Financial Structure, Managerial Incentives and Product Market Competition

by

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Abstract

This dissertation provides a contribution to the understanding of the interactions between the firm's financial structure and its operating decisions. The main idea is that financial structure impacts the payoff to the firm's decision-maker and that this impact on the managerial payoff will in turn affect his optimal response when confronted with different possible operating decisions. A particular focus is on the case where the manager's optimisation problem arises in a strategic environment in which the firm competes with rival firms in a product market.

The first main chapter reconsiders the strategic effect of debt, as first analysed by Brander and Lewis (1986), under the novel assumption that quantity choices are made by managers whose objective is to avoid bankruptcy. The basic result is that quantity choices, which are strategic substitutes under profit maximisation, may turn into strategic complements when the quantity choice is made by managers. This reversal in the nature of competition arises under reasonable assumptions on the firm's profit function. It allows debt to be used to sustain more collusive product market outcomes than in the benchmark case where firms maximise profits, thereby avoiding, and indeed reversing, the pro-competitive limited liability effect of debt, as described by Brander and Lewis (1986). Delegation of the quantity choice to a bankruptcy-averse manager is shown to occur in a dominant strategy equilibrium.

The next chapter analyses the effect of asymmetric information between a firm and its outside investors on the firm's competitive position in a model where first-period competition is followed by a financing stage à la Myers and Majluf (1984). Interim profit generated by the competition stage takes the role of financial slack and determines the extent to which external equity finance is required for a new investment opportunity. The full set of equilibria of the financing game is characterised and financial slack is formally analysed as a comparative statics variable. Using this the firm's first period objective is derived from first principles. In contrast to models of predatory behaviour, one finds that in the presence of an adverse selection problem the need to finance externally may provide a strategic benefit rather than a strategic disadvantage. The reason is that the adverse selection problem may induce speculative behaviour, which will make the firm more aggressive vis à vis its rival.

The last main chapter analyses a model where the firm's manager is asked to make an informed investment decision after evaluating the prospects of an investment project. In this model, which exhibits both moral hazard and hidden information on the part of the manager, different remuneration schemes are discussed and the optimal contract between financial investor and manager is derived. Assuming the manager is risk-neutral and protected by limited liability, a benefit from diversification is shown to exist, in that the right incentives can be provided more cheaply when the manager is supervising more than one project. This occurs even though the projects are technologically unrelated and choices made on one project do not constrain the choices on any other project.

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Contents

List of Figures

Chapter 1

Introduction

Traditionally, both Corporate Finance and Industrial Economics have considered the firm as the unit of analysis. The view was that, when faced with an investment opportunity, a firm would take projects with positive net present value. When firms compete with each other, again firms maximise profits. Economic Theory had nothing to say about how large a firm should be, what the consequences of limited liability are, or whether the fact that the firm has to raise finance externally can impact its operating decisions. Since then information economics, contract theory and game theory have been applied to take a view inside the firm and to analyse the interactions between the contractual structure inside the firm and the way the firm will behave in its product market. The aim of this thesis is to contribute to the understanding of the nature of these interactions with a particular focus on the links between the firm's financial structure and its product market behaviour. My thesis consists of three main chapters which are summarised below.

1.1 Managers, Debt, and Industry Equilibrium

This is the first of two chapters which examine linkages between the firm's financing decisions and the firm's strategic position in a product market. In particular, I reconsider the strategic commitment effect of debt in an environment of imperfect competition, as first analysed by Brander and Lewis (1986), under the novel assumption that quantity choices are made by managers whose objective is to avoid bankruptcy.

In their original analysis, Brander and Lewis (1986) study a Cournot game of imperfect competition with uncertainty. Firm owners may first issue debt against the future profits of the firm, before they move again to choose quantities. This introduces a limited liability effect. The idea is that, having issued debt, firm owners face an option-type payoff when they choose quantities. They do not care about bad states of nature, in which the firm is bankrupt and they face a flat payoff of zero, but only about good states of the nature, in which the firm is not bankrupt and they are residual claimants. Since firm owners choose quantities to maximise profit given good states of the world, debt will make owners more aggressive and will shift reaction functions out. In a symmetric equilibrium, both firms issue debt and both firms are more aggressive than they would otherwise have been. As a consequence, equilibrium quantities are larger and equilibrium profits are lower than in the Cournot model without a financing stage. The possibility of commitment through debt exacerbates the prisoners' dilemma inherent in quantity competition.

My work explores what happens to equilibrium quantities when the quantity decision is made by an agent who cares about bad states of the world. It is argued that managers have a strong incentive to avoid bankruptcy, since, as is well documented in the empirical literature, they face large negative wealth effects in the event of bankruptcy and may in addition lose private benefits and reputational capital.

The basic result is that, when quantity choices are made by managers who are bankruptcy averse, quantity choices, which are strategic substitutes under profit maximisation, may turn into strategic complements under reasonable assumptions on the profit function. Then, in contrast with the benchmark case of owner control over quantity choices, starting from the benchmark level of the single stage competition, owners will want to shift the manager's reaction function back, rather than out. As a result, equilibrium quantities will be less and equilibrium profits will be higher than in the benchmark case without a financing stage. The prisoners' dilemma inherent in quantity competition is softened. By employing a manager, shareholders not only avoid a limited liability effect of debt, but are able to achieve a more collusive outcome than in the simple model without a financing stage. ?

This result is robust when the decision to delegate is endogenised. The intuition is that, given a reversal from strategic substitutes to strategic complements, when one firm does not delegate its quantity choice to a manager, it will lose out against a very aggressive manager-controlled firm. Thus, delegation to a manager occurs in equilibrium and is associated with a positive ex ante value both on and off the equilibrium path. In contrast with Brander and Lewis (1986) and in line with the empirical evidence, in the equilibrium of our model positive leverage is associated with softer competition than in the standard oligopoly model without a financing stage. The model also implies that, given a contract domain including shares, options and bonus schemes, in equilibrium owners would choose simple bonus schemes for their managers. This provides some theoretical justification for the kind of managerial preferences assumed.

What about current vogue for stock-options?

I want large debt to precommit to aggressive behaviour, how credible is it to employ RA manager why don't I turn him into RL by giving him stock options and let that be known?

1.2 Equity Finance, Adverse Selection and Product Market Competition

This chapter also takes product market competition à la Brander and Lewis (1986) as a main building block. In contrast with chapter 2, which starts by making an assumption about managerial preferences, this chapter takes asymmetric information between the firm and its outside investors as the starting point. Also, this chapter is not primarily concerned with the commitment value of financing decisions and has a product market stage which precedes the financing stage.

Product market competition is as in Brander and Lewis (1986): two firms choose quantities in a environment of imperfect competition. The resulting interim profit is stochastic. It is a function both of the quantities chosen and of a random variable, which can be thought of as representing cost uncertainty or demand uncertainty. The financing stage follows Myers and Majluf (1984). One of the firms faces an investment opportunity. It may issue equity to finance why not debt? the investment. The amount it needs to raise depends on the interim profit (financial slack) generated by the product market competition stage. The crucial assumption here is that, at the interim date, there is asymmetric information between the firm and its outside investors about the value of the firm. This creates an underinvestment problem just as in Myers and Majluf (1984). The good firm may not find it in its interest to issue equity, since the market belief pools it with the bad firm, so that the good firm faces a dilution cost.

This chapter makes several contributions to the existing literature. In contrast to the original article by Myers and Majluf, it recognises that, for a range of values of interim profit, the financing stage game has multiple equilibria. With respect to the multiplicity of equilibria I formally analyse the role of financial slack as a comparative statics variable. I then argue for an equilibrium selection ? criterion which is based on the idea that realised profit may not be known to financial investors with certainty. Given this criterion, the equilibrium moves from a separating to a pooling equilibrium at some cut-off profit level.

I take this behaviour of the equilibrium in the financing stage and derive the firm's first-period objective from first principles. This improves on the approach taken in most of the literature, where an adverse selection problem à la ? Myers and Majluf is invoked to justify some ad hoc specification for the cost of external funds. Following my approach, one finds that, in the presence of the adverse selection problem, the firm will not simply maximise the expected value of profit, but will take into account the implication its quantity choice has on the probability that the profit generated exceeds the cut-off, above which both types invest in a pooling equilibrium. This will lead the firm to consider not only the first moment, but also the second moment of the profit distribution. When the cut-off is relatively high there is an incentive to increase the variance of the profit distribution by increasing output. The rival anticipates this and responds with a lower output, which will benefit the firm. When the cut-off is low there is an incentive to reduce the variance of the profit distribution by lowering output. The rival will take advantage of this and respond with a higher output, which will harm the firm.

One of the main insights of this model is that the fact that a firm has to finance externally does not necessarily worsen the firm's competitive position. In contrast to models where a financially constrained firm faces predation by a deep pocketed rival, in our model a particularly severe financing problem may actually help the firm in making it more aggressive vis à vis its rival.

1.3 Optimal Managerial Remuneration and Firm-level Diversification

This chapter addresses and relates two hitherto unrelated issues. In recent years managerial remuneration has been the focus of much public and academic debate. Contract theoretical analysis has centred on the case where the manager is assigned a single project and has provided justification for performance related contracts that are monotone in observed return. From a theoretical perspective, also, the observed diversification of firms into several lines of business is a puzzle, since standard portfolio theory would suggest that there is no gain to be had from firm-level diversification when investors can hold diversified portfolios themselves.

I analyse optimal managerial remuneration schemes when the manager supervises several identical projects. The underlying agency relationship is characterised by a combination of moral hazard and adverse selection. The manager is thought of as an insider who, having examined the projects, knows more about their likely returns than does the financial investor. Also, re-evaluating projects requires effort on the part of the manager. By use of a wage contract, financial investors are able to provide the manager with the right incentives to investigate project viability and to make investment decisions in their interest. Providing these incentives is costly, however. Given that the manager is protected by limited liability, the wage contract will have to over-compensate for the effort cost incurred, so that the manager receives an informational rent.

In this framework, the optimal wage contract is derived both for the case where the manager supervises a single project and for the case where he is assigned several identical projects. The main result is that assigning more projects to the manager will reduce the informational rent, and it is argued that this may provide a rationale for firm-level diversification. The optimal contract is such that the manager is rewarded highly when none of the projects he supervises fails. However, the manager has to take responsibility for project failure and optimally receives nothing (is sacked) when a single project in his portfolio fails to perform. Our results also suggest that, when managers are to provide effort as well as make investment decisions, optimal managerial wage contracts will in general not be monotone in aggregate returns when the manager supervises several projects and project outcomes are observable.

Chapter 2

Managers, Debt, and Industry Equilibrium

2.1 Introduction

In recent years there has been much interest in the way equilibria in oligopolistic markets may be affected when account is taken of the contractual structure inside the firm or of contractual ties with outside investors. This is usually modelled as a two stage game. In the case of Cournot competition, prior to the quantity setting stage, there is a stage in which firm owners can move to write contracts which may affect incentives at the later quantity setting stage. Examples of this literature are Brander and Lewis (1986), Fershtman and Judd (1987), Maksimovic (1990) and most recently Clayton and Jorgenson (1997). The common theme of all these papers is that, if goods are substitutes, and therefore are strategic substitutes when chosen by profit-maximising agents, the possibility of moving prior to the quantity setting stage will be used to commit the firm to more aggressive product market behaviour.

Brander and Lewis (1986) analyse the case, where firm owners can write debt contracts with investors in a perfect capital market, before they move again to choose quantities. When there is uncertainty about demand or cost conditions, debt introduces the possibility that the firm may go bankrupt. A positive debt level will therefore make the payoff of shareholders a convex function of the operating profit. Given any quantity choice, the shareholders payoff is flat for all realisations of the state of nature such that the firm is bankrupt, but is increasing linearly with profit for good states of nature. Under the assumptions that it is the firm owners who determine quantities and that marginal profit is an increasing function of the unobserved state of nature, it is shown by Brander and Lewis that a positive debt level will cause the firm's reaction function to move out. The intuition is that firm owners are only concerned with those states of nature that leave a positive payoff to them. Since these are the good states, and marginal profit is higher for good states, firm owners will choose higher quantities than they would if no debt had been issued. Given that quantities are strategic substitutes and reaction functions are therefore downward sloping, each firm has an incentive to move its reaction function out by issuing debt, in order to increase its profits, as its own reaction function slides along the rival's downward sloping reaction function. In equilibrium, debt levels are positive, quantities are larger, and profits are smaller than if the firms could not issue debt.

Both Fershtman and Judd (1987) and Sklivas (1987) study the case where quantities are chosen by managers and firm owners move first to design incentive contracts with their managers. They assume that these contracts can condition both on the realised profit and on sales and restrict the set of admissible contracts to linear combinations of those two variables, so that contracts have the form $b\left[\alpha\pi + (1-\alpha)S\right]$. Under these assumptions they find that the optimal α will be less than one. Managerial incentives are distorted away from profit maximisation towards sales maximisation. The intuition is that owners want to make their manager more aggressive. When positive weight is on sales, managers will take account less of the costs of an increase in quantities, than they would if their remuneration were based on profit alone. Therefore, reaction functions shift out as $(1-\alpha)$ increases and each owner has an incentive to choose $\alpha < 1$, since this will increase his profit, given that the other firm's reaction function slopes down. In equilibrium both owners choose $\alpha < 1$, so that quantities will be larger and profits will be smaller than if the owners could choose quantities themselves. The commitment available through the possibility of writing an incentive contract worsens the situation of the owners.

Maksimovic (1990) studies the strategic use of loan commitments. The assumption is that the production of current output has to be financed by a credit institution, which charges the market interest rate on funds used by the firm. Alternatively, the firms may strike an agreement with the credit institution prior to production, which commits the credit institution to lend out funds on demand at some lower interest rate, for which the credit institution is compensated with some upfront fixed fee. The idea is that such a loan commitment lowers the firm's marginal cost of production. Therefore, there is a strategic incentive to take out a loan commitment, since this will result in an outward shift of the firm's reaction function, committing the firm to a more aggressive product market behaviour. In a symmetric equilibrium both firms will do this, so that both reaction functions are moved out and again industry profits are lower than without the possibility of taking out a loan commitment.

Finally, Clayton and Jorgensen analyse a setting, where in a first stage each firm can take an equity position in the rival firm. Denoting by α the share acquired in the competitor's equity firm i will choose its output to maximise $\pi_i + \alpha\pi_j$. Clayton and Jorgensen show that, when the firms' products are substitutes, optimal cross holding involves a short position in the competitor's equity, that is, α is optimally negative. The intuition is that, when firm i has chosen a negative position in firm j, firm i gains when firm $j's$ profits are low. Increasing one's own output will now not only affect one's own profit but depress the competitors profit and therefore increase firm $i's$ payoff more than without crossholdings. By choosing a negative α each firm can give itself additional incentives to raise quantities. Again, reaction functions shift out and the equilibrium is characterised by larger quantities produced, and lower firm and industry profits.

In all of these papers the first stage action is used to commit the firm to a more aggressive output stance. However, since this commitment device is available to both firms, who take actions simultaneously, firms will end up with lower ex ante profits than they would enjoy if first stage actions could not be taken. The possibility of taking these first stage actions exacerbates the prisoner's dilemma, which is already present in the quantity setting stage, where both firms choose higher quantities than would be joint profit maximising.

In this chapter I will go back to the original analysis of Brander and Lewis and reconsider the case of commitment through debt. This case has attracted considerable interest, partly because the major predictions of the Brander and Lewis (1986) analysis have not been validated by the, albeit limited, empirical evidence, see e.g. Chevalier (1995), Kovenock and Phillips (1995), and Phillips (1995). These authors find that leverage increases in the 1980's led to softer product-market competition in the industries under study. Also, in the related empirical literature on management buyouts (MBOs), empirical research (Kaplan (1989) and Smith (1990)) has found increases in operating profits as well as firm value, rather than a decrease of these variables, as the Brander and Lewis (1986) analysis would suggest.

The Brander and Lewis (1986) model has been revisited before us by Glazer (1994), Showalter (1995), and Faure-Grimaud (1997). In a dynamic setting, Glazer (1994) offers some qualification of their basic result. In his model, equityholders choose quantities twice, before repayment of "long-term" debt is due. He shows that the behaviour in the first quantity setting stage may be quite different from the behaviour in the second stage. In the first stage, there is an incentive to reduce quantities, rather than increase quantities, beyond the Cournot level. The intuition is that, if the firm reduces its quantity in the first stupid.

stage, this will increase its rival's first stage profit, and thus reduce the net debt burden the rival takes into the second stage. In line with the basic insight of Brander and Lewis (1986), this reduction of indebtedness will make the rival a less aggressive second-stage competitor. Therefore long-term debt may lead to more collusive outcomes in the short-run, while the long-run as well as the average is still characterised by quantities above the Cournot-level.

Showalter (1995) replaces the assumption of Cournot competition by one of Bertrand Competition. When competition is in prices rather than quantities, the decision variables are strategic complements when chosen to maximise profit. The cross-partial of the profit function is positive, rather than negative, as was assumed in Brander and Lewis (1986). By assuming Bertrand competition, Showalter (1995) reverses yet another crucial assumption on the profit function. Under demand uncertainty, when firms compete in prices, marginal profit is lower, rather than higher for good states of nature. For the case of demand uncertainty, Showalter (1995) is then able to find positive debt levels in equilibrium which are associated with profits that are higher than for pure equity firms.

In Faure-Grimaud (1997) the financial investor can observe the quantity choice, but neither the realised state of nature nor the resulting profit. The terms of the contract are determined after the quantity has been chosen and are made conditional on the owner's announcement of the state of nature. To induce truthtelling the contract specifies a probability of granting a reward to the owner, which is increasing in the announced state of nature. When the owner has all the bargaining power vis à vis the investor, the investor has to break even ex ante. Thus both the truthtelling constraint and the break-even constraint are binding. The interplay between these two constraints makes owners choose quantities in equilibrium that are lower than if the owners were self-financed.

In all of these papers one major assumption of the Brander and Lewis (1986) analysis has been left unquestioned, which is that there is no conflict of interest between the owners and the manager who chooses quantities (throughout this chapter I will use the words "owners" and "shareholders" as synonyms). Recall that they assume that quantities are chosen by an agent, whose preferences are perfectly aligned with the owners or, equivalently, that owners choose quantities themselves after having issued debt. Instead, I want to follow up the idea that ownership and control over quantity choices may be separated and that therefore, quantity choices may be made by a manager whose objective differs from that of the owner. Specifically, in this chapter I ask what happens if quantity choices are made by a manager whose objective is to avoid bankruptcy. While it clearly is an extreme assumption that this is the only objective of managers in the real world, the threat of bankruptcy arguably is a real concern for managers, who, when their firm goes bankrupt, almost surely lose their job and most likely much of their reputation. Indeed, recent empirical studies support the presumption that bankruptcy may cause quite dramatic managerial wealth effects. For a sample of Swedish firms, Thorburn (1998) finds that for bankrupt firms the CEO turnover rate is 64% and that the median compensation loss is 40% over the two years following the filing for bankruptcy. Interestingly, the percentage drop in CEO compensation is shown to be independent of whether or not the CEO remains with the firm, which is consistent with the idea that bankruptcy significantly reduces rents derived from managerial reputation. Similarly, for a US sample, Gilson and Vetsuypens (1993) find that about half of the managers of firms facing financial distress are replaced and are not rehired by comparable exchange-listed firms for the following three years, and that those who are retained experience very large salary and bonus reductions. In this chapter it is argued, therefore, that having a manager, whose only objective is not to go bankrupt is at least as natural a starting point as to assume, as Brander and Lewis do, that managers preferences are perfectly aligned with the shareholders. Indeed, when the manager is risk-averse, or not sufficiently susceptible to monetary incentives, it may be impossible for the shareholders to write an incentive contract that perfectly aligns the manager's preferences to those of the owner.

In most settings, restrictions on contract design arising from these issues will tend to hurt the principal. One of the main results here will be that, in contrast, it may actually help the shareholders when quantity choices are made by a manager whose objectives differ from their own. A similar result has been obtained by Hirshleifer and Thakor (1992). The intuition there is that a manager who cares about his reputation may be more conservative with respect to project choices, which will alleviate the conflict of interest between shareholders and debtholders over the choice of investment portfolios, as described by Myers (1977). While in our setting, also, the manager will be more conservative than the shareholders, this is not what will eventually be driving the results. What is important in our case is the strategic interaction between manager controlled firms. To see the basic intuition, recall that, when goods are substitutes the choice of quantities is akin to a prisoners' dilemma situation. Both firms would like to reduce their quantities in order to enjoy larger profits. However, when the rival's quantity is low, it pays to increase one's own quantity since this increases sales, whereas the reduction in price is felt only on one's own share of the market. Consider therefore a standard prisoner's dilemma game, such as

$1\backslash 2$	c	d
c	$5,5$	$0,10$
d	$10,0$	$3,3$

where (d, d) is the only equilibrium. Assume then that the players (the prisoners) can now send agents (their lawyers) to play the game and that the lawyers get a private benefit, or a success premium, whenever the outcome is strictly bigger than a cut-off of, say 3. When both players send their lawyers, these will play the following game

$1\backslash 2$	c	d
c	b,b	$0,b$
d	$b,0$	$0,0$

In this example, if one lawyer cooperates, the other lawyer does not increase his payoff from moving to defect. Thus both are happy to play c, so that (c, c) becomes an equilibrium. This illustrates that more collusive and mutually beneficial outcomes can be sustained by delegating play to an agent whose preferences differs from one's own[1]. Of course, it then also becomes an issue which cut-off will be chosen and whether these agents are sent in equilibrium, if it is the player's choice to either play the game himself or to send an agent. These issues will be looked at more carefully in the framework of the model, below.

2.2 The Model

Consider two identical firms who compete in quantities in an output market. Each firms' profit is given by $\pi^i\left(\theta^i, q^i, q^j\right)$, where θ^i is an idiosyncratic shock, which can be thought of as representing demand or cost uncertainty, q^i is the quantity chosen by firm i and q^j is the quantity chosen by its rival. Shocks are distributed identically across firms[2]. More specifically, θ^i realises on an interval $\theta^i \in \left(\underline{\theta}, \overline{\theta}\right)$ according to some distribution function $F(\cdot)$ with density $f(\cdot)$.

In line with Brander and Lewis (1986), the profit function is assumed to have the following properties.

$$\forall \ \left(\theta^i, q^i, q^j\right) \ with \ \theta^i \in \left(\underline{\theta}, \overline{\theta}\right), q^i \geq 0, q^j \geq 0$$

incr in ~~state~~ world   decr in other

$$(i)\, \pi^i_\theta\left(\theta^i, q^i, q^j\right) > 0, (ii)\, \pi^i_j\left(\theta^i, q^i, q^j\right) < 0 \qquad \text{(A1)}$$

concave

$$(iii)\, \pi^i_{ii}\left(\theta^i, q^i, q^j\right) < 0, (iv)\, \pi^i_{ij}\left(\theta^i, q^i, q^j\right) < 0, (v)\, \pi^i_{\theta i}\left(\theta^i, q^i, q^j\right) > 0$$

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[1]This intuition has recently also been explored by Spagnolo (1998) who looks at the effect of managerial bankruptcy aversion in the context of a repeated oligopoly with debt à la Maksimovic (1988). He finds that managerial bankruptcy aversion makes it easier to support tacit collusion by use of trigger strategies.

[2]The model can accommodate any assumption on the correlation of the shocks across firms. For example, a positive correlation may be reasonable for certain types of demand uncertainty, whereas an independence assumption might be more reasonable for cost uncertainty.

Assumption (i) says that profit is increasing in the shock. This means that high realisations of θ^i result in high profits, and thus are 'good' states of the world. According to assumption (ii) profit of firm i is decreasing in the rival's output. Assumption (iii) is a standard concavity assumption while assumption (iv) determines the nature of competition between the two firms. It stipulates that quantities are strategic substitutes when both firms are maximising profit. When firm j increases its output, firm i has an incentive to decrease its output in response. Assumption (v) says that marginal profit is increasing in θ^i. According to this assumption, good states of the world are associated with higher marginal profits.

For future reference let us state here the equilibrium of the simple game in which owners move once to simultaneously choose quantities. This is given as the solution to

$$\int_{\underline{\theta}}^{\overline{\theta}} \pi_i^i \left(\theta^i, q^i, q^j\right) f\left(\theta^i\right) d\theta^i = 0$$

for firms i, j, and will be referred to as the Cournot equilibrium or the Cournot point (q^c, q^c).

In the model there is a financing stage, which precedes the quantity setting stage. In the financing stage, the owner of each firm can issue debt against the future earnings of the company. Owners can choose any face value $D \geq 0$. The choice of face value is made simultaneously. Once chosen, (D^i, D^j) becomes common knowledge.

After the financing stage, outputs are chosen by the agents who are in charge of making these decisions. Output decisions are taken before the realisation of (θ^i, θ^j) is known and are made simultaneously. It is assumed that the output decision taken by this agent is his private knowledge, but that realised operating profit is verifiable.

For debt to be relevant for the incentives of the decision-maker one needs nonsense!
that $\exists\ \theta^i \in (\underline{\theta}, \overline{\theta})$ *s.t.* $\pi^i\left(\theta^i, q^i, q^j\right) - D^i \overset{<}{\cancel{\neq}} 0\ \forall\ D^i > 0$ and $\forall\ (q^i, q^j)$ in a sufficiently large neighbourhood of (q^c, q^c). This guarantees that there is a risk

of default for all, even small, positive debt levels. It is assumed throughout that everybody in the model is risk neutral.

Two cases will be analysed. In the benchmark case, following Brander and Lewis (1986), quantities are chosen by the owners of the company. As an alternative I will consider the case, where the manager receives a private benefit when the firm is not bankrupt.

2.3 A Benchmark: Owner Control

Let us first analyse the case where owners choose quantities after having chosen debt levels at the financing stage. This case has been analysed by Brander and Lewis (1986) and it is reworked here for ease of reference. Consider the subgame that ensues after some arbitrary pair of debt face values, (D^i, D^j)has been fixed at the financing stage. In this subgame shareholders of firm i and firm j simultaneously choose quantities.

Given debt levels (D^i, D^j) the owner of firm i will choose q^i to maximise

$$S^i = \int_{\widehat{\theta}}^{\overline{\theta}} \left(\pi^i\left(\theta^i, q^i, q^j\right) - D^i\right) f\left(\theta^i\right) d\theta^i \tag{2.1}$$

where the lower bound of integration $\widehat{\theta}$ marks the threshold for bankruptcy and is defined implicitly by

$$\pi^i\left(\widehat{\theta}, q^i, q^j\right) - D^i = 0 \tag{2.2}$$

For given quantity choices the firm defaults for realisations of θ^i such that $\theta^i < \widehat{\theta}$. For these realisations the shareholders' payoff is zero, whereas it is $\pi^i\left(\theta^i, q^i, q^j\right) - D^i$ for all realisations such that $\theta^i > \widehat{\theta}$.

Differentiating one obtains the first-order condition for a maximum as

$$S_i^i = \int_{\widehat{\theta}}^{\overline{\theta}} \pi_i^i\left(\theta^i, q^i, q^j\right) f\left(\theta^i\right) d\theta^i - \frac{d\widehat{\theta}}{dq^i}\left(\pi^i\left(\widehat{\theta}, q^i, q^j\right) - D^i\right) f\left(\widehat{\theta}\right) = 0 \tag{2.3}$$

However, since

$$\pi^i\left(\widehat{\theta}, q^i, q^j\right) - D^i = 0$$

the second term vanishes and the first-order condition reduces to

$$S_i^i = \int_{\widehat{\theta}}^{\overline{\theta}} \pi_i^i \left(\theta^i, q^i, q^j\right) f\left(\theta^i\right) d\theta^i = 0 \tag{2.4}$$

which says that the expected or "average" marginal profit integrated over all non-default states must be zero.

The second-order condition for a maximum is

$$S_{ii}^i = \int_{\widehat{\theta}}^{\overline{\theta}} \pi_{ii}^i \left(\theta^i, q^i, q^j\right) f\left(\theta^i\right) d\theta^i - \frac{d\widehat{\theta}}{dq^i} \pi_i^i \left(\widehat{\theta}, q^i, q^j\right) f\left(\widehat{\theta}\right) < 0 \tag{2.5}$$

and is satisfied because of the concavity assumption $\pi_{ii}^i \left(\theta^i, q^i, q^j\right) < 0 \; \forall \theta^i$, made above and, for all but very large debt values, if one assumes that uncertainty is large, so that $f\left(\widehat{\theta}\right)$ is small. A Nash equilibrium of this subgame will then be characterised by the first-order condition holding both for firm i and for firm j. Under the assumptions made, the first-order condition implicitly defines a reaction function for the quantity setting stage, which is denoted by $q^i\left(q^j, D^i; s\right)$, where s stands for shareholder control.

Before going on to the financing stage, it is useful to analyse the behaviour of the reaction function in more detail.

Consider first the effect of a change of a firm's indebtedness on its optimal quantity choice for any given quantity choice of its rival. In a first step, note that by implicitly differentiating 2.2 one finds

$$\frac{d\widehat{\theta}}{dD^i} = \frac{1}{\pi_\theta^i \left(\widehat{\theta}, q^i, q^j\right)} > 0$$

which is intuitive. With a higher face value, the firm defaults for higher realisations θ^i, so that the threshold $\widehat{\theta}$ moves up with D^i. Implicitly differentiating the first-order condition 2.4 one has

$$\frac{\partial q_i}{\partial D^i} = -\frac{S_{iD^i}^i}{S_{ii}^i}$$

where the denominator is negative by the second-order condition 2.5. The numerator is

$$S^i_{iD^i} = -\pi^i_i\left(\widehat{\theta}, q^i, q^j\right) f\left(\widehat{\theta}\right) \frac{d\widehat{\theta}}{dD^i}$$

When evaluated at the optimum, by $\pi^i_{\theta i}\left(\theta^i, q^i, q^j\right) > 0$ and the first-order condition 2.4 one has that $\pi^i_i\left(\widehat{\theta}, q^i, q^j\right) < 0$; for "average" marginal profit to be zero, it must be that marginal profits are negative at the lower bound of integration. Therefore, $S^i_{iD^i} > 0$, and $\frac{\partial q_i}{\partial D^i} > 0$. This means that a higher debt level will shift the firm's reaction function out. Intuitively, for any quantity choice of the rival, with a higher debt level, states of negative marginal profits are discarded from the calculus, so that average marginal profits are positive and the quantity choice will increase.

Let us next consider the slope of the reaction functions. Firm *i's* optimal response to a change in the quantity of its rival can be found by implicitly differentiating the first-order condition 2.4to get

$$\frac{\partial q^i}{\partial q^j} = -\frac{S^i_{ij}}{S^i_{ii}}$$

where again the denominator is negative by the second-order condition. The overall effect will therefore have the same sign as the numerator, which can be evaluated as

$$S^i_{ij} = \int_{\widehat{\theta}}^{\overline{\theta}} \pi^i_{ij}\left(\theta^i, q^i, q^j\right) f\left(\theta^i\right) d\theta^i - \frac{d\widehat{\theta}}{dq^i}\pi^i_i\left(\widehat{\theta}, q^i, q^j\right) f\left(\widehat{\theta}\right)$$

One sees that there are two opposing effects. Since $\Pi^i_{ij}\left(\theta^i, q^i, q^j\right) < 0\ \forall\theta^i$ the first part of this expression is negative. It captures the usual intuition that, if goods are substitutes, quantity choice will be strategic substitutes. Observe, however, that the second part of this expression is positive. This can be established by noting again that $\pi^i_i\left(\widehat{\theta}, q^i, q^j\right) < 0$ and implicitly differentiating 2.2 to get

$$\frac{d\widehat{\theta}}{dq^i} = -\frac{\pi^i_j\left(\widehat{\theta}, q^i, q^j\right)}{\pi^i_\theta\left(\widehat{\theta}, q^i, q^j\right)} > 0$$

since $\pi^i_j\left(\widehat{\theta}, q^i, q^j\right) < 0$ and $\pi^i_\theta\left(\widehat{\theta}, q^i, q^j\right) > 0$.

The positive effect captures what goes on at the limit of integration. Note that its size depends on the distribution of θ^i. For $f\left(\widehat{\theta}\right)$ small enough over the relevant range, one will have a regular downward sloping curve. If there is a lot of uncertainty, so that the interval $\left(\underline{\theta},\overline{\theta}\right)$is large and $f(\theta)$ is small on average, then the positive effect is of second-order importance at least for small levels of debt and the first effect is likely to dominate. For these reasons I follow Brander and Lewis and assume that $S^i_{ij} < 0$. But isn't S^i_j endogenous?

Given the behaviour at the quantity stage, one can characterise equilibrium in debt levels. Since the debtholder pays the expected value of his claim to the shareholder, shareholders are concerned with maximising expected overall (debt +equity) value of the firm at the financing stage. One can then analyse the equilibrium in debt levels. Let us define

$$V^i\left(q^i,q^j\right)=\int_{\underline{\theta}}^{\overline{\theta}}\pi^i\left(\theta^i,q^i,q^j\right)f\left(\theta^i\right)d\theta^i$$

as the ex ante value of the firm. Equilibrium is characterised by a pair (D^i, D^j) such that

$$\max_{D^i} V^i\left(q^i,q^j\right)$$

$$\begin{aligned} s.t.\quad q^i &= q^i\left(q^j,D^i;s\right)\\ q^j &= q^j\left(q^i,D^j;s\right)\\ D^i &\geq 0 \end{aligned}$$

holds for both firms. Each firm owner chooses its firm's reaction function taking the reaction function of its rival as given. To characterise the equilibrium further recall that the Cournot point (q^c, q^c) is defined as the solution to

$$\int_{\underline{\theta}}^{\overline{\theta}}\pi^i_i\left(\theta^i,q^i,q^c\right)f\left(\theta^i\right)d\theta^i=0$$

for firms i and j. Consider the pair of reaction functions that go through (q^c, q^c). In the case of owner control the reaction function through (q^c, q^c) is given implicitly by

$$\int_{\underline{\theta}}^{\overline{\theta}} \pi_i^i \left(\theta^i, q^i, q^j\right) f\left(\theta^i\right) d\theta^i = 0$$

and is characterised by a zero level of debt, $(D^i, D^j) = (0,0)$. One can show that debt levels of zero do not constitute an equilibrium here, but that, starting with the reaction functions going through (q^c, q^c)reactions functions will be shifted out. Given that an increase in its own debt level shifts a firm's reaction function out, and that the rival firm's reaction function slopes down, a unilateral increase in debt will increase the firm's own quantity but decrease the rival's quantity, so that each firm has an incentive to raise its debt level above zero. To show this formally one can replace the constraints by the first-order conditions and linearise by totally differentiating the system of first -order conditions.

$$\begin{aligned} S_{ii}^i dq^i + S_{ij}^i dq^j + S_{iD^i}^j dD^i &= 0 \\ S_{ji}^j dq^i + S_{jj}^j dq^j + S_{jD^i}^j dD^i &= 0 \end{aligned}$$

Note that $S_{jD^i}^j = 0$. Using Cramer's rule one can establish that

$$\begin{aligned} \frac{dq^i}{dD^i} &= -\frac{S_{iD^i}^i S_{jj}^j}{S_{ii}^i S_{jj}^j - S_{ij}^i S_{ji}^j} > 0 \\ \frac{dq^j}{dD^i} &= \frac{S_{iD^i}^i S_{ji}^j}{S_{ii}^i S_{jj}^j - S_{ij}^i S_{ji}^j} < 0 \end{aligned}$$

when $S_{ii}^i S_{jj}^j - S_{ij}^i S_{ji} > 0$, which is the usual condition for reaction function stability, and assuming that $S_{ji}^j < 0$.

The total value of the firm is

$$V^i = \int_{\underline{\theta}}^{\overline{\theta}} \pi^i \left(\theta^i, q^i\left(D^i, D^j\right), q^j\left(D^i, D^j\right)\right) f\left(\theta^i\right) d\theta^i$$

where $\left(q^i\left(D^i, D^j\right), q^j\left(D^i, D^j\right)\right)$ is the solution to the pair of constraints for any pair $\left(D^i, D^j\right)$. Differentiating with respect to D^i one finds the first-order condition

$$V_{D^i} = \left[\int_{\underline{\theta}}^{\overline{\theta}} \pi_i^i \left(\theta^i, q^i\left(D^i, D^j\right), q^j\left(D^i, D^j\right)\right) f\left(\theta^i\right) d\theta^i\right] \frac{dq^i}{dD^i}$$

$$
\begin{aligned}
&+\left[\int_{\underline{\theta}}^{\overline{\theta}} \pi_j^i\left(\theta^i, q^i\left(D^i, D^j\right), q^j\left(D^i, D^j\right)\right) f\left(\theta^i\right) d\theta^i\right] \frac{dq^j}{dD^i} \\
= \;& 0
\end{aligned}
$$

Assume first that $D^i = D^j = 0$. Then quantities will be set at the Cournot level, $q^i = q^j = q^c$. At these levels of output the first bracket is zero. The second term is positive, however, since $\pi_j^i < 0$ and also $\frac{dq^j}{dD^i} < 0$. Therefore each firm wants to unilaterally increase its debt level. In a symmetric equilibrium therefore $D^i = D^j > 0$, which, looking back at the first-order condition (2.4), entails that $q^i = q^j > q^c$. Equilibrium quantities will be beyond the Cournot quantities. Note that this also implies that $V^i = V^j < V^c$. In equilibrium, owners will be worse off than they would if they could not issue debt.

2.4 Manager Control

Let us now consider the case where the output decision is delegated to a manager, whose objective is to avoid bankruptcy. I assume that the manager's quantity choice is unobservable to the owner, so that contracts forcing the manager to choose a particular quantity are impossible. For the main part of the analysis I also disallow any other contract which may condition on ex post profit, by assuming that the manager does not respond to monetary incentives. This means that, for the quantity decision, the manager's preferences cannot be driven away from the goal of avoiding bankruptcy. This assumption is made mainly to have a clear starting point and will be relaxed in a later section. Next, I assume that to produce any positive quantity $q^i > 0$ the manager has to spend some fixed, but small effort cost $\overline{e} > 0$, so that without any other incentives working on the manager the manager would choose $q^i = 0$. Once the debt level has been set and the strategic variable q^i is to be chosen, the threat of bankruptcy is the only thing that motivates the manager. In particular, the manager receives a private benefit b whenever the firm is not bankrupt and his payoff in bankrupt states is normalised to zero. This is without loss of generality, since one can alternatively think of b as a constant pay-off differential between bankrupt states and non-bankrupt states. I also assume $b >> \overline{e}$, so that the manager will choose to spend effort if debt has been issued and there is a positive probability of bankruptcy.

