowers have asymmetric information over their projects, a credit market is characterised by credit rationing. One problem with the analysis is that, even with symmetric information, it is a standard result that a non-price discriminating monopolist 'rations' its clients in its product markets. In other words, it is difficult to disentangle the two possible causes of credit rationing: the game theoretic set-up and the asymmetric information.

When conducting a SW type analysis in a competitive setting, credit contracts are traded after their issue, and the definition of a Walrasian equilibrium precludes credit rationing. The equilibrium interest rate is then not directly affected by the nominal coupon rate as in SW. However, there is a more complicated mechanism. The nominal coupon rate still sorts borrowers by their riskiness, and hence the quantity of credit at every coupon rate is dependent on the nominal coupon rate. Even though in equilibrium, the effective interest rate always adjusts to equilibrate demand and supply in the market, and, in this sense, there is no credit rationing, the different deliveries for different nominal rates imply that the equilibrium occurs at a different effective interest rate and a different quantity of credit traded. Indeed, if one were to constrain individuals in the quantity they could borrow, then it may happen that the credit market clears at a higher effective interest rate and a higher quantity of credit issued. In this sense, some of the intuition and mechanism behind the credit rationing result carries over to the competitive setting.

The analysis is extended to see how the credit market would react, if the propensity to lend deteriorates. The answer is that good risk borrowers would always be driven out of the market first.

Up to that point, the analysis only considers one-dimensional credit contracts. We then introduce collateral in an extension to the basic model, and show that separating equilibria can be constructed, similar to Bester (1985). In separating equilibria, projects are priced according to their riskiness, and no mispricing through pooling occurs. Moreover, as long as the separating equilibrium is upheld, the nominal coupon rate becomes insignificant. We also show that if the propensity to lend deteriorates, it will be the case that bad risk borrowers drop out of the market first, as their risk is properly priced and they need to pay a higher interest rate for the same expected return of the project.

1.3 Concluding Remarks

The articles on GEAA economies show that the underlying structure of general equilibrium models, price taking behaviour and the anonymity of contracts, can be used to analyse specific issues in specialised areas of economics. Financial markets, in particular, seem to lend themselves easily to this type of analysis. Using a general equilibrium approach ensures consistency and closedness of the models, and, as in the case of SW, isolates the informational restriction from the behaviour of the individuals in the economy.

Chapter 2

Pareto Improving Trade Restrictions in an Incomplete Markets Economy

2.1 Introduction

One of the most surprising results in General Equilibrium Theory with Incomplete Markets (GEI) concerns the inefficiency of a market economy. Geanakoplos-Polemarchakis (1986) and Geanakoplos-Magill-Quinzii-Drèze (1990) prove that, generically, a general equilibrium economy with at least two goods or at least three periods is *constrained Pareto inefficient.* Even if a 'central planner' is allowed to interfere with market allocations only once at the beginning of time, he can still improve upon the competitive allocation.

The reason for the inefficient behaviour of individuals is that, agents are not aware that their asset trades in previous periods induce a change in the distribution of wealth in the current period, which influences prices in the current period and hence total welfare. Since casual empiricism suggests that a market economy does not have a complete set of markets in the Arrow-Debreu sense, the apparent generality of the result is all the more striking.

However, fascinating though these assertions may be, they do not serve as a basis for a 'normative theory of inefficiency'. The information requirement on the central planner to *find* the Pareto improving allocation are extremely high: to intervene in the correct way, the central planner must know the agents' preferences and endowments. Geanakoplos-Polemarchakis (1990) show that if the individuals' assets *and* goods demands can be observed (in a multigood model), then preferences can be recovered. However, two limitations of their result seem important. One that to observe individuals' demands, there would have to be a period of observation before a central planner could intervene, and, secondly, that individuals' demands can usually not be deduced from aggregate demands.

In a recent paper, Herings-Polemarchakis (1997) show that a Pareto improvement allocation can be found by exogenously changing the prices of goods (in the setting of a multi-period good model). The strengthening with respect to previous results consists of allowing the central planner to intervene at the macro-level. However, it is not enough to intervene in the first period only, as required in Geanakoplos-Polemarchakis (1986) and Geanakoplos-Magill-Quinzii-Drèze (1990). In this respect, it remains unclear whether the

Herings-Polemarchakis (1997) contribution constitutes a weakening or not from the normative point of view of the inefficiency results. Kajii (1994) attacks the problem of the knowledge requirement of the central planner by proposing the concept of 'anonymous intervention'. In his set-up, the central planner suggests an intervention rule, but agents are allowed to choose whether to truthfully reveal their type or not. Kajii concludes that if agents are not allowed to retrade after the intervention, Pareto improvements are possible. However, if retrade is allowed, then agents will go back to the initial equilibrium and revert the changes of the central planner, essentially because of the maximum properties of the equilibrium allocation.

The two papers by Herings-Polemarchakis (1997) and Geanakoplos-Polemarchakis (1990) point towards the two routes that one can conceivably take to provide a basis for a normative theory of inefficiency. Either mechanisms must be found that reveal sufficient information to the central planner to enable him to intervene in a beneficial way, or alternative characterisations of Pareto inefficiency, and, correspondingly, simple intervention rules are called for. In spirit, this paper falls into the second category. The main finding is that we present a class of incomplete market economies for which an indiscriminate borrowing constraint on everyone in the time of need leads to a Pareto improvement. In terms of the 'central planner' analogy, a more appropriate term for our intervention would be to call it a 'non-discriminatory constraint mechanism'. The only knowledge requirement on the mechanism is to detect whether agents borrow out of need in a precise sense which will be defined below. Also, in contrast to the previous literature, we allow all prices to be endogenous, including prices in the period in which the mechanism is imposed on the economy, and prices in the first period. In this respect, our example is a generalisation of Geanakoplos-Polemarchakis (1986) and Geanakoplos-Magill-Quinzii-Drèze (1990). However, we confine our analysis to a highly parameterised economy, which is why we may call the model an 'example'.

The remainder of this paper is organised in five sections: Section 2.2 gives an intuitive explanation for our main assertion that an indiscriminate borrowing constraint can make everyone better off. Section 2.3 sets up the model. Section 2.4 contains the main result and Section 2.5 concludes the analysis. All the proofs are contained in the Appendix.

2.2 Intuition for the Effect

Our analysis builds on Geanakoplos and Polemarchakis (1988) and Geanakoplos, Magill, Quinzii, Drèze (1990). Their articles define and explore efficiency properties of incomplete market economies. As soon as an incomplete markets model has more than one good or lasts longer than two periods, a pecuniary externality effect arises which makes the equilibria generically constrained Pareto suboptimal. In other words, even a central planner who has not more securities available to himself than the market and, in addition, who is only allowed to intervene once, can still improve upon the market allocation.

In this paper we demonstrate that we can find easily characterisable and non-discriminating restrictions on agents' borrowing decisions in the time of need that trigger off a mechanism that produces the pecuniary externality needed in the proofs of the aforementioned theorems on market inefficiency. Moreover, all prices are endogenous. The intuition is as follows:

Since markets are incomplete, agents' marginal utilities of income in the same state are generally distinct: If it happens that in one state of nature the distribution of endowments is unequal, then, in this state, the poor agent has a high marginal utility of income, while the rich agent's marginal utility of income is low. Since in this state the poor agent wants to borrow and the rich agent is willing to lend, a reduction in the interest rate benefits the poor agent but hurts the rich agent. Symmetrically, if the distribution of endowments is reversed in a second state, a reduction in the interest rate in that state affects agents' marginal utility of income in the direction opposite to the first state.
However, since the marginal utility of income is low when an agent is a lender and high when he is a borrower, a reduction in the interest rate benefits an agent more when he borrows (low income/ high marginal utility) than it hurts him when he lends (high income/ low marginal utility). Thus, in expected terms, a reduction in the interest rate in a time period in which agents can be either rich or poor with some probability has the potential to improve the welfare of all agents.

The inequality of marginal valuations of income across agents due to the incompleteness of markets drives the Pareto improvement in the model.

What remains to show is how the introduction of a borrowing constraint leads to a fall in the interest rate, and why the agents could not 'find' an interest rate which made everyone better off in the competitive setting:

To produce the desired result we introduce agent heterogeneity. All agents have the same risk-averse, time separable utility functions, but their discount rates are different. The borrowing constraint in the time of need induces agents to change their behaviour in the preceding period. To ensure consumption in the state when they are poor, both agents would like to precautionarily save for one period. This is not possible, since one agent's savings decision is the other agent's lending decision. Who will be allowed to save in period one depends on the severity of the constraint imposed on agents. If they face the same constraint, the more patient agent will be allowed to save (ie. lend) in the previous period, since his marginal utility of consumption in the restricted state increases proportionately more with the introduction of the borrowing constraint. This agent will then arrive with more wealth in the next period. The price for the security in this period depends on total endowments in the period and the next period, 'weighted' by the discount rate of agents. If more weight is given to the patient agent (he arrives with a larger endowment), his preferences will determine the relative price for current consumption in terms of future consumption to a larger extent. Since future consumption is more important for the more patient agent, the price of future consumption will fall and the price of current consumption will rise. Hence the price of the security will rise, or equivalently, the interest rate will fall.

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One complication we have ignored so far is that when the patient agent is induced to save more prior to the period in which we introduce a borrowing constraint, then prices will turn against him, ie. the interest rate at which he can lend (=save) will fall. His utility will fall. We have to show that this fall in utility is more than compensated for by the utility increase due to the borrowing constraint.

Why do the agents not find the Pareto maximising prices, given the security constraints? After all, the claim is that utility maximisation does not find the utility maximum. The reason can be found in a *pecuniary externality*: Agents are not aware that their portfolio decisions today affect prices tomorrow by changing the distribution of wealth tomorrow, and that the new prices affect welfare. Although they have no *individual* market power, they *as a group* influence prices through the changes in the income distribution induced by the borrowing constraint.

The externality raises the question whether a competitive equilibrium is the right framework for this model. If agents understood that as a group they influence prices, they might be able to act strategically and induce a Pareto improving allocation. However, this analysis would depart from the assumption of infinitesimally small agents.

We would like to stress again that the following simple trade restriction, which drives the Pareto improvement in the economy, is not a general mechanism. In this sense, the following is more of an example than a theory.

2.3 The Economy

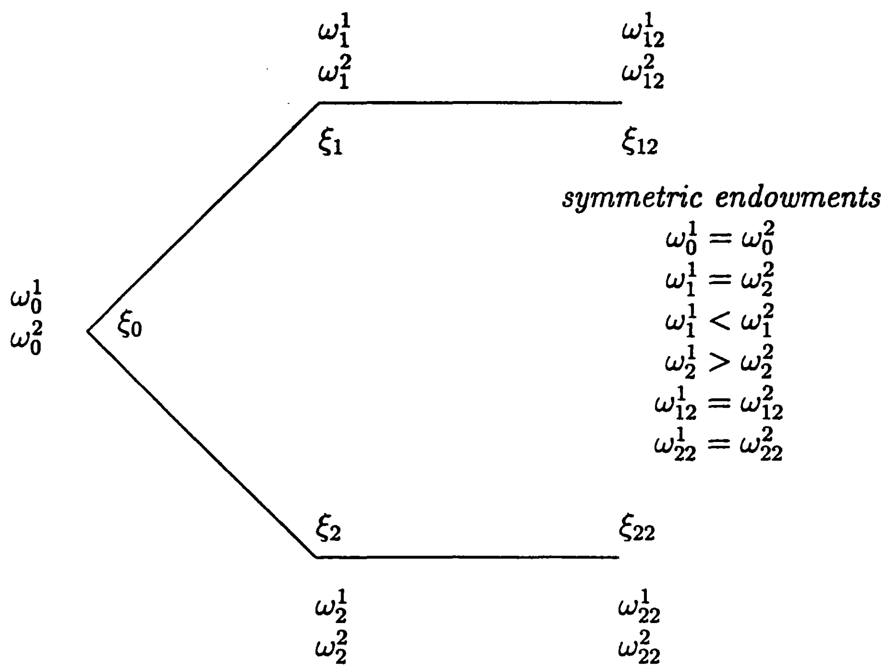
2.3.1 Agents, Endowments and Securities

Time and Uncertainty

The most simple model we can write has three time periods $t = 0, 1, 2$ and uncertainty only in period $t = 1$, represented by two states of nature ξ_1 and ξ_2, which happen with equal probability. The structure can be represented by a tree with five nodes, which we will refer to as $(\xi_0,(\xi_1,\xi_{12}),(\xi_2,\xi_{22}))$. One can think of the uncertainty as an endowment shock. Either agent 1 or agent 2 experiences an endowment shock with equal probability in period $t = 1$.

There is symmetric information throughout and agents form expectations rationally. Moreover, the probabilities of states ξ_1 and ξ_2 are objectively known.

Fig.1: Event Tree of the Economy $\mathcal{E}(U, \omega, V)$



Agents and Endowments

There are two types of agents, and a continuum of each type, such that each agent has Lebesgue-measure zero and the total measure of agents is two. Agents i have time-separable, state independent, strongly monotone and strictly quasi-concave preferences defined by a utility function

$$U^i : X \in I\!R^5 \longrightarrow I\!R$$

$$\begin{aligned} & U^i(x^i(\xi_0), x^i(\xi_1), x^i(\xi_{12}), x^i(\xi_2), x^i(\xi_{22})) \\ = {} & u^i(x_0(\xi_1)) + 1/2(\alpha_i u^i(x_i(\xi_1)) + \alpha_i^2 u^i(x_i(\xi_{12}))) \\ & +1/2(\alpha_i u^i(x_i(\xi_2)) + \alpha_i^2 u^i(x_i(\xi_{22}))) \end{aligned}$$

with $u^i\prime() > 0$ and $u^i\prime\prime() < 0$. α_i is the discount rate of agent i,with $\alpha_1 > \alpha_2$, ie agent 2 discounts future consumption more strongly (he is relatively impatient). Endowments are given by $\omega^i = (\omega^i(\xi_0), \omega^i(\xi_1), \omega^i(\xi_{12}), \omega^i(\xi_2), \omega^i(\xi_{22}))$. Agents have the same endowment in periods $t = 0$ and $t = 2$, but suffer from an endowment shock in one of the two states in $t = 1$. We model the endowment shock by setting $\omega^1(\xi_1) < \omega^2(\xi_1)$ and $\omega^1(\xi_2) > \omega^2(\xi_2)$. (see Fig.1)

Commodities and Securities

There is only one commodity in the model, whose price is normalised to one and which acts as the numéraire. We can interpret this commodity as 'income'. There is a structure of real securities V, meaning that they pay out in terms of the commodity. We assume that the security markets are incomplete. In a multi-period setting this implies that the number of actively traded securities is less than the number of states in at least one period of the model. In our simple setting, incomplete markets imply that there is only one security which has a non-zero payoff in period $t = 1$. For simplicity we assume that the only traded assets are one period bonds in states ξ_0,

ξ_1 and ξ_2, paying off one unit for sure of the commodity in the subsequent period. Focusing on short-lived and risk-free securities also allows us to disregard the well-known existence problem for multi-period security economies, first detected by Hart(1975). This problem only arises if securities can be traded before their date of maturity. Then the span of the security structure will depend on the - endogenously determined - prices of the securities. Non-existence arises if the span collapses discontinously at an equilibrium candidate. Since there is no interim trade in our securities, spanning between periods $t = 0$ and $t = 1$ is independent of prices.

Economy

The economy described above is denoted by $\mathcal{E}(U, \omega, V)$.

2.3.2 Equilibrium

Agents i maximise utility subject to their financial market budget constraint. Using q for the prices of securities and z^i for the porfolio choice of agent i, the budget constraint with three one period bonds 0, 1 and 2 issued at $t = 0$ and $t = 1$ and paying off one unit in the subsequent period (regardless of the state) is:

$$\begin{aligned} B^i(q, w^i, V) &= \qquad\qquad\qquad\qquad\qquad\qquad (2.1) \\ x^i(\xi_0) &= \omega^i(\xi_0) - q_0 z^i(\xi_0) \\ x^i(\xi_1) &= \omega^i(\xi_1) + z^i(\xi_0) - q_1 z^i(\xi_1) \end{aligned}$$

$$\begin{aligned} x^i(\xi_2) &= \omega^i(\xi_2) + z^i(\xi_0) - q_2 z^i(\xi_2) \\ x^i(\xi_{12}) &= \omega^i(\xi_{12}) + z^i(\xi_1) \\ x^i(\xi_{22}) &= \omega^i(\xi_{22}) + z^i(\xi_2) \end{aligned}$$

To illustrate the security structure more visibly, define the security matrix $W(q, V)$ as

$$W(q,V) = \begin{bmatrix} -q_0 & 0 & 0 \\ \cdots & \cdots & \cdots \\ 1 & -q_1 & 0 \\ 1 & 0 & -q_2 \\ \cdots & \cdots & \cdots \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Every row in the matrix corresponds to one node in the tree of the economy $\mathcal{E}$. The first row represents ξ_0, the second ξ_1, the third ξ_2, the fourth ξ_{12} and the fifth ξ_{22}. To better understand the structure, we have divided the different time periods by dotted lines, eg. q_0 in the first row indicates that there at node ξ_1 there is trade in security 1 at price q_0; following the first column down shows that this security pays off 1 at both nodes ξ_1 and ξ_2.

For subsequent analysis, it will be helpful to define the present value vector of an agent. The present value vector is the valuation that an agent gives to income in a certain state. It is defined as the vector of present values of the Lagrange multipliers of an agent's maximisation problem.

$$\bar{\pi}^i = (\bar{\pi}^i(\xi)) = \frac{\bar{\lambda}^i(\xi_s)}{\bar{\lambda}^i(\xi_0)}$$

where $\bar{\lambda}^i$ is the Lagrange multiplier of agent i.

We can now write the agents' maximisation program as a Lagrangean function

$$L^i(x^i, z^i, \lambda^i) = u^i - \lambda^i(x^i - \omega^i - W(q, V)z^i).$$

Given our assumptions on utility functions, the necessary and sufficient first-order conditions for the maximisation problem of an agent are:

$$\nabla L^i(\bar{x}^i, \bar{z}^i, \bar{\lambda}^i) = 0$$

Of particular interest to us is the gradient of $L^i()$ with respect to portfolios $z^i()$:

$$\nabla_z L^i(\bar{x}^i, \bar{z}^i, \bar{\pi}^i) = 0 \Leftrightarrow \lambda^i W(q, V) = 0 \tag{2.2}$$

which for our economy can be written as

$$\bar{\pi}^i(\xi)\bar{q}_\xi = \sum_{\xi' \in \xi^+} \bar{\pi}^i(\xi') \tag{2.3}$$

where ξ^+ are the successor nodes of ξ. (2.3) is the fundamental pricing equation for our model [1].

It asserts that in equilibrium the price of a security must be equal to the sum of its payoffs weighted by their marginal utilities. Equilibrium for the

[1]This first order condition is equivalent to a no-arbitrage condition. *see existence proof in Appendix*

values as parameters of the economy. The new economy we call $\mathcal{E}(U, \omega, \tilde{V})$. Then we impose a marginal positive change $dz^i(\xi_1), dz^i(\xi_2)$ [2] on portfolio holdings. The effect of the restriction is our main proposition and stated in the following theorem.

Proposition 1 (Pareto Improvement)
For a symmetric distribution of endowments, sufficiently small heterogeneity amongst agents' time preference and sufficiently low impatience of all agents a positive marginal portfolio change $dz^1(\xi_1), dz^2(\xi_2)$ in the economy $\mathcal{E}(U, \omega, \tilde{V})$, such that markets clear, is Pareto improving.

Proof: We organise the proof in a number of successive claims. Firstly we need to establish that the borrowing constraint is binding at the equilibrium:

Lemma 1 (Binding Borrowing Constraint)
The marginal restriction of trade $dz^i(\xi_1), dz^i(\xi_2)$ imposed on the parameterised economy $\mathcal{E}(U, \omega, \tilde{V})$ is binding.

We can now state the first claim, which will set the stage by revealing the variables which are affected by the borrowing constraint.

Claim 1 (Characterisation of Marginal Change in Utility)
The marginal change in utility for agent i induced by an exogenous change in portfolios dz_1^i, dz_2^i for the economy $\mathcal{E}(U, \omega, \tilde{V})$ is given by:

[2] When an agent borrows, he *sells* a security, ie. his portfolio holding $z^i(\xi)$ is negative. Hence a positive marginal change imposes a *borrowing* restriction.

It remains to demonstrate that there are parameterisation such first term of the fundamental equation (4) is small, and the Pareto improvement is not destroyed. The change in q_0, ie. the price elasticity of demand for security z_0 will depend on the precise specification of the utility function. However, the following claim indicates that suitable endowment/time-preference combinations are feasible to ensure that trade at period zero is small (ie. $-z_0^i$ small). The fact that it is not necessarily true that for every parameter values, a trading restriction leads to a Pareto improvement, bounds the generality of the borrowing constraint mechanism, even in the context of the special endowment and security structure which we use.

Parameter Restriction

A Pareto improvement requires that the first term in the marginal utility change equation (2.4), whose sign is negative for the patient agent, does not dominate the net effect of the sum of the last two terms. In other words, the product of the bond price change and the security trade in period $t = 0$ must be small relative to corresponding products in period $t = 1$. Only considering agent 1 (for agent 2 the price effect in period $t = 0$ is beneficial), equation (2.4) can be rewritten as

$$dq_0 < \frac{\bar{\pi}_1^1 dq_1(-\bar{z}_1^1) + \bar{\pi}_2^1 dq_2(-\bar{z}_2^1)}{-z_0^1} \tag{2.5}$$

By concavity of the utility functions a large difference in initial endowments for a similar utility function implies that net trades will be large. For $\alpha^1 - \alpha^2$ small and at node ξ_1: $\omega_1^1 <<< \omega_2^1$ (node ξ_2: $\omega_1^2 >>> \omega_2^2$) implies that $\bar{z}_1^1$ ($\bar{z}_2^1$) is large. Since $\bar{\pi}_1^1 > \bar{\pi}_2^1$, the difference of the products

$\bar{\pi}_1^1(-\bar{z}_1^1)+\bar{\pi}_2^1(-\bar{z}_2^1)$ (recall that $-\bar{z}_1^1 < 0$ and $-\bar{z}_2^1 > 0$) can be made arbitrarily large. In addition, large aggregate endowments and a discount factor smaller than one imply that it less costly in utility terms to transfer income across periods $t = 1$ and $t = 2$ than across $t = 0$ and $t = 1$. In combination with the assumption of 'symmetric endowments' this implies that $-\bar{z}_0^1$ is relatively small in absolute terms. In this way, parameter restriction can be found that control for the adverse price change of dq_0 for agent 1.

