The Structure of Organisation under Adverse Selection: Informed Principal and Collusion.

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Abstract.

This thesis presents new results in the theory of organisation under adverse selection, in particular in the theory of informed principal and of collusion.

In Chapter 1 we analyze a simple adverse selection model with one principal and one agent. They are both risk neutral and have private information about their type. We assume that the type of the principal is correlated with the one of the agent. The main result of the chapter is that the principal can extract a larger share of the surplus from the agent than in the case where her information is public.

In Chapter 2 we study a model of informed principal with private values where the principal is risk neutral and the agent is risk averse. We show that the principal gains by not revealing her type to the agent through the contract offer. Moreover the allocation chosen by the principal embodies more risk for the agent than the standard second best solution.

In Chapter 3 we study the delegation of a production process in a three-tier hierarchy. We allow the principal to costlessly monitor the communication, at the sub-contracting stage, between the lower levels of the hierarchy. We study two possible scenarios, one in which the principal observes the menu of sub-contracts offered by the first agent and the other in which she observes the report from the second agent. In both cases the monitoring damages the first agent and reduces production inefficiency. We then study how the agent can change the subcontract offer in an attempt to conceal the information that is monitored by the principal.

In Chapter 4 we study a simple model with adverse selection where one principal contracts with two agents that can write collusive agreements. We assume that the principal does not know the distribution of the bargaining power at the side-contracting stage. We show that the bargaining strength of the agents does not affect the collusion proof equilibrium and discuss some possible applications.

To GianPrimo, Mariuccia and Martina

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Introduction

Until the late 1960s firms and organisations were seen as black boxes by economic theorist, they were treated as single economic agents with a clearly defined maximising behaviour. One cannot say the same about other disciplines like sociology and also political science, that have always recognised that firms and organisations are very complex entities and by treating them as a single block a social scientist was missing in insight and running the risk of not understanding their role in our society.

The focus of economic research in the last 40 years has moved forward taking a closer look at firms and organisations: by now, organisational economics is an established branch of economics. Organisation theory has actually become a very complex subject that interacts with many other fields like industrial organisation, labour economics and institutional economics. The collaboration of these different fields of economics has allowed to give answers (at least preliminary ones) to open questions like the determinants of authority in organisation, or what determines the boundaries of firms, or how members of an organisation move between levels and build careers without using the external labour market.

This thesis belongs to another branch of organisation theory, the branch that studies how organisations deal with conflicting interests and private information, known as incentive theory or contract theory. This theory recognises that, in complex environments, delegation of tasks is a necessity and that the members of an organisation may tend to pursue different objectives. The issue of realigning the interests of the members with those of the organisation they belong to becomes a serious and difficult problem when there is imperfect information about agents' actions or characteristics.

The analytical instrument chosen to study these incentives problems is the "Principal-Agent" paradigm. In its basic form, an uninformed party (called the Principal) faces a privately informed party (called the Agent), whose information is important for the efficiency of the transaction. The paradigm allows the avoid issues of bargaining under asymmetric information by assuming that the Principal makes a "take-it-or-leave-it" offer to the agent.

We focus on problems where imperfect information is about some characteristics of the agent or the technology he uses, the case which is also known as adverse selection. The main problem when adverse selection is present is how to elicit the agent's private information, which is essential in order to use economic resources efficiently. This information can be extracted only by giving up some informational rent to the agent, which is costly to the principal. This costs adds up to the standard costs of performing a particular economic activity and is the cause of the distortions in the volume of trade achieved under asymmetric information. At the optimal second-best equilibrium, the principal optimally trades off between allocative efficiency and information rents given up to induce the revelation of information.

This thesis presents some new theoretical results in the theory of organisations when adverse selection is an issue. These results belong to branches that have modified slightly the standard principal-agent model to be able to describe more complex situations often observed in the real world: the theory of informed principal and the theory of collusion in organisation.

Part I presents two results in the theory of informed principal, which studies strategic interactions between two privately informed players, one of which (the

Principal) is assumed to have all the bargaining power when making a contract offer to the other one (the Agent).

This branch of information economics has been somehow neglected by most of the contract theory literature, the adverse selection one in particular, that uses the above mentioned Principal-Agent paradigm, where only the agent has private information, while the party that designs and proposes the contract is assumed to be uninformed.

Nonetheless many economic situations really fit in the double-sided asymmetric information setup that we are studying. With this we mean that, quite often, both parties involved in an economic relationship possess private information that is relevant to carry out a profitable transaction. We can find these situations almost in any field studied so far by contract theory including procurement settings, regulation problems, provision of public goods by the government or even in simple monopolist problems.

We believe that studying informed principal problems is quite interesting also because some of the insights provided by the standard principal-agent models no longer hold when double-sided asymmetric information is taken into account. This should help a deeper understanding of established theories by highlighting the robustness of some results or the weakness of others.

Informed principal problems are in general more complicated that the standard one-sided models because an informed principal, besides providing the right incentives to the agent, has to manage her private information in an optimal way.

In simple words, the principal has to decide what to do with her private information. She can either reveal it at the beginning through the contract offer, but then this is to be done in an incentive compatible way, or conceal it till a later stage, but then she has to be careful because any action that she takes during the game can signal something about her type to the agent.

A more complete review of the brief literature on the theory of informed principal is presented in the Introduction to Chapter 1, but here it is worth mentioning that we fall in the stream of literature which has begun with the seminal works by Maskin and Tirole [1990, 1992] that have set the basis for the non-cooperative analysis of informed principal problems.

The most related to this work is Maskin and Tirole [1990] where they study the *private values* framework, a model where the type of the principal does not directly influence the utility function of the agent. They assume that types are independently distributed and that payers have generic utility functions. They prove that the principal gains by not revealing her type through the contract offer and by waiting until a later stage to reveal it, simultaneously with the agent, right before contracts are implemented. Towards the end of the paper, they show that this gain disappear when both the principal and the agent have quasilinear utility functions.

In Chapter 1 of this thesis we analyse a simple two-sided adverse selection model with one principal and one agent. Both are risk neutral and have private information about their type. We also assume that the private information of the principal is correlated with the one of the agent. The main result of the chapter is that the principal can extract a larger share of the surplus from the agent if she does not reveal her type through the contract offer. The principal can design such a contract because she exploits the fact that her type is an informative signal on the agent's one. We fully characterise the equilibrium of the principal agent game in which different types of principal offer the same menu of contracts that leave the agent uninformed about the principal's type. This gives more freedom to the principal when setting the transfers, because the agent's constraints need to hold only at an interim stage. The principal gains from a peculiarity of the correlated environment: different types of agent have different beliefs about the probability distribution over the states of the world.

In the Chapter 2 we study a model of informed principal with private values where the principal is risk neutral and the agent is risk averse and types are independently distributed. We show that the principal, regardless of her type, gains by not revealing her type to the agent through the contract offer. Moreover the allocation chosen by the principal embodies more risk for the agent than the standard second best solution. In other words the optimal contract will result in an equilibrium allocation that involves a larger spread in the ex-post payoffs of the agent, forcing him to take up more risk when he accepts the contract.

We believe this is one example of the failure of some of the usual features of the one sided standard screening models when double sided asymmetric information is introduced. A risk neutral informed principal does not reduce any longer the risk that a risk averse agent has to face and the risk aversion of the agent plays a role on the solution of the trade-off between rents and incentives that defines the downward distortions in the physical production.

Part II moves away from the two players setting and studies frameworks where there are one principal and two agents and where it is assumed that agents can write binding agreements between themselves. They do it either because they collude against the principal, and try to jointly manipulate their reports, or because one has been put in charge of contracting with the other by the principal. The contracting between the agents is therefore "allowed" by the principal (delegation), but then the incentives to lie for the agent that directly deals with the principal are very similar to those present in a collusion framework.

In both collusion and delegation framework the principal needs to give incentives to coalitions of agents and not only to the single agents separately. This generally involves giving up more informational rents than when agents cannot write contracts between themselves and distorting production even more than in the second best.

This field of contract theory is also relatively recent and has borrowed ideas from the fields of economic and labour sociology. They had long recognised that members of organisations and hierarchies can form easily either vertical or horizontal "cliques" that can weaken the functioning of organisation itself. This work is in the stream of literature on collusion and delegation in hierarchies which started with Tirole [1986][1], who gave a clear cut to the way in which organisations and hierarchies were studied in economic theory. A distinctive feature, is that organisations are no longer considered as single blocks, but like networks of overlapping and nested principal-agent relationships where coalition formation and side-contracting are allowed.

In Chapter 3 we study the delegation of a production process in a three-tier hierarchy. The principal contracts directly only with the agent that produces the final good, leaving him in charge of the contract for the production of the intermediate good. Both agents have private information about their marginal costs of production. Delegation reduces the burden of communication on the principal but on the other hand it also introduces additional incentives problems and costs due to the loss of control over lower levels of the hierarchy. The principal can then try to reduce these costs by regaining some control over the "subordinates", one way to do so is monitoring these delegated relationships to acquire information.

We study two possible scenarios, one in which the principal observes the menu of sub-contracts offered by the first agent and the other in which she can observe the report from the second agent to the first one. In both cases, the monitoring damages the first agent who, as a consequence, reacts by modifying his sub-contracts in such a way that the principal's benefit from monitoring is greatly reduced or even nullified.

Finally, in Chapter 4 we study a simple model with adverse selection where one principal contracts with two agents that can write collusive agreements. We assume that the principal does not know the distribution of the bargaining power at the side-contracting stage.

[1] On collusion in hierarchies see also Tirole [1992] and Laffont and Martimort [1997, 2000].

The main result of the chapter is that bargaining powers are irrelevant, and their possible diversity, joint with the fact that the principal is uninformed about them do not affect the collusion proof equilibrium allocation. This result owes to the fact that the information about the two agents' types, and its transmission, is all that matters: the difference in bargaining power does not affect that information. We show that a Weak Collusion Proofness Principle still holds and that the constraints that ensure the equilibrium collusion proofness do not depend from the bargaining power at the collusive stage. Therefore, any distortion and asymmetry that may be present at the side-contracting stage disappears in the optimal collusion proof contract offered by the principal.

Part I

Informed Principal

Chapter 1

Informed Principal with Correlation

1.1 Introduction.

Most of the mechanism design theory is built around the hypothesis that there is an uninformed party (the principal) that contracts with an informed party (the agent). The principal offers a contract that the agent accepts or rejects, therefore the main problem for the principal is to find the optimal contractual way to elicit the agent's private information. Although this paradigm has been applied to many different contexts and has proved itself to be quite powerful in explaining many economic interactions, in some circumstances, the assumption of one-sided private information is sometimes restrictive. One example is the provision of a public good, where usually the lack of information rests with the government (principal) regarding citizens (agents) private evaluations. However it is likely that the government possesses superior information about the cost of supplying the good. Also in a regulated market the authority (principal) may have private information about the market demand for the regulated good even if it does not know the costs incurred by the firms (agents). Finally, a discriminating monopolist may have private information about the quality of the good it provides. In all these situations we face what is known in the literature as an "informed-principal" problem.

Once we assume double sided private information the problem that an informed principal has to solve is considerably more complicated. Rent extraction is not anymore the only worry when designing a contract, a principal has some private information that needs to be managed optimally. She has to decide whether to make it public, and, if so, she has to do it in a credible way. Interestingly, a principal might be worse off by having this private information; in fact in some circumstances she may not be able to use it to her benefit. The main problem in this respect comes from the signalling content that any action taken by the principal may have for the agent.

In this work we analyze a simple two sided adverse selection model with one principal and one agent. They both have private information about their types and a type-dependent quasilinear utility function. We also assume that the private information of the principal is correlated with the one of the agent. In other words after observing her private information the principal updates her beliefs on the agent characteristics. The same is true for the agent.

The main result of the paper is that the principal can extract a larger share of the surplus from the agent than in the case where her information is public. Intuitively, the principal can design such a contract because she exploits the fact that her type is an informative signal of the agent's one.

The existing literature, in particular Maskin and Tirole [1990], has explored this problem in details focusing on the risk sharing benefits to the principal of hiding her private information. They propose the interpretation of this principal-agent game as a fictitious exchange economy where the different types of principal trade the slackness on the agent's constraints. They show that there exists a Walrasian equilibrium of this economy and the main reason driving this exchange is risk sharing.

Different types of principal have different attitudes with respect to the risk of facing a "bad" type agent, therefore some arbitrage is mutually beneficial. Hence when the principal is risk averse there exists a strict gain from trade among the different types of principal. If the principal reveals her type when offering the contract, then the incentive compatibility and individual rationality constraints have to be satisfied for each type and hence no risk sharing possibility arises. If, instead, a risk averse principal hides her private information then she can capture the gains of sharing risk with other types of principal. Clearly, if the principal is risk neutral those gains are not present and hence there is no gain from hiding her type. It is for this reason that in their framework when the utility functions of both parties are assumed to be quasilinear there is no possibility of obtaining a higher payoff: if the different types of principal are risk neutral, they do not benefit from any risk-sharing activity.

In this paper we depart from this analysis by assuming that the principal is risk neutral and that her private information is correlated with the one of the agent. Obviously, in this case, there exist no gains from risk sharing. However, we show the principal can still gain from hiding her private information.

Since at the initial stage the agent remains uninformed about the principal's type, his constraints will have to hold only in expectation and this gives more freedom to the principal who can gain by relaxing these constraints in some states of the world. The peculiarity of this correlated environment is that even if the principal is risk neutral the relative costs of satisfying the agent's constraints is different across types of principal. When there is correlation even if principals of every type are all risk neutral the marginal rate of substitution between different states of the world is different across types, so there exists a relative price that makes the exchange beneficial. Obviously the principals do not trade to share risk, they trade because one values relatively more the slack on a constraint than the other does. In other words there exist market prices such that the principals buy something on the market that is cheaper than their private evaluation and sell something that they value relatively less than the market. In this way they can save on the transfers given to the agent.

This can be best seen in the characterisation of the equilibrium of the principal agent game when we show that if both types of principal make the same contract offer, they achieve a higher payoff since they manage to reduce the transfers, while the physical allocation remains the same. As a consequence the informational rent of the agent is reduced in favor of an increase in the expected payoff of the principal, who is able to extract more surplus.

We are certainly not the first to suggest that a principal that exploits the informativeness of a private information of individuals, other that the agent, can reduce the rent transferred to the agent (Crémer and McLean [1985,1988], McAfee and Reny [1992]). In particular these papers show that an uninformed principal facing many privately informed agents, whose types are correlated, can design a mechanism that extracts all the surplus from the agents. The principal can then achieve the first best because there is no more need to trade off rents with efficiency. The mechanism uses a sort of yardstick competition among the agents created by constructing lotteries that consist of payments conditional of the announcements of the other agents. The key difference between these papers and our analysis is in the incentive compatibility constraints for the various types of the principal. Indeed, the principal's information in our framework is private to her. This implies that any use the principal makes of this information in the optimal mechanism has to be incentive compatible. Satisfying these incentive compatibility constraints introduces an extra degree of complexity in our analysis and prevents the principal form extracting all the surplus form the agent as in Crémer and McLean [1985,1988] and McAfee and

Reny [1992].

The structure of the chapter is as follows. In Section 1.2 we discuss the literature on informed principal. In Section 1.3 we present the setup, we discuss the structure of the game and we study the benchmark of full information. In Section 1.4 we show that in our fictitious exchange economy there exists a set of equilibria that Pareto dominates the full-information outcome. In Section 1.5 we characterise the equilibrium of the principal-agent game. In Section 1.6 we present an example where, we believe, it is possible to appreciate the main results in a concise and transparent way. Section 1.7 concludes.

1.2 Related literature.

The pioneering work in this field is Myerson [1983], where the analysis encompasses both cooperative and non-cooperative approaches. The author lays down an axiomatic theory and defines which conditions an "inscrutable" mechanism needs to satisfy to be a reasonable selection for all the principal's types ("unblocked mechanism").

The papers that are closer to our work are Maskin and Tirole [1990,1992] that provide a full analysis of the non-cooperative theory of informed principal. Their first article studies what they have defined as the *private values* model, where the players have generic utility function and the type of the principal is not an argument of the agent's utility function, while in the second one, the *common values* model, the principal's type directly affects the agent's utility. Both models address the same question, but the distinction between private and common values has important consequences on the characteristics of the equilibrium contract. In both papers they study whether it is possible to achieve a Pareto efficient equilibrium from the point of view of all the types of principal. For this reason it is important to understand what is incentive compatible for the principal; unlike in other studies, here the principal is unable to commit, so that all the revealed information will have to satisfy some truthtelling condition. Then it will be fundamental to understand at what stage it is better for the principal to reveal her information.

The private values case (which fits the public good and the regulation examples) is much simpler to analyze. The main feature is that the optimal contract that each type of principal would offer in the case her type was publicly known (defined as the *full information* case) is incentive compatible and represents the lower bound to the expected payoff of the principal. The idea is to see whether the principal can secure herself a higher payoff when she has private information about her type. Their results prove that indeed this is possible, and that it is generically Pareto efficient for all types of principal to offer the same menu of contracts, in such a way that the agent does not learn anything from the offer and the principal gains by pooling his incentive and participation constraints. Signalling is not an issue because the different types of principal are not in "competition" with each other when dealing with the agent.

When we are dealing with a common values environment (the case of the discriminating monopolist falls in this category) some complications arise. First of all the type of the principal directly affects the utility function of the agent and therefore now there exist *good* and *bad* types of principal from the point of view of the agent. The agent can in fact be damaged by the principal's private information and as a consequence the full information contract may not be incentive compatible for all types of principal. It will be the case that bad types can gain from claiming to be good types. It is clear that there is some form of competition among the different types of principal and risk sharing is no longer the driving force in this kind of models. The main issue is signalling and Maskin and Tirole [1992] show that the equilibria in such framework will not necessarily be Pareto efficient.

Our setup clearly bears some similarities with the framework analyzed by Maskin and Tirole [1990], which are worth discussing, to understand the different forces driving somewhat similar results. First of all we assume private values and this means that the full-information allocation is incentive compatible for all the different types of principal. This has strong consequence of avoiding rivalry across principals. In the standard model with uncorrelated private types, the only link between the agent and the principal is provided by the contract which is contingent on both types. In our case, though, correlation of types creates some other connection in that one's type is informative on the other party's type. Still this does not make it a common values model, because each player remains directly affected only by his own private information. In common values the agent's utility function depends on the principal's type but not vice-versa, in our framework the new link created by correlation is perfectly symmetric: in the same way as the principal receives a signal on the agent's type so does the latter.

As mentioned in the introduction above, our work is related to the literature on mechanism design in the presence of correlated signals: Crémer and McLean [1985,1988], McAfee and Reny [1992] and Riordan and Sappington [1988]. Crémer and McLean [1985,1988] show that the principal can exploit the correlation of the agents' types and construct a mechanism which extracts all the agents' informational rents and achieves the complete information optimum. McAfee and Reny [1992] bring the analysis a step further by proving that the surplus extraction results hold in many mechanism design environments and also that the same results apply when the private information of the agents belongs to a continuum set. Finally Riordan and Sappington [1988] show that a principal can achieve the first best outcome if she can condition a contract on a signal correlated to the type of the agent and that will be made public ex-post.

The difference is that, in all these papers, the additional correlated signal is either known and verifiable at an ex-post stage (Riordan and Sappington [1988]) or is private information of other agents (Crémer and McLean [1985,1988], McAfee and Reny [1992]) but it is not private information of the principal. This implies that "...the principal is able to use the information that the buyers have about each other as if it were his own..."[1] without worrying about the principal's incentives to report her private information correctly. Unfortunately, as discussed above, this is not true in our framework and therefore the optimal mechanism we derive cannot extract the whole of the agent's surplus, or implement the first best.

1.3 The model.

We are going to compare our analysis to the results of Maskin and Tirole [1990] and also use some of their results, therefore the set-up and the notation will be kept as close as possible to the ones in their contribution.

1.3.1 Objective functions and information.

There are two players, a principal and an agent. The principal has a quasi-linear utility function $V^i = \phi^i(y) - t$, where y is an observable and verifiable action, t is a monetary transfer from the principal to the agent, and i is a parameter that represents the principal private information or type. $\phi^i(\cdot)$ is continuous, increasing and concave in y.

The agent has a quasi-linear utility function $U_j = t - \psi_j(y)$, where j represents the agent's type. It is worth noting that U does not depend from j, the principal's private information, this assumption is important and places our model in the private values framework. $\psi_j(\cdot)$ is increasing and convex in y. Moreover we assume that U_j decreases with j, this means that:

$$\psi_1(y) < \psi_2(y) \text{ for all } y.$$

[1] Crémer J. and R.P.McLean (1985), pg.346.

Moreover $\frac{\partial\psi_1(y)}{\partial y} \leq \frac{\partial\psi_2(y)}{\partial y}$ for all y.

The agent's reservation utility is normalised to zero.

This abstract set-up fits well the following real world situation: a buyer (the principal), with private information about her preferences for a good, offers a contract to a seller (the agent), that produces the good and has private information about his production costs. In that case y will be the quantity sold and t the price paid by the buyer.

In what follows we may use μ_j^i to indicate the pair (y, t) selected by the mechanism when the principal is of type i and the agent of type j.

To guarantee the existence of equilibrium, we assume that the feasible actions and transfers lie in compact and convex sets.

The parameters i and j are drawn from a joint discrete distribution which is common knowledge. We suppose that each parameter can assume only two values, therefore there are only four possible states of the world. The Principal's prior beliefs about the joint distribution of types are:

$$\begin{aligned} p_{11} &= \Pr(i = 1, j = 1) \\ p_{12} &= \Pr(i = 1, j = 2) \\ p_{21} &= \Pr(i = 2, j = 1) \\ p_{22} &= \Pr(i = 2, j = 2) \end{aligned}$$

The agent's prior beliefs are identical to those of the principal and denoted by π_{ij}.

We then define $\rho = p_{11}p_{22} - p_{12}^2 \neq 0$ as the correlation coefficient between the two player's information, when it is positive it means that it is relatively more likely that they are of the same type, when negative "mixed" pairings are more likely. For simplicity we also assume $p_{12} = p_{21}$.

We are therefore assuming that each player's type is an informative signal of the other's type. As a consequence conditional distribution of the agent's type are different for the two types of principal, the same is true for the agent.

As in Maskin and Tirole the limitation on the possible types for the players is not essential but simplifies the analysis and favors the intuition of the results.

1.3.2 The principal-agent game.

The timing of the principal-agent game is as follows:

1. The principal proposes a mechanism in the feasible set M to the agent. A mechanism m in M will specify i) a set of possible messages for each party and ii) for each pair of messages chosen simultaneously an allocation (y, t). Note that the set M includes the set of direct revelation mechanisms in which parties simultaneously announce their types, by invoking the revelation principle for Bayesian game we can restrict the attention to direct truthful mechanisms.[2]

2. The agent updates his prior (if he has learned something from the offer)[3], accepts or refuses the contract offered. If he refuses both players get zero utility and the game ends. If the agent accepts, the principal updates her beliefs, and the parties move to the last stage of the game.

3. Both parties announce their types and the proposed mechanism is implemented.

We will study the perfect Bayesian equilibria of the overall game.

[2]In this framework (as in Maskin and Tirole [1990]) the principle states that for any mechanism and for given beliefs any equilibrium of the mechanism is equivalent to an equilibrium of a direct revelation mechanism in which types are truthfully announced.

[3]We are denote by $\hat{\pi}_{ij}$ the updated beliefs of the agent

1.3.3 The case of full information.

As a benchmark we study the equilibrium when the principal's information is common knowledge. Maskin and Tirole call it the *full information* case (even if the principal does not know the agent's type) and it is nothing more than the standard screening model.

We know from the revelation principle that every equilibrium allocation of this game can be obtained as an equilibrium of a direct truthful mechanism. The outcome μ_j^i that will be implemented in equilibrium will have to satisfy two types of constraints individual rationality and incentive compatibility.

For every i the participation constraints are: $U_j\left(\mu_j^i\right) \geq 0$ for $j = 1, 2$. While the truthtelling constraints are: $U_j\left(\mu_j^i\right) \geq U_j\left(\mu_k^i\right)$ for all j,k.

Standard arguments apply, and in this context only two constraints are binding, the participation constraint of type 2 and the incentive compatibility of type 1.

Therefore in the case of full information a principal of type i proposes a contract $\{\mu_1^i, \mu_2^i\}$ that solves the following program:

$$(F^i)\begin{cases} \max\limits_{\{\mu_j^i\}} \sum\limits_{j=1}^{2} p_{ij} V^i\left(\mu_j^i\right) \text{ such that} & \\ \text{IR}^i : U_2\left(\mu_2^i\right) \geq 0 & (\rho^i) \\ \text{IC}^i : U_1\left(\mu_1^i\right) \geq U_1\left(\mu_2^i\right) & (\gamma^i), \end{cases}$$

where ρ^i and γ^i are the Lagrange multipliers for the IR and IC constraints.[4]

Given the specific functional forms chosen for the utility functions of the two players we can actually find the precise solution to this problem.

A principal of type i will offer the following decreasing schedule of output and the respective transfers, $(y_{i1}, y_{i2}, t_{i1}, t_{i2})$:

[4] A more detailed solution of the full-information problem can be found in the Appendix.

$$\begin{aligned}\phi^{i\prime}(y_{i1}) &= \psi_1'(y_{i1}) \text{ and } t_{i1} = \psi_1(y_{i1}) + (\psi_2(y_{i2}) - \psi_1(y_{i2})) \\ \phi^{i\prime}(y_{i2}) &= \psi_2'(y_{i2}) + \frac{p_{i1}}{p_{i2}}\left(\psi_2'(y_{i2}) - \psi_1'(y_{i2})\right) \text{ and } t_{i2} = \psi_2(y_{i2}).\end{aligned}$$

As one could expect, the solution preserves standard characteristics like the "no distortion at the top" property and no informational rent for the "bad" agent.

For future reference denote by $\left(\overline{\mu}^i_\cdot, \overline{\rho}^i, \overline{\gamma}^i\right)$ the solution to the full information program $\left(F^i\right)$ and let $\overline{v}^i \equiv \sum_j p_j V^i\left(\overline{\mu}^i_j\right)$ be the type i principal's payoff.

Moreover at the full-information allocation the ratios of the Lagrange multipliers of the two types of principal are different ($\frac{\overline{\gamma}^1}{\overline{\rho}^1} = \frac{p_{11}}{p_{12}+p_{11}} \neq \frac{\overline{\gamma}^2}{\overline{\rho}^2} = \frac{p_{12}}{p_{12}+p_{22}}$) meaning that the relative cost of fulfilling the individual rationality and incentive compatibility constraint is not the same across principals. This fact is going to be extremely important to prove the results in what follows.[5]

A feature, common to all the private values models, is that, regardless of the agent's information about the principal's type, $\overline{v}^i$ provides a lower bound to the type i principal's equilibrium payoff. This means that the full information contract is incentive compatible[6] for each type of principal. We are going to show that it is possible to find equilibria that improve on this payoff.

1.4 The fictitious exchange economy and the Walrasian equilibria.

The problem we are studying changes considerably if the type of the principal is private information. From the previous section we know that offering the full infor-

[5] The ratio of the Lagrange multipliers would be the same for the two types of principal if types were independently distributed.

[6] Each type of principal maximises his expected payoff over the same set of constraints, therefore they cannot do better by claiming to be of another type and implement this other type's contract.

mation contract is a possibility since it is incentive compatible. Another possibility for the principal is to hide his type until the implementation stage, in this way when the agent accepts the contract he would not know with certainty the state of the world. He would know his type and have a belief on the type of the principal which coincides with the prior probabilities if he has not learnt anything from the contract offer. We are going to show that for the different types of principal it is possible to design a mechanism that Pareto dominates the full information allocation. It is going to achieve this by "pooling" the agent's IR and IC constraints over the different types of principal, i.e. by having the constraints hold only in expectation rather than for each single type.

This methodology leads to the study of the Walrasian equilibria of the fictitious pure exchange economy where the traders are the two types of principal that exchange the slack variables of the agent binding constraints. When we are in the full information framework trade is not possible because the constraints have to be satisfied ex-post in every state of the world and slackness on them is not allowed (as if markets were totally absent). As soon as the agent does not know the type of the principal then his constraints have to hold in expectation, offering the principals (the different types) the possibility of exchanging slackness (as if markets were now open and complete).

The following proposition[7] introduces the idea of existing gains from the trade of slackness in our "economy".

Proposition 1.1 *When utility functions are quasilinear and there exists correlation between the information of the principal and the one of the agent, there exists an allocation that satisfies interim IR and IC constraints for the agent and that Pareto dominates the full-information allocation $\overline{\mu}$ (from the perspective of the different types of principal).*

[7]This is parallel to Proposition 1 in Maskin and Tirole [1990].

Proof. Consider the solution $(\overline{\mu}^i_\cdot, \overline{\rho}^i_\cdot, \overline{\gamma}^i_\cdot)$ to (F^i), we have shown already that $\frac{\overline{\gamma}^1}{\overline{\rho}^1} \neq \frac{\overline{\gamma}^2}{\overline{\rho}^2}$.

For an allocation $\mu^i_\cdot$, define now $r^i(\mu^i_\cdot)$ and $c^i(\mu^i_\cdot)$ as the negatives of the slack variables associated with the IR^i and IC^i constraints:

$$\begin{aligned} r^i(\mu^i_\cdot) &\equiv -U_2(\mu^i_2) \\ c^i(\mu^i_\cdot) &\equiv U_1(\mu^i_2) - U_1(\mu^i_1). \end{aligned}$$

In particular, $r^i(\overline{\mu}^i_\cdot) = 0$ and $c^i(\overline{\mu}^i_\cdot) = 0$; in fact in the full information problem the constraints have to be satisfied state by state therefore the slack variables in each of them have to be necessarily equal to zero. In case the offer of the contract is not fully revealing then, as we said before, then the constraints would have to be satisfied only in expectation. In terms of slack variables as we just have defined them the IR and IC constraints can be expressed as:

$$IR: \sum_i \dot{\pi}_{ij} r^i(\mu^i_\cdot) \leq 0 \text{ and } IC: \sum_i \dot{\pi}_{ij} c^i(\mu^i_\cdot) \leq 0.$$

Which says that the negatives of the slack variables need only be non-positive on average, and not for each type of principal. More precisely, in the case of only two principal's types, the above conditions are equivalent to:

$$r^2 = -\frac{\dot{\pi}_{21}}{\dot{\pi}_{22}} r^1 \text{ and } c^2 = -\frac{\dot{\pi}_{11}}{\dot{\pi}_{12}} c^1.$$

Consider now the following perturbed version of the full information program:

$$
(F_*^i)\begin{cases}\max\limits_{\{\mu_j^i\}}\sum\limits_{j=1}^{2} p_{ij}V^i\left(\mu_j^i\right) & \text{such that}\\ U_2\left(\mu_2^i\right)\geq -r^i \\ U_1\left(\mu_1^i\right)\geq U_1\left(\mu_2^i\right)-c^i\end{cases}
$$

It is evident that the only difference from the full-information program is that now there is some slack allowed on each constraint.

