Contract Renegotiation

under

Asymmetric Information;

On The Foundations of Incomplete Contracts

by

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Abstract

The dissertation explores the effect of limited contractual commitment on the form of contracts and studies its welfare implications. The main focus is on foundations of incomplete contracts. The thesis studies to what extent incompleteness of contracts can be linked to contract renegotiation. Particular emphasis is put onto explaining the absence of a contract from a relationship.

Chapter 1 reviews the literature on contract renegotiation and incomplete contracting.

Chapter 2 is based on a version of the hold-up problem. It shows that contracts that are vulnerable to renegotiation cannot provide better investment incentives than no contract. The main driving force is that investment, although beneficial from a total surplus point of view, has an ambivalent effect on the investing party's payoff. It increases the benefit of an efficient action and decreases the benefit of an inefficient action. An example is investment into human capital, such as additional job training. It increases personal satisfaction in a challenging job but may also increase the frustration from a job that consists only of repetitive tasks. If an exact job description is not feasible ex-ante and if the non-investing party has all the bargaining power ex-post, contracts cannot compensate for the cost of investment.

Chapter 3 formalizes the intuition that contracting involves a cost because a contract constitutes a less flexible status quo for ex-post bargaining than no contract. For this, asymmetric information is introduced. With asymmetric information contracting is potentially costly because an inefficient outcome is not necessarily undone by an ex-post bargain. For example, during the renegotiation of the contract between General Motors and Fisher Body, the latter adopted a cost intensive production technology in order to convince its partner to renege on the former agreement. In the model of this chapter, parties weigh the benefit of a contract against lost flexibility. If these effects are similar, no contract is written.

The possibility that a contract might be strictly dominated by no contract is explored in chapters 4 and 5. Such a strict dominance result is interesting because

it is a more forceful advocate for the incomplete contract assumption.

Chapter 4 contains a version of the durable good monopoly model with no discounting but costly contracting. These could be writing or legal costs. Early contracting is less costly than late contracting which highlights the idea that bargaining at a deadline is more costly. But also, early contracting suffers from the ratchet effect because it releases information. The main result says that the costs of the ratchet effect outweigh the cost savings, even if initial contracting costs are of order of magnitude smaller than late contracting costs. The seller strictly prefers to offer no contract.

In chapter 5, a sequential screening model endogenizes the fixed contracting cost. The buyer is privately informed about one part of the good's value but ignores the second part, which is revealed later. Early contracting is beneficial because it suffers less from asymmetric information than does late contracting. Nevertheless, if uncertainty with respect to the first variable is greater than uncertainty with respect to the second variable, the seller cannot take advantage of this fact and he strictly prefers to wait. Moreover, if this is not the case, contracts are partially incomplete because they are not conditioned on the second variable.

Finally, the thesis reports the new effect that all contracts are renegotiated in equilibrium. This is in contrast to the renegotiation proofness principle, which states that in models of contracting with renegotiation one can restrict attention to renegotiation proof contracts.

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Contents

Chapter 1

Introduction

The dissertation explores the effect of limited contractual commitment on the form of contracts and studies its welfare implications. The aim is to provide an understanding of why and to what extent contracts are incomplete. The problem of foundations of incomplete contracts has received attention in the literature on contract theory because it constitutes a major divide between two parallel streams of this theory, 'complete' and 'incomplete' contract theory.

1.1 Complete versus Incomplete Contracts

1.1.1 Complete Contracts

Complete contracting has developed traditionally by trying to offer contractual solutions to problems of asymmetric information. These can be broadly divided into three categories. One party has some private information to which another party has no access, the case of adverse selection or hidden information. A party can take an action unobserved by another party, the case of moral hazard or hidden action. Parties share the same information which is unverifiable by outsiders, in particular courts, the case of nonverifiable information. In what sense the described information asymmetries are an obstacle to reaching efficiency and what are optimal ways to cope with them is the focus of contract theory.

The literature on the first type of problem, hidden information, initiated with the mechanism design literature, which was pioneered by Hurwicz (1960), Clarke (1971) and Groves (1973), who study the optimal provision of a public good, and with literature on auction theory, (Vickrey (1961)). These approaches make decision rules contingent on the reports of agents about private information. It also developed in a principal-agent context as can be found for instance in Baron and Myerson (1982) or Maskin and Riley (1984). In this class of problems an uninformed principal screens the different possible types of an agent by offering contingent contracts. The symmetric problem, first studied by Spence (1971), where the informed party tries to signal his type, also shows how contingent contracts can help overcome informational problems. In the context of adverse selection, the main trade-off arises between providing incentives for truthful revelation of information and giving up informational rents to privately informed agents. In general, this leads to allocations that are less efficient than the first-best.

The paradigm of moral hazard, pioneered by Mirrlees (1975), Shavell (1979) and Holmstrom (1979), studies ways to provide incentives to an agent to exert effort. If this effort is unobservable, an employment contract cannot be made contingent on it. Instead, an incentive scheme will be based on the worker's performance, which is only an imperfect signal of his effort. Because performance is also dependent on variables that are not under the agent's control, such an incentive scheme introduces risk into the agent's wage. If he is risk averse, a contract has to strike a balance between incentives and insurance.

The case of non verifiability of information can be found when contracting parties have symmetric information but third parties, such as courts, have no access to this information. The implementation literature, which started with Maskin (1977), studies what allocations can be implemented when mechanisms are made contingent on agent's reports concerning non verifiable information. The main message that has emerged from this literature is that non verifiability of information does not pose a serious threat to implementability.

A particular problem that arises from non verifiability of information is the hold-up problem, which is close to the moral hazard paradigm. Parties engaging in a relationship are uncertain about future benefits of their relationship. If this information is non verifiable and message games as above are neglected, parties have to bargain ex-post over the split of the surplus. In this situation parties will underinvest in the relationship because they fear expropriation of the investment benefit by their partner in the transaction. Given the work on implementation theory the hold-up problem can in principle be solved in symmetric information environments if no further assumption restricting the set of feasible contracts are made. Similarly, Rogerson (1992) provides solutions to the hold-up problem in asymmetric information environments and concludes that it can be dealt with without loss of efficiency in a wide variety of circumstances.

Apart from the efficiency cost of asymmetric information, complete contract theory abstracts from any other cost of contracting.

1.1.2 Incomplete Contracts

Incomplete contracting on the other hand starts by assuming that either only very simple contracts can be written or that contracting is altogether impossible and studies the implication for organizational structures of economic institutions. The divide between incomplete contract and complete contract theory is not always obvious. Coming back to the moral hazard example, one could argue that this problem falls into the domain of incomplete contracts because the contract is not contingent on the effort variable. This is true, but in the moral hazard example, there is no way to include this variable into the contract because it is unobservable and *cannot be elicited.* In contrast, incomplete contract theory starts by assuming that it is rather some aspect of the agent's rationality, the legal framework or of the nature of information that prevents contracts from being complete. By limiting the possibility of contractual solutions to economic problems, this theory has allowed rich developments in the study of organizations and institutions. The initial focus, starting with the important contributions by Coase (1937) and Williamson (1975), was on the nature of the firm, see also Hart (1995). The central insight of this literature is that integration of firms is beneficial because it reduces the hold-up problem that arises between two independent entities. Similarly, a merger between firms reduces problems of asymmetric information in adverse selection problems. This theory can therefore trace the boundaries of firms, something which prior theories were unable to do. Consequently, the incomplete contract approach has enabled economic theory to conceptualize other important phenomena such as ownership, (see Grossman and Hart (1986), Hart and Moore (1990)), and authority, (Aghion and Tirole (1997)). Other applications have for instance been derived in corporate finance. The first articles, Aghion and Bolton (1992) and Hart and Moore (1994), explain shifts in control rights in financial contracts. Later articles study the role of diversified claims of investors in disciplining a firm's manager, see for example Dewatripont and Tirole (1994).

So far, the incomplete contract literature has developed mainly in symmetric information settings. Since the main focus of the complete contract literature is on asymmetric information settings, this difference does not help to clarify the nature of the divide. Moreover, independent of the debate on foundations of incomplete contracts, it seems important to assess the robustness of existing models and results of incomplete contracting to the extent of informational asymmetries. Therefore, the main focus of this thesis is on problems in which contracts may or may not be complete in the presence of asymmetric information.

1.1.3 An Illustrative Example

To illustrate the type of questions this thesis is going to focus upon consider the following example.

Example 1 *A buyer plans to purchase a software package for his computer. An* *upgrade of the software will become available in a year's time. At the time of the purchase the buyer is still uninformed about the exact value of the upgrade to him. This will for example depend on his use for a computer and on his specific needs concerning the software. Importantly, the value will be private information to the buyer, i.e. the seller will not be informed. The seller has the bargaining power in the negotiations. He sets the price in the form of a take-it-or-leave-it offer. How does the efficiency of the transaction depend on a contract? Does asymmetric information matter?*

For simplicity, assume that the seller's cost for the software is 0 *and that the upgrading service costs* 2. *The value of the software's use from now on is* 6. *In a year, it will have decreased to* 5 *whereas the upgraded version will have a value of either* 9 *or* 12, *that is, the upgrade will increase the software's usefulness by either* 4 *or* 7. *Assume that the buyer estimates the two possibilities as equally likely and that this information is common between both parties. Remark, that in either case the increase in the value of the software due to the upgrade lies above the seller's cost. It is therefore always beneficial to undertake the upgrade.*

Assume that the buyer purchases the software package today but that he and the seller do not agree on a price for an eventual upgrade. Instead they wait until the buyer has received his private information and bargain over the price of the service at that time. The following inefficiency might occur. The seller prefers to set a high incremental price of 7 for the service which is only accepted by the buyer with .5 probability instead of setting a low price of 4 which would guarantee the deal. That is, the seller wants to save on information rent that a buyer with high valuation would obtain. In the absence of a contractual arrangement concerning the upgrade, asymmetric information leads to an inefficient allocation.

Consider two scenarios. The buyer and seller write a complete contract in the form of a service contract that comes with the purchase of the software. With this contract the seller commits himself to provide the buyer with an upgrade of the software in a year's time. The buyer pays a price of 5.5 in advance. He accepts the deal because he receives 0 in expectation and he can't expect to obtain more by refusing this offer. The seller on the other hand is better off because he obtains $5.5 - 2 = 3.5$ instead of $.5 \times (7 - 2) = 2.5$. In this simple example, optimally, parties should write an early sales contract with a fixed price for the upgrade. Intuitively, by contracting early, they can circumvent the problem arising from asymmetric information.

Now assume, as could be the case in the incomplete contract paradigm, that a contract, stipulating that the buyer will receive an upgrade for his software after a year, is impossible. For instance, it might be difficult to specify in advance what exactly is entailed in the upgrade because technological progress in computer software is hard to predict. Following the incomplete contract approach, although an initial contract cannot be written, parties can jointly decide on different ownership structures for the basic version of the software package. If the software becomes the buyer's property, we are in the same situation as described above. After a year, the seller will ask a high price for the upgrade and the low valuation buyer will not buy. Alternatively, the buyer can rent the software for a year, such that the seller remains the owner. The rental price is set equal to 1, the value of the software's use over the next year. A year from now, the seller can sell the already upgraded version to the buyer for a price of either 9 or 12. Given his beliefs about the buyer's valuation, he prefers the lower price, because $9 - 2 > 0.5 \times (12 - 2)$. Efficiency is achieved. The buyer's outside option in the final bargaining is decreased by a shift in ownership and the seller can therefore commit himself to offering a better deal. While with complete contracts ownership plays no role, ownership matters in achieving efficiency in the incomplete contract approach.

A characteristic of the example is that there is no issue of ex-ante incentives. The upgrade becomes available after a year without any further investment of either of the contracting parties. The main part of the thesis considers similar problems. Hence it belongs to the branch in incomplete contract theory concerned with ex-post problems, as is for example Hart and Moore (1994). Usually, these ex-post problems are due to cash constraints limiting bargaining efficiency. The source of inefficiency in most of this thesis is novel as it stems from ex-post asymmetric information[1]. Another branch of the incomplete contract approach investigates ex-ante problems, as in Grossman and Hart (1986) and Hart and Moore (1990). The problem considered in chapter 2 falls within this category.

Remark, that in the above example both the complete and the incomplete contract yield the first-best. In this context there is therefore no loss in assuming that contracts are incomplete. The point here is that the particular form of contract incompleteness does not prevent parties from achieving efficiency. A 'simple' contract like ownership gets around this problem. In a more complex environment this is not necessarily true as the efficiency of the solution in general depends on the set of contracts that are assumed feasible.

1.1.4 Sensitivity of Results

One problem of the incomplete contract approach is the sensitivity of its predictions to modelling assumptions. In particular, the intuition that with simple contracts relatively robust results would be obtained has turned out to be misleading. The outcomes of incomplete contracting models tend to depend delicately on the specific assumptions about contractual possibilities and on the assumed extensive form game.

To see that the form of assumed contractual incompleteness matters, reconsider the above example. In the incomplete contract version it is assumed that a contract specifying a particular upgrade of the software package is infeasible. On the other hand, a rental agreement of the software is assumed to be perfectly enforceable. It is possible though, that a rental contract suffers from a different kind of incompleteness, namely from a commitment problem. If it is impossible to ensure that the buyer indeed hands the software back to the seller, he could for example retain a pirate

[1] A paper that considers ownership allocations in an incomplete contracting model with ex-post inefficiencies of this type is Matouschek (2000)

copy, such an agreement would be useless.

Several recent papers have tested the robustness of predictions derived from the incomplete contract approach with respect to the timing and assumed extensive form of the bargaining game. One important result in the incomplete contract literature for example is that asset ownership motivates investment. The intuition is that asset ownership enhances a party's bargaining power in ex-post negotiations by raising its disagreement payoff. Therefore, a larger share of the surplus can be obtained, which in turn increases returns from investment. This implies in particular that joint ownership is never optimal because none of the investing parties can protect its own investment.

DeMeza and Lockwood (1998) and Chiu (1998) point out that the first result is only true if a specific class of bargaining games is assumed for the ex-post negotiations, namely, bargaining games in which parties disagreement payoffs are taken to be *inside* rather than *outside* options. With inside options, i.e. when disagreement payoffs are realized during the bargaining period, a party's final payoff is indeed his disagreement payoff plus a fraction of the gains from agreement. In contrast, with outside options, a party's final payoff is either simply the payoff from his outside option or total surplus minus the value of the other party's outside option. Shifting ownership away from a party reduces his outside option which may then become non binding. The party therefore receives total surplus minus a constant, which provides optimal investment incentives. Although the central intuition that ownership matters for investment incentives still holds, with a bargaining game that is different from the one that is usually assumed in incomplete contracting models, asset ownership demotivates investment.

Halonen (1995) tests the role of asset ownership in a dynamic model in which contracting parties interact repeatedly and use trigger strategies to support equilibrium outcomes. In this setting, joint ownership might be optimal because it allows parties to punish severely in case of deviation from the equilibrium path. But joint ownership is never optimal in the static incomplete contract model, which points to a need to carefully examine the timing of such models.

1.2 Objective of the Thesis

The main question of the thesis is under what circumstances, incompleteness of contracts can be assumed without loss of generality. Put differently, when does contracting *not* matter?

Providing an answer to this questions is important because it lies at the heart of the criticism that has been levied on the incomplete contract approach. Its most substantial problem is the ad hoc assumption about what kinds of contracts are deemed possible. Although heuristic arguments are used to motivate incompleteness of contracts, they are rarely made precise and are often not tailored to the specific contracting problem under study. The discussion in section 1.1.4 points to the need to understand more clearly what the exact limitations of complete contracting are. Also, a link should be established between theoretically well founded assumptions and specific forms of contractual incompleteness.

The thesis aims at evaluating the inefficiencies of a particular limit to contracting, a commitment problem. More precisely, parties cannot commit not to renegotiate a contract if this is in their common interest. Renegotiation has been extensively studied in various contracting models and is by now a well accepted paradigm in the literature. It generally restricts the set of achievable outcomes and therefore makes contracting less valuable. In this thesis it is used to explain one special form of contract incompleteness, the absence of a contract. I consider various complete contracting frameworks in which the constraints on a contract imposed by incentive considerations and the possibility of contract renegotiation induces parties to refrain from writing a contract. The absence of a contract is of particular interest because many incomplete contracting models are based on the assumption that a contract cannot be written. Before going into details of the thesis's contribution I contrast the premise of my approach with existing assumptions about contractual incompleteness.

1.3 Existing Foundations

1.3.1 Transaction Costs

The main argument evoked to support contractual incompleteness are transaction costs. These are centered around three themes. First, possibly because of bounded rationality, contracting parties cannot foresee all relevant future contingencies and actions. Second, even if they could foresee them it would be inherently costly to include every single one of them in a contract. Finally, some form of legal authority must be able to read and interpret the terms of a contract and verify the contracted upon contingencies. I will discuss each of these in turn and indicate how far the theoretical literature has advanced in addressing these issues. Tirole (1999) provides an exhaustive survey of this literature.

1.3.2 Unforeseen Contingencies

Bounded rationality and the existence of unforeseen contingencies seem to be an intuitive argument to explain why contracts are incomplete. Nevertheless, it is quite difficult to capture these concepts in a theoretical model. For one, economic theory has so far not developed a concise notion of what constitutes boundedly rational behavior[2]. Similarly, only few models of unforeseen contingencies exist.

One way of modelling unforeseen contingencies is to assume that an individual has subjective, non additive beliefs over future states of the world. This is equivalent to assuming that, instead of using a single probability distribution to describe the world, an individual considers a whole range of possible distributions. Allowing for uncertainty over the exact distribution captures the idea that the individual is uncertain about the future. He is called ambiguity averse if, when evaluating a deci-

[2]For an introduction into the topic of bounded rationality, see Rubinstein (1998)

sion, he always considers the probability distribution that offers the lowest expected payoff. Mukerji (1998) uses this approach to explain incompleteness of contracts in a hold-up problem. Here, the role of a contract is to make a party's payoff sensitive to his investment. With two investing parties this can be achieved if different states of the world are affected differently by each party's investment. To induce party A, say, to invest, a contract needs to offer a high compensation in the state that is most affected by party A's investment. The effect of such a contract is to make a party's payoff volatile. With ambiguity aversion, similarly as with risk-aversion, this reduces a party's expected payoff and in certain circumstances, a contingent contract violates parties' participation constraints. A completely uncontingent contract that allows for a relatively equal distribution across states is the optimal contract. Unfortunately, it is quite difficult to distinguish this setting from one in which parties are risk averse.

The papers by Kreps (1979) and Dekel, Lipman, and Rustichini (2000) offer an alternative model of unforeseen contingencies. The authors consider a two stage decision problem of a single agent. In a first stage he chooses a set of actions from which he has to select one at the second stage. If he can foresee all future states of the world, he is able to foresee the consequences of every possible action. Therefore, if he is able to make a contingent choice at the first stage, that is, if he can write a plan that specifies an action in each future state of the world, the second stage is void. He does not gain anything from leaving his choices open. On the other hand, if he fears unforeseen contingencies, he might initially prefer a larger choice set so as to allow himself more flexibility at the second stage. The authors derive a representation theorem for the initial preferences over choice sets. If these preferences exhibit preference for flexibility, the derived utility over ex-post actions will be state dependent. The additional states are interpreted as 'unforeseen', see Kreps (1992). This set-up seems to capture the intuition of incomplete contracts quite well. If a large choice set is interpreted as an incomplete contract, the above model provides an explanation for a preference for incompleteness.

In this approach the agent can perfectly describe all possible future actions but is unable to come up with a complete list of future states of the world. He is therefore unable to forecast his future payoffs in a contractible way. In contrast, the notion of unforeseen contingencies put forward in the paper by Maskin and Tirole (1999) is that the physical attributes of a future state or an action is unforeseen so that they are indescribable *ex-ante* but payoffs resulting from these actions are perfectly foreseen.

The authors use this idea to provide a criticism to the argument that contracts are incomplete because of unforeseen contingencies. The authors point at a tension in existing incomplete contracting models between, on the one hand, infinitely rational parties who perfectly foresee all possible future payoffs, and on the other hand the assumption that parties are unable to describe the actions that will lead to these payoffs. In particular, the authors show that given the assumption that actions and states of nature are perfectly describable *ex-post*, an assumption which is usually made in incomplete contracting models, the problem of *ex-ante* indescribability can be overcome by writing 'payoff based' revelation mechanisms.

To see the intuition of their result, the software example is slightly modified. Assume, that the software can be developed along several possible lines. By how much each of these approaches will have advanced in a year is not yet known. Similarly, it is not known which one will finally turn out to be the most efficient alternative. Therefore, technical descriptions cannot be included in a contract. Nevertheless, assume that it is possible ex-post, i.e. once all relevant uncertainty is resolved, to costlessly describe all feasible upgrades. Furthermore, ex-ante contracting parties can foresee (or at least estimate) the values and costs of *all* possible future alternatives. Therefore, parties know what the utility profiles are that they want to implement. The following table summarizes costs and increments in value. Three feasible alternatives are considered.

	values		costs
	s_1	s_2	
α_1	4	7	2
α_2	5	8	4
α_3	1	4	0

The parties foresee three feasible upgrades with the above payoff structures, but there might be many more infeasible ones. State s_1 is the state in which the buyer has a low valuation for an upgrade, s_2 is the state in which he has a high valuation, whereas the seller's cost are the same in both states. Parallel to Example 1, I assume that s_1 and s_2 are equally likely. In addition, a state includes the physical description of the feasible upgrades with payoff profiles (α_1, α_2, α_3).

In this situation, regardless of the realization of s_i, the first set of payoffs α_1 should be implemented, because it guarantees the highest joint surplus. For example, the set of payoffs α_1 results in a surplus of 2 in state s_1, whereas profiles α_2 and α_3 result in a surplus of only 1. Similarly, in state s_2 the surplus in the profile α_1 is 5 compared to 4 for the other two profiles.

A contract along the following lines could be proposed: The buyer initially purchases the software. For an advance payment of 5.5, the seller commits himself to 'upgrade' the program in a year's time. The exact details of the upgrade are filled in later. More precisely, in a year the seller has to describe the physical details of three possible upgrades, X, Y, and Z, say, and propose one of them as the efficient one. That is, he must designate the upgrade that has utility profile α_1. Because the naming of upgrades is arbitrary, assume that he proposes upgrade X. The buyer can accept the proposal in which case the upgrade is undertaken by the seller. Alternatively, the buyer can challenge the seller's proposal. If he does so, the seller pays 10 to the buyer. The buyer can challenge in several ways

(*i*) He can claim that one of the proposed upgrades is not feasible, which is ex-post verifiable by assumption. If the seller has indeed lied in that way, the software is not upgraded and the buyer keeps the fine.

(*ii*) The buyer can claim that the seller has proposed the wrong upgrade amid the feasible ones. To prove his point he must choose a number $m \in \{1, 3.5\}$. The seller then has the choice between *a*) delivering X or *b*) paying m and not upgrading the software. The challenge is successful if $m = 3.5$ and the seller picks alternative *b*) or if $m = 1$ and the seller picks alternative *a*). Otherwise the challenge is unsuccessful. If the challenge is successful, the alternative chosen by the seller, either *a*) or *b*), is enforced. If the challenge is unsuccessful the buyer has to pay a fine of 20 to the court.

Step (*i*) ensures that the seller has no incentive to lie about the type of upgrades he can deliver because the fine he has to pay if the buyer challenges is larger than his disutility from implementing the efficient upgrade. It allows parties to fill in the ex-ante indescribable details ex-post and constitutes the central result of Maskin and Tirole. Step (*ii*) ensures that the efficient upgrade is chosen out of the three feasible ones and is standard in the literature on subgame perfect implementation. Assume for example, that the buyer wants to claim that the seller's proposed upgrade X in fact corresponds to profile α_2. The buyer can challenge and set $m = 3.5$. If the seller has indeed lied in that way, his true costs for the upgrade are 4 and he will prefer to pay m rather than undertake it. If he has told the truth on the other hand, he will prefer to deliver the upgrade. Similarly, the buyer might want to claim that the seller has proposed the upgrade with profile α_3. To expose such a lie, he sets $m = 1$. If the seller has lied, he will prefer the actual upgrade to the payment, and the opposite will be true if he has told the truth. This implies that it is possible to detect who has behaved untruthfully and the fines ensure that lying is too costly.

Remark, that for the mechanism to work the buyer needs to be informed about the seller's preferences because he must decide on the correct m. Maskin and Tirole only consider symmetric information settings, i.e. settings in which there is complete information between contracting parties. In specific circumstances, the proposed mechanism can be adopted to a setting with one-sided asymmetric information, as is the case in the software example. It is therefore irrelevant whether or not the seller is informed about the buyer's valuation. If there was two sided asymmetric information, i.e. the buyer was also uninformed about the seller's cost, such a finely tuned mechanism would no longer be feasible. In general, asymmetric information restricts the set of allocations that are implementable when payoffs are unverifiable. Nevertheless, it seems that as long as there is no asymmetry of information concerning the feasibility of ex-post actions, the central result of Maskin Tirole remains unaffected. What can be achieved by an ex-ante contract if ex-post only one of the parties is informed about the feasibility of some of the actions, is yet unknown and remains future work.

Two key reasons ensure that the above contract works. The first is the existence of large punishments. The seller pays 10 to the buyer if the buyer challenges and the buyer pays 20 to a third party if his challenge is unsuccessful. The contract fails if such punishments are not enforceable. Contract renegotiation may for example void the latter type of punishment because parties can jointly renegotiate contractual clauses that prescribe large payments to third parties. Maskin and Tirole show, that even with renegotiation it is possible to construct mechanisms that overcome the problem of indescribable actions. The restriction needed for the second result are albeit more stringent. More precisely, they show that with renegotiation it is more difficult to ensure that a contract is 'welfare neutral'. Welfare neutrality means that the contract implements the same utility profile in states of nature which are characterized by an identical set of utility profiles. This is a necessary requirement also in a setting without renegotiation because for two such states, any utility profile that emerges as an equilibrium in the first state will also be an equilibrium profile in the second state. In our example, states are differentiated by the type of upgrades that are feasible, but feasible upgrades all give rise to the same set of utility profiles (modulo states s_1 and s_2). The contract is welfare neutral because it requires implementation of the same profile α_1 in all these states. To find out how restrictive welfare neutrality is under renegotiation and under asymmetric information is another way in which to extend this line of research.

The contribution of Maskin and Tirole (1999) is to show that ex-ante indescribability of actions is no hindrance to contracting as long as actions are assumed to be describable ex-post. In contrast, the paper by Aghion, Dewatripont, and Rey (2000) considers the situation in which actions are unverifiable *both* ex-ante and ex-post. Intuitively, this limits the scope of revelation mechanisms such as the one above because a contract can no longer implement different actions depending on the parties' announcements. In particular, challenges as in points (*i*) and (*ii*) are no longer feasible. It is therefore not surprising that revelation mechanisms will have much less bite. In order to introduce scope for 'partial contracting' the authors consider situations, in which, although actions are not contractible, the control over an action can be either contracted upon or can be transferred from one party to another. They term the former situation contractible control actions and the latter transferable control actions.

In the first case the authors show that in a dynamic relationship, where co-operation between two parties is desirable and where there is sufficient strategic complementarity between actions, a partial (incomplete?) contract that implements a switch of control from one period to the next is the optimal contract. Thus, a simple contract dominates any complicated message game. The idea is that a switch in control allows parties to punish non cooperative play and reward cooperative play. In the second case it might be optimal for the party in charge of an action to relinquish its power if the other party has private information. This allows the latter to build up reputation in a dynamic game.

1.3.3 Writing Cost

The reason that contracts might be incomplete because of costs of writing them has been first addressed by Dye (1985). He assumes that each contractual clause involves a fixed cost and argues that the cost of including an additional clause should be weighted against its benefit. Consequently, relatively unimportant contingencies are optimally excluded from a contract. A main shortcoming of this paper is the assumed rigidity of the contracting language. For example, the rule $f(x) = x - 1$ is infinitely costly in his framework, because it specifies for each x a different value $f(x)$. But with a richer language the rule could be contained in a very simple statement.

Battigalli and Maggi (2000) develop an explicit language that is used to describe the contracting environment and parties' behavior. Their language is composed of simple statements describing either states of nature or actions which are linked by logical operators. A typical contractual clause in their framework would be 'If it is feasible to add a windows surface to the software, do so'. Writing down a statement is costly and these costs are assumed to take the form of a fixed cost c per simple statement. The above clause would for example cost $2c$ because it contains the description of a state of nature and the prescription of an action. With this assumption, contracts can be overly rigid, i.e. not as finely tuned as the first-best. Alternatively, they can be too loose, i.e. an agent has discretion over his behavior. The exact form of contractual incompleteness is endogenous and depends on the size of the writing costs and on the uncertainty in the environment. An interesting conclusion of the paper from the viewpoint of this thesis is that, depending on the form of contractual incompleteness, both overinvestment and underinvestment can be rationalized in the hold-up problem. If a contract is too rigid, overinvestment will occur, and the opposite is true if a contract is too loose. In contrast, the main intuition of the incomplete contract literature is that incompleteness of contracts will always lead an agent to underinvest. This result once more highlights the importance of studying sources of contractual incompleteness.

An issue missing in the above papers is the notion of complexity of a contract which is taken up by Anderlini and Felli (1994). The problem studied in this paper is a coinsurance problem and a contract prescribes a transfer payment conditional on a realized state of nature. In this context, the authors restrict complexity of the contracting language by assuming that a formal contract must correspond to a computable function, i.e. a function that can be calculated with an algorithm in a finite number of steps. This assumption alone does not restrain parties from approximating the first best. The rough intuition is that because parties' utilities are continuous in money terms, the first best contract must prescribe relatively 'smooth' transfers. These transfers can be approximated by step functions which are computable and can thus be prescribed with a computable contract. The paper proceeds by showing that, if the contract selection process is subject to a similar restriction, the resulting contract has features of incompleteness.

Retaining the assumption that formal contracts must correspond to computable functions, the authors study the impact of additional complexity costs on the form of contracts in Anderlini and Felli (1999), (2000). In Anderlini and Felli (1999) a computation involves a minimum cost c, so that a contract that contains n steps of calculation costs nc. In Anderlini and Felli (2000) the authors use an axiomatic approach to contracting costs, in which the complexity cost function needs to fulfill two axioms. First, the null contract (the contract that leaves parties utility levels unchanged for every state of nature) involves no costs. Second, a contract with bounded costs y, can only result in a finite set of outcomes. The axiomatic approach in Anderlini and Felli (2000) encompasses the direct modelling approach used in Anderlini and Felli (1999). The authors are then able to show that there exist contracting problems within the set of coinsurance problems, such that complexity costs generate incomplete contracts in the strong sense, that is, the optimal contract is the null contract. Intuitively, if the first-best contract is not far away from the null contract the extra restriction through complexity costs generates this result.

1.3.4 Legal Cost

A contract needs to be detailed enough for an outsider such as a court, to understand its terms. Furthermore, contingencies, on which the prescriptions of a contract are based, need to be verifiable. In the software example, if the contract stipulates that the seller is to deliver an 'appropriate' upgrade to the buyer, it must be possible for an outsider to decide what is meant by this term. It is likely that this is more difficult for a judge who is not familiar with this type of software. An expert might need to be paid to give his advice. Many incomplete contracting models make the simplifying assumption that it is impossible for an outsider to verify certain variables or that the cost of doing so is prohibitive. This has led to the term of 'observable but unverifiable information' which means that information is observable by the contracting parties themselves but not by outsiders such as courts or other legal institutions. Incomplete contracting models conclude that a contract cannot be made contingent on this nonverifiable information. This is in contrast to existing theory because parties messages concerning this information can be included into the contract. Implementation theory has provided a body of results showing that in a wide range of circumstances unverifiability of information is no serious obstacle[3].

1.3.5 Strategic Incompleteness

The idea that strategic considerations in conjunction with some form of transaction costs might lead to more contractual incompleteness than pure transaction costs alone is the focus of the papers by Spier (1992), Allen and Gale (1992) and Bernheim and Whinston (1998).

Spier (1992) considers a risk sharing contract in which a risk-averse principal is privately informed about the value of a project for which he hires a risk-neutral agent. A good principal, whose project has higher chances of success, might want to signal this information to the agent. Without asymmetric information, the agent should bear all the risk and receive a contingent wage. Suppose now that there are transaction costs any time a new contingency is included in the contract. Then, even if contracts are fully contingent under complete information, the presence of

[3] For a survey on this topic, see Moore (1992).

asymmetric information leads the good principal to offer a non-contingent contract in equilibrium. A non contingent contract prevents the transfer of risk to the risk neutral party. It is costly for the principal to do so, and more costly for the bad principal than for the good principal. Hence, there is a possibility of signalling.

In contrast to the separating result of Spier, Allen and Gale (1992) concentrate on a pooling equilibrium in which uncontingent contracts are offered by *all* principals in equilibrium. In their model, suppliers facing uncertainty about future costs want to insure against this risk by writing contingent supply contracts with their clients. Suppliers are differentiated by the quality (unobservable) of their product and not by the riskiness of their production technology as in Spier. There are neither writing nor legal costs but production costs must be apprehended by a measurement or accounting system. This in itself is not costly, but it is assumed that the principal can distort the output of the measurement system by incurring some cost. Importantly, a 'good' principal, who provides a high quality good, pays a higher cost for manipulating the system and therefore gains less from a contingent contract. This implies that he can credibly signal his type with an uncontingent contract. The authors argue in favor of the pooling equilibrium with uncontingent contracts because it is the unique stable equilibrium in the sense of Universal Divinity. Furthermore, all supplier types are better off in this equilibrium.

The above two papers are concerned with signalling properties of incomplete contracts. The paper by Bernheim and Whinston (1998) studies the *disciplinary* role of an incomplete contract. In a dynamic setting, an unverifiable and a verifiable action are undertaken in sequence by two agents. Contracts can be written to restrict the second agent's action set but cannot be conditioned on the first agent's action. Incomplete contracts, i.e. contracts in which the second agent's choice is unrestricted, emerge if actions are strategic complements, whereas complete contracts emerge if actions are substitutes. The intuition for the result is that, with complements, the second agent, having observed the first agent's action, tends to reward good behavior and punish bad behavior. He should therefore be given discretion. In contrast, with substitutes bad behavior is rewarded and good behavior is punished if parties have too much discretion. The central message is that it can be optimal to leave contractible actions unspecified in a contract because this ensures a better handling of the informal part of the relationship.

A related idea is explored in Dewatripont and Maskin (1995). The authors develop a dynamic model in which a contract can be made contingent on two variables which are set in sequence. The contract is a risk-sharing agreement between a risk neutral principal and a risk-averse agent who has private information. The difference with the model by Bernheim and Whinston (1998) is that no use of a transaction cost argument is made by assuming that one of the variables is unverifiable. Instead, another ingredient, renegotiation, interferes to create an incomplete contract. Renegotiation can occur in between the two stages at which the variables are chosen. Since the observation of the first variable may reveal information about the informed party's type, it can interfere with the second variable's risk-sharing role. Thus it may be optimal to leave the first variable unobservable.

