Essays on the Macroeconomics of Inequality

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Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others.

I warrant that this authorization does not, to the best of my belief, infringe the rights of any third party.

Abstract

This thesis contains three essays on the macroeconomics of inequality.

The first chapter analyses the effects of minimum wages on inequality. While there has been intense debate in the empirical literature about the effects of minimum wages on inequality in the US, its general equilibrium effects have been given little attention. In order to quantify the full effects of a decreasing minimum wage on inequality, I build a dynamic general equilibrium model, based on a two-sector growth model where the supply of high-skilled workers and the direction of technical change are endogenous. I find that a permanent reduction in the minimum wage leads to an expansion of low-skilled employment, which increases the incentives to acquire skills, thus changing the composition and size of high-skilled employment. These permanent changes in the supply of labour alter the investment flow into R&D, thereby decreasing the skill-bias of technology. The reduction in the minimum wage has spill-over effects on the entire distribution, affecting upper-tail inequality. Through a calibration exercise, I find that a 30 percent reduction in the real value of the minimum wage, as in the early 1980s, accounts for 15 percent of the subsequent rise in the skill premium, 18.5 percent of the increase in overall inequality, 45 percent of the increase in inequality in the bottom half, and 7 percent of the rise in inequality at the top half of the wage distribution.

In the second chapter, I construct a model, where the supply of skills and the skill premium can increase jointly, as occurred in the US over the past few decades. I highlight the importance of the joint determination of the direction of technical change and skill formation. There is a positive feedback between these two variables. Technological progress is driven by profit oriented R&D firms, where profits are increasing in the amount of labour that is able to use these technologies. Therefore, when the supply of high-skilled

labour increases, technology endogenously becomes more skill-biased. A more skill-biased technology leads to a higher skill premium, which increases the incentives to acquire education, and the supply of high-skilled labour rises. During the transition to the steady state, both quantities increase simultaneously. I map the dependence of the transition path of the economy on the initial skill supply and relative technology between the high- and the low-skilled sector. I find that, contrary to the previous literature, the skill premium and the skill supply can increase jointly even if the bias of technology is weak.

In the third chapter, I relate the degree of progressivity of the income tax scheme to the prevailing income inequality in the society. I find that, consistent with the data, more unequal societies implement more progressive income tax systems. I build a model of political coalition formation, where different income groups have to agree on a tax scheme to finance the public good. I show that, the greater income inequality is, i.e. the further away the rich are from the rest of the population, the less able they are to credibly commit to participating in a coalition. Therefore, as income inequality rises, the rich are increasingly excluded from the design of the income tax scheme. Consequently, the rich bear a larger fraction of the public good, and the tax system becomes more progressive.

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1.1 Introduction

It is well documented that income inequality has drastically increased in the United States over the past 30 years along several dimensions.[1] Inequality increased between workers with different educational levels: the college premium increased by 18 percent from 1981 to 2006. The distribution of wages also widened: the gaps between different percentiles of the wage distribution increased drastically. For example, in 2006 a worker at the 90th percentile of the wage distribution earned 283 percent more than a worker at the 10th percentile, whereas this figure was 190 percent in 1981.[2] These trends are illustrated in Figure 1.1.

The changes in the structure of wages fuelled an extensive debate on the forces driving them. One explanation focuses on changes in labour market institutions, and particularly, on a 30 percent decline in the real minimum wage that took place in the 1980s, since the

[1] See for example Eckstein and Nagypál (2004), Goldin and Katz (2007), and Goldin and Katz (2008).

[2] Calculations from Current Populations Survey (CPS) Merged Outgoing Rotation Group (MORG) data for years 1981 and 2006.

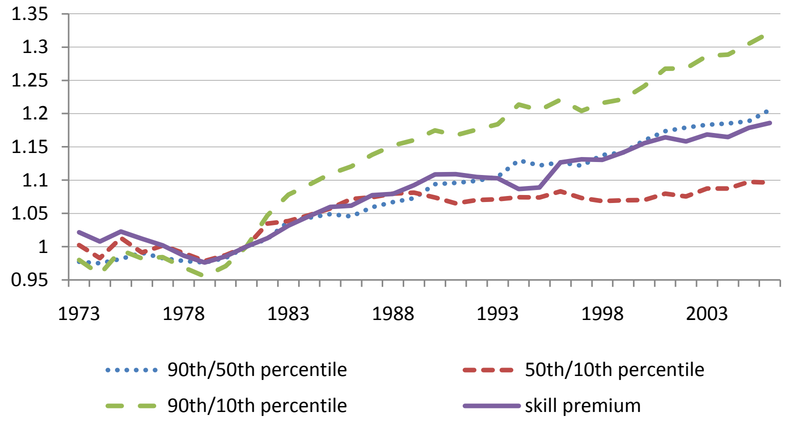

Figure 1.1: Wage inequality

Notes: Wages are calculated from CPS May Extracts and MORG supplements. Wages are the exponent of residuals from regressing log hourly wages on age, age squared, sex and race. The skill premium is the ratio of the average high-skilled wage to the average low-skilled wage. High school drop outs and high school graduates are low-skilled, everyone else is high-skilled.

biggest changes in wage inequality took place during this period (DiNardo, Fortin, and Lemieux (1996), Lee (1999), Card and DiNardo (2002)).

Despite the popularity of this hypothesis, there are, to my knowledge, no attempts in the literature to quantitatively assess the potential significance of falling minimum wages for wage inequality in the context of a general equilibrium model. People base their educational decisions on their potential job opportunities and earnings in different occupations. Hence, in general equilibrium, changes in the minimum wage could change the educational composition of the labour force at the aggregate level. Furthermore, the change in the educational composition of the labour force affects the profitability of R&D differentially across sectors. Therefore, the change in the educational composition of the labour force affects the choices firms make about which sectors to focus their R&D activity on, and this determines the direction of technical change. Thus, through educational decisions, the minimum wage influences the direction of technical change. Due to the links between minimum wages, education, and technological change, the quantitative general equilibrium effects of changes in the minimum wage on inequality could be quite different from what simple partial equilibrium reasoning may suggest.

In this chapter, I analyse the general equilibrium impact that lower minimum wages

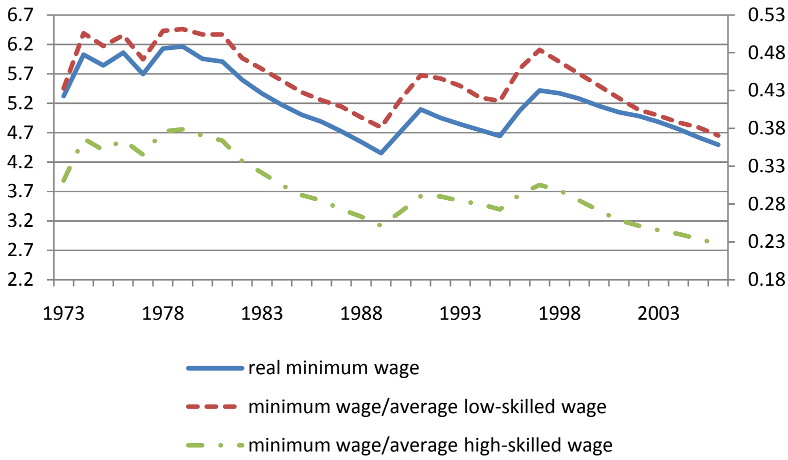

Figure 1.2: The decline in the real minimum wage

Notes: The real hourly minimum wage is the federal minimum wage in 2000$, calculated using the consumer price index (CPI) from the U.S. Bureau of Economic Analysis (BEA). Average wages are calculated using CPS MORG data. Wages and education categories are as in Figure 1.1. The left hand scale is the real value of the minimum wage, while the right hand scale is the ratio of the minimum wage compared to the average high- and low-skilled wage.

have on inequality. I consider two channels jointly: educational choices and the skill-bias of technology. I find that lower minimum wages increase wage inequality. This overall increase is the result of two opposing forces. On the one hand, the educational and ability composition of the labour force changes, leading to an increase in inequality. On the other hand, the relative supply of high-skilled labour decreases, which reduces the skill-bias of technology, and hence inequality.

By building a general equilibrium model with endogenous education and technology, and a binding minimum wage, this chapter bridges two of the most prominent explanations for increasing inequality in the literature.[3] Most of the theoretical literature on skill-biased technical change (SBTC) treats either technology or labour supply as exogenous. I contribute to this literature by allowing both technology and relative labour supply to adjust endogenously. I contribute to the literature on labour market institutions, by proposing a general equilibrium model – with endogenous education and technology – that allows the full quantitative analysis of the effects of falling minimum wages.

[3]Another prominent explanation for the increasing inequality – that my chapter does not relate to – is the increasing openness to trade, Goldberg and Pavcnik (2004) provide an extensive review of this literature.

To do this I build on and extend the two sector model of endogenous growth in Acemoglu (1998) by adding a binding minimum wage and allowing the supply of college graduates to be endogenous. As in Acemoglu (1998), the production side is a two sector Schumpeterian model of endogenous growth, with more R&D spending going towards technologies that are complementary with the more abundant factor. I explicitly model the labour supply side: workers, who are heterogeneous in their ability and time cost of education, make educational decisions optimally. I solve for the balanced growth path and calibrate the model to the US economy in 1981 in order to compare the transitional dynamics with the observed patterns of wages in the US over the subsequent thirty years.

I find that a decrease in the minimum wage increases the observed skill premium and the wage gaps between different percentiles of the wage distribution. According to the model, the 30 percent decline in the minimum wage accounts for about 15 percent of the observed increase in the skill premium in the US from 1981 to 2006. The fall in the minimum wage also explains almost one fifth of the observed increase in the 90/10 wage differential, and accounts for about one half of the increase in the 50/10 wage gap. In my model, the minimum wage also has some spill-over effects to the top end of the wage distribution, explaining 7 percent of the increase in the 90/50 wage gap.

The minimum wage affects inequality through several channels: through changes in the skill composition, in the ability composition and in directed technology.

The skill composition of the employed changes. As the minimum wage decreases, low ability workers flow into the low-skilled labour market.[4] This increases the skill premium in the short-run, thus increasing the incentives for acquiring education for higher ability workers. However, a lower minimum wage also makes it easier to find employment, reducing the role of education in avoiding unemployment. Educational attainment decreases at the lower end of the ability distribution and increases at the top end.

The ability composition of the labour aggregates changes, due to both the inflow from unemployment and the changing decision structure of skill acquisition. As the minimum

[4]The effects of minimum wages on unemployment are debated in the empirical literature. Brown, Gilroy, and Kohen (1982), Wellington (1991), Neumark and Wascher (1992) found negative employment effects, while Card (1992), Card and Krueger (1994) and Machin and Manning (1994) found no or small positive effects. These latter studies are controversial, see exchange between Neumark and Wascher (2000) and Card and Krueger (2000). Baker, Benjamin, and Stanger (1999) find a negative employment effect analyzing Canadian data.

wage decreases, lower ability workers flow into employment, thereby widening the range of abilities present among the employed. As both labour aggregates expand, the average ability in both sectors decrease. Since more low-ability individuals enter the low-skilled labour force, the average ability in the low-skilled sector decreases more. This composition effect reinforces the initial increase in the observed skill premium.

Finally, the direction of technology reacts to changes in the size of the low- and high-skilled labour aggregate. The direct effect of the minimum wage – the expansion of the low-skilled labour force – dominates, decreasing the relative supply of high-skilled labour. This implies that technology becomes less skill biased in the long run.

1.2 Related literature

The underlying causes of increasing inequality are highly debated among labour economists. There are two leading explanations, skill-biased technical change (SBTC) and labour market institutions. Many empirical studies concluded that SBTC is the driving force behind widening earnings inequality (Katz and Murphy (1992), Juhn, Murphy, and Pierce (1993), Krueger (1993), Berman, Bound, and Griliches (1994), Autor, Katz, and Krueger (1998)). This literature stems from the observation that the relative supply of high-skilled workers and the skill premium can only increase together if the relative demand for high-skilled workers also increases.[5]

Other authors have argued that the unprecedented increase in wage inequality during the 1980s cannot be explained by skill-biased technical change alone. DiNardo, Fortin, and Lemieux (1996) find that changes in labour market institutions – namely de-unionization and declining minimum wages – are as important as supply and demand factors in explaining increasing inequality. Lee (1999) uses regional variation in federal minimum wages to identify their impact on inequality, and finds that minimum wages can explain much of the increase in the dispersion at the lower end of the wage distribution. However, he also finds that the reduction in minimum wages is correlated with rising inequality at the top end of the wage distribution. This is seen by many as a sign that the correlation between declining

[5]Beaudry and Green (2005) find little support for ongoing skill-biased technological progress; in contrast, they show that changes in the ratio of human capital to physical capital conform to a model of technological adoption following a major change in technological opportunities.

minimum wages and increasing inequality is mostly coincidental (Autor, Katz, and Kearney (2008)). Card and DiNardo (2002) revise evidence for the claim that SBTC caused the rise in wage inequality and find that this view has difficulties accommodating the stabilization of wage inequality that occurred in the 1990s.

In the model presented here, the correlation between minimum wages and upper tail inequality is not coincidental: I provide a theoretical channel through which changes in minimum wages can affect inequality along the entire wage distribution. I find that minimum wages affect the bottom end of the wage distribution more, their impact on the top end is significant as well.

In my model, compositional effects play an important role in increasing inequality, as has been documented in the empirical literature. Lemieux (2006) finds that the compositional effects of the secular increase in education and experience explain a large fraction of the increased residual inequality. The study shows that increases in residual inequality and the skill premium do not coincide, implying that there must be other forces at play besides rising demand for high-skilled workers. Autor, Katz, and Kearney (2005) argue that even though compositional effects have had a positive impact on wage inequality, they mainly affect the lower tail, while the increase in upper tail inequality is mainly due to increasing wage differentials by education. Autor, Manning, and Smith (2009) assess the effects of minimum wages on inequality and find that minimum wages reduce inequality, but to a smaller extent, and that minimum wages also generate spill-over effects to parts of the wage distribution that are not directly affected by them.

In this study, minimum wages increase educational attainment at the low end of the ability distribution, while reducing educational attainment everywhere else through spill-over effects. In line with these findings, the empirical evidence on the effects of minimum wages on educational attainment is mixed. Neumark and Wascher (2003) and Neumark and Nizalova (2007) find that higher minimum wages reduce educational attainment among the young, and that individuals exposed to higher minimum wages work and earn less than their peers. Sutch (2010) finds that minimum wages induce more human capital formation.[6]

Theoretical explanations either rely on exogenous skill-biased technical change or on

[6] A related debate is on the effects of minimum wages on formal on-the-job training; see, for example, Acemoglu and Pischke (2003), Acemoglu (2003), Pischke (2005) and Neumark and Wascher (2001).

exogenously increasing relative supply of high-skilled workers; to my knowledge this is the first model where both the bias of technology and skill formation are endogenous.[7] Caselli (1999), Galor and Moav (2000) and Ábrahám (2008) allow for endogenous skill formation and explore the effects of exogenous skill-biased technical change. Heckman, Lochner, and Taber (1998) develop a general equilibrium model with endogenous skill formation, physical capital accumulation, and heterogeneous human capital to explain rising wage inequality. In this framework they find that skill-biased technical change explains the patterns of skill premium and overall inequality rather well. Explanations for the skill-bias of technology rely on exogenous shifts in the relative labour supplies. Acemoglu (1998) and Kiley (1999) use the market size effect in research and development, while Krusell, Ohanian, Ríos-Rull, and Violante (2000) rely on capital-skill complementarity and an increasing supply of high-skilled labour to account for the path of the skill premium.

1.3 The model

I begin by describing the model's production technologies, the R&D sector, the demographic structure and educational choices. Next I define the decentralised equilibrium, and finally, I analyse the balanced growth path and the transitional dynamics.

1.3.1 Overview

Time is infinite and discrete, indexed by $t = 0, 1, 2...$ The economy is populated by a continuum of individuals who survive from one period to the next with probability λ, and in every period a new generation of measure $1 - \lambda$ is born. Individuals are heterogeneous in two aspects: in their time cost of acquiring education and in their innate ability.

In the first period of his life every individual has to decide whether to acquire education or not, with the time to complete education varying across individuals. Those who acquire education become high-skilled. In my calibration I identify the high-skilled as having attended college. Those who opt out from education remain low-skilled. Workers with high

[7]This chapter more generally connects to the literature on the effects of labour market institutions on investments, which mainly focus on the differences in the European and American patterns (Beaudry and Green (2003), Alesina and Zeira (2006), Koeniger and Leonardi (2007)). Another strand of literature that relates to this chapter analyses the effects of labour market distortions on growth and educational attainment, for example Cahuc and Michel (1996) and Ravn and Sorensen (1999).

and low skills perform different tasks, are employed in different occupations, and produce different goods. The high-skilled sector includes skill-intensive occupations and production using high-skilled labour, while the low-skilled sector includes labour-intensive occupations and production using low-skilled labour. In equilibrium working in the high-skilled sector provides higher wages and greater protection from unemployment.

The government imposes a minimum wage in every period, and those who would receive a lower wage – depending on their skill and innate ability – cannot work and become unemployed. As soon as the minimum wage falls below their marginal productivity, they immediately become employed in the sector relevant to their skill.

There is a unique final good in this economy, which is used for consumption, the production of machines, and as an investment in R&D. It is produced by combining the two types of intermediate goods: one produced by the low- and the other by the high-skilled workers. Intermediate goods are produced in a perfectly competitive environment by the relevant labour and the machines developed for them.

Technological progress takes the form of quality improvements of machines that complement a specific type of labour, either high- or low-skilled. R&D firms can invest in developing new, higher quality machines. Innovators own a patent for machines and enjoy monopoly profits until it is replaced by a higher quality machine. There is free entry into the R&D sector, and more investment will be allocated to developing machines that are complementary with the more abundant labour type.

The economy is in a decentralised equilibrium at all times: all firms maximise their profits – either in perfect competition or as a monopoly – and individuals make educational decisions to maximise their lifetime income. I analyse how a permanent unexpected drop in the minimum wage affects the steady state and the transitional dynamics within this equilibrium framework.

1.3.2 Production

The production side of the model is a discrete time version of Acemoglu (1998). It is a two-sector endogenous growth model, where technological advances feature a *market size* effect, by which more R&D investment is allocated to develop machines complementary to the more abundant factor.

Final and intermediate goods

The unique final good is produced in perfect competition by combining the two intermediate goods:

$$Y = \left((Y^l)^\rho + \gamma(Y^h)^\rho\right)^{\frac{1}{\rho}},$$

where Y_l is the intermediate good produced by the low-skilled workers and Y_h is the intermediate good produced by high-skilled workers. The elasticity of substitution between the two intermediates is $1/(1-\rho)$, with $\rho \leq 1$. Perfect competition implies that the relative price of the two intermediate goods is:

$$p \equiv \frac{p^h}{p^l} = \gamma \left(\frac{Y^l}{Y^h}\right)^{1-\rho}. \tag{1.1}$$

Normalizing the price of the final good to one implies that the price of intermediate goods can be expressed as:

$$p^l = \left(1 + \gamma p^{\frac{\rho}{\rho-1}}\right)^{\frac{1-\rho}{\rho}}, \tag{1.2}$$

$$p^h = \left(p^{\frac{\rho}{1-\rho}} + \gamma\right)^{\frac{1-\rho}{\rho}}. \tag{1.3}$$

Intermediate good production is also perfectly competitive in both sectors $s \in \{l, h\}$. I simplify notation by allowing a representative firm:

$$Y^s = A^s(N^s)^\beta \quad \text{for} \quad s = \{l, h\}, \tag{1.4}$$

where $\beta \in (0, 1)$, N^s is the amount of effective labour employed and A^s is the technology level in sector s.[8] Productivity of labour is endogenous and depends on the quantity and quality of machines used. There is a continuum $j \in [0, 1]$ of machines used in sector s. High- and low-skilled workers use different technologies in the sense that they use a different set of machines. Firms decide the quantity, $x^{s,j}$ of a machine with quality $q^{s,j}$ to use. The productivity in sector s is given by:

$$A^s = \tfrac{1}{1-\beta} \int_0^1 q^{s,j}(x^{s,j})^{1-\beta} dj \quad \text{for} \quad s \in \{l, h\}.$$

[8]See Section 1.3.3 for exact definition of N^s.

Notice that even in the short run, productivity is not completely rigid. Productivity, A^s depends on the quality of machines and the *quantity* of each machine used. Producers of intermediate goods choose the quantity of machines ($x^{s,j}$) depending on the price and on the supply of effective labour it complements (N^s).

Since intermediate good production is perfectly competitive, industry demand for machine line j of quality $q^{s,j}$ and price $\chi^{s,j}$ is:

$$X^{s,j} = \left(\frac{p^s q^{s,j}}{\chi^{s,j}}\right)^{\frac{1}{\beta}} N^s \quad \text{for} \quad s = \{l, h\} \quad \text{and} \quad j \in [0,1]. \tag{1.5}$$

R&D firms

Technological advances are a discrete time version of Aghion and Howitt (1992). Investment in R&D produces a random sequence of innovations. Each innovation improves the quality of an existing line of machine by a fixed factor, $\overline{q} > 1$. The Poisson arrival rate of innovations for a firm k that invested z_k^j on line j is ηz_k^j. Denoting the total investments on line j by $\overline{z}^j \equiv \sum_k z_k^j$, the economy wide arrival rate of innovations in line j is $\eta\overline{z}^j$. Hence the probability that the quality of line j improves in one period is $(1 - e^{-\eta\overline{z}^j})$. In Section A.1.1 of the Appendix I show that the probability that the innovation is performed by firm k is $(1 - e^{-\eta\overline{z}^j})z_k^j/\overline{z}^j$. The cost of investing z_k^j units in R&D is Bqz_k^j in terms of final good. There are two key features to note: one is that the probability of success is increasing and concave in total investment, $\overline{z}^j$, the other is that the cost of investment is increasing in the quality of the machine line. The first feature guarantees the existence of an interior solution, while the second guarantees the existence of a steady state.

Notice that the probability of success for any single firm depends not only on their own R&D expenditure, but also on the total expenditure of other firms. There are many R&D firms, each of them small enough to take the total R&D spending as given when deciding how much to invest. There is free entry into the R&D sector: anyone can invest in innovation.

R&D firms with a successful invention have perpetual monopoly rights over the machine they patented. In Section A.1.2 of the Appendix I show that if quality improvements are sufficiently large, then even if the second highest quality machine were sold at marginal cost, firms would prefer to buy the best quality machine, the leading vintage at the monopoly price. I assume that this condition applies, therefore the price of the leading vintage in line j and sector s with quality q is:

$$\chi^{s,j} = \tfrac{q}{1-\beta} \quad \text{for} \quad s = \{l, h\} \quad \text{and} \quad j \in [0, 1].$$

Hence, if quality improvements are large enough, then each machine's productive life is limited. Once a higher quality machine is invented producers of intermediate goods switch to using the highest quality machine.

Monopoly pricing and industry demand (1.5) yield the following per period profit for the owner of the leading vintage in line j and sector s:

$$\pi^{s,j} = q^{s,j}\beta(1-\beta)^{\frac{1-\beta}{\beta}}(p^s)^{\frac{1}{\beta}}N^s \quad \text{for} \quad s = \{l, h\} \quad \text{and} \quad j \in [0, 1]. \tag{1.6}$$

The per period profit depends on the price of the intermediate good that the machine produces, and on the efficiency units of labour that can use the machine. A higher price of the intermediate good and a higher supply of effective labour, generates a greater demand for the machine. The second component drives the *scale effect* in R&D. A higher per period profit means a higher lifetime value from owning a patent, which implies more investment into improving that machine.

The value of owning the leading vintage is the expected discounted value of all future profits. This in turn depends on the per period profit and the probability that this quality remains the leading vintage in the following periods.

The value of owning the leading vintage of quality q in line j and sector s can be expressed as:

$$V_t^{j,s}(q) = \pi_t^{j,s}(q) + \tfrac{1}{1+r}(e^{-\eta\bar{z}_t^{j,s}(q)})V_{t+1}^{j,s}(q) \quad \text{for} \quad s = \{l, h\} \quad \text{and} \quad j \in [0, 1]. \tag{1.7}$$

Total R&D spending on line j in sector s of current quality q at time t is $\bar{z}_t^{j,s}(q)$, hence $e^{-\eta\bar{z}_t^{j,s}(q)}$ is the probability that quality q remains the leading vintage in line j in period $t+1$. The present value of owning the leading vintage of quality q in line j and sector s in period $t+1$ is $\frac{1}{1+r}V_{t+1}^{j,s}(q)$.

The value of owning a leading vintage is increasing in current period profit and in the continuation value of owning this vintage. It is decreasing in the amount of R&D spending targeted at improving quality in this line of machines.

Free entry into the R&D sector implies that all profit opportunities are exhausted. The expected return from R&D investment has to equal its cost for each firm.

$$\frac{E_t(V^{s,j}_{t+1}(q^{s,j}_t))}{1+r}(1-e^{-\eta\bar{z}^{j,s}_t(q^{s,j}_t)})\frac{z^{j,s}_k}{\bar{z}^{j,s}_t(q^{s,j}_t)} = Bq^{s,j}_t z^{j,s}_k \quad \text{for} \quad s=\{l,h\} \quad \text{and} \quad j\in[0,1]. \tag{1.8}$$

The left hand side is the expected return of investing $z^{j,s}_k$ in R&D, while the right hand side is the cost. The expected return depends on the discounted value of owning the leading vintage, and on the probability that firm k makes a successful innovation. Notice that both the expected return and the costs are proportional to the R&D investment of firm k. Hence, in equilibrium, only the total amount of R&D spending targeted at improving line j in sector s is determined.

Technology and prices

Given monopoly pricing the equilibrium production of intermediate goods is:

$$Y^s_t = (1-\beta)^{\frac{1-2\beta}{\beta}}(p^s_t)^{\frac{1-\beta}{\beta}} N^s_t Q^s_t \quad \text{for} \quad s=\{l,h\}. \tag{1.9}$$

Where $Q^s_t = \int_0^1 q^{j,s}_t dj$ is the average quality of the leading vintages in sector s. The average quality evolves according to the R&D targeted at improving the machines:

$$Q^s_{t+1} = \int_0^1 q^{j,s}_t \left((1-e^{-\eta\bar{z}^{j,s}_t(q^{j,s}_t)})\overline{q} + \left(e^{-\eta\bar{z}^{j,s}_t(q^{j,s}_t)}\right)\right) dj \quad \text{for} \quad s=\{l,h\}. \tag{1.10}$$

The growth rate of average quality in sector s is:

$$g^s_{t+1} = \frac{Q^s_{t+1}}{Q^s_t} \quad \text{for} \quad s=\{l,h\}.$$

Let $Q_t \equiv \frac{Q^h_t}{Q^l_t}$ denote the relative average quality or relative technology. This evolves according to:

$$Q_{t+1} = \frac{g^h_{t+1}}{g^l_{t+1}}\frac{Q^h_t}{Q^l_t} = \frac{g^h_{t+1}}{g^l_{t+1}}Q_t. \tag{1.11}$$

Combining (1.9) with the relative price equation (1.1) gives:

$$p_t = \gamma^{\frac{\beta}{(1-(1-\beta)\rho)}} \left(\frac{Q_t^h}{Q_t^l} \frac{N_t^h}{N_t^l} \right)^{-\frac{(1-\rho)\beta}{(1-(1-\beta)\rho)}} . \tag{1.12}$$

Note that the relative price – the price of the intermediate produced by the high-skilled compared to the one produced by the low-skilled – is decreasing in the relative supply of high-skilled labour and in the relative quality of the machines used by high-skilled workers. If the relative share of the high-skilled or the relative quality of the machines that complement them increases, then their production increases compared to the production of the low-skilled labour. This leads to a fall in the relative price of the intermediate produced by the high-skilled.

1.3.3 Labour supply

In this section I describe the labour supply side of the model. I assume that the only reason for unemployment is productivity below the minimum wage. I further assume that the only incentive for acquiring education is the higher lifetime earnings it provides. Education increases earnings potentially through two channels: a higher wage in periods of employment, and better employment opportunities for high- than for low-skilled individuals. These incentives and the minimum wage determine the optimal education decision of people, depending on their cost and return to education.

Individuals are heterogeneous in two aspects: in their cost of acquiring education, c and in their innate ability, a. Let $f(c, a)$ be the joint time invariant distribution of abilities and education costs at birth.[9] The demographic structure is as in Blanchard (1985): every period a new generation of mass $1-\lambda$ is born, while the probability of surviving from period t to $t+1$ is λ. These assumptions imply that both the size of the population and the joint distribution of costs and abilities are constant over time.

Each individual has to decide whether to acquire education in the first period of his life. Only those born in period t can enrol to study in period t. Completing education takes a fraction c_i of the first period of individual i's life, and during this time, he cannot participate

[9] I explain why I introduce heterogeneous time cost in Section 5.2.

in the labour market.[10] The time cost of education is idiosyncratic and is determined at birth. An individual who completes education becomes high-skilled and has the option of working in the high-skilled sector for life. High-skilled workers with ability a earn wage $w_t^h(a)$ in period t. Those who choose not to acquire education, remain low-skilled and can start working in the period they are born as low-skilled. The wage in period t for a low-skilled worker with ability a is $w_t^l(a)$.

I model innate ability as a factor that increases individual productivity. Each worker supplies one unit of raw labour inelastically, which translates to a units of efficiency labour for someone with ability a.

Using monopoly pricing and the implied demand for machines, the wage can be expressed in terms of the average quality of machines:

$$w_t^s(a) = a\beta(1-\beta)^{\frac{1-2\beta}{\beta}}(p_t^s)^{\frac{1}{\beta}}Q_t^s \equiv aw_t^s \quad \text{for} \quad s = \{l, h\}. \tag{1.13}$$

Since ability is equivalent to efficiency units of labour, it can be separated from other factors determining the wage. Let $w_t^s \equiv \beta(1-\beta)^{\frac{1-2\beta}{\beta}}(p_t^s)^{\frac{1}{\beta}}Q_t^s$ denote the wage per efficiency unit of labour in sector s in period t.

The government imposes a minimum wage $\underline{w}_t$ in every period. Nobody is allowed to earn less than the minimum wage, hence those with marginal product below the minimum wage in period t are unemployed in period t. People only remain unemployed while their marginal productivity is below the minimum wage.

This implies that for both skill levels, there is a cutoff ability in every period below which people become unemployed. This threshold is:

$$\underline{a}_t^s \equiv \frac{\underline{w}_t}{w_t^s} \quad \text{for} \quad s = \{l, h\}. \tag{1.14}$$

Workers with innate ability $a \geq \underline{a}_t^s$ work in sector s in period t.[11]

Individuals choose their education level to maximise the present value of their expected

[10] In the calibration exercise I set the length of a period to be five years.

[11] If the wage per efficiency unit for the high- and the low-skilled were equal, than some high skilled could work in the low-skilled sector. However, I later show that in equilibrium $w_t^h > w_t^l$ for all t.

lifetime utility from consuming the unique final good:

$$E_t \sum_{j=0}^{\infty} \left(\frac{\lambda}{1+r} \right)^j u_{t+j},$$

where u_{t+j} is their consumption of the final good, λ is the probability of staying alive until the next period, r is the discount rate, which is also the interest rate due to linear utility.

Consider the decision of an individual with ability a and cost c born in period t. Denote the expected present value of lifetime income by $\mathbf{W}_t^h(a,c)$ if high-skilled, and by $\mathbf{W}_t^l(a,c)$ if low-skilled; periods of zero income account for the possibility of unemployment. The optimal decision is then summarised by:

$$e(a,c)_t = \begin{cases} 1 & \text{if} \quad \mathbf{W}_t^h(a,c) \geq \mathbf{W}_t^l(a,c), \\ 0 & \text{if} \quad \mathbf{W}_t^h(a,c) < \mathbf{W}_t^l(a,c), \end{cases} \tag{1.15}$$

where $e(a,c)_t = 1$ if the individual acquires education and $e(a,c)_t = 0$ otherwise.

Let $d(a)_t^s$ be an indicator that takes the value one if an individual with skill s and ability a has marginal product higher than the minimum wage in period t, and zero otherwise. The lifetime earnings of an educated individual can be expressed as:

$$\mathbf{W}_t^h(a,c) = a \sum_{s=1}^{\infty} \left(\frac{\lambda}{1+r} \right)^s w_{t+s}^h d(a)_{t+s}^h + a(1-c) w_t^h d(a)_t^h. \tag{1.16}$$

Acquiring education takes a fraction, c, of the first period of an individual's life, implying that he can only work in the remaining fraction, $1-c$, of the first period. The lifetime earnings of a high-skilled individual are decreasing in c, the time acquiring education takes him. The more time he spends acquiring education, the less time he has to earn money.

The lifetime earnings of a low-skilled individual are:

$$\mathbf{W}_t^l(a,c) = a \sum_{s=0}^{\infty} \left(\frac{\lambda}{1+r} \right)^s w_{t+s}^l d(a)_{t+s}^l. \tag{1.17}$$

Notice that the lifetime earnings of a low-skilled worker do not depend on c, while the earnings of a high-skilled worker are decreasing in c. This gives rise to a cutoff rule in c for acquiring education.

Education is worth the investment for an individual with ability a and cost c if $W_t^h(a,c) > W_t^l(a,c)$. As described earlier, there are two channels through which education can increase lifetime earnings: either the wage per efficiency unit is higher for high-skilled than for low-skilled workers, or being high-skilled offers greater protection against unemployment. The second case arises when a is such that $aw_t^l < \underline{w}_t < aw_t^h$, which also requires that $w_t^l < w_t^h$. Hence the following remark:

Remark 1.1. *To have high-skilled individuals in a generation born in period t, there has to be at least one period $s \geq t$, such that the wage per efficiency unit of labour is higher for the high-skilled than for the low-skilled: $w_s^l < w_s^h$.*

This implies that the only reason for acquiring skills is the *skill premium*, a higher wage per efficiency unit in the high- than the low-skilled sector. Using the relative price of intermediates, (1.12) and the wage per efficiency unit, (1.13), the skill premium can be expressed as:

$$\frac{w^h(a)}{w^l(a)} = \gamma^{\frac{1}{1-(1-\beta)\rho}} \left(\frac{Q^h}{Q^l}\right)^{1-\frac{1-\rho}{1-(1-\beta)\rho}} \left(\frac{N^h}{N^l}\right)^{-\frac{1-\rho}{1-(1-\beta)\rho}}. \tag{1.18}$$

The above equation shows the ways in which education increases workers' wages. The first, represented by γ, arises because goods produced by high- and low-skilled workers are not weighed equally in final good production. If $\gamma > 1$, the high-skilled intermediate contributes more to the final good, and the overall productivity of the high-skilled, measured in units of final good is greater. The second source is the different quality machines: Q^h is the average quality in the high-skilled, and Q^l is the average quality in the low-skilled sector. If technology for the high-skilled is more advanced, then teaching workers to use these more advanced technologies makes workers more productive. The final source is decreasing returns in production: if the share of high-skilled workers is very low, their relative marginal productivity becomes very high.

The labour supply aggregates N_t^h and N_t^l are the total amount of high- and low-skilled efficiency units of labour available in period t:

$$N_t^l = (1-\lambda)\sum_{j=0}^{\infty} \lambda^j \int_{a_t^l}^{\infty} \int_c f(a,c)(1-e(a,c)_{(t-j)})d(a)_t^l dcda, \tag{1.19}$$

$$N_t^h = (1-\lambda)\int_{\underline{a}_t^h}^{\infty}\int_c f(a,c)(1-c)e(a,c)_{(t-j)}d(a)_t^h dcda \\ +(1-\lambda)\sum_{j=1}^{\infty}\lambda^j\int_{\underline{a}_t^h}^{\infty}\int_c f(a,c)e(a,c)_{(t-j)}d(a)_t^h dcda. \tag{1.20}$$

Recall that high-skilled workers born in period t only work for a fraction $(1-c)$ of period t, since they spend a fraction c studying.

1.4 Equilibrium

In this section I define the equilibrium of the economy and show that the steady state is fully characterised by two unemployment thresholds and a cutoff time cost for acquiring education. I also show that a lower minimum wage implies a shift in all three thresholds. These shifts lead to steady state changes in both the observed skill premium and the overall wage inequality. Inequality is affected mostly through composition: the *ability composition* in both skill groups and the *skill composition* at all percentiles along the wage distribution are altered.

The economy is in a decentralised equilibrium at all times; that is, all firms maximise profits and all individuals maximise their lifetime utility given a sequence of minimum wages.

Definition 1. *A* ***decentralised equilibrium*** *is a sequence of optimal education decisions* $\{e(a,c)_t\}_{t=0}^{\infty}$*, cutoff ability levels* $\{\underline{a}_t^h, \underline{a}_t^l\}_{t=0}^{\infty}$*, effective labour supplies* $\{N_t^h, N_t^l\}_{t=0}^{\infty}$*, discounted present values of expected lifetime income* $\{\mathbf{W}_t^h, \mathbf{W}_t^l\}_{t=0}^{\infty}$*, intermediate good prices* $\{p_t^h, p_t^l\}_{t=0}^{\infty}$*, average qualities* $\{Q_t^h, Q_t^l\}_{t=0}^{\infty}$*, investments into R&D* $\{\overline{z}_t^{j,h}, \overline{z}_t^{j,l}\}_{t=0}^{\infty}$ *and values of owning the leading vintage* $\{V_t^{j,h}, V_t^{j,l}\}_{t=0}^{\infty}$ *for all lines* $j \in [0,1]$*, where* $\{Q_0^h, Q_0^l, N_0^h, N_0^l\}$ *and* $\{\underline{w}_t\}_{t=0}^{\infty}$ *are given, such that the following conditions are satisfied:*

1. *the effective labour supplies satisfy* (1.20) *and* (1.19)*;*
2. *lifetime earnings are as in* (1.16) *and* (1.17)*, where* w_t^s *is as in* (1.13)*;*
3. *the average quality in sector* s *evolves according to* (1.10)*;*
4. *total R&D investment* $\overline{z}_t^{j,s}$ *satisfies* (1.8) *for all* $t \geq 0$ *and all* $j \in [0,1]$*;*
5. *the sequence* $\{V_t^{j,s}\}_{t=0}^{\infty}$ *satisfies* (1.7)*;*

6. *the price sequence* $\{p_t^h, p_t^l\}_{t=0}^{\infty}$ *satisfies* (1.2) *and the relative price,* p_t *satisfies* (1.12)*;*

7. *the optimal education decisions,* $\{e(a,c)_t\}_{t=0}^{\infty}$ *are as in* (1.15)*;*

8. *the cutoff abilities for unemployment,* $\{\underline{a}_t^h, \underline{a}_t^l\}_{t=0}^{\infty}$ *satisfy* (1.14)*.*

1.4.1 Steady state

As is standard in the literature, in this section I focus on steady states or balanced growth paths (BGP), which are decentralised equilibria, where all variables are constant or grow at a constant rate. In Section A.2 of the Appendix I solve for the BGP in detail, here I present a more informal discussion.

In the BGP the total R&D spending on all lines within a sector are equal, $\overline{z}^{j,s*} = \overline{z}^{s*}$ for $j \in [0,1]$ and $\overline{z}^{s*}$ is given by:

$$\beta(1-\beta)^{\frac{1-\beta}{\beta}}(p^{s*})^{\frac{1}{\beta}}N^{s*} = B\overline{z}^{s*}\frac{(1+r-e^{-\eta\overline{z}^{s*}})}{1-e^{-\eta\overline{z}^{s*}}} \quad \text{for} \quad s = \{l,h\}. \tag{1.21}$$

The above equation shows that R&D effort in a sector is increasing in the period profit from machine sales. These profits are higher if the price of the intermediate produced by it, p^{s*}, is higher, or if more effective labour, N^{s*}, uses this technology.