Let v_*^i be the maximised value of the maximand, by definition of the shadow prices $\overline{\rho}^i$ and $\overline{\gamma}^i$ it approximately equals $\overline{v}^i+\overline{\rho}^i r^i+\overline{\gamma}^i c^i$ for small values of r^i and c^i. Let μ_*^i be a solution to F_*^i.

Choose negative slack variables $\left(r^1,c^1\right)$ for the type 1 principal; then the slack variables for type 2 are defined according to the above conditions. Therefore we can now write:

$$
\begin{aligned}
v_*^1-\overline{v}^1 &\simeq \overline{\rho}^1 r^1+\overline{\gamma}^1 c^1\\
v_*^2-\overline{v}^2 &\simeq \overline{\rho}^2\left(-\frac{\dot{\pi}_{21}}{\dot{\pi}_{22}}r^1\right)+\overline{\gamma}^2\left(-\frac{\dot{\pi}_{11}}{\dot{\pi}_{12}}c_1\right)
\end{aligned}
$$

The left hand sides of the above equalities are both positive if:

$$
\begin{cases}\frac{\overline{\rho}^1}{\overline{\gamma}^1}>-\frac{c^1}{r^1} \\ -\frac{c^1}{r^1}>\frac{\overline{\rho}^2}{\overline{\gamma}^2}\frac{(\dot{\pi}_{12})^2}{\dot{\pi}_{11}\dot{\pi}_{22}}\end{cases}\quad \text{when } r^1>0\ ,
$$

and:

$$
\begin{cases}\frac{\overline{\rho}^1}{\overline{\gamma}^1}<-\frac{c^1}{r^1} \\ -\frac{c^1}{r^1}<\frac{\overline{\rho}^2}{\overline{\gamma}^2}\frac{(\dot{\pi}_{12})^2}{\dot{\pi}_{11}\dot{\pi}_{22}}\end{cases}\quad \text{when } r^1<0\ ,
$$

We know, from the solution to the full information case, that $\frac{\overline{\gamma}^1}{\overline{\rho}^1}=\frac{p_{11}}{p_{12}+p_{11}}$ and $\frac{\overline{\gamma}^2}{\overline{\rho}^2}=\frac{p_{12}}{p_{12}+p_{22}}$ and if we substitute these values in the above conditions (remembering that the priors were the same for principal and agent) we get that both sets are satisfied if:

$$\rho > 0 \text{ (for } r^1 > 0) \text{ and } \rho < 0$$

which hold by assumption.

We have therefore shown that μ_*^i solution to (F_*^i) Pareto dominates the full information allocation $\overline{\mu}^i$ from the perspective of both types of principal, when the correlation is non zero. ■

The intuition of Proposition 1 is relatively simple. In the full information case the agent's constraints have to be satisfied for each type i of principal; if we introduce a small amount of slack $-r^i$ and $-c^i$ on these constraints then the principal can obtain a payoff

$$\overline{v}^i + \overline{\rho}^i r^i + \overline{\gamma}^i c^i.$$

As long as $\sum_i \dot{\pi}_{ij} r^i \left(\mu_\cdot^i\right) \leq 0$ and $\sum_i \dot{\pi}_{ij} c^i \left(\mu_\cdot^i\right) \leq 0$, then the agent's constraints hold in expectation. We can choose $\left(r^1, c^1, r^2, c^2\right)$ in such a way that $v^i - \overline{v}^i$ is strictly positive for $i = 1, 2$. The allocation $\mu_{\cdot *}^{\cdot}$ corresponding to this choice then Pareto dominates $\overline{\mu}_\cdot^\cdot$.

A useful and fruitful interpretation is thinking of $\mu_{\cdot *}^{\cdot}$ as being generated by the different types of principal "trading" slack variables. In that case the full information allocation corresponds to autarchy.

The idea is that if different types of principal have different shadow values for the constraints that means that they value differently the relative slackness on the constraints and they can gain from trading it. This means that the two types of principal have different marginal rates of substitution between different state of the world, because given their type they attach different probability to each state of the world. They are allowed to trade slackness only if the constraints for the agent have to hold in expectation which can happen when the principal does not reveal her type at the contract offer stage. This amounts to having different marginal rates of substitution between two states of the world and not disclosing information allows to exploit advantageous trading opportunity, which would be unavailable otherwise.[8]

This Pareto improvement is available to the types of principal also when their utility function his quasilinear and this is due to the correlation between the private information of the principal and the one of the agent.

In the quasilinear case when types are independently distributed, as in Maskin and Tirole [1990], the ratio of the shadow values of the full information case is the same for both types of principal therefore no gain from trade exists in that exchange economy. But the equality of these ratio is not due only to the specific functional form of the utility function but also to the independence hypothesis. These ratios represent the probability of the agent being of type 1 given the type of the principal, because of correlation these ratios have to be different for different types of principal.

Therefore correlation allows principals of different types with a quasilinear utility function to benefit from the trade of slackness across different states of the world.

In the work Maskin and Tirole [1990] the quasilinear case represented a subset of the more general framework (with generic utility functions) in which the possibility for the principal of gaining from concealing their private information did not hold. As mentioned in the introduction, they trade for risk sharing reasons, so when the principal is risk neutral there are no gains to be made from risk sharing. Adding correlated types to the picture puts the quasilinear case back in line with their main results, even if the motivation for trading is different. The principals trade because they have a higher valuation then the market for the slackness they buy, and lower for the one they sell.

[8] In the literature this fact is also known as "Hirshleifer" effect (see Hirshleifer [1971]).

Let now $V_I^i\left(r^i, c^i\right)$ be the principals' indirect utility when there is slack $-r^i$ and $-c^i$ in the agent's participation and incentive compatibility constraints,respectively. Thus $V_I^i\left(r^i, c^i\right)$ is the value function of the perturbed full information problem $\left(F_*^i\right)$ we introduced in the Proof of Proposition 1 and that we can rewrite with our specific functional form as:

$$\left(F_*^i\right)\begin{cases} \max\limits_{\{y_{ij},t_{ij}\}} \sum\limits_{j=1}^{2} p_{ij}\left(\phi^i\left(y_{ij}\right)-t_{ij}\right) \text{ such that} \\ t_{i2}-\psi_2\left(y_{i2}\right)=-r^i \\ t_{i1}-t_{i2}-\left(\psi_1\left(y_{i1}\right)-\psi_1\left(y_{i2}\right)\right)=-c^i \end{cases}$$

The solution to this problem entails the same quantities as the full information case[9] but different transfers which depend on r^i and c^i, namely:

$$\begin{aligned} t_{i1} &= \psi_1\left(y_{i1}\right)+\left(\psi_2\left(y_{i2}\right)-\psi_1\left(y_{i2}\right)\right)-r^i-c^i \\ t_{i2} &= \psi_2\left(y_{i2}\right)-r^i. \end{aligned}$$

By implementing a contract with the quantities and different transfers the principal does not reduce the productive inefficiency of the full-information allocation (which is nothing but the "usual" second best solution), he just manages to improve his expected payoff by reducing the transfers given to the agent. By trading slackness they succeed in extracting more surplus from the agent.

The indirect utility function $V_I^i\left(r^i, c^i\right)$ already incorporates a maximisation over quantities and transfers, we can obtain its specific form by substituting the argmax of problem $\left(F_*^i\right)$. Since we are interested only in the effect of the slack variables we

[9]In fact the quantities are inplicitely defined by:

- $\phi^{i\prime}\left(y_{i1}\right)=\psi_1'\left(y_{i1}\right)$
- $\phi^{i\prime}\left(y_{i2}\right)=\psi_2'\left(y_{i2}\right)+\frac{p_{i1}}{p_{i2}}\left(\psi_2'\left(y_{i2}\right)-\psi_1'\left(y_{i2}\right)\right)$

can consider as constant everything in the value function which does not depend on r^i and c^i. We can therefore write:

$$V_I^i\left(r^i, c^i\right) = p_{i1}\left(K_{i1} + r^i + c^i\right) + p_{i2}\left(K_{i2} + r^i\right).$$

Suppose now that the type i principal can "buy" and "sell" slack in the agent's constraints at prices ρ and γ subject to the "budget" constraint that the value of the negative slack purchased be non-positive.[10] The principal would then take her trading decision through the solution of the following problem:

$$\left(D^i\right) = \begin{cases} \max\limits_{\{r^i, c^i\}} p_{i1}\left(K_{i1} + r^i + c^i\right) + p_{i2}\left(K_{i2} + r^i\right) \text{ subject to} \\ \rho r^i + \gamma c^i \leq 0 \end{cases}$$

We now have to check whether the conditions for the existence of a solution to this "consumer" problem are satisfied. The utility functions are linear, therefore also concave. Let $B^i(\rho, \gamma) \equiv \left\{r^i, c^i \text{ s.t. } \rho r^i + \gamma c^i \leq 0\right\}$ be the budget set of principal i, the following Lemma proves its compactness.

Lemma 1.1 *B^i is a compact set.*

Proof. By assumption feasible actions and transfers lie in a compact set. The function that maps action and transfers, y_{ij} and t_{ij}, into the space of "feasible" slack variables is linear. Therefore it is continuous and has a continuous inverse therefore also the space of r^i and c^i is compact. Then also $B^i(\rho, \gamma)$ is compact. ■

Therefore each principal maximises a linear (and concave) function over a compact set, then the solution to the program $\left(D^i\right)$ is a correspondence $D^i(\rho, \gamma)$ which we can interpret as the walrasian demand of slackness of principal i.

It can be shown (the Proof is in the Appendix) that a Walrasian Equilibrium of this economy exists and it is a pair of positive prices (ρ, γ) and a choice of negative

[10] This means that the value of the final slack "consumption" bundle should not exceed the value of her endowment, which for both types is zero since they start from the full information allocation.

slack variables (r^i, c^i) for each type i such that:

$$\begin{aligned}\sum_i \dot{\pi}_{ij} r^i\left(\mu_\cdot^i\right) &= 0 \text{ and } \sum_i \dot{\pi}_{ij} c^i\left(\mu_\cdot^i\right) = 0 \\ \left(r^i, c^i\right) &\in D^i(\rho, \gamma)\end{aligned}$$

The first set of conditions are "market clearing" requirements, which ensure that the average amount of slack demanded is equal to the average supply, i.e. zero.[11]

In Figure 1.1 we represent our exchange economy in a Edgeworth-like diagram. First note that the economy is not represented by a box, this is because there are no fixed endowments of slackness. In principle, as long as the market clearing conditions are satisfied we can have an infinite amount of slackness (e.g. if c^1 is very big and positive then c^2 will be very big and negative). The indifference curves of the two types of principal are straight lines with different slopes, they would have the same slope if the types were independently distributed.[12]

The origin of the axis is the endowment point, in fact the types of principal start trading having zero slackness on the constraints. While the area in the bottom right between the two thick indifference curves that go through the origin represents the possible gains from trade. That is in this area both types of principal would be on a higher indifference curve.

From the picture it is clear that we will have infinitely many possible equilibria, all in the bottom right region. To put it simply, we have two degrees of freedom, by

[11] This exactly amounts to satisfying the agent's constraints in expectation.

[12] The indifference curves of type 1, in the r^1, c^1 space are of the following type:

$$c^1 = -\left(1 + \frac{p_{12}}{p_{11}}\right) r^1,$$

while those of type 2 are:

$$c^1 = -\left(1 + \frac{p_{12}}{p_{11}} - \frac{\rho}{p_{11} p_{22}}\right) r^1.$$

If $\rho = 0$ then they would exactly coincide.

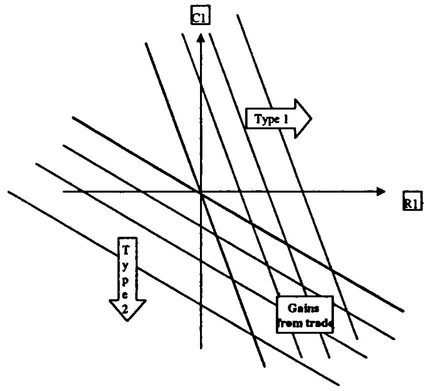

Figure 1.1: Gains from trade in the fictitious exchange ecnomy.

choosing a value for r_1 and c_1 (in that region) then we univocally define the values for r_2 and c_2 while the terms of the trade are going to be given by the slope of the line that goes through the equilibrium and the endowment.

It can be proven that these Walrasian equilibria possess a whole set of properties which carry the flavor of the First and Second welfare theorems, but for all these results refer to Maskin and Tirole [1990].

1.5 The equilibrium in the principal-agent game.

In the previous section we have shown that there exists many possible equilibria that secure to both types of principal a higher payoff when she is able to satisfy the agent's constraints in expectation. At this stage it is worth discussing the relationship between Walrasian equilibria of the fictitious exchange economy and the Bayesian equilibria of the principal-agent game. This is the goal of the following proposition.

Proposition 1.2 *For any Walrasian allocation of the fictitious exchange economy* *there exists a perfect Bayesian equilibrium where both types of principal propose the same contract and where the equilibrium outcome is this Walrasian allocation.*

Proof. Consider a Walrasian equilibrium $\left\{(\widehat{\rho},\widehat{\gamma}),(\widehat{r}^i,\widehat{c}^i)_{i=1,2}\right\}$ and let $\widehat{\mu}_{\cdot}^{\cdot}$ be the corresponding allocation. The equilibrium path is going to be the following: both principals propose the direct revelation mechanism $\widehat{\mu}_{\cdot}^{\cdot}$, the agent does not learn anything from this offer about the principal's type so his belief do not change. All types of agent accept the contract and both parties announce their types truthfully at the third stage.

To show that this is an equilibrium we proceed backward. At the third stage the agent will reveal his type truthfully because is interim IC constraints are satisfied by the Walrasian allocation. Because of the property of "no-envy" of the Walrasian allocation also the IC constraint of both types of principal are satisfied.

At the second stage the agent will accept the contract because at the third stage he will obtain at least his reservation utility.

It remains to show that at the first stage the principal does not want to offer a contract other than $\widehat{\mu}_{\cdot}^{\cdot}$. This can be done by choosing the appropriate off-equilibrium path strategies and beliefs. These are arbitrary, as cannot be derived with Bayes rule. These belief need to be chosen in such a way that if the principal proposes another mechanism all types of principal are no better off than with $\widehat{\mu}_{\cdot}^{\cdot}$. Suppose that a mechanism m is offered, and suppose that the agent has out of equilibrium beliefs such that $(\dot{\pi}_{1j}=1,\dot{\pi}_{2j}=0)$, the a type 1 principal will receive at most the full information payoff $\overline{v}^1$. Similarly if beliefs are $(\dot{\pi}_{1j}=1,\dot{\pi}_{2j}=0)$ then type 2 will at most obtain $\overline{v}^2$. From continuity and because $\widehat{\mu}_{\cdot}^{\cdot}$ is strongly Pareto optimal for the prior beliefs, then there exist intermediate beliefs for which both types do not prefer of deviate and offer m. ■

In this section our goal is to characterise such equilibria in the more standard principal-agent framework. More precisely we are going to study the contract offer that is going to lead to one of those equilibria.

First of all we need to ensure that the agent does not learn the type of the principal after she has offered the contract. That is why we are going to look for a "pooling offer" by the two types of principal. Pooling offer does not mean that in equilibrium different types of principal will implement the same allocation it just means the different types of principal will offer the same menu of contracts, knowing already that some of them (the ones contingent on types other that their true one) will never be implemented. In our particular setup each principal is going to offer four pairs, an action and a transfer, one for each possible state of the world knowing already that two of them will not be implemented. Offering a whole menu of contracts turns out to be a useful device for keeping the agent uninformed.

Secondarily we are going to make sure that the contract will be accepted by both types of agents and that it is incentive compatible. It has to be true in fact that, at the third stage when the players announce simultaneously their type, the agent reveals it truthfully.

Third, we are going to require that the contract is incentive compatible for the principal. That is, the contract offer has to be such that each principal prefers her possible allocations to the one of the other type. This will ensure a truthful announcement by the principal at the third stage.

Finally we want the different types of principal to profit from this game with respect to the full information case, that, in a private values framework, constitutes the lower bound to each principal's payoff.

Then, in a pooling offer equilibrium the contract proposed by the principal, regardless of her type, will be $(y_{ij}, t_{ij})_{i,j=1,2}$ such that it is a solution, for $i = 1, 2$, of:

$$
(P^i)\begin{cases}
\max_{\{y_{ij},t_{ij}\}} \sum_{j=1}^{2} p_{ij}\left(\phi^i\left(y_{ij}\right)-t_{ij}\right) \text{ such that} \\
IR_2: \sum_{i=1}^{2} \dot{\pi}_{i2}\left(t_{i2}-\psi_2\left(y_{i2}\right)\right)\geq 0 \\
IC_1: \sum_{i=1}^{2} \dot{\pi}_{i1}\left(t_{i1}-t_{i2}-\left(\psi_1\left(y_{i1}\right)-\psi_1\left(y_{i2}\right)\right)\right)\geq 0 \\
ICP^1: \sum_{j=1}^{2} p_{1j}\left(\phi^1\left(y_{1j}\right)-t_{1j}\right)\geq \sum_{j=1}^{2} p_{1j}\left(\phi^1\left(y_{2j}\right)-t_{2j}\right) \\
ICP^2: \sum_{j=1}^{2} p_{2j}\left(\phi^2\left(y_{2j}\right)-t_{2j}\right)\geq \sum_{j=1}^{2} p_{2j}\left(\phi^2\left(y_{1j}\right)-t_{1j}\right)
\end{cases}
$$

A solution to this problem is then incentive compatible for the principal and the agent and will be accepted by both types of agent.[13] In addition it can be shown that the incentive compatibility constraint of an agent of type 2 is not binding at the optimum (a proof can found in the Appendix).

After having found a solution to the above problem we will have to check that expected the payoff for each type of principal is higher at this solution than at the full information equilibrium, that is the following conditions must hold:

$$
\begin{aligned}
\lambda_1 &: \sum_{j=1}^{2} p_{1j}V^1\left(\widetilde{\mu}_j^1\right)\geq \sum_{j=1}^{2} p_{1j}V^1\left(\overline{\mu}_j^1\right) \\
\lambda_2 &: \sum_{j=1}^{2} p_{2j}V^2\left(\widetilde{\mu}_j^2\right)\geq \sum_{j=1}^{2} p_{2j}V^2\left(\overline{\mu}_j^2\right),
\end{aligned}
$$

where $\widetilde{\mu}_j^i$ is solution to P^i while $\overline{\mu}_j^i$ is the equilibrium allocation of the full information case.

We are now ready to state the following result, which characterises the equilibrium of the principal-agent game.

Proposition 1.3 *The following strategies are a candidate for an equilibrium of the principal agent game:*

[13] Standard considerations ensure the satisfaction of the participation constraint of a type 1 agent.

- *At date 1 both types of principal offer the same contract* $\widetilde{m} = (y_{ij}, t_{ij})_{i,j=1,2}$ *that satisfies the following conditions for* $i = 1, 2$*:*
  - $\phi^{i\prime}(y_{i1}) = \psi_1'(y_{i1})$
  - $\phi^{i\prime}(y_{i2}) = \psi_2'(y_{i2}) + \frac{p_{i1}}{p_{i2}}\left(\psi_2'(y_{i2}) - \psi_1'(y_{i2})\right)$
  - $\sum_{i=1}^{2} p_{i2}\left(t_{i2} - \psi_2(y_{i2})\right) = 0$
  - $\sum_{i=1}^{2} p_{i1}\left(t_{i1} - t_{i2} - \left(\psi_1(y_{i1}) - \psi_1(y_{i2})\right)\right) = 0$
- *At date 2 the beliefs of the agent are unchanged and all types of agent accept the proposed mechanism* $\widetilde{m}$*.*
- *At date 3 both parties announce their type truthfully and implement the mechanism.*

Proof. First note that any contract m which is a solution to problem $\left(P^i\right)$ is incentive compatible for the principal and agent and is individually rational for the agent. We need therefore to show that $\widetilde{m}$ is indeed a solution to $\left(P^i\right)$.

To begin with, note that $\widetilde{m}$ is a solution to the less constrained problem $\left(P_*^i\right)$ which is defined as:

$$\left(P_*^i\right) \begin{cases} \max\limits_{\{y_{ij}, t_{ij}\}} \sum\limits_{j=1}^{2} p_{ij}\left(\phi^i(y_{ij}) - t_{ij}\right) \text{ such that} \\ IR_2 : \sum\limits_{i=1}^{2} \hat{\pi}_{i2}\left(t_{i2} - \psi_2(y_{i2})\right) \geq 0 \quad (\widetilde{\rho}_i) \\ IC_1 : \sum\limits_{i=1}^{2} \hat{\pi}_{i1}\left(t_{i1} - t_{i2} - \left(\psi_1(y_{i1}) - \psi_1(y_{i2})\right)\right) \geq 0 \quad (\widetilde{\gamma}_i). \end{cases}$$

Program $\left(P_*^i\right)$ is the same as $\left(P^i\right)$ except that the incentive compatibility constraints for the two types of principal have been omitted.

The first order conditions for this problem are:

$$\frac{\partial L}{\partial y_{i1}} = p_{i1}\phi^{i\prime}(y_{i1}) + \dot{\pi}_{i1}\widetilde{\gamma}_i\psi_1'(y_{i1}) = 0$$
$$\frac{\partial L}{\partial y_{i2}} = p_{i2}\phi^{i\prime}(y_{i2}) - \dot{\pi}_{i1}\widetilde{\gamma}_i\psi_1'(y_{i2}) + \dot{\pi}_{i2}\widetilde{\gamma}_i\widetilde{\rho}_i\psi_2'(y_{i2}) = 0$$
$$\frac{\partial L}{\partial t_{i1}} = -p_{i1} - \widetilde{\gamma}_i\dot{\pi}_{i1} = 0$$
$$\frac{\partial L}{\partial t_{i2}} = -p_{i2} + \widetilde{\gamma}_i\dot{\pi}_{i1} - \widetilde{\rho}_i\dot{\pi}_{i2} = 0$$
$$\frac{\partial L}{\partial \widetilde{\rho}_i} = \sum_{i=1}^{2}\dot{\pi}_{i2}\left(t_{i2} - \psi_2(y_{i2})\right) = 0$$
$$\frac{\partial L}{\partial \widetilde{\gamma}_i} = \sum_{i=1}^{2}\dot{\pi}_{i1}\left(t_{i1} - t_{i2} - \left(\psi_1(y_{i1}) - \psi_1(y_{i2})\right)\right) = 0$$

From $\left(\frac{\partial L}{\partial y_{i1}}\right)$ and $\left(\frac{\partial L}{\partial t_{i1}}\right)$ we obtain the first condition which implicitly defines y_{i1}, while from $\left(\frac{\partial L}{\partial y_{i2}}\right)$ and $\left(\frac{\partial L}{\partial t_{i2}}\right)$ we get the definition of y_{i2}. Then $\left(\frac{\partial L}{\partial \widetilde{\rho}_i}\right)$ and $\left(\frac{\partial L}{\partial \widetilde{\gamma}_i}\right)$ give the last two conditions on the transfer. So $\widetilde{m}$ satisfies the first order conditions of problem $\left(P_*^i\right)$.

Now $\widetilde{m}$ is also a solution to $\left(P^i\right)$ if it is incentive compatible for both types of principal. To show this first note that each principal maximises her expected utility over the same set of constraints. Therefore the value at the optimum of the utility function cannot be higher if the principal then lies and chooses the optimal allocation chosen by the other type. More precisely, call $h(y_{ij}, t_{ij}) = 0$ the set of constraints of problem $\left(P_*^i\right)$ and let $\left(\widehat{y}_{1j}, \widehat{t}_{1j}\right)$ be the allocations that maximise $\sum_{j=1}^{2} p_{1j}\left(\phi^1(y_{ij}) - t_{ij}\right)$ over $h(y_{ij}, t_{ij}) = 0$ and $\left(\widehat{y}_{2j}, \widehat{t}_{2j}\right)$ the equivalent for a type 2 principal. It is evident that the following holds:

$$\sum_{j=1}^{2} p_{1j}\left(\phi^1(\widehat{y}_{1j}) - \widehat{t}_{1j}\right) \geq \sum_{j=1}^{2} p_{1j}\left(\phi^1(\widehat{y}_{2j}) - \widehat{t}_{2j}\right),$$

that is: each principal prefers her optimal allocations to the ones of the other type. If they were preferred, they would have been chosen in the first place because the set of constraints is the same.

So $\widetilde{m}$ is incentive compatible for both types of principal and is therefore a solution to (P^i). ■

The last proposition has characterised the strategies which can constitute an equilibrium of our principal-agent game. The contract offer is pooling, in the sense that both principals offer the same menu of four allocations, this does not allow the agent to learn anything about the type of the principal he is facing. The actions prescribed are the same as in the full information case while the transfers are potentially different. We are in fact left with two degrees of freedom in choosing the transfers (four unknowns and two equations), one possibility are the transfers of the full information case.[14] Our claim is though that both types of principal can do better than in the full information case so we can exploit these degrees of freedom in the constraints to make sure that the conditions for a higher equilibrium payoff are satisfied by the chosen transfers.

Proposition 1.4 *Contract $\widetilde{m}$, solution to (P^i), satisfies the following conditions:*

$$\begin{aligned}\lambda_1 &: \sum_{j=1}^{2} p_{1j} V^1\left(\widetilde{\mu}_j^1\right) \geq \sum_{j=1}^{2} p_{1j} V^1\left(\overline{\mu}_j^1\right) \\ \lambda_2 &: \sum_{j=1}^{2} p_{2j} V^2\left(\widetilde{\mu}_j^2\right) \geq \sum_{j=1}^{2} p_{2j} V^2\left(\overline{\mu}_j^2\right),\end{aligned}$$

and therefore is an equilibrium of the Principal-Agent game.

Proof. Both $\widetilde{\mu}_j^i$ and $\overline{\mu}_j^i$ prescribe the same actions, therefore the satisfaction of the above conditions will depend exclusively on the transfers chosen. The transfers of the full information case are: $t_{i1} = \psi_1(y_{i1}) + (\psi_2(y_{i2}) - \psi_1(y_{i2}))$ and $t_{i2} = \psi_2(y_{i2})$. While the ones of the pooling offer have to satisfy the following conditions:

$\sum_{i=1}^{2} \acute{\pi}_{i2}\left(t_{i2} - \psi_2(y_{i2})\right) = 0$

[14] The fact that the full information contract is still a solution when the information is not public is a typical feature of the private values case.

$\sum_{i=1}^{2} \dot{\pi}_{i1} \left(t_{i1} - t_{i2} - \left(\psi_1 \left(y_{i1} \right) - \psi_1 \left(y_{i2} \right) \right) \right) = 0.$

Let $\widetilde{t}_{i1}$ and $\widetilde{t}_{i2}$ be the transfers in the pooling offer $\widetilde{m}$, the conditions λ_1 and λ_2 then become respectively:

$$
\begin{aligned}
p_{11}t_{11} + p_{12}t_{12} &\geq p_{11}\widetilde{t}_{11} + p_{12}\widetilde{t}_{12} \\
p_{12}t_{21} + p_{22}t_{22} &\geq p_{12}\widetilde{t}_{21} + p_{22}\widetilde{t}_{22}.
\end{aligned}
$$

Remembering that when $\widetilde{m}$ is offered the posterior beliefs of the agent are unchanged and that the priors of principal and agent were identical, from the above equality constraints we can solve for:

$$
\widetilde{t}_{22} = \psi_2 \left(y_{22} \right) + \frac{p_{12}}{p_{22}} \psi_2 \left(y_{12} \right) - \frac{p_{12}}{p_{22}} \widetilde{t}_{12}
$$

and then using what we just found:

$$
\begin{aligned}
\widetilde{t}_{11} = \psi_1 \left(y_{11} \right) + \widetilde{t}_{12} - \psi_1 \left(y_{12} \right) - \frac{p_{12}}{p_{11}} \widetilde{t}_{21} + \frac{p_{12}}{p_{11}} \psi_1 \left(y_{21} \right) & \\
+ \frac{p_{12}}{p_{11}} \left(\psi_2 \left(y_{22} \right) + \frac{p_{12}}{p_{22}} \psi_2 \left(y_{12} \right) - \frac{p_{12}}{p_{22}} \widetilde{t}_{12} \right) - \frac{p_{12}}{p_{11}} \psi_1 \left(y_{21} \right).
\end{aligned}
$$

Substitute the values of full information transfers and those of $\widetilde{t}_{22}$ and $\widetilde{t}_{11}$ in the two inequalities, what we obtain are two linear inequalities in two unknowns:

$$
\begin{aligned}
\widetilde{t}_{21} - \widetilde{t}_{12} &\geq \psi_1 \left(y_{21} \right) - \psi_2 \left(y_{12} \right) + \psi_2 \left(y_{22} \right) - \psi_1 \left(y_{22} \right) \\
p_{12}\widetilde{t}_{21} - \frac{\rho}{p_{22}} \widetilde{t}_{12} &\geq p_{12} \left(\psi_1 \left(y_{21} \right) - \psi_2 \left(y_{12} \right) + \psi_2 \left(y_{22} \right) - \psi_1 \left(y_{22} \right) \right) - \frac{\rho}{p_{22}} \psi_2 \left(y_{12} \right)
\end{aligned}
$$

There exists infinite solutions to this system of inequalities, therefore it is possible to choose four transfers such that the conditions π_1 and π_2 are satisfied. ■

This proposition proves that for both types of principal it is possible to do better than in the full information case. They can achieve a higher payoff making a contract offer that does not reveal anything about their type to the agent. Moreover there are infinitely many contracts that allow a higher expected utility.[15]

In the case of no correlation all these equilibria would bear a payoff equal to the full information one, therefore the principal would be completely indifferent between revealing her information to the agent at the contract offer stage or keeping it secret until the third stage. Correlation allows to break this indifference.

1.6 A simple example.

In what follows we are going to apply our propositions in an extremely simplified framework and we are going to find a numerical solution so that it is going to be more evident that an informed principal can profit from having and concealing private information.

One principal wants to sell one unit of a good which he can produce at costs c_1 or c_2, with $c_1 < c_2$. The cost of producing the good is private information.

One agent wants to buy one unit of the same good and he values that unit v_1 or v_2, with $v_1 < v_2$. The valuation for the good is private information.

We also assume that $c_1 < v_1 < c_2 < v_2$, therefore a type 1 principal has always gains from trade while for type 2 gains from trade are conditional on the agent having a high valuation for the good.

The utility for the principal is:

$$V = t - c,$$

[15]Note that this is perfectly consistent with the result found in the previous section, also here we keep two degress of freedom.

and the one of the agent is:

$$U = v - t,$$

where t is the price paid for the good (i.e. a transfer form the agent to the principal).

We are therefore in a world of bilateral asymmetric information and our informed principal problem falls in the realm of the private values case because the agent does not care directly about the cost of production of the good (i.e. the type of the principal).