Claims (1) to (3) and the parameter restriction complete the proof of the proposition.□

2.5 Conclusion

In this paper we have given an example of a class of multiperiod incomplete market economies, for which a non-discriminating borrowing constraint leads to a Pareto improvement. The result supplements the literature on constrained Pareto inefficiency of general equilibria with incomplete markets. The knowledge requirement for the intervention is very low, and all prices are allowed to adjust, however, the economy is very special in its security structure, uncertainty structure, endowments and heterogeneity of agents. Because of these restrictions, the article serves as an illustration for a possible line of research on constrained Pareto inefficiency, namely the design of *easily implementable* mechanisms that generate Pareto improvements.

2.6 Appendix

Proof of Theorem (1)

We present a version of the proof which is based on the notion of a 'normalised no-arbitrage equilibrium'. The proof is standard.

For the notion of a no-arbitrage equilibrium we first state a standard lemma (without proof) that ensures that no arbitrage is equivalent to equation (2.2) and in turn guarantees the existence of positive state prices π.

Lemma 2

In the economy $\mathcal{E}(U,\omega,V)$, the following conditions are equivalent:
(i)The problem max $\{u^i(x^i) \mid x^i \in B^i(q,\omega^i,V)\}$ has a solution.
(ii)There are no arbitrage opportunities on the financial markets.
(iii) There is a vector of positive state prices $\pi \in \mathbb{R}^S_{++}$ such that $\pi W(q,V) = 0$.

Proof: see Magill and Quinzii (1996, p.73)

In order to transform the economy into one with state prices, first define the state prices for each node as:

$$\pi = (1, \pi(\xi_1), \pi(\xi_2), \pi(\xi_{12}), \pi(\xi_{22})) \in \mathbb{R}^5_{++} \tag{2.6}$$

such that (2.3) holds. Thus prices q can be written as:

$$q_0 = \pi_1 + \pi_2 \tag{2.7}$$

$$q_1 = \frac{\pi_{12}}{\pi_1}$$
$$q_2 = \frac{\pi_{22}}{\pi_2}$$

Inserting these equations into the budget constraint (2.1), and write the budget equations for ξ_1 and ξ_{12} and for ξ_2 and ξ_{22} in one equation.

$$\begin{aligned} \pi_1(x^i(\xi_1) - \omega^i(\xi_1)) + \pi_{12}(x^i(\xi^i_{12} - \omega^i(\xi_{12})) &= \pi_1 z^i(\xi_0) \\ \pi_2(x^i(\xi_2) - \omega^i(\xi_2)) + \pi_{22}(x^i(\xi^i_{22} - \omega^i(\xi_{22})) &= \pi_2 z^i(\xi_0) \end{aligned} \tag{2.8}$$

which can be written, using the 'successor box product', as:

$$\pi \underset{\xi_0^+}{\Box} (x^i - \omega^i) = \begin{bmatrix} \pi_1 \\ \pi_2 \end{bmatrix} z^i(\xi_0) \tag{2.9}$$

where

$$\pi \underset{\xi_0^+}{\Box} (x^i - \omega^i) = \sum_{\xi' \in \mathcal{D}} \pi(\xi')(x^i - \omega^i)(\xi')$$

In words, the box product is the vector of discounted values of net demand at each of the successors ξ' of ξ_0. Equivalently, (2.9) can be written as:

$$\pi \underset{\xi_0^+}{\Box} (x^i - \omega^i) \in \left\langle \begin{matrix} \pi_1 \\ \pi_2 \end{matrix} \right\rangle \tag{2.10}$$

where $\langle\rangle$ indicates the span. Using the state prices and the transformed budget equations (2.8), the original budget equation for date $t = 0$ can be rewritten as:

$$\pi(x^i - \omega^i) = 0$$

This completes the transformation of the budget set $B^i(q, \omega^i, V)$ into the state price budget set

$$B^i(\pi, \omega^i, V) = \left\{ x^i \in I\!R^{10}_+ \;\middle|\; \begin{array}{c} \pi(x^i - \omega^i) = 0 \\ \pi \; \square \; (x^i - \omega^i) \in \left\langle \begin{array}{c} \pi_1 \\ \pi_2 \end{array} \right\rangle \\ \xi_0^+ \end{array} \right\} \tag{2.11}$$

The transformation implies equivalence of the two budget sets. Since markets are incomplete between $t = 0$ and $t = 1$, there is an indeterminacy of state prices. We can hence use the 'Cass trick', and choose agent 1's present value vector to represent the security prices. Explicitly, by (2.3),

$$\bar{q} = \bar{\pi}^1 V \Longrightarrow \bar{\pi}^1 \in \{\pi \in I\!R^{\mathcal{D}}_{++} | \pi V = \bar{\pi} V\}$$

Consequently, agent 1's budget set reduces to

$$B^1(\pi, \omega^1) = \{x^1 \in I\!R^5_+ | \bar{\pi}(x^i - \omega^i) = 0\}$$

A no-arbitrage equilibrium for the economy $\mathcal{E}(U, \omega, V)$ can now be defined as a pair of state prices and allocations $(\bar{x}, \bar{\pi}) \in I\!R^{10}_+ \times I\!R^5_{++}$such that:
(i) $\bar{x}^1 = \arg\max \{u^1(x^1) | x^1 \in B^1(\bar{\pi}, \omega^1)\}$
(ii) $\bar{x}^2 = \arg\max \{u^2(x^2) | x^2 \in B^2(\bar{\pi}, \omega^2, V)\}$

(iii) $\sum_{i=1}^{i=2}(\bar{x}^i - \omega^i) = 0$

The reformulation allows to define demand functions (recall that $U^i(.)$ is strictly quasi-concave) for agents 1 and 2 which are functions of all the maximum elements for all possible state prices, ie..: $f^1(\pi)$ =arg max $\{u^1(x^1)|x^1 \in B^1(\pi,\omega^1)\}$ and $f_V^2(\pi)$ =arg max $\{u^2(x^2)|x^2 \in B^2(\pi,\omega^2,V)\}$. Since the budget correspondences are continuous and compact valued and convex valued correspondences, and $U^i(.)$ is strictly quasi-concave, by the Maximum Theorem the demand functions are continuous convex, compact and non-empty valued functions. Furthermore, the demand function for agents one and two satisfy $f^i(\alpha\pi) = f^i(\pi)$ for all $\alpha > 0$, for all $\pi \in I\!R^{\mathcal{D}}_{++}$ and $\pi f^i(\pi) = \pi\omega^i$ for all $\pi \in I\!R^{\mathcal{D}}_{++}$, and agent 1's demand satisfies the boundary condition that if $\pi^n \in I\!R^{\mathcal{D}}_{++}$ is such that $\pi^n \longrightarrow \pi \in \partial I\!R^{\mathcal{D}}_{++}$ and if $\pi\omega^1 > 0$, then $f^1(\pi^n) \longrightarrow \infty$ as $n \longrightarrow \infty$. The aggregate excess demand $Z_v(\pi) = f^1(\pi) - \omega^1 + f_V^2(\pi) - \omega^2$ inherits continuity, homogeneity, boundary behaviour, and Walras Law by the property of continuous compact correspondences. By Kakutani's theorem, a fixed point $Z_v(\pi) = 0$ of the economy $\mathcal{E}(U,\omega,V)$ exists. □.

Proof of Lemma (1)

The economy $\mathcal{E}(U,\omega,\tilde{V})$ is constructed by parameterising the economy $\mathcal{E}(U,\omega,V)$ with the equilibrium values of $\bar{z}^i(\xi_1)$ and $\bar{z}^i(\xi_2$. Since $u^i()$ is monotonically increasing and concave in x^i by assumption, it is also monotonically increasing and concave in z^i. Since the budget set is compact and u^i is continuous and strictly quasi-concave, the continuity is inherited in the solutions by the maximum theorem. The concavity of the maximisation problem guarantees that the constraints are binding at the maximum.□

Proof of Claim (1)

Given that there is only one good in the economy, we can write agent i's utility as the sum of the products of marginal utility times consumption:

$$u^i(x) = \bar\lambda_0^i x_0^i + \bar\lambda_1^i x_1^i + \bar\lambda_{12}^i x_{12}^i + \bar\lambda_2^i x_2^i + \bar\lambda_{22}^i x_{22}^i$$

where consumptions are given by the budget equations above. Using present value vectors, the total marginal change in utility induced by dz_1^i and dz_2^i at the equilibrium is:

$$\begin{aligned}
\frac{du^i}{\bar\lambda_0^i} &= dx_0^i + \bar\pi_1^i dx_1^i + \bar\pi_{12}^i dx_{12}^i + \bar\pi_2^i dx_2^i + \bar\pi_{22}^i dx_{22}^i \\
& \quad -dq_0 \bar z_0^i - q_0 dz_0^i + \\
&+ \bar\pi_1^i (dz_0^i - \bar q_1 dz_1^i - dq_1 \bar z_1^i) \\
&+ \bar\pi_{12}^i (dz_1^i) \\
&+ \bar\pi_2^i (dz_0^i - \bar q_2 dz_2^i - dq_2 \bar z_2^i) \\
&+ \bar\pi_{22}^i (dz_2^i)
\end{aligned}$$

which can be rewritten as

$$\begin{aligned}
&= -dq_0 \bar z_0^i - (\bar q_0 - \bar\pi_1^i - \bar\pi_2^i) dz_0^i \\
&- (\bar q_1 \bar\pi_1^i - \bar\pi_{12}^i) dz_1^i
\end{aligned}$$

$$
\begin{aligned}
&- \quad (\bar{q}_2\bar{\pi}_2^i - \bar{\pi}_{22}^i)dz_2^i \\
&- \quad \bar{\pi}_1^i dq_1 \bar{z}_1^i \\
&- \quad \bar{\pi}_2^i dq_2 \bar{z}_2^i
\end{aligned}
$$

Since we are considering marginal changes around the equilibrium, the first order conditions are still valid. Consequently the change in utility reduces to

$$\frac{du^i}{\bar{\lambda}_0^i} = dq_0(-\bar{z}_0^i) + \bar{\pi}_1^i dq_1(-\bar{z}_1^i) + \bar{\pi}_2^i dq_2(-\bar{z}_2^i) \qquad \square$$

Note that the constraint itself, dz_s^i for $i = 1, 2$, $s = 1, 2$ has no direct effect on utility, since the redistribution of wealth through the constraint is compensated in period $t = 2$ by $\bar{\pi}_{12}^i$. Since $q_s = \frac{\bar{\pi}_{s2}^i}{\bar{\pi}_s^i}$, the direct impact on wealth, $q_s\bar{\pi}_1^i dz_s^i$ is just equal to $\bar{\pi}_{s2}^i dz_s^i$. The only effect is through the portfolio reallocation in period $t = 0$, which, in turn changes the distribution of wealth in period $t = 1$ and induces reoptimisation at $t = 1$.

Proof of Claim (2)

Consider the version of the agents' maximisation problem which has the budget constraints substituted into the utility functions:

$$
\begin{aligned}
max \qquad & U^i(z, q) = u^i(w_0^i - q_0 z_0^i) \\
& +\tfrac{1}{2}(a_i u^i(w_1^i + z_0^i - q_1 z_1^i) + a_i^2 u^i(w_{12}^i + z_1^i)) \\
& +\tfrac{1}{2}(a_i u^i(w_2^i + z_0^i - q_2 z_2^i) + a_i^2 u^i(w_{22}^i + z_2^i))
\end{aligned}
$$

Collect the date 1 portfolio demands in a vector $z^i_{\mathbf{1}}$. Using this notation the restricted portfolio choice between date 0 and date 1 portfolios is $(z^i_0 - dz^i_0, z^i_{\mathbf{1}} + dz^i_{\mathbf{1}})$. We can measure the cost of the portfolio restriction by asking how much extra income agents need in period 0 to be just as well off with the restriction as without it. In other words, we want to solve the linear equation:

$$du^i_{z^i}(-dz^i_0, dz^i_{\mathbf{1}}) = 0 \tag{2.12}$$

Note that the borrowing constraint, viewed as a security, lies in the 'market subspace', ie.. the column span of $\langle V \rangle$. Since the differential is a linear functional, it can be represented using an inner product. For this purpose, define the marginal utility of an additional unit of portfolio as:

$$\mu^i_0(z^i) = \frac{\partial u^i(z^i)}{\partial z^i_0} \quad \textit{and} \quad \nabla_1 u^i(z^i) = (\frac{\partial u^i(z^i)}{\partial z^i_1}, \frac{\partial u^i(z^i)}{\partial z^i_2})$$

Then, equation (2.12) can be written as:

$$du^i_{z^i}(-dz^i_0, dz^i_{\mathbf{1}}) = [\mu^i_0(z^i), -dz^i_0] + [\nabla_1 u^i(z^i), dz^i_{\mathbf{1}}] = 0$$

Consequently the cost of marginal changes in portfolios in period 1 is:

$$c^i(dz^i_{\mathbf{1}}; x^i) = [\kappa^i_1(z^i), dz^i_{\mathbf{1}}]$$

where

$$\kappa^i_1(z^i) = \frac{\nabla_1 u^i(z^i)}{\mu^i_0(z^i)} = (\kappa^i_1, \kappa^i_2)$$

is agent i's present value of his portfolio holding.

The value of the trading restriction in $t = 0$ can then be written explicitly as:

$$c^i(dz^i_{\mathbf{1}}; x^i) = \frac{\partial u^i(\bar{z}^i)/\partial z^i_1}{\partial u^i(\bar{z}^i)/\partial z^i_0} dz^i_1 + \frac{\partial u^i(\bar{z}^i)/\partial z^i_2}{\partial u^i(\bar{z}^i)/\partial z^i_0} dz^i_2$$

Since the utility functions of the two types of agents are the same except for the discount factor, and states $s = 1$ and $s = 2$ are symmetric, the adjustment is higher for the patient agent. Since both agents solve the local maximisation problem

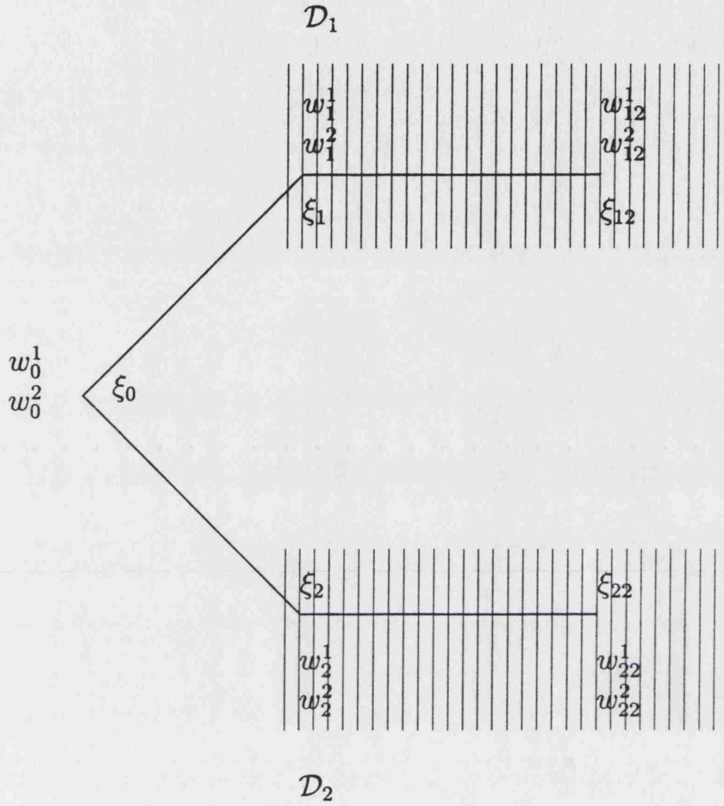
$$max \quad du^i_{z^i}(-dz^i_0, dz^i_{\mathbf{1}})$$

they both want to lend in period 0 (dz^i_0 is negative, they want to sell the security). However, only one agent can lend. The other one has to borrow. Therefore in equilibrium only the agent, whose cost is higher will lend. This agent, as we have shown, is the more patient agent. The price q_0 of the security will then adjust to equilibrate the marginal valuation of the bond income stream again. □

Proof of Claim (3)

To calculate the bond price change in period 1, we only need to look at the subeconomies $\mathcal{D}_1(\xi_1, \xi_{12})$ and $\mathcal{D}_2(\xi_2, \xi_{22})$, since the only determinants of bond prices in period 1 are preferences in period 1 and 2 and the distribution of endowments in those periods. In other words, for the purpose of price computations, the payoff of securities traded in period 0 influences the distribution of endowments in period 1 exogenously.

Fig.2 Subeconomies $\mathcal{D}_1(\xi_1, \xi_{12})$ and $\mathcal{D}_2(\xi_2, \xi_{22})$



Since $U^i(.)$ is strictly concave for both $i = 1, 2$, with utility of consumption in all states and interior aggregate endowments, $\omega^i(\xi_s) > 0$ for at least one i, the equilibrium in the subeconomies $\mathcal{D}_1(\xi_1, \xi_{12})$ is an interior equilibrium. By strict concavity, the slopes of the level sets of the functions: $v^i_{\mathcal{D}_1} = \alpha_i u^i(x_i(\xi_1)) + \alpha_i^2 u^i(x_i(\xi_{12})$ in the subeconomy $\mathcal{D}_1$ and $v^i_{\mathcal{D}_2} = \alpha_i u^i(x_i(\xi_2)) + \alpha_i^2 u^i(x_i(\xi_{22}))$ for $\mathcal{D}_2$ are, in equilibrium, equal for agents $i = 1, 2$ in each subeconomy. Since the argument is symmetric, restrict the analysis to one of the subeconomies, $\mathcal{D}_1$. Since markets are complete, and the bond from $t = 1$ to $t = 2$ pays out one unit, it is like an Arrow- security, and the equilibrium in the subeconomies is equivalent to a 'contingent market' equilibrium, meaning that the price of the bond is the relative price of consumption at $t = 1$. By the additive separability of the utility functions, a marginal increase in endowments in period $t = 1$ only has an effect on consumption in the same period, and not at $t = 2$. Take $s = 1$ as the starting point of an economy. Then the equilibrium in the subeconomy $\mathcal{D}_1$ can be represented by the following first-order conditions:

$$\begin{aligned} u'(\bar{x}^i(\xi_1))\bar{q} &= \alpha_i u'(\bar{x}^i(\xi_{12}) \quad i = 1, 2 \\ \bar{x}^i(\xi_1) + \bar{q}_1 \bar{x}^i(\xi_{12}) &= \omega^i(\xi_1) + \bar{q}_1 \omega^i(\xi_{12}) \quad i = 1, 2 \\ \bar{x}^1(\xi_j) + \bar{x}^2(\xi_j) &= \omega^1(\xi_j) + \omega^2(\xi_j) \quad j = 1, 12 \end{aligned} \tag{2.13}$$

Using the budget constraints and the pricing equations, the price of the bond can be written as:

$$q_1 = \frac{\frac{\alpha_1}{u'(\bar{x}^1(\xi_1))} + \frac{\alpha_2}{u'(\omega^1(\xi_1)+\omega^2(\xi_{12})-\bar{x}^1(\xi_0))}}{\frac{1}{u'(\bar{x}^1(\xi_{12}))} + \frac{1}{u'(\bar{x}^2(\xi_{12}))}} \tag{2.14}$$

By additive separability, the denominator will not change with a change in endowments at $t = 1$, and by market completeness the marginal utilities at $t = 1$ are equal across the two agents, $u'(\bar{x}^1)(\xi_1) = u'(\bar{x}^2)(\xi_1)$ Consequently, the marginal change in the price q_1,

$$dq_1 = \frac{\frac{\alpha_1}{u'(\bar{x}^1(\xi_1)+dx^1(\xi_1))} + \frac{\alpha_2}{u'(\omega^1(\xi_1)+\omega^2(\xi_{12})-\bar{x}^1(\xi_0)-dx^1(\xi_1)}}{\frac{1}{u'(\bar{x}^1(\xi_{12}))} + \frac{1}{u'(\bar{x}^2(\xi_{12}))}} \tag{2.15}$$

is positive, since $\alpha_1 > \alpha_2$. $\square$

Chapter 3

Signaling Credit Quality Independently of Contract Choice: a Non-Transactions Cost Approach to Swaps in Anonymous Markets

3.1 Introduction to the Problem

There is an informal argument in the literature that swaps can be used to signal future credit quality (eg. Litzenberger (1992)), when credit quality is unobservable by lenders. The verbal argument is simple: entrepreneurs prefer smooth repayment streams. Good risk entrepreneurs borrow short at every period and swap this variable rate repayment profile into a fixed repayment profile. Borrowing short means that entrepreneurs allow the market to evaluate their riskiness in every time period. If entrepreneurs have superior information of their own good future credit quality than the market, then they would like to give the market time to be able to reveal their good credit quality. Entering into a swap then allows borrowers to still retain a smooth consumption profile. Bad risk entrepreneurs, in contrast, will take out fixed rate finance from the start.

We construct a simple two-type, n-agent model for which this argument holds. However, it turns out that entrepreneurs could equally well signal their good credit quality by borrowing variable and then smoothing their consumption stream by acting as investors in the asset market. Consequently, we need an extra restriction on asset trades. The natural restriction is that entrepreneurs cannot at the same time be investors in the asset markets. Under this condition, entrepreneurs cannot smooth consumption over and above their original credit commitment.