1.3.6 Renegotiation

The first paper that draws a formal link between unverifiable information and contract renegotiation is Hart and Moore (1988). The authors define a contract as incomplete when it cannot be made contingent on a future state of nature, i.e. when the state is ex-post unverifiable. In spite of this restriction the overall outcome is dependent on the state of nature through an exchange of ex-post messages between the parties. The contract can structure this ex-post exchange which serves to complete the initially incomplete contract. The crux is that these messages can simultaneously be used to renegotiate the initial contract and therefore severe limitations are put on the initial contract. It is shown that these restrictions hinder parties from achieving the first-best in a version of the hold-up problem.

The papers by Che and Hausch (1999) and Segal (1999), Hart and Moore (1999) provide an even stronger result in the same type of problem by emphasizing different aspects of the contracting environments. The main result of these models is that under certain conditions contracting parties may be indifferent between writing an ex-ante agreement and simply relying on the ex-post bargaining game. Hence, an initial contract has no value in the relationship. The important assumption needed for this result is that contracts can be renegotiated. Here, a contract that is made indirectly contingent on the state of nature through parties' ex-post messages is not called incomplete. Rather, the null contract which emerges as the (weakly) best contract is called incomplete.

To understand the intuition I will explain the details of Che and Hausch (1999). The models by Segal (1999) and Hart and Moore (1999) provide a different explanation for contractual incompleteness and their contribution will be discussed extensively in the second chapter of the thesis. In contrast to most of the literature on the hold-up problem, Che and Hausch consider cooperative investments, that is, investments that not only benefit the investing party but also directly affect her partner's payoff.

Consider a version of the software example with purely cooperative investment.

Example 2 *When designing the software's upgrade, the seller can pay particular attention to the buyer's needs. He can for example take into account the buyer's wishes for a user friendly surface. This relationship specific investment involves a cost to the seller which is set to* 1. *The investment results in a deterministic increase in the buyer's valuation for the upgrade. The upgrade enhances the software's value to the buyer by* 7 *if the seller has undertaken the relationship specific investment, and by* 4 *otherwise. The cost of the actual upgrade is set equal to* 2. *The investment is desirable because* $7 - 4 > 1$. *The upgrade is desirable regardless of the seller's investment, because* $4 > 2$. *Investment costs are sunk when parties bargain over the price for the upgrade, in contrast to the upgrade's costs, which are only incurred when the upgrade is actually undertaken. Because the buyer's valuation is determined by the seller's investment, there is no asymmetric information ex-post. Furthermore, assume that the buyer has all the bargaining power in the ex-post negotiation.*

First consider the situation in which there is no contract. Because the buyer has all the bargaining power and investment costs are sunk, he will offer the seller a price of 2 for the upgrade regardless of whether the seller has invested or not. The seller has therefore no incentive to invest.

Now, consider the situation, in which the parties can commit themselves to a contract. An efficient solution to the above hold-up problem is the following contract. The purchase of the software includes the option for the buyer to upgrade the software for a price of 7 in a year from now. Because the buyer will exercise his option if and only if the seller has indeed undertaken the investment, this provides the seller with the right investment incentives. He obtains $7-2-1=4$ if he invests and 0 otherwise.

Now, assume that this option contract can be renegotiated. Namely, the buyer, if he rejects the upgrade, can make another offer to the seller. This offer will have the same form as if no contract had been written, the price will just be high enough to cover the production cost of 2. As it does not take the seller's sunk investment into account, the seller has no incentive to invest.

In the above example, even more general contracts than the simple option contract cannot provide the seller with investment incentives. Because the seller's payoff is not directly affected by his own investment a simple contract that enforces trade for a fixed price does not improve investment incentives. Instead, a contract must be made contingent on either of the parties' announcements about investment. Now, the seller has always an incentive to claim that he has invested whereas the buyer, having the bargaining power in contract renegotiation, always has an incentive to claim that he has not. Therefore, it is impossible to prevent both of them from lying. Che and Hausch show in a more general setting that, depending on the parties' respective bargaining powers and on the extent to which investments are cooperative, ex-ante contracts have no power.

Interestingly, renegotiation, although very powerful in the above example, is not necessarily harmful nor does it always destroy the value of contracting. In fact, adding asymmetric information to the above example will restore efficiency of an option contract even if contracts can be renegotiated. To see this, assume that the seller's cost for the actual upgrade are uncertain. They can be either 0 or 4, where these events are equally likely. The seller will find out whether the costs of the upgrade are high or low after investment has taken place and the investment cost are sunk, but shortly before trade. Assume, that the buyer is uninformed about the seller's cost realization.

In the absence of a contract the buyer will propose to purchase the upgrade from the seller for a price of 0 regardless of whether the seller has undertaken the investment or not. To see this, compute the buyer's expected utility for either case. In the case in which the seller has made the investment, the buyer obtains $0.5 \times (7-0) = 3.5$ if he offers the low price and $7-4=3$ if he offers the high price. In the case in which the seller has not invested, the buyer obtains $0.5 \times (4-0) = 2$ if he offers the low price and $4-4=0$ if he offers the high price. Therefore, a price offer of 0 is optimal for the buyer. Given this, the seller has no incentive to invest because he obtains a 0 payoff in either case but he has to bear the cost of investment.

Now, consider an ex-ante contract that gives the buyer the option to purchase the upgrade at a price of 3. Remark, that this price must lie below the price that is set when renegotiation is not an issue, because the buyer's outside option is strictly positive with renegotiation. With this contract, the buyer has an incentive to exercise the option if and only if investment has taken place. If the seller has invested, the buyer obtains $7-3=4$ from exercising the option, which is more than what he would get from not exercising the option and relying on renegotiation. If the seller has not invested, the buyer prefers not to exercise the option because $4-3=1$ is less than what he can expect from renegotiation. The seller has a (weak) incentive to invest because he expects to obtain $3-0.5\times 0-0.5\times 4-1=0$ if he invests and 0 if he does not invest.

The reason for the failure of the Che-Hausch result is that with asymmetric information the renegotiation game is not efficient. Therefore, contracting parties fearing a loss of surplus due to the inefficiency of the bargaining game are more likely to obey the rules' of a contract. Remark, that in both set-ups, although parties might not gain from writing a contract as is the case in the paper by Che and Hausch, there is no loss involved in contracting. The above option contract is always better or as good as no contract.

1.4 Organization of the Thesis

The aim of the thesis is to study how renegotiation affects contracting in asymmetric information environments and how this can be used to explain incompleteness of contracts. In addition to the theoretical interest of establishing foundations of incomplete contracts in such environments, there is another reason for this approach. Asymmetric information seems to be a necessary ingredient to explain a specific form of contractual incompleteness, the absence of a contract.

The papers by Che and Hausch (1999) and Segal (1999), Hart and Moore (1999) are concerned with this latter type of incompleteness. Because complete contracts are useless in these models, it is argued that they provide theoretical foundations for the incomplete contract approach.

There is one shortcoming of this explanation. If contracting parties are indifferent between writing a contract and not writing one, there remains a slight ambiguity. A still stronger advocate for the incomplete contracting front seems to be needed, namely, to show that contracting parties *strictly* prefer not to write a contract. If the null contract strictly dominates any more complicated contract, the basic assumption of most of the incomplete contracting models is on safe grounds.

In this thesis, a contract merely serves as a starting point for ex-post negotiations between contracting parties. That is, a contract decides on the status quo point of an ensuing out-of-contract bargaining game. The rules of this bargaining game are fixed, i.e. decided by nature. These negotiations can be interpreted as the renegotiation of the existing contract if a non trivial contract is in place. Then the question is under what circumstances parties strictly prefer the null contract as the uniquely optimal contract to serve as the status quo for this ex-post bargaining game.

What could be reasons for a strict preference? The above foundation models are 'only' concerned with incentive constraints. It is impossible to obtain a strong dominance result in their context because the outcome of the ex-post bargaining game under symmetric information can always be mirrored by an ex-ante contract.

Given this, asymmetric information seems to be a necessary ingredient. With asymmetric information, there is a potential for ex-post inefficiencies and therefore for harmful contracting. Contracts, by implementing an action or by changing parties's beliefs, affect the outcome of the ex-post bargaining game above a simple change in the division of surplus. On the other hand, in light of Example 2, it seems that asymmetric information might also serve as a disciplining device. Because it hinders parties from achieving ex-post efficiency, it might be easier to force them to adhere to a contract's rules.

In order to test and validate the hypothesis that asymmetric information can provide a strict dominance result, I proceed in three steps.

The second chapter of the thesis reconsiders the model by Hart and Moore (1999), hereafter the HM model. In their paper, the complexity of the contracting environment together with renegotiation imposes a large number of incentive constraints on contracts. Consequently, contracts have very limited scope. If the complexity of the environment, i.e. number of possible trading opportunities, increases without bounds, any complicated contract approaches the null contract. In Chapter 2, I simplify the HM model, and show that their result can be obtained without using a complexity argument[4]. Instead, the assumed nature of uncertainty makes it impossible to construct a revelation mechanism and at the same time provide investment

[4] I am grateful to Michele Picchione for inspriring this symmetric information version of the model.

incentives. The main driving force is that investment may have a positive as well as a negative effect on the investing party's payoff. Ex-post, parties have to decide on a joint action. They can only contract on this action to a limited extent because it is not known ex-ante which action is going to be efficient. Furthermore, this information is ex-post non verifiable. Early investment increases expected surplus if the most efficient action is chosen but decreases the surplus arising from an inefficient action. In equilibrium, the effect of investment is positive because parties will always renegotiate to the efficient action. But because the other party can always threaten to enforce an inefficient outcome she can expropriate the benefit from investment. The hold-up problem arises even if complete contracts can be written.

The simplification of the HM model allows me to introduce asymmetric information and to assess the consequences of this change more easily. This is done in Chapter 3. In the new setting, ex-ante considerations are neglected, i.e. there is no investment. Instead, I concentrate on ex-post inefficiencies created by the informational asymmetry which are similar to the ones in the introductory example. The intuition that I try to capture is that contracting might be costly because a contract is inflexible, i.e. constitutes a less flexible status quo for ex-post bargaining than no contract. More precisely, a contract might prescribe an inefficient outcome which has to be undone through ex-post bargaining. With the informational friction parties might get locked into this inefficient outcome.

For example, during the renegotiation of the contract between General Motors and Fisher Body, the latter adopted a cost intensive production technology in order to convince its partner to renege on the former agreement. Klein (1992) argues that this inefficient behavior was adopted by Fisher Body to credibly signal private information to General Motor. Importantly, the contract itself gave rise to the lock-in effect because it created an opportunity for Fisher Body to hold up General Motors.

Then, if the resulting inefficiencies of such lock-in effects are greater than the benefits of a contract, it might be strictly optimal to write no contract. This intuition is not confirmed by the model of this chapter. In contrast to the symmetric information version, parties do write complete contracts which produce the prescribed lock-in effects unless the costs of these effects are very close to the benefits of a contract. The intuition is that a contract can devise better punishment for misrepresentation of information because the ex-post bargaining game is inefficient under asymmetric information. Only in one instance, namely when costs of lock-in effects and benefits of contractual commitment are equal, there is no loss involved for the parties in adopting the null contract. At this point, it seems that asymmetric information has led us rather further away from showing the optimality of no contract.

In the last two chapters of the thesis I reexamine this conclusion with a new ingredient, individual rationality constraints. Two models are studied in which the interplay of incentive constraints, individual rationality constraints and the constraint imposed on contracts through renegotiation can actually make contracting harmful. Importantly, contracts do not decrease overall efficiency, but may reduce the payoff of the party who makes all the contracting offers. Hence, under certain conditions, the null contract is the strictly preferred alternative. The explanation is that a contract suffers from the ratchet effect because it releases information too early.

Chapter 4 contains a version of the durable goods monopoly model with no discounting but costly contracting. A seller with one unit of a good for sale faces a set of buyers with differentiated valuations. He can offer the good on two consecutive days on a market. To do so, he has to set up a stand and wait for customers. These actions are costly because the seller must pay for the stand and he incurs the opportunity costs of time spent waiting. These are the costs of contracting in this model. It is assumed that early contracting is less costly than late contracting. In our market place story, the price of a stand on the second day is higher than on the first. Alternatively, the seller might have to pay a fixed storage cost for the good over night. Importantly, the anonymous nature of the market makes it impossible for the seller to write a contract that fixes the price over the two periods. He can choose a price for the good today and is free to name a different price the next day, given his beliefs about the remaining buyers in the market. The main result says that the seller strictly prefers not to set up his stand on the first day even if the transaction costs he incurs on the first day are much smaller than the transaction costs he incurs on the second day. That is, he chooses the 'null contract', even if initial contracting costs are of order of magnitude smaller than late contracting costs. This result is interesting because it goes beyond a mere comparison of size in transaction costs. Second, I derive a parsimonious representation for the first stage contract that is solely due to incentive constraints. This representation is useful in the model of the following chapter.

In chapter 5, a sequential screening model endogenizes the fixed contracting cost. A seller and a buyer write a sales contract. The buyer is privately informed about one part of the good's value but is still uninformed about the second part, which is revealed later. Early contracting is beneficial because it suffers less from asymmetric information than does late contracting. If the seller can commit himself to a single contracting offer he will offer a sequential mechanism in form of a fixed initial fee and a price. The contract allows the buyer to decide on whether to purchase the good once the second parameter of his valuation is realized. Sequential price discrimination is common practice in a variety of circumstances, such as fidelity cards in cinemas, book clubs or air plane tickets. In this chapter it is assumed that commitment is not feasible. More precisely, whatever an early contract prescribes, the seller has always the opportunity of making one final renegotiation offer. Then, if the uncertainty concerning the second variable is smaller than the uncertainty concerning the first, early contracting has no benefit while still suffering from the ratchet effect. The null contract is therefore strictly dominant. Second, if this is not the case, some early contracting can be observed, but contracts are partially incomplete. It can be shown that any screening of the second variable does not affect final payoffs, and therefore this variable can be excluded.

1.5 Renegotiation-Proofness

The important assumption needed to derive a strict dominance result for the null contract is that the final bargaining game cannot be included into the contract. Indeed, if it were possible to write into the contract what is to be done in the bargaining game, i.e. to define a two stage contract with the renegotiation game as second stage, a strict dominance result could never be obtained. Parties are obviously indifferent between the outcome of such a contract, where the first stage is left blank, and the outcome of the 'one stage' null contract followed by the bargaining game itself.

The fact that renegotiation cannot be defined as part of a contract is not standard in the literature on contract renegotiation. In fact, it seems to contradict an important principle, the Renegotiation-Proofness-Principle, which has first been evoked by Dewatripont (1989) and Hart and Tirole (1988).

The underlying idea of the principle is that renegotiation is a bargaining game whose rules can be explicitly written into a contract. Hence, there is no need to resort to out-of-contract renegotiation in equilibrium. Similar to the revelation principle, which allows a planner to restrict his search for an optimal mechanism to the set of direct revelation mechanisms, the renegotiation-proofness principle allows us to restrict the set of contracts to renegotiation-proof contracts. As the revelation principle introduces incentive constraints, the renegotiation-proofness principle introduces renegotiation-proofness constraints. It therefore constitutes a convenient tool for studying contracting problems with renegotiation.

Although the renegotiation-proofness principle has found ample application, in contrast to the revelation principle it is less easy to provide a formal statement and proof for it. Consider two possible interpretations that are both expressed in the above paragraph:

I1 The renegotiation-proofness principle states that for every contract that is renegotiated in equilibrium, there is a renegotiation-proof contract that replicates the outcome of this contract.

I2 The renegotiation-proofness principle states that the set of mechanisms over which a planner optimizes can be taken to be the set of renegotiation-proof contracts.

First, I want to argue that there are situations in which interpretation I1 is in contrast with the assumption that contracts can be renegotiated. More precisely, the fact that renegotiation cannot be made part of a contract is a direct implication of the way renegotiation is modelled.

In this thesis I assume that *any* inefficiency of a contracting outcome is subject to renegotiation. This implies that parties cannot contractually commit to pay a transfer to third parties or to destroy part of jointly realized surplus in some other way. But more importantly it implies that a contract cannot stop at an outcome at which surplus is left unexploited. For example, it cannot end at a point at which parties do not trade although they are both aware that trade will raise total surplus.

In contrast, an out-of-contract bargaining game can stop at such a point. This assumption can be defended by thinking of the bargaining game as real time negotiations with an infinite time horizon and discounting. If these negotiations suffer from asymmetric information, inefficiencies occur because agreement is reached only with delay. Although I do not formally model the bargaining game as an infinite time horizon game and instead take it to be a take-it-or-leave-it offer by one party, the same intuition applies. After this offer is rejected, the size of surplus shrinks below parties' opportunity costs of time and they leave the bargaining table voluntarily. Importantly, this 'shrinking' cannot be recreated artificially by a contract. The only way to achieve it is by having parties effectively enter into the bargaining game. Because the outcome of the bargaining game itself is inefficient, it is impossible to include it as part of the contract.

I would like to stress that in symmetric information environments, this assumption does not interfere with the existence of renegotiation-proof contracts. In contrast, in asymmetric information environments this assumption does pose an obstacle to achieving renegotiation-proofness.

Second, there are situations in which although I1 applies, I2 is violated. This is the case if the renegotiation game must be effectively played as part of the contract as is the case in asymmetric information settings, and cannot be circumvented by simply adding its outcome as is the case in symmetric information settings. Then, I2 claims that the renegotiation game, as part of the contract, can be designed optimally subject to the constraint that parties do not want to 'renegotiate renegotiation'. Below, I will provide an example in which the set of renegotiation-proof contracts is bigger than the set of renegotiated contracts. Therefore I2 is violated. Intuitively, this is the case if the exogenously given renegotiation game is not interim efficient.

Instead of reviewing the extensive literature on contracting with renegotiation, I am going to demonstrate the renegotiation-proofness principle in its interpretations I1 and I2 in the context of two examples. I am then going to discuss my assumption of renegotiation and show how renegotiation-proofness fails in its interpretation I1. Finally, I am going to show in a Myerson-Satterthwaite type setting why even without this assumption there might be a problem with interpretation I2.

The first strand of literature on contracting with renegotiation is concerned with renegotiation in implementation problems of symmetric information, as pioneered by Maskin and Moore (1999). The discussed foundation models, Che and Hausch (1999), Segal (1999) and Hart and Moore (1999) are all problems of that kind. With symmetric information, one can always bypass the renegotiation stage by considering only renegotiation-proof contracts. Indeed, with symmetric information and in the absence of other sources of inefficiencies like cash constraints, any contract C_1 is renegotiated to $\tilde{C}_1(C_1)$, where, for every initial C_1, $\tilde{C}_1(\cdot)$ is pareto-efficient. If so, one can simply offer the initial contract $C_1' = \tilde{C}_1(C_1)$. By definition of $\tilde{C}_1(\cdot)$, C_1' is pareto efficient and therefore necessarily renegotiation-proof. Hence, one can restrict attention to initial contracts that are NOT renegotiated: a renegotiation-proofness principle.

The same feature can be found in environments with asymmetric information between parties. Typically, these are dynamic contracting problems, (see for example Dewatripont (1989) and Hart and Tirole (1988)), in which renegotiation arises naturally between two consecutive contracting dates. Take the following dynamic version of Example 1:

Example 3 *The seller can upgrade the software twice within a year at date* 1 *and date* 2*. At the time at which the seller proposes a sales contract for both upgrades, date* 0 *say, the buyer is already informed about his valuation. If he has a high use for the software, each upgrade will increase his valuation by* 7*, if he has a low use, his valuation will be increased by* 4*. The seller is uninformed about the buyer's exact preferences and he estimates the probability that the buyer is of either type with* 0.5*. The seller's costs for each upgrade are* 2*. There is no discounting between the dates.*

We know from Baron and Besanko (1984) that the optimal long term contract in the absence of renegotiation is the repetition of the optimal static contract offered at every period. Here, the seller should offer a price of 7 at every date and the buyer should buy at each date if and only if his valuation is equal to 7. Obviously if this price is rejected at date 1, the seller wants to renegotiate and if he cannot commit not to do so, he will then offer a price of 4 *for the next period.* However, in this framework, one can still focus on the set of contracts which are never renegotiated. For instance, the above contract that stipulates a repetition of the optimal static offer can be implemented by considering a mixed strategy for the buyer at date 1. Namely, the low valuation buyer rejects the first upgrade, whereas the high valuation buyer mixes between rejecting and accepting it. If, for example, he rejects the first upgrade with probability $\frac{2}{3}$, the seller, after a rejection at date 1, can still credibly maintain a high price of 7 at date 2. Nevertheless, this contract might not be optimal anymore. Instead, the optimal renegotiation-proof contract in this example is such that the first upgrade is sold to both buyer types for a price of 4 and the second upgrade is only sold to the high valuation buyer for a price of 7. The seller does not learn anything about the buyer's type from the first contractual arrangement and has therefore no incentive to renegotiate the second contract offer. Observe, that whatever the preferred contract is, no renegotiation takes place along the equilibrium path.

Hence, I have presented two settings where the renegotiation-proofness principle holds. One can restrict attention to contracts that are not renegotiated as any outcome which is achieved by a contract which is renegotiated can be replicated by "bringing backward", at the time of writing the contract, what renegotiation would do.

Consider now a static version of the previous dynamic example: suppose that only one upgrade can be sold. When parties meet to bargain over the price, the seller makes one single offer after which, if it is rejected, the value of trade decreases to zero. In this bargaining game the seller will make a price offer of 7.

Imagine now that the seller can try to alter the status quo of this bargaining game by offering an initial contract. Importantly, this contract is vulnerable to renegotiation, where renegotiation takes place according to the original bargaining game. Hence, if the contract prescribes and inefficient outcome it is subject to ex-post bargaining. Then, since the price offer of 7 is the optimal static offer for the seller, he has no incentive to write such a contract.

This result is in itself trivial but I believe that there are two ways of thinking about the implementation of this offer.

In the tradition of the renegotiation-proofness principle, one could say "Suppose no contract is written in the beginning, and the seller offers the upgrade for a price of 7 in the bargaining game. Instead, one could write an *initial* contract stipulating that the offer of 7 will be made in the bargaining game". This contract contains the renegotiation game and therefore the renegotiation-proofness principle holds.

I would like to argue that this is in contrast to assuming that a contract can be renegotiated. Namely, since the seller cannot contractually commit himself to stop bargaining as long as there are positive gains from trade and since the value of trade only decreases once parties have effectively spent time negotiating, it is impossible to circumvent out-of-contract negotiations.

Now, consider an example, in which the second interpretation I2 of the renegotiation-proofness principle is violated. Two agents contract over the sale of one unit of a good. The buyer is privately informed about his value v and the seller is privately informed about his cost c for this good. Both parameters are uniformly distributed on the unit interval. We know from Myerson and Satterthwaite (1983) that the second-best solution to this problem involves trade of the good if and only if the buyer's value and the seller's cost lie at least $\frac{1}{4}$ apart. This outcome can for example be implemented by a double auction as in Chatterjee and Samuelson (1983). Now assume that such an auction can be renegotiated and that at the renegotiation date one of the parties, the buyer say, has all the bargaining power. We can reinterpret this extension, which has the same timing as the above examples, in the following way. The good can be sold at either date 1 or at date 2. At date 0, parties can write a contract regulating the date 1 trade, whereas at date 2 the buyer can make a take-it-or-leave-it offer. First, remark that this two stage procedure, contract+renegotiation, cannot mimic the solution of the optimal static mechanism. To see this, assume that $v = \frac{1}{4}$. The optimal static mechanism would prevent trade regardless of the seller's cost. But this outcome can never be achieved in the contract+renegotiation setting, because if no trade occurred at date 1, the buyer would make an offer at date 2 and with some probability this offer would be accepted. More generally, because we know from Myerson and Satterthwaite (1983), that the contractual outcome of date 1 cannot be ex-post efficient, the buyer will always want to make another offer at date 2, a failure of the renegotiation proofness principle?

In the spirit of the renegotiation-proofness principle, interpretation I1, the date 0 contract could already prescribe what is to be done at date 1 and date 2. The buyer does not want to renegotiate this outcome if renegotiation can occur between dates 1 and 2 and this contract is therefore renegotiation-proof. But the question is why parties would ever want to write such a contract at date 0. More precisely, since the contract prescriptions for date 2 are not optimal, we cannot interpret this two stage contract as arising from an optimization procedure under an additional renegotiation-proofness constraint. Instead, the optimal contract arising from such an interpretation prescribes no trade at date 1 and the double auction at date 2. It is renegotiation-proof, even if renegotiation, in which the buyer has all the bargaining power, can occur between the two dates, but it implements a pareto superior outcome to the outcome of the above contract+renegotiation procedure. Therefore, the set of 'optimal, two stage, renegotiation-proof' contracts is larger than the set of 'optimal, one stage, renegotiated' contracts and I2 fails.

Chapter 2

Ambivalent Investment and the Hold-up Problem

2.1 Introduction

This chapter studies a simple version of the hold-up problem in which the underinvestment problem created by the expropriation of investment benefits by the other party cannot be alleviated by a contract. That is, investment is unaffected by any written ex-ante agreement if contracts are vulnerable to renegotiation.

The reason for this effect is neither the cooperative nature of investment as in Che and Hausch (1999), nor the complexity of the trading environment as in Segal (1999) and Hart and Moore (1999). Instead, it is due to the fact that investment may have a positive as well as a negative effect on the investing party's payoff. That is, I consider purely selfish investment that increases the investing party's expected payoff if an efficient action is undertaken ex-post and decreases her expected payoff if an inefficient action is undertaken. Although investment can have both a positive and a negative effect it is in fact riskless, provided that parties decide on the ex-post efficient action. That they will indeed coordinate on the right action is guaranteed by the assumption that there is symmetric information between contracting parties throughout. Therefore, it seems puzzling that investment should be unaffected by an ex-ante contract.

I call the type of investment considered in this chapter *ambivalent.* Compare this with the usual assumption about investment in the incomplete contracting literature. In Segal (1999) and Hart and Moore (1999), there is one efficient trading opportunity and N inefficient ones. Investment is selfish and has no effect on the surplus resulting from the ex-post inefficient trades, see the next section for a detailed discussion of the two papers. In Hart (1995), the efficient action is to trade with a longtime contracting partner and the inefficient action is to sell the product on the spot market. Here, investment has a smaller but positive benefit if the good is traded on the spot market.

In the model of this chapter, one party can make a non contractible investment with the described ambivalent effect on her own expected payoff. The parties would like to write an ex-ante contract that increases her investment incentives. Because investment is non contractible they have to contract on the ex-post action and distribution of surplus. This is only possible to a limited extent though, because it is not known ex-ante which action is going to be efficient. Furthermore, this information is ex-post non verifiable and the contract must therefore be made contingent on parties' announcements concerning this information. If parties can commit themselves not to renegotiate the contract ex-post, a contract can achieve first-best investment incentives. If, on the other hand, parties cannot commit themselves not to renegotiate the contract and if the positive and negative effect of investment on the investing party's payoff balance each other, any contract is a good as no contract.

To better understand the intuition for this result, consider the following example of *ambivalent* investment which is due to Hemingway. He writes in his book, 'Death in the afternoon':

'The chances are that the bullfight ... may not be a good bullfight artistically; for that to happen there must be good bullfighters and good bulls; artist bullfighters and poor bulls do not make interesting fights, for the bull fighter who has ability to do extraordinary things with the bull ... will not attempt them with a bull he can not depend on to charge; so, if the bulls are bad, that is only vicious rather than brave, undependable in their charges, ..., it is best that they be fought by bullfighters with knowledge of their profession, integrity, and years of experience rather than artistic ability. Such bull fighters will give a competent performance with a difficult bull ... However, if such a bullfighter ... without either genius or inspiration happens to receive in the ring a truly brave bull, one which charges in a straight line, which responds to all the cites of the bullfighter, ... and the bullfighter has only bravery and honest ability ... and nothing of the wrist magic and aesthetic vision, ... , then he fails completely, he gives an undistinguished, honest performance and he goes on lower down in the commercial ranking of bullfighting ...'.

In this example, the right match between a bullfighter and a bull produces a good fight, whereas the wrong match results in a bad fight. Assume that this effect can be measured in monetary terms because a good fight attracts a large crowd. The recipient of these monetary benefits is a capitalist who is the owner of the arena. The capitalist hires an organizer who is responsible for procuring the bull and engaging a matador. In contrast to the capitalist, the organizer is only interested in the private benefits he receives from associating with famous bullfighters and in his reputation for engaging the most artistic matador. That is, he always prefers to send a great bullfighter into the ring. Assume, that the capitalist's monetary benefits from the right match outweigh the private benefits and reputational effects of the organizer, so that the efficient action is to match the right pairs of opponents.

Imagine, that the organizer has to invest time and money in the search for a good bull. If he manages to find a 'truly brave' bull, his reputation will be increased significantly by sending a great matador into the ring, that is he will benefit from the efficient match. In contrast, the organizer's reputation will be harmed if a mediocre bullfighter is engaged, that is he suffers from the inefficient match. This situation is reversed if the organizer procures a bad bull. Then, his reputation suffers from the efficient match and is enhanced by the inefficient match.

What are the organizer's incentives to invest effort in the search for a bull? Can an ex-ante contract provide him with the right incentives? Assume, that every ex-ante contract between the organizer and the owner is open to ex-post renegotiation in which the owner has all the bargaining power. Then, the parties' announcements will be independent of the true states of nature, i.e. the type of bull. Namely, the owner always wants to claim that a mediocre bullfighter is needed. Either, this is in fact the efficient ex-post action or, as this is the organizer's least preferred option, the owner will be able to reap a high benefit from renegotiation. On the other hand, the organizer always wants to claim that a great bullfighter is needed because this gives him the highest benefits. There is no way in catching any of the two from lying.

Ambivalent investments are pervasive in other situations. For example, a pharmaceutical company could invest in the promotion of a newly discovered chemical substance that can be used to create a cheap substitute for an existing drug. If the campaign is successful in changing patients' preferences in favor of the new drug, the existing product becomes redundant. If the campaign is unsuccessful, it is not worthwhile producing the new drug at all. Another example is the expansion of a business project or a production operation. This corresponds to a proportionate increase in revenues if, following the expansion, an efficient project is undertaken but also in a proportionate decrease in revenues if an inefficient project is undertaken.

Finally, the result that no contract can improve over the null contract hinges on the fact that the investing party's payoffs from the efficient and inefficient action are perfectly negatively correlated. In our bullfighting example this translates into the assumption that the organizer's preference for the famous matador are unaffected by the quality of the bull. The second part of this chapter contains a model in which payoffs are not perfectly negatively correlated. I show that in such a model, contracts have some advantage. Nevertheless, as the correlation tends towards -1, contracts become less and less good in providing the right investment incentives.

The chapter is built as follows. Because my model is very close in structure to the model by Hart and Moore (1999), I will explain their model and result in the next section. It will become clear that my model is in a sense a simplification of their model, I will also discuss the intuition for this. Section 2.3 spells out my model in more details and shows that the result that no contract is optimal holds in this set-up. Section 2.4 checks for robustness of the result when cost are not perfectly negatively correlated and the final section concludes.

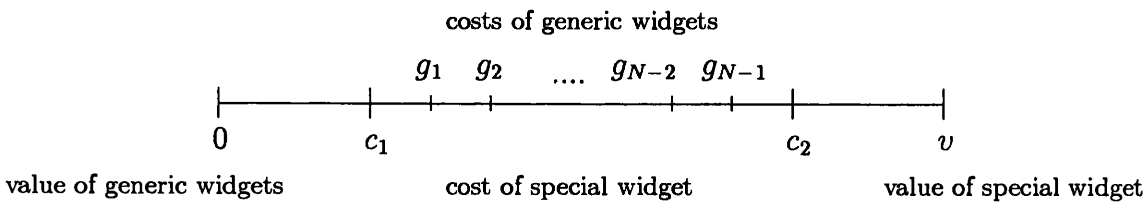
2.2 The Hart-Moore Model

To allow the comparison with the HM model more readily I will repeat their set-up and state their main result. I will then explain the difference to my set-up. It will be shown that their result follows directly from the change in the set-up without the need to resort to a 'complexity' argument.

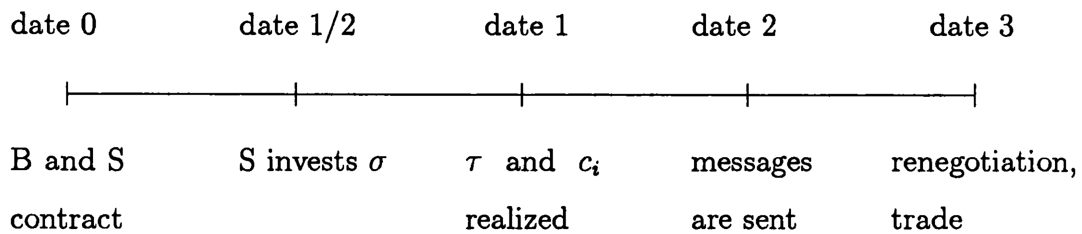
Two parties consider a future trade opportunity of one unit of a specific good, a widget. The widget's value v to the buyer is known in advance but its cost is uncertain ex-ante. There are two possible cost realizations c_1 and c_2, where $c_1 < c_2 < v$. This assumption implies in particular that trade is always efficient. At an interim stage before the contract is carried out the seller can make a relationship specific investment that lowers expected production cost. More specifically, $\pi(\sigma)$ is the probability with which production costs are c_1 and $1 - \pi(\sigma)$ is the probability with which they are c_2, where σ is the amount of the seller's investment. It is further assumed that $\pi'(\sigma) > 0$ for $\sigma > 0$, $\pi'(0) = \infty$ and $\pi''(\sigma) < 0$. To simplify, the investment cost is equal to the investment level σ. The seller's investment is observed by the buyer but not by outsiders. After the investment, the level of production cost is realized and becomes known to both parties.

Also, to capture the idea that it is difficult to contract on the exact nature of the good ex ante, there are $N-1$ other general purpose widgets in addition to the specific widget. Neither of these yield a positive surplus if they are traded. The generic widgets' costs are fixed and lie evenly distributed between the specific widget's low and high cost realization c_1 and c_2. Formally, these costs are $g_i = c_1 + \frac{i}{N}(c_2 - c_1)$. For simplicity, the generic widgets' values are set equal to zero, this is not a restrictive assumption. What is needed is that the value of a generic widget lies below its cost. Remark, that the seller's investment σ only affects the special widget's cost but not the generic widgets' cost. This assumption will be modified in this chapter.