The only tool available to shareholders to motivate their managers is to issue debt against the profits of the firm. Again the assumption is that shareholders choose the debt level so as to maximise the value of the firm. The assumption that the capital structure decision is taken by shareholders to discipline managers is in the spirit of Principal-Agent theories debt such as for example Jensen and Meckling (1976) and Hart and Moore (1995). These have recently been questioned by Zwiebel (1996), who argues that managers may be able to choose the capital structure themselves. Against this view one can argue that, unlike most operating decisions which are taken by chief executives alone, capital structure changes will normally require the approval of the board of directors and often also need to be approved by shareholders and other financial investors, such as creditors. Essentially, what is required for our model is that the capital structure decision is taken in a value maximising way, given the later play of the game. To guarantee this one can assume that the capital structure decision is taken by shareholders. This may be justifiable in particular for closely held firms, in which a venture capitalist plays an active role in designing the firm's financial structure. Alternatively, one can have the manager decide on the capital structure decision. Since the capital structure decision is observable, one can rely on efficient bargaining between the manager and financial investors such as shareholders to ensure that a the capital structure is chosen in a value-maximizing way.

In the subgame following the choice of debt levels the manager's objective is to maximise

$$B^i = \int_{\widehat{\theta}}^{\overline{\theta}} bf\left(\theta^i\right) d\theta^i \tag{2.6}$$

where again $\widehat{\theta}$ is given by

$$\pi^i\left(\widehat{\theta}, q^i, q^j\right) - D^i = 0 \tag{2.7}$$

This problem has first order condition

$$B_i^i = -bf\left(\widehat{\theta}\right)\frac{\partial\widehat{\theta}}{\partial q^i} = 0 \tag{2.8}$$

Implicitly differentiating 2.7 one finds

$$\frac{\partial\widehat{\theta}}{\partial q^i} = -\frac{\pi_i^i\left(\widehat{\theta}, q^i, q^j\right)}{\pi_\theta^i\left(\widehat{\theta}, q^i, q^j\right)} \tag{2.9}$$

and the first-order condition can be written as

$$B_i^i = bf\left(\widehat{\theta}\right)\frac{\pi_i^i\left(\widehat{\theta}, q^i, q^j\right)}{\pi_\theta^i\left(\widehat{\theta}, q^i, q^j\right)} = 0 \tag{2.10}$$

The second-order condition is

$$B_{ii}^i = -bf\left(\widehat{\theta}\right)\frac{\partial^2\widehat{\theta}}{\partial q^i\partial q^i} - bf'\left(\widehat{\theta}\right)\frac{\partial\widehat{\theta}}{\partial q^i}\frac{\partial\widehat{\theta}}{\partial q^i} < 0 \tag{2.11}$$

Again using 2.9 one has

$$\begin{aligned}
B_{ii}^i &= bf\left(\widehat{\theta}\right)\frac{\pi_\theta^i\left(\widehat{\theta}, q^i, q^j\right)\left[\pi_{ii}^i\left(\widehat{\theta}, q^i, q^j\right) + \pi_{i\theta}^i\left(\widehat{\theta}, q^i, q^j\right)\frac{\partial\widehat{\theta}}{\partial q^i}\right]}{\left(\pi_\theta^i\left(\widehat{\theta}, q^i, q^j\right)\right)^2} \\
&\quad + bf\left(\widehat{\theta}\right)\frac{-\pi_i^i\left(\widehat{\theta}, q^i, q^j\right)\left[\pi_{i\theta}^i\left(\widehat{\theta}, q^i, q^j\right) + \pi_{\theta\theta}^i\left(\widehat{\theta}, q^i, q^j\right)\frac{\partial\widehat{\theta}}{\partial q^i}\right]}{\left(\pi_\theta^i\left(\widehat{\theta}, q^i, q^j\right)\right)^2} \\
&\quad + bf'\left(\widehat{\theta}\right)\left(\frac{\pi_i^i\left(\widehat{\theta}, q^i, q^j\right)}{\pi_\theta^i\left(\widehat{\theta}, q^i, q^j\right)}\right)^2
\end{aligned}$$

and since $\pi_i^i\left(\widehat{\theta}, q^i, q^j\right) = 0$ by the first-order condition, the second-order condition reduces to

$$B_{ii}^i = -bf\left(\widehat{\theta}\right)\frac{\partial^2\widehat{\theta}}{\partial q^i\partial q^i} = bf\left(\widehat{\theta}\right)\frac{\pi_{ii}^i\left(\widehat{\theta}, q^i, q^j\right)}{\pi_\theta^i\left(\widehat{\theta}, q^i, q^j\right)} < 0 \tag{2.12}$$

One then sees that because $\pi^i_{ii}\left(\widehat{\theta}_i, q^i, q^j\right) < 0$ and $\pi^i_\theta\left(\widehat{\theta}_i, q^i, q^j\right) > 0$ by assumption, the required inequality holds. Thus, whenever the first-order condition holds the second-order condition will also be satisfied[3]. This implies that, for any given debt level and any given rival's output, the first-order condition uniquely defines the manager's optimal choice of q^i. The first-order condition therefore implicitly defines a function $q^i\left(q^j, D^i; m\right)$ which gives the manager's optimal output choice for any given rival's choice and for any given debt level.

It is useful at this point to compare the manager's problem with the one analysed in the benchmark case. The manager obtains a positive benefit only when the firm is not bankrupt. He is therefore interested in widening the interval $\left[\widehat{\theta}, \overline{\theta}\right)$ as much as possible, since this will minimise the probability of bankruptcy. The manager's problem is therefore equivalent to minimising $\widehat{\theta}$ by choice of q^i for any given debt level D^i and any given choice of q^j. Looking back at the first-order condition, it is worth noting that it implies that

$$\pi^i_i\left(\widehat{\theta}, q^i, q^j\right) = 0$$

holds at this minimised $\widehat{\theta}$. One can see the intuition for this by assuming that $\pi^i\left(\widehat{\theta}, q^i, q^j\right) - D^i = 0$ held for a given D^i, a given q^j, and some choice of q^i, and that $\pi^i_i\left(\widehat{\theta}, q^i, q^j\right) > 0$ for the implied $\widehat{\theta}$. Then the manager can increase profit by increasing q^i, which will make $\pi^i\left(\widehat{\theta}, q^i, q^j\right) > D^i$ at the old $\widehat{\theta}$. But this means that bankruptcy can be avoided for a realisation of θ^i below the old $\widehat{\theta}$. There will therefore be scope to decrease $\widehat{\theta}$ by increasing q^i, and the original choice of q^i can not have been optimal. A reverse argument can be made for the case that $\pi^i_i\left(\widehat{\theta}, q^i, q^j\right) < 0$. One therefore must have $\pi^i_i\left(\widehat{\theta}, q^i, q^j\right) = 0$. This means that the manager's choice of q^i is such that he is maximising profit at the minimised level of $\widehat{\theta}$. This is in contrast to the benchmark case where the shareholders were maximising profit over the interval $\left[\widehat{\theta}, \overline{\theta}\right)$.

As a first comparative static exercise, let us analyse how the manager's behaviour is influenced by the debt level chosen. One finds that, just as in the

[3] Note that this is true even though the manager's problem may not be globally concave.

benchmark case, the reaction function shifts out as the debt level increases. This is stated more formally as

Lemma 1 *In the subgame following the choice of debt levels, for given q^j, with manager control over quantities, a higher debt level D^i will induce the manager to choose a larger output q^i.*

Proof:

$$\frac{\partial q^i}{\partial D^i} = -\frac{B^i_{iD^i}}{B^i_{ii}}$$

Since the second-order condition holds, the sign of this will be the same as the sign of $B^i_{iD^i}$. One easily obtains

$$\begin{aligned}
B^i_{iD^i} &= -bf\left(\widehat{\theta}\right)\frac{\partial^2\widehat{\theta}}{\partial q^i \partial D^i} - bf'\left(\widehat{\theta}\right)\frac{\partial\widehat{\theta}}{\partial D^i}\frac{\partial\widehat{\theta}}{\partial q^i} \\
&= bf\left(\widehat{\theta}\right)\frac{\pi^i_\theta\left(\widehat{\theta},q^i,q^j\right)^i_{i\theta}\left(\widehat{\theta},q^i,q^j\right)\frac{d\widehat{\theta}}{dD^i} - \pi^i_i\left(\widehat{\theta},q^i,q^j\right)\pi^i_{\theta\theta}\left(\widehat{\theta},q^i,q^j\right)\frac{d\widehat{\theta}}{dD^i}}{\left(\pi^i_\theta\left(\widehat{\theta},q^i,q^j\right)\right)^2} \\
&\quad +bf'\left(\widehat{\theta}\right)\frac{1}{\pi^i_\theta\left(\widehat{\theta},q^i,q^j\right)}\frac{\pi^i_i\left(\widehat{\theta},q^i,q^j\right)}{\pi^i_\theta\left(\widehat{\theta},q^i,q^j\right)} \\
&= bf\left(\widehat{\theta}\right)\frac{\pi^i_{i\theta}\left(\widehat{\theta},q^i,q^j\right)\frac{d\widehat{\theta}}{dD^i}}{\pi^i_\theta\left(\widehat{\theta},q^i,q^j\right)}
\end{aligned}$$

again using that $\pi^i_i\left(\widehat{\theta},q^i,q^j\right) = 0$. All terms in the numerator of this last expression are positive. In particular, implicitly differentiating

$$\pi^i\left(\widehat{\theta},q^i,q^j\right) - D^i = 0$$

gives

$$\frac{d\widehat{\theta}}{dD^i} = \frac{1}{\pi^i_\theta\left(\widehat{\theta},q^i,q^j\right)} > 0$$

Hence

$$B^i_{iD^i} = bf\left(\widehat{\theta}\right)\frac{\pi^i_{i\theta}\left(\widehat{\theta},q^i,q^j\right)}{\left(\pi^i_\theta\left(\widehat{\theta},q^i,q^j\right)\right)^2} > 0$$

so that

$$\begin{aligned}\frac{\partial q^i}{\partial D^i} &= -\frac{B^i_{iD^i}}{B^i_{ii}} \\ &= -\frac{\pi^i_{i\theta}\left(\widehat{\theta}, q^i, q^j\right)}{\pi^i_\theta\left(\widehat{\theta}, q^i, q^j\right)\pi^i_{ii}\left(\widehat{\theta}, q^i, q^j\right)} > 0\end{aligned}$$

The intuition for this result starts by recalling that for any debt level the manager is minimising $\widehat{\theta}$ by choice of q^i. Call this minimised value $\widehat{\theta}^*$. It is clear that when $D^{i\prime} > D^i$, then also $\widehat{\theta}^{*\prime} > \widehat{\theta}^*$. For both levels of debt, the manager is maximising profit at the minimised $\widehat{\theta}$. Since marginal profit is increasing in θ^i, $\pi^i_{i\theta} > 0$, when profit is maximised at $\widehat{\theta}^{*\prime}$ a higher quantity is called for than when profit is maximised at the lower $\widehat{\theta}^*$. The quantity chosen will therefore be increasing in the debt level.

Since firm i' s output is increasing in its own debt level, both for the case where the manager makes decisions and for the benchmark case where quantities are chosen by the owners themselves, it may be interesting to compare quantity levels for given debt levels across regimes. The following result is easily obtained:

Proposition 1 *For given D^i and given rival's quantity q^j firm i's quantity choice will be smaller when taken by a manager than when taken by the firm's owner, $q^i(q^j, D^i; m) < q^i(q^j, D^i; s)$*

Proof: The manager chooses q^i at the minimised value $\widehat{\theta}^* = \widehat{\theta}\left(q^i(q^j, D^i; m), q^j, D^i\right)$ such that

$$\pi^i_i\left(\widehat{\theta}^*, q^i(q^j, D^i; m), q^j\right) = 0$$

is satisfied. Given the same debt level and rival quantity the owner's choice q^i would satisfy

$$\int_{\widehat{\theta}}^{\overline{\theta}} \pi^i_i\left(\theta, q^i(q^j, D^i; s), q^j\right) f\left(\theta^i\right) d\theta^i = 0$$

Clearly, in the latter expression $\widehat{\theta} \geq \widehat{\theta}^*$, since under owner control the lower bound of integration $\widehat{\theta}$ is not being minimised. Since $\pi^i_{i\theta}\left(\theta^i, q^i(q^j, D^i; s), q^j\right) > 0$ it then follows that $q^i(q^j, D^i; s) > q^i(q^j, D^i; m)$.

For any given debt level, the manager is less aggressive than the owner. The manager's objective is to avoid bankruptcy, so that he is looking at the marginal state, where marginal profit is low, whereas the owner will maximise profit over all non-bankrupt states $\theta^i \in \left[\widehat{\theta}, \overline{\theta}\right)$ where marginal profit is higher. This result confirms the intuition that the manager's output choice will be more conservative than the shareholder's output choice.

Since the analysis has been looking at the subgame only, however, this result cannot be taken to say that the overall equilibrium will be characterised by lower quantities when the manager is in charge of the quantity choice. The owner can choose the debt level before the manager chooses a quantity, so that in principle, the owners can counter the manager's reluctance to choose high quantities by pushing up the debt level at the financing stage.

Before one can characterise equilibrium in debt levels and quantities one needs to take into account the strategic interaction between managers. Recall that, when quantities are chosen by the owners, an increase in q_j always induces a decrease in q^i along a downward sloping reaction function for appropriate assumptions on the density $f\left(\theta^i\right)$. By contrast, under manager control this need not be the case. Depending on the exact functional form of the profit function the manager's reaction function may be downward sloping or upward sloping. More formally

Lemma 2 *In the subgame following the choice of debt levels, when the manager of the rival firm j chooses a higher quantity q^j, manager i's optimal quantity choice q^i may increase, stay the same, or decrease.*

For the proof note that:

$$\frac{\partial q^i}{\partial q^j} = -\frac{B^i_{ij}}{B^i_{ii}}$$

which again since $B^i_{ii} < 0$ will have the same sign as B^i_{ij}.

$$B^i_{ij} = bf\left(\widehat{\theta}\right)\frac{\pi^i_\theta\left(\widehat{\theta}, q^i, q^j\right)\left[\pi^i_{ij}\left(\widehat{\theta}, q^i, q^j\right) + \pi^i_{i\theta}\left(\widehat{\theta}, q^i, q^j\right)\frac{d\widehat{\theta}}{dq^j}\right]}{\left(\pi^i_\theta\left(\widehat{\theta}, q^i, q^j\right)\right)^2}$$

$$
\begin{aligned}
&+bf\left(\widehat{\theta}\right)\frac{-\pi_i^i\left(\widehat{\theta},q^i,q^j\right)\left[\pi_{j\theta}^i\left(\widehat{\theta},q^i,q^j\right)+\pi_{\theta\theta}^i\left(\widehat{\theta},q^i,q^j\right)\frac{d\widehat{\theta}}{dq^j}\right]}{\left(\pi_\theta^i\left(\widehat{\theta},q^i,q^j\right)\right)^2} \\
&+bf'\left(\widehat{\theta}\right)\frac{\pi_{ij}^i\left(\widehat{\theta},q^i,q^j\right)}{\pi_\theta^i\left(\widehat{\theta},q^i,q^j\right)}\frac{\pi_i^i\left(\widehat{\theta},q^i,q^j\right)}{\pi_\theta^i\left(\widehat{\theta},q^i,q^j\right)} \\
= \;& bf\left(\widehat{\theta}\right)\frac{\left[\pi_{ij}^i\left(\widehat{\theta},q^i,q^j\right)+\pi_{i\theta}^i\left(\widehat{\theta},q^i,q^j\right)\frac{d\widehat{\theta}}{dq^j}\right]}{\left(\pi_\theta^i\left(\widehat{\theta},q^i,q^j\right)\right)^2}
\end{aligned}
$$

since $\Pi_i^i\left(\widehat{\theta},q_i,q_j\right)=0$.

The sign of this is ambiguous. Note that it will be the same as the numerator, which since

$$\frac{d\widehat{\theta}}{dq^j}=-\frac{\pi_j^i\left(\widehat{\theta},q^i,q^j\right)}{\pi_\theta^i\left(\widehat{\theta},q^i,q^j\right)}>0$$

one can write as

$$\pi_{ij}^i\left(\widehat{\theta},q^i,q^j\right)-\pi_{i\theta}^i\left(\widehat{\theta},q^i,q^j\right)\frac{\pi_j^i\left(\widehat{\theta},q^i,q^j\right)}{\pi_\theta^i\left(\widehat{\theta},q^i,q^j\right)}$$

It follows from (A 1) that

$$\pi_{ij}^i\left(\widehat{\theta},q^i,q^j\right)<0$$

but that

$$-\pi_{i\theta}^i\left(\widehat{\theta},q^i,q^j\right)\frac{\pi_j^i\left(\widehat{\theta},q^i,q^j\right)}{\pi_\theta^i\left(\widehat{\theta},q^i,q^j\right)}>0$$

As can be seen from this, there are two effects at work.

The first term captures the usual strategic effect. If the other firm increases its quantity, manager i has an incentive to reduce his quantity, and vice versa. This is because, as pointed out before, the manager is maximising profit at some minimised level of $\widehat{\theta}$. At this level, the manager's response to a change in the rival's quantity will be profit-maximising and will therefore be of the same sign as when managers behave as shareholders would. Since $\pi_{ij}^i\left(\widehat{\theta},q^i,q^j\right)<0$, when the rival firm increases its output q^j, manager i has an incentive to reduce his choice of q^i in response.

On the other hand, and captured by the positive part of the expression, a change in q^j will move $\widehat{\theta}$. An increase in q^j will depress firm $i's$ profit, since $\pi^i_j\left(\widehat{\theta}, q^i, q^j\right) < 0$ and therefore move $\widehat{\theta}$ upward. When $\widehat{\theta}$ gets pushed up, this will call for a higher q^i, since marginal profit is higher at higher θ^i, $\pi^i_{i\theta}\left(\widehat{\theta}, q^i, q^j\right) > 0$. Therefore, when q^j goes up, the manager's response will be to increase his choice of q^i.

When the first effect dominates, quantities are strategic substitutes, as they are under profit-maximisation, and reaction functions slope downwards. When the second effect dominates, quantities, which are strategic substitutes under profit maximisation, become strategic complements when the probability of bankruptcy is being minimised, and reaction functions slope upwards. Loosely speaking, this is due to profit drain effect. When q^j goes up, this will put a profit drain on firm i. Under the pressure of this profit drain the manager of firm i will have to compete more aggressively, to keep up the odds of keeping the company out of bankruptcy. On the other hand, when q^j goes down, this will bolster firm $i's$ profit and relieve the pressure on the manager of firm i who will then respond by competing less aggressively, in order to increase the odds of keeping the company afloat.

Note that the direction of the overall effect no longer depends on the distribution of θ^i over its support. The density no longer enters the expression, and the sign of the expression will be the same for high and low degrees of uncertainty. Which of the two effects will dominate will solely depend on the exact shape of the profit function. From the expression one sees that quantities are more likely to become strategic complements when $\pi^i_{ij}\left(\theta^i, q^i, q^j\right)$ and $\pi^i_\theta\left(\theta^i, q^i, q^j\right)$are relatively small, but $\pi^i_j\left(\theta^i, q^i, q^j\right)$ and $\pi^i_{i\theta}\left(\theta^i, q^i, q^j\right)$are relatively large in absolute value. Another way of looking at this is to note that

$$\pi^i_{ij}\left(\widehat{\theta},q^i,q^j\right)-\pi^i_{i\theta}\left(\widehat{\theta},q^i,q^j\right)\frac{\pi^i_j\left(\widehat{\theta},q^i,q^j\right)}{\pi^i_\theta\left(\widehat{\theta},q^i,q^j\right)}\geq 0$$

translates into the following condition on the elasticities of the marginal effects of θ^i and q^j on firm profit

$$\frac{\pi^i_{\theta i}\left(\widehat{\theta},q^i,q^j\right)q^i}{\pi^i_\theta\left(\widehat{\theta},q^i,q^j\right)}\geq\frac{\pi^i_{ji}\left(\widehat{\theta},q^i,q^j\right)q^i}{\pi^i_j\left(\widehat{\theta},q^i,q^j\right)}$$

Reaction functions will slope upwards whenever the marginal effect of θ^i on firm profit is more elastic with respect to changes in q^i than the marginal effect of q^j. The intuition is that when the rival firm increases its quantity this will increase both q^j and $\widehat{\theta}$. When the manager increases his quantity in response this will enlarge the adverse effect on $\pi^i_j\left(\widehat{\theta},q^i,q^j\right)$ On the other hand it will have a positive impact on $\pi^i_\theta\left(\widehat{\theta},q^i,q^j\right)$. If the positive effect is stronger than the negative effect, the manager will optimally increase his quantity.

Whether reaction functions slope upwards or downwards will impact decisions at the financing stage. Equilibrium at the financing stage is given by

$$\max_{D^i} V^i\left(q^i,q^j\right)$$

$$\begin{aligned} s.t.\quad q^i &= q^i\left(q^j,D^i;m\right)\\ q^j &= q^j\left(q^i,D^j;m\right)\\ D^i &\geq 0\end{aligned}$$

Again replacing the constraints by the first-order conditions and linearising.

$$\begin{aligned} B^i_{ii}dq^i+B^i_{ij}dq^j+B^j_{iD^i}dD^i &= 0\\ B^j_{ji}dq^i+B^j_{jj}dq^j+B_{jD^i}dD^i &= 0\end{aligned}$$

Note that $B_{jD^i} = 0$. Using Cramer's rule one can establish that

$$\begin{aligned}
\frac{dq^i}{dD^i} &= -\frac{B^i_{iD^i}B^j_{jj}}{B^i_{ii}B^j_{jj} - B^i_{ij}B^j_{ji}} > 0 \\
\frac{dq^j}{dD^i} &= \frac{B^i_{iD^i}B^j_{ji}}{B^i_{ii}B^j_{jj} - B^i_{ij}B^j_{ji}} 0 > (=)\,(<)\,0
\end{aligned}$$

Assuming that the regularity condition $B^i_{ii}B^j_{jj} - B^i_{ij}B^j_{ji} > 0$ holds, one can sign the first derivative since $B^i_{iD^i} > 0$, as shown above and $B^j_{jj} < 0$ by the second-order condition.

Again under regularity condition $B^i_{ii}B^j_{jj} - B^i_{ij}B^j_{ji} > 0$, the sign of the second derivative will be the same as the sign of B^j_{ji}. This in turn will be of the same sign as

$$\pi^j_{ji}\left(\widehat{\theta}, q^i, q^j\right) - \pi^j_{j\theta}\left(\widehat{\theta}, q^i, q^j\right)\frac{\pi^j_i\left(\widehat{\theta}, q^i, q^j\right)}{\pi^j_\theta\left(\widehat{\theta}, q^i, q^j\right)}$$

As explained above, the sign of this expression is ambiguous.

Consider again the total value of the firm

$$V = \int_{\underline{\theta}}^{\overline{\theta}} \pi^i\left(\theta^i, q^i, q^j\right) f\left(\theta^i\right) d\theta^i$$

Differentiating with respect to D^i one finds the first-order condition

$$\begin{aligned}
V_{D^i} &= \left[\int_{\underline{\theta}}^{\overline{\theta}} \pi^i_i\left(\theta^i, q^i\left(D^i, D^j\right), q^j\left(D^i, D^j\right)\right) f\left(\theta^i\right) d\theta^i\right]\frac{dq^i}{dD^i} \\
&\quad + \left[\int_{\underline{\theta}}^{\overline{\theta}} \pi^i_j\left(\theta^i, q^i\left(D^i, D^j\right), q^j\left(D^i, D^j\right)\right) f\left(\theta^i\right) d\theta^i\right]\frac{dq^j}{dD^i} \\
&= 0
\end{aligned}$$

There will be positive debt levels $D^i = D^j > 0$ such that the managers' reaction functions intersect at the Cournot point (q^c, q^c). At the Cournot-level of output, the term in the first bracket is zero. Since $\pi^i_j\left(\theta, q^i, q^j\right) < 0$ the term in the second bracket is negative. The overall sign of the derivative will therefore depend on $\frac{dq^j}{dD^i}$.

If $B^j_{ji} < 0$, the rival's reaction function is downward sloping and one will have $\frac{dq^j}{dD^i} < 0$. Just as in the benchmark case there is an incentive to increase D^i, since this will lead the rival to reduce its quantity along its reaction curve. This incentive exists for both firms, so that in equilibrium quantities will be higher than Cournot, $(q^i, q^j) = (q^*, q^*) > (q^c, q^c)$, implying ex ante firm values less than Cournot, $V^i = V^j < V^c$. [4]

If $B^j_{ji} = 0$, the rival's reaction function is horizontal. The rival will produce q^c for any quantity firm i produces. Then also $\frac{dq^j}{dD^i} = 0$, and there is no incentive to change the debt level for strategic reasons. The equilibrium quantities will be the Cournot quantities, $(q^i, q^j) = (q^*, q^*) = (q^c, q^c)$. There is no limited liability effect and ex ante firm values will be the Cournot values, $V^i = V^j = V^c$.

If $B^j_{ji} > 0$, the rival's reaction function will be upward sloping and $\frac{dq^j}{dD^i} > 0$. There now is an incentive to decrease D^i, that is to move the own reaction function in, rather than out. This will imply that quantities will be lower than Cournot in equilibrium, $(q^i, q^j) = (q^*, q^*) < (q^c, q^c)$. One can also show that quantities will not be smaller than the joint profit maximising quantities[5], so that here, quantities will lie in between the joint profit maximising and the Cournot quantities. This implies that ex ante firm values will be higher than Cournot, $V^i = V^j > V^c$.

These results are summarised in the following

Proposition 2 *In a symmetric equilibrium in debt levels and quantities, when quantities are chosen by managers, equilibrium quantities may be less than,*

[4] Note that one may still have a more collusive quantity choice under manager control as compared with owner control. As shown in Appendix 2.1 at the end of this chapter, this will be the case whenever along the line $(q^i, q^j) = (q, q)$ with $(q, q) \geq (q^c, q^c)$ one has

$$-\frac{B^i_{ij}}{B^i_{ii}} > -\frac{S^i_{ij}}{S^i_{ii}}$$

[5] This is a consequence of the assumed reaction function stability and is proven in Appendix 2.2 at the end of this chapter.

greater than, or equal to Cournot quantities.

The case where equilibrium quantities are (weakly) less than Cournot is intriguing, since it highlights the possibility of sustaining a (weakly) more collusive outcome than would obtain in the simple one-shot game with straight equity value maximisation. The intuition for this case is that, at the Cournot levels of output, both firms want to decrease their debt levels in order to decrease the pressure on the rival firm's manager to generate profits. Less pressure on the rival firm will result in lower rival output and thus benefits the firm which decreases its debt level away from the Cournot level.

2.5 Examples

Under manager control, equilibrium quantities will be equal or below $(q^c, q^c)$when $B^j_{ji} \geq 0$. By symmetry this will be the case whenever the profit function satisfies

$$\pi^i_{ij}\left(\widehat{\theta}, q^i, q^j\right) - \pi^i_{i\theta}\left(\widehat{\theta}, q^i, q^j\right)\frac{\pi^i_j\left(\widehat{\theta}, q^i, q^j\right)}{\pi^i_\theta\left(\widehat{\theta}, q^i, q^j\right)} \geq 0$$

To illustrate that this may well be satisfied take the standard example of a linear demand function and weakly convex costs,

$$\pi^i\left(q^i, q^j\right) = \left[a - bq^i - \beta q^j\right] q^i - cq^{i\gamma}$$

where $0 \leq \beta \leq b$ and $\gamma \geq 1$. In the demand function I allow for the possibility that goods may not be perfect substitutes, in which case $\beta < b$. Costs are strictly convex when $\gamma > 1$, whereas they are linear when $\gamma = 1$. As it is, the profit function is deterministic. One can make it stochastic by letting its parameters be functions of θ^i. Let us start by looking at cost uncertainty. Replacing c by $c\left(\theta^i\right)$ with $c\left(\theta^i\right) > 0$ and $c'\left(\theta^i\right) < 0$ one arrives at a function

$$\pi^i\left(\theta^i, q^I, q^j\right) = \left[a - bq^i - \beta q^j\right] q^i - c\left(\theta^i\right) q^{i\gamma}$$

which satisfies A 1. One finds

$$\begin{aligned}
\pi_{ij}^{i}\left(\widehat{\theta},q^{i},q^{j}\right)-\pi_{i\theta}^{i}\left(\widehat{\theta},q^{i},q^{j}\right)\frac{\pi_{j}^{i}\left(\widehat{\theta},q^{i},q^{j}\right)}{\pi_{\theta}^{i}\left(\widehat{\theta},q^{i},q^{j}\right)} &= -\beta-\left(-c'\left(\theta^{i}\right)\gamma q^{i\gamma-1}\frac{-\beta q^{i}}{-c'\left(\theta^{i}\right)q^{i\gamma}}\right) \\
&= \beta\frac{\left(\gamma q^{i\gamma}-q^{i\gamma}\right)}{q^{i\gamma}}=\beta\left(\gamma-1\right)\geq 0
\end{aligned}$$

For the linear cost case, $\gamma = 1$, the two opposing effects exactly cancel. For any given debt level firm $i's$ response to any output of its rival will be the same fixed quantity, and likewise for firm j. As we have seen, in equilibrium then $(q^i, q^j) = (q^c, q^c)$ and $V^i = V^j = V^c$. When costs are strictly convex in output, $\gamma > 1$, the second effect dominates. Firm $i's$ response to a movement in the rival's quantity will go in the same direction as the rival firm's movement. In equilibrium this will lead to $(q^i, q^j) = (q^*, q^*) < (q^c, q^c)$ and $V^i = V^j > V^c$

For demand uncertainty one gets similar results. Let us start by analysing intercept uncertainty. Then a will be a function of θ^i and one will have

$$\pi^{i}\left(\theta^{i},q^{i},q^{j}\right)=\left[a\left(\theta^{i}\right)-bq^{i}-\beta q^{j}\right]q^{i}-cq^{i\gamma}$$

which satisfies A 1 when $a'\left(\theta^{i}\right) > 0$. For this function one finds

$$\begin{aligned}
\pi_{ij}^{i}\left(\widehat{\theta},q^{i},q^{j}\right)-\pi_{i\theta}^{i}\left(\widehat{\theta},q^{i},q^{j}\right)\frac{\pi_{j}^{i}\left(\widehat{\theta},q^{i},q^{j}\right)}{\pi_{\theta}^{i}\left(\widehat{\theta},q^{i},q^{j}\right)} &= -\beta-a'\left(\theta^{i}\right)\frac{\left(-\beta q^{i}\right)}{a'\left(\theta^{i}\right)q^{i}} \\
&= 0
\end{aligned}$$

so that again the two opposing effects exactly cancel. The net effect of a rival's move in quantities on the marginal benefit of a change in quantity is zero, so that when the rival's quantity changes this has no effect on manager $i's$ choice of quantity. Also, when firm i changes its debt level to move its reaction function, this will have no effect on the quantity chosen by the rival firm, so that, in equilibrium, debt levels will be chosen such that $(q^i, q^j) = (q^c, q^c)$ and $V^i = V^j = V^c$.

It remains to analyse slope uncertainty. One can think of θ^i entering b, the slope of firm $i's$ residual demand curve, or β, the degree of substitutability between the products. If $b = \beta$ one can analyse a mix of these two types of uncertainty. It turns out that the result is the same for all these cases and I present the analysis for the last of these possibilities only. In this case one has

$$\pi^i\left(\theta^i, q^i, q^j\right) = \left[a - b\left(\theta^i\right)\left(q^i + q^j\right)\right] q^i - cq^{i\gamma}$$

where $b'\left(\theta^i\right) < 0$ to fit assumption A 1. One easily finds

$$\begin{aligned}
& \pi^i_{ij}\left(\widehat{\theta}, q^i, q^j\right) - \pi^i_{i\theta}\left(\widehat{\theta}, q^i, q^j\right) \frac{\pi^i_j\left(\widehat{\theta}, q^i, q^j\right)}{\pi^i_\theta\left(\widehat{\theta}, q^i, q^j\right)} \\
= & -b\left(\theta^i\right) - \left(-b'\left(\theta^i\right)\left[2q^i + q^j\right]\right) \frac{-b\left(\theta^i\right) q^i}{-b'\left(\theta^i\right)\left[q^i + q^j\right] q^i} \\
= & -b\left(\theta^i\right) + b\left(\theta^i\right) \frac{2q^i + q^j}{q^i + q^j} > 0
\end{aligned}$$

For slope uncertainty a change in the rival's quantity has a positive net effect on marginal profit. As in the case of cost uncertainty with convex costs this will result in equilibrium quantities that are less than Cournot, $\left(q^i, q^j\right) = \left(q^*, q^*\right) < \left(q^c, q^c\right)$ and $V^i = V^j > V^c$.

2.6 Endogenous Control

So far it was assumed that owners of the firms have to rely on a manager to choose the firm's quantity and cannot choose quantities themselves. One traditional way of justifying such an assumption would be that ownership is dispersed and that free-rider problems lead to the need to employ an outsider to make business decisions on behalf of the shareholders. One could also assume that managers have special skills and expertise for making business decisions and that a manager has to be employed for this reason. Both these explanations are outside the realm of the model that is analysed here. In this section I want to drop the assumption that shareholders have to employ a manager. Instead I allow the owner a choice, as to whether he wants to employ a manager, or make the quantity decision himself. These decisions will again be taken in a non-cooperative fashion. They are modelled as a first stage which precedes the financing and quantity setting stages. At this first stage, owners simultaneously decide on whether they want to employ a manager to make the quantity decision for them, or whether they want to choose quantities themselves. After this first stage, as before, the owner can choose a debt level. Finally, quantities will then be chosen by the manager or the shareholder, depending on which decision was taken at the first stage of the game.

In a subgame perfect equilibrium, the later play of the game can be collapsed into the values associated with the equilibrium payoffs, resulting from the debt and quantity stages, for any pair of first-stage decisions. One therefore needs to analyse the following game

$i\backslash j$	m	s
m	$V^i(m,m), V^j(m,m)$	$V^i(m,s), V^j(m,s)$
s	$V^i(s,m), V^i(s,m)$	$V^i(s,s), V^j(s,s)$

where m denotes sending a manager and s means that quantities will be chosen by the owner (a shareholder) himself.

In order to characterise the equilibrium of this game one needs to make a further assumption. Given the results in the last section for the main part of this section I want to assume

$$\pi^i_{ij}\left(\widehat{\theta}, q^i, q^j\right) - \pi^i_{i\theta}\left(\widehat{\theta}, q^i, q^j\right) \frac{\pi^i_j\left(\widehat{\theta}, q^i, q^j\right)}{\pi^i_\theta\left(\widehat{\theta}, q^i, q^j\right)} \geq 0 \qquad \text{(A 2)}$$

Need explain better.

This ensures that, as in the examples given in the last section, the manager's reaction function is (weakly) upward sloping. One then has the following result.

Proposition 3 *When assumptions A1 and A2 hold, so that*

$$-\frac{B^i_{ij}}{B^i_{ii}} \geq 0$$

m is a dominant strategy and (m,m) is the unique Nash equilibrium of the game.

To see the intuition for this result, recall that under $A2$ profits are higher than Cournot if both firms send a manager, and lower than Cournot if both firms send a shareholder. It turns out that, if a manager-controlled firm is paired with a shareholder-controlled firm, it is the manager-controlled firm which will become more aggressive and will push its reaction function out and the shareholder-controlled firm which loses out against a more aggressive rival. In fact, one can show that in this case under $A2$ the manager-controlled firm becomes a Stackelberg leader and therefore has higher profits than Cournot, whereas the shareholder-controlled firm becomes a Stackelberg follower and will end up with lower profits than Cournot. Thus, when the other firm is sending a shareholder, the best response is to send a manager and become a Stackelberg leader, in order to enjoy higher than Cournot profits. When the other firm is sending the manager, the best response is again to send a manager, in order not to become a Stackelberg follower, but again to enjoy higher than Cournot profits. Given the choice between sending a manager and choosing quantities themselves shareholders will therefore want to send the manager, whatever choice is made by the rival firm. In equilibrium both firm owners will therefore employ managers. This will ensure a more collusive outcome in equilibrium than if they made the quantity choice themselves. Intuitively, a more collusive outcome is made possible here, since a manager-controlled firm is soft, when paired with another manager-controlled firm, but highly aggressive when paired with a shareholder-controlled firm. This allows the manager-controlled firm to credibly threaten to punish a deviation to shareholder-control. As a result both firms will use an agent and thus sustain a more collusive outcome in equilibrium. A graphical representation of the equilibria of the various subgames is provided in Figure 2.1.

For the proof, note first that for any pair of decisions made at the first stage, (a^i, a^j), $a^i \epsilon \{m, s\}$, $a^j \epsilon \{m, s\}$ the equilibrium of the financing stage can be characterized by

$$\max_{D^i} V^i\left(q^i, q^j\right)$$

$$\begin{aligned} s.t.\quad q^i &= q^i\left(q^j, D^i; a^i\right) \\ q^j &= q^j\left(q_i, D_j; a^j\right) \\ D^i &\geq 0 \end{aligned}$$

holding for both firms. At the financing stage, each firm chooses its own reaction function taking the rival's debt level and thus the rival's reaction function as given. Given the other firm's reaction function and the firm's choice of its own debt level, a pair of quantities $\left(q^i, q^j\right)$ results and determines the expected value of the firm.

Notice also that there is an alternative and more intuitive way of characterising the equilibrium. Whenever $D^i \geq 0$ is not binding, equilibrium quantities are solutions to

$$\max_{q^i} V^i\left(q^i, q^j\right)$$

$$s.t.\quad q^j = q^j\left(q^i, D^j; a^j\right)$$

for firms i and j. In equilibrium, each firm's quantity is value-maximising given the rival's reaction function. To see that this must hold, let the solution to this problem be q^{i*}. Recall also that firm i's quantity is continuously increasing in its debt level. It is then immediate that if firm $i's$ choice of debt level were to result in a quantity other than q^{i*} given firm $j's$ reaction function, it would have an incentive to change its debt level in order to move its quantity closer to q^{i*}. This means that one can characterise the equilibrium by a tangency condition of the firm's isovalue curve with the other firm's reaction function. If the rival's reaction function slopes downwards, the tangency will occur at the downward sloping part of the isovalue curve, so that Cournot quantities can no longer be an equilibrium. If the rival's reaction function slopes upwards, then Cournot quantities again are no longer an equilibrium, since the tangency must occur at the upward sloping branch of an isovalue curve. This implies that, as we have seen already, for $(a^i, a^j) = (m, m)$ equilibrium quantities will be less than Cournot, and profits will be higher than Cournot and that for $(a^i, a^j) = (s, s)$ equilibrium quantities are higher than Cournot and profits will be lower than Cournot[6].

