Strategic Agents in Voting Games

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Abstract

The first part of this Thesis asks whether we can devise voting rules that allow strategic voters to express the intensity of their preferences. As opposed to the *classical* voting system (one person - one decision - one vote), we first propose a *new* voting system where agents are endowed with a fixed number of votes that can be distributed freely between a predetermined number of issues that have to be approved or dismissed. Its novelty, and appeal, relies on allowing voters to express the intensity of their preferences in a simple manner. This voting system is optimal in a well-defined sense: in a setting with two voters, two issues and uniform independent priors, Qualitative Voting Pareto dominates Majority Rule and, moreover, achieves the only ex-ante (incentive compatible) optimal allocation. The result also holds true with three voters as long as the valuations towards the issues differ sufficiently. Experimental evidence is provided supporting equilibrium predictions and showing that Qualitative Voting is better able to replicate the efficient outcome than Majority Rule. More generally in a setting with an arbitrary number of voters and issues, we show: (1) that a mechanism is implementable only if it does not undertake interpersonal comparisons of utility; (2) the impossibility of implementing strategy-proof mechanisms that are sensitive to the voters' intensities of preferences and satisfy the unanimity property.

The second part of the Thesis studies the interaction between politicians' strategic behaviour and voters' turnout decision: politicians diverge to motivate citizens to vote and they adapt their policies to the most sensitive voters —thus less sensitive voters abstain on the grounds of perceiving *politicians being too similar.* Moreover, citizens in central/moderate positions abstain. We find support for our predictions using NES data: (1) a perceived low difference between the Democratic and Republican parties tends to decrease a citizen's probability to vote and (2) moderate citizens vote less.

Vengo no sé de dónde,

Soy no sé bien quién,

Muero no sé bien cuándo,

Voy no sé hacia adónde,

Me asombro de ser tan feliz.

Martinus von Biberach

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Preface

Voting leads to the aggregation of individual preferences into social preferences. Its analysis is central in Economics and justifies the contribution of economic theory in an area that traditionally belongs to Political Science. In addition, economists have attempted to envision economic policy decisions not as exogenous decisions but as generated by a complex institutional and political process which can be explicitly incorporated in formal models developed by economic theorists. Both aspects give rise to the area in which we are interested, Political Economics.

This Thesis is precisely concerned about analysing the strategic behaviour of agents involved in voting games. We divide our study in two parts. We first analyse how the **strategic interactions between voters** limit the possibilities of allowing them to express the intensity of their preferences when transfers are not permitted. Secondly, we elucidate the effect of **politicians' strategic behaviour** on voters' turnout decision. Apart from the obvious common objective of analysing strategic agents in voting games, both parts are also related in the sense that citizens are always modelled as heterogeneous agents with multidimensional preferences. The multidimensionality allows us to use economic theory tools in a setting that lacks monetary transfers but that has inherent gains from trade due to the heterogeneity of agents.

In both parts we accompany the theoretical analysis by an empirical one. We believe that this should be the way to proceed (when possible) to further emphasise the applicability and relevance of our theoretical speculations. In the first part, we run an experiment on Qualitative Voting and use its data to highlight how our theoretical predictions should be tested in subsequent and more exhaustive experiments. In the second part, we provide an empirical validation of our understanding of the relation between political competition and electoral participation using data from the United States.

In Chapter 1 we ask whether we can devise voting rules that allow voters to express the intensity of their preferences when monetary transfers are forbidden. We propose an answer in two stages.

First, as opposed to the *classical* voting system (one person - one decision - one vote), we propose a *new* voting system, Qualitative Voting, where each agent is endowed with a fixed number of votes that can be distributed freely between a predetermined number of issues that must be approved or dismissed. The novelty, and the appeal, of Qualitative Voting is that it allows voters to express the intensity of their preferences in a simple manner. We demonstrate that this voting system is optimal in a well-defined sense: in a setting with two voters, two issues and preference intensities uniformly and independently distributed across possible values, Qualitative Voting Pareto dominates Majority Rule and, moreover, achieves the only ex-ante incentive compatible optimal allocation. The result holds true whenever we introduce a third player as long as the possible preference intensities differ sufficiently.

Second and more generally, in a setting with an arbitrary number of voters and issues, we show that a social choice function is implementable only if it does not undertake interpersonal comparisons of utility (it should only be contingent on the voters' relative valuations between the issues). Following this characterisation we prove the impossibility of implementing strategy-proof mechanisms that are sensitive to the voters' intensities of preferences and that preserve unanimous wills (unanimity implies that any issue is approved or dismissed with certainty whenever all voters wish so). We end that Chapter by overcoming the impossibility result through dropping the unanimity property. We identify an infinite set of social choice functions that are strategy-proof and sensitive to the voters' intensity of preferences.

In Chapter 2 we design an experiment to contrast the theoretical predictions with the experimental evidence and to observe how voters behave in those situations where the theory remains silent. Ultimately, we want to compare the outcome and welfare achieved by our subjects through the use of Qualitative Voting with the ones that would obtain if Majority Rule was used.

Subjects are endowed with 30 votes and are matched in groups of two, three or six voters and vote over two, three or six, issues. We observe that subjects vote according to equilibrium predictions. In general, Qualitative Voting is better able to replicate the efficient outcome than Majority Rule (68% vs. 37% of the cases). In terms of welfare the gains are not so significant since Majority Rule already does pretty well. Most importantly, we construct the basic tools to be used in subsequent works when analysing Qualitative Voting and voting strategies in multidimensional settings. For this purpose we build three scores that capture (1) how close the voting behaviour is from *truthfully* revealing voters' preferences; (2) how close the voting profile is from distributing evenly the voting power across issues; and, (3) how close the voting strategy is from concentrating all the voting power in the most preferred issue.

Finally, in Chapter 3 we analyse the interaction between electoral competition and voters' decision to vote, as opposed to abstain. When voters weight the benefits of voting against the costs, we show that politicians offer differentiated policies to motivate citizens to vote and they adapt their policies to the most sensitive voters —thus less sensitive voters abstain on the grounds of perceiving *politicians as being too similar.* In a multidimensional policy space setting, this implies that citizens that only care about few issues do not vote. In our two-party model, citizens who position themselves relatively to the left or the right of the political spectrum vote for the party that is closer to them, while citizens who position themselves in the central/moderate positions abstain.

We test two implications of our model using data from the National Electoral Studies for 1972-2000 and find support for our predictions: (1) a perceived low difference between the platforms of the Democratic and Republican parties tends to decrease a citizen's probability to vote and (2) moderate citizens are less likely to vote. These results are robust to accounting for socioeconomic, demographic and political individual controls, state-level institutional controls, state and year fixed-effects, state-specific time trends and to the model specification.

Finally, it has been argued that studies using reported turnout may suffer from an over-reporting problem. We prove that false reports do not drive our results.

Contents

List of Figures

Part I

Strategic Voters

Chapter 1

Qualitative Voting

"The history of economic institutions shows a great deal of change, facilitating economic activities that would have earlier been impossible. No similar development and change has occurred in the political system; yet the need for such facilitation is undoubtedly equally great" James Coleman (1970)

1.1 Introduction

1.1.1 Motivation

Voting is the paradigm of democracy. It reflects the will of taking everyone's opinion into account instead of imposing, by different means, the decision of a particular individual. At its root lies the belief that people should be allowed to freely cast their votes and, above all, they should be treated equally.[1] Consequently, as opposed to many economic situations, voting is considered a situation where no side payments are allowed so that agents are treated in an ex-ante identical position and wealth effects play no role.

Despite the adequacy of different particular rules to different settings, Majority Rule (MR, hereafter) is almost uniquely the rule used. From an economist's perspective, and given that most of our work is built on the diverse behaviour of individuals with different marginal propensities to consume, produce, etc., the main concern is that MR does not capture the intensity of voters' preferences. Just as we contemplate the importance of

[1] See, for instance, Locke (1690).

the *willingness to pay* in the provision of public goods, we should take into account the *willingness to influence* in a voting situation. An increase in the overall efficiency should follow.

The fact that majorities usually impose their will regardless of the intensity of their preferences is known in the political science literature as "the problem of intensity". The answer to this puzzle has always been founded on an argument of equality: if we were to treat differently a very enthusiastic voter from a very apathetic one, equality would no longer hold.[2] Nevertheless this reasoning is too narrow. In this Chapter we show that we can build a very simple voting rule that allows voters to express intensity and reach a strictly Pareto superior allocation than the one achieved by MR; moreover, we characterise what can be implemented in multidimensional settings with no transfers.

Following Coleman's quote, we ultimately want to stimulate the current debate around the development that should occur in our political institutions to better represent and govern our societies. We want to consider voting systems where the concept of decision *preferred by most members* is replaced by decision *most preferred by members*; we want votes to have an embedded *quality* which is somehow associated to the intensity of the voters' preferences; ultimately, we want to show under which circumstances the strategic interactions between voters do not undermine the gains we expect from them expressing their *willingness to influence.*

In a setting with a closed agenda of N issues that have to be approved or dismissed, we first propose a *Qualitative Voting* rule (QV, hereafter) that allows voters to simultaneously and freely distribute a given number of votes among the issues. In this way we are providing voters with a broader set of strategies than the classical "one person — one decision — one vote" strategy and we are preserving the equality inherent in any voting procedure given that all individuals are endowed with the same ex-ante voting power.

Essentially, QV introduces two main improvements with respect to the usual voting rules. On the one hand, it answers the classical debate in the political science literature on "the problem of intensity" allowing strong minorities to decide over weak majorities. Secondly, it allows voters to trade off their voting power, adding more weight to the issues they most care about, and unlocks conflict resolution situations.

The latter intuition is best captured by the following situation. Imagine two voters with opposing views on two issues but such that the first voter mostly cares about the outcome on the first issue while the second voter mostly cares about the second issue. QV allows each of them to decide on their most preferred issue and hence non-cooperatively

[2] See Spitz (1984).

coordinate on the only Pareto optimal allocation that yields a strictly positive utility to both voters (in the sense that each one wins his most preferred issue and loses the least preferred one). We can devise many different instances in which such situations occur and where side payments may not be possible (or may be forbidden): a divorce settlement, an international dispute, a bilateral agreement in arms/pollution reduction, a country having the two chambers governed by opposing parties,[3] a clash between the management and the union of a particular firm, etc...[4]

The goal of the first part of the present Chapter is not only to compare QV to MR but also to assess its optimality. Hence we use a mechanism design approach that allows us to characterise the optimal allocations among the implementable ones.

In a setting with two voters, two issues and independent uniform priors on the voters' preferences, Theorem 1.1 tells us that QV reaches the only ex-ante optimal allocation. Moreover, Theorem 1.2 establishes that the result holds true whenever we introduce a third voter as long as the voters' valuations towards the issues differ sufficiently. The introduction of a third voter yields a departure from the pure conflict resolution situation so that we can assess the optimality of allowing minorities to decide over weak majorities. Theorem 1.2 tells us that it is ex-ante optimal to decide among divergent issues by means of QV as long as the minority's feeling towards a certain issue is stronger than the sum of the majority's feeling.

We present examples in Subsection 1.3.3 that illustrate the results and shed some light on the applicability of QV in the real world.

The dependence of the results on the independent uniform priors is shown to be critical in Section 1.3.4. The more skewed the priors are, the more strategically voters react and, consequently, the more difficult it is to achieve a truthful revelation of preferences. Hence, the strategic interactions between individuals may lead to the non-existence of

[3]The US Congress and Senate have repeatedly been in a situation where one chamber had a Republican majority and the other a Democratic one. Consequently, many bills have been vetoed by one chamber so that decisions have not been easily made. QV could have made the decision process more efficient allowing each party to support those bills which its electorate felt more strongly about. Money and Tsebelis (1997) claim that the gains we expect from the use of QV may already be observed through the existence of committees: "One essential assumption of distributive theories of Congress is that the policy space is multidimensional. This is how committee chairs and members extract gains from trade. They give up their positions in the less important dimension in order to gain in the more important one, their own jurisdiction."

[4]Our setting can be reinterpreted as an extension of the *Colonel Blotto Game* (two colonels are fighting over some regions and have to decide how to divide their forces; the one with larger forces wins the region and the winner of the battle is the one with the most won territory) taking into account that now the colonel is not indifferent between winning two different regions. Hence the payoff of the game is not only contingent on how many regions he has won or lost but precisely on which regions he has won or lost. Myerson (1993) refers also to the *Colonel Blotto Game* when analysing the incentives for candidates to create inequalities among voters by making heterogeneous campaign promises.

pure-strategy equilibria in the game induced by QV. This does not undermine the first part of this Chapter. There exist situations in which one can strictly Pareto improve the allocation achieved by MR through the simple mechanism QV.

The drawback above leads us to the second half of this Chapter where we assess which voting rules or general mechanisms are *robust* —that is, are implementable given any specification of the priors. In our setting *robustness* is equivalent to strategy-proofness. Hence, in the second half of the Chapter we move from Bayesian Nash implementation to dominant strategy implementation.[5]

We first characterise the set of strategy-proof mechanisms and present our main contribution to the mechanism design literature: any implementable mechanism should be homogeneous of degree zero on any player's declaration.[6] This means that all proportional types are treated equally and bunched together. In other words, the interim prospects are only sensitive to the relative valuation between the issues. The result can be interpreted as implying that there cannot be any direct interpersonal comparison of utilities (they are not incentive compatible) and any aggregation procedure should be preceded by an intrapersonal one. Intuitively, an apathetic voter and an enthusiastic one are essentially treated in the same manner provided that their relative valuations between any two issues coincide.

Following the characterisation of all implementable mechanisms we further require the usual condition of *unanimity* (in our setting *unanimity* requires an issue to be approved or dismissed with certainty whenever all players wish so) which leads to a very negative result: there are no mechanisms sensitive to the voters' intensity of preferences that are strategy-proof and satisfy the unanimity property.

The key intuition for this result lies on the fact that any strategy-proof mechanism that satisfies the unanimity property needs to be insensitive to the voters' intensities of preferences on those issues where unanimous wills exist. Consider now a strategy-proof *qualitative* mechanism that satisfies the unanimity property. It needs to be sensitive to the voters' intensities of preferences for some particular profiles but it cannot be so on those issues where unanimous wills exist. This renders the mechanism *asymmetric* with respect to the sensitiveness to preferences' intensities and implies that such mechanisms are not strategy-proof.

[5] In Bayesian Nash implementation it is required that players truthtell their preferences as a best response to the common knowledge prior distribution of their opponent's preferences. Instead, strategy-proof implementation (or implementation in dominant strategies) requires truthrevelation of preferences to be a dominant strategy for each player.

[6] A function $f : \mathbb{R}^N \to \mathbb{R}$ is homogeneous of degree zero if $f(\lambda \cdot v) = f(v)$, for all $\lambda \in \mathbb{R}^+$ and $v \in \mathbb{R}^N$.

We then proceed by dropping the unanimity requirement (alternatively we can restrict the set of preferences so that no unanimous wills exist) and we distinguish an infinite set of strategy-proof *qualitative* mechanisms satisfying the usual properties of anonymity and neutrality that are ex-post incentive efficient.

In the remaining of this section we review the existing literature and relate our model to this earlier work. Next, the Chapter is organised as follows: Section 1.2 introduces the model, Section 1.3 analyses the indirect mechanism QV and its optimality in a setting with uniform and independent priors, Section 1.4 provides the general analysis of the intensity problem in a scenario robust to any specification of the priors (strategy-proof) and, finally, Section 1.5 concludes.

1.1.2 Related literature

The fact that any implementable mechanism needs only to rely on the relative valuations between the issues stresses that intensity of preferences can play a role in voting games only when we move away from unidimensional settings. Furthermore, QV arises as a way to allow voters to trade-off their voting power. Indeed its gains come precisely from non-homogeneous preferences across issues.[7] Accordingly, our work belongs to a wider set of models with two key features: heterogeneity of preferences and a multidimensional setting. In fact, we are dealing with a simple comparative advantage argument, the key question is how to implement it: in the same way that each country should specialise in the production of the good in which it is relatively more productive, QV allows voters to decide on that issue they relatively care more about.

The two papers most closely related to our work are Jackson and Sonnenschein (2003) and Casella (2003). Jackson and Sonnenschein (2003) show that linking decisions normally leads to Pareto improvements. More specifically, they present a simple rule that achieves the ex-ante efficient allocation and that induces truthful revelation as we increase the number of decisions. Such rule is very simple in the sense that it just requires voters to match their voting profiles to the frequency of preferences across decisions according to the underlying distribution of preferences. The key differences with our work is that they propose an efficiency limiting result for a particular indirect mechanism and their action space depends on the prior distribution of preferences. Instead, we propose an indirect mechanism which does not depend on the prior distribution and characterise its

[7] Bowen (1943) has already pointed out that MR is an efficient mechanism whenever the intensity of the voters' preferences is disbributed symmetrically. In a similar way, Philipson and Snyder (1996) analyse an organised vote market and show that its efficiency gains (with respect to MR) are larger the more heterogeneous the preferences are.

optimality.[8]

Casella (2003) proposes a system of *Storable Votes* to be used in situations where voters have to decide over the same binary decision repeatedly over time. Such a voting system is proved to Pareto dominate MR in a particular setting. Our framework is different in the sense that voters simultaneously cast all their votes and know their full preference profile at the time of voting (no time dimension). Moreover, we undertake a mechanism design analysis which allows us not only to compare two particular voting rules but also to characterise all implementable allocations and, from them, identify the optimal ones. Our impossibility result on implementing voting rules that are sensitive to the voter's intensity of preferences (whenever we require *robustness* and unanimity) generalises also her conjecture that gains from *Storable Votes* may arise as long as priors are not too polarized.

Most of the literature on mechanism design without transfers (and most of the literature on voting) is built on a setting with ordinal utilities and where one alternative has to be elected out of many.[9] Within that literature, QV has the flavour of a *scoring rule* though there is a crucial distinction:[10] a *scoring rule* is used to elect one representative out of many, instead QV deals with a situation where N independent issues have to be approved or dismissed. Our setting is one of a repeated binary election thus we are implicitly restricting the domain of preferences (see the example in page 50).

There is also an incipient literature that addresses the problem of eliciting the intensity of preferences in different settings with no transfers. Eliaz, Ray and Razin (2004) analyse how voters may abstain from an election depending on their relative aversion towards disagreement; Borgers and Postl (2004) demonstrate in a setting where two agents have to elect a representative out of three that no efficient mechanism exists; and, finally, Abdulkadiroglu (2004) proposes a mechanism for the allocation of indivisible goods where intensity of preferences can be elicited and the allocation achieved is at least as good as the one achieved by *random serial dictatorship.*

[8] Assessing trade-offs between issues and extracting all possible gains from differences is also one of the main concerns of the negotiation analysis and the international relations literatures. See for instance Keeney and Raiffa (1991). Closer in spirit to our work, Shepsle and Weingast (1994, pg 156) assert that "The political solution is to create an institutional arrangement for exchanging support that is superior to a spot market". Likewise, Levy (2004) models political parties as being able to exploit the gains from differing relative valuations in a multidimensional policy space.

[9] The main references are Gibbard (1973) and Satterthwaite (1975). These works can be seen to parallel Arrow's Impossibility Theorem from a mechanism design perspective. We defer further discussion to this strand of the literature to Section 1.4.2 after the presentation of our impossibility result.

[10] "In a *scoring rule*, each voter's ballot is a vector that specifies some number of points that this voter is giving to each of the candidates (or parties) that are competing in the election. These vote-vectors are summed over all voters, to determine who wins the election ", Myerson (1999), pg 673-674.

Our result on the impossibility of implementing strategy-proof qualitative mechanisms that satisfy the unanimity property is related to the literature on social choice (e.g. Arrow 1951), on implementation (e.g. Gibbard 1973, Satterthwhaite 1975) and on the allocation of indivisible objects (e.g. Zhou 1990).

The literature on alternatives to MR is related to the present Chapter insofar as it proposes mechanisms which capture the intensity of the voters' preferences but their complexity undermines its applicability. On the one hand, Tideman and Tullock (1976) develops an application of the *Clarke-Groves* mechanism to a voting framework. Needless to say, this requires monetary transfers and thus fails to satisfy the equality property. On the other hand, Hylland and Zeckhauser (1979) propose a *Point Voting Rule* to be used for the contribution to public goods, with perfectly divisible points.[11] They focus on identifying an (arbitrary) social choice function that induces the truthful revelation of preferences. This function belongs to the set of strategy-proof functions we propose in Section 1.4.3.

When we imagine a way in which politicians give more weight to a particular position we immediately think of *logrolling* or *vote trading.* This occurs whenever two voters agree on voting against one's position on some non salient issues which are salient for the other voter. The result is that both voters will have gained support on their salient issues at the cost of losing non-salient ones. The relationship and gains of QV with respect to that particular way of expressing the *willingness to influence* are shortly discussed in Section 1.3.4.

1.2 The general model

A *voting game* is defined as a situation where I voters have to dismiss or approve N issues and no monetary transfers are allowed. Voters privately know their preference profile across the N issues and the prior distributions from which these preferences are drawn are common knowledge (note that this allows for deterministic priors or commonly known preferences). From a mechanism design perspective this is a problem where agents are characterised by multidimensional and multilateral asymmetric information and have no transfers.

Voters and issues are denoted $i \in \{1,2,...,I\}$ and $n \in \{1,2,...N\}$, respectively. Voter i's valuation towards issue n is θ_n^i. The preference vector of voter i is $\theta^i = \left(\theta_1^i,...,\theta_N^i\right) \in \Theta \subseteq \mathbb{R}^N, \forall i = 1 \div I$.

[11] Brams and Taylor (1996) propose a *Point Voting Rule* (the *Adjusted Winner Procedure*) that is essentially our voting system in a setting of a conflict resolution. Their weakness, though, is that they do not take into account the strategic interactions and restrict players to be truthful on their cast votes.

Preferences should be interpreted as follows. A positive type ($\theta_n^i > 0$) wishes the approval of the issue, a negative one ($\theta_n^i < 0$) wishes its dismissal and its absolute value ($|\theta_i^n|$) captures the intensity of the preference towards the approval or dismissal of that particular issue.

Voter i's payoff on a given voting procedure n is described as follows,

$$\begin{cases} \theta_n^i \text{ if the issue is approved} \\ -\theta_n^i \text{ if the issue is dismissed} \end{cases}$$

and the total payoff is the sum of the individual payoffs across the N issues.[12]

An allocation is a N-tuple of probabilities that corresponds to the probability of approving each of the N issues. The set of allocations is defined as $\mathcal{X} = \{(p_1, ..., p_N) : p_1, ..., p_N \in [0,1]\}$ where p_n is the probability that issue n is approved. Hence, a voter with preferences θ^i obtains the following utility for a given $p \in \mathcal{X}$:

$$u\left(p, \theta^i\right) := \sum_{n=1}^{N} p_n \theta_n^i + (1 - p_n)\left(-\theta_n^i\right) = \sum_{n=1}^{N} (2p_n - 1)\,\theta_n^i.$$

Note that we are in a setting of private values where each agent's utility depends only on his own type and utilities are multilinear.

1.3 A new voting rule

In this section we describe a particular voting rule, QV, that allows voters to distribute freely a certain number of votes between a prearranged number of issues. Our goal is two sided. First we want to compare the welfare properties of QV and MR. Second, we want to assess the optimality of the allocation achieved by QV when compared to all implementable voting rules.

The strategy space defined by QV are mappings from preference profiles to *voting profiles* $\mathcal{V}$

$$\mathcal{V} := \left\{(v_1, v_2) \in \{-V, ..., -1, 0^-, 0^+, 1, ..., V\}^2 : |v_1| + |v_2| = V\right\}$$

so that a positive (negative) vote indicates the voter's wish towards the approval (dis-

[12] The definition of the payoff is implicitly assuming that issues are independently valued. That is, there are no complementarities between the issues. Provided that issues are independently valued, results can be extended to any linear transformation of the payoffs.

missal) of the issue.[13]

QV also defines a particular way to aggregate the cast votes. An issue is approved whenever the sum of votes on that issue is strictly greater than zero, dismissed whenever it is strictly negative and a tie breaking rule is applied whenever the sum of votes is equal to zero. The importance of the tie breaking rule is explained in Section 1.3.1.2 below. For the moment we assume that ties are resolved applying the usual MR (that is, the will of the majority of voters is implemented and if no majority exists a fair coin is tossed). Briefly,

$$\begin{cases} v_n^1 + ... + v_n^I > 0 & \Rightarrow \text{ The issue is approved} \\ v_n^1 + ... + v_n^I < 0 & \Rightarrow \text{ The issue is dismissed} \\ v_n^1 + ... + v_n^I = 0 & \Rightarrow \text{ MR is applied} \end{cases}$$

for every $n = 1 \div N$.[14]

When assessing the optimality of QV we focus on a simplified setting with (i) two or three voters ($I = 2, 3$), (ii) two issues ($N = 2, n \in \{1, 2\}$), (iii) two valuations ($\theta_n^i \in \{\pm 1, \pm\theta\}, \theta \in (0, 1)$[15]) and (iv) uniform and pairwise independent priors:

$$\begin{cases} \Pr\left(\theta_n^i = 1\right) = \Pr\left(\theta_n^i = -1\right) = \Pr\left(\theta_n^i = \theta\right) = \Pr\left(\theta_n^i = -\theta\right) = \frac{1}{4} \\ \text{Pairwise independence across issues and voters.} \end{cases}$$

We define the set of a voter's preference profiles as $\Theta := \{\pm 1, \pm\theta\} \times \{\pm 1, \pm\theta\}$

1.3.1 The indirect mechanism

1.3.1.1 The two voters' case

The two voters' case introduces the main features of QV: it allows voters to trade-off their voting power. Specifically, when we consider two issues QV allows voters to rank the issues and reach the only ex-ante optimal allocation. The next example best captures this intuition:

[13] By means of a small abuse of notation, the action space is defined so that investing zero votes is informative about the wish of the voters' preferences towards the approval or dismissal of the issue (i.e. 0^+ and 0^- have positive and negative *sign*, respectively).

[14] MR requires an issue to be approved (dismissed) if the number of people wishing its approval is strictly higher (smaller) than the ones wishing its dismissal. In case of ties we assume that a fair lottery is played –i.e. the issue is approved with probability 1/2. Note that MR is just a particular case of QV with $V = 0$.

[15] Note that without loss of generality and in order to simplify the notation we have assumed the high issue to take a value equal to one. The analysis is totally analogous to the more general setting where $\theta_n^i \in \{\pm\bar{\theta}, \pm\underline{\theta}\}, \bar{\theta} > \underline{\theta} > 0$.

> **Example**: Two friends, Anna ($i = 1$) and John ($i = 2$), are to go out on a Friday night and have decided they will first go to dinner and then to the movies. It is their first date so, above all, they want to be together even if they do not come to an agreement. Anna wants to see a horror movie and would like to have dinner in a new Italian restaurant while John prefers a comedy and a sushi restaurant. Following the previous notation we could define issue 1 as being the decision of which movie to go to(where p_1 would be the probability of seeing an horror movie and $1 - p_1$ the probability of seeing a comedy) and issue 2 as being the restaurant choice (where p_2 would be the probability of choosing the Italian restaurant and $1 - p_2$ the probability of choosing the Japanese restaurant). If they vote on each of the issues nothing is decided and they have to stay at home (we assume that this option is not optimal for either of them). Additionally, suppose that Anna really cares about which restaurant to go to and John, instead, cares more about the movie (i.e. $\theta^1 = (\theta, 1)$ and $\theta^2 = (-1, -\theta)$). It seems sensible that, as good friends, each of them will give up on his/her least preferred option. That is, they will both go to the Italian restaurant and then watch the comedy yielding an overall utility of $(1 - \theta) > 0$.

From a game theoretic perspective, they are both coordinating on the only Pareto optimal allocation that yields a strictly positive utility to both players (in the sense that each one wins his/her most preferred issue and loses the least preferred one). QV is precisely a mechanism that allows voters to coordinate non-cooperatively on the only ex-ante optimal outcome. We turn now into the rigorous analysis of the two voters case.

Voters are endowed with $V > 0$ votes that can be freely distributed between the two issues. We assume that V is even so that voters can evenly split the votes between the two issues if necessary. The submitted votes can have a positive or negative value capturing the will of the voter towards the approval or dismissal of the issue.

The uniform and independent priors on the opponent's preferences imply that it is a dominant strategy to truthfully declare the true sign of the preferences. Notice that in the case where a voter loses one of the issues he definitely wins the remaining one. This is because of the binary nature of our setting with only two issues. Losing an issue means having opposing preferences to the opponent on that issue and having invested fewer votes than he did. This implies that the voter at hand has invested more votes in the remaining issue. It can be easily proved that it is optimal to ensure that a voter does not lose his most preferred issue and consequently the optimal strategy for a voter who is not indifferent between the two issues is to invest all votes in his most preferred issue.

Instead, a voter who is indifferent between the two issues is also indifferent between playing any of the strategies. We therefore assume that he splits his votes evenly. The adoption of this strategy can be seen as the middle point between the strategies followed by the mixed types and allows to reach the Pareto optimal allocation.

The *night out* example presented above highlights the fact that QV allows any voter to concede on his least preferred issue and whenever that issue is strongly preferred by his opponent, the opponent's will is implemented. It follows immediately that such a voting rule Pareto dominates the allocations achieved by MR. Moreover, when we analyse the direct mechanism we prove that QV is not only superior to MR but it reaches *the* optimal allocation. Below we formally characterise the equilibria of the indirect mechanism.

Without loss of generality, we analyse the optimal strategy of voter i whenever he has positive preferences. His payoff is:

$$\left(\frac{1}{2}+\frac{1}{2}\tilde{P}_1\left(v_1^i \mid \theta_1^j<0\right)\right)\theta_1^i+\left(\frac{1}{2}+\frac{1}{2}\tilde{P}_2\left(v_2^i \mid \theta_2^j<0\right)\right)\theta_2^i$$

where $v_2^i = V - v_1^i$.

The previous expression captures the property that unanimous preferences are implemented ($\frac{1}{2}=\frac{1}{2}\tilde{P}_1\left(v_1^i \mid \theta_1^j>0\right)$) and $\tilde{P}_n\left(v_n\right)$ is the expected value of $(2p_n-1)$ whenever voter i casts v_n votes. These expected values are defined as follows (conditional probabilities are omitted for notational simplicity):

$$\begin{aligned}\tilde{P}_1\left(v_1^i\right) &:= 2\left(\Pr\left(v_1^i-\left|v_1^j\right|>0\right)+\tfrac{1}{2}\Pr\left(v_1^i-\left|v_1^j\right|=0\right)\right)-1\\ \tilde{P}_2\left(v_2^i\right) &:= 2\left(\Pr\left(-\left(V-\left|v_1^j\right|\right)+v_2^i>0\right)+\tfrac{1}{2}\Pr\left(-\left(V-\left|v_1^j\right|\right)+v_2^i=0\right)\right)-1.\end{aligned}$$

Simple calculations allow us to rewrite the payoff of voter i as

$$\frac{1}{2}\theta_2^i+\left\{\Pr\left(v_1^j+v_1^i>0\right)+\frac{1}{2}\Pr\left(v_1^j+v_1^i=0\right)\right\}\left(\theta_1^i-\theta_2^i\right).$$

Voter i wants to maximise the expression inside the curly brackets whenever $\theta_1^i > \theta_2^i$ (i.e. $v_1^i = V$).[16] Conversely, voter i wants to minimise it when $\theta_1^i < \theta_2^i$ (i.e. $v_1^i = 0^+$). Finally, player i is indifferent on which strategy to play whenever he is indifferent between the two issues, $\theta_1^i = \theta_2^i$. In the latter case we assume that he splits evenly his votes (note that

[16] Voter i sets v_1^i is equal to V because he wants to set v_1^i strictly higher (if possible) than the absolute value of his opponent's invested votes on the first issue. Taking into account that player j plays accordingly, it follows that the only equilibrium has non-indifferent players investing all their voting power on their preferred issue.

this strategy can be seen as the limiting strategy of non-indifferent players and allows us to achieve the Pareto optimal allocation).

Summing up, the equilibrium strategies for a player with positive preferences are as follows:

$$\begin{aligned} \text{if } \theta_1^i &> \theta_2^i \text{ then } v^i = \left(V, 0^+\right) \\ \text{if } \theta_1^i &= \theta_2^i \text{ then } v^i = \left(\frac{V}{2}, \frac{V}{2}\right) \\ \text{if } \theta_1^i &< \theta_2^i \text{ then } v^i = \left(0^+, V\right) \end{aligned}$$

Hence, the allocation achieved by QV can be described as follows: whenever voters rank equally both issues or whenever both voters are indifferent, ties occur; instead, if voters rank issues differently, the individual that is not indifferent wins its preferred issue.