Along the BGP, relative quality in the two sectors, Q^*, has to be constant, which requires equal R&D spending in the two sectors: $\overline{z}^{h*} = \overline{z}^{l*} = \overline{z}^*$. From (1.21) R&D spending in the two sectors is equal if:

$$p^* = \frac{p^{h*}}{p^{l*}} = \left(\frac{N^{h*}}{N^{l*}}\right)^{-\beta}. \tag{1.22}$$

Combining the relative price (1.1),(1.22) with the intermediate output (1.9) gives:

$$Q^* = \frac{Q^{h*}}{Q^{l*}} = \gamma^{\frac{1}{1-\rho}}\left(\frac{N^{h*}}{N^{l*}}\right)^{\frac{\beta\rho}{1-\rho}}. \tag{1.23}$$

The above two equations are the key to understanding the dynamics of the skill premium. The skill premium, which is the ratio of the high- to low-skilled wage per efficiency unit, depends on the relative price of the intermediates and the relative quality in the two sectors. Since both of these ratios depend on the relative supply of skills, their interaction determines the effect of relative skill supply on the skill premium.

Equation (1.22) shows that the relative price of the two intermediates depends negatively on the relative supply of high-skilled workers. If there are more high-skilled workers, high-skilled intermediate production is greater, other things being equal. The technology effect reinforces this, since more R&D is directed towards the larger sector (from (1.23)), implying a higher relative quality, Q^*. Intuitively, having more high-skilled workers and better technologies, leads to more high-skilled intermediate production, and lowers the relative price of the intermediate.

Equation (1.23) shows that the relative quality level depends on the relative abundance of the two types of labour along the balanced growth path. The average quality in the high-skilled sector relative to the low-skilled sector depends positively on the relative supply of high-skilled workers. With more high-skilled workers, an innovation in the high-skilled sector is more profitable. Hence technology is more skill-biased – Q^* is greater, – if the relative supply of skills is higher.

Note that along the steady state, technological *change* is not biased towards either sector, the skill-bias of technology is constant, since both sectors are growing at the same rate. As pointed out earlier, total R&D investment in the two sectors is equal, hence the relative quality of the two sectors is constant along the balanced growth path.

The skill premium per efficiency unit of labour, using (1.13), is:

$$\frac{w_t^{h*}}{w_t^{l*}} = \left(\frac{p^{h*}}{p^{l*}}\right)^{\frac{1}{\beta}} \frac{Q_t^{h*}}{Q_t^{l*}} = \gamma^{\frac{1}{1-\rho}} \left(\frac{N^{h*}}{N^{l*}}\right)^{\frac{\beta\rho}{1-\rho}-1}. \tag{1.24}$$

The wage per efficiency unit of labour depends on two components: the price of the intermediate good and the average quality of machines in that sector. Since the relative price depends negatively, while the relative quality depends positively on the relative supply of skilled workers, the net effect depends on which influences the wages more.

This ultimately depends on the elasticity of substitution between the two intermediates. If the two intermediates are highly substitutable, ρ is higher, and relative output affects relative price less; hence the price effect is smaller. On the other hand, if they are not substitutable and ρ is low, the price effect is stronger than the quality effect.

If $(\beta\rho)/(1-\rho)-1>0$, then the skill premium per efficiency unit of labour is an increasing function of the relative supply of skills. In this case, the increase in relative quality more than compensates for the decrease in relative price. Hence, an increase in the relative supply of skills increases the skill premium, implying that technology is *strongly biased*. If $(\beta\rho)/(1-\rho)-1<0$ then the skill premium per efficiency unit of labour is decreasing in the relative supply, and technology is *weakly biased*: the technology effect does not compensate for the price effect.

The skill premium per efficiency unit of labour is not the same as the empirically observed skill premium. The observed skill premium is the ratio of the average wages:

$$\frac{\overline{w}_t^{h*}}{\overline{w}_t^{l*}} = \frac{w_t^{h*}}{w_t^{l*}}\frac{\overline{a}^{h*}}{\overline{a}^{l*}},$$

where $\overline{a}^{h*}$ is the average ability among the high-skilled and $\overline{a}^{l*}$ is the average ability among the low-skilled.

The skill premium per efficiency unit is constant from (1.24). From Remark 1.1, the skill premium has to be greater than one in at least one period. This implies that $w_t^{h*} > w_t^{l*}$ for all $t \geq 0$.

The threshold ability of unemployment for the low-skilled is defined in (1.14). Combining this with steady state wages yields:

$$\underline{w}_t = \underline{a}^{l*} w_t^{l*} = \underline{a}^{l*} \beta (1-\beta)^{\frac{1-2\beta}{\beta}} (p^{l*})^{\frac{1}{\beta}} Q_t^{l*}. \tag{1.25}$$

Note that for the existence of a BGP, it is required that the minimum wage grows at the same rate as the low-skilled wage per efficiency unit, g^*. Since the growth in average quality is driving wage growth, let $\widetilde{w}_t \equiv \frac{\underline{w}_t}{Q_t^l}$ denote the *normalised minimum wage*, which has to be constant for a steady state.

Given $\underline{a}^{l*}$, the cutoff ability for the high-skilled is given by:

$$\underline{a}^{h*} = \underline{a}^{l*} \frac{w_t^l}{w_t^h}. \tag{1.26}$$

As pointed out earlier, the skill premium is greater than one, implying that the threshold ability for unemployment for the low-skilled is higher than the threshold ability for the high-skilled: $\underline{a}^{h*} < \underline{a}^{l*}$. Acquiring skills through education, for instance learning how to use different machines, increases workers' productivity and protects them from unemployment.

Acquiring skills allows people with low ability to increase their marginal productivity above the minimum wage, and to find employment.

In the steady state everyone has a constant employment status: they are either unemployed or employed in the low- or high-skilled sector. Moreover, depending on their innate ability, a, everyone falls into one of the following categories: $a < \underline{a}^{h*}$, $a \in [\underline{a}^{h*}, \underline{a}^{l*})$ or $a \geq \underline{a}^{l*}$.

Consider an individual with ability $a < \underline{a}^{h*}$. He does not acquire education in equilibrium because he would be unemployed regardless of his skills.

Now consider an individual with ability $a \in [\underline{a}^{h*}, \underline{a}^{l*})$. If he does not acquire education, he becomes unemployed and earns zero income in every period. On the other hand, by completing his studies he earns the high-skilled wage. Since the opportunity cost of education is zero in this case, acquiring education to become high-skilled is the optimal decision.

Finally, consider an individual with ability $a \geq \underline{a}^{l*}$, who is always employed regardless of his skill level. Such an individual acquires education if the present value of his earnings as high-skilled (1.16) exceed his present value earnings as low-skilled (1.17).

Result 1.1. *Every individual with ability $a \geq \underline{a}^{l*}$ born in period t acquires education if his cost $c < c^*$, where c^* is the* cutoff time cost *implicitly defined by:*

$$c^* = \frac{1 - \frac{w_t^{l*}}{w_t^{h*}}}{1 - \frac{g^* \lambda}{1+r}}. \tag{1.27}$$

Proof. Combining (1.15) with (1.16) and (1.17) and using that in equilibrium $d_{t+k}^s(a) = 1$ for all $k \geq 0$, for $s = l, h$, and $a \geq \underline{a}^{l*}$, implies that the condition for acquiring education is:

$$a \sum_{s=0}^{\infty} \left(\frac{\lambda}{1+r} \right)^s w_{t+s}^{h*} - a \sum_{s=0}^{\infty} \left(\frac{\lambda}{1+r} \right)^s w_{t+s}^{l*} \geq a w_t^{h*} c.$$

This shows that the optimal education decision is equivalent to a threshold time cost, c_t^*. Using the fact that wages in both sectors grow at a constant rate g^*, and that the skill premium, w_t^{h*}/w_t^{l*} is constant, $c_t^* = c^*$ is constant and given by (1.27). □

The threshold time cost for acquiring education and consequently the fraction of high-skilled workers depends positively on the skill premium and on the growth rate of the average qualities. The threshold is increasing in the skill premium, since a higher skill premium implies a greater per period gain from working as high-skilled. The growth rate of wages also increases the threshold time cost; if wages grow at a higher rate, then for a given skill premium, future gains are greater.

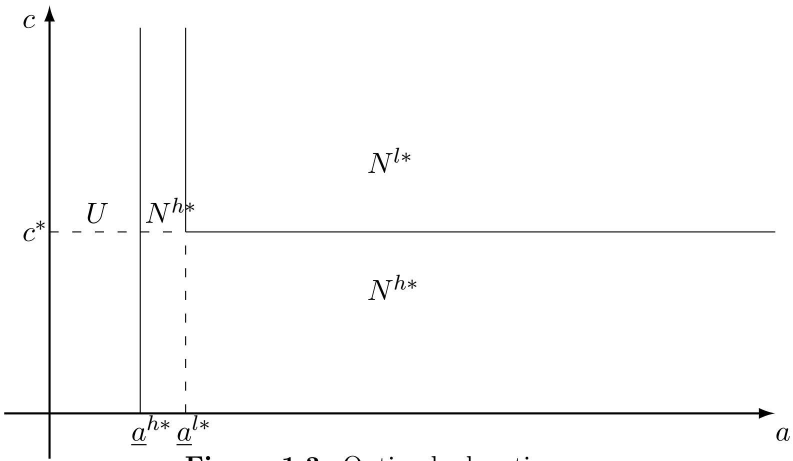

Figure 1.3: Optimal education

Notes: The horizontal axis represents the support of the ability distribution, and the vertical axis represents the support of the cost distribution.

Figure 1.3 depicts educational choices in the steady state. Individuals with ability lower than $\underline{a}^{h*}$ are unemployed and do not acquire education (U). Between the two thresholds, $\underline{a}^{h*} \leq a < \underline{a}^{l*}$, everyone acquires education and becomes high-skilled to avoid unemployment. Finally individuals with ability above $\underline{a}^{l*}$ acquire skills if their time cost is below c^*.

The three cutoff values determine the effective labour supplies, N^{h*} and N^{l*}. In turn, the effective labour supplies determine every other variable in the economy in steady state. Therefore, the steady state of the economy is characterised by the three thresholds $\underline{a}^{h*}, \underline{a}^{l*}$ and c^*. Furthermore, the three thresholds are also connected through the equilibrium condition (1.26). This condition relates the two cutoff values of unemployment through the skill premium.

Lemma 1.1. *The pair* $(\underline{a}^{l*}, c^*)$ *uniquely defines* $\underline{a}^{h*}$.

Proof. See Appendix A.2.4. □

The balanced growth path is defined by two key equations: the equilibrium c^* given the threshold for low-skilled unemployment (1.27) and the equilibrium $\underline{a}^{l*}$ given the cutoff time cost for acquiring education (1.25). Figure 1.4 graphs these two equations.

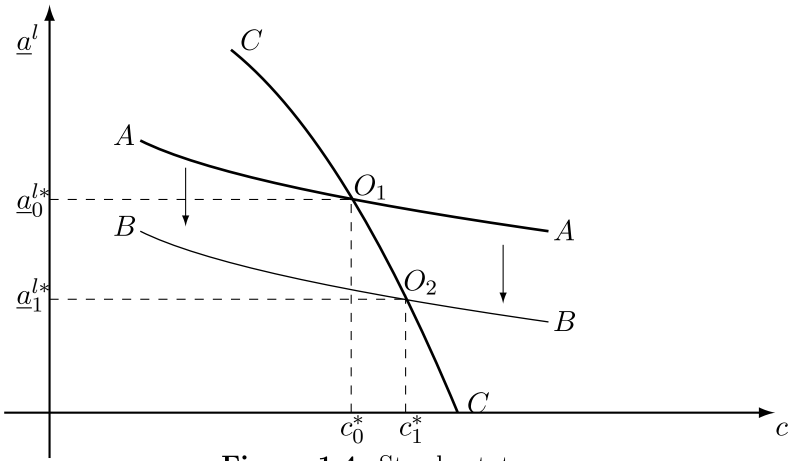

Figure 1.4: Steady state

The curve CC represents the equilibrium c^* for different values of $\underline{a}^l$ (1.27). The threshold ability for low-skilled unemployment affects c^* through two channels. The first is the growth rate: a higher $\underline{a}^l$ decreases the total amount of effective labour in the economy. Due to scale effects in R&D, this reduces the growth rate of the economy.[12] A lower growth rate implies a lower lifetime gain from being high-skilled, hence a lower c^*.

The second channel is the skill premium. A higher $\underline{a}^l$ reduces N^l and increases N^h, so the relative supply of high-skilled workers increases. A weak technology bias reduces the skill premium, and the gain from acquiring education; thus, a higher $\underline{a}^l$ reduces c^* both through its affect on growth and on the skill premium, so the curve represented by CC is downward sloping.

On the other hand if technology is strongly biased, then an increase in N^h/N^l increases the skill premium. The decreasing growth rate pushes c^* down, while the increasing skill premium pushes c^* up. The overall effect on the gain from education can be ambiguous if technology is strongly biased. For the range of values that are of interest, the overall effect is small and negative.

The curve AA represents the equilibrium unemployment threshold $\underline{a}^{l*}$ for different values of c (1.25). If c is higher, there are more high-skilled workers, and their production increases.

[12] See Appendix Section A.2.3 for the exact dependence of the growth rate on the supply of high- and low-skilled effective labour.

This, in turn, depresses the price of their intermediate, p^h, while the price of the low-skilled intermediate increases. A higher p^l allows workers with both lower ability and skills to participate in the market. Hence the threshold for unemployment for the low-skilled is a decreasing function of c, implying the downward sloping AA curve in Figure 1.4.

1.4.2 Lowering the minimum wage

To analyse the effects of minimum wage on inequality, I consider an unanticipated permanent decrease in the normalised minimum wage. A lower minimum wage excludes fewer people from the labour market, by lowering the unemployment threshold for both the high- and the low-skilled. Moreover, through endogenous R&D, the increase in the supply of effective labour raises the growth rate of the economy, thus increasing the incentives to acquire education, resulting in a higher cutoff cost for acquiring education. The shift of these three thresholds changes the *ability composition* in both sectors and the *skill composition* along the ability distribution. Average ability in both sectors decreases, with high-skilled average ability decreasing less. The fraction of high-skilled workers changes at every percentile in the wage distribution, increasing at the top end and decreasing at the bottom end, thereby increasing overall inequality.

The normalised minimum wage shifts curve AA and leaves curve CC unaffected. From (1.25) a lower $\widetilde{w}$ implies that a lower $\underline{a}^{l*}$ satisfies the equation for any c. Therefore, a higher normalised minimum wage shifts the curve up, and a lower value shifts the curve down.

Curve BB in Figure 1.4 represents the equilibrium unemployment threshold $\underline{a}^{l*}$ for any cutoff time cost of education for a lower $\widetilde{w}$. The steady state moves from O_1 to O_2. The new steady state features a lower threshold for unemployment, $\underline{a}_1^{l*}$ and a higher threshold for the time cost of education, c_1^*. The effect of these changes on the supply of high- and low-skilled effective labour are depicted in Figure 1.5.

The direct effect of an increase in c^* is to decrease N^{l*} and increase N^{h*}. A higher c^* implies that more people acquire education for higher wages. The fraction of low-skilled workers decreases while the fraction of high-skilled increases among those with ability greater than $\underline{a}_0^{l*}$.

A lower $\underline{a}^{l*}$ entails that fewer people acquire education to avoid unemployment. While

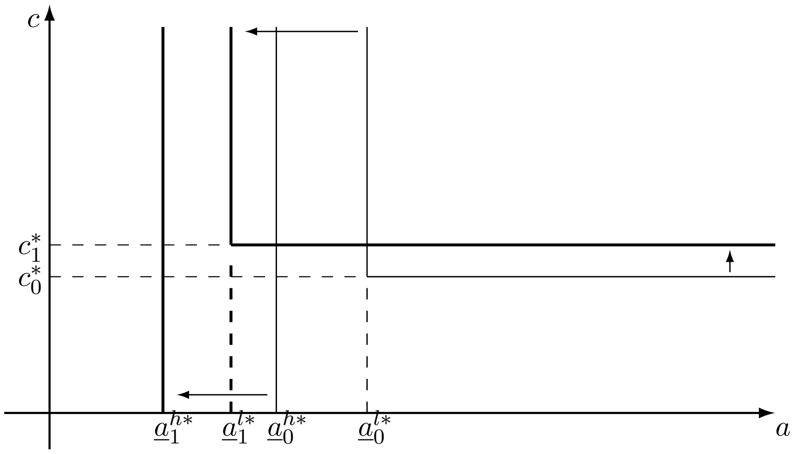

Figure 1.5: Change in the optimal education and labour market participation

Notes: In the graph I represent a case where $[\underline{a}_1^{h*}, \underline{a}_1^{l*}]$ and $[\underline{a}_0^{h*}, \underline{a}_0^{l*}]$ do not overlap. I chose to show such a case, since this is what I find in the calibration exercise.

previously everyone with ability, $a \in [\underline{a}_0^{h*}, \underline{a}_0^{l*})$ became high-skilled to avoid unemployment, now they would be employed regardless of their skill level. Only those with cost lower than c_1^* acquire education. This increases N^{l*} partly by reducing N^{h*} and partly by reducing unemployment.

A decrease in $\underline{a}^{l*}$ also implies a lower $\underline{a}^{h*}$, which increases N^h* by reducing unemployment. A lower unemployment cutoff for the high-skilled shifts down the range of abilities for which people acquire education to avoid unemployment.

The overall effect of a decrease in the minimum wage on the relative supply of skills depends on the elasticity of a^{l*} relative to the elasticity of c^*. The change in the supply of high and low skills governs the change in the skill premium as well.

In general, the effect of minimum wages on the supply of skills is ambiguous. However, numerical results suggest that a lower minimum wage increases the supply of high-skilled less than it increases the supply of low-skilled effective labour, leading to a decrease in the relative supply of skills. The calibration exercise presented in Section 5 yields that technology is strongly biased; hence, a reduction in the supply of skills decreases the skill premium per efficiency unit of labour.

Overall inequality in the economy, measured by the wage gap between different percentiles of the wage distribution, increases. With a lower minimum wage the range of abilities in the labour market widens, and the fraction of high-skilled increases at the top end of the ability distribution, and decreases at the bottom end. These forces both push towards greater inequality.

1.5 Calibration

I first present estimates of the parameters set outside the model. I then present maximum likelihood estimates of the ability and time cost of education distributions, based on the equilibrium conditions of the model. Finally, I calibrate the remaining parameters by globally minimizing the distance between data moments and steady state moments of the model.

1.5.1 Interest rate, lifespan and production technology

Three parameters, namely, the share of labour in the production function, β, the interest rate, r, and the survival probability, λ, can be set outside the model.

The intermediate good is produced by labour and machines, and the exponent on labour is β. This implies a wage bill of βY in the aggregate economy. Since the wage bill has been roughly constant at $\frac{2}{3}$ over long run US history, I set $\beta = \frac{2}{3}$.

The interest rate and the probability of survival depend on the length of a period in the model. Since people can spend only a fraction of their first period studying in the model, I set one period in the model to correspond to five years.[13]

Based on the real interest rate in the US, which has been about five percent annually, I set the interest rate for five years to be $r = 1.05^5 - 1$.

On average, since people spend 45 years working and studying, the rate of survival can be set to give an expected 9 periods of work, including the period of study.[14] This gives the value $\lambda = 1 - \frac{1}{9}$.

[13] A longer model period would also allow for completing education in one period. However, shorter periods provide richer transitional dynamics.

[14] The expected lifespan of someone who has a per period survival probability of λ is

$$E(j) = \sum_{j=1}^{\infty} j\lambda^{j-1}(1-\lambda) = \frac{1}{1-\lambda}.$$

Solving for $E(j) = 9$ gives $\lambda = 1 - \frac{1}{9}$.

1.5.2 Ability and cost distribution

Estimating the distribution of abilities and costs is a crucial part of the calibration exercise. Since ability and the cost of education are not directly observable, I combine equilibrium conditions of the model with observable characteristics such as wages, education levels and age to estimate these distributions.

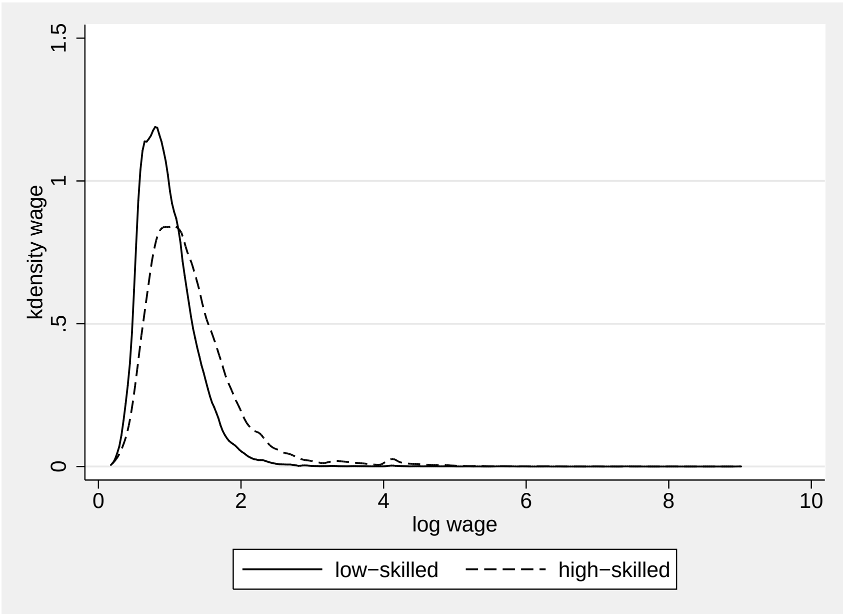

Figure 1.6: Hourly wages of the high- and low-skilled in 1981

Notes: Wages are calculated from the CPS MORG supplements. Wages are the exponent of the residuals from regressing log hourly wage on age, age square, sex and race. Those who attended college are high-skilled, everyone else is low-skilled. The lines represent the kernel density estimate produced by Stata.

Figure 1.6, which represents the hourly wages of high- and low-skilled individuals, offers a good starting point for identifying the ability and cost of education distributions. A striking feature in the figure is the significant overlap between the wages of the two educational groups. An appropriate distribution, therefore, must reproduce this pattern.

In general there are two components to the cost of education: a time cost and a consumption cost. The time cost arises because a person can work part-time at most while studying. The consumption cost is due to tuition fees and other expenses. Both these costs could be thought of as homogeneous or heterogeneous across individuals. For example, a model with credit constraints and differential endowments would yield a heterogeneous education cost in reduced form. I consider three cases—a homogeneous cost, a distribution of consumption costs and finally a distribution of time costs—and show that only heterogeneous time costs of education can reproduce the overlapping wages.[15] Therefore in the calibration and in the numerical results I assume that the cost of education is purely an idiosyncratic time cost.

First, consider the case with a homogeneous consumption cost of acquiring education. In this case, the returns to education are increasing in ability, while the cost is fixed. In equilibrium there is a cutoff ability above which people acquire education, and below which they do not. Since both ability and wage per efficiency unit are higher for high-skilled individuals, equilibrium choices imply higher wages for high-skilled individuals. Wage distributions in this setup would not overlap, contradicting the empirically observed pattern.[16]

Second, assuming a distribution of consumption costs does not fit the empirical pattern of overlapping wage distributions either. A distribution of consumption costs implies a cutoff cost for every ability level in equilibrium. Given the cutoff for an ability level, those with the respective ability and lower cost of education acquire education, while those with cost higher than the cutoff do not. The equilibrium cutoff cost is increasing in ability: people with higher ability, have higher returns from education and are willing to pay a higher consumption cost for education. This implies that the fraction of high-skilled is increasing in the ability level, implying a higher average ability among the high-skilled. As in the previous case, high-skilled individuals have higher wages due to a higher unit wage and higher average abilities, contradicting the overlapping wage distribution pattern.[17]

Third, assuming instead, that the cost of education is a time cost, the equilibrium cutoff cost for acquiring education is independent of ability. If the ability and cost distributions are independent, then the high-skilled have higher wages only because of higher unit wages,

[15]For sake of brevity in the discussion of the various cases I only consider the decision of those individuals, who acquire education for higher wages and not to avoid unemployment. In all cases, there would be a range of abilities at the very bottom end of the ability distribution, where some people would acquire education to avoid unemployment, while the rest would be unemployed.

[16]If the homogeneous cost was a time cost, everyone would need to be indifferent between acquiring education or not. Since both the cost and the returns to education are linearly increasing in ability, if people were not indifferent then either everyone would acquire education or nobody would. An equilibrium based on indifference cannot be estimated from the data, since the ability, and therefore the wages of high- and low-skilled individuals are indeterminate in equilibrium.

[17]This holds even when the ability and cost distributions are independent. With a negative correlation between ability and the consumption cost of education, the two wage distributions would overlap even less.

since the average ability in the two sectors are equal. The distribution of wages for the high-skilled is a shifted and compressed version of the distribution of wages for the low-skilled. Hence, in this case predictions on the distribution of wages in the high- and low-skilled sector match well with the pattern observed in Figure 1.6. Therefore in the calibration and in the numerical results I assume that the cost of education is purely an idiosyncratic time cost.

For simplicity I assume that ability and education costs are independently distributed. I assume a uniform time cost distribution on $[0, \overline{c}]$, with $\overline{c} \leq 1$, allowing a maximum of five years for studies if $\overline{c} = 1$. The probability density function is $g(c) = 1/\overline{c}$. I assume that ability is lognormally distributed, with probability density function $f(a) = \frac{1}{a\sigma}\phi(\frac{\ln(a)-\mu}{\sigma})$, where ϕ is the pdf of the standard normal distribution.

Since all variables of interest in the steady state calibration and in the quantitative assessment of the transition are invariant to the mean of the ability distribution, I normalise this mean to be one.[18]

In the model, the wage of an individual with ability a_i and education s is given by $w^s(a_i) = a_i w^s$, while the average wage in sector s is $\overline{w}^s = \overline{a}^s w^s$, where $\overline{a}^s$ is the average ability among those with education s, and w^s is the wage per efficiency unit in sector s. Based on this:

$$\frac{a_i}{\overline{a}^s} = \frac{w^s(a_i)}{\overline{w}^s} \equiv \widetilde{a}_i^s.$$

An individual's ability relative to the average ability in his education group is equal to his wage relative to the average wage in that sector. Since the education and wages of every respondent in the sample are recorded, I can infer *relative ability*, $\widetilde{a}_i^s$, from the data.

If the distribution of time costs and abilities is known, cutoff values for unemployment, $\underline{a}^{h*}, \underline{a}^{l*}$ and time cost c^* can be found by matching the fractions of unemployed, low- and high-skilled workers. The thresholds $\underline{a}^{h*}, \underline{a}^{l*}$ and c^*, and the parameters of the ability and cost distributions are sufficient to calculate the average ability in both education groups,

[18]This normalization is equivalent to:

$$E(a) = e^{\mu+\frac{1}{2}\sigma^2} = 1 \quad \Leftrightarrow \quad \mu = -\tfrac{1}{2}\sigma^2.$$

Furthermore, in any model, where agents are heterogeneous in ability, the mean of the ability distribution and the technology level are not separable along any observable measure. Since this setup does not require the absolute level of technology, or the mean of the ability distribution for any quantity of interest, this normalization is without loss of generality.

$\overline{a}^h, \overline{a}^l$ (see Figure 1.3 and Appendix A.3.1).

Multiplying the relative ability of a person by the average ability in his education group gives his ability level:

$$a_i = \frac{a_i}{\overline{a}^s}\overline{a}^s = \frac{w^s(a_i)}{\overline{w}^s}\overline{a}^s.$$

According to the model, if a high-skilled individual i's wage is lower than a low-skilled individual's wage, and since the skill premium is greater than one, it follows that his ability has to be lower as well. This implies the following:

$$k_i \equiv \arg\min_{j|w_i^h<w_j^l} w_j^l \quad a_i^h \leq a_{k_i}^l.$$

Similarly, the ability of any low-skilled individual has to be higher than the ability of all high-skilled individuals with a lower wage:

$$k_i \equiv \arg\max_{j|w_i^l>w_j^h} w_j^h \quad a_i^l \geq a_{k_i}^h.$$

A high-skilled individual has wage w_i^h if his ability is $a_i^h = \frac{w_i^h}{\overline{w}^h}\overline{a}^h$, and he acquired education either to avoid unemployment, or because his time cost is lower than the threshold, $c_i \leq c^*$. If he is in the first period of his life, his time cost of education must be lower than the maximum amount of time he could have spent studying. The probability of observing a high-skilled individal with wage w_i^h at age d is:

$$P(w_i^h, h, d) = \begin{cases} P(a = a_i^h) & \text{if} \quad a_i^h \in [\underline{a}^{h*}, \underline{a}^{l*}) \quad \& \quad d \geq 23 \\ P(a = a_i^h)P(c \leq \frac{d-18}{5}) & \text{if} \quad a_i^h \in [\underline{a}^{h*}, \underline{a}^{l*}) \quad \& \quad d < 23 \\ P(a = a_i^h)P(c_i < c^*) & \text{if} \quad a_i^h \geq \underline{a}^{l*} \quad \& \quad d \geq 23 \\ P(a = a_i^h)P(c_i < \min\{c^*, \frac{d-18}{5}\}) & \text{if} \quad a_i^h \geq \underline{a}^{l*} \quad \& \quad d < 23 \end{cases}$$

Since there is an upper bound on the ability a high-skilled individual can have, the likelihood of observing a given wage, w_i^h for a high-skilled person can be written as:

$$\mathfrak{L}(w_i^h, d; \sigma, \overline{c}) = \begin{cases} 0 & \text{if} \quad a_i^h < \underline{a}^{h*} \quad \text{or} \quad a_i^h > a_{k_i}^l \\ f(a_i^h) & \text{if} \quad a_i^h \in [\underline{a}^{h*}, \underline{a}^{l*}) \quad \& \quad a_i^h \leq a_{k_i}^l \quad \& \quad d \geq 23 \\ f(a_i^h)G(\frac{d-18}{5}) & \text{if} \quad a_i^h \in [\underline{a}^{h*}, \underline{a}^{l*}) \quad \& \quad a_i^h \leq a_{k_i}^l \quad \& \quad d < 23 \\ f(a_i^h)G(c^*) & \text{if} \quad a_i^h \geq \underline{a}^{l*} \quad \& \quad a_i^h \leq a_{k_i}^l \quad \& \quad d \geq 23 \\ f(a_i^h)G(\min\{c^*, \frac{d-18}{5}\}) & \text{if} \quad a_i^h \geq \underline{a}^{l*} \quad \& \quad a_i^h \leq a_{k_i}^l \quad \& \quad d < 23 \end{cases} \tag{1.28}$$

Similarly, a low-skilled individual earning wage w_i^l must have $a_i^l = \frac{w_i^l}{\overline{w}^l}\overline{a}^l$, and cost exceeding the cutoff time cost; $a_i^l \geq a_{k_i}^h$ must also hold. The probability of observing w_i^l is then:

$$P(w_i^l, l) = P\left(a = a_i^l\right) P(c_i \geq c^*).$$

The likelihood of observing wage w_i^l for a low-skilled individual is:

$$\mathfrak{L}(w_i^l; \sigma, \overline{c}) = \begin{cases} 0 & \text{if} \quad a_i^l < \underline{a}^{l*} \quad \text{or} \quad a_i^l < a_{k_i}^h \\ f(a_i^l)(1 - G(c^*)) & \text{if} \quad a_i^l \geq \underline{a}^{l*} \quad \& \quad a_i^l \geq a_{k_i}^h \end{cases} \tag{1.29}$$

I calculate the likelihood of observing the sample of wage and education pairs using (1.28) and (1.29). I maximise the likelihood by choosing parameters σ and $\overline{c}$.

I use the May and Outgoing Rotation Group supplements of the Current Population Survey for 1981. I choose 1981 as the initial steady state because from 1982 onwards, the minimum wage was not adjusted by inflation, and its real value started declining. I divide the population into high- and low-skilled based on college education: those who attended college are high-skilled, those who did not are low-skilled. I calculate the fraction of unemployed, low-skilled and high-skilled workers using the education and the employment status categories .[19] In order to capture only the effects of education and underlying ability, I use a cleaned measure of wage. This measure is the exponent of the residuals generated from regressing log hourly wages on age, age square, sex and race. The maximum likelihood yields $\sigma = 0.73$ and $\overline{c} = 0.82$, which corresponds to about four years.

[19] In the calibration I do not make a distinction in the educational attainment of the unemployed. In the steady state, only those who will be employed in the future should acquire education. In the data, half of the unemployed have some college education.

1.5.3 Final good production and R&D

I calibrate the remaining parameters to minimise the distance between moments of the initial steady state and the same moments from the data. It is common in calibration exercises to match n moments exactly by choosing n parameters, and use the remaining moments to test the goodness of fit of the model. In this method the parameters chosen depend heavily on which moments are matched, and the choice of these moments are rather arbitrary. The method I use, which is similar to a method of moments estimation, is to choose the values of 6 parameters to minimise the weighted distance from 9 moments of the data. The weight of the ith moment, is the estimated standard deviation of the ith moment in the data. I run a grid search over the set of parameter values and find the set that globally minimises the distance from the moments.

Table 1.1: Moments

Moment	Data	Model
L^u	0.0693	0.1023
L^l	0.5338	0.4923
L^h	0.3554	0.3964
g	0.0800	0.0798
$\overline{w}^h/\overline{w}^l$	1.3344	1.0518
$\overline{w}/w_{50}$	1.1072	1.2942
w_{90}/w_{50}	1.7060	2.4252
w_{50}/w_{10}	1.7006	2.0778
$\overline{w}^h/\overline{w}$	1.1796	1.0280

I chose three types of moments: moments that describe the skill-composition and fraction of unemployed in the economy, those that describe the wage distribution, and those that reflect the R&D process. Moments of the first type are important to match, as most of the movement in the model comes from changes in these aggregates. The second type is also crucial, since I analyse the effects of minimum wages on inequality. Finally, matching the growth rate, which is governed by the R&D process, determines the responsiveness of technology. The moments and the fit of the model are summarised in the Table 1.1.

I globally minimise the distance from the data moments by choosing $\rho, \gamma, \eta, \overline{q}, B$ and $\widetilde{w}$. The calibrated parameters are summarised in Table 1.2. Parameters η and B control the profitability of R&D activity, while $\overline{q}$, η and B together determine the growth rates. Parameter η determines how much R&D spending increases the Poisson arrival rate of innovations, while parameter B determines how costly R&D investments are in terms of the final good. The value of $\overline{q}$ determines the size of the improvement between two quality levels over a five year period. The weight of the high-skilled intermediate in the production of the final good is given by γ.

Table 1.2: Calibrated parameters

ρ	γ	η	$\overline{q}$	B	$\widetilde{w}$	β	λ	r	$\overline{c}$	σ
0.9	1.15	0.25	2.08	0.15	0.4	2/3	8/9	1.05^5	0.82	0.73

Parameter ρ controls the elasticity of substitution between the *intermediate goods* produced by the high- and low-skilled. This elasticity, $1/(1-\rho)$ cannot be estimated directly from the data. Note that the elasticity of substitution between the intermediate goods is not the same as the elasticity of substitution between high- and low-skilled workers, which has been estimated by several authors. However, their estimates are not comparable to ρ, since technology is usually modelled as exogenous, while in my model it is endogenous.[20]

1.6 Transitional dynamics

In this section I discuss the transitional dynamics following a reduction in the minimum wage. The transition takes relatively long as new generations have to replace older ones, as the new steady state features a different educational composition. During the transition, the average skill premium and the wage gaps between different percentiles in the wage distribution all increase. The increase is the most pronounced in the period of the announcement, due to the entry of previously unemployed workers into the labour force. Inequality measured by the skill premium and wage gaps continues to increase throughout the transition, as both the skill composition of the labour force and the ability composition of the two skill groups change.

[20]See Section A.3.2 of the Appendix for further details.

Initially the economy is in steady state. The minimum wage grows at the same rate as the wages and the quality in both sectors. The government unexpectedly announces a permanent decrease in the value of the normalised minimum wage. The normalised minimum wage drops to its new lower value in the period of the announcement, and stays there forever. Individuals and R&D firms have perfect foresight over the future sequence of the minimum wage, and form correct expectations about the future path of the average quality levels of machines and education acquisition of future generations. The economy is in a decentralised equilibrium along the transitional path from the initial BGP to the new one.

I use a second order approximation of the equations that have to hold throughout the transition to produce the transitional dynamics (see Appendix Section A.4 for details).[21]

Figure 1.2 shows that the real value of the minimum wage decreased by about 30 percent until the late 1980s, while the minimum wage compared to the average high- and low-skilled wage decreased by about 20 percent. In the transitional dynamics I mimic this pattern by a one-time 20 percent drop in the value of the normalised minimum wage. Since in the steady state the real minimum wage is not stationary, it is not possible to simulate a shock by changing its value while using perturbation methods. The change in the normalised minimum wage is not necessarily the same as the change in the minimum wage compared to the average wage, but the transition shows that it is sufficiently close.[22]

Figure 1.7 shows the transitional path from the original steady state to the new one, which features a 20 percent lower normalised minimum wage. The horizontal axis denotes the year, with the drop in the normalised minimum wage occurring in 1981.

The top two panels in Figure 1.7 show the path of the unemployment thresholds. At the moment of the announcement, both $\underline{a}^{h*}$ and $\underline{a}^{l*}$ drop almost to their new steady state

[21] I use the code designed in Schmitt-Grohe and Uribe (2004) to produce the transitional dynamics.

[22] Using the normalised minimum wage implies:

$$\widetilde{w}_1 \equiv \frac{w_t}{Q_t^l} = \underline{a}_t^l \beta (1-\beta)^{\frac{1-2\beta}{\beta}} (p_t^l)^{\frac{1}{\beta}},$$

while using the minimum wage compared to the average low-skilled wage implies:

$$\widetilde{w}_2 \equiv \frac{w_t}{\overline{w}_t^l} = \underline{a}_t^l \overline{a}_t^l.$$

These clearly do not imply the same dynamics for $\underline{a}_t^l$, but since the magnitude of the change in both p_t^l and $\overline{a}_t^l$ is small, their effect will be dominated by the drop in $\widetilde{w}$ throughout the transition.

