Economic Dynamics of Equality and Classes

by

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ABSTRACT

This dissertation analyses the dynamics of inequality and classes, from a positive and a normative viewpoint, focusing on two distinct, but related approaches – Analytical Marxism and the theory of equality of opportunity, – which raise significant philosophical, economic, and political issues. The importance of a dynamic perspective in the analysis of normative theories is emphasised as an essential tool in the process of theoretical construction. Indeed, this dissertation analyses some important *anomalies* of egalitarian and Marxian theories that arise in the dynamic context and suggest to reconsider our established views on inequality and classes.

First, the proper temporal unit of egalitarian (or Marxian) concern must be defined: agents' whole lives or selected parts of them. Egalitarian principles based on different units incorporate different normative concerns, both in the analysis of existing inequalities *and*, unlike in the static setting, in the definition of the egalitarian benchmark. No principle seems entirely satisfactory in the analysis of unequal distributions, but *corresponding segments egalitarianism* defines the appropriate intertemporal egalitarian benchmark.

Second, egalitarian theorists, since Rawls, have in the main advocated equalising some objective measure of individual well-being, rather than subjective welfare. This discussion, however, has assumed, implicitly, a static environment. In a dynamic context, equality of opportunity for some objective condition is incompatible with human development over time. This incompatibility can be resolved by equalizing opportunities for *welfare.*

Thus, 'subjectivism' seems necessary to obtain both equality of opportunities *and* the development of human capacity.

Finally, the modern theory of exploitation emphasises asset inequalities as the fundamental injustice of competitive economies. However, in dynamic equilibria with persistent asset inequalities and capital scarcity, exploitation tends to disappear. Asset inequality is therefore a normatively secondary (though causally primary) wrong. The analysis of the dynamic economy also raises doubts on the possibility of providing robust micro-foundations to Marxian concepts by means of Walrasian models.

A Vanda e Bruno.

For them, justice and equality have never been mere academic subjects.

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I am the sole responsible for any remaining errors and misdemeanours.

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Joanne Hay has had the bravery and patience to proof-read the thesis (except for this page) at various stages. She must have done it because of friendship and natural kindness, as I truly cannot see another reason.

Without my wife, I would have simply never got this far. This dissertation is – in a fundamental sense – also hers.

Contents

I hereby declare that the work presented in this Thesis is my own. With regard to the material presented in chapter 3 of this thesis, I hereby declare that it is based on work done in conjunction with John E. Roemer: the conceptualisation of the model is due to my co-author, while the derivation of the main results is mine.

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INTRODUCTION

This dissertation analyses the dynamics of inequality and classes, from both a positive and a normative viewpoint. In particular, two distinct, but related theoretical approaches to the analysis of inequalities and classes are considered: Analytical Marxism and the theory of equality of opportunity. Analytical Marxism represents one of the most controversial, analytically sophisticated, and thorough interpretations of Marx's theory. It proposes an original analysis of the Marxian concepts of exploitation, inequality, and classes. The theory of equality of opportunity provides a different perspective on inequality and classes within the liberal egalitarian tradition originated from Rawls's *A Theory of Justice* (1971). It raises deep philosophical issues – such as the relation between equality and responsibility – and it has significant political and economic implications. The relevance of both approaches can be measured by their interdisciplinary impact in the social sciences and by the vast literature they have generated.

Unlike most of the literature on both approaches, which has adopted a static framework, this dissertation emphasises the importance of a dynamic perspective in the evaluation of egalitarian (more generally, normative) theories. In particular, a dynamic analysis is an essential tool in the process of theoretical construction and in order to reach a *reflective equilibrium*. Indeed, this dissertation can be thought of as analysing some important *anomalies* (Kuhn, 1970, p.52) of egalitarian and Marxian theories, which arise in the dynamic context and possibly suggest the need to reconsider some established views on inequality and classes.

Chapter 1 sets the theoretical and methodological framework of the analysis. It provides a general introduction to Analytical Marxism and the theory of equality of opportunity and a comprehensive survey of the vast literature on both approaches. It also clarifies the methodology and scope of the analysis and indicates some directions for further research. Finally, it provides an independent contribution to the history of economic thought by highlighting the conceptual links, and theoretical lineage, between Analytical Marxism and the modern theory of equality of opportunity.

Chapter 2 analyses some conceptual problems of egalitarian theories which arise in the dynamic context. Since agents' lives extend over time, it is necessary to define the proper temporal unit of egalitarian concern: agents' whole lives or selected parts of them (e.g., focusing only on inequalities between all agents living in the same period). However, egalitarian principles based on different units incorporate different normative concerns and have different policy implications.

Thus, different intertemporal egalitarian principles provide different insights in the analysis of *inequalities* and in this context no principle seems entirely satisfactory: several views are possible, each of which seems plausible in some cases and implausible in others. However, an important theoretical and methodological distinction is emphasised: unlike in the static setting, intertemporal egalitarian principles also define different *egalitarian* states to reach. The two issues are connected but they should be kept conceptually distinct. This is even more evident for policy purposes since the definition of the ideal egalitarian distribution to reach and the transition to it raise different problems. It is argued that *Corresponding Segments Egalitarianism* (CSE) - which focuses on the corresponding stages of agents' lives (e.g., childhood, middle age, old age, etc.) - defines the appropriate intertemporal egalitarian benchmark.

The trade-offs between different egalitarian principles and other non-primarily-egalitarian ethical concerns, – namely, Rawls's maximin and utilitarianism – are also analysed. An overlapping generations model is set up to analyse intertemporal as well as intratemporal inequalities in the context of *all things considered* judgements. It is proved that the intertemporal maximin path tends to be incompatible with growth (Proposition 1), that intratemporal inequalities persist, but do not seem ethically relevant (Proposition 2), and that *CSE* has desirable properties in relation to both Rawlsian and utilitarian concerns (Proposition 3).

Chapter 3 extends the analysis of the dynamic implications of egalitarian views and of the relation between egalitarian and non egalitarian concerns, focusing on the theory of equality of opportunity (EOp). In order to avoid the conceptual problems discussed in chapter 2, it is assumed that agents live for one period, but the economic environment is considerably enriched by dropping the assumption of a representative agent in each generation, by allowing agents to care about *functionings* (and not only consumption), and by analysing educational investment. Hence, a larger set of issues can be explored, including the dynamics of intergenerational and intragenerational inequalities and classes, and the choice of the appropriate *equalisandum*, which are not discussed in chapter 2.

Three dynamic models are analysed under different assumptions on the relevant equalisandum. It is proved that if an objectivist equalisandum, such as *functionings*, is adopted, the intergenerational EOp path is inconsistent with sustained human development (Proposition 1), even if agents have altruistic preferences (Proposition 2). This suggests that the three desiderata: (i) protracted human development; (ii) equality of opportunity for some condition; and (iii) the condition be an objective characteristic of the individual, are inconsistent. This incompatibility can be resolved by equalising opportunities for *welfare* (Theorem 1), a result that suggests that 'subjectivism' may be necessary if we are to hope for a society which can both equalise opportunities and support the development of human capacity. Moreover, while the dynamics of *intra*generational inequalities and classes with an objectivist equalisandum cannot in general be determined, in the intergenerational 'subjectivist' EOp path, *intra*generational inequalities and classes disappear after a finite number of periods.

Chapters 4 and 5 analyse the dynamics of inequality and classes from a different perspective, focusing on John Roemer's (1982A, 1988A) theory of exploitation and classes. In chapter 4 a dynamic generalisation of Roemer's subsistence economy with labour-minimising agents is set up to analyse exploitation, inequalities, and classes. In particular, chapter 4 evaluates the causal and normative relevance of *Differential Ownership of Productive Assets* (DOSPA) in generating exploitation and classes as persistent features of a competitive economy; and the possibility of providing robust microfoundations to Marxian economics by means of neoclassical models, -

whereby the concepts of class and exploitation emerge as the product of constrained individual optimisation.

The conceptual issues raised in chapter 2 are proved to be relevant in this context, too, since in a dynamic framework two criteria to define exploitation and class emerge: one focuses on the agent's status in each period of her life, the other on the agent's whole life. The two criteria are equivalent only in an interior equilibrium in which no agent saves.

A dynamic generalisation of Roemer's theory is provided (Propositions 5 and 6) and exploitation and classes are proved to be persistent phenomena if agents discount future labour expended (Theorems 1 and 2). However, it is argued that the normative relevance of time preference is dubious and it is shown that, with no time preference, in equilibrium exploitation disappears in the long run, while asset inequalities and classes persist (Theorems 3 and 4). Roemer's results are derived in an essentially static environment in which agents face no intertemporal trade-offs: intertemporal credit markets are absent *and* savings are impossible. Chapter 4 proves that it is sufficient to allow agents to save to contradict Roemer's results. Hence, asset inequalities are argued to be normatively secondary, though causally primary in explaining exploitation, Roemer's definition of class based on the net amount of labour performed is questioned, and several doubts are raised on the possibility of providing robust microfoundations to Marx's concepts by means of Walrasian general equilibrium models.

Roemer's models essentially have a static environment where agents face no intertemporal trade-offs: intertemporal credit markets are absent *and* savings are impossible. Chapter 4 proves that it is sufficient to allow agents to save to contradict Roemer's results.

Chapter 5 extends the analysis of the dynamics of exploitation, inequality, and classes to economies with maximising agents and the possibility of capital accumulation. On the one hand, chapter 5 evaluates the robustness of the main conclusions of chapter 4 in a different analytical framework which incorporates an important feature of capitalist economies, namely capital accumulation. On the other hand, chapter 5 pursues one of the main substantive and methodological issues raised in chapter 4, namely the mechanisms generating exploitation, inequalities, and classes as persistent phenomena in a competitive economy. From this perspective, the model of an accumulating economy is extremely interesting, due to the role of differential ownership of *scarce* productive assets in the derivation of Roemer's results, and given that, unlike in static economies and in the subsistence model, it allows the modelling of two crucial features of a general theory of exploitation, namely technical progress and unemployment.

Chapter 5 analyses the role of DOSPA in generating persistent exploitation in a dynamic framework where agents maximise lifetime consumption opportunities and face a consumption-savings trade-off, so that capital accumulation is the outcome of optimal intertemporal choices. A dynamic generalisation of the *Fundamental Marxian Theorem* – which establishes that exploitation is synonymous with positive profits – is proved (Theorem 1). It is shown that without technical progress there is no equilibrium with persistent accumulation and persistent exploitation

(Proposition 2). Then, the conclusions reached in chapter 4 are strengthened (Theorem 3): if capitalists discount future consumption, there are equilibria in which revenues are entirely consumed in every period and exploitation persists. However, this result crucially depends on a strictly positive rate of time preference, rather than on unemployment or capital scarcity.

As concerns equilibria with capital accumulation, first, balanced growth paths – in which the whole economy grows at a uniform rate and reaches a steady state – are characterised. Next, it is proved that at a balanced growth path exploitation disappears, although DOSPA and classes persist (Theorem 4). Finally, it is proved that unlimited labour-saving technical progress may yield persistent exploitation by ensuring persistent unemployment in the labour market (Theorem 6), but in more general cases such result does not hold, and a more general analytical framework is advocated to analyse exploitation, inequalities, and classes.

CHAPTER 1. THE ECONOMIC DYNAMICS OF INEQUALITY AND CLASSES

> "If you can look into the seeds of time,
> And say which grain will grow and which will not,
> Speak then to me" (*Macbeth*, Act I, Scene III)

1.1. INTRODUCTION

This dissertation analyses the concepts of inequality and classes, their definition and implications, from both a positive and a normative viewpoint. It is primarily an exploration of both concepts, rather than a systematic defence of their relevance. Methodologically, the relevance of the notions of inequality and classes is supposed, it is an *a priori* of the analysis. However, a deeper understanding of both concepts should strengthen the case for their normative and positive importance.

In particular, two distinct, but as shown below, theoretically related approaches to inequality and classes are considered, namely Analytical Marxism (AM) and the theory of equality of opportunity. AM is one of the most controversial, analytically sophisticated, and thorough interpretations of Marx's theory – and one of the last "schools" of Marxist thought. It provides an original interpretation of the Marxian notions of exploitation, inequality, and classes. The theory of equality of opportunity provides a different perspective on inequality and classes within the liberal egalitarian tradition originated from Rawls's *A Theory of Justice* (1971). It raises deep philosophical issues – such as the relation between equality and responsibility – and it has far-reaching political and economic implications. The relevance of both approaches can be readily measured by their interdisciplinary impact in the social sciences and by the vast theoretical and empirical literature they have generated. However, most, if not all, of the vast literature has adopted a static framework.

This dissertation, instead, emphasises the importance of a dynamic analysis in the evaluation of egalitarian - more generally, normative - theories. There are some clear reasons why a dynamic approach may provide interesting insights from both a positive and a normative viewpoint: the problem analysed may be inherently dynamic (e.g., intergenerational justice or economic growth); a dynamic analysis may complement a static theory as a matter of generality; or it may provide crucial insights on the robustness or relevance of a theory even if the main object of analysis is not *per se* dynamic. (For instance, the main results of traditional Walrasian general equilibrium theory, such as existence and uniqueness, need not be analysed in a dynamic framework. However, arguably their theoretical – and even philosophical – relevance can be properly evaluated only with a dynamic analysis of price movements (see, e.g., McCloskey, 1991).)

This dissertation highlights a different role of dynamic analysis as a crucial tool in the process of theoretical construction and in order to reach a *reflective equilibrium*. In fact, a dynamic analysis may generate *anomalies* (Kuhn, 1970, p.52) that lead to reconsider a normative theory, as illustrated for instance, by the well-known difficulties in the application of utilitarianism in the intertemporal context – e.g., the issue of time preference (Sidgwick, 1907; Ramsey, 1928; Rawls, 1971), or the problems in the determination of optimal population (Parfit, 1984). Indeed, this dissertation can be thought of as analysing some important *anomalies* of egalitarian and Marxist theories which arise in the dynamic context, possibly suggesting the need of a major reconsideration of some established views on inequality and classes.

The aim of this chapter is threefold. First, it puts the research project in a broader perspective, by introducing the theories analysed and by providing a comprehensive survey of the relevant literature.[2] Second, it clarifies the methodology and scope of the analysis and it indicates some directions for further research, based on the results presented in this dissertation. Third, it provides an independent contribution to the history of economic thought by highlighting the significant conceptual links, and theoretical lineage, between AM and the modern theory of equality of opportunity.[3] Section 1.2 discusses AM, while Section 1.3 focuses on the theory of equality of opportunity.

1.2. EXPLOITATION, INEQUALITY, AND CLASSES

One of the two approaches analysed in this dissertation is Roemer's (1982A, 1982B, 1982C, 1982E, 1986A, 1988A) theory of exploitation and classes. Although the main focus of this dissertation is normative, Roemer's theory raises several methodological and positive issues. Sections 1.2.1 and 1.2.2 review the extensive literature on Analytical Marxism, as a general approach to Marx's philosophy (Elster, 1982A, 1985), economic theory

[2] This chapter focuses on general methodological and theoretical issues. Surveys focusing on specific aspects can be found in the following chapters.

[3] The only author who has explicitly analysed the theoretical link between market socialism (rather than AM) and the theory of equality of opportunity is Sugden (2004).

(Roemer, 1980, 1981; van Parijs, 1983), theory of international relations (Roemer, 1983A), theory of class stratification (Wright, 1984, 1994, 2000; van Parijs, 1986), and theory of class conflict and political struggles (Przeworski, 1985A). Section 1.2.3 provides a more specific review of the literature on Roemer's theory of exploitation and classes. Section 1.2.4 emphasises the normative aspects of Roemer's theory and the conceptual link with the theory of equality of opportunity.

1.2.1. ANALYTICAL MARXISM

Given the significant theoretical, methodological, and even political heterogeneity of Analytical Marxists, it is difficult to define the boundaries of AM, either theoretically or in terms of membership (Wood, 1989; Wright, 1989).[4] However, one of the main tenets of AM and its main departure from classical Marxism is the denial of a specific Marxist methodology[5] and the emphasis on the need to apply the tools of mainstream analytical philosophy, sociology, and economic theory to Marx's theory. More precisely, Wright (1989, pp.38-9) identifies four specific commitments that characterise AM.

DEFINITION 1. (*Weak AM*) AM is identified by

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[4] AM emerges during the decline of structuralist Marxism and the renaissance of liberal egalitarianism: G.A. Cohen's *Karl Marx's Theory of History* appears in 1978 and the annual seminars of the group in London start in 1979. See Carling (1986) and Wright (1989).

[5] "Orthodox Marxism ... does not imply the uncritical acceptance of the results of Marx's investigations. It is not the 'belief' in this or that thesis, nor the exegesis of a 'sacred' book. On the contrary, orthodoxy refers exclusively to method" (Lukacs, 1971, p.1).

C1. "A commitment to *conventional scientific norms* in the elaboration of theory and the conduct of research."

C2. "An emphasis on the importance of *systematic conceptualisation* [...]. This involves careful attention to both definitions of concepts and the logical coherence of interconnected concepts."

C3. "A concern with a relatively *fine-grained specification of the steps in the theoretical arguments linking concepts*."

C4. "The importance accorded to *the intentional action of individuals* within both explanatory and normative theories."

Definition 1 encompasses all Analytical Marxists; however, it is so general that it hardly identifies AM as a specific approach.[6] For instance, in principle virtually all Marxist mathematical economists – such diverse authors as Morishima, Steedman, Desai, etc. – could be included. Definition 1 does not capture the originality of AM and cannot really explain the controversy it has generated,[7] since in order to identify the minimum common denominator of all Analytical Marxists, it does not include the two most controversial axioms of AM, endorsed by its most prominent exponents,

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[6] As noted by Wright (1989, p.39), "it would be arrogant to suggest that Marxism lacked these elements prior to the emergence of Analytical Marxism as a self-conscious school".

[7] Basically all critics focus on the *strong definition* given below. In his review of Wright, Levine, and Sober (1992), who adopt instead the weak definition, Foley simply notes that the conclusions "are on the whole mild, sensible and, as the options are presented, persuasive" (Foley, 1993, p.298). He objects mostly to "the authors' addiction to philosophic and sociological jargon, extreme caution in the formulation of hypotheses, involuted prose, and painfully slow movement toward minimally exciting conclusions" (ibid.).

in particular Jon Elster and John Roemer. The definition of *strong AM*, also known as *rational choice Marxism* (Carling, 1986; Wood, 1989; Carver and Thomas, 1995; hereafter RCM), can be summarised as follows.

DEFINITION 2. (*Strong AM*, or *RCM*) RCM is defined by C2 and C3 plus

> C1'. A commitment to the use of "state of the arts methods of analytical philosophy and 'positivist' social science" (Roemer, 1986D, pp.3-4).
>
> C4'.(a) A commitment to *methodological individualism* as "the doctrine that all social phenomena – their structure and their change – are in principle explicable in ways that only involve subjects – their properties, their goals, their beliefs and their actions" (Elster, 1985, p.5);
>
> C4'.(b) A commitment to *rational actor models*.

Definition 2, and in particular C4', does not apply to all Analytical Marxists: Cohen's (1978, 1983A) reconstruction of Marx's theory of history is *functionalist*;[8] Van Parijs (1982, 1983) supports the search for Marxian microfoundations, but questions the "absolutism" of C4' admitting the possibility of alternative explanations; the weak "methodological individualism" endorsed by Levine, Sober, and Wright (1987) in line with Definition 1, has arguably little in common with C4'. However, Definition 2

[8] Elster distinguishes a *weak functional paradigm*, according to which "an institution or behavioural pattern often has consequences that are (a) beneficial for some dominant economic or political structure; (b) unintended by the actors; and (c) not recognized by the beneficiaries as owing to that behavior" (Elster, 1982A, p.454); and a *strong functional paradigm*, according to which "all institutions or behavioural patterns have a function that explains their presence" (ibid.).

captures the essential elements of originality of AM, – more precisely, RCM, – and it clarifies the main terms of the controversy.

First, the main *methodological* corollary of Definition 2 is that methodological individualism is the only legitimate foundation for social science (Elster, 1982A, p.463). Consequently, in a RCM perspective, the only parts of Marx's theory which "make sense" (Elster, 1985) are those that can be analysed within a methodological individualist perspective or more narrowly, with standard "rational choice models: general equilibrium theory, game theory and the arsenal of modelling techniques developed by neoclassical economics" (Roemer, 1986C, p.192). Elster argues that Marx was "committed to methodological individualism, at least intermittently" (Elster, 1985, p.7). However, largely due to the influence of Hegelian philosophy, Marx was not a consistent methodological individualist throughout his writings. Indeed, Elster reads various passages (especially those in the *Grundrisse* on the movement of capital and the subordinate explanatory role of competition) as an "explicit denial of methodological individualism" (ibid.). Then, he concludes that Marx was methodologically inconsistent or, more strongly, intellectually weak, since "it is difficult to avoid the impression that he often wrote whatever came into his mind, and then forgot about it as he moved on to other matters" (ibid., p.508).

Second, by adopting methodological individualism, AM typically reaches two kinds of *substantive* conclusions concerning Marxian concepts and propositions: some are considered either wrong or impossible to conceptualise in a rational choice framework, and thus are simply discarded.

Roemer (1981) disposes of the labour theory of value, the theory of the falling rate of profit, and the Marxian theory of crisis (see also Elster, 1985). After a long journey through Marx's theory, Elster concludes that "Today Marxian economics is, with a few exceptions, intellectually dead" (Elster, 1986, p.60).[9] Przeworski (1985A) and Elster (1982A) challenge the Marxian theory of the class struggle and the theory of the state.

Other concepts and propositions, instead, can be analysed in a rational choice framework, but need a substantial re-definition (this partly explains the AM emphasis on C2 and C3). Some intuitions on the symbiotic interaction between classes can be analysed in a game-theoretic framework (Elster, 1982A, pp.463-478; Przeworski, 1985A), although at the cost of a substantive shift in both meaning and political implications (see Burawoy, 1989, 1995). The concepts of class and exploitation can be derived as the product of agents' constrained optimisation (Roemer, 1982A); but this leads to the rejection of Marx's surplus value definition of exploitation, as a relevant positive and normative concept, in favour of the analysis of *differential ownership of scarce productive assets* (DOSPA). Similarly, Roemer provides microfoundations to the Marxian theory of unequal exchange (Roemer, 1983A) and outlines a micro-based Marxian political philosophy (Roemer, 1988B), thanks to (and possibly at the cost of) a reduction of Marx's theories to an almost exclusive emphasis on DOSPA.

[9] The main exception is the theory of technical change (Elster, 1986, p.188). According to Elster, scientific socialism, dialectical materialism, and the theory of productive forces and relations of production, too, are dead (ibid., p.186-200).

Given the scope and relevance of the issues analysed, it is not surprising that AM has generated a vast literature both on methodology and on substantive propositions.

1.2.2. METHODOLOGICAL ISSUES

> "We should look for precision in each class of things just so far as the nature of the subject admits" Aristotle, *Nichomachean Ethics*, bk. I: ch.3, 1094b 25.

A first set of methodological objections to AM concern the use of mathematics. According to some critics, mathematical models are inherently associated with bourgeois science and politics. Thus, in the struggle for socialism "any means-ends or cost-benefit calculation would tend to produce reformist solutions" (Kieve, 1986, p.574): the real issue is "not a question of quantitative, individualistic means-ends or petty or cost-benefit calculations, but a question of life and death" (ibid.).[10] This objection is not entirely convincing: as shown by Smolinski (1973), Marx studied pure mathematics and was convinced about the opportunity to apply it to the social sciences. Furthermore, the objection relies on the rather arbitrary claim that there exists no mathematical object (in a potentially infinite-dimensional space) that can be used to analyse any part of Marx's theory. This view seems as one-sided as the "mathematical fetishism" often attributed to Analytical Marxists.

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[10] Kieve (1986, p.574) quotes Rosa Luxemburg (1970, p.189): "in every individual act of the struggle so very many important economic, political and social, general and local, material and physical, factors react upon one another in such a way no single act can be arranged and resolved as if it were a mathematical problem." Then, Kieve argues that formal models and rational choice theory are inherently petty-bourgeois and counter-revolutionary.

More subtly, post-modern Marxists (Ruccio, 1988; Amariglio, Callari, and Cullenberg, 1989) do not reject *a priori* the use of mathematical models, but deflate their explanatory power – and their usefulness in general – to the vanishing point. According to them, mathematics is a "form of 'illustration.'" For Marxists, mathematical concepts and models can be understood as metaphors or heuristic devices" (Ruccio, 1988, p.36). However, it is unclear whether this argument can be supported by Marx's writings: Ruccio (1988) provides no textual evidence, while as already noted Smolinski's (1973) detailed analysis suggests a somewhat different view. Actually, rather than a specific interpretation of Marx, this view seems to reflect the adoption of a general post-modern epistemological stance, which reduces mathematical language, and indeed *all* (scientific) languages to mere "discourse." This position seems quite problematic since it is unclear how competing hypothesis and theories can be rationally evaluated, let alone tested.[11] Finally, the interpretation of mathematics as "illustration" reflects the post-modern anti-essentialist denial of the explanatory power of theoretical (and not only mathematical) abstractions. However, the emphasis on loosely defined "historically concrete social processes" does not seem to lead beyond either a focus on infinitesimally small phenomena or the formulation of vague, if not empty, general statements, such as the claim that "in order to be individuals (and individual needs), there has to be an infinity of other social processes that constitute their 'species-being'" (Ruccio, 1988, p.42). Or the claim that Marxian classes "can be analysed as the determinate result of the entire

[11] Unsurprisingly, econometrics, too, falls under Ruccio's (1988, p.18) axe.

constellation of social processes that can be said to make up a society or social formation at any point in time; in turn, it will be only one of the myriad determinants of those nonclass social processes" (ibid., p.38).

A different critique of the use of mathematics is based on the idea that the essential concepts of Marxian social science cannot be fully captured by formal models. An emphasis on formalism can obscure the more important theoretical and political issues and "enervate Marxist theory in the name of rigor" (Anderson and Thompson, 1988, p.228), while some critical facts about capitalist societies "can be established without mathematical proof" (Wood, 1989, p.466). This objection is more forceful, as acknowledged by Roemer himself (1981, pp.2-4), and cannot be dismissed *a priori*, but it does not necessarily entail a rejection of mathematical models.

As concerns the use of mathematics in the social sciences, and in particular in Marxist theory, the approach adopted in this dissertation is both *pluralist*, in that no exclusive or *a priori* primacy is assigned to mathematical modelling; and *minimalist* in that it is based on the idea that a model "says what it says" and a rigorous interpretation of assumptions and results, of their scope and limitations, is necessary.[12] However, it also recognises the usefulness of mathematics in theoretical analysis. "'Mathematics,' or models, cannot capture all that is contained in a theory. A model is necessarily one schematic image of a theory, and one must not be so myopic as to believe other schematic images cannot exist. Nevertheless this is not a reason not to

[12] "If we are going to be rigorous we should be rigorous, not rigorous about the proof and extremely sloppy about its range of applications" (McCloskey, 1991, p.10).

use mathematics in trying to understand a theory: for ... the production of different and contradicting models of the same theory can be the very process that directs our focus to the gray areas of the theory" (Roemer, 1981, p.3). From this perspective, the choice of the appropriate modelling tool is more important than the *a priori* discussion on the use of mathematics.

A second set of objections focus precisely on the *neoclassical models* used by AM, both general equilibrium (e.g., Roemer, 1981, 1982A, 1983A) and game theoretic models (e.g., Elster, 1982A; Roemer, 1982A, 1982C). The basic argument is that neoclassical models necessarily lead to neoclassical, or at least non-Marxian, conclusions and thus AM's mainly negative results are not surprising (Anderson and Thompson, 1988; Wood, 1989). This objection is theoretically relevant, since it is grounded in the difficulty of *inter-theoretic reduction*, a problem that is well-known in the philosophy of sciences (Sensat, 1988; Weldes, 1989). Indeed, its relevance is indirectly confirmed by the mainly negative results reached by AM. However, while it may be forcefully raised against the specific models set up by AM, its generality is less evident as it relies on a rather narrow, if unrealistic, description of neoclassical economics.[13] For instance, even the Marxian labour/labour-power distinction, whose absence is widely considered one of the main limits of Roemer's models, can be modelled within a broadly defined neoclassical framework (Bowles and Gintis, 1990, 1993). Moreover, game theory represents a vast and flexible arsenal of techniques, which do

[13] Neoclassical models "seem ill-suited to modeling anything but supply, demand, and technical relationships" (Anderson and Thompson, 1988, p.225).

not require any individualistic assumptions and can be fruitfully applied to Marxian economics, as suggested by various critics of AM (e.g., Lebowitz, 1988, pp.195-7; Sensat, 1988, p.215; Weldes, 1989, p.374).

Thus, although the objections concerning the non-neutrality of techniques and the problems of intertheoretic reduction are important, they do not seem sufficient to reject *a priori* all attempts at cross-fertilisation, especially if one considers neoclassical economics as a vast, heterogeneous arsenal of tools. Chapters 4 and 5 argue that the specific model set up by Roemer – a version of the Walrasian general equilibrium model, – may be inadequate to analyse Marxian economics, and that it is unclear that the standard "neoclassical model of a competitive economy is not a bad place for Marxists to start their study of idealized capitalism" (Roemer, 1986c, p.192). However, no general impossibility result is proved; instead, the analysis suggests alternative assumptions and formalisations.

Notwithstanding the relevance of the previous issues, the main methodological debate on AM focuses on C4' and on its relevance for Marxist social science. Although the AM critiques of teleological arguments and functionalism[14] are rather persuasive, the arguments in favour of methodological individualism *cum* rational choice and the critiques of Marx's theory on methodological grounds are less convincing. As for the latter issue, "nowhere does Elster show Marx committed to views that in principle deny microfoundational accounts" (Levine, 1986, p.726). Instead Elster (1985) finds Marx guilty of functionalism and teleological reasoning based on an

[14] Especially in its *strong* variant; see fn. 8 above for a definition.

arguably objectionable piecemeal reading of Marx's texts, with a propensity to extrapolate relevant passages from the context (Levine, 1986, pp.725-6; Sensat, 1988, pp.206-7; Wood, 1989, pp.475-6; Carver and Thomas, 1995, p.4), or as in the case of the theory of history – which generated the first AM debate on methodology (see the special issue of *Theory and Society*, 1982) – based on the identification of Marx's theory with Cohen's functionalist interpretation of it. At most, Elster's (1985) analysis shows that Marx does not support methodological individualism *cum* rational choice, and more specifically the neoclassical variant of the latter approach, hardly a startling result and arguably not enough to reject Marx's theory.[15]

As concerns methodological individualism *cum* rational choice, first, the AM critiques of functionalism do not automatically provide support for C4', and in particular its neoclassical variant: basically all critics of AM – including post-modern and post-structuralist Marxists,[16] – have rejected *both* functionalist arguments *and* the rather reductionist RCM view of individuals

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[15] Actually, Levine (1986, p.723) suggests that Elster's (1985) main methodological propositions would not stand up to his own procedure of critical assessment. Warren (1988) argues that Elster (1985) does not provide a single consistent definition of methodological individualism and often slips from one to another without proper justification.

[16] See, e.g., Ruccio (1988). Actually, post-modern Marxists have turned the accusation of functionalist reasoning against AM: "the reason why a full-blown functionalism is not needed is that the agents who comprise the economic structure are endowed initially with attributes which are functional to the system of exchange that AM imagines to constitute 'the economy'" (Amariglio, Callari and Cullenberg, 1989, p.362).

and of social science.[17] Thus, C4' should be evaluated *per se* as the proper explanatory strategy in the social sciences, rather than in opposition to functionalism and teleological reasoning, which sometimes appear in AM writings just as a rhetorical "straw man" (Foley, 1993, p.301). However, neither AM arguments nor the debates in the philosophy of science provide decisive support for C4'.

C4'.(a) encompasses two separate assumptions: the first one postulates the possibility of *intertheoretic reduction*, namely the reduction of macro-level theories to micro-level theories, without loss of meaning or explanatory content. A thorough analysis of this important issue of the philosophy of science goes beyond the limits of this dissertation. However, as forcefully shown by Sensat (1988) and Weldes (1989, p.363-6), even setting aside all doubts about methodological individualism, intertheoretic reduction is in general extremely problematic and may lead to "the complete *replacement* of the secondary theory, including its ontology, with the primary theory due to the transformation of both the meanings and the content of the secondary theory" (Weldes, 1989, p.365). As noted above, this is an important warning to identify the "hard core" (ibid., p.372) of Marx's theory when evaluating the adoption of neoclassical tools in Marxian economics.

[17] Sensat (1988) and Weldes (1989) thoroughly discuss alternative approaches to Marxist methodology from a general philosophical viewpoint.

The second assumption states that "ultimate ontological and explanatory priority is accorded to the individual" (ibid., p.356).[18] This is considered a self-evident "first principle," a defining principle for all good science, and thus it tends to be simply asserted by AM, by appealing to an alleged state-of-the-art scientific methodology. "The tension between individual and structural explanations is thus resolved (or dissolved), by fiat, by denying ontological and explanatory status to social structures" (Weldes, 1989, p.356; see also Howard and King, 1992, p.353, fn.38).

Yet, this assumption, too, is quite problematic. At the ontological level, it is unclear why the process of reduction should end at the level of the individual: first, individuals can be understood as structures liable of further decomposition in more elementary parts (e.g., cells; Howard and King, 1992, p.346). Second, even neoclassical economics admits supra-individual units, by only requiring that they be well-defined decision makers (e.g., the household). Indeed, Elster himself moves from a definition of methodological individualism as "the doctrine that all social phenomena – their structure and their change – are in principle explicable *only in terms of individuals*" (Elster, 1982A, p.453, italics added) to C4'.(a), where the emphasis is instead on more generic *subjects*.

More important, in any case "ontological reducibility (decomposability without remainder) does not *entail* explanatory reducibility" (Levine, 1986, p.724): not only macro-level theories might provide a satisfactory answer to

[18] Weldes argues that C4'.(a) also requires the additional assumption that "individuals should be construed as intentional actors" (Weldes, 1989, p.356).

certain questions, but it is not always true that the provision of a micro-mechanism would improve the understanding of a given phenomenon. Thus, "World War II was, in the sense in question, just an aggregation of subatomic particles in motion. But knowing all there is to know about these subatomic particles would not help us, in all likelihood, in knowing, say, the causes of World War II" (ibid., pp.724-5, fn.12).[19]

However, as regards the issues analysed in this dissertation, the most problematic aspects of methodological individualism concern the role of structural limits to individual choice and the atomistic conception of agents. The former issue relates to the problem of generalising individual-level predicates to group-level predicates: as is well-known in logic, if the individual property is not generalisable, a fallacy of composition may arise. As noted by Elster (1978, 1985), fallacies of composition are central to social science: "economic agents tend to generalize locally valid views into invalid global statements, because of a failure to perceive that causal relations that obtain *ceteris paribus* may not hold unrestrictedly" (Elster, 1985, p.19), leading to counterfinality and social contradictions (Elster, 1978, chapter 5). However, fallacies of composition and counterfinality imply that "the group as a whole faces a constraint that no individual member of the group faces" (Lebowitz, 1994, p.167), a property that suggests at least a refinement of methodological individualism. First, in general both individual and structural constraints shape individual choices. From this viewpoint, Przeworski's (1989) emphasis on abstract atomistic individual choice in the analysis of

[19] See also Sensat (1988, pp.201-3) and Howard and King (1992, pp.346-7).

classes and social conflict in advanced economies seems rather misleading and arguably misses Marx's point. Instead, chapter 4 below suggests that Roemer (1982A) can adopt a purely individualistic perspective, while retaining some crucial Marxian insights, only by focusing on a rather special case of general equilibrium in a static quasi-Walrasian economy where individual constraints severely limit agents' choices.[20]

Second, the existence of structural constraints implies that the analysis of the whole cannot be strictly reduced to the analysis of its parts. In the context of Marx's theory, this issue forcefully emerges in Cohen's (1983B) analysis of the structure of proletarian unfreedom. Cohen rejects the idea that proletarians are *forced* to remain in their class and stresses that they are individually free to improve their social conditions. However, to generalise such individual freedom would involve a fallacy of composition: since it is not possible for all proletarians to exit their class in a capitalist economy, each proletarian "is free only on condition that the others do not exercise their similarly conditional freedom" (Cohen, 1983B, p.11). Individual freedom coexists with collective unfreedom.[21] But then, fundamentally, knowledge of group-level properties and constraints "is prior in the explanatory order to

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[20] Thus, if "a person acquires membership in a certain class by virtue of choosing the best option available subject to the constraints she faces" (Roemer, 1988A, p.9), it is the latter part of the statement that should be emphasised, rather than the agents' free choice.

[21] Cohen (1983B) uses the famous example of ten workers in a locked room with a key on the floor that is assumed to work only once: each of them is free to exit the room, but only one of them can do it, and thus nine workers will remain in the room in any case.

understanding the conditional and contingent state of the individuals" (Lebowitz, 1994, p.167).

The second problem of methodological individualism (at least in the stronger versions) is that it requires an *asocial* view of individuals, whereby individuals are logically prior and individual attributes are not socially determined (Sensat, 1988, pp.197-9), or else structural features would play a fundamental explanatory role, via their effect on individuals' preferences and beliefs.[22] However, the very distinction between individual and social predicates is problematic, since at a general methodological level, "the individual-level predicates relied on by the individualist have built into them salient features of the relevant social context" (Weldes, 1989, p.361). Arguably, many AM assumptions, such as utility or profit maximisation, and the existence of enforceable property rights and of a labour market, incorporate certain social relations. More specifically, even within the context of given social relations, many individual attributes are socially determined, as acknowledged by AM's own emphasis on endogenous preference formation (see, e.g., Elster, 1978, 1979). For instance, Roemer rebuts the traditional neoclassical defence of DOSPA based on differential rates of time

[22] Sensat (1988, pp.195-6) identifies other three attributes of individualism: *psychologism* ("individual-level explanations of social phenomena must appeal to the operation of cognitive and motivational dispositions in specified settings" (ibid.)); *generality* ("the individual level must bring a transsocial generality to explanations of behavior in social settings" (ibid.)); and *cardinality* ("there is a small number *n* such that all social-scientific laws are derivable as applications to specific situations of (general, psychological) laws of interaction among *n* or fewer individuals" (ibid.)).

preference, because "it is a mistake to consider those differences to be a consequence of autonomous choices that people have made... Attitudes toward saving are shaped by culture, and cultures are formed by the objective conditions that their populations face" (Roemer, 1988A, p.62).

Although they acknowledge the importance of the social determination of individuals, and indeed the limits of individual rationality,[23] AM have essentially neglected these issues in their models,[24] which are instead based on a conventional view of individual agents and of instrumental rationality as in C4'.(b) (Levine, 1986, pp.726-7; Howard and King, 1992, pp.347-8).[25] Chapter 4 suggests, however, that a standard interpretation of economic agents and of their interaction may be unsuitable to analyse Marx's theory. Roemer provides Walrasian microfoundations to Marxian economics only by substantially moving away *both* from the Walrasian framework, since in his static models agents have a severely limited set of choices and their optimum is basically determined by their constraints; *and* from the Marxian

[23] Actually, much of Elster's work "rather paradoxically, shows the limitations of a rational choice paradigm" (Howard and King, 1992, p.347). See, e.g., Elster (1978, 1979).

[24] This leads to a seemingly inconsistent behaviour: Elster and Roemer "use models founded on neoclassical principles, and make claims about Marxism on this basis, but they do not believe that these principles are true" (Howard and King, 1992, p.349).

[25] According to Weldes, C4'.(b) is not a necessary requirement of methodological individualism but it is implied by the AM adoption of a conventional positivist and empiricist epistemology whereby "social scientific explanations must be deductive in order to achieve adequate predictive and explanatory power" (Weldes, 1989, p.357).

framework, since the agents' constraints, their interaction, and their class and exploitation status have no inherently social dimension.

The analysis of endogenous preferences (and structural constraints) seems a promising line for further research, which may be in contradiction with existing AM models, but not with a more general interpretation of the approach. To acknowledge that individuals are inherently social beings whose preferences, beliefs, and constraints are socially determined blurs the distinction between holism and methodological individualism and may provide some ground for dialogue. More important, the incorporation of structural constraints and endogenous preferences might lead to a more realistic and less one-sided (also, but not exclusively, from a Marxist viewpoint) relational conception of individuals as part of a social context (Weldes, 1989, pp.373-4). Indeed, it might lead to a more satisfactory "microfoundation" of Marx's theory (Sensat, 1988; Burawoy, 1989, 1995; Weldes, 1989; Bowles and Gintis, 1990), based on a concept of individual choice which escapes the dichotomy between abstract atomistic free choice and complete social determination of individual behaviour (Howard and King, 1992, p.348; Wood, 1989, pp.468-9). Although the normative analysis of exploitation is based on the historical determination of DOSPA, at the end of *Free to Lose*, Roemer notes that "there is a key dimension along which the autonomy of persons in capitalist society could be challenged; this I have not exploited. For if people's conceptions of welfare are themselves determined by the economic structure in which they live, then a welfare distribution might stand condemned for the further reason that the structure shaped those conceptions. Having a theory of how capitalism (or any economic structure) shapes preferences would add to the story" (Roemer, 1988A, p.177).

1.2.3. ROEMER ON EXPLOITATION AND CLASS

Many of the AM substantive propositions, too, have generated intense debate: from Cohen's (1978) analysis of forces and relations of production in Marx's theory of history (Foley, 1993), to Przeworski's (1985A) analysis of class conflict and the political process (Burawoy, 1989; Przeworski, 1989; Wood, 1989; Burawoy, 1995), to Elster's (1982A, 1985) analysis of class alliances, revolutionary motivations, and social change (Weldes, 1989). This section reviews the literature on Roemer's (1982A, 1982B, 1982C, 1988A) theory of exploitation and classes, which is examined in this dissertation.

Although Roemer's models are thoroughly analysed in chapters 4 and 5, it is opportune to briefly summarise them here. Roemer (1982A) assumes that there are N agents with identical preferences and equal access to the production technology of n goods. Production requires capital which is, in principle, unequally distributed. He defines *Marxian exploitation* as unequal exchange of labour: agent i is *exploited* (an exploiter) if she works more (less) time than is embodied in the consumption bundle she consumes.

First, Roemer (1982A, chapter 1) considers a pre-capitalist subsistence economy with labour minimising agents and no labour market (only physical goods, inputs and outputs, are traded). He proves that in equilibrium aggregate labour is equal to the amount of time embodied in the aggregate subsistence requirements, but given DOSPA labour time is not equally distributed: asset-rich agents are exploiters while asset poor agents are exploited. This, according to Roemer (ibid., Theorem 1.6, p.38), proves that exploitation can logically exist without the institution of labour exchange and thus without domination at the point of production.

Next, Roemer (ibid., chapter 2) introduces a labour market in the subsistence economy. This allows him to define classes in Marxian terms based on "the way in which an agent relates to the means of production – hiring labour power, selling labour power, working his own shop" (ibid., p.70). Roemer (ibid., Theorem 2.5, p.74) proves that classes emerge as the product of individual optimisation: in equilibrium asset rich agents are capitalists (net hirers of labour power), asset poor agents are proletarians (net sellers of labour power), and there exists a class of petty bourgeois who are self-employed. Finally, asset-rich agents are exploiters while asset-poor agents are exploited and the *Class Exploitation Correspondence Principle* (ibid., Theorem 2.7, p.79) holds: capitalists are exploiters, proletarians are exploited, and the petty bourgeois have an ambiguous exploitation status.

Then, Roemer (ibid., chapter 3) proves the functional equivalence of credit and labour markets. The subsistence economy with a labour market is isomorphic to an identical economy with a capital market: asset rich agents are exploiters and belong to the class of net lenders, while asset poor agents are exploited and belong to the class of net borrowers. According to Roemer, this proves that "there is nothing in the institution of the labour market intrinsically necessary for bringing about the Marxian phenomena of exploitation and class. [Instead] competitive markets and private, differential ownership of the means of production are the institutional culprits in producing exploitation and class" (ibid., p.93).

Having proved the positive and normative priority of DOSPA (similar results hold in accumulation economies with revenue-maximising agents (ibid., chapter 4)), Roemer highlights some analytical and conceptual problems of Marxian exploitation conceived as unequal exchange of labour: in an economy with a more general cone technology labour values cannot be defined prior to prices (ibid., chapter 5). Moreover, if agents' labour endowments (e.g. skills) or preferences over leisure are heterogeneous, "it is possible for some very wealthy producers to be exploited and for some very poor producers to be exploiters" (ibid., p. 175). Hence, according to Roemer, Marx's theory of exploitation should be abandoned as a problematic proxy for the normatively relevant phenomenon, namely DOSPA and the resulting welfare inequalities. Roemer (ibid., chapter 7) provides an alternative, game-theoretic definition of exploitation based on DOSPA which aims to generalise Marxian exploitation, capturing its essential normative content.[26]

Many critiques have been expounded on Roemer's definitions and his models, mainly based on issues of interpretation of Marx's theory. Lebowitz (1988) argues that Roemer gives logical priority to property relations over capitalist relations of production, while in Marx's theory the former are determined by the latter. Thus, Roemer does not realise that, according to

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[26] Roemer analyses exploitation mainly from a normative viewpoint, however exploitation also plays an important role from a positive viewpoint, e.g., in Roemer's interpretation of historical materialism (Roemer, 1982A, 1986B, 1988A, 1989).

Marx, "the situation in which the purchase of labor-power did not occur was explicitly pre-capitalist" (Lebowitz, 1988, p.206). For instance, in the subsistence economy with a capital market, what Roemer calls exploitation is more precisely defined as usury (see also Kieve, 1986, p.563).[27] Furthermore, Lebowitz (1988) argues that by assuming perfect information, perfectly enforceable contracts (especially in the purchase of labour-power), and no restrictions to the use of any technology, Roemer has effectively assumed away all possible effects of the relations of productions on the production function, and thus on profits and exploitation.

Anderson and Thompson (1988) also stress Roemer's neglect of the labour/labour-power distinction, which is equivalent to assuming that workers do not resist being exploited, and note that profits in his model are just a scarcity rent. Moreover, by neglecting the actual features of the labour process, Roemer's models cannot really capture the concept of class: lacking any direct social relation among workers, or between them and the capitalists, it is unclear how a sense of class comradeship could arise.

Foley (1989) criticises Roemer's concept of class as nothing more "than a static typology of equilibrium labor allocation and an associated inequality in control over social resources and consumption of social product" (Foley, 1989, p.191); and Roemer's definition of exploitation as "private and

[27] More generally, Kieve (1986) claims that Roemer's subsistence economies are models of advanced capitalism where some "theoretically troublesome" features "such as surplus value, surplus labor, capitalist class relations" (ibid., p. 561) have been abstracted away.

ahistorical" (ibid., p.189).[28] Instead, according to Foley, "the important historical aspect of class societies is that exploiting classes, through their control over social surplus production, shape the reproduction of the society and in particular the directions in which change can take place" (ibid., p.191). Based on this interpretation of Marx's theory, Foley finds Roemer's emphasis on the normative aspects of exploitation as an abstract measure of injustice misplaced (see also Wood, 1989, p.465). Furthermore, the exclusive stress on DOSPA inverts the understanding of exploitation and inequalities from Marx's viewpoint, whereby "exploitation is in the first instance an injury to the life of direct producers, because it removes from them … the control over a part of the fruits of their energies, talents, and efforts (ibid., p.192), and it is for this reason that it gives rise to inequalities.[29]

Dymsky and Elliot (1989) and Wood (1989) note that Roemer defines exploitation as unjust advantage, a form of inequality, – secondary rather than primary exploitation, according to Marx – and thus his conclusion that only DOSPA matters is not surprising.

These critiques are arguably relevant, but they do not seem conclusive. To be sure, interpretive and definitional issues are crucial in the evaluation of Roemer's theory, especially as an interpretation of Marx's theory. Indeed, most critics provide considerable textual evidence against Roemer's reading of Marx, which would suggest that his models– albeit interesting – are not

[28] The ahistorical character of Roemer's theory has been emphasised, to various degrees and extent, by all critics (see in particular Burawoy, 1990, pp.790-2; Wood, 1989, Section 3).

suitable to evaluate Marx's theory. Yet, as various endless debates show, very few issues in Marxist thought can be satisfactorily settled uniquely at the level of definition and interpretation. Moreover, an *a priori* rejection of Roemer's models can lead to "throw the baby away with the dirty water," both methodologically (see Section 1.2.2) and from a substantive viewpoint. As shown in chapters 4 and 5, a detailed critical analysis of Roemer's models suggests interesting directions for further research.

More important, fidelity to Marx's writings is not a major constraint for Roemer and Marx's concepts and definitions are substantially revised. So, a critique entirely based on textual evidence arguably misses the point. For instance, Anderson and Thompson (1988) argue that Roemer's subsistence economy with a labour market is equivalent to another economy in which the only non-produced input is a natural resource, say coal, and nobody works. But then, "we are forced by Roemer's logic to say that those who must sell coal-power are coal-exploited" (ibid., p.220). This is true; but the *Generalised Commodity Exploitation Theorem* (e.g., Roemer, 1988A, p. 53) is precisely one of Roemer's arguments to prove the irrelevance of the labour theory of value, and *a fortiori* of Marxian exploitation.

A priori critiques of the abstract and static nature of Roemer's models are not *per se* conclusive either. Roemer repeatedly acknowledges the static nature of his models and the importance of disequilibrium and dynamics (most explicitly in Roemer, 1982D). Yet, "constructing a model of capitalism

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[29] A general critique of Roemer's interpretation of Marxian exploitation along similar lines is advanced by Sensat (1984) and Reiman (1987).

that would reveal its essentially dynamic features is a different task from what mine was" (Roemer, 1992, p.150). The logical structure of Roemer's argument is different: "in the real world we observe X (DOPA), Y (coercion in the labour process), and Z (class and exploitation). We have, if you will, an 'empirical proposition' that $X + Y \Rightarrow Z$. Now I construct a model in which the following theorem holds: X + not $Y \Rightarrow Z$; from this I say that X is the 'fundamental' cause of Z in the real world, not Y" (ibid.).

But then a forceful critique of Roemer's core logical argument cannot be limited to noting that his models and definitions are ahistorical; that many empirically relevant features of the capital/labour relation are neglected; that the labour/labour-power distinction is not considered; that money, hard uncertainty, and institutions, including firms, are absent (Hodgson, 1989); or that unemployment and coercion in production are neglected, and that in general the description of production processes is simplistic (Devine and Dymsky, 1991). Arguably, these objections prove that "Roemer's inference is irrelevant for capitalism, because 'not Y' is false for capitalism" (Roemer, 1992, p.150), a point that does not challenge the basic logical argument. Moreover, given the lack of a formal analysis of Roemer's models, it is often *a priori* unclear whether the suggested changes would lead to significantly different conclusions.[30]

From a methodological viewpoint, the adoption of an abstract model, and indeed of the most abstract model in neoclassical economics, the

[30] For instance, as shown in chapter 4 below, contrary to Anderson and Thompson's (1988) claim, Roemer's assumption of *non benevolent capitalists* is not crucial for his results.

Walrasian model, is explained by Roemer's attempt to provide *general* microfoundations to exploitation and class. In his comment on Bowles and Gintis (1990) which aims to provide microfoundations to Marxian economics based on the concept of contested exchange (which incorporates issues of contractual incompleteness and conflicts of interest between the parties to the exchange), Roemer (1990) explicitly argues that market imperfections are a "thin thread" to provide general microfoundations to Marxian economics.[31]

In chapters 4 and 5, *a priori* issues of interpretation are left aside and Roemer's core logical argument is directly examined. Various dynamic extensions of Roemer's models are set up in order to evaluate the theoretical and analytical robustness of his methodological and substantive claims at the appropriate level of generality. Roemer's models are essentially static: they can be interpreted as describing either a succession of one-period economies or an infinitely lived generation, but in either case there are no intertemporal trade-offs: *intertemporal* credit markets and savings are ruled out, and the latter assumption in particular seems unduly restrictive. Thus, they do not seem suitable to analyse the persistence of exploitation and classes.

