University of London

Modelling of the Turkish Catastrophe Insurance Pool Data, 2000-2003

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A thesis submitted in fulfillment of the requirements for obtaining the degree of Doctor of Philosophy

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February 2007

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Acknowledgements

I am grateful to my supervisor Professor Henry P. Wynn for his encouragement, wisdom, knowledge, patience, kindness and support during all my studies.

My love and special thanks to my dearest mother, Mrs.Oya Manav Başbuğ, for her love, support and patience.

I would like to thank to Professor Öztaş Ayhan for his support, help and kindness during my Ph.D process.

I would like to thank to Assc. Prof. A. Sevtap Kestel for introducing me to the field of earthquakes, always supporting me in my career and being like a real sister to me.

Esther Heyhoe, Imelda Noble, Ulla Jakobsen, Lyn Grove and Thomas Hewitt at the London School of Economics, Department of Statistics made my life much easier during my PhD. I would like to thank to everybody at the Department of Statistics at the University of Warwick. Special thanks goes to Mrs. Paula Matthews for being a good friend and a great administrator. Thanks to Middle East Technical University Statistics department secretary Mrs. Neşe Bilal for all her administrative help during my leave abroad.

My dearest friends, Ms. Simla İçmez and Ms. Özlem Bay were always with me at the days full of stress, happiness and sadness for past five years. I love you both.

My very special thanks goes to Ms. Judith Anzures-Cabrera for always being there for me, for her great friendship and help with the submission of the thesis. Muchos muchos besos! Thanks to Ms. Chrysoula Dimitrou-Fakalou for being a very great friend full of wisdom and trust. Special thanks to Mr. Neil Course, Mr. Costas Kallis, Mr. Mahmut Kutlukaya, Mr. Ibrahim Erkan and Ms. Burçin Akgün for their valuable help during my studies. I would like to thank Sultan, Ieda, Shanshan, Bruno and Milena for all their support and hospitality. Thanks to Mr. Imran Ayoob and Mr. M. Gökhan Erdamar for their help during the printing and submission process.

I would like to thank to Mr. Selamet Yazici from the Undersecretariat of the Treasury, Prime Ministry of Turkey and Mr. Hüseyin Yunak from the Milli Re Ltd. for providing me the data of this study and sharing their expertise of the

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Turkish Catastrophe Insurance Pool. Thanks to Mrs. Katalin Demeter of the World Bank Institute (WBI) for her efforts to develop an online National Disaster Risk Management Program in Turkey. She provided permission to use some material of the WBI in Chapter 7 and the Glossary of this thesis.

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Basic research is what I am doing when I don't know what I am doing. —Werhner von Braun

Certum ex incertis (Certainty out of uncertainty). —The British Institute of Actuaries

Abstract

The devastating 1999 Marmara and Düzce earthquakes lead to a significant increase in the earthquake studies in Turkey in geological, engineering and financial aspects. The start of the Turkish Catastrophe Insurance Pool (TCIP) in September 2000 brought the mandatory earthquake insurance scheme in Turkey. Since then, many claims have been made after the earthquakes. In this study, the earthquake insurance claims data of the TCIP is used to model the number of claims, N_i, and the total claim size (amount), S_i, as response variables with time and other covariates considering earthquake risk zone 1 and zone 2 in Turkey. The special functions, which are the exponential and the power kernel functions, are used for the modelling purposes to represent the sudden jumps in the number of claims after a disaster. The methods to estimate the related model parameters are presented and the results are used in the modelling process. The total claim amount (or the aggregate claims) process, $S(t)$, is a main tool to calculate the risk process and the expectation of the total claim amount, $E(S(t)) = \mu\Lambda(t)$, gives an idea to calculate the necessary aount of the TCIP reserves. Therefore, the estimates of the suggested N_i and S_i models are used to predict the necessary reserves of the Turkish Catastrophe Insurance Pool for selected zones. Afterwards, some examples of existing disaster management programs in different countries are given and the features of the Turkish Catastrophe Insurance Pool are discussed. Then, a hypothetical financial vulnerability analysis for Turkey in 10-,50-,100- and 500- years is presented with suggested solutions in case of a financial gap.

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t, T: time.

$N(t)$: The claim number process up to time t.

$X(t)$: The claim amount (claim size) process up to time t.

N_i: The number of claims (bin count).

X_i: The claim amount in the corresponding bin ($X_i = X_{t_i}$).

S_i: The aggregate claim (total claim amount) in the corresponding bin.

μ_i: The mean of the aggregate claims S_i.

η_i: The mean of the raw claim amount X_i.

τ_i: The variance of the raw claim amount X_i.

$\lambda(t)$: The intensity (rate) of $N(t)$ process.

$\Lambda(t)$: The mean function (the intensity) of the whole process of $N(t)$ (aka the expected number of events by time t). This notation is used for non-bin case.

Λ_i: The intensity function Λ for binning case (also used as λ_i).

$S(t)$: The aggregate claims or the total claim amount process.

$R(t)$: The risk process of a company (aka surplus).

d: The deductible amount.

β: The non-linear parameter to represent the exponential decay (trend) in the earthquake risk zones in Turkey.

α_j: The coefficients representing the effect of earthquakes.

n: The number of observations (earthquake claims).

i: The index for the number of observations, $i = 1, \ldots, n$.

k: The number of the knots to replace the kernel function for the empirical earthquakes.

j: The index for the knots to replace the kernel function for the empirical earthquakes, $j = 1, \ldots, k$.

θ: Vector of the intensity function λ's, which is consisted of the α and β parameters.

i.i.d: independent identically distributed.

mgf: moment generating function.

Chapter 1

Introduction

The 1999 Marmara earthquake was a turn point in the earthquake research in many aspects (e.g building damage, socio-economic losses) in a highly earthquake-prone country like Turkey. The main question in people's mind since then is 'What happens if another earthquake strikes with a similar or bigger magnitude?'. Different scenarios are prepared to answer this question, especially for Istanbul and surroundings, which are situated in the earthquake risk zone 1 according to the classification of the Earthquake Region Map of Turkey (see the Appendix).

Most of the earthquake research are conducted by the civil engineers and geologists on fault structures, building structure and damage assessment, where psychologists and sociologists study the social impacts of disasters. In this thesis, we wanted to contribute to all these vital studies with a financial and statistical point of view. In what way can statistics science be included in such research rather than just keeping the basic numbers like the number of the earthquakes, the number of life losses.

There is a mandatory earthquake insurance scheme, the Turkish Catastrophe Insurance Pool (TCIP), in application in Turkey since 2000. The idea of applying some detailed statistical analysis for the ongoing earthquake research led us to the use of the TCIP data. This will bring a new look to the consequences of earthquake problem by the statistician's eye. Unfortunately, many people are waiting for a next big earthquake to see the efficiency of their estimates for different types of research. The work here will be a suggestion for the authorities to keep the TCIP reserves enough to be able to cope with the claims arriving after an earthquake.

This thesis studies the mandatory earthquake insurance claims data, which is collected in the Turkish Catastrophe Insurance Pool between December 2000 and July 2003. Chapter 2 gives information on the earthquakes, the hazard profile and earthquake history of Turkey and the most devastating earthquake of all times in the country, that is the 17/August/1999 Marmara (Kocaeli) earthquake. Chapter 3 is mainly the literature review of the distribution theory of the Poisson (homogeneous, inhomogeneous) process, the premium calculation, reinsurance, the moment generating and cumulant functions, the extreme value theory and its application to the data of this study. Chapter 4 introduces the likelihood function of the observations and time. The parameter estimates are given for Poisson likelihood of the number of claims model, N_i, and for Normal likelihood of the aggregate claims (total claim amount) S_i model since $\log S_i \sim$ Normal with the use of the special functions, which are the exponential and the power kernel functions.

Chapter 5 starts with the basic analysis of the thesis data. The models with time covariate, the parameter estimates and the related confidence intervals are given by risk zone 1 and zone 2 classification. In Chapter 6, the same methodology is used, when the magnitude and the number of residential buildings are added to the existing models as linear covariates (aka explanatory or regressor variables). Chapter 7 begins with an introduction to natural hazards, disaster insurance and application of disaster risk management programs in different countries. The detailed information about the insurance sector in Turkey and the features of the Turkish Catastrophe Insurance Pool (TCIP) are presented in this chapter. A major motivation for the thesis is to model in a straightforward way in order to be able to make conclusions relating to future earthquakes and the adequacy of the existing TCIP reserves. Thus, Chapter 7, in which these conclusions are made (summarised in Chapter 8), can be seen as the culmination of the analysis and modelling of the previous Chapters. A financial vulnerability analysis is presented using the Economic Commission for Latin America (ECLAC) methodology and the CATSIM (Catastrophe Simulation Model) of International Institute of Applied System Analysis (IIASA) introduced in the Financial Strategies module of the World Bank Institute online distance learning

Natural Disaster Risk Management Program. Moreover, some mitigation, response and recovery strategies are suggested for Turkey. The thesis ends with the Conclusion, the Glossary, Appendix and the Bibliography.

Chapter 2

Earthquakes

2.1 Historical view

Earthquakes are the results of the continuous reshaping of the Earth. In time, people tried to explain the shakings, which killed many of them and caused damage to their homes and lives. People did not have any knowledge of earthquakes scientifically, so they made up some legends, stories about monsters shaking the Earth. For instance, in ancient Japan, it was believed that Namazu, a big catfish, was living underground and when Namazu moved the ground was shaken. A God, Daimyojin, would control Namazu. When Daimyojin's attention was not on the catfish, the fish moved and that caused earthquakes. There were no answers to the questions in people's mind except such kind of legends, until Greek philosophers, like Strabo and Aristotle thought the earthquakes were caused by something going on physically underground [Bolt, 1988].

The earthquake catalogues, which consist of records about severe earthquakes, were created by the invention and spread of writing. The geological and seismological changes in our world are studied by these catalogues. The oldest one of these catalogues dates back to 3000 years, recorded by the Chinese. It includes moderate and large earthquakes in central China from 780 BC to present. The earliest earthquake, for which there is descriptive information about, occurred in China in 1177 BC [Bolt, 1988]. The Japanese also have an earthquake catalogue, which has records from 1600 AD onwards. It was not until mid-sixteenth century that there was some discrete information regarding the occurrence of earthquakes in Europe, although they are mentioned as early as 580 BC. The earliest known earthquakes in the American continent were in Mexico in the late fourteenth century and in Peru in 1471, but the records of them are not extensive. By the seventeenth century, descriptions of the effects of earthquakes started to be published around the world. In the recorded history of North America, there were a series of earthquakes, which occurred in 1811-1812 near New Madrid, Missouri. A big earthquake of magnitude 8.0 occurred on 16/December/1811. Another one occurred on 23/January/1812, and a third one, on 07/February/1812. The aftershocks of these earthquakes lasted for months [Bolt, 1988].

In 1906, one of the most destructive earthquakes throughout the recorded history of North America occurred in San Francisco. The earthquake itself and the following fires caused approximately 700 life losses and left the city in ruins. Year 2006 is the 100th anniversary of this earthquake. The Alaska earthquake of 27/March/1964 was greater than the San Francisco earthquake in magnitude, yet since the epicentre was far from the densely populated area, only 114 people died.

It is also noticeable and interesting that earthquakes destroyed the three of the Seven Wonders of the World in ancient times: the Mausoleum of Halicarnassus, the Colossus of Rhodes and the Pharos of Alexandria.

What is an earthquake?

Earthquakes are generally defined as the shaking of the ground resulting from the reshaping of the Earth. Our planet is still geologically in cooling process. There occurs some energy, which creates pressure in the Earth's surface during the cooling. The Earth's surface is formed by tectonic plates, which move very slowly over and under each other. During the horizontal movement of the tectonic plates, some of them touch the neighbouring plates and this action causes physical and chemical changes in their structure. Sometimes they are locked together and the accumulated energy needs to be released. When this energy reaches a very high level, it needs to find a way to be released and so it breaks the plates. This is the main reason for the earthquake occurrence. Therefore, an earthquake can be defined as the vibration of the Earth's surface due to the release of the accumulated energy in the Earth's crust [Bolt, 1988, Coburn and Spence, 1992].

Scientists specify different types of earthquakes. The most well-known earthquake is a '*tectonic earthquake*', which is defined above. Almost 90 % of the earthquakes are of that kind. The second one is a '*volcanic earthquake*', which occurs as a result of volcanic eruptions with the same mechanism to change the surface structure of the Earth as in tectonic ones. Another type of an earthquake is the one, which occurs in the underground caverns and mines, where the roof of the cavern or mine collapses. It is called a '*collapse earthquake*'. Sometimes, landslides can produce earthquakes. The last type is an '*explosion earthquake*'. There are nuclear test sites around the world. When there is a detonation of nuclear and chemical devices in these sites, a big amount of nuclear energy is released and this may cause earthquakes [Bolt, 1988, Coburn and Spence, 1992].

How does an earthquake occur?

Vibrations occur during the breakdown of the plates due to the accumulated energy in the Earth's crust. These vibrations are called '*seismic waves*'. Seismic waves travel outward from the source of the earthquake, a point from where the waves first flow out, along the surface and through the Earth at varying speeds depending on the material through which they move. The origin, or the source of the earthquake's energy is called the '*focus of the earthquake*'. In natural earthquakes, the focus is located below the ground; whereas, in artificial ones, such as caused by nuclear explosions, the focus is near the Earth's surface. Earthquakes with a depth of 70 kilometres (43.5 miles) from the surface are called '*shallow-focus earthquakes*', the ones with that of from 70 to 300 kilometres (43.5 to 186 miles) are called '*intermediate-focus earthquakes*' and those deeper than 300 kilometres are called '*deep-focus earthquakes*'. The depth may reach more than 700 kilometres (435 miles) in deep-focus ones. The focuses of most earthquakes are concentrated in the crust and in the upper mantle. The point on the ground surface just directly above the focus is called the '*earthquake epicentre*'. The location of the earthquake simply determined by its epicentre and the depth of its focus [Bolt, 1988, Coburn and Spence, 1992].

There are three types of waves:

P-wave : The P-wave is generated in a body of the rock and is faster than the other waves. It can move in solid rock, like granite mountains, with a speed of 6 km/sec and in liquid material, such as volcanic magma and oceans, with a speed of 2 km/sec. This characteristic of the P-wave is similar to normal sound waves. When P-waves occur, a fraction of them can be transmitted into the atmosphere like sound waves so that animals and humans can hear them. In most of the earthquakes, the P-waves are felt first.

S-wave : Being generated in a body rock like P-wave, the S-wave is slower than P-wave with a speed of 3 km/sec. It cuts the rock sideways at right angles to the direction of travel. They can not travel in the liquid areas of the Earth.

Surface wave : This is so-called since its movements are always near to the ground surface. It is like little waves, a light fretting of the surface of a liquid, as with movements on a lake. They move slower than the body (P and S) waves. There are two kinds of surface waves:

a- Love wave: The love wave moves the ground from side to side in a horizontal plane but at right angles to the direction of transmission. The horizontal movement of the love waves damages the foundations of structures. It affects only the surface water as the sides of lakes and oceans.

b- Rayleigh wave: The Rayleigh wave moves both horizontally and vertically in a vertical plane, in the direction of the transmitting waves. It moves slower than the love wave.

The types of the waves can be determined from their movements out of the earthquake source to the ground surface. The P and S waves have reflection and refractivity characteristics. Some of their energy can be changed into other waves after reflection. There is a key point about the waves. That is, their seismicity, the temporal statistics of earthquake occurrence and the geological distribution of the earthquakes, changes by the type of the soil and the topography of the area. It is much safer to make construction on solid surfaces rather than sand, water-saturated soil and alluvium.

The breaking in the Earth's crust, which can be observed as discontinuities in rock structure, is called a '*fault*'. Some of the faults ended the displacements thousands of years ago. Therefore, they are called '*inactive faults*'. It is the active faults, which cause earthquakes with sudden ruptures as they move very slowly by the time. Their yearly movements can be measured in millimetres. They can be both on the surface of the earth or under the sea. There are three types of faults determined by their movements: *Normal*, *strike* and *reverse* faults. If the fault plane moves downward by the tension, it is a '*normal fault*'. When the fault planes pass horizontally through one another, it is a '*strike fault*'. The '*reverse*' fault is where the wall of the fault moves up from the dip of the fault plane by compression. The special case of reverse fault is a '*thrust fault*', when the dip of the fault is small. Vertical displacements occur in normal and strike faults, which are called '*dip-slip faults*', whereas the horizontal ones along the strike of the fault are called '*strike-slips*' [Bolt, 1988, Coburn and Spence, 1992].

There is no guarantee that whenever an earthquake occurs along a fault there will not be another one in the future. There can always be some energy, which was not released by the latest earthquake. By using the results of the geological surveys, it is safer to make construction away from the fault lines. This mitigation effort can reduce the damage after the earthquakes. Furthermore, dip-slip faults can cause more damage than the strike-slip ones. It is the responsibility of the city-planners, the engineers and the central and local administration to decide where and what to build in settlement areas.

How to measure earthquakes?

For the first time, Chang Heng, a Chinese scholar, invented the device '*seismoscope*' about 132 AD, which was used to record the earthquakes. By the use of a seismoscope, it is only possible to obtain information about the direction of the main impulse in the earthquake. Later on, other devices were developed for the investigation on the earthquakes. One of them was the '*seismograph*', which was developed at the beginning of the twentieth century. The seismograph gives us the detailed record of an earthquake from the beginning to the end. The zigzag line record is called a '*seismogram*'. It shows the motion of the earthquake on a magnetic tape either photographically or electromagnetically. The path of the P, S and surface waves can be followed by the seismograms.

Mainly two terms are used to describe the size of earthquakes. The first one is '*intensity*'. Intensity measures the severity of the shaking of the ground at a specific location. By using the intensity scales developed through time, it can be guessed that how the earthquakes can affect the people and the environment. An Italian scientist, Michele Stefano de Rossi and Francois Forel of Switzerland developed the first modern intensity scale in the 1880s. Today, the Modified Mercalli (MM) Intensity Scale is one of the scales used to measure the intensity (see Table 2.2). Originally, Giuseppe Mercalli, the Italian seismologist and volcanologist constructed this scale in 1902. H. O. Wood and Frank Neumann had revised the scale in 1931. Later, in 1956, the American scientist, Charles F. Richter again revised it by using the masonry as indicator of intensity in a 12-point scale. The 12-point scale is commonly used in Unites States. The one generally used in Europe is the Medvedev-Sponheuer-Karnik (MSK) scale. In Japan, the Japanese Meteorological Agency (JMA) scale and 7-point scale in use. In China, they have their own scales related to their building types.

The other term, which many people heard of, is the '*magnitude of an earthquake*'. The magnitude is a measure of the size of the earthquake. There are some scales to measure magnitude, like intensity scales. In 1931, K. Wadati originally prepared the most famous of them in Japan. Later, in 1935, Charles F. Richter developed one at the California Institute of Technology and it is named after him, the *Richter Scale.* The magnitude is a logarithmic scale, based on the amplitude of the maximum seismic wave recorded on a standard seismograph at a distance of 100 kilometres from the epicentre of the earthquake. As the magnitude increases by 1-unit, the amplitude of the waves increases by 10-units due to the logarithmic scale. For instance, an earthquake with magnitude 7.0 generates 10-times more ground motion than an earthquake of magnitude 6.0. Moreover, a 1-unit increase in magnitude results in

32-times stronger energy, which is the destructive power of the earthquakes. The earthquakes with magnitude 5.0 or more are considered to cause damage. There is a power relation, which is used to explain the effect of earthquakes with different magnitudes. Here is an example:

How much bigger is an earthquake with magnitude 8.4 than the one with 5.2?

$$\frac{10^{8.4}}{10^{5.2}} = 10^{8.4-5.2} = 10^{3.2} = 1585$$

Therefore, an earthquake with magnitude 8.4 has 1585 times destructive effect than the one with magnitude 5.2. The table below gives energy information about the effects of the earthquakes with different magnitudes [Coburn and Spence, 1992] (Page 21).

Magnitude	Effect
less than 4.5	An earthquake with magnitude less than 4.5 generally does not cause damage. Approximately 108 kilojoules of energy (equivalent to 10 tons of TNT exploded underground) is released in an earthquake of magnitude 4.5.
4.5-6.0	Damage generally occurs after an earthquake of magnitude 5.0. It is estimated that 109 kilojoules of energy (equivalent to 1000 tons of TNT exploded underground) is released in an earthquake of magnitude 5.5.
6.0-7.0	With magnitude 6.0, 1010 kilojoules of energy (equivalent to 6000 tons of TNT exploded underground) is released. This is 1012 kilojoules for an earthquake of 7.0 magnitude.
7.0-8.9	It is terrifying that an earthquake of magnitude 8.0 releases 1013 kilojoules energy that is equal to the explosion of 400 atomic bombs underground.

Table 2.1: The magnitude effect of earthquakes.

Modified Mercalli Scale Degree	Description
I	Generally not felt.
II	Felt very slightly, especially on upper floors of buildings.
III	Felt indoors by few people, vibration is weak.
IV	Felt indoors by many people, outdoors by few. Windows, walls, floors creak. Furniture shakes.
V	Felt by almost all. Animals are to be uneasy.
VI	People get frightened and run outdoors. Heavy furniture moves.
VII	Everybody runs outdoors. Driving people can feel it. Chimneys and poorly built structures crash.
VIII	Heavy furniture moves and some overturns. Monuments, columns etc. fall.
IX	General panic. Underground pipes are broken.
X	Dams, bridges etc. show critical damage. Most masonry and well-built structures are destroyed.
XI	Catastrophe. Few, if any, structures remain. Earth slumps. Rails bent greatly.
XII	The Earth's surface changes. Objects thrown into air.

Table 2.2: The Modified Mercalli Scale Degree and corresponding effects. Source: [Bolt, 1988, Coburn and Spence, 1992]

2.2 Introduction to Turkish Earthquakes

2.2.1 Hazard Profile of Turkey

The country profile of Turkey is sourced from the CIA Country Factbook.

Total area: 780,580 sq km

Total population: 70,413,958 (July 2006 estimate)

GDP purchasing power parity: 572 USDb (2005 estimate)

GDP per capita: 8,200 USD (2005 estimate)

Unemployment rate: 10 % (plus underemployment of 4.0 %) (2005 estimate)

Table 2.3 summarises some social and economical indicators of Turkey. It is observed that the Gross National Product (GNP) growth rate drastically hits a negative value in 1999 (-6.1) due to the effect of the 1999 earthquakes.

Indicators	1995	1996	1997	1998	1999
GNP (USD b)	171.9	184.6	194.1	205.8	187.5
GNP per capita (USD)	2,841	3,005	3,110	3,247	2,914
GNP growth rate (%)	8	7.1	8.3	3.9	**-6.1**
Unemployment rate (%)	6.9	6.0	6.4	6.3	7.3
Population age structure (PAS/0-14)	32.3	31.7	31.2	30.7	30.5
PAS / 15-64	63	63.5	63.8	64.2	64
PAS / 65+	4.7	4.8	5	5.1	5.5
Infant mortality (in thousand)	44.4	42.2	39.5	38.9	36.8
Average life expectancy (ALE)	67.9	68.2	68.6	68.8	68.9
ALE (Female)	70.3	70.5	70.9	71.2	71.3
ALE (Male)	65.7	65.9	66.3	66.5	66.6

Table 2.3: Basic Social and Economical Indicators of Turkey. *Source:* The State Planning Organisation, State Institute of Statistics, [JICA, 2004]

The United Nations Development Programme (UNDP) announces Turkey as the third country after Iran and Yemen according to the number of deaths as a result of earthquakes. Earthquakes are the types of disasters, which occur with low frequency but high severity. The UNDP also ranks Turkey 35th among fifty five countries for the flood losses. In [Gurenko et al., 2006], the number of fatal earthquakes occurred in Turkey during the twentieth century is reported as 111 with total fatalities of 99,391. Floods, rock-falls, landslides and avalanches are the other types of disasters that the country faces (see the Appendix), where floods and landslides are mainly experienced in the Black Sea Region and the coastal areas. Table 2.4 gives the figures of different types of natural disasters in Turkey since 1990.

Event	Date	Killed	Injured	Homeless	Affected	Loss in $ m
Earthquake (Erzincan)	13/03/1992	653	3,850	95,000	250,000	750
Avalanches (S.Anatolia)	1992 (14 events)	328	53	11,600	30,000	25
Avalanches (S.& E. Anatolia)	1993 (31 events)	135	95	1,100	300	10
Mud flood (Senirkent-Isparta)	13/07/1995	74	46	2,000	10,000	65
Earthquake (Dinar)	01/10/1995	94	240	40,000	120,000	100
Flood (Izmir)	04/11/1995	63	117	6,500	300,000	1,000
Earthquake (Çorum-Amasya)	14/08/1996	0	6	9,000	17,000	30
Flood (W. Black Sea)	21/05/1998	10	47	40,000	1,200,000	1,000
Earthquake (Ceyhan)	27/06/1998	145	1,600	88,000	1,500,000	500
Earthquake (Marmara)	17/08/1999	17,480	43,953	675,000	15,000,000	16,000
Earthquake (Düzce)	12/11/1999	763	4,948	35,000	600,000	750
Earthquake (Sultandaği)	03/02/2002	42	327	30,000	222,000	95
Earthquake (Bingöl)	01/05/2003	177	520	45,000	245,000	135
Total		19,964	55,802	1,078,200	19,494,300	20,460

Table 2.4: Natural Disasters in Turkey between 1990-2004. *Source:* [JICA, 2004], GDDA

Table 2.5 is compiled from the data of the General Directorate of Disaster Affairs and the Project Implementation Unit of the Prime Ministry Turkey and gives the type of the disaster and the related building collapse since the beginning of the twentieth century.

The type of the disaster	The number of collapsed buildings
Earthquake	612,000
Landslide	65,551
Flood	61,000
Rockfall	30,000
Avalanche	5,500
Total	774,051

Table 2.5: The types of the disasters in Turkey and resulting building collapse between 1900-2003. *Source:* GDDA, PIU

2.2.2 The History of Earthquakes in Turkey

Turkey is a peninsula, which is a bridge between the continents of Europe and Asia. The country is one of the most earthquake-prone countries in the world with 96 % of the total land, 98 % of the total population, 90 % of the cities, 755 industrial complexes and 40 % of the dams being situated in the active zones [Özerdem and Barakat, 2000]. An Earthquake Region Map (see the Appendix), which divides Turkey into five risk zones, has been published in 1996 by the General Directorate of Disaster Affairs, Ministry of Public Works and Settlement. This map shows that 66 % of land area of Turkey is located in risk zones 1 and 2, where 70 % of the total population live in and 69 % of the industrial facilities are located. After the 1999 earthquakes, the Earthquake Map of Turkey is being revised by the scientists since the fault structures changed significantly.

Anatolia, the main land of Turkey, contains many active fault lines. The North Anatolian fault line (NAF) and the East Anatolian fault line (EAF) are the most important ones to cause the devastating earthquakes. The NAF is the expansion of the Alpine-Himalayan fault line, which restricts the Arabian-Eurosian tectonic plates. Many studies have been carried out to understand the structure and the behaviour of the North Anatolian fault. The fault line starts in Karliova-Bingöl in Eastern Turkey and continues to the west of the Marmara region with a length of 1000 kilometers. It separates North Anatolia from Middle Anatolia. The most important earthquakes in the history of the Anatolian Peninsula were due to the ruptures in the North Anatolian fault. The NAF has some similarities with the San Andreas fault, which is the cause of the 1994 Northridge earthquake, California, in movement (both from east to west), slip rate, age, length and straightness (see the Appendix) [KOERI, 1999, Bibbee et al., 2000, Erdik, 2000].

The 1939 Erzincan earthquake is the start of the chain of earthquakes along the North Anatolian fault. Between 1939-1944, the fault was ruptured 600 kilometers to the west. Afterwards, this movement slowed down and another rupture of 100 kilometers was recorded between 1957-1967. The 1999 Marmara and Düzce earthquakes filled the 100-150 kilometers gap of the previous ruptures [Bibbee et al., 2000].

Magnitude	1900-1932	1933-1966	1967-2004
8.0-9.9	0	0	0
7.0-7.9	3	13	5
6.0-6.9	6	14	18
5.0-5.9	6	28	27
Total	15	55	50
Total number of estimated deaths	4,926	48,410	28,522

Table 2.6: The significant earthquakes in Turkey between 1900-2004. *Source:* The General Directorate of Disaster Affairs (GDDA)

The scientists prepared various earthquake scenarios for Istanbul following the 1999 earthquake. These scenarios expect an earthquake of magnitude 7.6 along the main Marmara Fault of the North Anatolian Fault. The probability that this earthquake will occur in the next 10 and 30 years are 65 % and 20 %, respectively

[Erdik, 2003]. It is estimated that there will be 70,000 life losses, 520,000 injuries (400,000 of which is heavy) and a direct economic loss of USD 30b [Erdik, 2003].

2.3 17/August/1999 Marmara (Kocaeli) Earthquake

On 17/August/1999, at 03:01:37 a.m. local time, one of the most devastating earthquakes in the history of Turkey occurred as a result of the 120-kilometre rupture of the North Anatolian fault near Akyazi-Yalova region and lasted approximately 45 seconds. The epicentre was in 18 kilometres (10.5 miles) depth near to Gölcük, the town 11 kilometres (7 miles) to the southeast of the city of Izmit (Kocaeli) where the country's main naval base is located. The magnitude of the earthquake was 7.4 in Richter Scale and caused 2.7 metres (9 feet) right-lateral strike-slip movement on the fault. Preliminary field reports confirm this type of motion on the fault, and initial field observations indicate that the earthquake produced at least 60 kilometres (37 miles) of surface rupture. The more specific magnitude measurements are given below:

Surface Wave Magnitude: 7.8 (U.S. Geological Survey-USGS)
Body Wave Magnitude: 6.3 (USGS)
Duration Magnitude: 6.7 (Boğaziçi University Kandilli Observatory and Earthquake Research Institute)
Moment Magnitude: 7.4 (USGS, Kandilli Observatory and Earthquake Research Institute)
Epicentre: 40.702N, 29.987E (USGS)
Depth: 18 km. (USGS)

In [Bibbee et al., 2000, Özerdem and Barakat, 2000], it is mentioned that since the 1906 San Francisco and 1923 Kwanto, Tokyo-Japan earthquakes, there was no earthquake to cause heavy damage to such an industrialised region like Marmara. The earthquake area is very densely populated, which consists of the 23 % of the entire population of Turkey. The mainly affected cities (Kocaeli, Sakarya, Bolu, Yalova) have a share for the 7 % of the Gross Domestic Product (GDP) and 13.5

% of industrial value added. Together with the other affected, surrounding cities (Istanbul, Bursa, Eskisehir) in the region, those values increase to 34.7 % and 46.7 %, respectively [Erdik, 2000]. The Marmara Region counts for the 5 % of export and 15 % of import trade of Turkey. The per capita income of the region is the double of the national average [SPO, 1999].

The earthquake caused almost 18,000 deaths and 45,000 injured that approximately 20,000 of the injured were left permanently disabled. Small coastal towns (e.g. Değirmendere, Gölcük) were severely affected by the earthquake. Apart from the earthquake itself, tsunamis (tidal waves due to the sea floor earthquakes) hit the towns. Since it was summer time, the population of the coast towns was higher than that of in winter. Many were killed when they were asleep. There are no confirmed records on the losses caused by tsunamis.

Monetary losses:

Industrial facilities: USD 2b (most of them insured with an insured value of USD 15b)

Buildings: USD 5b (about 8 % insurance penetration)

Railways: USD 1b

Highways: USD 0.2b

Ports: USD 0.2b

Telecommunication: USD 75m

Energy transmission: USD 3m

Average total losses (physical and socio-economic): USD 16-20b (approximately 7-9 % of the GDP).

The damage to industry was estimated at USD 1.1 - 4.5b by the public and private sectors. The added-value loss stemming from that is about USD 700m, which results in a 1.6 % decrease in the growth of the production sector. The payments of the claims are to be estimated around USD 600-800m since most of the industry losses were covered by insurance [Erdik, 2000, SPO, 1999].

There are many plants in the earthquake area owned either by state or international firms (e.g. Good Year, Pirelli, Ford, Honda, Hyundai, Toyota, Isuzu, Renault, Fiat, Bridgestone, Pepsi Co, Castrol, Dow Chemical, BP, Du Pont, Phillips, La

Farge). The State Planning Organisation (SPO) estimates a total loss of USD 880m for the state-owned ones (Tüpraş, Tuvasaş, Igsaş, Petkim, Seka and Asil Çelik).

It is also estimated by the State Planning Organisation that there is a burden of USD 6.2b on public finance caused by the earthquake. USD 3.5b of this amount is used for the reconstruction of houses after the disaster. The government enacted special earthquake taxes and allowed a paid military service for men between certain ages, which have not yet done their military service for various reasons. A year after the earthquake, USD 3b was obtained by these temporary solutions. International resources like the European Union and the World Bank supplied USD 2.5b for the reconstruction and recovery period. There was a 5 % decline in the GDP during 1999, which was stopped in the first half of 2000. By the August 2000 report of the SPO, a 5 % increase in annual GDP was realised [Selçuk and Yeldan, 2001].

Apart from all the economic losses briefly summarised above, there are some psychological effects of the earthquake which will take a long time to recover. People lost their families, homes and jobs. As a result, there has been a high increase in suicide, depression, alcohol consumption, problems in the families and the divorce rate. Most people do not want to get married on the 17th August and do not celebrate birthdays on that day.

The damage reports show that most of the collapsed and heavily damaged buildings were 6-8 storey, some were under construction and some were built within last few years before the earthquake. In Turkey, there is a current Building Code of 1998 with necessary regulations for sophisticated earthquake-resistant buildings. This code is an adaptation of the Californian Uniform Building Code to the standards of Turkey. Although those damaged and collapsed multistorey buildings are supposed to be earthquake-resistant, they were not due to the following factors [Gülkan, 2000]:

1. Inadequate vertical and horizontal reinforcing steel and the widespread use of smooth (as opposed to deformed) reinforcing steel,

2. No verification by the design and structural engineers, employee of the contractor that the contractor has used the intent of the design drawings during

the construction,

3. The use of poor and inappropriate materials in construction,

4. Poor workmanship,

5. Allowance to make construction on active faults and in areas of high liquefaction potential,

6. The use of past experience instead of the engineering techniques to make buildings.

Building damage statistics after the Marmara earthquake are given in Table 2.7 below.