Let us now go on to characterise the equilibrium in the subgame following $(a^i, a^j) = (m, s)$. The claim is that this equilibrium is characterised by the Stackelberg quantities. To show this, look at the financing stage and assume that the shareholder controlled firm chooses a debt level such that its reaction function goes through the Cournot-level of output. This involves setting $D^j = 0$ in $q^j(q^i, D^j; s)$ so that $q^j(q^c, D^j; s) = q^c$. Given this reaction function of firm j firm i will choose its reaction function to

$$\max_{D^i} V^i\left(q^i, q^j\right)$$

$$\begin{aligned} s.t. \quad q^i &= q^i\left(q^j, D^i; m\right) \\ q^j &= q^j\left(q^i, 0; s\right) \\ D^i &\geq 0 \end{aligned}$$

[6]The observation that the equilibrium is characterised by a tangency condition between the firm's isovalue curve and the rival firm's reaction function also provides a necessary and sufficient condition for the existence of a symmetric equilibrium. This feature is shared by models which have a commitment stage which precedes the quantity setting stage. In all of Brander and Lewis (1986) Sklivas (1987), Fershtman and Judd (1987), Maksimovic (1990), as well as Showalter (1995) existence is predicated on such a condition, even though this is rarely discussed. For the case of Brander and Lewis (1986), as well as for the manager-control case here, the condition is that, along the line $\left(q^i, q^j\right) = (q, q)$, there is a point such that

$$-\frac{V_i^i\left(q^i, q^j\right)}{V_j^i\left(q^i, q^j\right)} = \frac{dq^j\left(q^i, D^j\left(q^j, q^i\right); a\right)}{dq^i}$$

where $D^j\left(q^j, q^i\right)$ is the inverse of $q^j\left(q^i, D^j; a\right)$ with respect to D^j.

Replacing reaction functions by the first-order conditions and linearising one has

$$
\begin{aligned}
B^i_{ii}dq^i + B^i_{ij}dq^j + B^i_{iD^i}dD^i &= 0 \\
S^j_{ji}dq^i + S^j_{jj}dq^j + S^j_{jD^i}dD^i &= 0
\end{aligned}
$$

from which one finds

$$
\begin{aligned}
\frac{dq^i}{dD^i} &= -\frac{B^i_{iD^i}S^j_{jj}}{B^i_{ii}S^j_{jj} - B^i_{ij}S^j_{ji}} > 0 \\
\frac{dq^j}{dD^i} &= \frac{B^i_{iD^i}S^j_{ji}}{B^i_{ii}B^j_{jj} - B^i_{ij}B^j_{ji}} < 0
\end{aligned}
$$

since $S^j_{ji} < 0$. Differentiating the value of firm i with respect to D_i one has

$$
\begin{aligned}
V_{D^i} &= \left[\int_{\underline{\theta}}^{\overline{\theta}} \pi^i_i\left(\theta^i, q^i\left(D^i, 0\right), q^j\left(D^i, 0\right)\right) f\left(\theta^i\right) d\theta^i\right] \frac{dq^i}{dD^i} \\
&\quad + \left[\int_{\underline{\theta}}^{\overline{\theta}} \pi^i_j\left(\theta^i, q^i\left(D^i, 0\right), q^j\left(D^i, 0\right)\right) f\left(\theta^i\right) d\theta^i\right] \frac{dq^j}{dD^i} \\
&= 0
\end{aligned}
$$

Start with a debt level D^i, such that firm $i's$ reaction function goes through $\left(q^c, q^c\right)$. Given this reaction function the first term is zero and the second term is positive, since

$$\frac{dq^j}{dD^i} < 0$$

and $\pi^i_j\left(\theta^i, q^i, q^j\right) < 0$. Firm i's best response to firm $j's$ reaction curve will therefore involve a larger than the hypothesised debt level. Therefore, starting from the Cournot reaction function, firm i will have an incentive to move its reaction function out.

Next one needs to check that firm j's choice of reaction function, $D^j = 0$, is a best response to firm $i's$ reaction function. Linearising the system of first-order conditions and differentiating with respect to firm $j's$ debt level one has

$$
\begin{aligned}
S^j_{jj}dq^j + S^j_{ij}dq^i + S^j_{jD^j}dD^j &= 0 \\
B^i_{ij}dq^j + B^i_{ii}dq^i + B^i_{iD^j}dD^j &= 0
\end{aligned}
$$

from which one finds

$$\begin{aligned}
\frac{dq^j}{dD^j} &= -\frac{S^j_{jD^j}B^i_{ii}}{S^j_{jj}B^i_{ii} - S^j_{ji}B^i_{ij}} > 0 \\
\frac{dq^i}{dD^j} &= \frac{S^j_{jD^j}B^i_{ij}}{S^j_{jj}B^i_{ii} - S^j_{ji}B^i_{ij}} \geq 0
\end{aligned}$$

since $B^i_{ij} \geq 0$ under A2. Differentiating the value of firm j with respect to its debt level one finds

$$\begin{aligned}
V_{D^j} &= \left[\int_{\underline{\theta}}^{\overline{\theta}} \pi^j_j \left(\theta^j, q^j\left(D^i, D^j\right), q_i\left(D^i, D^j\right)\right) f\left(\theta^j\right) d\theta^j\right] \frac{dq^j}{dD^j} \\
&\quad + \left[\int_{\underline{\theta}}^{\overline{\theta}} \pi^j_i \left(\theta^j, q^j\left(D^i, D^j\right), q^i\left(D^i, D^j\right)\right) f\left(\theta^j\right) d\theta^j\right] \frac{dq^i}{dD^j} \\
&= 0
\end{aligned}$$

If firm j has chosen its reaction curve to go through (q^c, q^c), and firm i has chosen any reaction curve, it must be that $q^i\left(D^i, D^j\right), q^j\left(D^i, D^j\right)$ is a point on firm $j's$ reaction curve. By definition for any such point the first term in brackets is zero. One therefore has

$$V^j_{D^j} \leq 0$$

since $\pi^j_i\left(\theta^j, q^j, q^i\right) < 0$ and $\frac{dq^i}{dD^j} \geq 0$. Because firm $i's$ reaction function is upward sloping, an increase in the debt level of firm j would decrease, rather than increase, firm $j's$ profits. Firm j therefore has no incentive to move its reaction function out. Setting $D^j = 0$ is indeed a best response of firm j to firm $i's$ reaction curve.

It remains to characterise the resulting equilibrium quantities and values. One needs to show that for firm i one finds $q^i > q^c$ and $V^i > V^c$, whereas for firm j one has $q^j < q^c$ and $V^j > V^j$.

Start with firm i. Firm i has a positive level of debt, so that the constraint $D^i \geq 0$ is not binding. Equilibrium quantities can therefore be characterised by

$$\max_{q^i} V^i\left(q^i, q^j\right)$$

$$s.t. \quad q^j = q^j\left(q^i, 0; s\right)$$

which is the programme for a Stackelberg leader. Substituting one has

$$\max_{q^i} V^i\left(q^i, q^j\left(q^i, 0; s\right)\right)$$

This problem has first-order condition

$$\begin{aligned} V_{q^i}\left(q^i, q^j\left(q^i, 0; s\right)\right) &= \left[\int_{\underline{\theta}}^{\overline{\theta}} \pi_i^i\left(\theta^i, q^i, q^j\right) f\left(\theta^i\right) d\theta^i\right] \\ &\quad + \left[\int_{\underline{\theta}}^{\overline{\theta}} \pi_j^i\left(\theta^i, q^i, q^j\right) f\left(\theta^i\right) d\theta^i\right] \frac{dq^j\left(q^i, 0; s\right)}{dq^i} \\ &= 0 \end{aligned}$$

which implies the well-known tangency condition. Looking at the derivative it is easy to see that when evaluated at $q^i = q^c$ one has $V_{q^i}\left(q^i, q^j\left(q^i, 0; s\right)\right) > 0$, since then the first term in brackets is zero and the second term is positive since $\pi_j^i\left(\theta^i, q^i, q^j\right) < 0$ and $\frac{dq^j\left(q^i,0;s\right)}{dq^i} < 0$. One therefore has $V_{q_i}\left(q^c, q^j\left(q^c, 0; s\right)\right) > 0$, which implies $q^i > q^c$ and $V^i > V^c$.

Moving on to firm j, recall that its quantity q^j is the solution to $q^j = q^j\left(q^i, 0; s\right)$, which is a downward sloping function. Taking this together with $q^j\left(q^c, 0; s\right) = q^c$ and $q^i > q^c$, one concludes that $q^j < q^c$. Also, since $q^i > q^c$ one has

$$\max_{q^j} V^j\left(q^j, q^i\right) < \max_{q^j} V^j\left(q^j, q^c\right)$$

which implies $V^j < V^c$.

This shows that $V^i\left(m, s\right) > V^c > V^j\left(m, s\right)$. To prove that m is a dominant strategy, it remains to invoke symmetry to get $V^j\left(m, s\right) = V^i\left(s, m\right)$, so that $V^i\left(m, s\right) > V^c > V^i\left(s, m\right)$. Taking this together with $V^i\left(m, m\right) \geq V^c$ and $V^i\left(s, s\right) < V^c$, one arrives at $V^i\left(m, m\right) > V^i\left(s, m\right)$, and $V^i\left(m, s\right) > V^i\left(s, s\right)$, q.e.d.

Intuitively, since a shareholder-controlled firm has downward sloping reaction functions, starting from the pair of reaction functions going through $\left(q^c, q^c\right)$, it pays the firm who has sent a manager for the quantity choice to increase its debt level, since this will lead the shareholder-controlled firm to decrease its quantity. On the other hand, it does not pay the firm who has sent a shareholder to increase its debt level, since this would lead to an increase rather than a decrease in the rival's quantity given that the rival is manager-controlled and has upward sloping reaction functions. Therefore only the manager-controlled firm will move its debt level, and it will move it up to the point where its reaction function cuts the reaction function of the shareholder-controlled rival in the Stackelberg point, which is value-maximising for the manager-controlled firm.

To complete the analysis, let us also briefly look at the case where $A2$ does not hold and reaction functions are downward sloping both under manager control and under shareholder control. In this case one may still find that delegation to a manager occurs in a dominant strategy equilibrium. As an intuitive extension to the case where the manager's reaction functions are upward sloping, when they are downward sloping, delegation can be shown to be dominant whenever under manager control reaction functions slope downwards less steeply than under shareholder control. More formally one has

Proposition 4 *Under assumption A 1, when*

$$-\frac{B^i_{ij}}{B^i_{ii}} < 0$$

m is a dominant strategy and (m, m) is the unique Nash equilibrium of the game, whenever along $(q^i, q^j) = (q, q) \geq (q^c, q^c)$ one has

$$-\frac{B^i_{ij}}{B^i_{ii}} > -\frac{S^i_{ij}}{S^i_{ii}}$$

To see the intuition behind this result, consider the condition on the relative slopes. Notice that it implies that, for any given increase in the rival's quantity, under manager control the firm will reduce its quantity by less than it would under shareholder control. When faced with a manager controlled firm the rival firm will therefore have less of an incentive to compete aggressively than when faced with a shareholder controlled firm. As a consequence, a manager-controlled firm will be better off than a shareholder controlled firm. Intuitively, since under manager control the firm's response to a rival's increase in quantity is "less elastic", there is less of strategic substitutability, and it pays the rival firm less to increase its quantity, either directly, or via an increase in its debt level. Note that in this case the equilibrium is less collusive than Cournot, but more collusive than it would be under shareholder control. For a proof see Appendix 2.3 at the end of this chapter.

2.7 Discussion

2.7.1 The nature of competition

One of the important underlying results of our analysis is that the quantity variables, which are strategic substitutes under shareholder control may under natural assumptions turn into strategic complements, when viewed from the manager's point of view. Under shareholder control, if the rival firm decreases its quantity this has a positive impact on the firm's marginal profit, so that shareholders will respond by increasing their output. The decision variables are therefore strategic substitutes in the terminology of Bulow, Geanakoplos and Klemperer (1985). Under manager control the effect on marginal profit may be dominated by the effect on total firm profit. If the rival firm decreases its output, this will raise total profit for all realisations of the state of the world. This will lower the probability of bankruptcy and allow the manager to compete less aggressively and to reduce the quantity produced. Thus, quantity variables may become strategic complements. The observation that agency problems can turn decision variables that are strategic substitutes under profit maximisation into strategic complements has recently also been made by Aghion, Dewatripont and Rey (1997). In their model of R&D competition, R&D effort decisions of two firms are strategic substitutes under profit maximisation. If one firm

OK. Nice. But so what? - Does this affect efficiency? - wealth of shareholders. Shareholders should be able to ... decision.

increases its research effort, this will make it more likely that both firms find the innovation, in which case the gain from the innovation will be competed away. Since this will reduce the marginal payoff to research effort, an increase in research effort of one firm will lead the other firm to respond by reducing effort. If, however, running the firm requires a large initial investment which is financed by an outside investor, the effort response may go the other way. The rival firm's increase in research effort will lower total expected profit. The agent running the firm may then have to commit contractually to a higher effort level, in order to increase total expected profit and to ensure that the outside investor still breaks even. Both here and in our model the reversal in the nature of competition stems from the impact the rival's decision has on total, rather than marginal profit. In Aghion, Dewatripont, and Rey (1997) total expected profit matters since the outside investor will want to be paid back his investment in expected terms. In our case total profit matters, due to the threshold in the manager's preferences that is drawn in by the bankruptcy level. In both cases the effect on total profit leads to a reversal of the strategic quality of the decision variables and turns strategic substitutes into strategic complements. Note that these results are possibly more general than they might seem at first glance. In our case, all one needs for the reversal to occur is that the pay-off to a variation in the decision variables varies as in A1 and A2. While quantity competition with linear demand and weakly convex costs is an example which fits these assumptions on the profit function, these assumptions may be taken as a reduced form description for a variety of other underlying games. For example, one could reinterpret the decision variable q as investment into plant and equipment, or indeed any other activity which exhibits strategic substitutability and model a subsequent stage of competition in prices or quantity. Whenever the payoff structure of such a game maps into the reduced form assumption made, the analysis will apply.

2.7.2 The value of delegation

The results also point toward the value of delegation in certain noncooperative environments. Here, in equilibrium, firm owners delegate strategic decisions to an agent whose objectives differ from their own. This alleviates the prisoners' dilemma quality of quantity competition and helps to sustain a more collusive equilibrium outcome. The idea that employing an agent with preferences different from the principal's can be valuable ex ante has been investigated in other contexts. In Schils (1996) delegated bargaining helps to alleviate a hold-up problem that arises when a firm undertakes a relationship with an outside research unit. When the price for an innovation can not be stipulated ex ante, there is an incentive for the firm owner to drive a tough bargain ex post and to extract as much of the surplus from the innovation as possible. Anticipating this, the research unit has less of an incentive to invest in innovation generating activity, so that research effort will be inefficiently low. When the firm owner employs a manager whose preferences differ from his own, this inefficiency is reduced. Similarly, in Dessi (1997) the firm-owner has an incentive to breach implicit (nonenforceable) agreements with the workforce to reward high effort whenever the short term gain of doing so exceeds the long term loss of reputation. Employing a manager who is incentivised by issuing short- and long term debt, this problem is reduced, because the marginal gain to the manager of breaching the implicit contract may be zero in situations in which the manager has enough cash to repay the short term debt. Related ideas can also be found in the literature on macroeconomic policy games, where it is suggested that pareto-superior outcomes can be sustained by delegating monetary policy to a conservative and independent central banker, cf. Rogoff (1985) and Walsh (1995). In all of these models it is valuable ex ante to employ an agent whose objectives will ex post be different from the principal's. The contribution of our results is to extend this idea to a symmetric setting with two competing vertical structures. In all of the cited papers there is a single vertical structure, with sequential moves along the structure. Here there are two rival structures that compete with each other in an output market. Delegation is shown to arise in an equilibrium of a simultaneous move game. Both firms would like to delegate play to a manager, since this is valuable ex ante in ensuring softer competition and a more collusive outcome. This can be sustained in equilibrium here, because in an off-equilibrium situation in which one of the firms did not employ a manager, it is the manager-controlled firm who will be aggressive and the shareholder-controlled firm who will lose out. Since deviations away from delegated play will be punished by more aggressive behaviour, delegation becomes sustainable as an equilibrium of a noncooperative simultaneous move game.

2.7.3 Contractual commitment and renegotiation

We have seen that with manager control, ex post the principal would choose a different quantity than the agent chooses. This feature is shared with most of the literature on contractual commitment in oligopoly. For example, in Brander and Lewis (1986) the investor, as a debtholder, would choose a different quantity than the shareholder. Likewise, in Sklivas (1987), ex post the owner would choose a different quantity than the manager who was incentivised to focus on sales. In each case contractual commitment prevents the principal from letting his preferences govern the quantity choice. The main difference here is that the ability to commit through contractual arrangements is actually valuable ex ante, in that it permits more collusive equilibrium outcomes, rather than less collusive outcomes.

One may still ask whether contracts are a good commitment device in our setting. Clearly, the shareholder would, after the manager is sent and the debt levels are chosen, seem to have an incentive to oust his manager and make the quantity choice himself. It is easy to see, however, that when the manager is ousted a conflict of interest will arise between debtholder and shareholder. The shareholder will want to increase the quantity, making the debt more risky. If before the firm had all the bargaining power vis a vis debtholders, then under manager control the debtholders would have broken even. Once the manager is ousted, debtholders will have a negative expected payoff. Anticipating the possibility that the shareholders will have an incentive to take over control from the manager, it is natural to assume that the original debt contract will have offered protection against this. Thus the debt contract will have contained a covenant that made it a condition that the manager would make the quantity decision. It may, of course be possible to renegotiate this debt contract. In a symmetric situation, however, this possibility should be open to both firms. Let us therefore consider an augmented game in which it is possible for both firms to oust their manager after the debt selection stage and then renegotiate the debt contract by making a take-it-or-leave-it offer to the debtholders. It is clear that in the equilibrium of this augmented game, none of the firms would want to oust their manager, since, just as before, this would be dominated, given the later play of the game. Thus, even though each principal would choose a different quantity than the agent chooses, given the choices of the other firm, the equilibrium obtained above clearly is renegotiation-proof when renegotiation is open to both firms and is modelled as a simultaneous move game.

2.7.4 Managerial entrenchment

In this model shareholders use capital structure to incentivise their manager and guide his quantity choice. If one thinks of the manager as having control over the company after the capital structure has been set, one might wonder whether the manager may not be able to change the capital structure and reduce the debt level in order to reduce the probability of bankruptcy. While he obviously has an incentive to reduce the debt level, it is easy to see that unless he uses his own personal wealth he will be unable to do so. This is because the capital structure that is in place is value maximising, given that a manager has been employed and given the reaction function of the rival firm. If the manager does not have any personal wealth, then, in order to buy back debt, the manager will have to raise the necessary funds by issuing equity. Since such a restructuring will change the manager's subsequent quantity choice this must diminish the value of the firm. It will therefore be impossible for the manager to raise sufficient funds for the purpose of buying back debt.

2.7.5 Wage contracts

So far I have thought of the manager as an agent who derives a private benefit from not going bankrupt, and who would not depart from the implied behaviour when offered a monetary incentive scheme. In the literature, by contrast, managers are often modelled as risk-neutral and highly susceptible to monetary incentives. One may ask therefore, whether our findings are robust to a switch to such an assumption. To examine this, consider a modified game in which as a first stage a managerial compensation scheme is chosen by the owner of each firm, after which, in a second stage, managers choose quantities. Let us restrict attention to contracts that condition on the firm's own profits, that is, let us assume that quantities, as well as rival profit are unobservable to the owner. I also want to restrict wage contracts to be either a profit share, an option contract with a weakly positive exercise profit, or a flat wage contract which conditions on some weakly positive cut-off profit level, i.e. a bonus contract.

$$w\left(\pi^{i}\right) \epsilon \left\{\alpha\pi^{i},\ \alpha \max\left[\pi^{i}-\overline{\pi},0\right],\ \overline{w}I\left(\pi^{i},\overline{\pi}\right)\right\}$$

where $\alpha \geq 0$, where $\overline{\pi} \geq 0$, and $I\left(\pi^{i},\overline{\pi}\right)$ is an indicator function with $I\left(\pi^{i},\overline{\pi}\right) = 1$ if $\pi^{i} \geq \overline{\pi}$ and $I\left(\pi^{i},\overline{\pi}\right) = 0$ otherwise. Note that if the reservation utility of the manager is not zero, one can always amend these schemes by paying the manager some fixed base wage, which can be adjusted to give the expected wage the manager requires. When $(A1)$ and $(A2)$ hold with respect to the earlier game and contracts are chosen simultaneously, it follows directly from our earlier analysis that in equilibrium owners will choose a bonus scheme. To see why, note that a bonus scheme is the only contract that will lead the manager's reaction function to slope upwards. Which cut-off is chosen will again be determined by the condition of tangency of the isovalue line of the owner and the reaction function chosen by the rival. When this condition is met, none of the owners has an incentive to switch to a different cut-off, or indeed to any other contract in the feasible set. This result suggests that low-powered incentive schemes, which are not as sensitive to the principal's pay-off, as they could, may be optimal when the manager's task is primarily to make strategic decisions. Note also that a bonus scheme is outside the contract domain considered in Fershtman and Judd (1987) and Sklivas (1987), which casts some doubt on the robustness of their results.

2.8 Conclusion

This chapter has reconsidered the strategic effect of debt under the assumption that quantity choices are made by managers whose objective is to avoid bankruptcy. The basic result is that quantity choices, which are strategic substitutes under profit maximisation, may turn into strategic complements under reasonable assumptions on the profit function. Then, in contrast with the benchmark case of owner control over quantity choices, starting from the Cournot level, shareholders will want to shift the manager's reaction function back, rather than out. As a result, equilibrium quantities will be less than the Cournot quantities. The prisoners' dilemma inherent in quantity competition is softened. By employing a manager, shareholders not only avoid a limited liability-effect of debt, but are able to support a more collusive equilibrium than in the simple model without a financing stage. We have seen that this result is robust when the decision to delegate is endogenised. Thus, delegation to a manager is not only valuable, but also supportable in equilibrium. The intuition is that, when one firm does not delegate its quantity choice, it will lose out against a rival who has delegated the quantity choice, but can credibly threaten to use a very

other financing contract?

aggressive debt policy when faced with a shareholder-controlled firm. In contrast with Brander and Lewis (1986) and in line with the empirical evidence, in the symmetric equilibrium of our model, in which both firms delegate the strategic decision, positive leverage is associated with softer competition and larger profits than in the standard oligopoly model without a financing stage. The model also implies that, given a contract domain including shares, options and bonus schemes, in equilibrium owners would choose simple bonus schemes for their managers, giving a theoretical justification for the kind of managerial preferences assumed.

Appendix 2.1

In this appendix I want to prove that, as claimed in section 2.4, the equilibrium under manager control is more collusive than the equilibrium under owner control whenever along $(q^i, q^j) = (q, q) \geq (q^c, q^c)$ one has

$$-\frac{B^i_{ij}}{B^i_{ii}} > -\frac{S^i_{ij}}{S^i_{ii}}$$

One can make use of the fact that, as discussed in section 2.5, both under manager control, and under shareholder control, equilibrium quantities are characterised by

$$\max_{q^i} V^i\left(q^i, q^j\right)$$

$$s.t. \quad q^j = q^j\left(q^i, D^j; a^j\right)$$

holding for both firms. Here $a = m$ for the case of manager control and $a = s$ for the case of owner-control. Substituting the constraint one has

$$\max_{q^i} V^i\left(q^i, q^j\left(q^i, D^j; a^j\right)\right)$$

from which one finds the first-order condition

$$\frac{dV^i}{dq^i} = V^i_i + V^i_j \frac{dq^j\left(q^i, D^j; a^j\right)}{dq^i} = 0$$

which can be rearranged to imply the tangency condition

$$-\frac{V^i_i}{V^i_j} = \frac{dq^j\left(q^i, D^j; a^j\right)}{dq^i}$$

Take the equilibrium quantities resulting from owner control and denote them by (q^s, q^s). They will satisfy

$$\begin{aligned} V^i_i + V^i_j \frac{dq^j\left(q^i, D^j; s\right)}{dq^i} &= \\ V^i_i + V^i_j\left(-\frac{S^j_{ji}}{S^j_{jj}}\right) &= 0 \end{aligned}$$

Since

$$-\frac{S^j_{ji}}{S^j_{jj}} < 0$$

one has

$$V^i_i\left(q^s, q^s\right) < 0$$

which implies that $\left(q^s, q^s\right) > \left(q^c, q^c\right)$.

If the same point $\left(q^s, q^s\right)$ were to result in the equilibrium under manager control, one would need

$$\begin{aligned} V^i_i + V^i_j \frac{dq^j\left(q^i, D^j; m\right)}{dq^i} &= \\ V^i_i + V^i_j\left(-\frac{B^j_{ji}}{B^j_{jj}}\right) &= 0 \end{aligned}$$

satisfied when evaluated at $\left(q^s, q^s\right)$.

If however at any point $\left(q^i, q^j\right) = (q, q)$ with $(q, q) \geq \left(q^c, q^c\right)$

$$-\frac{B^j_{ji}}{B^j_{jj}} > -\frac{S^j_{ji}}{S^j_{jj}}$$

then this is true at $\left(q^s, q^s\right)$. This implies

$$V^i_i + V^i_j\left(-\frac{B^j_{ji}}{B^j_{jj}}\right) < 0$$

at $\left(q^s, q^s\right)$ and one needs a reduction in the common quantity to make this hold as an equality, q.e.d.

Appendix 2.2

Here I want to show that, as claimed in section 2.4, under manager control equilibrium quantities are always strictly larger than the joint profit maximising quantities. Recall that, as discussed in section 2.5, equilibrium quantities are characterised by

$$\max_{q^i} V^i\left(q^i, q^j\right)$$

$$s.t. \quad q^j = q^j\left(q^i, D^j; m\right)$$

holding for both firms. Substituting the constraint one has

$$\max_{q^i} V^i\left(q^i, q^j\left(q^i, D^j; m\right)\right)$$

from which one finds the first-order condition

$$\frac{dV^i}{dq^i} = V_i^i + V_j^i \frac{dq^j\left(q^i, D^j; m\right)}{dq^i} = 0$$

which can be rearranged to imply the tangency condition

$$-\frac{V_i^i}{V_j^i} = \frac{dq^j\left(q^i, D^j; m\right)}{dq^i}$$

Since along $(q^i, q^j) = (q, q)$

$$-\frac{V_i^i}{V_j^i} < 0 \; if \; (q, q) > (q^c, q^c)$$

$$-\frac{V_i^i}{V_j^i} = 0 \; if \; (q, q) = (q^c, q^c)$$

$$-\frac{V_i^i}{V_j^i} > 0 \; if \; (q, q) < (q^c, q^c)$$

the tangency will occur at some $(q, q) < (q^c, q^c)$ only if reaction functions are upward sloping,

$$\frac{dq^j\left(q^i, D^j; m\right)}{dq^i} = -\frac{B_{ji}^j}{B_{jj}^j} > 0$$

Recall also that the intersection of the reaction functions is required to be stable, that is

$$B_{ii}^i B_{jj}^j - B_{ij}^i B_{ji}^j > 0$$

This is always satisfied for the case of vertical reaction curves with $B_{ij}^i = B_{ji}^j = 0$. If reaction curves are upward sloping, $B_{ij}^i = B_{ji}^j > 0$, this implies

$$-\frac{B_{ii}^i}{B_{ij}^i} > -\frac{B_{ji}^j}{B_{jj}^j}$$

which says that in $(q^i, q^j)-$ space at the intersection of the reaction curves the reaction curve of firm i is steeper than the reaction curve of firm j.

Next, note that the joint value maximizing output $(q^p, q^p)$is given as the solution to

$$\max_{q^i,q^j} V^i + V^j$$

with first-order conditions

$$\begin{aligned} V_i^i + V_i^j &= 0 \\ V_j^i + V_j^j &= 0 \end{aligned}$$

These imply the tangency condition

$$-\frac{V_i^i}{V_j^i} = -\frac{V_i^j}{V_j^j}$$

If the intersection of the reaction functions were to occur at this joint value maximising output one would have

$$-\frac{V_i^i}{V_j^i} = \left|\frac{dq^j}{dq^i}\right|_{q^j(q^i,D^j;m)} = -\frac{B_{ji}^j}{B_{jj}^j} = -\frac{B_{ii}^i}{B_{ij}^i} = \left|\frac{dq^j}{dq^i}\right|_{q^i(q_j,D^i;m)} = -\frac{V_i^j}{V_j^j}$$

The joint value maximising point is characterised by the tangency of the two isovalue functions. For this to be an equilibrium, reaction functions must be tangent to each other. However, since it is required that

$$-\frac{B_{ii}^i}{B_{ij}^i} > -\frac{B_{ji}^j}{B_{jj}^j}$$

this would contradict stability.

Next, consider a point $(q^i, q^j) = (q, q) < (q^p, q^p)$. At such a point one will have

$$\begin{aligned} V_i^i + V_i^j &> 0 \\ V_j^i + V_j^j &> 0 \end{aligned}$$

which implies

$$-\frac{V_i^i}{V_j^i} > -\frac{V_i^j}{V_j^j}$$

which means that the isovalue curve of firm i is steeper than the isovalue curve of firm j in $(q^i, q^j) -$ space. If the intersection of the reaction functions were to occur at such a point one would have

$$-\frac{V_i^i}{V_j^i} = \left|\frac{dq^j}{dq^i}\right|_{q^j(q^i,D^j;m)} = -\frac{B_{ji}^j}{B_{jj}^j} > -\frac{B_{ii}^i}{B_{ij}^i} = \left|\frac{dq^j}{dq^i}\right|_{q^i(q_j,D^i;m)} = -\frac{V_i^j}{V_j^j}$$

so that the reaction function of firm j would need to be steeper than the reaction function of firm i. This would again contradict

$$-\frac{B_{ii}^i}{B_{ij}^i} > -\frac{B_{ji}^j}{B_{jj}^j}$$

which is required for reaction function stability, q.e.d.

Appendix 2.3

Proof of Proposition 4:

According to Proposition 4, when

$$-\frac{B_{ij}^i}{B_{ii}^i} < 0$$

m is a dominant strategy and (m, m) is the unique Nash equilibrium of the game, whenever along $(q^i, q^j) = (q, q) \geq (q^c, q^c)$ one has

$$-\frac{B_{ij}^i}{B_{ii}^i} > -\frac{S_{ij}^i}{S_{ii}^i}$$

To prove $V^i(s, m) < V^i(m, m)$ consider the equilibrium under (m, m). This is characterised by

$$V_i^i + V_j^i\left(-\frac{B_{ji}^j}{B_{jj}^j}\right) = 0$$

and

$$V_j^j + V_i^j\left(-\frac{B_{ij}^i}{B_{ii}^i}\right) = 0$$

holding at the equilibrium quantities (q^m, q^m). Now consider firm i deviating to shareholder control. Since

$$-\frac{B_{ij}^i}{B_{ii}^i} > -\frac{S_{ij}^i}{S_{ii}^i}$$

at the old equilibrium point (q^m, q^m) one will have

$$V_i^i + V_j^i \left(-\frac{B_{ji}^j}{B_{jj}^j}\right) = 0$$

and

$$V_j^j + V_i^j \left(-\frac{S_{ij}^i}{S_{ii}^i}\right) > 0$$

This implies that firm i has no incentive to move its reaction function, whereas firm j, which is now facing a shareholder controlled firm, has an incentive to move its reaction function out. Firm j can do this by moving its debt level up. It follows that in the equilibrium under (s, m), firm i will have to be optimising along a reaction function of firm j which specifies a higher output for any quantity firm i chooses. Firm i must be worse off in the new equilibrium. This proves $V^i(s, m) < V^i(m, m)$.

To prove $V^i(m, s) > V^i(s, s)$ start with the equilibrium under (s, s). At the equilibrium quantities (q^s, q^s)

$$V_i^i + V_j^i \left(-\frac{S_{ji}^j}{S_{jj}^j}\right) = 0$$

and

$$V_j^j + V_i^j \left(-\frac{S_{ij}^i}{S_{ii}^i}\right) = 0$$

hold. Consider a deviation of firm i to manager control. Given

$$-\frac{B_{ij}^i}{B_{ii}^i} > -\frac{S_{ij}^i}{S_{ii}^i}$$

at (q^s, q^s) one now has

$$V_i^i + V_j^i \left(-\frac{S_{ji}^j}{S_{jj}^j}\right) = 0$$

and

$$V_j^j + V_i^j \left(-\frac{B_{ij}^i}{B_{ii}^i}\right) < 0$$

which implies that firm i has no incentive to move its reaction function and firm j has an incentive to move its reaction function in. Firm j can do this by reducing its debt level. It follows that in the equilibrium under (m, s), firm i will be optimising along a reaction function of firm j that specifies a lower output for any quantity firm i chooses. Firm i must be better off in the equilibrium under (m, s). This proves $V^i(m, s) < V^i(s, s)$ and completes the proof.

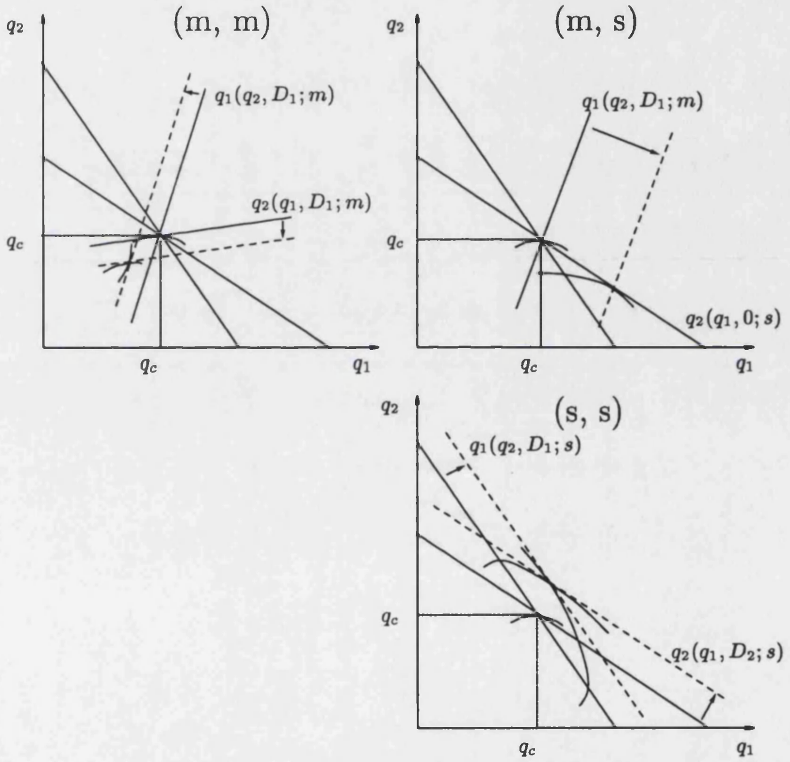

Figure 2.1: Endogenous Control, Subgames

Chapter 3

Equity Finance, Adverse Selection, and Product Market Competition

3.1 Introduction

Whereas the last chapter focused on the use of debt as a commitment device, this chapter analyses the implications of asymmetric information between a firm and its outside investors on the firm's strategic position in its product market. The model abstracts from issues of precommitment and has a financing stage which occurs after the product market stage. This captures the idea that the firm continuously interacts with its product market competitor, but then at some point in time may have to take recourse to the financial market. The financing stage is a version of the Myers and Majluf (1984) model of equity finance, in which the firm's management is assumed to have superior information on the value of the firm's assets in place. This gives rise to a lemons problem, in that there may be equilibria in which only bad firms may issue and invest. In their paper, Myers and Majluf stress the importance of financial slack to mitigate the adverse selection problem. They do not formally analyse the role of financial slack, however. Also, as has been pointed out by Giammarino and Lewis (1988) as well as Cadsby, Frank, and Maksimovic (1990), they do not formally analyse the full set of equilibria of their model. This chapter provides an analysis of both these issues for the particular version of the model. I then argue for an equilibrium selection such that the probability of the good firm investing is increasing in the amount of financial slack available and analyse the implications of this equilibrium play on the first-period competition. The idea is that first-period profit takes the role of financial slack and determines the amount the firm needs to raise externally. Under the assumption that first period profit is stochastic, I show that the firm will no longer maximise the expected value of first-period profit. In addition, it will care about the variance of the profit distribution and will seek to influence it by its output choice[1]. Depending on the severity of the adverse selection problem, this may make the firm a more aggressive or a less aggressive competitor. I identify situations in which the fact that the firm has to finance externally actually confers a strategic benefit on the firm, since it will have an incentive to compete more aggressively in order to increase the probability of investment.

A large part of the literature on adverse selection and product market rivalry has focused on formalising the idea that, if a firm has to finance externally under conditions of asymmetric information, this will make it vulnerable to predatory behaviour by rivals. This issue is explored in Poitevin (1989), Bolton and Scharfstein (1990) and Phillips (1993).

Poitevin (1989) argues that uncertainty about the value of an entrant may be larger than that of the incumbent firm. To signal its quality, the entrant may have to issue debt, whereas the incumbent can finance with equity. Debt financing renders the entrant vulnerable to predation through the possibility of

[1]In a set-up similar to our own, Raposo (1998) analyses the implications of an adverse selection problem on the firm's optimal risk-management. She does not, however, explore the incentives effects for a firm which competes in a strategic environment, which is the focus of the analysis here.

bankruptcy. This can be exploited by the incumbent predator, who may engage in a price war that decreases the entrants cash flow and increases his probability of bankruptcy.

In Bolton and Scharfstein (1990) the underlying agency problem is that first period profit is nonverifiable. To induce the entrant firm to truthfully reveal its profit there is a threat to terminate funding when reported profit is low. This, however, will encourage rivals to ensure that the firm's first period performance is poor. Bolton and Scharfstein model this by assuming that the rival has the option to increase the probability of low firm profit by taking an action that costs the rival some fixed amount. They derive the optimal contract, which, because of the trade-off between deterring predation and mitigating incentive problems, may or may not be designed to deter predation.

In a model set-up close to our own, Phillips (1993) analyses the case of two-period competition between two firms. Firms must make an investment at the end of the first period in order to stay in business in the second period. One of the firms has a deep pocket, whereas the other firm has to finance the investment through debt. There is asymmetric information regarding the firm's second period prospects, which under the assumption of debt financing creates an incentive to invest, even if the investment has negative net present value. To resolve this problem, some portion of the investment has to be financed by internal cash. This again creates incentives for the rival firm to compete more aggressively in the first period to reduce the firm's cash reserves and to force it to forgo the investment. Just as in our set-up, in the Phillips (1993) model internal cash is at the heart of the analysis. In contrast to our analysis, Phillips focuses on the rival's predatory incentives, from which I abstract. Another key difference to the model of this chapter is that the incentive problem which underlies the Phillips analysis comes about only because the firm is restricted to issue debt and would disappear if the firm could issue equity, as is assumed here.