Each player knows only the *a priori* distribution of the other player's type. The two types of the players are equally likely with $\Pr(v_i) = \Pr(c_i) = \frac{1}{2}$ with $i = 1, 2$. However they are not independently distributed with the conditional distributions being:

$$\begin{aligned}
\Pr(v_i \mid c_i) &= \Pr(c_i \mid v_i) = \frac{3}{4} \\
\Pr(v_i \mid c_j) &= \Pr(c_j \mid v_i) = \frac{1}{4}
\end{aligned}$$

with $i, j = 1, 2$ and $i \neq j$.

We are assuming therefore that there is higher probability of the two player's being of the same type with one's type acting like an informative signal on the other party's type.

After learning his cost of production, the principal offers a contract to the agent that specifies a price to be paid for the good in each state of the world. We are going to show that the principal will gain from not revealing her type at the contract offer stage. This means that both types of principal will offer the same menu of four prices (one for each state of the world). At that stage the principal already knows her type and so knows that two of those prices will never be implemented but by making this pooling contract-offer she does not allow the agent to learn anything new about her type. The agent will therefore accept the contract on the basis of the a priori distribution which he has not been able to update because no information has come from the offer. This therefore means that the participation and incentive constraints will have to hold only in expectation, leaving therefore more freedom to the principal when setting the prices.

At the third stage both principal and agent make an announcement about their type, we are going to show that the optimal contract which is also incentive compatible for the principal, in the sense that the principal will report truthfully his type.

At the final stage the contract is implemented and the transaction takes place at the chosen price.

We will also show that the principal is better off when concealing her information than in the case she reveals it from the very beginning. Finally we are going to show that correlation of information plays a big role in all this by showing that when types are independently distributed the principal cannot improve upon the full information payoff.

1.6.1 The full information case.

As we did before we are now going to study the contracts when the type of the principal is common knowledge, we are going to use it as a benchmark for evaluating the gains for the principal.

If the principal has cost of production c_1 then she offers a contract which consists of two prices, one for an agent that has valuation v_1 and one for an agent that has valuation v_2.

In order to have a lighter notation, simplify the analysis by making it more clear, we are going to assign the following specific values to valuations and costs: $v_2 = 3$, $v_1 = 1$, $c_2 = \frac{3}{2}$, $c_1 = 0$.[16]

[16]We chose these numbers in order that the assumed ranking was manatained, in fact it is still

The prices are determined through the following optimisation problem:

$$\max_{t(v_1,c_1),t(v_2,c_1)} E(V_1) = \frac{3}{4}t(v_1,c_1) + \frac{1}{4}t(v_2,c_1)$$

subject to the following participation and incentive constraints:

$$\begin{aligned}
IR_1 &: 1 - t(v_1,c_1) \geq 0 \\
IR_2 &: 3 - t(v_2,c_1) \geq 0 \\
IC_1 &: 1 - t(v_1,c_1) \geq 1 - t(v_2,c_1) \\
IC_2 &: 3 - t(v_2,c_1) \geq 3 - t(v_1,c_1).
\end{aligned}$$

It is clear that the only way to satisfy the incentive constraints is setting equal transfers for both types of agent. If the principal sets transfers:

$$t(v_1,c_1) = t(v_2,c_1) = 1,$$

then she will sell to both types of agent and his expect payoff will be:

$$E(V_1) = 1,$$

which is higher than what she would obtain by setting the transfers equal to 3 because that would ensure her a payoff of $\frac{3}{4}$[17]

Therefore both types of agent consume the good, and the agent with high valuation enjoys some rent (he pays a price which is well below his valuation and $E(U_2) = \frac{1}{2}$).

When the principal has high cost of production, then the two prices (always

true that: $c_1 < v_1 < c_2 < v_2$. Moreover we wanted: $v_1 - c_1 > \frac{1}{4}(v_2 - c_1)$ so that it is optimal for a type 1 principal to sell to both types of agent in the full-information case. This assumption ensures also efficiency.

[17]In that case a type 1 agent would refuse the contract.

contingent on the type of the agent) will solve the following problem:

$$\max_{t(v_2,c_2),t(v_1,c_2)} E(V) = \frac{3}{4}\left(t(v_2,c_2) - \frac{3}{2}\right) + \frac{1}{4}\left(t(v_1,c_2) - \frac{3}{2}\right)$$

subject to the following participation and incentive constraints:

$$\begin{aligned} IR_1 &: 1 - t(v_1,c_2) \geq 0 \\ IR_2 &: 3 - t(v_2,c_2) \geq 0 \\ IC_1 &: 1 - t(v_1,c_2) \geq 1 - t(v_2,c_2) \\ IC_2 &: 3 - t(v_2,c_2) \geq 3 - t(v_1,c_2). \end{aligned}$$

Again both transfers have to be equal and since setting $t(v_1,c_2) = t(v_2,c_2) = 1$ would gain the principal a negative expected payoff, then this time the transfers will be:

$$t(v_1,c_2) = t(v_2,c_2) = 3.$$

The principal will sell (and produce) the good only to a high valuation agent because the price is to high for a low valuation agent that prefers to enjoy his reservation utility. In this case the principal's equilibrium payoff is going to be:

$$E(V_2) = \frac{9}{8}.$$

Now both type of agents receive the same equilibrium payoff, even if one agent consumes and the other not.

Note also that, with the assumption that we made on the parameters, we obtain efficiency: the principal sells the good all the times that the valuation of the good by the agent is higher than her cost of production.

1.6.2 The pooling offer.

In this section we return to the case of privately informed principal and we show that the optimal contract is a menu of prices to be offered by both types of principal such that:

- it satisfies individual rationality and incentive compatibility constraints for both types of agent;
- it satisfies incentive compatibility for the principal;
- it secures a higher equilibrium expected payoff to both types of principal;
- it maintains efficiency (i.e. when the valuation is v_1 and the cost c_2 the good is not exchanged or produced).

This menu will be offered by both types of principal so that the agent does not learn any information from the contract offer so that his beliefs on the type of the principal coincides with the priors. This also means that his constraints will have to hold only in expectations (at an *interim* stage) leaving more freedom to the principal in setting the transfers.

At this stage we introduce four new variables $q_{ij} \in \{0,1\}$ with $i,j = 1,2$ which indicates whether a principal of type j will sell or not the good when she is paired with an agent of type i.

Then the expected utility for the different types of principal is going to be:

$$\begin{aligned} E(V_1) &= \frac{3}{4}t(v_1,c_1) + \frac{1}{4}t(v_2,c_1) \\ E(V_2) &= \frac{3}{4}\left(t(v_2,c_2) - \frac{3}{2}q_{22}\right) + \frac{1}{4}\left(t(v_1,c_2) - \frac{3}{2}q_{12}\right), \end{aligned}$$

The optimal contract will be the solution to the maximisation of these objective function with respect to $t(v_i,c_j)$ and q_{ij} provided that a set of constraints is satisfied.

This set includes participation constraints for the agent, incentive compatibility constraints for both agent and principal. Finally we are going to show also that the expected payoff when both types of principal offer this optimal contract will be higher than the one of the full-information case.

The individual rationality constraints that have to be satisfied are:

$$\begin{aligned} IR_1 &: \frac{3}{4}\left(q_{11} - t\left(v_1, c_1\right)\right) + \frac{1}{4}\left(q_{12} - t\left(v_1, c_2\right)\right) \geq 0 \\ IR_2 &: \frac{3}{4}\left(3q_{22} - t\left(v_2, c_2\right)\right) + \frac{1}{4}\left(3q_{21} - t\left(v_2, c_1\right)\right) \geq 0, \end{aligned}$$

The incentive compatibility constraints for the agent are:

$$\begin{aligned} IC_1 &: \frac{3}{4}\left(q_{11} - t\left(v_1, c_1\right)\right) + \frac{1}{4}\left(q_{12} - t\left(v_1, c_2\right)\right) \geq \frac{3}{4}\left(q_{21} - t\left(v_2, c_1\right)\right) + \frac{1}{4}\left(q_{22} - t\left(v_2, c_2\right)\right) \\ IC_2 &: \frac{3}{4}\left(3q_{22} - t\left(v_2, c_2\right)\right) + \frac{1}{4}\left(3q_{21} - t\left(v_2, c_1\right)\right) \geq \frac{3}{4}\left(3q_{12} - t\left(v_1, c_2\right)\right) + \frac{1}{4}\left(3q_{11} - t\left(v_1, c_1\right)\right). \end{aligned}$$

This time though we are going to require that the principal reveals truthfully her type at the third stage, therefore it has to be that, for each given type, she prefers "her prices" to the ones of other type. More precisely, incentive compatibility for the principal requires that the following two constraints are satisfied:

$$ICP^1 : \tfrac{3}{4}t\left(v_1, c_1\right) + \tfrac{1}{4}t\left(v_2, c_1\right) \geq \tfrac{3}{4}t\left(v_1, c_2\right) + \tfrac{1}{4}t\left(v_2, c_2\right)$$

$$ICP^2 : \tfrac{3}{4}\left(t\left(v_2, c_2\right) - \tfrac{3}{2}q_{22}\right) + \tfrac{1}{4}\left(t\left(v_1, c_2\right) - \tfrac{3}{2}q_{12}\right) \geq \tfrac{3}{4}\left(t\left(v_2, c_1\right) - \tfrac{3}{2}q_{21}\right) + \tfrac{1}{4}\left(t\left(v_1, c_1\right) - \tfrac{3}{2}q_{11}\right)$$

In addition, since we want to show that the principal benefit from concealing her type, then we require that the solution satisfies also the following inequalities are satisfied:

$$\begin{aligned} \pi_1 &: \frac{3}{4}t\left(v_1, c_1\right) + \frac{1}{4}t\left(v_2, c_1\right) \geq 1 \\ \pi_2 &: \frac{3}{4}\left(t\left(v_2, c_2\right) - \frac{3}{2}q_{22}\right) + \frac{1}{4}\left(t\left(v_1, c_2\right) - \frac{3}{2}q_{12}\right) \geq \frac{9}{8}. \end{aligned}$$

In other words the expected payoff for each type of principal have to be greater than what she could achieved by revealing her information, namely the full information payoff we computed in the previous section.

Arguments standard in this literature allow us to solve the programs with a binding participation constraint of a low valuation agent (IR_1) and a binding incentive compatibility constraint of a high valuation agent (IC_2).

As a result of optimisation we find that $q_{11} = q_{21} = q_{22} = 1$ and $q_{12} = 0$, this means that we are going to observe trade in all case but when the valuation of the agent is lower than the cost of production for the principal.

Now that we know the optimal values for the q's we can substitute them in the constraint and from the binding IR_1 we can derive the following:

$$t(v_1, c_1) = 1 - \frac{1}{3} t(v_1, c_2);$$

while from IC_2, after having plugged in $t(v_1, c_1)$, we obtain:

$$t(v_2, c_2) = \frac{10}{3} - \frac{1}{3} t(v_2, c_1) + \frac{8}{9} t(v_1, c_2).$$

We can now use these two expressions to simplify the remaining inequalities and obtain a system of linear inequalities in only two variables, $t(v_1, c_2)$ and $t(v_2, c_1)$. After the simplifications the constraints become:

$$\begin{cases} IR_2 : t(v_1, c_2) \leq \frac{3}{4} \\ IC_2 : t(v_2, c_1) \geq \frac{1}{4} - \frac{1}{3} t(v_1, c_2) \\ ICP^1 : t(v_2, c_1) \geq \frac{1}{4} + \frac{11}{3} t(v_1, c_2) \\ ICP^2 : t(v_2, c_1) \leq \frac{21}{8} + t(v_1, c_2) \\ \pi_1 : t(v_2, c_1) \geq 1 + t(v_1, c_2) \\ \pi_2 : t(v_2, c_1) \leq 1 + \frac{11}{3} t(v_1, c_2) \end{cases}$$

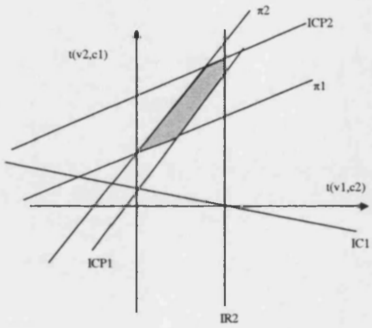

Figure 1.2: Set of equilibria of the pooling contract offer game.

We can graph the corresponding equations (Fig1.2) and look for a solution.

We can see that the shaded area in the graph satisfies all inequalities, that means that there exists infinite number of solutions, there are two degrees of freedom when choosing two of the prices, namely $t(v_2, c_1)$ and $t(v_1, c_2)$. The set of the solutions is also a Pareto set from the point of view of the types of principal, in fact the expected payoff of type 1 principal increase with $t(v_2, c_1)$ and decrease with $t(v_1, c_2)$ while the reverse holds for the expected payoff of a type 2 principal.

We can now pick a pair of prices inside the shaded area and verify that all the constraints are satisfied, for example: $t(v_1, c_2) = 0.6$ and $t(v_2, c_1) = 2.7$. From the binding constraints we derive that $t(v_1, c_1) = 0.8$ and $t(v_2, c_2) \simeq 2.966$. We can then verify in Table 1

Table 1	FULL-INFO	POOLING
$t(v_1, c_1)$	1	0.8
$t(v_1, c_2)$	0	0.6
$t(v_2, c_1)$	1	2.5
$t(v_2, c_2)$	3	2.96
rent A_1	0	0
rent A_2	0.5	0.10
π_1	1	1.275
π_2	$\frac{9}{8}$	1.249

that an agent who has low valuation obtains his reservation utility as in the full information case, while a high type agent enjoys a smaller expected rent of 0.100. We also get that IC_2 is binding while IC_1 is not. None of the incentive compatibility constraints for the principal is binding and she also enjoys higher expected profits, with respect to the full information case, whatever her type. In fact a type one principal gets 1.275 while in the former case she would get only 1; while a type two principal receives an expected payoff of 1.249 while it was only 1.125 when her information was known to the agent. It is therefore evident that by making the pooling offer the types of principal manages to extract some surplus from the agent. Some Crémer-McLean flavor emerges from the fact that with the new mechanism the principal raises the prices in the less likely states of the world therefore reducing the informational rent of the agent in those states.

1.6.3 The case of independent types.

To stress the importance of correlation in getting our result we are going to analyze the case in which the private information is independently distributed. We will show that even if the principals make a pooling offer she is not able to improve upon the full information outcome.

We maintain the assumption previously made that both types are equally likely with $\Pr(v_i) = \Pr(c_i) = \frac{1}{2}$ with $i = 1, 2$, but this time the conditionals distribution are going to be equal to the marginal ones:

$$\begin{aligned} \Pr(v_i \mid c_i) &= \Pr(c_i \mid v_i) = \frac{1}{2} \\ \Pr(v_i \mid c_j) &= \Pr(c_j \mid v_i) = \frac{1}{2}, \end{aligned}$$

this means precisely that one's type is non-informative signal of the other party's type.

The full information optimal contracts are the following pairs of prices and trade possibilities[18]:

$$\begin{aligned} t(v_2, c_j) &= 3 \text{ with } q_{2j} = 1 \\ t(v_1, c_j) &= 0 \text{ with } q_{1j} = 0, \end{aligned}$$

with $j = 1, 2$. This time both types of principal do not sell the good to a low valuation agent, therefore also leaving some gains from trade unexploited. The expected payoff for these contracts are respectively:

$$\begin{aligned} E(V_1) &= \frac{3}{2} \\ E(V_2) &= \frac{3}{4}. \end{aligned}$$

We now study the optimal contract when the different types of principal conceal their type at the offer stage. Optimal trade possibilities remain unchanged, with the low valuation agent not able to consume the good even when paired with a low cost principal.

[18] These are the solution to the following problems, with $i = 1, 2$:
$\max \frac{1}{2}(t(v_1, c_j) - q_{1j}c_j) + \frac{1}{2}(t(v_2, c_j) - q_{2j}c_j)$
subject to usual IC and IR constraints for the two types of agent.

At this stage the constraints have to be satisfied in expectation and if we let the individual rationality constraint of a low valuation agent and incentive compatibility of a high type be binding we obtain:

$$\begin{aligned} t(v_1, c_1) &= -t(v_1, c_2) \\ t(v_2, c_2) &= 6 - t(v_2, c_1). \end{aligned}$$

If we then substitute this values inside the other constraints we obtain the other constraints for the agent are always satisfied while those of the principal become:

$$\begin{aligned} ICP^1 &: t(v_2, c_1) \geq 3 + t(v_1, c_2) \\ ICP^2 &: t(v_2, c_1) \leq 3 + t(v_1, c_2) \\ \pi_1 &: t(v_2, c_1) \geq 3 + t(v_1, c_2) \\ \pi_2 &: t(v_2, c_1) \leq 3 + t(v_1, c_2). \end{aligned}$$

This time the shaded area where the solution to the system of constraints where lying collapses to a single straight line with all constraints strictly binding.

In Fig1.3 we can see that there is a continuum of equilibria of the game with pooling offer. All of them belong to the same line which is the locus of the transfers which leave the two types of principal at the same expected payoff level of the full information payoff. We have therefore shown that when types are independent the different types of principal are indifferent between revealing and concealing their private information because they neither gain nor lose from it.

1.7 Concluding remarks.

In this work we have shown how an informed principal with a quasilinear utility function and whose type is correlated with the one of the agent can improve his

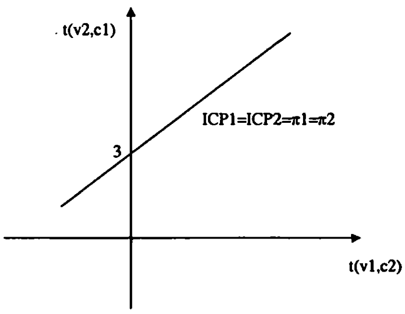

Figure 1.3: Set of equilibria of the pooling contract offer game when types are independent.

expected payoff with respect to the one he would obtain if he had no private information. The increase in payoff comes from pure redistribution of surplus that she manages to extract from the agent. It does not come, as elsewhere in the literature, from the elimination of risk. In this sense the efficiency of the economy as a whole is not improved, the principal, however, uses in a more efficient way the tools in her hands: the possibility of designing the contract.

Hence we have shown that the assumption of correlation between the information of the two parties is of great consequence in a world of private values. We believe it would be interesting to extend this analysis to the case of common values for which the literature offers less general and clear cut results.

This result could also be applied to the field of collusion under asymmetric information (e.g. models with one principal and two agents that can collude by signing a side-contract). Previous works in the literature have introduced a fictitious third party as collusion-contract designer to avoid problems related to bargaining under asymmetric information and informed principal problems. In these models the information is of the private value type and it would be interesting to study what would happen to the optimal contract designed by the principal when it is one of the agents who makes the collusive contract offer to the other and therefore acts as an informed principal vis-à-vis the other agent.

1.8 Appendix

1.8.1 The full information case.

In the full information framework each type of principal i solves the following problem:

$$\max_{\{y_{ij},t_{ij}\}} \sum_{j=1}^{2} p_{ij} \left(\phi^i(y_{ij}) - t_{ij}\right)$$

subject to:

$$\begin{aligned} t_{i1} - \psi_1(y_{i1}) &= t_{i2} - \psi_1(y_{i2}) \\ t_{i2} - \psi_2(y_{i2}) &= 0 \end{aligned}$$

Each Lagrangian would then be:

$$L^i = \sum_{j=1}^{2} p_{ij} \left(\phi^i(y_{ij}) - t_{ij}\right) - \gamma_i \left(t_{i1} - \psi_1(y_{i1}) - t_{i2} + \psi_1(y_{i2})\right) - \rho_i \left(t_{i2} - \psi_2(y_{i2})\right)$$

and maximizing it with respect to y_{ij}, t_{ij}, γ_i and ρ_i we obtain the following solution:

- $\phi^{i\prime}(y_{i1}) = \psi_1'(y_{i1})$ and $t_{i1} = \psi_1(y_{i1}) + (\psi_2(y_{i2}) - \psi_1(y_{i2}))$

- $\phi^{i'}(y_{i2}) = \psi_2'(y_{i2}) + \frac{p_{i1}}{p_{i2}}\left(\psi_2'(y_{i2}) - \psi_1'(y_{i2})\right)$ and $t_{i2} = \psi_2(y_{i2})$.

It is important to stress that the ratio of the Lagrange multipliers (i.e. the shadow value of the constraints) at the optimum is different across principals, more precisely:

$$\frac{\gamma_1}{\rho_1} = \frac{p_{11}}{p_{12}+p_{11}} \neq \frac{\gamma_2}{\rho_2} = \frac{p_{12}}{p_{12}+p_{22}}.$$

Had we been in a framework of independently distributed information these ratios would be the same because of the quasilinearity of the utility functions.

1.8.2 Existence of Walrasian Equilibrium in the fictitious exchange economy.

Textbook microeconomics tells us that to prove with "standard" theorems the existence of a Walrasian equilibrium in an exchange economy where the agents have strongly monotone utility functions we need the aggregate excess demand correspondence $z(p)$, defined for all price vectors $p \gg 0$, to satisfy the following properties:

1. $z(\cdot)$ is upper hemi-continuous;

2. $z(\cdot)$ is homogeneous of degree zero;

3. $pz(p) = 0$ for all p (Walras' law);

4. There is an $s > 0$ such that $z_l(p) > -s$ for every commodity l and all p;

5. $\lim_{p \to \partial\Delta} \inf \|z(p)\| \to \infty$.

One can easily check that the first four conditions are satisfied in our framework.

We have problems with the fifth property because of the assumption of compact choice sets (r^i and c^i belong to a compact set for $i = 1, 2$). In our case excess demands for both goods do not tend to infinity when their price tends to the boundary of the simplex because the choice set is bounded.

This inconvenience can be solved by removing the assumption of compact choice set for one principal[19]. We have therefore obtained that the aggregate excess demand for a commodity will go to infinity if the price of that commodity is zero. This allows to apply standard existence theorems to our framework.

The removal of the assumption of compactness does not cause any further problem because the market clearing conditions (which are the constraints of the agent in the principal agent game) will hold and together with the compactness assumption for the other principal will ensure that the equilibrium allocation will belong to a compact set.

1.8.3 IC2 is not binding at an optimum.

The argument of this proof is very similar to the one adopted in the proof of Lemma 1 in Maskin and Tirole [1990].

We need to show that IC constraint for type 2 agent is not binding at the optimum of program P_*^i, in other words that a solution of such program satisfies $IC2$.

If $\mu^i_\cdot$ is a solution to P_*^i then:

$$V^i\left(\mu_1^i\right) \geq V^i\left(\mu_2^i\right) \tag{*}$$

must hold because it if was violated then the polling allocation $\widetilde{\mu}$, defined so that for every i:

$$\widetilde{\mu}_1^i = \widetilde{\mu}_2^i = \mu_2^i$$

that also satisfies the constraints of P_*^i would generate higher values of the max-

[19]In general equilibrium theory with incomplete markets this procedure is known as the "Cass-trick".

imand.

If $\mu^{\cdot}_{\cdot}$ violates $IC2$ (that is type 2 strictly prefers $\mu^{\cdot}_{1}$ to $\mu^{\cdot}_{2}$), then define $\widehat{\mu}^{\cdot}_{\cdot}$ so that $\widehat{\mu}^{i}_{1} = \widehat{\mu}^{i}_{2} = \mu^{i}_{1}$ for all i. The allocation $\widehat{\mu}^{\cdot}_{\cdot}$ satisfies all the constraints of P^{i}_{*} and, from (*), generates at least as high a value of the maximand as $\mu^{\cdot}_{\cdot}$. But, because the type 2 agent strictly prefers $\mu^{\cdot}_{1}$ to $\mu^{\cdot}_{2}$, we can slightly reduce the transfer from the principal to the agent in $\widehat{\mu}^{\cdot}_{\cdot}$ without violating the constraints. But then $\widehat{\mu}^{\cdot}_{\cdot}$ generates a higher values of the maximand than $\mu^{\cdot}_{\cdot}$, a violation of $\mu^{\cdot}_{\cdot}$'s optimality.

Chapter 2

Risky Allocations from a Risk-Neutral Informed Principal.

2.1 Introduction

We are facing an informed principal problem when, in a principal agent relationship, the player that offers the contract possesses private information, and this is regardless of the agent having private information or not. When dealing with this kind of models the main question is to understand if the principal can gain from having private information and what is the optimal contract that had to be offered to the agent to maximise the principal surplus.

We study a model with a risk neutral informed principal who deals with a risk averse agent who has private information as well. We make the further assumptions that types are independently distributed and that the utility function of the agent does not depend from the type of the principal, the latter assumption implies that the model is one of *private values.* We show that the principal benefits from having private information in comparison to the payoff she would have received had her type been public. The optimal contract will involve a pooling offer that will leave the agent uninformed when he has to accept the contract.

The reason is that in models of private values signalling is not an issue, as Maskin and Tirole [1990] explain there is no competition among the different types of principal so no type of principal can loose from being pooled with another one[1]. In other words, concealing information has no costs because no type of principal can gain by separating herself from other types.

It can happen though that the principal is indifferent between revealing or not, that is there may not be benefits from hiding one's type. It has been shown that there are two reasons why the principal may wish to conceal her information to her benefit. Maskin and Tirole [1990] show that a risk averse informed principal benefits from not revealing her type through the contract offer for risk sharing reasons. They also show that this gain disappears when the principal has a quasilinear utility function. In Chapter 1 that even when both players are risk neutral the principal obtains a higher payoff by not revealing her type if there is correlation between the type of the agent and the one of the principal. Whether it is a different attitude towards risk, or a different belief about the probability of one particular type of agent, what happens in both models is that different types of principal have different relative costs of satisfying the agent's constraints[2]. If the principal makes a not-revealing contract offer then the agent remains uninformed and his constraints need to hold only at an interim stage and this gives more freedom to the principal. What will happen is that each type of principal will be able to relax the relatively more costly constraint while tightening the other. In this equilibrium ex-post constraints may

[1]See Maskin and Tirole [1992] for an analysis of common values model where sginalling causes more problems when looking for the optimal contract.

[2]These two circumstances can be both viewed as Hirshleifer effects (Hirshleifer [1971]). It has been shown in fact that early disclosure of information can destroy advantageous trading opportunities, which are present because agents have different attitudes towards the same risky event or because they have different beliefs about the probability of an event happening.

not be satisfied in every state of the world.

In this work we show that even when types are independent and the principal is risk neutral there are still benefits for the principal from concealing her information if the agent is risk averse.

Risk aversion on the part of the agent causes the relative costs of satisfying his incentive and participation constraints different for the two types of principal allowing the principal to make some profit from tightening and relaxing the ex-post constraints as long as the interim ones are satisfied.

The optimal contract results in a more risky final allocation, where the ex-post payoffs for the agent are more extreme than in the case where the type of the principal is public. The agent knows, when accepting the contract, that the spread of his ex-post utility will be bigger and being risk averse he will have to be paid some risk premium. Still, it is in the interest of the principal to do so meaning that the advantage of allowing each type of principal to relax the relatively more costly constraint is large enough to compensate for the higher expected payments that the principal is granting to the agent as risk premium.

The structure of the chapter is as follows. Section 2.2 presents the model. Section 2.3 analyses the contract when the type of the principal is known, which we use as benchmark. Section 2.4 studies the optimal contract when the principal is privately informed. Section 2.5 concludes.

2.2 The model.

2.2.1 Objective functions and information.

There are two players, one principal and one agent and both have a type-dependent utility function. Each agent has private information about his or her type but their utility function does not directly depend on the other player's type. This assumption sets our model in what the literature calls a *private values* framework. We are going to denote the type of the principal and the one of the agent with two parameters i and j (with $i = 1, 2$ and $j = 1, 2$)[3] respectively.

The principal is risk neutral with respect to transfers and her utility function takes the following quasi-linear form:

$$V^i = S^i(y) - t,$$

where y is an observable and verifiable action, t is a monetary transfer from the principal to the agent. $S^i(\cdot)$ is continuous, increasing and concave in y for every i.

The agent is risk averse and his utility function has the following functional form:

$$U_j = U(t - \theta_j y),$$

where $U(\cdot)$ is continuous, increasing and concave in y and, to simplify, independent from j. We assume that U_j is decreasing in j, implying therefore that: $\theta_1 < \theta_2$.

The agent's reservation utility is normalised to zero.

To guarantee the existence of equilibrium, we assume that the feasible actions and transfers lie in compact and convex sets.

We assume that the parameters i and j are drawn from independent common knowledge distributions. The parameters indicate the type of each player, i is known only to the principal and j to the agent. The prior beliefs of the agent on the type i of the principal are denoted by ϕ_i, type i can assume value 1 (resp. 2) with probability ϕ_1(resp.ϕ_2) such that $\phi_1 + \phi_2 = 1$. We denote by p_j the beliefs of the principal on the possible values of θ; $\theta_j = \theta_1$ and θ_2 with probabilities p_1 and p_2 (with $p_1 + p_2 = 1$).

[3] As in Maskin and Tirole [1990, 1992] the limitation on the possible types for the players is not essential but simplifies the analysis and favors the intuition of the results.

2.2.2 The principal-agent game.

The timing of the principal-agent game is as follows:

1. The principal proposes a mechanism in the feasible set M to the agent. A mechanism m in M will specify i) a set of possible messages for each party and ii) for each pair of messages chosen simultaneously an allocation (y, t). Note that the set M includes the set of direct revelation mechanisms in which parties simultaneously announce their types, by invoking the revelation principle for Bayesian game we can restrict the attention to direct truthful mechanisms.[4]

2. The agent updates his prior (if he has learned something from the offer), accepts or refuses the contract offered. If he refuses both players get zero utility and the game ends. If the agent accepts then parties move to the last stage of the game.

3. Both parties announce their types and the proposed mechanism is implemented.

We will study the perfect Bayesian equilibria of the overall game.

2.3 Benchmark: the full information allocation.

As a benchmark we study the equilibrium when the principal's information is common knowledge. Maskin and Tirole call it the *full information* case (even if the principal does not know the agent's type) and it is nothing more than the standard screening model.

We know from the revelation principle that every equilibrium allocation of this game can be obtained as an equilibrium of a direct truthful mechanism. The

[4] In this framework (as in Maskin and Tirole [1990]) the principle states that for any mechanism and for given beliefs any equilibrium of the mechanism is equivalent to an equilibrium of a direct revelation mechanism in which types are truthfully announced.

outcome $\left(y_j^i, t_j^i\right)$ that will be implemented in equilibrium will have to satisfy two types of constraints individual rationality and incentive compatibility.

For every i the participation constraints are: $U\left(t_j^i - \theta_j y_j^i\right) \geq 0$ for $j = 1, 2$. While the truthtelling constraints are: $U\left(t_j^i - \theta_j y_j^i\right) \geq U\left(t_k^i - \theta_j y_k^i\right)$ for all j,k.

Standard arguments apply, and in this context only two constraints are binding, the participation constraint of type 2 and the incentive compatibility of type 1.

Therefore in the case of full information a principal of type i proposes a contract $\left\{\left(y_1^i, t_1^i\right), \left(y_2^i, t_2^i\right)\right\}$ that solves the following program:

$$(F^i) = \begin{cases} \max\limits_{\left(y_j^i, t_j^i\right)} \sum\limits_{j=1}^{2} p_j \left(S^i\left(y_j^i\right) - t_j^i\right) \text{ such that} & \\ \mathrm{IR}^i : U\left(t_2^i - \theta_2 y_2^i\right) = 0 & (\rho^i) \\ \mathrm{IC}^i : U\left(t_1^i - \theta_1 y_1^i\right) = U\left(t_2^i - \theta_1 y_2^i\right) & (\gamma^i), \end{cases}$$

where ρ^i and γ^i are the Lagrange multipliers for the IR and IC constraints.