At the contracting stage it is not known to either of the parties which of the overall N widgets will turn out to be the efficient one. The parties have a uniform prior about this event, i.e. each widget has a probability of $\frac{1}{N}$ of being the special widget. For the remaining $N-1$ widgets the probability of being any of the other general purpose widgets is $\frac{N-1}{N} \times \frac{1}{N-1} = \frac{1}{N}$. At the same time at which the cost realization of the special widget becomes known, both parties observe the true configuration of widget types, i.e. there is symmetric information about the nature of the N widgets. Thus, a realization of the state of nature is a tuple (i, τ) where $i = 1, 2$ is the possible cost realization of the special widget, and τ is a permutation of the numbers 1 to N. There are $2N!$ possible states of nature. To clarify the assumptions about the various costs and values consider the following graph:



The timing of the model is the following:



The purpose of a contract in this set-up is to ensure the right investment incentives. The first-best level of investment, call it σ^*, is found by maximizing total expected surplus, i.e.

$$\sigma^* \in \arg\max_{\sigma} \quad \pi(\sigma)(v - c_1) + (1 - \pi(\sigma))(v - c_2) - \sigma. \tag{2.1}$$

Rearranging the first order condition of this maximization problem yields

$$\pi'(\sigma^*) = \frac{1}{c_2 - c_1}. \tag{2.2}$$

Second order conditions are satisfied because of the assumed strict concavity of the function $\pi(\sigma)$, moreover the solution σ^* is unique.

If the seller's investment is not observed by outsiders, a contract can only indirectly provide him with the right investment incentives. Call the seller's final expected payoff in the low cost state Π_1^S and in the high cost state Π_2^S. When choosing σ, the seller maximizes the expression

$$\pi(\sigma)\Pi_1^S + (1 - \pi(\sigma))\,\Pi_2^S - \sigma.$$

It follows that a first-best contract must ensure $\Pi_2^S - \Pi_1^S = -(c_2 - c_1)$. In fact, if parties can commit to a particular contract a very simple contract achieves this result. If parties sign a contract that allows the seller to make a take-it-or-leave-it offer to the buyer at date 3, the seller will indeed invest σ^* because he is the residual claimant of his investment.

The result relies heavily on the fact that there is no further interaction between the parties in case the buyer rejects the seller's take-it-or-leave-it offer. Hart and Moore continue in their analysis by assuming that it is impossible for parties to commit themselves to a contract. In particular, any inefficient contract outcome is subject to further bargaining between the parties. In this bargaining game the buyer has all the bargaining power. That is, the buyer reaps the entire surplus from pareto improving renegotiation. This assumption dramatically changes the situation. Take for example the contract in which the seller is supposed to make a take-it-or-leave-it offer to the buyer. The buyer has the option of rejecting this offer. He then gains all the surplus by just offering to pay the seller's production cost c_i, $i = 1, 2$. Expecting this outcome, the seller has no incentive to invest in cost reduction and sets $\sigma = 0$. This is also the outcome after the null contract, i.e. the contract that does not specify any contingencies in advance. Under this assumption the authors establish their main result

Proposition 2.1 *(Hart and Moore) If the parties cannot commit not to renegotiate a contract, then, as the number of widgets N tends to infinity, no contract can improve the seller's investment incentives over the null contract.*

The intuition behind this proposition is the following. A contract has to drive a wedge between the seller's payoff in the high cost and low cost state. Also, because c_1 and c_2 are not directly observable a contract has to provide the right incentives to the parties to truthfully reveal the state of nature. Both parties can announce a widget configuration τ and a cost realization i. A simple way to parametrize these announcements is by asking both parties how much each of the widgets 1 to N costs. For example, the claim of $(c_1, g_1,..., g_{N-2}, g_{N-1})$ fully describes the state of nature in which the first widget is the special widget with cost c_1 and the remaining 2 to N widgets are the generic widgets with cost g_1 to g_{N-1}. Obviously, because buyer and seller share the same information, their announcements should coincide in equilibrium. If their announcements differ the contract has to be designed carefully to punish them. It is here that renegotiation plays a crucial role. Because parties are assumed to be able to renegotiate inefficient outcomes, any form of punishment that involves large fines to outsiders is automatically voided. Regardless of the prescriptions of the contract the sum of the parties final payoffs is always equal to the total surplus $v - c_i$.

The situation is especially critical if the two parties' announcements differ in the following way. Assume that the claims are such that for each widget 1 to N the cost attributed to this widget by the seller lies just below the cost attributed to it by the buyer. If for example the seller claims that the state is $\underline{s} = (c_1, g_1, \ldots, g_{N-2}, g_{N-1})$ and the buyer claims that it is $\overline{s} = (g_1, g_2, \ldots, g_{N-1}, c_2)$, the difference that each claim imposes on the seller's cost is exactly $\frac{1}{N}(c_2 - c_1)$. Importantly, if N goes to infinity the two states $\underline{s}$ and $\overline{s}$, although they are distinct states of the world, become more and more similar in the sense that the seller's payoff from a physical action[1] is almost the same in both states. There are two possibilities in the case that announcements are as above. Either the buyer is lying or the seller is. Intuitively, in the above situation it is difficult to 'catch' the lying party because state $\underline{s}$ and $\overline{s}$ are nearly identical.

To see this more precisely consider the following argument. It is crucial to find an outcome o for any pair of states of nature $(\underline{s}, \overline{s})$ as above, such that the seller prefers the equilibrium outcome in state $\overline{s}$ over o, whereas the buyer prefers the equilibrium outcome in state $\underline{s}$ over o. That is, o is the punishment outcome if the seller announces $\underline{s}$ and the buyer announces $\overline{s}$. Formally,

$$\begin{aligned} \Pi^S(\overline{s}) &\geq \Pi^S(\overline{s}, o) \\ \Pi^B(\underline{s}) &\geq \Pi^B(\underline{s}, o), \end{aligned}$$

where $\Pi^S(\overline{s})$ denotes the seller's equilibrium payoff in state $\overline{s}$ and $\Pi^S(\overline{s}, o)$ denotes his payoff out-of-equilibrium when he announces $\underline{s}$ instead and outcome o is implemented. $\Pi^B(\underline{s})$ and $\Pi^B(\underline{s}, o)$ are similarly defined for the buyer. Because the buyer's and seller's payoffs always sum to a constant, one can add these two constraints to obtain

$$\Pi^S(\overline{s}) - \Pi^S(\underline{s}) \geq \Pi^S(\overline{s}, o) - \Pi^S(\underline{s}, o).$$

From our observation above, if N is very large, the right hand side is almost 0 regardless of the choice of the punishment outcome o. In particular, it cannot be smaller than $-\frac{1}{N}(c_2 - c_1)$. Because for each state s such as $\underline{s}$ a matching state s' such as $\overline{s}$ can be found, taking expectations over such pairs turns the left hand side of this expression into $\Pi_2^S - \Pi_1^S$ and the proposition follows.

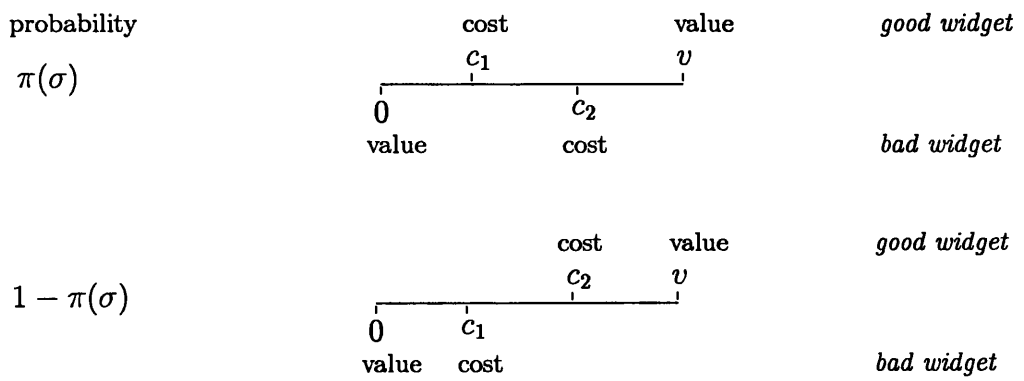
[1] A physical action is trade of one of the N widgets.

Importantly, through the increase in N the two states $\overline{s}$ and $\underline{s}$ become similar in the seller's payoff from any possible action that can be taken. In the next section I provide a set-up that directly assumes the existence of two such states.

2.3 A Simplification

The problem is very similar to the one in the preceding Section 2.2. Namely, there is a special widget that has either cost c_1 or cost c_2 and value v, with $c_1 < c_2 < v$. The seller can invest into cost reduction, i.e. he can make an investment σ that increases the probability of the low cost state c_1. In contrast to the model in Section 2.2 though, I merely assume that there is one other good, a 'bad' widget which yields a negative surplus from trade. For simplicity, its value is set equal to 0; again the value is known from the outset. The cost structure of the bad widget is the same as the cost of the efficient widget, cost can be low, c_1, or high, c_2, such that $0 < c_1 < c_2$. Furthermore, it is assumed, that the costs of both widgets are realized simultaneously. They are observed by both buyer and seller but not by any outsider.

In order to tie together the two extreme cases in which the right action is very beneficial and the wrong action is especially harmful the costs of the two goods are *negatively correlated.* If the good widget is especially cheap to produce, i.e. surplus from trade is very large at $v - c_1$, the bad widget is very expensive and thus trading it will result in the largest possible loss of surplus of $-c_2$ and vice versa. To clarify the assumptions about the cost structure, consider the following picture:



What the assumption about negatively correlated cost buys us will become apparent in the following section. If costs are interpreted as a widget's value to the seller a negative correlation is quite plausible. A good's value is influenced by exogenous conditions, such as personal tastes, fashion and the good's resale value in a market. These conditions are likely to be determinant for both widgets, where if one of them is especially fashionable and therefore preferred by the seller the other is likely to be unfashionable and of low value to him.

The information and time structure in the game is as in Section 2.2, namely: Ex-ante at date 0, there are two goods, only distinguishable by their physical attributes, such as color, size etc. A full description of these attributes is subsumed under a name, for example *widget X* and *widget Y*. Neither of the contracting parties knows, which of the two goods will be the good widget. They have a uniform prior about this event, i.e., each widget has a probability of $\frac{1}{2}$ of turning out to be either good or bad. In other words, there is a probability of $\frac{1}{2}$ that *widget X* has value v and *widget Y* has value 0 and vice versa. Also, costs are unknown. At date 1/2 the seller makes his relationship specific investment σ. At date 1, all relevant uncertainty is resolved. *Both* parties observe the configuration of widget types, as well as the cost realization. At date 2, messages are sent according to which the outcome prescribed by the contract agreed upon at date 0 is determined. Finally, parties can renegotiate any remaining inefficiencies before trade at date 3. The bargaining game at renegotiation is as in the above section, namely, the buyer makes a take-it-or-leave-it offer to the seller.

A state of nature is denoted by $s_{\tau i}$, $\tau = X, Y$, $i = 1, 2$, where the first index indicates which of the two widgets, *widget X* or *widget Y*, is the special widget and the second index stands for the cost realization of the good widget (which simultaneously determines how much the bad widget costs). To summarize, there are four possible states of the world:

state	*widget X*	*widget Y*	*probability*
	(value, cost)	(value, cost)	
s_{X1}	(v, c_1)	$(0, c_2)$	$\frac{1}{2}\pi(\sigma)$
s_{X2}	(v, c_2)	$(0, c_1)$	$\frac{1}{2}(1-\pi(\sigma))$
s_{Y1}	$(0, c_2)$	(v, c_1)	$\frac{1}{2}\pi(\sigma)$
s_{Y2}	$(0, c_1)$	(v, c_2)	$\frac{1}{2}(1-\pi(\sigma))$

Observe one important difference between this model and the Hart-Moore model: While in their model the seller's investment does not affect the generic widgets' cost, in this set-up, an increase in σ also increases the probability that the bad widget is very costly. This is the key assumption of the model.

Because assumptions concerning the special widget are unchanged, the first-best level of investment and the seller's investment in the absence of a contract are exactly as in the Hart-Moore model. The first-order condition in (2.2) defines the first-best level of investment σ^*, whereas the seller's investment after the null-contract is 0. The analysis of this set-up proceeds exactly as in the Hart-Moore model. As an introduction to the problem I will consider some straightforward contracting examples to show that none of them incites the seller to invest a positive amount.

First, take the contract in which the seller can choose a widget which is then traded for a fixed price p. In the model of the preceding section, for finite N this contract raises the seller's investment incentives slightly, because it guarantees a difference in his payoff between the high and the low cost state of $\frac{1}{N}(c_2 - c_1)$. This is also the case for the contract in which the buyer is allowed to choose a good. In the model of this section, with only two goods but negatively correlated costs, these two contracts are worthless. Take the contract in which the seller chooses a good. He always chooses the low cost good, regardless of whether it is the good or the bad widget. This gives him a payoff of $v - c_1$, because he gains nothing from renegotiation. Therefore he has no incentive to invest. For a similar reason the contract in which the buyer chooses the widget does not work either. The buyer always chooses the more expensive good. If the more expensive good is the special widget, there is no further renegotiation, the seller's payoff is $v - c_2$. If on the other hand, the more expensive good is the bad widget the buyer will obtain the good widget through renegotiation by asking the seller to pay an additional amount of $c_2 - c_1$. The seller is as well off as without renegotiation, he obtains $v - c_2$ in both states, which is again independent of his investment level.

Second, consider a specific performance contract in which one widget is designated ex ante to be traded for a fixed price p. If there are N goods ex-ante, with probability $\frac{1}{N}$ this is the special widget, in which case the seller's payoff differs from the low to the high cost state. If the widget is one of the generic widgets, which happens with probability $\frac{N-1}{N}$, his payoff is independent of his investment. Overall this contract raises the seller's incentives by $\frac{1}{N}(c_2 - c_1)$. Take now the situation with only two goods and negatively correlated cost. The seller's payoff from trade of *widget X* for example is

$$\frac{1}{2}(\pi(\sigma)(p - c_1) + (1 - \pi(\sigma))(p - c_2)) + \frac{1}{2}((1 - \pi(\sigma))(p - c_1) + \pi(\sigma)(p - c_2))$$
$$=$$
$$p - \frac{1}{2}c_1 - \frac{1}{2}c_2,$$

which is independent of his investment. The first term in the upper equation is his expected payoff if *widget X* turns out to be the good widget, the second term is his payoff when *widget X* is the bad widget.

It is very easy to see that this result holds for all possible contracts:

Proposition 2.2 *If parties cannot commit not to renegotiate, the seller will not invest regardless of the contract that they write.*

Proof.

Take the following announcement by the two parties

seller	buyer
s_{X1}	s_{Y2}

That is, the seller claims that *widget* X is the good widget with low cost whereas the buyer claims that *widget* Y is the good widget with high cost. Remark that these announcements imply that B and S attribute the same cost to both 'physical' widgets, namely, both claim that *widget* X has cost c_1 and *widget* Y has cost c_2. They only disagree on the identity of the special widget. Either the seller is lying and the state is indeed s_{Y2} or the buyer is and the state is s_{X1}. A contract should ensure that the seller has no incentive in state s_{Y2} to claim that the state is s_{X1} and the buyer should have no incentive in state s_{X1} to claim that the state is s_{Y2}.

In state s_{Y2}, call the seller's equilibrium payoff $p(s_{Y2}) - c_2$, where $p(s_{Y2})$ is the final payment that the buyer makes to him. It includes the payment prescribed by the contract plus the additional payment that the buyer offers for possible renegotiation. As information is symmetric, the good widget is always traded. Similarly the buyer obtains $v - p(s_{X1})$ in equilibrium in state s_{X1}. Assume that after the above announcements, the contract prescribes trade of *widget* X with probability α^X and trade of *widget* Y with probability α^Y. The transfer payment from buyer to seller that the contract prescribes is q. For the seller to tell the truth in state s_{Y2} it must be that

$$p(s_{Y2}) - c_2 \geq q - \alpha^X c_1 - \alpha^Y c_2,$$

whereas for the buyer to tell the truth in state s_{X1} it must be that

$$v - p(s_{X1}) \geq \alpha^X v + \alpha^Y (c_2 + v - c_1) + (1 - \alpha^X - \alpha^Y)(v - c_1) - q,$$

which is equivalent to

$$c_1 - p(s_{X1}) \geq \alpha^X c_1 + \alpha^Y c_2 - q.$$

Together the incentive constraints for seller and buyer imply

$$[p(s_{Y2}) - c_2] - [p(s_{X1}) - c_1] \geq 0. \tag{2.3}$$

The same reasoning can be applied for the pair of states s_{Y1} and s_{X2} to obtain

$$[p(s_{X2}) - c_2] - [p(s_{Y1}) - c_1] \geq 0. \tag{2.4}$$

Adding (2.3) and (2.4) and multiplying this expression by $\frac{1}{2}$ shows that the difference in the seller's expected payoff from the high cost states s_{X2} and s_{Y2} to the low cost states s_{X1} and s_{Y1} cannot be more than 0. Therefore, the seller has no incentive to invest regardless of the choice of the contract.

■

The intuition for this result is the following. For each state in which the special widget's costs are low there is a corresponding state in which its costs are high, such that the seller's payoff in both states from trading either of the widgets X or Y is the same. Therefore, it is impossible to write a contract in which the seller's equilibrium payoffs in these two states differ.

Remark that Proposition 2.2 does not depend on the assumption that it is only possible to trade one good. This assumption on technological feasibility is the driving force in the HM model. In their model, if all goods could be traded simultaneously, a contract prescribing trade of all goods would achieve first-best investment incentives. In the present model, a contract that forces trade of both *widget X* and *widget Y* is as good as no contract.

The crucial assumption is the negative correlation of the good and bad widget's cost. To see, how the result is changed when cost are positively correlated for example, consider the specific performance contract that enforces trade of *widget X*. The seller's payoff from such a contract would be

$$\frac{1}{2}(\pi(\sigma)(p - c_1) + (1 - \pi(\sigma))(p - c_2)) + \frac{1}{2}(\pi(\sigma)(p - c_1) + (1 - \pi(\sigma))(p - c_2))$$

$$=$$

$$p - \pi(\sigma)c_1 - (1 - \pi(\sigma))c_2$$

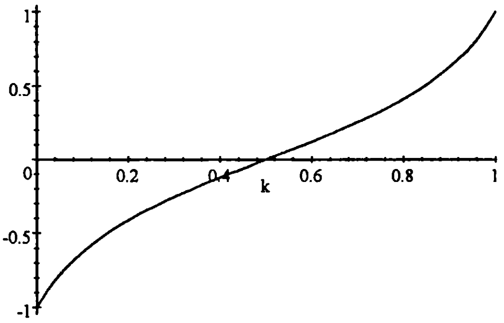
and this contract would be first-best. The next section shows that if we take some negative cost correlation between the two goods' cost which approaches -1, all contracts approach the null-contract.

2.4 Imperfect Correlation

Take the model in which the good and bad widget's costs are not perfectly negatively correlated. With probability k costs are the same for both widgets, with probability $1-k$ costs are different. The probability that the good widget's costs are low at c_1 is $\pi(\sigma)$, high costs c_2 occur with probability $1-\pi(\sigma)$. There are 8 possible states of nature $s_{\tau ij}$, $\tau = X, Y$, $i,j = 1,2$. Here, τ indicates which of the two widgets is the special one, i stands for the cost realization of the G widget, j stands for the cost realization of the B widget. The table of states is

state	*widget X*	*widget Y*	*probability*
	(value, cost)	(value, cost)	
s_{X11}	(v, c_1)	$(0, c_1)$	$\frac{1}{2}k\pi(\sigma)$
s_{X12}	(v, c_1)	$(0, c_2)$	$\frac{1}{2}(1-k)\pi(\sigma)$
s_{X21}	(v, c_2)	$(0, c_1)$	$\frac{1}{2}(1-k)(1-\pi(\sigma))$
s_{X22}	(v, c_2)	$(0, c_2)$	$\frac{1}{2}k(1-\pi(\sigma))$
s_{Y11}	$(0, c_1)$	(v, c_1)	$\frac{1}{2}k\pi(\sigma)$
s_{Y12}	$(0, c_2)$	(v, c_1)	$\frac{1}{2}(1-k)\pi(\sigma)$
s_{Y21}	$(0, c_1)$	(v, c_2)	$\frac{1}{2}(1-k)(1-\pi(\sigma))$
s_{Y22}	$(0, c_2)$	(v, c_2)	$\frac{1}{2}k(1-\pi(\sigma))$

The correlation coefficient is $corr(G,B) = \frac{-\pi(1-\pi)(1-2k)}{\sqrt{\pi(1-\pi)}\sqrt{\pi(1-\pi)(1-2k)^2+k(1-k)}}$, where the dependency of π on σ has been suppressed. Obviously, if $k = 1$, costs are positively correlated, if $k = 0$ cost are negatively correlated. If $0 < k < 1$, the correlation is as in the following graph:



Coefficient of Correlation for $\pi(\sigma) = 0.9$

The following can be shown

Proposition 2.3 *If k approaches 0, that is, costs become more negatively correlated, all contracts approach the null contract.*

Proof.

Following the proof of Proposition 2.2, the same argument can be applied to the pairs of states (s_{X12}, s_{Y21}) and (s_{Y12}, s_{X21}) to show that $[p(s_{Y21})-c_2]-[p(s_{X12})-c_1] \geq 0$ and $[p(s_{X21})-c_2]-[p(s_{Y12})-c_1] \geq 0$. To conclude the proof, the remaining states must be paired in the following way: $(s_{X11}, s_{Y22},)$ and (s_{Y11}, s_{X22}). For the first pair the set of incentive constraints is

$$p(s_{Y22}) - c_2 \geq q - \alpha^X c_2 - \alpha^Y c_2$$

and

$$c_1 - p(s_{X11}) \geq \alpha^X c_1 + \alpha^Y c_1 - q.$$

Making the right-hand-side as big as possible involves setting $\alpha^x = 1$ for example, which gives

$$[p(s_{Y22}) - c_2] - [p(s_{X11}) - c_1] \geq -(c_2 - c_1).$$

The same applies for the second pair of states. Conditional on the cost of the good widget being c_2 (c_1), the probability of states s_{Y21} and s_{X21} (s_{X12} and s_{Y12}) is $\frac{1}{2}(1-k)$ and the probability of states s_{Y22} and s_{X22} (s_{Y11} and s_{X11}) is $\frac{1}{2}k$.

Then, weighting the above expressions by the probabilities with which these states occur and adding the incentive constraints implies

$$\Pi_2^S - \Pi_1^S \geq -k(c_2 - c_1).$$

The smaller k, that is, the more negatively correlated the good and the bad widget's cost, the smaller is the gain from a contract. ■

The probability that the two goods have the same costs is k. In these states a contract can raise investment incentives over the null contract. With probability $1-k$, costs for the two goods differ and no contract can provide investment incentives. Intuitively therefore, the larger k the better a contract performs, the smaller k, the more useless it becomes.

2.5 Conclusion

This chapter has shown that in a specific trading environment in which investment can raise the surplus of a future transaction, contracts cannot improve investment incentives. The specificity of the trading environment is that each state of nature must have a 'mirror' image. That is, for each state of nature there must exist a second state in which the investing party's payoff from every action is the same, but where each action results in a different payoff to the non-investing party. This makes it impossible for a contract to effectively distinguish these two states of nature. If the party that does not invest has all the bargaining power at renegotiation, it is impossible for a contract to make the investing party's payoff dependent on the realization of cost. Therefore, beneficial investment will not be undertaken.

Chapter 3

Incomplete Contracts and Inefficient Renegotiation

3.1 Introduction

In this chapter, asymmetric information concerning the cost variable is introduced into the model of chapter 2. I abstract from the investment problem considered in the preceding chapter and concentrate solely on the inefficiency created in the buyer-seller model through asymmetric information. The goods' values, i.e. the widgets' types, remain symmetric information between both parties.

In the resulting model, contracting parties face an implementation problem that has both an aspect of asymmetric and symmetric information. In addition, due to the asymmetric information aspect, the renegotiation game is inefficient. This is one of the first models that studies inefficient ex-post renegotiation. Also, the joint aspect of symmetric and asymmetric information is new in an implementation problem.

The motivation for the analysis is to extend the foundation literature that is based on ex-post contract renegotiation into asymmetric information problems. As already noted in the introduction, it is not clear how far renegotiation, so powerful in symmetric information environments, can take us here. On the one hand, a contract has more power because parties expect to obtain less through inefficient renegotiation. On the other hand, a contract is weaker because the privately informed party has to be given an information rent and parties' announcements cannot be cross checked against each other. Potentially, these two effects could tilt the balance either in favor or against contingent contracts.

The main intuition that I try to capture is that contracts might be costly because they introduce inflexibility into the ex-post bargaining game. More precisely, under asymmetric information a formal contract may lead to an inefficient outcome that needs to be "undone" through ex post bargaining. Because with the information friction bargaining is inefficient, parties can get locked into the contractually specified inefficient outcome. Parties may therefore prefer to leave contracts incomplete because this provides a more flexible basis on which to conduct future business.

As an example of this type of lock-in effect consider the contract between General Motors and Fisher in 1919 (see Klein (1992)). The two parties wrote an exclusive dealing contract with a pre-specified price equal to variable cost plus a mark up. This exclusive dealing contract was in fact used by Fisher to 'hold-up' General Motors, taking advantage of the mark-up by adopting an inefficient, highly labor intensive technology and by refusing to locate its body-production plants adjacent to General Motors assembly plants. Thus, the contract resulted in a dissipation of real resources before General Motors was convinced to renege on the initial agreement. One possible explanation for the inefficiency of the renegotiation process is asymmetric information.

The above argument raises the question of why parties would ever want to commit to a detrimental outcome. One possible answer is that parties may simply be incapable of foreseeing all future consequences of their actions. This implies that they might find themselves in situations where their contractual agreement is no longer optimal and they try to renegotiate. But Maskin and Tirole (1999) show that most models in the contracting literature, incomplete contracting models in particular, are incompatible with the assumption of unforeseen contingencies.

The approach adopted in this chapter proposes a different explanation for contractual lock-in effects. At the date at which the contract is enforced all relevant information is known to at least one of the parties but it is unknown to outsiders. Therefore, a contract has to be designed to incite truthful revelation of this information. This introduces incentive considerations which will force parties to commit to a detrimental action on the equilibrium path. Ex-post renegotiation is then about undoing this negative outcome, which due to asymmetric information is not always feasible. On the other hand, not writing a contract leads to some surplus not being realized because of asymmetric information. Therefore a contract has some benefit (more trade of the good widget) and cost (trade of the bad widget).

Incentive considerations concern the seller's costs and the widget's types. The seller alone knows the costs whereas both parties know the widget configuration. The interplay between these two types of constraints constitutes the main analysis.

Revelation of the asymmetric information parameter, the widgets' costs, implies very stringent conditions for the contract (Lemma 3.2), which interfere with the incentive constraints imposed by the revelation of the goods' types, similarly as in the model of the preceding chapter. But the former constraints are stronger, i.e. a contract is weaker, in the asymmetric information context. Consequently one result of the model (Lemma 3.3) is that truthful revelation of the cost parameter cannot be achieved. This implies that for any contract the ensuing renegotiation will happen under asymmetric information. But also the position of a contract is stronger in the asymmetric information context, because the uninformed party, the buyer, is at a disadvantage in the renegotiation stage. It is thus easier for a contract to satisfy his incentive constraints with respect to the truthful revelation of the goods' types.

As noted above, these two forces could tilt the balance either towards or against contracting. But it turns out that there is no trade-off in this model. Generically, parties do write a contract that produces the kind of lock-in effect described above. So the cost of contracting is always smaller than the benefit. Only if the negative impact of the lock-in effect is very close to the positive effects of contractual commitment does the benefit of a contract vanish to zero. Therefore, one important implication of the model is that asymmetric information reinforces the position of complete contracting in the presence of ex-post renegotiation.

Second, the way renegotiation is modelled in this chapter makes it impossible to include the renegotiation game into the contract. The interplay of the incentive constraints implies that a contract is not ex-post efficient, similar to the result in Myerson and Satterthwaite (1983), and cannot separate cost types. Also, due to the nature of the assumed ex-post bargaining game, there is separation of cost types at renegotiation. Then, because *all* contractual inefficiencies are subject to ex-post renegotiation, parties will make use of renegotiation in equilibrium.

There is ample literature on contracting with asymmetric information and renegotiation. The first articles on renegotiation in dynamic contracting environments with adverse selection are Hart and Moore (1988) and Maskin and Moore (1999). In these papers renegotiation can occur *before* parties exchange messages, whereas in the model of this chapter renegotiation occurs *after* messages have been exchanged. The papers by Fudenberg and Tirole (1990) and Matthews (1995) study renegotiation in a static moral hazard problem. Renegotiation occurs after the unobservable effort has been chosen by the agent, but before the outcome has been realized. In Fudenberg and Tirole (1990), the principal offers a menu of renegotiation-proof contracts (one for each effort level). The authors argue in favor of contracts that are renegotiated as they guarantee uniqueness of a particularly desirable equilibrium. Matthews (1995) studies a similar set-up but concentrates on the emergence of straightforward sale contracts which are renegotiated in equilibrium. Menus of contracts are ruled out a priori due to complexity considerations. The papers most closely related are Beaudry and Poitevin (1993) and (1995). They study adverse selection problems (signalling and screening) in which principals can solicit further contracts after the first contract has been signed. Importantly, there is potentially an infinite round of renegotiation before the initial contract is carried out. In these contexts, the authors show that separation of types is achievable with an initial contract. The equilibrium is less efficient than if no renegotiation is allowed.

The following section introduces the model. Section 3.3.1 solves for the parties' strategies in the renegotiation game. Some simple contracting examples are considered in Section 3.3.2. Section 3.3.3 contains the main derivations. Section 3.4 summarizes the main results and contains a discussion. Section 3.5 provides the proofs of Lemmata 3.4 and 3.5.

3.2 The Model

The set-up is similar to the set-up of Section 2.3 in the preceding chapter, differing from it only in so far as I consider an asymmetric information problem. Therefore, I drop the assumption that the seller can make an investment in cost reduction but introduce the assumption that only the seller can observe the final realization of cost. The widget configuration on the other hand, is still observed by both parties. The cost of the good widget can be either low at c_1 or high at c_2, these two events occurring with the respective probabilities π and $1-\pi$. The value of the good widget is v and both cost realizations lie below the value. Costs of the bad widget are also either low or high, where the realizations are tied with the cost realizations of the good widget. That is, I only consider the case of perfectly negatively correlated cost. Therefore, the probability that the bad widget costs c_1 is $1-\pi$, the probability that it costs c_2 is π. Its value is fixed at 0.

The existence of two different goods in the model captures the idea that there are benefits and costs from acting. Namely, the benefits stem from trading the good widget which raises surplus, the cost come from exchanging the wrong widget which results in a loss of surplus. Therefore, a contract will have to be designed carefully as to ensure that the correct action is taken.

The time structure is as in the model of the last chapter except for the investment stage:

date 0	date 1	date 2	date 3
B and S contract	τ and c_i realized	messages are sent	renegotiation, trade

The information among the involved parties is as follows. At date 1, all relevant uncertainty is resolved. I assume that *both* parties observe the configuration of widget types, so that there is symmetric information about the nature of the 2 widgets. On the other hand the cost of the two goods are observed *only* by the seller, i.e. there is asymmetric information on the cost parameter.

It seems plausible that two partners in a relationship know the general direction of their common activity and therefore agree on the surplus maximizing action. In this model, the two parties agree on the type of widget that they want to trade. Nevertheless, the exact size of total surplus might not necessarily be known. In particular, the value to the buyer and the cost to the seller could well be private information of the concerned party only. In this set-up I simplify by assuming that only the costs are private information. Making also the value to the buyer uncertain and private information would not alter the general conclusions[1]. Nevertheless, as is common in most contracting models, outsiders have no information about the realization of either τ or c.

Call a state of nature $s_{\tau i}$. As in the preceding chapter, $\tau = X, Y$ indicates which of the two goods is the special widget, i stands for the cost realization of the special widget. For a full list of all possible states of nature consider Figure 1 of chapter 2. A contract written at date 0 can only be made indirectly contingent on the seller's and buyer's information. Remark, that, given the described information structure, the seller knows the whole state $s_{\tau i}$, whereas the buyer can only observe part of it.

[1]A further discussion on the implication of the assumption that the widget configuration is symmetric information can be found in Section 3.4.

With a slight abuse of notation I will call the buyer's information s_T. As the cost configuration i is private information to the seller, it will be identified with his type.

The problem I want to solve is whether trade of the special widget can be achieved in all four states of the world. Without going into details at that stage, observe that if parties can be made to truthfully reveal the widget configuration a contract could in principle achieve this goal[2]. Revelation of costs, on the other hand, is not a priori important. Finally, in what follows I will be only concerned with incentive constraints that a contract has to fulfill. An optimal contract is therefore a contract that maximizes total surplus.

3.2.1 Renegotiation

Because the model in this chapter is a contracting problem with both asymmetric and symmetric information I want to discuss the assumption of possible contract renegotiation at this point.

It seems plausible to assume that parties meet after the date at which they have exchanged messages and renegotiate any ex-post inefficient outcome. It is well known from implementation theory that, if arbitrarily large punishments off the equilibrium path are allowed and parties have to strictly abide to the contract's rules, nearly everything can be implemented. In particular, if in the current set-up both the buyer and the seller were asked to specify the type configuration and high punishments were levied on them in the case of disagreement, it would be very easy to enforce the first-best. But, given that the parties can not be prevented from communicating with each other after the mechanism has been played, they would not abide to it's prescriptions if there is room for pareto improvement. Then, as agents know that inefficient outcomes will be renegotiated to a pareto superior one, to employ these kinds of punishments becomes impossible.

This line of thought has been mainly explored within implementation problems

[2]A fixed price sale contract of the good widget is one possibility.

under symmetric information. To admit renegotiation in this context makes a lot of sense. In addition to the argument outlined above, renegotiation in symmetric information environments achieves pareto efficiency. It will not leave any surplus unexploited. This means that renegotiation does not need to be modelled explicitly. Any additional surplus realized through the renegotiation process is split between the involved parties according to an exogenously given fraction $(\lambda, 1-\lambda)$, indicating the parties' respective bargaining power. Thus, renegotiation can be seen as a cooperative game played after the mechanism has been carried out. The mechanism only serves as a status quo point from which parties move forwards. The two most important papers in this area are Hart and Moore (1988) and Maskin and Moore (1999).

Another strand of literature has been concerned with how the introduction of renegotiation into a dynamic game under asymmetric information changes the form of an optimal long term contract and information disclosure over time. Here renegotiation at the end of period t is over those terms in the contract that regulate the relationship in future time periods, $t+1$, $t+2$,... etc., but not over the outcome of the contract in period t itself. The first paper to study this issue is Dewatripont (1989).