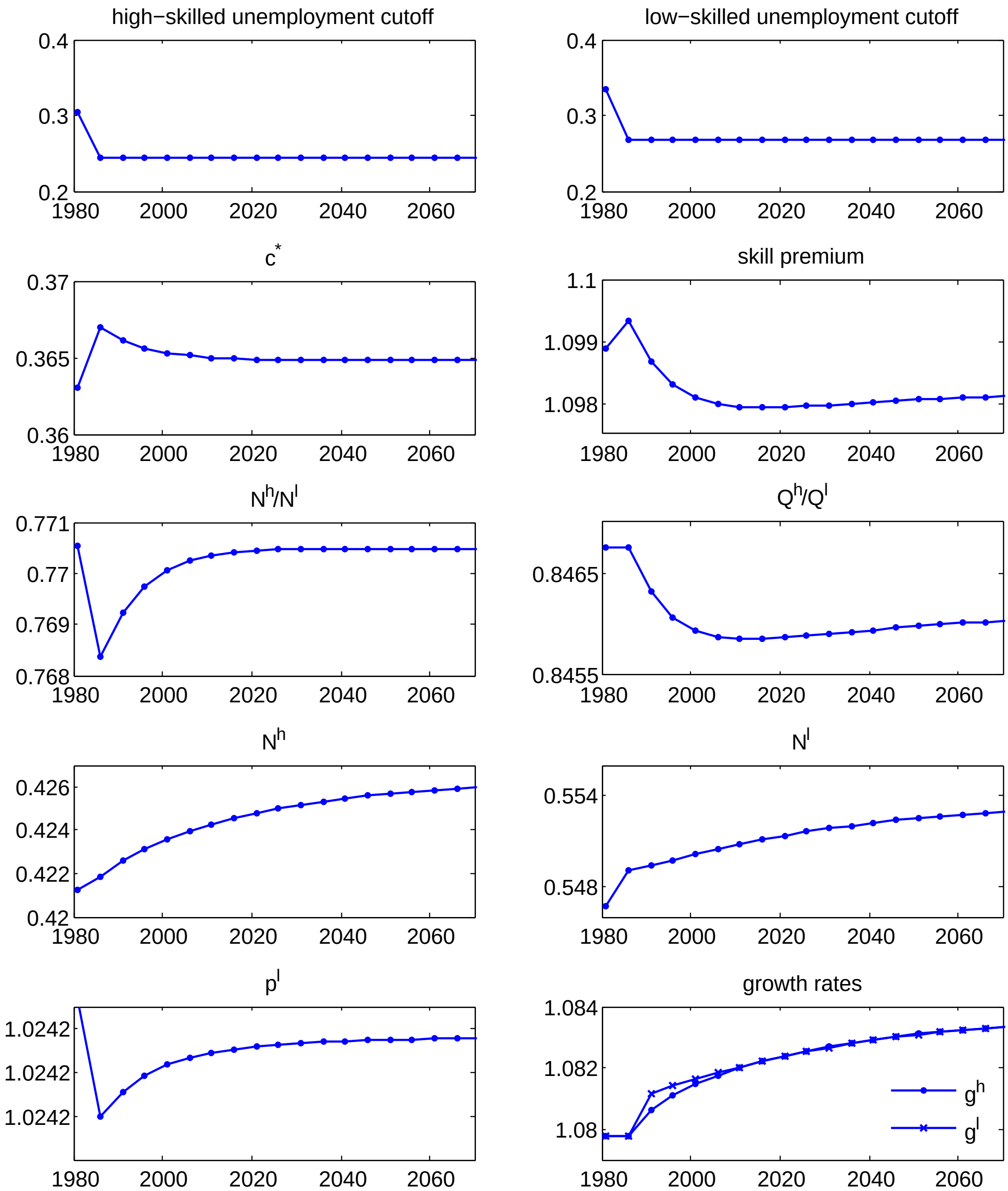

Figure 1.7: Transition of the main variables

value. It is not visible on the graphs, but the threshold ability for low-skilled unemployment initially stays above its steady state value and gradually falls towards it, while the threshold for high-skilled unemployment drops slightly below, then increases to its new steady state value. Equation (1.25) shows that only the price of the low-skilled intermediate affects the path of $\underline{a}^l$. As the bottom left panel of Figure 1.7 shows, the change in the steady state price is very small, which explains the seemingly immediate jump of $\underline{a}^l$ to its new steady state value. The movement of $\underline{a}^h$ can be understood from (1.26): $\underline{a}^h$ follows $\underline{a}^l w_t^l / w_t^h$, therefore the initial overshooting of the skill premium (second row, right panel in Figure 1.7) explains the undershooting of $\underline{a}^h$. The thresholds for unemployment do not change much after the initial drop because intermediate prices and the skill premium do not change much either.

Note that the new value of $\underline{a}^{l*}$ is lower than the initial $\underline{a}^{h*}$; this suggests that those who acquire education in order to avoid unemployment in the new steady state and during the transition have lower ability than those who did the same in the previous steady state.

The path of the cutoff time cost for acquiring education is shown in the left panel in the second row of Figure 1.7. This threshold c^* initially overshoots and then decreases monotonically towards its new steady state value, which is higher than the original one. This pattern can be understood by looking at the path of the skill premium (second row, right panel) and the path of the growth rates (bottom right panel). The initial jump in the skill premium drives the overshooting of c^*, then as the skill premium decreases, so does c^*. The monotone increase in the growth rate increases the present value gain of being high-skilled for a given skill premium, which keeps the new steady state value of c^* above the initial one.

Taking the path of the three cutoffs $\underline{a}^{h*}, \underline{a}^{l*}$ and c^* as given, the paths of the effective supply of high- and low-skilled labour (depicted in row 4 of Figure 1.7) can be understood. Figure 1.8 plots the effect of changes in the cutoffs on the high- and low-skilled effective labour supply and on the labour market participation of individuals. The initial steady state thresholds are denoted by $\underline{a}_0^{l*}, \underline{a}_0^{h*}, c_0^*$, while the new steady state values are denoted by $\underline{a}_1^{l*}, \underline{a}_1^{h*}, c_1^*$. The maximum value of c^*, which is reached in the period of the announcement is denoted by c^*_{max}.

The shift in the cutoffs lead to two types of changes: in the education decisions and in the labour force participation of individuals. These mostly affect the *new generations*:

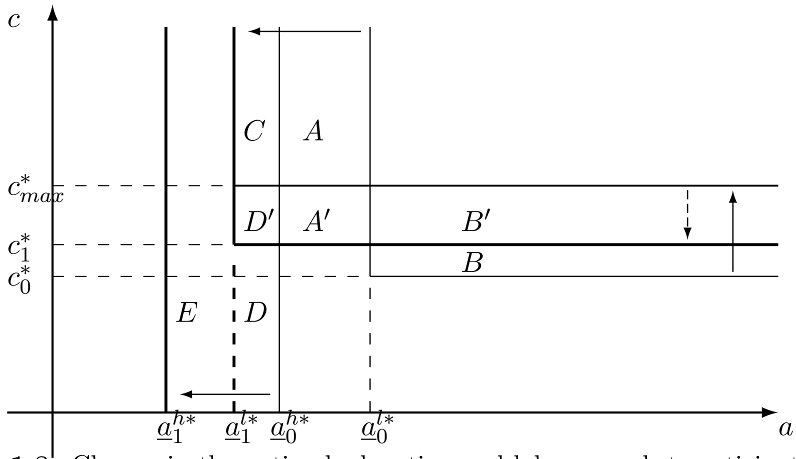

Figure 1.8: Change in the optimal education and labour market participation

Notes: Ability is on the horizontal axis, the time cost is on the vertical axis. The maximum of the time cost $\overline{c}$ is omitted from the graph, to make it less cluttered. The initial steady state cutoffs are: $\underline{a}_0^{h*}, \underline{a}_0^{l*}, c_0^*$, while the new ones are: $\underline{a}_1^{h*}, \underline{a}_1^{l*}, c_1^*$. I denote the maximum threshold time cost that is reached in the period of the announcement by c^*_{max}.

those born in the period of the announcement, and in subsequent generations. This is because the option of acquiring education is only available at birth, and individuals are not allowed to retrain themselves in later periods. Thus, the labour supplies adjust gradually, as new generations replace old ones, lengthening the transition period.

The only case where this is not true is for members of previous generations (for example person C, D or D')with ability between $\underline{a}_1^{l*}$ and $\underline{a}_0^{h*}$. They are low-skilled and have been unemployed until now, but in the period of the announcement they can immediately start working as low-skilled workers. Their *entry* into the workforce instantaneously increases the supply of low-skilled workers, which is reflected by the jump in N^l.

Members of the new generation with ability between $\underline{a}_1^{l*}$ and $\underline{a}_0^{h*}$ either start working as low-skilled, as C, or enrol in education at birth, as person D or D'. People with the same time cost as D' will only become high-skilled if they belong to generations born close to the initial shock, whereas people with time cost as D become high-skilled regardless of the generation they are born in. This implies that the initial increase in low-skilled labour supply will be diminished to some extent in future periods, as individuals similar to D become high-skilled instead of working as low-skilled. They replace some members of the older generations who went from unemployment into the low-skilled workforce. The *education* of individuals like D increases the supply of high-skilled workers while decreasing the supply of low-skilled workers gradually. This is reflected by the gradual increase in N^h.

Consider person E from one of the new generations. He would have been unemployed under the previous regime, but now can avoid unemployment by becoming high-skilled. This is true for all members with ability in $[\underline{a}_1^{h*}, \underline{a}_1^{l*})$ in the new generations. The *entry* of these individuals leads to a gradual increase in N^h.

Individuals similar to A and A' would have been high-skilled with the original, higher minimum wage in order to avoid unemployment. Under the new, lower minimum wage, they can work without acquiring education. Initially only individuals with time cost as high as A remain low-skilled. Gradually as c^* decreases to c_1^* individuals with time cost as A' also opt out from education. The change in the *education* of individuals with ability in $[\underline{a}_0^{h*}, \underline{a}_0^{l*})$ and high enough education time cost gradually increases the supply of low-skilled workers at the expense of the high-skilled workforce, reflected in the gradual increase in N^l.

Since the cutoff time cost initially overshoots and then decreases monotonically to its new steady state value, in generations closer to the announcement, more individuals become high-skilled among those with ability greater than $\underline{a}_1^{l*}$. Consider individual B'. If born in the period of the announcement, he acquires education. In the long run, however, it will only be individuals with time cost as B whose education choice is different from the choice of generations born before the change in the minimum wage. This implies that initially individuals with higher time cost acquire education than the new steady state implies. The *education* of these individuals gradually increases the supply of the high-skilled workforce.

The left panel in row 3 of Figure 1.7 shows the overall effect of these changes on the relative supply of skills, N^h/N^l: the relative supply of skills decreases on impact. This is the result of two forces. First there is mass entry from unemployment into the low-skilled labour force at the time of the announcement.The effect of this can be seen on the right panel in row 4 as N^l jumps up. Second, there is entry from unemployment into the high-skilled labour force, but the effect of this is offset to some extent by the exit of some ability levels, N^h initially increases only slightly (left panel in row 4).

As time passes the effect of the initial increase in the supply of low-skilled workers is diminished, the relative supply of high-skilled workers starts increasing more, and growth in the supply of low-skilled workers decreases. Row 4 of Figure 1.7 shows that both supplies increase gradually, and both measures rise above their initial level in the long run.

The skill premium per efficiency unit depends on two factors along the transitions: the relative supply of high-skilled workers and the relative technology available. The interaction of the two is shown in the right panel of row 2 in Figure 1.7: on impact the skill premium increases. The initial decline in the relative supply of skills increases the skill premium. This supply effect is not offset by technology, as depicted in the right panel in row 3. Even though technology becomes less biased towards the high-skilled workers in the long run, in the short run it does not have sufficient time to react to these changes. As the change in the relative supply is unanticipated, technology can only adjust from the next period onwards. This explains the initial increase in both the skill premium per efficiency unit of labour (right panel in row 2), and the average skill premium (top panel of Figure 1.9).

From the second period on, technology adapts according to the change in relative supplies, shown on the right panel in row 3. It reacts with a lag to the initial decline in relative supply by undershooting, and then gradually increasing to its new steady state value, which is slightly below the original steady state. As technology starts to react to the change in relative supply, the skill premium drops as well, undershooting its final steady state value. In the long run the skill premium converges to its new steady state value, which is slightly lower than its initial value.

The variables with empirically observable counterparts are the relative supply of high- and low-skilled raw labour, L^h/L^l, and the average skill premium, $\overline{w}^h/\overline{w}^l$. The relative raw labour supply is shown in the bottom panel of Figure 1.9. Its path is very similar to that of the effective labour supply, but the magnitude of change is quite different. This difference in magnitude is due to the difference in ability between those who join the low-skilled and the high-skilled labour market. The measure of people joining the low-skilled workforce is much larger than the measure of those joining the high-skilled workforce, reflected in the significant overall decline in the relative supply of raw high-skilled labour. On the other hand, the average ability of those joining the high-skilled workforce is higher than the average of those joining the low-skilled. This is demonstrated by the only slight long run decline in the relative supply of high-skilled effective labour. This implies that *compositional* changes play an important role in both the high-skilled and the low-skilled workforce. The average ability in both sectors decreases, but it decreases relatively more among the low-skilled than among the high-skilled workers.

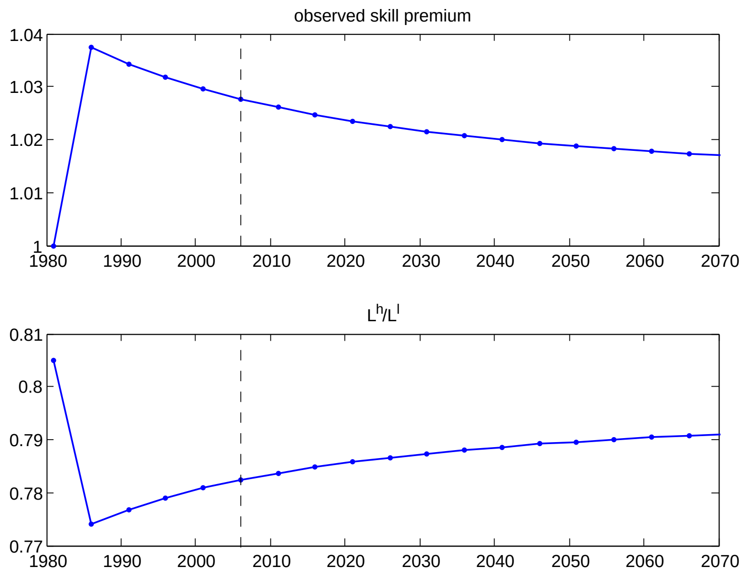

Figure 1.9: Average skill premium and relative raw labour supply

Notes: The vertical dashed line represents 2006, the year to which I am comparing the results to. The top panel represents the change in the observed skill premium compared to its initial value, while the bottom panel shows the path of the relative supply of raw high-skilled labour.

The top panel in Figure 1.9 represents the change in the observed skill premium compared to its initial value. The observed skill premium increases on impact and then decreases gradually, as does the skill premium per efficiency unit of labour. However, unlike the skill premium per efficiency unit, the average skill premium converges to a value higher than its initial value in the long-run. This is due to *compositional* effects: since the average ability in the low-skilled labour force decreases more than in the high-skilled labour force, the average skill premium increases relative to its initial value.

Between 1981 and 2006 the average skill premium increased by 18 percent (see Figure 1.1). In the model, twenty five years after the decline in the minimum wage (at the dashed vertical line), the increase is 2.7 percent, implying that the minimum wage accounts for 15 percent of the increase in the observed skill premium.

The widening wage inequality is well captured by the increasing gap between the wages of workers in the 90th, 50th and 10th percentile. Figure 1.10 shows the change in these measures during the transition. The dashed vertical line represents the year 2006.

These wage gaps increase due to two factors: changes in the skill premium per efficiency

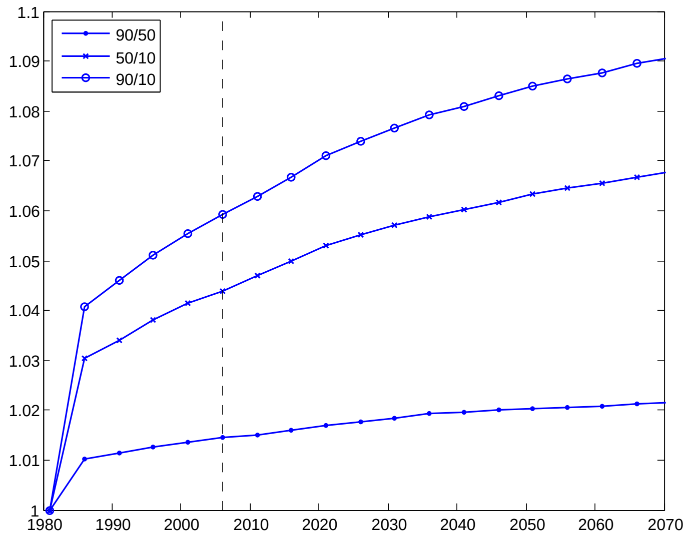

Figure 1.10: Wage gaps during the transition

Notes: The vertical dashed line represents 2006, the year to which I am comparing the results to.

unit, and compositional effects.

Changes in the skill premium only increase inequality in the period of the announcement; from the third period onwards these changes compress the wage distribution (see Figure 1.7 second row right panel).

Compositional forces always put an upward pressure on inequality. One component is the widening range of abilities present on the labour market. As the normalised minimum wage drops, the threshold abilities for unemployment decrease, increasing the range of abilities present on the labour market. As the range of abilities widens, the gap between the ability level at the 90th percentile gets further away from the ability level at the 50th percentile, which gets further from the 10th percentile. The second component is the changing ratio of high- to low-skilled workers at every percentile in the wage distribution. The fraction of high-skilled workers among the top 10 percent of earners increases, while their ratio at the bottom 10 percent decreases.

All three wage gaps increase the most in the period of the announcement, since the skill premium and the compositional effects both put an upward pressure on them in this period. After the first period, the wage gaps widen further, but at a slower rate. The 90/10 wage differential increases the most, while the 90/50 increases the least. This is expected, since most of the compositional changes affect the lower end of the wage distribution.

Note, however, that the the change in the minimum wage causes the top end of the wage distribution to widen as well. This is mostly due to the compositional changes both in ability and in skill levels, which affect the position of the 90th percentile and the 50th percentile eraner differentially.

The 90/10 wage gap increased by 32 percent between 1981 and 2006, the 90/50 wage gap increased by 21 percent, and the 50/10 wage gap increased by 10 percent (see Figure 1.1). The model is most successful at predicting the 50/10 wage gap - it explains about 45 percent of the observed increase, while it explains about 18.5 percent and 7 percent of the increase in the 90/10 and 90/50 wage gaps, respectively.

1.7 Decomposition

I consider three simplified versions of the model, in order to better understand the contributions of changing technology and education to the effects of minimum wages on the patterns of wage inequality. The first version is one where both educational attainment and technology are fixed. In the second version, the skill composition is endogenous, but technology is fixed. The third version features fixed educational attainment and endogenously directed technical change.

Comparing the transitional dynamics of the four models quantitatively shows that most of the initial effects are due to the inflow from unemployment into the labour market. The decomposition shows that in the case of endogenous education, compositional effects play an important role, and that the change in technology does not have a quantitatively big impact on overall inequality.

1.7.1 Exogenous education, exogenous technology

Consider a model, where the production side is as in the model, but technology and education are fixed. Technology in the low- and the high-skilled sector is growing at the same rate. There are high and low-skilled individuals, but the choice of acquiring education is fixed in other words, nobody can acquire additional education and nobody can opt out from education. I assume that the education and employment structure in the initial steady state is as in the full model.

If both education and technology are fixed, then lowering the minimum wage affects the wage distribution only through an expansion of low-skilled employment. A lower minimum wage allows people who have been previously unemployed, and are hence low-skilled, to enter the low-skilled labour market (see Section A.5.1 of the Appendix). With constant technology, this decreases the wage per unit of efficiency for the low-skilled, thereby increasing the skill premium. However, since education is fixed, this does not translate into an increase in the supply of high-skilled labour. The average ability in the low-skilled sector decreases, hence the observed skill premium increases more than the skill premium per efficiency unit.

In this setup there are no transitional dynamics, as low-skilled employment expands in the period of the announcement, the skill-premium responds, and there are no further adjustments. As a consequence of a fall in the minimum wage, the supply of low-skilled labour increases, the skill premium increases and wage gaps between different percentiles of the distribution also increase.

1.7.2 Endogenous education, exogenous technology

Now consider a model where educational choices are made optimally, but technology is fixed. As in the previous model, quality in the high- and the low-skilled sector is growing at the same rate. Since education changes endogenously, I model the labour market side exactly as in the full model. The key difference is that since growth is exogenous, there is no feedback from the effective labour supplies to the direction and rate of technological improvements. Therefore, the relative supply of skills only affects the skill premium through the price effect, as the market size effect is removed. Hence, in this setup, the skill premium per efficiency unit is always decreasing in the relative supply of skills:

$$\frac{w^h}{w^l} = \gamma^{\frac{1}{1-\rho+\beta\rho}} \left(\frac{N^{h*}}{N^{l*}}\right)^{-\frac{1-\rho}{1-\rho+\beta\rho}} \left(\frac{Q^h}{Q^l}\right)^{\frac{\beta\rho}{1-\rho+\beta\rho}}.$$

The unemployment cutoffs and the threshold for acquiring education are determined exactly as in the full model (see Appendix Section A.5.2 for details). The only differences are that the skill premium is always decreasing in the relative supply (see equation above) and the growth rate is exogenous and independent of the relative supply of skills.

In the Section A.5.2 of the Appendix, I show that the system can be reduced to two thresholds, $\underline{a}^{l*}$ and c^{*}, as in the full model, and the two equations defining the steady state are as in Figure 1.5. This also implies that as in the full model, a reduction in the minimum wage reduces the unemployment threshold in both sectors, and increases the threshold cost of acquiring education.

In the long-run, the supply of high- and low-skilled effective labour increases, with the relative supply of skills decreasing. This implies an increase in the skill premium per efficiency unit, unlike in the full model. Moreover, the average ability in the low skilled sector decreases more, which implies that the observed skill premium increases more that the skill premium per efficiency unit. The wage gaps between different percentiles also increase.

The transition takes a long time, as in the full model, since complete educational adjustment takes several generations.

1.7.3 Exogenous education, endogenous technology

Finally, consider an economy where education is fixed, but technology changes endogenously. In such a setup, a lower minimum wage increases the supply of low-skilled labour, thus increasing the skill premium. This does not lead to an increase in the supply of skills, as educational choices are fixed. The average ability in the low-skilled sector decreases, implying that the observed skill premium increases more than the skill premium per efficiency unit.

Transition takes time, as technology needs to adapt to the new relative labour supplies. In the long-run, technology becomes less skill-biased and the skill premium per efficiency unit falls below its original value.

1.7.4 Decomposition results

Figures 1.11 and 1.12 show the path of the observed skill premium and wage gaps between different percentiles in the distribution. The observed skill premium increases the most in the case of fixed education and technology, both in the short- and the long-run. This

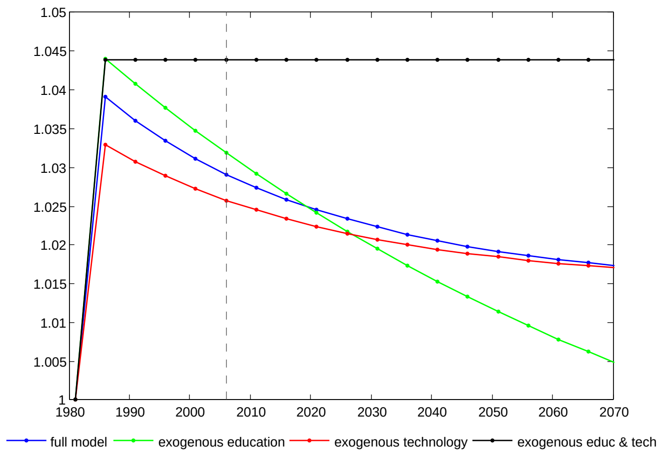

Figure 1.11: The role of education and technology in the average skill premium

Notes: The vertical dashed line represents 2006, which is the final year, to which I am comparing the results. The colours represent: blue – full model, red – exogenous technology, endogenous education, green – endogenous technology, exogenous education, black – exogenous technology, exogenous education.

is not true for the wage gaps: the wage gaps in the short-run increase the most in the case of fixed education and technology, but in the long-run, the effects are bigger when education is endogenous. The different pattern of the skill premium and the wage gaps suggest that the increase in the observed skill premium is driven by the expanding employment of the low-skilled.

The observed skill premium increases the most in the case of exogenous technology and exogenous education (see Figure 1.11), as the increase in the supply of low-skilled labour is the largest. With endogenous technology, the initial impact is the same, but is diminished in the long-run as technologies become less skill-biased. When education is endogenous, the initial impact of lowering the minimum wage is smaller. This is due to an expansion of high-skilled employment. As low-skilled workers enter the labour market and the skill premium increases, the incentives for acquiring education increase, leading to an expansion of the high-skilled labour force, thus diminishing the initial increase in the skill premium. The initial increase in the skill premium is larger when technology is endogenous, due to the higher growth rate of the economy. An expansion of the labour force leads to a higher growth rate in case of endogenous technology, which implies a higher lifetime gain from working in the high-skilled sector. Therefore, if technology is endogenous, the cutoff time

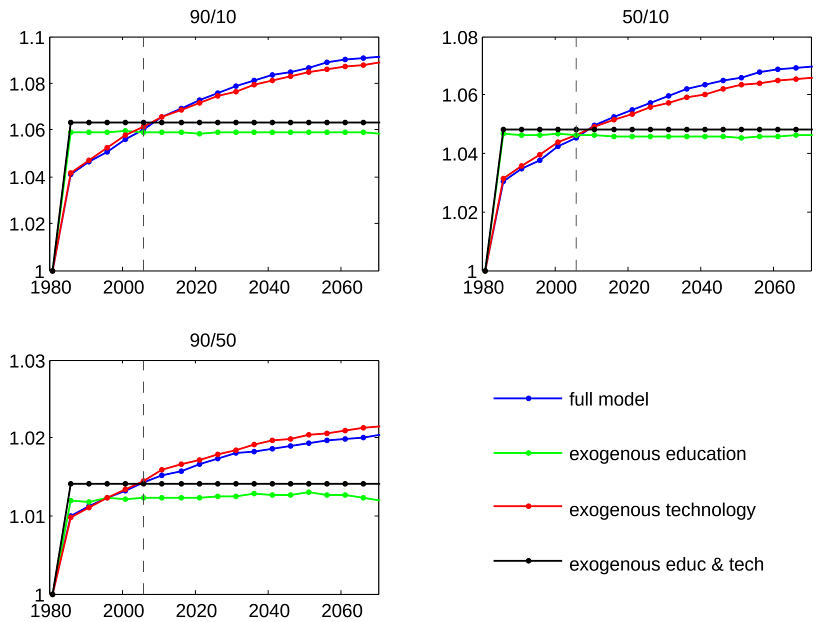

Figure 1.12: The role of education and technology in the wage gaps

Notes: The vertical dashed line represents 2006, the final year of data. The colours represent: blue – full model, red – exogenous technology, endogenous education, green – endogenous technology, exogenous education, black – exogenous technology, exogenous education.

cost for education increases more, leading to a larger change in average abilities and larger compositional effects.

Figure 1.12 shows the patterns of wage gaps. In all three graphs, the biggest initial impact is in the case of exogenous education, implying that most of the initial increase is due to the inflow of previously unemployed workers into the low-skilled labour market. In the long-run, the wage gaps increase the most in the case of endogenous education, suggesting that compositional effects play a significant role in the widening dispersion of wages.

1.8 Concluding remarks

There has been much debate about the contribution of the falling minimum wage to the widening wage inequality in the US. The real value of the minimum wage eroded over the 1980s, losing 30 percent of its initial value. At the same time - in the early 1980s -

there was an unprecedented surge in inequality. The wage gap widened between any two points in the wage distribution, and the college premium increased sharply. However, to my knowledge, there are no attempts in the literature to assess the quantitative significance of falling minimum wages for wage inequality in the context of a general equilibrium model.

In this chapter I propose a general equilibrium model to analyse the effects of a permanent decrease in the value of the minimum wage on inequality. This model incorporates minimum wages, endogenous educational choices and endogenous technological progress. All these components are relevant in their own right: minimum wages affect the educational decisions of individuals through their effect on job and earning opportunities; educational decisions shape the skill composition of the labour force and the ability composition of different skill groups; the supply of high- and low-skilled labour affects the direction of technological change and the direction of technological change affects the educational decision of individuals.

The analysis in general equilibrium reveals that a reduction in the minimum wage affects overall inequality through three channels. First, a reduction in the minimum wage widens the range of abilities present on the labour market, thereby increasing the difference between any two percentiles in the distribution. Second, it differentially affects the shares of high- and low-skilled workers at every percentile in the wage distribution, thus increasing overall inequality. A third channel is the reduction in the skill premium per efficiency unit, which reduces inequality. Therefore, a reduction in the minimum wage affects inequality at the top end of the wage distribution, even if only to a smaller extent.

The full effects of minimum wage reductions are only realised in the long run. Minimum wages affect the educational decisions of individuals in successive cohorts. New cohorts have to replace old ones for the new equilibrium to be reached. Through considering three simplified models, I show that the initial and highest increase in all measures of inequality is due to the inflow from unemployment in the period of the announcement. After this period, the observed skill premium contracts, while the widening of the wage distribution continues due to compositional changes in both ability and skills.

In this model, a reduction in the minimum wage reduces the skill-bias of technology, since the inflow from unemployment is mainly into the low-skilled sector. In future research I plan to test the robustness of the results to different labour market structures. More specifically the low-skilled sector should feature either monopsony or search frictions. In these scenarios the reduction of the minimum wage does not affect unemployment to the same extent, but it still triggers an expansion of the high-skilled labour force through the increase in the skill premium.

Chapter 2

Increasing Skill Premium and Skill Supply

Steady State Effects or Transition?

2.1 Introduction

In this chapter I challenge the existing literature that claims that strongly biased technology is necessary to observe a simultaneous increase in the skill supply and in the skill premium. Their joint increase throughout the past few decades is well-documented[1] and extensively researched. Theoretical explanations for this phenomenon either treat the increase in the supply of high-skilled labour or the increase in the skill-bias of technology as exogenous. When both are treated as endogenous, the skill bias of technology and the skill supply depend positively on each other. This positive dependence is crucial in understanding that the joint increase of these two variables can emerge during the transition to the steady state, independent of the strength of the bias in technology.

I present a model where both the relative technology and the relative supply of high-skilled labour is endogenous. I show that in such a framework the supply of high-skilled

[1]For example Katz and Murphy (1992), Juhn, Murphy, and Pierce (1993), Krueger (1993), Berman, Bound, and Griliches (1994), Richard B. Freeman (1995) Autor, Katz, and Krueger (1998), Berman, Bound, and Machin (1998), Katz and Autor (1999), Acemoglu (2003), Eckstein and Nagypál (2004), Goldin and Katz (2007 and 2008).

workers and the relative quality in the high-skilled sector change in the same direction during the transition to the steady state. I also characterise conditions under which the transition path to the steady state features an increase in the supply of skills *and* a parallel increase in the relative wages of high-skilled workers.

In the model technological progress is driven by profit oriented R&D firms, where profits are increasing in the amount of labour that is able to use these technologies. Hence when the relative supply of labour in one sector increases, the relative profitability of investing into that sector increases as well, thereby increasing the relative technology in that sector. This is referred to as the *bias of technology*: when a factor becomes more abundant, technology endogenously becomes more biased towards that factor. If this bias is large enough, then the increase in the relative technology more than offsets the negative effect of the higher relative supply, and the relative factor price rises in the long-run. This is termed *strong bias* of technology. On the other hand, if the effect of the increase in the relative technology is not large enough, then the relative factor price decreases, and technology displays a *weak bias*.

The supply of labour is determined by individual choices: everyone whose cost of education does not exceed the lifetime gains from working as high-skilled rather than low-skilled, acquires education, and becomes high-skilled. In such a setup, if the relative technology increases in the high-skilled sector, then the skill premium increases, thereby increasing the incentives to acquire education, and educational attainment increases.

It is a well-known fact that the supply of college graduates has been continuously increasing over the past few decades.[2] Human capital accumulation takes several generations, even if technology is fixed. However, in an environment where technology and human capital are evolving jointly, the transition process potentially takes longer, as both the skill supply and the technology adjusts more slowly. In this model, since I allow the supply of the different types of labour to be endogenous, it is natural to consider an economy that is on the transition path towards its steady state. I numerically map the dependence of this path on the initial values of relative quality and relative supply. I find that there is a set of initial values from which the transition features a continuously increasing supply of high-skilled workers, increasing relative quality and increasing skill premium. This feature

[2]See for example Goldin and Katz (2008).

persists even if the elasticity of substitution is low between the two sectors, although the set of such initial values shrinks.

There are two main strands of literature that relate to this chapter. The first strand is based on an exogenous technological progress, and the supply of high- and low-skilled labour adjusts endogenously. Caselli (1999) models the effects of skill-biased technological revolutions, where learning to use new machines is more costly than old ones. In such a scenario, new technologies are adopted slowly, there is a gradual shift of skills to new technologies and the skill premium increases. Ábrahám (2008) allows for endogenous skill formation in an overlapping generations model, with worker heterogeneity in ability. He explores the effects of an exogenous skill-biased technological shock on educational attainment, and finds that the slow adjustment in the supply of educated labour can result in a nonmonotonic pattern of the skill premium. His model, similarly to mine, features a slow adjustment in human capital, which is driven by the optimal educational choices of consecutive cohorts. Galor and Moav (2000) develop a model where an increase in the rate of technological progress increases the returns to education. In such a context, similarly to the chapter here, a feedback mechanism arises: with a higher supply of human capital, the rate of technological progress increases, and a higher rate of technological progress induces more human capital accumulation. The feedback works through a very different channel: it works through easier R&D and not more profitable. Heckman, Lochner, and Taber (1998) develop a general equilibrium model with endogenous skill formation, physical capital accumulation, and heterogeneous human capital to explain rising wage inequality. In this framework they find that skill-biased technical change explains the patterns of skill premium and overall inequality rather well.

The second strand takes the path of high-and low-skilled labour as given, while technological progress is endogenous. The most closely related papers are Acemoglu (1998 and 2002) and Kiley (1999), which study a model similar to the one presented here, and consider an exogenous increase in the supply of high-skilled labour. If the elasticity of substitution between the output of the different types of labour is sufficiently high, then the skill premium increases in the long run. Acemoglu (2007) studies the equilibrium bias of technology in a more general context and shows that if technologies are factor-augmenting, then the increase in the supply of a factor induces technical change to be relatively biased towards that factor. The condition under which this relative bias is strong enough to offset the price effect of increased supply is a sufficiently high elasticity of substitution between the different factors of production.

2.2 The model

The model is along the lines of the model in Chapter 1. There are two differences: there is no minimum wage, and individuals are only heterogeneous in their cost of acquiring education. The structure of this section follows the structure in the previous chapter. I begin by describing the model's production technologies, the R&D sector, the demographic structure and educational choices. Next I define the decentralised equilibrium, I analyse the balanced growth path, and finally, I analyse the transitional dynamics.

2.2.1 Overview

Time is infinite and discrete, indexed by $t = 0, 1, 2...$ The economy is populated by a continuum of individuals who survive from one period to the next with probability λ, and in every period a new generation of measure $1 - \lambda$ is born. Individuals are heterogeneous in their cost of acquiring education.

In the first period of his life every individual has to decide whether to acquire education or not, with the cost of education varying across individuals. Those who acquire education become high-skilled. Those who opt out from education remain low-skilled. Workers with high and low skills perform different tasks, are employed in different occupations, and produce different goods. The high-skilled sector includes skill-intensive occupations and production using high-skilled labour, while the low-skilled sector includes labour-intensive occupations and production using low-skilled labour. In equilibrium working in the high-skilled sector provides higher wages.

There is a unique final good in this economy, which is used for consumption, the production of machines, and as an investment in R&D. It is produced by combining the two types of intermediate goods: one produced by the low- and the other by the high-skilled workers. Intermediate goods are produced in a perfectly competitive environment by the relevant labour and the machines developed for them.

Technological progress takes the form of quality improvements of machines that complement a specific type of labour, either high- or low-skilled. R&D firms can invest in developing new, higher quality machines. Innovators own a patent for machines and enjoy monopoly profits until it is replaced by a higher quality machine. There is free entry into the R&D sector, and more investment will be allocated to developing machines that are complementary with the more abundant labour type.

The economy is in a decentralised equilibrium at all times: all firms maximise their profits – either in perfect competition or as a monopoly – and individuals make educational decisions to maximise their lifetime income. I analyse how the distribution of costs and the characteristics of the production function and the R&D process affect the steady state and the transitional dynamics within this equilibrium framework.

2.2.2 Production

The production side of the model is exactly the same as in Chapter 1. It is a two-sector endogenous growth model, where technological advances feature a *market size* effect, by which more R&D investment is allocated to develop machines complementary to the more abundant factor.

Final and intermediate goods

There is a unique final good, which is produced in perfect competition by combining the two intermediate goods:

$$Y = \Big((Y^l)^\rho + \gamma (Y^h)^\rho\Big)^{\frac{1}{\rho}},$$

where Y_l and Y^h is the intermediate good produced by the low- and high-skilled workers respectively. The elasticity of substitution between the two intermediate goods is $1/(1-\rho)$, where $\rho \leq 1$. In perfect competition the relative price of the two intermediates is:

$$p \equiv \frac{p^h}{p^l} = \gamma \left(\frac{Y^l}{Y^h}\right)^{1-\rho}. \tag{2.1}$$

I normalise the price of the final good to one, hence the price of intermediate goods is:

$$p^l = \left(1 + \gamma p^{\frac{\rho}{\rho-1}}\right)^{\frac{1-\rho}{\rho}}, \tag{2.2}$$

$$p^h = \left(p^{\frac{\rho}{1-\rho}} + \gamma\right)^{\frac{1-\rho}{\rho}}. \tag{2.3}$$

In both sectors intermediate good production is perfectly competitive. To simplify notation I allow a representative firm:

$$Y^s = A^s(N^s)^\beta \quad \text{for} \quad s = \{l, h\}, \tag{2.4}$$

where $\beta \in (0,1)$, N^s is the amount of labour employed and A^s is the level of technology in sector s. Each machine is sector specific in the sense that exclusively high- or low-skilled workers can operate it respectively. Firms decide the quantity, $x^{s,j}$ of the machine to use given the supply of labour, N^s, and the quality of a machine, $q^{s,j}$. Sector s productivity is given by:

$$A^s = \frac{1}{1-\beta}\int_0^1 q^{s,j}(x^{s,j})^{1-\beta}dj \quad \text{for} \quad s \in \{l, h\}.$$

Industry demand for machine line j of quality $q^{s,j}$ and price $\chi^{s,j}$ by the perfectly competitive intermediate good production is:

$$X^{s,j} = \left(\frac{p^s q^{s,j}}{\chi^{s,j}}\right)^{\frac{1}{\beta}} N^s \quad \text{for} \quad s = \{l, h\} \quad \text{and} \quad j \in [0,1]. \tag{2.5}$$

R&D firms

Investment in R&D stochastically produces innovations. Innovations improve the quality of an existing line of machine by a fixed factor, $\overline{q} > 1$. Innovations follow a Poisson process, with an arrival rate for firm k that invested z_k^j on line j is ηz_k^j. If total investments on line j is $\overline{z}^j \equiv \sum_k z_k^j$, the economy wide arrival rate of innovations in line j is $\eta\overline{z}^j$. The probability of an innovation in line j in one period is $(1 - e^{-\eta\overline{z}^j})$. The probability that the innovation is performed by firm k is $(1 - e^{-\eta\overline{z}^j})z_k^j/\overline{z}^j$. Investing z_k^j units in R&D costs Bqz_k^j in terms of final good, therefore a lower B implies less expensive innovation. There are two important things to note: one is that the probability of success is increasing and concave in total investment, $\bar{z}^j$, the other is that the cost of investment is increasing in the quality of the machine line. Due to the first feature there exists an interior solution, while due to the second one a steady state exists. There is free entry into the R&D sector.

Successful R&D firms become the monopolist owners of the machine they patented. As in Chapter 1, if quality improvements are sufficiently large, then in equilibrium only the best quality of any machine is sold at its monopoly price. I assume that this condition applies, hence the price of the leading vintage in line j and sector s with quality q is:

$$\chi^{s,j} = \frac{q}{1-\beta} \quad \text{for} \quad s = \{l, h\} \quad \text{and} \quad j \in [0,1].$$

The per period profit of the owner of the leading vintage using monopoly pricing and industry demand (2.5) can be expressed as:

$$\pi^{s,j} = q^{s,j}\beta(1-\beta)^{\frac{1-\beta}{\beta}}(p^s)^{\frac{1}{\beta}}N^s \quad \text{for} \quad s = \{l, h\} \quad \text{and} \quad j \in [0,1]. \tag{2.6}$$

The profit in each period is increasing in the price of the intermediate good, p^s, in the quality of the machine, $q^{s,j}$, and in the amount of labour that can use the machine, N^s. The value of owning the leading vintage is the expected discounted value of all future profits, and can be expressed recursively as:

$$V_t^{j,s}(q) = \pi_t^{j,s}(q) + \frac{1}{1+r}(e^{-\eta\bar{z}_t^{j,s}(q)})V_{t+1}^{j,s}(q) \quad \text{for} \quad s = \{l, h\} \quad \text{and} \quad j \in [0,1]. \tag{2.7}$$

Where $\bar{z}_t^{j,s}(q)$ is the total R&D spending on line j in sector s of current quality q at time t, and $\frac{1}{1+r}V_{t+1}^{j,s}(q)$ is the present value of owning the leading vintage of quality q in line j and sector s in period $t+1$. The probability that quality q remains the leading vintage in line j in period $t+1$ is $e^{-\eta\bar{z}_t^{j,s}(q)}$.