1.3.1.2 The three voters' case

We depart now from a pure conflict resolution situation and consider a setting with three voters. In the previous analysis any voter tries to counteract the votes invested by his opponents. This effect is still in place now but we have an additional element: in some situations some voters may not be pivotal.

In the case of only two voters the tie breaking rule had no welfare effects. Instead, with three voters the tie breaking rule plays a crucial role and has important welfare effects. We will assume that in case of ties issues should be decided through the usual MR. We defer the discussion about the optimal voting rule to the end of this subsection once we have characterised the equilibrium of the game.

Voters are endowed again with an even number of votes V. Provided the uniform and independent priors it is still a dominant strategy to declare the correct sign of their preferences. We will now focus on *symmetric* pure strategy equilibrium. *Symmetry* should be interpreted as usual in voting theory: the three voters play the same strategy.

We want to focus on the set of final allocations reached in equilibrium rather than the set of equilibria. For this purpose we introduce the term *essential* as an equivalence class of equilibria that reach the same allocation —notice that given the nature of our game there are many situations where some votes are not pivotal and hence can be placed anywhere without affecting the outcome.

The following Lemma proves first that the strategy followed by any voter is independent of the labelling of the issues. That is, the strategy of a non-indifferent voter is summarised by a parameter $\gamma \in \{0, 1, ..., V\}$ which should be interpreted (together with the corresponding positive or negative sign) as the number of votes invested in his most preferred issue. The votes invested in his least preferred issue are $(V - \gamma)$ or $(\gamma - V)$ depending on whether he desires the approval or dismissal of it. The Lemma also shows that in a symmetric equilibrium voters who are indifferent should divide equally their votes.

Lemma 1.1 *In a setting with two issues, three voters and uniform and independent priors, any symmetric pure strategy equilibrium satisfies the following two properties:*

1. *Non-indifferent voters use* ***essentially*** *the same strategy. That is, they invest the same number of votes in their most preferred strategy.*
2. *Indifferent voters* ***essentially*** *split their votes evenly. That is, they invest $\frac{V}{2}$ votes on each issue*

The proof (which is provided in the appendix) relies on showing that an equilibrium where the strategies depend on the labelling of the issues cannot be sustained. Imagine, for instance, that there exists an equilibrium where indifferent voters cast more votes on the first issue: $\gamma^*_{ind} > V/2$. Then any voter is better off by deviating and playing, for instance, the complementary strategy where he invests $\gamma_{ind} = V - \gamma^*_{ind}$ in the first issue. In this way, a voter shifts some votes from the first issue to the second and increases his pivotability.

Given the setting described above, an equilibrium to our game is uniquely defined by a number $\gamma^* \in \{0, ..., V\}$. The independent and uniform priors imply that the number of votes invested on a high valued issue should be at least as big as the number of votes invested on an issue whenever the voter is indifferent, i.e. $\gamma^* \geq \frac{V}{2}$. The next Proposition tells us which are *essentially* the three equilibria that one can find.

Proposition 1.1 *In a setting with two issues and three voters, there are* ***essentially*** *three symmetric pure strategy equilibria. These are:*

$$\begin{array}{rlll} & \gamma^* = V, & \gamma^*_{ind} = \frac{V}{2} & \text{—all votes into preferred issue.} \\ & \gamma^* = \frac{V}{2}, & \gamma^*_{ind} = \frac{V}{2} & \text{—equivalent to MR} \\ \text{for } \theta = \frac{1}{2}: & \gamma^* = \frac{3}{4}V, & \gamma^*_{ind} = \frac{V}{2} & \end{array}$$

where γ^* is the number of votes invested by non-indifferent voters in the most preferred issue and γ^*_{ind} is the number of votes invested by indifferent voters in issue one.

The proof of the proposition is quite tedious and is left to the appendix. Its difficulty lies on the *essential* aspect of it. This is because we can devise many possible combinations of votes where no individual is better off by deviating but where some votes are not pivotal and hence can be invested in any of the issues. The fact that these votes are not pivotal implies no changes on the final allocation.

The first equilibrium is the equilibrium we observed in the two voters case where non-indifferent voters invest all their votes in their preferred issue so that strong minorities impose their will over weak majorities. The second equilibrium replicates the MR allocation. For future reference they will be called Equilibrium QV (EqQV) and Equilibrium MR (EqMR), respectively.

Finally, the third equilibrium can be seen as a mid point between the other two where a member of a majority that feels stronger about the remaining issue just needs an indifferent voter to overcome a strong minority (instead of a voter with strong preferences as would be the case in the EqQV). The non-divisibility of the votes may imply that this equilibrium (and only this one) may not exist. Note that this equilibrium is not as relevant as the other equilibria since it only holds for a particular value of θ.[17]

Two relevant aspects are left to be considered. On the one hand, the fact that the Proposition holds for any number of votes indicates that it may also hold whenever we consider votes to be perfectly divisible.[18] On the other hand, the Proposition shows that QV has multiple equilibria and one of them replicates the outcome reached by MR. Henceforth, we focus our attention on the first equilibrium. It does not seem worth it to propose a slightly more complicated voting system than the traditional MR if it just replicates the same allocation and introduces no strictly positive gains.[19]

The Tie Breaking Rule

We said above that in the three voter's case the tie breaking rule plays a crucial role and has important welfare effects. Consider how pivotal is a voter under MR. Given the uniform priors assumption, a voter observes his will being implemented on any issue with

[17]This equilibrium disappears whenever we consider the continuous valuation of the issues (see Section 1.3.4). There are two reasons for this to be the case: (1) the relative intensity for which it holds has measure zero in the continuous case (given uniform preferences) and (2) the strategy followed by indifferent players is crucial for this equilibrium to hold and these voters have in general zero measure in the continuous case.

[18]In Section 1.3.4 below, we show that in the case with continuous valuation of issues and perfectly divisible votes, the EqQV and EqMR are the only equilibria.

[19]The multiplicity of equilibria when analysing different mechanisms is usually eluded by selecting the best equilibrium in each possible situation. Note that this approach would benefit our analysis because MR would never be able to do better than QV given that the latter also contemplates the allocation reached by the former. Therefore, focussing on the first equilibrium makes our optimality analysis more difficult.

probability $\frac{3}{4}$ since the issue can only be dismissed if the remaining two voters are opposed to him —that event has probability $\frac{1}{4}$. Imagine now, that the tie breaking rule under QV is the toss of a fair coin. That is, the issue is approved with probability $\frac{1}{2}$. This implies that any voter becomes much less pivotal ($\frac{1}{2} < \frac{3}{4}$) than he was under MR and it can be shown that QV is no longer optimal: MR does better.

The optimal tie breaking rule relies on preserving in an incentive compatible way how pivotal any player is under MR. In other words, in case of a tie issues should be decided through the usual MR. QV becomes a voting rule that allows issues to be decided on the grounds of the total intensity of preferences. In case the intensity of preferences is not decisive, the issue is approved on the basis of overall support (MR). QV happens to be a natural extension of the usual voting rule where voters declare their position with respect to the approval or dismissal of an issue and then invest extra votes to reflect their *willingness to influence.*

1.3.2 The direct mechanism

We want now to characterise the optimality properties of QV in the previous setting (that is, two issues and two or three voters). In order to do that we first need to characterise the whole set of implementable mechanisms in our setting.

The *Revelation Principle* allows us, without any loss of generality, to restrict the analysis to the study of *direct revelation mechanisms.* A *direct revelation mechanism* is a function (p) that maps any revelation of the agents types into an allocation. Such mapping is known as a Social Choice Function (SCF).

$$p : \Theta^I \rightarrow \mathcal{X}$$

$$\text{i.e. } p\left(\theta^1, ..., \theta^I\right) = \left(p_1\left(\theta^1, ..., \theta^I\right), p_2\left(\theta^1, ..., \theta^I\right)\right).$$

As standard in the literature, we want to focus our analysis on the set of SCFs that preserve unanimous wills (an issue is approved or dismissed with certainty if all players wish so), have no systematic tendency towards the approval or dismissal of any of the issues (neutrality) and treat all individuals in the same manner (anonymity). Moreover, given that we are in a multidimensional setting we want to extend these properties accordingly. On the one hand, we want the SCF to be *neutral across issues* in the sense that it should be invariant with respect to the particular labelling on each of the remaining issues. On the other hand, we want every issue to be treated analogously. It will be useful to define a SCF as being *reasonable* whenever it satisfies the previous five properties.

Definition 1.1 *A SCF $p : \Theta^I \to X$ is* ***reasonable*** *if and only if it satisfies*

1. ***Unanimity:*** $p_n\left(\theta^1,...,\theta^I\right) = \begin{cases} 1 & \text{if } \theta_n^i > 0, \forall i = 1 \div I \\ 0 & \text{if } \theta_n^i < 0, \forall i = 1 \div I \end{cases}, \forall n = 1 \div N.$
2. ***Anonymity:*** $p_n\left(\theta^1,...,\theta^I\right) = p_n\left(\theta^{\sigma(1)},...,\theta^{\sigma(I)}\right), \forall n = 1 \div N, \forall \sigma \in S_I.$
3. ***Neutrality:*** $p_n\left(\theta^1,...,\theta^I\right) = 1 - p_n\left(-\theta^1,...,-\theta^I\right), \forall n = 1 \div N.$
4. ***Neutrality across issues:*** $\forall n = 1 \div N$ *and* $\xi_m \in \{+1,-1\}, m = 1 \div N$
   $p_n\left(\theta^1,...,\theta^I\right) = p_n\left(\left(\xi_1 \cdot \theta_1^1,...,\theta_n^1,...,\xi_N \cdot \theta_N^1\right),...,\left(\xi_1 \cdot \theta_1^I,...,\theta_n^I,...,\xi_N \cdot \theta_N^I\right)\right).$
5. ***Symmetry across issues:*** $\forall n = 1 \div N, \forall \sigma \in S_N,$
   $p_n\left(\theta^1,...,\theta^I\right) = p_{\sigma(n)}\left(\left(\theta_{\sigma(1)}^1,...,\theta_{\sigma(N)}^1\right),...,\left(\theta_{\sigma(1)}^I,...,\theta_{\sigma(N)}^I\right)\right).$

where S_k denotes the set of all possible permutations of k elements.

It is trivial to check that the set of *reasonable* SCFs that are implementable is not empty. For instance MR is one of them.

1.3.2.1 Implementable mechanisms

We want to characterise all Bayesian Nash implementable allocations. Thus, we are interested in the SCFs that induce truthful revelation at the interim stage —the point where each agent privately knows his own type (but only holds beliefs on his opponents' types) and he has to reveal his type in the direct mechanism or cast his votes in the indirect mechanism. The *interim utility* of a voter that declares $\hat{\theta}^i$ while his type is θ^i, is defined as:

$$u\left(\hat{\theta}^i,\theta^i\right) := E_{\theta_{-i}}\left\{u\left(p\left(\hat{\theta}^i,\theta^{-i}\right),\theta^i\right)\right\}$$

where, $\theta^{-i} := \left(\theta^1,...,\theta^{i-1},\theta^{i+1},...,\theta^I\right)$. Note that this is simply his expected utility taking into account that his opponents truthfully reveal their type. To simplify the notation let us also define the *interim prospect* on issue n as:[20]

$$P_n\left(\hat{\theta}^i\right) := E_{\theta^{-i}}\left\{2p_n\left(\hat{\theta}^i,\theta^{-i}\right) - 1\right\}.$$

Hence, the interim utility is: $u\left(\cdot,\theta\right) = P_1\left(\cdot\right)\theta_1 + P_2\left(\cdot\right)\theta_2$.

[20] Note that the interim prospect is the expectation of a linear transformation of the SCF, hence it is not a well defined probability. In particular, its domain lies on $[-1,1]$.

In order to characterise all implementable SCFs we just need to impose the *Incentive Compatibility* constraints (IC) according to which it should be optimal for each voter to reveal his true type. Restricting the analysis to the set of *reasonable* SCFs, together with the uniform and independent priors we assumed, imply that we just need to analyse the ICs from the perspective of a positive valued issue. That is, we just need to look at the interim prospects of approving the first issue whenever the declarations are $(1,\theta)$, $(\theta,1)$, $(1,1)$ and (θ,θ). The utilities of each of the three types of voter given truthful revelation are:

- A non-indifferent type: $P(1,\theta)\cdot 1 + P(\theta,1)\cdot\theta$
- A high type: $P(1,1)\cdot 1 + P(1,1)\cdot 1$
- A low type: $P(\theta,\theta)\cdot\theta + P(\theta,\theta)\cdot\theta$

Note that we have dropped the interim prospects subscripts —i.e. $P\left(\hat{\theta}^i\right) = P_1\left(\hat{\theta}^i\right)$.

The next Proposition tells us which are the conditions that any *reasonable* SCF should satisfy in order to be implementable.

Proposition 1.2 *A reasonable SCF* $p:\Theta^I \to X$ *is implementable if and only if the next four conditions are satisfied*

$$\textit{1.}\quad P(1,1) = P(\theta,\theta) \qquad \textit{3.}\quad P(1,1) \geq \frac{P(\theta,1)+P(1,\theta)}{2}$$
$$\textit{2.}\quad P(1,\theta) \geq P(\theta,1) \qquad \textit{4.}\quad P(1,1) \leq \frac{P(\theta,1)\,\theta + P(1,\theta)}{1+\theta}.$$

The proof of the Proposition is an immediate consequence of imposing the conditions for truthtelling. For instance, the first condition is a consequence of requiring that a high type does not have an incentive to deviate by declaring he is a low type together with a low type not having incentives to deviate by declaring he is a high type. The rest of the conditions follow from considering the remaining deviations.

There are few interesting things to say about the previous result which will be generalised in Section 1.4.1. First of all, observe that the SCF treats exactly in the same way an enthusiastic and an apathetic voter ($P(1,1) = P(\theta,\theta)$).[21] This highlights the fact that the first best allocation (the one that maximizes the sum of ex-ante utilities)

[21] The *symmetry across issues* property plays a relevant role for this result to hold true. The next example shows that dropping such property may be critical in the case with discrete preferences:

> *There is only one voter ($i = 1$), and there only two issues ($n = 1,2$). The player's valuation θ_1^1 and θ_2^1 are stochastically independent and uniformly distributed on $\{1,2\}$. The following SCF is strategy-proof but is not HD0 because it allocates a different outcome to the players*

can never be achieved since it requires interpersonal comparisons of utility. That is, it requires favouring those voters with stronger preferences and this can never be incentive compatible.

The remaining three conditions imply that the interim utilities should be convex. In particular, they require the interim prospect on an issue to be weakly increasing on the declaration on that issue, i.e. $P(1,1) > P(\theta,1)$ and $P(1,\theta) > P(\theta,\theta)$.

Finally, note that the Proposition holds for any number of voters as long as they are deciding over two issues.

1.3.2.2 Is qualitative voting optimal?

From the viewpoint of the designer of the mechanism it is reasonable to ask if the voting rule he would like to implement is the best one under the "veil of ignorance". That is, if by weighting all the possible combinations of types (given the prior distributions of them) the voting rule reaches the best possible allocation.

As Holmstrom and Myerson (1983) first pointed out, "the proper object for welfare analysis in an economy with incomplete information is the decision rule, rather than the actual decision or allocation ultimately chosen [...] a decision rule is efficient if and only if no other feasible decision rule can be found that may make some individuals better off without ever making any other individuals worse off." In our setting this means that we do not have to compare the set of final allocations but the set of implementable mappings from preference profiles to allocations (that is, implementable SCFs). It would be useless to provide a welfare analysis regardless of incentive compatibility because strategic manipulation of privately held information will almost surely lead to a different allocation than the expected one.

Henceforth we adopt the criteria that any optimality analysis is made out of the set of implementable SCFs. We denote this set $\mathcal{P}$ (i.e. $\mathcal{P} := \{p : \Theta^I \to X : p \text{ is implementable}\}$).

The welfare criteria we are interested in is the set of SCFs that reach a Pareto optimal

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$(1,1)$ *and* $(2,2)$:

$$p_1(1,1) = 1 \quad p_2(1,1) = 0 \quad p_1(0,1) = 0 \quad p_2(0,1) = 1$$
$$p_1(1,0) = 1 \quad p_2(1,0) = 0 \quad p_1(2,2) = 0 \quad p_2(2,2) = 1$$

I am indebted to Tilman Borgers for bringing this fact to my attention.

In Section 1.4.1 we show that the "equal treatment of proportional voters" holds in general whenever we have a continuous support.

allocation at the ex-ante stage.[22] First, a Definition for the ex-ante utility for voter i given the SCF p:

Definition 1.2 *We the define the ex-ante utility of player i given the implementable SCF $p \in \mathcal{P}$ as,*

$$u^i(p) := E_{\theta^i}\left\{E_{\theta^{-i}}\left\{u\left(p\left(\theta^i, \theta^{-i}\right), \theta^i\right)\right\}\right\}$$

Definition 1.3 *An **ex-ante efficient** SCF $p : \Theta^I \to X$ is an implementable SCF such that there does not exist any other implementable SCF such that makes some voters better off without worsening off any other, that is:*

$$p \text{ is } \textbf{ex-ante efficient} \Leftrightarrow \nexists \hat{p} \in \mathcal{P} \text{ such that } u^i(\hat{p}) \geq u^i(p) \text{ for all } i = 1 \div I \text{ and } u^i(\hat{p}) > u^i(p) \text{ for some } i \in \{1, ..., I\}.$$

Definition 1.4 *A voting mechanism is **optimal** if its associated direct revelation mechanism $p : \Theta^I \to X$ is reasonable and ex-ante efficient.*

It is essential to consider SCFs that are ex-ante efficient so that they are stable in the sense that voters will never want to jointly deviate and jointly choose a different decision rule. This argument also holds for the interim stage: we want mechanisms to be robust once agents privately know their types. It can be proved that ex-ante efficiency implies interim efficiency, hence our welfare criteria will also imply the stability of the voting rule at the interim stage.

The *night out* example described above illustrates that MR is in some cases not interim efficient. In that example, John and Anna had incentives to concede on their least preferred issue and both go to the Italian restaurant and the comedy. It follows that MR is not ex-ante efficient and that both friends may unanimously agree on resolving their dissenting issues through alternative methods.

The assumption that the intensity of the preferences towards each issue can only take two values (θ and 1) becomes now crucial. It allows us to write the interim prospects in terms of a finite number of parameters and, given that we restricted the analysis to reasonable SCFs, the number of parameters is treatable. The optimal SCFs are simply those that maximise the ex-ante utility of any single voter subject to the four constraints in Proposition 1.2 (the detailed analysis of the resulting linear program is in the appendix).

[22]Our definition of ex-ante efficiency corresponds to the notion of *ex-ante incentive efficient* in Holmstrom and Myerson (1983).

Theorem 1.1 *In a setting with two issues and two voters, QV is optimal. Moreover, MR is not optimal.*

QV is replicating the only ex-ante efficient and reasonable SCF but it is not the only indirect mechanism that can do so: QV is just one possible alternative. No other voting mechanism can do better.

In the three voters case we have seen that QV reaches two equilibria: one that replicates the MR outcome and one that allows strong minorities to decide over weak majorities. The next Theorem tells us when is the second equilibria ex-ante efficient.

Theorem 1.2 *In a setting with two issues and three voters, whenever the values of the various issues are "different enough" (i.e. $\theta \in \left(0, \frac{1}{3}\right)$), QV is ex-ante optimal. Moreover, in that case MR is not optimal.*

What do we mean by issues being *"different enough"*? Recall that when we described the simplified model we denoted the relative valuation of a low issue with respect to a high one as θ. The Theorem above is telling us that QV is optimal whenever the valuation of the high issue is at least three times the one of the low issue —$\theta \in \left(0, \frac{1}{3}\right)$. In other words, it is optimal to implement the will of an enthusiastic minority as long as the majority does not oppose the preference of the minority too strongly —agents want to commit to use such a rule before knowing their preferences so that their possibly strong views are not silenced by indifferent majorities.[23]

The main argument for proposing an alternative voting rule to allow voters to express their *willingness to influence* the final decision implicitly assumed that gains can only be possible as long as voters differed on which issue is the most relevant. And as long as their relative valuation towards the issues was different enough. Theorem 1.2 reinforces this idea and shows precisely that the optimality of QV relies now on a particular range of values of the parameter θ in contrast to the case with only two voters.

[23]In the interval $\theta \in \left(\frac{1}{3}, \frac{1}{2}\right)$ the allocation achieved by the third equilibrium replicates the optimal allocation –note though that the third equilibrium only exists for $\theta = \frac{1}{2}$. For $\theta \in \left(\frac{1}{2}, 1\right)$ MR achieves the optimal allocation. Proofs are provided in the appendix. Note that the costs of the incentive compatibility are captured precisely in the interval $\theta \in \left(\frac{1}{3}, \frac{1}{2}\right)$: from an ex-ante perspective (and regardless of incentive constraints) it is optimal for a strong minority to decide over a weak majority when $\theta \in \left(\frac{1}{3}, \frac{1}{2}\right)$.

1.3.3 Two examples

1.3.3.1 The two voters' case: *conflict resolution*

A more realistic version of the *night out* example may take the shape of a conflict resolution situation. In this case, two parties that have agreed on all concurring issues are to resolve their conflict on some dissenting ones. In this context it seems sensible not to expect the amicable behaviour we observed in the previous example. Now, parties may see any concession as a loss and (given the sequential nature of bargaining) may never truthfully declare their preferred alternatives leading to the deferring of any decision.[24]

Imagine a family enterprise that, after being badly managed for two generations, is in a very delicate situation and decides to hire a manager or CEO to redirect their business. The new CEO's team carries out a comprehensive analysis of the situation and concludes that the image of the firm has to be updated and two proposals are made. On the one hand a restyling of the logo will change the consumer's perception of their brand at a very low cost. On the other hand, a structural improvement of their main product line would also be beneficial to consumers' perceptions and, furthermore, it will gain the attention of the press.

The owners are against any change in their product because this is, from their point of view, the essence of their business. Similarly, they cannot contemplate a restyling of their logo because it was designed by one of their ancestors and they feel emotionally attached to it.

The negotiations between both parties are at a deadlock and, as was highlighted before, any concession is seen as a loss. Furthermore, the parties rank the issues differently. The CEO realises that the first policy is interesting given its low costs but it will have no persistent effect on the public and he sees the latter as the essential move to refloat the firm. Instead, the family owners realise that something has to change but would not like to be unfaithful to their ancestor so, above all, want to keep their logo. This is a *Prisoner's Dilemma* situation: whatever the opponent does any voter is always better off by not conceding and declaring both issues to be equally important (it is *dominant* to do so). And, as it is always the case, the unique equilibrium is a Pareto dominated one.

QV allows the voters to unlock the negotiation and non-cooperatively choose the Pareto optimal allocation. Let us analyse its logic: the CEO and the family are endowed with

[24] The social psychology literature has largely focussed on the problem of people not declaring what they perceive as less important because there exists the risk that they will lose that issue without any compensation. See for instance Rubin *et al* (1986).

V votes each and invest all votes in their preferred issue. The reason being that, given the binary nature of the situation, winning one issue implies losing the remaining one. Hence, the optimal strategy is to make sure that the most preferred issue is not lost

Note that a particular feature of the conflict resolution situation with two issues (where voters' preferences are opposed) is that it is robust to any possible prior in the voters' preferences —that is, it is dominant for a non-indifferent voter to invest all his voting power on his preferred issue. In other words, Theorem 1.1 is strategy-proof whenever both voters have opposing preferences. The optimal outcome has each player deciding on their most preferred issue (if they both prefer a different one) or ties occurring in both issues.

1.3.3.2 The three voters' case: *a committee meeting*

Imagine now a religious association which is composed of three factions with the same voting power at the annual committee. In that committee they need to update the association's position in two major biological scientific advances: human cloning and the use of stem cells. Imagine that each of the members of the committee has no clue about their opponents' preferences but privately know their own. The most progressive faction has no strong position on any of the issues but it is mostly in favour of both. Each of the other two strongly opposes one of the two issues and recognises that the positive aspects of the other one outweighs their moral prejudices and hence favours it. The next diagram captures their positions:

	Human cloning	**Use of stem cells**
F1	*agree*	*agree*
F2	*strongly disagree*	*agree*
F3	*agree*	*strongly disagree*

If they vote through MR, both issues are approved: a weak majority imposes its will over a strong minority. Is that situation *optimal*? We have just shown that from an ex-ante perspective (that is, before voters know what they are going to vote) the MR outcome may not be optimal. If the difference between the strength of the *strongly disagree* and the *agree* positions is wide enough, it is optimal to allow the enthusiastic minorities decide over the apathetic majorities. QV is again a system where agents are able to increase the probability of winning their preferred issue investing all their votes on that issue.

Following the analysis above, the first faction evenly splits its votes, the second invests all of them in the first issue and the third does the same in the second one (as depicted in the table below).

	Human cloning	**Use of stem cells**
F1	$\frac{V}{2}$	$\frac{V}{2}$
F2	$-V$	0
F3	0	$-V$

The outcome is now the opposite to the one before, both issues are dismissed and the overall welfare is strictly higher than the one obtained through MR.

1.3.4 Discussion

The equilibrium of the voting game is not driven by the non-divisibility of points or the binary nature of preferences. Whenever we consider preferences to belong to the interval $[-1, 1]$ with independent and uniform priors, voters still follow the described strategy: they invest all votes in their most preferred issue or, only in the case with three voters, they evenly split their votes.[25]

Conversely, all optimality analysis rested heavily on the binary nature of the preferences and the uniform and pairwise independent priors. It seems natural to relax the latter assumptions and check whether the main optimality results are affected by such a change. A more precise knowledge of the opponents' preferences may lead to the non-existence of pure strategy equilibria in the game induced by QV. The intuition is the following. For the voting profiles to be an equilibrium in a complete information framework, no voter should invest a single vote in an issue he is going to lose; consequently, a single vote should be sufficient to win any issue and overcoming the single vote invested by an opponent will occur almost surely.[26] Hence, relaxing the priors may lead to some critical problems in the applicability of QV and in its optimality properties.[27]

[25] Formal proofs of these statements can be found in the Appendix.

[26] In general it is also true that the situation where ties occur in all issues is not an equilibria.

[27] This may contrast with the intuition derived from Cremer and McLean (1988) that correlation allows the attainment of an efficient allocation. The result does not follow in our setting because correlation enhances the strategic interaction between individuals without introducing *penalties associated with lying* (recall that we are not allowing transfers). Jackson and Sonnenschein (2003) provide an example that illustrates how the correlation on the intensity between the issues affects the gains we expect from linking decisions: perfect positive correlation collapses the problem into a one-dimensional one; conversely, perfect negative correlation is the best possible scenario for QV. Note also that Milgrom and Weber (1985) ensures the existence of pure strategy equilibrium in the general game with I issues, N players and V votes as long as the informational structure of the game satisfies some mild conditions. Whenever we consider the more general case with perfectly divisible votes Simon and Zame (1990) generally characterise the conditions under which equilibria exists.

Briefly, we have seen that more skewed priors may lead to voters becoming more strategic. Consequently, it is more difficult to achieve truthful revelation of preferences and the costs of inducing truthful revelation may outweigh the welfare gains we expect from the use of QV. This contrasts with the behaviour we observe under MR where voters always declare truthfully their type. In other words, MR is robust to any possible specification on the preferences' prior distributions.

It is largely the above observation that leads us to the general analysis of eliciting the intensity of preferences in Section 1.4 where we characterise the implementable SCFs that allow voters to express the intensity of their preferences under any specification of the priors.

There are a couple of aspects of QV that we should discuss before proceeding to the analysis of robust mechanisms in general *voting games.* On the one hand, we should comment on how QV relates to the most usual way political parties express the intensity of their preferences, i.e. logrolling. And, on the other hand, we should also comment on the importance of the agenda in our setting.

Logrolling is defined as the exchanging of votes among legislators to achieve the approval or dismissal of the issues that are of interest to one another. Heuristically we could say that QV is related to logrolling in the same way monetary economies are related to barter. It eases the ways through which agents can express their *willingness to influence* given that it does not require a double coincidence of wants. Furthermore, it seems reasonable to expect that this increased freedom in the available strategies should prevent agents trading their votes because under QV a vote for an issue has always a value given that it can be *moved* to a more relevant issue.

The problem of modelling theoretically such phenomena relies on the fact that it usually occurs in a situation where a certain knowledge of the opponent's preferences exists but there is still scope for the understatement of one's preferences and, of course, the violation of the agreement once it is made.[28] The latter can be easily overcome through some kind of reputation argument but the former generates major difficulties and remains an area of interest for future research.

The selection of the agenda is shown to be an important matter that arises when analysing QV and is one of the most important problems that arises in any negotiation. The introduction of a new bill can drastically change the action taken by a particular individual, as is the case with QV. Namely, how, by whom and when should the issues be selected?

[28] The lack of a satisfactory theoretical treatment of *logrolling* supports such an assertion. The most relevant work is by Wilson (1969) where agents interact in an exchange economy framework with votes being tradeable and perfectly divisible.

There is a clear incentive to manipulate the agenda in order to induce particular outcomes and bundle issues that benefit some particular groups.[29] Nevertheless, the literature lacks tractable models of agenda setting given the somehow dubious knowledge of the opponent's preferences that is needed to correctly manipulate it. In our case, we take the agenda as exogenous: after some unmodelled negotiations an agenda is agreed by all voters.

1.4 General analysis of eliciting the intensity of preferences

All results from the previous section have rested on the assumption of uniform and pairwise independent priors. It seems natural to relax such assumptions and check whether the main optimality results are affected by such change. Section 1.3.4 above has called attention to the fact that a more precise knowledge of the opponents' preferences may lead to non-existence of equilibrium in the game induced by QV. Relaxing the priors may also lead to some critical problems in the optimality properties of QV —this is indeed a long standing critique to the whole literature on Bayesian Nash implementation.

Driven by the fact that MR induces truthful revelation given any possible specification of the priors, we want to characterise the set of SCFs that are *robust* to any specification of the priors and are also sensitive to the voters' intensity of preferences.[30]

Bergemann and Morris (2004) show that requiring a SCF to be *robust* to any specification of the priors (interim implementation for all possible type spaces) in private value environments is equivalent to *ex-post implementation* and is also equivalent to *dominant strategy implementation* or *strategy-proofness.* Hence in the remaining of the section we use the standard notion of strategy-proof. In a setting with I players and N issues, we generally show that a social choice function is implementable only if it does not undertake interpersonal comparisons of utility (it should only be contingent on the voters' relative valuations between the issues). Following this characterisation we find the impossibility

[29] For an example of the scope of such a problem see Metcalfe (2000). In the context of criminalising bribery at an international level between OECD countries, he shows how the setting of the agenda monopolised the negotiations for twelve years. He also emphasizes the perverse effect that the introduction of a *divisive* issue has in a negotiation: it creates a conflict between two factions that strongly disagree on the outcome of such issue and prevents any agreement being reached on the remaining ones. In a different setting Dutta *et al.* (2003) define and prove the existence of an *equilibrium for agenda formation* when one alternative has to be selected out of many -other references can be found therein.

[30] A voting game has been defined as a situation where N independent binary decisions have to be made. Trivially, it is always optimal for any voter to truthfully reveal whether he wishes the approval or dismissal of each of the issues. See Dasgupta and Maskin (2003) for a further defense of the robustness of MR in a standard Social Choice framework.

of implementing strategy-proof (or *robust*) mechanisms that are sensitive to the voters' intensities of preferences and satisfy the unanimity property.

The impossibility result is consistent with both literatures on social choice and implementation. The former has exposed the impossibility of producing rational aggregators (in the sense that the social preference relation is transitive) whenever we consider universal preference domains. The latter has shown that the strategic interaction between voters that arises from the fact that individual's preferences are not publicly observable also leads to impossibility results (e.g. Gibbard-Satterthwhaite Theorem).

At the end of this Section we drop the unanimity requirement and propose a set of SCFs that satisfy some appealing conditions.