Given the specific functional forms chosen for the utility functions of the two players we can actually find the precise solution to this problem.

A principal of type i will offer the following decreasing schedule of output and the respective transfers, $\left\{\left(y_1^i, t_1^i\right), \left(y_2^i, t_2^i\right)\right\}$:

$$\begin{aligned} \text{type 1} \quad &: \quad S^{i\prime}\left(y_1^i\right) = \theta_1 \text{ and } t_1^i = \theta_1 y_1^i + \Delta\theta y_2^i \\ \text{type 2} \quad &: \quad S^{i\prime}\left(y_1^i\right) = \theta_2 + \frac{p_1}{p_2}\Delta\theta \text{ and } t_2^i = \theta_2 y_2^i. \end{aligned}$$

As one could expect, the solution preserves standard characteristics like the "no distortion at the top" property and no informational rent for the "low-type" agent. It preserves also the feature that the risk aversion on the part of the agent does not influence the solution In particular, the downward distortion in the production requested to a type 2 agent is the same that one would observe if the agent was risk neutral.

For future reference denote by $\left(\overline{y}_{\cdot}^{i}, \overline{t}_{\cdot}^{i}, \overline{\rho}^{i}, \overline{\gamma}^{i}\right)$ the solution to the full information program $\left(F^{i}\right)$. Let $\overline{v}^{i} \equiv \sum_{j} p_{j} V^{i}\left(\overline{y}_{j}^{i}, \overline{t}_{j}^{i}\right)$ be the type i principal's payoff.

At this stage it is essential to look at the ratio of the Lagrange multipliers of the full information optimisation problem; they are different across types of principal and have the following expression:

$$\frac{\rho^{i}}{\gamma^{i}}=\frac{U'\left(t_{1}^{i}-\theta_{1} y_{1}^{i}\right)}{p_{1} U'\left(t_{2}^{i}-\theta_{2} y_{2}^{i}\right)}=\frac{U'\left(\Delta \theta y_{2}^{i}\right)}{p_{1} U'(0)}.$$

This implies that the relative cost of fulfilling the individual rationality and incentive compatibility constraint is different for each type of principal[5].

As Maskin and Tirole [1990] have shown, as long as the relative costs of the incentive and participation constraints is not the same for both types of principal there are some gains coming from the fact one type of principal can relax the constraint that is relatively more costly for her, while enforcing the one that is relatively less costly. This result can be achieved only if the constraints of the agent can be satisfied at an interim stage (and not ex-post as in the full information case) and they need to hold only on average exactly when the agent does not know the type of the principal, in other words when the offer is not revealing.

2.4 The pooling offer.

As anticipated in the previous section we now return to the case in which the type of the principal is not known to the agent, and we are going to show that not revealing her type to the agent until the third stage of the game will allow any type of principal to obtain a payoff higher than the full-information one.

[5] The ratio of the Lagrange multipliers would be the same for the two types of principal if also the agent was risk neutral (see Maskin and Tirole [1990].

First of all it is important to bear in mind that the full information allocation can still be implemented when the type of the principal is not known to the agent. This is true because we are in the private values framework, as Maskin and Tirole [1990] explain, there is no rivalry between different types of principal and therefore revealing the type through the contract offer at the first stage is incentive compatible for the principal.

An informed principal has the alternative choice of not revealing her type, and we are going to show that indeed the full-information allocation can never be an equilibrium when the type of the principal is private information to her.

First of all we show that it is dominated and then we characterise the new equilibrium.

Proposition 2.1 *The full information allocation* $\left(\overline{y}^i_\cdot, \overline{t}^i_\cdot\right)$ *is dominated from the point of view of every type of principal* $i = 1, 2$ *by the allocation* $\left(\overline{y}^i_{\cdot *}, \overline{t}^i_{\cdot *}\right)$ *, which is the solution to the following program:*

$$\left(F^i_*\right) = \begin{cases} \max\limits_{\left(y^i_j, t^i_j\right)} \sum\limits_{j=1}^{2} p_j \left(S^i\left(y^i_j\right) - t^i_j\right) \text{ such that} \\ IR^i : U\left(t^i_2 - \theta_2 y^i_2\right) = -r^i \\ IC^i : U\left(t^i_1 - \theta_1 y^i_1\right) - U\left(t^i_2 - \theta_1 y^i_2\right) = -c^i. \end{cases}$$

Proof. It is evident that the only difference from the full-information program is that now there is some slack allowed on each constraint.

Let v^i_* be the maximised value of the maximand, by definition of the shadow prices $\overline{\rho}^i$ and $\overline{\gamma}^i$ it approximately equals $\overline{v}^i + \overline{\rho}^i r^i + \overline{\gamma}^i c^i$ for small values of r^i and c^i. Let μ^i_* be a solution to F^i_*.

Choose negative slack variables $\left(r^1, c^1\right)$ for the type 1 principal; then the slack variables for type 2 are defined as: $r^2 = -\frac{\phi_1}{\phi_2} r^1$ and $c^2 = -\frac{\phi_1}{\phi_2} c^1$. We can then write:

$v^1_* - \overline{v}^1 \simeq \overline{\rho}^1 r^1 + \overline{\gamma}^1 c^1$

$v^2_* - \overline{v}^2 \simeq -\frac{\phi_1}{\phi_2} \left(\overline{\rho}^2 r^1 + \overline{\gamma}^2 c^1\right).$

Both the above left hand sides can be positive for $\left(r^1, c^1\right)$ if and only if $\frac{\overline{p}^1}{\overline{\gamma}^1} \neq \frac{\overline{p}^2}{\overline{\gamma}^2}$, as it is in our case. ■

We have therefore shown that when the each type of principal is allowed some slackness on the agent's constraints then they can achieve a higher payoff than in the full information case.

The principal, whatever her type, will offer the same contract which will consist of a menu of four allocations, one for each possible state of the world. If she does so then the agent will not be able to infer anything about her type and accept the contract still on the basis of the prior probability distribution. What really happens is that the principal offers a contract that contains allocations that will never be implemented (e.g. a type1-1principal offers also those that are designed for a type-2 principal), but she does so because this will keep the agent ignorant about her type. This means that agent's constraints, individual rationality and incentive compatibility, will have to hold only in expectation leaving the principal the freedom of not satisfying some of the ex-post ones. We have already shown that each type of principal gains from being able to not satisfy the constraint that is relatively more expensive for her, in what follows we are going to characterise the equilibrium contract.

In a pooling offer each type of principal will offer a contract $\left(y_j^i, t_j^i\right)_{i,j=1,2}$ such that it is a solution, for $i = 1, 2$, of:

$$
\left(P^i\right) = \begin{cases}
\max\limits_{\{y_j^i, t_j^i\}} \sum\limits_{j=1}^{2} p_j \left(S^i\left(y_j^i\right) - t_j^i\right) \text{ such that} \\
IR_2 : \sum\limits_{i=1}^{2} \phi_i U\left(t_2^i - \theta_2 y_2^i\right) = 0 \\
IC_1 : \sum\limits_{i=1}^{2} \phi_i \left[U\left(t_1^i - \theta_1 y_1^i\right) - U\left(t_2^i - \theta_1 y_2^i\right)\right] = 0 \\
ICP^1 : \sum\limits_{j=1}^{2} p_j \left(S^1\left(y_j^1\right) - t_j^1\right) \geq \sum\limits_{j=1}^{2} p_j \left(S^1\left(y_j^2\right) - t_j^2\right) \\
ICP^2 : \sum\limits_{j=1}^{2} p_j \left(S^2\left(y_j^2\right) - t_j^2\right) \geq \sum\limits_{j=1}^{2} p_j \left(S^2\left(y_j^1\right) - t_j^1\right)
\end{cases}
$$

A solution to this problem is then incentive compatible for the principal and the agent and will be accepted by both types of agent.[6]

We can now state the following result, which characterises the equilibrium of the principal-agent game.

Proposition 2.2 *In a perfect Bayesian equilibrium of the principal agent game any type of principal will offer the same contract* $\widetilde{m} = \left(y_j^i, t_j^i\right)_{i,j=1,2}$. *The agent's beliefs are unchanged and accepts the contract. At the last stage both parties announce their type truthfully and the contract is implemented.*

Contract $\widetilde{m} = \left(y_j^i, t_j^i\right)_{i,j=1,2}$ *that satisfies the following conditions for* $i = 1, 2$*:*

1. $S^{i\prime}\left(y_1^i\right) = \theta_1$
2. $S^{i\prime}\left(y_2^i\right) = \theta_2 + \frac{p_1}{p_2}\Delta\theta \frac{U'\left(\left(t_2^i - \theta_1 y_2^i\right)\right)}{U'\left(\left(t_1^i - \theta_1 y_1^i\right)\right)}$
3. $\sum_{i=1}^{2} \phi_i U\left(t_2^i - \theta_2 y_2^i\right) = 0$
4. $\sum_{i=1}^{2} \phi_i \left[U\left(t_1^i - \theta_1 y_1^i\right) - U\left(t_2^i - \theta_1 y_2^i\right)\right] = 0$
5. $-\frac{U\left(t_1^i - \theta_1 y_1^i\right) - U\left(t_2^i - \theta_1 y_2^i\right)}{U\left(t_2^i - \theta_2 y_2^i\right)} = \frac{p_1}{p_2}\frac{U'\left(\left(t_1^i - \theta_1 y_1^i\right)\right)}{U'\left(\left(t_2^i - \theta_2 y_2^i\right)\right)} + \frac{U'\left(\left(t_2^i - \theta_1 y_2^i\right)\right)}{U'\left(\left(t_2^i - \theta_2 y_2^i\right)\right)}.$

Proof. First note that any contract m which is a solution to problem $\left(P^i\right)$ is incentive compatible for the principal and agent and is individually rational for the agent. We need therefore to show that $\widetilde{m}$ is indeed a solution to $\left(P^i\right)$.

To begin with, note that $\widetilde{m}$ is a solution to the less constrained problem $\left(P_*^i\right)$ which is defined as:

[6] Standard considerations ensure the satisfaction of the participation constraint of a type 1 agent and of the incentive compatibility constraints of a type 2 agent.

$$
(P_*^i) = \begin{cases} \max_{\{y_j^i, t_j^i\}} \sum_{j=1}^{2} p_j \left(S^i \left(y_j^i \right) - t_j^i \right) \text{ such that} \\ IR_2 : \sum_{i=1}^{2} \phi_i U \left(t_2^i - \theta_2 y_2^i \right) = 0 & (\widetilde{\rho}^i) \\ IC_1 : \sum_{i=1}^{2} \phi_i \left[U \left(t_1^i - \theta_1 y_1^i \right) - U \left(t_2^i - \theta_1 y_2^i \right) \right] = 0 & (\widetilde{\gamma}^i) \end{cases}
$$

Program (P_*^i) is the same as (P^i) except that the incentive compatibility constraints for the two types of principal have been omitted.

The first order conditions for this problem are:

$$
\begin{aligned}
\frac{\partial L}{\partial y_1^i} &= p_1 S^{i\prime} \left(y_1^i \right) + \phi_1 \widetilde{\gamma}^i \theta_1 U' \left(t_1^i - \theta_1 y_1^i \right) = 0 \\
\frac{\partial L}{\partial y_2^i} &= p_2 S^{i\prime} \left(y_2^i \right) - \phi_1 \widetilde{\rho}^i \theta_2 U' \left(t_2^i - \theta_2 y_2^i \right) + \phi_1 \widetilde{\gamma}^i \theta_1 U' \left(t_2^i - \theta_1 y_2^i \right) = 0 \\
\frac{\partial L}{\partial t_1^i} &= -p_1 - \phi_1 \widetilde{\gamma}^i U' \left(t_1^i - \theta_1 y_1^i \right) = 0 \\
\frac{\partial L}{\partial t_2^i} &= -p_2 + \phi_1 \widetilde{\rho}^i U' \left(t_2^i - \theta_2 y_2^i \right) - \phi_1 \widetilde{\gamma}^i U' \left(t_2^i - \theta_1 y_2^i \right) = 0 \\
\frac{\partial L}{\partial \widetilde{\rho}^i} &= \sum_{i=1}^{2} \phi_i U \left(t_2^i - \theta_2 y_2^i \right) = 0 \\
\frac{\partial L}{\partial \widetilde{\gamma}^i} &= \sum_{i=1}^{2} \phi_i \left[U \left(t_1^i - \theta_1 y_1^i \right) - U \left(t_2^i - \theta_1 y_2^i \right) \right] = 0
\end{aligned}
$$

From $\left(\frac{\partial L}{\partial y_1^i} \right)$ and $\left(\frac{\partial L}{\partial t_1^i} \right)$ we obtain the first condition which implicitly defines y_1^i, while from $\left(\frac{\partial L}{\partial y_2^i} \right)$ and $\left(\frac{\partial L}{\partial t_2^i} \right)$ we get the definition of y_2^i. Then $\left(\frac{\partial L}{\partial \widetilde{\rho}^i} \right)$ and $\left(\frac{\partial L}{\partial \widetilde{\gamma}^i} \right)$ give the agent's constrains (IR and IC). So $\widetilde{m}$ satisfies the first order conditions of problem (P_*^i).

Now $\widetilde{m}$ is also a solution to (P^i) if it is incentive compatible for both types of principal. To show this first note that each principal maximises her expected utility over the same set of constraints. Therefore the value at the optimum of the utility function cannot be higher if the principal then lies and chooses the optimal allocation chosen by the other type. More precisely, call $h\left(y_j^i, t_j^i\right) \geq 0$ the set of constraints of problem $\left(P_*^i\right)$ and let $\left(\widehat{y}_j^1, \widehat{t}_j^1\right)_{j=1,2}$ be the allocation that maximises $\sum_{j=1}^{2} p_j \left(S^1\left(y_j^1\right) - t_j^1\right)$ over $h\left(y_j^i, t_j^i\right) \geq 0$ and $\left(\widehat{y}_j^2, \widehat{t}_j^2\right)$ the equivalent for a type 2 principal. It is evident that the following holds:

$$\sum_{j=1}^{2} p_j \left(S^1\left(\widehat{y}_j^1\right) - \widehat{t}_j^1\right) \geq \sum_{j=1}^{2} p_j \left(S^1\left(\widehat{y}_j^2\right) - \widehat{t}_j^2\right),$$

that is: each principal prefers her optimal allocations to the ones of the other type. If the other's one were preferred, they would have been chosen in the first place because the set of constraints is the same.

So $\widetilde{m}$ is incentive compatible for both types of principal and is therefore a solution to $\left(P^i\right)$.

Finally, note that the ratio of the Lagrange multipliers is:

$$\frac{\widetilde{\rho}^i}{\widetilde{\gamma}^i} = \frac{p_1}{p_2}\frac{U'\left(\left(t_1^i - \theta_1 y_1^i\right)\right)}{U'\left(\left(t_2^i - \theta_2 y_2^i\right)\right)} + \frac{U'\left(\left(t_2^i - \theta_1 y_2^i\right)\right)}{U'\left(\left(t_2^i - \theta_2 y_2^i\right)\right)}.$$

This can be used to rewrite condition 5 as:

$$\widetilde{\rho}^i r^i + \widetilde{\gamma}^i c^i = 0^7$$

where $r^i = -U\left(t_2^i - \theta_2 y_2^i\right)$ and $c^i = -\left[U\left(t_1^i - \theta_1 y_1^i\right) - U\left(t_2^i - \theta_1 y_2^i\right)\right]$.

In addition from the agent's constrains we know that: $r^2 = -\frac{\phi_1}{\phi_2} r^1$ and $c^2 = -\frac{\phi_1}{\phi_2} c^1$, which together with the above implies that also the following must hold:

$$\widetilde{\rho}^k r^i + \widetilde{\gamma}^k c^i = 0, \text{ with } k \neq i.$$

Since in general, r^i and c^i are different from zero, this implies that in equilibrium:

[7] This ensures that "the value" of the slackness (positive and negative) for each type of principal is the same. Morevoer it is equal to zero, the value of the slackness of the full-information acquisition.

$$\frac{\widetilde{\rho}^1}{\widetilde{\gamma}^1} = \frac{\widetilde{\rho}^2}{\widetilde{\gamma}^2}.$$

The above means that at the equilibrium the relative cost of satisfying the constraints is the same for the two types of principal, there are therefore no more gains to be obtained from relaxing further some constraints.

It remains to show that no type of principal can gain by deviating by offering a contract different from $\widetilde{m}$. This can be done by choosing the appropriate off-equilibrium path beliefs, that are arbitrary. As Maskin and Tirole [1990] argue these belief need to be chosen in such a way that if the principal proposes another mechanism all types of principal are no better off than with $\widetilde{m}$. Suppose that a mechanism m is offered, and suppose that the agent has out of equilibrium beliefs such that $\left(\dot{\phi}_1 = 1, \dot{\phi}_2 = 0\right)$, the a type 1 principal will receive at most the full information payoff $\overline{v}^1$. Similarly if beliefs are $\left(\dot{\phi}_1 = 1, \dot{\phi}_2 = 0\right)$ then type 2 will at most obtain $\overline{v}^2$. From continuity and because $\widetilde{m}$ is strongly Pareto optimal for the prior beliefs, then there exist intermediate beliefs for which both types do not whish to deviate and offer m. ■

We have therefore characterised the allocation that a risk neutral informed principal, regardless of her type, will offer to a risk averse agent. The main feature of this allocation is that it does not satisfy all the agent's ex-post constraints, but only the interim ones. As a consequence the risk aversion of the agent plays a role in determining the optimal downward distortion in the quantity of a low-type agent. Efficiency for the high-type is preserved. Finally, in equilibrium, the ratios of the lagrange multipliers of the two types of principal are the same. This implies that no more gains can be reaped by tightening and relaxing the constraints.

To understand more clearly what goes on in the equilibrium we need to specify which principal gains from relaxing which constraint. Let's assume that $S^1(\cdot) > S^2(\cdot)$ for every y (and that $S^{1\prime}(\cdot) > S^{2\prime}(\cdot)$). This enables us to say that the full information quantities can be ordered as follows:

$$\overline{y}_1^1 > \overline{y}_1^2 \text{ and } \overline{y}_2^1 > \overline{y}_2^2.$$

Which in turn tells us that the informational rent enjoyed by an efficient agent is greater when the principal is of type 1 (the informational rent is $\Delta\theta\overline{y}_2^i$, for every i). This means that when the type of principal is not known if he offers the full information contract to the agent then the expected payoff of the two types of agent would be agent would be:

$$\phi_1 U\left(\Delta\theta\overline{y}_2^1\right) + \phi_2 U\left(\Delta\theta\overline{y}_2^2\right), \text{ for a type 1 agent,}$$

and

$$\phi_1 U\left(0\right) + \phi_2 U\left(0\right), \text{ for a type 2 agent.}$$

As we have shown above the full-information allocation is not optimal for the principal. In particular:

$$\frac{U'\left(\Delta\theta\overline{y}_2^1\right)}{p_1 U'\left(0\right)} = \frac{\overline{\rho}^1}{\overline{\gamma}^1} > \frac{\overline{\rho}^2}{\overline{\gamma}^2} = \frac{U'\left(\Delta\theta\overline{y}_2^2\right)}{p_1 U'\left(0\right)},$$

meaning that a type 1 principal finds relatively more costly to satisfy the individual rationality constraint than the type 2. In the pooling offer type 1 principal is going to relax the individual rationality constraint, i.e.:

$$U\left(t_2^1 - \theta_2 y_2^1\right) < 0,$$

but because the constraints have to hold in expectation the former inequality implies the following:

$$U\left(t_2^2 - \theta_2 y_2^2\right) > 0.$$

On the other side a type 2 principal will relax the incentive compatibility constraint:

$$U\left(t_1^2 - \theta_1 y_1^2\right) < U\left(t_2^2 - \theta_1 y_2^2\right),$$

which implies that for a type 1 principal the following must hold:

$$U\left(t_1^1 - \theta_1 y_1^1\right) < U\left(t_2^1 - \theta_1 y_2^1\right).$$

This constraints relaxing and tightening have, obviously, an effect on the marginal utility of the agent and the term which defines the downward distortion of an inefficient agent will be different for each type of principal. In particular:

$$\frac{U'\left(\left(t_2^1 - \theta_1 y_2^1\right)\right)}{U'\left(\left(t_1^1 - \theta_1 y_1^1\right)\right)} < 1 \text{ and } \frac{U'\left(\left(t_2^2 - \theta_1 y_2^2\right)\right)}{U'\left(\left(t_1^2 - \theta_1 y_1^2\right)\right)} > 1,$$

while in the full information allocation they were both equal to 1. As a consequence y_2^1 is now less distorted that in the full information allocation ($y_2^1 > \overline{y}_2^1$)and y_2^2 is more distorted ($y_2^2 < \overline{y}_2^2$), therefore widening the gap between the quantities produced under the two types of principal.

This has a direct effect on the expected payoff of both types of agent that are now facing more risk, for a type one agent (the efficient type) the difference in informational rent has become larger:

$$\Delta\theta y_2^1 - \Delta\theta y_2^2 > \Delta\theta\overline{y}_2^1 - \Delta\theta\overline{y}_2^2,$$

increasing therefore the riskiness of the ex-ante expected rent/payoff:

$$\phi_1 U\left(\Delta\theta y_2^1\right) + \phi_2 U\left(\Delta\theta y_2^2\right).$$

While a type 2 agent, who in the full information allocation does not face any ex-ante risk (getting his reservation utility with any type of principal), will get a positive rent in one state of the world and negative in the other:

$$\phi_1 U\big(\underset{-}{t_2^1 - \theta_2 y_2^1}\big) + \phi_2 U\big(\underset{+}{t_2^2 - \theta_2 y_2^2}\big).$$

What is surprising about this result then is that a risk neutral principal prefers to implement an allocation that is more risky to for the agent, if compared to the full information allocation, instead of eliminating any risk. The agent must of course be given a risk premium if he has to accept a more risky contract, still the principal, whatever her type, gains from the new pooling offer. This means that the benefit of relaxing the relatively more expensive constraint gives an expected benefit which is higher then the risk premium that needs to be paid to the agent.

2.5 Concluding remarks.

We have shown that in a model with a risk neutral informed principal and a risk averse agent the principal gains by making a not revealing contract offer that will keep the agent uninformed about her type until the implementation stage.

The optimal contract will result in an equilibrium allocation that involves a larger spread in the ex-post payoffs of the agent, forcing him to take up more risk when he accepts the contract.

We believe this highlights how interesting is to study principal agent relationships with an informed principal because some of the usual features the one sided standard screening models disappear. A risk neutral principal does not reduce any longer the risk that a risk averse agent has to face and the risk aversion of the agent plays a role on the solution of the trade-off between rents and incentives that defines the downward distortions in the physical production.

Part II

Collusion and Delegation

Chapter 3

Monitoring of Delegated Contracting.

3.1 Introduction.

> In any organisational structure, the nature of communication can be important to the efficiency of the transactions governed by that form.
> (O. Williamson [1975]).

Delegation of economic activity and subcontracting are widely observed phenomena. The need of exploiting the gains from specialisation is one of the reasons for their diffusion, which has been favored by the high improvement of communication systems and by the increased sophistication of the available forms of contracts.

What is often observed are hierarchical structures, where each level is linked to the lower one by a contract ruling one or more economic activities. Hierarchical decentralisation involves therefore gains from specialisation, but brings also extra-costs due to the loss of control over lower levels of these hierarchies. The head of the hierarchy can then try to reduce these costs by regaining some control over the "subordinates": one way to do so is monitoring these delegated relationships to acquire information.

While most of the incentive theory literature has taken information constraints as given, we make an attempt towards endogenising the informational structure. We see how incentives are affected by the strategic interaction of the members of a hierarchy when the principal can monitor the lower levels.

We study the contracting over a production process in a very simple delegated environment. More precisely, we look at a three-tier hierarchy where a principal wants a final good which is produced by an agent (A_1) using an intermediate good provided by another agent (A_2). Both agents have private information about their marginal costs of production, this sets us in an adverse selection world.

We assume that the principal contracts over the quantity desired of final good directly with A_1 and lets him free to contract with A_2 about the provision of the intermediate good, i.e. the principal cannot contract directly with the second agent. Two contracts will have to be studied, a grand-contract between the principal and the first agent and a sub-contract between the two agents. Given that hierarchies and delegation are widely observed phenomena, we study some features of the agents' strategic behavior in this specific environment. We therefore impose a delegated structure to our model, without asking what are the reasons that lead to such an organisational mode or comparing it with a centralised one.

Delegation reduces the burden of communication and information processing on the principal, but on the other hand it also introduces additional incentives problems. These come from the fact that, even if the principal contracts directly only with one agent, she would like to condition the menu of contracts also on the type of the second agent. Therefore, when offering the contract to A_1 the principal has to give incentives to this agent to truthfully report not only his own type, but also the type of the second one, which he will have learned at the sub-contracting stage. In other words, the cost of delegation comes from the fact that the informational rents paid to the agent with whom the principal deals directly can be quite high; this happens because the principal has to reimburse A_1 for the informational rent he has paid to A_2 (after all, these are costs for A_1) and then give him incentives to truthfully report two pieces of information.

It would then seem that monitoring at no cost the communication between the two agents would be profitable for the principal because she would get to know at no cost a piece of information for which she would have to pay otherwise. The direct effect coming from this activity is indeed a gain for the principal, due to a reduction in the transfer she makes to the first agent. But to evaluate thoroughly the effect of monitoring on the profits of the principal we need to study the possibility of profitable deviations of the first agent who is the one loosing the most from this activity.

We consider two monitoring possibilities: in the first case the principal observes the report that the second agent makes to the first one; in the second framework the principal observes the menu of subcontracts offered by the first agent to the second one.

When the principal monitors the report from A_2 to A_1 she becomes informed about the type of the second agent, but it is the first agent who is made worse off by the monitoring activity even if he still earns a positive rent in some states of the world. The agent can then try to nullify the effect of this monitoring by eliminating the communication with the other agent. A way to do so is to offer a "pooling" contract that does not require any report from A_2. The possibility of this reaction introduces a new constraint for the principal: she has to give the right incentives to the agent to screen properly the types of the second one. This new constraint introduces some elements of moral hazard and puts back in the framework some inefficiencies that had disappeared with the monitoring, so that the gain for the principal from monitoring is greatly reduced or disappears completely.

When the principal monitors the sub-contract offer she may get to know the type of the first agent who is then left with no informational rent whatsoever. The first agent, when dealing with the agent at the bottom of the hierarchy, is an "informed principal",[1] since his marginal cost of production is private information to him. Previous results in the literature show that A_1 is indifferent between revealing or not his type to A_2, but the monitoring activity of the principal makes him prefer a not-revealing contract offer.

In other words, as an attempt to neutralise the monitoring activity, A_1 can try to delay the revelation of his own type by offering a menu of two pairs of contracts, each pair designed for one of the possible types of A_2 (but that pools across the types of the first agent, leaving for the moment the second agent uninformed). Then, before production takes place, he will reveal his type and it will be clear which contract, inside the chosen menu, is going to be implemented. The principal then obtains no information and she does not gain from monitoring the first agent's offer of sub-contract.

This work is in the stream of literature on collusion and delegation in hierarchies which started with Tirole [1986][2], who gave a clear cut to the way in which organisations and hierarchies were studied in economic theory. They were no longer considered single blocks but networks of overlapping and nested principal-agent relationships where coalition formation and side-contracting are allowed. Since then many articles have been published on this topic, trying to model the additional incentive problems that delegation and collusion can cause even in very simple hierarchies.

More precisely, the set-up of our model is taken from Laffont and Martimort [1998] where they compare decentralised and centralised organisation of a production process when there are limits on communication.

[1] See Maskin and Tirole [1990] for a clear definition of the informed principal problem.

[2] On collusion in hierarchies see also Tirole [1992] and Laffont and Martimort [1997, 2000].

An analysis very similar to ours is carried out in Baron and Besanko [1992], but they do not consider both monitoring activities and do not model the possibility of reaction by the agents. In fact what we do is trying to endogenise the informational structure in a hierarchy. Another work in this direction is Dequiedt and Martimort [2002], they study a hierarchy where the middle agent can choose whether to learn the information of the bottom agent through fixed cost monitoring or via arm's length contracting. The choice affects the overall costs of information acquisition and the distribution of rents in the hierarchy. They then study how the optimal contract, designed by the principal, changes with the cost of monitoring. They also have some moral hazard in the model because the preferences over the information acquisition methods of the principal and the agent may not be aligned.

Also very related to this, the work by Melumad, Mookherjee and Reichelstein [1995] compares centralised and decentralised structures with similar timing, but agents do not supply strictly complementary inputs as in our setting. Other related works are Felli [1996] and Faure-Grimaud, Laffont and Martimort [2002] but in their settings the first agent is unproductive and plays only a supervisory role.

The structure of the chapter is as follows. Section 3.2 presents the model, utility functions and contracts. Section 3.3 derives the optimal delegation proof contract in the benchmark case. Section 3.4 studies the same organisational structure but allows for the monitoring of the report into the sub-contract. Section 3.5 allows instead for the monitoring of the contract offered. Section 3.6 concludes. All proofs are in the appendix (section 3.7).

3.2 The Model

The principal P wants to buy a quantity q of final good. The first agent A_1 produces a quantity $q_1$of the final good using the intermediate good q_2 which is produced by the second agent A_2. Production uses a Leontieff technology such that $q = q_1 = q_2$, in other words the production process is "componetised".[3] As we said before, the organisational structure is decentralised, so the principal contracts directly only with the first agent, leaving him the freedom of contracting with the second one.

Each agent A_i $(i = 1, 2)$ faces a constant marginal cost θ_i of producing good i. These marginal costs are independently drawn from the same common knowledge distribution with discrete support $\Theta_i = \Theta = \{\underline{\theta}, \overline{\theta}\}$, and $\Delta\theta = \overline{\theta} - \underline{\theta} > 0$. With probability ν the agent is efficient, i.e. $\theta_i = \underline{\theta}$. With probability $(1 - \nu)$ the agent is inefficient, i.e. $\theta_i = \overline{\theta}$.

Each agent knows only its own cost and not that of the other agent. The principal is uninformed on both agents' costs.

The principal maximises her revenue minus the monetary transfers to the first agent:

$$W = S(q) - t$$

with $S'(\cdot) > 0$, $S''(\cdot) < 0$.

The first agent's utility is given by the monetary transfer received by the principal minus the total costs:

$$U_1 = t - \theta_1 q - y_2$$

where y_2 is the transfer he makes to the second agent at the subcontracting stage.