Due to free entry into the R&D sector all profit opportunities are exhausted. Therefore for each firm the expected return from R&D investment has to equal its cost:

$$\frac{E_t(V_{t+1}^{s,j}(q_t^{s,j}))}{1+r}(1 - e^{-\eta\bar{z}_t^{j,s}(q_t^{s,j})})\frac{z_k^{j,s}}{\bar{z}_t^{j,s}(q_t^{s,j})} = Bq_t^{s,j}z_k^{j,s} \quad \text{for} \quad s = \{l, h\} \quad \text{and} \quad j \in [0,1]. \tag{2.8}$$

In equilibrium, only the total amount of R&D spending targeted at improving line j in sector s is determined, since both the expected return and the costs are proportional to the R&D investment of firm k.

Technology and prices

The equilibrium production of intermediate goods given monopoly pricing is:

$$Y_t^s = (1-\beta)^{\frac{1-2\beta}{\beta}} (p_t^s)^{\frac{1-\beta}{\beta}} N_t^s Q_t^s \quad \text{for} \quad s = \{l, h\}, \tag{2.9}$$

where $Q_t^s = \int_0^1 q_t^{j,s} dj$ is the average quality of the leading vintages in sector s. This evolves according to the R&D investments targeted at improving the machine in sector s:

$$Q_{t+1}^s = \int_0^1 q_t^{j,s} \left((1 - e^{-\eta \bar{z}_t^{j,s}(q_t^{j,s})})\bar{q} + \left(e^{-\eta \bar{z}_t^{j,s}(q_t^{j,s})} \right) \right) dj \quad \text{for} \quad s = \{l, h\}. \tag{2.10}$$

The average quality in sector s grows at rate:

$$g_{t+1}^s = \frac{Q_{t+1}^s}{Q_t^s} \quad \text{for} \quad s = \{l, h\}.$$

I denote the relative average quality or relative technology by $Q_t \equiv \frac{Q_t^h}{Q_t^l}$. This evolves according to:

$$Q_{t+1} = \frac{g_{t+1}^h}{g_{t+1}^l} \frac{Q_t^h}{Q_t^l} = \frac{g_{t+1}^h}{g_{t+1}^l} Q_t. \tag{2.11}$$

The relative prices of intermediates can be expressed by combining (2.9) with (2.1):

$$p_t = \gamma^{\frac{\beta}{(1-(1-\beta)\rho)}} \left(\frac{Q_t^h}{Q_t^l} \frac{N_t^h}{N_t^l} \right)^{-\frac{(1-\rho)\beta}{(1-(1-\beta)\rho)}}. \tag{2.12}$$

The relative price is decreasing in the relative supply of high-skilled labour and in the relative quality of the machines used by high-skilled workers. This is because if the relative share of the high-skilled or the relative quality of the machines that complement them increases, then the production of the high-skilled sector increases compared to the production of the low-skilled sector. This leads to a fall in the relative price.

2.2.3 Labour supply

Individuals are heterogeneous in their cost of acquiring education, c. The total cost of acquiring education is cw_t^h, where w_t^h is the high-skilled wage in period t, and c is the idiosyncratic cost drawn from the time invariant distribution of education costs, $f(c)$. The crucial part of the assumption is that the cost is proportional to one of the wage rates in the economy, without this assumption the economy would not have a steady state.[3] This assumption is reasonable: the cost of education is partly a time cost, thereby involving foregone earnings, moreover the tuition fees and other expenses incurred while studying are likely to depend on the wage rates in the economy as well.

The demographics follow the perpetual youth model: every period a new generation of mass $1-\lambda$ is born, while the probability of surviving from period t to $t+1$ is λ. Hence both the size of the population and the distribution of costs are constant over time.

In the first period of his life each individual decides whether to acquire education, those born in period t can enrol to study in and only in period t. Acquiring education involves a cost cw_t^h, where c is idiosyncratic, determined at birth and the total cost is paid upon enrollment into education. Individuals who complete education become high-skilled, work in the high-skilled sector and earn wage w_t^h in period t. Those who choose not to acquire education, work as low-skilled for wage w_t^l in period t.

Monopoly pricing and the industry demand for machines implies a wage:

$$w_t^s = \beta(1-\beta)^{\frac{1-2\beta}{\beta}}(p_t^s)^{\frac{1}{\beta}}Q_t^s \quad \text{for} \quad s = \{l,h\}. \tag{2.13}$$

The wage in sector s is increasing in the price of intermediate good s and the average quality in sector s. Individuals choose their education level to maximise the present value of their expected lifetime utility:

$$\max_{e(c)_t} E_t \sum_{j=0}^{\infty} \left(\frac{\lambda}{1+r}\right)^j u_{t+j},$$

where u_{t+j} is their consumption of the final good, λ is the probability of staying alive until

[3]If the steady state features growth, wages grow, hence if the costs of education remain the same, more and more people would have an incentive to acquire education.

the next period, r is the discount rate, which is also the interest rate due to linear utility. Since utility is linear, lifetime utility is increasing in lifetime earnings. Therefore individuals make educational decisions to maximise the expected present value of lifetime income.

Let $\mathbf{W}_t^h(c)$ denote the expected present value of lifetime income of an individual with cost c born in period t if high-skilled, and $\mathbf{W}_t^l(c)$ denote the same if low-skilled. The optimal decision is:

$$e(c)_t = \begin{cases} 1 & \text{if} \quad \mathbf{W}_t^h(c) \geq \mathbf{W}_t^l(c), \\ 0 & \text{if} \quad \mathbf{W}_t^h(c) < \mathbf{W}_t^l(c), \end{cases} \tag{2.14}$$

where $e(c)_t = 1$ if the individual acquires education and $e(c)_t = 0$ otherwise.

The lifetime earnings of an educated individual can be expressed as:

$$\mathbf{W}_t^h(c) = \sum_{s=0}^{\infty} \left(\frac{\lambda}{1+r} \right)^s w_{t+s}^h - w_t^h c. \tag{2.15}$$

The lifetime earnings of a high-skilled individual are decreasing in his cost of acquiring education c.

Whereas the lifetime earnings of a low-skilled individual are unaffected by the costs of education:

$$\mathbf{W}_t^l(c) = \sum_{s=0}^{\infty} \left(\frac{\lambda}{1+r} \right)^s w_{t+s}^l. \tag{2.16}$$

Education is worth the investment for an individual with cost c if $W_t^h(c) > W_t^l(c)$. This requires that the wage for high-skilled is higher than for low-skilled workers. Hence the following remark,

Remark 2.1. *To have high-skilled individuals in a generation born in period t, there has to be at least one period $s \geq t$, such that the wage is higher for the high-skilled than for the low-skilled: $w_s^l < w_s^h$.*

The only reason for acquiring education is the *skill premium*, a higher wage in the high- than in the low-skilled sector. Using the relative price of intermediates, (2.12) and the wage, (2.13), the skill premium can be expressed as:

$$\frac{w_t^h}{w_t^l} = \gamma^{\frac{1}{1-(1-\beta)\rho}} \left(\frac{Q_t^h}{Q_t^l} \right)^{1-\frac{1-\rho}{1-(1-\beta)\rho}} \left(\frac{N_t^h}{N_t^l} \right)^{-\frac{1-\rho}{1-(1-\beta)\rho}}. \tag{2.17}$$

Education increases workers' wages potentially through three channels: γ, Q_t^h/Q_t^l and N_t^h/N_t^l. The first two increases the skill premium, as they imply either a higher contribution of high-skilled intermediates to the final good (γ), or better quality machines in the high-skilled sector (Q_t^h/Q_t^l). The last term decreases the skill premium, as there are decreasing returns in production.

The labour supply aggregates N_t^h and N_t^l are the total amount of high- and low-skilled labour available in period t:

$$N_t^h = (1-\lambda)\sum_{j=0}^{\infty}\lambda^j \int_c f(c)e(c)_{(t-j)}dc, \tag{2.18}$$

$$N_t^l = (1-\lambda)\sum_{j=0}^{\infty}\lambda^j \int_c f(c)(1-e(c)_{(t-j)})dc = 1 - N_t^h. \tag{2.19}$$

2.3 Equilibrium

All firms maximise profits and all individuals maximise their lifetime utility at all times, that is the economy is in a decentralised equilibrium.

Definition 2. *A **decentralised equilibrium** is a sequence of optimal education decisions* $\{e(c)_t\}_{t=0}^{\infty}$*, labour supplies* $\{N_t^h, N_t^l\}_{t=0}^{\infty}$*, discounted present values of expected lifetime income* $\{\mathbf{W}_t^h, \mathbf{W}_t^l\}_{t=0}^{\infty}$*, intermediate good prices* $\{p_t^h, p_t^l\}_{t=0}^{\infty}$*, average qualities* $\{Q_t^h, Q_t^l\}_{t=0}^{\infty}$*, investments into R&D* $\{\overline{z}_t^{j,h}, \overline{z}_t^{j,l}\}_{t=0}^{\infty}$ *and values of owning the leading vintage* $\{V_t^{j,h}, V_t^{j,l}\}_{t=0}^{\infty}$ *for all lines* $j \in [0,1]$*, where* $\{Q_0^h, Q_0^l, N_0^h, N_0^l\}$*, such that the following conditions are satisfied:*

1. *the labour supplies satisfy* (2.18) *and* (2.19)*;*
2. *lifetime earnings are as in* (2.15) *and* (2.16)*;*
3. *the average quality in sector* s *evolves according to* (2.10)*;*
4. *total R&D investment* $\overline{z}_t^{j,s}$ *satisfies* (2.8) *for all* $t \geq 0$ *and all* $j \in [0,1]$*;*
5. *the sequence* $\{V_t^{j,s}\}_{t=0}^{\infty}$ *satisfies* (2.7)*;*
6. *the price sequence* $\{p_t^h, p_t^l\}_{t=0}^{\infty}$ *satisfies* (2.2) *and the relative price,* p_t *satisfies* (2.12)*;*

7. the optimal education decisions, $\{e(c)_t\}_{t=0}^{\infty}$ *are as in* (2.14).

2.3.1 Steady state

In this section I identify the steady states or balanced growth paths (BGP) of this economy, which are decentralised equilibria, where all variables are constant or grow at a constant rate. The solution of the steady state follows that in Chapter 1, here I present a more informal discussion.

In the steady state the total R&D spending on all lines within a sector are equal, $\overline{z}^{j,s*} = \overline{z}^{s*}$ for $j \in [0,1]$ and $\overline{z}^{s*}$ is given by:

$$\beta(1-\beta)^{\frac{1-\beta}{\beta}}(p^{s*})^{\frac{1}{\beta}}N^{s*} = B\overline{z}^{s*}\frac{(1+r-e^{-\eta\overline{z}^{s*}})}{1-e^{-\eta\overline{z}^{s*}}} \quad \text{for} \quad s=\{l,h\}. \tag{2.20}$$

Hence R&D effort in a sector is increasing in the period profit from machine sales. As discussed earlier, these profits are increasing in the price of the intermediate, p^{s*}, and in the amount of labour, N^{s*}, which uses this technology.

Relative quality, Q^*, has to be constant along the BGP, which requires equal R&D spending in the two sectors: $\overline{z}^{h*} = \overline{z}^{l*} = \overline{z}^*$. From (2.20) this holds if:

$$p^* = \frac{p^{h*}}{p^{l*}} = \left(\frac{N^{h*}}{N^{l*}}\right)^{-\beta}. \tag{2.21}$$

The relative quality in the steady state can be expressed by combining the relative price (2.1),(2.21) with the intermediate output (2.9):

$$Q^* = \frac{Q^{h*}}{Q^{l*}} = \gamma^{\frac{1}{1-\rho}}\left(\frac{N^{h*}}{N^{l*}}\right)^{\frac{\beta\rho}{1-\rho}}. \tag{2.22}$$

Since the skill premium depends on the relative quality and the relative price, the above two equations are key in understanding the dynamics of the skill premium. These ratios both depend on the relative supply of skills, therefore their interaction determines the response of the skill premium to relative skill supply.

From (2.22) the relative quality level depends on the relative abundance of the two types of labour along the balanced growth path. If there are more high-skilled workers, an innovation in the high-skilled sector is more profitable. Hence technology is more skill-biased – Q^* is greater, – if the relative supply of skills is higher.

The relative price of intermediate depends negatively on the relative supply of high-skilled workers (from (2.21)). Intuitively, more high-skilled workers and better technologies leads to more high-skilled intermediate production, which lowers the relative price of the intermediate. Moreover, since more R&D is directed towards the larger sector (from (2.22)), more high-skilled workers implies a higher relative quality, Q^*.

Along the steady state, technological *change* is not biased towards either sector, since both sectors are growing at the same rate, implying that the skill-bias of technology is constant.

The skill premium using (2.17), (2.22) and (2.21) can be expressed as:

$$\frac{w_t^{h*}}{w_t^{l*}} = \left(\frac{p^{h*}}{p^{l*}}\right)^{\frac{1}{\beta}} \frac{Q_t^{h*}}{Q_t^{l*}} = \gamma^{\frac{1}{1-\rho}} \left(\frac{N^{h*}}{N^{l*}}\right)^{\frac{\beta\rho}{1-\rho}-1} . \tag{2.23}$$

The skill premium depends on two components: the relative price and the relative quality. Since the relative price depends negatively, while the relative quality depends positively on the relative supply of skilled workers, the net effect depends on which influences the wages more.

If the two intermediates are highly substitutable (ρ is higher), the price effect is smaller and is dominated by the effect of relative quality. On the other hand, if the elasticity of substitution is low (low ρ), the price effect is stronger than the quality effect in the steady state.

For sufficiently high ρs (if $(\beta\rho)/(1-\rho) - 1 > 0$) the skill premium is an increasing function of the relative supply of skills. In this case, the increase in relative quality more than compensates for the decrease in relative price. This is what Acemoglu (1998) termed as *strong relative bias* in technology, as increase in the relative supply of skills increases the skill premium. On the other hand if $(\beta\rho)/(1-\rho) - 1 < 0$ then the skill premium is decreasing in the relative supply, and technology displays *weak relative bias*: the technology effect does not compensate for the price effect.

The skill premium is constant in the steady state (from (2.23)), and from Remark 2.1 the skill premium has to be greater than one in at least one period. This implies that

$w_t^{h*} > w_t^{l*}$ for all $t \geq 0$.

Result 2.1. *Every individual born in period t acquires education if his cost $c < c^*$, where c^* is the* cutoff cost *implicitly defined by:*

$$c^* = \frac{1 - \frac{w^{l*}}{w^{h*}}}{1 - \frac{g^* \lambda}{1+r}}. \tag{2.24}$$

Proof. Combining (2.14) with (2.15) and (2.16), implies that the condition for acquiring education is:

$$\sum_{s=0}^{\infty} \left(\frac{\lambda}{1+r}\right)^s w_{t+s}^{h*} - \sum_{s=0}^{\infty} \left(\frac{\lambda}{1+r}\right)^s w_{t+s}^{l*} \geq w_t^{h*} c.$$

This shows that the optimal education decision is equivalent to a threshold time cost, c_t^*. Using the fact that wages in both sectors grow at a constant rate g^*, and that the skill premium, w_t^{h*}/w_t^{l*} is constant, $c_t^* = c^*$ is constant and given by (2.24). $\square$

The supply of high-skilled workers using the previous result and (2.18) can be expressed as:

$$N^{h*} = F(c^*) = F\left(\frac{1 - \frac{w^{l*}}{w^{h*}}}{1 - \frac{g^* \lambda}{1+r}}\right), \tag{2.25}$$

where $F(\cdot)$ is the cumulative distribution function of the cost of acquiring education. The threshold cost for acquiring education and consequently the fraction of high-skilled workers depends positively on the skill premium and on the growth rate of the average qualities. The threshold is increasing in the skill premium, since a higher skill premium implies a greater per period gain from working as high-skilled. The growth rate of wages also increases the threshold cost; if wages grow at a higher rate, then for a given skill premium, future gains are greater.

The growth rate of the economy depends on the amount of R&D spending, z^*, which can be expressed as (using (2.2) and (2.21)):

$$B\overline{z}^* \frac{(1 + r - e^{-\eta\overline{z}^*})}{1 - e^{-\eta\overline{z}^*}} = \beta(1-\beta)^{\frac{1-\beta}{\beta}} \left(\gamma {N^{h*}}^{\frac{\beta\rho}{1-\rho}} + {N^{l*}}^{\frac{\beta\rho}{1-\rho}}\right)^{\frac{1-\rho}{\beta\rho}}. \tag{2.26}$$

The right hand side is the steady state per period profit from owning the leading vintage normalised by the quality of the vintage. The profit is increasing in both N^{h*} and N^{l*}. If the labour supply increases, then any unit of investment into R&D has a higher expected return, since there are more people who are able to use it. The left hand side is increasing in z^*.[4] This implies that the steady state R&D spending, and hence the steady state growth rate is increasing in the labour supplies. The growth rate of the economy is given by:

$$g^* = 1 + (\overline{q} - 1)(1 - e^{-\eta \overline{z}^*}). \tag{2.27}$$

This completes the identification of the steady state. The cutoff cost for acquiring education determines N^{h*}. In turn, the supply of high-skilled labour, N^{h*}, determines every other variable in the economy in the steady state. From (2.26) N^{h*} determines the optimal investment into R&D, z^*. This pins down the growth rate, g^*, through (2.27). The supply of high-skilled workers also determines the skill premium, w^{h*}/w^{l*}, through (2.23). On the other hand, these variables (w^{h*}/w^{l*} and g^*) pin down the steady state cutoff cost for acquiring education, c^*, which pins down the level of N^{h*} through (2.25). The possible steady state high-skilled labour supplies of the economy are thus the fixed points of the function $F(h(\cdot))$, and the steady state of the economy is fully characterised by the supply of high-skilled labour, N^{h*}:

$$N^{h*} = F(h(N^{h*})), \quad \text{where} \tag{2.28}$$

$$h(x) \equiv \frac{1 - \frac{w^{l*}}{w^{h*}}(x)}{1 - \frac{g^*(x)\lambda}{1+r}}. \tag{2.29}$$

The function $h : (0, 1) \to \mathbb{R}$ is defined as the optimal cutoff value c^* for a given supply of high-skilled workers N^h, where the skill premium is given by (2.23), and the growth rate is given by (2.27). The steady state of the economy is the fixed point of function $F(h(N^h))$,

[4] To see this, take the derivative:

$$\frac{\partial z^* \left(1 + \frac{r}{1-e^{-\eta z^*}}\right)}{\partial z^*} = 1 + \frac{z^*}{1 - e^{-\eta z^*}} \left(1 - \frac{r\eta z^* e^{-\eta z^*}}{1 - e^{-\eta z^*}}\right).$$

A sufficient condition for this derivative to be positive is $1 - \frac{r\eta z^* e^{-\eta z^*}}{1-e^{-\eta z^*}} \geq 0$. This can be rearranged to the following inequality:

$$1 \geq e^{-\eta z^*}(1 + r\eta z^*).$$

For $z^* = 0$ this holds with equality, while the right hand side is decreasing in z^*. QED as shown in Figure 2.1.

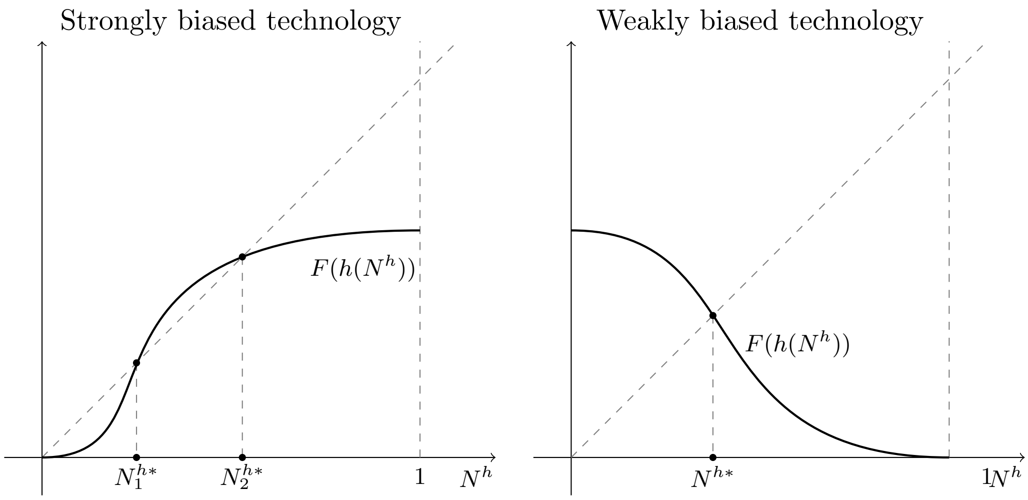

Figure 2.1: Steady states

The panel on the left shows the case of a strongly biased technology, while the panel on the right shows a weakly biased technology. Whether $F(h(N^h))$ is increasing or decreasing in N^h depends on whether $h(N^h)$ increases or decreases in N^h. The optimal c^* depends on N^h through the growth rate, g, and through the skill premium, w^h/w^l. Hence the sign of $h'(N^h)$ depends on the net effect from these two channels.

The effect of N^h on the growth rate depends on the elasticity of substitution between the two intermediate goods. If the elasticity of substitution is not too high ($\rho < 1/(1+\beta)$), then the growth rate is increasing until it reaches its maximum at $N^h = 1/(1+\gamma^{\frac{1-\rho}{\beta\rho-(1-\rho)}})$, and then it decreases as N^h increases further. The intuition for this result is that when the elasticity is low, then similar amounts are required from the two goods, and hence it is not good to specialise in neither high- nor low-skilled intermediates. On the other hand, if the elasticity of substitution is higher ($\rho > 1/(1+\beta)$), then the two goods can be easily substituted, and the best is to specialise in either high- or low-skilled intermediate production. In this case the growth rate is decreasing until $N^h = 1/(1+\gamma^{\frac{1-\rho}{\beta\rho-(1-\rho)}})$, where the growth rate is the lowest, and then starts increasing as N^h increases further.

The elasticity of substitution also determines the effect of N^h on the skill premium. Recall that, with an increase in the (relative) supply of high-skilled labour the steady state relative quality increases (from equation (2.22)), since when a larger labour force works in a sector, the demand for machines in that sector, and hence profits on machines increases. However, parallel to the increase in the relative quality, the relative price of the intermediate good produced by the high-skilled workers decreases, as the supply of high-skilled intermediates increases. The effect of an increase in N^h on the skill premium depends on the strength of these two responses. As discussed earlier, when the two intermediates are easily substitutable, $\rho > 1/(1+\beta)$, then the effect of the relative quality dominates, and the technology is strongly biased. In this case the skill premium is increasing in N^h. On the other hand, when the two intermediates cannot be substituted that easily, $\rho < 1/(1+\beta)$, then the relative price effect dominates, and technology is weakly biased. The skill premium decreases with N^h in such cases.

For most parameter values, however, the effect of N^h on the growth rate is relatively small, and is dominated by the effect of N^h on the skill premium. This implies that when technology is strongly biased, the skill premium is increasing in the supply of high-skilled workers, and $h'(N^h) > 0$, and hence the $F(h(N^h))$ curve is upward sloping. Conversely, when technology is weakly biased, the skill premium is decreasing in the supply of high-skilled workers, then $h'(N^h) < 0$ and the $F(h(N^h))$ curve is downward sloping.

In the case of a weakly biased technology there is maximum one steady state, depicted in the right panel of Figure 2.1 by N^{h*}. The graph suggests that this steady state is stable, as for high-skilled labour supplies lower than N^{h*}, a higher fraction of the new cohort would acquire skills than N^{h*}, and the converse is true for high-skilled labour supplies higher than N^{h*}. However, the conditions that govern $F(h(N^h))$ only hold in the steady state, so to fully ascertain the stability of the steady state, an analysis of the transitional dynamics is required.

In the case of a strongly biased technology multiple steady states are possible, as depicted in the left panel of Figure 2.1 by N_1^{h*} and N_2^{h*}. The graph suggests, that steady states where $F(h(N^h))$ crosses the 45 degree line from below are unstable (like N_1^{h*}), whereas the steady states where it crosses it from above are stable (like N_2^{h*}).

2.4 Comparative dynamics

In this section I analyse the characteristics of the transition path. I look at two types of transitions. In the first section, I do not assume that the economy is in a steady state, but I analyse the transition path from different initial points to the steady state. In the second, I assume that the economy is in the steady state, introduce a change in of the parameters, and follow the transition path to the new steady state. In both cases, throughout the transition the economy is in a decentralised equilibrium, and the transitional dynamics are governed by the initial value of the state variables and the final steady state. I calculate the transition paths using second order approximation of the decentralised equilibrium around the final steady state.[5]

2.4.1 Initial values

Available data shows that the supply of high-skilled workers and the educational attainment of consecutive cohorts has been steadily increasing over time, however, the growth rate of educational attainment has significantly slowed down over the last few years.[6] This evidence suggests that the developed economies have been in a transition towards their steady state. Therefore in this section I do not formulate a hypothesis about a change in steady states. Instead I analyse the dependence of the transition path on the initial values of the economy.

This analysis shows, that an increasing skill supply, increasing relative quality and increasing skill premium can arise during the transition to the steady state regardless of whether the technology is strongly or weakly biased.

I consider two baseline set of parameter values for the steady state, one that features weakly biased technology, and one that features strongly biased technology.[7] I choose the sets of baseline parameters to provide reasonable steady state values: a final skill supply of 45 percent, a final skill premium of around 40 percent, and an annual growth rate of around 2 percent.[8] I analyse the transition path to the steady state from all possible initial

[5]See Section B.1 of the Appendix for the equations that have to hold during the transition.
[6]See for example OECD Factbook 2005-2010, various editions of OECD Education at a Glance.
[7]I take β, λ and r to be the same as in the previous chapter. The parameter-values are:

	ρ	γ	η	$\overline{q}$	B	μ	σ	β	λ	r
weak bias	0.5	1.15	0.04	2.08	0.3	1	6	2/3	8/9	1.05^5
strong bias	0.7	1.15	0.04	2.08	0.16	1	6	2/3	8/9	1.05^5

[8]Of course, *reasonable* is not an easily judgeable concept here, since I assume that none of the advanced

skill supply and relative quality pairs. This exercise shows that the steady state can be reached with increasing skill supply and *increasing* skill premium in case of both weakly and strongly biased technologies.

This is an important result, as it implies that observing increasing relative supplies and increasing relative wages can be the result of the economy's normal transition process while building up human capital. On the one hand, the relative quality depends positively on the supply of skills. On the other, the skill premium, which determines the change in the skill supply, depends positively on the relative quality. Therefore, it is not surprising that during the transition the skill supply, the relative quality, and potentially the skill premium increase together. If the economy is not in the steady state, the explanation of this phenomenon does not require exogenously skill biased technological progress nor a strong, endogenous bias in technology. Only two conditions are necessary for this to happen. First, the relative quality has to increase as a response to an increase in the relative supply. If R&D is modeled as a profit driven activity, and profits are increasing in the demand, then this is a natural result. The second necessary condition is that the quality in the high-skilled sector is not too high compared to the quality in the low-skilled sector. If the initial relative quality is also the result of some form of optimization, and the supply of high-skilled workers is low, then again, this is a natural result of a profit maximizing R&D sector.

Based on numerical solutions to the transition path Figure 2.2 shows how the type of transition depends on the initial values.[9] The AOB curve shows the border where the direction of change in the relative quality, Q, changes. Below the curve, relative quality is increasing, whereas above the curve, relative quality is decreasing. The left panel in Figure 2.2 shows a strongly biased technology, while the right panel shows a weakly biased technology. The AOB curve is much steeper in the left panel, i.e. for higher ρs, indicating that for low values of N^h, a lower relative quality is desirable if ρ is higher, and the converse is true for above steady state values of N^h. If the elasticity of substitution is higher, relative prices are less sensitive to the relative output of the two sectors (see (2.1)), which translates into less sensitivity of monopolist profits. Hence, with higher ρ it is less worthwhile to invest into the high-skilled sector if N^h is low, while it is more worthwhile if N^h is high.

economies are in their steady state yet.

[9]See Section B.2 of the Appendix for Matlab graphs.

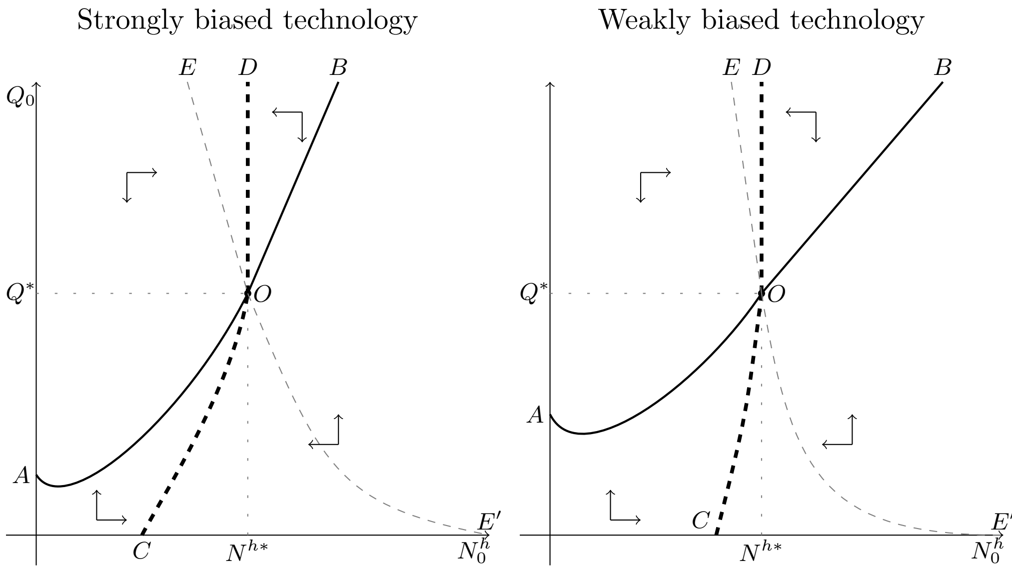

Figure 2.2: Phase diagram

Therefore, since the steady state is almost the same in the two cases depicted on Figure 2.2, and the AOB curve is bound to be less steep for lower ρs, the relative quality decreases for more initial values below N^h and for fewer initial values above N^h.

The dashed curve COD shows the border where the direction of change in the skill supply, N^h, changes. To the left of the curve, N^h is increasing, while to the right, it is decreasing. Comparing curves COD, which determine the movement of N^h, the implication is that for a given value of Q a lower supply of high-skilled labour is desirable. This can be understood from (2.17), which shows that for a higher ρ, the skill premium is more sensitive to the relative quality than to the relative supply of skills. Therefore for a higher ρ with the same Q a lower N^h is necessary.

The steady state, denoted by O, is at the intersection of curves AOB and COD, where neither Q nor N^h changes. Numerical solutions show that there are only two ways to reach the steady state, O: either from above right, where both Q and N^h are decreasing, or from below left, where both Q and N^h are increasing. From the left side of the EOE' curve, the economy transitions to the steady state from below, whereas from the right of this curve the economy transitions from above.

If the economy starts from the area bounded by AOE, then while the economy stays in this area the relative quality continuously decreases, while the supply of high-skilled workers increases. This is due to the fact that the initial relative quality is too high compared to the relative supply of skilled workers, therefore it is more worthwhile to invest in improving the quality in the unskilled sector, leading to a decline in the relative quality. On the other hand, the skill premium is quite high, therefore the new cohorts keep acquiring more education than previous ones, and the supply of skills increases.

If the economy starts from the COE' area, then the supply of skills decreases, while the relative quality increases as long as the economy stays in this region. Here the relative supply of skills is too high compared to the relative quality, therefore the new cohorts acquire less education than previous ones, and the supply of skills decreases. Meanwhile, since the supply of skills is high, the R&D sector focuses investment into the skilled sector, and the relative quality continuously increases.

From both of these regions, the economy eventually moves into the AOC area. Here, the relative quality is neither too high, nor too low compared to the supply of skills, and hence both the supply of skills and the relative quality increase together to the steady state, O.

In the $E'OB$ area the supply of skills is too high compared to the relative quality. Therefore, the new cohorts acquire less education, and the skill supply decreases, while the R&D sector invests into the high-skilled sector, and the relative quality increases.

If the economy starts in the EOD region, then the relative quality is too high compared to the supply of high-skilled workers. Therefore, the relative quality decreases, as there will be more investment into the unskilled sector, while the supply of skills increases, as the skill premium is high, and new cohorts acquire more education.

From both the $E'OB$ and EOD area the economy moves into the DOB region, where the relative quality is neither too high nor too low compared to the supply of skills, and both decrease together to the steady state, O.

This shows, that if the supply of high-skilled workers approaches its steady state from below, then the transition path can only feature decreasing relative quality at the beginning of the transition, but as the economy gets closer to the steady state, then eventually the relative quality increases. Therefore, technological change can only be *unskill-biased* at the beginning of the transition, and it is necessarily *skill-biased* while approaching the steady state. On the other hand, if the supply of skills reaches its steady state value from above, then the relative quality decreases for most of the transition, apart from some of the initial periods. Thus technological change is *unskill-biased* for most of the transition. Therefore the skill supply and the relative quality tend to move together during the transition. This is due to the positive dependence of these two variables on each other. If the relative quality increases in the future, the skill premium increases as well, which leads to an increase in the skill supply. If the skill supply increases in the future, then there are more gains to be made from investing in high-skilled machines, and hence relative quality increases as well. Therefore the joint increase of the skill supply and the relative quality should not be surprising. However, this does not automatically imply that the skill supply and the skill premium should move hand-in-hand as well.

There are two aspects of the skill premium that are of interest: its change in the short run, and its change in the long run. The long run change in the skill premium is the change between the initial skill premium and the final, steady state skill premium. From (2.23) the skill premium increases in the long run if:

$$Q_0 \leq \left(\frac{N_0^h}{N^{h*}}\right)^{\frac{1-\rho}{\beta\rho}} \left(\frac{1-N^{h*}}{1-N_0^h}\right)^{\frac{1-\rho}{\beta\rho}} Q^* \equiv s(N_0^h).$$

The function $s(N_0^h)$ is depicted in Figure 2.3 by the blue curve, and the above inequality implies that if the initial point is below the blue curve, then the skill premium increases in the long-run. There are two things to note from this inequality. First, that $s(N_0^h)$ is upward sloping. If the initial skill supply is higher, the initial relative quality can be higher, and the skill premium still increases in the long run. The intuition for this is that the relative supply and the relative quality have opposite effects on the skill premium: while the former decreases it, the latter increases it, thus leaving it unchanged. Second, that the blue curve in the strongly biased technology case (left panel) is flatter: for low initial skill supplies it is higher and for high initial skill supplies it is lower. If ρ is higher, then the effect of the relative quality on the skill premium is larger, and the effect of the relative supply is lower. Therefore, a very low skill supply does not imply such a high skill premium if ρ is larger, while a very high skill supply does not imply such a low skill premium. This implies that for higher ρs the $s(N_0^h)$ curve is flatter.

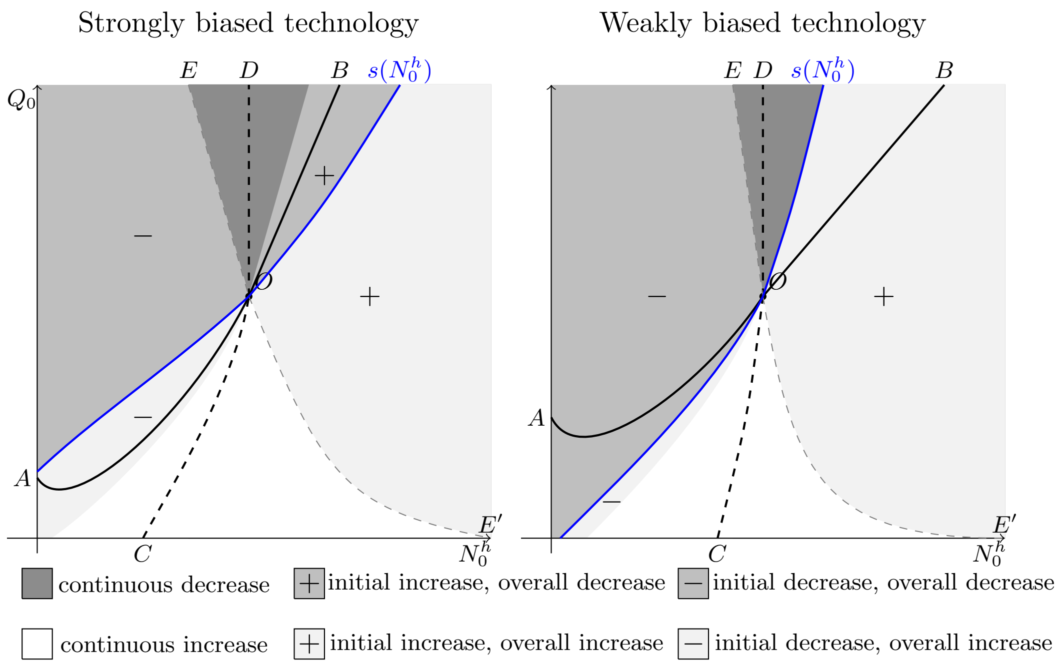

Figure 2.3: The path of the skill premium

In Figure 2.3 the different shades of gray represent the different paths the skill premium can take throughout the transition. The two lighter colours represent the initial points from which the skill premium increases in the long-run, while the two darker grays represent the initial points from the skill premium decreases in the long-run.

The short-run change in the skill premium depends on the magnitude of the change in the relative quality and the relative supply of skilled workers. From (2.17), we get:

$$\frac{\frac{w_t^h}{w_t^l}}{\frac{w_{t-1}^h}{w_{t-1}^l}} = \left(\frac{Q_t}{Q_{t-1}}\right)^{\frac{\beta\rho}{1-(1-\beta)\rho}} \left(\frac{\frac{N_t^h}{N_t^l}}{\frac{N_{t-1}^h}{N_{t-1}^l}}\right)^{-\frac{1-\rho}{1-(1-\beta)\rho}} . \tag{2.30}$$

An increase in the relative skill supply reduces the skill premium, while an increase in the relative quality increases it. The greater the increase in the relative quality compared to the increase in the relative supply, the more likely it is that the skill premium also increases. From this equation it is easy to see that a higher ρ makes the skill premium more responsive to changes in the relative quality and less responsive to changes in the relative skill supply. Intuitively, this is because for more substitutable intermediates, as the price effect is smaller, the effect of the relative quality on the skill premium is stronger than the effect of the relative supply. Hence, when ρ is higher a smaller increase in Q leads to a greater increase in the skill premium.

If the economy is in the DOA area, the relative quality decreases, while the relative supply increases (from Figure 2.2). From (2.30), this leads to an unambiguous decrease in the skill premium. The opposite holds for an economy that is in the BOC area, leading to an unambiguous increase in the skill premium. This is depicted by the $+$ and $-$ signs in Figure 2.3.

In the AOC area both the skill supply and the relative quality is increasing, hence in general, the overall effect on the skill premium is ambiguous. It is clear, that the closer is the economy to the AO curve, the less likely it is that the skill premium increases, as the relative quality hardly changes initially at these points. In the case of strongly biased technologies, as discussed earlier, the skill premium is more responsive to changes in the relative quality, and less responsive to changes in the relative supply. Therefore, for most part of the AOC area the skill premium increases (shown in white), and only for a smaller fraction does it decrease (shown in the lightest gray). The situation is different if the technology is weakly biased. In this case the skill premium only increases for a smaller part of the AOC area (again shown in white), and for the rest, the skill premium decreases.

