MAKING MONETARY POLICY:

CAUTION, CONSERVATISM AND

THE PUBLIC SUPPLY OF LIQUIDITY

by

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ABSTRACT

This dissertation offers two perspectives on the making of monetary policy under uncertainty. The first two chapters examine the consequences of uncertainty for the macroeconomic function of the central bank - the stabilisation of macroeconomic variables of interest around socially desirable targets. The third chapter examines the consequences of uncertainty for the central bank's microeconomic function - the public supply of liquidity.

The first chapter asks whether society benefits from the delegation of monetary policy to cautious and conservative central bankers. We offer a critical view on the delegation literature and relax seemingly innocuous assumptions about uncertainty and preferences. First, caution improves credibility but does not obviate the need for central-bank conservatism. Second, previous models of delegation have focused on suboptimal forms of conservatism. We derive optimal concepts of conservatism that mitigate, or eliminate, any residual problem of credibility. Third, we rationalize why credible monetary policy may be conducive to stable inflation and output.

The second chapter examines the implications of instrument uncertainty for optimal monetary policy following the introduction of non-quadratic preferences. We

investigate both symmetric and asymmetric preferences and discuss the consequences for caution, gradualism and the optimal delegation of monetary policy.

The third chapter examines the microeconomic role of the central bank. We develop a rationale for the provision of public liquidity based on an incomplete contracting framework. The model illustrates to what extent wealth-constrained entrepreneurs are leveraged by collateralized debt contracts and examines the consequences of costly collateral liquidation and aggregate asset price uncertainty for the provision of external finance.

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TABLE OF CONTENTS

LIST OF FIGURES

Figures in Chapter One

Figures in Chapter Two

INTRODUCTION

"Everything is vague to a degree you do not realize till you have tried to make it precise." (Bertrand Russell, The Philosophy of Logical Atomism)

Uncertainty about the world we live in has always been a good source for discussion and differences of opinion. In the field of central banking, matters are no different. Consider, for example, the ongoing debate on whether money matters, whether monetary institutions matter and, more recently, whether inflation isn't dead after all. Uncertainty - an elegant metaphor for our lack of understanding - is a paramount feature of the world the policy maker lives in.

Uncertainty is both a challenge and an opportunity. A challenge, because uncertainty complicates the implementation of policy, given a set of well-specified objectives. An opportunity, because uncertainty may create a rationale for policy, legitimizing a particular set of objectives. The role and implementation of monetary policy under uncertainty is the central theme of this thesis.

Traditionally, the functional repertoire of a central bank has been categorised into a macroeconomic objective (namely the stabilisation of variables of interest - such as inflation and output - around socially desirable targets) and a microeconomic objective (namely the provision of liquidity to safeguard the health of the financial system). Following this traditional distinction, the thesis offers two perspectives on the consequences of uncertainty for the making of monetary policy.

At the macroeconomic level, we examine how uncertainty interferes with the implementation of macroeconomic objectives. The type of uncertainty that we consider here is instrument uncertainty, which disturbs the transmission of a given stance of monetary policy. The issues that we examine in this context are twofold. First, in relation to the conduct of monetary policy, we study how instrument uncertainty gives rise to caution and gradualism in the setting of the policy maker's instrument. ('Caution' here refers to a tendency to adopt more neutral policies; 'gradualism' reflects a tendency to smooth an instantaneous policy adjustment into smaller ones over time.) Second, in relation to the design of monetary policy, we study how instrument uncertainty interacts with the delegation of monetary policy to individuals with socially unrepresentative preferences. We explore various notions of central-bank conservatism and show how the consequences of uncertainty become endogenous to the policy regime. These issues form the subject of Chapters 1 and 2.

At the microeconomic level, we examine how uncertainty may legitimize a rationale for policy, more specifically a role for the public supply of liquidity to the banking system. The environment that we study here features aggregate asset price uncertainty. We show how aggregate uncertainty disrupts the feasibility of contractual arrangements between banks and entrepreneurs who are wealth-constrained. Furthermore, we look into the desirability of interest rate smoothing and the likelihood of currency crises in economies that lack sufficient financial development. These issues form the subject of Chapter 3.

It is useful to briefly motivate how each individual chapter fits into the thesis as a whole. For the discussion of related literature we refer to the introduction of each chapter.

CHAPTER ONE

The first chapter, titled *'Caution and Conservatism in the Making of Monetary Policy'*, takes the following question as a starting point: Does society benefit from the delegation of monetary policy to cautious and conservative central bankers? Our interest in this question is motivated by the following two observations.

First, as Mervyn King pointed out in a recent talk, "non-economists will always point to the dangers of delegating decisions to the so-called experts... And the cult of the amateur is still revered by many."[1] In relation to monetary policy,

[1]On his appointment as Financial Secretary to the UK Treasury in the 1930s, Duff Cooper wrote: "I had feared that my limited acquaintance with political economy and my ignorance of finance would prove serious handicaps, but within a week of my appointment I had to wind-up a debate on currency in the House of Commons and, speaking without the slightest knowledge of the subject, I was able, by drawing attention to the discrepancies in the remedies proposed by the previous speakers, all of them experts, to create a favourable opinion and to earn many congratulations." (Quoted from King's address to the joint luncheon of the American Economic Association and the American Finance Association at the Boston Marriott, 7 January 2000).

the popular media continues to hold a perception that delegation to cautious and conservative central bankers is costly. At the same time, empirical economists have found that the delegation of monetary policy to central banks (with the alleged features) seems more like a free lunch, leading to lower inflation at no real costs. However, we do not yet have a theoretical understanding of how caution and conservatism can be reconciled with the free lunch result of delegation.[2]

Second, observers and practicioners in the field of central banking have repeatedly expressed their dissatisfaction about the lack of convergence between the theory and practice of monetary policy. The credibility literature of monetary policy and the delegation literature, in particular, have been the subject of much scrutiny recently. Alan Blinder forcefully suggests two exceptions in his celebrated book 'Central Banking in Theory and Practice' when he reviews the issues where academics and policy makers have actually come together.[3] The first is the so-called Brainard

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[2] Appraising the tension between public perceptions and policy optimality, Charles Goodhart concluded his Keynes Lecture at the British Academy (29 October 1998) by indicating that "there is an absolute yawning gap between the general perception of non-economist outsiders that reversals of policy, changes of mind, are to be deplored and castigated as evidence of error, irresolution and general incompetence, and the apparent findings from our economic models that such reversals should optimally occur some four, or so, times more frequently than they do in practice. Maybe our models are missing something important. If not, we have then singularly failed to explain to the world at large how policy should be carried out. Either way, there is still an enormous amount of work to be done."

[3] See Blinder (1998, pp. 11-12 and pp. 46-47).

conservatism principle, which explains why policy makers may want to err on the side of caution.[4] The second is the Rogoff conservative-central-banker approach, which explains why delegation to conservative central banks may improve social welfare.[5]

This chapter offers a critical view on the delegation literature and relaxes seemingly innocuous assumptions which this literature has continued to make about uncertainty and preferences. As to uncertainty, most of the literature assumes that the transmission mechanism is either deterministic or only subject to additive uncertainty. To generate a motive for caution, we wish to depart from certainty-equivalence. This is achieved in the simplest possible manner by introducing multiplicative instrument uncertainty. As to preferences, most of the literature follows Rogoff in assuming a particular form of conservatism, namely weight-conservatism, which is based on quadratic preferences. Weight-conservatism refers to a stronger preference for stable inflation than for stable output (around respective targets) but results in suboptimal output stabilisation. In contrast, we generalise the standard quadratic paradigm and derive new notions of conservatism that do not distort output stabilisation.

The implications of this chapter are threefold. First, the credibility problem of monetary policy may have been vastly exaggerated once it is recognised that policy makers have internal incentives to err on the side of caution. Second, previous models of delegation have focused on suboptimal notions of central-bank conservatism. We

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[4] See Brainard (1967).

[5] See Rogoff (1985).

derive optimal concepts of conservatism (termed 'target-conservatism' and 'stability-conservatism') that mitigate, or eliminate, any residual problem of credibility. We also offer interpretations in terms of the practice of inflation targeting and the penchant of central banks for stability. Third, we rationalize why credible monetary policy may be conducive to stable inflation and output. In contrast to previous work, we find that conservatism need not necessarily lead to greater output variability and that the delegation to cautious and conservative central bankers may be consistent with a strong version of the free luch result of monetary policy delegation.

CHAPTER TWO

The second chapter, titled *'The Brainard Conservatism Principle with Non-Quadratic Objectives'*, is based on joint work with Jagjit Chadha (University of Cambridge). The central issue addressed here is the robustness of the Brainard conservatism principle with respect to the specification of the policy maker's loss function. In a dynamic context recall that this principle suggests that in the case of multiplicative instrument uncertainty the policy instrument is moved incompletely (or cautiously) and smoothly (or gradually).

We examine the implications for the optimal interest rate rule that follow from relaxing the assumption that the loss function is quadratic. In particular, we are interested in whether, and how, alternative loss functions give rise to caution and gradualism. We deviate from the quadratic framework in the following two respects. First, while keeping with the assumption of symmetry, we examine the optimal interest rule that is obtained when the policy maker's preferences are given by a split constant absolute risk aversion function. In contrast to the simple quadratic, the split-CARA allows us to flexibly change the curvature of the objective function. Second, we examine the consequences of asymmetric policy preferences for the optimal interest rule. This is achieved with a linear exponential objective function that allows us to parameterise the degree of asymmetry in a straightforward fashion.

We argue that non-quadratic preferences, per se, are neither sufficient nor necessary to generate the Brainard conservatism principle. First, if the uncertainty is additive, deviating from quadratics, while keeping with symmetry, does not buy us anything new: the optimal rule remains the same, and only the policy maker's dead-weight losses are different. Thus the question of curvature of the objective function, by itself, remains orthogonal to that of whether the policy maker should exhibit caution or gradualism in the conduct of policy.

Second, if the loss function represents a view that the policy maker's implicit weights to downside and upside risk differ, the asymmetry tends to affect the optimal rule under both additive and multiplicative uncertainty but results in interest rate paths observationally similar, and in some cases equivalent, to those implied by a shifted quadratic.

We further explore the use of asymmetric loss functions in relation to how monetary policy is delegated to goal-dependent central bankers. We find that an inflation-targeting central bank that is required to be goal dependent should also be required to pursue the delegated goal in a symmetric way. We also argue that an asymmetry in the social costs of inflation can be translated into the level of the mandated target, without requiring that this target should be pursued asymmetrically.

CHAPTER THREE

The third, and last, chapter, titled *'Costly Collateral and the Public Supply of Liquidity'*, starts from the observation that credit market imperfections, resulting from informational asymmetries between borrowers and lenders, may lead to an underprovision of external finance. Credit constraints clearly come at a cost: they prevent productive activity that is socially worthwhile. Society has dealt with such problem by requiring collateral or net worth as a condition to the provision of finance.

The central issue that is explored in this chapter concerns the pros and cons of collateral requirements as an element of optimal contracts between entrepreneur-borrowers and banks. The model relies on a problem of extreme moral hazard where entrepreneurs cannot commit the returns from their productive activity as these are assumed to be non-verifiable to courts. Collateralization is necessary (because of the information asymmetry) but generally incomplete (because of a liquidation cost). The assumption of costly collateral liquidation forms a first element in this model that contributes to the possibility of a collapse of financial intermediation.

We examine how leverage and collateralization interact and find that collateralized finance may be highly susceptible to interest rate shocks if liquidation costs are high. We discuss two implications that emerge from this result. First, interest rate smoothing may be more desirable in countries that lack sufficient financial development. Second, costly collateral contributes to our understanding of the link between the degree of financial development and the susceptibility of an economy to speculative currency attacks. We argue that the model provides micro-foundations for currency crisis models with self-fulfilling features.

The second element in our model that may cloud the beneficial role of collateral is aggregate asset price uncertainty.[6] An issue of particular interest concerns the consequences of aggregate uncertainty for credit-constrained entrepreneurs, whose assets are maximally collateralized. It turns out that, from the point of view of the entrepreneur, an uncontingent contract (where collateralization varies countercyclically to offset fluctuation in liabilities) is dominated by a contingent contract (where liabilities vary with the asset price). However, from the point of view of banks that are poorly capitalized, aggregate uncertainty about collateral values

[6]This issue has also been raised in the IMF's latest World Economic Outlook: "To the extent that falling stock and property prices affect the solvency of household and corporate borrowers, they tend to raise the share of non-performing loans in the portfolios of financial institutions, thereby undermining the banks' capital position and lending capacity. Under generalized asset price deflation, these effects are reinforced by the falling value of collateral, which banks can usually recover in the case of outright defaults." (IMF, 2000, p. 127).

may introduce too much bank profit variability. If the bank's solvency condition binds, entrepreneurs will be (further) deprived from credit, leading to a lower level of welfare. Under the conditions of poor bank and borrower capitalization, we find that state-contingent government provision of liquidity enhances the level of welfare in this economy.

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CHAPTER ONE

Caution and Conservatism in the Making of Monetary Policy

> *"An important reason to expose central bankers to elected officials is that, just as the latter may have an inflationary bias, the former may easily develop a deflationary bias. Shielded as they are from public opinion, cocooned within an anti-inflationary temple, central bankers can all too easily deny ... that cyclical unemployment can be reduced by easing monetary policy." (Stanley Fischer, 1994. p. 293)*

Introduction

What principles should motivate the conduct and design of monetary policy? Uncertainty about what monetary policy can do and disagreement about what it should do have caused significant controversy on the practical resolution of this question. Uncertainty about the transmission mechanism and disagreement about the optimal form of delegation, in particular, have always complicated the making of policy and have led to a variety of policy regimes observed over time and across countries.

Despite the ongoing debates, policy makers need to take a preliminary stance, though, on how to implement policy. And it seems, more often than not, that their practical response has been one of caution and conservatism. As reflected in the above quote, this immediately raises the concern whether caution and conservatism are desirable from a social welfare point of view. And, if they are, there is still the legitimate question whether actual policy makers conduct policy in an excessively cautious and conservative fashion. But in order to answer the latter question, a benchmark is needed, and therefore we first need to answer the former, more fundamental, question: do caution and conservatism improve the making of monetary policy? This is the central theme of the chapter.

Before proceeding let us first specify what we actually mean with caution and conservatism. Caution emerges from the interaction between *uncertainty* and *preferences*, and is meant to reflect a more neutral, or less activist, policy stance. Conservatism refers to *preferences* that are unrepresentative from a social point of view. Central-bank conservatism typically refers to a stronger emphasis on inflation than on output.

Our interest in the normative underpinnings of caution and conservatism is motivated by two observations. First, there is an unresolved tension between the popular perception that caution and conservatism are costly and the empirical finding that delegation to independent central bankers is beneficial. Given that central bankers are so often depicted as being cautious and conservative, the latter empirical finding (also known as the 'free lunch result of delegation') seems to suggest that these features are in fact desirable qualities. But is it really true that caution and conservatism are the blessings that generate this result? In order to arrive at such conclusions, one must first come up with a social welfare benchmark.

Second, policy makers have felt somewhat uneasy with the descriptive realism of the proposals suggested by the 'credibility literature'. This is reflected, for example, in the call by McCallum to improve the "interpretive *mappings* between analytical constructs and real-world institutions".[7] The uneasiness about the lack of convergence between models and realities is probably best exemplified by Blinder (1998). However, in reviewing the literature, Blinder mentions two notable exceptions where minds have actually come together. The first is the so-called Brainard conservatism principle that rationalizes why policy makers may want to err on the side of caution.[8] The second is the Rogoff conservative-central-banker approach that explains why policy makers with unrepresentative preferences may do things better.[9] There is thus considerable independent interest in improving the models we use to describe policy makers. And, it is our intention to make a step into that direction with the analysis of both caution and conservatism.

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[7]See McCallum (1995, p. 207), no emphasis added.

[8]See Brainard (1967).

[9]See Rogoff (1985).

This chapter jointly analyzes the optimal conduct and design of monetary policy, and examines the desirability of caution and conservatism. The framework we propose has the following two features: 'multiplicative instrument uncertainty' and 'generalized quadratic preferences'. First, in order to generate a motive for caution, we need to break certainty-equivalence. We do so in the simplest possible manner: we follow Brainard (1967) in assuming that monetary policy transmission is subject to multiplicative instrument uncertainty. Second, in order to discuss conservatism as an element of optimal monetary policy delegation, we need to deviate from the commonly assumed quadratic objective function. As we will show, the simple quadratic objective function does not allow us to generate *optimal* notions of conservatism. We propose a 'generalized quadratic objective function' that enables us to derive new notions of conservatism.

We will offer a critical perspective on the credibility literature of monetary policy. First of all, the problem of credibility may have been exaggerated if policy makers have internal incentives to conduct policy cautiously. We show that a stronger degree of caution, as a result of increased uncertainty, may actually lead to an improvement in social welfare.

Second, we introduce multiplicative uncertainty into the Rogoff (1985) model. We restrict the notion of conservatism to the suboptimal concepts used in the literature and show that, under these circumstances, an interesting relationship arises between the extent of uncertainty in the economy and the degree of conservatism of the central banker. Economies characterized by a large credibility problem will benefit from delegation to central bankers who become increasingly 'ultra-conservative' in the face of greater uncertainty.

Third, we argue that previous work has focused on suboptimal notions of conservatism. We derive optimal notions of conservatism that reproduce the best feasible equilibrium. In its extreme form, our approach will be interpreted as an application of Mundell's (1968) 'principle of effective market segmentation'. In its weaker form, we offer interpretations in terms of the practice of inflation targeting and the penchant of central banks for stability.

Finally, the model sheds some light on the interaction between credibility and nominal and real stability. The model formalizes that "monetary policy can prevent money itself from being a major source of uncertainty" (Friedman, 1968, p. 12) and implies that the *credibility* of monetary policy, too, can help reduce the variability of inflation and output. The model therefore generates a (strong) version of the free lunch result of delegation.

At this stage, it is useful to discuss how our contribution relates to the literature. The uncertainties surrounding the making of monetary policy received considerable attention during the 1960's and early 1970's. Important contributions include those of Brainard (1967) on the effectiveness of policy under multiplicative uncertainty, Friedman (1968) on the merits of fixed rules when lags are long and variable, and Poole (1970) on the choice of an intermediate target under additive uncertainty. The question how policy makers operate under uncertainty has recently received renewed interest. Most of this literature, however, ignores the the credibility problem of monetary policy. Notable exceptions are Swank (1994), Letterie (1997) and Pearce and Sobue (1997). There is also a growing body of research on learning and optimal control theory in dynamic environments with multiplicative uncertainty[10] but none of these contributions addresses the role of preferences and monetary policy delegation.

With regard to optimal monetary policy delegation, the credibility literature has offered a convenient framework. This literature traditionally features a role for policy that is clouded by various policy conflicts, such as the temptation to use monetary policy for the wrong reasons and the trade-off between inflation and output variability. The analysis of monetary policy delegation gained much impetus with the application of the notion of time-inconsistency (Kydland and Prescott, 1977) to monetary economics (Barro and Gordon, 1983a), which led to the discovery of a credibility problem in the form of an inflationary bias. Much of the subsequent literature has then looked for possible mechanisms that reduce or remove such a credibility problem without compromising the flexibility needed

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[10]See, for example, Bertocchi and Spagat (1993), Wieland (1996), Estrella and Mishkin (1998), Sack (1998), Svensson (1999). Interestingly, this literature has also illustrated the possibility of uncertainty leading to more aggressive policy. See Craine (1979), Sargent (1998) and Onatski and Stock (1998).

for output stabilisation. Barro and Gordon (1983b) and Canzoneri (1985) suggest reputational forces that may restore the economy's best feasible equilibrium. One prominent approach suggests the delegation of monetary policy to an independent central banker with *incentives* distinct from those of the government. Walsh (1995a) and Persson and Tabellini (1993) have argued that the apparent trade-off between credibility and flexible output stabilisation arises because the delegation mechanism is restricted to ad-hoc incentive structures. If instead an inflation contract ensuring an optimal incentive structure were introduced, the best feasible equilibrium would prevail with full credibility and flexibility simultaneously.[11] Our approach does bear some similarity to Walsh (1995a), who highlighted how restrictive assumptions on the delegation mechanism may generate a credibility-flexibility trade-off. This branch of the credibility literature, however, does not have anything to say about central-bank conservatism. Moreover, as is the case for most of the credibility literature, the issue of caution is ignored: most studies assume that the transmission mechanism is either deterministic or subject only to additive uncertainty.

Instead, we draw on another branch of the credibility literature. Rogoff (1985) proposes the delegation of monetary policy to central banks with divergent *preferences*

[11] Side payments in the form of a linear penalty on inflation would do the trick but implementation may present practical difficulties. Critical discussions can be found in Blinder (1998), Canzoneri, Nolan and Yates (1997), Goodhart and Viñals (1994), King (1997) and Walsh (1995b). The incentives implied by the optimal contract could also be implemented with a dismissal rule (Walsh, 1995c). For further extensions, see Fratianni et al. (1997).

and shows that the appointment of a 'weight-conservative' central banker improves the problem of imperfect credibility.[12] However, given that the notion of weight-conservatism refers to the relative preference for inflation versus output stabilisation, complete removal of the inflationary bias would entail too high a cost in terms of output variability. As a result, a suboptimal equilibrium is obtained and this has been the reason why attention has for some time shifted away from delegation mechanisms based on conservatism.

Recently, a few studies have re-established a role for weight-conservatism by enriching the environment in which the central bank conducts policy. Herrendorf and Lockwood (1997) and Svensson (1997a) suggest that weight-conservatism may be useful when the inflationary bias is state-contingent and the delegation decision is not. Rather than enriching the environment so that a role for weight-conservatism re-emerges, our approach suggests a review of the notion of weight-conservatism itself.

An interesting contribution that also reconsiders the concept of conservatism is Svensson (1997b). He shows that conservatism in the form of a lower inflation *target* may lead to the best feasible equilibrium, where inflation settles down at its socially optimal level. However, as suggested by King (1997), this proposal raises doubts as it is implied that central banks should target inflation rates that are anticipated to be missed systematically.

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[12] See also Flood and Isard (1989), Lohmann (1992), Waller (1992), Waller and Walsh (1996) and Obstfeld (1997).

The organization of this chapter is as follows. Section 1 develops a baseline model of caution in the conduct of policy. Section 2 addresses the role of conservatism in the delegation of monetary policy under quadratic preferences. Section 3 introduces generalized preferences and derives optimal notions of conservatism. Section 4 applies the framework to the free lunch result of delegation. The last section concludes.

1. Caution in the Making of Monetary Policy

This section examines the role of caution in the conduct of policy. In later sections, we will consider the role of central-bank conservatism.