The model in this chapter departs from both approaches in so far as it considers renegotiation of a contract under asymmetric information in what is basically a static game. The major departure from renegotiation in symmetric information environment is that agents are not necessarily able to achieve a pareto efficient outcome. In comparison with common models of contract renegotiation in asymmetric information environments two things can be said. First, the usual ratchet effect and slow information revelation is observed as well. That is, although there is no discount factor which discounts payoffs from different dates in the model, some trade is concluded through the initial contract, some only through renegotiation. Second, in contrast to other models, in this static model renegotiation occurs on the equilibrium-path. That is, no contract is renegotiation-proof. This result is discussed at length in the introduction and conclusion.

The renegotiation game considered is as in the preceding chapter. It consists of one stage in which one of the parties can make an offer to the other party who can accept or reject it. To exploit possible inefficiencies arising from the absence of a contract it is the uninformed party who makes the offer, i.e., the buyer is assumed to be having all the bargaining power.

3.3 Analysis

Because this is a model of contracting under asymmetric information and ex-post renegotiation, we need to separate the parties' behavior under the contract, i.e. at the message sending stage (date 2), and at the ex-post bargaining game (date 3). The buyer's strategy in the overall game is a message concerning τ at date 2 and a renegotiation offer at date 3. The seller's strategy in the overall game is a message concerning i and τ at date 2 paired with either the rejection or the acceptance of the buyer's renegotiation offer at date 3. The player's strategy at date 2 will be explained in more details when I turn to the analysis of general mechanisms in section (3.3.3). The next section deals with the analysis of the renegotiation game.

3.3.1 The Renegotiation Game

After parties' have exchanged messages and the outcome of a contract has been determined, the buyer can make a final take-it-or-leave-it offer to the seller to overcome remaining inefficiencies. Bargaining takes place under asymmetric information over the widgets's costs and symmetric information over the widgets' types.

There are two possible types of inefficiencies at that stage. First, the outcome of the contract is no trade, i.e. trade of the good widget has not been concluded. Second, the contract prescribes trade of the bad widget. In the first instance, renegotiation is only about concluding the efficient trade. The buyer must decide between offering a price q_G for the good widget of either c_1 or c_2. In the second case, in addition to bargaining about the price of the good widget, the buyer will try to undo the bad trade. The optimal solution for the buyer for these a priori *inter*dependent problems is a direct revelation mechanism. The mechanism, conditional on the seller's type announcement, prescribes a probability of trading the good widget and a probability of returning the bad widget together with a transfer payment from the buyer to the seller. In fact, the outcome of the optimal revelation mechanism can be implemented more easily. The buyer makes two *in*dependent price offers, q_G for the good widget and q_B for the bad widget, after which the seller chooses if he wants to sell the good widget at price q_G and if he wants to 'buy back' the bad widget at price q_B. The seller's decisions on these two offers are independent as well. This result is a direct implication of the assumption that technically, simultaneous trade of both widgets is feasible.

To prove this formally, some notation is needed. As the buyer's beliefs about the seller's type is influenced by the seller's message at the message sending stage, call μ the updated probability that the seller is of type 1, i.e. has low cost for the good widget. Symmetrically, $1-\mu$ is the probability that he is of type 2. The dependance of this probability on the seller's equilibrium strategy at the message sending stage will be detailed in section 3.3.3, when general mechanisms are considered.

The above discussion is summarized in the following Lemma.

Lemma 3.1 *If the status quo of the ex-post bargaining game is no trade, the buyer will offer a price q_G for the good widget. This offer can be rejected or accepted by the seller. If the status quo is trade of the bad widget, the buyer will in addition ask a price q_B for the bad widget. The seller decides independently on the two offers. He can either accept both offers, reject both offers or accept only one of them. The renegotiation offers can be classified according to the size of μ.*

- *If $\mu < \frac{v-c_2}{v-c_1}$, the buyer sets $q_G = c_2$ and both seller types agree to trade. If $\mu \geq \frac{v-c_2}{v-c_1}$, the buyer offers to purchase the good widget at a price of $q_G = c_1$. This offer is only accepted by the type 1 seller.*

- *If $\mu < \frac{c_1}{c_2}$, the buyer returns the bad widget to both seller types for a payment of $q_B = c_1$. If $\mu \geq \frac{c_1}{c_2}$, the buyer asks $q_B = c_2$ for the return of the bad widget. This offer is only accepted by the type* 1 *seller.*

Proof. The claim is trivial if the status quo is no trade.

If the bad widget is traded under the contract, the buyer solves the following constrained maximization problem at renegotiation:

$$\max_{(\phi_i,\varphi_i,g_i,b_i)_{i=1,2}} \mu(\phi_1(v-g_1)+\varphi_1 b_1)+(1-\mu)(\phi_2(v-g_2)+\varphi_2 b_2) \qquad \text{s.t.} \tag{T}$$

$$\phi_1(g_1-c_1)+\varphi_1(c_2-b_1) \;\geq\; \phi_2(g_2-c_1)+\varphi_2(c_2-b_2) \tag{IC1}$$

$$\phi_2(g_2-c_2)+\varphi_2(c_1-b_2) \;\geq\; \phi_1(g_1-c_2)+\varphi_1(c_1-b_1) \tag{IC2}$$

$$\phi_i(g_i-c_i)+\varphi_i(c_j-b_i) \;\geq\; 0 \qquad j \neq i, \text{ (IRi)},$$

where for seller type $i = 1, 2$, ϕ_i is the probability that the seller has to provide the good widget, for which he is paid g_i, and φ_i is the probability that the bad widget is returned to the seller (or not produced by the seller), for which he has to pay b_i. The index i is the screening parameter, i.e. the seller self selects his intended renegotiation offer. (IC1) and (IC2) ensure that he has no incentive to select the wrong proposal.

Using a standard argument we can dispense with IR1, the individual rationality constraint of type 1: the right-hand-side of the IC1-constraint is larger than the left-hand-side of the IR2-constraint and thus by ensuring IC1 and IR2, IR1 is automatically fulfilled. Similarly, we can ignore IC2, the incentive compatibility constraint of type 2, and solve for the parameters when IC1 and IR2 are binding. If IR2 is not binding, we can simply lower g_2 or raise b_2 which does not interfere with IC1. Equally, if IC1 is slack, we can lower g_1 or raise b_1 without violating IR2. The objective function then becomes

$$\phi_1\mu(v-c_1) \;+\; \varphi_1\mu c_2 \;+\; \phi_2(v-c_2-\mu(v-c_1)) \;+\; \varphi_2(c_1-\mu c_2) \tag{3.1}$$

From (3.1), the buyer's decision on the good and the bad widget are independent of each other. The first two terms in (3.1) are positive and φ_1 and ϕ_1 are optimally set to 1 independent of the buyer's beliefs. In contrast, the solutions for ϕ_2 and φ_2 and the payments g_i and b_i between buyer and seller do depend on the buyer's beliefs. In particular, the solutions for ϕ_2 and φ_2 are set equal to either 1 or 0, depending on whether the last terms in expression (3.1) are positive or negative. The transfer payments are found by substituting φ_i and ϕ_i into IC1 and IR2. ■

The offers detailed in Lemma 3.1 can result in four possible different outcomes of the renegotiation game. First, the seller accepts both prices and the outcome is first-best, that is, the good widget is sold to the buyer and the bad widget is returned to the seller. Second, the seller accepts only the price of the good widget, in which case both widgets become the buyer's property. Third, the seller accepts only the bad widget's price, in which case, the bad widget is returned but the good widget is not traded. The second and third alternative are mutually exclusive, i.e. their existence depends on the parameter configuration of the model. Finally, the seller rejects both prices, in which case the good widget is not traded and the bad widget remains the property of the buyer. This is the worst outcome.

Definition 3.1 *There are three types of renegotiation offers*

(*a*) *A type* (*a*) *renegotiation offer is such that* $q_G = c_2$ *and* $q_B = c_1$. *It is made if* $\min[\frac{v-c_2}{v-c_1}, \frac{c_1}{c_2}] > \mu$.

(*b*) *Two mutually exclusive cases must be distinguished*

- (*bi*) *This case applies if* $v - c_2 > c_1$. *A type* (*bi*) *renegotiation offer is such that* $q_G = q_B = c_2$. *It is made if* $\frac{v-c_2}{v-c_1} > \mu \geq \frac{c_1}{c_2}$.
- (*bii*) *This case applies if* $v - c_2 < c_1$. *A type* (*bii*) *renegotiation offer is such that* $q_G = q_B = c_1$. *It is made if* $\frac{c_1}{c_2} > \mu \geq \frac{v-c_2}{v-c_1}$.

(*c*) *A type* (*c*) *renegotiation offer is such that* $q_G = c_1$ *and* $q_B = c_2$. *It is made if* $\mu \geq \max[\frac{v-c_2}{v-c_1}, \frac{c_1}{c_2}]$.

The buyer's offers are ranked in order of increasing efficiency. A type (a) renegotiation offer is fully efficient, whereas a type (c) offer is the most inefficient.

With no contract in place, the buyer's ex-post belief coincides with the initial probability distribution on seller types, i.e. $\mu = \pi$. In order to exclude the trivial case, where there is no need for a contract because ex-post bargaining achieves efficiency, I will make the following additional assumption:

Assumption 3.1 $\pi > \frac{v-c_2}{v-c_1}$.

Because we are interested in finding a situation in which contracting can be harmful and produce lock-in effects which are not reneged upon, we need to exclude the possibility that bad trade is always undone by the ex-post bargaining game. This motivates the following assumption.

Assumption 3.2 $\pi > \frac{c_1}{c_2}$

Assumptions 3.1 and 3.2 imply that for the initial belief the renegotiation game is of type (c), i.e. the most inefficient one.

In light of Assumption 3.1, let us reconsider the assumed bargaining procedure. As much of what is to come depends on the assumption that the buyer can only make a take-it-or-leave-it offer at the renegotiation stage, it is important to investigate this assumption further. Clearly, as the model contains only one-sided asymmetric information, the seller's information could be exploited by making him an active player in the renegotiation game which would increase efficiency. In fact, if the seller was allowed to make the offer, he would offer to sell the good widget for a price of v, which would be accepted by the buyer and the first-best could be achieved. But then, the contracting problem would be void and the null contract would trivially be the (weakly) preferred alternative. This result extends to a more general split of bargaining power such as $(\lambda, 1-\lambda)$, where this notation means that with probability λ the seller makes the take-it-or-leave-it offer and with probability $1-\lambda$ the buyer makes the offer. As long as $\lambda < 1$, the null contract remains the optimal contract under the given restrictions on the parameter space. Moreover, the described lock-in effects can still be found. What cannot be allowed, is for parties to design their renegotiation game as part of a contract[3]. In that sense the assumption about renegotiation is restrictive. What I have in mind is a situation, where a contract can design outcomes in a formal meeting such as a trial in court, but an informal meeting between the two parties afterwards cannot be prevented and cannot be subject to any form of ex-ante agreement. Then, even in this simple situation of one-sided-asymmetric information there is potential for inefficiencies, as long as the uninformed party is involved in the bargaining.

Similarly, in light of Assumption 3.2, let us reconsider the assumption about negatively correlated cost. If instead costs are perfectly correlated, i.e. the good and bad widget are either both cheap or both expensive, an easy first-best solution to the contracting problem exists. A fixed price sales contract of either of the two widgets X or Y achieves efficiency. If the traded widget turns out to be the good one, there will be no further renegotiation and ex-post surplus is maximized. If it turns out to be the bad widget the buyer will simply offer to exchange the two goods without any further payments. As the two goods are equally valuable to the seller he will agree. The bad trade is undone and at the same time the efficient trade is undertaken.

3.3.2 Some Contract Examples

The above section establishes that the absence of a contract involves an efficiency loss because the good widget is not traded in all states of the world. Similarly, surplus can be lost by a contract that enforces harmful trade. To better understand this trade-off, I will describe three contracting examples. These contracts highlight

[3]The effect of renegotiation design on a contractual solution to the hold-up problem has been studied by Aghion, Dewatripont, and Rey (1994). The authors show that, if a contract can monitor renegotiation by assigning bargaining power to one of the agents and specifying default options in the event that renegotiation breaks down, contracts can in general solve the hold-up problem.

various results that will be established in greater generality in the following section.

Contract 1: A possible simple contract specifies the trade of *widget X* for a fixed price of p. This contract will raise surplus if *widget X* turns out to be the good widget and will decrease surplus if it turns out to be the bad widget. According to Lemma 3.1 this contract only affects joint surplus in the states $s_{\tau 2}$, $\tau = X, Y$, where each occurs with probability $\frac{1}{2}(1-\pi)$. In state s_{X2}, X is the good widget and costs c_2, in which case the contract raises efficiency by $v - c_2$. In state s_{Y2}, X is the bad widget and costs c_1, in which case the bad trade is not undone by the renegotiation game and c_1 of the total surplus is lost. Therefore, the contract affects total surplus by

$$\frac{1}{2}(1-\pi)(v-c_2) - \frac{1}{2}(1-\pi)c_1.$$

Clearly, it depends on the sign of $v - c_2 - c_1$ whether this contract will be chosen over the null contract. Namely, if the change in surplus is greater than zero this contract performs better in expected terms than the null contract, if it is smaller than or equal to zero this contract does worse.

Contract 1 is the first example of a contract with a lock-in-effect. The bad widget is traded on the equilibrium-path and this trade cannot be reversed by ex-post bargaining, given the constraint of asymmetric information. It can be shown (Lemma 3.5) that the optimal contract has the same feature as long as $v - c_2 \geq c_1$. Intuitively, the increase in surplus from the good trade outweighs the decrease in surplus from the bad trade

Contract 2: Let us examine a contract in which the buyer is allowed to choose a widget for trade at a fixed price of p. If he chooses the good widget he will obtain a payoff of $v - p$, if he chooses the bad one, given the analysis in section 3.3.1 and Assumptions 3.1 and 3.2, he will obtain $\pi(v + c_2 - c_1) - p$: Under the contract he pays p for a widget with no value to him. At the renegotiation stage, he offers to sell the bad widget back to the seller for a price of c_2 and to buy the good widget for a price of c_1. Thus, he offers to exchange goods for a negative payment of $(c_2 - c_1)$.

Only a type 1 seller accepts. For this contract to fail, we need the assumption that

Assumption 3.3 $\pi > \frac{v}{v-c_1+c_2}$

Contract 2 highlights a second aspect of the model. If the probability π of low cost for the good widget is very high, the buyer has a strong position at the renegotiation stage. He expects to obtain the good widget at a low price and to eventually resell the bad widget at a high price. Reaching an agreement under a contract is therefore of little interest to him. A slightly stronger version of Assumption 3.3 implies in particular, that no contract can have a fully separating equilibrium (Lemma 3.3).

Contract 3: As a last example I want to consider a contract that gives the seller the right to pick a widget. This is a signalling game in which the seller's choice under the contract signals his type to the buyer[4]. First notice, that a type 2 seller would never choose the good widget. This would result in a payoff of $p - c_2$ to him. But he does better by picking the cheap bad widget which costs only c_1 regardless of the renegotiation offer by the buyer. Consider now a type 1 seller: Should he pick the good widget which is cheap or the more expensive bad widget? Remark, that choosing the good widget implies a separating equilibrium whereas choosing the bad widget would mean that the equilibrium involves pooling.

In a separating equilibrium, the buyer would correctly infer the seller's type from the latter's behavior at date 2 and adjust his renegotiation offer accordingly. Observing the seller select the bad widget the buyer would conclude that the seller is of type 2 and offer to exchange widgets for a payment of $c_2 - c_1$. But then a type 1 seller would obtain a payoff of $(p - c_2) + (c_2 - c_1) + (c_2 - c_1)$ if he mimicked the equilibrium behavior of type 2. The first term is the seller's payoff under the

[4]The underlying assumption is that both the buyer and the seller are present at the revelation stage. This implies that the buyer observes the seller's strategy at date 2. The situation can be thought of as a trial in court as opposed to an anonymous revelation procedure where each party sends unobservable messages to a social planner. A discussion of this assumption can be found in Section 3.4

contract itself, the second term is the cost saving that will result from exchanging the relatively more expensive bad widget against the cheap good widget at renegotiation, the third is the additional payment he receives from the buyer at renegotiation. This total payoff is obviously greater than $p - c_1$, his own equilibrium payoff. Thus, a fully separating equilibrium does not exist in this game.

A pooling equilibrium in which both seller types choose the same widget is equally infeasible. The only candidate for a pooling equilibrium is the one in which both seller types choose the bad widget. Now, in a pooling equilibrium the buyer does not update his beliefs and his renegotiation offer will consequently be of type (c). A type 1 seller choosing the bad widget will thus receive a payoff of $p - c_2$ as he will reject the unfavorable renegotiation offer at the renegotiation stage. As trading the good widget will give him an overall payoff of $p - c_1$ he will prefer to do so.

In fact, a range of semi-separating equilibria exists in this game, in which the type 1 seller mixes between choosing the good and the bad widget. Depending on the parameter configuration of benefits and costs of trade $(v,\, c_i,\, i = 1, 2)$ this contract will be either better than or equivalent to the null contract. I will not expand upon this point further but rather consider general mechanisms in the next section.

Contract 3 is illustrative of several results that will be obtained for general contracts. First, it highlights the fact that cost separation is impossible in this model. In a separating equilibrium a type 1 seller gains too much from imitating a type 2 seller's equilibrium strategy. Second, only semi-separating equilibria exist.

3.3.3 Revelation Mechanisms

Now consider the question of general contracts more formally. In a revelation mechanism the seller announces $s_{\tau i}$ and the buyer announces s_τ after which a contract specifies an action and some money transfer. If parties *agree* on τ[5], let β_i be the probability that the good widget (widget τ) is traded if the seller has announced

[5]We will also refer to this situation as *on-the-equilibrium-path.*

type $s_{\tau i}$. Similarly, let α_i be the probability that the bad widget is traded and call p_i the transfer from buyer to seller. If parties *disagree* on τ, i.e., we are off-the-equilibrium-path, let γ_i^b be the probability that the widget the buyer claims to be the efficient one is traded. Define γ_i^s in a similar way and let q_i be the transfer payment. The contract is dependent on the seller's announcement i about cost, but dependency on the announcement τ is suppressed. This is without loss of generality as taking expectations over the states $s_{\tau i}$, $\tau = X, Y$, which occur with equal probability, eliminates any possible dependency. To summarize:

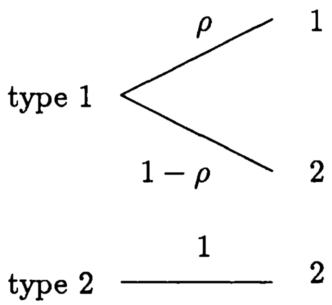
on the equilibrium path	good widget	bad widget	transfer
seller: $s_{\tau i}$ buyer: s_τ	β_i	α_i	p_i
off the equilibrium path	good widget (buyer)	good widget (seller)	transfer
seller: $s_{\tau i}$ buyer: $s_{\tau'}$	γ_i^b	γ_i^s	q_i

Then, a contract G is defined as $G := (\beta_i,\ \alpha_i,\ \gamma_i^b,\ \gamma_i^s,\ p_i,\ q_i)_{i=1,2}$, $\beta_i, \alpha_i, \gamma_i^b, \gamma_i^s \in [0,1]$, $p_i, q_i \in R$. A revelation mechanism of this form is the most general type of contract in this set-up. Although it is impossible to apply the Revelation Principle directly because of the subsequent renegotiation stage, a modified version of the Principle does indeed apply. First, consider the revelation of the widget configuration. Because this information is shared between the two parties, beliefs do not play any role. Therefore truthful revelation of this part of the state of nature can be considered without loss of generality. Only the incentive constraints resulting from truthful revelation of τ need to be studied. On the other hand, i is private information and it is not straightforward to show that attention can be restricted to direct revelation of i. Appendix A of chapter 4 proves formally that there is no loss in generality in considering contracts in which the seller mixes over announcements of his private information. The seller's strategy can be simplified further by studying equilibria in which only one type mixes.

I will proceed by studying the seller's behavior under a contract concerning the revelation of his type (Subsection 3.3.4) and then move to the simultaneous revelation of τ by the buyer and the seller (Subsection 3.3.5).

3.3.4 Revelation of Cost

The seller's mixed strategy is of the following form:



Here, ρ is taken to vary between 0 and 1, such that this semi-separating equilibrium encompasses the two situations of a fully separating equilibrium ($\rho = 1$) and a pooling equilibrium ($\rho = 0$)[6].

The buyer's equilibrium beliefs follow from Bayes' Rule

$$\begin{aligned} \mu(1 \mid 1) &= 1 \\ \mu(1 \mid 2) &= \tfrac{\pi - \pi\rho}{1 - \pi\rho}, \end{aligned} \tag{3.2}$$

where $\mu(1 \mid i)$ denotes the probability that the buyer attaches to seller type 1 following announcement i. The opposite beliefs $\mu(2 \mid i)$ are given by $1 - \mu(1 \mid i)$. For ease of notation, set $\mu := \mu(1 \mid 2)$.

These beliefs result in one of the renegotiation offers (a), (b) or (c) in Definition 3.1. Obviously, these offers are only made if the contract does not lead to trade

[6]A mixed-strategy equilibrium, in which the type 1 seller always reports 1 and a type 2 seller mixes between announcements 1 and 2 is similar to a type (a) equilibrium as both involve separation of seller types in the renegotiation game. It will be shown in Lemma 3.3 that type (a) equilibria do not exist if π is large enough (Assumption 3.4). For the same reason, the above mixed equilibria fail to exist and we disregard them to simplify the exposition.

of the good widget or if it leads to trade of the bad widget. From Lemma 3.1, the buyer will make a type (c) renegotiation offer after an announcement of 1 by the seller. The type of renegotiation offer made after an announcement of 2 by the seller, depends on the probability with which a type 1 seller mixes between his two announcements, i.e. on the size of ρ. Therefore, we can classify the equilibria in the message sending stage according to the parameter ρ in the seller's mixed strategy.

Definition 3.2 *There are three types of mixed strategy equilibria for the seller*

(a) *A type* (a) *equilibrium has* $\rho > \max\left[\frac{\pi(v-c_1)-(v-c_2)}{\pi(c_2-c_1)}, \frac{\pi c_2-c_1}{\pi(c_2-c_1)}\right]$.

(b) *A type* (bi) *equilibrium has* $\frac{\pi c_2-c_1}{\pi(c_2-c_1)} < \rho \leq \frac{\pi(v-c_1)-(v-c_2)}{\pi(c_2-c_1)}$. *A type* (bii) *equilibrium has* $\frac{\pi(v-c_1)-(v-c_2)}{\pi(c_2-c_1)} < \rho \leq \frac{\pi c_2-c_1}{\pi(c_2-c_1)}$.

(c) *A type* (c) *equilibrium has* $\rho \leq \min\left[\frac{\pi(v-c_1)-(v-c_2)}{\pi(c_2-c_1)}, \frac{\pi c_2-c_1}{\pi(c_2-c_1)}\right]$.

I now turn to the seller's incentive constraints. The mixed strategy above must be the optimal strategy for him.

First, a type 2 seller's incentive constraint is independent of the subsequent renegotiation stage. A type 2 seller has high cost for the good widget and low cost for the bad widget and can never expect to benefit from eventual renegotiation. He will be either indifferent between accepting or rejecting the buyer's renegotiation offer (the (a) and part of the (b) offer) or he will strictly prefer to reject it (the (c) and part of the (b) offer). Therefore, we can write his constraints as

$$p_2 - \beta_2 c_2 - \alpha_2 c_1 \geq p_1 - \beta_1 c_2 - \alpha_1 c_1.$$

A type 1 seller's incentive constraint on the other hand, is dependent on the buyer's renegotiation offer and will thus be different for the three kinds of offers defined in

Definition 3.1:

$$p_1 - \beta_1 c_1 - \alpha_1 c_2 = \left\{ \begin{array}{ll} p_2 - \beta_2 c_1 - \alpha_2 c_2 + (1-\beta_2)(c_2 - c_1) + \alpha_2(c_2 - c_1). & (a) \\ & \\ p_2 - \beta_2 c_1 - \alpha_2 c_2 + (1-\beta_2)(c_2 - c_1) & (bi) \\ p_2 - \beta_2 c_1 - \alpha_2 c_2 + \alpha_2(c_2 - c_1) & (bii) \\ & \\ p_2 - \beta_2 c_1 - \alpha_2 c_2 & (c) \end{array} \right\}$$

The equality constraint comes from the fact that in a mixed strategy equilibrium the type 1 seller has to be indifferent between announcing either type 1 or 2. The expression $p_2 - \beta_2 c_1 - \alpha_2 c_2$ is the type 1 seller's payoff under the contract if he announces type 2. Whether he will obtain an additional payment at renegotiation depends on the type of renegotiation offer the buyer will make.

In a type (a) offer the seller will be offered a high price of c_2 for the good widget if the good widget was not traded under the contract, i.e., with probability $1 - \beta_2$. This increases his payoff by the difference in price and cost, that is, by $c_2 - c_1$. Similarly, if the bad widget was traded under the contract he will be allowed to buy it back for a low price of c_1. This will also raise his payoff by the difference in costs.

Consider next a type (c) offer. It involves a low price for the good widget and a high price for the bad widget. The seller's gain from this offer is 0. As he is indifferent, he will accept. The payoffs involving a type (b) offer can be understood in a similar way.

Combining the constraints of the two seller types we arrive at the following Lemma.

Lemma 3.2 *A given contract G allows type (a), (b), (c) equilibria with associated renegotiation offers only if*

(a)

$$\begin{aligned} \beta_1 &= 1, \quad \alpha_1 = 0 \\ p_1 &= p_2 - \alpha_2 c_1 + (1-\beta_2)c_2. \end{aligned} \tag{3.3}$$

(b)

$$\begin{aligned} (i)\quad 0 &\leq \beta_1 - 1 - \alpha_1 + \alpha_2, \\ p_1 &= p_2 + (\beta_1 - 1)c_1 + (1 - \beta_2 + \alpha_1 - \alpha_2)c_2, \\ (ii)\quad 0 &\leq \beta_1 - \beta_2 - \alpha_1, \\ p_1 &= p_2 + (\beta_1 - \beta_2 - \alpha_2)c_1 + \alpha_1 c_2, \end{aligned} \tag{3.4}$$

(c)

$$\begin{aligned} 0 &\leq \beta_1 - \beta_2 - \alpha_1 + \alpha_2 \\ p_1 &= p_2 + (\beta_1 - \beta_2)c_1 + (\alpha_1 - \alpha_2)c_2. \end{aligned} \tag{3.5}$$

Lemma 3.2 shows that the more information is revealed through a contract, i.e., the more efficient is the renegotiation game, the more stringent are the conditions on a contract arising from the interplay between incentive constraints and renegotiation.

Compare the above constraints to the ones imposed by incentive considerations in a situation where renegotiation can be prevented. The revelation principle applies and without loss of generality one can consider truthful revelation of costs (in our model this would be a type (a) equilibrium). The incentive constraints in this situation can be written as

$$(\beta_2 - \beta_1)c_2 + (\alpha_2 - \alpha_1)c_1 \;\leq\; p_2 - p_1 \;\leq\; (\beta_2 - \beta_1)c_1 + (\alpha_2 - \alpha_1)c_2.$$

These constraints are similar to the constraints in a type (c) equilibrium with renegotiation. Nevertheless, when renegotiation is possible, the more the equilibrium involves separation of cost types the tighter are the constraints imposed on a contract. In a type (a) equilibrium only very few degrees of freedom for a contract remain. In the following chapter I show, that in a continuous type setting it is in fact impossible to separate types through a contract.

3.3.5 Revelation of the Configuration

The analysis has so far been centered on the revelation of costs. The revelation of τ, the widget configuration, implies a new set of incentive constraints. Revelation of τ is necessary because a general mechanism makes use of this information. In particular, β_i and α_i are the probabilities that the good and bad widgets are traded on the equilibrium path. As the widget configuration is common knowledge between the two parties a contract can elicit this information from both the buyer and the seller. This implies incentive constraints for the two seller types, 1 and 2, and the buyer.

As noted, for the revelation of τ, attention can be restricted to direct revelation in which τ is announced truthfully by both parties. Nevertheless, the amount of information revelation concerning i affects the incentive constraints with respect to τ. Therefore, the tree types of equilibria (a), (b) and (c) need to be analyzed separately.

3.3.5.1 Type (a) Equilibria

If a contract with a type (a) equilibrium exists, it achieves the first-best regardless of the contractual details. This is, because starting from any status quo point renegotiation is ex-post efficient. It will be shown that, under certain conditions, this type of equilibrium fails to exist. To show this, only the buyer's and the type 2 seller's incentive constraints need to be considered.

The buyer's incentive constraint is

$$\begin{aligned} v - p_1 \;\geq\; & (v - c_2) + \pi\rho(c_2 - c_1) \\ & + \pi\rho\left[\gamma_1^b c_2 + \gamma_1^s c_1 - q_1\right] \\ & + (1 - \pi\rho)\left[\gamma_2^b c_1 + \gamma_2^s c_2 - q_2\right]. \end{aligned}$$

To understand this inequality, first consider the buyer's equilibrium payoff. Because of (3.3) he obtains $v - p_1$ with probability $\pi\rho$, the probability with which a type 1 seller announces that he is type 1. With probability $1 - \pi\rho$, an announcement of 2 is made by the seller. This case comprises a type 1 seller who announces that he is type 2 and the type 2 seller. The buyer then obtains $\beta_2 v - p_2$ under the contract and $\alpha_2 c_1 + (1-\beta_2)(v-c_2)$ through renegotiation. Using the constraint in (3.3), one can show that this is equal to $v - p_1$ as well. The buyer's out-of-equilibrium payoff is composed of two components. With probability $\pi\rho$ the buyer receives $\gamma_1^s v - q_1$ under the contract and achieves $\gamma_1^b c_2 + (1-\gamma_1^s)(v-c_1)$ through his renegotiation offer. With probability $1-\pi\rho$, the buyer receives $\gamma_2^s v - q_2$ through the contract and $\gamma_2^b c_1 + (1-\gamma_2^s)(v-c_2)$ through his renegotiation offer. Adding these payoffs weighted by their probabilities explains the right-hand-side of the above inequality.

The type 2 seller's incentive constraint for truthful revelation of τ is

$$p_2 - \beta_2 c_2 - \alpha_2 c_1 \geq \max_j q_j - \gamma_j^b c_2 - \gamma_j^s c_1. \tag{3.6}$$

The left-hand side of this expression is his equilibrium payoff if he announces τ truthfully. The right-hand side is his out-of-equilibrium payoff when he lies about τ and simultaneously chooses his type announcement j (possibly his true type) to maximize this out-of-equilibrium payoff. It is based on a type (c) renegotiation game. The types (a) and (b) are neglected. That is, I restrict attention to equilibria that are supported by out-of-equilibrium beliefs for the buyer of the form $\mu > \max[\frac{v-c_2}{v-c_1}, \frac{c_1}{c_2}]$. In fact, this is not restrictive for our purposes. The more inefficient the out-of-equilibrium renegotiation game, the more scope there is for contracting, i.e. the less stringent are incentive constraints. As the aim is to show that type (a) equilibria do not exist, such a result is the more forceful, the more power we give to contracting.

Remark, that the buyer's out-of-equilibrium beliefs, and therefore his renegotiation offer, differ in the two cases when the buyer lies about τ as opposed to when the seller lies. In the first situation, the beliefs are given by the equilibrium beliefs, in the second, they are not determined and we are free to choose the best beliefs from the viewpoint of the contract. This follows from the definition of a Perfect Bayesian Equilibrium, see for instance chapter 8 of Fudenberg and Tirole (1991).

To combine the two incentive constraints we write (3.6) as

$$p_2 - \beta_2 c_2 - \alpha_2 c_1 \geq \pi\rho \left[q_1 - \gamma_1^b c_2 - \gamma_1^s c_1\right] + (1-\pi\rho)\left[q_2 - \gamma_2^b c_2 - \gamma_2^s c_1\right], \tag{3.7}$$

which allows us to add the two constraints. Then, by using (3.3) we obtain

$$-\pi\rho(c_2 - c_1) \geq (1-\pi\rho)(\gamma_2^s - \gamma_2^b)(c_2 - c_1).$$

Making the right-hand-side of this expression as small as possible involves setting $\gamma_2^s = 0$ and $\gamma_2^b = 1$, which implies that we must have

$$\rho \leq \frac{1}{2\pi}. \tag{3.8}$$

Condition (3.8) is inconsistent with the definition of a type (a) equilibrium in Definition 3.2, if

Assumption 3.4 $\pi > \max\left[\frac{v-c_2}{v-c_1} + \frac{c_2-c_1}{2(v-c_1)}, \frac{c_1}{c_2} + \frac{c_2-c_1}{2c_2}\right].$

This condition is slightly stronger than Assumptions 3.1 and 3.2. Observe, that, if $v = c_1 + c_2$ is assumed, Assumption 3.4 and Assumption 3.3 are equivalent. Similarly, Assumptions 3.1 and 3.2 are the same. This completes this section.

Lemma 3.3 *Under Assumption 3.4 it is impossible to write a contract with a type (a) equilibrium. In particular, there is no contract with a fully separating equilibrium. If Assumption 3.4 does not hold, contracts with type (a) equilibria exist. They achieve the first-best.*

The intuition for Lemma 3.3 is simple. It is a generalization of the result obtained for Contract 3 in Section 3.3.2. A type (a) equilibrium reveals information to the buyer which he can exploit at the renegotiation stage. It is therefore difficult to satisfy his incentive constraint with respect to the truthful revelation of the widget configuration τ. This is particularly difficult if π, the probability that the seller is of type 1, is very large because the buyer expects to gain the most from renegotiation.

Also, in such an equilibrium a type 1 seller has much to gain from making the buyer believe that he is in fact a type 2 seller. This will make it very hard for a contract to satisfy the seller's incentive constraint with respect to revelation of cost (see Lemma 3.2). The interplay of both incentive constraints makes it impossible for such an equilibrium to arise if π is large.

3.3.5.2 Type (b) Equilibria

For the remainder of the chapter I assume that Assumption 3.4 holds. The value of a contract with a type (bi) equilibrium depends on one of its parameters only, namely, on the probability with which it prescribes trade of the bad widget after an announcement of 2 by the seller in equilibrium, i.e. on α_2. The larger is α_2, the smaller is the benefit of the contract, because a type 2 seller will reject the buyer's renegotiation offer concerning the bad widget and the wasteful trade is enforced. Only if $\alpha_2 = 0$ is the contract first best. The other parameters β_1, β_2 and α_1 play no role for efficiency because renegotiation is ex-post efficient with respect to the good widget independent of the type announcement and it is ex-post efficient with respect to the bad widget after announcement 1.