It is easy to see that if either of the two trading restrictions - asymmetric information or the one-side constraint - is not present, then the swap contract would be redundant:

If there is no asymmetric information, then agents will, by the maximum property of their portfolio choice problem, always trade their best portfolio at the initial date [1]. With our assumptions of consumption smoothing

[1]In a different context see Kajii (1995).

over time and the structure of endowments, both types of entrepreneurs will choose the fixed bond contract at their respective publicly observable likelihood of default.
If there are no one-side restrictions and entrepreneurs are allowed to trade as investors at $t = 1$, then good risk entrepreneurs will choose the variable rate bond and smooth their repayments through additional investment in period $t = 1$, while bad risk entrepreneurs will opt for the fixed coupon bond contract.

It could be argued that, in our context, the assumption of a one-side constraint is a natural one to make. There is only one class of projects available, so that in the case that entrepreneurs decided to invest as well as borrow in period $t = 1$, they would finance their own projects, which seems somewhat artificial. A situation in which we would observe asset positions of this kind (on both sides of the market) is usually one where arbitrage opportunities exist. Precisely the problem of arbitrage opportunities also arises in our theoretical setting of competitive markets with asymmetric information. The way by which we make the competitive model compatible with asymmetric information is to introduce one-side constraints and pool securities. Thus the one-side constraint solves the arbitrage problem present in the competitive set-up, and, at the same time, gives rise to the non-redundancy of swaps. Since, in this way, it is a requirement of the theory employed, it appears much more palatable as an assumption for the functioning of a swap market.

3.1.1 Related Literature

The literature on the existence of swaps is often informal and, if models are introduced, almost always focuses on the bilateral exchange nature of swaps. There has been a steady development in the sophistication of the argument put forward for the existence of swaps, and, given the enormous size of the swap market, there is the perception of a need to explain why swaps are traded beyond the obvious, if intuitive, assertion that they save on transactions costs.

The first idea for the existence of swaps was 'comparative advantage' (Wichmann (1988), Simons (1989)). For some exogenous reason, one type of entrepreneur is seen as having a borrowing advantage in a fixed rate market, while another entrepreneur has an advantage in a variable rate market. If the preferences of these firms are the reverse of their respective comparative advantages, then they can profitably swap their coupon payments. In particular, it was observed that firms with a good reputation in financial markets are often the only ones that can issue fixed long bonds. If these firms are also the ones which are less averse to 'payment smoothing', then they can profitably swap their finance with a smaller or lesser known firm. It was soon realised that this explanation relies on some kind of market imperfection [2]. Without any frictions, borrowers could exploit their comparative advantage directly.

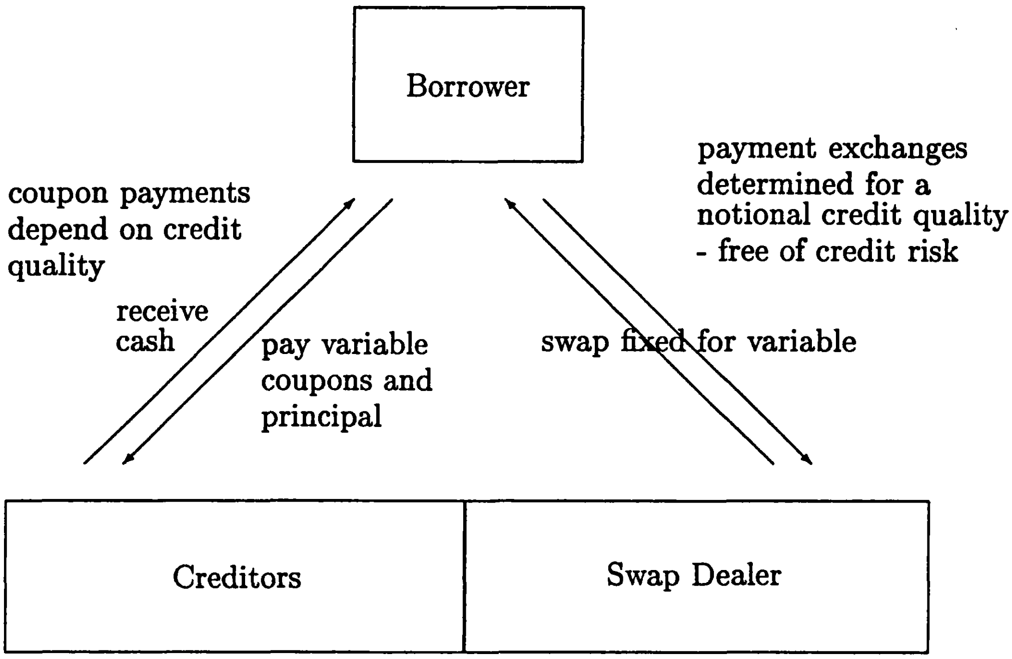
[2]There is another strand of literature on swaps, which emphasises default arrangements (Cooper and Mello (1991), Duffie and Huang (1996)).

A fundamental friction in the credit market is the asymmetric information between borrowers and lenders. The swaps literature has shifted its attention to these asymmetries, and recent articles view swaps as a contract choice that signals credit quality (Simmons (1993), Litzenberger (1992)). The present model is written along these lines of the recent development in the literature, and uses swaps to allow a good risk borrower the signaling of his good credit quality.

However, one essential difference to the established literature is that we take a completely unilateral and anonymous view of the swap market. There is a swap dealer, who, at the same time, is also the pool of investors providing project finance. In this view, swaps are regarded as not being very different from bonds. They promise a repayment stream with a future type addition that they pay more if the credit quality of the borrower deteriorates. According to this view, the 'price' of a fixed-for-variable swap is the interest rate on a fixed payment profile which sets the market valuation of that payment stream equal to the market valuation of a variable rate payment stream, for the same duration, and *for the same credit quality.* The variable counterparty is then obliged to pay, to its own variable creditors, any additional payments that result out of a deterioration of its credit quality.

In the model, the underlying agreed swap rate is the fixed rate of the good borrowers. Should the borrower turn out to be bad, then he has to make an additional payment to the lenders, over and above his payment to the swap dealer. For simplicity, the two coincide, so that, in our specific setting, they effectively buy a credit contract which has a fixed 'floor' and the option to top up payments, should credit quality deteriorate. Figure (3.1.1) illustrates the view of the swap market in the article.

Fig.(3.1.1) A uni-lateral View of the Swap Market



In an extension to the model we explicitly introduce a swap counterparty that issues fixed bonds and swaps them for variable bonds. The counterparty is constructed in such a way that it is indifferent between entering into the swap and issuing the variable bonds directly. The extension is written with the purpose to illustrate that concentrating on one party in a swap contract is not a necessity, and is done to emphasise the anonymity of the market and a unilateral view of swap transactions.

In the following, the next section sets up the uncertainty structure of the model. Section (3.2) describes the uncertainty and security structure, the agents, and the equilibrium concept. Section (3.3) provides a formal development of an equilibrium with swaps and consists of our main proposition. Section (3.4) extends the model to take account of a counterparty. Section (3.5) concludes. Proofs are contained in the appendix.

3.2 Structure of the Model

3.2.1 Uncertainty

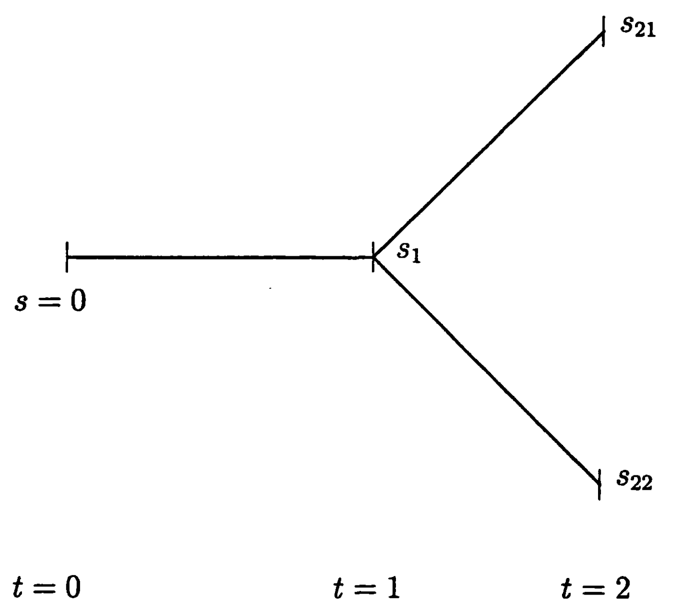
The model we are using is based on the general equilibrium model of asymmetric information, as, for example, in Bisin, Geanakoplos, Gottardi, Minelli and Polemarchakis (1998). The economy is a pure finance economy with one physical good, whose price is normalised to one. There are investors and entrepreneurs with projects, over which they have asymmetric information. Investors lend to entrepreneurs (or borrowers) to finance their projects. There are three time periods, $t = 0, 1, 2$ and two states in period $t = 2$ for every project. There are two types of entrepreneurs, denoted by $\theta \in \{\theta_1, \theta_2\}$, and the number of entrepreneurs of each type is countably infinite, with an entrepreneur of a given type indexed by $n = 1, \cdots, \infty$. An entrepreneur is then identified by the tuple $\{\theta, n\}$. The proportion of entrepreneurs of a type θ in the economy is called λ^{θ}, and, since there are only two types, $\lambda^{\theta_1} = 1 - \lambda^{\theta_2}$. Every entrepreneur $\{\theta, n\}$ has a project which requires an investment of one unit of the numéraire commodity in $t = 0$ and which has a safe interim payoff of K_1 units at $t = 1$ and an uncertain payoff of K_2 units if successful in period $t = 2$, where $K = \{K_1, K_2\}$ is such that the value of the project is positive. Types are identified by the riskiness of their projects. Projects are treated implicitly. Entrepreneurs need to raise one unit of *endowment* in the first period to finance the project. If they choose to do so, they will obtain payouts (K_1, K_2) in periods $t = 1$ and $t = 2$ respectively, if the project is successful. Entrepreneurs have no utility of consumption in period $t = 0$, and are therefore only concerned about the repayment stream on the financing of the project.

When describing the states in period $t = 2$, individual states need to be distinguished from social (or aggregate) states. Individual states refer to the outcomes of the individual projects of every agent. Social or aggregate states are then all possible combinations of outcomes of the individual projects. Aggregate states will be discussed below.

The two individual states in period $t = 2$ are referred to as the *success* state s_{21} and the *default* state s_{22}. Define a payoff in a state s as R_s. Then $R_{21} = K_2$ is the payoff in the success state, while $R_{22} = 0$ is the payoff in the default state. This description defines a random variable $\tilde{R} : \Omega \longrightarrow I\!R_+$ with finite support $\{R_{21}, R_{22}\}$ on a probability space $(\Omega, \mathcal{F}, P)$. The payoffs in an individual state are the same for the two types of entrepreneurs, and types are only distinct in the likelihood that a certain state occurs. Consequently the measure $p_s(\theta)$ is type-specific. In order to clarify the dependence of the distribution of the random variable $\tilde{R}$ on θ and n, we use the shorthand $\tilde{R}^{\theta,n}$.

To complete the temporal structure of the model, we collect all the states s in the set $S = \{s_0, s_1, s_{21}, s_{22}\}$ and introduce a set of partitions $\mathbb{F} = \{\mathbb{F}_1, \mathbb{F}_2\}$ on the set of states S. The temporal structure from the point of view of entrepreneurs can be depicted in the following tree:

Fig.1 Uncertainty Structure for Entrepreneurs



We order the two types of entrepreneurs θ_1 and θ_2 in the following simple way, and describe the information structure by assumption (2):

Assumption 1 (Riskiness of Projects in the Economy)
Projects of type θ_1 are the 'good risk projects', ie.

$$p_{s_{21}}(\theta_1) > p_{s_{21}}(\theta_2) \tag{3.1}$$

The probability measures are such that the net present value of the projects is positive.

Assumption 2 (Asymmetric Information)
The probabilities $p_{s_{21}}(\theta_1)$, $p_{s_{21}}(\theta_2)$ are private information of the entrepreneurs at $t = 0$. However, they become publicly known at $t = 1$.

We now have to describe how the random variables for every individual entrepreneur $\{\theta, n\}$ are combined to form aggregate states.

Essentially, we have to make assumptions on the correlation between different projects. We assume the following:

Assumption 3 (Correlation of Project Payoffs)
$\{\tilde{R}^{\theta,n}\}_{\theta,n}$ are mutually independent, and for every θ, $\{\tilde{R}^{\theta,n}\}_n$ are identically and independently distributed across n.

Assumption (3) allows to invoke the law of large numbers. In the limit for the number of projects of each type approaching infinity, it holds that:

$$\frac{1}{N}\sum_{n=1}^{N} \overset{d}{\longrightarrow} E[\tilde{R}^{\theta,n}] = \mu \tag{3.2}$$

where μ is the vector $[\mu(\theta_1), \mu(\theta_2)]$.

The law of large numbers ensures that if infinitely many projects with the characteristics of assumption (3) are pooled together, then the payoff of

$\tilde{R}_\sigma$ is constant across all states. From the point of view of investors, there is no uncertainty in the economy. All the states are equivalent and can hence be written as one single state. This state is denoted by σ.

3.2.2 Project Finance

As we have stated before, entrepreneurs want to finance a project that costs one unit of the numéraire good in $t = 0$ and pays out K_1 units of the good in $t = 1$ for sure and K_2 units at $t = 2$ if successful. We impose certain restrictions on the model that allow us to get explicit results regarding the use of swaps.

The project is indivisible. We choose utility functions such that all entrepreneurs undertake the project, and only the method of finance remains as a choice variable for them. There are three forms of financing available: a two-period bond, which the entrepreneur issues at $t = 0$ and which pays the same coupon at $t = 1$ and in the success state s_{21} (together with repayment of the principal in that state); two one-period bonds, which are issued at $t = 0$ and $t = 1$, respectively, and which pay coupons and principal at $t = 1$ and in state s_{21}; and two one-period bonds and a fixed-for-variable interest rate swap, in which case the bonds are issued at $t = 0$ and $t = 1$, the principal is repaid at $t = 1$ and in s_{21}, but the coupons paid out at $t = 1$ and in state s_{21} are swapped and hence the same for both states. The three methods of finance are abbreviated as f for fixed, v for variable, and sw for swapped.

In contrast to the usual discount formulation of the price of financial securities, here we take the interest payments as variable, while the issue price is fixed and equal to one[3]. A coupon payment in state s is denoted by V_s^j, and the portfolio holding in a bond for a type θ is $z^{\theta,j}$ where $j = f, v, sw$. For the two standard methods of finance, the following relations hold: $V_{s_1}^f(\theta) = V_{s_{21}}^f(\theta)$ and $V_{s_1}^v \neq V_{s_{21}}^v(\theta)$.

While variable and fixed coupon finance are conventional, swap finance warrants more explanation: the way we define a fixed-for-variable swap is that it is a promise at $t = 0$ to pay a fixed coupon based on a presumed probability of success $p_{s_{21}}(\theta_1)$. Should the true probability of success turn out to be lower, then the issuer tops up the payment by the difference between the variable rate for that period for type θ_2, and the fixed rate.

Definition 1 (Swap Finance)

Swap finance consists of the coupons payments:

$$\left\{V_{s_1}^{sw} = V_{s_1}^f(\theta_1)\right\} \quad (3.3)$$

$$\left\{\begin{array}{ll} V_{s_{21}}^{sw}(\theta) = V_{s_{21}}^f(\theta_1) & if \ p_{s_{21}}(\theta) = p_{s_{21}}(\theta_1) \\ V_{s_{21}}^{sw}(\theta) = V_{s_{21}}^f(\theta_1) + \left(V_{s_{21}}^v(\theta_2) - V_{s_{21}}^f(\theta_1)\right) & if \ p_{s_{21}}(\theta) = p_{s_{21}}(\theta_2) \end{array}\right\}$$

The way we look at swap finance here is completely unilateral from the point of view of the entrepreneur. This is made possible by the set-up of anonymous markets and pool securities, which are discussed below. Essentially, we look at the net pay-off of swaps. From this point of view, swaps simply introduce an 'future' type element into the bond contract: should the

[3]In other words, all bonds are sold at par.

basis for the swap (the variable coupon rate) not turn out to be true, then the issuer must make excess payments.

To ensure that agents are price takers, we need to view the securities as already existing in the market. All three types of securities are available for the financing decision of entrepreneurs, but not all of them will be traded in equilibrium.

3.2.3 Utility Functions and Optimisation Problems

Entrepreneurs

Entrepreneurs are assumed to have utility of consumption in periods $t = 1$ and $t = 2$ only. We make the following assumption on entrepreneurs' utility functions. Since all entrepreneurs of the same type take the same action[4], the superscript 'n' will be omitted.

Assumption 4 (Assumption on Entrepreneurs' Utility Functions) *For each state $s \in S \setminus \{s_0\}$ the utility function u_s^θ is increasing, linear, and time independent. Furthermore, U^θ is additively separable. Explicitly:*

$$U^\theta(c^\theta) = u^\theta(c^\theta_{s_1}) + u^\theta(c^\theta_{s_{21}}) + u^\theta(c^\theta_{s_{22}}) \tag{3.4}$$

[4]Contrary to BGGMP, the 'θ, n' construction is mainly used to generate logically consistent pool securities. All the asymmetric information is contained in the type, and not, as in their work, in different members of the same type.

Entrepreneurs are assumed to have no endowments at $t = 0$, and sufficiently high project payouts at $t = 1$ and in s_{21} to repay interest on the financing of their project. Since entrepreneurs' projects are sold for one unit of investment in period $t = 0$ and pay out K_2 units in state s_{21}, entrepreneurs' optimisation programs are as follows [5]:

$$\max_{z^\theta} \quad U^\theta(c^\theta) \tag{3.5}$$

$$\begin{aligned}
c^\theta_{s_1} &= K_1 + V^f_{s_1}(\theta) z^{\theta,f} + V^v_{s_1} z^{\theta,v} + V^{sw}_{s_1}(\theta) z^{\theta,sw} \\
c^\theta_{s_{21}} &= K_2 + (V^f_{s_{21}}(\theta)+1) z^{\theta,f} + (V^v_{s_{21}}(\theta)+1) z^{\theta,v} + (V^{sw}_{s_{21}}(\theta)+1) z^{\theta,sw} \\
c^\theta_{s_{22}} &= 0
\end{aligned}$$

Let the budget constraint be denoted by $B^\theta(K, V^\theta)$ where V^θ is the matrix of security returns. We assume that project finance cannot be split, so that all the porfolio variables z^θ either take on the value zero or minus one. In addition, we impose a particularly strong form of a one-side constraint:

Assumption 5 (No retrade at $t = 1$ for entrepreneurs)
$z^{\theta,j} = z^{\theta,j}_{s_0} = z^{\theta,j}_{s_1}$, $j = f, v, sw$, which implies that entrepreneurs cannot refinance their projects at $t = 1$, and $z^{\theta,j} \leq 0$, entrepreneurs are only on the issue side of the asset market.

The way we model entrepreneurs may seem overly restrictive at first sight. However, it is a natural multiperiod extension of an Allen and Gale (1991,

[5]The form of the entrepreneurs' optimisation program is developed explicitly in the appendix.

1994) type set-up. In Allen and Gale, entrepreneurs have an uncertain income stream in the second period of the model, which they want to sell to maximise their utility in the first period. In fact, entrepreneurs disappear completely after the first period. Here, the assumption of linear utilities of entrepreneurs combined with the 'no retrade' restriction and the construction that entrepreneurs do not consume in period $t = 0$ serves a similar purpose. It implies that entrepreneurs want to issue the security which has the lowest coupon payments, ie. which allows them to invest one unit in period $t = 0$ with the minimum reduction in consumption in periods $t = 1$ and $t = 2$, regardless of the payment profile.

Investors

We assume that lenders want to smooth their consumption over time and have sufficient endowments to buy the contracts offered by entrepreneurs. Since there is no uncertainty for investors, there is no loss of generality in only considering one aggregate state σ. There are countably infinite identical investors with utility functions given by:

Assumption 6 (Investors' Utility Functions)
Investors i have utility functions $U^i(c^i)$, which are continuous and strictly quasi-concave in every period, time and state independent and additively separable. The relevant states for investors are the equivalent aggregate states σ. Thus:

$$U^i(c^i) = u^i(c_0^i) + u^i(c_1^i) + u^i(c_\sigma^i) \tag{3.6}$$

The aim of the paper is to model swaps in an anonymous market. In particular, we want to abstract from the notion that a swap is a bilateral arrangement. Instead, we think of swaps as trades in risk profiles. We model the agent who offers these trades as the pool of investors. In addition to swap finance, the pool of investors also buys the fixed and variable bond finance described in the a preceding paragraph. Thus it acts as a counterparty to all borrowers. The pool is hence the swapdealer and the counterparty at the same time.

Investors invest into pools. They buy the project finance of entrepreneurs in period one, in return for which they get a coupon payment at $t = 1$ and in the success state s_{21}, and their principal is returned to them in s_{21}. However, they do not buy individual securities, but rather a share in the pool of all the securities issued by borrowers. A pool security is constructed in the following way: the coupons of the individual securities $V^j(\theta)$, $j = f, v, sw$, are paid into a pool (a separate pool for each risk class). Depending on the riskiness of entrepreneurs, the deliveries into the pool differ. The actual deliveries then define the coupon in the respective pool securities $V^{p,j}(\theta)$, $j = f, v, sw$. In other words, the coupons of pool securities is determined by the actual deliveries of entrepreneurs, whereas the value of the individual securities is determined by demand and supply in the securities market. Using assumption (3), in the limit for the number of entrepreneurs of every type approaching infinity, the coupons on pool securities are[6]:

[6]Strictly speaking, the payoff of the pool security stated is only the limit payoff for the number of projects $n \longrightarrow \infty$, ie. equation (3.7) should read $V_k^{p,j}$ $a.s.$.

$$V_k^{p,j}(\theta) = \frac{\sum_\theta \lambda^\theta z^{\theta,j}\left(\sum_s p_s V_s^j(\theta)\right))}{\sum_\theta \lambda^\theta z^{\theta,j}} \quad j = f, v, sw \;\; k = 1, \sigma \;\; s = s_{21} \qquad (3.7)$$

If only one type of security is pooled, the vector of payoffs is simply:

$$V^{p,j}(\theta_k) = \left[\begin{array}{cc} V_{s_1}^j(\theta_k) & p_{s_{21}} V_{s_{21}}^j(\theta_k) \end{array}\right] \qquad k = 1, 2 \qquad (3.8)$$

We also need to assume rational expectations on the delivery rates of the coupons, in other words, that investors know the proportion of entrepreneurs who default.