Instead, from a methodological viewpoint, a formal dynamic model proves extremely useful in the analysis of the possibility of providing neoclassical (more specifically, Walrasian) microfoundations to Marxian economics. In particular, a model that aims to provide microfoundations to

[31] In one of the (surprisingly) few economic models in the AM tradition, Yosihara (1998) attempts to provide a synthesis of Roemer (1982A) and Bowles and Gintis (1990). However,

Marx's concepts of exploitation and class must be able to account for their persistence, since, according to Marx, they are inherent features of a capitalist economy (see, Roemer, 1982A, p.6). From a substantive viewpoint, a dynamic model allows one to assess the causal and moral relevance of DOSPA, focusing on its role in generating exploitation and classes as persistent features of a competitive economy in which agents can save and the distribution of productive assets can change over time.

The advantage of this strategy is twofold. First, the robustness of Roemer's core arguments can be directly evaluated at the appropriate level of generality. Thus, the results derived in chapters 4 and 5 can be interpreted as follows. Let D denote the dynamic features of the economy (agents living for more than one period, savings, intertemporal optimisation). Roemer proves that X + not Y + not $D \Rightarrow Z$ *as a persistent phenomenon*; instead chapters 4 and 5 prove that X + not $Y + D \Rightarrow Z$ is *not persistent*. Second, by avoiding an *a priori* rejection and focusing on specific issues arising from Roemer's models, this dissertation provides ground for dialogue and for further cross-fertilisation, and it suggests various interesting lines for further research.

1.2.4. EXPLOITATION AND DISTRIBUTIVE JUSTICE

Inter alia, chapters 4 and 5 raise doubts on Roemer's interpretation of Marx's theory of exploitation as "a kind of resource egalitarianism" (Roemer, 1994A, p.2) and on the claim that his own definition of exploitation based on

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the model is not entirely convincing given some *ad hoc* assumptions (for instance, he assumes a competitive economy with only one firm hiring labour).

DOSPA generalises Marx's theory capturing its essential normative content – which is interpreted as requiring "an egalitarian distribution of resources in the external world" (ibid., p.3). However, Roemer's property rights definition of exploitation is arguably interesting *per se*.

Let $e^i = (\phi^i, \omega^i, \sigma^i, \nu^i)$ be the endowment of agent i, living under feudalism: ϕ^i denotes i's degree of feudal privilege, where $\phi^i > 0$ if i is a feudal lord, while $\phi^i < 0$ if i is a serf, and ϕ^i is normalised so that $\Sigma_{i \in N} \phi^i = 0$; ω^i is the vector of i's private alienable assets; σ^i is i's vector of labour skills; and ν^i is the vector of i's needs, where ν^i is normalised so that $\nu_j^i = 0$ implies that i has a normal level of need j, while $\nu_j^i > 0$ indicates freedom from need j enjoyed by i, and if $\nu_j^i < 0$ then i is needy in j. The following taxonomy of exploitation can be derived (Roemer, 1986B): a coalition S is *feudally exploited* at a given allocation if there is an hypothetically feasible alternative such that by withdrawing with $e^S = (0, \Sigma_{i \in S}\omega^i, \Sigma_{i \in S} \sigma^i, \Sigma_{i \in S} \nu^i)$, i.e. with its per capita share of feudal privilege and its own private property of everything else, the welfare of S improves; a coalition S is *capitalistically exploited* if it would improve by withdrawing with $e^S = (0, (S/N)\Sigma_{i \in S}\omega^i, \Sigma_{i \in S} \sigma^i, \Sigma_{i \in S} \nu^i)$, i.e. with its per capita share of both feudal privilege and alienable property; a coalition S is *socialistically exploited* if it would improve by withdrawing with $e^S = (0, (S/N)\Sigma_{i \in S}\omega^i, (S/N)\Sigma_{i \in S} \sigma^i, \Sigma_{i \in S} \nu^i)$, i.e. with its per capita share of all assets, alienable and inalienable; a coalition S is *needs exploited* if it would improve by withdrawing with $e^S = (0, (S/N)\Sigma_{i \in S}\omega^i, (S/N)\Sigma_{i \in S} \sigma^i, \mathbf{0})$, i.e. with its per capita share of all assets and needs. Whereas the elimination of socialist exploitation implies equality of income (skills are socialised *ex-*

post), the elimination of needs exploitation requires an unequal distribution of income, as in the famous dictum: "To each according to his needs."

Based on this taxonomy of exploitation Roemer suggests an original interpretation of historical materialism, according to which "history progresses by the successive elimination of dynamically socially unnecessary forms of exploitation" (Roemer, 1986B, p.146).[32] Thus, for instance, when feudal exploitation becomes a fetter to the development of the forces of production, class struggle leads to its elimination and to the passage from a feudal to a capitalist mode of production.

More importantly, for the purposes of this dissertation, this theory naturally leads to an original normative approach by defining an *historical materialist ethical imperative* which requires the elimination of (socially unnecessary) exploitation in order to promote the self-actualisation of men and of man as the "unquestionable good" (ibid., p.147).[33] But also on distributive grounds, since the exploitation-free society provides a natural egalitarian benchmark, with an unequal distribution of transferable resources to compensate for unequal endowments of skills and needs. Nevertheless, Roemer's asset-based theory of exploitation is not a complete theory of distributive justice: "The injustice of an exploitative allocation depends upon the injustice of the initial distribution [of alienable and inalienable assets]" (Roemer, 1988A, p.57). While the moral arbitrariness of the distribution of

[32] For a discussion of the notion of statically and dynamically socially necessary exploitation, see, e.g., Roemer (1986B, pp.143-5).

[33] For a definition of self-actualisation of men and man, see chapter 3, fn.1, below.

feudal privilege is rather clear, the same is not true for skills, needs, and most importantly, physical assets (see, e.g., Nozick, 1974).

In *Free to Lose* (1988A), Roemer explicitly discusses the morality of DOSPA (and *a fortiori* of capitalist exploitation) and suggests that it is morally objectionable since it is typically the product either of immoral original accumulation - "robbery and plunder" (ibid., pp.58-9) - or of morally arbitrary factors, such as socially determined differential rates of time preference and entrepreneurial abilities or sheer luck (ibid., pp.60-9). In the latter case, even if asset inequalities have arisen in morally respectable ways, Roemer briefly suggests that the resulting exploitation can still be condemned on grounds of *equality of opportunity* (ibid.), even though no comprehensive discussion is provided. From this perspective, Roemer's asset-based theory of exploitation can be seen as an equality of opportunity approach *in nuce*, which suggests the (fairly controversial) view that the Marxist ethical imperative requires the progressive elimination of different forms of exploitation *in order to equalise opportunities*.[34] This establishes an interesting unexplored link between AM and the modern theories of equality of opportunity which are the object of the next section.

1.3. EQUALITY OF OPPORTUNITY

"The law, in its majestic equality, forbids rich and poor alike to sleep under bridges, beg in the streets or steal bread." (Anatole France, *The Red Lily*, chapter 7).

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[34] Roemer (1994B) proposes *market socialism* as the best way to achieve equality of opportunity as a core socialist objective. (For a critique, see Arneson, 1994; Levine, 1994).

The other egalitarian approach adopted to analyse the dynamics of inequality and classes in this dissertation is the theory of equality of opportunity (EOp) proposed by Arneson (1989) and Cohen (1989), and formalised by Roemer (1996, 1998, 2002). In particular, by adopting an EOp perspective, the dynamics of inequality and classes are analysed assuming different variables of egalitarian concern. This section surveys the three main strands of literature relevant for the analysis. Section 1.3.1 presents the general EOp approach and Roemer's specific proposal and surveys the extensive philosophical and economic literature on both. Section 1.3.2 focuses on the contributions on the appropriate currency for distributive justice and on the intertemporal egalitarian (more precisely, maximin) paths.

1.3.1. OUTCOMES AND OPPORTUNITIES

Since Rawls's (1971) rejuvenation of egalitarianism, individual choice and responsibility have played an increasingly important role in egalitarian thinking. Arguably, they are present *in nuce* in Rawls's (1971) and Sen's (1980, 1985) emphasis on, respectively, primary goods or functionings as the appropriate *equalisandum*, given their role as a means for agents to reach their freely chosen ends. However, they are put at the centre of the stage only in Dworkin's (1981A,B) theory of 'equality of resources', according to which distributive justice requires that agents be not compensated for the outcomes of their autonomous choices and preferences. According to Cohen, Dworkin "has performed for egalitarianism the considerable service of incorporating within it the most powerful idea in the arsenal of the antiegalitarian right: the idea of choice and responsibility" (Cohen, 1989, p.933).[35]

In recent years, many authors have proposed theories of distributive justice in which individual choice and responsibility play a crucial role, and which can be broadly classified as *theories of equality of opportunity*.[36] This dissertation focuses in particular on Arneson's (1989, 1990) and Cohen's (1989) theories, as developed and formalised by Roemer (1993, 1995A, 1996, 1998, 2002). However, in order to examine the main philosophical tenets of this approach, a general framework of analysis is provided first.[37]

Let N be the set of agents in the economy, indexed by $i = 1, \ldots, N$. Each agent i is characterised by a vector $\theta^i \in \Theta \subseteq \Re^n$ denoting i's personal features and actions. Let $o^i \in O \subseteq \Re$ be the level of the relevant *outcome* attained by i: o^i could denote income, welfare, functionings, etc.[38] Let Ψ be the set of feasible allocations of transferable resources: a *policy* is a function $R: N \to \Psi$, such that $r = (r^1, \ldots, r^N)$ describes the allocation of resources to all

[35] Roemer (1996, chapters 6-8) interprets the EOp approach as the latest stage of a progressive evolution of egalitarian thinking since *A Theory of Justice* (1971). For a succinct exposition, see Arneson (2000C), Sugden (2004), Roemer (2001A). For a critique of this interpretation, see Daniels (1990), Wolff (1998, p.116ff) and Anderson (1999, p.290ff).

[36] Alternative definitions include "luck egalitarianism" (Anderson, 1999; Arneson, 2000B), "responsibility-sensitive egalitarianism" (Mason, 2001), "liberal egalitarianism" (Levine, 1999), or, with a pejorative connotation, "conservative egalitarianism" (Fleurbaey, 2001).

[37] In addition to the authors mentioned in the main text one may include Kymlicka (1990), Nagel (1991), Rakowski (1991), Van Parijs (1991, 1995).

[38] See section 1.3.2. In principle, o^i can be a vector of outcomes, rather than a single variable.

agents. The first postulate of responsibility-sensitive egalitarianism is captured by the following assumption.

ASSUMPTION 1. Θ can be partitioned in two subsets Θ_c and Θ_r such that $\theta_c^i \in \Theta_c$ denotes i's arbitrary factors – her circumstances, – while $\theta_r^i \in \Theta_r$ denotes the variables she is deemed responsible for. Then, $\theta^i = (\theta_c^i, \theta_r^i)$.

The next assumption states that i's outcome level depends on government policies and on i's circumstances and choices.[39]

ASSUMPTION 2. There is a function o: $\Psi \times \Theta_c \times \Theta_r \to O$ such that, for all i, $o^i = o(r^i, \theta_c^i, \theta_r^i)$.

A general formulation of EOp can now be stated.

DEFINITION (GENERAL EOP; Roemer, 2002, p.456). Egalitarianism seeks to equalise individual outcomes to the extent that they are due to differences in arbitrary factors θ_c, but allow differences in outcomes to the extent that they are due to differences in responsible factors θ_r.

Thus, if an opportunity is an "access to advantage" (Cohen, 1989, p.907), the EOp promotes "equal access but the individual is responsible for turning that access into actual advantage" (Roemer, 1998, p.24).

This general framework is suitable to describe different EOp theories focusing on two coordinates: the choice of outcome and the θ_c/θ_r partition.

[39] So far, no uncertainty is assumed (although the variables may be the result of random processes); however with a slight abuse of notation θ_r^i can be interpreted as the outcome of a gamble deliberately chosen by agent i. See below for a discussion of luck.

Thus, according to Dworkin (1981A,B), θ_r should include agents' choices and preferences (as long as they identify with the latter) and θ_c should include *extended* resources (both transferable and nontransferable, such as talents and handicaps). However, he does not define the opportunity equalisandum *o* and proposes instead that bundles of extended resources be equalised.[40] More precisely, he proposes that taxation systems should mimic the functioning of an ideal insurance market where agents are placed under a thin veil of ignorance (they are assumed to know their preferences) and have the same budget to buy goods and to insure against an adverse realisation of the lottery of talents and handicaps.

Arneson (1989, 1990) proposes equality of opportunity for *welfare* and criticises Dworkin's partition, – "Dworkin's cut" (Cohen, 1989) – arguing that involuntarily acquired tastes may call for compensation. He argues that θ_c should include all characteristics that are *beyond* an agent's control, while θ_r are the variables *within* her control.[41] Then, "equal opportunity for welfare obtains among persons when all of them face equivalent decision trees – the expected value of each person best (most prudent) choice of options, second-

[40] Thus, Dworkin's resource egalitarianism is different from an equality of opportunity theory, narrowly conceived (Dworkin, 1981B, p.307). However, the definition of general EOp provided above encompasses his approach, too.

[41] Arneson (2000A, 2000C) states that different EOp theories advocate different "divisions of responsibility" between individuals and society. However, this terminology is slightly misleading as it conflates responsibility as *accountability* (for individuals) and responsibility as *moral obligation* (for society).

best … n-best is the same. The opportunities persons encounter are ranked by the prospects for welfare they afford" (Arneson, 1989, pp.85-6).

Cohen (1989) suggests *advantage* as the appropriate outcome – which includes welfare but is closer to Sen's (1980, 1985) functionings, – but proposes a similar θ_c/θ_r partition. However, the precise definition of advantage and the actual mechanism to implement EOp are not specified.

This dissertation focuses on Roemer's (1998, 2002) theory, which is the most comprehensive and rigorous attempt to provide an operational definition of the EOp. Roemer's proposal is captured by the following additional assumptions. Let the *type* of an agent be her circumstances θ_c and let T be the set of types in the population, with cardinality $T \leq N$: agents with the same circumstances belong to the same type, denoted by $t = 1, \ldots, T$, so that if agent i is of type t her circumstances can be denoted as θ_c^t.[42]

ASSUMPTION 3. Let $\Theta_r \subseteq \Re_+$ and interpret θ_r as *effort*.[43]

The policy r allocates resources to agents of different types based on their effort: $r^t(\theta_r)$ is the amount of resources received by an agent of type t if she expends effort θ_r. However, θ_r^i will in turn depend on the government's policy r, which will generate a distribution of effort levels for each type. Thus, let P denote the set of probability distributions on $\Re_+$.

[42] Formally, the function t: $N \rightarrow T$ defines a partition of N into types, such that if i belongs to $t(i)$ then her circumstances are $\theta_c^{t(i)}$. The notation in the text is used for simplicity.

[43] The assumption that effort is unidimensional can be replaced by (Roemer, 2002, p.461):

ASSUMPTION 4. There is a mapping F: T x $\Psi \rightarrow P$ which associates to any government policy r, a distribution of effort $F(\theta_r | r^t, t)$ for each type.

Assumption 4 entails a potential contradiction: since effort distributions are type-specific, θ_r^i is partially determined by i's circumstances,[44] and thus the effort *level* θ_r^i is not a satisfactory index of agents' responsible choices. Assumption 5 suggests to focus on the agents' *degree* of effort as measured by their rank, the quantile π, in the effort distribution of their type.

ASSUMPTION 5 (Roemer, 2001A, p.8). (*i*) Two agents of type t have tried equally hard if and only if they sit at the same rank of their type distribution of effort F_r^t; (*ii*) Two agents of different types have tried equally hard if and only if they sit at the same rank of their *respective* distributions of effort F_r^t.

Assumption 5 provides a plausible level-comparable inter-type measure of effort – a measure of "sterilized effort" (ibid., p.18), – which incorporates the intuition that each agent's effort – conceived as volition rather than disutility (Risse, 2002, p.726; Roemer, 1998, pp.21-2) – should be measured "on her own hook."[45] Hence, if $\theta_r^i(\pi, r^t)$ is agent i's effort level,

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ASSUMPTION 3'. There is a set of increasing functions f^t: $\Theta_r \rightarrow \Re$, $t = 1, ..., \tau$, which represent indices of effort, such that O is increasing in f^t, all t.

[44] Alternatively, one may interpret the model as being based on the profile (θ'_c, θ'_r), which is a proxy for the true vector (θ_c, θ_r). See Fleurbaey (1998, p.220) for a thorough discussion.

[45] Roemer (1998, p.15) justifies Assumption 5 with the *assumption of charity* (AC), which states that (i) beneath their circumstances agents possess a deep individuality, including a propensity to expend effort, and (ii) the distribution of this propensity is the same in all types. Hurley (2002A) and Risse (2002) argue that, from a philosophical viewpoint, AC is

corresponding to the quantile π of the conditional distribution F_r^t, then $F(\theta_r^i(\pi, r^t)| r, t) = \pi$. Hence, assuming the mapping F to be known, it is possible to compute the *indirect outcome functions* $v^t(\pi, r^t) = o(r^t(\theta_r^i(\pi, r^t)), \theta_r^i(\pi, r^t), \theta_c^t)$ which give "the level of [o] for individuals of type t at the π^{th} quantile of the effort distribution for type t when the policy is [r]" (Roemer, 2002, p.458). Roemer's version of EOp can now be stated.

DEFINITION (ROEMER'S EOP). The ideal EOp policy is defined as follows.

$r^\pi = \text{Arg Max}_r \text{ Min}_t \, v^t(\pi, r)$, for all π.

Although they are rather intuitive, EOp theories (both the general EOp and Roemer's specific proposal) raise deep economic, philosophical, and political issues. The purpose of this dissertation is not to provide an articulate defence of the EOp, but rather to analyse its implications. However, in the remainder of this section a thorough review of the debate on the EOp is provided. This discussion aims to clarify the essential features of the EOp, the main differences with respect to other theories, and the reasons why it is a promising line for further research in liberal egalitarian thinking.

A first set of critiques of EOp theories concern the implementation of the proposal. On the one hand, there is an issue of feasibility: it may be difficult to gather all the necessary information to determine (a) the components of θ within and those beyond the agents' control; and (b) every agent's vector θ_c (or, equivalently, her type), especially if the possibility of

unsatisfactory and possibly leads to a fundamental incoherence. However, AC is by no means essential: for an alternative, less controversial justification, see e.g. Roemer (2001A).

manipulation is considered (Fleurbaey, 1995A, p.30). Moreover, in general the implementation of the EOp may require a vast, expensive bureaucracy (Solow, 1995; Epstein, 1995).

Even if the set of types is correctly specified, θ_r (say, effort) may be unobservable or extremely difficult to measure, especially if it is a vector, if it is not monotonically related to outcome, and if uncertainty plays an important role. Moreover, in modern economies conceived as cooperative endeavours, various market failures (externalities, market incompleteness, asymmetric information, etc.) might make it difficult to implement effort-based distributions (Levine, 1999).[46]

On the other hand, even if feasible, the application of the EOp may require an intolerable intrusion in people's lives in order to evaluate how genuine their choices are (Fleurbaey, 1995A, pp.46-7) and their full set of circumstances (Fox-Genovese, 1995).[47] Moreover, if preferences are determined by factors beyond agents' control, including their family situation, this may motivate a substantial intrusion in people's lives and in parental choices on EOp grounds (Daniels, 1990, p.291).

These issues should be taken into account when designing EOp policies, but they do not question the *theoretical* foundations of the EOp. Moreover, at the *empirical* level, Roemer's proposal is not particularly vulnerable to them:

[46] In economies with high unemployment, job incumbency is (at least to a degree) morally arbitrary and there are agents who would like to expend effort but cannot (Levine, 1999).

[47] This problem is even more evident if the equalisandum is (or includes, as in Cohen's notion of advantage) welfare. As Cohen (1989, p.910) puts it: "Hi! I'm from the Ministry of Equality. Are you, by any chance, unusually happy today?"

although the number of types may quickly become very large (Solow, 1995),[48] in empirical applications (e.g., Roemer *et al.*, 2003) a suitably small set of circumstances can be chosen and the EOp policy can be interpreted as proving that even a minimal egalitarian commitment leads to a significant amount of redistribution (Fleurbaey, 2001).

A second set of problems relate to the existence of the EOp policy. Let O^i be the set of outcomes that i can reach by an appropriate choice of θ_r^i; that is, $O^i = \{o^i | \, o^i = o(r^i, \theta_c^i, \theta_r^i), \theta_r^i \in \Theta_r\}$. A natural formalisation of the general EOp (Fleurbaey, 1995A, p.29) may require that the allocation of resources r equalises the choice sets (O^i, θ_r^i), for all $i = 1, .., N$. However, unless a fairly strong separability requirement is imposed on the function o, the EOp solution will not exist (ibid.). In general, various axioms formalising the two principles underlying the EOp have been proposed (namely, "the principle of compensation" and "the principle of natural reward" defined below; see, e.g., Fleurbaey, 1995A, 1998; Maniquet, 2004) and "the economic analysis has revealed that in most contexts, there is a substantial conflict between [them]" (Fleurbaey, 2001, p.509).[49]

This problem is even clearer in Roemer's EOp. Suppose that there is a continuum of agents in every type: in general it is not possible to equalise something for an infinite number of populations at the same time, and the set

[48] Solow (1995) raises also the important issue of the appropriate choice of the egalitarian time unit – whole lives or shorter spans, - which is discussed in chapter 2 below.

[49] The issue of existence of the EOp policy is raised by Roemer (1996, chapter 8) in his discussion of Arneson's (1989) equivalence condition of different decision trees.

$\{r^{\pi} \mid \pi \in [0, 1]\}$ solving Roemer's EOp will normally consist in a continuum of different policies (Roemer, 1998, p. 27). Thus, Roemer suggests to adopt a second-best approach whereby "the objective function of each effort slice of the population ... is weighted by its size" (Roemer, 2002, p.459).[50]

ROEMER'S REVISED EOP: The EOp policy is defined as follows.

$$r^{EOp} = \text{Arg Max}_r \int_0^1 \text{Min}_t \, v^t(\pi, r) \, d\pi.$$

Fleurbaey (1998, pp.221-2) notes that as the number of types decreases, Roemer's revised EOp approaches the utilitarian objective function, which contradicts the EOp view that agents should bear the consequences of their responsible actions.[51] However, this criticism is not convincing since it neglects the second-best nature of Roemer's revised EOp.

The next two sets of critiques relate to general theoretical issues and highlight the differences between the EOp and other theories of distributive justice. The EOp consists of two logically separate principles (Fleurbaey, 1998, pp.210-9; 2001, pp.506-12): the *principle of compensation*, which states that equality of outcomes should prevail if responsibility is absent (e.g., Cohen, 1989, p.914); and the *principle of natural reward*, which states that agents must bear the consequences of their responsible actions (e.g., Arneson,

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[50] Alternatively, the EOp policy could be defined as the average of the r^{π} policies (Roemer, 2002, p.459, fn.1) or, more simply, one may focus only on one effort slice (say, the median) of the population (Roemer, 1993, 1998).

[51] Fleurbaey (1998, pp.221-2) argues that if instead the revised EOp is defined as focusing only agents sitting at a certain quantile π (say, the median) of their type's distribution of effort, then the EOp policy may require giving all the resources to them.

1989, p.86; Cohen, 1989, p.913). "Distributive justice does not recommend any intervention by society to correct inequalities that arise through the voluntary choice or fault of those who end up with less, so long as it is proper to hold the individuals responsible for the voluntary choice or faulty behavior that gives rise to the inequalities" (Arneson, 1989, p.176).

Due to the principle of compensation, the EOp differs from theories of formal equality of opportunity based on the *merit principle* (or *non-discrimination principle*), according to which agents "should be recruited to positions in society according to their merits, ... that is, according to the attributes they have that are relevant to performing the tasks of the position in question" (Roemer, 1998, p.84). This principle contains a negative and a positive prescription (Mason, 2001): the negative prescription requires that no one be discriminated against, due to characteristics that are irrelevant for the competition (e.g., race, gender, religion, etc.); the positive prescription requires that the most qualified candidate obtains the position.

The principle of compensation requires to implement policies to "level the playing field" among individuals who compete for positions. Thus, the EOp encompasses the negative prescription of the merit principle, but goes beyond it, since it advocates compensatory transfers for circumstances beyond agents' control and for which they should not be held accountable, such as their race, gender, family's socio-economic status, but also, more importantly, inborn natural talents. Thus, as noted by Roemer (1998) and Mason (2001), the EOp is in conflict with the *positive* prescription of the merit principle, since it may require to allocate positions based on effort rather than merit, which, at least in some situations, - e.g., in surgical schools or in basketball teams - seems undesirable.

Yet, the differences between the EOp and the merit principle do not seem sufficient to abandon the former. In its negative part, the merit principle is a very basic requirement of fairness, which advocates *open* – rather than *equal* – opportunities (Hansson, 2004, p.315), but it is unsatisfactory as a complete conception of justice. In fact, it lacks independent justification, since a broader concept of justice is necessary to define what constitutes a legitimate qualification for a position (see, Mason 2001; and references therein). Moreover, "it says nothing about the size of the rewards that can be justly attached to various positions" (Scanlon, 1995; see also Hansson, 2004; Mason, 2001; and *contra* Flew, 1981). Finally, the merit view reflects a purely procedural conception of justice (Hansson, 2004, p.315) which – paraphrasing Anatole France – requires that the well-nourished and educated rich and the destitute poor have the same right to compete for relevant social positions.[52] Hence, it is not surprising that a theory of distributive justice may be in conflict with the merit view.

Instead, in its positive part, it can be understood as incorporating an intuitive social efficiency condition for allocating individuals to at least some socially relevant positions. Thus, the inconsistency with the EOp can be interpreted just as the specific form that the equity vs. efficiency trade-off

[52] This criticism echoes Marx's famous critique of equality of rights in his *Critique of the Gotha Programme*. For a thorough discussion, see Wood (1979B). See also Rawls's analysis of the "system of natural liberty" (Rawls, 1971, §12).

takes in this context. Therefore, although distributive justice may require social positions to be allocated according to the EOp, considerations of merit should play an important role in the determination of the scope and extent to which opportunities should be equalised (Roemer, 1998, chapter 12) in the context of *all things considered judgments* (Temkin, 1993)

Due to the principle of natural reward, the EOp differs from *outcome egalitarianism*: even if opportunities are equalised, the actual levels of the equalisandum could differ considerably across agents. This allows the EOp to avoid some problems of pure outcome egalitarianism (e.g., the *levelling down objection* or the allocation of enormous amounts of resources to "irresponsible" agents). However, it has been argued that in the EOp the core of the egalitarian idea is lost, due to the adoption of the non-egalitarian (if not *anti*-egalitarian; Fleurbaey, 2001, pp.509-10) principle of natural reward.

According to some critics, the EOp may be in conflict with the absolute value of equal respect, equal social status, and equal participation which should lie at the heart of egalitarianism (Anderson, 1999, p.295ff; Wolff, 1998, p.105ff). "First, its rules for determining who shall be included among the blamelessly worst off fail to express concern for everyone who is worse off" (Anderson, 1999, p.303), since it refuses aid to the victims of bad option luck (see below), no matter what the amount of the loss incurred. "Second, the reasons it offers for granting aid to the worst off are deeply disrespectful of those to whom the aid is directed" (ibid.): agents receive compensation due to lack of talents or handicaps or other personal characteristics which are considered to make them inferior.[53] Thus EOp conditional transfers are likely to decrease the "respect-standing" (Wolff, 1998, p.107) of the agents who receive them. Finally, the EOp does not promote a culture of solidarity, since the main motivation for compensation relies on pity – for the disadvantaged – and on envy – for the lucky ones (Anderson, 1999, pp.306-7), while altruistic and morally motivated acts are treated as voluntarily cultivated expensive preferences which entail no compensation.

These issues are quite relevant and should be taken into account when implementing EOp policies. However, they do not question the theoretical core of the EOp. Besides, it is unclear whether competing approaches, including Wolff's (1998) and Anderson's (1999) ideal of *democratic equality* (see also Daniels, 1990), would fare better than the EOp in practice, beyond a rather vague general emphasis on the equal moral worth of persons as human beings, as agents in a system of cooperative production, and as citizens.

A more worrying set of critiques suggest that EOp policies may be in contradiction with egalitarian considered judgments. First, according to the EOp, "When deciding whether or not justice (as opposed to charity) requires redistribution, the egalitarian asks if someone with a disadvantage could have avoided it or could now overcome it. If he could have avoided it, he has no claim to compensation, from an egalitarian viewpoint" (Cohen, 1989,

[53] "When racial and sexual prejudice have been reduced, we shall still be left with the great injustice of the smart and the dumb, … the talented and the untalented, or even the beautiful and the ugly" (Nagel, 1979, p.105).

p.920).[54] This view seems unacceptably unforgiving – and much tougher than existing legal and welfare state systems, – *from an egalitarian viewpoint*, since it is insensitive to the amount of loss and does not take into account the possibility of changes in preferences over time (Anderson, 1999, pp.295-302; Fleurbaey, 1995A, pp.40-2; Lippert-Rasmussen, 2001, pp.549, 557-9).[55] As acknowledged by Arneson, the EOp "is blind to results once equal opportunities have been provided ... [but] in some circumstances the refusal to tender more resources is unfair" (Arneson, 1994, p.225).[56]

Second, consider a stochastic environment. EOp theories distinguish two concepts of luck: "Option luck is a matter of how deliberate and calculated gambles turn out – whether someone gains or loses through accepting an isolated risk he or she should have anticipated and might have declined. Brute luck is a matter of how risks fall out that are not in that sense deliberate gambles" (Dworkin, 1981B, p.293). The general EOp is usually construed to entail compensation for the outcomes of brute luck, but not for the consequences of deliberately chosen gambles. However, on the one hand, it is unclear that if brute luck is neutralised and opportunities equalised, there should be no further egalitarian objection to outcome inequalities. Suppose that two agents face effectively equivalent sets of options and both opt for the

[54] See also Arneson (1989, pp.85-6). Both Dworkin (1981B, pp.293-5) and Rakowski (1991, 69-81 and *passim*) take a similar, if not stronger position.

[55] According to Arneson (1990, p.179), there is just a "canonical moment" of passage to adulthood, after which people are entirely responsible for their choices.

[56] According to Levine (1999, p.404), in the limit, according to the EOp there would be no egalitarian objection to slavery, if it is the result of deliberately taken gambles.

most prudent course of action (with the same expected value by assumption). As argued by Lippert-Rasmussen (1999, pp.482-4; see also Lippert-Rasmussen, 2001), it is difficult to maintain that if one of them ends up badly off and the other well off, there is no egalitarian objection because such inequalities are the product of choice. By the same token, it is unclear that inequality of opportunity is a sufficient condition for an outcome to be bad with regard to inequality. Suppose that two agents face almost equivalent opportunity sets, which differ only for the (expected) outcome of an irrelevant and remotely possible course of action. If both choose their most prudent outcome and end up equally well off, it seems difficult to claim that the distribution is objectionable due to the initial differences in opportunities (Lippert-Rasmussen, 1999, pp.484-6; see also Christiano, 1991).

Third, arguably, the EOp fails according to the principle that "To count as egalitarian, a doctrine must, for some X, favor relatively more equal patterns of distribution of X over relatively less equal patterns of X, other things equal" (Hurley, 2001, p.52; see also Lippert-Rasmussen, 2001). From an EOp viewpoint, an allocation with large outcome inequalities is equivalent to another allocation in which agents end up equally well off, if in both cases outcome levels are the product of agents' responsible choices. Actually, an EOp policy may yield greater outcome inequalities if the existing allocation does not reflect agents' responsible choices. Moreover, according to the EOp, a policy that *removes* differential circumstances (e.g., by eliminating slums, if they negatively affect agents' outcomes) is in principle equivalent to another policy that *compensates* for them (e.g., by providing subsidies to slum dwellers), a conclusion that most egalitarians would reject (Lippert-Rasmussen, 2001, pp.575-9). The problem is that *per se* the notion of responsibility is not a relational concept and the EOp is consistent with any view on the default distribution of outcome (Hurley, 2001, 2002A).[57]

As acknowledged by EOp theorists (Arneson, 1994, 1999, 2000B), these objections cannot be easily dismissed. Arneson (2000B) suggests that by abandoning the idea of responsibility altogether one may end up spending enormous amounts of resources on irresponsible individuals. It remains true, though, that no satisfactory egalitarian theory can be completely insensitive to outcome inequalities, even if opportunities are equalised (Lippert-Rasmussen, 2001). This suggests an interesting line for further research aimed at integrating the EOp with outcome-egalitarian concerns. However, in this dissertation, these issues can be left aside. First, Roemer's EOp aims to provide an algorithm to implement any egalitarian view as defined by the *a priori* choice of the θ_c/θ_r partition: at one extreme, libertarians can be taken to advocate $\theta = \theta_r$, which implies that there is only one type in the population and no redistributive policy is needed; at the other extreme, outcome egalitarians may be seen as defining $\theta = \theta_c$.[58] Second, as already noted, the empirical literature (Roemer *et al.*, 2003) suggests an interpretation of

[57] Hurley (2001, 2002A) argues that the aim to neutralise luck (in the form, e.g., of different circumstances) can at most indicate *what* should be distributed but not *how*.

[58] However, libertarians may be described as denying the normative relevance of the θ_c/θ_r distinction altogether, based on the view that choice is uncaused (Risse, 2002, p.728) and that self-ownership entails a right to the revenues accruing from θ_c (Levine, 1999, p.406).

Roemer's EOp as a pragmatic theory for the egalitarian planner (Roemer, 1993) which proves that even a minimal egalitarian commitment leads to significant redistribution. Finally, in line with Roemer's approach, in chapter 3 it is assumed that the outcome function o is continuous – so that small mistakes lead to small losses – and that uncertainty plays no role so that agents can predict the outcomes of their responsible choices.

Actually, thanks to the latter assumption, in chapter 3 the controversial distinction between brute and option luck need not be discussed, which is the object of another set of criticisms. First, option luck requires that the risk (i) should have been anticipated; (ii) might have been declined; and (iii) is isolated.[59] Conditions (i)-(iii) are crucial to make the distinction normatively appealing: for instance, as noted above, if the risk is not isolated the EOp may be too unforgiving.[60] However, the more stringent (i)-(iii) are, the larger the set of cases that fall in the brute option luck category, making the distinction *practically* irrelevant. For instance, Lippert-Rasmussen (1999, p.483) argues that in order to rule out some paradoxical examples agents should have at least one effectively equivalent riskless option, a requirement that would severely limit the cases where option luck prevails.

Second, the *theoretical* distinction between option and brute luck is a matter of degree along all three dimensions (Vallentyne, 2002; see also

[59] "A risk of a certain event occurring is an isolated risk if, and only if, its occurrence will only have a minor impact on one's life" (Lippert-Rasmussen, 2001, p.558).

[60] Actually, condition (iii) may take care of *some* of the examples where the EOp seems most unforgiving. However, as argued by Lippert-Rasmussen (2001, p.562), it does so by incorporating *sufficientarian*, rather than egalitarian concerns.

Cohen, 1989; Arneson, 2001); for instance, whether a risk is avoidable or not depends on how costly it is to insure against it. However, this blurs the distinction between outcomes that do or do not call for compensation.

Third, from an egalitarian viewpoint, the main issue is not to determine whether the outcome level of an agent is the result of option or brute luck, but rather "whether an inequality between two persons is a matter of differential option luck or a matter of differential brute luck" (Lippert-Rasmussen, 2001, p.562). However, it is rather difficult to provide a satisfactory definition of differential option luck (not discussed by Dworkin, 1981A,B), which should identify outcome inequalities stemming from voluntary choices of agents with equal opportunity sets. In fact, it is unclear what probabilities should be used to evaluate available gambles: from an egalitarian viewpoint, outcome inequalities due to mistakes in the subjective evaluation of risks (for reasons beyond the agents' control) seem objectionable. One possibility is to define opportunities to be equalised "only when, first, opportunities are equal in the sense of objective probabilities and, second, they would have been obtained if the individuals had had subjective probabilities equal to the objective probabilities" (Fleurbaey, 2001, p.515).[61] Based on this definition, however, it becomes very difficult to measure opportunities and to assess whether a given inequality is the product of option or brute luck.[62]

[61] Lippert-Rasmussen (2001, p.568) uses ideal subjective (instead of objective) probabilities and proposes the stronger requirement that actual subjective probabilities be equal.

[62] For a thorough critical analysis of the EOp concept of luck (and in particular of the idea of the natural lottery of constitutions) and the notion of responsibility, see Hurley (2002B).

These critiques are quite relevant and they are the object of debate (for a reply, see, e.g., Arneson, 1999, 2000B, 2001). Roemer himself (1996, p.250) seems to question the distinction between option and brute luck, since he identifies cases where differential option luck should lead to equality of outcomes. Indeed, as noted by Sugden (2004), Roemer's effort-based scheme entails *ex-post* compensation to neutralise option luck so that identical people expending the same amount of effort get the same outcome. Nevertheless, the relevance of these issues for Roemer's EOp is unclear since he provides no thorough treatment of uncertainty. This suggests an interesting line for further research; however, these issues are not essential to expose the problems which lie at the core of this dissertation and therefore they are abstracted away in chapter 3.

Another set of critiques focus on the relation between the EOp and efficiency. First, since the EOp view is non-welfarist, it is not surprising that "it may violate the most basic welfarist principle, namely, Pareto efficiency" (Fleurbaey, 1995A, p.34). For instance, if "agents are responsible for their preferences (viewed as factors of well-being), the principle of natural reward ... [may] require the allocation to be independent of their preferences, and the allocation rule could not then be Pareto-efficient" (Fleurbaey, 1998, p.212). However, such incompatibility does not always hold (ibid.; and references therein) and in any case, *ceteris paribus*, by rewarding responsible choices (e.g., effort) the EOp seems to provide a more satisfactory answer to incentive problems than more traditional approaches.

More relevant is Sugden's (2004) argument that Roemer's EOp cannot be implemented unless some vital market process is disabled. Sugden (2004, p.221ff) shows that a EOp equilibrium can be implemented in a Walrasian economy (Walras island) thanks to a double tatônnement process managed by the auctioneer. However, in an economy characterised by *division of knowledge* (Hayek island), markets generate successes and failures which can be attributed *ex-post* to circumstances beyond the agents' control, – such as differences in beliefs due to arbitrary circumstances, – but which cannot be compensated without seriously hampering the coordinating role of markets.[63] This interesting argument confirms that the analysis of (Roemer's) EOp in a stochastic environment should be one of the next steps in the EOp research program. However, arguably it does not represent an impossibility result and in chapter 3 it is ignored by assuming that the EOp does provide an ethically viable normative view. Furthermore, as already noted, uncertainty is not necessary to expose the problems analysed in chapter 3 and thus Sugden's problem is assumed away by focusing on a deterministic environment.

Lastly, a set of critiques focus on the EOp notion of responsibility. Fleurbaey distinguishes two concepts. "*Responsibility by control* is assigned to an agent on a particular variable when this agent has full control over the value of this variable. … *Responsibility by delegation* … is assigned to an agent on a particular variable when the rest of society decides not to spend

[63] EOp *ex-ante* compensation for differences in beliefs is impossible, while if feasible, an effort-conditional insurance scheme (*ex-post* compensation) would interfere with markets by rewarding effort independently of where it is expended (Sugden, 2004, pp.227-8).

any resource on the outcome obtained by the individual for this variable" (Fleurbaey, 1995B, p.684). By adopting the first concept, – assigning responsibility over factors, θ_r, – EOp theorists reject *hard determinism* and endorse a *compatibilist* view of free will.[64] However, this approach makes distributive justice theory hostage of metaphysics since the philosophical foundations and the normative relevance of the EOp ultimately rest on the resolution of the free will debate (Fleurbaey, 1995A, pp.38-9; Cowen, 2002). Thus, the very possibility of a philosophically meaningful, widely agreed θ_c/θ_r partition is called into question (Scheffler, 1995).

A further complication arises since, as acknowledged by EOp theorists (e.g., Arneson, 1989, p.86), responsibility is a matter of degree. This may provide a (partial) way out of the free will problem by avoiding the need of "an absolute distinction between presence and absence of genuine choice" (Cohen, 1989, p.934), but it raises clear practical and definitional problems. Consider the related notion of opportunity: the availability of an option does not tell much about how difficult it is for an agent to access it, and thus how responsible she is for reaching a certain outcome.[65] Thus, it would be more appropriate to define opportunities in terms of a *degree* of access to advantage and to define the EOp in terms of *effectively* equivalent options (Arneson, 1989, p.86). Roughly, "if I have an opportunity to get X amount of

[64] More precisely, EOp theorists are agnostic on *hard determinism*, however the latter must be false for EOp to differ from outcome egalitarianism (Lippert-Rasmussen, 1999, pp.481-2). For a very good discussion of these concepts, see Risse (2002).

[65] For a discussion of the concept of opportunity, see Hansson (2004, especially p.309-10).

welfare, then there is a [reasonably] prudent course of action available to me that will yield me X if I take it, and if I do not, the responsibility for my lower welfare level lies with me" (Arneson, 2001, p.82).[66]

The concept of individual responsibility is even more elusive in the presence of externalities since opportunity sets are interrelated and other people's actions may affect my degree of access to some opportunities (Fleurbaey, 1995A, pp.36-7). Actually, this raises the issue of the distinction between a person's circumstances and other people's choices: "are there legitimate claims to compensation if, because of other people's morally impeccable choices, an individual's life goes worse than it would had those people acted differently?" (Risse, 2002, p.734).[67]

These critiques are philosophically relevant and they are still the object of debate. However, Roemer's EOp does not seem vulnerable to them. First, it is "not metaphysical in the sense of trying to solve the deep problem of what *actually* is beyond a person's control; it is political in the sense that it depends on the current views of the society in question" (Roemer, 1995A). Second, Roemer's (1998, 2002) proposal to measure the degree of effort by

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[66] "Your opportunity for welfare is the welfare level you would reach if you tried prudently to advance your own welfare without violating strict obligations of law and morality and if you pursued this prudent aim as effectively as it would be reasonable to expect of you, when taking into account your choice-making and choice-executing abilities and the difficulty and pain you would have to overcome to live prudently" (Arneson, 2000A, p.507).

[67] Lippert-Rasmussen (2001, pp.571-5) also argues that in an uncertain environment the notion of responsibility does not help to determine legitimate claims for compensation, since it is unclear whether agents can be considered responsible for differential option luck.

the individual's ranking in the effort distribution of her type can be seen as a way to capture the degree of access to advantage (and *a fortiori*, the degree of responsibility) as a statistical issue, which significantly deflates Roemer's proposal of complex metaphysical implications. Third, as argued by Fleurbaey (1995, 1998), in principle, the θ_c/θ_r partition need not be based on responsibility and Roemer's proposal itself does not crucially rely on the latter view. Instead, Roemer's EOp can be naturally interpreted as a desert-based, rather than responsibility-based theory, advocating a just reward for effort (see, e.g., Roemer, 1993, 1994B; Hurley, 2002A).

To summarise: the EOp is the object of ongoing controversy and, as acknowledged by EOp theorists (most notably Arneson, 1999, 2000B, 2001), it may need some refinements. However, the EOp, and specifically Roemer's version, is an interesting approach and a promising research program in the liberal egalitarian tradition, which is worth exploring further. First, it offers various insights on important issues in egalitarian thinking, including the relation between equality and responsibility. Second, by leaving the θ_c/θ_r partition undetermined, Roemer's EOp is inherently pluralistic, in that it can accommodate different views on the scope and extent of egalitarian policies and therefore it provides fruitful common ground for discussion. However, third, and most important, as shown by the empirical literature, it is a fruitful policy-oriented framework, which proves that significant redistribution may be needed even if only a minimal egalitarian commitment is endorsed.

1.3.2. OBJECTIVISM, SUBJECTIVISM, AND GROWTH

Section 1.3.1 surveys various controversial issues raised by the EOp that are debated in the economic and philosophical literature, and suggests some interesting lines for further research within the EOp paradigm. However, the main contribution of this dissertation to the EOp research program focuses in particular on two arguably crucial issues for any theory of distributive justice: the choice of the appropriate equalisandum, and intergenerational justice and growth. As concerns the former issue, in Section 1.3.1, the EOp is discussed without specifying the variable of egalitarian concern. Partly, this is due to the fact that the general EOp is proposed by many authors with different views on the relevant currency of egalitarian justice. Partly, the interpretation of Roemer's EOp as a pragmatic egalitarian theory naturally leads to endorse a "spherical" interpretation of distributive justice with different outcomes in different "spheres of justice" (Walzer, 1983; Roemer, 2001A) and to eschew the discussion of the appropriate equalisandum (Risse, 2002). However, at the theoretical level, the latter issue is crucial if the EOp is to provide a general theory of distributive justice (see, e.g. Arneson, 2000A).

Inter alia, the analysis of the dynamics of inequality and classes in chapter 3 aims to contribute to the EOp research program by offering various interesting insights on the issue of the appropriate equalisandum. Although no argument is provided to support an egalitarian, or more specifically, an EOp approach – the analysis starts from the *assumption* that the EOp provides an ethically viable theory of distributive justice, – the choice of the appropriate equalisandum is crucial in the determination of the ethical appeal of an egalitarian theory. The EOp is non-welfarist or more precisely, *non-*

outcomist, since other information – e.g. concerning agents' responsible choices and their available options – is necessary to evaluate a distribution, in addition to the level of the relevant outcome attained by all agents (Fleurbaey, 1995A, p.34). However, this does not imply that welfare cannot be the appropriate *opportunity* equalisandum.

Since Rawls's (1971) critique of utilitarianism, the choice of the appropriate currency of egalitarian justice has become one of the major foci of discussion in egalitarian theory.[68] Rawls (1971) forcefully criticises the *subjectivist* dimension of utilitarianism, i.e. the idea that the normatively relevant variable is utility, which can only be measured knowing the utility function of the individual in question, and can only be compared interpersonally if an interpersonally comparable unit scale exists. Instead, he endorses an *objectivist* view, i.e. the view that the equalisandum should be something which is measurable independently of the views of the individuals who have it; whence his focus on primary goods.

Rawls's critique of subjectivism has been very influential and although several qualifications have been put forth as to what the equalisandum should be, most, although not all, participants in the discussion have advocated an objectivist equalisandum, such as *functionings* (Sen, 1980), *resources* (Dworkin, 1981), or *advantage* (Cohen 1989). Interestingly, although Arneson (1989, 1990) has argued that *opportunity for welfare* should be the

[68] A thorough survey of the vast economic and philosophical literature on this issue goes far beyond the scope of this chapter. For excellent surveys, see Roemer (1996), Clayton and Williams (1999), and Maguain (2000).

equalisandum,[69] he has progressively moved from a subjective conception of welfare as preference satisfaction (albeit with a number of qualifications to meet objectivist critiques) to a perfectionist, "objective list" conception of welfare as the appropriate opportunity equalisandum (more precisely, *distribuendum*; see Arneson, 1999, 2000A). None of the major writers advocates subjective welfare as the appropriate variable.

In this dissertation, the issue of the appropriate equalisandum is analysed in the context of *all things considered judgments*, rather than directly in the context of the requirements of distributive justice. More specifically, in chapter 3 it is stipulated that in addition to equality of opportunity for some condition among members of society, most egalitarians would hold that that society is best which promotes human development over time, and the consistency of these objectives is analysed.

In order to analyse human development, the static framework usually adopted in the debate on the appropriate equalisandum is abandoned and equality of opportunity for some condition, becomes equality of opportunity among all adults who ever live. More precisely, the EOp assumes that individuals have different circumstances and exert different efforts. In chapter 3, differential effort is assumed away while a person's circumstances are assumed to be her parent's socio-economic status (measured by her parent's wage, w) - a summary of the environmental factor which affects the child's education and therefore her wage when adult - and the date t at which

[69] 'Opportunity for welfare' is, in general, quite different from 'welfare' as an equalisandum. That difference is due to differential effort, which is abstracted away in chapter 3.

she is born. Children are taken as 'adults in formation.' Then, since effort is nugatory, the EOp prescribes to allocate educational resources to maximize the minimal level of some condition *o* among all adults across types, where an adult's type is a pair (*w*, *t*). In other words, the EOp requires to find the maximin intertemporal allocation.

The analysis of intertemporal maximin paths and their consistency with growth has started immediately after the publication of *A Theory of Justice* (1971). Arrow (1973A) and Dasgupta (1974A, 1974B) analyse the dynamic properties of the difference principle in an economy with an infinite number of non-overlapping generations living for one period. They assume a representative agent with a well-behaved utility function which depends on consumption of the only good produced with a linear technology. They prove first, that if agents are *egoistic* and care only about their own consumption, the maximin path leads to no savings and "the economy would be imprisoned in perpetual poverty if it begins in poverty" (Dasgupta, 1974A, p.408; see also Arrow, 1973A, p.325), confirming the result obtained by Solow (1974A) in a continuous time model.[70]

Second, they prove that if agents have additively separable utility functions displaying *paternalistic altruism*,[71] the maximin does not lead to growth. In fact, beyond the altruistic horizon *T*, the maximin path "leads to

[70] The issue of whether Rawlsian intergenerational justice requires the use of the difference principle or the just savings principle (Rawls, 1971, §44), is not relevant here. See, e.g., Arrow (1973A, p325); Dasgupta (1974A, p.408); Phelps and Riley (1978, p.104 and fn.4).

[71] Preferences are said to display *paternalistic altruism* if each agent's utility depends on her own consumption and on the consumption of a finite number *T* of her descendants

periodic repetition of the solution with a period equal to that of the horizon" (Arrow, 1973A, p.333). Worse still, the maximin path is intertemporally inconsistent (Dasgupta, 1974A, Proposition 3), a rather worrying feature for a principle of intergenerational justice.

Leininger (1985) proves that similar results hold in economies with more general technologies and utility functions, unless initial capital is higher than the "golden rule" level, which maximises steady state consumption.

Instead, Calvo (1978) posits a simple form of *non paternalistic* altruism whereby every generation's utility is additively separable and depends on their consumption and the next generation's *utility*. Then, he proves that, under fairly general assumptions on technology and initial capital, the maximin path is time consistent and leads to capital accumulation. The intuition is that "non paternalistic altruism prevents time-inconsistency ... by letting each generation recognize the altruism of its children and thereby removing a source of intergenerational conflict" (Asheim, 1988, p.469). Calvo's (1978) result is extended by Rodriguez (1981) to economies with general utility aggregators of the form $U_t = V(c_t, U_{t+1})$; and by Asheim (1988) to an economy with non-renewable resources.

Similar conclusions on the consistency of growth and maximin justice are reached by Phelps and Riley (1978), in the context of an overlapping generations model in which labour is a productive input in a general concave technology and leisure enters the agents' utility. They prove that if labour supply is fixed, the Rawlsian economy reaches a stationary state after one period of adjustment,[72] while if labour supply can vary, in the maximin path there may be capital accumulation but welfare is constant. Welfare growth is possible only if nonpaternalistic altruism is postulated (ibid., pp.115-6).