1.4.1 Implementability result

Without any loss of generality we restrict the analysis to the study of *direct revelation mechanisms.* That is, *Social Choice Functions* (SCF) that map any possible preference profile into an allocation.

$$p : \Theta^I \to \mathcal{X}$$
$$\text{i.e. } p\left(\theta^1, ..., \theta^I\right) = \left(p_1\left(\theta^1, ..., \theta^I\right), ..., p_N\left(\theta^1, ..., \theta^I\right)\right).$$

We want to characterise all feasible allocations under the *universal domain assumption* —the SCFs that induce truthful revelation when $\Theta = \mathbb{R}^N$. It will be useful to define the *prospect* of issue n being approved for the present case of dominant strategy implementation analogously to what we did in the previous section:

$$P_n(\theta) := 2p_n(\theta) - 1, \theta \in \Theta^I$$

Hence, the indirect utility of a type θ^i who declares being $\hat{\theta}^i$ whenever the remaining voters truthfully reveal their type is:

$$\begin{aligned} u\left(\hat{\theta}^i, \theta\right) \quad : & = u\left(p\left(\hat{\theta}^i, \theta^{-i}\right), \theta^i\right) \\ & = \sum_{n=1}^{N} P_n\left(\hat{\theta}^i, \theta^{-i}\right) \cdot \theta^i_n = P\left(\hat{\theta}^i, \theta^{-i}\right) \cdot \theta^i \end{aligned}$$

where, $\theta^{-i} := \left(\theta^1, ..., \theta^{i-1}, \theta^{i+1}, ..., \theta^I\right)$.

Strategy-proof mechanisms are those SCFs that satisfy *Incentive Compatibility* constraints (IC) —that is, it should be optimal for each voter to reveal his true type given any profile of preferences:

$$\theta^i \in \arg\max_{\hat{\theta}^i \in \Theta} u\left(\hat{\theta}^i, \theta\right), \quad \forall i = 1 \div I, \forall \theta \in \Theta^I$$

We define $u^i(\theta) := u\left(\theta^i, \theta\right)$ as the utility of voter i in equilibria.

It follows that a strategy-proof SCF needs to satisfy the necessary first and second order conditions for truthtelling for all voters:

$$\begin{cases} \frac{\partial}{\partial \hat{\theta}^i} u\left(\hat{\theta}^i, \theta\right)\Big|_{\hat{\theta}^i = \theta^i} = 0 \\ \frac{\partial^2}{\partial\left(\hat{\theta}^i\right)^2} u\left(\hat{\theta}^i, \theta\right)\Big|_{\hat{\theta}^i = \theta^i} \text{ is negative semidefinite} \\ \text{for } i = 1, ..., I. \end{cases}$$

The next Proposition is just an extension of the usual technique used in one-dimensional screening problems due to Mirrlees (1971). It is the first step to simplify the first and second order conditions above in order to characterise all the implementable allocations.

Proposition 1.3 *The SCF $p : \Theta^I \to \mathcal{X}$ is strategy-proof if and only if the voters' induced utilities are convex and its gradients are equal to the interim prospects.*

$$i.e.: \quad \theta^i \in \arg\max_{\hat{\theta}^i \in \Theta} u\left(\hat{\theta}^i, \theta\right) \Longleftrightarrow \begin{cases} \nabla_{\theta^i} u^i(\theta) = P(\theta) \text{ for all } \theta \in \Theta^I \\ u^i \text{ is convex on } \theta^i \in \Theta \text{ for all } \theta \in \Theta^I \end{cases}$$

$$\text{where } \nabla_{\theta^i} u^i(\theta) := \left(\frac{\partial u^i(\theta)}{\partial \theta^i_1}, ..., \frac{\partial u^i(\theta)}{\partial \theta^i_N}\right)$$

Proof. **Sufficiency**. The envelope Theorem directly implies that $\frac{\partial u\left(\hat{\theta}^i, \theta\right)}{\partial \theta}\Big|_{\hat{\theta}^i = \theta^i} = \nabla_{\theta^i} u^i(\theta) = P^i(\theta)$. Given that the FOC is satisfied for all $\theta \in \Theta^I$ it can be differentiated with respect to θ yielding: $\frac{\partial^2 u\left(\theta^i, \theta\right)}{\partial \hat{\theta}^2} + \frac{\partial^2 u\left(\theta^i, \theta\right)}{\partial \hat{\theta} \partial \theta} = 0$. The SOC implies that the first matrix is negative semidefinite, hence the second one should be positive semidefinite.

Necessity. One can easily reverse the previous reasoning to get the local conditions. We just need to prove that the conditions are global. A continuously differentiable function $u^i : \Theta^I \mapsto \mathbb{R}$ is convex on θ^i if and only if $u^i(\theta) \geq u^i\left(\hat{\theta}\right) + \nabla_{\theta^i} u^i(\hat{\theta})\left(\theta - \hat{\theta}\right), \forall \theta, \hat{\theta} = \left(\hat{\theta}^i, \theta^{-i}\right) \in \Theta$. Using the fact that $\nabla_{\theta^i} u^i(\theta) = P(\theta)$ and the Definition of $u^i(\theta)$ we can get the global condition:

$$\begin{aligned} u^i(\theta) &\geq u^i\left(\hat{\theta}\right) + P(\hat{\theta})\left(\theta - \hat{\theta}\right), \forall \theta, \hat{\theta} \in \Theta^I \\ P(\theta)\cdot\theta &\geq P\left(\hat{\theta}\right)\cdot\hat{\theta} + P(\hat{\theta})\cdot\left(\theta - \hat{\theta}\right), \forall \theta, \hat{\theta} \in \Theta^I \\ P(\theta)\cdot\theta &\geq P(\hat{\theta})\cdot\theta, \forall \theta, \hat{\theta} \in \Theta^I \\ u^i\left(\theta^i, \theta\right) &\geq u^i\left(\hat{\theta}^i, \theta\right), \forall \hat{\theta}, \theta \in \Theta^I \end{aligned}$$

This concludes the proof. ■

An analogous result can be found in Rochet and Chone (1998) in the presence of transfers. Note that the convexity condition implies that the prospects are, *ceteris paribus*, weakly increasing in the type of each voter on the relevant issue (i.e. $\frac{\partial P_n(\theta)}{\partial \theta^i_n} \geq 0, \forall n, i$). Moreover, the convexity condition implies that the utility function of each player is differentiable almost for all preference profile. Hence we require no regularity condition on the set of solutions of our problem but instead these are derived from the IC constraints.[31]

Note that the equality $\nabla_{\theta^i} u^i(\theta) = P(\theta)$ in Proposition 1.3 together with the Definition of $u^i(\cdot)$ imply that any implementable SCF should satisfy the following linear first-order partial differential equation:

$$\nabla_{\theta^i} u^i(\theta)\cdot\theta = u^i(\theta). \tag{1.1}$$

Euler's Theorem implies that the former equality is satisfied if $u^i(\cdot)$ is homogeneous of degree one on $\theta^i \in \Theta$ (HD1: $u^i(\theta^1, ..., \lambda\cdot\theta^i, ..., \theta^N) = t\cdot u^i(\theta), \lambda \in \mathbb{R}, \lambda > 0$). Furthermore, Euler's Theorem on homogeneous functions is invertible, that is, only homogeneous functions of degree one satisfy equation (1.1).[32] The next result follows:

Theorem 1.3 *The SCF $p : \Theta^I \to \mathcal{X}$ is strategy-proof if and only if the voters' induced utilities are HD1 and convex on their own preferences. That is, $u^i(\theta)$ is HD1 and convex on θ^i for all $\theta \in \Theta^I$.*

The homogeneity of degree one on the interim utilities implies that the interim prospects

[31] See Rochet (1985) for a detailed proof of the fact that implementability implies differentiability for almost all preference profiles. Note the equality $\nabla_{\theta^i} u^i(\theta) = P(\theta)$ in Proposition 1.3 should be stated in terms of "for almost all $\theta^i \in \Theta^i$.

[32] See Lemma 1.4 in the Appendix.It is worth noting that this Lemma can only be applied wherever the function is differentiable. Nevertheless, the fact that $u^i(\cdot)$ is convex implies that it is continuous and differentiable almost everywhere. Thus, Lemma 1.4 can be applied wherever the function is differentiable and by continuity we can extend the homogeneity result to those points where the function may not be differentiable (the kink points that may exist in the convex function $u^i(\cdot)$).

are homogeneous of degree zero (HD0).[33] This means that all proportional types are treated equally and bunched together. In other words, the interim prospects are only sensitive to the relative valuation between the issues. The result can be interpreted as implying that there cannot be any direct interpersonal comparison of utilities and any aggregation procedure should be preceded by an intrapersonal one. Intuitively, an apathetic voter and an enthusiastic one are essentially treated in the same manner provided that their relative valuations between any two issues coincide. This extends the equality argument embedded on any voting game and presented in the introduction: not only it is the case that wealth effects can play no role in a voting game, but neither can the preference endowment of each individual. Whilst the former argument is an axiomatic one (imposed by ethical or practical reasons) the latter is an equilibrium result, a necessary condition for the voting game to be implementable.

Note that we consider a multidimensional mechanism design problem with multilateral asymmetric information without transfers. The main difficulty (and main contribution with respect to the existing literature) lies exactly in the fact that transfers are not allowed. Consequently, we introduce an endogeneity problem in the sense that we can no longer associate a high transfer to a high type declaration in order to induce truthful revelation of the preferences. Therefore, when a voter declares that an issue is highly preferred, the SCF should not only increase the probability of winning that issue but the associated cost should be formulated in terms of a decrease in the probability of him winning any other issue.[34] Intuitively, this complicates the analysis. However, as opposed to what one would expect, having no transfers simplifies the analysis because the first order partial differential equation that arises from imposing truthful revelation (IC) is now solvable.

To illustrate such a property, imagine a setting with only two issues. For a SCF to be implementable it should only depend on the direction of the preference vector and should be invariant to its modulus.[35] As a result, we have reduced the dimensionality of our problem to one dimension. Furthermore, if the setting is unidimensional, the HD0 implies that interim prospects should be invariant with respect to the intensity of the preferences and should only depend on its sign (that is, whether the voter wants the approval or the dismissal of the issue).

Hence, the argument usually endorsed by political scientists that "the introduction of an

[33]The partial derivative of a HD1 function is a HD0 function.

[34]Precisely, this intuition is at the heart of the particular voting rule we described in Section 1.3: QV endows agents with a given number of votes such that whenever an agent wishes to strengthen his position on a particular issue he does so at the cost of lowering his voting power on the remaining ones.

[35]Such concept becomes clearer if we consider polar coordinates. In that setting, the interim prospects should only care about the angular coordinate (angular coordinates if the setting has more than two issues) and neglect the radial coordinate.

intensity dimension attacks political equality in ways not permissible within the context of democratic theory" is questionable.[36] Intensity can be taken into account as long as we broaden the usual limits and we bundle together the voting of more than one issue. In other words, allowing agents to express the intensity of their preferences whenever they vote goes hand-in-hand with the argument of analysing voting games in multidimensional settings.

It is worth saying that Theorem 1.3 applies to any general setting as long as voters have quasi-linear von Neumann Morgenstern utilities (i.e. cardinal utilities) and there are no transfers. Hence it does applies to electing representatives, allocating private goods to individuals, etc.... We may, in each case, add extra feasibility constraints on the sum of probabilities across individuals or issues. Note also that our result is stronger than the standard result that voters' incentives in any game are not changed if the von Neumann Morgenstern utilities are multiplied by a constant. Indeed it is the case from the IC constraints that the expected utility of a voter when he declares θ^i or $\lambda \cdot \theta^i$ $(\lambda > 0)$ coincides, yet this is a weaker statement than requiring the SCF on *every* single issue to remain unchanged when the voter's declaration is multiplied by a positive scalar.[37] Moreover, our result is not solely a necessary condition for implementability but, together with the convexity condition, it is also a characterisation of *all* implementable SCFs.

It is clear from Theorem 1.3 that increasing the number of issues should relax the implementability constraints which in turn allow us to reinterpret the main result in Jackson and Sonnenschein (2003) —incentive costs diminish as we increase the number of issues we consider and first best can be arbitrarily approached. The HD0 result implies that the SCF can only be sensitive to declarations that have one lower dimension than the preference space. Consequently, the constraints that truthtelling impose on the implementable SCFs are less binding the higher the dimensionality of the preference space; at the limit, these constraints tend not to bind and the first best can be arbitrarily approached.

Theorem 1.3 also extends naturally to a Bayesian Nash implementation setting. We just need to be aware that the conditions are then imposed on the interim utilities rather than the ex-post ones. It is worth pointing out that in that case the Theorem is general in the sense that it allows for any prior on the opponents preferences. That is, it allows for correlation between issues, individuals, etc. The drawback is that it would be stated in terms of the interim prospects and consequently the necessary and sufficient conditions for a SCF to be implementable critically depend on such priors.

[36]Spitz (1984), pg 30.

[37]It could be the case that the allocation (i.e. the n-dimensional vector of probabilities) changes when we multiply the declaration of player i by a positive scalar though keeping his expected payoff constant. This may ease the achievement of truthtelling of player j and/or could have an effect on the ex-ante total welfare achieved by a particular SCF.

Before we state two immediate consequences of Theorem 1.3 in the form of corollaries we want to stress the fact that the implementable SCFs have a very *balanced* structure. This is best captured by applying Schwarz's Theorem (the order of the differentiation does not alter the result) to voter i's induced utilities:

$$\frac{\partial P_n(\theta)}{\partial \theta^i_m} = \frac{\partial P_m(\theta)}{\partial \theta^i_n} \quad \begin{cases} \forall n, m = 1 \div N \\ \forall i = 1 \div I \\ \forall \theta \in \Theta^I \end{cases}$$

That is, the marginal change of issue n's prospect to a variation on voter i's preference on issue m should coincide with the correspondent change on issue m's prospect to a variation on issue n. Note that this should hold for every preference profile and any voter.

Corollary 1.1 *The utilitarian first best allocation can never be reached.*

The utilitarian first best allocation requires approving an issue whenever the sum of utilities is higher than zero and dismissing it whenever it is lower than zero; needless to say, this requires interpersonal comparisons of utilities thus cannot be truthfully implementable.

Theorem 1.3 also implies that any voter is indifferent between declaring his own preferences or declaring his own preferences normalised by, say, the L_1 norm (that is, such that the sum of the absolute value of its components adds up to 1). This line of reasoning leads to the following Corollary.

Corollary 1.2 *Any strategy-proof mechanism in a multidimensional setting with no transfers can be replicated by a point-voting mechanism where voters are endowed with a given number of votes that can be distributed freely among the issues.*

This last result can be read as a *taxation principle* in our environment. In the same way that the *revelation principle* allowed us to restrict our attention to direct revelation mechanisms we can now go back from any direct mechanism to an indirect mechanism where players are endowed with one perfectly divisible point that can be split among the issues.

This Corollary offers a *new* rational for the existence of fiat money in the following sense: any mechanism we can devise in a setting with no monetary transfers can be replicated by a mechanism where we introduce a numeraire that has no value. It has no value, first, because it does not enter the utility function of agents and, second, because it is useless outside the framework where it is defined (that is, it can only be used to express the voters' preferences in a particular voting game). Hence, the only possible use it may have is on smoothing transactions, on allowing the mechanism to elicit the voters' intensities of preferences when deciding which allocation to implement. Money is in our model a *useless* token that plays three main roles: (1) allows the mechanism to compare the voters' valuation (*unit of account*); (2) allows agents to trade-off their voting power among the issues (if we gave our model a temporal reinterpretation this could be considered the usual *storage of value* property); and (3) allows agents to extract gains from their different relative valuations towards the issues (*medium of exchange*).

The analogy with prices allows a better understanding of Theorem 1.3 above. Just in the same way as a consumer requires that his marginal rate of substitution equals the price ratio of goods when he maximizes his utility, the ratio between the allocated votes on each issue should be equal to the relative valuation between them.

1.4.2 Impossibility result

We have characterised all strategy-proof SCFs as those that induce indirect utilities that are HD1 and convex. The set of such functions is not empty. Indeed, any voting rule that is not sensitive to the voters' intensity of preferences is implementable. MR, dictatorial rules or rules that implement a particular allocation regardless of the voters declared preferences induce truthtelling. Obviously, we would like to impose minimal requirements on the set of SCFs we want to analyse so as to avoid the latter ill-behaved rules.

In this section we pay no attention to the usual requirements of anonymity and neutrality (that is, invariance of the mechanism with respect to the labelling of individuals and issues) but, instead, we require unanimous wills to be implemented.

The *unanimity* condition is a very mild requirement but leads to the central result of this section, an impossibility result. This condition is also known in the social choice literature as a weak form of efficiency; it requires an issue to be approved (alt. dismissed) with certainty when all player wish so.

Definition 1.5 *The SCF* $p : \Theta^I \to \mathcal{X}$ *satisfies the unanimity property if*

$$p_n\left(\theta^1, ..., \theta^I\right) = \begin{cases} 1 & \text{if } \theta_n^i > 0, \forall i = 1 \div I \\ 0 & \text{if } \theta_n^i < 0, \forall i = 1 \div I \end{cases}, \forall n = 1 \div N.$$

We also define a mechanism as being *qualitative* whenever it is sensitive to the voters' intensities of preferences —that is, it implements different allocations when some players vary the intensity of their preferences (but do not vary their wish towards the approval or dismissal of any of the issues). Note, for instance, that MR is not qualitative in the sense that it is only sensitive to the sign of the voters' preferences and it is not sensitive to the particular relative intensities.

Definition 1.6 *The SCF $p : \Theta^I \to \mathcal{X}$ is qualitative if there exists two preference profiles ($\theta, \tilde{\theta} \in \Theta^I$) such that $sign(\theta) = sign\left(\tilde{\theta}\right)$ and $p(\theta) \neq p\left(\tilde{\theta}\right)$.*[38]

The following Lemma shows that without loss of generality we can restrict our attention to those qualitative mechanisms that are sensitive to the intensity of the preference of a particular voter on a single issue. If the SCF was never sensitive to the intensity of a particular voter on a single issue we could construct an iterative process where we would change the valuation of an individual in a specific issue until we reached all possible profiles. The SCF would not change in the whole process thus it will fail to be qualitative.

Lemma 1.2 *If $p : \Theta^I \to \mathcal{X}$ is qualitative then there exists a voter j, an issue m and two preference profiles ($\theta, \tilde{\theta} \in \Theta^I$) such that*

$$\begin{cases} \exists j, m \text{ such that } sign\left(\theta_m^j\right) = sign\left(\tilde{\theta}_m^j\right) \\ \theta_n^i = \tilde{\theta}_n^i, \forall i \neq j, n \neq m \\ p(\theta) \neq p\left(\tilde{\theta}\right) \end{cases}$$

Proof. We know that for any qualitative SCF p there exists $\theta, \tilde{\theta} \in \Theta^I$ such that $sign(\theta) = sign\left(\tilde{\theta}\right)$ and $p(\theta) \neq p\left(\tilde{\theta}\right)$. Now consider an iterative process where change the initial preference profile θ into $\tilde{\theta}$ by varying at each stage a single value.

To illustrate this process we rewrite the preference profiles into the following form:

$$\begin{aligned} \theta &= (a_1, ..., a_{I*N}) \in \mathbb{R}^{I*N} \\ \tilde{\theta} &= (b_1, ..., b_{I*N}) \in \mathbb{R}^{I*N} \end{aligned}$$

Now consider the profile $\varphi_k = (b_1, ..., b_k, a_{k+1}, ..., a_{I*N})$ for $k = 0 \div I * N$.

[38] The operator *sign* should be interpreted as a vector of minus ones, zeros and ones according to the sign of each coordinate.

During the iterative process we compare the allocation achieved by φ_k with the one achieved φ_{k+1}. Given that $p(\theta) = p(\varphi_0) \neq p(\varphi_{I*N}) = p\left(\tilde{\theta}\right)$ we know that during that process the SCF modifies the implemented allocation at least once. In other words, the allocation implemented by p changes when we vary the preference of at least one individual on a single issue. ■

The key intuition of this section lies on the fact that any strategy-proof SCF that satisfies the unanimity property needs to be insensitive to the voters' intensities of preferences on those issues where unanimous wills exist. Alternatively, the SCF can no longer be strategy-proof: any voter has incentives to *save* resources on that issue where unanimous wills exist thus strengthening his position on the remaining issues. Recall that we have seen, as a consequence of Proposition 1.3, that the prospect of approving any issue is weakly increasing on the declaration of any player on that issue ($\frac{\partial P_n(\theta)}{\partial \theta_n^i} \geq 0$). Hence any player is willing to increase his valuation on any particular issue if this has no cost to him.

Lemma 1.3 *If $p : \Theta^I \to \mathcal{X}$ is strategy-proof and satisfies the unanimity property then, whenever $sign\left(\theta_n^i\right) = sign\left(\theta_n^j\right)$ $\forall i, j = 1 \div I$,*

$$\begin{aligned} p\left(\left(\theta_1^i, ..., \theta_n^i, ..., \theta_N^i\right), \theta^{-i}\right) &= p\left(\left(\theta_1^i, ..., \underset{(n)}{x}, ..., \theta_N^i\right), \theta^{-i}\right), \forall x \in \mathbb{R} \\ \textit{such that } sign\left(x\right) &= sign\left(\theta_n^i\right). \end{aligned}$$

Proof. Without loss of generality we assume that at $\tilde{\theta} \in \Theta^I$ all voters wish the approval of issue one, in particular voter i has preferences $\tilde{\theta}_1^i > 0$. Given that there are unanimous wills on issue one, the probability of approving it is one and should not change whenever voter i slightly varies the strength of his preference towards that issue,

$$\left.\frac{\partial p_1(\theta)}{\partial \theta_1^i}\right|_{\theta=\tilde{\theta}} = 0.$$

Moreover, that probability should still remain unchanged whenever voter i varies his declaration on any of the remaining issue. Hence, given Schwarz's Theorem, we have that the next equalities should hold:

$$\left.\frac{\partial p_1(\theta)}{\partial \theta_n^i}\right|_{\theta=\tilde{\theta}} = \left.\frac{\partial p_n(\theta)}{\partial \theta_1^i}\right|_{\theta=\tilde{\theta}} = 0, \forall n = 2 \div N.$$

Therefore $\left.\frac{\partial p(\theta)}{\partial \theta_1^i}\right|_{\theta=\tilde{\theta}} = (0, ..., 0)$ for all $\tilde{\theta}_1^i > 0$. ■

The proof is just using extensively the fact that the probability of approving an issue where unanimous wills exist can not change as long as unanimous wills are in place.

The Lemma implies that whenever unanimous wills exist, the implementability conditions should apply to the remaining declarations. That is, if agents are deciding over N issues and there are unanimous wills on one issue then the implementability conditions should apply to the remaining $N-1$ issues. In particular, if there are two issues and voters unanimously agree on one, no intensities of preferences can be considered at all because the HD0 applies to the single remaining issue.

Consider now a strategy-proof qualitative mechanism that satisfies the unanimity property. It needs to be sensitive to the voters' intensities of preferences for some particular profiles but it cannot be so on those issues where unanimous wills exist. This places a very *asymmetric* restriction on how sensitive to the intensity of preferences the mechanisms are —this is why such mechanisms cannot be strategy-proof. The next example sheds some light on this reasoning.

Example: Imagine a situation with two issues, two voters and a strategy-proof qualitative mechanism. We can find three positive parameters $(a, b, \alpha > 0)$ such that $\theta = ((a,1),(-b,-1))$ and $\tilde{\theta} = ((\alpha,1),(-b,-1))$ implement a different allocation (i.e. $p(\theta) = p\left(\tilde{\theta}\right)$). Without loss of generality assume that $a > \alpha$ thus $p_1(\theta) > p_1\left(\tilde{\theta}\right)$ (hence $p_2(\theta) < p_2\left(\tilde{\theta}\right)$).

Now consider the preference profile $\varphi = ((a,-1),(-b,-1))$. Note that it only changes the sign of player 1's preferences on issue 2. The SCF needs now to satisfy unanimous wills hence $p_2(\varphi) = 0$. The SCF is now left with evaluating the voters' preferences on only one issue, hence it can no longer be sensitive to their intensities. In other words, $p((x,-1),(-y,-1)) = (\bar{p}_1, 0)$ for any $x, y > 0$.

For x large enough and $y = b$ we need $\bar{p}_1$ to be at least as high as $p_1(\theta)$ for the SCF to be strategy-proof —i.e. $\bar{p}_1 \geq p_1(\theta)$. Instead, for y large enough and $x = \alpha$ we need $\bar{p}_1$ to be at least as low as $p_1\left(\tilde{\theta}\right)$ for the SCF to be strategy-proof —i.e. $\bar{p}_1 \leq p_1\left(\tilde{\theta}\right)$.

Indeed, the condition $p_1(\theta) > p_1\left(\tilde{\theta}\right)$ tells us that the former two conditions cannot be satisfied.

The proof of the following (impossibility) Theorem is basically an extension of the former example to the general case with I voters and N issues.

Theorem 1.4 *There exists no strategy-proof qualitative SCF that satisfies the unanimity* *property.*

Proof. We prove the Theorem by induction on the number of issues.

When $N = 1$ there exists no strategy-proof qualitative mechanism (note that the unidimensional case makes no use of the unanimity property).

Suppose now that there exists no strategy-proof qualitative mechanism for $N = k$ and, instead, suppose the opposite for $N = k + 1$.[39] That is, imagine that there exists a strategy-proof mechanism that satisfies the unanimity property and is sensitive to the voters intensity of preferences in the $(k+1)$-dimensional case:

$$\exists \theta, \varphi \in \Theta \text{ such that } sign(\theta) = sign(\varphi) \text{ and } p(\theta) \neq p(\varphi)$$

where no unanimous wills are present in any of the issues of θ or $\tilde{\theta}$.

By Lemma 1.2 we can assume without loss of generality that θ and φ differ only on the valuation of voter one in the first issue. That is,

$$\begin{cases} \theta = \left(\left(a, \theta_2^1, ..., \theta_N^1\right), ..., \theta^I\right) \\ \varphi = \left(\left(b, \theta_2^1, ..., \theta_N^1\right), ..., \theta^I\right) \end{cases}$$

where $a > b \geq 0$. Strategy-proofness implies that $p_1(\theta) > p_1(\varphi)$.

Define the set of voters that wish the dismissal of the first issue as $J := \left\{j \in I : \theta_1^j < 0\right\} \subsetneq I$. This set is not empty given that there are no unanimous wills. Denote $k := \#J$ and $J = \{j_1, ..., j_k\}$.[40]

We consider now an iterative process on the elements of J similar to the one described in the proof of Lemma 1.2 where we sequentially switch the negative valuations towards the first issue of voters in J into neutral valuations.

Define the following preference profiles where voter j_1's preference towards the first issue is set to zero:

$$\begin{cases} \bar{\theta} = \left(\left(a, \theta_2^1, ..., \theta_N^1\right), ..., \left(0, \theta_2^{j_1}, ..., \theta_N^{j_1}\right), ..., \theta^I\right) \\ \bar{\varphi} = \left(\left(b, \theta_2^1, ..., \theta_N^1\right), ..., \left(0, \theta_2^{j_1}, ..., \theta_N^{j_1}\right), ..., \theta^I\right). \end{cases}$$

Strategy-proofness implies that $p_1\left(\bar{\theta}\right) \geq p_1(\theta)$ and $p_1(\bar{\varphi}) \geq p_1(\varphi)$. Three things can

[39] The remaining of the proof consists on showing that the proposed SCF cannot be qualitative and strategy proof and satisfy the unanimity property when $N = k + 1$. Thus by contradiction we show that the inductive argument is satisfied.

[40] The sign # denotes the cardinality of a set, the number of elements the set contains.

happen: $p_1(\bar{\theta}) > p_1(\bar{\varphi})$, $p_1(\bar{\theta}) = p_1(\bar{\varphi}) < 1$ and $p_1(\bar{\theta}) = p_1(\bar{\varphi}) = 1$.

If $p_1(\bar{\theta}) > p_1(\bar{\varphi})$ or $p_1(\bar{\theta}) = p_1(\bar{\varphi}) < 1$ we move into the second stage of the iterative process now with j_2 and starting from the resulting preference profiles $\bar{\theta}$ and $\bar{\varphi}$ from the precedent stage. We keep on repeating the process until we reach the third possible scenario where $p_1(\bar{\theta}) = p_1(\bar{\varphi}) = 1$. The unanimity property ensures that such allocation is achieved and the process should end in at most k ($k < \infty$) stages (say it ends in stage κ).

The fact that $p_1(\theta) > p_1(\varphi)$ implies that $p_1(\bar{\varphi}) > p_1(\varphi)$. Moreover, the inductive hypothesis implies the allocation achieved by $\bar{\theta}$ and $\bar{\varphi}$ coincide with the allocation where the first issue is approved and the intensity of preferences are not taken into account. This is immediate if $k = \kappa$. Instead, if $k < \kappa$ we can finish our iterative process by switching the preference of all voters that wish the dismissal on the first issue and hence end up in a situation where unanimous wills towards the first issue exist (note that the SCF does not change in this process). Hence, by Lemma 1.3 the achieved allocation is equivalent to one where the dimensionality is reduced in one and hence the inductive hypothesis applies and the SCF can only consider the sign of the preferences.

We now show that there exists a particular preference profile of voter j_κ for which truthtelling cannot be an equilibria.

Once again, strategy-proofness implies that $p_1(\bar{\varphi}) > p_1(\varphi)$ and that for least one $m \in \{2, ..., N\}$, one of the two following conditions holds

$$\begin{cases} p_m(\bar{\varphi}) < p_m(\varphi) \text{ and } \theta_m^{j_\kappa} > 0 \\ p_m(\bar{\varphi}) > p_m(\varphi) \text{ and } \theta_m^{j_\kappa} < 0. \end{cases}$$

Imagine $p_m(\bar{\varphi}) < p_m(\varphi)$ and $\theta_m^{j_\kappa} > 0$. We can find preference profiles for voter j_κ for which he has no incentives to tell the truth.

Given that the allocation achieved by $\bar{\varphi}$ does not take into account the intensity of the preferences, for any profile of voter j_κ such that the sign of his preferences does not change the SCF implements the same allocation. In particular, the SCF should implement the same allocation for any positive value of voter j_κ's preference on issue m. For a big enough value it cannot be optimal to tell the truth given that $p_m(\bar{\varphi}) < p_m(\varphi)$.

An analogous argument applies whenever $p_m(\bar{\varphi}) > p_m(\varphi)$ and $\theta_m^{j_\kappa} < 0$. ∎

We have seen that any mechanism that is robust to any possible specification on the priors and satisfies the unanimity property cannot take into account the intensity of the voters preferences. This impossibility result is consistent with the social choice literature and, in particular, the Gibbard-Satterthwhaite (G-S) Theorem where we see that the strategic interactions between individuals do not allow to propose mechanisms that are implementable in dominant strategies and satisfy some appealing properties.

A version of the G-S Theorem states that in an election with three or more outcomes and where we assume a universal domain in the voters' preferences, the only strategy-proof and onto SCFs are dictatorial. This result is more restricting than ours because we only claim the impossibility of implementing qualitative mechanisms. The reason why we get a distinct result is because we are implicitly restricting the domain of preferences. This is best captured by considering the following example. Consider a voting game with two issues and map all possible outcomes into the G-S framework with four alternatives:

$$\begin{cases} \text{Alternative } A \text{ is defined as approving both issues} \\ \text{Alternative } B \text{ is defined as approving the first issue and denying the second.} \\ \text{Alternative } C \text{ is defined as denying the first issue and approving the second.} \\ \text{Alternative } D \text{ is defined as denying both issues} \end{cases}$$

Clearly, not all strict preferences can be assumed in the set of outcomes $\{A, B, C, D\}$ —for example, the strict preference $A \succ D \succ B \succ C$ can never be observed. Most related to our work, Hylland (1980) proves that even in the case of *cardinal* and unrestricted preference profiles the random dictatorship was the only strategy-proof mechanism that satisfied the unanimity property.

Given the impossibility of implementing qualitative mechanisms that are strategy-proof and satisfy the unanimity property we are left with the question of which mechanism may be optimal. May's Theorem (1952) could be extended to our setting if we added a stronger condition than unanimity, namely, *positive responsiveness*.[41] In that case we obtain that the only SCF that is strategy-proof, anonymous, neutral and positive responsive is MR.

In the spirit of the literature following Arrow's Impossibility result we should identify ways to overcome our result. One way to get through this result is relaxing the equilibrium criteria from dominant strategies to Bayesian Nash. We have seen in Section 1.3 that this particular line of research, together with an appropriate restriction on the domain of preferences, proves to be very successful and we can characterise some situations where there exists a very simple mechanism (QV) that allows the expression of interest by the voters and is not only superior to the MR but also optimal.

[41] "By this [*positive responsive*] we mean that if the group decision is indifference or favorable to x, and if the individual preferences remain the same except that a single individual changes in a way favorable to x, then the group decision becomes favorable to x." May (1952) pg 682.

The second way to provide some positive results consists on dropping the unanimity requirement. This immediately implies that we are not able to achieve ex-post efficiency (regardless of Incentive Compatibility), yet we are able to characterise an infinite set of strategy-proof SCFs. The next subsection develops this approach.