There are two more articles which are related to our analysis. In Rotemberg and Scharfstein (1990) managers maximise a weighted average of expected profits and the stock price. While the authors do not provide a formal argument, they argue that such behaviour may come out of a model in which the firm anticipates to issue equity in the future. They analyse a two period model in which demand and cost conditions are uncertain, but correlated across periods and the stock market tries to infer these from the firm's and its rivals' realised profits. Kovenock and Phillips (1995) invoke the pecking order theory of finance to argue that external finance is more costly than internal finance. In a model in which capacity has to be financed before revenues are earned, they analyse the incentive to reduce financial slack by issuing debt for both price and quantity competition. They find such an incentive to increase financing costs for price competition, but not for quantity competition. Their result can be viewed as complementary to the result obtained by Maksimovic (1990), who has found a strategic value in reducing the variable cost of borrowing, and thus an incentive to decrease financing costs, for the case of quantity competition[2]. In contrast to both Rotemberg and Scharfstein (1990) and Kovenock and Phillips (1995), who argue informally that the objective function they assume may be justified by costs of external funds, I derive the firm's first period objective function from first principles, taking asymmetric information between the manager and the financial investor as a starting point. In some ways this can be viewed as complementary to the approach taken in chapter 2, where the driving force of the model is an assumption on the manager's preferences. The other main difference to the previous chapter is that I consider an asymmetric setting in which only one of the firms, firm i, faces an investment opportunity which needs to be funded externally. This can be justified by saying that firm j either does not happen to have a positive NPV project, or that it has sufficient internal cash to finance it without recourse to the capital market has a deep pocket. Yet another way of justifying this is to say that there is less uncertainty on

[2]See the discussion of this paper in the introduction to chapter 2.

firm j, perhaps because it is better known by financial investors. In terms of modelling strategy, the main implication of the assumption is that one can be sure that firm j is a straight expected profit maximiser. Thus, issues arising from a possible reversal of the nature of competition, which have been the focus of the previous chapter, will not arise here, since firm j has a standard downward sloping reaction function.

3.2 The Model

The model has two periods. At the start of the first period, at $t = 0$, a firm, which is called firm i, competes with another firm, which is labelled firm j, as one of two duopolists in a product market. Each firm has to choose some strategic variable which affects both its own profit and the profit of the rival. As in the previous chapter, this variable can be thought of as the firm's output choice. In addition, profits are affected by a random variable $\widetilde{z}$, which may in general be a vector and has positive support on some set Z. The profit function is identical for both firms. For firm i it is given by $\widetilde{\pi}^i = \pi^i\left(\widetilde{z}^i, q^i, q^j\right)$. The profit function is assumed to satisfy[3]:

$$\forall \ \left(z^i, q^i, q^j\right) \ \ with \ z\epsilon Z, \ q^i \geq 0, q^j \geq 0$$

$$(i)\, \pi^i_{ii}\left(z^i, q^i, q^j\right) < 0, (ii)\, \pi^i_j\left(z^i, q^i, q^j\right) < 0, (iii)\, \pi^i_{ij}\left(z^i, q^i, q^j\right) < 0 \qquad \text{(A1*)}$$

$$\begin{aligned} (iv)\, z' &> z \Rightarrow \pi^i\left(z', q^i, q^j\right) > \pi^i\left(z, q^i, q^j\right) \\ (v)\, z' &> z \Rightarrow \pi^i_i\left(z', q^i, q^j\right) \geq \pi^i_i\left(z, q^i, q^j\right) \end{aligned}$$

The random variables $\widetilde{z}^i$ and $\widetilde{z}^j$ realise, and become publicly known at $t = 1$, after q^i and q^j have been chosen.

[3]This assumption is slightly more general than A 1 of the previous chapter and will enable me to analyse both the specification of Brander and Lewis (1988), as well as an alternative profit function, which combines demand uncertainty with some additional additive uncertainty.

At $t = 1$ a second random variable realises, which affects the value of firm $i's$ assets in place. These assets are thought of as unrelated to the product market in which the firm competes in the first period. The value of the assets in place is best thought of as the liquidation value of the firm. Firm i can be of two types. If the firm is of type H, the value of its assets in place is s_H. If the firm is of type L, its assets in place are worth s_L, where $s_H > s_L$. Ex ante (at $t = 0$) the firm is of the high type H with probability r and it is of the low type L with probability $1-r$. I assume that the firm's type is privately revealed to the firm's owner at $t = 1$. At this stage outside investors only know the ex ante probability r.

At $t = 1$ an investment opportunity opens up to the firm which requires an initial outlay of I and returns an expected payoff of x. Both these values are publicly known. Again, I want to assume that the investment opportunity is unrelated to the product market in which the firm competes in the first period. One can think of it as an investment into some new line of business or as funds required for a diversifying acquisition. It is assumed that this investment is profitable, so that $x > I$. Therefore when $\pi^i \geq I$ the firm will always invest, since in this case it need not (and will not) raise any external funds. When first period profits fall short of the required outlay, the firm can raise $I - \pi^i$ externally. Following the original analysis of Myers and Majluf (1984), I assume that the firm is constrained to issue outside equity[4]. Therefore, for any realisation of first period profit π^i such that $\pi^i < I$ a continuation game ensues. In this game the

[4]It is well known that financing via rights issued to existing shareholders solves the adverse selection problem. To rule out rights issues, one can think of the firm as being owner-managed and that the owner does not have any funds other than π. Allowing the firm to issue risky debt rather than equity would make the financing problem less severe, but would not change the main conclusions of the model, as long as some adverse selection remains and is reflected in the default premium that has to be paid by the firm. Since the focus here is not on solving the financial adverse selection problem, but on the implications of an adverse selection problem for a firm which competes in a product market, I have chosen to consider the market for equity, just as in the original paper by Myers and Majluf (1984).

firm can either raise $I - \pi^i$ from outside investors by selling off a fraction α of the firm's equity and invest, or forgo the investment opportunity. For the main part of the analysis it is assumed that financial investors are able to observe the realised profit and thus the financing need of the firm with certainty. Also, the market for outside equity is assumed to be competitive. Thus, in the spirit of Myers and Majluf (1984), it is assumed that there is an auction for the firm's equity, which ensures that the price for the firm's equity is bid down to the point where financial investors just break even.

At $t = 2$ the investment pays off, the value of the assets in place become publicly known, the firm is liquidated and all claims are settled[5].

As in the previous chapter, everybody in the model is risk neutral.

3.3 Second-Stage Equilibria

I solve the game backwards and start at the beginning of the second period, $t = 1$. For any realisation of first period profit π such that $\pi < I$ the firm may either decide to raise $I - \pi$ from an outside investor by selling off a fraction α of the firm's equity and invest, or forgo the investment opportunity[6]. Denote the firm's decision by $d \epsilon \{1, 0\}$, where $d = 1$ when the firm decides to raise $I - \pi$ and invests and $d = 0$ when the firm does not invest. In order to capture that the firm can make this decision conditional on its type let $\delta_H = \Pr[d = 1 \mid H]$ and $\delta_L = \Pr[d = 1 \mid L]$. Outside investors can expect to break even, given their beliefs about the firm. Let ρ be the probability attached to the possibility that the firm is of the high type.

[5]The timing of the model has been chosen to disentangle the financing stage from the product market stage as much as possible. If one had the firm know its value from the start, for example, the main conclusions of the model would be preserved. One would have an additional issue, however, in that high type and low type firms would behave differently at the product market stage, so that the financial market could try and draw inferences from observed profit.

[6]In both this and the next section I focus on firm i and drop the superscript throughout.

An equilibrium of this game is defined as follows.

why should it be?

Definition 1 *For any realized* π *such that* $\boxed{\pi < I}$ *an equilibrium of the game is a quadruple* $(\delta_H, \delta_L, \rho, \alpha)$ *such that*

1. *Beliefs are updated using Bayes' rule*

$$\rho = \frac{r\delta_H}{r\delta_H + (1-r)\,\delta_L}$$

2. α *is defined by the equation*

$$I - \pi = \alpha\left[\rho s_H + (1-\rho)\, s_L + x\right]$$

ensuring that the financial investors just break even, given equilibrium beliefs.

3. *For any* $k\epsilon\{H, L\}$

Prob type k invests = 1 if ... retained share x firm value.

$$\delta_k = 1 \; if \qquad s_k + \pi < (1-\alpha)\left[s_k + x\right]$$

$$\delta_k \epsilon\,[0,1] \; if \qquad s_k + \pi = (1-\alpha)\left[s_k + x\right]$$

$$\delta_k = 0 \; if \qquad s_k + \pi > (1-\alpha)\left[s_k + x\right]$$

Analysing the set of equilibria of this game, one arrives at the following

Lemma 3 *There is a cut-off level of realised first period profits* $\widehat{\pi}$, *such that*

a) for $\pi < \widehat{\pi}$ *the unique equilibrium of the continuation game is a separating equilibrium in which* $\delta_H = 0$, $\delta_L = 1, \rho = 0$, *and*

Low π => high dilution. adv select investor problem.

$$\alpha = \frac{I - \pi}{s_L + x}$$

b) for $\pi \geq \widehat{\pi}$ *there exists a pooling equilibrium in which* $\delta_H = 1$, $\delta_L = 1, \rho = r$, *and*

$$\alpha = \frac{I - \pi}{\left[r s_H + (1-r)\, s_L + x\right]}$$

The cut-off $\widehat{\pi}$ *is implicitly defined as the solution to*

$$s_H + \pi = \left(1 - \frac{I - \pi}{\left[r s_H + (1-r)\, s_L + x\right]}\right)\left[s_H + x\right]$$

A proof is provided in Appendix 3.1 at the end of this chapter.

It is useful at this point to analyse the cut-off profit level $\widehat{\pi}$. One can find an explicit expression for it by rearranging the condition under which the high type will invest.

$$s_H + \pi \leq \left(1 - \frac{I - \pi}{[r s_H + (1-r) s_L + x]}\right) [s_H + x]$$

$\Longleftrightarrow$

$$s_H + \pi \leq s_H + x - \frac{I - \pi}{[r s_H + (1-r) s_L + x]} [s_H + x]$$

$\Longleftrightarrow$

$$\pi \leq x - \frac{I - \pi}{[r s_H + (1-r) s_L + x]} [s_H + x]$$

$\Longleftrightarrow$

$$I \frac{[s_H + x]}{[r s_H + (1-r) s_L + x]} - x \leq \pi \left(\frac{[s_H + x]}{[r s_H + (1-r) s_L + x]} - 1\right)$$

$\Longleftrightarrow$

$$\left(I \frac{[s_H + x]}{[r s_H + (1-r) s_L + x]} - x\right) \left(\frac{[s_H + x]}{[r s_H + (1-r) s_L + x]} - 1\right)^{-1} \leq \pi$$

$\Longleftrightarrow$

$$\left(I \frac{[s_H + x]}{[r s_H + (1-r) s_L + x]} - x\right) \frac{[r s_H + (1-r) s_L + x]}{[s_H + x] - [r s_H + (1-r) s_L + x]} \leq \pi$$

$\Longleftrightarrow$

$$I - (x - I) \frac{[r s_H + (1-r) s_L + x]}{[s_H + x] - [r s_H + (1-r) s_L + x]} \leq \pi$$

$\Longleftrightarrow$

$$I - (x - I) \left(\frac{[s_H + x]}{[r s_H + (1-r) s_L + x]} - 1\right)^{-1} \leq \pi$$

For $\pi = \widehat{\pi}$ this holds as an equality and one has

$$I - (x - I) \left(\frac{[s_H + x]}{[r s_H + (1-r) s_L + x]} - 1\right)^{-1} = \widehat{\pi}$$

By inspection, one sees that the profit level required for a pooling equilibrium to exist is increasing in the required investment outlay I. One also finds that it is decreasing in the net present value of the project[7]. For lower net present value, a higher amount of internal funds is required for it to be optimal to go ahead with the project. This is because in the pooling equilibrium the high type faces a dilution cost. Since the firm receives $(I-\pi)$, but pays

$$\alpha\left[s_H+x\right]=\frac{I-\pi}{\left[rs_H+(1-r)\,s_L+x\right]}\left[s_H+x\right]$$

this cost is equal to the difference which can be written as

$$(I-\pi)\left(\frac{\left[s_H+x\right]}{\left[rs_H+(1-r)\,s_L+x\right]}-1\right)$$

The dilution cost is decreasing in π. Therefore, for low net present value projects to be acceptable, the dilution cost has to be low, which requires a higher π.

Finally, the required profit level is increasing in the dilution factor

$$\left(\frac{\left[s_H+x\right]}{\left[rs_H+(1-r)\,s_L+x\right]}-1\right)$$

The larger is this factor, the higher is the level of internal funds needed to make investment attractive.

It is worth pointing out that, as long as the project has a strictly positive net present value, $\widehat{\pi}<I$. Recall also that for $\pi\geq I$ both types of firm will invest, since then there is no need to raise external funds, Therefore, given that the pooling equilibrium is played whenever it exists, one has that for $\pi<\widehat{\pi}$ only the low type invests and for $\pi\geq\widehat{\pi}$ both types will invest.

For the main part of the analysis I am going to assume that the pooling equilibrium is played whenever it exists. Before doing so, let us explore how much generality is lost by such an assumption[8]. One finds

[7]This result is derived algebraically in the later section 3.5.

[8]Myers and Majluf (1984) do not provide a formal characterisation of the full set of equilibria of their model. In a model set-up similar to ours the issue of multiple equilibria has first been addressed by Cadsby, Frank, and Maksimovic (1990). The main difference between their analysis and the analysis here is that they assume financial slack to be zero, whereas I focus on first-period profit as the main variable of interest.

Lemma 4 *There is a cut-off level of realised first period profits* $\hat{\hat{\pi}}$ *such that*

a) for $\pi \leq \hat{\hat{\pi}}$ *a separating equilibrium exists in which* $\delta_H = 0$, $\delta_L = 1, \rho = 0$, *and*

$$\alpha = \frac{I - \pi}{s_L + x}$$

b) for $\pi > \hat{\hat{\pi}}$ *the unique equilibrium is a pooling equilibrium in which* $\delta_H = 1$, $\delta_L = 1, \rho = r$, *and*

$$\alpha = \frac{I - \pi}{[r s_H + (1 - r) s_L + x]}$$

The cut-off $\hat{\hat{\pi}}$ *is implicitly defined as the solution to*

$$s_H + \pi = \left(1 - \frac{I - \pi}{s_L + x}\right) [s_H + x]$$

For a proof see the Appendix 3.1.

Putting Lemma 3 and Lemma 4 together it is immediate that $\hat{\hat{\pi}} \geq \hat{\pi}$. In fact one can show that $\hat{\hat{\pi}}$ is strictly larger than $\hat{\pi}$. This can be easily established by deriving an explicit expression for $\hat{\hat{\pi}}$. The condition for the separating equilibrium to exist is

$$s_H + \pi \geq \left(1 - \frac{I - \pi}{s_L + x}\right) [s_H + x]$$

This can be rearranged to give

$$s_H + \pi \geq s_H + x - \frac{I - \pi}{s_L + x} [s_H + x]$$

$\Longleftrightarrow$

$$\pi \geq x - \frac{I - \pi}{s_L + x} [s_H + x]$$

$\Longleftrightarrow$

$$\frac{I}{s_L + x} [s_H + x] - x \geq + \frac{\pi}{s_L + x} [s_H + x] - \pi$$

$\Longleftrightarrow$

$$\frac{I}{s_L + x} [s_H + x] - x \geq \pi \left(\frac{s_H + x}{s_L + x} - 1\right)$$

$\Longleftrightarrow$

$$\left(I \frac{s_H + x}{s_L + x} - x\right) \left(\frac{s_H + x}{s_L + x} - 1\right)^{-1} \geq \pi$$

$\iff$

$$\left(I\frac{s_H + x}{s_L + x} - x\right)\left(\frac{s_L + x}{s_H + x - [s_L + x]}\right) \geq \pi$$

$\iff$

$$I - (x - I)\frac{s_L + x}{s_H + x - [s_L + x]} \geq \pi$$

$\iff$

$$I - (x - I)\left(\frac{s_H + x}{s_L + x} - 1\right)^{-1} \geq \pi$$

For $\widehat{\widehat{\pi}}$ this holds as an equality and one has

$$I - (x - I)\left(\frac{s_H + x}{s_L + x} - 1\right)^{-1} = \widehat{\widehat{\pi}}$$

To see that $\widehat{\pi} < \widehat{\widehat{\pi}}$ note that

$$\left(\frac{[s_H + x]}{[rs_H + (1 - r)s_L + x]} - 1\right)^{-1} > \left(\frac{s_H + x}{s_L + x} - 1\right)^{-1}$$

$\iff$

$$\left(\frac{[s_H + x]}{[rs_H + (1 - r)s_L + x]} - 1\right) < \left(\frac{s_H + x}{s_L + x} - 1\right)$$

It remains to characterise semiseparating equilibria. These equilibria are such that the low type always invests, whereas the high type is just indifferent between investing and not investing, given beliefs. The high type randomises and beliefs are consistent with the probability of the high type investing. One finds

Lemma 5 *A semiseparating equilibrium exists if and only if* $\pi \epsilon \left[\widehat{\pi}, \widehat{\widehat{\pi}}\right]$. *In a semiseparating equilibrium* $\delta_L = 1$, $\delta_H \epsilon [0, 1]$, *and*

$$\rho(\delta_H) = \frac{r\delta_H}{r\delta_H + (1 - r)}$$

For given parameters δ_H *is the solution to*

$$s_H + \pi = \left(1 - \frac{I - \pi}{[\rho(\delta_H)s_H + (1 - \rho(\delta_H))s_L + x]}\right)[s_H + x]$$

For a proof see the appendix.

Across the semiseparating equilibria one finds that

$$\frac{d\delta_H}{d\pi} < 0$$

That is, as the high type has more cash on hand, the probability of the high type investing falls. This may seem counterintuitive, in particular since, as one moves from separating equilibria to pooling equilibria, an increase in profit is associated with an increase in δ_H. The intuition for the case of the semiseparating equilibria is that for these an indifference condition has to hold. As one lets δ_H increase ρ increases, so that dilution costs decrease. Dilution cost are decreasing in π. To keep the high type indifferent between investing and not investing, dilution costs have to increase through a decrease in π.

One can summarise Lemmas 3-5 in the following

Corollary 1 *For $\pi < \widehat{\pi}$ the unique equilibrium is a separating equilibrium. For $\pi \epsilon \left[\widehat{\pi}, \widehat{\widehat{\pi}}\right]$ separating, semiseparating and pooling equilibria exist. For $\widehat{\widehat{\pi}} < \pi$ the unique equilibrium is a pooling equilibrium.*

For $\pi \epsilon \left[\widehat{\pi}, \widehat{\widehat{\pi}}\right]$ one has multiple equilibria[9]. Figure 3.1. gives a graphical representation of the equilibrium correspondence. Which equilibrium is played for

[9] Standard equilibrium refinements like the Intuitive Criterion (Cho and Kreps(1987)) do not help to reduce the number of equilibria here. These refinements are based on the notion that players should have reasonable beliefs on off-equilibrium behaviour. The idea is that in a given equilibrium some off-equilibrium action may not be undertaken because it is assigned an unreasonable belief. There are two reasons, why such arguments do not work in the Myers and Majluf model. First, the action space is not rich enough: there are only two possible actions, to issue equity, and not to issue equity. Second, the belief assigned to not issuing equity is not payoff- relevant, precisely because no equity is being issued. In the separating and the semiseparating equilibria both possible actions are on the equilibrium path. In the pooling equilibrium, the profitability of an off-equilibrium move to not issuing equity does not vary with the belief that is assigned to this action. Standard refinements therefore do not have any bite.

each π will affect the first period objective of firm i and will therefore affect our conclusions on the outcome of the first period competition. I would like to motivate an equilibrium selection such that $\rho(\pi)$ is weakly increasing in π. This condition rules out semiseparating equilibria being played on $\left[\widehat{\pi}, \widehat{\widehat{\pi}}\right]$. It also rules out an equilibrium selection such that the equilibrium moves from a pooling equilibrium to a separating equilibrium as π increases. The condition has some intuitive appeal since one can view lack of internal funding as the source of the inefficiency that arises as the high type forgoes investment. Intuitively, this problem should become less severe and the market should place a higher probability on the firm being of the high type as the amount of external funding which the firm asks for becomes smaller.

In deriving the set of equilibria for the financing game, it was assumed that the value of the assets in place is the only source of asymmetric information and that the financial investor knows all other variables including the realisation of first-period profit with certainty. As one moves away from this assumption and introduces some uncertainty as regards the first-period profit, one would expect financial investors to take a larger equity issue as a sign of the firm being more likely of the bad type. The reason is that the bad type has a dilution gain which is increasing in the issue size, whereas the high type has a dilution cost which is increasing in the issue size. Bad firms will therefore have a stronger incentive to overstate their financing needs and should therefore be thought of as more likely to issue larger amounts.

For a continuous distribution of first-period profit it is difficult to state these ideas formally. To motivate the equilibrium selection condition that $\rho(\pi)$ is weakly increasing in π, I move to a situation where first period profit can be either high or low. Consider a profit distribution such that $\pi \epsilon \{\underline{\pi}, \overline{\pi}\}$, where the probability of the high realisation is $\Pr[\pi = \overline{\pi}] = p$, independent of the firm's type. I want both profit realisations to be in the region of multiple equilibria, that is $\widehat{\widehat{\pi}} > \overline{\pi} > \underline{\pi} > \widehat{\pi}$. I assume now that financial markets are unable to observe profits. When the profit realisation is $\overline{\pi}$, the firm has three possibilities. It can issue $I - \overline{\pi}$ worth of equity, or it can understate its profit realisation and issue a larger amount of equity equalling issue size $I - \underline{\pi}$, or it can forgo investment. When the firm issues the larger amount, it is assumed that it is able to pay the original owners a dividend equalling the surplus cash. Equivalently one can assume that when $\overline{\pi}$ realises the owners are able to eat up the amount $\overline{\pi} - \underline{\pi} = \varepsilon$ without the financial investors being able to observe this. When the profit realisation is $\underline{\pi}$, the firm has two possibilities, as before: it can either issue $I - \underline{\pi}$ or forgo the investment[10]. Denote by $\delta_H(\overline{\pi})$ the probability that the high type issues $I - \overline{\pi}$, given a profit realisation of $\overline{\pi}$ and by $\mu_H(\overline{\pi})$ the probability that the high type issues the larger amount $I - \underline{\pi}$, given a profit realisation of $\overline{\pi}$. Likewise, denote by $\delta_L(\overline{\pi})$ the probability that the low type issues $I - \overline{\pi}$, given a profit realisation of $\overline{\pi}$ and by $\mu_L(\overline{\pi})$ the probability that the low type understates its profit and issues the larger amount $I - \underline{\pi}$, given a profit realisation of $\overline{\pi}$. Finally, let $\delta_H(\underline{\pi})$ and $\delta_L(\underline{\pi})$be the probabilities that the high type and the low type issue $I - \underline{\pi}$, respectively, given that the profit realisation is $\underline{\pi}$. These probabilities satisfy

$$\begin{aligned} \delta_H(\overline{\pi}) + \mu_H(\overline{\pi}) &\leq 1 \\ \delta_L(\overline{\pi}) + \mu_L(\overline{\pi}) &\leq 1 \end{aligned}$$

and

$$\begin{aligned} \delta_H(\underline{\pi}) &\leq 1 \\ \delta_L(\underline{\pi}) &\leq 1 \end{aligned}$$

For any action the firm takes, financial investors have a belief on the type of the firm. Let $\rho(\overline{\pi})$ be the probability that the firm is of the high type when the

[10]There is no point in assuming that the firm has a third possibility of issuing a smaller amount than it needs for investment since financial investors are able to observe whether the investment is undertaken or not, and would be able to demand their money back if the firm did not invest after having issued equity.

issue size $I - \overline{\pi}$ is observed and $\rho(\underline{\pi})$ be the probability attached to the firm being of the high type, given that the larger issue size of $I - \underline{\pi}$ is observed. In an equilibrium in which the amount $I - \underline{\pi}$ is issued with positive probability one will have

$$\rho(\underline{\pi}) = \frac{r\left[(1-p)\,\delta_H(\underline{\pi}) + p\mu_H(\overline{\pi})\right]}{r\left[(1-p)\,\delta_H(\underline{\pi}) + p\mu_H(\overline{\pi})\right] + (1-r)\left[(1-p)\,\delta_L(\underline{\pi}) + p\mu_L(\overline{\pi})\right]}$$

In an equilibrium in which the amount of $I - \overline{\pi}$ is issued with positive probability one will have

$$\rho(\overline{\pi}) = \frac{rp\delta_H(\overline{\pi})}{rp\delta_H(\overline{\pi}) + (1-r)\,p\delta_L(\overline{\pi})} = \frac{r\delta_H(\overline{\pi})}{r\delta_H(\overline{\pi}) + (1-r)\,\delta_L(\overline{\pi})}$$

While destroying some equilibria, the introduction of additional asymmetric information regarding the realisation of first period profit will in general generate further equilibria that are supportable by particular choices of out-of-equilibrium beliefs. To justify the equilibrium selection criterion I want to focus on equilibria in which, as in the case of symmetric information with respect to profit, both issue sizes occur with positive probability on the equilibrium path. To illustrate how the introduction of uncertainty changes the set of equilibria of the game, let us first ask whether the semiseparating equilibrium on both $\underline{\pi}$ and $\overline{\pi}$ survives the introduction of uncertainty. This would be so if for neither type it pays to understate its profit, given the equilibrium beliefs. Let us first check the high type. Given $\overline{\pi}$ the high type would want to deviate to $\mu_H(\overline{\pi}) = 1$, if

$$\begin{aligned}
&\left(1 - \frac{I - \overline{\pi} + \varepsilon}{\left[\rho(\underline{\pi})\, s_H + (1-\rho(\underline{\pi}))\, s_L + x\right]}\right)[s_H + x] + \varepsilon \\
> &\left(1 - \frac{I - \overline{\pi}}{\left[\rho(\overline{\pi})\, s_H + (1-\rho(\overline{\pi}))\, s_L + x\right]}\right)[s_H + x]
\end{aligned}$$

But

$$\begin{aligned}
&\left(1 - \frac{I - \overline{\pi} + \varepsilon}{\left[\rho(\underline{\pi})\, s_H + (1-\rho(\underline{\pi}))\, s_L + x\right]}\right)[s_H + x] + \varepsilon \\
= &\left(1 - \frac{I - \underline{\pi}}{\left[\rho(\underline{\pi})\, s_H + (1-\rho(\underline{\pi}))\, s_L + x\right]}\right)[s_H + x] + \varepsilon \\
= &\ s_H + \underline{\pi} + \varepsilon
\end{aligned}$$

$$
\begin{aligned}
&= s_H + \overline{\pi} \\
&= \left(1 - \frac{I - \overline{\pi}}{[\rho(\overline{\pi}) s_H + (1 - \rho(\overline{\pi})) s_L + x]}\right) [s_H + x]
\end{aligned}
$$

so that for the high type, understating its profit does not increase its payoff, given equilibrium beliefs. The high type is just indifferent as to whether to play its equilibrium strategy or to deviate to a larger issue size. Intuitively, the high type gains from issuing a larger amount, since this will lead to a more favourable belief. This is so since $\rho(\underline{\pi}) > \rho(\overline{\pi})$ in the semiseparating equilibrium. On the other hand, the dilution cost incurred by the high type is larger for larger issue sizes. These two effects exactly cancel out. The high type therefore does not have an incentive to deviate to larger issue size, but is just as happy playing the equilibrium strategy. For the low type, on the other hand, a deviation to $\mu_L(\overline{\pi}) = 1$ is profitable, since

$$
\begin{aligned}
&\left(1 - \frac{I - \overline{\pi} + \varepsilon}{[\rho(\underline{\pi}) s_H + (1 - \rho(\underline{\pi})) s_L + x]}\right) [s_L + x] + \varepsilon \\
> &\left(1 - \frac{I - \overline{\pi}}{[\rho(\overline{\pi}) s_H + (1 - \rho(\overline{\pi})) s_L + x]}\right) [s_L + x]
\end{aligned}
$$

$\Longleftrightarrow$

$$
\begin{aligned}
&\varepsilon \left(1 - \frac{[s_L + x]}{[\rho(\underline{\pi}) s_H + (1 - \rho(\underline{\pi})) s_L + x]}\right) \\
> &\frac{[I - \overline{\pi}][s_L + x]}{[\rho(\underline{\pi}) s_H + (1 - \rho(\underline{\pi})) s_L + x]} - \frac{[I - \overline{\pi}][s_L + x]}{[\rho(\overline{\pi}) s_H + (1 - \rho(\overline{\pi})) s_L + x]}
\end{aligned}
$$

which is satisfied since $\rho(\underline{\pi}) > \rho(\overline{\pi})$ so that the LHS is strictly positive and the RHS is strictly negative. Intuitively, again the low type gains from more favourable equilibrium beliefs associated with the larger issue size. In addition, its dilution gain is increasing in the issue size. Both effects work in the same direction here and make it profitable for the low type to choose the larger issue size. The low type, therefore, does have an incentive to understate its profit. The semiseparating equilibrium does not survive the introduction of additional uncertainty since beliefs are such that $\rho(\underline{\pi}) > \rho(\overline{\pi})$. In fact, one can show

Lemma 6 *In any equilibrium in which both issue sizes occur with positive probability,* $\rho(\underline{\pi}) \leq \rho(\overline{\pi})$.

Proof: Assume $\rho(\underline{\pi}) > \rho(\overline{\pi})$. Then $\mu_L(\overline{\pi}) = 1$, since

$$\begin{aligned} & \left(1 - \frac{I - \overline{\pi} + \varepsilon}{[\rho(\underline{\pi}) s_H + (1 - \rho(\underline{\pi})) s_L + x]}\right)[s_L + x] + \varepsilon \\ > & \left(1 - \frac{I - \overline{\pi}}{[\rho(\overline{\pi}) s_H + (1 - \rho(\overline{\pi})) s_L + x]}\right)[s_L + x] \end{aligned}$$

as shown above. $\mu_L(\overline{\pi}) = 1$ implies $\delta_L(\overline{\pi}) = 0$, so that one has

$$\rho(\underline{\pi}) > \frac{r\delta_H(\overline{\pi})}{r\delta_H(\overline{\pi}) + (1 - r)\delta_L(\overline{\pi})} = 1$$

This completes the proof.

Notice that the set of equilibria in which both issue sizes occur with positive probability includes all those equilibria in which there is no incentive to understate, i.e. in which $\mu_L(\overline{\pi}) = \mu_H(\overline{\pi}) = 0$. The latter set of equilibria is a subset of all possible combinations of the equilibria with perfect information regarding profit. One can easily show that of these combinations, only four survive the introduction of uncertainty. These are separating on both $\underline{\pi}$ and $\overline{\pi}$, separating on $\underline{\pi}$ and semiseparating on $\overline{\pi}$, semiseparating on $\underline{\pi}$ and pooling on $\overline{\pi}$, and separating on $\underline{\pi}$ and pooling on $\overline{\pi}$. As we have seen, semiseparating on both $\underline{\pi}$ and $\overline{\pi}$ does not survive since under the equilibrium beliefs the low type has an incentive to $\mu_L(\overline{\pi}) = 1$, whereas the high type has no such incentive.

3.4 The First-Period Objective

Let us now go on to construct the objective of firm i in the first-period competition. This will depend on which equilibrium is played for given first-period profit. In line with the arguments in the last section, I will assume that the equilibrium selection is such that $\rho(\pi)$ is weakly increasing in π. This implies that the equilibrium moves from the separating equilibrium to the pooling equilibrium at some cut-off. For concreteness, let us start with the assumption that a pooling equilibrium is played whenever it exists. This means that for $\pi < \widehat{\pi}$ the equilibrium will be separating and for $\pi \geq \widehat{\pi}$ the equilibrium will be pooling as in Lemma 3.

Guess: if $\hat{\pi}$ [illegible] $<$ Cournot then get Cournot or [illegible]

or if $\hat{\pi} >$ Cournot then [illegible]

With probability r the firm is type H.

In this case it will not invest for realisation of π, such that $\pi < \widehat{\pi}$ and its payoff will be

$$s_H + \pi$$

For realisations of π, such that $i \geq \pi \geq \widehat{\pi}$ the firm invests and has payoff

$$\left(1 - \frac{I-\pi}{[rs_H + (1-r)\, s_L + x]}\right)[s_H + x]$$

$$= s_H + x - \frac{I-\pi}{[rs_H + (1-r)\, s_L + x]}[s_H + x]$$

$$= s_H + x - (I-\pi)\frac{[s_H + x]}{[rs_H + (1-r)\, s_L + x]}$$

$$= s_H + x - (I-\pi)\left(\frac{[s_H + x]}{[rs_H + (1-r)\, s_L + x]} - 1\right) + [-(I-\pi)]$$

$$= s_H + \pi + x - I - (I-\pi)\left(\frac{[s_H + x]}{[rs_H + (1-r)\, s_L + x]} - 1\right)$$

The firm receives $s_H + \pi$. It also receives the net present value of the project, but loses the dilution cost of

$$(I-\pi)\left(\frac{[s_H + x]}{[rs_H + (1-r)\, s_L + x]} - 1\right)$$

Finally, for realisation of π such that $\pi > I$ the firm's payoff is

$$s_H + \pi + x - I$$

With probability $1 - r$ the firm is the low type L.

In this case the firm always invests.

For realisations such that $\pi < \widehat{\pi}$ its payoff will be

$$\left(1 - \frac{I-\pi}{s_L + x}\right)[s_L + x]$$

$$= s_L + \pi + x - I$$

For realisations such that $I > \pi \geq \widehat{\pi}$ its payoff will be

$$\left(1 - \frac{I - \pi}{[rs_H + (1-r)\, s_L + x]}\right)[s_L + x]$$

$$s_L + x - (I - \pi)\frac{s_L + x}{rs_H + (1-r)\, s_L + x}$$

$$s_L + x - (I - \pi)\left(\frac{s_L + x}{rs_H + (1-r)\, s_L + x} - 1\right) - (I - \pi)$$

$$s_L + \pi + x - I - (I - \pi)\left(\frac{s_L + x}{rs_H + (1-r)\, s_L + x} - 1\right)$$

$$s_L + \pi + x - I + (I - \pi)\left(1 - \frac{s_L + x}{rs_H + (1-r)\, s_L + x}\right)$$

It gets $s_L + \pi$ and the net present value of the project. In addition, it gets a dilution gain of

$$(I - \pi)\left(1 - \frac{s_L + x}{rs_H + (1-r)\, s_L + x}\right) > 0$$

Finally, for realisations of π such that $\pi > i$ the firm's payoff is

$$s_L + \pi + x - I$$

It is important to realise that for any realisation of π the expected dilution cost is zero. This follows directly from the fact that the financial investor breaks even in equilibrium. It can be verified algebraically by noting that

$$\begin{aligned}
& -r\,(I - \pi)\left(\frac{[s_H + x]}{[rs_H + (1-r)\, s_L + x]} - 1\right) \\
& +(1-r)\,(I - \pi)\left(1 - \frac{s_L + x}{rs_H + (1-r)\, s_L + x}\right) \\
= \ & r\,(I - \pi) + (1-r)\,(I - \pi) \\
& -(I - \pi)\left[r\frac{[s_H + x]}{[rs_H + (1-r)\, s_L + x]} + (1-r)\frac{s_L + x}{rs_H + (1-r)\, s_L + x}\right] \\
= \ & 0
\end{aligned}$$

For given π the expected payoff is therefore

$$(1-r)\left[s_L+\pi+x-I\right]+r\left[s_H+\pi\right] \; if \; \pi<\widehat{\pi}$$

~~$(1-r)s_L+x-I$~~ $\pi+(1-r)(x-I)$

and

$$(1-r)\left[s_L+\pi+x-I\right]+r\left[s_H+\pi+x-I\right] \; if \; \pi\geq\widehat{\pi}$$

$\pi+x-I$

So better off if have made π become otherwise distribution so high that will forgo pos NPV market

Across realisations of π the expected payoff can then be written down as

$$\int_{-\infty}^{\widehat{\pi}}\left\{(1-r)\left[s_L+\pi+x-I\right]+r\left[s_H+\pi\right]\right\}f(\pi)\,d\pi$$
$$+\int_{\widehat{\pi}}^{\infty}\left\{(1-r)\left[s_L+\pi+x-I\right]+r\left[s_H+\pi+x-I\right]\right\}f(\pi)\,d\pi$$

$$=\int_{-\infty}^{\widehat{\pi}}\left\{(1-r)s_L+rs_H+\pi+(1-r)(x-I)\right\}f(\pi)\,d\pi$$
$$+\int_{\widehat{\pi}}^{\infty}\left\{(1-r)s_L+rs_H+\pi+(1-r)(x-I)+r(x-I)\right\}f(\pi)\,d\pi$$

$$=(1-r)s_L+rs_H+(1-r)(x-I)+\int_{-\infty}^{\infty}\pi f(\pi)\,d\pi+\int_{\widehat{\pi}}^{\infty}r(x-I)f(\pi)\,d\pi$$

$$=(1-r)s_L+rs_H+(1-r)(x-I)+\int_{-\infty}^{\infty}\pi f(\pi)\,d\pi+(1-F(\widehat{\pi}))r(x-I)$$

$$=(1-r)s_L+rs_H+(x-I)+\int_{-\infty}^{\infty}\pi f(\pi)\,d\pi-F(\widehat{\pi})r(x-I)$$

$$=E[\widetilde{s}]+(x-I)+E[\widetilde{\pi}]-F(\widehat{\pi})r(x-I)$$

Discarding constants, the firm's first period objective is therefore

$$E[\widetilde{\pi}]-F(\widehat{\pi})r(x-I)$$

O.K. So ∴ incentive to ~~[illegible]~~ produce aggressively

In addition to the expected value of profit, there is a second term. It is the loss in net present value, which occurs when the firm is of the high type and the profit realisation is too low for the pooling equilibrium to exist, so that the high type will not invest. This is multiplied by the probability that the profit realisation is below the cut-off $\widehat{\pi}$, above which the pooling equilibrium is played.

It is easy to see that for any other assumption on equilibrium selection satisfying $\rho(\underline{\pi}) \leq \rho(\overline{\pi})$ there will again be a cut-off $\pi \epsilon \left[\widehat{\pi}, \widehat{\widehat{\pi}}\right]$ such that the high type invests for profit realisations larger than the cut-off. The first period objective can then be found by replacing $\widehat{\pi}$ with the cut-off chosen.