1.1. Description of the Model

We begin with a description of the economic environment. Aggregate supply is represented by a standard surprise supply function:

$$y = y^* + b(\pi - \pi^e) + \varepsilon \qquad b > 0, \tag{1}$$

where y is log of output, y^* is log of natural output, π is inflation, π^e is expected inflation, and ε is a temporary aggregate supply shock with mean 0 and variance σ_ε^2.

Aggregate demand is assumed to be primarily influenced by a policy maker, who can generate inflationary surprises. Denote the planned deviation of the policy maker's single instrument from its neutral level by i^p. Due to multiplicative instrument uncertainty, the policy maker controls inflation imperfectly:

$$\pi = s\, i^p\,, \tag{2}$$

where s is a multiplicative instrument shock with mean 1 and variance σ_s^2. All variances in the model are strictly positive and finite. For analytical convenience, supply and control shocks are independent of each other. The assumption of multiplicative instrument uncertainty marks a first departure from the literature which generally assumes that transmission is either deterministic or subject only to additive uncertainty.

In practice, randomness in the relation between policy instrument and policy goal is of course the result of various, possibly conflicting, forces. Also, control typically becomes more difficult depending on whether the policy maker wants to affect instruments, operating targets, intermediate targets or ultimate policy goal variables. Shocks to the interest elasticities of money demand and aggregate demand are examples of factors that constrain the policy maker's ability to control inflation in an accurate manner.

The multiplicative nature of the shock is meant to reflect that policy makers become more agnostic about the consequences of their actions, the larger the policy deviation they wish to introduce. In particular, the specification implies that inflation is more variable and less predictable when it is higher, a feature that has strong empirical foundations (see, for example, Okun, 1971; Taylor, 1981; Ball and Cecchetti, 1990). A number of authors, including Okun and Ball and Cecchetti, have argued that this relationship is motivated through the effect of inflation on policy. When inflation is low, a concensus typically arises that it should be kept low. As a result, inflation is stable and predictable. However, in the case of moderate or high inflation, disagreement may arise about the necessity of reducing it, and so inflation becomes more variable and difficult to predict. Judd and Scadding (1982) offer an alternative explanation based on financial innovation. They argue that the most likely cause of the observed instability in the demand for money (after 1973) has been innovation in financial arrangements. These innovations allowed agents to economize on their holdings of transactions balances and appeared to have been triggered, to a large extent, by high inflation rates. Thus, high inflation, through its effect on financial innovation, may lead to more difficult monetary control. Holland (1993) argues that the strong (postwar) link between the rate of inflation and the degree of inflation uncertainty. may have been due to the uncertainty of forecasters about the impact of money growth on the price level. He presents evidence indicating that this has been the case. As long as the impact of money growth on the price level remains unpredictable, then even predictable money growth will cause inflation uncertainty.

The description of the monetary policy game is standard. There are two players: a private sector and a policy maker. Before locking itself into a nominal wage contract, the private sector formulates a prediction (π^e) about the increase in the price level during the duration of the contract. The strategy of the policy maker is to choose the degree of policy intervention (i^p). The timing is as follows. At time one, the private sector optimally chooses π^e. At time two, a supply shock ε is realized. At time three, the policy maker optimally chooses instrument i^p. At time four, a control shock s is realized and inflation, output and the payoffs of the players are determined. The information set of the private sector at time one only includes the structure of the model, whereas that of the policy maker at time three also includes the realization of the supply shock. At the times of their respective decisions, both players are uninformed about the future realization of the control shock.

The private sector's objective is to minimize forecast errors. Optimal prediction requires $\pi^e = E[\pi]$, where $E[\pi]$ denotes the mathematical expectation over the inflation rate, conditional on the private sector's information set at time one.

The description of the policy maker's objective function marks a second departure from previous work, which generally assumes regular quadratic preferences. We propose the following extension of the quadratic objective function:

$$\begin{aligned} \Omega &= \mu_1 (E[\pi])^2 + \theta_1 Var[\pi] \\ &+ \mu_2 (E[y] - ky^*)^2 + \theta_2 Var[y] , \end{aligned} \tag{3}$$

where μ_1, μ_2, θ_1 and $\theta_2 \geq 0$ and $k > 1$. We will term (3) the 'generalized quadratic objective function'. The policy maker is assumed to be concerned about inflation and output. The objective function reflects, for each variable of interest, the cost of a mean-squared bias (MSB) around a target and the cost of variability around the mean. The inflation target has been set to zero. The output target equals ky^* and exceeds the natural rate (as $k > 1$). The policy maker's concern for systematic underproduction may arise from the presence of labor market distortions or from political economy considerations. In what follows, the gap between the natural rate and the target rate of output will be denoted by $z \equiv (k-1)y^* > 0$.

The novelty of this objective function lies in the separation of the costs of expected and unexpected deviations in inflation and output. Parameters μ_1 and μ_2 measure the intensity of the policy maker's aversion to systematically missing the inflation and output target. Parameters θ_1 and θ_2 measure the policy maker's preference for nominal and real stability. Note that the simple quadratic objective function obtains as a special case of (3). The quadratic objective function imposes the technical restriction that $\mu_1 = \theta_1$ and $\mu_2 = \theta_2$. For example, let $\mu_1 = \theta_1 = \alpha$ and $\mu_2 = \theta_2 = 1$. We then obtain:

$$\tilde{\Omega} = E\left[\alpha\pi^2 + (y - ky^*)^2\right], \tag{4}$$

where α measures the relative aversion to inflation variability versus output variability around their respective targets.

We can think of a normative and a political economy justification for the proposed objective function. From a normative perspective, it may well be the case that society values expected versus unexpected deviations asymmetrically (resulting in $\mu_i \neq \theta_i$ for some $i \in \{1, 2\}$). For example, if shoe-leather costs of inflation are primarily associated with expected inflation and the costs of relative price distortion with unexpected inflation, society may find expected inflation relatively more costly ($\mu_1 > \theta_1$) if shoe-leather costs are relatively larger. All we need is that expected and unexpected deviations produce different types or magnitudes of costs.

From a political economy perspective, even if the preferences of society, or the government for that matter, cannot be represented by the generalized objective function, there may still be an interest in delegating monetary policy to a different agent whose behavior accords to a generalized preference structure. Society could instruct the agent to conduct policy *as if* the agent's preferences were given by (3).

The generalized objective function serves a double purpose in this chapter. First, with regard to the analysis of caution, it will make sense to focus not only on 'risk' (measured by σ_s^2) but also on the 'price of risk' (measured by θ_1 and θ_2). Second, with regard to the role of conservatism, policy maker heterogeneity in terms of μ_1, μ_2, θ_1 and θ_2 will prove helpful in the design of optimal delegation mechanisms.

1.2. Equilibrium

Given the set-up outlined above, we now look for a time-consistent equilibrium. Consider, as a description of the policy maker's strategy, the following linear policy reaction function:

$$i^p = \lambda_1 + \lambda_2 \epsilon.$$

Because of the linear-quadratic framework, we can focus without loss of generality on policy reaction functions of this form.Given the reaction function, the inflation outcome will be:

$$\pi = s\left(\lambda_1 + \lambda_2 \epsilon\right). \tag{5}$$

Taking rational expectations over (5), the private sector's optimal strategy is to set $\pi^e = \lambda_1$.

The policy maker's optimal choice of λ_1 and λ_2 minimizes (3) subject to the strategy of the private sector, the specification of uncertainty, and Equations (1) and (2). Substituting the relevant equations into the objective function and evaluating expectations at time one yields:

$$\begin{aligned}\Omega\left(\lambda_1, \lambda_2\right) &= \mu_1 \left\{\lambda_1^2\right\} \\ &+ \mu_2 \left\{\left(b\lambda_1 - b\pi^e - z\right)^2\right\} \\ &+ \theta_1 \left\{\sigma_\epsilon^2 \lambda_2^2 + \sigma_s^2 \left(\lambda_1^2 + \sigma_\epsilon^2 \lambda_2^2\right)\right\} \\ &+ \theta_2 \left\{\sigma_\epsilon^2 \left(1 + b\lambda_2\right)^2 + b^2 \sigma_s^2 \left(\lambda_1^2 + \sigma_\epsilon^2 \lambda_2^2\right)\right\}, \end{aligned} \tag{6}$$

where the first two lines display the respective mean-squared biases for inflation and output and the last two lines their respective variances.

It is instructive to see how the four terms in (6) are affected by λ_1 and λ_2. The *credibility* part of the policy rule (λ_1) shows up in all four terms. The *stabilisation* part of the policy rule (λ_2) matters only for the variance terms. As in the standard literature, optimal stabilisation policy trades off the benefit of lower output variability against the cost of higher inflation variability. But now, with multiplicative uncertainty, the policy maker also needs to take into account the consequences of policy non-neutrality ($\lambda_1 \neq 0$ or $\lambda_2 \neq 0$) for the variability in inflation and output.

The first-order conditions for λ_1 and λ_2 are:

$$\mu_1\lambda_1 + \sigma_s^2\left(\theta_1 + b^2\theta_2\right)\lambda_1 = b\mu_2\left(z + b\pi^e - b\lambda_1\right) \times \left(1 - \frac{\partial \pi^e}{\partial \lambda_1}\right) ; \tag{7}$$

$$\left(1 + \sigma_s^2\right)\left(\theta_1 + b^2\theta_2\right)\sigma_\epsilon^2\lambda_2 = -\, b\,\theta_2\sigma_\epsilon^2 , \tag{8}$$

where π^e is to be evaluated at λ_1.

Equation (7) illustrates the problem of time-inconsistency. If a formal commitment technology exists, the policy maker could commit to fully take into account the endogeneity of expected inflation with respect to the policy regime (i.e. $\partial\pi^e/\partial\lambda_1 = 1$). The solution under commitment (c) would then be given by $i^p = \lambda_1^c + \lambda_2^c\epsilon$ with

$$\lambda_1^c = 0 ;$$

$$\lambda_2^c = -\frac{b\,\theta_2}{\left(1+\sigma_s^2\right)\left(\theta_1 + b^2\theta_2\right)} .$$

However, if no formal commitment technology exists, the endogeneity of expected inflation is not internalized ($\partial\pi^e/\partial\lambda_1 = 0$). Optimal policy is then time-inconsistent and time-consistent policy is suboptimal (Kydland and Prescott, 1977) with:

$$\lambda_1 = \frac{b\,\mu_2\,z}{\mu_1 + \sigma_s^2\left(\theta_1 + b^2\theta_2\right)} ; \tag{9}$$

$$\lambda_2 = -\frac{b\,\theta_2}{\left(1+\sigma_s^2\right)\left(\theta_1 + b^2\theta_2\right)} , \tag{10}$$

where the λ's without superscripts refer to the no-commitment solution. Note that $\lambda_1 > \lambda_1^c$ and $\lambda_2 = \lambda_2^c$. The equilibrium policy reaction function is then given by:

$$i^p = \frac{b\,\mu_2\,z}{\mu_1 + \sigma_s^2\,(\theta_1 + b^2\theta_2)} - \frac{b\,\theta_2}{(1+\sigma_s^2)\,(\theta_1 + b^2\theta_2)}\,\epsilon\,. \tag{11}$$

Finally, the equilibrium realizations of inflation and output equal:

$$\begin{aligned} \pi &= s\,(\lambda_1 + \lambda_2\epsilon)\,, \\ y &= y^* + (s-1)\,b\lambda_1 + (1 + sb\lambda_2)\,\epsilon\,, \end{aligned}$$

where λ_1 and λ_2 are given by (9) and (10).

1.3. Properties of Equilibrium

In equilibrium, expected inflation equals:

$$E\,[\pi] = \frac{b\,\mu_2\,z}{\mu_1 + \sigma_s^2\,(\theta_1 + b^2\theta_2)}\,. \tag{12}$$

Average inflation exceeds the zero target rate of inflation. The inflationary bias arises because of the policy maker's systematic desire to surprise the private sector so as to achieve real output objectives ($z > 0$). In equilibrium, $E[y] = y^*$ because the inflationary bias exactly offsets the policy maker's temptation to surprise. Not surprisingly, a weaker preference for the inflation target relative to the output target (a lower μ_1/μ_2) leads to more inflation in equilibrium. One further mechanism arises due to the presence of multiplicative uncertainty.

PROPOSITION 1. *Caution enhances the credibility of monetary policy.*

The temptation to inflate is moderated by two other factors: the amplification of inflation and output variability through policy non-neutrality (as $\sigma_s^2 > 0$) and the aversion to such variability (measured by $\theta_1 > 0$ and $\theta_2 > 0$). The interaction between uncertainty and preferences results in cautious policy making. Recall that caution here refers to a more neutral, or less activist, policy stance.[13] The private sector rationally understands the policy maker's motive for caution and this will reduce the inflationary bias.[14]

The equilibrium variance of inflation is given by:

$$Var[\pi] = \left(\frac{b\,\theta_2}{\theta_1 + b^2\theta_2}\right)^2 \frac{\sigma_\epsilon^2}{1+\sigma_s^2} + \left(\frac{b\,\mu_2\, z}{\mu_1 + \sigma_s^2\left(\theta_1 + b^2\theta_2\right)}\right)^2 \sigma_s^2. \tag{13}$$

To interpret this expression, imagine an economy without multiplicative shocks ($\sigma_s^2 = 0$). The second term then disappears and the variability of inflation would

[13]Note that we have assumed that the consequences of multiplicative uncertainty are minimized when the policy deviation is set to zero. A cautious policy maker will want to set the instrument closer to this neutral threshold.

[14]The statement that *uncertainty* may improve credibility is not new. Devereux (1987) proposes an indirect mechanism: uncertainty induces wage setters to index nominal contracts, which reduces the effectiveness of surprise inflation and also the temptation of the policy maker to surprise. Swank (1994) and Pearce and Sobue (1997) show that uncertainty induces caution into policy making, directly constraining the temptation to surprise. This model focuses explicitly on the interaction between *uncertainty* and *preferences* to explain caution and will address delegation mechanisms which enhance the degree of caution.

crucially depend on θ_1/θ_2. For a given degree of supply volatility, the policy maker will be increasingly reluctant to vary the inflation rate if nominal stability is preferred to real stability ($\theta_1 > \theta_2$). Multiplicative uncertainty ($\sigma_s^2 > 0$) affects inflation variability in two ways. First, the policy maker becomes cautious and therefore less willing to stabilise output (the first term). Second, caution leads to more neutral policy and therefore smaller control errors (the second term). The overall effect is ambiguous, as the second channel may offset the first for relatively small degrees of uncertainty ($\sigma_s^2 < \mu_1/(\theta_1 + b^2\theta_2)$).

The equilibrium variance of output is given by:

$$Var[y] = \left(\left(\frac{\theta_1}{\theta_1 + b^2\theta_2}\right)^2 + \sigma_s^2\right) \frac{\sigma_\epsilon^2}{1+\sigma_s^2} \; + \; b^2 \left(\frac{b\,\mu_2\, z}{\mu_1 + \sigma_s^2\,(\theta_1 + b^2\theta_2)}\right)^2 \sigma_s^2. \qquad (14)$$

Without multiplicative shocks, the variability of output primarily depends on θ_1/θ_2. Multiplicative uncertainty affects output variability in two ways. First, output variability rises as the policy maker becomes cautious (the first term). Second, increased policy neutrality reduces the size of control errors that feed into output fluctuations (the second term). The overall effect is again ambiguous.

What are the overall implications for the policy maker's welfare?

PROPOSITION 2: *In the absence of an underproduction problem, multiplicative uncertainty unambiguously reduces welfare. In the absence of a stabilisation problem, multiplicative uncertainty unambiguously raises welfare. If both underproduction and stabilisation are an issue, the effect on welfare is ambiguous.*

To see this, substitute the mean-square biases and the variances of inflation and output into the objective function and differentiate with respect to σ_s^2:

$$\frac{\partial \Omega}{\partial \sigma_s^2} > 0 \quad \Leftrightarrow \quad \mu_2 z < \left(\frac{\mu_1 + \sigma_s^2 (\theta_1 + b^2\theta_2)}{(1+\sigma_s^2)(\theta_1 + b^2\theta_2)} \right) \theta_2 \sigma_\epsilon . \tag{15}$$

First, underproduction may not be a problem either because there is no underproduction ($z = 0$) or because underproduction is not valued in the policy maker's objective function ($\mu_2 = 0$). From (15), it is clear that multiplicative uncertainty would then reduce welfare. Intuitively, this results from the fact that higher uncertainty makes stabilisation more cautious, thereby distorting the policy maker's previously preferred balance between inflation and output variability. Output stabilisation may not be problematic either because there are no productivity shocks to stabilise ($\sigma_\epsilon = 0$) or because output stability is not valued by the policy maker ($\theta_2 = 0$). From (15), it follows that multiplicative uncertainty would then increase welfare. The intuition here is that uncertainty reduces the inflationary bias without creating additional distortions. In the intermediate case, the welfare effect is ambiguous and depends on the condition in (15).

The result that uncertainty may improve welfare is an application of the old idea that the introduction of an additional distortion in a second-best world does not necessarily reduce welfare. Of course, this need not imply that such uncertainty should be increased deliberately. There are more efficient ways to improve on the welfare properties of equilibrium and it is to these that we now turn our attention.

2. Monetary Policy Delegation with Quadratic Preferences

We now consider the delegation of monetary policy to an independent agent ('the central bank') with preferences distinct from those of the principal ('the government'). We ask whether central bankers who conduct policy cautiously, as a result of uncertainty, should be required to also conduct policy conservatively. To put it differently, does uncertainty obviate the need for conservatism? And, if it does not, are uncertainty and conservatism substitutable or complementary?

In addressing these questions, we assume regular quadratic preferences for the moment. The objective functions of the government and the central bank are respectively given by:

$$\begin{aligned} \tilde{\Omega} &= E\left[\, \alpha\pi^2 + (y - ky^*)^2 \,\right] \; ; \\ \tilde{\Omega}^* &= E\left[\, \alpha^*\pi^2 + (y - ky^*)^2 \,\right], \end{aligned}$$

where only α and α^* are allowed to differ. If $\alpha^* > \alpha$, the central banker attaches a stronger weight to the stabilisation of inflation than to that of output around the respective targets. The central banker is then said to be *weight-conservative.*

In considering mechanisms indexed by α^* only, we restrict attention to constrained-optimal delegation mechanisms. The setting therefore closely corresponds to the Rogoff (1985) model. Later on, we will use generalized preferences and look for optimal delegation mechanisms based on conservatism.

2.1. The Rogoff Conservative Central Banker Revisited

It is well-known that delegation to a weight-conservative central banker ($\alpha^* > \alpha$) improves the welfare properties of the discretionary equilibrium. But does the same conclusion continue to hold in an environment with multiplicative uncertainty? Appointment of a weight-conservative central banker results in an equilibrium with:

$$\begin{aligned} E[\pi] &= \frac{bz}{\alpha^* + \sigma_s^2(\alpha^* + b^2)}\,; \\ Var[\pi] &= \left(\frac{b}{\alpha^* + b^2}\right)^2 \frac{\sigma_\epsilon^2}{1+\sigma_s^2} + \left(\frac{b\,z}{\alpha^* + \sigma_s^2\,(\alpha^* + b^2)}\right)^2 \sigma_s^2\,; \\ Var[y] &= \left(\left(\frac{\alpha^*}{\alpha^* + b^2}\right)^2 + \sigma_s^2\right) \frac{\sigma_\epsilon^2}{1+\sigma_s^2} + b^2 \left(\frac{b\,z}{\alpha^* + \sigma_s^2\,(\alpha^* + b^2)}\right)^2 \sigma_s^2 \end{aligned}$$

and $E[y] = y^*$. These expressions are obtained as special cases of (12)-(14) with $\mu_1 = \theta_1 = \alpha^*$ and $\mu_2 = \theta_2 = 1$. Substituting them into the loss function of the government, $\tilde{\Omega} = \alpha\,(E[\pi])^2 + \alpha Var[\pi] + (E[y] - ky^*)^2 + Var[y]$, and differentiating with respect to α^* yields the following result:

PROPOSITION 3: *In an economy with multiplicative instrument uncertainty, delegation to an independent central banker is welfare improving as long as the central banker is weight-conservative but not excessively weight-conservative.*

Some algebra results in the following expression for government welfare:

$$\tilde{\Omega} = \left(\frac{\alpha b^2 + \alpha^{*2} + \sigma_s^2\,(\alpha^* + b^2)^2}{(\alpha^* + b^2)^2}\right) \frac{\sigma_\epsilon^2}{1+\sigma_s^2} + \frac{\alpha + \sigma_s^2(\alpha + b^2)}{(\alpha^* + \sigma_s^2(\alpha^* + b^2))^2} b^2 z^2 + z^2\,,$$

The optimal degree of weight-conservatism follows from inspection of:

$$\frac{\partial\tilde{\Omega}}{\partial\alpha^*} = -\frac{(\alpha+\sigma_s^2(\alpha+b^2))\,(1+\sigma_s^2)}{(\alpha^*+\sigma_s^2(\alpha^*+b^2))^3}b^2z^2 + \frac{(\alpha^*-\alpha)}{(1+\sigma_s^2)\,(\alpha^*+b^2)^3}b^2\sigma_\varepsilon^2\,. \tag{16}$$

First, note that, for $0 \le \alpha^* \le \alpha$, the first term in (16) is strictly negative while the second one is only weakly negative. As a result, $\partial\tilde{\Omega}/\partial\alpha^* < 0$. Second, the sign of $\partial\tilde{\Omega}/\partial\alpha^*$ must become positive for large values of α^*. To see this, note that the first term in (16) is negative while the second term is positive (for $\alpha^* > \alpha$). Both terms converge to 0 as α^* approaches $+\infty$. The first term converges at rate α^{*-3}, while the second term converges only at rate α^{*-2}. Consequently, $\partial\tilde{\Omega}/\partial\alpha^*$ must become positive as $\alpha^* \to +\infty$.