Similarly, the value of a contract with a type (bii) equilibrium depends on the probability with which it prescribes trade of the good widget after an announcement of 2 by the seller, i.e. on β_2. The smaller is β_2, the smaller is the benefit of the contract because a type 2 seller will reject the buyer's renegotiation offer concerning the good widget. The parameters β_1, α_1 and α_2 are of no importance because renegotiation with a type 1 seller is always ex-post efficient and trade of the bad widget is undone through a type (bii) renegotiation offer even with a type 2 seller.

It can be shown that in both cases, in order to fulfill incentive constraints, the critical parameter must be set to its least optimal level. More precisely,

Lemma 3.4 *Under Assumption 3.4, for $v - c_2 \geq c_1$, a contract with a type (bi) must have $\alpha_2 = 1$. It (weakly) dominates the null contract. The equilibrium is unique with $\rho = \frac{\pi c_2 - c_1}{\pi(c_2 - c_1)}$. For $v - c_2 \leq c_1$, a contract with a type (bii) equilibrium* *must have* $\beta_2 = 0$. *It is equivalent to the null contract. The equilibrium is unique with* $\rho = \frac{\pi(v-c_1)-(v-c_2)}{\pi(c_2-c_1)}$.

A discussion of this result in conjunction with the result for type (c) equilibria can be found at the end of the following section.

3.3.5.3 Type (c) Equilibria

The final candidate for a contract allows only type (c) equilibria. For efficiency, two parameters are important, the probability of trade of the good widget and the probability of trade of the bad widget after an announcement of 2 by the seller. The type (c) renegotiation offer is rejected by a type 2 seller and therefore the optimal contract has β_2 as large and α_2 as small as possible.

It is shown that type (c) equilibria are the most efficient because parties can be punished the most heavily off-the-equilibrium-path. Indeed, it turns out that

Lemma 3.5 *Under Assumptions 3.4, the optimal contract has only type* (c) *equilibria. If* $v-c_2 \geq c_1$, *the good widget is traded with certainty and the bad widget is traded with a probability smaller than* 1 *after an announcement of* 2 *on-the-equilibrium-path. Off-the-equilibrium-path, after an announcement of* 2 *by the seller, no widget is traded.*

$$\beta_2 = 1, \qquad \alpha_2 = \frac{c_2-\theta}{\pi c_2 - c_1},$$
$$\gamma_2^b = 0 \qquad \gamma_2^s = 0$$

If $v-c_2 \leq c_1$, *after an announcement of* 2, *the good widget is traded with a probability smaller than* 1 *and the bad widget is not traded on the equilibrium path. Off-the-equilibrium path both widgets are traded.*

$$\beta_2 = (1-\pi)\frac{c_1+c_2-v}{c_1+c_2-2\theta}, \qquad \alpha_2 = 0,$$
$$\gamma_2^b = 1 \qquad \gamma_2^s = \frac{\pi c_2 - \theta}{c_1+c_2-2\theta}$$

Lemma 3.5 has an intuitive explanation. The benefit of a contract consists in raising the probability of trading the good widget when its costs are high. The benefit can therefore be roughly measured by $v - c_2$. On the other hand, trading the bad widget results in a loss of overall surplus of $-c_1$ in the high cost state because renegotiation breaks down and the bad widget is kept by the buyer.

If the cost of taking the inefficient action is relatively low compared to the increase in surplus resulting from the efficient trade, i.e. if $v - c_2 \geq c_1$, the threat of taking the bad action off-the-equilibrium path has no bite ($\gamma_2^b = \gamma_2^s = 0$). On the other hand, if costs of the bad action compared to the benefits of the good action are very high, i.e. $v - c_2 \leq c_1$, the parties can be forced to reveal the configuration τ truthfully by being threatened with trade of the bad widget off the equilibrium path ($\gamma_2^b, \gamma_2^s > 0$).

To see this more clearly consider the two off-the-equilibrium-path options for a contract. Either it prescribes trade of the widget that the buyer claims to be the good widget or it prescribes trade of the widget that the seller designates. The first option is indeed good for the buyer's incentives because he receives the bad widget out-of-equilibrium which he will not be able to resell to the seller. Joint surplus is reduced by c_1. On the other hand this is bad for overall incentives because, if it is instead the seller who is lying about τ, this results in trade of the good widget which raises the parties joint payoff out-of-equilibrium by $v - c_2$. Next, consider the option of trading the widget that the seller claims to be the good widget. If it is the buyer who is lying this raises overall surplus by $v - c_2$, if the seller is lying, surplus is reduced by c_1. In either case total surplus is increased by such actions by $v - c_2 - c_1$. The best solution therefore depends on the sign of this expression.

If $v - c_2 \geq c_1$, it is best to prescribe no trade off-the-equilibrium-path. In contrast, on-the-equilibrium-path, it is beneficial to enforce both actions. In particular, the bad action should be taken. This is a generalization of the result that is obtained for Contract 1 in Section 3.3.2. Trading the bad widget on the equilibrium path provides incentives for truthful revelation because the buyer expects to obtain a high price for it in the renegotiation game. With probability π he can sell it back to the seller for a price of c_2. A type 2 seller also prefers to provide the bad widget because it is very cheap.

The difference to the result in Lemma 3.4 is that in a type (c) equilibrium the probability of trading the bad widget does not need to be raised as high, i.e. $\alpha_2 < 1$. To understand why this is so, compare the buyer's out-of-equilibrium payoffs when he lies about τ in the two types of equilibria. In a type (c) equilibrium the buyer expects that a total surplus of $\pi(v - c_1)$ is realized off-the-equilibrium-path, whereas his expectations are $\pi(v - c_1) + (1 - \pi)(v - c_2)$ in a type (bi) equilibrium. Therefore, it is more difficult to satisfy the buyer's incentive constraints in a (bi) equilibrium and his compensation in equilibrium (trade of the bad widget) must be higher.

If $v - c_2 \leq c_1$, it is costly to enforce trade of the bad widget on-the-equilibrium-path. In contrast, from the above discussion parties can be threatened with trade of both widgets off-the-equilibrium-path, which will lower joint surplus compared to the surplus realized in equilibrium.

Finally, the situation when $v - c_2 = c_1$ is such that costs and benefits of a contract balance each other. Neither the threat of inefficient off-the-equilibrium-path trade, nor the enticement of inefficient on-the-equilibrium-path trade can be used to make the parties' incentive constraints less binding. Thus, any contract with a type (c) equilibrium is as good as the null contract.

Lemma 3.6 *Under Assumption 3.4, as $v - c_2$ approaches c_1, a contract with a type (c) equilibrium achieves less and less. Finally, at $v - c_2 = c_1$, no contract with a type (c) equilibrium can raise efficiency above the null contract situation.*

3.4 Results

This section summarizes the above results. The first main result is the following.

Proposition 3.1 *If the benefit of contractual commitment outweighs the loss in flexibility due to contracting, i.e. $v - c_2 \geq c_1$, contractual lock-in effects are a necessary implication of asymmetric information.*

Proposition 3.1 is the collective result of Lemmata 3.2, 3.3, 3.4 and 3.5. Lemma 3.2 shows that precise information revelation about costs implies very strong constraints on a contract. Lemma 3.3 proves that full revelation of costs under a contract is impossible and that consequently renegotiation is inefficient. Therefore, c_1 measures the cost of contracting or the loss in flexibility. It is the loss in total surplus when the wrong action is enforced under a contract. On the other hand, $v - c_2$ is the benefit of contracting. It is the increase in total surplus resulting from trade of the good widget which could not be realized in the absence of a contract. Lemmata 3.4 and 3.5 show that if $v - c_2 \geq c_1$, parties can be induced to truthfully reveal the configuration by contractually forcing them to undertake the bad action in equilibrium. Intuitively, the bad action involves a very small loss in total surplus at the same time as being very desirable to both the buyer and a type 2 seller.

The second main result concerns incompleteness of contracts.

Proposition 3.2 *Contracting is the more beneficial the larger (or the smaller) is the benefit of contractual commitment compared to the loss in flexibility due to contracting, i.e.* $v - c_2 >> c_1$ $(v - c_2 << c_1)$*. As* c_1 *approaches* $v - c_2$*, no contract is better than the null-contract. Therefore, contracts can be expected to be incomplete if costs and benefits of trade are similar.*

When costs and benefits are close contracts have very little screening options. On the one hand, parties cannot be punished with the bad action if it is found that they are lying, on the other hand, they cannot be enticed with the good action if they tell the truth. Thus, Proposition 3.2 claims that if stakes are not very high contracts can achieve very little. More precisely, if ex-ante an action can have a similarly negative or positive effect on total surplus, writing a contract, and in a sense committing to an action, is of little value. This result is intuitive if one considers commitment at a point at which the true consequences of an action are not known. In other words, if only simple, non-contingent contracts in the same vein as *Contract 1* in Section 3.3.2 are allowed, commitment to trade might well be detrimental to total surplus. More surprisingly though, this intuition carries through to situations where an action is enforced at a point at which all necessary information is known.

Furthermore, if π, the probability of low costs for the good widget, is large, it is difficult to fulfill the buyer's incentive constraints. Because he can exploit his strong bargaining position in the renegotiation stage, reaching an agreement under a contract is of little value to him. Thus, the proposition implies that, if the overall benefit of a contract to a party that has a strong position without a contractual agreement is very small, it is unlikely that a contractual solution will be reached.

The final result that I want to stress concerns the fact that for all feasible contracts, on-the-equilibrium-path-renegotiation can not be avoided.

Proposition 3.3 *Under Assumption 3.4, any feasible contract involves renegotiation on the equilibrium path.*

This result has been already discussed in the introduction. The reason that in this model contract renegotiation occurs in equilibrium is that renegotiation happens in a static game under asymmetric information. As renegotiation is the last stage of the game it separates seller types. But because separation under a contract cannot occur in equilibrium (Lemma 3.3), including renegotiation into a contract as the Renegotiation-Proofness-Principle suggests is impossible. In addition, the outcome of a contract is not ex-post efficient and so the two parties will make use of the possibility of renegotiation in equilibrium.

I have assumed a very crude bargaining procedure for renegotiation: The buyer is allowed one proposal which the seller can accept or reject. In particular, bargaining might end even though there is still surplus left unexploited, in which case it can be argued that renegotiation should reopen. Assume instead that renegotiation consists of two rounds of sequential offers by the buyer, where payoffs in the second round are discounted by some positive discount factor smaller than 1. Assume further that the parameter configuration of the model is such, that the buyer screens seller types intertemporally, i.e., trade of the good widget occurs in the first round at a low price with a type 1 seller and in the second round at a high price with a type 2 seller. Then, although all surplus is exhausted ex-post, inefficiency arises through delayed trade. Nevertheless, the reasoning behind the last result is still valid. I conjecture therefore, that for any finite bargaining game that is not fully efficient contracts will be renegotiated. What results are obtained when the bargaining procedure at renegotiation is infinite could be subject for further research.

Finally, let us investigate some of the assumptions made throughout the model. First, for an extension into a model with continuous type space, the reader is referred to the following chapter. It is shown that a contract cannot be made dependent on types, i.e. there exists essentially only a pooling contract in a continuous type framework.

Second, consider the specificity of the assumption that information is asymmetrically distributed only on one side of the relationship. Why should the seller have superior information about the buyer's value of the two goods? This assumption is not stringent. In fact, the results would not be altered by the introduction of multiple buyer types and two-sided asymmetric information. What is important for the derivation is that both parties must be aware which of the two goods is the good widget and which one is the bad widget. Imagine for example that the seller does not know the widget configuration. Consider the following contract: The seller is asked to announce his type and the widget that he indicates as the cheapest is traded for a fixed price of p. This is a separating equilibrium if he tells the truth and his payoff in equilibrium will be $p - c_1$. He will not obtain additional surplus from renegotiation even if the cheaper widget is not the efficient one. On the other hand, if he does not play according to his equilibrium strategy and points out the more expensive good he will obtain $p - c_2$, any possible renegotiation offer from the buyer will be rejected. Thus, as this is less than what he would obtain in equilibrium, a separating equilibrium in this context exists and the proposed contract achieves first-best.

Why the same is not true when the seller is informed about the widget configuration is explained in Section 3.3.2, *Contract 3*. Here, we have an example of the observation that superior information of one of the parties is actually hurtful from an efficiency point of view. This is not straightforward. If the widget configuration is symmetric information between the two parties, a contract can use the announcement of one of the parties as a check against the other parties' announcement and this should reinforce the position of a contract. But, on the other hand, the seller's superior knowledge increases his gain from lying about the other parameter of the model, his cost. Thus, the positive effect of an additional instrument for the contract is off-set by the possibility for the seller to behave strategically when he reveals his cost.

To conclude, this chapter has provided a possible interpretation of the term 'transaction costs' and its link to contractual incompleteness. Transaction costs arise in this setting because incentive considerations force contracting parties to undertake some negative action. The more far reaching step would be to investigate a model in which these kind of transaction costs lead contracts to be strictly worse than no contract. If the negative effect on total surplus from the bad action is too large the null-contract, which trivially fulfills all incentive constraints as it forces no action, might dominate any more contingent contract. Intuitively, 'acting' is worse and less flexible than 'not acting'. Whether this result can be obtained in such a model is an open question. Another form of incompleteness is found in this model in the sense that all contracts are renegotiated. Here, we have the result that *all* contracts *must* be renegotiated. An interesting question is whether there are instances in which some contracts are renegotiation-proof but the optimal contract is not.

3.5 Appendix A

Proof of Lemma 3.4

Proof.

First, I consider the case where $v - c_2 \geq c_1$. The proof proceeds similarly to the analysis of type (a) equilibria. But in addition to the buyer's and the type 2 seller's incentive constraints for truthful revelation of τ, we need to consider also a type 1 seller's constraint. In a type (bi) equilibrium the buyer's incentive constraint is

$$\begin{aligned}\beta_2 c_2 + \alpha_2 \pi c_2 - p_2 \geq\ & \pi\rho(c_2 - c_1) \\ & + \pi\rho\left[\gamma_1^b c_2 + \gamma_1^s c_1 - q_1\right] \\ & + \pi(1-\rho)\left[\gamma_2^b c_2 + \gamma_2^s c_2 - q_2\right] \\ & + (1-\pi)\left[\gamma_2^s c_2 - q_2\right].\end{aligned}$$

A type 2 seller's incentive constraint is as in (3.7). Assume that the contract is such that the type 2 seller, when lying about τ, is indifferent between his two possible cost announcements 1 and 2. Formally this means

$$q_1 - \gamma_1^b c_2 - \gamma_1^s c_1 = q_2 - \gamma_2^b c_2 - \gamma_2^s c_1. \tag{3.9}$$

This is without loss of generality, because q_2 can be adjusted to fulfill the above equality. As to derive the results, we add the incentive constraints of buyer and seller, the transfer payments cancel.

Then, a type 1 seller's out-of-equilibrium payoff, when lying about τ, depends on his announcement concerning his type. If he announces 1 his payoff is $q_1 - \gamma_1^b c_1 - \gamma_1^s c_2$, if he announces 2 it is $q_2 - \gamma_2^b c_1 - \gamma_2^s c_2$. Due to (3.9), the former expression is (weakly) greater than the latter if

$$\gamma_1^b - \gamma_1^s \geq \gamma_2^b - \gamma_2^s. \tag{3.10}$$

Assume that (3.10) holds, the symmetric case when $\gamma_1^b - \gamma_1^s < \gamma_2^b - \gamma_2^s$ proceeds along the same lines.

Then by using (3.4) and (3.9), a type 1 seller's incentive constraint can be written as

$$\begin{aligned} p_2 - \beta_2 c_2 - \alpha_2 c_2 + (c_2 - c_1) \ \geq \ & \pi\rho\left[q_1 - \gamma_1^b c_1 - \gamma_1^s c_2\right] \\ & +(1-\pi\rho)\left[q_2 - \gamma_2^b c_2 - \gamma_2^s c_1 + (\gamma_1^b - \gamma_1^s)(c_2 - c_1)\right]. \end{aligned}$$

This allows us to add the buyer's incentive constraint to both seller types' incentive constraints to obtain the set

$$\alpha_2(\pi c_2 - c_1) \ \geq \tag{3.11}$$
$$\pi\rho(c_2 - c_1) + (1-\pi\rho)\gamma_2^s(c_2 - c_1) - (1-\pi)\gamma_2^b c_2$$
$$(c_2 - c_1) - \alpha_2(1-\pi)c_2 \ \geq \tag{3.12}$$
$$\pi\rho(c_2 - c_1) + (1-\pi\rho)\gamma_2^s(c_2 - c_1) - (1-\pi)\gamma_2^b c_2 + (\gamma_1^b - \gamma_1^s)(c_2 - c_1),$$

where the first (second) line is the sum of the buyer's and type 2 (1) seller's incentive constraints. In order to meet (3.12), it is best to set $(\gamma_1^b - \gamma_1^s)$ as small as possible and (3.10) implies $\gamma_1^b - \gamma_1^s = \gamma_2^b - \gamma_2^s$. Also, both (3.11) and (3.12) are satisfied the easier the smaller is ρ. Substituting for the smallest value of ρ consistent with a type (bi) equilibrium, i.e. $\rho = \frac{\pi c_2 - c_1}{\pi(c_2 - c_1)}$, the two constraints become

$$\begin{aligned} \alpha_2(\pi c_2 - c_1) \ &\geq \ \pi c_2 - c_1 - (\gamma_2^b - \gamma_2^s)(1-\pi)c_2 \\ -\alpha_2(1-\pi)c_2 \ &\geq \ -(1-\pi)c_2 + (\gamma_2^b - \gamma_2^s)(\pi c_2 - c_1). \end{aligned}$$

The objective is to minimize α_2 while at the same time fulfilling the two above constraints. Given Assumption 3.4, this is done by setting $\gamma_2^b = \gamma_2^s$ and it follows that $\alpha_2 = 1$. If Assumption 3.4 does not hold, α_2 can be set equal to 0 and the contract can be made first best.

Next, I consider the case where $v - c_2 \leq c_1$. The three parties' incentive constraints can be constructed using similar arguments as in the above demonstration.

In a type (*bii*) equilibrium the buyer's incentive constraint is

$$\begin{aligned}\beta_2\theta + \alpha_2 c_1 - p_2 &\geq +\pi\rho\left[\gamma_1^b c_2 + \gamma_1^s c_1 - q_1\right] \\ &\quad +\pi(1-\rho)\left[\gamma_2^b c_1 + \gamma_2^s c_1 - q_2\right] \\ &\quad +(1-\pi)\left[\gamma_2^b c_1 + \gamma_2^s v - q_2\right],\end{aligned}$$

where $\theta := \pi c_1 + (1-\pi)v$. Remark, that $c_1 < \theta < c_2$ from Assumption 3.1.

A type 2 seller's incentive constraint is as in (3.7). Assume that (3.9) and (3.10) hold. Then the type 1 seller's incentive constraint is

$$\begin{aligned}p_2 - \beta_2 c_1 - \alpha_2 c_1 &\geq \pi\rho\left[q_1 - \gamma_1^b c_1 - \gamma_1^s c_2\right] \\ &\quad +(1-\pi\rho)\left[q_2 - \gamma_2^b c_2 - \gamma_2^s c_1 + (\gamma_1^b - \gamma_1^s)(c_2 - c_1)\right].\end{aligned}$$

Adding the buyer's incentive constraint to both seller types' incentive constraints and, as above, substituting for the smallest values of $(\gamma_1^b - \gamma_1^s) = (\gamma_2^b - \gamma_2^s)$ and $\rho = \frac{\pi(v-c_1)-(v-c_2)}{\pi(c_2-c_1)}$ yields

$$\begin{aligned}\beta_2(\theta - c_2) &\geq -(\gamma_2^b - \gamma_2^s)(\theta - c_1) \\ \beta_2(\theta - c_1) &\geq (\gamma_2^b - \gamma_2^s)(\theta - c_2)\end{aligned}$$

The objective is to maximize β_2 under the two constraints above. Under Assumption 3.4, $c_2 - \theta > \theta - c_1$ and therefore $\beta_2 = 0$ and $\gamma_2^b = \gamma_2^s$. If Assumption 3.4 does not hold, β_2 can be set equal to 1 and the contract can be made first best. ■

Proof of Lemma 3.5

Proof.

The argument is as in the proof of Lemma 3.4 and we can write the sum of the buyer's and the two seller types' incentive constraints as

$$\beta_2(\theta - c_2) + \alpha_2(\pi c_2 - c_1) \geq \gamma_2^s(\theta - c_1) + \gamma_2^b(\pi c_2 - c_2) \quad (3.13)$$

$$\beta_2(\theta - c_1) + \alpha_2(\pi c_2 - c_2) \geq \gamma_2^s(\theta - c_2) + \gamma_2^b(\pi c_2 - c_1) \quad (3.14)$$

As $\theta < c_2$ and $\pi c_2 > c_1$, raising β_2 above 0 is only possible if *either* α_2 is sufficiently large *or* if the right-hand-side of (3.13) is negative. At the same time, the solution should respect (3.14).

Let us consider the first option. Because a positive α_2 involves a true loss of efficiency we set it just large enough to offset the negative impact of β_2 on the left-hand-side in (3.13):

$$\alpha_2 = \beta_2 \frac{c_2 - \theta}{\pi c_2 - c_1}.$$

For simplicity set all the γ_j^i equal to 0. The increase in expected surplus from such a contract is

$$(1-\pi)(\beta_2(v - c_2) - \alpha_2 c_1).$$

Substituting the obtained identity for α_2 and rearranging this expression we obtain that the expected gain in surplus is equal to

$$\beta_2(1-\pi)\pi(c_2 - c_1)\frac{v - c_2 - c_1}{\pi c_2 - c_1}.$$

The sign of $v - c_2 - c_1$ determines whether such a contract is beneficial to the two parties or not. If $v - c_2 \geq c_1$, this is the case and optimally $\beta_2 = 1$. Then, (3.14) holds automatically.

Next, assume that γ_2^s, $\gamma_2^b \geq 0$ are chosen such that $\gamma_2^s(\theta - c_1) + \gamma_2^b(\pi c_2 - c_2)$ is negative and set $\alpha_2 = 0$. The maximal β_2 consistent with (3.13) is

$$\beta_2 = \frac{\gamma_2^b(c_2 - \pi c_2) - \gamma_2^s(\theta - c_1)}{c_2 - \theta}. \tag{3.15}$$

This β_2 must also satisfy (3.14) and therefore a contract has to ensure that

$$\frac{\gamma_2^b(c_2 - \pi c_2) - \gamma_2^s(\theta - c_1)}{c_2 - \theta}(\theta - c_1) \geq \gamma_2^s(\theta - c_2) + \gamma_2^b(\pi c_2 - c_1),$$

which is equivalent to

$$\gamma_2^b \leq \gamma_2^s \frac{c_1 + c_2 - 2\theta}{\pi c_2 - \theta}.$$

Both the numerator and the denominator of the fraction on the left-hand side are positive due to Assumptions 3.4 and 3.3. Also, this fraction is smaller than or equal to 1 if $v - c_2 \geq c_1$, and it is greater than or equal to 1 if $v - c_2 \leq c_1$.

Consider the first case. To make the right-hand side of (3.13) as negative as possible, γ_2^b should be made as large as possible. Therefore, setting $\gamma_2^s = 1$ and replacing the above expression with equality in (3.15), we obtain

$$\beta_2 = (1-\pi)\frac{c_1 + c_2 - v}{\pi c_2 - \theta} \leq 0.$$

Consequently, in this case, making the right hand side of (3.13) negative does not result in a positive β_2.

If $v - c_2 \leq c_1$ on the other hand, $\gamma_2^b = 1$ and $\gamma_2^s = \frac{\pi c_2 - \theta}{c_1 + c_2 - 2\theta}$ allows us to compute β_2 in (3.15)

$$\beta_2 = (1-\pi)\frac{c_1 + c_2 - v}{c_1 + c_2 - 2\theta} \geq 0.$$

Due to Assumption 3.3, $\beta_2 < 1$.

■

Chapter 4

Costly Contracting

4.1 Introduction

This chapter identifies a strategic reason for incompleteness of contracts. As in chapter 3, a contract merely serves as a starting point for negotiations between contracting parties, which are governed by an exogenously given, costly bargaining procedure. In the model of the preceding chapter a complete contract can constitute a worse status quo point for such ex-post negotiations because it can lead to wasteful trade. In contrast, in the model of this chapter no such bad outcome exists. Rather, it is the informational status quo that is changed by a contract. The uninformed party, by learning something about his contracting partner through a complete contract, is not able to credibly keep a 'tough' stance in the ex-post bargaining game. This party therefore prefers the contract that releases the least information, i.e. the null contract.

In order to make this point I modify the buyer seller story. The chapter abstracts from the issue of widget types that has been at the heart of the preceding two chapters. It concentrates instead on a more standard contracting problem, the sale of a single, indivisible good. In order to allow an easy comparison with the durable goods monopoly literature, the buyer-seller relationship is reversed. In this model the seller makes the contracting offer and is uninformed about the buyer's valuation.

He also has all the bargaining power in the final bargaining game. Furthermore, a continuous type space for the buyer's valuation is considered. But most importantly, I study a problem in which information is already asymmetrically distributed at the time at which parties contract. Thus, the individual rationality constraint of the informed party imposes an additional constraint on contracts and introduces strategic considerations on the part of the seller. The last assumption delivers the strict dominance result. The seller strictly prefers not to offer an early contract. Instead, he moves straight into the bargaining game.

Similarly to the preceding chapters, the ex-post bargaining game is modelled as a take-it-or-leave-it offer by the seller. From the seller's ex-ante viewpoint, this game is constrained efficient, given the asymmetry of information. To introduce some further inefficiency, I assume that the seller has to pay a fixed fee ε_2 in order to enter into the bargaining game. The fee has to be paid in advance to cover the opportunity cost of time spent in a meeting between the parties.

Before the negotiations the seller can decide to offer an initial contract to the buyer to possibly save on some of ε_2. A contract offer in itself involves costs ε_1, which have to be born by the seller as well. They can for example be regarded as the legal costs of drawing up a contract.

To model the benefit of an early contract over the later renegotiation game, it is assumed that the renegotiation offer is more costly than the initial contracting offer, i.e. $\varepsilon_2 > \varepsilon_1$. Apart from this cost differential there is no further loss in waiting, i.e. there is no discounting. One reason for this exogenous increase in costs is that actual bargaining, where parties meet around a table, is more costly than the writing of a contract, which can be drafted by third parties, such as lawyers for example. Alternatively, it can be interpreted as resulting from storage costs because the good, if it is not sold at the first possible date through a contract, must be kept until the renegotiation period.

The disadvantage of an early contract is that it releases information before the final meeting between the parties. Through the messages that are sent according to the contract the seller learns about the buyer's type and has an incentive to adapt his behavior at renegotiation. This is the ratchet effect. Because there is an incentive for the seller, once he has observed a rejection of a high price offer by the buyer, to lower his price at the final stage, strong restrictions are imposed on an initial contract. These restrictions are particularly strong in this model because of the assumption of no discounting. This has interesting implications for the form of the initial contract, which is almost uniquely determined by incentive considerations. In fact, only one degree of freedom is left in choosing contract prescriptions. This results in a parsimonious representation for *every* feasible initial contract, which greatly simplifies the seller's problem of choosing the *optimal* contract.

In this context, two results obtain. First, assuming that initial contracting costs are negligible one can show that, although the benefit of early contracting outweighs the cost, these effects are of same order of magnitude. This result can be seen as a first step towards explaining incompleteness of contracts because of bounded rationality. If contracting parties base their reasoning on a rough cost-benefit analysis, there is no strong reason in this context to write a contract. Parties are 'indifferent'. Second, if initial contracting costs are strictly positive, the null contract is strictly preferred even if those initial costs are of order of magnitude smaller than the renegotiation costs.

The intuition goes as follows. One can show that any early contract is equivalent to a so called *simple contract* with a partition equilibrium that separates buyer types into two groups. Trade is concluded contractually with the first group of high valuation buyers, whereas the second group moves into the bargaining round. This simple contract saves on the bargaining cost ε_2 for the first group. On the other hand, by revealing information too early it imposes a 'cost' on the seller through the ratchet effect. This cost is a function of the size of ε_2. Intuitively, the larger ε_2, the larger the optimal size of the first group, the lower the final price offer after the simple contract compared to the final price offer after the null contract. So, when ε_2 decreases, both the benefit of a contract decreases because the optimal size of the first group of buyers decreases, and the cost from information revelation shrinks. Through a form of envelope theorem argument one can show that benefits fall more quickly than costs.

The papers most closely related to this chapter are Fudenberg and Tirole (1983), Hart and Tirole (1988) and Hart (1989). Fudenberg and Tirole (1983) study a two-period bargaining game with discounting, in which a decreasing price path is found. Hart and Tirole (1988) explore short term and long-term contracting in a T-period version of this model. Instead of assuming a given bargaining procedure, they derive the optimal contracting structure explicitly. Finally, Hart (1989) studies the effect on the length of disagreement in a sequential bargaining game if one party faces a crunch, i.e. if its value decreases sharply at a fixed point in time. The main difference to these models is that there is no discounting but a fixed cost of contracting in this paper. With discounting, a strict preference for waiting can never be obtained.

A paper which is similar in spirit is Anderlini and Felli (2001) who study an infinite horizon bargaining game with symmetric information and transaction costs. The authors argue that the equilibrium in which these costs are not paid and consequently an agreement is never reached is a pervasive equilibrium. It is the unique equilibrium if either these costs are sufficiently high or if parties in the course of the game have the option to renegotiate inefficient outcomes.

The chapter is divided into four sections. The following section contains the set-up of the model with fixed contracting cost and solves for the benchmark contract under full commitment. Section 4.3 derives the optimal contract when renegotiation is allowed. The final section concludes. Appendix A deals with the question of whether the revelation principle applies in a set-up with renegotiation. I prove a result which I call *Revelation Principle with Renegotiation*[1]. It establishes that without loss of generality, in this set-up, the set of feasible contracts can be taken to

[1] A recent paper, Bester and Strausz (2000), studies this problem with a finite type space in a more general framework. Since the type space considered in this paper is a continuum the same techniques cannot be applied.

be the set of direct revelation mechanisms in which the informed party *mixes* over announcements of his type. Appendix B contains some minor derivations.

4.2 The Model

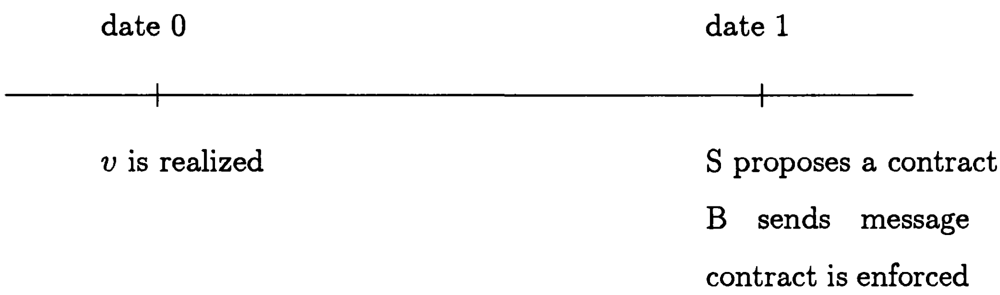
Consider a buyer-seller model in which a seller has one unit of an indivisible good for sale. The seller's production cost is fixed and normalized to 0. The buyer's valuation for the good is randomly drawn according to some distribution function $F(\cdot)$ with continuous, strictly positive density $f(\cdot)$ on a closed interval $V = [\underline{v}, \overline{v}]$. I further assume that $\underline{v} > 0$, so that trade is beneficial for all possible realizations of the buyer's value. If parties agree to exchange the good for a price p, payoffs to the buyer and the seller respectively are given by

$$\begin{aligned} u^b(v) &= v - p \\ u^s &= p, \end{aligned}$$

hence, parties are risk neutral.

Assume that the seller has all the bargaining power in the relationship, that is, he can choose the price at which he is willing to sell the good. If he is informed about the buyer's valuation, he will ask a price of v. The buyer being indifferent will accept.

If the seller is uninformed about the buyer's type, there is some potential for inefficiency due to the asymmetry of information. As a reference point I will first describe the standard second best contract under the assumption that the seller is bound by his contractual offer. That is, contracts cannot be renegotiated. Such a contract will typically entail some ex-post inefficiency which it would be in the common interest to renegotiate. Nevertheless, from an ex-ante viewpoint the seller weighs this inefficiency against the lower price he would have to accept in order to increase efficiency and chooses the contract that maximizes his own payoff. The problem is a standard contract design problem. For later reference I include a time line at this point:



4.2.1 The Benchmark: Full Commitment

From the revelation principle the seller can restrict attention to a contract in which the buyer announces his type v. The contract specifies a probability of trading the good together with a payment from buyer to seller conditional on the buyer's announcement. Call these $\beta(v)$ and $p(v)$. The seller maximizes his expected payoff subject to the relevant incentive and individual rationality constraints for all buyer types. That is, the optimal contract $(\beta^*(v),\ p^*(v))_{v\in V}$ for the seller is given by the solution to the program

$$\max_{(\beta(v),\ p(v))_{v\in V}} \int_V p(v)\,dF(v) \tag{4.1}$$

$$\begin{aligned} \beta(v)v - p(v) &\geq \beta(v')v - p(v') && \forall v,\ v' \in V \quad \text{(IC)} \\ \beta(v)v - p(v) &\geq 0 && \forall v \in V \quad\ \ \text{(IR)} \end{aligned}$$

The solution to this problem is a simple cut-off level v^*[2], such that

$$(\beta^*(v),\ p^*(v)) = \begin{cases} (1,\ v^*) & v \geq v^* \\ (0,0) & \text{otherwise.} \end{cases} \tag{4.2}$$

[2]See for example Fudenberg and Tirole (1991), chp 7, Section 7.3.

The level v^* solves

$$\max_q \; q(1 - F(q)),$$

which implies a first-order condition

$$1 - F(v^*) - v^* f(v^*) = 0. \tag{4.3}$$

For v^* to be uniquely defined through (4.3), assume that the seller's expected surplus $S(q) := q(1 - F(q))$ is strictly concave in q. This offer is not first-best if v^* lies in the interior of V because the good will not be sold to low buyer types. For there to be an inefficiency created by the asymmetric information I introduce the following assumption

Assumption 4.1 $v^* > \underline{v}$

4.2.2 Renegotiation

Imagine now, that after the prescriptions of the contract have been carried out, there is time for the seller to make a further offer to the buyer. If the good was not exchanged under the contract ($\beta(v) < 1$), the seller can make a second proposal to the buyer[3]. In particular, assume that this offer is made after randomization, in the case in which $0 < \beta(v) < 1$, has taken place[4]. The time line is as follows:

date 0	date 1	date 2
v is realized	S proposes a contract B sends message, contract enforced	renegotiation, final offer

[3]Because $\underline{v} > 0$ the seller will never offer to *buy* the good back from the buyer.