Although the model presented in chapter 3 may be considered an intellectual descendent of this literature, some important differences should be highlighted. First, the models just reviewed provide important insights on intertemporal maximin paths, but they are not suitable to analyse the dynamics of inequalities and classes, due to the representative agent assumption. Instead, chapter 3 focuses on what *inter*generational equality requires with respect to *intra*generational wage differentials, a question that none of the authors mentioned above poses. Unlike in the previous literature, two types of individuals, – two socio-economic classes – are assumed to exist at least at the early dates, and the dynamics of intragenerational class differences is analysed in relation to intergenerational justice.

Second, in the previous literature the choice of the equalisandum is not discussed and a straightforward utility-based approach is adopted, whereby agents are typically assumed to care only about (theirs and possibly their descendants') consumption.[73] In chapter 3, a more general approach is adopted to analyse the differences between an objectivist and a subjectivist view. As concerns the former, chapter 3 focuses on *functionings*, which are defined to include both consumption and the wage: the wage is a measure of

[72] Phelps and Riley (1978, Theorem 2.1). See also Theorem 1 in chapter 2 below.

[73] As noted above, Phelps and Riley (1978) assume that leisure also enters the utility function, but this does not alter the essence of the argument.

an agent's level of human capital and it is assumed that individuals derive welfare directly from their human capital. Moreover, functioning involves a degree of self-esteem and self-realization, and these arguably depend positively on an individual's level of human capital. Formally, the main difference is that in the previous literature the planner has only one instrument each period, whereas in our model she has two instruments, income tax and educational resources.

Finally, since chapter 3 analyses the dynamics of inequality and classes in a EOp perspective, special attention is devoted to the intertemporal and intragenerational role of education. Therefore, unlike in previous models, the emphasis is on investment in education, rather than physical capital.

Nevertheless, the results derived in chapter 3 are qualitatively similar to the ones discussed above: it is proved that the three desiderata: (*i*) protracted human development; (*ii*) equality of opportunity for some condition; and (*iii*) the condition be an objective characteristic of the individual; are inconsistent. Only if the equalisandum is non-paternalistic, altruistic welfare – a non-objectivist concept, – equality of opportunity is consistent with human development. If this inconsistency is correct, then egalitarians are faced with a choice: either dropping their advocacy of equality (of opportunity), or of human development, or of objectivist equalisanda. Dropping the third desideratum seems the obvious choice.

However, there are three important *caveats* that qualify the above conclusions and, at the same time, indicate some lines for further research. First, in chapter 3 it is assumed that the EOp (for whatever condition) is an ethically viable conception in a multi-generation world, and that in such a context, it calls for equalizing opportunities across all types (t, w). However, even setting aside the problems discussed in Section 1.3.1, the idea that justice requires that a person fare no better than another simply by virtue of being born at a different date is not uncontroversial. As argued in chapter 2, there are various possible views on the proper temporal unit of egalitarian concern, and according to some equality of condition among living persons is all that an egalitarian ethic requires. One rationale is that self-esteem is affected by comparing one's condition to those of contemporaries.

In chapter 3, some intuitions that might justify the adoption of the EOp in the intergenerational context are briefly discussed; however from a philosophical viewpoint, providing a proper motivation should be one of the next steps of the EOp research program.

Second, this inquiry does not show that *justice* requires that subjectivism be endorsed. For at most it suggests to drop objectivism because of its inconsistency with equality of opportunity and human development, and while the 'equality of opportunity' part of that compound phrase refers to a state of justice, the 'human development' part does not. That is, chapter 3 does not prove that justice requires human development, or even, more weakly, that justice requires human development in an environment where it is possible. Human development over time seems an obvious good, but it is unclear what to call the state of a society which has it, the way a society with equality of opportunity is in a state of justice.

But then, it is crucial to investigate the robustness of the inconsistency result. The model examined in chapter 3 is quite general and, at least in some respects, fairly standard. Furthermore, the results presented are in line with most of the literature. However, and this is the third *caveat*, chapter 3 does not prove a general impossibility result. Actually, Silvestre (2002) provides a counterexample showing that "the conflict between non altruistic maximin and progress is not universal" (ibid., p.2), based on an overlapping generations model with (*i*) a positive intergenerational stock externality and (*ii*) a bound to feasible transfers from young to old agents living in the same period.[74] The actual relevance of the example is unclear (for instance, it involves only two generations and five dates) and the formal results derived by Silvestre (2002, Theorems 1 and 2) in a more general model only prove that conditions (*i*) and (*ii*) are *necessary* for maximin and growth to be compatible. Furthermore, the relevance of condition (*ii*) may be disputable. However, chapter 3 and Silvestre (2002) suggest an interesting line for research on the dynamics of inequality and classes in the EOp perspective, aimed at providing a general characterisation of intertemporal maximin paths.

[74] Interestingly, Silvestre (2002) posits an index of well-being which may be an objectivist variable such as *functionings*. However, he does not consider intragenerational issues.

CHAPTER 2. INTERTEMPORAL EGALITARIAN PRINCIPLES

2.1. INTRODUCTION.

Many of the crucial debates on egalitarianism, and especially those on the foundations of egalitarian theory, have been carried out within the confines of a static environment. The choice of the appropriate equalisandum has been explored in a "model" with a single generation (e.g., Rawls, 1971; Sen, 1980; Dworkin, 1981; Arneson 1989; Cohen, 1989; Roemer, 1998; see chapter 3 below). Similarly, the analysis of different measures of inequality has typically focused on the distribution of the relevant variable in a single period (e.g., Sen 1973, 1992; Temkin 1993). Even when distributive dynamics have been considered, the complex economic and philosophical implications of the fact that agents' lives develop over time have often been overlooked. In a seminal article which has generated a growing literature across the disciplinary borders of philosophy and economics (e.g., Temkin, 1992, 1993; Daniels, 1993; Kappel, 1997; McKerlie, 2001), McKerlie (1989) noted that since agents' lives extend over time, a sound egalitarian analysis requires the definition of the proper unit of egalitarian concern, i.e. whole lives or selected parts of them.[1] Egalitarian principles based on different units incorporate different moral concerns and have different policy implications.

This chapter analyses three intertemporal principles that incorporate what may be considered the most relevant egalitarian considered judgements,

[1] In chapter 4, it is shown that similar issues may be relevant in the theory of exploitation.

proposed by McKerlie (1989) and Temkin (1993): according to *complete lives egalitarianism* (*CLE*), agents' lives, taken as a whole, are the proper unit of egalitarian concern. If one adopts *corresponding segments egalitarianism* (*CSE*), inequalities must be measured between corresponding stages of agents' lives – e.g., childhood, early adulthood, middle age, etc. age. Finally, according to *simultaneous segments egalitarianism* (*SSE*), only inequalities between contemporaries are morally relevant.

Different views have been advanced to identify the appropriate intertemporal egalitarian principle. On the one hand, as convincingly argued by Temkin (1993), in the analysis of inequalities no principle is entirely satisfactory: "several views are possible, each of which seems plausible in some cases and implausible in others" (ibid., p.291). However, an important distinction has been overlooked in the literature, which is a peculiar feature of the intertemporal context. Unlike in the static setting, apart from differing in the analysis of unequal distributions, intertemporal egalitarian principles also define different egalitarian states to reach. The two issues are connected but they should be kept conceptually distinct in the choice of the appropriate principle. This is even more evident for policy purposes, - e.g., from the viewpoint of a government concerned with equality, - since the definition of the ideal "steady-state" egalitarian distribution and the design of the transition process to that state raise different problems. In order to implement an egalitarian strategy, in addition to a correct analysis of the *status quo*, it is necessary to define the appropriate egalitarian benchmark.

This chapter focuses on the latter issue. Section 2.2 briefly reviews the main results of the existing literature on the properties of the three egalitarian views in the evaluation of unequal distributions. Then, the methodological and philosophical distinction between the evaluation of existing inequalities and the definition of the appropriate egalitarian distribution is introduced, and it is argued that, as regards the distribution to establish, *CSE* defines the appropriate intertemporal egalitarian benchmark.

Since the evaluation of a distribution, e.g., for policy purposes, is influenced by more than one normative concern, in Section 2.3, a formal analysis of the trade-offs between the different egalitarian principles and other normative views is presented, which aims to provide a formal basis for *all things considered judgements* (Temkin, 1993). In particular, the relations of *CLE*, *CSE*, and *SSE* with two non-primarily-egalitarian normative concerns, Rawls's maximin and utility,[2] are analysed. To be specific, a stylised model is set up, which generalises Arrow (1973A) and Dasgupta (1974A). The main substantive difference is that overlapping generations are assumed here, so that at each date there are two types of individual, young and elderly, while Arrow and Dasgupta worked with a representative agent. This allows us to analyse *inter*temporal as well as *intra*temporal equality. It is proved that the maximin solution yields *CSE* and *CLE*, but not *SSE*, and if the assumptions of the model are relaxed, *CLE* remains the egalitarian principle that can best accommodate rawlsian or utilitarian concerns, and it is easier to

[2] For a discussion of the relation between the difference principle and egalitarianism, see Temkin (1993) and Cohen (1997).

reconcile these concerns with *CSE* than with *SSE*. It is also worth noting that despite the formal differences the results presented here confirm and extend Arrow's and Dasgupta's conclusions on Rawls's maximin principle.

2.2. THREE EGALITARIAN PRINCIPLES COMPARED.

Let x be the relevant egalitarian variable, which shall be called 'welfare' (x could be income, utility, opportunities, primary goods, etc.). Assume that agents' lives can be divided into an equal number T of well-defined periods of equal length. Let $X_{(T)} = \{\mathbf{x}_i^t = (x_{i1}^t, x_{i2}^{t+1}, \ldots, x_{iT}^{t+T-1}),\ x_{ij}^t \in \Re\}$ be the set of vectors describing the attainment of x_{ij}^t by agent i, at date t, in period j, $1 \leq j \leq T$, of her life. For the sake of simplicity, assume x to be interpersonally and intertemporally comparable, and additive along agents' lives, so that $x_i = \sum_{j=1}^{T} x_{ij}^{t+j-1}$ is the lifetime attainment of x by agent i. These assumptions make the analysis comparable with McKerlie (1989) and Temkin (1993), and they are quite natural if x is a variable such as income or an index of primary goods. On the other hand, if a subjective variable like utility is considered, these assumptions give the opportunity to compare the egalitarian principles *in vitro*, as a first step towards a more satisfactory and realistic analysis.

The three egalitarian principles can be interpreted as different ways of evaluating distributions of the $\mathbf{x}_i^t$ vectors. Let D_1, D_2, and D_3 denote inequality measures associated with *CLE*, *CSE*, and *SSE*, respectively. Formally, D_y: $X_{(T)} \times X_{(T)} \times \ldots \times X_{(T)} \rightarrow \Re$, $y = 1, 2, 3$. Without loss of generality, let $D_y = 0$, $y = 1, 2, 3$, denote the egalitarian distributions corresponding to the three principles. Given the definitions in Section 1, $D_1 =$

0 if and only if $x_i = x_h$, for all agents i, h; $D_2 = 0$ if and only if $x_{ij}^t = x_{hj}^\tau$ for all agents i, h, dates t, τ, and corresponding life stages j; and $D_3 = 0$ if and only if $x_{ij}^t = x_{hz}^t$ for all agents i, h, life stages j, z, and simultaneous dates t.

In order to focus on the implications of the three egalitarian principles, - rather than on the features of specific measures, - no further restrictions are imposed on the D_y's. As in the static setting, where the problems of inequality measurement are reflected by the existence of several measures capturing different aspects of inequality (e.g., Gini index, Atkinson's measure, etc.; see the discussion in Temkin, 1993), in principle there are many possible ways of measuring inequalities *according to each criterion*, that is, there are various specifications of every D_y. Actually, in the intertemporal context the choice of the appropriate inequality measure associated to each criterion is more complex, since the D_y's should rank distributions of vectors rather than distributions of real numbers.

However, as convincingly argued by Temkin (1993), one of the specific features of intertemporal analysis is that, unlike in the atemporal context, even assuming a unique possible D_y associated to each principle, the issue of inequality measurement would not be solved: different egalitarian principles highlight different kinds of inequalities and no principle, *CLE*, *CSE*, or *SSE*, seems completely satisfactory in the analysis of unequal distributions.[3]

The main problem of *CLE*, first noted by McKerlie (1989), is that it leads to "changing places egalitarianism" (CPE). If whole lives are the unit of

[3] For a thorough discussion of the three principles in the context of inequality analysis, the reader is referred directly to Temkin (1993, chapter 8)

egalitarian concern, in a "situation involving differential treatment of equally deserving people – no matter how significant, … and even perverse those differing treatments are – there can be no egalitarian objection as long as the roles of the equally deserving people are interchanged so that each receives an equivalent share of the treatments meted out" (Temkin, 1993, p.236).

CSE and *SSE* rule out CPE, since they do not allow present inequalities to compensate for past ones. Yet, they do not represent entirely satisfactory alternatives in the evaluation of unequal distributions. By focusing only on inequalities in selected portions of the agent's lives, both principles can lead to the paradoxical conclusion that a distribution exhibiting CPE is as objectionable as one in which the agents' roles are not interchanged and one agent is worse off in every relevant segment. Hence, it is legitimate to conclude that in the evaluation of existing inequalities, "several views are possible, each of which seems plausible in some cases and implausible in others" (ibid., p.291), and that it may be opportune to use the information conveyed by all principles rather than adopting only one of them.

However, these arguments do not extend to the choice of the appropriate intertemporal *egalitarian benchmark*, which is quite a different issue from the analysis of past and present *inequalities*. In the static context, while the measurement of inequalities can be controversial, the definition of egalitarian states is uncontroversial: different inequality measures give the same answer if the distribution is egalitarian.[4] In the intertemporal context, it is slightly

[4] See, e.g., the measures discussed in Sen (1973, 1992) and in Temkin (1993, chapter 5). Note that the distinctions between outcome egalitarianism, opportunity egalitarianism,

misleading to say that different views can "be regarded as built around ways of measuring the inequality between lives" (McKerlie, 1989, p.487). The three principles stress different aspects of existing inequalities, but they also define different egalitarian states to reach, as shown by the fact that, unlike in the static context, in general $D_y = 0$ does not imply $D_{y'} = 0$, $y \neq y'$. The two issues are connected, but should be kept conceptually distinct in the choice of the appropriate egalitarian principle. For instance, while CPE arises in a *CL*-egalitarian distribution, Temkin's (1993) analysis of *SSE* and *CSE* focuses on unequal distributions and therefore it provides little information as to the features of the egalitarian distributions associated with them.[5]

The difference between the two perspectives is even more evident for policy purposes, since the definition of the ideal "steady-state" egalitarian distribution and the design of the transition process to that state raise different issues. In order to implement an egalitarian strategy, in addition to a correct analysis of the *status quo* (involving the evaluation of existing inequalities and claims for compensation of past ones), it is necessary to define the appropriate intertemporal egalitarian benchmark.

Consider the three principles from the point of view of the distributions with $D_y = 0$, $y = 1, 2, 3$. As noted above, CPE raises serious doubts about

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maximin egalitarianism, which define different egalitarian states to reach even in a static setting need not concern us here (see chapters 1 and 3 for a discussion).

[5] Neither McKerlie (1989) nor Temkin (1993) explicitly distinguishes the two sets of issues so that the scope of their conclusions is sometimes unclear. For instance, McKerlie (1989) discusses the choice of the egalitarian benchmark, but his arguments are based mainly on the analysis of the claims for compensation of past inequalities implied by the different views.

CLE as the intertemporal egalitarian benchmark: for instance, a feudal system in which the roles of nobles and peasants are interchanged so as to equalise their overall welfare is not objectionable from a *CL*-egalitarian viewpoint.

A first puzzling feature of *SSE* is reflected in the time-dependency of D_3 and in particular in its sensitivity to changes in the agents' date of birth. In principle, *for given allocations of* x, it is sufficient a "slight" shift in the date of birth of an agent to change dramatically the value of D_3 and the egalitarian judgement. However, it is hard to see why if an agent is born, say, ten years later, or earlier, the judgement about an otherwise identical (and possibly *CL*- and *CS*-egalitarian) distribution should change. This is more evident the shorter the stages in which agents' lives are divided.[6]

Second, according to *SSE*, only inequalities between contemporaries are ethically relevant, and therefore $D_3 = 0$ whenever agents' lives do not overlap. However, let $T = 4$ and consider the following example.

Example 1 (E1)

$$\mathbf{x}^t_i = (1, 2, 3, 4), \text{ for all } i, t,$$

where $D_1 = D_2 = 0$, while any D_3 would definitely be positive. Suppose that the only available action to reach $D_3 = 0$ is the construction of a nuclear plant that will explode in $t = 10$ yielding the following welfare distribution.

Example 2 (E2)

[6] However, in the determination of the appropriate length of the stages, a trade-off arises between the robustness of the results (which tends to increase with the length of periods) and their relevance (since in the limit only whole lives matter).

$$\mathbf{x}^t_i = (t + 100, t + 101, t + 102, t + 103), \text{ for } t \leq 6, \text{ for all } i,$$

$$\mathbf{x}^7_i = (107, 108, 109, 0), \text{ for all } i,$$

$$\mathbf{x}^8_i = (108, 109, 0, 0), \text{ for all } i,$$

$$\mathbf{x}^9_i = (109, 0, 0, 0), \text{ for all } i, \text{ etc.}$$

According to *SSE*, if future generations' welfare is uniformly affected in each t, no other egalitarian consideration is necessary to evaluate a policy: the distribution in E2 is strictly preferable to that in E1 and it raises no egalitarian objection. Therefore the nuclear plant should be built. This conclusion would be rejected by most egalitarians and it raises serious doubts on *SSE* as the appropriate egalitarian benchmark.[7]

E1 and E2 also show a more general point: the requirement of *CL*-equality cannot be abandoned without generating unappealing results (from an egalitarian perspective). This suggests that the analysis of intertemporal egalitarian benchmarks should focus on the choice of the most appropriate restriction on *CLE*. Indeed, only in the context of inequality analysis the "views are independent of each other, in the sense that each of their judgments may be in agreement or disagreement depending on the particular case in question" (Temkin, 1993, p.242). Instead, if egalitarian distributions are analysed, it is misleading to ask whether "the whole lives view [should] be rejected entirely, and replaced by some combination of the simultaneous and corresponding segments views" (ibid., p.238). Neither *CSE* nor a

[7] It is worth noting that E2 does not represent a variant of the *levelling down objection*: what is objectionable is not that *SSE* leads to a lower welfare level in E2 than in E1, but rather that according to *SSE*, E2 must be considered better than E1 *from an egalitarian viewpoint*.

simultaneous segments restriction on *CLE* (discussed below) replaces the latter. Actually, in order to avoid CPE, any restriction on *CLE* should require all agents belonging to the same generation to have identical patterns of x during their lives. Hence, for a given x_i equal for all i, alternative restrictions will differ only in the admissible patterns of x for agents belonging to different generations.

One possibility, suggested by McKerlie (1989, p.484), is to impose *SS*-equality in addition to *CL*-equality. This version of *SSE* (hereafter, SSE_2) is subject to the same time-dependency problem faced by the unconstrained *SSE* (hereafter, SSE_1). Moreover, the emphasis on simultaneity as the relevant egalitarian restriction on the allocation of x along agents' lives is not entirely convincing. SSE_2 removes CPE between agents belonging to the same generation, but the requirement of equality in the overlapping segments of the lives of agents belonging to different generations seems less compelling. According to SSE_2, the distribution in E1 - in which agents are treated identically regardless of the generation they belong to - is definitely non-egalitarian, while the following distribution is SS_2- egalitarian.

Example 3

$\mathbf{x}^t_i = (1, 2, 3, 4)$, for all i, and $t = 4d$, $\quad d = 0, 1, 2, \ldots$

$\mathbf{x}^t_i = (2, 3, 4, 1)$, for all i, and $t = 1 + 4d$, $d = 0, 1, 2, \ldots$

$\mathbf{x}^t_i = (3, 4, 1, 2)$, for all i, and $t = 2 + 4d$, $d = 0, 1, 2, \ldots$

$\mathbf{x}^t_i = (4, 1, 2, 3)$, for all i, and $t = 3 + 4d$, $d = 0, 1, 2, \ldots$

In E3, only agents born every four periods have the same pattern during their lives. However, unless agents are assumed to be myopic and to care only about the inequalities that they can actually observe in every t, it is hard to see why a distribution exhibiting such a cyclical pattern should be desirable from an egalitarian perspective, and indeed why it should be strictly preferable to E1. Notice that the egalitarian intuition behind SSE_2 is not the same as that behind SSE_1: in the latter case, the idea is that inequalities between contemporaries are worse than inequalities between removed generations - e.g., between the present generation and people living in the middle age. Instead, given the same total level of x, the only role played by simultaneity in SSE_2 is to constrain its allocation during agents' lives.

Another possibility is to adopt *CSE*: since the distributions with $D_2 = 0$ are a strict subset of those with $D_1 = 0$, *CSE* can be naturally interpreted as a restriction on *CLE*.[8] Moreover, unlike *CLE* and SSE_2, *CSE* fully incorporates the egalitarian intuition that identical agents should be treated exactly in the same way, since in *CS*-egalitarian distributions they have an identical welfare allocation along their lives. Formally, unlike D_1 and D_3, $D_2 = 0$ if and only if $\mathbf{x}_i = \mathbf{x}_h$, for all i, h; that is, $D_2 = 0$ if and only if the vectors describing the pattern of the egalitarian variable along agents' lives are identical. Thus, all distributions in the class with $D_2 = 0$ can be simply described as follows.

Example 4

$$\mathbf{x}_i = (p, q, r, s), \text{ for all } i,$$

regardless of agent i's date of birth. Unlike the distributions with $D_1 = 0$ or $D_3 = 0$, - as CPE and E2 respectively show, - those belonging to the class with

[8] If the duration of agents' lives is uncertain, neither *CSE* nor SSE_2 necessarily implies *CLE* *ex-post*, but the above arguments still hold *ex-ante*, if applied to expected welfare.

$D_2 = 0$ are equivalent *from an egalitarian point of view*, as the comparison of any pair of *CS*-egalitarian distributions shows.

However, it is not necessarily true that $p = q = r = s$, and thus *CSE* allows potentially great inequalities between people living in the same period belonging to different age cohorts. For instance, a *CS*-egalitarian distribution could imply that in every period there are happy young people, while the elderly live in despair. However undesirable such a distribution may be, if $D_2 = 0$ – that is, if, when they were young, the elderly were treated as the current young - there should be no egalitarian objection to it, since identical people have an identical pattern of x during their lives.[9]

To be sure, there may be non-egalitarian objections to the latter allocation and in general distributions with $D_2 = 0$ are not equivalent *all things considered*. For instance, distributions with a higher overall welfare or without unbalanced welfare allocations along agents' lives may be preferred. Actually, as shown by E4, if *CSE* is adopted, egalitarian and non-egalitarian concerns can be clearly distinguished in the evaluation of a distribution. The former reduce to the requirement $\mathbf{x}_i = \mathbf{x}$, all i, while the latter are related to the features of $\mathbf{x}$, that is, the desirable pattern of the egalitarian variable along agents' lives. *All things considered* a distribution with, say, $p > q = r = s$ may be rejected because of the unbalanced welfare allocation along agents' lives.

[9] If identity changed during an agent's life, there might be an egalitarian objection to the distribution. However a similar critique can be moved to any intertemporal egalitarian principle, since it amounts to saying that the principle is analysed in the wrong context. Once the agents' identity is correctly specified, all the arguments in this chapter remain valid.

However, this is an argument regarding the welfare pattern *along an agent's life* and not how she fares *relative to others* and therefore it is not an egalitarian reason to reject the distribution. A smoother welfare profile would probably be preferable but this would be the outcome, e.g., of the adoption of some kind of maximin principle applied to portions of an agent's life.

2.3. EGALITARIANISM, UTILITY AND THE MAXIMIN.

In the static context, given the relevant equalisandum, different egalitarian views can "be regarded as built around ways of measuring the inequality between lives" (McKerlie, 1989, p.487) and their implications can be appreciated only in the analysis of unequal distributions. Instead, as noted above, the egalitarian state to reach is unambiguously defined. As a result, the differences between the various views in relation with other normative principles can be shown in *unequal* distributions, but not if one evaluates the desirability of reaching the common egalitarian state in relation, e.g., to utilitarian concerns. This is not true in the intertemporal context: different principles yield different trade-offs between egalitarian and non-egalitarian concerns also in *egalitarian* distributions. Since the evaluation of a distribution, e.g., for policy purposes, is influenced by more than one ethical concern, it is important to analyse these trade-offs in a systematic way.

In this Section, *CLE*, *CSE*, and *SSE* are analysed in relation to two non-primarily-egalitarian normative principles, namely Rawls's (1971) difference principle and utilitarianism. If, as argued in Section 2.2, it is appropriate to impose a restriction on *CLE*, then it is important to analyse whether this implies a welfare loss, whether different restrictions have different effects on welfare, and what are the consequences for the worst-off generation.

The problem is modelled in a stark way. We generalise Arrow (1973A) and Dasgupta (1974A), in which the maximin criterion is examined in a dynamic framework. There is a society that exists for an infinite number of generations. Population is stationary, there is no technical progress and only one good that can be consumed or invested. Utility is the relevant egalitarian variable, and thus $x_j^t = u_j^t = u(c_j^t)$, where the subscript i denoting different agents belonging to the same age cohort is dropped in order to focus on *inter*generational inequalities. Assume that $T = 2$ and $j = 1, 2$ (youth and old age), and agents have identical additively-separable utility functions:

$$W(c_1^t, c_2^{t+1}) = u(c_1^t) + \beta u(c_2^{t+1}),$$

where c_1^t is consumption of the young in t, c_2^{t+1} is consumption of the elderly in $t + 1$, $0 < \beta \leq 1$ is the subjective discount factor and u satisfies $u(0) = 0$, $u'(c_j^t) \equiv \mathrm{d}u/\mathrm{d}c_j^t > 0$, $\lim_{c\to 0} u'(c) = \infty$, and $u''(c_j^t) \equiv \mathrm{d}^2(u)/\mathrm{d}(c_j^t)^2 < 0$.

Production possibilities can be represented by a production function $F(K, L)$, where K is the stock of capital, and L is labour supply. F is continuous and homogeneous of degree one. L^t is proportional to population and it is normalised to one. Thus, if $k^t \equiv K^t/L^t$, then $F(K^t, L^t) = f(K^t/L^t, 1) = f(k^t)$. The function f satisfies $f(0) = 0, f' > 0, f'' < 0$, and Inada conditions.

For any variable z, let $\{z^t\}_{t = 0,1,\ldots}$ denote an infinite sequence of values of z. The maximin program can be written as follows.

$$\max_{\{c_1^t, c_2^{t+1}\}_{t=0,1,\ldots}} \min_t W(c_1^t, c_2^{t+1}),$$

subject to $k^{t+1} - k^t + c_1^{\,t} + c_2^{\,t} = f(k^t)$, all $t \geq 0$, (MP)

given k^0 and $c_2^{\,0}$.

For any variable z, let $\mathrm{d}z = z' - z$ denote a change in z. Propositions 1 and 2 provide necessary conditions for a maximin solution.

PROPOSITION 1: *At the solution to (MP), $W(c_1^{\,t}, c_2^{\,t+1}) = W(c_1^{\,t+1}, c_2^{\,t+2})$, all t.*

Proof. Let W^* be the value of MP and suppose that, contrary to the statement, $W(c_1^{\,0}, c_2^{\,1}) > W^*$. By continuity, there is a sufficiently small $\mathrm{d}c_1^{\,0} < 0$, such that $W(c'^{\,0}_1, c_2^{\,1}) > W^*$, $-\,\mathrm{d}k^1 = \mathrm{d}c_1^{\,0}$ and the amount of resources available in $t = 1$ increases by $[1 + f'(k^1)]\mathrm{d}k^1$. Let $\mathrm{d}c_1^{\,1} = f'(k^1)\mathrm{d}k^1 > 0$ and $\mathrm{d}k^2 = \mathrm{d}k^1 > 0$ and repeat the procedure for all $t \geq 2$ so that $\mathrm{d}c_1^{\,t} = f'(k^t)\mathrm{d}k^t > 0$, $\mathrm{d}k^{t+1} = \mathrm{d}k^t > 0$, and $W(c'^{\,t}_1, c_2^{\,t+1}) > W^*$, all t, a contradiction. The proof of the case with $W(c_1^{\,t}, c_2^{\,t+1}) > W^*$, some $t > 0$, is similar. ∎

In other words, a welfare distribution must satisfy *CLE* in order to be the maximin solution. In this sense the maximin criterion poses an efficiency restriction on *CLE*: the maximin solution is the *CL*-egalitarian distribution with the highest level of equal welfare.

PROPOSITION 2: *At the solution to (MP), $u'(c_1^{\,t})/u'(c_2^{\,t+1}) = \beta(1 + f'(k^{t+1}))$, all t.*

Proof. Suppose not. Then there is a $\mathrm{d}c_1^{\,t}$, $\mathrm{d}c_2^{\,t+1}$ such that $\mathrm{d}c_2^{\,t+1} = -\,[1 + f'(k^{t+1})]\mathrm{d}c_1^{\,t}$ and $u'(c_1^{\,t})\mathrm{d}c_1^{\,t} + \beta u'(c_2^{\,t+1})\mathrm{d}c_2^{\,t+1} > 0$. By the concavity of W, this implies $W(c'^{\,t}_1, c'^{\,t+1}_2) > W(c_1^{\,t}, c_2^{\,t+1})$ leaving unmodified $c_1^{\,j}$, all $j \neq t$ and k^j, $c_2^{\,j}$, all $j \neq t + 1$, violating Proposition 1. ∎

By Proposition 2, agents born in t attain the highest $W(c_1{}^t, c_2{}^{t+1})$ given k^{j+1}, $c_1{}^j$ and $c_2{}^{j+1}, j \neq t$. This suggests that the condition in Proposition 2 can be derived as the first order condition of a constrained optimisation problem, defining the optimal consumption allocation along an agent's life. Given the assumptions on u and f, Proposition 2 implies that the maximin solution is unique, while none of the egalitarian criteria identifies *per se* a particular welfare distribution. However, the main implication of Proposition 2 in the analysis of the egalitarian views is that in general $u(c_1{}^t) \neq \beta u(c_2{}^{t+1})$ so that the maximin allocation will not be *SS*-egalitarian.

Let $c = c_1{}^t + c_2{}^t$, all t, denote the generic constant (total) consumption program and let $c^m \equiv f(k^0)$, all t.

LEMMA 1: *c^m is the maximum sustainable aggregate constant consumption.*

Proof. Consider $c > c^m$. At $t = 0$, $k^1 < k^0$, and thus $k^2 - k^1 = k^1 - k^0 + f(k^1) - f(k^0) < 0$, and $k^2 - k^1 < k^1 - k^0$, i.e. $|k^2 - k^1| / |k^1 - k^0| > 1$, and, by induction, $|k^{t+1} - k^t| / |k^t - k^{t-1}| > 1$. Therefore $k^t = 0$ for t finite, and c is not sustainable. ■

Lemma 1 provides a natural benchmark for the maximin path. Let $c_2{}^m = c_2{}^0$ and $c_1{}^m = c^m - c_2{}^m$; for any given $c_2{}^0$, no distribution in which $W(c_1{}^t, c_2{}^{t+1}) < W^m = W(c_1{}^m, c_2{}^m)$, some t, can solve (MP).[10] Hence, let $R^t = f(k^t) + k^t - c_2{}^t$

[10] Alternatively, the benchmark path could be the solution to the following problem:

$$\max_{c_1, c_2} \quad u(c_1) + \beta u(c_2),$$

$$\text{subject to } c_1 + c_2 = f(k^0).$$

In this case, the assumption of a given $c_2{}^0$ would be dropped, and the constraint $c_2{}^0 \geq c_2$ would be necessary to guarantee equal treatment of the generation born in $t = -1$. This choice denote the resources available to the generation born in t: if $R^t = R^0$ then *all* generations from t onwards can reach at least W^m. Consider the following sequence of maximisation programs.

$$\max_{c_1^t, c_2^{t+1}} u(c_1^t) + \beta u(c_2^{t+1}),$$

$$\text{subject to } k^{t+1} + c_1^t \leq R^t, \qquad (\mathrm{P}_t)$$

$$f(k^{t+1}) + k^{t+1} - c_2^{t+1} \geq R^{t+1},$$

$$\text{given } R^t, R^{t+1}.$$

Let (c_1*, c_2*, k*) be the solution of P_t with $R^t = R^{t+1} = R^0$, where in general $k^* \neq k^0$. Let $V(R^t, R^{t+1})$ denote the maximum function associated with P_t. Let $W^* \equiv W(c^*_1, c^*_2) = V(R^0, R^0)$. The main theorem can now be proved.

THEOREM 1: *Let c_2^0 be given. The maximin solution corresponds to the vector (c^*_1, c^*_2, k^*) for each generation.*

Proof. 1. The existence and uniqueness of (c_1^*, c_2^*, k^*) is guaranteed by the assumptions on u and f. Note also that (c_1^*, c_2^*, k^*) satisfies the conditions in Propositions 1 and 2.

2. Suppose it is possible to raise the welfare of all generations above W^*. Consider P_0: by construction the first generation's welfare can increase over W^* if and only if $R^1 < R^0$. Consider now generation 2: clearly $V(R^1, R^0) < W^*$. Moreover, $V(R^t, R^{t+1})$ is concave and its iso-welfare contours have slope $[1 + f'(k(R^t, R^{t+1}))]$, where $k(R^t, R^{t+1})$ is the

would include generation $t = -1$ in the definition of the just path, allowing for an explicit treatment of the transition to justice, instead of taking its past consumption choices as given. However, the main results of this chapter would not change.

optimum value of k^{t+1} from P_t. Hence, $W(c_1^{\ 1}, c_2^{\ 2}) > W^*$ implies $R^2 < R^0$, with $|R^2 - R^0| > [1 + f'(k(R^0, R^0))]|R^1 - R^0|$. Iterating the argument, $W(c_1^{\ t}, c_2^{\ t+1}) > W^*$ implies $|R^{t+1} - R^0|/|R^t - R^0| > [1 + f'(k(R^0, R^0))]$, all t, and the path violates the non-negativity of R^t in some finite t.█

Theorem 1 states that although the maximin principle and *CSE* represent different restrictions on *CLE*, they coincide in the economy described, since at the solution to MP, agents have the same consumption - and welfare - allocation during their lives.[11] Thus, if the egalitarian social planner also adopts an intergenerational maximin criterion, Theorem 1 proves that the two objectives would not be in contradiction if *CLE* or *CSE* are adopted, while if SSE_1 (or SSE_2) is chosen, a trade-off between the two concerns arises.

Moreover, since the maximin solution coincides with the allocation that maximises agents' utility under a *CLE* constraint, the model allows us to introduce some utilitarian concern in the analysis. Consider, for instance, classical (average or total) utilitarianism. By Proposition 2, it is more difficult to reconcile a utilitarian concern with SSE_2 than with *CLE* or *CSE*, since SSE_2 does not allow a constrained welfare-maximising allocation along agents' lives. Instead, if SSE_1 is adopted, in principle it is possible for infinitely many generations to reach a higher welfare level than at the maximin, with only a finite number of generations falling below it in order to start capital accumulation. Thus, due to the infinite gain in utility, a utilitarian would

[11] It can be proved that with a finite horizon this is not true. However, the adoption of the infinite horizon hypothesis is implied by the very nature of the problem, as there is no reason to restrict the analysis of a normative principle to an arbitrary, finite number of generations.

prefer the latter distribution to the maximin/*CSE* solution. In general, such a distribution might be appealing (as opposed to *CSE* or *CLE* distributions) not because it is *SS*-egalitarian but because some *CS*, or even *CL* inequalities can be outweighed by an infinite gain in utility, *all things considered.* In this sense, SSE_1 is the only intertemporal egalitarian principle compatible with sustained welfare growth and thus the principle that can best accommodate utilitarian concerns (although *SS*- equality could still imply some welfare loss with respect to unconstrained utility maximisation). However, this result derives from the exclusive focus of SSE_1 on *intra*temporal inequalities, and thus it should not be seen as a solution to the equality/growth dilemma, but rather as a way of escaping it.

The model presented is highly stylised and some caution is necessary in interpreting the results. While the analysis of SSE_1 does not depend on any particular assumptions, in more general settings, *CLE* and *CSE* will not be equivalent as concerns their relations with other normative principles and the maximin solution will be neither *CS*- nor *CL*-egalitarian.[12] However, despite its simplified structure, the model does captures *in vitro* some inherent features of the egalitarian views. As concerns utilitarianism, since *CSE* and SSE_2 distributions are strict subsets of those with $D_1 = 0$, the *CLE* welfare level will always be at least as high as the SSE_2 and *CSE* levels. Moreover, from Proposition 2, it is legitimate to infer that even in more general settings

[12] However, while Theorem 1 is more sensitive to changes in the assumptions, heterogeneous, non additive or non concave preferences, technical progress or more general production functions would leave Proposition 1 basically unchanged.

the *CLE* welfare would be at least as high as the SSE_2 welfare, since SSE_2 does not allow agents to allocate consumption optimally along their lives. Similarly, as regards rawlsian concerns, the above results suggest that in a more general setting, if the maximin solution was not egalitarian, the *CLE* level would be at least as close to it as the *CSE* level, and the latter in turn would be at least as close to the maximin as the SSE_2 level.

2.4. CONCLUSION

In this chapter three egalitarian views are analysed in the intertemporal context. Once the static setting is abandoned, egalitarian principles - apart from differing in the analysis of existing inequalities, - also define different ideal egalitarian distributions. While it may be important to use the different information conveyed by every criterion in the analysis of existing inequalities, when the egalitarian distributions associated with them are analysed, *CLE* and *SSE* have undesirable features while *CSE* represents the appropriate egalitarian benchmark.

The relations between the three egalitarian principles and other moral ideals, namely maximin and utilitarianism, are also analysed. As regards the maximin principle, Propositions 1-2 and Theorem 1 show that, unlike with *CLE* and *CSE*, the adoption of *SSE* implies a trade off between egalitarianism and a concern for the worst off. As regards utility, the same conclusion holds if one interprets *SSE* as a restriction on *CLE*, since it yields a lower egalitarian welfare level. This is not true if *SSE* is analysed *per se*, but this is just because in this case the *SSE* is a strictly *intra*temporal principle.

CHAPTER 3. EQUALITY OF OPPORTUNITY AND TIME

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3.1. INTRODUCTION

Egalitarians - and more specifically, socialists - have long cherished two ideals: that that society is best which promotes human development over time, and equality of condition among members of society.[1] More recently, since Rawls's rejuvenation of egalitarian studies, several qualifications have been put forth as to what the equalisandum should be. Most, although not all, participants in the discussion have advocated what we call an objectivist view, that the equalisandum should be something which is measurable independently of the views of the individuals who have it - primary goods, functionings, or resources (Rawls, 1971; Sen, 1980; and Dworkin, 1981; respectively). The principal non-objectivist equalisandum is, of course, welfare or utility, which can only be measured knowing the utility function of the individual in question, and can only

[1] Socialists have said (before consciousness about gender neutral language) that in the good society there will be 'self-realisation of man' and 'self-realisation of men.' The latter means that, over the course of a life, a person becomes self-realised, in the sense of developing her capacities. The former means that, over generations, human beings become more knowledgeable and developed. Here, we take human development to mean 'self-realisation of man.'

be compared interpersonally if an interpersonally comparable unit scale exists. None of the major writers advocates equality of welfare as an ethic.

Moreover, in recent years, various theories of equal opportunity have been proposed including Arneson (1989), Cohen (1989), and Roemer (1998), and we would say that Dworkin's (1981) 'equality of resources' is indeed an 'equal opportunity' theory as well. So we might well say that egalitarians advocate, as well as human development, equality of opportunity for some condition. That condition could be something objective like functionings or primary goods, or the subjective welfare.

What we argue in this chapter is that the three desiderata

i. protracted human development,

ii. equality of opportunity for some condition, and

iii. the condition be an objective characteristic of the individual,

are inconsistent. Because the first desideratum makes sense only in a dynamic context, equality of condition, or equality of opportunity for some condition, becomes equality (of opportunity) among all adults who ever live. Our claim says that if the equalisandum is objective - something like functioning - then achieving such equality implies the absence of human development over time. It is only by taking the equalisandum to be welfare of a particular kind, a non-objectivist concept, that equality of opportunity is consistent with human development. If our claimed inconsistency is correct, then egalitarians are faced with a choice: either dropping their advocacy of equality (of opportunity), or of human development, or of objectivist equalisanda. We think that the most attractive choice is to drop the objectivist view.

In other words, we claim to show that, if we move away from the static thought experiments imagined by Rawls and the objectivist writers heretofore, then objectivism ceases to be attractive (if it ever was). We must say, however, that our inquiry does not show that *justice* requires that we endorse subjectivism (the view that welfare is what must count for an egalitarian). For we advocate dropping objectivism because of its inconsistency with equality of opportunity *and human development,* and while the 'equality of opportunity' part of that compound phrase refers to a state of justice, the 'human development' part does not. That is, we do not claim that justice requires human development, or even, more weakly, that justice requires human development in an environment where it is possible. Human development over time is, for us, an obvious good, but we do not know what to call the state of a society that has it, the way a society with equality of opportunity is in a state of justice.

The rest of the chapter is structured as follows. Section 3.2 sets up the dynamic environment. Sections 3.3 and 3.4 show that if either an objectivist equalisandum or a paternalistic welfarist approach is adopted equality of opportunity is incompatible with human development. Section 3.5 proves that a non paternalistic welfarist approach is consistent with intra- and intergenerational equality and human development. Section 3.6 focuses on conclusions, while all lemmas and the induction step of the proof of Theorem 1 are in the Appendix.

3.2. THE DYNAMIC ENVIRONMENT

We model the problem in a stark way. There is a society that exists for an infinite number of generations. At each generation there are adults and children. Each adult has one child, and so the population size is constant. Adults, at least at the beginning date zero (0), have different wage rates - indeed, we shall seek simplicity by declaring that only two wage rates exist at date 0. We suppose that an adult's wage is a measure of her family's socio-economic status (SES), where SES has an impact on the docility – in the classical sense, educability, – of children. More specifically, the economic outcome of educating a child is the wage she will earn as an adult, and it takes more educational resources to bring a low SES child up to a given (adult) wage rate than it does a high SES child. We take the view that all children have identical inborn talent, and that the wage a child earns as an adult is a function of her talent, the educational resources invested in her, and the SES status of her parent, our summary of the environmental factor. To be specific, let $\mathfrak{R}_+$ denote the nonnegative real numbers: we suppose there are two functions h: $\mathfrak{R}_+ \rightarrow \mathfrak{R}_+$ and g: $\mathfrak{R}_+ \rightarrow \mathfrak{R}_+$, such that a child of a parent who has a wage of w will, as an adult, earn a wage of $h(x)g(w)$, if x is the fraction of GNP per capita that is invested in her through the educational process. In particular, we assume:

Aw: h and g are continuous and strictly increasing. Moreover, $h(0) = g(0) = 0$.

Our economic environment dispenses with two important aspects of reality - that children are differentially talented, and that children expend differential effort[2] - since we think they are unnecessary to expose the problem we want to concentrate upon.

At each generation, taxation of adult income is used to redistribute income among adults, as well as to finance education of that generation's children, and tax revenues, in the form of educational finance, must be distributed between the two types of children, those from low wage parents and those from high wage parents. The result of that education will be adults at the next date who have (perhaps) two wage levels, and the problem repeats itself. All children of a given SES receive the same educational investment, and hence have the same wage as adults. To be specific, we suppose that taxation takes the following form. First, all adult incomes are pooled, and each adult receives the average income. Then each adult pays the same fraction of her income as a tax. At date 0, a fraction f_L of the adults earn the low wage, w_L^0, and a fraction f_H earn the high wage, w_H^0, with $f_L + f_H = 1$. We define mean income at date 0 as $\mu^0 = f_L w_L^0 + f_H w_H^0$. If the tax rate is τ^0, then the net income of every adult is $(1 - \tau^0)\mu^0$.

[2] One is, of course, free to interpret the difficulty in educating low SES children as due to their lower talent. This is formally equivalent to our model, yet it might lead to different ethics. (Some would say that it is alright for low talent people to earn less than high talent people, although it is not alright for children from disadvantaged backgrounds to earn less than equally talented children from advantaged backgrounds.)

We wish to abstract from incentive problems; in particular, taxation does not alter labour supply, nor does anticipation of their future net income affect how hard children work in school. These would be poor assumptions if we were interested in advising policymakers, but our investigation here is of a different kind. We are interested in exposing certain logical inconsistencies in a conception of 'the good society,' and it is appropriate for this inquiry to assume that citizens are almost perfectly cooperative. We limit their cooperative spirit only by assuming that private incentives would come into play if we redistributed adult income so that low wage earners ended up with *more* income than high wage earners. (The best we can do is to equalize all net incomes.)

In the theory of equal opportunity (see Roemer, 1998) it is assumed that individuals have different circumstances and exert different efforts. Here, we abstract away from differential effort. A person's circumstances - those characteristics beyond her control that influence her outcome - are two in number, the SES (wage) of her parent, and the date at which she is born. We take children as 'adults in formation,' and are concerned with equalizing opportunities among adults for some condition X, which we shall call 'welfare.' Since effort is nugatory, the theory of equal opportunity expounded in Roemer (1998) says that our objective is to maximize the minimal level of 'welfare' among all adults across types, where an adult's type is a pair (w, t), w being her parent's wage, and t being the date at which she is born. Informally speaking, the SES of a child's parents and the date at which she is born are circumstances beyond her control, and equality of opportunity requires that we equalize, so far as possible, the welfare of individuals with such different circumstances.

Thus, our problem is to maximize the least level of 'welfare' across all adults who ever live. At each date the instruments we have available are a tax rate of adult income, τ, and, if there are adults with two wage levels (there are never more than two), an allocation of educational finance (r_L, r_H) among children of the two types, where $f_L\, r_L + f_H\, r_H = 1$. A child from an L family receives educational investment in the amount $\tau\mu r_L$ and a child from an H family receives $\tau\mu r_H$. Thus, if w_L and w_H were the parents' wages, then the children will earn, as adults, $h(\tau r_L)g(w_L)$ and $h(\tau r_H)g(w_H)$.

Let $\Re_+^2 = \Re_+ \times \Re_+$ and let $S \subseteq \Re_+^2$. We define an adult's level of *functioning* as a function F: $S \to \Re$ of her wage, w, and consumption (net income), y. We attempt to capture Sen's (1980) idea of functioning, which Cohen (1993) has characterized as 'midfare,' something midway between consumption and welfare. To wit, we imagine that a person's wage is a measure of her level of human capital and individuals derive welfare directly from their human capital. Moreover, functioning involves a degree of self-esteem and self-realization, and these, we propose, depend positively on an individual's level of human capital. In particular, we assume:

AF: Let $\underline{F} \equiv \inf_{w,y \in S} F(w, y)$. F is continuous and strictly increasing in both arguments. Moreover, $\lim_{w\to 0} F(w, y) = \underline{F}$, all y, and $\lim_{y\to 0} F(w, y) = \underline{F}$, all w.

We define *human development* as *an increase in functioning level of adults over time*. We believe this is consistent with the standard concept of human development, which is not an increase in welfare as such, but rather an increase in human capacity. Capacity, in our stark model, is a function of consumption and the wage, or more directly, of consumption, self-esteem, and self-realization. The wage is important as the reflection of education; in addition, it can be argued that self-esteem is a capacity enhancer, and that, too, is captured by the wage. Children embody the knowledge of past generations, through the educational process, and we have attempted to capture this in our specification of the educational technology.

This model has similarities to Arrow (1973A) and Dasgupta (1974A), in which the maximin criterion was examined in a dynamic framework. The main substantive difference is that we posit two types of individual, at least at the early dates, while Arrow and Dasgupta worked with a representative agent. Thus, we are interested in what *inter*generational equality requires with respect to *intra*generational wage differentials, a question that neither Arrow nor Dasgupta posed.

3.3. EQUALITY OF OPPORTUNITY FOR FUNCTIONING: MODEL I

Let $w^t = (w_L^t, w_H^t)$, all t, and let $\{w^t\}_{t=0}^{\infty}$ denote the infinite sequence $(w^0, w^1, \ldots)$: for any given $w^0 = (w_L^0, w_H^0)$, by Aw wages are given recursively by

$$w_J^{t+1} = h(\tau^t r_J^t)g(w_J^t), \qquad J = L, H. \tag{1}$$

Therefore the set of wage sequences feasible from a given w^0 is $\Pi(w^0) = \{\{w^t\}_{t=0}^{\infty} : w_J^{t+1} = h(\tau^t r_J^t)g(w_J^t), J = L, H, \tau^t \in [0, 1]$, and $r_L^t \in [0, 1/f_L]$, all $t\}$. Our first exercise is to take the 'welfare' of an adult to be her functioning level. Thus, our problem is to[3]

$$\max_{\{w^t\}_{t=0}^{\infty} \in \Pi(w^0)} \min_t (F_L^0, F_H^0, F_L^1, F_H^1, \ldots), \tag{2}$$

where $F_J^t = F(w_J^t, (1 - \tau^t)\mu^t)$ is the functioning level of adults in the 'J dynasty' at date t. The 'low dynasty' is the set of persons consisting of the low wage adults at date 0 and all their descendants; likewise for the 'high dynasty.' It is important to note that, at some date, the wages of the two adult types may be equalised, and if that is the case, then we stipulate that, thereafter, since there is only one type of child, there is no longer any decision concerning how to allocate educational finance - all children receive the same investment. We need not consider the possibility that a child in the H dynasty has a wage lower than one in the L dynasty at a given date, for that will never be an aspect of an optimal solution. It thus follows that at any date, the functioning level of L adults will be less than or equal to the functioning level of H adults (where L and H refer to the *dynasties*, not to the wages of particular adults), because the two types have the same consumption. Hence, the equality of opportunity program takes the form:

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[3] In Sections 3.3 - 3.4, we assume that the value of the program is attained. Similar results can be proved in the general case, but at the cost of a substantial increase in technicalities, with no

$$\max_{\{w^t\}_{t=0}^{\infty} \in \Pi(w^0)} \min_t (F_L^0, F_L^1, \ldots), \qquad \text{subject to } w_H^{t+1} \geq w_L^{t+1}, \text{ all } t. \qquad (2')$$

PROPOSITION 1: *Under Aw and AF, at the solution to (2'), $F_L^0 = F_L^t$, all t.*

Proof: 1. By Aw and AF, $0 < \tau^t < 1$, all t.

2. Let m be the value of program (2'). Suppose $F_L^0 > F_L^{t'}$, some t'. By Part 1, increase τ^0, so that, by (1) and Aw, w_L^t increases, for all $t > 0$. By AF, F_L^t increases for all $t > 0$, and the change in τ^0 can be small enough so that F_L^0 is still above m, a contradiction. Hence, $F_L^0 = m$.

3. Suppose $F_L^{t'} > F_L^0$, some t'. Let $t' = \min \{t: F_L^t > F_L^0\}$. By Part 1, decrease $\tau^{t'-1}$ and increase $\tau^{t'}$ so that, by Aw, w_J^t does not decrease, all $t > t'$ and $J = L, H$. By AF, $F_L^{t'-1}$ is increased above m, while changes in tax rates can be small enough so that $F_L^{t'}$ is still above m. Iterating backwards, the result follows from Part 2. ■

Proposition 1 proves that equality of opportunity for functioning is inconsistent with human development, in the sense that a fraction f_L of adults at every date remain at the level of functioning of date 0 L adults. If, as is reasonable, $f_L > .5$, then the majority of all adults are held to a low level of human capacity. (H adults do not necessarily get reduced to F_L^0 over time. If consumption is very important in functioning, it may pay to keep w_H^t above w_L^t in order to bring about a relatively high mean income.)