It is useful to establish this result also graphically. To interpret Figure 1, first rewrite condition (16) as

$$\alpha^* \;=\; \alpha \;+\; \frac{(\alpha+\sigma_s^2(\alpha+b^2))\,(1+\sigma_s^2)^2\,(\alpha^*+b^2)^3}{(\alpha^*+\sigma_s^2(\alpha^*+b^2))^3} \times \left(\frac{z}{\sigma_\varepsilon}\right)^2 \;\equiv\; F\,(\alpha^*)\,. \tag{17}$$

where the left-hand side is the 45-degree line and the right-hand side is a complicated function denoted by $F(\alpha^*)$. The Appendix establishes the following properties for $F\,(\alpha^*)$: (i) both $F\,(0)$ and $F\,(+\infty)$ are finite quantities exceeding α; (ii) $\partial F\,(\alpha^*)\,/\partial\alpha^* < 0$; (iii) $\partial^2 F\,(\alpha^*)\,/\partial\alpha^{*2} > 0$. These properties ensure that shape of $F(\alpha^*)$ conforms with the way it is drawn in Figure 1. As $F\,(\alpha^*)$ monotonically decreases in α^* and is bounded above and below by quantities larger than α, optimal weight-conservatism (α^{*opt}) is uniquely determined. (Other solutions to the fourth-order polynomial are two complex conjugates and a real root that is negative,

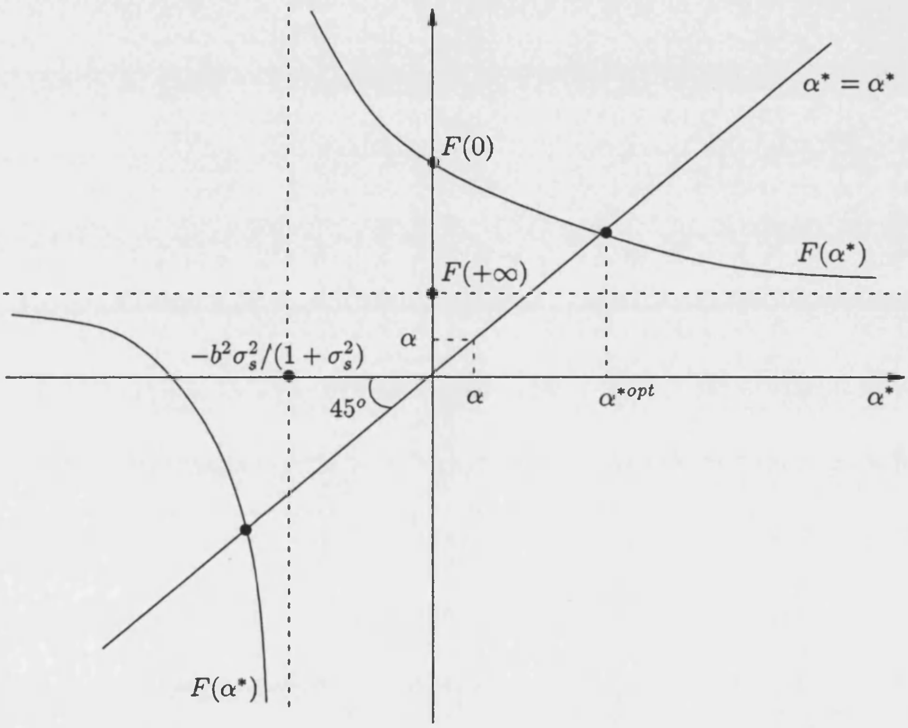

Figure 1. The Optimal Degree of Conservatism

given the location of the of the vertical asymptote at $-b^2\sigma_1^2/(1+\sigma_1^2)$.) Note that α^{*opt} is bounded above and below. The finite upper bound is given by $F(0)$. The lower bound is given by $F(+\infty)$, which is strictly larger than α.

Rogoff's (1985) analysis on the desirability of weight-conservatism thus seems robust to more general forms of uncertainty. Intuitively, weight-conservatism reduces the inflationary bias problem in a first-order way while only affecting the variance of output (and inflation) in a second-order way, thereby improving overall welfare. It is still impossible to restore full credibility, as a complete removal of the inflationary bias problem would continue to require more weight-conservatism than is optimal from the point of view of output stabilisation.

2.2. Conservatism, Ultra-Conservatism and Uncertainty

What are the consequences of an exogenous increase in the degree of multiplicative instrument uncertainty for the optimal degree of weight-conservatism?

PROPOSITION 4: *When optimal delegation suggests a moderate amount of conservatism, higher uncertainty reduces the optimal degree of conservatism. When optimal delegation requires ultra-conservatism, higher uncertainty increases the optimal degree of conservatism. Ultra-conservatism is defined by* $\alpha^* > \omega$*, where* $\omega \equiv 3\alpha + 2b^2\sigma_s^2/(1+\sigma_s^2)$.

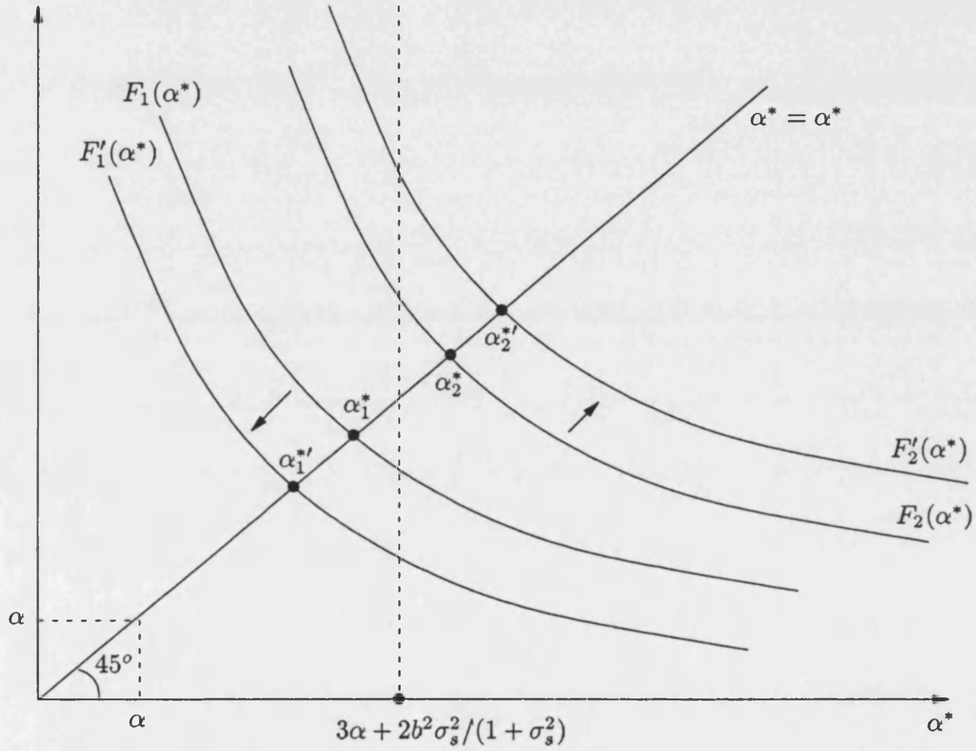

Figure 2. The Ambiguous Trade-Off between Uncertainty and Conservatism

The Appendix provides a proof.

This result can also be illustrated graphically. Consider the consequences of an increase in σ_s^2. Figure 2 is divided into two Regions. In Region 1, moderate conservatism ($\alpha^* < \omega$) was optimal before the change in σ_s^2. In Region 2, ultra-conservatism ($\alpha^* > \omega$) was optimal initially. The curves $F_1(\alpha^*)$ and $F_2(\alpha^*)$ are examples of environments consistent with Region 1 and Region 2. What happens when σ_s^2 increases? The left-hand side of (17) is not affected by σ_s^2. Moreover, the Appendix establishes that $\partial F(\alpha^*)/\partial\sigma_s^2 < 0$ for $\alpha^* < \omega$ and vice versa. As a result, $F_1(\alpha^*)$ will shift down to $F_1'(\alpha^*)$ and $F_2(\alpha^*)$ will shift up to $F_2'(\alpha^*)$. Optimal weight-conservatism therefore decreases from α_1^* to $\alpha_1^{*\prime}$ in Region 1 and increases from α_2^* to $\alpha_2^{*\prime}$ in Region 2.

While weight-conservatism remains welfare-improving under multiplicative uncertainty, its optimal level becomes dependent on the degree of such uncertainty. Intuitively, this follows from the observationally similar consequences of uncertainty and conservatism for the policy maker's equilibrium reaction function:

$$i^p = \frac{bz}{\alpha^* + \sigma_s^2(\alpha^* + b^2)} - \frac{b}{(1+\sigma_s^2)(\alpha^* + b^2)}\epsilon \ .$$

Note that both α^* and σ_s^2 reduce the systematic and feedback part in the policy rule. In environments where output stabilisation is a major issue and where therefore ultra-conservatism can never be optimal (see Proposition 3), uncertainty and conservatism are partial substitutes. If output stabilisation is only a minor issue, ultra-conservatism may be optimal initially and uncertainty and conservatism are partial complements.

3. Monetary Policy Delegation with Generalized Preferences

Weight-conservatism generally improves, but does not remove, the credibility problem of monetary policy. But why should the meaning of conservatism be artificially restricted to the particular notion of weight-conservatism? Why does the government not design a scheme that requires the central banker to behave conservatively in some *optimal* fashion? In answering these questions, we will now consider generalized preferences for the government and the central bank:

$$\begin{aligned}\Omega &= \mu_1 (E[\pi])^2 + \mu_2 (E[y] - ky^*)^2 + \theta_1 Var[\pi] + \theta_2 Var[y] \;; \\ \Omega^* &= \mu_1^* (E[\pi])^2 + \mu_2^* (E[y] - ky^*)^2 + \theta_1^* Var[\pi] + \theta_2^* Var[y] \;,\end{aligned}$$

where stars refer to the central bank's objective function. Note, however, that all the results would carry through if the government had regular quadratic preferences.

3.1. The Principle of Effective Market Classification

To begin with, consider delegation mechanisms where only μ_1^* and μ_2^* are allowed to vary. The preference space of the central banker is therefore restricted to $(\mu_1^*, \mu_2^*, \theta_1, \theta_2)$.

PROPOSITION 5: *Delegation of monetary policy to a central banker who disregards underproduction ($\mu_2^* = 0$) leads to the second-best.*

The proof is straightforward. Optimization subject to $\mu_2^* = 0$ leads to the following policy reaction function:

$$i^p = -\frac{b\,\theta_2}{(1+\sigma_s^2)\,(\theta_1 + b^2\theta_2)}\varepsilon, \tag{18}$$

which corresponds to commitment. The equilibrium in this economy is characterized by full credibility ($E[\pi] = 0$) and efficient stabilisation:

$$\begin{aligned} Var[\pi] &= \left(\frac{b\,\theta_2}{\theta_1 + b^2\theta_2}\right)^2 \frac{\sigma_\epsilon^2}{1+\sigma_s^2}\,; & (19) \\ Var[y] &= \left(\left(\frac{\theta_1}{\theta_1 + b^2\theta_2}\right)^2 + \sigma_s^2\right) \frac{\sigma_\epsilon^2}{1+\sigma_s^2}\,. & (20) \end{aligned}$$

This mechanism resembles what several authors have suggested informally. To quote Alan Blinder, for example, "a disarmingly simple solution to the Kydland-Prescott problem [is to] *direct* the central bank to behave as if it prefers [y^*] rather than [ky^*]".[15] This does not require, of course, that there would be no underproduction. All that is required, is that the central bank does not value the cost of underproduction, which can be nicely formalized with the generalized objective function by setting $\mu_2^* = 0$. The proposal can also be interpreted as an application of Mundell's 'principle of effective market classification'. This principle suggests that "an instrument should be matched with the target on which it exerts the greatest

[15] Blinder (1998, p. 43), no emphasis added. See also McCallum (1995, p. 208-9): "All that is needed for avoidance of the inflationary bias ... is for the CB to recognize the futility of continually exploiting expectations ..., and to recognize that its objectives would be more fully achieved on average if it were to abstain from attempts to exploit these temporarily-given expectations."

relative influence".[16] In this context, it is clear that monetary policy is not the right tool to deal with underproduction. If underproduction arises from labor market distortions, for example, structural labor market policy would be the right way to proceed.

A conceptually related mechanism can be derived if we restrict attention to delegation mechanisms where only θ_1^* and θ_2^* can be varied. The central banker's preference space is then restricted to $(\mu_1, \mu_2, \theta_1^*, \theta_2^*)$. Recall that θ_1^* and θ_2^* measure the preference intensity of the central bank for nominal and real stability. If we wish to retain an optimal mix of output and inflation stabilisation, it must be the case that $\theta_1^*/\theta_2^* = \theta_1/\theta_2$. This implies that $\theta_1^* = \chi^*\theta_1$ and $\theta_2^* = \chi^*\theta_2$, where $\chi^* \geq 0$ measures the central bank's overall concern for stability.

PROPOSITION 6: *Delegation of monetary policy to a central banker with an exclusive concern for stability ($\chi^* \to \infty$) leads to the second-best.*

If $\chi^* \to \infty$, the central bank's optimization problem ignores the MSB in inflation and output. Consequently, there will be no problem of credibility and, by construction, stabilisation will be optimal. This will correspond to commitment. The policy reaction function is given by (18), the variances of inflation and output by (19) and (20). Expected inflation equals zero and expected output is at the natural rate.

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[16] See Mundell (1968, p. 203).

Again, this is an application of Mundell's principle. If monetary policy, unlike structural policy, is a relatively swift tool for short-frequency stabilisation, optimal policy assignment implies that monetary policy should focus on what it is good at.[17]

3.2. Target-Conservatism and Stability-Conservatism

A valid criticism to any discretionary delegation scheme is that it may be overturned after the private sector has locked itself into nominal contracts. It is indeed odd that a government can perfectly commit to an institutional regime it puts into existence but not to the optimal monetary policy rule in the first place. This point, put forward by McCallum (1995), has been formalized by Jensen (1998), who shows that

[17]Propositions 6 and 7 seem identical but they are not. A central banker who disregards underproduction ($\mu_2^* = 0$) still cares about the mean level of inflation. A central banker with an exclusive concern for stability ($\chi^* \to \infty$) does not. Both approaches are equivalent in that the second-best equilibrium is obtained. But the reasons why zero average inflation is chosen differ. In the first case, the central banker suffers a direct loss from nonzero average inflation. In the second case, the central banker suffers an indirect loss: if $\sigma_s^2 > 0$, even the slightest systematic policy non-neutrality (i.e. $\lambda_1 > 0$) is amplified to an infinite extent. The central banker will therefore choose zero average inflation out of a concern for nominal and real stability. Note that the χ^*-approach only works in conjunction with multiplicative uncertainty. If $\sigma_s^2 = 0$, the equilibrium inflationary bias would be indeterminate.

the traditional second-best mechanisms break down when the cost of changing the monetary regime is not prohibitive.[18]

From a practical perspective, the identification of an institutional commitment problem should not be interpreted as a denial of the usefulness of delegation mechanisms in *reducing* the problem of credibility. To see this, consider that the monetary policy has been assigned to what it is good at and that the optimality requirements $\mu_2^* = 0$ or $\chi^* \to \infty$ have been formalized in the constitution of the central bank. The credibility of the constitutional regime will then depend on the cost of changing the constitution. In case of a prohibitive cost, $\mu_2^* = 0$ or $\chi^* \to \infty$ is renegotiation-proof. In the absence of a cost, $\mu_2^* = 0$ or $\chi^* \to \infty$ is not renegotiation-proof. Time-consistency would in that case require $\mu_2^* = \mu_2$ or $\chi^* = \chi \equiv 1$ and we are back at the discretionary solution. In what follows, we will discuss the intermediate case where the cost is nonzero but finite.

Target-Conservatism. Consider for example the case where the central banker has a stronger *relative* disregard for the output target versus the inflation target than the government has. This would correspond to $\mu_1^*/\mu_2^* > \mu_1/\mu_2$ and is

[18] McCallum (1995) argues that delegation merely relocates the time-inconsistency problem from one commitment problem (to a rule) into another one (to a regime). Al-Nowaihi and Levine (1996) show that the Walsh contract does not solve the time-inconsistency problem but relocates the credibility problem as a renegotiation problem. Jensen (1998) shows that a reputational solution to the credibility problem may still exist, although its likelihood becomes smaller when reappointment costs are positive.

termed the degree of 'target-conservatism'. Note that such a regime will produce a lower inflationary bias without distorting output stabilisation (as $\theta_1^*/\theta_2^* = \theta_1/\theta_2$ continues to hold). A natural empirical counterpart of an institutional set-up which induces an increase in the degree of target-conservatism is a regime of inflation targeting, where attention is diverted from the inconsistent output objective towards the inflation target. The concept of target-conservatism is in concert with the claim by practitioners that inflation targeting need not imply that output is ignored (Bernanke and Mishkin, 1997; King, 1997). As we have shown, target-conservatism does not distort output stabilisation.

Stability-Conservatism. Consider next the possibility where the central banker has a stronger *overall* preference for stability than the government has. This would correspond to $\chi^* > \chi \equiv 1$ and is termed the degree of 'stability-conservatism'. Once again, this leads to a lower inflationary bias without creating additional stabilisation distortions (as χ^* does not affect $\theta_1^*/\theta_2^* = \theta_1/\theta_2$). The notion of stability-conservatism seems to accord with successful monetary policy often being attributed to the penchant of central banks for stability. The model in this chapter shows that this feature may indeed be a desirable one. A penchant for stability leads to a lower inflationary bias provided that the economic environment features multiplicative uncertainty. Because of the central bank's stronger preference for nominal and real stability, uncertainty is now χ^* times more costly and this will lead to greater caution in the conduct of policy. This in turn reduces the inflationary bias (cf. Proposition 1).

4. Application: The Free Lunch Result of Delegation

Empirical studies have suggested that the delegation of monetary policy to an independent central bank is like a free lunch: it lowers inflation without increasing the variability of output.[19] At the theoretical level, this has created an anomaly in the Rogoff (1985) model, which predicts higher output variability in response to weight-conservatism. Subsequent research has shown that the free lunch result may be explained by (i) the offsetting interaction between higher 'economic variability' due to increased weight-conservatism and lower 'political variability' due to better insulation from the political business cycle (Alesina and Gatti, 1995); (ii) a positive correlation between the degree of central bank independence and the ability to stabilise or the degree of fiscal discipline (Fischer, 1995); (iii) the presence of a second-best delegation scheme (Svensson, 1997b). We offer an alternative explanation.

PROPOSITION 7. *In the presence of multiplicative instrument uncertainty, target-conservatism and stability-conservatism enhance nominal and real stability.*

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[19] See Alesina and Summers (1993), Debelle and Fischer (1994) and Fischer (1995). One caveat applies to the result that central bank independence *causes* low inflation at no real cost. Posen (1993, 1995) argues that correlations between institutions and economic outcomes may be spurious. In the context of this paper, for example, a period of economic instability may trigger a stronger aversion to variability (a higher χ^*), thereby making the development of institutions supporting that aversion more likely.

Consider first the consequences of target- and stability-conservatism for output variability. The variances of output will be respectively given by:

$$\begin{aligned} Var[y] &= \left(\left(\frac{\theta_1}{\theta_1 + b^2\theta_2}\right)^2 + \sigma_s^2\right) \frac{\sigma_\epsilon^2}{1+\sigma_s^2} + b^2 \left(\frac{b\,\mu_2^* z}{\mu_1^* + \sigma_s^2\,(\theta_1 + b^2\theta_2)}\right)^2 \sigma_s^2\,; \\ Var[y] &= \left(\left(\frac{\theta_1}{\theta_1 + b^2\theta_2}\right)^2 + \sigma_s^2\right) \frac{\sigma_\epsilon^2}{1+\sigma_s^2} + b^2 \left(\frac{b\,\mu_2 z}{\mu_1 + \chi^*\sigma_s^2\,(\theta_1 + b^2\theta_2)}\right)^2 \sigma_s^2. \end{aligned}$$

Note that, in both cases, the first term is unaffected while the second term will be lower. The first term is unaffected precisely because target- and stability-conservatism are *optimal* forms of conservatism. The second term decreases because both target- and stability-conservatism are conducive to a more neutral average policy stance. A stronger tendency towards policy neutrality reduces the nuisance of multiplicative random shocks. As a result, the variability of output will be lower relative to an environment with no delegation mechanism in place that reduces, or removes, the inflationary bias. The consequences for inflation variability are entirely analogous.

With reference to the earlier quote from Friedman (1968) on page 26, this result implies that the *credibility* of monetary policy, too, can help preventing money from being a source of variability. The implications for the free lunch result of delegation are thus twofold. First of all, delegation to a conservative central banker does not entail suboptimal output stabilisation if conservatism is not arbitrarily restricted to the notion of weight-conservatism. Alternative forms of conservatism, such as target-conservatism and stability-conservatism, reduce the inflationary bias without distorting the stabilisation of output. Second, *any* delegation scheme which improves or removes the credibility problem of monetary policy reduces at the same time the variability of output (and of inflation), if the transmission of monetary policy is subject to multiplicative uncertainty.

The overall theoretical implication is thus, surprisingly, that delegation based on optimal notions of conservatism should not only lead to lower inflation but also to less variable output. Strictly speaking, the empirical finding that delegation does not affect output variability could then be taken as evidence that the delegation schemes in place are not optimal. Observers may in fact argue that central banks favor nominal stability to real stability ($\theta_1^*/\theta_2^* > \theta_1/\theta_2$), leading to suboptimal output stabilisation but possibly identical degrees of output variability across institutional regimes.

Concluding Remarks

This chapter has addressed the question whether society benefits from the delegation of monetary policy to cautious and conservative central bankers. The framework that we have used extends the credibility literature with a more general description of preferences and uncertainty. We have made three points. First, the credibility problem of monetary policy may be exaggerated if there is multiplicative uncertainty about the effectiveness of policy. The tendency of policy makers to err on the side of caution creates internal incentives that compensate for the lack of precommitment. In principle, uncertainty may therefore improve welfare. We have also highlighted an interesting trade-off between uncertainty and weight-conservatism. Second, and this is the key insight, the chapter suggests a reconsideration of the role of conservatism in the delegation of monetary policy. Previous models have focused on suboptimal notions of central-bank conservatism. With a more flexible specification of preferences, new notions of conservatism are derived which in principle lead to the best feasible equilibrium. Third, the conservative-central-banker approach may not be inconsistent with the free lunch result of delegation. In fact, conservatism may not only lead to lower inflation but also to a lower variability of output.

We close with some limitations of the model and ideas for future work. For reasons of comparability, we preferred to keep with the credibility literature and therefore chose the simplest possible description of monetary policy transmission.

Adding a dynamic structure to the transmission mechanism would be a worthwhile extension. Another limitation is that the model features purely exogenous transmission uncertainty and that it abstracts from the issue of learning. We have not developed the model in this direction. Nevertheless, as suggested by Caplin and Leahy (1996), the possibility of learning should be kept in mind, especially if systematic search behavior of the policy maker influences the response of the private sector to policy. Furthermore, the model may be extended to address the issue of transparency. Observers have often argued that transparency is desirable, not so much because central banks possess valuable private information about the economy, but rather because the public may be uncertain about the preferences of policy makers. This chapter suggest another rationale for transparency: transparency clarifies the perceptions of policy makers. If the public fully appreciates the uncertainties facing the policy maker, average inflation may be lower. Finally, the model abstracts from the endogeneity of preferences to economic outcomes. Future work could analyze whether the interaction between variability and aversion to variability leads to monetary arrangements designed to foster stability in the future.

