FINANCIAL MARKETS, MONETARY POLICY, AND THE REAL ECONOMY

by

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Abstract

This thesis studies the interactions between financial markets, monetary policy, and the real economy. It analyses the role of financial markets in business cycle fluctuations and explores issues concerning systemic financial stability.

Chapter One develops a dynamic general equilibrium model in which firms and banks face financial frictions in obtaining external funds. The model exhibits an unconventional bank capital channel as monetary policy affects the economy partly via its effect on bank capital. We show that the dynamic interactions between bank capital, firm net worth and asset price amplify and propagate the effect of a monetary shock in the macroeconomy.

Chapter Two empirically investigates the importance of financial markets in the monetary transmission. The analysis is based on the argument that the real money stock serves as a proxy for the relative yields of various non-money assets that matter for aggregate demand. Using Thailand data, we find that the two-asset assumption is biased and that this problem can be ameliorated by introducing an explicit role for money into standard macroeconomic models.

Chapter Three develops a numerically-solvable version of our general model [Goodhart, Sunirand, and Tsomocos (2003)] to analyse financial fragility. The model incorporates heterogeneous agents and therefore leads to different simulation results from those obtained when using standard representative agent models; the effect of a shock depends on the part of the economy on which it falls and can generally shift the distribution of income and welfare between agents.

Chapter Four proposes a general equilibrium model incorporating three heterogeneous banks. This allows us to study not only the interactions between any two individual banks, but also their inter-relationship with the rest of the banking sector. The model is calibrated against real UK banking data and therefore can be implemented as a risk assessment tool for financial regulators and central banks.

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Introduction

This thesis studies the interactions between financial markets, monetary policy, and the real economy. It consists of two main parts. The first part (Chapters One and Two) explores such interactions *over time*. It analyses the role of financial markets in real business cycle fluctuations, focusing on its implications for the monetary policy transmission mechanism. The second part (Chapters Three and Four) addresses issues concerning *systemic* financial stability. Its main emphasis is on the interactions between *heterogeneous* banks in the financial sector and their inter-relationship with *heterogeneous* private agents which in turn could produce real contagion effects in the real economy.

The idea that financial markets are not just a 'veil', and therefore play an important role in business cycle fluctuations, has long been the key standpoint of the credit view of the monetary transmission mechanism. Instead of assuming that all non-money financial assets are *perfect* substitutes and that they can be aggregated into a single composite asset called 'bond', the credit view argues that macroeconomic models for monetary policy need to depart from this so-called 'two asset world' assumption by introducing certain forms of market imperfections in financial markets. This, in turn, would break the 'perfect substitutability' property across different non-money assets, thereby generating a non-trivial role for financial factors in the monetary transmission process.

There are three broad forms of imperfect substitutability between non-money assets proposed in the literature, each of which represents a distinct credit channel. The first, the so-called bank lending channel, assumes that loans and securities are imperfect substitutes, both for borrowers and banks. Put differently, at the heart of the bank lending channel is the lack of perfect substitutes for bank loans on the part of borrowers and the lack of perfect substitutes for (reservable) deposit liabilities on the liability side of the banking sector's balance sheet. The second form, the so-called balance sheet channel, emphasises the imperfect substitutability between firms' internal and external sources of finance, i.e. firms' debt and equity. In the past decade, this balance sheet channel has been formalised in the context of dynamic general equilibrium models to study the interactions between firm net worth, monetary policy, and business cycle fluctuations. [1]

The third channel, the most recently identified one thus far, is the bank capital channel. In this case it is argued that a monetary policy shock can affect bank lending and aggregate economic activities through its *direct* impact on bank capital.[2] One key assumption underpinning this channel is that banks' debt and equity are imperfect substitutes. In the literature, this usually arises from the presence of an exogenously given regulatory capital requirement, e.g. Van den Heuvel (2001) and Bolton and Freixas (2000). In Chapter One, we show that a form of imperfect substitutability between banks' debt and equity can also emerge from an agency problem between banks and their depositors. Since this implies that these banks face market-based, as opposed to regulatory-based, capital requirements, we are able to identify an *unconventional* bank capital channel.[3] Moreover, the standard bank capital channel of monetary transmission has only been studied in the context of partial equilibrium models of banks' asset and liability management.[4] This implies that the dynamic interactions between bank capital, monetary policy, and aggregate macro variables such as aggregate output, investment and prices have not been formally analysed.

In Chapter One, we embed a 'double' costly state verification (CSV) problem into the otherwise standard dynamic general equilibrium model with price stickiness. It is 'double' because both firms and banks face endogenous financial frictions in obtaining external finance. This, in turn, implies that a wedge between the internal and external costs of funds exists for both firms and banks, thereby motivating an endogenous role for firms' and banks' inside capital in the model. The novel feature of this double CSV approach is that, while retaining rigorous microfoundations developed in the theory

[1] Recent contribution on this front includes Bernanke, Gertler, and Gilchrist (1999), Carlstrom and Fuerst (2001), and Kiyotaki and Moore (1997). For a comprehensive survey on this issue, see Bernanke, Gertler, and Gilchrist (1999). A survey and overview of the bank lending and balance sheet channels in general can be found in Kashyap and Stein (1994), Freixas and Rochet (1997), among others.

[2] In the literature, several papers argue that bank capital also plays a distinct role in the monetary policy transmission by affecting the strength of the 'bank lending' channel (see, for example, Van den Heuvel (2002), Jayaratne and Morgan (2000), Kishan and Opiela (2000), and Gambacorta and Mistrulli (2004)). According to the bank lending channel, a contractionary monetary policy affects bank lending because the decrease in *reservable* deposits cannot be completely offset by issuing *non-reservable* liabilities, i.e. lack of perfect substitutes for reservable deposits. Since the market for bank debt is not frictionless and non-reservable liabilities are typically not insured, investors would demand a 'lemon's premium'. In this case, banks with less capital are more exposed to agency problems and have to face a higher cost of non-reservable funding. Thus, the bank lending channel is stronger for banks with lower levels of capital. This role for bank capital, however, differs from that played in the bank capital channel in that, while the former treats bank capital as given in response to monetary policy shocks, the latter explicitly takes into account the direct effect of such shocks on bank capital. For example, as argued by Van den Heuvel (2001), given that banks' assets have a longer maturity than their liabilities, they are exposed to interest rate risk. Thus, their profitability and capital decline in response to a monetary policy tightening. A drop in bank capital then feeds back to constrain bank lending and economic activities.

[3] The standard bank capital channel, as defined by Van den Heuvel (2001), relies on the following three hypotheses. First, there is an imperfect market for bank capital, i.e. banks cannot costlessly issue new equity. Second, banks are subject to interest rate risk since their assets typically have longer maturity than their liabilities. Third, banks have to meet regulatory capital requirements set by financial regulators. The bank capital channel identified in Chapter One differs from the standard one in that the third hypothesis is replaced by the assumption that banks face an agency problem in obtaining external funds from depositors.

[4] Chapter One provides a detailed literature review on the issue.

of banking literature, it is simple and analytically tractable enough to be readily embedded into the standard Dynamic New Keynesian (DNK) model for monetary policy. This enables us to study, both qualitatively and quantitatively, how the dynamic evolution of bank capital operates to enrich the transmission mechanism of monetary policy by augmenting the dynamics of other aggregate macro variables, including aggregate investment, output and asset price.

We conduct a simulation analysis, where the steady state equilibrium is calibrated to match U.S. data, in order to study the quantitative importance of bank capital in business cycle fluctuations in response to a monetary policy shock. The results highlight a 'financial accelerator' effect in that the endogenous evolution of bank capital, and its dynamic interplay with that of firm net worth and asset prices, operates to amplify and propagate the effect of a monetary shock in the macroeconomy. Banks' and firms' inside capital plays a principal role here because lower bank capital and firm net worth means that both firms and banks have less inside capital to contribute to the firms' investment projects. This implies that the agency problem faced by depositors, who are the ultimate lenders, is intensified. As compensation, they therefore require a higher external finance premium in the form of higher deposit rates. Since this directly imposes a greater cost of borrowing on banks, lending rates rise. Given a higher external cost of funds for firms, aggregate investment and output have to decrease compared with their corresponding levels as implied by conventional *frictionless* models. To the extent that bank capital and firm net worth are procyclical, an external finance premium will be countercyclical and thus operates as a propagation mechanism to the model's dynamics.

As mentioned, the way that financial markets are usually entered in the monetary transmission process relies on the validity of the two-asset world assumption. If such assumption is invalid, as the proponents of the credit view would argue, the effect of monetary policy from its initial impulse to the ultimate responses on real aggregate macro variables involves changes in *relative* yields of various non-money financial assets. One way of taking macroeconomic models away from the two-asset assumption is by explicitly introducing some forms of imperfect substitutability among different financial assets in the models. This is the approach pursued in Chapter One. Though analytically insightful, the extent of complexity of such an approach can become overwhelmingly high when *multiple* financial assets are simultaneously introduced. Another approach, proposed by Meltzer (2001), is to incorporate an explicit role of the real money stock in the models as an auxiliary *proxy* for unidentified monetary transmission channels which arise from changes in relative yields of a wide array of non-money assets. The real money stock is, arguably, a reasonable proxy in this case because the demand for money is generally a function of these yields.

Chapter Two presents an empirical investigation of the validity of the two-asset assumption based on Meltzer's theoretical argument. Using Thailand data, we tested for the significance of the role for the real money stock as an *explicit* determinant of aggregate demand under a number of different specifications of IS equations. We found that lagged real money growth enters all of these IS equations positively, sizably, and significantly, even when the short-term risk free rate is explicitly controlled for. This implies that the real money stock has information content concerning aggregate demand fluctuations *over and above* that captured by the short-term interest rate. This finding is consistent with those found for developed countries such as the U.K. and U.S.[5]

Methodologically this chapter also has something new to offer. In particular, the analysis is also based on *hybrid* IS equations which essentially allow for both forward looking and backward looking behaviour of rational agents. We argue that this enables us to identify separately the two distinct forms of changes in relative yields of financial assets that money is conventionally found to proxy; one being the changes along the term structure of interest rate (the term structure effect) and the other being the changes in relative risk premia among different kinds and classes of assets (the risk premium effect). Given that the risk premium effect is found to be strong and statistically significant, we argue that the two-asset world assumption, which has long underpinned conventional macroeconomic models, including the class of models with microfoundations, becomes inherently distorting. This problem can be ameliorated by introducing an explicit role for money into the model. This is because the real money stock may serve as a reasonable *stand-in* for the relative yields of various risky assets which are important for aggregate demand fluctuations.

The underlying assumption of the models used in Chapters One and Two is that each sector in the economy has agents which behave identically. This 'representative agent' assumption greatly reduces the technical complexity of the models to the extent that they can be presented in an infinite-horizon format. This, in turn, enables us to analyse the dynamic interactions between financial markets, monetary policy and the real economy *over time*. However, the representative agent assumption is no longer justifiable when applying to a model whose aim is to address issues concerning systemic financial stability such as bank inter-linkages, financial contagion and crises. This is because such an assumption obscures many of the economic and behavioural inter-relationships among agents within each sector, particularly in the banking sector. By assuming homogeneity in the banking sector, either all banks fail or they all survive in face of some assumed common shock. In such a case, financial contagion among banks and their inter-linkages cannot, almost by definition, occur.

So, in order to study systemic financial stability we need a model which incorporates *heterogeneous* banks. There must also be active markets wherein banks interact with each other and the

[5] A literature review on the issue is provided in Chapter Two.

household sector, e.g. via the interbank and credit markets. The existence of these markets, in turn, allows a process of contagion to take place in response to some assumed shock. We also need default to exist, since if there were no default, there would be no crises. Moreover, financial markets cannot be complete. Otherwise banks can always hedge themselves against all kinds of eventualities, in which case, there would, again, be no crises. In addition, equilibria in incomplete markets are constrained inefficient, implying that policy matters and can be welfare improving. We also need a Central Bank and a regulator who respectively conduct monetary and regulatory policies so that the dynamic interactive effects of such policies on the banking sector and the real economy can be analysed. Since the focus here is not to study the interactions of agents over time, we can simplify the analysis, without loss of generality, by assuming a finite (two period) horizon model. In Goodhart, Sunirand and Tsomocos (2003), we developed a general equilibrium model with an endowment economy incorporating all these features.

In Goodhart, Sunirand and Tsomocos (2003), we have shown that an equilibrium exists and that financial fragility, characterised by reduced aggregate bank profitability and increased aggregate default, occurs as an equilibrium phenomenon. However, owing to the scale of the model which contains B heterogeneous banks, H households, S possible states in the second period, a variety of financial assets, and default, it is impossible to find a numerical solution to this general model. This implies that the model cannot be readily applied to study the behavioural responses of each economic agent and their possible inter-linkages through various interactive contagion channels in face of some assumed shocks.

So Chapter Three of this thesis presents simplified, and thus numerically solvable, versions of the general model. The economy consists of two heterogeneous banks, each of which is distinguished by a unique risk/return preference, and different initial capital endowments, and three heterogeneous households who either borrow from or deposit with the banks. Moreover, a Central Bank conducts monetary policy through open market operations, and a regulator fixes the bankruptcy codes for households and banks as well as sets the capital adequacy requirements for banks. We assume that there are five types of competitive markets wherein agents actively interact; commodity, consumer loan, deposit, interbank and asset (Arrow security) markets. For asymmetric information standard reasons, we assume a limited participation in the consumer loan markets.[6] We also allow agents to default in some of the financial markets subject to the penalty set by the regulator. This setting was chosen as our base-line specification because it is the simplest possible given that we need at least two

[6] In Bhattacharya, Goodhart, Sunirand, and Tsomocos (2003) we show that restricted participation in the loan market can also emerge as an equilibrium outcome if we incorporate a relative performance criterion in banks' objective functions, i.e. banks have a preference to outperform their competitors.

heterogeneous banks in order to analyse the *intra*-sector contagion effect within the banking sector via their interaction in the interbank and asset markets, and the possible *inter*-sector contagion effect involving the real sector via the credit, deposit, asset and commodity markets. Moreover, such a setting allows default in one market, e.g. a consumer loan market, to produce additional source of contagion to the others and the rest of the economy.

Given the arbitrarily chosen values of the exogenous variables/parameters in the model, e.g. the Central Bank's and the regulator's policy choices, we solve numerically for an initial equilibrium. We then conduct a series of comparative statics analysis by perturbing the values of the exogenous variables one at a time and tracing the new equilibria of the simulations. This allows us to study how the multiple markets and the agents' choice variables interact, and how the many system-wide effects determine prices, interest rates and real allocations. In general, the simulation results highlight the importance of the main innovative feature of the model that the real world is heterogeneous; agents and banks are not all alike. We found that the effect of a shock may depend on the particular agent, and part of the economy, on which it falls and can generally shift the distribution of income, and welfare, between agents in a complex way. For example, a positive shock which is concentrated in one part of the economy may produce adverse negative contagion effects onto others and therefore does not necessarily improve the welfare and overall payoff of everyone in the economy. We found that monetary policy also has a distributional effect on the economy, implying that a trade-off between economic efficiency and financial stability exists not only for regulatory policies but also for monetary policy. Moreover, we found that regulatory policy can be seen as a mirror image of monetary policy since it affects banks' credit extension via their portfolio decisions. Finally, agents who have more investment opportunities can deal with negative shocks more effectively since they are able to transfer negative externalities to others.

Although we show that the model presented in Chapter Three can be solved numerically, its major disadvantage is that the solutions obtained are based on an arbitrarily chosen initial condition. The outcome, therefore, is a somewhat artificial construct of our own assumed inputs and may not correspond to the particular economic regime of the economy whose systemic risk and financial fragility we want to study. In Chapter Four, we take a step further by attempting to calibrate an alternative version of the general model against *real* UK banking data. Its primary objective is to construct a model which can be implemented by central banks and financial regulators as a tool to assess risk for banks. The main innovation of the model is that, unlike existing models for this purpose which focus almost entirely on *individual* institutions[7], it allows endogenous interaction

[7] See, for example, Hoggarth, Logan, and Zicchino (2004).

between banks, recognising that the actual risk to which an individual bank is exposed also depends on its interaction with other banks and other private sector agents.

A banking system in reality generally comprises more than two banks. In order to study interactions between these multiple banks, we need at least a setting of *three* heterogeneous banks so that we can study dynamic inter-relationships between any two specific banks as well as their interactions with the rest of the banking sector, which is represented by the third bank. Moreover, given the lack of *disaggregated* household and private investors' portfolio data[8], we model household behaviour via reduced-form equations which relate their actions to a variety of economic variables such as GDP, interest rates, and aggregate credit supply.[9] In this sense, the model presented in Chapter Three is a partially-microfounded general equilibrium model. However, the main aspects of equilibrium analysis such as market clearing, rational expectation, and agent optimisation are maintained. Moreover, contagion effects between the banking sector and the real economy still operate actively in equilibrium via the reduced-form equations. Thus, we adhere to the general equilibrium spirit of our general model presented in Goodhart et al. (2003) and the model presented in Chapter Three.

We show that the model can be used as a stress-testing tool for the UK banking sector. In particular, we address the impact of monetary policy and regulatory policy as well as GDP and capital shocks in the UK banking sector and the real economy. The focus of our stress-testing exercises is on adjustments in the interbank market wherein banks have mutual exposures, and in the relative interest rates on deposits and loans. Hence, in part via changes in bank margins, this feeds back into changes in bank profitability, capital and capital adequacy requirements. Moreover, changes in aggregate credit supply in the banking sector via each bank's portfolio adjustment decision affect aggregate output and household default probability in the economy.

Our simulation results identify at least two key channels of contagion which operate actively in our model. The first is the interbank rate channel whereby the effect of a shock, which is initially concentrated in a particular group of banks, produces contagion effects to the rest of the banks in the banking sector via the adjustment in the interbank rate. This is because such adjustment directly affects the cost of interbank borrowing for every bank which has net exposure in the interbank market. The second is the consumer loan default channel. The main idea of this channel is that a higher default probability of a particular group of households in the loan markets increases the default risk to which banks are exposed. Since the loan markets are perfectly competitive, these banks are

[8]For example, the monetary and good endowment of each bank's borrowers and depositors.

[9]As will be shown in Chapter Three, household and investor optimisation can be introduced. However, because one of the objectives of this chapter is to take our model to real data, we chose not to follow that route.

forced to cut back their loan supply not only to the group of agents whose default probability is higher but also to the others. Put differently, this latter channel operates by aggravating the severity of 'credit crunch' in the economic system. Our simulation results also highlight the importance of central banks' monetary policy instrument choices in achieving financial stability. In particular, whether monetary authorities target base money or the interbank interest rate in response to shocks has different implications with respect to financial stability.

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Chapter 1

The Role of Bank Capital and the Transmission Mechanism of Monetary Policy

Abstract

This chapter studies the transmission mechanism of monetary policy in the presence of an endogenous role of bank capital. The basic framework is a standard Dynamic New Keynesian model modified so that firms and banks face endogenous financial frictions in obtaining external funds. The model exhibits an unconventional '*bank capital*' channel in which monetary policy affects aggregate economic activities partly via its effect on bank capital. The simulation results highlight a financial accelerator effect in that endogenous evolution of bank capital, and its dynamic interplay with that of entrepreneurial net worth and asset price, operates to amplify and propagate the effect of a monetary shock in the macroeconomy.

1.1 Introduction

The goal of this chapter is to study the transmission mechanism of monetary policy in the presence of an endogenous role of bank capital. This is motivated by the observation that conventional macro models for monetary policy, both with and without the explicit role of entrepreneurial net worth, abstract completely from the role of bank capital.[1] This consensus practice would be a justifiable

[1] For macroeconomic models in which the transmission mechanism of monetary policy works only through the conventional interest rate channel, see, amongst others, Clarida, Gali and Gertler (1999). For those with an explicit role of entrepreneurial net worth, thus incorporating the balance sheet channel, see, amongst others, Bernanke, Gertler and Gilchrist (1999) and Carlstrom and Fuerst (2001).

simplifying assumption only if one of the following conditions holds: 1) an unexpected monetary shock does not affect bank capital, or 2) if it does, changes in the dynamics of bank capital must have no major effect on that of other important aggregate macroeconomic variables.

One of the main functions that banks perform is the transformation of securities with short maturities, offered to depositors, into securities with long maturities that borrowers desire (Freixas and Rochet, 1997). This maturity mismatch on banks' balance sheets implies that lending rates are relatively stickier compared to deposit rates in response to unanticipated aggregate shocks. Consequently, in response to an unanticipated increase in the risk free (policy) rate, banks' interest rate cost rises faster than their interest rate revenue, thereby depleting their inside capital. This invalidates the prior first condition; an unexpected monetary shock can theoretically affect bank capital.

Turning to the second condition, there are many empirical findings which lend support to the importance of the role of bank capital in constraining bank lending and aggregate economic activities. Amongst others, Bernanke and Lown (1991), Furlong (1992) and more recently, Peek and Rosengren (1997) and Ito and Sasaki (1998) find that the capital position of banks has had positive and statistically significant effects on bank lending. Moreover, Hubbard, Kuttner, and Palia (2002) find that higher bank capital lowers the rate charged on loans, even after controlling for borrower characteristics, other bank characteristics and loan contract terms. Given these findings, the second condition is also violated. Thus excluding bank capital from a model's dynamics can distort our understanding of the monetary policy transmission.

The basic framework employed in this chapter is an extension of the financial accelerator model of Bernanke, Gertler, and Gilchrist (1999). Here we propose the *'double'* costly state verification (Double CSV) approach as the principal modification.[2] It is *'double'* because, in addition to firms, banks also face endogenous financial frictions in obtaining external funds. This implies that a wedge between internal and external costs of funds exists, thereby motivating an endogenous role of firms' and banks' inside capital in the model. The novel feature of the approach is that, while retaining rigorous microfoundations found in the theory of banking literature, the approach is simple and tractable enough to be readily embedded into the otherwise standard Dynamic New Keynesian (DNK) model with price stickiness. This allows us to study, both qualitatively and quantitatively, how the dynamic evolution of bank capital operates to enrich the transmission mechanism of monetary policy by augmenting the dynamics of other aggregate macro variables, including aggregate investment, output and asset prices.

[2] The one-sided CSV problem was first introduced by Townsend (1979).

The organisation of the chapter is as follows. Section 1.2 discusses some related literature. Section 1.3 presents the partial equilibrium model of financial contract, the double CSV approach. Section 1.4 embeds the key equations derived in section 1.3 into the otherwise standard DNK model with price stickiness. Section 1.5 gives the definition of the equilibrium and describes the model in a completely log-linearised form. Section 1.6 discusses the calibration and presents the simulation results. Section 1.7 concludes the chapter.

1.2 Related Literature

Bernanke et al. (1999) and Carlstrom and Fuerst (2001) examine the role of credit market frictions in business cycle fluctuations. Firms in these models face financial frictions in borrowing from banks, which causes their net worth to become the key element in determining their debt capacity. This allows monetary policy to have an independent effect on entrepreneurial net worth, the so-called balance sheet channel. However, banks do not need to hold any inside capital in equilibrium as they are assumed to have a *perfectly diversified portfolio of bank loans.* This implies that any idiosyncratic risk associated with firms' investment return is completely diversified at the bank level and therefore *is not* passed on to ultimate depositors. Given that depositors are risk averse, they can therefore be guaranteed an equivalently riskless rate of return.

Another set of literature focuses on an explicit role of bank capital in the model of bank's asset and liability management. Van den Heuvel (2001) examines the role of bank capital in the transmission mechanism of monetary policy. Banks in his model hold their inside capital to satisfy exogenous capital adequacy regulations. This, together with a maturity mismatch on banks' balance sheets, gives rise to a 'bank capital' channel in which monetary policy affects bank lending through its impact on bank capital.[3] Schneider (1999) studies the relationship between bank's borrowing constraint and the observed heterogeneity in borrowing and lending behaviours across banks. The holding of bank capital in his model is endogenous as it can alleviate the moral hazard problem associated with strategic defaults by entrepreneurial bankers. However, these models do not consider an independent role of entrepreneurial net worth and therefore do not exhibit its dynamic interplay with bank capital. Moreover, they are not fully general equilibrium models in the sense that they abstract from consumption, investment and aggregate demand effects relating to price stickiness.

[3] According to Van den Heuvel (2001), the bank capital channel relies on the following three hypotheses. First, there is an imperfect market for bank capital, i.e. banks cannot costlessly issue new equity. Second, banks are subject to interest rate risk since their assets typically have longer maturity than their liabilities. Third, banks have to meet regulatory capital requirements set by financial regulators. As will be illustrated, the bank capital channel identified in this chapter differs from Van den Heuvel's primarily in that the third hypothesis is replaced by the assumption that banks face the CSV problem in obtaining external funds from depositors.

The last set of papers endogenises both entrepreneurial net worth and bank capital. Bolton and Freixas (2000) analyse the transmission mechanism of monetary policy in the context where direct and indirect finance coexist. Given the presence of an exogenous capital requirement, asymmetric information on the value of bank capital implies the existence of an endogenous cost in raising outside equity capital. The monetary policy transmission implied by their model exhibits an amplification effect on bank lending through its effect on bank capital. Cantillo (1997) adopted the CSV approach to study coexistence between direct and indirect finance. In his model, both firms and banks face financial frictions in obtaining external funds. They, therefore, hold limited inside capital in equilibrium. However, as these models have only two periods, the issue of dynamics cannot be disentangled. Moreover, similar to Van den Heuvel (2001) and Schneider (1999), they are not fully general equilibrium models.

Chen (2001) studies dynamic interaction between entrepreneurial net worth, bank capital and real economic activities by extending Holmstrom and Tirole's (1997) model into a dynamic general equilibrium setting. The moral hazard problem both at the firm and bank levels is assumed in order to motivate an endogenous role for firms' and banks' inside capital. However, since the model has no role for monetary authorities and price stickiness, it cannot be used to study the transmission mechanism of monetary policy.[4]

1.3 The Partial Equilibrium Model of Financial Contract: Double Costly State Verification (Double CSV)

1.3.1 Basic Assumptions and the Structure of the Model

There are five types of agents in the economy: entrepreneurs, banks, households (depositors), retailers and the Central Bank.[5] As this section discusses the financial contracting problem among entrepreneurs, banks and depositors, only the basic structure of these sectors which are relevant to the contracting problem will be addressed in the following subsections. The rest will be discussed in section 1.4.

[4]Moreover, as argued by Cantillo (1997), the role of banks in resolving the moral hazard problem as assumed in Chen's (2001) and Holstrom and Tirole's (2001) models is a less appealing description of what banks in reality do. In particular, since banks in these models perform a *monitoring* role to ensure that firms do not choose a 'bad' project in pursuit for higher 'private' benefit, they are more suitably described as *equity* intermediaries such as venture capitalists who are quite involved in the day-to-day activities of the companies that they fund. In contrast, banks or other *debt* intermediaries in reality do not actively involve with non-bankrupt firms. Since, as will be explained, banks under the CSV approach only monitor bankrupt firms (by paying a verification cost), using the CSV framework may be more realistic.

[5]As the main focus of this chapter is on the transmission of monetary policy, we shall abstract from the role of government and therefore fiscal policy in the model.

1.3.1.1 Entrepreneurial Sector

Entrepreneurs are assumed to be risk neutral and are the only type of agent in the economy with access to investment technology which involves the transformation of capital together with hired labour (from the household sector) into wholesale goods. A representative entrepreneur, say entrepreneur i, operates firm i.[6] At the end of period t, firm i purchases capital, K_t^i. The unit price of capital is given by Q_t. All capital is homogeneous. In addition, capital purchased at the end of period t cannot be used in production until the end of period $t+1$. The gross rate of return from investing in capital is denoted by $\varpi_{i,t+1}R_{t+1}^K$, where R_{t+1}^K and $\varpi_{i,t+1}$ are the non-idiosyncratic and idiosyncratic components of firm i's rate of return to capital, respectively.[7]

The random variable $\varpi_{i,t+1}$ is assumed to be log normally distributed with mean unity, $E(\varpi_{i,t+1}) = 1$, and variance τ^2, and is independently and identically distributed (*i.i.d.*) across time and firms.[8] Formally, the distribution of the random variable $\varpi_{i,t+1}$ can be summarised as follows:

$$\ln \varpi_{i,t+1} \overset{i.i.d.}{\sim} N(-\frac{1}{2}\sigma^2, \sigma^2), \text{ where } \sigma^2 = \ln(1+\tau^2)$$

In addition to idiosyncratic risk, firm i also encounters aggregate risk. This is because the non-idiosyncratic component of return associated with period-t capital investment, R_{t+1}^K, will not be realised until the end of period $t+1$. The timeline of the model will be discussed in detail in subsection 1.3.1.4.

We assume that firm i can borrow external funds from a representative bank, say bank j, to partially finance its capital investment. All financial contracts, including both loan and deposit contracts, are assumed to have one period maturity. Following the CSV literature, the realisation of idiosyncratic component of return, $\varpi_{i,t+1}$, is private information and bank j has to pay a verification cost in order to observe its value. This, as mentioned earlier, motivates entrepreneur i to hold his inside capital as the existence of a verification cost drives a wedge between internal and external costs of funds. Moreover, following Krasa and Villamil (1992), we assume that the realisation of $\varpi_{i,t+1}$ is *privately* revealed only to the agent who requests CSV technology. This assumption is essential to the analysis since if all information could be made public *ex post* there would be no need

[6] Firm i and entrepreneur i will be used interchangeably throughout the chapter.

[7] Throughout the chapter, the time subscript denotes the period in which the value of an underlying variable is realised.

[8] Denote $F(\varpi_{i,t+1})$ and $f(\varpi_{i,t+1})$ as *c.d.f.* and *d.f.* of $\varpi_{i,t+1}$, respectively, and let $h(\varpi_{i,t+1}) \equiv \frac{f(\varpi_{i,t+1})}{1-F(\varpi_{i,t+1})}$ be the hazard rate, the assumption that $\omega_{i,t+1}$ is log-normally distributed implies that the restriction $\frac{\partial(\varpi_{i,t+1}h(\varpi_{i,t+1}))}{\varpi_{i,t+1}} > 0$ holds. This regularity condition is a relatively weak restriction as it is satisfied by most conventional distributions (Bernanke et al., 1999).

for depositors to pay a verification cost to observe banks' return on their portfolio of loans.[9]

Denote firm i's inside capital held at the end of period t by W_t^i. Given that the total outlay of the investment is $Q_t K_t^i$, loans borrowed from bank j, L_t^i, is defined as follows:

$$L_t^i \equiv Q_t K_t^i - W_t^i \tag{1.1}$$

Using the fact that the total return from investing $Q_t K_t^i$ is $\varpi_{i,t+1} R_{t+1}^K Q_t K_t^i$, firm i's threshold value of $\varpi_{i,t+1}$, $\overline{\varpi}_{i,t+1}^F$, is defined such that it satisfies the following equation:

$$\overline{\varpi}_{i,t+1}^F Q_t R_{t+1}^K K_t^i = r_{i,t}^L L_t^i \tag{1.2}$$

where $r_{i,t}^L$ is defined as the non-default loan rate associated with the loan contract between firm i and bank j signed in period t. For the realisation of idiosyncratic component below the threshold level, $\varpi_{i,t+1} < \overline{\varpi}_{i,t+1}^F$, firm i's realised revenue from investing $Q_t K_t^i$ is strictly less than the amount required to fulfil its loan contract with bank j. Thus firm i declares bankruptcy and faces liquidation. In contrast, when $\varpi_{i,t+1} \geq \overline{\varpi}_{i,t+1}^F$, firm i does not go bankrupt as its realised return in period $t+1$ is sufficient to repay its debt obligations to bank j.[10]

1.3.1.2 Banking Sector

Banks in this economy operate under a perfectly competitive environment. Similar to entrepreneurs, they are assumed to be risk neutral. They function as financial intermediary, i.e. they borrow from a representative depositor and lend to a representative entrepreneur.

As commonly shown in the conventional *one-sided* costly state verification literature[11], given that $\omega_{i,t+1}$ is identically and independently distributed (*i.i.d.*) across firms, the idiosyncratic risk associated with each investment project is *completely* diversified in the *infinitely* large portfolio of bank loans, by virtue of the law of large numbers. Thus, depositors can be guaranteed an equivalently riskless rate of return and banks have no incentive to hold any inside capital.[12] However, as argued by Krasa and Villamil (1992), the diversification would *not* be complete if the portfolio is of a *finite* size, in which case the idiosyncratic risk associated with firms' investment projects remains *at the*

[9] This assumption is consistent with institutional features which characterise most lending arrangements. As Diamond (1984, p.395) illustrated, "Financial intermediaries in the world monitor much information about their borrowers in enforcing loan covenants, but typically do not directly announce this information or serve an auditor's function."

[10] As $\overline{\varpi}_{i,t+1}^F$ is realised in period $t+1$, its value is contingent on the *ex-post* realisation of R_{t+1}^K.

[11] Diamond (1984), Gale and Hellwig (1985) and Williamson (1986)), amongst others.

[12] This assumption is taken in the macro models which incorporate the balance sheet channel of monetary policy but completely ignore the role of bank capital, e.g. Bernanke et al. (1999).

bank level and therefore is passed on to ultimate depositors. This gives an incentive for depositors to monitor banks and thus motivate an explicit role of bank capital.[13]

In general, given that the size of bank loan portfolio is finite, the distribution of return within *each* individual bank's loan portfolio becomes essential to the analysis.[14] To maintain the model's tractability, we assume that a bank can only lend to *one* entrepreneur. This assumption is essentially tantamount to the case in which a bank can finance multiple firms but the return on firms' investment projects is assumed to be perfectly correlated within a bank but *i.i.d.* across banks (Holstrom and Tirole (1997) and Chen (2001)).[15] Although this assumption is obviously unrealistic, it is meant to avoid the equally unrealistic conclusion that banks can never collapse and its intermediation service can be carried out without any inside capital.

Bank j can borrow external funds from a representative depositor, say depositor m, to partially finance its lending to firm i. Given the assumption that a bank can only borrow from one entrepreneur, the idiosyncratic risk associated with firm i's investment is passed on directly to bank j's returns on its lending. As firm i's return on investment is private information, so is the return on bank j's loans. Given the CSV problem at the bank level, depositor m has to pay a verification cost if he or she wishes to observe the return on bank j's lending. This creates an external finance premium for bank j in obtaining external funds from depositor m, thereby motivating the bank to hold inside capital. So, the holding of bank capital in this model is a market-based, as opposed to a regulatory-based, requirement.

In period t, a representative bank which finances its lending to firm i (L_t^i) by its own inside capital (A_t^i) and deposits acquired from depositor m (D_t^i) has the following balance sheet identity:[16] [17]

[13]Thus the CSV problem becomes *two*-sided. On the one hand, banks act as delegated monitors on firms' investment projects. On the other hand, in the terminology of Krasa and Villamil (1992), depositors perform the role of 'monitoring the monitor'.

[14]For example, the contract term and the aggregation process depend on the distribution of an individual bank's portfolio of risky loans.

[15]Technically, the assumption that each individual bank could only lend to *one* firm while $\varpi_{i,t+1}$ is allowed to be *i.i.d.* across time and firms gives the same result as the case in which each bank can lend to multiple firms but $\varpi_{i,t+1}$ is assumed to be perfectly correlated within a bank and is *i.i.d.* across banks. They imply that the idiosyncratic risk is fully diversified at the aggregate level, *but not at the bank level.*

[16]As mentioned, similar to loan contracts, we assume that all deposit contracts have only one period maturity.

[17]Compared to the asset side of the balance sheet of a typical bank in reality, there are three important elements that this model lacks, namely short term bonds, cash and equipments. Because banks have to maintain the convertibility commitment with depositors at *any* point in time and loans are a relatively illiquid kind of asset, banks have an incentive to hold short-term government bonds as a means of reducing their exposure to liquidity risk. However, the model at hand is not rich enough to accommodate the prevalence of liquidity risk.

Banks also have to hold cash or reserves (the liabilities of central banks) owing to their inability to forecast perfectly their payment flows and to arrange transactions in the interbank market throughout the day so as to maintain settlement balances constant at a balance greater than, or equal to, zero (Goodhart, 2000). As central banks have monopoly power in issuing 'cash', this gives them the power to set the short-term risk free rate. However, as emphasised by Woodford (2000), this power does *not* depend on the *size* of the banks' cash holding. To simplify the analysis, we therefore assume that the size of cash holding by banks is negligibly small, i.e. approaching zero.

Lastly, we assume for simplicity that the value of equipments required in conducting banking service is zero.

$$L_t^i = D_t^i + A_t^i \tag{1.3}$$

The verification cost that bank j has to pay in the event that firm i declares bankruptcy is assumed to equal a proportion 'θ^B' of the realised gross return to firm i's investment ($\theta^B \varpi_{i,t+1} R_{t+1}^K Q_t K_t^i$). In such event, bank j would receive the net liquidation revenue from firm i equivalent to $(1-\theta^B)\varpi_{i,t+1}$ $Q_t R_{t+1}^K K_t^i$. Given this value, *bank j's threshold* value of $\varpi_{i,t+1}$, $\overline{\varpi}_{i,t+1}^B$, is defined such that it satisfies the following equation:

$$(1-\theta^B)\overline{\varpi}_{i,t+1}^B \; Q_t R_{t+1}^K K_t^i = r_{i,t+1}^D D_t^i \tag{1.4}$$

where $r_{i,t+1}^D$ denotes the non-default deposit rate realised in period $t+1$ associated with the deposit contract between bank j and depositor m signed in period t. When $\varpi_{i,t+1} \geq \overline{\varpi}_{i,t+1}^B$, the bank's *net* revenue received from liquidating firm i is sufficient to fulfil the deposit contract. In contrast, when $\varpi_{i,t+1} < \overline{\varpi}_{i,t+1}^B$, the bank declares bankruptcy as its net liquidation revenue is insufficient to repay its debt obligation to the depositor.[18]

1.3.1.3 Depositor (Household)

Depositors invest their savings by depositing their money with banks. Unlike entrepreneurs and banks, *they are neutral to idiosyncratic risk but are averse to aggregate risk.* This implies that aggregate risk inherited in the firms' project has to be completely absorbed by entrepreneurs and banks. As can be seen from equations (1.2) and (1.4), unlike the non-default lending rate which is determined instantaneously once the loan contract is signed, the non-default deposit rate associated with the deposit contract signed in period t will not be realised until period $t+1$. Consequently, as period $t+1$ arrives and aggregate risk associated with period-t capital investment is uncovered, in response to a lower than expected realised return on non-idiosyncratic component of return to firm's i investment in period t ($R_{t+1}^K < E_t(R_{t+1}^K)$), depositor m will be compensated with a higher non-default deposit rate, so they are completely hedged against any plausible realisation of aggregate risk. Crucially, this assumption implies that the adjustment of the lending rate will be relatively stickier in response to a monetary shock compared to that of the deposit rate. As discussed in the Introduction, this proxies realistically the effect of having a *maturity mismatch* in the bank's balance

[18] Similar to $\overline{\varpi}_{i,t+1}^F$, $\overline{\varpi}_{i,t+1}^B$ is realised in period $t+1$, implying that its value is contingent on the *ex-post* realisation of R_{t+1}^K.

sheet.[19]

Because the return on bank j's portfolio is private information, depositor m has to pay a verification cost if he wishes to observe its realisation. The verification cost is assumed to equal a proportion θ^D of the gross liquidated return that can be recovered from his debtor. Thus, in the event that bank j declares bankruptcy, the verification cost that depositor m has to pay amounts to $\theta^D(1-\theta^B)\varpi_{i,t+1}\ Q_t R^K_{t+1} K^i_t$. However, if depositor m lends directly to firm i, the verification cost that he has to pay in the event that firm i declares bankruptcy is $\theta^D \varpi_{i,t+1}\ Q_t R^K_{t+1} K^i_t$. The '*special*' role of bank j as a *delegated verifier* can be summarised by the following assumption:

$$(\theta^D - \theta^B) \int_0^{\overline{\varpi}^F_{i,t}} \varpi_{i,t} f(\varpi_{i,t}) d\varpi_{i,t} > (1-\theta^B)\theta^D \int_0^{\overline{\varpi}^B_{i,t}} \varpi_{i,t} f(\varpi_{i,t}) d\varpi_{i,t}$$

Intuitively, the left hand side (the right hand side) is the expected benefit (cost) from having banks in the economy. The expected benefit arises from the fact that bank j can verify the outcome of the project relatively *cheaper* compared to depositor m, i.e. θ^B is sufficiently lower than θ^D. On the contrary, the expected cost arises from the fact that depositor m has to pay an *extra* cost of monitoring 'the monitor' in certain states of the world. Thus, having bank j as a financial intermediary dominates the one-sided financial contract between firm i and depositor m because the aggregate expected verification cost is lower.

1.3.1.4 The Timeline

To summarise the structure of the financial contract model, its timeline is shown in Figure 1.1.

At the end of period t, entrepreneur i chooses his optimal demand for capital (K^i_t). To partially finance his investment, he engages in a loan contract with bank j and thus borrows L^i_t. The non-default lending rate associated with the loan contract ($r^L_{i,t}$) is *simultaneously* determined. To finance its lending to firm i, bank j also engages in a deposit contract with depositor m from whom it borrows D^i_t.

As time approaches the end of period $t+1$, the non-idiosyncratic component of firm i's return on its period-t investment (R^K_{t+1}) is realised. The deposit rate associated with the deposit contract signed in period t ($r^D_{i,t+1}$) is then realised, which, as emphasised before, implies that the depositor is perfectly hedged against any plausible aggregate risk.

After the aggregate risk (but not idiosyncratic risk) associated with period-t financial contract

[19] As all the financial contracts in the model have only one period maturity, maturity mismatch in banks' balance sheet cannot be *explicitly* modelled. The assumption concerning the risk profile of depositors is a modelling device which is meant to *implicitly* deliver the effect of having maturity mismatch in the model while, at the same time, maintaining the model's tractability.

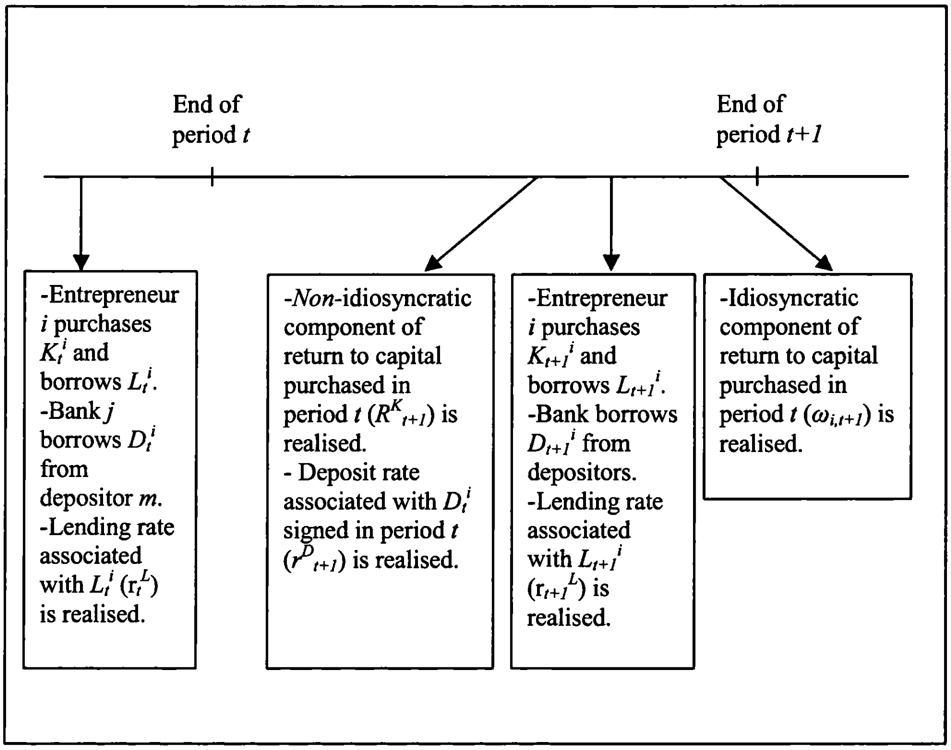

Figure 1.1: The timeline of the model

is uncovered, firm i decides on its optimal purchase of capital and its borrowing from bank j in period $t+1$, K^i_{t+1} and L^i_{t+1} respectively. The corresponding lending rate ($r^L_{i,t+1}$) is simultaneously determined. The bank then borrows D^i_{t+1} from the depositor.

Lastly, the idiosyncratic return to firm i's investment in period t ($\varpi_{i,t+1}$) is realised. In the event that the entrepreneur(the banker) goes bankrupt, he pays whatever is left to his debtor and departs from the scene.

1.3.2 The Contract Term

Given that all firms and banks are subject to limited liability clauses, as shown by Gale and Hellwig (1985), optimal financial contracts with the presence of CSV become those of risky debt contracts. In this section, we study an optimal financial contract amongst firm i, bank j, and depositor m, and derive the firm's optimal demand for capital. We proceed by finding the agents' expected profit functions in the following subsection 1.3.2.1. Subsection 1.3.2.2 then uses these functions to solve for firm i's demand for capital.

1.3.2.1 The Agents' Expected Profit Functions

Firm i's Expected Profit Function: As can be seen from equation (1.2), when $\varpi_{i,t+1} \geq \overline{\varpi}^F_{i,t+1}$, the return to firm i's project is sufficient to repay its debt obligation to bank j, $r^L_{i,t}L^i_t$. So the firm does not default. It pays the contractual amount and retains the remaining profit, $[\varpi_{i,t+1}Q_tR^K_{t+1}K^i_t - r^L_{i,t}L^i_t]$. However, when $\varpi_{i,t+1} < \overline{\varpi}^F_{i,t+1}$, firm i declares a default, liquidates its asset, and retains nothing. Given that firm i's opportunity cost of funds is the real risk free rate (r^f_{t+1}), its expected profit function in period $t+1$ conditional solely on the realisation of idiosyncratic risk ($\pi^F_{i,t+1}$) is given by[20]:

$$\pi^F_{i,t+1} = \int_{\overline{\varpi}^F_{i,t+1}}^{\infty} [\varpi_{i,t+1}Q_tR^K_{t+1}K^i_t - r^L_{i,t}L^i_t]f(\varpi_{i,t+1})d\varpi_{i,t+1} - W^i_t r^f_{t+1} \tag{1.5}$$

Substituting equation (1.2) in equation (1.5), we obtain:

$$\pi^F_{i,t+1} = [\int_{\overline{\varpi}^F_{i,t+1}}^{\infty} \varpi_{i,t+1}f(\varpi_{i,t+1})d\varpi_{i,t+1} - (1 - F(\overline{\varpi}^F_{i,t+1})\overline{\varpi}^F_{i,t+1}]Q_tR^K_{t+1}K^i_t - W^i_t r^f_{t+1} \tag{1.6}$$

Bank j's Expected Profit Function: Bank j's expected profit function, unlike that of firm i, depends in general on the relative values of $\overline{\varpi}^F_{i,t+1}$and $\overline{\varpi}^B_{i,t+1}$. However, under the restriction that $\overline{\varpi}^F_{i,t+1} > \overline{\varpi}^B_{i,t+1}$[21], Appendix A of this chapter shows that the expected profit function for bank j in period $t+1$, conditional on the realisation of idiosyncratic risk, is given by:

$$\begin{aligned}\pi^B_{i,t+1|\overline{\varpi}^F_{i,t+1}>\overline{\varpi}^B_{i,t+1}} &= \int_{\overline{\varpi}^B_{i,t+1}}^{\overline{\varpi}^F_{i,t+1}} [(1-\theta^B)\varpi_{i,t+1}Q_tR^K_{t+1}K^i_t - r^D_{i,t+1}D^i_t]f(\varpi_{i,t+1})d\varpi_{i,t+1} \\ &\quad +[1 - F(\overline{\varpi}^F_{i,t+1})][r^L_{i,t}L^i_t - r^D_{i,t+1}D^i_t] - A^i_t r^f_{t+1}\end{aligned} \tag{1.7}$$

Depositor m's Expected Profit Function: Similar to that of bank j, depositor m's expected profit function depends, in general, on the relative values of the threshold $\overline{\varpi}^F_{i,t+1}$and $\overline{\varpi}^B_{i,t+1}$. However, under the restriction that $\overline{\varpi}^F_{i,t+1} > \overline{\varpi}^B_{i,t+1}$, it can be seen from equations (1.2) and (1.4) that when $\varpi_{i,t+1} < \overline{\varpi}^B_{i,t+1}$, both firm i and bank j declare bankruptcy. So after paying the verification cost, depositor m retains $(1-\theta^D)(1-\theta^B)\varpi_{i,t+1}Q_tR^K_{t+1}K^i_t$. When $\varpi_{i,t+1} \geq \overline{\varpi}^B_{i,t+1}$, bank j does not go bankrupt and therefore does not default on its debt obligation with the depositor. In this case, the depositor gets $r^D_{i,t+1}D^i_t$. The depositor's expected profit function in period $t+1$, conditional on the

[20] In period $t+1$, aggregate risk associated with period-t capital investment has been resolved. Therefore, expectation is taken solely over the remaining idiosyncratic risk.

[21] Appendix A of this chapter shows that this restriction holds under two assumptions, both of which are satisfied under a reasonable parameterisation. Descriptively, this restriction implies that, in the event of no bankruptcy, bank j's total revenue from its lending to firm i has to be sufficiently larger than its total repayment cost to depositor m. In other words, the bank's profit must be sufficiently large in the event that both firm i and bank j do not go bankrupt.

realisation of idiosyncratic risk, is therefore given by:

$$\pi^D_{t+1|\overline{\varpi}^F_{i,t+1}>\overline{\varpi}^B_{i,t+1}} = \int_0^{\overline{\varpi}^B_{i,t+1}} [(1-\theta^D)(1-\theta^B)\varpi_{i,t+1}f(\varpi_{i,t+1})d\varpi_{i,t+1}]R^K_{t+1}Q_tK^i_t + [1-F(\overline{\varpi}^B_{i,t+1})]r^D_{i,t+1}D^i_t - D^i_t r^f_{t+1} \quad (1.8)$$

where the last term represents the depositor's opportunity cost of funds from depositing his money with the bank. For notation simplicity, $\forall j \in \{F, B\}$, we define:

$$\Gamma(\overline{\varpi}^j_{i,t+1}) \equiv \int_0^{\overline{\varpi}^j_{i,t+1}} \varpi_{i,t+1}f(\varpi_{i,t+1})d\varpi_{i,t+1} + [1-F(\overline{\varpi}^j_{i,t+1})]\overline{\varpi}^j_{i,t+1} \quad (1.9)$$

$$G(\overline{\varpi}^j_{i,t+1}) \equiv \int_0^{\overline{\varpi}^j_{i,t+1}} \varpi_{i,t+1}f(\varpi_{i,t+1})d\varpi_{i,t+1} \quad (1.10)$$

Using the notations defined above together with equations (1.2) and (1.4), after some algebraic manipulations, the expected profit functions of firm i (equation 1.6), bank j (equation 1.7) and depositor m (equation 1.8) can be rewritten as:

$$\pi^F_{t+1} = [1-\Gamma(\overline{\varpi}^F_{i,t+1})]R^K_{t+1}Q_tK^i_t - W^i_t r^f_{t+1} \quad (1.11)$$

$$\pi^B_{t+1|\overline{\varpi}^F_{i,t+1}>\overline{\varpi}^B_{i,t+1}} = \left[\Gamma(\overline{\varpi}^F_{i,t+1}) - (1-\theta^B)\Gamma(\overline{\varpi}^B_{i,t+1}) - \theta^B G(\overline{\varpi}^F_{i,t+1})\right] R^K_{t+1}Q_tK^i_t - A^i_t r^f_{t+1} \quad (1.12)$$

$$\pi^D_{t+1|\overline{\varpi}^F_{i,t+1}>\overline{\varpi}^B_{i,t+1}} = (1-\theta^B)\left[\Gamma(\overline{\varpi}^B_{i,t+1}) - \theta^D G(\overline{\varpi}^B_{i,t+1})\right] R^K_{t+1}Q_tK^i_t - D^i_t r^f_{t+1} \quad (1.13)$$

1.3.2.2 Optimal Demand for Capital

Thus far, we have derived the expected profit functions for firm i, bank j and depositor m, *where expectation is conditional solely on idiosyncratic risk*, as given in equations (1.11)-(1.13). This subsection employs these equations to derive firm i's optimal demand for capital, K^i_t.