The second agent's utility is given by:

$$U_2 = y_2 - \theta_2 q$$

If we had a centralised structure (where the principal directly contracts with

[3] As in Baron and Besanko [1992] we use the word *componetised* in the sense that the good is formed by putting together components in fixed proportions. The components are produced by different firms or organizational units. As an example we can think of a producer of electricity and a distributor of electricity.

each agent[4]) we would have obtained the following second best quantities and rents:

- $S'(q(\underline{\theta},\underline{\theta})) = 2\underline{\theta}$
- $S'\left(q\left(\underline{\theta},\overline{\theta}\right)\right) = S'\left(q\left(\overline{\theta},\underline{\theta}\right)\right) = S'(\widehat{q}) = \underline{\theta} + \overline{\theta} + \frac{\nu}{1-\nu}\Delta\theta$
- $S'\left(q\left(\overline{\theta},\overline{\theta}\right)\right) = 2\overline{\theta} + \frac{2\nu}{1-\nu}\Delta\theta$
- $U_1\left(\overline{\theta},\theta_i\right) = U_2\left(\theta_i,\overline{\theta}\right) = 0$
- $U_1(\underline{\theta},\theta_i) = U_2(\theta_i,\underline{\theta}) = \Delta\theta\left(\nu\widehat{q} + (1-\nu)\, q\left(\overline{\theta},\overline{\theta}\right)\right)$

The agents are therefore treated symmetrically by the principal and obtain a positive informational rent only when they are efficient.

3.2.1 The contracts

As we mentioned in the previous section the organisation of the productive activity is decentralised, the principal contracts with A_1 and then the latter contracts with A_2. Therefore we will have to study two contracts, which will be offered by the parties at different stages.

The principal proposes a grand contract, GC, to the first agent that specifies a quantity to be produced and a transfer, i.e. a pair $\left\{q\left(\widehat{\theta}_1,\widehat{\theta}_2\right), t\left(\widehat{\theta}_1,\widehat{\theta}_2\right)\right\}$, where $\widehat{\theta}_1$ and $\widehat{\theta}_2$ are the reported types[5]. This contract takes therefore a form which is standard in the adverse selection literature, the menu offered by the principal to the first agent is composed by four pairs, a quantity and a transfer, one for each possible state of the world. Which pair will actually be implemented depends on the report of A_1 about both agents' types.

[4] Laffont and Martimort [1997] show that this outcome is also collusion proof.

[5] The Revelation Principle applies in this framework so we focuse only on direct mechanims, both for the grand contract and for the sub-contract.

At a later stage, A_1, who is the one allowed to communicate with A_2, offers a side contract, SC, to the second agent that consists of a *manipulation-function*[6] of reports and a transfer, i.e. $\left\{\Phi\left(\theta_1, \widetilde{\theta}_2\right), y_2\right\}$, where $\widetilde{\theta}_2$ is the report from A_2 to A_1. This sub-contract is then an agreement between the two agents on how A_1 shall report the information to the principal and how much A_2 receives for each of the possible reports. Therefore while the schedule for q is decided by the contracting between P and A_1, how much will be produced is determined at the subcontracting stage when the two agents fix the manipulation function given their true types. The manipulation function acts therefore as a commitment device for the first agent: if it was not part of the sub-contract then A_1 could have incentive to renege the agreement reached with A_2 over the reports to be made to the principal.[7]

Throughout the paper we assume that sub-contracting is not contractible, that is the contract between the principal and the first agent cannot specify a particular sub-contract between the two agents.

In order to simplify notation, denote $t\left(\overline{\theta}, \overline{\theta}\right) = \overline{t}$; $t\left(\underline{\theta}, \overline{\theta}\right) = \widehat{t}_1$; $t\left(\overline{\theta}, \underline{\theta}\right) = \widehat{t}_2$; $t\left(\underline{\theta}, \underline{\theta}\right) = \underline{t}$ and use a similar notation for $q(\cdot)$.

3.2.2 The timing.

The timing of the game is the following:

1. Nature draws θ_i each agent learns his cost.

2. P proposes the grand contract M to A_1.

3. A_1 offers SC to A_2.

[6] This is a function that to any true pair of types assigns a pair of messages to be delivered to the principal $\Phi : \Theta^2 \to M_1 \times M_2$ (we do not allow random messages). Then because of the Revelation Principle the relevant range for $\Phi(\theta_1, \theta_2)$ will be Θ^2.

[7] In that case the analysis would be much more complicate, various additional incentive constraints should be considered. Unless, of course, the report to be made is included in the sub-contract in another way.

4. A_2 accepts or refuses the other agent's offer, if he refuses the game ends and both agents get their reservation utility.

5. A_2 reports to A_1.

6. A_1 accepts or refuses M, if he refuses the game ends.

7. A_1 reports to P according to the manipulation function $\Phi\left(\theta_1, \widetilde{\theta}_2\right)$.

8. Output and monetary transfers are implemented. t to A_1 according to M. y_2 to A_2 according to SC.

With this timing then the first agent accepts the grand contract only at the very end and more important after getting to know the type of the second agent. This means that individual rationality constraints will have to be satisfied ex-post for A_1. Another possibility would have been to consider interim individual rationality constraints for A_1, this means that the first agent accepts the grand contract when he still does not know A_2's type. This variation would amount to inserting stage 6 in our timing between stages 2 and 3. The principal is better off when she has to satisfy these constraints only in expected terms because she can "play" with the slackness of the constraints in different states of the world and since the agent is risk neutral this does not bring any extra-cost. Actually the second-best can be achieved in this case and delegation has no cost since it implements the same equilibrium as under centralisation. This is because at the time of accepting the contract the principal and A_1 are in the same situation vis-à-vis A_2, none of them knows his type and this is enough to enable the principal to align the interest of the first agent to his own[8]. In our setting instead, at the moment he has to accept the grand contract, A_1 has an advantage over the principal vis-à-vis A_2. He has two pieces of private

[8]This is a well established result (see for example Laffont and Martimort [1998]). Laffont and Martimort [1997] obtain the same result in a centralized framework with collusion under asymmetric information with non-anonymous transfers. There the two agents enter into the side contract offered by an third party without knowing more on each other than the principal does.

information and this will cause the even larger distortions in output compared to the second-best.

Therefore by choosing this particular timing we set ourselves in a framework where delegation is truly costly.[9]

3.3 Delegation without monitoring (benchmark).

In this section we study what can be considered the benchmark case for our analysis, a simple delegation model with no monitoring.[10]

3.3.1 The side contract.

The overall game has two stages so we can solve it backwards starting at the sub-contract stage. When agent A_1, being of type θ_1, offers the sub-contract to the bottom agent he maximises his expected utility with respect to a manipulation function and a transfer to the other agent. Since contracting takes place under asymmetric information A_1 has to ensure that A_2 participates and truthfully reveals his type, so standard constraints have to be considered when solving the following problem, $SC(\theta_1)$[11]:

[9] Alternatively one could assume risk aversion for the first agent as in Faure-Grimaud, Laffont and Martimort [2002] and Faure-Grimaud and Martimort [2001].

[10] The analysis of this section follows an extension of Laffont and Martimort [1998].

[11] Since the first agent has private infromation but act as a principal when contracting with the bottom agent we are in an informed principal framework. As Maskin and Tirole [1990] have shown when utility function are quasilinear the principal cannot gain from concealing her private information. Therefore A_1 does not loose from making a revealing offer.

$$SC(\theta_1) = \begin{cases} \max\limits_{\substack{\Phi(\theta_1,\theta_i) \\ y_2(\theta_1,\theta_i)}} E_{\theta_2}[U_1(\theta_1)] = \nu\left(t\left(\Phi\left(\theta_1,\underline{\theta}\right)\right) - y_2\left(\theta_1,\underline{\theta}\right) - \theta_1 q\left(\Phi\left(\theta_1,\underline{\theta}\right)\right)\right) \\ \qquad\qquad + (1-\nu)\left(t\left(\Phi\left(\theta_1,\overline{\theta}\right)\right) - y_2\left(\theta_1,\overline{\theta}\right) - \theta_1 q\left(\Phi\left(\theta_1,\overline{\theta}\right)\right)\right) \\ \text{s.t.} \\ y_2\left(\theta_1,\overline{\theta}\right) - \overline{\theta} q\left(\Phi\left(\theta_1,\overline{\theta}\right)\right) = 0 \\ y_2\left(\theta_1,\underline{\theta}\right) - \underline{\theta} q\left(\Phi\left(\theta_1,\underline{\theta}\right)\right) = y_2\left(\theta_1,\overline{\theta}\right) - \underline{\theta} q\left(\Phi\left(\theta_1,\overline{\theta}\right)\right) \end{cases} \tag{3.1}$$

Where the two constraints are the participation constraint of an inefficient second agent and the incentive compatibility constraint of an efficient one, the others are trivially satisfied if the schedule of outputs is monotonic. From the binding constraints above we can derive the transfers for the bottom agent:

$$y_2\left(\theta_1,\overline{\theta}\right) = \overline{\theta} q\left(\Phi\left(\theta_1,\overline{\theta}\right)\right) \tag{3.2}$$

$$y_2\left(\theta_1,\underline{\theta}\right) = \underline{\theta} q\left(\Phi\left(\theta_1,\underline{\theta}\right)\right) + \Delta\theta q\left(\Phi\left(\theta_1,\overline{\theta}\right)\right) \tag{3.3}$$

These transfers are conditional on the report to the principal and leave some rent to the efficient type and, by assumption, in our framework they cannot be observed by the principal otherwise she would be able to infer the cost parameter of the first agent.

3.3.2 The Grand Contract.

When offering the grand contract the principal faces some constraints caused by the asymmetry of information between him and the agents; she has to give incentives to the agent she is contracting with, namely A_1, to truthfully report all the valuable information. Since we are in a delegated environment, the first agent has not only to report his own type, but also the second agent's one. This means that the incentive compatibility constraints will be different with respect to those that have to be satisfied in a normal one agent-one principal adverse selection model. In other words, we apply the *Delegation-Proofness Principle*, that says that there is no loss of generality in restricting the analysis to the study of grand mechanisms which are unchanged through the process of delegation, i.e. such that the sub-contract is equal to the "null sub-contract" that is the one where it is optimal for the agents not to manipulate the reports and the manipulation function is the identity function[12], $(\Phi\left(\theta_1,\widetilde{\theta}_2\right)=\left(\theta_1,\widetilde{\theta}_2\right))$.

Lemma 3.1 *A grand contract, GC, is delegation proof if the following incentive compatibility constraints are satisfied:*

$$t\left(\underline{\theta},\underline{\theta}\right)-2\underline{\theta}q\left(\underline{\theta},\underline{\theta}\right)\geq t\left(\theta_1,\theta_2\right)-2\underline{\theta}q\left(\theta_1,\theta_2\right) \tag{3.4}$$

$$t\left(\overline{\theta},\underline{\theta}\right)-\left(\overline{\theta}+\underline{\theta}\right)q\left(\overline{\theta},\underline{\theta}\right)\geq t\left(\theta_1,\theta_2\right)-\left(\overline{\theta}+\underline{\theta}\right)q\left(\theta_1,\theta_2\right) \tag{3.5}$$

$$t\left(\underline{\theta},\overline{\theta}\right)-\left(\underline{\theta}+\overline{\theta}+\frac{\nu}{1-\nu}\Delta\theta\right)q\left(\underline{\theta},\overline{\theta}\right)\geq t\left(\theta_1,\theta_2\right)-\left(\underline{\theta}+\overline{\theta}+\frac{\nu}{1-\nu}\Delta\theta\right)q\left(\theta_1,\theta_2\right) \tag{3.6}$$

$$t\left(\overline{\theta},\overline{\theta}\right)-\left(2\overline{\theta}+\frac{\nu}{1-\nu}\Delta\theta\right)q\left(\overline{\theta},\overline{\theta}\right)\geq t\left(\theta_1,\theta_2\right)-\left(2\overline{\theta}+\frac{\nu}{1-\nu}\Delta\theta\right)q\left(\theta_1,\theta_2\right) \tag{3.7}$$

$\forall\left(\theta_1,\theta_2\right)\in\Theta\times\Theta$.

The above constraints give the conditions that the transfers from the principal to the middle agent has to satisfy to have truthful report in the grand contract in each possible state of nature. The reports are going to be ex-post efficient only for pairs that involve an efficient second agent, in the other two cases there is some

[12] As is becoming common in the works on delegation we loosely borrow from the collusion literature and the concept of collusion proofness, for a definition see Tirole [1992]. In the collusion framework the null side-contract involves also no transfers between the agent, this of course cannot happen in delegation models where transfers are legitimate.

inefficiency due to the asymmetric information at the sub-contract stage. In particular a coalition of the kind $(\overline{\theta},\underline{\theta})$ is more efficient[13] than a coalition of the kind $(\underline{\theta},\overline{\theta})$ therefore the former has an incentive to mimic the latter. This difference is due to the fact that A_1, when he is facing an inefficient second agent, has even more incentives to distort his report because of the informational rent he has to pay to an efficient one[14]. This obviously applies also to a coalition of two inefficient agents.

In solving the problem the principal can think of facing a single individual who can be of four different types in a decreasing (or increasing) efficiency order.

The problem the principal is facing is therefore:

$$\begin{aligned} \max E_{\theta_1,\theta_2}[W] &= \nu^2\left(S\left(\underline{q}\right)-\underline{t}\right)+\nu\left(1-\nu\right)\left(S\left(\widehat{q}_1\right)-\widehat{t}_1\right)+ \\ &+\nu\left(1-\nu\right)\left(S\left(\widehat{q}_2\right)-\widehat{t}_2\right)+\left(1-\nu\right)^2\left(S\left(\overline{q}\right)-\overline{t}\right) \end{aligned} \tag{3.8}$$

Subject to incentive compatibility constraints (3.4-3.7) and the following individual rationality constraints:

$$\begin{aligned} \underline{t}-2\underline{\theta}\underline{q}+-\Delta\theta\widehat{q}_1 &\geq 0 \\ \widehat{t}_1-\left(\underline{\theta}+\overline{\theta}\right)\widehat{q}_1 &\geq 0 \\ \widehat{t}_2-\left(\underline{\theta}+\overline{\theta}\right)\widehat{q}_2-\Delta\theta\overline{q} &\geq 0 \\ \overline{t}-2\overline{\theta}\overline{q} &\geq 0 \end{aligned}$$

These four participation constraints are *ex-post* constraints because the first agent accepts or refuses the grand-contract after he has received the report by the second agent, so he knows exactly what is the state of the world. He has a double

[13] In other words, the virtual type of a coalition $(\overline{\theta},\underline{\theta})$ is lower then the virtual type of a coalition $(\underline{\theta},\overline{\theta})$; where the virtual type is the relevant type for the principal when she chooses production assignments and it is given by the actual type plus the informational rent.

[14] Remember that the informational rent for an efficient second agent is $\Delta\theta q\left(\Phi\left(\theta_1,\overline{\theta}\right)\right)$ so distorting upward the report will reduce the quantity prescribed for a pair $\left(\theta_1,\overline{\theta}\right)$, but also cause a decrease of the informational rent paid in the other two possible situations $(\theta_1,\underline{\theta})$.

informative advantage with respect to the principal.

The binding constraints are the first three upward IC constraints[15] and the individual rationality constraint of a pair of inefficient agents:

$$\underline{t} - 2\underline{\theta}\underline{q} = \widehat{t}_2 - 2\underline{\theta}\widehat{q}_2$$

$$\widehat{t}_2 - \left(\underline{\theta} + \overline{\theta}\right)\widehat{q}_2 = \widehat{t}_1 - \left(\underline{\theta} + \overline{\theta}\right)\widehat{q}_1$$

$$\widehat{t}_1 - \left(\underline{\theta} + \overline{\theta} + \frac{\nu}{1-\nu}\Delta\theta\right)\widehat{q}_1 = \overline{t} - \left(\underline{\theta} + \overline{\theta} + \frac{\nu}{1-\nu}\Delta\theta\right)\overline{q}$$

$$\overline{t} - 2\overline{\theta}\overline{q} = 0$$

If we solve for the transfers and substitute them in the objective function we can then maximise with respect to the optimal quantities.

Proposition 3.1 *The optimal delegation proof contract has the following properties:*

- *for* $\nu < \nu^*$
  - *It implements a decreasing schedule of outputs* $\underline{q} > \widehat{q}_2 > \widehat{q}_1 > \overline{q}$ *where the prescribed quantities are implicitly defined by:*
    - $S'\left(\underline{q}\right) = 2\underline{\theta}$
    - $S'\left(\widehat{q}_2\right) = \overline{\theta} + \underline{\theta} + \frac{\nu}{1-\nu}\Delta\theta$
    - $S'\left(\widehat{q}_1\right) = \overline{\theta} + \underline{\theta} + \frac{\nu(2-\nu)}{(1-\nu)^2}\Delta\theta$
    - $S'\left(\overline{q}\right) = 2\overline{\theta} + \frac{\nu(2-\nu)(1-2\nu)}{(1-\nu)^3}\Delta\theta$
  - *The informational rents granted to the agents are the following:*
    - $U_1\left(\underline{\theta},\underline{\theta}\right) = \Delta\theta\left(\widehat{q}_2 - \widehat{q}_1\right) + \frac{\nu}{1-\nu}\Delta\theta\widehat{q}_1 + \frac{1-2\nu}{(1-\nu)}\Delta\theta\overline{q}$
    - $U_1\left(\underline{\theta},\overline{\theta}\right) = \Delta\theta\overline{q} + \frac{\nu}{1-\nu}\Delta\theta\left(\widehat{q}_1 - \overline{q}\right)$
    - $U_1\left(\overline{\theta},\underline{\theta}\right) = \frac{\nu}{1-\nu}\Delta\theta\left(\widehat{q}_1 - \overline{q}\right)$

[15]The other constraints are satisfied if the quantity schedule is monotonic.

* $U_1\left(\overline{\theta},\overline{\theta}\right)=0$
* $U_2\left(\underline{\theta},\underline{\theta}\right)=\Delta\theta\widehat{q}_1$
* $U_2\left(\overline{\theta},\underline{\theta}\right)=\Delta\theta\overline{q}$
* $U_2\left(\theta_i,\overline{\theta}\right)=0$

- *for* $\nu \geq \nu^*$
  - *It implements a decreasing schedule of outputs with some bunching* $\underline{q} > \widehat{q}_2 > \widetilde{q} = \widehat{q}_1 = \overline{q}$ *where the prescribed quantities are implicitly defined by:*
    * $S'\left(\underline{q}\right)=2\underline{\theta}$
    * $S'\left(\widehat{q}_2\right)=\overline{\theta}+\underline{\theta}+\frac{\nu}{1-\nu}\Delta\theta$
    * $S'\left(\widetilde{q}\right)=2\overline{\theta}+\frac{\nu}{(1-\nu)}\Delta\theta$
  - *The informational rents granted to the agents are the following:*
    * $U_1\left(\underline{\theta},\underline{\theta}\right)=\Delta\theta\widehat{q}_2$
    * $U_1\left(\underline{\theta},\overline{\theta}\right)=\Delta\theta\widetilde{q}$
    * $U_1\left(\overline{\theta},\underline{\theta}\right)=U_1\left(\overline{\theta},\overline{\theta}\right)=0$
    * $U_2\left(\underline{\theta},\underline{\theta}\right)=U_2\left(\overline{\theta},\underline{\theta}\right)=\Delta\theta\widetilde{q}$
    * $U_2\left(\theta_i,\overline{\theta}\right)=0$

These quantities are more distorted downwards than the second best ones; the amounts of informational rent is more than double and consequently the principal optimally trades off some productive efficiency. Comparing these quantities to the second best schedule reveals that the further distortions are in the quantities prescribed to pairs where an inefficient second agent is present, this is due to the extra incentive that A_1 must be given to truthfully report the pair of types after he has paid the informational rent to A_2. Hence there is a cost for the principal of not being able to communicate directly with one agent. This is exactly what is meant by the cost of delegation, the first agent accepts the contract offered by the principal only after he has learned the type of the second agent. He has therefore a double informational advantage with respect to the principal and he is given double informational rent plus a "reimbursement" for the rent he has paid to the second agent.

By comparing the informational rents instead, we can see that the bottom agent is treated as in the second best contract: he gets positive rents only when he is efficient. What is different is what happens to the equilibrium payoff of the first agent when $\nu < \nu^*$, in this sub-case he obtains a positive rent also when he is inefficient but he is paired with an efficient second agent, this is due to the ex-post acceptance of the grand contract that gives him a double informative advantage when deciding about participation in the grand contract. When $\nu \geq \nu^*$ the probability of facing an efficient agent increases therefore the principal gains by bunching the contracts which involve an inefficient second agent.

3.4 Monitoring the report.

We now suppose that the principal can costlessly and perfectly monitor the communication between the agents (the report that A_2 makes to A_1).The *collusive* behavior of A_1 (misreporting two types) is not feasible anymore, and this impossibility is exogenously imposed, it does not stem from the optimizing behavior of the agents. What we now observe is a mismatch between the organisational and informational structures; the first agent still contracts with the second one but, when reporting to the principal, he cannot manipulate the information about the other because the principal knows it already. In other words we have delegation of production and of contracting but not informational delegation.

The sub-contracting between A_1 and A_2 is not directly affected by this monitoring activity, and since the revelation principle is still valid[16] we can restrict attention to the set of direct mechanisms, as we did in the previous section, without the fear of losing in generality. Hence the incentive compatible transfers between the two agents are still given by:

$$y_2\left(\theta_1,\overline{\theta}\right)=\overline{\theta}q\left(\Phi\left(\theta_1,\overline{\theta}\right)\right)$$

$$y_2\left(\theta_1,\underline{\theta}\right)=\underline{\theta}q\left(\Phi\left(\theta_1,\underline{\theta}\right)\right)+\Delta\theta q\left(\Phi\left(\theta_1,\overline{\theta}\right)\right)^{17}$$

What changes instead is the contracting between the principal and the first agent: when offering the grand contract P has to give incentives to A_1 to reveal only one piece of information, his own type, because he already knows the type of the second agent. Since the agent cannot misreport the other's type, incentive compatibility needs to hold over two separate pairs of contracts, each pair pools across the types of the second agent.

Lemma 3.2 *When the principal can monitor the report from A_2 to A_1, a grand contract is incentive compatible if the following constraints are satisfied:*

$$\underline{t}-2\underline{\theta}\underline{q}=\widehat{t}_2-2\underline{\theta}\widehat{q}_2 \tag{3.9}$$

$$\widehat{t}_1-\left(\underline{\theta}+\overline{\theta}+\frac{\nu}{1-\nu}\Delta\theta\right)\widehat{q}_1=\overline{t}-\left(\underline{\theta}+\overline{\theta}+\frac{\nu}{1-\nu}\Delta\theta\right)\overline{q} \tag{3.10}$$

The above are the relevant incentive compatibility constraints, namely the ones of an efficient first agent paired respectively with an efficient and inefficient second

[16] The intuition of why this is so goes as follows: any inference that A_1 can make on the type of A_2 can be made also by the principal, hence allowing also indirect mechanisms does not change the results.

[17] Now the side contract includes a trivial version of the manipulation function which is now $\Phi\left(\theta_1,\theta_2\right)=\left(\widehat{\theta}_1,\theta_2\right)$, in other words only the type of the first agent can be distorted via a non truthful report.

agent. Now only pairs which include an efficient first agent will be given incentives to report truthfully, exactly because the type of the agent is known to the principal.

These constraints are ex-post because when A_1 reports to the principal they both already know A_2's type. The first agent knows it because the side-contracting stage, and the report it entails, precedes the moment he has to report to the principal. The principal in turn is allowed to listen to the truthful report that A_2 makes to A_1.

The principal must also ensure the participation of the first agent into the grand contract, therefore the following individual rationality constraints have to be satisfied:

$$
\begin{aligned}
\underline{t} - 2\underline{\theta}\underline{q} + -\Delta\theta\widehat{q}_1 &\geq 0 \\
\widehat{t}_1 - \left(\underline{\theta} + \overline{\theta}\right)\widehat{q}_1 &\geq 0 \\
\widehat{t}_2 - \left(\underline{\theta} + \overline{\theta}\right)\widehat{q}_2 - \Delta\theta\overline{q} &\geq 0 \\
\overline{t} - 2\overline{\theta}\overline{q} &\geq 0
\end{aligned}
$$

These are the usual participation constraints for A_1 and in the benchmark case only one was binding. Instead now, that the principal monitors and gets to know the type of A_2, an inefficient first agent will be left with his reservation utility irrespectively of the type of second agent he is matched with. Namely:

$$
\widehat{t}_2 - \left(\underline{\theta} + \overline{\theta}\right)\widehat{q}_2 - \Delta\theta\overline{q} = 0 \tag{3.11}
$$

$$
\overline{t} - 2\overline{\theta}\overline{q} = 0 \tag{3.12}
$$

This is because the principal is extracting only one piece of information, she knows the type of A_2 therefore she is not giving any *extra* rent to A_1 to reveal that the second agent is efficient.

We are now ready to characterise the contract offered.

Proposition 3.2 *When the principal can costlessly and perfectly monitor the report of the second agent into the sub-contract the optimal grand contract has the following characteristics:*

- *It implements a decreasing schedule of output* $\underline{q} > \widehat{q} > \overline{q}$ *(where* $\widehat{q} = \widehat{q}_1 = \widehat{q}_2$*) implicitly defined by:*
  - $S'(\underline{q}) = 2\underline{\theta}$
  - $S'(\widehat{q}) = (\underline{\theta} + \overline{\theta}) + \frac{\nu}{1-\nu}\Delta\theta$
  - $S'(\overline{q}) = 2\overline{\theta} + \frac{2\nu}{1-\nu}\Delta\theta$
- *The informational rents granted to the agents are the following:*
  - $U_1(\underline{\theta}, \underline{\theta}) = \Delta\theta\overline{q}$
  - $U_1(\underline{\theta}, \overline{\theta}) = \frac{\nu}{1-\nu}\Delta\theta\widehat{q} + \cdot\frac{1-2\nu}{(1-\nu)}\Delta\theta\overline{q}$
  - $U_1(\overline{\theta}, \theta_i) = 0$
  - $U_2(\underline{\theta}, \underline{\theta}) = \Delta\theta\widehat{q}$
  - $U_2(\overline{\theta}, \underline{\theta}) = \Delta\theta\overline{q}$
  - $U_2(\theta_i, \overline{\theta}) = 0$

In each state of the world the quantities produced are equal to those that would be produced in a centralised organisation, this means that if the principal is allowed to monitor the report made into the sub-contract the second best can be achieved.[18] The principal though, cannot do better than the second best even if she gets to know a piece of information because she receives this information when the second agent is reporting to the first one after he has been given the right rents and incentives to do so. These in turn are costs for A_1 that the principal has to reimburse if she

[18] Note in fact that $\widehat{q}_1 = \widehat{q}_2$, symmetry is back in the model because the principal can avoid paying the extra-rent so that the two pairs $(\underline{\theta}, \overline{\theta})$ and $(\overline{\theta}, \underline{\theta})$ can now be treated equally as in a centralized organization.

wants to ensure the participation of A_1 in the production process. In other words, in the organisation overall two pieces of information have to be extracted, one by A_1 and one by the principal, exactly like in a centralised setting where both pieces are extracted by the principal.

With the monitoring what disappears is the extra-cost of delegation compared to centralisation, but nothing more: even if we do not have informational delegation anymore we still face two agents that have private information and this keeps us in a second best world. The expected rents that the principal has to pay are exactly the second best ones, what is different is their distribution: when the principal monitors, an efficient first agent gets more rent when paired with an inefficient second one and less when the other agent is efficient than in the centralised organisation.

If we compare the equilibrium payoffs of all the players we can see that the principal gains from the monitoring while the first agent is worse off, he receives lower informational rents in two states of the world. The agent at the bottom of the hierarchy, instead, is unaffected by the monitoring because he still receives the right incentives to reveal his type when he sub-contracts with the middle agent. It is only after the sub-contract has been offered and accepted that the principal gets to know his type.

3.4.1 Can the agent profitably deviate?

We have seen that monitoring is useful to the principal, when this possibility is available, in fact, she can achieve the second best. She reduces distortions and lowers the transfers with respect to the benchmark case. This improvement happens at the expenses of A_1 who gets lower utility in two states of the world, the ones in which he is paired with an efficient second agent.

We argue that to be complete the analysis of this environment should consider the possibility of the agent to deviate to the monitoring activity carried over by the head of the hierarchy, we will then see whether any profitable deviation exists and if this affects the overall equilibrium of the game.

Since the principal gets an advantage when she listens to the communication between the two agents at the sub-contracting stage a possible reaction is to reduce or eliminate the communication that is monitored[19]. In other words A_1 could offer a contract that does not require a report, that is independent from A_2's type. We call this a pooling sub-contract, because it does not separate the types of A_2. More precisely the first agent will offer a set of transfers to the second one as if he was always inefficient, namely:

$$y_2(\theta_1, \theta_2) = \overline{\theta} q(\Phi(\theta_1, \theta_2))$$

By paying always the high marginal cost of production he ensures that both types of A_2 are willing to participate in fact their individual rationality constraints are satisfied:

$$\overline{U}_2 = 0$$

$$\underline{U}_2 = \Delta\theta q(\Phi(\theta_1, \theta_2)) > 0$$

Because A_2 can only be of two types these ex-post payoffs are the same as in the previous cases, when the incentive compatibility of the efficient type was binding making him indifferent between telling the truth and claiming to be inefficient.

Note that the transfer $y_2(\cdot)$ and the quantities to be produced are apparently still dependent on both types, this is to be more consistent with what we have done so far and have a more homogeneous notation. In theory in this case the message space for the first agent when reporting to the principal is larger than before (it is equal to the one in the benchmark case), when, given the monitoring, A_1 was

[19] We assume that the action space of A_1 is given by the set of possible sub-contracts he can offer to A_2.

restricted to the message space $\{m_1, \theta_2\}$ (he had to report the true θ_2). Now the message space is in fact $\{m_1, m_2\}$ but the pooling contract implies that $m_2 = \overline{\theta}$ always. As a consequence of this pooling contract the manipulation function is reduced once again to a function of one variable: $\Phi(\theta_1, \theta_2) = \left(\widehat{\theta}_1, \overline{\theta}\right)$.

If we restrict ourselves to non-random contracts given that agents can be only of two possible types then this pooling contract is the only possible deviation, no partial pooling (or semi-separating) is possible, and it is also consistent with the idea of concealing some information to the principal[20].

We solve this game backwards so at the time A_1 has to offer the sub-contract now the decision is more complex, first for a given grand contract he must decide whether to offer a pooling or a separating sub-contract. Then, given this choice, the principal, who will anticipate the agent's behavior, will make her offer of the grand contract.

More precisely given a grand contract $GC = \{\underline{t}, \underline{q}, \widehat{t}_1, \widehat{q}_1, \widehat{t}_2, \widehat{q}_2, \overline{t}, \overline{q}\}$, A_1 will choose the type of subcontract that maximises his expected utility[21].