APPENDIX

The function $F(\alpha^*)$ has the following properties:

$$\begin{aligned}
F(0) &= \alpha + \frac{(\alpha + \sigma_s^2(\alpha + b^2))(1+\sigma_s^2)^2}{\sigma_1^6} \times \left(\frac{z}{\sigma_\varepsilon}\right)^2 > \alpha; \\
F(+\infty) &\equiv \lim_{\alpha^* \to +\infty} F(\alpha^*) = \alpha + 6\frac{\alpha + \sigma_s^2(\alpha + b^2)}{1+\sigma_s^2} \times \left(\frac{z}{\sigma_\varepsilon}\right)^2 > \alpha; \\
\frac{\partial F(\alpha^*)}{\partial \alpha^*} &= -3\,\frac{b^2(\alpha + \sigma_s^2(\alpha + b^2))(\alpha^* + b^2)^2(1+\sigma_s^2)^2}{(\alpha^* + \sigma_s^2(\alpha^* + b^2))^4} \times \left(\frac{z}{\sigma_\varepsilon}\right)^2 < 0
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial^2 F(\alpha^*)}{\partial \alpha^{*2}} &= 6b^2\left(\alpha + \sigma_s^2(\alpha + b^2)\right)\left(\alpha^* + b^2\right)\left(1+\sigma_s^2\right)^2 \times \\
&\quad \frac{(\alpha^* + b^2)(1+\sigma_s^2) + b^2}{(\alpha^* + \sigma_s^2(\alpha^* + b^2))^5} \times \left(\frac{z}{\sigma_\varepsilon}\right)^2 > 0.
\end{aligned}$$

To prove Proposition 4, rewrite (16) as

$$\Phi \equiv \frac{(\alpha + \sigma_s^2(\alpha + b^2))(\alpha^* + b^2)^3(1+\sigma_s^2)^2}{(\alpha^* + \sigma_s^2(\alpha^* + b^2))^3} \times \left(\frac{z}{\sigma_\varepsilon}\right)^2 + \alpha - \alpha^* = 0.$$

Implicit differentiation yields:

$$\frac{d\alpha^*}{d\sigma_s^2} = -\frac{\Phi_{\sigma_s^2}}{\Phi_{\alpha^*}},$$

where $\Phi_{\sigma_s^2}$ and Φ_{α^*} are given by:

$$\begin{aligned}
\frac{\partial \Phi}{\partial \sigma_s^2} &= \frac{b^2(1+\sigma_s^2)(\alpha^* + b^2)^3\left[(\alpha^* - 3\alpha)(1+\sigma_s^2) - 2b^2\sigma_s^2\right]}{(\alpha^* + \sigma_s^2(\alpha^* + b^2))^4} \times \left(\frac{z}{\sigma_\varepsilon}\right)^2; \\
\frac{\partial \Phi}{\partial \alpha^*} &= \frac{-3b^2(\alpha + \sigma_s^2(\alpha + b^2))(\alpha^* + b^2)^2(1+\sigma_s^2)^2}{(\alpha^* + \sigma_s^2(\alpha^* + b^2))^4} \times \left(\frac{z}{\sigma_\varepsilon}\right)^2 - 1.
\end{aligned}$$

These partial derivatives can be signed as follows:

$$\Phi_{\sigma_s^2} \;\propto\; (\alpha^* - 3\alpha)\,(1+\sigma_s^2) - 2b^2\sigma_s^2\;;$$

$$\Phi_{\alpha^*} \;<\; 0.$$

Hence,

$$\frac{d\alpha^*}{d\sigma_s^2} \;<\; 0 \;\Leftrightarrow\; \alpha^* \;<\; 3\alpha + \frac{2b^2\sigma_s^2}{1+\sigma_s^2} \;\equiv\; \omega,$$

which is the condition found in the main text.

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CHAPTER TWO
The Brainard Conservatism Principle with Non-Quadratic Objectives

"Academic macroeconomists tend to use quadratic loss functions for reasons of mathematical convention, without thinking much about their substantive implications. The assumption is not innocuous... I believe that both practical central bankers and academics would benefit from more serious thinking about the functional form of the loss function." Alan Blinder (1997, p.6)

Introduction

Ever since the inception of the inception of the Tinbergen-Theil framework in the 1950s, there has been an uneasy acceptance of the quadratic loss function for the purpose of analysing monetary policy. Both Tinbergen (1954, pp. 49-51) and Theil (1966) themselves were aware of the potential limitations of quadratic losses in terms of the description of risk and the lack of robustness of results.[20] Brainard (1967),

[20] For example, Theil writes (p. 19): "There is no particular reason to assume that the loss function should always be quadratic...th[e] asssumption [is a] convenient first approximation. When we try to

in his realistic extensions[21] to the standard framework, recognised this potential limitation[22], which makes the lack of attention paid in the subsequent literature surprising. Moreover, one of the most respected of academics cum policy maker, Alan Blinder (1997), has recently asked for a similar consideration, and it is the intention of this chapter to provide a response.

This chapter re-examines the celebrated 'Brainard conservatism principle' (Blinder, 1997, p. 11) under non-quadratic objective functions. This principle prescribes that the optimal policy response to a shock (that drives a policy objective away from target) should be muted if the economy is subject to multiplicative uncertainty. A muted response derives from the policy maker's concern about not only the expected deviation of the objective from target but also the variance of the target variable. The Brainard conservatism principle suggests that the policy instrument should be moved incompletely and gradually. Incompletely, because the policy maker is cautious about generating too much policy-induced variability. Gradually, because

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generalise...it appears that the results become much more complicated...it turns out frequently that the results become completely unmanageable. This is undoubtedly why the quadratic loss function has such a prominent place in several fields...".

[21] See Tobin's (1990) appreciation of the 1967 Brainard paper.

[22] Brainard writes (p. 413) that "the assumption of a quadratic is, of course, subject to the objection that it treats positive and negative deviations from target as equally important. The use of a fancier utility function would provide additional reasons for departing from certainty equivalence."

the policy maker prefers to smooth an instantaneous policy change into smaller changes over time.

The adoption of quadratic losses would seem to be an important part of the Brainard conservatism principle as the quadratic suggests a particular, and possibly perverse, attitude to risk. One where, for example, the policy maker is indifferent between a one-period *under*shoot of a particular inflation objective by 4% and a four-period *over*shoot by 2% (assuming, of course, that there is no discounting). Also, the use of the quadratic involves the implicit assumption of symmetry and it is worth examining how possible asymmetries would interact with the presence of additive and multiplicative uncertainty.

It seems quite plausible that if the characterisation of the policy maker's behaviour were made in a more appealing manner than quadratic utility then the specific generation of the Brainard conservatism principle, in response to multiplicative uncertainty alone, may be overturned. In fact, much recent work in both consumption theory and applied finance has involved examining of the integration of newer concepts of utility to older pricing puzzles.[23] Again, given the influence of this healthy literature it is surprising how little impact this has made on the analysis of optimal policy. And it is the examination of the robustness of the

[23]See Deaton (1992) for recent developments in consumption and Shiller (1998) for a signpost to the next generation of applied finance work in the non-expected utility paradigm.

Brainard conservatism principle to the deviations from quadratic losses that will be the focus of this chapter.

The rest of the chapter is structured as follows. Section 1 examines the impact on the optimal interest rule when the loss function reflects constant absolute risk aversion (CARA) in the face of additive uncertainty, and by analogy other classes of risk aversion (such as CRRA). Section 2 examines the impact on the optimal interest rate rule of both additive and multiplicative uncertainty when preferences are asymmetric. Section 3 analyses simulations of the resulting optimal rules in four different cases and provides a graphical general solution to the time path of interest and inflation rates following an inflation shock. The last section offers concluding remarks, discusses some implications for the optimal delegation of monetary policy and suggests some possible further work.

1. Deviations from Quadratics: Other Attitudes to Risk

The Brainard conservatism principle arises due to the interaction of multiplicative uncertainty with quadratic preferences. Alternatively, one could take the view that the resulting smoothing of interest rates is simply caused by a form of risk aversion (with respect to inflation volatility) other than the one implied by quadratics. For example, in terms of risk, two well-known properties of the quadratic are that the coefficient of relative risk aversion equals unity and that its third derivative is zero: the former implies that the elasticity of the policy maker's marginal loss with respect to inflation is always 1 and the latter implies that the variability of inflation does not affect marginal loss. The use of non-quadratics might be analogous to agents smoothing consumption in response to temporary income shocks. One might expect that the introduction of loss functions which deliver such smoothing in a consumption setting will also produce interest rate gradualism in a setting of monetary policy making. This is, however, not necessarily true. It is shown below that caution and gradualism may not follow from non-quadratic preferences as long as losses are symmetric and uncertainty is simply additive.

1.1. The Framework

Consider the following simple control problem:

$$\min_{\{i_t\}_{t=0}^{+\infty}} \; \Omega \equiv E_0 \left[\sum_{t=0}^{+\infty} \delta^{t+1} L\left(\pi_{t+1}; \pi^*\right) \right] \tag{1}$$

subject to

$$\pi_{t+1} - \bar{\pi} = a\left(\pi_t - \bar{\pi}\right) - b\left(i_t - \bar{i}\right) + e_{t+1} \tag{2}$$

with $e_{t+1} \sim N(0, \sigma_e^2)$, i.i.d., and where π_t, $\bar{\pi}$ and π^* refer respectively to the inflation rate at time t, the unconditional mean of inflation and the socially optimal rate of inflation; δ is the discount factor; a measures the persistence of the inflation process; b is the policy multiplier; $i_t - \bar{i}$ captures the deviation of the policy instrument from its natural level; e_{t+1} is an additive shock at time $t+1$.

Equations (1) and (2) tell us that the policy maker sets a path of interest rates such that future deviations of inflation from its target are minimised subject to an inflation relationship and a particular specification of preferences. The reduced-form process for inflation in (2) is kept deliberately simple as the emphasis of this framework is on the specification of the preferences of the policy maker.[24] The minimal features we require are persistence in inflation (ensuring a role for policy) and uncertainty (of the additive and later the multiplicative form). Inflation is described as an autoregressive process with a long-run mean equal to $\bar{\pi}$. Inflationary persistence is captured by parameter a where $0 \leq a < 1$. As well as the additive shock, e_{t+1}, inflation can be influenced by deviations of the policy instrument i_t from its neutral

[24]In a model that does incorporate a private sector with forward-looking expectations, the sluggishness in Equation (2) could be derived from the existence of nominal rigidities caused by menu costs or overlapping contracts.

level $\bar{i}$. For convenience, we assume throughout that $\bar{i} = 0$. Policy multiplier b (where $b > 0$) translates policy actions into inflation outcomes and is assumed to be non-stochastic in this section. The only source of uncertainty is the additive shock, e_{t+1}, which is normal and i.i.d. with mean 0 and variance σ_e^2. Note that the instrument is set at the beginning of each period, whereas the shock occurs at the end of each period. As a result, a shock has one-for-one first-round effects on inflation during the current period, but stabilisation policy can offset its second-round effects in subsequent periods. Finally, the intertemporal loss function in (1) consists of the infinitely discounted sum of per-period losses $L(\pi_{t+1}; \pi^*)$. Discount factor δ takes some value between 0 and 1.

Let us now turn to the specification of the per-period loss function. Natural candidates for a richer description of the policy makers behaviour towards risk would be the exponential (or CARA) and the isoelastic (or CRRA) loss functions:

$$L^{cara}(\pi_{t+1}; \pi^*) = \exp[\beta(\pi_{t+1} - \pi^*)] - 1 \tag{3}$$

where $\beta > 0$, and

$$L^{crra}(\pi_{t+1}; \pi^*) = \frac{(1 + \pi_{t+1} - \pi^*)^{1-\rho} - 1}{1 - \rho} \tag{4}$$

where $\rho > 0$ but $\rho \neq 1$, and

$$L^{crra}(\pi_{t+1}; \pi^*) = \ln(1 + \pi_{t+1} - \pi^*) \tag{4'}$$

for $\rho = 1$. As the name suggests, the CARA loss function is characterised by constant *absolute* risk aversion (equal to β), whereas CRRA implies constant *relative* risk aversion (equal to ρ). Recall that quadratic losses,

$$L^q(\pi_{t+1}; \pi^*) = \frac{(\pi_{t+1} - \pi^*)^2}{2},$$

imply *increasing absolute* risk aversion.

In order to substantiate our claim that symmetric non-quadratic preferences (and thus other descriptions of risk aversion than implied by the simple quadratic) may not deliver policy-caution or policy-gradualism per se, we will focus subsequently, without loss of generality, on the CARA loss function.

There is however one important caveat before proceeding. In the consumption literature, smoothing occurs due to the interaction between risk aversion and an intertemporal budget constraint ensuring an intertemporal trade-off between consumption today and consumption in the future. In the Tinbergen-Theil setting, however, there is no natural constraint on the inter-temporal behaviour of inflation - higher inflation does not necessarily imply lower inflation tomorrow. Due to the absence of a properly defined resource constraint, optimisation under CARA preferences will yield unrealistic solutions for the setting of interest rates. If the inflation target were for example equal to zero, optimality would require the interest rate to be set at plus infinity because the resulting negative rates of inflation imply policy gains. While the introduction of an output term in the loss function could certainly offset some of this perverse tendency, we have opted for a modification of the CARA function such that it incorporates the concept of a target. This will also allow for a more natural and direct comparison with the quadratic paradigm.

1.2. The Optimal Interest Rule

We now introduce the symmetric two-part CARA loss function:

$$L(\pi_{t+1};\pi^*) = \begin{cases} \exp\left[-\beta_1\ (\pi_{t+1}-\pi^*)\right]-1 & \text{for } \pi_{t+1}<\pi^* \\ \exp\left[\beta_2\ (\pi_{t+1}-\pi^*)\right]-1 & \text{otherwise} \end{cases} \tag{5}$$

where we impose $\beta_1 = \beta_2 \equiv \beta$ for symmetry. This two-part function is displayed in Figure 1. Using the indicator function, the loss function can be re-written as follows:

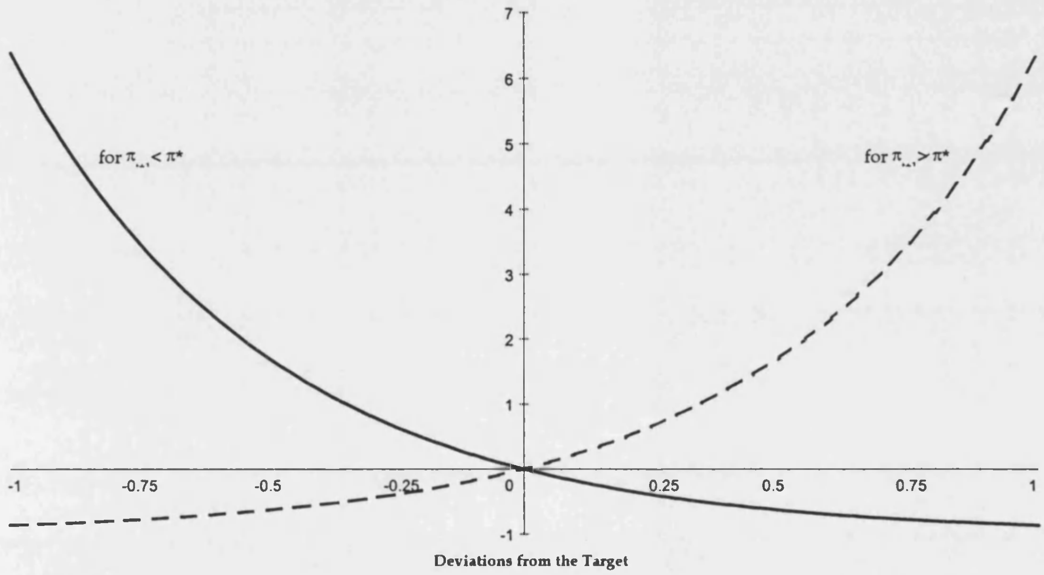
$$\begin{aligned} L(\pi_{t+1};\pi^*) &= I_{t+1}\exp\left[-\beta\ (\pi_{t+1}-\pi^*)\right] \\ &+ (1-I_{t+1})\exp\left[\beta\ (\pi_{t+1}-\pi^*)\right]\ -\ 1\ , \end{aligned} \tag{6}$$

where I_{t+1} takes the value 1 for inflation draws below target ($\pi_{t+1} < \pi^*$) and 0 for draws above. Equation (1) can therefore be re-written as:

$$\begin{aligned} \min_{\{i_t\}_{t=0}^{+\infty}} \quad & E_0\sum_{t=0}^{+\infty}\delta^{t+1}\, I_{t+1}\exp\left[-\beta\ (\pi_{t+1}-\pi^*)\right] \\ + \quad & E_0\sum_{t=0}^{+\infty}\delta^{t+1}\ (1-I_{t+1})\exp\left[\beta\ (\pi_{t+1}-\pi^*)\right]\ -\ 1 \end{aligned} \tag{7}$$

As a result of the simple dynamic structure in Equation (2), the multi-period control problem in (7) can be reduced to a series of one-period problems.

FIGURE 1
A Symmetric Two-Part CARA Loss Function



The decision problem at time t is therefore given by:

$$\begin{aligned}\min_{i_t} \quad & E_t\{\ I_{t+1}\ \exp[-\beta\ (\bar{\pi} + a\,(\pi_t - \bar{\pi}) - bi_t + e_{t+1} - \pi^*)\,]\ \} \\ & +\ E_t\{\ (1 - I_{t+1})\ \exp[\,\beta\ (\bar{\pi} + a\,(\pi_t - \bar{\pi}) - bi_t + e_{t+1} - \pi^*)\,]\ \}\ -\ 1 \quad (8)\end{aligned}$$

where $\bar{i}$ has been conveniently set to 0.

In order to evaluate the probability of the additive shock being on one side of the split-distribution or the other, we need to examine the probability of the inflation draw being greater or less than the current period inflation rate being less than or equal to the target. For notational convenience, let X_t denote $\pi^* - \bar{\pi} - a\,(\pi_t - \bar{\pi}) + bi_t$. The expectations over the indicator functions are then given by:

$$\begin{aligned} E_t\,[I_{t+1}] &= \Pr\ [e_{t+1} < X_t] = \Phi\left(\frac{X_t}{\sigma_e}\right) \\ E_t\,[1 - I_{t+1}] &= \Pr\ [e_{t+1} \geq X_t] = 1 - \Phi\left(\frac{X_t}{\sigma_e}\right), \end{aligned} \quad (9)$$

where $\Phi(\cdot)$ is the cumulative density of $N(0,1)$, the standard normal. The argument in (8) then becomes:

$$\begin{aligned} & \Phi\left(\frac{X_t}{\sigma_e}\right) \int_{-\infty}^{X_t} \exp\left[\beta\,(X_t - e_{t+1})\right] \phi\,(e_{t+1})\, de_{t+1} \\ & +\ \left(1 - \Phi\left(\frac{X_t}{\sigma_e}\right)\right) \int_{X_t}^{+\infty} \exp\left[\beta\,(-X_t + e_{t+1})\right] \phi\,(e_{t+1})\, de_{t+1} \end{aligned} \quad (10)$$

where $\phi(\cdot)$ is the probability density of the standard normal. Note that expectations are taken over the intervals $[-\infty, X_t]$ and $[X_t, +\infty]$ respectively. Evaluating Equation

(10) gives us:

$$\Phi\left(\frac{X_t}{\sigma_e}\right)\Phi\left(\beta\sigma_e+\frac{X_t}{\sigma_e}\right)\exp\left(\frac{\beta^2\sigma_e^2}{2}+\beta X_t\right)$$
$$+\left(1-\Phi\left(\frac{X_t}{\sigma_e}\right)\right)\left(1-\Phi\left(\beta\sigma_e+\frac{X_t}{\sigma_e}\right)\right)\exp\left(\frac{\beta^2\sigma_e^2}{2}-\beta X_t\right). \tag{11}$$

Because of global convexity of (5) we can see that this function attains its global minimum when X_t equals 0, i.e. when interest rates are set to close the gap between current inflation and target completely and immediately. To sum up, we have:

$$X_t = 0 \quad \Leftrightarrow \quad i_t = \frac{a}{b}(\pi_t - \bar{\pi}) + \frac{1}{b}(\bar{\pi} - \pi^*). \tag{12}$$

The expression in (12) is exactly the same as the one obtained under quadratic losses. It says that the deviation of the optimal interest rate from its neutral level (recall that the neutral level has been set to zero here) is a function of two components. The first component is essentially a simple feedback rule, implying that the interest rate response depends on how far last period's inflation was away from its long-run mean. The second component derives from the possibility that the inflation target does not necessarily correspond with the long-run mean of the autoregressive inflation process. If the inflation target is such that inflation will have to be sustained above (below) its long-run level, then interest rates need to be permanently lower (higher). Note that when the inflation target coincides with the unconditional mean of the inflation process, then the optimal rule is simply given by $i_t = (a/b)(\pi_t - \bar{\pi})$.

From (12) an important (and well-known) conclusion can be derived for the making of monetary policy: deviations from quadratics do not affect the optimal rule as long as the loss function is symmetric and uncertainty is additive. Interest rates will still be set so as to offset completely any shock to inflation last period. This result goes back to the work of Tinbergen (1952) and Theil (1964,1966) on optimal control theory in simple linear models, which can be further generalized for any symmetric loss function and any symmetric shock distribution. In an econometric context, Granger (1969), has shown that the conditional mean (leading to the optimal rule described above) continues to be the optimal predictor under more general conditions. The results therefore imply that richer descriptions of risk aversion (to that implied by quadratic losses) are irrelevant if the maintained hypothesis of additive uncertainty and symmetric preferences is not violated. To put it differently, risk aversion merely affects dead-weight losses.

Recall that nothing can be done about the first-round effects of an additive shock to inflation. Only the second-round effects to the next, and subsequent, period's inflation rate can be stabilised. We have shown that stabilisation will be complete and immediate: there is no element of policy-caution or policy-gradualism. This is what is meant with risk aversion, *per se*, being irrelevant for the *optimal rule.*

Of course, the extent of risk aversion is not irrelevant for the *value of the loss*, which equals:

$$\frac{1}{2}\left(1+\int\limits_{-\beta\sigma_e}^{\beta\sigma_e}\phi\left(e_{t+1}\right)de_{t+1}\right)\exp\left(\frac{\beta^2\sigma_e^2}{2}\right) \tag{13}$$

if the optimal policy is implemented. Note that the value of the loss in (13) increases both with the level of additive variability (σ_e) and the extent of the policy maker's aversion to risk (β). Intuitively, an increase in risk aversion, for example, means that a particular level of additive variability becomes more costly as the first-round inflationary effects cannot be undone. As a result, at the time that the policy maker can act upon the shock (i.e. the next period), the loss has occurred and is dead-weight. Note the analogy with finance theory: choosing a risk aversion parameter may alter the price of risk but as additive uncertainty is uncorrelated with policy risk, there is no impact on the insurable quantity of risk. This means that the optimal plan does not alter.