Investors are assumed to have endowments in period $t = 0$ only. They buy shares in whichever pool securities are issued. We assume that the number of investors is the same as the number of entrepreneurs, so that every investor invests a total of one unit into the pool securities. Combining the security structure and the utility functions of investors, their optimisation program becomes:

$$\begin{aligned} \underset{z^i}{max} \quad & U^i(c^i) & (3.9) \\ c_{s_0}^i &= \omega_{s_0}^i - z_p^{i,f} - z_p^{i,v} - z_p^{i,sw} \\ c_{s_1}^i &= V_1^{p,f}(\theta) z_p^{i,f} + V_1^{p,v}(\theta) z_p^{i,v} + V_1^{p,sw}(\theta) z_p^{i,sw} \\ c_\sigma^i &= V_\sigma^{p,f}(\theta) z_p^{i,f} + V^{p,v}(\theta)_\sigma z_p^{i,v} + V_\sigma^{p,sw}(\theta) z_p^{i,sw} + 1 \end{aligned}$$

where $V_k^{p,j}(\theta) = \left[\begin{array}{cc} V_k^{p,j}(\theta_1) & V_k^{p,j}(\theta_2) \end{array} \right]$ are row vectors, $z_p^{i,j} = \left[\begin{array}{cc} z_p^{i,j(\theta_1)} & z_p^{i,j(\theta_2)} \end{array} \right]'$ are column vectors, for $j = f, v, sw$, $k = 1, \sigma$. The portfolios at a time period add to one and are all positive.

Analogous to the 'no retrade' restriction we imposed on entrepreneurs, we make a similar assumption for investors:

Assumption 7 (No retrade at $t = 1$ for investors)
No asset retrade is permitted at $t = 1$ for investors.

With this assumption, there is essentially 'autarky' between periods $t = 1$ and $t = 2$. In this way, the only security trade in the economy is that entrepreneurs finance their project by issuing one unit of one of the three securities. Their consumption stream will be $\left(0, K_1 - V_{s_1}^j(\theta), K_2 - V_{s_{21}}^j(\theta)\right)$, $j = f, v, sw$. Investors' consumption stream is $\left(\omega_{s_0}^i - z_p^{i,j}, V_{s_1}^{j,p}(\theta), V_{s_{21}}^{j,p}(\theta)\right)$, $j = f, v, sw$. Since the project has a positive net present value, and entrepreneurs' utility functions are such that they are indifferent between income in the different periods, they will not want to issue more securities in period $t = 0$.

Autarky between $t = 1$ and $t = 2$ means that the marginal utilities of income in a state, or even for the same consumption streams, are not forced into equality in equilibrium. This allows the agents to have different marginal valuations of the same income stream, ie. the same security.

Market Clearing

Since we have strictly separated the market into its supply and demand sides, no additional concerns arise out of the introduction of asymmetric information. The market clearing conditions are that the total supply of individual asymmetric information securities by entrepreneurs is equal to the total demand by the pool of investors, and that the deliveries on the contracts are feasible.

$$z^{i,j} = \sum_{\theta} \lambda^{\theta} z^{\theta,j} \quad j = f, v, sw \tag{3.10}$$

implies

$$\sum_{\theta} \lambda^{\theta} V_s^{\theta,j} \leq \sum_{\theta} \lambda^{\theta} K_s \quad s = s_1, s_{21} \tag{3.11}$$

By our assumption on endowments of entrepreneurs in states $s_1, 21$, this condition is trivially satisfied.

3.3 A Separating Equilibrium with Swaps

There are two issues when establishing the separating equilibrium with swaps. The first issue is to determine the securities which are traded in equilibrium. Then, given these securities, it must be shown that the existence of an equilibrium can be established. Which securities are traded is determined by the incentive compatibility constraints in the separating equilibrium.

We attack the problem in the following way: we describe a separating equilibrium with swaps by a set of conditions. We then argue that an equilibrium exists if these conditions are met. In the next section, and our main proposition, we proceed to show that given the structure of the model, the conditions of the separating equilibrium are indeed fulfilled.

Define $V^{p,j} = \begin{bmatrix} V_1^{p,j} & V_\sigma^{p,j} \end{bmatrix}$, where $V^{p,j} = \begin{bmatrix} V^{p,f} \\ V^{p,v} \\ V^{p,sw} \end{bmatrix}$, as the matrix of pool security payoffs, and $V^j = \begin{bmatrix} V_1^j & V_\sigma^j \end{bmatrix}$, where $V^j = \begin{bmatrix} V^f \\ V^v \\ V^{sw} \end{bmatrix}$, as the matrix of individual security payoffs. Denote vectors of consumption by $c^i = \begin{bmatrix} c_0^i & c_{s_1}^i & c_{s_{21}}^i \end{bmatrix}$, and $c^\theta = \begin{bmatrix} c_0^\theta & c_1^\theta & c_\sigma^\theta \end{bmatrix}$, and vectors of portfolios by $z_p^i = \begin{bmatrix} z_p^{i,f} & z_p^{i,v} & z_p^{i,sw} \end{bmatrix}$ and $z^\theta = \begin{bmatrix} z^\theta & z^{\theta,v} & z^{\theta,sw} \end{bmatrix}$.

A separating equilibrium with swaps is a collection $((c^i, c^\theta); (z^i, z^\theta); V^{p,j}, V^j) \in \mathbb{R}_+^{3\theta} \times \mathbb{R}_+^3 \times \mathbb{R}^{3\theta} \times \mathbb{R}^3 \times \mathbb{R}_+^6 \times \mathbb{R}_+^6$ for all i, for all θ s.t.

agents choose portfolios to maximise their utility of consumption

(i)$(c^i, z^i) \in \arg\max \{U^i(c^i) | (c^i, z^i) \in B^i(\omega^i, V^i)\}$

(ii) $(c^\theta, z^\theta) \in \arg\max \{U^\theta(c^\theta) | (c^\theta, z^\theta) \in B^\theta(K, V^j)\}$

security markets clear and deliveries are feasible

(iii) $z^{i,j} = \sum_\theta \lambda^\theta z^{\theta,j} \quad j = f, v, sw \Longrightarrow \sum_\theta V_s^{\theta,j} \leq \sum_\theta K_s \quad s = s_1, s_{21}$

θ_1 issues swaps and θ_2 issues fixed rate finance

(iv) $z^{\theta_1,v} = z^{\theta_1,f} = z^{\theta_2,v} = z^{\theta_2,sw} = 0$

(v) $z^{\theta_1,sw} = z^{\theta_2,f} = -1$

(vi) $V^{p,sw(\theta_1)} = V^{p,f(\theta_2)}$

Points (iv)-(vi) are restrictions on agents' trades; given these restrictions, asymmetric information disappears, since the distinct trades of the two types θ_1 and θ_2 reveals the riskiness of their projects. Then, the pool security construction generates two different pool securities for the two agents and the model is transformed into a standard general equilibrium problem, which can be shown to exist by standard arguments.

3.3.1 Structure of Traded Securities

It remains to demonstrate that the restrictions on securities traded encompassed in (iv) to (vi) is the equilibrium choice of the agents, in other words it satisfies (i) to (iii). In terms of agency theory, (ii) is the incentive compatibility constraint in the model. We need to show that if (iv) to (vi) hold, then (ii) (as well as the investors' maximisation program and the market clearing condition) is satisfied :

Proposition 1 *In the economy, the only equilibrium is separating. The good risk borrowers θ_1 sell the variable/fixed-for-variable swap finance, while the bad risk borrowers θ_2 issue fixed bonds.*

Proof:

If a separating equilibrium exists, then it will reveal entrepreneurs' types θ_1 and θ_2, and the probabilities $p_s(\theta)$ $\forall s, \theta = \theta_1, \theta_2$ become public at $t = 0$. This, and the independence assumption on the projects, allows us to rewrite the *type specific probabilities* set-up in an equivalent *common probabilities* framework. Since there are two types in the original economy with projects that have payouts with different probabilities in the two individual states s_{21} and s_{22}, the model can be rewritten as a four state model in which the two types of entrepreneurs issue different securities and the probabilities of the states are *binomial.* The following lemma states that this is indeed the case.

Lemma 1

The binomial model with common probabilities and the model with type specific probabilities are equivalent.

Investors evaluate issued and unissued securities using their own valuation of pooled securities. Then entrepreneurs, who only care about minimising the expected value of repayments using their own non-smoothing preferences, issue that security which has a higher valuation by investors. We show that this security is the fixed bond security.

Lemma 2

With assumption (6) and the one-side restriction of assumption (7), investors have a higher valuation for project finance with a lower variance.

Given that investors dislike variance, it needs to be established that the coupons of the individual fixed projects do yield a lower variance in the pool securities.

Lemma 3

Fixed project finance yields a lower variance of the pool repayments.

It is then required to show that, given the variance dislike of investors and the security structure, entrepreneurs prefer to issue fixed rate bonds.

Lemma 4

All entrepreneurs prefer to issue fixed rate bonds.

It remains to show that good risk borrowers issue the swap, while bad risk borrowers issue fixed rate finance, and that bad risk borrowers have no incentive to imitate.

Lemma 5

θ_1 type borrowers issue the swap, while θ_2 type borrowers issue fixed rate finance.

•

Hence the equilibrium is separating and fulfills conditions (iv) to (vi) in section (3.3). Thus we have characterised the separating equilibrium with swaps of section (3.3). □

3.4 Extension: Explicit Modeling of the Swap Counterparty

Our model takes a simplified unilateral view of the swap market. There is no explicit counterparty to the swap which borrows fixed and then swaps variable for fixed. The swap is seen as a normal bond contract, but with the option of increased payments to investors should the borrowers' credit quality be worse than expected in period $t = 0$.

It is straightforward to extend the model to take into account a variable-for-fixed counterparty for one particular case. This case is the one in which the variable-for-fixed counterparty is a 'known' borrower, meaning a borrower who does not suffer from asymmetric information over his project. It could be argued that this is a relevant case in practice: variable-for-fixed swap counterparties are often governments or state owned banks with a known high credit rating.

We make the following assumptions on the preferences of the variable-for-fixed counterparty. Let the superscript τ denote the actions of that party.

Assumption 8 (Preference of Variable-for-Fixed Swap Counterparty)

For each state $s \in S \setminus \{s_0\}$ the utility function u^{τ}_{s} is continuous, increasing, strictly concave, time independent and additively separable:

$$U^{\tau}(c^{\tau}) = u^{\tau}_{s_1}(c^{\tau}_{s_1}) + u^{\tau}_{s_{21}}(c^{\tau}_{s_{21}}) + u^{\tau}_{s_{22}}(c^{\tau}_{s_{22}}) \tag{3.12}$$

Counterparty swap finance is defined in the same way as the swap finance of definition one. However, we make the assumption *that the swap counterparty has a project with a known probability of success, ie.* $p_{s_{21}}(\tau)$ *is known at* $t = 0$. To simplify even further, we assume that the probability of success of the counterparty project is the same as the probability of success of the good risk firm. We assume that $p_{s_{21}}(\tau) = p_{s_{21}}(\theta_1)$. This ensures that the pricing for any security issued by the party and the counterparty have the same present value in a separating equilibrium and that, consequently they can swap the coupon payments without any alteration.

Introducing the counterparty brings one further complication: there are now two effects of default. Firstly, bondholders lose their principal and the interest to which they are entitled to, but, additionally, the swap closes out prematurely. Like all financial securities, by no arbitrage a swap is an exchange of payments such that the net present value of the transaction is zero. From a *bilateral* viewpoint, the difference with bonds is that there is no intertemporal element. Both parties pay and receive payments in all periods. Hence, when one of the parties defaults and swap payments have already been made, the net present value of the outstanding payments is generally different from zero. This must be taken into account in the pricing of the swap. In our set-up, in addition to the exchanges of coupon payments in period $t = 1$, one of the parties must make a payment which is just equal to the net present value of its gain should one of the parties default. We have shown that the fixed coupon payment is higher than the variable coupon payment in period $t = 1$. Given that the net present values of the two payment streams, under the probability measure of the investors, must be equal, this implies that the fixed coupon payment in $t = 2$ is smaller than the variable coupon payment. Thus, in a swap agreement, the fixed-for-variable party *receives* a larger fixed payment at $t = 1$ and must *make* a larger payment at $t = 2$. If any one of the counterparties defaults at $t = 2$, it is the fixed-for-variable party that stands to gain. When entering into the swap, it must pay the present value of the expected gain of default to the variable-for-fixed party. The relevant state prices for the calculation of this swap default premium are the consumption smoothing state prices of the swap counterparty. Given our set-up, the swap premium is $\frac{\pi^{\tau}_{s22}}{\pi^{\tau}_{s_1}}\left(V^v_{s_{21}}(\theta_1) - V^f(\theta_1)\right)$. In period $t = 2$, the fixed-for-variable party's repayment is lowered by the probability that default occurs multiplied by the difference of the variable minus the fixed coupon. The repayment is reduced by $p_{s_{21}}(\tau)(V^v_{s_{21}}(\theta_1) - V^f(\theta_1))$[7]. For the fixed-for-variable party to still enter into the swap, we need that $q^f(\theta_1) + \left[\frac{\pi^{\tau}_{s22}}{\pi^{\tau}_{s_1}} - p_{s_{21}}(\tau)\right]\left(V^v_{s_{21}}(\theta_1) - V^f(\theta_1)\right) < q^v_{s_0} + q^v_{s_1}(\theta_1)$.

Modified swap finance is:

$$\left\{V^{sw}_{s_1}(\theta_1) = V^f_{s_1}(\theta_1) + \frac{\pi^{\tau}_{s22}}{\pi^{\tau}_{s_1}}\left(V^v_{s_{21}}(\theta_1) - V^f(\theta_1)\right)\right\} \tag{3.13}$$

$$\begin{cases} V^{sw}_{s_{21}}(\theta_1) = V^f_{s_{21}}(\theta_1) - p_{s_{21}}(\tau)(V^v_{s_{21}}(\theta_1) - V^f(\theta_1)) & p_{s_{21}}(\theta) = p_{s_{21}}(\theta_1) \\ V^{sw}_{s_{21}}(\theta_1) = V^f_{s_{21}}(\theta_1) + \left(V^v_{s_{21}}(\theta_2) - V^f_{s_{21}}(\theta_1)\right) - p_{s_{21}}(\tau)(V^v_{s_{21}}(\theta_1) - V^f(\theta_1)) & p_{s_{21}}(\theta) = p_{s_{21}}(\theta_2) \end{cases}$$

Counterparty swap finance becomes:

Definition 2 (Counterparty Swap Finance)

[7]Implicitly, the swaps constructed here are between pools of borrowers.

Counterparty swap finance consists of the coupon payments:

$$\left\{V_{s_1}^{sw}(\tau) = V_{s_1}^{v} + \frac{\pi_{s_{22}}^{\tau}}{\pi_{s_1}^{\tau}}\left(V_{s_{21}}^{v}(\theta_1) - V^f(\theta_1)\right)\right\} \tag{3.14}$$

$$\left\{V_{s_{21}}^{sw}(\tau) = V_{s_{21}}^{v}(\theta_1) + p_{s_{21}}(\theta_1)(V_{s_{21}}^{v}(\theta_1) - V^f(\theta_1))\right\}$$

The separating equilibrium essentially remains unchanged for the asymmetric information entrepreneurs, as long as the additional restriction given above is satisfied.

If the swap counterparty has sufficiently strong preferences for smooth consumption, then if the project payoffs are such that

$$var\left(K_1 - V_{s_1}^{sw}(\tau), K_2 - (V_{s_{21}}^{sw}(\tau) + 1)\right) < var\left(K_1 - V_{s_1}^{f}(\theta_1), K_2 - (V_{s_{21}}^{f}(\theta_1) + 1)\right)$$

the effect of higher coupons for variable financing described in lemma (3) may be outweighed, and the counterparty has an incentive to issue fixed rate finance and swap into variable bonds (or may at least be indifferent between variable bond finance and variable-for-fixed swap finance).

Our construction of the swap counterparty is only intended to serve as an illustration of the kinds of characteristics that would be needed for such a party to exist, and to demonstrate that the unilateral view of the swap market is taken to simplify the analysis. Perhaps more intuitively, the swap counterparty could be constructed as the pooling of heterogeneous borrowers by a swapdealer. The pool would then have to have characteristics that induce the swapdealer to synthetically construct a variable-for-fixed counterparty. In this paper, however, the main thrust of the argument is the construction of a model in which, in an anonymous market, swaps will be issued by asymmetrically informed entrepreneurs.

3.5 Conclusion

In this article, we have described a model in which swaps are used by asymmetrically informed entrepreneurs, who cannot reoptimise their financing in interim periods. We have shown that both these conditions must hold for swaps to be traded in an anonymous market. If there is no asymmetric information, all agents choose their optimal finance immediately by the maximum property of their portfolio choice problem. If retrade is allowed in the interim period of the model, the consumption smoothing achieved through swaps can equally well be attained by issuing variable rate bonds and buying or selling additional units of these bonds. If no reoptimisation is allowed, then swaps, in a precise limited sense, allow entrepreneurs to signal their credit quality to the market and still have their preferred consumption stream, given the trading restriction. In this way the article unifies two strands of literature on swaps: the literature which emphasizes that swaps allow the refinancing of liabilities when no such arrangement was previously agreed on, and the work which stresses the possibility to use swaps to signal future credit quality.

3.6 Appendix

Optimisation Program of Entrepreneurs

In its most general form, the optimisation program of entrepreneurs is:

$$
\begin{aligned}
\max_{z^\theta} \quad & U^\theta(c^\theta) \\
c^\theta_{s_0} &= -q^f_{s_0} z^{\theta,f}_{s_0} - q^v_{s_0} z^{\theta,v}_{s_0} - q^{sw}_{s_0} z^{\theta,sw}_{s_0} \\
c^\theta_{s_1} - K_1 &= \left(V^f(\theta) + q^f_{s_1}\right) z^{\theta,f}_{s_0} - q^f_{s_1} z^{\theta,f}_{s_1} \\
&\quad + V^v_{s_1} z^{\theta,v}_{s_0} - q^v_{s_1} z^{\theta,v}_{s_1} \\
&\quad + V^{sw}_{s_1}(\theta) z^{\theta,sw}_{s_0} - q^{sw}_{s_1} z^{\theta,sw}_{s_1} \\
c^\theta_{s_{21}} - K_2 &= V^f(\theta) z^{\theta,f}_{s_1} + V^v_{s_{21}}(\theta) z^{\theta,v}_{s_1} + V^{sw}_{s_{21}}(\theta) z^{\theta,sw}_{s_1} - 1 \\
c^\theta_{s_{22}} &= 0
\end{aligned}
\tag{3.15}
$$

The differences between the three methods of financing are that the long bond could be sold at $t = 1$, and that the coupons have the characterisations $V^f_{s_1}(\theta) = V^f_{s_{21}}(\theta)$, and $V^v_{s_1} \neq V^v_{s_{21}}(\theta)$.

Using assumption (5) the optimisation program simplifies to:

$$
\begin{aligned}
\max_{z^\theta} \quad & U^\theta(c^\theta) \\
c^\theta_{s_0} &= -q^f_{s_0} z^{\theta,f} - q^v_{s_0} z^{\theta,v} - q^{sw}_{s_0} z^{\theta,sw}
\end{aligned}
\tag{3.16}
$$

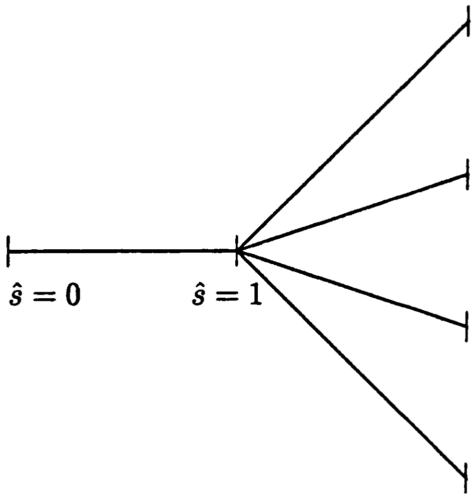
$$
\begin{aligned}
c^{\theta}_{s_1} - K_1 &= V^f(\theta) z^{\theta,f} \\
&\quad + (V^v_{s_1} + 1) z^{\theta,v} - q^v_{s_1} z^{\theta,v} \\
&\quad + (V^{sw}_{s_1} + 1) z^{\theta,sw} - q^{sw}_{s_1} z^{\theta,sw} \\
c^{\theta}_{s_{21}} - K_2 &= (V^f(\theta) z^{\theta,f} + 1) + (V^v_{s_{21}}(\theta) + 1) z^{\theta,v} + (V^{sw}_{s_{21}}(\theta) + 1) z^{\theta,sw} \\
c^{\theta}_{s_{22}} &= 0
\end{aligned}
$$

The difference in the securities can be read from here: both the fixed and swapped finance pay fixed coupons, but the swapped finance is based on borrowing short. By the normalisation of all bond prices to one, and the construction that the project requires one unit of investment at $t = 0$, we arrive at the budget constraint stated in (3.5).

Proof of Lemma (1)

Let the states in this reformulation be denoted by a hat. The transformed uncertainty structure is illustrated in Fig. 2.

Fig.1 Binomial reformulation of the Model when Types are revealed



	θ_1	θ_2	*Bin.Prob.*
$\hat{s} = 21$	K_2	K_2	$p_{s_{21}}(\theta_1)p_{s_{21}}(\theta_2)$
$\hat{s} = 22$	K_2	0	$p_{s_{21}}(\theta_1)p_{s_{22}}(\theta_2)$
$\hat{s} = 23$	0	K_2	$p_{s_{22}}(\theta_1)p_{s_{21}}(\theta_2)$
$\hat{s} = 24$	0	0	$p_{s_{22}}(\theta_1)p_{s_{22}}(\theta_2)$

$t = 0$ $\quad t = 1$ $\quad t = 2$

The column vectors below the θ's stand for the payoffs of the original projects in the transformed economy.

It is required to show that the two formulations are in fact equivalent. We do this by proving that the values of the projects are the same under any probability measure. This construction will then allow us to introduce state prices for the explicit valuation of the projects.