1.4.3 A way to overcome the impossibility result

We restrict our analysis to the two voters case for ease of presentation. At the end of the section we show how our main result extends to the general case with I voters.

As indicated above, we now drop the unanimity requirement to avoid the impossibility result. Equivalently we could restrict the domain of preferences such that there are no unanimous wills or assume that there has been a previous stage were all unanimous wills have been implemented.

In order to proceed in a meaningful way and avoid trivial mechanisms such as constant or dictatorial ones we have to require further conditions. Recall that the neutrality condition requires no systematic tendency towards the approval or dismissal of any of the issues $(p_n(\theta^1, \theta^2) = 1 - p_n(-\theta^1, -\theta^2), \forall n = 1 \div N)$. In the two voters case, any neutral qualitative mechanism satisfies a rationality constraint in the sense that it never does worst than MR.[42] The proof of the latter result follows from the implementability conditions:

$$u(\theta^i, \theta) \geq u\left(\hat{\theta}^i, \theta\right) = P\left(\hat{\theta}^i, \theta^j\right) \cdot \theta^i, \forall \hat{\theta}^i, \theta^i, \theta^j \in \Theta.$$

Whenever voter i declares the opposite preference than voter j $(-\theta^j)$, neutrality together with anonimity imply that the SCF should implement the MR outcome. Thus, for every possible preference profile the allocation implemented by truthfully reporting the type is at least as good as the one achieved by MR.

From the observation above we know that the utility induced by any neutral and implementable SCF should be non-negative since MR never generates a negative payoff. We also know that any implementable SCF generates utilities that are convex and HD1. Hence any neutral and strategy-proof SCF induces a seminorm in the space of preferences of each voter.[43] In other words, the indirect utilities are functions defined in the space of preferences of each voter that satisfy the nonnegative property $(u(\theta^i, (\theta^i, \theta^j)) \geq 0)$, the

[42] In the two players scenario MR is analogous to Unanimity: it implements unanimous will when they are in place and ties issues when players have opposing views.

[43] A seminorm on a real vector space V is a function p from V to the non-negative real numbers satisfying the scaling property $(p(ax) = |a| p(x))$ and the subadditive property $(p(x+y) \leq p(x) + p(y))$.

scaling property $\left(u\left(a\cdot\theta^{i},\left(a\cdot\theta^{i},\theta^{j}\right)\right)=|a|\cdot u\left(\theta^{i},\left(\theta^{i},\theta^{j}\right)\right)\right)$ and the triangular (or subaditive) property $\left(u\left(\theta^{i},\left(\theta^{i},\theta^{j}\right)\right)+u\left(\varphi^{i},\left(\varphi^{i},\theta^{j}\right)\right)\geq u\left(\theta^{i}+\varphi^{i},\left(\theta^{i}+\varphi^{i},\theta^{j}\right)\right)\right)$. Hence the pair $\left(\Theta,u\left(\cdot,\left(\cdot,\theta^{j}\right)\right)\right)$ is a seminormed space for all $\theta^{j}\in\Theta$ (and for all $i,j\in\{1,2\},i\neq j$). Guided by this idea and the fact that the implementable SCFs should have a well defined structure as suggested by Schwarz's Theorem, we can propose the following infinite countable set of implementable SCFs:

$$P_n\left(\theta^1,\theta^2\right)=\frac{1}{2}\left(\left(\frac{\theta_n^1}{\left\|\theta^1\right\|_k}\right)^{k-1}+\left(\frac{\theta_n^2}{\left\|\theta^2\right\|_k}\right)^{k-1}\right)$$

where k is any positive even number and $\|\cdot\|_k$ denotes the usual k-norm.[44] The prospect of approving any issue is well defined between -1 and 1. We can extend this set for any real number k greater than one just by carefully adapting the previous formula to avoid complex solutions:

$$P_n\left(\theta^1,\theta^2\right)=\frac{1}{2}\left(sign(\theta_n^1)\cdot\left(\frac{\left|\theta_n^1\right|}{\left\|\theta^1\right\|_k}\right)^{k-1}+sign(\theta_n^2)\cdot\left(\frac{\left|\theta_n^2\right|}{\left\|\theta^2\right\|_k}\right)^{k-1}\right),k>1. \quad (1.2)$$

Besides strategy-proofness, this infinite uncountable set of strategy-proof functions satisfy some appealing properties such as neutrality, anonymity, symmetry across issues and neutrality across issues.[45] It is also interesting to observe that the SCF tends to the MR outcome whenever we let $k\to 1$ —in that case, the exponent $(k-1)$ tends to zero thus tends to put equal weight on all issues and voters. Instead, whenever $k\to\infty$, the SCF tends to be almost equivalent to a SCF that requires voters to rank issues and only uses the information about the highest ranked issue —that is, puts a weight equal to one on the most preferred issue of any voter and zero on the remaining issues.

There are still a couple of properties worth mentioning. On the one hand, the defined SCFs are *ex-post incentive efficient* in the sense of Holmstrom and Myerson (1983). That is, there is no implementable SCF that makes some voters better off without worsening off some other voters.[46]

On the other hand, we know that any linear combination of strategy-proof mechanisms is also strategy-proof. Moreover, any convex combination of SCFs characterised by (1.2) is also neutral, anonymous, symmetric across issues, neutral across issues and ex-post

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[44] The k-norm on $\mathbb{R}^N$ is defined for any real number $k\geq 1$ as follows: $\|x\|_k=\left(|x_1|^k+...+|x_N|^k\right)^{\frac{1}{k}}$.

[45] See Definition 1.1 for a precise definition of these terms.

[46] Note that the concept of ex-post incentive efficiency differs from the one of ex-post efficiency. As was highlighted above, the sole fact of not satisfying the unanimity property implies that ex-post efficiency can not be achieved.

incentive efficient. This leads us to conjecture that the whole set of SCFs satisfying the previous properties is described by (1.2).[47]

In the alternative in which we restrict the set of preferences such that no unanimous wills exist, we drop the coefficient $\frac{1}{2}$ in front of (1.2) given that the value of the term inside the brackets is now included in $[-1,1]$ instead of $[-2,2]$.[48] Finally the analysis extends immediately to a setting with I voters defining:

$$P_n\left(\theta^1,...,\theta^I\right) = \left(sign(\theta_n^1)\cdot\left(\frac{|\theta_n^1|}{\|\theta^1\|_k}\right)^{k-1} + ... + sign(\theta_n^I)\cdot\left(\frac{|\theta_n^I|}{\|\theta I\|_k}\right)^{k-1}\right), k > 1.$$

1.5 Conclusion

In the first half of this Chapter we have proposed an alternative to the usual voting rule which is simple and allows voters to express their *willingness to influence.* A mechanism which seems the most natural extension to MR and that is proved to be not only superior to MR but also a mechanism that achieves the best possible allocation and induces truthful revelation of the voters' preferences in some general settings. Its essence relies on almost allowing for transferable utilities without introducing money; players can freely move their voting power across issues to strengthen their position in some issues. Following our initial quote we have extended the use of purely economic concepts into the political system. Yet, the results in the second half of the Chapter show that such development is not exempt from difficulties and we have shown that it is impossible to allow the willingness to influence to play a role in general settings where unanimity is satisfied. James Coleman best captured, once again, the rationale behind our reasoning:

> *"Clearly a system of power that was parallel in all respects to a monetary system could not be devised, because of the different nature of private goods and public policies. Yet it is equally absurd to believe, as the lack of political innovations seems to imply, that political power must be as different from economic power in its organizations as is the case in existing political systems."*[49]

The main findings of this Chapter can be summarised through its four theorems: (1) QV unlocks conflict resolution situations allowing each of the opponents to trade off their

[47] We have found no counterexample nor formal proof of such statement.

[48] In the two voters' case, this restriction corresponds to requiring the two voters to have opposing preferences. In this case, QV replicates the allocation of the provided SCF in (1.2) when $k \to \infty$. In particular, QV is strategy-proof and ex-post incentive efficient.

[49] Coleman (1970) pg 1082.

voting power between the various divergent issues; (2) in a situation with more than two voters, QV allows *very* enthusiastic minorities to decide on those issues that the majorities are mostly indifferent towards; (3) whenever a public decision has to be made and no transfers are allowed, only the agents' relative intensities between the issues can be considered; and (4) there exists no mechanism that sensitive to the voters' intensity of the preferences, satisfies the unanimity property and is *robust* to any specification of the priors.

The driving force of our results and our main contribution to the existing literature relies on forbidding any kind of transfers between voters. This has been assumed on the grounds of an *equal* argument so that no endowment effects can ever play a role in voting games. Furthermore, we have extended such a concept when analysing QV by imposing the *anonymity* property: any aggregating device should not benefit any particular individual. Finally, departing from these axiomatic properties we derived an equilibrium result (Theorem 1.3) that concludes that a further condition of *equality* has to be satisfied: no direct interpersonal comparison of utility can be undertaken. It is not solely because a voter values one issue more strongly than another voter that he should be given more voting power on it. In other words, *preference endowments* should not play a role either.

Precisely, the *equality* argument in the three forms expressed above is crucial to ensure the stability of any aggregating mechanism as it is stated in the following quote:

> "I do not believe, and I never have believed, that in fact men are necessarily equal or should always be judged as such. But I do believe that, in most cases, political calculations which do not treat them *as if* they were equal are morally revolting."[50]

Likewise, Dahl (1956) endorses the view that intensity of the voters' preferences should be taken into account in order to ensure the stability of political institutions.

Given how the complexity of the problem escalates when we consider more general settings, we are actually working on experimenting with QV in a more complex setting with diverse issues and voters to realise how people may react to different information structures.[51] It seems sensible to expect that, the more issues or voters, the more dispersed the information about the opponents' preferences will be. Consequently, similar results to the ones stated in this Chapter should follow. This is congruent with the notion that voters

[50] Robbins (1938) pg 635.

[51] See the following Chapter..

may not be able to react rationally to some complex situations given their lack of time, knowledge or aptitude to do so. Hence we may observe a less strategic misrepresentation of preferences and voters may use their private information almost truthfully. Be that as it may, this and further considerations are analysed in Chapter 2.

QV also introduces a new ingredient in the debate around the institutional adjustment that should occur in an extended European Union (EU). "There is a widespread conviction that the system established by the Treaty of Rome cannot function effectively in a Union of 25 to 30 members"[52] . *Logrolling* is a common feature in the present EU where countries (almost publicly) exchange their votes depending on the issue at hand. In an enlarged EU such system can not be efficient and QV could be the answer. Countries would be free to intensify their votes regardless of side agreements and, hence, obscure side payments.

[52] http://europa.eu.int

1.6 Appendix

Proof of Part 1 of Lemma 1.1.

Given the uniform and independent priors we can restrict our attention without loss of generality to voters with positive preferences.

Assume that there is an equilibrium where non-indifferent voters use different strategies. That is, where a voter that prefers the first issue invests v on his preferred issue and a voter that prefers the second issue invests w on his preferred issue ($v \neq w$). Finally, an indifferent voter invests v_i on the first issue (without loss of generality we assume that $v_i \geq \frac{V}{2}$.). Once again, the described priors imply that $v, w \geq \frac{V}{2}$ and $v \geq v_i, w \geq V - v_i$. We now show that the described equilibrium cannot be so because an indifferent voter always has incentives to deviate.

Any voter can face thirty six possible situations on each issue depending on the strategy played by both his opponents. In some situations the votes cast by his opponents are higher or equal than zero in which case, regardless of his strategy, the issue is approved. Similarly, if the invested votes are smaller or equal than $-V$ the issue is dismissed. The table below depicts such situations with a positive and negative sign, respectively. The remaining cells capture the total number of votes cast by voters two and three:

ISSUE 1

v	$+$	$+$	$+$	$+$	$+$	$+$
v_i	$v_i - v$	$+$	$+$	$+$	$+$	$+$
$(V-w)$	$V - v - w$	$V - w - v_i$	$+$	$+$	$+$	$+$
$-(V-w)$	$-V + w - v$	$-V + w - v_i$	$-2(V-w)$	$+$	$+$	$+$
$-v_i$	$-$	$-$	$-V + w - v_i$	$V - w - v_i$	$+$	$+$
$-v$	$-$	$-$	$-V + w - v$	$V - v - w$	$v_i - v$	$+$
	$-v$	$-v_i$	$-(V-w)$	$(V-w)$	v_i	v

ISSUE 2

w	$+$	$+$	$+$	$+$	$+$	$+$
$(V-v_i)$	$V - w - v_i$	$+$	$+$	$+$	$+$	$+$
$(V-v)$	$V - w - v$	$v_i - v$	$+$	$+$	$+$	$+$
$-(V-v)$	$-V + v - w$	$-2V + v + v_i$	$-2(V-v)$	$+$	$+$	$+$
$-(V-v_i)$	$-V + v_i - w$	$-2(V-v_i)$	$-2V + v + v_i$	$v_i - v$	$+$	$+$
$-w$	$-$	$-V + v_i - w$	$-V + v - w$	$V - v - w$	$V - w - v_i$	$+$
	$-w$	$-(V-v_i)$	$-(V-v)$	$(V-v)$	$(V-v_i)$	w

We can now compute the final allocation in each possible situation whenever voter one follows the three possible strategies. That is, whenever he invests $(v_i, V - v_i)$, $(v, V - v)$ or $(V - w, w)$. In order to compute the expected interim payoffs we define the following parameters:

$$\begin{array}{lll} a = 1 & \Leftrightarrow & V - v - w + v_i \geq 0 \\ a = -1 & \Leftrightarrow & V - v - w + v_i < 0 \end{array} \qquad \begin{array}{lll} b = 1 & \Leftrightarrow & -2V + 2w + v_i > 0 \\ b = -1 & \Leftrightarrow & -2V + 2w + v_i \leq 0 \end{array}$$

$$\begin{array}{lll} c = 1 & \Leftrightarrow & 2V - w - 2v_i \geq 0 \\ c = -1 & \Leftrightarrow & 2V - w - 2v_i < 0 \end{array} \qquad \begin{array}{lll} d = 1 & \Leftrightarrow & 2V - v - w - v_i \geq 0 \\ d = -1 & \Leftrightarrow & 2V - v - w - v_i < 0 \end{array}$$

$$\begin{array}{lll} e = 1 & \Leftrightarrow & -V + 2v - v_i > 0 \\ e = -1 & \Leftrightarrow & -V + 2v - v_i \leq 0 \end{array} \qquad \begin{array}{lll} B = 1 & \Leftrightarrow & -2V + v + 2w > 0 \\ B = -1 & \Leftrightarrow & -2V + v + 2w \leq 0 \end{array}$$

$$\begin{array}{lll} C = 1 & \Leftrightarrow & 2V - 2v - w \geq 0 \\ C = -1 & \Leftrightarrow & 2V - 2v - w < 0 \end{array}$$

Weighting each possible situation by its probability[53] we have that the expected payoffs when playing the three possible strategies are

$$\begin{cases} \Pi(v_i, V - v_i) \cdot 64 = 58 + 2a + b + 4c + 2d + e \\ \Pi(v, V - v) \cdot 64 = 60 - 4a + 4d - 3e + B + 2C \\ \text{and} \\ \Pi(w, V - w) \cdot 64 = 63 + 4a - 4b - 4c - 4d - 2B - C \end{cases}$$

Now we just need to consider all possible combinations of parameters to realise whether it is strictly better to deviate. The following inequalities show that not all parameter combinations are possible[54]

$$\underbrace{V - v - w + v_i}_{a} \geq \underbrace{2V - 2v - w}_{C} \geq \underbrace{2V - v - w - v_i}_{d} \geq \underbrace{2V - w - 2v_i}_{c}$$

$$\underbrace{-2V + v + 2w}_{B} \geq \underbrace{-2V + 2w + v_i}_{b}$$

Whenever $a = -1$ an indifferent voter is strictly better off by playing $(v, V - v)$. Hence, for the proposed strategies to be an equilibrium a should be equal to one.

[53] Given the uniform and independent priors, all columns (alternatively rows) occur with probability $\frac{1}{8}$ except columns two and five which occur with probability $\frac{1}{4}$.

[54] For instance, $a = -1 \Longrightarrow C = d = c = -1$.

Repeating the previous reasoning for $d = -1$ we can also see that an indifferent voter has incentives to deviate by playing $(V - w, w)$. Thus, $C = d = 1$.

Now assume that $c = -1$. In that case, the expected interim payoffs are equal to $\Pi(v_i, V - v_i) \cdot 64 = 58 + b + e$ and $\Pi(w, V - w) \cdot 64 = 66 - 4b - 2B$. Note that it is not strictly better to deviate only when $b = e = B = 1$. It can be easily shown that $d = 1$ and $b = 1$ imply that $w > v$, but $d = 1$ and $e = 1$ imply that $V + v - w - 2v_i > 0$. The latter inequality cannot hold when $w > v$. Hence, in equilibrium, $a = C = d = c = 1$.

Suppose now that $B = 1$. First note that in that situation e should be equal to -1 because $e = 1$ implies (together with $d = 1$) that $w > v$ and this is not compatible with $V + v - w - 2v_i > 0$ (this inequality results from combining $C = 1$ and $B = 1$). Thus $e = -1$. Nevertheless, in that situation a non-indifferent voter that prefers issue 2 is better off by deviating and playing $(v_i, V - v_i)$. Hence, in equilibrium $B = b = -1$.

$e = 1$ implies, as before, that a non-indifferent voter that prefers issue 2 is better off by deviating and playing $(v_i, V - v_i)$. And finally, $e = -1$ achieves an allocation which is identical to have all voters splitting evenly their voting power (hence it is essentially a situation where $v = w = v_i$ and all values are close enough to $\frac{V}{2}$, i.e. $2V - 3v \geq 0$) ■

Proof of Part 2 of Lemma 1.1.

The proof is analogous to the previous one. Assume that there is an equilibrium (v, v_i) such that indifferent voters do not evenly split their voting power. That is, such that it reaches a different allocation to $\left(v, \frac{V}{2}\right)$. Without loss of generality we assume that $v_i > \frac{V}{2}$. Given that the only equilibrium with $v = v_i$ is $\left(\frac{V}{2}, \frac{V}{2}\right)$ we have that $v > v_i > \frac{V}{2}$. As before, the uniform and independent priors allow us to do our analysis from the perspective of voter one and we assume that he has positive preferences (that is, he desires the approval of both issues).

The table below depicts the thirty six possible situations that a voter can face on each issue depending on the strategy played by both his opponents.

ISSUE 1

v	$+$	$+$	$+$	$+$	$+$	$+$
v_i	$v_i - v$	$+$	$+$	$+$	$+$	$+$
$(V - v)$	$V - 2v$	$V - v - v_i$	$+$	$+$	$+$	$+$
$-(V - v)$	$-$	$-V + v - v_i$	$-2(V - v)$	$+$	$+$	$+$
$-v_i$	$-$	$-$	$-V + v - v_i$	$V - v - v_i$	$+$	$+$
$-v$	$-$	$-$	$-$	$V - 2v$	$v_i - v$	$+$
	$-v$	$-v_i$	$-(V - v)$	$(V - v)$	v_i	v

ISSUE 2

v	$+$	$+$	$+$	$+$	$+$	$+$
$(V - v_i)$	$V - v - v_i$	$+$	$+$	$+$	$+$	$+$
$(V - v)$	$V - 2v$	$v_i - v$	$+$	$+$	$+$	$+$
$-(V - v)$	$-$	$-2V + v + v_i$	$-2(V - v)$	$+$	$+$	$+$
$-(V - v_i)$	$-$	$-2(V - v_i)$	$-2V + v + v_i$	$v_i - v$	$+$	$+$
$-v$	$-$	$-$	$-$	$V - 2v$	$V - v - v_i$	$+$
	$-v$	$-(V - v_i)$	$-(V - v)$	$(V - v)$	$(V - v_i)$	v

We can now compute the final allocation in each possible situation whenever voter one follows the proposed strategy and whenever he unilaterally deviates and invests $(V - v_i)$ votes in the first issue. As noted in the main text, we want to consider a deviation where voter one, realising that both his opponents invest more voting power on the first issue, deviates and casts more votes on the second one. Furthermore, the considered deviation does not change his payoff when he faces non-indifferent voters. In order to compute the expected interim payoffs we define the following parameters:

$$\begin{aligned} a = 1 &\Leftrightarrow V - 2v + v_i \geq 0 \qquad & c = 1 &\Leftrightarrow 2V - v - 2v_i \geq 0 \\ a = -1 &\Leftrightarrow V - 2v + v_i < 0 \qquad & c = -1 &\Leftrightarrow 2V - v - 2v_i < 0 \end{aligned}$$

$$\begin{aligned} b = 1 &\Leftrightarrow -2V + 2v + v_i > 0 \qquad & d = 1 &\Leftrightarrow -2V + 3v_i \leq 0 \\ b = -1 &\Leftrightarrow -2V + 2v + v_i \leq 0 \qquad & d = -1 &\Leftrightarrow -2V + 3v_i > 0 \end{aligned}$$

Weighting each possible situation by its probability we have that the expected payoffs when non-deviating and deviating are respectively

$$\begin{cases} \Pi := \frac{1}{64}\left[(27-2b+4c-a)+(31+2a+b)\right] \\ \text{and} \\ \Pi_d := \Pi + \frac{1}{64}\left[8-4c+4d\right]. \end{cases}$$

Now we just need to consider all possible combinations of parameters to realise whether it is strictly better to deviate.

Whenever $d=1$ or $c=-1$ it is strictly better to deviate. Instead, when $d=-1$ and $c=1$ both strategies yield the same expected payoff. Nevertheless in that situation some of the hypotheses are violated (for $b=-a=1$ and $b=-a=-1$, (v,v_i) is *essentially* equal to $\left(v,\frac{V}{2}\right)$; instead, when $a=b=1$, (v,v_i) does not constitute an equilibrium because a non-indifferent voter that prefers issue one is strictly better off playing v_i votes on the first issue; finally, the case $a=b=-1$ can never happen). ∎

Proof of Proposition 1.1.

Given that indifferent voters invest $\frac{V}{2}$ votes in each issue we have that all possible combinations of cast votes in any of the issues by two voters that follow the strategy $\left(v,\frac{V}{2}\right)$ are depicted in the matrix below:

v	$+$	$+$	$+$	$+$	$+$	$+$
$\frac{V}{2}$	$\frac{V}{2}-v$	$+$	$+$	$+$	$+$	$+$
$(V-v)$	$V-2v$	$\frac{V}{2}-v$	$+$	$+$	$+$	$+$
$-(V-v)$	$-$	$-\frac{3}{2}V+v$	$-2(V-v)$	$+$	$+$	$+$
$-\frac{V}{2}$	$-$	$-$	$-\frac{3}{2}V+v$	$\frac{V}{2}-v$	$+$	$+$
$-v$	$-$	$-$	$-$	$V-2v$	$\frac{V}{2}-v$	$+$
	$-v$	$-\frac{V}{2}$	$-(V-v)$	$(V-v)$	$\frac{V}{2}$	v

As we did before, we define the following four parameters:

$$\begin{array}{llll} a=1 \Leftrightarrow \bar{v}\geq 2v-V & & c=1 \Leftrightarrow \bar{v}>\frac{3}{2}V-v \\ a=-1 \Leftrightarrow \bar{v}<2v-V & & c=-1 \Leftrightarrow \bar{v}\leq\frac{3}{2}V-v \end{array}$$

$$\begin{array}{llll} b=1 \Leftrightarrow \bar{v}\geq v-\frac{V}{2} & & d=1 \Leftrightarrow \bar{v}>2V-2v \\ b=-1 \Leftrightarrow \bar{v}<v-\frac{V}{2} & & d=-1 \Leftrightarrow \bar{v}\leq 2V-2v \end{array}$$

where $\bar{v}$ indicates the number of votes invested in issue one by the remaining voter. Without loss of generality we assume that this voter has positive preferences and strictly prefers the first issue.

$\left(v, \frac{V}{2}\right)$ is an equilibrium if and only if it is optimal for the remaining voter to invest exactly v votes on the first issue (that is, $\bar{v} = v$ should be optimal).

The way to proceed is to define all possible cases so that the conditions that define the four parameters are well ordered. For instance, whenever $v > \frac{5}{6}V$ we have that $0 \leq 2V - 2v \leq v - \frac{V}{2} \leq \frac{3}{2}V - v \leq 2v - V \leq V$ and it can easily be shown that $\bar{v} = v$ is an optimal response for voter one. Hence, $\left(v, \frac{V}{2}\right)$ is a symmetric equilibrium as long as $v \in \left(\frac{5}{6}V, V\right]$. This set of equilibria are *essentially* identical to $\left(V, \frac{V}{2}\right)$.

A further analysis shows that there exists no symmetric equilibrium where $v \in \left(\frac{3}{4}V, \frac{5}{6}V\right]$. The case in which $v = \frac{3}{4}V$ implies that $0 < v - \frac{V}{2} < 2V - 2v = 2v - V < \frac{3}{2}V - v < V$ and a symmetric equilibrium can be sustained if and only if $\theta = \frac{1}{2}$. If $\theta < \frac{1}{2}$, voter one prefers investing more voting power on his preferred issue and, inversely, he prefers to split his votes more equally whenever $\theta > \frac{1}{2}$. Hence we conclude that $\left(\frac{3}{4}V, \frac{V}{2}\right)$ is an equilibrium if and only if $\theta = \frac{1}{2}$. Moreover, that equilibrium can be sustained by any $v \in \left(\frac{2}{3}V, \frac{3}{4}V\right]$.

Finally, $v \in \left(\frac{V}{2}, \frac{2}{3}V\right)$ can constitute a symmetric equilibrium only when $\theta \geq \frac{1}{2}$; when $\theta < \frac{1}{2}$, a non-indifferent voter knows that by deviating and investing all of his voting power on his preferred issue he gains that issue when he is confronted with an indifferent voter and a low one (instead he loses it if he invests v votes). This equilibrium reaches the same allocation as MR. In fact, $\left(\frac{V}{2}, \frac{V}{2}\right)$ is clearly an equilibrium for any θ because any voter is equally pivotal with any number of votes (in particular with $v = \frac{V}{2}$). ■

Proof of Theorem 1.1.

Any direct mechanism is defined by 512 parameters. That is, all possible combinations of both voters' types multiplied by the number of issues we are considering. Restricting the analysis to *reasonable* SCFs renders the problem tractable and simplifies the analysis into six parameters; we need to define the SCF only on a particular issue when both voters' preferences on that issue are opposed and this can be done regardless of the sign of the remaining issue.

More precisely, the *neutrality* property defines the value of the SCF whenever voters have analogous preferences (that is, whenever both voters coincide on how strongly they prefer each issue) and allows us to focus on positively valued issues (when the agent we analyse wants the approval of the issue). The *symmetry across issues* allows us to focus on a particular issue (say, issue one) and the *neutrality across issues* property reduces the possible types we have to analyse to four because the SCF has to be invariant with respect to the sign of the remaining issue. Finally, *unanimity* implies that we only have to consider the cases when the opponent wants the dismissal of issue one. The next table depicts the six parameters that uniquely define any SCF given the properties above:

$(1,\theta)$	$\frac{1}{2}$	A	B	C
$(\theta,1)$		$\frac{1}{2}$	D	E
$(1,1)$			$\frac{1}{2}$	F
(θ,θ)				$\frac{1}{2}$
	$(-1,\theta)$	$(-\theta,1)$	$(-1,1)$	$(-\theta,\theta)$

Note that these parameters are probabilities of approving an issue, hence they lie in the interval $[0,1]$.

We define the interim prospects given the four possible declarations as $P(1,\theta), P(\theta,1)$, $P(1,1)$ and $P(\theta,\theta)$. For instance,

$$P(1,\theta) = 2\left\{E_{\tilde{\theta}}\left(p\left((1,\theta),\left(\tilde{\theta}\right)\right)\right)\right\} - 1 = 2\cdot\frac{1}{8}\left\{\frac{1}{2}+A+B+C+4\right\} - 1.$$

The optimal (*reasonable* and ex-ante efficient) SCF is the one that maximises the ex-ante utility subject to the truthtelling constraints (Proposition 1.2) and the feasibility ones (the six parameters need to belong to the interval $[0,1]$). The program reads as follows

$$\max_{A,B,C,D,E,F\in[0,1]} u^i(p) = 8\left[3+A+C-D+F+(4-A-B+E+F)\theta\right]$$

$$\text{subject to} \begin{cases} \mathbf{1.} & -B+C-D+E+2F-1=0 \\ \mathbf{2.} & 2A+B+C-D-E-1\geq 0 \\ \mathbf{3.} & -6B-2C-6D-2E+4F+6\geq 0 \\ \mathbf{4.} & -A-2B-C-D+F+2+(A-B-2D-E+F+1)\theta\leq 0 \end{cases}$$

Solving this linear program we get that $A=C=B=1, D=E=0$ and $F=1/2$. Note that this allocation is the same than the one achieved by QV, hence QV is optimal.[55] ■

[55] Note that IC implies that players that are indifferent between the issues should be treated analogously at the interim stage whether they hold strong or weak preferences. We have now proved that this is not only the case at the interim stage but also at the ex-post stage. In other words, the optimal implementable SCF does not undertake ex-post interpersonal comparisons of utility.

Proof of Theorem 1.2.

Any direct mechanism is now defined by 8192 parameters. Restricting the analysis to *reasonable* SCFs renders the problem tractable and simplifies the analysis into 44 parameters belonging to the interval $[0, 1]$. The following tables define such parameters depending on the preferences of each individual. Note that given that we have three voters the final allocation should be a three dimensional table. Hence, in order to depict it we provide four tables each one corresponding to a different preference profile of voter one (as we assume throughout, voter one has positive preferences towards both issues).