1.3. Caution and Gradualism

Introducing multiplicative uncertainty (about multiplier b) affects the optimal rule since certainty-equivalence no longer holds. The reason is simply that the actions of the policy maker bring about an additional source of variability into the loss function.

Thus, the dead-weight loss argument no longer applies. As in the quadratic case, this will make optimal policy cautious and gradualist.

In a framework with symmetric preferences and both additive and multiplicative uncertainty there are now two interactions going on. First of all, there is the earlier result that for a given level of additive variability risk aversion increases the dead-weight losses due to first-round effects on inflation. This does not affect, however, the optimal rule. Second, risk aversion will amplify the costs from a given degree of multiplicative uncertainty, when the policy maker tries to stabilise the second-round effects on inflation. The more risk averse the policy maker is, the more cautious and gradualist policy will be.[25] In contrast to the interaction between risk aversion and additive variability, risk aversion will not affect the dead-weight losses through the multiplicative uncertainty channel because this channel operates when interest rates are moved. Since in this model interest rate actions tomorrow cannot offset the first-round effects on inflation today, risk aversion does not amplify the dead-weight losses through this mechanism.

The split-CARA framework becomes analytically untractable when multiplicative uncertainty is introduced. But, in any case, the framework has served our purpose, in that we show formally that risk aversion is irrelevant in a setting of additive variability and symmetric preferences. If one wishes to examine issues

[25] The consequences of risk aversion for caution in the setting of the policy instrument have been discussed in Chapter 1, where we introduced the generalised quadratic objective function.

of policy-caution and policy-gradualism, it seems that non-quadratic symmetric preferences (and their implications for risk aversion), *per se*, are not sufficient. Moreover, they are not necessary as one can easily examine these issues in a quadratic framework.

2. Deviations from Quadratics: Asymmetry

The quadratic paradigm is sometimes criticised because positive and negative deviations from the target are treated symmetrically. In this section, we explore the implications arising from the assumption of asymmetric losses in a setting of monetary policy making. The analysis will show that a non-quadratic loss function around a particular target may be observationally similar to a quadratic loss function around a different target, even if we allow for a rich description of the stochastic nature of the economy.

Asymmetric losses may be an interesting way of characterising the policy maker's attitude to policy outcomes, *if* such attitudes reflect either (i) a view about the social welfare function or (ii) an exogenous view of the policy maker about the embarrassment costs of positive as compared to negative deviations from target or both. Note that even though the latter political economy line of thought may not be applicable to goal dependent inflation-targeting central banks with a symmetric inflation remit, the possibility remains that the government, when determining the level of the inflation target, has taken into account possible asymmetries to the social cost of inflation. As a consequence, the level of a symmetric inflation target may internalise possible asymmetries in the social cost of deviations of inflation from that target.

Varian (1975), in his discussion on the losses faced by property valuers, suggests an asymmetric loss function which rises linearly on one side of zero and rises exponentially on the other side.[26] It is this loss function, the so-called LINEX (Linear Exponential), which we employ to examine the impact of both additive and multiplicative uncertainty on the optimal path of interest rates.[27]

2.1. Introducing the LINEX Function

Varian (1975) introduced the following convex loss function:

$$L(x) = \alpha \exp(\gamma x) - \beta x - \alpha \tag{14}$$

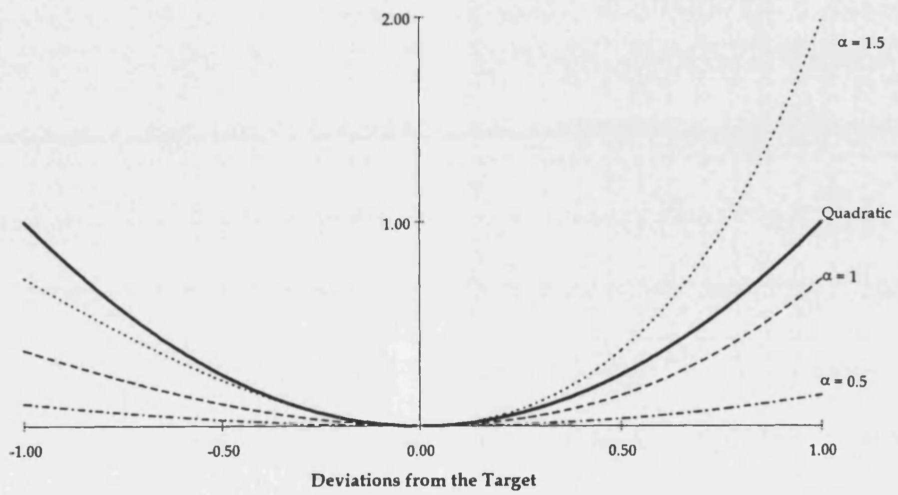
where $\alpha > 0$, $\beta > 0$ and $\gamma \neq 0$. Let x be the deviation of the policy objective from target.[28] We can see that $L(0) = 0$ and that for a minimum to exist at $x = 0$ we

[26] The argument used by Varian was that underassessment of property values led to approximately linear losses whereas overassessment may result in appeals, litigation and other costs. Zellner (1986) suggests an even clearer example by pointing out that in the construction of dams underestimations of peak flows is much more serious than overestimation.

[27] The results that we obtain with this particular asymmetric objective function can be easily generalized to other asymmetric objective functions. For example, if uncertainty is merely additive, Granger (1969) has shown that, for any asymmetric loss function and for a normal shock distribution, the optimal predictor will equal the conditional mean plus an additive term.

[28] In a related vein, Christofferson and Diebold (1997) study the optimal prediction problem under general asymmetric loss functions.

FIGURE 2
LINEX and Quadratic Losses Compared



must have $\gamma\alpha = \beta$.[29] Equation (14) can therefore be re-written as:

$$L(x) = \alpha\left[\exp(\gamma x) - \gamma x - 1\right], \tag{15}$$

where $\alpha > 0$ and $\gamma \neq 0$.

Note that γ determines the extent of the asymmetry in the LINEX function and that α scales the losses. Figure 2 shows the LINEX function for $\alpha = 1$ and for $\gamma = 0.5, 1.0, 1.5$. For comparison the quadratic losses are also plotted. The x-axis plots the deviation x. Note that for small losses the difference between the LINEX and quadratic appear small and, in fact, if we expand $\exp(\gamma x) = 1 + \gamma x + \frac{1}{2}(\gamma x)^2$, we find that $L(x) = \frac{1}{2}(\gamma x)^2$. But, of course, for larger values of x, the differences in losses become more substantial. Appendix A discusses some related points on the LINEX function.

2.2. The Optimal Interest Rule

The intertemporal maximisation problem with policy being subject to asymmetric preferences and both additive and multiplicative instrument uncertainty can be summarised as:

$$\min_{\{i_t\}_{t=0}^{+\infty}} \Omega \equiv E_0\left[\sum_{t=0}^{+\infty}\delta^{t+1}L(\pi_{t+1};\pi^*)\right] \tag{16}$$

[29]This is simply found by differentiating (14) with respect to x and solving for β.

with

$$L\left(\pi_{t+1};\pi^{*}\right)=\exp\left[\gamma\left(\pi_{t+1}-\pi^{*}\right)\right]-\gamma\left(\pi_{t+1}-\pi^{*}\right)-1, \tag{17}$$

where $\gamma > 0$, subject to:

$$\pi_{t+1}-\bar{\pi}=a\left(\pi_{t}-\bar{\pi}\right)-b_{t+1}i_{t}+e_{t+1}\ ,$$

and

$$\begin{pmatrix} b_{t+1} \\ e_{t+1} \end{pmatrix} \sim N\left[\begin{pmatrix} \bar{b} \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_b^2 & 0 \\ 0 & \sigma_e^2 \end{pmatrix}\right].$$

Note that we also have $0 \leq a < 1$, $\bar{b} > 0$ and b_{t+1} and e_{t+1} i.i.d.

The preferences of the policy maker are described by the LINEX loss function. If for some reason overshooting the inflation target (π^*) is more costly than undershooting it, we can restrict γ to be strictly positive. This will imply that undershooting is penalised in an approximately linear fashion, whereas the marginal losses from overshooting are increasing in next period's inflation rate. Of course, the following analysis could also be completed for the case where negative deviations imply exponential losses and positive deviations imply linear losses. But for the remainder of this chapter, we require, without loss of generality, γ to be strictly positive.

The aim of the exercise is to find the optimal interest rate path which will minimise the intertemporal loss function subject to the relationships in the above equations. Note that control is imperfect due to both additive and multiplicative uncertainty. Both sources of uncertainty are assumed to follow a normal distribution and to be independent from each other (i.e. $\sigma_{be} = 0$). The parameter of inflation persistence is assumed to be a known constant.

As to the precise nature of the uncertainty, the authorities may believe that the parameters of the model are random variables with a particular positive variance. Alternatively, they may regard the true (population) parameter values as being non-random quantities in the underlying model but put some margin of error on their estimated (sample) values. In what follows, we assume that the underlying additive shocks are genuine random variables (which will ensure a role for stabilisation policy) and that the multiplicative uncertainty derives from imperfect inference (which will deliver a cautious and gradualist setting of policy).[30]

Again, the multi-period control problem can be conveniently reduced to a series of one-period problems. The decision problem at date t is then given by:

$$\begin{aligned} \min_{i_t} \quad & E_t \; \{ \; \exp \left[\gamma \; (\bar{\pi} + a \, (\pi_t - \bar{\pi}) - b_{t+1} i_t + e_{t+1} - \pi^*) \right] \; \} \\ & - \; E_t \; \{ \; \gamma \; (\bar{\pi} + a \, (\pi_t - \bar{\pi}) - b_{t+1} i_t + e_{t+1} - \pi^*) \; \} \; - \; 1 \end{aligned} \tag{18}$$

[30]For a discussion, see Brainard (1967, p. 413-4). The assumption that multiplicative uncertainty derives from estimation error is made for mathematical convenience. Note that, in Chapter 1, we assumed that multiplicative shocks are genuinely random.

subject to

$$\begin{pmatrix} b_{t+1} \\ e_{t+1} \end{pmatrix} \sim N \left[\begin{pmatrix} \bar{b} \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_b^2 & 0 \\ 0 & \sigma_e^2 \end{pmatrix} \right].$$

The first-order condition of this optimisation problem implicitly defines the optimal interest rate setting:

$$\begin{aligned} i_t &= \frac{a}{\bar{b}} (\pi_t - \bar{\pi}) + \frac{1}{\bar{b}} (\bar{\pi} - \pi^*) + \frac{\gamma \sigma_e^2}{2\bar{b}} \\ &\quad + \frac{\gamma \sigma_b^2}{2\bar{b}} i_t^2 + \frac{1}{\gamma \bar{b}} \ln \left(1 - \frac{\gamma \sigma_b^2}{\bar{b}} i_t \right). \end{aligned} \tag{19}$$

The mathematical derivation of this expression is referred to Appendix B.

It is analytically impossible to get a reduced-form solution for the optimal rule in this general case. However, there are some interesting simple special cases.

Special Case 1. Symmetry and No Multiplicative Uncertainty ($\gamma \to 0$ and $\sigma_b^2 = 0$).

If the preferences of the policy maker tend to symmetry and there is no multiplicative instrument uncertainty, then the optimal rule collapses to:

$$i_t = \frac{a}{\bar{b}} (\pi_t - \bar{\pi}) + \frac{1}{\bar{b}} (\bar{\pi} - \pi^*). \tag{20}$$

This corresponds to the result that was also obtained in the previous section. To interpret this expression, assume that a one-off additive shock has occurred at time t, producing an overshooting of the inflation target by $x\%$ points. Nothing can be done about the initial boost in inflation, but as long as there is some persistence in inflation ($a > 0$), the second-round effects of the shock to inflation at time $t+1$ (another deviation from the inflation target by $ax\%$ points) will be fully neutralised.

Special Case 2. Asymmetry and No Multiplicative Uncertainty ($\gamma > 0$ and $\sigma_b^2 = 0$).

If preferences are asymmetric and there is no multiplicative uncertainty, the optimal rule is given by:

$$i_t = \frac{a}{\bar{b}}(\pi_t - \bar{\pi}) + \frac{1}{\bar{b}}(\bar{\pi} - \pi^*) + \frac{\gamma\sigma_e^2}{2\bar{b}}. \tag{21}$$

The asymmetry assumption produces an upward bias in the optimal rule if overshooting is considered to be more costly than undershooting. It is clear that the interest rate 'premium' due to risk aversion increases with the extent of additive variability as well as with the degree of asymmetry in the preferences of the policy maker.

Special Case 3. Symmetry and Multiplicative Uncertainty ($\gamma \to 0$ and $\sigma_b^2 > 0$).

If preferences approach symmetry and there is multiplicative uncertainty, then L'Hôpital's rule delivers the optimal rule:

$$i_t = \frac{1}{1 + \left(\sigma_b/\bar{b}\right)^2} \left[\frac{a}{\bar{b}}(\pi_t - \bar{\pi}) + \frac{1}{\bar{b}}(\bar{\pi} - \pi^*) \right]. \tag{22}$$

The derivation is found in Appendix C. This expression corresponds to Brainard's result (1967, p. 414) that one would obtain under quadratic losses. If the coefficient of variation ($\sigma_b/\bar{b}$) exceeds zero, the optimal interest rate response will be such that the gap with the inflation target is not entirely closed.

We can also solve for the long-run steady-state values of inflation and interest rates. This will be illustrated for Special Cases 1 and 3 (default and uncertainty), where there is no asymmetry. Analytically, Equations (20) and (22) need to be matched with the steady-state condition:

$$i = \frac{a-1}{\bar{b}} (\pi - \bar{\pi}) \tag{23}$$

which follows from Equation (2).

For the default case (Special Case 1), the steady-state values for the inflation and interest rate are respectively:

$$\pi^{SS} = \pi^* \tag{24}$$

$$i^{SS} = \frac{a-1}{\bar{b}} (\pi^* - \bar{\pi}). \tag{24'}$$

Inflation thus settles down at the inflation target and, unless the long-run mean is equal to the inflation target, policy is continually non-neutral ($i^{SS} \neq 0$).

For the uncertainty case (Special Case 3), the steady state can be characterised by:

$$\pi^{SS} = \lambda\pi^* + (1-\lambda)\bar{\pi} \tag{25}$$

$$i^{SS} = \frac{1}{1+\left(\sigma_b/\bar{b}\right)^2} \left[\frac{a\lambda - 1}{b} (\pi^* - \bar{\pi})\right], \tag{25'}$$

where

$$\lambda = \frac{1}{1+(1-a)\left(\sigma_b/\bar{b}\right)^2}.$$

Equation (25) shows that the long-run steady state in the uncertainty case can be represented as a weighted average of the long-run mean of the inflation process and the inflation target. If there is no multiplicative uncertainty, then λ equals 1 (as $\sigma_b^2 = 0$) and long-run inflation will hit the inflation target. The other extreme is the case of infinite multiplicative uncertainty which delivers λ equal to 0 (as $\sigma_b^2 \to +\infty$) and a long-run inflation rate which reverts to the long-run mean of the process. Similarly, Equation (25) shows that the degree of activism is inversely related to the degree of multiplicative uncertainty. This is what we mean with policy-caution in this particular setting: because of multiplicative uncertainty, the long-run response in interest rate is biased towards its neutral level; as a result, inflation will settle down closer to its mean. Note as well that if the long-run mean and the inflation target coincide, then the issue of caution entirely evaporates: inflation settles down at its target and interest rates at their neutral level.

Returning to the most general case (i.e. multiplicative instrument uncertainty and asymmetric preferences), note that the last two terms in Equation (19) result from the introduction of multiplicative uncertainty into the asymmetry case. An interesting issue is to what extent these terms lead to substantially different results compared to the introduction of such uncertainty in the default case. In order to answer this question let us turn to some simulation results.

3. Simulation Results

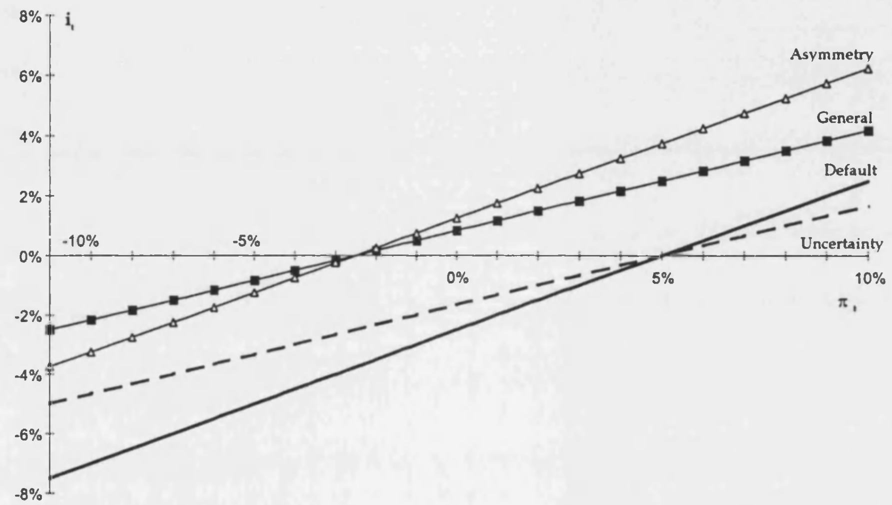
Section 2 derived expressions for the general form of the optimal rule, Equation (19), and for three relevant special cases: Case 1 being the default case (D), Case 2 the asymmetry case (A) and Case 3 the uncertainty case (U). This section examines numerically the implication of the optimal rule for the setting of interest rates both as a static first-period choice and as a dynamic path. We are also able to derive graphically the solution to the choice of optimal interest rates for all four cases. The final result allows us to answer the question of whether asymmetric preferences are sufficient to deliver gradualist interest rate responses and whether cautious interest rate responses are delivered.

For expositional purposes, we have made one modification to the general expression in Equation (19): the long-run mean of the inflation process has been set to zero (i.e. $\bar{\pi} = 0$). This will give further insights on the interaction between a long-run mean and an inflation target, which are not necessarily equal.

3.1. The Initial Interest Rate Response

Figure 3 examines the initial interest rate response to inflation shocks under the four cases. The size of inflation shocks (on the x-axis) is allowed to vary from -10% to +10% and the choice of optimal interest rates in the first period is shown on the y-axis.

FIGURE 3
Initial Interest Rate Responses to Inflation Shocks



We chose the following parameter values for the simulations: $\gamma = 1.5$, $a = 0.5$, $\bar{b} = 1$, $\pi^* = 2.5$, $\sigma_e^2 = 0.05$ and $\sigma_b^2 = 0.5$. This parameter choice is explained as follows: as γ is the extent of asymmetry in the loss function, which tends to symmetry as $\gamma \rightarrow 0$, a value of 1.5 from Figure 2 would seem a fair degree of asymmetry. Parameter a is the extent of non-policy related inflation persistence in the economy and is set to be something below the observed persistence - which includes policy reaction - typically found for modern industrialised economies.[31] Parameter $\bar{b}$ measures the mean impact on inflation of an interest movement and is set to allow for full pass through. Further, π^* has been set equal to the inflation target of the Bank of England and σ_e^2 is the variance of additive shocks, which is set to a small number less than σ_b^2 (the variance of multiplicative uncertainty). This will ensure that the state-independent bias in interest rates (the third term on the r.h.s. of Equation (19)) does not swamp the interaction terms between asymmetry and multiplicative uncertainty (the last two terms in Equation (19)).

Figure 3 shows that, for this range of single inflation draws, the optimal initial interest rate response rises linearly in the value of the inflation draw. The initial interest rate response rises at the rate $a/\bar{b}$ in both the default and asymmetry cases, at rate $a/(\bar{b}+\sigma_b^2/\bar{b})$ in the uncertainty case and at approximately the same rate

[31]The estimated sample persistence post-Bretton Woods has been in the order of $0.5 - 1.0$ for OECD countries. Of course, the non-policy related persistence parameter will in the real world include some expectation of likely policy accommodation.

in the general case. We find that in both the default and uncertainty case the optimal initial interest rate response from a five percent inflation draw is zero but that in both the asymmetry and the general case, reflecting the asymmetry bias, an optimal initial interest rate response of zero occurs when the inflation draw is $\pi^*/a - \gamma\sigma_e^2/(2a)$. This means that in comparison to the symmetric cases the optimal initial interest rate response with an inflation draw equal to target is biased up by an intercept amount of $\gamma\sigma_e^2/(2b)$ in the asymmetry case but something less in the general case because of the interaction between risk aversion and uncertainty. This is reflected by the last two terms in Equation (19).[32]

What happens to the initial interest rate choice under increasing parameter uncertainty? Figure 4 examines the initial response of interest rates to a given 10% inflation shock when the variance of b - the extent of multiplicative uncertainty - is allowed to increase from 0 to the implausible level of 2.[33] For obvious reasons, the initial interest response is state-independent of the level of multiplicative uncertainty in both default and asymmetry cases. The introduction of multiplicative uncertainty makes the initial response of the optimal interest rule in the general case similar to the default case when the variance of b is set at 1.5. One way of thinking about

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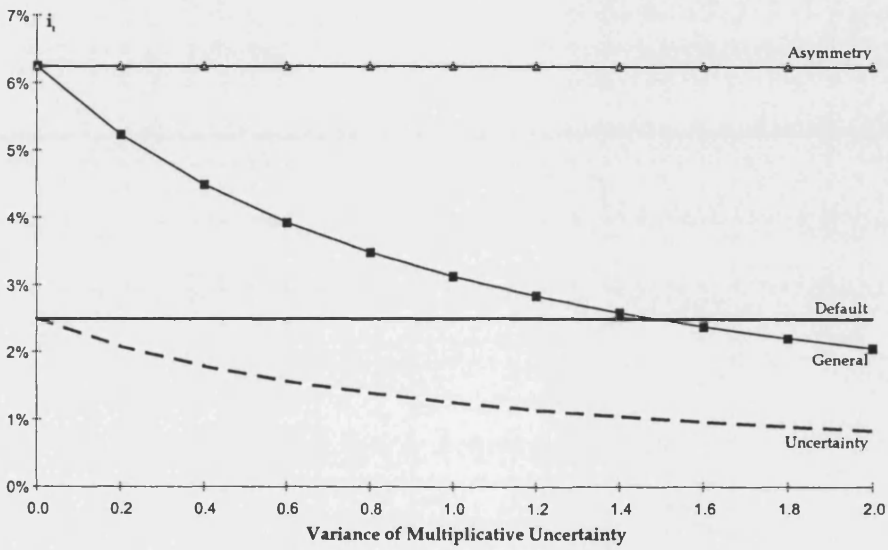
[32]Note again that, for convenience, we have assumed that the inflation target exceeds the long-run mean of inflation.