When offering a pooling sub-contract the expected utility of A_1 when his type is θ_1 is:

$$U_P(\theta_1) = t\left(\Phi\left(\theta_1, \overline{\theta}\right)\right) - \left(\overline{\theta} + \theta_1\right) q\left(\Phi\left(\theta_1, \overline{\theta}\right)\right) \ ,$$

the expected utility of a separating offer is instead:

$$\begin{aligned} U_1(\theta_1) \quad = \quad & \nu\left(t\left(\Phi\left(\theta_1, \underline{\theta}\right)\right) - \underline{\theta} q\left(\Phi\left(\theta_1, \underline{\theta}\right)\right) - \Delta\theta q\left(\Phi\left(\theta_1, \overline{\theta}\right)\right) - \theta_1 q\left(\Phi\left(\theta_1, \underline{\theta}\right)\right)\right) + \\ & (1-\nu)\left(t\left(\Phi\left(\theta_1, \overline{\theta}\right)\right) - \overline{\theta} q\left(\Phi\left(\theta_1, \overline{\theta}\right)\right) - \theta_1 q\left(\Phi\left(\theta_1, \overline{\theta}\right)\right)\right) \end{aligned}$$

What A_1 will prefer depends on the grand contract, which in turn will depend

[20] Also indirect mechanisms are not of any use, any information that they would convey could be monitored by the principal.

[21] With expectations taken over the possible types of A_2 since the offer is made before knowing the type of the second agent.

on the goals that the principal wishes to achieve through the contract. The principal will want truthful revelation and a separating contract offer because this allows her to obtain the second best which is by definition the best she can reach given the asymmetric information setting and the monitoring.

As we saw before the constraints that need to be satisfied to obtain truthtelling by A_1 are:

$$\begin{aligned} \underline{t} - 2\underline{\theta}\underline{q} &= \widehat{t}_2 - 2\underline{\theta}\widehat{q}_2 \\ \widehat{t}_1 - \left(\underline{\theta} + \overline{\theta} + \frac{\nu}{1-\nu}\Delta\theta\right)\widehat{q}_1 &= \overline{t} - \left(\underline{\theta} + \overline{\theta} + \frac{\nu}{1-\nu}\Delta\theta\right)\overline{q} \end{aligned}$$

We can, without loss of generality, limit the analyses to the case of an efficient first agent because an inefficient one will receive his reservation utility regardless of the type of sub-contract offered. If the principal wants the first agent to screen among the different types of the other agent, it has to be that the separating contract gives A_1 a higher expected utility than the pooling one, that is:

$$\nu U_1\left(\underline{\theta},\underline{\theta}\right) + (1-\nu)\,U_1\left(\underline{\theta},\overline{\theta}\right) \geq U_P^*\left(\underline{\theta}_1\right) \tag{3.13}$$

where $U_1\left(\underline{\theta},\underline{\theta}\right)$ and $U_1\left(\underline{\theta},\overline{\theta}\right)$ are the rents earned by an efficient first agent who is paired with and efficient and inefficient second agent respectively when he offers a separating sub-contract and truthfully reports to the principal. While $U_P^*\left(\underline{\theta}_1\right)$ is the maximum utility that can be achieved by an efficient first agent that offers a pooling sub-contract, and it is defined as:

$$U_P^*\left(\underline{\theta}_1\right) = \max_{\Phi} t\left(\Phi\left(\underline{\theta}_1,\overline{\theta}\right)\right) - \left(\overline{\theta}+\underline{\theta}_1\right) q\left(\Phi\left(\underline{\theta}_1,\overline{\theta}\right)\right)$$

Constraint (3.13) is in fact a moral hazard constraint, when designing the contract the principal has to give incentives to the first agent to do her preferred action which in this case is offering a screening contract.

It is worth noting that $U_P^*(\underline{\theta}_1)$ could be achieved by truthtelling but also by any other report, the following Remark is of some help in this direction.

Remark 3.1 *If a Grand Contract is incentive compatible when the sub-contract offer is separating then it is incentive compatible if the offer is pooling and the expected utility of* A_1 *is:*

$$U_P^*(\underline{\theta}_1) = \widehat{t}_1 - \left(\underline{\theta} + \overline{\theta}\right)\widehat{q}_1.$$

Having calculated the maximum a first agent can get under any of the two possible sub-contract offers we are now ready to check which will be the preferred choice given the optimal contract designed by the principal.

Proposition 3.3 *When the principal can monitor the report of the bottom agent into the sub-contract and the first agent is free to choose the type of sub-contract the optimal contract has the following characteristics:*

- *if* $\nu < \nu^{**}$ *then the principal implements the following decreasing schedule of output* $\underline{q} > \widehat{q}_2 > \widehat{q}_1 > \overline{q}$ *defined by:*

  - $S'\left(\underline{q}\right) = 2\underline{\theta}$
  - $S'\left(\widehat{q}_2\right) = \overline{\theta} + \underline{\theta}$
  - $S'\left(\widehat{q}_1\right) = \overline{\theta} + \underline{\theta} + \frac{\nu(2-\nu)}{(1-\nu)^2}\Delta\theta$
  - $S'\left(\overline{q}\right) = 2\overline{\theta} + \frac{\nu\left(2-4\nu+\nu^2\right)}{(1-\nu)}\Delta\theta$

  and it leaves the following informational rents to the first agent:

  - $U_1\left(\underline{\theta},\underline{\theta}\right) = U_1\left(\underline{\theta},\overline{\theta}\right) = \Delta\theta\overline{q} + \frac{\nu}{(1-\nu)}\Delta\theta\left(\widehat{q}_1 - \overline{q}\right)$
  - $U_1\left(\overline{\theta},\overline{\theta}\right) = U_1\left(\overline{\theta},\underline{\theta}\right) = 0$

- *if $\nu \geq \nu^{**}$ the optimal contract entails some bunching and the implemented schedule $\underline{q} > \widehat{q}_2 > \widetilde{q}\,(= \widehat{q}_1 = \overline{q})$ is defined by:*
  - $S'(\underline{q}) = 2\underline{\theta}$
  - $S'(\widehat{q}_2) = \overline{\theta} + \underline{\theta} + \frac{\nu}{(1-\nu)}\Delta\theta$
  - $S'(\widetilde{q}) = 2\overline{\theta} + \frac{\nu}{(1-\nu)}\Delta\theta$

  and it leaves the following informational rents to the first agent:
  - $U_1(\underline{\theta}, \underline{\theta}) = \Delta\theta\widehat{q}_2$
  - $U_1(\underline{\theta}, \overline{\theta}) = \Delta\theta\widetilde{q}$
  - $U_1(\overline{\theta}, \overline{\theta}) = U_1(\overline{\theta}, \underline{\theta}) = 0.$

If we take into consideration the possibility for the first agent of choosing a pooling subcontract then we introduce also some moral hazard constraint which in one case is binding. For low values of ν, the moral hazard constraint is active and reduces the equilibrium payoff of the principal when she monitors. In this case the second best is not attainable anymore, but the optimal contract is still more efficient than the one without monitoring. There is no distortion at the top for two pairs[22], $\widehat{q}_1$ is equal to the no-monitoring benchmark and $\overline{q}$ is higher than the one in the no-monitoring case. Informational rents for the first agent are lower than in the no-monitoring case and above those with monitoring but without the moral-hazard constraint. For high values of ν the optimal contract does not screen between a pair of two inefficient agents and the mixed one in which the first agent is efficient exactly as it happens in the case of no monitoring, also quantities and rents are the same.

In other words, for any value of ν, the threat of deviation of the first agent in presence of monitoring is effective. The possibility of him offering a pooling contract

[22] This is because the first upward incentive constraints is not binding anymore due to the presence of the moral hazard constraint.

to the second agent introduces an additional constraint that greatly reduces, and in some case nullifies, the gains obtained by the principal through monitoring. It is like if we were reintroducing some sort of incentive compatibility constraint, the one in which the principal gives incentives to the first agent to report that the other one is efficient. This constraint had disappeared from the program because of the monitoring.

3.5 Monitoring the sub-contract.

In the previous section we supposed that the principal could monitor the response by the second agent to the contract offer, more precisely she could listen to a report or observe the choice of a particular sub-contract but always without observing the contract itself. We now study what will happen in the hierarchy when the principal can observe the part of communication at the sub-contracting stage that comes from the first agent, she will therefore observe, perfectly and at no cost, the menu of contracts offered by A_1 to A_2 .

As we noted earlier each possible type of A_1 offers a menu of contracts that depends on his own type (in other words the offer is revealing), therefore the principal will be able to know the type of the first agent and she will obtain this information at no cost.

This monitoring does not directly affect the side-contracting stage so, if there is no reaction from the agents, the optimal transfers that induce participation and truthful revelation by A_2 are still given by:

$$\begin{aligned} y_2(\theta_1,\overline{\theta}) &= \overline{\theta}q(\Phi(\theta_1,\overline{\theta})) \\ y_2(\theta_1,\underline{\theta}) &= \underline{\theta}q(\Phi(\theta_1,\underline{\theta}))+\Delta\theta q\left(\Phi(\theta_1,\overline{\theta})\right) \end{aligned}$$

As we just said, these are observed by the principal that gets to know θ_1 with certainty, so there is no more asymmetric information between P and A_1 concerning the type of the latter, what is left is the asymmetric information (at the report stage) between P and A_1 vis-a-vis A_2 . The principal has to give incentives to A_1 only to report what he has learned from the other agent.

Because of the information acquired by the principal, the first agent cannot misrepresent his type anymore and incentive compatibility needs to hold over two sets of two contracts each, instead of over the four original contracts (now only one piece of information needs to be reported). This is similar to what was happening to the possibility of misreporting the second agent's type in the previous section, the corresponding Lemma holds.

Lemma 3.3 *When the principal can monitor the sub-contract offer a grand contract is incentive compatible if the following constraints are satisfied:*

$$\underline{t} - 2\underline{\theta}\underline{q} = \widehat{t}_1 - 2\underline{\theta}\widehat{q}_1 \tag{3.14}$$

$$\widehat{t}_2 - \left(\underline{\theta} + \overline{\theta}\right)\widehat{q}_2 = \overline{t} - \left(\underline{\theta} + \overline{\theta}\right)\overline{q}^{23} \tag{3.15}$$

In other words the relevant (and binding) incentive compatibility constraints are only those of a coalition made by two efficient agents and a mixed pair with an efficient second agent. The individual rationality constraints for the first agent will be binding regardless of his type, moreover the coalition participation constraints will be binding when there is an inefficient second agent in the pair:

$$\overline{t} - \overline{\theta}\overline{q} = 0 \tag{3.16}$$

$$\widehat{t}_1 - \left(\underline{\theta} + \overline{\theta}\right)\widehat{q}_1 = 0 \tag{3.17}$$

We can think about this situation as if the principal could see through A_1, we are now able to characterise the contract.

Proposition 3.4 *When the principal can costlessly and perfectly monitor the side-contract offered to the second agent the optimal contract implements has the following characteristics:*

- *It implements a decreasing schedule of output* $\underline{q} > \widehat{q}_2 > \widehat{q}_1 > \overline{q}$, *where the prescribed quantities are implicitly defined by:*
  - $S'\left(\underline{q}\right) = 2\underline{\theta}$
  - $S'\left(\widehat{q}_2\right) = \left(\underline{\theta} + \overline{\theta}\right)$
  - $S'\left(\widehat{q}_1\right) = \left(\underline{\theta} + \overline{\theta}\right) + \frac{\nu}{1-\nu}\Delta\theta$
  - $S'\left(\overline{q}\right) = 2\overline{\theta} + \frac{\nu}{1-\nu}\Delta\theta$
- *The informational rents granted to the agents are the following:*
  - $U_1\left(\underline{\theta}, \underline{\theta}\right) = U_1\left(\underline{\theta}, \overline{\theta}\right) = U_1\left(\overline{\theta}, \underline{\theta}\right) = U_1\left(\overline{\theta}, \overline{\theta}\right) = 0$
  - $U_2\left(\underline{\theta}, \underline{\theta}\right) = \Delta\theta\widehat{q}_1$
  - $U_2\left(\overline{\theta}, \underline{\theta}\right) = \Delta\theta\overline{q}$
  - $U_2\left(\overline{\theta}, \overline{\theta}\right) = U_2\left(\underline{\theta}, \overline{\theta}\right) = 0$

It is interesting to note that at this stage the first two quantities are efficient, we observe the "no distortion at the top" condition each time the second agent is efficient, also the schedule of output is monotonic with respect to θ_2. This is because when the principal monitors the contracts offer the asymmetry of information with respect to A_1 disappears, in fact the principal can improve upon the second best, which by definition is the upper bound when you have two pieces of private information. For the very same reason, the first agent does not earn any informational rent, while because the second one earns a rent when he is efficient because sub-contracting happens as if no monitoring was taking place.

3.5.1 The agent's reaction.

The first agent loses all the advantages coming from his private information thanks to the monitoring activity of the principal. A reaction on his part should go in the direction of concealing in some way that part of information that he transmits via the contract offer.

In order not to reveal any important information to the principal that observes the menu of contracts he can offer to A_2 two menus of two contracts each. He asks for truthful revelation from A_2, but postpones the disclosure of his private information until the last stage (at least after he has reported to the principal). The second agent will know later the type of A_1: before production takes place, he will find out what amount of the two prescribed by the chosen side-contract he has to produce. In fact the quantity that the principal demands is dependent on both agents' types.

At this stage it is important to discuss what happens to the principal's beliefs about A_1's type when she observes the sub-contract offer. We are going to assume that if the offer is not revealing then the principal does not update and the beliefs coincide with the prior probability distribution over possible types.

More precisely the sub-contracts take the following form:

$$SC(\theta_1, \underline{\theta}) = \{y_2(\theta_1, \underline{\theta}), \Phi(\theta_1, \underline{\theta}); \quad \theta_1 \in \Theta\}$$

$$SC\left(\theta_1, \overline{\theta}\right) = \left\{y_2\left(\theta_1, \overline{\theta}\right), \Phi\left(\theta_1, \overline{\theta}\right); \quad \theta_1 \in \Theta\right\}$$

These two contracts are designed for an efficient and an inefficient second agent (respectively) but are conditioned on the type of the first agent as well, any type of the second agent will choose the contract designed for himself and wait until a later stage to find out exactly what price-quantity pair of the possible two will be implemented.

Moreover, A_2 is aware of the fact that both transfer and quantity will be later conditioned on the type of A_1 also, but at the acceptance and report stage he does not possess this information (and learns nothing from the sub-contract offered) therefore all his constraints, participation and incentive compatibility, will take an *interim* form. We, in addition, assume that the beliefs of the second agent remain equal to the prior if the sub-contract offer pools across the types of A_1.

In particular the relevant interim constraints are:

$$\nu\left(y_2\left(\underline{\theta},\overline{\theta}\right)-\overline{\theta}q\left(\Phi\left(\underline{\theta},\overline{\theta}\right)\right)\right)+(1-\nu)\left(y_2\left(\overline{\theta},\overline{\theta}\right)-\overline{\theta}q\left(\Phi\left(\overline{\theta},\overline{\theta}\right)\right)\right)=0 \tag{3.18}$$

$$\begin{aligned}&\nu\left(y_2\left(\underline{\theta},\underline{\theta}\right)-\underline{\theta}q\left(\Phi\left(\underline{\theta},\underline{\theta}\right)\right)\right)+(1-\nu)\left(y_2\left(\overline{\theta},\underline{\theta}\right)-\underline{\theta}q\left(\Phi\left(\overline{\theta},\underline{\theta}\right)\right)\right)= \\ = \ &\nu\left(y_2\left(\underline{\theta},\overline{\theta}\right)-\underline{\theta}q\left(\Phi\left(\underline{\theta},\overline{\theta}\right)\right)\right)+(1-\nu)\left(y_2\left(\overline{\theta},\overline{\theta}\right)-\underline{\theta}q\left(\Phi\left(\overline{\theta},\overline{\theta}\right)\right)\right)\end{aligned} \tag{3.19}$$

where (3.19) is the individual rationality constraint of an inefficient second agent and (3.18) is the incentive compatibility constraint of an efficient second agent.

The standard techniques employed to solve for the optimal side-contract would require, at this stage, to solve for the transfers first but from the equations above it is evident that our system is underidentified (we have four unknowns and only two equations), so there is some leeway in determining the optimal transfers between the agents. One possibility is to break up the interim constraints and impose the ex-post ones, so that we get an equal number of equations and unknowns.

A more formal justification comes from Mookherjee and Reichelstein [1992] that tells us how in this environment (linear utilities and constant marginal costs) we can equivalently implement this Bayesian allocation in dominant strategies without incurring in any loss for the "principal" (in this case the first agent). But requiring dominant strategy implementation amounts to breaking up the constraints as we have just explained above.

Another justification for proceeding in this way comes from a subtle application of the theorems found in Maskin and Tirole [1990]: they prove that, in a informed-principal relationship with private values and quasilinear utilities, the principal is indifferent between revealing or not his information to the agent. This finding applies to our setting but some more considerations need to be done. If it is true that the first agent (the "informed-principal" in our case) does not gain nor loose vis-à-vis A_2 by revealing his type, then the same holds for the latter. Therefore A_1 guarantees to A_2 the satisfaction of the ex-post constraints even if he need not to; this because he is not trying to improve upon the contracting with A_2, he wants to regain some power at the grand-contract stage and he does it by postponing the revelation of his type. Then, if the contract is offered as we proposed above and if the transfers are computed using dominant strategy, the second agent participates and truthfully reveals his information[24] but the principal does not observe anything (and cannot force A_1on his reservation utility).

The benefit to A_1 from not revealing his type through the contract offer does not come from the additional surplus he gets from A_2 but from the increase in rents he manages to obtain from the principal when he is offered the grand contract.

For all these reasons we can then use the dominant strategy constraints, and note that they are the same ones calculated in the previous section where no monitoring was going on. If transfers in the side contract are the same, then also incentive compatibility and participation constraints for A_1, when contracting with P, are identical to the previous ones. This means that the optimisation problem that the principal solves does not change and the optimal contract is the one derived in Proposition 2. This enables us to state the following.

Proposition 3.5 *The principal does not gain in monitoring the menu of sub-contracts*

[24]This happens because the contract is identical in the transfers and quantities to the one that would have been offered if the principal was not monitoring. The only difference is that the revelation of the private information of the first agent is slightly delayed.

that the first agent offers to the second one.

Proof. If the first agent offers the following pair of side contracts:

$$\begin{aligned} SC(\theta_1,\underline{\theta}) &= \{y_2(\theta_1,\underline{\theta}),\Phi(\theta_1,\underline{\theta});\quad \theta_1\in\Theta\} \\ SC(\theta_1,\overline{\theta}) &= \{y_2(\theta_1,\overline{\theta}),\Phi(\theta_1,\overline{\theta});\quad \theta_1\in\Theta\} \end{aligned}$$

then second agent does not learn the type of the first agent and his constraints have to hold in expectation, at an interim stage. The relevant ones are (3.18) and (3.19):

$$\nu\left(y_2(\underline{\theta},\overline{\theta})-\overline{\theta}q\left(\Phi(\underline{\theta},\overline{\theta})\right)\right)+(1-\nu)\left(y_2(\overline{\theta},\overline{\theta})-\overline{\theta}q\left(\Phi(\overline{\theta},\overline{\theta})\right)\right)=0$$

$$\begin{aligned} &\nu\left(y_2(\underline{\theta},\underline{\theta})-\underline{\theta}q\left(\Phi(\underline{\theta},\underline{\theta})\right)\right)+(1-\nu)\left(y_2(\overline{\theta},\underline{\theta})-\underline{\theta}q\left(\Phi(\overline{\theta},\underline{\theta})\right)\right)= \\ = \;&\nu\left(y_2(\underline{\theta},\overline{\theta})-\underline{\theta}q\left(\Phi(\underline{\theta},\overline{\theta})\right)\right)+(1-\nu)\left(y_2(\overline{\theta},\overline{\theta})-\underline{\theta}q\left(\Phi(\overline{\theta},\overline{\theta})\right)\right) \end{aligned}$$

This constraints are satisfied by the ex-post transfers of our benchmark model. This leaves the bottom agent indifference between one setting or another. But if the sub-transfers are the ex-post ones then the problem that A_1 solves when offering the sub-contract is exactly like problem (3.1) in the benchmark case. This implies that also the delegation proof constraints that the grand contract has to satisfy for the first agent to reveal truthfully the two types are exactly those of the benchmark model. Since the objective function of the principal is unchanged, due to the fact that her beliefs about the type of the first agent remain the prior after a not revealing contract offer, the solution of the optimisation problem that defines the grand contract is the same. Quantities and transfers will be the same of the benchmark case. ■

What we have learned from the above analysis is that, in a delegation model, the principal does not gain if she monitors, even at no cost, the menu of subcontracts offered by agents in lower levels of the hierarchy because of the reaction of the agent that looses the most from this activity. In fact, the latter manages to offer a sub-contract that allows him to reveal his private information only before production takes place, and all this happens without affecting the welfare of the agent at the bottom of the hierarchy.

3.6 Concluding remarks.

We have seen that not every type of monitoring is equally beneficial to the head of a hierarchy because of the possible deviations by the lower levels. The monitoring that gives a higher utility as a direct effect, the monitoring of contracts, can always be nullified by an action of the agent that looses the most. Monitoring of reports can be beneficial in some cases, nonetheless it always triggers a deviation from the agent proposing the sub-contract in the attempt of concealing some information to the principal.

This analysis was carried out in the spirit of considering hierarchies as networks of agents that interact through contracts. We therefore wanted to take a closer look at the strategic interactions of members of hierarchical organisations and see whether strategic behavior could at least weaken some "results" that hold in standard one principal-one agent models.

Despite our attempt to generalise the analysis our setup was quite specific, it is probably worth extending the analysis to a different production function (some degree of substitution allowed) and to a setting where there is correlation between the types of the agents (the monitoring could even have some stronger effect).

3.7 Appendix

Proof of Lemma 3.1. If we substitute the optimal side transfers (3.3) and (3.2) into the first agent's expected utility function (??) we get:

$$\begin{aligned} \max E_{\theta_2}[U_1] &= \nu\left(t\left(\Phi\left(\theta_1,\underline{\theta}\right)\right)-\underline{\theta}q\left(\Phi\left(\theta_1,\underline{\theta}\right)\right)-\Delta\theta q\left(\Phi\left(\theta_1,\overline{\theta}\right)\right)-\theta_1 q\left(\Phi\left(\theta_1,\underline{\theta}\right)\right)\right)+ \\ & \quad (1-\nu)\left(t\left(\Phi\left(\theta_1,\overline{\theta}\right)\right)-\overline{\theta}q\left(\Phi\left(\theta_1,\overline{\theta}\right)\right)-\theta_1 q\left(\Phi\left(\theta_1,\overline{\theta}\right)\right)\right) \end{aligned}$$

now we can check for incentive compatibility for any of the possible "coalition" (there are four of them).

Then the condition for the optimality of $\Phi\left(\underline{\theta},\underline{\theta}\right)=\left(\underline{\theta},\underline{\theta}\right)$ is the following:

$\nu\left(t\left(\underline{\theta},\underline{\theta}\right)-2\underline{\theta}q\left(\underline{\theta},\underline{\theta}\right)-\Delta\theta q\left(\Phi\left(\underline{\theta},\overline{\theta}\right)\right)\right)+(1-\nu)\left(t\left(\Phi\left(\underline{\theta},\overline{\theta}\right)\right)-\left(\overline{\theta}+\underline{\theta}\right)q\left(\Phi\left(\underline{\theta},\overline{\theta}\right)\right)\right)\geq$

$\nu\left(t\left(\theta_1,\theta_2\right)-2\underline{\theta}q\left(\theta_1,\theta_2\right)-\Delta\theta q\left(\Phi\left(\underline{\theta},\overline{\theta}\right)\right)\right)+(1-\nu)\left(t\left(\Phi\left(\underline{\theta},\overline{\theta}\right)\right)-\left(\overline{\theta}+\underline{\theta}\right)q\left(\Phi\left(\underline{\theta},\overline{\theta}\right)\right)\right).$

For $\Phi\left(\underline{\theta},\overline{\theta}\right)=\left(\underline{\theta},\overline{\theta}\right)$ is:

$\nu\left(t\left(\Phi\left(\underline{\theta},\underline{\theta}\right)\right)-2\underline{\theta}q\left(\Phi\left(\underline{\theta},\underline{\theta}\right)\right)-\Delta\theta q\left(\underline{\theta},\overline{\theta}\right)\right)+(1-\nu)\left(t\left(\underline{\theta},\overline{\theta}\right)-\left(\underline{\theta}+\overline{\theta}\right)q\left(\underline{\theta},\overline{\theta}\right)\right)\geq$

$\nu\left(t\left(\Phi\left(\underline{\theta},\underline{\theta}\right)\right)-2\underline{\theta}q\left(\Phi\left(\underline{\theta},\underline{\theta}\right)\right)-\Delta\theta q\left(\theta_1,\theta_2\right)\right)+(1-\nu)\left(t\left(\theta_1,\theta_2\right)-\left(\underline{\theta}+\overline{\theta}\right)q\left(\theta_1,\theta_2\right)\right).$

For $\Phi\left(\overline{\theta},\underline{\theta}\right)=\left(\overline{\theta},\underline{\theta}\right)$ is:

$\nu\left(t\left(\overline{\theta},\underline{\theta}\right)-\left(\overline{\theta}+\underline{\theta}\right)q\left(\overline{\theta},\underline{\theta}\right)-\Delta\theta q\left(\Phi\left(\overline{\theta},\overline{\theta}\right)\right)\right)+(1-\nu)\left(t\left(\Phi\left(\overline{\theta},\overline{\theta}\right)\right)-2\overline{\theta}q\left(\Phi\left(\overline{\theta},\overline{\theta}\right)\right)\right)\geq$

$\nu\left(t\left(\theta_1,\theta_2\right)-\left(\overline{\theta}+\underline{\theta}\right)q\left(\theta_1,\theta_2\right)-\Delta\theta q\left(\Phi\left(\overline{\theta},\overline{\theta}\right)\right)\right)+(1-\nu)\left(t\left(\Phi\left(\overline{\theta},\overline{\theta}\right)\right)-2\overline{\theta}q\left(\Phi\left(\overline{\theta},\overline{\theta}\right)\right)\right).$

For $\Phi\left(\overline{\theta},\overline{\theta}\right)=\left(\overline{\theta},\overline{\theta}\right)$ is:

$\nu\left(t\left(\Phi\left(\overline{\theta},\underline{\theta}\right)\right)-\left(\overline{\theta}+\underline{\theta}\right)q\left(\Phi\left(\overline{\theta},\underline{\theta}\right)\right)-\Delta\theta q\left(\overline{\theta},\overline{\theta}\right)\right)+(1-\nu)\left(t\left(\overline{\theta},\overline{\theta}\right)-2\overline{\theta}q\left(\overline{\theta},\overline{\theta}\right)\right)\geq$

$\nu\left(t\left(\Phi\left(\overline{\theta},\underline{\theta}\right)\right)-\left(\overline{\theta}+\underline{\theta}\right)q\left(\Phi\left(\overline{\theta},\underline{\theta}\right)\right)-\Delta\theta q\left(\theta_1,\theta_2\right)\right)+(1-\nu)\left(t\left(\theta_1,\theta_2\right)-2\overline{\theta}q\left(\theta_1,\theta_2\right)\right).$

Simplifying we obtain constraints (3.4)-(3.7) . This are the conditions for truthtelling, what any coalition gets in the contract leaves it better off than anything else they could have gotten misreporting their types.

Note that we have used first agent's expected utility because the manipulation function is part of the side-contract that is offered before getting to know the second agent's type, then incentive compatibility constraints are ex-post because by the time A_1 reports to the principal he knows the true type of A_2. ■

Proof of Proposition 3.1. We can solve the system of equations of the binding constraints and obtain the incentive compatible and individually rational transfers:

$\underline{t} = 2\underline{\theta}\underline{q} + \Delta\theta\widehat{q}_2 + \frac{\nu}{1-\nu}\Delta\theta\widehat{q}_1 + \frac{1-2\nu}{1-\nu}\Delta\theta\overline{q}$

$\widehat{t}_2 = \left(\underline{\theta} + \overline{\theta}\right)\widehat{q}_2 + \frac{\nu}{1-\nu}\Delta\theta\widehat{q}_1 + \frac{1-2\nu}{1-\nu}\Delta\theta\overline{q}$

$\widehat{t}_1 = \left(\underline{\theta} + \overline{\theta} + \frac{\nu}{1-\nu}\Delta\theta\right)\widehat{q}_1 + \frac{1-2\nu}{1-\nu}\Delta\theta\overline{q}$

$\overline{t} = 2\overline{\theta}\overline{q}$

We can substitute them in the principal's objective function and then maximise with respect to $\underline{q}$, $\widehat{q}_1$, $\widehat{q}_2$ and $\overline{q}$, we then obtain the decreasing schedule of output in the first part of Proposition 1.

But we need to ensure that monotonicity is satisfied, and $\widehat{q}_1 > \overline{q}$ is true only when:

$$\overline{\theta} + \underline{\theta} + \frac{\nu(2-\nu)}{(1-\nu)^2}\Delta\theta < 2\overline{\theta} + \frac{\nu(2-\nu)(1-2\nu)}{(1-\nu)^3}\Delta\theta.$$

The above is satisfied when $\nu < \nu^*$ where ν^* is a root of:

$$(1-\nu)^3 - \nu^2(2-x) = 0$$

which is $\nu^* = \frac{3}{2} - \frac{1}{2}\sqrt{5} \simeq .38197$.