	Number of housing units	Number of business premises
Total collapse-destroyed	66,441	10,901
Moderate damage	67,242	9,927
Light damage	80,160	9,712
Total	213,843	30,540

Table 2.7: The building damage after 1999 Marmara earthquake. *Source*: Government Crisis Centre, [Bibbee et al., 2000]

It is sad to observe that most of the collapsed buildings were the newest ones indicating the deterioration in the quality of design, construction and building control of modern structures. It is mainly the responsibility of the local authorities to check the application of the building code and regulations before and during the construction. Unfortunately, the control system does not work efficiently. On the other hand, many buildings in the most heavily damaged areas survived without significant damage because of the fact that they were designed with earthquake-resistant features with good quality materials and on firm ground or rock base.

Earthquake Protection and Preparedness

Earthquake itself, following fires, landslides, mudslides, avalanches, tsunamis, seiches, floods from dams and soil liquefaction are the important factors, which cause casualties and damages and increase the total losses. It is possible to reduce the losses of earthquakes by some mitigation methods and effective disaster risk preparedness and management plan. The following are some basic steps for individuals to do before, during and after an earthquake [Bolt, 1988]:

1. **Before an earthquake**:

i) Have a flash-light and a first-aid kit in your home. Each household should know where they are kept.

ii) Learn first-aid.

iii) Do not keep heavy objects on the shelves.

2. **During an earthquake**:

i) Keep calm. Stay wherever you are, indoors or outdoors. A large number of deaths are due to the heart attacks and jumping from balconies or windows due to panic.

ii) If near to an exit or can reach to exit easily, leave the building as soon as possible. Do not stay to collect belongings and valuables.

iii) If indoors (upstairs), stand against a wall near the centre of the building. Do not get close to the windows, balconies and outdoors. Do not try to run away through the elevators or stairs. If possible, stay close to heavy furniture like washing machine or dishwasher since they can provide a space for you to survive until the rescue teams reach.

iv) If outdoors, stay in the open. Keep away from overhead electric wires or anything that may fall.

v) Do not use candles, matches or any other kind of flames.

vi) If you are in a moving car, stop away from the overpasses and bridges and remain inside the car until the shaking stops.

3. **After an earthquake**:

i) Check yourself and others nearby for injuries. Provide first-aid if needed.

ii) Check water, gas and electric lines. If damaged, close the valves.

iii) Check for gas leaks. If detected, open all the windows and outdoors, leave immediately and report it to the authorities.

iv) Turn on the radio for emergency instructions. Do not keep the telephone lines busy as there may be some urgent calls.

v) Stay out of damaged buildings.

vi) Do not use lifting machinery, bulldozers or mechanical diggers to move the rubble even if they are available. It is highly possible survivors could be killed under the rubble if you use them. Use manual labour.

vii) Think of where people could be trapped under the rubble. Listen for people inside the rubble replying or making noise for help.

viii) Do not expose yourself and your rescue team to unnecessary risks. A collapsed building is highly unstable and dangerous.

Chapter 3

Distribution Theory

In this chapter, the available literature is revised on the claim number process, the claim arrival times, the total claim amount (the aggregate claims) process, the use of the aggregate claims in the actuarial context, the moment generating functions and the application of the extreme value theory with the use of extreme events data.

3.1 Poisson Process

3.1.1 Homogeneous and Inhomogeneous Poisson Process

The claim number process is a stochastic process, which has a very wide use in the insurance and actuarial studies in estimating the risk process of a company. Let $N(t)$ be the process of the number of claims in the interval (0,t]. In classical risk theory, the claim number process is generally assumed to have independent increments. If the intensity λ of claim frequency is independent of time t, then $N(t)$ is a homogeneous Poisson process with intensity $\lambda > 0$ [Bühlmann, 1970, Cox and Miller, 1965, Rolski et al., 1999]. Then the probability density function of the distribution of $N(t)$ is Poisson

$$Pr(N(t) = n) = \frac{(\lambda t)^n e^{-\lambda t}}{n!} \quad (n = 0, 1, 2 \ldots),$$

with the following properties

1. $N(0) = 0$,

2. For any time points $t_0 = 0 < t_1 < t_2 < \ldots < t_n$, the process increments $N(t_1) - N(t_0), N(t_2) - N(t_1), \ldots, N(t_n) - N(t_{n-1})$ are independent Poisson random variables and stationary (i.e. all increments of intervals of the same length have the same distribution).

The moments of the claim number process $N(t)$ are given as follows by using the properties of the Poisson distribution (mean=variance)

$$E(N(t)) = \lambda t,$$

and

$$Var(N(t)) = \lambda t.$$

The claim number process $N(t)$ is assumed to be a *counting process* with random time

$$t_n = W_1 + \ldots + W_n \quad (n \geq 1).$$

The differences $t_n = W_{n+1} - W_n$ are called event (claim) interoccurrence (interarrival) time. Any counting process generated by an independent identically distributed sum process t_n is also called as a *renewal counting process.* Since a Poisson process restarts itself at any point in time, there is no memory of the past (renewal) and this differs the Poisson process from other renewal processes.

One can also define $N(t)$ as a pure jump process with sample paths in $D[0, \infty)$ that increase to ∞ as $t \to \infty$ with jumps of height 1 at the random times t_n. Since any process with independent, stationary increments and simple paths in $D[0, \infty)$ is a *Lévy process*, $N(t)$ satisfies this definition and also entitled to be a *Lévy process.* By all of the properties it has, the claim number process $N(t)$ is [Embrechts et al., 1997]:

1. A homogenous Poisson process,

2. A renewal process,

3. A jump process,

4. A Lévy process.

The rate λ in a Poisson process $N(t)$ is the proportionality constant in the probability of an event occurring during an arbitrarily small interval. It is relevant to consider the rate λ as time varying, that is $\lambda = \lambda(t)$, in many applications. In that case, the process $N(t)$ is an *inhomogeneous (non-homogeneous)* or *non-stationary* with time. When $N(t)$ is an inhomogeneous Poisson process with rate $\lambda(t)$ (generalisation of a Poisson Process), then the increment $N(t) - N(0)$, given the number of events in an interval $(0, t]$, has a Poisson distribution with parameter $E[N(t)] \equiv \Lambda_0(t) = \int_0^t \lambda(r)dr$ and the increments over disjoint intervals are independent random variables [Cox and Miller, 1965, Cinlar, 1975, Daley and Vere-Jones, 2003]. In other words, the intensity λ is replaced by a mean value function (or the cumulative intensity function) $\Lambda(t)$ and $N(t)$ is said to be an inhomogenous Poisson process with the mean value function $\Lambda(t)$. This is called the *Cox* or *doubly stochastic* Poisson process, when $\Lambda(t)$ is itself random [Bühlmann, 1970, Basu and Dassios, 2002, Ruggeri and Sivaganesan, 2002, Ruggeri et al., 2000, Ruggeri and Pievatolo, 2002, Rolski et al., 1999]. An inhomogeneous Poisson process allows some events to occur more likely at certain times than during other times [Ross, 2003, Cox and Lewis, 1966, Lewis, 1972].

When one uses the real data on the number of claims arriving in a certain time interval, it is not always suitable to use the Poisson assumption. In many cases, negative binomial distribution is preferred, which can be obtained by mixing the Poisson distribution with a gamma distribution (mixed Poisson process). In the claim arrival processes, more variability is expected. In a mixed Poisson process, this variability decreases as time goes by. By a similar argument above, to stop the decrease in the variability, Cox processes are used with the cumulative intensity measure $\Lambda(a, b] = \int_a^b \lambda(r)dr$ (aka integrated rate function). One example of the mixed Poisson processes is the mixture of a Poisson process and Gamma distribution, which is also known as 'Pólya' or 'Pascal' process.

Remark 1: For an inhomogeneous Poisson process, there exists a non-decreasing, right-continuous function $\Lambda(t)$ and the number of events in an interval $(a, b]$ has a Poisson distribution with parameter $\Lambda(b) - \Lambda(a)$ [Vere-Jones, 1970].

Remark 2: The time-dependent intensity $\lambda(t)$ of an inhomogeneous Poisson process $N(t)$ is approximated by $\Lambda(t)$ as an expansion form by the integral over $\lambda(t)$ [Vere-Jones and Ozaki, 1982, Cox and Miller, 1965]. Therefore $N(t) \sim \text{Poisson}(\Lambda(t))$. This implies [Daykin et al., 1994b]:

1. $E(N(t)) = \Lambda(t)$,
2. $Var(N(t)) = \Lambda(t) \rightarrow$ standard deviation $= \sqrt{\Lambda(t)}$,
3. $\gamma(N(t)) = \frac{1}{\sqrt{\Lambda(t)}}$, where γ stands for the skewness,
4. $\gamma_2(N(t)) = \frac{1}{\Lambda(t)}$, where γ_2 stands for the kurtosis.

The use of the intensity $\Lambda(t)$ in modelling chapters of this thesis will be based on the Poisson binning approach. That is, the rate $\lambda(t)$ is assumed to be a constant over each bin, which tells us that the Poisson count for the bin is equal to $\Lambda(t) = \int_a^b \lambda(r)dr$, where Λ represents the expected number of events by time t.

Remark 3: By Remark 1, if $\int_a^b \lambda(r)dr = \Lambda(b) - \Lambda(a)$ holds, it implies that $\lambda(r)$ is the intensity function of Λ.

Remark 4: An important property of an inhomogeneous Poisson process is the additivity property of the intensities of the two independent inhomogeneous Poisson processes. This applies and easily proved for the case of regular Poisson distribution. It states in [Rolski et al., 1999] that suppose $N_1(t)$ and $N_2(t)$ are two independent inhomogeneous Poisson processes with intensity functions $\lambda_1(t)$ and $\lambda_2(t)$, respectively. The superposition $N(t)$, where $N(t) = N_1(t) + N_2(t)$ is an inhomogeneous Poisson process with intensity function $\lambda(t) = \lambda_1(t) + \lambda_2(t)$.

Let W_0 be the time of the first claim event to occur in $(0, t]$ and $N(t)$ be an inhomogeneous Poisson process with intensity λ. Then

$$\begin{aligned} Pr(W_0 > t) &= Pr(N(t) = 0) = Pr(\text{no claim occurs in } (0, t]) = p \\ &= Pr(\text{no event in}(0, t_1))Pr(\text{no event in}(t_1, t_2)) \\ &\quad \ldots Pr(\text{no event in}(t_{n-1}, t_n)), \end{aligned}$$

and

$$p \approx (1-\lambda(r)dr)(1-\lambda(r)dr)\dots(1-\lambda(r)dr) \approx \prod_t \Big(1-\lambda(r)dr\Big)$$

$$\log p \approx \log\Big(\prod_t (1-\lambda(r)dr\Big)$$

$$\log p \approx \sum_t \log\Big(1+(-\lambda(r)dr)\Big) \approx -\sum_t \lambda(r)dr,$$

by using $\log(1-x) \cong -x + o(x^2)$, where o is a function of x,

$$\log p \approx -\int_0^t \lambda(r)dr$$

$$p \approx e^{-\int_0^t \lambda(r)dr},$$

and the cumulative distribution function is

$$F_0(t) = Pr(W_0 \leq t) = 1 - Pr(W_0 > t) = 1 - e^{-\int_0^t \lambda(r)dr}.$$

As explained in [Bühlmann, 1970], Markov processes are the generalisation of the processes with independent increments. Since the process above is Markovian, the times $W_1, W_2 \dots$ form a renewal (inhomogeneous Poisson) process and we can, by a similar argument to the above, show that conditional on W_i, the distribution of the time to the next claim event, $t_i = W_{i+1} - W_i$ has the following distribution function [Cox and Lewis, 1966]

$$F_i(t|W_i) = 1 - e^{-\Lambda_i(t)},$$

where

$$\Lambda_i(t) = \int_{W_i}^{W_{i+1}} \lambda(r)dr, \tag{3.1}$$

and by differentiation, the probability density function is [Cox and Lewis, 1966]:

$$f_i(t) = \lambda(W_{i+1})e^{-\Lambda_i(t)}.$$

In the modelling chapters of this thesis, special kernel functions will be used: the exponential and the power kernel functions. It might be interesting to search for the type of the distribution of the first event occurrence time, $f_0(t)$, for these kernel choices before we continue with the total claim amount process. The exponential and the power kernel functions simply have the following forms, respectively:

$$\log \lambda(t) = \alpha e^{-\beta t},$$

and

$$\log \lambda(t) = \alpha t^{\beta}.$$

If the exponential kernel is substituted in $f_0(t)$ with $\lambda(t) = e^{\alpha e^{-\beta t}}$, we have

$$f_0(t) = e^{\alpha e^{-\beta t}} e^{-\int_0^t e^{\alpha e^{-\beta t}} dr}.$$

The question arises here is to find the recognisable distribution of this form. For $\beta = 1$, the exponential kernel takes the form $\log \lambda(t) = \alpha e^{-t}$ and this implies $\lambda(t) = e^{\alpha e^{-t}}$. By the same argument above

$$f_0(t) = e^{\alpha e^{-t}} e^{-\int_0^t e^{\alpha e^{-t}} dr}.$$

If the integral above is solved, we have $e^{\alpha e^{-t}(1-t)}$ and this might suggest the distribution of the first event occurrence time by the use of the exponential kernel function.

When $\beta = 1$, the use of the power kernel results in a familiar distributional form. That is, for $\beta = 1$, $\log \lambda(t) = \alpha t$, which is log-linear case itself and $\lambda(t) = e^{\alpha t}$. Therefore

$$f_0(t) = e^{\alpha t} e^{-\int_0^t e^{\alpha t} dr} = e^{\alpha t} e^{-e^{\alpha t} t} = e^{\alpha t} e^{-t e^{\alpha t}}.$$

Let $y = e^t$, then $\log y = t$ with the Jacobian $|\frac{\partial t}{\partial y}| = |\frac{1}{y}|$. Then, the distribution of the occurrence of first event with the use of the power kernel is

$$f_Y(y) = |\frac{1}{y}| y^{\alpha} y^{-y^{\alpha}},$$

which is a form of Weibull distribution. If $Y \sim$ Weibull then $\log Y \sim$ logWeibull (aka the Fisher-Tippett distribution) appears as a distribution of the occurrence of first event with the use of the power kernel by $\log y = t$ transformation.

If we use the form $\log \lambda(t) = \alpha t^{-\beta}$ and substitute $\beta = 1$, then $\log \lambda(t) = \frac{\alpha}{t}$. If we check the plot of the intensity $\lambda(t)$ for this case when $t = 0$, the result is infinity for both $\lambda(t)$ and $\Lambda(t)$, where $\Lambda(t)$ is the integral of $\lambda(t)$ over the given interval. Either the form $t^{-\beta}$ or $-t^{\beta}$ gives the decreasing power kernel function as we expect it to happen to represent the claim decay pattern. We will use the form of $t^{-\beta}$ for the rest of the analysis to keep consistency with the use of the exponential kernel function.

This will not cause a big gap in the analysis, since we use the + part of the $t_i - s_j$ difference (in notation $(t_i - s_j)|_+$), where we never have $t_i = s_j$ case (please note the use of t_i for the actual event, s_j to represent the site of the kernel knot).

3.2 The total claim amount (aggregate claims) process

In the insurance context, the total claim amount process, $S(t)$, has a very wide use. In this process, the claim amount X_i, which corresponds to each policy, is summed up over a given time period. The calculation of $S(t)$ depends on the claim size (amount) X_i and the claim number process $N(t)$ in a given time period and that leads to the calculation of the net (pure) premium [Embrechts et al., 1997, Bühlmann, 1970]. A claim is called 'large', when it consumes a large portion of the total claim amount [Rolski et al., 1999].

When the claim amount distributions are independent but not necessarily identical, the total claim amount process is called as the '*individual model*'. The aggregate claims model, which is often used to approximate the individual claims model [Kaas et al., 2001], is called the '*collective model*' [Rolski et al., 1999] and defined with the following equation, assuming that the claim size X_i's and the claim number process $N(t)$ are independent and also X_i's are independently identically distributed (i.i.d)

$$S(t) = \sum_{i=1}^{N(t)} X_i \quad (t \geq 0),$$

where $S(t) = 0$ if $N(t) = 0$.

In the collective risk model, the insurance portfolio is thought to be a process, which generates claims over time [Kaas et al., 2001]. In both the individual and collective models, the total claims on a portfolio of insurance contracts is the random variable of interest [Kaas et al., 2001]. The total claim amount $S(t)$ is a random variable, when the claim number process $N(t)$ and the claim amount process $X(t)$ are stochastic, time-dependent random variables. Here, both the claim number process $(N(t))$ and the claim amount process $(X(t))$ are assumed to be time-varying as given in their notation. If the number of claims in the portfolio is large and rare, it is reasonable to approximate from individual model to collective model, because collective models are computationally more efficient to work with [Kaas et al., 2001]. In further computations, the aggregate claims are assumed to be exempted from the effects of the micro/macro economical indicators (e.g. inflation, unemployment rate, interest rate, taxes) of the country and the management expenses (e.g. cost of office equipment, salaries of employees, commissions to agents, rental of offices) [Booth et al., 1999, Rolski et al., 1999, Hogg and Klugman, 1984].

The claim number process $N(t)$ can be chosen as Poisson, binomial, Pascal (negative binomial), geometric or other types of appropriate distributions. In actuarial analysis, mainly the three types (Poisson, Pascal and geometric) are of interest. When the claim number process $N(t)$ is an inhomogeneous Poisson process with the mean function $\Lambda(t)$, the total claim amount process $S(t)$ is a '*Compound Poisson process*'. Also, if $N(t)$ is Negative Binomial, then $S(t)$ has a compound negative binomial distribution [Kaas et al., 2001]. This compound case is a special interest in the actuarial context and the total claim amount has the following distribution function at time t [Rolski et al., 1999, Daykin et al., 1994b, Kaas et al., 2001]

$$F_{S(t)}(s) = Pr(S(t) \leq s) = Pr(\sum_{i=1}^{N(t)} X_i).$$

It is almost impossible to derive an explicit formula for this distribution function. Therefore by using the law of total probability and under the assumption that $N(t)$ and X_i are independent where X_i are independent identically distributed, the distribution function F of $S(t)$ is written in the following (3.2) by using the convolution (see the Appendix) of X's

$$\begin{aligned} G^k(x) &= Pr(X_1 + \ldots + X_k \leq s) \\ &= \int_{-\infty}^{\infty} G^{k-1}(x-s)dG(s), \end{aligned}$$

with

$$G^0(x) = 1 \text{ for } x \geq 0 \text{ and } 0 \text{ for } x < 0.$$

Then

$$
\begin{aligned}
F_{S(t)}(s) &= Pr(S(t) \leq s) \\
&= \sum_{k=0}^{\infty} Pr\Big(\sum_{i=1}^{N(t)} X_i \leq s \mid N(t) = k\Big) Pr(N(t) = k) \\
&= \sum_{k=0}^{\infty} \frac{\Lambda(t)^k e^{-\Lambda(t)}}{k!} G^k(s),
\end{aligned}
\tag{3.2}
$$

where $G^k(s)$ is the k^{th} fold convolution of the X distribution.

Remark 5: The necessary condition to be a Lévy process is to be infinitely divisible. The total claim amount $S(t)$ is also a Lévy process since the Poisson distribution satisfies this condition.

The expectation of the aggregate claims, $E(S(t))$, is used as the net premium in actuarial calculations. With the assumption of independent identically distributed X_i's, let $E(X_i) = \eta$ and $Var(X_i) = \tau$. Then, the central moments (mean and variance) of $S(t)$ can be obtained for an inhomogeneous process $N(t)$ with rate $\Lambda(t)$ [Embrechts et al., 1997, Karlin and Taylor, 1994] by conditioning on the number of claims as

$$
\begin{aligned}
E(S(t)) &= \sum_{k=0}^{\infty} E(S(t) \mid N(t) = k) Pr(N(t) = k) \\
&= \sum_{k=1}^{\infty} E(X_1 + \ldots + X_{N(t)} \mid N(t) = k) Pr(N(t) = k) \\
&= \sum_{k=1}^{\infty} \eta N(t) Pr(N(t) = k) \\
&= \eta \sum_{k=1}^{\infty} N(t) Pr(N(t) = k) = \eta E(N(t)) = \eta \Lambda(t) \\
&= E(X(t)) E(N(t)).
\end{aligned}
\tag{3.3}
$$

Also

$$
\begin{aligned}
Var(S(t)) &= E\Big((S(t) - \eta\Lambda(t))^2\Big) \\
&= E\Big((S(t) - N(t)\eta + N(t)\eta - \Lambda(t)\eta)^2\Big) \\
&= E\Big((S(t) - N(t)\eta)^2\Big) + E\Big(\eta^2(N(t) - \Lambda(t))^2\Big) \\
&\quad + 2E\Big(\eta(S(t) - N(t)\eta)(N(t) - \Lambda(t))\Big),
\end{aligned}
\tag{3.4}
$$

$$
\begin{aligned}
E\Big((S(t)-N(t)\eta)^2\Big) &= \sum_{k=0}^{\infty} E\Big((S(t)-N(t)\eta)^2 \mid N(t)=k\Big) Pr_{N(t)}(k) \\
&= \sum_{k=1}^{\infty} E\Big((X_1+\ldots+X_{N(t)}-k\eta)^2 \mid N(t)=k\Big) Pr_{N(t)}(k) \\
&= \tau \sum_{k=1}^{\infty} k Pr_{N(t)}(k) \\
&= \tau \Lambda(t),
\end{aligned}
\tag{3.5}
$$

and

$$
\begin{aligned}
E\Big(\eta^2(N(t)-\Lambda(t))^2\Big) &= \eta^2 E\Big((N(t)-\Lambda(t))^2\Big) \\
&= \eta^2 \Lambda(t),
\end{aligned}
\tag{3.6}
$$

while

$$
\begin{aligned}
E\Big(\eta(S(t)-N(t)\eta)(N(t)-\Lambda(t))\Big) &= \eta \sum_{k=0}^{\infty} E\Big((S(t)-k\eta)(k-\Lambda(t)) \mid N(t)=k\Big) Pr_{N(t)}(k) \\
&= \eta \sum_{k=0}^{\infty} (k-\Lambda(t)) E\Big((S(t)-k\eta) \mid N(t)=k\Big) Pr_{N(t)}(k) \\
&= 0 \\
&\quad (\text{since } E\Big((S(t)-k\eta) \mid N(t)=k\Big) = \\
&\qquad E(X_1+\ldots+X_{N(t)}-k\eta)=0).
\end{aligned}
\tag{3.7}
$$

When (3.5), (3.6) and (3.7) are substituted in (3.4), the variance of the total claim amount process $S(t)$ is (the famous variance decomposition rule)

$$
\begin{aligned}
Var(S(t)) &= E(N(t))Var(X(t)) + Var(N(t))(E(X(t)))^2 \\
&= \Lambda(t)\tau + \eta^2\Lambda(t) + 0 = \Lambda(t)(\tau+\eta^2).
\end{aligned}
$$

We shall see a way of obtaining all moments in Section 3.3.1 below. A simple example of the use of the mean and the variance of the total claim amount is given next.

Numerical Example

1- i) For an aggregate claims model, which is based on 5000 claims, a claim frequency (N_i) model has a Poisson distribution with 10 claims per month. Individual losses (X_i) follow a lognormal distribution with mean 10 and variance 15. Then the mean and variance of the aggregate claims are found as

$$E(S(t)) = E(N)E(X) = 10(10) = 100,$$

which means the expectation of the total claim amount is 100 YTL (Turkey New Lira) per month. And the variance is

$$Var(S(t)) = E(N(t))Var(X(t)) + Var(N(t))(E(X(t)))^2 = 10(15) + 10(10)^2 = 1150,$$

For instance, if Normal approximation is used to estimate the probability S is greater than 150 YTL

$$\begin{aligned} Pr(S > 150) &= Pr(S > \frac{150 - E(S)}{\sqrt{Var(S)}}) = Pr(Z > \frac{150 - 100}{\sqrt{1150}}) \\ &= Pr(Z > 1.47) = 1 - 0.9292 = 0.0708, \end{aligned}$$

which says the probability that the aggregate claims will be more than 150 YTL per month is 7 %.

3.2.1 The Risk Reserve and Premiums

The early studies in the risk theory started with life insurance and individual risk units, that is the number of insured people. Later, the studies of Lundberg, Cramér and other Swedish researchers initiated a new period of development of risk theory. This is called the 'collective theory of risk', which is mentioned previously. The claim occurrences are studied on the basis of collectivity, not on individual claims [Daykin et al., 1994b].

The collective model of the total claim amount process, $S(t)$, in the classical model for the insurance risk $R(t)$-(surplus, or free reserves)-is given as follows [Asmussen, 2000, Albrecher and Asmussen, 2005, Daykin et al., 1994b, Rolski et al., 1999]:

$$R(t) = u + \Pi(t) - S(t),$$

where u = initial surplus (capital), $\Pi(t) = ct$ (aggregate premiums), c is the loaded premium rate at time t.

The stability of an insurer is studied in the ruin models [Kaas et al., 2001]. The capital of the company is assumed to increase linearly in time by fixed annual premiums starting with capital u at time $t = 0$, but whenever a claim occurs with the jump effect, the capital decreases [Kaas et al., 2001]. When $R(t)$ is 0 or less, ruin occurs for an insurance company, which probably happens due to wrong investment. Some of the suggestions to stop ruin might be to buy more reinsurance, arrange the premium ratings or to increase the initial capital [Kaas et al., 2001]. The mean of the risk process is

$$E(R(t)) = u + ct - E(S(t)), \tag{3.8}$$

and if (3.3) is replaced in (3.8) we have

$$E(R(t)) = u + ct - \mu\Lambda(t). \tag{3.9}$$

Moreover, the probability of ruin is calculated as

$$\Psi(u) = Pr(R(t) < 0, \text{for some time } t > 0 \mid \text{initial surplus} = u), \tag{3.10}$$

which denotes the probability that the surplus will reach below zero given a level of initial surplus (capital) u.

One special model commonly used in calculating the ruin probabilities in financial/actuarial mathematics is the '*Sparre Andersen model*'. Consider the claim number process $N(t)$ as a renewal counting process and let the claim amount X_i's be independent identically distributed (iid) with a distribution function F_X. If $U_i, i = 1, \dots,$ denote the iid interclaim time random variables with a common distribution F_U, then the Sparre Andersen model is also given by [Rolski et al., 1999, Gerber and Shiu, 2005]

$$R(t) = u + \Pi(t) - S(t),$$

where the initial surplus $u > 0$.

Premium

The existence and survival of the insurance industry depend on the willingness of people to pay a price for being insured [Kaas et al., 2001]. Basically, premium means an adequate price to insure risks. The $\Pi(t)$ is calculated by some methods to guarantee the solvability of the portfolio. The premiums can not be low because

$\Pi(t)$ should provide enough money to the insurers to be able to cope with incoming claims. Also, the amount of $\Pi(t)$ should not be very high because it might result in losing the current clients to the rival insurance companies if the others keep the premiums lower to attract your customers. Some important properties of the premium Π for risks X and Y are summarised below as [Rolski et al., 1999]

1. No unjustified safety loading if, for all constants $c \geq 0$, $\Pi(c) = c$,

2. Proportionality if for all constants $c \geq 0$, $\Pi(c) = c$,

3. Additivity if $\Pi(X + Y) = \Pi(X) + \Pi(Y)$ (valid for independent risks and depends on the joint distribution of $X + Y$),

4. Subadditivity if $\Pi(X + Y) \leq \Pi(X) + \Pi(Y)$ (policyholders can not gain advantage when the risk is split),

5. Consistency if, for all $c \geq 0$, $\Pi(X + c) = \Pi(X) + c$,

6. Preservation of stochastic order if $X \leq_{st} Y \Rightarrow \Pi(X) \leq \Pi(Y)$.

A premium principle assigns a real number to the risk as a financial compensation for the risk taker. The most commonly used premium calculation is called the 'expected value principle', that is [Rolski et al., 1999, Kaas et al., 2001]

$$\Pi(S) = (1 + c)E(S),$$

where $E(S) < \infty$.

If $c = 0$ above, $\Pi(S) = E(S)$ and it is called the 'net premium principle'. The difference of $\Pi(S) - E(S)$ is called the 'safety loading', which is supposed to be positive for the survival of a company.

As these two principles do not take the variability of the risk S into account, it is more risky to the insurers. Therefore, the following principles are also used in the premium calculations for some constant c [Kaas et al., 2001, Rolski et al., 1999]:

1. Variance Principle: $\Pi(S) = E(S) + cVar(S)$,

2. Standard Deviation Principle: $\Pi(S) = E(S) + c\sqrt{Var(S)}$,

3. Exponential Principle: $\Pi(S) = \frac{1}{c}\log E(e^{cS})$.

Referring to the numerical example in Page 50:

ii) Using the Normal approximation, what is the minimum premium that should be charged to ensure that the probability of a loss (that is the aggregate losses are greater than the premium) is less than 10 %?

The expected value principle is used to solve this question and the premium rate c is obtained by

$$Pr\Big(S > (1+c)E(S)\Big) = 0.10,$$

after standardisation

$$Pr\Big(Z > \frac{cE(S)}{\sqrt{Var(S)}}\Big) = 0.90,$$

and from the Standard Normal Table

$$\frac{cE(S)}{\sqrt{Var(S)}} = 1.282 \rightarrow c = \frac{(1.282)\sqrt{1150}}{100} = 0.435,$$

so the pure premium with no deductible is

$$\text{Premium} = cE(S) = 0.435(100) = 43.5.$$

In the actuarial context, the information of the average claim payment gives an idea to the insurer about how much the cost will be in case of a disaster. This has a very important use for the preparation of the insurance contracts [Hogg and Klugman, 1984]. For the data of this study (n=4297 and the total claim payment in thousand YTL is 7,329), the average payment per claim [Bühlmann, 1970] is

$$E\big(S(t)\big) = \frac{\text{total claim payments}}{\text{total number of claims}} = \frac{7,329,000.00}{4297} = 1,71 \text{ YTL}, \qquad (3.11)$$

which is quite a significant amount.

The average payment per claim for the Turkish Catastrophe Insurance Pool data by the figures of 24/May/2006 (claims gathered in the Pool from December 2000-current) is

$$E\big(S(t)\big) = \frac{\text{total claim payments}}{\text{total number of claims}} = \frac{15,943,264.89}{8263} = 1,929.48 \text{ YTL}. \qquad (3.12)$$

3.2.2 Insurance/Reinsurance

It is better to remind the leading definitions of risk before giving the details of the idea of insurance/reinsurance. Risk is simply the uncertainty concerning the occurrence of a loss. In terms of disasters, risk is the probability of expected losses (deaths, injuries, environmental damage and economic activity disruptions) resulting from interaction between natural and man-made hazards and vulnerable conditions.

Insurance is a risk transfer mechanism. It is a way of spreading risk. The insurance companies (insurers) mainly insure pure risk, which has a possibility of *loss* or *no loss.*

Basic Characteristics of Insurance

According to [Rejda, 2003]:

1. *Pooling of losses:* It means the sharing of the losses and constructs the heart of the insurance. By considering the few over the entire group of portfolio, the actual loss is substituted with the average loss.

2. *Payment of fortuitous losses:* It is a payment of the unexpected loss, which occurs by pure chance.

3. *Risk transfer:* The transfer of pure risk from the insured to the insurer, who is financially much stronger to cover the possible losses.

4. *Indemnification:* The repositioning of the financial situation of the insured prior to the occurrence of a loss.

Basic Parts of an Insurance Contract

1. *Declarations:* Information about the property or the activity to be insured.

2. *Definitions:* This section defines the meaning of the key words and phrases in the policy to ease the understanding of the coverage. For instance "you", "your", "we" and "us" are used in this part.

3. *Insurance agreement:* Summarises the major promises of the insurer. It has two parts: 'named-perils coverage' and "all-risks" policy.

4. *Exclusions:* It provides the information on the exclusions of the policy like excluded perils, excluded losses and excluded property.

5. *Conditions:* These are the provisions in the policy to limit the promises of the insurer.

6. *Miscellaneous provisions:* This part includes some provisions like the cancellation terms, reinstatement of lapsed policy, misstatement of age.

The ideal expectation of the insurers is to sell insurance for non-catastrophic losses. One way to cope with the catastrophic risks is to reinsure the risk that is on the insurers [Rejda, 2003].

Therefore, reinsurance is defined as the insurance of insurers against high risk. Reinsurance is needed by the insurers, when there occurs:

1. a very large individual claim (like TÜPRAŞ oil refinery losses of the Marmara earthquake),

2. an event like a flood, an earthquake, which results simultaneously in claims from a large number of separate policies.

An insurance company transfers its risk to another insurance company so that large risk is splitted into small portions and shared between different companies. The first company, who transfer that risk, is called a 'cedant'. An insurance company seeks for reinsurance, when it wants to reduce the possible huge losses, which might be a threat for the survival of the company. These big losses may come from a large number of claims as a result of earthquakes, hurricanes etc., excessively large claims arriving after severe accidents etc. like an explosion in a nuclear plant, legal restrictions or changes in premium collections due to cases like a sudden change in inflation [Rolski et al., 1999]. When a reinsurer thinks the risk is too large for him as well, he insures himself, which is called a 'retrocession'.

Types of Reinsurance

The insurer keeps some portion of the aggregate claims, $S(t)$, for himself considering the capacity of its portfolio and reinsures the remaining part. The amount, which the insurer agrees to pay himself, is called the 'deductible' amount and is denoted as $d(S(t))$. The difference of $S(t) - d(S(t))$ is the 'reinsured part'. The following are some kinds of reinsurance [Rolski et al., 1999]:

1. *Proportional or quota-share reinsurance*: This is a very popular type of reinsurance especially with starting small companies, in which a certain amount a of the total portfolio is reinsured. In this case, as a special case $d(s) = as$,

$$d(S(t)) = aS(t) = a\sum_{i=1}^{N(t)} X_i = \sum_{i=1}^{N(t)} d(X_i).$$

   Then, the distribution of $d(S(t))$ is

$$Pr(d(S(t)) \leq s) = Pr(S(t) \leq \frac{s}{a}) = F_{S(t)}(\frac{s}{a}).$$

2. *Excess-loss reinsurance*: This type of reinsurance is determined by a positive number b, which is called the 'retention level'. Then, the reinsured amount is $\sum_{i=1}^{N(t)}(X_i - b)_+$. Excess-loss reinsurance limits the liability of the cedant and has a wide use in motor-liability and windstorm reinsurance as it is especially used in small number of risks and when the individual claim sizes are heavy-tailed.