$\theta^1 = (1, \theta)$,

$(1,\theta)$	A	B	C	D	1	1	1	1
$(\theta,1)$	E	F	G	H	1	1	1	1
$(1,1)$	I	J	K	L	1	1	1	1
(θ,θ)	M	N	O	P	1	1	1	1
$(-\theta,\theta)$	1-M	Q	R	S	P	L	H	D
$(-1,1)$	1-I	T	U	R	O	K	G	C
$(-\theta,1)$	1-E	V	T	O	N	J	F	B
$(-1,\theta)$	1-A	1-E	1-I	1-M	M	I	E	A
	$(-1,\theta)$	$(-\theta,1)$	$(-1,1)$	$(-\theta,\theta)$	(θ,θ)	$(1,1)$	$(\theta,1)$	$(1,\theta)$

$\theta^1 = (\theta, 1)$,

$(1,\theta)$	E	F	G	H	1	1	1	1
$(\theta,1)$	1-V	a	b	c	1	1	1	1
$(1,1)$	1-T	d	e	f	1	1	1	1
(θ,θ)	1-Q	g	h	i	1	1	1	1
$(-\theta,\theta)$	1-N	1-g	j	k	i	f	c	H
$(-1,1)$	1-J	1-d	l	j	h	e	b	G
$(-\theta,1)$	1-F	1-a	1-d	1-g	g	d	a	F
$(-1,\theta)$	1-B	1-F	1-J	1-N	1-Q	1-T	1-V	E
	$(-1,\theta)$	$(-\theta,1)$	$(-1,1)$	$(-\theta,\theta)$	(θ,θ)	$(1,1)$	$(\theta,1)$	$(1,\theta)$

$\theta^1 = (1,1)$,

$(1,\theta)$	I	J	K	L	1	1	1	1
$(\theta,1)$	1-T	d	e	f	1	1	1	1
$(1,1)$	1-U	1-l	n	o	1	1	1	1
(θ,θ)	1-R	1-j	p	q	1	1	1	1
$(-\theta,\theta)$	1-O	1-h	1-p	r	q	o	f	L
$(-1,1)$	1-K	1-e	1-n	1-p	p	n	e	K
$(-\theta,1)$	1-G	1-b	1-e	1-h	1-j	1-l	d	J
$(-1,\theta)$	1-C	1-G	1-K	1-O	1-R	1-U	1-T	I
	$(-1,\theta)$	$(-\theta,1)$	$(-1,1)$	$(-\theta,\theta)$	(θ,θ)	$(1,1)$	$(\theta,1)$	$(1,\theta)$

$\theta^1 = (\theta,\theta)$,

$(1,\theta)$	M	N	O	P	1	1	1	1
$(\theta,1)$	1-Q	g	h	i	1	1	1	1
$(1,1)$	1-R	1-j	p	q	1	1	1	1
(θ,θ)	1-S	1-k	1-r	s	1	1	1	1
$(-\theta,\theta)$	1-P	1-i	1-q	1-s	s	q	i	P
$(-1,1)$	1-L	1-f	1-o	1-q	1-r	p	h	O
$(-\theta,1)$	1-H	1-c	1-f	1-i	1-k	1-j	g	N
$(-1,\theta)$	1-D	1-H	1-L	1-P	1-S	1-R	1-Q	M
	$(-1,\theta)$	$(-\theta,1)$	$(-1,1)$	$(-\theta,\theta)$	(θ,θ)	$(1,1)$	$(\theta,1)$	$(1,\theta)$

Similarly to the proof of Theorem 1.1, we just need to compute the interim prospects in terms of these parameters and maximise the ex-ante utility of any of the voters subject to the truthtelling constraints. The interim prospects are proportional to:

$$\begin{aligned}
P(1,\theta) = {} & -9 + A + 2D + 2B + 2C + 2F + 2G + 2H + 2J + 2K + \\
& + 2L + 2N + 2O + 2P + 2Q + 2R + S + 2T + U + V. \\
P(\theta,1) = {} & 2 + 2E - B + 2G + 2H - 2J - 2N - 2Q - 2T - 2V + \\
& + 2i + a + 2b + 2c + 2f + 2h + 2j + l + k + 2e. \\
P(1,1) = {} & 9 + 2I + 2J + 2L - 2T + 2d + 2f - 2U - 2l + n + \\
& + 2o - 2R - 2j + 2q - 2O - 2h - 2G - C - b + r. \\
P(\theta,\theta) = {} & 12 - 2L - 2f - 2R - 2j + 2O - D - 2H - 2Q - 2S + \\
& + 2h + 2N - 2k + 2g - c - o + 2p + 2M + s - 2r.
\end{aligned}$$

The optimal (*reasonable* and ex-ante efficient) SCF is the one that maximizes the ex-ante expected utility subject to the truthtelling constraints and the feasibility ones (that is, the forty parameters need to belong to the interval [0, 1]).

$$\max u^i(p)$$
$$\text{subject to} \begin{cases} P(1,1) = P(\theta,\theta) \\ P(1,\theta) \geq P(\theta,1) \\ P(1,1) \geq \dfrac{P(\theta,1) + P(1,\theta)}{2} \\ P(1,1) \leq \dfrac{P(\theta,1)\underline{\theta} + P(1,\theta)\bar{\theta}}{\bar{\theta} + \underline{\theta}}. \end{cases}$$

The end of the proof relies on writing the program in terms of the forty parameters and then, step by step, assuming whether or not any of the constraints is binding. Once this is done we are just left with some tedious (though trivial) linear programs. And it can be proved that for different values of θ the corner solution varies. More specifically, all parameters are equal to one except those specified below:

- $\theta \in \left(0, \frac{1}{3}\right)$: $R = S = U = b = c = j = k = l = 0.$
- $\theta \in \left(\frac{1}{3}, \frac{1}{2}\right)$: $Q = R = S = T = U = j = k = l = r = 0.$
- $\theta \in \left(\frac{1}{2}, 1\right)$: $Q = R = S = T = U = V = j = k = l = r = 0.$

A proper analysis of such allocations tells us that they coincide with the allocations achieved by the strategies where a non-indifferent voter invests V, $\frac{3}{4}V$ and $\frac{V}{2}$ votes on his preferred issue, respectively. ■

Equilibria with continuous preferences and divisible votes (2 players).

We restrict the analysis to pure strategy equilibrium. Remember that Theorem 1 tell us that the optimal strategy is only contingent on the relative intensities of the preferences and, moreover, it is well behaved (monotonic) with respect to them. In order to simplify the analysis we assume a uniform distribution on the relative intensities rather than on the preferences themselves i.e.

$$\theta_n^i \in \{\pm 1, \pm\theta\} : \begin{cases} \Pr\{|\theta_n^i| = 1\} = \Pr\{\theta_n^i > 0\} = \frac{1}{2} \\ \theta \sim U[0,1] \\ \text{Pairwise independence across issues and voters.} \end{cases}$$

We analyse the equilibrium from the perspective of a voter with positive preferences. The interim expected payoff of voter i when he invests $v^i \in [0, V] \subset \mathbb{R}$ votes on the first issue is:

$$\tilde{P}_1\left(v^i\right)\cdot\theta_1^i+\tilde{P}_2\left(V-v_1^i\right)\cdot\theta_2^i$$

$$\text{where, }\begin{cases}\tilde{P}_1\left(v^i\right)=\Pr\left(v^i+v^j>0\mid\theta_1^j<0\right)+\frac{1}{2}\Pr\left(v^i+v^j=0\mid\theta_1^j<0\right)\\ \tilde{P}_2\left(1-v^i\right)=\Pr\left(v^i+v^j<0\mid\theta_2^j<0\right)+\frac{1}{2}\Pr\left(v^i+v^j=0\mid\theta_2^j<0\right)\end{cases}$$

Simple calculations allow us to rewrite the interim expected payoff of voter i as:[56]

$$\frac{1}{2}\theta_2^i+\left\{\Pr\left(v^i+v^j>0\right)+\frac{1}{2}\Pr\left(v^i+v^j=0\right)\right\}\cdot\left(\theta_1^i-\theta_2^i\right).$$

Hence, an indifferent voter is indifferent between playing any of the strategies (as was done in the binary case, we assume that he plays the undominated strategy $v_i=\frac{V}{2}$) and a non-indifferent voter (say he prefers issue one) wants to maximise the expression inside the curly brackets. In the case where $v^j(\cdot)$ induces an atomless distribution on $[0,V]$ it is dominant for voter one to set $v^i=V$. Otherwise, if the induced distribution on the invested votes by voter j on issue one is not atomless, v_i will always be strictly higher (if possible) than the absolute value of the lowest possible value of v^j. Thus, the only equilibrium has non-indifferent voters investing all their voting power on their preferred issue.

Finally note that the proof can also be applied to the case of continuous preferences and non-divisible votes. We just need to restrict the set of strategies of voter i. ■

Equilibria with continuous preferences and divisible votes (3 players).

The setting is analogous to the one described in the proof above. We just need to add the restriction that we focus our analysis on *symmetric* equilibrium (that is, the three voters play the same strategy) and (as was done in Section 4.2) we further assume that voters behave equivalently regardless of the labelling or the sign of the issue.

This proof is a bit more complicated than the one above because now we need to consider whether each of them is in favour or against the approval of each of the issues in order to assign the appropriate sign to the cast votes. Once we take this into account we have

[56] Conditional probabilities are omitted for simplicity.

that the interim prospects read as follows ($v^j, v^k \geq 0$):

$$\begin{aligned}
\tilde{P}_1\left(v^i\right) = & \ \tfrac{1}{2}\Pr\left(v^j + v^k < v^i \mid \theta_1^j, \theta_1^k < 0\right) + \Pr\left(v^j - v^k \leq v^i \mid \theta_1^j, -\theta_1^k < 0\right) - \tfrac{1}{2} \\
\tilde{P}_2\left(1 - v^i\right) = & \ \tfrac{1}{2}\Pr\left(v^j + v^k > V + v^i \mid \theta_1^j, \theta_1^k < 0\right) \\
& + \Pr\left(v^j - v^k \leq V - v^i \mid \theta_1^j, -\theta_1^k < 0\right) - \tfrac{1}{2}.
\end{aligned}$$

Note that the tie breaking rule is now playing a role because voter i just needs to equate the sum of his opponents votes whenever only one of them desires the dismissal of the issue. Given the assumption that voters play equivalently regardless of the sign of his preferences we have that v^j and $\left(1 - v^j\right)$ have the same induced distribution (the same can be said about voter k's strategy). That implies that v^j is symmetrically distributed around $\frac{V}{2}$. In order to simplify the notation we define $X := v^j + v^k$ (which, accordingly, is symmetrically distributed around V i.e. $\Pr(X < k) = \Pr(X > 2V - k)$ for $k \in [0, 2V]$). Using such symmetry and the fact that $\left(v^j + \left(1 - v^k\right)\right)$ is distributed as X, we can write the interim expected payoff for a voter that prefers issue one as follows

$$ct + \frac{1}{2}\Pr\left(X < v^i\right) \cdot \left\{\frac{1}{2} - \theta\right\} + \Pr\left(X \leq V + v^i\right) \cdot \left\{1 - \frac{1}{2}\theta\right\}.$$

First note that whenever both opponents are splitting their voting power evenly (the case of MR), voter i is indifferent between playing any of the strategies. In particular $v^i = \frac{V}{2}$ is a best response. Hence, a symmetric equilibrium has all voters always splitting their voting power equally among both issues.

In the remainder of the proof we show that there exists only one more (and only one) equilibrium which corresponds to the one in which non-indifferent voters invest all their voting power on their preferred issue.[57]

Any other equilibrium will have non-indifferent voters investing more than $\frac{V}{2}$ votes on their preferred issue. Consequently, any voter with $\theta \in \left[0, \frac{1}{2}\right)$ clearly invests all his voting power on his preferred issue. Suppose now that there are some voters with $\theta \in \left[\frac{1}{2}, 1\right]$ such that $v^i < V$. Theorem 1 tell us that the optimal strategy is a well behaved function (decreasing with respect to θ) thus we can consider a parameter $\tilde{\theta} \in \left[\frac{1}{2}, 1\right]$ such that any voter with $\theta^+ > \tilde{\theta}$ invests strictly less votes on his preferred issue ($v^i\left(\theta^+\right) < V$) and any voter with $\theta^- < \tilde{\theta}$ sticks to the strategy $v^i = V$.

[57]The behaviour of indifferent voters does not need to be specified because they have zero measure. Nevertheless, it can be shown that their best response to any of the equilibria is splitting their voting power evenly.

Given that both are acting optimally we have that the next two inequalities should hold:

$$\begin{aligned}&\left(\Pr\left(X < V\right) - \Pr\left(X < v^i\left(\theta^-\right)\right)\right)\cdot\left\{\theta - \tfrac{1}{2}\right\} \leq \\ &\qquad \leq \left(\Pr\left(X \leq 2V\right) - \Pr\left(X \leq V + v^i\left(\theta^-\right)\right)\right)\cdot\left\{2 - \theta^-\right\} \\ &\left(\Pr\left(X < V\right) - \Pr\left(X < v^i\left(\theta^-\right)\right)\right)\cdot\left\{\theta^+ - \tfrac{1}{2}\right\} \geq \\ &\qquad \geq \left(\Pr\left(X \leq 2V\right) - \Pr\left(X \leq V + v^i\left(\theta^-\right)\right)\right)\cdot\left\{2 - \theta^+\right\}\end{aligned}$$

Given that the optimal function is decreasing we have that we should consider two possible cases: (1) the function is smooth at $\tilde{\theta}$ (i.e.$\lim_{\varepsilon\to 0} v^i\left(\tilde{\theta}+\varepsilon\right) = V$) and (2) there is a discontinuity (i.e.$\lim_{\varepsilon\to 0} v^i\left(\tilde{\theta}+\varepsilon\right) = \bar{v} < V$). Consequently, taking limits as θ^- and θ^+ tend to θ in the previous inequalities lead to two possible equalities depending on the behaviour of the optimal strategy at $\tilde{\theta}$:

1- $\left(\Pr\left(X < V\right) - \Pr\left(X < V\right)\right)\cdot\left\{\tilde{\theta} - \frac{1}{2}\right\} = \left(\Pr\left(X \leq 2V\right) - \Pr\left(X < 2V\right)\right)\cdot\left\{2 - \tilde{\theta}\right\}$.

2- $\left(\Pr\left(X < V\right) - \Pr\left(X < \bar{v}\right)\right)\cdot\left\{\tilde{\theta} - \frac{1}{2}\right\} = \left(\Pr\left(X \leq 2V\right) - \Pr\left(X \leq V + \bar{v}\right)\right)\cdot\left\{2 - \tilde{\theta}\right\}$.

Clearly, the first equality cannot be met because there is a positive measure of types playing the non-diversification strategy thus $\Pr\left(X = 2V\right) > 0$. The second case also leads to a contradiction given the following inequalities and the fact that one of them will always be strict:

$$2\tilde{\theta} - 1 \leq 2 - \tilde{\theta}$$

$$\Pr\left(X < V\right) - \Pr\left(X < \bar{v}\right) \leq 2\cdot\left(\Pr\left(X \leq 2V\right) - \Pr\left(X \leq V + \bar{v}\right)\right).$$

The second inequality needs some clarification. The term in brackets on the RHS accounts for all those cases in which both opponents are investing strictly more than $(V + \bar{v})$ votes (i.e. $X \in (V + \bar{v}, 2V]$). That is, those cases in which both voters have a type belonging to the interval $\left[0, \tilde{\theta}\right)$. Hence this occurs with probability ρ^2 where $\rho := \Pr\left\{\theta \in \left[0, \tilde{\theta}\right)\right\}$. Instead, the LHS accounts for those cases in which X belongs to $[\bar{v}, V)$. A necessary condition for that event is that none of the voters should invests V votes. That is, it occurs with a probability lower than $1 - \rho$. Given that θ is uniformly distributed, we know that $\rho \geq \frac{1}{2}$.

Finally, we just need to see that the second inequality is strict for $\rho > \frac{1}{2}$ and the first one is strict for $\rho = \frac{1}{2}$. ∎

Lemma 1.4 *$f:\mathbb{R}^N \longrightarrow \mathbb{R}$ is homogeneous of degree k if and only if the following first order partial differential equation is satisfied:*

$$\nabla f(x)\cdot x = k\cdot f(x).$$

Proof. Sufficiency. (Euler's Theorem) Given that f is homogeneous of degree k we have that for all $\lambda > 0$ and all $x \in \mathbb{R}^N$ the following holds: $f(\lambda x) = \lambda^k \cdot f(x)$. Differentiating the equality with respect to λ we obtain:

$$x_1 \cdot \frac{\partial f}{\partial x_1}(\lambda x) + ... + x_N \cdot \frac{\partial f}{\partial x_N}(\lambda x) = k \cdot \lambda^{k-1} \cdot f(x).$$

For $\lambda = 1$ we get our result.

Necessity.[58] Define $\xi(\lambda) := \lambda^{-k}\cdot f(\lambda x) - f(x)$ and differentiate such expression,

$$\xi'(\lambda) := -k\cdot\lambda^{-k-1}\cdot f(\lambda x) + \lambda^{-k}\cdot\left[x_1 \cdot \frac{\partial f}{\partial x_1}(\lambda x) + ... + x_N \cdot \frac{\partial f}{\partial x_N}(\lambda x)\right].$$

Using the fact that $\nabla f(x)\cdot x = k\cdot f(x)$ we have that $\xi'(\lambda) = 0$. Hence, $\xi(\lambda)$ is constant. Moreover, $\xi(1) = 0$, thus $\xi(\lambda) = 0$ for all $\lambda > 0$ which proves that f is homogeneous of degree k. ■

[58] This part of the proof is extracted from Martin J. Osborne webpage (www.chass.utoronto.ca/~osborne)

Chapter 2

A First Experiment on Qualitative Voting

2.1 Introduction

2.1.1 Motivation

Voting is the most common tool for collective decision-making. It is the way we normally aggregate individual preferences into social decisions whenever we want to treat citizens equally (in particular, we do not allow monetary transfers so that endowment play no role in the decision process). Regardless of its relevance, not much research has been done trying to assess the optimality of different voting rules to different situations.

It is precisely on the grounds of equality that voting rules do not usually consider the intensities of the voters' preferences. In the first Chapter we propose an alternative voting rule, Qualitative Voting (QV, hereafter), that is sensitive to the voters intensities of preferences and preserves an equal treatment of all voters. We consider a voting rule where the concept of decision *preferred by most members* is replaced by the concept of decision *most preferred by members.* We want votes to have an embedded *quality* which is somehow associated to the intensity of the voters' preferences. Ultimately, we want to show under which circumstances the strategic interactions between voters do not undermine the gains we expect from them expressing their *willingness to influence.*

We see QV as a natural extension to the Majority Rule (MR, hereafter) where a group of voters that have to decide over the approval or dismissal of a predetermined set of issues is endowed with a fixed number of votes that can be distributed freely among the issues. Equality is preserved given that all voters are endowed with the same voting power. The main difference with respect to MR is that now voters do not need to attach the same voting power to each issue and can, consequently, trade off their voting power.

The fact that voters can intensify their votes on those issues they mostly care about allows two main improvements with respect to standard voting rules. On the one hand, it allows relatively strong minorities to decide over weak majorities —the fact that majorities usually impose their will regardless of the intensity of their preferences is known in the political science literature as "the problem of intensity" and motivates the first part of this Thesis. On the other hand, in conflict resolution situations where two parties need to decide over various conflicting issues, QV allows each party to unilaterally unlock the negotiation.

Theoretical properties of QV are studied in Chapter 1. We show that in a setting with two voters, two issues and preference intensities uniformly distributed across possible values, QV Pareto dominates MR and, moreover, achieves the only ex-ante optimal (incentive compatible) allocation. The result also holds true with three voters as long as the possible preference intensities differ substantially.

The novelty of QV lies in its simplicity, thus it seems natural to design a series of experiments to further examine its properties. First we want to contrast the theoretical predictions with the experimental evidence. Second, we want to observe how voters behave in those situations where the theory remains silent. Ultimately, we want to evaluate the welfare achieved by our subjects through the use of QV.

In this Chapter we show preliminary evidence from a pilot experiment ran last June 2004 with eighteen subjects in the *Laboratori d'Economia Experimental* at *University Pompeu Fabra.* Needless to say, this Chapter is just a first step towards a more exhaustive analysis of QV and, most importantly, proposes the basic tools to be used in subsequent work. The main two variables we want to vary in this first experiment are the number of issues that need to be decided and the size of the group of voters that need to jointly decide over the previous issues. The informational structure is also very simple: voters are announced their own payments/valuations but know nothing about their opponents'. Our analysis is divided in three parts.

First we compare the outcome achieved in the experiment with the one that would have been achieved if MR was used.[1] As a reference benchmark we compare both results to

[1] Note that the MR outcome is immediate given that there are only two choices for each issue (approval or dismissal) and no strategic voting can occur. That is, it is dominant for any player to truthfully declare whether he wishes the approval or dismissal of any of the issues.

the outcome that maximises the sum of utilities —hereafter, the efficient outcome. Our data shows that QV is better able to replicate the efficient outcome than MR: QV reaches the efficient outcome in 68% of the issues, while MR only does so in 37% of them.

Second, we evaluate the different outcomes in terms of welfare and remark that the gains of QV with respect to MR are not as noticeable as above. The reason for this to be the case is that, in our data, the MR outcome does very well whenever there are more than two voters per group. In any case, QV still does 20% better than MR. The more relevant gains are observed precisely in the *conflict resolution* situations (that is, whenever two voters need to decide over various issues on which they have opposing views) where MR does not allow any decision to be made and ties occur in all issues and QV displays all of its strength in allowing voters to trade-off their interest across issues and unlocking the MR outcome.

We then analyse the subjects' voting behaviour in order to understand what is driving our former results. Within this analysis we first identify what we call *elementary errors*. That is, voting profiles where voters cast most of their votes in issues that are not their most preferred ones. This is clearly non-optimal given that they know nothing about their opponent's payments. We then turn to the more precise analysis of the subjects' voting behaviour. On the one hand, we observe that players vote according to equilibrium predictions—that is, they invest most votes in their most preferred issue whenever they need to decide over two issues. On the other hand, in order to anlayse what happens where the theory still remains silent we compute three scores associated with the observed voting decisions.[2] These scores capture (1) how close the voting behaviour is from *truthfully* revealing voters' preferences (a subject is truthful whenever the ratio between his votes coincides with his relative intensities towards the issues); (2) how close the voting profile is from distributing evenly the voting power across issues; and, (3) how close the voting strategy is from concentrating all the voting power in the most preferred issue.

We want to use the scores to test two hypothesis. If voters act according to how pivotal their votes are we expect that an increase in the number of players per group (that is, as we increase the total number of votes that are cast in the decision of a given set of issues) should lead voters to a higher concentration of their voting power in their most preferred issue —this should be captured by our third score. Instead, an increase in the number of issues distributes the total number of votes across more issues hence each vote becomes now more pivotal. Therefore, voters should diversify their voting power across issues increasing their chances to be pivotal in more issues —this should be captured by

[2]We define these scores by computing the minimal angle between a hypothetical voting profile and the observed one. As a result, the smaller the angle is the closer the subject's behaviour is to the one captured by the hypothetical voting profile.

the second score.

Our data shows none of these effects. The only certain effect we observe is that whenever voters face many issues (in our case six), they tend to drop the two or three issues with lowest payments and focus their attention (and votes) on the remaining issues. This is consistent with an explanation based on computational complexity and is, indeed, endorsed by some of the answers in the questionnaire that was handed at the end of the experiment.

In the remainder of this Section we relate our work to the existing literature. In Section 2.2 we introduce the theoretical model and its predictions. Section 2.3 introduces the experimental design and presents two clarifying examples. Preliminary results based on the data of our experiment and, more importantly, the tools through which subsequent experiments need to be analysed are contained in Section 2.4. Finally, Section 2.5 discloses our view on how further experiments on QV need to be conducted. Section 2.6 concludes the Chapter.

2.1.2 Related literature

The lack of simple alternative voting rules that capture the intensity of the voters's preferences justify the lack of experimental evidence around our subject.

First and most related to our work is the experimental study of *storable votes* by Casella, Gelman and Palfrey (2003). *Storable votes* is a mechanism to be used in situations where voters have to decide over the same binary decision repeatedly over time. It allows voters to abstain and store that vote for further meetings when the intensity of his preference may be stronger. In their experimental analysis they show that the welfare achieved by subjects is remarkably close to theoretical predictions even when players do not exactly follow the theoretical equilibrium predictions.

Secondly, McKelvey and Ordeshook (1981) propose a setting analogous to ours where five issues have to be approved or dismissed. They allow subjects to consider bills either sequentially or simultaneously as a package. The latter allows for the intensity of preferences to intervene in a similar fashion as committees or parties bundle issues together —see Shepsle and Weingast (1994) and Levy (2004). Their main message is that whenever players are given ordinal information about their opponents' preferences, strategic interactions make subjects worse off with respect to the no information situation.

Finally, Yuval (2002) also proposes a first empirical investigation of the mechanism of

Sequential Voting by Veto. The core of his analysis relies on showing that agents are more likely to act sincerely whenever the size of the agenda increases. That is, whenever the calculation of strategic voting becomes increasingly more complex.

2.2 The model

A *voting game* is defined as a situation where I voters have to dismiss or approve N issues and no monetary transfers are allowed. Voters privately know their preference profile across the N issues and the prior distributions from which these preferences are drawn are common knowledge. From a mechanism design perspective this is a multidimensional problem with multilateral asymmetric information and no transfers.

Voters and issues are denoted $i \in \{1, 2, ..., I\}$ and $n \in \{1, 2, ...N\}$, respectively. Voter i's valuation towards issue n is θ_n^i. The preference vector of voter i is $\theta^i = (\theta_1^i, ..., \theta_N^i) \in \Theta \subseteq \mathbb{R}^N$. Preferences should be interpreted as follows: a positive type ($\theta_n^i > 0$) wishes the approval of the issue, a negative one ($\theta_n^i < 0$) wishes its dismissal and the type's absolute value ($|\theta_i^n|$) captures the intensity of the preference towards that particular issue.

Voter i's payoff on a given voting procedure n is described as follows,

$$\begin{cases} \theta_n^i \text{ if the issue is approved} \\ -\theta_n^i \text{ if the issue is dismissed} \end{cases}$$

and the total payoff is the sum of the individual payoffs across the N voting procedures.[3]

An allocation is a N-tuple of probabilities that corresponds to the probability of approving each of the N issues. The set of allocations is defined as

$$\mathcal{X} = \{(p_1, ..., p_N) : p_1, ..., p_N \in [0, 1]\}$$

where p_n is the probability of issue n to be approved. In general p_n can take any value in the interval $[0, 1]$ but note that a voting rule yields only three outcomes: an issue is approved, dismissed or tied. The latter can be seen as delaying the issue until the following meeting and, in our risk neutral framework, it is equivalent to approving the issue with probability $\frac{1}{2}$.

[3] In the definition of payoffs we implicitly assume that issues are independently valued. That is, there are no complementarities between the issues. Provided that issues are independently valued, results can be extended to any linear transformation of the payoffs.

A voter with preferences θ^i obtains the following utility from an allocation $p \in \mathcal{X}$:

$$u\left(p, \theta^i\right) := \sum_{n=1}^{N} p_n \theta_n^i + (1 - p_n)\left(-\theta_n^i\right) = \sum_{n=1}^{N} (2p_n - 1)\, \theta_n^i.$$

Players are endowed with V votes that can be freely distributed between the issues. The votes can have a positive or negative value capturing the will of the voter towards the approval or dismissal of the issue. The action space is the collection of voting profiles:

$$\mathcal{V} := \left\{(v_1, ..., v_N) \in \left\{-V, ..., -1, 0^-, 0^+, 1, ..., V\right\}^N : |v_1| + ... + |v_N| = V\right\}$$

An issue is approved through QV whenever the sum of votes towards that issue is positive. It is dismissed whenever the sum is negative. Finally, if the sum of votes is zero the will of the majority of voters is implemented or otherwise the issue is tied (alternatively, the issue is approved or dismissed with equal probability). That is,

$$\text{QV} \ (n = 1 \div N): \begin{cases} v_n^1 + ... + v_n^I > 0 & \Rightarrow \ \text{The issue is approved} \\ v_n^1 + ... + v_n^I < 0 & \Rightarrow \ \text{The issue is dismissed} \\ v_n^1 + ... + v_n^I = 0 & \Rightarrow \ \begin{cases} \text{The will of the majority is implemented.} \\ \text{Otherwise, a tie occurs.} \end{cases} \end{cases}$$

We can further define the outcome of MR in this context as the one where all voters evenly split their voting power among all issues. Formally,

$$\text{MR} \ (n = 1 \div N): \begin{cases} \#\left\{i : v_n^i > 0\right\} > \#\left\{i : v_n^i < 0\right\} & \Rightarrow \ \text{The issue is approved} \\ \#\left\{i : v_n^i > 0\right\} < \#\left\{i : v_n^i < 0\right\} & \Rightarrow \ \text{The issue is dismissed} \\ \#\left\{i : v_n^i > 0\right\} = \#\left\{i : v_n^i < 0\right\} & \Rightarrow \ \text{The issue is delayed.} \end{cases}$$

The relevant results in from the previous Chapter for our experiment are best summarised through Theorems 1.1 and 1.2. They regard the use of QV as an alternative voting rule that would allow the expression of the voters' *willingness to influence.*

The first theorem states that in a setting with two voters, two issues, discrete preferences (i.e. $\theta_n^i \in \left\{\pm\bar{\theta}, \pm\underline{\theta}\right\}, \bar{\theta} > \underline{\theta} > 0$) and independent uniform priors on the other voter's preferences, QV reaches the only ex-ante optimal allocation. That allocation corresponds to the one where non-indifferent voters invest all voting power on their most preferred issue and indifferent voters evenly split their voting power. Therefore, QV unlocks conflict resolution situations. It allows voters to simultaneously and non-cooperatively declare which issue they prefer and hence extract all possible gains from heterogeneous preferences —that is, each voter decides on his preferred issue whenever it does not coincide with the preferred issue of the other voter.

The second theorem asserts that the previous result holds true whenever we introduce a third voter and as long as the possible preference intensities differ sufficiently (specifically, $\underline{\theta}/\bar{\theta} < 1/3$). The introduction of a third voter yields a departure from the pure conflict resolution situation so that we can characterise when it is incentive compatible for *very* enthusiastic minorities to decide over weak majorities.

2.3 Experimental design

Eighteen subjects participate in the experiment.[4] The experiment was programmed and conducted with the software z-Tree (Fischbacher 1999).[5]

To induce the appropriate preferences we simply assign each player different payments in case each of the issues is approved or dismissed: a high payment induces an intense preference and, on the contrary, a low payment induces a weak preference. Payments are integer values uniformly and independently distributed between 1 and 100 and players are told so.

We partition the 18 players into three sets of six players so as to obtain three independent observations.[6] At each period, players from each partition are randomly matched in groups of two, three or six members. Subjects are simply told that they are randomly matched into groups of different size at each period. At each period, players are endowed with 30 votes to be distributed among the issues.

The structure of the experiment is summarised in the following table.

[4]The pilot experiment was conducted last June 2004 in the *Laboratori d'Economia Experimental* (University Pompeu Fabra)

[5]The data, programme code and instructions for the experiment are available upon request.

[6]We observe no significant difference between the three independent observations therefore, in the subsequent analysis, we will merge the data.

Subjects	Group Size	Number of Issues
18	2	2
18	2	3
18	2	6
18	3	2
18	3	3
18	3	6
18	6	2
18	6	3
18	6	6

Each row is repeated 3 times. Hence subjects play a total of 27 voting games.

Given that this first experiment was just a pilot, we paid a flat rate to our participants. In subsequent experiments we should randomly select a certain number of voting games and pay subjects according to their payoff on those periods.

In our experimental design we distinguish between the *conflict resolution* and the *committee meeting* situations. In the former, two voters need to decide over various issues on which they have opposing views. In the latter, three or six subjects need to vote on some discerning issues and there is an equal chance that any of the voters wish the approval or dismissal of any of the issues —hence unanimous wills may exist over some issues.

At the beginning of each period subjects are told whether they are facing a conflict resolution or a committee meeting situation (in the latter case they are also told the size of their group —3 or 6 voters). They are then informed of their payments and asked to cast their votes.[7] Once all have done so, the programme computes the outcome for each group and announces the following information to players: (i) their payments for each of the issues; (ii) how many people wished the approval or dismissal on each of the issues; (iii) the sum of votes on each issue; (iv) the final outcome on each of the issues; (v) the precise voting profiles cast by each member in their group; and finally (vi) their payment for that period.

[7]The programme requires any single entrance to be an integer between 0 and 30 and the sum of them to be 30. For simplicity, voters do not need to attribute a sign to their vote -the programme automatically plays the dominant strategy of assigning a positive sign when the voter wishes the approval and a negative sign when he wishes its dismissal.

2.3.1 Conflict resolution example

There are two players. Each one is endowed with 30 votes. Players are announced their payments and know that their opponent wishes the opposite they wish (that is, one player wishes the dismissal of all issues and the other their approval). They are then asked to submit their voting profiles. The following table summarises the information and their actions (Figure A1 in the appendix captures the screen shot for player 1).

	Payments	Voting profile
Player 1	$(87, 38, 13)$	$(20, 10, 0)$
Player 2	$(-1, -48, -100)$	$(0, 5, 25)$

The outcome is that the first two issues are approved and the third one is dismissed (that is, the outcome is $(1, 1, -1)$). Payments are 112 and 51 for players 1 and 2 respectively —see the outcome screen shot for player 1 in Figure A2. Note that if voters used MR ties would occur in all issues and both players get a zero payment.

2.3.2 Committee meeting example

There are three players. Each one is endowed with 30 votes. Players are announced their payments and are asked to submit their voting profiles. The following table summarises the information and their actions (Figure A3 captures the screen-shot for player 1).

	Payments	Voting profile
Player 1	$(-38, 71, 78)$	$(0, 15, 15)$
Player 2	$(-53, -59, 33)$	$(13, 13, 4)$
Player 3	$(57, -24, 15)$	$(15, 10, 5)$

The outcome is that issues one and three are approved and issue two is dismissed. Payments are -31, 39 and 96 for players 1, 2 and 3 respectively —see the outcome screen shot for player 1 in Figure A4.

If voters used MR the first two issues would be dismissed and the third one approved. Payments would be: 45, 145 and 18. Note that the realised outcome of QV does not Pareto improve the MR outcome in this case —nor it dominates it in utilitarian terms.

2.4 Analysis

2.4.1 Realised outcomes

The upper bound in terms of welfare is given by the efficient outcome (EffO, hereafter), the one that maximizes the sum of utilities. Specifically, an issue is approved when the sum of utilities towards that issue is positive, and dismissed when it is negative.

The ultimate goal of our analysis is to compare QV with MR. To do so we need to know the outcome that would have been obtained if subjects used MR. To compute this outcome we just need to count how many individuals wish the approval or the dismissal of each issue and assign the majoritarian will to each issue. We refer to this outcome as MRO.

Finally we define the outcome achieved in our experiment as the QVO.

During our experiment there were a total of 594 issues that were approved, dismissed or tied. In the next table we compare the QVO with the efficient and MR ones. We compute the percentage of outcomes for which the different outcomes coincide. For instance, row one counts the number of issues for which all outcomes coincide (that is, EffO=MRO=QVO). Instead, row two counts the number of issues for which the MRO and QVO coincide but are different from EffO.