If $\nu \geq \nu^*$ the the optimal contract requires some pooling. This means that two different pairs will be offered the same contract $\widehat{t}_1 = \overline{t} = \widetilde{t}$ and $\widehat{q}_1 = \overline{q} = \widetilde{q}$ and the constraints become:

$$\begin{aligned}
\widetilde{t} - 2\overline{\theta}\widetilde{q} &= 0 \\
\underline{t} - 2\underline{\theta}\underline{q} &= \widehat{t}_2 - 2\underline{\theta}\widehat{q}_2 \\
\widehat{t}_2 - \left(\underline{\theta} + \overline{\theta}\right)\widehat{q}_2 &= \widetilde{t} - \left(\underline{\theta} + \overline{\theta}\right)\widetilde{q}
\end{aligned}$$

If we solve for the transfers, substitute in the objective function and then maximise with respect to $\underline{q}$, , $\widehat{q}_2$ and $\widetilde{q}$ we obtain the implicit definitions of the second part of the proposition. ∎

Proof of Lemma 3.2. as in the case with no monitoring we want the grand contract to be delegation proof, i.e. $\Phi(\theta_1, \theta_2) = (\theta_1, \theta_2)$ but because of the monitoring the agent cannot misreport anymore the type of the second agent and the manipulation function boils down to a trivial version of the previous one $\Phi(\theta_1, \theta_2) = \left(\widehat{\theta}_1, \theta_2\right)$. Given this and the fact that each agent can be only of two types, for each possible coalition, the coalition they could mimic, it is uniquely defined (for example $(\overline{\theta}, \underline{\theta})$ can pretend to be only $(\underline{\theta}, \underline{\theta})$). Therefore applying the same methodology of the proof of Lemma 1, the "coalition" incentive constraints are:

$$\underline{t} - 2\underline{\theta}\underline{q} \geq \widehat{t}_2 - 2\underline{\theta}\widehat{q}_2$$
$$\widehat{t}_1 - \left(\underline{\theta} + \overline{\theta} + \tfrac{\nu}{1-\nu}\Delta\theta\right)\widehat{q}_1 \geq \overline{t} - \left(\underline{\theta} + \overline{\theta} + \tfrac{\nu}{1-\nu}\Delta\theta\right)\overline{q}$$
$$\widehat{t}_2 - \left(\underline{\theta} + \overline{\theta}\right)\widehat{q}_2 \geq \underline{t} - \left(\underline{\theta} + \overline{\theta}\right)\underline{q}$$
$$\overline{t} - \left(2\overline{\theta} + \tfrac{\nu}{1-\nu}\Delta\theta\right)\overline{q} \geq \widehat{t}_1 - \left(2\overline{\theta} + \tfrac{\nu}{1-\nu}\Delta\theta\right)\widehat{q}_1$$

It is straightforward to show that the only relevant constraints are the downward ones (namely the first two) that are binding at the optimum, the other two will be satisfied if the prescribed schedule of output is monotonic. ∎

Proof of Proposition 3.2. Considering the binding constraints (3.9), (3.10), (3.11) and (3.12) allows us to determine the incentive compatible and individually rational transfers, namely:

$$\underline{t} = 2\underline{\theta}\underline{q} + \Delta\theta\widehat{q}_2 + \Delta\theta\overline{q}$$
$$\widehat{t}_2 = \left(\underline{\theta} + \overline{\theta}\right)\widehat{q}_2 + \Delta\theta\overline{q}$$
$$\widehat{t}_1 = \left(\underline{\theta} + \overline{\theta} + \tfrac{\nu}{1-\nu}\Delta\theta\right)\widehat{q}_1 + \tfrac{1-2\nu}{1-\nu}\Delta\theta\overline{q}$$
$$\overline{t} = 2\overline{\theta}\overline{q}$$

We can plot them in the principal's objective function and then maximise with respect to $\underline{q}$, $\widehat{q}_1$, $\widehat{q}_2$ and $\overline{q}$, we then obtain the decreasing schedule of output of Proposition 2. ■

Proof of Remark 3.1. Since,when offering a pooling sub-contract, A_1 will always report $\theta_2 = \overline{\theta}$ the relevant incentive constraint is the one of a pair $\left(\underline{\theta}, \overline{\theta}\right)$ that is:

$\widehat{t}_1 - \left(\underline{\theta} + \overline{\theta}\right)\widehat{q}_1 \geq \overline{t} - \left(\underline{\theta} + \overline{\theta}\right)\overline{q}$

which is for sure satisfied whenever the incentive compatibility constraint for the same pair under separating sub-contract offer is satisfied. In fact (3.6) can be rewritten as:

$\widehat{t}_1 - \left(\underline{\theta} + \overline{\theta}\right)\widehat{q}_1 \geq \overline{t} - \left(\underline{\theta} + \overline{\theta}\right)\overline{q} + \frac{\nu}{1-\nu}\Delta\theta\left(\widehat{q}_1 - \overline{q}\right).$

Hence when the principal satisfies incentives constraints under separation, the utility of an agent that offers a pooling sub-contract is maximised by truthful report and $U_P^*\left(\underline{\theta}_1\right) = \widehat{t}_1 - \left(\underline{\theta} + \overline{\theta}\right)\widehat{q}_1$. ■

Proof of Proposition 3.3. The optimal contract, $GC = \left\{\underline{t}, \underline{q}, \widehat{t}_1, \widehat{q}_1, \widehat{t}_2, \widehat{q}_2, \overline{t}, \overline{q}\right\}$, now it is a solution to a program that maximises the principal expected utility subject to the following constraints:

$$\begin{aligned}
\widehat{t}_2 - \left(\underline{\theta} + \overline{\theta}\right)\widehat{q}_2 - \Delta\theta\overline{q} &\geq 0 \\
\overline{t} - 2\overline{\theta}\overline{q} &\geq 0 \\
\underline{t} - 2\underline{\theta}\underline{q} &\geq \widehat{t}_2 - 2\underline{\theta}\widehat{q}_2 \qquad (3.20) \\
\widehat{t}_1 - \left(\underline{\theta} + \overline{\theta} + \frac{\nu}{1-\nu}\Delta\theta\right)\widehat{q}_1 &\geq \overline{t} - \left(\underline{\theta} + \overline{\theta} + \frac{\nu}{1-\nu}\Delta\theta\right)\overline{q} \\
\nu U_1\left(\underline{\theta}, \underline{\theta}\right) + (1-\nu)U_1\left(\underline{\theta}, \overline{\theta}\right) &\geq U_P^*\left(\underline{\theta}_1\right) \qquad (3.21)
\end{aligned}$$

where the last constraint is the moral-hazard constraint which rewrites as:

$$\underline{t} - 2\underline{\theta}\underline{q} - \Delta\theta\widehat{q}_1 \geq \widehat{t}_1 - \left(\underline{\theta} + \overline{\theta}\right)\widehat{q}_1$$

It is standard to set the first two individual rationality constraint binding, then the second incentive compatibility constraint is binding as well. The problem is to decide which one between (3.20) and (3.21) is binding. If we consider the optimal contract without this constraint (the one described in Prop.2) then the binding one is (3.21), the moral hazard constraint. We can then solve for transfers and optimise with respect to the quantities.

The schedule is always monotonic for $\nu < \nu^{**}$, where ν^{**} is a root of $3-9\nu+6\nu^2-\nu^3=0$. If values of ν are higher then the treshold we need to bunch together two pairs of agent and set $\widehat{t}_1 = \overline{t} = \widetilde{t}$ and $\widehat{q}_1 = \overline{q} = \widetilde{q}$. Once we set these two quantities equal the moral hazard constraint is not the binding one anymore. After substituting the transfer in the objective function and maximise we obtain the quantities implicitly define as in the second part of the Proposition. ■

Proof of Lemma 3.3. The proof here parallels that of Lemma 2, with just minor changes. Now A_1 cannot misreport his own type therefore the new version of the manipulation function is: $\Phi(\theta_1,\theta_2) = \left(\theta_1, \widehat{\theta}_2\right)$. The incentive constraints are uniquely determined as well:

$$\underline{t} - 2\underline{\theta}\underline{q} \geq \widehat{t}_1 - 2\underline{\theta}\widehat{q}_1$$
$$\widehat{t}_2 - \left(\underline{\theta}+\overline{\theta}\right)\widehat{q}_2 \geq \overline{t} - \left(\underline{\theta}+\overline{\theta}\right)\overline{q}$$
$$\widehat{t}_1 - \left(\underline{\theta}+\overline{\theta}+\tfrac{\nu}{1-\nu}\Delta\theta\right)\widehat{q}_1 \geq \underline{t} - \left(\underline{\theta}+\overline{\theta}+\tfrac{\nu}{1-\nu}\Delta\theta\right)\underline{q}$$
$$\overline{t} - \left(2\overline{\theta}+\tfrac{\nu}{1-\nu}\Delta\theta\right)\overline{q} \geq \widehat{t}_2 - \left(2\overline{\theta}+\tfrac{\nu}{1-\nu}\Delta\theta\right)\widehat{q}_2$$

where the relevant and binding ones are the first two, while the other "upward" constraints will be automatically satisfied if the schedule of output is monotonic. ■

Proof of Proposition 3.4. Taking the binding constraints (3.14), (3.15), (3.17) and (3.16) (and remembering that participation constraints are binding in any state of the world) we determine the transfers:

- $\underline{t} = 2\underline{\theta}\underline{q} + \Delta\theta\widehat{q}_1$
- $\widehat{t}_2 = \left(\underline{\theta}+\overline{\theta}\right)\widehat{q}_2 + \Delta\theta\overline{q}$

- $\widehat{t}_1 = \left(\underline{\theta} + \overline{\theta}\right)\widehat{q}_1$
- $\overline{t} = 2\overline{\theta}\overline{q}$

We can plot them in the principal's objective function and then maximise with respect to $\underline{q}$, $\widehat{q}_2$, $\widehat{q}_1$ and $\overline{q}$, we then obtain the decreasing schedule of output of Proposition 4. ∎

Chapter 4

The Irrelevance of Bargaining Power in Side-Contracting.

4.1 Introduction.

When dealing with models of collusion under asymmetric information a stream of literature (see Laffont and Martimort [1997,2000]) has adopted the modelling device of the "ring-master" to design the side-contract. This player is a third party who is uninformed about the colluding parties' private information and that offers them the collusive contract. It is recognised as an acceptable shortcut to isolate the impact of asymmetric information on collusion and to avoid more complicated issues, such as bargaining under asymmetric information and informed principal problems (that involves a signalling problem when the contract is offered).

Usually this benevolent (towards the agents) third party maximises the sum of the colluding agents' expected utilities under the standard incentive and participation constraints. Other underlying assumptions are that the bargaining powers of the agents are equal (in the maximisation problem the utility functions enter with equal weight) and known to the principal.

We believe that if the collusive contract is written at the expenses and "behind the back" of the principal it would be quite strange for the principal to know exactly the relative strength of the agents at the collusive stage.

We consider a model with one principal and two privately-informed agents that can collude with the aid of a collusion designer. We assume that the principal does not know the, potentially different, bargaining powers of the agents at the collusive stage. This assumption differentiates out model from the one of Laffont and Martimort [2000] where they analyse collusion-proof equilibria in an environment where the types of the agent are correlated.

The main result of the paper is that bargaining powers are irrelevant, and their diversity joint with the fact that the principal is uninformed about them do not affect the collusion proof equilibrium allocation.

This result is due to the fact that the information about the two agents' types, and its transmission, is all that matters and the difference in bargaining power does not affect that information. We show that a Weak Collusion Proofness Principle still holds and that the constraints that ensure the equilibrium is collusion proof do not depend from the bargaining power at the collusive stage. Therefore any distortion and asymmetry that the ring-master can introduce with the side-contract disappears in the optimal collusion proof contract offered by the principal.

The only other work that has studied collusion issues with varying bargaining power is Faure-Grimaud, Laffont and Martimort [2002], in a model with supervision and collusion they also obtain a result of irrelevance but in a setup where there is only one-sided asymmetric information at the collusive stage and the bargaining power is known to the principal.

The result of this paper sheds more light on the power of the contractual instruments that are available to the principal even when the agents are prone to collusion. Even if she does not know the true distribution of the bargaining power, she can offer a symmetric collusion proof grand contract and eliminate any asymmetry

Moreover we provide a theoretical justification for the assumption generally made in the literature of equal bargaining power known to the principal.

The structure of the chapter is as follows. Section 4.2 introduces the model. Section 4.3 solves the model and presents the results. Section 4.4 contains discussion and conclusions. All proofs are in the appendix (section 4.5).

4.2 The model.

The principal P wants to buy a quantity q of final good. The first agent A produces a quantity q_A of the final good using the intermediate good q_B which is produced by the second agent B. Production uses a Leontieff technology such that $q = q_A = q_B$, in other words the production process is "componetised".

Each agent has private information about his constant marginal cost θ_i. These marginal costs are drawn from a joint common knowledge distribution with discrete support $\Theta = \{\theta_1, \theta_2\}$ with $\Delta\theta = \theta_2 - \theta_1 > 0$, which we denote by:

$$\begin{aligned}
p_{11} &= \Pr\left(\theta^A = \theta_1 \text{ and } \theta^B = \theta_1\right) \\
p_{12} &= \Pr\left(\theta^A = \theta_1 \text{ and } \theta^B = \theta_2\right) \\
p_{21} &= \Pr\left(\theta^A = \theta_2 \text{ and } \theta^B = \theta_1\right) \\
p_{22} &= \Pr\left(\theta^A = \theta_2 \text{ and } \theta^B = \theta_2\right)
\end{aligned}$$

Let $\rho = p_{11}p_{22} - p_{12}p_{21}$ be the measure of correlation which is assumed to be non-negative. For simplicity we restrict the analysis to the symmetric case where $p_{12} = p_{21} \neq 0$.

Each agent's utility function is given by:

$$U^k = t^k - \theta_i q, \quad k = A, B \text{ and } i = 1, 2$$

where t^k is the monetary transfer received from the principal.

The utility function of the principal takes the following form:

$$W = S(q) - \left(t^A + t^B\right)$$

with $S'(\cdot) > 0$, $S''(\cdot) < 0$.

In the paper we will simplify the notation as follows: we are using a double subscript to indicate the type of coalition both for quantities and transfers; the first digit refers to the type of the agent A while the second one to the type of B, (e.g. q_{21} will denote the quantity produced when the A is inefficient and the B is efficient.

The complete-information social optimum,the first best outcome, is the following decreasing schedule of output:

$$S'(q_{11}^*) = 2\theta_1$$

$$S'(q_{12}^*) = S'(q_{21}^*) = S'(\widehat{q}) = \theta_1 + \theta_2$$

$$S'(q_{22}^*) = 2\theta_2$$

4.2.1 The contracts.

The principal proposes a grand mechanism G to the agents, G maps any pair of messages (m_A, m_B) belonging to the product message space $M = M_A \times M_B$ (where M_k denotes the message space used by each agent) into a triplet $\left\{q, t^A, t^B\right\}$.We therefore denote the grand mechanism by $G = \left\{q(\cdot), t^A(\cdot), t^B(\cdot)\right\}$.

At a subsequent stage an uninformed third party, T, proposes a side mechanism $S = \left\{\phi(\cdot), y_k(\cdot)_{k\in\{A,B\}}\right\}$ to the agents to induce their collusive behavior.

Where $\phi(\cdot)$ is a collective manipulation of the messages sent to the principal and $y_k(\cdot)_{k\in\{A,B\}}$ is a pair of side-transfers. The third party is not a source of money and therefore the coalition's budget balance constraint must hold: $\sum\limits_{k=A,B} y_k\left(\theta^A, \theta^B\right) = 0$

for all $(\theta^A, \theta^B) \in \Theta^2$.

Due to the Revelation Principle it is without loss of generality that we assume that S is a direct mechanism. Therefore $\phi(\cdot)$ and $y_k(\cdot)$ $(k \in \{A, B\})$ map Θ^2 respectively into the set of measures on M and the set of balanced side transfers.

Lastly, the third party maximises a weighted sum of the agents' utilities:

$$\alpha U^A + (1-\alpha) U^B.$$ [1]

An asymmetry could therefore be present at the side-contracting stage, even if the agents are equal, the payoff of A, for example, could be weakly more important at the collusion stage. In this way it is as if agent A had more bargaining power than the second one.

The principal does not know the true value of α, she just knows the distribution of the parameter and its expected value $\overline{\alpha}$.

4.2.2 The timing.

The timing of the overall game of contract offer and coalition formation is as follows:

1. Agents learn their type.
2. The principal proposes a grand mechanism G. If an agent refuses it then all agents get their reservation utility (which is normalised to zero).
3. The third party proposes a side mechanism S to the agents and a noncooperative continuation play of G if anyone refuses this side contract. If both agents accept the collusive offer, agents report their types to the third party who

[1] This expression could be seen as a monotone transformation of the Nash product:

$$\left(U^A\right)^{\alpha}\left(U^B\right)^{1-\alpha}.$$

recommends reports into the grand mechanisms and who commits to enforce the corresponding side transfers.

4. Reports are sent into the grand mechanism. Production and transfers take place accordingly.

4.3 The collusion proof equilibrium.

In this regulated duopoly framework,we are going to carry out an analysis similar to the one developed by Laffont and Martimort [2000][2], this type of setting allows a good understanding of the effects of collusion under asymmetric information on both allocative efficiency and the distribution of rents.

We study a model with one principal and two agents whose types are privately known and jointly distributed and, as it is now standard in the collusion literature, we assume that the principal has not perfect control over the communication technology so that she cannot prevent the agents from colluding.

Collusion is modeled as in Laffont and Martimort [1997][3] in a reduced form through the shortcut of the uninformed third party that designs and proposes the collusive agreement to the agents.

When agents cannot collude, an established result tells us that, in this correlated environment, the optimal mechanism achieves the first-best when the implementation concept is Bayesian-Nash equilibrium (Crémer and McLean [1988]). This is because B's report (which is truthful in equilibrium) acts as a signal correlated with A's type and allows to condition his transfers on this information. The flexibility in transfers, due to the participation constraint in expected terms and the risk neu-

[2]They deal with the provision of a public good, but the nature of the problems to be solved remains the same.

[3]The 2000 paper complements and extends the 1997 one by highlighting the importance of correlated information as a direct casue of the strength of the coalition and the relevance of the collusive stake. Moreover in the more recent work they do not restrict the class of mechanism available to the principal to the class of anonymous one as previously done.

trality of the agents, allows to "stochastically" and costlessly deter any incentive to lie.

Since the agents earn zero rent from the mechanism proposed by the principal it should not come as a surprise that they are willing to collude and coordinate their messages. Moreover this stake for collusion is created endogenously by this mechanism á-la Crémer-McLean that manages to successfully play one agent against the other by exploiting even the smallest correlation between their private information.

A *weak collusion-proofness principle* holds even in this correlated environment so that to fully evaluate the effects of collusion on the efficiency of the organisation it is sufficient to study only the mechanisms that are weakly collusion-proof. More precisely, G is weakly collusion-proof if and only if it is a truth telling direct mechanism and the null side contract, $S_0^* = \left\{\phi^* = Id, (y_k^* = 0)_{k \in \{A,B\}}\right\}$, is a BNE at the collusion stage.[4]

We can therefore state the weak collusion-proof principle.

Weak Collusion-Proof Principle *Any Bayesian perfect equilibrium of the two stages game of contract offer cum coalition formation $G \circ S$ can be achieved by a truthtelling mechanism offered by the principal such that the best response of the third party is to offer the null side-contract, i.e. no manipulation of reports and no side-transfers.*

The intuition behind the weak collusion-proof principle is similar to that underlying the well known revelation principle: any equilibrium allocation of the overall game can be obtained with a direct grand mechanism offered by the principal himself. In the latter case the mechanism is designed in such a way that the coalition still forms but then finds it optimal to report truthfully, so collusion does not happen in equilibrium. In other words the principal offers a grand-contract that gives the

[4]For a more rigorous analysis and a discussion of the equilibrium beliefs that sustain the equilibrium refer to Laffont and Martimort [2000].

agents the same equilibrium payoffs they would get through collusion. Agents are then indifferent between truthtelling and misreporting.

Since this principle holds we can solve the game backwards and focus at first on the direct side-mechanism that the third party offers to the colluding agents. The ring-master maximises the weighted average of the agents utilities with respect to a pair of joint reports to be made to the principal and a pair of side transfers, more precisely:

$$\max_{\phi_{ij},y_k(\cdot)} \sum_{ij} p_{ij}[\alpha\left(t^A\left(\phi_{ij}\right)-\theta_i q\left(\phi_{ij}\right)+y_A\left(\theta^A,\theta^B\right)\right)$$
$$+(1-\alpha)\left(t^B\left(\phi_{ij}\right)-\theta_j q\left(\phi_{ij}\right)+y_B\left(\theta^A,\theta^B\right)\right)].$$

The solution to this optimisation will have to satisfy also the usual Bayesian individual incentive compatibility and individual rationality constraints, because the third party is uninformed and collusion takes place under asymmetric information (only the relevant agent knows his own type). Note that the reservation utility in this case is given by the utility that each type of agent would obtain by playing non-cooperatively the grand contract.

From the first order conditions of the third party problem we can derive the conditions for the optimality of the null side contract, $S_0^*=\left\{\phi^*=Id,(y_k^*=0)_{k\in\{A,B\}}\right\}$. The following result introduces the collusion-proof constraints, also called coalition incentive compatibility constraints.

Proposition 4.1 *A grand mechanism G is Collusion Proof if and only if there exists* $(\epsilon_A^{\overline{\alpha}},\epsilon_B^{\overline{\alpha}})\in[0,1)^2$ *such that:*

$$t_{11}^A+t_{11}^B-2\theta_1 q_{11}\geq t^A\left(\widetilde{\theta}^A,\widetilde{\theta}^B\right)+t^B\left(\widetilde{\theta}^A,\widetilde{\theta}^B\right)-2\theta_1 q\left(\widetilde{\theta}^A,\widetilde{\theta}^B\right) \qquad (4.1)$$

$$t_{12}^{A}+t_{12}^{B}-\left(\theta_2+\theta_1+\frac{p_{11}}{p_{12}}\epsilon_A^{\overline{\alpha}}\Delta\theta\right)q_{12}$$
$$\geq t^{A}\left(\widetilde{\theta}^{A},\widetilde{\theta}^{B}\right)+t^{B}\left(\widetilde{\theta}^{A},\widetilde{\theta}^{B}\right)-\left(\theta_2+\theta_1+\frac{p_{11}}{p_{12}}\epsilon_A^{\overline{\alpha}}\Delta\theta\right)q\left(\widetilde{\theta}^{A},\widetilde{\theta}^{B}\right) \quad (4.2)$$

$$t_{21}^{A}+t_{21}^{B}-\left(\theta_2+\theta_1+\frac{p_{11}}{p_{12}}\epsilon_B^{\overline{\alpha}}\Delta\theta\right)q_{12}$$
$$\geq t^{A}\left(\widetilde{\theta}^{A},\widetilde{\theta}^{B}\right)+t^{B}\left(\widetilde{\theta}^{A},\widetilde{\theta}^{B}\right)-\left(\theta_2+\theta_1+\frac{p_{11}}{p_{12}}\epsilon_B^{\overline{\alpha}}\Delta\theta\right)q\left(\widetilde{\theta}^{A},\widetilde{\theta}^{B}\right) \quad (4.3)$$

$$t_{22}^{A}+t_{22}^{B}-\left(2\theta_2+\frac{p_{12}\left(\epsilon_A^{\overline{\alpha}}+\epsilon_B^{\overline{\alpha}}\right)}{p_{22}+\frac{\rho\epsilon_B^{\overline{\alpha}}}{p_{12}}}\Delta\theta\right)q_{22}$$
$$\geq t^{A}\left(\widetilde{\theta}^{A},\widetilde{\theta}^{B}\right)+t^{B}\left(\widetilde{\theta}^{A},\widetilde{\theta}^{B}\right)-\left(2\theta_2+\frac{p_{12}\left(\epsilon_A^{\overline{\alpha}}+\epsilon_B^{\overline{\alpha}}\right)}{p_{22}+\frac{\rho\epsilon_B^{\overline{\alpha}}}{p_{12}}}\Delta\theta\right)q\left(\widetilde{\theta}^{A},\widetilde{\theta}^{B}\right) \quad (4.4)$$

$\forall\left(\widetilde{\theta}^{A},\widetilde{\theta}^{B}\right)\in\Theta^2$.

These are the conditions for coalition truthtelling and are derived from the first order conditions of the third party optimisation problem. If a grand contract has to be collusion-proof then these constraints need to be satisfied, ensuring that the optimal manipulation function chosen by the collusion designer will be the identity function. These constraints define how high need the transfer of each pair of agents be in order to deter any incentive to lie.

It is worth noting that we cannot without loss of generality restrict our attention to symmetric mechanisms and equilibria, our aim being the desire to study the equilibria when some asymmetry is introduced at the collusive stage. For the moment then the two mixed coalitions, namely (θ_1,θ_2) and (θ_2,θ_1), have to be considered as two different ones.

For this reason we have two different discount factors $\epsilon_A^{\overline{\alpha}}$ and $\epsilon_B^{\overline{\alpha}}$. They capture the fact that collusion takes place under asymmetric information and the third party has to induce truthtelling into the side-contract. Individual incentives and participation constraints are costly to provide and the true costs must be replaced by virtual costs, which are normally higher[5].

It is important to stress that these discount factors depend on the Lagrange multipliers (the price of the constraints) of the third party problem and therefore can be "indirectly" chosen by the principal[6]. Moreover they are the only terms that depend on the expected bargaining powers $\overline{\alpha}$ and $(1-\overline{\alpha})$.

Once coalition incentive compatibility constraints are characterised, the difficulty is to understand which are the relevant ones. We need therefore to derive the necessary monotonicity conditions for the implementability of a schedule of outputs.

Proposition 4.2 *For a weak correlation, $\rho \leq (p_{12}+p_{22})\left(p_{12}^2/p_{11}\right)$, the schedule of implementable outputs is decreasing ($q_{11} > q_{12} > q_{21} > q_{22}$) for all $\left(\epsilon_A^{\overline{\alpha}}, \epsilon_B^{\overline{\alpha}}\right) \in [0,1)^2$. For a strong correlation, $\rho > (p_{12}+p_{22})\left(p_{12}^2/p_{11}\right)$, the schedule of implementable outputs is non-monotonic ($q_{11} > q_{22} > q_{12} \geq q_{21}$) if and only if*

$$\psi\left(\epsilon_A^{\overline{\alpha}}, \epsilon_B^{\overline{\alpha}}\right) = 1 + \frac{p_{11}\epsilon_A^{\overline{\alpha}} + p_{11}\epsilon_B^{\overline{\alpha}}}{p_{22}} - \frac{p_{11}}{p_{12}}\epsilon_B^{\overline{\alpha}} < 0$$

i.e. for $\epsilon_A^{\overline{\alpha}}$ and $\epsilon_B^{\overline{\alpha}}$ large enough; otherwise it remains increasing.

While a standard monotonicity condition is needed in the case of low correlation, the peculiarity of this proposition is that the principal can implement non-monotonic schedule of output if she chooses $\epsilon_A^{\overline{\alpha}}$ and $\epsilon_B^{\overline{\alpha}}$ sufficiently close to one in the case of strong correlation. This is because it can happen that virtual costs may not be ranked as the true ones[7], collusion under asymmetric information introduces

[5] This shows that already some distortions are due to the fact that at the collusive stage agents are asymmetrically informed.

[6] In fact the third party maximises for any given grand contract and, for any optimisation problem, there is a set of multipliers at the optimum. The principal choses the grand contract also having this in mind.

[7] When positive correlation is almost perfect the probability agents are both efficient is very large, therefore the incentive constraint of a θ_1 agent willing to mimic a θ_2 one is also very costly

therefore some countervailing incentives in the problem[8].

Before considering each of the two problems in more detail it is useful to write our constraints in term of ex-post rents of the agents, i.e. $u_{ij}^A = t_{ij}^A - \theta_i q_{ij}$ and $u_{ij}^B = t_{ij}^B - \theta_j q_{ij}$. We introduce eight new variables, one for each agent in each of the possible four states of nature. (e.g. u_{21}^A is the rent enjoyed by agent A when he is inefficient and the other agent is efficient).

We can now discuss which constraints are the relevant ones when looking for the optimal collusion proof contract in the low correlation case.

Proposition 4.3 *In case of weak correlation, $\rho \leq (p_{12} + p_{22}) \left(p_{12}^2 / p_{11}\right)$, the optimal weakly collusion proof grand mechanism is symmetric and the relevant constraints are the following*[9]*:*

$$2\underline{u} \geq \widehat{u}_1 + \widehat{u}_2 + \Delta\theta\widehat{q} \tag{CIC1a}$$

$$\widehat{u}_1 + \widehat{u}_2 \geq 2\overline{u} + \Delta\theta\overline{q} + \frac{p_{11}}{p_{12}}\epsilon_A^{\overline{\alpha}}\Delta\theta\left(\widehat{q} - \overline{q}\right) \tag{CIC2a}$$

$$p_{11}\underline{u} + p_{12}\widehat{u}_1 \geq p_{11}\widehat{u}_2 + p_{12}\overline{u} + \Delta\theta\left(p_{11}\widehat{q} + p_{12}\overline{q}\right) \tag{BIC}$$

$$p_{12}\widehat{u}_2 + p_{22}\overline{u} \geq 0 \tag{BIR}$$

Moreover it is optimal for the principal to set $\epsilon_A^{\overline{\alpha}}$ and $\epsilon_B^{\overline{\alpha}}$ both equal to zero.

These are all the constraints that have to be satisfied by a collusion-proof grand

for the third party from an ex-ante point of view. Truthelling in the coalition comes at the cost of large distortions of the optimal manipulation of reports. This may cause the costs of a $(\theta_1, \theta_2\rangle$ pair to be higher than those of a (θ_2, θ_2) coalition.

[8] We are facing countervailing incentives when the agent has an incentive to understate his private information for some of its realisations, and to overstate it for others. Lewis and Sappington [1989] and Maggi and Rodriguez-Clare [1995] have shown that the optimal contract is not monotonic in the type of the agent and it may involve some bunching. Here the interplay of correlation and the collusion proof constraints cause the virtual types of the coalitions to be ranked differently from the true types.

[9] A clarification on the notation: $\underline{u}$ is the rent enjoyed by any agent which is part of a coalition of two efficient agent, $\widehat{u}_1$ is the rent of the efficient agent in a mixed coalition while $\widehat{u}_2$ is the rent of the inefficient one, finally $\overline{u}$ is the rent of an inefficient agent when paired with another inefficient one. Similarly $\widehat{q}$ and $\overline{q}$ are the quantities produced by a mixed pair and one made of two ineffcient agents.

contract in the weak correlation case. The first two inequalities are the two downward coalition incentive compatibility constraints, which basically say that an efficient pair does not gain from claiming that it is a mixed pair and the latter does not gain from telling the principal that it is inefficient. The last two are the standard Bayesian incentive compatibility constraint of an efficient agent and the participation constraint of an inefficient one.

The first best cannot be obtained anymore due to the fact that the largely positive and negative transfers of the no-collusion setup cannot be used here because the would violate the coalition incentive constraints. Agents obtain some informational rents and, at the optimum, downward output distortions will be implemented. These distortions are the product of the conflict between coalition incentive and individual participation constraints.

When correlation is low it is optimal to treat equally the mixed coalitions and since the virtual costs represented the only asymmetry left in the model there is no problem in restricting attention to symmetric mechanisms. The schedule is monotonic implying that the coalition incentive compatibility constraints need to hold in the standard way mentioned above.

Note also that it is optimal to set both discount factors, $\epsilon_A^{\overline{\alpha}}$ and $\epsilon_B^{\overline{\alpha}}$, equal to zero to save on the rents to be given to mixed coalitions. The binding collusion-proofness constraints take the form of those that we would observe had collusion been under symmetric information[10], in this particular case virtual costs are equal to true costs.

We can now focus our attention to the case of strong correlation, where it is very likely that agents are of the same type. In this case the principal can obtain a higher payoff by implementing a non-monotonic schedule of output that significantly reduces the output for the very unlikely mixed coalitions.

The following result holds in the case of strong correlation.

[10]This is because when agents' collude under symmetric information they know each other's type and there's no need to give any rent to reveal truthfully in the side-contract.

Proposition 4.4 *In the case of strong correlation, $\rho > (p_{12} + p_{22})\left(p_{12}^2/p_{11}\right)$, the optimal weakly collusion proof mechanism is symmetric and the relevant constraints are the following:*

$$2\underline{u} \geq 2\overline{u} + 2\Delta\theta\overline{q} \tag{CIC1b}$$

$$\widehat{u}_1 + \widehat{u}_2 \geq 2\overline{u} + \Delta\theta\overline{q} + \frac{p_{11}}{p_{12}}\epsilon_B^{\overline{\alpha}}\Delta\theta\left(\widehat{q} - \overline{q}\right) \tag{CIC2b}$$

$$p_{11}\underline{u} + p_{12}\widehat{u}_1 \geq p_{11}\widehat{u}_2 + p_{12}\overline{u} + \Delta\theta\left(p_{11}\widehat{q} + p_{12}\overline{q}\right) \tag{BIC}$$

$$p_{12}\widehat{u}_2 + p_{22}\overline{u} \geq 0 \tag{BIR}$$

Moreover it is optimal for the principal to set $\epsilon_A^{\overline{\alpha}}$ and $\epsilon_B^{\overline{\alpha}}$ both arbitrarily close to one.