3.5 First-Period Competition

Let us now go on to analyse the first-stage game in which each of the two firms chooses a strategic variable to maximise its first period objective. In the general framework profits are given by

$$\widetilde{\pi}^i = \pi\left(\widetilde{z}, q_i, q_j\right)$$

I want to make this more specific in two different ways.

3.5.1 Profit function à la Brander and Lewis

In line with Brander and Lewis (1986) and adopting the framework of chapter 2, let us first assume that uncertainty can be represented by a scalar variable, i.e.

$$\widetilde{\pi}^i = \pi^i\left(\widetilde{z}^i, q^i, q^j\right) = \pi^i\left(\widetilde{\theta}^i, q^i, q^j\right)$$

where $\widetilde{\theta}^i \epsilon \left(\underline{\theta}, \overline{\theta}\right)$. The distribution of $\widetilde{\theta}$ is given by $F(\theta)$ which is assumed to have a density $f(\theta)$. Further, the following assumptions hold

$$\forall \;\left(\theta^i, q^i, q^j\right) \; with \; \theta^i \in \left(\underline{\theta}, \overline{\theta}\right), q^i \geq 0, q^j \geq 0$$

$$(i)\, \pi^i_{ii}\left(\theta^i, q^i, q^j\right) < 0, (ii)\, \pi^i_j\left(\theta^i, q^i, q^j\right) < 0, (iii)\, \pi^i_{ij}\left(\theta^i, q^i, q^j\right) < 0 \qquad \text{(A1)}$$

$$(iv)\, \pi^i_\theta\left(\theta^i, q^i, q^j\right) > 0, (v)\, \pi^i_{i\theta}\left(\theta^i, q^i, q^j\right) > 0,$$

Here assumptions (iv) and (v) are versions of the more general assumptions in A1*, for the case of a profit function which is differentiable with respect to a scalar random variable $\widetilde{z}^i = \widetilde{\theta}^i$. I also make two additional assumptions. First, I assume that $\widetilde{\theta}$ is uniformly distributed. This guarantees that $f'(\theta) = 0$. Second, I want to assume that $\pi^i_{\theta\theta} = 0$[11].

Consider first the benchmark of single stage competition. Equivalently, assume that $x - I$ is negative, so that neither the low type nor the high type will invest in the second stage. Then firm $i's$ objective is to maximise

$$E\left[\widetilde{\pi}^i\right] = \int_{\underline{\theta}}^{\overline{\theta}} \pi^i\left(\theta^i, q^i, q^j\right) f\left(\theta^i\right) d\theta^i$$

The equilibrium is given by the unique intersection of the firms' reaction functions. These are implicitly defined by

$$\begin{aligned}
\int_{\underline{\theta}}^{\overline{\theta}} \pi^i_i\left(\theta^i, q^i, q^j\right) f\left(\theta^i\right) d\theta^i &= 0 \\
\int_{\underline{\theta}}^{\overline{\theta}} \pi^j_j\left(\theta^j, q^i, q^j\right) f\left(\theta^j\right) d\theta^j &= 0
\end{aligned}$$

The intersection will yield equilibrium quantities $(q^i, q^j) = (q^c, q^c)$which will give equilibrium profit

$$\int_{\underline{\theta}}^{\overline{\theta}} \pi^i\left(\theta^i, q^c, q^c\right) f\left(\theta^i\right) d\theta^i \equiv \pi^c$$

Now assume that $\boxed{x - i}$ is positive. Then firm $i's$ objective is

$$\begin{aligned}
V^i &= E\left[\widetilde{\pi}^i\right] - F\left(\widehat{\pi}\right) r\left(x - I\right) \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \pi^i\left(\theta^i, q^i, q^j\right) f\left(\theta^i\right) d\theta^i - F\left(\widehat{\theta}\right) r\left(x - I\right)
\end{aligned}$$

where $\widehat{\theta}$ is implicitly defined by

So $\hat{\theta}$ is a fn of q^i & q^j

$$\pi^i\left(\widehat{\theta}, q^i, q^j\right) - \widehat{\pi} = 0$$

[11]These assumptions, as well as the formal analysis in this section, are similar to Brander and Lewis (1988). They assume an objective function which is similar to the one we derive by speculating that the firm has debt in its capital structure and that the firm maximises profits minus a fixed exogenous bankruptcy cost, which is incurred whenever profits fall short of the debt obligation.

By assumption, only firm i faces an investment opportunity. Firm $j's$ objective therefore is

$$V^j = E\left[\widetilde{\pi}^j\right] = \int_{\underline{\theta}}^{\overline{\theta}} \pi^j\left(\theta^j, q^i, q^j\right) f\left(\theta^j\right) d\theta^j$$

as before.

Differentiating firm $i's$ objective with respect to q^i now yields the first-order condition

$$\int_{\underline{\theta}}^{\overline{\theta}} \pi_i^i\left(\theta^i, q^i, q^j\right) f\left(\theta^i\right) d\theta^i - f\left(\widehat{\theta}\right) \frac{\partial \widehat{\theta}}{\partial q_i} r\left(x - I\right) = 0$$

The second term of the derivative can be analyzed further. Using the implicit function theorem one finds

$$-f\left(\widehat{\theta}\right) \frac{\partial \widehat{\theta}}{\partial q_i} r\left(x - I\right) = f\left(\widehat{\theta}\right) \frac{\pi_i^i\left(\widehat{\theta}, q^i, q^j\right)}{\pi_\theta^i\left(\widehat{\theta}, q^i, q^j\right)} r\left(x - I\right)$$

The sign of this expression is ambiguous. It will be positive when $\pi_i^i\left(\widehat{\theta}, q^i, q^j\right) > 0$ and negative when $\pi_i^i\left(\widehat{\theta}, q^i, q^j\right) < 0$. Notice however, that the sign of $\pi_i^i\left(\widehat{\theta}, q^i, q^j\right)$ will depend on the position of $\widehat{\pi}$. For higher $\widehat{\pi}$ it is more likely to be positive. To see this note that $\pi_{i\theta}^i\left(\widehat{\theta}, q^i, q^j\right) > 0$ and that

$$\frac{\partial \widehat{\theta}}{\partial \widehat{\pi}} = \frac{1}{\pi_\theta^i\left(\widehat{\theta}, q^i, q^j\right)} > 0$$

Therefore the sign of $\pi_i^i\left(\widehat{\theta}, q^i, q^j\right)$ will be positive for high $\widehat{\pi}$ and negative for low $\widehat{\pi}$. Moreover, when, as assumed $f'\left(\theta\right) = 0$ and $\pi_{\theta\theta}^i = 0$, one can show a monotone relationship between the size of the second term of the derivative and the position of $\widehat{\pi}$.

$$\begin{aligned}
& \frac{\partial}{\partial \widehat{\pi}}\left(-f\left(\widehat{\theta}\right) \frac{\partial \widehat{\theta}}{\partial q^i} r\left(x - I\right)\right) \\
= & -f\left(\widehat{\theta}\right) r\left(x - I\right) \frac{\partial \widehat{\theta}}{\partial q^i \partial \widehat{\pi}} - f'\left(\widehat{\theta}\right) r\left(x - I\right) \frac{\partial \widehat{\theta}}{\partial \widehat{\pi}} \frac{\partial \widehat{\theta}}{\partial q^i} \\
= & \ f\left(\widehat{\theta}\right) r\left(x - I\right) \frac{\pi_{i\theta}^i\left(\widehat{\theta}, q^i, q^j\right) \frac{\partial \widehat{\theta}}{\partial \widehat{\pi}} \pi_\theta^i\left(\widehat{\theta}, q^i, q^j\right) - \pi_{\theta\theta}^i\left(\widehat{\theta}, q^i, q^j\right) \frac{\partial \widehat{\theta}}{\partial \widehat{\pi}} \pi_i^i\left(\widehat{\theta}, q^i, q^j\right)}{\left(\pi_\theta^i\left(\widehat{\theta}, q^i, q^j\right)\right)^2}
\end{aligned}$$

$$= f\left(\widehat{\theta}\right) r(x-I) \frac{\pi^i_{i\theta}\left(\widehat{\theta}, q^i, q^j\right) \frac{\partial \widehat{\theta}}{\partial \widehat{\pi}} \pi^i_\theta\left(\widehat{\theta}, q^i, q^j\right)}{\left(\pi^i_\theta\left(\widehat{\theta}, q^i, q^j\right)\right)^2} > 0$$

Thus under the assumptions made, the size of the second term is increasing in $\widehat{\pi}$.

The second order condition is

$$\frac{\partial}{\partial q^i}\left\{\int_{\underline{\theta}}^{\overline{\theta}} \pi^i_i\left(\theta^i, q^i, q^j\right) f\left(\theta^i\right) d\theta^i + f\left(\widehat{\theta}\right) \frac{\pi^i_i\left(\widehat{\theta}, q^i, q^j\right)}{\pi^i_\theta\left(\widehat{\theta}, q^i, q^j\right)} r(x-I)\right\}$$

$$= \int_{\underline{\theta}}^{\overline{\theta}} \pi^i_{ii}\left(\theta^i, q^i, q^j\right) f\left(\theta^i\right) d\theta^i + \frac{\partial}{\partial q^i}\left\{f\left(\widehat{\theta}\right) \frac{\pi^i_i\left(\widehat{\theta}, q^i, q^j\right)}{\pi^i_\theta\left(\widehat{\theta}, q^i, q^j\right)} r(x-I)\right\} < 0$$

The first term is negative since $\pi^i_{ii} < 0$. Given that $f'\left(\widehat{\theta}\right) = 0$ the second term is equal to

$$f\left(\widehat{\theta}\right) \frac{\left[\pi^i_{ii}\left(\widehat{\theta}, q^i, q^j\right) + \pi^i_{i\theta} \frac{\partial \widehat{\theta}}{\partial q^i}\right] \pi^i_\theta\left(\widehat{\theta}, q^i, q^j\right)}{\pi^i_\theta\left(\widehat{\theta}, q^i, q^j\right)^2} r(x-I)$$

$$-f\left(\widehat{\theta}\right) \frac{\left[\pi^i_{\theta i}\left(\widehat{\theta}, q^i, q^j\right) + \pi^i_{\theta\theta}\left(\widehat{\theta}, q^i, q^j\right) \frac{\partial \widehat{\theta}}{\partial q^i}\right] \pi^i_i\left(\widehat{\theta}, q^i, q^j\right)}{\pi^i_\theta\left(\widehat{\theta}, q^i, q^j\right)^2} r(x-I)$$

Assume that $\pi^i_i\left(\widehat{\theta}, q^i, q^j\right) > 0$ at the point at which the first order condition holds. This implies that $\frac{\partial \widehat{\theta}}{\partial q^i} < 0$ so that given $\pi^i_{\theta\theta} = 0$ this expression has a negative sign and the second-order condition holds. When $\pi^i_i\left(\widehat{\theta}, q^i, q^j\right) < 0$, so that $\frac{\partial \widehat{\theta}}{\partial q^i} > 0$ at the point at which the first order condition holds, this expression cannot be signed. In order for the second-order condition to hold, one has to make appropriate assumptions on the relative sizes of the first and the second term. One can guarantee that the second order condition holds, by assuming that $f\left(\widehat{\theta}\right)$ is small.

Next consider equilibrium quantities. When $\widehat{\pi}$ is high enough, such that $\pi^i_i\left(\widehat{\theta}, q^c, q^c\right) > 0$ one will have

$$V^i_i = \int_{\underline{\theta}}^{\overline{\theta}} \pi^i_i\left(\theta^i, q^c, q^c\right) f\left(\theta^i\right) d\theta^i + f\left(\widehat{\theta}\right) r(x-I) \frac{\pi^i_i\left(\widehat{\theta}, q^c, q^c\right)}{\pi^i_\theta\left(\widehat{\theta}, q^c, q^c\right)} > 0$$

Provided that the intersection of the reaction functions is stable, which requires that $V_{ii}^i V_{jj}^j - V_{ij}^i V_{ji}^j > 0$, this implies that $q^i > q^c$ and $q^j < q^c$ in equilibrium. On the other hand, when $\widehat{\pi}$ is low, such that $\pi_i^i\left(\widehat{\theta}, q^c, q^c\right) < 0$ one will have

$$V_i^i = \int_{\underline{\theta}}^{\overline{\theta}} \pi_i^i\left(\theta^i, q^c, q^c\right) f\left(\theta^i\right) d\theta^i + f\left(\widehat{\theta}\right) r\left(x - I\right) \frac{\pi_i^i\left(\widehat{\theta}, q^c, q^c\right)}{\pi_\theta^i\left(\widehat{\theta}, q^c, q^c\right)} < 0$$

so that given reaction function stability one has $q^i < q^c$ and $q^j > q^c$ in equilibrium.

For a proof assume first that $\widehat{\pi}$ is such that $\pi_i^i\left(\widehat{\theta}, q^c, q^c\right) = 0$ and then consider a variation in $\widehat{\pi}$. Totally differentiating the system of first-order conditions one has

$$\begin{aligned} V_{ii}^i dq_i + V_{ij}^i dq_j + V_{i\widehat{\pi}}^i d\widehat{\pi} &= 0 \\ V_{ji}^j dq_i + V_{jj}^j dq_j + V_{j\widehat{\pi}}^j d\widehat{\pi} &= 0 \end{aligned}$$

Note that $V_{j\widehat{\pi}}^j = 0$. Then one can solve for comparative statics effects by using Cramer's rule to get

$$\begin{aligned} \frac{dq_i}{d\widehat{\pi}} &= -\frac{V_{i\widehat{\pi}}^i V_{jj}^j}{V_{ii}^i V_{jj}^j - V_{ij}^i V_{ji}^j} > 0 \\ \frac{dq_j}{d\widehat{\pi}} &= \frac{V_{i\widehat{\pi}}^i V_{ji}^j}{V_{ii}^i V_{jj}^j - V_{ij}^i V_{ji}^j} < 0 \end{aligned}$$

using

$$V_{i\widehat{\pi}}^i = \frac{\partial}{\partial \widehat{\pi}} \left(-f\left(\widehat{\theta}\right) \frac{\partial \widehat{\theta}}{\partial q_i} r\left(x - I\right) \right) > 0$$

as shown above.

Let us summarize these findings in the following

$\frac{\partial \pi_i}{\partial q_i}$ at $\hat{\theta}$.

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Proposition 5 *When $\widehat{\pi}$ is high enough such that $\pi_i^i\left(\widehat{\theta}, q^c, q^c\right) > 0$ one will have $q^i > q^c$ and $q^j < q^c$ in equilibrium. When $\widehat{\pi}$ is such that $\pi_i^i\left(\widehat{\theta}, q^c, q^c\right) = 0$ one will have $\left(q^i, q^j\right) = \left(q^c, q^c\right)$. When $\widehat{\pi}$ is low enough such that $\pi_i^i\left(\widehat{\theta}, q^c, q^c\right) < 0$ one will have $q^i < q^c$ and $q^j > q^c$ in equilibrium.*

This is the main result of this chapter and is shown graphically in Figure 3.2. The intuition for this result is the following. First, observe that, when $\pi^i_\theta\left(\theta^i, q^i, q^j\right) > 0$ and $\pi^i_{\theta i}\left(\theta^i, q^i, q^j\right) > 0$, an increase in q_i induces an increase in the spread or variance of $\widetilde{\pi}^i$. For larger q^i, a given swing in θ^i will translate into a larger swing in $\widetilde{\pi}^i$. This is true under the assumptions made on the way uncertainty enters the profit function, and is plausible. It says that, as the firm increases output, it exposes itself more to the underlying demand or cost uncertainty. To see how this follows from the assumptions, note that

$$\begin{aligned} var\left[\widetilde{\pi}^i\right] &= E\left[\left(\widetilde{\pi}^i - E\left[\widetilde{\pi}^i\right]\right)^2\right] \\ &= E\left[\left(\widetilde{\pi}^i\right)^2\right] - \left[E\left[\widetilde{\pi}^i\right]\right]^2 \end{aligned}$$

so that

$$\begin{aligned} \frac{\partial var\left[\widetilde{\pi}^i\right]}{\partial q^i} &= 2E\left[\widetilde{\pi}^i\widetilde{\pi}^i_i\right] - 2E\left[\widetilde{\pi}^i\right]E\left[\widetilde{\pi}^i_i\right] \\ &= 2cov\left[\widetilde{\pi}^i, \widetilde{\pi}^i_i\right] \end{aligned}$$

Both $\pi^i\left(\widehat{\theta}, q^i, q^j\right)$ and $\pi^i_i\left(\widehat{\theta}, q^i, q^j\right)$ are increasing in θ since by assumption $\pi^i_\theta\left(\theta^i, q^i, q^j\right) > 0$ and $\pi^i_{i\theta}\left(\theta^i, q^i, q^j\right) > 0$. Therefore $cov\left(\widetilde{\pi}^i, \widetilde{\pi}^i_i\right) > 0$, which implies that the variance of $\widetilde{\pi}^i$ is increasing in q^i.

The firm's objective function can be written as

$$E\left[\widetilde{\pi}^i\right] - F\left(\widehat{\pi}\right) r\left(x - I\right)$$

or equivalently as

$$(1-r)\left[E\left[\widetilde{\pi}^i\right] + x - I\right] + r\left[E\left[\widetilde{\pi}^i\right] + \left(1 - F\left(\widehat{\pi}\right)\right)(x - I)\right]$$

The firm obtains the benefit of being able to invest as a high type for realisations of $\widetilde{\pi}^i$ such that $\pi^i > \widehat{\pi}$.

When $\widehat{\pi}$ is in the right tail of the distribution of $\widetilde{\pi}^i$, the firm can increase the probability of investment by increasing the variance of $\widetilde{\pi}^i$. This creates an incentive to increase q^i.

On the other hand, when $\widehat{\pi}$ is low enough to be in left tail of the distribution of $\widetilde{\pi}^i$, the firm benefits from reducing the variance of $\widetilde{\pi}^i$, since in this case it is a reduction in variance that will increase the probability of realisations larger than $\widehat{\pi}$. Therefore, in this case there is an incentive to reduce q^i

Comparative statics

In our setup $\widehat{\pi}$ is an endogenous variable. It is therefore interesting to explore how the product market equilibrium is influenced by the factors determining $\widehat{\pi}$. Recall that

$$I-(x-I)\left(\frac{[s_H+x]}{[rs_H+(1-r)\,s_L+x]}-1\right)^{-1}=\widehat{\pi}$$

It was pointed out before that $\widehat{\pi}$ is increasing in the dilution factor

$$\left(\frac{[s_H+x]}{[rs_H+(1-r)\,s_L+x]}-1\right)\equiv T$$

Note that the size of the dilution factor depends on the uncertainty associated with the value of the assets in place. Consider subjecting the distribution of $\widetilde{s}$ to a mean preserving spread such that s_H is increased by some ε and s_L is decreased by $\frac{r}{(1-r)}\varepsilon$. This will leave $E\left[\widetilde{s}\right]=\left[r\left(s_H+\varepsilon\right)+(1-r)\left(s_L-\frac{r}{(1-r)}\varepsilon\right)+x\right]$ unchanged but will increase the numerator to $[s_H+x]+\varepsilon$. Larger uncertainty in the sense of a mean preserving spread will therefore increase $\widehat{\pi}$. The cut-off will move towards the right tail of the profit distribution, creating a stronger incentive to increase the variance. This will cause firm i to compete more aggressively and will result in a larger q^i and a smaller q^j. Thus one can see that larger uncertainty may actually benefit firm i, in that it leads to a lower rival quantity. Whether this competitive benefit of increased uncertainty is outweighed by the reduction in the probability of investment will depend on the exact parameter specification of the model.

Next consider an increase in the net present value of the project. The net present value is $x-I$. It can increase through an increase in x or a decrease in

I. These effects are best analysed separately. Consider first an increase in x.

$$\begin{aligned}
&\frac{\partial}{\partial x}\left\{I-(x-I)\left(\frac{[s_H+x]}{[rs_H+(1-r)\,s_L+x]}-1\right)^{-1}\right\}=\frac{\partial}{\partial x}\left\{I-(x-I)\,T\,(x)^{-1}\right\}\\
=&\ -T\,(x)^{-1}+xT\,(x)^{-2}\,\frac{\partial}{\partial x}T\,(x)\\
=&\ -T\,(x)^{-1}+xT\,(x)^{-2}\,\frac{rs_H+(1-r)\,s_L-s_H}{[rs_H+(1-r)\,s_L+x]^2}<0
\end{aligned}$$

Next

$$\begin{aligned}
&\frac{\partial}{\partial I}\left\{I-(x-I)\left(\frac{[s_H+x]}{[rs_H+(1-r)\,s_L+x]}-1\right)^{-1}\right\}\\
=&\ 1+\left(\frac{[s_H+x]}{[rs_H+(1-r)\,s_L+x]}-1\right)^{-1}>0
\end{aligned}$$

Therefore

$$\Delta\,(x-I)>0\Longrightarrow\Delta\widehat{\pi}<0$$

which is intuitive. When the net present value of the project is larger, it will exceed the dilution cost for a smaller cut-off. Whether an increase in the net present value of the project will lead firm i to compete more or less aggressively will not only depend on the effect on the cut-off. Recall that the term causing deviations from the Cournot equilibrium is

$$-f\left(\widehat{\theta}\right)\frac{\partial\widehat{\theta}}{\partial q^i}r\,(x-I)$$

There will therefore be a direct and an indirect effect.

$$\frac{\partial}{\partial x}\left\{-f\left(\widehat{\theta}\right)\frac{\partial\widehat{\theta}}{\partial q^i}r\,(x-I)\right\}=-f\left(\widehat{\theta}\right)\frac{\partial\widehat{\theta}}{\partial q^i}r-f\left(\widehat{\theta}\right)r\,(x-I)\,\frac{\partial\widehat{\theta}}{\partial q^i\partial\widehat{\pi}}\frac{\partial\widehat{\pi}}{\partial x}$$

We know that

$$-f\left(\widehat{\theta}\right)r\,(x-I)\,\frac{\partial\widehat{\theta}}{\partial q^i\partial\widehat{\pi}}>0$$

Hence

$$-f\left(\widehat{\theta}\right)r\,(x-I)\,\frac{\partial\widehat{\theta}}{\partial q^i\partial\widehat{\pi}}\frac{\partial\widehat{\pi}}{\partial x}<0$$

When

$$-f\left(\widehat{\theta}\right)\frac{\partial\widehat{\theta}}{\partial q^i}r<0$$

which will be the case when $q^i < q^c$ both effects go in the same direction. An increase in x will cause a further reduction in q^i. Intuitively, the cut-off moves further into the left tail of the profit distribution and the benefit from being above the cut-off is greater. Both give an incentive to reduce the variance at the expense of expected profit so that firm i will compete less aggressively in equilibrium.

When

$$-f\left(\widehat{\theta}\right)\frac{\partial\widehat{\theta}}{\partial q^i}r > 0$$

which will be the case when $q^i > q^c$ the net effect is ambiguous. Start from a situation where the cut-off is in the right tail of the distribution. As the net present value of the project increases, the benefit from being above the cut-off increases, which will give an incentive to increase the variance and cause firm i to compete more aggressively. On the other hand, as x increases the cut-off moves in and there is a reduced incentive to increase the variance so that firm i will have an incentive to compete less aggressively.

Finally, let us look at an increase in r. One finds

$$\begin{aligned}
&\frac{\partial}{\partial r}\left\{I-(x-I)\left(\frac{[s_H+x]}{[rs_H+(1-r)\,s_L+x]}-1\right)^{-1}\right\} \\
=\ &(x-I)\left(\frac{[s_H+x]}{[rs_H+(1-r)\,s_L+x]}-1\right)^{-2}\frac{\partial}{\partial r}\left(\frac{[s_H+x]}{[rs_H+(1-r)\,s_L+x]}-1\right)<0
\end{aligned}$$

which says that an increase in r causes the cut-off $\widehat{\pi}$ to shift in. Again, however, there is a competing direct effect, so that the comparative static results are qualitatively the same as for an increase in x.

I have derived comparative statics results under the assumption that the pooling equilibrium is played whenever it exists. Let us briefly move to the assumption that the separating equilibrium is played whenever it exists. This means that up to a profit level of $\widehat{\widehat{\pi}}$, only the bad type invests. Recall that

$$I-(x-I)\left(\frac{s_H+x}{s_L+x}-1\right)^{-1}=\widehat{\widehat{\pi}}$$

Since we know that $\widehat{\widehat{\pi}} > \widehat{\pi}$ it is clear that firm $i's$ equilibrium quantity will be larger and firm $j's$ equilibrium quantity will be smaller than under the original assumption. Notice that this effect will work against the usual notion that the pooling equilibrium is superior to the separating equilibrium. In our framework, lost investment efficiency will be partly recovered by more aggressive product market behaviour, causing a reduction in rival output and thus benefiting firm i.

The comparative static effects with respect to an increase in the uncertainty regarding s and with respect to an increase in the net present value are qualitatively unchanged. Notice that $\widehat{\widehat{\pi}}$ is not a function of r. Intuitively this comes about since $\widehat{\widehat{\pi}}$ is the profit level such that the high type would invest, even if the market believed that the firm was low type with probability one. An increase in r would therefore only have a direct effect, which would reinforce the deviation of firm $i's$ quantity from the Cournot level.

3.5.2 An alternative profit function

To explore further the intuition that the cut-off profit level creates incentives to manipulate the variance of the profit distribution, I want to explore a different specification for the profit function of both firms. In particular, I want to assume that profit is represented by

$$\pi^i\left(\widetilde{z}^i, q^i, q^j\right) = \pi^i\left(\widetilde{a}, \widetilde{\theta}^i, q^i, q^j\right) = \left(\widetilde{a} - b\left(q^i + q^j\right)\right) q^i - cq^i + \widetilde{\theta}^i$$

There is demand uncertainty, which is assumed to have a two-point distribution. Demand can be high or low, $\widetilde{a}\epsilon\left\{\underline{a}, \overline{a}\right\}$ and the probability that it is high is denoted by $\Pr\left[a = \overline{a}\right] = p$. In addition, there is some additive noise which I assume to have a zero mean normal distribution, $\widetilde{\theta}^i \sim N\left(0, \sigma\right)$[12].

Firm $i's$ objective is to maximise

$$V^i = E\left[\widetilde{\pi}^i\right] - F\left(\widehat{\pi}\right) r \left(x - I\right)$$

[12] It is easiliy seen that this specification satisfies A1*.

Denote the conditional means of the profit distribution given a high demand and a low demand realisation, respectively by

$$\begin{aligned} \overline{\pi} &= \left(\overline{a} - b\left(q^i + q^j\right)\right) q^i - cq^i \\ \underline{\pi} &= \left(\underline{a} - b\left(q^i + q^j\right)\right) q^i - cq^i \end{aligned}$$

One can then write down $F\left(\widehat{\pi}\right)$ explicitly as

$$\begin{aligned} F\left(\widehat{\pi}\right) &= pF\left(\widehat{\pi} \mid \overline{\pi}\right) + (1-p)\, F\left(\widehat{\pi} \mid \underline{\pi}\right) \\ &= p\int_{-\infty}^{\widehat{\pi}} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(v-\overline{\pi})^2}{2\sigma^2}} dv + (1-p)\int_{-\infty}^{\widehat{\pi}} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(v-\underline{\pi})^2}{2\sigma^2}} dv \end{aligned}$$

The first-order condition for firm $i's$ problem is

$$V_i^i = \frac{\partial}{\partial q^i}\left\{E\left[\widetilde{\pi}^i\right] - F\left(\widehat{\pi}\right) r\left(x - I\right)\right\} = 0$$

which can be written as

$$p\left(1 - \frac{\partial F\left(\widehat{\pi} \mid \overline{\pi}\right)}{\partial \overline{\pi}} r\left(x - I\right)\right) \frac{\partial \overline{\pi}}{\partial q^i} + (1-p)\left(1 - \frac{\partial F\left(\widehat{\pi} \mid \underline{\pi}\right)}{\partial \underline{\pi}} r\left(x - I\right)\right) \frac{\partial \underline{\pi}}{\partial q^i} = 0$$

One finds

$$\frac{\partial F\left(\widehat{\pi} \mid \overline{\pi}\right)}{\partial \overline{\pi}} = \int_{-\infty}^{\widehat{\pi}} \frac{1}{\sigma\sqrt{2\pi}} \frac{(v - \overline{\pi})}{\sigma^2} e^{\frac{-(v-\overline{\pi})^2}{2\sigma^2}} dv$$

Since $\widetilde{\pi}^i = \overline{\pi} + \widetilde{\theta}^i$ one can change the variable of integration to $\theta = v - \overline{\pi}$ to get

$$\begin{aligned} \frac{\partial F\left(\widehat{\pi} \mid \overline{\pi}\right)}{\partial \overline{\pi}} &= \int_{-\infty}^{\widehat{\pi}-\overline{\pi}} \frac{1}{\sigma\sqrt{2\pi}} \frac{\theta}{\sigma^2} e^{\frac{-\theta^2}{2\sigma^2}} d\theta \\ &= -\int_{-\infty}^{\widehat{\pi}-\overline{\pi}} \frac{1}{\sigma\sqrt{2\pi}} \frac{-\theta}{\sigma^2} e^{\frac{-\theta^2}{2\sigma^2}} d\theta \\ &= \left| -\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-\theta^2}{2\sigma^2}} \right|_{-\infty}^{\widehat{\pi}-\overline{\pi}} \\ &= -\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(\widehat{\pi}-\overline{\pi})^2}{2\sigma^2}} = -f\left(\widehat{\pi} \mid \overline{\pi}\right) \end{aligned}$$

Similarly, one finds

$$\frac{\partial F\left(\widehat{\pi} \mid \underline{\pi}\right)}{\partial \underline{\pi}} = -\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(\widehat{\pi}-\underline{\pi})^2}{2\sigma^2}} = -f\left(\widehat{\pi} \mid \underline{\pi}\right)$$

so that one has

$$\begin{aligned} V_i^i &= p\left(1+\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(\widehat{\pi}-\overline{\pi})^2}{2\sigma^2}}r(x-I)\right)\frac{\partial\overline{\pi}}{\partial q^i} \\ &\quad +(1-p)\left(1+\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(\widehat{\pi}-\underline{\pi})^2}{2\sigma^2}}r(x-I)\right)\frac{\partial\underline{\pi}}{\partial q^i} \\ &= p\left(1+f\left(\widehat{\pi}\mid\overline{\pi}\right)r(x-I)\right)\frac{\partial\overline{\pi}}{\partial q^i}+(1-p)\left(1+f\left(\widehat{\pi}\mid\underline{\pi}\right)r(x-I)\right)\frac{\partial\underline{\pi}}{\partial q^i} \end{aligned}$$

Similarly, one can show that second-order condition can be written as

$$\begin{aligned} V_{ii}^i &= p\left(1+f\left(\widehat{\pi}\mid\overline{\pi}\right)r(x-I)\right)\frac{\partial^2\overline{\pi}}{\partial q^{i2}}+(1-p)\left(1+f\left(\widehat{\pi}\mid\underline{\pi}\right)r(x-I)\right)\frac{\partial^2\underline{\pi}}{\partial q^{i2}} \\ &\quad -pf'\left(\widehat{\pi}\mid\overline{\pi}\right)r(x-I)\left(\frac{\partial\overline{\pi}}{\partial q^i}\right)^2-(1-p)f'\left(\widehat{\pi}\mid\underline{\pi}\right)r(x-I)\left(\frac{\partial\underline{\pi}}{\partial q^i}\right)^2<0 \end{aligned}$$

Again, it is assumed that this is satisfied.

Let us go back to the first-order condition and consider equilibrium quantities. Start by evaluating $\overline{\pi}$ and $\underline{\pi}$ at the Cournot-point (q^c, q^c). Then note that (q^c, q^c) is such that

$$p\frac{\partial\overline{\pi}}{\partial q^i}+(1-p)\frac{\partial\underline{\pi}}{\partial q^i}=0$$

Therefore $V_i^i > 0$ and firm i will have an incentive to increase its quantity beyond the Cournot level when $f(\widehat{\pi}\mid\overline{\pi}) > f(\widehat{\pi}\mid\underline{\pi})$ at the Cournot-point. There will be no incentive to deviate from the Cournot level, $V_i^i = 0$, when $f(\widehat{\pi}\mid\overline{\pi}) = f(\widehat{\pi}\mid\underline{\pi})$ and it will have an incentive to reduce its quantity, $V_i^i < 0$, when $f(\widehat{\pi}\mid\overline{\pi}) < f(\widehat{\pi}\mid\underline{\pi})$. Rearranging the condition one finds

$$f\left(\widehat{\pi}\mid\overline{\pi}\right)>f\left(\widehat{\pi}\mid\underline{\pi}\right)$$

$\Longleftrightarrow$

$$\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(\widehat{\pi}-\overline{\pi})^2}{2\sigma^2}}>\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(\widehat{\pi}-\underline{\pi})^2}{2\sigma^2}}$$

$\Longleftrightarrow$

$$\frac{-\left(\widehat{\pi}-\overline{\pi}\right)^2}{2\sigma^2}>\frac{-\left(\widehat{\pi}-\underline{\pi}\right)^2}{2\sigma^2}$$

$\Longleftrightarrow$

$$\left(\widehat{\pi}-\overline{\pi}\right)^2<\left(\widehat{\pi}-\underline{\pi}\right)^2$$

$\Longleftrightarrow$

$$|\widehat{\pi}-\overline{\pi}|<|\widehat{\pi}-\underline{\pi}|$$

When $\widehat{\pi}=\frac{1}{2}(\overline{\pi}+\underline{\pi})$ one has $|\widehat{\pi}-\overline{\pi}|=|\widehat{\pi}-\underline{\pi}|$, so that $f(\widehat{\pi}\mid\overline{\pi})=f(\widehat{\pi}\mid\underline{\pi})$. Thus when the cut-off is exactly halfway between $\underline{\pi}$ and $\overline{\pi}$, conditional densities are the same and there is no incentive to change the quantity.

When $\widehat{\pi}>\frac{1}{2}(\overline{\pi}+\underline{\pi})$ one has $|\widehat{\pi}-\overline{\pi}|<|\widehat{\pi}-\underline{\pi}|$ and $f(\widehat{\pi}\mid\overline{\pi})>f(\widehat{\pi}\mid\underline{\pi})$. The density conditional on a realization of $\overline{\pi}$ is larger than the density conditional on a realization of $\underline{\pi}$. An increase in output will therefore increase the probability of the profit realization being above the cut-off.

Finally, when $\widehat{\pi}<\frac{1}{2}(\overline{\pi}+\underline{\pi})$ one has $|\widehat{\pi}-\overline{\pi}|>|\widehat{\pi}-\underline{\pi}|$ and $f(\widehat{\pi}\mid\overline{\pi})<f(\widehat{\pi}\mid\underline{\pi})$, so that in this case there is an incentive to reduce the quantity. One therefore has

Proposition 6 *Evaluate $\overline{\pi}$ and $\underline{\pi}$ at the Cournot-point (q^c,q^c). When $\widehat{\pi}>\frac{1}{2}(\overline{\pi}+\underline{\pi})$ one will have $q^i>q^c$ and $q^j<q^c$ in equilibrium. When $\widehat{\pi}=\frac{1}{2}(\overline{\pi}+\underline{\pi})$ one will have $(q^i,q^j)=(q^c,q^c)$. When $\widehat{\pi}<\frac{1}{2}(\overline{\pi}+\underline{\pi})$ one will have $q^i<q^c$ and $q^j>q^c$ in equilibrium.*

To complete the proof, one only needs to reemploy Cramer's Rule for the system of first-order conditions, just as we did in the last section.

The intuition is that when $\widehat{\pi}>\frac{1}{2}(\overline{\pi}+\underline{\pi})$, it pays to increase the spread or variance of the profit distribution. Since at (q^c,q^c) one has $\frac{\partial\underline{\pi}}{\partial q^i}<0$ and $\frac{\partial\overline{\pi}}{\partial q^i}>0$ moving q^i up will decrease $\underline{\pi}$ and increase $\overline{\pi}$, which will make it more likely that $\pi>\widehat{\pi}$. Thus the key property of the profit function under study is again that the variance of the profit distribution is increasing in q^i. This is clear from the fact that the profit distribution satisfies

$$\begin{aligned}
z' &> z\Rightarrow\pi^i\left(z',q^i,q^j\right)>\pi^i\left(z,q^i,q^j\right)\\
z' &> z\Rightarrow\pi_i^i\left(z',q^i,q^j\right)\geq\pi_i^i\left(z,q^i,q^j\right)
\end{aligned}$$

so that

$$\frac{\partial var\left[\widetilde{\pi}\right]}{\partial q^i}=2cov\left[\widetilde{\pi}^i,\widetilde{\pi}_i^i\right]\geq 0$$

as was argued above. It can also be seen more directly by noting that

$$\widetilde{\pi}^i = \left(\widetilde{a}^i - b\left(q^i + q^j\right)\right) q^i - cq^i + \widetilde{\theta}^i$$

implies

$$var\left[\widetilde{\pi}^i\right] = var\left[\widetilde{a}^i\right] q^{i2} + var\left[\widetilde{\theta}^i\right]$$

so that the variance of $\widetilde{\pi}^i$ is increasing in q^i. When the cut-off is in the right tail of the distribution, it will pay the firm to increase the variance of the distribution in a speculative attempt to increase the probability of the profit realisation exceeding the cut-off. This speculative behaviour has two effects. First, it will make it more likely that the firm invests in the second stage. Second, it will let the rival firm decrease its quantity in response to the more aggressive behaviour of firm i.

Comparative statics

As for the comparative statics with respect to $\widehat{\pi}$ one finds unambiguous results when $\overline{\pi} \geq \widehat{\pi} \geq \underline{\pi}$. In this case one finds

$$\begin{aligned}
&\frac{\partial^2}{\partial q^i \partial \widehat{\pi}} \left\{E(\pi) - F(\widehat{\pi}) r(x - I)\right\} \\
= \;& p\frac{1}{\sigma\sqrt{2\pi}} \frac{-(\widehat{\pi} - \overline{\pi})}{\sigma^2} e^{\frac{-(\widehat{\pi}-\overline{\pi})^2}{2\sigma^2}} r(x - I) \frac{\partial \overline{\pi}}{\partial q^i} \\
& + (1 - p) \frac{1}{\sigma\sqrt{2\pi}} \frac{-(\widehat{\pi} - \underline{\pi})}{\sigma^2} e^{\frac{-(\widehat{\pi}-\underline{\pi})^2}{2\sigma^2}} r(x - I) \frac{\partial \underline{\pi}}{\partial q^i} \\
= \;& pf'(\widehat{\pi} \mid \overline{\pi}) r(x - I) \frac{\partial \overline{\pi}}{\partial q^i} + (1 - p) f'(\widehat{\pi} \mid \underline{\pi}) r(x - I) \frac{\partial \underline{\pi}}{\partial q^i} > 0
\end{aligned}$$

It is easy to see that the first-order condition implies that $\frac{\partial \overline{\pi}}{\partial q^i} > 0$ and $\frac{\partial \underline{\pi}}{\partial q^i} < 0$. Therefore when $\overline{\pi} \geq \widehat{\pi} \geq \underline{\pi}$ both terms are positive since the density $f(\cdot \mid \overline{\pi})$ has positive slope at $\widehat{\pi}$, whereas the density $f(\cdot \mid \underline{\pi})$ has negative slope at $\widehat{\pi}$. The overall comparative static effect is therefore positive. Just as in the last section it will therefore again be the case that more severe asymmetric information will lead firm i to compete more aggressively and firm j to respond by competing less aggressively.