Total observations			594
EffO	*MRO*	*QVO*	36%
EffO	*MRO*		5%
EffO		*QVO*	39%
EffO			20%
	MRO	*QVO*	4%
	MRO		55%
		QVO	21%

MR reaches the efficient outcome in 41% (36% + 5%) of the occasions. In those situations it makes no sense to propose an alternative voting rule to MR given that this one is achieving the first best outcome. Nevertheless, we can see that QV manages to do at least as good as MR in almost 90% of the cases (36% out of 41%). Whenever MR does not reach the efficient outcome, QV does so about two thirds of the times.[8] Instead, MR

[8]Note that MR does not reach the EffO a 59% (100%-36%-5% or 4%+55%) of the times. In the

reaches the efficient outcome whenever QV does not in 5% of the issues.

Overall, QV does much better than MR and it does very well replicating the efficient outcome: QV reaches the efficient outcome in three fourths of the issues as opposed to the 41% of MR.

Recall that MR was only able to replicate the efficient outcome 37% of the times.

The following table disaggregate the data above by the number of issues that were being voted ($N = 2, 3, 6$) and the number of players that were being grouped ($I = 2, 3, 6$).

			N = 2	N = 3	N = 6	I = 2	I = 3	I = 6
Total observations			108	162	324	297	198	99
EffO	*MRO*	*QVO*	37%	39%	34%	0%	78%	58%
EffO	*MRO*		5%	4%	6%	0%	12%	9%
EffO		*QVO*	31%	39%	42%	67%	6%	23%
EffO			27%	19%	18%	33%	5%	10%
	MRO	*QVO*	5%	3%	5%	6%	5%	0%
	MRO		55%	54%	55%	94%	6%	33%
		QVO	28%	19%	19%	27%	12%	19%

The first three columns do not show any significant difference but we can observe that the gains we observe from allowing players to express their *willingness to influence* increase as we increase the number of issues. In particular we see that QV reaches the Efficient outcome whenever MR does not in 31, 39 and 42 percent of the times when they are voting towards 2, 3, or 6 issues, respectively. As we could have predicted, the gains from trade that arise when we consider multidimensional settings increase when we increase the opportunities of trade through the inclusion of more issues.

Finally, the last three columns show that the strengths of QV lie mainly in the two issues case (QV reaches the efficient outcome a 67% of the times when MR never does so). When we consider three issues, instead, we can see that there is not much scope of improvement with respect to the MR —the MR reaches the efficient outcome a 90% of the times. That can be explained by the fact that whenever there are three players any player is 'very' pivotal given that it requires both his opponents to be against his position for his will not to be implemented. In any case, QV manages to reach the efficient outcome 84% of the times.

situations where MR does not reach the EffO, QV does so a 36% of the times. Hence, it is an approximate 66% of the times (39/59).

We turn next to quantifying these results in terms of voters' welfare.

2.4.2 Welfare analysis

The experiment consists of 18 subjects playing 27 voting games thus we have a total of 486 observations on voters' welfare. The table below shows the realised average payoff and compares it with the one obtained when using MR. Using the efficient outcome as our benchmark we obtain the *scores* in the last column and we see that QV is able to improve upon the welfare obtained through MR —it does 20% better.

	average	score
Efficient Payoff	72.82	100
Realised Payoff	55.11	76
MR Payoff	46.11	63

Note that the efficient outcome constitutes a non achievable upper bound to our analysis. In Chapter 1 we proved that the efficient outcome is in general not implementable. This is because it requires interpersonal comparison of utilities and we show that for a mechanism to be implementable it needs to be sensitive only to the relative intensities between the issue. In other words, it cannot treat very enthusiastic voters better than apathetic ones. Otherwise, those with weaker preferences will have an incentive to pretend they are also very enthusiastic. Obviously, the EffO implies such comparisons.[9]

We can disaggregate the *Realised* and *MR scores* by the number of issues being bundled and the size of the committee that had to decide. We expect that a higher number of issues (increase in N) would benefit the use of QV because voters have more possibilities to trade off their voting power across issues.[10] Conversely, an increase in the number of members in the committee (increase in I) should decrease the gains we expect from allowing voters to express their *willingness to influence* given that players become less pivotal thus their role through any voting rule is diluted.

[9] The fact that the efficient outcome is in general not implementable is best captured in the following example:

> Consider a conflict resolution situation with two issues and imagine that the two players' payments/preferences profiles are : $(90, 60)$ and $(-10, -40)$. The efficient allocation requires both issues to be approved and the efficient utilitarian social welfare is 100. Nevertheless, approving both issues is not incentive compatible. The implementable allocation that maximises the sum of the voter's utilities consists of approving the first issue and dismissing the second one.

[10] Jackson and Sonnenschein (2003) show theoretically that an increase in the number of issues being bundled implies, at the limit, the achievement of the efficient outcome.

The following table captures the scores of QV (coefficient above) and MR (coefficients below) by group size (I) and number of issues (N).

	N = 2	N = 3	N = 6
I = 2	30	55	71
	0	0	0
I = 3	87	94	81
	94	98	93
I = 6	91	71	79
	69	83	79

The induced preferences in the conflict resolution case ($I = 2$) are such that one subject wishes the approval of all issues and his opponent wishes their dismissal —that is, they have opposing preferences. In that situation MR yields ties and obtains a zero score in all issues. QV improves upon that allocation and, as noted above, the gains increase with the number of issues we consider. The extremely positive effects we observe in the conflict resolution situation contrast with the weaker results of the committee meeting examples. The small gains from the use of QV are particularly evident in the case of group size three or six.

When $I = 3$, MR does extremely well. This is because under such a rule any player is *very* pivotal —it needs to be the case that both his opponents are against his position for the issue to be decided against his will. Hence, there is not much scope for improvement. Moreover, QV needs a more complex reasoning and may introduce errors in the voting profile. Overall QV reaches a slightly lower welfare than MR in this framework.

The argument for which MR does very well when there are only three players per group fails to hold true when we consider bigger groups. We can see that the MR scores when $I = 6$ are always below the scores obtained when $I = 3$. As opposed to the situation with group size three, QV does almost 50% better than MR when six subjects are only voting upon two issues. Instead,QV does not manage to do better than MR in situations with more than two issues.

The committee meeting examples do not exhibit any longer increasing gains of the use of QV when voters decide upon more issues. Indeed, a larger number of issues increases the *trading possibilities* but it also adds more complexity to the subjects' strategy space. The latter effect may induce lower than optimal responses. Precisely, some respondents to the questionnaire that was distributed after the experiment described that whenever they were confronted with six issues they dropped the two or three issues with lower payments and focused their attention (and votes) on the remaining ones. This aspect is treated within the next section where we analyse the individuals' voting behaviour.

2.4.3 Voting behaviour

2.4.3.1 Elementary errors

Subjects may act more or less strategically according to the situation they are in (that is, group size and number of issues), their history in the experiment and their individual characteristics. Before analysing how they vote depending on their induced preferences we want to check whether they understood the game they were playing. For this we compute what we consider *elementary errors* (EE).

Recall that voters do not know the payments of any of their opponents and just know that in the conflict resolution situation their opponent has opposing preferences. Therefore, the issue where the highest number of votes is invested should coincide with the one of highest payment. In light of this, we classify as EE the voting profiles where the former does not happen. The next table summarises the total number of such errors per subject:

subject	# EE	subject	# EE	subject	# EE
1	17	7	3	13	2
2	3	8	11	14	0
3	5	9	4	15	0
4	0	10	0	16	1
5	17	11	0	17	6
6	8	12	1	18	0

There are few reasons why these errors might occur. First, and most importantly, the instructions may not be clear enough (we plan to simplify the experimental design in the future). Second, it may be the case that some people only made EE in the first periods but then learned how to play in subsequent periods. Finally, some errors may not be very big in the sense that they may have assigned a single extra vote to an issue that was just given a slightly lower payment. We have checked in detail the distribution of EE and found no clear evidence of the latter two reasons. For instance player 1 played the voting profile $(3, 7, 20)$ when he was given the payments $(44, 57, 45)$ and did *not* make EE in periods 1, 4, 6, 12, 14, 15, 20, 25, 26 and 27.

Most importantly, the presence of EE cannot fully explain the non-substantial welfare obtained by QV in the committee meeting situations.

2.4.3.2 Strategic vs. truthful behaviour

We emphasised in Section 2.4.2 that some players may move away from a truthtelling strategy driven by a higher complexity of the game. This puts forward a question regarding what we understand by strategic behaviour as opposed to truthtelling behaviour. It is indeed the case that we can identify situation where a player is deviating from truthful behaviour. These deviations may be driven by a more complex reasoning. Conversely, deviations from truthful behaviour may be driven by a naïve strategy caused by the increased complexity of the setting —the latter may not be regarded as strategic behaviour.[11] Without entering this debate, we call strategic behaviour any departure from truthful behaviour.

We want to understand whether the departure from truthful behaviour leads towards a *diversifying* or *intensifying* strategy. In other words, we want to know if subjects try to increase their stakes in most issues or, alternatively, they invest most of their voting power in only a few issues.

Our multidimensional setting implies that payments in a given period or voting profiles are N-tuples (or vectors) of integer values. In order to identify how close a voter is from being truthful we need to identify how closely his voting profile replicates his payment profile. For that purpose we compute a score defined as the minimal angle formed by these two vectors. We do so using the expression of the dot product of two N-dimensional vectors $v = (v_1, ..., v_N)$ and $w = (w_1, ..., w_N)$:

$$v \cdot w = \|v\| \cdot \|w\| \cdot \cos \alpha.$$

Thus the angle between the two vectors v and w is,

$$\begin{aligned} \alpha &= \arccos \left(\frac{v \cdot w}{\|v\| \cdot \|w\|} \right) \\ &= \arccos \left(\frac{v_1 \cdot w_1 + ... + v_N \cdot w_1}{\sqrt{v_1^2 + ... + v_N^2} \cdot \sqrt{w_1^2 + ... + w_N^2}} \right). \end{aligned}$$

[11] For instance McKelvey and Ordeshook (1981) show precisely in a related experiment that more information makes the strategic interaction too difficult and drives players to concentrate their behaviour in fewer issues. Instead, Yuval (2002) shows that increased complexity induces players being more truthful –nevertheless his argument holds because players are confronted with unidimensional decisions at a time in a *sequential voting by veto* framework.

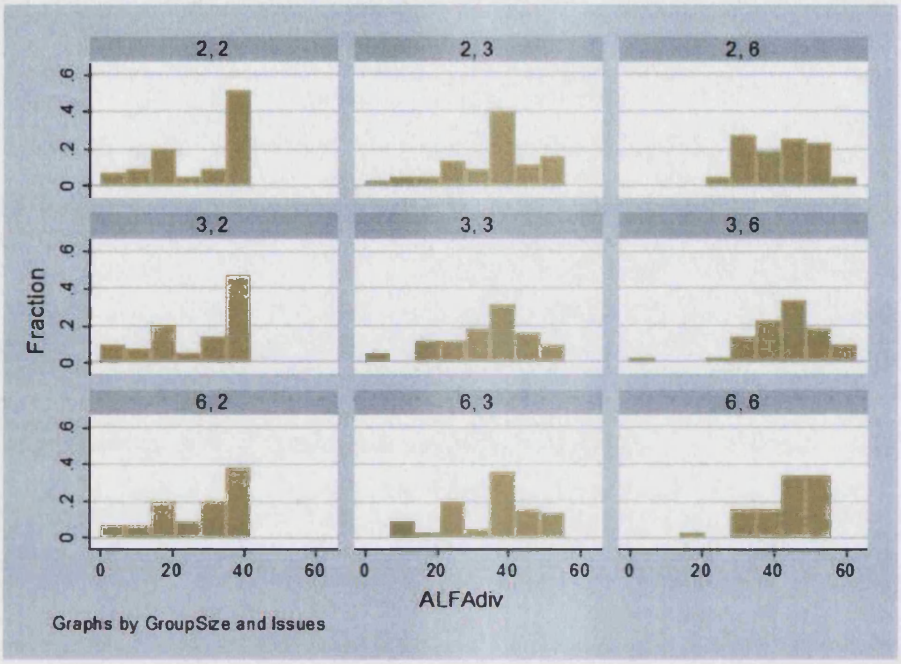

Figure 2.1: α_{div}

This way we can define α_{truth} as the angle between the cast voting profile (v) and the vector of payments (or induced preferences). The smaller this angle is, the more truthful the voter behaviour is.

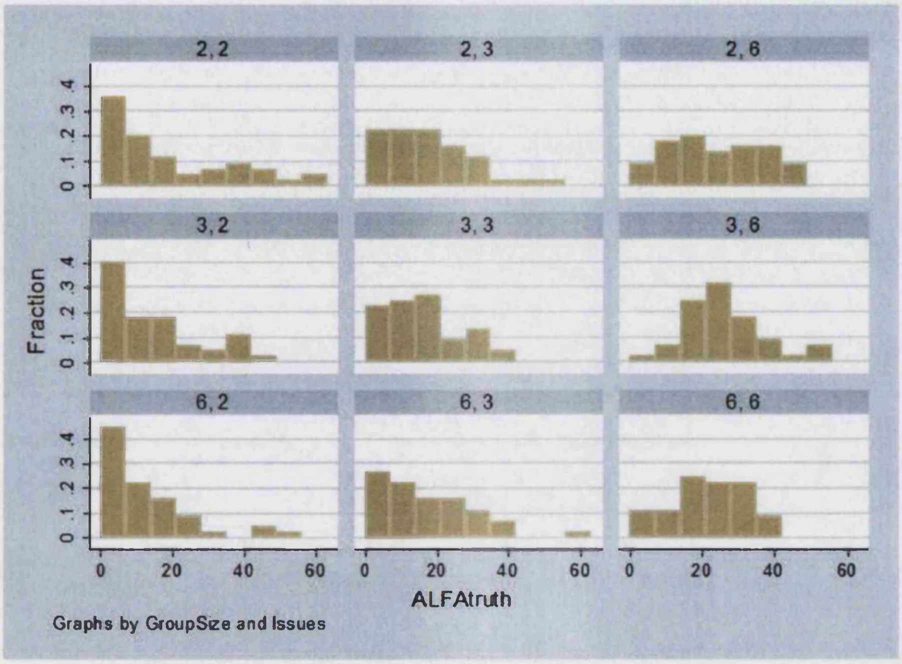

Figure 2.2: α_{truth}

how much the strategies followed by our subjects differ from the MR strategy —when α_{div} is zero it denotes that the cast voting profile coincides with perfect diversification of voting power across issues. It is clear that players are using QV to intensify their voting power on certain issues and very seldom they replicate the MR strategy. The pike we observe around the 45° in the two and three issues denotes precisely that players are playing according to the predicted equilibrium behaviour. That is, they are investing most of their resources in only one issue.

Figure 2.1 just captures how much players differ from evenly splitting their voting power but this is obviously a partial analysis. To better understand their behaviour we should take into account their preferences. We first show in Figure 2.2 whether players depart from behaving truthfully.

The most relevant fact we observe in Figure 2.2 is that as we increase the number of issues (as we move right) subjects tend to be less truthful. Instead we do not really see a significant difference in their behaviour as we vary the group size (as we move vertically). Any realisation above 45° in the two issue situation is what we have defined above as a *essential errors*. Note that we can observe a learning effect because *elementary errors* in

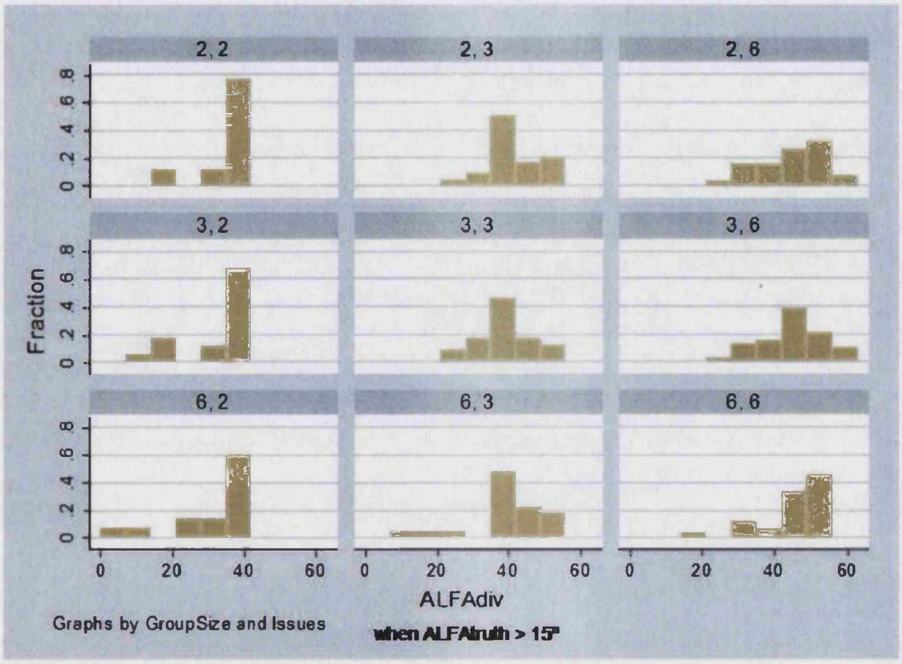

Figure 2.3: α_{div} (when $\alpha_{truth} > 15°$)

the two issues cases diminish as the group size increases (group-size-two voting games are played at the beginning of the experiment for all subjects).

Overall there is a clear tendency to play truthfully. This tendency is stron

are fewer issues. We believe that the observed behaviour in the six issues case is mostly driven by the fact that players tend to disregard some of the issues when they face very complex situations. We come back to this point below.

Next we try to understand the behaviour of subjects when they depart from truthtelling. That is, whenever $\alpha_{truth} > 15°$.

Figure 2.3 shows what we presumed from Figure 2.1: there is no tendency to diversify. Moreover, in all cases but one we observe no occurrence where a subject diversifies completely (angle α_{di} , smaller than 10°) when he is departing from truthtelling. We do not observe more diversification either when subjects are voting towards a higher number of

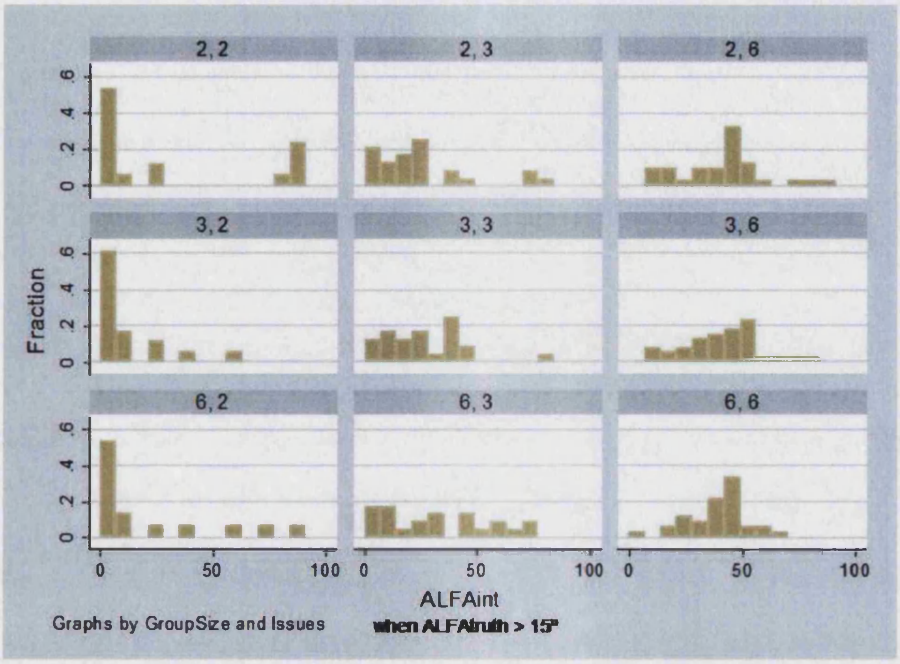

Figure 2.4: α_{int} (when $\alpha_{truth} > 15°$)

difference as we vary the group size.

We can contrast our theoretical predictions with Figure 2.4: whenever subjects are acting strategically in the case of two issues and two or three voters per group they tend to invest all votes in their most preferred issue. We can conclude that the negative results we observed from the use of QV in the three players and two issues case is not due to non-optimal strategies but instead to the fact in this case MR renders each voter *very* pivotal (whenever intensities are uniformly and independently distributed).

In the conflict resolution situation with two issues there is a positive mass of voters with α_{int} around 90°. That is, there are subjects investing most of their votes in only one issue as predicted by the theory, but this issue is the least preferred, a reflection of an *elementary error*. Once again we can observe some learning since the probability of this kind of error diminishes when we move away from the early stages of the game.

As long as we increase the number of issues, subjects move away from the intensification strategy and this behaviour is not accompanied by a diversification strategy (as observed in Figure 2.3). Both details provide evidence towards the assertion that, when confronted with many issues, subjects drop a few of them and concentrate on the subset of issues that is most valuable to them —as observed by McKelvey and Ordeshook (1981).

Finally, we do not observe any effect in Figure 2.4 of an increase in group size. Once again, we expected that an increase in the group size would induce subjects to invest most of their voting power on their most preferred issue(s) given that their probability of being pivotal diminished.

The analysis of the last two figures has tried to control for the subject no being truthful ($\alpha_{truth} > 15°$) and to identify a diversified or intensified behaviour that was not simply driven by the induced preferences.

2.5 A second experiment on qualitative voting

The above analysis sets out the following step in our experimental QV research agenda. In further experiments we need to increase the number of subjects in order to obtain a larger data set that would allow us a more accurate quantitative analysis. Similarly, we need to adapt our experimental design to overcome some of the observed weaknesses in the present pilot.

Regarding the latter aspect we need all observations to be useful for our analysis and it is obviously futile to propose an alternative to MR when this one is already replicating the first best allocation. In a first stage of our analysis we need to check whether QV is able to improve upon the allocation achieved by MR and this is only possible when the latter is not efficient. The second stage would then consist of characterising when MR is not efficient. In future research we need to tackle the first stage inducing the appropriate preferences where MR is not efficient instead of allowing the computer to draw them from a uniform distribution.

We also plan to normalise the players' payments so that their sum is constant. The purpose of this is twofold. On the one hand, we would ensure that the observations are comparable avoiding framing effects.[12] On the other hand, it introduces a constraint imposed by incentive compatibility into the computation of the efficient outcome —thus making our upper bound in welfare (the efficient payoff) closer to what is implementable. The next example best captures the latter phenomenon.

[12] Framing effects imply that voters may behave differently when they are assigned payments $(1,2)$ or $(50,100)$. We want to abstract from such framing issues which have been broadly analysed in many different settings –see the seminal reference Kahneman and Tversky (1983).

Example: Consider a conflict resolution situation with two issues and imagine that the two players' payments/preferences profiles are : $(90, 60)$ and $(-10, -40)$. The efficient allocation requires both issues to be approved and the efficient utilitarian social welfare is 100. Nevertheless, approving both issues is not incentive compatible and the implementable allocation that maximises the sum of the voter's utilities consists of approving the first issue and dismissing the second one (that is, it requires not undertaking interpersonal comparisons of utility and considering instead which is the most preferred issue by each player).

Normalisation implies that the payments are $(60, 40)$ and $(-20, -80)$. Hence the efficient outcome coincides with the implementable efficient one.

In future work we may also consider relaxing the informational assumptions. We may first consider disclosing how many voters are in favour or against each of the issues before subjects cast their votes. This may make our treatment more appropriate for certain situations where committee members almost certainly know the stand of their opponents but may not know the intensity of their preferences. This information may also overcome the inefficiency of QV in the present setting where voters may be *wasting* some votes on issues were a unanimous will exists. Subsequent experiments may disclose additional information about the opponents' intensity of preferences.[13]

The variation in the group size never seemed to be relevant in our data. This may be due to the fact that the information about group size was only conveyed through a small number in a corner of the computer screen. We may be able to disclose this information in a more explicit way without manipulating subjects' behaviour.

2.6 Conclusion

This Chapter is a first step towards a thorough experimental analysis of the voting system we call Qualitative Voting. As opposed to majority rule, QV allows voters to express their relative preferences and hence their *willingness to influence*. In Chapter 1, we have

[13] The experimental literature on voting has mostly focussed on how information affects the strategic behaviour of voters. For instance Eckel and Holt (1989) or McKelvey and Ordeshook (1981) emphasize that more information may complicate the game hence reduce strategic behaviour. Another strand of the literature has pointed out, instead, that information may act as a coordination device Forsythe *et al* (1993). In our companion theoretical paper we have proved that QV is very sensitive to the informational structure. Namely, it is not possible to find a mechanism that is sensitive to the voters' intensity of preferences, satisfies the unanimity property and is robust to any specification of the priors. Be that as it may, there is still scope to design an experiment with complete information to realise how subjects react to such environment.

shown that in a setting with two or three voters, two issues and uniform independent priors, QV Pareto dominates majority rule. Furthermore, it achieves the only ex-ante incentive compatible optimal allocation. The experiment in this Chapter tries to build on the theoretical findings from the first Chapter. For this, we provide each of our 18 experiment participants with an endowment of 30 votes to allocate freely across issues in a variety of settings (two, three, and six issues to be voted in groups of two, three, or six members).

We can conclude that subjects who belong to groups of size two or three play according to equilibrium predictions when voting towards two issues. That is, they invest most of their voting power in their most preferred issue.

More generally and in terms of welfare, the gains of QV with respect to MR are very noticeable in the conflict resolution situation but are diluted for potentially different reasons in the committee meeting examples. We also observe that players tend to focus their attention on a few number of issues when they face the decision over various issues. The welfare implications of this fact may vary a great deal depending on the preferences' and their prior distribution and open the question of the effect of agenda setting in our environment.

This Chapter suggests the basic measures to be used in subsequent work when analysing QV and, more generally, voting in multidimensional settings. A way to recognise how truthful voters are and, whenever they are not, whether they are diversifying or intensifying their voting power.

2.7 Appendix

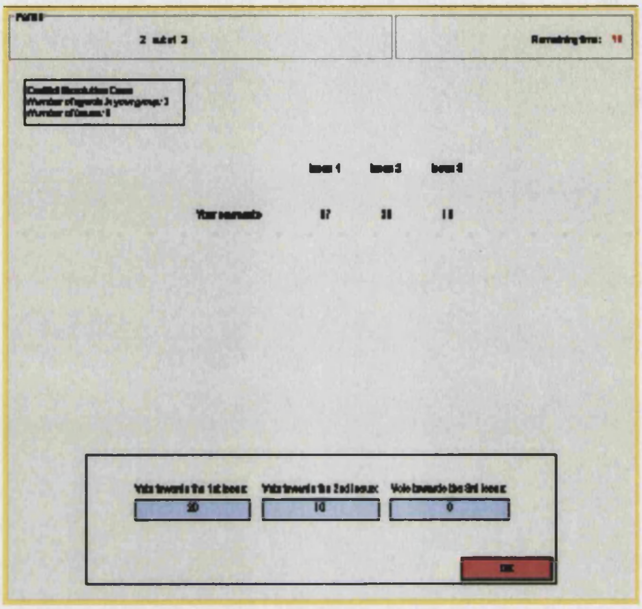

Figure A1

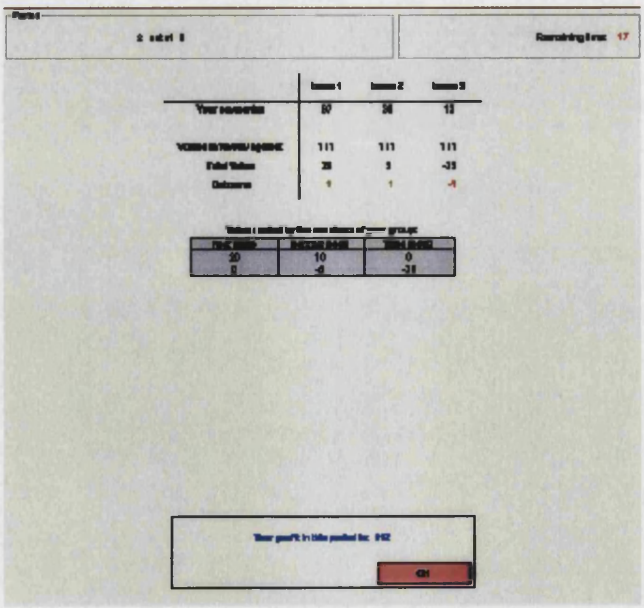

Figure A2

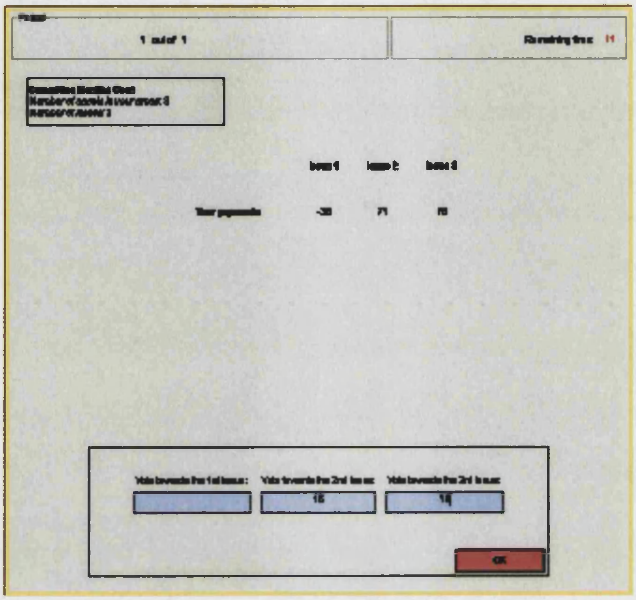

Figure A3

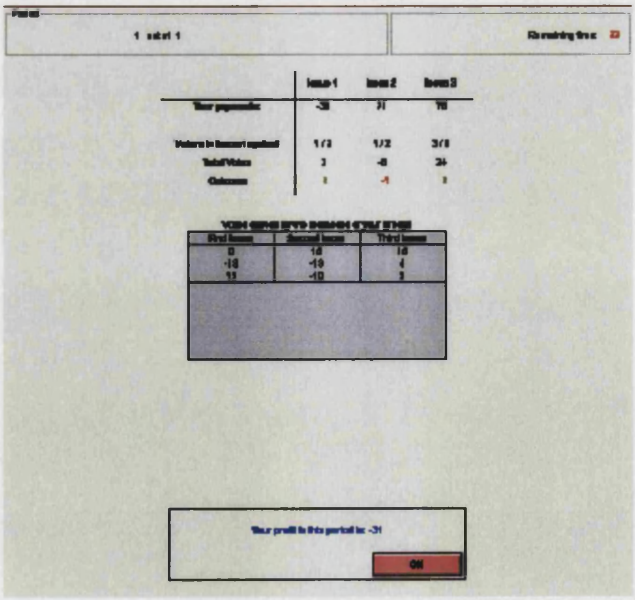

Figure A4

Part II

Strategic Politicians

Chapter 3

Voter Turnout and Electoral Competition

joint with Berta Esteve-Volart

3.1 Introduction

3.1.1 Motivation

Why do individuals vote, and how can political parties influence the individuals' decision to go to the polls? This Chapter aims at providing an answer to this question. In particular, we analyse how politicians anticipate the voters' decision of turning out to the polls and how they strategically react to this decision. In order to do so we offer a comprehensive theory of turnout and electoral competition. The underlying assumption in this Chapter is that one of the causes of the decline in turnout in established democracies can be found in the nature of electoral competition.[1]

Following the seminal work by Downs (1957), models of electoral competition have shown that politicians have incentives to move towards the median voter in order to attract at least half of the electorate. As a result, considering that rational agents turn out to vote only if the benefits of doing so outweigh its costs, we have that more electoral competition leads, *ceteris paribus*, to lower voter turnout.

[1] In the same way that competition between firms may erode their own profits, we show how competition between parties erodes their electoral support and hence decreases turnout.

We challenge this view by proposing a simple model *a la* Wittman (1977) where policy motivated candidates spatially compete to implement their announced policy. In our model, citizens only vote if candidates offer platforms that differ enough. If both candidates' platforms converge to the median voter's most preferred platform there are no gains from selecting one candidate over another, thus, no one turns out to vote given that voting is costly. As a consequence politicians offer differentiated policies (away from the median voter) to provide incentives for some voters to turn out at the polls — politicians anticipate abstention and they offer divergent policies. The main intuition behind our model is that the more the individuals' perceptions about the two candidates' platforms differ (with respect to their own policy preferences), the higher is the voter's incentive to vote. A related implication from the model is that moderate (as opposed to liberal or conservative) individuals tend to vote less.