The first two constraints are the coalition incentive compatibility constraints, they are not the usual downward ones because of the countervailing incentives created by the strong correlation of types. The last two are the standard individual constraints.

In this case the whole set of constraints that needs to be satisfied at the optimum has some peculiar features due to the asymmetric information at the collusion stage, $\epsilon_A^{\overline{\alpha}}$ and $\epsilon_B^{\overline{\alpha}}$ are both set very large[11] in a way that changes the ranking of the virtual costs. It is exactly this that allows the principal to implement a non-monotonic schedule of outputs. This is convenient because distorting a lot the quantity in the very unlikely event of a mixed coalition does not lower much her expected payoff. Moreover when p_{12} is very small, if $\widehat{q} \leq \overline{q}$, then the right hand side of CIC2b is very negative and as a consequence relaxes significantly the constraint. Rents for a member of a mixed coalition can be quite negative[12].

Moreover also in presence of strong correlation the principal can gain by setting the virtual costs of the two mixed coalitions equal, therefore the problem regains

[11] Virtual costs are effectively different from the true ones.

[12] This is of course possible because the agents are risk neutral.

symmetry and a pair of agents of different types will be treated in the same way regardless of who is the efficient member.

We are now ready to state the main result of the paper, namely that the bargaining powers at side-contracting have no influence on the final solution whatsoever. In fact the term $\overline{\alpha}$ remained only in the coalition incentive constraints through $\epsilon_A^{\overline{\alpha}}$ and $\epsilon_B^{\overline{\alpha}}$, but we then have shown that it was convenient for the principal to set this "virtual" terms equal so neglecting any asymmetry that could have been in place at the collusive stage.

Proposition 4.5 *In a framework of collusion under asymmetric information with correlation asymmetric bargaining powers in side-contracting are irrelevant.*

Proof. It follows immediately from the above propositions. The set of constraints that a collusion proof grand-contract has to satisfy does not depend on $\overline{\alpha}$, the expected bargaining power, and both agents are treated symmetrically. ■

The irrelevance of bargaining power at the side-contracting stage may seem a puzzling result at first sight. Since one may think that in order to neutralise a coalition it is important to know how it formed and how the surplus will be split between members. Some more discussion on the concept of collusion proofness may help in clarifying one's perplexity.

Collusion proofness is related to how information is transmitted to the principal via the manipulation function fixed in the side-contract, what is relevant for the principal problem are the two agents' true marginal costs and we have shown that the conditions for a truthful joint report do not depend from the bargaining powers. How they would split the gain from collusion is not relevant for the principal, who in fact does not need to know the true bargaining powers to offer the collusion proof grand contract and avoid collusion on the equilibrium path.

Another intuitive explanation comes from traditional bargaining theory even if collusion proofness is a concept which belongs to the framework of non-cooperative game theory, where agents do only what it is in their interest to do. The only way for a principal to avoid side-contracting is to give the agents the same informational rents they would obtain if they were colluding and misreporting, so in a collusion-proof equilibrium it does not really matter how the gains from colluding would be split since there are no more gains. Therefore in equilibrium it is like in Nash bargaining theory when there is no surplus to split, then bargaining powers are irrelevant and the outcome is given by the status-quo payoffs (the reservation utilities of the collusive stage, which are the equilibrium rents offered by the principal with the collusion proof contract). With the grand contract the principal is able to choose these payoffs, which are then the reservation utilities at the collusion stage. What a collusion proof symmetric grand-contract does it to bring the agents on the Pareto frontier in the utility space, but it does so in a symmetric way so any asymmetric preference that the third party might have does not change anything in equilibrium.

4.4 Discussion and concluding remarks.

Our result has shown that, in a framework where agents can set up collusive agreements once a grand-contract is offered, the principal does not need to know their true bargaining powers to deter collusion. If the agents secretly collude, it would be unreasonable to assume that the principal knew the exact terms of the deal.

We have shown that the assumption quite widespread in the literature of known and equal bargaining power does not affect the results and can be made without loss of generality.

We believe that this proves that the principal has one more "weapon" available as a contract designer, if she has no reason or advantage in considering the two agents as different[13] then she can avoid to do it even if their relative importance at

[13]The result carries through even in a more asymmetric framework with a risk averse privately informed non-productive supervisor and a gent, as in Faure-Grimaud, Laffont and Martimort [2002].

the side-contracting stage is (or is believed to be) different. It is enough to treat them as equal and the internal incentives of the contract will make them achieve the same payoff in equilibrium.

This can be a useful tool in some applied frameworks. For example our model could represent a procurement problem, where the state agency is the principal. In this case it can happen that the two contracting firms are equal with respect to the specific project but very different in other fields, one could be a big multinational firm with a stronger power when deciding collusive agreements because of size, experience, ability or other exogenous reasons.

The same thing could happen in a regulation model, with the principal being the regulator and the two agents being firms in a duopolistic market. Maybe the firms are identical for what concerns the regulatory discipline in that particular market but in other fields they can be quite different and one may be stronger during the bargaining at the collusive stage.

In both cases the principal can re-establish the symmetry if she has no advantage in doing otherwise. If the utility function of the principal was different and the agent's products were not complements then she may prefer to treat the agents in an asymmetric way. In fact when the principal does not gain from treating the agents differently she can avoid it via the offer of a symmetric grand contract which will establish the outside options. In this way she sets the minimum utility that the "weak" agent must get from collusion therefore giving him the strength to refuse too uneven side-contracts.

This powerful tool in the hands of the head of the organisation could find an application also internally, among employees. Think for example of an old employee and a younger one (or more and less experienced) that need to be considered equal with respect to performing a specific task, then also the collusion proof result will be symmetric even if when forming the "horizontal-clique" (Dalton [1959]) one of the two has more power.

This result sheds some more light on an aspect of collusion and side-contracting previously ignored, some others need to be studied more deeply: first of all the true role of the third party as a side-contract designer and the informed-principal problem that we are avoiding through such modeling device.

4.5 Appendix

Proof of Proposition 4.1. The collusion proof truthtelling constraints (also called coalition incentive compatibility) are derived from the first order conditions of the optimisation problem of the third party at the side-contracting stage. One of the condition for collusion proofness is that it must be best response by the third party to offer the null side contract and report truthfully into the grand mechanism. This is because it is the third party that, for any given grand contract and any pair of types, gives recommendation on how to jointly report information to the principal.

The principal does not know the true value of α, but she knows that its expected value is $\overline{\alpha}$ and since the objective function of the third party is linear in α the following holds:

$$E_{\alpha}\left[\sum_{ij} p_{ij}\left[\alpha U^{A}+(1-\alpha) U^{B}\right]\right]=\left[\sum_{ij} p_{ij}\left[\overline{\alpha} U^{A}+(1-\overline{\alpha}) U^{B}\right]\right].$$

Therefore, from the point of view of the principal, the ring-master solves the following problem:

$$\max_{\phi_{ij}, y_{k}(\cdot)} \sum_{ij} p_{ij}[\overline{\alpha}\left(t^{A}\left(\phi_{ij}\right)-\theta_{i} q\left(\phi_{ij}\right)+y_{A}\left(\theta^{A}, \theta^{B}\right)\right)$$
$$+(1-\overline{\alpha})\left(t^{B}\left(\phi_{ij}\right)-\left(\theta_{j}\right) q\left(\phi_{ij}\right)+y_{B}\left(\theta^{A}, \theta^{B}\right)\right)]$$

subject to the following constraints:

◇budget balance:

$$y_A\left(\theta^A,\theta^B\right)+y_B\left(\theta^A,\theta^B\right)=0; \tag{BB}$$

◇Bayesian incentive compatibility constraints for agents A and B when they are of type 2:

$$\begin{aligned}&p_{11}\left[t^A\left(\phi_{11}\right)+y_A\left(\theta_1,\theta_1\right)-\theta_1 q\left(\phi_{11}\right)\right]+p_{12}\left[t^A\left(\phi_{12}\right)+y_A\left(\theta_1,\theta_2\right)-\theta_1 q\left(\phi_{12}\right)\right]\\ &\geq p_{11}\left[t^A\left(\phi_{21}\right)+y_A\left(\theta_2,\theta_1\right)-\theta_1 q\left(\phi_{21}\right)\right]+p_{12}\left[t^A\left(\phi_{22}\right)+y_A\left(\theta_2,\theta_2\right)-\theta_1 q\left(\phi_{22}\right)\right]\end{aligned} \tag{BIC-A}$$

$$\begin{aligned}&p_{11}\left[t^B\left(\phi_{11}\right)+y_B\left(\theta_1,\theta_1\right)-\theta_1 q\left(\phi_{11}\right)\right]+p_{12}\left[t^B\left(\phi_{21}\right)+y_B\left(\theta_2,\theta_1\right)-\theta_1 q\left(\phi_{21}\right)\right]\\ &\geq p_{11}\left[t^B\left(\phi_{12}\right)+y_B\left(\theta_1,\theta_2\right)-\theta_1 q\left(\phi_{12}\right)\right]+p_{12}\left[t^B\left(\phi_{22}\right)+y_B\left(\theta_2,\theta_2\right)-\theta_1 q\left(\phi_{22}\right)\right];\end{aligned} \tag{BIC-B}$$

◇Bayesian individual rationality for both agents of both types:

$$\begin{aligned}&p_{11}\left[t^A\left(\phi_{11}\right)+y_A\left(\theta_1,\theta_1\right)-\theta_1 q\left(\phi_{11}\right)\right]+p_{12}\left[t^A\left(\phi_{12}\right)+y_A\left(\theta_1,\theta_2\right)-\theta_1 q\left(\phi_{12}\right)\right]\\ &\geq \left(p_{11}+p_{12}\right)U_A^G\left(\theta_1\right)\end{aligned} \tag{BIR-A1}$$

$$\begin{aligned}&p_{11}\left[t^B\left(\phi_{11}\right)+y_B\left(\theta_1,\theta_1\right)-\theta_1 q\left(\phi_{11}\right)\right]+p_{12}\left[t^B\left(\phi_{21}\right)+y_B\left(\theta_2,\theta_1\right)-\theta_1 q\left(\phi_{21}\right)\right]\\ &\geq \left(p_{11}+p_{12}\right)U_B^G\left(\theta_1\right)\end{aligned} \tag{BIR-B1}$$

$$p_{21}\left[t^A(\phi_{21})+y_A(\theta_2,\theta_1)-\theta_2 q(\phi_{21})\right]+p_{22}\left[t^A(\phi_{22})+y_A(\theta_2,\theta_2)-\theta_2 q(\phi_{22})\right]$$
$$\geq (p_{21}+p_{22})\,U_A^G(\theta_2) \quad \text{(BIR-A2)}$$

$$p_{21}\left[t^B(\phi_{12})+y_B(\theta_1,\theta_2)-\theta_2 q(\phi_{12})\right]+p_{22}\left[t^B(\phi_{22})+y_B(\theta_2,\theta_2)-\theta_2 q(\phi_{22})\right]$$
$$\geq (p_{21}+p_{22})\,U_B^G(\theta_2) \quad \text{(BIR-B2)}$$

Call $\mu(\theta_A,\theta_B)$, δ_A, δ_B, ν_{A1}, ν_{B1}, ν_{A2} and ν_{B2} the multipliers associated with the above constraints.

Even if we have substituted the true value of α with its expectation $\overline{\alpha}$ this has not effect on the conditions for a maximum.

Maximizing with respect to $y_1(\cdot,\cdot)$ and $y_2(\cdot,\cdot)$ yields the following:

$\mu(\theta_1,\theta_1)=p_{11}[\delta_A+\nu_{A1}+\alpha]$ and $\mu(\theta_1,\theta_1)=p_{11}[\delta_B+\nu_{B1}+(1-\alpha)]$

$\mu(\theta_1,\theta_2)=p_{12}[\delta_A+\nu_{A1}+\alpha]$ and $\mu(\theta_1,\theta_2)=p_{12}(\nu_{B2}+(1-\alpha))-p_{11}\delta_B$

$\mu(\theta_2,\theta_1)=p_{21}(\nu_{A2}+\alpha)-p_{11}\delta_A$ and $\mu(\theta_2,\theta_1)=p_{12}[\delta_B+\nu_{B1}+(1-\alpha)]$

$\mu(\theta_2,\theta_2)=p_{22}(\nu_{A2}+\alpha)-p_{12}\delta_A$ and $\mu(\theta_2,\theta_2)=p_{22}(\nu_{B2}+(1-\alpha))-p_{12}\delta_B$

Maximizing with respect to ϕ_{11}, ϕ_{12}, ϕ_{21} and ϕ_{22} we find the following conditions:

$\phi_{11}^* \in \arg\max_{\phi_{11}}\left\{t^A(\phi_{11})+t^B(\phi_{11})-2\theta_1 q(\phi_{11})\right\}$

$\phi_{12}^* \in \arg\max_{\phi_{12}}\left\{t^A(\phi_{12})+t^B(\phi_{12})-\left(\theta_1+\theta_2+\frac{p_{11}}{p_{12}}\epsilon_A^{\overline{\alpha}}\Delta\theta q(\phi_{12})\right)\right\}$

where $\epsilon_A^{\overline{\alpha}}=\frac{\delta_B}{\overline{\alpha}+\delta_A+\nu_{A1}}$.

$\phi_{21}^* \in \arg\max_{\phi_{21}}\left\{t^A(\phi_{21})+t^B(\phi_{21})-\left(\theta_1+\theta_2+\frac{p_{11}}{p_{12}}\epsilon_B^{\overline{\alpha}}\Delta\theta q(\phi_{21})\right)\right\}$

where $\epsilon_B^{\overline{\alpha}}=\frac{\delta_A}{(1-\overline{\alpha})+\delta_B+\nu_{B1}}$.

$\phi_{22}^* \in \arg\max\left\{t^A(\phi_{22})+t^B(\phi_{22})-\left(2\theta_2+\frac{p_{12}\left(\epsilon_A^{\overline{\alpha}}+\epsilon_B^{\overline{\alpha}}\right)}{p_{22}+\frac{\rho\epsilon_B^{\overline{\alpha}}}{p_{12}}}\Delta\theta q(\phi_{22})\right)\right\}$ ■

Proof of Proposition 4.2. We are going to analyse the case in which $\alpha \in \left(\frac{1}{2}, 1\right]$ this is just to simplify the analysis and it is without loss of generality with respect to the final results. Applying a revealed preference argument to constraints (4.1) and (4.2) we get:

$$t_{11}^A + t_{11}^B - 2\theta_1 q_{11} \geq t_{12}^A + t_{12}^B - 2\theta_1 q_{12}$$

$$t_{12}^A + t_{12}^B - \left(\theta_2 + \theta_1 + \frac{p_{11}}{p_{12}}\epsilon_A^{\overline{\alpha}}\Delta\theta\right) q_{12} \geq t_{11}^A + t_{11}^B - \left(\theta_2 + \theta_1 + \frac{p_{11}}{p_{12}}\epsilon_A^{\overline{\alpha}}\Delta\theta\right) q_{11}$$

summing up these two inequalities we obtain:

$$\Delta\theta\left(1 + \frac{p_{11}}{p_{12}}\epsilon_A^{\overline{\alpha}}\right)(q_{11} - q_{12}) \geq 0$$

which is satisfied if $q_{11} \geq q_{12}$.

Using (4.2) and (4.3) we obtain:

$$t_{12}^A + t_{12}^B - \left(\theta_2 + \theta_1 + \frac{p_{11}}{p_{12}}\epsilon_A^{\overline{\alpha}}\Delta\theta\right) q_{12} \geq t_{21}^A + t_{21}^B - \left(\theta_2 + \theta_1 + \frac{p_{11}}{p_{12}}\epsilon_A^{\overline{\alpha}}\Delta\theta\right) q_{21}$$

$$t_{21}^A + t_{21}^B - \left(\theta_2 + \theta_1 + \frac{p_{11}}{p_{12}}\epsilon_B^{\overline{\alpha}}\Delta\theta\right) q_{21} \geq t_{12}^A + t_{12}^B - \left(\theta_2 + \theta_1 + \frac{p_{11}}{p_{12}}\epsilon_B^{\overline{\alpha}}\Delta\theta\right) q_{12}$$

summing up these two inequalities we get:

$$\left(\epsilon_B^{\overline{\alpha}} - \epsilon_A^{\overline{\alpha}}\right)(q_{12} - q_{21}) \geq 0$$

which is satisfied when $q_{12} \geq q_{21}$ provided that $\epsilon_B^{\overline{\alpha}} \geq \epsilon_A^{\overline{\alpha}}$.

Finally consider the constraints (4.3) and (4.4) from the revealed preference argument we obtain:

$$t_{21}^A + t_{21}^B - \left(\theta_2 + \theta_1 + \frac{p_{11}}{p_{12}}\epsilon_B^{\overline{\alpha}}\Delta\theta\right) q_{21} \geq t_{22}^A + t_{22}^B - \left(\theta_2 + \theta_1 + \frac{p_{11}}{p_{12}}\epsilon_B^{\overline{\alpha}}\Delta\theta\right) q_{22}$$

$$t_{22}^A + t_{22}^B - \left(2\theta_2 + \frac{p_{12}\left(\epsilon_A^{\overline{\alpha}} + \epsilon_B^{\overline{\alpha}}\right)}{p_{22} + \frac{\rho\epsilon_B^{\overline{\alpha}}}{p_{12}}} \Delta\theta \right) q_{22} \geq t_{21}^A + t_{21}^B - \left(2\theta_2 + \frac{p_{12}\left(\epsilon_A^{\overline{\alpha}} + \epsilon_B^{\overline{\alpha}}\right)}{p_{22} + \frac{\rho\epsilon_B^{\overline{\alpha}}}{p_{12}}} \Delta\theta \right) q_{21}$$

summing these last two inequalities we get:

$$\Delta\theta \left(1 + \frac{p_{12}^2\left(\epsilon_A^{\overline{\alpha}} + \epsilon_B^{\overline{\alpha}}\right)}{p_{12}p_{22} + \epsilon_B^{\overline{\alpha}}\left(p_{11}p_{22} - p_{12}^2\right)} - \frac{p_{11}}{p_{12}}\epsilon_B^{\overline{\alpha}} \right) (q_{21} - q_{22}) \geq 0$$

which is satisfied if $q_{21} \geq q_{22}$ when the first parenthesis contains a positive number, and it is satisfied when $q_{21} \leq q_{22}$ if the term in the first parenthesis is negative.

Define:

$\psi\left(\epsilon_A^{\overline{\alpha}}, \epsilon_B^{\overline{\alpha}}\right) = \Delta\theta \left(1 + \frac{p_{12}^2\left(\epsilon_A^{\overline{\alpha}} + \epsilon_B^{\overline{\alpha}}\right)}{p_{12}p_{22} + \epsilon_B^{\overline{\alpha}}\left(p_{11}p_{22} - p_{12}^2\right)} - \frac{p_{11}}{p_{12}}\epsilon_B^{\overline{\alpha}} \right)$,

this term is positive for low correlation and negative for high correlation and $\epsilon_A^{\overline{\alpha}}$ and $\epsilon_B^{\overline{\alpha}}$ sufficiently close to one.

To show it we need to investigate some properties of the $\psi\left(\epsilon_A^{\overline{\alpha}}, \epsilon_B^{\overline{\alpha}}\right)$ on the relevant domain. The function is in fact quasiconcave because the determinant of the bordered Hessian is positive, which is a sufficient condition for quasi-concavity and pseudoconcavity.

The bordered Hessian is given by:

$$\widetilde{H} = \begin{bmatrix} 0 & \frac{p_{12}^2}{p_{12}p_{22}+\epsilon_B^{\overline{\alpha}}\left(p_{11}p_{22}-p_{12}^2\right)} & \frac{p_{12}^2\left(p_{12}p_{22}-\epsilon_A^{\overline{\alpha}}\left(p_{11}p_{22}-p_{12}^2\right)\right)}{\left(p_{12}p_{22}+\epsilon_B^{\overline{\alpha}}\left(p_{11}p_{22}-p_{12}^2\right)\right)^2} - \frac{p_{11}}{p_{12}} \\ \frac{p_{12}^2}{\left(p_{12}p_{22}+\epsilon_B^{\overline{\alpha}}\left(p_{11}p_{22}-p_{12}^2\right)\right)^2} & 0 & \frac{-p_{12}^2\left(p_{11}p_{22}-p_{12}^2\right)}{\left(p_{12}p_{22}+\epsilon_B^{\overline{\alpha}}\left(p_{11}p_{22}-p_{12}^2\right)\right)^2} \\ \frac{p_{12}^2\left(p_{12}p_{22}-\epsilon_A^{\overline{\alpha}}\left(p_{11}p_{22}-p_{12}^2\right)\right)}{\left(p_{12}p_{22}+\epsilon_B^{\overline{\alpha}}\left(p_{11}p_{22}-p_{12}^2\right)\right)^2} - \frac{p_{11}}{p_{12}} & \frac{-p_{12}^2\left(p_{11}p_{22}-p_{12}^2\right)}{\left(p_{12}p_{22}+\epsilon_B^{\overline{\alpha}}\left(p_{11}p_{22}-p_{12}^2\right)\right)^2} & \frac{-2p_{12}^2\left(p_{11}p_{22}-p_{12}^2\right)\left(p_{12}p_{22}-\epsilon_A^{\overline{\alpha}}\left(p_{11}p_{22}-p_{12}^2\right)\right)}{\left(p_{12}p_{22}+\epsilon_B^{\overline{\alpha}}\left(p_{11}p_{22}-p_{12}^2\right)\right)^2} \end{bmatrix}$$

and its determinant is $2p_{11}p_{12}^3 \frac{p_{11}p_{22}-p_{12}^2}{\left(p_{12}p_{22}+\epsilon_B^{\overline{\alpha}}\left(p_{11}p_{22}-p_{12}^2\right)\right)^3}$. The determinant is always positive because $p_{11}p_{22} - p_{12}^2 = \rho$ that is positive by assumption. Being the Ψ function quasiconcave it will reach a it minimum at the extremes of the domain, therefore we study its positivity in those for points (remember that both $\epsilon_A^{\overline{\alpha}}$ and $\epsilon_B^{\overline{\alpha}}$ belong to $[0, 1)$).

$\Psi(0,0) = 1$, always positive.

$\Psi(1,0) = 1 + \frac{p_{12}^2}{p_{12}p_{22}}$, always positive.

$\Psi(1,1) = 1 + \frac{2p_{12}^2}{p_{12}p_{22}+\rho} - \frac{p_{11}}{p_{12}}$, positive if $\rho < (p_{12}+p_{22})\frac{p_{12}^2}{p_{11}}$.

$\Psi(0,1) = 1 + \frac{p_{12}^2}{p_{12}p_{22}+\rho} - \frac{p_{11}}{p_{12}}$, positive if $\rho < \frac{p_{22}p_{12}^2}{p_{11}}$.

So for weak correlation the monotonicity condition is $q_{21} \geq q_{22}$. For strong correlation and $\epsilon_B^{\overline{\alpha}}$ close enough to 1 the monotonicity condition is reversed, and therefore it must be that $q_{22} \geq q_{21}$. ∎

Proof of Proposition 3. Considering monotonicity and the global constraints we can restrict attention to the following:

$$u_{11}^1 + u_{11}^2 \geq u_{12}^1 + u_{12}^2 + \Delta\theta q_{12}$$

$$u_{12}^1 + u_{12}^2 \geq u_{21}^1 + u_{21}^2 + \frac{p_{11}}{p_{12}}\epsilon_A^{\overline{\alpha}}\Delta\theta\left(q_{12} - q_{21}\right)$$

$$u_{21}^1 + u_{21}^2 \geq u_{22}^1 + u_{22}^2 + \Delta\theta\overline{q} + \frac{p_{11}}{p_{12}}\epsilon_B^{\overline{\alpha}}\left(q_{21} - \overline{q}\right)$$

It is evident that the principal will set both $\epsilon_A^{\overline{\alpha}}$ and $\epsilon_B^{\overline{\alpha}}$ equal to zero in order to save on the rents to be given to the two mixed coalitions. But then if $\epsilon_A^{\overline{\alpha}} = \epsilon_B^{\overline{\alpha}} = 0$ there is no more difference between the virtual costs of these two coalitions, namely (θ_1, θ_2) and (θ_2, θ_1) (there was already no difference in the "real" costs). Therefore there is no need to set different quantities and distort more one of the two, and: $q_{12} = q_{21} = \widehat{q}$.

Therefore also in this case we can focus on symmetric mechanism with $t_{11}^A = t_{11}^B$, $t_{22}^A = t_{22}^B$, but also $t_{12}^A = t_{21}^B = \widehat{t}_1$ and $t_{21}^A = t_{12}^B = \widehat{t}_2$, (it matters only wether one is the efficient or inefficient part of a mixed coalition).

Moreover the symmetry allows us to write just one IC constraint for an efficient agent and one IR for an inefficient agent. As it is standard in this literature we can temporarily neglect the IC for an inefficient agent and IR for an efficient one. ∎

Proof of Proposition 4. When correlation is strong p_{12} is so small that allocative distortions in this state of nature loose importance for the payoff of the principal. It is therefore optimal to offer a contract that implements a non-monotonic schedule of outputs.

The relevant collusion proof constraints therefore become:

$$u_{11}^1 + u_{11}^2 \geq u_{22}^1 + u_{22}^2 + 2\Delta\theta\overline{q}$$

$$u_{12}^1 + u_{12}^2 \geq u_{21}^1 + u_{21}^2 + \frac{p_{11}}{p_{12}}\epsilon_A^{\overline{\alpha}}\Delta\theta\left(q_{12} - q_{21}\right)$$

$$u_{21}^1 + u_{21}^2 \geq u_{22}^1 + u_{22}^2 + \Delta\theta\overline{q} + \frac{p_{11}}{p_{12}}\epsilon_B^{\overline{\alpha}}\left(q_{21} - \overline{q}\right)$$

notice that this time the value of the difference $(q_{21} - \overline{q})$ in the last constraint is negative, it is therefore convenient for the principal to set and $\epsilon_B^{\overline{\alpha}}$ as close to one as possible so that she can save on rents to the mixed coalition (where the first agent is inefficient). Moreover the principal has some interest in setting $q_{12} = q_{21}$ because then the second constraint above would be relaxed, but then again if it is profitable to make both mixed coalitions produce the same quantity then also their virtual costs must be set equal and the principal will set $\epsilon_A^{\overline{\alpha}} = \epsilon_B^{\overline{\alpha}}$ close to one.

Since both $\epsilon_A^{\overline{\alpha}}$ and $\epsilon_B^{\overline{\alpha}}$ tend to one the relevant treshold for defining a situation of high correlation is the one used in Corollary 1, namely: $(p_{12} + p_{22})\frac{p_{12}^2}{p_{11}}$.

Then as for the previous case standard method applies and one can restrict attention to IC constraint for an efficient agent and one IR for an inefficient one. ∎

Bibliography

[1] Baron, D.P. and D. Besanko (1992) "Information, Control, and Organizational Structure", *Journal of Economics & Management Strategy*, 1:237-275.

[2] Crémer, J. and R.P. McLean (1985) "Optimal Selling Strategies Under Uncertainty for a Discriminating Monopolist when Demands are Interdependent", *Econometrica*, 53:345-361.

[3] Crémer, J. and R.P. McLean (1988) "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions", *Econometrica*, 56:1247-1257.

[4] Dalton, M. (1959) *Men Who Manage*, Wiley, New York

[5] Dequiedt V. and D. Martimort (2002) "Delegated Monitoring versus Arm's Length Contracting", mimeo.

[6] Faure-Grimaud, A., J.J. Laffont and D. Martimort (2002) "Collusion, Delegation and Supervision with Soft Information", forthcoming *Review of Economic Studies.*

[7] Faure-Grimaud and D. Martimort (2001) "On some Agency Costs of Intermediated Contracting", *Economic Letters*, 71:75-81.

[8] Felli, L. (1996) "Preventing Collusion through Discretion", STICERD Theoretical Economics discussion paper No. TE/96/303

[9] Hirshleifer, J. (1971) "The Private and Social Value of Information and the Reward to Inventive Activity", *American Economic Review*, 61:561-574.

[10] Laffont, J.J. *Incentives and Political Economy*, Clarendon Lectures in Economics, Oxford University Press, Oxford

[11] Laffont, J.-J. e D. Martimort (2002), *The Theory of Incentives*, Princeton University Press, Princeton NJ, USA.

[12] Laffont, J.J. and D. Martimort (2000) "Mechanism Design with Collusion and Correlation", *Econometrica*, 68:238-263.

[13] Laffont, J.J. and D. Martimort (1998) "Collusion & Delegation", *Rand Journal of Economics*, 29:280-305.

[14] Laffont, J.J. and D. Martimort (1997) "Collusion Under Asymmetric Information", *Econometrica*, 65:875-911.

[15] Lewis, T.R. and D.E.M. Sappington (1989) "Countervailing Incentives in Agency Problems", *Journal of Economic Theory*, 49:294-313.

[16] Maggi, G. and A. Rodriguez-Clare (1995) "On Countervailing Incentives", *Journal of Economic Theory*, 66:238-263.

[17] Maskin, E. and J. Tirole (1990) "The Principal-Agent relationship with an Informed Principal: the Case of Private Values", *Econometrica*, 58:379-409.

[18] Maskin, E. and J. Tirole (1992) "The Principal-Agent Relationship with an Informed Principal, II: Common Values.", *Econometrica*, 60:1-42.

[19] McAfee, R.P. and P.J. Reny (1992), "Correlated Information and Mechanism Design.", *Econometrica*, 60:395-421.

[20] Melumad, D.M., D.Mookherjee and S. Reichelstein (1995) "Hierarchical Decentralization of Incentives Contract", *Rand Journal of Economics*, 26:654-672.

[21] Mookherjee, D. and S. Reichelstein (1992) "Dominant Strategy Implementation of Bayesian Incentive Compatible Rules", *Journal of Economic Theory*, 56:378-399.

[22] Myerson, R.B. (1983) "Mechanism Design by an Informed Principal", *Econometrica*, 51:1767-1797.

[23] Riordan, M.H. and D.E.M. Sappington (1988) "Optimal Contracts with Public Ex Post Information", *Journal of Economic Theory*, 45:189-199.

[24] Tirole, J. (1986) "Hierarchies & Bureaucracies: On the Role of collusion in Organizations", *Journal of Law, Economics & Organizations*, 2:181-214.

[25] Tirole, J. (1992) "Collusion and the Theory of Organization", in J.J. Laffont (ed.), *Advances in Economic Theory*, Cambridge University Press, Cambridge.