Given the assumption that depositors are *completely averse to aggregate risk*, depositors' optimisation requires that their expected profit functions *conditional only on idiosyncratic risk* be equal to zero. Thus, from equation (1.13), the optimal zero expected profit condition for depositor m is given by:

$$(1-\theta^B)\left[\Gamma(\overline{\varpi}^B_{i,t+1}) - \theta^D G(\overline{\varpi}^B_{i,t+1})\right] R^K_{t+1}Q_tK^i_t - D^i_t r^f_{t+1} = 0 \quad (1.14)$$

Unlike depositors, banks are risk neutral and therefore are willing to bear both aggregate and idiosyncratic risks. Given that banks operate under a perfectly competitive environment, optimality conditions for banks require that their expected profit functions conditional on *both aggregate and idiosyncratic risks* be equal to zero. Thus, from equation (1.12), the optimal zero profit condition for bank j is given by:

$$E_t\left[\left\{\left[\Gamma(\overline{\varpi}^F_{i,t+1})-(1-\theta^B)\Gamma(\overline{\varpi}^B_{i,t+1})-\theta^B G(\overline{\varpi}^F_{i,t+1})\right]R^K_{t+1}\right\}Q_tK^i_t-A^i_t r^f_{t+1}\right]=0 \tag{1.15}$$

where $E_t(\cdot)$ denotes rational expectation taken as of time t.

It is important to examine equations (1.14) and (1.15) carefully. The assumption that depositors will not bear any aggregate risk implies that as the risk free rate, r^f_{t+1}, rises unexpectedly, ceteris paribus, bank j's threshold $\overline{\varpi}^B_{i,t+1}$ will instantaneously increase via equation (1.14). Consequently, the non-default deposit rate associated with the deposit contract signed in period t, $r^D_{i,t+1}$, has to increase correspondingly via equation (1.4) in order to compensate depositor m for an unexpected rise in his opportunity cost of funds. In contrast, banks are risk neutral and therefore are willing to bear aggregate risk. The non-default lending rate associated with the loan contract signed in period t, $r^L_{i,t}$, will be determined as of period t via equations (1.15) and (1.2). As a result, unlike the deposit rate, the lending rate associated with period-t loan contract has been predetermined as of period $t+1$ and therefore will not respond instantaneously to an unexpected monetary shock. Importantly, the result that the lending rate adjusts to an unexpected rise in the risk free rate relatively slower compared to the deposit rate implies that bank j's inside capital has to deplete as its interest rate cost rises relatively faster compared to its interest rate revenue. This, as we shall see, underpins the operational mechanism of the bank capital channel of the monetary policy transmission in the model.

As firm i is risk neutral, it maximises its expected profit function, where the expectation is conditional on both aggregate and idiosyncratic risks, subject to bank j's balance sheet identity (equation (1.3)) and the zero expected profit conditions of depositor m and bank j (equations (1.14) and (1.15), respectively). The maximisation problem taken as given the values of W^i_t, A^i_t, $E_t(R^K_{t+1})$, $E_t(r^f_{t+1})$ and Q_t[22], can be written as follows:

$$\max_{K^i_t, r^D_{i,t}, r^L_{i,t}} E_t\sum_{j=0}^{\infty}\{[1-\Gamma(\overline{\varpi}^F_{i,t+1+j})]Q_{t+j}R^K_{t+1+j}K^i_{t+j}-W^i_{t+j}r^f_{t+1+j}\}$$

[22]These variables are endogenised in the next section.

subject to

$$(1-\theta^B)\left[\Gamma(\overline{\varpi}^B_{i,t+1})-\theta^D G(\overline{\varpi}^B_{i,t+1})\right]R^K_{t+1}Q_tK^i_t - D^i_t r^f_{t+1} = 0 \tag{a}$$

$$E_t\{\left[\Gamma(\overline{\varpi}^F_{i,t+1})-(1-\theta^B)\Gamma(\overline{\varpi}^B_{i,t+1})-\theta^B G(\overline{\varpi}^F_{i,t+1})\right]R^K_{t+1}\}Q_tK^i_t - A^i_tE_t(r^f_{t+1}) = 0 \tag{b}$$

$$Q_tK^i_t - W^i_t - A^i_t = D^i_t \tag{c}$$

The maximisation problem implies that firm i chooses its optimal demand for capital, non-default lending and deposit rates[23] so that it maximises its expected profits, given that bank j and depositor m are paid just enough to participate in the contracts and that bank j's balance sheet is balanced. Put differently, because loan and deposit markets are competitive, bank j must offer the contracts which maximise the expected profits of firm i and assure depositor m with his expected reservation payoff. Otherwise, other banks would offer an alternative contract.

The solution to the maximisation problem is given in Appendix B of this chapter. Define $k^i_t \equiv \frac{Q_tK^i_t}{(W^i_t+A^i_t)}$, $s_t \equiv E_t\left(\frac{R^K_{t+1}}{r^f_{t+1}}\right)$, $u_{t+1} \equiv \frac{R^K_{t+1}}{E_t(R^K_{t+1})}\frac{E_t(r^f_{t+1})}{r^f_{t+1}}$, where u_{t+1} captures the source of aggregate risk in the model, the first order necessary conditions from the maximisation problem given that $k^i_t > 1$[24] yield the following optimal demand for capital:

$$k^i_t = \Psi_t(s_t, A^i_t/(W^i_t+A^i_t)), \text{ where } \frac{\partial\Psi_t(\cdot)}{\partial s_t} > 0,\ \frac{\partial\Psi_t(\cdot)}{\partial[A^i_t/(W^i_t+A^i_t)]} < 0 \tag{1.16}$$

Equation (1.16) describes the key relationship in the model as it crucially implies that the Modigliani-Miller (1958) theorem does not hold. In particular, it effectively relates the *financial* positions of the agents to the *real* capital investment decision of the firm. The rationale underlying the strictly positive sign of the first derivative is as follows. Other things constant, a higher expected discounted return on capital investment (s_t) decreases the expected default probability of the firm and the bank. This attracts more savings from the depositor and allows the entrepreneur to expand the size of the firm. The second derivative is strictly negative because, other things constant, a higher $A^i_t/(W^i_t+A^i_t)$ implies that the agency problem is relatively more severe at the firm level, as opposed to the bank level. The bank in turn has to impose a higher interest rate margin between

[23]The choice of the non-default lending and deposit rates together with the realisation of aggregate risk then determine the state-contigent values of $\overline{\varpi}^F_{i,t}$ and $\overline{\varpi}^B_{i,t}$.

[24]Otherwise the sum of the firm's and bank's inside capital would be sufficient to finance the firm's investment outlay, in which case the bank does not need to obtain any deposits from the depositor. To illustrate, when $W^i_t/(W^i_t+A^i_t) < k^i_t \leq 1$, entrepreneur i's inside capital is insufficient to finance his investment project. He therefore has to borrow from bank j. However, bank j's inside capital alone is sufficient to finance the demand for loans by entrepreneur i. Thus the source of external finance premium in this case draws solely from financial frictions at the firm level. Essentially, this case is consistent with Bernanke et al. (1999). If $k^i_t < W^i_t/(W^i_t+A^i_t)$, entrepreneur i's inside capital is sufficient to finance his own investment outlay. In this case, there will be no external finance premium, $s_t = 1$. This is the standard case for models in which financial frictions are absent.

the non-default lending and deposit rates. As the firm faces a higher cost of borrowing, its capital demand declines. However, under a reasonable parameterisation, the magnitude of the latter derivative is so small that ignoring the effect of changes in $A_t^i/(W_t^i + A_t^i)$ on the firm's optimal demand for capital does not affect the dynamics of the model.[25] Thus, equation (1.16) can be approximately written as:

$$k_t^i \simeq \psi(s_t) \ , \ \psi'(\cdot) > 0 \tag{1.17}$$

Alternatively equation (1.17) can be rearranged to express the following inverse demand for capital:

$$s_t = \psi^{-1}\left[k_t^i\right] \ , \ \frac{\partial \psi^{-1}(\cdot)}{\partial k_t^i} > 0 \tag{1.18}$$

Equation (1.18) implies that as firm i and bank j increase their leverage, or equivalently their financial positions worsen, the expected discounted return to capital –which can also be interpreted as *the external finance premium*– has to increase. The key to understand this relationship is to recognise the link between the agents' financial positions and the market interest rates which is captured by equations (1.2), (1.4), (1.14) and (1.15). Other things constant, a higher demand for capital relative to the sum of the firm's and bank's inside capital ($\uparrow k_t^i$) implies that depositor m is exposed to a higher agency problem. This implies, via equations (1.4) and (1.14), that a higher non-default deposit rate is required ($\uparrow r_{i,t+1}^D$) in order to induce him to supply more savings. Given rational expectation, bank j anticipates a higher borrowing cost. The non-default loan rate, therefore, has to increase ($\uparrow r_{i,t}^L$), via equations (1.2) and (1.15), in order to satisfy the bank's zero expected profit condition. This directly imposes a greater cost of borrowing on firm i which in turn implies that a higher external finance premium ($\uparrow s_t$) is required. Thus, it is the relationship '$\uparrow k_t^i \Rightarrow \uparrow r_{i,t+1}^D \Rightarrow \uparrow r_{i,t}^L \Rightarrow \uparrow s_t$' that underpins the positive sign of the derivative shown in equation (1.18).

1.3.3 Aggregation

In general, when the demand for capital depends on the financial position of agents, aggregation becomes difficult as it depends on the distribution of wealth among firms (similarly for banks). However, owing to the assumption of constant returns to scale throughout the chapter, a firm's demand for capital is proportional to its net worth with the factor of proportionality being the same

[25]By taking into account the effect of $A_t^i/(W_t^i + A_t^i)$ on k_t in the simulation analysis (not reported), the result in terms of the dynamic responses of the key variables is virtually the same as the case when such effect is ignored.

for all firms (Bernanke et al., 1999). In other words, firms will have the same leverage ratio.

$$\frac{Q_t K_t^i}{W_t^i} = \frac{Q_t K_t^j}{W_t^j} = ... = \frac{Q_t K_t}{W_t} \tag{1.19}$$

where the variables without superscript denote aggregate variables. Similarly, each competitive bank will optimally choose its lending in the same proportion to its inside capital.

$$\frac{L_t^i}{A_t^i} = \frac{L_t^j}{A_t^j} = ... = \frac{L_t}{A_t} \tag{1.20}$$

Given equations (1.19) and (1.20), the ratio $\frac{W_t+A_t}{Q_t K_t}$ is the same across firms.[26] This implies that the aggregation of entrepreneurs' demand for capital, equation (1.17), is straightforward.[27] Thus, the aggregate demand for capital as a positive function of the external finance premium can be given as follows:

$$\frac{Q_t K_t}{W_t + A_t} = \psi(E_t \left[\frac{R_{t+1}^K}{r_{t+1}^f}\right]) \ , \psi'(E_t \left[\frac{R_{t+1}^K}{r_{t+1}^f}\right]) > 0 \tag{1.21}$$

Because the ratios $\frac{W_t^i+A_t^i}{Q_t K_t^i}$ and $\frac{A_t^i}{Q_t K_t^i}$ are the same for all i, the zero expected profit conditions for depositors and banks, given in equation (1.14) and (1.15) respectively, hold in aggregate. As a result, via equations (1.2) and (1.4), the non-default lending rate (non-default deposit rate) charged to different firms (banks) will be the same. The intuition is as follows. Since all firms have the same leverage ratio (equation (1.20)), they possess the same degree of risk *ex ante*. This implies that banks will charge all firms the same non-default lending rate. Similarly, as all banks have the same leverage ratio, depositors are exposed to the same degree of risk *ex ante*. As a compensation, they would thus universally charge the same non-default deposit rate to all banks.

Given the above argument, the *aggregate* zero expected profit conditions for depositors and banks as well as the *economy-wide* non-default lending and deposit rates can be given by:

$$(1-\theta^B)\left[\Gamma(\varpi_{t+1}^B) - \theta^D G(\varpi_{t+1}^B)\right] R_{t+1}^K Q_t K_t - (Q_t K_t - W_t - A_t) r_{t+1}^f = 0 \tag{1.22}$$

$$E_t\{\left[\Gamma(\varpi_{t+1}^F) - (1-\theta^B)\Gamma(\varpi_{t+1}^B) - \theta^B G(\varpi_{t+1}^F)\right] R_{t+1}^K\} Q_t K_t - A_t E_t(r_{t+1}^f) = 0 \tag{1.23}$$

$$\varpi_{t+1}^F Q_t R_{t+1}^K K_t = r_t^L L_t \tag{1.24}$$

$$(1-\theta^B)\varpi_{t+1}^B \ Q_t R_{t+1}^K K_t = r_{t+1}^D D_t \tag{1.25}$$

[26] $\frac{W_t+A_t}{Q_t K_t} = \frac{W_t}{Q_t K_t} + (\frac{A_t}{L_t})(\frac{L_t}{Q_t K_t}) = \frac{W_t}{Q_t K_t} + (\frac{A_t}{L_t})(1 - \frac{W_t}{Q_t K_t})$

[27] Note that the aggregation would remain straightforward for the exact form of firm i's optimal demand for capital (equation 1.16). This is because, given the assumption of constant returns to scale technologies, $\frac{A_t^i}{W_t^i+A_t^i}$ is the same for all i.

In the next section, equations (1.21)-(1.25) will be embedded into the general equilibrium setting. As we shall see, they add the source of financial imperfection both in the loan and deposit markets into the otherwise frictionless dynamic general equilibrium model and therefore underpin the operational mechanism of the balance sheet and bank capital channels of the monetary policy transmission within the model.

1.4 General Equilibrium

In this section, we embed the partial equilibrium analysis developed in section 1.3 into the otherwise standard DNK model. This allows us to endogenise the risk free interest rate, return to capital, price of capital, entrepreneurial net worth and bank capital, all of which were taken as given in the previous section.

As mentioned earlier, there are five types of agents in this economy; entrepreneurs, households, banks, retailers and the Central Bank. Section 1.3 explains the basic set-up of the first three sectors, addressing only the issues relevant to the financial contract problem. This section completes the task by explaining the remaining, e.g. the production function and the household sector's optimal consumption choice, and illustrating the basic set-up of the retailer and the Central Bank. However, as only entrepreneurial and banking sectors are non-standard in the DNK literature, the emphasis will be placed on them.

1.4.1 Entrepreneurial sector

In order to motivate the coexistence between aggregate and idiosyncratic risks, as discussed in section 1.3, entrepreneurs cannot instantaneously use their purchased capital to produce wholesale goods. Thus, capital purchased in period $t-1$ will be used in production, together with labour hired in period t, to produce wholesale output in period t. Assuming a constant returns to scale production technology, the aggregate production function is given by:

$$Y_t = T_t K_{t-1}^{\alpha} H_t^{1-\alpha} \tag{1.26}$$

T_t is an exogenous technology parameter. K_{t-1} is the aggregate amount of capital purchased in period $t-1$. H_t is labour input hired in period t. Y_t is the aggregate wholesale output.[28]

[28]Notice here that idiosyncratic risk is completely diversified in aggregate. This stems from the assumption that $\varpi_{i,t+1}$ is identically and independently distributed across firms and time.

Entrepreneurs are assumed to sell their wholesale output only to retailers. Let $\frac{1}{X_t}$ be the relative price of wholesale goods. Equivalently, X_t is the gross mark-up of retail goods over wholesale goods. The gross return from holding capital from period $t-1$ to t is given by:

$$R_t^K = \left[\frac{\frac{1}{X_t}\frac{\alpha Y_t}{K_{t-1}} + (1-\varphi)Q_t}{Q_{t-1}} \right] \tag{1.27}$$

where $\frac{1}{X_t}\frac{\alpha Y_t}{K_{t-1}}$ is the rental payment paid to capital as implied by the Cobb-Douglas production technology, φ is the capital depreciation rate, Q_t is the price of capital in period t. In sum, the non-idiosyncratic component of return from holding capital from period $t-1$ to t, R_t^K, is the sum of rental revenue and capital gain netting off depreciated capital stock.

Equation (1.27) represents the standard downward sloping demand for capital. In particular, owing to diminishing returns, the return to capital falls as more capital is demanded. Essentially, this equation has endogenised the non-idiosyncratic component of return to capital, R_t^K, which was taken as given in the previous section.

In contrast to the conventional literature, entrepreneurs in this economy do not have access to unlimited amount of funds at the sole opportunity cost equivalent to the risk free rate. Rather, equation (1.21) which expresses entrepreneurs' aggregate demand for capital as a function of the external finance premium captures the source of financial imperfection in the model. In particular, firms as well as banks can acquire more external funds only at a higher external finance premium. Put differently, given their inside capital, they face an upward sloping cost of external finance.

We now turn to find the evolution of capital. We follow the standard literature by assuming the following equation:[29]

$$K_t = \Omega(\frac{I_t}{K_{t-1}})K_{t-1} + (1-\varphi)K_{t-1} \tag{1.28}$$

where I_t denotes aggregate investment expenditures and φ denotes the depreciation rate.

Following the standard literature, we assume that there are increasing marginal adjustment costs in the production of capital, captured by assuming that aggregate investment expenditures (I_t) yields a gross output of new capital goods $\Omega(\frac{I_t}{K_{t-1}})K_{t-1}$, where $\Omega(\cdot)$ is increasing and concave and $\Omega(0) = 0$. The introduction of the adjustment cost is made in order to permit a variable price of capital (a variable asset price) which in turn will further enrich the model dynamics through the asset price channel. In equilibrium, given the adjustment cost function, the price of a unit of capital

[29]See, among others, King and Wolman (1996), Gertler (2000) and Bernanke et al. (1999) for detail.

in terms of the numeraire goods, Q_t, is given by:[30]

$$Q_t = \left[\Omega'(\frac{I_t}{K_{t-1}})\right]^{-1} \tag{1.29}$$

Next, we proceed to endogenise aggregate entrepreneurial net worth. As a technical matter, we are required to start entrepreneurs off with some net worth in order to allow them to begin operation. For simplicity, we assume that, in each period, each entrepreneur is endowed with a small endowment, e^F. Moreover, in order to prevent entrepreneurs from accumulating sufficient wealth to become self financed, we assume that each entrepreneur faces a constant probability of dying equal to γ^E. The dying entrepreneurs simply consume their remaining net worth (C_t^E) and depart from the scene. We also assume that a new generation of entrepreneurs is born, where the birth rate is such that the population of entrepreneurs remains constant over time.

We can write the evolution of aggregate entrepreneurial net worth and dying entrepreneurs' consumption as follows:

$$W_t = (1-\gamma^E)[V_t^E + e^F] \tag{1.30}$$

$$C_t^E = \gamma^E[V_t^E + e^F] = (\frac{\gamma^E}{1-\gamma^E})W_t \tag{1.31}$$

where W_t is the expected aggregate entrepreneurial net worth available in period t right before period-t capital decision is made. V_t^E is the expected entrepreneurial gross return from investing in capital. The expectation is taken solely on the unrealised idiosyncratic risk associated with the last period capital investment since the corresponding aggregate risk component has been resolved. From equation (1.14), V_t^E can be written as:

$$V_t^E = [1 - \Gamma(\overline{\omega}_t^F)]Q_{t-1}R_t^K K_{t-1} \tag{1.32}$$

Substituting equation (1.32) into equation (1.30), we obtain the following evolution of entrepreneurial net worth equation:

$$W_t = (1-\gamma^E)\{[1-\Gamma(\overline{\omega}_t^F)]Q_{t-1}R_t^K K_{t-1} + e^F\} \tag{1.33}$$

[30] Following Gertler (2000), there are capital-producing firms which use final goods (I_t) together with rented capital to produce new capital goods via the production function $\Omega(\frac{I_t}{K_{t-1}})K_{t-1}$. They then sell the newly produced capital to wholesale good producers at the price Q_t. Capital good firms therefore maximise their profit, $Q_t\Omega(\frac{I_t}{K_{t-1}})K_{t-1} - I_t - Z^k K_{t-1}$, where Z^k is the rental cost. FOC with respect to I_t is given by equation (1.29). Gertler (2000) shows that, via FOC with respect to K_{t-1}, the value of Z^k, and therefore the capital goods firms' profit will be approximately zero in the neighbourhood of the steady state.

Lastly, analogous to capital, the return to labour (real wage) is equal to the marginal product of labour. This demand for labour condition is given by the following equation:

$$\frac{N_t}{P_t} = \left[\frac{1}{X_t}\frac{(1-\alpha)Y_t}{H_t}\right] \tag{1.34}$$

where $\frac{N_t}{P_t}$ denotes real wage in period t.

1.4.2 Banking Sector

In section 1.3, we have established equilibrium conditions for most of the key variables which are relevant to the banking sector treating as given only bank capital. In this section, we endogenise this variable.

We assume that the cost of raising bank capital directly is prohibitively costly. Thus bank capital can only be accumulated via retained earnings. Similar to the evolution of entrepreneurial net worth, we need to start off banks with some endowment so that they can begin their operation. We therefore assume that each bank is given a small endowment equal to e^B. Moreover, to prevent banks from accumulating sufficient capital to become self financed, we assume that they face a constant probability of dying, γ^B. The dying banks simply consume all of their remaining capital and depart from the scene. At the same time, new banks are established. Their birth rate is such that the number of banks in the economy remains constant over time.

We can write the evolution of aggregate bank capital and banks' consumption, respectively, as follows:

$$A_t = (1-\gamma^B)[V_t^B + e^B] \tag{1.35}$$

$$C_t^B = \gamma^B[V_t^B + e^B] = \left(\frac{\gamma^B}{1-\gamma^B}\right) A_t \tag{1.36}$$

where A_t is expected aggregate bank capital in period t. V_t^B is expected banks' gross profits excluding their opportunity cost. Again, the expectation is taken conditional solely on the unrealised idiosyncratic risk associated with the capital investment in the previous period. From equation (1.23), V_t^B is given as follows:

$$V_t^B = \left[\Gamma(\overline{\varpi}_t^F) - (1-\theta^B)\Gamma(\overline{\varpi}_t^B) - \theta^B G(\overline{\varpi}_t^F)\right] R_t^K Q_{t-1} K_{t-1} \tag{1.37}$$

Substituting equation (1.37) into equation (1.35), we obtain the following evolution of bank capital equation:

$$A_t = (1-\gamma^B)[\Big[\Gamma(\overline{\varpi}_t^F) - (1-\theta^B)\Gamma(\overline{\varpi}_t^B) - \theta^B G(\overline{\varpi}_t^F)\Big] R_t^K Q_{t-1} K_{t-1} + e^B] \tag{1.38}$$

1.4.3 Retail sector

This sector is introduced solely as a means to present some form of nominal price stickiness into the model without complicating the aggregation process.[31] This section follows Appendix B of Bernanke et al. (1999).

Monopolistic competition is assumed at the retail level. Retailers purchase wholesale output from entrepreneurs, slightly modify and resell them in the form of CES aggregate to households. Let $Y_t(z)$ be the quantity of output sold by retailer z, measured in units of wholesale goods, and let $P_t(z)$ be the nominal price of retail goods z. Total final usable goods, Y_t^f, is the following composite of individual retail goods:

$$Y_t^f = \left[\int_0^1 Y_t(z)^{(\epsilon-1)/\epsilon} dz\right]^{\frac{\epsilon}{\epsilon-1}} \tag{1.39}$$

with the elasticity of substitution $\epsilon > 1$. The corresponding price index is given by:

$$P_t = \left[\int_0^1 P_t(z)^{(1-\epsilon)} dz\right]^{\frac{1}{1-\epsilon}} \tag{1.40}$$

Final output may either be transformed into a single type of consumption goods, invested, or used up in verifying costs.[32] In particular, the economy wide resource constraint is given by:

$$\begin{aligned} Y_t^f &= C_t + C_t^E + C_t^B + I_t \\ &+ \left[\theta^B \int_0^{\overline{\varpi}_t^F} \varpi_t f(\varpi_t) d\varpi_t + (1-\theta^B)\theta^D \int_0^{\overline{\varpi}_t^B} \varpi_t f(\varpi_t) d\varpi_t\right] Q_{t-1} R_t^K K_{t-1} \end{aligned} \tag{1.41}$$

where C_t^E is dying entrepreneurs' consumption, C_t^B is dying bankers' consumption, $\left[\theta^B \int_0^{\overline{\varpi}_t^F} \varpi_t f(\varpi_t) d\varpi_t + (1-\theta^B)\theta^D \int_0^{\overline{\varpi}_t^B} \varpi_t f(\varpi_t) d\varpi_t\right] Q_{t-1} R_t^K K_{t-1}$ is aggregate resource used up as the verification cost.

Given the index that aggregates individual retail goods into final goods, equation (1.39), the

[31] Had the retailers not been introduced, entrepreneurs would have to be price setters themselves, thereby having to face a downward sloping demand curve. This would have added non-linearities in entrepreneurs' demand for capital as a function of entrepreneurial net worth and bank capital. Consequently, the aggregation process would have been complicated.

[32] In general, aggregate final output, Y_t^f, differs from aggregate wholesale output, Y_t. However, as shown by Gertler (2000), they are approximately the same in the neighbourhood of the steady state. Hence, in the simulation analysis they will be treated as the same.

demand curve faced by each retailer is given by:

$$Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\epsilon} Y_t^f \tag{1.42}$$

To introduce price stickiness in the model, we assume that retailers can adjust their selling prices only with probability $1-\rho$ in a given period (Calvo, 1983). Let P_t^* denote the price set by retailers who are able to change prices at t, and let $Y_t^*(z)$ denote the demand corresponding to this price. Retailer z chooses his price, $P_t(z)$, to maximise the expected discounted profit taken as given the demand curve and the price of wholesale goods, P_t^w. The retailers' expected discounted profit is given by:

$$\sum_{k=0}^{\infty}\rho^k E_{t-1}\left[\Lambda_{t,k}\frac{P_t^* - P_{t+k}^w}{P_{t+k}}Y_{t+k}^*(z)\right] \tag{1.43}$$

where the discount rate $\Lambda_{t,k} \equiv \varkappa\frac{C_t}{C_{t+k}}$ is households' intertemporal marginal rate of substitution[33] and $P_t^w \equiv \frac{P_t}{X_t}$ is the nominal price of wholesale goods.

The optimal price setting is obtained by differentiating the objective function, equation (1.43), with respect to P_t^*. This implies that the optimally-set price satisfies the following equation:

$$\sum_{k=0}^{\infty}\rho^k E_{t-1}\left[\Lambda_{t,k}(\frac{P_t^*}{P_{t+k}})^{-\epsilon}Y_{t+k}^*(z)\left(P_t^* - \frac{\epsilon}{(\epsilon-1)}P_{t+k}^w\right)\right] = 0 \tag{1.44}$$

Intuitively, retailers set their prices so that the expected discounted marginal revenue equals the expected discounted marginal cost, given the constraint that the nominal price is fixed in period k with probability ρ^k. Given that the fraction ρ of retailers do not change their price in period t, the aggregate price evolves according to the following equation:

$$P_t = \left[\rho P_{t-1}^{1-\epsilon} + (1-\rho)\left(P_t^*\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{1.45}$$

where P_t^* satisfies equation (1.44).

Equations (1.44) and (1.45) form the evolution of aggregate price.

1.4.4 Household sector

Households in this model are standard. They consume CES aggregate of retail goods, save, and supply their labour. They save by depositing their money with banks, given that the opportunity cost of funds is equal to the risk free rate. In addition, because retailers are monopolistic competitors,

[33] As will be seen in the next section, we assume that households own retail firms.

they will earn positive profit in equilibrium. We assume that such profit is transferred to households. Put differently, we assume that households own retail firms. A representative household's problem is given by:

$$\max_{C_t,H_t,D_t} E_t\{\sum_{j=0}^{\infty}\varkappa^j\left[\ln(C_{t+j})+\varrho\ln\left(1-H_{t+j}\right)\right]\}$$

subject to

$$C_{t+1}=\frac{N_{t+1}}{P_{t+1}}H_{t+1}+R^D_{t+1}D_t-D_{t+1}+\Pi_{t+1}$$

where $\varkappa$ is households' coefficient of relative impatience, C_t households' consumption, D_t is interest-rate-earning deposits (in real term) held at the bank in period t, $\frac{N_t}{P_t}$ is real wage, H_t is household labour, Π_t is dividends received from owning retail firms. R^D_{t+1} is the actual rate of return on depositing money with banks which will not be realised until period $t+1$.[34]

Although depositors are perfectly hedged against any realisation of aggregate risk, they are still exposed to idiosyncratic risk. Importantly, as of period t, the expectation of the actual rate of return on deposits ($E_t(R^D_{t+1})$) conditioning on the realisation of aggregate and idiosyncratic risks must be equal to the expected real risk free rate, $E_t(r^f_{t+1})$, in order to satisfy the depositors' zero expected profit function as implied by equation (1.22). As a result, the solution to the above optimisation problem yields the following two standard first order conditions:

$$\frac{1}{C_t} = E_t\left[\varkappa\frac{r^f_{t+1}}{C_{t+1}}\right] \quad (1.46)$$

$$\frac{N_t}{P_t}\frac{1}{C_t} = \varrho\frac{1}{1-H_t} \quad (1.47)$$

Equation (1.46) is a standard inter-temporal consumption Euler equation. Equation (1.47) is a standard intra-temporal Euler equation between households' consumption and their labour supply.

[34]Note here that the budget constraint is evaluated at the end of period $t+1$. Thus real consumption, and new deposit contract are financed by realised return on deposit invested last period (period t), real labour income, and profit redistributed from retailers.

1.4.5 The Central Bank

The Central Bank sets the nominal risk free rate via a variant Taylor-type rule.[35] Define the nominal gross risk free rate, r_t^{nf}, as follows:

$$r_t^{nf} \equiv r_t^f E_t(\frac{P_{t+1}}{P_t}) \tag{1.48}$$

A form of Taylor's rule is given by:

$$r_t^{nf} = f(r_{t-1}^{nf}, \frac{P_t}{P_{t-1}}, ...) \tag{1.49}$$

1.5 Equilibrium and The Completely Log-linearised Version of the Model

Equilibrium is defined as an allocation $\{Y_t, C_t, C_t^E, C_t^B, I_t, K_t, W_t, A_t, H_t\}_{t=0}^{\infty}$ together with a vector of price variables $\{\overline{\omega}_t^F, \overline{\omega}_t^B, R_t^K, Q_t, r_t^f, r_t^{nf}, r_t^L, r_t^D, X_t, P_t, P_t^*, N_t\}_{t=0}^{\infty}$ satisfying equations (1.21)-(1.25), (1.26)-(1.29), (1.31), (1.33)-(1.34), (1.36), (1.38), (1.41), (1.44)-(1.49), given a sequence of the initial values of a vector of the model's state variables $\{Q_{-1}, K_{-1}, W_{-1}, A_{-1}, r_{-1}^L, r_{-1}^{nf}, P_{-1}\}$, a sequence of a vector of exogenous process $\{T_t\}_{t=0}^{\infty}$ and a sequence of interest rate shock $\{\varepsilon_t^r\}_{t=0}^{\infty}$.

In order to study the dynamic response of the model to a monetary shock, we log-linearise the model around a unique stationary steady state equilibrium. Define $\pi_t \equiv P_t - P_{t-1}$ and let the variables with a tilda ($\tilde{}$) denote percentage deviations from the steady state and those without a time subscript denote the steady state values, the completely log-linearised version of the model around the steady state can be given by the following 20 equations in 20 variables. They are divided into 5 blocks of equations: 1) aggregate demand; 2) aggregate supply; 3) financial markets; 4) evolution of state variables; and 5) monetary policy rule and exogenous process.

[35] The Central Bank has power to set the nominal risk free rate due to the assumption that banks have to hold cash, though the amount is assumed to be approaching zero (see footnote 17).

1) Aggregate demand

$$\widetilde{Y}_t = \frac{C}{Y}\widetilde{C}_t + \frac{C^E}{Y}\widetilde{C}^E_t + \frac{C^B}{Y}\widetilde{C}^B_t + \frac{I}{Y}\widetilde{I}_t + a_1[\widetilde{Q}_{t-1} + \widetilde{K}_{t-1} + \widetilde{R}^K_t] + a_2\widetilde{\overline{\varpi}}^F_t + a_3\widetilde{\overline{\varpi}}^B_t \tag{L1}$$

$$E_t(\widetilde{C}_{t+1}) = \widetilde{C}_t + E_t(\widetilde{r}^f_{t+1}) \tag{L2}$$

$$\widetilde{C}^E_t = \widetilde{W}_t \tag{L3}$$

$$\widetilde{C}^B_t = \widetilde{A}_t \tag{L4}$$

$$\widetilde{R}^K_t + \widetilde{Q}_{t-1} = (1-b_1)[\widetilde{Y}_t - \widetilde{X}_t - \widetilde{K}_{t-1}] + b_1\widetilde{Q}_t \tag{L5}$$

$$\xi[E_t(\widetilde{R}^K_{t+1}) - E_t(\widetilde{r}^f_{t+1})] = \{\widetilde{Q}_t + \widetilde{K}_t - \left(\frac{W}{W+A}\right)\widetilde{W}_t - \left(\frac{A}{W+A}\right)\widetilde{A}_t\} \tag{L6}$$

$$\widetilde{Q}_t = \epsilon^Q[\widetilde{I}_t - \widetilde{K}_{t-1}] \tag{L7}$$

2) Aggregate Supply

$$\widetilde{Y}_t = \widetilde{T}_t + \alpha\widetilde{K}_{t-1} + (1-\alpha)\widetilde{H}_t \tag{L8}$$

$$\widetilde{Y}_t = \widetilde{C}_t + \widetilde{X}_t + (1 + \frac{1}{\epsilon^H})\widetilde{H}_t \tag{L9}$$

$$\widetilde{\pi}_t = \varkappa E_t(\widetilde{\pi}_{t+1}) - u\widetilde{X}_t \tag{L10}$$

3) Financial Markets

$$(\frac{K}{W+A} - 1)[\widetilde{r}^f_t - \widetilde{R}^K_t] + \widetilde{Q}_{t-1} + \widetilde{K}_{t-1} - \frac{W}{W+A}\widetilde{W}_{t-1} - \frac{A}{W+A}\widetilde{A}_{t-1} - l_1\widetilde{\overline{\varpi}}^B_t = 0 \tag{L11}$$

$$(\frac{K}{W+A} - 1)[\widetilde{R}^K_t + \widetilde{\overline{\varpi}}^B_t - \widetilde{r}^D_t] - (\widetilde{Q}_{t-1} + \widetilde{K}_{t-1}) + \frac{W}{W+A}\widetilde{W}_{t-1} + \frac{A}{W+A}\widetilde{A}_{t-1} = 0 \tag{L12}$$

$$j_1 E_t(\widetilde{\overline{\varpi}}^F_{t+1}) - j_2 E_t(\widetilde{\overline{\varpi}}^B_{t+1}) + E_t(\widetilde{R}^K_{t+1}) + \widetilde{Q}_t + \widetilde{K}_t - \widetilde{A}_t - E_t(\widetilde{r}^f_{t+1}) = 0 \tag{L13}$$

$$\widetilde{\overline{\varpi}}^F_t - (\frac{1}{\frac{K}{W} - 1})[\widetilde{Q}_{t-1} + \widetilde{K}_{t-1}] + \widetilde{R}^K_t = \widetilde{r}^L_{t-1} - (\frac{1}{\frac{K}{W} - 1})\widetilde{W}_{t-1} \tag{L14}$$

4) Evolution of state variables

$$\widetilde{K}_t = \varphi\widetilde{I}_t + (1-\varphi)\widetilde{K}_{t-1} \tag{L15}$$

$$\widetilde{W}_t = c_1(\widetilde{Q}_{t-1} + \widetilde{K}_{t-1} + \widetilde{R}^K_t) + c_2\widetilde{\overline{\varpi}}^F_t \tag{L16}$$

$$\widetilde{A}_t = d_1(\widetilde{Q}_{t-1} + \widetilde{K}_{t-1} + \widetilde{R}^K_t) + d_2\widetilde{\overline{\varpi}}^F_t + d_3\widetilde{\overline{\varpi}}^B_t \tag{L17}$$

5) Monetary policy rule and exogenous processes

$$\widetilde{r}_t^f = \widetilde{r}_t^{nf} - E_t(\widetilde{\pi}_{t+1}) \tag{L18}$$

$$\widetilde{r}_t^{nf} = g_1 \widetilde{r}_{t-1}^{nf} + g_2 \widetilde{\pi}_{t-1} + \varepsilon_t^r \tag{L19}$$

$$\widetilde{T}_t = v_2 \widetilde{T}_{t-1} \tag{L20}$$

where,

$a_1 \equiv [\theta^B G(\overline{\varpi}^F) + (1-\theta^B)\theta^D G(\overline{\varpi}^B)] R^K \frac{K}{Y}$

$a_2 \equiv \theta^B \overline{\varpi}^F G'(\overline{\varpi}^F) R^K \frac{K}{Y}$

$a_3 \equiv (1-\theta^B)\theta^D \overline{\varpi}^B G'(\overline{\varpi}^B) R^K \frac{K}{Y}$

$b_1 \equiv \frac{(1-\varphi)}{\frac{\alpha}{X}\frac{Y}{K}+(1-\varphi)}$

$c_1 \equiv (1-\gamma^E) R^K \frac{K}{W}[1-\Gamma(\overline{\varpi}^F)]$

$c_2 \equiv -(1-\gamma^E) R^K \frac{K}{W} \overline{\varpi}^F \Gamma'(\overline{\varpi}^F)$

$d_1 \equiv (1-\gamma^B) R^K \frac{K}{A}[\Gamma(\overline{\varpi}^F)$
$-(1-\theta^B)\Gamma(\overline{\varpi}^B) - \theta^B G(\overline{\varpi}^F)]$

$d_2 \equiv (1-\gamma^B) R^K \frac{K}{A} \overline{\varpi}^F [\Gamma'(\overline{\varpi}^F) - \theta^B G'(\overline{\varpi}^F)]$

$d_3 \equiv -(1-\gamma^B)(1-\theta^B) R^K \frac{K}{A} \overline{\varpi}^B \Gamma'(\overline{\varpi}^B)$

$\xi \equiv \frac{\psi'(\frac{R^K}{r^f})}{\psi(\frac{R^K}{r^f})} \frac{R^K}{r^f}$

$u \equiv \frac{(1-\rho)}{\rho}(1-\varkappa\rho)$

$\epsilon^Q \equiv \frac{\partial Q}{\partial \frac{I}{K}} \frac{\frac{I}{K}}{Q}$

$\epsilon^H \equiv \frac{\partial H}{\partial \frac{N}{P}} \frac{\frac{N}{P}}{H}$

$j_1 \equiv \left[\frac{\Gamma'(\overline{\varpi}^F) - \theta^B G'(\overline{\varpi}^F)}{\Gamma(\overline{\varpi}^F) - (1-\theta^B)\Gamma(\overline{\varpi}^B) - \theta^B G(\overline{\varpi}^F)}\right] \overline{\varpi}^F$

$j_2 \equiv \left[\frac{(1-\theta^B)\Gamma'(\overline{\varpi}^B)}{\Gamma(\overline{\varpi}^F) - (1-\theta^B)\Gamma(\overline{\varpi}^B) - \theta^B G(\overline{\varpi}^F)}\right] \overline{\varpi}^B$

$l_1 \equiv \left[\frac{\Gamma'(\overline{\varpi}^B) - \theta^D G'(\overline{\varpi}^B)}{\Gamma(\overline{\varpi}^B) - \theta^D G(\overline{\varpi}^B)}\right] \left(\frac{K}{W+A} - 1\right) \overline{\varpi}^B$

Equation (L1) is the log-linearised version of equation (1.41), the economy wide resource constraint. The variation in aggregate output depends on the variation in consumption, investment, dying entrepreneur's and dying banker's consumption and the aggregate expected verification cost.[36] Equation (L2) is the log-linearised version of equation (1.46), a standard forward-looking consumption Euler equation. Equations (L3) and (L4) are the log-linearised version of equations (1.31) and (1.36), respectively. They imply that the variation in (dying) entrepreneur's and (dying) banker's consumption depends on the variation in the respective values of their inside capital.

Equations (L5)-(L7) characterise investment demand. They are the log-linearised version of equations (1.27), (1.21) and (1.29), respectively. Equations (L5) and (L7) are conventional in the DNK literature. While the former implies a standard downward sloping demand for capital, the latter relates investment demand to the price of capital. Equation (L6) is unconventional in the frictionless monetary model. It implies that the variation in the external finance premium increases as the variation in the aggregate demand for capital is higher compared to that of aggregate sum of entrepreneurial net worth and bank capital.

Equations (L8)-(L10), all of which are standard in the DNK literature, represent the aggre-

[36] However, under a reasonable parameterisation, the weight given to the variation in dying entrepreneur's consumption, dying banker's consumption and the verification cost will be very small.

gate supply block of the model. Equation (L8) is the log-linearised version of equation (1.26), the production function. Equation (L9) characterises the labour market equilibrium. It is obtained by equating the log-linearised aggregate labour demand (equation (1.34)) to the log-linearised aggregate labour supply (equation (1.47)). Equation (L10) combines the log-linearised version of equations (1.44) and (1.45). It captures the source of price stickiness in the model and therefore underpins the effectiveness of monetary policy in affecting real variables.

Equations (L11)-(L14) constitute the equilibrium in the financial markets. Equations (L11) and (L12) are the log-linearised version of the aggregate depositors' zero profit condition (equation (1.22)) and the equation which characterises the equilibrium banks' threshold value of idiosyncratic component ($\tilde{\bar{\varpi}}_t^B$) (equation (1.25)), respectively. Equation (L11) implies that, ceteris paribus, an increase in the variation of aggregate capital demand relative to the sum of the aggregate entrepreneurial net worth and bank capital in the previous period will result in a higher variation of the current value of $\tilde{\bar{\varpi}}_t^B$. This implies, via equation (L12), that the variation in the non-default deposit rate in this period has to rise. Thus, equations (L11) and (L12) together imply that the variation in deposit rate will respond positively to an increase in the variation of the leverage ratio of firms and banks.

Equations (L13) and (L14) are the log-linearised version of the aggregate banks' zero profit condition (equations (1.23)), and the equation which characterises the equilibrium firms' threshold value $\tilde{\bar{\varpi}}_t^F$ (equation (1.24)), respectively. They together imply that, ceteris paribus, the variation in the non-default lending rate is a positive function of the variation in the non-default deposit rate. This is because a higher deposit rate imposes a greater borrowing cost on banks. In order to maintain their optimal zero expected profit condition, they must increase their lending rate correspondingly. However, it is very crucial to notice from these two equations that the response of non-default lending rate to an *unanticipated* increase in the deposit rate will be subject to a one period lag. This implies that the variation in the non-default lending rate will be relatively stickier compared to that of the non-default deposit rate in response to any unanticipated aggregate shock.

Equation (L15) is the log-linearised version of equation (1.28), the standard evolution of capital equation. Equations (L16) and (L17) are the log linearised version of equations (1.33) and (1.38), respectively. They are the transition equations for the aggregate entrepreneurial net worth and bank capital. Equation (L16) implies that the variation in the aggregate entrepreneurial net worth depends positively on the variation in the last-period price of capital, the return to capital, the demand for capital and negatively on the firms' threshold value $\tilde{\bar{\varpi}}_t^F$. Equation (L17) implies that the aggregate bank capital is a positive function of the last period price of capital, return to capital, demand for capital, the firms' threshold value $\tilde{\bar{\varpi}}_t^F$ and is a negative function of the banks' threshold value $\tilde{\varpi}_t^B$.

It should now become clear how adding financial imperfection at the firm and bank levels works to enrich the dynamic of the model. Equations (L11)-(L14), which represent the financial market block of the model, effectively link entrepreneurial net worth and bank capital to the equilibrium non-default lending and deposit rates. These links underpin the mechanism in which the *financial* position of firms and banks works to augment the *real* investment decision of firms. This mechanism, which is completely absent in frictionless models, is captured by equation (L6).[37] Equations (L16) and (L17) then characterise the evolution of firm's and bank's financial positions, i.e. entrepreneurial net worth and bank capital, respectively.

Equations (L18) and (L19) are the log linearised version of equations (1.48) and (1.49), respectively. The former relates the nominal risk free rate to the real risk free rate whereas the latter is a variant Taylor-type interest rate rule. Following Bernanke et al. (1999), we consider the rule in which the Central Bank sets the current nominal risk-free interest rate as a function of lagged inflation and lagged nominal interest rate. Essentially this implies that the Central Bank puts zero weight on the output stabilisation objective.[38] This is intended to highlight the financial accelerator effect. As Bernanke et al. (1999) emphasised, the greater the extent to which monetary policy can stabilise output, the smaller is the role of any kind of propagation mechanism in amplifying and propagating business cycles.

Lastly, equation (L20) characterises the exogenous process of the variation in technology. As the main focus of this chapter is on the transmission mechanism of monetary policy, we will not analyse the dynamic responses of the model to a technology shock.

1.6 Model Simulation

1.6.1 Calibration

The model is calibrated at a quarterly frequency. The values assigned to most of the parameters relevant to preference, technology and price stickiness are standard in the DNK literature. The

[37] A rise in capital investment demand compared to the sum of aggregate entrepreneurial net worth and bank capital, i.e. both firms and banks become more leveraging, implies, via equations (L11) and (L12), that the non-default deposit rate has to be higher in order to induce depositors to be willing to supply more of their savings. A higher non-default deposit rate implies a rise in the cost of borrowing for banks. In order to maintain their zero expected profit condition, they in turn have to raise their non-default lending rate (via equations (L13) and (L14)). Given a higher opportunity cost of external funds, i.e. a higher lending rate, firms have to require higher return to capital in order to justify their investment. Thus the external finance premium $(E_t(\tilde{R}_{t+1}^K) - E_t(\tilde{r}_{t+1}^f))$ rises as firms and banks become more leveraging. This is captured by equation (L6).

[38] In other words, this implies that the Central Bank adheres to a strict form of inflation targeting. (Svensson, 1995)

discount factor, $\varkappa$, is set to be 0.99, which implies an annualised real interest rate of about 4 percent. The depreciation rate, φ, is set to 2.5 percent. We select the steady state capital share, α, to be 0.35. We choose the labour supply elasticity, ϵ^H, to be 3 and, following Bernanke et al. (1999), the elasticity of the price of capital with respect to the investment capital ratio, ϵ^Q , to be 0.25. The elasticity of substitution, ϵ, is set so as the steady state mark-up price X is equal to 1.05. The probability that a retail firm does not change its price in a given period, ρ, is chosen to be 0.75, implying an average price duration of one year. The autoregressive parameters in the policy rule, g_1 and g_2, are set to 0.9 and 0.11, respectively.

The unconventional choices of parameterisation are those relevant to the financial contract problem. The endowment given to each firm and bank as a proportion of their inside capital, $\frac{e^E}{W}$ and $\frac{e^B}{A}$, is arbitrarily set to be 0.01. Its magnitude is meant to be so small that it does not affect the model's dynamics. We then treat the proportional factors of the verification cost paid by a bank (θ^B), and a depositor (θ^D), the death rate for a firm (γ^E) and bank (γ^B), and the standard deviation of $\ln \varpi$ (σ) as unobservable and choose their values to match the following steady state outcomes: [1] an annualised risk spread, $R^k - r^f$, of 200 basis points, approximately the historical average spread between the prime lending rate and the six-month Treasury bill rate (Bernanke et al., 1999, and Carlstrom and Fuerst, 2001); [2] an annualised business failure rate, $F(\overline{\varpi}^F)$, of 3 percent, the approximate value in the data (Bernanke et al., 1999); [3] an annualised bank failure rate, $F(\overline{\varpi}^B)$, of 0.5 percent, the average value for commercial banks insured by Federal Deposit Insurance Corporation (FDIC) during 1988-2002[39]; [4] a leverage ratio of entrepreneurial net worth to capital ($\frac{W}{K}$) of 0.4, the approximate value found in the empirical literature for small firms in the U.S.[40]; [5] a bank's capital to asset ratio, $\frac{A}{L}$, of 0.12, the average value of $(\frac{\text{tier1 capital+tier2 capital}}{\text{risk-weighted asset}})$ for U.S. commercial banks during 1990-1999 [41]. To satisfy [1]-[5], the values of $\theta^B, \theta^D, \gamma^E, \gamma^B$ and σ are 0.01, 0.4, 0.03, 0.02, and 0.2, respectively.

1.6.2 The Transmission Mechanism of Monetary Policy: The Role of Bank Capital

Figure 1.2 shows the response of the model to an unanticipated rise in the nominal risk-free policy rate by 1 percent from the steady state. Owing to price stickiness, the real interest rate rises

[39] Source: Quarterly Banking Profile, FDIC.

[40] Using the data from the 1993 National Survey of Small Business Finances, Gibson (2002) found that equity accounts for approximately 40 percent of U.S. small firms' overall source of finance. It should be noted that the emphasis is placed on small firms, as opposed to large firms, because they are relatively more bank dependent which in turn can be more appropriately rationalised in this model where bank loans are the only source of external finance.

[41] Source: OECD, Bank Profitability.

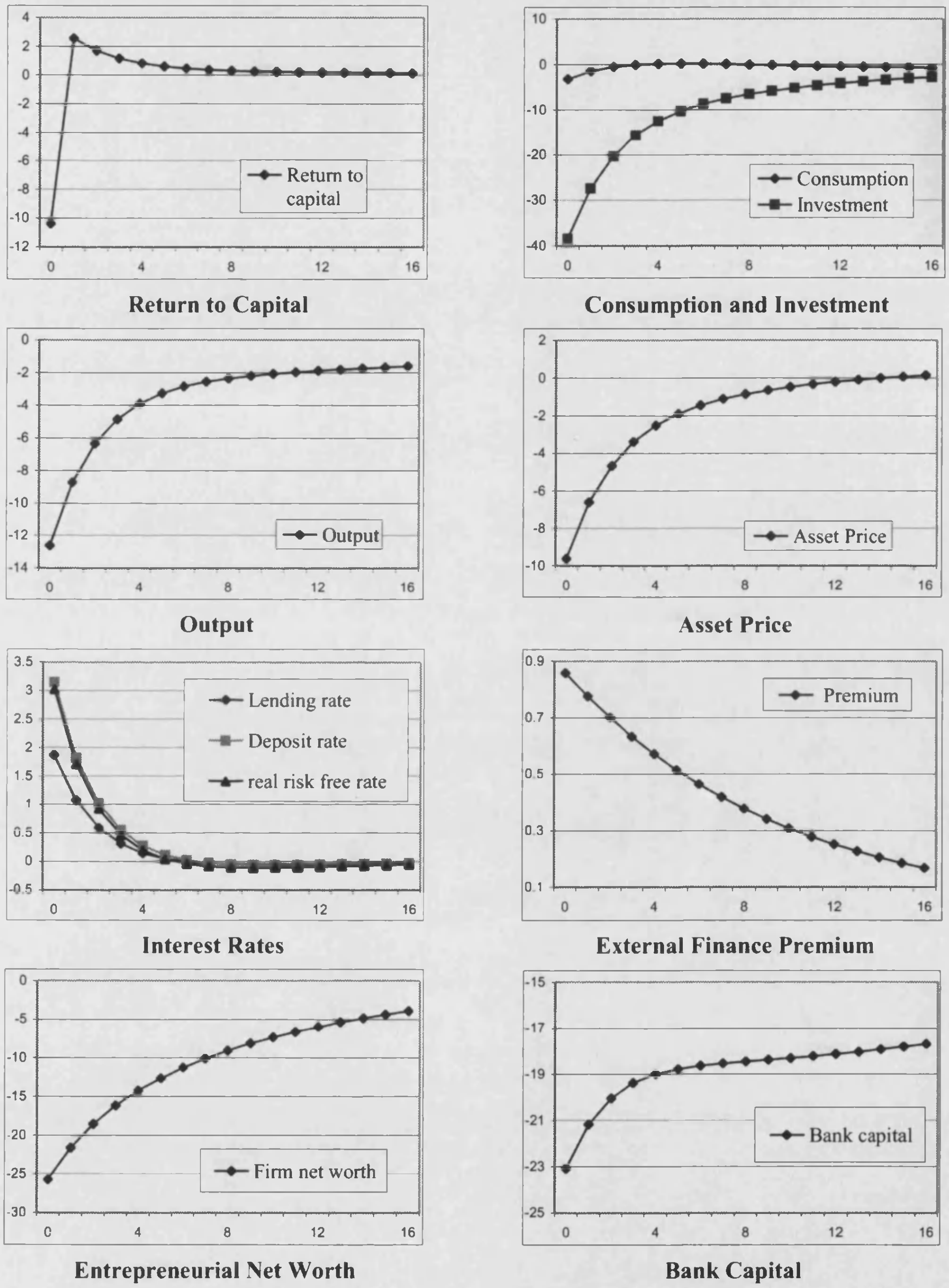

Figure 1.2: Responses to a monetary shock (All panels: time horizon in quarters)

correspondingly. Consumption and thus aggregate demand fall via the standard intertemporal substitution effect. Because *capital stock has to be purchased one period in advance*, an unexpected decline in aggregate demand causes the return to capital purchased *last period* to fall. As depositors are completely hedged against any realisation of aggregate risk (recall that they are completely risk averse to aggregate risk by assumption), the non-default deposit rate associated with deposit contracts signed *last period* has to rise instantaneously in order to compensate them for the *lower-than-expected* realisation of return to capital, $R_t^K < E_{t-1}(R_t^K)$, as well as the *higher-than-expected* realisation of their opportunity cost of funds, $r_t^f > E_{t-1}(r_t^f)$. This directly imposes a higher cost of borrowing on banks. However, the lending rate associated with loan contracts signed last period was determined in period t-1 (recall that banks are risk neutral and therefore are willing to bear aggregate risk). This in turn implies that banks' interest rate cost has to rise relatively faster compared to their revenue counterpart, thereby depleting banks' inside capital. Moreover, a lower than expected return to capital decreases entrepreneurial net worth directly. The decline in both firms' and banks' inside capital means that both firms and banks have less to contribute to firms' investment projects which in turn implies that depositors are exposed to greater agency cost. As a compensation, the non-default deposit rate associated with deposit contracts signed in period t (which will not be realised until period $t+1$) has to rise. Rational expectation then implies that banks will have to increase their non-default lending rate immediately, in anticipation of the higher expected future interest rate cost. Given the instantaneous increase in the non-default lending rate, firms face a higher cost of borrowing, implying that the external finance premium, $E_t(\frac{R_{t+1}^K}{r_{t+1}^f})$, must rise. The demand for capital, investment and asset price have to fall as firms' external cost of borrowing becomes more expensive. A kind of *multiplier* effect then arises as a higher non-default lending rate, together with a lower asset price, decrease entrepreneurial net worth further in the subsequent period. Moreover, in response to the initial fall, bank capital slowly accumulates back to trend at the rate equivalent to the real risk free rate. Given that the accumulation process is slow enough, bank capital will be persistently below trend. The negative effect of persistent decline in bank capital and entrepreneurial net worth then feeds into the subsequent periods by aggravating the deposit rate, the lending rate, and therefore the external finance premium, which in turn works to depress the demand for capital, investment and aggregate output further.