3. *Stop-loss reinsurance*: It is defined as $(\sum_{i=1}^{N(t)} X_i - b)_+ = (S(t) - b)_+$. Here, small claims have a lot of influence on the total claim amount. Stop-loss reinsurance is simple to apply and does not require high administrative expenses. It is commonly used for windstorm and hail reinsurance and in some occasions for fire reinsurance.

4. *Le Traité d'Excédent du Coût Moyen Relatif (ECOMOR)*: When the large claims are considered, it is suitable to look for the largest claim treaties in the portfolio. The French actuary Thépaut first introduced the idea of ECOMOR

in 1950. If the order statistics of the claim size $(X_1 \ldots X_{N(t)})$ is denoted as $(X_{(1)} \ldots X_{(N(t))})$, then we can define a nice form of the reinsurance treaty as

$$Z(t) = \sum_{i=1}^{r} X_{(N(t)-i+1)} - rX_{(N(t)-r)} = \sum_{i=1}^{N(t)} (X_i - X_{N(t)-r})_+.$$

The amount $Z(t)$ covers only the r largest claims that exceeds the random retention level $X_{N(t)-r}$. If the counting process $N(t) \leq r$, then $X_{N(t)-r} = 0$. ECOMOR is similar to the type excess-of-loss but with a random retention at a large claim.

3.3 The Distribution of the Aggregate Claims

The distribution of the aggregate claims is an interesting subject in the insurance context. There have been numerical problems in the direct calculation of such distribution by convolution. Therefore, some approximation methods are used to approach the distribution of the total claim amount for large t. Since this distribution approximation is related to the moment generating and cumulant functions, these are explained next. The general form of the distribution of $S(t)$ is already stated in (3.2).

3.3.1 Moment Generating Function (mgf)

In Statistics, two important characteristics are needed to determine a type of a distribution: the mean and the variance. The mean is the first moment and the variance is the second moment of a distribution, which can be obtained by using the moment generating function.

The moment generating function, except for a sign in ϑ, is the Laplace transform of the distribution function. The mgf transform uniquely determines a distribution of probability, since it is one-to-one transform and so every cumulative distribution function (cdf) has exactly one mgf [Kaas et al., 2001]. As mentioned earlier, it is not easy to determine the distribution function of the aggregate claims. Therefore, the moment generating function is one way to help to find such distribution. The moment generating function of the Compound Poisson total claim amount S, when $N(t)$ is Poisson $(\Lambda(t))$, has the following form by using (3.2)

$$M_S(\vartheta) = E(e^{\vartheta s}) = \int_0^\infty e^{\vartheta s} f(s)ds = \int_0^\infty e^{\vartheta s} dF_S(s)$$

$$\begin{aligned} M_S(\vartheta) &= \int_0^\infty e^{\vartheta s} d(\sum_{k=0}^\infty \frac{\Lambda(t)^k e^{-\Lambda(t)}}{k!} G^k(s)) \\ &= \sum_{k=0}^\infty \frac{\Lambda(t)^k e^{-\Lambda(t)}}{k!} \int_0^\infty e^{\vartheta s} d(G^k(s)) \\ &= \sum_{k=0}^\infty \frac{\Lambda(t)^k e^{-\Lambda(t)}}{k!} (M_X(\vartheta))^k = e^{-\Lambda(t)} \sum_{k=0}^\infty \frac{(\Lambda(t) M_X(\vartheta))^k}{k!} \\ &= e^{\Lambda(t)(M_X(\vartheta)-1)}, \end{aligned} \tag{3.13}$$

which is true for any X distribution.

The mean and the variance of the total claim amount S can be derived using the moment generating function method. If interested, higher moments can also be derived. (3.13) states

$$M_S(\vartheta) = E(e^{\vartheta s}) = e^{\Lambda(t)(M_X(\vartheta)-1)}.$$

So, if $X \sim \mathrm{N}(\eta, \tau)$, then the moment generating function of the total claim amount is

$$M_S(\vartheta) = e^{\Lambda(t)(e^{\eta\vartheta + \frac{\tau\vartheta^2}{2}} - 1)}.$$

If we differentiate this moment generating function with respect to the parameter ϑ

$$M_S'(\vartheta) = e^{\Lambda(t)(e^{\eta\vartheta + \frac{\tau\vartheta^2}{2}} - 1)} \Lambda(t) e^{\eta\vartheta + \frac{\tau\vartheta^2}{2}} (\eta + \vartheta\tau).$$

If $\vartheta = 0$ is substituted in $M_S'(\vartheta)$

$$M_S'(0) = e^{\Lambda(t)(e^0 - 1)} \Lambda(t)\eta = \eta\Lambda(t),$$

which is the mean of the total claim amount (aka the aggregate mean) and if the moment generating function is differentiated twice with respect to the corresponding parameter

$$\begin{aligned} M_S''(\vartheta) &= e^{\Lambda(t)(e^{\eta\vartheta + \frac{\tau\vartheta^2}{2}} - 1)} \Big(\Lambda(t) e^{\eta\vartheta + \frac{\tau\vartheta^2}{2}} (\eta + \vartheta\tau)\Big)^2 \\ &\quad + e^{\Lambda(t)(e^{\eta\vartheta + \frac{\tau\vartheta^2}{2}} - 1)} \Big(e^{\eta\vartheta + \frac{\tau\vartheta^2}{2}} (\eta + \vartheta\tau)\Lambda(t)(\eta + \vartheta\tau) + e^{\eta\vartheta + \frac{\tau\vartheta^2}{2}} \Lambda(t)\tau\Big), \end{aligned}$$

and if $\vartheta = 0$ is substituted

$$M_S''(0) = \Lambda(t)^2\eta^2 + \eta^2\Lambda(t) + \Lambda(t)\tau.$$

Therefore, the variance of the total claim amount process, $S(t)$, is obtained as before

$$Var(S(t)) = M_S''(0) - (M_S'(0))^2 = \Lambda(t)^2\eta^2 + \eta^2\Lambda(t) + \Lambda(t)\tau - (\eta\Lambda(t))^2 = \Lambda(t)(\eta^2 + \tau).$$

The general form of the moment generating function of S in terms of the mgf for any chosen X distribution is

$$M_S^n(\vartheta) = e^{\Lambda(t)(M_X(\vartheta)-1)}\Big(\frac{\partial}{\partial\vartheta}\Lambda(t)M_X(\vartheta)\Big)^n + e^{\Lambda(t)(M_X(\vartheta)-1)}\frac{\partial^n}{\partial^n\vartheta}(\Lambda(t)M_X(\vartheta)).$$

Cumulant generating function:

The natural logarithm of the moment generating function is called the 'cumulant generating function'. The cumulants of the probability distribution of a random variable S for the aggregate claims is defined in [Seal, 1969] as

$$E(e^{\vartheta S}) = e^{(\sum_{k=0}^{\infty}\frac{\kappa_k\vartheta^k}{k!})}.$$

The cumulants help us to determine the central moments of a distribution or to characterise a random variable of that distribution (e.g say the first cumulant $\kappa_1 = \mu$ and the second cumulant $\kappa_2 = \sigma^2$). Basically, differentiating the cumulant function three times and setting $\vartheta = 0$, the mean, the variance and the third moment of the random variable of the interest are obtained [Kaas et al., 2001]. Cumulants were first introduced in 1889 by the Danish astronomer, actuary, mathematician and statistician Thorvald N. Thiele and were called as 'half-invariants'. The actual name 'cumulants' is first used in a 1931 paper 'The derivation of the pattern formulae of two-way partitions from those of simpler patterns', Proceedings of the London Mathematical Society, Series 2, Volume 33, pp. 195-208 by Sir Ronald Fisher and John Wishart.

By using (3.13), the cumulant function of the Compound Poisson total claim amount $S(t)$ is

$$\kappa_S(\vartheta) = \Lambda(t)(M_X(\vartheta) - 1).$$

When this cumulant function is differentiated with respect to ϑ,

$$\kappa_S'(\vartheta) = \Lambda(t)(M_X'(\vartheta)),$$

and by substituting $\vartheta = 0$, the mean of the total claim amount $S(t)$ is

$$E(S(t)) = \kappa_S'(0) = \Lambda(t)(M_X'(0)) = \Lambda(t)\eta,$$

where $M_X'(0) = \eta$ is the mean of the chosen raw claim amount X distribution (the variance of X is denoted with τ).

The second derivative of the cumulant function with respect to ϑ is

$$\kappa_S''(\vartheta) = \Lambda(t)(M_X''(\vartheta)),$$

and this gives the variance of the total claim amount process at $\vartheta = 0$ as:

$$Var(S(t)) = \kappa_S''(0) = \Lambda(t)(M_X''(0)) = \Lambda(t)(\tau + \eta^2),$$

where $M_X''(0) = E(X^2) = Var(X) + (E(X))^2$. The mean and variance of the total claim amount are already computed in Pages 49 and 50 previously. The work above shows that the use of the cumulant generating function is a simple alternative to compute the mean and variance of the total claim amount.

One can show the moment-cumulant relation of the total claim amount $S_i = \sum_{i=1}^{N(t)} X_i$ by conditioning on the claim number N [Daykin et al., 1994b, Ross, 2003]

$$M_S(\vartheta) = E_{N(t)}E[e^{\vartheta \sum_{i=1}^{N(t)} X_i} \mid N(t) = n].$$

By using the independency of $N(t)$ and X_i and i.i.d X_i's, the moment generating function can be written as

$$M_S(\vartheta) = E_{N(t)}E[e^{\vartheta X_1}e^{\vartheta X_2}\dots e^{\vartheta X_n} \mid N(t) = n] = E_{N(t)}[M_X(\vartheta)]^{N(t)}.$$

Exponentiating the inside of the expectation gives

$$M_S(\vartheta) = E\left(e^{N \ln M_X(\vartheta)}\right) = M_N(\ln M_X(\vartheta)) = M_N(\kappa_X(\vartheta)),$$

and

$$\kappa_S(\vartheta) = \ln\left(M_N(\kappa_X(\vartheta))\right) = \kappa_N(\kappa_X(\vartheta)),$$

where κ_X denotes the cumulant function of the claim amount distribution and κ_N is the cumulant function of the claim number distribution. Details of the cumulant function is explained later in this section.

Next, some examples are given with the choice of Gamma, Normal and Poisson distributions for the claim amount X_i to show the calculation of the moment generating function of the total claim amount process $S(t)$. The moment generating function of $S(t)$ for some other distribution choices of the individual claim size are given in the Appendix. The relation between the moment generating functions, the cumulant functions and the use of the mean function $\Lambda(t)$ of the claim number process by considering the use of the exponential and the power kernel functions, is explained in detail in Chapter 4, Section 4.5.

1. If the claim amount X_i is chosen to be distributed as Gamma with parameters α_g and β_g and X_i is time-dependent itself ($X_i(t)$), then the moment generating function of X is

$$M_{X_{t_i}}(\vartheta) = (1 - \frac{\vartheta}{\beta_{gt_i}})^{-\alpha_{gt_i}} \quad (\vartheta < \beta_g).$$

By using (3.13), the moment generating function of $S(t)$ with the convolution of X is

$$\begin{aligned}
M_{S_1}(\vartheta) &= E(e^{\vartheta s}) = \int_0^\infty e^{\vartheta s} f(s) ds = \int_0^\infty e^{\vartheta s} d(G^k(s)) \\
&= \sum_{k=0}^{\infty} \frac{e^{-\Lambda(t)}\Lambda(t)^k}{k!} \int_0^\infty e^{\vartheta s} \frac{\beta_{gt_i}^{\alpha_{gt_i}}}{\Gamma(\alpha_{gt_i})} s^{\alpha_{gt_i}-1} e^{-s\beta_{gt_i}} ds \\
&= \sum_{k=0}^{\infty} \frac{e^{-\Lambda(t)}\Lambda(t)^k}{k!} (1 - \frac{\vartheta}{\beta_{gt_i}})^{-\alpha_{gt_i} k} \qquad (3.14) \\
&= e^{-\Lambda(t)} \sum_{k=0}^{\infty} \Big(\frac{\Lambda(t)}{k!} (1 - \frac{\vartheta}{\beta_{gt_i}})^{-\alpha_{gt_i}} \Big)^k \\
&= e^{-\Lambda(t)} e^{\Lambda(t)(1-\frac{\vartheta}{\beta_{gt_i}})^{-\alpha_{gt_i}}} = e^{\Lambda(t)\Big((1-\frac{\vartheta}{\beta_{gt_i}})^{-\alpha_{gt_i}} - 1\Big)},
\end{aligned}$$

and if we take the natural logarithm of this moment generating function, the cumulant function is

$$\kappa_{S_1}(\vartheta) = \Lambda(t)\Big((1 - \frac{\vartheta}{\beta_{gt_i}})^{-\alpha_{gt_i}} - 1\Big), \qquad (3.15)$$

which is a special case of (3.13).

2. If X_i is distributed as Normal with parameters (η, τ), the moment generating function of the total claim amount will be

$$M_{S_2}(\vartheta) = e^{\Lambda(t)\left(e^{\eta t_i \vartheta + \frac{\vartheta^2 \tau}{2}} - 1\right)}, \tag{3.16}$$

and the cumulant function is

$$\kappa_{S_2}(\vartheta) = \Lambda(t)\left(e^{\eta t_i \vartheta + \frac{\vartheta^2 \tau}{2}} - 1\right). \tag{3.17}$$

3. If X_i follows a Poisson distribution with parameter (α_p)

$$M_{S_3}(\vartheta) = e^{\Lambda(t)\left(e^{\alpha p_{t_i}(e^{\vartheta}-1)} - 1\right)}, \tag{3.18}$$

the cumulant function is

$$\kappa_{S_3}(\vartheta) = \Lambda(t)\left(e^{\alpha p_{t_i}(e^{\vartheta}-1)} - 1\right), \tag{3.19}$$

which are other special cases of (3.13) when Normal and Poisson distributions are used.

Note 1: For some random variables with heavy tail, like Cauchy, the moment generating function does not exist [Kaas et al., 2001]. Also, the mgf of the Pareto distribution does not exist [Bühlmann, 1970].

Note 2: For lognormal and Beta distributions, the moment generating functions are not integrable in closed form [Bühlmann, 1970].

Approximate Distribution of the aggregate claims $S(t)$

As mentioned before, it is not always easy to define the distribution of the aggregate claims and some approximation techniques are suggested. One method is to use the moment generating functions or related transforms like characteristic functions, probability generating functions or cumulant generating functions. If the moment generating function or the cumulant function (since they are one-to-one functions) are in a recognisable form of a specific type of distribution, then this defines the required distribution [Kaas et al., 2001]. Moreover, if the aggregate claims $S(t)$ is considered as the sum of a large number of variables (big claim sizes), by using the Central Limit Theorem, the approximate distribution of $S(t)$ is assumed to be Normal.

By using the moment generating and cumulant function technique, (3.13) states that $M_S(\vartheta) = e^{\Lambda(t)\left(M_X(\vartheta)-1\right)}$. Then the corresponding cumulant function is $\kappa_S(\vartheta) = \Lambda(t)\left(M_X(\vartheta) - 1\right)$ and it can be expressed in words that:

cumulants of $S = \Lambda(t) \times$ (*non-central moments of individual claim distribution* X)

The following diagram summarises the relation between the moment generating and the cumulant functions of the individual claim distribution X and the aggregate claims $S(t)$.

Moments of X	$\leftrightarrow$	Moments of S
Moments of X	$\leftrightarrow$	Cumulants of X
Moments of S	$\leftrightarrow$	Cumulants of S
Cumulants of S	$\leftrightarrow$	Moments of X

Moment-cumulant relationships are well-known and here are the first few of them

$$\begin{aligned}
\kappa_0 &= 0 \\
\kappa_1 &= \mu_1 \\
\kappa_2 &= \mu_2 - \mu_1^2 \\
\kappa_3 &= \mu_3 - 3\mu_1\mu_2 + 2\mu_1^3.
\end{aligned}$$

Conversely

$$\begin{aligned}
\mu_0 &= 1 \\
\mu_1 &= \kappa_1 \\
\mu_2 &= \kappa_2 + \mu_1^2 = \kappa_2 + \kappa_1^2 \\
\mu_3 &= \kappa_3 + 3\mu_1\mu_2 - 2\mu_1^3 = \kappa_3 + 3\kappa_1(\kappa_2 + \mu_1^2) - 2\kappa_1^3 \\
&= \kappa_3 + 3\kappa_1\kappa_2 + 3\kappa_1\kappa_1^2 - 2\kappa_1^3 \\
&= \kappa_3 + 3\kappa_1\kappa_2 + \kappa_1^3.
\end{aligned}$$

By (3.13), the cumulant function is obtained as $\kappa_S(\vartheta) = \Lambda(t)(M_X(\vartheta) - 1)$. If we generalise the cumulant-moment relation above with the structure in (3.13) for the Compound Poisson case, for example, the non-central moments of S and X are

$$\begin{aligned}
\mu_0^{(s)} &= 1 \\
\mu_1^{(s)} &= \kappa_1^{(s)} = \Lambda(t)\mu_1 \\
\mu_2^{(s)} &= \kappa_2^{(s)} + (\kappa_1^{(s)})^2 = \Lambda(t)\mu_2 + (\Lambda(t)\mu_1)^2 = \Lambda(t)\mu_2 + \Lambda(t)^2\mu_1^2 \\
\mu_3^{(s)} &= \kappa_3^{(s)} + 3\kappa_1^{(s)}\kappa_2^{(s)} + (\kappa_1^{(s)})^3 \\
&= \Lambda(t)\mu_3 + 3\Lambda(t)\mu_1\Lambda(t)\mu_2 + (\Lambda(t)\mu_1)^3 \\
&= \Lambda(t)\mu_3 + 3\Lambda(t)^2\mu_1\mu_2 + \Lambda(t)^3\mu_1^3 \\
&= \vdots
\end{aligned}$$

where the cumulants of a Compound Poisson $S(t)$ is [Kaas et al., 2001]

$$\begin{aligned}
\kappa_1^{(s)} &= \Lambda(t)\mu_1 \\
\kappa_2^{(s)} &= \Lambda(t)\mu_2 \\
\kappa_3^{(s)} &= \Lambda(t)\mu_3 \\
&= \vdots
\end{aligned}$$

These cumulants helps us to find the mean, variance, skewness, kurtosis (or higher moments) of the related distribution. For the Compound Poisson total claim amount $S(t)$ [Kaas et al., 2001] states

$$\begin{aligned}
E(S(t)) &= \Lambda(t)\mu_1 \\
Var(S(t)) &= \Lambda(t)\mu_2 \\
\gamma(S(t)) &= \frac{\kappa_3}{\sigma^3} = \frac{\Lambda(t)\mu_3}{(\Lambda(t)\mu_2)^{\frac{3}{2}}} = \frac{\mu_3}{\sqrt{\Lambda(t)}(\mu_2)^{\frac{3}{2}}} \\
&= \vdots
\end{aligned}$$

where γ is used for the skewness term.

The Edgeworth approximation

A distribution with arbitrary given cumulants can be approximated by some series expansions, of one of which is the Edgeworth series. As mentioned earlier, it is difficult to obtain the exact distribution of the total claim amount $S(t)$. The Edgeworth expansion is important for this thesis work in the following sense. If the mean intensity function $\Lambda(t)$ can be modelled against time and covariates (magnitude, residential building number), and the assumption of independence of the $N(t)$ process and the X_i is justified, then the expansions provide a simple tool for the estimation and prediction of the distribution of the total loss process $S(t)$.

One way to find an approximate distribution of S by using an Edgeworth approximation is to use the cumulants. The first step is simply to standardise the total claim amount $S(t)$

$$Z(t) = \frac{S(t) - E(S(t))}{\sqrt{Var(S(t))}} = \frac{S(t) - \mu_1^{(s)}}{\sqrt{Var(S(t))}},$$

where $\mu_1^{(s)} = \kappa_1^{(s)} = \Lambda(t)\mu_1$ and $Var(S(t)) = \kappa_2^{(s)} = \Lambda(t)\mu_2$. Then

$$Z(t) = \frac{S(t) - \Lambda(t)\mu_1}{\sqrt{\Lambda(t)\mu_2}}.$$

Therefore

$$\begin{aligned} M_Z(\vartheta) = E(e^{z\vartheta}) &= E(e^{\left(\frac{S(t)-\Lambda(t)\mu_1}{\sqrt{\Lambda(t)\mu_2}}\right)\vartheta}) \\ &= e^{-\frac{\Lambda(t)\mu_1}{\sqrt{\Lambda(t)\mu_2}}\vartheta} M_S(\frac{\vartheta}{\sqrt{\Lambda(t)\mu_2}}) \\ &= e^{-\frac{\Lambda(t)\mu_1}{\sqrt{\Lambda(t)\mu_2}}\vartheta} e^{\Lambda(t)\left(M_X(\frac{\vartheta}{\sqrt{\Lambda(t)\mu_2}})-1\right)}, \end{aligned}$$

so the corresponding cumulant function is

$$\kappa_Z(\vartheta) = -\frac{\Lambda(t)\mu_1}{\sqrt{\Lambda(t)\mu_2}}\vartheta + \Lambda(t)\Big(M_X(\frac{\vartheta}{\sqrt{\Lambda(t)\mu_2}}) - 1\Big),$$

and by expansion

$$\begin{aligned} \kappa_Z(\vartheta) &= -\frac{\Lambda(t)\mu_1}{\sqrt{\Lambda(t)\mu_2}}\vartheta + \Lambda(t)\Big(1 + \frac{\mu_1\vartheta}{\sqrt{\Lambda(t)\mu_2}} + \frac{\mu_2\vartheta^2}{2!\Lambda(t)\mu_2} + \frac{\mu_3\vartheta^3}{3!(\Lambda(t)\mu_2)^{\frac{3}{2}}} + \frac{\mu_4\vartheta^4}{4!(\sqrt{\Lambda(t)\mu_2})^4} \\ &\quad + \ldots - 1\Big) \\ &= \frac{\vartheta^2}{2!} + \frac{\mu_3}{(\Lambda(t)\mu_2)^{\frac{1}{2}}}\frac{\vartheta^3}{3!} + \frac{\mu_4}{(\Lambda(t)\mu_2)^2}\frac{\vartheta^4}{4!} + \ldots . \end{aligned}$$

The probability density function of Z can be obtained by using the Hermite polynomials. The polynomials orthogonal with respect to the normal distribution e^{-z^2} are the Hermite polynomials [Andrews et al., 1999, Barndorff-Nielsen and Cox, 1989]. The first few Hermite polynomials are [Barndorff-Nielsen and Cox, 1989]

$$H_0(z) = 1, H_1(z) = z, H_2(z) = z^2 - 1, H_3(z) = z^3 - 3z, \ldots.$$

If $\phi(z)$ is distributed N(0,1), the density function of the total claim amount $S(t)$ by using the Edgeworth expansion is approximately Normal as [Asmussen, 2000] states

$$f(z) \sim \phi(z)\Big(1 + \frac{\mu_3}{(\Lambda(t)\mu_2)^{\frac{3}{2}}} H_3(z) + \frac{\mu_4}{(\Lambda(t)\mu_2)^2} H_4(z) \ldots\Big),$$

where $H_r(z)$ is the r^{th} Hermite polynomial. The basic definition of $H_r(z)$ is given in [Barndorff-Nielsen and Cox, 1989] as

$$\phi(z)H_r(z) = (-1)^r H_r(z)\phi(z),$$

where $\phi(z) = \frac{1}{\sqrt{2\pi}}(e^{-\frac{z^2}{2}})$. The derivation of the Edgeworth expansion uses the important inverse property and so the Hermite polynomials can be associated with the moment generating function as

$$\int_{-\infty}^{+\infty} e^{\vartheta z}\phi(z)H_r(z)dz = E(e^{\vartheta Z}H_r(Z)) = \vartheta^r e^{\frac{\vartheta^2}{2}}.$$

The justification of the $(N(t), X_i)$ independence assumption will be discussed in more detail in the Copula section and the estimation-prediction of the aggregate losses will be conducted by using the modelling chapters and later used in the chapter on the Turkish Catastrophe Insurance Pool.

3.4 Extreme Value Theory

In the past decades, the effect of large claims due to the natural disasters became a much of an interest of the scientists. Extremal events are quantifiable in monetary units and this is an advantage for mathematical modelling of insurance and finance context. In insurance studies, heavy-tailed distributions are widely used as standard models for the claim amount X_i.

Extreme Value Theory (EVT) studies the pattern of the extreme events data like major insurance claims, flood levels of rivers, changes in the value of the stock market, wind speed and wave height during a hurricane or storm [Embrechts et al., 1997].

Basically, the characteristics and the behaviour of the tail of a distribution are studied in the extreme value theory. Since natural disasters are extreme events, EVT is a good way for the inference of the data, which is the earthquake insurance claims data for the case of Turkey in this study.

Let's consider a sequence $X_1, \ldots, X_n$ of independent identically distributed (iid) random variables with unknown cumulative distribution function $F(x) = Pr(X \leq x)$. The upper tail of F is important here since the occurrence of the extreme observations are highly probable in the tails. The sequence of n random variables is considered to be independent and follows as in [Embrechts et al., 1997, Reiss and Thomas, 2001, Woo, 1999]

$$Pr(max(X_1, \ldots, X_n) \leq x) = \prod_{i}^{n} Pr(X_i \leq x) = F^n(x).$$

Since the extreme value distribution should be stable as n increases, the F^n should asymptotically converge to some fixed distribution G and then it is said that F is in the *domain of attraction of* G. There are three main extreme value distributions; Gumbel, Fréchet and Weibull. The Generalised Extreme Value Distribution (GEV), $G(x)$, provides a good representation of these distributions [Embrechts et al., 1997]

$$G(x) = \begin{cases} e^{-(1+\xi\frac{x-\mu}{\sigma})^{-\frac{1}{\xi}}} & \text{if } \xi \neq 0; \\ e^{-e^{-\frac{x-\mu}{\sigma}}} & \text{if } \xi = 0. \end{cases}$$

where μ is the location parameter, σ is the scale parameter and ξ is the shape parameter, which is the main parameter used to characterise the tail of the distribution.

The classification of $G(x)$ with the restrictions on the shape parameter ξ concludes that

$\xi = 0 \Rightarrow G(x)$ is Gumbel distribution,

$\xi > 0 \Rightarrow G(x)$ is Fréchet distribution,

$\xi < 0 \Rightarrow G(x)$ is Weibull distribution.

Medium-tailed distributions like Normal, gamma, exponential and lognormal are within the domain of attraction of Gumbel distribution; light-tailed distributions like beta and uniform are in that of Weibull and the heavy-tailed distributions like Pareto, log-gamma and Cauchy distributions are in that of Fréchet distribution.

There is an alternative way to the use of GEV to approach extremal events. Instead using the annual maxima, the large events exceeding some threshold value are modelled. Let h be the threshold and X be the exceedances of h during a given period, the distribution of excess values of X over threshold h is [Gençay et al., 2001, Wallis and Hosking, 1987]

$$F_h(y) = Pr(X - h \leq y | X > h) = \frac{F(y+h) - F(h)}{1 - F(h)},$$

which is the probability that the value of X exceeds the threshold h by at most an amount y given that X exceeds the threshold h. For sufficiently high threshold h, the given distribution function is approximated by the distribution function G, which is the standard generalised Pareto distribution (GPD). Then

$$G_\xi(x) = \begin{cases} 1 - (1+\xi x)^{-\frac{1}{\xi}} & \text{if } \xi \neq 0; \\ 1 - e^{-x} & \text{if } \xi = 0. \end{cases}$$

when $\xi = 0$, it is the Exponential distribution function with parameter $\lambda_e = 1$, the shape parameter ξ and $x \geq 0$, $\xi \geq 0$, $0 \leq x \leq -\frac{1}{\xi}$.

One can also define the location-scale family $G_{\xi;\beta_{ev},\nu}$ by replacing x in the equations above with $\frac{x-\nu}{\beta_{ev}}$ for $\nu \epsilon R, \beta_{ev} > 0$. $G_{\xi;\beta_{ev},\nu}$ is also the generalised Pareto distribution and when $\nu = 0$, the following distribution function is used to estimate the parameters $\hat{\xi}$ and $\hat{\beta_{ev}}$ of the GPD [Embrechts et al., 1997]

$$G_{\xi;\beta_{ev}} = \begin{cases} 1 - (1+\xi \frac{x}{\beta_{ev}})^{-\frac{1}{\xi}} & \text{if } \xi \neq 0; \\ 1 - e^{-\frac{x}{\beta_{ev}}} & \text{if } \xi = 0. \end{cases}$$

For $\xi > 0$, the generalised Pareto distribution takes the form of ordinary Pareto distribution, which is heavy-tailed. $\xi = 0$ indicates Exponential distribution as mentioned before and when $\xi < 0$, it is Pareto II type distribution. As explained next, for the GPD, when $\xi > -0.5$, the distribution can also be classified as heavy-tailed.

In [Wallis and Hosking, 1987], the case $\xi > -0.5$, where there occurs a heavy-tailed distribution, it is shown that maximum likelihood regularity conditions are satisfied and the maximum likelihood estimators (MLEs) are asymptotically normally distributed. By that argument, the MLEs $\hat{\xi}$ and $\hat{\beta_{ev}}$ of the generalised Pareto distribution can be obtained by using the following density and likelihood function [Embrechts et al., 1997]

$$g(x) = \frac{\partial G_{\xi;\beta_{ev}}}{\partial x} = \frac{1}{\beta_{ev}}(1+\xi\frac{x}{\beta_{ev}})^{-(\frac{1}{\xi}+1)},$$

where $-\infty < x < +\infty$.

$$L(x_i;\xi,\beta_{ev}) = \prod_{i=1}^{n} g(x_i) = \prod_{i=1}^{n} \frac{1}{\beta_{ev}}(1+\xi\frac{X_i}{\beta_{ev}})^{-(\frac{1}{\xi}+1)}$$
$$= \frac{1}{\beta_{ev}^n}\prod_{i=1}^{n}(1+\xi\frac{X_i}{\beta_{ev}})^{-(\frac{1}{\xi}+1)},$$

and the following log-likelihood is used to the maximise the parameters [Embrechts et al., 1997]

$$\log(L(x_i;\xi,\beta_{ev})) = -n\log\beta_{ev} - (\frac{1}{\xi}+1)\sum_{i=1}^{n}\log(1+\xi\frac{X_i}{\beta_{ev}}). \qquad (3.20)$$

It is possible to get the estimates of ξ and β_{ev} by some numerical methods from this log-likelihood.

The tail estimator of the unknown cumulative distribution function F is given in [Embrechts et al., 1997, Gençay et al., 2001] as

$$\hat{F}(x) = 1 - \frac{E_h}{n}(1+\hat{\xi}\frac{x-h}{\hat{\beta_{ev}}})^{-\frac{1}{\hat{\xi}}},$$

where h is the threshold, E_h is the number of exceedances over this threshold and n is the sample size. The quantile estimator $\hat{x}_p$, which also applies in Value-at-Risk (VAR) studies, for a given probability $p > F(p)$ is

$$\hat{x}_p = h + \frac{\hat{\beta_{ev}}}{\hat{\xi}}\Big((\frac{n}{E_h}(1-p))^{-\hat{\xi}} - 1\Big).$$

There is another estimator for the shape parameter ξ, which is named after Hill (1975) [Gençay et al., 2001, Drees et al., 2000]

$$\hat{\alpha_H} = \frac{1}{k-1}\sum_{i=1}^{k-1}\ln X_{i,B} - \ln X_{k,B} \quad \text{for } k \geq 2, \qquad (3.21)$$

where k is the upper order statistics (the number of exceedances), B is the sample size and $\alpha_H = \frac{1}{\xi}$ is the tail index.

One other commonly used method in the Extreme Value Theory to estimate the shape parameter of the EVT distributions is called the 'Peaks Over Threshold (POT)', which assumes [Embrechts et al., 1997]:

i) the excesses of an independent identically distributed sequence over a high threshold, say h, occur at the times of a Poisson process,

ii) the corresponding excesses over h are independent and have a GPD,

iii) excesses and exceedance times are independent of each other.

It is also possible to fit GPD models with time-dependent parameters like $\xi_{ev}(t)$, $\beta_{ev}(t)$ by using the POT method [Embrechts et al., 1997], which can be suggested as part of a future work.

Remark 6: The following interpretations can be made out of all given above:

1. Excesses over high thresholds can be modelled by the generalised Pareto distribution,

2. The number of exceedances of a high threshold follows a Poisson process,

3. An appropriate value of high threshold can be found by plotting the empirical mean excess function (the sum of the excesses over the threshold u divided by the number of data points that exceeds the threshold),

4. The distribution of the maximum of Poisson number of independent identically distributed excesses over a high threshold is the Generalised Extreme Value Distribution.

3.4.1 Moment Generating Function of the Extreme Value Distributions

In this section, the moment generating functions of the extreme value distributions, Gumbel, Weibull and Fréchet are derived. These distributions have many applications in insurance, finance and engineering studies and the derived moment generating functions might have a use in future studies.

Gumbel distribution:

One of the famous extreme value distributions is the Gumbel distribution. The Generalised Extreme Value distribution takes the form of the Gumbel distribution when the shape parameter $\xi = 0$ (see Page 68). Therefore, the distribution function of the Gumbel is given as

$$F(x) = e^{-e^{-x}}, \quad \forall x,$$

and the corresponding density function is

$$f(x) = e^{-e^{-x}} e^{-x}, \quad \forall x.$$

The moment generating function of the Gumbel distribution is obtained by using the definition of mgf as

$$M_X(\omega) = E(e^{x\omega}) = \int_0^{\infty} e^{x\omega} e^{-e^{-x}} e^{-x} dx. \tag{3.22}$$

If we substitute $e^{-x} = u$ then $-e^{-x}dx = du$, where if $x = 0 \rightarrow u = 1$ and if $x = \infty \rightarrow u = 0$. This substitution gives us the suggested form of the Gumbel moment generating function

$$M_X(\omega) = \int_0^1 e^{-u} u^{-w} du, \tag{3.23}$$

which is the Gamma function $\Gamma(1-w)$.