First note that the expectation under the binomial probability measures do indeed give the same value as under the type-specific probabilities:

$$\begin{aligned} E[\tilde{R}]_{p_s(\theta_1)} &= p_{s_{21}}(\theta_1)p_{s_{21}}(\theta_2)K_2 + p_{s_{21}}(\theta_1)(1-p_{s_{21}})K_2 \qquad (3.17) \\ &= p_{s_{21}}K_2 \end{aligned}$$

Now let ρ_s^θ, $\theta = 1,2$ denote the vectors of some alternative probability measures for the original economy, ie.

$$\rho_s(\theta) = [\rho_{s_{21}}(\theta)\, \rho_{s_{22}}(\theta)]^T \qquad (3.18)$$

where $\rho_s(\theta) > 0$ and $\sum_s \rho_s(\theta) = 1$ for $\theta = 1,2$. Then define a change of measure vector $[\rho_s(\theta) - p_s(\theta)]$

$$[\rho_s(\theta) - p_s(\theta)] = \begin{bmatrix} \rho_{s_{21}}(\theta) - p_{s_{21}}(\theta) \\ \rho_{s_{22}}(\theta) - p_{s_{22}}(\theta) \end{bmatrix} \qquad (3.19)$$

For the type specific economy, the expectation of the project payoff under the original measures $p_s(\theta)$ are:

$$E[\tilde{R}]_{p_s}(\theta) = p_s(\theta)^T R^\theta \qquad for\ \theta_1, \theta_2 \qquad (3.20)$$

Under alternative measures $\rho_s(\theta)$, the expectations are:

$$E[\tilde{R}]_{\rho_s}(\theta) = \left(p_s(\theta)^T + [\rho_s(\theta) - p_s(\theta)]^T\right) R^\theta \qquad \theta_1, \theta_2 \qquad (3.21)$$

In the transformed economy, define the vector of binomial probabilities as

$$\hat{p}_s = \begin{bmatrix} p_{s_{21}}(\theta_1)p_{s_{21}}(\theta_2) \\ p_{s_{21}}(\theta_1)p_{s_{22}}(\theta_2) \\ p_{s_{22}}(\theta_1)p_{s_{21}}(\theta_2) \\ p_{s_{22}}(\theta_1)p_{s_{22}}(\theta_2) \end{bmatrix} \tag{3.22}$$

and similarly for the changed measure $\hat{\rho}_s$.

Taking the alternative measures for the original economy $(\rho_s(\theta_1), \rho_s(\theta_2))$ and writing the expectation in the transformed economy yields:

$$\begin{aligned} E[\tilde{R}]_{\hat{\rho}_s} &= \hat{\rho}_s^T R_{\hat{s}} \qquad (3.23) \\ &= [\hat{\rho}_{s\hat{2}1}\ \hat{\rho}_{s\hat{2}2}\ \hat{\rho}_{s\hat{2}3}\ \hat{\rho}_{s\hat{2}4}]\, R_{\hat{s}} \\ &= \begin{bmatrix} \rho_{s_{21}}(\theta_1)\rho_{s_{21}}(\theta_2) \\ \rho_{s_{21}}(\theta_1)\rho_{s_{22}}(\theta_2) \\ \rho_{s_{22}}(\theta_1)\rho_{s_{21}}(\theta_2) \\ \rho_{s_{22}}(\theta_1)\rho_{s_{22}}(\theta_2) \end{bmatrix}^T R_{\hat{s}} \end{aligned}$$

This expression must be equal to transforming the economy first and then changing its measure. The expectation of the transformed economy computed with the original probabilities is:

$$E[\tilde{R}^\theta]_{\hat{p}_s} = \hat{p}_s^T R^\theta \tag{3.24}$$

Define a change of measure vector for the transformed economy as:

$$[\hat{\rho}_s - \hat{p}_s] = \begin{bmatrix} \rho_{s_{21}}(\theta_1)\rho_{s_{21}}(\theta_2) - p_{s_{21}}(\theta_1)p_{s_{21}}(\theta_2) \\ \rho_{s_{21}}(\theta_1)\rho_{s_{22}}(\theta_2) - p_{s_{21}}(\theta_1)p_{s_{22}}(\theta_2) \\ \rho_{s_{22}}(\theta_1)\rho_{s_{21}}(\theta_2) - p_{s_{22}}(\theta_1)p_{s_{21}}(\theta_2) \\ \rho_{s_{22}}(\theta_1)\rho_{s_{22}}(\theta_2) - p_{s_{22}}(\theta_1)p_{s_{22}}(\theta_2) \end{bmatrix} \quad (3.25)$$

Then the expectation in the transformed economy under the alternative measure is:

$$\begin{aligned} E[\tilde{R}^{\theta}]_{\hat{\rho}_s} &= \left(\hat{p}_s^T + [\hat{\rho}_s - \hat{p}_s]\right) R_{\hat{s}} \quad (3.26) \\ &= \hat{\rho}_s^T R_{\hat{s}} \end{aligned}$$

This equation is the same as transforming the changed-measure expectation directly. We have thus shown that if a separating equilibrium exists, state prices for the redefined individual economy can be found.□

Proof of Lemma (2)

The first order conditions of investors are:

$$\begin{aligned} \nabla_{c^i_\sigma} U^i(\vec{c}^i) &= \bar{\lambda}^i_\sigma \quad (3.27) \\ -\bar{\lambda}^i_0 + \bar{\lambda}^i_1 V_1^{p,j}(\theta) + \bar{\lambda}^i_\sigma V_\sigma^{p,j}(\theta) &= 0 \qquad j = f, v, sw \\ \vec{c}^i_0 &= \omega^i_0 - \bar{z}^{i,f}_p - \bar{z}^{i,v}_p - \bar{z}^{i,sw}_p \\ \vec{c}^i_1 &= V_1^{p,f}(\theta)\bar{z}^{i,f}_p + V_1^{p,v}(\theta)\bar{z}^{i,v}_p + V_1^{p,sw}(\theta)\bar{z}^{i,sw}_p \end{aligned}$$

$$\bar{c}^i_\sigma = V^{p,f}_\sigma(\theta)\bar{z}^{i,f}_p + V^{p,v}_\sigma(\theta)\bar{z}^{i,v}_p + V^{p,sw}_\sigma(\theta)\bar{z}^{i,sw}_p$$

Defining a state price (the price of an Arrow security) as

$$\pi^i_s = \frac{\lambda^i_s}{\lambda^i_0} \tag{3.28}$$

the pricing equations can be rewritten as:

$$1 = \bar{\pi}^i_1 V^{p,j}_1(\theta) + \bar{\pi}^i_\sigma V^{p,j}_\sigma(\theta) \tag{3.29}$$

Since the repayments on the bond is the only source of consumption for investors, (3.29) becomes:

$$1 = \bar{\pi}^i_1 c^i_1 + \bar{\pi}^i_\sigma c^i_\sigma \tag{3.30}$$

We need to show that $u^i(\bar{c}^i_{s_1}) + u^i(\bar{c}^i_\sigma)$ is decreasing in the variance of consumption. In general, concavity of the utility function does not imply variance-aversion. However, in a two 'state'- model, this is indeed the case. The variance of the consumption stream in $t = 1$, $t = 2$ can be written as:

$$\begin{aligned} var(\bar{c}^i_{s_1}, \bar{c}^i_\sigma) &= \frac{1}{2}(\bar{c}^i_{s_1} - E[\bar{c}^i_{s_1}, \bar{c}^i_\sigma])^2 + \frac{1}{2}(\bar{c}^i_\sigma - E[\bar{c}^i_{s_1}, \bar{c}^i_\sigma])^2 \\ &= \frac{1}{2}(\bar{c}^i_{s_1})^2 + \frac{1}{2}(\bar{c}^i_\sigma)^2 - \bar{c}^i_{s_1}\bar{c}^i_\sigma \end{aligned} \tag{3.31}$$

which implies that the variance is increasing in the difference $\bar{c}^i_{s_1} - \bar{c}^i_\sigma$. To ensure that we stay at the same mean utility, ie. for $\bar{c}^i_{s_1} + \bar{c}^i_\sigma$, choose

$dc^i_{s_1} = -dc^i_\sigma > 0$. Then the total change in utility from a marginal increase in the variance of consumption is:

$$\begin{aligned} dU^i(\bar{c}^i_{s_1}, \bar{c}^i_\sigma) &= U_{\bar{c}^i_1} dc^i_{s_1} - U_{\bar{c}^i_\sigma} dc^i_\sigma \\ &= \left(U_{\bar{c}^i_{s_1}} - U_{\bar{c}^i_\sigma}\right) dc^i_{s_1} \end{aligned} \tag{3.32}$$

which is negative by concavity.□

Proof of Lemma (3)

Since $p_{s_{21}}(\theta) < 1$, $V^v_{s_{21}}(\theta) > V^v_{s_1}(\theta)$, and since there is no aggregate risk, it must hold that $V^{v,p}_{s_1}(\theta) = V^{v,p}_\sigma(\theta) \equiv V^{v,p}(\theta)$. Correspondingly, if individual coupons are constrained to be the same across periods, then, since $p_{s_{21}}(\theta) < 1$, $V^{f,p}_{s_1}(\theta) > V^{f,p}_\sigma(\theta)$. Under these conditions, and with the reasonable assumption that $V^{j,p} < 1$, we need to show that $var\,(V^{v,p}(\theta), V^{v,p}(\theta) + 1) > var\left(V^{f,p}_{s_1}(\theta), V^{f,p}_\sigma(\theta) + 1\right)$. For variable coupons the variance is:

$$\begin{aligned} var\,(V^{v,p}(\theta), V^{v,p}(\theta) + 1) &= \sum_{t=1}^{t=2} \frac{1}{2} \left[V^{v,p}_t(\theta) - E[V^{j,p}(\theta)]\right]^2 \\ &= \frac{1}{2} \end{aligned} \tag{3.33}$$

For fixed coupons, it is:

$$var\left(V^{f,p}_{s_1}(\theta), V^{f,p}_{\sigma}(\theta)+1\right) = \left(\frac{1}{2}\left(V^{f,p}_{s_1}(\theta) - V^{f,p}_{\sigma}(\theta)\right) - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\left(V^{f,p}_{\sigma}(\theta) - V^{f,p}_{s_1}(\theta)\right) + \frac{1}{2}\right)^2 \tag{3.34}$$

which is smaller that $\frac{1}{2}$, since $0 < V^{f,p}_{s_1}(\theta) - V^{f,p}_{\sigma}(\theta) < 2$. We have thus shown that the fixed coupon finance has a lower variance in the pool and consequently will be issued by entrepreneurs.□

Proof of Lemma (4)

By assumptions (5) and (7) on the one-side constraints, entrepreneurs and investors will in general have different valuations of the same securities. The view we take is that investors evaluate rank pooled project finance given their 'preference for smooth consumption' utility functions. Given this ranking, entrepreneurs take the repayments V as given and evaluate them using their own 'non-smoothing preferences'. Entrepreneurs issue those securities that have a lower individual valuation for them. Denote the individual valuation of project finance j by entrepreneurs θ as $q^j(\theta)$. Recall that the budget constraint is:

$$\begin{aligned} \underset{z^\theta}{max} \quad & U^\theta(c^\theta) \\ c^\theta_{s_0} \quad &= \quad -q^f_{s_0} z^{\theta,f} - q^v_{s_0} z^{\theta,v} - q^{sw}_{s_0} z^{\theta,sw} \\ c^\theta_{s_1} - K_1 \quad &= \quad V^f(\theta) z^{\theta,f} \\ & \quad\quad +(V^v_{s_1}+1) z^{\theta,v} - q^v_{s_1} z^{\theta,v} \end{aligned} \tag{3.35}$$

$$
\begin{aligned}
& +(V^{sw}_{s_1}(\theta)+1)z^{\theta,sw} - q^{sw}_{s_1} z^{\theta,sw} \\
c^{\theta}_{s_{21}} - K_2 &= (V^f(\theta)z^{\theta,f}+1) + (V^v_{s_{21}}(\theta)+1)z^{\theta,v} + (V^{sw}_{s_{21}}(\theta)+1)z^{\theta,sw} \\
c^{\theta}_{s_{22}} &= 0
\end{aligned}
$$

This yields the following pricing equations:

$$
\begin{aligned}
q^f(\theta) &= \pi(\theta)V^f(\theta) + \pi(\theta)\left(V^f(\theta)+1\right) \\
q^v_{s_0}(\theta) + q^v_{s_1}(\theta) &= \pi(\theta)V^v_{s_1} + \pi(\theta)\left(V^v_{s_{21}}(\theta)+1\right)
\end{aligned}
\tag{3.36}
$$

Non-smoothing implies that the state prices are the same and independent of consumption in that state. By the risk-averse pricing of investors, $2V^f(\theta) < V^v_{s_1} + V^v_{s_{21}}(\theta)$. Together with (3.36) this implies that the valuation of fixed finance is lower. Therefore all entrepreneurs prefer issuing fixed finance. □

Proof of Lemma(5)

In a *separating* equilibrium, since, by assumption (1), $p_{s_{21}}(\theta_1) > p_{s_{21}}(\theta_2)$, a simple argument shows that the coupons for the same type of security for types θ_2 must be larger:

$$
\begin{aligned}
V^{p,j}(\theta_1) &= V^{p,j}(\theta_2) \\
\Leftrightarrow p_{s_{21}}(\theta_1)V^j(\theta_1) &= p_{s_{21}}(\theta_2)V^j(\theta_2)
\end{aligned}
\tag{3.37}
$$

$$\Rightarrow V^j(\theta_1) \quad < \quad V^j(\theta_2) \quad j = f, v, sw$$

For a *pooling* equilibrium, recall that investors require the return $V_{PE}^{p,j}$, $j = f, v, sw$, where PE denotes 'pooling equilibrium'. The default probabilities are pooled, such that it can be deduced from assumption (1) that $p_{s_{21}}(\theta_1) \geq p_{s_{21}}(\theta_{PE}) \geq p_{s_{21}}(\theta_2)$, where at least one inequality must be strict. It follows that

$$V^j(\theta_1) \leq V_{PE}^j \leq V^j(\theta_2) \quad j = f, v, sw \tag{3.38}$$

with at least one inequality strict.

Therefore, good risk entrepreneurs θ_1 always have an incentive to reveal their type. However, if they choose the variable coupon finance, they will lose some of their gain of their revelation, since they prefer a fixed income stream to a variable income stream. Hence, with variable and fixed coupon securities, a separating equilibrium cannot be guaranteed for all parameter values, and pooling equilibria may exist. With a swap, however, the cost of signaling disappears, since entrepreneurs can have a fixed coupon stream, while still signaling their type. Therefore type θ_1 borrowers prefer swap finance.

Suppose entrepreneurs θ_1 enter into the fixed-for-variable swap. Then the swap is a promise to pay a low coupon rate, and should the probability of default $p_{s_{22}}(\theta_2)$ at $t = 1$ turn out to be higher than envisioned at $t = 0$, when the swap was written, to top up the coupon rate by the difference to the variable rate given the true probability. (recall definition (1)). Since entrepreneurs θ_1 are the good risk entrepreneurs, they will be revealed to be good at $t = 2$ and end up paying fixed finance correctly priced for their type θ_1. Suppose now that entrepreneurs θ_2 also entered into the swap. By definition (1), they would end up paying $V^f_{s_1}(\theta_1)$ and $V^v_{s_{21}}(\theta_2)$. Since $p_{s_1} = 1$, but $p_{s_{21}}(\theta_1) < 1$ and $V^f_{s_1}(\theta) = V^f_{s_{21}}(\theta)$, it follows that $V^f(\theta_1) > V^v_{s_1}$. This implies that the coupons $V^v_{s_1}$ and $V^v_{s_{21}}(\theta_2)$ would be preferred. However, by lemma (4), $(V^f(\theta_2), V^f(\theta_2) + 1)$ is preferred to $(V^v_{s_1}, V^v_{s_{21}}(\theta_2) + 1)$. Therefore, imitation would leave bad risk borrowers with a more expensive payment stream, and, by the linearity and time independence of their utility function, with less utility than if they paid the fixed coupons conditioned on their own bad credit quality $V^f(\theta_2)$. Thus the equilibrium is separating. $\square$

Chapter 4

Anonymous Corporate Bond Markets with Asymmetric Information

4.1 Introduction

This paper studies an anonymous credit market which is characterised by asymmetric information. The market is anonymous in the sense that credit contracts are traded by many borrowers and lenders at a price which only reflects the information in the market, and that this price is taken as given by all participating traders. The market we intend to represent with this model is the market for corporate bonds. Corporate bonds are debt instruments which are issued by companies and are bought by private or institutional investors. On the whole, even though financial engineering has meant that there now is a proliferation of different debt structures available to companies, corporate bonds are normally tradable fixed income debt contracts.

A corporate bond contract is a promise to pay a coupon at pre-specified payment dates over a pre-arranged period. At the time of writing the contract, the borrower receives money from the lender, which he pledges to return at maturity, and on which he promises to make coupon payments at regular intervals. The lender is the buyer of the credit contract, and he can sell the contract to other traders in financial markets before maturity. Tradability of the contract means that the contract not only has a nominal coupon rate and a face value, but that it also trades at a market price, which is determined by the demand and supply conditions for that specific contract and by the overall conditions prevalent in the credit market.

It is usually argued that debt markets suffer from a fundamental asymmetry of information between borrowers and lenders. Borrowers are thought of as being better informed regarding either the likelihood of success or the size of the payouts of the projects which they undertake. Indeed, there is a whole literature on the design of different credit contracts that addresses this issue (Freixas and Rochet, 1997). Usually, a distinction is drawn between two types of asymmetric information: adverse selection and moral hazard. The distinction relates to the timing of the private information (Hart and Holmström, 1987). In adverse selection economies, agents have private information over either the distribution or the realisation of their payout and trade on this information. With moral hazard, agents can affect the state which will be realised.

In this model, we chose to introduce asymmetric information in its 'adverse selection' incarnation. Agents have a project, which they need to finance. They have private information over the probability of success of the project, and they trade bond contracts dependent on their private information. In particular, depending on their riskiness, they prefer some bond contracts over others, and, depending on the market price for these contracts, would want to engage in the trade of some but not others. For lenders, we assume for simplicity, that the only public information available is that all projects have the same mean payoff for all risk classes of agents.

A second important feature of credit markets, which is incorporated in the present model, is the possibility of default. If a project is not successful and the payoff does not cover the required repayments on the corporate bond which was issued to finance it, a firm is said to default. If default happens, the lender will not receive the full repayment of the loan. In that sense, the possibility of default corresponds to limited liability for the borrower.

The possibility of default makes adverse selection bite in our set-up. Default, or rather limited liability, introduces a non-linearity in the payoff function of borrowers. Should a project be unsuccessful, then the borrower is only required to return the value of the project, and not the full value of the loan. For his payoff position this implies that as long as the value of the project is below the repayment on the bond, his payoff is zero, *but it can never be negative.* Once the payout of the project surpasses the value of repayments on the loan, the borrower starts to get a positive payout. Indeed, what we have just described is the payoff of a long position in a call option on one's own project. As is well known from the option pricing literature, the non-linearity of the payoff implies that the valuation of a call option is a positive function of the volatility of the underlying security. Since, in our model, the volatility of the underlying (of the project) is private information, borrowers can be thought of as trading on their own idiosyncratic *volatility*.

In the following we describe how the interaction of private information on the borrowers' own project volatility, limited liability, and the anonymity of the market interact to the effect that the nominal coupon rate influences the market price and quantity traded indirectly through the sorting of borrowers.

This effect is the central result of this paper and stands in contrast to a symmetric information environment, in which the nominal coupon rate plays no role. The result carries over from a non-competitive asymmetric information model like the SW-model. However, just like in a symmetric competitive environment, a market clearing interest rate always exists, and there is no credit rationing.

In section (4.6) we provide an important extension to our basic set-up, by allowing borrowers to put up collateral at the time of writing the contract. In this way, we make the bond contracts multidimensional. We show that with collateral separating equilibria can exist, and we characterise one such equilibrium for the special case, when there are only two types of risk classes.

In this case, it still holds that the characteristics of the credit contracts sort borrowers, while the preferences of investors determine the demand and supply of credit. However, it is no longer the case that good risk borrowers drop out of the market first.

Section (4.2) introduces the set-up verbally and gives an intuition for our main result. For reference purposes, section (4.3) briefly reviews the Stiglitz-Weiss (SW) model (Stiglitz-Weiss, 1981) of credit rationing, which pioneered the limited liability uncertainty structure which we use. Section (4.4) translates the SW set-up into a general competitive model with asymmetric information and describes the model structure and equilibrium concept in detail. Section (4.5) presents the results of the basic model in a series of propositions. Section (4.6) extends the basic model to take into account borrower heterogeneity and collateral. Section (4.7) concludes. The appendix contains all the proofs.

4.2 The Separation of Nominal Bond Coupon Rates and Market Prices

There are many investors and many borrowers with projects in the economy. Borrowers have asymmetric information over the probability of success of their projects. They have no endowments at the beginning of time, which forces them to borrow in order to finance their projects. They borrow by selling limited liability bonds to investors. The issuing process is not modeled.

Instead, we look at economies in which some assets are traded that already exist.

Even though all projects have the same mean payoff, the possibility of default combined with the private information of borrowers means that different types of agents value the limited liability bonds differently. It is standard to show that given any coupon rate, riskier agents have a higher valuation of the bonds.

The reduced valuation for borrowers of higher coupon bonds, however, does not mean that these bonds feature a higher return for lenders. The reason is that the bond also has a market price. The market price adjusts the payout on the bond in such a way, that, should we live in an economy with symmetric information and risk neutrality, the equilibrium valuation of the bond, defined as its payoff divided by its price, is the same for all borrowers. Here, however, asymmetric information lets borrowers have different valuations for the same income stream. The precise effect of one market price for different types of contracts is that, as soon as contracts feature pooling of different types of borrowers, good borrowers always have a lower expected return on the project than bad borrowers. Indeed, this is just the common occurrence in asymmetric information models, that good agents 'subsidise' bad agents in pooling equilibria, and that in an extreme case, the market may unravel completely, either breaking down, or only leaving the worst agents willing to trade.