As a theoretical counterpart to the result shown in Figure 1.2, Figure 1.3 summarises how the transmission mechanism of monetary policy works within the model. Crucially, it exhibits the unconventional transmission channel in which a monetary shock affects real economic activities partly via its effect on bank capital; *the bank capital channel.* This channel and its dynamic interplay with the balance sheet channel (via entrepreneurial net worth), and asset price channel (price of capital) work to 'augment' the otherwise standard investment decision of firms via exacerbating the *two-sided* agency problem, the *double* CSV problem. We turn now to the issue of amplification and propagation properties implied by the model.

1.6.3 Amplification and Propagation Mechanism

The importance of the role of bank capital in *amplifying* as well as *propagating* responses of aggregate economic activities to a monetary shock can be seen from Figure 1.4. In the figure, we compare the dynamic responses of the model to a negative monetary shock with those of the frictionless model (FLM) and the model with bank capital channel turned off (NBM). In the FLM, the role of entrepreneurial net worth and bank capital is completely shut off and firms can borrow external funds at the sole opportunity cost equivalent to the risk free rate. Thus the transmission mechanism of monetary policy in the FLM relies solely on the conventional interest rate channel. For the NBM, we ignore the assumption that depositors are averse to aggregate risk but assume instead that they are risk neutral when engaging in financial contract decisions. As a result, similar to banks, they are willing to bear aggregate risk. This eliminates the operational mechanism of the bank capital channel that we discussed in the previous section as the adjustment of the lending rate to aggregate shock is no longer stickier compared to that of deposit rate. This implies that, in response to a negative monetary shock, the interest rate cost may not necessarily rise *faster* compared to its revenue counterpart and thus bank capital may not decline. Put differently, *turning off the bank capital channel is analogous to assuming away the effect of having maturity mismatch in banks' balance sheet.*

As can be seen from the figure, adding the bank capital channel amplifies as well as propagates the effect of a negative monetary shock on aggregate output and investment. The initial response of investment is twice as great compared to that of FLM and approximately 25 percent greater compared to that of NBM. As for output, the effect is approximately 65 percent greater compared to that of FLM and 20 percent greater compared to that of NBM. Moreover, the persistence of the real effects implied by the model is significantly greater compared to those of FLM and NBM. Evidently, the responses of investment and output persist the least under FLM, e.g. investment reverts back to trend only after 9 quarters while those of the other two models remain well below trend even after four years. This is because, as can be seen from the figure, the role of entrepreneurial net worth and bank capital are passive under FLM. This implies that the role of the external finance

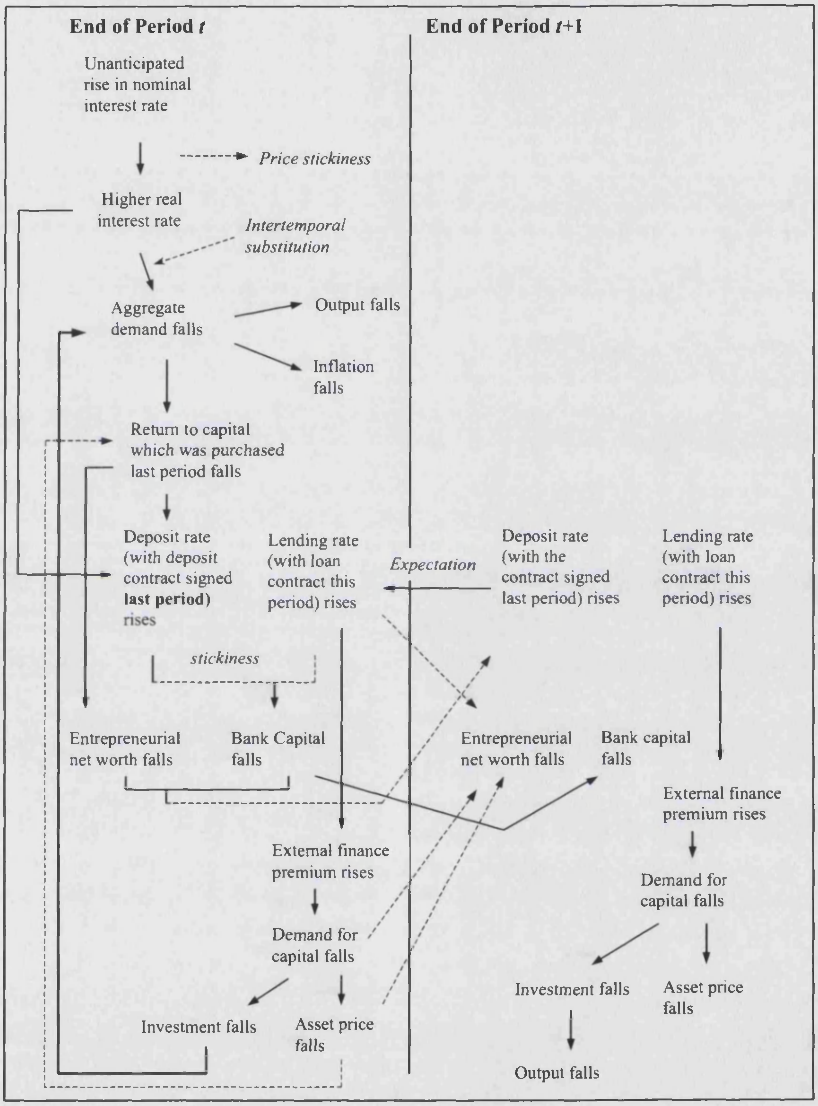

Figure 1.3: The monetary transmission mechanism in the model

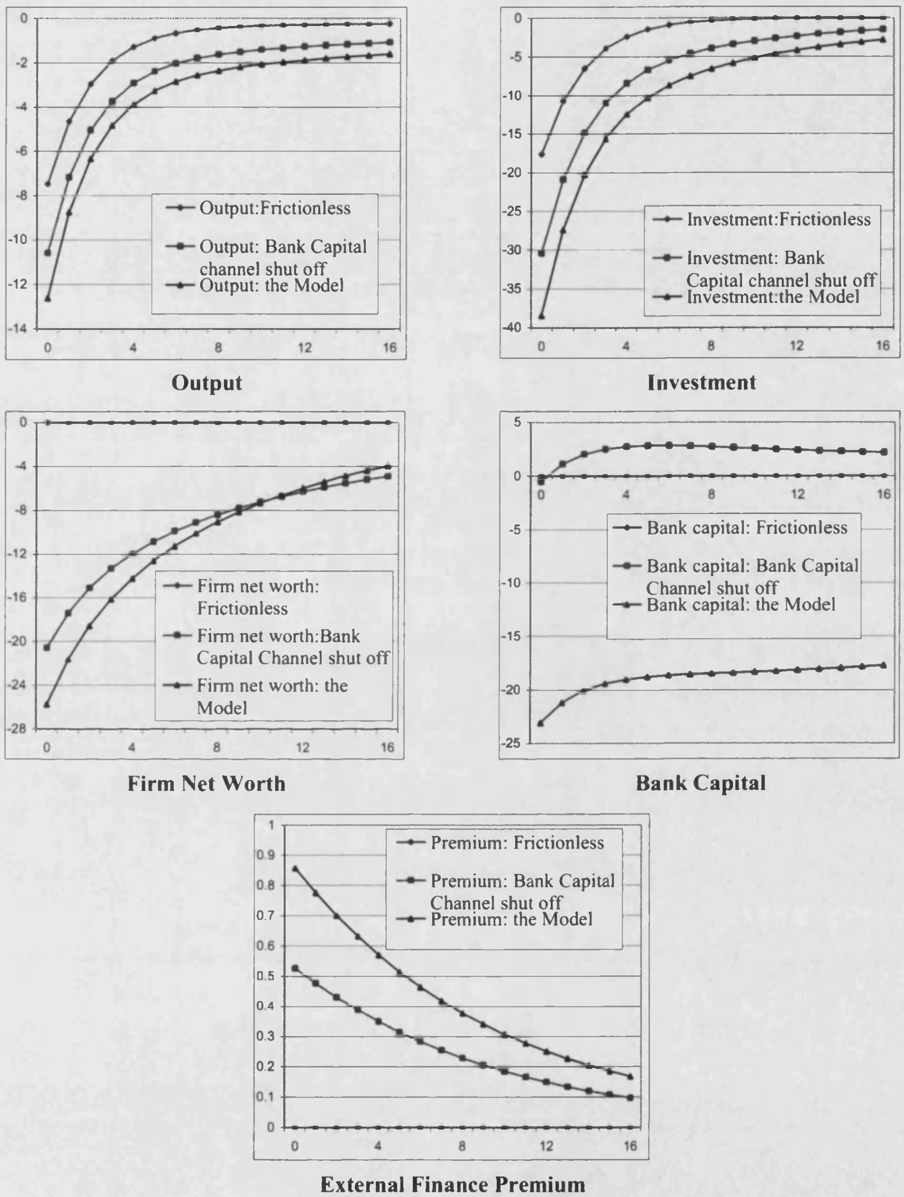

Figure 1.4: Responses to a monetary shock: baseline model vs. FLM vs. NBM (All panels: time horizon in quarters)

premium, which operates to constrain the demand for capital and thus future investment and output, is nullified. More interestingly, when the bank capital channel is turned on, in comparison with the NBM, the persistence of the real effect is evidently larger; e.g., relative to trend, investment and output in the full model after 10 quarters are about where they are in the NBM after only 6 quarters. Although the responses of entrepreneurial net worth are not markedly different under the two models, the opposite is true for the responses of bank capital. When the bank capital channel is shut off, i.e. the deposit rate no longer adjusts to a monetary shock relatively faster compared to the lending rate[42], the immediate response of bank capital is to decline slightly and turn positive in the subsequent periods. Thus unlike the role of bank capital in the full model, the response of bank capital in the model when the bank capital channel is turned off operates to *lessen* the agency problem arisen as a result of persistently declining entrepreneurial net worth. The relatively more active role of bank capital in *magnifying* the agency problem in the full model is mirrored by a substantially stronger response of the external finance premium. Crucially, the strongest response of the external finance premium in the full model compared to those in the other two models serves as the main amplifying and propagating mechanism in the model.

All in all, in the terminology of Bernanke et al. (1999), the simulation result shows that the transmission mechanism of monetary policy implied by the model exhibits *a financial accelerator effect* in that endogenous evolution of bank capital, together with entrepreneurial net worth and asset price, works to amplify as well as propagate the effect of a monetary shock in the macroeconomy.

1.7 Conclusion

In this chapter, we propose a model to illuminate how the monetary policy transmission process from its initial impulse to the ultimate response on aggregate economic activities can get amplified and propagated through its effect on bank capital; the bank capital channel. This channel and its dynamic interplay with the balance sheet channel and the asset price channel work to augment the real investment decision of firms by magnifying the two-sided agency problem, the double Costly State Verification. The simulation results confirm the *quantitative* importance of the financial accelerator effect in that endogenous evolution of bank capital operates to amplify and propagate the effect of a monetary shock in the macroeconomy.

To keep the model as simple as possible, the banking sector in this model has been highly simplified. Amongst others, it abstracts from the fact that banks in reality hold other assets besides

[42] Thus, the interest rate cost (paid to depositors) does not rise faster compared to the interest rate revenue (collected from firms).

loans, and acquire external funds from other sources besides short-term deposits. Relaxing these simplifying assumptions would allow us to identify other transmission channels in which monetary policy affects the real macroeconomy via the banking system. This task is left for future research.

Appendix

Appendix A

This appendix shows the assumptions which ensure that $\overline{\varpi}^F_{i,t+1}$ is strictly greater than $\overline{\varpi}^B_{i,t+1}$ in equilibrium. In general, 3 scenarios are plausible concerning the relative values of $\overline{\varpi}^F_{i,t+1}$ and $\overline{\varpi}^B_{i,t+1}$. For notation simplicity, we ignore the time subscript in this appendix.

To begin, we first re-state equations (1.2) and (1.4) in the text:

$$\overline{\varpi}^F_i QR^K K^i = r^L_i L^i \tag{A1}$$

$$(1-\theta^B)\overline{\varpi}^B_i \; QR^K K^i = r^D_i D^i \tag{A2}$$

Scenario 1: $\overline{\varpi}^F_i \leq (1-\theta^B)\overline{\varpi}^B_i$

Under this scenario, equations (A1) and (A2) imply that $r^L_i L^i \leq r^D_i D^i$. Given a strictly positive opportunity cost for bank j, $A^i r^f > 0$, the bank will always go bankrupt as its revenue from lending can never cover its cost. Thus we can dismiss this scenario as a potential equilibrium solution.

Scenario 2: $(1-\theta^B)\overline{\varpi}^B_i < \overline{\varpi}^F_i \leq \overline{\varpi}^B_i$

When $\varpi_i < \overline{\varpi}^F_i$, the firm will go bankrupt. After paying the verification cost, bank j receives $(1-\theta^B)\varpi_i QR^K K^i$ as its net liquidation revenue. Since this revenue is less than the bank's obligation to repay depositor m, i.e. $(1-\theta^B)\varpi_i QR^K K^i < (1-\theta^B)\overline{\varpi}^B_i QR^K K^i = r^D_i D^i$, the bank will go bankrupt.

When $\varpi_i \geq \overline{\varpi}^F_i$, firm i is able to pay back to bank j according to the contract. Since the bank's revenue in this case is enough to fulfil the deposit contract, $r^L_i L^i = \overline{\varpi}^F_i QR^K K^i > (1-\theta^B)\overline{\varpi}^B_i QR^K K^i = r^D_i D^i$, the bank does not default and pockets the profit equivalent to $r^L_i L^i - r^D_i D^i$. Given the opportunity cost of funds equivalent to the real risk-free rate, r^f, the bank's expected profit function conditional only on the realisation of idiosyncratic risk is given by:

$$\pi^B_{|(1-\theta^B)\overline{\varpi}^B_i < \overline{\varpi}^F_i \leq \overline{\varpi}^B_i} = \int_{\overline{\varpi}^F_i}^{\infty} [r^L_i L^i - r^D_i D^i] f(\varpi_i) d\varpi_i - A^i r^f \tag{A3}$$

Using equations (A1) and (A2), equation (A3) can be rewritten as:

$$\pi^B_{|(1-\theta^B)\overline{\varpi}^B_i < \overline{\varpi}^F_i \leq \overline{\varpi}^B_i} = (\overline{\varpi}^F_i - (1-\theta)\overline{\varpi}^B_i)[1 - F(\overline{\varpi}^F_i)]QR^K K^i - A^i r^f \tag{A4}$$

Scenario 3: $\overline{\varpi}_i^F > \overline{\varpi}_i^B$

When $\varpi_i < \overline{\varpi}_i^B$, firm i goes bankrupt. Since bank j's revenue after paying the verification cost is insufficient to fulfil the deposit contract, i.e. $(1-\theta^B)\varpi_i R^K QK^i < (1-\theta^B)\overline{\varpi}_i^B R^K QK^i = r_i^D D^i$, after declaring a default, the bank passes all its revenue to the depositor and retains nothing. When $\overline{\varpi}_i^F > \varpi_i \geq \overline{\varpi}_i^B$, the firm remains bankrupt. However, the bank's revenue netting off the verification cost is now enough to fulfil the deposit contract, $(1-\theta^B)\varpi_i R^K QK^i \geq (1-\theta^B)\overline{\varpi}_i^B R^K QK^i = r_i^D D^i$. Hence the bank pockets $(1-\theta^B)\varpi_i R^K QK^i - r_i^D D^i$. Lastly, when $\varpi_i \geq \overline{\varpi}_i^F$, both the bank and the firm do not declare bankruptcy. In this case, the bank pockets $r_i^L L^i - r_i^D D^i$.

The bank's expected profit conditional on the realisation of idiosyncratic return ϖ_i in this case is given by:

$$\pi^B_{|\overline{\varpi}_i^F > \overline{\varpi}_i^B} = \int_{\overline{\varpi}_i^B}^{\overline{\varpi}_i^F} [(1-\theta^B)\varpi_i R^K QK^i - r_i^D D^i] f(\varpi_i) d\varpi_i + [1 - F(\overline{\varpi}_i^F)][r_i^L L^i - r_i^D D^i] - A^i r^f \quad \text{(A5)}$$

Using the simplifying notations given in the text (equations (1.9)-(1.10)) together with equations (A1) and (A2), equation (A5) can be rewritten as:

$$\pi^B_{|\overline{\varpi}_i^F > \overline{\varpi}_i^B} = \left[\Gamma(\overline{\varpi}_i^F) - (1-\theta^B)\Gamma(\overline{\varpi}_i^B) - \theta^B G(\overline{\varpi}_i^F)\right] R^K QK^i - A^i r^f \quad \text{(A6)}$$

Although scenario 1 can be ruled out, the equilibrium value of $\overline{\varpi}_i^F$ can in general fall onto either scenarios 2 or 3. However, we will show below that under certain assumptions, we can restrict the equilibrium $\overline{\varpi}_i^F$ to lie strictly in scenario 3. For notation simplicity, we first define R^B such that the following equation holds:

$$\pi^B = R^B R^K QK^i - A^i r^f \quad \text{(A7)}$$

From equations (A3) and (A6), using equation (A7), the value of R^B can be given as follows:

$$\begin{array}{rcll} & \leq & 0 & \text{;scenario 1} \\ R^B & = & (\overline{\varpi}_i^F - (1-\theta^B)\overline{\varpi}_i^B)[1 - F(\overline{\varpi}_i^F)] & \text{;scenario 2} \\ & = & \left[\Gamma(\overline{\varpi}_i^F) - (1-\theta^B)\Gamma(\overline{\varpi}_i^B) - \theta^B G(\overline{\varpi}_i^F)\right] & \text{;scenario 3} \end{array}$$

As mentioned, scenario 1 can be immediately ruled out as $R^B \leq 0$, implying that $\pi^B \leq 0$.

First, we take limit of R^B at $\overline{\varpi}_i^F = \overline{\varpi}_i^B$:

$$\lim_{\overline{\varpi}_i^F \to \overline{\varpi}_i^{B+}} R^B = \lim_{\overline{\varpi}_i^F \to \overline{\varpi}_i^{B+}} \left[\Gamma(\overline{\varpi}_i^F) - (1-\theta^B)\Gamma(\overline{\varpi}_i^B) - \theta^B G(\overline{\varpi}_i^F)\right] = \theta^B \overline{\varpi}_i^B [1 - F(\overline{\varpi}_i^B)]$$

$$\lim_{\overline{\varpi}_i^F \to \overline{\varpi}_i^{B-}} R^B = \lim_{\overline{\varpi}_i^F \to \overline{\varpi}_i^{B-}} (\overline{\varpi}_i^F - (1-\theta^B)\overline{\varpi}_i^B)[1 - F(\overline{\varpi}_i^F)] = \theta^B \overline{\varpi}_i^B [1 - F(\overline{\varpi}_i^B)]$$

So we have shown the following:

$$\lim_{\overline{\varpi}_i^F \to \overline{\varpi}_i^{B+}} R^B = \lim_{\overline{\varpi}_i^F \to \overline{\varpi}_i^{B-}} R^B = \theta^B \overline{\varpi}_i^B [1 - F(\overline{\varpi}_i^B)] \geq 0 \tag{A8}$$

Next, we take the partial derivative of R^B with respect to the threshold value $\overline{\varpi}_i^F$ for both scenarios 2 and 3 to obtain:

$$\frac{\partial R^B}{\partial \overline{\varpi}_i^F}_{|(1-\theta^B)\overline{\varpi}_i^B < \overline{\varpi}_i^F \leq \overline{\varpi}_i^B} = [1 - F(\overline{\varpi}_i^F)][1 - (\overline{\varpi}_i^F - (1-\theta^B)\overline{\varpi}_i^B)h(\overline{\varpi}_i^F)] \tag{A9}$$

$$\frac{\partial R^B}{\partial \overline{\varpi}_i^F}_{|\overline{\varpi}_i^F > \overline{\varpi}_i^B} = \Gamma'(\overline{\varpi}_i^F) - \theta^B G'(\overline{\varpi}_i^F) = [1 - F(\overline{\varpi}_i^F)][1 - \theta^B \overline{\varpi}_i^F h(\overline{\varpi}_i^F)] \tag{A10}$$

where, as defined in the text, $h(\varpi_i) \equiv \frac{f(\varpi_i)}{1-F(\varpi_i)}$ is the hazard rate.

Then by taking limit of the two derivatives at $\overline{\varpi}_i^F = \overline{\varpi}_i^B$, we obtain:

$$\lim_{\overline{\varpi}_i^F \to \overline{\varpi}_i^{B-}} \frac{\partial R^B}{\partial \overline{\varpi}_i^F}_{|(1-\theta^B)\overline{\varpi}_i^B < \overline{\varpi}_i^F \leq \overline{\varpi}_i^B} = [1 - F(\overline{\varpi}_i^B)][1 - \theta^B \overline{\varpi}_i^B h(\overline{\varpi}_i^B)] \tag{A11}$$

$$\lim_{\overline{\varpi}_i^F \to \overline{\varpi}_i^{B+}} \frac{\partial R^B}{\partial \overline{\varpi}_i^F}_{|\overline{\varpi}_i^F > \overline{\varpi}_i^B} = [1 - F(\overline{\varpi}_i^B)][1 - \theta^B \overline{\varpi}_i^B h(\overline{\varpi}_i^B)] \tag{A12}$$

As ϖ_i is log normally distributed, it satisfies an increasing hazard rate restriction (see footnote 8). This implies that R^B reaches a global maximum at a unique $\overline{\varpi}_i^*$ and is an increasing and concave function for $\overline{\varpi}_i^F < \overline{\varpi}_i^*$.[43] Put differently, we have:

[43] From equation (A10), $\frac{\partial^2 R^B}{\partial(\overline{\varpi}_i^F)^2}_{|\overline{\varpi}_i^F > \overline{\varpi}_i^B} = -[f(\overline{\varpi}_i^F)(1-\theta^B\overline{\varpi}_i^F h(\overline{\varpi}_i^F)) + \theta^B(1-F(\overline{\varpi}_i^B))\frac{\partial \overline{\varpi}_i^F h(\overline{\varpi}_i^F)}{\partial \overline{\varpi}_i^F}]$, where $f(\overline{\varpi}_i^F)$ and $h(\overline{\varpi}_i^F)$ are *df* and the hazard rate of $\overline{\varpi}_i^F$, respectively. The increasing hazard rate restriction, as shown in footnote 8 in the text, implies $\frac{\partial^2 R^B}{\partial(\overline{\varpi}_i^F)^2}_{|\overline{\varpi}_i^F > \overline{\varpi}_i^B} < 0$. Thus, the function R^B is concave.

$$\frac{\partial R^B}{\partial \varpi_i^F} \begin{array}{ccl} = & 0 & \text{for } \varpi_i^F = \varpi_i^* \\ > & 0 & \text{for } \varpi_i^F < \varpi_i^* \\ < & 0 & \text{for } \varpi_i^F > \varpi_i^* \end{array}$$

Since $\varpi_i^F > \varpi_i^*$ can never be an equilibrium, in order to restrict the equilibrium value of ϖ_i^F to be greater than that of ϖ_i^B as given by scenario 3, it must be the case that $\varpi_i^B < \varpi_i^*$. To achieve this, we must assume that $\frac{\partial R^B}{\partial \varpi_i^F}$ evaluated at $\varpi_i^F = \varpi_i^B$ be greater than zero. From equations (A11) and (A12), this implies the following assumption A1.

Assumption A1: $[1 - F(\varpi_i^B)][1 - \theta^B \varpi_i^B h(\varpi_i^B)] > 0$

From equation (A8), another assumption to ensure that equilibrium ϖ_i^F will be greater than ϖ_i^B is given as follows:

Assumption A2: $\theta^B \varpi_i^B [1 - F(\varpi_i^B)] < \frac{A^i r^f}{R^K Q K^i}$

Assumption A2 implies that the normalised opportunity cost of funds for the bank, $\frac{A^i r^f}{R^K Q K^i}$, is *greater* than the bank's expected revenue when $\varpi_i^F = \varpi_i^B$. Thus, assumption A2 means that $\varpi_i^F = \varpi_i^B$ cannot be an equilibrium as the opportunity cost of funds outweighs the expected revenue. Assumption A1 means that at $\varpi_i^F = \varpi_i^B$, the slope of the R^B curve is positive. This together with the assumption of increasing hazard rate imply that equilibrium ϖ_i^F must lie within the range (ϖ_i^B, ϖ_i^*) given that the bank's opportunity cost is not too high, i.e. $\frac{A^i r^f}{R^K Q K^i} \leq R^B{}_{|\varpi_i^F = \varpi_i^*}$. If the bank's opportunity cost is too high, $\frac{A^i r^f}{R^K Q K^i} > R^B{}_{|\varpi_i^F = \varpi_i^*}$, the firm is rationed. However, we will focus only on the non-rationing equilibrium.[44]

All in all, we have shown that, assumptions A1 and A2 are sufficient to ensure that equilibrium ϖ_i^F will be strictly greater than ϖ_i^B.

Appendix B

In this appendix, we derive the optimality conditions for the firm's demand for capital. For notation simplicity, we drop the i subscript.

First, we define the following variables:

$$k_t \equiv \frac{Q_t K_t}{W_t + A_t}, s_t \equiv E_t\left(\frac{R_{t+1}^K}{r_{t+1}^f}\right), u_{t+1} \equiv \frac{R_{t+1}^K}{E_t(R_{t+1}^K)} \frac{E_t(r_{t+1}^f)}{r_{t+1}^f}$$

[44] In order to ensure non-rationing equilibrium, it must be the case that $\frac{\partial R^B}{\partial \varpi_i^F}{}_{|\varpi_i^F > \varpi_i^B} = \Gamma'(\varpi_i^F) - \theta^B G'(\varpi_i^F) > 0$.
This restriction holds under the parameterisation taken in this chapter.

where u_{t+1} captures the source of aggregate risk in the model.

We restate firm i's optimisation problem given in the text as follows:[45]

$$\max_{k_t, r^D_{t+1}, r^L_t} E_t \sum_{j=0}^{\infty} \{[1 - \Gamma(\overline{\omega}^F_{t+1+j})] u_{t+1+j} s_{t+j} k_{t+j}\}$$

subject to

$$(1-\theta^B)\left[\Gamma(\overline{\omega}^B_{t+1}) - \theta^D G(\overline{\omega}^B_{t+1})\right] u_{t+1} s_t k_t - (k_t - 1) = 0$$

$$E_t\{\left[\Gamma(\overline{\omega}^F_{t+1}) - (1-\theta^B)\Gamma(\overline{\omega}^B_{t+1}) - \theta^B G(\overline{\omega}^F_{t+1})\right] u_{t+1}\} s_t k_t - \frac{A_t}{W_t + A_t} = 0$$

where

$$\overline{\omega}^F_{t+1} = \frac{r^L_t (k_t - \frac{W_t}{W_t + A_t})}{r^f_{t+1} k_t s_t u_{t+1}} \tag{B1}$$

$$\overline{\omega}^B_{t+1} = \frac{r^D_{t+1}(k_t - 1)}{(1-\theta^B) r^f_{t+1} k_t s_t u_{t+1}} \tag{B2}$$

Using Dynamic Lagrangian, we can write the dynamic optimisation problem as follows:

$$\begin{aligned} L = \; & E_t\{\sum_{j=0}^{\infty}([1 - \Gamma(\overline{\omega}^F_{t+1+j})] u_{t+1+j} s_{t+j} k_{t+j} \\ & + \lambda^1_{t+j}\{\left[\Gamma(\overline{\omega}^F_{t+1+j}) - (1-\theta^B)\Gamma(\overline{\omega}^B_{t+1+j}) - \theta^B G(\overline{\omega}^F_{t+1+j})\right] u_{t+1+j} s_{t+j} k_{t+j} - \tfrac{A_{t+j}}{W_{t+j} + A_{t+j}}\} \\ & + \lambda^2_{t+j}\{(1-\theta^B)\left[\Gamma(\overline{\omega}^B_{t+j}) - \theta^D G(\overline{\omega}^B_{t+j})\right] u_{t+j} s_{t-1+j} k_{t-1+j} - (k_{t-1+j} - 1)\})\} \end{aligned}$$

The resulting first order conditions are as follows:

$$r^D_{t+1} : \frac{\lambda^2_{t+1}}{\lambda^1_t} = \frac{\Gamma'(\overline{\omega}^B_{t+1})}{\Gamma'(\overline{\omega}^B_{t+1}) - \theta^D G'(\overline{\omega}^B_{t+1})} \tag{B3}$$

$$r^L_t : \lambda^1_t = \frac{E_t[\Gamma'(\overline{\omega}^F_{t+1}) \frac{\partial \overline{\omega}^F_{t+1}}{\partial r^L_t} u_{t+1}]}{E_t[(\Gamma'(\overline{\omega}^F_{t+1}) - \theta^B G'(\overline{\omega}^F_{t+1})) \frac{\partial \overline{\omega}^F_{t+1}}{\partial r^L_t} u_{t+1}]} \tag{B4}$$

where λ^1_t is the *ex-ante* value of the Lagrange multiplier on the constraint that bank j earns zero expected profit *prior* to the realisation of aggregate risk and λ^2_{t+1} is the *ex-post* value of the Lagrange multiplier on the constraint that depositor m earns zero expected profit *after* the realisation

[45] Here $k_t \equiv \frac{Q_t K_t}{W_t + A_t}$ is the choice variable. This is because the production functions are constant returns to scale.

of aggregate risk.

$$k_t : JJ_t(s_t, r_t^L, r_{t+1}^D) \equiv E_t\left[\wp_{t+1} u_{t+1} s_t - \lambda_{t+1}^2\right] = 0 \tag{B5}$$

$$\begin{aligned}\text{where } \wp_{t+1} \;\equiv\; & [1-\Gamma(\varpi_{t+1}^F)] + \lambda_t^1[\Gamma(\varpi_{t+1}^F) - (1-\theta^B)\Gamma(\varpi_{t+1}^B) - \theta^B G(\varpi_{t+1}^F)] \\ & +\lambda_{t+1}^2(1-\theta^B)\left[\Gamma(\varpi_{t+1}^B) - \theta^D G(\varpi_{t+1}^B)\right]\end{aligned}$$

$$\lambda_{t+1}^2 :\; (1-\theta^B)\left[\Gamma(\varpi_{t+1}^B) - \theta^D G(\varpi_{t+1}^B)\right] u_{t+1} s_t k_t - (k_t - 1) = 0 \tag{B6}$$

$$\lambda_t^1 : E_t\{\left[\Gamma(\varpi_{t+1}^F) - (1-\theta^B)\Gamma(\varpi_{t+1}^B) - \theta^B G(\varpi_{t+1}^F)\right] u_{t+1}\} s_t k_t - \frac{A_t}{W_t + A_t} = 0 \tag{B7}$$

From equations (B1)-(B7), there are 7 equations in 7 variables ($\varpi_{t+1}^F, \varpi_{t+1}^B, r_{t+1}^D, r_t^L, k_t, \lambda_{t+1}^2, \lambda_t^1$), taken as given the value of s_t and $\frac{A_t}{W_t+A_t}$. Thus, in equilibrium, it is possible to write k_t solely as a function of s_t and $\frac{A_t}{W_t+A_t}$; $k_t = \Psi(s_t, \frac{A_t}{W_t+A_t})$.

In what follows, we will show that $\frac{\partial\Psi(s_t,\frac{A_t}{W_t+A_t})}{\partial s_t} > 0$ and $\frac{\partial\Psi(s_t,\frac{A_t}{W_t+A_t})}{\partial(\frac{A_t}{W_t+A_t})} < 0$.

From equations (B1) and (B2), given that $k_t > 1$[46],

$$\frac{\partial\varpi_{t+1}^F}{\partial r_t^L} = \frac{(k_t - \frac{W_t}{W_t+A_t})}{r_{t+1}^f k_t s_t u_{t+1}} > 0 \tag{B8}$$

$$\frac{\partial\varpi_{t+1}^B}{\partial r_{t+1}^D} = \frac{(k_t - 1)}{(1-\theta^B) r_{t+1}^f k_t s_t u_{t+1}} > 0 \tag{B9}$$

Substituting equation (B8) into equation (B4), the latter can be rewritten as:

$$\lambda_t^1 = \frac{E_t[\frac{\Gamma'(\varpi_{t+1}^F)}{r_{t+1}^f}]}{E_t[\frac{(\Gamma'(\varpi_{t+1}^F) - \theta^B G'(\varpi_{t+1}^F))}{r_{t+1}^f}]} \tag{B10}$$

Given the increasing hazard rate assumption and the result from Appendix A that $\varpi_i^* > \varpi_i^F > \varpi_i^B$, $\left[\Gamma'(\varpi_{t+1}^F) - \theta^B G'(\varpi_{t+1}^F)\right]$ and $\left[\Gamma'(\varpi_{t+1}^B) - \theta^D G'(\varpi_{t+1}^B)\right]$ are strictly positive. These imply, via equations (B3) and (B10), that $\lambda_t^1 > 0$ and $\lambda_{t+1}^2 > 0$.

From equation (B10), we take the partial derivative with respect to ϖ_{t+1}^F to obtain:

$$\frac{\partial\lambda_t^1}{\partial\varpi_{t+1}^F} = \frac{\theta^B[E_t(\frac{\Gamma'(\varpi_{t+1}^F)}{r_{t+1}^f})E_t(\frac{G''(\varpi_{t+1}^F)}{r_{t+1}^f}) - E_t(\frac{\Gamma''(\varpi_{t+1}^F)}{r_{t+1}^f})E_t(\frac{G'(\varpi_{t+1}^F)}{r_{t+1}^f})]}{\{E_t(\frac{\Gamma'(\varpi_{t+1}^F)}{r_{t+1}^f}) - \theta^B E_t(\frac{G'(\varpi_{t+1}^F)}{r_{t+1}^f})\}^2}$$

[46]Otherwise the sum of bank capital and entrepreneurial net worth would be sufficient to finance the firm's investment outlay, in which case the bank does not need to obtain any deposits from the depositor.

Using the fact that $\Gamma'(\overline{\omega}^F_{t+1}) = 1 - F\left(\overline{\omega}^F_{t+1}\right)$, and $G'(\overline{\omega}^F_{t+1}) = \overline{\omega}^F_{t+1} f(\overline{\omega}^F_{t+1})$, where $F\left(\overline{\omega}^F_{t+1}\right)$ and $f(\overline{\omega}^F_{t+1})$ are *cdf* and *df* of $\overline{\omega}^F_{t+1}$ respectively, we can then write $\frac{\partial \lambda^1_t}{\partial \overline{\omega}^F_{t+1}}$ as follows:

$$\frac{\partial \lambda^1_t}{\partial \overline{\omega}^F_{t+1}} = \frac{\theta^B[E_t(\frac{1-F(\overline{\omega}^F_{t+1})}{r^f_{t+1}})E_t(\frac{\partial(\overline{\omega}^F_{t+1}f(\overline{\omega}^F_{t+1}))}{\partial \overline{\omega}^F_{t+1}}\frac{1}{r^f_{t+1}}) + E_t(\frac{f(\overline{\omega}^F_{t+1})}{r^f_{t+1}})E_t(\frac{\overline{\omega}^F_{t+1}f(\overline{\omega}^F_{t+1})}{r^f_{t+1}})]}{\{E_t(\frac{1-F(\overline{\omega}^F_{t+1})}{r^f_{t+1}}) - \theta^B E_t(\frac{\overline{\omega}^F_{t+1}f(\overline{\omega}^F_{t+1})}{r^f_{t+1}})\}^2}$$

In general, the value of $\frac{\partial(\overline{\omega}^F_{t+1}f(\overline{\omega}^F_{t+1}))}{\partial \overline{\omega}^F_{t+1}}$ could be either positive or negative. However, under a reasonable parameterisation, including the one used for calibration in this chapter, it will be strictly positive in the neighbourhood of the steady state, in which case $\frac{\partial \lambda^1_t}{\partial \overline{\omega}^F_{t+1}} > 0$.

From equation (B3), take the partial derivative with respect to $\overline{\omega}^F_{t+1}$and $\overline{\omega}^B_{t+1}$, we obtain:

$$\frac{\partial \lambda^2_{t+1}}{\partial \overline{\omega}^F_{t+1}} = \frac{\Gamma'(\overline{\omega}^B_{t+1})}{\Gamma'(\overline{\omega}^B_{t+1}) - \theta^D G'(\overline{\omega}^B_{t+1})}\frac{\partial \lambda^1_t}{\partial \overline{\omega}^F_{t+1}} = \frac{\lambda^2_{t+1}}{\lambda^1_t}\frac{\partial \lambda^1_t}{\partial \overline{\omega}^F_{t+1}} > 0$$

$$\frac{\partial \lambda^2_{t+1}}{\partial \overline{\omega}^B_{t+1}} = \frac{\theta^D[\Gamma'(\overline{\omega}^B_{t+1})G''(\overline{\omega}^B_{t+1}) - \Gamma''(\overline{\omega}^B_{t+1})G'(\overline{\omega}^B_{t+1})]}{\left[\Gamma'(\overline{\omega}^B_{t+1}) - \theta^D G'(\overline{\omega}^B_{t+1})\right]^2}\lambda^1_t = \frac{\theta^D[1-F(\overline{\omega}^B_{t+1})]^2\frac{\partial \overline{\omega}^B_{t+1}h(\overline{\omega}^B_{t+1})}{\partial \overline{\omega}^B_{t+1}}}{\left[\Gamma'(\overline{\omega}^B_{t+1}) - \theta^D G'(\overline{\omega}^B_{t+1})\right]^2}\lambda^1_t$$

Due to the assumption of an increasing hazard rate, $\frac{\partial \overline{\omega}^B_{t+1}h(\overline{\omega}^B_{t+1})}{\partial \overline{\omega}^B_{t+1}} > 0$, where $h(\overline{\omega}^B_{t+1}) \equiv \frac{f(\overline{\omega}^B_{t+1})}{1-F(\overline{\omega}^B_{t+1})}$ is the hazard rate evaluated at $\overline{\omega}^B_{t+1}$, it follows directly that $\frac{\partial \lambda^2_{t+1}}{\partial \overline{\omega}^B_{t+1}} > 0$.

Thus far, we have established that $\lambda^1_t, \lambda^2_{t+1}, \frac{\partial \lambda^1_t}{\partial \overline{\omega}^F_{t+1}}, \frac{\partial \lambda^2_{t+1}}{\partial \overline{\omega}^F_{t+1}}, \frac{\partial \lambda^2_{t+1}}{\partial \overline{\omega}^B_{t+1}}, \frac{\partial \overline{\omega}^F_{t+1}}{\partial r^L_t}, \frac{\partial \overline{\omega}^B_{t+1}}{\partial r^D_{t+1}}$ are all positive for $\overline{\omega}^F_{t+1} \in (\overline{\omega}^B_{t+1}, \overline{\omega}^*_{t+1})$.

To show that $\frac{\partial k_t}{\partial s_t} > 0$, take the derivative of $JJ_t(s_t, r^L_t, r^D_{t+1})$ (the first order condition with respect to capital, equation (B5)) with respect to s_t.

$$\frac{\partial JJ_t}{\partial s_t} + \frac{\partial JJ_t}{\partial r^L_t}\frac{\partial r^L_t}{\partial s_t} + \frac{\partial JJ_t}{\partial r^D_{t+1}}\frac{\partial r^D_{t+1}}{\partial s_t} + [\frac{\partial JJ_t}{\partial r^L_t}\frac{\partial r^L_t}{\partial k_t} + \frac{\partial JJ_t}{\partial r^D_{t+1}}\frac{\partial r^D_{t+1}}{\partial k_t}]\frac{\partial k_t}{\partial s_t} = 0$$

and rearrange to obtain:

$$\frac{\partial k_t}{\partial s_t} = -\frac{[\frac{\partial JJ_t}{\partial s_t} + \frac{\partial JJ_t}{\partial r^L_t}\frac{\partial r^L_t}{\partial s_t} + \frac{\partial JJ_t}{\partial r^D_{t+1}}\frac{\partial r^D_{t+1}}{\partial s_t}]}{[\frac{\partial JJ_t}{\partial r^L_t}\frac{\partial r^L_t}{\partial k_t} + \frac{\partial JJ_t}{\partial r^D_{t+1}}\frac{\partial r^D_{t+1}}{\partial k_t}]} \tag{B11}$$

From equation (B5),

$$\begin{aligned}\frac{\partial JJ_t}{\partial s_t} &= E_t(\wp_{t+1}u_{t+1}) > 0\\ \frac{\partial JJ_t}{\partial r_t^L} &= E_t[-\Gamma'(\overline{\omega}_{t+1}^F)\frac{\partial \overline{\omega}_{t+1}^F}{\partial r_t^L}u_{t+1} + \lambda_t^1(\Gamma'(\overline{\omega}_{t+1}^F) - \theta^B G'(\overline{\omega}_{t+1}^F))\frac{\partial \overline{\omega}_{t+1}^F}{\partial r_t^L}u_{t+1}]s_t\\ &\quad + E_t[\frac{\partial \lambda_t^1}{\partial r_t^L}\left[\Gamma(\overline{\omega}_{t+1}^F) - (1-\theta^B)\Gamma(\overline{\omega}_{t+1}^B) - \theta^B G(\overline{\omega}_{t+1}^F)\right]u_{t+1}]s_t\\ &\quad + E_t\left[\frac{\partial \lambda_{t+1}^2}{\partial r_t^L}\left((1-\theta^B)\left[\Gamma(\overline{\omega}_{t+1}^B) - \theta^D G(\overline{\omega}_{t+1}^B)\right]u_{t+1}s_t - 1\right)\right]\end{aligned} \quad \text{(B12)}$$

Equation (B6) can be rearranged to obtain:

$$\left((1-\theta^B)\left[\Gamma(\overline{\omega}_{t+1}^B) - \theta^D G(\overline{\omega}_{t+1}^B)\right]u_{t+1}s_t - 1\right) = -\frac{1}{k_t} \quad \text{(B13)}$$

Equation (B7) can be rearranged to obtain:

$$E_t\{\left[\Gamma(\overline{\omega}_{t+1}^F) - (1-\theta^B)\Gamma(\overline{\omega}_{t+1}^B) - \theta^B G(\overline{\omega}_{t+1}^F)\right]u_{t+1}\}s_t = \frac{A_t}{W_t + A_t}\frac{1}{k_t} \quad \text{(B14)}$$

From equation (B3),

$$\frac{\partial \lambda_{t+1}^2}{\partial r_t^L} = \frac{\lambda_{t+1}^2}{\lambda_t^1}\frac{\partial \lambda_t^1}{\partial r_t^L} \quad \text{(B15)}$$

Substituting equations (B4), (B13), (B14) and (B15) into equation (B12), we obtain:

$$\frac{\partial JJ_t}{\partial r_t^L} = E_t\left(\frac{\partial \lambda_{t+1}^2}{\partial r_t^L}\frac{1}{k_t}\left[\frac{A_t}{W_t + A_t}\frac{\lambda_t^1}{\lambda_{t+1}^2} - 1\right]\right) \quad \text{(B16)}$$

Because $\frac{\partial \lambda_{t+1}^2}{\partial r_t^L} > 0$, $\frac{A_t}{W_t + A_t} \leq 1$, and $\frac{\lambda_t^1}{\lambda_{t+1}^2} < 1$, it must be that $\frac{\partial JJ_t}{\partial r_t^L} < 0$.

From equation (B5),

$$\begin{aligned}\frac{\partial JJ_t}{\partial r_{t+1}^D} &= E_t[-\lambda_t^1(1-\theta^B)\Gamma'(\overline{\omega}_{t+1}^B)\frac{\partial \overline{\omega}_{t+1}^B}{\partial r_{t+1}^D}u_{t+1}\\ &\quad + \lambda_{t+1}^2(1-\theta^B)[\Gamma'(\overline{\omega}_{t+1}^B) - \theta^D G'(\overline{\omega}_{t+1}^B)]\frac{\partial \overline{\omega}_{t+1}^B}{\partial r_{t+1}^D}u_{t+1}]s_t\\ &\quad + E_t[\frac{\partial \lambda_{t+1}^2}{\partial r_{t+1}^D}\{(1-\theta^B)\left[\Gamma(\overline{\omega}_{t+1}^B) - \theta^D G(\overline{\omega}_{t+1}^B)\right]u_{t+1}s_t - 1\}]\end{aligned}$$

Using equations (B3) and (B13), we can write:

$$\frac{\partial JJ_t}{\partial r_{t+1}^D} = -E_t[\frac{\partial \lambda_{t+1}^2}{\partial r_{t+1}^D}\frac{1}{k_t}] < 0 \quad \text{(B17)}$$

From equation (B6), we take the partial derivative with respect to s_t and k_t, respectively to obtain:

$$\frac{\partial r_{t+1}^D}{\partial s_t} = -\frac{\left[\Gamma(\varpi_{t+1}^B) - \theta^D G(\varpi_{t+1}^B)\right]}{\left[\Gamma'(\varpi_{t+1}^B) - \theta^D G'(\varpi_{t+1}^B)\right] \frac{\partial \varpi_{t+1}^B}{\partial r_{t+1}^D} s_t} < 0 \tag{B18}$$

$$\frac{\partial r_{t+1}^D}{\partial k_t} = \frac{\left(1 - (1-\theta^B)\left[\Gamma(\varpi_{t+1}^B) - \theta^D G(\varpi_{t+1}^B)\right] u_{t+1} s_t\right)}{(1-\theta^B)\left[\Gamma'(\varpi_{t+1}^B) - \theta^D G'(\varpi_{t+1}^B)\right] \frac{\partial \varpi_{t+1}^B}{\partial r_{t+1}^D} u_{t+1} s_t k_t}$$

Using equation (B13), $\frac{\partial r_{t+1}^D}{\partial k_t}$ can be rewritten as:

$$\frac{\partial r_{t+1}^D}{\partial k_t} = \frac{k_t^{-1}}{(1-\theta^B)\left[\Gamma'(\varpi_{t+1}^B) - \theta^D G'(\varpi_{t+1}^B)\right] \frac{\partial \varpi_{t+1}^B}{\partial r_{t+1}^D} u_{t+1} s_t k_t} > 0 \tag{B19}$$

From equation (B7), we take the partial derivative with respect to s_t and k_t, respectively, to obtain:

$$\frac{\partial r_t^L}{\partial s_t} = \frac{E_t\left(\begin{array}{c} (1-\theta^B)\Gamma'(\varpi_{t+1}^B)\frac{\partial \varpi_{t+1}^B}{\partial r_{t+1}^D}\frac{\partial r_{t+1}^D}{\partial s_t} s_t k_t u_{t+1} \\ -\left[\Gamma(\varpi_{t+1}^F) - (1-\theta^B)\Gamma(\varpi_{t+1}^B) - \theta^B G(\varpi_{t+1}^F)\right] u_{t+1} k_t \end{array}\right)}{E_t\{\Gamma'(\varpi_{t+1}^F) - \theta^B G'(\varpi_{t+1}^F))\frac{\partial \varpi_{t+1}^F}{\partial r_t^L} u_{t+1}\} s_t k_t}$$

By substituting equation (B18) into the above equation, we obtain:

$$\frac{\partial r_t^L}{\partial s_t} = -\frac{E_t\left(\begin{array}{c} (1-\theta^B)\frac{\lambda_{t+1}^2}{\lambda_t^1}\left[\Gamma(\varpi_{t+1}^B) - \theta^D G(\varpi_{t+1}^B)\right] k_t u_{t+1} + \\ \left[\Gamma(\varpi_{t+1}^F) - (1-\theta^B)\Gamma(\varpi_{t+1}^B) - \theta^B G(\varpi_{t+1}^F)\right] k_t u_{t+1} \end{array}\right)}{E_t\{\Gamma'(\varpi_{t+1}^F) - \theta^B G'(\varpi_{t+1}^F))\frac{\partial \varpi_{t+1}^F}{\partial r_t^L} u_{t+1}\} s_t k_t} < 0 \tag{B20}$$

$$\frac{\partial r_t^L}{\partial k_t} = \frac{E_t\left(\begin{array}{c} (1-\theta^B)\Gamma'(\varpi_{t+1}^B)\frac{\partial \varpi_{t+1}^B}{\partial r_{t+1}^D}\frac{\partial r_{t+1}^D}{\partial k_t} s_t k_t u_{t+1} \\ -\left[\Gamma(\varpi_{t+1}^F) - (1-\theta^B)\Gamma(\varpi_{t+1}^B) - \theta^B G(\varpi_{t+1}^F)\right] u_{t+1} s_t \end{array}\right)}{E_t\{\Gamma'(\varpi_{t+1}^F) - \theta^B G'(\varpi_{t+1}^F))\frac{\partial \varpi_{t+1}^F}{\partial r_t^L} u_{t+1}\} s_t k_t}$$

By substituting equations (B19) and (B14) into the above equation, we obtain:

$$\frac{\partial r_t^L}{\partial k_t} = \frac{1}{k_t}\frac{E_t\{\frac{\lambda_{t+1}^2}{\lambda_t^1} - \frac{A_t}{W_t + A_t}\}}{E_t\{\Gamma'(\varpi_{t+1}^F) - \theta^B G'(\varpi_{t+1}^F))\frac{\partial \varpi_{t+1}^F}{\partial r_t^L} u_{t+1} s_t k_t\}} \tag{B21}$$

Because, $\frac{\lambda_{t+1}^2}{\lambda_t^1} > 1$ and $\frac{A_t}{W_t + A_t} \leq 1$, it must be the case that $\frac{\partial r_t^L}{\partial k_t} > 0$.