Weibull distribution:

Another widely used extreme value distribution is the Weibull distribution. It is often used in survival and reliability analysis and provides a close approximation for the distribution of lifetime. It occurs when the shape parameter ξ is smaller than 0 for the generalised extreme value distribution (see Page 68). The distribution function of the Weibull type with parameters $(\alpha_w, \beta_w = 1)$ is

$$G(y) = 1 - e^{-y^{\alpha_w}}, \; 0 \leq y < \infty,$$

and the probability density function by differentiation will be in the form of

$$g(y) = \alpha_w y^{\alpha_w - 1} e^{-y^{\alpha_w}}; \quad \alpha_w > 0, \; 0 \leq y < \infty.$$

In Page 45, the distribution of the occurrence of the first event with the use of the power kernel function and some transformation was also obtained in the form of

Weibull distribution. Therefore, the Weibull distribution can be named as one of the possible distributions to explain the disaster (extreme) claims data including shock pattern. In terms of the moment generating function of this case, it only exists when $\alpha_w \geq 1$.

Fréchet distribution:

The Fréchet distribution is generated from the generalised extreme value distribution, when ξ is greater than 0. The distribution of the Fréchet type is

$$H(z) = e^{(-z)^{-\alpha_f}},$$

for $0 \leq z < \infty$ and the probability density function is

$$h(z) = \alpha_f(-z)^{-(1+\alpha_f)} e^{(-z)^{-\alpha_f}}; \quad (z \geq 0).$$

The moment generating function of this density follows as

$$M_H(\eta) = \int_0^{\infty} e^{z\eta} \alpha_f(-z)^{-(1+\alpha_f)} e^{(-z)^{-\alpha_f}} dz. \tag{3.24}$$

Let $e^{(-z)^{-\alpha_f}} = u$ and so $z = -e^{\frac{-\ln(\ln u)}{\alpha_f}}$ then the moment generating function has the form

$$M_H(\eta) = -\int_0^1 e^{\frac{-\eta \ln(\ln u)}{\alpha_f}} dz = \frac{\eta}{\alpha_f}\left(e^{\frac{-\eta \ln(\ln u)}{\alpha_f}}\right)\Big|_0^1 = \frac{\eta}{\alpha_f}(e^{\frac{-\eta}{\alpha_f}} - 1). \tag{3.25}$$

Note 3: The moment generating function of the Weibull distribution exists only if the parameter $\alpha \geq 1$, which is said to be not useful in [Bühlmann, 1970] and also for the moment generating function of the Gumbel case, it seems not to exist since there is $\ln(-1)$ term in the calculations.

3.4.2 Explanatory EVT Data Analysis of the Turkish Catastrophe Insurance Pool data between 2000-2003

In this section, a basic explanatory data analysis of the extreme value theory literature on the claim amount X_i of the Turkish Catastrophe Insurance Pool (TCIP) data is presented. The analysis is conducted to suggest a possible distributional fit to the TCIP claims data. There are $n_2 = 4297$ observations in total in the Turkish Catastrophe Insurance Pool, which are recorded between 15/December/2000-31/July/2003. Since most of the claims arrive from risk zone 1, we will study with zone 1 earthquake claims data $n_{\text{zone 1}} = 3602$. The raw (without log transformation) data by week classification is used for more specific analysis.

The extreme movements due to extreme events cause a significant effect on the financial markets. The tail behaviour of the data is the main interest in the extreme value analysis. Since the total claim amount process $S(t)$ and $E(S(t))$ has a wide use in the insurance and financial markets, the use of extreme value distributions for the claim amount distribution might suggest some new ideas for the portfolio of the insurance companies.

There are many studies on the application of the extreme value modelling for large claims data. The Danish fire insurance claims (n_1=2167 observations in millions of Danish Kroner) data between 1980-1990 is used in many of these studies. Extreme Value Theory analysis is available in S-Plus package with EVIS (Extreme Value in S-Plus), in Matlab with EVIM (Extreme Value in Matlab) and in a software called the Extremes.

In this study, Extreme Value in S-Plus is used to derive some basic information of the Turkish earthquake insurance claims. EVIS is a library with built-in functions in S-Plus environment to run the Extreme Value analysis. It is developed by Alexander McNeil at ETH Zurich. It allows the users to check the fit of the extreme data like the large number of insurance claims to the extreme value distributions such as Pareto, Weibull. There are some estimates special to the Extreme Value analysis like the mean excess plot and the Hill plot. EVIS provides these diagnostic plots and the estimates, which ease the users to follow the heavy-tailed or not pattern of their data. The generalised Pareto distribution (GPD) is commonly used to model the exceedances over some threshold values. The comparison of the parameters of these models for the Danish fire and the Turkish earthquake claims are given in Table 3.1.

The estimate of the shape parameter, $\hat{\xi}$, of the generalised Pareto distribution is calculated in EVIS by using (3.20). According to those calculations, in Table 3.1, it is observed that the parameter estimate of the Danish fire claims is $\hat{\xi} = 0.638$. Since this match with the classification in Pages 67 and 68, it indicates an ordinary type Pareto heavy-tailed distribution. Turkish zone 1 earthquake claims data gives a value of $\hat{\xi} = -0.329$ of the shape parameter, which less than 0. By using the condition in Page 68 about the value of ξ, it can considered to denote Pareto II type distribution. However by the MLE assumption in [Wallis and Hosking, 1987], if $\hat{\xi} > -0.5$, for zone 1 earthquake claim data, the idea of a heavy-tailed distribution still holds. It is also observed in Table 3.1 that the Turkish zone 1 earthquake insurance claims data has a lower variance than that of Danish fire insurance claims.

	Danish	Turkish
threshold value h	17.068	23.597
number of exceedances	50	51
parameter estimates	$\hat{\xi} = 0.638$	$\hat{\xi} = -0.329$
variance of estimates	$\text{var}(\hat{\xi}) = 0.049$	$\text{var}(\hat{\xi}) = 0.014$

Table 3.1: The results of the GPD application to the Danish fire and the Turkish zone 1 earthquake insurance claims data for exceedances over sample size 50.

Graphical Analysis

In this section, some graphical analysis of the Danish fire insurance claims and the Turkish earthquake claims data are given in comparison in the Extreme Value Theory context. The mean excess, the Quantile-Quantile, the Hill and the shape parameter plots are presented in the given order.

The Mean Excess Plot:

In Extreme Value analysis, the mean excess function is a very useful tool to distinguish the type of the distribution in tail. It is calculated as $\frac{\sum \text{excesses over the threshold } h}{\text{the number of observations exceeding } h}$, that is $e_n = \frac{\sum_{i=1}^{n}(X_i - h)}{\sum_{i=1}^{n} I_{X_i > h}}$, where $I = 1$ if $X_i > h$ and 0 otherwise and e_n are the mean excess values [Gençay et al., 2001]. If the mean excess plot tends to infinity and has an upward slope, the distribution is said to be heavy-tailed. If the mean plot is close to a straight line with a positive slope, then the underlying distribution can be modelled as Pareto-like distributions. Sometimes, median excess plots are used instead of the mean excess plots for more robustness. Next, in Figure 3.1, both the Danish fire and Turkish zone 1 earthquake claims data, which has more dense observations in the upper tail, show an upward trend and indicates a heavy-tailed distribution fit as expected.

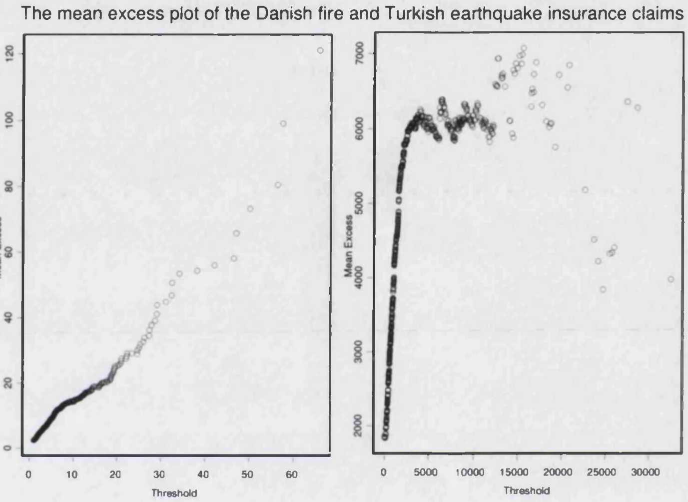

Figure 3.1: The mean excess plot of 50 exceedances: left: Danish fire insurance claims, right: Turkish zone 1earthquake insurance claims.

The Hill Plot

In the Extreme Value Analysis, the Hill plot is generated with the assumption that the data comes from a heavy-tailed distribution. It plots the Hill estimate of the tail index against the k upper order statistics and it is mainly used in determining the threshold of the data for further modelling and analysis [Gençay et al., 2001]. The calculation of the Hill estimate is given in (3.21)

$$\hat{\alpha_H} = \frac{1}{k-1} \sum_{i=1}^{k-1} \ln X_{i,B} - \ln X_{k,B} \text{ for } k \geq 2,$$

where k is the upper order statistics (the number of exceedances), B is the sample size and $\alpha_H = \frac{1}{\xi}$ is the tail index.

A Hill plot is constructed such that the estimated $\hat{\xi}$ is plotted as a function of either the k upper order statistics or the threshold h [Gençay et al., 2001]. In Figure 3.2, the estimate is plotted against order statistics with a 95 % confidence. Both of the plots represent a stable structure.

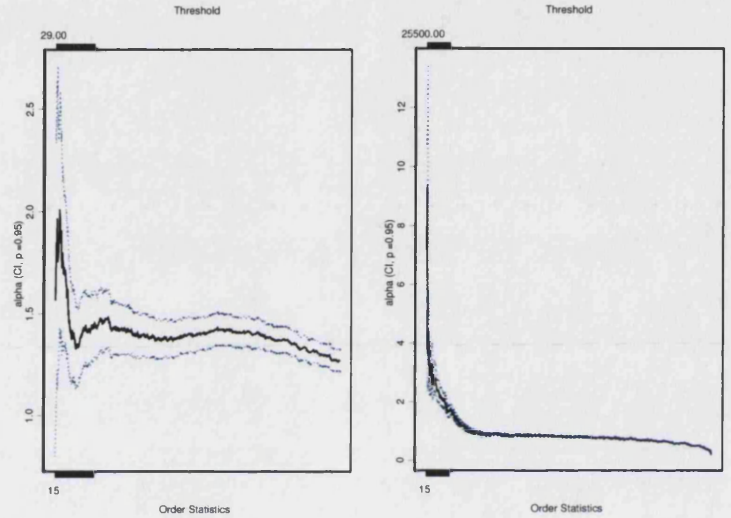

Figure 3.2: The Hill plot of 50 exceedances: left: Danish fire insurance claims, right: Turkish zone 1 earthquake insurance claims.

In [Drees et al., 2000], it is mentioned that the traditional Hill plot is the most effective way, when the underlying distribution is exactly Pareto or close to Pareto. In fact, the Hill plot can be used as a good way to give information about the real data. One subplot of the Hill plot is called the altHill plot. It is suggested in [Drees et al., 2000] that it is better to plot both Hill and altHill graphs and compare them when analysing real data. The altHill plot can provide more precise information, when the traditional Hill plot is not enough itself to make interpretation of the data. The detailed explanation (e.g. mean, variance computations of the Hill plot) can be followed in [Adler et al., 1998] (pages 283-310).

The Quantile-Quantile Plot

The Quantile-Quantile (QQ) plot, which is plotted against the standard exponential quantiles, is another basic graphical tool to check the distribution of the data, whether it belongs to a specific distribution or not. In the context of the Extreme Value Theory, if a concave-shaped QQ-plot is observed, this indicates a heavy-tailed distribution; whereas a convex structure is considered to be a sign of light-tailed distribution.

In the following Figure 3.3, it can be concluded that both of the QQ-plots back up the heavy-tailedness of the distribution. There are some extreme observations towards the edge of the distribution. Some statistical techniques are available to check of the fit of some other kind of distributions to the data, if some strange/extreme pattern is observed in the QQ-plots.

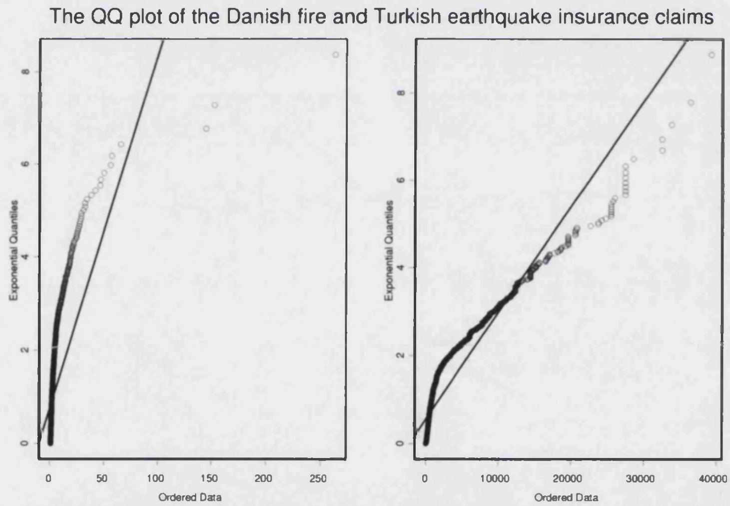

Figure 3.3: The QQ-plot of 50 exceedances: left: Danish fire insurance claims, right: Turkish earthquake insurance claims.

The Shape Parameter ξ

When fitting the generalised Pareto distribution to the excesses over some threshold value, the shape parameter ξ is the defining element to check if the data follows a Generalised Pareto Distribution or not. Next, Figure 3.4 gives the plot of the maximum likelihood estimates of the shape parameter $\hat{\xi}$, which is derived as the solution of (3.20) by some numerical methods in EVIS built-in function, with 95 % confidence interval at different level of exceedances. While the maximum likelihood parameter of the Turkish earthquake insurance claims data shows figures with deep jumps, the Danish fire insurance claims data follows a more consistent/stable path. The sudden and sharp jumps of the shape parameter in the Turkish earthquake claims data represent the extreme and large earthquake claim sizes due to the effect of the big earthquakes.

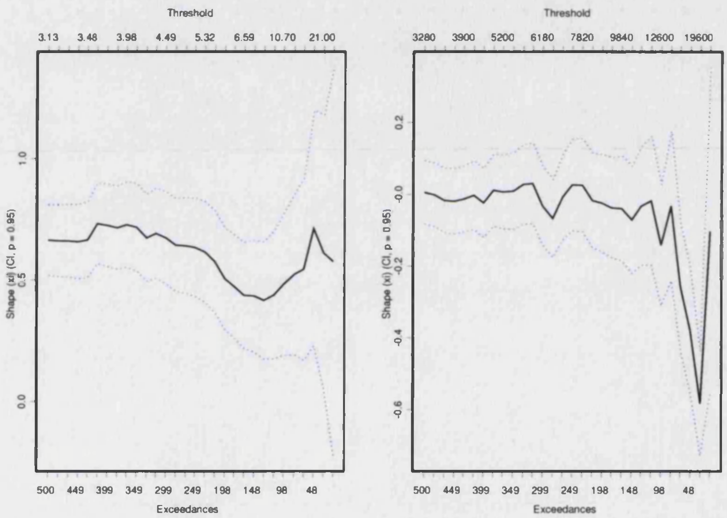

Figure 3.4: The plot of the maximum likelihood estimate of the shape parameter ξ with 95 % confidence band for 50 exceedances: left: Danish fire insurance claims, right: Turkish zone 1 earthquake insurance claims.

The extreme value analysis in S-Plus by EVIS concludes that zone 1 mandatory earthquake insurance claims data of the Turkish Catastrophe Insurance Pool (TCIP) suggest a reasonable fit in terms of the common well-known specifications of the generalised Pareto distribution. The parameter estimates $\hat{\xi}$ and the related extreme value plots support this suggestion. Previously, in another study, the similar analysis was conducted for the Marmara earthquake reinsurance claims of the industrial facilities and similar results were obtained in terms of graphical analysis.

It should be noted here that the Turkish Catastrophe Insurance Pool data only consists the earthquake insurance claims of the residential building. This residential-industrial claim separation might be a reason for the difference in the stability of the analysis. Time Series Analysis (e.g ARMA, ARCH-GARCH Processes), regular variation, other types of estimators for the shape parameter ξ (e.g. Pickand's Estimator, the Dekkers-Einmahl-de Haan Estimator), records and return period are some topics that the researchers can do research on in the Extreme Value context.

In the modelling process of this thesis, the claim amount is assumed to be lognormal distributed. Lognormal distribution is in the family of medium-tailed distributions, which are in Gumbel distribution in the Extreme Value context as explained in Page 68. By using the results of the graphical tools and Table 3.1, we can still suggest that the earthquake claims data of zone 1 is in the family of heavy-tailed distribution as it is more likely to happen for the case of extreme events data. Generally, it is preferred to work with high quality data (easier to check the heavy-tailedness of the distribution) to obtain better results in the extreme value analysis. In the case of Turkish earthquake claims data, the future work will provide stronger analysis with the improvements in the data recording.

Chapter 4

Likelihoods

A likelihood function $L(x;\theta)$ is defined as the probability or the probability density for the occurrence of a sample of observed response values $x_1 \ldots x_n$ given the probability density function $f(x;\theta)$ considered as a function of the unknown parameter vector θ, which is

$$L(x;\theta) = \prod_{i=1}^{n} f(x_i;\theta) = f(x_1;\theta)\ldots f(x_n;\theta).$$

In this chapter, the likelihood of the data (observations x_i), which consists of the earthquake insurance claims, event time, the likelihood of time for an inhomogeneous Poisson process of the counts, the generalised linear models, the likelihood and the Hessian of the Poisson count models and Normal distribution for the aggregate claims S (note that $\log S \sim$ Normal) are studied. The distribution of the claim size and the distribution of event times are needed to obtain the likelihood of the observations. The Normal, Weibull and Gamma distributions are chosen for the claim amount and three different methods are suggested for the likelihood of time.

The maximum likelihood estimation (MLE) technique is convenient to use as it provides an easy adaptation to the changes in model structure [Coles, 2001]. Some effort is spent on the estimation of the non-linear parameter β (exponential decay) for the choice of the exponential and the power kernel special functions in the time likelihood of an inhomogeneous Poisson process. The non-linear parameter (β) gives us information about the behaviour of the earthquake claim arrivals in different risk zones in Turkey.

4.1 Likelihood of Observations

Our main aim is to fit a point process model for the claims data of this thesis. The likelihood of the model data has two parts: the likelihood of the aggregate claims S_i and the likelihood of the time for the claim arrivals. The assumption used here is the independency of the claim number process $N(t)$ and the raw claim amount X_i, where the event times distribution is conditional given the number of Poisson counts. Then, the likelihood can be written as follows conditional on the number of claims $N(t) = n$

$$L(S, t; \theta) = \prod_{i=1}^{n} f(S_i \mid N(t) = n) f(t_i \mid N(t) = n) f(N(t) = n), \tag{4.1}$$

where all the parameters of the distributions are given in the form of θ parameter.

4.1.1 Likelihood of time

Here, three different types of likelihood for the event times of an inhomogeneous Poisson process are examined.

Method 1:

In Chapter 2, the distribution of the interarrival times is given with the probability density function $\lambda(t_i)e^{-\Lambda_i(t)}$, where $\Lambda_i(t) = \int_{W_i}^{W_{i+1}} \lambda(r)dr = \Lambda(W_{i+1}) - \Lambda(W_i)$. By differentiating (3.1), conditional on W_i, the likelihood is [Cox and Lewis, 1966]

$$\begin{aligned} L_1(t_1, \ldots, t_n; \theta) &= \prod_{i=1}^{n} \lambda(t_i; \theta) e^{-\Lambda_i(t_i)} \\ &= \lambda(t_1)e^{-\Lambda(t_1)}\lambda(t_2)e^{-(\Lambda(t_2)-\Lambda(t_1))} \ldots \lambda(t_n)e^{-(\Lambda(t_n)-\Lambda(t_{n-1}))} \\ &= \prod_{i=1}^{n} \lambda(t_i; \theta) e^{-\Lambda(t_n)}, \end{aligned} \tag{4.2}$$

as the Λ terms in the power of the exponentials cancel. Since the claim number process $N(t) = n$ is fixed, the likelihood takes the form $\prod_{i=1}^{n} f(t_i \mid \theta)$.

Method 2:

The idea of this method is very similar to the censoring concept in the survival analysis. Suppose that $N(t) = n$ claims arrive in a time interval of $(0, t_N)$. If we condition on event time t_{N+1}, the interval of $(t_{N+1} - t_N)$ is the survival period until the arrival of the next claim. If there occurs another claim after the fixed time, that will cause censoring for the claim arrivals. The likelihood to forecast the next failure time is derived by using the likelihood in Method 1 and the part for the censoring, conditional on the number of claims. This is formulated as

$$\begin{aligned} L_2(t_1, \dots, t_n; \theta) &= L_1(\theta) e^{-(\Lambda(t_{N+1}) - \Lambda(t_N))} \\ &= \prod_{i=1}^{n} \lambda(t_i; \theta) e^{-\Lambda(t_N)} e^{-(\Lambda(t_{N+1}) - \Lambda(t_N))} = \prod_{i=1}^{n} \lambda(t_i; \theta) e^{-\Lambda(t_{N+1})}. \end{aligned} \tag{4.3}$$

Method 3:

The third likelihood is built on the idea of using the bins (an interval into which a given data point does or does not fall) with independent Poisson counts. Lets assume the claims arrive in the time interval $(0, t]$. If we divide this interval of length t into m equal subintervals, the length of each subinterval will be $\frac{t}{m}$. Then the intervals will be $(0, \frac{t}{m}], (\frac{t}{m}, \frac{2t}{m}], \dots, (\frac{(m-1)t}{m}, t]$. The Poisson parameter for interval i is $\Lambda_i = \int_{\frac{(i-1)t}{m}}^{\frac{it}{m}} \lambda(r) dr$. Let N_i be the number of events in the i^{th} interval, because it is known that the claim number process N has independent increments. Then the third likelihood of time is based on the independent Poisson counts (see **Remark 2**, where it says $N(t) \sim \text{Pois}(\Lambda(t))$)

$$L_3(t_1, \dots, t_n; \theta) = \prod_{i=1}^{m} \frac{e^{-\Lambda_i} \Lambda_i^{N_i}}{N_i!}. \tag{4.4}$$

When $m \to \infty$, $L_3(\theta)$ behaves like $L_1(\theta)$. The simplest way to prove this behaviour is to use the log-likelihood of both cases as in (4.2) and (4.4). That is

$$\log L_1(t_1, \dots, t_n; \theta) = -\Lambda(t_n) + \sum_{i=1}^{n} \log \lambda(t_i),$$

and

$$\log L_3(t_1, \dots, t_n; \theta) = -\sum_{i=1}^{m} \Lambda(t_i) + \sum_{i=1}^{m} N_i \log \Lambda_i - \sum_{i=1}^{m} \log(N_i!).$$

First let (non-binning case)

$$-\sum_{i=1}^{m} \Lambda(t_i) = \int_0^t \lambda(r)dr = \Lambda(t).$$

In the binning case, the Poisson parameter is assumed to be $\Lambda_i = \int_{\frac{(i-1)t}{m}}^{\frac{it}{m}} \lambda(r)dr$ in the interval $(0, t)$ with a subinterval length of $\frac{t}{m}$. Then, for some d_i in the interval of $[\frac{(i-1)t}{m}, \frac{it}{m}]$, by the integral mean value theorem

$$\Lambda_i = \int_{\frac{(i-1)t}{m}}^{\frac{it}{m}} \lambda(r)dr = \lambda(d_i)\frac{t}{m}.$$

This implies

$$\sum_{i=1}^{m} N_i \log \Lambda_i = \sum_{i=1}^{m} N_i \log(\lambda(d_i)\frac{t}{m}) = \sum_{i=1}^{m} N_i \log \lambda(d_i) + \sum_{i=1}^{m} N_i \log(\frac{t}{m}).$$

As a next step, if N_i is fixed, as $m \to \infty$, there will be at least one event, so one t_i in each interval and this makes $\sum_{i=1}^{m} N_i \log \lambda(d_i)$ approach to $\sum_{i=1}^{m} \log \lambda(t_i)$. For fixed N_i, also, $\sum_{i=1}^{m} N_i \log(\frac{t}{m}) = N \log(\frac{t}{m})$ and can be written as $(\frac{t}{m})^N$ when exponentiated and as $m \to \infty$, $(\frac{t}{m})^N \to 0$. For sufficiently large m, $\sum_{i=1}^{m} \log(N_i!) \to 0$. Then it concludes that $L_3(\theta) \to L_1(\theta)$ as $m \to \infty$.

4.2 The use of shock kernels

The big earthquakes are generally followed by some small earthquakes, which occur close to the original earthquake epicentre. In seismology, the big initial earthquake is called the 'mainshock' and the following small ones are called the 'aftershocks'. Many scientists study the combination of both cases, which is the 'mainshock-aftershock pattern'. It is important to understand and have some idea of this relation, because it might contribute to a successful disaster protection plan (e.g effective rescue operations, lessening the anxiety of the residents of the disaster area and minimising the possible damage to occur due to a future earthquake) [Chen et al., 2004, Ogata, 1988].

The aftershock occurrences are usually considered as an inhomogeneous process. There exists 'Omori's law' that the aftershocks are believed to follow. It shows the empirical relation for the temporal decay of aftershock rates [Utsu, 1961]

$$\lambda^* = (\frac{K}{t+c}),$$

where λ^* is an aftershock frequency (the number of earthquakes measured at certain time t), which is measured over a certain interval time. K and c are constants, where parameter K generally depends on the amount of the aftershock denoting the decay rate and c is the time offset parameter, which is less than 0.1 days.

[Utsu, 1961] suggested a modified version of the Omori's law and it is called the 'Modified Omori's Law (MO Formula)'

$$\lambda^* = (\frac{K}{(t+c)^p}),$$

where p is a constant and it can be related to the frictional heat. When $p = 1$, the MO Formula is simple Omori's law. The value of p changes in the interval of 0.7-1.5 but it is normally 1 or slightly larger.

Gutenberg-Richter Formula:

The big earthquakes are low frequency, high severity events. As the magnitude of the aftershocks increase, the number of these aftershocks declines exponentially. This is called the 'Gutenberg-Richter Relation' and can be expressed as [JERC, 1998, Vere-Jones, 1970]

$$\log \lambda^*(M) = a - bM,$$

or exponentiating with the log 10-base

$$\lambda^*(M) = 10^{a-bM},$$

where M is the magnitude, $\lambda^*(M)$ is the number of aftershocks bigger than magnitude M and a,b are constants, which express the level of overall aftershock activity. The number of aftershocks for a shock in $[M, M + \delta M]$ is denoted as $\lambda^*(M)\delta M$.

For this frequency-magnitude relation, if $\Lambda^*(M)$ is denoted as the 'total cumulative number of aftershocks larger than M'

$$\Lambda^*(M) = \int_{\infty}^{M} \lambda^*(x)dx = \int_{\infty}^{M} 10^{a-bx}dx,$$

where $\lambda^*(x) = 10^{a-bx}$, then

$$\Lambda^*(M) = 10^a \int_{\infty}^{M} e^{-\beta^* x} dx = \frac{10^{a-bM}}{\beta^*},$$

where $\beta = b \ln 10$ and ln is the natural logarithm. If the logarithm of both sides is taken

$$\log \Lambda^*(M) = a - bM - \log \beta^*,$$

and if $A = a - \log \beta^*$

$$\log \Lambda^*(M) = A - bM.$$

Let M_m denote the magnitude of the mainshock and M is the variable, which describes the magnitude of the aftershocks. Then, the aftershock sequence can be rewritten as [JERC, 1998, Reasenberg and Jones, 1989]

$$\log \Lambda^*(M) = A - b(M - M_m).$$

Generally the difference of the mainshock and aftershock magnitude is expected to be positive since the mainshock magnitude should be bigger than the aftershock magnitude. Therefore, the terms in the parentheses can be switched and it does not change the basic idea

$$\log \Lambda^*(M) = A + b(M_m - M).$$

Here, A is thought to represent the 'producing power' of the aftershock sequence and it is dependent on the mainshock. If the value of A is high, it can be interpreted that many aftershocks can be produced given the size of the mainshock. The b value is related to the magnitude distribution.

The analysis of an inhomogeneous Poisson process, a family of log-linear functions, whose logarithms are linear in coefficients, are very useful. We will continue the analysis by using the binning case, where we count for the number of claims arriving to each bin in the given time period. The notation for the intensity function $\lambda(t)$ of non-binning case will change to Λ_i in the binning case (the approximation of $\lambda(t)$ to Λ_i), where $i = 1, \ldots, n$.

Therefore, the exponential kernel function, which is used in the number of claims ($N_i \sim$ Pois(Λ_i)) likelihood calculations and modelling sections of this thesis, has basically the following form

$$\log \Lambda_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+} \longrightarrow \Lambda_i = e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+}},$$

where $s_j, j = 1 \ldots k$ is the kernel knots (sites) that the earthquake takes place with α_j effect. If the knot positions are parameters to be estimated, then it is a non-linear regression problem to solve (See Section 4.4.1). In the next page, Figure 4.1 represents the jump behaviour of the exponential kernel function. The power kernel on the other hand has the form

$$\log \Lambda_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j((t_i - s_j)|_+)^{-\beta} \longrightarrow \Lambda_i = e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j((t_i - s_j)|_+)^{-\beta}}.$$

The power kernel function can be named as a power-law shock kernel. Following the discussion about the form of the power kernel function in Page 46, if $\beta = 1$

$$\log \Lambda_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j((t_i - s_j)|_+)^{-1} \longrightarrow \Lambda_i = e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j((t_i - s_j)|_+)^{-1}}.$$

This is a case where the model can be expressed as a piecewise pure log-linear Poisson case. The jumps occur at significant bins as a flexibility of using an in-homogeneous Poisson process. For a single power kernel, the intensity function of the process is $\Lambda_i = e^{\alpha_j((t_i - s_j)|_+)}$. In both the exponential and power kernel cases, α_0 corresponds to the producing power A value as mentioned previously and it is used to represent the ordinary claims arriving in ordinary time period. The daily tremors and small earthquakes of magnitude less than 5 can also cause damage to the buildings, building contents and infrastructure. The claims arriving as a result of these events is represented by α_0 in the special kernel forms.

The + sign of the $(t_i - s_j)$, the difference between the actual event time and the kernel knot, denotes the heavy-side function to represent the shock effect of the earthquake occurrence. The α_j parameters have an additional effect to α_0 existence. They have a reasonable explanation that they represent the big claim size earthquakes, where big claims arrive as a result of big magnitude.

The β parameter represents the fix characteristic of each risk region and scientifically it is the universal constant of the rate of the decay. Also, in our case, it will change with the magnitude of the earthquake and is used to indicate the exponential decay after the mainshock. The β value changes with the amount of the incoming claims from region to region in Turkey. One advantage of using a single β is that the different kernels can be added and the same value of β is obtained with different α_j weights. This means the rate $\log \lambda$ is piecewise exponential or power (piecewise pure log-linear).

In this thesis, in time period $(0, t]$, the ordinary claims follow a routine path. If there is a sudden event like an earthquake, flood or any other type of man-made or natural disaster, the number of claims arriving to the insurance companies increase very significantly. The insurance companies decide to give priority to some of these claims to be able to deal with huge amount of claims in a short period of time. One way to do so is to decide on a threshold value by the company's own criteria (disaster area, the damage level of the property or the income level etc.) and they start processing the payments in order by giving priority to urgent and big claims. Then, things go to a normal period again until the next disaster strike or an extreme event occurrence. The plots showing the jump in the number of claims for the Turkish earthquake claims data are presented in Section 5.2. Considering the Modified Omori's Law and the magnitude-frequency-time relations above, the exponential and power kernel functions seem to reflect the jump behaviour of the claim arrival process.

It is interesting to observe that the power kernel function seems to be a generalisation of the Gutenberg-Richter form, which was given in Page 85 as $\Lambda^*(M) = 10^{a-bM}$. The simplest case of the suggested power kernel function (say considering the one bin), that is $\Lambda = e^{\alpha_0+\alpha_1 t^\beta}$, can explain the reason to choose these kernels to show the decreasing structure of the claim arrivals after main shock. In some cases of the big earthquakes, when $\beta = 1$ and α_1 is negative (i.e $\Lambda = e^{\alpha_0-\alpha_1 t}$), the power kernel function match with the main/after shock relation expressed with the Gutenberg-Richter Formula. The following plots give the idea behind the construction and the use of these types of kernel function analysis.

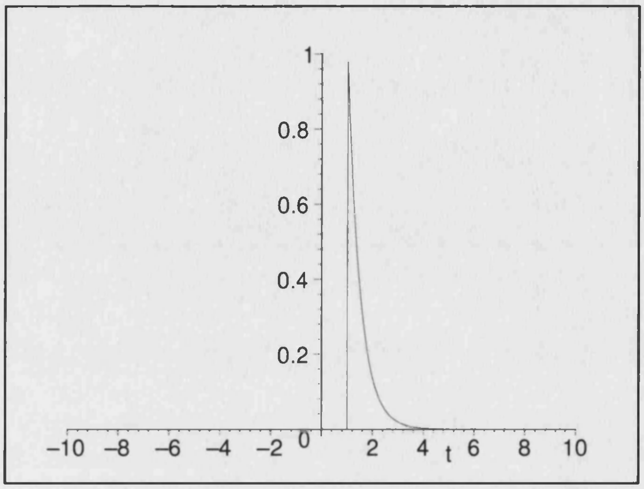

Figure 4.1: The plot of the exponential kernel

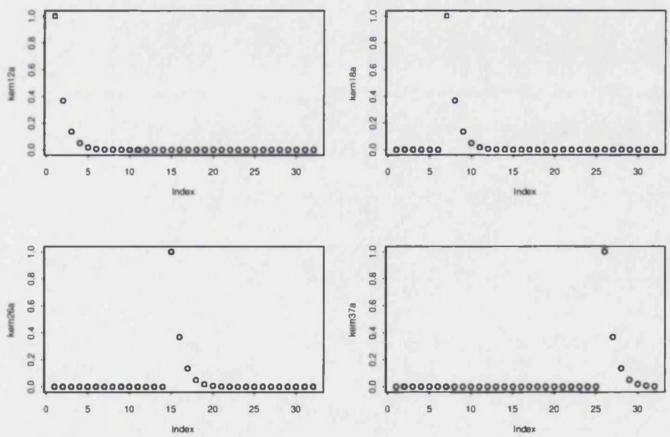

Figure 4.2: The exponential kernel function representing the jump behaviour of the large number of claims due to a sudden extreme event in all risk zones in terms of months

In the next page, Figure 4.3 shows how the power kernel function represents the decay of the claim arrivals with negative α_1 and positive β estimates. Figure 4.4 again denotes the decreasing shape of the power kernel function choice for the earthquake claim arrivals in the given time period (please refer to Page 45 for the use of the power kernel form). In this case, the α_1 is positive and β is negative. These graphs support the idea of the choice of the exponential and the power kernel special forms to work with in expressing the decrease in the claim arrivals after an earthquake strike for a given time period. It is observed that the sign of the α_0 (denoting the ordinary/very small claim arrivals in an ordinary time period) does not affect the increasing or decreasing shape of the kernel functions.