The different valuation of contracts by borrowers is the counterimage of their different delivery rates. As all the investors in our model are identical and risk-averse, they will hold the market portfolio, or, equivalently, a share in the 'pool' of all traded bonds. Since different types of borrowers have different delivery rates into the pool, and since the probability of delivery changes with the 'strike price' of the bond, the price of the bonds varies with the nominal coupon rate in a way which not only reflects the changed deliveries required by a changed nominal coupon rate, but also by the mix of borrowers. We show that with our assumption of mean-preserving spreads, there are two possibilities. Either an increase in the coupon rate raises the effective interest rate on the limited liability bond, or it is lowered. If the effective interest rate goes up, some borrowers may leave the market. These are the good risk borrowers, since the bad risk borrowers always have a higher valuation of the limited liability bond.

The pooling of deliveries of different types of agents and the resulting price determination of the limited liability bonds is the fundamental force behind the model, and the reason why introducing an anonymous market into a Stiglitz-Weiss setting is not superfluous. Indeed, with respect to the Stiglitz-Weiss model, the introduction of tradability and the use of a general equilibrium set-up allows to 'close' the model, in the precise sense that all assets are priced consistently in a market. The central theme of Stiglitz-Weiss, 'credit rationing' disappears in this way, while some of the characteristics, for example that good risk borrowers drop out of the market first, both if the strike price increases and if the supply of credit contracts, still hold up.

For all contracts that can be issued, higher risk agents always have a higher valuation. This is true for every contract and consequently makes contract choice independent of the demand and supply conditions in the market. In this way, the coupon rate sorts borrowers by their riskiness (in the sense that a coupon rate gives a lower limit for the riskiness of borrowers who still apply for credit), while the preferences of investors and borrowers and the riskiness of projects determine the average return on any one credit contract.

In our model, a higher coupon rate *may* worsen the quality of the pool of borrowers who still want to finance the project. Moreover, the coupon rate is independent of the demand and supply of credit, and there is no contract which is valued higher by good risk borrowers than by bad risk borrowers, this result also implies that if credit contracts, good borrowers will always drop out of the market first.

When extending the model to take into account collateral, we show in a simplified two borrower type set-up, that separating equilibria exist. In separating equilibria, the driving force behind the different valuations of the same credit contracts, namely the pooling of different delivery rates, breaks down. The bond prices correctly reflect the delivery rates of the different types of borrowers. However, the good risk type must signal his good risk quality to the market through collateral and, in equilibrium, will have to put up a quantity of collateral that makes the bad risk type just indifferent between the cost of putting up collateral and the benefit of pooled pricing. This cost of collateral prevents the separating equilibrium from being unconstrained efficient.

An immediate consequence of the separation is that, in equilibrium, higher risk borrowers pay higher interest rates than lower risk borrowers. Since all the projects are identical with respect to the mean payoff, this also means that, should credit contract and the interest rate required by investors rise, bad credits will leave the market first.

Another consequence of separation is that, at least for the party who does not have to signal its good credit quality, the strike price and the nominal coupon rate on the contract become irrelevant just like in a symmetric information setting.

One could argue that the Stiglitz-Weiss credit rationing result is, in our set-up, reflected by the pooling of deliveries, and the resulting 'mispricing' of the bonds. Once a separating equilibrium is established, the mispricing disappears in the same way that credit rationing disappears in the Stiglitz-Weiss model (Bester, 1985).

4.3 The Limited Liability Set-up of Stiglitz-Weiss

For reference purposes, and to facilitate understanding, we briefly recap the SW-model of credit rationing. Our model is essentially a general equilibrium with asymmetric information version of the SW-model. The objective of the SW model is to show that credit markets can be characterised by the rationing of credit for ex-ante indistinguishable borrowers. Instead of charging a higher interest rate, banks may prefer to turn down creditors who would be willing to pay the same interest rate as others who are given credit. This is similar to our model in the sense that it is useless to change the nominal coupon rate if there is an excess demand for bonds. However, in our model the market price regulates the demand and supply of loans, while in SW the unwillingness of the bank to charge a higher interest rate *may* result in 'credit rationing'.

In SW, the agents are banks and borrowers. They are all risk neutral. Borrowers have an indivisible project of fixed size, whose mean payoff is known, but whose riskiness is private information. Let θ denote the riskiness of the project, R its gross return, $F(R, \theta)$ the cumulative distribution function of the returns with associated density function $f(R, \theta)$. Then, project θ_1 is riskier than θ_2 in the sense of *mean preserving spreads*, if, for θ_1 riskier than θ_2

$$\int_0^{\infty} Rf(R, \theta_1)\, dR = \int_0^{\infty} Rf(R, \theta_2)\, dR \tag{4.1}$$

then, for $y \geq 0$

$$\int_0^y F(R, \theta_1)\, dR \geq \int_0^y F(R, \theta_2)\, dR \tag{4.2}$$

The borrowers have to put up an amount of collateral C. Crucially, they have limited liability. If B is the amount borrowed, and $\hat{r}$ is the interest rate charged by the bank, then borrowers are said to default if

$$C + R \leq B(1 + \hat{r}) \tag{4.3}$$

In the case of default, the bank receives the collateral. Consequently the net return to the borrower is:

$$\pi(R, \hat{r}) = max\,(R - (1 + \hat{r})B, -C) \tag{4.4}$$

The net return to the bank is then:

$$\delta(R, \hat{r}) = min\,(R + C, B(1 + \hat{r})) \tag{4.5}$$

Note that the net return to the borrower is a kinked, convex function in R, while the net return to the bank is a kinked, concave function in R. Even though banks compete for deposits, they are not price takers. Banks set the interest rate $\hat{r}$ to maximise their profits.

Fig.(1) Borrowers' Profits in Stiglitz-Weiss

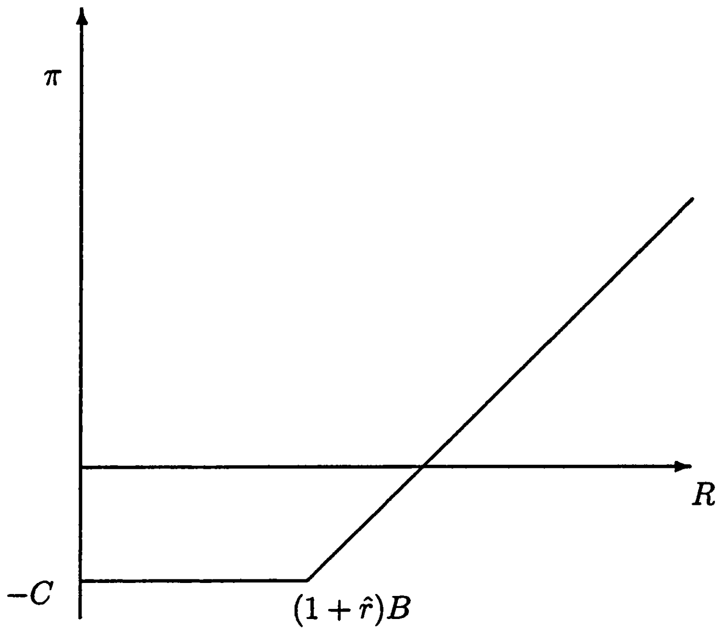
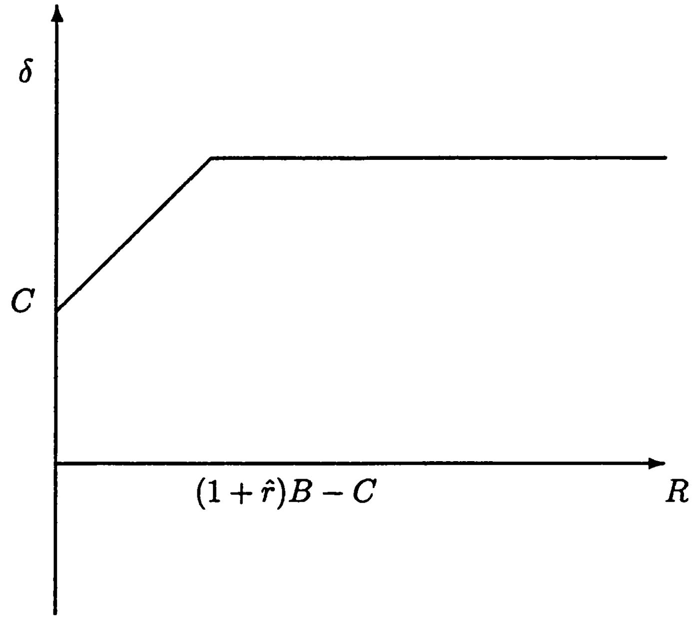


Fig.(2) Return to the Bank in Stiglitz-Weiss



With this setup, Siglitz-Weiss establish the possibility of credit rationing through a succession of five theorems. We state the theorems without proofs.

The first theorem establishes that the expected value of the project, which determines the ability to repay by the assumption of risk-neutrality, is increasing in the riskiness of the project θ.

Theorem 1 (Stiglitz-Weiss 1)
For a given $\hat{r}$, there is a critical value of $\hat{\theta}$ s.t. a firm borrows from the bank if and only if $\theta > \hat{\theta}$.

The second theorem states that agents will choose riskier projects if the interest rate is higher.

Theorem 2 (Stiglitz-Weiss 2)
As the interest rate increases, the critical value of θ, below which individuals do not apply for loans, increases.

Theorem 3 is the analogue of theorem 1 for the bank.

Theorem 3 (Stiglitz-Weiss 3)
The expected return on a loan to a bank is a decreasing function of the riskiness of the loan.

Theorem 4 establishes non-monotonicity of the bank's profits, which underpins the result on credit rationing in theorem 5.

Theorem 4 (Stiglitz-Weiss 4)
If there are a discrete number of potential borrowers (or types of borrowers), each with a different θ, $\bar{\delta}(\hat{r})$ will not be a monotonic function of $\hat{r}$, since as each successive group drops out of the market, there is a discrete fall in $\bar{\delta}$. ($\bar{\delta}(\hat{r})$ is the mean return to the bank from the set of applicants at interest rate $\hat{r}$)

Theorem 5 (Stiglitz-Weiss 5)
Whenever $\bar{\delta}(\hat{r})$ has an interior mode, there exist supply functions of funds s.t. competitive equilibrium[1] entails credit rationing.

[1] The notion of 'competitive equilibrium' used in SW is one of a bank setting interest rates for many potential borrowers and quantities of deposits for potential depositors. In contrast, competitive equilibrium in the current paper is used in the strict sense of price taking agents.

SW note that even when there is a bank's profit maximising interest rate which entails credit rationing, there is always another 'Walrasian' interest rate, which is higher and which has the market clearing property that demand for loanable funds equal to supply of loanable funds.

4.4 Modeling Stiglitz-Weiss in General Equilibrium

There appear to be three main issues when attempting to conduct a competitive credit market analysis based on a SW structure. The first is that the uncertainty structure must be reformulated. The second refers to the modeling of limited liability. The third consists of the treatment of competition itself.

Uncertainty in SW is described by the continuous distribution function $F(.)$ and its associated density $f(.)$. To avoid continuous state spaces these functions need to be 'discretised' in an appropriate manner.

SW rely heavily on the possibility of limited liability. No limited liability is possible in a standard general equilibrium model (see, however, Zame (1993) and Dubey, Geanakoplos and Shubik (1997)). To introduce the characteristics of default, we introduce contracts that have the same payoffs as limited liability contracts.

As regards the issue of competition, even though competition is not explicitly modeled in SW, the implicit game theoretic setup is that banks are price setters in the credit market and quantity setters in the deposit market. They simultaneously choose a demand for deposits and a nominal loan rate to maximise their profits, taking as given the return demanded by depositors and the loan rates set by other banks. In our model, bond contracts are traded on an anonymous credit market. In order to gain insights in the spirit of SW into the functioning of the anonymous market, we conduct comparative statics analysis on bond contracts with different nominal coupon rates.

4.4.1 Discrete Uncertainty Structure

The model we are using is based on the general equilibrium model with asymmetric information, as exemplified by Bisin, Geanakoplos, Gottardi, Minelli and Polemarchakis (1998). In the economy there are agents with projects, over which they have private information (they will be called borrowers), and there are investors who lend to these borrowers. There are two periods, which are denoted by $t = 0, 1$, and finitely many states in period $t = 1$. To use the same notation as SW, there are $\theta \in \Theta = \{1, \cdots, \Theta\}$ types of borrowers, and countably infinite borrowers of each type, $n = 1, \cdots, \infty$. A borrower is then identified by the tuple $\{\theta, n\}$. The proportion of borrowers of a type θ in the economy is called $\lambda^\theta = \frac{1}{\Theta}$. Every borrower $\{\theta, n\}$ has a project which requires one unit of investment in period one and has an uncertain payoff in period $t = 1$. The project pays off in the single consumption good, whose price is normalised to one. In order to simplify the model, projects are not explicit, but rather manifest themselves in the endowments of borrowers. Borrowers have an endowment of zero at $t = 0$. If they borrow, they will receive a random endowment in period $t = 1$. If not, they will end up with zero endowments in both periods. Different types of borrowers are identified with the riskiness of their projects.

To obtain discrete analogues of (4.1) and (4.2) it is first necessary to describe and order the payoffs of projects in period $t = 1$. We make a distinction between individual and aggregate payoffs. Individual payoffs refer to the outcomes of the individual projects of every agent. Social or aggregate payoffs are then all possible combinations of outcomes of the individual projects. Aggregate payoffs will be discussed below.

Without loss of generality we choose to rank the payoffs by an ascending order. Define a payoff in a state s as $R_s \geq 0$, then the ordering is $R_1 < R_2 < \ldots < R_S$. Therefore $\tilde{R} : \Omega \longrightarrow \mathbb{R}_+$ is a random variable with finite support $\{R_1, R_2, \ldots, R_S\}$, defined on a probability space $(\Omega, \mathcal{F}, P)$. The payoffs in a particular state are the same for all agents, and types are distinct only in the likelihood that a certain payoff occurs. Consequently the image measure $p_s(\theta) = P\{\omega \in \Omega | \tilde{R} = R_s\}$ is type-specific, implying that the random variables have different probability measures on the same support. In order to clarify the dependence of the distribution of the random variable $\tilde{R}$ on θ and n, we use the shorthand $\tilde{R}^{\theta,n}$.

Differences in the riskiness of projects are defined in the sense of *mean* *preserving spreads.*

Assumption 1 (Mean Preserving Spread)

Type θ_1 has a riskier project than θ_2, if, for

$$\sum_{s=1}^{S} R_s p_s(\theta_1, \hat{n}) = \sum_{s=1}^{S} R_s p_s(\theta_2, \hat{n}) \tag{4.6}$$

then, for all $1 < s < S$ and for all $R_s < y < R_{s+1}$ (and define $R_0 = 0$ and $R_{S+1} = \infty$)

$$\sum_{s=1}^{s=y} (R_y - R_{s-1}) p_s(\theta_1) \geq \sum_{s=1}^{s=y} (R_y - R_{s-1}) p_s(\theta_2) \tag{4.7}$$

It is now necessary to describe how the individual random variables are combined to form aggregate payoffs. For this purpose, we must make assumptions on the correlation of different individual projects.

Assumption 2 (Correlation of Project Payoffs)

$\{\tilde{R}^{\theta,n}\}_{\theta,n}$ are mutually independent and have the same mean μ, and for every θ, $\{\tilde{R}^{\theta,n}\}_n$ are identically and independently distributed across n.

Assumption (2) allows to invoke the law of large numbers. In the limit for the number of projects of each type approaching infinity, it holds that:

$$\frac{1}{N} \sum_{n=1}^{N} \xrightarrow{d} E[\tilde{R}^{\theta,n}] = \mu \tag{4.8}$$

The construction ensures that if infinitely many projects are pooled together, aggregate payoffs are constant and equal to μ.

It will be helpful to sum up the uncertainty set-up. The probability distribution of an individual project is a function of the private information risk parameter θ. However, to make sense of a notion of 'the average payoff of a project' as in SW theorem (4), many projects of each type have to be introduced. Consequently, the probability distribution of *aggregate* payoffs are functions not only of the riskiness of projects, but also of the nature of the correlation between projects of the same type and across types. We have made an appropriate assumption on the correlation between different projects to exploit averaging properties of the pooling of payoffs. The result is that in the limit for the number of projects approaching infinity, all aggregate payoffs are equivalent, since all idiosyncratic risk is averaged out.

There is a subtle difference to the Gottardi and Bisin (1997) use of an $n \times \theta$ structure. In their case, under asymmetric information the n agents of every type have private information over their index. This generates problems of averaging, exemplified in the possible existence of arbitrage. Here, private information occurs exclusively for the types θ, and the n agents of each type only guarantee a consistent description of pooling via the law of large numbers.

4.4.2 Security Structure and Optimisation Programs

Security Structure

Borrowers' optimisation

In SW, borrowers have an uncertain project and finance this uncertain project by taking out a bank loan of the 'standard debt contract' form. They partially secure the loan by collateral and repay only if the project is successful. In general equilibrium, all agents have unlimited liability for all their obligations. In order to mimic the payoff of a limited liability debt contract, we split up the SW contract into two parts [2]: a *sale of the project with unlimited liability* and the *purchase of a call option on one's own project.* We postulate that these two contracts can only be executed jointly.

Firstly, agents sell their project (ie. equity). With assumptions (1) and (2) on utility functions the price of this contract in period $t = 0$ must be equal to $\beta^* \, E[\tilde{R}^{\theta,n}]$, where β^* is the equilibrium discount factor. Since projects all have the same mean this price is equal across all agents $\{\theta, n\}$ and can be written as $\beta^* \mu$. The payoff of this contract is just the payoff of the project, ie. it is R_s for $s = 1, \cdots, S$, and again, it is ex ante the same for all agents of all types[3].

[2]Note that there is only one good in the economy, whose price is normalised to one, and the securities pay off in this good; also, in the basic model, we consider only the case of non-collateralised loans, meaning that we set $C = 0$ throughout (Section 4.6 provides an extension to the basic model which introduces collateral). To simplify further, we only consider bonds of size $B = 1$.

[3]Recall that the individual component of the payoffs are the type specific probabilities

Secondly, borrowers buy a call option on their own project, ie. a contract which pays off zero in states that have a lower payoff than $(1+\hat{r})$, and $R_s - (1+\hat{r})$ in states with a payoff exceeding $(1+\hat{r})$. $(1+\hat{r})$ is the strike price of the call option and will be denoted K. Let the price of the call option be denoted by q^c.

Netting the payoffs of the two contracts gives a payoff equivalent to a limited liability bond of size 1 with coupon rate $(1+\hat{r})$.

$$Min[\tilde{R}, 1+\hat{r}] = \tilde{R} - Max[\tilde{R} - (1+\hat{r}), 0)] \tag{4.9}$$

Let the price of the limited liability bond be q^b. Then

$$q^b = \beta^* \mu - q^c \tag{4.10}$$

Call z^θ the position of agents θ in the limited liability bond contract. Borrowers issue, ie. sell the limited liability bond contract, which means that, in conjunction with the sign convention we use, they take a long position in the call option. Crucially, only the combination of equity and the call option together generates the limited liability contract of SW. Since the equity part of the contract is the same for all agents, it will be convenient in the analysis that follows to focus on the call option part of the contract only.

over the states, and not the payoffs in a state.

The security construction can be better visualised when we write the optimisation program of borrowers. We assume that all borrowers are identical in their preferences (but not in their endowments). Since ex ante borrowers only differ in their private information parameter θ, all n borrowers of the same type take the same actions, conditional on θ. Therefore the superscript n can be omitted. Borrowers' utility functions are:

Assumption 3 (Utility functions of borrowers)
The utility functions of borrowers are given by

$$U^{\theta}(c_0^{\theta}, c_1^{\theta}) = c_0^{\theta} + \beta^b E[c_1^{\theta}] \tag{4.11}$$

where $\beta^b \in (0,1)$ *is the common discount factor of borrowers,* $U^{\theta}(.)$ *is smooth and strictly increasing and expectations are taken with respect to each borrower's type specific probability measure* $p(\theta)$.

Combining the utility functions of borrowers, the project characteristics and the security structure, the optimisation program of borrowers becomes[4]:

$$\begin{aligned} & max \;\; c_0^{\theta} + \beta^b E[c_1^{\theta}] \qquad (4.12) \\ c_0^{\theta} &= [\beta^* \mu - q^c]\, z^{\theta} \\ c_s^{\theta} &= \begin{cases} 0 & if \;\; R_s \leq (1+\hat{r}) \\ [R_s - (1+\hat{r})]z^{\theta} & if \;\; R_s > (1+\hat{r}) \\ & for \;\; s = 1, \cdots, S \end{cases} \end{aligned}$$

[4]Recall that the project pays off R_s which is equivalent to the equity contract, so that for payoffs below the strike price, the payout position of borrowers is $(R_s - R_s)z^{\theta} = 0$, while for project payoffs exceeding the strike price it is: $[R_s - R_s + R_s - (1+\hat{r})]z^{\theta} = [R_s - (1+\hat{r})]z^{\theta}$.