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further insights gained. In Section 3.5, where we adopt specific functional forms, we prove that

The maximin social welfare function is sometimes criticized for spending huge amounts of resources to raise the level of welfare of a very small group of individuals who are very poor welfare producing machines. Let us note this criticism does not apply here. Nobody is extremely handicapped in our environment - there are no terribly inefficient 'welfare' creating individuals. It is true, however, that L adults at date 0 comprise an arbitrarily small fraction of the adults who have lived up to date T, as T becomes large, and all L adults are held to *their* level of functioning. This is surely a form of 'extremism' of maximin, although it has a different character from the form of extremism we referred to in the first sentence of this paragraph. If we contemplate sacrificing the L adults at date 0, we are led to ask, why do they have less than an equal right to welfare than those at later dates? The answer 'Because it is too costly to their descendants *not* to sacrifice them' invites sacrificing the L adults, or indeed all adults, at any finite number of dates beginning at date 0. After all, this group, too, constitutes an arbitrarily small fraction of all adults who shall ever live.

3.4. EQUALITY OF OPPORTUNITY FOR WELFARE: MODEL II

We now suppose that, at each t, J adults, $J = L, H$, care about the functioning levels of their children, F_J^{t+1}, as well as their own, F_J^t, so that a J adult's utility depends on her own and her child's wage (w_J^t, w_J^{t+1}), and consumption (y^t, y^{t+1}). To be specific, we define a function u: $S \times S \rightarrow \Re$ such

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the supremum is attained.

that the utility of a J adult at date t can be written, as a shorthand notation, as $u_J^t = u(F_J^t, F_J^{t+1})$, $J = L, H$.

We let $\underline{u} \equiv \inf_{(w_J^t, y^t, w_J^{t+1}, y^{t+1}) \in S \times S} u(F_J^t, F_J^{t+1})$, $J = L, H$, and assume u to be continuous and strictly increasing in all arguments. Therefore, if $\lim_{w_J^t \to 0} u(F_J^t, F_J^{t+1})$ is finite, we shall assume, without loss of generality, that u is defined at $w_J^t = 0$; likewise for y^t, w_J^{t+1}, or y^{t+1}. Finally, we rule out an extreme form of altruism by assuming:

Au: $\lim_{F_J^{t+1} \to \underline{F}} u(X, F_J^{t+1}) \geq \lim_{F_J^t \to \underline{F}} u(F_J^t, X)$, for all X, and $J = L, H$.[4]

Our 'equality of opportunity' program is now to

$$\max_{\{w^t\}_{t=0}^{\infty} \in \Pi(w^0)} \min_t (u_L^0, u_L^1, \ldots), \qquad \text{subject to } w_H^{t+1} \geq w_L^{t+1}, \text{ all } t. \tag{3}$$

PROPOSITION 2: *Under Aw, AF, and Au, at the solution to (3), (i) $u_L^0 \leq u_L^t$, all t, and (ii) there are no two consecutive dates t and $t + 1$ such that $u_L^t > u_L^0$ and $u_L^{t+1} > u_L^0$.*

Proof: 1. By AF, Aw, and u's monotonicity: (a) $\tau^t > 0$, all t, and at any adjacent periods either $\tau^t < 1$, or $\tau^{t+1} < 1$, or both; (b) if $\lim_{F_L^t \to \underline{F}} u(F_L^t, F_L^{t+1}) = \underline{u}$ then $\tau^t < 1$ all t.

2. Let m be the value of (3). Suppose $u_L^0 > m$. If $\tau^0 < 1$, increase τ^0 a little. This raises u_L^t, all $t > 0$, and does not lower u_L^0 to m. If $\tau^0 = 1$, by Part 1.(b)

[4] Au shortens the proof of Proposition 2 considerably, but the main result does not depend on Au.

$u_L^0 = u(\underline{F}, F_L^1)$. Since $u_L^0 > m$, then by Au, $u_L^1 > m$, all τ^1, τ^2. Hence, by Part 1.(a), increase τ^1 a little: u_L^0 and u_L^1 remain above m, while u_L^t increases, all $t \geq 2$, a contradiction.

3. Suppose $u_L^1 > u_L^0$ and $u_L^2 > u_L^0$. (The same argument holds, by iterating backward, for any consecutive u_L^t and u_L^{t+1}, all $t \geq 1$.) By Part 1.(a), decrease τ^1, which increases u_L^0 above m. If $\tau^2 < 1$, increase τ^2 so that w_J^t, and u_J^t, $J = L, H$, do not decrease, for all $t \geq 3$. If $\tau^2 = 1$, by Part 1.(b) $u_L^2 = u(\underline{F}, F_L^3)$ and since $u_L^2 > m$, by Au it follows that $u_L^3 > m$, all τ^3, τ^4. Hence, by Part 1.(a), increase τ^3 so that w_J^t, and u_J^t, $J = L, H$, do not decrease, all $t \geq 4$. In both cases, the changes in tax rates can be small enough so that u_L^1 and u_L^2 (and u_L^3 in the latter case) remain above m, while u_L^0 is now above m, and Proposition 2.(ii) follows by Part 2. ∎

If each adult cares about her child's and her grandchild's level of functioning, then the same argument shows that $u_L^0 \leq u_L^t$, all t, and no *three* consecutive utilities can be greater than u_L^0. Thus, allowing parents to care about the functioning levels of a finite sequence of descendents does not enable us to escape the conclusion that protracted human development fails to occur. For it is clear that if the utility level of the L dynasty returns to u_L^0 periodically, then the functioning level of one generation must return, periodically, to F_L^0 or F_L^1 or lower. In this society, history repeats itself, condemning every n^{th} generation to the level of human development of the primeval ancestor.

It is worth noting that u can be any continuous monotonic utility function. In particular, an adult may well prefer that her child functions at a higher level than she, in the sense that, for all X and small $\varepsilon > 0$, $u(X - \varepsilon, X + \varepsilon) > u(X, X)$. This is perhaps somewhat surprising: even if adults *want* their children to function at a higher level than themselves, there is no protracted human development in the optimum.

3.5. EQUALITY OF OPPORTUNITY FOR WELFARE: MODEL III

We now suppose that adults care about their own level of functioning and their child's *utility*. In particular, we suppose that there is a concept of utility such that

$$u_J^t = F_J^t + \beta u_J^{t+1}, \qquad \text{all } t, \text{ and } J = L, H, \tag{4}$$

where $0 < \beta < 1$. Thus, if $\lim_{N\to\infty} \sum_{t=0}^{N} (\beta)^t F_J^t$ is bounded above for all w^0 and $\{w^t\}_{t=0}^{\infty} \in \Pi(w^0)$ - a condition that, as shown below, is satisfied in our model, - then we can set $\lim_{N\to\infty} (\beta)^{N+1} u^{N+1} = 0$ and write u_J^t recursively as

$$u_J^t = \sum_{i=t}^{\infty} (\beta)^{i-t} F_J^i, \qquad \text{all } t, \text{ and } J = L, H. \tag{5}$$

Thus, the utility of any adult born in period t is the discounted sum of her dynasty's levels of functioning. Caring about the *welfare* of your child forces you, implicitly, to care about the *functioning* of your descendents, all the way down. It is reasonable to suppose that this formulation is psychologically accurate. Are we parents content if our children are functioning well, or does our contentment depend upon their *happiness*, where their happiness derives from the happiness of *their* children?

Our 'equal opportunity for welfare' program is stated again as (3), where the notation now refers to the new concept of utility. In order to reach more definite results we add more structure to the model, replacing A*F* and A*w* by:

A*F'*: $F(w, y) = \gamma \log w + (1 - \gamma) \log y$, *where* $0 < \gamma < 1$.

A*w*.1: $h(x) = kx^{c_1}$, $g(w) = w^{c_2}$, *where* $k > 0$, *and* c_1, $c_2 > 0$.

In addition, as regards the educational technology, we assume:

A*w*.2: *Non-increasing returns to scale:* $c_1 + c_2 \leq 1$.

Assumption A*w*.2 is reasonable given our broad interpretation of human capital as reflecting self-esteem and self-realization (and not only productive human capital or knowledge), and given the role played by the SES status of the children's parents in the educational technology. Furthermore, A*w*.2 significantly enhances the tractability of the dynamic optimisation problem by guaranteeing some important regularity conditions.

Let W denote the state space with generic element $w = (w_L, w_H)$. By A*w*.1-A*w*.2, we can define a vector $w' = (w_L', w_H')$, with $w_J' \geq [k/(f_J)^{c_1}]^{1/(1-c_2)}$, $J = L$, H, and restrict the state space to $W = \{w \in \Re_+^2$: $w_L \leq w_L'$ and $w_H \leq w_H'\}$ without loss of generality. Let Γ: $W \rightarrow W$ denote the *feasibility correspondence*: $\Gamma(w)$

describes the set of feasible values for the state next period, $\hat{w}$, if the current state is w. By (1) and Aw.1, we have

$\Gamma(w) = \{\ \hat{w} \in W\!:\ 0 \leq \frac{f_H}{k^{1/c_1}}\frac{(\hat{w}_H)^{1/c_1}}{(w_H)^{c_2/c_1}} + \frac{f_L}{k^{1/c_1}}\frac{(\hat{w}_L)^{1/c_1}}{(w_L)^{c_2/c_1}} \leq 1$, and $\hat{w}_L \leq \hat{w}_H\ \}$,

with $\Gamma(w) \neq \varnothing$, all $w \in W$. Let $A = \{(w, \hat{w}) \in W \times W\!: \hat{w} \in \Gamma(w)\}$ be the graph of Γ. By AF', (1), and Aw.1, we can write the *one-period return function* $\phi\!: A \to \Re$ at t as:

$$\phi(w^t, w^{t+1}) = \gamma \log w_L^t + (1-\gamma)\log\left(1 - \left[\frac{f_H (w_H^{t+1})^{1/c_1}}{k^{1/c_1}(w_H^t)^{c_2/c_1}} + \frac{f_L (w_L^{t+1})^{1/c_1}}{k^{1/c_1}(w_L^t)^{c_2/c_1}}\right]\right) + (1-\gamma)\log[f_L w_L^t + f_H w_H^t]$$

so that ϕ is bounded above by $\phi(w', 0, 0)$, continuously differentiable, and under Aw.2, as shown in the Appendix, it is strictly concave. Then, noting that at the solution to (3) it must be $u_L{}^0 \leq u_L{}^t$, all t,[5] program (3) is equivalent to the sequence problem

$$v^*(w^0) = \max_{\{w^t\}_{t=0}^{\infty} \in \Pi(w^0)} u_L^0 = \max_{\{w^t\}_{t=0}^{\infty} \in \Pi(w^0)} \sum_{t=0}^{\infty} (\beta)^t \phi(w^t, w^{t+1}), \tag{6}$$

where $\Pi(w^0) = \{\{w^t\}_{t=0}^{\infty}\!: w^{t+1} \in \Gamma(w^t)$, all $t\}$ and v^* denotes the *supremum function*.

As a first step, consider the *single wage* problem, with $w_H{}^0 = w_L{}^0$ and only one type of adult, so that all children receive an equal per capita share of

[5] Since under AF', Aw.1, and Aw.2, the infinite sum is bounded above for every feasible wage sequence, at the solution to (3), it must be $\tau^t < 1$, all t. Hence, if $u_L{}^0 > u_L{}^t$, some t, then increase τ^0 a little.

educational investment: the state space is $W = \{w \in \Re_+ : w \leq w'\}$, where $w' \geq k^{1/(1-c_2)}$, and at any t, the only control variable is τ^t. Hence, (6) becomes:

$$v^*(w^0) = \max_{\{w^t\}_{t=0}^{\infty} \in \Pi(w^0)} \sum_{t=0}^{\infty} (\beta)^t \left[\log w^t + (1-\gamma) \log\left(1 - \frac{1}{k^{1/c_1}} \frac{(w^{t+1})^{1/c_1}}{(w^t)^{c_2/c_1}}\right)\right], \qquad (6')$$

where $\Pi(w^0) = \{\{w^t\}_{t=0}^{\infty} : w^{t+1} \in [0, k(w^t)^{c_2}], \text{ all } t\}$, given w^0. Then:

PROPOSITION 3: *Let $w_L^0 = w_H^0 = w^0$. Let $\tau^* \equiv [\beta c_1/((1 - \beta c_2)(1 - \gamma) + \beta c_1)]$. Under AF', Aw.1, and Aw.2, the function $v^*(w^0) = \alpha + [1/(1 - \beta c_2)]\log w^0$ solves (6'), where*

$$\alpha = \frac{(1-\gamma)}{(1-\beta)} \log \frac{(1-\gamma)(1-\beta c_2)}{(1-\gamma)(1-\beta c_2) + \beta c_1} + \frac{\beta}{(1-\beta)(1-\beta c_2)} \log k + \frac{\beta c_1}{(1-\beta)(1-\beta c_2)} \log \frac{\beta c_1}{(1-\gamma)(1-\beta c_2) + \beta c_1}, \qquad (7)$$

and the optimal policy is

$$w^{*t+1} = k(\tau^*)^{c_1} (w^{*t})^{c_2}, \qquad \text{all } t. \qquad (8)$$

Proof: The Euler equations and transversality condition deriving from (6') are:

$$\frac{(1-\gamma)(w^{t+1})^{1/c_1}}{c_1 k^{1/c_1} (w^t)^{c_2/c_1}} \Big/ \left(1 - \frac{(w^{t+1})^{1/c_1}}{k^{1/c_1} (w^t)^{c_2/c_1}}\right) = \beta + \frac{\beta(1-\gamma)c_2}{k^{1/c_1} c_1} \frac{(w^{t+2})^{1/c_1}}{(w^{t+1})^{c_2/c_1}} \Big/ \left(1 - \frac{(w^{t+2})^{1/c_1}}{k^{1/c_1} (w^{t+1})^{c_2/c_1}}\right), \qquad (9)$$

$$\lim_{t\to\infty} (\beta)^t \left[\frac{1}{w^t} + (1-\gamma) \frac{1}{k^{1/c_1}} \frac{(w^{t+1})^{1/c_1}}{(w^t)^{c_2/c_1}} \Big/ w^t \left(1 - \frac{1}{k^{1/c_1}} \frac{(w^{t+1})^{1/c_1}}{(w^t)^{c_2/c_1}}\right)\right] w^t = 0 \cdot \qquad (10)$$

Since $0 < \beta < 1$ and $\gamma < 1$, then $0 < \tau^* < 1$, and it is easy to show that (8) satisfies (9) and (10). Therefore, by the strict concavity of the one-period return function, (8) is the optimal policy, with

$w^{*t} = (k)^{\frac{1-(c_2)^t}{1-c_2}} (\tau^*)^{\frac{c_1[1-(c_2)^t]}{1-c_2}} (w^0)^{(c_2)^t}$, all $t \geq 1$. Substituting the latter expression into (6'), $v^*(w^0)$ is obtained. ∎

Let $\rho \equiv w_H/w_L$. We can now analyse the general case with $w_L^0 \neq w_H^0$, i.e. $\rho^0 > 1$, and $W \subset \Re_+^2$. Our strategy to solve (6) is to find the function v: $W \rightarrow \Re$ that solves

$$v(w) = \max_{\hat{w} \in \Gamma(w)} [\phi(w, \hat{w}) + \beta v(\hat{w})], \tag{11}$$

then we shall prove that $v = v^*$ and verify that at the solution to (6), $u_L^0 \leq u_L^t$, all t. To be specific, we shall find an infinite sequence of intervals $(\bar{\rho}_n, \bar{\rho}_{n+1}]$, disjoint sets $W_n = \{w \in W: \rho \in (\bar{\rho}_n, \bar{\rho}_{n+1}]\}$ with $\bigcup_{n=0}^{\infty} W_n = W$, and functions v_n: $W_n \rightarrow \Re$, $n \geq 0$, such that the function v: $W \rightarrow \Re$ defined as $v(w) = v_n(w)$ if $w \in W_n$, solves (11). By Proposition 3, we let $\bar{\rho}_0 = 1$ and conjecture that there is a $\bar{\rho}_1 > 1$ such that if $\rho^0 \in (1, \bar{\rho}_1]$, then it is optimal to set $\rho^1 = 1$, and thus $\rho^t = 1$, all $t > 1$.

Consider the Euler Equations deriving from (6), in terms of the controls τ and r_L, and the wage ratio ρ. At time t, in an interior solution:[6]

$$(1-\gamma)\frac{\tau^t}{1-\tau^t} = \beta c_1 + \beta c_2 (1-\gamma)\frac{\tau^{t+1}}{1-\tau^{t+1}}, \tag{12}$$

$$\frac{\tau^t}{1-\tau^t}(1-f_L r_L^t) = \frac{\beta c_1 (f_H)^{1-c_1}(1-f_L r_L^t)^{c_1}(\rho^t)^{c_2}}{f_L (r_L^t)^{c_1} + (f_H)^{1-c_1}(1-f_L r_L^t)^{c_1}(\rho^t)^{c_2}} + \beta c_2 \frac{\tau^{t+1}}{1-\tau^{t+1}}(1-f_L r_L^{t+1}). \tag{13}$$

[6] Notice that (12) is identical to (9) if the latter is expressed in terms of the controls τ^t and τ^{t+1}.

Let $t = 0$: substituting for $r_L{}^0$ and $r_L{}^1$ in (13) from (1), and noting that $\rho^1 = 1$ implies $\rho^2 = 1$, a necessary condition for $\rho^1 = 1$ to be optimal is

$$-\frac{\tau^0}{1-\tau^0}\frac{f_H}{f_H + f_L(\rho^0)^{c_2/c_1}} + \beta c_1 f_H + \beta c_2 \frac{\tau^1}{1-\tau^1} f_H \leq 0. \tag{13'}$$

By Proposition 3, if $\rho^1 = 1$ then in the optimum $\tau^1 = \tau^* \equiv \beta c_1/[(1 - \gamma)(1 - \beta c_2) + \beta c_1]$. Hence, by (12) we conjecture that $\tau^0 = \tau^1 = \tau^*$ and (13') becomes:

$$\rho^0 \leq \left[\frac{\gamma(1-\beta c_2)}{f_L[(1-\beta c_2)(1-\gamma)+\beta c_2]}+1\right]^{c_1/c_2} \equiv \bar{\rho}_1,\ \bar{\rho}_1 > 1. \tag{14}$$

Thus, we let $W_0 = \{w \in W: (w_H/w_L) \in (1, \bar{\rho}_1]\}$ and define v_0: $W_0 \rightarrow \Re$ as

$$v_0(w) = \alpha + \gamma \log w_L + (1-\gamma)\log(f_L w_L + f_H w_H) + \frac{\beta c_1}{1-\beta c_2}\log\frac{(w_H)^{c_2/c_1}(w_L)^{c_2/c_1}}{f_L(w_H)^{c_2/c_1} + f_H(w_L)^{c_2/c_1}}, \tag{15}$$

where α is given by (7): v_0 is strictly increasing and continuously differentiable in both variables;[7] if $w_H = w_L$, it coincides with the single wage solution; and, under Aw.2, it is strictly concave. Thus, given the strict concavity of ϕ and the convexity of Γ (see Appendix 3.1), it is immediate to verify that v_0 solves (11) on W_0 at the corner solution given by the control functions $\tau_0 : (1, \bar{\rho}_1] \rightarrow [0,1]$ and $r_0 : (1, \bar{\rho}_1] \rightarrow \Re_+$, defined as $\tau_0(\rho) = \tau^*$ and $r_0(\rho) = (\rho)^{c_2/c_1}/[f_H\ [\pi_0(\rho)]^{1/c_1} + f_L(\rho)^{c_2/c_1}]$, where π_0: $(1, \bar{\rho}_1] \rightarrow \Re_+$ gives the conjectured optimal $\hat{\rho}$ if $\rho \in (1, \bar{\rho}_1]$,

[7] Differentiating v_0 one obtains

$$\frac{\partial v_0(w)}{\partial w_L} = \frac{\gamma}{w_L} + \frac{(1-\gamma)f_L}{[f_L w_L + f_H w_H]} + \frac{\beta c_2}{1-\beta c_2}\frac{1}{w_L}\frac{f_L(w_H)^{c_2/c_1}}{f_L(w_H)^{c_2/c_1} + f_H(w_L)^{c_2/c_1}},$$

$$\frac{\partial v_0(w)}{\partial w_H} = \frac{(1-\gamma)f_H}{[f_L w_L + f_H w_H]} + \frac{\beta c_2}{1-\beta c_2}\frac{1}{w_H}\frac{f_H(w_L)^{c_2/c_1}}{f_L(w_H)^{c_2/c_1} + f_H(w_L)^{c_2/c_1}}.$$

and $\pi_0(\rho) = 1$ all $\rho \in (1, \bar{\rho}_1]$. Given $\tau_0(\rho)$ and $r_0(\rho)$, the wage functions $\varpi_{0,L}$: $W_0 \to \Re_+$, and $\varpi_{0,H}$: $W_0 \to \Re_+$ can be derived, which provide the conjectured optimal $\hat{w}$ if $w \in W_0$.

Next, we conjecture that there exists a $\bar{\rho}_2$ such that if $\rho^0 \in (\bar{\rho}_1, \bar{\rho}_2]$, it is optimal to set $\rho^1 \in (1, \bar{\rho}_1]$, and thus $\rho^t = 1$, all $t \geq 2$. Assuming v_0 to be the value function on W_0, in order for $\hat{w}$ in the interior of W_0 to solve (11), the following conditions are necessary:

$$\frac{(1-\gamma)}{\hat{w}_L}\frac{f_L}{c_1 k^{1/c_1}}\frac{(\hat{w}_L)^{1/c_1}}{(w_L)^{c_2/c_1}} \Bigg/ \left[1-\left(\frac{f_H(\hat{w}_H)^{1/c_1}}{k^{1/c_1}(w_H)^{c_2/c_1}}+\frac{f_L(\hat{w}_L)^{1/c_1}}{k^{1/c_1}(w_L)^{c_2/c_1}}\right)\right] = \beta\frac{\partial v_0(\hat{w})}{\partial \hat{w}_L}, \tag{16}$$

$$\frac{(1-\gamma)}{\hat{w}_H}\frac{f_H}{c_1 k^{1/c_1}}\frac{(\hat{w}_H)^{1/c_1}}{(w_H)^{c_2/c_1}} \Bigg/ \left[1-\left(\frac{f_H(\hat{w}_H)^{1/c_1}}{k^{1/c_1}(w_H)^{c_2/c_1}}+\frac{f_L(\hat{w}_L)^{1/c_1}}{k^{1/c_1}(w_L)^{c_2/c_1}}\right)\right] = \beta\frac{\partial v_0(\hat{w})}{\partial \hat{w}_H}. \tag{17}$$

By substituting for $\partial v_0(\hat{w})/\partial \hat{w}_L$ and $\partial v_0(\hat{w})/\partial \hat{w}_H$,[8] and expressing (16) and (17) in terms of the controls and the wage ratio, it follows that $\tau = \tau^*$ and

$$\frac{1}{(1-\beta c_2)}\frac{f_H(\hat{\rho})^{1/c_1}}{[f_L(\rho)^{c_2/c_1}+f_H(\hat{\rho})^{1/c_1}]} = \frac{(1-\gamma)f_H\hat{\rho}}{[f_L+f_H\hat{\rho}]}+\frac{\beta c_2}{(1-\beta c_2)}\frac{f_H(\pi_0(\hat{\rho}))^{1/c_1}}{[f_L(\hat{\rho})^{c_2/c_1}+f_H(\pi_0(\hat{\rho}))^{1/c_1}]},$$

[8] Note that the two derivatives in the previous footnote can also be expressed as:

$$\frac{\partial v_0(w)}{\partial w_L} = \frac{\gamma}{w_L}+\frac{(1-\gamma)f_L}{[f_L w_L+f_H w_H]}+\frac{(1-\gamma)c_2}{w_L c_1}\frac{f_L[\varpi_{0,L}(w)]^{1/c_1}}{k^{1/c_1}(w_L)^{c_2/c_1}} \Bigg/ \left[1-\left(\frac{f_H[\varpi_{0,H}(w)]^{1/c_1}}{k^{1/c_1}(w_H)^{c_2/c_1}}+\frac{f_L[\varpi_{0,L}(w)]^{1/c_1}}{k^{1/c_1}(w_L)^{c_2/c_1}}\right)\right],$$

$$\frac{\partial v_0(w)}{\partial w_H} = \frac{(1-\gamma)f_H}{[f_L w_L+f_H w_H]}+\frac{(1-\gamma)c_2}{w_H c_1}\frac{f_H[\varpi_{0,H}(w)]^{1/c_1}}{k^{1/c_1}(w_H)^{c_2/c_1}} \Bigg/ \left[1-\left(\frac{f_H[\varpi_{0,H}(w)]^{1/c_1}}{k^{1/c_1}(w_H)^{c_2/c_1}}+\frac{f_L[\varpi_{0,L}(w)]^{1/c_1}}{k^{1/c_1}(w_L)^{c_2/c_1}}\right)\right].$$

which implicitly defines a function $\lambda_1 : (1, \bar{\rho}_1) \to \Re_+$, where $\rho = \lambda_1(\hat{\rho})$ is the conjectured current wage ratio that makes it optimal to choose $\hat{\rho} \in (1, \bar{\rho}_1)$. After some algebra:

$$\lambda_1(\hat{\rho}) = \frac{(\hat{\rho})^{1/c_2}}{(f_L)^{c_1/c_2}} \left[\frac{[f_L + f_H \hat{\rho}][f_L(\hat{\rho})^{c_2/c_1} + f_H(\pi_0(\hat{\rho}))^{1/c_1}]}{(1-\beta c_2)(1-\gamma)\hat{\rho}[f_L(\hat{\rho})^{c_2/c_1} + f_H(\pi_0(\hat{\rho}))^{1/c_1}] + \beta c_2 [f_L + f_H \hat{\rho}](\pi_0(\hat{\rho}))^{1/c_1}} - f_H \right]^{\frac{c_1}{c_2}}$$

so that $\lambda_1(\hat{\rho}) > 0$, all $\hat{\rho} \in (1, \bar{\rho}_1)$, and $\lim_{\hat{\rho}\to 1} \lambda_1(\hat{\rho}) = \bar{\rho}_1$, while as proved in Lemma 3, Appendix 3.2, λ_1 is differentiable with $d\lambda_1(\hat{\rho})/d\hat{\rho} > 1$, all $\hat{\rho} \in (1, \bar{\rho}_1)$. Hence, we define $\bar{\rho}_2 = \lim_{\hat{\rho}\to\bar{\rho}_1} \lambda_1(\hat{\rho})$, with $\bar{\rho}_2 - \bar{\rho}_1 \geq \bar{\rho}_1 - 1$, and $\pi_1 = \lambda_1^{-1} : (\bar{\rho}_1, \bar{\rho}_2] \to (1, \bar{\rho}_1]$, where $\pi_1(\rho)$ is the conjectured optimal $\hat{\rho}$ if $\rho \in (\bar{\rho}_1, \bar{\rho}_2]$: π_1 is strictly increasing, continuous, and differentiable with $\lim_{\rho\to\bar{\rho}_1} \pi_1(\rho) = 1 = \pi_0(\bar{\rho}_1)$, so that patching π_0 and π_1 one obtains an increasing and continuous function.

Thus, we let $W_1 = \{w \in W: (w_H/w_L) \in (\bar{\rho}_1, \bar{\rho}_2]\}$ and define the conjectured optimal control functions $\tau_1 : (\bar{\rho}_1, \bar{\rho}_2] \to [0,1]$, as $\tau_1(\rho) = \tau^*$, and $r_1 : (\bar{\rho}_1, \bar{\rho}_2] \to \Re_+$, as $r_1(\rho) = (\rho)^{c_2/c_1}/[f_H\,[\pi_1(\rho)]^{1/c_1} + f_L(\rho)^{c_2/c_1}]$, from which the wage functions $\varpi_{1,L}$: $W_1 \to \Re_+$ and $\varpi_{1,H}$: $W_1 \to \Re_+$ can be derived. The function v_1: $W_1 \to \Re$, defined as $v_1(w) = \phi(w, \varpi_{1,L}(w), \varpi_{1,H}(w)) + \beta\, v_0(\varpi_{1,L}(w), \varpi_{1,H}(w))$ solves (11) on W_1 by construction: v_1 is strictly increasing, continuously differentiable,[9] and as proved

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[9] With

$$\frac{\partial v_1(w)}{\partial w_L} = \frac{\gamma}{w_L} + \frac{(1-\gamma)f_L}{[f_L w_L + f_H w_H]} + \frac{(1-\gamma)c_2}{w_L c_1} \frac{f_L[\varpi_{1,L}(w)]^{1/c_1}}{k^{1/c_1}(w_L)^{c_2/c_1}} \Bigg/ \left[1 - \left(\frac{f_H[\varpi_{1,H}(w)]^{1/c_1}}{k^{1/c_1}(w_H)^{c_2/c_1}} + \frac{f_L[\varpi_{1,L}(w)]^{1/c_1}}{k^{1/c_1}(w_L)^{c_2/c_1}}\right)\right],$$

in Lemma 4, Appendix 3.2, strictly concave, with $\lim_{w \to \overline{w}_1} v_1(w) = v_0(\overline{w}_1)$ and $\lim_{w \to \overline{w}_1} \partial v_1(w)/\partial w_J = \partial v_0(\overline{w}_1)/\partial w_J$, $J = L, H$, where $\overline{w}_1$ is any $w \in W_1$ such that $\rho = \overline{\rho}_1$.

By iterating the latter procedure, it is possible to derive by induction (see Appendix 3.2) an infinite sequence of intervals $(\overline{\rho}_n, \overline{\rho}_{n+1}]$, disjoint sets $W_n = \{w \in W$: $(w_H/w_L) \in (\overline{\rho}_n, \overline{\rho}_{n+1}]\}$, and functions $\varpi_{n,L}$: $W_n \to \Re_+$, $\varpi_{n,H}$: $W_n \to \Re_+$, and v_n: $W_n \to \Re$, such that (i) $\lim_{n\to\infty} \overline{\rho}_n = \infty$; (ii) the strictly increasing, continuously differentiable, and strictly concave function v: $W \to \Re$ defined as $v(w) = v_n(w)$, if $w \in W_n$, $n \geq 0$, solves (11); and (iii) at the solution to (11), if $\rho^0 \in (\overline{\rho}_n, \overline{\rho}_{n+1}]$, then $\rho^{t+1} \in (\overline{\rho}_{n-t-1}, \overline{\rho}_{n-t}]$, $0 \leq t < n$, and $\rho^{t+1} = 1$, $t \geq n$. Then:

THEOREM 1: *Consider an inegalitarian economy in which $w_L^0 \neq w_H^0$. Let $\rho^t \equiv w_H^t/w_L^t$. Under AF', Aw.1, and Aw.2, for any finite ρ^0, at the solution to program (6), equality is reached in a finite number of periods. Once equality is reached, wages grow according to (8) and eventually converge to*

$$w_\infty^* = (k)^{1/(1-c_2)}\left(\frac{\beta c_1}{(1-\gamma)(1-\beta c_2)+\beta c_1}\right)^{c_1/(1-c_2)}.$$

Proof: We shall prove that $v = v^*$.

$$\frac{\partial v_1(w)}{\partial w_H} = \frac{(1-\gamma)f_H}{[f_L w_L + f_H w_H]} + \frac{(1-\gamma)c_2}{w_H c_1}\frac{f_H[\varpi_{1,H}(w)]^{1/c_1}}{k^{1/c_1}(w_H)^{c_2/c_1}} \Bigg/ \left[1 - \left(\frac{f_H[\varpi_{1,H}(w)]^{1/c_1}}{k^{1/c_1}(w_H)^{c_2/c_1}} + \frac{f_L[\varpi_{1,L}(w)]^{1/c_1}}{k^{1/c_1}(w_L)^{c_2/c_1}}\right)\right].$$

1. Let $\rho^0 \in (\bar{\rho}_n, \bar{\rho}_{n+1}]$ and let $\{w^{*t}\}_{t=0}^{\infty}$ be the path of the states in the proposed solution with $\tau^t = \tau^*$, all t, $r_L^t = (\rho^t)^{c_2/c_1}/[f_H\,[\pi_{n-t}(\rho^t)]^{1/c_1} + f_L(\rho^t)^{c_2/c_1}]$, all $t \leq n$, and $r_L^t = 1$, all $t > n$. Let $\{w^t\}_{t=0}^{\infty} \in \Pi(w^0)$ be a feasible path of the states and let $D \equiv \lim_{T\to\infty} \sum_{t=0}^{T} (\beta)^t [\phi(w^{*t}, w^{*t+1}) - \phi(w^t, w^{t+1})]$. Let $\phi_{w_J^i}$ (w^t, w^{t+1}) $\equiv \dfrac{\partial \phi(w^t, w^{t+1})}{\partial w_J^i}$, $J = L, H$ and $i = t, t+1$. By the strict concavity of ϕ,

$$D > \lim_{T\to\infty} \sum_{t=0}^{T} (\beta)^t [\phi_{w_L^t}(w^{*t}, w^{*t+1})(w^{*t}_L - w_L^t) + \phi_{w_L^{t+1}}(w^{*t}, w^{*t+1})(w^{*t+1}_L - w_L^{t+1})]$$
$$+ \lim_{T\to\infty} \sum_{t=0}^{T} (\beta)^t [\phi_{w_H^t}(w^{*t}, w^{*t+1})(w^{*t}_H - w_H^t) + \phi_{w_H^{t+1}}(w^{*t}, w^{*t+1})(w^{*t+1}_H - w_H^{t+1})]$$

.

By construction, $\phi_{w_J^{t+1}}(w^{*t}, w^{*t+1}) + \beta\phi_{w_J^{t+1}}(w^{*t+1}, w^{*t+2}) = 0$, $J = L, H$, all $t < n$, and $\rho^n \in (1, \bar{\rho}_1]$. Next, it is easy to verify that $w_L^{*t} = w_H^{*t}$, all $t > n$, and $\tau^t = \tau^*$, all t, imply

$$\phi_{w_L^{t+1}}(w^{*t}, w^{*t+1}) + \beta\phi_{w_L^{t+1}}(w^{*t+1}, w^{*t+2}) = -[\phi_{w_H^{t+1}}(w^{*t}, w^{*t+1}) + \beta\phi_{w_H^{t+1}}(w^{*t+1}, w^{*t+2})] \geq 0,$$

for all $t \geq n$, given $\rho^n \in (1, \bar{\rho}_1]$. Hence, given $w_L^{*0} = w_L^0$, and $w_H^{*0} = w_H^0$:

$$D > \lim_{T\to\infty} \sum_{t=n}^{T-1} (\beta)^t [\phi_{w_L^{t+1}}(w^{*t}, w^{*t+1}) + \beta\phi_{w_L^{t+1}}(w^{*t+1}, w^{*t+2})](w_H^{t+1} - w_L^{t+1})$$
$$+ \lim_{T\to\infty} (\beta)^T \phi_{w_L^{T+1}}(w^{*T}, w^{*T+1})(w^{*T+1}_L - w_L^{T+1}) + \lim_{T\to\infty} (\beta)^T \phi_{w_H^{T+1}}(w^{*T}, w^{*T+1})(w^{*T+1}_H - w_H^{T+1})$$

Since $\phi_{w_J^{T+1}}(w^{*T}, w^{*T+1}) \leq 0$, and $\phi_{w_J^{T+1}}(w^{*T}, w^{*T+1}) w^{*T+1}_J = -f_J\,\beta/(1-\beta c_2)$, $J = L, H$, it follows that $D > 0$ all $w^0 \in W$ and $\{w^t\}_{t=0}^{\infty} \in \Pi(w^0)$. Convergence to w_∞^* along $\{w^{*t}\}_{t=0}^{\infty}$ follows from (8).

2. Since $\lim\sup_{t\to\infty}(\beta)^t\, v(w^t) \leq \lim\sup_{t\to\infty}(\beta)^t\, v(w') = 0$, for all $w^0 \in W$ and $\{w^t\}_{t=0}^{\infty} \in \Pi(w^0)$, and by Part 1, $\lim_{t\to\infty}(\beta)^t\, v(w^{*t}) = 0$, then by recursive dynamic optimisation theory (e.g., Stokey and Lucas, 1989, p.72-5), $v = v^*$.■

In other words, the optimal path involves equating the wages of the contemporaneous members of the two dynasties in a finite number of periods: if $\rho^0 \in (\bar{\rho}_{n-1}, \bar{\rho}_n]$, convergence occurs in n periods. Once equality is reached, human development continues forever.

3.6. CONCLUSION

Earlier, we remarked on the similarity between the present chapter, Arrow (1973A), and Dasgupta (1974A). The main differences between the latter models and ours are that in Arrow (1973A) and Dasgupta (1974A): (i) there is a representative agent each period, and so the only issue is to maximin welfare of that agent's descendents across time, whereas in our model there is an issue of *intragenerational* as well as *intergenerational* justice; (ii) agents care only about consumption, not about functioning (i.e., not about the wage per se); (iii) investment is modelled as capital, rather than educational, investment. Formally, the main difference is that the planner has only one instrument each period in Arrow and in Dasgupta, whereas in our model she has two instruments. (This is, of course, due to difference (i) above.) Nevertheless, Arrow's and Dasgupta's results are qualitatively similar to ours: an increase in consumption over time is compatible with maximin only if the equalisandum is welfare, in which case parents care about the consumption stream of their entire dynasty. Thus, the present chapter may be considered an intellectual descendent of Arrow (1973A) and Dasgupta (1974A).

Our concern with intragenerational inequality, not expressed in the earlier literature, led us to deduce that, as long as individuals value their human capital as well as their consumption, then the maximin program will eventually equalize the levels of human capital of all individuals. We remark, however, that this result may well depend on our assumption A*w*.2, of nonincreasing returns in the educational technology.

Let us recapitulate. One of the major foci of discussion in egalitarian theory of the last thirty years has been the nature of the equalisandum. The main participants in the discussion have moved away from taking welfare as that equalisandum, although it is important to note that Arneson (1989) has argued for choosing *opportunity for welfare* as the equalisandum. ('Opportunity for welfare' is, in general, quite different from 'welfare' as an equalisandum. That difference is due to differential effort, which in the present chapter, does not appear.) However, this debate has been carried out within the confines of a static environment, a 'model' with a single generation. Here, we have maintained that equality of opportunity, for whatever kind of condition, is an ethically viable conception in a multi-generation world, and that in such a context, it calls for equalizing opportunities across all types of adult, where an adult's type is characterized by the date at which he is born and the SES of the family in which he grew up. It is beyond this chapter's scope to *argue* that justice requires that a person fare no better than another simply by virtue of being born at a different date.[10] An asymmetric version of this principle is familiar in discussions of sustainable development and environmental preservation: we should leave to future generations a world as bountiful as the one left to us by our ancestors. But the other part is, we believe, just as compelling: we are under no ethical mandate to leave our descendents a world more bountiful than our own, *although we may decide to do so if that increases our welfare* by contemplating the happiness it will bring our children, and their children...

In studying the multi-generation world, we have learned that, if we choose what we call an objectivist equalisandum - we have taken 'functioning' as an appealing one - then equality of opportunity for that condition implies there is no protracted human development, where human development is conceived of not as an increase in human welfare, but rather in human capacities to function. Thus, two major characteristics of what comprises the good society, as it has been conceived of by egalitarians for several hundred years, are incompatible. We showed that if we equalize opportunities for welfare, where an adult's welfare depends upon her own level of functioning and the functioning levels of

[10] This is contestable. Some argue that equality of condition among living persons is all that an egalitarian ethic requires. One rationale is that self-esteem is affected by comparing one's condition to those of contemporaries, not to the dead, or to those not yet born.

a finite stream of her descendents, the unpleasant inconsistency continues to hold. If, however, we choose a thorough-going kind of welfare as the condition for which opportunities should be equalized - one which declares that an individual's welfare depends not just on his capacities and the capacities of his children, but rather on his own capacities and his child's *welfare* - then human development and equality of opportunity are mutually consistent.

The most appealing solution to the unpleasant inconsistency is, we believe, to drop the objectivist requirement.[11] It is opportunities for welfare that we should advocate equalizing. This, incidentally, conforms to Arneson's (1989) recommendation, although the reasons brought to bear here are entirely different from those he presents. But we must add that this escape from the inconsistency is predicated upon a psychological premise - that adults care about their own functioning, and the *welfare* of their children.

[11] Before agreeing with us, however, the reader should consult Silvestre (2002), who works with a different economic environment from ours, in which, he shows, an increase in welfare over time and egalitarianism are consistent, even when adults do not care about the welfare of their children.

APPENDIX 3.1. SOME PRELIMINARY LEMMAS

LEMMA 1: *Under Aw.2, the one-period return function ϕ is strictly concave.*

Proof: Let $K = \dfrac{(w^{t+1})^{1/c_1}}{(w^t)^{c_2/c_1}}$. The Hessian of K has entries:

$$\frac{\partial^2 K}{\partial (w^{t+1})^2} = \frac{1-c_1}{(c_1)^2}\frac{(w^{t+1})^{(1-2c_1)/c_1}}{(w^t)^{c_2/c_1}}, \qquad \frac{\partial^2 K}{\partial (w^t)^2} = \frac{c_2(c_2+c_1)}{(c_1)^2}\frac{(w^{t+1})^{1/c_1}}{(w^t)^{(c_2+2c_1)/c_1}},$$

$$\frac{\partial^2 K}{\partial w^t \partial w^{t+1}} = \frac{\partial^2 K}{\partial w^{t+1}\partial w^t} = -\frac{c_2}{(c_1)^2}\frac{(w^{t+1})^{(1-c_1)/c_1}}{(w^t)^{(c_2+c_1)/c_1}}.$$ Let D_i denote the principal minor of order i: clearly $D_1 \geq 0$, while, $D_2 \geq 0 \Leftrightarrow (1 - c_1)c_2(c_1 + c_2) - (c_2)^2 \geq 0 \Leftrightarrow c_1 + c_2 \leq 1$. Hence, K is convex and ϕ is strictly concave. ■

Remark: the same argument proves that in the single-wage problem

$$\phi(w^t, w^{t+1}) = \log w^t + (1-\gamma)\log\left(1 - \frac{1}{k^{1/c_1}}\frac{(w^{t+1})^{1/c_1}}{(w^t)^{c_2/c_1}}\right)$$ is strictly concave.

LEMMA 2: *Under Aw.2, Γ is convex in the sense that, for any w^t, $\widetilde{w}^t \in W$, and $\theta \in [0, 1]$, $w^{t+1} \in \Gamma(w^t)$ and $\widetilde{w}^{t+1} \in \Gamma(\widetilde{w}^t)$ implies $w^{t+1}(\theta) \in \Gamma(w^t(\theta))$, where for any $\theta \in [0, 1]$, $w^i(\theta) = (w_L^i(\theta), w_H^i(\theta)) \equiv \theta w^i + (1 - \theta)\widetilde{w}^i$, $i = t, t + 1$.*

Proof: Clearly, $w_H^{t+1}(\theta) \geq w_L^{t+1}(\theta)$, and $w_H^{t+1}(\theta)$, $w_L^{t+1}(\theta) \geq 0$. Finally, $\theta + (1 - \theta) \geq$

$$\left[\theta\frac{f_H(w_H^{t+1})^{1/c_1}}{k^{1/c_1}(w_H^t)^{c_2/c_1}} + (1-\theta)\frac{f_H(\widetilde{w}_H^{t+1})^{1/c_1}}{k^{1/c_1}(\widetilde{w}_H^t)^{c_2/c_1}} + \theta\frac{f_L(w_L^{t+1})^{1/c_1}}{k^{1/c_1}(w_L^t)^{c_2/c_1}} + (1-\theta)\frac{f_L(\widetilde{w}_L^{t+1})^{1/c_1}}{k^{1/c_1}(\widetilde{w}_L^t)^{c_2/c_1}}\right],$$

for any $\theta \in [0, 1]$, while, as in Lemma 1, it is easy to show that under Aw.2 the right hand side of the latter expression is greater or equal to

$$\left[\frac{f_H(w_H^{t+1}(\theta))^{1/c_1}}{k^{1/c_1}(w_H^t(\theta))^{c_2/c_1}} + \frac{f_L(w_L^{t+1}(\theta))^{1/c_1}}{k^{1/c_1}(w_L^t(\theta))^{c_2/c_1}}\right].$$ ■

APPENDIX 3.2. THE VALUE FUNCTION

We now extend the analysis of (11) proceeding by induction. Let $\bar{\rho}_0 = 1$. We assume that the functions λ_n, τ_n, r_n, π_n, $\varpi_{n,L}$ $\varpi_{n,H}$, and v_n, $n \geq 1$, can be defined as in Section 5.3. Let ρ and $\hat{\rho}$ denote, respectively, the current value of the wage ratio and its value next period and let a similar notation hold for w. We define the function $\lambda_n : (\bar{\rho}_{n-1}, \bar{\rho}_n] \to (\bar{\rho}_n, \bar{\rho}_{n+1}]$, $n \geq 1$, as

$$\lambda_n(\hat{\rho}) = \frac{(\hat{\rho})^{1/c_2}}{(f_L)^{c_1/c_2}} \left[\frac{[f_L + f_H \hat{\rho}][f_L(\hat{\rho})^{c_2/c_1} + f_H(\pi_{n-1}(\hat{\rho}))^{1/c_1}]}{(1-\beta c_2)(1-\gamma)\hat{\rho}[f_L(\hat{\rho})^{c_2/c_1} + f_H(\pi_{n-1}(\hat{\rho}))^{1/c_1}] + \beta c_2 [f_L + f_H \hat{\rho}](\pi_{n-1}(\hat{\rho}))^{1/c_1}} - f_H \right]^{\frac{c_1}{c_2}}$$

where $\bar{\rho}_n > \bar{\rho}_{n-1}$, and $\pi_{n-1} : (\bar{\rho}_{n-1}, \bar{\rho}_n] \to (\bar{\rho}_{n-2}, \bar{\rho}_{n-1}]$ is differentiable, with $\pi_{n-1}(\bar{\rho}_n) = \bar{\rho}_{n-1}$. Let $\lim_{\hat{\rho} \to \bar{\rho}_{n-1}} \lambda_n(\hat{\rho}) = \bar{\rho}_n$, $d\lambda_n(\hat{\rho})/d\hat{\rho} > 0$ and $\frac{d\lambda_n(\hat{\rho})}{d\hat{\rho}} \frac{\hat{\rho}}{\lambda_n(\hat{\rho})} \geq 1$, all $\hat{\rho} \in (\bar{\rho}_{n-1}, \bar{\rho}_n]$, and $\bar{\rho}_{n+1} = \lambda_n(\bar{\rho}_n)$, so that $\bar{\rho}_{n+1} > \bar{\rho}_n$ and $\hat{\rho}/\lambda_n(\hat{\rho}) < 1$, all $\hat{\rho} \in (\bar{\rho}_{n-1}, \bar{\rho}_n]$. Therefore, $d\lambda_n(\hat{\rho})/d\hat{\rho} > 1$, all $\hat{\rho} \in (\bar{\rho}_{n-1}, \bar{\rho}_n]$ and $\bar{\rho}_{n+1} - \bar{\rho}_n > \bar{\rho}_n - \bar{\rho}_{n-1}$. Let $\pi_n = \lambda_n^{-1} : (\bar{\rho}_n, \bar{\rho}_{n+1}] \to (\bar{\rho}_{n-1}, \bar{\rho}_n]$.

Let $W_n = \{w \in W: \rho \in (\bar{\rho}_n, \bar{\rho}_{n+1}]\}$. Define $\tau_n : (\bar{\rho}_n, \bar{\rho}_{n+1}] \to [0,1]$, $\tau_n(\rho) = \tau^*$, and $r_n : (\bar{\rho}_n, \bar{\rho}_{n+1}] \to \Re_+$, $r_n(\rho) = (\rho)^{c_2/c_1}/[f_H\, [\pi_n(\rho)]^{1/c_1} + f_L(\rho)^{c_2/c_1}]$, and the wage functions $\varpi_{n,L}$: $W_n \to \Re_+$, $\varpi_{n,L}(w) = k(\tau^*)^{c_1} [r_n(\rho)]^{c_1} (w_L)^{c_2}$, and $\varpi_{n,H}$: $W_n \to \Re_+$, $\varpi_{n,H}(w) = k(\tau^*)^{c_1} [(1 - f_L r_n(\rho))/f_H]^{c_1} (w_H)^{c_2}$. Let $\varpi_n(w) = (\varpi_{n,L}(w), \varpi_{n,L}(w))$. Define v_n: $W_n \to \Re$ as

$$v_n(w) = \gamma \log w_L + (1-\gamma)\log[f_L w_L + f_H w_H] + (1-\gamma)\log\left[1 - \left(\frac{f_H[\varpi_{n,H}(w)]^{1/c_1}}{k^{1/c_1}(w_H)^{c_2/c_1}} + \frac{f_L[\varpi_{n,L}(w)]^{1/c_1}}{k^{1/c_1}(w_L)^{c_2/c_1}}\right)\right]$$
$$+ \beta v_{n-1}(\varpi_n(w))$$

where v_{n-1}: $W_{n-1} \to \Re$, with $W_{n-1} = \{w \in W: (w_H/w_L) \in (\bar{\rho}_{n-1}, \bar{\rho}_n]\}$. Let v_n be strictly increasing, strictly concave, and continuously differentiable on W_n with

$$\frac{\partial v_n(w)}{\partial w_L} = \frac{\gamma}{w_L} + \frac{(1-\gamma)f_L}{[f_L w_L + f_H w_H]} + \frac{(1-\gamma)c_2}{w_L c_1}\frac{f_L[\varpi_{n,L}(w)]^{1/c_1}}{k^{1/c_1}(w_L)^{c_2/c_1}} \Bigg/ \left[1 - \left(\frac{f_H[\varpi_{n,H}(w)]^{1/c_1}}{k^{1/c_1}(w_H)^{c_2/c_1}} + \frac{f_L[\varpi_{n,L}(w)]^{1/c_1}}{k^{1/c_1}(w_L)^{c_2/c_1}}\right)\right],$$

$$\frac{\partial v_n(w)}{\partial w_H} = \frac{(1-\gamma)f_H}{[f_L w_L + f_H w_H]} + \frac{(1-\gamma)c_2}{w_H c_1}\frac{f_H[\varpi_{n,H}(w)]^{1/c_1}}{k^{1/c_1}(w_H)^{c_2/c_1}} \Bigg/ \left[1 - \left(\frac{f_H[\varpi_{n,H}(w)]^{1/c_1}}{k^{1/c_1}(w_H)^{c_2/c_1}} + \frac{f_L[\varpi_{n,L}(w)]^{1/c_1}}{k^{1/c_1}(w_L)^{c_2/c_1}}\right)\right].$$

Finally, let $\bar{w}_n$ be any $w \in W_n$ such that $w_H/w_L = \bar{\rho}_n$ and let $\lim_{w \to \bar{w}_n} v_n(w) = v_{n-1}(\bar{w}_n)$, and $\lim_{w \to \bar{w}_n} \partial v_n(w)/\partial w_J = \partial v_{n-1}(\bar{w}_n)/\partial w_J$, $J = L, H$.