3.6 Dividends

So far I have analysed a model where the firm did not actively seek to commit itself to a particular output strategy. The equilibrium outputs differ from the Cournot output simply because both firms anticipate firm i to face a financing problem in the future[13]. It should be pointed out, however, that an ex ante commitment to reduce financial slack may be valuable to firm i. Such a commitment could be brought about by the firm entering into a debt contract at $t = 0$, before it chooses its quantity. It could also be brought about by a commitment to a certain dividend policy. Thus imagine that the firm can, before it chooses its quantity, commit to pay out an amount d to existing shareholders at $t = 1$. Let us assume that the pooling equilibrium is played whenever it exists. Then, when the firm is of the high type, it will invest only if $\pi - d \geq \widehat{\pi}$, i.e. when $\pi \geq \widehat{\pi} - d$. A dividend payout of d increases the cut-off by that same amount. This will have two effects. First, it will reduce the probability that the investment is taken. This negative effect has to be traded off against a positive effect. This comes about since with a higher cut-off, the firm has more of an incentive to increase the variance of the profit distribution by increasing its output. The dividend commits firm i to a more aggressive product market stance, which will lead its rival firm to reduce its quantity and thus benefit firm i. This trade-off may or may not lead to a strictly positive choice of d, depending on the exact parameter specification of the model.

3.7 Conclusion

I have analysed a two-period model where firms first compete in a product market and one of the firms then finances an investment opportunity under conditions of asymmetric information. Special care has been taken to analyse the

[13]This is in contrast to Brander and Lewis (1988) where debt financing precedes the product market competition stage and debt is issued for its commitment value only.

full set of equilibria of the financing game and to motivate an equilibrium selection such that the probability that the firm invests is increasing in the amount of financial slack it has on hand. This introduces a cut-off into the firm's objective function, since it is only for profit realisations above the cut-off that the high type firm will issue and invest. Under these conditions, the firm will not simply maximise the expected value of profit. Rather, it will take into account the consequences its choice of the strategic variable has on the probability that the profit generated exceeds the cut-off. This will lead the firm to consider not only the first moment, but also the second moment of the profit distribution. When the cut-off $\widehat{\pi}$ is high, there is an incentive to speculate and to increase the variance of the profit distribution by increasing its output. The rival anticipates this and responds with a lower output, which will benefit the firm. When the cut-off $\widehat{\pi}$ is low, there is an incentive to hedge and to reduce the variance of the profit distribution by lowering output. The rival will take advantage of this and respond with a higher output, which will harm the firm. One of the main insights of this model is that the fact that a firm has to finance externally does not necessarily worsen the firm's competitive position. In contrast to models where a financially constrained firm faces predation by a deep pocketed rival, in our model a particularly severe financing problem may actually help the firm in making it more aggressive vis à vis its rival. This is an implication of the main comparative statics result of the model, which says that a larger degree of uncertainty, and thus a more severe adverse selection problem, will make the firm a more aggressive competitor as the firm strives to increase the probability of investment. This result may also lend itself to empirical testing. To the extent that smaller firms are surrounded by a larger degree of uncertainty than larger firms, so that size can be taken as a proxy for uncertainty, the model suggests that smaller firms should be more aggressive competitors than larger, more established firms. The results also suggest that smaller firms may be able to survive in an environment in which they are competing against larger firms,

in what way.

precisely because they face a more severe adverse selection problem. Finally, the implications of the model seem consistent with the empirical finding that conglomerates are trading at a discount when measured against "focused" firms[14]. One could argue that in contrast to conglomerates, focused firms have to face the external capital market more often, as there is less scope for cross-subsidisation. In addition, there may be a larger degree of uncertainty surrounding the smaller, focused firm than there is surrounding an established conglomerate. This would imply that more focused firms are the more aggressive competitors and may be one reason why they are more valuable.

[14]See for example Lang and Stulz (1994).

Appendix 3.1

Proof of Lemma 3:

For part a) note first that for all beliefs $\rho \epsilon [0, 1]$ it is optimal for the low type L to set $d = 1$ and invest. To see this, note that the low type will set $d = 1$ whenever

$$s_L + \pi < \left(1 - \frac{I - \pi}{\rho s_H + (1 - \rho) s_L + x}\right) [s_L + x]$$

By inspection the RHS of this condition is increasing in ρ. For $\rho = 0$ the condition reduces to

$$s_L + \pi < \left(1 - \frac{I - \pi}{[s_L + x]}\right) [s_L + x]$$

$\Longleftrightarrow$

$$s_L + \pi < s_L + x - (I - \pi)$$

$\Longleftrightarrow$

$$0 < x - I$$

and is satisfied because the investment opportunity has positive net present value by assumption. Hence $\delta_L = 1$ for all beliefs $\rho \epsilon [0, 1]$. In any equilibrium the low type will invest.

Next note that this implies an upper bound on equilibrium belief ρ.

$$\rho = \frac{r\delta_H}{r\delta_H + (1 - r)\,\delta_L} = \frac{r\delta_H}{r\delta_H + (1 - r)} \leq r$$

Finally, consider the high type H. It is optimal for the high type to invest only if

$$s_H + \pi \leq \left(1 - \frac{I - \pi}{[\rho s_H + (1 - \rho) s_L + x]}\right) [s_H + x]$$

Given the upper bound on ρ, and employing the fact that the RHS of this condition is increasing in ρ one has

$$\left(1 - \frac{I - \pi}{[\rho s_H + (1 - \rho) s_L + x]}\right) [s_H + x] \leq \left(1 - \frac{I - \pi}{[r s_H + (1 - r) s_L + x]}\right) [s_H + x]$$

for all possible equilibrium beliefs $\rho \epsilon [0, r]$. For $\pi < \widehat{\pi}$ one also has

$$s_H + \pi > \left(1 - \frac{I - \pi}{[r s_H + (1 - r) s_L + x]}\right) [s_H + x]$$

so that

$$s_H + \pi > \left(1 - \frac{I - \pi}{[\rho s_H + (1-\rho)\, s_L + x]}\right)[s_H + x]$$

for all possible equilibrium beliefs $\rho \epsilon [0, r]$. This implies that whenever $\pi < \widehat{\pi}$ it will not be optimal for the high type to invest. Hence $\delta_H = 0$ in any equilibrium with $\pi < \widehat{\pi}$. Hence the unique equilibrium has $\delta_H = 0$ and $\delta_L = 1$, which implies $\rho = 0$ and α as shown.

For part b) note first that the argument regarding the equilibrium behaviour of the low type given for part a) goes through regardless of π. When $\pi \geq \widehat{\pi}$, therefore, again the low type will set $\delta_L = 1$ for all $\rho \epsilon [0, 1]$. In the proposed pooling equilibrium the high type also always invests, so that $\delta_H = 1$. Given $\delta_L = 1$ and $\delta_H = 1$, the equilibrium belief must be $\rho = r$. When $\pi \geq \widehat{\pi}$ one has

$$s_H + \pi \leq \left(1 - \frac{I - \pi}{[r s_H + (1-r)\, s_L + x]}\right)[s_H + x]$$

so that it is indeed optimal for the high type to invest given beliefs. This completes the proof.

Proof of Lemma 4:

For part a) recall that $\delta_L = 1$ is optimal for any beliefs. It therefore suffices to show that $\delta_H = 0$ is optimal given $\rho = 0$ and $\pi < \widehat{\widehat{\pi}}$. To see this, one needs to note only that when $\pi < \widehat{\widehat{\pi}}$ one has

$$s_H + \pi \geq \left(1 - \frac{I - \pi}{s_L + x}\right)[s_H + x]$$

For part b) I need to show that the pooling equilibrium is unique. Given that $\delta_L = 1$ is uniquely optimal for all beliefs and all profit levels, one needs to show only that whenever $\pi > \widehat{\widehat{\pi}}$ holds $\delta_H = 1$ is the only optimal choice for the high type for any belief $\rho \epsilon [0, 1]$.

$\delta_H = 1$ is uniquely optimal when

$$s_H + \pi < \left(1 - \frac{I - \pi}{[\rho s_H + (1-\rho)\, s_L + x]}\right)[s_H + x]$$

When $\rho = 0$ this reduces to

$$s_H + \pi < \left(1 - \frac{I - \pi}{s_L + x}\right)[s_H + x]$$

which is satisfied since $\pi > \widehat{\widehat{\pi}}$. When $\rho > 0$

$$\left(1 - \frac{I - \pi}{[\rho s_H + (1 - \rho) s_L + x]}\right) > \left(1 - \frac{I - \pi}{s_L + x}\right)$$

and the condition for unique optimality is again satisfied. Therefore $\delta_H = 1$ whenever $\pi > \widehat{\widehat{\pi}}$ which then implies that $\rho = r$ and α as shown. This completes the proof.

Proof of Lemma 5:

The equation which determines δ_H will deliver a solution $\delta_H \epsilon [0, 1]$ if and only if $\pi \epsilon \left[\widehat{\pi}, \widehat{\widehat{\pi}}\right]$. To see this, note first that with $\delta_H = 0$ one has

$$s_H + \pi = \left(1 - \frac{I - \pi}{[s_L + x]}\right)[s_H + x]$$

whereas with $\delta_H = 1$ one has

$$s_H + \pi = \left(1 - \frac{I - \pi}{[r s_H + (1 - r) s_L + x]}\right)[s_H + x]$$

We know

$$\rho(\delta_H) = \frac{r\delta_H}{r\delta_H + (1 - r)}$$

Hence

$$\rho'(\delta_H) = \frac{r[r\delta_H + (1 - r)] - r\delta_H r}{[r\delta_H + (1 - r)]^2} = \frac{r(1 - r)}{[r\delta_H + (1 - r)]^2} > 0$$

Totally differentiating the indifference condition with respect to π and δ_H one has

$$\left(1 - \frac{[s_H + x]}{[\rho(\delta_H) s_H + (1 - \rho(\delta_H)) s_L + x]}\right) d\pi$$

$$-(I - \pi)[s_H + x] \frac{(s_H - s_L)\rho'(\delta_H)}{[\rho(\delta_H) s_H + (1 - \rho(\delta_H)) s_L + x]^2} d\delta_H = 0$$

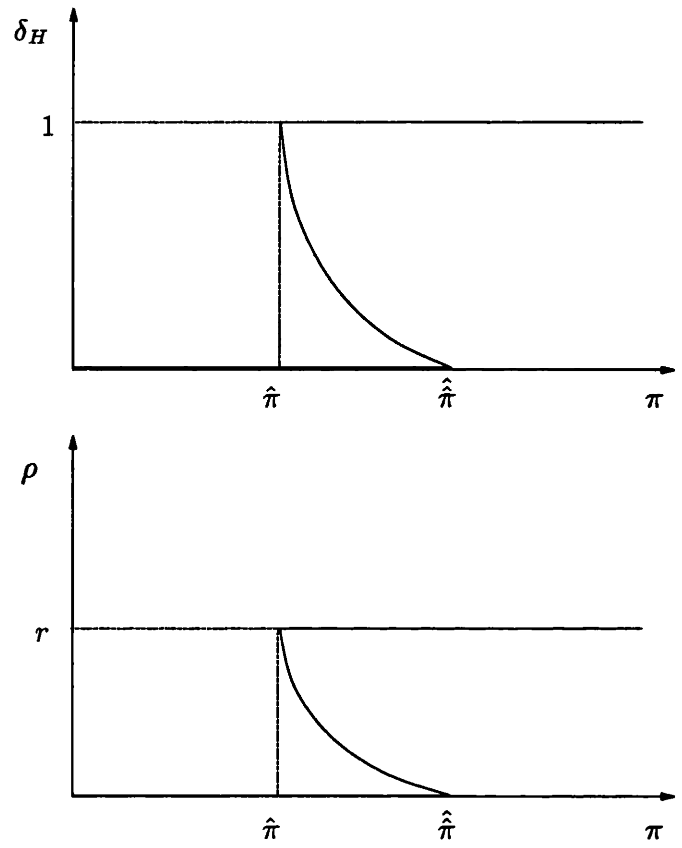
This implies

$$\frac{d\pi}{d\delta_H} = \frac{(I-\pi)\left[s_H + x\right]\frac{(s_H - s_L)\rho'(\delta_H)}{[\rho(\delta_H)s_H + (1-\rho(\delta_H))s_L + x]^2}}{\left(1 - \frac{[s_H + x]}{[\rho(\delta_H)s_H + (1-\rho(\delta_H))s_L + x]}\right)}$$

Since the denominator is negative one has

$$\frac{d\pi}{d\delta_H} < 0$$

This completes the proof.



$$\rho = \frac{r\delta_H}{r\delta_H + (1-r)}$$

Figure 3.1: Second-Stage Equilibria

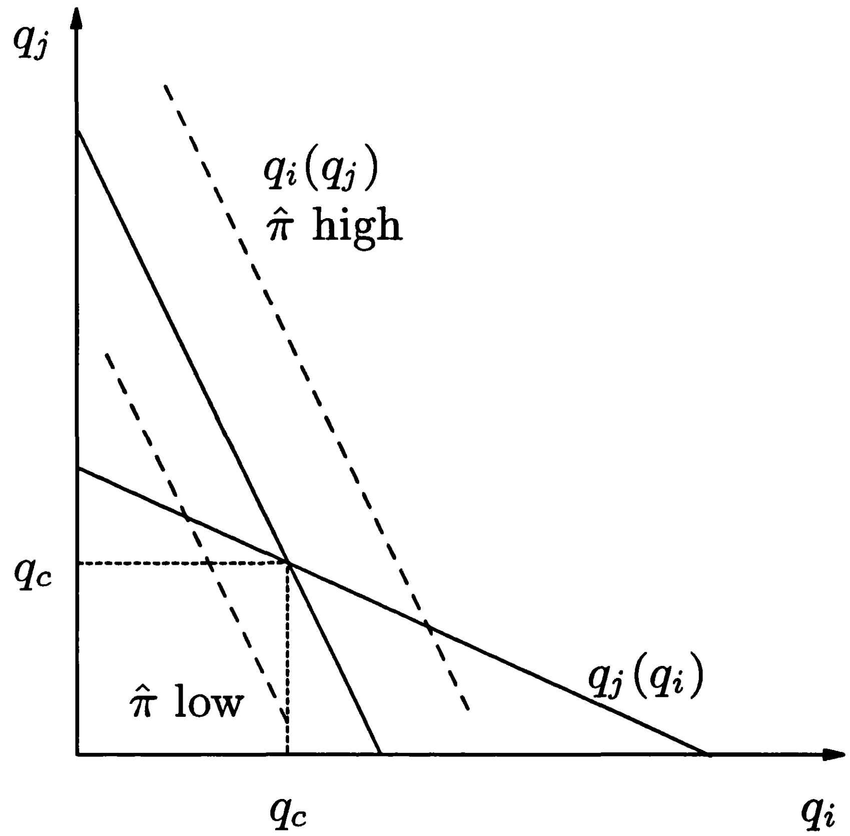


Figure 3.2: The First-Period Equilibrium

Chapter 4

Optimal Managerial Remuneration and Firm-level Diversification

4.1 Introduction

The main motivation for this chapter is to identify a set of circumstances under which firm-level diversification may be beneficial. It is well known that, in a world of risk-averse investors, diversification as such creates benefits by reducing portfolio variance. It is less clear, however, why diversification should be observed at the firm level. After all, if investors are able to hold well - diversified portfolios themselves, why should firm-level diversification add any further value. Traditional arguments which come to mind rely on economies of scope between projects (synergies), a benefit of an internal capital market (Williamson (1975), pp. 147-148), or a reduced probability of incurring bankruptcy costs (Lewellen (1971)).

I will analyse a model which abstracts from all of these arguments and instead takes a contracting problem between financial investors and managers as the starting point. This contracting problem arises from an agency problem which involves both moral hazard and hidden information on the part of the manager. As in the previous chapter, asymmetric information between the firm manager and financial investors is thought of as an important feature of the relationship between financial investor and manager. In the model of this chapter, the manager is able to obtain private information on the profitability of the investment[1]. Financial investors will therefore want the manager to spend effort to investigate the project and then make an informed investment decision in their interest. By designing an appropriate wage contract, financial investors may be able to provide the right incentives for both these managerial decisions. Providing these incentives will, however, be costly; the manager will receive an informational rent over and above the compensation for his effort. I will be able to show that, if the manager is risk-neutral and protected by limited liability, this informational rent will be driven down as one assigns more projects to the manager. This provides a rationale for firm-level diversification which is distinct from the arguments that have traditionally been put forward.

The second motivation for this chapter is to characterize the structure of the optimal incentive scheme for a manager who is in charge of several projects, and whose task is to decide which, if any, of the available projects should be invested in. This is an interesting question in its own right, in that managers may be in charge of several projects for reasons other than the rent-reduction effect identified in this chapter. Thus, for example, one of the main tasks of the CEO of a large company is to decide which projects should be pursued further and which projects should be terminated. The optimal incentive scheme for such a manager is of interest, even if the company is diversified for reasons unrelated to rent reduction. Another example is a fund managers, who necessarily makes

[1]The presence of this kind of hidden information will make the model relevant especially for R&D- intensive industries, such as chemicals and drugs. In these industries managers typically gain private information about the likely return of a project in the course of product development. This information will often be "soft", in the sense that it cannot be verifiably communicated to investors.

joint decisions on a number of assets. In recent years, much research has been devoted to the shape of optimal managerial remuneration schemes in the presence of agency problems. Most of the results give a justification for contracts that are monotone in the return of the project and have led theorists to argue that contracts which involve awarding shares or options, may be taken as approximations of the contracts which are derived as optimal. However, there is an issue of whether these contracts are incentive compatible if manager's task is to make investment decisions on the basis of his private information and when the manager supervises more than one project at the same time[2].

The arguments in this chapter contribute to two strands of literature, one concerned with optimal managerial remuneration schemes and the other concerned with explaining firm-level diversification in a Principal-Agent context. Some authors (e.g. Haubrich (1994)), have taken the simple moral hazard problem with a risk-averse agent, as analysed for example in Grossman and Hart (1983), to be informative on optimal managerial remuneration. In this model, there is a single-dimensional effort choice to be made by the agent, which is stochastically related to observed output. If more effort is put in, output is more likely to be higher in the sense of first-order stochastic dominance. Since the principal wants to elicit effort, the contract will reward high profit outcomes and will therefore typically be monotone in observed returns. When the principal is risk-neutral but the agent is risk-averse, there is scope for insurance. Providing wage insurance will, however, blunt incentives and is therefore costly, which creates the main trade-off in this model.

Holmstrom and Ricart-i-Costa (1986) were one of the first to argue that this model might not fully capture the incentive problems between manager and

[2]Most of the existing Principal-Agent literature assumes that the agent has a single project. Notable exceptions are Holmstrom and Milgrom (1991) on multi-task Principal-Agent analysis, Aghion and Tirole (1994) on real and formal authority, and Diamond (1984), and Williamson (1986) on diversified financial intermediaries.

financial investor. Financial investors, they argue, may be more worried about how effective managers are at making decisions. Stochastic managerial ability is introduced and the focus of the analysis is on managerial career concerns. In a similar spirit, Bhattacharya and Pfleiderer (1985) analyse a model of delegated portfolio management. Financial investor employ a manager for an investment decision in order to make use of the manager's ability to forecast returns. Again, the focus is on unobserved heterogeneity in managerial abilities and optimal screening devices are derived.

Lambert (1988) abstracts from managerial heterogeneity and instead introduces more-dimensional decisions. As in the model of this chapter, the manager first expends effort on gathering information and then makes an investment decision based on his private information. A similar set-up is analysed by Huang and Suarez (1996), who derive a contract which can be interpreted as an option contract assuming risk-neutrality and limited liability, rather than risk-aversion on the part of the manager, as does Lambert (1988). In this chapter, I follow Lambert (1988) and Huang and Suarez (1996) and argue that the relationship between manager and financial investor is not adequately captured by a simple moral hazard problem with a monotone stochastic relationship between an unobservable effort variable and the observable project return. Instead, I analyse a model where hidden information on the part of the manager is a crucial ingredient. The manager is thought of as an insider who, by expending effort, may acquire superior information on the likely return of the project. Then, financial investors want the manager to make an investment decision in the light of this information. The main contribution to the existing literature is that I extend the analysis to the case, where the manager is in charge of several investment projects, rather than a single project, and the manager's task is to make decisions simultaneously on a all of the projects he supervises. This will make the analysis relevant in particular for the manager of a large corporation.

There are two papers which provide a rationale for firm-level diversification in a Principal-Agent context: Aron (1988) and Hermalin and Katz (1996). Aron (1988) analyses a moral hazard problem with a risk-averse manager. The manager is asked to choose an effort variable, which has a noisy, but positive impact on the returns of a production process. Financial investors use realised return as a signal for the effort level chosen. When there are two projects, the manager still chooses a single effort variable which now becomes an input into both processes. This enables financial investors to observe two independent signals of the manager's effort choice, so that the precision of their inference is improved. Aron obtains an optimal extent of diversification by diseconomies of scale in production. Hermalin and Katz (1996) also analyse a pure moral hazard problem, where again the manager is asked to choose an effort level. Diversification is thought of as splitting this effort variable and letting the fractions enter two activities. Again this will under certain conditions improve the informativeness of the observed returns. In both Aron (1988) and Hermalin and Katz (1996), diversification is driven by the fact that projects are technologically related. In Aron (1988) the single effort choice becomes a common input into two different processes, whereas in Hermalin and Katz (1996) it is split in a known ratio and then enters both projects. In contrast to both these papers, in the model of this chapter the projects which the manager is asked to investigate are technologically unrelated. The manager's choices concerning one project do not constrain his choices on any other project. Also, I abstract from insurance issues and assume that both financial investors and manager are risk-neutral. As in Huang and Suarez (1996) the main impediment to first-best contracting here is the fact that the manager is protected by limited liability, so that one cannot impose arbitrarily large punishments to provide incentives.

4.2 The General Framework, N projects

There is one manager who becomes associated with some number N of indivisible projects, indexed by $i = 1, .., N$. Each project requires a financial outlay of I.

There is one financial investor who is endowed with funds sufficient to finance the projects, whereas both the manager's initial wealth and his reservation wage are normalised to zero. Both the financial investor and the manager are assumed to be risk-neutral. The manager is, however, protected by limited liability, so that his wage can never be negative.

The information structure is as follows. The ex ante distribution of the project returns is known to all parties. I assume that returns are distributed independently and identically across projects. For each project the manager can privately choose to expend some nonpecuniary effort to investigate the project. If he spends effort on a project he becomes privately informed whether the project investigated is good or bad. Since this information remains private, the manager will at an interim stage have more information than the financial investor, if he chooses to investigate the project. Thus there will be interim adverse selection or "hidden information". There is moral hazard as well, in that the manager's effort choice is assumed to be unobservable to investors. It turns out to be immaterial, whether the subsequent investment decision itself is assumed to be observable to the financial investor, since for each project he can infer it perfectly from the realised project return, which is both observable and verifiable.

Let us come to the timing of the model. When time starts, the financial investor offers at take-it-or-leave-it contract to the manager, which the manager can accept or reject. If he accepts, the financial investor hands over sums of NI to the manager.

Then the manager privately chooses a vector of effort levels $e = \left(e^1..., e^N\right)$, where it is assumed that for each project the effort level can only take two values, $e^i \in \{0, 1\}$; either the manager investigates a given project, $e^i = 1$, or he does not, $e^i = 0$. The effort cost associated with e is given by $C(e)$ and it is assumed that costs rise linearly with effort, $C(e) = \sum_{i=1}^{N} ce^i$, where $c \in R^+$ is the cost of investigating one project.

Next, a random vector of the projects' prospects $\widetilde{\pi} = \left(\widetilde{\pi}^1, .., \widetilde{\pi}^N\right)$ realises. I assume that $\widetilde{\pi}^i$ is independently and identically distributed across projects and can take two realisations, $\pi^i \in \{\underline{\pi}, \overline{\pi}\}$, where $\overline{\pi} > \underline{\pi}$. I also define $\Pr\left(\pi^i = \overline{\pi}\right) = p$ and $\Pr\left(\pi^i = \underline{\pi}\right) = (1-p)$. The realised π^i has the interpretation of a success probability for project i. With probability p the project is good and has a high probability of success, $\pi^i = \overline{\pi}$, with probability $1-p$ the project the project is bad and has a low probability of success, $\pi^i = \underline{\pi}$.

The manager then privately receives a signal $s\left(e\right) = \left(s^1, ...s^N\right)$ of the success probabilities realised. Its precision depends on the effort expended. In particular, it is assumed that $s^i = \pi^i$ if $e^i = 1$ and $s^i = 0$ if $e^i = 0$, that is, the manager receives a perfect signal on project i if he has expended effort on it, whereas he receives no information pertaining to i, if he did not spend effort on investigating it.

Having observed the signal, the manager makes an investment decision on each project, which is summarised by $d = \left(d^1, ..., d^N\right)$. I want to restrict attention to $d^i \in \{0, 1\}$, the manager either invests, $d^i = 1$, or he does not invest, $d^i = 0$ into a given project. Note that investment involves spending a financial outlay of I, but does not cause further nonpecuniary costs to the manager. Also, it is assumed that the effort choice does not constrain the investment decision, so that "blind" investment is possible[3].

Finally, and observable to both the financial investor and the manager, the vector of project (gross) returns $\widetilde{x} = \left(\widetilde{x}^1, ..., \widetilde{x}^N\right)$ realizes. I assume $x^i \in \{0, R\}$ if $d^i = 1$ and $x^i = I$ if $d^i = 0$. In accordance with the earlier interpretation for π, let $\Pr\left[x^i = R \mid d^i = 1\right] = \pi^i$ and $\Pr\left[x^i = 0 \mid d^i = 1\right] = (1-\pi^i)$, where, recall, $\pi^i \in \{\underline{\pi}, \overline{\pi}\}$. Realised project returns x are handed over from the manager to the investor, who in turn pays the manager a wage w , the size of which can depend on the observed vector x of project returns. Note that if the manager did not

[3] Under the alternative assumption that effort spent is necessary for investment the qualitative results remain the same. The main difference is that some (IC_d) constraints become binding.

invest in a project, he will just return I to the investor, so that the investment decision on any project can be perfectly inferred from its observed return.

As for the profitability of the projects, I assume that $\overline{\pi}R > I$, but $\underline{\pi}R < I$, so that it is efficient to invest at the interim stage, if and only if the project is good. It is also assumed that $p\overline{\pi}R + (1-p)\,I - c > \max\{I, p\overline{\pi}R + (1-p)\,\underline{\pi}R\}$, so that, given the interim investment decision is made efficiently, it is also efficient to investigate each project. One can then distinguish two cases. If $p\overline{\pi}R + (1-p)\,\underline{\pi}R > I$, the project could profitably be undertaken without the manager re-evaluating the project at the interim stage. On the other hand, if $p\overline{\pi}R + (1-p)\,\underline{\pi}R < I$, the manager's job of re-evaluating the project and aborting it, if it turns out to be bad, is necessary for the project to be profitable ex ante.

Resulting from this set-up, given any contract between the financial investor and the manager, the manager has a number of strategies available to him. Let us denote these by $(e, d(\cdot))$. He can choose which, if any, projects to look at, $e \in \{0,1\}^N$, and choose any function mapping the set of possible signals received into the set of investment decisions, $d(\cdot) : \{0, \underline{\pi}, \overline{\pi}\}^N \rightarrow \{0,1\}^N$. Since the financial investor is assumed to have all the bargaining power, he will be interested in implementing the efficient strategy $(e^*, d^*(\cdot))$. This is defined by $e^i = 1\ \forall i$ and $d^{i*} = 1$ if $s^i = \overline{\pi}$ and $d^{i*} = 0$ if $s^i = \underline{\pi}\ \forall i$, i.e. investigate all projects and invest only if the signal is favourable. Let us assume for now that the investor wants to give the manager incentives to choose this strategy and that he wants to do this as cheaply as possible. The investor's problem is then to choose a wage schedule $w(\cdot)$ to maximise return net of wages, making sure that the manager's expected wage compensates for the effort costs incurred, that the wage schedule induces the manager to voluntarily choose the efficient strategy, and finally, that the manager never receives a negative wage.

One can write down the investor's problem as follows:

$$\max_{w(\cdot)} E\left[\widetilde{x} - w(\widetilde{x}) \mid \widetilde{s} = \widetilde{\pi}, d = d^*(\widetilde{\pi})\right]$$

s.t.

(IC_d) :

$$E\left[w\left(\widetilde{x}\right) \mid s=\pi,\ d=d^{*}\left(\pi\right)\right] \geq$$
$$E\left[w\left(\widetilde{x}\right) \mid s=\pi,\ d=d\left(\pi\right)\right]\ \forall\pi,\forall d\left(\cdot\right)$$

(IC_e) :

$$E\left[w\left(\widetilde{x}\right) \mid \widetilde{s}=\widetilde{\pi},d=d^{*}\left(\widetilde{\pi}\right)\right]-Nc \geq$$
$$E\left[w\left(\widetilde{x}\right) \mid \widetilde{s}=\widetilde{s}\left(e\right),d=d\left(\widetilde{s}\right)\right]-\textstyle\sum_{i=1}^{N} ce^{i}\ \ \forall e,\forall d\left(\cdot\right)$$

(IR) :

$$E\left[w\left(\widetilde{x}\right) \mid \widetilde{s}=\widetilde{\pi},d=d^{*}\left(\widetilde{\pi}\right)\right]-Nc \geq 0$$

(NNW) :

$$w\left(x\right) \geq 0\ \forall x$$

If the manager chooses the efficient strategy, so that $\widetilde{s}=\widetilde{\pi}$ and $d=d^{*}\left(\widetilde{\pi}\right)$, this will induce a certain ex ante distribution over returns and thus over wages. Taking this distribution as given, the financial investor maximises his payoff by minimising the expected wage to be paid to the manager. This distribution will, however, only obtain, if the manager voluntarily chooses the efficient strategy. This is what is ensured by the (IC_d) and (IC_e) constraints. The (IC_d) constraint ensures that, given the manager has investigated all projects, he makes efficient interim investment decisions, whereas the (IC_e) constraint ensures that, ex ante, investigating all projects and then choosing $d^{*}\left(\cdot\right)$ is superior to any other strategy in terms of expected wage net of effort costs. (IR) then makes sure that, again ex ante, the manager gets compensated for his effort cost if he chooses the efficient strategy. Lastly, the non-negative wage (NNW)constraint is a limited liability constraint that forces all wages to be non-negative.

Rather than proceeding directly to a derivation of the general solution to this problem, it seems interesting to see what can be learned from the basic case, where the manager is given a single project.

4.3 One Project per Manager, N=1

In this case there is a single indivisible project which requires a financial investment I and yields a gross return $x \in \{0, R\}$. With probability p the project is good, i.e. $\Pr[x = R] = \overline{\pi}$ and $\Pr[x = 0] = (1 - \overline{\pi})$, and with probability $(1 - p)$ the project is bad , i.e. $\Pr[x = R] = \underline{\pi}$ and $\Pr[x = 0] = 1 - \underline{\pi}$, where $\overline{\pi} > \underline{\pi}$. The project is profitable, if it is good, $\overline{\pi}R > I$, but unprofitable, if it is bad, $\underline{\pi}R < I$. It is worthwhile ex ante to investigate the project, given that investment efficiently conditions on the signal, $p\overline{\pi}R + (1 - p)\, I - c > \max\{I, p\overline{\pi}R + (1 - p)\, \underline{\pi}R\}$.

The financial investor is interested in implementing the efficient strategy. That is, he would like the manager to first investigate the project, $e^* = 1$, and that, if he sees the favourable signal, to go ahead with the investment, $d^*(\overline{\pi}) = 1$, whereas if sees a bad signal, to abort the project, $d^*(\underline{\pi}) = 0$, and to return the outlay I back to the financial investor. To implement this behaviour the investor solves the following problem:

$$\max_{w(0), w(I), w(R)} p\left[\overline{\pi}\left(R - w(R)\right) + (1 - \underline{\pi})\left(-w(0)\right)\right] + (1 - p)\left[I - w(I)\right] - I$$

$$s.t.$$

$$(IC_d)\ \ (d(\overline{\pi}) = 1)\ \ \overline{\pi}w(R) + (1 - \overline{\pi})\, w(0) \geq w(I)$$

$$(IC_d)\ \ (d(\underline{\pi}) = 0)\ \ w(I) \geq \underline{\pi}w(R) + (1 - \underline{\pi})\, w(0)$$

$$(IC_e)$$

$$(1)\ \ p\left[\overline{\pi}w(R) + (1 - \overline{\pi})\, w(0)\right] + (1 - p)\, w(I) - c \geq w(I)$$

$$(2)\ \ p\left[\overline{\pi}w(R) + (1 - \overline{\pi})\, w(0)\right] + (1 - p)\, w(I) - c \geq$$
$$p\left[\overline{\pi}w(R) + (1 - \overline{\pi})\, w(0)\right] + (1 - p)\left[\underline{\pi}w(R) + (1 - \underline{\pi})\, w(0)\right]$$

$$(IR)\ \ p\left[\overline{\pi}w(R) + (1 - \overline{\pi})\, w(0)\right] + (1 - p)\, w(I) - c \geq 0$$

$$(NNW)\ \ w(0) \geq 0, w(I) \geq 0, w(R) \geq 0$$

In this basic problem, the manager has two possible investment decisions available to him. The (IC_d) constraints ensure that, given that the manager has spent effort and thus received a signal $s \in \{\overline{\pi}, \underline{\pi}\}$, he makes the right investment decision in response to the signal. Next, if the manager does not spend effort, and accordingly receives $s = 0$, he again has two possible options. He can either abstain from investing and return I to the investor, or invest blindly. (IC_e) (1) and (2) ensure that both these options are less worthwhile to the manager than following the efficient strategy. Next, (IR) makes sure that the expected wage induced by the efficient strategy is larger than the effort cost c. Finally, all wages have to be non-negative (NNW).

The optimal contract is

$$\begin{aligned} w(R) &= \frac{c}{p\overline{\pi} + (1-p)A - A} \\ w(I) &= \frac{Ac}{p\overline{\pi} + (1-p)A - A} \\ w(0) &= 0 \end{aligned}$$

where $A = p\overline{\pi} + (1-p)\underline{\pi}$ is the expected success probability.

The proof of this is straightforward and outlined here. Since there are three variables, three constraints will be binding First of all, note that $w(0) = 0$ will be one of the binding constraints. To see why, assume $w(0) > 0$. Then one can decrease $w(0)$ by some $\varepsilon \leq w(0)$ and increase $w\ (R)$ by $\frac{(1-\overline{\pi})}{\overline{\pi}}\varepsilon$. This will leave all constraints satisfied, can be done costlessly and relaxes (IC_d) $(d(\underline{\pi}) = 0)$, as well as $(IC_e)(2)$ since

$$\underline{\pi}\frac{(1-\overline{\pi})}{\overline{\pi}}\varepsilon - (1-\underline{\pi})\varepsilon < 0$$

which is true, since $\overline{\pi} > \underline{\pi}$. Also, one sees that (IR) will not be binding, since it is implied by $(IC_e)(1)$ and $w(I) \geq 0$. In fact $w(I) > 0$ necessarily, because otherwise $w(R) = w(0) = 0$ from (IC_d) $(d(\underline{\pi}) = 0)$ and (NNW). One also sees that (IC_d) $(d(\overline{\pi}) = 1)$ must be slack, since if it were binding $(IC_e)(1)$ would be violated. Likewise (IC_d) $(d(\underline{\pi}) = 0)$ will not be binding, since if it were, $(IC_e)(2)$ would be violated. Next, $w(R) > 0$, since otherwise one would have $(1-\overline{\pi})w(0) \geq w(I) \geq (1-\underline{\pi})w(0)$, combining$(IC_d)$ $(d(\overline{\pi}) = 1)$ and (IC_d) $(d(\underline{\pi}) = 0)$. This could be true only if $w(I) = w(0) = 0$, contradicting $w(I) > 0$. This leaves $(IC_e)(1)$ and (2) as the only possible further binding constraints. Substituting $w(0) = 0$ into these two constraints one finds

$$\begin{aligned} & p\overline{\pi}w(R) + (1-p)\,w(I) - c \\ = \;& w(I) \\ = \;& [p\overline{\pi} + (1-p)\,\underline{\pi}]\,w(R) \end{aligned}$$

which is easily solved for the remaining two wages.

As a first observation, notice that the optimal wage schedule is monotone in return,

$$0 = w(0) < w(I) < w(R).$$

Observe also that, under the optimal contract, the manager's expected wage payment exceeds his effort cost c. This excess payment can be interpreted as his informational rent and it can be read off from the RHS of the binding $(IC_e)\,(1)$ constraint as being equal to $w(I)$. Thus,

$$rent = w(I) = \frac{Ac}{p\overline{\pi} + (1-p)\,A - A}$$

The rent arises because of the interplay between the moral hazard and the hidden information components of the agency problem. Notice in particular, that the financial investor has to reward a gross return of I with a positive wage. Otherwise the manager would always invest, and never spend any effort.

The informational rent would not arise in the benchmark case, where s is publicly observable. One could then pay the manager a wage $w(x,s)$. Clearly then, $w(I,\underline{\pi}) = w(R,\overline{\pi}) = w(0,\overline{\pi}) = c$, $w(I,\overline{\pi}) = w(R,\underline{\pi}) = w(0,\underline{\pi}) = 0$, and $w(x,0) = 0\ \forall x$ would implement the efficient strategy and would involve a zero rent. One may ask, however, whether the wage contract derived above can be improved upon by letting the manager make explicit and verifiable claims about the signal he received. The contract could then condition both on the return x and the announced signal $\widehat{s}$, resulting in a wage schedule $w(x,\widehat{s})$. It is shown in the appendix (Appendix 4.1) that this is not the case. The wage contract derived above remains optimal, when one allows the manager to make those explicit claims to the financial investor and d is viewed as an observable and hence contractible variable. This result establishes that the wage contract derived above is equivalent to a direct revelation mechanism and hence is optimal in the class of all possible mechanisms implementing the efficient effort and investment choice.