This simple rationale is the key to our analysis. We first propose a simple model formalising that idea and we then analyse data from the United States' biennial National Electoral Studies (NES). We use the NES Cumulative Data File (Sapiro *et al* 2001), which allows us to explore elections held during the 1972-2000 period. Our empirical evidence supports our model's prediction; namely, that a perceived low difference between the platforms of both the Democratic and Republican parties tends to decrease an agent's probability to vote. In particular, we find that an increase from zero perceived difference to the maximum perceived difference between the two parties' ideology increases the turnout probability of about 10 percentage points. We find that this result is robust to the inclusion of a series of socioeconomic, demographic, and political controls, state level institutional controls, state and year fixed-effects, state-specific time trends, and to the model specification. This effect is larger if we consider only the perceived differences between presidential candidates: in that case, an increase from zero perceived difference to the maximum perceived difference between the two parties' ideology increases the turnout probability of about 14 percentage points. A lower but statistically significant effect also appears for differences in the perception of the parties' positions on several particular issues, such as guaranteed jobs, aid to African-Americans, and cooperation with the (now ex) USSR. Using the same set of controls, we also confirm that moderate individuals have a lower probability to turn out to vote. Finally, given the well-known overreporting concern for declared turnout in surveys such as the NES, we further check for the robustness of our results using only the set of validated votes —in all cases, our results do not change.

This Chapter introduces a straightforward and, to our knowledge, not yet analysed model of how political competition may affect turnout. Moreover, we hope that it helps rationalise the increasing lack of interest towards politics. In a complex political scenario with many different dimensions, political competition leads to the perception of only slight variations among platforms in each single dimension. Consequently, only voters who care about most of the issues, or that are politically educated enough, can perceive a difference between platforms that is large enough for their vote to be cast; the rest of the electorate simply abstains on the grounds that *all politicians are/offer the same stuff* or, as George Wallace put it in 1968:[2]

"There isn't a dime's worth of difference between the Democratic and Republican parties"

We are thus left with an inherent contradiction. A wealthy political system leads parties to compete and hence offer very similar platforms. In turn, this competition depresses voter turnout, which ultimately hinges on the legitimacy of the political system — given that a society's representatives are then being elected by only a minority of the population.

The rest of the Chapter is organised as follows. In Section 3.1.2, we relate this study to the more relevant existing literature. Section 3.2 introduces a model that explores parties' strategies under electoral competition facing the possibility of the electorate's abstention. In Section 3.3, we describe the data and in particular the variables more relevant to our analysis. In Section 3.4 we turn to the empirical evidence. Finally, Section 3.5 concludes and discusses some policy implications.

3.1.2 Related literature

The purpose of this Section is not to offer a comprehensive review on the theoretical and empirical literature on turnout and political competition. We therefore just highlight the most relevant papers for our analysis.

As mentioned above, it was Downs (1957) who introduced the most basic notion of spatial electoral competition and derived the well-known convergence result. Since then, the literature has offered many different ways to explain the divergence of political candidates: introducing politicians' uncertainty about voters' preferences (Calvert 1985 and Wittman 1983), considering the threat of a third party entering the election (Palfrey 1984), having parties determining the national policy when candidates compete in different constituencies (Eyster and Kittsteiner 2004, Callander 2003), considering dynamic incentives (Alesina 1988), admitting that any citizen could run for election (Besley and Coate 1997) or assuming an incumbency advantage (Bernhardt and Ingberman 1985). Our work can also be seen as an alternative way to overcome the convergence result by

[2] George Wallace was a third-party presidential candidate in 1968.

adding the turnout decision of the electorate to the spatial model of electoral competition. Nonetheless, below we offer no empirical evidence to support our model above any other model of party divergence. Elements from all of them could be at work.

Few authors introduce the turnout decision into a model of electoral competition. On the one hand, some authors proposed *probabilistic voting models* as a means to allow voters that are indifferent between the platforms to abstain with some probability (Hinich and Ordeshook 1969, Hinich, Ledyard and Ordeshook 1972). This assumption smoothens the candidates' payoffs and ensures the general existence of two-candidate electoral equilibria. Nevertheless, the probability of abstention is not microfounded. Moreover, in equilibria both parties converge. On the other hand, Myerson (2000) proposes the *theory of large Poisson games* where the size of the electorate is supposed to be a random variable —with the drawback that leads to platform convergence and hence zero turnout whenever there is a unique policy that maximizes the electorate aggregate utility.

The empirical literature on voters' turnout spans several decades and countries. Related to our analysis in this Chapter, there exists a substantial literature on policy voting in US elections (see Sapiro for a review on literature using data from the NES). These papers try and assert whether the electorate is sensitive to difference in platforms —'issue difference'. While early work by Campbell *et al* (1960) and others argued that voters were not able to perceive differences between candidates' policies, subsequent evidence has been more favourable to the existence of such sensitivity (Aldrich *et al* 1989, Pomper 1972, Page and Brody 1972 with regards to the Vietnam war, and Palfrey and Poole 1987). There are several features that distinguish our Chapter from these previous studies. First, they usually focus on one election year, second, many of them do not address turnout but rather focus on the choice among Democratic and Republican candidates, and third, most of them do no control for other individual characteristics. Of these, Palfrey and Poole (1987) is the closest in spirit to this Chapter's findings' that more informed agents perceive platforms to be more different across candidates, and that more informed agents tend to vote more. However, Palfrey and Poole (1987) does not explore the connection between both features, which is the focus of our Chapter. In fact, our results suggest that controlling for information, individuals who perceive political parties to be more different are more likely to vote. That is, information is not the only link between perception about parties and vote.

3.2 The model

We consider a standard model of spatial electoral competition with two politicians that are policy driven. That is, they only care about the implemented policy by the government.

Regarding the voting decision of the electorate we consider the standard Downsian approach where citizens vote only if the benefits from doing so outweigh the costs. Hence, only if the following inequality holds,

$$p \cdot A \geq C$$

where p denotes de probability of being pivotal, A denotes the utility difference between the most preferred politician and the remaining one and C denotes the cost of voting (that is, the opportunity costs of going to the polls). We assume $p \in (0,1)$ and $C > 0$ to avoid trivial solutions.

There is a large debate in the literature about considering perfectly rational voters that compute their probability of being pivotal in a given election. Not only such an assumption is controversial but also implies infinitessimal benefits from voting when considering large electorates and hence turnout tends to zero (see, for instance, Palfrey and Rosenthal 1983). The fact that empirically there is an increase in turnout when elections are perceived to be close shows that there is indeed an effect, but the high turnout observed in the real world suggests that agents are not perfectly rational —or, putting it in Daniel Kahneman's words, they may be led by intuition and impulse rather than reason.[3]

We avoid such debate here by assuming that the probability of being pivotal is exogenously given and hence focus our attention on the simple rationale behind our analysis. Nevertheless, all our qualitative results would follow if we relaxed such assumption.

3.2.1 A unidimensional policy space

Consider a compact and convex unidimensional policy space $\mathcal{P} = [-1, 1]$. There is a continuum of voters whose preferred policy is uniformly distributed along $\mathcal{P}$. Voter i's preferred policy is denoted g^i. Whenever a policy is implemented, voter i gets a disutility equal to the distance between the implemented policy and her preferred one:[4] $U_i(g) =$

[3] See Daniel Kahneman's 2002 Nobel Prize lecture.

[4] The assumption that an agent's utility function is linear in the distance between the policy implemented and the agent's most preferred policy is essential to obtain theoretical predictions that are independent of the priors considered. Assuming concave utility functions in an electoral model with turnout decision leads politicians to shift their offered policies to those points where a mass of the electorate is

$-d(g,g^i) = -|g-g^i|$.

The policies are implemented by an elected politician. There are two polarised candidates L (left) and R (right) that compete in order to implement their announced policies. Candidates only care about the policy implemented by the government as first proposed by Wittman (1977). Left and right candidates' preferred polices are equal to -1 and 1, respectively.

The described setting contrasts with the Downsian model of electoral competition where politicians are solely driven by their desire of winning office. This adds an additional component to the analysis given that politicians do not solely want to offer a policy that attracts at least half of the electorate but they care about the offered policy. Having policy-driven polarised candidates introduces a trade-off because politicians need to move towards the median voter to attract most of the electorate but this has an embedded cost because they are moving away from their ideal policy. Below we detail the robustness of our results to the case of politicians that are also rent-seeking.

Both candidates simultaneously announce a platform to the electorate ($g^\kappa, \kappa = L, R$) which is credible in the sense that they are able to commit to implement such policy if they are elected. The winner is decided by plurality voting. That is, the winner is the candidate that obtains the highest share of votes. In case of ties a fair lottery is played and both announced policies are implemented with equal probability. We can define the expected payoff of candidate L as

$$P_L(g^L,g^R)\cdot U_L(g^L) + P_R(g^L,g^R)\cdot U_L(g^R)$$

where $U_L(g) = -d(g,-1)$ and $P_\kappa(g^L,g^R)$ is the probability of candidate κ winning the election given the announced platforms (g^L,g^R). Note that $P_L(g^L,g^R) = 1 - P_R(g^L,g^R)$ for all $(g^L,g^R) \in \mathcal{P}\times\mathcal{P}$. The expected payoff of candidate R is defined analogously.

Next we describe the second stage of the game. In this stage, agents need to decide whether to abstain, vote for candidate L, or vote for candidate R once politicians have announced their respective platforms.

For any agent i the benefits from voting are

$$p\cdot\left|U_i(g^L) - U_i(g^R)\right|$$

where p is the probability of voter i being pivotal. Given that p is exogenous we have

concentrated (i.e. in general, the set of agents voting for a candidate is in this case a closed set strictly included in $\mathcal{P}$).

that an agent votes if and only if

$$\left|U_i(g^L) - U_i(g^R)\right| \geq \frac{C}{p} =: c.$$

Whenever an agent votes she does so in favour of her preferred candidate —the one closest to her bliss point. In case she is indifferent, she votes for each candidate with equal probability.

The former description induces a two-stage extensive form game. Solving it by backward induction we can easily prove the following result.

Proposition 3.1 *There exists a unique equilibrium in which the candidates' announced policies diverge;* $(g^L, g^R) = \left(\frac{-c}{2}, \frac{c}{2}\right)$.

Citizens to the left of $\frac{-c}{2}$ *vote for candidate* L*, those to the right of* $\frac{c}{2}$ *vote for candidate* R*, and those in between abstain.*

Proof. In equilibria, $P_L(g^L, g^R) = P_R(g^L, g^R) = \frac{1}{2}$. This follows immediately from realising that if, say, candidate R wins the election with certainty then candidate L is better off by proposing a platform infinitesimally to the left of g^R. These offered platforms induce no turnout hence both candidates win the election with equal probability thus candidate L is better off.

Given that candidates are policy driven there is a *centrifugal* force that will tend to separate them as much as possible preserving the condition that $P_L = P_R = \frac{1}{2}$.[5] That is, the distance between both platforms is in equilibrium equal to c, or $d(g^R, g^L) = c$. Moreover, given our distributional assumptions (the electorate is uniformly distributed on $\mathcal{P}$) we have that the equilibria should be symmetric—both candidates need to be equidistant from the median voter.[6] Hence, the only announced policies by candidates in equilibrium are $(g^L, g^R) = \left(\frac{-c}{2}, \frac{c}{2}\right)$.

The proof ends with the observation that the former pair of announced policies is indeed an equilibrium. Note that a deviation towards the median voter by any of the candidates entails a cost while does not change her probability of winning; conversely, a deviation

[5] In spatial models of electoral competition a *centrifugal* force (as opposed to a *centripetal* force) is usually understood as an incentive for politicians to offer differentiated platforms or platforms away from the median voter.

[6] This result heavily relies on the assumed symmetric distribution. For instance, a non-symmetric distribution implies asymmetric offered platforms.

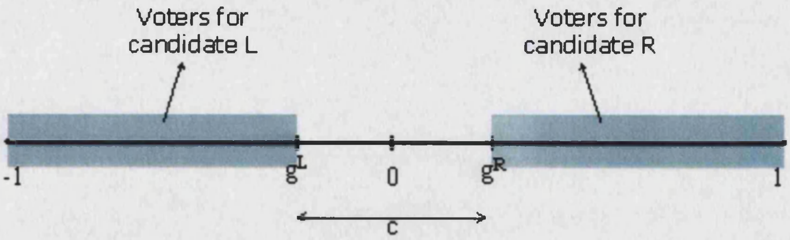

Figure 3.1: The unidimensional case

towards the extremes makes the deviating candidate lose the election with probability one. ■

Figure 3.1 depicts the equilibrium policies and the equilibrium behaviour of individuals. It is relevant to realise that voters in centrist or moderate positions abstain given that politicians are considered to be too similar for them to be worth voting. The voter with bliss point $g^i = 0$ sees both platforms as equally apart from her most preferred one and hence gets null benefit from going to the polls. Note also that both candidates extract all possible gains from the abstention by pulling their offered platforms towards their most preferred ones (that is, towards the extremes). Abstention introduces a centrifugal force that counteracts the centripetal force that arises from electoral competition and candidates diverge exactly by c inducing a proportion of $\frac{c}{2}$ of the electorate to abstain. A higher voting cost increases abstention but also increases divergence between platforms.

There has been a persistent concern in the existing literature about the convergence result obtained in the standard model of electoral competition. The simple model above can be seen as a new way to explain divergence in a model of electoral competition without the need of introducing uncertainty. Regarding the set of voters that abstain, the result is also consistent with US data on elections where we can observe that non-voters tend to have more centrists preferences than voters do, we come back to this point in Section 3.4.4.[7]

Note also that whenever politicians are only motivated by winning office the strategies described in Proposition 3.1 are still an equilibrium given that no candidate can improve unique, any pair of policies in the interval $\left[\frac{-c}{2}, \frac{c}{2}\right]$ is an equilibrium. This is because a party that offers a policy on the former interval ensures that the opposing party can never attract more than half of the electorate. Introducing an infinitessimal preference towards the implemented policy and assuming that politicians are sufficiently polarised leads to our uniqueness result because candidates select the described equilibrium out of all possible pairs from the interval $\left[\frac{-c}{2}, \frac{c}{2}\right]$.

3.2.2 Heterogeneous voters

As a first extension from the previous model assume that there are two class of voters: those with high opportunity costs of voting and those with low opportunity costs ($c^h > c^l > 0$). We further assume that the distribution of voting costs and policy preferences across the population are independent.

Given the above description of voters, we can reinterpret high cost voters as voters who are less sensitive to the difference between platforms. Recall that a high opportunity cost voter turns out to vote if and only if $\left|U_i(g^L) - U_i(g^R)\right| \geq c^h$. Equivalently, she turns out to vote if and only if $\lambda \cdot \left|U_i(g^L) - U_i(g^R)\right| \geq c^l$ where $\lambda = \frac{c^l}{c^h} < 1$. This allows us to reinterpret heterogeneous voters as citizens with equal voting costs who derive different marginal utilities from the distance between platforms. Thus, agents with higher costs are less sensitive to the distance between platforms.

A similar reasoning as before shows that there exists a unique equilibrium in which politicians diverge by a distance c^l. To show that this is the case imagine there is an equilibrium such that the distance is strictly larger than c^l. In that situation one of the candidates can unilaterally move inwards by an $\varepsilon > 0$ small enough such that the distance between platforms is still higher than c^l. Clearly there still is a positive mass of agents turning out to vote, but the candidate that deviated is now strictly closer to the median voter than her opponent —and hence wins the election with certainty. Consequently the only equilibrium looks like the one with homogeneous voters but the distance between platforms is now the minimal distance that makes some voters turn out to vote. Note that this result is independent of the mass of low opportunity costs voters. The following Proposition summarises this result.

Proposition 3.2 *There exists a unique equilibrium in which the candidates' announced policies diverge;* $(g^L, g^R) = \left(\frac{-c^l}{2}, \frac{c^l}{2}\right)$.

Low cost citizens to the left of $\frac{-c^l}{2}$ *vote for candidate L, low cost citizens to the right of* $\frac{c^l}{2}$ *vote for candidate R, and citizens in between abstain. All high cost citizens abstain.*

We can therefore conclude that electoral competition leads politicians to target only the most sensitive groups. The situation where the whole population is targeted does not constitute an equilibrium because politicians can unilaterally deviate towards the median voter making the whole set of less sensitive population abstain and capture more than half of the most sensitive voters. The deviating party wins the election at an infinitessimal cost and overall we observe a decrease in turnout.

Corollary 3.1 *Electoral competition leads politicians to target only the most sensitive voters, hence less sensitive voters abstain.*

There is an immediate consequence of this simple model that contrasts with the existing literature. In case that the two sets of voters have sufficiently different median points the model leads to non-existence of equilibria. Hence we have a unidimensional model with single-peaked preferences with the slight modification of introducing the turnout decision of heterogeneous voters that leads, as opposed to most of the literature in unidimensional spatial electoral competition, to non-existence of equilibria.

Heuristically we have provided a rationalisation for most stylised facts in voter turnout. It has been found (see Section 3.1.2) that a greater involvement in social institutions or a higher level of education or income increases the likelihood of citizens turning out to vote. We can build a bridge between these facts and our model by assuming that voters that are more involved with social institutions, more educated or richer tend to be more sensitive to the offered policies by politicians and hence, following Corollary 3.1, tend to vote more given that political competition tends to target these groups of citizens.

Next we turn to extending the rationale of Corollary 3.1 to a multidimensional policy space where its applicability and relevance to the real world becomes clearer.

3.2.3 A multidimensional policy space

Politicians do not solely offer policies on a single dimension. Party programmes have reached a level of complexity that very few voters (if any) can grasp. Pennings (2002) compares the *manifestos* of most political parties in the European Union and shows that the policy space is composed by strictly more than two dimensions. Hence the unidimensional case, though useful in some instances, cannot be treated as a mirror of the complex political arena. Building further on such a model is also consistent with the belief that parties can no longer be classified only through a unidimensional left-right scale.

The problem of a multidimensional model from a theoretic perspective is that ensuring the existence of equilibria requires very strong conditions on the distribution of voters.[8] In the present Chapter we avoid such questions by choosing a uniform distribution.

As important as the dimensionality of the policy space is the fact that most voters are only interested in a few issues. And most importantly, different voters may care about different issues. That is precisely the driving force of our analysis. Given that electoral competition targets only the most sensitive voters, we have that in a multidimensional world the distance between policies is the minimal distance that attracts those voters. Consequently, the perceived distance between platforms by a voter who only cares about a few issues does not generally compensate for her opportunity cost of voting.

Electoral competition in a multidimensional world provides the right incentives to those individuals who care about the whole set of issues. The rest perceive "politicians as being all alike" and, as a result, show no interest towards politics and ultimately abstain.

Consider an extension of the model above to best capture this intuition. Assume a bidimensional policy space ($\mathcal{P} = [-1, 1]^2$) and the preferred policies of L and R politicians to be $(-1, -1)$ and $(1, 1)$, respectively. Moreover suppose that the first issue regards some aspect of the welfare state that all voters care about. The second issue, instead, regards some aspect that only concerns a proportion $\alpha \in (0, 1)$ of the population (e.g. an issue that only concerns particular interest groups such as abortion laws, university tuition fees or the use of stem cells). More generally we could consider that the relative intensity across issues varies among different groups of voters.

The most sensitive voters in this scenario are those who care about both issues. Their disutility from an implemented policy g is computed according to the following distance:[9]

$$U_i^h(g) = -d(g, g^i) = -\left(|g_1 - g_1^i| + |g_2 - g_2^i|\right).$$

The less sensitive voters' preferences are computed as in the unidimensional setting:

$$U_i^l(g) = -d(g, g^i) = -|g_1 - g_1^i|.$$

Assume all voters have an identical cost of voting $c > 0$, g is uniformly distributed on $\mathcal{P}$

[8] Intuitively, for an equilibrium to exist in the multidimensional space we need the distribution of preferred policies to have a median point (i.e. a point that belongs to all hyperplanes that separate the space in two equal parts given the provided prior distribution). For further reference, see Austen-Smith and Banks (1999) and references therein.

[9] This distance is induced by the norm sub one or taxicab norm.

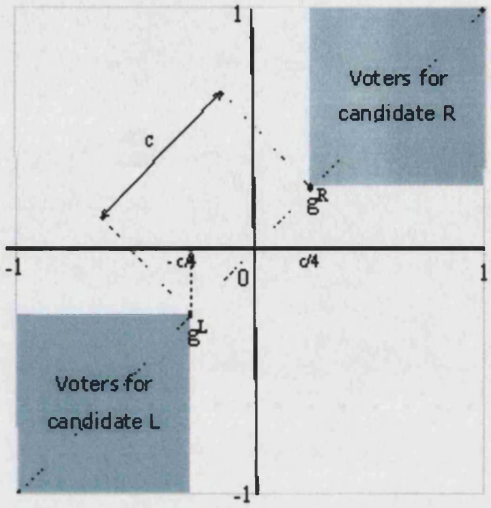

Figure 3.2: The multidimensional case

and the probability of being a voter that cares about both issues (α) is independent of the policy preferences. The following Proposition characterises the unique equilibrium.

Proposition 3.3 *There exists a unique equilibrium in which the candidates' announced policies diverge;* $(g^L, g^R) = \left\{\left(\frac{-c}{4}, \frac{-c}{4}\right), \left(\frac{c}{4}, \frac{c}{4}\right)\right\}$.

Citizens who care only about one issue abstain while citizens who care about both issues vote for L if and only if $g < \left(\frac{-c}{4}, \frac{-c}{4}\right)$, *and for R if and only if* $g < \left(\frac{-c}{4}, \frac{-c}{4}\right)$.[10]

The proof is analogous to the proof of Proposition 3.2. Once again we see that politicians adapt their policy to the most sensitive voters and only voters to either side of the offered platforms vote for their closest party. Voters in between as well as less sensitive voters abstain (see Figure 3.2).

At this point we can propose a new explanation for the lack of interest towards politics. Political parties have increasingly targeted their policies on special interest groups introducing new issues into the political agenda in order to attract those voters who only care about these issues. This increase in the number of issues, together with the fact that political competition tends to respond only to the most sensitive voters (as pointed out in Corollary 3.1), makes politicians differ only marginally on each issue. As a consequence, only those voters who care about most of the issues or only those who are politically well

[10]The vector inequalities should be interpreted componentwise (i.e. $g > 0 \Leftrightarrow g_n \geq 0, \forall n$).

educated can perceive enough difference between political parties relative to their own political preferences to show up at the polls. The rest of the population cannot perceive any big difference between political parties and abstain on the grounds that *politicians are all the same* and hence there is no real difference between voting for any one of them.

3.3 Data and descriptive analysis

We test the model with data from repeated cross-sections from the American biennial National Electoral Studies (NES), in particular the NES 1948-2002 Cumulative Data File (Sapiro *et al* 2001).

The data base contains detailed information on voter turnout and participation, their perceptions about political parties, their ideology, their support of the political system, and demographic characteristics of individuals. Our broad usable sample spans the 1972-2000 period. We construct our variables of interest from information about the individuals' positioning on specific issues, and about how individuals perceive the Democratic and Republican parties in the United States with respect to those issues.[11] The issues on which the respondent is interviewed are: defense spending, health care, guaranteed jobs, aid to African-Americans, rights of the accused, women's equal rights, government services/spending, cooperation with the USSR, urban unrest, and school busing. These questions are asked in different periods which determines a different sample size in each case. Additionally, there is a general question on the respondent's ideology.[12]

In Table 1 we provide descriptive statistics about the scores for all these variables.

Some information about the intensity of the respondent's preferences is given as well.[13] The list of options and a description of how we mapped them to the set of issues described above is in the Data Appendix. We also have information on the degree of partisanship of respondents from several questions.[14] Similarly, we include a proxy for the degree to

[11] For instance, the question regarding the issue of defense spending reads as follows: *Some people believe that we should spend much less money for defense. Where would you place yourself on a* [seven point] *scale, or haven't you thought much about this? Where would you place the Democratic (Republican) party on this scale?* In this given example, scores number one and seven correspond to *greatly decrease defense spending* and *greatly increase defense spending*, respectively.

[12] *We hear a lot of talk these days about liberals and conservatives. Here is a seven-point scale on which the political views that people might hold are arranged from extremely liberal* [score equal to one] *to extremely conservative* [score equal to seven]. *Where would you place yourself on this scale, or haven't you thought much about this?*

[13] *What do you think that are the most important problems facing this country? Of all you've told me, what would you say is the single most important problem the country faces?*

[14] In one of the questions regarding partisanship respondents are asked: *Generally speaking, do you usually see yourself as a Republican, a Democrat, an Independent, or what?* again, a scale is given that

which respondents consider voting a duty.[15] As a further political control, we use an index of political participation.[16]

Other variables that we consider in the analysis are newspaper information,[17] the income and education levels of the respondent, and demographic variables such as racial/ethnic group, age, gender, religious affiliation, frequency of church attendance, marital status, and number of children in the household. Table 2 shows descriptive statistics for this second set of variables.

Finally, we use some state-level variables from Besley and Case (1993) as controls. As institutional controls, we introduce the type of registration that voters can have (that is,, whether an individual can register during the polling day or not, or whether conventional registration is available only), whether citizens' initiatives are permitted, and whether the state has restrictions on corporate campaign contributions. Additionally, we also control for voting age population.

3.4 Empirical analysis

3.4.1 Basic results

The empirical analysis explores how the different individuals' perceptions about political party platforms have impacted on voter turnout. In our main specification we estimate the following Probit regression:

$$v_{ist} = \alpha_s + \gamma_t + \tau_s + \beta A_{ist} + \phi X_{ist} + \varphi W_{st} + \varepsilon_{ist}$$

ranges between one (*strong Democrat*) and seven (*strong Republican*), with Independents in the middle.

[15] They are asked whether they agree or disagree with the following statement: *If a person doesn't care how an election comes out then that person shouldn't vote in it.* We interpret those who disagree as having a higher sense of duty, as opposed to those who agree with the statement.

[16] Respondents are asked the following questions. 1. *During the campaign, did you talk to any people and try to show them why they should vote for or against one of the parties or candidates?* 2. *Did you go to any political meetings, rallies, fund raising dinners, or things like that in support of a particular candidate?* 3. *Did you do any (other) work for one of the parties or candidates?* 4. *Did you wear a campaign button, put a campaign sticker on your car, or place a sign in your window or in front of your house?* 5. *Did you give money to a political party during this election year? Did you give money to an individual candidate running for public office?* 6. *Have you ever written a letter to any public officials giving them your opinion about something that should be done?* Each answer gets one or zero points depending on whether the respondent answers yes or not respectively. We then use the sum of the points in all six questions as an index of the political participation of that individual.

[17] *Did you read about the campaign in any newspaper?* For 1988, and from 1992 onwards, this question was formulated as *[If the respondent has read a daily newspaper in the past week:] Did you read about the campaign in any newspaper?* For other years, the question was not restricted to a specific period of time.

where v_{ist} is the voter turnout variable for individual i in state s and year t (which is equal to one if the individual declares that she has voted, and equal to zero if she declares that she has not), α_s is a state fixed effect, γ_t is a year fixed effect, τ_s is a state-specific time trend, A_{ist} is the perceived distance between party platforms, X_{ist} is the vector of individual demographic and socioeconomic controls, and W_{st} is a vector of state controls.

In all regressions we cluster our standard errors by state so as to avoid potential problems with autocorrelation (Bertrand *et al* 2003).

While the theoretical section clearly distinguishes between the unidimensional and multidimensional policy spaces, our data base does not allow for such a clear-cut distinction. We show below how our results are consistent with both dimensional spaces.

The most straightforward way to test the implications of our model is to use the policy dimension that most closely summarises the policy preferences of our agents. We have information on how agents perceive their own preferences and the preferences of the Democratic and Republican parties respectively on a number of issues: first, a broader liberal/conservative scale, and second, on a list of more specific aspects such as government health insurance, guaranteed jobs, aid to African-Americans, women's rights, government services/spending, cooperation with the USSR, and defense spending (see more information about these categories and scores in Section 3.3). We now focus on the first characteristic, which we may consider more ideological, and construct our main variable of interest, A, as follows. Recall that in the unidimensional setting, individual i votes whenever $\left|U_i(g^L) - U_i(g^R)\right| \geq c$, that is, whenever the utility distance between the two platforms is large enough. Accordingly, we define

$$A_i = \left|\left|s_i^D - s_i\right| - \left|s_i^R - s_i\right|\right|$$

where s_i is the score given by individual i about her preference on that issue, and s_i^κ is the score given by i about party κ's policy position. Thus, A_i is a measure of i's perceived distance between party platforms with respect to her preferences.

In Table 3 we present our main empirical findings. Our goal is to show that the perceived distance between party ideological platforms with respect to i's preferences can significantly affect the probability of turning out at the polls. For this purpose we run several Probit regressions. In columns 1 and 2 we present the estimated coefficients without controls, with just fixed effects and state-specific time trends. Our variable of interest is statistically significant at the 1 percent level. In column 3, we introduce our main political, socioeconomic and demographic individual controls: the perceived ideological distance between the two parties' platforms is still significant at the 1 percent level. To have an idea of the magnitude of the effect, the same column tells us that the fact that somebody has read in the newspaper about the campaign increases the probability of her voting of 11 percentage points. This result is consistent with previous evidence that information matters (e.g. Palfrey and Poole 1987).[18] We find that the fact that a person perceives both ideological platforms as being very different (take the maximum distance which equals six) implies that, *ceteris paribus*, her probability of voting is 10 percentage points larger than that of a person who sees no difference between the two platforms. In short, the effect of maximum perceived distance is of about the same order of magnitude as the effect of having read about the campaign on the newspaper. Other interesting, although not new, results follow: individuals vote more often in presidential elections, the probability of voting increases significantly with the degree of partisanship, older, richer, and more educated individuals tend to vote more, while divorced individuals tend to vote less. We also find that the more often a person goes to church, the higher her probability of turning out at the polls. Consequently we might interpret church attendance as part of an individual's social or community involvement.

We refer to the result in column 3 as our benchmark estimate. In Table 4 we proceed to include further controls. In column 1, we introduce our proxy for voting as a sense of duty. This variable has a positive and statistically significant effect on turnout; however, our variable of interest is still significant at the 1 percent level and its estimated effect is of about the same size as in Table 3. In column 2, we introduce our index of political participation: our results also hold true. Finally, column 3 includes a series of state controls from Besley and Case (1993). Although contrary to intuition and previous evidence, most of these variables appear to have negative signs in this regression—we will come back to their effect once we evaluate validated turnout (instead of only focusing on reported turnout, as has been the case so far).

3.4.2 Robustness checks

3.4.2.1 Validated turnout

It has been argued that studies using reported turnout, such as the NES, may suffer from an overreporting problem (Burden 2000). In our case, we calculate real turnout to be 4 to 17 percentage points lower than reported one depending on the vote validation method used (Table 2). There is a possibility that these could be affecting our results: other

[18] In our case, this could be due to either agents being more informed about parties' platforms through the newspaper (and this is an effect our variable of interest, A, is presumably picking up), or due to the fact that reading about the campaign reminds voters of the upcoming election.

studies suggest that voters who falsely report their turnout tend to be different from the population at large, that is, more educated, and older (Silver *et al* 1986).

In order to discard the possibility that false reports are driving our results, in Table 5 we re-estimate columns 3 in Table 3 and Table 4 using only *validated* turnout. Table A1 gives information on the real voting patterns of individuals interviewed by the NES. There are mainly three possibilities: 1) that a person's record was found and that it was confirmed that she did vote (60% of observations), 2) that the person's registration record was found, without a record of that person voting (19% of observations), and 3) that a registration record was not found, and neither a record of her vote.[19]

In our first method of vote validation, we consider votes under 1) to be valid, and votes under 2) to be false, and hence we classify the latter as abstentions. Votes under 3) remain as missing observations. That is, this method considers only votes for which we have complete information. In our second method of validation, we suppose that votes under 3) are also false votes. The rationale of these two methods is that the reality is probably somewhere in between these two hypotheses. Finding that our results do not change under any of these two extreme possibilities would therefore give more credit to our results. Table 5 shows that our results are robust to the inclusion of only validated votes under either validated method. In fact, our key estimated coefficient is very similar to our benchmark coefficient (column 3 in Table 3). In sum, overreporting does not seem to be driving our results. We have checked all our regressions with only the validated samples and our results are still preserved. We have included some of these checks in the Tables in the Appendix. The negative signs for some of the institutional variables are actually positive once we only consider validated turnout: for instance, *ceteris paribus*, states that allow for polling day registration and citizens' initiatives tend to have higher turnout, while restrictions on corporate campaign contribution now has a negative effect on turnout (see Table A2).