[33]For a value of b equal to 1, a variance of 2 may be considered to be implausible because there would be an approximately 20% chance of an increase in interest rates leading to a perverse response in inflation.

FIGURE 4
Initial Interest Rate Responses under Increasing Uncertainty



this result (if there is agreement on the other parameters) is to argue that if the policy maker thinks that the optimal initial step in interest rates following a 10% inflation shock is 2.5%, they either live in a default world or a general world with relatively large multiplicative uncertainty. Also note that if multiplicative uncertainty rises to implausibly large levels (i.e. greater than 2) then the initial interest rate response looks similar for the uncertainty and general cases. Or, if the policy maker considers that the economic structure is chronically uncertain with other factors tending to be outweighed, the initial interest rate response will tend to zero.[34]

3.2. The Dynamic Interest Rate Path

We are now ready to solve for the dynamic path of interest rates (and simultaneously for inflation) following the calculation of the initial response. We assume that the inflation rate is the beginning of period rate and the interest rate is the end of period rate. Following the initial inflation draw (π_t) and optimal interest rate response (i_t), the economy's inflation relationship, Equation (2) with $\bar{\pi} = \bar{i} = 0$, delivers a new inflation level (π_{t+1}). This in turn leads to a second optimal interest rate response (i_{t+1}) and so on until the steady-state values are reached.

[34]This is analogous to the Friedman (1951) argument for what might be termed as 'policy passivism'.

FIGURE 5a
Impulse Response Paths of Interest Rates

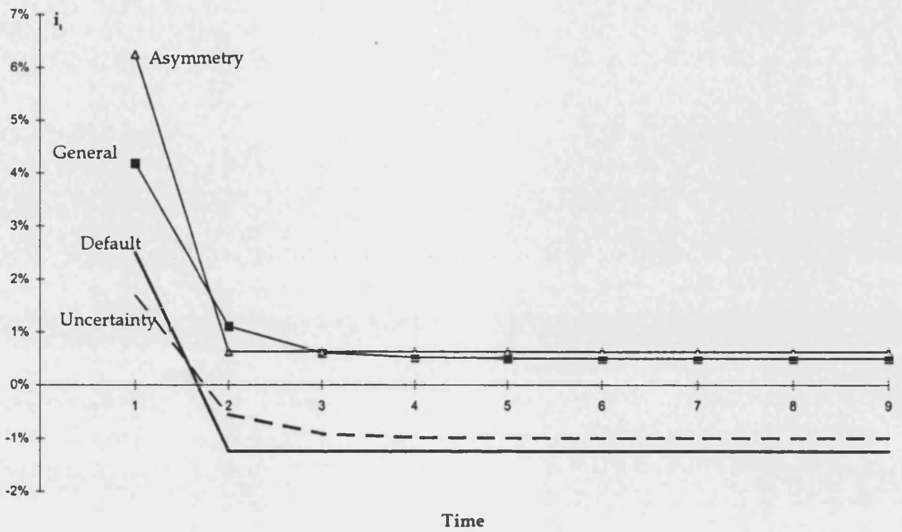


FIGURE 5b
Impulse Response Paths of Interest and Inflation Rates

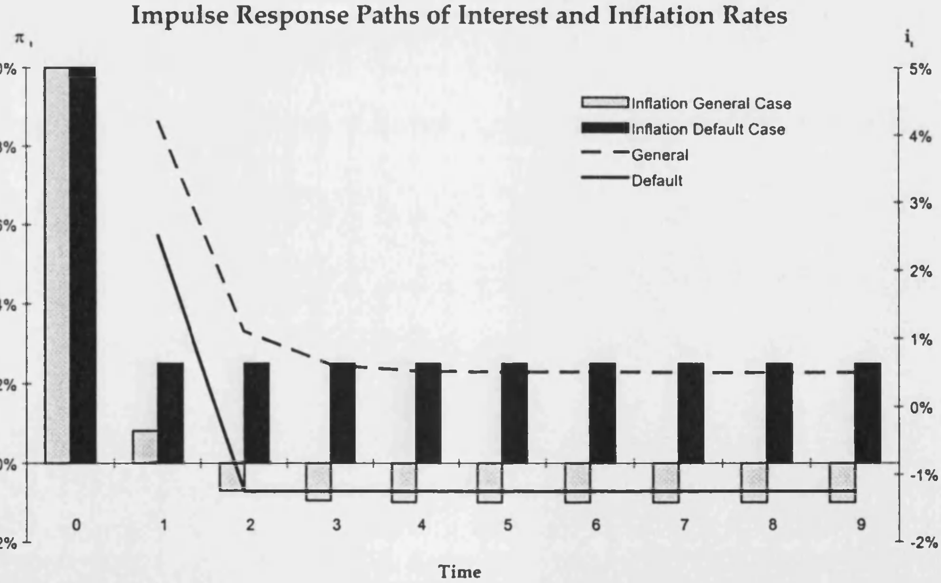


Figure 5a plots the response of interest rates over time to an inflation shock of 10% under the four cases. The first point to note is that the level of steady state interest rates is different in the four cases and so then is the steady-state inflation rate (or implied target).[35] Second, note that in the default and asymmetry cases interest rates return to their steady-state path at the end of the second period. In other words, there is no gradualism. In the two cases involving uncertainty the return to the steady state occurs by the end of the fourth period - i.e. it is gradualist. Finally, as long as the long-run mean of the inflation process is not equal to the inflation target, the gradualist response also delivers one which is cautious, in the sense that the long-run steady-state value of the interest rate will be closer to its neutral level.

Figure 5b plots the dynamic response of interest rates and inflation to a 10% inflation shock in the default and general cases where the explicit inflation target has been set to 2.5%. For the default case, in the absence of gradualism, interest rates and inflation arrive at their steady-state values after one period.[36] In the general case, the economy is close to its steady-state at the end of the fourth period. In the general case, because of both uncertainty and risk aversion, the LINEX loss function forces the optimal policy maker to drive the economy towards a lower inflation target

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[35]Note from the discussion in Section 3 that the implicit target is identical in the default and uncertainty case if the explicit inflation target (π^*) equals the long-run mean of inflation ($\bar{\pi}$).

[36]Interest rates are a negative deviation from base in this example because the inflation target is set above the zero long-run mean of inflation.

than explicitly stated. It is this implicit modification to the explicit target, down to some -1% in this case, and to the long-run mean projected by the economy's inflation relationship in Equation (2) which leads to the negative bias in the long-run inflation rate and the analogous positive bias to interest rates.

3.3. The Graphical Solution

Figure 6 plots a graphical solution to the simulations presented in Figures 3-5, namely the steady-state locus and the initial interest rate response. From the steady-state solution to Equation (2), we find that the steady-state locus passes through the origin with slope $(a-1)/\bar{b}$ and cuts the initial interest rate responses at the steady state locus of inflation and interest rates. This means for the four cases shown that the default, uncertainty, general and asymmetry cases imply successively lower inflation targets and higher steady-state interest rate. Just as the dynamic paths in Figure 4 showed different steady-state interest rates, Figure 6 shows the same steady-state in inflation/interest rate space. The rankings of the implied inflation targets in terms of their deviations from default case are parameter dependent but from Figure 6 we are able to say that, for positive asymmetry, (i) the non-default cases have lower implied inflation targets and (ii) the implied inflation target for the general case will always be *lower* than that for the uncertainty case.

FIGURE 6
The Steady State Values of Inflation and Interest Rates

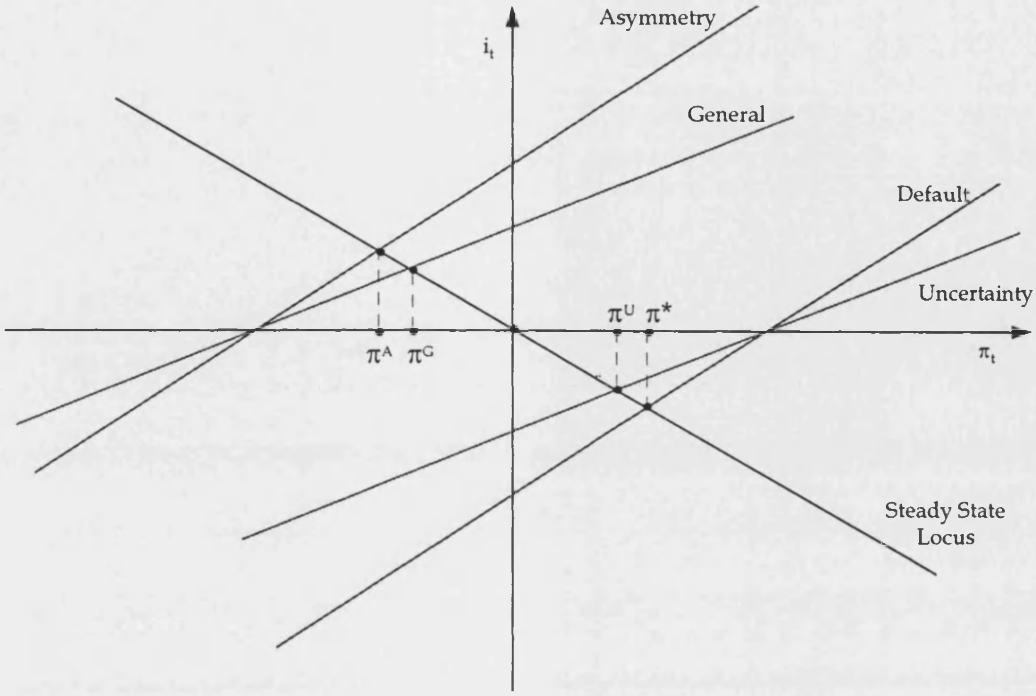

FIGURE 7
Response Paths in The Interest Rate Space

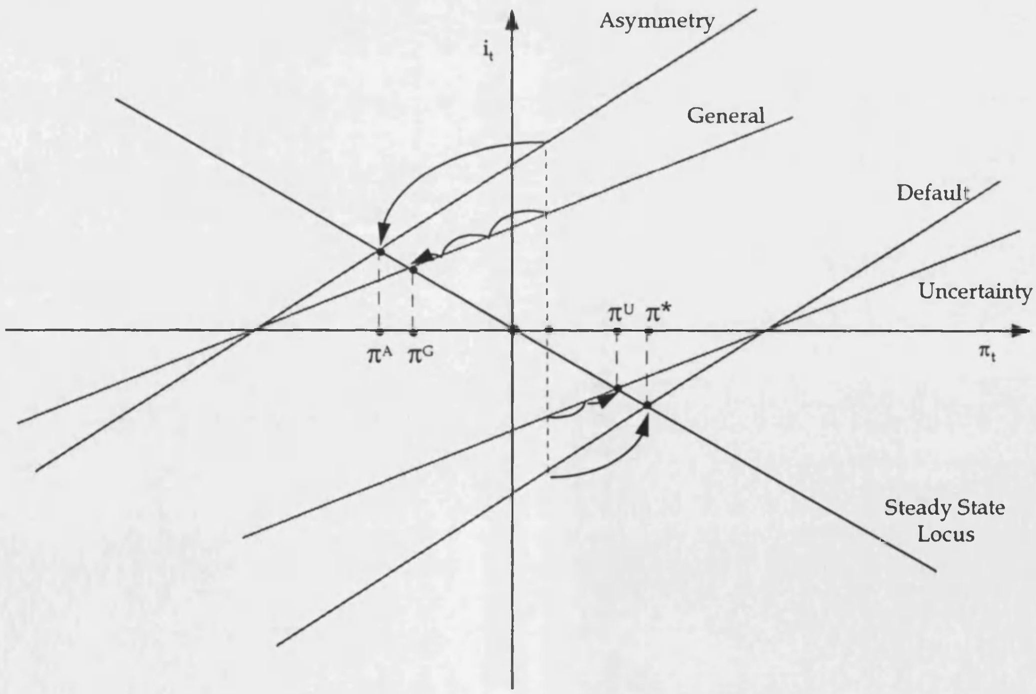

Figure 7 plots the dynamic response of interest rates in the inflation/interest rate space. To find the dynamic response to an initial inflation draw, a vertical line is displayed from the inflation draw to the initial interest rate response for one of the four cases. This vertical line shows the jump variable property of interest rates in the first period. In the cases without multiplicative uncertainty the next step (indicated by a little arrow) is the final one and represents the move back to the steady-state locus. In the cases involving uncertainty, the next steps involve exponentially decaying movements along the initial response line back to the steady-state locus (this is visualised by the periodic intersection of the arrow with the relevant response path). From the graph we can also see that the gradualist response is cautious: the intersection of the response path and the steady-state locus is closer to the origin when there is multiplicative uncertainty.

So why is the inflation target lower in the non-default case? There are three separate reasons. The easiest way is to first examine the move from the default case, with no multiplicative uncertainty, to the uncertainty case, we can see that arithmetically the bias follows from the (squared) coefficient of variation in the denominator. Intuitively, this means that the lack of perfect control over the economy makes the optimal policy maker choose to base interest rate decisions around an implicit inflation target somewhat lower than the explicit inflation target. Because the long-run mean of inflation embedded in the Phillips curve relationship is not identical to that of the explicit inflation target, the achievement of the inflation target actually requires some policy initiative towards which, in the presence of control uncertainty, the policy maker minimises expected losses by aiming too low.

Second, if we introduce asymmetry to the default case, this creates a bias to the explicit inflation target because losses due to positive deviations are magnified and the rational policy maker is able to mitigate them by aiming for a point that is smaller than the minimum of the LINEX function.

Finally, combining asymmetry and multiplicative uncertainty in the general case increases, for the parameters chosen, the inflation target. This is because of two separate effects: (i) the dichotomy between the explicit inflation target and the economy's long-run inflation mean mitigates the policy action in the asymmetry case and (ii) the tendency of the policy maker to choose neutral policy in the presence of uncertain control over the economy. For the cases involving uncertainty the lower inflation target results in interest rate smoothing which, with cautious responses, delivers lower inflation. This is so because the interest rate response is in two periods given the persistence of inflation. In the asymmetry case, the lower inflation target simply implies an activist setting of the first-period interest rate.

The main impact of asymmetric preferences is not to over-turn the Brainard conservatism principle. It is still parameter (or what we might to think of as control) uncertainty that leads to gradual responses. With either symmetric or asymmetric preferences, risk aversion in itself does not deliver smoothing when shocks are additive. We can find biases in interest rate setting for additive, as well as multiplicative uncertainty, in the case of asymmetric preferences. But these seem to occur in a very Brainard way - the implicit quadratic is simply shifted aside.

Concluding Remarks

This chapter re-examines the Brainard conservatism principle under non-quadratic policy objective functions. We deviate from the quadratic framework in the following two respects. First, while keeping with the assumption of symmetry, we examine the optimal interest rule that is obtained when the policy maker's preferences are given by a split constant absolute risk aversion function. In contrast to the simple quadratic, the split-CARA allows us to flexibly parameterise risk aversion and thereby change the curvature of the objective function. Second, we examine the consequences of asymmetric policy preferences for the optimal interest rule. This is achieved with a linear exponential objective function that allows us to parameterise the asymmetry in a straightforward fashion.

Changing the curvature of a symmetric loss function - for example, by introducing constant absolute (or relative) risk aversion - is shown not to matter for the optimal rule as long as uncertainty is additive. As a result, certainty equivalence also applies to non-quadratic loss functions provided that these are symmetric. So if the uncertainty is additive, deviating from quadratics does not buy us anything new: the optimal rule remains the same, and only the policy maker's dead-weight losses are different.

As with the quadratic case under additive uncertainty, welfare losses will be minimised at an inflation rate set equal to target. And so it continues to make sense to hit this target as soon and as closely as possible: there will thus be no case for gradualism or caution. In order to examine gradualism and caution, non-quadratic preferences, *per se*, are not sufficient as one needs to introduce multiplicative uncertainty. Moreover, non-quadratic preferences are not necessary as one can easily examine these issues in a quadratic framework. This brings us to the conclusion that the analytically convenient assumption of quadratic losses may not be that unreasonable after all.

When we introduce an asymmetry in the loss function (with a LINEX function) we find that the optimal interest rate rule is biased in a state-independent way, if uncertainty is merely additive. Asymmetric preferences then result in an interest rate path which is equivalent to that implied by a shifted quadratic loss function. If upward risks are considered more (less) costly than downward risks, then the minimum of the quadratic loss function is smaller (larger) than that of the non-quadratic. In our framework, this means that the implied inflation target (which internalises the asymmetry) is smaller (larger) than the stated target.

With multiplicative uncertainty, the asymmetry does not yield qualitatively different conclusions from changing the curvature of a symmetric loss function: gradualism and caution only obtain when uncertainty is multiplicative. Moreover, simulations of the optimal rule under asymmetry and multiplicative uncertainty show that the interest rate paths are very similar to those implied by a shifted quadratic. As for caution, we have also established that multiplicative uncertainty is not sufficient.

An additional requirement is that long-run policy interventions are necessary. This latter feature is illustrated in our rather simple model by letting the inflation target and the long-run mean of the inflation process differ.

With reference to the delegation of monetary policy, the use of asymmetric loss functions leads to a number of important insights. First of all, if the government requires the central bank to be goal dependent, then the central bank should also be required to pursue the delegated goal in a symmetric way. This result is consistent with the inflation remits of many central banks operating in inflation-targeting regimes. Second, if there is, for some social welfare or political economy reason, an asymmetry in the loss function of the government, this need not require the loss function of the central bank to be asymmetric as well. The asymmetry in the government's loss function would simply increase or decrease the mandated target (depending on the nature of the asymmetry) without necessarily altering the symmetry of the central bank's objectives.

Perhaps Alan Blinder (1998) had himself come to a conclusion similar to the one suggested by this chapter because in the year following his plea quoted in the Introduction he wrote "Sceptics often object to certainty equivalence on the grounds that...there is no particular reason to think that the objective function is quadratic...[but] policy makers almost always will be contemplating changes in policy instruments that can be expected to lead to small changes in macroeconomic variables, For such changes...any convex objective function is approximately quadratic".

As to future research, there may be considerable interest in exploring the implications of the results when the long-run mean and the inflation target are allowed to gradually coincide under some process of learning. To do so, the next step is to incorporate our results in a more realistic setting with agents whose expectations about inflation influence actual inflation outcomes. In addition, we might suggest at least three other possible uses of asymmetric loss functions: in the field of examining non-quadratic adjustment costs, for example, in models of investment; applications in explaining the excess returns in financial markets (i.e. that prices of assets may be biased); and finally, with respect to the maintenance of fixed exchange rate zones, where there is large asymmetry in the policy makers preferences at either limit of the exchange rate band.

APPENDIX A

The Arrow-Pratt coefficient of absolute risk aversion for this LINEX loss function is (we take the positive value of the second over the first derivative for $x < 0$ and the negative value for $x > 0$):

$$r_A(x) = -\frac{\gamma \exp(\gamma x)}{\exp(\gamma x) - 1}.$$

This coefficient has the property that there is risk neutrality at $x = 0$ and that $r'_A(x) > 0$ and $r''_A(x) < 0$ for $x \neq 0$.

The expectation of the following LINEX function:

$$L(\pi\,;\,\tilde{\pi}) = \exp\left[\gamma\,(\pi - \tilde{\pi})\right] - \gamma(\pi - \tilde{\pi}) - 1$$

is given by:

$$E_\pi L(\pi\,;\,\tilde{\pi}) = \exp(-\gamma\tilde{\pi}) E_\pi \exp(\gamma\pi) - \gamma(E_\pi \pi - \tilde{\pi}) - 1, \tag{A1}$$

where $\tilde{\pi}$ is the implicit inflation target. Minimizing (A1) with respect to π yields:

$$\tilde{\pi} = \frac{1}{\gamma} \ln\left[E_\pi \exp(\gamma\pi)\right],$$

which can be evaluated analytically when π has a normal probability density function (say with mean μ and variance σ_π^2). We then have:

$$E_\pi \exp(\gamma\pi) = \exp\left(\gamma\mu - \frac{\gamma^2\sigma_\pi^2}{2}\right).$$

This in turn gives:

$$\tilde{\pi} = \mu - \frac{\gamma\sigma_\pi^2}{2} \tag{A2}$$

Equation (A2) tells us that the expectation of the loss function tends to move away from the quadratic as γ and σ_π^2 differ from zero.

APPENDIX B

We first look for the solution to the following non-linear expectation:

$$\begin{aligned}
& E_t\left[b_{t+1}\exp\left(-\gamma b_{t+1}i_t\right)\right] \\
= & \int_{-\infty}^{+\infty} b_{t+1}\exp\left(-\gamma b_{t+1}i_t\right)\frac{1}{\sqrt{2\pi}\sigma_b}\exp\left[-\frac{\left(b_{t+1}-\bar{b}\right)^2}{2\sigma_b^2}\right]db_{t+1} \\
= & \int_{-\infty}^{+\infty} b_{t+1}\frac{1}{\sqrt{2\pi}\sigma_b}\exp\left[\frac{-b_{t+1}^2+2\left(\bar{b}-\gamma\sigma_b^2 i_t\right)b_{t+1}-\bar{b}^2}{2\sigma_b^2}\right]db_{t+1} \\
= & \exp\left(-\bar{b}\gamma i_t+\frac{\gamma^2\sigma_b^2}{2}i_t^2\right)\int_{-\infty}^{+\infty} b_{t+1}\frac{1}{\sqrt{2\pi}\sigma_b}\exp\left[-\frac{\left(b_{t+1}-\left(\bar{b}-\gamma\sigma_b^2 i_t\right)\right)^2}{2\sigma_b^2}\right]db_{t+1} \\
= & \left(\bar{b}-\gamma\sigma_b^2 i_t\right)\exp\left(-\bar{b}\gamma i_t+\frac{\gamma^2\sigma_b^2}{2}i_t^2\right).
\end{aligned}$$

This will help us to derive the optimal rule in the general case.