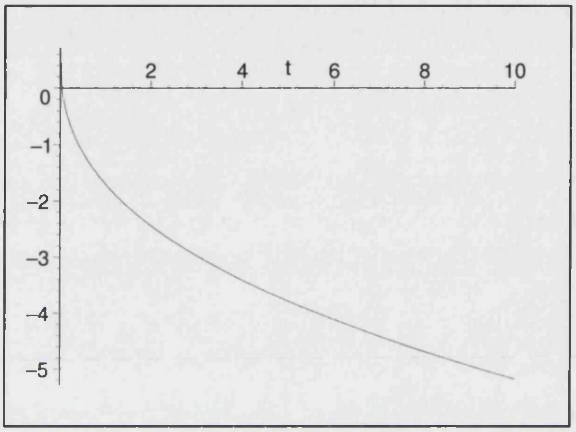

Figure 4.3: The power kernel form to represent the decreasing claim arrival process if $\alpha_1 < 0$, $\beta > 0$.

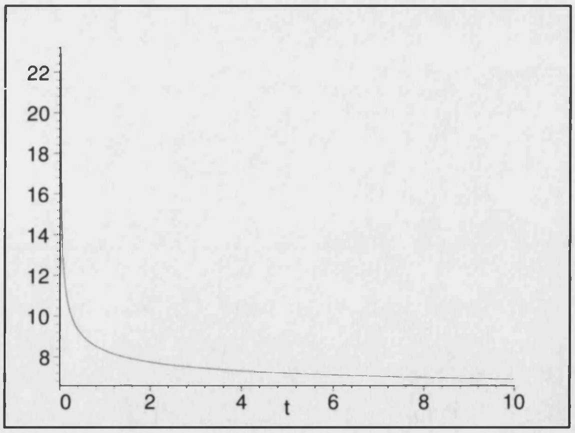

Figure 4.4: The power kernel form to represent the decreasing claim arrival process if $\alpha_1 > 0$, $\beta < 0$.

4.2.1 The use of the exponential kernel in the time likelihood

In the next two sections, the likelihood of time is derived by using (4.2) of Method 1 with the use of the exponential and the power kernel functions. The aim of this analysis is to show the procedure to use Method 1 (conditioning on time) in the likelihood of observations and parameter estimation. The detailed work will be explained for the bin case (see **Remark 2**, $N_i \sim \text{Pois}(\Lambda_i)$) by using Method 3 in the likelihood and parameter estimation, where the earthquake events are used as the counts in the modelling sections. Here, the way to estimate the non-linear parameter β in both the exponential and power kernel forms are given only without going into much detail. The estimation method for the α_j parameters are presented under the Section 4.4 of non-linear models.

The first choice for the kernel function is the exponential form. The kernel function includes the non-linear parameter β, whose values are estimated in the modelling sections in Chapter 5 and 6. By using Method 1 for the time likelihood, which represents the pure non-binning case of the claim arrivals in time period $(0, t]$, the intensity has the following suggested form

$$\log \lambda(t) = \begin{cases} \alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t-s)} & \text{if} \quad t \geq s; \\ 0 & \text{if} \quad t < s. \end{cases}$$

where $\lambda(t) = e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t-s)}}$ with $j = 1, \ldots, k$ knots, at which the kernels are replaced. When using the idea of binning as explained in Method 3 (Page 83), we have $N_i \sim \text{Poisson}(\Lambda_i)$ and in this case

$$\log \Lambda_i = \begin{cases} \alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|+} & \text{if} \quad t_i \geq s_j; \\ 0 & \text{if} \quad t_i < s_j. \end{cases}$$

where t_i is the beginning of the i^{th} bin. In the 'bin' case of Method 3, each count is attached to the whole bin, i.e. it is assumed that $\lambda(t)$ is a constant over each bin and approximated by Λ_i instead (see **Remark 2**).

Next, we will present some initial steps for the estimation of β, if Method 1 is used for the exponential kernel function with Gamma, Normal and Weibull distributions and later on for the power kernel function cases.

First, the value of Λ_i (as an approximation to λ since we are binning) above is replaced in the time likelihood of Method 1 in (4.2) as

$$L_1(t_1,\ldots,t_n;\theta,\alpha,\beta) = \prod_{i=1}^{n} e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|_+}} e^{-\Lambda(t_n)}, \tag{4.5}$$

where $\theta = (\alpha_0, \alpha_1, \ldots, \alpha_k; \beta)$ is the vector of the unknown parameters. Then, the log-likelihood is

$$\begin{aligned}
\log L_1(t_1,\ldots,t_n;\theta,\alpha,\beta) &= \log\left(\prod_{i=1}^{n} e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|_+}} e^{-\Lambda(t_n)}\right)\\
&= \sum_{i=1}^{n}\left(\log\left(e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|_+}} e^{-\Lambda(t_n)}\right)\right)\\
&= \sum_{i=1}^{n}\left(\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|_+} - \Lambda(t_n)\right),
\end{aligned} \tag{4.6}$$

where the mean rate function is $\Lambda(t_n) = \int_0^{t_n}\lambda(r)dr$. The next step is to differentiate the log-likelihood function in (4.6) with respect to the non-linear parameter β.

$$\begin{aligned}
\frac{\partial \log L_1}{\partial\beta} &= \frac{\partial}{\partial\beta}\sum_{i=1}^{n}\left(\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|_+} - \Lambda(t_n)\right)\\
&= \sum_{i=1}^{n}\frac{\partial}{\partial\beta}\left(\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|_+} - \Lambda(t_n)\right)\\
&= \sum_{i=1}^{n}\left(\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|_+}(-t_i+s_j)|_+ - \frac{\partial}{\partial\beta}\Lambda(t_n)\right).
\end{aligned} \tag{4.7}$$

Let's find the derivative of the mean function Λ at the last time point t_n.

$$\Lambda(t_n) = \int_0^{t_n}\lambda(r)dr = \int_0^{t_n} e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|_+}} dr$$

$$\begin{aligned}
\frac{\partial\Lambda(t_n)}{\partial\beta} &= \int_0^{t_n} e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|_+}} \sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|_+}(-t_i+s_j)|_+ dr\\
&= \sum_{j=1}^{k}\alpha_j\int_0^{t_n} e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|_+}} e^{-\beta(t_i-s_j)|_+}(-t_i+s_j)|_+ dr.
\end{aligned} \tag{4.8}$$

If (4.8) is replaced in (4.7), the following (4.9) is obtained. Numerical solution for this equation suggests the maximum likelihood estimate of the non-linear parameter

β for the use of the exponential kernel in the likelihood of time in Method 1.

$$\begin{aligned} \frac{\partial \log L_1}{\partial \beta} = \sum_{i=1}^{n} \Bigg(& \sum_{j=1}^{k} \alpha_j e^{-\beta (t_i - s_j)|_+} (-t_i + s_j)|_+ \\ & - \sum_{j=1}^{k} \alpha_j \int_0^{t_n} e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+}} e^{-\beta (t_i - s_j)|_+} (-t_i + s_j)|_+ dr \Bigg). \end{aligned} \tag{4.9}$$

4.2.2 The use of the power kernel in the time likelihood

In the calculation of the time likelihood, the second choice of the kernel function is the power kernel. The same procedure with the use of exponential kernel function is applied in this section. For the binning case, the log-rate of the process is

$$\log \Lambda i = \begin{cases} \alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta} & \text{if} \quad t_i \geq s_j; \\ 0 & \text{if} \quad t_i < s_j. \end{cases}$$

which gives the value of the rate itself $\Lambda_i = e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta}}$. The likelihood function is then

$$L_1(t_1, \ldots, t_n; \theta, \alpha, \beta) = \prod_{i=1}^{n} \lambda(t_i; \theta) e^{-\Lambda(t_n)} = \prod_{i=1}^{n} e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta}} e^{-\Lambda(t_n)}, \tag{4.10}$$

and the log-likelihood function is

$$\begin{aligned} \log L_1(t_1, \ldots, t_n; \theta, \alpha, \beta) &= \log \left(\prod_{i=1}^{n} \lambda(t_i; \theta) e^{-\Lambda(t_n)} \right) = \prod_{i=1}^{n} e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta}} e^{-\Lambda(t_n)} \\ &= \sum_{i=1}^{n} \log \left(e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta}} e^{-\Lambda(t_n)} \right) \\ &= \sum_{i=1}^{n} \left(\alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta} - \Lambda(t_n) \right). \end{aligned} \tag{4.11}$$

If (4.11) is differentiated with respect to β

$$\begin{aligned} \frac{\partial \log L_1}{\partial \beta} &= \frac{\partial}{\partial \beta} \left(\sum_{i=1}^{n} \left(\alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta} - \Lambda(t_n) \right) \right) \\ &= \sum_{i=1}^{n} \left(\sum_{j=1}^{k} -\alpha_j ((t_i - s_j)|_+)^{-\beta} \log (t_i - s_j)|_+ - \frac{\partial}{\partial \beta} \Lambda(t_n) \right). \end{aligned} \tag{4.12}$$

The next step is to find $\frac{\partial}{\partial\beta}\Lambda(t_n)$

$$\Lambda(t_n) = \int_0^{t_n} \lambda(r)dr = \int_0^{t_n} e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta}} dr, \tag{4.13}$$

and

$$\frac{\partial}{\partial\beta}\Lambda(t_n) = \int_0^{t_n} e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta}} \sum_{j=1}^{k} -\alpha_j ((t_i - s_j)|_+)^{-\beta} \log(t_i - s_j)|_+ dr \tag{4.14}$$

If (4.14) is replaced in (4.12) and solved for β, the maximum likelihood estimator of the non-linear parameter β can be obtained by solving the following equation with some numerical methods

$$\begin{aligned}\frac{\partial \log L_1}{\partial\beta} = \sum_{i=1}^{n} \Bigg(& \sum_{j=1}^{k} -\alpha_j ((t_i - s_j)|_+)^{-\beta} \log(t_i - s_j)|_+ \\ & + \int_0^{t_n} e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta}} \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta} \log(t_i - s_j)|_+ dr \Bigg)\end{aligned} \tag{4.15}$$

If of an interest, the maximum likelihood estimates of the α_j parameters can be derived for the time likelihood by using the similar estimation procedure above.

4.3 Generalised Linear Models

In the model selection process, generally the first attempt is to check the plots of the data and make some assumptions to provide a good fit. Then, by using the selected model and the data itself, the unknown parameters of the model are estimated. As a final step, the model is assessed to see if the analysis is appropriate for the data. The model can always be calibrated with the change of information of the data or for further research interest. These basic steps of the model building process are summarised as

$$\boxed{\text{Model selection}} \\ \downarrow \\ \boxed{\text{Model fitting}} \\ \downarrow \\ \boxed{\text{Model validation}}$$

Generalised linear model is an important tool to solve many actuarial statistics problems since the variable of the interest mostly does not fit the assumption of normally distributed data [Kaas et al., 2001]. The generalised linear model (glm) and the Cox proportional hazards model are two examples of the statistical models, which have a wide use in regression analysis, especially when the data is non-normally distributed. The generalised linear model is often used in applied statistics in demography, economics, geography, geology, history, medicine and other subjects. Especially, the milestone in the development of count data regression methods is the development of the idea of the generalised linear models. The term is first introduced in 1972 by Nelder and Wedderburn [Nelder and Wedderburn, 1972]. The name 'generalised linear models' comes from the idea that the glm generalises the classical linear models on the normal distribution with two methods [Lindsey, 1997, Montgomery et al., 2001a]:

1. These models involve a variety of distributions selected from a special family, e.g. exponential family (which includes Poisson, normal, exponential and gamma distributions),
2. They involve transformations of the mean, through a 'link function', which links the regression part to the mean of one of these distributions.

The generalised linear model is a unification of linear and non-linear models. The response (dependent) variable strictly needs to be a member of exponential family. Transformation of a response variable is generally very effective to deal with non-normality [Montgomery et al., 2001a]. The generalised linear model basically has three parts:

i) a random component: independent observations (responses $Y_1, \ldots, Y_i$),

ii) a systematic component: the linear predictor, which is denoted by η_i,

iii) a link between the random and systematic components by the use of a link function g, that is $g(\mu_i) = \eta_i$, where μ_i is the expected response. A link function g explains how the expected response is linked to the explanatory variables.

In statistics, linear models are the first choice to fit to the data for modelling purposes. But, recently, log-linear and logistic models started to be used in the modelling of discrete and categorical data. The inference and the emphasis on the likelihood function is a very important success of the use of the generalised linear models.

One way to describe the behaviour of a series of events like how and when these events occurred is 'counting processes'. A counting process, $N(t)$, is a random variable over time, which gives a simple count of the number of events occurring up to time t. In log-linear regression analysis, the response variable is explained with the suitable explanatory variables, which have some information on the dependence of an event on the past. The intensity of events, which is the rate $\lambda(t)$ in this study, may change over time and can show a dependence structure on the previous events [Andersen et al., 1993].

A Poisson distribution is considered to be a good choice to fit a model for the counts if the assumption of the Poisson Processes are valid [Kaas et al., 2001]. It is a special case of generalised linear models. The idea of Poisson distribution with the log-link as log-linear models is brought into statistical modelling in 1963 by Birch. As Poisson distribution is one of the most famous discrete distributions, which is a member of the exponential family, it also satisfies the condition of being in a family of distributions in the generalised linear model concept (see (4.38)).

4.3.1 Non-linear models

In statistical analysis, there are mainly two types of modelling, which are used to explain the behaviour of the data. The main aim is to estimate the parameters of the suggested models as much as possible so that the response variable is explained with maximum information. The first one of these models is *linear models*, which are often used when there is not enough theoretical model suggestions for the data. The general form of the linear models is [Faraway, 2005, Bierens and Gallant, 1997, Seber and Wild, 1989]

$$y \approx \beta_0 + \beta_1 X_1 + \ldots + \beta_n X_n + \epsilon,$$

or

$$y \approx \beta_0 + \beta_1 X_1^2 + \beta_2 X_1 X_2,$$

where y is the response (dependent) variable, X's are the explanatory (independent) variables, β's are the parameters to be estimated to explain the model and ϵ is the error term.

Any model, which is not linear in the unknown parameters, is called a '*non-linear regression model*'. Non-linear models have a range of different applications. They can be used to explain the problems even when the explanation is possible with the linear models. The non-linear behaviour of the data can be explained by non-linear modelling. In both cases, the choice of β is the consideration so that the systematic part of the model gives the maximum amount of information about the variables. For non-linear models, the change in the deviance is needed for a proper explanation and choice of the models. This argument is used during the modelling process and it is observed that when the covariates are used, the deviance of the models reduce.

4.3.2 Estimation of the non-linear parameter β

The method of least squares is one way to estimate the model parameters in regression analysis. The necessary assumption on the error terms is that the errors have zero mean and a constant variance, that is, $E(\epsilon_i) = 0$ and $Var(\epsilon_i) = \sigma^2$. Also, the error terms are uncorrelated regardless of the distribution. The normality assumption of the errors and so the response variable is needed to be able to conduct the hypothesis testing and to construct the confidence intervals. In non-linear regression, it is difficult to solve the resulting normal equations of the least square estimation method. Under the assumption that the errors are normally and independently distributed (i.e. $\epsilon_i \sim \mathrm{NID}(0, \sigma^2)$), the least square estimators are the same as the maximum likelihood estimators. The maximum likelihood estimators of the model parameters can be derived in linear regression [Montgomery et al., 2001a].

Even the assumption of $\epsilon_i \sim \mathrm{NID}(0, \sigma^2)$ holds in non-linear regression, finding the exact tests and confidence interval is not the case. Statistical inference requires large sample or asymptotic results in non-linear regression, as the large n theory applies in both normal and non-normal error distributions.

Poisson regression

An 'event count' means the number of times an event occurs. In the regression analysis, the response variable (dependent variable) is a non-negative integer-valued random variable [Cameron and Trivedi, 1998]. The interarrival time of an event, that is the length of the period between events, is dual of the event count [Cameron and Trivedi, 1998]. When the response variable is a count, the most suitable probability model is often the Poisson distribution, which was derived from the Binomial distribution by Poisson. The classic study of Bortkiewicz on the number of deaths caused by the kick of the mules in the Prussian army is an example of the applications of the Poisson distribution [Cameron and Trivedi, 1998]. If the intensity parameter of the Poisson distribution is let to depend on the covariates, the related regression model is called the 'Poisson regression model' [Cameron and Trivedi, 1998].

In log-linear regression analysis, suppose there is a canonical location parameter θ, which is equal to the other parameters in the linear form as [Lindsey, 1997]

$$\theta(\varsigma) = X\beta,$$

where ς is the mean of the Poisson distribution, β is a vector of unknown parameters (different than the non-linear parameter β of the models, which is used to represent the exponential decay of the claim arrivals due to some fix characteristic of the earthquake region) and X is a set of unknown explanatory variables (aka 'design' or 'model' matrix). Here, $X\beta$ forms the linear structure. This model can be rewritten by using other functions of the mean, $\eta(.)$

$$\eta(\varsigma) = X\beta,$$

which is the linear predictor. In this case, the model has linear and non-linear components together. If $\theta_i = \eta_i$, the relationship between the mean of the i^{th} observation ($Y_i \sim$ Poisson(ς_i)) and its linear predictor is given by a 'link function', $g_i(.)$

$$\eta_i = g_i(\varsigma_i) = x_i^T\beta.$$

Therefore, the counts data is modelled as log-linear Poisson distribution with log-rate, $g(\varsigma) = \log(\varsigma)$, and with generalised linear models. Some simple hierarchy can be created here [Neter et al., 1996]:

$$\boxed{\text{generalised linear models}} \longrightarrow \boxed{\text{nonlinear regression model}} \longrightarrow \boxed{\text{Poisson regression}}$$

The parameter β here is linked with non-normality (being Poisson) and non-linearity. It might also be suitable to call such case as generalised non-linear model (GNLM).

As the claim number process $N(t)$ is assumed to be an inhomogeneous Poisson process with the rate $\lambda(t)$ (as an approximation of the mean function $\Lambda(t)$), log-linear Poisson regression is appropriate for our case to model the number of counts [Lindsey, 1995, Howard, 2002, Lawless, 1987]. In the general canonical form, the model can be shown as

$$\log(\varrho_i) = \sum_j \beta_j x_{ij},$$

with a simple example

$$\log(\varrho_i) = \beta_0 + \beta_1 x_i,$$

where the mean ϱ refers to the rate $\lambda(t)$ of an inhomogeneous claim number process $N(t)$ and x_i simply refers to the counts (considering the calendar time in months/weeks) in our case.

The probability density function of Poisson distribution is

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-\lambda}e^{x\log\lambda}}{x!} = \frac{e^{x\log\lambda-\lambda}}{x!}; \quad x > 0. \tag{4.16}$$

The exponential family, which has the probability function as in the following Equation will next be linked to the Poisson case

$$f(y) = e^{\frac{y\theta+c(\theta)}{\phi}+d(y,\phi)}, \tag{4.17}$$

where θ is the canonical parameter as a natural estimator for the distribution.

Poisson regression represents the likelihood function for any type of distribution from the exponential family for frequency data. The method of maximum likelihood is used to estimate the parameters in Poisson regression [Ozaki, 1979, Vere-Jones and Ozaki, 1982].

The logarithmic form of the Poisson density given in (4.16) is

$$\log f(x) = x \log \lambda - \lambda - \log(x!),$$

and the terms match with the exponential family in (4.17) as $\log \lambda = \theta$, $c(\theta) = -\lambda = -e^{\theta}$, $\phi = 1$ and $d(y, \phi) = -\log(x!)$.

Since we are using the idea of binning when modelling the number of claims as Poisson counts, we denote the rate of the process as $\log \Lambda_i$ and then the likelihood of the Poisson distribution can be written as

$$L(x, \theta) = \prod_{i=1}^{n} \frac{e^{x_i \log \Lambda_i - \Lambda_i}}{x_i!} = \frac{e^{\sum_{i=1}^{n}(x_i \log \Lambda_i - \Lambda_i)}}{\prod_{i=1}^{n}(x_i!)},$$

where x_i (here and in Poisson likelihood computations x_i is used for the counts) denotes the counts in the i^{th} bin given as N_i in Method 3 (4.4).

Then the log-likelihood is

$$\log L(x; \theta) = \sum_{i=1}^{n}(x_i \log \Lambda_i - \Lambda_i) - \log(\prod_{i=1}^{n} x_i!) = \sum_{i=1}^{n}(x_i \log \Lambda_i - \Lambda_i) - \sum_{i=1}^{n} \log(x_i!). \tag{4.18}$$

Please note that the logarithms are to base e (natural logarithm) in further calculations. As a next step, the exponential and power kernels are substituted in the intensity of the process and the estimation of the non-linear parameter β and the linear α_j parameters are worked through. The α_j parameters are assumed to be non-normal (non-Gaussian) and the β parameter is non-normal (non-Gaussian) and also non-linear as stated earlier. The maximum likelihood estimate $\hat{\beta}$ is assumed to be distributed normal, that is $\hat{\beta} \sim N\Big(\beta, (I(\hat{\beta}))^{-1}\Big)$, where I is the observed information matrix.

4.4 Estimation of the model parameters for Poisson likelihood

4.4.1 β derivatives

Exponential kernel: As stated earlier, in log-linear Poisson regression, rather than the rate itself, the log-rate is modelled to explain the number of claims N_i.

Therefore the estimation of the non-linear parameter β uses the exponential kernel function form of the rate of bins, Λ_i, in the modelling process. Here, it is assumed that ordinary claims occur from small tremors, wall cracks due to these tremors etc. and these claims are represented with a constant α_0 in the intensity. In the pure log-linear case

$$\log \Lambda_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+}, \tag{4.19}$$

and the intensity of the process will be $\Lambda_i = e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i-s_j)|_+}}$.

In (4.18), the log-likelihood of the process is given as follows with the intensity function $\log \Lambda_i$ [Ozaki, 1979, Vere-Jones and Ozaki, 1982]

$$\log L(x;\beta,\alpha) = \sum_{i=1}^{n} \left(x_i \big(\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i-s_j)|_+}\big) - e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i-s_j)|_+}} \right) - \sum_{i=1}^{n} \log(x_i!). \tag{4.20}$$

If this log-likelihood is differentiated with respect to non-linear parameter β

$$\frac{\partial \log L}{\partial \beta} = \frac{\partial}{\partial \beta} \sum_{i=1}^{n} (x_i \log \Lambda_i - \Lambda_i) - \frac{\partial}{\partial \beta} \sum_{i=1}^{n} \log(x_i!),$$

the Fisher's score function (also called 'Fisher's score vector' if there is more than one parameter) $U(\beta)$ is obtained as

$$\begin{aligned} \frac{\partial \log L}{\partial \beta} = U(\beta) = \sum_{i=1}^{n} \Bigg(& - x_i \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i-s_j)|_+} (-t_i + s_j)|_+ \\ & - \Big(e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i-s_j)|_+}} \Big(\sum_{j=1}^{k} \alpha_j e^{-\beta(t_i-s_j)|_+} (-t_i + s_j)|_+ \Big) \Big) \Bigg). \end{aligned}$$

If the score function, $U(\beta)$, is differentiated with respect to β again, the $\beta\beta$ entry of the Hessian (matrix of second derivatives of the log-likelihood function) is obtained as

$$\frac{\partial^2 \log L}{\partial \beta^2} = \frac{\partial U(\beta)}{\partial \beta} = \sum_{i=1}^{n} \Bigg(- x_i \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+} (-t_i + s_j)|_+ (-t_i + s_j)|_+$$
$$- \Big(e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+}} \Big(\sum_{j}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+} (-t_i + s_j)|_+ \Big)$$
$$\Big(\sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+} (-t_i + s_j)|_+ \Big)$$
$$+ \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+} (-t_i + s_j)|_+ (-t_i + s_j)|_+ e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+}} \Big) \Bigg). \tag{4.21}$$

if (4.21) is rewritten

$$\frac{\partial^2 \log L}{\partial \beta^2} = \frac{\partial U(\beta)}{\partial \beta} = \sum_{i=1}^{n} \Bigg(- x_i \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+} ((-t_i + s_j)|_+)^2$$
$$- \Big(e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+}} \Big(\sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+} (-t_i + s_j)|_+ \Big)^2$$
$$+ e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+}} \Big(\sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+} ((-t_i + s_j)|_+)^2 \Big) \Big) \Bigg),$$

and the estimated approximate variance of the non-linear parameter β is obtained via the observed information matrix $I(\alpha_0, \alpha_1, \ldots, \alpha_k; \beta)$, by using the expectation of the Hessian matrix

$$I(\alpha_0, \alpha_1, \ldots, \alpha_k; \beta)) = E(-H(\alpha_0, \alpha_1, \ldots, \alpha_k; \beta))).$$

Since the expectation is sometimes hard to obtain in this form, $-H$ is called 'the observed information matrix I' and we can use $I(\alpha_0, \alpha_1, \ldots, \alpha_k; \beta)) = -H(\alpha_0, \alpha_1, \ldots, \alpha_k; \beta))$ instead. The inversion $[H]^{-1}$ is used to get the confidence interval for the β and α_j parameters. In that case, the corresponding entry of the observed information matrix for the approximate β variance is

$$
\begin{aligned}
I(\beta) = \sum_{i=1}^{n} \Bigg(& - x_i \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+} ((-t_i + s_j)|_+)^2 \\
& + \Big(e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+}} \Big(\sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+} (-t_i + s_j)|_+ \Big)^2 \\
& - e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+}} \Big(\sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+} ((-t_i + s_j)|_+)^2 \Big) \Big) \Bigg).
\end{aligned}
\tag{4.22}
$$

Power kernel: Instead of the exponential kernel, if the power kernel function is used in the calculations above to obtain the form for the estimates and the confidence interval of the model parameters, the log-rate of the bin case is

$$
\log \Lambda_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j (t_i - s_j)|_+^{-\beta}, \tag{4.23}
$$

which implies $\Lambda_i = e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta}}$.

The log-likelihood is the same as in (4.18) and if (4.23) is replaced in it, the new log-likelihood is

$$
\log L(x; \beta, \alpha) = \sum_{i=1}^{n} \Big(x_i (\alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta}) - e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta}} \Big) - \sum_{i=1}^{n} \log x_i!.
$$

If we differentiate this log-likelihood with respect to the non-linear β parameter, the score function will be

$$
\begin{aligned}
\frac{\partial \log L}{\partial \beta} = U(\beta) = \sum_{i=1}^{n} \Bigg(& x_i \big(\sum_{j=1}^{k} -\alpha_j ((t_i - s_j)|_+)^{-\beta} \big) \log(t_i - s_j)|_+ \\
& + \Big(e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta}} \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta} \log(t_i - s_j)|_+ \Big) \Bigg).
\end{aligned}
$$

Then, the second derivative of the log-likelihood with respect to β is

$$\frac{\partial^2 \log L}{\partial \beta^2} = \frac{\partial U(\beta)}{\partial \beta} = \sum_{i=1}^{n} \left(x_i \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta} (\log(t_i - s_j)|_+)^2 \right.$$
$$- \left(e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta}} \left(\sum_{j=1}^{k} -\alpha_j ((t_i - s_j)|_+)^{-\beta} (\log(t_i - s_j)|_+) \right)^2 \right.$$
$$\left. \left. + \, e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta}} \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta} (\log(t_i - s_j)|_+)^2 \right) \right), \tag{4.24}$$

and

$$I(\beta) = -H(\beta) = \sum_{i=1}^{n} \left(- x_i \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta} (\log(t_i - s_j)|_+)^2 \right.$$
$$+ \left(e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta}} \left(\sum_{j=1}^{k} -\alpha_j ((t_i - s_j)_+)^{-\beta} (\log(t_i - s_j)|_+) \right)^2 \right.$$
$$\left. \left. - \, e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta}} \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta} (\log(t_i - s_j)|_+)^2 \right) \right). \tag{4.25}$$

This $(k+2, k+2)$ entry of the inverse information matrix is the variance estimate of the β parameter and used to construct the confidence interval of β in the following form:

$$\hat{\beta} \pm \text{Critical value} \times \text{standard error}(\hat{\beta}).$$

The use of the non-linear parameter β is to explain some special characteristics (geographical structure like closeness to the fault line, population, earthquake history of the area etc.) of the earthquake risk area and the α_j parameters denote the effect of the big, empirical earthquakes as later shown in Table 5.1 (the events with more than 100 claims, $N_i > 100$). The score function $U(L(\beta; \alpha_0, \alpha_1, \ldots, \alpha_k))$, is denoted in the matrix notation for the β and α parameters as follows:

$$\mathbf{U}(\beta;\alpha_0,\alpha_1,\ldots,\alpha_k) = \begin{bmatrix} \frac{\partial \log L(\beta;\alpha_0\ldots\alpha_k)}{\partial\alpha_0} \\ \frac{\partial \log L(\beta;\alpha_0\ldots\alpha_k)}{\partial\alpha_1} \\ \frac{\partial \log L(\beta;\alpha_0\ldots\alpha_k)}{\partial\alpha_2} \\ \vdots \\ \frac{\partial \log L(\beta;\alpha_0\ldots\alpha_k)}{\partial\alpha_k} \\ \frac{\partial \log L(\beta;\alpha_0\ldots\alpha_k)}{\partial\beta} \end{bmatrix}$$

and the Hessian matrix (the matrix of second derivatives) will be in the form:

$$\mathbf{H}(\beta;\alpha_0,\alpha_1,\ldots,\alpha_k) = \begin{bmatrix} \frac{\partial^2 \log L(\beta;\alpha_0,\alpha_1\ldots\alpha_k)}{\partial\alpha_0\partial\alpha_0} & \frac{\partial^2 \log L(\beta;\alpha_0,\alpha_1\ldots\alpha_k)}{\partial\alpha_0\partial\alpha_1} & \cdots & \frac{\partial^2 \log L(\beta;\alpha_0,\alpha_1\ldots\alpha_k)}{\partial\alpha_0\partial\beta} \\ \vdots & \vdots & & \vdots \\ \frac{\partial^2 \log L(\beta;\alpha_0,\alpha_1\ldots\alpha_k)}{\partial\alpha_j\partial\alpha_0} & \frac{\partial^2 \log L(\beta;\alpha_0,\alpha_1\ldots\alpha_k)}{\partial\alpha_j\partial\alpha_1} & \cdots & \frac{\partial^2 \log L(\beta;\alpha_0,\alpha_1\ldots\alpha_k)}{\partial\alpha_j\partial\beta} \\ \frac{\partial^2 \log L(\beta;\alpha_0,\alpha_1\ldots\alpha_k)}{\partial\beta\partial\alpha_0} & \frac{\partial^2 \log L(\beta;\alpha_0,\alpha_1\ldots\alpha_k)}{\partial\beta\partial\alpha_1} & \cdots & \frac{\partial^2 \log L(\beta;\alpha_0,\alpha_1\ldots\alpha_k)}{\partial\beta\partial\beta} \end{bmatrix}$$

The diagonal entries of the inverted observed information matrix (variance-covariance matrix) are the variances of the corresponding parameters and will be used in confidence interval construction.

4.4.2 α derivatives

In the log-linear Poisson modelling, the estimation of the linear parameter α_j's, which is used to represent the big earthquakes, is shown by using the exponential and power kernels with the same argument as in the estimation of the non-linear parameter β.