[4]If this is not the case, the seller has to take the buyer's random participation constraint $\beta(v)v - p(v)$ into account when designing the renegotiation offer.

If there is a contract in place which includes an exchange of messages at stage 1, the seller obtains additional information about the buyer's type and can use this information to tailor his renegotiation offer. The whole game becomes a signalling game in which the informed party (buyer) signals his type to the uninformed party (seller) through his strategy at stage 1. In this setting it is therefore impossible to employ the revelation principle in order to analyze the set of contracts.

Instead, a contract defines more generally a measurable message space (M, S), where M is a set of messages and S is a σ-algebra of its subsets[5], together with an outcome function (β, p), where $\beta : M \to [0,1]$ and $p : M \to R$ are real-valued functions on M. If message m is sent by the buyer, $\beta(m)$ is the probability with which the good is contractually traded and $p(m)$ denotes the price that the buyer pays. A possibly mixed strategy equilibrium for the buyer parametrized by $(\mu_v)_{v \in V}$ is considered, where for all buyer types v, μ_v defines a probability measure on M. So, $\mu_v(m)$ denotes the probability with which type v sends message m in equilibrium. Define $V_m := [\underline{v}_m, \overline{v}_m]$, where $\underline{v}_m$ $(\overline{v}_m)$ is the infimum (supremum) of all buyer type v with $\mu_v(m) > 0$.

Note first, that in this setting the seller can reach the same utility as in the game in which commitment is possible. Renegotiation does not restrict his opportunity set because he can simply refrain from proposing a contract at stage 1[6]. The second-best outcome is implemented at stage 2 through the 'renegotiation' offer v^*. On the other hand, he cannot do any better than in the game with commitment because contract plus renegotiation can be viewed as a two stage mechanism with commitment after the second stage. From the revelation principle, it is known that any outcome that can be achieved with such a sequential procedure can be reached through a simple revelation mechanism. The utility the seller obtains in his most preferred revelation mechanism therefore constitutes an upper bound on the payoffs he can expect in

[5] S is assumed to contain the elements m of M itself.

[6] Alternatively, G can be constructed by setting $M = \{m\}$, $\beta(m) = 0$, $p(m) = 0$, $\mu_v(m) = 1$ for all v.

any two stage mechanism, and in particular in the mechanism+renegotiation game studied here.

4.2.3 Contracting Cost

To introduce some friction in the model between the two different dates at which offers are made, I want to assume that each time the seller makes an offer to the buyer he has to pay a small fee. Offers are made at date 1 and 2, which leads us to distinguish two different costs. Call the cost of the first offer ε_1, the cost of the second ε_2[7].

The first fee ε_1 can be regarded as the cost of a phone call or the legal cost of drawing up a contract. The second type of cost ε_2 can be interpreted as the opportunity cost of time spent in a meeting between the two parties when they meet for the bargaining round. It can also be viewed as the cost of delayed trade. When trade happens through renegotiation rather than through the initial contract the good is exchanged at a later stage in the game and some additional cost might be incurred. Modelling ε_2 as a fixed cost rather than a proportional cost such as a common discount factor for example, has the advantage that in the former case the analysis of the incentive constraints for the stage 1 contract is very clear-cut and allows us to derive a very strong result, see Proposition 4.1.

With this change, a friction between the different dates of the model is introduced. Contracting at date 1 is different from bargaining at date 2 and it is not obvious how big the advantage is of one action over the other. More precisely, when comparing the null contract to a more complex contract the seller faces the following trade-off: Not writing a contract at date 1 and proceeding directly to the final offer at date 2 saves on the initial contracting costs ε_1 but comes at the cost of paying ε_2 for the final stage offer. Roughly, the seller has to compare ε_1 with ε_2.

In what follows, we will be interested in the case in which $\varepsilon_1 < \varepsilon_2$, so bargaining

[7] ε_1 and ε_2 are positive.

at the deadline is more costly. It is for example more likely that bargaining ends without a satisfactory conclusion simply because time is up. If the deadline is some official deadline as in an internet auction for example, costs are increased because of congestion, phone lines are more likely to be blocked and there is an increased probability that the final offer does not even reach the receiver. Similarly, legal cost could be increased because parties are more eager to reach a conclusion and lawyers can exploit their superior position by raising their fees. Another interpretation is that the seller has to pay a cost to store the good from one period to the other.

4.3 Contracting

Given these definitions, the analysis proceeds by backward induction. I solve for the Perfect Bayesian Nash Equilibrium of the game. A strategy for the seller consists of a contract offer $G = [(M, S), (\beta, p)]$[8] at stage 1 and a set of renegotiation offers $(q(m))_{m \in M}$ at stage 2. A strategy for the buyer consists of a probability distribution $(\mu_v)_{v \in V}$ on M at stage 1 and a decision to either accept or reject the seller's renegotiation offer $q(m)$ at stage 2.

4.3.1 Renegotiation Stage

First, I solve the game by setting ε_2 equal to 0. To derive the buyer's response to a proposed price offer $q(m)$ by the seller is straightforward. The buyer will accept this offer if and only if $v \geq q(m)$. The seller's optimal renegotiation offer is therefore given by a similar expression as (4.2), where the cut-off level is found by using the seller's posterior beliefs about the buyer's type. The analysis depends on the contract put into place and the message sent by the buyer at date 1. For each

[8]For simplicity, I assume here that the seller offers only contracts that are accepted by the buyer, i.e. contracts that fulfill the buyer's participation constraints.

message m the optimal renegotiation price offer $q(m)$ is found by solving

$$\max_{q \in V_m} q(1 - F(q \mid m)), \tag{4.4}$$

where $F(q \mid m)$ denotes the seller's updated beliefs about the buyer's value[9][10]. With a slight abuse of notation set

$$\mu(m) := \int_{\underline{v}}^{\overline{v}} \mu_v(m)\, dF(v). \tag{4.5}$$

The seller's beliefs can then be stated formally using Bayes' Rule

$$F(q \mid m) = \frac{1}{\mu(m)} \int_{\underline{v}}^{q} \mu_v(m)\, dF(v). \tag{4.6}$$

For the integral in (4.6) to exist, $\mu_v(m)$ must be a measurable function with respect to the measure $F(\cdot)$ on V. Remark that $q(m)$, the optimal renegotiation price after message m, depends on the equilibrium strategy $(\mu_v)_{v \in V}$ of the buyer. If for a given m the function $(\mu_v(m))_{v \in V}$ is continuous in v, the first order condition of the maximization problem in (4.4) is obtained by substituting (4.6) into (4.4) and taking the derivative with respect to $q(m)$:

$$\int_{q(m)}^{\overline{v}} \mu_v(m)\, dF(v) - q(m) f(q(m)) \mu_{q(m)}(m) = 0. \tag{4.7}$$

There is no need to consider corner solutions: the constraint in (4.7) is always a strict equality constraint. For all messages m that are sent in equilibrium by a set of buyer types with positive mass, $q(m) = \underline{v}_m$ is never optimal. Also, if $q(m) = \overline{v}_m$, given the definition of $\overline{v}_m$, the seller's objective function cannot be increasing at that point.

Turning now to the question of contracting cost, an additional assumption regarding the renegotiation cost ε_2 is needed. If ε_2 is very high and discourages the

[9]For $\beta(m) = 1$, the seller's 'renegotiation' offer is not determined. In this case, $q(m)$ is the unique limit when $\beta(m) \to 1$.

[10]If a message m is sent with probablity 0 in equilibrium by all buyer types, i.e. $\mu_v(m) = 0$ for all v, it is eliminated from the message space. This implies that the seller's beliefs are always determined by Baye's Rule and there is no problem with multiplicity of equilibria due to the freedom of choosing out-of-equilibrium beliefs.

seller from making a further offer at date 2, this additional cost could serve as a form of commitment device. In order to ensure that this is not the case and renegotiation is an issue, the following assumption is needed:

Assumption 4.2 $\varepsilon_2 \leq \underline{v}$

Assumption 4.2 guarantees that even for the most pessimistic beliefs about the buyer's type the seller prefers to trade and pay the fee ε_2 at stage 2 rather than walk away from trade. Given Assumption 4.2, the analysis of the equilibrium behavior of seller and buyer in the last stage of the game is exactly as detailed above.

4.3.2 Message Sending Stage

Having solved for stage 2 of the game, we now turn to the analysis of stage 1. In choosing his equilibrium strategy $(\mu_v)_{\in V}$, a type v buyer solves the following maximization problem

$$\max_{\mu_v'} \int_m \mu_v'(\beta(m)v - p(m) + (1-\beta(m))\max[v - q'(m), 0])\, dm, \tag{4.8}$$

where $q'(m)$ is the seller's renegotiation offer, which depends on μ_v'.

Take an equilibrium strategy of the buyer $(\mu_v)_{\in V}$ that maximizes (4.8) and a resulting set of renegotiation prices $(q(m))_{m\in M}$. The maximization procedure in (4.8) imply constraints for the contract that the seller must have proposed[11]. Namely, the buyer must be indifferent between any message in the support of his equilibrium strategy and he must weakly prefer such a message to any message which is not in the support. Importantly, this depends again on the seller's stage 2 renegotiation offer. Although this seems to imply a quite complicated interdependency it turns out that a contract is almost fully defined by these constraints. Furthermore, it must be of a surprisingly simple form. Either it must involve complete pooling or it can involve only a very restricted amount of separation.

[11]Contracting costs play no role in the analysis of this stage because they are borne entirely by the seller.

Because the seller's renegotiation offer determines the type of contract that is feasible at stage 1, we separate the two cases of pooling and separating. The following two possibilities span the whole set of renegotiation offers.

Condition 4.1 *The renegotiation offer is independent of the buyer's equilibrium strategy at stage* 1, *i.e.*

$$q(m) \equiv q \quad \textit{for all } m \in M.$$

Condition 4.2 *The renegotiation offer is dependent on the buyer's equilibrium strategy at stage* 1. *That is, there exists an equilibrium* $(\mu_v)_{v \in V}$ *of the stage* 1 *contract, such that*

$$q(m) \neq q(m') \quad \textit{for some } m,\ m' \in M$$

where m *and* m' *are in the support of* $(\mu_v)_{v \in V}$ *for some* v.

I first characterize the feasible set of contracts given Condition 4.1. The following Lemma provides the first step in the characterization.

Lemma 4.1 *Under Condition 4.1,* $q \equiv v^*$.

Proof.

Consider the seller's maximization problem in (4.4), which implies that

$$q \int_q^{\overline{v}} \mu_v(m)\, dF(v) \geq r \int_r^{\overline{v}} \mu_v(m)\, dF(v) \qquad \forall m \in M \text{ and } \forall r \in V.$$

Taking expectation over M, this expression simplifies to

$$q(1 - F(q)) \geq r(1 - F(r)) \qquad \forall r \in V.$$

But this problem has as a unique solution the cut-off level v^*, and therefore $q = v^*$.

■

The main result concerning the stage 1 contract under Condition 4.1 is given in the proposition below:

Proposition 4.1 *Full Pooling: Under Condition 4.1*

$$\beta(m) \equiv \beta \in [0,1] \qquad p(m) \equiv p \in R,$$

for all messages $m \in M$, for which $F(\{v : \mu_v(m) > 0\}) > 0$, except for a set of messages of zero measure.

Proof.

First, I show that for all messages m, buyer types above and below the unique renegotiation price v^* send m with positive probability in equilibrium. That is, I prove that for all m, there are types v_1 and v_2, $v_1 \leq v^* < v_2$ with $\mu_{v_1}(m) > 0$ and $\mu_{v_2}(m) > 0$.

First, if there was no such $v_2 > v^*$, the seller would optimally decrease his renegotiation price to capture a set of buyers with valuations below v^* who would accept the new price. This proves the first part.

Second, assume that for some m there is no such v_1 as above. Divide the message space M into two disjoint subsets $\underline{M}$ and $\overline{M}$. $\underline{M}$ is defined such that for all $m \in \underline{M}$, buyer types below and above v^* send message m. In particular, there exists $v < v^*$ with $\mu_v(m) > 0$. $\overline{M}$ is defined such that for all $m \in \overline{M}$, only buyer types above v^* send m, i.e. $\mu_v(m) = 0$ for all $v < v^*$. The aim is to show that $\overline{M}$ is empty or has measure 0. Assume the contrary.

Because v^* is the seller's optimal renegotiation offer after every message m, it must be that

$$v^* \int_{v^*}^{\overline{v}} \mu_v(m)\, dF(v) \geq (v^* - \alpha) \int_{v^*-\alpha}^{\overline{v}} \mu_v(m)\, dF(v) \qquad \forall m \in M,\ \alpha > 0. \tag{4.9}$$

Call $x := \int_{v^*}^{\overline{v}} \int_{\underline{M}} \mu_v(dm)\, dF(v)$. Because a set with positive mass of buyer types above v^* must send messages in $\overline{M}$ and $\overline{M}$ is assumed to have a positive mass in M, $\int_{\underline{M}} \mu_v(dm) < 1$ for those types. It follows that $x < 1 - F(v^*)$. By taking expectation over $\underline{M}$ in (4.9) and changing the order of integration we obtain

$$v^* x \geq (v^* - \alpha) \left(x + \int_{v^*-\alpha}^{v^*} \int_{\underline{M}} \mu_v(dm)\, dF(v) \right) \qquad \forall \alpha > 0,$$

Given the definition of $\underline{M}$, this expression is equivalent to

$$v^*x \geq (v^* - \alpha)\,(x + F(v^*) - F(v^* - \alpha)) \qquad \forall \alpha > 0,$$

which is equivalent to

$$x \geq v^*G(\alpha) - (F(v^*) - F(v^* - \alpha)),$$

where $G(\alpha) := \frac{F(v^*)-F(v^*-\alpha)}{\alpha}$. Now, as α approaches 0, this inequality is violated. The first part of the right-hand side approaches $v^*f(v^*)$, which is, substituting for the definition of v^*, equal to $1 - F(v^*)$, whereas the second part approaches 0. Therefore, the right-hand side approaches $1 - F(v^*) > x$.

This shows that for all m there are buyer types v_1 and v_2, with $v_1 \leq v^* < v_2$, who both play m with positive probability in equilibrium. Take any two messages m and m' with such buyer types $v_1 \leq v^* < v_2$ and $v_1' \leq v^* < v_2'$. In equilibrium, type v_1 must weakly prefer to send message m over m' and type v_2' must weakly prefer to send m' over m which leads to the following constraints:

$$\beta(m)v_1 - p(m) \geq \beta(m')v_1 - p(m') \tag{4.10}$$

and

$$\beta(m')v_2' - p(m') + (1 - \beta(m'))(v_2' - v^*) \geq \beta(m)v_2' - p(m) + (1 - \beta(m))(v_2' - v^*). \tag{4.11}$$

(4.10) and (4.11) together imply the following:

$$(\beta(m) - \beta(m'))v_1 \geq p(m) - p(m') \geq (\beta(m) - \beta(m'))v^*,$$

i.e. $\beta(m) \leq \beta(m')$. The same is true for buyer types v_1' and v_2, where the roles of m and m' are reversed implying similar constraints as (4.10) and (4.11) and thus

$$(\beta(m) - \beta(m'))v^* \geq p(m) - p(m') \geq (\beta(m) - \beta(m'))v_1'.$$

Thus, it must be that $\beta(m) \geq \beta(m')$, which finally implies that $\beta(m) = \beta(m')$ and therefore $p(m) = p(m')$. ■

Proposition 4.1 says that, if the renegotiation offer is independent of the buyer's message under the contract, the contract itself must be independent of his message, i.e. it must be a pooling contract.

The proof proceeds in two steps. First one shows that under Condition 4.1, every message must be sent by buyer types above and below the unique renegotiation price v^*. This is easy to see for buyer types above v^* since the seller expects to capture a positive surplus with his renegotiation offer. To see that this is also the case for buyer types below, assume that there is a message $\hat{m}$ for which this is not the case. Since v^* is the optimal renegotiation offer for the seller knowing m for all m, it must also be the optimal offer knowing $M \setminus \hat{m}$. In words, the seller sets v^*, when he faces the entire pool of buyer types except a set of types $v > v^*$, namely those who send $\hat{m}$. But this is in contradiction with the definition of v^*, which is defined as the renegotiation offer when the seller faces all buyer types. The second step is to show that this implies that the outcome function must be independent of the message. Incentive compatibility necessitates the familiar monotonicity condition. Since here, types below and above v^* send a message m, the monotonicity condition must hold in both directions.

Finally, the following proposition deals with the case when the renegotiation offers differ according to the messages sent at stage 1, i.e. when Assumption 4.2 holds. It makes use of the following definition.

Definition 4.1 *Define q^* as the lowest renegotiation price, that is,*

$$q^* = \min_m q(m). \tag{4.12}$$

Define $M^* \subseteq M$ as the set of messages which result in renegotiation price q^*, that is,

$$M^* := \{m : q(m) = q^*\}.$$

Now, we are ready to state the following proposition and corollary.

Proposition 4.2 *Separation: Under Condition 4.2,*

$$\beta(m) \equiv 1 \qquad p(m) \equiv p(m^*) + (1 - \beta(m^*))q^*$$

for all messages $m \notin M^*$, for which $F(\{v : \mu_v(m) > 0\}) > 0$, and for all messages $m^* \in M^*$.

Proof. Consider the two renegotiation offers $q^* < q(m)$ with corresponding messages $m^* \in M^*$ and $m \notin M^*$. Then, there must exist a buyer type v_1, with $q^* \leq v_1 < q(m)$, who plays m^* in equilibrium with positive probability and a type v_2, with $q(m) \leq v_2$, who plays m with positive probability. To see this, assume first that there is no such type v_2. But then, the seller obtains an expected payoff of 0 from his renegotiation price $q(m)$, whereas he will obtain a strictly positive expected payoff if he decreases his offer sufficiently. Next, if there is no v_1 in between the two renegotiation prices q^* and $q(m)$, such that $\mu_{v_1}(m^*) > 0$, the seller can raise q^* to $q(m)$ without affecting the probability with which his offer is accepted while increasing his expected payment. Consider now two values v_1 and v_2 as above. Given a contract $G = (M, \beta, p)$ with an equilibrium probability measure $(\mu_v)_{v \in V}$ on M, the incentive constraints for the two buyer types v_1 and v_2 are

$$\beta(m^*)\, v_1 - p(m^*) + (1 - \beta(m^*))(v_1 - q^*) \geq \beta(m)\, v_1 - p(m)$$

and

$$\beta(m)\, v_2 - p(m) + (1 - \beta(m))(v_2 - q(m)) \geq \beta(m^*) v_2 - p(m^*) + (1 - \beta(m^*))(v_2 - q^*)$$

Taken together they imply

$$(1 - \beta(m))v_1 - (1 - \beta(m^*))q^* \geq p(m^*) - p(m) \geq (1 - \beta(m))q(m) - (1 - \beta(m^*))q^*$$

and thus $\beta(m) = 1$ and $p(m) = p(m^*) + (1 - \beta(m^*))q^*$.

■

Proposition 4.2 shows that even with differentiated renegotiation prices, only a small level of screening is achieved. It allows the seller to separate buyer types into a maximum of two groups according to whether they send messages in M^* or not. For those who send a message that leads to a renegotiation price above the minimum price q^* the contract is completely determined. It must enforce trade at a fixed price, i.e. the outcome must be $[1,p]$. To see this intuitively, observe that any buyer type above q^* can ensure himself $[1,p]$ by sending a message m^* in M^*. Call the outcome after a message $m \notin M^*$, x. Now, x must be weakly preferred to $[1,p]$ by those buyer types who send a message not in M^*. On the other hand, a type above q^* who sends m^* in M^* also obtains $[1,p]$ and must therefore weakly prefer it to x. This shows that $x = [1,p]$.

Finally, the outcome function after messages in M^* need to be determined. The following corollary provides a characterization

Corollary 4.1 *Proposition 4.2 implies the following for buyer types $v < q^*$.*

(i) If there exists a buyer type $v < q^*$, who sends message $m \notin M^*$ in equilibrium, then also $\beta(m^*) = 1$ for all $m^* \in M^*$.

(ii) All buyer types $v < q^*$ send message $m^* \in M^*$, for which $m^* = \arg\min_{m'^*} \beta(m'^*)$

Proof.

To see (i) assume that there is a buyer type $v < q^*$ who sends message $m \notin M^*$ in equilibrium, that is $\mu_v(m) > 0$. For such a strategy to be optimal for type v, sending message m must weakly dominate sending some message $m^* \in M^*$. Using Proposition 4.2, this implies that

$$v - p(m^*) - (1 - \beta(m^*))q^* \geq \beta(m^*)v - p(m^*),$$

hence

$$(1 - \beta(m^*))(v - q^*) \geq 0,$$

which is only guaranteed if $\beta(m^*) = 1$.

To see (ii), observe that from Proposition 4.2 it follows that for two messages $m^*, m'^* \in M^*$

$$\beta(m^*)q^* - p(m^*) = \beta(m'^*)q^* - p(m'^*).$$

But then a buyer type $v \leq q^*$ obtains

$$\begin{aligned}\beta(m^*)v - p(m^*) &= \beta(m^*)v - p(m'^*) - (\beta(m^*) - \beta(m'^*))q^* \\ &= \beta(m'^*)v - p(m'^*) + (\beta(m^*) - \beta(m'^*))(v - q^*).\end{aligned}$$

So he will send $m^* \in M^*$ if and only if $\beta(m^*) \leq \beta(m'^*)$ for all $m'^* \in M^*$.

■

Propositions 4.1 and 4.2 highlight the limited scope of 'early' contracting in this set-up when contracts are vulnerable to renegotiation. Basically, with renegotiation a contract has only one degree of freedom, namely to fix one level of trade and one price. This is in sharp contrast to the limitation that incentive constraints impose on contracts in the case when renegotiation can be prevented. Then, for two announcements v and v' to be incentive compatible we need

$$(\beta(v) - \beta(v'))v \geq p(v) - p(v') \geq (\beta(v) - \beta(v'))v',$$

and therefore

$$sign(\beta(v) - \beta(v')) = sign(v - v').$$

The decision rule needs to be monotonic but β can take as many values as v and also the price is not constrained to two levels only. Because the set of possible contracts is strongly restricted we can expect that contracting is much less valuable with renegotiation than without.

Finally, Propositions 4.1 and 4.2 go beyond what is needed in this section. In fact, the seller, to maximize his payoff, only needs to consider single price offers at stage 1. Nevertheless, most prior papers on durable goods monopoly and bargaining with asymmetric information do not derive the incentive constraints on a general mechanism in a set-up without commitment. Mostly, a given bargaining game is assumed[12]. The above analysis allows us to solve the situation for other objective functions than the seller's payoff and allows us to incorporate the above simple model into a more complicated one. The latter will be especially useful in the sequential screening model of the following chapter.

[12] An exception is Hart and Tirole (1988).

4.3.3 Initial Contracting Stage

We are ultimately interested in the contract that the seller chooses within the set of implementable contracts, i.e. among the ones that satisfy the conditions of either Proposition 4.1 or 4.2.

Formally the seller's maximization problem in (4.1) must be adapted to include the possibility of renegotiation and the two contracting cost. The seller's problem is

$$\max_{(\beta(m),\, p(m))_{m\in M}} \int_M \left(p(m)\mu(m) + q(m)(1-\beta(m)) \int_{q(m)}^{\overline{v}} \mu_v(m)\, dF(v) \right) dm \tag{4.13}$$

$$-\varepsilon_1 I^C - \varepsilon_2 \int_M (1-\beta(m))\mu(m)\, dm \tag{4.14}$$

subject to the constraints in either Proposition 4.1 or 4.2

subject to the buyer's individual rationality constraints

The first term in this expression is the total expected price the seller receives for the good under the contract and the renegotiation offer. The contracting costs ε_1 are only incurred if the seller makes a first stage offer. Then, I^C is the indicator function, where $I^C = 1$ if a contract is signed initially and $I^C = 0$ if no contract is signed. Finally, in the last term, the expression under the integration operator is the probability that trade does not occur under the contract after message m. The seller computes the expectation of 'no trade' over M.

Pooling Contracts. We first investigate the problem using the constraints detailed in Proposition 4.1. It is easy to see that among the contracts that satisfy these conditions the optimal contract for the seller is the null-contract, i.e. $\beta = 0$ and consequently $p = 0$. The seller's maximization problem in (4.13) is simply

$$\max_{\beta,\, p} \; p + v^*(1-\beta)(1-F(v^*)) - \varepsilon_1 I^C - \varepsilon_2(1-\beta) \tag{4.15}$$

s.t.

$$\beta v - p \geq 0 \qquad \forall v.$$

Because the individual rationality constraints must be met for all buyer types, the seller optimally sets $p = \beta \underline{v}$. Therefore, 4.15 becomes

$$\max_{\beta} \beta \underline{v} + (1-\beta)[v^*(1-F(v^*)) - \varepsilon_2] - \varepsilon_1 I^C$$

This is a linear expression in β and is therefore maximized by setting β equal to 1 or 0. The following assumption guarantees that, independent of the size of the initial contracting cost ε_1, $\beta = 1$ is never optimal.

Assumption 4.3 $\varepsilon_2 \leq v^*(1-F(v^*)) - \underline{v}$

Because it is impossible with such a contract to separate the buyer types above and below the cut-off level v^*, the seller can either sell to everybody for a low price at stage 1 or wait until stage 2 and sell only to a proportion of buyers. If waiting is not too costly, i.e. ε_2 satisfies the above assumption, the neutral stance of 'doing-nothing' is strictly optimal.

Separating Contracts. Next, we turn to the constraints detailed in Proposition 4.2. Define $m^* := \arg\min_{m'^* \in M^*} \beta(m'^*)$. Write β^* and p^* for the probability of trade and the price after message m^*. Because of the constraints imposed on a contract in Proposition 4.2 and Corollary 4.1, part (*ii*), there is no loss in generality in restricting attention to so called *simple contracts*. The message space of a *simple contract* contains only two messages, namely m^* and one other generic message m. After message m is sent by the buyer, the contract enforces trade of the good at a price $p = p^* + (1-\beta^*)q^*$ as detailed in Proposition 4.2. Denote by q the renegotiation price after message m.[13]

Assumption 4.3 implies that the null contract strictly dominates a separating contract in which buyer types below q^* send message m. This follows from Corollary 4.1, part (*i*). Therefore, the only type of contract that the seller might prefer has only equilibria in which all $v < q^*$ send message m^*. This is summarized in the following figure:

[13]Remark that this price plays no role in the following analysis because there is no scope for renegotiation after message m.

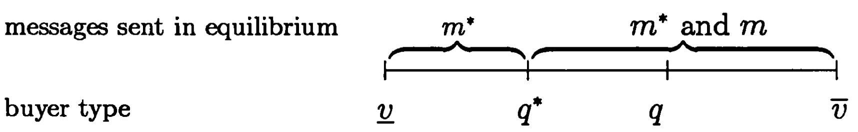


Figure 1

The seller's maximization problem (4.13) reduces to

$$\max_{\beta^*,\, p^*} \; p^* + (1-\beta^*)(1-F(q^*))q^* - \varepsilon_1 I^C - \varepsilon_2(1-\beta^*)\mu(m^*) \tag{4.16}$$

s.t.

$$\begin{aligned} v - p^* - (1-\beta^*)q^* &\geq 0 \qquad \forall v > q^* \\ \beta^* v - p^* &\geq 0 \qquad \forall v \leq q^*, \end{aligned}$$

where $\mu(m^*)$ was defined in (4.5). The second set of individual rationality constraints in (4.16) is the most stringent and it follows that $p^* = \beta^*\underline{v}$. The buyer's problem can be simplified even further to

$$\max_{\beta^*} \; \beta^*\underline{v} + (1-\beta^*)[(1-F(q^*))q^* - \mu(m^*)\varepsilon_2] - \varepsilon_1 I^C.$$

This expression is a linear function in the choice parameter β^*. As shown above, setting $\beta^* = 1$ leaves the seller with a suboptimal payoff of $\underline{v} - \varepsilon_1$. When $\beta^* = 0$ (which implies $p^* = 0$) on the other hand, the seller obtains $q^*(1-F(q^*)) - \mu(m^*)\varepsilon_2 - \varepsilon_1$. This payoff has to be compared with the payoff $v^*(1-F(v^*)) - \varepsilon_2$ that the seller can secure by ignoring date 1, i.e. by not writing a contract initially, and by making his optimal static offer at date 2. From our assumption about the uniqueness of the optimal offer v^*, $q^*(1-F(q^*))$ is strictly smaller than $v^*(1-F(v^*))$ for all $q^* \neq v^*$. It can be shown that $q^* = v^*$ is not an optimal response for the buyer after having observed message m^*, given that the seller mixes as detailed in Figure 1 above. In fact, q^* must be strictly smaller than v^*. The proof proceeds along the same lines a the proof of Proposition 4.2 and is relegated to Appendix B.

To see the intuition for this result consider the following reasoning. Call the set of buyer types who send message m with positive probability in equilibrium $V(m)$.

Formally, $V(m) := \{v : \mu_v(m) > 0\}$. If $V(m)$ has measure 0, i.e. if $F(V(m)) = 0$, the only possible renegotiation offer given the definition of v^* is $q^* = v^*$. Neglecting initial contracting cost, contracts of this form are equivalent to the null contract because all parties receive the same overall payoff as under the null contract. The larger the measure of buyer types that have m in the support of their equilibrium strategy, i.e. the larger $F(V(m))$, the smaller must be q^* and the larger is the disutility of the contract to the seller modulo the contracting costs.

To gain an intuition about the relationship between q^* and the measure of $V(m)$, consider a particular equilibrium of the set of equilibria detailed in Figure 1:

Example 4 *Partition Equilibrium*[14]
There is a buyer type $\hat{v} > q^*$, such that the equilibrium behavior of the buyer at stage 2 is given by:

$$\mu_v(m) = 1 \quad (\mu_v(m^*) = 0) \quad v > \hat{v}$$
$$\mu_v(m) = 0 \quad (\mu_v(m^*) = 1) \quad v \leq \hat{v}.$$

This equilibrium partitions the set of buyer types into two subintervals, such that the seller trades contractually with all high valuation buyers in $V(m) = \{v : v > \hat{v}\}$ and offers to trade with the low valuation buyers only at renegotiation. The measure of the set of buyer types for whom trade occurs under the contract is given by $F(V(m)) = 1 - F(\hat{v})$. For q^* to be a consistent renegotiation offer given the buyer's equilibrium strategy, q^* must maximize

$$\tilde{q}(1 - F(\tilde{q} \mid m^*)),$$

[14]In fact, any other equilibrium of a separating contract is equivalent to a partition equilibrium in the sense that the seller's and buyer's payoffs are identical. Indeed, for a given equilibrium of such a contract and a minimal renegotiation price q^*, define $\hat{v}$ by

$$F(\hat{v}) := F(q^*) + q^* f(q^*).$$

Because $q^* < v^*$ and $S(q)$ is strictly concave, $\hat{v}$ is well defined. It is easy to verify that parties' payoffs are the same in both equilibria.

where

$$F(\tilde{q} \mid m^*) = \begin{cases} 1 & \tilde{q} > \hat{v} \\ \frac{F(\tilde{q})}{F(\hat{v})} & \tilde{q} \leq \hat{v} \end{cases} .$$

The first-order-condition of this problem is

$$F(\hat{v}) - F(q^*) - q^* f(q^*) = 0. \tag{4.17}$$

Implicit differentiation yields

$$\frac{dq^*}{d\hat{v}} = -\frac{f(\hat{v})}{-2f(q^*) - q^* f'(q^*)}. \tag{4.18}$$

The denominator of this expression is the second-order-condition of the above maximization problem. The whole expression must therefore be nonnegative. That is, the optimal renegotiation offer q^* is weakly de*creasing in the measure $F(\hat{v})$ of buyer types for whom trade occurs under the contract.*

If the difference between the null contract and a more complex contract is represented by the measure of the set of buyer types for whom trade occurs under the contract, i.e. by the measure $F(V(m))$ of types who send message m in equilibrium, one can state the result in Example 4 in the following form:

The closer a contract is to the null-contract, the better it performs. The null contract strictly dominates any complicated contract. Nevertheless, its outcome can be approximated arbitrarily closely by a sequence of contracts that converges to the null contract. Convergence means that the best equilibrium for the seller in such a sequence of contracts has $F(V(m)) \to 0$.

If the seller's strategy is given by the partition equilibrium of Example 4, the renegotiation cost ε_2 are only incurred for buyer types below $\hat{v}$. A simple contract with partition point $\hat{v}$ yields a payoff of

$$\Pi^{SC}(\varepsilon_1, \varepsilon_2) = q^*(\hat{v})(1 - F(q^*(\hat{v}))) - F(\hat{v})\varepsilon_2 - \varepsilon_1 \tag{4.19}$$

to the seller. On the other hand, the null contract yields

$$\Pi^{NC}(\varepsilon_2) = v^*(1 - F(v^*)) - \varepsilon_2. \tag{4.20}$$

Comparing Π^{SC} in (4.19) with Π^{NC} in (4.20), the advantage of the null contract is that it saves on the initial contracting cost ε_1. A simple contract on the other hand saves on renegotiation cost ε_2, but comes at the disadvantage of lowering the overall price that the seller can ask for the good. Which of the two contracts the seller prefers obviously depends on the relative size of ε_2 and ε_1. Trivially, if the cost of writing a contract at the initial stage is very large compared to the cost of recontracting, the null contract will be preferred. In contrast, the seller prefers to write a contract at stage 1 if ε_1 is very small compared to ε_2.

Two more interesting points can be made. First, assume that initial contracting cost are 0 but that there exist a non negligible cost of recontracting, i.e. $\varepsilon_2 > 0$. It can be shown that, although the cost saving effect on ε_2 outweighs the information effect on price in a simple contract, these two effects have the same order of magnitude. Second, initial contracting cost of order of magnitude smaller than the recontracting cost ε_2 support the null contract as the strictly optimal contract.

Formally, define $\underline{\varepsilon}_1(\varepsilon_2)$ as the initial cost on contracting such that a simple contract yields the same payoff to the seller as the null contract

$$\underline{\varepsilon}_1(\varepsilon_2) := \varepsilon_2(1 - F(\hat{v})) - (v^*(1 - F(v^*)) - q^*(1 - F(q^*))). \tag{4.21}$$

Proposition 4.3 *(i) Neglecting initial contracting cost, benefit and cost of a simple contract are of the same order of magnitude*

$$\varepsilon_2(1 - F(\hat{v})) = O(v^*(1 - F(v^*)) - q^*(1 - F(q^*))).$$

(ii) The minimum cost on contracting $\underline{\varepsilon}_1(\varepsilon_2)$ that is needed to support the null contract as the strictly dominant contract, is of order of magnitude smaller than the recontracting cost ε_2. That is

$$\underline{\varepsilon}_1(\varepsilon_2) = o(\varepsilon_2).$$

Proof.