Note that when the optimal contract is used to implement the efficient strategy, the rent accruing to the manager may be so large that the financial investor will not want to implement the efficient strategy. This will be the case if and only if

$$p\overline{\pi}R + (1-p)\,I - c - \frac{Ac}{p\overline{\pi} + (1-p)\,A - A} < \max\left\{I, p\overline{\pi}R + (1-p)\,\underline{\pi}R\right\}$$

where the LHS is the expected return net of wage costs from implementing the efficient strategy. To understand the RHS, note first that since $\underline{\pi}R < I$ implies $p\underline{\pi}R + (1-p)\,I < I$, it does not make sense to implement inefficient investment, $d\left(\underline{\pi}\right) = 1$ and $d\left(\overline{\pi}\right) = 0$, even if this could be implemented at zero wage costs. One is therefore left with implementing either no investment, $d\left(s\right) = 0$ $\forall s$, or blind investment, $d\left(s\right) = 1$ $\forall s$. Observe that none of $d\left(s\right) = 0$ $\forall s$ and $d\left(s\right) = 1$ $\forall s$ can be implemented in conjunction with $e = 1$, since there is no way of telling whether the manager has chosen $e = 1$ or $e = 0$. Since neither $d = 1$ nor $d = 0$ involves any cost to the manager, both of these two possibilities are optimally implemented by offering the manager a flat wage of zero. Therefore, if $p\overline{\pi}R + (1-p)\,\underline{\pi}R > I$ and implementing the efficient strategy is not viable, the financial investor will implement blind investment, $e = 0$ and $d = 1$, whereas if $p\overline{\pi}R + (1-p)\,\underline{\pi}R < I$ the financial investor will implement no investment $e = 0$ and $d = 0$. Therefore, when the above inequality holds, the financial investor will not implement the efficient strategy, since he is better off implementing either no investment or blind investment. Conversely, if the inequality is reversed, the financial investor will implement the efficient strategy, rather than no investment or blind investment.

Note that the RHS can also be interpreted as what the financial investor can guarantee himself without employing a manager. Therefore, another way of viewing the result is that, if the financial investor does employ a manager, he will never implement anything other than the efficient strategy.

Before going on to the case of more than one project, let us briefly analyse the possibility of approximating the efficient contract with some of the incentive tools which are used in practice, such as bonus schemes, options and shares. It seems interesting, in particular, which of these mechanisms are incentive compatible and will therefore be able to implement the efficient strategy.

Consider first offering the manager a flat wage $\overline{w}$ and a bonus b for a return of R. Then the wage schedule will be $w(0) = \overline{w}$, $w(I) = \overline{w}$, $w(R) = \overline{w} + b$. One sees immediately, that under such a contract $\underline{\pi} w(R) + (1 - \underline{\pi}) w(0) > w(I)$, so that both (IC_d) $(d(\underline{\pi}) = 0)$ and $(IC_e)(2)$ are violated. The manager will invest even after observing that the project is bad. What is more, the manager will always prefer to invest blindly, rather than spending effort on becoming informed. The bonus scheme is therefore not incentive compatible and can be seen to induce excessive risk-taking by the manager. The reason is that it does not adequately reward no investment for a return of I.

Next, consider an option contract. In practice, managers are often awarded options with a strike equal to the expected return of the firm. If the option is made exercisable when x has realised, one would have $w(x) = \delta(x - E^*[x])^+$, where $E^*[x] = p\overline{\pi}R + (1-p)I$. Since $E^*[x] > I > 0$, the manager will only exercise his option when $x = R$, so that the wage is again flat for realisations other than R. Since $w(I) = w(0) = 0$, but $w(R) > 0$ one again finds both IC_d $d(\underline{\pi}) = 0$ and $IC_e(2)$ violated. Again, under such a contract, one would expect to see excessive risk-taking by the manager. Note that both the option contract and the bonus scheme would be incentive compatible in a model where there is a simple increasing relationship between the manager's effort choice and the expected project return, as would be the case in a pure moral hazard model. One should note, therefore, that such a model might be misleading if the manager's task is to make an informed investment decision in the interest of the financial investor, as it is assumed here.

Finally, consider the possibility to promise the manager a certain fraction α of the ex post return, that is, to award the manager shares in the company. Then the return contingent payment to the manager is just $w(x) = \alpha x$. Given that $\overline{\pi}R > I$ and that $\underline{\pi}R < I$, one sees immediately that both (IC_d) constraints are satisfied for any $\alpha \in [0, 1]$. The (IC_e) constraints reduce to

(1) $\alpha[p\overline{\pi}R + (1-p)I] - c \geq \alpha I$

(2) $\alpha[p\overline{\pi}R + (1-p)I] - c \geq \alpha\ [p\overline{\pi}R + (1-p)\underline{\pi}R]$

implying a lower bounds on α, which is given by

$$\alpha \geq \frac{c}{\min\{p(\overline{\pi}R - I),\ (1-p)(I - \underline{\pi}R)\}}$$

If α is larger than this lower bound, the share contract will be incentive compatible, so that, in principle, it is possible to implement the efficient strategy using a share contract. Observe, however, that under a share contract the financial investor can expect a net return of at most $(1-\alpha)E^*[x] - I$. This expression may well be negative, in which case the efficient strategy cannot be profitably implemented using a share contract, whereas it could possibly be profitably implemented using the optimal contract.

4.4 Two Projects per Manager, N=2

Let us now examine the case where the manager is assigned two identical projects. The contract can then condition on the vector of observable returns, so that the following matrix of wages needs to be determined:

$$\begin{array}{ccc} w(0,0) & w(0,I) & w(0,R) \\ w(I,0) & w(I,I) & w(I,R) \\ w(R,0) & w(R,I) & w(R,R) \end{array}$$

However, since the projects are *iid*, it is clear that the optimal wage schedule will exhibit symmetry, in that $w(x,y) = w(y,x) \equiv w(x+y)$ so that only the following six wages need to be found[4]:

$$w(0), w(I), w(R), w(2I), w(I+R), w(2R)$$

.If the manager chooses the efficient strategy, this will induce a probability distribution over returns and thus over wages. The financial investor will receive an expected return and will have to pay an expected wage which is given by

$$\begin{aligned}
&E^*[w] \equiv \\
&p^2\left[(1-\overline{\pi})^2 w(0) + 2\overline{\pi}(1-\overline{\pi}) w(R) + \overline{\pi}^2 w(2R)\right] \\
&+2p(1-p)\left[(1-\overline{\pi}) w(I) + \overline{\pi} w(I+R)\right] \\
&+(1-p)^2 w(2I)
\end{aligned}$$

The financial investor's problem is to minimise this expression by choice of $\{w(0), .., w(2R)\}$. This minimisation is subject to a number of constraints, which will be introduced as we go along.

A first subset of constraints is given by the (IC_d) constraints. Given that the manager has spent effort on both projects, these constraints make sure that the manager makes the right investment decision for each possible vector of signals received, $(\overline{\pi}, \overline{\pi})$ $(\overline{\pi}, \underline{\pi})$ $(\underline{\pi}, \overline{\pi})$ $(\underline{\pi}, \underline{\pi})$. Given any of these vectors of signals, exactly one of the possible four strategies $(0,0), (0,1), (1,0), (1,1)$ is efficient. To ensure efficient investment, one must have that, for each signal, the expected wage under the efficient investment strategy is weakly larger than under any other strategy [5]:

[4]This notation can be used for the two-project case, as long as $R \neq 2I$. An alternative is to count the number of $I's$ and the number of $R's$, which is the convention used for general case. Then $w(0)$, $w(I)$, $w(R)$, $w(2I)$, $w(I+R)$, $w(2R)$ translates into $w(0,0)$, $w(0,1)$, $w(1,0)$, $w(0,2)$, $w(1,1)$, $w(2,0)$, where the first argument gives the number of $R's$ and the second argument gives the number of $I's$ in the return vector.

[5]It is easily checked that the constraint for $(\underline{\pi}, \overline{\pi})$ is exactly the same as for $(\overline{\pi}, \underline{\pi})$ and it is therefore omitted. As for the notation, inequality signs are understood to relate to the top line.

(IC_d)

$(d(\overline{\pi},\overline{\pi}) = (1,1))$:

$$(1-\overline{\pi})^2 w(0,0) + 2\overline{\pi}(1-\overline{\pi}) w(0,R) + \overline{\pi}^2 w(R,R)$$

$(1) \geq w(2I)$

$(2) \geq (1-\overline{\pi}) w(0,I) + \overline{\pi} w(I,R)$

$(d(\overline{\pi},\underline{\pi}) = (1,0))$:

$$(1-\overline{\pi}) w(0,I) + \overline{\pi} w(I,R)$$

$(1) \geq w(2I)$

$(2) \geq (1-\underline{\pi}) w(I) + \underline{\pi} w(I+R)$

$(3) \geq (1-\underline{\pi})(1-\overline{\pi}) w(0) + [(1-\underline{\pi})\overline{\pi} + (1-\overline{\pi})\underline{\pi}] w(R) + \overline{\pi}\underline{\pi} w(2R)$

$(d(\underline{\pi},\underline{\pi}) = (0,0))$:

$$w(2I)$$

$(1) \geq (1-\underline{\pi}) w(I) + \underline{\pi} w(I+R)$

$(2) \geq (1-\underline{\pi})^2 w(0) + 2(1-\underline{\pi})\underline{\pi} w(R) + \underline{\pi}^2 w(2R)$

Let us now come to the set of (IC_e) constraints. These ensure that the wage net of effort costs the manager can expect when he chooses to spend effort on both projects, $e = (1,1)$, and then invests efficiently, is larger than the wage net of effort costs the manager can expect under any other effort choice $(0,0)$, $(0,1)$, $(1,0)$ and all possible subsequent investment strategies. Let us start with the set of constraints that discourage $e = (0,0)$. Given this effort choice, the manager does not receive any information on the projects, $s = (0,0)$, and may choose among

$$(d(0,0)) \ \epsilon \ \{(0,0)(0,1)(1,0)(1,1)\}$$

Since the wage contract is symmetric $d(0,0) = (0,1)$ and $d(0,0) = (1,0)$ will result in the same expected payoff, so that one can write down the following three constraints:

(IC_e) $(e \neq (0,0))$:

$E^{*}[w]-2c$

$(1) \geq w(2I)$

$(2) \geq p[(1-\overline{\pi})w(I)+\overline{\pi}w(I+R)]+(1-p)[(1-\underline{\pi})w(I)+\underline{\pi}w(I+R)]$

$(3) \geq p^{2}\left[(1-\overline{\pi})^{2}w(0)+2\overline{\pi}(1-\overline{\pi})w(R)+\overline{\pi}^{2}w(2R)\right]$
$+2p(1-p)[(1-\underline{\pi})(1-\overline{\pi})w(0)+[(1-\underline{\pi})\overline{\pi}+(1-\overline{\pi})\underline{\pi}]w(R)+\overline{\pi}\underline{\pi}w(2R)]$
$+(1-p)^{2}\left[(1-\underline{\pi})^{2}w(0)+2(1-\underline{\pi})\underline{\pi}w(R)+\underline{\pi}^{2}w(2R)\right]$

One will also have to discourage strategies that involve the manager becoming informed on one of the two projects only, that is, the manager being lazy on one project. By symmetry, the constraints discouraging $e=(0,1)$ and $e=(1,0)$ will be identical and one can focus on $e=(1,0)$. Given this effort choice, the manager will receive a signal $s \epsilon \ \{(\underline{\pi},0)(\overline{\pi},0)\}$ and can therefore condition his investment decision on the signal received. Thus, if $e=(1,0)$, say, the following strategies are possible : $\binom{d(\overline{\pi},0)}{d(\underline{\pi},0)}\epsilon$

$$\begin{matrix}
\begin{pmatrix}0,0\\0,0\end{pmatrix}, & \begin{pmatrix}0,0\\1,0\end{pmatrix}, & \begin{pmatrix}1,0\\0,0\end{pmatrix}, & \begin{pmatrix}1,0\\1,0\end{pmatrix}\\
\begin{pmatrix}0,0\\0,1\end{pmatrix}, & \begin{pmatrix}0,0\\1,1\end{pmatrix}, & \begin{pmatrix}1,0\\0,1\end{pmatrix}, & \begin{pmatrix}1,0\\1,1\end{pmatrix}\\
\begin{pmatrix}0,1\\0,0\end{pmatrix}, & \begin{pmatrix}0,1\\1,0\end{pmatrix}, & \begin{pmatrix}1,1\\0,0\end{pmatrix}, & \begin{pmatrix}1,1\\1,0\end{pmatrix}\\
\begin{pmatrix}0,1\\0,1\end{pmatrix}, & \begin{pmatrix}0,1\\1,1\end{pmatrix}, & \begin{pmatrix}1,1\\0,1\end{pmatrix}, & \begin{pmatrix}1,1\\1,1\end{pmatrix}
\end{matrix}$$

This matrix of possible investment strategies which are associated with deviations to $e=(1,0)$ will result in sixteen constraints. To illustrate and for ease of reference, let us write out the constraints associated with these strategies explicitly.

The first row translates into the following four constraints.

$(IC_{e})\,(e \neq (1,0))$

$E^{*}[w]-2c$

$(1) \geq w(2I){-}c$

$(2) \geq p^2 w(2I)$
$+p(1-p)w(2I)$
$+(1-p)p[(1-\underline{\pi})w(I)+\underline{\pi}w(I+R)]$
$+(1-p)^2[(1-\underline{\pi})w(I)+\underline{\pi}w(I+R)]-c$

$(3) \geq p^2[(1-\overline{\pi})w(I)+\overline{\pi}w(I+R)]$
$+p(1-p)[(1-\overline{\pi})w(I)+\overline{\pi}w(I+R)]$
$+(1-p)pw(2I)$
$+(1-p)^2 w(2I)-c$

$(4) \geq p^2[(1-\overline{\pi})w(I)+\overline{\pi}w(I+R)]$
$+p(1-p)[(1-\overline{\pi})w(I)+\overline{\pi}w(I+R)]$
$+(1-p)p[(1-\underline{\pi})w(I)+\underline{\pi}w(I+R)]$
$+(1-p)^2[(1-\underline{\pi})w(I)+\underline{\pi}w(I+R)]-c$

The constraints associated with the second row are

$(IC_e)\ (e \neq (1,0))$
$E^*[w]-2c$
$(5) \geq p^2 w(2I)$
$+p(1-p)w(2I)$
$+(1-p)p[(1-\overline{\pi})w(I)+\overline{\pi}w(I+R)]$
$+(1-p)^2[(1-\underline{\pi})w(I)+\underline{\pi}w(I+R)]-c$

$(6) \geq p^2 w(2I)$
$+p(1-p)w(2I)$
$+(1-p)p[(1-\underline{\pi})(1-\overline{\pi})w(0)+[(1-\underline{\pi})\overline{\pi}+(1-\overline{\pi})\underline{\pi}]w(R)+\overline{\pi}\underline{\pi}w(2R)]$
$+(1-p)^2\left[(1-\underline{\pi})^2 w(0)+2(1-\underline{\pi})\underline{\pi}w(R)+\underline{\pi}^2 w(2R)\right]-c$

$(7) \geq p^2[(1-\overline{\pi})w(I)+\overline{\pi}w(I+R)]$
$+p(1-p)[(1-\overline{\pi})w(I)+\overline{\pi}w(I+R)]$
$+(1-p)p[(1-\overline{\pi})w(I)+\overline{\pi}w(I+R)]$
$+(1-p)^2[(1-\underline{\pi})w(I)+\underline{\pi}w(I+R)]-c$

$(8) \geq p^2 \left[(1-\overline{\pi}) w(I) + \overline{\pi} w(I+R) \right]$

$+p(1-p) \left[(1-\overline{\pi}) w(I) + \overline{\pi} w(I+R) \right]$

$+(1-p)p \left[(1-\underline{\pi})(1-\overline{\pi}) w(0) + \left[(1-\underline{\pi})\overline{\pi} + (1-\overline{\pi})\underline{\pi} \right] w(R) + \overline{\pi}\underline{\pi} w(2R) \right]$

$+(1-p)^2 \left[(1-\underline{\pi})^2 w(0) + 2(1-\underline{\pi})\underline{\pi} w(R) + \underline{\pi}^2 w(2R) \right] - c$

The constraints associated with the third row are

$(IC_e)\ (e \neq (1,0))$

$E^*[w] - 2c$

$(9) \geq p^2 \left[(1-\overline{\pi}) w(I) + \overline{\pi} w(I+R) \right]$

$+p(1-p) \left[(1-\underline{\pi}) w(I) + \underline{\pi} w(I+R) \right]$

$+(1-p) p w(2I)$

$+(1-p)^2 w(2I) - c$

$(10) \geq p^2 \left[(1-\overline{\pi}) w(I) + \overline{\pi} w(I+R) \right]$

$+p(1-p) \left[(1-\underline{\pi}) w(I) + \underline{\pi} w(I+R) \right]$

$+(1-p)p \left[(1-\underline{\pi}) w(I) + \underline{\pi} w(I+R) \right]$

$+(1-p)^2 \left[(1-\underline{\pi}) w(I) + \underline{\pi} w(I+R) \right] - c$

$(11) \geq p^2 \left[(1-\overline{\pi})^2 w(0) + 2\overline{\pi}(1-\overline{\pi}) w(R) + \overline{\pi}^2 w(2R) \right]$

$+p(1-p) \left[(1-\underline{\pi})(1-\overline{\pi}) w(0) + \left[(1-\underline{\pi})\overline{\pi} + (1-\overline{\pi})\underline{\pi} \right] w(R) + \overline{\pi}\underline{\pi} w(2R) \right]$

$+(1-p) p w(2I)$

$+(1-p)^2 w(2I) - c$

$(12) \geq p^2 \left[(1-\overline{\pi})^2 w(0) + 2\overline{\pi}(1-\overline{\pi}) w(R) + \overline{\pi}^2 w(2R) \right]$

$+p(1-p) \left[(1-\underline{\pi})(1-\overline{\pi}) w(0) + \left[(1-\underline{\pi})\overline{\pi} + (1-\overline{\pi})\underline{\pi} \right] w(R) + \overline{\pi}\underline{\pi} w(2R) \right]$

$+(1-p)p \left[(1-\underline{\pi}) w(I) + \underline{\pi} w(I+R) \right]$

$+(1-p)^2 \left[(1-\underline{\pi}) w(I) + \underline{\pi} w(I+R) \right] - c$

Finally, there are four constraints associated with deviations to strategies given by the last row.

$(IC_e)\ (e \neq (1,0))$

$E^*[w] - 2c$

$$(13) \geq p^2 \left[(1-\overline{\pi})\, w(I) + \overline{\pi} w(I+R)\right]$$
$$+p(1-p)\left[(1-\underline{\pi})\, w(I) + \underline{\pi} w(I+R)\right]$$
$$+(1-p)\,p\left[(1-\overline{\pi})\, w(I) + \overline{\pi} w(I+R)\right]$$
$$+(1-p)^2\left[(1-\underline{\pi})\, w(I) + \underline{\pi} w(I+R)\right] - c$$

$$(14) \geq p^2 \left[(1-\overline{\pi})\, w(I) + \overline{\pi} w(I+R)\right]$$
$$+p(1-p)\left[(1-\underline{\pi})\, w(I) + \underline{\pi} w(I+R)\right]$$
$$+(1-p)\,p\left[(1-\underline{\pi})(1-\overline{\pi})\, w(0) + \left[(1-\underline{\pi})\overline{\pi} + (1-\overline{\pi})\underline{\pi}\right] w(R) + \overline{\pi}\underline{\pi} w(2R)\right]$$
$$+(1-p)^2\left[(1-\underline{\pi})^2 w(0) + 2(1-\underline{\pi})\underline{\pi} w(R) + \underline{\pi}^2 w(2R)\right] - c$$

$$(15) \geq p^2 \left[(1-\overline{\pi})^2 w(0) + 2\overline{\pi}(1-\overline{\pi})\, w(R) + \overline{\pi}^2 w(2R)\right]$$
$$+p(1-p)\left[(1-\underline{\pi})(1-\overline{\pi})\, w(0) + \left[(1-\underline{\pi})\overline{\pi} + (1-\overline{\pi})\underline{\pi}\right] w(R) + \overline{\pi}\underline{\pi} w(2R)\right]$$
$$+(1-p)\,p\left[(1-\overline{\pi})\, w(I) + \overline{\pi} w(I+R)\right]$$
$$+(1-p)^2\left[(1-\underline{\pi})\, w(I) + \underline{\pi} w(I+R)\right] - c$$

$$(16) \geq p^2 \left[(1-\overline{\pi})^2 w(0) + 2\overline{\pi}(1-\overline{\pi})\, w(R) + \overline{\pi}^2 w(2R)\right]$$
$$+p(1-p)\left[(1-\underline{\pi})(1-\overline{\pi})\, w(0) + \left[(1-\underline{\pi})\overline{\pi} + (1-\overline{\pi})\underline{\pi}\right] w(R) + \overline{\pi}\underline{\pi} w(2R)\right]$$
$$+(1-p)\,p\left[(1-\underline{\pi})(1-\overline{\pi})\, w(0) + \left[(1-\underline{\pi})\overline{\pi} + (1-\overline{\pi})\underline{\pi}\right] w(R) + \overline{\pi}\underline{\pi} w(2R)\right]$$
$$+(1-p)^2\left[(1-\underline{\pi})^2 w(0) + 2(1-\underline{\pi})\underline{\pi} w(R) + \underline{\pi}^2 w(2R)\right] - c$$

To ensure that the manager accepts the contract, there will again be an (IR) constraint.

(IR) :

$$E^*[w] - 2c \geq 0$$

It remains to state the limited liability constraints

(NNW) :

$$w(0) \geq 0, w(I) \geq 0, w(R) \geq 0, w(2I) \geq 0, w(I+R) \geq 0, w(2R) \geq 0$$

The optimal contract implementing the efficient strategy has six constraints binding. These are $w(0) = 0, w(I) = 0, w(R) = 0$ and (IC_e) $(e \neq (0,0))$ $(1), (2),$ and (3). Substituting the binding NNW–constraints into (IC_e) $(e \neq (0,0))$ $(1), (2),$ and (3) one has:

$$
\begin{aligned}
& p^2\overline{\pi}^2 w(2R) + 2p(1-p)\overline{\pi} w(I+R) + (1-p)^2 w(2I) - 2c \\
= \;& w(2I) \\
= \;& [p\overline{\pi} + (1-p)\underline{\pi}]\, w(I+R) \\
= \;& \left[p^2\overline{\pi}^2 + 2p(1-p)\overline{\pi}\underline{\pi} + (1-p)^2\underline{\pi}^2\right] w(2R)
\end{aligned}
$$

which one can solve for the closed form solution of the remaining wages as

$$
\begin{aligned}
w(2R) &= \frac{2c}{[p\overline{\pi} + (1-p)A]^2 - A^2} \\
w(I+R) &= \frac{A2c}{[p\overline{\pi} + (1-p)A]^2 - A^2} \\
w(2I) &= \frac{A^2 2c}{[p\overline{\pi} + (1-p)A]^2 - A^2}
\end{aligned}
$$

where again, $A = p\overline{\pi} + (1-p)\underline{\pi}$.

For a formal proof of the optimality of this contract, the reader is referred to the proof of the general case, which is given in the appendix. Let us here give some intuition on how this contract works.

First, turning to the $NNW-$constraints, one can see why $w(0) = 0$, $w(I) = 0$, $w(R) = 0$ at the optimum by noting that, whenever these wages are paid, the manager has returned a zero return on one of the projects. A return of zero is more likely to occur when the manager has deviated from the efficient strategy than under the efficient strategy. Therefore, if, say, $w(0) > 0$ one can decrease $w(0)$ and at the same time increase $w(2R)$ in such a way as to leave the manager's equilibrium payoff unchanged, but making deviations from the efficient strategy less attractive. This can be achieved by reducing $w(0)$ by some $\varepsilon \leq w(0)$ and increasing $w(2R)$ by $\varepsilon\frac{(1-\overline{\pi})^2}{\overline{\pi}^2}$. The reader can easily check that this will leave all constraints satisfied, but will relax all those constraints, in which $w(0)$ enters as multiplied by off-equilibrium conditional probabilities $(1-\underline{\pi})^2$or $(1-\underline{\pi})(1-\overline{\pi})$. Likewise, if $w(R) > 0$ one can reduce $w(R)$ by some $\varepsilon \leq w(R)$

and increase $w(2R)$ by $\varepsilon\frac{2\overline{\pi}(1-\overline{\pi})}{\overline{\pi}^2}$ to leave all constraints satisfied, but relaxing all constraints that contain $w(R)$ premultiplied by off-equilibrium conditional probabilities of $2(1-\underline{\pi})\underline{\pi}$ or $[(1-\underline{\pi})\overline{\pi}+(1-\overline{\pi})\underline{\pi}]$. Finally, if $w(I)>0$ one can reduce $w(I)$ by some $\varepsilon \leq w(I)$ and at the same time increase $w(I+R)$ by $\varepsilon\frac{(1-\overline{\pi})}{\overline{\pi}}$. Again, this operation will leave expected wages unchanged if the manager chooses the efficient strategy, but will make deviations less worthwhile, relaxing all constraints which contain $w(I)$ as premultiplied by the off-equilibrium path probability $(1-\underline{\pi})$ on their RHS. Therefore $w(0)=0$, $w(I)=0$, $w(R)=0$.

Turning to the remaining three binding constraints, note that since (IC_e) $(e\neq(0,0))$ (1), (2), and (3) all hold with equality, the manager is made indifferent between not investing at all, blindly investing in one project, and blindly investing in both projects, when he did not investigate either of the two projects. Also, when $w(0)=0$, $w(I)=0$, $w(R)=0$, and $w(2I)=Aw(I+R)=A^2$ $w(2R)$, as implied by the binding constraints, the manager is made indifferent as to whether to blindly invest or not, given that he investigated one of the two projects, but did not investigate the other, for any strategy he chooses to pursue on the project he did investigate; in terms of the matrix of possible investment strategies for $e=(1,0)$, one can show that each investment strategy in the same column of that matrix will give the manager the same expected wage. Thus, looking at the constraints associated with the first column, the RHSs of (IC_e) $(e\neq(1,0))$ $(1),(5),(9)$, and (13) all reduce to the same expected utility. Likewise the RHSs of (IC_e) $(e\neq(1,0))$ $(2),(6),(10)$, and (14) are the same, and the same is true for the third column and its associated constraints (IC_e) $(e\neq(1,0))$ $(3),(7),(11)$, and (15). Finally, all of (IC_e) $(e\neq(1,0))$ $(4),(8),(12)$, and (16) share the same value on the RHS. Thus, whatever strategy the manager is planning to follow for the project he will become informed on, he is indifferent as to investing, or not investing, in the project he did not look at. Going on from there, one can see that in terms of the matrix of investment strategies, the strategies in the third column yield the highest payoff, which is intuitive, since those are the ones that specify efficient investment on the project the manager did investigate. By substituting the expressions for the positive wages, one can finally show that these strategies, which have the manager investigate only one project, and subsequently invest efficiently on the one project he did investigate, leave the manager with a lower expected utility than if he does not investigate either of the two projects, $e = (0,0)$, and hence also with a lower ex ante expected utility than the manager could obtain if he followed the efficient strategy.

Lastly, one can check that the (IC_d) constraints are satisfied when $w(0) = 0$, $w(I) = 0, w(R) = 0$, and $w(2I) = Aw(I+R) = A^2\, w(2R)$ as implied by the optimal contract.

Since $(IC_e)\,(e \neq (0,0))\,(1)$ is binding, one can read off the rent accruing to the manager from its RHS. It is given by

$$rent = w(2I) = \frac{A^2 2c}{[p\overline{\pi} + (1-p)A]^2 - A^2}$$

One can show that

$$\frac{Ac}{p\overline{\pi} + (1-p)A - A} + \frac{Ac}{p\overline{\pi} + (1-p)A - A} > \frac{A^2 2c}{[p\overline{\pi} + (1-p)A]^2 - A^2}.$$

This means that the rent arising from employing two managers, one for each project, is higher than the rent arising from employing one manager for both projects. In fact, one can also show that[6]

$$\frac{Ac}{p\overline{\pi} + (1-p)A - A} > \frac{A^2 2c}{[p\overline{\pi} + (1-p)A]^2 - A^2}$$

which says that if one gives an additional project to the manager, his informational rent will decrease. Thus the financial investor can implement project investigation and efficient re-evaluation more cheaply by allocating two projects to one manager.

The intuition behind this result is that, given any wage contract, the manager faces a joint problem when he is given two projects. As is apparent from the

[6] For a proof the reader is referred to the general case, which is discussed in the next section.

strategies available to the manager, the manager will not decide what to do on one project independently of what he does on the second project. Rather, these decisions are linked. Since the manager will make a joint decision on both projects, the wage contract should take account of this fact and condition not on each of the project returns separately, but on the whole vector of returns. The optimal contract does just that. Notice in particular, that under the optimal contract the manager receives a zero wage whenever the return on any one of the two projects is zero, that is whenever the vector of returns contains a 0. On the other hand, when the vector of returns does not contain a zero, wages are positive and increasing in the number of times the vector contains an R. key pt.

Thus the manager is punished and rewarded, not on the basis of his average performance, but using both return observations in a particular way. To see why this helps reducing the rent, recall that in the one-project case the manager was punished and received a wage of zero only if he returned a zero gross return on the project, but was paid positive wages for returns of I or R. If the manager has two projects, he is rewarded more highly when the return vector does not contain a zero, but punished with a zero wage whenever he returns zero on any one of the two projects, even if on the other project he returns I or R. Thus, with two projects one can push the manager's wage down to the limited liability constraint more often. Another way of seeing this is to say that with two projects, since wages for a return vector containing a 0 are zero and wages for a vector containing $I's$ and $R's$ are pushed up to compensate, there is a greater spread between the positive wages and zero. Intuitively, since the manager has more to lose when he deviates, it becomes easier to provide incentives for the manager to choose the efficient strategy.

Even when the optimal contract is used to implement the efficient strategy, the manager still receives a positive rent, so that the financial investor may not find it in his interest to implement the efficient strategy on the two projects.

This will be the case, if and only if

$$p\overline{\pi}R + (1-p)\,I - c - \frac{A^2 c}{[p\overline{\pi} + (1-p)\,A]^2 - A^2} < \max\{I, p\overline{\pi}R + (1-p)\,\underline{\pi}R\}$$

To see this, notice that the financial investor will never implement the efficient strategy on one project only and either blind investment or no investment on the other project. While both blind investment and no investment can be implemented at a zero wage, if efficient investment is implemented on one project only, this is optimally done by using the contract for the one-project case. This will, however, involve an even larger rent than implementing efficient investment on both projects, since

$$\frac{Ac}{p\overline{\pi} + (1-p)\,A - A} > \frac{2A^2 c}{[p\overline{\pi} + (1-p)\,A]^2 - A^2}$$

Thus efficient investment is implemented, either on both projects, or on none of them. In the latter case, either blind investment, or no investment, is implemented on both projects, depending on which is more profitable.

4.5 General Case, N Projects per Manager

Let us now proceed to analyse the general problem of implementing the efficient strategy when the manager is given N projects. Recall that the financial investor's problem can be written down as

$$\max_{w(\cdot)} E\left[\widetilde{x} - w\left(\widetilde{x}\right) \mid \widetilde{s} = \widetilde{\pi}, d = d^*\left(\widetilde{\pi}\right)\right]$$

s.t.

(IC_d) :

$$E\left[w\left(\widetilde{x}\right) \mid s = \pi,\ d = d^*\left(\pi\right)\right] \geq$$
$$E\left[w\left(\widetilde{x}\right) \mid s = \pi,\ d = d\left(\pi\right)\right]\ \forall \pi, \forall d\left(\cdot\right)$$

(IC_e) :

$$E\left[w\left(\widetilde{x}\right) \mid \widetilde{s}=\widetilde{\pi}, d=d^{*}\left(\widetilde{\pi}\right)\right]-Nc \geq$$
$$E\left[w\left(\widetilde{x}\right) \mid \widetilde{s}=\widetilde{s}\left(e\right), d=d\left(\widetilde{s}\right)\right]-\sum_{i=1}^{N} ce^{i} \quad \forall e, \forall d\left(\cdot\right)$$

$(IR):$

$$E\left[w\left(\widetilde{x}\right) \mid \widetilde{s}=\widetilde{\pi}, d=d^{*}\left(\widetilde{\pi}\right)\right]-Nc \geq 0$$

$(NNW):$

$$w\left(x\right) \geq 0 \, \forall x$$

Note first that since all projects are identical, the optimal contract will exhibit *symmetry* in the following sense. For two return vectors x and x', such that x contains k times R, $N-k-J$ times 0, and J times I, and x' also contains k times R, $N-k-J$ times 0, and J times I, wages will be equal, i.e. $w\left(x\right)=w\left(x'\right)$. For the general case it is useful to move to the notation $w\left(x\right)=w\left(x'\right)\equiv w\left(k,J\right)$, where the first argument denotes the number of R's, the second argument the number of I's, and the number of $0'$s are being suppressed. Given that the manager faces such a symmetric contract, the manager's optimal strategy will also be symmetric, in the sense that, for any investment strategy $d\left(\cdot\right)$ and for any signal $s\left(e\right)=\left(s_{1},...s_{N}\right)$, any permutation of s will induce the same permutation on investment decisions $d\left(s\right)$. Using this, one can rewrite $\left(IC_{d}\right)$ and $\left(IC_{e}\right)$ in terms of the explicit distribution over wages induced by any strategy (these expressions are shown in Appendix 4.2 at the end of this chapter). Having spent effort on all projects and having received a signal $s=\pi$, the manager's strategy decision can then be seen as equivalent to a sampling problem. He has to decide, in how many projects which he sees to have success probability $\pi^{i}=\overline{\pi}$ to invest in, and in how many projects he sees to have success probability $\pi^{i}=\underline{\pi}$ he invests in. The financial investor wants the manager to invest only in those projects, for which $\pi^{i}=\overline{\pi}$, which gives rise to the set of $\left(IC_{d}\right)$ constraints. Also, the manager has to decide ex ante, how many projects he wants to become informed on. Denote the number of projects the manager does not investigate (is lazy on) by L. If the manager chooses not to become informed on some number of projects, he can still invest blindly into those projects, knowing only their expected success probability. The financial investor wants the manager to gain information on all projects, $L = 0$, and then to invest efficiently. The manager's expected utility from this will therefore have to be greater than the utility from any other effort choice $L = 1, .., N$ and any subsequent investment strategy, (IC_e). Lastly, this expected utility must be greater than zero, (IR). and wages must never be negative, (NNW). The wage contract that optimally implements the efficient strategy can then be characterised as follows.

Proposition 7 *$\forall N, \forall p, c, \overline{\pi}, \underline{\pi}$, s.t. $\overline{\pi} > \underline{\pi}$, the following contract optimally implements the efficient strategy:*

a)

$$w(k, N-K) = 0 \;\; \forall k < K, \; \forall K = 0, .., N$$

b)

$$w(K, N-K) = \frac{A^{N-K} Nc}{[p\overline{\pi} + (1-p)A]^N - A^N} \;\; \forall K = 0, .., N$$

where $A = p\overline{\pi} + (1-p)\underline{\pi}$.

For a proof, see the appendix (Appendix 4.2).

Part a) of the proposition says that the manager optimally receives a zero wage whenever the return on any of the projects is zero. The intuition is that such an event is more likely to occur when the manager deviated, than when he invested efficiently. In particular, assume that the manager has decided to invest on K projects, but not to invest on $N - K$ projects. On the equilibrium path, all of the K projects will have had a conditional success probability of $\overline{\pi}$, that is the $K-$vector of success probabilities for those projects would have been $\pi_K^* = (\overline{\pi}, ..., \overline{\pi})$. Off the equilibrium path, however, the manager may sometimes invest into K projects, even though the vector of success probabilities for the K projects is $\pi'_K \neq \pi^*_K$, that is when this vector contains one or more $\underline{\pi}'s$. This will occur when the manager invests into projects he knows to be bad, but also when the manager invests blindly into bad projects. Both on and off the equilibrium path the manager will receive a wage $w \in \{w(0, N-K), w(1, N-K) ..., w(K, N-K)\}$, depending on how many $R's$ he returns. It is shown in the appendix, that

$$\frac{a^*}{b^*} = \frac{\Pr[w = w(K, N-K) \mid \pi^*_K]}{\Pr[w = w(k, N-K) \mid \pi^*_K]} > \frac{\Pr[w = w(K, N-K) \mid \pi'_K]}{\Pr[w = w(k, N-K) \mid \pi'_K]} = \frac{a'}{b'}$$

for all $k < K$, for any such π'_K and for any fixed $N - K$. Hence, one can costlessly relax constraints by reducing $w(k, N-K)$ by ε and increasing $w(K, N-K)$ by $\varepsilon\frac{b^*}{a^*}$, since

$$-b^*\varepsilon + \frac{b^*}{a^*}a^*\varepsilon = 0,$$

so that on-equilibrium path conditional expected wages are left unchanged, but

$$-b'\varepsilon + \frac{b^*}{a^*}a'\varepsilon < 0,$$

so that conditional expected wages from deviating from efficient investment are reduced.

Apart from the binding (NNW) constraints given in part a), the solution has the set of constraints $(IC_e)(L = N)$ binding. Incorporating the binding (NNW) constraints, these constraints are given by

$$\sum_{K=0}^{N} \binom{N}{K} p^K (1-p)^{N-K} \overline{\pi}^K w(K, N-K) - Nc$$

$$\geq A^B w(B, N-B)$$

$$\forall B = 0, 1, .., N$$

For N projects, this set comprises $N + 1$ constraints: when the manager did not investigate any project, he still has the choice of investing blindly into any number $B = 0, 1, ..., N$ of projects. Under the optimal contract, all of these constraints hold with equality, so that, given that he did not investigate any project, the manager is made indifferent as to the number of projects he invests in blindly. That is, given $L = N$, for any B the manager's expected wage is the same. This expected wage from not investigating any project and then possibly investing blindly is equal to the wage, net of effort cost, from efficiently investigating all projects, $L = 0$ and then investing efficiently. Any other strategy, involving an interior number of projects being evaluated, $L = 1, .., N-1$, and arbitrary subsequent investment strategies, can be shown to give the manager a lower expected utility.