Exponential kernel: If the exponential kernel function is used for modelling of the rate Λ_i for the binning case, the log-likelihood in (4.18) changes to

$$\log L(x;\beta,\alpha) = \sum_{i=1}^{n}\left(x_i\big(\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|+}\big)-e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|+}}-\sum_{i=1}^{n}\log(x_i!)\right). \tag{4.26}$$

Then the general form of the first derivative of this likelihood, or the score function with respect to the parameter α_j will be

$$\frac{\partial \log L}{\partial \alpha_j} = \sum_{i=1}^{n}\Bigg(x_i \sum_{j=1}^{k} e^{-\beta(t_i - s_j)|+} - e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i-s_j)|+}} \sum_{j=1}^{k} e^{-\beta(t_i - s_j)|+}\Bigg), \tag{4.27}$$

and the Hessian of the log-likelihood is

$$\begin{aligned}\frac{\partial^2 \log L}{\partial \alpha_j^2} &= \sum_{i=1}^{n}\Bigg(- e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i-s_j)|+}} \sum_{j=1}^{k} e^{-\beta(t_i - s_j)|+} \sum_{j=1}^{k} e^{-\beta(t_i - s_j)|+}\Bigg) \\ &= -\sum_{i=1}^{n}\Bigg(e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i-s_j)|+}} (\sum_{j=1}^{k} e^{-\beta(t_i - s_j)|+})^2\Bigg),\end{aligned} \tag{4.28}$$

and the $\alpha_j\alpha_j$ (e.g $\frac{\partial^2 \log L}{\partial\alpha_1\partial\alpha_1}$ or $\frac{\partial^2 \log L}{\partial\alpha_2\partial\alpha_2}$ ...) entries of the information matrix is obtained by

$$I(\alpha_j) = +\sum_{i=1}^{n}\Bigg(e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i-s_j)|+}} (\sum_{j=1}^{k} e^{-\beta(t_i - s_j)|+})^2\Bigg). \tag{4.29}$$

For the entries $\alpha_j\alpha_{j'}$ (e.g $\frac{\partial^2 \log L}{\partial\alpha_1\partial\alpha_2}$ or $\frac{\partial^2 \log L}{\partial\alpha_2\partial\alpha_1}$ ...), the direct second partial derivative for $j = j' = 1, \ldots, k$ is

$$\frac{\partial^2 \log L}{\partial \alpha_j \partial \alpha_{j'}} = \sum_{i=1}^{n}\Bigg(- e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i-s_j)|+}} e^{-\beta(t_i - s_{j'})|+} e^{-\beta(t_i - s_j)|+}\Bigg). \tag{4.30}$$

The score function with respect to α_0 by (4.26) is

$$\frac{\partial \log L}{\partial \alpha_0} = \sum_{i=1}^{n}\Bigg(x_i - e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i-s_j)|+}}\Bigg), \tag{4.31}$$

and the corresponding Hessian is

$$\frac{\partial^2 \log L}{\partial \alpha_0^2} = \sum_{i=1}^{n}\Bigg(- e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i-s_j)|+}}\Bigg). \tag{4.32}$$

By (4.31), the Hessian with parameters α_0, α_j is obtained as

$$\frac{\partial^2 \log L}{\partial \alpha_0 \partial \alpha_j} = -\sum_{i=1}^{n}\Bigg(e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i-s_j)|+}} (\sum_{j=1}^{k} e^{-\beta(t_i - s_j)|+})\Bigg). \tag{4.33}$$

Power kernel: If the power kernel function replaces the exponential kernel, the log-likelihood in (4.18) changes into

$$\log L(x;\beta,\alpha) = \sum_{i=1}^{n}\left(x_i\big(\alpha_0+\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{-\beta}\big)-e^{\alpha_0+\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{-\beta}}-\sum_{i=1}^{n}\log(x_i!)\right). \tag{4.34}$$

The general form of the first derivative of this likelihood with respect to α_j is

$$\begin{aligned}\frac{\partial \log L}{\partial \alpha_j} = \sum_{i=1}^{n}\Bigg(& x_i\big(\sum_{j=1}^{k}((t_i-s_j)|_+)^{-\beta}\big) - e^{\alpha_0+\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{-\beta}} \\ & \big(\sum_{j=1}^{k}((t_i-s_j)|_+)^{-\beta}\big)\Bigg),\end{aligned} \tag{4.35}$$

and the Hessian is

$$\begin{aligned}\frac{\partial^2 \log L}{\partial \alpha_j^2} &= \sum_{i=1}^{n}\Bigg(- e^{\alpha_0+\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{-\beta}}\big(\sum_{j=1}^{k}((t_i-s_j)|_+)^{-\beta}\big) \\ &\qquad \big(\sum_{j=1}^{k}((t_i-s_j)|_+)^{-\beta}\big)\Bigg) \\ &= -\sum_{i=1}^{n}\left(e^{\alpha_0+\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{-\beta}}\big(\sum_{j=1}^{k}((t_i-s_j)|_+)^{-\beta}\big)^2\right),\end{aligned} \tag{4.36}$$

which gives

$$I(\alpha_j) = +\sum_{i=1}^{n}\left(e^{\alpha_0+\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{-\beta}}\big(\sum_{j=1}^{k}((t_i-s_j)|_+)^{-\beta}\big)^2\right). \tag{4.37}$$

Similar to the exponential kernel case, the second partial derivative is ($j = j' = 1,\ldots,k$)

$$\frac{\partial^2 \log L}{\partial \alpha_j \partial \alpha_{j'}} = \sum_{i=1}^{n}\left(- e^{\alpha_0+\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{-\beta}}((t_i-s_{j'})|_+)^{-\beta}\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{-\beta}\right). \tag{4.38}$$

By using (4.34), the score function with respect to α_0 is

$$\frac{\partial \log L}{\partial \alpha_0} = \sum_{i=1}^{n}\left(x_i - e^{\alpha_0+\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{-\beta}}\right), \tag{4.39}$$

and the corresponding Hessian is

$$\frac{\partial^2 \log L}{\partial \alpha_0^2} = \sum_{i=1}^{n} \left(- e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta}} \right). \tag{4.40}$$

By (4.39), the Hessian of parameters α_0, α_j with the use of power kernel is

$$\frac{\partial^2 \log L}{\partial \alpha_0 \alpha_j} = - \sum_{i=1}^{n} \left(e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta}} (\sum_{j=1}^{k} ((t_i - s_j)|_+)^{-\beta}) \right). \tag{4.41}$$

The following gives the confidence interval for the α_j parameters by using the $\alpha_j \alpha_j$ entries of the Hessian matrix as the variance estimates of α_j's:

$$\hat{\alpha_j} \pm \text{Critical value} \times \text{standard error}(\hat{\alpha_j}).$$

4.4.3 α β derivatives

The entries for $\alpha_j \beta$ and $\beta \alpha_j$ of the Hessian matrix is the same by the symmetry. Next, the derivation of these entries are explained.

Exponential kernel: If the log-intensity is modelled with the exponential kernel of the form $\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta (t_i - s_j)|_+}$ by using (4.39)

$$\begin{aligned} \frac{\partial \log L}{\partial \beta} = \sum_{i=1}^{n} \Bigg(& - x_i (\sum_{j=1}^{k} \alpha_j e^{-\beta (t_i - s_j)|_+}) (-t_i + s_j)|_+ \\ & - \left(e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta (t_i - s_j)|_+}} \left(\sum_{j=1}^{k} \alpha_j e^{-\beta (t_i - s_j)|_+} (-t_i + s_j)|_+ \right) \right) \Bigg), \end{aligned}$$

and the second derivative with respect to α_j is

$$\begin{aligned} \frac{\partial^2 \log L}{\partial \beta \partial \alpha_j} = \sum_{i=1}^{n} \Bigg(& - x_i \sum_{j=1}^{k} e^{-\beta (t_i - s_j)|_+} (-t_i + s_j)|_+ \\ & - \Bigg(e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta (t_i - s_j)|_+}} \sum_{j=1}^{k} e^{-\beta (t_i - s_j)|_+} \sum_{j=1}^{k} \alpha_j e^{-\beta (t_i - s_j)|_+} (-t_i + s_j)|_+ \\ & + \sum_{j=1}^{k} e^{-\beta (t_i - s_j)|_+} (-t_i + s_j)|_+ e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta (t_i - s_j)|_+}} \Bigg) \Bigg). \end{aligned} \tag{4.42}$$

If the log-likelihood is first differentiated with respect to α_j and then with respect to β

$$\frac{\partial \log L}{\partial \alpha_j} = U(\alpha_j) = \sum_{i=1}^{n} \Bigg(- x_i(\sum_{j=1}^{k} e^{-\beta(t_i-s_j)|_+}) - e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|_+}} \sum_{j=1}^{k} e^{-\beta(t_i-s_j)|_+} \Bigg),$$

and

$$\begin{aligned}\frac{\partial^2 \log L}{\partial \alpha_j \partial \beta} = \sum_{i=1}^{n} \Bigg(x_i \sum_{j=1}^{k} e^{-\beta(t_i-s_j)|_+}(-t_i+s_j)|_+ - \Big(e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|_+}} \\ \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i-s_j)|_+}(-t_i+s_j)|_+ (\sum_{j=1}^{k} e^{-\beta(t_i-s_j)|_+}) \\ + \sum_{j=1}^{k} e^{-\beta(t_i-s_j)|_+}(-t_i+s_j)|_+ e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|_+}} \Big) \Bigg).\end{aligned} \tag{4.43}$$

It is observed that (4.42) and (4.43) are the same.

For the $\frac{\partial^2 \log L}{\partial \beta \partial \alpha_0}$ entry of the score function and the Hessian, if the score of the log-likelihood of (4.20) is used

$$\begin{aligned}\frac{\partial^2 \log L}{\partial \beta \partial \alpha_0} &= \sum_{i=1}^{n} \Bigg(- e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|_+}} (\sum_{j=1}^{k} \alpha_j e^{-\beta(t_i-s_j)|_+}(-t_i+s_j)|_+) \Bigg) \\ &= - \sum_{i=1}^{n} \Bigg(e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|_+}} (\sum_{j=1}^{k} \alpha_j e^{-\beta(t_i-s_j)|_+}(-t_i+s_j)|_+) \Bigg),\end{aligned}$$

and by the symmetry the $\frac{\partial^2 \log L}{\partial \alpha_0 \partial \beta}$ entry is the same value.

Power kernel: In this case, if the power kernel of the form $\alpha_0 + \sum_{j=1}^{k} \alpha_j((t_i - s_j)|_+)^{-\beta}$ is used during the modelling process

$$\begin{aligned}\frac{\partial \log L}{\partial \beta} = \sum_{i=1}^{n} \Bigg(x_i(\sum_{j=1}^{k} -\alpha_j((t_i-s_j)|_+)^{-\beta}) \log(t_i-s_j)|_+ \\ + \Big(e^{\alpha_0+\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{-\beta}} \sum_{j=1}^{k} \alpha_j((t_i-s_j)|_+)^{-\beta} \log(t_i-s_j)|_+ \Big) \Bigg),\end{aligned}$$

and the second derivative of the log-likelihood with respect to α_j is

$$\frac{\partial^2 \log L}{\partial\beta\partial\alpha_j} = \sum_{i=1}^{n}\Bigg(x_i\sum_{j=1}^{k} -((t_i - s_j)|_+)^{-\beta}\log(t_i - s_j)|_+ \\ - \Big(e^{\alpha_0+\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{-\beta}}(\sum_{j=1}^{k}((t_i - s_j)|_+)^{-\beta})\sum_{j=1}^{k} -\alpha_j((t_i - s_j)|_+)^{-\beta} \\ \log(t_i - s_j)|_+ + \sum_{j=1}^{k} -((t_i - s_j)|_+)^{-\beta}\log(t_i - s_j)|_+ e^{\alpha_0+\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{-\beta}}\Big)\Bigg), \tag{4.44}$$

and if the order of the parameter derivatives changes

$$\frac{\partial \log L}{\partial\alpha_j} = \sum_{i=1}^{n}\Bigg(x_i(\sum_{j=1}^{k}((t_i - s_j)|_+)^{-\beta}) - e^{\alpha_0+\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{-\beta}}(\sum_{j=1}^{k}((t_i - s_j)|_+)^{-\beta})\Bigg),$$

then the second derivative will be

$$\frac{\partial^2 \log L}{\partial\alpha_j\partial\beta} = \sum_{i=1}^{n}\Bigg(x_i\sum_{j=1}^{k} -((t_i - s_j)|_+)^{-\beta}\log(t_i - s_j)|_+ \\ - \Big(e^{\alpha_0+\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{-\beta}}\sum_{j=1}^{k} -\alpha_j((t_i - s_j)|_+)^{-\beta}\log(t_i - s_j)|_+ \\ (\sum_{j=1}^{k}((t_i - s_j)|_+)^{-\beta}) + \sum_{j=1}^{k} -((t_i - s_j)|_+)^{-\beta}\log(t_i - s_j)|_+ \\ e^{\alpha_0+\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{-\beta}}\Big)\Bigg). \tag{4.45}$$

(4.44) and (4.45) are the same as well for the second partial derivative of the parameter estimates for the likelihood in (4.18) when the power kernel is used. The $\frac{\partial^2 \log L}{\partial\beta\partial\alpha_0}$ entry of the Hessian with the use of the power kernel is

$$\frac{\partial^2 \log L}{\partial\beta\partial\alpha_0} = \sum_{i=1}^{n}\Bigg(- e^{\alpha_0+\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{-\beta}}\sum_{j=1}^{k} -\alpha_j((t_i - s_j)|_+)^{-\beta}\log(t_i - s_j)|_+\Bigg) \\ = \sum_{i=1}^{n}\Bigg(+ e^{\alpha_0+\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{-\beta}}\sum_{j=1}^{k}\alpha_j((t_i - s_j)|_+)^{-\beta}\log(t_i - s_j)|_+\Bigg),$$

and again by the symmetry of the variance-covariance matrix, $\frac{\partial^2 \log L}{\partial\alpha_0\partial\beta}$ entry has the same value.

4.5 Estimation of the model parameters for Normal likelihood

Normal distribution is a member of the exponential family like the Poisson distribution. Therefore, generalised linear models are applicable in the modelling of the total claim amount (aggregate claims) in the bin case as $S_i = \sum_{i=1}^{N_i} X_i$ in this study. The argument that if $\log S_i \sim$ Normal then $S_i \sim$ lognormal is used in the further calculations, since the natural logarithm of the aggregate claims data in the corresponding bin is used in the analysis. In general, if a random variable has a Normal distribution with parameters μ and σ^2, where σ^2 is assumed to be known, the probability density function of say, X, is [Dobson, 1990]

$$f(x;\mu) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}; \quad -\infty < x < +\infty.$$

This is written in the form of the exponential family as

$$f(x;\mu) = e^{-\frac{x^2}{2\sigma^2}+\frac{x\mu}{2\sigma^2}-\frac{\mu^2}{2\sigma^2}-\frac{1}{2}\log(2\pi\sigma^2)}; \quad -\infty < x < +\infty.$$

Then for our case, the likelihood function of this density can be written as

$$L(x;\mu) = \prod_{i=1}^{n} e^{-\frac{x_i^2}{2\sigma^2}+\frac{x_i\mu}{2\sigma^2}-\frac{\mu^2}{2\sigma^2}-\frac{1}{2}\log(2\pi\sigma^2)},$$

where here x_i represents the aggregate claims of each corresponding bin. The log-likelihood to derive the score function and second partial derivatives is

$$\log L(x;\mu) = \sum_{i=1}^{n} \Big(-\frac{x_i^2}{2\sigma^2} + \frac{x_i\mu}{2\sigma^2} - \frac{\mu^2}{2\sigma^2} - \frac{1}{2}\log(2\pi\sigma^2)\Big). \qquad (4.46)$$

4.5.1 β derivatives

Here, the exponential and the power kernel functions are substituted for the mean μ, which is the parameter of the normal distributed aggregate claims.

Exponential kernel:

For the exponential kernel representation, instead the Poisson parameter Λ_i of the bins, the mean μ_i of the aggregate claims S_i is used as

$$\mu_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+}.$$

If this is replaced in (4.46), the log-likelihood is

$$\log L(x;\mu) = \sum_{i=1}^{n} \Big(-\frac{x_i^2}{2\sigma^2} + \frac{x_i(\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i-s_j)|_+})}{2\sigma^2} - \frac{(\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i-s_j)|_+})^2}{2\sigma^2} - \frac{1}{2}\log(2\pi\sigma^2)\Big). \tag{4.47}$$

If (4.47) is differentiated with respect to β, the score function is

$$\begin{aligned}\frac{\partial \log L}{\partial \beta} = \sum_{i=1}^{n} \Bigg(-\frac{1}{2}\Big(&\frac{x_i t_i \sum_{j=1}^{k} \alpha_j e^{\beta s_j}}{\sigma^2 e^{\beta t_i}} + \frac{x_i \sum_{j=1}^{k} \alpha_j s_j e^{\beta s_j}}{2\sigma^2 e^{\beta t_i}} + \frac{\alpha_0 t_i \sum_{j=1}^{k} \alpha_j e^{\beta s_j}}{\sigma^2 e^{\beta t_i}} \\ &- \frac{\alpha_0 \sum_{j=1}^{k} \alpha_j s_j e^{\beta s_j}}{\sigma^2 e^{\beta t_i}} + \frac{t_i (\sum_{j=1}^{k} \alpha_j e^{\beta s_j})^2}{\sigma^2 (e^{\beta t_i})^2} - \frac{(\sum_{j=1}^{k} \alpha_j e^{\beta s_j})(\sum_{j=1}^{k} \alpha_j s_j e^{\beta s_j})}{\sigma^2 (e^{\beta t_i})^2}\Big)\Bigg),\end{aligned} \tag{4.48}$$

and the second partial derivative of the log-likelihood with respect to β is

$$\begin{aligned}\frac{\partial^2 \log L}{\partial \beta^2} = \sum_{i=1}^{n} \Bigg(\frac{1}{2}\Big(&\frac{x_i t_i^2 \sum_{j=1}^{k} \alpha_j e^{\beta s_j}}{\sigma^2 e^{\beta t_i}} - \frac{x_i t_i \sum_{j=1}^{k} \alpha_j s_j e^{\beta s_j}}{2\sigma^2 e^{\beta t_i}} + \frac{x_i \sum_{j=1}^{k} \alpha_j s_j^2 e^{\beta s_j}}{2\sigma^2 e^{\beta t_i}} \\ &- \frac{\alpha_0 t_i^2 \sum_{j=1}^{k} \alpha_j e^{\beta s_j}}{\sigma^2 e^{\beta t_i}} + \frac{2\alpha_0 t_i (\sum_{j=1}^{k} \alpha_j s_j e^{\beta s_j})}{\sigma^2 e^{\beta t_i}} - \frac{\alpha_0 \sum_{j=1}^{k} \alpha_j s_j^2 e^{\beta s_j}}{\sigma^2 e^{\beta t_i}} \\ &- \frac{2t_i^2 (\sum_{j=1}^{k} \alpha_j e^{\beta s_j})^2}{\sigma^2 (e^{\beta t_i})^2} + \frac{4t_i (\sum_{j=1}^{k} \alpha_j e^{\beta s_j})(\sum_{j=1}^{k} \alpha_j s_j e^{\beta s_j})}{\sigma^2 (e^{\beta t_i})^2} \\ &- \frac{(\sum_{j=1}^{k} \alpha_j s_j e^{\beta s_j})^2}{\sigma^2 (e^{\beta t_i})^2} - \frac{(\sum_{j=1}^{k} \alpha_j e^{\beta s_j})(\sum_{j=1}^{k} \alpha_j s_j^2 e^{\beta s_j})}{\sigma^2 (e^{\beta t_i})^2}\Big)\Bigg).\end{aligned} \tag{4.49}$$

Power kernel:

In this case, the power kernel function of the form

$$\mu_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta},$$

is substituted for mean μ in the log-likelihood of (4.46) for the binning case as

$$\begin{aligned}\log L(x;\mu) = \sum_{i=1}^{n}\Big(&-\frac{x_i^2}{2\sigma^2}+\frac{x_i(\alpha_0+\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{-\beta})}{2\sigma^2}\\&-\frac{(\alpha_0+\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{-\beta}|_+)^2}{2\sigma^2}-\frac{1}{2}\log(2\pi\sigma^2)\Big).\end{aligned}\tag{4.50}$$

Then the score function is obtained as

$$\begin{aligned}\frac{\partial\log L}{\partial\beta} = \sum_{i=1}^{n}\Bigg(\frac{1}{2}\Big(&\frac{x_i(\sum_{j=1}^{k}-\alpha_j((t_i-s_j)|_+)^{-\beta})\log((t_i-s_j)|_+)}{\sigma^2}\\&-\frac{\alpha_0(\sum_{j=1}^{k}-\alpha_j((t_i-s_j)|_+)^{-\beta}|_+\log((t_i-s_j)|_+))}{\sigma^2}\\&-\frac{(\sum_{j=1}^{k}-\alpha_j((t_i-s_j)|_+)^{-\beta}|_+)(\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{-\beta}\log((t_i-s_j)|_+))}{\sigma^2}\Big)\Bigg),\end{aligned}\tag{4.51}$$

with the corresponding Hessian

$$\begin{aligned}\frac{\partial^2\log L}{\partial\beta^2} = \sum_{i=1}^{n}\Bigg(\frac{1}{2}\Big(&\frac{x_i(\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{\beta})\log((t_i-s_j)|_+)^2}{\sigma^2}\\&-\frac{\alpha_0(\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{\beta}|_+\log((t_i-s_j)|_+)^2)}{\sigma^2}\\&-\frac{(\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{\beta}|_+\log((t_i-s_j)|_+))^2}{\sigma^2}\\&-\frac{(\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{\beta}|_+)((\sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{\beta})\log((t_i-s_j)|_+)^2)}{\sigma^2}\Big)\Bigg).\end{aligned}\tag{4.52}$$

4.5.2 α derivatives

Here, the score and the Hessian functions are obtained by differentiating (4.47) with respect to parameter α_j. These functions are presented below, respectively, for the exponential and the power kernel cases by using the idea of binning.

Exponential kernel:

$$\frac{\partial\log L}{\partial\alpha_j} = \sum_{i=1}^{n}\Bigg(\frac{1}{2}\Big(\frac{x_i\sum_{j=1}^{k}e^{\beta s_j}}{\sigma^2 e^{\beta t_i}}-\frac{\alpha_0\sum_{j=1}^{k}e^{\beta s_j}}{\sigma^2 e^{\beta t_i}}-\frac{(\sum_{j=1}^{k}\alpha_j e^{\beta s_j})(\sum_{j=1}^{k}e^{\beta s_j})}{\sigma^2(e^{\beta t_i})^2}\Big)\Bigg),\tag{4.53}$$

and the corresponding Hessian is

$$\frac{\partial^2 \log L}{\partial \alpha_j^2} = \sum_{i=1}^{n} \left(- \frac{(\sum_{j=1}^{k} e^{\beta s_j})^2}{\sigma^2 (e^{\beta t_i})^2} \right). \tag{4.54}$$

If we differentiate (4.53) with respect to α_0, the second partial derivative of the parameters α_0 and α_j is

$$\frac{\partial^2 \log L}{\partial \alpha_j \partial \alpha_0} = \sum_{i=1}^{n} \left(- \frac{\sum_{j=1}^{k} e^{\beta s_j}}{\sigma^2 e^{\beta t_i}} \right). \tag{4.55}$$

If (4.47) is differentiated with respect to α_0 twice, the score and the Hessian function are derived as

$$\frac{\partial \log L}{\partial \alpha_0} = -\frac{n\alpha_0}{\sigma^2} + \left(\sum_{i=1}^{n} \Big(\frac{1}{2} \frac{x_i}{\sigma^2} - \frac{\sum_{j=1}^{k} \alpha_j e^{\beta s_j}}{\sigma^2 e^{\beta t_i}} \Big) \right), \tag{4.56}$$

and

$$\frac{\partial^2 \log L}{\partial \alpha_0^2} = -\frac{n}{\sigma^2}. \tag{4.57}$$

When $j = j' = 1, \ldots, k$

$$\frac{\partial^2 \log L}{\partial \alpha_j \partial \alpha_{j'}} = \left(- \sum_{i=1}^{n} \frac{e^{-\beta (t_i - s_j)|_+} e^{-\beta (t_i - s_{j'})|_+}}{\sigma^2} \right). \tag{4.58}$$

Power kernel:

In this case, (4.50) is differentiated with respect to α_j and α_0 parameters to obtain the required score and Hessian functions, where the score function

$$\begin{aligned} \frac{\partial \log L}{\partial \alpha_j} = \sum_{i=1}^{n} \Bigg(\frac{1}{2} \Big(& \frac{x_i (\sum_{j=1}^{k} ((t_i - s_j)|_+)^{-\beta})}{\sigma^2} - \frac{\alpha_0 (\sum_{j=1}^{k} ((t_i - s_j)|_+)^{-\beta}|_+)}{\sigma^2} \\ & - \frac{(\sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta}|_+)(\sum_{j=1}^{k} ((t_i - s_j)|_+)^{-\beta})}{\sigma^2} \Big) \Bigg), \end{aligned} \tag{4.59}$$

and the corresponding Hessian is

$$\frac{\partial^2 \log L}{\partial \alpha_j^2} = \sum_{i=1}^{n} \left(- \frac{(\sum_{j=1}^{k} ((t_i - s_j)|_+)^{-\beta})^2}{\sigma^2} \right). \tag{4.60}$$

If (4.59) is differentiated with respect to α_0

$$\frac{\partial^2 \log L}{\partial \alpha_j \alpha_0} = \sum_{i=1}^{n} \left(- \frac{\sum_{j=1}^{k} ((t_i - s_j)|_+)^{-\beta}}{\sigma^2} \right). \tag{4.61}$$

For the α_0 parameter, the score and the Hessian are, respectively:

$$\frac{\partial \log L}{\partial \alpha_0} = -\frac{n\alpha_0}{\sigma^2} + \left(\sum_{i=1}^{n} \Big(\frac{1}{2}\frac{x_i}{\sigma^2} - \frac{\sum_{j=1}^{k} \alpha_j((t_i - s_j)|_+)^{-\beta}}{\sigma^2} \Big) \right), \tag{4.62}$$

and

$$\frac{\partial^2 \log L}{\partial \alpha_0^2} = -\frac{n}{\sigma^2}. \tag{4.63}$$

When $j = j' = 1, \dots, k$, the Hessian is

$$\frac{\partial^2 \log L}{\partial \alpha_j \partial \alpha_{j'}} = \left(-\sum_{i=1}^{n} \frac{((t_i - s_j)|_+)^{-\beta}((t_i - s_{j'})|_+)^{-\beta}}{\sigma^2} \right). \tag{4.64}$$

4.5.3 α β derivatives

Here, the entries of the Hessian matrix are presented in terms of α and β parameters.

Exponential kernel:

If (4.48) is differentiated with respect to α_0, the Hessian is

$$\frac{\partial^2 \log L}{\partial \beta \partial \alpha_0} = \sum_{i=1}^{n} \Big(\frac{t_i \sum_{j=1}^{k} \alpha_j e^{\beta s_j}}{\sigma^2 e^{\beta t_i}} - \frac{\sum_{j=1}^{k} \alpha_j s_j e^{\beta s_j}}{\sigma^2 e^{\beta t_i}} \Big). \tag{4.65}$$

By the symmetry, $\frac{\partial^2 \log L}{\partial \alpha_0 \partial \beta}$ entry is the same. The derivative of the score function in (4.48) with respect to α_j gives

$$\begin{aligned} \frac{\partial^2 \log L}{\partial \beta \partial \alpha_j} = \sum_{i=1}^{n} \Bigg(& -\frac{1}{2} \Big(\frac{x_i t_i \sum_{j=1}^{k} e^{\beta s_j}}{\sigma^2 e^{\beta t_i}} + \frac{x_i \sum_{j=1}^{k} s_j e^{\beta s_j}}{2\sigma^2 e^{\beta t_i}} + \frac{\alpha_0 t_i \sum_{j=1}^{k} e^{\beta s_j}}{\sigma^2 e^{\beta t_i}} \\ & - \frac{\alpha_0 \sum_{j=1}^{k} s_j e^{\beta s_j}}{\sigma^2 e^{\beta t_i}} + \frac{2t_i (\sum_{j=1}^{k} \alpha_j e^{\beta s_j})(\sum_{j=1}^{k} e^{\beta s_j})}{\sigma^2 (e^{\beta t_i})^2} \\ & - \frac{(\sum_{j=1}^{k} \alpha_j s_j e^{\beta s_j})(\sum_{j=1}^{k} \alpha_j e^{\beta s_j})}{\sigma^2 (e^{\beta t_i})^2} - \frac{(\sum_{j=1}^{k} \alpha_j e^{\beta s_j})(\sum_{j=1}^{k} s_j e^{\beta s_j})}{\sigma^2 (e^{\beta t_i})^2} \Big) \Bigg). \end{aligned} \tag{4.66}$$

The $\alpha_j \beta$ entry of the Hessian has the same value by the symmetry.

If (4.62) is differentiated with respect to β, the corresponding Hessian is obtained as

$$\frac{\partial^2 \log L}{\partial \alpha_0 \partial \beta} = \sum_{i=1}^{n} \Big(-\frac{\sum_{j=1}^{k} -\alpha_j((t_i - s_j)|_+)^{-\beta} \log((t_i - s_j)|_+)}{\sigma^2} \Big), \tag{4.67}$$

and for the α_j β parameters, (4.59) is differentiated with respect to β parameter, that is

$$\frac{\partial^2 \log L}{\partial \alpha_j \partial \beta} = \sum_{i=1}^{n} \left(\frac{1}{2} \Big(\frac{x_i(\sum_{j=1}^{k} -((t_i - s_j)|_+)^{-\beta} \log((t_i - s_j)|_+)}{\sigma^2} - \frac{\alpha_0(\sum_{j=1}^{k} -((t_i - s_j)|_+)^{-\beta}|_+ \log((t_i - s_j)|_+))}{\sigma^2} - \frac{(\sum_{j=1}^{k} -\alpha_j((t_i - s_j)|_+)^{-\beta}|_+ \log((t_i - s_j)|_+)(\sum_{j=1}^{k}((t_i - s_j)|_+)^{-\beta})}{\sigma^2} - \frac{(\sum_{j=1}^{k} \alpha_j((t_i - s_j)|_+)^{-\beta}|_+)(\sum_{j=1}^{k} -((t_i - s_j)|_+)^{-\beta} \log((t_i - s_j)|_+))}{\sigma^2} \Big) \right). \tag{4.68}$$

The relation between the estimated model parameters and the moment generating function of the total claim amount

The expectation of the total claim amount process is given in (3.3) with $E(S(t)) = \mu\Lambda(t)$. This expectation is used as the reserve amount and in pure premium calculations in the insurance context. It helps to decide how much the insurance company should charge the insured to keep their reserve in positive value. The main aim of an insurance contract is to provide profit for both the insured and the insurer. The expected utility model is a good choice to explain the existence of the insurers. This model assumes that the insured is a risk averse, which means that the insured prefers a fixed loss against a random loss with the same expected value. The insurer is willing to pay more than the expected value of his/her claims to be in a safe financial situation by using the Jensen's inequality, that is $E(f(Y)) \geq f(E(Y))$ where $f(x)$ is a convex function and Y is a random variable [Kaas et al., 2001]. Let's say, A: The amount the insurer agrees to pay in an insurance contract and B: The amount the insured agrees to pay, in case of a disaster, or any other event. The optimal situation where both parties will profit is

$$A < E(S(t)) < B.$$

Therefore the calculation of the E(S(t)) plays a crucial role for the insurance companies. The regulations on the premiums, which flow into the Turkish Catastrophe Insurance Pool, are very important for the case of Turkey since the social, economic and geographical conditions vary a lot from place to place. The reserves and the capacity of the TCIP always should be kept enough to be able to cover a huge amount of claims if a big earthquake occurs again. The average payment per claim, $E(S(t))$ is given in (3.11) and (3.12) in Chapter 3.

After the use of the binning idea for the claim number process, the intensity of the process is expressed with $\Lambda_i(t) = \int_{W_i}^{W_{i+1}} \lambda(r)dr$ (see **Remark 2**). Then the rate Λ is equal to $\Lambda_i = e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|+}}$ when the exponential kernel is used and $\Lambda_i = e^{\alpha_0+\sum_{j=1}^{k}\alpha_j((t_i-s_j)|+)^{-\beta}}$ when the power kernel is used. If the exponential kernel function is replaced in (3.3), the expected value of the total claim amount process $S(t)$ will be

$$E(S(t)) = \eta\Lambda_i(t) = \eta \int_{w_i}^{w_{i+1}} e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|+}} dr, \tag{4.69}$$

where μ is the mean of the chosen claim amount distribution and for our case the exponential kernel function substitutes μ with the same significant earthquakes and same parameter notation

$$E(S(t)) = \eta\Lambda(t) = \left(\alpha_0 + \sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|+}\right) \int_{w_i}^{w_{i+1}} e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|+}} dr. \tag{4.70}$$

The similar argument for the use of the power kernel changes the equation above to

$$E(S(t)) = \left(\alpha_0 + \sum_{j=1}^{k}\alpha_j((t_i-s_j)|_+)^{-\beta}\right) \int_{w_i}^{w_{i+1}} e^{\alpha_0+\sum_{j=1}^{k}\alpha_j((t_i-s_j)|+)^{-\beta}} dr. \tag{4.71}$$

The value of the $E(S(t))$ with the replacement of the parameter estimates will effect the risk process $E(R(t))$ in (3.8) as

$$E(R(t)) = u + ct - E(S(t)) = u + ct - \mu\Lambda(t), \tag{4.72}$$

and by first replacing the exponential kernel form in the rate Λ, the mean function of the risk process is

$$\begin{aligned} E(R(t)) = u + ct - E(S(t)) = u + ct - \Bigg(\left(\alpha_0 + \sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|+}\right) \\ \int_{w_i}^{w_{i+1}} e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|+}} dr \Bigg), \end{aligned} \tag{4.73}$$

and then (4.73) is rewritten as follows with the use of the power kernel function

$$E(R(t)) = u + ct - E(S(t)) = u + ct - \left(\left(\alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta} \right) \int_{w_i}^{w_{i+1}} e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta}} dr \right). \tag{4.74}$$

The premium ratings, which give the loaded premium rate c and the choice of the claim amount distribution for the value of mean μ, should be revised very carefully. Also, the expected value of the total claim amount process $S(t)$ and the risk process $R(t)$ will increase or decrease with the effect of the value of the β and α_j estimates.

Chapter 5

The Data and Modelling

In the next two chapters, the modelling of the mandatory earthquake insurance claims data of the Turkish Catastrophe Insurance Pool is presented. Two variables are of interest in the analysis, the number of claims N_i and the aggregate claims (total claim amount) S_i. The binning approach is used, which means we model the number of claims and the aggregate claims corresponding to each bin. The number of claims has a Poisson distribution ($N_i \sim$ Pois(Λ_i)) and modelled by using log-linear Poisson regression by generalised linear models (glm) in S-Plus. We know that the aggregate claims, S_i in the binning case, is defined as $S_i = \sum_{i=1}^{N_i} X_i$. Under the assumption that if $\log S_i \sim$ Normal then $S_i \sim$ lognormal and the i.i.d X_i's with mean η_i and variance τ_i of the individual claim amount, we modelled the aggregate claims S_i as Gaussian in glm by S-Plus. The estimates of the non-linear parameter β and the linear parameter α's, which are used to represent the effect of the big earthquakes, are later used in Chapter 7 to suggest an estimation of the Turkish Catastrophe Insurance Pool reserves. Next, first some basic statistical information of the data and plots of the variables are presented and then the modelling process follows.

5.1 Turkish Earthquake Insurance Claims Data

The data of this study, which comes from the Turkish Catastrophe Insurance Pool (TCIP), is obtained from the Milli Re Ltd. (National Reinsurance Company of Turkey). The collection of the claims data started after the introduction of the

TCIP in September 2000. The data consists of the earthquake insurance claims, which the TCIP received from December 2000 until July 2003. Since we base the analysis on risk zone 1 and zone 2, according to the data, in risk zone 1, the first earthquake event occurred on the 15/December/2000 and the last event did on the 26/July/2003. In risk zone 2, the first event happened on the 29/May/2001 and the last event did on the 01/May/2003.

The time of the event (earthquake), the time of the claim, the paid claim amount, the magnitude of the earthquake, the amount of loss-which the experts estimated, the final loss amount-which is calculated by the experts of the TCIP, the reason of loss (the earthquake itself, fire or landslide following the earthquake), the type of loss (light, medium or heavy), the earthquake risk zone (1 indicating the highest, 5 indicating the lowest risk regions) and the deductible amount are the main variables of the data. According to this information, the total and residential building numbers of the earthquake town/city is obtained from the State Institute of Statistics (figures of 2000) and used as another regressor (explanatory variable).