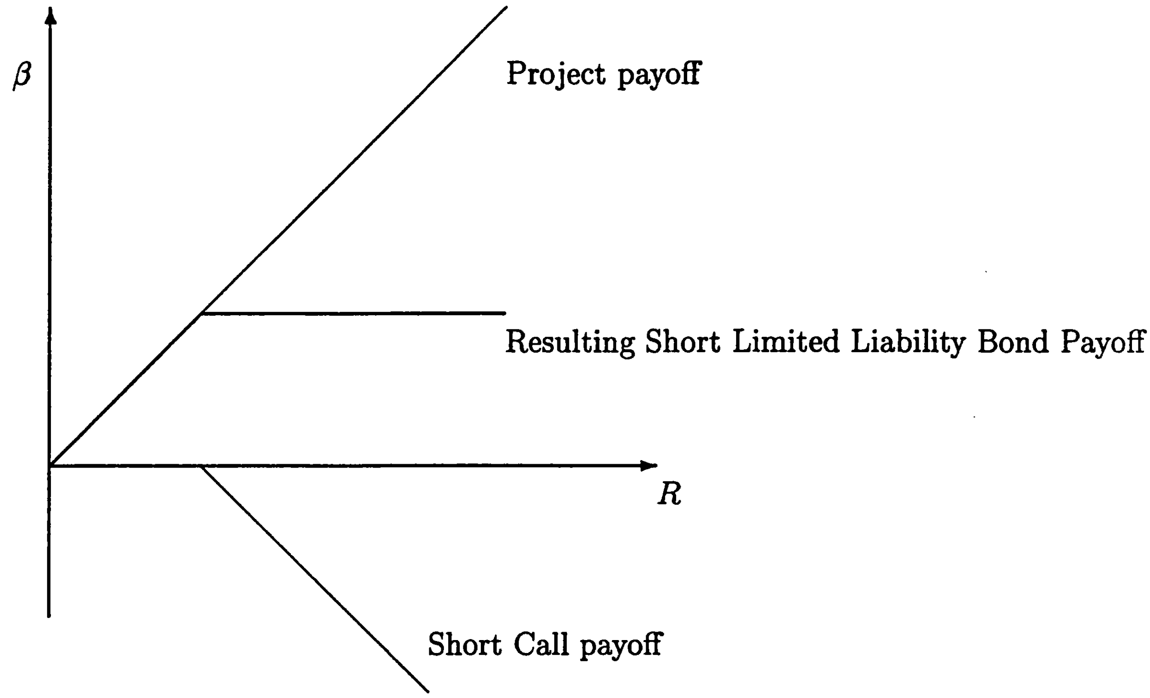
Borrowers are assumed to take long positions only in the call option, and, since the project is indivisible, $z^\theta \in (0, 1)$. The budget set of borrowers will be denoted by $B^\theta(q^c, \tilde{R})$.

Investors' optimisation

Since for borrowers, issuing the limited liability bond is equivalent to selling their project and buying the call option contract described above, investors - who act as counterparties to the borrowers - buy the projects and sell the call contracts. The result is that the counterparties have a payment profile equivalent to a short position in the limited liability bond contract, and, in this sense, the final position again corresponds to the SW structure.

Figure (3) shows graphically the payoff position of investors:

Fig.(3) Payoff position of investors



For each individual project, investors' payoff such that markets clear in period $t = 1$ must be:

$$\begin{cases} R_s & if \ R_s \leq (1+\hat{r}) \\ (1+\hat{r}) & if \ R_s > (1+\hat{r}) \\ & for \ \ s = 1, \cdots, S \end{cases} \tag{4.13}$$

To see this, add the delivery of the borrowers' contract to the contract just stated. The result is R_s for $s = 1, \ldots, S$, which is just the payoff of the project.

The θ, n-structure of the model greatly simplifies the investors' optimisation problem. We assume that investors are all the same and they are all risk-averse. The risk in the economy is embedded in the asymmetric information that borrowers have over their own projects. By the anonymity of the market, all options sell at the same price, but, in general, each (θ, n) option will have different delivery rates, since θ is the riskiness of the project, and n reflects the idiosyncratic risk of an individual borrower. Since, by the law of large numbers there is no aggregate uncertainty, risk-averse agents, in equilibrium, will ensure themselves against the idiosyncratic risks of different individual delivery rates by holding a share of every project offered in the market. Therefore, there is no loss in generality to model investors' diversification of idiosyncratic risk directly as the purchase of shares in an explicit asset pool. Since the equity part of the contract pays off the same for every project, only the call option part of the contract requires pooling.

The payoff of the pooled call option, $r^{c,p}$ is the average payoff of the call option, where the simple average can be used because of the law of large numbers.

$$r^{c,p} = \frac{\sum_\theta \lambda^\theta z^\theta \left(\sum_{R_s \geq (1+\hat{r})} p(\theta)(R_s - (1+\hat{r}))\right)}{\sum_\theta \lambda^\theta z^\theta} \tag{4.14}$$

is the average payoff of the pooled call option for $n \longrightarrow \infty$.

In order to introduce some consistency into the equilibrium concept, we need to make an assumption on the expectation of the delivery rates into the pool. The least stringent, yet consistent assumption would be that, in equilibrium, investors rationally expect *aggregate deliveries* in the limit for the number of members n of each type θ going to infinity. Since, for investors, all options are ex-ante the same, the truly expected aggregate deliveries will then be used to find the equilibrium price of all call options.

Assumption 4 (Rationally expected aggregate delivery rates)
Let the superscript 'p' denote the pool of call options. Then the expected average deliveries into the pool, or equivalently the expected average payoff of the pooled call option, $E[r^{c,p}]$ is, in equilibrium, just equal to the true payoff of the pooled call option, $r^{*c,p}$.

The construction of the asset pool and the law of large numbers allow us to write the utility function of investors in a simplified way as the utility of consumption today and one aggregate state σ at $t = 1$ only. Since investors do not have an asymmetric information project, we simply index them by i. There are $i \in I$ investors in the economy, where I is a countably infinite set.

Assumption 5 (Utility function of investors)
The utility functions of all investors are identical and are given by

$$U^i(c_0^i, c_\sigma^i) = v^i(c_0^i) + \beta^i v_\sigma^i(c_\sigma^i) \tag{4.15}$$

where $v_0^i, v_\sigma^i : \mathbb{R}_+^2 \longrightarrow \mathbb{R}$ are smooth and strictly increasing functions on $\mathbb{R}_{++}$ and we assume $v_t^{i'}(c) > 0$, $v_t^{i''}(c) < 0$, $\forall c \in \mathbb{R}_{++}$, $v_t^{i'}(c) \to \infty$ as $c \to 0$, and agents have 'pure time preference', ie.

$$\frac{\beta^i v_\sigma^{i'}(c)}{v_0^{i'}(c)} < 1 \qquad \forall c > 0$$

The fact that for the pooled call option the repayments are endogenous and determine the return on the contract, makes it possible to have several normalisations for the price of the pooled call. One such normalisation would be to set the price equal to one. Another one would be to set the price equal to the price of the individual call option. We choose the latter for consistency reasons of the pricing of the individual and the pooled call option. Therefore, the optimisation program of investors is:

$$\begin{aligned} & max \;\; v_0^i(c_0^i) + \beta^i v_1^i(c_\sigma^i) \\ c_0^i &= \omega_0^i - [\beta^* \mu - q^{c,p}]\, z^i \\ c_\sigma^i &= [\mu + r^{c,p}]\, z^i \end{aligned} \tag{4.16}$$

For investors we restrict z^i to be positive, and assume that they have sufficiently large endowments to cover their purchase of the project in $t = 0$, ie. we assume that each investor has $\omega_0^i \geq \beta^* \mu$. The budget set of investors is denoted by $B^i(q^{c,p}, \omega^i, r^{c,p})$.

Lower and upper bounds for the nominal coupon rate $\hat{r}$

Our results depend on a strictly positive nominal coupon rate $(1+\hat{r}) > 0$, since only a positive strike price K of the call option mimics the limited liability nature of the bonds we want to model. As an upper bound for the nominal coupon rate, we only allow for $\hat{r}$'s, whose mean repayments are smaller than the mean payoff of the project, ie the maximum strike price $K = (1 + \hat{r})$ cannot exceed the mean payoff of the project μ.

Market Clearing

Market clearing in the call option requires that the supply of call option contracts by investors equals the demand of call option contracts by borrowers, and that the deliveries of call option payoffs is feasible.

$$z^i = \sum_{\theta} \lambda^{\theta} z^{\theta} \tag{4.17}$$

implies

$$z^i r^{c,p} = \sum_{\theta} \lambda^{\theta} z^{\theta} r^c(\theta) \tag{4.18}$$

This condition is trivially satisfied, since borrowers simultaneously sell their projects to investors and buy the call option on it, such that there are always sufficient endowments to pay the borrowers.

Competition

As mentioned in the introduction to section (4.2), the SW model is set in a game theoretic framework. Here, we describe the competitive environment by allowing borrowers to trade on their private information θ, but they cannot influence the prices of the contract. Likewise, we allow investors to sell the call option at a uniform price for borrowers of the same observable quality, but they are not in a position to set the interest rate on these contracts. The price setting behaviour of the SW bank is modeled by conducting comparative statics analysis on changes in $\hat{r}$. The analysis involving different nominal coupon rates is thus transformed into one that analyses an equilibrium with different credit contracts.

By performing comparative statics analysis of different types of contracts via the nominal coupon rate, the price taking assumption can still be used.

The central contribution of the paper is that, in contrast to standard models of symmetric information, unlimited liability and individual contract trading, the coupon rate is not just another description of the price of the bond. The market price of the bond is determined in equilibrium by the return requirements of investors on the pool of these bonds. The deliveries in the pool, in turn, depend on the nominal coupon rate, since the nominal coupon rate sorts borrowers by their riskiness. Consequently, the relationship between the coupon rate and the price of the contracts is more complex than the usual relationship

$$1 + r^j = \frac{1}{q^j} \Longrightarrow 1 + r_e^j = (1 + \hat{r})\frac{1}{q^j} \tag{4.19}$$

for a standard one period security, where r_e^j is the normalised effective return on one unit, ie when $\frac{1}{q^j}$ units of the security are purchased at price q^j. In our model, it is the case that $\hat{r}$ describes the call option contract, which in turn induces an equilibrium return on the *pooled* call option. $r^{*call,p}$. So (4.19) changes to

$$(1 + r_e^c) = (1 + r^{c,p}))\,\frac{1}{q^{c,p}} \tag{4.20}$$

As we will show, which risk classes decide to trade and deliver into the pool depends on the nominal coupon rate. Since different risk classes have different average deliveries, the pooled return required to leave the effective return unchanged, does not change in a one-to-one way with changes in the nominal coupon rate.

Recall that pooling is generated endogenously from the asymmetric information set-up, and that the asymmetric information set-up has bite, since we have modeled limited liability. In this way, (4.20) is - in a nutshell - the difference of introducing an anonymous market into the Stiglitz-Weiss model [5].

4.4.3 Existence of a General Credit Market Equilibrium

Since contracts are only one-dimensional and the private information is over the same parameter, only pooling equilibria exist. In the following, we prove the existence of a general credit market equilibrium for any strike price in the range given above. A competitive equilibrium of this economy is a triple consisting of actions, a price and the associated pool deliveries for the call

[5] The fact that the simple relationship expressed in (4.19) is no longer true in our set-up, also strengthens the view that borrowers can issue securities without, somehow implicitly, becoming pricesetters in the asset markets.

option $((c^*, z^*), q^{*c}, r^{*c,p}) \in X \subset \mathbb{R}_+^{\theta(S+1)+2I} \times \mathbb{R}^{\theta \times I} \times \mathbb{R}_+ \times \mathbb{R}_+$ such that [6]:

(i) $(c^{*i}, z^{*i}) \in \arg\max \{U^i(c_0^i, c_1^i) \,|(c^i, z^i) \in B^i(q^{c,p}, \omega_0^i, r^{c,p})\}$

(ii) $(c^{*\theta}, z^{*\theta}) \in \arg\max \{U^\theta(c_0^\theta, c_1^\theta) \,\Big|(c^\theta, z^\theta) \in B^\theta(q^c, \tilde{R})\}$

(iii) $z^i = \sum_\theta \lambda^\theta z^\theta \Longrightarrow z^i r^{*c,p} = \sum_\theta \lambda^\theta z^\theta r^c$

(iv) $r^{*c,p} = \frac{\sum_\theta \lambda^\theta z^{*\theta}\left(\sum_{s=\tilde{R}^{-1}(K(\hat{r}))}^{R_s=R_S} p_s(*\theta)((1+\hat{r})-R_s)\right)}{\sum_\theta \lambda^{*\theta} z^{*\theta}}$

$$(v)\ r^{*c} = \begin{cases} 0 & if\ \ R_s \leq (1+\hat{r}) \\ (R_s - (1+\hat{r}))z^\theta & if\ \ R_s > (1+\hat{r}) \\ & for\ \ s = 1, \cdots, S \end{cases}$$

(vi) $q^c \equiv q^{c,p}$

(vii) $z^i \leq 0$

(viii) $z^\theta \in \{0, 1\}$

Verbally, the equilibrium price determination can be thought of in the following way: borrowers perceive a price for the call option q^c. At this price, they decide whether to buy or not. Since they are risk neutral, and by the indivisibility of the project, they will either demand one unit, or zero. Since all n agents of a given type θ take the same actions, there are then θ different demands for the call option. All the deliveries of the call option are pooled. The price of the pooled call is, by (vi), the same as the price of the individual call. The price - pool delivery combination is observed by investors and they supply a quantity of call options for this combination. If the quantity supplied and the quantity demanded coincide, there exists an

[6]Since all n borrowers of a type θ take the same actions, we omit the superscript n

equilibrium.

Theorem 6 (Existence)
A general credit market equilibrium (GCME) exists.

4.5 Characterising the Equilibrium

We proceed in the following way. Firstly, we show that for every coupon rate contracts are relatively more valuable for higher risk types. The counterimage of the first proposition is that individual delivery rates, for every coupon rate, are a decreasing function of the riskiness of the project. We deduce that deliveries into the pool are dependent on the strike price. Since, by a standard argument, the equilibrium price will adjust if the deliveries into the pool change to equilibrate demand and supply of the call option, the individual call option price will change with both the strike price and the different mix of borrowers. With mean preserving spreads the effective interest rate on the call option may rise or fall with an increase in the strike price. However, by proposition (1) it will always be true that bad risk borrowers value the call option more, independently of the mix of borrowers. Consequently, if the strike price change induces an adverse change in the mix of borrowers, then good risk borrowers will drop out of the market first.

Next we assert that a reduced willingness to lend will increase the equilibrium price of the call option, for every strike price. We deduce that this leads to a deterioration in the quality of the borrowers. We argue that the equilibrium price of the call option must rise proportionately more than the increase in the unwillingness to trade. However, as long as the market does not unravel to a no trade equilibrium, we can still always find a - higher - equilibrium price independently of the contract chosen, which yields the same required average return. We will then deduce that an increased unwillingness to lend has the effect that the average return on bonds increases for all contracts, ie. independently of the coupon rate, and that, if the same contracts are used, the equilibrium volume of credit goes down.

It is first necessary to establish that the real return on the asset is positive. Otherwise, all projects would always be undertaken.

Lemma 1 (Positive Effective Return)
The equilibrium real return on the pooled call option is positive.

Even though the market, from the point of view of the agents, is effectively 'complete', the asymmetric information over borrowers' projects, combined with the short sale constraints (vii) and (viii), imply that the valuation of the call option will remain different for different borrowers. The next proposition states how the riskiness of a type of borrower relates to their valuation of the call option.

Proposition 1 (Borrowers' Call Option Valuation)
For a positive strike price $K = (1 + \hat{r})$, borrowers with a higher θ have a higher valuation of the call option.

We deduce from this proposition that the preferred contract for everyone is the one with the lowest possible strike price, ie. with $K = 0$. In this case, all borrowers have the same valuation of the limited liability bond. Once the strike price is strictly positive, the mix of borrowers induced by the strike price starts playing a role.

For the reverse side of the market, we can state a similar proposition, namely that for every individual call option, the payout is inversely related to the riskiness of the borrower.

Proposition 2 (Payout of Individual Call Option)
The payout r^c of the individual call option is a decreasing function of the riskiness of the projects.

All the deliveries go to the pool, and the deliveries in the pool determine the price of the pooled call option and consequently the price of the individual call option by the arguments in section (4.4.3). Since all the call options are pooled, the mix of borrowers who buy the call option, which by proposition (1) is dependent on the strike price K, as well as the repayment characteristic of the call option itself, are decisive for the deliveries of the pool. These deliveries, in turn, determine the price of the pooled call option. The next proposition states that, by a standard argument, the price of the pooled call option adjusts in such a way that markets clear.

Proposition 3 (Adjustment of Pooled Call)

When the strike price changes, the equilibrium price of the pooled call option adjusts such that markets clear.

However, it is still the case that the nominal coupon rate $\hat{r}$ plays a role. It still sorts borrowers adversely by their riskiness (Proposition (1)), and, more strongly, in the sense that they will be the first not to demand the call option should a change in the strike price induce an adverse mix. Hence the strike price has an influence on the demand for the bond. However, it is important to note that the proposition is weaker than in the SW set-up: the assumption of a mean preserving spread does not necessarily imply that the riskiness of the pool always deteriorates with an increase in the coupon rate.

Proposition 4 (Sorting of Borrowers)
As the nominal coupon rate $\hat{r}$ of the bond increases, the critical value of θ, below which borrowers do not buy the bond, may increase. Borrowers are sorted by their riskiness, in the sense that if borrowers drop out of the market, they will be the good risk ones. The nominal coupon rate influences the demand for bonds.

We state as a corollary:

Corollary 1 (Pool Deliveries)
The deliveries into the pool are not a monotonic function of the nominal coupon rate $\hat{r}$. Therefore the effective interest rate on the call option may go up or down with changes in the strike price.

4.5.1 Comparative Statics of a Credit Crunch

An increase in the strike price is only one way in which good borrowers are forced out of the market. The other possibility refers to a situation, in which the propensity to lend falls. Then, the following proposition shows , that the price of the call option must increase. Since lower risk borrowers have a lower valuation for all call options with a positive strike price K it is then implied that, should credit contract, good borrowers will be driven out of the market first.

The situation we want to depict is the following. For some external reasons, the supply of credit is low (in our model creditors are unwilling to lend). We ask whether in this situation lenders should raise the interest rate at which they lend to borrowers. The answer is that it is not useful to use the nominal coupon rate as a device to adjust the volume of credit, since independently of the supply of credit it will always sort the riskiness of the borrowers. A lower volume of credit may actually worsen the adverse selection effect, in the sense that good borrowers will be driven out of the market. However, even if this corresponds to the SW credit rationing argument, our point here is that the problem of the return on pool securities is unchanged by the volume of credit. The contracts used will not be changed.

We model reduced willingness to lend money by a decrease in the discount factor β^i.

Proposition 5 (Fall in Propensity to Lend)
The equilibrium price for the call option is a decreasing function in β^i.

A reduced willingness to lend on the part of investors drives up the equilibrium price and consequently drives out those borrowers that have a lower valuation for the insurance offered by the call option. By proposition (1), these are the low risk borrowers. Consequently the risk profile of borrowers deteriorates. However, this does not imply that the return on the pool security falls, since the price of the call option rises as well. Since proposition (3) on the equilibrating mechanism of the market price is independent of the level of the market price, all contracts with different strike prices K and different nominal coupon rates $(1+\hat{r})$ will adjust accordingly.

4.5.2 A Comparison with Credit Rationing in Stiglitz-Weiss

Since the uncertainty structure and securities are very similar to the set-up in SW, it may be of interest to compare their result of credit rationing to our model. Naturally, since the definition of Walrasian equilibrium does not permit the existence of excess demand for credit at positive prices in equilibrium, the concept of credit rationing needs to be reformulated in an appropriate manner. One way to achieve comparable predictions is to introduce borrowing constraints on the individuals level, and characterise the credit market equilibrium in the presence of these constraints.

The change we need to introduce is to consider divisible projects, and impose a borrowing constraint on the equilibrium demand for credit. Risk-neutrality on the part of borrowers simplifies the process. Assume that the upper bound on the size of a project is one unit of the numéraire commodity in period $t = 0$. By risk neutrality of borrowers, they will either demand zero or one unit of the security. Consequently, a binding borrowing constraint must be of size less than one.

If the propensity to lend of investors is unchanged, the borrowing constraint implies that there is excess supply of credit at the old equilibrium price. Consequently, the price of credit falls. Since, by proposition (5), good risk borrowers drop out of the market first, conversely it must hold that new entrants into the market will be better risk borrowers. As a consequence, by corollary (1) average pool deliveries *may* rise, and the price for the individual call option *could* fall, while the volume of credit dispensed *could* rise. The resulting new equilibrium, even though it necessarily features a lower individual price for the call option, may have a higher effective interest rate for investors and/or a higher volume of credit.

Compared to SW, there is a remarkable similarity in the slant of the argument. Although a borrowing constraint will not directly change the return for investors as in SW, by attracting lower risk borrowers it may decrease the effective interest rate for borrowers and hence increase the volume of credit at the same rate of return for borrowers. Market clear in both situations, but the borrowing constraint may be beneficial for lenders. This outcome could be interpreted as corresponding to credit rationing in the SW set-up.

4.6 An Extension to the Basic Model: Introducing Collateral

Credit contracts are generally multidimensional. Perhaps the most important feature apart from the interest rate is collateral. Collateral is used both to secure a loan and to recover the losses in case of default. We introduce collateral into a simplified version of the model with two types only, θ_1 and θ_2, where θ_1's project is a mean preserving spread of θ_2.

Collateral is collateral of the consumption good, with the restriction that it cannot be consumed by investors at $t = 0$. Collateral is *stored* from $t = 0$ to $t = 1$ (it cannot be invested), and, in the case of default by firms, is consumed by investors at $t = 1$. On the other hand, if the project is successful, the collateral will return to borrowers for consumption in $t = 1$. Since we want to consider risky loans only, we assume that the collateral which can be call up to secure the loan is less than the outstanding value of the loan. All projects cost one in period $t = 0$, so that this condition implies that the maximum collateral must be smaller than one. The way we restrict collateral is by assuming that firms have an endowment smaller than one in period $t = 0$.