Following the assumption that in a simple contract the best equilibrium from the seller's viewpoint is played, $\hat{v}$ maximizes expression (4.19). It solves the first order condition

$$(1 - F(q^*))\frac{dq^*}{d\hat{v}} - q^* f(q^*)\frac{dq^*}{d\hat{v}} - \varepsilon_2 f(\hat{v}) = 0.$$

By using the definition of q^* in (4.17) and the expressions for $\frac{dq^*}{d\hat{v}}$ in (4.18) of Example 4, it is possible to simplify further

$$[1 - F(\hat{v}) + \varepsilon_2 S''(q^*)]\, f(\hat{v}) = 0, \tag{4.22}$$

where $S''(q)$ stands for the second derivative of the seller's surplus function $S(q) = q(1 - F(q))$. Expression (4.22) defines $\hat{v}$ implicitly as a function of the renegotiation cost ε_2. If $S(q)$ is a strictly concave function, $\hat{v}$ will lie in the interior of V.

To prove the statement in (i) we need to show that there exists a constant K, such that

$$\frac{\varepsilon_2(1 - F(\hat{v}))}{v^*(1 - F(v^*)) - q^*(1 - F(q^*))} \to K \qquad\qquad \text{as } \varepsilon_2 \to 0.$$

Both the numerator and the denominator tend to zero. We only need to show the latter which follows from $q^* \to v^*$. To see this, note that because $f(v) > 0$ for all v, expression (4.22) can be divided by $f(\hat{v})$. Then, because S'' is bounded, $\varepsilon_2 S'' \to 0$ for $\varepsilon_2 \to 0$ and therefore $\hat{v} \to \overline{v}$. Comparing (4.17) and (4.3), it follows immediately that q^* approaches v^*.

Using Hôpital's Rule:

$$\lim_{\varepsilon_2 \to 0} \frac{\varepsilon_2(1 - F(\hat{v}))}{q^*(1 - F(q^*)) - v^*(1 - F(v^*))} = \lim_{\varepsilon_2 \to 0} \frac{1 - F(\hat{v}) - \varepsilon_2 f(\hat{v})\frac{d\hat{v}}{d\varepsilon_2}}{-S'(q^*)\frac{dq^*}{d\hat{v}}\frac{d\hat{v}}{d\varepsilon_2}} \tag{4.23}$$

To obtain the rate of change of $\hat{v}$ when ε_2 changes, we use the Implicit Function Theorem and equation (4.22):

$$\frac{d\hat{v}}{d\varepsilon_2} = \frac{S''(q^*)}{f(\hat{v}) + \varepsilon_2 S'''(q^*)\frac{dq^*}{d\hat{v}}}. \tag{4.24}$$

Exploiting the expressions for $\frac{d\hat{v}}{d\varepsilon_2}$ in (4.24), $\frac{dq^*}{d\hat{v}}$ in (4.18) and (4.17), we see that the limes in (4.23) is equivalent to

$$\lim_{\varepsilon_2 \to 0} 1 + \varepsilon_2 \left(\frac{S'''}{S''} - S''\right).$$

The first part of Proposition 4.3 follows with $K = 1$ from the strict concavity of S and the assumption that all derivatives of S are bounded in V.

Next, part (*ii*) follows almost immediately. To prove it, we need to show that

$$\frac{\varepsilon_2(1 - F(\hat{v})) - (v^*(1 - F(v^*)) - q^*(1 - F(q^*)))}{\varepsilon_2} \rightarrow 0 \qquad \text{as } \varepsilon_2 \rightarrow 0. \tag{4.25}$$

First, note that $\underline{\varepsilon}_1(\varepsilon_2) > 0$ from the definition of $\hat{v}$. Next, the first term $\frac{\varepsilon_2(1-F(\hat{v}))}{\varepsilon_2}$ in (4.25) tends to 0 because $\hat{v}$ goes to $\overline{v}$. Then, it follows that the second term $\frac{v^*(1-F(v^*))-q^*(1-F(q^*))}{\varepsilon_2}$ in (4.25) must also tend to 0 because it is always positive but smaller than the first.

■

To gain an intuition for Proposition 4.3, consider the case when $F(\cdot)$ is the uniform distribution on $[\frac{1}{2}, \frac{3}{2}]$. Then, $v^* = \frac{3}{4}$, $\hat{v} = \frac{3}{2} - 2\varepsilon_2$ and $q^* = v^* - \varepsilon_2$.

With a simple contract and partition equilibrium indexed by $\hat{v}$ the savings on the renegotiation cost is $2\varepsilon_2^2$, whereas the loss to the seller due to a lower price is equal to ε_2^2. Therefore, benefit and cost of a contract are of same order of magnitude. Also, it follows that the boundary on initial contracting cost ε_1, such that the null contract is strictly preferred to all other contracts is $\underline{\varepsilon}_1(\varepsilon_2) = \varepsilon_2^2 = o(\varepsilon_2)$.

Intuitively, even if the cost ε_1 of writing a contract is smaller than the cost ε_2 of recontracting, the seller prefers to incur these higher cost because they come with commitment. Paying ε_1 at date 1 does not preclude the additional payment of ε_2 for at least some buyer types. Furthermore, through an initial contract the seller learns something about the buyer and will contract with the remaining buyer types date 2 for a price that lies below the optimum. Together these two forces imply that for very small levels of renegotiation costs ε_2 he is prepared to wait, even if they are higher than the initial contracting cost ε_1.

But the strength of Proposition 4.3 lies in the fact that it goes beyond a mere comparison of two different levels of contracting costs. It makes a strong point in favor of the null contract because the null contract is chosen by the seller even though the *order of magnitude of it's savings on contracting cost (ε_1)* is smaller than the *order of magnitude of it's loss due to higher bargaining cost (ε_2).* Models that have tried to explain incompleteness of contracts by evoking writing costs have been vulnerable to criticism. The difficulty to measure these costs empirically is one of the main arguments against such explanations. Proposition 4.3 is an example in which the absence of a contract is explained by writing costs without the need to resolve to a pure comparison of size.

Compare the obtained result to the result generated under full commitment. The second best contract yields a total surplus of $v^*(1 - F(v^*)) - \varepsilon_1$. If $\varepsilon_1 < \varepsilon_2$, this amount is greater than $v^*(1-F(v^*))-\varepsilon_2$, the total surplus that the null-contract plus renegotiation delivers under no commitment. Under these conditions renegotiation is harmful and the situation is in that respect similar to other models that consider renegotiation in dynamic contract environments.

Some parallels can be drawn between these results and the ones in Hart (1989). The author tries to explain strikes 'of reasonable length' by studying a bargaining model between a firm (uninformed) and its workers (informed). A first ingredient is that real time elapses between any two of the firm's offers and that this is costly because the firm is inactive during the strike. A second ingredient is that the firm faces a crunch at some point in time, that is, if the strike has continued up to a certain point, the firm's value decreases dramatically after that. It is shown, that these two forces can yield strikes of considerable length up to and beyond the crunch line. These results are very similar to what is shown in this paper, which takes them to the extreme. There is no discounting and the crunch line is in fact the end of time, i.e., the good's value decreases to zero. The implications are therefore also stronger. I find no contracting before the deadline, whereas there is some positive probability that a strike ends before the deadline in Hart's paper. Also, in his paper the type distribution is discrete and a strict 'waiting' result cannot be found with a discrete distribution.

Hart and Tirole (1988) study contracting in a dynamic durable goods monopoly model with two types. They derive optimal long term and short term contracts in the rental and sales version of the model with T periods. The difference between long term and short term contracts does not arise in a two period model and their sales model, whether with long term or short term contracts, is therefore very similar to the one in this chapter. The difference lies in the fact that in Hart and Tirole (1988), there is discounting. Also, in this chapter, a continuous type version is studied. Interestingly, as in the current chapter, the authors study the optimal long term contract, i.e., they do not restrict themselves to a given bargaining procedure.

4.4 Discussion and Conclusion

This chapter has taken a first step towards establishing strict dominance of the null contract over more complicated contracts. In a durable goods monopoly model, where the seller cannot commit not to renegotiate an initial contract offer, it is shown that he might strictly prefer not to make this offer at all.

Three ingredients are necessary for this result. First, no real time passes between the time at which the initial contract is carried out and the final renegotiation offer, i.e. there is no discounting. With discounting, there is always some amount of early contracting, i.e. trade, as can be seen in the earlier literature on the durable goods monopoly and in related models of bargaining with asymmetric information (see for example Stokey (1981), Gul, Sonnenschein, and Wilson (1986) and Fudenberg and Tirole (1983)).

Second, we need a continuous type setting. With continuous types, the bargaining offer varies continuously with the amount of information revealed beforehand through a contract. In contrast, in discrete type settings a small amount of information revelation, i.e. early contracting, does not necessarily result in a lower renegotiation offer (see for example Fudenberg and Tirole (1983)).

Finally, the fact that an early contract can only regulate the starting point of the final bargaining game and cannot already prescribe what is to be done at renegotiation is crucial. If this was possible, the seller would pay ε_1 for an initial contract specifying that the optimal static offer v^* is to be made *in the final bargaining game.* Such a contract, if feasible, is indeed preferable over the actual bargaining, which involves paying the higher costs ε_2.

I want to argue that such a contract is in contradiction with the assumption that contracts can be renegotiated. Namely, the way renegotiation is modelled in this thesis is to assume that any inefficiency in a contract is subject to renegotiation. Trying to include the renegotiation game into the contract fails, because the renegotiation game itself suffers from inefficiencies.

To understand this point, it is useful to picture the exact timing of the game. The bargaining game is such that parties must sit together at a table. Once they have sat down, the clock starts ticking and there is just time for one take-it-or-leave-it offer. The value of the transaction declines sharply after this offer and is then in fact smaller than the opportunity cost of the parties' time. They therefore prefer to leave the table without having concluded the trade. This bargaining situation cannot be recreated in a contract. On the one hand, a contract that simply mimics the outcome by specifying that the buyer can buy the good at the price v^* and that no follow-up offer will be made once this offer has been rejected, is vulnerable to renegotiation. On the other hand, a contract that tries to recreate the bargaining game by bringing parties together at the bargaining table, in addition to costing ε_1, suffers from the same ε_2 costs.

Finally, it would be interesting to model the bargaining game with an infinite horizon, in which payoffs are discounted by a factor δ. Take for example the one-sided offer game which is studied in Gul, Sonnenschein, and Wilson (1986). If $\delta = 0$, this is exactly the bargaining game assumed in this chapter. But for other discount factors, this game leads to bargaining over several periods in which the good is sold at a decreasing price to buyers with decreasing valuations. In such a model one could dispense with the exogenous bargaining cost ε_2. In fact, the closer δ is to 1, i.e. the more efficient is the bargaining game, the more costly it is from the seller's point of view. In this model, would an ex-ante contract increase his bargaining position?

4.5 Appendix A

This Appendix formalizes the idea that it is possible to restrict attention to mechanisms for which the message space over which a buyer type v randomizes is taken to be the set of types itself. More formally:

Remark 4.1 *(Revelation Principle with Renegotiation) Suppose that a mechanism G with message space M and allocation function $y(\cdot) = [\beta(\cdot), p(\cdot)]$ has a Bayesian equilibrium $(\mu_v)_{v\in V}$. Then it is possible to construct a mechanism $\hat{G}$, for which the message space is the space of types V, with allocation function $\hat{y}(\cdot) = [\hat{\beta}(\cdot), \hat{p}(\cdot)]$ that has a Bayesian equilibrium $(\hat{\mu}_v)_{v\in V}$, which gives rise to the same levels of utility to the seller and all buyer types. Therefore, one can restrict attention to mechanisms in which the buyer types randomize over announcements of their valuation.*

The proof proceeds by construction. If the two spaces M and V are isomorphic, i.e. there exists a bijective mapping $\tau : V \to M$, the mechanism $\hat{G}$ has $\hat{y}(\cdot) := y(\tau(\cdot))$. The equilibrium strategies of the different buyer types are given by $\hat{\mu}_v(\cdot) = \mu_v(\tau(\cdot))$ for all v. As τ is one-to-one, $\hat{\mu}_v(\cdot)$ is a measure on V for all v[15]. Obviously, $\hat{G}$ together with $(\hat{\mu}_v)_{v\in V}$ thus defined ensure that each party obtains the same level of utility as with G and $(\mu_v)_{v\in V}$.

Suppose now that M and V are not isomorphic. There are two possibilities.

Either no mapping $\tau : V \to M$ is one-to-one, that is, for each such mapping τ there are at least two buyer types v and v', such that $\tau(v) = \tau(v') \equiv m$. Then, add an additional message m' to M and proceed as follows. Redefine $\tau(v) := m$, $\tau(v') := m'$, $y(m') := [0,0]$ and $\mu_v(A \cup \{m'\}) := \mu_v(A)$ for all measurable sets A in the sigma algebra of M. By repeating this procedure for all v in the kernel of the original mapping τ, a new message space M which is isomorphic to V and an isomorphism τ are constructed. The thus obtained $(\mu_v)_{v\in V}$ is an equilibrium strategy for the buyer in the new mechanism, given that the seller's renegotiation

[15]In particular, $\int_V \hat{\mu}_v(\tilde{v})\, d\tilde{v} = \int_V \mu_v(\tau(\tilde{v}))\, d\tilde{v} = \int_M \mu_v(m)\, dm = 1$ for all v.

offer in equilibrium is taken to be his offers in the original game for the original messages m and $q(m') = \overline{v}$ for all new messages m'. It is easy to see that the new mechanism G gives rise to the same levels of utility as the original contract. $\hat{G}$ and $(\hat{\mu}_v)_{v\in V}$ can then be obtained as above.

The second possibility is that no mapping $\tau : V \rightarrow M$ is onto, that is for all such mappings τ, there always exists some message $\bar{m}$ such that for no $v \in V$, $\tau(v) = \bar{m}$. Then, for at least two messages m and m' (possibly different from $\bar{m}$), the renegotiation price offered by the seller at stage 3 is the same, i.e. $q(m) = q(m') \equiv q$. To see this, assume the contrary. This implies that the mapping $\iota : M \rightarrow V$ defined by $\iota(m) = q(m) \in V$ is one-to-one and can be inverted. Then, $\iota^{-1} : V \rightarrow M$ is onto, which is in contradiction with our assumption above. Now, a 'smaller' message space and mechanism can be constructed by simply 'deleting' message m', say, from M. The buyer's equilibrium strategy is adapted by setting $\mu_v(m) = \mu_v(m) + \mu_v(m')$ for all v and m and m' as above, the buyer's equilibrium strategy remains the same. Finally, if $m' \neq \bar{m}$, we set $\tau^{-1}(m') := \tau^{-1}(\bar{m})$. The new mapping is onto.

We need to check whether this transformation constitutes an equilibrium in the new mechanism and finally whether it leaves all involved players' final payoffs unaffected. Because q is the renegotiation offer after both announcements m and m' in the original mechanism, it remains optimal for the seller to offer q after the announcement of m in this changed set-up[16]. Furthermore, because the buyer type q has to send both messages with positive probability in the equilibrium of the original game, by taking his incentive constraints with respect to the two messages m and m', we obtain the constraint

$$(\beta(m) - \beta(m'))q = p(m) - p(m'). \tag{4.26}$$

[16]Simply note, that the first order condition (4.7) must hold at q for both m and m', implying that also

$$-\int_q^{\overline{v}} (\mu_v(m) + \mu_v(m'))\, dF(v) - qf(q)(\mu_q(m) + \mu_q(m')) = 0.$$

W.l.o.g we can assume that either there are buyer types $\tilde{v}, \tilde{v}' < q$ with m in the support of $\mu_{\tilde{v}}$ and m' in the support of $\mu_{\tilde{v}'}$ or that no buyer type $\tilde{v} < q$ sends m' in equilibrium[17].

Assume that the first of these possibilities holds. Then, by taking the incentive constraints with respect to m and m' of types $\tilde{v}$ and $\tilde{v}'$ and by combining them with (4.26) we conclude that

$$\beta(m) = \beta(m') \text{ and } p(m) = p(m').$$

In that case, the incentive and participation constraints of all buyer types as well as everybody's final payoff are unaffected by the above 'deletion' of message m'.

Assume now that there is no such $\tilde{v}$ as above. Then, m' is sent only by buyer types above q and he obtains

$$\beta(m')v + (1 - \beta(m'))(v - q) - p(m')$$

after sending message m' under the original mechanism plus renegotiation, which because of (4.26) is the same as

$$\beta(m)v + (1 - \beta(m))(v - q) - p(m),$$

the amount he would receive by sending message m. Therefore, his payoff is unchanged by the deletion of m'. Every other buyer type has $\mu_v(m') = 0$ and his equilibrium strategy as well as his payoff is unaffected by the transformation. As the final allocation of the good is unaltered in all states of nature and this is a zero-sum game, the seller also receives the same payoff in the new game.

[17]The other alternative is to have $\mu_{\tilde{v}}(m') > 0$ and $\mu_{\tilde{v}}(m) = 0$ for some $\tilde{v} < q$. But then we can simply interchange the roles of the two messages m and m' in the preceeding argument.

4.6 Appendix B

This appendix finalizes the argument of Section 4.3. We need to show that in a separating contract, where the buyer's equilibrium strategy is as detailed in Figure 1, the seller's lowest renegotiation offer must lie below the ex-post optimal price offer, i.e. $q^* < v^*$. For this, assume that instead $q^* \geq v^*$. Because q^* maximizes the seller's payoff at renegotiation after message m^*, it must be that

$$q^* \int_{q^*}^{\overline{v}} \mu_v^* \, dF(v) \geq v^* \left(\int_{q^*}^{\overline{v}} \mu_v^* \, dF(v) + F(q^*) - F(v^*) \right),$$

where I have used the specific form of the equilibrium of a simple contract. But also, given the definition of v^*

$$v^*(1 - F(v^*)) \geq q^*(1 - F(q^*)).$$

These two inequalities together imply

$$(q^* - v^*) \int_{q^*}^{\overline{v}} \mu_v^* \, dF(v) \geq (q^* - v^*) \int_{q^*}^{\overline{v}} dF(v). \tag{4.27}$$

Because a positive mass of seller types above q^* send message m in equilibrium, $\int_{q^*}^{\overline{v}} \mu_v^* \, dF(v) < \int_{q^*}^{\overline{v}} dF(v)$ and the inequality in (4.27) can only hold for $q^* = v^*$. But if v^* maximizes the buyer's payoff at renegotiation a similar argument as in the proof of Proposition 4.1 shows that

$$\int_{q^*}^{\overline{v}} \mu_v^* \, dF(v) \geq \frac{F(v^*) - F(v^* - \alpha)}{\alpha}(v^* - \alpha) \qquad \forall \alpha > 0. \tag{4.28}$$

The right hand side tends towards $f(v^*)v^*$, which because of (4.3) is equal to $1 - F(v^*)$. Therefore, the inequality in (4.28) is violated for α close to 0 and q^* must be strictly smaller than v^*.

Finally, we need to prove that all other equilibria of a simple contract yield the same payoff as the partition equilibrium in Example 4. Take any such equilibrium as in Figure 1 of Section 4.3 with corresponding q and q^*. In what follows I will refer to this equilibrium as a 'non-partition' equilibrium to distinguish it from the partition equilibrium. By the Intermediate Value Theorem there must exist a level $\hat{v}$, $q^* < \hat{v} < \overline{v}$, such that $\int_{q^*}^{\overline{v}} \mu_v \, dF(v) = 1 - F(\hat{v})$. Because $\mu_v + \mu_v^* = 1$, this implies that $\int_{q^*}^{\overline{v}} \mu_v^* \, dF(v) = F(\hat{v}) - F(q^*)$. Take the partition equilibrium of a simple contract that has cut-off level $\hat{v}$ and call $\hat{q}^*$ the seller's renegotiation offer after message m^*. We show that the seller obtains the same payoff in both equilibria. Then, without loss of generality one can choose the equilibrium that gives all buyer types the highest payoff. So, the two equilibria are equivalent.

First, assume that $q^* \geq \hat{q}^*$. The renegotiation offer q^* is optimal for the seller in the non-partition equilibrium after message m^* and therefore,

$$q^* \int_{q^*}^{\overline{v}} \mu_v^* \, dF(v) \geq \hat{q}^* \left(\int_{q^*}^{\overline{v}} \mu_v^* \, dF(v) + F(q^*) - F(\hat{q}^*) \right), \tag{4.29}$$

which is equivalent to

$$q^* \left(F(\hat{v}) - F(q^*)\right) \geq \hat{q}^* (F(\hat{v}) - F(\hat{q}^*)). \tag{4.30}$$

In the partition equilibrium, $\hat{q}^*$ maximizes the seller's payoff after message m^*. In particular,

$$\hat{q}^* (F(\hat{v}) - F(\hat{q}^*)) \geq q^* \left(F(\hat{v}) - F(q^*)\right). \tag{4.31}$$

Together, (4.30) and (4.31) imply that the payoffs to the seller in the non-partition equilibrium and in the partition equilibrium are the same.

Last, it is shown that $q^* < \hat{q}^*$ is impossible. (4.30) is amended to give

$$q^* \left(F(\hat{v}) - F(q^*)\right) \geq \hat{q}^* \int_{\hat{q}^*}^{\overline{v}} \mu_v^* \, dF(v).$$

Combining this inequality with (4.31), we obtain

$$F(\hat{v}) - F(\hat{q}^*) \geq \int_{\hat{q}^*}^{\overline{v}} \mu_v^* \, dF(v),$$

which is equivalent to

$$\int_{\hat{q}^*}^{\overline{v}} (1 - \mu_v^*) \, dF(v) \geq 1 - F(\hat{v}) = \int_{q^*}^{\overline{v}} (1 - \mu_v^*) \, dF(v).$$

This inequality cannot hold if $q^* < \hat{q}^*$.

Chapter 5

Sequential Screening and Renegotiation

5.1 Introduction

In this chapter I consider a sequential screening problem in which information about the buyer's type is released over time. Some part of his value for the good is realized at date 0 and therefore already privately known to the buyer at date 1 when he contracts with the seller. A second state of nature, which further influences the good's valuation, is realized at date 1.5 shortly before trade occurs. Its realization is again only observed by the buyer. Finally, trade can take place any time between dates 2 and 3. This set-up opens the black box of fixed contracting costs that were assumed in the preceding chapter. In this set-up contracting is costless. Instead, early contracting is beneficial for the seller because it softens the buyer's participation constraints. The seller prefers to contract early because there is less asymmetric information at the initial stage which makes contracting more efficient.

If the seller can commit himself to a single contracting offer, he will offer a sequential mechanism at date 1 in form of a fixed initial fee and a price. In paying the initial fee the buyer purchases the option to buy the good at a reduced price once the second parameter of the valuation is realized, i.e. at date 2. Typically, a higher access fee is paired with a lower final price. Sequential price discrimination is common practice in a variety of circumstances such as fidelity cards in cinemas, book clubs or air plane tickets.

In this chapter, it is assumed that commitment is not feasible. More precisely, whatever an early contract prescribes concerning the good's purchase between dates 2 and 3, as long as it is not ex-post efficient, the seller has the opportunity to make one final renegotiation offer at date 3. Here, an alternative interpretation of the idea that renegotiation cannot be included in a contract is used compared to the preceding chapter. I assume that in order for a contract to be made contingent on parties' messages concerning their preferences, these messages must be sent in a verifiable way. More precisely, I assume that dates are contractible and that therefore a contract can completely specify what should be done at dates 2 and 3. Parties' messages, on which contractual prescriptions are based, must take the form of letters, emails or conversations in the presence of a third party such as a judge[1]. Assume, that the writing of verifiable messages takes time and that they must be sent well in advance to the actual trading date. For instance, a meeting with a third party cannot be scheduled necessarily at the same time at which trade should occur. Similarly, there is a possible delay between the receipt of a letter and the execution of the contract. This time gap leaves an opportunity for renegotiation. In contrast, messages that parties exchange as part of a private bargaining game need not be verifiable. As such, renegotiation does not necessitate a formal message system. It can for example take place in an informal telephone call or in a private meeting between the two parties. Therefore, it is instantaneous and allows parties to exchange information directly before trade. A contract that contains a prescription of what is to be done at the last possible moment, i.e. at date 3, in reality necessitates the sending of messages some time before date 3, date 2.5 say. This contract is therefore vulnerable to a 'last minute' renegotiation in between dates 2.5 and 3.

[1]In Hart and Moore (1988), there is an interesting comparison between different message systems and the impact they have on the efficiency of a contractual agreement.

Under this assumption an initial contract must have a very simple structure. The optimal contract from the seller's viewpoint is either no contract or a simple sales contract that concludes trade with only the highest initial buyer types. The null contract is the strictly superior alternative if uncertainty concerning the first variable is large compared to uncertainty concerning the second variable. Intuitively, under this condition an early contract provides only a small benefit while the ratchet effect is relatively severe. The null contract is therefore strictly dominant and the seller prefers to regulate trade in an informal ex-post bargaining game. If on the other hand, uncertainty concerning the first variable is small relative to uncertainty concerning the second variable, a simple trade contract is optimal. This contract allows the seller to conclude trade with initial high buyer types for sure before they learn their exact valuations. In addition, such a contract provides a commitment to the seller to offer a low price at the final bargaining round. Ex-ante, the seller can extract the benefit of this low price through an initial flat payment from every buyer. Remark, that this payment has to satisfy the individual rationality constraint of the lowest initial valuation buyer. At the final bargaining round, intermediate buyer types buy the good.

Literature on screening when information is released sequentially include Courty and Li (2000) in static contracting environments, and Baron and Besanko (1984), in dynamic environments. These papers study the case when commitment is feasible and make use of the revelation principle. A paper that studies a buyer seller relationship with time varying valuations and contract renegotiation is by Blume (1998), which is an extension to Hart and Tirole (1988). The author retains a persistent component for the buyer's valuation and in addition introduces a transient component. By assumption the seller does not want to screen this transient component. This is in contrast to the model in this chapter, where the seller has always an incentive to ex-post screen the total valuation of the buyer.

The next section contains the set-up. Section 5.3 solves for the optimal contract in the benchmark case when commitment is feasible. The following section solves for the optimal contract when renegotiation can occur. The final section concludes.

5.2 The Model

The buyer's valuation for the good is influenced by two states of the world which are realized sequentially. For simplicity, an additive structure for the valuation, $v = v_1 + v_2$, is assumed[2]. At the time at which the first contract is signed v_1 is already private information to the buyer. I will call v_1 the buyer's first stage type as opposed to v, the buyer's second stage or final type. The second variable is privately revealed to the buyer shortly before trade occurs. Both variables are drawn independently[3] according to commonly known distribution functions $F_i(\cdot)$ with strictly positive densities $f_i(\cdot)$, $i = 1, 2$. The supports are denoted by $[\underline{v}_i, \overline{v}_i]$. The seller's production costs are fixed and normalized to 0 and we assume that the $\underline{v}_i > 0$, $i = 1, 2$. The seller makes all contracting offers.

In order to simplify on necessary notation we extend $F_i(\cdot)$ and $f_i(\cdot)$ over the borders of $[\underline{v}_i, \overline{v}_i]$ such that

$$F_i(v_i) = \begin{cases} 0 & v_i < \underline{v}_i \\ F_i(v_i) & v_i \in [\underline{v}_i, \overline{v}_i] \\ 1 & v_i > \overline{v}_i \end{cases}$$

and similarly

$$f_i(v_i) = \begin{cases} 0 & v_i < \underline{v}_i \\ f_i(v_i) & v_i \in [\underline{v}_i, \overline{v}_i] \\ 0 & v_i > \overline{v}_i. \end{cases}$$

[2]To extend the analysis to a framework in which the valuation is a more general function of two consecutively realized states of nature is beyond the scope of this paper. It might be interesting material for future research.

[3]I conjecture that the dominance result is reinforced if the variables are correlated. If there is perfect correlation for example, the buyer is informed about v_2 from the beginning, which is basically the situation of the preceding chapter.

Then, the distributions on the individual v_i's induce a probability distribution over the final value v on the interval $[\underline{v}_1 + \underline{v}_2, \overline{v}_1 + \overline{v}_2]$, which can be written as

$$F(v) = \int_{\underline{v}_1}^{v-\underline{v}_1} \int_{\underline{v}_2}^{v-v_1} dF_2(v_2)\, dF_1(v_1). \tag{5.1}$$

5.3 The Benchmark: Full Commitment

Consider first the set-up with full commitment. The revelation principle tells us that we can restrict attention to a contract in which the seller asks the buyer to truthfully reveal his first stage type which determines what kind of contract the buyer receives. Once the second variable is realized, the buyer is asked to announce its value and obtains the good with a certain probability and pays a price, both of which are functions of his *two* announcements. Price and probability of trade are chosen such as to maximize the seller's expected surplus subject to the constraint that the buyer tells the truth about both v_1 and v_2 and, conditional on v_1, receives at least 0 in expectation.

Consider the following time line:

stage 0	stage 1	stage 1.5	stage 2
v_1 realized	S proposes contract	v_2 realized	contract enforced

For future reference let us consider what happens if no contract is signed at stage 1 and the seller relies on ex-post bargaining at stage 2 to sell his product. He faces a continuum of types v distributed on $[\underline{v}_1 + \underline{v}_2, \overline{v}_1 + \overline{v}_2]$ and he maximizes $v(1 - F(v))$, where $F(v)$ is given by (5.1). Call the solution to this problem v^* in line with the notation of chapter 4.

Let us now turn to the full problem. It turns out that a relatively simple contract in which the buyer initially pays a fee $A(v_1)$ dependant on his type and then, once

v_2 is known to him, can decide whether or not to trade at a price $p(v_1)$ is optimal for the seller.

Lemma 5.1 *With full commitment the seller offers a contract of the following form* $C = [A(v_1), p(v_1)]_{v_1 \in [\underline{v}_1, \overline{v}_1]}$. *The buyer selects a pair* $(A(v_1), p(v_1))$ *at the first stage and then decides at the second stage whether he wants to trade at the predetermined price* $p(v_1)$.

Set $U(v_1, v_2) = \beta(v_1, v_2)(v_1 + v_2) - p(v_1, v_2)$.

Formally, the principal's program can be written as

$$\max_{\beta, U} \int_{\underline{v}_1}^{\overline{v}_1} \int_{\underline{v}_2}^{\overline{v}_2} [\beta(v_1, v_2)(v_1 + v_2) - U(v_1, v_2)] \, dF_2(v_2) \, dF_1(v_1) \tag{5.2}$$

s.t.

$$(IC_{II}) \quad U(v_1, v_2) \geq \beta(v_1, v_2')(v_1 + v_2) - p(v_1, v_2'), \qquad \forall\, v_1, v$$

$$(IC_I) \quad \int_{\underline{v}_2}^{\overline{v}_2} U(v_1, v_2) \, dF_2(v_2) \geq \int_{\underline{v}_2}^{\overline{v}_2} [\beta(v_1', \hat{v}_2)(v_1 + v_2) - p(v_1', \hat{v}_2)] \, dF_2(v_2), \qquad \forall\, v_1, v'$$

$$\text{where } \hat{v}_2 \; (= \hat{v}_2(v_1, v_1', v_2)) = \arg\max_{\tilde{v}_2} \beta(v_1', \tilde{v}_2)(v_1 + v_2) - p(v_1', \tilde{v}_2)$$

$$(IR) \quad \int_{\underline{v}_2}^{\overline{v}_2} U(v_1, v_2) \, dF_2(v_2) \geq 0 \qquad \forall\, v_1$$

The first set of constraints are the incentive constraints at the final screening stage. Given that at the preceding stage a v_1 buyer has selected the right contract, he should also truthfully announce his second type v_2.

The second set of constraints is concerned with the announcement of the buyer's initial type v_1. The left-hand side is his payoff when he announces both his types truthfully in the two consecutive revelation stages. The right-hand side is his payoff when he lies in the first stage and then chooses his optimal announcement in the second stage. Given that he lied in the first stage, he does not necessarily tell the truth in the second period. Instead, his announcement $\hat{v}_2$ is a function of his true type, $v_1 + v_2$, and his first stage announcement v_1'.

Finally, the last line formalizes the buyer's individual rationality constraint. He must receive at least his reservation utility in expectation if he tells the truth in both stages.

Proof.

First, let us consider the (IC_{II}) constraints. By a familiar argument, which makes use of the envelope theorem, it can be shown that the buyer's utility at the final stage can be written as

$$U(v_1, v_2) = U(v_1, \underline{v}_2) + \int_{\underline{v}_2}^{v_2} \beta(v_1, \tilde{v}_2)\, dF_2(\tilde{v}_2). \tag{5.3}$$

Second, the level of $U(v_1, \underline{v}_2)$ for each first stage type v_1 needs to be determined which will be done by looking at the first stage incentive constraints (IC_I).