We conjecture that there is a $\bar{\rho}_{n+2}$ such that if $\rho \in (\bar{\rho}_{n+1}, \bar{\rho}_{n+2}]$, then $\hat{\rho} \in (\bar{\rho}_n, \bar{\rho}_{n+1}]$. Assuming v_n to be the value function on W_n, in order for $\hat{w}$ in the interior of W_n to solve (11), the following conditions are necessary:

$$\frac{(1-\gamma)}{\hat{w}_L}\frac{f_L}{c_1 k^{1/c_1}}\frac{(\hat{w}_L)^{1/c_1}}{(w_L)^{c_2/c_1}} \Bigg/ \left[1 - \left(\frac{f_H(\hat{w}_H)^{1/c_1}}{k^{1/c_1}(w_H)^{c_2/c_1}} + \frac{f_L(\hat{w}_L)^{1/c_1}}{k^{1/c_1}(w_L)^{c_2/c_1}}\right)\right] = \beta\frac{\partial v_n(\hat{w})}{\partial \hat{w}_L},$$

$$\frac{(1-\gamma)}{\hat{w}_H}\frac{f_H}{c_1 k^{1/c_1}}\frac{(\hat{w}_H)^{1/c_1}}{(w_H)^{c_2/c_1}} \Bigg/ \left[1 - \left(\frac{f_H(\hat{w}_H)^{1/c_1}}{k^{1/c_1}(w_H)^{c_2/c_1}} + \frac{f_L(\hat{w}_L)^{1/c_1}}{k^{1/c_1}(w_L)^{c_2/c_1}}\right)\right] = \beta\frac{\partial v_n(\hat{w})}{\partial \hat{w}_H}.$$

Manipulating the latter expressions as in Section 5.3, one obtains $\tau^0 = \tau^*$, and it is possible to define a function $\lambda_{n+1}: (\bar{\rho}_n, \bar{\rho}_{n+1}) \to \Re_+$ as

$$\lambda_{n+1}(\hat{\rho}) = \frac{(\hat{\rho})^{1/c_2}}{(f_L)^{c_1/c_2}}\left[\frac{[f_L + f_H\hat{\rho}][f_L(\hat{\rho})^{c_2/c_1} + f_H(\pi_n(\hat{\rho}))^{1/c_1}]}{(1-\beta c_2)(1-\gamma)\hat{\rho}[f_L(\hat{\rho})^{c_2/c_1} + f_H(\pi_n(\hat{\rho}))^{1/c_1}] + \beta c_2[f_L + f_H\hat{\rho}](\pi_n(\hat{\rho}))^{1/c_1}} - f_H\right]^{\frac{c_1}{c_2}},$$

(A.1)

so that λ_{n+1} is a continuous function with $\lambda_{n+1}(\hat{\rho}) > 0$, for all $\hat{\rho} \in (\bar{\rho}_n, \bar{\rho}_{n+1})$, and $\lim_{\hat{\rho} \to \bar{\rho}_n} \lambda_{n+1}(\hat{\rho}) = \bar{\rho}_{n+1} = \lambda_n(\bar{\rho}_n)$.

LEMMA 3: *Under AF', Aw.1, and Aw.2,* $\frac{d\lambda_{n+1}(\hat{\rho})}{d\hat{\rho}}\frac{\hat{\rho}}{\lambda_{n+1}(\hat{\rho})} \geq 1$ *and* $\frac{d\lambda_{n+1}(\hat{\rho})}{d\hat{\rho}} > 1$, *all* $\hat{\rho} \in (\bar{\rho}_n, \bar{\rho}_{n+1})$. *Thus, if* $\bar{\rho}_{n+2} = \lim_{\hat{\rho}\to\bar{\rho}_{n+1}} \lambda_{n+1}(\hat{\rho})$, *then* $\bar{\rho}_{n+2} - \bar{\rho}_{n+1} > \bar{\rho}_{n+1} - \bar{\rho}_n$.

Proof: From (A.1) we have

$$\frac{d\lambda_{n+1}(\hat{\rho})}{d\hat{\rho}} = \frac{(\hat{\rho})^{\frac{1}{c_2}-1}}{c_2(f_L)^{c_1/c_2}}\left\{\frac{[f_L+f_H\hat{\rho}][f_L(\hat{\rho})^{c_2/c_1}+f_H(\pi_n(\hat{\rho}))^{1/c_1}]}{(1-\beta c_2)(1-\gamma)\hat{\rho}[f_L(\hat{\rho})^{c_2/c_1}+f_H(\pi_n(\hat{\rho}))^{1/c_1}]+\beta c_2[f_L+f_H\hat{\rho}](\pi_n(\hat{\rho}))^{1/c_1}} - f_H\right\}^{\frac{c_1}{c_2}-1}$$

$$\times\frac{\{A_{n+1}-B_{n+1}-C_{n+1}\}}{\left[(1-\beta c_2)(1-\gamma)\hat{\rho}[f_L(\hat{\rho})^{c_2/c_1}+f_H(\pi_n(\hat{\rho}))^{1/c_1}]+\beta c_2[f_L+f_H\hat{\rho}](\pi_n(\hat{\rho}))^{1/c_1}\right]^2} \tag{A.2}$$

where all terms but $\{A_{n+1} - B_{n+1} - C_{n+1}\}$ are positive and

$$A_{n+1} =$$
$$[(1+c_2)(f_L)^2(\hat{\rho})^{c_2/c_1} + (1+c_1)(f_H)^2\hat{\rho}(\pi_n(\hat{\rho}))^{1/c_1} + (1+c_1+c_2)f_Lf_H(\hat{\rho})^{\frac{c_2}{c_1}+1} + f_Lf_H\hat{\rho}\frac{d\pi_n(\hat{\rho})}{d\hat{\rho}}\pi_n(\hat{\rho}))^{\frac{1}{c_1}-1} + f_Lf_H\pi_n(\hat{\rho}))^{1/c_1}$$
$$+(f_H)^2(\hat{\rho})^2\frac{d\pi_n(\hat{\rho})}{d\hat{\rho}}\pi_n(\hat{\rho}))^{\frac{1}{c_1}-1}]\bullet\left[(1-\beta c_2)(1-\gamma)\hat{\rho}[f_L(\hat{\rho})^{c_2/c_1}+f_H(\pi_n(\hat{\rho}))^{1/c_1}]+\beta c_2[f_L+f_H\hat{\rho}](\pi_n(\hat{\rho}))^{1/c_1}\right]$$

$$B_{n+1} = [(1-\beta c_2)(1-\gamma)(c_1+c_2)f_L(\hat{\rho})^{\frac{c_2}{c_1}} + [(1-\beta c_2)(1-\gamma)+\beta c_2]c_1f_H(\pi_n(\hat{\rho}))^{1/c_1} + \beta c_2f_L(\pi_n(\hat{\rho}))^{\frac{1}{c_1}-1}\frac{d\pi_n(\hat{\rho})}{d\hat{\rho}}$$
$$+[(1-\beta c_2)(1-\gamma)+\beta c_2]f_H\hat{\rho}(\pi_n(\hat{\rho}))^{\frac{1}{c_1}-1}\frac{d\pi_n(\hat{\rho})}{d\hat{\rho}}]\times\left[(f_L)^2(\hat{\rho})^{\frac{c_2}{c_1}+1}+[(f_H)^2\hat{\rho}+f_Lf_H]\hat{\rho}(\pi_n(\hat{\rho}))^{1/c_1}+f_Lf_H(\hat{\rho})^{\frac{c_2}{c_1}+2}\right]$$

$$C_{n+1} = f_H\times\left[(1-\beta c_2)(1-\gamma)\hat{\rho}[f_L(\hat{\rho})^{c_2/c_1}+f_H(\pi_n(\hat{\rho}))^{1/c_1}]+\beta c_2[f_L+f_H\hat{\rho}](\pi_n(\hat{\rho}))^{1/c_1}\right]^2$$

Grouping all terms according to the exponents of $\hat{\rho}$, after some algebra:

$$A_{n+1} - B_{n+1} - C_{n+1} =$$

$$(1-\beta c_2)(1-\gamma)[1-(1-\beta c_2)(1-\gamma)](f_L)^2f_H(\hat{\rho})^{2\frac{c_2}{c_1}+2} + (1-c_1)(1-\beta c_2)(1-\gamma)(f_L)^3(\hat{\rho})^{2\frac{c_2}{c_1}+1}$$
$$+2[(1-c_1)(1-\beta c_2)(1-\gamma)+(1+c_2)\beta c_2-\beta c_2(1-\beta c_2)(1-\gamma)]f_H(f_L)^2(\hat{\rho})^{\frac{c_2}{c_1}+1}(\pi_n(\hat{\rho}))^{\frac{1}{c_1}}$$
$$+[2(1-\beta c_2)(1-\gamma)-2(1-\beta c_2)^2(1-\gamma)^2-2\beta c_2(1-\beta c_2)(1-\gamma)+(1+c_2)\beta c_2]f_L(f_H)^2(\hat{\rho})^{\frac{c_2}{c_1}+2}(\pi_n(\hat{\rho}))^{\frac{1}{c_1}}$$

$$+[(1-c_1)(1-\beta c_2)(1-\gamma)+2\beta c_2-2\beta c_2((1-\beta c_2)(1-\gamma)+\beta c_2)]f_L(f_H)^2\hat{\rho}(\pi_n(\hat{\rho}))^{\frac{2}{c_1}}$$
$$+[(1-\beta c_2)(1-\gamma)+\beta c_2][1-((1-\beta c_2)(1-\gamma)+\beta c_2)](f_H)^3(\hat{\rho})^2(\pi_n(\hat{\rho}))^{\frac{2}{c_1}}$$
$$+\beta c_2(1-\beta c_2)(f_L)^2f_H(\pi_n(\hat{\rho}))^{\frac{2}{c_1}}$$

$$+(1+c_2)\beta c_2(f_L)^3(\hat{\rho})^{\frac{c_2}{c_1}}(\pi_n(\hat{\rho}))^{\frac{1}{c_1}}-\beta c_2 f_L(f_H)^2\frac{d\pi_n(\hat{\rho})}{d\hat{\rho}}(\pi_n(\hat{\rho}))^{\frac{1}{c_1}-1}(\hat{\rho})^{\frac{c_2}{c_1}+3}$$

$$-2\beta c_2 f_H(f_L)^2\frac{d\pi_n(\hat{\rho})}{d\hat{\rho}}(\pi_n(\hat{\rho}))^{\frac{1}{c_1}-1}(\hat{\rho})^{\frac{c_2}{c_1}+2}-\beta c_2(f_L)^3\frac{d\pi_n(\hat{\rho})}{d\hat{\rho}}(\pi_n(\hat{\rho}))^{\frac{1}{c_1}-1}(\hat{\rho})^{\frac{c_2}{c_1}+1}$$

Under A*w*.1 and A*w*.2, all terms apart from the last three are strictly positive, for all $\hat{\rho}\in(\overline{\rho}_n,\overline{\rho}_{n+1})$, and in order for $d\lambda_{n+1}(\hat{\rho})/d\hat{\rho}>0$ to hold for all $\hat{\rho}\in(\overline{\rho}_n,\overline{\rho}_{n+1})$, it is sufficient to have $\frac{d\pi_n(\hat{\rho})}{d\hat{\rho}}\frac{\hat{\rho}}{\pi_n(\hat{\rho})}\leq 1$, all $\hat{\rho}\in(\overline{\rho}_n,\overline{\rho}_{n+1}]$. However, the latter condition holds, since $\pi_n=\lambda_n^{-1}$ and by assumption the elasticity of λ_n is greater than one over its entire domain. Hence, we define $\overline{\rho}_{n+2}=\lim_{\hat{\rho}\to\overline{\rho}_{n+1}}\lambda_{n+1}(\hat{\rho})$, with $\overline{\rho}_{n+2}>\overline{\rho}_{n+1}$, and $\hat{\rho}/\lambda_{n+1}(\hat{\rho})<1$, all $\hat{\rho}\in(\overline{\rho}_n,\overline{\rho}_{n+1}]$. Next, notice that:

$$\frac{d\lambda_{n+1}(\hat{\rho})}{d\hat{\rho}}\frac{\hat{\rho}}{\lambda_{n+1}(\hat{\rho})}=$$

$$\frac{(\hat{\rho})^{\frac{1}{c_2}-1}}{c_2(f_L)^{c_1/c_2}}\times\left\{\frac{[f_L+f_H\hat{\rho}][f_L(\hat{\rho})^{c_2/c_1}+f_H(\pi_n(\hat{\rho}))^{1/c_1}]}{(1-\beta c_2)(1-\gamma)\hat{\rho}[f_L(\hat{\rho})^{c_2/c_1}+f_H(\pi_n(\hat{\rho}))^{1/c_1}]+\beta c_2[f_L+f_H\hat{\rho}](\pi_n(\hat{\rho}))^{1/c_1}}-f_H\right\}^{\frac{c_1}{c_2}-1}$$

$$\times\frac{1}{\left[(1-\beta c_2)(1-\gamma)\hat{\rho}[f_L(\hat{\rho})^{c_2/c_1}+f_H(\pi_n(\hat{\rho}))^{1/c_1}]+\beta c_2[f_L+f_H\hat{\rho}](\pi_n(\hat{\rho}))^{1/c_1}\right]^2}\times\{A_n-B_n-C_n\}$$

$$\times(\hat{\rho})^{-\frac{1}{c_2}+1}(f_L)^{c_1/c_2}*\left\{\frac{[f_L+f_H\hat{\rho}][f_L(\hat{\rho})^{c_2/c_1}+f_H(\pi_n(\hat{\rho}))^{1/c_1}]}{(1-\beta c_2)(1-\gamma)\hat{\rho}[f_L(\hat{\rho})^{c_2/c_1}+f_H(\pi_n(\hat{\rho}))^{1/c_1}]+\beta c_2[f_L+f_H\hat{\rho}](\pi_n(\hat{\rho}))^{1/c_1}}-f_H\right\}^{-\frac{c_1}{c_2}}\geq 1$$

is equivalent to

$$(1-c_2)(1-\beta c_2)(1-\gamma)[1-(1-\beta c_2)(1-\gamma)](f_L)^2 f_H(\hat{\rho})^{2\frac{c_2}{c_1}+2}+(1-c_1-c_2)(1-\beta c_2)(1-\gamma)(f_L)^3(\hat{\rho})^{2\frac{c_2}{c_1}+1}$$

$$+2[(1-c_1-c_2)(1-\beta c_2)(1-\gamma)+\beta c_2-(1-c_2)\beta c_2(1-\beta c_2)(1-\gamma)]f_H(f_L)^2(\hat{\rho})^{\frac{c_2}{c_1}+1}(\pi_n(\hat{\rho}))^{\frac{1}{c_1}}$$

$$+\{2(1-c_2)(1-\beta c_2)(1-\gamma)+\beta c_2-2(1-c_2)(1-\beta c_2)(1-\gamma)[(1-\beta c_2)(1-\gamma)+\beta c_2]\}f_L(f_H)^2(\hat{\rho})^{\frac{c_2}{c_1}+2}(\pi_n(\hat{\rho}))^{\frac{1}{c_1}}$$

$$+[(1-c_1-c_2)(1-\beta c_2)(1-\gamma)+2(1-c_2)\beta c_2-2(1-c_2)\beta c_2((1-\beta c_2)(1-\gamma)+\beta c_2)]f_L(f_H)^2\hat{\rho}(\pi_n(\hat{\rho}))^{\frac{2}{c_1}}$$

$$+(1-c_2)[(1-\beta c_2)(1-\gamma)+\beta c_2][1-((1-\beta c_2)(1-\gamma)+\beta c_2)](f_H)^3(\hat{\rho})^2(\pi_n(\hat{\rho}))^{\frac{2}{c_1}}$$

$$+(1-c_2)\beta c_2(1-\beta c_2)(f_L)^2 f_H(\pi_n(\hat{\rho}))^{\frac{2}{c_1}}+\beta c_2(f_L)^3(\hat{\rho})^{\frac{c_2}{c_1}}(\pi_n(\hat{\rho}))^{\frac{1}{c_1}}-\beta c_2 f_L(f_H)^2\frac{d\pi_n(\hat{\rho})}{d\hat{\rho}}(\pi_n(\hat{\rho}))^{\frac{1}{c_1}-1}(\hat{\rho})^{\frac{c_2}{c_1}+3}$$

$$-2\beta c_2 f_H(f_L)^2\frac{d\pi_n(\hat{\rho})}{d\hat{\rho}}(\pi_n(\hat{\rho}))^{\frac{1}{c_1}-1}(\hat{\rho})^{\frac{c_2}{c_1}+2}-\beta c_2(f_L)^3\frac{d\pi_n(\hat{\rho})}{d\hat{\rho}}(\pi_n(\hat{\rho}))^{\frac{1}{c_1}-1}(\hat{\rho})^{\frac{c_2}{c_1}+1}\geq 0 \quad \text{(A.3)}$$

Thus, again, by Aw.1 and Aw.2, and given the assumptions on the elasticity of λ_n, then $\frac{d\lambda_{n+1}(\hat{\rho})}{d\hat{\rho}}\frac{\hat{\rho}}{\lambda_{n+1}(\hat{\rho})} \geq 1$ for all $\hat{\rho} \in (\overline{\rho}_n, \overline{\rho}_{n+1}]$. Hence, given $\hat{\rho}/\lambda_{n+1}(\hat{\rho}) < 1$, all $\hat{\rho} \in (\overline{\rho}_n, \overline{\rho}_{n+1}]$, it follows that $\mathrm{d}\lambda_{n+1}(\hat{\rho})/\mathrm{d}\hat{\rho} > 1$, all $\hat{\rho} \in (\overline{\rho}_n, \overline{\rho}_{n+1}]$, and thus $\overline{\rho}_{n+2} - \overline{\rho}_{n+1} > \overline{\rho}_{n+1} - \overline{\rho}_n$.■

Remark: Since $\mathrm{d}\pi_0(\hat{\rho})/\mathrm{d}\hat{\rho} = 0$, all $\hat{\rho} \in [1, \overline{\rho}_1]$, then $\mathrm{d}\lambda_1(\hat{\rho})/\mathrm{d}\hat{\rho} > 1$, all $\hat{\rho} \in [1, \overline{\rho}_1]$.

Thus, let $\pi_{n+1} = \lambda_{n+1}^{-1} : (\overline{\rho}_{n+1}, \overline{\rho}_{n+2}] \to (\overline{\rho}_n, \overline{\rho}_{n+1}]$: $\pi_{n+1}(\rho)$ is the conjectured optimal wage ratio next period, if the current value is $\rho \in (\overline{\rho}_{n+1}, \overline{\rho}_{n+2}]$; π_{n+1} is strictly increasing, continuous, and differentiable, with $\lim_{\rho \to \overline{\rho}_{n+1}} \pi_{n+1}(\rho) = \pi_n(\overline{\rho}_{n+1})$. Let $\tau_{n+1} : (\overline{\rho}_{n+1}, \overline{\rho}_{n+2}] \to [0,1]$, $\tau_{n+1}(\rho) = \tau^*$, and $r_{n+1} : (\overline{\rho}_{n+1}, \overline{\rho}_{n+2}] \to \Re_+$, $r_{n+1}(\rho) = (\rho)^{c_2/c_1}/[f_H\,[\pi_{n+1}(\rho)]^{1/c_1} + f_L(\rho)^{c_2/c_1}]$. Let $W_{n+1} = \{w \in W\colon (w_H/w_L) \in (\overline{\rho}_{n+1}, \overline{\rho}_{n+2}]\}$ and define the wage functions $\varpi_{n+1,L}\colon W_{n+1} \to \Re_+$, $\varpi_{n+1,L}(w) = k(\tau^*)^{c_1}\,[r_{n+1}(\rho)]^{c_1}(w_L)^{c_2}$, and $\varpi_{n+1,H}\colon W_{n+1} \to \Re_+$, $\varpi_{n+1,H}(w) = k(\tau^*)^{c_1}\,[(1 - f_L r_{n+1}(\rho))/f_H]^{c_1}(w_H)^{c_2}$. Let $\varpi_{n+1}(w) = (\varpi_{n+1,H}(w), \varpi_{n+1,H}(w))$, and define $v_{n+1}\colon W_{n+1} \to \Re$, by

$$v_{n+1}(w) = \gamma \log w_L + (1-\gamma)\log[f_L w_L + f_H w_H] + (1-\gamma)\log\left[1 - \left(\frac{f_H[\varpi_{n+1,H}(w)]^{1/c_1}}{k^{1/c_1}(w_H)^{c_2/c_1}} + \frac{f_L[\varpi_{n+1,L}(w)]^{1/c_1}}{k^{1/c_1}(w_L)^{c_2/c_1}}\right)\right]$$

$$+ \beta v_n(\varpi_{n+1}(w)),$$

so that v_{n+1} is strictly increasing and continuously differentiable on W_{n+1} with

$$\frac{\partial v_{n+1}(w)}{\partial w_L} = \frac{\gamma}{w_L} + \frac{(1-\gamma)f_L}{[f_L w_L + f_H w_H]} + \frac{(1-\gamma)c_2}{w_L c_1}\frac{f_L[\varpi_{n+1,L}(w)]^{1/c_1}}{k^{1/c_1}(w_L)^{c_2/c_1}} \Bigg/ \left[1 - \left(\frac{f_H[\varpi_{n+1,H}(w)]^{1/c_1}}{k^{1/c_1}(w_H)^{c_2/c_1}} + \frac{f_L[\varpi_{n+1,L}(w)]^{1/c_1}}{k^{1/c_1}(w_L)^{c_2/c_1}}\right)\right]$$

,

$$\frac{\partial v_{n+1}(w)}{\partial w_H} = \frac{(1-\gamma)f_H}{[f_L w_L + f_H w_H]} + \frac{(1-\gamma)c_2}{w_H c_1}\frac{f_H[\varpi_{n+1,H}(w)]^{1/c_1}}{k^{1/c_1}(w_H)^{c_2/c_1}} \Bigg/ \left[1 - \left(\frac{f_H[\varpi_{n+1,H}(w)]^{1/c_1}}{k^{1/c_1}(w_H)^{c_2/c_1}} + \frac{f_L[\varpi_{n+1,L}(w)]^{1/c_1}}{k^{1/c_1}(w_L)^{c_2/c_1}}\right)\right]$$

.

Let $\bar{w}_{n+1}$ be any $w \in W_{n+1}$ such that $w_H/w_L = \bar{\rho}_{n+1}$: $\lim_{w \to \bar{w}_{n+1}} v_{n+1}(w) = v_n(\bar{w}_{n+1})$, and $\lim_{w \to \bar{w}_{n+1}} \partial v_{n+1}(w)/\partial w_J = \partial v_n(\bar{w}_{n+1})/\partial w_J$, $J = L, H$.

LEMMA 4: *Under AF', Aw.1, and Aw.2, v_{n+1} is strictly concave.*

Proof: Consider any w^0, $\tilde{w}^0 \in W_{n+1}$. Let $w^1 = (\varpi_{n+1,L}(w^0), \varpi_{n+1,H}(w^0))$ and $\tilde{w}^1 = (\varpi_{n+1,L}(\tilde{w}^0), \varpi_{n+1,H}(\tilde{w}^0))$. For any $\theta \in [0, 1]$, let $w^t(\theta) \equiv \theta w^t + (1 - \theta)\tilde{w}^t$, $t = 0, 1$, and notice that $w^0(\theta) \in W_{n+1}$ and by Lemma 2, $w^1(\theta) \in \Gamma(w^0(\theta))$. Thus

$$\begin{aligned}
v_{n+1}(w^0(\theta)) &\geq \phi(w^0(\theta), w^1(\theta)) + \beta v_n(w^1(\theta)) \\
&> \theta\phi(w^0, w^1) + (1-\theta)\phi(\tilde{w}^0, \tilde{w}^1) + \beta v_n(w^1(\theta)) \\
&> \theta\phi(w^0, w^1) + (1-\theta)\phi(\tilde{w}^0, \tilde{w}^1) + \beta\theta v_n(w^1) + \beta(1-\theta)v_n(\tilde{w}^1) \\
&= \theta v_{n+1}(w^0) + (1-\theta)v_{n+1}(\tilde{w}^0)
\end{aligned}$$

by the strict concavity of ϕ and v_n, and by the definition of w^1 and $\tilde{w}^1$. ■

Remark: Since v_0 is strictly concave, by Lemma 4 v_1 is strictly concave.

Lemma 4 completes the induction step for all $n > 1$. Since $W_i \cap W_j = \varnothing$, all $i \neq j$, and, by Lemma 3, $\lim_{n \to \infty} \bar{\rho}_n = \infty$, it follows that $\bigcup_{n=0}^{\infty} W_n = W'$. Moreover, let $\bar{w}_n$ denote any $w \in W_n$ such that $w_H/w_L = \bar{\rho}_n$; since $\lim_{\rho \to \bar{\rho}_n} \pi_n(\rho) = \pi_{n-1}(\bar{\rho}_n)$, then $\lim_{w \to \bar{w}_n} \varpi_{n,J}(w) = \varpi_{n-1,J}(\bar{w}_n)$, $J = L, H$, $\lim_{w \to \bar{w}_n} v_n(w) = v_{n-1}(\bar{w}_n)$, and $\lim_{w \to \bar{w}_n} \partial v_n(w)/\partial w_J = \partial v_{n-1}(\bar{w}_n)/\partial w_J$, $J = L, H$.

Therefore, we define (i) the strictly increasing, continuously differentiable, and strictly concave function v: $W \to \Re$, such that $v(w) = v_n(w)$, for $w \in W_n$, $n \geq 0$; and (ii) the continuously differentiable wage functions ϖ_L: $W \to \Re_+$ and ϖ_H: $W \to \Re_+$, such that $\varpi_J(w) = \varpi_{n,J}(w)$, for $w \in W_n$, $n \geq 0$, and $J = L, H$. By substituting the function v into (11) and considering the first order conditions, it is not difficult to show that v is the value function, such that if $\rho^0 \in (\bar{\rho}_n, \bar{\rho}_{n+1}]$ then, in the optimum $\rho^1 \in (\bar{\rho}_{n-1}, \bar{\rho}_n]$, and ϖ_L and ϖ_H represent the corresponding optimal policies.

APPENDIX 3.3. AN ALTERNATIVE PROOF OF PROPOSITION 3

Recall that if $w_H^0 = w_L^0$, at any date t t^t is the only control and the state space is $W \subseteq \Re_+$, $W = \{w \in W: w \leq w', w' \geq k^{1/(1-c_2)}\}$. The feasibility correspondence is $\Gamma(w) = [0, k(w)^{c_2}]$, so that given $w^0 \in W$, $\Pi(w^0) = \{\{w^t\}_{t=0}^{\infty}: w^{t+1} \in [0, k(w^t)^{c_2}]$, all $t\}$, and the one-period return function is $\phi(w^t, w^{t+1}) = \log w^t + (1-\gamma)\log\left(1 - \frac{1}{k^{1/c_1}}\frac{(w^{t+1})^{1/c_1}}{(w^t)^{c_2/c_1}}\right)$. In this appendix we provide an alternative proof of Proposition 3 based on Bellman's functional equation:

$$v(w^0) = \max_{w^1 \in [0, k(w^0)^{c_2}]}\left[\log w^0 + (1-\gamma)\log\left(1 - \frac{(w^1)^{1/c_1}}{k^{1/c_1}(w^0)^{c_2/c_1}}\right) + \beta v(w^1)\right], \tag{A.4}$$

where v denotes the *value function*. To be specific, first, we prove that $v(w) = \alpha + \delta \log w$ solves (A.4), where α and δ are unknown constants to be determined. Then, we prove that $v = v^*$.

Proof of Proposition 3: 1. By substituting our postulated solution into (A.4)

$$v(w^0) = \max_{w^1 \in [0, k(w^0)^{c_2}]}\left[\log w^0 + (1-\gamma)\log\left(1 - \frac{(w^1)^{1/c_1}}{k^{1/c_1}(w^0)^{c_2/c_1}}\right) + \beta\alpha + \beta\delta \log w^1\right].$$

The first order condition for this problem is

$$w^1 = k\left(\frac{\beta\delta c_1}{(1-\gamma) + \beta\delta c_1}\right)^{c_1}(w^0)^{c_2}.$$

Hence, $v(w) = \alpha + \delta \log w$ solves (A.5) if

$\alpha + \delta \log w^0 =$

$$\beta\alpha + (1 + \beta\delta c_2)\log w^0 + (1-\gamma)\log\frac{(1-\gamma)}{(1-\gamma) + \beta\delta c_1} + \beta\delta \log k + \beta\delta c_1 \log\frac{\beta\delta c_1}{(1-\gamma) + \beta\delta c_1},$$

for all w^0, which holds if $\delta = 1/(1 - \beta c_2)$, and α is given by (7).

2. We need to show that $v(w^0) = v^*(w^0)$. First, note that since $v(w) \le v(w')$, all $w \in W$, then $\lim_{t\to\infty}(\beta)^t\, v(w^t) \le \lim_{t\to\infty} (\beta)^t\, v(w') = \lim_{t\to\infty} (\beta)^t\{\alpha + [1/(1 - \beta c_2)]\log w'\} = 0$, all $w^0 \in W$ and $\{w^t\}_{t=0}^{\infty} \in \Pi'(w^0)$.

Next, the optimal policy is $\hat{w} = k(\tau^*)^{c_1}(w)^{c_2}$, all t. As in Proposition 3 it is not difficult to show that, for each $w^0 \in W$ and $\{w^t\}_{t=0}^{\infty} \in \Pi'(w^0)$, $\sum_{t=0}^{\infty}(\beta)^t\phi(w^{*t}, w^{*t+1}) \ge \sum_{t=0}^{\infty}(\beta)^t\phi(w^t, w^{t+1})$, while $\lim_{t\to\infty} (\beta)^t\, v(w^{*t}) = 0$. Hence, by the theorems on dynamic optimization (see e.g. Stokey and Lucas, 1989, pp.72-5), $v = v^*$. ∎

CHAPTER 4. EXPLOITATION AND TIME

4.1. INTRODUCTION

Since *A General Theory of Exploitation and Class* (1982A), John Roemer has developed an original interpretation of Marx's economic theory (Roemer, 1982B, 1986A, 1988A). From a methodological point of view, Roemer's main contribution concerns the possibility (and, indeed, the necessity) of providing microfoundations to Marxian economics. The concepts of class and exploitation are modelled as the product of individual optimisation, and the full class and exploitation structures of a society are derived from agents' constrained rational choices. From a substantive point of view, Roemer rejects Marx's definition of exploitation based on surplus value as a relevant normative concept. According to him, all relevant moral information is conveyed by the analysis of *Differential Ownership of Productive Assets* (DOPA) and the resulting welfare inequalities. Roemer develops an alternative game theoretical definition of exploitation based on DOPA which is meant to be a generalisation of Marx's theory that captures its essential normative content.

Due to the scope and relevance of the issues analysed, Roemer's theory has generated a vast literature. Several critiques have been expounded on his methodology and on his conclusions, mainly based on issues of interpretation of Marx's theory (e.g., Reiman, 1987; Foley, 1989; Hodgson, 1989; Howard and King, 1992; Lebowitz, 1988, 1994), but surprisingly little attention has been devoted to his models.[1] In this chapter *a priori* problems of interpretation are left aside, while both methodological and substantive issues are discussed by means of an intertemporal generalisation of Roemer's subsistence economies.

From a methodological viewpoint, a formal dynamic model is extremely useful in the analysis of the *possibility* of providing neoclassical (and more specifically, Walrasian) microfoundations to Marxian economics. In particular, a model that aims to provide microfoundations to Marx's concepts of exploitation and class must be able to account for their persistence, since, according to Marx, they are inherent features of a capitalist economy.

Roemer himself acknowledges this; "The economic problem for Marx, in examining capitalism, was to explain the *persistent* accumulation of wealth by one class and the *persistent* impoverishment of another, in a system characterized by voluntary trade" (Roemer, 1982A, p.6, italics added). However, his models (both subsistence and accumulating economies) are essentially static in that there are no intertemporal trade-offs; they can be interpreted as describing either a succession of one-period economies (ibid., p.45) or an infinitely lived generation, but in either case *intertemporal* credit markets are absent *and* savings are impossible. Thus, they do not seem suitable for analysing the persistence of exploitation and classes in a competitive economy. In particular, while the absence of intertemporal credit markets is consistent with the subsistence hypothesis, the impossibility of

[1] Devine and Dymski's (1991) article represents a partial exception. However, the lack of a formal model makes some of their arguments not entirely compelling (see fn.2 below).

savings seems very restrictive. Moreover, savings and the intertemporal allocation of labour are particularly relevant, both because of the positive and normative importance of inter-class mobility, and because the introduction of a savings decision enlarges the set of choices available to agents. Thus, a dynamic model with savings is more realistic and it offers a more general framework to evaluate the possibility of providing microfoundations to Marxian economics.

From a substantive viewpoint, a dynamic model allows one to assess the *causal and moral relevance of DOPA*, focusing in particular on its role in generating exploitation and classes as persistent features of a competitive economy in which agents can save and the distribution of productive assets can change over time.

Given the importance of dynamics, the focus on Roemer's subsistence economies (in which agents minimise labour expenditure, provided they reach a minimum amount of consumption), rather than accumulating economies (in which agents maximise revenues) might seem contradictory. However, first, despite the lack of an explicit analysis of capital scarcity, the results obtained in Roemer's static economies depend on differential ownership of *scarce* productive assets (Skillman, 1995, 2001). Hence, it is not surprising *per se* that exploitation may disappear when accumulation is allowed (Devine and Dymski, 1991). Focusing on subsistence economies allows one to abstract from the issue of capital scarcity.[2]

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[2] See Roemer's (1992) reply to Devine and Dymski (1991), and the discussion in chapter 5 below. For a brief discussion of capital scarcity, see Roemer (1982A, pp. 9-11; 1988A, p. 23).

Secondly, and more importantly, Roemer's main theoretical conclusions do not depend on accumulation models. On the contrary, one of his most relevant results is precisely that "exploitation emerges logically prior to accumulation" (Roemer, 1982B, p.264). Thus, not only the analysis of subsistence economies gives the opportunity to examine the role of DOPA in a context where capital scarcity persists, it is also theoretically crucial in order to evaluate Roemer's fundamental claim that "differential distribution of property and competitive markets are sufficient institutions to generate an exploitation phenomenon, under the simplest possible assumptions" (Roemer, 1982A, p.43).

The rest of the chapter is structured as follows. In Section 4.2, a dynamic extension of the subsistence economy with a labour market is set up. In Sections 4.3 and 4.4, it is shown that in a dynamic framework two criteria to define exploitation and class emerge: one focuses on the agent's status in each period of her life, the other on the agent's whole life.[3] It is proved that the two criteria are equivalent in an interior equilibrium (in which agents do not save) and, more generally, that Roemer's model can be interpreted as a special case of the intertemporal model with no savings. In Section 4.5, a dynamic generalisation of Roemer's theory is provided. It is proved that in an economy with a strictly positive rate of time preference, exploitation and classes persist. However, the normative relevance of time preference is put into question, both in general and in the context of Marx's theory, and it is

[3] For a discussion of *single periods* and *whole lives* definitions of normative concepts in the context of egalitarian theories (McKerlie, 1989; Temkin, 1993), see chapter 2.

proved that, with no time preference, asset inequalities and classes persist in an interior equilibrium, while exploitation disappears in the long run. Hence, asset inequalities are normatively secondary, though causally primary in explaining exploitation, Roemer's definition of class based on the net amount of labour performed is questioned, and several doubts are raised on the possibility of providing robust microfoundations to Marx's concepts by means of Walrasian general equilibrium models. Section 4.6 focuses on conclusions. The existence of an equilibrium is proved in Appendix 1, and Roemer's game-theoretic model of exploitation is analysed in Appendix 2.

4.2. THE INTERTEMPORAL MODEL

The economy consists of a sequence of nonoverlapping generations, each with $\nu = 1, \ldots, N$, identical producers, living for T periods, and indexed by the date of birth kT, $k = 0, 1, 2, \ldots$ In every period t, each agent ν requires a $n \times 1$ vector of commodities b for subsistence, where $b >> \mathbf{0}$,[4] $\mathbf{0} = (0, \ldots, 0)'$, and can operate any activity of a given fixed coefficient technology (A, L), where A is a productive $n \times n$ input matrix and L is a $1 \times n$ vector of direct labour coefficients. As concerns A and L, the following assumption is made for the sake of simplicity.

ASSUMPTION 1: A is indecomposable and $L >> \mathbf{0}'$.[5]

[4] Given two vectors x, y in $\Re^n$, the following notation holds: $x \geq y$ if $x_i \geq y_i$, all $1 \leq i \leq n$; $x > y$ if $x \geq y$ and $x \neq y$; and $x >> y$ if $x_i > y_i$, all i.

[5] A matrix A is *decomposable* if there is a permutation matrix P such that $P'AP$ is upper block triangular with square matrices on the main diagonal. If A is *indecomposable* then it has at least one non-zero off-diagonal entry in every row and column.

In every period t, (p_t, w_t) denotes the $1 \times (n + 1)$ price vector, where w_t is the nominal wage; x_t^ν denotes the $n \times 1$ vector of activity levels that ν operates as a self-employed producer; y_t^ν denotes the $n \times 1$ vector of activity levels that ν hires others to operate; $z_t^\nu \in \Re_+$ denotes ν's labour supply; ω_t^ν denotes ν's $n \times 1$ vector of perfectly storable productive endowments, where ω_{kT}^ν is the vector of endowments inherited by ν, born in kT. The market value of ν's endowments, ν's wealth, in t is $W_t^\nu \equiv p_t\omega_t^\nu$. Finally, s_t^ν denotes ν's $n \times 1$ vector of net savings. As concerns credit markets and savings, the following assumption holds.

ASSUMPTION 2: *No credit market.* Productive assets must be bought with current wealth, while consumption and savings must be financed out of current revenue.

First, as in Roemer (1982A, 1988A),[6] Assumption 2 rules out fully developed intertemporal credit markets and thus the possibility of intertemporal trade *between agents*, consistently with the subsistence hypothesis. Second, due to the possibility of saving, Assumption 2 allows for intertemporal trade-offs in the allocation of labour *during an agent's life.* Thus, it is consistent with a dynamic setting in which agents' lives are divided into more than one period and it represents a genuine dynamic extension of Roemer's models, in which agents cannot save.

Let $x^\nu = \{x_t^\nu\}_{t=kT,\ldots,(k+1)T-1}$ denote producer ν's lifetime plan of activity levels operated as a self-employed producer, and let a similar notation hold

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[6] In "capital market island" (Roemer, 1982A, pp.87ff) credit markets operate *within* periods.

for y^ν, z^ν, s^ν, and ω^ν. Let $(p, w) = \{p_t, w_t\}_{t=kT,\dots,(k+1)T-1}$ denote the intertemporal path of the price vector during the lifetime of a generation.[7] Thus, let $\xi^\nu = (x^\nu, y^\nu, z^\nu, s^\nu)$ denote a generic intertemporal plan for ν. Let $\Lambda_t^\nu \equiv Lx_t^\nu + z_t^\nu$: Λ_t^ν denotes total labour expenditure by agent ν in period t. Let $0 < \beta \leq 1$ be the time preference factor. Given initial endowments ω_{kT}^ν, each ν is assumed to choose ξ^ν to solve the minimisation programme MP, whose value is $V(\omega_{kT}^\nu)$.[8]

$$\text{MP: } V(\omega_{kT}^\nu) = \min_{\xi^\nu} \sum_{t=kT}^{(k+1)T-1} \beta^t \Lambda_t^\nu,$$

subject to: $p_t(I - A)x_t^\nu + [p_t(I - A) - w_tL]y_t^\nu + w_tz_t^\nu \geq p_tb + p_ts_t^\nu$, (1)

$p_tA(x_t^\nu + y_t^\nu) \leq p_t\omega_t^\nu$, (2)

$Lx_t^\nu + z_t^\nu \leq 1$, (3)

$\omega_{t+1}^\nu = \omega_t^\nu + s_t^\nu$, (4)

$\omega_{(k+1)T}^\nu \geq \omega_{kT}^\nu$, (5)

$x_t^\nu, y_t^\nu, \omega_t^\nu \geq \mathbf{0}$, and $z_t^\nu \geq 0$.

Thus, agent ν is assumed to minimise lifetime labour, both when self employed and when working for somebody else, subject to the constraints that in every t: (1) net revenues are sufficient to reach subsistence and for savings plans; (2) wealth is sufficient for productive plans; (3) labour performed cannot exceed the working day, normalised to one; (4) the dynamic path of productive endowments is determined by net savings.

[7] The index k is not included in x^ν, y^ν, z^ν, s^ν, and (p, w) in order to simplify the notation and because, as shown below, the equilibrium will be interpreted as a stationary state.

[8] As shown below, there are price vectors such that the value of MP is indeed attained.

Finally, agents are required not to deplete their resources at the end of their lives and, in particular, (5) they are constrained to bequeath to the following generation at least as many resources as they inherited.

Let $O^{\nu}(p, w) \equiv \{\xi^{\nu} \text{ solves MP}\}$ denote the set of individually optimal ξ^{ν}. Let $\Omega_{kT} \equiv (\omega_{kT}^{1}, \omega_{kT}^{2}, \ldots, \omega_{kT}^{N})$; let $E(A, L, b, \Omega_{kT})$, or as a shorthand notation $E(\Omega_{kT})$, denote the economy described by technology (A, L), subsistence vector b, and distribution of endowments Ω_{kT}. Let $x_t = \sum_{\nu=1}^{N} x_t^{\nu}$; and likewise for y_t, z_t, s_t, ω_t.

DEFINITION 1. A *reproducible solution* (RS) for $E(\Omega_{kT})$ is an intertemporal profile (p, w) of the price vector and an associated set of actions such that

(i) $\xi^{\nu} = (x^{\nu}, y^{\nu}, z^{\nu}, s^{\nu}) \in O^{\nu}(p, w)$, for all ν;

(ii) $(x_t + y_t) \geq A(x_t + y_t) + Nb + s_t$, for all $t = kT, \ldots, (k+1)T\text{-}1$;

(iii) $A(x_t + y_t) \leq \omega_t$, for all $t = kT, \ldots, (k+1)T\text{-}1$;

(iv) $Ly_t = z_t$, for all $t = kT, \ldots, (k+1)T\text{-}1$;

(v) $\omega_{(k+1)T} \geq \omega_{kT}$.

Condition *(i)* requires that every agent be optimising; *(ii)* and *(iii)* require that in every period, there are enough resources for consumption and saving plans, and for production plans, respectively; *(iv)* requires the labour market to be in equilibrium in every period; *(v)* is the intertemporal *reproducibility condition*: every generation must leave to the following at least as many resources as they have inherited. By *(ii)* and *(v)*, Roemer's one-period reproducibility condition (Roemer, 1982A, Definition 2.1.*(ii)*, p.64) is significantly relaxed. In order to simplify the notation, let "for all t" stand for "for all t, $t = kT, \ldots, (k+1)T$-1".

DEFINITION 2. An *interior reproducible solution* (IRS) for $E(\Omega_{kT})$ is a RS such that $s_t^{\nu} = \mathbf{0}$, for all ν, t.

Let $\Lambda^{\nu} = \sum_{t=kT}^{(k+1)T-1} \Lambda_t^{\nu}$. As in Roemer (1982A, 1988A), in order to avoid an excess of uninteresting technicalities, it is assumed that agents who are able to reproduce themselves without working use just the amount of wealth strictly necessary to reach subsistence and to satisfy the reproducibility constraint (ν). In a subsistence economy, wealthy agents have no reason to accumulate or to consume more than b; hence, by stating that they do not "waste" their capital, Assumption 3 endows them "with embryonic capitalist behavior" (Roemer, 1982A, p.65).

ASSUMPTION 3: Let (p, w) be a RS for $E(\Omega_{kT})$. If there is a $\xi^{\nu} \in O^{\nu}(p, w)$ such that $\Lambda^{\nu} = 0$, then agent ν chooses y^{ν}, s^{ν} to minimise capital outlay.

In the remainder of this section some preliminary results of the static model are extended to the dynamic setting. First, at a RS the net revenues constraint (1) binds, for all agents, in every period t.

LEMMA 1. *Let (p, w) be a RS for $E(\Omega_{kT})$. Then $p_t(I - A)x_t^{\nu} + [p_t(I - A) - w_t L]y_t^{\nu} + w_t z_t^{\nu} = p_t b + p_t s_t^{\nu}$, all t, ν.*

Proof. Suppose $p_t(I - A)x_t^{\nu} + [p_t(I - A) - w_t L]y_t^{\nu} + w_t z_t^{\nu} > p_t b + p_t s_t^{\nu}$, some t, ν.

If $\Lambda^{\nu} = 0$, then it is possible to reduce capital outlay without destroying feasibility, thus contradicting Assumption 3. If $\Lambda^{\nu} > 0$, two cases may occur. *Case 1*: $\Lambda_t^\nu > 0$. It is feasible to decrease either x_t^ν or z_t^ν, contradicting optimality. *Case 2*: $\Lambda_t^\nu = 0$. Let $\tau = \min \{j | \Lambda_j^\nu > 0, (k+1)T-1 \geq j > t\}$. It is feasible to increase s_t^ν, with $x_j^\nu = \mathbf{0}$ and $z_j^\nu = 0$, all j, $\tau - 1 \geq j \geq t$, making the net revenue constraint slack in τ, and *Case 1* obtains. The proof of *Case 2* with $\Lambda_\tau^\nu > 0$, $t > \tau \geq kT$, is similar. ∎

Next, wealth constraints (2) bind at all t, for all ν who work at a RS.

LEMMA 2. *Let (p, w) be a RS for $E(\Omega_{kT})$ such that $p_t > p_t A + w_t L$, all t. If $\Lambda^\nu > 0$ for all $\xi^\nu \in O^\nu(p, w)$, then $p_t A(x_t^\nu + y_t^\nu) = p_t \omega_t^\nu$, all t.*

Proof. Suppose $p_t A(x_t^\nu + y_t^\nu) < p_t \omega_t^\nu$, some t. Let A_i denote the i-th column of A and let L_i be the i-th component of L. Then it is possible to choose a sector i with $p_{it} > p_t A_i + w_t L_i$ and increase y_{it}^ν making the net revenue constraint slack in t. By Lemma 1, given that $\Lambda^\nu > 0$, $\xi^\nu \notin O^\nu(p, w)$. ∎

The next results characterise RS's with $Nb + s_t > \mathbf{0}$, all t. This condition is imposed only for analytical convenience and it implies no significant loss of generality: it can be interpreted as a condition on capital scarcity and at an IRS, it reduces to $Nb > \mathbf{0}$, which is true by assumption. Lemma 3 proves that in equilibrium, profits are nonnegative and the price vector is strictly positive at all t.

LEMMA 3. *Let (p, w) be a RS for $E(\Omega_{kT})$ such that $Nb + s_t > \mathbf{0}$, all t. Then, for all t, (i) $p_t \geq p_t A + w_t L$, and (ii) $w_t > 0$ and $p_t >> \mathbf{0}'$.*

Proof. Part (i). As in (Roemer, 1982A, Lemma 2.2, p.66), for every period t.

Part (ii). At every t, if $w_t = 0$ then $z_t^\nu = 0$, all ν, while by Lemma 1, at the solution to (MP), $y_t^\nu > \mathbf{0}$, for all ν with $p_t\omega_t^\nu > 0$, so that $Ly_t > z_t = 0$. Hence $w_t > 0$ and the result follows since $p_t \geq w_tL(I-A)^{-1} >> \mathbf{0}'$. ■

The profit rate of sector i at time t is defined as $\pi_{it} = [p_t(I-A) - w_tL]_i/p_tA_i$, where $[p_t(I-A) - w_tL]_i$ is the i-th component of the vector $[p_t(I-A) - w_tL]$ and A_i is the i-th column of A. In every t, let $\pi_t \equiv \max_i \pi_{it}$ be the maximal profit rate. The next result proves that at a RS, the profit rate is equalised across sectors, in all t.

PROPOSITION 1. *Let (p, w) be a RS for $E(\Omega_{kT})$ such that $Nb + s_t > \mathbf{0}$, all t. Then $\pi_{it} = \pi_t$, all i, t, and $p_t = (1 + \pi_t)p_tA + w_tL$, all t.*

Proof. First, notice that if $(x^\nu, y^\nu, z^\nu, s^\nu) \in O^\nu(p, w)$ then $(\bar{x}^\nu, \bar{y}^\nu, \bar{z}^\nu, s^\nu) \in O^\nu(p, w)$, whenever $\bar{x}_t^\nu + \bar{y}_t^\nu = x_t^\nu + y_t^\nu$ and $\bar{z}_t^\nu + L\bar{x}_t^\nu = z_t^\nu + Lx_t^\nu$, all t. Thus, consider solutions of the form $(\mathbf{0}, \bar{y}^\nu, \bar{z}^\nu, s^\nu)$, and suppose that $\pi_{it} < \pi_t$, some i, t: for all $\xi^\nu \in O^\nu(p, w)$, it must be $\bar{y}_{it}^\nu = 0$, all ν, and thus $\bar{y}_{it} = 0$, or else it would be possible to increase revenues, or to reduce capital outlay, by reducing $\bar{y}_{it}^\nu$ and increasing $\bar{y}_{jt}^\nu$, where $\pi_{it} < \pi_{jt}$. However, by Assumption 1, if $\bar{y}_{it} = 0$ there can be no RS. Hence $\pi_{it} = \pi_t$, all i, t, and the price equations follow from the definition of π_{it}. ■

Let λ denote the $1 \times n$ vector of labour values, $\lambda = L(I-A)^{-1} >> \mathbf{0}'$. The next result derives aggregate activity levels and aggregate labour performed, both in every period t and during the lifetime of generation k.

PROPOSITION 2. *Let (p, w) be a RS for $E(\Omega_{kT})$. Then:*

(i) $x_t + y_t = (I - A)^{-1} (Nb + s_t)$, *all t;*

(ii) $\Lambda_t = Lx_t + z_t = N\lambda b + \lambda s_t$, *all t;*

(iii) $\sum_{t=kT}^{(k+1)T-1} (x_t + y_t) = (I - A)^{-1} NTb$;

(iv) $\Lambda = \sum_{\nu=1}^{N} \Lambda^{\nu} = NT\lambda b.$

Proof. Part (i)-(ii). See Roemer (1982A, Theorem 2.1, p.67). *Part (iii)-(iv).*

Given Assumption 3, by optimality, $\omega^{\nu}_{(k+1)T} = \omega^{\nu}_{kT}$ and thus $\sum_{t=kT}^{(k+1)T-1} s^{\nu}_t = \mathbf{0}$, all ν. ■

Two properties of a RS should be noted, which allow one to simplify the notation considerably. First, since at the solution to MP, $\omega^{\nu}_{(k+1)T} = \omega^{\nu}_{kT}$, all ν, then if (p, w) is a RS for $E(\Omega_{kT})$, it is also a RS for $E(\Omega_{(k+1)T})$. Hence, (p, w) can be interpreted as a stationary solution and in what follows the case $k = 0$ is considered without loss of generality. Second, let $\mathbf{1} = (1, \ldots, 1)'$: since at a RS $w_t > 0$, all t, by Proposition 1, prices can be normalised choosing labour as the numeraire, setting $w_t = 1$, all t, and in what follows reproducible solutions of the form $(p, \mathbf{1})$ are considered without loss of generality.

4.3. EXPLOITATION

In order to analyse exploitation in the intertemporal context, first of all, it is necessary to extend the concept of *Socially Necessary Labour Time.*

DEFINITION 3. *Socially Necessary Labour Time in t* is the amount of labour time that is needed by an agent to reproduce herself in t: $SNLT_t \equiv \lambda b$.

Aggregate Socially Necessary Labour Time in t is the amount of time that is needed by society to reproduce itself in *t*: $ASNLT_t \equiv N\lambda b$. Similarly, considering whole lives, *Socially Necessary Labour Time* and *Aggregate Socially Necessary Labour Time* are defined, respectively, as $SNLT \equiv T\lambda b$ and $ASNLT \equiv TN\lambda b$.

Thus, unlike in the static model, there are two different criteria to define an agent's exploitation status, focusing on the amount of labour performed either in each period of her life, or during her whole life. Let $\Delta^\nu = \sum_{t=0}^{T-1}(\Lambda_t^\nu - \lambda b)$.