Thus far we have shown that $\frac{\partial JJ_t}{\partial r_t^L}$, $\frac{\partial JJ_t}{\partial r_{t+1}^D}$, $\frac{\partial r_{t+1}^D}{\partial s_t}$, and $\frac{\partial r_t^L}{\partial s_t}$ are strictly negative and that $\frac{\partial JJ_t}{\partial s_t}$, $\frac{\partial r_{t+1}^D}{\partial k_t}$, and $\frac{\partial r_t^L}{\partial k_t}$ are strictly positive. Plugging these values into equation (B11) implies that $\frac{\partial k_t}{\partial s_t}$ is strictly positive as required.

We turn now to show that $\frac{\partial k_t}{\partial \frac{A_t}{W_t+A_t}} < 0$. From equation (B5), take the derivative of JJ_t with respect to $\frac{A_t}{W_t+A_t}$ to obtain:

$$\frac{\partial JJ_t}{\partial r_t^L}\frac{\partial r_t^L}{\partial \frac{A_t}{W_t+A_t}} + [\frac{\partial JJ_t}{\partial r_t^L}\frac{\partial r_t^L}{\partial k_t} + \frac{\partial JJ_t}{\partial r_{t+1}^D}\frac{\partial r_{t+1}^D}{\partial k_t}]\frac{\partial k_t}{\partial \frac{A_t}{W_t+A_t}} = 0$$

$$\frac{\partial k_t}{\partial \frac{A_t}{W_t+A_t}} = -\frac{[\frac{\partial JJ_t}{\partial r_t^L}\frac{\partial r_t^L}{\partial \frac{A_t}{W_t+A_t}}]}{[\frac{\partial JJ_t}{\partial r_t^L}\frac{\partial r_t^L}{\partial k_t} + \frac{\partial JJ_t}{\partial r_{t+1}^D}\frac{\partial r_{t+1}^D}{\partial k_t}]} \tag{B22}$$

In order to obtain $\frac{\partial r_t^L}{\partial \frac{A_t}{W_t+A_t}}$, take the derivative of equation (B7) with respect to $\frac{A_t}{W_t+A_t}$ and rearrange to obtain:

$$\frac{\partial r_t^L}{\partial \frac{A_t}{W_t+A_t}} = \frac{1}{E_t\{\Gamma'(\overline{\omega}_{t+1}^F) - \theta^B G'(\overline{\omega}_{t+1}^F))\frac{\partial \overline{\omega}_{t+1}^F}{\partial r_t^L}u_{t+1}\}s_t k_t} > 0 \tag{B23}$$

As $\frac{\partial JJ_t}{\partial r_t^L}$ and $\frac{\partial JJ_t}{\partial r_{t+1}^D}$ are strictly negative and $\frac{\partial r_t^L}{\partial \frac{A_t}{W_t+A_t}}, \frac{\partial r_t^L}{\partial k_t}, \frac{\partial r_{t+1}^D}{\partial k_t}$ are strictly positive, by substituting these values into equation (B22), it must be that $\frac{\partial k_t}{\partial \frac{A_t}{W_t+A_t}} < 0$.

In sum, we have shown the following:

$$k_t = \Psi(s_t, \frac{A_t}{W_t + A_t}) \tag{B24}$$

$$\text{where } \frac{\partial k_t}{\partial s_t} > 0, \ \frac{\partial k_t}{\partial \frac{A_t}{W_t+A_t}} < 0$$

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Chapter 2

The Role of Money and the Transmission Mechanism of Monetary Policy: Evidence from Thailand

Abstract

This chapter shows that the scope of changes in relative yields of various assets for which money is conventionally argued to proxy can be empirically segregated into (i) changes in relative prices along the term structure (term-structure effect) and (ii) changes in relative risk premia component of different kinds/classes of assets (risk-premium effect). Using Thailand data, we found that both effects are significant. We argue from this finding that standard macro models which are based on the two-asset assumption are distorting and that the problem can be alleviated by introducing an explicit role of money in these models.

2.1 Introduction

The current trend for downgrading the role of money in small-scale macroeconomic models for monetary policy is indeed widespread.[1] As emphasised by King (2002) and Meyer (2001), this trend is no longer just an academic phenomenon since it has already been popularised in large scale macro-

[1] To name a few, these models range from the forward-looking models with microfoundations of McCallum and Nelson (1999) and Rotemberg and Woodford (1997) to the pure backward-looking model without microfoundations of Rudebusch and Svensson (2002).

econometric models employed by various leading central banks, including the Fed and the Bank of England.

The main goal of this chapter is to examine whether this trend may have major disadvantages in neglecting important channels of the monetary policy transmission mechanism; specifically, the channels which operate through changes in relative yields on a wide array of assets (Meltzer, 2001b). In doing so, we use Thailand quarterly data as the basis of investigation. As related evidence on this issue is all from developed countries, it should be interesting to see whether consistent results would be obtained for a developing country such as Thailand.[2]

Although Nelson (2002) attempted a similar type of empirical exercise for the U.S. and U.K., we argue below that his empirical methodology does not allow for the optimal forward-looking consumption behaviour typically encapsulated in models with microfoundations. In particular, the novel feature of the analysis in this chapter is that we test for the significance of the role of the real monetary stock in a *hybrid* IS equation, which essentially allows for both forward looking and backward looking behaviours of rational agents. As we shall argue, this allows us to identify separately the two distinct forms of changes in relative yields of assets that money is conventionally found to proxy; one being changes along the term structure of interest rate (the term-structure effect) and the other being changes in relative risk premia amongst different kinds and classes of assets (the risk premium effect). Given that the risk premium effect is found to be strong and statistically significant, the two-asset world assumption which has long underpinned conventional macro models, including the class of models with microfoundations, becomes inherently distorting. This problem can be ameliorated by introducing an explicit role of money into the model.

The organisation of this chapter is as follows. Section 2.2 illustrates a literature review on the independent role of money in the monetary transmission mechanism. Section 2.3 discusses the theoretical background. Section 2.4 illustrates the empirical methodology while section 2.5 shows the empirical evidence. Section 2.6 provides policy implications and concluding remarks.

[2]The fact that financial markets in developing countries are immaturely established implies that the transmission process in these countries should rely less on the standard channel via money and bond markets and may therefore rely more on the non-standard channels via changes in relative prices of various assets. As will be elaborated below, it is precisely these non-standard channels for which Meltzer (2001b) argues that money may serve as an auxilary proxy. In this light, it is natural to expect strong evidence in support of an independent role of money in a developing country such as Thailand.

2.2 A Literature Review

2.2.1 A conventional macro model with no explicit role of money

A version of the standard closed economy macro model with microfoundations in the spirit of Clarida, Gali and Gertler (1999) can be described as follows:

$$\widetilde{y}_t = -a_1[R_t - E_t(\pi_{t+1})] + E_t\widetilde{y}_{t+1} + \varepsilon_t^y \quad (2.1)$$

$$\pi_t = b_1\widetilde{y}_t + b_2E_t(\pi_{t+1}) + \varepsilon_t^\pi \quad (2.2)$$

$$R_t = c_1\widetilde{y}_t + c_2(\pi_t - \pi^*) + \varepsilon_t^i \quad (2.3)$$

$$m_t = d_1 + d_2[R_t - E_t(\pi_{t+1})] + d_3\widetilde{y}_t + \varepsilon_t^m \quad (2.4)$$

where $\widetilde{y}_t$ denotes the output gap[3], π_t denotes inflation, π^* denotes the inflation target, R_t denotes the (nominal) short-term interest rate, m_t denotes the real money stock, $\varepsilon_t^y, \varepsilon_t^\pi, \varepsilon_t^i, \varepsilon_t^m$ are *i.i.d.* disturbance terms with zero mean and σ^2 variance, and $E_t(\cdot)$ is the rational expectation operator conditional on the information available in period t.

Equations (2.1) and (2.2) are a standard forward-looking IS equation and a standard forward-looking Phillips curve equation, respectively. Equation (2.3) is a Taylor-type rule. Equation (2.4) is the derived real money demand equation. The transmission mechanism of monetary policy in this typical model works as follows. The Central Bank sets the nominal policy rate via equation (2.3). Due to nominal rigidity in price setting, an increase in the policy rate increases the real interest rate. Consequently, rational agents demand more bonds and less money, and reduce aggregate consumption and output. The equilibrium money stock is supplied by the Central Bank to satisfy the demand for money (equation (2.4)).

This transmission mechanism represents the standard interest rate channel. Apparently, equations (2.1) to (2.3) sufficiently determine the dynamic behaviour of output, inflation and interest rate without requiring further information on the money stock. In other words, the LM curve, equation (2.4), is not part of the simultaneous structure of the model and the real money stock, therefore, does not play an independent role beyond that summarised by the interest rate.

In the literature, several arguments have been proposed concerning the plausibility that money may have an independent role in the monetary transmission process. These are the real balance effect, the transaction cost effect and the argument that money serves as an auxiliary proxy for

[3] In the standard optimisation-based IS-LM framework, the output gap is defined as the deviation of output from its natural level, which in turn is defined as the output level at the flexible price.

unidentified transmission channels.

2.2.2 The real balance effect

The idea that the money stock is part of agents' net wealth can be traced back to Pigou (1943) and Patinkin (1965). The underlying idea is that, with the presence of nominal rigidity in price adjustment, an increase in the nominal money stock also increases the 'real' money stock. As the real money stock, which is part of agents' net wealth, increases aggregate output should expand by more than what the conventional interest rate channel suggests. In other words, equation (2.1) is misspecified as it should have incorporated the real money stock as one of the right hand side variables. Ireland (2001b) has formalised 'the real balance effect' into an otherwise standard dynamic stochastic general equilibrium model. The main result shows that there is no liquidity trap and monetary policy remains effective through the real balance effect even when the nominal interest rate hits the zero bound.

Although the real balance effect is likely to prevail, its magnitude is arguably small. As pointed out by King (2002), the only part of money supply which constitutes the economy's net wealth is monetary base. Since it accounts for a very small fraction of financial wealth, the quantitative significance of the real balance effect is likely to be of second order importance. Moreover, as argued by Metzler (1951), a monetary expansion usually requires an exchange of money for bonds. As bonds are also part of agents' financial wealth, the initial real balance effect may therefore be mitigated.[4]

2.2.3 The transaction-cost effect and non-additive separability in the utility function

McCallum (2001) and Ireland (2001a) have formalised the idea that holding money helps reduce the transaction cost into the otherwise standard macro model with microfoundations. While McCallum (2001) captures the idea by explicitly adding a transaction cost term in the representative household's budget constraint, Ireland (2001a) and Svensson (2001) relax the standard assumption of additive separability between money and consumption in the representative agent's utility function. After some algebraic manipulation, it could be shown that a real money term enters the derived IS equation explicitly.

However, McCallum (2001) argues that a reasonable parameterisation in the utility function leads to an insignificantly small value of the coefficient of the real money stock in the derived IS

[4]However, the wealth status of bonds has later been challenged by the literature on Ricardian equivalence.

equation. Ireland's (2001a) empirical finding, using M2 as the measure of money, lends support to McCallum's conclusion: the transaction cost effect is arguably small.

2.2.4 Money as an auxiliary proxy for unidentified monetary transmission channels

> "The transmission of monetary policy from initial impulse to final effect involves changes in many relative prices of assets and output. That last statement may seem obvious to many of you, but it is inconsistent with most, if not all, recent work on quarterly, dynamic models of monetary policy" (Meltzer, 2001a, pp.30)

One critical assumption underlying standard macro models is that assets other than money, both financial and real, are *perfect* substitutes. This implies that all these assets can be treated as a single composite goods and the interest rate on the short-term government bonds is a perfectly accurate *stand-in* for all other yields. Agents in these models can therefore be perceived as if they were living in the two-asset world, money and the short-term riskless bonds.

However, owing to the fact that most assets in agents' portfolio are *gross* substitutes, not *perfect* substitutes, Meltzer (2001b), in line with Friedman and Schwartz (1982), Brunner and Meltzer (1993), argues, as the quote above suggests, that monetary policy operates by changing the *relative* yields of these assets. As the short-term riskless yield is no longer an adequate stand-in for all other yields, the assumption that monetary policy operates within the two-asset world may mask important monetary policy transmission channels. Because the demand for money is generally a function of these yields[5], the monetary stock could arguably serve as a good proxy for these unidentified monetary transmission channels.

Meltzer (2001b), following Koenig (1990), tested a two-stage backward looking model of changes in consumption, with changes in real money balances, real interest rates, income, and other variables as arguments of the consumption function using U.S. quarterly data. Similar to Koenig's result, he finds that changes in real money balances have a positively significant effect on changes in consumption even after the short-term interest rate is included as one of the explanatory variables. Meltzer (2001b) concludes from his finding that money plays an independent role in determining aggregate demand even when the role of the *short-term* interest rate has been taken into account. He further argues that the evidence lends support to the idea that money serves as a proxy for relative prices of other assets that are relevant to aggregate demand.

[5] By virtue of the portfolio theory of money demand, see, amongst others, Friedman (1956).

Nelson (2002) employs variant versions of Rudebusch and Svensson's (2002) pure backward-looking IS equation to test for the independent role of money using U.S. and U.K. quarterly data. His specification for the U.S. is given as follows:

$$\begin{aligned} \widetilde{y}_t &= \psi_1 + \psi_2 \widetilde{y}_{t-1} + \psi_3 \widetilde{y}_{t-2} + \psi_3 r_{t-1} + \sum_{j=1}^{4} [\psi_{4,j} \triangle m_{t-j}] + \varepsilon_t \\ \varepsilon_t &\sim N(0, \sigma^2) \end{aligned} \tag{2.5}$$

where $\widetilde{y}_t$ is the output gap, r_t is the real interest rate, and $\triangle m_t$ is real monetary base growth.

Nelson finds that lags of real monetary base growth enter equation (2.5) sizably, positively and significantly even when the *short term* interest rate has been explicitly controlled for.[6] In Nelson's terminology, real monetary base growth has a *'direct effect'* on aggregate demand. Although his result implies that conventional *backward-looking* IS equations, e.g. equation (2.5) with no money terms, are clearly misspecified, he argues that forward-looking IS equations derived from the standard optimisation-based framework, e.g. equation (2.1), are *not*, provided that a portfolio adjustment cost is introduced.[7] However, we shall argue on the contrary; IS equations which are based on the two-asset world assumption, whether or not they have allowed for the forward-looking behaviour of rational agents, are misspecified. In other words, the result that money terms enter equation (2.5) significantly cannot be *fully* rationalised even within the modified optimising IS-LM framework proposed by Nelson (2002). This, as we shall argue below, is owing to the empirical significance of the 'risk premium' effect.

2.3 Theoretical Background: The term structure and risk premium effects

According to Meltzer, unidentified monetary transmission channels that the real money stock might be proxying are the channels which arise from changes in relative prices of a wide array of assets. There are two distinct aspects of changes in these relative prices; one being the changes along the term structure (the term-structure effect) and the other being the changes in relative risk premia (the risk premium effect) amongst different kinds and classes of assets.

[6] For the case of the U.S., in line with Bernanke and Blinder's (1992) conclusion, Rudebusch and Svensson (2002) report that, using M2 as a proxy, real money growth terms enter the backward-looking IS equation insignificantly. However, Nelson (2002) finds that the conclusion does not hold when the monetary base is used as an alternative proxy.

[7] Given this modification, the derived demand for money becomes a function of both short and long term interest rates. This in turn implies that money growth is highly correlated with the long rate.

2.3.1 The term-structure effect

The term structure effect captures the fact that an initial monetary impulse, i.e. a change in the short-term policy rate, changes relative yields along the term structure of interest rate. This implies that aggregate spending should also be a function of longer-term real interest rates, in addition to the real short term rate. Importantly, this effect partially captures the expectation channel of the monetary policy transmission.

To elaborate, when the Central Bank decreases its short-term policy rate, the ultimate effect on aggregate spending, ceteris paribus, depends on agents' beliefs about the persistence of the initial impulse. If agents believe the impulse to be transitory, a decrease in the short rate would not lead to a significant decline in the long-term rate and hence the effect on aggregate spending will not be as strong as it would have been had the policy been believed to be permanent.[8] In this light, the typical backward looking IS equation is misspecified and, as the work of Nelson (2002) shows, the statistical significance of the real monetary base growth term in equation (2.5) could, in one and *only one* respect, be interpreted as evidence in support of the term structure effect.

2.3.2 The risk-premium effect

In addition to the term structure effect, monetary policy also operates through changes in the risk premia component of relative prices of various assets. This effect encompasses several monetary policy transmission channels commonly known in the literature, e.g. the balance sheet channel, the asset price channel, the expectation channel etc., all of which are absent in conventional macro models simply because all assets besides money are assumed to be perfect substitutes.

The balance sheet channel: An unanticipated increase in the short-term policy rate impairs firms' financial position (e.g. through higher interest rate expenses, and unexpectedly lower return on prior investment). As their net worth deteriorated, a higher external financial premium may be required to compensate lenders (i.e. banks) as the default probability increases. Thus lending rates may not increase on a *one-to-one* basis with the policy rate as the *relative* riskiness of bank loans has been altered. This channel has not been incorporated as part of the transmission process in conventional macro models as risky bank loans are treated as perfect substitutes for riskless bonds.

The asset price and expectation channels: In one scenario, rational agents could interpret a reduction in the short-term policy rate as a demand stimulus and thus a signal for future growth. This

[8] In fact, the impact on longer-term yields could go either way. This is because they are influenced by current and expected short-term yields. The outcome therefore depends upon the direction and the extent of the impact of the policy rate changes on the expectation of the future path of interest rates.

unexpectedly good news leads to a reduction in the relative risk premia of various risky financial assets, e.g. equities. A decrease in relative yields of equities (higher prices) compared to those of riskless bonds could in turn produce an additional wealth effect, further stimulating aggregate demand. In contrast, rational agents could interpret a reduction in the short-term rate as a sign of the authority being pre-emptive against future recession. As the relative risk premia increases, stock prices decline, the negative wealth effect could therefore work to attenuate the initial stimulus effect on aggregate demand.

2.3.3 Separating the risk-premium effect from the term-structure effect

Although Nelson's (2002) and Rudebusch and Svensson's (2002) empirical IS specifications can be employed to test for the direct effect of the money stock on aggregate demand, they *cannot* be used to separately identify the risk premium effect from the term structure effect. This is because their specifications are of a pure backward looking type and therefore include only the real short-term interest rate. Based on the theoretical argument of Meltzer (2001b), this implies that the existence of real monetary base growth in equation (2.5) could proxy either real longer-term rates (the term structure effect) or relative yields of other risky assets (the risk premium effect).

The distinction between the two is important to our understanding of the monetary transmission mechanism. To understand this, iterate equation (2.1) forward to obtain:

$$\begin{aligned} \widetilde{y}_t &= E_t \sum_{j=0}^{\infty} [-a_1(R_{t+j} - \pi_{t+j})] \\ &= -a_1 E_t \sum_{j=0}^{\infty} [(R_{t+j} - \pi_{t+j})] \end{aligned} \tag{2.6}$$

Applying the expectation theory of the term structure of interest rates, equation (2.6) can be rewritten as:

$$\widetilde{y}_t = -a_1 r_t^l \tag{2.7}$$

where r_t^l is defined as the real long-term interest rate.

The above equation emphasises the point stressed by Rotemberg and Woodford (1999) and Clarida, Gali and Gertler (1999) in that it is the real *long-term* interest rate that matters for aggregate demand in optimisation-based forward looking macro models. Hence, except for the pure backward looking type, conventional macro models have already *implicitly* taken into account the term structure effect. However, as all assets other than money are treated as perfect substitutes in these models, the risk premium effect has not been incorporated as part of the monetary policy transmission mechanism.

In order to identify the risk premium effect, we base our analysis on the aforementioned Meltzer's argument and estimate an equation which adds money terms into the otherwise standard *hybrid* IS equation. As the hybrid IS equation allows for both forward and backward looking behaviours of rational agents, following the above line of argument, if the money terms enter the hybrid IS equation sizably and significantly, we may interpret the results as evidence in support of the prevalence of the risk premium effect.

The above line of argument, as equations (2.6) and (2.7) clearly show, depends largely on the validity of the expectation theory of the term structure. As its empirical justification is largely controversial[9], to ensure the validity of our result, we also *explicitly* control for the term structure effect by adding a proxy for the real long-term interest rate into the hybrid IS specification. The detail will be given in the next section.

Indeed, if the risk premium effect is found to be empirically *in*significant, we could then infer that the interest rate channel currently identified in conventional macro models is sufficient to capture the main transmission process. Moreover, the widely adopted two-asset world assumption would be a justifiable simplifying assumption. Another implication is that real monetary growth would have no independent role in the *forward-looking* class of models as the term structure effect has already been encapsulated.

In contrast, if the risk premium effect is found to be empirically and sizably significant, the validity and completeness of conventional macro models which are based on the two-asset world assumption become seriously doubtful, particularly in light of its being a tool to identify and understand the transmission mechanism of monetary policy. As typical IS equations derived from the standard optimising agent framework are based on the two-asset world assumption, they are misspecified. Thus, the claim made by Nelson (2003) and McCallum and Nelson (1999) that *"while recognizing many distinct assets 'is clearly correct for some purposes..., disaggregation provides benefits but also costs, so two-asset models will often prove convenient and satisfactory'"* (McCallum and Nelson, 1999, page 298-299) would become unjustified and that the problem imposed by the two-asset world assumption could be ameliorated by explicitly taking into account the independent role of money in the model.

[9] See, amongst others, Thornton (2000).

2.4 Empirical Methodology

2.4.1 The backward looking IS specifications: *The direct effect of the money stock*

We first estimate a version of pure backward looking IS equations along the line of Nelson (2002) in order to investigate whether the conclusion that he obtained for the U.S. and U.K., i.e. the real money stock contains information content over and above that captured by the real *short-term* rate, holds when using Thailand data. The sample covers from the period 1993:Q1 to 2002:Q2.[10][11] However, as Thailand is a small-open economy, the baseline specification has to be modified in order to allow for open-economy factors. Specifically, we estimate the following backward looking IS equation:

$$\widetilde{y}_t = \beta_1 + \beta_2 \widetilde{y}_{t-1} + \beta_3 \widetilde{y}_{t-2} + \beta_4 \widetilde{y}_{t-3} + \beta_5 r_{t-1} + \beta_6 \Delta y^w_{t-1} + \beta_7 \Delta q_{t-1} + \beta_8 \Delta m_{t-1} + \varepsilon_t \tag{2.8}$$

$$r_t \equiv \frac{1}{4}\left(\sum_{i=0}^{3} R_{t-i}\right) - \Delta_4 p_t \tag{2.9}$$

$$\Delta y^w_t \equiv \frac{tr_{J,t}}{tr_{J,t} + tr_{US,t} + tr_{S,t}} \Delta y^J_t + \frac{tr_{US,t}}{tr_{J,t} + tr_{US,t} + tr_{S,t}} \Delta y^{US}_t + \frac{tr_{S,t}}{tr_{J,t} + tr_{US,t} + tr_{S,t}} \Delta y^S_t \tag{2.10}$$

$$m_t = M_t - p_t \tag{2.11}$$

$$\varepsilon_t \sim N(0, \sigma^2)$$

$\widetilde{y}_t$ is Thailand output gap, defined as the deviation of (log) seasonally adjusted real GDP of Thailand (y_t) from the potential output (y^*_t). As the potential output is not observable and alternative detrending filters may plausibly extract different types of information from the data (Canova, 1998), we use four methods to estimate y^*_t (and therefore $\widetilde{y}_t$) as a means to check for robustness. These are linear detrending (LT), quadratic detrending (QT), Hodrick-Prescott filtering (HP), and Beveridge

[10] This is owing to the limited availability of Thailand's quarterly GDP data. One common way of dealing with the relatively short sample size of the GDP series is to use Manufacturing Production Index (MPI), which is available from 1989 in monthly frequency, as a proxy for real GDP. However, based on a simple correlation statistics, we found that the ability of such index to proxy real GDP is quite poor. To elaborate, we used Hodrick-Prescott (HP) Filtering method to estimate the output gap series from 1989:M1-2002:M6, where real GDP is proxied by MPI index. The series is then quarterly averaged and compared with that estimated using the original quarterly real GDP data over the period 1993:Q1-2002:Q2. We found that the correlation of the two series is only 0.56 over this period. One possible reason for this finding may be that Thailand's MPI is compiled based on a restricted sample size (255 producers). As noted by Jiwaskapimat (2000), owing to the limitation of staff and legal power, the index accounts for 62.4% of the total manufacturing sector in 1995. We also found that if the same correlation is computed using the Industrial Production Index (IPI) and real quarterly GDP of the U.S., which has comparatively more resources and expertise in compiling the IPI, the correlation becomes 0.81. Given the limitation of Thailand's MPI, the analysis throughout this chapter is based on the original quarterly real GDP series, bearing in mind when interpreting the results that such analysis is based on a relatively short sample.

[11] Unless stated otherwise, the source of data is from the Bank of Thailand.

and Nelson's (1981) decomposition (BN) methods.[12] Figure 2.1 shows the results of estimated y_t^* and $\widetilde{y}_t$ obtained from the four detrending methods. r_t is the real short-term interest rate which is explicitly defined in equation (2.9). R_t is the short-term policy rate, defined as the quarterly averaged RP14 rate. $\triangle_4$ is the fourth-difference operator and p_t is (log) core consumer price index. Thus the real short-term interest rate, r_t, that we use here, following Nelson (2002) and Rudebusch and Svensson (2002), is a smoothed version of the pseudo-real interest rate. $\triangle y_t^w$ is a proxy for the world output growth, and is defined in equation (2.10). Specifically, $\triangle y_t^w$ is the weighted average of the first difference of (log) seasonally adjusted real GDP of the top-three trading partners of Thailand, namely Japan (J), U.S. (US) and Singapore (S).[13][14] $tr_{i,t}$ is the total value of export plus import between Thailand and country i, where $i = J, US, S$. $\triangle q_t$ is the first difference of (log) Thailand real effective exchange rate (REER), where an increase in q_t indicates a real appreciation in Thai baht.[15] M_t is the (log) quarterly average of the (seasonally adjusted) monetary stock. We use three proxies for this variable, namely M0 (monetary base), M1 and M2, as an additional means to check for robustness. Given these definitions, equation (2.8) is estimated using Ordinary Least Square (OLS) method.[16]

As shown in Table 2.1, Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests indicate that the null hypothesis of the series being nonstationary I(1) can be rejected at the 10 percent level of significance for all variables included in equation (2.8) except for r_t. In the case of r_t, the result is ambiguous as ADF test could reject the null that the series is I(1) while PP test could not. To ensure that the empirical result obtained from equation (2.8) is not sensitive to the ambiguous stationarity property of r_t, we also regress the following equation, equation (2.12), using OLS method where r_{t-1} in equation (2.8) is replaced by its first difference, $\triangle r_{t-1}$.

$$\begin{aligned}
\widetilde{y}_t &= \gamma_1 + \gamma_2\widetilde{y}_{t-1} + \gamma_3\widetilde{y}_{t-2} + \gamma_4\widetilde{y}_{t-3} + \gamma_5\triangle r_{t-1} + \gamma_6\triangle y_{t-1}^w + \gamma_7\triangle q_{t-1} \\
&\quad + \gamma_8\triangle m_{t-1} + \varepsilon_t \qquad (2.12) \\
\varepsilon_t &\sim N(0, \sigma^2)
\end{aligned}$$

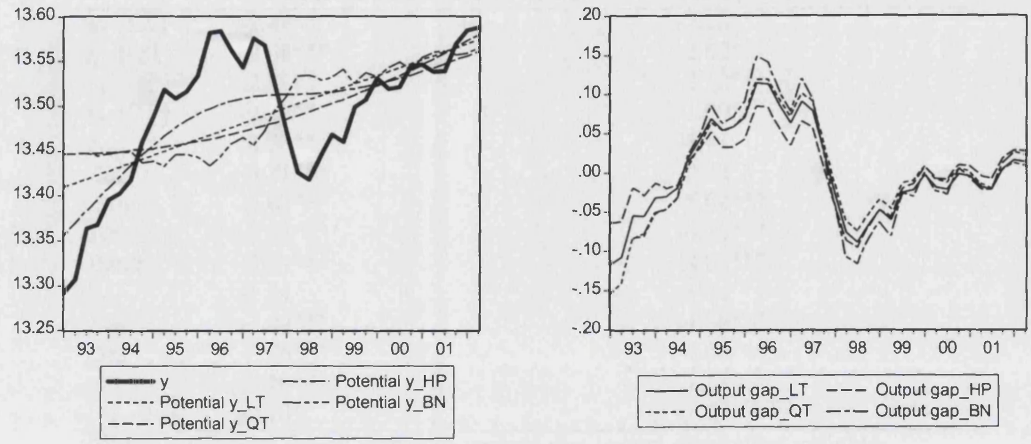
[12]For the BN method, we use a quick computational procedure proposed by Cuddington and Winters (1987) where the initial value of the potential output is taken to be that estimated by the QT method. Various ARMA(p,q) models are initially estimated on the changes in log seasonally adjusted real GDP up to ARMA(3,3) and the Akaike Info Criterion is used to select the best model, which turned out to be ARMA(2,2).

[13]The value of trade (export+import) between Thailand and the top three trading partners accounts, on average, for approximately 45 percent of Thailand's total external trading.

[14]The source of real quarterly GDP data for U.S. and Japan is IMF International Financial Statistics and that for Singapore is Singapore Department of Statistics.

[15]We use $\triangle q_t$ instead of q_t in equation (2.8) because, as Table 2.1 shows, the null hypothesis that q_t is I(1) cannot be rejected at the 10 percent level of significance.

[16]Higher order lags of $\triangle m_t$ are included in the preliminary regressions analogous to equation (2.8) (not reported), but are found to be statistically insignificant in most specifications. They are therefore dropped.



Note: LT, QT, HP, and BN denote Linear Detrending, Quardratic Detrending, Hodrick-Prescott Filtering, and Beveridge-Nelson Decomposition, respectively.

Figure 2.1: Estimated potential output and output gap from four detrending methods

As will be shown in section 2.5, the main results obtained from regressing equations (2.8) and (2.12) are not sensitive to the specification of the interest rate term. Thus, throughout the rest of the chapter, we shall assume that r_t is a stationary series.

2.4.2 The Hybrid IS Specifications: *The risk premium effect*

In order to investigate the risk premium effect, a version of the small-open economy hybrid IS equation is employed. Following Gali and Monacelli (2002) and Clarida, Gali and Gertler (2001), a small-open economy IS equation with microfoundations can be written as follows[17]:

$$\widetilde{y}_t = E_t(\widetilde{y}_{t+1}) + \delta_1[R_t - E_t(\pi_{t+1})] + \delta_2 E_t(\triangle y^w_{t+1}) \tag{2.13}$$

where, as usual, $\widetilde{y}_t$ is the domestic-economy output gap, R_t is the nominal short term rate, π_t is CPI inflation, and y^w_t is the (log) world output.

Equation (2.13) is similar to a standard optimisation-based IS equation found in its closed-economy counterpart (i.e. Clarida, Gali and Gertler, 1999) except that elements representing 'the rest of the world' are factored in.[18] More specifically, the coefficients in the open-economy equi-

[17]Other open economy optimisation-based models include Svensson (2000) and Obsfield and Rogoff (2000), among others.

[18]For simplicity, we assume that the discount factor of a representative agent in the domestic economy is equal to unity and the domestic production technology parameter follows a random noise process.

	Augmented Dickey-Fuller (ADF) test	Phillips-Perron (PP) test
$\tilde{y}_t$ (LT)	-2.47**	-2.26**
$\tilde{y}_t$ (QT)	-2.68***	-2.52**
$\tilde{y}_t$ (HP)	-2.48**	-2.13**
$\tilde{y}_t$ (BP)	-2.15**	-1.90*
r_t	-3.26**	-1.41
Δr_t	-3.31***	-2.43**
$\Delta m0$	-5.60***	-5.64***
$\Delta m1$	-3.84***	-5.35***
$\Delta m2$	-3.31**	-4.00***
q_t	-1.94	-1.54
Δq_t	-4.49***	-4.56***
Δy_t^w	-3.76***	-4.65***
r_t^l	-3.10**	-1.18
$(r_t^l - r_t)/r_t$	-2.78*	-1.30

Note: ADF tests for all variables include one lagged dependent variable. For PP tests, the number of lag truncation is set to 3. Except for $\tilde{y}_t$ (LT, QT, HP,BN) and Δr_t , all tests include a constant term. ***, **,* indicate significance at the 1, 5, and 10 percent level.

Table 2.1: Unit root tests

librium, δ_1 and δ_2, also depend on parameters that are specific to the open economy (the index of openness and the elasticity of substitution between domestic and foreign goods), while fluctuations in world output also matter for domestic aggregate demand. The key assumption underpinning the derivation of equation (2.13) is that the uncovered interest parity (UIP) relationship holds. Given this assumption, the real exchange rate term does not *explicitly* appear as a determinant of the output gap in the reduced-form IS equation, equation (2.13). In particular, the UIP relationship is used to substitute the term away, and the effect of the real exchange rate on aggregate demand is *implicitly* captured by the coefficient of the real interest rate, δ_1. Given that the Central Bank uses the interest rate as its instrument in conducting monetary policy operation, the real exchange rate becomes *endogenously* determined in the model and its movement is *fully* dictated by the UIP relationship.[19] However, the empirical evidence on the UIP relationship has been mostly discouraging (see amongst others, Froot and Thaler, 1990). This implies that, as the Central Bank changes the interest rate, its effect on aggregate demand via the exchange rate channel may not be *fully* captured by the coefficient of the real interest rate term and the standard 'imported inflation' effect via CPI inflation (π_{t+1}). To account for the remaining effect, we *explicitly* add the expected lead of real exchange rate growth ($E_t(\triangle q_{t+1})$) as one of the determinants of the output gap. This is given in the following equation:

[19] Similarly, if the Central Bank uses the exchange rate as the instrument, the model implies that the interest rate will become endogenous via the UIP relationship.

$$\widetilde{y}_t = E_t(\widetilde{y}_{t+1}) + \sigma_1[R_t - E_t(\pi_{t+1})] + \sigma_2 E_t(\triangle q_{t+1}) + \sigma_3 E_t(\triangle y^w_{t+1}) \tag{2.14}$$

Furthermore, to improve the empirical fit, we assume that the output gap is a convex combination of lagged output gap and the right hand side of equation (2.14). This gives the following *hybrid* IS equation:

$$\widetilde{y}_t = \varpi \widetilde{y}_{t-1} + (1-\varpi)\Big[E_t(\widetilde{y}_{t+1}) + \sigma_1[R_t - E_t(\pi_{t+1})] + \sigma_2 E_t(\triangle q_{t+1}) + \sigma_3 E_t(\triangle y^w_{t+1})\Big] \tag{2.15}$$

In order to test for the existence of the risk premium effect, we add both contemporaneous and lagged real money growth terms in equation (2.15). After some algebraic manipulation, the following specification is obtained:

$$\begin{aligned} \widetilde{y}_t &= \phi_1 \widetilde{y}_{t-1} + \phi_2 E_t(\widetilde{y}_{t+1}) + \phi_3[R_t - E_t(\pi_{t+1})] + \phi_4 E_t(\triangle q_{t+1}) + \phi_5 \triangle m_t \\ &\quad + \phi_6 \triangle m_{t-1} + \phi_7 E_t(\triangle y^w_{t+1}) + \varepsilon^y_t \\ \varepsilon^y_t &\sim N(0, \sigma^2) \end{aligned} \tag{2.16}$$

Assume for simplicity that $[R_t - E_t(\pi_{t+1})]$ is approximately equal to the 'pseudo' real short-term interest rate r_t defined earlier, equation (2.16) can be written as:

$$\widetilde{y}_t = \phi_1 \widetilde{y}_{t-1} + \phi_2 \widetilde{y}_{t+1} + \phi_3 r_t + \phi_4 \triangle q_{t+1} + \phi_5 \triangle m_t + \phi_6 \triangle m_{t-1} + \phi_7 \triangle y^w_{t+1} + v_t \tag{2.17}$$

where $v_t \equiv \phi_2[E_t(\widetilde{y}_{t+1}) - \widetilde{y}_{t+1}] + \phi_4[E_t(\triangle q_{t+1}) - \triangle q_{t+1}] + \phi_7[E_t(\triangle y^w_{t+1}) - \triangle y^w_{t+1}] + \varepsilon^y_t$ is the linear combination of the forecast errors of the output gap, the forecast errors of lead real exchange rate growth, the forecast errors of world output growth, and the exogenous random disturbance (ε^y_t). The disturbance term v_t is correlated with two of the regressors ($\widetilde{y}_{t+1}, \triangle y^w_{t+1}$), hence standard least square estimators become biased and inconsistent. Moreover, v_t suffers from serial correlation problems. This in turn invalidates any statistical inference made using typical (uncorrected) least squared standard errors. To account for these problems, we use General Method of Moments (GMM) as a means of estimation.[20] Let Z_t be the vector of variables within agents' information set at time t that are orthogonal to the disturbance term v_t. Plausible elements in Z_t include any lagged variables

[20] A GMM estimator in this case is a consistent (though not necessarily efficient) estimator. It is tantamount to a Two Stage Least Square (TSLS) estimator but standard errors are corrected to allow for the plausibility of autocorrelation problems by using Newey and West's (1987) Heteroskedastic and Autocorrelation Consistent (HAC) estimator of the asymptotic covariance matrix.

that help forecast the output gap. Since $E_t[v_t \mid Z_t]=0$, equation (2.17) implies the following set of orthogonality conditions that we exploit for estimation:

$$E_t[\widetilde{y}_t - \phi_1\widetilde{y}_{t-1} - \phi_2\widetilde{y}_{t+1} - \phi_3 r_t - \phi_4\triangle q_{t+1} - \phi_5\triangle m_t - \phi_6\triangle m_{t-1} - \phi_7\triangle y^w_{t+1} \mid Z_t] = 0 \qquad (2.18)$$

The instrument set Z_t includes two lags of each variable in equation (2.17).[21] Since the potential instrument set-and hence the number of orthogonality conditions-exceeds the number of parameters to be estimated, the model is over-identified, in which case we employ Hansen's (1982) J-statistic to test for the validity of the over-identifying restriction. If the null hypothesis is violated, it implies that the hypothesis of the model that had led to the moment equations in the first place is incorrect and at least some of the sample moment conditions are systematically violated. It is worth noting that the J-test is based on an asymptotic property. As the sample size taken in this chapter is not very large, the interpretation of the test result must be done with this caution in mind.

As mentioned, identifying the risk premium effect by estimating equation (2.17) relies on the validity of the expectation theory of the term structure. To guard our result against the plausibility that the theory may not hold, we also *explicitly* incorporate a proxy for the real long-term interest rate into equation (2.17) as an *additional* control for the term structure effect. Ideally, the yields of riskless long term government bonds, e.g. 7 or 10 year riskless T-bonds issued by the Thai government, should be used. However, the series on such yields only began in 1999:Q3, which is obviously too short to be used in any empirical work.[22] We therefore use the quarterly average of state-enterprise (SE) bond yield series released by the Bank of Thailand as an alternative.[23] Although SE bonds are not as default free as T-bonds, judging from the fact that most SE bonds are fully guaranteed by the government and that their yields are highly correlated with those of 7 year T-bonds over the available sample periods[24], we argue that they could serve as a reasonably good proxy for the riskless long term yields.[25]

Analogous to the definition of the real short term rate, the 'pseudo' real SE bond yield is defined as $r^l_t \equiv \frac{1}{4}(\sum_{i=0}^{3} R^l_{t-i}) - \triangle_4 p_t$, where R^l_t is the nominal SE bond yield. In the preliminary regressions

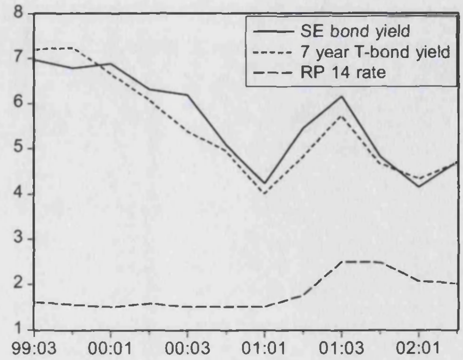
[21] Specifically, $Z_t = [\widetilde{y}_{t-1}, \widetilde{y}_{t-2}, r_{t-1}, r_{t-2}, \triangle m_{t-1}, \triangle m_{t-2}, \triangle y^w_{t-1}, \triangle y^w_{t-2}, \triangle q_{t-1}, \triangle q_{t-2}]$.

[22] One primary reason is that the Thai government had not issued any new government bonds from 1990 to 1997, owing to the long and continuous period of government budget surplus. In 1997, in response to the breakdown of the financial crisis, the Thai government has begun to re-issue government bonds. However, the secondary bond market has not been formally developed until 1999.

[23] The series is calculated by weighted averaging the yields of new issues in each month, where the weight is taken to be the face value of the issuance. Almost all state-enterprise bonds have initial maturities of 3-10 years (mode is equal to 7).

[24] The correlation coefficient = 0.95, see Figure 2.2.

[25] As of 2001, approximately 85 percent of the outstanding values of state-enterprise bonds are completely guaranteed by the government (Source: Thai Bond Dealer Club).



Note: Sample period is dictated by the availability of the 7 year riskless T-bond series. Values on the y-axis are in percentage.

Figure 2.2: State Enterprise (SE) bond yield, 7 year T bond yield, and RP14 rate

(not reported), we directly include r_t^l as an additional regressor in equation (2.17). However, the results suffer from the multicollinearity problem as r_t^l is highly correlated with r_t.[26] To attenuate the problem, we use percentage deviations of r_t^l from r_t, $\left[\frac{(r_t^l - r_t)}{r_t}\right]$, as an alternative *additional* control for the term structure effect. More specifically, the following equation is regressed using GMM method;

$$\begin{aligned}
\widetilde{y}_t &= \lambda_1 \widetilde{y}_{t-1} + \lambda_2 \widetilde{y}_{t+1} + \lambda_3 r_t + \lambda_4 \triangle q_{t+1} + \lambda_5 \triangle m_t + \lambda_6 \triangle m_{t-1} \\
&\quad + \lambda_7 \triangle y_{t+1}^w + \lambda_8 \left[\frac{(r_t^l - r_t)}{r_t}\right] + \xi_t \qquad (2.19) \\
\xi_t &\equiv \lambda_2 [E_t(\widetilde{y}_{t+1}) - \widetilde{y}_{t+1}] + \lambda_4 [E_t(\triangle q_{t+1}) - \triangle q_{t+1}] + \lambda_7 [E_t(\triangle y_{t+1}^w) - \triangle y_{t+1}^w] + \varepsilon_t^y
\end{aligned}$$

As before, the instrument set, Z_t, composes of two lags of all variables in equation (2.19).

2.5 Empirical Results

This section reports the estimation results of various equations outlined in the previous section.[27] For each equation, the results of 12 specifications using four different detrending methods (LT, QT, HP, BN) and 3 proxies for the monetary stock (M0 (monetary base), M1, and M2) are shown.

[26] Their correlation over the sample is equal to 0.95.

[27] Throughout the chapter, ***, ** and * denote statistical significance at the 1, 5, and 10 percent level, respectively.

2.5.1 The backward-looking specifications: *The direct effect of the monetary stock*

Table 2.2 reports the estimation results of equation (2.8). For comparison, the first block shows the results for the baseline equation which does not include any monetary term. The three remaining blocks give the results when lagged real money growth is added. The values of the reported R^2 in all specifications which include lagged real money growth are reasonably high and are noticeably higher compared to their baseline counterparts. This implies that adding lagged real money growth in the baseline specification significantly improves the overall goodness of fit. Using Ramsey's RESET test, the null hypothesis of model misspecification cannot be rejected at the 5 percent level in all specifications. Moreover, based on the Ljung-Box Q statistic, the estimation results using OLS method are statistically efficient as no serial correlation is found up to 16 lags. As the sample includes the financial crisis period, structural changes in the IS equation may be suspected. We therefore conduct CUSUM tests and the results indicate that the null hypothesis of no structural instability cannot be rejected at the 5 percent level in all specifications (see Figure 2.3 for the results of the CUSUM test).

One crucial feature that can be seen from Table 2.2 is that the coefficients of lagged real money growth are statistically significant at the 5 percent level in *all* specifications. On the contrary, though correctly signed (negative), the coefficients of lagged real short term rate (r_{t-1}) are statistically significant only in 5 out of 16 specifications. Moreover, their magnitude is much smaller compared to those of the lagged money growth terms. Turning to the variables which represent 'open-economy' factors, the coefficients of $\triangle y^w_{t-1}$ are positive and statistically significant at the 10 percent level in 8 out of 16 specifications. The coefficients of lagged real exchange rate growth ($\triangle q_{t-1}$) are statistically significant at the 10 percent level in approximately 70 percent of the specifications. However, they are of the wrong sign (positive).[28]

[28] One plausible explanation for this adverse result could be because the sample taken in this chapter includes the crisis period. As Figure 2.4 shows, the Bank of Thailand decided to float Thai baht exchange rate in 1997:Q3, which had led to an instantaneous sharp depreciation in the real value of Thai Baht. The exchange rate 'shock' coincided with a subsequent fall in aggregate demand. Statistically, the pairwise correlation between the output gap $\widetilde{y}_t$(HP) and the (log) real effective exchange rate q_t, over the full sample is 0.356. If one restricts the sample to the post-crisis period, 1998:Q1-2002:Q2, for which the exchange rate arguably behaved less abnormally, the correlation becomes correctly signed, -0.651.

It is worth emphasising that, even though the sample covers the period in which the exchange rate movement is irregular, particularly owing to the shift from the fixed to the flexible exchange rate regime, based on the result of CUSUM test discussed earlier, this does not affect the overall structural stability property of the estimated IS equation in a statistically significant way.

Estimating $\tilde{y}_t = \beta_1 + \beta_2\tilde{y}_{t-1} + \beta_3\tilde{y}_{t-2} + \beta_4\tilde{y}_{t-3} + \beta_5 r_{t-1} + \beta_6\Delta y^w_{t-1} + \beta_7\Delta q_{t-1} + \beta_8\Delta m_{t-1} + \epsilon_t$ [Equation (2.8)]																
	Baseline model (no money stock)				$m = m0$ (monetary base)				$m = m1$				$m = m2$			
	LT	QT	HP	BN	LT	QT	HP	BN	LT	QT	HP	BN	LT	QT	HP	BN
cons	0.007 (0.008)	0.006 (0.008)	0.009 (0.008)	0.01 (0.012)	0.001 (0.001)	-0.001 (0.008)	0.003 (0.007)	0.001 (0.011)	-0.001 (0.007)	-0.004 (0.007)	0.0001 (0.007)	-0.001 (0.011)	0.004 (0.008)	0.003 (0.008)	0.006 (0.007)	0.004 (0.011)
$\tilde{y}_{t-1}$	1.151 (0.17)***	1.164 (0.17)***	1.056 (0.17)***	1.111 (0.18)***	1.001 (0.17)***	1.011 (0.17)***	0.912 (0.17)***	0.966 (0.18)***	1.011 (0.15)***	1.039 (0.15)***	0.922 (0.15)***	0.955 (0.17)***	0.978 (0.17)***	1.005 (0.17)***	0.881 (0.17)***	0.873 (0.19)***
$\tilde{y}_{t-2}$	-0.304 (0.27)	-0.299 (0.27)	-0.317 (0.26)	-0.268 (0.28)	-0.242 (0.24)	-0.239 (0.25)	-0.256 (0.23)	-0.239 (0.26)	-0.327 (0.23)	-0.319 (0.23)	-0.34 (0.21)	-0.296 (0.24)	-0.262 (0.25)	-0.263 (0.25)	-0.272 (0.24)	-0.155 (0.26)
$\tilde{y}_{t-3}$	0.054 (0.18)	0.032 (0.18)	0.114 (0.19)	0.029 (0.2)	0.109 (0.17)	0.096 (0.16)	0.156 (0.17)	0.115 (0.18)	0.262 (0.17)	0.215 (0.162)	0.33 (0.17)*	0.277 (0.19)	0.148 (0.17)	0.128 (0.17)	0.183 (0.17)	0.066 (0.18)
r_{t-1}	-0.001 (0.001)	-0.001 (0.001)	-0.001 (0.001)	-0.001 (0.001)	-0.001 (0.001)	-0.001 (0.001)	-0.001 (0.001)*	-0.001 (0.001)	-0.001 (0.001)	-0.001 (0.001)	-0.001 (0.001)	-0.001 (0.001)	-0.002 (0.001)**	-0.002 (0.001)*	-0.002 (0.001)**	-0.003 (0.001)**
Δy^w_{t-1}	0.505 (0.51)	0.536 (0.52)	0.493 (0.48)	0.663 (0.72)	0.983 (0.5)*	1.037 (0.5)**	0.913 (0.46)*	1.332 (0.703)*	0.875 (0.44)*	0.918 (0.46)*	0.822 (0.41)*	1.162 (0.64)*	0.43 (0.47)	0.474 (0.48)	0.385 (0.44)	0.582 (0.65)
Δq_{t-1}	0.136 (0.07)*	0.133 (0.07)*	0.118 (0.07)*	0.191 (0.1)*	0.106 (0.06)	0.103 (0.06)	0.088 (0.06)	0.151 (0.09)	0.162 (0.06)***	0.16 (0.06)**	0.141 (0.06)**	0.233 (0.085)**	0.149 (0.06)**	0.148 (0.07)**	0.127 (0.06)**	0.217 (0.09)**
Δm_{t-1}					0.274 (0.1)**	0.279 (0.1)**	0.258 (0.1)**	0.373 (0.15)**	0.406 (0.12)***	0.396 (0.12)***	0.388 (0.11)***	0.547 (0.17)***	0.734 (0.3)**	0.704 (0.3)**	0.698 (0.28)**	1.215 (0.47)**
R^2	0.874	0.875	0.824	0.849	0.9	0.902	0.862	0.88	0.914	0.912	0.881	0.891	0.897	0.896	0.858	0.881
Adj-R^2	0.847	0.848	0.787	0.814	0.874	0.876	0.826	0.846	0.891	0.889	0.851	0.861	0.87	0.869	0.821	0.848
RESET test (p-value)	0.715	0.846	0.756	0.889	0.73	0.962	0.995	0.959	0.996	0.991	0.792	0.917	0.764	0.472	0.698	0.343
Note: OLS estimates; Sample (unadjusted): 1993Q1-2002Q2; Numbers in parentheses are standard errors; ***, **, * indicate significance at the 1, 5, and 10 percent, respectively.																

Table 2.2: Estimation result: equation (2.8)

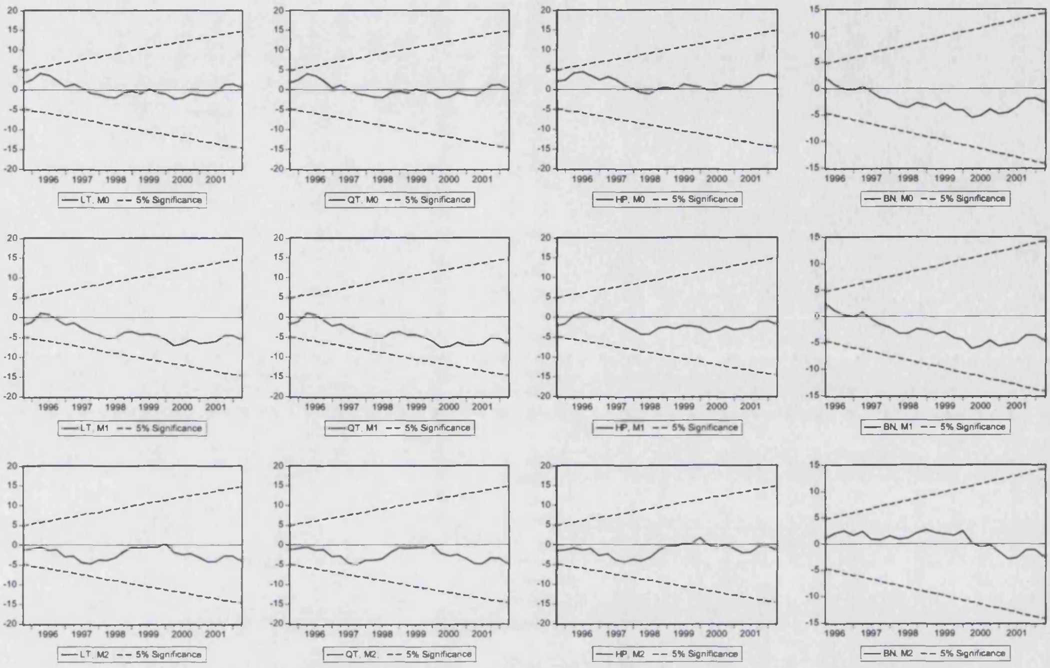
The next table, Table 2.3, shows the estimation results of equation (2.9) where r_{t-1} in equation (2.8) is replaced by $\triangle r_{t-1}$.[29] Compared to the results shown in Table 2.2, the coefficients of lagged real money growth remain positively significant at the 5 percent level in the $m = m0$ and $m = m1$ specifications and is positively significant at the 10 percent level in the $m = m2$ specification. Crucially, this confirms that the evidence found in Table 2.2 that lagged real money growth enters the aggregate demand equation sizably, positively and significantly is not sensitive to the ambiguous stationarity property of the real interest rate. On the contrary, in none of the specifications does $\triangle r_{t-1}$ enter significantly at the 10 percent level.[30] [31] For $\triangle y^{w}_{t-1}$, its effect on aggregate demand is much less strong compared to that shown in Table 2.2 as its coefficient is statistically significant at the 10 percent in only one specification. Lastly, $\triangle q_{t-1}$ remains wrongly signed and statistically significant at the 10 percent level in most specifications.