Also, given part a) of the proposition, the (IC_d) constraints reduce to

$$\begin{aligned} w(K, N-K) &\geq \underline{\pi} w(K+1, N-K-1) \ \forall K = 0, .., N-1 \\ \overline{\pi} w(K, N-K) &\geq w(K-1, N-K+1) \ \forall K = N, .., 1 \end{aligned}$$

where the first constraint discourages overinvestment, while the second forestalls underinvestment. Both are clearly met when

$$w(K, N-K) = Aw(K+1, N-K-1)$$

as under the schedule given in part b) of the proposition, so that the manager interim has an incentive to invest efficiently.

As a first observation on the optimal contract, let us make the following

Remark 1 : *For $N \geq 2$, the optimal wage schedule will in general not be monotone and will thus be neither concave nor convex in aggregate (or average) returns.*

Proof:$w(N-1, 0) = 0 < w(0, N)$, but $(N-1)R > NI$ for N large enough.

Notice that this already pertains to the case of $N = 2$, where we found $w(R) = 0$ and $w(2I) > 0$, whereas it may well be that $R > 2I$. This is in contrast to the basic problem with $N = 1$, where we found a monotone wage schedule, and is in contrast also to most results found in the literature (exceptions include Innes (1990)). Here, the non-monotonicity results from the interplay between parts a) and b) of the proposition. As long as the manager presents a vector that contains only $I's$ and $R's$, the wage is increasing more than proportionately in the number of $R's$. However, whenever the vector contains one or more $0's$, the wage drops down to zero. While this scheme will provide the right incentives for any given number of projects, one can show that it works better as the number of projects rises.

From the binding (IC_e) constraints one can read off the size of the rent accruing to the manager. It is

$$r_N = w(0, N) = \frac{A^N N c}{[p\overline{\pi} + (1-p)A]^N - A^N}$$

Let us define $\{C_N\}_{N=1}^{\infty}$ to be the sequence of optimal contracts as specified in Proposition 1 and then define$\{r_N\}_{N=1}^{\infty}$ as the sequence of associated rents. Analysing this sequence one arrives at the following

Corollary 2 : *The sequence of rents* $\{r_N\}_{N=1}^{\infty}$ *is*

a) *strictly decreasing,* $r_N > r_{N+1}$ *and*

b) *converging to xero,* $\lim_{N \to \infty} r_N = 0$

Proof: To prove part a), one writes down the inequality in terms of its explicit expressions,

$$\frac{A^N N c}{[p\overline{\pi} + (1-p)A]^N - A^N} > \frac{A^{N+1}(N+1)c}{[p\overline{\pi} + (1-p)A]^{N+1} - A^{N+1}},$$

Letting $D \equiv [p\overline{\pi} + (1-p)A]$, cancelling common terms and rearranging one gets

$$N\left(D^{N+1} - A^{N+1}\right) > (N+1)\left(D^N - A^N\right)A$$

or

$$K(D) \equiv N\left(D^{N+1} - A^{N+1}\right) - (N+1)\left(D^N - A^N\right)A > 0$$

To see that this must hold, one needs to note only that $K(A) = 0$ and that

$$\frac{\partial K(D)}{\partial D} = (N+1) N D^N - (N+1) N D^{N-1} A > 0$$

since $D > A$.

To prove part b) one proves $\lim_{N\to\infty} \frac{1}{r_N} = \infty$, which is equivalent to $\lim_{N\to\infty} r_N = 0$, since $r_N > 0 \; \forall N$. But since

$$\frac{1}{r_N} = \frac{D^N - A^N}{A^N N c} = \frac{1}{Nc}\left(\frac{D}{A}\right)^N - \frac{1}{Nc}\left(\frac{A}{A}\right)^N$$

one immediately has $\lim_{N\to\infty} \frac{1}{r_N} = \infty + 0 = \infty$, again using that $D > A$.

The result that the rent accruing to the manager is decreasing as he is assigned more and more projects and that it will vanish in the limit, can be taken to provide a rationale for firm-level diversification. Let us state one immediate implication of the fact that the rent is strictly decreasing with N as the following

Corollary 3 *There exists a smallest number of projects $\underline{N}$, such that for $N \geq \underline{N}$, the financial investor will want to implement the efficient strategy on all projects. If $\underline{N} > 1$, then for all N such that $0 < N < \underline{N}$ the financial investor either implements blind investment or no investment on all projects.*

To see this, note that since

$$p\overline{\pi}R + (1-p) I - c > \max\{I, p\overline{\pi}R + (1-p)\underline{\pi}R\}$$

there exists a smallest number of projects $\underline{N}$, such that for all $N \geq \underline{N}$

$$p\overline{\pi}R + (1-p) I - c - \frac{A^N c}{D^N - A^N} \geq \max\{I, p\overline{\pi}R + (1-p)\underline{\pi}R\}$$

so that it will eventually become profitable to implement the efficient strategy, rather than no investment or blind investment on all projects. Notice that, for any N, either the efficient strategy, or no investment or blind investment is implemented on all projects. Given the manager has N projects, the financial investor will never implement the efficient strategy on a subset of $N' < N$

projects and blind investment or no investment on the remaining projects, since this is optimally done by using the contract $C_{N'}$ for the projects on which the efficient strategy is to be implemented and by paying a zero wage on the remaining projects. This will involve a higher rent than implementing the efficient strategy on all projects using C_N. Thus, for any N,the efficient strategy will be implemented on all projects, or on none of them. For $N \geq \underline{N}$ it will be implemented on all projects. If $\underline{N} > 1$, then for $N < \underline{N}$, either blind investment or no investment is implemented on all projects depending on which of $p\overline{\pi}R + (1-p)\,\underline{\pi}R$ and I is larger. Both can be implemented at a zero wage. Implementing no investment is of course equivalent to not financing the projects. Notice, therefore, that it is possible in this model that small firms will not be financed, whereas large firms always will.

In fact, of course, since the rent from implementing the efficient strategy on all projects is decreasing and converging to zero, in this basic model the optimal firm size is infinite. This is due to the simplifying assumptions, that the manager's effort endowment is unbounded and that effort costs increase linearly with the number of projects. These issues will be looked at more closely in the next section.

First, however, let us come back to the basic intuition for why firm-level diversification is beneficial in our set-up. It is not, as in Aron (1988) and Hermalin and Katz (1996), due to the fact that diversification provides additional independent signals of the manager's single effort choice[7]. In our setup the projects are technologically unrelated. Unlike in Aron (1988) and Hermalin and

[7]Also, it should be noted that the intuition here is distinct from the rationale for diversification in the theory of banking, as in Diamond (1984) or Williamson (1986). In contrast to the analysis in this chapter, in these papers the contract between client and bank is a simple debt contract for any size of the bank. Expected bankruptcy costs decrease with the number of firms the bank takes on because of a diversification effect. The optimal size of the bank is infinite, since, if the bank exerts monitoring effort, the moneys received per firm cease to be stochastic as the bank takes on more and more firms.

Katz (1996), decisions one project do not constrain the decisions on any other project. Projects become related only through the fact that, as is apparent from the description of the strategies available to the manager, the manager makes joint decisions on the full set of projects. The optimal contract takes this into account and conditions on the vector of project returns rather than on each return observation separately. In the model, it was assumed that limited liability is the source of the contracting problem, i.e. the fact that one cannot impose unbounded punishment on the manager to give him incentives. Thus, when the manager is given one project, he is punished with a zero wage only if he returns zero on the project and he is paid more than zero if he returns I or R.. Looking back at this basic problem, one sees that the non-negative-wealth constraint is indeed responsible for the manager receiving a rent. Without it, one could find $w(0) < 0$, $w(R) > 0$, and $w(I) = 0$ that satisfy all constraints and leave the manager with an expected wage equal to his effort cost. When the manager has more projects, he still is protected by the (NNW) constraint, so that a wage of zero is still the worst one can do to the manager. However, this punishment can now be used more and more often, as the manager gets zero whenever he returns zero on any one of the projects he supervises. This allows $w(0, N)$ to shrink down to zero as N becomes large. Thus, one can think of the diversification effect as coming about through "relaxing" the limited liability constraint. Another way of viewing it would be to say that diversification relaxes the assumption that the manager is endowed with zero wealth. The incentive problem would not arise, if the manager could finance the project himself, and would be mitigated if he could put in at least some inside equity. When the manager has more projects, the manager's expected compensation for his effort cost can act as a substitute for inside equity, since he stands to lose it if he deviates from efficient decisions..

more discussion

4.6 Extensions

4.6.1 Bounded effort endowment

The assumption made throughout in the analysis is that the manager has an unbounded effort endowment, so that one manager can handle any number of projects. This may be a rather bold assumption[8]. One might think that relaxing it will give us an upper bound on the firm size. Let any manager's endowment be $\overline{E}$. Then, if one is constrained to employ one manager, the firm size will be bounded by $\overline{N} \leq \overline{E}$. However, one might also be able to offer a contract as specified above to a coalition of managers. Consider the extreme case where $\overline{E} = 1$, so that one manager can at most handle one project. If one were to offer the wage contract for N projects to a coalition of N managers and the aggregate wage comes subsequently to be shared equally by all managers, then, if one assumes that each manager can costlessly monitor the effort choice, information and investment decision of one other manager, the coalition will accept the contract and enforce the efficient strategies on each of the N projects. It is easy to see, then, that under this arrangement the optimal size of a firm is again infinite. Note, however, that while it was implicitly assumed that it is prohibitively costly for a financial investor to monitor a manager, the optimality of the arrangement described above relies on monitoring being costless inside the firm.

4.6.2 Effort cost a function of N

A straightforward extension of the basic model is to make the effort cost per project a function of N. Thus instead of a per project cost c, one now has $c(N)$. An assumption that $c(N)$ is increasing in N may be justified by overheads increasing more than proportionately with N. One may also think that, if there is a coalition of managers, as N rises, it may become more and more costly to

[8] It has of course been made in the literature before, cf. e.g. Diamond (1984)

enforce an internal monitoring scheme, as outlined in the previous subsection. For any of these reasons, let us assume that $c(N)$ is an increasing sequence which is unbounded above. Since the wage schedule offered in the main text is linear in c, for any N, the optimal wage schedule implementing $e^i = 1$ and $d^i(\underline{\pi}) = 0$ and $d^i(\overline{\pi}) = 1$ can be found by replacing c everywhere by $c(N)$. Assuming that the manager's task of re-evaluating the project is necessary for the project to be profitable, the condition for it to be profitable to implement efficient investment is

$$[p\overline{\pi}R + (1-p)I] - I - c(N) - \frac{A^N c(N)}{D^N - A^N} \geq 0$$

Clearly then, under the assumptions made, the optimal firm size must be finite: even if $\frac{A^N c(N)}{D^N - A^N}$ were to converge to zero, the increasing costs ensure that the LHS will eventually become negative and stay negative for larger N.

4.7 Conclusion

In an environment which exhibits both moral hazard and adverse selection on the part of the manager, I have characterised the optimal contract implementing the efficient investment rule when the number of projects, with respect to which both information asymmetries pertain, is arbitrary.

The main result is that increasing the number of projects helps to alleviate the incentive problem between financial investor and manager. With more projects, the financial investor is able to provide the right incentives more cheaply. The reason is that the optimal contract conditions on the vector of returns, rather than each return observation separately, exploiting the fact that the manager makes joint decisions on the full set of projects. Under the optimal remuneration scheme, when the manager is assigned more projects, he is more and more unlikely to receive larger and larger positive rewards and more and more likely to be punished with a zero wage. For any fixed N, the manager is rewarded highly when all projects turn out I or R, while he receives nothing when he returns a 0. The intuition is that assigning more projects relaxes the limited liability constraint, which limits the size of the punishment that can be imposed on the manager. With more projects the punishment of a zero wage can be applied more often. Alternatively, one can think of additional projects as acting as a substitute for inside equity. With more projects the manager has more to lose if one of the projects fails. This can be used to improve incentives, so that the manager's informational rent decreases as he supervises more projects.

The main other conclusion of this chapter is that the shape of the optimal managerial remuneration scheme may look rather different depending on whether the manager is in charge of a single, or of more than one project.. I find that, in the case of a single project, the optimal remuneration scheme is monotone in observed return. This feature is not preserved, however, when one looks at the case of a manager supervising more than one project at the same time. Our results therefore suggests that, when managers are to provide effort as well as make investment decisions, optimal managerial wage contracts will in general not be monotone in aggregate returns. In general, the optimal contract is such that the manager is rewarded highly when none of the projects he supervises fails. However, the manager has to take responsibility for project failure and optimally receives nothing when a single project in his portfolio fails to perform.

While this may seem a very stark prediction and may at first sight not seem to square with what one sees we in practice, one can argue that what the optimal contract is suggesting has some intuitive appeal. If a manager is in charge of project selection and his role is to spot bad projects, then one would expect the manager to receive relatively little when one project turns out badly, even though overall things are going fine. A similar idea can be found in the theory of teams, where team members are paid with reference to their relative performance, as compared with the average[9]. There, however, the result is predicated on the assumption that the noise is correlated across team members, whereas our result holds in a world of stochastically independent projects that become linked only because a decision is made on all of them jointly.

Our results also suggests that managers of large corporations ought to be rewarded more highly, not only because their job presumably takes more effort, but also because they bear more "responsibility" in choosing among a large number of potential projects. Then, however, if things go wrong with one of the selected projects, the manager ought also be forced to take that responsibility and be given little, if any, reward. Notice that shares and options may well come back into the picture when one tries to implement such a scheme. A package of shares and options can easily be used to generate a salary that is more than proportionately increasing in company returns. This package has, however, to be bundled with the threat of being sacked and losing all these benefits altogether when one of the manager's projects goes wrong, to keep the manager from overinvesting into bad projects. One empirical implication of such a reinterpretation would be that one ought to observe large and diversified firms to have a higher turnover of chief executive officers than small and undiversified firms. This implication appears to be consistent with the available empirical evidence. Thus, in their study of determinants of board turnover, Franks, Mayer and Renneboog (1997) find a positive relationship between size as measured by market capitalisation and board turnover rates for a sample of 250 publicly quoted UK companies. Another empirical implication would be that inside a hierarchical organisation, one should observe higher turnover rates at higher echelons of the organisation, since staff in the higher echelons typically supervise more projects than those at the bottom of the hierarchy.

[9] cf. Holmstrom (1982)

Appendix 4.1

A Direct Revelation Mechanism

This part of the appendix will analyse a direct revelation mechanism implementing efficient investment for $N = 1$.

Let us assume that the manager can make an announcement about the signal he received, $\widehat{\pi} \epsilon \{\overline{\pi}, \underline{\pi}, 0\}$. Let us also view the investment decision as contractible, so that the contract can specify the investment decision to be made, given the announced signal. $d(\widehat{\pi}) : \{\overline{\pi}, \underline{\pi}, 0\} \rightarrow \{0,1\}$. Wages can then condition both on the realised return and on the announced signal, so that one needs to find $w(x, \widehat{\pi})$. It is clear that $w(x, 0) = 0$ for any function $d(\widehat{\pi})$ to be implemented, since if the manager claims not to have spent any effort, one will not reward him. Since $w(x, \widehat{\pi}) \geq 0$, if one wants to implement $e = 1$ and some $d(\widehat{\pi},)$ one can therefore restrict attention to $\widehat{\pi} \epsilon \{\overline{\pi}, \underline{\pi}\}$; the manager is never going to admit that he did not spend effort. Defining

$$W(\pi, \widehat{\pi}) = (1 - d(\widehat{\pi}))\, w(I, \widehat{\pi}) + d(\widehat{\pi})\left(\pi w(R, \widehat{\pi}) + (1 - \pi)\, w(0, \widehat{\pi})\right)$$

the following constraints will have to be satisfied:

$W(\overline{\pi}, \overline{\pi}) \geq W(\overline{\pi}, \underline{\pi})$

$W(\underline{\pi}, \underline{\pi}) \geq W(\underline{\pi}, \overline{\pi})$

$pW(\overline{\pi}, \overline{\pi}) + (1 - p)\, W(\underline{\pi}, \underline{\pi}) - c \geq 0$

$pW(\overline{\pi}, \overline{\pi}) + (1 - p)\, W(\underline{\pi}, \underline{\pi}) - c \geq pW(\overline{\pi}, \underline{\pi}) + (1 - p)\, W(\underline{\pi}, \underline{\pi})$

$pW(\overline{\pi}, \overline{\pi}) + (1 - p)\, W(\underline{\pi}, \underline{\pi}) - c \geq pW(\overline{\pi}, \overline{\pi}) + (1 - p)\, W(\underline{\pi}, \overline{\pi})$

$w(x, \widehat{\pi}) \geq 0\ \forall (x, \widehat{\pi})$

When $d(\overline{\pi}) = 1$ and $d(\underline{\pi}) = 0$ these constraints reduce to

$\overline{\pi} w(R, \overline{\pi}) + (1 - \overline{\pi})\, w(0, \overline{\pi}) \geq w(I, \underline{\pi})$

$w(I, \underline{\pi}) \geq \underline{\pi} w(R, \overline{\pi}) + (1 - \underline{\pi})\, w(0, \overline{\pi})$

$p\left[\overline{\pi} w(R, \overline{\pi}) + (1 - \overline{\pi})\, w(0, \overline{\pi})\right] + (1 - p)\, w(I, \underline{\pi}) - c \geq 0$

$p\left[\overline{\pi} w(R, \overline{\pi}) + (1 - \overline{\pi})\, w(0, \overline{\pi})\right] + (1 - p)\, w(I, \underline{\pi}) - c \geq w(I, \underline{\pi})$

$p\left[\overline{\pi} w(R, \overline{\pi}) + (1 - \overline{\pi})\, w(0, \overline{\pi})\right] + (1 - p)\, w(I, \underline{\pi}) - c \geq$

$p\left[\overline{\pi} w(R, \overline{\pi}) + (1 - \overline{\pi})\, w(0, \overline{\pi})\right] + (1 - p)\left[\underline{\pi} w(R, \overline{\pi}) + (1 - \underline{\pi})\, w(0, \overline{\pi})\right]$

$$w(0,\overline{\pi}) \geq 0, w(I,\underline{\pi}) \geq 0, w(R,\overline{\pi}) \geq 0$$

which are exactly the constraints for the programme given in the main text for $N = 1$.

Appendix 4.2

Proof of Proposition 7

To prove that the contract specified in Proposition 7 is optimal, one needs to first introduce some further notation. Partition the set $\mathfrak{N}$ of N projects into two subsets, $\mathfrak{N} = \{\mathfrak{C}, \mathfrak{L}\}$, where $\mathfrak{C}$ is the set of projects the manager investigates (is curious about), and $\mathfrak{L}$ is the subset of projects the manager does not investigate (is lazy on). Denote the number of projects in these sets by C and L, respectively, where $C = N - L$. Introduce two partitions of $\mathfrak{C}$: $\mathfrak{C} = \{\mathfrak{J}, \mathfrak{K}\}$ and $\mathfrak{C} = \{\mathfrak{I}, \mathfrak{T}\}$, where $\mathfrak{J}$ is the subset of projects with low success probability, $\mathfrak{K}$ is the number of projects with high success probability, $\mathfrak{I}$ is the subset of $\mathfrak{C}$ the manager invests into, and $\mathfrak{T}$ is the subset of projects in $\mathfrak{C}$ the manager aborts. Denote the number of projects in these subsets by J and K for the first partition and I and T for the second. Next, partition the set of projects $\mathfrak{I}$ into which the manager knowingly invests into two subsets, $\mathfrak{I} = \{\mathfrak{I}_J, \mathfrak{I}_K\}$, where e.g. $\mathfrak{I}_J = \mathfrak{I} \cap \mathfrak{J}$ is the set of projects the manager knowingly invests in, even though success probabilities are low. Next denote the number of elements in these sets by I_J and I_K, so that one has $I_J + I_K \equiv I$.

Given the distributional assumptions made, one can then write down (IC_d) as

$$\sum_{k=0}^{K} \binom{K}{k} \overline{\pi}^k (1 - \overline{\pi})^{K-k} w(k, N - K)$$

$$\geq \sum_{k=0}^{I} \sum_{j=0}^{k} \binom{I_J}{j} \binom{I_K}{k-j} \underline{\pi}^j (1 - \underline{\pi})^{I_J - j} \overline{\pi}^{k-j} (1 - \overline{\pi})^{I_K - (k-j)} w(k, N - I)$$

$$\forall \ (I_J, I_K) \ s.t. \ 0 \leq I_K \leq K, \ 0 \leq I_J \leq N - K \ \ \forall \ K \ s.t. \ 0 \leq K \leq N$$

(Note that by convention $\binom{a}{b} = 0$ for $b > a$).

In order to be able to state the set of (IC_e) constraints, one needs to introduce one more piece of notation. If the manager did not investigate L projects, let $\mathfrak{B}$

$\subseteq \mathfrak{L}$ be the set of projects the manager invests in blindly and let B be the number of these projects. Let K' be the number of projects in $\mathfrak{B}$ which, unknown to the manager, have high success probability and J' be the number of projects in $\mathfrak{B}$ which, unknown to the manager, have low success probability. The set of (IC_e) constraints can then be written down as

$$\sum_{K=0}^{N}\binom{N}{K}p^K(1-p)^{N-K}\binom{K}{k}\overline{\pi}^k(1-\overline{\pi})^{K-k}w(k,J)-Nc$$

$$\geq\sum_{K=0}^{N-L}\binom{N-L}{K}p^K(1-p)^{N-L-K}\sum_{K'=0}^{B}\binom{B}{K'}p^{K'}(1-p)^{B-K'}$$

$$\sum_{k=0}^{I}\sum_{j=0}^{k}\binom{I_J}{j}\binom{I_K}{k-j}\underline{\pi}^j(1-\underline{\pi})^{I_J-j}\overline{\pi}^{k-j}(1-\overline{\pi})^{I_K-(k-j)}$$

$$\sum_{k'=0}^{B}\sum_{j'=0}^{k'}\binom{B-K'}{j'}\binom{K'}{k'-j'}\underline{\pi}^{j'}(1-\underline{\pi})^{B-K'-j'}\overline{\pi}^{k'-j'}(1-\overline{\pi})^{K'-(k'-j')}$$

$$w(k+k',N-B-I)-(N-L)c$$

$\forall\, L=1,...,N; \forall\, K \longmapsto (I_J,I_K,B)$ *s.t.* $0\leq I_K\leq K,\ 0\leq I_J\leq N-L-K,\ \ 0\leq B\leq L$

Proof of part a):

In order to prove that $w(k,N-K)=0\ \forall k<K,\ \forall K=0,..,N$ as in a) is indeed optimal, fix some $K>0$ and look at the subset of wages $\{w(k,N-K)\}_{k=0}^{N-K}$. On the equilibrium path, whenever the manager invests in K projects, but does not invest in $N-K$ projects, the $K-$vector of success probabilities for those projects will be $\pi_K^*=(\overline{\pi},...,\overline{\pi})$, containing K times $\overline{\pi}$. Off the equilibrium path, the manager invests in K projects and does not invest in $N-K$ projects when the vector of success probabilities for the K projects he invests in is $\pi'_K\neq\pi_K^*$, that is, when this vector contains one or more $\underline{\pi}'s$. Let the number of $\underline{\pi}'s$ in this vector be $I_J+J'\equiv I_{J'}$, and the number of $\overline{\pi}'s$ be $I_K+K'\equiv I_{K'}$, where $I_{J'}+I_{K'},=K$. On the equilibrium path one has

$$\Pr\left[w=w(K,N-K)\mid I_{J'}=0,I_{K'}=K\right]=\overline{\pi}^K$$

whereas for a given $k < K$

$$\Pr\left[w = w\left(k, N-K\right) \mid I_{J'} = 0, I_{K'} = K\right] = \binom{K}{k}\overline{\pi}^{k}\left(1-\overline{\pi}\right)^{K-k}$$

Off the equilibrium path one has

$$\Pr\left[w = w\left(K, N-K\right) \mid I_{J'} > 0, I_{K'} < K\right] = \underline{\pi}^{I_{J'}}\overline{\pi}^{I_{K'}} = \underline{\pi}^{I_{J'}}\overline{\pi}^{K-I_{J'}}$$

whereas one can write

$$\Pr\left[w = w\left(k, N-K\right) \mid I_{J'} > 0, I_{K'} < K\right]$$

$$= \sum_{j=0}^{k}\binom{I_{J'}}{j}\binom{I_{K'}}{k-j}\underline{\pi}^{j}\left(1-\underline{\pi}\right)^{I_{J'}-j}\overline{\pi}^{k-j}\left(1-\overline{\pi}\right)^{I_{K'}-(k-j)}$$

$$= \sum_{j=0}^{k}\binom{I_{J'}}{j}\binom{K-I_{J'}}{k-j}\underline{\pi}^{j}\left(1-\underline{\pi}\right)^{I_{J'}-j}\overline{\pi}^{k-j}\left(1-\overline{\pi}\right)^{K-I_{J'}-(k-j)}$$

Given these expressions, we are now ready to establish that

$$\frac{\Pr\left[w = w\left(K, N-K\right) \mid I_{J} = 0, I_{K} = K\right]}{\Pr\left[w = w\left(k, N-K\right) \mid I_{J} = 0, I_{K} = K\right]}$$

$$\geq \frac{\Pr\left[w = w\left(K, N-K\right) \mid I_{J'} > 0, I_{K'} < K'\right]}{\Pr\left[w = w\left(k, N-K\right) \mid I_{J'} > 0, I_{K'} < K'\right]}$$

Substituting and rearranging one finds

$$\frac{\overline{\pi}^{K}}{\binom{K}{k}\overline{\pi}^{k}\left(1-\overline{\pi}\right)^{K-k}}$$

$$> \frac{\underline{\pi}^{I_{J'}}\overline{\pi}^{K-I_{J'}}}{\sum_{j=0}^{k}\binom{I_{J'}}{j}\binom{K-I_{J'}}{k-j}\underline{\pi}^{j}\left(1-\underline{\pi}\right)^{I_{J'}-j}\overline{\pi}^{k-j}\left(1-\overline{\pi}\right)^{K-I_{J'}-(k-j)}}$$

$\Longleftrightarrow$

$$\frac{\overline{\pi}^{K}\overline{\pi}^{k}\left(1-\overline{\pi}\right)^{K-k}\sum_{j=0}^{k}\binom{I_{J'}}{j}\binom{K-I_{J'}}{k-j}\underline{\pi}^{j}\left(1-\underline{\pi}\right)^{I_{J'}-j}\overline{\pi}^{-j}\left(1-\overline{\pi}\right)^{j-I_{J'}}}{\underline{\pi}^{I_{J'}}\overline{\pi}^{K-I_{J'}}\binom{K}{k}\overline{\pi}^{k}\left(1-\overline{\pi}\right)^{K-k}} > 1$$

$\iff$

$$\frac{\sum_{j=0}^{k}\binom{I_{J'}}{j}\binom{K-I_{J'}}{k-j}\underline{\pi}^{j}\left(1-\underline{\pi}\right)^{I_{J'}-j}\overline{\pi}^{-j}\left(1-\overline{\pi}\right)^{j-I_{J'}}}{\underline{\pi}^{I_{J'}}\overline{\pi}^{-I_{J'}}\binom{K}{k}}>1$$

$\iff$

$$\frac{\sum_{j=0}^{k}\binom{I_{J'}}{j}\binom{K-I_{J'}}{k-j}\underline{\pi}^{j-I_{J'}}\left(1-\underline{\pi}\right)^{I_{J'}-j}\overline{\pi}^{I_{J'}-j}\left(1-\overline{\pi}\right)^{j-I_{J'}}}{\binom{K}{k}}>1$$

But

$$\underline{\pi}^{j-I_{J'}}\left(1-\underline{\pi}\right)^{I_{J'}-j}\overline{\pi}^{I_{J'}-j}\left(1-\overline{\pi}\right)^{j-I_{J'}}=\left(\frac{\left(1-\underline{\pi}\right)\overline{\pi}}{\left(1-\overline{\pi}\right)\underline{\pi}}\right)^{I_{J'}-j}\geq 1$$

with strict inequality for $j<I_{J'}$. Also, as a straightforward application of Vandermond's identity, one has $\sum_{j=0}^{k}\binom{I_{J'}}{j}\binom{K-I_{J'}}{k-j}=\binom{K}{k}$, which establishes the inequality. As explained in the main text, this result allows to set

$w\left(k,N-K\right)=0\ \forall k<K,\ \forall K=1,..,N.$

Proof of part b):

Incorporating a) one can write down a simplified set of (IC_e) constraints as

$$\sum_{K=0}^{N}\binom{N}{K}p^{K}\left(1-p\right)^{N-K}\overline{\pi}^{K}w\left(K,N-K\right)-Nc$$

$$\geq\sum_{K=0}^{N-L}\binom{N-L}{K}p^{K}\left(1-p\right)^{N-L-K}\sum_{K'=0}^{B}\binom{B}{K'}p^{K'}\left(1-p\right)^{B-K'}\underline{\pi}^{I_J}\overline{\pi}^{I_K}\underline{\pi}^{B-K'}\overline{\pi}^{K'}$$

$$w\left(I+B,N-B-I\right)-\left(N-L\right)c$$

$\forall\, L=1,...,N;\forall\, K\longmapsto\left(I_J,I_K,B\right)$ *s.t.* $0\leq I_K\leq K,\ 0\leq I_J\leq N-L-K,\ \ 0\leq B\leq L$

$\Leftrightarrow$

$$\sum_{K=0}^{N}\binom{N}{K}p^{K}\left(1-p\right)^{N-K}\overline{\pi}^{K}w\left(K,N-K\right)-Nc$$

$$\geq\sum_{K=0}^{N-L}\binom{N-L}{K}p^{K}\left(1-p\right)^{N-L-K}\underline{\pi}^{I_J}\overline{\pi}^{I_K}A^{B}w\left(I+B,N-I-B\right)-\left(N-L\right)c$$

$\forall\, L=1,...,N;\forall\, K\longmapsto\left(I_J,I_K,B\right)$ *s.t.* $0\leq I_K\leq K,\ 0\leq I_J\leq N-L-K,\ \ 0\leq B\leq L$

The wage schedule stated under b) is derived from the subset of (IC_e) that has $L=N$ (so that $I=0$), with $B=0,1,..,N$. Writing these out one has

$$\sum_{K=0}^{N} \binom{N}{K} p^K (1-p)^{N-K} \overline{\pi}^K w(K, N-K) - Nc$$

$$\geq A^B w(B, N-B)$$

$$\forall B = 0, 1, .., N$$

from which, after imposing the equality and substituting, the closed form schedule

$$w(K, N-K) = \frac{A^{N-K} Nc}{[p\overline{\pi} + (1-p) A]^N - A^N} \; \forall K = 0, ..N$$

is easily obtained. To prove that this wage schedule is optimal, one needs to show that (i) under this schedule all other constraints are satisfied and that (ii) the choice of binding constraints is optimal.

(i) Start with the (IC_e) constraints. In a first step, one can show that under the proposed schedule, for any L and any (I_J, I_K) the manager is indifferent as regards the number of projects he invests into blindly. To see this, note that under the proposed schedule

$$w(I+B, N-I-B) = A^{N-I-B} w(N, 0)$$

so that (IC_e) further simplifies to

$$\sum_{K=0}^{N} \binom{N}{K} p^K (1-p)^{N-K} \overline{\pi}^K w(K, N-K) - Nc$$

$$\geq \sum_{K=0}^{N-L} \binom{N-L}{K} p^K (1-p)^{N-L-K} \underline{\pi}^{I_J} \overline{\pi}^{I_K} A^B A^{N-I-B} w(N, 0) - (N-L)c$$

$$\forall L = 1, ..., N; \forall K \longmapsto (I_J, I_K, B) \;\; s.t. \; 0 \leq I_K \leq K, \; 0 \leq I_J \leq N-L-K, \;\; 0 \leq B \leq L$$

from which the claimed indifference of the manager regarding B is immediate. In the following analysis, one can therefore let $B = 0$ without loss of generality and only consider

$$\sum_{K=0}^{N} \binom{N}{K} p^K (1-p)^{N-K} \overline{\pi}^K w(K, N-K) - Nc$$

$$\geq \sum_{K=0}^{N-L} \binom{N-L}{K} p^K (1-p)^{N-L-K} \underline{\pi}^{I_J} \overline{\pi}^{I_K} A^{N-I} w(N,0) - (N-L)c$$

$$\forall\ L = 1, ..., N; \forall\ K \longmapsto (I_J, I_K)\ s.t.\ 0 \leq I_K \leq K,\ \ 0 \leq I_J \leq N-L-K$$

Note next that since $A > \underline{\pi}$, it cannot be optimal for the manager to have $I_J > 0$. Also, since $\overline{\pi} > A$ it cannot be optimal to have $I_K < K$, so that optimally $(I_J, I_K) = (0, K)\ \forall\ K$. Under the proposed wage schedule, given any L and any K, the manager will have an incentive to invest efficiently as regards the projects he investigated. It remains to verify that the proposed wage schedule satisfies

$$\sum_{K=0}^{N} \binom{N}{K} p^K (1-p)^{N-K} \overline{\pi}^K w(K, N-K) - Nc$$

$$\geq \sum_{K=0}^{N-L} \binom{N-L}{K} p^K (1-p)^{N-L-K} \overline{\pi}^K A^{N-K} w(N,0) - (N-L)c$$

$$\forall\ L = 1, ..., N$$

Substituting $A^{N-K} w(N,0)$ for $w(K, N-K)$ on the LHS, defining

$$D \equiv p\overline{\pi} + (1-p)A$$

and then substituting

$$w(N,0) = \frac{Nc}{[p\overline{\pi} + (1-p)A]^N - A^N} = \frac{Nc}{D^N - A^N}$$

on both sides one obtains

$$\sum_{K=0}^{N} \binom{N}{K} p^K (1-p)^{N-K} \overline{\pi}^K A^{N-K} \frac{Nc}{D^N - A^N} - Nc$$

$$\geq \sum_{K=0}^{N-L} \binom{N-L}{K} p^K (1-p)^{N-L-K} \overline{\pi}^K A^{N-K} \frac{Nc}{D^N - A^N} - (N-L)\, c$$

$$\forall\, L = 1, ..., N$$

This can be further simplified to read

$$D^N \frac{Nc}{D^N - A^N} - Nc \geq D^{N-L} A^L \frac{Nc}{D^N - A^N} - (N-L)\, c \;\; \forall\, L = 1, ..., N$$

Rearranging one finally obtains

$$D^N N - \left(D^N - A^N\right) N - D^{N-L} A^L N + \left(D^N - A^N\right)(N-L) \geq 0$$

$$\forall\, L = 1, ..., N$$

$\Longleftrightarrow$

$$\left(A^N - D^{N-L} A^L\right) N + \left(D^N - A^N\right)(N-L) \geq 0$$

$$\forall\, L = 1, ..., N$$

Note that for $L = N$ (the manager does not investigate any project) one has

$$\left(A^N - A^N\right) N + \left(D^N - A^N\right)(N-N) = 0$$

as one would expect given that $L = N$ is binding.

Next define

$$H_L(D) = \left(A^N - D^{N-L} A^L\right) N + \left(D^N - A^N\right)(N-L)$$

and note that

$$H_L(A) = \left(A^N - A^{N-L} A^L\right) N + \left(A^N - A^N\right)(N-L) = 0$$

But

$$\frac{\partial H_L}{\partial D} = -(N-L)\,ND^{N-L-1}A^L + N(N-L)\,D^{N-1} > 0$$

$\forall L = 1,..,N-1$. Hence, since $D > A$ the inequality will be strictly satisfied for $L = 1,..,N-1$, so that the solution satisfies all (IC_e) constraints.

Next one can verify that all IC_d-constraints are satisfied. Incorporating a) these can be written down as

$$\overline{\pi}^K w(K, N-K)$$

$$\geq \underline{\pi}^{I_J}\overline{\pi}^{I_K} w(I, N-I)$$

$$\forall\ (I_J, I_K)\ s.t.\ I_J + I_K = I,\ \ 0 \leq I \leq N\ \ \forall\ K\ s.t.\ 0 \leq K \leq N$$

It is easily seen that these can more succinctly but equivalently be written down as

$$\overline{\pi}^K w(K, N-K) \geq \underline{\pi}\overline{\pi}^K w(K+1, N-K-1)\ \forall K = 0,..,N-1$$

$$\overline{\pi}^K w(K, N-K) \geq \overline{\pi}^{K-1} w(K-1, N-K+1)\ \forall K = N,..,1$$

The first set of constraints ensures that the manager does not overinvest, while the second set forestalls underinvestment. Both are clearly met when $w(K, N-K) = Aw(K+1, N-K-1)$ as under the schedule given in part b).

(ii) To establish optimality of the candidate wage schedule one has to show that the Kuhn-Tucker-conditions are satisfied. It is well known that, for a linear problem, these reduce to the requirement that the gradient of the objective lies inside the cone generated by the normals of the supposedly binding constraints. Given that $w(k, J) = 0\ \forall k < N-J,\ \forall J = 0,..,N$, the minimand can be written as

$$\sum_{K=0}^{N} \binom{N}{K} p^K (1-p)^{N-K}\, \overline{\pi}^K w(K, N-K) - Nc$$

The binding (IC_e) constraints are given by

$$\sum_{K=0}^{N} \binom{N}{K} p^K (1-p)^{N-K} \overline{\pi}^K w(K, N-K) - Nc \geq A^B w(B, N-B)$$

$$\forall B = 0, 1, .., N$$

Defining

$$\left[\binom{N}{K} p^K (1-p)^{N-K} \overline{\pi}^K \right]_{K=X} = G_X$$

and stacking the gradient vector and constraint matrix with the top element pertaining to $w(0, N)$ and the first column of the constraint matrix pertaining to $B = 0$, one has

$$\begin{pmatrix} G_0 \\ G_1 \\ G_2 \\ \\ G_N \end{pmatrix} = \begin{pmatrix} G_0 - 1 & G_0 & G_0 & G_0 \\ G_1 & G_1 - A & G_1 & G_1 \\ G_2 & G_2 & G_2 - A^2 & G_2 \\ \\ G_N & G_N & G_N & G_N - A^N \end{pmatrix} * \begin{pmatrix} \lambda_0 \\ \lambda_1 \\ \lambda_2 \\ \\ \lambda_N \end{pmatrix}$$

where for optimality one requires that $\lambda_l > 0$ for $l = 0, ..., N$.

It is easily checked, that

$$\lambda_l = \frac{\left[\binom{N}{K} p^K (1-p)^{N-K} \overline{\pi}^K A^{N-K} \right]_{K=l}}{[p\overline{\pi} + (1-p) A]^N - A^N} = \frac{G_l A^{N-l}}{[p\overline{\pi} + (1-p) A]^N - A^N}$$

so that one has $\lambda_l > 0$ for $l = 0, ..., N$, q.e.d.

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