DEFINITION 4. Agent ν is *exploited within period t*, or WP_t *exploited*, if $\Lambda_t^\nu > \lambda b$; a WP_t *exploiter* if $\Lambda_t^\nu < \lambda b$; and WP_t *exploitation-neutral* if $\Lambda_t^\nu = \lambda b$. Similarly, agent ν is *exploited during her whole life*, or *WL exploited*, if $\Delta^\nu > 0$; a *WL exploiter* if $\Delta^\nu < 0$; and *WL exploitation-neutral* if $\Delta^\nu = 0$.[9]

The *WP* and the *WL* definitions (as shown below, a similar distinction holds for classes) incorporate different normative concerns. An analysis based on the *WL* definition reflects the intuition that, from an *individual's* viewpoint, to be exploited in every period is certainly worse than being exploited only in some periods. However, the *WL* criterion leads to the rather counterintuitive conclusion that there would be no Marxist objection to "*changing places capitalism*," i.e. to a capitalist economy in which exploitation, - no matter how significant and widespread, - existed in every

[9] The *WL* definition encompasses the case with T infinite.. If T is finite, agent ν is *WL exploited*, if $\Lambda^\nu > T\lambda b$; a *WL exploiter* if $\Lambda^\nu < T\lambda b$; and *WL exploitation-neutral* if $\Lambda^\nu = T\lambda b$.

period, but the agents' status changed over time so as to equalise the amount of exploitation suffered by every individual.[10]

Instead, the *WP* definition captures Marx's idea that the existence of exploitation in the economy is morally relevant *per se*, and even a society with significant upward and downward social mobility is not necessarily just. Indeed, from a Marxian perspective, "we might want to consider exploitation as a property of the economy as a whole, not just of individuals" (Elster, 1985, p.176), and as a qualitative as well as a quantitative condition, so that *society* should not want anyone to be in a relationship of exploiter or exploited with respect to anyone else. Therefore, although both criteria convey normatively relevant information, and both are discussed below, the main conclusions of this chapter focus on the *WP* definition,[11] which seems also more natural in an intertemporal setting, since it gives the opportunity to analyse the dynamics of exploitation.

Proposition 3 derives labour expended by each ν in every t.

PROPOSITION 3. *Let (p, 1) be a RS for $E(\Omega_0)$. Then* $\Lambda_t^\nu = \max\{0, [p_t b + p_t s_t^\nu - \pi_t p_t \omega_t^\nu]\}$, *all t, ν.*

Proof. Case 1: $\Lambda^\nu > 0$ for all $\xi^\nu \in O^\nu(p, 1)$. By Lemma 1, constraint (1) holds as an equality: by Proposition 1, it can be written as $\pi_t p_t A(x_t^\nu + y_t^\nu) +$

[10] A similar problem emerges in the context of intertemporal egalitarianism (see chapter 2).

[11] These arguments apply to Roemer's concept of exploitation as "a *property* of individuals or of whole economies, not primarily as a relation between individuals" (Elster, 1985, p.173). If a "relational" definition is adopted, the theoretical importance of *WP* exploitation is even clearer.

$(Lx_t^{\nu} + z_t^{\nu}) = p_t b + p_t s_t^{\nu}$, and by constraint (2), $\Lambda_t^{\nu} = p_t b + p_t s_t^{\nu} - \pi_t p_t \omega_t^{\nu} \geq 0$, all t. *Case 2*: there is a $\xi^{\nu} \in O^{\nu}(p, \mathbf{1})$ such that $\Lambda^{\nu} = 0$. Then $\Lambda_t^{\nu} = 0$ and, by Lemma 1, $p_t b + p_t s_t^{\nu} - \pi_t p_t \omega_t^{\nu} \leq 0$, all t. ∎

From Proposition 3, the following corollary is immediately derived.[12]

COROLLARY 1. *Let $(p, 1)$ be an IRS for $E(\Omega_0)$. Then* $\Lambda_t^{\nu} = \max\{0, p_t[b - \pi_t \omega_0^{\nu}]\}$, *all t, ν.*

Thus, if agents save, Roemer's theory of exploitation based on asset inequalities does not seem immediately generalisable to the dynamic context. In fact, given the linearity of MP and the optimality of $\sum_{t=0}^{T-1} s_t^{\nu} = \mathbf{0}$, all ν, an agent can be a WP_t exploiter while being WP_{t+j} exploited, $j \neq 0$, depending on the dynamic paths of savings and wealth (and thus only indirectly on ω_0^{ν}).[13] However, such changes in WP exploitation status do not necessarily convey any morally relevant information: the fact that in a non-interior RS a relatively wealthy agent might optimally work more than λb in t, in order to accumulate more assets and minimise labour in future periods, does not seem to raise serious moral concerns. This point is more evident if one notes that

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[12] From Corollary 1 it follows that $p_t b \leq 1$, all t, is a sufficient condition for $Lx_t^{\nu} + z_t^{\nu} \leq 1$, for all ν, t, to hold in an IRS. It is also necessary if there are agents with $\omega_0^{\nu} = \mathbf{0}$.

[13] Consider, as an illustration, a two-period economy. Take agent ν with $W_0^{\nu} = (p_0 b - \lambda b)/\pi_0$, who is exploitation-neutral in the static model: by Proposition 3 $\Lambda_0^{\nu} = p_0 s_0^{\nu} + \lambda b$ and $\Lambda_1^{\nu} = p_1 b + p_1 s_1^{\nu} - \pi_1 p_1 \omega_1^{\nu}$. Then, since $s_0 = - s_1$ in the optimum, in a non-interior RS ν's WP exploitation status may change in the two periods. Indeed, there is a continuum of wealth values around $W_0^{\nu} = (p_0 b - \lambda b)/\pi_0$, such that an agent's WP exploitation status may change.

by Proposition 2.(*ii*), if $s_t \neq \mathbf{0}$ then $\Lambda_t \neq ASNLT_t$ and there is no conceptual equivalence between *WP* exploitative and inegalitarian solutions: only in an IRS, if an agent works less than λb, there must be another agent working more than λb.[14]

As concerns interior RS's, the next proposition proves a necessary condition for $s_t^{\nu} = \mathbf{0}$, to be optimal, for all ν and t.

PROPOSITION 4. *Let (p, 1) be an IRS for* $E(\Omega_0)$. *Then* $p_t = \beta(1 + \pi_{t+1})p_{t+1}$, *all t.*

Proof. By Proposition 2, consider ν such that $\Lambda^{\nu} > 0$ for all $\xi^{\nu} \in O^{\nu}(p, \mathbf{1})$, and take j such that $\Lambda_{j+1}^{\nu} > 0$. By Corollary 1, $\Lambda_j^{\nu} = p_j b - \pi_j p_j \omega_0^{\nu} \geq 0$ and $\Lambda_j^{\nu} + \beta\Lambda_{j+1}^{\nu} = p_j b + \beta p_{j+1} b - \pi_j p_j \omega_0^{\nu} - \beta\pi_{j+1} p_{j+1}\omega_0^{\nu}$. A necessary condition for $\omega_t^{\nu} = \omega_0^{\nu}$, all t, to be optimal is that there exists no ξ'^{ν} such that $\omega_{j+1}'^{\nu} \neq \omega_0^{\nu}$, $0 \leq j \leq T - 2$, $\omega_t'^{\nu} = \omega_0^{\nu}$, all $t \neq j + 1$, and $\Lambda_j'^{\nu} + \beta\Lambda_{j+1}'^{\nu} < \Lambda_j^{\nu} + \beta\Lambda_{j+1}^{\nu}$. In particular, consider a one-period perturbation $(s_j'^{\nu}, s_{j+1}'^{\nu})$ of the putatively optimal ω^{ν} such that $\omega_{j+1}'^{\nu} = \omega_0^{\nu} + s_j'^{\nu}$, $\omega_{j+2}'^{\nu} = \omega_0^{\nu} = \omega_{j+1}'^{\nu} + s_{j+1}'^{\nu}$, and thus $s_j'^{\nu} = - s_{j+1}'^{\nu}$. Suppose $p_{ij} < \beta(1 + \pi_{j+1})p_{ij+1}$, for some sector i. By Proposition 1:

$$\Lambda_j'^{\nu} + \beta\Lambda_{j+1}'^{\nu} = (p_j + \beta p_{j+1})b + p_j s_j'^{\nu} - \pi_j p_j \omega_j^{\nu} - \beta\pi_{j+1} p_{j+1}\omega_{j+1}'^{\nu} + \beta p_{j+1} s_{j+1}'^{\nu}.$$

or, $\Lambda_j'^{\nu} + \beta\Lambda_{j+1}'^{\nu} = \Lambda_j^{\nu} + \beta\Lambda_{j+1}^{\nu} + [p_j - \beta(1 + \pi_{j+1})p_{j+1}]s_j'^{\nu}$. Hence $s_{ij}'^{\nu} > 0$ yields $\Lambda_j'^{\nu} + \beta\Lambda_{j+1}'^{\nu} < \Lambda_j^{\nu} + \beta\Lambda_{j+1}^{\nu}$. And likewise if $p_{ij} > \beta(1 + \pi_{j+1})p_{ij+1}$. ■

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[14] This argument does not apply to the *WL* definition of exploitation: the existence of a general monotonic relationship between initial wealth and *WL* exploitation at a RS where agents save is an interesting issue for further research.

In other words, if $p_{it} < \beta(1 + \pi_{t+1})p_{it+1}$, some i, then by setting $s_{it}^{\nu} > 0$ and $s_{it+1}^{\nu} = - s_{it}^{\nu}$, Λ_t^{ν} increases by $p_{it}s_{it}^{\nu}$, but Λ_{t+1}^{ν} decreases by a larger amount given by $\pi_{t+1}p_{it+1}s_{it}^{\nu}$, due to the possibility of hiring more people, and by $p_{it+1}s_{it+1}^{\nu}$, due to the decumulation of the additional resources. The opposite holds if $p_{it} > \beta(1 + \pi_{t+1})p_{it+1}$ and in general, if $p_t \neq \beta(1 + \pi_{t+1})p_{t+1}$ it is not optimal to set $s_t^{\nu} = \mathbf{0}$.

Let $W_t^* \equiv (p_t b - \lambda b)/\pi_t$: by Corollary 1, it is immediate to verify that at an IRS, W_t^* is the level of wealth at t, associated with a working time of $\Lambda_t^{\nu} = \lambda b$ for its possessor. The next result proves that at an IRS, the *WL* and *WP* definitions of exploitation are equivalent and it extends Roemer's theory of exploitation based on the agents' initial wealth W_0^{ν} to the dynamic context. This suggests that the static model can be interpreted as a special case of the intertemporal model under the *assumption* that $s_t^{\nu} = \mathbf{0}$, all ν.

PROPOSITION 5. *Let (p, 1) be an IRS for $E(\Omega_0)$ with $\pi_0 > 0$. Then $\Delta^{\nu} > 0$ and $\Lambda_t^{\nu} > \lambda b$, all t, if and only if $W_0^{\nu} < W_0^*$; $\Delta^{\nu} = 0$ and $\Lambda_t^{\nu} = \lambda b$, all t, if and only if $W_0^{\nu} = W_0^*$; and $\Delta^{\nu} < 0$ and $\Lambda_t^{\nu} < \lambda b$, all t, if and only if $W_0^{\nu} > W_0^*$.*

Proof. 1. For all $t \geq 0$, $W_t^{\nu} = W_t^*$ is equivalent to $\pi_t W_t^{\nu} = [p_t(I - A) - L](I - A)^{-1}b$, or by Proposition 1, to $p_t\omega_0^{\nu} = p_t A(I - A)^{-1}b$. By Proposition 4, the latter expression implies $p_{t+1}\omega_0^{\nu} = p_{t+1}A(I - A)^{-1}b$, and therefore $W_t^{\nu} = W_t^*$ implies $W_{t+1}^{\nu} = W_{t+1}^*$, for all t. Similarly, by Proposition 4, $W_t^{\nu} > W_t^{\mu}$ implies $W_{t+1}^{\nu} > W_{t+1}^{\mu}$, for any ν, μ, and all $t \geq 0$.

2. By Corollary 1 and the strict monotonicity of $p_t[b - \pi_t\omega_0^{\nu}]$ in wealth, it follows that, for all $t \geq 0$, $\Lambda_t^{\nu} > \lambda b$ if and only if $W_t^{\nu} < W_t^*$, $\Lambda_t^{\nu} = \lambda b$ if

and only if $W_t^v = W_t^*$, and $\Lambda_t^v < \lambda b$ if and only if $W_t^v > W_t^*$. Hence, by part 1 $\Lambda_0^v > \lambda b$ implies $\Lambda_t^v > \lambda b$, all $t > 0$, and thus $\Delta^v > 0$. Conversely, if $\Delta^v > 0$, it must be $\Lambda_t^v > \lambda b$, for at least some $t \geq 0$. However, as just shown, *WP* exploitation status cannot change over time, and thus $\Lambda_t^v > \lambda b$, all $t \geq 0$. The other two cases are proved similarly.∎

4.4. CLASSES

Let $X^v \equiv \sum_{t=0}^{T-1} x_t^v$, $Y^v \equiv \sum_{t=0}^{T-1} y_t^v$, $Z^v \equiv \sum_{t=0}^{T-1} z_t^v$; let $\Gamma^v \equiv \{(X^v, Y^v, Z^v) | \xi^v \in O^v(p, w)\}$ and $\Gamma_t^v \equiv \{(x_t^v, y_t^v, z_t^v) | \xi^v \in O^v(p, w)\}$; and let (a_1, a_2, a_3) be a vector where $a_i = \{+, \mathbf{0}\}$, $i = 1, 2$, and $a_3 = \{+, 0\}$, and "+" means a non-zero vector in the appropriate place. Since agents live for more than one period, there are two possible dynamic extensions of Roemer's definition of class.

DEFINITION 5. Let (p, w) be a RS for $E(\Omega_0)$. Agent v is said to be a member of *WP class* (a_1, a_2, a_3) in t, if there is a $\xi^v \in O^v(p, w)$ such that (x_t^v, y_t^v, z_t^v) has the form (a_1, a_2, a_3) in t. Similarly, agent v is said to be a member of *WL class* (a_1, a_2, a_3), if there is a $\xi^v \in O^v(p, w)$ such that (X^v, Y^v, Z^v) has the form (a_1, a_2, a_3).

Although there are seven possible classes (a_1, a_2, a_3) for each definition, the theoretical relevance of classes $(+, +, +)$ and $(\mathbf{0}, +, +)$ is rather unclear from a Marxian viewpoint. Instead, a more specific definition of the remaining five classes can be provided. According to the *WL* definition:

$C^1 = \{ v \,|\, \Gamma^v$ contains a solution $(\mathbf{0}, +, 0)\}$,

$C^2 = \{ v \,|\, \Gamma^v$ contains a solution $(+, +, 0)$, but not one of form $(+, \mathbf{0}, 0)\}$,

$C^3 = \{\, \nu \mid \Gamma^\nu$ contains a solution $(+, \mathbf{0}, 0)\}$,

$C^4 = \{\, \nu \mid \Gamma^\nu$ contains a solution $(+, \mathbf{0}, +)$, but not one of form $(+, \mathbf{0}, 0)\}$,

$C^5 = \{\, \nu \mid \Gamma^\nu$ contains a solution $(\mathbf{0}, \mathbf{0}, +)\}$.

WP classes C_t^1 to C_t^5 are specified similarly, by replacing Γ^ν with Γ_t^ν. As in Roemer (1982A, 1988A), agents belonging to classes C^1 to C^5 (C_t^1 to C_t^5) are defined, respectively, *WL* (*WP*) *big capitalists*, *small capitalists*, *petty bourgeois*, *semiproletarians*, and *proletarians*. As a first step in the analysis of classes, Lemmas 4 and 5 extend a result of the static model.

LEMMA 4. *Let (p, 1) be a RS for $E(\Omega_0)$. Let $(x_t^\nu, y_t^\nu, z_t^\nu) \in \Gamma_t^\nu$ be such that ν is a WP member of* $(+, +, +)$ *or* $(\mathbf{0}, +, +)$ *in t: if $Ly_t^\nu > z_t^\nu$ then $\nu \in (+, +, 0)$, in t; if $Ly_t^\nu = z_t^\nu$ then $\nu \in (+, \mathbf{0}, 0)$, in t; and if $Ly_t^\nu < z_t^\nu$ then $\nu \in (+, \mathbf{0}, +)$, in t.*

Proof. Let ν's solution be ξ^ν with $(x_t^\nu, y_t^\nu, z_t^\nu, s_t^\nu)$ in t. The rest of the proof is as in Roemer (1982A, *Lemma 2.4*, p.75), for every t, given s^ν. ∎

In other words, every *WP* member of $(+, +, +)$ or $(\mathbf{0}, +, +)$ in t is also a *WP* member of either $(+, \mathbf{0}, +)$, or $(+, \mathbf{0}, 0)$, or $(+, +, 0)$, in t. Therefore C_t^1 to C_t^5 are sufficient to fully describe the *WP* class structure of the economy in t.

LEMMA 5. *Let (p, 1) be a RS for $E(\Omega_0)$. Then (i) if ν is a WL member of* $(+, +, 0)$ *then ν is a WP member of either* $(+, +, 0)$, *or* $(+, \mathbf{0}, 0)$, *or* $(\mathbf{0}, +, 0)$, *all t; (ii) if ν is a WL member of* $(+, \mathbf{0}, +)$ *then ν is a WP member of* $(+, \mathbf{0}, +)$ *or* $(+, \mathbf{0}, 0)$ *or* $(\mathbf{0}, \mathbf{0}, +)$, *all t; (iii) $\nu \in C^1$ if and only if $\nu \in C_t^1$, all t, $\nu \in C^3$ if and only if $\nu \in C_t^3$, all t, and $\nu \in C^5$ if and only if $\nu \in C_t^5$, all t.*

Proof. Straightforward, given Lemma 4, $x_t^\nu, y_t^\nu \geq \mathbf{0}$, and $z_t^\nu \geq 0$. ∎

Lemmas 4 and 5 highlight some limitations of Roemer's definition of classes based on the net amount of labour performed by an agent. Consider, for instance, ν such that $\Lambda^{\nu} > 0$. By Lemma 1 and rearranging ν's net revenues constraint in t, it is not difficult to show, as in Section 4.3, that, given the linearity of MP and the optimality of $\sum_{t=0}^{T-1} s_t^{\nu} = \mathbf{0}$, in a non-interior RS, the sign of $z_t^{\nu} - Ly_t^{\nu}$ and thus, by Lemma 4, *WP* class status can change over time, depending on dynamic paths of the price vector, optimal savings, and wealth. However, again, such change in *WP* class status does not necessarily reflect genuine inter-class mobility and may simply be the product of intertemporal labour trade-offs with little normative content.

As for the *WL* criterion, Lemmas 4 and 5 imply that in general the five *WL* classes are not exhaustive: agents whose *WP* class status switches, e.g., from C_t^2 to C^4_{t+j}, $j > 0$, do not belong to any of C^1 to C^5 and form instead a *WL* class whose members have a solution (+, +, +), or (**0**, +, +) in Γ^{ν}. However, as already noted, the interpretation of the latter classes (a potentially large portion of the society) in Marxian terms is unclear, raising doubts on the *WL* criterion and, *a fortiori*, on Roemer's definition of classes based on the net amount of labour performed.

Moreover, due to the crucial role of savings, there is no general relation between *WP* and *WL* classes and initial wealth. Thus, for instance, $\omega_0^{\nu} = \mathbf{0}$ does not necessarily imply $x_t^{\nu} = y_t^{\nu} = \mathbf{0}$, all t: agents remain *WP* proletarians after $t = 0$ only if $p_t \geq \beta p_{t+1}(1 + \pi_{t+1})$.[15] If time is added, the element of lack of freedom (intended as a severely limited set of available options) that is important in Marx's definition of a proletarian is lost. While in the static economy this element is incorporated by the initial conditions, in the intertemporal setting, ω_0^ν represents a much weaker constraint on the agents' sets of options and ω_t^ν is a choice variable.

As concerns IRS's, Lemma 6 proves that the set of optimal activity levels $x_t^\nu + y_t^\nu$ does not change over time, for all ν who work at the optimum.

LEMMA 6. *Let* $A_t^\nu(p, w) \equiv \{(x_t^\nu + y_t^\nu) \in \Re^n \mid \xi^\nu \in O^\nu(p, w)\}$. *Let* $(p, 1)$ *be an IRS for* $E(\Omega_0)$. *If* $\Lambda^\nu > 0$ *for all* $\xi^\nu \in O^\nu(p, 1)$, *then* $A_t^\nu(p, 1) = A_{t+1}^\nu(p, 1)$, *all* t.

Proof. Consider constraint (2) in t and $t + 1$. By Proposition 4, the set of feasible $x_j^\nu + y_j^\nu$ is identical in $j = t, t + 1$. Moreover, since $\pi_{it} = \pi_t$, all i, t, every vector $x_t^\nu + y_t^\nu$ that exhausts W_t^ν is part of an optimal solution, and the set of such vectors is identical in $j = t, t + 1$. ■

Given Lemma 6, Proposition 6 can be derived, which generalises Roemer's theory of classes to the dynamic context: at an IRS, *WL* and *WP* class structures coincide, both *WP* classes C_t^1-C_t^5 and *WL* classes C^1-C^5 are pairwise disjoint and exhaustive, and the correspondence between class and exploitation status (based on the agents' initial wealth) holds for both the *WL* and the *WP* definitions of classes and exploitation.

[15] Actually, they may be unable to save if $p_t b = 1$, i.e. if the real wage is set at the subsistence level (this is a common interpretation of Marx's theory of real wage determination).

PROPOSITION 6. *Let (p, 1) be an IRS for $E(\Omega_0)$ with $\pi_0 > 0$. Then (i) for all $1 \leq i < j \leq 5$, $C_t^i \cap C_t^j = \{\varnothing\}$ and if $\nu \in C_t^i$ and $\mu \in C_t^j$, then $\Lambda_t^\mu > \Lambda_t^\nu$, all t; (ii) For all j, if $\nu \in C_0^j$ then $\nu \in C_t^j$, all t, and $\nu \in C^j$; conversely, if $\nu \in C^j$ then $\nu \in C_t^j$ all t; (iii) (Class-Exploitation Correspondence Principle) If $\nu \in C_0^1 \cup C_0^2$, and thus $\nu \in C^1 \cup C^2$, then $\Delta^\nu < 0$ and $\Lambda_t^\nu < \lambda b$, all t while if $\nu \in C_0^4 \cup C_0^5$, and thus $\nu \in C^4 \cup C^5$, then $\Delta^\nu > 0$ and $\Lambda_t^\nu > \lambda b$, all t.*

Proof. Part (i). As in Roemer (1982A, Theorem 2.5, p.74). In particular, in every t: if $Ly_t^\nu > z_t^\nu$ all $(x_t^\nu, y_t^\nu, z_t^\nu) \in \Gamma_t^\nu$, then $\nu \in C_t^2$; if there is a $(x_t^\nu, y_t^\nu, z_t^\nu) \in \Gamma_t^\nu$ such that $Ly_t^\nu = z_t^\nu$, then $\nu \in C_t^3$; if $Ly_t^\nu < z_t^\nu$ all $(x_t^\nu, y_t^\nu, z_t^\nu) \in \Gamma_t^\nu$, then $\nu \in C_t^4$. Exactly one of these holds for every $\nu \notin C_t^1 \cup C_t^5$.

Part (ii). First, at an IRS, if $\nu \in C_t^5$ then $\nu \in C_{t+1}^5$, all t, and therefore $\nu \in C^5$. Conversely, $\nu \in C^5$ implies $\nu \in C_t^5$, all t. Next, if $p_0\omega_0^\nu > (p_0b)/\pi_0$ but $p_t\omega_0^\nu < (p_tb)/\pi_t$, some t, clearly $s_0^\nu = \mathbf{0}$ cannot be optimal. Hence, if $\nu \in C_0^1$ then $\nu \in C_t^1$, all t, and $\nu \in C^1$. Conversely, $\nu \in C^1$ implies $\nu \in C_t^1$, all t. Finally, consider $\nu \in C_t^j$, j = 2, 3, 4. By Lemma 1 and Proposition 4, in any two adjacent periods:

$z_t^\nu - Ly_t^\nu = p_t[b - (I - A)(x_t^\nu + y_t^\nu)]$,

$z_{t+1}^\nu - Ly_{t+1}^\nu = p_t\,[b - (I - A)(x_{t+1}^\nu + y_{t+1}^\nu)]/\beta(1 + \pi_{t+1})$.

By Lemma 6, if $Ly_t^\nu > z_t^\nu$ for all $(x_t^\nu, y_t^\nu, z_t^\nu) \in \Gamma_t^\nu$ then $Ly_{t+1}^\nu > z_{t+1}^\nu$ for all $(x_{t+1}^\nu, y_{t+1}^\nu, z_{t+1}^\nu) \in \Gamma_{t+1}^\nu$. Similar arguments hold if $Ly_t^\nu < z_t^\nu$ for all $(x_t^\nu, y_t^\nu, z_t^\nu) \in \Gamma_t^\nu$, or if there is a $(x_t^\nu, y_t^\nu, z_t^\nu) \in \Gamma_t^\nu$ such that $Ly_t^\nu = z_t^\nu$. Hence, by part (*i*), $\nu \in C_t^j$ implies $\nu \in C_{t+1}^j$, all t, and thus $\nu \in C^j$.

Conversely, suppose that $\nu \in C^j$: since $\nu \in C_t^j$ implies $\nu \in C_{t+1}^j$, all t, and by Lemma 5, part (*ii*) is proved.

Part (*iii*). By Proposition 5, $\Lambda_t^\nu = \lambda b$, all t, and $\Delta^\nu = 0$ if and only if $W_0^\nu = W_0^*$, while if $W_0^\nu = W_0^*$, by setting $y_0^\nu = (I - A)^{-1}b$, it is easy to verify that Γ_0^ν contains a solution with $Ly_0^\nu = z_0^\nu$ so that by part (*i*), $\nu \in C_0^3$. Next, since by part (*i*) Λ_t^ν is monotonic in WP_t class status, if $\nu \in C_0^1 \cup C_0^2$ then $\Lambda_0^\nu < \lambda b$, while if $\nu \in C_0^4 \cup C_0^5$ then $\Lambda_0^\nu > \lambda b$. Then the result follows from part (*ii*) and Proposition 5. ∎

4.5. EXPLOITATION, ASSET INEQUALITY, AND TIME

Given Propositions 2, 5, and 6, it is natural to focus on IRS's in order to analyse the links between exploitation, class, and wealth in the intertemporal context. The next results characterise the conditions under which Roemer's (1982A, 1988A) theory of exploitation can be extended to the intertemporal context, and at the same time highlight the conceptual links and differences between his definition of exploitation and neoclassical welfare inequalities.

Let $1/(1 + \tilde{\pi})$ be the Frobenius eigenvalue of A: by Assumption 1 and the productivity of A, $\tilde{\pi} > 0$.

THEOREM 1. *Assume $1 > \beta > 1/(1 + \tilde{\pi})$. Let $\pi' = (1 - \beta)/\beta$ and let p' denote the associated price vector. If $p_t = p'$, all t, and $p'b \leq 1$ then for all ν, $s_t^\nu = \mathbf{0}$ all t, is optimal and if T is finite, then $V(\omega_0^\nu) = \max\{0, (1 - \beta^T)[p'b\beta/(1 - \beta) - W_0^\nu]/\beta\}$, while if $T \to \infty$, then $V(\omega_0^\nu) = \max\{0, p'b/(1 - \beta) - W_0^\nu/\beta\}$.*

Proof. 1. Suppose $W_0^\nu \geq p'b\beta/(1 - \beta)$. The vector ξ^ν such that $s_t^\nu = \mathbf{0}$, all t, and $y_t = y'$ all t, with $\pi'p'Ay' = p'b$ is optimal and $\Lambda_t^\nu = 0$, all t.

2. Suppose $W_0^\nu < p'b\beta/(1 - \beta)$, so that $\Lambda^\nu > 0$ for all $\xi^\nu \in O^\nu(p, \mathbf{1})$. First, let us write MP using recursive dynamic optimisation theory. Let $W \subseteq \Re_+^n$ be the state space with generic element ω. Let Ψ: $W \to W$ be the feasibility correspondence: $\Psi(\omega_t^\nu)$ describes the set of feasible values for the state next period, ω_{t+1}^ν, if the current state is ω_t^ν. Thus, $\Psi(\omega_t^\nu) = \{\omega_{t+1}^\nu \in W$: $\omega_{t+1}^\nu \geq \mathbf{0}$ and $p_t\omega_{t+1}^\nu \leq 1 - p_tb + p_t\omega_t^\nu + \pi_t p_t\omega_t^\nu\}$. Let $\Pi(\omega_0^\nu) = \{\omega^\nu$: $\omega_{t+1}^\nu \in \Psi(\omega_t^\nu)$, all t, $\omega_T^\nu \geq \omega_0^\nu$, and ω_0^ν given$\}$ be the set of feasible sequences ω^ν. Let $\Phi = \{(\omega_t^\nu, \omega_{t+1}^\nu) \in W \times W$: $\omega_{t+1}^\nu \in \Psi(\omega_t^\nu)\}$ be the graph of Ψ. The *one-period return function* F: $\Phi \to \Re_+$ at t is $F(\omega_t^\nu, \omega_{t+1}^\nu) = p_tb + p_t(\omega_{t+1}^\nu - \omega_t^\nu) - \pi_t p_t\omega_t^\nu$: Then, MP can be written as

$$\text{MP} \qquad V(\omega_0^\nu) = \min_{\omega^\nu \in \Pi(\omega_0^\nu)} \sum_{t=0}^{T-1} \beta^t[p_tb + p_t(\omega_{t+1}^\nu - \omega_t^\nu) - \pi_t p_t\omega_t^\nu].$$

If $p_tb - \pi_t p_t\omega_t^\nu \leq 1$, all t, then $\Psi(\omega_t^\nu) \neq \varnothing$, all $\omega_t^\nu \in W$. Then, since F is continuous and bounded, MP is well defined for all T.[16]

2. If $p_t = p'$, all t, then $p_tb - \pi_t p_t\omega_t^\nu \leq 1$, all t, ν, and MP becomes:

$$V(\omega_0^\nu) = \min_{\omega^\nu \in \Pi(\omega_0^\nu)} \sum_{t=0}^{T-1} \beta^t p'b + \beta^{T-1}p'\omega_T^\nu - (1 + \pi')p'\omega_0^\nu.$$

Therefore, for all T, any feasible ω^ν such that $\omega_T^\nu = \omega_0^\nu$ (or $\lim_{T\to\infty}\omega_T^\nu = \omega_0^\nu$, if $T \to \infty$) is optimal and $V(\omega_0^\nu)$ immediately follows.

3. The last part of the statement is straightforward. ∎

Given Theorem 1, the next result characterises welfare inequalities and exploitation at an IRS, if agents discount future labour.

[16] If $T \to \infty$, then appropriate terminal condition is $\lim_{T\to\infty} \omega_T^\nu \geq \omega_0^\nu$.

THEOREM 2. *Let $1 > \beta$. Let $(p, 1)$ be an IRS for $E(\Omega_0)$ with $\pi_t = (1 - \beta)/\beta$, all t. Then (i) for all ν and μ, if $W_0^\mu < p'b\beta/(1 - \beta)$ then $V(\omega_0^\nu) < V(\omega_0^\mu)$ if and only if $W_0^\nu > W_0^\mu$. Moreover, (ii) for all ν, there is a constant number k^ν such that $\Lambda_t^\nu - \lambda b = k^\nu$, all t.*

Proof. Part (i). The result follows from Theorem 1, since $V(\omega_0^\nu) = 0$ if and only if $W_0^\nu \geq p'b/\pi'$; while if $V(\omega_0^\nu) > 0$ then $V(\omega_0^\nu) - V(\omega_0^\mu) = (1 - \beta^T)[W_0^\mu - W_0^\nu]/\beta$, if T is finite, while $V(\omega_0^\nu) - V(\omega_0^\mu) = [W_0^\mu - W_0^\nu]/\beta$, if $T \to \infty$.

Part (ii). Straightforward, given Corollary 1. ■

Theorems 1 and 2 complete the intertemporal generalisation of Roemer's theory of exploitation:[17] the dynamic economy with discounting displays exactly the same pattern of *WP* and *WL* exploitation as the *T*-fold repetition of the static economy, and both *WP* and *WL* exploitation are persistent. Furthermore, unlike in the static model, the introduction of time preference clarifies - at the *WL* level - the difference between Roemer's interpretation of Marxian exploitation as an objectivist measure of inequalities – "the exploitation–welfare criterion" (Roemer, 1982A, p.75) – and subjectivist neoclassical welfare inequalities, which instead depend on β. Theorems 1 and 2 prove that the two perspectives coincide at an IRS; yet, they show that in principle they are conceptually distinct.

However, the previous results crucially depend on the assumption that $\beta < 1$. By Propositions 1 and 4, if $\beta = 1$, the only constant price vector that

[17] Together with Theorem A.1 in the appendix, which proves the existence of the IRS.

satisfies the condition in Proposition 4 is the labour values vector, leading to an egalitarian and non-exploitative – according to Roemer's definition – IRS. This is quite unsatisfactory: first, the moral relevance of pure time preference is far from being widely accepted, even in non-Marxian approaches (e.g., Sidgwick, 1907; Ramsey, 1928; Rawls, 1971), and a theory of persistent inequalities that crucially depends on time preference seems objectionable. This is particularly relevant in this model since, by Theorems 1 and 2 not only the persistence of exploitation and inequalities, but also, *ceteris paribus*, their magnitude depend on time preference. Given the positive *ceteris paribus* relation between the profit rate and inequalities-exploitation, the higher β, the lower the profit rate in the RS with constant prices, and thus the lower exploitation, *ceteris paribus*.

Second, and more important, although the above results highlight the conceptual links with the neoclassical analysis of welfare inequalities, Roemer's theory is intended to be an interpretation and generalisation of Marx's theory of exploitation. Arguably, time preference plays no essential role in the latter and thus an explanation of persistent exploitation based on exogenous time preference is far from Marx's.

Theorem 3 describes the dynamics of profits, labour performed, and the net labour supply at an IRS of an economy with no discounting.

THEOREM 3. *Let* $\beta = 1$. *Let* $(p, 1)$ *be an IRS for* $E(\Omega_0)$ *with* $\pi_0 > 0$. *Then* (*i*) $\pi_t > \pi_{t+1}$, *all t. Moreover, for all* ν *such that* $\Lambda^\nu > 0$, *all* $\xi^\nu \in O^\nu(p, 1)$, *at all t:* (*ii*) *if* $W_t^\nu < W_t^*$ *then* $\Lambda_t^\nu > \Lambda_{t+1}^\nu$, *if* $W_t^\nu = W_t^*$ *then* $\Lambda_t^\nu = \Lambda_{t+1}^\nu$, *and if* $W_t^\nu > W_t^*$ *then* $\Lambda_t^\nu < \Lambda_{t+1}^\nu$;

(iii) for all $x_t^\nu + y_t^\nu \in A_t^\nu(p, 1)$ *and all* $x_{t+1}^\nu + y_{t+1}^\nu = x_t^\nu + y_t^\nu$, *at the solution to MP: if* $z_t^\nu - Ly_t^\nu > 0$ *then* $z_t^\nu - Ly_t^\nu > z_{t+1}^\nu - Ly_{t+1}^\nu$*; if* $z_t^\nu - Ly_t^\nu = 0$ *then* $z_t^\nu - Ly_t^\nu = z_{t+1}^\nu - Ly_{t+1}^\nu$*; if* $z_t^\nu - Ly_t^\nu < 0$ *then* $z_t^\nu - Ly_t^\nu < z_{t+1}^\nu - Ly_{t+1}^\nu$.

Proof. Part (i). The result follows since by Proposition 1, p_t is a continuous, strictly increasing function of π_t, all t, while by Proposition 4, $p_t >> p_{t+1}$.

Part (ii). By Proposition 5, if $W_t^\nu = W_t^*$ then $\Lambda_t^\nu = \Lambda_{t+1}^\nu = \lambda b$, all t. By Corollary 1, $\Lambda_{t+1}^\nu - \Lambda_t^\nu = (p_{t+1} - p_t)b + (\pi_t p_t - \pi_{t+1} p_{t+1})\omega_0^\nu$, or, $\Lambda_{t+1}^\nu - \Lambda_t^\nu = (p_{t+1} - p_t)b + [\pi_t - \pi_{t+1}/(1 + \pi_{t+1})]p_t\omega_0^\nu$. Therefore the result follows from part (*i*) and the monotonicity of the right hand side of the latter expression in W_t^ν.

Part (iii). Straightforward from Lemma 6 and Proposition 4. ∎

Theorem 3 is rather counterintuitive, at least in a Marxian framework. In the equilibrium that preserves the class and exploitation structure of the competitive economy, profits (Theorem 3.(*i*)) and *WP* exploitation (Theorem 3.(*ii*)) decrease over time: *WP* exploiters work more, while *WP* exploited agents work less, even if neither accumulates. The simple possibility of saving implies a decrease in the dispersion of agents' labour times around λb, due to the decrease in profits. Similarly, given Roemer's definition of class in terms of the net amount of labour performed, and given that the result holds for every $(x_t^\nu + y_t^\nu) \in A_t^\nu(p, w)$, Theorem 3.(*iii*) can be interpreted as showing a tendential decrease in the dispersion around the *WP* middle classes, regardless of the specific intertemporal path $\{x_t^\nu + y_t^\nu\}_{t = 0,\ldots, T-1}$. The next result analyses the long-run behaviour of prices and profits, and the persistence of exploitation, classes, and asset inequalities.

THEOREM 4. *Let $T \to \infty$. Let $(p, 1)$ be an IRS for $E(\Omega_0)$ with $\pi_0 > 0$. If $\beta = 1$:*

(i) $p_t \to \lambda$ and $\pi_t \to 0$, as $t \to \infty$;

(ii) $C^1 = \{\varnothing\}$ and $C_t^1 = \{\varnothing\}$, all t. Furthermore, for all $2 \leq j \leq 5$, if $\nu \in C_0^j$ then $\nu \in C_t^j$, as $t \to \infty$;

(iii) $\Lambda_t^\nu \to \lambda b$ and $W_t^\nu \to \lambda\omega_0^\nu$, all ν, as $t \to \infty$.

Proof. Part (i). By Propositions 1 and 4, it follows that $p_{t+1} = p_tA + L$, which implies $p_t = [p_0 - L[I - A]^{-1}]A^t + L[I - A]^{-1}$ so that by Assumption 1 and the productivity of A, $p_t \to \lambda$, and by Proposition 1, $\pi_t \to 0$, as $t \to \infty$.

Part (ii). First, at an IRS if $\Lambda^\nu = 0$ then $p_t\omega_0^\nu \geq (p_tb)/\pi_t$, all t. Hence, by part (*i*), $C^1 = \{\varnothing\}$ and by Proposition 6, $C_t^1 = \{\varnothing\}$, all t. Second, if $\nu \in C_0^5$ then $\nu \in C_t^5$, as $t \to \infty$. Third, consider $\nu \in C_0^2$: if $z_0^\nu < Ly_0^\nu$ all $(x_0^\nu + y_0^\nu) \in A_0^\nu(p, 1)$, then, as $t \to \infty$, by Lemma 6, $\lim_{t\to\infty} z_t^\nu - Ly_t^\nu < 0$, all $(x_t^\nu + y_t^\nu) \in A_t^\nu(p, 1)$ and $\nu \in C_t^2$, as $t \to \infty$. A similar argument holds if $z_0^\nu > Ly_0^\nu$, all $(x_0^\nu + y_0^\nu) \in A_0^\nu(p, 1)$, or if there is a solution in Γ_0^ν such that $Ly_0^\nu = z_0^\nu$. Hence, the result follows as in Proposition 6.(*i*).

Part (iii). Straightforward, from part (*i*), part (*ii*), and Corollary 1.■

Thus, at an IRS, if $T \to \infty$, profits and *WP* exploitation decrease over time and disappear in the long run. The *WP* class structure tends to become, loosely speaking, more just, due to the decrease in the dispersion of *WP* classes around the petty bourgeois (Theorem 3.(*iii*)) and to the absence of big capitalists (Theorem 4.(*ii*)). However, there is no full convergence and wealth inequalities and classes persist. Thus, in the limit, the *Class-Exploitation Correspondence Principle*, the "most important analytical result" (Roemer, 1982A, p.15) of the subsistence economy ceases to hold within periods.

These results have several *methodological* implications. In the static models two assumptions hold: (a) incomplete markets (namely, the impossibility of intertemporal trade between agents), as in Assumption 2; and (b) the impossibility of savings, unlike in Assumption 2. By proving that it is sufficient to drop (b) to make *WP* exploitation transitory (thus suggesting a sort of paradoxical Marxist justification for laissez-faire policies), Theorem 4 shows that Roemer's models do not provide robust microfoundations to persistent exploitation and thus they may be unsuitable to formalise Marx's concept of exploitation as an inherent feature of a capitalist economy.

More generally, the results presented suggest that Roemer's claim that "the neoclassical model of a competitive economy is not a bad place for Marxists to start their study of idealized capitalism" (Roemer, 1986C, p.192) should at least be qualified. In particular, although the model does not exhaust the possibilities for modelling Marxian concepts in a Walrasian framework, it seems legitimate to say that the results raise the methodological issue of the possibility of providing robust and theoretically convincing microfoundations by means of "standard general equilibrium models" (ibid., p.193). Theorems 3 and 4 imply that Roemer's static models do not provide convincing support to this claim, since the main results depend crucially on *both* (a) and (b), which represent substantial departures from a Walrasian framework. Moreover, (b) is extremely restrictive and does not seem theoretically salient, since it is not implied by the subsistence hypothesis and it captures no feature of Marx's theory. On the contrary, in a subsistence economy, too, savings are essential to analyse the persistence of exploitation and classes and the dynamics of inter-class mobility. Theorem 4 proves that it is sufficient to drop (b), the most restrictive assumption, to make *WP* exploitation transitory, even though the economy is still far from the Walrasian benchmark, due to the significant market incompleteness incorporated in Assumption 2.

From a *substantive* viewpoint, Theorem 4 shows that *WP* exploitation tends to disappear even if wealth inequalities remain an inherent equilibrium feature of the economy and, unlike in accumulation models (Devine and Dymsky, 1991), capital scarcity, - whether defined in physical or in economic terms (e.g., as the requirement that "the total supply of productive assets is limited, relative to current demand" (Skillman, 2001, p.1, fn.1)) - persists. At a RS where no agent accumulates and capital scarcity persists, DOPA is necessary to generate exploitation, but it is not sufficient for the latter to persist. This provides a formal proof of Cohen's claim that "the asset distribution is unjust because it enables or makes possible an unjust flow" (Cohen, 1995, p.207), but it does not necessitate such flow. Thus, the persistence of inequalities in the ownership of productive assets is not a sufficient statistic of the unfairness of the labour/capital relations (and more generally, of the society) from a Marxist perspective.

Theorem 4 provides strong support to the argument that asset inequalities are "a normatively secondary (though causally primary) wrong"

(Cohen, 1995, p.199). In fact, if, as in Roemer's theory, productive assets are important only because of their role in production (e.g., no satisfaction results from ownership *per se*), in order to maintain that DOPA is normatively primary it must be proved that DOPA and exploitation are equivalent. However, even adopting Roemer's arguably narrow interpretation of Marxian exploitation as reflecting a specific kind of welfare inequalities,[18] DOPA is not necessary and sufficient to generate persistent exploitation, even if capital scarcity persists, and thus an emphasis on inequalities in the ownership of productive assets while exploitation disappears seems misplaced.

It should be noted, however, that even if $\beta = 1$, *WL* exploitation does not disappear, which may be a sufficient reason for policies aimed at removing DOPA. If the condition in Proposition 4 holds, the monotonic relationship between initial wealth and *WL* exploitation status is preserved. Thus, from a mathematical viewpoint, the model might be interpreted as providing a generalisation of Roemer's theory of exploitation under the *WL* definition.

However, this does not affect the main conclusions of the chapter. First, given the theoretical relevance of the *WP* definition discussed above, Marxian exploitation should arguably be microfounded as a persistent *WP* phenomenon. Second, the tendential disappearance of *WP* exploitation is not only disturbing *per se* for a model that aims to provide microfoundations to Marx's theory; it also implies that - due to the simple possibility of savings and as a result of the inherent mechanisms of the competitive economy, -

[18] See, e.g. Cohen (1995, chapter 8) for a broader definition and a thorough discussion.

ceteris paribus, *WL* exploitation, too, is lower in the dynamic model with agents living for T periods than in the T-fold iteration of the static model.

Let $E_s(\Omega_0)$ be the static counterpart of $E(\Omega_0)$, with the same technology, agents, subsistence vector, and asset distribution. Let p_s, and the associated π_s, be a RS for $E_s(\Omega_0)$; let $\Lambda_s^{\nu}(p_s)$ be the labour expended by ν in $E_s(\Omega_0)$ at $(p_s, 1)$. Theorem 1 (together with Theorem A.1 in Appendix A.1) can be read as identifying the value of β that makes $p_t = p_s$, all t, an IRS of the dynamic economy such that $E(\Omega_0)$ corresponds to the T-fold iteration of $E_s(\Omega_0)$, whereby $\pi_t = (1 - \beta)/\beta = \pi_s$ and $\Lambda_t^{\nu} = \Lambda_s^{\nu}(p_s)$, all t. However, if $\beta = 1$ this is no longer true: *WL* exploitation is lower in $E(\Omega_0)$ than in the T-fold iteration of $E_s(\Omega_0)$, as shown by Corollary 2, which directly follows from Theorem 3.

COROLLARY 2. *Let* $\beta = 1$. *Let* $(p, 1)$ *be an IRS for* $E(\Omega_0)$ *with* $\pi_0 = \pi_s$. *If* $W_0^{\nu} < W_0^{*}$ *then* $\Lambda_t^{\nu} \leq \Lambda_s^{\nu}(p_s)$, *all t; if* $W_0^{\nu} = W_0^{*}$ *then* $\Lambda_t^{\nu} = \Lambda_s^{\nu}(p_s)$, *all t; if* $W_t^{\nu} > W_t^{*}$ *then* $\Lambda_t^{\nu} \geq \Lambda_s^{\nu}(p_s)$, *all t. If* $\pi_s > 0$, *the inequalities are strict for all* $t > 0$.

Finally, it is worth noting that the model is still far from the Walrasian benchmark and, e.g., the introduction of intertemporal credit markets would likely strengthen the results. Skillman (1995, 2001) suggests that a Walrasian model including exogenous growth in the labour force, heterogeneous saving preferences (e.g. different time preferences), and/or labour-saving technical progress might provide microfoundations to persistent exploitation. Although this is an interesting line for further research, the main conclusions of this chapter would not change. It would remain true that DOPA and competitive markets are not sufficient to yield persistent *WP* exploitation, while arguably, as noted by Roemer himself (e.g., Roemer, 1988A, p.6), an explanation of persistent exploitation critically relying on such exogenous factors (whose analytical and theoretical relevance in a subsistence economy is not obvious) would be hardly distinguishable from the neoclassical account of inequalities, and its normative relevance in a Marxian perspective would be unclear.

4.6. CONCLUSION

In this chapter an intertemporal model of a subsistence economy is set up to analyse exploitation and class formation in a dynamic context, to evaluate the causal and moral relevance of *Differential Ownership of (Scarce) Productive Assets*, and to assess the possibility of providing neoclassical microfoundations to Marxian models. It is proved that if agents save in equilibrium, Roemer's (1982A, 1986A, 1988A) definitions of exploitation and class do not necessarily convey morally relevant information, and there is no clear-cut relation between agents' initial wealth and their class and exploitation status.

In the equilibrium in which agents do not save, Roemer's theory of exploitation and classes can be extended to the intertemporal setting and in an economy with positive time preference, exploitation and classes are proved to be persistent. However, the normative and theoretical relevance of time preference is questioned and, absent time preference, it is proved that while asset inequalities and classes are persistent features of the economy, *WP* exploitation decreases over time and disappears in the long run. Hence, asset inequalities are proved to be normatively secondary, though causally primary in explaining exploitation and the normative relevance of asset inequalities *per se* is put into question. Moreover, Roemer's definition of class based on the net amount of labour performed is questioned, and several doubts are raised on the possibility of providing robust microfoundations to Marx's concepts by means of Walrasian models.

These results suggest two main lines for further research. One concerns the appropriate interpretation of exploitation and in particular the choice between a *surplus value* definition and Roemer's *property rights* definition. "The legitimacy of Roemer's reformulation depends in large part on the validity of his claims concerning the role of DOPA in capitalist exploitation" (Skillman, 1995, p.311). However, since DOPA is proved to be necessary but not sufficient to generate persistent Marxian exploitation, even if no agent accumulates, Roemer's game-theoretic definition should be seen as incorporating a different moral concern, rather than as a generalisation of Marx's definition based on surplus value. More generally, the question arises whether DOPA should be a basic moral concern, both in itself and in a theory of exploitation, or rather a different role of DOPA should be stressed as a causally primary, but normatively secondary wrong.

Secondly, given the limitations general equilibrium models, it might be opportune to explore alternative approaches to model exploitation and classes in a Marxian perspective. The above analysis suggests that the *property rights theory of the firm* (Grossman and Hart, 1986; Hart and Moore, 1990) may provide a promising analytical and theoretical framework to analyse Marxian exploitation, given its concern with power and the emphasis on the role of physical assets in explaining hierarchical relations and the existence of firms. Thus, although the dynamic implications of these models are yet to be fully explored, especially as concerns capital accumulation, the property rights approach might provide a framework to model exploitation consistent with the idea that asset inequalities are causally primary, but normatively secondary, in the explanation of exploitative relations and low wages, given various sources of contractual incompleteness (e.g., Marx's labour/labour-power distinction).

APPENDIX 4.1: EXISTENCE OF A REPRODUCIBLE SOLUTION

Due to the linearity of MP, it is difficult to derive general conditions for the existence of a RS if $p_t \neq \beta(1 + \pi_{t+1})p_{t+1}$, some t, since all agents (except, possibly, big capitalists and proletarians) tend to change their labour supply in the same direction, so that the existence of a RS may depend on *ad hoc* restrictions on Ω_0. This appendix proves the existence of RS's with $p_t = \beta(1 + \pi_{t+1})p_{t+1}$, all t. Although only IRS's are considered, given their theoretical relevance, similar arguments hold for RS's with $s_t^\nu \neq \mathbf{0}$, some t, ν.

With regard to IRS's, let $\omega^* \equiv A(I - A)^{-1}Nb$: ω^* is the minimum aggregate amount of initial endowments necessary for a RS to exist. In the static subsistence economy, if $\omega_0 = \omega^*$; a continuum of equilibria exist, while if $\omega_0 > \omega^*$ only isolated solutions emerge (Roemer, 1982A, Corollary 3.8 and Theorem 3.9, pp.100-1). Roemer argues that the former is a singular case arising from a particular combination of parameter values. However, in the intertemporal context, $\omega_0 = \omega^*$ is arguably the theoretically relevant case: if the RS is interpreted as a steady state, as in Roemer (1982A, 1988A), then given the subsistence assumption, it is natural to assume that total capital in the economy has converged to the amount just necessary for reproduction. Furthermore, the assumption $\omega_0 = \omega^*$ incorporates the strongest form of capital scarcity. Hence, although Theorem A.1 can be extended to the case $\omega_0 > \omega^*$, if $\beta < 1$, the existence of an IRS is here analysed assuming $\omega_0 = \omega^*$.