In the case of strongly biased technologies, as the $s(N_0^h)$ curve is above the AO curve, the skill premium increases in the long-run for all initial values in area AOC. This implies that even if the skill premium decreases initially for economies in the light gray area, the transition takes the economy into the white region, where the skill premium increases continuously, finally increasing above its original value. If the technology is weakly biased, then as the $s(N_0^h)$ curve is below the AO curve, for some values (shown in the darker gray), the skill premium decreases in the long-run, and for only a smaller set of initial points does it increase in the long run after an initial decline (lighter gray area).

If the economy is in the DOB area, the change in the skill premium is ambiguous, as both the skill supply and the relative quality decreases. Again, the closer is the initial point to the OB curve, the more likely it is that the skill premium initially increases. This is due to the fact that close to the OB curve the change in the relative quality is small, and hence the decrease in the relative supply can potentially dominate its effect. In case of strongly biased technologies, from (2.30), the skill premium is more responsive to changes in the relative quality. Therefore, the change in the relative supply can dominate the effect of decreasing technology for a smaller set of initial points (shown in the second darkest gray), and the skill premium decreases for most values (darkest gray). In case of weakly biased technologies the skill premium increases for a larger set from the DOB area (shown in light gray), since the skill premium is more responsive to changes in the supply of high-skilled workers.

Since the $s(N_0^h)$ curve is below the OB curve for strongly biased technologies, the skill premium decreases in the long run for the entire set of initial values in the DOB region, while for a large part of the DOB area in case of weakly biased technologies the skill premium increases in the long-run (shown in the lightest gray).

To summarise, the darkest gray represents the area where the skill premium continuously decreases throughout the entire transition. In this area, the relative quality is much higher than what is profitable given the current supply of skills and the future decreasing path of skills, therefore the relative quality decreases drastically, while the relative supply of skills decreases at a slower rate (or even increases from area EOD) as the skill premium is relatively high. Therefore the skill premium decreases continuously until it reaches its steady state value. For higher ρs this area is wider, as the skill premium responds more to changes in the relative quality.

The white areas contain the initial points from where the skill premium continuously increases throughout the transition. These are points, where the relative quality is low compared to the current supply and the future increasing path of high-skilled workers. If the sub-optimality of the relative quality is sufficiently large, then it increases at such a high rate, that it dominates the slowly increasing supply of skilled workers (or for the initially decreasing supply from area COE'). Therefore from the white area the skill premium continuously increases until reaching the steady state. Again, this area is wider for higher values of ρ, as the skill premium is less sensitive to supply effects.

If the initial point is in one of the medium gray areas and $N_0^h < N^{h*}$, then the skill premium initially decreases (denoted by the $-$ sign in the area), and then increases until the steady state is reached, whereas if $N_0^h > N^{h*}$, then it increases initially (denoted by the $+$ sign in the area), and then decreases to the steady state. This suggests that the stable arm lies in the white area if the initial point is to the left of the EOE' curve, and it lies in the darkest gray area if the initial point is to the right of the EOE' curve. [10] If the initial point is in the second darkest area, then the skill premium decreases compared to its initial value in the long run. The lightest gray area contains the initial points for which the skill premium increases in the long run.

In light of this analysis, the fact that the skill supply and the skill premium have been growing together over the last few decades should not be surprising. The developed economies had to start with a sufficiently low relative quality in the high-skilled sector, and the skill supply and the skill premium had to increase together. Moreover, an unexpected increase in N^h, for example due to bigger cohort sizes or other reasons for enrollment into higher education (for example to avoid the draught), would push the economy towards the right. This would reduce the skill premium immediately, while possibly shifting the economy into the white region. An important implication of the joint analysis of skill supply and relative technologies, is that as ρ decreases, this area shrinks, but it does not disappear. Therefore, the relative bias of technology does not have to be strong in order to observe increasing skill premium and increasing skill supply.

2.4.2 Parameters

In this section I consider the dynamic effects of changes in various parameters of the model.[11] I analyse how the steady state changes and the characteristics of the transition path. It is important to note that the exact path of the transition, as discussed in the previous section, is determined by the region in which the old steady state falls compared to the new steady state in terms of Figure 2.3.

The steady state is affected by parameters in two ways. The distribution of education costs affect the steady state supply of high-skilled workers by changing the function $F(\cdot)$. All other parameters, which are either connected to the production of goods or to the R&D process affect the steady state via changing the function $h(\cdot)$.

The effects of the parameters of the R&D process are the most straightforward to assess. These parameters only affect the steady state through their effect on the growth rate. If a

[10] The final increase or decrease in the skill premium is at times hardly noticeable, implying that for most of the transition the economy stays in one of the medium gray areas.

[11] The final steady state parameters are the same as in the previous section.

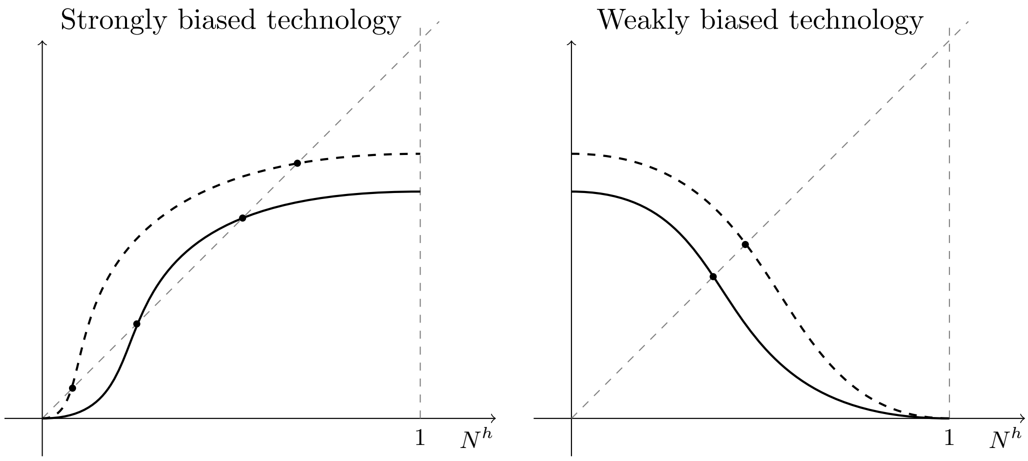

Figure 2.4: Higher $\overline{q}, \eta, \gamma$ or lower B, μ

change in a parameter increases the growth rate of the economy, then from equation (2.1) the steady state gain from working as high-skilled relative to low-skilled increases. This implies an upward shift in the $F(h(N^h))$ curve, as $h(N^h)$ increases for every N^h.

Figure 2.4 demonstrates the effects of an upward shift of $F(h(N^h))$ on the steady states. In case of a weakly biased technology, the steady state N^{h*} unambiguously increases. For strongly biased technologies the situation is more complicated. Steady states where the $F(h(N^h))$ curve crosses the 45 degree line from below shift down, while steady states where the $F(h(N^h))$ curve crosses the 45 degree line from above shift up. However, the *stable* steady state is where $F(h(N^h))$ crosses from above, and in these cases, similarly to the weakly biased technology case, the stable steady state N^{h*} unambiguously increases.

Parameter η controls the effectiveness of R&D spending (through the Poisson arrival rate of innovations), and $\overline{q}$ controls the quality improvement per innovation. An increase in either of these parameters increases the growth rate. Since B is the price of investing one unit into innovation in terms of final good, B increases the cost of the R&D activity, and hence decreases the equilibrium growth rate. Therefore an increase in either η or $\overline{q}$ as well as a decrease in B increases the steady state supply of high-skilled workers unambiguously regardless of whether the technology is strongly or weakly biased. However, from (2.23) in case of a strongly biased technology this unambiguously implies a higher final skill premium, whereas with a weakly biased technology, this implies a lower final skill premium. Figure

2.5 shows the transition paths for a change in the parameter η.[12]

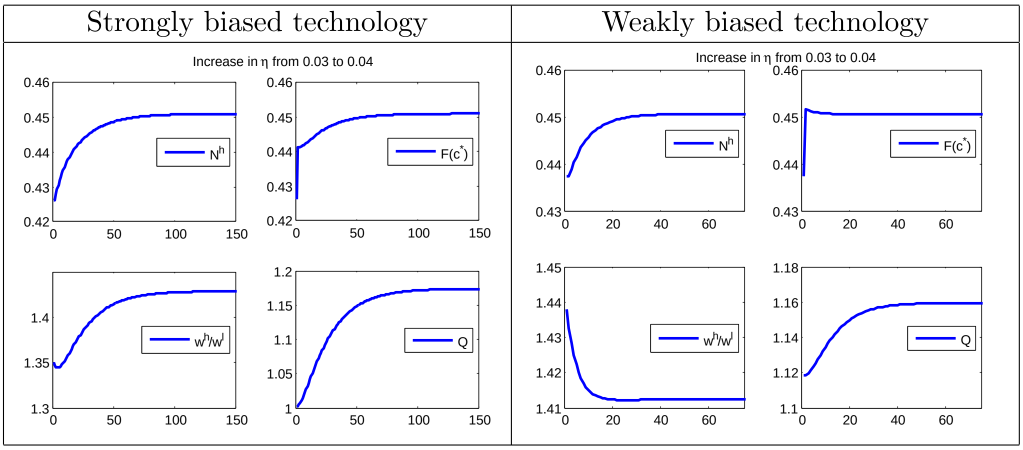

Figure 2.5: Change in the R&D parameters

An increase in η increases the growth rate of the economy immediately, which increases the present value gain from acquiring education. Therefore there is a jump in the educational attainment of new cohorts, as can be seen on the top right panel for both types of technology. There is a difference though in the consequent path of $F(c_t^*)$: in case of a strongly biased technology, it continues to increase, whereas for weakly biased technologies, it declines after its initial increase. This is due to the differential response of the skill premium to the increase in the relative supply and relative quality. The initial response of the skill premium in both cases is a decline, as relative quality does not change, while the skill supply increases. In case of a weakly biased technology, the skill premium continues to decrease, thereby offsetting some of the increase in the present value gain from acquiring education, whereas in case of a strongly biased technology, the skill premium starts to increase, this way further increasing the present value gain from acquiring education. The skill supply and the relative quality continuously increases for both types of technology, although the increase is more pronounced in case of a strongly biased technology.

In terms of Figure 2.2, this implies that the initial steady state was in the *AOC* region compared to the new steady state. In the case of strongly biased technologies, based on the path of the skill premium the initial point fell into the light gray region within *AOC* in

[12]Since the transition paths look very similar in case of an increase in $\overline{q}$ or a decrease in B, I do not include them in the main text, they can be found in the Section B.3 of the Appendix.

Figure 2.3, and the economy almost immediately crossed over to the white region during the transition. In case of a weakly biased technology the skill premium almost continuously falls, there is just a slight increase before reaching the steady state, thus the initial point fell into the dark gray region within AOC in Figure 2.3, and the economy crossed over to the white region just before reaching the steady state.

The next set of parameters I consider are related to the production of the final good, γ and ρ. First consider γ, the weight of the high-skilled intermediate in the production of the final good. Intuitively an increase in this parameter increases the value of the high-skilled intermediate and thus increases the returns to acquiring education as well. This intuition is supported by equation (2.23), which shows that an increase in γ increases the skill premium. At the same time, γ also affects the steady state through its effect on R&D. From equation (2.26), an increase in γ increases the returns to investment into R&D, and hence increases the growth rate.[13] Both of these shift the $h(\cdot)$ function up, and therefore an increase in γ has similar effects as depicted in Figure 2.4.

Figure 2.6 shows the transition from the old steady state to the new one in case of an unexpected increase in γ. An increase in γ immediately increases the skill premium and the growth rate, thereby increasing the present value gain from acquiring education. This leads to an immediate jump in the education acquisition of new cohorts (as can be seen on the top right panels). The skill premium (bottom left panels) continues to increase in both cases, though to a much smaller extent in case of weakly biased technologies. For weakly biased technologies, the increase in the supply of high-skilled workers and in the relative quality reduces the skill premium. However, the increase in γ has a direct positive effect on the skill premium by increasing the weight of the skilled intermediate in the production of the final good (see (2.23)). Therefore in case of weakly biased technologies, the overall effect depends on the magnitude of the two opposing effects. In the example below, the skill premium continues to increase in the weakly biased case, even though to a smaller extent than in the case of strongly biased technologies.[14] In this case both initial steady states fall

[13]Note that an increase in γ increases the returns to R&D in both the high- and the low-skilled sector. This is the case, as γ besides measuring the relative importance of high- and low-skilled intermediates in the production of final good, also measures the absolute contribution of high-skilled intermediates. An increase in γ increases the final output for any combination of inputs, i.e. it makes production more efficient.

[14]The strength of the effect of γ on the skill premium through N^{h*} depends on ρ. The closer is ρ to $1/(1+\beta)$, the more likely it is that the direct effect of γ dominates.

into the white region of AOC and hence during the transition the skill supply, the relative quality and the skill premium all increase together.

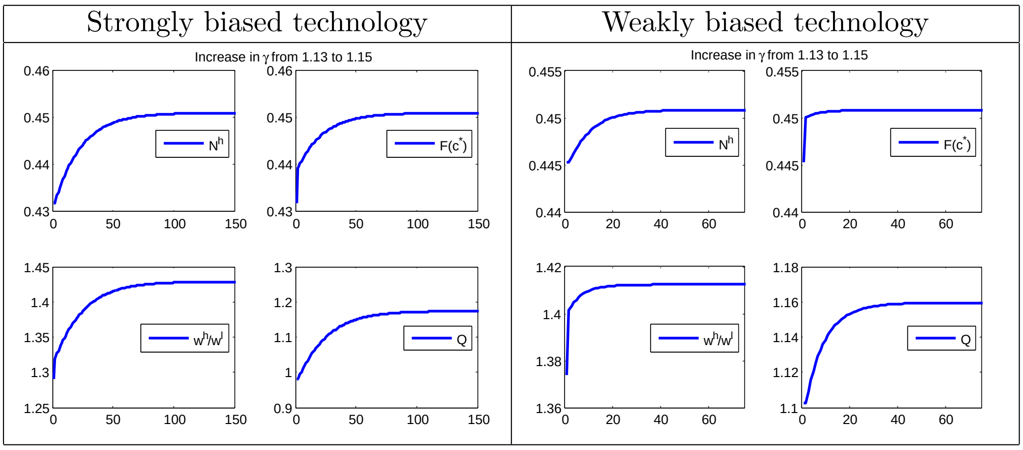

Figure 2.6: Increase in γ

The effects of ρ on the steady state high-skilled labour supply are more complex. This parameter controls the elasticity of substitution between the high- and the low-skilled intermediate good. This way, it affects the lifetime gain from acquiring education through both the growth rate and the skill premium. The growth rate depends negatively on ρ (from equation (2.26)), implying that when the intermediate goods are more substitutable with each other, the growth rate is lower. The responsiveness of the skill premium to the supply of high-skilled workers depends on the relation between ρ and $1/(1+\beta)$. When $\rho = 1/(1+\beta)$, then the skill premium does not change in response to a change in the supply of high-skilled workers, as the price effect and the technology effect exactly offset each other. Thus, the closer is ρ to $1/(1+\beta)$, the less the steady state skill premium responds to changes in the supply of high-skilled workers (see equation (2.23)). Thus, for weakly biased technologies, higher substitutability implies a flatter skill premium, one that responds less to extreme values of N^h. In case of a weakly biased technology the steady state skill premium is a decreasing function of the supply of skilled labour. Therefore a higher ρ implies a lower skill premium for low N^hs and a higher skill premium for high N^hs. Thus, the $F(h(N^h))$ curve for higher ρs goes below the one for lower ρs for low values of N^h, and goes above it for high values of N^h. In case of strongly biased technologies the steady state skill premium is an increasing function of the supply of skilled labour. In this case a higher elasticity implies a steeper skill premium: one that is lower for low values of N^h and higher for high values of N^h. The overall effect of a change in ρ thus depends on the other parameter values and on the magnitude of change. Figure 2.7 shows the transition path for higher elasticities of substitution, for cases where the new steady state features a lower supply of high-skilled workers.

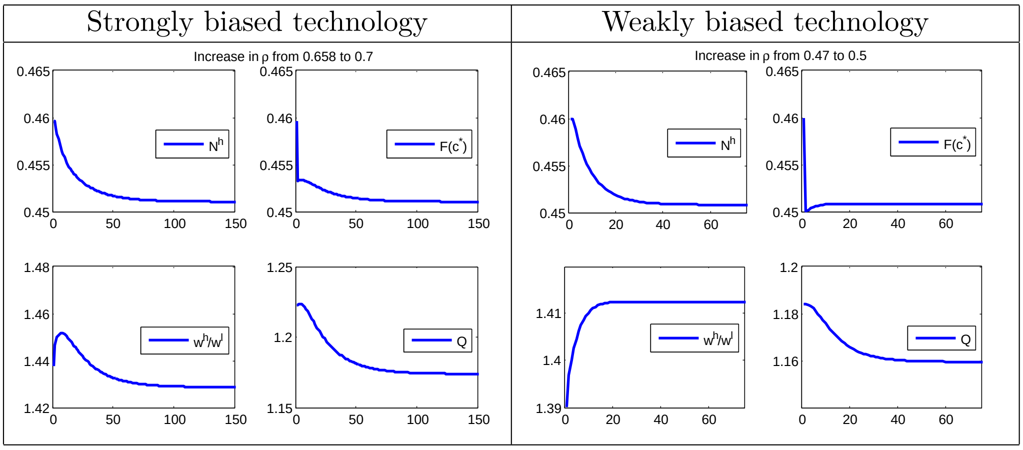

Figure 2.7: An increase in ρ

The increase in ρ lowers the growth rate, this way reducing the gain from working as high-skilled, and leading to the downward jump in the educational attainment of new cohorts. This reduces initially the skill premium, since the supply of skills increases, while technology stays the same. As the skill supply and relative quality decline throughout the transition, the skill premium increases further for weakly biased technologies. This leads to a slight increase in the educational attainment of new cohorts. On the other hand, for strongly biased technologies the skill premium, after its initial increase, decreases below its original level in the long run. The educational attainment of new cohorts thus continues to decrease. The initial steady state in the case of the strongly biased technology fell into the darker gray part of the BOE' region in Figure 2.3, as the relative quality and the skill premium slightly increase initially, and the economy crosses over to the dark gray region in DOB at the beginning of the transition. In the case of weakly biased technologies, the relative quality continuously declines together with the supply of skills. Therefore the initial steady state was in the BOE' region, and the economy crossed over to the darkest gray area of DOB just before reaching the steady state.

The last parameters to consider are the mean and the variance of the distribution of educational costs. A distribution with a lower mean, μ, can be represented by the dashed curve in Figure 2.4, while the higher mean distribution can be represented by the solid curve. This is due to the fact that in a lower mean distribution, there is more mass below any given point, than in the higher mean distribution. Hence, for any present value gain from acquiring education, it is worthwhile for more people to acquire education if the mean cost is lower. Therefore, the stable steady state with lower mean costs of education features higher N^h, which is quite intuitive: where education is cheaper more people acquire education in the long run.[15] Figure 2.8 shows the transition after an unexpected drop in the mean cost of education.

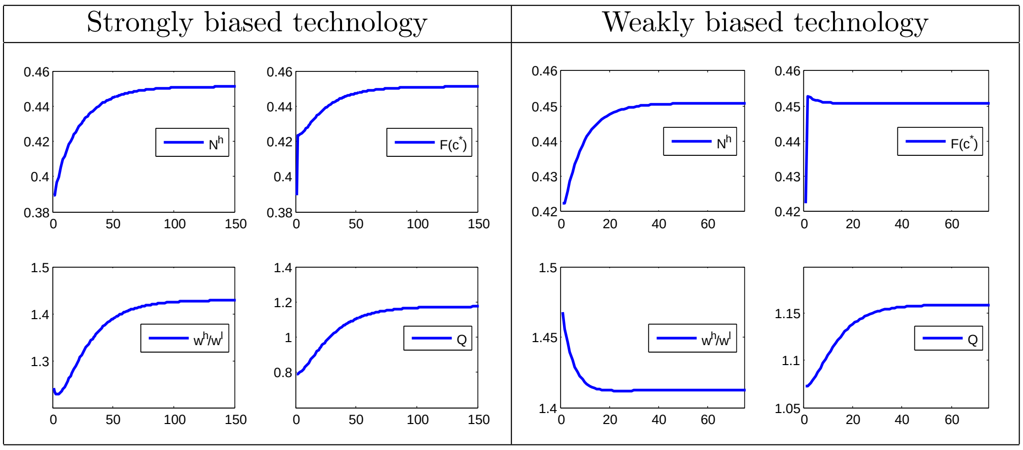

Figure 2.8: A decline in the mean cost of education, μ

The decline in the mean cost of acquiring education leads to a jump in the educational attainment of new cohorts, since even with the same gain from working as high-skilled, it is worthwhile for a larger fraction of the population. The skill premium in both cases decreases initially, as there is a larger supply of high-skilled workers, while technology does not adjust immediately. The skill premium continues to decline in case of weakly biased technologies, as the effects of the increase in the relative supply are not compensated by the increase in the relative quality. Therefore, the educational attainment of consecutive cohorts declines, but stays above its original level. In case of strongly biased technologies the joint increase in

[15]In general it is true that if a distribution F first order stochastically dominates distribution G, then G can be represented with the dashed curve, while F can be represented with the solid curve, and hence the steady state under G has more skilled workers.

the relative supply and the relative quality lead to an increase in the skill premium, leading to a continuous increase in the educational attainment of new cohorts.

Finally I consider is the variance of the cost of education. A higher variance implies that there are more people with low costs, up until the median costs, while for costs above the median there are more people with higher costs, i.e. fewer people with lower costs. Hence, as long as in the steady state less than half of the population acquires education, the steady state N^h is higher when the costs of education are more dispersed. Figure 2.9 shows the transition path after an increase in the dispersion of the costs of education.

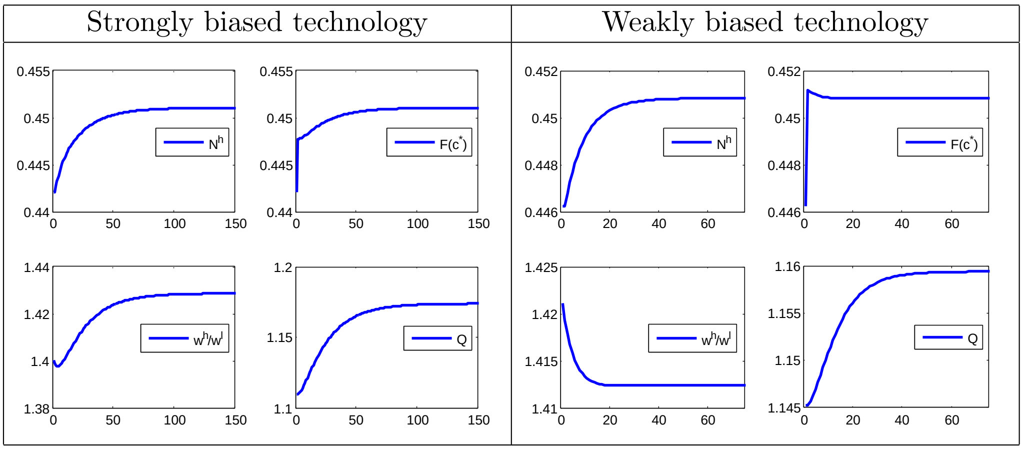

Figure 2.9: An increase in the dispersion of the costs of education, σ

As the variance of costs increases, a larger fraction of the new cohort acquires education (as long as the present value gain is below the median cost). The transition path and the intuition for the adjustment of the variables is exactly the same as in the case of a lower mean cost of education.

To summarise, for all of the parameter changes considered, the path of the supply of high-skilled workers and the path of the relative quality in the two sectors are similar in case of weakly and strongly biased technologies. However, the path of the skill premium and of the educational attainment of new cohorts is dramatically different for all but one parameter change. The only exception is an increase in parameter γ, where all four variables follow similar paths for the two types of technologies. Except for this case, the skill premium and the educational attainment of new cohorts always moves in opposite directions. This is due to the fact that for weakly biased technologies the increasing relative quality compensates less for the negative effect of the increasing skill supply on the skill premium. Therefore, in most cases the skill premium decreases if the skill supply is increasing, and hence the incentives of acquiring education are reduced for newer cohorts. The opposite holds for strongly biased technologies: as the skill premium continuously increases, the incentives to acquire education increases for newer cohorts.

2.5 Concluding remarks

In this chapter I challenge the view that a strong relative bias in the technology is necessary for the simultaneous increase of the skill supply and the skill premium. Assuming, consistently with the data, that the developed economies are not in their steady state, and considering explicitly the transition to the steady state, the model shows that the joint increase in the skill supply and the skill premium can arise regardless of the bias in the technology.

I propose a model where the direction of technical change and the supply of skilled labour is endogenous. Technological change is driven by R&D firms, which invest more into developing technologies for bigger markets. Therefore when the supply of high-skilled labour increases, technology becomes more biased towards high-skilled workers. The increase in the skill-bias of technology increases the skill premium, however, this is offset to some extent by the negative effect of increasing skill supply on the skill premium. If the overall effect is an increase in the skill premium, then technology is strongly biased, whereas if the overall effect is a decline in the skill premium, then technology is weakly biased. On the other hand, the supply of skilled labour is determined by individual decisions whether to acquire education or not, therefore a higher skill premium leads to a larger supply of skilled labour. The positive dependence of these two variables on each other are crucial in understanding the dynamics.

I analyse the steady state of this model and its dependence on parameter values. This exercise shows, that for most steady state shifts that arise due to a parameter-change, a strongly biased technology is necessary to observe a long-run increase in both the skill supply and the skill premium.

I conduct a thorough analysis of the transitional dynamics, and its dependence on the initial value of the skill supply and the relative quality. The analysis shows, that if initially the relative quality is not too high compared to the supply of high-skilled workers, then the transition can feature a joint continuous increase in the supply of high-skilled labour and the skill premium. I highlight the importance of transitional dynamics by showing that this pattern can emerge independent of whether technology is weakly or strongly biased.

Chapter 3

Income Inequality and the Progressivity of Taxes in a Coalition Formation Model

3.1 Introduction

The widespread progressivity of income taxes is a puzzling phenomenon for economists. The normative literature is inconclusive on the optimality of progressive income taxes. The results depend on the equity-efficiency trade-off, and hence are very sensitive to the social welfare function and the elasticity of labour supply. In the positive literature, self-interested citizens, politicians or parties are the central element. When modeled as a classical Downsian competition, additional restrictions on policies or preferences have to be implemented, otherwise the typical problem of voting cycles arises in the multi-dimensional setting. Even in models with policy-motivated politicians the conclusions about the progressivity of the tax scheme is ambiguous. The question, however, has not been analyzed in the context of endogenous coalition formation, which is a natural framework to think about redistributive issues. In this chapter, I relate the *degree* of progressivity of the income tax scheme to the prevailing income inequality in the society. I find, that as in the data, more unequal societies implement more progressive income tax systems.

This chapter contributes to the discussion on progressivity from both an empirical and a theoretical perspective. In order to understand how uniform the progressivity of income taxes is in advanced economies, I calculate the progressivity index of income taxes for 17 OECD countries. I find that there is a substantial variation in the index of progressivity, and that more unequal countries have more progressive income tax schemes in place. From a theoretical perspective, I present a model of political coalition formation in which the tax scheme is determined. The society has to decide on how to share the burden of financing a given level of public good. I show that in such a model, as inequality increases, the representative of the rich group becomes less able to participate in any coalition, and the equilibrium tax scheme shifts the tax burden towards the rich, thus increasing the progressivity of taxes.

In this chapter, I analyze an economy where the citizens have to decide on how to raise funds to provide a given level of public good. The society has to select a tax scheme that raises a fixed amount of revenue, and the different income groups have conflicting interests on which groups to tax more heavily. In the elections, each income group is represented by a politician, whose interests coincide with his group's. The representatives have to decide whether to run alone, or to form a coalition with another representative. Each representative or coalition chooses a tax scheme to offer, and citizens then vote on the candidate or party that offers the tax scheme that maximises their utility. As in the citizen-candidate model of Besley and Coate (1997), single representatives can only offer their ideal policy, as they cannot credibly commit to the implementation of any other platform. On the other hand, coalitions offer a commitment mechanism for parties: due to the internal conflict of party members, the party can credibly commit to any policy that is in the Pareto set of its members. I identify the stable coalitions and the equilibrium winning platforms.

I find that when income inequality is low, then a coalition of the poor and the rich wins and implements a tax system that puts a large fraction of the tax burden on the middle income group. When income differences are small, then the middle income group's preferred policy is the median policy. This implies that in the absence of coalitions, the middle income group wins and implements its ideal policy. Thus, the rich and the poor can credibly commit to cooperate and choose a platform that is better for both than the middle income group's ideal policy. As inequality increases, the policy role of tax rates on higher levels of income increases: more revenue can be raised by a marginal increase in the tax rates. This implies that there is more room to trade-off tax rates on middle and high incomes. This gives a lot of power to the poor, as the preferences of the middle income and the rich are very different. If inequality is moderately high, then the poor are not powerful enough to stop the middle income group and the rich from forming a coalition. This coalition implements policies which feature high taxes on low levels of income and moderate taxes on higher levels of income. When inequality is very high, the poor can prevent the rich and the middle income group from forming a stable coalition, and hence implement a highly progressive tax scheme.

I calculate the pre- and post-tax Gini coefficients, the progressivity index of the personal income tax, and the average tax rate for 17 OECD countries. I proxy the distribution of income by the employment share and average earnings of the main occupation groups based on labour force surveys. I find that low inequality countries have a less progressive income tax scheme. Hence, contrary to common belief, the progressivity index is relatively low in the Nordic countries, and it is relatively high for Southern European countries. However, the average tax rate, which also contributes to redistribution, is higher in countries with lower inequality. The overall redistributive effect of the personal income tax scheme also increases as inequality increases. Therefore the model I present here is in line with the data in predicting that as inequality increases in a society, the implemented tax scheme becomes more progressive.

3.2 Related literature

One strand of models of voting on the progressivity of income taxes is in the Downsian tradition: parties or politicians only care about holding office, and can perfectly commit to implementing any policy platform. Snyder and Kramer (1988) were one of the first to address the progressivity of income taxes from a political economy perspective. Under the restriction that parties can only offer policies that are ideal for some citizens, and citizens optimally allocate their time between taxable and non-taxable activities, they show that marginal rate progressivity emerges due to the desire of middle-income citizens to reduce their own tax burden. Cukierman and Meltzer (1991) analyze the question in cases when a Condorcet winner exists, over quadratic tax schemes, but only succeed in showing the prevalence of progressive taxes under very strong restrictions. Marhuenda and Ortuño-

Ortín (1995) relax the requirement of the existence of a Condorcet winner, and show that a marginal rate progressive tax always defeats a marginal rate regressive tax, if the median income is below the mean income. Hindriks (2001) shows under similar conditions, that for any tax scheme there exists a less progressive one, which is supported by a majority of voters, thus demonstrating that the demand for progressivity cannot be derived from the standard Downsian framework. These voting cycles arise, because to analyze the progressivity of the tax scheme, the policy space has to be at least two-dimensional. In a multi-dimensional policy space pure strategy Nash-equilibrium of the standard two-party game generally does not exist. Carbonell-Nicolau and Ok (2007) identify mixed strategy equilibria and find that in an unconstrained policy space there is an equilibrium which is not marginal rate progressive. Carbonell-Nicolau (2009) circumvents the problem of voting cycles by allowing politicians to reveal their policy platforms gradually in more than one period, and shows that the tax scheme benefits the most populous groups, and puts the burden of taxation on groups with fewer voters. Therefore in log-normal income distributions the income tax scheme is not progressive, as the tax burden is on the rich *and* the poor.

This chapter is closer to the non-Downsian strand of the literature which assumes that politicians have some preferences over the policy to be implemented. Roemer (1999) introduces a new equilibrium concept (Party Unanimity Nash Equilibrium, PUNE), one which is based on the idea that parties have internal conflicts: some members only care about winning the election, whereas others care about the policy that will be implemented. In such a setup, he shows that from the set of quadratic tax functions in a two-party election, both parties propose a progressive tax scheme. This paper is similar to mine in the sense that the existence of parties and the internal conflict allows the parties to offer policy platforms that a single candidate (either office- or policy-motivated) would not be able to offer. However, in my model the candidates only care about the policy that is implemented, and party formation is endogenous.

Carbonell-Nicolau and Klor (2003) analyze a similar setup to the one presented here: there is an exogenous set of parties, who have preferences over after-tax inequality. Each party decides whether to enter the election with a candidate or not. Voters vote sincerely in order to minimise their expected tax payment. In such a setup they characterise the conditions under which a strong Nash equilibrium exists, and show that these equilibria feature increasing marginal rates. Their setup is similar to the one of this chapter in the sense that the citizens vote on representatives, and that the representatives have preferences over the implemented policy. However, in my model, the representatives are citizens as well and they have preferences over their own after-tax income, and not over the after-tax income inequality as in the paper of Carbonell-Nicolau and Klor (2003). Moreover, in my model, candidates are allowed to form coalitions and this way improve their chances of implementing a platform that increases their utility.

Most of the positive literature has omitted the equity-efficiency trade-off, which is at the heart of the normative literature. This trade-off arises, as progressive taxes provide a mechanism for the state to redistribute income from the rich to the poor, however, high marginal tax rates have efficiency costs. Since the seminal paper by Mirrlees (1971), this literature has developed significantly, but as Saez (2001) notes, its implications for policy are still very limited.[1]

Donder and Hindriks (2003) is a notable exception in the positive literature in the sense that they explicitly consider labour supply choices. They show through simulation results that under less demanding equilibrium concepts than majority winner, progressive tax schemes are more likely to arise, and they are the only possibility if the distribution of abilities is sufficiently concentrated at the middle.

From an empirical perspective only a few papers have quantified the progressivity of the income tax schemes across countries. Kakwani (1977), who introduced the progressivity index that I use in this chapter, calculates the redistributive effect and the progressivity of the tax scheme in the US, Canada, the UK and Australia. Suits (1977), introduces a similar measure of progressivity and calculates the change in the progressivity of different US taxes. A more recent study by Wagstaff et al (1999) calculates and decomposes the redistributive effect of income tax schemes in 12 OECD countries.

[1] Mirrlees (1971) shows that the tax rates have to be non-negative and below full taxation. The most well-known result is that when the income distribution is bounded, the top marginal tax rate should be zero, Sadka (1976) and Seade (1977) show this result. Seade (1977) showed that the marginal rate at the bottom should also be zero if everyone in the society works.

3.3 Tax progressivity in 17 OECD countries

3.3.1 Measuring progressivity

It is generally accepted that the progressivity of taxes at a given income level depends on the elasticity of the tax function with respect to income. If the elasticity is equal to unity, then the tax is exactly proportional at that income level, if this elasticity exceeds 1, then the tax is progressive, while if it is below 1, then it is regressive at that income level. Based on this definition, one can look at tax systems and determine the parts of the income distribution where the tax system is regressive, progressive and proportional. However, this is a rather tedious exercise to compare the progressivity of tax systems across countries.

To measure the progressivity of taxes, I use an index proposed by Kakwani (1977).[2] This measure is based on the above definition of progressivity, but characterises an entire tax system with a single index. This index is essentially the difference between the inequality of income and the inequality of tax payments. Figure 3.1 shows this measure.

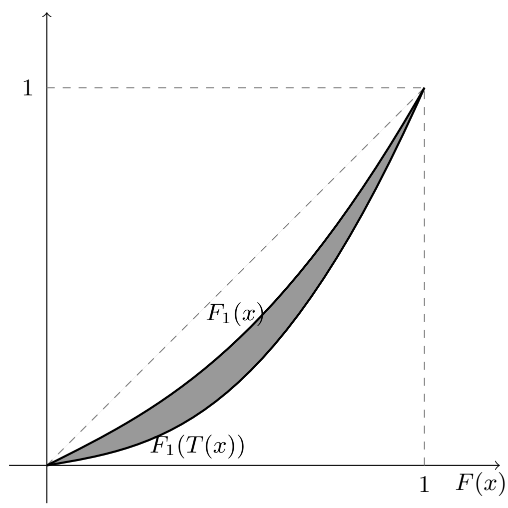

Figure 3.1: Index of progressivity

The cumulative density function of income, $F(x)$ is plotted on the horizontal axis, where x is the income level. The vertical axis represents two other cumulative density functions. The one closer to the 45 degree line is the accumulated fraction of total income of those

[2] This is quite similar, although not the same as the one developed in Suits (1977). For a comparison of the two see Formby, Seaks, and Smith (1981).

who have income less or equal to x, denoted by $F_1(x)$. This is the Lorenz curve of income and the Gini coefficient, G is twice the area between the 45 degree line and $F_1(x)$. The line further from the 45 degree line is the accumulated fraction of the total tax burden of those who have income less than or equal to x, this is denoted by $F_1(T(x))$. This curve is the *concentration curve* of taxes. The concentration index of taxes, C is defined as twice the area between the 45 degree line and $F_1[T(x)]$. Kakwani's measure of progressivity is $P = C - G$, which is equivalent to twice the integral of $(F_1(x) - F_1(T(x)))$, which is twice the shaded area on Figure 3.1.

If this measure is positive (as in Figure 3.1), then the tax system is said to be progressive, conversely a negative measure implies a regressive system, while a zero value implies a proportional system. However, since this measure captures the progressivity of a system in a single number, the same value P can be assigned to quite different tax systems, just as different income distributions can have the same Gini coefficient.

Consider for example a perfectly proportional tax system, where those who earn k percent of total income bear exactly k percent of the total tax burden, and hence the $F_1(T(x))$ curve and the $F_1(x)$ curve are perfectly aligned. This implies a progressivity index equal to zero, $P = 0$. Note however, that a progressivity index of zero can be reached in other ways, for example by $F_1(T(x))$ below $F_1(X)$ for low values of x and above it for higher values of x. This system is only neutral on average, on some parts of the income distribution it is progressive, whereas on other parts it is regressive.

This is a drawback that is bound to arise with any measure that captures progressivity in a single index, since progressivity depends on the entire income distribution. However, when considering tax systems already in place, this drawback is not so severe, since the shape of the curve $F_1(x) - F_1(T(x))$ is similar in all countries considered.[3] As the shape is in general similar, similar values of P truly reflect a similar degree of progressivity.

An alternative and widespread measure of tax progressivity is the difference between the pre-tax (G) and the post-tax (G^*) Gini coefficients, which was introduced by Musgrave and Thin (1948). Kakwani (1977) shows, that while this difference measures the *redistributive effect* of a tax system, it captures not only the progressivity of the system (P), but also the

[3]See graphs in Section C.1 of the Appendix.

effects of the average tax rate ($\overline{\tau}$).[4] The decomposition is the following:

$$RE = G - G^{*} = \frac{\overline{\tau}}{1-\overline{\tau}}P.$$

This shows, that the redistributive effect is increasing both in the average tax rate and in the progressivity of the tax scheme. For example, by doubling all tax rates, the progressivity of the system does not change, but the average tax rate doubles, implying that the Musgrave-Thin measure increases as well.

In what follows, I present both Kakwani's measure P of progressivity and Musgrave-Thin's measure of the redistributive effect of tax systems for 17 OECD countries, and a similar pattern emerges for both measures.

3.3.2 Data sources

To calculate the progressivity index across countries I need data on the income distribution and the tax scheme of these countries.