Write $\bar{U}(v_1, v_1') := \int_{\underline{v}_2}^{\overline{v}_2} (\beta(v_1', \hat{v}_2)(v_1 + v_2) - p(v_1', \hat{v}_2))\, dF_2(v_2)$, where $\hat{v}_2$ is defined in the second line of the (IC_I) constraint. The buyer chooses his announcement v_1' to maximize this expression and (IC_I) ensures that $v_1' = v_1$ is optimal, i.e.

$$\frac{\partial \bar{U}}{\partial v_1'} = \int_{\underline{v}_2}^{\overline{v}_2} \left[\left(\frac{\partial \beta}{\partial v_1'}(v_1 + v_2) - \frac{\partial p}{\partial v_1'} \right) + \left(\frac{\partial \beta}{\partial \hat{v}_2}(v_1 + v_2) - \frac{\partial p}{\partial \hat{v}_2} \right) \frac{\partial \hat{v}_2}{\partial v_1'} \right] dF_2(v_2) \bigg|_{v_1' = v_1} = 0$$

Now, the second term in round brackets is equal to 0, because it is the first order condition of the buyer's maximization problem with respect to $\hat{v}_2$. Using this, one can write the buyer's expected utility $\bar{U}(v_1) := \bar{U}(v_1, v_1)$ at the first stage conditional on his type v_1 as

$$\bar{U}(v_1) = \int_{\underline{v}_2}^{\overline{v}_2} U(v_1, v_2)\, dF_2(v_2) = \bar{U}(\underline{v}) + \int_{\underline{v}_2}^{\overline{v}_2} \int_{\underline{v}_1}^{v_1} \beta(\tilde{v}_1, v_2)\, dF_1(\tilde{v}_1)\, dF_2(v_2). \tag{5.4}$$

Remark that, given that the buyer announces v_1 truthfully, (IC_{II}) implies that $\beta(\tilde{v}_1, \hat{v}_2) = \beta(\tilde{v}_1, v_2)$. Obviously, the seller would like to keep the lowest first stage buyer type on his reservation level and therefore sets $\bar{U}(\underline{v}) = 0$. Combining (5.3) and (5.4), one obtains

$$U(v_1, v_2) = \int_{\underline{v}_2}^{\bar{v}_2} \left[\int_{\underline{v}_1}^{v_1} \beta(\tilde{v}_1, \breve{v}_2)\, dF_1(\tilde{v}_1) - \int_{\underline{v}_2}^{\breve{v}_2} \beta(v_1, \tilde{v}_2)\, dF_2(\tilde{v}_2) \right] dF_2(\breve{v}_2) + \int_{\underline{v}_2}^{v_2} \beta(v_1, \tilde{v}_2)\, dF_2(\tilde{v}_2).$$

This expression can be substituted into the seller's objective function in (5.2) to obtain

$$\max_{\beta} \int_{\underline{v}_1}^{\bar{v}_1} \int_{\underline{v}_2}^{\bar{v}_2} \left[\beta(v_1, v_2)(v_1 + v_2) - \int_{\underline{v}_2}^{\bar{v}_2} \int_{\underline{v}_1}^{v_1} \beta(\tilde{v}_1, \breve{v}_2)\, dF_1(\tilde{v}_1)\, dF_2(\breve{v}_2) \right.$$
$$\left. + \int_{\underline{v}_2}^{\bar{v}_2} \int_{\underline{v}_1}^{\breve{v}_2} \beta(v_1, \tilde{v}_2)\, dF_2(\tilde{v}_2)\, dF_1(\breve{v}_2) - \int_{\underline{v}_2}^{v_2} \beta(v_1, \tilde{v}_2)\, dF_2(\tilde{v}_2) \right] dF_2(v_2)\, dF_1(v_1).$$

Remark, that the last line is equal to 0, because the first term is independent of v_2 and the second term, once the integral with respect to v_2 is included, is identical to the first (just replace $\breve{v}_2$ with v_2). Therefore, the above expression is just equal to

$$\max_{\beta} \int_{\underline{v}_1}^{\bar{v}_1} \int_{\underline{v}_2}^{\bar{v}_2} \left[\beta(v_1, v_2)(v_1 + v_2) - \int_{\underline{v}_1}^{v_1} \beta(\tilde{v}_1)\, dF_1(\tilde{v}_1) \right] dF_2(v_2)\, dF_1(v_1),$$

where with some slight abuse of notation $\beta(\tilde{v}_1)$ denotes the expectation of $\beta(\tilde{v}_1, \breve{v}_2)$ with respect to its second argument. A familiar argument based on integration by parts shows that the above expression is equal to

$$\max_{\beta} \int_{\underline{v}_1}^{\bar{v}_1} \int_{\underline{v}_2}^{\bar{v}_2} \beta(v_1, v_2) \left(v_1 + v_2 - \frac{1 - F_1(v_1)}{f_1(v_1)} \right) dF_2(v_2)\, dF_1(v_1).$$

Therefore, the optimal solution is a cut-off rule, such that

$$\beta(v_1, v_2) = \begin{cases} 1 & \text{if} \quad v_1 + v_2 \geq \frac{1 - F_1(v_1)}{f_1(v_1)} \\ 0 & \text{otherwise.} \end{cases} .$$

The buyer's first stage utility level $\bar{U}(v_1)$ is given by (5.4). Finally, it is easy to see that this allocation can be achieved by the following contract. At the first stage, the buyer pays $A(v_1)$ for the option to buy the good for a price $p(v_1)$, which he can decide to exercise or not once he has learned his complete type. The final price is set $p(v_1) = \frac{1-F_1(v_1)}{f_1(v_1)}$ and the expression for $A(v_1)$ follows from (5.4).

■

The solution to this contracting problem has some interesting features. First, remark that the solution is bang-bang although there is some non-linearity introduced through the expectation operator in the incentive constraint (IC_I). The solution is similar to a static problem in which the good is sold if and only if $v \geq \frac{1-F(v)}{f(v)}$. In the sequential model total valuation must lie above $\frac{1-F_1(v_1)}{f_1(v_1)}$, the hazard rate of the first variable's distribution function because only the first variable is known at the time of the contract. Also, the allocation depends on the realization of the second variable although the price does not. This is in contrast to what can be done without commitment as will be seen in the next section.

The model is a special case of Courty and Li (2000), a version of which they discuss as an example. The authors study general sequential screening problems with commitment when the buyer has some private information with respect to the distribution of his total valuation. Here, he has no better information about the distribution but he knows the support. In Courty and Li (2000) the support is fixed and therefore a slightly different proof must be employed.

5.4 Sequential Screening with No Commitment

Finally, we consider the situation when the seller is not committed by his stage 1 contract. More precisely, I am interested in the case when the contract renegotiation is effected after all prescriptions of the contract written at stage 1 have been carried out. That is, if the good has not been traded according to the contract, the seller can make a final offer to the buyer after which the good becomes obsolete and time ends. The time line is

stage 0	stage 1	stage 1.5	stage 2	stage 3
v_1 realized	S proposes contract	v_2 realized	contract enforced	final offer

We will be interested in the Perfect Bayesian Nash Equilibria of the game that consists of the contract offer at stage 1, the message game played at stages 1 and 2 and the final renegotiation offer.

Parallel to Section 4.2.2, a contract consists of two message spaces M_1 and M_2[4] with elements m_1 and m_2. Message m_1 is sent by the buyer after the contract offer at stage 1, message m_2 is sent by him at stage 2 once he has learned v_2. Given messages (m_1, m_2), the contract enforces trade with probability $\beta(m_1, m_2)$ at price $p(m_1, m_2)$. A buyer's strategy can be written as $\mu_1(m_1 \mid v_1)$ at stage 1 and $\mu_2(m_2 \mid v_1, v_2, m_1)$ at stage 2. The latter for example is the probability with which a buyer with value $v = v_1 + v_2$ who has sent message m_1 at stage 1 sends message m_2 at stage 2. Following messages (m_1, m_2), there is a final offer at stage 3 by the seller which will be denoted by $q(m_1, m_2)$.

Finally, it is convenient to write the problem as in the above section. We define the utility that a final buyer type $v = v_1 + v_2$ receives under the contract, i.e. *before* the final renegotiation offer, as

$$U(m_1, m_2, v) := \beta(m_1, m_2)v - p(m_1, m_2).$$

We can now state the final proposition which will be proved using several lemmata.

Proposition 5.1 *If uncertainty concerning v_1 is much larger than uncertainty concerning v_2, any contract is strictly dominated by the null contract. If uncertainty concerning v_1 is smaller than uncertainty concerning v_2, then the optimal contract*

[4]The message space M_2 could depend on the earlier message m_1. For simplicity this dependency is suppressed. The following analysis is independent of this simplification.

is a simple contract in which trade only occurs for high stage 1 buyer types at a relatively low fixed price. The seller conlcudes trade with internediate final buyer types for a higher price at renegotiation.

The proof relies on arguments already encountered in section 4.3. It is divided into several lemmata, which are concerned with the constraints imposed on a contract through the buyer's equilibrium strategy in the message sending game and the resulting beliefs by the seller.

5.4.1 Message Sending Stages

The first lemma is the counterpart of Proposition 4.1 in subsection 4.3.2. It states that the contract must involve complete pooling, if the buyer's equilibrium strategy in the induced message game does not affect the seller's beliefs, i.e. if it does not affect his renegotiation offer:

Lemma 5.2 *If $q(m_1, m_2) \equiv q \ \forall (m_1, m_2)$, then $q = v^*$ and a stage* 1 *contract must have $\beta(m_1, m_2) \equiv \beta$ and $p(m_1, m_2) = p$, $\forall (m_1, m_2)$.*

Proof.

Write $\mu(m_1, m_2) := \int_{\underline{v}_1}^{\overline{v}_1} \mu_1(m_1 \mid v_1) \left[\int_{\underline{v}_2}^{\overline{v}_2} \mu_2(m_2 \mid v_1, v_2, m_1)\, dF_2(v_2)\right] dF_1(v_1)$ as the probability that message pair (m_1, m_2) is sent. The seller's belief after messages (m_1, m_2) is

$$F(v \mid m_1, m_2) \ :=$$

$$\frac{1}{\mu(m_1,m_2)} \int_{\underline{v}_1}^{v-\underline{v}_1} \mu_1(m_1 \mid v_1) \left[\int_{\underline{v}_2}^{v-v_1} \mu_2(m_2 \mid v_1, v_2, m_1)\, dF_2(v_2)\right] dF_1(v_1).$$

Assume that $q \neq v^*$. The fact that q is the seller's renegotiation offer after message pair (m_1, m_2) implies that

$$(1 - F(q \mid m_1, m_2))q \geq (1 - F(v^* \mid m_1, m_2))v^* \qquad \forall (m_1, m_2).$$

Taking expectations over M_2, then over M_1 implies that this expression is equivalent to

$$(1 - F(q))q \geq (1 - F(v^*))v^*,$$

which contradicts the fact that v^* is the unique maximizer of $(1 - F(v))v$.

Next, I show that for every message pair (m_1, m_2), $\beta(m_1, m_2) \equiv \beta$ and $p(m_1, m_2) \equiv p$. The proof proceeds in two steps. First, it is shown that $\beta(m_1, m_2) = \beta(m_1, m_2')$ for all messages m_2, m_2', second that $\beta(m_1, m_2) = \beta(m_1', m_2)$ for all messages m_1, m_1'.

To see the first point, we need to consider the second stage incentive constraints. The same argument as in the proof of Proposition 4.1 is used. First, it is shown that every message m_2 must be sent by second stage buyer types below *and* above the final renegotiation offer v^* : Because v^* maximizes $(1 - F(v \mid m_1, m_2))v$ for all message pairs (m_1, m_2), v^* must also maximize $(1 - F(v \mid m_1))v$; simply take the expectation with respect to m_2. But then the exact same reasoning as in the proof of Proposition 4.1, where $F(v \mid m_1)$ replaces $F(v)$ and $F(v \mid m_1, m_2)$ replaces $F(v \mid m)$ shows the first claim. Second, for two given messages m_2 and m_2', the incentive constraints of buyer types below and above v^* who send these messages in equilibrium finalizes the argument. We can then simplify notation by suppressing the dependency of $\beta(m_1, m_2)$, $p(m_1, m_2)$ and $U(m_1, m_2, v)$ on m_2.

To see the second step in the proof, we need to turn to the incentive constraints of a first stage buyer type who sends message m_1. For this, we need to compute the expected utility that he receives by sending this message. A second stage buyer type $v > v^*$ who has sent message m_1 obtains a payoff of

$$\beta(m_1)v + (1 - \beta(m_1))(v - v^*) - p(m_1) = (v - v^*) + U(m_1, v^*),$$

whereas a second stage buyer type $v < v^*$ who has sent message m_1 obtains a payoff of

$$\beta(m_1)v - p(m_1) = \beta(m_1)(v - v^*) + U(m_1, v^*).$$

Therefore, a first stage buyer type v_1 expects the following payoff from message m_1:

$$\begin{aligned}\bar{U}(m_1, v_1) &= \beta(m_1)\int_{\underline{v}_2}^{v^*-v_1}(v_1+v_2-v^*)\,dF_2(v_2)\\ &\qquad +\int_{v^*-v_1}^{\overline{v}_2}(v_1+v_2-v^*)\,dF_2(v_2)+U(m_1,v^*)\\ :&= \beta(m_1)\xi(v_1)+\zeta(v_1)+U(m_1,v^*).\end{aligned}$$

Note, that $\xi(v_1) < 0$ and $\zeta(v_1) > 0$ and that both are monotonically increasing in v_1.

Take first the situation in which two different first stage buyer types v_1', v_1 *both* send the two messages m_1' and m_1 in equilibrium. This would imply that

$$\begin{aligned}\beta(m_1')\xi(v_1')+\zeta(v_1')+U(m_1',v^*) &= \beta(m_1)\xi(v_1')+\zeta(v_1')+U(m_1,v^*)\\ \beta(m_1')\xi(v_1)+\zeta(v_1)+U(m_1',v^*) &= \beta(m_1)\xi(v_1)+\zeta(v_1)+U(m_1,v^*)\end{aligned}$$

and consequently that

$$(\beta(m_1')-\beta(m_1))\xi(v_1') = (\beta(m_1')-\beta(m_1))\xi(v_1).$$

Because $\xi(v_1') \neq \xi(v_1)$ if $v_1' \neq v_1$, this is only possible if $\beta(m_1') = \beta(m_1)$.

Next, consider the situation in which no two buyer types send the same two messages m_1' and m_1. So, there are disjoint sets $V(m_1')$, $V(m_1) \subseteq [\underline{v}_1, \overline{v}_1]$, s.t. all buyer types in $V(m_1')$ send only message m_1' and all buyer types in $V(m_1)$ send only message m_1. Then, either $\beta(m_1') = \beta(m_1)$ or $V(m_1')$ and $V(m_1)$ must be connected. Assume for example that $V(m_1')$ is not connected. Then there must exist three types $v_1' < v_1 < v_1''$ with $v_1', v_1'' \in V(m_1')$ and $v_1 \in V(m_1)$. The incentive constraints of v_1' and v_1 yield

$$(\beta(m_1')-\beta(m_1))\xi(v_1') \geq (\beta(m_1')-\beta(m_1))\xi(v_1)$$

and $\beta(m_1') \leq \beta(m_1)$. The incentive constraints of types v_1 and v_1'' together imply that

$$(\beta(m_1')-\beta(m_1))\xi(v_1'') \geq (\beta(m_1')-\beta(m_1))\xi(v_1),$$

which is only satisfied if $\beta(m_1') \geq \beta(m_1)$. Therefore, $\beta(m_1') = \beta(m_1)$ if $V(m_1')$ is not connected.

Finally, consider the situation in which $V(m_1')$ and $V(m_1)$ are two disjoint intervals with $V(m_1')$ 'below' $V(m_1)$. But then it is impossible that v^* simultaneously maximizes $(1 - F(v \mid m_1'))v$ and $(1 - F(v \mid m_1))v$. The v that maximizes $(1 - F(v \mid m_1'))v$ should in fact lie strictly below the one that maximizes $(1 - F(v \mid m_1))v$. Therefore, the fact that $V(m_1')$ and $V(m_1)$ are two disjoint intervals is inconsistent with our initial assumption that the renegotiation offer is the same after every message. This proves that $\beta(m_1') = \beta(m_1)$ from which it follows that $U(m_1', v^*) = U(m_1, v^*)$ and also $p(m_1') = p(m_1)$. ■

It remains to investigate the situation in which the seller, depending on the buyer's messages, sets different prices at the renegotiation stage. For this, it is sufficient to consider two different prices. The argument extends in a similar fashion to the possibility of more than two prices. Before we start some definitions are needed. Define

$$q^*(\hat{v}_1) := \arg\max_q (1 - \hat{F}(q))q, \tag{5.5}$$

where

$$\hat{F}(q) := \int_{\underline{v}_1}^{q-\underline{v}_1} \int_{\underline{v}_2}^{v-v_1} dF_2(v_2)\, d\hat{F}_1(v_1)$$

and

$$\hat{F}_1(v_1) = \begin{cases} 1 & \text{if} \quad v_1 > \hat{v}_1 \\ \frac{F_1(v_1)}{F_1(\hat{v}_1)} & \text{if} \quad v_1 \in [\underline{v}_1, \hat{v}_1] \\ 0 & \text{if} \quad v_1 < \underline{v}_1 \end{cases} .$$

That is, $\hat{F}(\cdot)$ is the distribution function of the final valuation, given that the first stage type v_1 lies below a certain threshold $\hat{v}_1$. The definition will become important for the type of equilibria that will emerge in the following contract. The next lemma mirrors Proposition 4.2, subsection 4.3.2:

Lemma 5.3 *Assume that the seller sets two different renegotiation prices $q^* < q$ depending on the buyer's messages at stages 1 and 2. Without loss of generality, i.e.*

without affecting final payoffs, we can set

$$\beta(m_1, m_2') = \beta(m_1, m_2) \equiv \beta(m_1), \qquad \forall m_1 \text{ and } \forall m_2, m_2'.$$

that is, the second message can be deleted.
Furthermore, if q^* is the renegotiation price after message m_1^* and q is the price after message m_1, then two situations can occur. Either $\beta(m_1^*) = \beta(m_1) = 1$ and $p(m_1^*) = p(m_1)$, or the buyer's equilibrium strategy can be described by a partition equilibrium, i.e. there must exist a first stage buyer type $\hat{v}_1$, such that all first stage buyer types below $\hat{v}_1$ send message m_1^* and all first stage buyer types above $\hat{v}_1$ send message m_1. In the latter case $\beta(m_1^*) \leq \beta(m_1)$.

Proof.

Assume that there are two different prices at renegotiation $q^* < q$. Following a first message m_1 there are three possibilities: *i)* Either there are two different messages m_2' and m_2 such that the renegotiation price after message m_2' is q^* and the price after message m_2 is q_2 or the renegotiation offer is either *ii)* q^* or *iii)* q, regardless of the second stage message. Let us investigate these possibilities in turn.

i) In the first case, similarly to the proof of Proposition 4.2, we can use the incentive constraints of final buyer types v' and v, with $q^* \leq v' < q$ and $q \leq v$, who send messages m_2' and m_2 respectively, to show that $\beta(m_1, m_2) = 1$ and $p(m_1, m_2) = p(m_1, m_2') + (1 - \beta(m_1, m_2'))q^*$. Therefore, we can suppress the dependency on m_2', m_2. Then, regardless of which message m_2 is sent after message m_1[5], a final stage buyer type v with $v > q^*$ receives

$$v - q^* + U(m_1, q^*)$$

and a final stage buyer type $v < q^*$ receives

$$\beta(m_1)(v - q^*) + U(m_1, q^*).$$

[5]Assume here, that there are only two possible messages m_2 and m_2'. If there are more than two messages that result in renegotiation price q^*, the case is as in the following subcase *ii)*.

ii) Turning to the second case, the renegotiation price is q^* after any two messages m_2' and m_2. The incentive constraints of two final stage buyer types v', $v > q^*$ who send messages m_2' and m_2 respectively, imply that

$$p(m_1, m_2) - p(m_1, m_2') = (\beta(m_1, m_2) - \beta(m_1, m_2'))q^*.$$

Therefore, $U(m_1, m_2', q^*) = U(m_1, m_2, q^*)$ and we can suppress the dependency of U on m_2. If a final stage buyer type $v < q^*$ sends message m_2', he obtains

$$\begin{aligned}\beta(m_1, m_2')v - p(m_1, m_2') &= \beta(m_1, m_2')(v - q^*) + \beta(m_1, m_2)q^* - p(m_1, m_2) \\ &= \beta(m_1, m_2')(v - q^*) + U(m_1, q^*).\end{aligned}$$

whereas sending message m_2 yields

$$\begin{aligned}\beta(m_1, m_2)v - p(m_1, m_2) &= \beta(m_1, m_2)(v - q^*) + \beta(m_1, m_2')q^* - p(m_1, m_2') \\ &= \beta(m_1, m_2)(v - q^*) + U(m_1, q^*),\end{aligned}$$

Therefore, a last stage buyer type $v < q^*$ sends message m_2, such that $m_2 = \arg\min_{\tilde{m}_2} \beta(m_1, \tilde{m}_2)$. Set $\beta(m_1, m_2) = \beta(m_1)$ and we obtain the same overall utility levels as in case *i)*.

iii) The latter case is similar to case *ii)* but the final levels of utility are

$$v - q + U(m_1, q)$$

for final buyer types $v > q$ and

$$\beta(m_1)(v - q) + U(m_1, q)$$

for final buyer types $v < q$.

This finalizes the proof of the first statement.

When analyzing the incentive constraints for a first stage buyer type v_1, we now only need to consider two possible messages[6]: m_1^* and m_1 with expected payoffs

$$\begin{aligned}\bar{U}(m_1^*, v_1) &= \beta(m_1^*) \int_{\underline{v}_2}^{q^*-v_1} (v_1 + v_2 - q^*)\, dF_2(v_2) \\ &\qquad + \int_{q^*-v_1}^{\overline{v}_2} (v_1 + v_2 - q^*)\, dF_2(v_2) + U(m_1^*, q^*) \\ &= \beta(m_1^*) \int_{\underline{v}_2}^{\overline{v}_2} \min(v_1 + v_2, q^*)\, dF(v_2) + \int_{q^*-v_1}^{\overline{v}_2} (v_1 + v_2 - q^*)\, dF_2(v_2) - p(m_1^*) \\ &= \beta(m_1^*)\xi(v_1, q) + \varsigma(v_1, q^*) - p(m_1^*). \\ &\quad \text{and} \\ \bar{U}(m_1, v_1) &= \beta(m_1)\xi(v_1, q) + \varsigma(v_1, q) - p(m_1).\end{aligned}$$

First, assume that there are two first stage buyer types $v_1' < v_1$ who both send messages m_1^* and m_1. It will be shown that then $\beta(m_1^*) = \beta(m_1) = 1$. Both types must be indifferent between sending either message and therefore

$$\begin{aligned}\beta(m_1^*)\xi(v_1', q^*) + \varsigma(v_1', q^*) - p(m_1^*) &= \beta(m_1)\xi(v_1', q) + \varsigma(v_1', q) - p(m_1) \\ \beta(m_1^*)\xi(v_1, q^*) + \varsigma(v_1, q^*) - p(m_1^*) &= \beta(m_1)\xi(v_1, q) + \varsigma(v_1, q) - p(m_1).\end{aligned}$$

These two together imply that

$$\beta(m_1^*)(\xi(v_1', q^*) - \xi(v_1, q^*)) + \varsigma(v_1', q^*) - \varsigma(v_1, q^*) = \beta(m_1)(\xi(v_1', q) - \xi(v_1, q)) + \varsigma(v_1', q) - \varsigma(v_1, q).$$

This expression is equivalent to

$$(1 - \beta(m_1^*)) \int_{q^*-v_1}^{q^*-v_1'} F_2(v_2)\, dv_2 = (1 - \beta(m_1)) \int_{q-v_1}^{q-v_1'} F_2(v_2)\, dv_2. \tag{5.6}$$

To see this, we compute $\varsigma(v_1', q^*) - \varsigma(v_1, q^*)$:

$$\begin{aligned}&\textstyle\int_{q^*-v_1'}^{\overline{v}_2} (v_1' + v_2 - q^*)\, dF_2(v_2) - \int_{q^*-v_1}^{\overline{v}_2} (v_1 + v_2 - q^*)\, dF_2(v_2) \\ &\quad = (v_1' - v_1) + (q^* - v_1')F_2(q^* - v_1') - (q^* - v_1)F_2(q^* - v_1) - \textstyle\int_{q^*-v_1}^{q^*-v_1'} v_2\, dF_2(v_2) \\ &\quad = (v_1' - v_1) + \textstyle\int_{q^*-v_1}^{q^*-v_1'} F_2(v_2)\, dv_2,\end{aligned}$$

[6]The case with more than two initial messages can be treated similarly as in Lemma 5.2.

where the second line is obtained through integration by parts. Next, a similar calculation shows that $\xi(v_1', q^*) - \xi(v_1, q^*) = -\int_{q^*-v_1}^{q^*-v_1'} F_2(v_2)\,dv_2$. The right-hand side of (5.6) is obtained symmetrically. Because $F_2(\cdot)$ is strictly increasing, it follows that $\int_{q^*-v_1}^{q^*-v_1'} F_2(v_2)\,dv_2 < \int_{q-v_1}^{q-v_1'} F_2(v_2)\,dv_2$ for $q > q^*$ and $v_1 > v_1'$ and therefore $\beta(m_1^*) = \beta(m_1) = 1$.

Next, assume that there are two disjoint subsets $V(m_1^*)$ and $V(m_1)$ of first stage buyer types who send messages m_1^* and m_1 respectively. We want to show that unless $V(m_1^*)$ and $V(m_1)$ are two connected intervals, as above, $\beta(m_1^*) = \beta(m_1) = 1$. Assume w.l.o.g. that $V(m_1^*) = [\underline{v}_1, v_1^1] \cup [v_1^2, \overline{v}_1]$ and $V(m_1) = [v_1^1, v_1^2]$. Then the two border types v_1^1 and v_1^2 must be indifferent between sending messages m_1^* and m_1 and the same argument as above can be applied.

Finally we have the possibility that $V(m_1^*)$ and $V(m_1)$ are two connected, disjoint intervals, where because $q^* < q$ we can write $V(m_1^*) = [\underline{v}_1, \hat{v}_1]$ and $V(m_1) = [\hat{v}_1, \overline{v}_1]$. In this instance, the renegotiation price $q^* = q^*(\hat{v}_1)$ is defined through $\hat{v}_1$ as in (5.5). Because the first stage buyer type $\hat{v}_1$ must be indifferent between messages m_1^* and m_1 we have

$$\beta(m_1^*)\xi(\hat{v}_1, q^*) + \varsigma(\hat{v}_1, q^*) - p(m_1^*) = \beta(m_1)\xi(\hat{v}_1, q) + \varsigma(\hat{v}_1, q) - p(m_1). \tag{5.7}$$

Therefore, the incentive constraints of a type $v_1 > \hat{v}_1$ together with the incentive constraint of a type $v_1' < \hat{v}_1$ imply that

$$\begin{aligned}
\beta(m_1)\xi(v_1, q) - \beta(m_1^*)\xi(v_1, q^*) + \varsigma(v_1, q) - \varsigma(v_1, q^*) &\geq \\
\beta(m_1)\xi(\hat{v}_1, q) - \beta(m_1^*)\xi(\hat{v}_1, q^*) + \varsigma(\hat{v}_1, q) - \varsigma(\hat{v}_1, q^*) &\geq \\
\beta(m_1)\xi(v_1', q) - \beta(m_1^*)\xi(v_1', q^*) + \varsigma(v_1', q) - \varsigma(v_1', q^*).&
\end{aligned} \tag{5.8}$$

Also, we can conclude from (5.6) that $\beta(m_1^*) \leq \beta(m_1)$. ■

5.4.2 Contract Offer Stage

Last, we need to compute the contract that maximizes the seller' expected payoff. We first consider a contract as detailed in Lemma 5.2. It is optimal to put the lowest first stage buyer type on his reservation utility. The reservation utility of a type v_1 buyer is the expected payoff he receives from no contract at stage 1 followed by the seller's final offer. It is therefore not necessarily equal to 0. It can be expressed as

$$R(v_1) = \int_{v^*-v_1}^{\overline{v}_2} (v_1 + v_2 - v^*)\, dF_2(v_2).$$

Then, the lowest first stage buyer type's participation constraint can be written as

$$\beta\left((\underline{v}_1 + E[v_2]) - \int_{v^*-\underline{v}_1}^{\overline{v}_2} (\underline{v}_1 + v_2 - v^*)\, dF_2(v_2)\right) - p = 0.$$

Substituting for p in the seller's objective function we obtain

$$\beta\left((\underline{v}_1 + E[v_2]) - \int_{v^*-\underline{v}_1}^{\overline{v}_2} (\underline{v}_1 + v_2 - v^*)\, dF_2(v_2)\right) + (1-\beta)(1-F(v^*))v^*.$$

The first part of this expression is the price that he receives under a contract, the second part is the expected price he receives at renegotiation. If for example $(\underline{v}_1 + E[v_2]) < (1-F(v^*))v^*$, this expression is maximized by setting $\beta = p = 0$.

Now, let us consider a contract as detailed in Lemma 5.3.

The seller's objective function is

$$\int_{\underline{v}_1}^{\hat{v}_1} \left(p(m_1^*) + (1-\beta(m_1^*)) \int_{q^*-v_1}^{\overline{v}_2} q^*\, dF_2(v_2)\right) dF_1(v_1)$$
$$+ \int_{\hat{v}_1}^{\overline{v}_1} \left(p(m_1) + (1-\beta(m_1)) \int_{q-v_1}^{\overline{v}_2} q\, dF_2(v_2)\right) dF_1(v_1).$$

The first part of (5.9) is the expected payment he receives from first stage buyer types below $\hat{v}_1$. They send message m_1^* and pay a price $p(m_1^*)$ under the contract. With probability $1-\beta(m_1^*)$ trade does not occur before the final offer and all final buyer types $v_1 + v_2 > q^*$ accept the renegotiation offer q^*. The second part of (5.9) is the expected payment that the seller receives from first stage buyer types above $\hat{v}_1$. Using (5.7), we can write (5.9) as

$$p(m_1^*) + \int_{\hat{v}_1}^{\overline{v}_1} (\beta(m_1)\xi(\hat{v}_1, q) - \beta(m_1^*)\xi(\hat{v}_1, q^*) + \varsigma(\hat{v}_1, q) - \varsigma(\hat{v}_1, q^*))\, dF_1(v_1)$$
$$+(1-\beta(m_1^*)) \int_{\underline{v}_1}^{\hat{v}_1} \int_{q^*-v_1}^{\overline{v}_2} q^*\, dF_2(v_2)\, dF_1(v_1) + (1-\beta(m_1)) \int_{\underline{v}_1}^{\hat{v}_1} \int_{q-v_1}^{\overline{v}_2} q\, dF_2(v_2)\, dF_1(v_1).$$

From the lowest first stage buyer type's binding participation constraint we obtain

$$\beta(m_1^*)\xi(\underline{v}_1, q^*) + \varsigma(\underline{v}_1, q^*) - p(m_1^*) = \varsigma(\underline{v}_1, v^*)$$

and substituting this into the seller's objective function we derive the following linear expression in $\beta(m_1^*)$ and $\beta(m_1)$

$$\begin{aligned}
&\beta(m_1^*)\left(\xi(\underline{v}_1, q^*) - \xi(\hat{v}_1, q^*)(1 - F_1(\hat{v}_1)) - q^* \int_{\underline{v}_1}^{\hat{v}_1} (1 - F_2(q^* - v_1))\, dF_1(v_1)\right) + \\
&\beta(m_1)\left(\xi(\hat{v}_1, q)(1 - F_1(\hat{v}_1)) - q \int_{\underline{v}_1}^{\hat{v}_1} (1 - F_2(q - v_1))\, dF_1(v_1)\right) + \\
&\qquad \varsigma(\underline{v}_1, q^*) - \varsigma(\underline{v}_1, v^*) + (\varsigma(\hat{v}_1, q) - \varsigma(\hat{v}_1, q^*))(1 - F_1(\hat{v}_1)) + \\
&\qquad q^* \int_{\underline{v}_1}^{\hat{v}_1} (1 - F_2(q^* - v_1))\, dF_1(v_1) + q \int_{\underline{v}_1}^{\hat{v}_1} (1 - F_2(q - v_1))\, dF_1(v_1).
\end{aligned}$$

Now, the only possible solution is to set $\beta(m_1^*) = 0$ and $\beta(m_1) = 1$[7]. This simplifies the above expression to

$$\begin{aligned}
&\varsigma(\underline{v}_1, q^*) - \varsigma(\underline{v}_1, v^*) + q^* \int_{\underline{v}_1}^{\hat{v}_1} (1 - F_2(q^* - v_1)\, dF_1(v_1) \\
&\quad + \int_{\hat{v}_1}^{\overline{v}_1} \int_{\underline{v}_2}^{\overline{v}_2} \min(\hat{v}_1 + v_2, q^*)\, dF_2(v_2)\, dF_1(v_1)
\end{aligned} \tag{5.9}$$

Here, one can see two possible benefits of contracting for the seller. Because the contract allows the seller to credibly commit to a lower final price offer q^* after message m^*, he can ex-ante extract the possibly positive rent $\varsigma(\underline{v}_1, q^*) - \varsigma(\underline{v}_1, v^*)$ that the lowest first stage buyer type obtains from this decrease. Next, a contract might ensure trade with some high first stage buyer types who would not have traded for sure without a contract. First stage buyer types above $\hat{v}_1$ pay a lower price than without a contract because $\min(\hat{v}_1 + v_2, q^*) < v^*$, but more buyer types accept trade.

The next conditions are sufficient to guarantee that both benefits are 0. The conditions are far from being necessary, because we only need that the expression in (5.9) lies below $v^*(1 - F(v^*))$.

[7]From incentive constraints $\beta(m_1^*) \leq \beta(m_1)$. Then, setting both parameters equal to 0 violates (5.8), setting both equal to 1 is inconsistent with profit maximizing on the part of the seller.

Assume that uncertainty about v_1 is much larger than uncertainty about v_2 in the sense that $\overline{v}_2 - \underline{v}_2 << \overline{v}_1 - \underline{v}_1$. This makes it easier for the following assumption to be true

Assumption 5.1 *It exists a $\hat{\underline{v}}_1 < \overline{v}_1$, such that $\underline{v}_1 + \overline{v}_2 < q^*(\hat{v}_1) < \hat{v}_1 + \underline{v}_2$ for all $\hat{v}_1 \in [\hat{\underline{v}}_1, \overline{v}_1]$,*

The assumption implies two things. First, the lowest first stage buyer type can never expect to benefit from the reduced renegotiation offer that a contract entails because the offer always lies above his highest possible total valuation. Then, the first possible benefit of a contract is 0. Second, a set of high first stage buyer types obtains trade for sure even without a contract. Therefore, the second possible benefit of a contract is 0. Then, only the ratchet effect kicks in: The above expression (5.9) simplifies to

$$q^* \int_{\underline{v}_1}^{\overline{v}_1} (1 - F_2(q^* - v_1) \, dF_1(v_1) = q^*(1 - F(q^*)) < v^*(1 - F(v^*))$$

for all $\hat{v}_1 \in [\hat{\underline{v}}_1, \overline{v}_1]$. Therefore, within this interval the above expression is maximized for $\hat{v}_1 = \overline{v}_1$. Finally, if $q^*(\hat{\underline{v}}_1)(1 - F(q^*(\hat{\underline{v}}_1)))$ lies sufficiently below $v^*(1 - F(v^*))$, $\hat{v}_1 = \overline{v}_1$ is also the global maximum.

5.5 Conclusion

This last chapter models the fixed contracting costs assumed in the preceding chapter and derives a strict dominance result of the null contract. The main assumption is that messages must be verifiable in order to be included in a contract. Because, such a message system introduces a time gap between information revelation and execution of the contract, parties will make use of pareto improving renegotiation.

This last model leaves some open questions. The sequential screening model has a simple additive structure and the two variables are independent. A first question is of whether the results are robust and in how far we can generalize them. A more general structure such as in Courty and Li (2000) could be used. Also, the idea that correlation between v_1 and v_2 could strengthen the result, because the ratchet effect is reinforced, would be interesting to explore. Another way of extending the result is to assume different reasons for contracting than the ones considered here, such as risk-sharing or investment, and see whether the ratchet effect still can make contracting obsolete. It seems unlikely to me at this stage, because the proofs of Lemmata 5.2, 5.3 and Proposition 5.1 rely heavily on the linearity of the problem (i.e. risk neutrality and no discounting) which would be destroyed by assuming risk aversion or investment (except perhaps if investment increases the surplus by a fixed amount).

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