Let $1/(1 + \tilde{\pi})$ be the Frobenius eigenvalue of A. Lemma A.1 states that if parts (*i*), (*ii*), and (*v*) of Definition 1 are satisfied, so are parts (*iii*) and (*iv*).

LEMMA A.1: *Let* $\omega_0 = \omega^*$. *Let* $(p, 1)$ *be such that* $\pi_t \in D = [0, \tilde{\pi})$, *all t. Let* $\xi^\nu \in O^\nu(p, 1)$ *be such that* $s_t^\nu = 0$, *all t,* ν, *and* $x_t + y_t = (I - A)^{-1}Nb$, *all t. Then* $(p, 1)$ *is an IRS for* $E(\Omega_0)$.

Proof. First, $x_t + y_t = (I - A)^{-1}Nb$, all t, implies $A(x_t + y_t) = A(I - A)^{-1}Nb \equiv \omega^*$, all t, and $p_t(I - A)(x_t + y_t) = Np_tb$, all t. Moreover, given $\xi^\nu \in O^\nu(p, 1)$, all ν, by summing the agents' constraints (1), it follows that $z_t - Ly_t = 0$. ■

Lemma A.2 identifies a relevant interval of profit rate values.

LEMMA A.2: *Let* $\lambda b < 1$. *There is a* $\pi^m > 0$ *such that for all* $\pi \in [0, \pi^m] \subset D$, $0 \le pb - \pi p\omega_0^\nu \le 1$, *all* ν.

Proof. Let $\rho^\nu(\pi) \equiv pb - \pi p\omega_0^\nu$. By Proposition 1, $\rho^\nu(\pi)$ is a continuous function and $\rho^\nu(0) = \lambda b$, all ν. Since $0 < \lambda b < 1$, there is a largest interval $[0, \pi^m]$ such that if $\pi \in [0, \pi^m]$ then $0 \le \rho^\nu(\pi) \le 1$, all ν. ■

Let $O_s^\nu(p, \mathbf{1}) = \{\xi^\nu \ge \mathbf{0} \mid s_t^\nu = \mathbf{0}, Lx_t^\nu + z_t^\nu = p_tb - \pi_tp_t\omega_0^\nu$, and $p_tA(x_t^\nu + y_t^\nu) = p_t\omega_0^\nu$, all $t\}$. Let $\beta < 1$. By Theorem 1, if $\pi_t = \pi' = (1 - \beta)/\beta$, all t, and $0 \le p'b - \pi'p'\omega_0^\nu \le 1$, all ν, then $O_s^\nu(p, \mathbf{1}) \subseteq O^\nu(p, \mathbf{1})$, all ν. Lemma A.3 proves a similar result under the assumption $\beta = 1$.

LEMMA A.3: *Let* $\beta = 1$. *Let* $(p, 1)$ *be such that* $0 \le p_tb - \pi_tp_t\omega_0^\nu \le 1$, *all t,* ν, *and* $p_t = (1 + \pi_{t+1})p_{t+1}$, *all t. Then* $O_s^\nu(p, 1) \subseteq O^\nu(p, 1)$, *all* ν.

Proof. 1. If $\pi_t = 0$, all t, both premises of the Lemma are satisfied and the result immediately follows.

2. Let $\pi_0 > 0$. Since $0 \le p_tb - \pi_tp_t\omega_0^\nu \le 1$, all t, then $\Lambda_t^\nu = p_tb + p_ts_t^\nu - \pi_tp_t\omega_t^\nu$, and $p_tA(x_t^\nu + y_t^\nu) = p_t\omega_t^\nu$, all t. Hence, as in Theorem 1, MP can

be written as $V(\omega_0^\nu) = \min_{\omega^\nu \in \Pi(\omega_0^\nu)} \sum_{t=0}^{T-1} p_t b + p_{T-1}\omega_T^\nu - (1+\pi_0)p_0\omega_0^\nu$, and s_t^ν

$= \mathbf{0}$, all t, is indeed feasible and optimal.■

Let π^m be defined as in Lemma A.2. Theorem A.1 proves the existence of the IRS's analysed in this chapter.

THEOREM A.1: *Let $\omega_0 = \omega^*$ and $\lambda b < 1$.*

1. If $1 > \beta \geq 1/(1 + \pi^m)$, then the vector $(p, 1)$ with $\pi_t = \pi' = (1 - \beta)/\beta$, all t, is an IRS for $E(\Omega_0)$.

2. If $\beta = 1$, then for all p_0 such that $\pi_0 \in [0, \pi^m]$, the vector $(p, 1)$ determined by $p_t = (1 + \pi_{t+1})p_{t+1}$, all t, is an IRS for $E(\Omega_0)$.

Proof. Part (i). Since both premises of Lemma A.3 hold, consider $\xi^\nu \in O_s^\nu(p, \mathbf{1})$, all ν: summing over ν, $p_tA(x_t + y_t) = p_t\omega^* = p_tA(I - A)^{-1}Nb$, all t. Then consider ξ'^ν, constructed as follows: at all t, partition $(I - A)^{-1}Nb$ into $\{x'^\nu_t + y'^\nu_t\}$ such that $Lx'^\nu_t + z'^\nu_t = p_tb - \pi_t p_t\omega_0^\nu$, $x'^\nu_t + y'^\nu_t \geq \mathbf{0}$, and $p_tA(x'^\nu_t + y'^\nu_t) = p_tA(I - A)^{-1}Nb$, all ν. Since $\xi'^\nu \in O_s^\nu(p, \mathbf{1})$ and $x'_t + y'_t = (I - A)^{-1}Nb$, all t, the result follows from Lemma A.1.

Part (ii). Consider any p_0 such that $\pi_0 \in [0, \pi^m]$. If $\pi_0 \in [0, \pi^m]$ and $p_t = p_{t+1}(1 + \pi_{t+1})$ then by Proposition 1 and Theorem 3.(*i*) it follows that $\pi_t \in [0, \pi^m]$, all t. Then, by Lemma A.3, consider $\xi^\nu \in O_s^\nu(p, \mathbf{1})$, all ν: the rest of the proof is as in part (*i*).■

APPENDIX 4.2. GAME THEORY AND EXPLOITATION

This Appendix analyses Roemer's (1982A) game-theoretical approach to exploitation. According to Roemer (1982A, pp.194-195), a coalition J which is part of a larger society N is exploited if and only if three conditions hold: (1) there is an hypothetically feasible alternative in which J would be better off than in its present situation; (2) under this alternative, the complement to J, the coalition $N - J = J'$, would be worse than at present; (3) J' is in a relation of dominance to J.

Conditions (1)-(3) are fairly general and are meant to capture various kinds of exploitation, including Marxian exploitation, by specifying different hypothetically feasible alternatives - more precisely, different withdrawal rules. Let $\{V^1, \ldots, V^N\}$ describe the agents' payoffs at the existing allocation: in Roemer's game-theoretical framework, it is natural to consider $\{V^1, \ldots, V^N\}$ as *WL* values. Thus, for instance, at an RS for $E(\Omega_0)$, $V^1 = - V(\omega_0^1), \ldots, V^N = - V(\omega_0^N)$. Next, let $P(N)$ denote the power set of N and let K: $P(N) \to \Re_+$ be a *characteristic function* which assigns to every coalition J of agents in the economy an aggregate payoff $K(J)$ in the case it withdraws.

DEFINITION A.1. Coalition J is exploited at allocation $\{V^1, \ldots, V^N\}$ with respect to alternative K if and only if J' is in a relation of dominance to J and

$$\sum_{\nu \in J} V^\nu < K(J), \tag{A.1}$$

$$\sum_{\nu \in J'} V^\nu > K(J'). \tag{A.2}$$

Three points should be noted about Definition A.1. First, by (A.1) it implicitly requires that it be possible for J to distribute $K(J)$ to all its members so that $V^\nu < K^\nu$, all $\nu \in J$. Second, formally, there is a relation between Definition A.1 and the *core* of an economy: under fairly general conditions, the set of nonexploitative allocations coincides with the core of the game described by K (ibid., Theorem 7.1, p.198).

Third, the precise definition of exploitation depends on the specific function K chosen to identify the hypothetical alternative to the existing allocation: different functions K define different concepts of exploitation. Thus, a coalition is *feudally exploited* at a given allocation if it can improve by withdrawing from society with its own endowments and arranging production on its own. If $E(\Omega_0)$ is considered - no explicit model of a feudal economy is provided here, - feudally nonexploitative allocations coincide with the *private ownership core*, which can be formally defined as follows (ibid., pp.45-49). First of all, a coalition J is viable if it has enough assets to reproduce itself if it secedes from the parent economy.[19]

DEFINITION A.2. Let N be the set of producers. Let $J \subseteq N$ be any subset of N. Coalition J is *viable* if $\sum_{\nu \in J} \omega_0^\nu \geq JA(I-A)^{-1}b$.

A reproducible allocation is a set of (not necessarily optimal) actions of all agents in $E(\Omega_0)$, that satisfy the feasibility and reproducibility constraints.

DEFINITION A.3. A *reproducible allocation* (RA) for $E(\Omega_0)$ is a set of actions $\xi^\nu = (x^\nu, y^\nu, z^\nu, s^\nu)$, for all ν, such that

(i) $Lx_t^\nu + z_t^\nu \leq 1$, all ν, t;

[19] With a slight abuse of notation, the same symbols are used here to denote both the sets J and N and their cardinalities J and N.

(ii) $A(x_t + y_t) \leq \omega_t$, all t;

(iii) $(x_t + y_t) \geq A(x_t + y_t) + Nb + s_t$, all t;

(iv) $\omega_{t+1} = \omega_t + s_t$, all t;

(v) $\omega_T \geq \omega_0$.

Let $\{\xi^\nu\}_{\nu=1,\ldots,N}$ be a RA. A viable coalition J can block $\{\xi^\nu\}_{\nu=1,\ldots,N}$ if there is a reproducible allocation $\{\xi'^1, \ldots, \xi'^J\}$ for the smaller economy that yields higher welfare to its members.

DEFINITION A.4. A viable coalition J can *block* a RA $\{\xi^\nu\}_{\nu=1,\ldots,N}$ if there is a vector $\{\xi'^1, \ldots, \xi'^J\}$ such that

(i) $\sum_{t=0}^{T-1} \beta^t \Lambda'^\nu_t < \sum_{t=0}^{T-1} \beta^t \Lambda^\nu_t$, for all $\nu \in$ J;

(ii) $A \sum_{\nu \in J} x_t^\nu \leq \sum_{\nu \in J} \omega_t^\nu$, all t;

(iii) $(I - A) \sum_{\nu \in J} x_t^\nu = Jb + \sum_{\nu \in J} s_t^\nu$, all t;

(iv) $\sum_{\nu \in J} \omega^\nu_{t+1} = \sum_{\nu \in J} \omega^\nu_t + \sum_{\nu \in J} s^\nu_t$, all t;

(v) $\sum_{\nu \in J} \omega_T^\nu \geq \sum_{\nu \in J} \omega_0^\nu$.

The *private ownership core* of $E(\Omega_0)$ is the set of RA's which no coalition can block.

DEFINITION A.5. A RA is in the private ownership core of $E(\Omega_0)$ if and only if no coalition can block it.

The characteristic function K that defines feudal exploitation is the one associated to the private ownership core, "which defines the payoff to the coalition J as what it could achieve by cooperative arrangements on its own, availing itself of the private endowments of its members" (ibid., p.219). The next theorem proves the absence of feudal exploitation in $E(\Omega_0)$.

THEOREM A.2: *Let $\beta \leq 1$. The IRS's of $E(\Omega_0)$ lie in its private ownership core and thus display no feudal exploitation.*

Proof. 1. If $\pi_t = 0$, all t, then the result is trivial. Hence, assume $\pi_0 > 0$.

2. Suppose that there is a coalition J that can block the IRS. By Definition A.4.(*i*), no pure capitalist can be part of J; thus, by Lemmas 1 - 2 and Corollary 1, at an IRS $\pi_t p_t \omega_0^\nu = p_t b - \Lambda_t^\nu$, all t and all $\nu \in J$. Summing over $\nu \in J$ and t, $\sum_{t=0}^{T-1} \beta^t \pi_t p_t \sum_{\nu \in J} \omega_0^\nu = \sum_{t=0}^{T-1} \beta^t J p_t b - \sum_{t=0}^{T-1} \beta^t \sum_{\nu \in J} \Lambda_t^\nu$, where by Proposition 4, $\sum_{t=0}^{T-1} \beta^t \pi_t p_t \sum_{\nu \in J} \omega_0^\nu = [(1 + \pi_0)p_0 - \beta^{T-1} p_{T-1}] \sum_{\nu \in J} \omega_0^\nu$.

3. If J can block the IRS, pre-multiplying Definition A.4.(*iii*), by $\beta^t \lambda$ and summing over t, $\sum_{t=0}^{T-1} \beta^t \sum_{\nu \in J} \Lambda_t^{\prime\nu} = \sum_{t=0}^{T-1} \beta^t J \lambda b + \sum_{t=0}^{T-1} \beta^t \lambda \sum_{\nu \in J} s_t^\nu$. By Definition A.4.(*i*) and part 2: $\sum_{t=0}^{T-1} \beta^t J(p_t - \lambda)b - \sum_{t=0}^{T-1} \beta^t \lambda \sum_{\nu \in J} s_t^\nu > [(1 + \pi_0)p_0 - \beta^{T-1} p_{T-1}] \sum_{\nu \in J} \omega_0^\nu$.

4. If J can block the IRS, by Definition A.4.(*ii*)-(*iii*), $A(I - A)^{-1}(Jb + \sum_{\nu \in J} s_t^\nu) \leq \sum_{\nu \in J} \omega_t^\nu$, all t; pre-multiplying both sides of the latter expression by $\beta^t \pi_t p_t$ and using Proposition 1, $\beta^t(p_t - \lambda)Jb - \beta^t \lambda \sum_{\nu \in J} s_t^\nu \leq \beta^t \pi_t p_t \sum_{\nu \in J} \omega_t^\nu - \beta^t p_t \sum_{\nu \in J} s_t^\nu$, all t. Summing over t and using Definition A.4.(*iv*), the latter expression becomes $\sum_{t=0}^{T-1} \beta^t (p_t - \lambda)Jb -$

$$\sum_{t=0}^{T-1} \beta^t \lambda \sum_{\nu \in J} s_t^\nu \leq \sum_{t=0}^{T-1} \beta^t [(1 + \pi_t) p_t \sum_{\nu \in J} \omega_t^\nu - p_t \sum_{\nu \in J} \omega_{t+1}^\nu].$$

Then, using $\beta(1 + \pi_{t+1})p_{t+1} = p_t$, all t, $\sum_{t=0}^{T-1} \beta^t (p_t - \lambda) Jb - \sum_{t=0}^{T-1} \beta^t \lambda \sum_{\nu \in J} s_t^\nu \leq (1 + \pi_0) p_0 \sum_{\nu \in J} \omega_0^\nu - \beta^{T-1} p_{T-1} \sum_{\nu \in J} \omega_T^\nu$.

5. The latter inequality and the inequality in part 3 can both hold only if $\beta^{T-1} p_{T-1} \sum_{\nu \in J} \omega_T^\nu < \beta^{T-1} p_{T-1} \sum_{\nu \in J} \omega_0^\nu$. However, this is impossible, given $p_{T-1} >> \mathbf{0}'$ and Definition A.4.(v). ■

In the context of Roemer's interpretation of historical materialism, which predicts the progressive disappearance of various forms of exploitation (see Section 1.2.4), Theorem A.2 proves that capitalist relations of production eliminate feudal exploitation in $E(\Omega_0)$.[20] However, a different specification of K is necessary to define *capitalist* exploitation. Let $\omega_0^J \equiv (J/N)\omega_0$; ω_0^J is coalition J's per-capita share of aggregate initial assets. Clearly, given the assumptions on the technology all coalitions are viable if they withdraw with ω_0^E. Then, a coalition can communally block a RA if it can increase the welfare of its members by withdrawing with ω_0^E.

DEFINITION A.6. A coalition J can *communally block* a RA ξ^ν if there is a vector $\{\xi'^1, \ldots, \xi'^J\}$ such that

(i) $\sum_{t=0}^{T-1} \beta^t \Lambda_t'^\nu < \sum_{t=0}^{T-1} \beta^t \Lambda_t^\nu$, for all $\nu \in J$;

(ii) $A \sum_{\nu \in J} x_t^\nu \leq \sum_{\nu \in J} \omega_t^\nu$, all t;

[20] It also clarifies the neoclassical claim concerning the absence of exploitation in a competitive economy: there is no *feudal* exploitation (Roemer, 1982A, pp.205-8).

(iii) $(I - A)\sum_{\nu \in J} x_t^{\nu} = Jb + \sum_{\nu \in J} s_t^{\nu}$, all t;

(iv) $\sum_{\nu \in J} \omega_{t+1}^{\nu} = \sum_{\nu \in J} \omega_t^{\nu} + \sum_{\nu \in J} s_t^{\nu}$, all t;

(v) $\sum_{\nu \in J} \omega_T^{\nu} \geq \omega_0^{E}$.

The communal core of $E(\Omega_0)$ consists of the set of reproducible allocations which no coalition can communally block.

DEFINITION A.7. A reproducible allocation ξ^{ν} is in the *communal core* of $E(\Omega_0)$ if and only if no coalition can communally block it.

A coalition is *capitalistically exploited* if it can communally block the allocation and an allocation is *capitalist nonexploitative* if it lies in the communal core of the economy. The next theorem proves that Marxian exploitation and capitalist exploitation coincide in $E(\Omega_0)$ at an IRS.

THEOREM A.3: *Let $\beta \leq 1$. At an IRS, a coalition is WL Marxian exploited if and only if it is capitalistically exploited.*

Proof. If a coalition J is Marxian exploited, $\sum_{t=0}^{T-1} (\sum_{\nu \in J} \Lambda_t^{\nu} - J\lambda b) > 0$. But then by Proposition 5, at an IRS $\sum_{t=0}^{T-1} \beta^t(\sum_{\nu \in J} \Lambda_t^{\nu} - J\lambda b) > 0$, and J can communally block the allocation. The converse is proved similarly. ∎

CHAPTER 5. ACCUMULATION, INEQUALITY, AND EXPLOITATION

5.1. INTRODUCTION

In chapter 4, a dynamic extension of Roemer's (1982A) subsistence economy with labour-minimising agents is set up to analyse the *substantive* claim that DOPA "and competitive markets are sufficient institutions to generate an exploitation phenomenon, under the simplest possible assumptions" (Roemer, 1982A, p.43); and the *methodological* claim that robust microfoundations to Marxian economics can be provided by means of Walrasian general equilibrium models. The main results raise doubts on both claims by proving that it is sufficient to allow agents to save to make exploitation transitory, while asset inequalities persist in equilibrium.

This chapter extends the analysis of the dynamics of exploitation, inequality, and classes to economies with maximising agents and capital accumulation. Although the logical core of Roemer's theory is properly evaluated in subsistence economies, the study of accumulation economies strengthens the main methodological and substantive conclusions of chapter 4: by assuming a more simplified class structure, the tendential disappearance of exploitation, together with the persistence of DOPA and classes, is proved to be a general equilibrium feature of a larger class of economies.

More importantly, a thorough analysis of the dynamics of exploitation and inequalities in capitalist economies arguably requires a proper treatment of accumulation. Indeed, despite the lack of a formal analysis of capital scarcity, Roemer's results depend on differential ownership of *scarce* productive assets. In the accumulating economy with a Leontief technology (Roemer, 1982A, chapter 4), exploitation persists only if agents consume all net revenues: even if the economy is in equilibrium with positive profits in a period, accumulation would drive profits to zero in the next period. This knife-edge property derives "from the stark specification of the model" (ibid., p.120), but it is unclear whether labour-constrained equilibria, with profits falling to zero, are a general property of Roemer's accumulating economies. Devine and Dymski (1991) show that if the "static model is allowed to run for many periods, the accumulation of capital will eventually drive the profits to zero" (Roemer, 1992, p.150). However, as noted by Roemer, "this hardly requires a response. Constructing a model of capitalism that would reveal its essentially dynamic features is a different task from what mine was" (ibid.). The T-fold iteration of a static model is not necessarily a satisfactory way of modelling a dynamic economy with intertemporal decisions.

In this chapter, a fully specified dynamic framework with optimising agents is set up. The model generalises Roemer's (1981, 1982A) economies with profit or revenue-maximising agents, since agents face a consumption-savings trade-off, while revenue and profit maximisation and capital accumulation are the outcome of optimal choices. Thus, first, it allows us to evaluate the robustness of Roemer's substantive and methodological claims, and the role of DOSPA in generating persistent exploitation, from another perspective. Second, unlike in static and subsistence economies, the model can also be extended to include technical progress and disequilibrium in the labour market, two key issues in the analysis of the mechanisms that guarantee the persistent abundance of *labour* in a capitalist economy and, in general, of the relation between economic inequality, growth, and relative factor scarcity, a crucial and long-debated issue in economics.

The chapter is structured as follows. In Section 5.2, a dynamic economy with maximising agents is set up. A dynamic *Fundamental Marxian Theorem* is proved, which states that exploitation is synonymous with positive profits.[1] It is then shown that without technical progress, there is no equilibrium with persistent accumulation and persistent exploitation. Section 5.3 confirms the conclusions of chapter 4: if revenues are entirely consumed at all t, in equilibrium the economy displays persistent exploitation and, possibly, unemployment. However, the persistence of exploitation crucially depends on a strictly positive rate of time preference. Section 5.4 analyses equilibria with accumulation. First, labour-constrained equilibria are ruled out if agents discount the future. Next, *balanced growth paths* are characterised, in which the economy grows at a uniform rate and reaches a steady state, and again, the persistence of exploitation is proved to depend on time preference. Section 5.5 proves that labour-saving technical progress may yield persistent exploitation by ensuring persistent abundance of labour. However, this result depends on technical progress being unbounded, which suggests that further departures from the Walrasian framework, in addition to disequilibrium in

[1] See, e.g., Okishio (1963) and Morishima (1973). A microfounded version of the theorem is proved by Roemer (1981, Theorems 1.1 and 2.11). The result is interesting given its "prominent place in the modern formulation of Marxian economics" (Roemer, 1981, p.16).

the labour market, may be necessary to provide a satisfactory model of exploitation. Section 5.6 is devoted to the conclusions.

5.2. THE INTERTEMPORAL MODEL

The economy consists of a sequence of nonoverlapping generations, with $\nu = 1, \ldots, N_c$, capitalists, and $\eta = 1, \ldots, N_w$, workers, living for T periods and indexed by the date of birth kT, $k = 0, 1, 2, \ldots$ In every period t, each capitalist ν can operate any activity of the Leontief technology (A, L) described in section 4.2, satisfying:

ASSUMPTION 1: A is indecomposable and $L >> \mathbf{0}'$.

Let (p_t, w_t) denote the $1 \times (n + 1)$ price vector in t, where w_t is the nominal wage. As concerns capitalist ν: y_t^ν is the $n \times 1$ vector of activity levels that ν hires workers to operate at t; ω_{kT}^ν is the $n \times 1$ vector of perfectly storable productive endowments inherited, when born in kT, and ω_t^ν is the vector of endowments at t; s_t^ν is the $n \times 1$ vector of net savings at t; $c_t^\nu = \theta_t^\nu b$ is the $n \times 1$ consumption vector at t, where $b >> \mathbf{0}$ is a given subsistence vector. As for worker η: z_t^η is η's labour supply at t, while $\zeta_t^\eta = \chi_t^\eta b$ is η's consumption vector at t. Without significant loss of generality, it is assumed that $\theta_t^\nu \geq 0$ and $\chi_t^\eta \geq 1$, all t: this assumption incorporates the idea that capitalists are not essential and, together with the assumption that $\omega_0^\eta = \mathbf{0}$, all η, it starkly outlines class differences. Thus, as in von Neumann-Morishima models, "workers are like farm animals, and capitalists are simply the self-service stands for capital" (Morishima, 1969, p.95), whose main role is to drive accumulation.[2] As shown below, this assumption allows us to introduce disequilibrium in the labour market.

The assumption that consumption vectors move along rays reflects the theoretical focus on class-related consumption *possibilities*, rather than individual consumer *choice*.[3] Indeed, it is assumed that there is a continuous, increasing, and homogeneous of degree one function $\phi: \Re_+^n \rightarrow \Re_+$, such that $\phi(\chi_t^\eta b) = \chi_t^\eta \phi(b)$ and $\phi(\theta_t^\nu b) = \theta_t^\nu \phi(b)$ describe, respectively, worker η's and capitalist ν's consumption possibilities in t.[4]

As in chapter 4, intertemporal trade *between agents* is ruled out, consistently with the lack of a pure accumulation motive – that is, the desire to maximise capital accumulation *per se*, which is often assumed in standard Marxist models (e.g., Morishima, 1969; Roemer, 1981). However, Roemer's (1981, 1982A) static models are generalised by allowing intertemporal trade-offs *during an agent's life*.

ASSUMPTION 2: *No credit market.* Productive assets are bought with current wealth, while consumption and savings are financed with current revenue.

Let $(p, w) = \{p_t, w_t\}_{t=kT,..., (k+1)T-1}$ denote the path of the price vector during the lifetime of a generation; let $y^\nu = \{y_t^\nu\}_{t=kT,..., (k+1)T-1}$ denote ν's

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[2] In a less schematic model, if profits fall below some level, capitalists would start to work.

[3] Similar assumptions are made in Sraffian models (e.g. Kurz and Salvadori, 1995, p.102).

[4] Silvestre (2005) makes similar assumptions in the construction of an index of primary goods. Actually, given Roemer's normative interpretation of exploitation theory, the n produced goods analysed in this model can be naturally interpreted as primary goods. The function ϕ might also be interpreted as a neoclassical homothetic utility function.

lifetime plan of activity levels; let $z^{\eta} = \{z_t^{\eta}\}_{t=kT,\ldots,(k+1)T-1}$ be η's lifetime labour supply; and let the same notation hold for θ^{ν}, s^{ν}, and χ^{η}.[5] As a shorthand notation, let "all t" stand for "all t, $t = kT, \ldots, (k+1)T-1$." Let $0 < \beta \leq 1$ be the agents' subjective time preference factor.

Capitalist ν chooses $\xi^{\nu} = (y^{\nu}, \theta^{\nu}, s^{\nu})$ to maximise lifetime consumption opportunities subject to the constraint that (*i*) net revenues are sufficient for consumption and savings, all t; (*ii*) wealth is sufficient for production plans, all t; (*iii*) the evolution of productive assets is determined by net savings, all t; (*iv*) ν's descendants receive at least as many resources as she inherited. Let $C(\omega_{kT}^{\nu})$ be the value of the optimisation program. Formally, ν solves:

$$\text{MP}_{\nu} \qquad C(\omega_{kT}^{\nu}) = \max_{\xi^{\nu}} \sum_{t=kT}^{(k+1)T-1} \beta^t \phi(\theta_t^{\nu} b),$$

subject to: $\quad [p_t(I - A) - w_t L]y_t^{\nu} \geq \theta_t^{\nu} p_t b + p_t s_t^{\nu}$, all t, $\qquad$ (*i*)

$p_t A y_t^{\nu} \leq p_t \omega_t^{\nu}$, all t, $\qquad$ (*ii*)

$\omega_{t+1}^{\nu} = \omega_t^{\nu} + s_t^{\nu}$, all t, $\qquad$ (*iii*)

$\omega_{(k+1)T}^{\nu} \geq \omega_{kT}^{\nu}$, $\qquad$ (*iv*)

$y_t^{\nu}, \omega_t^{\nu} \geq \mathbf{0}$, and $\theta_t^{\nu} \geq 0$, all t. $\qquad$ (*v*)

Worker η chooses $\xi^{\eta} = (z^{\eta}, \chi^{\eta})$ to maximise lifetime consumption opportunities subject to the constraint that at all t, revenues are sufficient for η's consumption, subsistence is reached, and working time does not exceed the length of the working period (normalised to one). Formally, η solves:

[5] The index k is not included in y^{ν}, s^{ν}, etc. in order to avoid notational confusion.

$$\text{MP}_\eta \qquad C^\eta = \max_{\xi^\eta} \sum_{t=kT}^{(k+1)T-1} \beta^t \phi(\chi_t^\eta b),$$

subject to: $w_t z_t^\eta \geq \chi_t^\eta p_t b$, all t, *(i)*

$1 \geq z_t^\eta \geq 0$, and $\chi_t^\eta \geq 1$, all t. *(ii)*

Given the absence of capital markets and bequests, MP_ν is a natural dynamic generalisation of the static profit or revenue maximisation program (Roemer, 1981, 1982A). Thus, let $O^\alpha(p, w) \equiv \{\xi^\alpha \text{ solves } \text{MP}_\alpha\}$, $\alpha = \nu, \eta$, be the set of individually optimal plans for α. Let $\Omega_{kT} \equiv (\omega_{kT}^1, \omega_{kT}^2, \ldots, \omega_{kT}^{N_c})$. Let $E(A, L, N_c, N_w, b, \Omega_{kT}, \phi)$, or as a shorthand notation $E(\Omega_{kT})$, denote the economy described by technology (A, L), population (N_c, N_w), subsistence vector b, distribution of endowments Ω_{kT}, and index function ϕ. Let $y_t = \sum_{\nu=1}^{N_c} y_t^\nu$, $z_t = \sum_{\eta=1}^{N_w} z_t^\eta$, and let θ_t, s_t, ω_t, and χ_t be similarly defined.

DEFINITION 1: An *unconstrained reproducible solution (RS)* for $E(\Omega_{kT})$ is an intertemporal profile (p, w) of the price vector such that

(i) $\xi^\alpha \in O^\alpha(p, w)$, for all $\alpha = \nu, \eta$;

(ii) $y_t \geq Ay_t + \theta_t b + \chi_t b + s_t$, all t;

(iii) $Ay_t \leq \omega_t$, all t;

(iv) $Ly_t = z_t$, all t;

(v) *(Reproducibility)* $\omega_{(k+1)T} \geq \omega_{kT}$.

Condition *(i)* requires individual optimisation; *(ii)* and *(iii)* require that there are enough resources for consumption and saving plans, and for production plans, respectively, at all t; *(iv)* states that the labour market clears at all t, *(v)* states that resources should not be depleted.

Let $\hat{z}_t^\eta$ be worker η's effective labour supply at t and let $\hat{z}_t = \sum_{\eta=1}^{N_w} \hat{z}_t^\eta$.

Although Definition 1 is an important benchmark, in this chapter RS's with $\hat{z}_t^\eta \neq z_t^\eta$, some t, η, and unemployment are not excluded.[6]

DEFINITION 2: The vector (p, w) is a *constrained* RS at t' for $E(\Omega_{kT})$ if

(i) $\xi^\nu \in O^\nu(p, w)$, for all ν;

(ii) $y_t \geq Ay_t + \theta_t b + \chi_t b + s_t$, all t;

(iii) $Ay_t \leq \omega_t$, all t;

(iv) $Ly_t \leq z_t$, all t, with $z_{t'} > \hat{z}_{t'} = Ly_{t'}$, some t';

(v) *(Reproducibility)* $\omega_{(k+1)T} \geq \omega_{kT}$.

Since workers are identical, if a RS is *constrained* at t, it is assumed that they work an equal amount of time and they are all able to reach subsistence. Given the absence of a subsistence sector and of the public sector, this seems an appropriate way to capture unemployment in this model.

ASSUMPTION 3: If a RS is *constrained* at t, $\hat{z}_t^\eta = Ly_t/N_w$ and $\chi_t^\eta = 1$, all η.

Next, an interior reproducible solution (IRS) is defined.

DEFINITION 3: An IRS for $E(\Omega_{kT})$ is a RS such that $s_t^\nu = \mathbf{0}$, all ν and t.

Finally, the next definition captures the idea of capital scarcity as requiring that "the total supply of productive assets is limited, relative to current demand" (Skillman, 2000, p.1, fn.1).[7]

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[6] Since $\theta_t^\nu \geq 0$, the case with excess labour demand need not be considered.

[7] I am indebted to Gil Skillman for Definition 4.

DEFINITION 4: Let (p, w) be a RS for $E(\Omega_{kT})$. The economy $E(\Omega_{kT})$ is said to exhibit *capital scarcity* at (p, w), in period t, if $p_tAy_t^{\nu} = p_t\omega_t^{\nu}$, all ν. If $p_tAy_t^{\nu} < p_t\omega_t^{\nu}$, some ν, then capital is said to be *abundant* at (p, w), in period t.

Since at the solution to MP$_\nu$, $\omega^{\nu}_{(k+1)T} = \omega^{\nu}_{kT}$, all ν, if (p, w) is a RS for $E(\Omega_{kT})$, then it is also a RS for $E(\Omega_{(k+1)T})$. Hence, it is possible to interpret (p, w) as a steady state solution and to focus on $E(\Omega_0)$ without loss of generality.

It is immediate to show that at a RS for $E(\Omega_0)$, in every period, revenue constraints are binding for all agents, workers work and consume as much as possible, and the wealth constraints of all capitalists are binding.

LEMMA 1: *Let (p, w) be a RS for $E(\Omega_0)$. Then, for all t,*

(i) $[p_t(I - A) - w_t L]y_t^{\nu} = \theta_t^{\nu}p_tb + p_ts_t^{\nu}$, *all* ν;

(ii) $z_t^{\eta} = 1$, *all* η;

(iii) $w_t\hat{z}_t^{\eta} = \chi_t^{\eta}p_tb$, *all* η, *where* $\hat{z}_t^{\eta} = \min\,[Ly_t/N_w, 1]$;

(iv) *if* $p_t > p_tA + w_tL$, *then* $p_tAy_t^{\nu} = p_t\omega_t^{\nu}$, *all* ν.

The following result proves that at a RS, wages and prices are positive, and profits are non-negative at all t.

LEMMA 2: *Let (p, w) be a RS for $E(\Omega_0)$. Then $w_t > 0$, $p_t >> 0'$, and $p_t \geq p_tA + w_tL$, all t.*

Proof: 1. At a RS $p_t > \mathbf{0}'$ or else $y_t << Ay_t + \theta_tb + \chi_tb + s_t$. Hence, $w_t > 0$, all t, or else workers could not reach subsistence.

2. If $p_{it} = 0$, some i, t, then $y_{it}^{\nu} = 0$, all ν, for all $\xi^{\nu} \in O^{\nu}(p, w)$, and thus $y_{it} = 0$. However, $A_iy_t + \theta_tb_i + \chi_tb_i + s_{it} > 0$, contradicting feasibility.

3. If $p_{it} < p_tA_i + w_tL_i$, all i, then $y_{it}^{\nu} = 0$, all i, ν, for all $\xi^{\nu} \in O^{\nu}(p, w)$, and $y_t = \mathbf{0}$, which is not possible at a RS since workers would not reach subsistence. Then, the possibility that $p_{it} < p_tA_i + w_tL_i$, some i, t, is ruled out as in part 2 of the proof, noting that capitalists' wealth will be used only to activate maximum profit rate processes. ■

As in chapter 4, the profit rate of sector i at t is $\pi_{it} = [p_t(I - A) - w_tL]_i/p_tA_i$. Lemma 3 proves that at a RS, profit rates are equalised in all i.

LEMMA 3: *Let (p, w) be a RS for $E(\Omega_0)$. Then $\pi_{it} = \pi_t$, all i, t, and $p_t = (1 + \pi_t)p_tA + w_tL$, all t.*

Proof: By Lemma 2, $\pi_t \geq 0$, all t. Hence, if $\pi_{it} < \pi_t$, some i, t, then $y_{it}^{\nu} = 0$, for all $\xi^{\nu} \in O^{\nu}(p, w)$, all ν, and $y_{it} = 0$. Thus, since capitalists' wealth will be used only to activate maximum profit rate processes, $A_iy_t + \theta_tb_i + \chi_tb_i + s_{it} > 0$, which contradicts feasibility. The second part of the Lemma follows from the definition of the profit rate. ■

By Lemmas 2 and 3 labour can be chosen as the numeraire, setting $w_t = 1$, all t, and RS's of the form $(p, \mathbf{1})$ can be considered. Then, capitalists' consumption expenditure at all t can be derived.

PROPOSITION 1: *Let $(p, 1)$ be a RS for $E(\Omega_0)$: $\theta_t^{\nu}p_tb = \pi_tp_t\omega_t^{\nu} - p_ts_t^{\nu}$, all t, ν.*

Proof: By Lemma 3 and Lemma 1.(i), $\pi_tp_tAy_t^{\nu} = \theta_t^{\nu}p_tb + p_ts_t^{\nu}$, all ν, t. By Lemma 2, $\pi_t \geq 0$: if $\pi_t = 0$ the result follows immediately. If $\pi_t > 0$ it follows from Lemma 1.(iv). ■

Let λ be the $1 \times n$ vector of labour values. Let $y = \sum_{t=0}^{T-1} y_t$ and likewise for θ and χ. Roemer's (1981, 1982A) definitions of *Socially Necessary Labour Time* and exploitation can be extended to the intertemporal context.[8]

DEFINITION 5: *Socially Necessary Labour Time at t* is the amount of labour embodied in the worker's consumption bundle, $\lambda\chi_t b$. Similarly, considering the whole life of a generation, *Socially Necessary Labour Time* is $\lambda\chi b$.

DEFINITION 6: The *within-period (WP) exploitation rate at t* is $e_t = (Ly_t - \lambda\chi_t b)/\lambda\chi_t b$, while the *whole-life (WL) exploitation rate* is $e = (Ly - \lambda\chi b)/\lambda\chi b$.

As argued in chapter 4, both definitions convey morally relevant information, but the *WP* definition is more pertinent in a Marxian approach and it is more interesting in a dynamic context.

The *Dynamic Fundamental Marxian Theorem* can now be proved.

THEOREM 1 (*Dynamic FMT*): *Let (p, 1) be a RS for $E(\Omega_0)$. Then, (i) at all t, $e_t > 0$ if and only if $\pi_t > 0$. Furthermore, (ii) $e > 0$ if and only if $\pi_t > 0$, some t.*

Proof: *Part (i).* Consider any t. By Lemma 1.(*ii*), at a RS $Ly_t = \hat{z}_t \leq z_t = N_w$.

Thus, by Lemma 1.(*iii*), summing over η, $Ly_t = \hat{z}_t = \chi_t p_t b$. Then, by Lemmas 2 and 3, $Ly_t > \chi_t \lambda b$ if and only if $\pi_t > 0$.

Part (ii). The result follows from part (*i*), since $Ly_t - \lambda\chi_t b \geq 0$, all t. ■

Theorem 1 suggests that there is no RS with persistent accumulation and persistent exploitation. In fact, note that if $e_t > 0$, all t, by Theorem 1,

[8] For a discussion of various definitions of the exploitation rate, see Desai (1979, p.48).

Lemma 1.(*iv*), and Lemma 2, at a RS $Ly_t = LA^{-1}\omega_t$, all t. Since $z_t = N_w$, all t, then $e_t > 0$, all t, is possible only if $LA^{-1}\omega_t \leq N_w$, all t. Hence, if $\omega_{t+1} >> \omega_t$, all t, $T-1 > t \geq 0$, then $LA^{-1}\omega_t < N_w$, all t, $T-1 > t \geq 0$, which implies by (A.3) $\chi_t^{\eta} = 1$, all η, and $p_t b = LA^{-1}\omega_t/N_w$, all t, $T-1 > t \geq 0$. By Lemma 1.(*i*)-(*ii*) and Lemma 2, at a RS $(\mathrm{I}-A)y_t = s_t + \theta_t b + \chi_t b$, all t, which implies $s_t = (\mathrm{I}-A)A^{-1}\omega_t - \theta_t b - \chi_t b$, all t, or using the previous results, $\omega_{t+1} = A^{-1}\omega_t - \theta_t b - N_w b$, all t, $T-1 > t \geq 0$.

Given the linearity of MP$_\nu$, there is at most one period in which at the solution to MP$_\nu$, both savings and consumption are positive at a constrained RS with accumulation.[9] Hence, given that capitalists are identical there is a τ such that $\theta_t = 0$ all $t \geq \tau$ and $\omega_{t+1} = A^{-1}\omega_t - N_w b$, all $t \geq \tau$, which implies $\omega_t = (A^{-1})^{t-\tau}[\omega_\tau - \omega^S] + \omega^S$, all $t \geq \tau$, where $\omega^S \equiv N_w A(\mathrm{I}-A)^{-1}b$. Thus, by ($A$.1) and the productivity of A, given that workers' subsistence requires $\omega_t \geq \omega^S$, all t, if T is sufficiently big, labour demand exceeds supply after a finite number of periods, driving π_t and e_t to zero. This can be summarised as follows.

PROPOSITION 2: *For all $T \in \Re_+$, there is a $T' > T$ such that there is no RS with $\omega_{t+1} >> \omega_t$, all $T'-1 > t \geq 0$, and $e_t > 0$, all t.*

5.3. INEQUALITIES, EXPLOITATION, AND TIME PREFERENCE

This section analyses the dynamic foundations of exploitation in the economy with maximising agents, focusing on interior RS's. This is not only due to their theoretical relevance, discussed in chapter 4, but also because in

[9] This is proved rigorously below; see e.g. the analysis of MP$_\nu$ in the proof of Theorem 4.

this economy IRS's represent a benchmark solution whereby the labour market clears at all t. Let $\overline{p}_t \equiv \beta(1 + \pi_{t+1})(p_{t+1}/p_{t+1}b)$ - (p_t/p_tb). Lemma 4 provides a necessary condition for the existence of an IRS.

LEMMA 4: *Let (p, 1) be an IRS for $E(\Omega_0)$ with $\pi_t > 0$, all t. Then $\overline{p}_t = \mathbf{0}'$, all t.*

Proof: 1. Suppose that $s_t^{\nu} = \mathbf{0}$, all t, ν, but $\overline{p}_{ij} > 0$, some $i, j, j < T - 1$. By Proposition 1, if $s_j^{\nu} = s_{j+1}^{\nu} = \mathbf{0}$, then $\theta_j^{\nu} > 0$, $\theta_{j+1}^{\nu} > 0$, and $\theta_j^{\nu} + \beta\theta_{j+1}^{\nu} = [\pi_j p_j/p_j b + \beta\pi_{j+1}p_{j+1}/p_{j+1}b]\omega_0^{\nu}$.

2. Consider a one-period perturbation $s_j'^{\nu}$, $s_{j+1}'^{\nu}$ such that $\omega_{j+1}'^{\nu} = \omega_0^{\nu} + s_j'^{\nu}$; $\omega_t'^{\nu} = \omega_0^{\nu}$, all $t \neq j + 1$. In the perturbed path $\theta_j'^{\nu} + \beta\theta_{j+1}'^{\nu} = \theta_j^{\nu} + \beta\theta_{j+1}^{\nu} + \overline{p}_j s_j'^{\nu}$, and there is a sufficiently small $s_{ij}'^{\nu} > 0$ such that $\theta_j'^{\nu} \geq 0$ and $\theta_j'^{\nu} + \beta\theta_{j+1}'^{\nu} > \theta_j^{\nu} + \beta\theta_{j+1}^{\nu}$, a contradiction. A similar argument holds if $\overline{p}_{ij} < 0$, some i, j. ■

Intuitively, if $\overline{p}_{it} > 0$, some i, t, capitalists optimise by saving at t and producing good i at $t + 1$, while if $\overline{p}_t = \mathbf{0}'$ they are indifferent. This result can be extended to an economy where workers can save, forming a class of petty bourgeois, who own capital but also work for a wage, with a revenue constraint equal to $[p_t(I - A) - L]y_t^{\eta} + z_t^{\eta} = p_t\chi_t^{\eta}b + p_t s_t^{\eta}$, all t.

Let $1/(1 + \tilde{\pi})$ be the Frobenius eigenvalue of A: by (A.1) and the productivity of A, $\tilde{\pi} > 0$. Theorem 2 analyses program MP_{ν}.

THEOREM 2. (*i*) *Let $1 > \beta > 1/(1 + \tilde{\pi})$. If $\pi_t = \pi' = (1 - \beta)/\beta$, all t, then for all ν and all $\xi^{\nu} \in O^{\nu}(p', 1)$, $s_t^{\nu} = \mathbf{0}$, all t, solves MP_{ν}. Moreover, if T is finite,*

$C(\omega_0^v) = \phi(b)(1 - \beta^T)p'\omega_0^v/\beta p'b$, while if $T \to \infty$, $C(\omega_0^v) = \phi(b)p'\omega_0^v/\beta p'b$.

(ii) Let $\beta \leq 1$. If $\pi_t = 0$, all t, then for all v and all $\xi^v \in O^v(p', 1)$, $s_t^v = 0$, all t, solves MP_v, and $C(\omega_0^v) = 0$.

Proof. Part (i). 1. Write MP_v using dynamic recursive optimisation theory. Let $W \subseteq \Re_+^n$ be the state space with generic element ω. Let $\Psi: W \to W$ be the feasibility correspondence: $\Psi(\omega_t^v) = \{\omega_{t+1}^v \in W: \omega_{t+1}^v \geq \mathbf{0}$ and $p_t\omega_{t+1}^v \leq (1 + \pi_t)p_t\omega_t^v\}$. Let $\Pi(\omega_0^v) = \{\omega^v: \omega_{t+1}^v \in \Psi(\omega_t^v)$, all t, $\omega_T^v \geq \omega_0^v$, and ω_0^v given$\}$. Let $\Phi = \{(\omega_t^v, \omega_{t+1}^v) \in W \times W: \omega_{t+1}^v \in \Psi(\omega_t^v)\}$ be the graph of Ψ. The one-period return function $F: \Phi \to \Re_+$ at t is $F(\omega_t^v, \omega_{t+1}^v) = \phi(b)[(1 + \pi_t)p_t\omega_t^v - p_t\omega_{t+1}^v]/p_tb$. Then, MP_v can be written as

$$MP_v \qquad C(\omega_0^v) = \max_{\omega^v \in \Pi(\omega_0^v)} \sum_{t=0}^{T-1} \phi(b)\beta^t[(1 + \pi_t)p_t\omega_t^v - p_t\omega_{t+1}^v]/p_tb.$$

Since $\Psi(\omega_t^v) \neq \varnothing$, all $\omega_t^v \in W$, and F is continuous, concave, and bounded below by 0, the program MP_v is well defined.

2. Since $\pi' \in [0, \tilde{\pi})$, let $p' = (1 + \pi')p'A + L$. By construction, the condition in Lemma 4 is satisfied at all t, and MP_v reduces to

$$(MP_v) \qquad C(\omega_0^v) = \max_{\omega^v \in \Pi(\omega_0^v)} \phi(b)\left[\frac{(1+\pi')p'\omega_0^v}{p'b} - \beta^{T-1}\frac{p'\omega_T^v}{p'b}\right].$$

Therefore, for any T, any feasible ω^v such that $\omega_T^v = \omega_0^v$ is optimal and $C(\omega_0^v)$ follows by noting that $\beta < 1$.

Part (ii). The result follows from MP_v, given $\omega_T^v \geq \omega_0^v$. ■

Let $H = \{\omega \in \Re^n \mid \omega = \gamma N_w A(I - A)^{-1}b, \gamma \geq 1\}$; in what follows, it is assumed that $\omega_0 \in H$. This restriction is imposed mainly for analytical convenience, given the linearity of MP$_\nu$ and MP$_\eta$ and the assumptions on consumption patterns. No theoretical conclusion depends on this restriction, which in any case – given $\gamma \geq 1$ – allows us to consider a rather large set of economies. The existence of an IRS can now be proved.

THEOREM 3: *Let* $\omega_0 = \gamma_0 N_w A(I - A)^{-1}b$, $\gamma_0 \geq 1$. *Let* $\lambda b < 1$.

(i) If $\gamma_0 = 1$, *the only RS for* $E(\Omega_0)$ *requires* $\pi_t = 0$ *and* $s_t = \mathbf{0}$, *all t;*

(ii) Let $\gamma_0 > 1$ *and* $\gamma_0\lambda b < 1$. *Let* π' *be defined by* $\gamma_0\lambda b = L[I - (1 + \pi')A]^{-1}b$. *If* $\beta(1 + \pi') = 1$, *there is an IRS for* $E(\Omega_0)$ *with* $\pi_t = \pi'$, *all t;*

(iii) Let $\gamma_0 > 1$ *and* $\gamma_0\lambda b = 1$. *Let* π' *be defined by* $1 = L[I - (1 + \pi')A]^{-1}b$. *For all* $\beta \in [1/(1 + \pi'), 1)$ *there is an IRS for* $E(\Omega_0)$ *with* $\pi_t = (1 - \beta)/\beta$, *all t;*

(iv) Let $\gamma_0 > 1$ *and* $\gamma_0\lambda b \leq 1$. *If* $\beta = 1$, *there is an IRS for* $E(\Omega_0)$ *with* $\pi_t = 0$, *all t. Moreover, if* $\beta = 1$, *at any IRS there is at most one period t with* $\pi_t > 0$.

Proof. Part (i). 1. Existence. Since $\pi_t = 0$, all t, given the terminal condition $\omega_T^\nu \geq \omega_0^\nu$, any ξ^ν such that $s_t^\nu = \mathbf{0}$, $\lambda A y_t^\nu = \lambda\omega_0^\nu$, and $\theta_t^\nu = 0$, all t, is an optimal solution for all ν. Hence, assign actions $\{y_t^\nu\}$ to all ν such that $y_t = A^{-1}\omega_0$ at all t: the capital goods market clears at all t. Then $Ly_t = LA^{-1}\omega_0 = N_w\lambda b < N_w$ and $\hat{z}_t = N_w\lambda b$, all t. Then assign actions $\hat{z}_t^\eta = \lambda b$, all η, t, which implies $\chi_t^\eta = 1$, all η, t, consistently with (A.3). Finally, the market for final goods clears at all t since $(I - A)y_t = N_w b$, all t.

2. Uniqueness. Consider $t = 0$. First, since, $\hat{z}_0 = Ly_0 = N_w\lambda b$, it follows that $\pi_0 = 0$ or else $\lambda b < p_0 b$ and workers could not reach subsistence. Next, at an RS, $(I - A)y_0 \geq \theta_0 b + \chi_0 b + s_0$ and $y_0 \leq A^{-1}\omega_0$. By (A.1), by

pre-multiplying the latter expression by $(\mathrm{I} - A)$, at a RS $(\mathrm{I} - A)A^{-1}\omega_0 \geq \theta_0 b + N_w b + s_0$, or $N_w b \geq \theta_0 b + N_w b + s_0$. The latter inequality implies $\mathbf{0} \geq \theta_0 b + s_0$, which in turn implies $\mathbf{0} \geq s_0$ by the nonnegativity of θ_0. However, since $\omega = N_w A(\mathrm{I} - A)^{-1}b$ is the minimum sustainable amount of capital that guarantees workers' subsistence at all t, then $s_0 \geq \mathbf{0}$. Therefore at a RS, $s_0 = \mathbf{0}$, and the reasoning can be iterated.

Part (ii). 1. (Optimal ξ^ν.) By the Perron-Frobenius theorem π' exists and $\pi' \in (0, \tilde{\pi})$. Let p' be the price vector associated with π'. If $\beta = 1/(1 + \pi')$ and $\pi_t = \pi'$, all t, by Theorem 2, any ξ^ν such that $s_t^\nu = \mathbf{0}$, $p'Ay_t^\nu = p'\omega_0^\nu$, and $\theta_t^\nu p'b = \pi' p'\omega_0^\nu$, all t, solves MP$_\nu$, for all ν.

2. (Capital market.) Hence, by Lemma 3, at all t, it is possible to assign a vector y_t^ν to all ν such that $p'Ay_t^\nu = p'\omega_0^\nu$, all ν, and $y_t = A^{-1}\omega_0$.