I use the harmonised European Union Labour Force Survey (ELFS) from 2005, supplemented by earnings data from the Eurostat Structure of Earnings Survey 2006 (SES) to create the income distribution for 17 OECD countries.[5] I proxy the distribution of incomes with a discrete categorization of the workforce into occupation groups. Since finer than first-digit occupational data is not available for all countries, I use first-digit occupation groups from the International Standard Classification of Occupations (1988 - ISO-88(COM)), which divides the workforce into nine occupations. The ELFS contains information on the number of employees and self-employed individuals for these nine main occupation groups, however, data on earnings is not available in this survey due to anonymity requirements.

Earnings information is taken from the SES, which, like most earnings surveys only records employees, since earnings data for self-employed individuals is generally not reliable.

[4]The decomposition by Kakwani assumes that individuals with equal income pay equal tax, i.e. that there exists a tax function, $T(x)$ which takes the same value for everyone with income x. Aronson, Johnson, and Lambert (1994) show that the difference between the pre- and the post-tax Gini coefficients depends on other factors as well, if there is unequal treatment of equals, for example by treating incomes differently depending on the source of the income. Here, since I do not have such detailed data on incomes, I omit this analysis.

[5]Austria, Belgium, Germany, Denmark, Spain, Finland, France, Greece, Iceland, Ireland, Italy, Luxembourg, Netherlands, Norway, Portugal, Sweden, United Kingdom. I use the ELFS 2005 and 2008 for Italy, as Italy is missing from the 2005 survey.

Table 3.1: Relative earnings and self-employment across occupations

occupation category	relative earnings	share in workforce	share of self-employed
Legislators, senior officials & managers	1.87	0.09	0.37
Professionals	1.41	0.15	0.11
Technicians & associate professionals	1.08	0.16	0.09
Clerks	0.83	0.11	0.02
Craft & related trades workers	0.85	0.13	0.15
Plant & machine operators & assemblers	0.82	0.08	0.07
Service, shop & market sales workers	0.69	0.15	0.06
Skilled agricultural & fishery workers	0.67	0.04	0.52
Elementary occupations	0.64	0.09	0.04

Notes: The first column contains the occupation categories. The second column contains the cross-country average of relative earnings of each occupation category relative to the average earnings within that country, for the 17 countries based on data from the SES 2006 data. The third column contains the share of the workforce working in that specific occupation across 16 countries from the EU LFS 2005 data (excluding Italy, as data is not available in the 2005 survey). The fourth column contains the average share of self-employed across 16 countries. Including Italy using the 2003 and 2008 surveys does not significantly change the values reported here.

As can be seen in Table 3.1, in some occupations a significant part of the workforce is self-employed, hence excluding them when analyzing the distribution of earnings would lead to significant discrepancies. Therefore, I include the self-employed when constructing the income distributions. Since data on the earnings of self-employed is not available, I assume that their average earnings are the same as the average earnings of employees within each occupation category.[6] For some countries the average earnings in certain occupation groups are missing, I interpolate these values from the average earnings in the other occupations groups.[7]

The data on tax structures is taken from the OECD's Taxing Wages 2006 publication, where each country's tax code is described in detail. In all calculations I only take into account personal income taxes at all levels of government. I calculate the income tax

[6]Several studies show (for example Pissarides and Weber (1989), Clotfelter (1983), Slemrod (1985), Feinstein (1991), Andreoni, Erard, and Feinstein (1998), Slemrod (2007), Feldman and Slemrod (2007), Kleven, Knudsen, Kreiner, Pedersen, and Saez (2011), Hurst, Li, and Pugsley (2011)), that on average, self-employed individuals have lower earnings than employees, and that they consume a significantly higher fraction of their earnings than employees. This can be explained either by self-selection and preferences (i.e. self-employed enjoy the freedom of setting their own hours more and have a lower savings rate) or by tax evasion, whereas the self-employed earn the same amount, but declare a lower fraction of it, which explains the discrepancy both in earnings, and in saving rates. Here I take the stand, that self-employed earn the same amount, just declare a lower fraction of their earnings.

[7]Robustness checks on interpolation methods shows that the results are not sensitive to the different methods.

function for all countries in my sample. Most tax schemes provide tax brakes or tax credits after dependents, varying with the number of earners in the household. For simplicity, I treat everyone in my sample as a single earner without any dependents.

3.3.3 Progressivity indices

Table 3.2 contains the Gini coefficient, the progressivity index, the redistributive effect and the average tax rate calculated in the above described way. It is important to note that all countries in my sample implement a progressive income tax scheme. Note that Denmark, Norway, and Sweden, which are typically regarded as countries with very progressive tax systems, all have relatively *low* progressivity indices $(0.04, 0.06, 0.07)$. Their progressivity indices are so low, that even with relatively high average tax rates, the redistributive effect of the personal income tax is below average. On the other hand, Greece, Portugal, the Netherlands and Ireland have very high progressivity indices $(0.38, 0.34, 0.29, 0.25)$, and relatively low average tax rates $(0.10, 0.13, 0.12, 0.12)$, resulting in a high redistributive effect.

Figure 3.2 presents the correlation between income inequality and the progressivity of taxes and the redistributive effect. The left panel of this Figure shows the progressivity index of the personal income tax, P, which is the integral of the difference between the cumulative share of total income and the cumulative share of total taxes with respect to $F(x)$ plotted against the Gini index. The right panel in the same figure shows the redistributive effect of the personal income tax, RE, again plotted against the Gini coefficient. Note that both graphs show a clear positive correlation: countries with a higher Gini coefficient tend to have more progressive tax systems, that achieve more redistribution. Since $P = C - G$, if C, the concentration index of taxes was unrelated to inequality, then one would expect a negative relationship between P and G. However, the linear trend line shows a positive correlation that is significant at 1.3%: higher income inequality and more progressivity seem to go together. The right panel shows the change in the Gini coefficient due to the income tax. The positive correlation implies that the income tax has a larger redistributive effect in more unequal societies. However, the coefficient of correlation is significantly smaller than one, implying that countries where gross earnings are more unequal, the net earnings are more unequal as well, even though to a smaller extent.

Table 3.2: Gini coefficients, progressivity and redistributive effect

country	Gini	Progressivity	Redistributive effect	Average tax rate
Austria	0.21	0.17	0.05	0.21
Belgium	0.20	0.10	0.04	0.28
Denmark	0.12	0.04	0.02	0.38
Finland	0.17	0.11	0.04	0.27
France	0.17	0.10	0.02	0.16
Germany	0.18	0.13	0.04	0.22
Greece	0.20	0.38	0.04	0.10
Ireland	0.17	0.25	0.04	0.12
Iceland	0.15	0.07	0.02	0.24
Italy	0.23	0.15	0.04	0.19
Luxembourg	0.24	0.22	0.04	0.16
Netherlands	0.15	0.29	0.04	0.12
Norway	0.12	0.06	0.02	0.22
Portugal	0.33	0.34	0.05	0.13
Spain	0.20	0.16	0.03	0.15
Sweden	0.12	0.07	0.02	0.26
UK	0.23	0.11	0.03	0.20
average	0.19	0.16	0.03	0.20

Notes: The first column contains the countries, the second column contains the Gini coefficients, the third contains the progressivity indices, the fourth column contains the redistributive effect, and the fifth column the average personal income tax rates. Author's own calculations from ELFS 2003, 2005 and 2008, SES 2006, OECD Taxing Wages 2006.

In the next section I present a theoretical explanation for why more unequal countries have a more progressive income tax scheme. The basic idea is that the more unequal the society is, the further are the rich from the rest of the population. The further are the rich, the less likely it is that they participate in setting the tax scheme, and hence the more likely it is that a significant fraction of the tax burden will fall on them.

3.4 Model

The society consists of three groups, who have to decide on how to finance the public good, g. They set three tax rates, made to resemble the most commonly observed bracket-type tax structure, with the marginal rates given for each bracket. The trade-off is clear: those with lower income aim to have high marginal rates at the top, whereas those with high income aim to have high marginal rates at the bottom.

The coalition formation model is based on the model developed by Levy (2004). Each

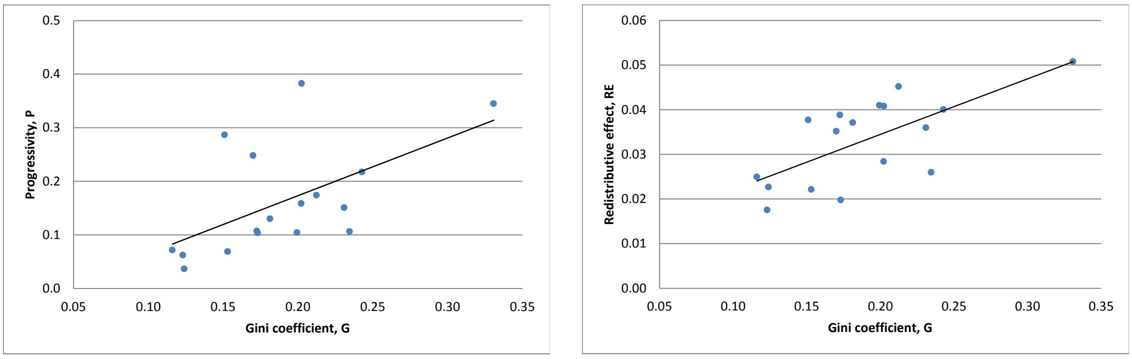

Figure 3.2: Progressivity of the income tax, redistributive effect and Gini coefficients

Notes: For the progressivity index the linear trendline is given by $P = 1.0763G - 0.042$, where the coefficient on G is significant at 1.3%. The redistributive effect's trendline $RE = 0.1244G + 0.0096$, where the coefficient on G is significant at 0.1%. The progressivity indices and the redistributive effects are the author's own calculations as described in the text. Data sources: OECD Taxing wages 2006, ELFS 2005 (2003 and 2008 for Italy), SES 2006.

of the three groups is represented by one candidate, who decides whether to run alone, and offer his ideal policy, or to run in coalition with another candidate, in which case they offer a policy from their Pareto set. Each citizen votes on the candidate or party that offers the policy that is best for him. The equilibrium of the game is a partition of the candidates into parties and the policy platform that each offers.

3.4.1 Admissible policies and preferences

Citizens differ in their level of income: people are either poor, middle income, or rich, with the following income levels: $y^P < y^M < y^R$. The size of group $i \in \{P, M, R\}$ is α^i, and the total population is normalised to one, $\sum_i \alpha^i = 1$.

A public good, g has to be financed by taxes. The tax structure is the following: everyone pays τ^P fraction of their income below y^P, τ^M fraction of income between y^P and y^M, and τ^R fraction of income above y^M. The balanced budget condition is:

$$\tau^P y^P + (\alpha^M + \alpha^R)\tau^M(y^M - y^P) + \alpha^R \tau^R (y^R - y^M) = g \tag{3.1}$$

Let $\Delta(g) \subseteq \mathbb{R}^3$ denote the set of *admissible policies*, which consists of tax triples $\theta = \{\tau^P, \tau^M, \tau^R\}$, for which the balanced budget condition is met and all tax rates are between 0 and 1. Taking g as exogenous leaves two free variables: any two tax rates uniquely determine the third.

Given a tax structure, θ, the utility of the poor, middle income and rich individuals are the following:

$$U^P(\theta) = H(g) + (1 - \tau^P)y^P, \quad (3.2)$$

$$U^M(\theta) = H(g) + (1 - \tau^P)y^P + (1 - \tau^M)(y^M - y^P), \quad (3.3)$$

$$U^R(\theta) = H(g) + (1 - \tau^P)y^P + (1 - \tau^M)(y^M - y^P) + (1 - \tau^R)(y^R - y^M). \quad (3.4)$$

Citizens derive utility $H(g)$ from the public good, g. For simplicity I assume that the utility is linear in disposable income. This simplification does not affect the results qualitatively, as the shape of the utility function only has a quantitative effect on how the different groups value the different taxes and the trade-offs between them.

3.4.2 Ideal policies and indifference curves

Within an income group agents have identical preferences. The *ideal policy* (or the set of ideal policies) of a group is the policy platform that maximises their utility from the admissible set. In general each group wants to reduce their own tax payment. This implies that the poor want as low τ^P as possible, the middle want τ^P and τ^M to be low, while the rich would prefer all tax rates to be as low as possible. However, since the public good has to be financed from taxes, there is a trade-off between the tax rates, and the different income groups value this trade-off differently. The poor only care about the level of τ^P, and their utility increases as τ^P decreases. Hence their ideal policy or set of ideal policies are those where τ^P is minimal within $\Delta(g)$. The middle income care about τ^M and τ^P, hence their ideal policy is where τ^R is the highest possible. If expropriating all income above y^M does not cover the public good, the middle income prefer to increase τ^P, as one unit of revenue is financed by all groups, whereas increasing τ^M would only increase the burden on the rich and themselves. Finally, the rich would like to share as much of the financing of the public good as possible with the other members of the society, hence they favour to first increase τ^P as much as possible, and then τ^M.

Figure 3.3 shows the ideal policy of each group and their indifference curves.[8] Note that each group has linear indifference curves, as utility only depends on disposable income, which is a linear function of the tax rates.

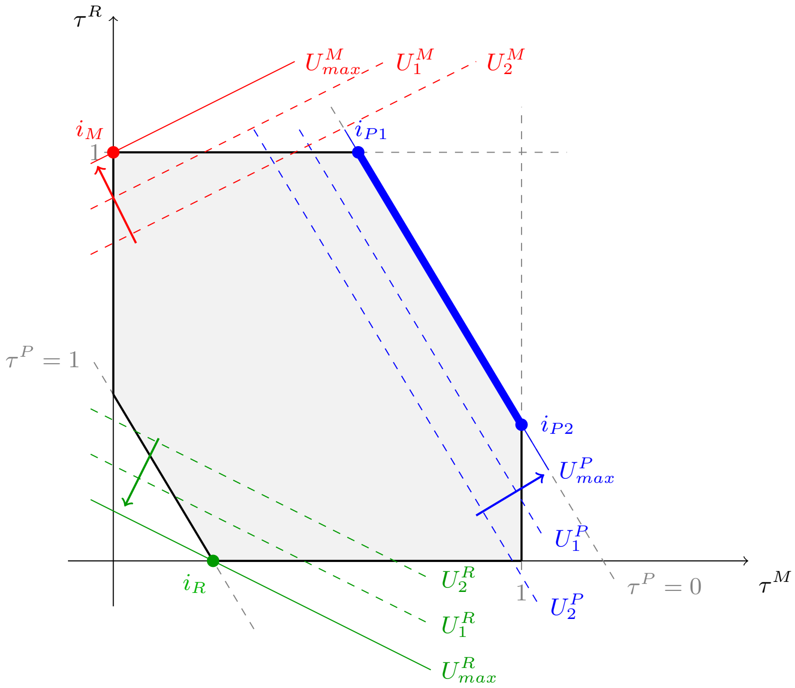

Figure 3.3: Ideal policies and indifference curves

The horizontal axis represents the middle tax rate, τ^M, whereas on the vertical axis is τ^R, the top tax rate. As mentioned earlier, the third tax rate, τ^P is implicitly defined as one that allows the budget to be balanced. The grey dashed lines represent the boundaries on the minimum and maximum possible rates ($\tau^i = 0$ and $\tau^i = 1$). The set of admissible policies, $\Delta(g)$, is the shaded hexagon bounded by the horizontal ($\tau^R = 0$) and vertical ($\tau^M = 0$) axis, the $\tau^R = 1$ and $\tau^M = 1$ line and finally by the $\tau^P = 0$ and $\tau^P = 1$ lines.[9]

The set of ideal policies of the poor are represented by the thick blue line, at the top-right part of $\Delta(g)$. The poor want to finance as little as possible of the public good from τ^P, hence their ideal policy is where τ^P is minimal. In this graph the ideal policies of the

[8]See Section C.2 of the Appendix for the analytical solution of the ideal policies.

[9]$\Delta(g)$ is not necessarily a hexagon: as the $\tau^P = 0$ line shifts out, the top corner of the square will not be cut off, while as the $\tau^P = 1$ line shifts down, the bottom corner will not be cut off, thus leaving $\Delta(g)$ a pentagon or quadrangle.

poor are the platforms where $\tau^P = 0$. If the required amount of tax would be higher, the $\tau^P = 0$ line would shift out, gradually reducing the measure of ideal policies of the poor, up to the point of leaving the poor with one ideal policy, where the other two tax rates reach their maximum level, $\tau^M = \tau^R = 1$.

The indifference curves of the poor are depicted by the dashed blue lines parallel to the $\tau^P = 0$ line. Utility is increasing along the blue arrow, and maximum utility is achieved on the solid blue line. As the poor only care about the level of τ^P, their indifference curves are sets of points for which τ^P is constant. It is straightforward that these indifference curves are downward sloping in the $\{\tau^M, \tau^R\}$ space: if τ^M increases, a lower τ^R is enough to balance the budget.[10]

The ideal policy of the middle income group is represented by the red dot at the top left corner of the admissible policies. The middle income prefer to finance the public good primarily by τ^R, then by τ^P and want as low τ^M as possible. In this graph this platform is where $\tau^R = 1$, $\tau^M = 0$ and τ^P is such that the tax requirement is met. For lower levels of public good, the $\tau^P = 0$ line shifts down, eventually until the ideal policy of the middle income would consist of $\tau^M = \tau^P = 0$ and $\tau^R \leq 1$ to meet the tax requirement.[11] On the other hand, with the level of public good increasing, the $\tau^P = 1$ line would shift out, leading to the ideal policy of the middle moving along the $\tau^R = 1$ line, with positive τ^M.

The dashed red lines represent the middle income group's indifference curves, with utility increasing along the red arrow, and maximum utility reached at the solid red line. An increase in τ^M reduces the utility of middle income individuals by more than how much it improves the budget. To be kept indifferent they need a decrease in τ^P, which would leave the budget in deficit unless τ^R increases as well. This implies that their indifference curves are upward sloping.

Finally, the ideal policy of the rich is represented by the green dot in the bottom left part of the admissible policies. The rich prefer to finance as much as possible of the public good by τ^P, and as little as possible by τ^R. If the tax requirement is higher, the $\tau^P = 1$ line shifts out, thus moving the ideal policy of the rich along the $\tau^R = 0$ line.

The indifference curves of the rich are represented by the dashed green lines, with utility

[10] See Section C.2 of the Appendix for the analytical expression of the slope of the indifference curves.

[11] In this case the ideal policy of the middle income would be an ideal policy for the poor as well.

increasing along the green arrow, and the highest utility achieved at the solid green line. To understand why the indifference curves are downward sloping, consider an upward sloping line in the $\{\tau^M, \tau^R\}$ space. An increase in τ^M or τ^R hurts rich individuals more, than by how much it improves the balance of the budget. Hence, the magnitude of reduction they require in τ^P would leave the budget unbalanced. Therefore their indifference curves are downward sloping. It is easy to see, that their indifference curves are less steep than those of the poor. Rich individuals are hurt more by an increase in τ^R, than the poor, hence a decrease in τ^M has to be financed by an increase in both τ^P and τ^R. (As opposed to the poor, who can be compensated by solely increasing τ^R.)

3.4.3 Policy selection

The society consists of three very distinct groups of citizens, who are divided on the issue of how to finance the public good. Through a political process the citizens choose a policy to be implemented from the set $\Delta(g)$. I adapt the political model introduced by Levy (2004).

The most important assumption is that parties or single candidates can only offer *credible policies*, policies that are in the Pareto set of their members. Parties are the union of one or more representatives. Therefore, if a representative of the poor, middle income or rich group runs on his own, his only option is to offer his (and his group's) ideal policy. Underlying this assumption is the idea that politicians cannot credibly commit to implementing a policy. The citizens understand that once elected, the politician has the freedom to implement whatever he wishes, which leads to the implementation of his own ideal policy. In such a setup parties serve as a commitment device. Since members of parties have potentially opposing interests, once they offer a policy from their Pareto set, they cannot agree on implementing something else, as this would make some member of the party worse off.

I assume that each group has one representative, who I denote by P, M and R, and each representative's ideal policy coincides with his group's. Consider a partition on the set of politicians.[12] For instance $PM|R$ represents the case where the representative of the poor and the middle income form a party, and compete in the election with a joint platform

[12] I do not consider the possibility of a full coalition, i.e. when all candidates join in one party.

against the representative of the rich. On the other hand, for example, $P|M|R$ represents the case where all representatives run as individual candidates.

For now, assume that the partition of politicians into parties is given. Each party decides whether to run in the election or not, and if running, decides which policy to offer from its Pareto set. Running in the election entails a small cost, $\varepsilon > 0$. Citizens then vote on the policy that maximises their utility. The election is won by the party or candidate who receives the highest vote share, and this party then implements the policy they offered.[13]

Given a partition, a set of policy platforms is an equilibrium, if taking the other parties' actions as given, no party has an incentive to alter its action. A party can alter its action by switching to another platform, by withdrawing from or by joining the electoral competition. The party has an incentive to do this, if this action improves the utility of all of its members. Given the partition of representatives into parties, the set of equilibrium winning platforms can be found, which are the platforms that are implemented given a set of equilibrium strategies.[14]

Finally, the last step is to identify the *stable political outcomes*. Stability is defined in a recursive way as in Ray and Vohra (1997). Representatives start from some coalition structure and are allowed to break this structure up into finer ones. Deviations can be done by one or more representatives jointly. Credible threats are deviations to finer partitions which are stable themselves, since deviators take into account future deviations. In this setup, since there is either one two-member coalition, or everyone is running as a single representative (which is stable by nature), the only deviation to consider is a member leaving the coalition and thus reducing the game to three single representatives. Representatives, when considering splitting from the coalition, take into account the outcome of the single representative election. I identify the stable partitions together with their equilibrium winning platforms.

[13]In case of a tie, all parties that tied have equal probabilities of winning. If none of the parties decides to run in the election, a status quo is implemented. I assume, as Levy (2005) that the status quo is worse for everyone than any other outcome. This ensures that in equilibrium, at least one party runs and some platform is chosen.

[14]Given a set of platforms, in general, there is only one platform that receives the highest vote share in pure strategies. I focus on pure strategy equilibria when they exist.

3.5 Equilibrium

In this section I present the equilibrium outcome of the coalition formation game. The results suggest that when inequality is low, the poor and the rich representative form a coalition and win the election with a platform that has a high middle tax rate and low tax rates on the bottom and top parts of income. At intermediate inequality levels all or some of the two-representative coalitions can be stable. Finally, at high levels of inequality, the poor run alone and win the elections with a highly progressive tax scheme.

First, I present the equilibrium outcomes in the absence of coalitions, as this is the outside option if a coalition member decides to split from his party. Then I present the equilibrium partitions and strategies for the full model.

3.5.1 Equilibrium without coalitions

Consider the case when the representatives cannot form coalitions. In such a setup, the only question is who will compete in the election and with which platform. As Figure 3.3 shows, the representative of the middle income group and of the rich each have one ideal point, i_M and i_R. The representative of the poor can offer any point from the $i_P = [i_{P1}, i_{P2}]$ set, as all these points give him equal utility.

Figure 3.4 depicts the different regimes. The colour red is assigned to the middle income group: the red dot indicates their ideal policy, i_M, while the dashed red line is one of their indifference curves. The colour green is assigned to the rich, while blue indicates the poor. The equilibrium winning strategy (or set of strategies) is indicated in black. It is important to note, that the poor always weakly prefer the policy of the middle income group to that of the rich, since i_R contains the highest possible τ^P, and hence the worst payoff for them.

In Regime 1, the rich prefer the ideal policy of the middle income group to the entire set of ideal policies of the poor. All three candidates entering the electoral competition is not an equilibrium, as the representative of the group with the highest population share wins, and either one of the other representatives have an incentive to drop out, as that does not change the outcome and saves the cost ε. Similarly, two candidates running does not constitute an equilibrium, since the losing candidate has an incentive to withdraw from the competition. The only possible equilibria are those where only one candidate is running,

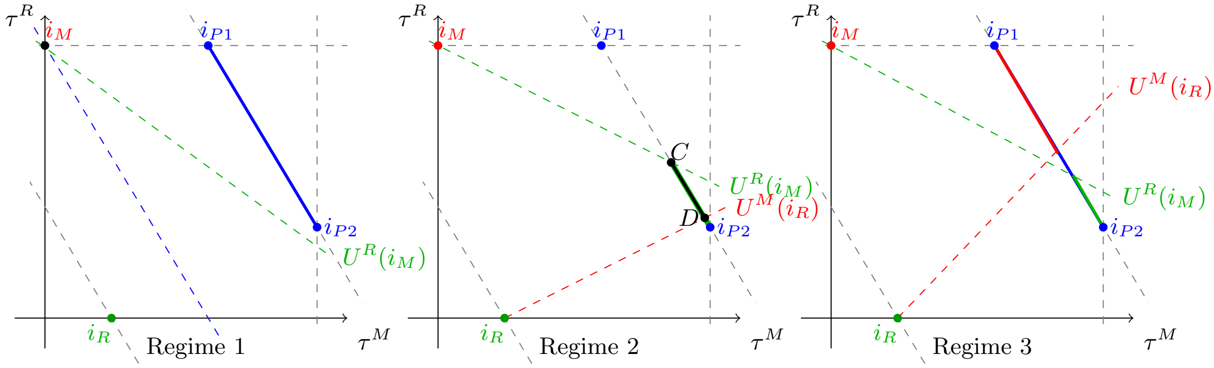

Figure 3.4: Winning party and platform

and this can only be the middle income group's representative. If any other candidate is running on his own, the middle representative can enter and win the election, since the third group (the rich or the poor) will vote for him in the election. The only equilibrium strategy in this case is: $\{\emptyset, i_M, \emptyset\}$, and the equilibrium winning platform is i_M. This is represented by the black dot at the ideal policy of the middle income group.

In Regime 2 and in Regime 3, the rich prefer some policies of the poor to the the ideal policy of the middle (the policies below $U^R(i_M)$), and the middle prefer some policies of the poor to the ideal policy of the rich (the policies above $U^M(i_R)$).

In Regime 2 these two sets have an intersection: the black segment of i_P contains those ideal policies of the poor, which are preferred by the rich to the middle income group's ideal policy, and by the middle income to the rich group's ideal policy. Similarly to the previous case, more than one candidate running cannot constitute an equilibrium. In these cases the equilibrium strategy set has P running uncontested with a policy from the black line segment and winning the election: $\{\theta_P \in (C, D), \emptyset, \emptyset\}$. The equilibrium winning platform can be any policy from the set (C, D), and in expectation it will be: $E = (C + D)/2$.

In Regime 3, none of the ideal policies of the poor are preferred both by the rich to the middle income's ideal policy and by the middle income to the rich group's ideal policy: $i_P \cap U^R(i_M)^+ \cap U^M(i_R)^+ = \emptyset$. In these cases, there is no equilibrium where one representative runs uncontested and wins the election, since none of the ideal policies is a

Condorcet winner.[15] The equilibrium in this case features mixed strategies, and depends on which group is the largest in the society. In what follows I present the results; for details of mixing probabilities and complete characterization of the equilibria see Section C.3 of the Appendix.

If the rich constitute the largest part of the society ($\alpha^R > \alpha^M, \alpha^P$), then P mixes between running with i_{P1} and not running, M mixes between running and not running, while R runs with i_R. The expected equilibrium winning platform, E in this case is close to i_R, as this is implemented if both or neither of the other representatives enters the electoral competition.

When the middle income is the largest group, then P plays i_{P2} and M and R are mixing between running and not. In this case the expected platform E is close to i_{P2}, as if R does not enter, then i_{P2} is implemented.

Finally if P is the largest group, then all representatives mix: P between i_{P1} and i_{P2}, M and R between running and not.[16] The expected equilibrium winning platform in this case is an interior point of i_P, as in most cases one of the ideal points of the poor is implemented.

Inequality and indifference curves

To understand what determines whether the economy is in Regime 1, Regime 2 or Regime 3, the forces that shape the indifference curves and the admissible policies have to be analyzed. The differences in income levels, the shares of the different income groups and the level of public good are the factors that have to be considered. The differences in income levels are crucial, since these govern how efficient the three tax rates are in raising revenue.

As $y^R - y^M$ decreases, τ^R looses its role as a policy tool: an increase in τ^R changes the revenue collected by less, and hence allows a smaller reduction in the burden on middle and

[15]To see this consider first the poor representative running alone with a policy from i_P below $U^M(i_R)$, from the green part of i_P. The rich group has an incentive to enter and win the election with policy i_P. If P runs with a policy from i_P and above $U^R(i_M)$, from the red part of i_P, then the middle income group can enter and win the election with i_M. If P runs with a policy from the blue part of i_P, then both M and R can enter and win the election. If M runs alone, then P can beat him for example by i_{P2}, while R can beat beaten by M or by P if running with i_{P1}.

[16]When P is mixing between the extremes, it could be possible that it is better to mix between some interior points. However, numerical tests show that P achieves the highest payoff when mixing between i_{P1} and i_{P2}.

low incomes. As the role of τ^R is reduced the conflict between M and P becomes sharper. A reduction in τ^P cannot be offset so easily by increasing τ^R, and implies a larger increase in τ^M. The set of ideal policies of the poor, i_P is reduced, and in particular all policies will feature high τ^M rates. At the same time, i_M will feature a high τ^P. In addition, a low $y^R - y^M$ implies that the effect of τ^R on the utility of the rich is relatively small. Given this, the rich will prefer the ideal policy of the middle income group. So as $y^R - y^M$ decreases, it is more likely that the economy is in Regime 1, where the median policy is i_M.

A small difference between y^M and y^P erodes the role of τ^M as a policy tool. This leads to an increased conflict of interest between the poor and the rich: they can only achieve their goals at the expense of one and other. The set of ideal policies of the poor is small in this case as well. However, with low $y^M - y^P$, all policies in i_P feature relatively high τ^R rates. This in turn makes the rich prefer the ideal policy of the middle income group, as they prefer increasing τ^P to increasing τ^M. Again, as $y^M - y^P$ decreases, the likelihood of the economy being in Regime 1 increases.

Finally consider the case of a very low y^P. In this case, there is not much to gain from increasing τ^P, and it is the rich and the middle income whose goals are in sharp contrast. In such a case, P can offer a policy from i_P, which both the rich and the middle income prefer to each others ideal policy. So as y^P decreases, it is more likely that the economy is in a situation depicted in Regime 2.

When none of these measures is small, a situation arises when there is no Condorcet winner among the ideal policies. The ideal policy of M is not better than all the ideal policies of P in the eyes of the rich, as i_P contains some policies with low enough τ^R. However, these policies have too high τ^M, so the middle income prefer i^R, which is characterised by a lower τ^M and a high τ^P. On the other hand, the middle prefer some ideal policies of the poor to i_R, since there are some with high τ^R and low τ^P. Compared to these policies, the rich prefer the ideal policy of the middle income. In such cases, the economy is in Regime 3.

To summarise, Regime 1 depicts economies where $y^R - y^M$ or $y^M - y^P$ is small, Regime 2 shows economies where y^P is small or $y^R - y^M$ or $y^M - y^P$ is higher, and Regime 3 shows a situation, when all these measures are intermediate.

The total amount of revenue to be collected, g, also affects the incidence of the different cases. For a very low g, it is relatively easier to finance the public good, both the $\tau^P = 0$ and the $\tau^P = 1$ lines shift down. This implies, that Regime 1 arises for fewer combinations of parameters: P can always offer a platform with a low τ^R and τ^P, which the rich prefer to i_M. Regime 3 is also less likely, as i_P can contain platforms with moderate τ^M and τ^R, something that both the rich and the middle income prefer to the other's ideal policy. On the other hand, as g increases, the $\tau^P = 0$ and the $\tau^P = 1$ lines both shift out, thus reducing the ideal policies of the poor. This implies that it will be less likely that P can offer a policy that the rich prefer to i_M, hence making Regime 1 more likely.

3.5.2 Equilibrium with coalitions

The definition of a stable political outcome immediately implies that a representative who would win in the absence of parties cannot be a member of a party in a stable political outcome. If a winning representative were a member of a party, he could break up this coalition, thus returning to the individual candidate election, run and win. This would guarantee him the highest possible payoff, making a partition with him in a coalition unstable with any platform.

In Regime 1, when the middle income group's ideal policy is a Condorcet winner, the only real coalition that can be part of a stable political outcome consists of the representative of the poor and the rich. Therefore, the only partitions that can be stable are $\{PR|M\}$ and $\{P|M|R\}$.

A party consisting of P and R can offer anything from their Pareto set, which is depicted by the black line, on the border of $\Delta(g)$ between i_{P2} and i_R. Any point below the dashed green line is better for the rich than i_M, while any point to the right of the dashed blue line is better for the poor than i_M. The shaded area contains the platforms that are better for both the rich and the poor than i_M, and hence any platform from this area would receive the votes of both groups. The platforms that can be *winning equilibrium platforms* are indicated by the thick black line, and are at the intersection of the shaded area and the Pareto set of R and P. Since the indifference curves of the rich are less steep than the indifference curves of the poor, this set is never empty.

Thus the party of the rich and the poor can win by advocating policies which are characterised by heavy taxation on incomes between y^P and y^M, and low taxes on income

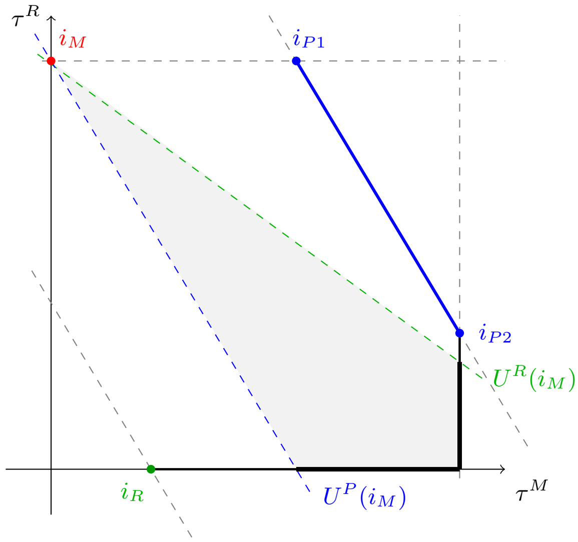

Figure 3.5: Poor-rich coalition

below y^P and above y^M. Against competition from the middle income, these policies attract the votes of the groups it represents. The party is also stable as neither the rich nor the poor want to break it. When the economy is in Regime 1, when the middle income are not sufficiently different from either the rich or the poor, a coalition comprising the rich and the poor will be stable. This is the case, as when the middle income group is very similar to either the rich or the poor, then their ideal policy constitutes a median policy, which gives the group too much power, thus disabling them from credibly committing to any coalition. However, since the middle income group is still different from the other two groups, the others can agree on at least one aspect: to put a disproportionate burden on middle income levels, this way achieving relatively low tax rates at both extremes.

In Regime 2, the only possibly stable coalition is between M and R, since P wins the single candidate election. Whether M and R can form a stable coalition and offer a pure strategy winning equilibrium platform depends on whether there is a platform that beats the set of ideal policies of the poor for both their groups.

Figure 3.6 shows the two possible cases. The left panel, Regime 2a, the platforms from the shaded area are preferred both by the rich and by the middle income group to the *entire set* of ideal policies of the poor. Any platform from this area is preferred by all

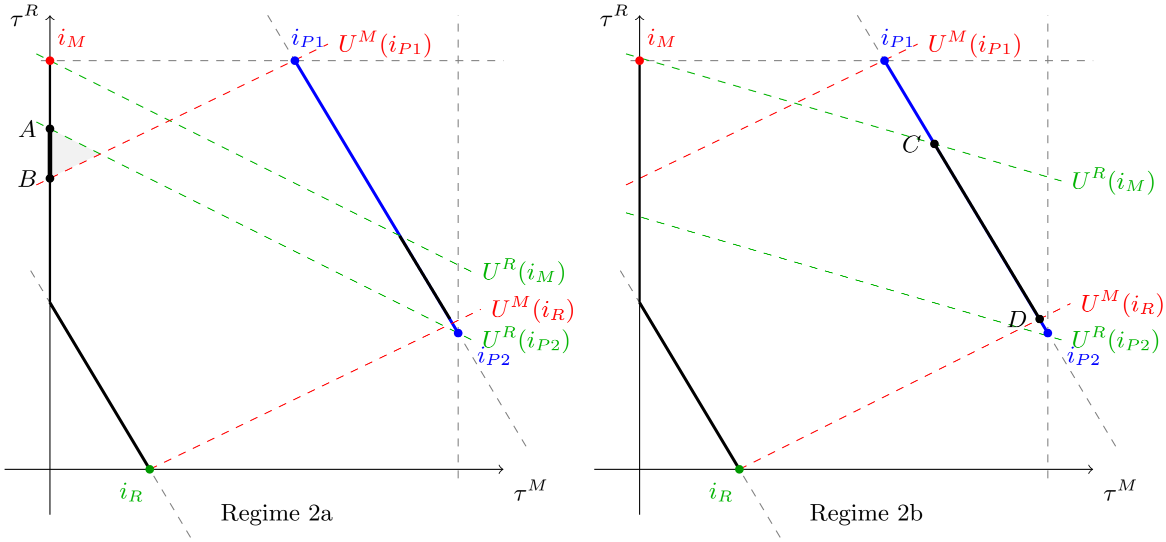

Figure 3.6: Middle-rich coalition

members of the rich and the middle income group to any point from i_P. The Pareto set of the middle income group and the rich are the black line running along the border of $\Delta(g)$ between i_M and i_R. The MR coalition can run uncontested with a platform from $[A, B]$. This guarantees the stability of the coalition, as neither the rich, nor the middle income want to split, and both groups vote for the MR coalition regardless of the policy P would run with. In such scenarios, the MR coalition will offer policies that tax the lowest part of the income very heavily, and have low to medium tax rates on higher parts of income. The expected equilibrium winning platform is $E = (A + B)/2$.

In Regime 2b, on the right panel of 3.6 there are no platforms in the set of admissible policies that both the rich and the middle income prefer to all of the poor's ideal policies. This implies that the MR coalition cannot offer anything in pure strategies, since P could then offer something from i_P that would be preferred either by R or by M to the coalition's platform, and win. In this case the coalition members would be better off by not running in the election, thus reducing the game to the single representative competition and having P win with $p \in [C, D]$. This implies that in such cases the only stable political outcome is the partition $\{P|M|R\}$ with strategies $\{p \in [C, D], \emptyset, \emptyset\}$, that is P runs uncontested and wins the election with a platform from $[C, D]$.[17] The expected equilibrium winning platform,

[17]There are cases in which the MR coalition could be stable with a mixed strategy. However, the stability

$E = (C + D)/2$, has very low tax rates on low income levels, and medium to high rates on higher income levels.

Whether the economy is in Regime 2a or in Regime 2b depends on whether M and R can find a platform that is better for both of them than any policy in the ideal set of P. This crucially depends on how sharp the conflict of interest is between the two groups. When y^P is relatively high, the conflict of the two groups is less pronounced. They can agree on increasing τ^P, and when y^P is relatively high, this action sufficiently reduces the amount of public good that has to be financed from τ^M and τ^R, mitigating the intensity of conflict. On the other hand, if y^P is low, even if they agree on taxing it very heavily, this does not allow a satisfactory reduction in *both* τ^M *and* τ^R. Another important factor is the size of $y^R - y^M$. If this difference is high, then i_M and i_{P1} are close, which makes finding a policy better than i_{P1} for M a hard task. Hence, when y^P is relatively high and $y^R - y^M$ is not very high, M and R can find a platform that is better for both groups than any policy from i_P.

In Regime 3, the model does not uniquely predict the winning party. Since the outside option is always a mixed strategy equilibrium, the expected payoff from breaking the coalition is in general a weighted average of some of the ideal policies. Let E denote the expected equilibrium winning platform of the single candidate game, which is an internal point of $\Delta(g)$.