Since the distribution of the aggregate claims S_i is assumed to be lognormal, the transformed distribution of $\log S_i$ is assumed to be Normal. Figures 5.1 and 5.2 shows that indeed the $\log S_i$ values follow a bell-shaped structure, which represents Normal distribution, in both risk zones. Since we are using the aggregate claims data in each bin, the normality assumption is still valid to use with the total claim amount S_i. By this argument, after initial graphical analysis, the natural logarithm of the claim amount is used in the modelling purposes of the total claim amount S_i. The actual event time is assigned in S-Plus in terms of months by denoting the first event month as '12', which is December 2000 and onwards. Also, the data is organised in terms of weeks, as 'week 1' for 15/December/2000 in zone 1 and for 29/May/2001 in zone 2 and as 'week 138' for 31/July/2003 in zone 1 and 'week 101' for 01/May/2003 in zone 2 in the given time period. The number of claims in each of these weeks/months is totalled and used as N_i variable [Lindsey, 1995, TCIP, 2006]. The average of the aggregate claims corresponding to the number of claims at these weeks/months is used as S_i variable. The delays in the claim payment process assumed to be ignored and the actual event time is used. This is to prevent the effect of the intervening events like the inflation, unemployment rate on the behaviour of the claim distributions [Hogg and Klugman, 1984]. The magnitude of the earthquake and the number of residential buildings (as the TCIP only insures residential buildings) are also been averaged at the corresponding weeks/months and used as covariates during the modelling process.

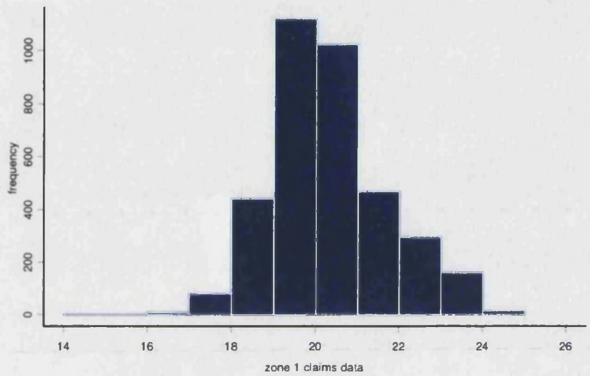

Figure 5.1: The histogram of the log of aggregate claims in risk zone 1

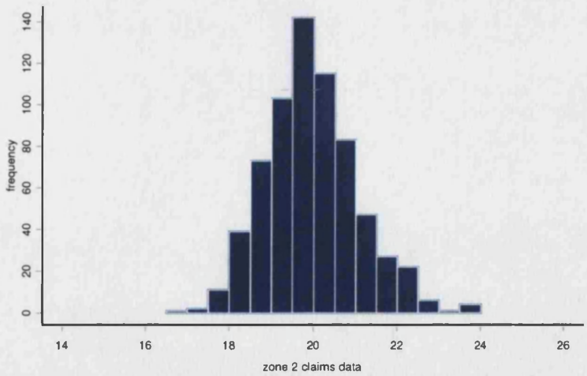

Figure 5.2: The histogram of the log of aggregate claims in risk zone 2

5.2 Explanatory Analysis of the TCIP Data

In this section, the relation between the number of the residential buildings in the earthquake effected area, the magnitude of an earthquake and the claim amount are presented in all risk zones (1-4) and separately for zone 1-zone 2 effect in terms of weeks and months data. The pairwise scatter plots in Figure 5.3 and Figure 5.4 are obtained by using the raw claims data (without any transformation) of zone 1 and zone 2 claims.

In earthquake studies, earthquakes of magnitude 5 or more are considered to cause serious damage. Figures 5.5, 5.6 and 5.7 give the relation between the average total claim amount, magnitude and the residential building number in all risk zones in terms of time in weeks and in terms of months. At weeks 60, 122, 123, 125 and at months 26, 37, 38, 40, the accumulation of the large number of claims is observed. These claims generally occur as a result of magnitude 5 or greater earthquakes.

It is a fact in Turkey that if there are many buildings in the earthquake affected area and the magnitude of an earthquake is small, the total losses can still be high due to the volume of the small claims arriving to the insurance companies. This is a serious problem in the country and unfortunately the towns/cities are settled very close to the fault lines in the past without any consideration and awareness of the possible losses due to possible earthquakes. It can also be stated that more claims might arrive from small settlement areas, where there are fewer buildings (up to 100,000). The most important point is, regardless of the building number, population, building type, risk zone etc., most of the losses depend on the application of the use of the valid building code (the Building Code of 1998 is currently use in Turkey) during the construction. It is known that there are minimum 506 and maximum 298,841 buildings in the earthquake affected areas and the magnitude of the recorded earthquakes range from 3.6 to 6.5 in the Richter Scale.

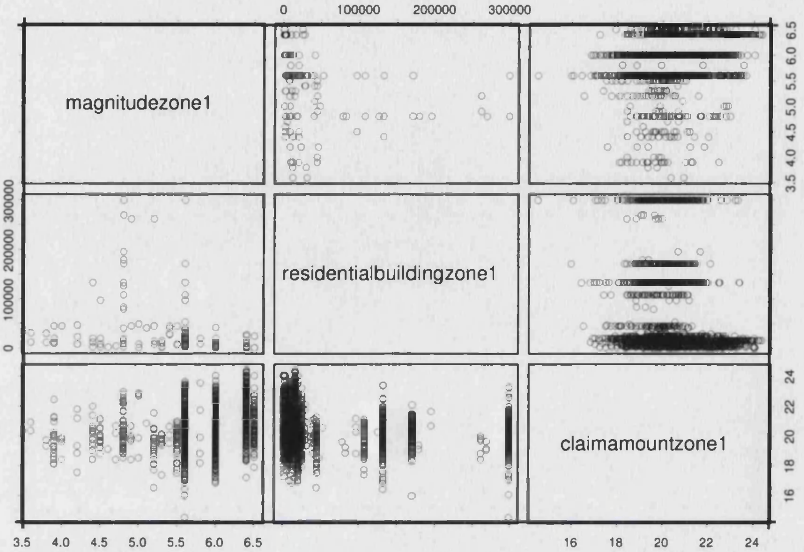

Figure 5.3: The scatter matrix of the variables of zone 1 claim data

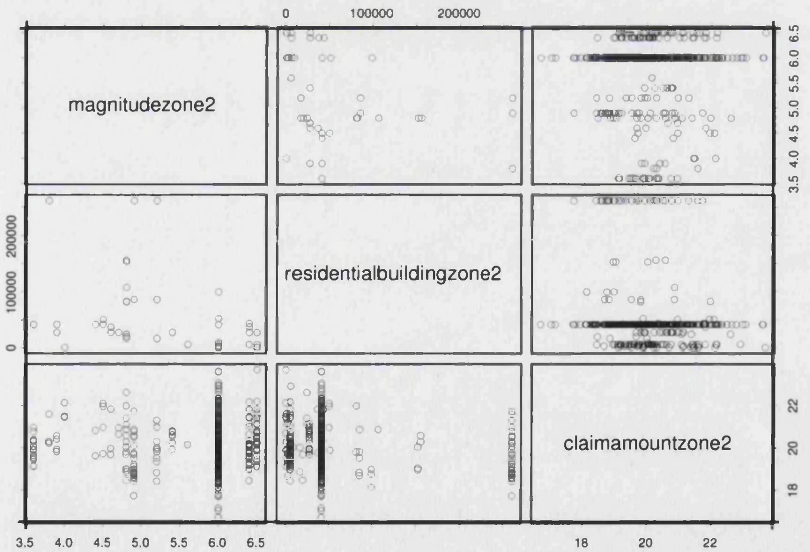

Figure 5.4: The scatter matrix of the variables of zone 2 claims data

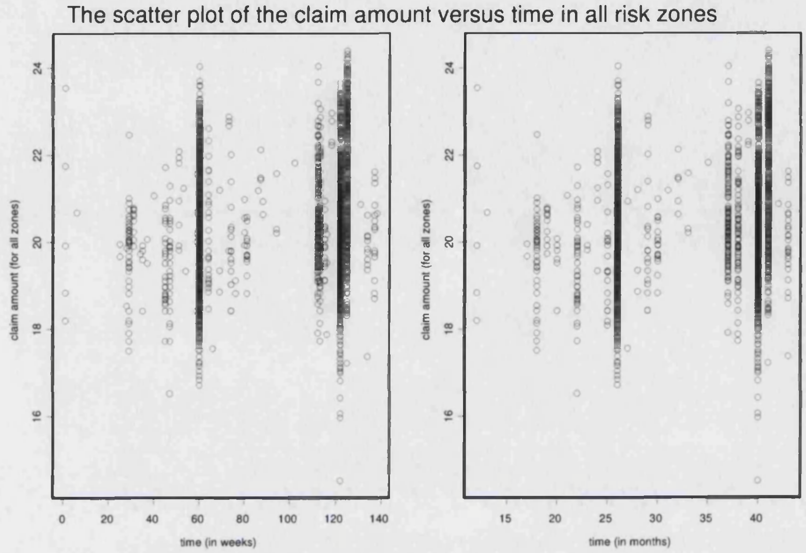

Figure 5.5: The plot of the claim amount versus time (left: weeks, right: months) in all risk zones

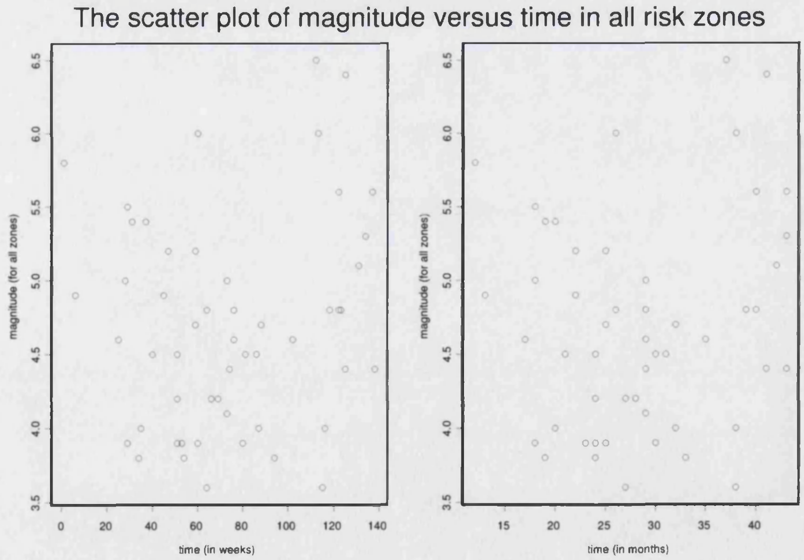

Figure 5.6: The plot of magnitude versus time (left: weeks, right: months) in all risk zones

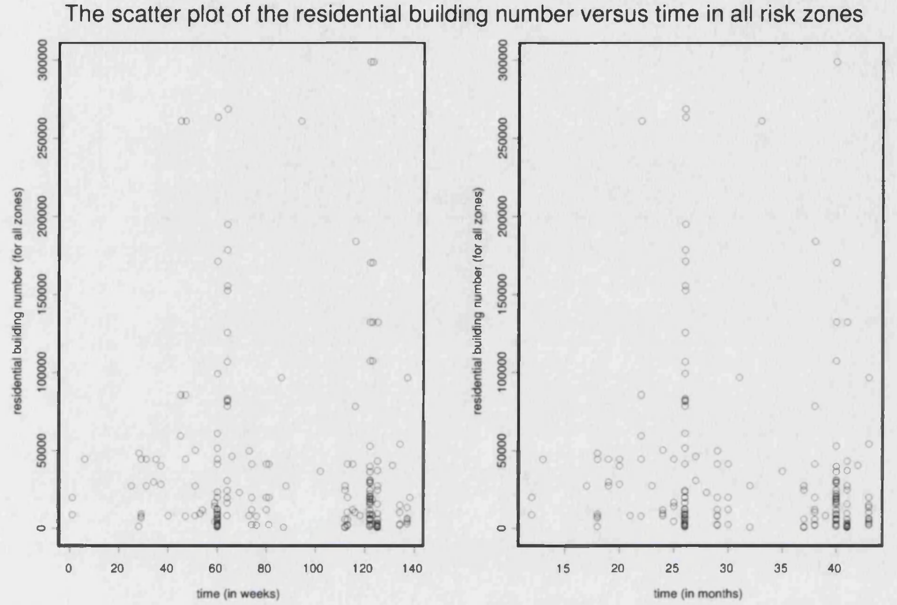

Figure 5.7: The plot of the residential building number versus time (left: weeks, right: months) in all risk zones

The main interest of the thesis work is to show and represent the jump behaviour of the earthquake claims when a big earthquake strikes the country. The claim number and the total claim amount change mainly with the size of the earthquake (measured with magnitude). Next, Figure 5.8 and Figure 5.9 show the relation between the claim number, N_i, and time in terms of weeks/months by classification of risk zones 1-2 and the jumps in the data, which are caused due to the big earthquake claims, can be observed in both plots. The ordinary claims arriving from different risk zones as a result of small tremors are followed by sudden jumps, when there occurs an earthquake shock as explained in Section 4.2. The special kernel functions of the exponential and power kernel form are used in the modelling sections to represent the jump feature of these claims. The empirical selection of the knots (sites) with jumps occurring at big earthquakes are used in the model parameter (α_j) estimation.

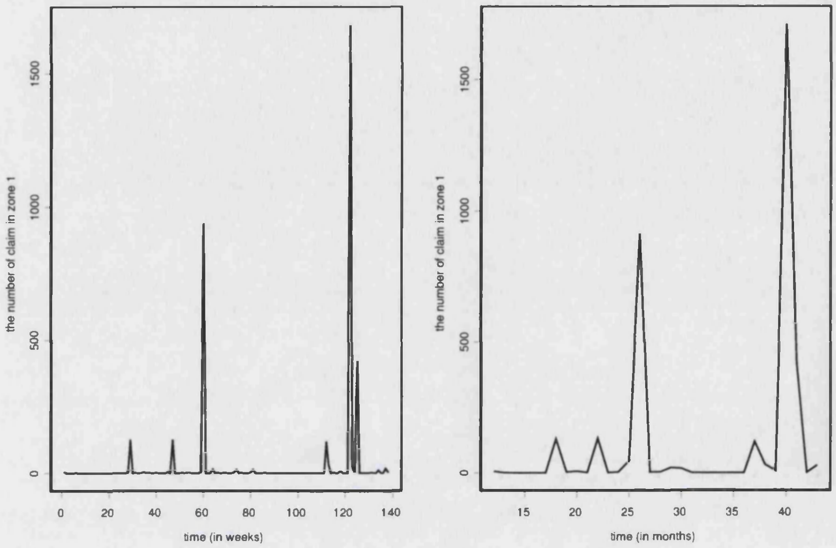

Figure 5.8: The number of claims versus time in zone 1

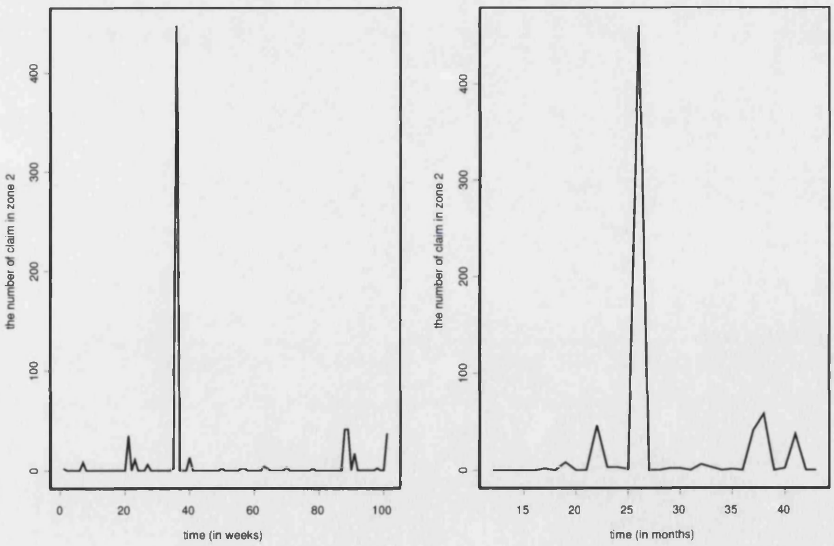

Figure 5.9: The number of claims versus time in zone 2

In the following Table 5.1, the empirical list of the big earthquakes of the thesis data with more than 100 claims is given. It is observed that these earthquakes are realised (picked) by their big α_j coefficients during the modelling.

Event date	Corresponding month (weeks) in S-Plus	Place	Magnitude	Risk Zone	The number of claims
25/06/2001	18 (28-29)	Osmaniye	5.5	1	130
03/02/2002	26 (36)	Sultandaği	6	2	461
27/01/2003	37 (112)	Pülümür	6.5	1	120
10/04/2003	40 (122-123)	Urla	5.6	1	1708
01/05/2003	41 (125)	Bingöl	6.4	1	423

Table 5.1: The significant earthquake claims data from the Turkish Catastrophe Insurance Pool

Zone 1 claims data plots

Figures 5.10, 5.11 and 5.12 denote the relation between the claim amount, magnitude and the number of the residential buildings in the earthquake effected area in risk zone 1, respectively and in terms of weeks and months data.

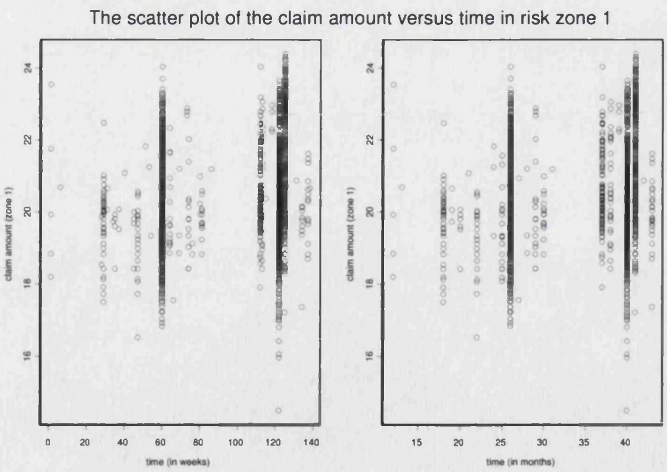

Figure 5.10: The plot of the claim amount versus time (left: weeks, right: months) in risk zone 1

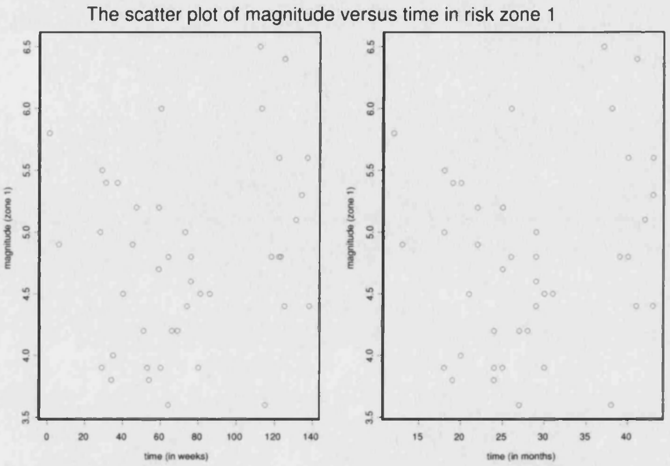

Figure 5.11: The plot of magnitude versus time (left: weeks, right: months) in risk zone 1

The accumulation of the claims arriving at weeks 60, 122, 123, 125 is observed in zone 1 data. Some of the earthquakes, which cause these claims, occur in areas where there are 300000 residential buildings. The death toll can be very high in case of a significant earthquake strike in these areas.

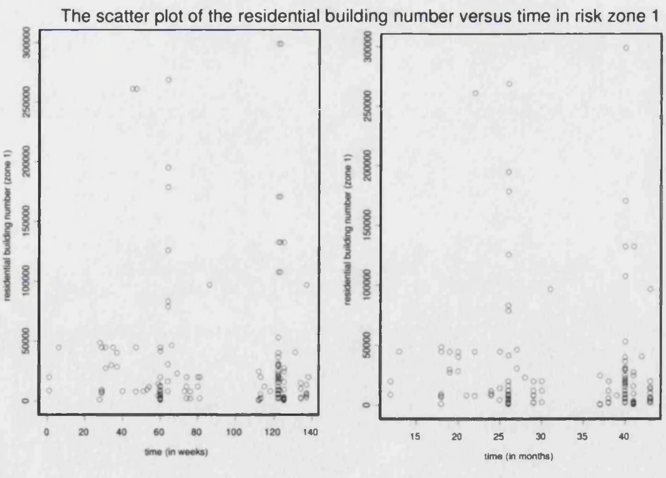

Figure 5.12: The plot of the residential building number versus time (left: weeks, right: months) in risk zone 1

Zone 2 claims data plots

Accordingly, Figures 5.13, 5.14 and 5.15 are plotted to show the relation between the claim amount, magnitude and the number of the residential buildings in the earthquake effected area in risk zone 2 in terms of weeks and months data. In this case, it is observed that at weeks 21, 36, 88, 89, 91 and 101 and at months 22, 26, 37, 38 and 41 more claims arrived to the Pool.

The earthquakes of magnitude of more than 5 in Richter Scale cause significant damage, if the buildings were not constructed by using earthquake resistant materials as in months case more claims were observed as a result of big earthquakes in zone 2. Especially the claims at weeks 88-89 come from areas, where there are 50000 or less residential buildings, indicate that different levels of damage (light/heavy) or total collapse were observed in most of the buildings in the earthquake area.

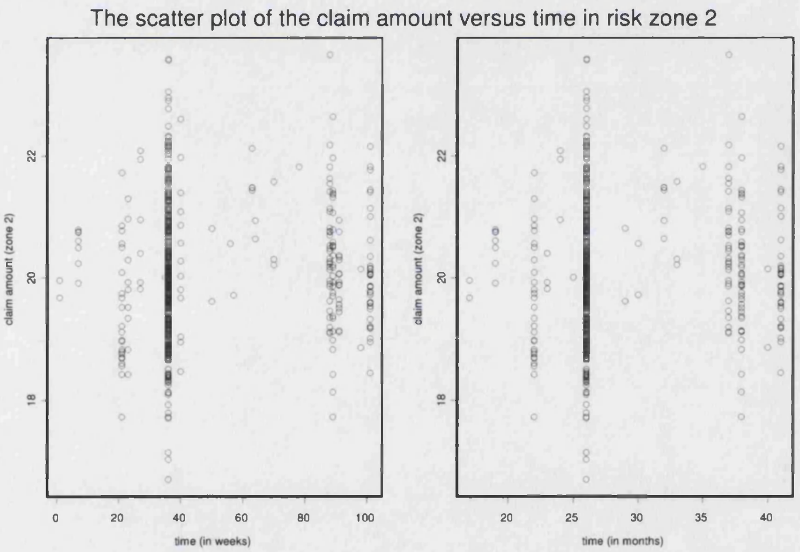

Figure 5.13: The plot of the claim amount versus time (left: weeks, right: months) in risk zone 2

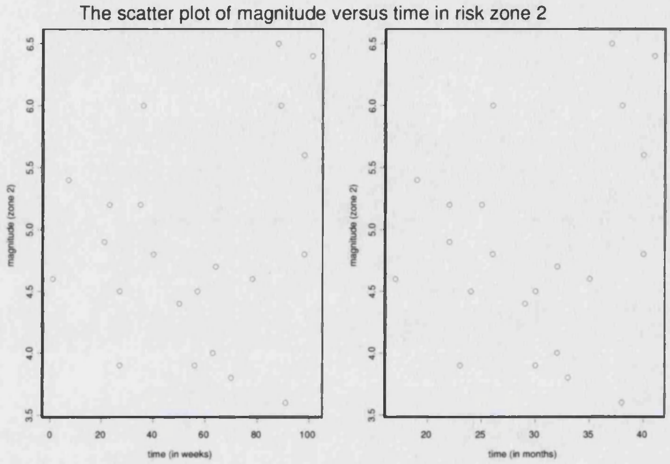

Figure 5.14: The plot of magnitude versus time (left: weeks, right: months) in risk zone 2

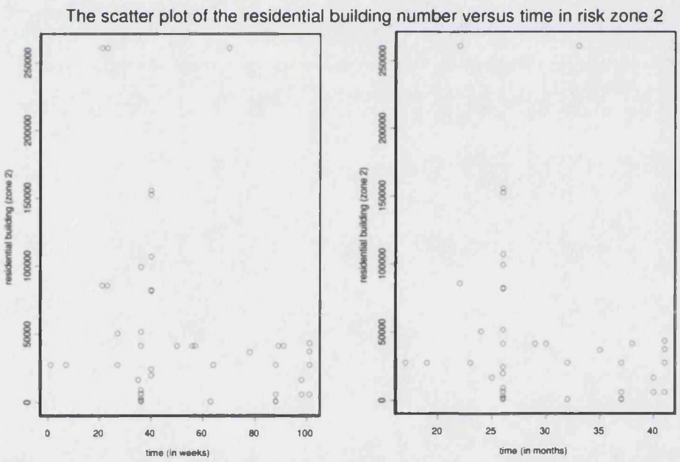

Figure 5.15: The plot of the residential building number versus time (left: weeks, right: months) in risk zone 2

As there are few observations ($n < 30$) in other risk zones (3-4-5), some of the plots and the models were not sufficient enough to represent the characteristics of those regions and are not presented in the thesis.

5.3 Modelling over time

In this section, the number of claims N_i ($N_i \sim \text{Pois}(\Lambda_i)$) and the aggregate claims S_i are modelled by using time as a covariate in generalised linear models. Method 3 in Page 83 allows us to use time as a variable with an approximation from discrete calendar to continuous calendar time. The additive model is presented here and further research might investigate other types of modelling (like multiplicative, intersection or quadratic). The data of this study is the earthquake insurance claims of the Turkish Catastrophe Insurance Pool (TCIP), which arrived to the pool between 15/December/2000 and 31/July/2003. The significant earthquakes with large number of claims (> 30) arriving at months 18, 22, 25, 26, 37, 38, 40, 41 (weeks 29, 47, 60, 112, 113, 122 and 125) in zone 1 and at months 22, 26, 37, 38, 41 (weeks 21, 36, 88, 89, 138) in zone 2 are used as an empirical selection in some cases together with other ordinary event occurring times. The aggregate claims S_i is modelled using the lognormal distribution (under the assumption that $\log S_i \sim$ Normal) and the claim number N_i is modelled as ordinary Poisson counts with a log-link function. The Poisson regression is very appropriate to use with discrete and large rare response events. Here, the models are suggested for the different earthquake risk zones (Zone 1 and Zone 2) of Turkey by using the binning approach (see also Method 3 in Page 83).

Our main aim is to give some estimates for the non-linear parameter β and the linear parameters α's to represent the shock effects of the earthquakes with jumps of the process at the time of the earthquake event. The estimate of the non-linear parameter β represents some fixed characteristics of the earthquake region and gives an idea of the effect of an earthquake in these regions. The use of this estimate leads to the calculation of the necessary reserve amount ($E(S(t))$) to keep the insurance pool safe. α_0 behaves like a nuisance parameter, which already exists at the ordinary time period with the ordinary claim arrivals and α_j parameters represent the big earthquakes, which result in big claims. The following algorithm is followed in S-Plus for the modelling purposes.

Algorithm 1:

1. Choose on the empirical event times (where large earthquakes happen causing large claims) and the corresponding kernel knots (one-sided kernel to represent the jump effect ahead in the time period),

2. Use the exponential kernel to model claim number N_i with rate $\Lambda_i = e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|+}}$ and to model the total claim amount S_i with rate $\mu_i = \alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|+}$ and use the power kernel correspondingly with rates $\Lambda_i = e^{\alpha_0+\sum_{j=1}^{k}\alpha_j(t_i-s_j)|_+^{-\beta}}$ and $\mu_i = \alpha_0 + \sum_{j=1}^{k}\alpha_j(t_i - s_j)|_+^{-\beta}$,

3. Use additive generalised linear models,

4. Model the number of claims N_i with a Poisson log-link function as counts ($N_i \sim \text{Pois}(\Lambda_i)$) and model the total claim amount (aggregate claims), S_i, as Gaussian by using the selected empirical kernel knots,

5. Use the non-linear parameter β as a representative of the characteristics of earthquake risk zones 1 and 2 and use the α_j parameters as the coefficients picking the significant earthquakes with big number of claims,

6. Set an initial value for β (random),

7. Run step-wise regression,

8. Choose the β value, which corresponds to the minimum deviance since the non-linear models are mainly based on deviance (see the Appendix for deviance). Note that, the formulae in Chapter 4 for the estimation of $\hat{\beta}$ is used as a check as well,

9. Run the model again with the fixed β,

10. Obtain the values of α_j estimates,

11. Construct the Hessian matrix H by using the second partial derivatives of the log-likelihood, which are derived in Chapter 4 and the parameter estimates and invert the Hessian,

12. Construct the confidence interval of β parameter by using the corresponding diagonal value of the inverse observed information matrix I,
13. If interested, by the similar argument in the previous step, construct the confidence interval of α_j,
14. By using the parameter estimates of the models, calculate the expected total claim amount $E(S(t))$ and other statistical measures.

In the tables of the modelling parts, we present the estimates values for the non-linear parameter β and some examples of the empirically selected α_j coefficients, at which we observe the big earthquakes of big claims.

The claim number (N_i) Models:

In this section, the results of the modelling process are presented by months and weeks classification and by using the exponential and power kernel function. The S-Plus analysis lead to the given tables, which present the selected models of the best possible fit to explain the claim number N_i with the replacement of the empirical kernel knots at the significant earthquakes. Algorithm 1 is used in the whole process of the model selection. It is observed that as expected the α_j parameters of the models pick the big earthquakes, where the kernel knots sit on the empirical sites.

Here, we model the number of claims N_i as an ordinary log-link Poisson count regression with the use of the exponential and the power kernel functions with the unknown parameter vector $\theta = (\beta; \alpha_0, \alpha_1, \ldots, \alpha_k)$ as

$$\log \Lambda_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+},$$

and in the form of power kernel function

$$\log \Lambda_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta},$$

where $i = 1, \ldots, n$, $j = 1, \ldots, k$, α_0 is the coefficient for the ordinary claim arrivals in the given time period and α_j's represent the significant claims arriving after an earthquake strike. Basically, the suggested model in this case is

$$N_i \sim \text{Pois}(\Lambda_i).$$

The following table gives the number of claims considering all risk zones (zone 1 to zone 4) from 15/December/2000 to 31/July/2003. It is observed that at the specific months, ordinary claims due to small tremors etc. show a sudden increase and then goes back to normal routine. When modelling the categorical data by using log-linear models, the sampling zeros can be observed like in our case here and they are included in the data. The tables of the number of claims by different zones are given in the Appendix.

Months	Frequency	Months	Frequency
12	6	28	3
13	1	29	22
14	0	30	19
15	0	31	1
16	0	32	6
17	2	33	3
18	130	34	0
19	11	35	1
20	6	36	0
21	1	37	161
22	179	38	94
23	3	39	8
24	6	40	1711
25	46	41	460
26	1384	42	1
27	2	43	30

The use of the exponential kernel:

By months data:

In Table 5.2, Model 1 suggests the estimates of the non-linear parameter β and the corresponding 95 % confidence interval in risk zone 1 and zone 2 with the use of the exponential kernel. These β estimates and the related α_j estimates of the models are used to estimate the expected value of the total claim amount $E(S(t))$ and the corresponding risk value for a company given the premium loading coefficient. The amount of the reserve $E(S(t))$ to keep the Turkish Catastrophe Insurance Pool steady will change by the value of the $\hat{\beta}$ and $\hat{\alpha_j}$. More money will be needed for risk zone 1 to cover the losses in case of a possible earthquake, which is already expected. Unfortunately, in risk zone 1, there are highly populated cities and a lot of buildings under the threat of earthquakes any time (e.g. Istanbul).

Model 1	$\hat{\beta}$	$\hat{\alpha_j}$	Residual deviance	95 % CI for $\hat{\beta}$
Zone 1	0.370	$\hat{\alpha_{26}} = 5.268$ $\hat{\alpha_{40}} = 7.622$	187.905	(0.369,0.372)
Zone 2	0.325	$\hat{\alpha_{26}} = 7.995$ $\hat{\alpha_{41}} = 4.135$	176.069	(0.323,0.327)

Table 5.2: The estimate of the selected model parameters and 95 % $\hat{\beta}$ confidence interval for risk zones 1-2 in Turkey

The high values of the given α_j coefficients correspond to month 26, month 40 and 41 claim arrivals. They reflect the reasonable choice of the exponential kernel to represent the jump pattern of the claims after significant earthquakes. Figure 5.16 shows how the estimate of the non-linear parameter β is realised for the claim number model for zone 1 claims data by using the exponential kernel in modelling process.

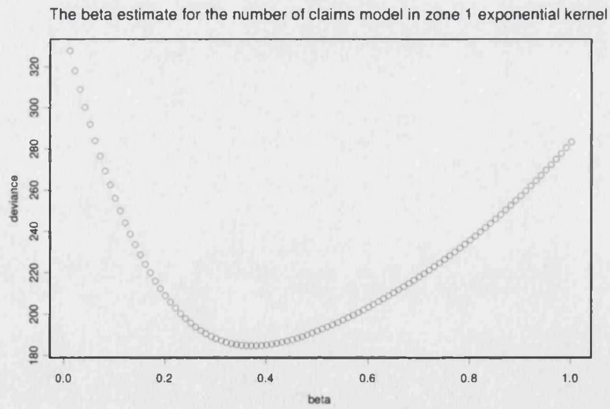

Figure 5.16: The plot of β selection in zone 1 versus deviance values by the exponential kernel use for the number of claims model

The selection criteria during whole modelling is based on the idea of the maximum likelihood estimation, where we make the normality assumption for the estimate (see Page 104). Same argument applies for zone 2 claims data. Next, the diagnostic plots are checked to see if the selected models actually provide a good fit of the data or not. In Figures 5.17 and 5.18, the consistency pattern between the actual claim numbers and fitted values of the claim number model can be observed.