3.4.2.2 Alternative models

We perform a further robustness check by making sure that our results are not driven by the Probit model. Columns 1 and 2 in Table 6 respectively provide estimates under the Logit and linear probability models. Both models throw positive significant relationships between our variable of interest and turnout.[20]

[19] There were also a number of observations in which individuals reported not voting while the validation apparently confirmed their voting. We have interpreted these as missing values. However, our results do not change if we consider those observations as "validated votes".

[20] Table A3 in the appendix provides the respective validated turnout regressions.

3.4.3 Other policy dimensions

In Table 7 we take advantage of information on the individuals' perception on the distance between parties across other issues.[21] Table 7 shows Probit regressions with other A_i's which correspond to other specific dimensions, such as government health insurance. We would expect to see an effect both if there is enough variability in the way that people perceive distances between the parties in these dimensions, and if these issues are of importance to the population. We do find a statistically significant effect in three of these additional issues: guaranteed jobs, aid to African-Americans, and cooperation with the (now ex) USSR.

In Table 8, we run similar regressions but now only focusing on the individuals' perceptions about *presidential candidates'* (instead of political parties') different platforms with respect to their preferences. In column 1, we use our benchmark liberal/conservative dimension: we estimate a somewhat larger effect than in the case of political parties: while the effect was an approximate 10 percentage point increase from no difference to maximum difference, now this is estimated to be about 15 percentage points. This could be either due to the fact that people vote differently in the case of a presidential election, or to the fact that people have a more accurate perception of platform difference (given that platforms are mainly represented by a sole candidate instead of different candidates per state). Consistent with this larger broad effect, we find larger effects in the other dimensions too (in fact, differences in the perception of how candidates see government services/spending, and defense spending, seem to also matter for turnout if we only consider presidential candidates).

In sum, the evidence from these two tables confirms that there is a relationship between perceptions of platforms and voting behaviour in more than a pure ideological sense.

If the premise that citizens vote on the grounds of a multidimensional policy space is true, we would expect to find a higher effect when we only consider each individual's most important issue instead of the more general liberal/conservative scale —indeed, an individual's voting decision should react more with respect to the problem she considers most important. This is what we consider next.

The NES contains information on what every respondent considers the most important problem (*What do you think are the most important problems facing this country? Of all you've told me, what would you say is the single most important problem the country faces?*). Ideally, we would like to have a list of options that relates the answers to this

[21] Correlations between A on the liberal/conservative scale and the As in these other dimensions are in the range 24-36%.

question to the list of policy issues in various dimensions that we have seen in Tables 6 and 7 and for which we have information on perceptions of individuals.However, the list of answers that respondents were posed with does not provide nearly as good a mapping of subjects.[22]

Table 9 shows results considering the seven-point scale score reported by the individual in (what we map as) her described most important problem.[23] Columns 1-3 in Table 9 show that there is a positive significant effect between either of these three measures of A for the most important problem, and voter turnout. However, we also observe that the estimated effect is smaller than our benchmark effect (Column 3, Table 3). This is surprising: if our measure is well constructed, we would expect that taking into account the intensity of preferences would make our estimated coefficient larger —that is, in view of Section 3.2.2 we would expect the probability of an agent's voting to be more sensitive to the issue she considers most crucial. We think that the reason for this relatively low coefficient may be two-fold. First, it is possible that our less-than-ideal mapping generates measurement errors and hence the key estimates in Table 9 suffer from attenuation bias. Second, and perhaps more importantly, we think that it might be the case that the liberal/conservative dimension may already represent every individual's most important problem better than our mapping: in what would be an example of *framing*.[24] It is possible that for example a mother concerned with the quality of public education for her children may find the country's most important problem to be public education, and identifies her views of the parties' political ideological positions under that frame (that is, categorising the Democratic party as very liberal if she thinks that the Democrats substantially promote public education, while categorising the Republican party as very conservative if she thinks that the Republicans do not promote public education). This view is consistent with the unidimensional model in Section 3.2.1 and can also represent a framing of the intensity of preferences consistent with the multidimensional model in Section 3.2.3.

[22]For instance, "agriculture" was an answer option for the most important problem however, we do not have a seven-point scale to capture the positioning and perception of the respondents. Similarly, we have scores for different issues corresponding all to the same most important problem category. For example, the public order category includes the issues of urban unrest, rights of accused, and women's equal rights. The mapping of most important problem categories and seven-point scales is described in the Data Appendix.

[23]In the cases that we have more than one A for a given most important problem, we have calculated the regressor in three ways. First, we use the maximum of the, say, several seven-point scales differences, second, we use the minimum, and finally, we take the average of As.

[24]There is a burgeoning literature in both (behavioural) economics and psychology that finds that the way in which questions or situations are posed critically influences an outcome. See e.g. McNeil *et al* (1982) for an application to medicine and Kahneman and Tversky (1983) for an application to decisions involving risk and monetary payoffs.

3.4.4 Voting patterns of moderates

Another implication of the model is that individuals who are in the centre of the political spectrum (see Figures 3.1 or 3.2) tend to vote less than individuals who are in either extreme. In order to test this, we classify voters into moderate and non-moderate according to two criteria.[25]

Table 10 shows that under both classifications we find that moderates tend to vote less than non-moderates.[26]

3.5 Conclusion

This Chapter tries to build on the existing knowledge on voter turnout but analysing the interaction between electoral competition and voters' turnout decision. Models of electoral competition have since Downs (1957) shown that political parties have incentives to converge on policy platforms in order to attract at least half of the electorate. In this context, more electoral competition would lead to lower voter turnout.

The theoretical section in this Chapter challenges this setting by providing a simple model *a la* Wittman (1977) where policy driven candidates spatially compete to implement their announced policy. In our model, citizens only vote if candidates offer platforms different enough (if both candidates converge to the median voter's preferred platform there are no gains from selecting one candidate over another, thus, no one turns out to vote given that voting has a cost). If politicians' preferences are polarised they strategically choose divergent policy platforms. In our two-party model, citizens who position themselves relatively to the left or right of the political spectrum vote for the party that is closer to them, while citizens who position themselves in the central, more moderate, positions abstain. Allowing for heterogenous voters does not alter this finding. In a refinement of this unidimensional model, we develop a multidimensional policy space model that allows for additional dimensions of interest to voters. In the same spirit as the unidimensional model, we find that voters who are more sensitive to the policy issues at stake vote. Similarly, moderate voters decide to abstain.

[25] We use information from the question: *When it comes to politics, do you usually think of yourself as 1) extremely liberal, 2) liberal, 3) slightly liberal, 4) moderate or middle of the road, 5) slightly conservative, 6) extremely conservative?* Under the first criterion, we classify first strict moderates as those individuals who position themselves in category 4), and then broad moderates as those who position themselves in categories 3), 4) or 5), that is, we include both slight liberals and slight conservatives in our second measure of moderate voters.

[26] Table A4 in the appendix provides the corresponding validated turnout regressions.

Thus, the main intuition derived from our model is that the bigger is the difference in the individuals' perceptions of the two candidates' platforms (relative to the voter's policy preferences), the higher the incentive to vote. A related implication from the model is that moderate (as opposed to liberal or conservative) individuals tend to vote less.

We test these two implications of our model using data from the United States' National Electoral Studies for 1972-2000. These data base contains respondents' perceptions about the Democratic and Republican (as well as their own) spatial positions in the political spectrum.

Our empirical evidence supports our model's prediction: namely, that a perceived low difference between the platforms of both the Democratic and Republican parties tends to decrease an agent's probability to vote. We find that an increase from zero perceived difference to the maximum perceived difference between the two parties' ideology increases the probability of turnout of about 10 percentage points. This is similar in size to the effect of newspaper information on voter turnout, as measured in the same data base. Our result is robust to the inclusion of a series of socioeconomic, demographic, and political individual controls, state-level institutional controls, state and year fixed-effects, state-specific time trends, and to the model specification.

We also find that the effect is larger if we consider only the perceived differences between presidential candidates. Moreover, a lower but statistically significant positive effect also arises for perceptions about the parties' positions on several additional issues. Finally, using the same set of controls, we also confirm that moderate individuals have a lower probability to turn out to vote.

All our results are also robust to potential overreporting by survey respondents. We have checked all our regressions using only the set of validated votes and find no change in our findings.

In sum, this Chapter analyses an interaction, that of electoral competition and voter turnout. Although the effect that we find in the perceptions of party platforms on empirical voter turnout is significant, we do not claim that it is sufficient to fully explain voter's turnout. Indeed, and as other authors have pointed out, our understanding of voter turnout is still very limited and further research is needed.

What policy implications do we learn from this Chapter? We consider democratic societies to be fairer when they represent their individuals better. Therefore, society should care about extending voter turnout in order to improve representability of all citizens. In light of the findings in this Chapter, our political systems should favour institutional policies that 1) work towards decreasing the voting cost of citizens (for example, allowing for polling day registration), and 2) make party platforms more transparent to citizens, so that they are able to evaluate the distance between political parties' platforms more accurately. Possible ways in which to tackle the latter problem include establishing a board consisting of objective evaluators of party policies (extending the methodology already used in various areas of economic policy —see for instance the information provided by the American Evaluation Association), media regulation that encourages objectivity in party policy assessments, and last but not least, dealing with campaign spending limitations in such a way that perceptions about party platforms' differences improve. In this sense, further research should take a close look at the link between campaign spending regulation and the perceived difference between party programmes.

3.6 Data appendix

Descriptions of the questions our data draws from are in Section 4. This Data Appendix provides the mapping that we have used between the possible responses to the most important problem question and the list of policies given under the seven-point scales that we have used to construct the perceived difference between party platforms.

- Most important problem responses → Policy Issues:

  1. Agricultural → [None]
  2. Economics → [None]
  3. Foreign affairs → Defense spending; Cooperation with USSR
  4. Government functioning → [None]
  5. Labour → Guaranteed jobs
  6. Natural resources → [None]
  7. Public order → Urban unrest; Rights of accused; Women's equal rights
  8. Racial problems → Aid to African-Americans
  9. Social welfare → Health care; Guaranteed jobs; Government services; School busing

Table 1. Individuals' preferences and their perception of political parties' platforms

	Mean Scores		
	Individual (1)	*Democratic* (2)	*Republican* (3)
Government health insurance	3.84 (2.14)	3.04 (1.55)	4.83 (1.56)
Jobs guaranteed	4.35 (1.87)	3.21 (1.48)	4.83 (1.48)
Aid to African-Americans	4.45 (1.81)	3.18 (1.46)	4.49 (1.48)
Rights of the accused	4.28 (2.10)	3.37 (1.53)	4.09 (1.58)
Urban unrest	3.38 (1.98)	3.13 (1.49)	4.17 (1.52)
Women's equal rights	2.76 (1.96)	2.99 (1.41)	3.74 (1.57)
Government services/spending	3.88 (1.62)	3.01 (1.37)	4.69 (1.46)
Cooperation with the USSR	4.06 (1.83)	3.35 (1.36)	4.38 (1.51)
Defense spending	3.95 (1.59)	3.63 (1.41)	5.09 (1.33)
Liberal/Conservative	4.26 (1.37)	3.23 (1.43)	4.97 (1.40)

Note: standard deviations in parentheses. Scores are given by respondents on a seven-point scale, where the most liberal option gets a score equal to one and the most conservative option gets a score equal to seven. The first column reports the mean of the declared scored by the individual about herself. The second (third) column reports the mean of the scored that the respondent has assigned to the Democratic (Republican) party. See Section 3 for more details.

Table 2. Voter turnout and demographic and socioeconomic characteristics – descriptive statistics

Variable	*Mean*	*Standard deviation*
Turnout (%)	74.1	43.8
Turnout (validated)[1] (%)	70.4	45.7
Turnout (validated)[2] (%)	57	49.5
Newspaper information (=1 if read, =0 otherwise)	0.66	0.47
Partisanship (=1 if partisan, =0 otherwise)	0.65	0.48
Duty (=1 if people should vote, =0 otherwise)	0.50	0.50
Political participation (from 1 to 6)	1.56	0.95
Age	45.6	17.7
Female (%)	56.0	49.6
White (%)	80.5	39.6
African-American (%)	11.5	32.0
Asian (%)	1.05	10.2
Native American (%)	2.41	15.3
Hispanic (%)	4.34	20.4
Protestant (%)	62.6	48.4
Catholic (%)	24.2	42.8
Jewish (%)	2.19	14.6
Other religion or none (%)	11.0	31.3
Attends church every week (%)	26.9	44.3
Attends church almost every week (%)	11.1	31.5
Attends church once or twice a month (%)	13.5	34.2
Attends church a few times a year (%)	23.8	42.6

Married and living with spouse (%)	58.8	49.2
Never married (%)	15.4	36.1
Divorced (%)	9.7	29.6
Separated (%)	3.33	17.9
Widowed (%)	11.1	31.4
Partners; not married (%)	1.7	12.8
Number of children (%)	0.78	1.15
Grade school or less (%)	11	31.4
High school (%)	46	50.0
Some college (%)	22	41.7
College or advanced degree (%)	20	40.1
Income category 0 to 16 percentile (%)	17	37.1
Income category 17 to 33 percentile (%)	17	37.2
Income category 34 to 67 percentile (%)	34	47.2
Income category 68 to 95 percentile (%)	28	44.9
Income category 95 to 100 percentile (%)	52	22.1

1/The first validation method only includes information for which the registration record was found.
2/The second validation method considers observations for which a registration was not found as if that person had not voted.

Table 3. Voter turnout and the perceived distance between parties' platforms (with respect to the individual's preferences) – Liberal/Conservative

Dependent variable: Voter Turnout (=1 if voted, =0 if not voted)				
Probit		(1)	(2)	(3)
A Liberal/Conservative		*0.034*	*0.034*	*0.016*
		(7.62)	*(7.65)*	*(3.44)*
Newspaper information (=1 if yes, =0 if not)				0.11
				(5.65)
Presidential election				0.40
				(7.28)
Female				0.02
				(1.20)
Partisan:	Leaning independent			0.10
				(4.20)
	Weak partisan			0.09
				(4.09)
	Strong partisan			0.15
				(6.25)
Age:	30 – 39			0.07
				(4.69)
	40 – 60			0.11
				(5.47)
	More than 60			0.16
				(7.83)
Education:	High school			0.06
				(2.57)
	Some college			0.11
				(4.19)
	College or advanced			0.15
				(5.74)
Income level:	17 – 33 percentile			0.03
				(1.37)
	34 – 67 percentile			0.08
				(3.29)
	68 – 95 percentile			0.08
				(2.62)
	95 – 100 percentile			0.09
				(2.56)
Race:	African-American			0.01
				(0.18)
	Asian			-0.02
				(0.35)
	Native American			-0.11
				(3.06)
	Hispanic			-0.001
				(0.05)

	Other or none			0.09
				(3.35)
Church attendance:	Almost every week			-0.03
				(1.57)
	Once/twice a month			-0.05
				(2.56)
	Few times a year			-0.07
				(3.26)
	Never			-0.12
				(5.55)
	No religious preference			-0.24
				(4.58)
Marital status:	Never married			0.01
				(0.33)
	Divorced			-0.04
				(1.95)
	Separated			-0.04
				(1.13)
	Widowed			-0.02
				(0.76)
	Partners			0.01
				(0.17)
	Number of children			0.01
				(0.84)
	State effects	YES	YES	YES
	Year effects	YES	YES	YES
	State specific time trends	NO	YES	YES
	Number of observations	17082	17082	7754

Note: t-statistics calculated with robust standard errors clustered at the state level in parentheses. See Section 4 for details about the estimation procedure. Data details are in Section 3.

Table 4. Voter turnout and the perceived distance between parties' platforms (with respect to the individual's preferences) – Liberal/Conservative – Other controls

Dependent variable: Voter Turnout (=1 if voted, =0 if not voted)

Probit	(1)	(2)	(3) [State controls]
Δ Liberal/Conservative	*0.019*	*0.012*	*0.018*
	(4.43)	*(2.84)*	*(3.98)*
Newspaper information (=1 if yes, =0 if not)	0.16	0.11	0.14
	(8.48)	(6.53)	(8.13)
Duty	0.09		
	(6.98)		
Political participation index		0.09	
		(12.7)	
Conventional registration			0.05
			(1.27)
Polling day registration possible			-0.25
			(2.82)
Voting age population			-0.01
			(0.44)
Citizens' initiatives permitted			-1.00
			(10.5)
Restriction on corporate campaign contributions			0.01
			(6.74)
Socioeconomic and demographic controls	YES	YES	YES
State effects	YES	YES	YES
Year effects	YES	YES	YES
State specific time trends	YES	YES	YES
Number of observations	4276	7239	7233

Notes: t-statistics calculated with robust standard errors clustered at the state level in parentheses. See Section 4 for details about the estimation procedure. Data details are in Section 3.

Table 5. *Validated* voter turnout and the perceived distance between parties' platforms (with respect to the individual's preferences) – Liberal/Conservative dimension

Dependent variable: Validated Voter Turnout			
Probit	(1) Validation 1	(2) Validation 2	(3) Validation 1
A Liberal/Conservative	*0.015* *(3.20)*	*0.021* *(3.44)*	*0.015* *(3.18)*
Newspaper information (=1 if yes, =0 if not)	0.10 (4.60)	0.15 (6.11)	0.10 (4.63)
Socioeconomic and demographic controls	YES	YES	YES
State controls	NO	NO	YES
State effects	YES	YES	YES
Year effects	YES	YES	YES
State specific time trends	YES	YES	YES
Number of observations	3726	4132	3726

Notes: the first validation method only includes information for which the registration record was found. The second validation method considers observations for which a registration was not found as if that person had not voted. t-statistics calculated with robust standard errors clustered at the state level in parentheses. See Section 4 for details about the estimation procedure. Data details are in Section 3. An extended version of this table (Table A2) in the appendix provides estimates for all variables in the socioeconomic and demographic controls and state controls groups.

Table 6. Logit and linear probability models

Dependent variable: Voter Turnout (=1 if voted, =0 if not voted)		
	(1) Logit	(2) OLS
A Liberal/Conservative	*0.10* *(3.88)*	*0.01* *(3.73)*
Newspaper information	0.72 (8.01)	0.13 (8.26)
Socioeconomic and demographic controls	YES	YES
State effects	YES	YES
Year effects	YES	YES
State specific time trends	YES	YES
Number of observations	7240	7240

Notes: t-statistics calculated with robust standard errors clustered at the state level. A presidential election dummy have also been included. See Section 4 for details about the estimation procedure. Data details are in Section 3.

Table 7. Voter turnout and the perceived distance between parties' platforms (with respect to the individual's preferences) – All dimensions

Dependent variable: Voter Turnout (=1 if voted, =0 if not voted)									
Probit	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
A Liberal/ Conservative	0.016 (3.44)								
A Government health insurance		0.008 (1.03)							
A Jobs guaranteed			0.01 (2.62)						
A Aid to African-Americans				0.01 (2.47)					
A Rights of the accused					-0.001 (0.52)				
A Women's equal rights						0.006 (1.02)			
A Government services/ spending							0.008 (1.42)		
A Cooperation with the USSR								0.01 (1.88)	
A Defense spending									0.002 (0.62)
Socioeconomic and demographic controls	YES	YES	YES	YES	YES	YES	YES	YES	YES
State effects	YES	YES	YES	YES	YES	YES	YES	YES	YES
Year effects	YES	YES	YES	YES	YES	YES	YES	YES	YES
State specific time trends	YES	YES	YES	YES	YES	YES	YES	YES	YES
Number of observations	7754	1881	5675	6526	1138	3791	5896	2954	6867

Note: t-statistics calculated with robust standard errors clustered at the state level in parentheses. Newspaper information and a presidential dummy have also been included. See Section 4 for details about the estimation procedure. Data details are in Section 3.

Table 8. Voter turnout and the perceived distance between presidential candidates' platforms (with respect to the individual's preferences) – All dimensions

Dependent variable: Voter Turnout (=1 if voted, =0 if not voted)

Probit	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
A Liberal/ conservative	*0.024* *(5.22)*								
A Government health insurance		0.007 (1.30)							
A Jobs guaranteed			0.01 (1.89)						
A Aid to African-Americans				0.01 (3.37)					
A Rights of the accused					-0.001 (0.52)				
A Women's equal rights						-0.001 (0.21)			
A Government services/spending							0.009 (2.40)		
A Cooperation with the USSR								0.02 (3.34)	
A Defense spending									0.009 (2.49)
Socioeconomic and demographic controls	YES	YES	YES	YES	YES	YES	YES	YES	YES
State effects	YES	YES	YES	YES	YES	YES	YES	YES	YES
Year effects	YES	YES	YES	YES	YES	YES	YES	YES	YES
State specific time trends	YES	YES	YES	YES	YES	YES	YES	YES	YES
Number of observations	3320	818	910	3226	1138	1914	2925	2971	3895

Note: t-statistics calculated with robust standard errors clustered at the state level in parentheses. Newspaper information and a presidential dummy have also been included. See Section 4 for details about the estimation procedure. Data details are in Section 3.

Table 9. Voter turnout and the perceived distance between parties' platforms (with respect to the individual's preferences) – Most Important Problem

Dependent variable: Voter Turnout (=1 if voted, =0 if not voted)			
	(1)	(2)	(3)
A Most important problem (maximum A)	0.007 (1.90)		
A Most important problem (minimum A)		0.009 (2.25)	
A Most important problem (mean A)			0.008 (2.16)
Newspaper information	0.14 (9.09)	0.14 (9.01)	0.14 (9.04)
Socioeconomic and demographic controls	YES	YES	YES
State effects	YES	YES	YES
Year effects	YES	YES	YES
State specific time trends	YES	YES	YES
Number of observations	6851	6851	6851

Note: t-statistics calculated with robust standard errors clustered at the state level in parentheses. Newspaper information and a presidential dummy have also been included. See Section 4 for details about the estimation procedure. Data details are in Section 3. See the Data Appendix for the construction of the mapping between most important problem responses and policy issues.

Table 10. Voting patterns of moderates

Dependent variable: Voter Turnout (=1 if voted, =0 if not voted)		
	(1)	(2)
Strict moderate voters	-0.033 (2.33)	
Broadly moderate voters		-0.040 (3.48)
Newspaper information	0.11 (7.55)	0.06 (5.67)
Socioeconomic and demographic controls	YES	YES
State effects	YES	YES
Year effects	YES	YES
State specific time trends	YES	YES
Number of observations	7545	7545

Note: t-statistics calculated with robust standard errors clustered at the state level in parentheses. *Strict moderates* includes only those individuals who on a seven-point scale where, have reported four points ("moderate"). *Broadly moderates* includes those individuals who on a seven-point scale where, have reported either three ("slightly liberal"), four ("moderate"), or five ("slightly conservative") points. Newspaper information and a presidential dummy have also been included. See Section 4 for details about the estimation procedure. Data details are in Section 3.

Table A1. Vote validation

Vote Validated	Observations	Percent
Yes	7219	60
Registration record found, no record of voting	2241	18.6
No registration record found, no record of voting	2564	21.3

Notes: See Section 4 for details about the estimation procedure. Data details are in Section 3.

Table A2. *Validated* voter turnout and the perceived distance between parties' platforms (with respect to the individual's preferences) – Liberal/Conservative dimension (extended Table 5)

Dependent variable: Validated Voter Turnout				
Probit		(1) Validation 1	(2) Validation 2	(3) Validation 1
Δ Liberal/Conservative		*0.015*	*0.021*	*0.015*
		(3.20)	*(3.44)*	*(3.18)*
Newspaper information (=1 if yes, =0 if not)		0.10	0.15	0.10
		(4.60)	(6.11)	(4.63)
Presidential election		0.08	-0.11	0.06
		(4.97)	(3.08)	(2.83)
Female		0.02	0.03	0.04
		(1.71)	(1.81)	(2.08)
Partisan:	Leaning independent	0.06	0.08	0.06
		(2.84)	(3.31)	(2.83)
	Weak partisan	0.04	0.07	0.04
		(2.08)	(3.04)	(2.08)
	Strong partisan	0.09	0.15	0.09
		(4.44)	(5.88)	(4.45)
Age:	30 – 39	0.05	0.08	0.05
		(3.95)	(4.95)	(3.94)
	40 – 60	0.08	0.14	0.08
		(5.18)	(7.49)	(5.17)
	More than 60	0.11	0.19	0.11
		(6.90)	(9.64)	(6.88)
Education:	High school	0.03	0.04	0.03
		(2.60)	(1.41)	(1.44)
	Some college	0.05	0.09	0.05
		(2.21)	(3.06)	(2.18)
	College or advanced	0.10	0.15	0.10
		(3.74)	(4.88)	(3.72)
Income level:	17 – 33 percentile	0.03	0.04	0.03
		(2.60)	(1.93)	(2.65)
	34 – 67 percentile	0.06	0.10	0.03
		(3.08)	(3.81)	(2.65)
	68 – 95 percentile	0.07	0.11	0.07
		(4.03)	(4.20)	(4.05)
	95 – 100 percentile	0.07	0.11	0.07
		(3.46)	(4.59)	(3.47)
Race:	Black	-0.03	-0.07	-0.03
		(1.60)	(2.80)	(1.52)
	Asian	-0.05	-0.18	-0.05
		(0.96)	(2.16)	(0.96)
	Native American	-0.09	-0.13	-0.09
		(1.98)	(2.61)	(1.90)

	Jewish	0.03	0.02	0.03
		(0.83)	(0.52)	(0.93)
	Other or none	0.03	0.03	0.03
		(0.63)	(0.56)	(0.66)
Church attendance:	Almost every week	-0.07	-0.07	-0.07
		(2.50)	(2.28)	(2.52)
	Once/twice a month	-0.09	-0.12	-0.09
		(3.18)	(3.54)	(3.20)
	Few times a year	-0.08	-0.11	-0.08
		(4.07)	(4.50)	(4.06)
	Never	-0.14	-0.17	-0.14
		(4.62)	(5.09)	(4.60)
	No religious preference	0.03	-0.17	-0.14
		(0.63)	(2.44)	(2.57)
Marital status:	Never married	0.02	-0.01	0.02
		(1.00)	(1.22)	(1.03)
	Divorced	-0.07	-0.08	-0.07
		(3.65)	(3.65)	(3.68)
	Separated	-0.03	-0.07	-0.03
		(1.01)	(1.72)	(1.00)
	Widowed	-0.03	-0.06	-0.03
		(1.28)	(2.01)	(1.30)
	Partners	-0.07	-0.08	-0.07
		(1.68)	(1.41)	(1.63)
Number of children		0.01	0.01	0.01
		(0.13)	(0.91)	(0.14)
State controls:	Conventional registration			0.09
				(1.33)
	Polling day registration possible			0.99
				(4.94)
	Voting age population			0.01
				(1.73)
	Citizens' initiatives permitted			1.00
				(5.45)
	Restriction on corporate campaign contributions			-0.01
				(33.0)
State effects		YES	YES	YES
Year effects		YES	YES	YES
State specific time trends		YES	YES	YES
Number of observations		3726	4132	3726

Notes: the first validation method only includes information for which the registration record was found. The second validation method considers observations for which a registration was not found as if that person had not voted. t-statistics calculated with robust standard errors clustered at the state level in parentheses. See Section 4 for details about the estimation procedure. Data details are in Section 3.

Table A3. Logit and linear probability models – Validated Turnout

Dependent variable: Voter Turnout (=1 if voted, =0 if not voted)	(1) Logit Validation 1	(2) Logit Validation 2	(3) OLS Validation 1	(4) OLS Validation 2
Δ Liberal/Conservative	*0.14*	*0.14*	*0.013*	*0.015*
	(2.94)	*(3.29)*	*(2.78)*	*(2.95)*
Newspaper information	0.78	0.89	0.11	0.14
	(4.63)	(6.04)	(4.39)	(6.02)
Socioeconomic and demographic controls	YES	YES	YES	YES
State effects	YES	YES	YES	YES
Year effects	YES	YES	YES	YES
State specific time trends	YES	YES	YES	YES
Number of observations	3726	4132	3726	4132

Note: t-statistics calculated with robust standard errors clustered at the state level in parentheses. *Strict moderates* includes only those individuals who on a seven-point scale where, have reported four points ("moderate"). *Broadly moderates* includes those individuals who on a seven-point scale where, have reported either three ("slightly liberal"), four ("moderate"), or five ("slightly conservative") points. Newspaper information and a presidential dummy have also been included. See Section 4 for details about the estimation procedure. Data details are in Section 3.

Table A4. Voting patterns of moderates – Validated Turnout

Dependent variable: Voter Turnout (=1 if voted, =0 if not voted)

	(1) Validation 1	(2) Validation 2	(3) Validation 1	(4) Validation 2
Strict moderate voters	-0.02 (1.74)	-0.03 (2.12)		
Broadly moderate voters			-0.031 (2.32)	-0.033 (2.39)
Newspaper information	0.08 (0.87)	0.10 (3.95)	0.01 (0.82)	0.10 (3.92)
Socioeconomic and demographic controls	YES	YES	YES	YES
State effects	YES	YES	YES	YES
Year effects	YES	YES	YES	YES
State specific time trends	YES	YES	YES	YES
Number of observations	3027	3149	3027	3149

Note: t-statistics calculated with robust standard errors clustered at the state level in parentheses. *Strict moderates* includes only those individuals who on a seven-point scale where, have reported four points ("moderate"). *Broadly moderates* includes those individuals who on a seven-point scale where, have reported either three ("slightly liberal"), four ("moderate"), or five ("slightly conservative") points. Newspaper information and a presidential dummy have also been included. See Section 4 for details about the estimation procedure. Data details are in Section 3.

Conclusion

In the first part of this Thesis we propose an alternative to the usual voting rule which is simple and allows voters to express their *willingness to influence*. A mechanism which seems the most natural extension to Majority Rule and that is proved to be not only superior to it but also a mechanism that achieves the best possible allocation and induces truthful revelation of the voters' preferences in some general settings. Its essence relies on almost allowing for transferable utilities without introducing money; players can freely move their voting power across issues to strengthen their position in some issues.

Later, in Chapter 2 we offer the first step towards a thorough experimental analysis of the voting system we call Qualitative Voting. For this, we provide each of our 18 experiment participants with an endowment of 30 votes to allocate freely across issues in a variety of settings (two, three, and six issues to be voted in groups of two, three, or six members). We observe that the welfare gains of Qualitative Voting with respect to Majority Rule are very noticeable in the conflict resolution situation but are diluted for potentially different reasons in the committee meeting examples. We also observe that players tend to focus their attention on a few number of issues when they face the decision over various issues. The welfare implications of this fact are open for further analysis.

The difficulties of extending Qualitative Voting to more general settings are best captured by the results in the second half of Chapter 1. In there we show that it is impossible to allow the *willingness to influence* to play a role in general settings where unanimity needs to be satisfied.

The second part of this Thesis builds, instead, on the interaction between electoral competition and voters' turnout decision. We present a simple model where policy driven candidates spatially compete to implement their announced policy. In our model, citizens only vote if candidates offer platforms different enough (if both candidates converge to the median voter's preferred platform there are no gains from selecting one candidate over another, thus, no one turns out to vote given that voting has a cost).

The main intuition derived from our model is that the bigger is the difference in the individuals' perceptions of the two candidates' platforms (relative to the voter's policy preferences), the higher the incentive to vote. A related implication from the model is that moderate (as opposed to liberal or conservative) individuals tend to vote less. We test these two implications of our theoretical model using data from the United States' National Electoral Studies for 1972-2000 finding support for both predictions.

In light of the findings in Chapter 3, our political systems should favour institutional policies that make party platforms more transparent to citizens, so that they are able to evaluate the distance between political parties' platforms more accurately. Possible ways in which to tackle this problem include establishing a board consisting of objective evaluators of party policies, media regulation that encourages objectivity in party policy assessments, and last but not least, dealing with campaign spending limitations in such a way that perceptions about party platforms' differences improve.

Overall, this Thesis is characterised by the analysis of the strategic behaviour of different actors in voting games. The one of voters and the one of politicians. Nevertheless there is a rather different approach in both parts.

The first part takes a normative approach offering a *new* voting rule that allows voters to express their *willingness to influence*, characterising its properties and answering in a general manner the classical political science debate about the *intensity problem*. The second part, instead, has a positive slant and tries to understand the widely analysed topic of voters' turnout in elections from a different perspective. We take an insider view into the paradox of voter turnout and look at the effects of political competition on turnout.

Both approaches are necessary but indeed the positive approach should just be (in our view) the initial step towards the normative one. As social scientists we are given the gift of interpreting what is happening around us thus we have the responsibility to try to improve or, at least, question our social institutions. Very deep in our hearts there should be the belief that our work will improve the world we live in.

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