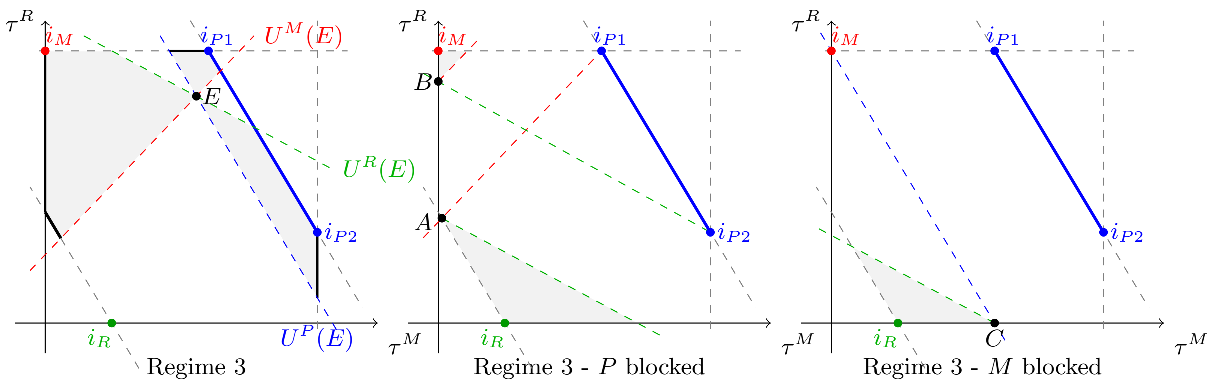

Figure 3.7: Any coalition

of a coalition with mixed strategies is questionable, therefore I omit the discussion here.

In the left panel of Figure 3.7 the shaded areas show the platforms that M and R prefer to E (the area to the left of E), that M and P prefer to E (the area above) and that P and R prefer to E (the area to the right). It is easy to see, that due to the slope of the indifference curves, whenever E is strictly inside $\Delta(g)$, any two-representative party has a segment in their Pareto set, which is better than E for both party members.[18]

The final thing to consider is whether the third representative is able to offer a platform which captures the votes of one of the coalition member's group, this way winning against the coalition. If this is the case, then the other coalition member does not have an incentive to enter the coalition, because the expected policy E is better for him than the ideal policy of the third representative. This happens, if the policies preferred to E and the policies preferred to the third representative's ideal policy by both coalition members do not have an intersection with the Pareto set of the coalition.

The MP coalition always has an equilibrium winning platform, that cannot be beaten by i_R, since that is worse for both M and P than their entire Pareto set.

The middle panel in Figure 3.7 shows the areas that are blocked by P in case of an MR coalition. Consider a case when the expected policy E is in the top left shaded area. If MR were to run with a policy they both prefer to policy E, this would have to be from $[i_m, B]$. However, this could not be a winning platform, since P could run with i_{P2} and win, since the rich and poor both vote for P. In this case M would not have an incentive to join coalition, since E is better for M than i_{P2}. Hence, if the expected platform of the game in the absence of coalitions is in the top left shaded area, then P can *block* the MR coalition from winning the election. Similarly, if E is in the bottom left shaded area, then P can capture the votes of the middle income by running with i_{P1}, and R would not want to participate in the coalition.

The right panel in Figure 3.7 shows the area blocked by M if P and R were to form a coalition. Imagine that E is in the shaded area. PR would have to run with a policy from $[i_R, C]$ to be better off than E. In this case, if M enters the electoral competition with i_M, the poor would vote for him, and he would win the election. In this case, the PR coalition is better off by withdrawing from the coalition.

[18]In the case that $\alpha^M > \alpha^R, \alpha^P$, as $\varepsilon \to 0$ the implemented platform tends to i_{P2}, which is in the border. In such a case P cannot be a member of a stable coalition.

In Regime 3 in general, more than one partition can be stable. In these cases, the prediction of the model on the equilibrium winning platforms are in expected terms, which usually gives an internal point as the expected equilibrium winning platform. It is hard to evaluate the progressivity implied by these expected platforms, as it is in fact an expected progressivity index, one that is never realised. The evaluation of these indices would be difficult, however, none of the 17 countries falls into Regime 3.

3.6 Predictions and data

In this section I summarise the model's predictions about the relation between inequality and the implied progressivity between regimes as well as within regimes. I also show the progressivity indices the model predicts for the sample countries and compare these predictions to the data.

3.6.1 Progressivity in the model

When the income difference between the middle income and either the poor or the rich is low to moderate, then the taxes are set either by a coalition of the rich and the poor, or by the middle income group. The coalition sets relatively high tax rates on middle income levels, and low rates on both extremes. This implies a tax system that is not very progressive. It is progressive moving from low to middle incomes, but for higher income levels it is actually regressive. On the other hand, if the middle income group sets the tax scheme, then the rates show the opposite pattern: high rates on top income levels, low rates on middle income levels, and low to medium rates on the lowest part of income. This tax scheme shows more progressivity, as the marginal rates on high incomes are the largest. This implies that for low income inequality countries, if the PR coalition sets the tax rates, then the system will show little progressivity, whereas if M is setting the tax scheme, then the system will be quite progressive.

If the PR coalition is setting the tax scheme within these regimes, for a higher $y^R - y^M$, the system becomes more progressive, while for a higher $y^M - y^P$ the system becomes less progressive. This is because when the rich are relatively richer, the objectives of the poor and the middle income are more aligned. Hence, even when the poor are in coalition with the rich, the rich have to bear a larger burden of the public good. On the other hand, when $y^M - y^P$ is higher, then a vast part of the public good can be financed by taxing middle incomes, which makes the system less progressive.

If M is setting the tax scheme, then in terms of $y^R - y^M$ a similar pattern emerges. If $y^R - y^M$ is high, then most of the public good is financed by taxes levied on R. However, the progressivity of the tax system does not depend too much on the difference in income levels between the middle income and the poor, since the taxes on middle incomes are kept low anyway.

To summarise, within Regime 1, progressivity increases as inequality increases between the rich and the middle income. Overall progressivity and its dependence on the inequality between the middle income and the poor, depends on who is setting the tax rates.

On the other extreme, when the rich are very rich, or the poor are very poor, Regime 2b is implemented, and it is the poor who regulate the tax system. The system set by the poor is characterised by no taxation for low income individuals. All the public good is financed by the middle income and the rich. These systems are very progressive, as the low income individuals do not (or hardly) contribute to the public good. Within these regimes, as $y^M - y^P$ increases, the progressivity of the implemented regime decreases. This is due to the fact that when $y^M - y^P$ is higher, it is relatively easy to finance the public good by taxing middle incomes.

Regime 2a is implemented for relatively low levels of $y^M - y^P$ and intermediate levels of y^R, and in these cases either the MR coalition or P sets the tax scheme. If P is setting the regime, then the same holds as for Regime 2b. When MR is setting the tax scheme, they put high taxes on low incomes, low taxes on the middle income levels, and intermediate taxes on high income levels. These regimes typically are not progressive, and progressivity is increasing as $y^M - y^P$ is decreasing, since a higher fraction of the public good has to be financed by τ^R.

Finally, the predictions of the model in Regime 3 are less precise and harder to interpret. Without coalitions, the equilibrium is in mixed strategies, whereas with coalitions, there is always more than one stable coalition. However, as presented in the next section, for the countries in my sample, none fall in the region where Regime 3 would be implemented.

3.6.2 Model predictions and data

In this section I present the model's predictions about the progressivity index in the 17 OECD countries. In general, the model does quite well: more unequal countries implement more progressive tax schemes. Even though the model over-predicts the level of progressivity, it gives a good prediction of the magnitude of increase in progressivity with income inequality.

There are a few difficulties in translating the data into the parameters of the model. First of all, I have data on nine occupation categories, which I have to transform into three groups, the poor, the middle income and the rich. Second, I have to fix the amount of revenue that needs to be collected.

I divide the nine income categories into three income groups in two ways for each country in my sample. First, I group the three richest occupations into the rich group, the three middle income occupations into the middle income group and the three lowest paid occupations into the poor group (Panel A). The second is a more sophisticated division: country-by-country I cut the nine groups into three categories depending on the distance in income between the different occupations (Panel B).[19]

Taking the level of public good to be provided from the data raises several issues. One problem is what part of government expenditure to take as the public good, g. Taking the entire government expenditure is problematic, as not all of it is financed from personal income taxes. Also, the public good has to be measured in terms of economy-wide personal income (or average income): $g = \eta\overline{y}$. From this point of view, it is not clear what to take as $\overline{y}$, which in the model is economy-wide income. Since the earnings data is only on earnings from employment and self-employment, the best measure would be to take total personal income or labour earnings, however, this data is not available. Another way of getting η from the data is through the average tax rate. In the model, the required amount of public good pins down the average tax rate: $\overline{\tau} = \frac{g}{\overline{y}} = \eta$, and since I have data on the average tax rate for each country, I use these as a proxy for η.

Figure 3.8 plots the predicted progressivity indices against the actual progressivity indices for the 17 countries. The top row contains the first type of division (Panel A), with

[19] See Section C.4 of the Appendix for division.

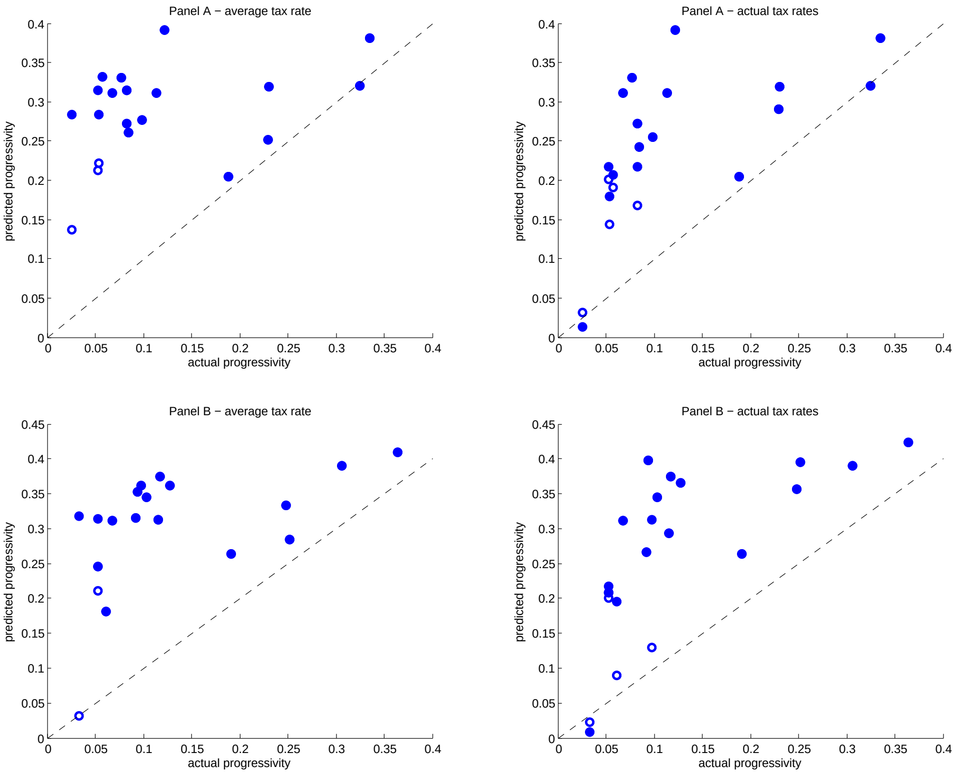

Figure 3.8: Actual and predicted progressivity indices

Notes: Actual and predicted progressivity index for the 17 OECD countries with different divisions of occupation groups into income categories (Panel A and Panel B) and different levels of public good (actual average tax rate for each country and their cross country average). The actual progressivity indices are calculated using the same division of occupation groups into income categories.

three occupations in each income group, while the second row has the country specific division (Panel B), based on the difference in income levels. The left column shows the predictions when the cross country average of the country-specific average income tax rate is used as a proxy for η, whereas the right panel shows the predictions using for each country their own average tax rate as a proxy for η. In all cases I calculated the progressivity index of the implied equilibrium winning platform for all 17 countries, using their actual income and population shares. The full circles show the progressivity implied by the single candidate equilibrium, while if a stable coalition exists, I indicate the progressivity implied by their equilibrium winning platform as well, with empty circles.

The model predicts the existence of stable equilibrium platforms for only a few countries, where income inequality is low. The stable equilibrium platforms that emerge in these cases are mostly PR platforms in Regime 1 or MR in Regime 2a. This can be seen in Figure 3.8, as the empty circles are in most cases *below* the full circles, in line with the observation that PR coalitions implement a less progressive tax scheme than the one that representative M would implement, and that MR coalitions implement a less progressive system than the one that representative P would implement.

Note that in all cases the model over-predicts the progressivity indices obtained from the data. The reason for this is that the model predicts that in the majority of cases it is the representative of the poor, who wins the election (Regime 2b). In this case, the implemented policy is one that is better for the rich than the ideal policy of the middle income group, and is better for the middle than the ideal policy of the rich. These policies implemented by the poor are always progressive, but the degree of progressivity depends on the income of the other two groups. As inequality increases, the implemented policy taxes the rich more heavily, and hence is more progressive.

The degree of over-prediction of the model is smaller if the actual tax rates are used. This is the case, since countries with lower inequality tend to have higher average tax rates. A higher average tax rate in the data implies a higher public good provision in the model. This in turn implies, that Regime 2b will be implemented in fewer cases, thus leading to lower progressivity, especially for the low inequality countries, where the over-prediction of the model is the highest. Hence, using the actual average tax rates rather than their cross-country average improves the fit of the model.

3.7 Conclusion

In this chapter I present a model of political coalition formation, where a society has to decide how to share the burden of providing the public good. I show that in such a model, more unequal societies implement a more progressive system. This is due to two factors: the more unequal a society is, the more power the poor have and the less likely it is that the rich can be in a winning coalition. Thus depending on income inequality different regimes are in place, which implement different type of tax schemes. Second, within a regime, more inequality increases the progressivity of the system. If inequality is higher because the rich are further away from the middle and low income individuals, then it is relatively easy to tax the rich.

I test the predictions of the model by applying it to the income distribution of 17 OECD countries. By comparing the model predictions to the data on the progressivity of the tax system, I find that even though the model over-predicts the degree of progressivity, it predicts, in line with real world tax schemes, that more unequal societies implement more progressive tax schemes. The predicting power of the model is improved if the division of occupation groups into the three income categories is country-specific and if country level average tax rates are used as a proxy for $g/\overline{y}$.

The main dispersion in the model's prediction on the progressivity of income tax systems is driven by within regime variation, as I find that in a majority of the countries the representative of the poor sets the tax scheme. Where stable coalitions emerge, in low inequality countries, the fit of the model is significantly improved, by reducing the predicted progressivity.

These findings imply that the predictive power of the model could be improved by obtaining better data on the distribution of income. My proxy for the distribution of earnings is very coarse: I combine two labour force surveys, the ELFS and the SES, to obtain the share and relative earnings of the nine main occupation groups. The relative average earnings of the occupation groups might be too coarse to capture the full dispersion income dispersion in reality, and might over-predict the power of poorest groups. An alternative would be to use household surveys for all countries, this way having a better proxy for the distribution of income. Parallel to improving the data on the distribution of income, the income distribution could also be refined in the model. Allowing for more than three income groups would potentially reduce the power of the poorest group, and open up the possibility to more diverse coalition formation, which seems to be the key in matching the progressivity indices.

Dealing with data from household surveys would also allow a more extensive analysis of the progressivity of the tax scheme of these countries. Since household surveys contain data on earnings from various sources, number of dependants and consumption, an analysis could be conducted on the progressivity of the different type of taxes (personal income tax, capital tax, consumption tax, social security contributions) as well as of the tax scheme as a whole. Looking at the progressivity of the tax scheme as a whole would allow a better proxy for the required revenue from the tax scheme, for example government spending net of changes in government debt. On the other hand, while the personal income tax has a clear redistributive role, this is not true for social security contributions and consumption taxes, which might be determined through another process.

Appendix A

Appendix to Chapter 1

A.1 R&D

A.1.1 Probability of successful innovation for a given R&D firm

The Poisson arrival rate of innovation for all firms indexed by $k = 1, 2, ...$ when spending z_k units on R&D is ηz_k. Since Poisson processes are additive, the economy wide arrival rate of innovation is $\eta \sum_{k=1}^{\infty} z_k$ if $\sum_{k=1}^{\infty} z_k \equiv \overline{z} < \infty$. In this case the probability that there is at least one innovation until the end of the period is:

$$\int_0^1 \eta \overline{z} e^{-\eta \overline{z} t} dt = 1 - e^{-\eta \overline{z}}.$$

I assume that once a firm has a successful innovation, that firm receives the patent and innovation on that line is finished for that period. Then the probability that matters is the probability that a given firm has the first innovation. The probability that firm k has the first innovation at time t is:

$$\eta z_k e^{-\eta z_k t} (e^{-\eta(\overline{z} - z_k)t}) = \eta z_k e^{-\eta \overline{z} t}.$$

The probability that firm k has the first successful innovation until the end of the period is just:

$$\int_0^1 \lambda z_k e^{-\eta \overline{z} t} dt = \frac{z_k}{\overline{z}} (1 - e^{-\eta \overline{z}}).$$

Which is what I wanted to show.

A.1.2 Monopoly pricing

Lemma A.1. *If $\overline{q} > (1-\beta)^{-\frac{1-\beta}{\beta}}$ then at any moment in time only the best quality of any machine will be bought at its monopoly price.*

Proof. When the marginal cost of producing a machine of quality q_1 is q_1, then given the demand in (1.5) the monopoly price of this machine is $\chi_1 = \frac{q_1}{1-\beta}$. If an intermediate good producing firm uses this machine his profit is:

$$\pi_1 = (p^s)^{\frac{1}{\beta}} N^s q_1 \left((1-\beta)^{\frac{1-\beta}{\beta}-1} - (1-\beta)^{\frac{1-\beta}{\beta}}\right).$$

If the firm instead uses a lower $q_2 = \frac{q_1}{\overline{q}^k}$ quality machine at the price of its marginal cost $\chi_2 = q_2$, then his profit is:

$$\pi_2 = (p^s)^{\frac{1}{\beta}} N^s q_1 \left(\frac{1}{\overline{q}^k}(1-\beta)^{-1} - \frac{1}{\overline{q}^k}\right).$$

If $\pi_1 > \pi_2$ for all $k > 0$ integers, then only the best quality of any machine will be bought in equilibrium at its monopoly price.
The $\pi_1 > \pi_2$ condition is equivalent to:

$$(1-\beta)^{\frac{1-\beta}{\beta}-1} - (1-\beta)^{\frac{1-\beta}{\beta}} > \frac{1}{\overline{q}^k}(1-\beta)^{-1} - \frac{1}{\overline{q}^k}.$$

With some algebra we get that this is equivalent to:

$$\overline{q}^k > (1-\beta)^{-\frac{1-\beta}{\beta}}.$$

Since the above holds for $k = 1$ and $\overline{q} > 1$, it holds for all $k \geq 1$. □

A.2 Steady State

A.2.1

Since the total size of the population is constant, both N^{h*} and N^{l*} are constant along the BGP. The supply of effective labour, N^{h*} and N^{l*} can only be constant if the threshold abilities for unemployment, $\underline{a}^l, \underline{a}^h$ and the optimal education decision $e(a,c)$ for all a and c are constant. The cutoff abilities for unemployment are defined by:

$$\underline{w}_t = \underline{a}^l \beta (1-\beta)^{\frac{1-2\beta}{\beta}} (p_t^{l*})^{\frac{1}{\beta}} Q_t^{l*},$$
$$\underline{w}_t = \underline{a}^h \beta (1-\beta)^{\frac{1-2\beta}{\beta}} (p_t^{h*})^{\frac{1}{\beta}} Q_t^{h*}.$$

Hence along the steady state where both $\underline{a}^h$ and $\underline{a}^l$ are constant

$$\frac{\underline{a}^l}{\underline{a}^h}\frac{p_t^{h*}}{p_t^{l*}}^{-\frac{1}{\beta}} = \frac{Q_t^{h*}}{Q_t^{l*}}.$$

The relative price of the intermediate goods depends on the relative quality and the relative labour supply in the two groups. Combining the above with (1.12) gives:

$$\frac{Q_t^{h*}}{Q_t^{l*}}^{1-\frac{(1-\rho)}{(1-(1-\beta)\rho)}} = \frac{\underline{a}^l}{\underline{a}^h}\gamma^{-\frac{1}{(1-(1-\beta)\rho)}}\left(\frac{N^{h*}}{N^{l*}}\right)^{\frac{(1-\rho)}{(1-(1-\beta)\rho)}}. \tag{A.1}$$

Since $\beta \neq 0$ the relative quality, $Q_t^* = Q_t^{h*}/Q_t^{l*}$ is constant in the steady state. This also immediately implies that the relative price of the intermediates, $p^* = p_t^{h*}/p_t^{l*}$ is constant in the steady state. Since the price of the final good is normalized to one, this also implies that p^{h*} and p^{l*} are constant.

If prices of intermediate goods are constant, and the supply of both types of effective labour is constant, then from (1.6), the per period profit from owning a leading vintage of quality q is constant as well. In the next section I show that constant period profits imply that steady state R&D investments on a line j in sector s are independent of the quality of the leading vintage in that line.

A.2.2 R&D spending

Using that the steady state profits in sector s are constant:

Lemma A.2. *The total R&D spending on any line for a given quality is constant along the BGP:* $\overline{z}_t^{j,s*}(q) = \overline{z}_{t+T}^{j,s*}(q) = \overline{z}^{j,s*}(q)$ *for all* $t, T \geq 0$*.*

Proof. The R&D spending on each line has to be either constant or growing at a constant rate along the balanced growth path. This implies that the equilibrium total R&D spending on line j in sector s can be written as: $\overline{z}_{t+T}^{j,s*}(q) = \gamma^T \overline{z}_t^{j,s*}(q)$. Where $\gamma > 0$ is the growth rate of the R&D spending on line j in sector s for a given quality q. In what follows I denote $\overline{z}_t^{j,s*}(q)$ by z_t. Conditional on quality q, the per period profit is constant, $\pi^{s*}q$, since both N^{s*} and p^{s*} are constant along the BGP. Iterating forward (1.7), the value of owning the leading vintage on line j with quality q at time $t+T$ can be written as:

$$V_{t+T}(q) = q\pi^{s*}\sum_{\tau=0}^{\infty}\frac{e^{-\eta z_t \gamma^T \frac{\gamma^\tau - 1}{\gamma - 1}}}{(1+r)^\tau}.$$

Given $V_{t+T}(q)$ the equilibrium level of R&D spending is z_{t+T} if (1.8) is satisfied:

$$\frac{1}{1+r}\frac{(V_{t+T}(q))}{z_{t+T}}(1-e^{-\eta z_{t+T}})=q.$$

This has to hold for all $T>0$, implying that

$$\begin{aligned}&\textstyle\sum_{k=0}^{\infty}\frac{e^{-\eta z_t\frac{\gamma^k-1}{\gamma-1}}}{(1+r)^k}(1-e^{-\eta z_t})=\\&\textstyle\frac{1}{\gamma}\sum_{k=0}^{\infty}\frac{e^{-\eta z_t\gamma\frac{\gamma^k-1}{\gamma-1}}}{(1+r)^k}(1-e^{-\eta z_t\gamma})=...\\&\textstyle\frac{1}{\gamma^T}\sum_{k=0}^{\infty}\frac{e^{-\eta z_t\gamma^T\frac{\gamma^k-1}{\gamma-1}}}{(1+r)^k}(1-e^{-\eta z_t\gamma^T}).\end{aligned}$$

To simplify notation denote $a_k\equiv\frac{\gamma^k-1}{\gamma-1}$ and $\eta z_t\equiv b$. Since the above should hold for any $T>0$, this implies that the difference between two consecutive terms should be zero. Taking logarithm and derivative with respect to T yields the following condition:

$$0=\ln\gamma\left(-1+\left(\frac{b\gamma^Te^{-b\gamma^T}}{1-e^{-b\gamma^T}}-\frac{b\gamma^T\sum_{k=0}^{\infty}\frac{a_ke^{-b\gamma^Ta_k}}{(1+r)^k}}{\sum_{k=0}^{\infty}\frac{e^{-b\gamma^Ta_k}}{(1+r)^k}}\right)\right).\tag{A.2}$$

This has to hold for all $T>0$, even as $T\to\infty$. There are three cases: $\gamma>1$, $\gamma<1$ and $\gamma=1$. For $\gamma=1$ the above trivially holds for all $T>0$.

For $\gamma>1$ taking the limes yields:

$$\begin{aligned}&\lim_{T\to\infty}\left(\frac{b\gamma^Te^{-b\gamma^T}}{1-e^{-b\gamma^T}}-\frac{b\gamma^T\sum_{k=0}^{\infty}\frac{a_ke^{-b\gamma^Ta_k}}{(1+r)^k}}{\sum_{k=0}^{\infty}\frac{e^{-b\gamma^Ta_k}}{(1+r)^k}}\right)=\\&0-\lim_{T\to\infty}\frac{b\gamma^T\sum_{k=0}^{\infty}\frac{a_ke^{-b\gamma^Ta_k}}{(1+r)^k}}{\sum_{k=0}^{\infty}\frac{e^{-b\gamma^Ta_k}}{(1+r)^k}}<0.\end{aligned}$$

Where the second term is non-negative, implying a negative value as T grows very large. Hence, for $\gamma>1$ (A.2) does not hold for all $T>0$.

For $\gamma<1$ I will show that the second term in the brackets is strictly smaller than 1, except in the limit. Denote $x\equiv b\gamma^T$, then as $T\to\infty$, $x\to 0$. The first term is smaller than 1 for any $x>0$:

$$\frac{xe^{-x}}{1-e^{-x}}<1\Leftrightarrow e^{-x}(1+x)<1.$$

For $x=0$, $e^{-x}(1+x)=1$. The derivative of the left hand side is $-e^{-x}x$, which is negative for all $x>0$, implying that for any $x>0$ the above inequality strictly holds.

The second term in the brackets is strictly positive for all $T>0$ and finite. This implies that the term in the brackets is strictly smaller than 1 for any finite T. Hence (A.2) does not hold for any

$T > 0$. $\square$

Therefore in the steady state $\overline{z}^{j,s*}$ is constant for a line with quality q. This also implies that the value of owning the leading vintage with quality q in line j and sector s is constant in the steady state. Its value can be expressed from iterating (1.7) forward and using the above lemma as:

$$V_t^{j,s*}(q) = \frac{q\beta(1-\beta)^{\frac{1-\beta}{\beta}}(p^{s*})^{\frac{1}{\beta}}N^{s*}}{1 - \frac{e^{-\eta\overline{z}^{j,s*}(q)}}{1+r}}.$$

Note that the value of owning a leading vintage is proportional to its quality level. This observation leads to the following corollary:

Corollary A.1. *In the steady state the total R&D spending on each line within a sector is constant and equal:* $\overline{z}_t^{j,s*} = \overline{z}_{t+v}^{k,s*} = \overline{z}^{s*}$ *for all* $j, k \in s$ *and all* $v \geq 0$.

Proof. Using (1.8) and the steady state value of owning a leading vintage, the total amount of R&D spending on line j in sector s with quality q is implicitly defined by:

$$\beta(1-\beta)^{\frac{1-\beta}{\beta}}(p^{s*})^{\frac{1}{\beta}}N^{s*} = B\overline{z}^{j,s*}(q)\frac{(1+r-e^{-\eta\overline{z}^{j,s*}(q)})}{1-e^{-\eta\overline{z}^{j,s*}(q)}}.$$

The left hand side only depends on sector specific variables, hence the total amount of R&D spending on improving line j in sector s is independent of the current highest quality, q on that line. Since it is only the quality level that distinguishes the lines from each other within a sector the corollary follows. $\square$

A.2.3

Therefore, the total amount of R&D spending on each line within a sector is equal and constant over time. This equilibrium R&D spending is given by (1.21). In the steady state $\overline{z}^{h*} = \overline{z}^{l*} = \overline{z}^*$ and the growth rate is $g^* = 1 + (\overline{q} - 1)(1 - e^{-\eta\overline{z}^*})$.

The price of the intermediates can be expressed from substituting the steady state relative price (1.22) into the intermediate good prices (1.2):

$$p^{l*} = \left(1 + \gamma\left(\frac{N^{h*}}{N^{l*}}\right)^{\frac{\beta\rho}{1-\rho}}\right)^{\frac{1-\rho}{\rho}}, \tag{A.3}$$

$$p^{h*} = \left(\left(\frac{N^{h*}}{N^{l*}}\right)^{-\frac{\beta\rho}{1-\rho}} + \gamma\right)^{\frac{1-\rho}{\rho}}. \tag{A.4}$$

Using the steady state relative price and the steady state R&D investment:

$$B\overline{z}^* \frac{(1 + r - e^{-\eta \overline{z}^*})}{1 - e^{-\eta \overline{z}^*}} = \beta(1-\beta)^{\frac{1-\beta}{\beta}} \left(\gamma N^{h*\frac{\beta\rho}{1-\rho}} + N^{l*\frac{\beta\rho}{1-\rho}} \right)^{\frac{1-\rho}{\beta\rho}}. \tag{A.5}$$

The right hand side is the steady state per period profit from owning the leading vintage normalized by the quality of the vintage. This profit is increasing in both N^{h*} and N^{l*}. If the labour supply increases, then any unit of investment into R&D has a higher expected return, since there are more people who are able to use it. This implies that the steady state R&D spending and the steady state growth rate is increasing in the effective labour supplies.

A.2.4 Proof of Lemma 2

Proof. To see that $\underline{a}^{l*}$ and c^* uniquely define $\underline{a}^{h*}$ consider equation (1.26), making use of (1.24):

$$\underline{a}^{h*} = \underline{a}^{l*} \gamma^{-\frac{1}{1-\rho}} \left(\frac{N^{l*}}{N^{h*}} \right)^{\frac{\beta\rho}{1-\rho} - 1}.$$

N^{h*} is decreasing in $\underline{a}^{h*}$. If $\frac{\beta\rho}{1-\rho} - 1 < 0$ then the right hand side is decreasing in $\underline{a}^{h*}$, while the left hand side is increasing, hence there is a unique $\underline{a}^{h*}$ that satisfies the equation.
If $\frac{\beta\rho}{1-\rho} - 1 > 0$, then both the right and the left hand side is increasing in $\underline{a}^{h*}$. The derivative of the left hand side is 1, while the derivative of the right hand side is:

$$\frac{\partial \underline{a}^{l*} \gamma^{-\frac{1}{1-\rho}} \left(\frac{N^{l*}}{N^{h*}} \right)^{\frac{\beta\rho}{1-\rho} - 1}}{\partial \underline{a}^{h*}} = \underline{a}^{h*} \left(\frac{\beta\rho}{1-\rho} - 1 \right) \frac{\left((1-\lambda) \int_{\underline{c}}^{\overline{c}} (1-c) g(c) dc + \lambda \right) \underline{a}^{h*} f(\underline{a}^{h*})}{N^{h*}}.$$

The second two terms are smaller than one, and the first term is also smaller than one for any $\underline{a}^{h*}$ that gives a sensible unemployment rate. This implies that in the region of interest there is a unique solution. □

A.3 Calibration

A.3.1 Ability and Cost Distribution

Given the assumptions on the distribution of a and c, and the thresholds $\underline{a}^{l*}, \underline{a}^{h*}$ and c^* the high- and low-skilled effective labour supplies are:

$$\begin{aligned} N^{h*} = & \left((1-\lambda)\int_0^{\overline{c}}(1-c)g(c)dc+\lambda\right)\int_{\underline{a}^{h*}}^{\underline{a}^{l*}} af(a)da + \\ & \left((1-\lambda)\int_0^{c^*}(1-c)g(c)dc+\lambda G(c^*)\right)\int_{\underline{a}^{l*}}^{\infty} af(a)da, \end{aligned} \tag{A.6}$$

$$N^{l*} = (1-G(c^*))\int_{\underline{a}^{l*}}^{\infty} af(a)da. \tag{A.7}$$

Where $f(\cdot)$ is the probability density function of the ability distribution and $G(\cdot)$ is the cumulative distribution function of the cost distribution. The above expressions account for the fact that those members of the new generation who choose to acquire education only work $1-c$ fraction of the first period of their life.

Note that the effective supply of labour is not equivalent to the measure of high- and low-skilled individuals, the difference being that the former counts the total *ability* available, while the latter counts the number of people. The measure of high-skilled, low-skilled and unemployed is given by:

$$\begin{aligned} L^{h*} = & \left((1-\lambda)\int_0^{\overline{c}}(1-c)g(c)dc+\lambda\right)\int_{\underline{a}^{h*}}^{\underline{a}^{l*}} f(a)da + \\ & \left((1-\lambda)\int_0^{c^*}(1-c)g(c)dc+\lambda G(c^*)\right)G(c^*)\int_{\underline{a}^{l*}}^{\infty} f(a)da, \\ L^{l*} = & (1-G(c^*))\int_{\underline{a}^{l*}}^{\infty} f(a)da, \\ L^{u*} = & \int_0^{\underline{a}^{l*}} f(a)da. \end{aligned}$$

The cutoff ability of unemployment for the low-skilled is found by matching the fraction of unemployed:

$$U = \int_0^{\underline{a}^{h*}} f(a)da \quad \Leftrightarrow \quad \underline{a}^{h*} = e^{(\sigma\Phi^{-1}(U)+\mu)}. \tag{A.8}$$

The cutoff time cost is found by matching the fraction of low-skilled:

$$L^{l} = (1-G(c^*))\int_{\underline{a}^{l*}}^{\infty} f(a)da, \tag{A.9}$$

where $\underline{a}^{l*}$ satisfies (using (1.26)):

$$\underline{a}^{l*} = \underline{a}^{h*} \frac{w^{h*}}{w^{l*}} = \underline{a}^{h*} \frac{\overline{w}^h}{\overline{w}^l} \frac{\overline{a}^l}{\overline{a}^h},$$

and $\overline{a}^h, \overline{a}^l$ are the average abilities and $\overline{w}^h, \overline{w}^l$ are the average wages in the two education groups. The average ability in a sector is the ratio of the supply of efficiency units of labour to the supply of raw labour in that sector: $\overline{a}^s = N^s/L^s$. The supply of high- and low-skilled raw labour, L^h and L^l are observed from the data, but N^h and N^l have to be calculated using (A.6).

This way for any cost and ability distribution $\underline{a}^{l*}, \underline{a}^{h*}$ and c^* is given as a function of the fraction of unemployed and low-skilled workers. Finally note that the three thresholds and the parameters of the ability and cost distribution are sufficient to calculate the average ability in both education groups.

A.3.2 Elasticity of Substitution

The consensus value is around 1.4 based on the paper by Katz and Murphy (1992). This original estimate was based on 25 data points, and Goldin and Katz (2008) updated this estimate by including more years and found an elasticity of 1.64. The estimating equation is:

$$\log \frac{w^h}{w^l} = \alpha_1 + \alpha_2 \log \frac{H}{L}. \tag{A.10}$$

These estimates typically adjust for productivity differentials within a skill-group, but do not adjust for differentials between skill groups. Hence the labour aggregates H and L are between the measure of effective labour and raw labour. The parameter estimate $\widehat{\alpha}_2$ is interpreted as the inverse of the elasticity of substitution between the two types of labour. I cannot use these estimates directly for several reasons.

First of all, the interpretation of $\widehat{\alpha}_2$ is different depending on the assumptions. To see this note that the skill premium per efficiency unit can be expressed as

$$\frac{w^h}{w^l} = \gamma^{\frac{1}{1-\rho}} \frac{N^h}{N^l}^{\frac{\beta\rho}{1-\rho}-1},$$

along the balanced growth path, while it can be measured as

$$\frac{w^h}{w^l} = \gamma^{\frac{1}{1-(1-\beta)\rho}} \left(\frac{N^h}{N^l}\right)^{-\frac{1-\rho}{1-(1-\beta)\rho}} \left(\frac{Q^h}{Q^l}\right)^{\frac{\beta\rho}{1-(1-\beta)\rho}}.$$

in the transition. Thereby, the interpretation along the BGP is $\widehat{\alpha}_2 = \beta\rho/(1-\rho) - 1$, while along the transition it is $\widehat{\alpha}_2 = -(1-\rho)/(1-(1-\beta)\rho)$. However, the estimate of $\widehat{\alpha}_2$ in the transition will be biased due to the lack of a good measure of average quality in the two sectors. Second, as noted before, the measure of labour supply aggregates used inKatz and Murphy (1992) are not the effective supply of labour, which in the model determines wages. Moreover, the measure of skill premium is not the skill premium per efficiency unit w^h/w^l of the model, it is probably closer to the average skill premium. Due to these reasons, reinterpreting the implications of the value of $\widehat{\alpha}_2$ for ρ is not sufficient to use these estimates in my calibration.

A.4 Transitional Dynamics

To use perturbation methods all variables have to be stationary in the steady state. Two variables are not stationary in the steady state, the value of owning a leading vintage, and the present value gain from working as high-skilled rather than low-skilled. The value of owning the leading vintage, $V_t^s(q)$, is proportional to the quality of that machine. Let v_t^s denote the normalized value of owning the leading vintage in sector s at time t:

$$v_t^h = \tfrac{V_t^h(q)}{q} \quad v_t^l = \tfrac{V_t^l(q)}{q}.$$

In the steady state the discounted expected present value of working as high-skilled rather than low-skilled starting from period t is proportional to the wages in period t, which is proportional to the average quality. Let Δ_t denote the normalized present value gain per unit of effective labour from acquiring education conditional on being employed in every future period (normalized by the current quality in the low-skilled sector):

$$\Delta_t = \sum_{j=0}^{\infty} \left(\frac{\lambda}{1+r}\right)^j \frac{w_{t+j}^h - w_{t+j}^l}{Q_t^l}.$$

The equations that hold throughout the transition in terms of these normalized variables are:

$$
\begin{array}{lll}
v^s_{t+1} = & B\frac{(1+r)\overline{z}^s}{1-e^{-\eta\overline{z}^s_t}} & s=l,h \\
v^s_t = & \beta(1-\beta)^{\frac{1-\beta}{\beta}}(p^s_t)^{\frac{1}{\beta}}N^s_t - \frac{e^{-\eta\overline{z}^s_t}}{1+r}v^s_{t+1} & s=l,h \\
g^s_{t+1} = & 1+(\overline{q}-1)(1-e^{-\eta\overline{z}^s_t}) & s=l,h \\
p^h_t = & \left(\gamma+\gamma^{\frac{\beta\rho}{(1-(1-\beta)\rho)(1-\rho)}}\left(Q_t\frac{N^h_t}{N^l_t}\right)^{-\frac{\beta\rho}{(1-(1-\beta)\rho)}}\right)^{\frac{1-\rho}{\rho}} & \\
p^l_t = & \left(1+\gamma^{1-\frac{\beta\rho}{(1-(1-\beta)\rho)(1-\rho)}}\left(Q_t\frac{N^h_t}{N^l_t}\right)^{\frac{\beta\rho}{(1-(1-\beta)\rho)}}\right)^{\frac{1-\rho}{\rho}} & \\
\widetilde{w} = & \underline{a}^l_t\beta(1-\beta)^{\frac{1-2\beta}{\beta}}(p^l_t)^{\frac{1}{\beta}} & \\
\widetilde{w} = & \underline{a}^h_t\beta(1-\beta)^{\frac{1-2\beta}{\beta}}(p^h_t)^{\frac{1}{\beta}}Q_t & \\
Q_{t+1} = & \frac{g^h_{t+1}}{g^l_{t+1}}Q_t & \\
\Delta_t = & c^*_t\beta(1-\beta)^{\frac{1-2\beta}{\beta}}(p^h_t)^{\frac{1}{\beta}}Q_t & \\
\Delta_t = & \beta(1-\beta)^{\frac{1-2\beta}{\beta}}\left((p^h_t)^{\frac{1}{\beta}}Q_t-(p^l_t)^{\frac{1}{\beta}}\right)+\frac{\lambda}{1+r}\frac{1}{g^l_{t+1}}\Delta_{t+1} & \\
N^h_t = & \lambda N^h_{t-1}-\lambda(1-\lambda)\frac{\overline{c}-\frac{\overline{c}^2}{2}}{\overline{c}}\int_{\underline{a}^h_{t-1}}^{\underline{a}^l_{t-1}}af(a)da & \\
 & +\lambda(1-\lambda)\int_{\underline{a}^h_{t-1}}^{\underline{a}^l_{t-1}}af(a)da & \\
 & -\lambda(1-\lambda)\frac{c^*_{t-1}-\frac{{c^*_{t-1}}^2}{2}}{\overline{c}}\int_{\underline{a}^l_{t-1}}^{\infty}af(a)da & \\
 & +\lambda(1-\lambda)\frac{c^*_{t-1}}{\overline{c}}\int_{\underline{a}^l_{t-1}}^{\infty}af(a)da & \\
 & +\lambda\frac{c^*_t-\frac{{c^*_t}^2}{2}}{\overline{c}}\int_{\underline{a}^l_t}^{\infty}af(a)da & \\
 & +\lambda\frac{\overline{c}-\frac{\overline{c}^2}{2}}{\overline{c}}\int_{\underline{a}^h_t}^{\underline{a}^l_t}af(a)da+\lambda(1-\lambda)\int_{\underline{a}^h_t}^{\underline{a}^h_{t-1}}af(a)da & \\
N^l_t = & \lambda N^l_{t-1}+(1-\lambda)\frac{\overline{c}-c^*_t}{\overline{c}}\int_{\underline{a}^l_t}^{\infty}af(a)da &
\end{array}
$$

A.5 Decomposition

I denote the initial steady state by a subscript 0 and the new steady state by a subscript 1.