Consider the following optimisation problem:

$$\begin{aligned}
\min_{i_t} \quad & E_t\ \left\{\ \exp\left[\gamma\left(\bar{\pi}+a\left(\pi_t-\bar{\pi}\right)-b_{t+1}i_t+e_{t+1}-\pi^*\right)\right]\ \right\} \\
& -\ E_t\ \left\{\ \gamma\left(\bar{\pi}+a\left(\pi_t-\bar{\pi}\right)-b_{t+1}i_t+e_{t+1}-\pi^*\right)\ \right\}\ -\ 1
\end{aligned}$$

subject to:

$$\begin{pmatrix} b_{t+1} \\ e_{t+1} \end{pmatrix} \sim N\left[\begin{pmatrix} \bar{b} \\ 0 \end{pmatrix},\begin{pmatrix} \sigma_b^2 & 0 \\ 0 & \sigma_e^2 \end{pmatrix}\right].$$

The first-order condition to this problem is:

$$E_t \left\{ \gamma b_{t+1} \exp \left[\gamma \left(\bar{\pi} + a \left(\pi_t - \bar{\pi} \right) - b_{t+1} i_t + e_{t+1} - \pi^* \right) \right] \right\} = \gamma \bar{b}.$$

Since b_{t+1} and e_{t+1} are assumed to be independent, we have:

$$E_t \left\{ \gamma b_{t+1} \exp \left[-\gamma \, b_{t+1} i_t \right] \right\} \; E_t \left\{ \exp \left[\gamma \, e_{t+1} \right] \right\} = \frac{\gamma \bar{b}}{\exp \left[\gamma \left(\bar{\pi} + a \left(\pi_t - \bar{\pi} \right) - \pi^* \right) \right]}.$$

Evaluating the nonlinear expectations using our earlier result gives:

$$\left(\bar{b} - \gamma \sigma_b^2 i_t \right) \exp \left(-\bar{b} \gamma i_t + \frac{\gamma^2 \sigma_b^2}{2} i_t^2 \right) \exp \left(\frac{\gamma^2 \sigma_e^2}{2} \right) = \frac{\gamma \bar{b}}{\exp \left[\gamma \left(\bar{\pi} + a \left(\pi_t - \bar{\pi} \right) - \pi^* \right) \right]}.$$

Taking logarithms delivers the final result mentioned in the main text:

$$\begin{aligned} i_t \;\; = \;\; & \frac{a}{\bar{b}} \left(\pi_t - \bar{\pi} \right) \; + \; \frac{1}{\bar{b}} \left(\bar{\pi} - \pi^* \right) \; + \; \frac{\gamma \sigma_e^2}{2 \bar{b}} \\ & + \; \frac{\gamma \sigma_b^2}{2 \bar{b}} i_t^2 \; + \; \frac{1}{\gamma \bar{b}} \ln \left(1 - \frac{\gamma \sigma_b^2}{\bar{b}} i_t \right). \end{aligned}$$

APPENDIX C

The derivation of the optimal rule when preferences tend to symmetry is as follows:

$$\begin{aligned}
i_t &= \lim_{\gamma\to 0}\left[\frac{a}{\bar{b}}\left(\pi_t-\bar{\pi}\right)+\frac{1}{\bar{b}}\left(\bar{\pi}-\pi^*\right)+\frac{\gamma\sigma_e^2}{2\bar{b}}+\frac{\gamma\sigma_b^2}{2\bar{b}}i_t^2+\frac{1}{\gamma\bar{b}}\ln\left(1-\frac{\gamma\sigma_b^2}{\bar{b}}i_t\right)\right]\\
&= \frac{a}{\bar{b}}\left(\pi_t-\bar{\pi}\right)+\frac{1}{\bar{b}}\left(\bar{\pi}-\pi^*\right)+\lim_{\gamma\to 0}\left[\frac{1}{\gamma\bar{b}}\ln\left(1-\frac{\gamma\sigma_b^2}{\bar{b}}i_t\right)\right]\\
&= \frac{a}{\bar{b}}\left(\pi_t-\bar{\pi}\right)+\frac{1}{\bar{b}}\left(\bar{\pi}-\pi^*\right)-\lim_{\gamma\to 0}\left(\frac{\sigma_b^2 i_t}{\bar{b}^2-\bar{b}\gamma\sigma_b^2 i_t}\right)\\
&= \frac{a}{\bar{b}}\left(\pi_t-\bar{\pi}\right)+\frac{1}{\bar{b}}\left(\bar{\pi}-\pi^*\right)-\frac{\sigma_b^2}{\bar{b}^2}i_t.
\end{aligned}$$

This latter expression yields the result mentioned in the main text:

$$i_t=\frac{1}{1+\left(\sigma_b/\bar{b}\right)^2}\left[\frac{a}{\bar{b}}\left(\pi_t-\bar{\pi}\right)+\frac{1}{\bar{b}}\left(\bar{\pi}-\pi^*\right)\right].$$

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CHAPTER THREE
Costly Collateral and The Public Supply of Liquidity

"The potentially disruptive impact of asset price fluctuations on the balance sheets of financial institutions underscores the need for a highly capitalized and well-supervised financial system. Fragile financial systems have a reduced capacity for channeling funds from savers to borrowers, raising the cost of capital and restricting the access of innovative entrepreneurs to liquid funds which, in turn, hampers investment and economic growth." (International Monetary Fund, 2000, pp. 127-8)

Introduction

Credit market imperfections, resulting from informational asymmetries between borrowers and lenders, create moral hazard and adverse selection problems that make lenders reluctant to lend and leave borrowers deprived from credit. Credit constraints disrupt the essential function of the financial system: to channel funds to those with the most productive investment opportunities. If this role is not performed well, the economy does not operate efficiently and economic growth will be hampered. One important way for society to deal with such problems has been to require collateral or net worth as a condition to the provision of finance.

The central issue explored in this chapter is that the positive role of collateral in alleviating informational asymmetry needs to be balanced with two potential complications: (i) the possibility that the seizure of collateral results in sizable liquidation costs and (ii) the problem that collateralization may enhance vulnerability to asset price volatility.

The model that we present here develops an incomplete contracting framework where wealth-constrained entrepreneurs cannot commit project returns since these are assumed to be non-verifiable to courts. The information asymmetry creates an extreme form of moral hazard that makes the use of collateral a necessary condition for the provision of finance. However, because of costly asset liquidation, optimal collateralization is generally incomplete. The model illustrates the interaction between leverage and collateralization and shows under what conditions underinvestment and credit constraints occur. We also show that collateralized finance may be highly susceptible to interest rate shocks.

The model also addresses the impact of aggregate asset price uncertainty on the borrower-lender relationship. Particularly when borrowers are poorly capitalized, aggregate uncertainty affects the nature of the optimal contract (leading to a stronger degree of asset price contingency). We discuss the case where banks are also poorly capitalized and show how aggregate uncertainty may amplify the extent to which wealth-constrained entrepreneurs are deprived from credit.

Furthermore, we argue that there is scope for publicly supplied liquidity when banks face tight solvency constraints and collateral is subject to much asset price volatility. We will argue that the type of state-contingent government intervention that arises from this model may also be interpreted as countercyclical monetary policy.

We conclude this chapter with a brief discussion of a number of policy-related applications of the model. It is argued that our framework may be used to study the desirability of interest rate smoothing and to provide micro-foundations for second-generation currency crisis models.

How does our contribution relate to the literature? There is a large body of research that has focused on the microeconomic role of collateral in alleviating moral hazard and adverse selection problems. A common theme of this literature is that collateral requirements lower the hurdle to external finance by limiting the loss of the lender in the case of default. In the presence of *adverse selection*, collateral may be used by lenders to sort borrowers according to their risk profile. Borrowers with a high default probability reveal themselves by rejecting contracts with low loan rates and high collateral requirements. As a result, high risks pay a higher loan rate and are not required to put down any collateral. Low risks put down collateral but pay a lower loan rate (see, for example, Bester, 1985).

If *moral hazard* is the concern, collateral may serve as an incentive device that motivates borrowers to behave diligently. In this context, Boot, Thakor and Udell (1991) study the relation between collateral and borrower risk in the presence of a repossession cost. The availability of collateral in their paper is taken as exogenous. Our work corresponds most closely to this branch of thinking. We assume an extreme form of moral hazard, enabling the borrower to divert all of the project returns in the case of success. A major difference however is that we deviate from the constant project size assumption that is common in this literature. This enables us to study the consequences of moral hazard and liquidation costs for the optimal or, if credit-constrained, feasible scale of operation. Hart and Moore (1994) develop a theory of debt that is also based on an extreme moral hazard problem. They assume that the entrepreneur-borrower has special skills (so that he cannot be costlessly replaced) and that he can credibly threaten to repudiate the contract by withdrawing his human capital. As a result, project returns will be perfectly divertable. In contrast, our model derives the divertability of returns from the assumption that returns are not verifiable to courts. A further difference is that the Hart-Moore set-up is entirely deterministic, whereas our model addresses the consequences of uncertainty about the value of collateral for the ex ante design of the optimal contract.

Recently, a growing body of research has emerged on the macroeconomics of collateral. This new literature has studied how collateral (and net worth) requirements, as a result of microeconomic credit market imperfections, may make the macroeconomy more vulnerable to unanticipated shocks. One branch of research has focused on *aggregation problems*, showing how the financial sector may lose its ability to aggregate, and efficiently reallocate, resources in response to a shock. Holmstrom and Tirole (1997, 1998) show how shocks may lead to credit rationing when financial intermediaries face a moral hazard problem or when the aggregate collateral of private agents does not satisfy their liquidity needs. Caballero and Krishnamurthy (1999) introduce a distinction between domestic and international collateral and show how the aggregate insufficiency of international collateral may cause financial turmoil.

A second branch of research has focused on *level problems* associated with shocks. The level of borrowing, or the degree of leverage, may become highly susceptible to shocks that either directly or indirectly affect the value of collateral or net worth. This branch of research has shown that small shocks may lead to persistent and amplified business cycle fluctuations. In a seminal article, Bernanke and Gertler (1989) develop a framework where endogenous procyclical movements in net worth magnify investment and output fluctuations. An income-accelerator effect on investment emerges as income increases relax borrowing constraints. Kiyotaki and Moore (1997, 1999) construct a dynamic general equilibrium model with flow and stock effects on credit-constrained entrepreneurs. Not only does higher profit alleviate credit constraints in the future, but the resulting asset price response also magnifies the degree to which credit constraints bind today. Our model fits into this line of research. We also focus on level problems linked to collateralized external finance. A major difference, however, concerns the fact that our contribution highlights the role of bank balance sheets in constraining the ability of entrepreneurs to find project finance. Unlike previous work, we link the nature of the optimal contract to the interaction between the balance sheets of banks and the degree of asset price uncertainty. It is this interaction that leads to a further tightening of credit constraints, creating scope for welfare-enhancing policy intervention.

1. A Model of Costly Asset Collateralization

The purpose of this section is to set up a framework where collateral requirements emerge as an element of an optimal loan contract between banks and entrepreneurs. The model addresses both benefits and costs associated with the use of collateral. On the positive side, collateral requirements reduce problems of asymmetric information. In this model, collateral serves as incentive device that eliminates the desire of borrowers to strategically default. On the negative side, collateral requirements also create a number of independent problems. One complication that we explore in this section is the fact that liquidation of collateral is typically costly.

The model identifies productive assets as the source of collateral. Assets serve both as security against loan default and as input in the process of production. The borrower in this model wants to be leveraged to reach the optimal scale of operation. Leverage and asset collateralization are mutually reinforcing. Higher leverage expands the asset base of the borrower that can be collateralized. Higher collateralization contributes to loan security and thereby creates the very possibility of leveraging.

We first discuss the basic model. We then revisit the welfare effects of the crucial assumptions of the model. To conclude, we examine a number of implications of the model, namely underinvestment, credit constraints and the susceptibility of the optimal contract to interest rate shocks.

1.1. The Basic Set-Up

Consider an economy that is populated with a continuum of identical risk neutral banks and entrepreneur-borrowers. The representative bank competes for loans and deposits in a perfectly competitive market. Bank loans are assumed to be the entrepreneur's only source of external finance. Banks have access to an abundant supply of funds at the riskless net rate r. As a result, (i) the net return to depositors equals r, (ii) the expected profit of the entrepreneur-borrower is maximized subject to break-even and informational constraints and (iii) the expected profit of the bank equals zero.

The entrepreneur has access to a risky productive technology. An entrepreneur with asset base of size k can start a project at date 0 that returns $\tilde{R}k$ at date 1 where $\tilde{R} = R$ in the case of success and $\tilde{R} = 0$ in the case of failure. Throughout we assume that R is large. The probability of success is given by p. Assets in this economy could refer to durables such as land, project returns to nondurables such as fruit. We take nondurables as the numeraire and denote the relative price at times 0 and 1 by q_0 and q_1. Operation of the project is subject to a utility cost given by $ck^2/2$ (with $c > 0$). The utility cost is also expressed in terms of the numeraire.

The entrepreneur is wealth-constrained. He is endowed with an asset base of size w which does not enable the maximum possible extraction of surplus from the productive technology he has access to. The entrepreneur therefore wants to leverage the scale of production by borrowing from the bank. If the optimal project size is k, the entrepreneur needs to finance a purchase of $k - w$ assets by borrowing a sum of $b \equiv q_0(k - w)$. The fraction of the asset base that is externally financed is denoted by $f \equiv (k - w)/k$ with $0 \leq f \leq 1$. Autarky corresponds to $f = 0$ with $b = 0$ and $k = w$. Full external finance corresponds to $f = 1$ with $b = q_0 k$ and $w = 0$.

Two critical assumptions affect the nature of resource transfers in this economy between banks and entrepreneurs.

A1 (Incomplete Contracts). *Returns $\tilde{R}$ are not verifiable to courts.*

This assumption motivates why contracts cannot be written contingent on the realization of the productivity shock. Since $\tilde{R}$ is not verifiable to courts, returns cannot be pledged in the case of success. This assumption creates a role for an incentive device such as the security of collateral.

A2 (Cost of Specificity). *Asset liquidation is costly.*

A cost of specificity arises in the valuation of the asset. It is assumed that the inside value of the asset exceeds its outside value. This wedge is measured by $1 - \beta$ with $0 < \beta < 1$ and referred to as the liquidation cost. Such a cost may arise, for example, due to limited asset redeployability (Shleifer and Vishny, 1992) or due to inefficiencies associated with the transfer of ownership (e.g. as a result of bankruptcy costs). This assumption identifies a cost for the provision of incentives.

The optimal contractual arrangement between bank and entrepreneur maximizes the social surplus in this economy. The contract will specify (i) the degree of leverage, (ii) the net loan rate and (iii) the degree of asset collateralization. The degree of leverage determines asset base k given an endowment of w. The net loan rate is denoted by α and determines what the entrepreneur is expected to voluntarily repay out of the non-verifiable returns of the project. The degree of asset collateralization is denoted by γ (with $0 \leq \gamma \leq 1$) and determines the fraction of the asset base which the bank is entitled to seize if the entrepreneur defaults. At time 0, the entrepreneur receives a sum of $b = q_0(k - w)$. At time 1, the entrepreneur repays $(1 + \alpha)b$. If the entrepreneur defaults, the bank liquidates γ of the asset base yielding $\beta\gamma q_1 k$ in proceeds.

The problem to solve is then given by:

$$\max_{k,\alpha,\gamma} \quad p\,(R + q_1)\,k \;-\; p\,(1+\alpha)\,q_0(k-w) \;+\; (1-p)\,(1-\gamma)\,q_1 k \\ -\,(1+r)\,q_0 w \;-\; \frac{c}{2}k^2 \tag{1}$$

subject to

$$(1+\alpha)\,q_0(k-w) \quad \leq \quad \beta\gamma q_1 k\;,$$

$$p\,(1+\alpha)\,q_0(k-w) \;+\; (1-p)\,\beta\gamma q_1 k \quad \geq \quad (1+r)\,q_0(k-w) \;\;;$$

$$p\,(R+q_1)\,k \;-\; p\,(1+\alpha)\,q_0(k-w) \;+\; (1-p)\,(1-\gamma)\,q_1 k \\ -\,(1+r)\,q_0 w \;-\; \frac{c}{2}k^2 \;\geq\; (pR+q_1)\,w \;-\; (1+r)q_0 w \;-\; \frac{c}{2}w^2 \quad \geq \quad 0,$$

where the constraints are respectively the incentive compatibility (IC) constraint of the enrepreneur and the individual rationality (IR) constraints of the bank and the entrepreneur. Implicit there are two other constraints: (i) the loan rate repayment should not exceed R and (ii) the degree of collateralization should not exceed unity. We assume throughout that the first condition is always implicitly satisfied (this requires R to be sufficiently large). For the moment, we also assume that the second condition is satisfied. We will revisit the collateral constraint at the end of this section.

The maximand expresses the expected utility of the entrepreneur obtained under a contract with parameters (k, α, γ) and consists of the following components. In the case of success, the entrepreneur earns the return, keeps the asset and repays the loan. In the case of failure, the entrepreneur earns no return and keeps the part of the asset base that is not collateralized. The entrepreneur also foregoes the opportunity to earn the riskless rate on his endowment and incurs operational costs $ck^2/2$. Because of the non-verifiability of returns, the entrepreneur could always strategically default and claim failure in the case of success. The IC constraint rules out this possibility. The value of the outstanding debt can never exceed the value of liquidated collateral to the bank, otherwise the entrepreneur would be able to renegotiate a lower loan rate. The IR constraint of the bank requires that the expected revenue from loan-making covers the cost of deposit-taking. The sources of income are loan repayment $(1 + \alpha)\, q_0(k - w)$ and liquidation proceeds $\beta\gamma q_1 k$. The IR constraint of the entrepreneur requires signing the contract to be at least as profitable as remaining in autarky. Of course, for the entrepreneur to invest in autarky, production must be more profitable than depositing the endowment.

Since the supply of funds is perfectly elastic at r, competition among banks drives the expected return from a loan down to the cost of borrowing. The bank's IR constraint therefore binds and, after rearranging, we obtain:

$$\begin{aligned} 1+\alpha &= \frac{1+r}{p} - \beta\frac{1-p}{p}\frac{q_1}{q_0}\gamma\frac{k}{k-w} \\ &= \frac{1+r}{p} - \beta\frac{1-p}{p}\frac{q_1}{q_0}\frac{\gamma}{f}\,, \end{aligned} \tag{2}$$

where q_1/q_0 is the gross capital gain and γ/f is termed the degree of relative collateralization. This latter ratio measures the extent to which collateralization exceeds the fraction of external finance and is an important determinant of the cost of external finance. Substitution of (2) into the maximand yields the following problem:

$$\max_{k,\gamma}\ (pR+q_1)k - (1+r)q_0k - (1-p)(1-\beta)\gamma q_1 k - \frac{c}{2}k^2\,, \tag{3}$$

subject to the IC and IR constraints of the entrepreneur. From this formulation it is clear that the use of collateral ($\gamma > 0$) as an incentive device is costly. Therefore, no more collateral will be used than is necessary to provide incentives. Thus, the IC constraint binds and determines the optimal degree of collateralization:

$$\gamma = \frac{(1+\alpha)}{\beta}\frac{q_0}{q_1}\frac{(k-w)}{k}.$$

Substituting this into (2) gives $\alpha = r$. As a result, we have

$$\begin{aligned} \gamma &= \frac{(1+r)}{\beta}\frac{q_0}{q_1}\frac{(k-w)}{k} ; \qquad (4) \\ \frac{\gamma}{f} &= \frac{(1+r)}{\beta}\frac{q_0}{q_1}. \end{aligned}$$

This latter expression has the following interpretation: interest rate increases, asset price depreciation and costly liquidation all lead to larger fraction of the entrepreneur's equity stake being at risk.

Substituting (4) into (3) leads to the following problem:

$$\max_k \ (pR+q_1)k - (1+r)q_0k - \frac{(1-p)(1-\beta)(1+r)}{\beta}q_0(k-w) - \frac{c}{2}k^2 ,$$

subject to the entrepreneur's IR constraint. We will from now assume that external finance is not too costly so that the entrepreneur's IR constraint will not be violated. The optimal contract can then be characterised by:

$$\begin{aligned} k^* &= \frac{pR+q_1-(1+r)q_0}{c} - \frac{(1-p)(1-\beta)\,(1+r)\,q_0}{\beta c} \\ \alpha^* &= r \\ \gamma^* &= \frac{(1+r)}{\beta}\frac{q_0}{q_1}\frac{(k^*-w)}{k^*}. \end{aligned}$$

Substituting k^*, α^* and γ^* into the entrepreneur's objective function (1) yields

$$\begin{aligned} W^* &= \frac{1}{2c}\left(pR+q_1-(1+r)q_0 - \frac{(1-p)(1-\beta)\,(1+r)\,q_0}{\beta}\right)^2 \\ &\quad + \frac{(1-p)(1-\beta)\,(1+r)}{\beta}q_0w. \end{aligned}$$

1.2. Welfare Implications

Non-verifiability of returns and costly asset liquidation are the crucial features of the environment we studied. To examine their welfare implications, we now look at what the world would look like if these assumptions did not hold.

If returns were verifiable, it is obvious from (3) that the optimal contract does not involve asset collateralization. The distortionary cost of collateral liquidation could be avoided by relying entirely on loan rate repayment. Consequently, the scale of production equals its first-best level. From (2) it follows that the loan rate would include a default premium. The optimal contract can thus be summarized by:

$$\begin{aligned} k &= \frac{pR + q_1 - (1+r)q_0}{c} \\ \alpha &= \frac{1+r}{p} - 1 \\ \gamma &= 0. \end{aligned}$$

Welfare equals the first-best level and is given by:

$$W = \frac{1}{2c}\left(pR + q_1 - (1+r)q_0\right)^2.$$

If asset liquidation were not costly, loan rate repayment and collateral liquidation are perfect substitutes. Changes in the degree of collateralization in that case would no longer lead to changes in welfare. The optimal contract in this environment is then given by:

$$
\begin{aligned}
k &= \frac{pR + q_1 - (1+r)q_0}{c} \\
\alpha &= r \\
\gamma &= (1+r)\frac{q_0}{q_1}\frac{(k-w)}{k}.
\end{aligned}
$$

Welfare again equals its first-best level.

If asset liquidation were not costly and returns were verifiable, the first best would again obtain but the loan rate and degree of collateralization would be indeterminate.

1.3. Underinvestment and Credit Constraints

One implication of the model is that non-verifiability, in conjunction with costly asset liquidation, leads to underinvestment compared to the first-best economy:

$$
k^* < \frac{pR + q_1 - (1+r)q_0}{c}.
$$

Note that the liquidation cost has a non-linear effect on production. This is caused by two interacting forces. On the one hand, because of the zero profit condition of the bank, the ultimate burden of the liquidation cost falls on the entrepreneur. The higher the liquidation cost, the more the entrepreneur will loose per unit of collateral in the case of default. On the other hand, a higher liquidation cost necessitates the bank to require more collateral so that the proceeds from liquidation remain constant. It is these two reinforcing mechanisms that make the entrepreneur want to decrease external borrowing.