All in all, the results obtained from Tables 2.2 and 2.3 signify the strong prevalence of the so-called 'direct effect' of lagged real monetary growth on aggregate demand. This implies that the monetary stock has information content concerning aggregate demand fluctuations *over and above* that captured by the short-term interest rate. In contrast, the real *short-term* interest rate (both in level and its first difference) performs much poorly as a direct determinant of aggregate demand. These results are consistent and if anything more forceful than those found by Nelson (2002) for the U.S. and U.K. as the results shown here are robust against alternative detrending methods and different proxies for real monetary growth.

[29] Similar to Table 2.2, the overall goodness of fit and all relevant diagnostic checking tests [RESET test, Ljung-Box Q statistics, CUSUM test] justify the validity of statistical inference made using the estimated results shown in Table 2.3.

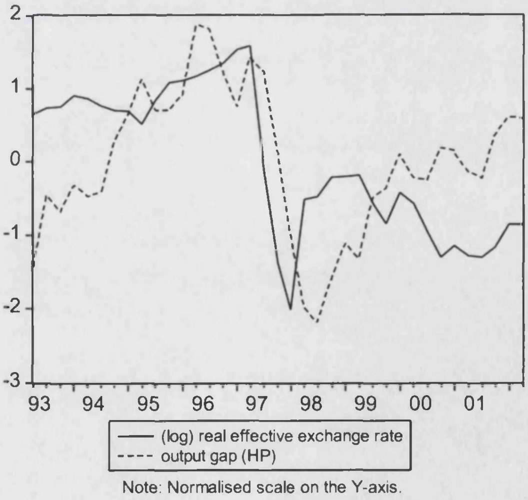
[30] The evidence that interest rate terms generally enter IS equations insignificantly is indeed consistent with what have been found for the U.S. and U.K. In particular, Nelson (2002) reported that, when removing the pre-1982 sample, the interest rate terms enter his backward looking IS specification insignificantly and wrongly signed. This conclusion holds for both the U.S. and U.K. economies.

[31] Although the real short term rate (the Central Bank's monetary policy instrument) is consistently found to have very little, in several cases no, *direct* effect on aggregate demand, monetary policy could at least in principle remains effective in affecting aggregate demand via its *indirect* effect on the relative yields of other assets. To the extent that money serves as a good proxy for these yields, the evidence found in this chapter suggests that it is this *'indirect effect'* that matters for aggregate demand.



Note: LT, QT, HP, and BN denote linear detrending, quadratic detrending, Hodrick-Prescott filtering, and Beveridge-Nelson's decomposition.

Figure 2.3: CUSUM test; equation (2.8)



Note: Normalised scale on the Y-axis.

Figure 2.4: (Log) Real effective exchange (q_t) rate and output gap ($\tilde{y}_t$)

Estimating $\tilde{y}_t = \gamma_1 + \gamma_2\tilde{y}_{t-1} + \gamma_3\tilde{y}_{t-2} + \gamma_4\tilde{y}_{t-3} + \gamma_5\Delta r_{t-1} + \gamma_6\Delta y^w_{t-1} + \gamma_7\Delta q_{t-1} + \gamma_8\Delta m_{t-1} + \epsilon_t$ [eq. (2.9)]

	Baseline model (no money stock)				$m = m0$ (monetary base)				$m = m1$				$m = m2$			
	LT	QT	HP	BN	LT	QT	HP	BN	LT	QT	HP	BN	LT	QT	HP	BN
cons	-0.0003 (0.005)	-0.0001 (0.005)	-0.0001 (0.005)	0.0002 (0.007)	-0.006 (0.005)	-0.006 (0.005)	-0.006 (0.005)	-0.008 (0.007)	-0.008 (0.005)	-0.007 (0.005)	-0.007 (0.004)	-0.01 (0.007)	-0.007 (0.006)	-0.007 (0.006)	-0.006 (0.006)	-0.01 (0.009)
$\tilde{y}_{t-1}$	1.182 (0.17)***	1.179 (0.17)***	1.113 (0.17)***	1.147 (0.18)***	1.021 (0.17)***	1.014 (0.17)***	0.954 (0.16)***	0.991 (0.18)***	1.039 (0.15)***	1.051 (0.15)***	0.957 (0.15)***	0.988 (0.17)***	1.08 (0.17)***	1.071 (0.17)***	1.023 (0.17)***	1.011 (0.19)***
$\tilde{y}_{t-2}$	-0.241 (0.27)	-0.247 (0.27)	-0.233 (0.26)	-0.22 (0.29)	-0.177 (0.25)	-0.183 (0.25)	-0.168 (0.24)	-0.184 (0.26)	-0.287 (0.23)	-0.297 (0.23)	-0.272 (0.22)	-0.261 (0.25)	-0.179 (0.26)	-0.182 (0.26)	-0.173 (0.25)	-0.117 (0.28)
$\tilde{y}_{t-3}$	-0.021 (0.17)	-0.024 (0.17)	0.073 (0.18)	-0.046 (0.19)	0.046 (0.16)	0.053 (0.15)	0.131 (0.17)	0.053 (0.17)	0.196 (0.16)	0.181 (0.15)	0.316 (0.17)*	0.208 (0.18)	-0.005 (0.16)	0.003 (0.16)	0.085 (0.18)	-0.076 (0.18)
Δr_{t-1}	-0.003 (0.003)	-0.002 (0.003)	-0.005 (0.003)	-0.002 (0.005)	-0.003 (0.003)	-0.003 (0.003)	-0.005 (0.003)	-0.003 (0.004)	-0.001 (0.003)	-0.001 (0.002)	-0.004 (0.003)	-0.001 (0.004)	-0.004 (0.003)	-0.004 (0.003)	-0.006 (0.003)	-0.003 (0.004)
Δy^w_{t-1}	0.302 (0.53)	0.373 (0.52)	0.2 (0.5)	0.468 (0.75)	0.794 (0.51)	0.888 (0.51)*	0.634 (0.47)	1.124 (0.73)	0.752 (0.47)	0.854 (0.47)*	0.593 (0.43)	1.025 (0.68)	0.147 (0.51)	0.212 (0.51)	0.029 (0.49)	0.308 (0.73)
Δq_{t-1}	0.127 (0.07)*	0.125 (0.07)*	0.111 (0.07)	0.183 (0.1)*	0.095 (0.07)	0.092 (0.06)	0.078 (0.06)	0.139 (0.09)	0.158 (0.06)**	0.158 (0.06)**	0.134 (0.06)**	0.227 (0.09)**	0.133 (0.07)*	0.131 (0.07)*	0.116 (0.07)*	0.197 (0.1)*
Δm_{t-1}					0.283 (0.1)**	0.289 (0.1)***	0.27 (0.1)***	0.383 (0.15)**	0.409 (0.12)***	0.401 (0.12)***	0.401 (0.11)***	0.553 (0.18)***	0.523 (0.29)*	0.549 (0.29)*	0.475 (0.28)*	0.832 (0.46)*
R^2	0.87	0.874	0.818	0.843	0.898	0.902	0.859	0.876	0.91	0.911	0.88	0.887	0.884	0.889	0.836	0.861
Adj-R^2	0.843	0.847	0.779	0.807	0.872	0.877	0.822	0.841	0.887	0.888	0.848	0.855	0.854	0.86	0.794	0.822
RESET test (p-value)	0.85	0.89	0.665	0.931	0.644	0.833	0.376	0.671	0.877	0.987	0.34	0.783	0.553	0.116	0.244	0.273

Note: OLS estimates; Sample (unadjusted): 1993Q1-2002Q2; Numbers in parentheses are standard errors; ***, **, * indicate significance at the 1, 5, and 10 percent, respectively.

Table 2.3: Estimation result: equation (2.9)

2.5.2 The hybrid IS specifications: *The risk premium effect*

The empirical results reported in this section show that the finding of the strong existence of a direct effect found in the previous section can partly be attributed to the 'risk-premium' effect. Table 2.4 shows the estimation results of equation (2.17). The evidence of high values of the reported R^2 across all specifications indicates that the specification of the hybrid IS equation is appropriately reasonable.[32] Moreover, the reported value of J-statistic implies that the overidentifying restrictions can be rejected at the 1 percent level in all specifications. This implies that the moment conditions specified and exploited for estimation are reasonably appropriate, though it has to be stressed that the results are based on a large-sample property. The appropriateness of the *hybrid* IS specification is also confirmed by the result that the coefficients of both *lead* and *lagged* output gap are statistically significant in all specifications and that their sum is approximately equal to 1, which is consistent with the value suggested by the theory.[33] Importantly, the significance of the lead terms and the reasonably large values of their coefficients across all specifications, with the value ranging from 0.284 to 0.558, could be interpreted as evidence in favour of the existence of the term structure effect, i.e. agents are reasonably forward looking and therefore take into account the expected future path of interest rates in formulating their decisions.

[32]On the surface, it may appear that inconsistency has arisen as the two basic specifications of aggregate demand, namely the pure backward looking IS specification [equation (2.8)] and the hybrid IS specification [equation (2.17)], simultaneously exhibit a high degree of goodness of fit. However, with the assumption that rational agents form their expectation for future and contemporaneous variables based on the past data (i.e. via the instrument set), the hybrid version of aggregate demand nests the backward looking version. In particular, it implies that rational agents utilise past relevant variables not only as determinants of aggregate demand but also as indicators for forecasting future variables. For example, rational agents' consumption may be a function of lagged interest rates. On the one hand, it could be interpreted that the agents use lagged interest rates as an indicator for their opportunity cost of funds (backward looking argument). On the other hand, it could mean that the agent use lagged interest rates as part of their information set in determining the future interest rate path (forward looking argument).

[33]From equation (2.17), $\phi_1 + \phi_2 = 1$.

Estimating $\tilde{y}_t = \phi_1 \tilde{y}_{t-1} + \phi_2 \tilde{y}_{t+1} + \phi_3 r_t + \phi_4 \Delta q_{t+1} + \phi_5 \Delta m_t + +\phi_6 \Delta m_{t-1} + \phi_7 \Delta y^w_{t+1} + v_t$ [eq. (2.17)]												
	$m = m0$ (monetary base)				$m = m1$				$m = m2$			
	LT	QT	HP	BN	LT	QT	HP	BN	LT	QT	HP	BN
$\tilde{y}_{t-1}$	0.673 (0.08)***	0.652 (0.09)***	0.671 (0.08)***	0.636 (0.1)***	0.568 (0.09)***	0.577 (0.11)***	0.554 (0.06)***	0.565 (0.08)***	0.487 (0.08)***	0.486 (0.08)***	0.508 (0.05)***	0.498 (0.09)***
$\tilde{y}_{t+1}$	0.311 (0.13)**	0.336 (0.14)**	0.283 (0.15)*	0.302 (0.17)*	0.371 (0.14)**	0.357 (0.19)*	0.365 (0.1)***	0.351 (0.13)**	0.558 (0.13)***	0.577 (0.15)***	0.498 (0.12)***	0.49 (0.15)***
r_t	-0.001 (0.0003)**	-0.001 (0.0003)*	-0.001 (0.0003)*	-0.001 (0.0004)*	-0.001 (0.0004)	-0.001 (0.0004)	-0.001 (0.0003)*	-0.001 (0.001)	-0.001 (0.001)	-0.001 (0.001)	-0.001 (0.001)	-0.001 (0.001)*
Δq_{t+1}	-0.033 (0.09)	-0.031 (0.1)	-0.036 (0.1)	-0.065 (0.14)	-0.107 (0.15)	-0.121 (0.19)	-0.119 (0.12)	-0.149 (0.19)	0.068 (0.07)	0.078 (0.08)	0.037 (0.06)	0.064 (0.11)
Δm_t	0.066 (0.09)	0.053 (0.09)	0.077 (0.09)	0.058 (0.15)	-0.058 (0.07)	-0.064 (0.07)	-0.056 (0.06)	-0.019 (0.1)	-0.37 (0.21)*	-0.365 (0.2)*	-0.248 (0.23)	-0.24 (0.33)
Δm_{t-1}	0.171 (0.04)***	0.168 (0.04)***	0.176 (0.03)***	0.241 (0.06)***	0.133 (0.04)***	0.133 (0.04)***	0.153 (0.03)***	0.181 (0.05)***	0.465 (0.11)***	0.46 (0.1)***	0.464 (0.13)***	0.68 (0.16)***
Δy^w_{t+1}	0.174 (0.36)	0.143 (0.37)	0.128 (0.37)	0.367 (0.48)	0.199 (0.65)	0.274 (0.77)	0.168 (0.48)	0.414 (0.83)	0.3 (0.35)	0.266 (0.37)	0.271 (0.33)	0.52 (0.61)
R^2	0.947	0.95	0.916	0.93	0.945	0.944	0.918	0.927	0.942	0.942	0.923	0.934
Adj-R^2	0.935	0.939	0.898	0.914	0.932	0.931	0.899	0.91	0.93	0.93	0.906	0.919
J-statistics	0.079	0.086	0.077	0.089	0.107	0.101	0.108	0.109	0.083	0.081	0.081	0.085
p-value	0.442	0.404	0.452	0.401	0.303	0.327	0.3	0.31	0.42	0.43	0.432	0.422
Note: GMM estimates; Sample (unadjusted): 1993Q1-2002Q2; Numbers in parentheses are standard errors; ***, **, * indicate significance at the 1, 5, and 10 percent, respectively; Instrument set includes $\tilde{y}_{t-1}$, $\tilde{y}_{t-2}$, r_{t-1}, r_{t-2}, Δm_{t-1}, Δm_{t-2}, Δy^w_{t-1}, Δy^w_{t-2}, Δq_{t-1}, Δq_{t-2}; Standard errors are corrected for autocorrelation problem using Newey and West's (1987) asymptotic covariance matrix (Bandwidth=3 as suggested by Newey and West).												

Table 2.4: Estimation result: equation (2.17)

Consistent with the evidence found by Nelson (2002) for the U.S. and U.K., *contemporaneous* real money growth enters the IS equation insignificantly in most specifications. However, the same is not true for lagged real money growth. The coefficients of *lagged* real money growth are sizably positive and highly significant (at the 1 percent level) in all specifications. The coefficients of the contemporaneous real interest rate term are statistically significant at the 10 percent level in 6 out of 12 specifications, and are correctly signed (negative). The coefficients of lead world output growth ($\triangle y^w_{t+1}$) and those of lead real exchange rate growth ($\triangle q_{t+1}$) are not statistically significant in all specifications.

In all, given that the term structure effect has been *implicitly* controlled for under the *hybrid* IS specification, the results reported in Table 2.4 may be interpreted as evidence in support of the strong prevalence of 'the risk premium effect'; *lagged real money growth may serve as an indicative proxy for changes in the risk premia component of relative prices of various kinds and classes of assets.*

Table 2.5 shows the estimation results of equation (2.19) which essentially adds percentage deviations of r^l_t from r_t in equation (2.17) as an *additional and explicit* control for the term structure effect.[34] The results are generally the same as those reported in Table 2.4. Most importantly, the coefficients of lagged money growth remain statistically significant at the 1 percent level in all specifications. Their magnitude is reasonably large and invariably comparable to that reported in Table 2.4. Moreover, the coefficients of $\left(\frac{r^l_t - r_t}{r_t}\right)$ are found to be insignificant in most specifications. As the term structure effect has been at least partially captured by the lead output gap term (which is reported to be consistently significant in most specifications), the insignificance of this *additional* control should not be interpreted as evidence against the importance of the term structure effect.

Although using three proxies for the monetary stock and four alternative detrending methods provide reasonable means for robustness checking, it is interesting to see if the results obtained thus far are sensitive to the inclusion of the financial crisis period in the sample. To examine this, we re-estimate equation (2.19), restricting the sample period to 1997:Q3-2002:Q2.[35] The result is shown in Table 2.6.

[34]Note that the results from the formal unit root tests on $\frac{(r^l_t - r_t)}{r_t}$ are ambiguous (see Table 2.1). However, as the estimation results reported in Table 2.2 are insensitive to the stationarity property of the interest rate series. We shall assume that $\frac{(r^l_t - r_t)}{r_t}$ is I(0).

[35]Because the analysis here is based on a very restricted sample size, the result should be interpreted with this caution in mind.

Estimating $\tilde{y}_t = \lambda_1 \tilde{y}_{t-1} + \lambda_2 \tilde{y}_{t+1} + \lambda_3 r_t + \lambda_4 \Delta q_{t+1} + \lambda_5 \Delta m_t + +\lambda_6 \Delta m_{t-1} + \lambda_7 \Delta y^w_{t+1} + \lambda_8 [\frac{r^l_t - r_t}{r_t}] + \xi_t$ [eq. (2.19)]												
	$m = m0$ (monetary base)				$m = m1$				$m = m2$			
	LT	QT	HP	BN	LT	QT	HP	BN	LT	QT	HP	BN
$\tilde{y}_{t-1}$	0.7 (0.1)***	0.689 (0.12)***	0.721 (0.09)***	0.667 (0.12)***	0.558 (0.05)***	0.572 (0.06)***	0.558 (0.05)***	0.502 (0.06)***	0.563 (0.06)***	0.548 (0.06)***	0.557 (0.05)***	0.532 (0.07)***
$\tilde{y}_{t+1}$	0.332 (0.17)*	0.352 (0.2)*	0.273 (0.17)	0.306 (0.2)	0.503 (0.06)***	0.528 (0.07)***	0.459 (0.06)***	0.53 (0.06)***	0.35 (0.1)***	0.394 (0.11)***	0.303 (0.08)***	0.393 (0.12)***
r_t	-0.001 (0.0003)**	-0.001 (0.0003)**	-0.001 (0.0003)**	-0.001 (0.0004)*	-0.0002 (0.0002)	-0.0003 (0.0002)	-0.0002 (0.0002)	-0.0002 (0.0003)	-0.002 (0.001)***	-0.001 (0.001)***	-0.002 (0.0004)***	-0.002 (0.001)**
Δq_{t+1}	-0.007 (0.1)	-0.003 (0.12)	-0.021 (0.09)	-0.038 (0.15)	0.01 (0.04)	0.026 (0.05)	-0.01 (0.04)	0.025 (0.06)	0.013 (0.05)	0.024 (0.05)	-0.003 (0.05)	0.014 (0.08)
Δm_t	0.09 (0.14)	0.082 (0.16)	0.106 (0.14)	0.164 (0.24)	0.008 (0.06)	-0.002 (0.06)	0.035 (0.06)	-0.026 (0.08)	0.151 (0.21)	0.026 (0.2)	0.217 (0.19)	0.129 (0.37)
Δm_{t-1}	0.173 (0.04)***	0.173 (0.05)***	0.171 (0.05)***	0.279 (0.07)***	0.172 (0.04)***	0.163 (0.04)***	0.175 (0.03)***	0.256 (0.05)***	0.614 (0.12)***	0.574 (0.12)***	0.617 (0.13)***	0.742 (0.19)***
Δy^w_{t+1}	0.025 (0.24)	0.016 (0.29)	0.104 (0.21)	-0.083 (0.39)	-0.058 (0.18)	-0.009 (0.17)	-0.104 (0.2)	-0.181 (0.27)	0.421 (0.24)*	0.397 (0.27)	0.401 (0.2)*	0.435 (0.33)
$(r^l_t - r_t)/r_t$	-0.00004 (0.001)	-0.0002 (0.001)	-0.0001 (0.001)	-0.001 (0.002)	-0.001 (0.001)	-0.001 (0.001)	-0.001 (0.001)*	-0.001 (0.001)	-0.001 (0.001)	-0.001 (0.001)	-0.001 (0.001)	-0.002 (0.001)
R^2	0.942	0.94	0.91	0.925	0.955	0.949	0.935	0.936	0.961	0.957	0.941	0.943
Adj-R^2	0.925	0.923	0.884	0.903	0.941	0.934	0.916	0.917	0.949	0.944	0.924	0.926
J-statistics	0.082	0.089	0.077	0.072	0.156	0.166	0.149	0.146	0.084	0.108	0.07	0.077
p-value	0.455	0.416	0.485	0.51	0.172	0.149	0.189	0.198	0.44	0.328	0.525	0.481

Note: GMM estimates; Sample (unadjusted): 1993Q1-2002Q2; Numbers in parentheses are standard errors; ***, **, * indicate significance at the 1, 5, and 10 percent, respectively; Instrument set includes $\tilde{y}_{t-1}, \tilde{y}_{t-2}, r_{t-1}, r_{t-2}, \Delta m_{t-1}, \Delta m_{t-2}, \Delta y^w_{t-1}, \Delta y^w_{t-2}, \Delta q_{t-1}, \Delta q_{t-2}, \frac{(r^l_{t-1} - r_{t-1})}{r_{t-1}}, \frac{(r^l_{t-2} - r_{t-2})}{r_{t-2}}$; Standard errors are corrected for autocorrelation problem using Newey and West's (1987) asymptotic covariance matrix (Bandwidth=3 as suggested by Newey and West).

Table 2.5: Estimation result: equation (2.19)

Estimating (subsample) $\tilde{y}_t = \lambda_1 \tilde{y}_{t-1} + \lambda_2 \tilde{y}_{t+1} + \lambda_3 r_t + \lambda_4 \Delta q_{t+1} + \lambda_5 \Delta m_t + +\lambda_6 \Delta m_{t-1} + \lambda_7 \Delta y^w_{t+1} + \lambda_8 [\frac{r^l_t - r_t}{r_t}] + \xi_t$ [eq. (2.19)]												
	$m = m0$ (monetary base)				$m = m1$				$m = m2$			
	LT	QT	HP	BN	LT	QT	HP	BN	LT	QT	HP	BN
$\tilde{y}_{t-1}$	0.701 (0.06)***	0.72 (0.06)***	0.647 (0.05)***	0.637 (0.04)***	0.643 (0.05)***	0.67 (0.05)***	0.61 (0.05)***	0.41 (0.05)***	0.613 (0.04)***	0.612 (0.05)***	0.551 (0.02)***	0.557 (0.05)***
$\tilde{y}_{t+1}$	-0.086 (0.19)	-0.13 (0.14)	0.029 (0.22)	0.351 (0.21)	0.047 (0.13)	-0.057 (0.12)	0.044 (0.13)	0.956 (0.19)***	0.173 (0.11)	0.101 (0.14)	0.161 (0.08)*	0.31 (0.11)**
r_t	-0.001 (0.0004)***	-0.001 (0.0003)***	-0.001 (0.001)**	-0.001 (0.001)	-0.001 (0.0004)**	-0.001 (0.0003)**	-0.001 (0.0004)**	0.001 (0.001)*	-0.003 (0.001)***	-0.003 (0.001)***	-0.004 (0.0004)***	-0.003 (0.001)**
Δq_{t+1}	-0.193 (0.07)**	-0.197 (0.05)***	-0.145 (0.08)*	-0.048 (0.08)	-0.21 (0.06)***	-0.234 (0.06)***	-0.201 (0.06)***	0.021 (0.09)	0.006 (0.04)	-0.009 (0.05)	0.105 (0.05)**	0.024 (0.06)
Δm_t	0.163 (0.08)*	0.166 (0.05)**	0.118 (0.09)	0.042 (0.09)	0.183 (0.05)***	0.216 (0.04)***	0.19 (0.05)***	-0.204 (0.11)*	0.273 (0.18)	0.115 (0.17)	0.654 (0.12)***	0.299 (0.27)
Δm_{t-1}	0.229 (0.03)***	0.227 (0.03)***	0.213 (0.04)***	0.271 (0.03)***	0.279 (0.02)***	0.288 (0.03)***	0.281 (0.03)***	0.263 (0.05)***	0.776 (0.15)***	0.753 (0.15)***	1.228 (0.16)***	0.889 (0.27)***
Δy^w_{t+1}	-0.154 (0.27)	0.092 (0.27)	-0.181 (0.26)	-0.434 (0.34)	-0.272 (0.24)	-0.121 (0.28)	-0.344 (0.23)	-0.092 (0.4)	0.934 (0.24)***	1.026 (0.27)***	0.797 (0.16)***	0.897 (0.44)*
$(r^l_t - r_t)/r_t$	-0.003 (0.001)**	-0.002 (0.001)***	-0.001 (0.001)	-0.001 (0.001)	-0.004 (0.001)***	-0.004 (0.001)***	-0.003 (0.001)***	0.001 (0.002)	-0.002 (0.001)**	-0.002 (0.001)*	-0.004 (0.001)***	-0.003 (0.001)*
R^2	0.843	0.833	0.87	0.855	0.854	0.834	0.862	0.864	0.931	0.914	0.927	0.901
$Adj\text{-}R^2$	0.743	0.727	0.787	0.763	0.761	0.728	0.775	0.777	0.886	0.86	0.881	0.838
J-statistics	0.124	0.112	0.138	0.12	0.149	0.13	0.158	0.193	0.09	0.082	0.116	0.084
p-value	0.5	0.547	0.454	0.515	0.417	0.482	0.392	0.3	0.633	0.671	0.53	0.661

Note: GMM estimates; Sample (unadjusted): 1997Q3-2002Q2; Numbers in parentheses are standard errors; ***, **, * indicate significance at the 1, 5, and 10 percent, respectively; Instrument set includes $\tilde{y}_{t-1}, \tilde{y}_{t-2}, r_{t-1}, r_{t-2}, \Delta m_{t-1}, \Delta m_{t-2}, \Delta y^w_{t-1}, \Delta y^w_{t-2}, \Delta q_{t-1}, \Delta q_{t-2}, \frac{(r^l_{t-1} - r_{t-1})}{r_{t-1}}, \frac{(r^l_{t-2} - r_{t-2})}{r_{t-2}}$; Standard errors are corrected for autocorrelation problem using Newey and West's (1987) asymptotic covariance matrix (Bandwidth=3 as suggested by Newey and West).

Table 2.6: Estimation result: equation (2.19); subsample

The results show that the coefficients of lagged real money growth remain statistically significant at the 1 percent level in *all* specifications and their magnitude is greater compared to that shown in the previous two tables in most specifications. This implies that the strong evidence in favour of the existence of the risk premium effect found thus far is robust, and if anything stronger, when the pre-crisis period is excluded from the sample. Three additional points are worth noting from the results of this subsample regression. First, the coefficients of the lead output gap term turn out to be statistically significant in only 3 out of 12 specifications. If anything, this casts doubt on the robustness of the existence of the term structure effect. Second, the coefficients of the real short-term interest rate turns out to be statistically significant at the 5 percent level as well as correctly signed (negative) in most specifications (10 out of 12). This strengthens our prior conclusion on the sign and the significance of the real interest rate terms in that they appear to be much less consistent across different specifications (and subsamples) compared to those of the money growth terms. Third, while the exchange rate term enters the IS equation insignificantly in all specifications in the previous two tables, when the subsample period in which the exchange rate arguably behaved less abnormally is considered, it turns out to be statistically significant as well as correctly signed (negative) in 6 out of 12 specifications.

2.6 Policy Implications and Concluding Remarks

Using Thailand data, this chapter provides another piece of empirical evidence favouring the independent role of money as one of the determinants of aggregate demand. The key finding is that the effect of lagged real monetary growth on aggregate demand remains positive, sizable and statistically significant even when one controls for the term structure effect, both implicitly and explicitly. Thus the scope of changes in relative prices that money is conventionally found to proxy in the empirical literature is not limited to the changes in relative prices along the term structure of interest rate (the term structure effect). Instead, it may also extend to the changes in relative risk premia among different kinds and classes of asset (the risk premium effect). This implies that the two-asset world assumption typically assumed in standard macro models, both with and without microfoundations, is distorting.

Given that the two-asset world assumption is not empirically justified, the transmission mechanism of monetary policy in reality becomes far more complicated than has traditionally been implied by standard macro models. To understand more on how the transmission process works, future work is needed to embed more microfoundations into the optimisation-based macro model so that imperfect substitution amongst multiple assets takes place in equilibrium. These additional frictions would take the model away from the two-asset assumption as assets other than money can no longer be treated as a single composite goods and monetary policy in this world would operate by changing the relative risk premia of these assets. Until this ambitious task is fully developed, the results in this chapter suggest that taking into account the explicit role of money in these models can mitigate the problem. This is because the transmission process from its initial impulse to its ultimate response involves various changes in relative yields of various financial assets, and real monetary growth may serve as a justifiable *stand-in* for these relative yields. At the very least, the monetary stocks should be monitored closely and the information attained has to be utilised as important informative indicators in the conduct of monetary policy. As John Taylor emphasised, *"it is useful for central bank to keep track of money supply... even when they are using interest rules as a guideline."* (Taylor, 1999, p. 661)

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Chapter 3

A Model to Analyse Financial Fragility: Applications

Abstract

The purpose of our work is to explore contagious financial crises. To this end, we use simplified, thus numerically solvable, versions of our general model [Goodhart, Sunirand and Tsomocos (2003)]. The model incorporates heterogeneous agents, banks and endogenous default, thus allowing various feedback and contagion channels to operate in equilibrium. Such a model leads to different results from those obtained when using a standard representative agent model. For example, there may be a trade-off between efficiency and financial stability, not only for regulatory policies, but also for monetary policy. Moreover, agents who have more investment opportunities can deal with negative shocks more effectively by transferring 'negative externalities' onto others.

3.1 Introduction

The purpose of this work[1] is to explore contagious financial crises. To this end we need a model of heterogeneous banks with differing portfolios; if all banks were identical, or there was only one bank, there would be no interbank market and no contagion by definition. We also require a set-up in which default exists, and can be modelled. Otherwise there would be no crises. Similarly financial markets cannot be complete, since, if they were, all eventualities could be hedged. Finally, since we are concerned with financial crises, there must be an inherent role for money, banks and interest rates. We have constructed a rational expectations, forward-looking dynamic general equilibrium model along these lines in Goodhart, Sunirand, and Tsomocos (2003).

[1] This chapter is based on a joint paper with Charles A.E. Goodhart and Dimitrios P. Tsomocos. The paper is published in *Journal of Financial Stability* (forthcoming).

Default is modelled as in Shubik and Wilson (1977) and Dubey, Geanakoplos and Shubik (2000). By varying the penalties imposed on default from 0 to infinity, we can model 100% default (0 penalty), no default (infinite penalty) or an equilibrium default level between 0 and 100%. The main financial imperfection is that we assume that individual bank borrowers are assigned during the two periods of our model, by history or by informational constraints, to borrow from a single bank.[2] Money is introduced by a cash in advance constraint, whereby a private agent needs money to buy commodities from other agents; commodities cannot be used to buy commodities. Similarly we assume that agents needing money can always borrow more cheaply from their (assigned) bank than from other agents; banks have an informational (and perhaps scale) advantage that gives them a role as an intermediary.

In our general model (Goodhart, Sunirand and Tsomocos (2003)) there are a set of heterogeneous private sector agents with initial endowments of both money and commodities; it is an endowment model without production. There is also a set of heterogeneous banks, who similarly have differing initial allocations of capital (in the form of government bonds)[3]. There are two other players, a Central Bank which can inject extra money into the system, e.g. by buying an asset or lending, and a Financial Supervisory Agency, which can set both liquidity and capital minimum requirements and imposes penalties on failures to meet such requirements and on defaults. We do not seek to model the actions of these official players. They are strategic dummies.

The game lasts two periods. Period one involves trading in bank loans, bank deposits (including interbank deposits), a potential variety of other financial assets, e.g. an Arrow-type security or bank equity, and commodities. Such trading is done in anticipation that nature will randomly select a particular state, $s \in S = \{i, ii, ..., S\}$. In period 2, dependent on the state actually selected, there is further trading in commodities; all loans, including interbank loans, are repaid, subject to any defaults, which are then penalised, and the banks are in effect wound-up. The timeline of this model is shown in Figure 3.1.

In Goodhart et al. (2003) we demonstrate that such a model has an equilibrium and that financial fragility emerges naturally as an equilibrium phenomenon. In our model financial fragility is characterised by reduced aggregate bank profitability and increased aggregate default as in Tsomocos (2003 and 2004). Whenever such financial fragility is present in the economy, the role of economic

[2]Restricted participation can also arise as an outcome of banks aiming to outperform each other by introducing a relative performance criterion into their objective functions. For more on this, see Bhattacharya, Goodhart, Sunirand and Tsomocos (2003).

[3]Commercial banks are modelled as in Shubik and Tsomocos (1992). The modelling of banks is akin to Tobin (1963 and 1982).

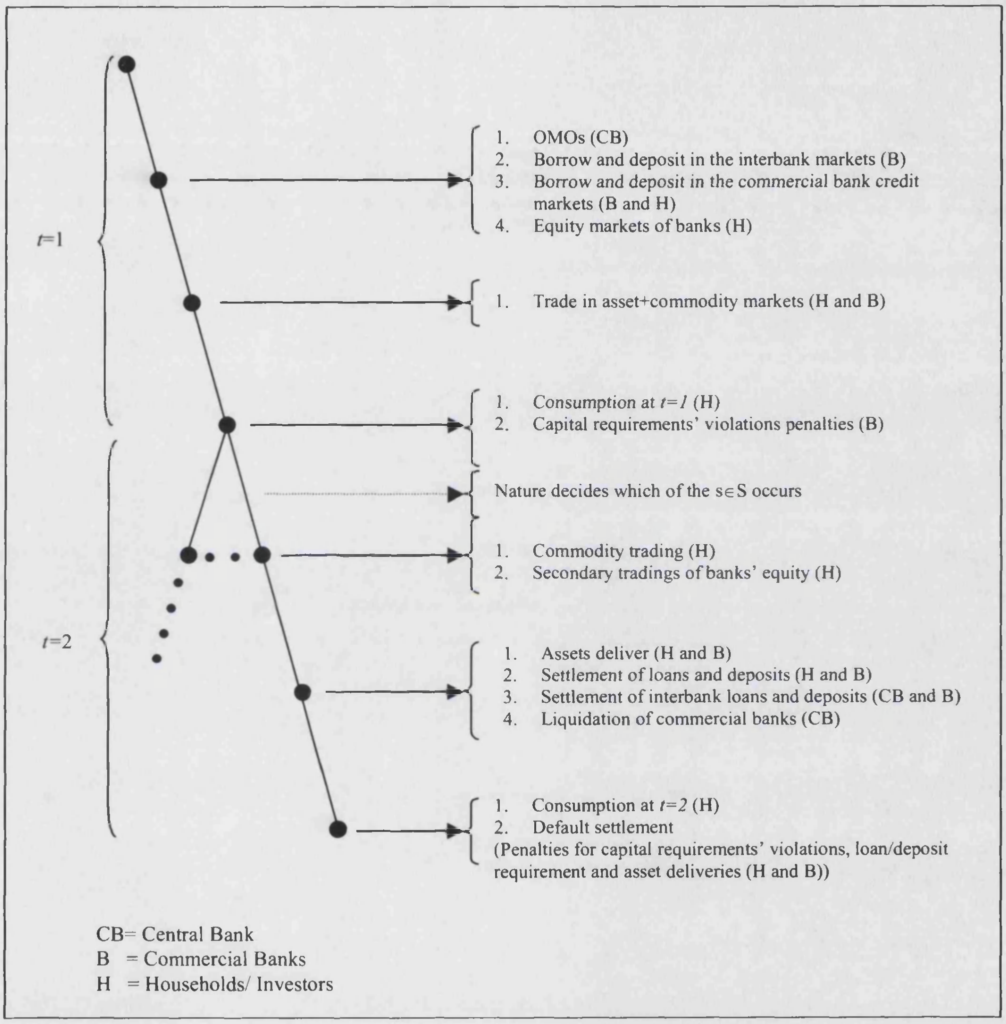

Figure 3.1: The time structure of the model

policy is justified. Regulatory and monetary policies are shown to be non-neutral due to the lack of the classical dichotomy between the real and nominal sectors of the economy. We also show that a non-trivial quantity theory of money holds, and the liquidity structure of interest rates depends both on aggregate liquidity and the risk of default in the economy. Finally, we address formally the Modigliani-Miller proposition, and establish the conditions that cause its failure. In particular, it fails *either* due to limited participation *or* incomplete (financial) markets *or* different risk preferences among banks. Given the scale of the model with B heterogeneous banks, H private sector agents, S states, a variety of financial assets, default and default penalties, and a variety of non-linearities, it is impossible to find either a closed-form or a numerical solution to the general model. The purpose of this chapter, therefore, is to present a smaller, specific version of this model which can be numerically solved.

3.2 The Base-line Model

We first simplify the general model fully developed in Goodhart et al. (2003) to the case of three households, $h \in H = \{\alpha, \beta, \phi\}$, and two banks, $b \in B = \{\gamma, \delta\}$, with a Central Bank which conducts monetary policy through open market operations (OMOs) and a regulator, which fixes the bankruptcy code for households and commercial banks as well as sets the capital-adequacy requirements for banks. The decisions of households and banks are endogenous in the model, whereas the Central Bank and the regulator are treated as strategic dummies with pre-specified strategies. The time horizon extends over two periods ($t \in T = \{1, 2\}$) and three possible states ($s \in S = \{i, ii, iii\}$) in the second period.

Given the cash-in-advance constraint, money is essential in the model.[4] There are 4 active markets in this economy: commodity, consumer credit, interbank, and financial asset markets. In period 1, the commodity, asset, credit and interbank markets meet. At the end of this period consumption and settlement, including any bankruptcy and capital requirements' violation penalties, take place. In period 2, the commodity market opens again, loans are settled and assets are delivered. At the end of this period consumption and settlement for default and capital requirements' violations take place. Also, commercial banks are liquidated.

In order to show inter-connections between banks, we need at a minimum two banks. One bank, bank γ, is relatively poor at $t = 1$ and therefore has to seek external funds to finance its loans. As

[4] The cash-in-advance constraint can be traced back at least to Clower (1965 and 1967). The modeling of money in this chapter is similar to the models developed by Dubey and Geanakoplos (1992) and Drèze and Polemarchakis (2000).

in the general model, we assume a limited participation assumption in the consumer loan market. Thus, bank γ lends to its *nature-selected* borrower, Mr. α. Bank γ can raise its funds either by borrowing from the *default free* interbank market[5] or selling its securities. In general, there are a variety of financial assets, besides deposits and bank loans, that we can introduce into the model, but owing to the size of the system, amounting to over 60 equations, we can only do so one at a time. In our first base-line model, we introduce an Arrow-type security, which the weaker bank (bank γ) can issue. This pays out 1 in state i in period 2, and nothing in any other states. In this way, state i is regarded as the 'good' state whereas the other two states, states ii and iii, are treated as 'bad' states.[6] Bank γ can be thought of as a typical straightforward small commercial bank. Its assets comprise only its credit extension to the consumer loan market. This way we can focus on the effects of policies on banks that cannot quickly restructure their portfolios by diversifying their asset investments, perhaps due to inaccessibility of capital and asset markets, during periods of financial adversity. The other bank, bank δ, is a large and relatively rich investment bank which, in addition to its lending activities to its *nature-selected* borrower, Mr. β, has a portfolio consisting of deposits in the interbank market and investment in the asset market (i.e. purchasing bank γ's Arrow security). Its richer portfolio allows it to diversify quickly and more efficiently than bank γ. As we shall see later, this extended opportunity set enables bank δ to transfer the negative impact of adverse shocks to the rest of the economy without necessarily reducing its profitability.

Given our restriction that agents can borrow only from a single bank, we need three agents, two who want to borrow in period 1 (Mr. α and β), because they are comparatively more constrained in money and goods in period 1 relative to period 2, and want to smooth consumption over time. The third agent, Mr. ϕ, is richer in both goods and money in period 1, relative to period 2, and hence deposits money with the banks in period 1 and sells goods to the borrowers. He deposits money with bank γ, which in equilibrium offers the highest *default free* deposit rate, and buys Arrow securities to transfer wealth from $t = 1$ to $t = 2$, and thus smooth his consumption. In a sense, Mr. α and β represent the household sector of the economy in which their main activity is borrowing for present consumption in view of future expected income. On the other hand, Mr. ϕ represents the investors' sector, with a more diversified portfolio consisting of deposits and investments in the asset market, in order to smooth his intertemporal consumption. At this stage we assume that the deposit rate

[5] In section 3.3, we relax this assumption, allowing default both in the interbank and deposit markets.

[6] Note that there are three states, two unconstrained assets (interbank investment and the Arrow security), loans with short-selling constraints for banks, and limited participation in the loan markets for households. Thus, markets are incomplete and equilibria are constrained inefficient. This in turn implies that there is scope for welfare improving economic policy (both regulatory and monetary).

	Agent			Bank	
	α	β	ϕ	γ	δ
Period 1	Poor in goods/money	Poor in goods/money	Rich in goods/money	Weakly capitalised	Well capitalised
Period 2	Richer in goods	Richer in goods	Poorer in goods		

State *i*	State *ii*	State *iii*
Good state (Arrow Security pays-off)	Bad state	Bad state

Table 3.1: The structure of the base-line model

is always equal to the lending rate offered by bank γ i.e. perfect financial intermediation.[7] We summarise the structure of our base-line model in Table 3.1.

We have chosen to begin with this specification since it is the simplest version possible given that we need at least two *heterogeneous* banks in order to analyse the *intra-sector* contagion effect within the banking sector via their interaction in the interbank and asset markets and the possible *inter-sector* contagion effect involving the real sector via the credit, deposit, asset and commodity markets. Moreover, by allowing two separate *defaultable* consumer loan markets, default in one market can produce an additional channel of contagion to the other and to the rest of the economy; a 'consumer loan contagion' channel.

In the following section (3.2.1) we formally summarise the agents' optimisation problem and the market clearing conditions. Section (3.2.2) then explains the resulting initial equilibrium given the exogenous parameters. Section (3.2.3) shows the results of a number of comparative statics exercises.

3.2.1 The Agents' Optimisation Problems and Market Clearing Conditions

3.2.1.1 Household α's and β's Optimisation Problem

Each consumer $h \in \{\alpha, \beta\}$ maximises his payoff, which is his utility of consumption minus the (non-pecuniary) default penalty he incurs if he does not pay back his loans. He also observes his cash-in-advance and quantity constraints in each period. These constraints are consistent with the timeline of the model.

As in Goodhart et al. (2003) and Tsomocos (2003 and 2004), we assume that asset and loan markets clear automatically via a background clearinghouse whereas commodity markets are more

[7] An assumption that we shall relax in section 3.3.

sluggish. Put differently, agents cannot use contemporaneous receipts from commodities to engage in other purchases.

The maximisation problem of households α and β is as follows:

$$\max_{\{b_0^h, q_s^h, v_{sb}^h\}, s\in S} \Pi^h = [\chi_0^h - c_0^h(\chi_0^h)^2] + \sum_{s\in S}[\chi_s^h - c_s^h(\chi_s^h)^2] - \sum_{s\in S}\lambda_{sb}^h \max[0, \mu^{h^b} - v_{sb}^h \mu^{h^b}]$$

subject to

$$b_0^h \leq \frac{\mu^{h^b}}{(1+r^\gamma)} \tag{3.1}$$

(i.e. expenditure for commodity $\leq$ borrowed money from the consumer loan market)

$$\chi_0^h \leq \frac{b_0^h}{p_0} \tag{3.2}$$

(i.e. consumption $\leq$ amount of goods purchased)

$$v_{sb}^h \mu^{h^b} \leq \Delta(3.1) + p_s q_s^h + m_s^h, \ s \in S \tag{3.3}$$

(i.e. loans repayment $\leq$ money at hand + receipts from sales of commodity + initial private monetary endowment in state s)

$$0 \leq e_s^h, \ s \in S \tag{3.4}$$

(i.e. $0 \leq$ endowments of commodities)

$$\chi_s^h \leq e_s^h - q_s^h, \ s \in S \tag{3.5}$$

(i.e. consumption $\leq$ initial endowment - sales)

where,

$\Delta(x) \equiv$ the difference between RHS and LHS of inequality (x),

$b_s^h \equiv$ amount of fiat money spent by $h \in H$ to trade in the market of commodity, $s = \{0\} \cup S$,

$q_s^h \equiv$ amount of commodity offered for sales by $h \in H$, $s = \{0\} \cup S$,

$\mu^{h^b} \equiv$ amount of fiat money agent $h^b \in H^b = \{\alpha^\gamma, \beta^\delta\}$ chooses to borrow from his nature selected bank b[8],

[8] i.e. Mr. α borrows from bank γ whereas Mr. β borrows from bank δ.

v^h_{sb} ≡ the corresponding rates of repayment in the loan market by household h^b to his nature-selected bank b in states $s \in S$,

χ^h_s ≡ commodity consumption by $h \in H$ in state $s \in \{0\} \cup S$,

λ^h_{sb} ≡ (non-pecuniary) penalties imposed on h when contractual obligations in the consumer loan market are broken,

r^b ≡ lending rate offered by bank b,

p_s ≡ commodity price in $s = \{0\} \cup S$,

m^h_s ≡ monetary endowment of household h in states $s = \{0\} \cup S$,

e^h_s ≡ commodity endowment of household h in states $s = \{0\} \cup S$, and

c^l_s ≡ exogenous parameters in the utility/profit functions of agent l where $l \in H \cup B$.

3.2.1.2 Household ϕ's Optimisation Problem

Mr. ϕ's maximisation problem is as follows:

$$\max_{\{q_0^\phi, b_s^\phi, b_j^\phi, d_\gamma^\phi\}, s\in S} \Pi^\phi = [\chi_0^\phi - c_0^\phi(\chi_0^\phi)^2] + \sum_{s\in S}[\chi_s^\phi - c_s^\phi(\chi_s^\phi)^2]$$

$$b_j^\phi + d_\gamma^\phi \leq m_0^\phi \tag{3.6}$$

(i.e. expenditures for the Arrow securities + bank deposits ≤ initial private monetary endowments)

$$q_0^\phi \leq e_0^\phi \tag{3.7}$$

(i.e. sales of commodity ≤ endowments of commodity)

$$\chi_0^\phi \leq e_0^\phi - q_0^\phi \tag{3.8}$$

(i.e. consumption ≤ initial endowment - sales)

$$b_i^\phi \leq \Delta(3.6) + p_0 q_0^\phi + d_\gamma^\phi(1 + r^\gamma) + \frac{b_j^\phi}{\theta} \tag{3.9}$$

(i.e. expenditures for commodity in state i ≤ cash at hand + receipts from sales of commodity from period $t = 1$ + deposits and interest payment + asset deliveries)

$$b_s^\phi \leq \Delta(3.6) + p_0 q_0^\phi + d_\gamma^\phi(1 + r^\gamma), \; s = \{ii, iii\} \tag{3.10}$$

(i.e. expenditures for commodity in states ii and $iii \leq$ cash at hand + receipts from sales of commodity from period $t = 1$ + deposits and interest payment)

$$\chi_s^\beta \leq \frac{b_s^\phi}{p_s} \tag{3.11}$$

(i.e. consumption $\leq$ purchases)

where,

$b_j^\phi \equiv$ amount of money placed by Mr. ϕ in the Arrow security market,

$d_\gamma^\phi \equiv$ amount of money that Mr. ϕ deposits with bank γ, and

$\theta \equiv$ asset price.

3.2.1.3 Bank γ's Optimisation Problem

Bank γ (similarly for bank δ) maximises its profit in $t = 2$ and suffers a capital requirement violation penalty proportional to its capital requirement violation. Moreover, it observes its liquidity constraints as described in the timeline of the model in Figure 3.1.

Bank γ's optimisation problem is as follows:

$$\max_{\{\mu^\gamma, \overline{m}^\gamma, q_j^\gamma\}} \Pi^\gamma = \sum_{s \in S} \pi_s^\gamma - \sum_{s \in S} \lambda_{ks}^\gamma \max[0, \overline{k} - k_s^\gamma]$$

subject to

$$\overline{m}^\gamma \leq \frac{\mu^\gamma}{(1+\rho)} + d_\gamma^\phi + \theta q_j^\gamma \tag{3.12}$$

(i.e. credit extension $\leq$ interbank loans + consumer deposits + receipt from asset sales)

$$\mu^\gamma + q_j^\gamma + (1 + r^\gamma) d_\gamma^\phi \leq \Delta(3.12) + v_{i\gamma}^\alpha (1 + r^\gamma) \overline{m}^\gamma + e_i^\gamma \tag{3.13}$$

(i.e. interbank loan repayment + expenditure for asset deliveries + deposit repayment $\leq$ money at hand + loan repayment + initial capital endowment in state i)

$$\mu^\gamma + (1 + r^\gamma) d_\gamma^\phi \leq \Delta(3.12) + v_{s\gamma}^\alpha (1 + r^\gamma) \overline{m}^\gamma + e_s^\gamma, \ s = \{ii, iii\} \tag{3.14}$$

(i.e. interbank loan repayment + deposit repayment $\leq$ money at hand + loan repayment + initial capital endowment in state $s = \{ii, iii\}$)

where,

$\pi_s^\gamma = \triangle(3.13)$ for $s = i$, and $\triangle(3.14)$ for $s = \{ii, iii\}$

$k_s^\gamma = \frac{e_s^\gamma}{\overline{\omega} v_{s\gamma}^{\alpha}(1+r^\gamma)\overline{m}^\gamma}$, $s \in S$,

$\overline{k} \equiv$ capital adequacy requirement set by the regulator,

$\lambda_{ks}^b \equiv$ capital requirements' violation penalties on bank $b \in B$ in state $s \in S$ set by the regulator,

$\overline{\omega} \equiv$ risk weight for consumer loans,

$\overline{m}^b \equiv$ amount of credit that bank $b \in B$ extends,

$q_j^\gamma \equiv$ bank γ's quantity supply of Arrow securities,

$e_s^b \equiv$ initial capital endowment of bank $b \in B$ in state $s = \{0\} \cup S$,

$\rho \equiv$ interbank rate, and

$\mu^\gamma \equiv$ amount of money that bank γ borrows from the interbank market.

3.2.1.4 Bank δ's Optimisation Problem

Bank δ's optimisation problem is as follows:

$$\max_{\{d^\delta, \overline{m}^\delta, b_j^\delta\}} \Pi^\delta = \sum_{s \in S} \pi_s^\delta - \sum_{s \in S} \lambda_{ks}^\delta \max[0, \overline{k} - k_s^\delta]$$

subject to

$$d^\delta \leq e_0^\delta \tag{3.15}$$

(i.e. deposits in the interbank market $\leq$ initial capital endowment)

$$\overline{m}^\delta + b_j^\delta \leq \triangle(3.15) \tag{3.16}$$

(i.e. credit extension + expenditure for asset $\leq$ money at hand)

$$0 \leq \triangle(3.16) + \frac{b_j^\delta}{\theta} + v_{i\delta}^\beta \overline{m}^\delta (1 + r^\delta) + d^\delta (1 + \rho) + e_i^\delta \tag{3.17}$$

(i.e. $0 \leq$ money at hand + money received from asset payoffs + loan repayments in state i + interbank deposits and interest payment + initial capital endowment in state i)

$$0 \leq \triangle(3.16) + v_{s\delta}^\beta \overline{m}^\delta (1 + r^\delta) + d^\delta (1 + \rho) + e_s^\delta,\ s = \{ii, iii\} \tag{3.18}$$

(i.e. $0 \leq$ money at hand + loan repayments in state $s = \{ii, iii\}$ + interbank deposits and interest payment + initial capital endowment in state $s = \{ii, iii\}$)

where,

$\pi_s^\delta = \triangle(3.17)$ for $s = \{i\}$, and $\triangle(3.18)$ for $s = \{ii, iii\}$,

$k_i^\delta = \frac{e_i^\delta}{\overline{\omega} v_{i\delta}^\beta (1+r^\delta)\overline{m}^\delta + \omega d^\delta (1+\rho) + \widetilde{\omega}\frac{b_j^\delta}{\theta}}$,

$k_s^\delta = \frac{e_s^\delta}{\overline{\omega} v_{s\delta}^\beta (1+r^\delta)\overline{m}^\delta + \omega d^\delta (1+\rho)}$, for $s = \{ii, iii\}$,

ω ($\widetilde{\omega}$) $\equiv$ risk weights for interbank market deposits (the Arrow security),

$b_j^\delta \equiv$ amount of money placed by bank δ in the market of the Arrow security,

$d^\delta \equiv$ bank δ's interbank deposits, and

$M^{CB} \equiv$ money supply.