A.5.1 Exogenous education, exogenous technology

Since the total supply of high-skilled effective and raw labour is constant $N^{h*}_0 = N^{l*}_1 = N^{h*}$. The equations that define the new steady state are:

$$N^{l*}_1 = \int_{\underline{a}^{l*}_1}^{\underline{a}^{h*}_0} af(a)da + N^{l*}_0.$$

Note that this adjustment only takes place if $\underline{a}^{l*}_1 < \underline{a}^{h*}_0$, that is if the decrease in $\widetilde{w}$ is large enough. When the change in the minimum wage is small, then the decline only implies that some people should not get educated, because they would be productive enough even without acquiring skills. However, since education is fixed, this would imply no adjustments in the economy.

$$\underline{a}_1^{l*}(p_1^{l*})^{\frac{1}{\beta}} = \widetilde{w}_1.$$

$$p_1^{l*} = \left(1 + \gamma^{1-\frac{\beta\rho}{(1-\rho+\beta\rho)(1-\rho)}} \left(\frac{N^{h*}}{N_1^{l*}}\right)^{\frac{\beta\rho}{1-\rho+\beta\rho}} Q^{\frac{\beta\rho}{1-\rho+\beta\rho}}\right)^{\frac{1-\rho}{\rho}},$$

where $Q = Q^h/Q^l$ and $Q^s = \frac{1}{\beta}\int_0^1 (q^{s,j})^{\frac{1}{\beta}} (\chi^{s,j})^{\frac{1-\beta}{\beta}} dj$. I do not explicitly model the pricing of the machines, I denote the price of a machine with quality q^s in line j by $\chi^{s,j}$. The assumption that technology is exogenous boils down to having Q^h and Q^l growing at the same constant rate. If the pricing of machines would follow monopoly pricing or competitive pricing, then this would be equivalent to a constant growth rate in the quality of each line.

Since education and technology are fixed, the new steady state is reached in the moment of the announcement. The lower bound of unemployment for the low-skilled, which implies the adjustment in the size of the low skilled labour force. The new skill premium is:

$$\frac{w_1^{h*}}{w_1^{l*}} = \gamma^{\frac{1}{1-\rho+\beta\rho}} \left(\frac{N^{h*}}{N_1^{l*}}\right)^{-\frac{1-\rho}{1-\rho+\beta\rho}} Q^{\frac{\beta\rho}{1-\rho+\beta\rho}},$$

which is higher than before.

A.5.2 Endogenous education, exogenous technology

The supply of high- and low-skilled workers in the new steady state are as in (A.6) and (A.7), while through the transition they are governed by the same equations as in section D of the Appendix. The threshold for low- and high-skilled unemployment are given exactly as in (1.26) and (1.25) (again the transition is as in section D of the Appendix, except for $Q_t = Q$ here, since technology is exogenous). The cutoff time cost for acquiring education is given by:

$$c^* = \frac{1 - \frac{w_t^{l*}}{w_t^{h*}}}{1 - \frac{g\lambda}{1+r}},$$

where g is the exogenous growth rate of the economy. The skill premium is given by:

$$\frac{w^h}{w^l} = \gamma^{\frac{1}{1-\rho+\beta\rho}} \left(\frac{N^{h*}}{N^{l*}}\right)^{-\frac{1-\rho}{1-\rho+\beta\rho}} Q^{\frac{\beta\rho}{1-\rho+\beta\rho}}.$$

The price of intermediates is given by:

$$p_t^{h*} = \left(\gamma^{\frac{\beta\rho}{(1-\rho+\beta\rho)(1-\rho)}} \left(\frac{N_t^{h*}}{N_t^{l*}} \right)^{\frac{-\beta\rho}{1-\rho+\beta\rho}} Q^{\frac{-\beta\rho}{1-\rho+\beta\rho}} + \gamma \right)^{\frac{1-\rho}{\rho}},$$

$$p_t^{l*} = \left(1 + \gamma^{1-\frac{\beta\rho}{(1-\rho+\beta\rho)(1-\rho)}} \left(\frac{N_t^{h*}}{N_t^{l*}} \right)^{\frac{\beta\rho}{1-\rho+\beta\rho}} Q^{\frac{\beta\rho}{1-\rho+\beta\rho}} \right)^{\frac{1-\rho}{\rho}}.$$

It is straightforward that Lemma 1.1 applies in this setup as well. The only thing left to show is that the two curves are both downward sloping, with the curve which gives $\underline{a}^{l*}$ for different values of c being flatter. This curve is downward sloping as before: a higher c implies an increase in the fraction of high skilled and a decrease in the fraction of low-skilled, implying an increase in p_t^{l*}. This from (1.25) implies a lower $\underline{a}^{l*}$. The other curve, which defines the optimal c^* for any value of $\underline{a}^l$ is also downward sloping. To see this, consider an increase in $\underline{a}^l$, which increases the relative supply of skills, as $\underline{a}^l$ shifts up, the population between $\underline{a}^h$ and $\underline{a}^l$ get a bigger weight in the relative supply of skills. An increase in the relative supply decreases the skill premium, which in turn decreases c^*.

A.5.3 Exogenous education, endogenous technology

The supply of high and low skilled workers evolves the same way as in section E.1 of the Appendix. The main difference is that the intermediate price in the new steady state is given by:

$$p_1^{l*} = \left(1 + \gamma \left(\frac{N^{h*}}{N_1^{l*}} \right)^{\frac{\beta\rho}{1-\rho}} \right)^{\frac{1-\rho}{\rho}}.$$

Appendix B

Appendix to Chapter 2

B.1 Transitional dynamics

I calculate the transition using second order perturbations, for which all equations have to be defined in terms of variables that are stationary in the steady state. Let v_t^s denote the normalized value of owning the leading vintage in sector s at time t:

$$v_t^h = \frac{V_t^h(q)}{q} \quad v_t^l = \frac{V_t^l(q)}{q}.$$

Let Δ_t denote the normalized present value gain from acquiring education (normalized by the current quality in the low-skilled sector):

$$\Delta_t = \sum_{j=0}^{\infty} \left(\frac{\lambda}{1+r} \right)^j \frac{w_{t+j}^h - w_{t+j}^l}{Q_t^l}.$$

The transitional path is fully characterized by the initial values N_0^h and Q_0 and the following equations:

$$
\begin{aligned}
v^s_{t+1} &= B^s \tfrac{(1+r)\overline{z}^s}{1-e^{-\eta\overline{z}^s_t}} && s = l,h \\
v^s_t &= \beta(1-\beta)^{\frac{1-\beta}{\beta}} (p^s_t)^{\frac{1}{\beta}} N^s_t - \tfrac{e^{-\eta\overline{z}^s_t}}{1+r} v^s_{t+1} && s = l,h \\
g^s_{t+1} &= 1 + (\overline{q}-1)(1-e^{-\eta\overline{z}^s_t}) && s = l,h \\
p^h_t &= \left(\gamma + \gamma^{\frac{\beta\rho}{(1-(1-\beta)\rho)(1-\rho)}} \left(Q_t \tfrac{N^h_t}{N^l_t}\right)^{-\frac{\beta\rho}{(1-(1-\beta)\rho)}}\right)^{\frac{1-\rho}{\rho}} \\
p^l_t &= \left(1 + \gamma^{1-\frac{\beta\rho}{(1-(1-\beta)\rho)(1-\rho)}} \left(Q_t \tfrac{N^h_t}{N^l_t}\right)^{\frac{\beta\rho}{(1-(1-\beta)\rho)}}\right)^{\frac{1-\rho}{\rho}} \\
Q_{t+1} &= \tfrac{g^h_{t+1}}{g^l_{t+1}} Q_t \\
\Delta_t &= c^*_t \beta(1-\beta)^{\frac{1-2\beta}{\beta}} (p^h_t)^{\frac{1}{\beta}} Q_t \\
\Delta_t &= \beta(1-\beta)^{\frac{1-2\beta}{\beta}} \left((p^h_t)^{\frac{1}{\beta}} Q_t - (p^l_t)^{\frac{1}{\beta}}\right) + \tfrac{\lambda}{1+r} g^l_{t+1} \Delta_{t+1} \\
N^h_t &= \lambda N^h_{t-1} + (1-\lambda) F(c^*_t) \\
N^l_t &= 1 - N^h_t.
\end{aligned}
$$

B.2 Initial values

B.2.1 $dQ = 0$ and $dN^h = 0$

The Figure below shows the border where the regions where the state variables are increasing (in black) and decreasing (in white), the border between the regions is where the state variable stays constant. The intersection of the two borders is the steady state.

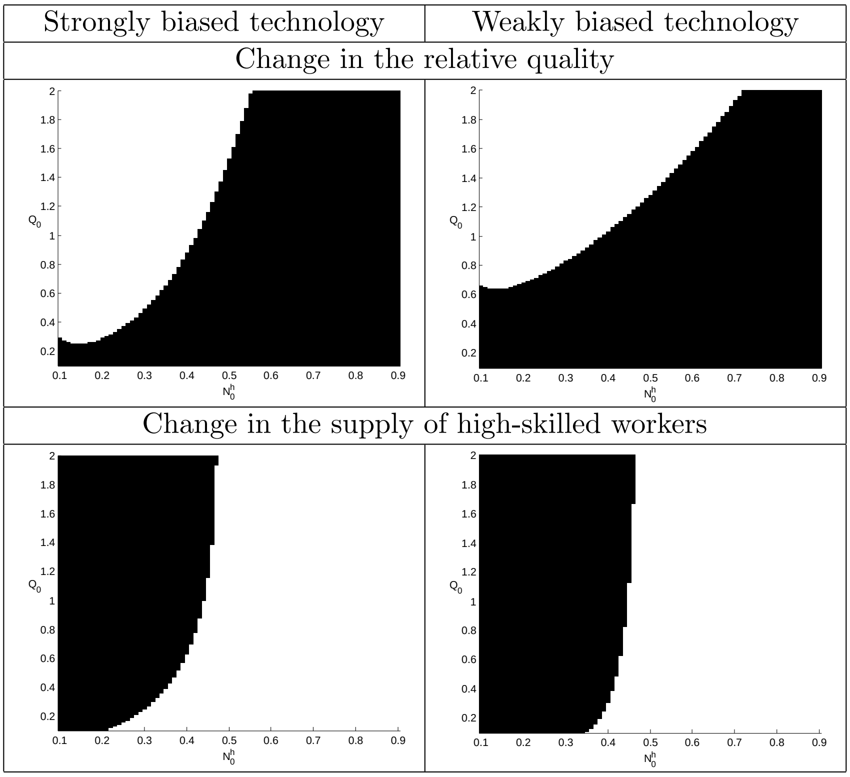

Figure B.1: Phase diagram source

B.2.2 Short-run and long-run change in the skill premium

The Figure below shows the immediate change in the skill premium and the overall change in the skill premium for each initial point. As before black indicates an increase and white indicates a decrease.

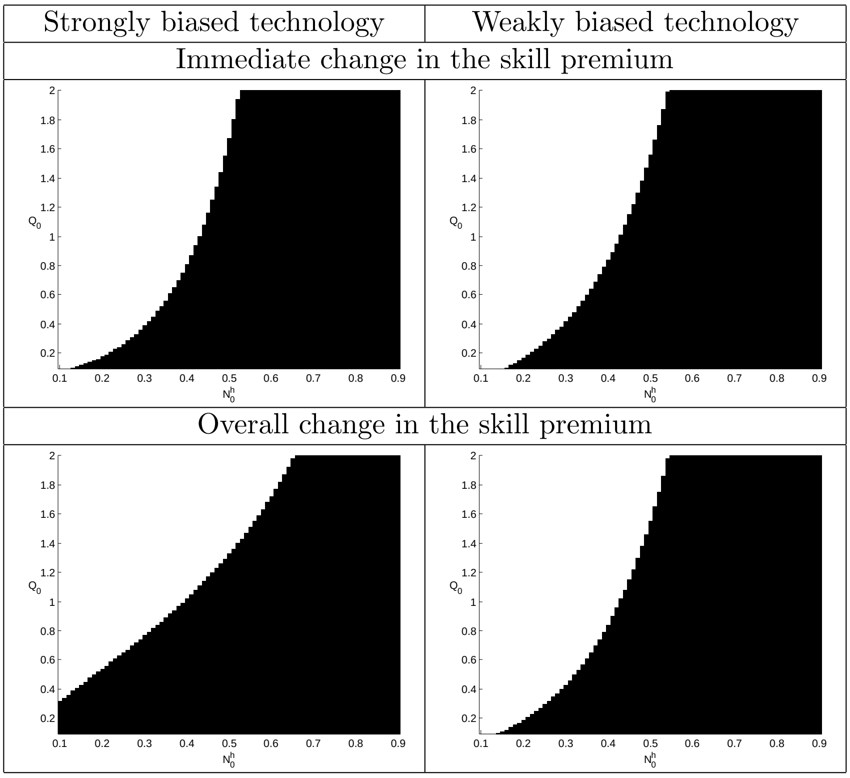

Figure B.2: Skill premium change source

B.2.3 Saddle path

The following Figure shows how the entire transition of the skill premium and the relative quality. Black shows continuous increase, darker gray shows points where there is an overall increase, but the path is not monotonic, lighter gray shows non-monotonic overall decrease, and white shows continuous decrease.

Since to the left of N^{h*} for most part Q decreases, the fact that Q doesn't continuously decrease to the steady state implies that the transition takes the economy into the black region in the top row of Figure B.1. Similarly, for initial points above N^h there is a large part where Q initially increases, but does not increase until the steady state, as the area is in the darker gray. This implies that the transition takes the economy up into the white region in the top row of Figure B.1. This suggests, that the stable arm to the steady state is a path, where either both Q and N^h increases, or they both decrease.

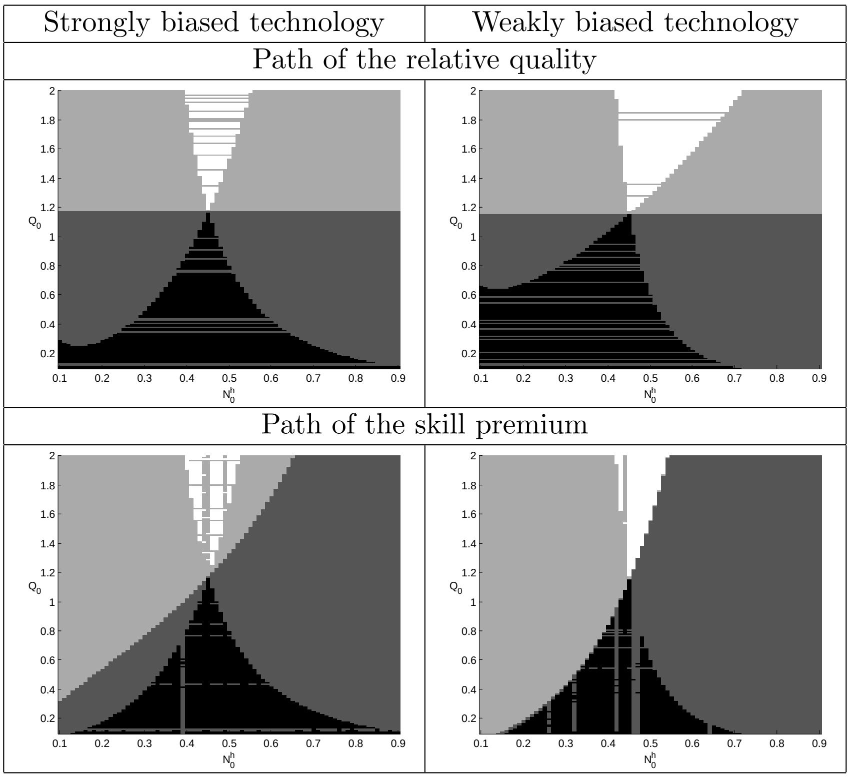

Figure B.3: Stable arm source

B.3 Parameters

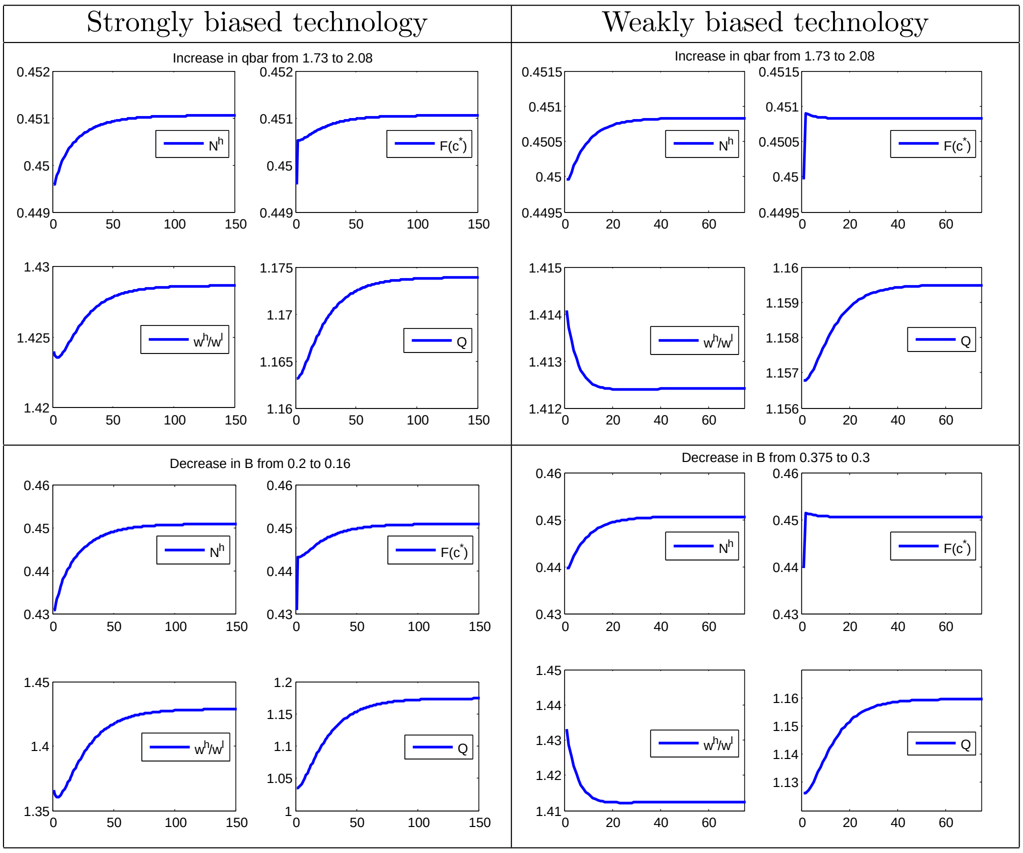

Figure B.4: Change in the R&D parameters 2

Appendix C

Appendix to Chapter 3

C.1 Progressivity curves

Figure C.1 plots the *progressivity curves*, the difference between the Lorenz curve of income and the concentration curve of taxes $(F_1(x) - F_1(T(x)))$ against F(x) for the income tax. Notice, that the progressivity curves follow similar patterns across countries. All of the progressivity curves in all countries are upwards sloping for lowest deciles of the income distribution, start declining between the 6th, 7th or 8th decile, but remain positive. This implies, that the non-invertibility of the P index is not a major problem when looking at the progressivity of existing tax systems. The fact that the progressivity curves are upward sloping for the lowest deciles means that the income tax shows some degree of progressivity at least at the very bottom of the distribution.

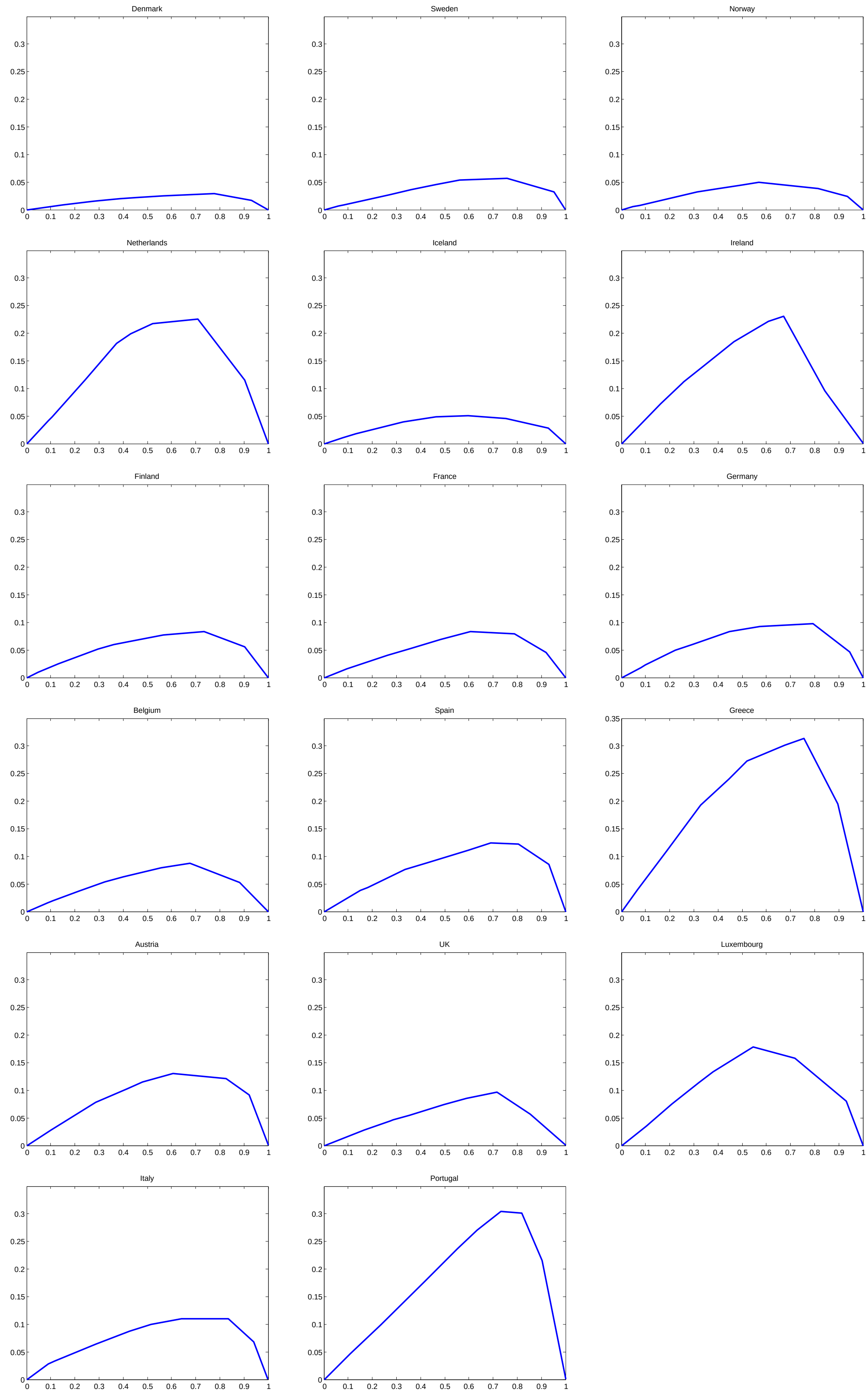

Figure C.1: Progressivity curves

C.2 Ideal policies and indifference curves

The ideal policy of a group can be found by solving a constrained maximisation problem: each group maximises their utility, (3.2), subject to the balanced budget condition, (3.1). This maximisation leads to the following ideal policies:

$$
\begin{aligned}
i_{P1} &= \{\max(0, \tfrac{g-\alpha^R(y^R-y^M)-(\alpha^R+\alpha^M)(y^M-y^P)}{y^P}), \min(1, \max(0, \tfrac{g-\alpha^R(y^R-y^M)}{(\alpha^R+\alpha^M)(y^M-y^P)})), \min(1, \tfrac{g}{\alpha^R(y^R-y^M)})\} \\
i_{P2} &= \{\max(0, \tfrac{g-\alpha^R(y^R-y^M)-(\alpha^R+\alpha^M)(y^M-y^P)}{yp}), \min(1, \tfrac{g}{(\alpha^R+\alpha^M)(y^M-y^P)}), \min(1, \max(0, \tfrac{g-(\alpha^R+\alpha^M)(y^M-y^P)}{\alpha^R(y^R-y^M)}))\} \\
i_M &= \{\min(1, \max(0, \tfrac{g-\alpha^R(y^R-y^M)}{y^P})), \max(0, \tfrac{g-\alpha^R(y^R-y^M)-y^P}{(\alpha^M+\alpha^R)(y^M-y^P)}), \min(1, \tfrac{g}{\alpha^R(y^R-y^M)})\} \\
i_R &= \{\min(1, \tfrac{g}{y^P}), \min(1, \max(0, \tfrac{g-y^P}{(\alpha^M+\alpha^R)(y^M-y^P)})), \max(0, \tfrac{g-y^P-(\alpha^M+\alpha^R)(y^M-y^P)}{\alpha^R(y^R-y^M)})\}
\end{aligned}
$$

Each group is indifferent over a hyperplane, which is defined by $U_a^i = T \in \mathbb{R}^3 s.t. U^i(T) = a$. However, the relevant part of these hyperplanes is their intersection with the set of admissible policies, $\Delta(g)$. The indifference set is reduced to a line segment within $\Delta(g)$. Figure 3.3 shows these indifference lines. The slope of the indifference lines can be then found by solving $U_a^i = T \in \mathbb{R}^3 s.t. U^i(T) = a \wedge T \in \Delta(g)$. This yields the following slopes:

$$
\begin{aligned}
P: \quad \frac{d\tau^R}{d\tau^M} &= -\frac{(\alpha^M+\alpha^R)(y^M-y^P)}{\alpha^R(y^R-y^M)} \\
M: \quad \frac{d\tau^R}{d\tau^M} &= \frac{\alpha^P(y^M-y^P)}{\alpha^R(y^R-y^M)} \\
R: \quad \frac{d\tau^R}{d\tau^M} &= -\frac{\alpha^P(y^M-y^P)}{(1-\alpha^R)(y^R-y^M)}
\end{aligned}
$$

C.3 Equilibria without coalitions

Lemma C.1. *In the absence of coalitions the following equilibria exist:*

1. *If $U^R(i_M) \geq U^R(i_{P2})$, then the representative of the middle income group runs alone and wins with the platform i_M.*

2. *If $\exists p \in [i_{P1}, i_{P2}]$ such that $U^M(p) \geq U^M(i_R)$ and $U^R(p) \geq U^R(i_M)$, the representative of the poor runs alone with such a platform p from the set of his ideal policies and wins.*

3. *If $U^R(i_M) < U^R(i_{P2})$, but $[i_{P1}, i_{P2}] \cap \{p | U^M(p) \geq U^M(i_R) \vee U^R(p) \geq U^R(i_M)\} = \emptyset$,*

   - *$\alpha^R > \alpha^M, \alpha^P$: R runs, M mixes between running (with probability δ^M), P mixes between i_{P1} (with probability δ^P) and not running. The mixing probabilities are:*

$$
\begin{aligned}
\delta^M &= \frac{U^P(i_P)-U^P(i_R)-\varepsilon}{U^P(i_P)+U^P(i_M)-2U^P(i_R)} \\
\delta^P &= \frac{U^M(i_M)-U^M(i_R)-\varepsilon}{U^M(i_M)+U^M(i_{P1})-2U^M(i_R)}
\end{aligned}
$$

- $\alpha^M > \alpha^R, \alpha^P$: *P runs with i_{P2} and M mixes between running (with probability δ^M), and R mixes between running (with probability δ^R) and not running. The mixing probabilities are:*

$$\begin{aligned}
\delta^M &= \tfrac{U^R(i_R)-U^R(i_{P2})-\varepsilon}{U^R(i_R)-U^R(i_M)} \\
\delta^R &= \tfrac{\varepsilon}{U^M(i_M)-U^M(i_R)}
\end{aligned}$$

- $\alpha^P > \alpha^R, \alpha^M$: *P mixing between i_{P1} (with probability δ^P) and i_{P2}, M and R mixing between running (with probability δ^M and δ^R) and not. The mixing probabilities are:*

$$\begin{aligned}
\delta^R &= \frac{(U^P(i_P)-U^P(i_M))\sqrt{\frac{U^P(i_P)-U^P(i_R)}{U^P(i_P)-U^P(i_M)}\frac{U^R(i_R)-U^R(i_{P2})}{U^M(i_R)-U^M(i_{P2})}\frac{U^M(i_M)-U^M(i_{P1})}{U^R(i_M)-U^R(i_{P1})}}}{(U^P(i_P)-U^P(i_M))\sqrt{\frac{U^P(i_P)-U^P(i_R)}{U^P(i_P)-U^P(i_M)}\frac{U^R(i_R)-U^R(i_{P2})}{U^M(i_R)-U^M(i_{P2})}\frac{U^M(i_M)-U^M(i_{P1})}{U^R(i_M)-U^R(i_{P1})}}+U^P(i_P)-U^P(i_R)} \\
\delta^M &= \frac{\sqrt{\frac{U^P(i_P)-U^P(i_R)}{U^P(i_P)-U^P(i_M)}\frac{U^R(i_R)-U^R(i_{P2})}{U^M(i_R)-U^M(i_{P2})}\frac{U^M(i_M)-U^M(i_{P1})}{U^R(i_M)-U^R(i_{P1})}}}{1+\sqrt{\frac{U^P(i_P)-U^P(i_R)}{U^P(i_P)-U^P(i_M)}\frac{U^R(i_R)-U^R(i_{P2})}{U^M(i_R)-U^M(i_{P2})}\frac{U^M(i_M)-U^M(i_{P1})}{U^R(i_M)-U^R(i_{P1})}}} \\
\delta^P &= \frac{\delta^R(U^M(i_R)-U^M(i_{P2}))+\varepsilon}{(U^M(i_R)-U^M(i_{P2}))\delta^R+(U^M(i_M)+U^M(i_{P1}))(1-\delta^R)}
\end{aligned}$$

Proof. 1. See in text.

2. See in text.

3. When $U^R(i_M) \leq U^R(i_{P2})$ and $[i_{P1}, i_{P2}] \cap \{p | U^M(p) \geq U^M(i_R) \vee U^R(p) \geq U^R(i_M)\} = \emptyset$, then as explained in the text, there aren't any pure strategy equilibria. Dividing the cases based on which group is the largest and going through each case yields the identification of the equilibria as described in the Lemma.

   (a) $\alpha^R > \alpha^M, \alpha^P$

   The only equilibria is P mixing between running with i_{P1} probability δ_P and not running with probability $1-\delta^P$, M mixes between running with probability δ^M and not running with probability $1-\delta^M$, and R running for sure.

   The mixing probabilities are:

$$\begin{aligned}
\delta^M &= \tfrac{U^P(i_P)-U^P(i_R)-\varepsilon}{U^P(i_P)+U^P(i_M)-2U^P(i_R)} \\
\delta^P &= \tfrac{U^M(i_M)-U^M(i_R)-\varepsilon}{U^M(i_M)+U^M(i_{P1})-2U^M(i_R)}
\end{aligned}$$

   The equilibrium expected payoffs are:

$$\begin{aligned}
E(U^P) &= \delta^M U^P(i_R) + (1-\delta^M)U^P(i_P) \\
E(U^M) &= \delta^P U^M(i_R) + (1-\delta^P)U^M(i_M) \\
E(U^R) &= \delta^M(1-\delta^P)U^R(i_M) + (1-\delta^M)\delta^P U^R(i_P1) + (1-\delta^P-\delta^M+2\delta^M\delta^P)U^R(i_R)
\end{aligned}$$

The expected equilibrium winning platform, E in this case is:

$$E = (\delta^P \delta^M + (1-\delta^P)(1-\delta^M))i_R + \delta_M(1-\delta^P)i_M + \delta^P(1-\delta^M)i_{P1}$$

This expected platform will be quite close to i_R, as i_R has at least 1/2 weight.

(b) $\alpha^M > \alpha^R, \alpha^P$

The only equilibrium is P playing i_{P2} and $M(R)$ playing $i_M(i_R)$ with probability $\delta^M(\delta^R)$ and not running with probability $1-\delta^M(1-\delta^R)$. The mixing probabilities are:

$$\begin{aligned} \delta^M &= \tfrac{U^R(i_R)-U^R(i_{P2})-\varepsilon}{U^R(i_R)-U^R(i_M)} \\ \delta^R &= \tfrac{\varepsilon}{U^M(i_M)-U^M(i_R)} \end{aligned}$$

The equilibrium expected payoffs are:

$$\begin{aligned} E(U^P) &= \delta^R U^P(i_M) + (1-\delta^R)U^P(i_P) - \varepsilon \\ E(U^M) &= \delta^R U^M(i_R) + (1-\delta^R)U^M(i_{P2}) \\ E(U^R) &= U^R(i_{P2}) \end{aligned}$$

The expected equilibrium winning platform, E in this case is:

$$E = \delta^M \delta^R i_M + (1-\delta^M)\delta^R i_R + (1-\delta^R)i_{P2}$$

As $\varepsilon \to 0$, $\delta^R \to 0$ and the implemented platform is i_{P2}. Hence it is likely that only an MR coalition is feasible.

(c) $\alpha^P > \alpha^M, \alpha^R$

Only equilibrium: P mixing between i_{P1} and i_{P2}, M and R mixing between running and not. The mixing probabilities satisfy:

$$\begin{aligned} \delta^R(U^P(i_P) - U^P(i_R)) - \delta^M(U^P(i^P) - U^P(i_M)) &= \delta^M\delta^R(U^P(i_M) - U^P(i_R)) \\ \delta^P(1-\delta^R)(U^M(i_M) - U^M(i_{P1})) &= \delta^R(1-\delta^P)(U^M(i_R) - U^M(i_{P2})) + \varepsilon \\ \delta^P\delta^M(U^R(i_M) - U^R(i_{P1})) + \varepsilon &= (1-\delta^P)(1-\delta^M)(U^R(i_R) - U^R(i_{P2})) \end{aligned}$$

The solution of these three equations yields the probabilities in the Lemma. The expected payoffs are:

$$\begin{aligned}
E(U^P) &= \delta^M(1-\delta^R)U^P(i_M) + (1-\delta^M+\delta^M\delta^R)U^P(i_P)\\
E(U^M) &= \delta^R\delta^P U^M(i_{P1}) + (1-\delta^R)\delta^P U^M(i_M) + (1-\delta^P)U^M(i_{P2})\\
E(U^R) &= \delta^M(1-\delta^P)U^R(i_{P2}) + (1-\delta^M)(1-\delta^P)U^R(i_R) + \delta^P U^R(i_{P1})
\end{aligned}$$

The expected equilibrium winning platform is:

$$\begin{aligned}
E = \quad & \delta^P(\delta^R + (1-\delta^R)(1-\delta^M))i_{P1} + (1-\delta^P)(\delta^M + (1-\delta^R)(1-\delta^M))i_{P2} +\\
& (1-\delta^P)(1-\delta^M)\delta^R i_R + \delta^P\delta^M(1-\delta^R)i_M
\end{aligned}$$

This platform is close to an internal point of i_P. As P wins the election in most of the cases.

$\square$

C.4 Division of occupation groups into income categories

Table C.1: Relative earnings of occupation groups within countries and categorization B

country	1	2	3	4	7	8	5	6	9
Austria	2.12(R)	1.43(R)	1.15(R)	0.92	0.77	0.80	0.63(P)	0.62(P)	0.61(P)
Belgium	1.75(R)	1.37(R)	0.91	0.81	0.72	0.76	0.67(P)	0.64(P)	0.61(P)
Germany	1.96(R)	1.45(R)	1.04	0.95	0.83	0.81	0.64(P)	0.62(P)	0.62(P)
Denmark	1.59(R)	1.25(R)	1.04	0.87	0.98	0.95	0.73(P)	0.83	0.78(P)
Spain	2.25(R)	1.48(R)	1.14(R)	0.82	0.84	0.85	0.71(P)	0.78	0.64(P)
Finland	1.84(R)	1.27(R)	0.98	0.81	0.89	0.88	0.71(P)	0.66(P)	0.70(P)
France	1.82(R)	1.46(R)	1.07	0.80	0.81	0.83	0.74(P)	0.73(P)	0.67(P)
Greece	1.89(R)	1.47(R)	0.96	0.80	0.93	0.81	0.70(P)	0.66(P)	0.63(P)
Ireland	1.38(R)	1.47(R)	1.07	0.79	0.92	0.77	0.65(P)	0.69(P)	0.68(P)
Iceland	1.88(R)	1.22(R)	1.07	0.84	0.95	0.86	0.76	0.72(P)	0.69(P)
Italy	2.46(R)	1.62(R)	1.09	0.94	0.73(P)	0.79	0.75	0.60(P)	0.45(P)
Luxembourg	2.08(R)	1.39(R)	1.18(R)	0.83	0.69(P)	0.72	0.61(P)	0.49(P)	0.37(P)
Netherlands	1.49(R)	1.28(R)	1.04	0.83	0.97	0.93	0.70(P)	0.63(P)	0.61(P)
Norway	1.59(R)	1.25(R)	1.13(R)	0.86	0.92	0.92	0.76(P)	0.80(P)	0.74(P)
Portugal	2.79(R)	1.96(R)	1.32(R)	0.83	0.58(P)	0.67	0.59(P)	0.52(P)	0.57(P)
Sweden	1.71(R)	1.22(R)	1.07	0.84	0.91	0.90	0.81	0.76(P)	0.74(P)
United Kingdom	1.68(R)	1.58(R)	1.10	0.73	0.87	0.72	0.56(P)	0.63(P)	0.58(P)

Notes: The values indicate the average earnings within that occupation compared to the average earning in the country. The occupations are: 1 - Legislators, senior officials & managers; 2 - Professionals; 3 - Technicians & associate professionals; 4 - Clerks; 7 - Craft & related trades workers; 8 - Plant & machine operators & assemblers; 5 - Service, shop & market sales workers; 6 - Skilled agricultural & fishery workers; 9 - Elementary occupations. An (R) behind the value indicates that this occupation belongs to the rich group in Panel B, a (P) implies that it belongs the the poor, nothing implies that it belongs to the middle income group. Values calculated from SES 2006, averages calculated with weights from the ELFS 2005 (2003 and 2008 for Italy).

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