When we interpret the liquidation cost as a metaphor for weak contractual enforcement, the previous finding relates to the observation made by Krishnamurthy (1999) that collateral is likely to be scarcer in less developed economies. Using a simple measure of financial aggregate collateral (the ratio of domestic credit to GDP), he finds values of 0.4 for less developed economies such as India or Turkey and 1.25 for more developed economies such as UK or USA. This can be explained by the following circular problem. On the one hand, weak enforcement complicates the writing of contracts. As a result, the use of collateral is more desirable. On the other hand, weak enforcement also leads to a reduced supply of collateral. If property rights, for example, are not respected, not even land may qualify as collateral.

Another, potentially more harmful, implication is that some agents will be credit-constrained, in the sense that projects with positive net present value will not be fully funded. In the context of this model, non-verifiability is a necessary condition for credit constraints. Given that returns are non-verifiable, a high liquidation cost makes it more likely that some agents with limited endowments will be credit-rationed. To see this, note that the upper bound on the degree of collateralization ($\gamma^{\max}$) is unity. This derives from the assumption that only assets can be credibily committed as a security. Recall that the entrepreneur's binding IC constraint and the bank's binding IR constraint imply:

$$(1+r)\,q_0(k-w) = \beta\gamma q_1 k.$$

As a consequence, for a given degree of leverage, high interest rates, high liquidation costs or depreciating asset prices must be compensated for by stronger collateral requirements. Otherwise the entrepreneur cannot be motivated to repay in the case of success and the bank would not be able to break even. However, it is possible that the required collateral requirement hits its upper bound:

$$\frac{(1+r)}{\beta}\frac{q_0}{q_1}\frac{(k-w)}{k} = \gamma^{\max} = 1.$$

If this is the case, the bank reduces its exposure in order to break even. The maximum fraction of external finance consistent with the bank breaking even is given by:

$$f^{\max} = \frac{(k^{\max}-w)}{k^{\max}} = \frac{\beta}{1+r}\frac{q_1}{q_0}.$$

Whether or not the entrepreneur is credit-constrained is determined by the following lower bound on his endowment:

$$\begin{aligned} w^{\min} &= \left(1-\frac{\beta}{1+r}\frac{q_1}{q_0}\right)k^* \\ &= \left(1-\frac{\beta}{1+r}\frac{q_1}{q_0}\right)\left(\frac{pR+q_1-(1+r)q_0}{c} - \frac{(1-p)(1-\beta)\,(1+r)\,q_0}{\beta c}\right). \end{aligned}$$

If $w \geq w^{\min}$, the entrepreneur will not be credit-constrained. He can borrow $b = q_0(k^*-w)$ and achieve his preferred scale of production k^*. However, if $w < w^{\min}$, the entrepreneur will be credit-constrained with the scale of production equal to:

$$k = \frac{q_0 w}{q_0 - \beta q_1/(1+r)}.$$

The effect of liquidation costs on the minimum endowment requirement is ambiguous. On the one hand, costly liquidation lowers the desired production scale for the unconstrained entrepreneur. This lowers the minimum endowment requirement. On the other hand, costly liquidation reduces the maximum possible degree of leverage for the constrained entrepreneur. It may thus be possible that an increase in the liquidation cost makes external finance so expensive that the entrepreneur is no longer credit-constrained. In this case, the entrepreneur simply *chooses* to borrow less.

1.4. Susceptibility to Interest Rate Shocks

Let us now examine the response of production to an increase in the riskless rate. The prediction implied by this framework is that small changes in r may have large consequences when liquidation costs are substantial. To see this, note that

$$\begin{aligned}
\frac{\partial k^*(r;\beta)}{\partial r} &= \frac{\partial}{\partial r}\left(\frac{pR + q_1 - (1+r)q_0}{c} - \frac{(1-p)(1-\beta)\,(1+r)\,q_0}{\beta c}\right) \\
&= -\left(1 + \frac{(1-p)(1-\beta)}{\beta}\right)\frac{q_0}{c}
\end{aligned}$$

is decreasing in the liquidation cost.

The intuition that drives this result is as follows. As the cost of funds increases, the bank needs to raise more revenue. However, the possibilities of raising revenue are restricted. Due to the non-verifiability of returns, the bank is forced to equalize revenue across realizations of the productivity shock. The IC constraint dictates equality between loan rate repayment and collateral liquidation proceeds. Therefore, in order to boost revenue without compromising incentives, collateralization must increase. And, the more costly collateral liquidation is, the more strongly collateralization needs to be increased. This in turn motivates the entrepreneur to reduce the scale of production more drastically.

Due to costly collateral liquidation, interest rate changes have powerful effects on the feasibility of the optimal contract. Especially when liquidation costs are high, a small increase in the interest rate may violate the entrepreneur's IR constraint as leveraging becomes too costly. Collateral is beneficial as an incentive device but at the same time makes the feasibility of the contract more susceptible to changes in interest rates.

2. Aggregate Uncertainty

The previous section examined the optimal contractual arrangement between banks and entrepreneur-borrowers. It was shown how non-verifiability and costly asset liquidation necessarily lead to the bank breaking even whatever the realization of the productivity shock. Due to the non-verifiability assumption, the proceeds from collateralization must be at least as much as those from loan rate repayment. Due to the costly asset liquidation assumption, this weak inequality turns into an equality. Therefore, regardless of idiosynchratic uncertainty about the productivity of the project, banks always break even in this economy.

This section revisits the optimal contract in an environment with aggregate asset price uncertainty. Depending on the contractual form, banks may no longer break even in every possible state of the world. An issue of particular interest is the interaction between credit constraints and the optimal contractual form under aggregate uncertainty.

Aggregate uncertainty is introduced as follows. We assume that, for exogenous reasons, the date-1 asset price equals q_H in the good state of nature and q_L in the bad state (with $q_H > q_L$). The good state occurs with probability π and the bad state with probability $1 - \pi$. Let $q_1 \equiv \pi q_H + (1 - \pi) q_L$, which is the expected asset price as of date 0.

2.1. Contracting under Aggregate Uncertainty

We are looking for a contract between bank and entrepreneur that specifies (i) the degree of leverage (determined by k), (ii) the net loan rate in the good and the bad state (α_H and α_L) and (iii) the degree of asset collateralization in the good and the bad state (γ_H and γ_L).

The problem to solve is given by:

$$\max_{k,\alpha_H,\alpha_L,\gamma_H,\gamma_L} \quad \pi \; [\; p(R+q_H)\,k \;-\; p(1+\alpha_H)\,q_0(k-w) \;+\; (1-p)\,(1-\gamma_H)\,q_H k\;]$$
$$+\;(1-\pi)\;[p(R+q_L)\,k \;-\; p(1+\alpha_L)\,q_0(k-w) \;+\; (1-p)\,(1-\gamma_L)\,q_L k]$$
$$-\;(1+r)\,q_0 w \;-\; \frac{c}{2}k^2$$

subject to the incentive constraints for the good and the bad state ($\mathrm{IC_H}$ and $\mathrm{IC_L}$):

$$(1+\alpha_H)\,q_0(k-w) \quad \leq \quad \beta\gamma_H q_H k\;,$$

$$(1+\alpha_L)\,q_0(k-w) \quad \leq \quad \beta\gamma_L q_L k\;,$$

the IR constraint of the bank:

$$\pi\;[\,p(1+\alpha_H)\,q_0(k-w) \;+\; (1-p)\,\beta\gamma_H q_H k\,]$$
$$+\;(1-\pi)\;[\,p(1+\alpha_L)\,q_0(k-w) \;+\; (1-p)\,\beta\gamma_L q_L k\,]$$
$$\geq \quad (1+r)\,q_0(k-w)\;,$$

and the IR constraint of the entrepreneur:

$$\begin{aligned}
&\pi\ [\ p\,(R+q_H)\,k\ -\ p\,(1+\alpha_H)\,q_0(k-w)\ +\ (1-p)\,(1-\gamma_H)\,q_H k\ ] \\
&+\ (1-\pi)\,[\ p\,(R+q_L)\,k\ -\ p\,(1+\alpha_L)\,q_0(k-w)\ +\ (1-p)\,(1-\gamma_L)\,q_L k\ ] \\
&-\ (1+r)\,q_0 w\ -\ \frac{c}{2}k^2 \\
\geq\quad & (pR+q_1)\,w\ -\ (1+r)q_0 w\ -\ \frac{c}{2}w^2\ \ \geq\ 0.
\end{aligned}$$

Note that there are now two incentive constraints ($\mathrm{IC_H}$ and $\mathrm{IC_L}$) to ensure that strategic default is prevented in both states of nature. Since collateral liquidation is costly, we know that the optimal contract will economize as much as possible on the use of collateral. $\mathrm{IC_H}$ and $\mathrm{IC_L}$ will therefore both be binding. Due to competition the bank's IR constraint will also be binding. Substituting the IC constraints into the bank's IR constraint yields:

$$\pi\,(\beta\gamma_H q_H k)\ +\ (1-\pi)\,(\beta\gamma_L q_L k)\ =\ (1+r)\,q_0(k-w)\,. \tag{5}$$

We explore two types of contract that may emerge from this problem: a contingent contract and an uncontingent contract. In both cases we restrict attention to renegotiation proof contracts. We show that both types of contract lead to the same welfare level. For the moment, we also restrict attention to the case where the collateral constraint does not bind. Binding collateral constraints will be discussed in the next section.

2.2. Contingent Contract

State-contingency in this context refers to dependence of the entrepreneur's liabilities on the state of nature. This means that the loan rate and/or the *value* of the collateralized asset base depend on the realization of the asset price. In other words, α_H and α_L may differ from each other and so may $\gamma_H q_H$ and $\gamma_L q_L$.

Without loss of generality, we focus on the case where $\gamma_H = \gamma_L \equiv \gamma$.[37] From (5) it then follows that

$$\gamma = \frac{(1+r)}{\beta}\frac{q_0}{q_1}\frac{(k-w)}{k},$$

where $q_1 \equiv \pi q_H + (1-\pi)q_L$.

Using this latter expression as well as the binding IC constraints, the problem can be written in the following convenient way:

$$\max_k \ (pR+q_1)k - (1+r)q_0 k - \frac{(1-p)(1-\beta)(1+r)}{\beta} q_0(k-w) - \frac{c}{2}k^2 \ , \qquad (6)$$

subject to the entrepreneur's IR constraint. Note that this expression coincides with what we derived in the absence of aggregate uncertainty. The only difference is that q_1 now refers the expected date 1 asset price. If the entrepreneur's IR constraint is not violated, the optimal contingent contract (among the contracts that we focus on)

[37] Alternative state-contingent contracts where the degree of collateralization does depend on the asset price result in exactly the same level of welfare.

is characterised by:

$$\begin{aligned}
k &= \frac{pR + q_1 - (1+r)q_0}{c} - \frac{(1-p)(1-\beta)\,(1+r)\,q_0}{\beta c} \\
\alpha_H &= (1+r)\,\frac{q_H}{q_1} - 1 \\
\alpha_L &= (1+r)\,\frac{q_L}{q_1} - 1 \\
\gamma_H &= \gamma_L = \frac{(1+r)}{\beta}\frac{q_0}{q_1}\frac{(k-w)}{k}
\end{aligned}$$

The level of welfare is the same as if there were no aggregative uncertainty:

$$\begin{aligned}
W &= \frac{1}{2c}\left(pR + q_1 - (1+r)q_0 - \frac{(1-p)(1-\beta)\,(1+r)\,q_0}{\beta}\right)^2 \\
&\quad + \frac{(1-p)(1-\beta)\,(1+r)}{\beta} q_0 w.
\end{aligned}$$

Note that the contingent contract results in bank profit variability. The bank now makes a profit in the good state and a loss in the bad state.

2.3. Uncontingent Contract

The state-uncontingent contract insulates the liabilities of the entrepreneur from asset price volatility. This is achieved by setting $\alpha_H = \alpha_L = \alpha$ and $\gamma_H q_H = \gamma_L q_L$. From (5), we can derive γ_H and γ_L. Note that $\gamma_H < \gamma_L$ since $q_H > q_L$. Again, it turns out that the objective function can be conveniently rewritten as (6). If the entrepreneur's

IR constraint is not violated, the optimal uncontingent contract is characterised by:

$$
\begin{aligned}
k &= \frac{pR + q_1 - (1+r)q_0}{c} - \frac{(1-p)(1-\beta)(1+r)\,q_0}{\beta c} \\
\alpha_H &= \alpha_L = r \\
\gamma_H &= \frac{(1+r)}{\beta}\frac{q_0}{q_H}\frac{(k-w)}{k} \\
\gamma_L &= \frac{(1+r)}{\beta}\frac{q_0}{q_L}\frac{(k-w)}{k}.
\end{aligned}
$$

The level of welfare is again the same as in the case without aggregative uncertainty. Note that the uncontingent contract does not cause bank profit variability. Collateral requirements vary countercyclically with asset prices so that stable revenue is maintained across states of nature.

3. Credit Constraints, Bank Solvency and The Public Supply of Liquidity

The previous section identified costly asset liquidation as a first complication for the provision of finance. We now study a second complication: contracts with collateral clauses introduce relative price risk. Aggregate uncertainty about collateral values may under certain conditions be a source of credit constraints. We will establish that aggregate uncertainty may be particularly troublesome when both banks and borrowers are poorly capitalized. We will also identify a role for the public supply of liquidity.

3.1. Credit Constraints under Aggregate Uncertainty

We now examine the consequences of aggregate uncertainty for agents that are credit-constrained. Consider first the uncontingent contract. Since the credit-constrained entrepreneur will want to be leveraged to the maximum possible extent, the degree of collateralization hits its upper bound. In the presence of aggregate uncertainty, this upper bound binds in the bad state of nature. This is because the uncontingent contract requires a higher degree of collateralization when the asset price is low. As a result, we have

$$\gamma_H = \frac{(1+r)}{\beta}\frac{q_0}{q_H}\frac{(k-w)}{k}$$

and

$$\gamma_L = \frac{(1+r)}{\beta}\frac{q_0}{q_L}\frac{(k-w)}{k} = 1.$$

The latter condition will pin down the feasible scale of production:

$$k = \frac{q_0 w}{q_0 - \beta q_L/(1+r)}.$$

which is lower than in the case without aggregate uncertainty since $q_L < q_1$.

In contrast, the contingent contract does not require the degree of collateralization to vary countercyclically. The contingent contract we studied before simply sets $\gamma_H = \gamma_L \equiv \gamma$. Maximum collateralization implies:

$$\gamma_H = \gamma_L = 1,$$

leaving the feasible scale of production unchanged at:

$$k = \frac{q_0 w}{q_0 - \beta q_1/(1+r)}.$$

In sum, the equivalence of welfare between the contingent and uncontingent contract breaks down when the entrepreneur is credit-constrained. Since the degree of leverage is not affected by asset price volatility, the contingent contract now dominates the uncontingent contract.

3.2. Bank Solvency under Aggregate Uncertainty

We are now ready to examine the consequences of bank solvency constraints for the provision of external finance to credit-constrained entrepreneurs. We show that bank solvency considerations affect the nature of contracting by limiting the degree of asset price contingency and thereby creating an independent source for credit constraints.

Let ϕ denote the capital-to-assets ratio of the representative bank in this economy. Solvency requires that the losses in the bad state of nature do not exceed the bank's net worth:

$$\phi(1+r)q_0(k-w) \geq (1+r)q_0(k-w) - \beta q_L \gamma_L k$$

or

$$\beta q_L \gamma_L k \geq (1-\phi)(1+r)q_0(k-w).$$

Bank solvency considerations will come into play when asset price volatility is high relative to the bank's net worth. The precise condition under which bank solvency will matter is given by:

$$q_L < \left(\frac{(1-\phi)\pi}{\phi + (1-\phi)\pi}\right) q_H. \tag{7}$$

If this condition holds, then financing a credit-constrained entrepreneur up to scale $\tilde{k} \equiv q_0 w/(q_0 - \beta q_1/(1+r))$ will push the bank into insolvency in the bad state of nature. To see this, note that $\beta q_L \gamma_L \tilde{k} < (1-\phi)(1+r)q_0(\tilde{k}-w) = (1-\phi)\beta q_1 \gamma_L \tilde{k}$ for $\gamma_L = 1$ and $q_L < (1-\phi)q_1$.

If condition (7) holds, the solvency constraint binds and can be rewritten as:

$$1 = \gamma_L = (1-\phi)\frac{1+r}{\beta}\frac{q_0}{q_L}\frac{k-w}{k}. \tag{8}$$

Substituting the latter expression into the bank's binding IR constraint,

$$\pi\left(\beta\gamma_H q_H\right) + (1-\pi)\left(\beta q_L \gamma_L\right) = (1+r)\, q_0 \frac{k-w}{k},$$

yields the optimal degree of collateralization in the good state of nature:

$$\gamma_H = \frac{q_L}{q_H}\left(\frac{\phi}{(1-\phi)\pi} + 1\right).$$

From (8) we can derive the optimal project size:

$$k = \frac{q_0 w}{q_0 - \beta q_L / \left[(1-\phi)(1+r)\right]} .$$

Another way to determine whether the solvency condition binds is in terms of the capital-to-asset ratio. Let

$$\phi^* \equiv \frac{\pi(q_H - q_L)}{q_L + \pi(q_H - q_L)} ,$$

with $0 < \phi^* < 1$. We can then summarize our results as follows. If $\phi = 0$, the bank has no capital cushion and cannot afford to suffer any losses resulting from asset price depreciation. The optimal contract therefore corresponds to the uncontingent contract and the project size will equal:

$$k = \frac{q_0 w}{q_0 - \beta q_L/(1+r)}.$$

If $0 < \phi < \phi^*$, the bank can withstand a certain degree of asset price volatility but still has to reduce its exposure in order to survive in the bad state of nature. The optimal contract corresponds to a contingent contract with $\gamma_H < \gamma_L = 1$. The scale of operation is given by:

$$k = \frac{q_0 w}{q_0 - \beta q_L/\left[(1-\phi)(1+r)\right]}.$$

If $\phi \geq \phi^*$, the bank is immune to the asset price volatility in this economy and the optimal contract corresponds to a contingent contract with $\gamma_H = \gamma_L = 1$. The project size equals:

$$k = \frac{q_0 w}{q_0 - \beta q_1/(1+r)}.$$

These results can also be more generally related to agents that are initially not credit-constrained. If the uncontingent contract remains feasible given a particular degree of asset price volatility, then bank solvency constraints are irrelevant. If the uncontingent contract is not feasible for that degree of asset price volatility, then credit constrants occur depending on the extent of bank capitalization.

3.3. The Public Supply of Liquidity

The combination of poor bank and borrower capitalization may lead to credit tightening in the face of aggregate uncertainty. We have shown that non-verifiability may produce credit constraints which in turn lead to a trade-off between high leverage and low bank profit variability. A credit-constrained entrepreneur in this economy will want to sign a contingent contract to ensure maximum leverage. At the same time, the contingent contract produces asset volatility that may be too high from the perspective of the bank. In response, poorly capitalized banks will reduce their exposure and this leads to a further welfare loss.

Since the uncertainty in this model is taken to be aggregate, there is no scope for private insurance. As a result, a role for public insurance emerges in the form of government supplied liquidity (as in Holmstrom and Tirole, 1998). This is particularly so if the environment is prone to a great deal of asset price volatility and if it is difficult to improve on the capitalization of banks. In the context of this model, the government could tax banks in the good states of nature and provide them with liquidity in the bad states. The extent to which the government needs to provide liquidity can be measured by $\phi^* - \phi$, where ϕ is the bank's capital-to-asset ratio and ϕ^* is the threshold value beyond which the solvency constraint does not bind.

This is somewhat reminiscent of the government acting as a co-signer. As has been highlighted by Besanko and Thakor (1987), the practice of co-signing serves to increase the availability of collateral. And this is exactly what credit-constrained entrepreneurs need. The state-contingent provision of liquidity could also be interpreted as countercyclical monetary policy, where monetary policy is lenient when asset prices are low (and liquidity needs are the highest) and monetary policy is tight when asset prices are high.

The beneficial role of government supplied liquidity (or, alternatively, countercyclical monetary policy) must be balanced with potential dangers that the model does not address. For example, as has been pointed out by Krugman (1998), Corsetti, Pesenti and Roubini (1998) and others, implicit government guarantees may create moral hazard problems that lead to excessive indebtedness and overinvestment. In addition, asset price collapse may cause a problem of regulatory forebearance. Governments may be tempted to compromise on regulation standards and to delay financial restructuring reforms. Finally, financial fragility may also undermine long-term price stability by triggering monetary policy forebearance, that is to adopt, in the face of asset price collape, a looser policy stance than is justified by macroeconomic conditions.[38]

[38]See IMF (2000, p. 128).

Concluding Remarks

This chapter has highlighted two complications associated with collateral requirements in the provision of external finance. We have argued that costly asset liquidation may sharply reduce the extent to which wealth-constrained entrepreneurs are leveraged. We have shown that aggregate asset price uncertainty may leave borrowers deprived from credit when banks are poorly capitalized. Of course, due to the non-verifiability of returns, collateral has to be used despite the presence of such costs, and will be used if the entrepreneur wishes to be leveraged. As the Chinese proverb says, it is better to light a candle than to curse the darkness! Finally, we have also suggested that state-contingent provision of liquidity by the government, also interpreted as countercyclical monetary policy, may be useful in the environment that we studied here.

We conclude with a brief discussion of two possible avenues for future research. We found that the degree of leverage is highly susceptible to interest rate changes when the liquidation is high. A first possible application of this result relates to the desirability of interest rate smoothing. Interest rate smoothing can be rationalized if there are convex adjustment costs in the scale of production. Since adjustment costs increase with the size of the response of production, interest rate smoothing may be more desirable when the cost of asset liquidation is high.

A second application concerns the micro-foundations of second-generation currency crisis models. These models are based on the notion that defending a fixed exchange rate is a matter of trade-offs. The self-fulfilling nature of a crisis hinges on the requirement that the costs of defending a peg outweigh the benefits *in the event of* a speculative attack (see, for example, Obstfeld, 1986 and 1994). The implication of our model is that countries with high liquidation costs may be more prone to speculative pressure. This is because the cost of withstanding an attack in terms of reduced financial intermediation is much higher when the liquidation cost is high. Speculators who understand the increased reluctance of policy makers to raise rates therefore attack sooner rather than later.

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