3.2.1.5 Market Clearing Conditions

There are 8 markets in the model (one commodity in $t = 1$ and three in $t = 2$, one asset, the interbank and two consumer loan markets). Each of these markets determines a price that equilibrates demand and supply in equilibrium.[9]

$$p_0 = \frac{b_0^\alpha + b_0^\beta}{q_0^\phi} \quad \text{(i.e. commodity market at } t = 1 \text{ clears)} \tag{3.19}$$

$$p_s = \frac{b_s^\phi}{q_s^\alpha + q_s^\beta},\ s \in S \quad \text{(i.e. commodity market at } t = 2,\ s \in S \text{ clears)} \tag{3.20-3.22}$$

$$1 + \rho = \frac{\mu^\gamma}{M^{CB} + d^\delta} \quad \text{(i.e. interbank market clears)} \tag{3.23}$$

$$1 + r^\gamma = \frac{\mu^{\alpha^\gamma}}{\overline{m}^\gamma} \quad \text{(i.e. bank } \gamma\text{'s loan market clears)} \tag{3.24}$$

$$1 + r^\delta = \frac{\mu^{\beta^\delta}}{\overline{m}^\delta} \quad \text{(i.e. bank } \delta\text{'s loan market clears)} \tag{3.25}$$

$$\theta = \frac{b_j^\delta + b_j^\phi}{q_j^\gamma} \quad \text{(i.e. asset market clears)} \tag{3.26}$$

3.2.1.6 Equilibrium

Let $\sigma^h = (b_0^h, q_s^h, v_{sb}^h) \in R \times R^3 \times R^3$ for $h \in \{\alpha, \beta\}$; $\sigma^\phi = (q_0^\phi, b_s^\phi, b_j^\phi, d_\gamma^\phi) \in R \times R^3 \times R \times R$; $\sigma^\gamma = (\mu^\gamma, \overline{m}^\gamma, q_j^\gamma) \in R^3$; $\sigma^\delta = (d^\delta, \overline{m}^\delta, b_j^\delta) \in R^3$. Also, let $\eta = (p_0, p_1, p_2, p_3, \rho, r^\gamma, r^\delta, \theta)$, $B^h(\eta) = \{\sigma^h :$

[9] The price formation mechanism is identical to the offer-for-sale mechanism in Dubey and Shubik (1978). The denominator of each of the expressions (3.19-3.26) represents the supply side whereas the numerator divided by the price corresponds to the demand. Note that this price formation mechanism is well-defined both in, and out of, equilibrium.

$(3.1)-(3.5)$ hold$\}$ for $h \in \{\alpha, \beta\}$, $B^{\phi}(\eta) = \{\sigma^h : (3.6)-(3.11)$ hold$\}$, $B^{\gamma}(\eta) = \{\sigma^{\gamma} : (3.12)-(3.14)$ hold$\}$, $B^{\delta}(\eta) = \{\sigma^{\delta} : (3.15)-(3.18)$ hold$\}$. We say that $(\sigma^{\alpha}, \sigma^{\beta}, \sigma^{\phi}, \sigma^{\gamma}, \sigma^{\delta}; p_0, p_1, p_2, p_3, \rho, r^{\gamma}, r^{\delta}, \theta)$ is a monetary equilibrium with commercial banks and default iff:

(*i*) (a) $\sigma^h \in \underset{\sigma^h \in B^h(\eta)}{Argmax} \Pi^h(\chi^h)$, $h \in \{\alpha, \beta, \phi\}$

(b) $\sigma^b \in \underset{\sigma^b \in B^b(\eta)}{Argmax} \Pi^b(\pi^b)$, $b \in \{\gamma, \delta\}$

and

(*ii*) All markets (3.19)-(3.26) clear.

3.2.2 Exogenous Parameters and Initial Equilibrium

The values of the exogenous variables are summarised in Table 3.A.1 of Appendix A. The numbers chosen are mostly illustrative at this stage.[10] Thus, of itself a simulation of this kind is not particularly interesting, though it was, because of the size of the system, technically quite difficult. However, of greater interest are the comparative statics arising from varying the chosen inputs to the system. Armed with the propositions of the general model, we can trace the equilibria of the simulations and study how the multiple markets and choice variables interact. In turn, we can see how the many system-wide effects determine prices, interest rates and allocations.

The values of commodity and monetary endowments of households are chosen so that Mr. α and β (Mr. ϕ) are poor (rich) at $t = 1$, and therefore are net borrowers (lender). Similarly, the selected value of capital endowments of banks ensures that bank γ is relatively poor at $t = 1$ and has to borrow from the interbank and asset markets, and vice versa for bank δ. Furthermore, the value of regulatory capital adequacy requirement is chosen to be sufficiently high (0.4) in order to ensure that all banks violate their capital requirements and thus are penalised accordingly. The risk weight for consumer loans is set to 1, while that of interbank loans and assets is set to 0.5, to reflect the fact that loans are defaultable and therefore riskier than the other two types of assets. The rest of the exogenous variables/parameters are chosen to ensure a reasonable initial Monetary Equilibrium with Commercial Banks and Default (MECBD). The values of the initial equilibrium are shown in Table 3.A.1 in Appendix A. In particular, they are chosen to ensure that the values of all the repayment rates are realistic, and the interbank interest rate is lower than both the interest rates charged by both banks since interbank loans are assumed to be default free and thus do *not* include a default premium. Finally, the loan rate of bank γ is higher than that of bank δ so that Mr. ϕ chooses bank γ to deposit.

[10] As will be seen, Chapter Four attempts to calibrate an alternative version of our general model against real data.

3.2.3 Results

This section shows the effects of changes in the exogenous variables/parameters of the model. Table 3.A.2 of Appendix A describes the directional effects on endogenous variables of changing various parameters listed in the first column. We solve the model using *Mathematica*. We first guessed the initial equilibrium described in Table 3.A.1 of Appendix A. Then using Newton's method, we calculated numerically how the initial equilibrium changes as we vary each parameter at a time.

The analysis is conducted using the principles derived in Goodhart et al. (2003). Besides the non-neutrality of both regulatory and monetary policies, we have also established the following results:

(i) Liquidity Structure of Interest Rates:

Since base money is fiat and the horizon is finite, in the end no household will be left with fiat money. Thus, all households will finance their loan repayments to commercial banks via their private monetary endowment and the initial capital endowments of banks (recall that banks' profits are distributed to their shareholders). However, since we allow for defaults, the total amount of interest rate repayments is adjusted by the corresponding anticipated default rates. In sum, aggregate *ex post* interest rate payments adjusted for default to commercial banks is equal to the total amount of outside money (i.e. sum of private monetary and initial commercial banks' endowments). In this way, the overall liquidity of the economy and endogenous default co-determine the structure of interest rates.

(ii) Quantity Theory of Money Proposition:

The model possesses a non-mechanical quantity theory of money. Velocity will always be less than or equal to one (one if all interest rates are positive). However, since quantities supplied in the markets are chosen by agents (unlike the representative agent model's *sell-all* assumption), the real velocity of money, that is how many real transactions can be moved by money per unit of time, is endogenous. The upshot of the analysis is that nominal changes (i.e. changes in monetary policy) affect both prices and quantities.

(iii) Fisher Effect:

The nominal interest rate is equal to the real interest rate plus the expected rate of inflation.

We conclude this section by highlighting the key results that we obtain from this numerical exercise.

3.2.3.1 An Increase in Money Supply

Let the Central Bank engage in expansionary monetary policy by increasing the money supply (M^{CB}) in the interbank market (or equivalently lowering the interbank interest rate(ρ)) (see row 1 of Table 3.A.2 in Appendix A). Lowering the interbank rate induces bank γ to borrow more from the interbank market and therefore to increase its supply of loans to Mr. α, pushing down the corresponding lending rate r^{γ}. Consequently, agent ϕ reduces his deposits in bank γ and switches his investment to the asset market, pushing the asset price up slightly. Given lower expected rates of return from investing in the interbank and asset markets, bank δ invests less in these markets and switches to supply more loans to Mr. β, causing the corresponding lending rate r^{δ} to decline.

Since more money chases the same amount of goods, by the quantity theory of money proposition, prices in both periods and all states increase. Prices in state i increase the most, since Mr. ϕ has increased his demand for Arrow securities and therefore has more income to spend on commodities in state i. Lower interest rates make trade more efficient, since the increase in liquidity results in lower default rates for both Mr. α and Mr. β, especially in state i where Arrow securities pay off.[11] Thus aggregate consumer default falls.

Turning now to capital requirements' violation, both banks break their capital requirement constraints more than before, particularly bank δ. Higher repayment rates and credit extension overcompensate for the decrease in interest rates and thus, for given capital, risk weighted total assets increase. Bank δ, which is relatively richer than bank γ, violates its requirements even more, since the marginal benefit of the increased profits is greater than the marginal cost of the capital requirement violation. Thus, given an initially adverse capital requirement position (and also banks' inability to access capital markets to raise new equity), expansionary monetary policy worsens their capital adequacy condition. The reason is that the extra profit effect dominates the capital requirement violation cost.

Both regulatory and monetary policies affect credit extension. In addition, default and capital requirements' violation have different marginal costs (due to the different penalties). So, there exists a trade-off between earning a greater excess return through interest receipts and the cost of capital requirements' violation. Thus, the interaction of the capital adequacy ratio and credit extension should be analysed contemporaneously in order to determine the optimal composition of banks' assets. We also note that lower defaults on consumer borrowing does not necessarily improve capital

[11] Since our model is transaction based, lower interest rates generate lower 'transaction' costs to agents who borrow. In principle, default therefore falls. In the limit, when interest rates are equal to zero and markets are complete, full pareto optimality is obtained (see Corollary 2 of Tsomocos (2003) for further discussion).

assets' ratios since profit-maximising banks will respond by lending even more.

As far as the welfare of the agents is concerned, the utility of Mr. α and the profits of bank γ improve whereas the profits of bank δ deteriorate. The welfare of Mr. β and Mr. ϕ remains almost unaffected (slight improvement). The welfare improvement of Mr. α results from lower interest rates, (and consequently a higher repayment rate on his loans and thus lower default penalties). The higher expected prices in period 2 also contribute to the higher repayment rates, since higher prices imply higher expected income from selling commodities. Thus, as predicted by the Fisher effect, higher prices imply lower real interest rates at $t = 1$ since nominal interest rates fall. The profitability of bank γ increases, mainly due to lower consumer default which dominates the higher cost of capital requirements' violations. However, the positive spillover effect of lower consumer default for bank δ fails to dominate the lower revenue, due to lower interest rates, whose profitability therefore decreases along with higher capital requirements violations.

In sum, even though expansionary monetary policy improves aggregate consumer default rates, it does not necessarily induce less financial fragility. Higher liquidity provides an incentive for profit-maximising commercial banks to expand without necessarily improving their capital requirements condition.

3.2.3.2 An Increase in the Loan Risk Weights applied to Capital Requirements

An assessment of the effect of an increase in the risk weights on loans for both banks ($\overline{\omega}$) (see row 3 of Table 3.A.2 in Appendix A) underscores the argument that those agents who have more investment opportunities, and therefore greater flexibility, can mitigate the effect of a negative shock by restructuring their portfolios. In this simulation the initial condition of the economy is adverse in the sense that capital requirements are binding and there is no access to the capital markets to raise new equity. Bank δ will further reduce credit extension to avoid the extra cost of the additional capital requirements' violation penalty, and bank γ in particular will increase its violation since it cannot switch its investments to maintain its profitability. Consequently, its payoff will be severely affected both from reduced interest rate receipts and also the higher penalties for capital requirements' violation. In contrast, bank δ reduces investments in both the loan and interbank markets and increases its investment in the asset market.

Bank γ, anticipating the higher expected capital requirement violation penalty, will increase its credit extension to lessen its profit reduction, by borrowing more from the asset market and thus lowering the asset price. Since bank γ will charge lower interest rates in order to increase credit extension, deposits from Mr. ϕ decrease and, given lower asset prices, he switches to invest more in the asset market. In contrast, bank δ, which diversifies away from the loan market, increases its interest rates. Moreover, reduced investments in the interbank market by bank δ increase the interbank market interest rate. Tighter credit reduces commodity prices in all periods, except state i where the Arrow security pays and there is extra liquidity in the economy. This in turn implies higher default rates, except in the case where the Arrow security pays off (i.e. state i). So, default by both agents increases on average, (even though both of them maintain higher repayment rates in state i), because of tighter credit market conditions for Mr. β and lower expected income for both Mr. α and Mr. β.

The profitability of bank γ is reduced substantially, whereas bank δ's ability to restructure its portfolio generates slightly positive profits, even though the aggregate profit of the banking industry is reduced. Paradoxically, though, Mr. α's welfare is improved. Because in effect bank γ follows a countercyclical policy in response to the higher risk weights, so lower interest rates help Mr.α to borrow more cheaply and increase his consumption in period 1, thus slightly improving his utility. However, Mr. β is hurt by the higher interest rate charged by bank δ. Finally, Mr. ϕ's utility is almost unchanged (with ambiguous sign), since the lower purchasing power resulting from lower bank deposit rates is more than offset by a higher return on his asset investment.

Regulatory policy may be seen as a mirror image of monetary policy, since it directly affects credit extension via the capital requirements' constraint. Moreover, banks without well-diversified portfolios, and thus not so many investment opportunities, follow a countercyclical credit extension policy that hurts them, but benefits their respective clients. The countercyclical credit extension policy of not-well-diversified banks may also be thought of as a built-in-stabilizer in the economy when regulatory policy becomes tighter and the economy faces a danger of multiplicative credit contraction. On the contrary, banks that can quickly restructure their portfolios transfer the negative externalities of higher risk weights to their clients. Thus, restrictive regulatory policy in periods of economic adversity may enhance financial fragility by inducing lower profitability, higher default and further capital requirement violations.[12]

3.2.3.3 Summary of the Base-line Model Results

All the results of the various comparative statics are tabulated in Table 3.A.2 of Appendix A. Their interpretation and analysis can be undertaken using the principles we have used so far. Here

[12]As shown in Catarineu-Rabell, Jackson and Tsomocos (2004), if banks are allowed to choose the risk-weights of their assets, they would opt for countercyclical risk-weight setting. In this way, they would lessen the profit reduction induced by falling loan opportunities in the economy. And if they are not allowed to do so by the regulatory authorities, then they would choose procyclical weights rather than forward looking ones, thus exacerbating credit contraction in the economy.

we recapitulate the key results obtained from these comparative statics. First, in an economic environment in which capital constraints are binding, which may be viewed as representing adverse economic conditions, expansionary monetary policy can aggravate financial fragility since the extra liquidity injected by the Central Bank may be used by certain banks to expand, and in some senses to 'gamble for resurrection', worsening their capital position, and therefore the overall financial stability of the economy. Thus, a trade-off between efficiency and financial stability need not exist only for regulatory policies, but also for monetary policy.

Second, agents who have more investment opportunities can deal with negative shocks more effectively by using their flexibility to restructure their investment portfolios quickly as a means of transferring 'negative externalities' to other agents with a more restricted set of investment opportunities. This result has various implications. Among others, banks which have no well-diversified portfolios tend to follow a countercyclical credit extension policy in face of a negative regulatory shock in the loan market (e.g. tighter loan risk weights). In contrast, banks which can quickly restructure their portfolio tend to reallocate their portfolio away from the loan market, thus following a procyclical credit extension policy. Moreover, regulatory policies which are selectively targeted at different groups of banks can produce very *non-symmetric* results, e.g. an increase in capital requirement penalty of bank γ vs. bank δ (see rows 6 and 7 of Table 3.A.2 in Appendix A). When the policy is aimed at banks which have more investment opportunities, e.g. bank δ, much less contagion to the rest of the economy occurs since those banks simply restructure their portfolios between interbank and asset markets without greatly perturbing the credit market, thus not affecting substantially interest rates and prices in the economy. On the contrary, when the same policy is targeted at banks which have relatively limited investment opportunities, e.g. bank γ, they are forced to 'bite the bullet' by altering their credit extension. This produces changes in a series of interest rates, and therefore the cost of borrowing for agents. This in turn produces a contagion effect to the *real* sector in the economy.

Third, an improvement such as a positive productivity shock, which is concentrated in one part of the economy, does not necessarily improve the overall welfare and profitability of the economy. The key reason for this lies in that our model has *heterogeneous* agents and therefore possesses various feedback channels which are all active in equilibrium. Thus, a positive shock in one specific sector can produce a negative contagion effect in others, even possibly causing the welfare and/or profitability of the whole economy to fall. For example, if the commodity endowment of household α increases in state i (see row 9 of Table 3.A.2 in Appendix A), his increased revenue leads him to increase his repayment rate on his loans. This in turn pushes bank γ's lending rate down considerably. This results in lower profitability for bank γ, because higher repayments are outweighed by lower interest rate payments. Moreover, the fall of commodity prices also adversely affects Mr. β whose income from commodity sales in state i drops.

3.3 Extension: Endogenous Defaults in the Interbank and Deposit Markets

The comparative static results shown in the previous section can be varied to incorporate a different set of assets. So, we next, briefly, describe an extended version of the base-line model. In addition to our attempt to examine the robustness of our results in the previous section, this extension aims at illuminating how the effect of various shocks can generate contagion effects via the interbank and deposit markets. To that end, we modify the structure of the model given in section 3.2.

First, we allow endogenous defaults in the interbank market, i.e. bank γ can default on its interbank loans. Second, we allow separated deposit markets.[13] Moreover, Mr. ϕ has a choice to deposit his money with either bank. Bank γ's deposit rate differs from that of bank δ since it is allowed to default on its deposit obligation to Mr. ϕ. Bank γ may also default on its loans from the interbank market. Third, in order to incorporate these additional complexities while retaining the model tractability, we simplify the model by removing the Arrow security. Finally, we assume that the cost of default in the interbank and deposit markets is quadratic. This in turn implies that the marginal cost of default in these markets is greater as the size of borrowing is larger. The detailed optimisation problems are given in Appendix B. Moreover, Tables 3.C.1 of Appendix C summarises the values of exogenous parameters and the resulting initial equilibrium.

3.3.1 Results

Table 3.C.2 of Appendix C describes the directional effects on endogenous variables of increasing various parameters listed in the first column.

3.3.1.1 An Increase in Money Supply

As in section 2.3.1, let the Central Bank engage in expansionary monetary policy by increasing the money supply (M^{CB}) in the interbank market (or equivalently lowering the interbank market rate (ρ)) (see row 1 of Table 3.C.2 in Appendix C). Given a lower rate of return on interbank

[13] Recall that in the previous comparative static we assume perfect financial intermediation, i.e. a perfectly elastic demand for deposits by bank γ at the rate of interest equal to its lending rate.

market investment, bank δ borrows less from the deposit market and switches to invest more in the consumer loan market by supplying more credit to Mr. β. Thus bank δ's deposit and lending rates both decrease. As the deposit rate of bank δ falls, Mr. ϕ, who now has the option to diversify his deposits between banks γ and δ, choose to deposit more with bank γ, causing its deposit rate to decline as well. Moreover, given a lower cost of borrowing in the deposit and interbank markets, bank γ borrows more from these markets and increases its credit extension, thus lowering its lending rate offered to Mr. α.

Due to the fact that bank γ borrows more both from the interbank and deposit markets and the default penalty is now quadratic, it increases its repayment rates in these markets. Given increased liquidity in the economy, all prices increase in both periods, however more in the first period when monetary policy loosens.[14] This in turn generates more income to households, including those who sell their commodities in the second period. Thus, they all increase their repayment rates in the consumer loan markets. Bank γ violates more capital requirements because its risk-weighted assets increase and it does not have access to equity markets. Their risk-weighted assets increase because the effects of higher credit extension and higher borrowers' repayment rates dominate the effect of lower lending rates. In contrast, bank δ violates less capital requirements since the effect of lower lending rates coupled with the effect of lower interbank market investment dominate the effects of higher credit extension and higher borrowers' repayment rates.

As far as welfare is concerned, both borrowers, namely Mr. α and β, improve their payoffs due to lower borrowing costs and lower default penalties since they increase their repayment. However, the creditor who in our case is Mr. ϕ suffers from lower deposit rates, thus his expected income falls. This causes him to reduce his consumption in period 2. Similarly, both banks end up with a lower payoff. This is because the negative effect of lower lending rates dominates the positive effect of higher repayment rates by both Mr. α and β.

In sum, since there are separate deposit markets and default in the deposit market is also allowed, the effects we observed in subsection 3.2.3.1 are now accentuated in the banking sector where the profitability of both banks is reduced and financial fragility is further increased.

3.3.1.2 An Increase in the Loan Risk Weights applied to Capital Requirements

A tightening of regulatory policy by increasing risk weights on loans of both banks (see row 3 of Table 3.C.2 in Appendix C) will have similar effects as in subsection 3.2.3.2. However, some differences

[14] Note that in most models with liquidity constraints, there is always an overshooting phenomenon in the period when a policy change occurs. For the same phenomenon in an international context, see Geanakoplos and Tsomocos (2002).

will be noticeable particularly in the banking sector because we now allow for default in both the interbank and deposit markets, and deposit and lending markets are now separated.

As before, tighter regulatory policy is a mirror image of contractionary monetary policy, and so the interbank rate increases. Bank δ will further reduce its credit extension to avoid capital requirements' violation penalties, whereas bank γ whose portfolio is limited increases its credit extension to maintain its profitability. So, bank γ's lending rate decreases and bank δ's increases. However, bank δ has less flexibility than before because interbank loans are now defaultable and the deposit/lending spread is variable. In other words, both bank γ and the depositor can adjust their behaviour in the light of bank γ's action.

We introduce quadratic default penalties that imply that the marginal cost of defaulting is increasing. Thus, as regulation tightens, bank γ not only reduces its borrowing from the interbank market, but also lowers its repayment rate to support its profitability. Bank δ rationally expects higher defaults, and thus lowers its deposits in the interbank market, pushing the interbank rate even higher. Given higher interbank rates, bank γ increases its deposit demand by offering higher deposit rate to Mr. ϕ who in turn deposits more with bank γ and less with bank δ. This pushes up the deposit rate of bank δ. Finally, since bank γ increases its deposit demand and reduces its interbank loans, it increases its repayment of deposits while reduces its repayment of interbank borrowing, given quadratic default penalties.[15]

Since both deposit rates increase, Mr. ϕ receives more income from his investments. He is the buyer of commodities in period 2 and since more money chases the same quantity of goods, by the quantity theory of money proposition, prices increase in the second period. Note that this is in contrast with what happened in the previous comparative static where there was no separated deposit market, which in turn implied that deposit and lending rates were, by definition, restricted to be the same, since tighter credit was automatically translated to lower income to depositors as well. Here we face a wealth redistribution from the banks to their depositors.

Mr. α and β, anticipating higher expected income from their commodity sales, increase their repayment rates on their respective loans. Finally, both banks, bank γ in particular, increase their capital requirements' violation. Again the bank with the richer portfolio will follow a procyclical credit extension policy, whereas the one with the more restricted portfolio will follow a countercyclical policy.

[15]Endogenous default and the ensuing penalties can be seen as altering the effective payoff of banks' liabilities which therefore form an optimal liability portfolio, given their risk preferences.

Turning to the welfare of the economy, Mr. α's welfare is improved as before. However, unlike previously, Mr. β's welfare remains unaffected since higher prices in the second period allow him to pay back his loans without increasing his commodity sales. Similarly, Mr. ϕ's welfare remains unaltered since the positive effect from higher deposit interest payments is offset by the negative effect from higher commodity prices in the second period. As before, bank γ is hurt by higher capital requirements' violation penalties. The main difference, however, lies in the reduced profitability of bank δ. This occurs because bank γ now has a default option and the separate deposit markets allow more room for Mr. ϕ to diversify his deposits. Thus, we see that what matters is the number of financial instruments available to an agent relative to others. In other words, when a wide array of instruments such as the default option and separate deposit markets are available to everybody, then banks with stronger and more diversified portfolios cannot simply transfer the negative impact of shocks to the rest of the economy. Indeed, they must bear some of it themselves.

To summarise, as regulatory policy tightens in times of adverse economic conditions, bank profitability is further affected. In addition, default may also increase in the interbank market, thus increasing financial fragility in the economy.

3.3.1.3 Summary of the Extended Model Results

The rest of the results are tabulated in Table 3.C.2 of Appendix C and can be analysed along the same lines. In principle, they reinforce the conclusion reached in section 3.2. Expansionary monetary policy may enhance financial fragility in the short run, and banks with more investment opportunities can cope with negative shocks more effectively, thus limiting their profit losses.

When the commodity endowment of Mr. α increases in state i (see row 9 of Table 3.C.2 in Appendix C), bank δ supplies less credit to Mr. β and switches to increase its investment in the interbank market. This is so because Mr. β is adversely affected in state i by lower commodity prices and thus defaults more to bank δ. Meanwhile, bank γ has a lower cost of borrowing, and does not decrease its deposit rate commensurately. Indeed, the presence of the deposit markets provides an extra degree of freedom to banks to vary optimally the deposit and lending spread and thus depositors can diversify their deposits. Put differently, this is testimony that a wider array of financial markets, typically, improves economic welfare. Thus, unlike previously, although this shock which directly improves the welfare of one agent may worsen that of the others, aggregate welfare now improves.

Regulatory policies targeted at the relatively more flexible bank δ, e.g. an increase in capital requirement penalty of bank δ in state i (see row 7 of Table 3.C.2 in Appendix C), now have more real effects in the economy. This is so because bank δ does not any longer have the opportunity to invest in the asset market and consequently changes more forcefully its credit extension policy. Credit extension changes have more direct effects on the real economy since credit multipliers are typically greater than asset multipliers. Put differently, given our initial condition, changes in credit extension work through the budget constraints of agents who, in turn, decide how to spend their extra liquidity. However, changes in the asset investment portfolio of banks affects not only the liquidity of the suppliers (i.e. agents), but also generates a price effect. Thus, the real effect of asset portfolio changes is mitigated as contrasted to the credit extension changes.

The contagion effects of a positive shock now depend largely on where the shock is initiated. Financial fragility in the interbank and deposit markets now depends on the agent who was first affected by the shock. For example, when we conduct money financed fiscal transfer (i.e. an increase in an agent's money endowment) or a productivity shock (i.e. an increase in an agent's commodity endowment) to Mr. α in state i (see rows 2 and 9 of Table 3.C.2 in Appendix C, respectively), average default in the interbank and deposit markets falls. This is so because bank γ, whose client is Mr. α, borrows from the interbank market. However, the opposite is true when the shocks emanate from Mr. β since his nature-selected bank (bank δ) is a net lender in the interbank market (see rows 10 and 18 of Table 3.C.2 in Appendix C).

In conclusion, policies must be context specific since one size does *not* fit all objectives in heterogeneous models. In particular, real business cycle models that rely heavily on the representative agent hypothesis are not able to address policy effects in multi-agent economies. As most of our experiments make clear, contagion and its impact to the various sectors of the economy depends on the origin of the shock.

3.4 Conclusion

Large, and non-linear, models, such as Goodhart, et al. (2003), normally do not have closed-form solutions. They have to be solved numerically. This chapter provides numerical simulations of simplified versions of the above more general model.

The ability to do this shows that, in some senses, the model 'works'. Moreover, it can be made to 'work' in a massively wide variety of initial starting conditions, e.g. depending on which asset markets are included in each variant of the model, and of comparative static exercises to be run. Indeed, the exercises and results reported in Sections 3.2.3 and 3.3.1 are a hugely boiled-down version, a precis, of the full set of exercises, both those that we have done, and, even more so, those that we, in principle, could do. We selected a small sub-set of starting conditions, and of comparative static exercises, with the aim of being both, (relatively), simple and illustrative.

What then have we illustrated? These insights fall into two general categories. First there are those characteristics of a monetary model which not only hold here, but should hold in any well-organised model. We have emphasised three. The first is what we have termed the 'quantity theory of money', whereby monetary changes feed through into price and quantity changes, both in the current and future period ($t = 1, 2$). We have assumed an endowment economy, so the volume of goods is, by definition, fixed. But more, or less, everything else 'real' in the system does change, distributions between agents, 'real' interest rates, bank profitability, default penalties, etc., etc. The system (and the 'real world') is non-neutral.

As noted, our model allows for non-zero expectations of future price inflation. Our model also incorporates the Fisher effect, whereby nominal rates (at $t = 2$) are a function of 'real' rates and inflation expectations. Finally 'real' rates, and *rate differentials*, are a function of the temporal, and distributional, pattern of endowments (time preference), liquidity (i.e. the amount of money injected into the system), and default risk (the greater the risk, the higher the required rate).

The second set of insights relates to the implications of the main innovative feature of our model, which is that the real world is heterogeneous; agents and banks are not all alike. This has some, fairly obvious, implications. The result of a shock may depend on the particular agent, part of the economy, on which it falls. The response of a bank to a regulatory change will generally depend sensitively on the particular context in which that bank finds itself, and will vary as that context changes. The result of a shock can often shift the distribution of income, and welfare, between agents in a complex way, which is hard to predict in advance.

In short, heterogeneity leads to greater complexity. What we lose, by including it in our model, is simplicity; what we hope to gain is greater reality. In this latter respect, however, simulations, such as these, are always somewhat lacking. We have chosen the initial conditions, and so the outcome is the somewhat artificial construct of our own assumed inputs. The next step will be to take our model, adjusted as may be necessary, to the actual data, to calibrate inter-actions between existing banks and (sets of) agents. This is one of the main focuses of the analysis in Chapter Four.

Appendix

Appendix A

Exogenous variables					
Coefficient of	Endowment		Penalty		
risk aversion	Commodities	Money	Capital	Default	CAR
$c_0^\alpha = 0.032$	$e_0^\alpha = 0$	$m_0^\alpha = 0$	$e_0^\gamma = 0.1$	$\lambda^\alpha_{1\gamma} = 0.2$	$\lambda^\gamma_{1k} = 5$
$c_1^\alpha = 0.046$	$e_1^\alpha = 18$	$m_1^\alpha = 0.028$	$e_1^\gamma = 2$	$\lambda^\alpha_{2\gamma} = 0.1$	$\lambda^\gamma_{2k} = 4$
$c_2^\alpha = 0.048$	$e_2^\alpha = 18$	$m_2^\alpha = 0.007$	$e_2^\gamma = 2$	$\lambda^\alpha_{3\gamma} = 0.05$	$\lambda^\gamma_{3k} = 3.71$
$c_3^\alpha = 0.045$	$e_3^\alpha = 18$	$m_3^\alpha = 0.037$	$e_3^\gamma = 2$	$\lambda^\beta_{1\delta} = 0.2$	$\lambda^\delta_{1k} = 2.49$
$c_0^\beta = 0.026$	$e_0^\beta = 0$	$m_0^\beta = 0$	$e_0^\delta = 16$	$\lambda^\beta_{2\delta} = 0.1$	$\lambda^\delta_{2k} = 0.93$
$c_1^\beta = 0.037$	$e_1^\beta = 20$	$m_1^\beta = 1.65$	$e_1^\delta = 1.5$	$\lambda^\beta_{3\delta} = 0.05$	$\lambda^\delta_{3k} = 0.93$
$c_2^\beta = 0.037$	$e_2^\beta = 20$	$m_2^\beta = 2.368$	$e_2^\delta = 1.5$	Others	
$c_3^\beta = 0.037$	$e_3^\beta = 20$	$m_3^\beta = 2.147$	$e_3^\delta = 1.5$	$M^{CB} = 0.19$	
$c_0^\phi = 0.035$	$e_0^\phi = 25$	$m_0^\phi = 1.707$		$\overline{\omega} = 1$	
$c_1^\phi = 0.021$	$e_1^\phi = 0$	$m_1^\phi = 0$		$\omega = 0.5$	
$c_2^\phi = 0.019$	$e_2^\phi = 0$	$m_2^\phi = 0$		$\widetilde{\omega} = 0.5$	
$c_3^\phi = 0.033$	$e_3^\phi = 0$	$m_3^\phi = 0$		$A_j = [1\ 0\ 0]'$	
				$\overline{k} = 0.4$	

Initial Equilibrium				
Prices	Commodities	Capital/Asset ratio	Repayment rate	Loans/deposits
$p_0 = 1$	$b_0^\alpha = 8.01$	$k_1^\gamma = 0.19$	$v^\alpha_{1\gamma} = 0.94$	$\overline{m}^\gamma = 8.01$
$p_1 = 1.1$	$q_1^\alpha = 9.5$	$k_2^\gamma = 0.193$	$v^\alpha_{2\gamma} = 0.927$	$\overline{m}^\delta = 10$
$p_2 = 1.18$	$q_2^\alpha = 8.81$	$k_3^\gamma = 0.194$	$v^\alpha_{3\gamma} = 0.923$	$d^\phi_\gamma = 1.45$
$p_3 = 1.34$	$q_3^\alpha = 7.7$	$k_1^\delta = 0.085$	$v^\beta_{1\delta} = 0.9$	$d^\delta = 1.56$
	$b_0^\beta = 10$	$k_2^\delta = 0.102$	$v^\beta_{2\delta} = 0.89$	$\mu^{\alpha^\gamma} = 11.19$
Interest rates/	$q_1^\beta = 9.5$	$k_3^\delta = 0.103$	$v^\beta_{3\delta} = 0.88$	$\mu^{\beta^\delta} = 13.49$
Asset price	$q_2^\beta = 8.2$			$\mu^\gamma = 2.1$
$r^\gamma = 0.4$	$q_3^\beta = 7.29$	Assets		
$r^\delta = 0.35$	$q_0^\phi = 18.02$	$b_j^\phi = 0.26$		
$\rho = 0.197$	$b_1^\phi = 20.9$	$b_j^\delta = 4.45$		
$\theta = 0.278$	$b_2^\phi = 20.02$	$q_j^\gamma = 16.9$		
	$b_3^\phi = 20.02$			

Table 3.A.1: Exogenous variables and Initial equilibrium: the base-line model

	p_0	p_1	p_2	p_3	r^γ	r^δ	ρ	θ	$\overline{m}^\gamma$	$\overline{m}^\delta$	d_γ^ϕ	d^δ	μ^{α^γ}	μ^{β^δ}	μ^γ	k_1^γ	k_2^γ	k_3^γ	k^γ	k_1^δ	k_2^δ	k_3^δ	k^δ	k	$v_{1\gamma}^\alpha$	$v_{2\gamma}^\alpha$	$v_{3\gamma}^\alpha$	v_γ^α
M	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$-$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$-$	$\underset{\approx}{-}$	$\approx$	$\approx$	$\underset{\approx}{+}$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\approx$	$\approx$	$\approx$	$\approx$	$\underset{\approx}{-}$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$
m_1^α	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$-$	$+$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$-$	$+$	$\approx$	$\approx$	$+$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$
$\overline{\omega}$	$\approx$	$+$	$-$	$-$	$-$	$+$	$+$	$-$	$+$	$-$	$-$	$-$	$\approx$	$\underset{\approx}{+}$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$+$	$-$	$-$	$\underset{\approx}{-}$
$\lambda_{1\gamma}^\alpha$	$\approx$	$\underset{\approx}{+}$	$-$	$-$	$-$	$+$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$-$	$+$	$-$	$\approx$	$+$	$-$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$+$	$+$	$\underset{\approx}{+}$	$+$	$+$	$+$	$+$
$\lambda_{1\delta}^\beta$	$\underset{\approx}{+}$	$-$	$+$	$+$	$+$	$-$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$+$	$-$	$\approx$	$-$	$-$	$+$	$-$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\approx$	$-$	$+$	$+$	$\underset{\approx}{-}$
λ_{1k}^γ	$\approx$	$-$	$+$	$+$	$+$	$-$	$-$	$\underset{\approx}{+}$	$-$	$+$	$+$	$-$	$\approx$	$\approx$	$-$	$+$	$-$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\approx$	$-$	$+$	$+$	$\underset{\approx}{+}$
λ_{1k}^δ	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$-$	$\approx$	$\approx$	$-$	$\approx$	$\approx$	$\approx$	$\approx$	$-$	$+$	$+$	$+$	$+$	$\approx$	$\approx$	$\approx$	$\approx$
e_0^ϕ	$-$	$+$	$-$	$-$	$-$	$-$	$-$	$\underset{\approx}{+}$	$\approx$	$\approx$	$-$	$+$	$-$	$-$	$+$	$-$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$+$	$+$	$+$
e_1^α	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$\underset{\approx}{-}$	$+$	$-$	$+$	$-$	$\approx$	$\underset{\approx}{+}$	$-$	$-$	$-$	$-$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$-$	$+$	$+$	$+$	$+$
e_1^β	$\underset{\approx}{+}$	$-$	$+$	$+$	$+$	$-$	$+$	$-$	$-$	$+$	$+$	$-$	$\approx$	$\approx$	$-$	$+$	$-$	$-$	$\underset{\approx}{+}$	$-$	$-$	$-$	$-$	$\underset{\approx}{+}$	$-$	$+$	$+$	$\underset{\approx}{-}$
e_1^γ	$\approx$	$-$	$+$	$+$	$+$	$-$	$-$	$\underset{\approx}{+}$	$-$	$+$	$+$	$-$	$\approx$	$\approx$	$-$	$+$	$-$	$-$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$-$	$+$	$+$	$\underset{\approx}{+}$
e_1^δ	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$-$	$\approx$	$\approx$	$-$	$\approx$	$\approx$	$\approx$	$\approx$	$+$	$+$	$+$	$+$	$+$	$\approx$	$\approx$	$\approx$	$\approx$
c_1^α	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$\underset{\approx}{-}$	$+$	$-$	$+$	$-$	$\approx$	$\underset{\approx}{+}$	$-$	$-$	$-$	$-$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$-$	$+$	$+$	$+$	$+$

Note: $+(-)$: substantial increase (decrease), $\underset{\approx}{+}\left(\underset{\approx}{-}\right)$: weak increase (decrease), $\approx$: approximately equal, $+/-$:ambiguous effect

$k^\gamma \equiv (k_1^\gamma + k_2^\gamma + k_3^\gamma)/3, k^\delta \equiv (k_1^\delta + k_2^\delta + k_3^\delta)/3, k \equiv (k^\gamma + k^\delta)/2$

$v_\gamma^\alpha \equiv (v_{1\gamma}^\alpha + v_{2\gamma}^\alpha + v_{3\gamma}^\alpha)/3, v_\delta^\beta \equiv (v_{1\delta}^\beta + v_{2\delta}^\beta + v_{3\delta}^\beta)/3, v \equiv (v_\gamma^\alpha + v_\delta^\beta)/2$

Table 3.A.2: Simulation results: the base-line model (Directional effects of an increase in exogenous parameters on endogenous variables)

	$v^{\beta}_{1\delta}$	$v^{\beta}_{2\delta}$	$v^{\beta}_{3\delta}$	v^{β}_{δ}	v	b^{α}_0	q^{α}_1	q^{α}_2	q^{α}_3	b^{β}_0	q^{β}_1	q^{β}_2	q^{β}_3	q^{ϕ}_0	b^{ϕ}_1	b^{ϕ}_2	b^{ϕ}_3	b^{ϕ}_j	b^{δ}_j	q^{γ}_j	U^{α}	U^{β}	U^{ϕ}	U^{γ}	U^{δ}	U^{H}	U^{B}
M	+	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	≈	≈	+ ≈	+ ≈	≈	≈	≈	+ ≈	+ ≈	+ ≈	+	− ≈	≈	+ ≈	≈	≈	+ ≈	− ≈	≈	− ≈
m^{α}_1	+ ≈	− ≈	− ≈	≈	+ ≈	+ ≈	+ ≈	− ≈	− ≈	− ≈	+ ≈	− ≈	− ≈	≈	+ ≈	− ≈	− ≈	+	− ≈	+ ≈	+ ≈	≈	≈	+ ≈	+ ≈	≈	+ ≈
$\overline{\omega}$	+	−	−	− ≈	− ≈	+	+ ≈	− ≈	− ≈	−	+ ≈	− ≈	− ≈	≈	+	−	−	+	+	+	+ ≈	− ≈	+/	−	+ ≈	≈	−
$\lambda^{\alpha}_{1\gamma}$	+ ≈	−	−	−	+	+ ≈	+	− ≈	− ≈	− ≈	+ ≈	− ≈	− ≈	≈	+	−	−	+	−	+ ≈	+	− ≈	+ ≈	+	− ≈	+ ≈	≈
$\lambda^{\beta}_{1\delta}$	+	+	+	+	+	− ≈	− ≈	+ ≈	+ ≈	+ ≈	+	+ ≈	+ ≈	− ≈	−	+	+	−	+/	−	− ≈	+ ≈	+ ≈	−	+ ≈	+ ≈	≈
λ^{γ}_{1k}	−	+	+	+ ≈	+ ≈	−	− ≈	+ ≈	+ ≈	+	− ≈	+ ≈	+ ≈	≈	−	+	+	−	+	− ≈	− ≈	+ ≈	+/	−	−	≈	−
λ^{δ}_{1k}	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	+	+	≈	≈	≈	≈	−	≈	−
e^{ϕ}_0	+	+	+	+	+	− ≈	+ ≈	− ≈	− ≈	+ ≈	+ ≈	− ≈	− ≈	+	+	−	−	+	−	− ≈	+	+	+	+	− ≈	+	≈
e^{α}_1	−	+	+	−	≈	+	+	+ ≈	+ ≈	−	−	+ ≈	+ ≈	−	−	+	+	−	+	+	+	−	+	−	+ ≈	+ ≈	− ≈
e^{β}_1	≈	+	+	+	≈	−	−	+ ≈	+ ≈	+	+	+ ≈	+ ≈	−	−	+	+	−	+/	−	−	+ ≈	+	−	+ ≈	+ ≈	≈
e^{γ}_1	−	+	+	+ ≈	+ ≈	−	− ≈	+ ≈	+ ≈	+	− ≈	+ ≈	+ ≈	≈	−	+	+	−	+	−	≈	≈	≈	+	≈	≈	+ ≈
e^{δ}_1	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	+	+	≈	≈	≈	≈	+ ≈	≈	+ ≈
c^{α}_1	−	+	+	−	≈	+	+	+ ≈	+ ≈	−	−	+ ≈	+ ≈	−	−	+	+	−	+	+	−	−	+	−	+ ≈	− ≈	− ≈

Note: $U^H \equiv (U^{\alpha} + U^{\beta} + U^{\phi})$, $U^B \equiv U^{\gamma} + U^{\delta}$

Table 3.A.2 (CONTINUE)

Appendix B

In this Appendix, we describe the banks' and agents' optimisation problems for the extended model. However, we only describe the problems of Mr. ϕ and the two banks since those of Mr. α and β remain the same as in the base-line model.

Household ϕ

$$\max_{\{q_0^\phi, b_s^\phi, d_\gamma^\phi, d_\delta^\phi\}, s \in S} U^\phi = [\chi_0^\phi - c_\phi(\chi_0^\phi)^2] + \sum_{s \in S} [\chi_s^\phi - c_{\phi,s}(\chi_s^\phi)^2]$$

subject to

$$d_\delta^\phi + d_\gamma^\phi \le m_0^\phi \tag{B1}$$

$$q_0^\phi \le e_0^\phi \tag{B2}$$

$$\chi_0^\phi \le e_0^\phi - q_0^\phi \tag{B3}$$

$$b_s^\phi \le \Delta(\text{B1}) + p_0 q_0^\phi + v_{s\phi}^\gamma d_\gamma^\phi(1 + r_d^\gamma) + d_\delta^\phi(1 + r_d^\delta), \ s \in S \tag{B4}$$

$$\chi_s^\phi \le \frac{b_s^\phi}{p_s}, \ s \in S \tag{B5}$$

where,

$d_b^\phi \equiv$amount of money that Mr. ϕ deposits with bank b, $b \in B$

$r_d^b \equiv$deposit rate offered by bank $b \in B$, and

$v_{s\phi}^\gamma \equiv$bank γ's repayment rates in the deposit market in state $s \in S$.

Bank γ

$$\max_{\{\mu^\gamma, \mu_d^\gamma, \overline{m}^\gamma, v_s^\gamma, v_{s\phi}^\gamma\}, s \in S} U^\gamma = \sum_{s \in S}[\pi_s^\gamma] - \sum_{s \in S}\left[\lambda_{ks}^\gamma \max[0, \overline{k} - k_s^\gamma] - \lambda_s^\gamma[\ \mu^\gamma - v_s^\gamma \mu^\gamma]^2 - \lambda_{s\phi}^\gamma[\ \mu_d^\gamma - v_{s\phi}^\gamma \mu_d^\gamma]^2\right]$$

subject to

$$\overline{m}^\gamma \le \frac{\mu^\gamma}{(1+\rho)} + \frac{\mu_d^\gamma}{(1 + r_d^\gamma)} \tag{B6}$$

$$v_s^\gamma \mu^\gamma + v_{s\phi}^\gamma \mu_d^\gamma \le \Delta(\text{B6}) + v_{s\gamma}^\alpha(1 + r^\gamma)\overline{m}^\gamma + e_s^\gamma, \ s \in S \tag{B7}$$

where,

$\pi_s^\gamma = \Delta(\text{B7})$,

$k_s^\gamma = \frac{e_s^\gamma}{\overline{\omega} v_{s\gamma}^\alpha (1+r^\gamma)\overline{m}^\gamma}$, $s \in S$,

$v_s^\gamma \equiv$ bank γ's repayment rate in the interbank market in state $s \in S$,

$\mu_d^b \equiv$ deposit demand by bank b, $b \in B$,

$\lambda_s^\gamma \equiv$ default penalty in the interbank market imposed on bank γ in state $s \in S$, and

$\lambda_{s\phi}^\gamma \equiv$default penalty in the deposit market imposed on bank γ in state $s \in S$

Bank δ

$$\max_{\{d^\delta, \mu_d^\delta, \overline{m}^\delta\}} U^\delta = \sum_{s\in S}[\pi_s^\delta] - \sum_{s \in S}\lambda_{ks}^\delta \max[0, \overline{k} - k_s^\delta]$$

subject to

$$d^\delta \leq e_0^\delta \tag{B8}$$

$$\overline{m}^\delta \leq \triangle(\text{B8}) + \frac{\mu_d^\delta}{(1+r_d^\delta)} \tag{B9}$$

$$\mu_d^\delta \leq \triangle(\text{B9}) + v_{s\delta}^\beta \overline{m}^\delta(1+r^\delta) + v_s^\gamma d^\delta(1+\rho) + e_s^\delta \tag{B10}$$

where,

$\pi_s^\delta = \triangle(\text{B10})$, and

$k_s^\delta = \frac{e_1^\delta}{\overline{\omega} v_{s\delta}^\beta (1+r^\delta)\overline{m}^\delta + \omega v_s^\gamma d^\delta (1+\rho)}$.

Market Clearing Conditions

$$p_0 = \frac{b_0^\alpha + b_0^\beta}{q_0^\phi} \quad \text{(i.e. commodity market at } t=1 \text{ clears)} \tag{B11}$$

$$p_s = \frac{b_s^\phi}{q_s^\alpha + q_s^\beta},\ s \in S \text{ (i.e. commodity market at } t=2,\ s \in S \text{ clears)} \tag{B12}$$

$$1+\rho = \frac{\mu^\gamma}{M^{CB} + d^\delta} \quad \text{(i.e. interbank market clears)} \tag{B13}$$

$$1+r^\gamma = \frac{\mu^{\alpha^\gamma}}{\overline{m}^\gamma} \quad \text{(i.e. bank } \gamma\text{'s loan market clears)} \tag{B14}$$

$$1+r^\delta = \frac{\mu^{\beta^\delta}}{\overline{m}^\delta} \quad \text{(i.e. bank } \delta\text{'s loan market clears)} \tag{B15}$$

$$1+r_d^\gamma = \frac{\mu_d^\gamma}{d_\gamma^\phi} \quad \text{(i.e. bank } \gamma\text{'s deposit market clears)} \tag{B16}$$

$$1+r_d^\delta = \frac{\mu_d^\delta}{d_\delta^\phi} \quad \text{(i.e. bank } \delta\text{'s deposit market clears)} \tag{B17}$$

Equilibrium is defined similarly to that given in subsection 3.2.1.6.