Assumption 6 (Collateral of Firms)
Firms have an endowment of $\omega_0^\theta < 1$ *units of consumption good in* $t = 0$ *and can call up at most* $\gamma^\theta \leq \omega_0^\theta$ *units of collateral.*

The optimisation problem of borrowers is altered in an obvious way:

$$\begin{aligned} & max \;\; v^{\theta}(c_0^{\theta}) + \beta\, E[c_1^{\theta}] \\ c_0^{\theta} &= \omega_0^{\theta} + \left[\beta^{*}\mu - \gamma^{\theta} - q^{c}\right] z^{\theta} \\ c_1^{\theta} &= \begin{cases} 0 & if \;\; R_s \le (1+\hat{r}) - \gamma^{\theta} \\ (R_s - (1+\hat{r}))z^{\theta} & if \;\; R_s > (1+\hat{r}) - \gamma^{\theta} \\ & for \;\; s = 1, \cdots, S \end{cases} \end{aligned} \tag{4.21}$$

Since the strike price is changed, we will denote the strike price with collateral by $K^c = (1+\hat{r}) - \gamma^{theta}$. To construct the payoff of the pooled call option, again we first look at the payout of the corresponding individual call.

$$\begin{cases} R_s + \gamma^{\theta} & if \;\; R_s \le (1+\hat{r}) - \gamma^{\theta} \\ (1+\hat{r}) & if \;\; R_s > (1+\hat{r}) - \gamma^{\theta} \\ & for \;\; s = 1, \cdots, S \end{cases} \tag{4.22}$$

Following assumption (5) again on the price of the pooled security, and using the law of large numbers, the payoff of the pooled call option is now:

$$r^{c,p} = \sum_{\theta} \lambda^{\theta} z^{\theta} \left(\left(\sum_{s=1}^{R_s=K-1} p_s(\theta)\gamma^{\theta} + \sum_{R_s=K}^{R_s=R_S} p_s(\theta)(1+\hat{r}) - R_s \right) \right) \tag{4.23}$$

and the investors' optimisation program is unchanged:

$$\begin{aligned} & max \;\; v^{i}(c_0^{i}) + \beta^{i} E[c_1^{i}] \\ c_0^{i} &= \omega_0^{i} - \left[\beta^{*}\mu - q^{c,p}\right] z^{i} \\ c_{\sigma}^{i} &= r^{c,p} z^{i} \qquad\qquad \forall \sigma \end{aligned} \tag{4.24}$$

Note that the collateral 'disappears' in $t = 0$. It is subtracted from the utility of borrowers but does not appear in the utility function of investors. This is the most simple formulation for the notion that collateral can only be stored, but not consumed. Looking at the net payoff of a long position in the call option and a short position in the call option,

$$\begin{cases} R_s + \gamma^\theta & if \;\; R_s \leq (1+\hat{r}) - \gamma^\theta \\ R_s & if \;\; R_s > (1+\hat{r}) - \gamma^\theta \end{cases} \tag{4.25}$$

collateral reappears in the payoff. Thus, it is implicit in the investment in the call option.

Equilibrium determination in this variation of the model is unchanged. Investors require a return on the pool security depending on their preferences of consumption today over consumption tomorrow. Individual securities then for every combination of a coupon rate and a level of collateral then sell at the price which makes their deliveries just equal to the expected return required by lenders.

However, there is now a fundamental difference in pricing. Since a separating equilibrium reveals the true θ's, the securities will be priced 'correctly'. The pool does not longer consist of a mixture of different types with different probability distributions of success. Rather there exist two pools now, whose prices reflect the probabilities of success of each type. A direct consequence is that the more complex price/return relationship in equation (4.20) is replaced by the standard price/return relationship of (4.19). Thus, in a separating equilibrium, the price *or* the return of a security characterise it completely and the introduction of the anonymous market does not add to the analysis. Interestingly, with collateral it is also the case that separating equilibria, should they exist, solve the problem of credit rationing in the Stiglitz-Weiss model (Bester (1985)). Consequently, in a certain way, the credit rationing of Stiglitz-Weiss is reflected in the pooling of different risk types in the anonymous market price in our model. Credit rationing can be expressed as 'mispricing' in the credit market.

4.6.1 Characterisation of a Separating Equilibrium with Collateral

We will now state the proposition regarding the separation with collateral:

Proposition 6 (Separating Equilibrium in Economy with Collateral)

In the economy with collateral, the following separating equilibrium exists: low risk borrowers θ_2 buy a call option with more collateral and a lower coupon rate than high risk borrowers θ_1.

4.6.2 Comparative Statics of a Credit Crunch with Collateralised Bonds

Our main interest is in the behaviour of borrowers in a situation in which credit contracts. Without collateral we have shown that the average credit quality always deteriorates, and that firms have no incentives to offer higher coupon rates since the market price always adjusts.

The result we present is that good borrowers can outstay bad borrowers in the market.

Corollary 2 (Fall in Propensity to Lend in Economy with Collateral)

If, in an economy with collateral, the propensity to lend falls, and all assets have limited liability, the bad risk firm θ_1 may be the first one not to apply for credit.

4.7 Conclusion

In this paper we have demonstrated that if borrowers have asymmetric information over projects and can issue limited liability bonds, then, while the nominal coupon rate on bonds has no *direct* effect on the equilibrium price, it always sorts borrowers by their riskiness at any strike price. The sorting mechanism implies that borrowers have different delivery rates for ex-ante identical contracts. If the deliveries are pooled, or, as in the present model, investors hold the market portfolio, then different average delivery rates are reflected in the dependency of the bond price on the nominal coupon rate. Therefore, there is an *indirect* effect of the nominal coupon rate on both the volume and price of the contracts traded.

The model we have chosen is a perfectly competitive price taking market. By the equilibrium definition of this model, Stiglitz-Weiss type credit rationing cannot exist. Rationing is avoided through the adjustment of the market price of credit. However, a result which is similar in spirit to Stiglitz-Weiss can be obtained, if borrowers face individual borrowing constraints.

Once collateral is introduced, separating equilibria which solve the asymmetric information problem may exist. In such an equilibrium, borrowers pay for the true riskiness of their projects plus a possible signaling cost. In this setting, the nominal coupon rate and collateral not only serve to sort borrowers and are important for the characteristics of the equilibrium, like in the version without collateral, but they also have a direct effect on price, since the riskiness of borrowers is revealed.

4.8 Appendix

Proof of Theorem (6)

For investors there is no aggregate risk, since the only payoff is the payoff of the project security, which is constant across aggregate states. By the risk-neutrality of borrowers, only c_0^i and $E[\tilde{R}]$ matter. Consequently the individual call option (q^c, r^c) can be thought of as financing consumption transfers from c_0^θ to $E[c_s^\theta]$, while the pooled call option $(q^{c,p}, r^{c,p})$ finances transfers from c_0^i to c_σ^i. An equilibrium occurs when the quoted price q^c induces aggregate deliveries $r^{c,p}$ by borrowers such that at the combination $(q^c \equiv q^{c,p}, r^{c,p})$ the supply of the pooled call is just equal to the sum of individual calls. By conditions (vi) and (vii), and the positivity of prices, the budget correspondences: $B^i : (q^{c,p}, \omega_0^i, r^{c,p}) \longrightarrow X$ and $B^\theta : (q^c, \tilde{R}) \longrightarrow X$ are compact-valued, convex-valued, and continuous correspondences. Define the demand correspondences $\Phi^i(q^c) = argmax\{U^i(c_0^i, c_1^i) \,|(c^i, z^i) \in B^i(q^{c,p}, \omega_0^i, r^{c,p})\}$ and $\Phi^\theta(q^{c,p}, r^{c,p}) = argmax\{U^{\theta,n}(c_0^{\theta,n}, c_1^{\theta,n}) \,\big|(c^{\theta,n}, z^{\theta,n}) \in B^{\theta,n}(q^c, \tilde{R})\}$, where the demand correspondences are shorthand for 'c^* are the elements which maximise $U(.)$ over the budget set and z^* finances c^*'. Using the Maximum Theorem, by the linearity and smoothness of $U^\theta(.)$, $\Phi^\theta(.)$ is an upper- hemicontinuous (uhc) correspondence. By the strict concavity and smoothness of $U^i(.)$, $\Phi^i(.)$ is a continuous function. Since the asset pays in the numéraire commodity, doubling prices doubles demand and income, so that $\Phi^\Theta(.)$, Φ^i are homogeneous of degree zero, $\Phi(\alpha q^c) = \Phi(q)$ for all $\alpha > 0$, for all $q^c \in I\!R_{++}$. By strict monotonicity, and recalling that the price of the numéraire commodity is normalised to one, $c_0 - \omega_0 = -q^c\Phi(.), c_s = R_s^c\Phi(.)\ \forall s$, for all q^c. Since $z^\theta \in \{0, 1\}$ for every i, $\Phi(.)$ is bounded. Demand 'tends away' from the boundary, since agents have interior endowments (resources), ie if $(q^c)^n \longrightarrow q^c \in \partial I\!R_{++}$ and $\omega_0 > 0$, $\Phi(.) \longrightarrow \infty$ as $n \longrightarrow \infty$. An equilibrium exists where the aggregate excess demand function $Z(q^c) = \sum_i \Phi^i(.) + \sum_\theta \Phi^\theta(.)$ has a zero. By standard arguments on the sum of continuous correspondences, $Z(q^c)$ inherits boundedness, uhc, homogeneity, Walras' Law that $q^c Z(q^c) = 0$ for all q^c, and boundary behaviour. Since the borrowers only exist through the projects (if $z^\theta = 0$ for all zero, there are no borrowers), their equilibrium demand must lie in the set $\{0, 1\}$. Since $z^\theta = 0$ is also an equilibrium candidate, by the intermediate value theorem, an equilibrium must exist. □

Proof of Lemma (1)

By the risk neutrality of borrowers and the constancy of payoffs, and hence resources, in the aggregate economy, projects carry no risk premium. Thus the equilibrium effective return on the pooled call option can be written as $1 + r^{*e,p} = \frac{1+r^{*c,p}}{q^{*c}}$. It is required to show that $r^{*e,p} > 0$. Projects are viable, $E[\tilde{R}] > 1$ and by the monotonicity of $U^\theta(.)$, borrowers prefer positive consumption to zero consumption, and will always prefer trade at $1 + r^{*e,p} > 0$ to no trade. For investors, pure time preference implies that $v_\sigma^{i'}(c) < v_0^{i'}(c)$ $\forall c > 0$. $r^{*e,p} < 0$ contradicts this assumption and consequently investors would not lend. Hence the real return on the call option must be strictly positive. □

Proof of Proposition(1)

Consider the optimisation program of the borrowers. Identifying the Lagrange multipliers by λ and equilibrium values by a star, the first order conditions are:

$$\begin{aligned}
\nabla U^{\theta}(c^{*\theta}) &= \lambda^{*\theta} \qquad (4.26)\\
-\lambda_0^{*\theta} q^{*\theta} + \sum_{R_s=K}^{R_s=R_S} \lambda_s^{*\theta}(R_s-(1+\hat{r})) &= 0\\
c_0^{*\theta,n} &= [\beta^{*}\mu - q^{*}]\, z^{*\theta}\\
c_1^{*\theta,n} &= \begin{cases} 0 & if\ R_s \le (1+\hat{r})\\ (R_s-(1+\hat{r}))\bar{z}^{\theta} & if\ R_s > (1+\hat{r})\\ & for\ s=1,\cdots,S \end{cases}
\end{aligned}$$

Since all agents are risk neutral, the first line can be expressed in terms of the type-specific probabilities of the agents:

$$\nabla U^{\theta}(c^{*\theta}) = \frac{1}{\beta^{b}}(p_1(\theta),\cdots,p_S(\theta)) \tag{4.27}$$

Use this to rewrite the equilibrium valuation of the call option as:

$$\hat{q}^{\theta}(R) = \sum_{R_s=1}^{R_s=K-1} \frac{1}{\beta^{b}} p_s(\theta) * 0 + \sum_{R_s=K}^{R_s=R_S} \frac{1}{\beta^{b}} p_s(\theta)(R_s-(1+\hat{r})) \tag{4.28}$$

For a positive strike price K, the price of the call option is non-decreasing convex in the project payout $\{R_s\}_{s=1}^{R_s=R_S}$. Since a mean-preserving spread is equivalent to second-order stochastic dominance, it follows that for two different risk classes θ_1 and θ_2, with θ_1 riskier than θ_2,

$$\sum_{i=1}^{i=S} q^{\theta_1}(R)(R_s - R_{i-1})p_i(\theta_1) \geq \sum_{i=1}^{i=S} q^{\theta_2}(R)(R_s - R_{i-1})p_i(\theta_2) \tag{4.29}$$

which by the assumption of the same mean of R implies that

$$q^{\theta_1}(R) \geq q^{\theta_2}(R) \tag{4.30}$$

□

Proof of Proposition (2)

The proof is the mirror image of the proof to proposition (1).

Proof of Proposition (3)

We distinguish two cases: one that deliveries into the pool rise to $r^{+c,p}$, the other that deliveries fall to $r^{-c,p}$, such that $r^{+c,p}$ and $r^{-c,p}$ are in the neighbourhood of the equilibrium (ie in an open set which contains these points which is a subset of every set that contains the equilibrium). We need to show that $(r^{-c,p}, q^{*c,p}, (c^*, z^*))$ and $(r^{-c,p}, q^{*c,p}, (c^*, z^*))$ are not GCME. This will always be the case if GCMEs are regular, the condition for which is that $\partial Z(q^c)/\partial q^c$ (Z being the excess demand correspondence) is non-zero at the equilibrium, which is the case given our utility functions and endowment and uncertainty structure. □

Proof of Proposition (4)

By proposition (1), borrowers with a higher θ have a higher valuation of a call option for any positive strike price K. For a given type $\hat{\theta}$, the valuation of the option is:

$$q^{\theta}(R) = \frac{1}{\beta^b}\left(\sum_{R_s=R_1}^{R_s=K-1} p_s(\hat{\theta}) * 0 + \sum_{R_s=K}^{R_s=R_S} p_s(\theta)(R_s - (1+\hat{r}))\right) \tag{4.31}$$

$p_s > 0$ and that the second sum is decreasing in $\hat{r}$ imply that for a larger K the equilibrium valuation of a given type $\hat{\theta}$ falls. Assumption (1) permits two cases: one in which the valuation falls relatively more for a high risk type, and one in which it falls relatively more for a low risk type (as long as proposition (1) holds). To see this, consider two risk classes, θ_1 and θ_2.

By assumption (1),

$$\sum_{s=1}^{s=K-1} (R_{K-1} - R_{s-1})\,[p_s(\theta_1) - p_s(\theta_2)] \geq 0 \tag{4.32}$$

integrating by parts yields:

$$\begin{aligned}(F(K,\theta_1) - F(K,\theta_2))\,K &= \qquad\qquad (4.33)\\ \left(\sum_{s=1}^{s=K-1} [p_s(\theta_1) - p_s(\theta_2)]\right) K &\geq \sum_{s=1}^{s=K-1} R_s p_s(\theta_1) - \sum_{s=1}^{s=K-1} R_s p_s(\theta_2)\end{aligned}$$

by the assumption of the same mean for every θ, it follows that:

$$\left(\sum_{s=1}^{s=K-1} [p_s(\theta_1) - p_s(\theta_2)]\right) K \leq \sum_{s=K}^{s=S} R_s p_s(\theta_1) - \sum_{s=K}^{s=S} R_s p_s(\theta_2) \tag{4.34}$$

Note that the right hand side is just the difference of the type-specific valuations of the call option. Proposition (1) assures that the right hand side is always positive, for every K and every pair of θ's. However, the left hand side could be positive or negative, and consequently the size of the difference on the right hand side is not determined by the mean preserving spread. Geometrically, the intuition is the following: the criterion of the mean preserving spread is a statement about the area under the distribution functions for different risk types. However, it still allows (indeed requires) that the distribution functions itself cross an odd number of times. Depending on the strike price, either the θ_1 or the θ_2 distribution function may lie above the other. When $F(R, \theta_1)$ lies above $F(R, \theta_2)$ for a particular strike price, the difference or ratio of call option valuations expands. When $F(R, \theta_1)$ lies below $F(R, \theta_2)$, the difference or ratio of call option valuation contracts. □

Proof of Corollary (1)

Proposition (4) implies that depending on the strike price of the call option, the differences in valuation across different risk types can expand or contract. Since valuations are just the mirror image of deliveries into the pool, different strike prices imply different average pool deliveries. Since by proposition (3) bond prices always adjust, the effective return on the bond as defined in section (4.4.2) may be higher or lower. Since for borrowers, the individual effective return on the bond is their delivery rate divided by the price of the call, a different mix of borrowers will buy the call option, depending on the strike price. Consequently deliveries into the pool are non-monotonic in the strike price. □

Proof of Proposition (5)

Since the two sides of the market are completely separate, a change in β^i is 'shift' upwards in the demand function. By the strict concavity of U^i, its level surfaces are convex and smooth. An increase in β^i means that $-\frac{\partial U^i(.)/\partial c^i_\sigma}{\partial U^i(.)/\partial c^i_0}$ decreases, meaning that a higher c^i_σ is required for indifference. Hence the 'supply' function $\Phi^i(.)$ moves to the right, and the equilibrium price will increase. □

Proof of Proposition (6)

The good type borrowers call up collateral up to the point at which the bad type borrowers are just indifferent between imitating and paying the higher interest rate in a separating equilibrium. If at this point the incentive compatibility conditions of borrowers are still met, then a separating equilibrium exists.

The first order conditions for the borrowers' program with collateral is:

$$\begin{aligned}
\nabla U^\theta(\bar{c}^\theta) &= \bar{\lambda}^\theta \qquad (4.35)\\
-\bar{\lambda}^\theta_0 \bar{q}^\theta + \sum_{R_s=K}^{R_s=R_S} \bar{\lambda}^\theta_s (R_s - (1+\hat{r})) &= 0 \\
\bar{c}^{\theta,n}_0 &= \omega^\theta_0 + \left[\beta^* \mu - \bar{q} - \gamma^\theta\right] \bar{z}^\theta
\end{aligned}$$

$$\bar{c}_1^{\theta,n} = \begin{cases} 0 & if\ R_s \leq (1+\hat{r}) - \gamma^\theta \\ (R_s - (1+\hat{r})B)\bar{z}^\theta(\theta) & if\ R_s > (1+\hat{r}) - \gamma^\theta \\ & for\ s = 1, \cdots, S \end{cases}$$

Using risk-neutrality, the gradient vector can be expressed in terms of the probabilities of the states:

$$\nabla U^\theta(\bar{c}^\theta) = \frac{1}{\beta^b}(p_1(\theta), \cdots, p_S(\theta)) \tag{4.36}$$

Use this to rewrite the equilibrium valuation of the call option as:

$$\hat{q}^\theta = \sum_{s=K^c}^{R_s=S} \frac{1}{\beta^\theta} p_s(\theta)(R_s - (1+\hat{r})) \tag{4.37}$$

Compare the first order conditions and equation (4.37) with the first order conditions for the program without collateral and the pricing equation (4.27). The difference of introducing collateral into the model is that the strike price shifts 'down' at a cost of γ units of consumption in period $t = 0$.

The value of the collateral can be explicitly calculated, using the linearity property of the expectations operator:

$$\begin{aligned} \hat{q}_c^\theta &= \sum_{s=K^c}^{s=S} \frac{1}{\beta^b} p_s(\theta)(R_s - (1+\hat{r})) - \sum_{s=K^c}^{s=S} \frac{1}{\beta^b} p_s(\theta)(R_s - (1+\hat{r})) \qquad (4.38) \\ &= \sum_{s=K^c}^{s=K^c+\gamma^\theta} \frac{1}{\beta^b} p_s(\theta)(R_s - (1+\hat{r})) \end{aligned}$$

Using proposition (1), it can be deduced that the value of collateral is decreasing in the riskiness θ of the projects. This implies that the good risk firm θ_2 has a lower cost of collateral than the firm θ_1.

The bad risk firm takes on the cost of collateral as long as the benefit from pooling exceeds the cost of collateral. The benefit from pooling for the high risk firm is the 'subsidy' they gain by being pooled with low risk agents who deliver more into the pool security than they themselves. Denoting by e the effective interest rate, and writing p for a pooling allocation and s for a separating allocation, the benefit in terms of a normalised effective interest rate is:

$$r_s^e - r_p^e = \hat{r}\left(\frac{1}{q^{c,p}} - \frac{1}{q^c}\right) \tag{4.39}$$

The incentive compatibility constraints are then the obvious differences between the cost of collateral and the benefit of pooling, and the cost of collateral and the cost of pooling. They are:

(i') $\hat{q}_c^{\theta_1} \leq r_s^e - r_p^e$
(ii') $\hat{q}_c^{\theta_2} \geq r_s^e - r_p^e$

Whether these conditions are fulfilled depends on the risk parameters θ and the proportions λ^θ in the economy. A description of an equilibrium would then consist of actions and prices for the call option $((c^*, z^*, \gamma^*), q^{*c}(\hat{r}_1), q^{*c}(\hat{r}_2)) \in X \subset I\!R_+^{\theta(S+1)+2I} \times I\!R^{I+\theta} \times I\!R_+^{\theta} \times I\!R_+ \times I\!R_+$ such that:

(i) $(c^{*i}, z^{*i}) \in \arg\max \{U^i(c_0^i, c_1^i) \,|(c^i, z^i) \in B^i(q^c, \omega_0^i)\}$
(ii) $(c^{*\theta}, z^{*\theta}, \gamma^{*\theta}) \in \arg\max \{U^\theta(c_0^\theta, c_1^\theta) \,\big|(c^\theta, z^\theta, \gamma^\theta) \in B^\theta(q^c, \tilde{R})\}$

(iii) $\sum_n \lambda^{\theta_k} z^{\theta_k,n} = z^i$ for $k = 1, 2$ and for both call contracts.

(iv) $r_1^{*c} = \begin{cases} 0 & if \;\; R_s \leq (1+\hat{r}_1) \\ (R_s - (1+\hat{r}_1))z^\theta & if \;\; R_s > (1+\hat{r}_1) \\ & for \;\; s = 1, \cdots, S \end{cases}$

(v) $r_2^{*c} = \begin{cases} 0 & if \;\; R_s \leq (1+\hat{r}_2) \\ (R_s - (1+\hat{r}_2))z^\theta & if \;\; R_s > (1+\hat{r}_2) \\ & for \;\; s = 1, \cdots, S \end{cases}$

(vii) $z^i \leq 0$

(viii) $z^\theta \in \{0, 1\}$

and conditions (i') and (ii'). If the incentive compatibility conditions are met, the problem turns into a standard equilibrium problem with symmetric information and prices for the individual call options can be found by standard arguments. □

Proof of Corollary (2)

The corollary is an immediate consequence of the existence of separating equilibria. In separating equilibria, types are revealed and bad types pay higher interest rates at positive strike prices than good types. Since all projects have the same mean return, for positive strike prices bad borrowers will find the net payoff of the project less valuable than good borrowers and consequently will be the first to drop out of the market should credit contract.
□

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