3. (Labour market and constrained ξ^η.) Since $Ly_t = \gamma_0\lambda b N_w < N_w$, all t, by (A.3) assign actions $z_t^\eta = \gamma_0\lambda b$, all t, η; then by construction $\gamma_0\lambda b = p'b$, and thus $\chi_t^\eta = 1$, all t, η, and $\chi_t = N_w$, all t.

4. (Final goods market.) The goods market clears at all t since $(I - A)y_t = \gamma_0 N_w b$ while $\chi_t b = N_w b$ and $\theta_t p'b = \pi' p'\omega_0$, or $\theta_t p'b = \gamma_0 N_w[p' - \lambda]b$, which implies $\theta_t b = N_w(\gamma_0 - 1)b$.

Part (iii). 1. (Optimal ξ^ν.) By the Perron-Frobenius theorem π' exists and $\pi' \in (0, \tilde{\pi})$. Thus $\pi_\beta = (1 - \beta)/\beta \in (0, \tilde{\pi})$. Let p_β be the price vector associated with π_β. If $\pi_t = \pi_\beta$, all t, by Theorem 2, any ξ^ν such that $s_t^\nu = \mathbf{0}$, $p_\beta A y_t^\nu = p_\beta\omega_0^\nu$, and $\theta_t^\nu p_\beta b = \pi_\beta p_\beta \omega_0^\nu$, all t, solves MP$_\nu$, for all ν.

2. (Capital market.) Hence, by Lemma 3, at all t, it is possible to assign a vector y_t^ν to all ν such that $p_\beta A y_t^\nu = p_\beta \omega_0^\nu$, all ν, and $y_t = A^{-1}\omega_0$.

3. (Labour market and optimal ξ^η.) Since $Ly_t = N_w$, all t, assign actions $z_t^\eta = 1$ and $\chi_t^\eta = 1/p_\beta b$, all t, to all η, so that $\chi_t = N_w/p_\beta b$, all t. Since $\pi_\beta \in (0, \pi']$ then $1/\lambda b > \chi_t^\eta \geq 1$, all t, η. Hence, these actions satisfy the subsistence requirement and are optimal for all η, with $Ly_t = z_t$, all t.

4. (Final goods market.) By the previous arguments the goods market clears at all t since $(I - A)y_t = \gamma_0 N_w b$ while $\chi_t b = N_w b/p_\beta b$ and $\theta_t p_\beta b = \pi_\beta p_\beta \omega_0$, or $\theta_t p_\beta b = \gamma_0 N_w[p_\beta - \lambda]b$.

Part (iv). 1. If $\gamma_0\lambda b = 1$, existence is proved as in part (*iii*) with $z_t^\eta = 1$ and $\chi_t^\eta = 1/\lambda b$, all η, t. If $\gamma_0\lambda b < 1$, existence is proved as in part (*ii*) with $y_t = (1/\gamma_0)A^{-1}\omega_0$ and $Ly_t = \lambda b N_w$, all t, $z_t^\eta = \lambda b$ and $\chi_t^\eta = 1$, all η, t.

2. By Lemma 4, there can be no two adjacent periods with $\pi_t > 0$ and $\pi_{t+1} > 0$. A similar argument rules out $\pi_t > 0$ and $\pi_{t+j} > 0$, for $j > 1$. ■

Remarks: From Lemma 4, it follows that Theorem 3.(*ii*)-(*iii*) identifies the only IRS with $\pi_t > 0$ all t. Note that in Theorem 3 there is no restriction on T.

Theorem 3 provides another dynamic generalisation of Roemer's theory of exploitation and strengthens the results in chapter 4. Unless assets are just sufficient to guarantee workers' subsistence (Theorem 3.(*i*)), the dynamic economy with maximising agents displays persistent exploitation – and possibly persistent unemployment, – if revenues are consumed at all *t and* capitalists discount future consumption (Theorem 3.(*ii*)-(*iii*)). However, as in chapter 4, the persistence of exploitation at an IRS crucially depends on a strictly positive rate of time preference, rather than on unemployment or capital scarcity (Theorem 3.(*iv*)). Moreover, if $\gamma_0\lambda b = 1$, the magnitude of inequalities and exploitation will depend on β.[10]

Interestingly, Theorems 2-3 also characterise inter-capitalist inequalities as a different phenomenon from exploitation. In fact, at an IRS with $\pi_t = (1 - \beta)/\beta > 0$, all t, by Theorem 2 for any two capitalists ν and μ, $C(\omega_0^\nu) > C(\omega_0^\mu)$ if and only if $p'\omega_0^\nu > p'\omega_0^\mu$. Instead, if $\pi_t = 0$, all t, $C(\omega_0^\nu) = 0$, all ν.

5.4. BALANCED GROWTH AND DISTRIBUTION

Sections 5.2 and 5.3 show that persistent growth and exploitation are inconsistent and that, even if the economy does not grow, persistent exploitation is possible only if $\beta < 1$. This section explores further the relation between exploitation, time preference, and growth, by focusing on balanced growth paths in which all sectors grow at the same rate *and* the economy eventually reaches a steady state.

DEFINITION 7: A *balanced growth path* (BGP) for $E(\Omega_0)$ is a RS such that $\omega_{t+1} = (1 + g_t)\omega_t$, for all $t < t'$, and $\omega_{t+1} = \omega_t$, all t, $T - 1 > t \geq t'$.

In order to analyse BGP's, a technically convenient restriction on b is imposed which implies no significant loss of generality, given the theoretical focus on consumption opportunities, rather than consumer choice.

ASSUMPTION 4: There is a positive scalar K such that $b = KA(I - A)^{-1}b$.

[10] As noted in chapter 4, these results raise doubts on Roemer's *property rights* definition of exploitation as a generalisation of Marx's *surplus value* definition. It is worth noting that,

By (A.4), b is uniquely determined up to a positive scalar. Lemma 5 characterises the dynamics of capital under (A.4).

LEMMA 5: *Let (p, 1) be a RS for* $E(\Omega_0)$ *such that the economy exhibits capital scarcity at t. Under (A.4), if* $\omega_t \in H$ *then* $\omega_{t+1} \in H$.

Proof: Assume that $\omega_t \in H$. By Lemma 2 and Lemma 1.(i)-(iii), given capital scarcity, at a RS $(I - A)y_t = \theta_t b + \chi_t b + s_t$ and $y_t = A^{-1}\omega_t$. Hence at a RS, $\gamma_t N_w b = \theta_t b + \chi_t b + s_t$, or $s_t = (\gamma_t N_w - \theta_t - \chi_t)b$, and by ($A$.4), $\omega_{t+1} \in H$.■

Given Lemma 5, the next result confirms the relevance of IRS's as a theoretical benchmark: only at an IRS can equilibrium in the labour market *and* exploitation exist at all t.

LEMMA 6: *Let* $\omega_0 \in H$. *Let (p, 1) be an unconstrained RS for* $E(\Omega_0)$ *such that the economy exhibits capital scarcity at all t. Under (A.4),* $\gamma_t \lambda b = 1$, *all t.*

Proof: By Lemma 2, at a RS with capital scarcity at all t, $y_t = A^{-1}\omega_t$, all t. Hence, by Lemma 5, $Ly_t = \gamma_t N_w \lambda b$, all t, and by Lemma 1.(ii), $Ly_t = z_t = N_w$, all t, if and only if $\gamma_t \lambda b = 1$, all t.■

In general, by Lemma 6 if a RS is unconstrained *from t'*, then $\gamma_t \lambda b = 1$, all $t \geq t'$, and thus IRS's are a benchmark for all accumulation paths with persistent capital scarcity, which lead to a stationary state with equilibrium in the labour market. Instead, if $\omega_t \in H$ and $\gamma_t \lambda b < 1$, the economy is constrained at t. Given the focus on accumulation, we assume $\gamma_0 \lambda b \leq 1$. The next result rules out paths where capital becomes abundant.

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trivially, no coalition of capitalists or workers alone can block a RS (see Appendix 4.2).

PROPOSITION 3: *Assume (A.4). Let* $\omega_0 \in H$, *with* $\gamma_0 > 1$ *and* $\gamma_0 \lambda b \leq 1$. *If* $\beta < 1$, *there is no RS with capital scarcity at all periods until t but* $LA^{-1}\omega_{t+1} > N_w$.

Proof. 1. Suppose not, so that $LA^{-1}\omega_t \leq N_w$ but $LA^{-1}\omega_{t+1} > N_w$, some t. By Lemma 5, $\pi_t > 0$; instead, $\pi_{t+1} = 0$ since capital is abundant at $t + 1$.

2. For all ν, $\theta_t^{\nu} = \pi_t p_t \omega_t^{\nu}/p_t b - p_t s_t^{\nu}/p_t b$ and $\theta_{t+1}^{\nu} = -\lambda s_{t+1}^{\nu}/\lambda b$, and thus $\lambda s_{t+1}^{\nu} \leq 0$. If $\lambda s_{t+1}^{\nu} < 0$, some ν, then since $\bar{p}_{it} < 0$ at least some i (given $\beta < 1$), there is a feasible perturbation of the savings path with $\mathrm{d}s_{it}^{\nu} = -\mathrm{d}s_{it+1}^{\nu} < 0$, which increases ν's consumption, contradicting optimality.

3. Let $\lambda s_{t+1}^{\nu} = 0$, all ν, so that the reasoning in part 2 does not hold. Since $\lambda s_{t+1} = 0$ then $\lambda\omega_{t+2} = \lambda\omega_{t+1}$, so that $\pi_{t+2} = 0$ and $\bar{p}_{t+1} << \mathbf{0}'$. Again, if $\lambda s_{t+2}^{\nu} < 0$, there is a feasible perturbation of the savings path with $\mathrm{d}s_{it}^{\nu} < 0$, some i, which yields an increase in ν's consumption opportunities. Therefore $\lambda s_{t+2}^{\nu} = 0$, all ν, and $\pi_{t+3} = 0$; and so on.

4. By assumption and by Lemma 5, $\omega_{t+1} >> \omega_0$. Hence, individual optimality implies $\sum_{l=t+1}^{T-1} s_l << \mathbf{0}$, and thus by (*A*.1), $\lambda\sum_{l=t+1}^{T-1} s_l < 0$, which contradicts $\lambda s_l^{\nu} = 0$, for all ν and all $T - 1 \geq l \geq t + 1$. ■

Thus, overaccumulation is not an equilibrium because the fall of the profit rate to zero would rather lead capitalists to anticipate consumption, if $\beta < 1$. Indeed, Proposition 3 confirms the importance of time preference for the persistence of exploitation in Roemer's theory: if $\beta = 1$, overaccumulation and profits falling to zero are not ruled out.

Given Proposition 3, Theorem 4 characterises balanced growth paths.

THEOREM 4: *Assume (A.4). Let* $\omega_0 \in H$ *with* $\gamma_0 > 1$. *Let (p, 1) be a BGP for* $E(\Omega_0)$ *such that* $LA^{-1}\omega_t \leq N_w$, *all t. Define* $g'_t = [(\gamma_t - 1)N_w - \theta_t]\tilde{\pi}/\gamma_t N_w$.

(i) $\omega_{t+1} = (1 + g'_t)\omega_t$, *all* $t \leq t' - 1$, *and* $p_{t+1}b = (1 + g'_t)p_t b$, *all* $t < t' - 1$. *Furthermore, if* $\beta < 1$ *then* $g'_t = \pi_t$, *all* $0 < t < t' - 1$, *while if* $\beta = 1$ *then* $g'_t = \pi_t$, *all* $t < t' - 1$.

(ii) If $\beta < 1$ *and* $\pi_t > 0$ *all t,* $T - 2 \geq t \geq t'$, *then* $\overline{p}_t = \mathbf{0'}$, *all t,* $T - 2 \geq t \geq t'$. *If* $\beta = 1$, *there is no BGP with* $\pi_t > 0$ *and* $\pi_{t+j} > 0$, *any t,* $T - 2 \geq t \geq t'$ *and* $j > 0$.

Proof: Part (ii). 1. Consider capitalist ν's program MP_ν recursively: at all t, Bellman's functional equation is $C(\omega_t^\nu) = \max_{\omega_{t+1}^\nu \in \Psi(\omega_t^\nu)} F(\omega_t^\nu, \omega_{t+1}^\nu) + \beta C(\omega_{t+1}^\nu)$. At $T-1$, since $C(\omega_T^\nu) = 0$, optimality requires $\omega_T^\nu = \omega_0^\nu$ and $C(\omega_{T-1}^\nu) = \phi(b)[(1 + \pi_{T-1})p_{T-1}\omega_{T-1}^\nu - p_{T-1}\omega_0^\nu]/p_{T-1}b$. Therefore at $T-2$, $C(\omega_{T-2}^\nu) = \max_{\omega_{T-1}^\nu \in \Psi(\omega_{T-2}^\nu)} \phi(b)[(1 + \pi_{T-2})p_{T-2}\omega_{T-2}^\nu - p_{T-2}\omega_{T-1}^\nu]/p_{T-2}b + \beta C(\omega_{T-1}^\nu)$.

2. Suppose $\beta < 1$: given $\pi_{T-2} > 0$, if $\overline{p}_{T-2} \neq \mathbf{0'}$ then, since all capitalists are alike, $\omega_{T-1}^\nu \neq \omega_{T-2}^\nu$, all ν, and $\omega_{T-1} \neq \omega_{T-2}$. Hence, $\overline{p}_{T-2} = \mathbf{0'}$ and $C(\omega_{T-2}^\nu) = \phi(b)[(1 + \pi_{T-2})p_{T-2}\omega_{T-2}^\nu/p_{T-2}b - \beta p_{T-1}\omega_0^\nu/p_{T-1}b]$. Iterating backwards, if $\omega_{t+1} = \omega_t$, all t, $T - 2 \geq t \geq t'$, then $\overline{p}_t = \mathbf{0'}$, all t, $T - 2 \geq t \geq t'$, which implies $C(\omega_{t'}^\nu) = \phi(b)[(1 + \pi_{t'})p_{t'}\omega_{t'}^\nu/p_{t'}b - \beta^{T-1-t'}(p_{T-1}\omega_0^\nu/p_{T-1}b)]$.

3. Suppose $\beta = 1$. Suppose, contrary to the statement, that at a BGP $\pi_t > 0$ and $\pi_{t+j} > 0$, for some t, $T - 2 \geq t \geq t'$, and $j > 0$. Since $\pi_t > 0$, then $\theta_t^\nu = 0$, all ν, is not possible, or else $\omega_{t+1} \neq \omega_t$, and since $\pi_{t+j} > 0$ then $(1 + \pi_{t+j})p_{it+j}/p_{t+j}b > p_{it}/p_t b$, at least some i. Hence, as in Lemma 4, there is a

feasible perturbation $ds_{it} = -ds_{it+j} > 0$, with $ds_l = \mathbf{0}$ all $l \neq t, t+j$, that increases ν's consumption opportunities, contradicting optimality.

Part (i). 4. By Proposition 1, at a BGP $\theta_t p_t b = (\pi_t - g_t) p_t \omega_t$, all t. Since $\omega_t \in H$, all t, by (*A*.4) the latter expression implies $\theta_t = (\pi_t - g_t)\gamma_t N_w / \tilde{\pi}$, all t, or $g_t = [\pi_t - (\theta_t \tilde{\pi} / \gamma_t N_w)]$, all t.

5. By Lemma 3, $(p_t - \lambda) = \pi_t p_t A(\mathrm{I} - A)^{-1}$, all t. Post-multiplying the latter expression by b and using (*A*.4), $\pi_t = \tilde{\pi}(p_t - \lambda)b/p_t b$, all t. Hence, given $\omega_t \in H$, all t, by (*A*.3) at a BGP with $\gamma_t \lambda b \leq 1$, all t, $p_t b = \gamma_t \lambda b < 1$, all $t \leq t' - 1$. Then, the expression for g'_t follows from $\pi_t = \tilde{\pi}(\gamma_t - 1)/\gamma_t$, all $t \leq t' - 1$, and $p_{t+1} b = (1 + g'_t) p_t b$, all $t < t' - 1$.

6. Suppose $\beta < 1$. At $t = t' - 1$, $C(\omega_{t'-1}^{\nu}) = \max_{\omega_{t'}^{\nu} \in \Psi(\omega_{t'-1}^{\nu})} \phi(b)[(1 + \pi_{t'-1}) p_{t'-1} \omega_{t'-1}^{\nu} - p_{t'-1} \omega_{t'}^{\nu}]/p_{t'-1} b + \beta C(\omega_{t'}^{\nu})$, where $C(\omega_{t'}^{\nu})$ is as in part 2 above. Hence, at a BGP $\overline{p}_{t'-1} = u_1 p_{t'-1}/p_{t'-1} b$, some $u_1 \geq 0$, or else $\omega_{it'}^{\nu} = 0$, some i, all ν. If $u_1 > 0$, then $p_{t'-1} \omega_{t'}^{\nu} = (1 + \pi_{t'-1}) p_{t'-1} \omega_{t'-1}^{\nu}$, all ν, and $g_{t'-1} = \pi_{t'-1}$. If $u_1 = 0$, then $\beta(1 + \pi_{t'}) = 1$, and $g_{t'-1}$ is undetermined. In either case, $C(\omega_{t'-1}^{\nu}) = \phi(b)[(1 + u_1)(1 + \pi_{t'-1}) p_{t'-1} \omega_{t'-1}^{\nu}/p_{t'-1} b - \beta^{T-t'}(p_{T-1} \omega_0^{\nu} / p_{T-1} b)]$, all ν.

7. Consider $t = t' - 2$. Again, at a BGP, it must be $[\beta(1 + u_1)(1 + \pi_{t'-1}) p_{t'-1}/p_{t'-1} b - p_{t'-2}/p_{t'-2} b] = u_2 p_{t'-2}/p_{t'-2} b$, some $u_2 \geq 0$, and $C(\omega_{t'-2}^{\nu}) = \phi(b)[(1 + u_2)(1 + \pi_{t'-2}) p_{t'-2} \omega_{t'-2}^{\nu}/p_{t'-2} b - \beta^{T-t'+1}(p_{T-1} \omega_0^{\nu} / p_{T-1} b)]$, all ν. If $u_2 = 0$, then $\beta(1 + \pi_{t'-1}) \leq 1$: but then since by part (*i*) at a BGP $p_{t+1} b > p_t b$, all $t < t' - 1$, by Lemma 3 it follows that $\beta(1 + \pi_{t'-2}) < 1$. However, if $t' > 2$, by considering Bellman's functional equation at $t' - 3$ this cannot be a BGP

since it violates $\bar{p}_{t'-3} \geq \mathbf{0}'$. Therefore, it must be $u_2 > 0$, $p_{t'-2}\omega_{t'-1}{}^{\nu} = (1 + \pi_{t'-2})p_{t'-2}\omega_{t'-2}{}^{\nu}$, all ν, and $g_{t'-2} = \pi_{t'-2}$. This argument can be iterated backwards for all t, $0 < t < t' - 1$, showing that $p_t\omega_{t+1}{}^{\nu} = (1 + \pi_t)p_t\omega_t{}^{\nu}$, all ν, and all t, $0 < t < t' - 1$, and thus $g_t = \pi_t$, all t, $0 < t < t' - 1$.

8. Suppose $\beta = 1$. A similar argument as in parts 6 and 7 applies noting that $\pi_t > 0$, all $t \leq t' - 1$, implies $u_i > 0$, all $i \geq 2$, given part (*ii*). ■

Remark: if $\beta < 1$, by Proposition 3 the assumption $LA^{-1}\omega_t \leq N_w$ is redundant.

Theorem 4 shows some interesting links between the present model and the literature on inequalities, classes, and growth. On the one hand, the model may be interpreted as providing microfoundations to traditional Sraffa/von Neumann models, which derive a negative relationship between capitalists' consumption and growth, given workers' subsistence. In fact, the balanced growth rate g'_t can be shown to coincide with the uniform growth rate of Sraffian models (see, e.g., Kurz and Salvadori, 1994, p.102ff). On the other hand, Theorem 4 proves that the growth rate coincides with the profit rate - at least in some periods - as in the so-called Cambridge equation. However, these results are derived as equilibrium features of an accumulating economy with agents who explicitly solve a dynamic optimisation problem.

The previous results confirm the main theoretical and methodological conclusions of chapter 4 and section 5.3. Only if $\beta < 1$ can overaccumulation – leading to labour scarcity and the disappearance of exploitation – be ruled out in equilibrium (Proposition 3).[11] Moreover, if $\beta = 1$, exploitation and

[11] Thus, Devine and Dymsky's (1991) result can only be an equilibrium if $\beta = 1$.

profits may well disappear after a finite number of periods, both at an IRS (Theorem 3) and at a BGP (Theorem 4), even if capital remains scarce. Instead, if agents discount the future, exploitation can be persistent even in paths with capital accumulation (Theorem 4). The crucial role of time preference, as opposed, e.g., to capital scarcity, is further confirmed by the fact that if $\beta < 1$, the steady state value of the profit rate (and thus the rate of exploitation) is a positive function of β (Theorem 4.(*ii*)).

As noted in chapter 4, Skillman (2000) suggests that exogenous growth in the labour force, heterogeneous preferences, and/or labour-saving technical progress might make *WP* exploitation persistent. Although, as argued in chapter 4, the main methodological and substantive conclusions on Roemer's theory of exploitation would not change, the relation between inequalities, exploitation, and growth is theoretically crucial and the model presented here provides a promising analytical framework to analyse it. The next section takes on Skillman's suggestion, focusing on technical progress.

5.5. THE ONE-GOOD ECONOMY AND TECHNICAL PROGRESS

The previous sections complete the analysis of the relationship between exploitation, time preference, and growth in the *n*-good economy. This section modifies the basic model by introducing exogenous labour-saving technical progress. However, for the sake of simplicity a one-good economy is considered. In fact, the linearity of MP_ν and MP_η makes it difficult to analyse the equilibria of the *n*-good model, - especially because it would be more realistic to assume different rates of technical progress in different sectors. Given the theoretical focus of this chapter, this simplification implies no significant loss of generality.

Thus, let $c_t^{\nu} \geq 0$ be capitalist ν's consumption, all t; let $\zeta_t^{\eta} \geq b$ be worker η's consumption, all t; and let ϕ be the identity function. As a first step in the analysis of the one-good economy, capitalists' optimal saving paths with accumulation are characterised and the existence of a BGP is proved, in the economy with no technical progress. Let $\pi_\beta \equiv (1 - \beta)/\beta$.

PROPOSITION 4: *Let $(p, 1)$ be such that $\pi_t > \pi_\beta$, all $t \leq \tau$, and $\pi_t = \pi_\beta$, all $T - 1 \geq t \geq \tau + 1$, for some τ, $T - 1 \geq \tau \geq 0$. The path $\omega_{t+1}^{\nu} = (1 + \pi_t)\omega_t^{\nu}$, all $t \leq \tau - 1$, $\omega_{t+1}^{\nu} = (1 + g_t)\omega_t^{\nu}$, $g_t \in [0, \pi_\beta]$, all t, $T - 2 \geq t \geq \tau$, and $\omega_T^{\nu} = \omega_0^{\nu}$, solves MP_ν, all ν, and $C(\omega_0^{\nu}) = \left[\beta^{\tau} \prod_{i=0}^{\tau} (1 + \pi_i) - \beta^{T-1}\right]\omega_0^{\nu}$, all ν.*

Proof. 1. Since the state space is $W \subseteq \Re_+$, the feasibility correspondence is $\Psi(\omega_t^{\nu}) = \{\omega_{t+1}^{\nu} \in W: 0 \leq \omega_{t+1}^{\nu} \leq (1 + \pi_t)\omega_t^{\nu}\}$ and the *one-period return function* is $F(\omega_t^{\nu}, \omega_{t+1}^{\nu}) = [(1 + \pi_t)\omega_t^{\nu} - \omega_{t+1}^{\nu}]$. Then, MP_ν becomes

$$MP_\nu \qquad C(\omega_0^{\nu}) = \max_{\omega^{\nu} \in \Pi(\omega_0^{\nu})} \sum_{t=0}^{T-1} \beta^t[(1 + \pi_t)\omega_t^{\nu} - \omega_{t+1}^{\nu}].$$

2. Consider MP_ν recursively. At $T - 1$, since $C(\omega_T^{\nu}) = 0$, then $\omega_T^{\nu} = \omega_0^{\nu}$ is optimal and $C(\omega_{T-1}^{\nu}) = [(1 + \pi_{T-1})\omega_{T-1}^{\nu} - \omega_0^{\nu}]$. At $T - 2$, $C(\omega_{T-2}^{\nu}) = \max_{\omega_{T-1}^{\nu} \in \Psi(\omega_{T-2}^{\nu})} [(1 + \pi_{T-2})\omega_{T-2}^{\nu} - \omega_{T-1}^{\nu} + \beta C(\omega_{T-1}^{\nu})]$. Hence, if $\pi_{T-1} = \pi_\beta$ then $\omega_{T-1}^{\nu} = \omega_{T-2}^{\nu}$ is optimal and $C(\omega_{T-2}^{\nu}) = [(1 + \pi_{T-2})\omega_{T-2}^{\nu} - \beta\omega_0^{\nu}]$. Iterating backwards, if $\pi_t = \pi_\beta$, all t, $T - 1 \geq t \geq \tau + 1$, then $\omega_{t+1}^{\nu} = \omega_t^{\nu}$, all t, $T - 2 \geq t \geq \tau$, is optimal and $C(\omega_\tau^{\nu}) = [(1 + \pi_\tau)\omega_\tau^{\nu} - \beta^{T-\tau-1}\omega_0^{\nu}]$. If $\tau = 0$, the result is proved.

3. If $\tau > 0$, consider $\tau - 1$. Since $C(\omega_{\tau-1}{}^{v}) = \max_{\omega_\tau^v \in \Psi(\omega_{\tau-1}^v)} [(1 + \pi_{\tau-1})\omega_{\tau-1}{}^{v} - \omega_\tau{}^{v} + \beta C(\omega_\tau{}^{v})]$ and $\pi_\tau > \pi_\beta$, at the solution to MP_v, $\omega_\tau{}^{v} = (1 + \pi_{\tau-1})\omega_{\tau-1}{}^{v}$ and $C(\omega_{\tau-1}{}^{v}) = [\beta(1 + \pi_\tau)(1 + \pi_{\tau-1})\omega_{\tau-1}{}^{v} - \beta^{T-\tau}\omega_0{}^{v}]$. Iterating backwards, if $\pi_t > \pi_\beta$,, all $t \leq \tau$, at the solution to MP_v, $\omega_{t+1}{}^{v} = (1 + \pi_\tau)\omega_\tau{}^{v}$, all $t \leq \tau - 1$, and the expression for $C(\omega_0{}^{v})$ follows.█

Let π' be defined as in Theorem 3.(*iii*). Let the sequence $\{\bar{\gamma}_\tau\}_{\tau=0}^{T-1}$ be defined by $\bar{\gamma}_o = 1/\lambda b$ and $\bar{\gamma}_{\tau+1} = (\bar{\gamma}_\tau + \tilde{\pi})/(1 + \tilde{\pi})$. The sequence is decreasing and finite, if T is finite. By (*A*.1) and the productivity of A, the size of the intervals $[\bar{\gamma}_\tau, \bar{\gamma}_{\tau-1})$ decreases with τ and tends to zero, with $\bar{\gamma}_\tau \to 1$ as $\tau \to \infty$. Theorem 5 proves the existence of a BGP.

THEOREM 5: *(Existence of a BGP). Let $\lambda b < 1$. Let $\beta \in [1/(1 + \pi'), 1]$. Let $\omega_0 = \gamma_0 N_w A(I - A)^{-1}b$, with $\gamma_0 > 1$. If $\gamma_0 \in [\bar{\gamma}_{\tau+1}, \bar{\gamma}_\tau)$ and $\bar{\gamma}_\tau > \beta\tilde{\pi}/[\beta(1 + \tilde{\pi}) - 1]$, if $\tau \geq 1$, the path (p, 1) with $\pi_t = \tilde{\pi}(\gamma_t - 1)/\gamma_t$, all t, $\tau \geq t \geq 0$, and $\pi_t = \pi_\beta$, all t, $T - 1 \geq t \geq \tau + 1$, is a BGP for $E(\Omega_0)$ such that $\omega_{t+1} = (1 + \pi_t)\omega_t$, all $t \leq \tau - 1$, $\omega_{\tau+1} = (1 + g_\tau)\omega_\tau$, with $g_\tau \in (0, \pi_\tau]$, and $\omega_t = \omega_{\tau+1}$, all t, $T - 1 \geq t \geq \tau + 1$.*

Proof. 1. Suppose $\gamma_0 \in [\bar{\gamma}_{\tau+1}, \bar{\gamma}_\tau)$. First, at all $t \leq \tau$, if $\gamma_t \in [\bar{\gamma}_{\tau+1-t}, \bar{\gamma}_{\tau-t})$ and $\pi_t = \tilde{\pi}(\gamma_t - 1)/\gamma_t$, then by construction $\gamma_{t+1} = (1 + \pi_t)\gamma_t$ implies $\gamma_{t+1} \in [\bar{\gamma}_{\tau-t}, \bar{\gamma}_{\tau-t-1})$; moreover if $\gamma_t \in [\bar{\gamma}_1, \bar{\gamma}_0)$ and $\pi_t = \tilde{\pi}(\gamma_t - 1)/\gamma_t$, there is a $g'_t \in (0, \pi_t]$ such that $\gamma_{t+1} = (1 + g'_t)\gamma_t$ implies $\gamma_{t+1} = 1/\lambda b$. Second, suppose $\tau \geq 1$: if $\gamma_0 \in [\bar{\gamma}_{\tau+1}, \bar{\gamma}_\tau)$ and $\bar{\gamma}_\tau > \beta\tilde{\pi}/[\beta(1 + \tilde{\pi}) - 1]$ then $\pi_1 = \tilde{\pi}(\gamma_1 - 1)/\gamma_1 > \pi_\beta$, for all $\gamma_1 \in [\bar{\gamma}_\tau, \bar{\gamma}_{\tau-1})$.

2. (Optimal ξ^{ν}; reproducibility.) By part 2 and Proposition 4, $\omega_{t+1}{}^{\nu} = (1 + \pi_t)\omega_t{}^{\nu}$, all $t \leq \tau - 1$, $\omega_{\tau+1}{}^{\nu} = (1 + g'_{\tau})\omega_{\tau}{}^{\nu}$, with $g'_{\tau} \in (0, \pi_{\tau}]$, $\omega_t{}^{\nu} = \omega_{\tau+1}{}^{\nu}$, all t, T- $1 \geq t \geq \tau + 1$, $\omega_T{}^{\nu} = \omega_0{}^{\nu}$, and $y_t{}^{\nu} = A^{-1}\omega_t{}^{\nu}$, all t, solves MP$_{\nu}$, all ν.

3. (Capital market) At the proposed path, $y_t = A^{-1}\omega_t$ and the capital market clears at all t. (Labour market) At all $t \leq \tau$, $Ly_t = LA^{-1}\omega_t < N_w$. Hence, assign actions $z_t{}^{\eta} = \gamma_t\lambda b$, all η, consistently with $(A.3)$, all $t \leq \tau$. At all $t \geq \tau + 1$, $Ly_t = LA^{-1}\omega_t = N_w$, while $z_t{}^{\eta} = 1$, all η, and the labour market clears. (Final goods market) At all $t \leq \tau$, by construction $\gamma_t\lambda b = p_t b$, and thus $\zeta_t{}^{\eta} = b$, all η, consistently with $(A.3)$, while $c_t{}^{\nu} + s_t{}^{\nu} = \pi_t\omega_t{}^{\nu}$, all ν. Thus, aggregate demand is $c_t + s_t + \zeta_t = \pi_t\omega_t + N_w b$ and substituting for π_t and ω_t, $c_t + s_t + \zeta_t = \gamma_t N_w b$; aggregate supply is $(1 - A)y_t = (1 - A)A^{-1}\omega_t = \gamma_t N_w b$. At all $t \geq \tau + 1$, by individual optimisation $p_t\zeta_t{}^{\eta} = 1$, all η, while $c_t{}^{\nu} + s_t{}^{\nu} = \pi_t\omega_t{}^{\nu}$, all ν. Thus, aggregate demand is $c_t + s_t + \zeta_t = \pi_t\omega_t + N_w/p_t$, all $t \geq \tau + 1$: substituting for the proposed values of π_t and p_t, and noting that at the proposed path $\gamma_t = 1/\lambda b$, all $t \geq \tau + 1$, it follows that $c_t + s_t + \zeta_t = N_w b/\lambda b$, all $t \geq \tau + 1$. Aggregate supply is $(1 - A)y_t = (1 - A)A^{-1}\omega_t = \gamma_t N_w b = N_w b/\lambda b$, all $t \geq \tau + 1$. Therefore, the goods market clears at all t.■

Remarks: If $\beta \to 1$ then $\beta\tilde{\pi}/[\beta(1 + \tilde{\pi}) - 1] \to 1$ and the higher β, the larger the set of $\bar{\gamma}_{\tau}$ such that the BGP described in Theorem 5 exists. However, Theorem 5 only provides sufficient conditions for the existence of a BGP.

In the BGP described in Theorem 5, given an initial level of capital, the economy accumulates at the maximum rate and reaches the steady state in a finite number of periods. In the first τ periods, profits and labour expended increase over time and workers' consumption remains at the subsistence level. At the steady state, full employment prevails, profits remain constant, and workers' consumption exceeds subsistence. If $\beta < 1$, exploitation is a persistent phenomenon; if $\beta = 1$, it disappears.

As in Proposition 2, this pattern is due to an initial excess supply of labour which is then rapidly absorbed due to accumulation. As noted above, this raises the issue of the factors that may generate a persistent excess supply of labour: in what follows, labour-saving technical progress is analysed. The intuition is that by substituting capital for labour, technical progress may allow labour supply to be persistently higher than labour demand. Formally, the labour input coefficient is assumed to decline geometrically over time.

ASSUMPTION 5: For all t, $L_{t+1} = \delta L_t$, $\delta < 1$, with $L_0 > 0$ given.

Under (A.5), all the results in Section 5.2 (plus Lemmas 4-6 and the FMT) and Proposition 4 hold, once L_t is substituted for L at all t. Then, Theorem 6 provides sufficient conditions for the existence of a RS with persistent exploitation in the economy with technical progress.

THEOREM 6: *Assume (A.5). Let $\omega_0 = \gamma_0 N_w A(1-A)^{-1}b$, with $\gamma_0 > 1$ and $\gamma_0\lambda_0 b \leq 1$. If $\delta(1 + \tilde{\pi}) \leq 1$ and $\beta[1 + \tilde{\pi}(\gamma_0 - 1)/\gamma_0] \geq 1$. The path (p, 1) with $\pi_0 = \tilde{\pi}(\gamma_0 - 1)/\gamma_0$ and $\pi_{t+1} = \pi_t(1 + \tilde{\pi})/(1 + \pi_t)$, all t, $T - 2 \geq t \geq 0$, is a RS for $E(\Omega_0)$ with $L_t y_t < N_w$, all $t > 0$, and $\omega_{t+1} = (1 + \pi_t)\omega_t$, all t, $T - 2 \geq t \geq 0$.*

Proof. 1. (Optimal ξ^ν; reproducibility.) By construction, $\pi_t > \pi_\beta$, all $t > 0$, and

thus by Proposition 4 $y_t^\nu = A^{-1}\omega_t^\nu$, all t, $\omega_{t+1}^\nu = (1 + \pi_t)\omega_t^\nu$, all t, $T - 2 \geq t$

≥ 0, and $\omega_T^\nu = \omega_0^\nu$, solves MP$_\nu$, all ν. Hence, $\omega_{t+1} = (1 + \pi_t)\omega_t$, all t, $T - 2 \geq t \geq 0$, which in turn implies $\pi_t = \tilde{\pi}\,(\gamma_t - 1)/\gamma_t$, all t, by construction.

2. (Capital market) The capital market clears at all t, since $y_t = A^{-1}\omega_t$, all t. (Labour market) By parts 1 and 2, and (A.5), at all t, $L_t y_t = L_t A^{-1}\omega_t = \delta(1 + \pi_{t-1})L_{t-1}y_{t-1}$. Hence, given $L_0 y_0 \leq N_w$, $\delta(1 + \tilde{\pi}) \leq 1$, and $\pi_t \leq \tilde{\pi}$, all t, at the proposed path $L_t y_t < N_w$, all t. Then, $z_t = \gamma_t \lambda b N_w$, all t, and by (A.3) assign actions $z_t^\eta = \gamma_t \lambda b$, all η, t.

3. (Final goods market) At the proposed path, $p_t b = \gamma_t \lambda b$, all t, and thus $\zeta_t^\eta = b$, all η, t, and $\zeta_t = N_w b$, all t, consistently with (A.3). Moreover, $c_t^\nu + s_t^\nu = \pi_t \omega_t^\nu$, all ν, t, and thus $c_t + s_t = \pi_t \omega_t$, all t. Therefore aggregate demand is $\zeta_t + c_t + s_t = \pi_t \omega_t + N_w b$, all t. Substituting for ω_t and π_t in the latter expression $\zeta_t + c_t + s_t = \gamma_t N_w b$, all t. Aggregate supply is $(1 - A)y_t = (1 - A)A^{-1}\omega_t = \gamma_t N_w b$, all t, and thus the goods market clears at all t.■

Theorem 6 is encouraging: with labour-saving technical progress, the economy settles on a "golden rule" growth path with persistent exploitation even if $\beta = 1$. The increase in productivity ensures that labour remains in excess supply even along a path with maximal accumulation. However, there are at least two features of the result that seem doubtful. Firstly, Theorem 6 relies on the arguably strong assumption of unbounded technical progress: if there is a lower bound to the labour input requirement, the result does not hold. More importantly, in the RS with persistent exploitation, both prices and labour expended by workers decrease over time and tend to zero, due to the increase in productivity, a rather unappealing feature in a model of exploitation, especially given that there is no disutility of labour. This suggests that Theorem 6 is but a first step in the analysis of the relation between exploitation, inequalities, and growth.

5.6. CONCLUSION

In this chapter, an intertemporal model of an economy with maximising agents is set up to analyse the relation between exploitation, inequality, and growth. A dynamic generalisation of the Fundamental Marxian Theorem is proved, then it is shown that there is no equilibrium with persistent accumulation and exploitation. It is also proved that *both* at a stationary equilibrium with no savings and along a balanced growth path (along which the economy eventually settles on a steady state), capital scarcity and DOSPA persist, but exploitation disappears after a finite number of periods, if agents do not discount the future. For exploitation to be persistent, the agents' rate of time preference must be positive. This confirms the main theoretical conclusions reached in chapter 4. Actually, the analysis of the economy with technical progress shows that persistent exploitation can emerge if there is a mechanism that yields persistent unemployment, a non-Walrasian feature. Although the model with technical progress is not entirely convincing, it indicates a promising line for further research, which, as argued in chapter 4, should incorporate further departures from the traditional Walrasian framework in order to model persistent exploitation.

CONCLUSIONS

This dissertation analyses the dynamics of inequality and classes, from a positive and a normative viewpoint. In particular, two distinct, but – as shown in chapter 1 – related theoretical approaches to equality and classes are analysed from a dynamic perspective; namely, Analytical Marxism (AM) and the theory of equality of opportunity (EOp). Methodologically, this dissertation shows the importance of a dynamic analysis in the evaluation of egalitarian (more generally, normative) theories, as an essential tool in the process of theoretical construction. Indeed, the main results presented can be thought of as illustrating some *anomalies* of egalitarian and Marxian theories, which arise in the dynamic context and which suggest the need to reconsider some established views on inequality, exploitation, and classes.

Chapter 2 analyses various intertemporal egalitarian principles, based on different temporal units of concern – whole lives or selected portions of them – which incorporate different normative views and yield different policy implications. The principles provide different insights in the analysis of *inequalities* and in this context no principle seems entirely satisfactory. However, unlike in the static setting, they also define different *egalitarian* states to reach. From this viewpoint, *corresponding segments egalitarianism* (*CSE*) – which focuses on the corresponding stages of agents' lives – arguably defines the appropriate egalitarian benchmark.[1]

[1] In an overlapping generations model, *CSE* seems more convincing than other intertemporal egalitarian principles in the context of *all things considered* judgements, too, since it has desirable properties in relation to both Rawlsian and utilitarian concerns.

Chapters 4 and 5 show that similar issues arise in the context of exploitation theory: two criteria to define exploitation and class in a dynamic context are discussed, one focusing on the agent's status in each period of her life, the other on the agent's whole life. Although both criteria convey morally relevant information, the *within-period* definition is arguably more interesting in a dynamic context and more in line with a Marxian approach. However, the analysis in chapters 4 and 5 is based on the rather strong simplifying assumption that all agents living in the same period belong to the same age cohort, so that the distinction between *CS* and *simultaneous segments* (*SS*) views can be set aside: the *WP* definition of exploitation captures both.

This assumption obscures some interesting issues for further research. Consider Roemer's definition of Marxian exploitation as unequal exchange of labour, analysed in chapter 4. As noted by Elster (1985), the definition is non-relational: it states that agent A is either exploiting or exploited (or neither), but not that agent A exploits another agent B (or viceversa).[2] Under this interpretation, exploitation is an objectivist measure of inequality; thus, the *CS*/*SS* distinction and the analysis in chapter 2 are relevant, which would suggest to generalise the model in chapters 4 and 5 by adopting an overlapping generations framework.

However, if exploitation cannot be reduced to a form of inequality, the relevance of the *CS* view, as opposed to the *SS* view, is less evident within

[2] Apart from the trivial observation that, at an equilibrium with no savings, if A works more than is socially necessary, there is *some* agent B who works less.

exploitation theory: it is unclear why different amounts of exploitation suffered by, e.g., adults belonging to different generations, and whose lives never overlap, should be of primary normative concern. If exploitation is a relational concept, according to which an agent (or group of agents) exploits another agent (or group of agents), it seems natural to restrict its application to contemporaneous agents. If correct, on the one hand, this casts further doubts on Roemer's interpretation of Marxian exploitation and it raises the issue of the proper definition of exploitation (see below). On the other hand, it suggests to extend the analysis in chapter 2 to the difference between relational and non-relational egalitarian approaches in the dynamic context.

Chapter 2 also proves that in an overlapping generations economy, the intertemporal maximin path tends to be incompatible with growth, a well-known property of Rawls's difference principle. This conclusion is confirmed and strengthened in chapter 3 which extends the analysis of the dynamic implications of egalitarian approaches, focusing on the EOp view. In chapter 3, agents are assumed to live for one period, but the economic environment is enriched by considering heterogeneous agents in each generation, by allowing them to care about *functionings* (and not only consumption), and by analysing educational investment. If an objectivist equalisandum (e.g., *functionings*) is adopted, the intergenerational EOp path is inconsistent with sustained human development, even with altruistic agents. This incompatibility can be resolved by equalising opportunities for *welfare*, which suggests that 'subjectivism' may be necessary if we are to hope for a society which can both equalise opportunities and support the development of human capacity.

The results presented in chapters 2 and 3 are quite general and in line with most of the literature, but no general impossibility result is proved. Given the relevance of the difference principle in contemporary egalitarian approaches, it is crucial to investigate the robustness of the inconsistency result. Silvestre (2002) provides a counterexample to show that "the conflict between … maximin and progress is not universal" (ibid., p.2). As argued in chapter 1, the actual relevance of the example is unclear, but given the results in chapter 3, it suggests some interesting lines for further research.

First, it would be interesting to provide a general, rigorous characterisation of the existence of intertemporal maximin paths, and of their properties, in a recursive optimisation framework. In the models analysed in chapters 2 and 3, the maximin path exists, thanks to the assumptions on the productivity of the economy which allows for welfare growth. However, the issue of existence is not trivial and it is quite important from an egalitarian perspective, as shown by the analysis of Rawls's principle in economies with non-renewable resources (Solow, 1974A, 1974B).

The analysis of the relation between growth and the maximin is the object of work in progress (joint with John Roemer), which extends the intergenerational EOp approach to deal with international justice and environmental issues. We model an intergenerational society, with two countries, called the North and the South, who must deplete a renewable resource, a forest, from which they derive utility, to produce consumption goods. In each country, there is a representative agent at each date, although these two agents may be of different sizes (capturing different populations in the North and South). The South has a technology for producing goods out of trees that is inferior to the North's. The forest is a global commons, which each country can freely harvest: it is a global public good, since the citizens of both countries enjoy without congestion what is not harvested.

We postulate that each country is concerned with implementing a just intergenerational allocation for its citizens. Three solutions to the problem of *international* relations are studied: the *non-cooperative Nash solution,* where the North and South play strategically against each other; a *bargaining solution,* where the North and South enter into cooperative relations, but where each country remains interested only in justice for its own citizens; and the *cosmopolitan solution,* which implements the maximin solution for the world, ignoring national boundaries. We are, in particular, interested in the intertemporal path of forest and welfare growth at these three solutions. The preliminary results obtained are encouraging: in all three frameworks, it is possible to have welfare growth along the intertemporal maximin path (although this is more likely to happen in the cosmopolitan solution). This is due to the presence of the state variable – the stock of the forest – in the objective function, a result that seems liable of further generalisation.

Chapters 4 and 5 analyse the dynamics of inequality and classes, focusing on Roemer's (1982A, 1988A) theory of exploitation and classes. In chapter 4 an dynamic generalisation of Roemer's subsistence economy with labour-minimising agents is set up to evaluate the causal and normative relevance of *Differential Ownership of Productive Assets* (DOPA) in generating exploitation and classes as persistent features of a competitive economy; and the possibility of providing robust microfoundations to Marxian economics by means of neoclassical models.

A dynamic generalisation of Roemer's theory is provided: exploitation and classes are persistent phenomena, if agents discount the future. However, the normative relevance of time preference is dubious and, with no time preference, in equilibrium exploitation disappears in the long run, even if DOPA and classes persist. Chapter 4 proves that it is sufficient to allow agents to save – unlike in the static models in which agents face no intertemporal trade-offs – to contradict Roemer's results. Hence, asset inequalities seem normatively secondary, though causally primary in explaining exploitation and several doubts are raised on the possibility of providing robust microfoundations to Marx's concepts by means of Walrasian general equilibrium models.

Chapter 5 extends the analysis of exploitation, inequality, and classes to economies with maximising agents and, possibly, unemployment and capital accumulation. First, the main conclusions of chapter 4 are strengthened and generalised: if capitalists discount the future, there are equilibria with no accumulation and persistent exploitation. However, this result depends on a strictly positive rate of time preference, rather than unemployment or capital scarcity. Second, chapter 5 pursues one of the main substantive and methodological issues raised in chapter 4, namely the mechanisms generating exploitation, inequalities, and classes as persistent features of a competitive economy. In fact, the role of capital scarcity is more properly evaluated in an accumulating economy, which, unlike static and subsistence models, can naturally accommodate two arguably crucial features of a general theory of exploitation, namely technical progress and unemployment.

Chapter 5 shows that without technical progress, there is no equilibrium with persistent accumulation and exploitation. Moreover, along balanced growth paths – in which the whole economy grows at a uniform rate and reaches a steady state – exploitation disappears, although DOPA and capital scarcity persist. Instead, unbounded labour-saving technical progress may yield persistent exploitation by ensuring persistent unemployment in the labour market. This is encouraging, but there are at least two features of the result that seem dubious. First, it relies on the arguably strong assumption of unbounded technical progress, whereby in the limit no labour is necessary to produce goods. Second, in the equilibrium with persistent exploitation, both prices and labour expended tend to zero, due to the increase in productivity, a rather unappealing feature in a theory of exploitation. This suggests that the model presented in chapter 5 is but a first step in the analysis of the relation between exploitation, inequalities, and growth.

More generally, chapters 4 and 5 suggest two main lines for further research. From a substantive viewpoint, they raise the issue of the appropriate definition of exploitation and the distinction between exploitation and welfare inequalities. The results presented raise doubts on Roemer's interpretation of Marx's theory of exploitation as "a kind of resource egalitarianism" (Roemer, 1994A, p.2) and on the claim that his *property* *relations* (PR) definition of exploitation based on DOPA generalises Marx's theory capturing its essential normative content – interpreted as requiring "an egalitarian distribution of resources in the external world" (ibid., p.3). In fact, "The legitimacy of Roemer's reformulation depends in large part on the validity of his claims concerning the role of DOPA in capitalist exploitation" (Skillman, 1995, p.311). However, since DOPA is proved to be necessary but not sufficient to generate persistent Marxian exploitation, Roemer's PR definition should be seen as incorporating a different moral concern, rather than as a generalisation of Marx's labour-based definition. In general, the question arises whether DOPA should be a basic moral concern, both in itself and in a theory of exploitation, or rather a different role of DOPA should be stressed as a causally primary, but normatively secondary wrong.

On the other hand, it is unclear whether Roemer's non-relational definition of exploitation as unequal exchange of labour captures the Marxian notion of exploitation, or indeed a notion of exploitation essentially different from welfare inequalities. In chapter 4, the unequal exchange definition of exploitation is shown to be different from subjective welfare inequalities, if agents discount the future, due to its more objectivist leaning. Yet, the two concepts are strictly related, and it is legitimate to wonder why "should Marxists be interested in exploitation." Interestingly, Roemer's own PR definition provides some suggestions for further research. In fact, not only can the same arguments as for the unequal exchange definition be used to show that the PR definition is related to the notion of welfare inequalities, but does not coincide with it; the PR definition also differs from welfare inequalities due to its inherently relational nature. In particular, dominance condition (3) - briefly analysed in Appendix 4.2 - which is not defined by Roemer, is not just necessary "to rule out some bizarre examples" (Roemer, 1982A, p.195) and it might play a more prominent role in a theory of exploitation as a feature of relations between people.

From a methodological viewpoint, it might be opportune to explore non-walrasian approaches to model exploitation and classes in a Marxian perspective. Chapters 4 and 5 suggest that the *property rights theory of the firm* (Grossman and Hart, 1986; Hart and Moore, 1990) may provide a promising framework to analyse Marxian exploitation, given its concern with power and the emphasis on the role of physical assets in explaining hierarchical relations and the existence of firms. Thus, although the dynamic implications of these models are yet to be fully explored, they might provide a framework to model exploitation consistent with the idea that asset inequalities are causally primary, but normatively secondary, in the explanation of exploitative relations, given various sources of contractual incompleteness (e.g., Marx's labour/labour-power distinction).

Finally, although chapters 4 and 5 suggest that the definition of Marxian exploitation as unequal exchange of labour does not necessarily capture the essence of exploitative relations *within* a country, it may be a useful concept to understand some features of economic relations *between* countries. In recent work in progress, we set up a dynamic general equilibrium model of an international economy with capital flows, which generalises Roemer (1983A). Countries maximise their national product and differ both in wealth levels and – unlike in chapters 4 and 5 – in their labour endowment. In the dynamic equilibrium, countries can be partitioned based on their position on the international credit market (net borrowers, net lenders, neither) and a phenomenon of unequal exchange emerges as an equilibrium feature of a perfectly competitive economy. The status of each country in the system of international relations can also be derived: more advanced countries are net lenders and benefit from unequal exchange, while less advanced countries are net borrowers and suffer from unequal exchange.

From a theoretical viewpoint, unequal exchange emerges due to profit maximising behaviour and differential levels of development and wealth across countries. Thus, it can arise in a perfectly competitive environment where international economic relations are mediated only by markets, while non-competitive phenomena are not necessary to understand (and possibly condemn) international inequalities. However, consistently with the results in chapters 4 and 5, perfectly competitive markets and differences in development and wealth do not provide foundations to unequal exchange as a persistent phenomenon. This suggests that non-competitive practices may be crucial to understand the persistence of unfair international relations.

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