Appendix C

Exogenous variables					
Coefficient of	Endowment		Penalty		
risk aversion	Commodities	Money	Capital	Default	CAR violation
$c_0^\alpha = 0.011$	$e_0^\alpha = 0$	$m_0^\alpha = 0$	$e_0^\gamma = 9$	$\lambda_{1\gamma}^\alpha = 0.2$	$\lambda_{1k}^\gamma = 3$
$c_1^\alpha = 0.111$	$e_1^\alpha = 27$	$m_1^\alpha = 2.947$	$e_1^\gamma = 1.728$	$\lambda_{2\gamma}^\alpha = 0.1$	$\lambda_{2k}^\gamma = 2$
$c_2^\alpha = 0.073$	$e_2^\alpha = 27$	$m_2^\alpha = 3.015$	$e_2^\gamma = 1.411$	$\lambda_{3\gamma}^\alpha = 0.05$	$\lambda_{3k}^\gamma = 1$
$c_3^\alpha = 0.067$	$e_3^\alpha = 27$	$m_3^\alpha = 1.902$	$e_3^\gamma = 1.116$	$\lambda_{1\delta}^\beta = 0.2$	$\lambda_{1k}^\delta = 4$
$c_0^\beta = 0.010$	$e_0^\beta = 0$	$m_0^\beta = 0$	$e_0^\delta = 21.5$	$\lambda_{2\delta}^\beta = 0.1$	$\lambda_{2k}^\delta = 2$
$c_1^\beta = 0.015$	$e_1^\beta = 27$	$m_1^\beta = 3.397$	$e_1^\delta = 2.047$	$\lambda_{3\delta}^\beta = 0.05$	$\lambda_{3k}^\delta = 1$
$c_2^\beta = 0.105$	$e_2^\beta = 27$	$m_2^\beta = 1.852$	$e_2^\delta = 1.656$	$\lambda_1^\gamma = 1.576$	Others
$c_3^\beta = 0.071$	$e_3^\beta = 27$	$m_3^\beta = 2.543$	$e_3^\delta = 1.316$	$\lambda_2^\gamma = 1.123$	
$c_0^\phi = 0.040$	$e_0^\phi = 45$	$m_0^\phi = 9$		$\lambda_3^\gamma = 0.936$	$M^{CB} = 0.5$
$c_1^\phi = 0.007$	$e_1^\phi = 0$	$m_1^\phi = 0$		$\lambda_{1\phi}^\gamma = 1.685$	$\varpi = 1$
$c_2^\phi = 0.009$	$e_2^\phi = 0$	$m_2^\phi = 0$		$\lambda_{2\phi}^\gamma = 1.404$	$\omega = 0.5$
$c_3^\phi = 0.011$	$e_3^\phi = 0$	$m_3^\phi = 0$		$\lambda_{3\phi}^\gamma = 1.203$	$\overline{k} = 0.1$

Initial Equilibrium					
Prices	Interest rates	Loans/ deposit	Capital/Asset ratio	Repayment rate	Commodities
$p_0 = 1$	$r^\gamma = 0.65$	$\overline{m}^\gamma = 19.05$	$k_1^\gamma = 0.06$	$v_{1\gamma}^\alpha = 0.919$	$b_0^\alpha = 19.04$
$p_1 = 1.1$	$r^\delta = 0.6$	$\overline{m}^\delta = 21$	$k_2^\gamma = 0.05$	$v_{2\gamma}^\alpha = 0.9$	$q_1^\alpha = 23.5$
$p_2 = 1.2$	$\rho = 0.48$	$d_\gamma^\phi = 4.31$	$k_3^\gamma = 0.04$	$v_{3\gamma}^\alpha = 0.89$	$q_2^\alpha = 21$
$p_3 = 1.3$	$r_d^\gamma = 0.48$	$d_\delta^\phi = 4.69$	$k_1^\delta = 0.06$	$v_{1\delta}^\beta = 0.899$	$q_3^\alpha = 20$
	$r_d^\delta = 0.4$	$d^\delta = 5.24$	$k_2^\delta = 0.05$	$v_{2\delta}^\beta = 0.87$	$b_0^\beta = 20.95$
		$\mu^{\alpha^\gamma} = 31.37$	$k_3^\delta = 0.04$	$v_{3\delta}^\beta = 0.866$	$q_1^\beta = 24.355$
		$\mu^{\beta^\delta} = 33.58$		$v_1^\gamma = 0.963$	$q_2^\beta = 22.82$
		$\mu^\gamma = 8.5$		$v_2^\gamma = 0.95$	$q_3^\beta = 20.4$
		$\mu_d^\gamma = 6.38$		$v_3^\gamma = 0.94$	$q_0^\phi = 40$
		$\mu_d^\delta = 6.58$		$v_{1\phi}^\gamma = 0.954$	$b_1^\phi = 52.67$
				$v_{2\phi}^\gamma = 0.944$	$b_2^\phi = 52.61$
				$v_{3\phi}^\gamma = 0.935$	$b_3^\phi = 52.55$

Table 3.C.1: Exogenous variables and Initial equilibrium: the extended model

	p_0	p_1	p_2	p_3	r^γ	r^δ	ρ	r_d^γ	r_d^δ	$\overline{m}^\gamma$	$\overline{m}^\delta$	d_γ^ϕ	d_δ^ϕ	μ_d^γ	μ_d^δ	d^δ	μ^{α^γ}	μ^{β^δ}	μ^γ	$v_{1\gamma}^\alpha$	$v_{2\gamma}^\alpha$	$v_{3\gamma}^\alpha$	v_γ^α
M^{CB}	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$-$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$-$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$+$	$-$	$\underset{\approx}{-}$	$\approx$	$\approx$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$
m_1^α	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$-$	$+$	$+$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$-$	$-$	$+$	$\underset{\approx}{-}$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$+$	$+$	$\approx$	$\approx$	$+$
$\overline{\omega}$	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$+$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$+$	$-$	$+$	$-$	$-$	$\approx$	$\approx$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$
$\lambda_{1\gamma}^\alpha$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\approx$	$\approx$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$+$	$+$	$+$	$+$
$\lambda_{1\delta}^\beta$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\approx$	$\approx$	$\underset{\approx}{+}$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$
λ_{1k}^γ	$\approx$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$+$	$-$	$-$	$-$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$-$	$+$	$-$	$\underset{\approx}{+}$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$-$	$\approx$	$\approx$	$\approx$	$\approx$
λ_{1k}^δ	$\approx$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$-$	$+$	$-$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$
e_0^ϕ	$-$	$\approx$	$\approx$	$\approx$	$+$	$+$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$+$	$+$	$\approx$	$+$	$+$	$+$	$+$
e_1^α	$\underset{\approx}{+}$	$-$	$\approx$	$\approx$	$-$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$+$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$+$
e_1^β	$\underset{\approx}{+}$	$-$	$\approx$	$\approx$	$+$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$-$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$-$
e_1^γ	$\approx$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$+$	$-$	$-$	$-$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$-$	$+$	$-$	$\underset{\approx}{+}$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$-$	$\approx$	$\approx$	$\approx$	$\approx$
e_1^δ	$\approx$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$-$	$+$	$-$	$-$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$-$	$+$	$-$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$
c_1^α	$\underset{\approx}{+}$	$-$	$\approx$	$\approx$	$-$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$+$	$-$	$-$	$+$	$+$	$-$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$+$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$
e_0^δ	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$-$	$+$	$-$	$+$	$\underset{\approx}{+}$	$\approx$	$+$	$+$	$+$	$+$	$+$
e_0^γ	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$	$-$	$+$	$+$	$-$	$+$	$-$	$+$	$-$	$\approx$	$\underset{\approx}{+}$	$-$	$+$	$+$	$+$	$+$
λ_1^γ	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$+$	$-$	$+$	$-$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$-$	$\approx$	$\approx$	$\approx$	$\approx$
$\lambda_{1\phi}^\gamma$	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$-$	$+$	$-$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$
m_1^β	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$-$	$+$	$+$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$
m_0^ϕ	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$-$	$+$	$-$	$+$	$\underset{\approx}{+}$	$\approx$	$+$	$+$	$+$	$+$	$+$

Note: $+(-)$: substantial increase (decrease), $\underset{\approx}{+}\left(\underset{\approx}{-}\right)$: weak increase (decrease), $\approx$: approximately equal,

$\approx$: approximately equal, $+/-$:ambiguous effect

$v_\gamma^\alpha \equiv (v_{1\gamma}^\alpha + v_{2\gamma}^\alpha + v_{3\gamma}^\alpha)/3, v_\delta^\beta \equiv (v_{1\delta}^\beta + v_{2\delta}^\beta + v_{3\delta}^\beta)/3, v \equiv (v_\gamma^\alpha + v_\delta^\beta)/2$

Table 3.C.2: Simulation results: the extended model (Directional effects of an increase in exogenous parameters on endogenous variables)

	$v_{1\delta}^{\beta}$	$v_{2\delta}^{\beta}$	$v_{3\delta}^{\beta}$	v_{δ}^{β}	v	v_1^{γ}	v_2^{γ}	v_3^{γ}	v^{γ}	$v_{1\phi}^{\gamma}$	$v_{2\phi}^{\gamma}$	$v_{3\phi}^{\gamma}$	v_{ϕ}^{γ}	b_0^{α}	q_1^{α}	q_2^{α}	q_3^{α}	b_0^{β}	q_1^{β}	q_2^{β}	q_3^{β}
M^{CB}	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	≈	≈	≈	+	≈	≈	≈
m_1^{α}	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	≈	≈	≈	− ≈	≈	≈	≈
$\overline{\omega}$	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	− ≈	− ≈	− ≈	− ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	≈	≈	≈	− ≈	≈	≈	≈
$\lambda_{1\gamma}^{\alpha}$	− ≈	≈	≈	− ≈	+	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+	≈	≈	− ≈	− ≈	≈	≈
$\lambda_{1\delta}^{\beta}$	+	+	+	+	+	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	≈	≈	+ ≈	+	≈	≈
λ_{1k}^{γ}	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	≈	≈	≈	+ ≈	≈	≈	≈
λ_{1k}^{δ}	− ≈	− ≈	− ≈	− ≈	− ≈	+ ≈	+ ≈	+ ≈	+ ≈	− ≈	− ≈	− ≈	− ≈	+ ≈	≈	≈	≈	− ≈	≈	≈	≈
e_0^{ϕ}	+	+	+	+	+	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈
e_1^{α}	−	+ ≈	+ ≈	−	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+	+	≈	≈	−	− ≈	≈	≈
e_1^{β}	+	− ≈	− ≈	+	+ ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	−	− ≈	≈	≈	+	+	≈	≈
e_1^{γ}	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	≈	≈	≈	+ ≈	≈	≈	≈
e_1^{δ}	− ≈	− ≈	− ≈	− ≈	− ≈	+ ≈	+ ≈	+ ≈	+ ≈	− ≈	− ≈	− ≈	− ≈	+ ≈	≈	≈	≈	− ≈	≈	≈	≈
c_1^{α}	−	+ ≈	+ ≈	− ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+	+	≈	≈	−	− ≈	≈	≈
e_0^{δ}	+	+	+	+	+	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+	+ ≈	+ ≈	+ ≈	+	+ ≈	+ ≈	+ ≈
e_0^{γ}	+	+	+	+	+	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	+	+ ≈	+ ≈	+ ≈	+	+ ≈	+ ≈	+ ≈
λ_1^{γ}	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+	− ≈	− ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	≈	≈	≈	− ≈	≈	≈	≈
$\lambda_{1\phi}^{\gamma}$	+ ≈	≈	≈	+ ≈	≈	+ ≈	+ ≈	+ ≈	+ ≈	+	− ≈	− ≈	+ ≈	+ ≈	≈	≈	≈	− ≈	≈	≈	≈
m_1^{β}	+	+ ≈	+ ≈	+	+	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	−	≈	≈	≈	+	≈	≈	≈
m_0^{ϕ}	+	+	+	+	+	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+	+ ≈	+ ≈	+ ≈	+	+ ≈	+ ≈	+ ≈

Note: $v^{\gamma} \equiv (v_1^{\gamma} + v_2^{\gamma} + v_3^{\gamma})/3, v_{\phi}^{\gamma} \equiv (v_{1\phi}^{\gamma} + v_{2\phi}^{\gamma} + v_{3\phi}^{\gamma})/3$

Table 3.C.2 (CONTINUE)

	q_0^ϕ	b_1^ϕ	b_2^ϕ	b_3^ϕ	U^α	U^β	U^ϕ	U^γ	U^δ	U^H	U^B	k_1^γ	k_2^γ	k_3^γ	k^γ	k_1^δ	k_2^δ	k_3^δ	k^δ	k
M^{CB}	≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	≈	− ≈	−	≈	− ≈	− ≈	− ≈	− ≈	− ≈	+ ≈	+ ≈	+ ≈	+ ≈	− ≈
m_1^α	≈	+ ≈	+ ≈	+ ≈	+	− ≈	≈	+ ≈	+ ≈	+ ≈	+ ≈	−	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	−
$\overline{\omega}$	≈	+ ≈	+ ≈	+ ≈	+ ≈	≈	≈	−	− ≈	≈	− ≈	−	−	−	−	−	−	−	−	−
$\lambda_{1\gamma}^\alpha$	− ≈	≈	≈	≈	+	− ≈	+ ≈	− ≈	≈	+	≈	− ≈	≈	≈	− ≈	+ ≈	− ≈	− ≈	+ ≈	− ≈
$\lambda_{1\delta}^\beta$	− ≈	≈	≈	≈	− ≈	+	+ ≈	≈	≈	+	≈	+ ≈	≈	≈	+ ≈	− ≈	+ ≈	+ ≈	− ≈	+ ≈
λ_{1k}^γ	≈	− ≈	− ≈	− ≈	− ≈	+ ≈	≈	+ ≈	− ≈	≈	− ≈	+ ≈	+ ≈	+ ≈	+ ≈	− ≈	− ≈	− ≈	− ≈	+ ≈
λ_{1k}^δ	≈	− ≈	− ≈	− ≈	+ ≈	− ≈	≈	− ≈	− ≈	≈	− ≈	+ ≈	+ ≈	+ ≈	+ ≈	−	+	+	+	≈
e_0^ϕ	+	≈	≈	≈	+	+	+ ≈	≈	≈	+	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈
e_1^α	−	−	+	+	+	−	+	− ≈	+ ≈	+ ≈	≈	−	≈	≈	−	+	− ≈	− ≈	+	− ≈
e_1^β	− ≈	≈	≈	≈	−	+	+	+ ≈	− ≈	+ ≈	≈	+	≈	≈	+	−	+ ≈	+ ≈	−	+ ≈
e_1^γ	≈	− ≈	− ≈	− ≈	− ≈	+ ≈	≈	+	− ≈	≈	+	+	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+
e_1^δ	≈	− ≈	− ≈	− ≈	+ ≈	− ≈	≈	− ≈	+	≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+	− ≈	− ≈	+	+
c_1^α	− ≈	≈	≈	≈	−	−	+ ≈	− ≈	≈	− ≈	≈	−	≈	≈	−	+	− ≈	− ≈	+ ≈	− ≈
e_0^δ	+ ≈	+	+	+	+	+	≈	−	+	+ ≈	+	−	−	−	−	−	−	−	−	−
e_0^γ	+ ≈	+	+	+	+ ≈	+ ≈	≈	+	−	+ ≈	+	−	−	−	−	− ≈	− ≈	− ≈	− ≈	−
λ_1^γ	≈	+ ≈	+ ≈	+ ≈	+ ≈	− ≈	≈	− ≈	+ ≈	≈	+ ≈	− ≈	− ≈	− ≈	− ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈
$\lambda_{1\phi}^\gamma$	≈	+ ≈	− ≈	− ≈	+ ≈	− ≈	≈	− ≈	+ ≈	≈	+ ≈	− ≈	+ ≈	+ ≈	− ≈	−	−	−	−	−
m_1^β	− ≈	+ ≈	+ ≈	+ ≈	− ≈	+	≈	+	+	+ ≈	+	− ≈	− ≈	− ≈	− ≈	−	− ≈	− ≈	− ≈	−
m_0^ϕ	+ ≈	+	+	+	+	+	− ≈	−	+	+ ≈	+	−	−	−	−	−	−	−	−	−

Note: $U^H \equiv (U^\alpha + U^\beta + U^\phi)$, $U^B \equiv U^\gamma + U^\delta$
$k^\gamma \equiv (k_1^\gamma + k_2^\gamma + k_3^\gamma)/3, k^\delta \equiv (k_1^\delta + k_2^\delta + k_3^\delta)/3, k \equiv (k^\gamma + k^\delta)/2$

Table 3.C.2 (CONTINUE)

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Chapter 4

A Risk Assessment Model for Banks

Abstract

The objective of this chapter is to propose a model to assess risk for banks. Its main innovation is to incorporate endogenous interaction between banks, recognising that the actual risk to which an individual bank is exposed also depends on its interaction with other banks and other private sector agents. To this end, we develop a two-period general equilibrium model with three active heterogeneous banks, incomplete markets, and endogenous default. The setting of three heterogeneous banks allows us to study not only interaction between any two individual banks, but also their interaction with the rest of the banks in the banking system. We show that the model is analytically tractable and can be calibrated against real UK banking data and therefore can be implemented as a risk assessment tool for financial regulators and central banks. We address the impact of monetary and regulatory policy as well as credit and capital shocks in the real and financial sectors.

4.1 Introduction

The objective of this chapter[1] is to propose a model to assess risk for banks. Existing models for this purpose, e.g. stress-testing models, focus almost entirely on *individual* institutions.[2] The major flaw of these models is that they fail to recognise that the actual risk to which an individual bank is exposed also depends on its interaction with other banks and other private sector agents and therefore is *endogenous*. This endogeneity aspect of risk may matter enormously in times of financial crises; a negative shock which is *ex ante* specific to a particular bank can produce a

[1]This chapter is based on a joint paper with Charles A.E. Goodhart and Dimitrios P. Tsomocos. The paper is published in *Annals of Finance* (forthcoming).

[2]An exception is, for example, an approach developed in Elsinger, Lehar and Summer (2003).

serious strain on others through a series of interactive contagion effects. The main innovation of our approach, therefore, is to take into account such *endogenous* interaction between banks, particularly through their mutual exposures in the interbank market. To this end, we require a set-up which can incorporate heterogeneous banks, each with a unique risk/return portfolio; if they were identical, there would be no interbank market by definition. We also need default to exist, since if there were no default, there would be no crises. Moreover, financial markets cannot be complete. Otherwise banks can always hedge themselves against all kinds of shocks, in which case, there would, again, be no crises. We have constructed a two-period general equilibrium model along these lines in Goodhart, Sunirand, and Tsomocos (2003).

We show in Goodhart et al. (2003) that an equilibrium exists in such model and that financial fragility emerges naturally as an equilibrium phenomenon. However, given the scale of the model which contains B heterogeneous banks, H private sector agents, S possible states, a variety of financial assets and default, it is impossible to find either a closed-form or a numerical solution to this general model. In Chapter Three, we therefore present a *simplified* version of our general model and show that it can be solved *numerically*. This implies that the model, in some sense, 'works' and can, in principle, be used to assess various policies for crisis management. However, the model was solved on the basis of an arbitrarily chosen set of initial conditions. The outcome, therefore, is a somewhat artificial construct of our own assumed inputs. In this chapter, we take a step further by attempting to calibrate an alternative version of our general model against *real* UK banking data. Thus, we argue that our model is not only rich enough to incorporate endogenous interaction between banks, but it is also sufficiently flexible to be implemented as a risk assessment tool for financial regulators and central banks.

The model presented in Chapter Three has two heterogeneous banks. A banking system, however, generally comprises multiple banks. Thus, although the model can be used to study interaction between *any* two individual banks, its level of complexity is not sufficient to incorporate contagion effects arising from their interaction with the rest of the banking sector. In this chapter, we therefore introduce an additional bank, which can be thought of as the aggregation of the remaining banks in the system. Given the lack of disaggregated household and investors' portfolio data, we model household behaviour via reduced-form equations which relate their actions to a variety of economic variables such as GDP, interest rates, and aggregate credit supply etc. In this sense, our model is a partially-microfounded general equilibrium model.[3] However, the main aspects of equilibrium

[3] As is shown in Chapter Three, household and investor optimisation problem can be introduced. However, owing to the limited availability of disaggregated household data, we chose not to follow that route.

analysis which are market clearing, rational expectations, and agent optimisation are maintained. Moreover, contagion effects between the banking sector and the real economy still operate actively in equilibrium via the reduced-form equations. Thus, we adhere to the general equilibrium spirit of our models presented in Goodhart et al. (2003) and Chapter Three. The upshot of our modelling framework is that financial decisions generate real effects in the rest of the economy. Our model is therefore amenable to welfare analysis.

The rest of the chapter is structured as follows. The next section presents the model. Section 4.3 then explains how the model is calibrated against UK banking data. Section 4.4 provides a stress-testing analysis for the UK banking sector. Section 4.5 presents a robustness check of the result obtained in section 4.4. The final section provides concluding remarks.

4.2 The Model

The model has three heterogeneous banks, $b \in B = \{\gamma, \delta, \tau\}$, four private sector agents, $h \in H = \{\alpha, \beta, \theta, \phi\}$, a Central Bank and a regulator. The time horizon extends over two periods, $t \in T = \{1, 2\}$ and two possible states in the second period, $s \in S = \{i, ii\}$. We assume that state i is a normal/good state whereas state ii represents an extreme/crisis event. The probability that state i will occur is denoted by p.

As in Goodhart et al. (2003) and in Chapter Three, we assume that individual bank borrowers are assigned during the two periods, by history or by informational constraint, to borrow from a single bank (i.e. a limited participation assumption).[4] Given this assumption, together with our set-up of a system of three heterogeneous banks, we need at least three borrowers. We therefore assume that agents α, β, and θ borrow from banks γ, δ, and τ, respectively. The remaining agent, Mr. ϕ, represents the pool of depositors in this economy which supplies funds to every bank. This implies that we have multiple active markets for deposits (by separate bank) and for loans (by borrower and bank). In addition, we also assume a single, undifferentiated, interbank market where deficit banks are allowed to borrow from surplus banks, and wherein the Central Bank conducts open market operations (OMOs).

The time structure of the model is presented in Figure 4.1. At $t = 1$, loan, deposit and interbank markets open. Banks decide how much to lend/borrow in each market, expecting rationally any one of the two possible future scenarios to be realised. Moreover, the Central Bank also conducts

[4]In Bhattacharya, Goodhart, Sunirand and Tsomocos (2003) we show that restricted participation in the loan market can also arise as an equilibrium outcome given that the objective functions of banks also include a relative performance criterion, i.e. a preference to outperform their competitors.

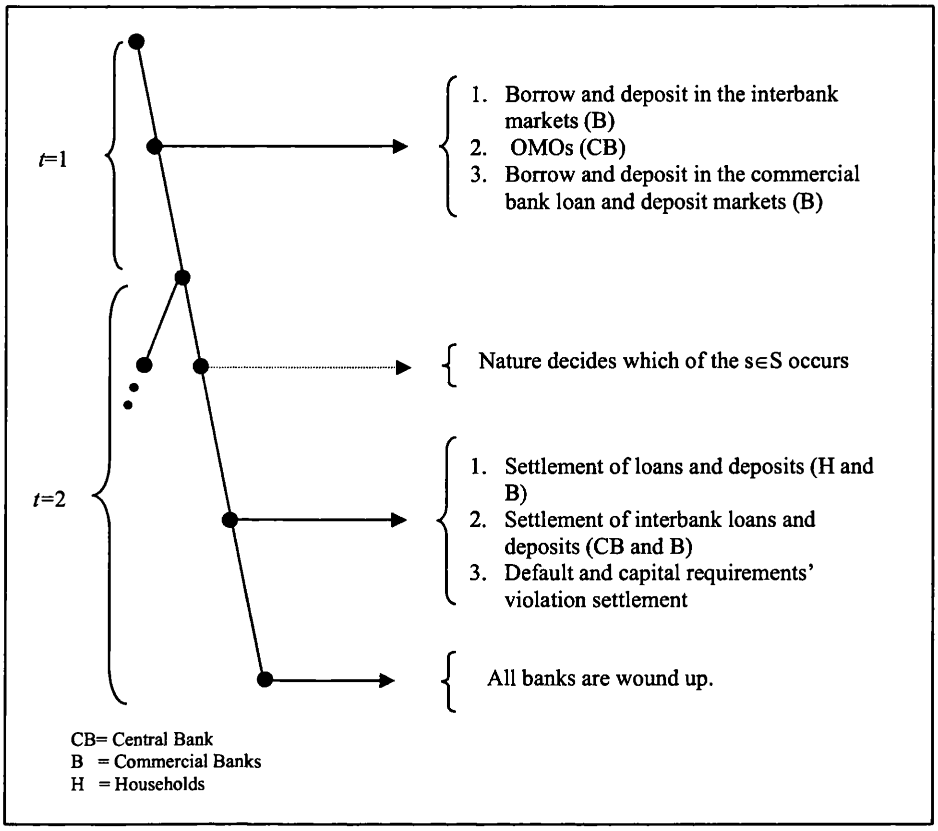

Figure 4.1: The time structure of the model

OMOs in the interbank market. At $t = 2$, depending on the state which actually occurs, all financial contracts are settled, subject to any defaults and/or capital requirements' violations, which are then penalised. At the end of the second period, all banks are wound up.

4.2.1 Banking Sector: UK banks

Without loss of generality, our specification of the banking sector is based on the UK banking sector, which we assume to comprise of seven largest UK banks; Lloyds, HSBC, Abbey National, HBOS, Barclays, Royal Bank of Scotland, and Standard Chartered. Banks γ and δ can represent any two of these individual banks, whereas bank τ represents the aggregation of the remaining banks. As will be explained more below, in our calibration exercise, banks γ and δ are chosen *specifically* to represent two of these actual UK banks. However, for data confidentiality reason, we do not reveal their identities.

All banks in the model, $b \in B = \{\gamma, \delta, \tau\}$, are assumed to operate under a perfectly competitive environment (i.e. they take all interest rates as exogenously given when making their optimal portfolio decisions). The structure of their balance sheets is given below;

Assets	Liabilities
Loans to agents	Deposits from Mr.ϕ
Interbank deposits	Interbank borrowing
Market book	Equity
	Others

We assume that all banks endogenise their decisions in the loan, deposit and interbank markets.[5] The remaining variables are treated as exogenous.[6] We further assume that banks in our model can default on their financial obligations, subject to default penalties set by the regulator. Thus, by varying the penalties imposed on default from 0 to infinity, we can model 100% default, no default or an equilibrium level of default between 0 and 100%.[7] At first sight, this 'continuous' default rate approach may seem problematic since in reality banks either repay in full at the due date or are forced to close down. However, we interpret a bank's default rate in our model as a *probability* that such bank *chooses* to shut down, and hence in the short run to default *completely* on its financial obligations. For example, a default rate of 4 percent implies that there is roughly a 4 percent chance of a shut down and a 96 percent chance that the bank will repay in full and continue its normal operation. Therefore, a bank's decision to increase its default rates is isomorphic to its decision to adopt a riskier position in pursuit of higher expected profitability.[8] Finally, as in Bhattacharya et al. (2003), we make a simplifying assumption by assuming that banks' default rates in the deposit and interbank markets are the same, i.e. banks are restricted to repay all their creditors similarly.

Analogous to the modelling of default, banks can violate their capital adequacy requirement, subject to capital requirement violation penalties set by the regulator. In principle, each bank's *effective* capital to asset ratios may not be binding, (i.e. their values may be above the regulator's requirement), in which case they are not subject to any capital requirement penalty. However, in our calibration exercise, we assume for simplicity that each bank wants to keep a buffer above the required minimum, so that there is a non-pecuniary loss of comfort and reputation as capital

[5] The modelling of the banking sector follows Shubik and Tsomocos (1992) and Tsomocos (2003 and 2004).

[6] Given the present set-up, we cannot endogenise banks' decisions on market book or equity. This is because the model has two states in the second period and one unconstrained asset (i.e. an asset that banks can either go infinitely short or long), which is the interbank market investment. By adding another unconstrained asset, markets would be complete. In principle, our model can be extended to incorporate additional states in the second period and therefore can be used to study the economic effects on the market (trading) book. For example, we can disaggregate the market book into two components according to their riskiness (or rating) and endogenise banks' decisions on these variables. This would allow us to study the endogenous response of risk premia on corporate debt to a series of shocks. However, we face a practical problem on this front since there are insufficient data on the composition of the market book by category, e.g. rating maturity, and currency.

[7] This modelling of default follows Shubik and Wilson (1977).

[8] For more on this issue, see work in progress by Tsomocos and Zicchino (2004).

declines; in this sense the ratios are *always* binding. Put differently, we assume that banks' self-imposed ideal capital holdings are always above the actual values of all banks' capital to asset ratios. Given this assumption, we can rule out corner equilibria and therefore focus our analysis entirely on well-defined interior solutions whereby banks violate their enhanced capital requirements. We assume that penalties are linear as capital declines from its ideal level. In practice, there will be some non-linearity as capital falls below its required minimum, but this is just too complex to model at this stage.

As will be elaborated in section 4.3, our calibration exercise is based on the data of UK banks at the end of 2002. At that point in time, bank δ is a net lender whereas banks γ and τ are net borrowers in the interbank market.[9] Given this setting, we describe the optimisation problems of these banks below.

4.2.1.1 The interbank net borrowers' (banks γ and τ) optimisation problems

Bank $b \in \{\gamma, \tau\}$ maximises its payoff, which is a quadratic function of its expected profitability in the second period minus non-pecuniary penalties that it has to incur if it defaults on its deposit and interbank obligations. It also suffers a capital violation penalty proportional to its capital requirement violation. Formally, the optimisation problem of bank $b \in \{\gamma, \tau\}$ is as follows:

$$\max_{\overline{m}^b, \mu^b, \mu^b_d, v^b_s, s \in S} \Pi^b = \sum_{s \in S} p_s [\frac{\pi^b_s}{10^{10}} - c^b_s \left(\frac{\pi^b_s}{10^{10}}\right)^2] - \sum_{s \in S} p_s \begin{bmatrix} \lambda^b_{ks} \max[0, \overline{k}^b - k^b_s] + \\ \frac{\lambda^b_s}{10^{10}}[\ \mu^b - v^b_s \mu^b] + \frac{\lambda^b_s}{10^{10}}[\ \mu^b_d - v^b_s \mu^b_d] \end{bmatrix}$$

subject to

$$\overline{m}^b + A^b = \frac{\mu^b}{(1+\rho)} + \frac{\mu^b_d}{(1+r^b_d)} + e^b_0 + Others^b \tag{4.1}$$

$$v^b_s \mu^b + v^b_s \mu^b_d + Others^b + e^b_0 \leq v^{h^b}_{sb}(1+r^b)\overline{m}^b + (1+r^A)A^b,\ s \in S \tag{4.2}$$

where,

[9] As noted earlier, we have chosen banks γ and δ to represent specifically two of the seven largest UK banks in our calibration exercise. Bank τ then represents the aggregation of the remaining five banks.

$$\pi_s^b = \Delta(4.2) \tag{4.3}$$

$$e_s^b = e_0^b + \pi_s^b,\ s \in S \tag{4.4}$$

$$k_s^b = \frac{e_s^b}{\overline{\omega} v_{sb}^{h^b}(1+r^b)\overline{m}^b + \widetilde{\omega}(1+r^A)A^b},\ s \in S \tag{4.5}$$

$\Delta(x) \equiv$ the difference between RHS and LHS of inequality (x)

$p_s \equiv$ probability that state $s \in S$ will occur,

$c_s^b \equiv$ coefficient of risk aversion in the utility function of bank $b \in B$,

$\lambda_{ks}^b \equiv$ capital requirements' violation penalties imposed on bank $b \in B$ in state $s \in S$,

$\overline{k}^b \equiv$ capital adequacy requirement for bank $b \in B$,

$\lambda_s^b \equiv$ default penalties on bank $b \in B$,

$\mu^b \equiv$ amount of money that bank $b \in \{\gamma, \tau\}$ owes in the interbank market,

$\mu_d^b \equiv$ amount of money that bank $b \in B$ owes in the deposit market,

$v_s^b \equiv$ repayment rates of bank $b \in B$ to all its creditors in state $s \in S$,

$\overline{m}^b \equiv$ amount of credit that bank $b \in B$ extends in the loan market,

$A^b \equiv$ the value of market book held by bank $b \in B$,

$e_s^b \equiv$ amount of capital that bank $b \in B$ holds in state $s \in \{0\} \cup S$,

$Others^b \equiv$ the 'others' item in the balance sheet of bank $b \in B$,

$r^b \equiv$ lending rate offered by bank $b \in B$,

$r_d^b \equiv$ deposit rate offered by bank $b \in B$,

$\rho \equiv$ interbank rate,

$r^A \equiv$ the rate of return on market book,

$v_{sb}^{h^b} \equiv$ repayment rates of agent $h^b \in H^b = \{\alpha^\gamma, \beta^\delta, \theta^\tau\}$ to his nature-selected bank $b \in B$ in the consumer loan market,

$\overline{\omega} \equiv$ risk weight on consumer loans, and

$\widetilde{\omega} \equiv$ risk weight on market book.

Equation (4.1) implies that, at $t = 1$, the assets of bank $b \in \{\gamma, \tau\}$, which consist of its credit extension and market book investment, must be equal to its liabilities obtained from interbank and deposit borrowing and its initial equity endowment, where '$Others^b$' represents the residual. Equations (4.2) and (4.3) then show that, dependent on which of the $s \in S$ actually occurs, the profit that bank b incurs in the second period is equal to the difference between the amount of money that it receives from its asset investment and the amount that it has to repay on its liabilities, adjusted appropriately for default in each market. As shown in equation (4.4), this profit earned is then added to its initial capital, which in turn becomes its capital in the second period. Finally, equation (4.5) implies that the capital to asset ratio of bank b in state $s \in S$ is equal to its capital in state s divided by its risk-weighted assets in the corresponding state.

4.2.1.2 The interbank net lender's (bank δ) optimisation problem

Bank δ, unlike the other two banks, is a net lender in the interbank market. Thus it suffers only a default penalty in the deposit market. Formally, bank δ's optimisation problem is as follows:

$$\max_{\overline{m}^\delta, d^\delta, \mu_d^\delta, v_s^\delta, s\in S} \Pi^\delta = \sum_{s\in S} p_s [\frac{\pi_s^\delta}{10^{10}} - c_s^\delta \left(\frac{\pi_s^\delta}{10^{10}}\right)^2] - \sum_{s\in S} p_s \left[\lambda_{ks}^\delta \max[0, \overline{k}^\delta - k_s^\delta] + \frac{\lambda_s^\delta}{10^{10}} [\ \mu_d^\delta - v_s^\delta \mu_d^\delta]\right]$$

subject to

$$A^\delta + d^\delta + \overline{m}^\delta \quad = \quad e_0^\delta + \frac{\mu_d^\delta}{(1+r_d^\delta)} + Others^\delta \qquad (4.6)$$

$$v_s^\delta \mu_d^\delta + Others^\delta + e_0^\delta \quad \leq \quad v_{s\delta}^{\beta^\delta} \overline{m}^\delta (1+r^\delta) + A^\delta(1+r^A) + \widetilde{R}_s d^\delta (1+\rho) \qquad (4.7)$$

where,

$$\pi_s^\delta \quad = \quad \Delta(4.7) \qquad (4.8)$$

$$e_s^\delta \quad = \quad e_0^\delta + \pi_s^\delta \qquad (4.9)$$

$$k_s^\delta \quad = \quad \frac{e_s^\delta}{\overline{\omega} v_{s\delta}^{\beta^\delta} (1+r^\delta)\overline{m}^\delta + \omega \widetilde{R}_s d^\delta (1+\rho) + \widetilde{\omega}(1+r^A) A^\delta} \qquad (4.10)$$

$d^\delta \equiv$ bank δ's investment in the interbank market,

$\widetilde{R}_s \equiv$ the rate of repayment that bank δ expects to get from its interbank investment, and

$\omega \equiv$ risk weight on interbank investment.

The budget set of bank δ is similar to those of the other two banks except that it invests in, instead of borrows from, the interbank market. Moreover, its risk-weighted assets in the second period, as shown in equation (4.10), also includes bank δ's expected return on its interbank investment.

4.2.2 Central Bank and Regulator

The Central Bank's and the regulator's decisions are exogenous. The Central Bank and the regulator may, but need not, be a single institution. The Central Bank conducts monetary policy by engaging in open market operations in the interbank market. We assume as our base-line specification that the Central Bank sets its base money (M) as its monetary policy instrument, allowing the interbank rate to be determined endogenously. However, as will be seen in section 4.3, we also consider an alternative instrument targeting regime whereby the Central Bank fixes the interbank rate and lets its base money adjust endogenously to clear the interbank market. As will be explained below, the simulation results depend crucially on which monetary policy instrument the Central Bank employs. Therefore, whether monetary authorities target base money or the interbank rates in response to shocks has different implications with respect to financial stability.

The regulator sets capital adequacy requirements for all banks ($\overline{k}^b$, $b \in B$) as well as imposes penalties on their failures to meet such requirements (λ^b_{ks}, $b \in B$, $s \in S$) and on default on their financial obligations in the deposit and interbank markets (λ^b_s, $b \in B$, $s \in S$). Finally, he also sets the risk weights on consumer loan, interbank and market book investment ($\overline{\omega}, \omega, \widetilde{\omega}$).

4.2.3 Private agent sector

Each household borrower, $h^b \in H^b = \{\alpha^\gamma, \beta^\delta, \theta^\tau\}$, demands consumer loans from his nature-selected bank b and chooses whether to default on his loans in state $s \in S$. The remaining agent, Mr. ϕ, supplies his deposits to each bank $b \in B$. As mentioned, we do not explicitly model the optimisation problems of households. The reason is that it is very difficult, if at all possible, to find real data for the (heterogeneous) private agent sector, e.g. the monetary and good endowment of *each* banks's borrowers and depositors. This latter is particularly important since one of the key objectives of this chapter is to take our model to *real* data. So, instead of explicitly providing microfoundations for households' decisions, we 'artificially' endogenise them by assuming the following reduced-form equations.

4.2.3.1 Household Borrowers' Demand for Loans

Because of the limited participation assumption in every consumer loan market, each household's demand for loans is a negative function of the lending rate offered by his nature-selected bank. In addition, his demand for loans also depends positively on the expected GDP in the subsequent period. Put differently, we implicitly assume that household borrowers rationally anticipate GDP in both states of the next period, which then determines their expected future income, and adjust their loan demand in this period accordingly in order to smooth their consumption over time. As in Goodhart et al. (2003) and Chapter Three, our money demand function manifests the standard Hicksian elements whereby it responds positively to current and expected income and negatively to interest rates. In particular, we assume the following functional form of household h^b's loan demand from his nature-selected bank b, $\forall h^b \in H^b$, and $b \in B$:

$$\ln(\mu^{h^b}) = a_{h^b,1} + a_{h^b,2}\ln[p(GDP_i) + (1-p)GDP_{ii}] + a_{h^b,3}r^b \qquad (4.11)$$

where,

$\mu^{h^b} \equiv$ amount of money that agent $h^b \in H^b$ chooses to owe in the loan market of bank $b \in B$, and

$GDP_s \equiv$ Gross Domestic Product in state $s \in S$ of the second period.

4.2.3.2 Mr. ϕ's Supply of Deposits

Unlike the loan markets, we do not assume limited participation in the deposit markets. This implies that Mr. ϕ can choose to diversify his deposits with every bank. Thus, Mr. ϕ's deposit supply with bank b depends not only on the deposit rate offered by bank b but also on the rates offered by the other banks. Moreover, since banks in our model can default on their deposit obligations, the expected rate of return on deposit investment of Mr. ϕ with bank b has to be adjusted appropriately for its corresponding expected default rate. Finally, Mr. ϕ's deposit supply is a positive function of the expected GDP in the subsequent period.

In sum, since his deposit decisions determine his investment portfolio, given the expected rates of return, he diversifies among the existing deposit markets. Mr. ϕ's deposit supply function with bank b, $\forall b \in B$, is as follows:

$$\begin{aligned}\ln(d_b^{\phi}) &= z_{b,1} + z_{b,2}\ln[p(GDP_i) + (1-p)GDP_{ii}] + z_{b,3}[r_d^b(pv_i^b + (1-p)v_{ii}^b)] \\ &\quad + z_{b,4}\sum_{\acute{b}\neq b\in B}[r_d^{\acute{b}}(pv_i^{\acute{b}} + (1-p)v_{ii}^{\acute{b}})]\end{aligned} \qquad (4.12)$$

where,

$d_b^{\phi} \equiv$ amount of money that agent ϕ chooses to deposit with bank $b \in B$.

4.2.3.3 Households' Loan Repayment Rates

We assume that each household's repayment rate on his loan obligation to his nature-selected bank in state $s \in S$ is a positive function of the corresponding GDP level as well as the *aggregate* credit supply in the economy. The latter variable captures the effect of 'credit crunch' in the economy whereby a fall in the overall credit supply in the economy aggravates the default probability of every household.[10] Specifically, the functional form of the repayment rate of household h^b, $\forall h^b \in H^b$, to his nature-selected bank $b \in B$, in state $s \in S$ is as follows:

$$\ln(v_{sb}^{h^b}) = g_{h^b,s,1} + g_{h^b,s,2}\ln(GDP_s) + g_{h^b,s,3}[\ln(\bar{m}^{\gamma}) + \ln(\bar{m}^{\delta}) + \ln(\bar{m}^{\tau})] \tag{4.13}$$

4.2.4 GDP

As can be seen from equations (4.11) to (4.13), we have assumed that households' actions depend on their expected GDP in the second period. So, in this section we endogenise GDP in both states of the second period. We assume that GDP in each state is a positive function of the aggregate credit supply available in the *previous* period. Since the Modigliani-Miller proposition does not hold in our model[11], higher credit extension as a result of loosening monetary policy, or any other shocks, increases private agents' liquidity which in turn raises consumption demand and ultimately GDP. In particular, the following functional form for GDP in state $s \in S$ of the second period (GDP_s) holds:

$$\ln(GDP_s) = u_{s,1} + u_{s,2}[\ln(\bar{m}^{\gamma}) + \ln(\bar{m}^{\delta}) + \ln(\bar{m}^{\tau})] \tag{4.14}$$

4.2.5 Market Clearing Conditions

There are seven active markets in the model (three consumer loan, three deposit and one interbank markets). Each of these markets determines an interest rate that equilibrates demand and supply

[10] Higher interest rates, given that households are liquidity constrained, ultimately increase their debt obligations in the future. Hence, defaults rise.
[11] See Goodhart et al. (2003) for an extensive discussion.

in equilibrium.[12]

$$1+r^b = \frac{\mu^{h^b}}{\overline{m}^b},\ h^b \in H^b, \forall b \in B \qquad \text{(i.e. bank } b\text{'s loan market clears)} \quad (4.15)$$

$$1+r_d^b = \frac{\mu_d^b}{d_b^\phi}, \forall b \in B \qquad \text{(i.e. bank } b\text{'s deposit market clears)} \quad (4.16)$$

$$1+\rho = \frac{\mu^\gamma + \mu^\tau}{M + d^\delta} \qquad \text{(i.e. interbank market clears)} \quad (4.17)$$

We note that these interest rates, i.e. r^b, r_d^b, and $\rho, b \in B$, are the *ex ante* nominal interest rates that incorporate default premium since default is permitted in equilibrium. Their effective (*ex post*) interest rates have to be suitably adjusted to account for default in their corresponding markets.[13]

4.2.6 Equilibrium

Let $\sigma^b = \{\overline{m}^b, \mu^b, \mu_d^b, v_s^b, \pi_s^b, e_s^b, k_s^b\} \in R_+ \times R \times R_+ \times R_+^2 \times R^2 \times R^2 \times R^2$ for $b \in \{\gamma, \tau\}$; $\sigma^\delta = \{\overline{m}^\delta, d^\delta, \mu_d^\delta, v_s^\delta, \pi_s^\delta, e_s^\delta, k_s^\delta\} \in R_+ \times R \times R_+ \times R_+^2 \times R^2 \times R^2 \times R^2$; $\sigma^{h^b} \equiv (\mu^{h^b}, v_{sb}^{h^b}) \in R_+ \times R_+^2$ for $h^b \in H^b$ and ; $\sigma^\phi \equiv (d_b^\phi) \in R_+$ for $b \in B$; and $GDP_s \in R^2$. Also, let $\eta \in \{r^\gamma, r^\tau, r^\delta, r_d^\gamma, r_d^\tau, r_d^\delta, \rho\}$, $B^b(\eta) = \{\ \sigma^b : (4.1) - (4.2) \text{ hold }\}$, and $B^\delta(\eta) = \{\ \sigma^\delta : (4.6) - (4.7) \text{ hold }\}$. We say that $((\sigma^b)_{b\in\{\gamma,\tau\}}, \sigma^\delta, \eta, (\sigma^{h^b})_{h^b \in H^b}, \sigma^\phi, (GDP_s)_{s\in S})$ is a monetary equilibrium with commercial banks and default for the economy

$$E\{(e_0^b, Others^b, A^b)_{b\in B}; M; (\overline{k}^b, \lambda_s^b, \lambda_{ks}^b, \overline{\omega}, \omega, \widetilde{\omega})_{b\in B, s\in S}; r^A; p\}$$

iff:

(i) (a) $\sigma^b \in \underset{\sigma^b \in B^b(\eta)}{Argmax}\ \Pi^b(\pi^b)$, $b \in \{\gamma, \tau\}$
(b) $\sigma^\delta \in \underset{\sigma^\delta \in B^\delta(\eta)}{Argmax} \Pi^\delta(\pi^\delta)$
(i.e. all banks optimise.)

(ii) All markets (4.15)-(4.17) clear.

(iii) $\widetilde{R}_s = \dfrac{\sum_{b\in\{\gamma,\tau\}} v_s^b \mu^b}{\sum_{b\in\{\gamma,\tau\}} \mu^b}$, $s \in S$

[12] The interest rate formation mechanism is identical to the offer-for-sale mechanism in Dubey and Shubik (1978). The denominator of each of the expressions (4.15-4.17) represents the supply side whereas the numerator divided by $(1+r)$, $r \in \{r^b, r_d^b, \rho\}$, $b \in B$ corresponds to the demand. Note that this interest rate formation mechanism is well-defined both in, and out of, equilibrium.

[13] For more on the method of calculating the *ex post* interest rates, see Shubik and Tsomocos (1992).

(i.e. bank δ is correct in its expectation about the repayment rates that it gets from its interbank investment.)

We emphasise here that the equilibrium conditions $(i)-(iii)$ are consistent with the defining properties of a competitive equilibrium with rational expectations.

(iv) σ^{h^b}, σ^{ϕ} and GDP_s, for $h \in H$ and $s \in S$ satisfy the reduced-form equations (4.11)-(4.14).

(i.e. loan demand, deposit supply, repayment rates, and GDP in both states satisfy the reduced-form equations (4.11)-(4.14).)

4.3 Calibration

Excluding the Lagrange multipliers, conditions $(i)-(iv)$ in the previous section imply that we have a system of 56 equations in 135 unknown variables, 79 of which are exogenous variables/parameters in the model. This implies that there are 79 variables whose values have to be chosen in order to obtain a *numerical* solution to the model. Thus, they represent the degrees of freedom in the system and can either be set appropriately or calibrated against the real data. It is important to note that these variables, which are exogenous when solving the *system* of simultaneous equations, do *not* necessarily have to be those which are exogenous in the *model*.[14] We report the values of exogenous parameters/variables in the model and the resulting initial equilibrium in Table 4.1. The table also summarises whether the value of each variable reported is (1) calibrated against real data, (2) arbitrarily selected, or (3) endogenously solved. We note, however, that, owing to the data confidentiality reason, we suppress those numbers which are based on the calibrated balance sheet data of UK banks and replace them by '*xxx*' in Table 4.1. Unless stated otherwise, the values of all the nominal variables reported therein, e.g. all bank balance sheet items, are normalised by 10^{10}.

The values of all banks' balance sheet items in the initial period, i.e. $\{\overline{m}^b, \mu_d^b, Others^b, A^b\}_{b\in B}$, $\{\mu^b\}_{b\in\{\gamma,\tau\}}$, and $\{d^\delta\}$, are calibrated using the 2002 annual account data for seven largest UK banks. Based on this source of data, we also calibrate the values of private agents' loan repayment rates to their nature-selected banks in the good/normal state, i.e. $(v_{ib}^{h^b})_{\{h^b\in H^b\}}$, using the data of each bank b's ratio of provision at the end of the year to total customer loans. However, since there are no data available for crisis/extreme events, the default rates of all private agents in the bad state (state ii) are arbitrarily set to 0.1.

[14] For example, the Central Bank in our model fixes its base money and lets the interbank interest rate adjust endogenously, i.e. base money is exogenous in the model. However, in solving for a numerical solution, we can first choose the value of the interbank rate and let the system of simultaneous equations determine endogenously the value of base money that supports the preset value of the interbank rate.

	Initial Equilibrium			Exogenous variables in the model	
Endogenously solved	r^{γ}=0.0601 r^{δ}=0.069 r^{τ}=0.0615 r_d^{γ}=0.04 r_d^{δ}=0.0357 r_d^{τ}=0.04 $\mu_d^{\gamma}=xxx$ $\mu_d^{\delta}=xxx$ $\mu_d^{\tau}=xxx$ k_i^{γ}=0.098 k_{ii}^{γ}=0.061	k_i^{δ}=0.155 k_{ii}^{δ}=0.147 k_i^{τ}=0.119 k_{ii}^{τ}=0.089 π_i^{γ}=0.25 π_{ii}^{γ}=-0.357 π_i^{δ}=0.327 π_{ii}^{δ}=-0.136 π_i^{τ}=2.1 π_{ii}^{τ}=-1.173 e_i^{γ}=1.425	e_{ii}^{γ}=0.818 e_i^{δ}=3.894 e_{ii}^{δ}=3.431 e_i^{τ}=10.58 e_{ii}^{τ}=7.307 $\widetilde{R}_i$=0.999 $\widetilde{R}_{ii}$=0.95 $\mu^{\alpha^{\gamma}}=xxx$ $\mu^{\beta^{\delta}}=xxx$ $\mu^{\theta^{\tau}}=xxx$	$Other^{\gamma}$=2.31 $Other^{\delta}$=-1.142 $Other^{\tau}$=27.064 $g_{\alpha^{\gamma},i,1}$=-0.75 $g_{\alpha^{\gamma},ii,1}$=-1.04 $g_{\beta^{\delta},i,1}$=-0.76 $g_{\beta^{\delta},ii,1}$=-1.04 $g_{\theta^{\tau},i,1}$=-0.75 $g_{\theta^{\tau},ii,1}$=-1.04 $u_{i,1}$=3.64 $u_{ii,1}$=3.6 M=6.928	$a_{\alpha^{\gamma},1}$=-3.85 $a_{\beta^{\delta},1}$=-3.35 $a_{\theta^{\tau},1}$=-2.08 $z_{\gamma,1}$=1.56 $z_{\delta,1}$=2.53 $z_{\tau,1}$=3.3 c_i^{γ}=0.214 c_{ii}^{γ}=0.129 c_i^{δ}=0.159 c_{ii}^{δ}=0.351 c_i^{τ}=0.024 c_{ii}^{τ}=0.042
Calibrated	ρ=0.04 $\overline{m}^{\gamma}=xxx$ $\overline{m}^{\delta}=xxx$ $\overline{m}^{\tau}=xxx$ $d_{\gamma}^{\phi}=xxx$	$d_{\delta}^{\phi}=xxx$ $d_{\tau}^{\phi}=xxx$ $\mu^{\gamma}=xxx$ $d^{\delta}=xxx$ $\mu^{\tau}=xxx$	$v_{i\gamma}^{\alpha^{\gamma}}=xxx$ $v_{i\delta}^{\beta^{\delta}}=xxx$ $v_{i\tau}^{\theta^{\tau}}=xxx$ GDP$_i$=104.3	$e_0^{\gamma}=xxx$ $e_0^{\delta}=xxx$ $e_0^{\tau}=xxx$ $a_{h^b,2}=1.41, \forall h \in H^b$ $a_{h^b,3}=-0.68, \forall h \in H^b$	$\overline{\omega}$=1 $\omega(\widetilde{\omega})$=0.2 $A^{\gamma}=xxx$ $A^{\delta}=xxx$ $A^{\tau}=xxx$
Arbitrarily selected	v_i^{γ}=0.999 v_{ii}^{γ}=0.95 v_i^{δ}=0.999	v_{ii}^{δ}=0.955 v_i^{τ}=0.999 v_{ii}^{τ}=0.95	$v_{ii\gamma}^{\alpha^{\gamma}}$=0.9 $v_{ii\delta}^{\beta^{\delta}}$=0.9 $v_{ii\tau}^{\theta^{\tau}}$=0.9 GDP$_{ii}$=100.2	$\lambda_{ks}^{b}=0.1, \forall b \in B, s \in S$ $\lambda_i^{b}=0.9, \forall b \in B$ $\lambda_{ii}^{b}=1.1, \forall b \in B$ $u_{s,2}=0.1, \forall s \in S$ $z_{b,2}=0.19, \forall b \in B$ $z_{b,3}=0.5, \forall b \in B$ $z_{b,4}=-0.1, \forall b \in B$ $g_{h,i,2}=0.05, \forall h \in H^b$ $g_{h,ii,2}=0.05, \forall h \in H^b$ $g_{h,i,3}=0.05, \forall h \in H^b$ $g_{h,ii,3}=0.07, \forall h \in H^b$	$\overline{k}^{\gamma}$=0.11 $\overline{k}^{\delta}$=0.16 $\overline{k}^{\tau}$=0.13 r^A=0.045 p=0.95

Table 4.1: Exogenous variables and the initial equilibrium

The probability that state *ii* will occur, $1-p$, is chosen to be 0.05, given that it reflects an extreme event. Since banks rarely default on their debt obligations in the good state, the corresponding repayment rates in the deposit and interbank markets for all banks, i.e. $v_i^b, b \in B$, are set to 0.999. In state *ii*, the bad state, we arbitrarily set the analogous repayment rates to 0.95 for banks γ and τ and 0.955 for bank δ. These values are selected to be relatively higher compared with households' repayment rates in state *ii* (0.9) since in reality the probability that banks would default on their financial obligations is smaller than that of households. Note also that the chosen value for bank δ's repayment rate is slightly greater than those of the other two banks because its deposit rate, whose values are determined endogenously, is slightly smaller in equilibrium. This may suggest at first glance that we are assuming what we need to estimate, i.e. a bank's willingness to run a risky position, which could lead to enforced shut down. Not quite so, since each chosen value for a bank's chosen default rate relates to an equivalent subjective default penalty. If you give us, the model builders, some guidance on banks' aversion to default penalties, i.e. the size of the $\lambda_s^b, b \in B, s \in S$, we can adjust the default probabilities accordingly.

We choose the value of the interbank interest rate, ρ, to be 4 percent to match with the actual value of UK RP rate in December 2002. The value of risk weight for loans is set to 1 whereas the corresponding values for market book and interbank lending are 0.2. The value of capital to asset requirement set by the regulator for each bank ($\overline{k}^b, b \in B$) is chosen to be slightly higher, but almost equal to, its corresponding value in state i so that all banks always violate their capital requirement.[15]

The values of default and capital violation penalties (λ_s^b and $\lambda_{ks}^b, b \in B, s \in S$) reflect both the tightness of the regulator's regulatory policy and the (subjective) aversion of banks' managements to putting themselves at risk of default and/or regulatory violations, and can, in principle, be treated as inputs given by the practitioner users of this model. Their values are, however, unobservable and therefore have to be chosen somehow. We have chosen them in this example to be consistent with the following outcomes. First, the resulting endogenously-solved banks' lending rates are such that all banks earn positive profit in state i, whereas they suffer a loss in state *ii*. This in turn implies that banks' capital at $t = 2$ deteriorates if the bad state (*ii*) occurs. Second, all banks' coefficients of risk aversion ($c_s^b, b \in B, s \in S$) are positive, implying that banks' utility functions are well-behaved, i.e. concave. Lastly, the rate of return on market book is arbitrarily chosen to be 4.5 percent.

[15] As mentioned in section 4.2.1, this is a simplifying assumption. Recall that capital requirements' violation penalty enters banks' objective functions as '$\max[0, \overline{k}^b - k_s^b]$'. However, given our assumption that banks always violate their capital requirement, we can restrict the optimisation problem to $\overline{k}^b - k_s^b > 0$, thus avoiding 'corner' equilibria.

We calibrate the value of GDP in the good state (GDP_i) to match with the actual UK (annual) GDP in 2002. We set the value of GDP in the bad state (GDP_{ii}) to represent a 4% fall from its corresponding value in the good state. The values of coefficients $a_{h^b,2}$ and $a_{h^b,3}$, $\forall h^b \in H^b$, in the reduced-form equation (4.11) are calibrated, respectively, using the values of the long-run income and interest rate elasticities of UK household sector estimated by Chrystal and Mizen (2001).

To our knowledge, we do not know any empirical study which estimates deposit supply and default probability functions for UK household/private sectors. Although this can, in principle, be done, such an exercise is beyond the scope of this chapter. So, we arbitrarily choose the appropriate values of $z_{b,2}, z_{b,3}, z_{b,4}$, $\forall b \in B$, in equation (4.12), and the values of $g_{h^b,s,2}$ and $g_{h^b,s,3}$, $\forall h^b \in H^b$, $s \in S$, in equation (4.13). As can be seen from Table 4.1, the values of $g_{h^b,i,3}$ is chosen to be greater than the corresponding values of $g_{h^b,ii,3}$, $\forall h^b \in H^b$, implying that the effect of a 'credit crunch' is assumed to be stronger in the bad state.

The remaining parameters for which their values have to be chosen are the coefficients $u_{s,2}$, $\forall s \in S$, in the reduced-form equation (4.14). We set them to be equal to 0.1. Because the value of these coefficients capture the inter-relationship between real and nominal variables in the economy, they are therefore important in determining the strength of the 'amplification' effect when a shock hits the economic system. For this reason, as will be seen in section 4.5, we conduct a robustness check by considering alternative initial values of these two parameters in our simulation exercise. At this point we note that the above specifications rely on the monetary and regulatory non-neutrality properties of our model. We formally prove these propositions in Goodhart et al. (2003).

Given the chosen values of the variables mentioned above, we are left with the system of 56 simultaneous equations in 56 unknown variables. By solving such system, the values of all the remaining variables are specified and a numerical solution to the model is obtained.

4.4 Comparative Static Analysis: Stress Testing UK Banks

In this section we show how the model can be used as a risk assessment tool for UK banks. Given that the initial equilibrium has been found, we conduct a series of comparative statics by perturbing each of the variables which are exogenous in the model and studying how the initial equilibrium changes.[16] Table 4.A.1 in the Appendix reports the *directional* responses of all endogenous variables in the model given that we *increase* the values of the variables listed in the first column one at a time. Recall that these comparative statics are based on the base-line specification, which assumes that

[16]The calculation is carried out using a version of Newton's method in *Mathematica*.

	Interest rates			π_i^b	π_{ii}^b	e_i^b	e_{ii}^b	k_i^b	k_{ii}^b	v_i^b	v_{ii}^b	GDP$_i$	GDP$_{ii}$
	r_d^b	r^b	ρ										
δ	-0.9	-0.9	-0.8	$\frac{-4}{10^3}$	$\frac{-5}{10^3}$	$\frac{-3}{10^4}$	$\frac{-2}{10^4}$	-0.1	-0.1	$\frac{6}{10^3}$	0.01	0.05	0.05
γ	-0.8	-1		$\frac{-9}{10^3}$	-0.01	$\frac{-2}{10^3}$	$\frac{-5}{10^3}$	-0.1	-0.2	$\frac{-2}{10^3}$	$\frac{3}{10^3}$		
τ	-0.8	-1		$\frac{-2}{10^3}$	$\frac{-2}{10^3}$	$\frac{-3}{10^4}$	$\frac{-3}{10^4}$	-0.1	-0.1	$\frac{-1}{10^3}$	$\frac{4}{10^3}$		

Table 4.2: Responses of key variables (percentage changes from the initial equilibrium) to a positive shock in base money

the Central Bank's monetary policy instrument is its base money. In what follows, we also conduct an alternative set of comparative statics whereby the Central Bank is assumed to set the interbank rate as its instrument and let its base money adjust endogenously.[17] Table 4.A.2 in the Appendix reports the results of this alternative set of comparative statics in an analogous format to Table 4.A.1.

4.4.1 An expansionary monetary policy

We first analyse the case when the Bank of England engages in an expansionary monetary policy. As expected, we found that the result is exactly the same regardless of whether the Bank uses its base money or the interbank rate as its monetary policy instrument (i.e. the Bank increases its base money and allows the interbank rate to be determined endogenously, produces the same result as the case when the Bank decreases the interbank rate and lets its base money adjust endogenously). We summarise the percentage changes in the values of certain key variables in response to an increase in the Bank's base money from 69.28 to 71.28 trillion pounds (approximately 2.9%) in Table 4.2. As can be seen from the table, the interbank rate decreases by 0.8%. Given a lower rate of return on interbank market investment, other things constant, bank δ invests less in this market (recall that bank δ is the net lender in this market). It demands less funds from the deposit market and it increases its loan supply to its nature-selected customer, Mr. β. This portfolio adjustment of bank δ produces a negative pressure on both its deposit and lending rates. Unlike bank δ, banks γ and τ are the net borrowers in the interbank market. Thus they respond to a lower cost of interbank borrowing by reducing their demand for deposits, borrowing more from the interbank market, and lending more to their nature-selected customers, Mr. α and θ, respectively. This, in turn, causes a negative pressure on these two banks' deposit and lending rates.

All banks rationally anticipate that their greater credit extension would increase the overall supply of credit in the economy, thus causing the probability of household default to decline. This

[17]More specifically, the interbank rate becomes an exogenous parameter in the model under this alternative specification.

is because greater aggregate credit supply not only directly increases households' liquidity but also increases their income in both states of the subsequent period. As can be seen from Table 4.2, GDP increases by 0.05% in both states. Thus, the expected rate of return from extending loans increases for all banks, implying that their willingness to supply more credit rises even further.

Given higher expected GDP in both states, every household borrower (i.e. Mr. α, β, and θ) demands more loans, imposing a positive pressure on the lending rates offered by their respective nature-selected banks. However, this 'crowding-out' effect is dominated by the corresponding negative pressure from greater credit supply by all banks. Thus, we observe that their lending rates decline (0.9% for bank δ, and 1% for banks γ and τ). We also find that the deposit rates offered by all banks decrease (i.e. 0.9% for bank δ and 0.8% for banks γ and τ). This is not only because all banks demand less funds from the deposit markets but also because Mr. ϕ responds to higher expected GDP by supplying more deposits to every bank.

Banks in our model choose their optimal expected level of profitability by equating the derived marginal benefit with the corresponding marginal cost. On the one hand, higher profitability not only directly increases their utility but also raises their capital to asset ratios, allowing them to suffer less capital violation penalties. This latter source of marginal benefit is lower the higher the value of banks' risk-weighted assets.[18] On the other hand, in order to achieve higher profitability, other things constant, they take more risk and therefore suffer higher cost in the form of higher expected default penalties.[19] In this comparative static exercise, since the default probability of all household borrowers decreases in both states, the corresponding values of every bank's risk-weighted assets increase. This leads all banks to revise the trade off between the relative marginal benefit and cost in such a way that they are willing to achieve a marginally lower level of profitability in both states, compared with the corresponding initial equilibrium value, in pursuit of suffering less default penalties.[20] Consequently, their capital declines slightly in both states compared with the initial equilibrium. This, together with the fact that the values of their risk-weighted assets increase, causes all banks to suffer greater capital violation penalties.

[18]For example, if the profit of a bank increases by 1 dollar, given that its risk-weighted asset is 100 dollar, its capital to asset ratio would increase by $\frac{1}{100}$. However, if the value of the risk-weighted assets is 1000 dollar, the bank's capital to asset ratio would increase by much less, i.e. by $\frac{1}{1000}$.

[19]Recall that we interpret a bank's continuous default rate as isomorphic to the probability that the bank will shut down, and therefore, at least in the very short run, completely default on its obligations. Thus, when banks choose higher default rates, this implies that it adopts a riskier position.

[20]More precisely, higher risk-weighted assets in both states for all banks cause their marginal benefit of achieving higher profitability in terms of suffering less capital violation penalties to decrease. Given that the marginal benefit of achieving higher profitability is now lower than the corresponding marginal cost, they reduce their optimal desired level of profits.

	Interest rates			π_i^b	π_{ii}^b	e_i^b	e_{ii}^b	k_i^b	k_{ii}^b	v_i^b	v_{ii}^b	GDP$_i$	GDP$_{ii}$
	r_d^b	r^b	ρ										
δ	-2.7	-2.9	-2.3	-0.03	-0.04	$\frac{-3}{10^3}$	$\frac{-2}{10^3}$	-0.8	-0.9	0.03	0.03	0.16	0.16
γ	-2.3	-3.2		-0.03	-0.04	$\frac{-5}{10^3}$	-0.02	-0.4	-0.5	$\frac{-6}{10^3}$	$\frac{9}{10^3}$		
τ	-2.3	-3.1		$\frac{-5}{10^3}$	$\frac{-6}{10^3}$	$\frac{-9}{10^4}$	$\frac{-9}{10^4}$	-0.4	-0.4	$\frac{-3}{10^3}$	0.01		

Table 4.3: Responses of key variables (percentage changes from the initial equilibrium) to a positive deposit supply shock on bank δ; Central Bank sets base money

Given that banks operate under a perfectly competitive environment, they treat all interest rates and the households' default probability as given when making their optimal decisions. However, in response to shocks, these variables have to adjust endogenously to satisfy market clearing conditions, where the direction and extent of the adjustment depend on how banks and households adjust their portfolios. This '*portfolio reallocation*' effect produces pressures on banks' profitability and their willingness to take risk.[21] As for bank δ in both states and banks γ and τ in the bad state, this portfolio reallocation effect is relatively weak, causing them to simply adopt a more conservative position in response to their lower targeted level of profitability. However, for banks γ and τ in the good state, the portfolio reallocation effect produces a relatively strong negative pressure on their profitability so that they end up adopting a slightly riskier position in order to achieve their targeted level of profits.

4.4.2 A positive deposit supply shock to bank δ in the initial period

We next turn to the scenario where there is a positive deposit supply shock to bank δ in the initial period. In particular, we increase the (log) autonomous deposit supply of Mr. ϕ with bank δ ($z_{\delta,1}$) by approximately 0.8%. We first assume that the Bank fixes base money as its monetary policy instrument, allowing the interbank rate to be determined endogenously. We report the percentage changes of certain key variables in response to the shock in Table 4.3. Not surprisingly, the results are almost qualitatively identical to that of a change in the Bank's injection of funds, since both represent a change in the overall broad money supply. The difference is that here we assume that the shock is concentrated in bank δ.

Major differences arise, however, when we analyse the effects of the shock in the context whereby the Bank fixes the interbank rate as its monetary policy instrument. Key results are summarised in Table 4.4. In the previous case, even though the effect of the shock is initially concentrated in

[21] For example, a bank may respond to a positive shock by supplying more loans, thereby imposing a negative pressure on the lending rate. Similarly, the shock may cause household borrowers to demand more loans from such a bank, causing a positive crowding-out pressure on the lending rate. The relative strength of the pressures caused by the bank's and the borrower's portfolio adjustment depends in general on the relative elasticities of demand and supply in such a market.

	Interest rates			π_i^b	π_{ii}^b	e_i^b	e_{ii}^b	k_i^b	k_{ii}^b	v_i^b	v_{ii}^b	GDP$_i$	GDP$_{ii}$
	r_d^b	r^b	ρ										
δ	$\frac{4}{10^3}$	-0.01		-0.02	-0.02	$\frac{-2}{10^3}$	$\frac{-1}{10^3}$	-0.5	-0.5	$\frac{8}{10^3}$	$\frac{-3}{10^3}$		
γ	0	$\frac{-3}{10^3}$	0	0	0	0	0	$\frac{-8}{10^4}$	$\frac{-9}{10^4}$	0	0	$\frac{3}{10^4}$	$\frac{3}{10^4}$
τ	0	$\frac{-3}{10^3}$		0	0	0	0	$\frac{-7}{10^4}$	$\frac{-8}{10^4}$	0	0		

Table 4.4: Responses of key variables (percentage changes from the initial equilibrium) to a positive deposit supply shock on bank δ; Central Bank sets the interbank rate

bank δ, we observe that the other two banks also benefit from increased overall liquidity through their interactions in the interbank market. In particular, a positive deposit supply shock in bank δ causes the bank to supply more liquidity in the interbank market. Such higher liquidity is then passed on to banks γ and τ in the form of a lower cost of interbank borrowing. However, in this case, the Bank's intervention to maintain the interbank rate at its original level precisely shuts down this interbank rate channel, causing the dominant channel of contagion to become the one which operates via changes in the household sector's default probability in the loan market instead. The chain of contagion of this 'consumer loan default' channel begins with bank δ's decision to supply more credit. This in turn causes the *overall* credit supply in the economy to increase. This implies that every household benefits directly from greater liquidity as well as from higher income (GDP) in both states i and ii of the subsequent period. Thus, the default probability of *every* household in the consumer loan market decreases, causing the expected rate of return from extending more loans to increase not only for bank δ but also for banks γ and τ. Consequently, their respective lending rates fall. Since the cost of interbank borrowing is fixed regardless of the amount demanded, the two banks finance their greater credit extension by borrowing more from the interbank market.

As can be seen clearly by comparing the results presented in Tables 4.3 and 4.4, the contagion effects when the Bank's monetary instrument is the interbank rate are much weaker than those observed when the Bank fixes its base money. This is because fixing the interbank rate not only directly shuts down the interbank rate contagion channel *but also* weakens the extent of contagion effects which operate through the consumer loan default channel. As mentioned, the contagion effects which operate via the latter channel arise from bank δ's decision to extend more credit. However, the Bank's intervention to fix the interbank rate implicitly increases the attractiveness of the interbank investment for bank δ since its rate of return does not diminish as it invests more. Thus, even though we observe that bank δ increases its credit extension when the Bank fixes the interbank rate, the extent of such increase is not as strong compared to the case when the rate is allowed to adjust endogenously. Put differently, the money supply multiplier is larger when the Bank does not target interest rates since besides their direct effect due to increased deposits, a second-order effect from

	Interest rates			π_i^b	π_i^b	e_i^b	e_{ii}^b	k_i^b	k_{ii}^b	v_i^b	v_{ii}^b	GDP$_i$	GDP$_{ii}$
	r_d^b	r^b	ρ										
δ	-0.9	-0.9	-0.8	-0.01	-0.01	5.1	5.8	4.9	5.5	0.03	$\frac{5}{10^3}$	0.05	0.05
γ	-0.8	-1		$\frac{-9}{10^3}$	-0.01	$\frac{-2}{10^3}$	$\frac{-5}{10^3}$	-0.1	-0.2	$\frac{-2}{10^3}$	$\frac{3}{10^3}$		
τ	-0.8	-1		$\frac{-2}{10^3}$	$\frac{-2}{10^3}$	$\frac{-3}{10^4}$	$\frac{-3}{10^4}$	-0.1	-0.1	$\frac{-1}{10^3}$	$\frac{4}{10^3}$		

Table 4.5: Responses of key variables (percentage changes from the initial equilibrium) to a positive shock on bank δ's capital at $t = 1$; Central Bank sets base money

allowing the interbank rate to change enhances credit supply of the entire banking sector.

4.4.3 A positive bank capital shock to bank δ in the initial period

In this comparative static exercise, we increase the capital endowment of bank δ in the initial period by approximately 5.6%. As before, we first consider the case when the Bank sets its base money as its instrument. The percentage changes in the values of certain key variables in response to such shock is summarised in Table 4.5. We observe that the directional responses of most of the variables are the same as the case when we assume that there is a positive deposit supply shock to bank δ. This is because these two shocks both result in more available funds for bank δ to invest. However, the exception is the response of bank δ's capital, and capital to asset ratios, in the second period. Unlike the case of the positive deposit supply shock, here we observe that bank δ's capital position and its capital to adequacy ratios improve in both states of the world. The reason for this lies in the fact that a positive endowment shock imposes a direct positive effect on the capital of bank δ in the second period, thus reducing its capital requirements' violation penalty.

We now turn to analyse the effects of the same shock but this time under the assumption that the Bank fixes the interbank rate. Key results are reported in Table 4.6. As in the case when there is a positive deposit supply shock to bank δ in state ii, we found in this case that the main contagion effects operate via the consumer loan default channel. However, the major difference is that here the effects of liquidity injection in bank δ produces *negative* contagion effects onto the rest of the banks in the banking sector. Recall that the Bank's sterilisation policy in the interbank market increases the relative attractiveness of interbank investment, implying that bank δ responds to higher capital by increasing its investment in the interbank market. Unlike the results found in section 4.4.2, however, the extent of such increase is so large that bank δ has to switch part of its investment away from the loan market. This in turn puts a *downward* pressure on the overall credit supply in the economy, aggravating the probability of default in the consumer loan markets. This negative contagion effect depresses the other two banks' expected return on their credit extension. Thus,

	Interest rates			π^b_i	π^b_{ii}	e^b_i	e^b_{ii}	k^b_i	k^b_{ii}	v^b_i	v^b_{ii}	GDP_i	GDP_{ii}
	r^b_d	r^b	ρ										
δ	-0.02	0.05	0	$\frac{-6}{10^3}$	$\frac{-7}{10^3}$	5.1	5.8	5	5.7	0.03	$\frac{-5}{10^3}$	$\frac{-2}{10^3}$	$\frac{-2}{10^3}$
γ	0	0.02		0	0	0	0	$\frac{4}{10^3}$	$\frac{4}{10^3}$	0	0		
τ	0	0.02		0	0	0	0	$\frac{3}{10^3}$	$\frac{4}{10^3}$	0	0		

Table 4.6: Responses of key variables (percentage changes from the initial equilibrium) to a positive shock on bank δ's capital at $t = 1$; Central Bank sets the interbank rate

they extend less credit, causing the aggregate output to fall in both states. This in turn worsens the severity of credit crunch in the economic system even further. Moreover, unlike before, we observe that banks γ and τ now violate 'less' capital adequacy requirements and therefore suffer less capital violation penalties. This is because a higher default probability of every household causes the values of banks γ's and τ's risk-weighted assets to decrease. In sum, we see that emergency liquidity assistance in the form of bank capital injection, when implemented under interest rate targeting regime, may engender adverse effects to financial stability. Banks may radically restructure their portfolios in a way that depresses credit extension in the loan markets.

One of the key implications from our results presented thus far is that, as far as financial stability is concerned, there is *no* clear-cut answer as to whether a Central Bank should set the interbank rate or base money as its monetary policy instrument. Although all the contagion effects which operate through the interbank rate channel are completely sterilised away when the Central Bank's instrument is the interbank rate, such a sterilisation policy may mitigate the extent of contagion effects which operate via *other* channels of contagion. As the results in this section show, the direction of contagion effects can even be reversed, thereby producing negative rather than positive contagion effects to the rest of the economic system. Disentangling the issue is beyond the scope of this chapter. Since our main objective here is to study financial contagion and banks' inter-linkages, we have chosen to focus our analysis in the remainder of this chapter on our base-line specification (i.e. the Bank's monetary policy instrument is base money). This allows all contagion channels to operate actively in equilibrium.

4.4.4 An increase in default penalties imposed on all banks in the bad state

We now turn to analyse the case where the regulator engages in a restrictive regulatory policy by increasing the default penalties imposed on all banks in the bad state from 1.1 to 1.12 (approximately 1.8%). This implies that adopting a riskier position in the bad state is more costly for all banks since by doing so they have to suffer greater default penalties in this particular state. Thus, as

	Interest rates			π_i^b	π_{ii}^b	e_i^b	e_{ii}^b	k_i^b	k_{ii}^b	v_i^b	v_{ii}^b	GDP_i	GDP_{ii}
	r_d^b	r^b	ρ										
δ	0.3	0.3	-0.4	$\frac{3}{10^4}$	-20.9	$\frac{3}{10^5}$	-0.8	$\frac{9}{10^3}$	-0.8	$\frac{3}{10^3}$	0.1	$\frac{-3}{10^3}$	$\frac{-3}{10^3}$
γ	-0.4	-0.08		$\frac{1}{10^4}$	-21.6	$\frac{2}{10^5}$	-9.4	$\frac{2}{10^3}$	-9.4	$\frac{8}{10^3}$	0.6		
τ	-0.4	-0.08		$\frac{2}{10^5}$	-20.2	$\frac{3}{10^6}$	-3.3	$\frac{2}{10^3}$	-3.3	$\frac{9}{10^3}$	0.3		

Table 4.7: Responses of key variables (percentage changes from the initial equilibrium) to a rise in default penalty for all bank in state ii; Central Bank sets base money

reported in Table 4.7, all banks optimally choose a significantly lower desired level of profitability in the bad state, allowing them to adopt a much more conservative position and therefore to mitigate the extent of default penalties that they have to face in this particular state. Moreover, bank δ increases its investment in the interbank market which is relatively 'safer' in the bad state.[22] In doing so, it borrows more from the deposit market, and invests less in the loan market. This portfolio adjustment of bank δ imposes a negative pressure on the interbank rate and produces a positive pressure on its lending and deposit rates. In response, Mr. ϕ supplies more deposits with bank δ and less with the other two banks.

Unlike bank δ, banks γ's and τ's initial positions in the interbank market are net borrowers. Thus, even though interbank investment is now relatively safer, switching its 'net' position in such market would result in a relatively more severe negative portfolio reallocation effect on these banks' overall payoff. Anticipating this, they 'gamble to resurrect' by extending more loans, expecting that such an action would increase their profitability in the good state, which is not subject to higher default penalties, and therefore to reduce the extent of the decline in their overall payoff which is due mainly to the significant fall in their profitability in the bad state. This causes their lending rates to decrease both by 0.08%. Given a lower interbank rate, they also adjust their portfolios by switching away from the deposit market and borrowing more from the interbank market, causing their deposit rates to decrease by 0.4%.

We observe that the extent of decrease in bank δ's credit extension is larger than the extent of increase in banks γ's and τ's credit supply combined, causing the overall supply of credit in the economy to fall. This directly decreases households' liquidity, which in turn imposes a downward pressure on the probability that households will repay their loan obligations in full. This pressure is further exacerbated since lower aggregate credit supply depresses GDP in both states of the second period (i.e. by 0.003% in both states), causing the corresponding income of every household to fall.

As mentioned, because of the direct first-order effect of the initial shock (i.e. higher default penalty in state ii), all banks are willing to obtain lower profitability in the bad state, compared

[22] More specifically, the probability with which banks (γ, τ) choose to default completely on their interbank obligations is less than the corresponding probability of Mr. β on his loans.

	Interest rates			π_i^b	π_{ii}^b	e_i^b	e_{ii}^b	k_i^b	k_{ii}^b	v_i^b	v_{ii}^b	GDP_i	GDP_{ii}
	r_d^b	r^b	ρ										
δ	-0.02	$\frac{6}{10^3}$	$\frac{-6}{10^3}$	$\frac{1}{10^5}$	0.9	0	0.04	$\frac{2}{10^4}$	0.04	$\frac{6}{10^4}$	$\frac{-4}{10^3}$	$\frac{-8}{10^4}$	$\frac{-8}{10^4}$
γ	$\frac{-6}{10^3}$	$\frac{5}{10^3}$		$\frac{2}{10^5}$	1.6	0	0.7	$\frac{4}{10^4}$	0.7	$\frac{5}{10^4}$	-0.05		
τ	$\frac{-6}{10^3}$	$\frac{-1}{10^3}$		0	0.2	0	0.04	$\frac{1}{10^4}$	0.04	$\frac{1}{10^4}$	$\frac{-4}{10^3}$		

Table 4.8: Responses of key variables (percentage changes from the initial equilibrium) to a rise in CAR penalty for all bank in state ii; Central Bank sets base money

with the corresponding initial equilibrium level, and therefore to adopt a more conservative position in the bad state, causing their corresponding profitability and capital to decrease so much that they suffer more capital violation penalties in this particular state. In state i, because households' default probability is higher, the values of their risk-weighted assets in this particular state decrease. This changing condition alters the trade off between the benefit derived from higher profitability and the corresponding cost in terms of suffering more default penalties in such a way that it favours the former. Thus, all banks optimally choose to target a slightly higher level of profitability in the good state, causing the corresponding value of their capital to increase marginally. Even though the targeted level of profits is higher, owing to the relatively strong and positive portfolio reallocation effect, all banks can adopt a slightly more conservative position in the good state. Finally, since the values of risk-weighted assets decrease for all banks in the good state and that the corresponding values of their capital increase, they suffer slightly less capital violation penalties in this particular state.

4.4.5 An increase in capital violation penalties for all banks in the bad state

We next turn to another restrictive regulatory policy. Let the regulator increase the capital violation penalties for all banks in the bad state from 0.1 to 0.12 (approximately 20%). We summarise the percentage changes of some of the key variables in response to such shock in Table 4.8.[23]

Since violating capital requirement in state ii is now more costly for all banks, in response, they engage in the following actions in an attempt to increase the values of their capital to asset ratios in the bad state. First, they choose to increase their optimal profitability level in this particular

[23] As can be seen from the results reported in Tables 4.7 and 4.8, the effects of an increase in default penalty are much stronger than the case when we increase the capital requirements violation penalty by a comparable magnitude. This is owing to our choice of parameterisation which assumes that banks care relatively more about them being penalised from defaulting on their financial obligations, rather than violating more their capital requirements. This is a reasonable assumption since banks in our model compare their actual level of capital with their self-imposed ideal capital holding, not the (Pillar 1) *minimum* capital requirements. Thus, as mentioned earlier, the cost of violating capital requirements in our model largely represents a non-pecuniary loss of comfort and reputation. Such cost should be of less concern to the banks as contrasted with the cost that they bear from breaching their financial obligations.

state by adopting a riskier position. The higher profitability, other things constant, improves their capital position in the bad state, allowing them ultimately to suffer fewer capital violation penalties. Second, banks γ and δ adjust their portfolios such that the size of their risk-weighted assets in the bad state decreases, thereby alleviating the extent of their capital requirements' violation even further. For bank γ, it reduces its overall investment by demanding less funds, both from the deposit and interbank markets, and invests less in the loan market. This causes both the interbank rate and its deposit rate to decrease and its lending rate to increase. Consequently, Mr. ϕ supplies less deposits with bank γ and Mr. α demands less funds from bank γ. Bank δ also demands less funds from the deposit market, causing its deposit rate to fall and Mr. ϕ to supply less deposits with bank δ. Moreover, it reduces its investment both in the loan and interbank markets. The extent of decline in bank δ's interbank investment is further aggravated since the interbank rate is now lower. Because bank δ's action produces an upward pressure on its lending rate, Mr. β demands less consumer loans.

Even though a higher capital violation penalty in state *ii* produces an upward pressure on the cost of extending more credit for bank τ, such a pressure is marginally outweighed by the negative pressure arising from a cheaper cost of interbank borrowing. Thus, bank τ borrows less from the deposit market, switches to borrow more from the interbank market, and extends slightly more loans to Mr. θ. Thus, both its deposit and lending rates decrease by 0.006% and 0.001%, respectively.

Because the extent of decrease in credit extension by banks γ and δ over-compensates the extent of increase in the credit supply of bank τ, the aggregate supply of credit in the economy decreases. This causes GDP to decline in both states (i.e. by 0.0008% in both states), which in turn results in a higher default probability of every household.

4.4.6 A positive shock in GDP in the bad state

Our final comparative static exercise studies the effect of a positive shock in the autonomous component of GDP in the bad state. In particular, we increase the value of u_1 by 0.6%.[24] Table 4.9 summarises certain key results.

The shock directly increases the expected aggregate output in the second period. This simultaneously raises all individual borrowers' demand for loans, and the probability that they will repay their loans in full, as well as increases Mr. ϕ's supply of deposits with every bank. Given a higher loan demand by Mr. α and θ, the lending rates offered by their respective nature-selected banks (γ and τ) increase by 1.1%. Moreover, a decrease in the default probability of Mr. α and θ further

[24]This is tantamount to an increase in the autonomous GDP in the bad state by 2.02%.

	Interest rates			π_i^b	π_{ii}^b	e_i^b	e_{ii}^b	k_i^b	k_{ii}^b	v_i^b	v_{ii}^b	GDP$_i$	GDP$_{ii}$
	r_d^b	r^b	ρ										
δ	2.4	1.1	1	$\frac{-4}{10^3}$	$\frac{-9}{10^3}$	$\frac{-4}{10^3}$	$\frac{-4}{10^4}$	-0.1	-0.2	-0.02	0.06	0.01	2.03
γ	2	1.1		$\frac{-7}{10^3}$	-0.02	$\frac{-1}{10^3}$	$\frac{-7}{10^3}$	-0.1	-0.2	$\frac{1}{10^3}$	$\frac{1}{10^3}$		
τ	2	1.1		$\frac{-1}{10^3}$	$\frac{-2}{10^3}$	$\frac{-2}{10^3}$	$\frac{-4}{10^4}$	-0.1	-0.2	$\frac{-1}{10^3}$	0.1		

Table 4.9: Responses of key variables (percentage changes from the initial equilibrium) to a rise in GDP in state *ii*; Central Bank sets base money

raises the expected rates of return on credit extension for banks γ and τ. Thus, these banks supply more credit, and demand more funds from both the deposit and interbank markets. Even though Mr. ϕ's decision to deposit more with these banks imposes a negative pressure on banks γ's and τ's deposit rates, such a pressure is relatively weak when compared to the positive effect from these banks' greater demand for deposits. Thus, we observe that the interbank rate and the deposit rates offered by the two banks increase.

For the same reason as banks γ and τ, bank δ supplies more loans to Mr. β. Unlike the other two banks, however, bank δ is the net lender in the interbank market. So, it responds to a higher interbank rate by investing more in the interbank market. To finance its greater investment, bank δ demands more funds from the deposit market, pushing up its deposit rate by 2.4%.

As mentioned, the initial shock directly increases the probability that an individual household will repay his loans in full. The extent of such increase is further magnified since the overall credit supply in the economy and the aggregate output in both states increase.[25] Consequently, the values of all banks' risk-weighted assets rise. This in turn implies that the marginal benefit of higher profitability in terms of suffering less capital violation penalties for all banks is lower. Thus, banks revise their optimal level of profits downward in order to suffer lower default penalties. This directly worsens their capital position. Given lower profitability, bank γ is able to adopt a more conservative position in both states. The same is true for banks δ and τ in the bad state. However, the negative portfolio reallocation effect on banks δ's and τ's profitability in the good state is so strong that they have to adopt a riskier position to be able to achieve the targeted profitability level.

[25]As can be seen from Table 4.9, GDP in the bad state rise by 2.03%, 2.02% of which is from the direct effect and 0.01% of which is from the indirect effect via higher aggregate supply of credit in the economy.

4.5 Robustness Check: The effect of an expansionary monetary policy under alternative initial conditions

In section 4.4, we analysed the effect of an expansionary monetary policy on the economic system based on a given value of the parameters $u_{s,2}$, $\forall s \in S$, in the reduced-form equation (4.14). We have already noted that these parameters are important because they capture the degree of responsiveness of aggregate economic activities, as measured by GDP, to changes in *financial* variables of the model. Put differently, they represent the *multiplier effect* of changes in the overall credit supply in the economy with respect to aggregate output in the subsequent period. Thus, in this section, we provide a robustness check for the result of one of the comparative statics that we obtained in the previous section, i.e. a positive monetary policy shock, by studying the sensitivity of the impact of the shock on the economy as we simultaneously vary the value of the parameters $u_{s,2}$, $\forall s \in S$. In doing so, we re-do another two independent comparative static exercises analogous to that shown in section 4.4.1, assuming instead that the value of the two parameters is 0 in the first and 0.05 in the second. Moreover, in order to maintain the same 'initial' equilibrium values of all *endogenous* variables across the these comparative statics, the values of autonomous part of (log) GDP (i.e. $u_{s,1}$, $\forall s \in S$) are adjusted accordingly. In this way, the initial equilibrium values of GDP in both states are the same across all comparative static exercises. Figure 4.2 reports the percentage changes in key variables in response to an expansionary monetary policy (i.e. an increase in base money from 69.28 to 71.28 trillion pounds) under different values of $u_{s,2}$, $\forall s \in S$, i.e. 0, 0.05, and 0.1.

Assuming the same positive monetary policy shock, the remainder of this section analyses how the response of key endogenous variables changes as we simultaneously *increase* the value of $u_{s,2}$, $\forall s \in S$. Given a higher value of these parameters, other things constant, an increase in same amount of aggregate credit supply results in a larger increase in GDP in both states of the subsequent period, causing, in turn, households' default probability in the loan market to decrease more precipitously. This implies that, a higher value of these two parameters, other things constant, results in a higher expected *rate of return* from extending consumer loans for all banks. Thus, all banks increase their credit supply even more, causing the extent of increase in the aggregate credit supply in the economy to rise. This implies that extent of increase in GDP and the probability that households will repay in full in both states increase.

Since all banks supply more credit, they need more funds to finance their higher investment. Thus, although, as explained in section 4.4.1, all banks respond to a positive monetary policy shock by decreasing their demand for deposits, the rate of such decrease falls. This in turn implies that

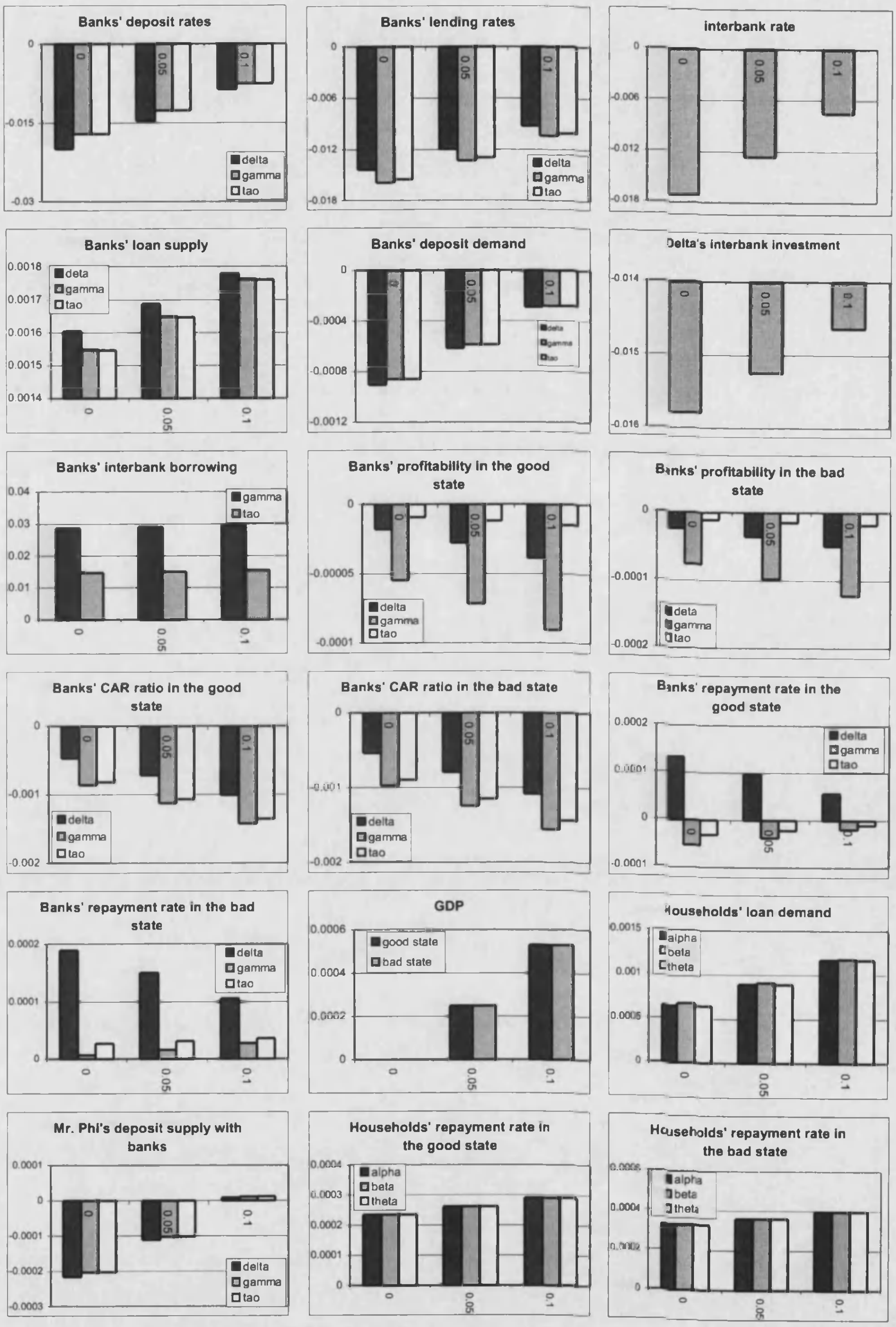

Figure 4.2: Percentage changes in key variables in response to an expansionary monetary policy under different values of $u_{s,2}$, $\forall s \in S$ (i.e. 0, 0.05, 0.1).

the extent of decline in every bank's deposit rates has to be smaller as well. Moreover, banks γ and τ, which are net borrowers in the interbank market, demand an even greater amount of interbank borrowing. This causes a smaller decline in the interbank rate.

Because of the bigger multiplier effect of GDP with respect to aggregate credit extension in both states, the increase in all household borrowers' loan demand is greater. This effect is strong enough to reduce the rate of decrease in every banks' lending rates. Moreover, Mr. ϕ increases his deposit supply to banks, thus increasing their available investment funds. This causes not only the percentage increase in bank δ's investment in the loan market to be higher but also the percentage decrease in its interbank investment to be lower.

Because the extent of decrease in households' default probability in both states is bigger, the values of every bank's risk-weighted assets grow by an even larger amount. This reduces the incentive for banks to achieve higher profitability even further. Consequently, all banks choose to achieve an even lower optimal level of profits, exacerbating the extent of decline in the capital violation penalties (recall that profits enter the numerator of the capital adequacy requirements). Given that the targeted level of profits decreases more substantially, banks γ and τ choose a relatively more conservative portfolio, though such position is still riskier compared to their initial equilibrium positions. We observe, however, that the percentage increase in bank δ's probability of default declines. This is because the negative pressure from the portfolio reallocation effect is relatively stronger for bank δ.

In sum, we observe that the effect of an expansionary monetary policy on the economic system is, in general, directionally the same under alternative values of $u_{s,2}$, $\forall s \in S$. However, the magnitude of its effect on the banking sector and the real economy is found to be stronger as we increase the value of the parameters. This is because, in such a case, the 'multiplier' effect from changing aggregate credit condition to default and ultimately to aggregate output is stronger.

4.6 Conclusion

This is the first attempt at calibration, to bring the model to real (UK) data, that we have made; the programme of work that we are following is just beginning to span the chasm between pure theory and practical empirical modelling. Nevertheless there is a long way yet to go. For example, this remains a two-period model only, in which banks take decisions in period one, e.g. determining their holdings of loans and deposits, on the basis of expectations of the potential states in period two; they then are stuck with their decisions in period two. In a longer period model, the outcome in period two would cause banks to revise their expectations of states in period three (e.g. as in a Markov switching model), thereby causing them dynamically to revise their loans/deposits in period two, and so on. Also, for a variety of reasons, mostly connected with data availability, we took the value of each bank's trading books (its investments) as a constant throughout.

Although we have based this exercise on real UK data, it remains a simulation. It does not provide an independent check whether our model can capture the main time series properties of the major UK banks. Trying to do this latter, and also to work out some way to lengthen the number of time periods in the model, without making the model far too unwieldy to solve are priorities for future research. In particular, this analysis reveals how complex, and complicated, default is as an institution, which is one reason why it rarely figures in other models.

Given these constraints, the focus of this exercise was on adjustments in the interbank market, and in the relative interest rates on deposits and loans; hence, in part via changes in bank margins, this fed back into changes in bank profits, capital and CARs. Like most other empirical research in this field, we do not find much serious contagion occurring via the interbank market with our arbitrarily chosen set of banks. Note that we could re-do this exercise for any other pair of UK banks (with a third residual banking sector). One result that practitioners will have expected, but not perhaps academics, is that contagion is much diminished if the Central Bank targets interest rates rather than a fixed time path for base money. We intend to write this latter up as a separate academic article.

Perhaps the most striking result is the sensitivity of banking profitability (in a bad state) to changes in the 'default penalty' on banks, see Table 4.7. We think of this as measuring the general risk aversion of banks, which is a function of the banks' own conservatism, and concern for regulation, interacting with externally-imposed discipline from markets and from the regulatory/supervisory regime. The greater the risks that bank managers are prepared to run, in pursuit of profit, the greater the probability of default in a bad state.

No doubt a glimpse of the obvious, but the problem is that such risk aversion is not objectively measurable. Indeed, but it is crucial, at least in our models. What this means is that to run models where default matters, the model builders will have to depend on the regulators/supervisors to give them some input, e.g. in the form of rankings, on the relative appetite for risk of the various banks involved. It is not, however, clear whether this is a 'good' result, since it would require all concerned to focus on the really important issues, or a 'bad' result since it reveals just how difficult quantification and modelling continues to be in this field.

Appendix

	r^δ	r^γ	r^τ	r_d^δ	r_d^γ	r_d^τ	ρ	$\overline{m}^\delta$	$\overline{m}^\gamma$	$\overline{m}^\tau$	μ_d^δ	μ_d^γ	μ_d^τ	d^δ	μ^γ	μ^τ	$\widetilde{R}_i$	$\widetilde{R}_{ii}$	v_i^δ	v_{ii}^δ	v^δ	v_i^γ	v_{ii}^γ	v^γ	v_i^τ	v_{ii}^τ	v^τ
M	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$-$	$-$	$-$	$-$	$+$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$
$z_{\delta 1}$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$-$	$-$	$+$	$+$	$+$	$\underset{\approx}{-}$	$+$	$+$	$+$	$+$	$\underset{\approx}{-}$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$+$	$\underset{\approx}{-}$
$z_{\gamma 1}$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$-$	$+$	$-$	$-$	$-$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$
e_0^δ	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$+$	$+$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$
λ_{ii}^b	$+$	$-$	$-$	$+$	$-$	$-$	$-$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$-$	$-$	$+$	$+$	$+$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$
λ_{kii}^b	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\approx$
$\overline{\omega}$	$\underset{\approx}{+}$	$-$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$
u_1	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$\underset{\approx}{-}$	$+$	$-$	$+$	$-$	$\underset{\approx}{+}$	$+$	$+$	$\underset{\approx}{-}$	$+$	$\underset{\approx}{+}$

Note: +(-): substantial increase (decrease), $\underset{\approx}{+}\left(\underset{\approx}{-}\right)$: weak increase (decrease), $\approx$: approximately equal, $+/-$:ambiguous effect

; An increase in $\lambda_{ii}^b(\lambda_{kii}^b)$ denotes an increase in the default (CAR) penalty for all banks; $v^b \equiv pv_i^b + (1-p)v_{ii}^b, b \in \{\gamma, \delta, \tau\}$

Table 4.A.1: Simulation results (Directional effects of an increase in exogenous parameters on endogenous variables); Central Bank sets base money

	Π^δ	Π^γ	Π^τ	Π^B	π_i^δ	π_{ii}^δ	π_i^γ	π_{ii}^γ	π_i^τ	π_{ii}^τ	e_i^δ	e_{ii}^δ	e_i^γ	e_{ii}^γ	e_i^τ	e_{ii}^τ	k_i^δ	k_{ii}^δ	k^δ	k_i^γ	k_{ii}^γ	k^γ	k_i^τ	k_{ii}^τ	k^τ	k
M	$+$	$-$	$-$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$z_{\delta 1}$	$+$	$-$	$-$	$+$	$-$	$-$	$-$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$z_{\gamma 1}$	$+$	$-$	$-$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
e_0^δ	$+$	$-$	$-$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$+$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$+$	$+$	$+$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
λ_{ii}^b	$-$	$+$	$+$	$+$	$\underset{\approx}{+}$	$-$	$\underset{\approx}{+}$	$-$	$\underset{\approx}{+}$	$-$	$\underset{\approx}{+}$	$-$	$\underset{\approx}{+}$	$-$	$\underset{\approx}{+}$	$-$	$\underset{\approx}{+}$	$-$	$-$	$\underset{\approx}{+}$	$-$	$-$	$\underset{\approx}{+}$	$-$	$-$	$-$
λ_{kii}^b	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{+}$	$+$	$\approx$	$+$	$\approx$	$\underset{\approx}{+}$	$\approx$	$+$	$\approx$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$
$\overline{\omega}$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$-$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
u_1	$-$	$+$	$+$	$-$	$\underset{\approx}{-}$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$

Note: $k^b \equiv pk_i^b + (1-p)k_{ii}^b$, $b \in \{\gamma, \delta, \tau\}$; $k \equiv (k^\delta + k^\gamma + k^\tau)/3$; $\Pi^B \equiv (\Pi^\delta + \Pi^\gamma + \Pi^\tau)/3$

Table 4.A.1 (CONTINUE)

	$\mu^{\beta^{\delta}}$	$\mu^{\alpha^{\gamma}}$	$\mu^{\theta^{\tau}}$	d^{ϕ}_{δ}	d^{ϕ}_{γ}	d^{ϕ}_{τ}	$v^{\alpha}_{i\gamma}$	$v^{\alpha}_{ii\gamma}$	$v^{\beta}_{i\delta}$	$v^{\beta}_{ii\delta}$	$v^{\theta}_{i\tau}$	$v^{\theta}_{ii\tau}$	GDP_i	GDP_{ii}
M	$+$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
$z_{\delta 1}$	$+$	$+$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
$z_{\gamma 1}$	$+$	$+$	$+$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{+}$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
e^{δ}_0	$+$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
λ^{b}_{ii}	$-$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$
λ^{b}_{kii}	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$
$\overline{\omega}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$
u_1	$+$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{+}$	$+$

Table 4.A.1 (CONTINUE)

	r^{δ}	r^{γ}	r^{τ}	r^{δ}_d	r^{γ}_d	r^{τ}_d	M	$\overline{m}^{\delta}$	$\overline{m}^{\gamma}$	$\overline{m}^{\tau}$	μ^{δ}_d	μ^{γ}_d	μ^{τ}_d
$z_{\delta 1}$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\approx$	$\approx$	$-$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$
$z_{\gamma 1}$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$-$	$\approx$	$\approx$	$\approx$	$\approx$	$+$	$\approx$
e^{δ}_0	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\approx$	$\approx$	$-$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$
λ^{b}_{ii}	$+$	$+$	$+$	$+$	$\approx$	$\approx$	$-$	$-$	$-$	$-$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$
λ^{b}_{kii}	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\approx$	$\approx$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\approx$	$\approx$
$\overline{\omega}$	$\underset{\approx}{-}$	$-$	$\underset{\approx}{-}$	$+$	$\approx$	$\approx$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$
u_1	$-$	$-$	$-$	$+$	$\approx$	$\approx$	$+$	$+$	$+$	$+$	$+$	$+$	$+$

	d^{δ}	μ^{γ}	μ^{τ}	$\tilde{R}_i$	$\tilde{R}_{ii}$	v^{δ}_i	v^{δ}_{ii}	v^{δ}	v^{γ}_i	v^{γ}_{ii}	v^{γ}	v^{τ}_i	v^{τ}_{ii}	v^{τ}
$z_{\delta 1}$	$+$	$+$	$\underset{\approx}{+}$	$\approx$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{-}$	$+$	$\approx$	$\underset{\approx}{+}$	$\approx$	$\approx$	$\underset{\approx}{+}$	$\approx$
$z_{\gamma 1}$	$\approx$	$-$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$
e^{δ}_0	$+$	$-$	$\underset{\approx}{-}$	$\approx$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$	$\approx$
λ^{b}_{ii}	$+$	$-$	$-$	$\underset{\approx}{+}$	$+$	$\underset{\approx}{-}$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$+$	$+$	$\underset{\approx}{+}$	$+$	$+$
λ^{b}_{kii}	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\approx$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\approx$
$\overline{\omega}$	$\underset{\approx}{-}$	$+$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$	$\underset{\approx}{+}$
u_1	$-$	$+$	$+$	$\underset{\approx}{-}$	$+$	$-$	$+$	$\underset{\approx}{-}$	$\underset{\approx}{-}$	$+$	$\underset{\approx}{+}$	$\underset{\approx}{-}$	$+$	$\underset{\approx}{+}$

Note: $+(-)$: substantial increase (decrease), $\underset{\approx}{+}\left(\underset{\approx}{-}\right)$: weak increase (decrease), $\approx$: approximately equal, $+/-$:ambiguous effect

; An increase in $\lambda^{b}_{ii}(\lambda^{b}_{kii})$ denotes an increase in the default (CAR) penalty for all banks; $v^{b} \equiv pv^{b}_i + (1-p)v^{b}_{ii}$, $b \in \{\gamma, \delta, \tau\}$

Table 4.A.2: Simulation results (Directional effects of an increase in exogenous parameters on endogenous variables); Central Bank sets the interbank rate

	Π^δ	Π^γ	Π^τ	Π^B	π_i^δ	π_{ii}^δ	π_i^γ	π_{ii}^γ	π_i^τ	π_{ii}^τ	e_i^δ	e_{ii}^δ	e_i^γ	e_{ii}^γ	e_i^τ	e_{ii}^τ	k_i^δ	k_{ii}^δ	k^δ	k_i^γ	k_{ii}^γ	k^γ	k_i^τ	k_{ii}^τ	k^τ	k
$z_{\delta 1}$	−	− ≈	≈	−	−	−	≈	≈	≈	≈	− ≈	− ≈	≈	≈	≈	≈	−	−	−	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	−
$z_{\gamma 1}$	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈	≈
e_0^δ	+	+ ≈	+ ≈	+	− ≈	− ≈	≈	≈	≈	≈	+	+	≈	≈	≈	≈	+	+	+	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+
λ_{ii}^b	−	+	+	+	+ ≈	−	+ ≈	−	+ ≈	−	≈	−	≈	−	≈	−	+	−	+	+	−	−	+	−	−	−
λ_{kii}^b	+ ≈	+ ≈	+ ≈	+ ≈	≈	+	≈	+	≈	+	≈	+ ≈	≈	+	≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈
$\overline{\omega}$	−	−	−	−	−	−	−	−	−	−	− ≈	− ≈	−	−	− ≈	− ≈	−	−	−	−	−	−	−	−	−	−
u_1	−	−	−	−	−	−	−	−	−	−	− ≈	− ≈	−	−	− ≈	− ≈	−	−	−	−	−	−	−	−	−	−

Note: $k^b \equiv pk_i^b + (1-p)k_{ii}^b$, $b \in \{\gamma, \delta, \tau\}$; $k \equiv (k^\delta + k^\gamma + k^\tau)/3$; $\Pi^B \equiv (\Pi^\delta + \Pi^\gamma + \Pi^\tau)/3$

Table 4.A.2 (CONTINUE)

	μ^{β^δ}	μ^{α^γ}	μ^{θ^τ}	d_δ^ϕ	d_γ^ϕ	d_τ^ϕ	$v_{i\gamma}^\alpha$	$v_{ii\gamma}^\alpha$	$v_{i\delta}^\beta$	$v_{ii\delta}^\beta$	$v_{i\tau}^\theta$	$v_{ii\tau}^\theta$	GDP_i	GDP_{ii}
$z_{\delta 1}$	+ ≈	+ ≈	+ ≈	+	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈
$z_{\gamma 1}$	≈	≈	≈	≈	+	≈	≈	≈	≈	≈	≈	≈	≈	≈
e_0^δ	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈
λ_{ii}^b	−	−	−	+ ≈	− ≈	− ≈	− ≈	−	− ≈	−	− ≈	−	−	−
λ_{kii}^b	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈	− ≈
$\overline{\omega}$	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈	+ ≈
u_1	+	+	+	+ ≈	+ ≈	+ ≈	+ ≈	+	+ ≈	+	+ ≈	+	+ ≈	+

Table 4.A.2 (CONTINUE)

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