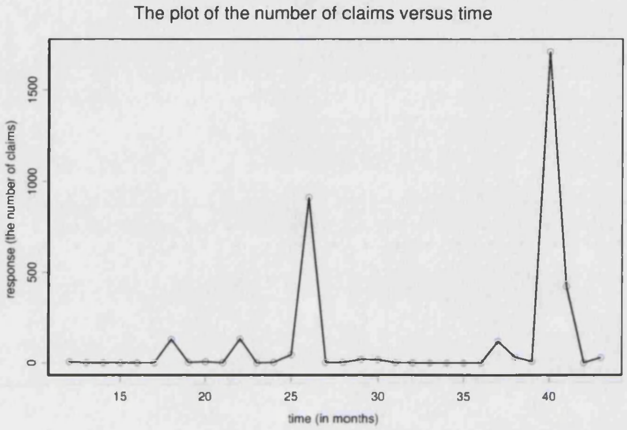

Figure 5.17: The plot of the number of claims versus time (in months) in zone 1 by the exponential kernel use

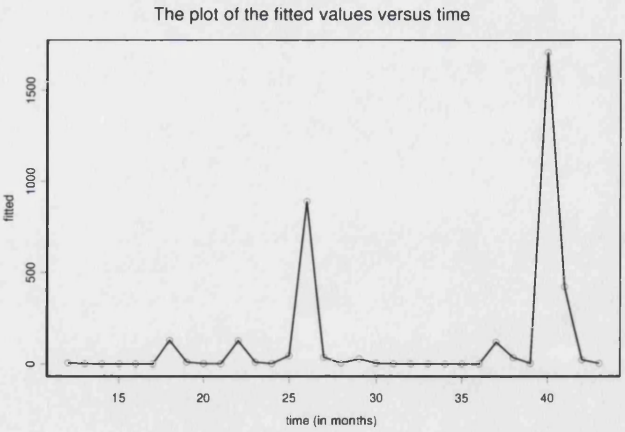

Figure 5.18: The plot of the fitted values versus time (in months) in zone 1 by the exponential kernel use

In Figure 5.19 (log tranformed residuals), the 45° pattern between the actual claim numbers and the fitted values supports the validity of the model. Residuals are the differences between the actual observations and the values predicted for these observations by the model. Generally, residuals are checked to decide if a model is good enough and where it can be improved [Kaas et al., 2001]. In Figure 5.20, the residual plot suggests that the chosen model is good enough to represent the claim number models in risk zone 1 with the use of the exponential kernel function.

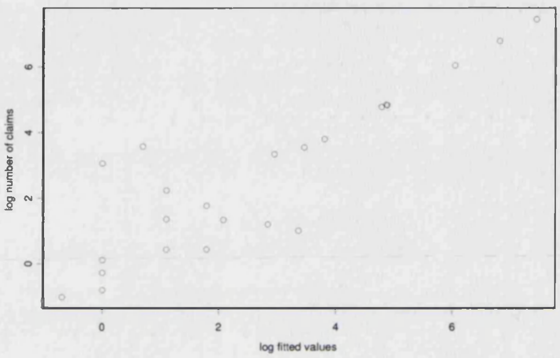

Figure 5.19: The plot of log number of claims versus log fitted values in zone 1 by the exponential kernel use

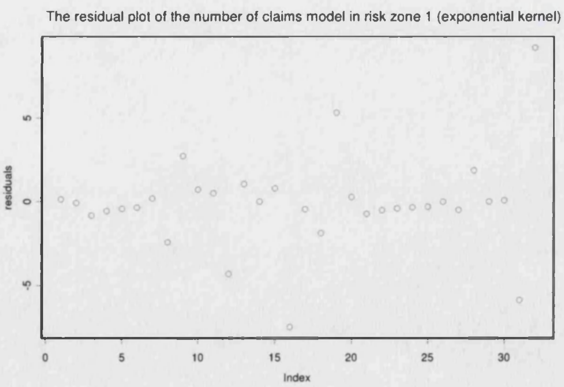

Figure 5.20: The plot of the residuals of the number of claim model in zone 1 by the exponential kernel use

The diagnostic plots of the zone 2 claims data are presented in Figures 5.21, 5.22, 5.23 and 5.24. The plots indicate an adequate model, especially when we check the close match of Figures 5.21 and 5.22 for the actual and fitted observations.

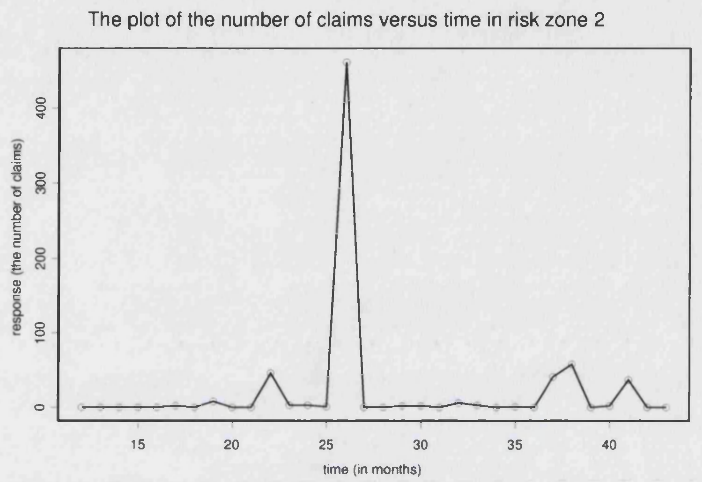

Figure 5.21: The plot of the number of claims versus time (in months) in zone 2 by the exponential kernel use

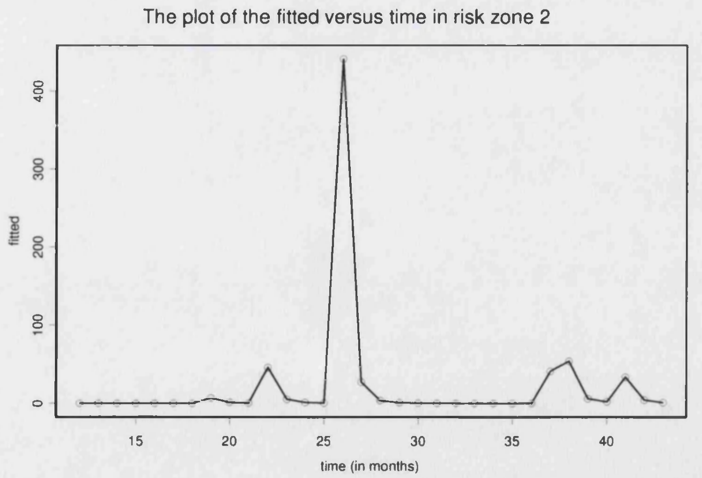

Figure 5.22: The plot of the fitted values versus time (in months) in zone 2 by the exponential kernel use

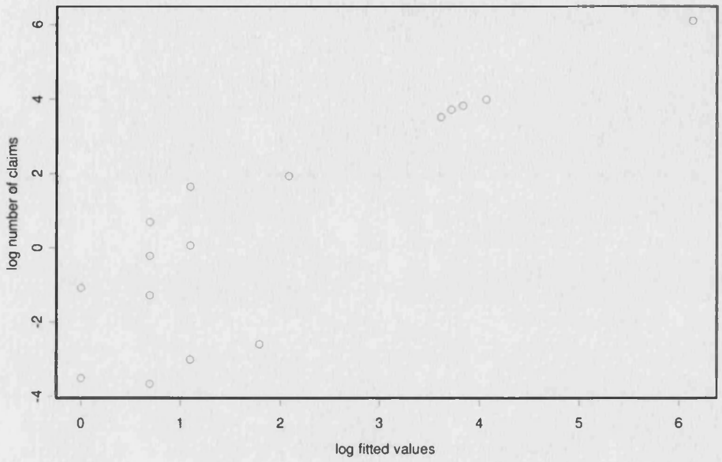

Figure 5.23: The plot of log number of claims versus log fitted values in zone 2 by the exponential kernel use

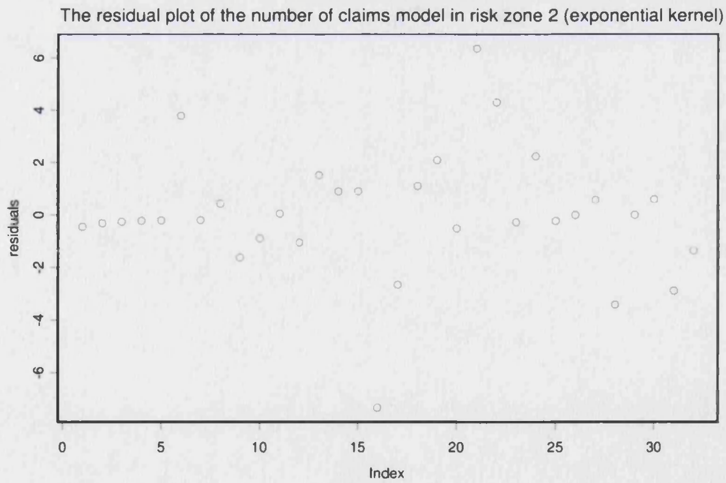

Figure 5.24: The plot of the residuals of the number of claim model in zone 2 by the exponential kernel use

By weeks data:

In weeks case, the same model is used, that is $N_i \sim \text{Pois}(\Lambda_i)$, but the time covariate is based on weeks. Table 5.3 is suggested by using the weeks data. The value $\hat{\beta}$ is smaller than the corresponding values in Table 5.2 in both zones. The deviance values are not very reasonable in weeks data compared to the months case. In the next chapter, it is observed that the models are calibrated by the addition of the covariates to the model. A narrow confidence interval is observed here in zone 1. On the other hand, a much wider confidence interval compared to the case in Table 5.2 is obtained for zone 2 data. This is probably due to the different effect of the significant earthquakes occurring at different times at these two risk zones.

Model 2	$\hat{\beta}$	$\hat{\alpha}_j$	Residual deviance	95 % CI for $\hat{\beta}$
Zone 1	0.2	$\hat{\alpha_{60}} = 7.06$ $\hat{\alpha_{122}} = 13.29$	1048.398	(0.19953,0.20057)
Zone 2	0.3	$\hat{\alpha_{36}} = 5.75$ $\hat{\alpha_{88}} = 5.18$	691.175	(0.2383,0.3617)

Table 5.3: The estimate of the selected model parameters and 95 % $\hat{\beta}$ confidence interval for risk zones 1-2 in Turkey

$\hat{\beta}$ values are not much different in Tables 5.2 and 5.3, when month-week comparison is made. The only difference is that the estimate value for zone 1 is higher than the value of zone 2 in the months case, and it is the other way around in weeks data. This is again related to the occurrence of the big earthquakes in the corresponding bin. The main idea of the models is verified in both months and weeks cases, that is the big earthquakes are picked by their $\hat{\alpha}_j$ coefficients. The big earthquake occurrence time can be realised by the $\hat{\alpha}_j$ values and the different earthquake risk zones have slightly different β values. That is $\hat{\beta}_{\text{zone 1}} = 0.37$ and $\hat{\beta}_{\text{zone 2}} = 0.325$, where normally more claims are expected in zone 1. The positive values of β parameter backs up the idea of the decreasing kernel function idea to represent the decay in the claim number following the jump due to an earthquake shock.

Figures 5.25 and 5.26 confirm the validity of the models with a nice presentation of the suggested zone 1 models, including the low number of claims.

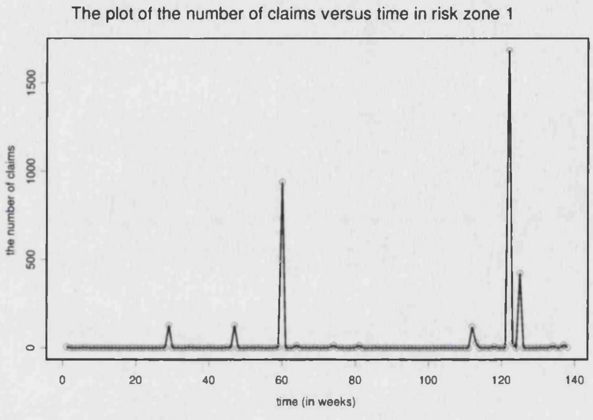

Figure 5.25: The plot of the number of claims versus time (in weeks) in zone 1 by the exponential kernel use

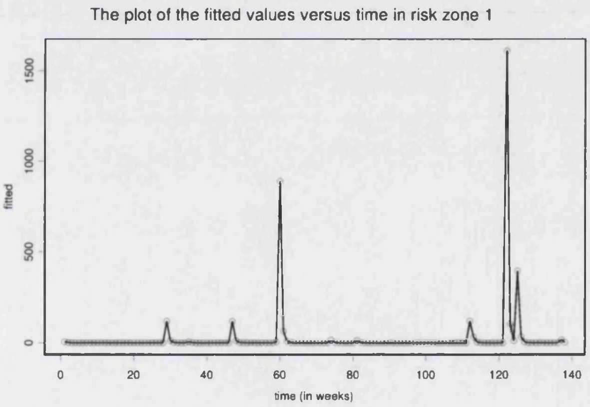

Figure 5.26: The plot of the fitted values versus time (in weeks) in zone 1 by the exponential kernel use

Figures 5.27 and 5.28 support the suggested model fit for zone 2 claims data and the exponential kernel use by weeks.

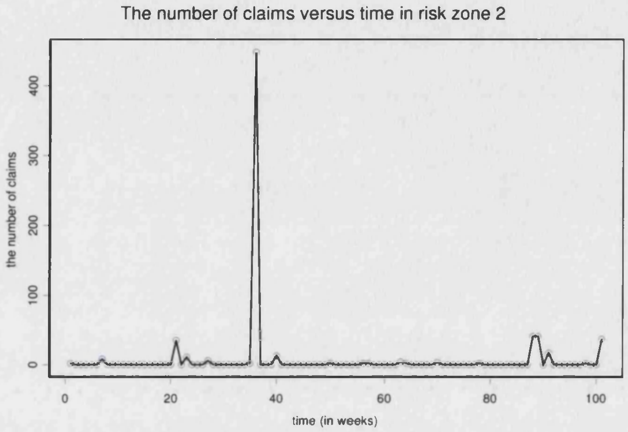

Figure 5.27: The plot of the number of claims versus time (in weeks) in zone 2 by the exponential kernel use

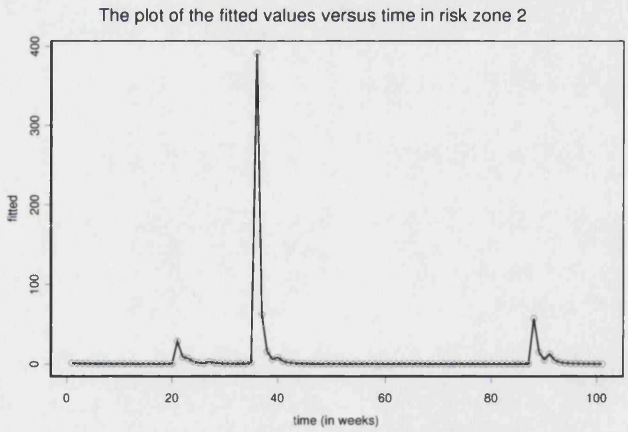

Figure 5.28: The plot of the fitted values versus time (in weeks) in zone 2 by the exponential kernel use

The use of the power kernel:

By months data:

Table 5.4 gives the estimate of the non-linear parameter β for risk zones 1 ($\hat{\beta}_{\text{zone 1}} = 0.43$) and 2 ($\hat{\beta}_{\text{zone 2}} = 0.52$) by the use of the power kernel function. In Model 3, both $\hat{\beta}$ estimates are higher than that of the values in Table 5.2 and Table 5.3, which are obtained by the use of the exponential kernel. In power kernel case, the residual deviance for zone 1 does not indicate a very preferable model. However, as the $\hat{\beta}$ values are closer to 1 than the exponential kernel models, the idea of pure log-linear modelling of the number of claims as Poisson counts is supported more.

Model 3	$\hat{\beta}$	$\hat{\alpha_j}$	Residual deviance	95 % CI for $\hat{\beta}$
Zone 1	0.43	$\hat{\alpha_{26}} = 6.1$ $\hat{\alpha_{40}} = 3.27$	1237.05	(0.4169,0.4431)
Zone 2	0.52	$\hat{\alpha_{26}} = 10.73$ $\hat{\alpha_{41}} = 12.93$	239.17	(0.519966,0.520033)

Table 5.4: The estimate of the selected model parameters and 95 % $\hat{\beta}$ confidence interval for risk zones 1-2 in Turkey

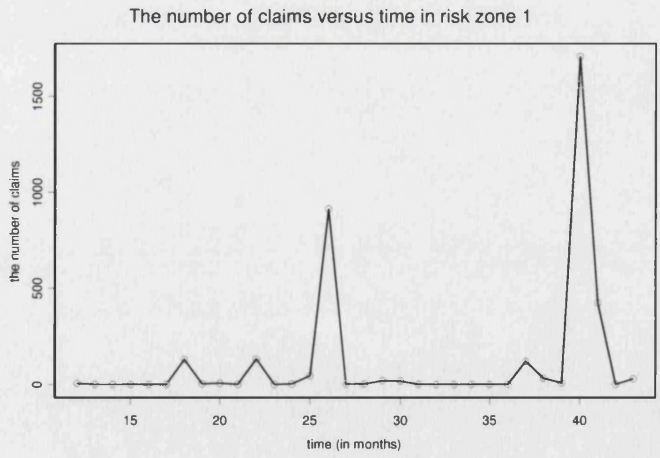

Figure 5.29: The plot of the number of claims versus time (in months) in zone 1 by the power kernel use

In Figure 5.29 and Figure 5.30, the plots of the actual observations and fitted model values of zone 2 claims data versus time (in months) with the use of the power kernel function are observed.

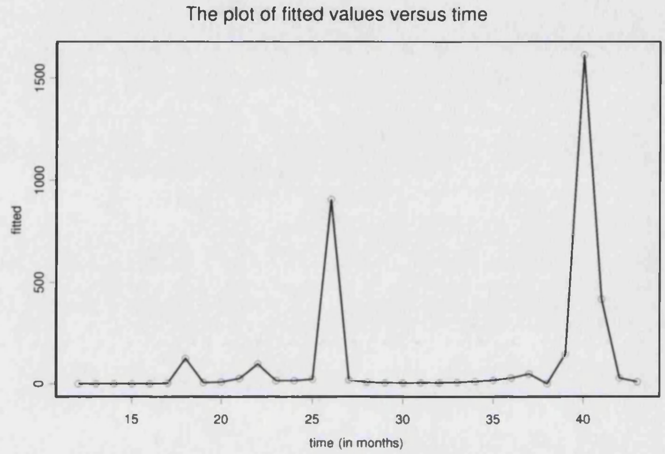

Figure 5.30: The plot of the fitted values versus time (in months) in zone 1 by the power kernel use

Figures 5.31 and 5.32 are presented to show the reasonability of the model fit in this case.

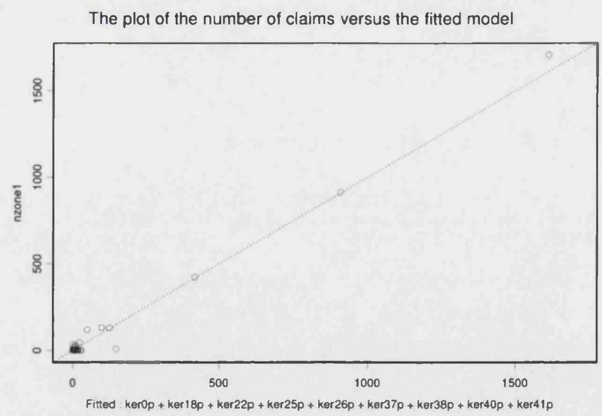

Figure 5.31: The plot of the number of claims versus fitted values in zone 1 by the power kernel use

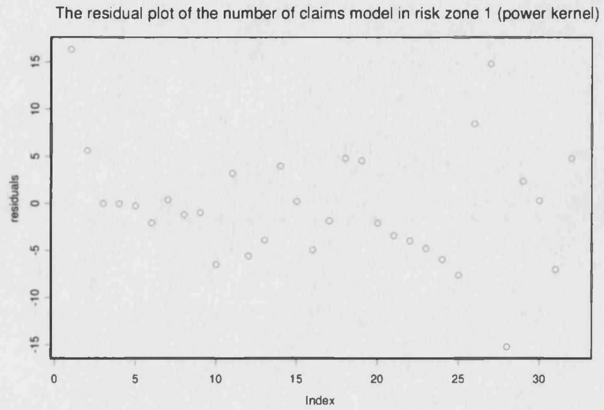

Figure 5.32: The plot of the residuals of the number of claim model in zone 1 by the power kernel use

By weeks data:

In weeks based data, the estimates of Model 4 are obtained with a similar analysis of the modelling of the number of claims with the use of the exponential kernel function. The value $\hat{\beta} = 0.2$ is the same in zone 1 exponential kernel weeks data case (see Table 5.3), yet the estimate for zone 2 is slightly higher. The residual deviance values do not suggest a very reasonable fit, especially with a large value for zone 1. The confidence intervals of the non-linear parameter β are very narrow for both zones, which increases the validity of the $\hat{\beta}$ values. Weeks data here suggests a stronger effect of the β (exponential decay) parameter, where it represents some fix characteristics of the earthquake area. For instance, β might represent the reason how the decay in the claims arrivals is linked with the effect of the distance to the fault line, or the age of the fault line, and then these factors play an important role to expect smaller or larger number of claims due to a disaster in that zone.

Model 4	$\hat{\beta}$	$\hat{\alpha_j}$	Residual deviance	95 % CI for $\hat{\beta}$
Zone 1	0.2	$\hat{\alpha_{60}} = 11.3$ $\hat{\alpha_{122}} = 7.76$	4518.727	(0.199129,0.2000871)
Zone 2	0.4	$\hat{\alpha_{36}} = 4.35$ $\hat{\alpha_{88}} = 1.74$	1286.833	(0.3999914,0.400086)

Table 5.5: The estimate of the selected model parameters and 95 % $\hat{\beta}$ confidence interval for risk zones 1-2 in Turkey

The similar graphical back up applies here for the choice of these β values as in the previous cases, where we use minimum deviance as a selection criteria. The general observation of the number of claims models is that the effect of α_0, which denotes the ordinary claim arrival period until the occurrence of a big earthquake, is found to be significant and α_0 has an obvious effect on the α_j estimates considering the arrival of the claims. The claims arrive in a routine period, then an earthquake happens and the number of claims increases suddenly. It is interesting to observe that the next day following the earthquake, people apply for the compulsory earthquake insurance. Our choice of the kernel let the models pick the jump behaviour of the claims at the significant earthquakes. The diagnostic check of the models also support the reasonability of the models.

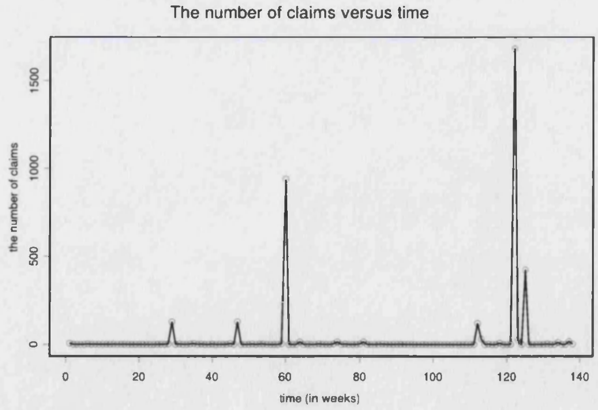

Figure 5.33: The plot of the number of claims versus time (in weeks) in zone 1 by the power kernel use

Figure 5.33 and Figure 5.34 are presented to represent the actual jump behaviour of the claims data in zone 2 and the jump pattern of the fitted values in the suggested model, respectively.

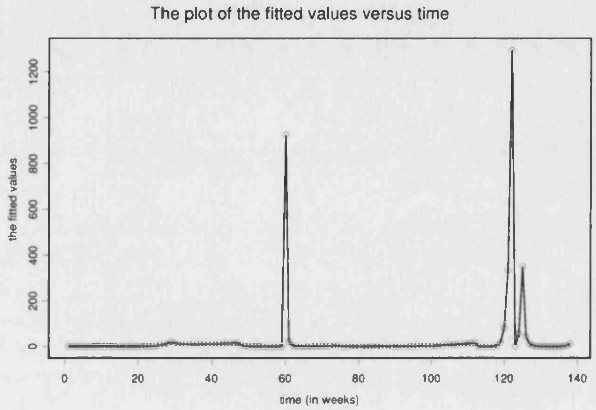

Figure 5.34: The plot of the fitted values versus time (in weeks) in zone 1 by the power kernel use

Figures 5.35 and 5.36 are to check the fit of zone 2 claims data.

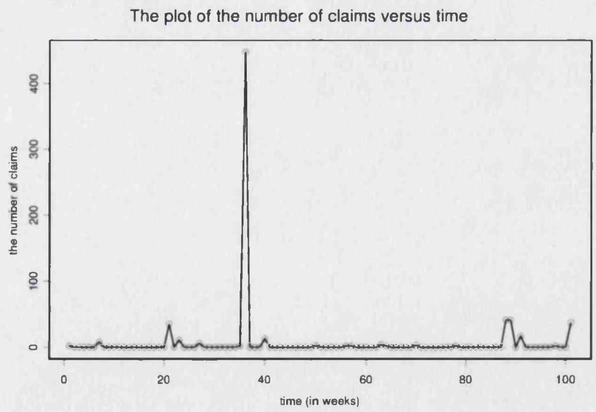

Figure 5.35: The plot of the number of claims versus time (in weeks) in zone 2 by the power kernel use

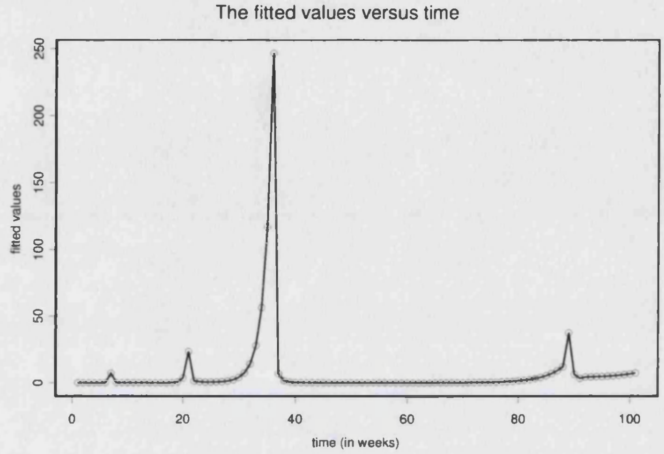

Figure 5.36: The plot of the fitted values versus time (in weeks) in zone 2 by the power kernel use

In the claim number N_i modelling, the exponential kernel function seems to have a better representation for the models of the zone 1 claims and leads to the idea that the effect of time is more significant in zone 1 regions in terms of earthquake claim arrivals. In zone 2 risk areas, the power kernel function takes the role of the exponential kernel of zone 1 and the time variable gains more importance for the claim related models.

The total claim amount (S_i) Models:

Two ways can be suggested when modelling the claim amount data. One way is to model the raw claim amount X_i, whose distribution is not lognormal, with mean η_i and variance τ_i. The second way is to model the aggregate claims S_i (total claim amount) with the assumption of the underlying X_i's are i.i.d. In this way, we can check for the double effect of taking the aggregate claims S_i with underlying raw claims are i.i.d. The idea of binning is used in the construction of kernel functions (exponential and power) in the modelling process.

Here we let the claims in bin i, $X_{i1}, \ldots, X_{iN_i}$, which are the raw claims with mean η_i and variance τ_i and where N_i is the number of claims in this bin. We model the aggregate claims S_i in the corresponding count N_i in that bin by using

$$S_i = \sum_{i=1}^{N_i} X_{ij}.$$

We want to justify the use of the same kernel function form for the claim number N_i and the aggregate claims S_i models by using the following computations to show the relation between those two. Under the assumption of i.i.d X_{ij}'s and with the argument of conditional on the bin counts N_i

$$E(S_i|N_i) = E(\sum_{i=1}^{N_i} X_{ij}|N_i) = N_i\eta_i,$$

which gives $E(\frac{S_i}{N_i}) = \eta_i$ is a constant and the variance of the total claim amount is

$$Var(S_i|N_i) = N_i\tau_i.$$

By using the second approach above,we assume to model the aggregate claims S_i over each bin. If we use a new random variable $Y = \log S_i$,

$$Y_i = \log S_i \sim N(\mu_i, \sigma_i^2).$$

The relation between the parameters μ_i, σ_i^2 and the claim number N_i, the parameters η_i and τ_i of the aggregate claims can be constructed as follows by using the mean and variance of the lognormal distribution for the aggregate claims, where we condition on the claim number N_i

$$E(S_i|N_i) = N_i\eta_i = e^{\mu_i+\frac{\sigma_i^2}{2}}, \tag{5.1}$$

and

$$Var(S_i|N_i) = N_i\tau_i = e^{2\mu_i+\sigma_i^2}(e^{\sigma_i^2} - 1). \tag{5.2}$$

The variance is not affected by the number of claims since the number of claims is constant and constant term has no effect on variance. If we take the log in (5.1), the mean of the aggregate claims will be

$$\mu_i = \log(N_i) + \log(\eta_i) - \frac{1}{2}\sigma_i^2. \tag{5.3}$$

By substituting (5.3) in (5.2), conditional on the number of claims N_i

$$\begin{aligned} Var(S_i|N_i) = N_i\tau_i &= e^{2(\log(N_i)+\log(\eta_i)-\frac{1}{2}\sigma_i^2)+\sigma_i^2}(e^{\sigma_i^2}-1) \\ N_i\tau_i &= e^{2\log(N_i)+2\log(\eta_i)}(e^{\sigma_i^2}-1). \end{aligned} \tag{5.4}$$

If we log the both sides

$$\begin{aligned} \log(N_i)+\log(\tau_i) &= 2(\log(N_i)+\log(\eta_i))+\log(e^{\sigma_i^2}-1) \\ \log(N_i)+\log(\tau_i)-2\log(N_i)-2\log(\eta_i) &= \log(e^{\sigma_i^2}-1) \\ \log(\frac{\tau_i}{N_i})-\log(\eta_i^2) &= \log(e^{\sigma_i^2}-1) \\ \log(\frac{\frac{\tau_i}{N_i}}{\eta_i^2}) &= \log(e^{\sigma_i^2}-1) \\ \frac{\tau_i}{\eta_i^2 N_i} &= e^{\sigma_i^2}-1 \\ \frac{\tau_i}{\eta_i^2 N_i}+1 &= e^{\sigma_i^2} \\ \sigma_i^2 &= \log(1+\frac{\tau_i}{\eta_i^2 N_i}). \end{aligned} \tag{5.5}$$

Replacing the result of (5.5) in (5.3), we get

$$\mu_i = \log(N_i)+\log(\eta_i)-\frac{1}{2}\log(1+\frac{\tau_i}{\eta_i^2 N_i}). \tag{5.6}$$

If one is modelling the total claim amount with $E(\log(S_i)) = \mu_i$, the behaviour of the model will be as in (5.6).

The claim number N_i can be replaced by the intensity Λ_i of the claim number process, when $\frac{\tau_i}{\eta_i^2}$ is small and N_i is large. By the series expansion $\log(1+x) \approx x$

$$\log(1+\frac{\tau_i}{\eta_i^2 N_i}) \approx \frac{\tau_i^2}{\eta_i^2 N_i}.$$

Thus (5.3) can be rewritten as

$$\mu_i \approx \log(N_i)+\log(\eta_i) \approx \log(\Lambda_i)+\log(\eta_i).$$

By the similar argument

$$\sigma_i^2 \approx \frac{\tau_i}{\eta_i^2 N_i},$$

and

$$N_i \sigma_i^2 \approx \frac{\tau_i}{\eta_i^2}.$$

Therefore, the total claim amount S_i is just the model for the claim number N_i, which is approximately shifted by $\log N_i$ instead.

The raw claim amount is totalled over each bin and logged. Then the log data is modelled with a Gaussian model under the assumption of $\log S \sim$ Normal (see Figure 5.1). The mean of the total claim amount is of interest here, which is $E(\log(S_i)) = \mu_i$. We can represent the mean of the aggregate claims in the form of the special kernel functions, which are also used in the modelling of the claim number N_i. Then the suggested model can be shown as

$$\log S_i \sim \mathrm{N}(\mu_i, \sigma_i^2),$$

where $\mu_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+} + \log N_i$ for the use of the exponential kernel and $\mu_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta} + \log N_i$ where the power kernel replaces the exponential kernel and $i = 1, \ldots, n$, $j = 1, \ldots, k$. The variance of the aggregate claims is approximated by $\sigma_i^2 \approx \frac{\tau_i}{\eta_i^2 N_i}$. Then basically the variance depends on the claim number N_i, the mean η_i and the variance τ_i of the underlying claim amount X_i's. Since we are conditioning on the claim number N_i as in (5.1) and (5.2), the N_i part of the variance is given. When modelling the aggregate claims, the ideal way is to iterate and weight to make a use of the variance τ_i since it depends on the claim number N_i. However, we will assume that the τ_i behaves like a constant under the conditioning on the claim number N_i and we did not do any reweighed variance analysis. The claim amount model is simply for the mean η_i but we need to transfer the model to the parameter μ_i of the aggregate claims since our aim is to model the aggregate claims in the related bin and see the double effect of the raw and aggregate claims. In the modelling, we only correct for the claim number N_i part, see also (Chapter 7).

The use of the exponential kernel:

By months data:

In terms of $\hat{\beta}$ values, S_i model suggests a lower value for zone 1 and slightly larger value in zone 2 than the claim number N_i models for the use of the exponential kernel. The confidence intervals are obtained by the values of the Hessian, where the Hessian is constructed via the estimation of the model parameters by using the Normal likelihood for the claims data.

Model 5	$\hat{\beta}$	$\hat{\alpha_j}$	Residual deviance	95 % CI for $\hat{\beta}$
Zone 1	0.29	$\hat{\alpha_{26}} = 0.71$ $\hat{\alpha_{40}} = 0.06$	5.35	(0.2898,0.2902)
Zone 2	0.4	$\hat{\alpha_{26}} = 0.67$ $\hat{\alpha_{41}} = 0.74$	0.05	(0.39998,0.4002)

Table 5.6: The estimate of the selected model parameters and 95 % $\hat{\beta}$ confidence interval for risk zones 1-2 in Turkey

The aggregate claims models result in lower deviance values compared to the claim number models. The big empirical earthquake selection is still observed by checking the values of α_j values. The models here will have a use in financial estimation, it can be concluded that there needs to be enough money in the Turkish Catastrophe Insurance Pool during the normal claim arrival period so that the TCIP can respond to the high demand for the claim payments due to an earthquake strike, especially in zone 1. Figures 5.37, 5.38, 5.39 and 5.40 are presented to back up the adequacy of this suggested total claim amount model in zone 1. The diagnostic plots for zone 2 also support the same argument.

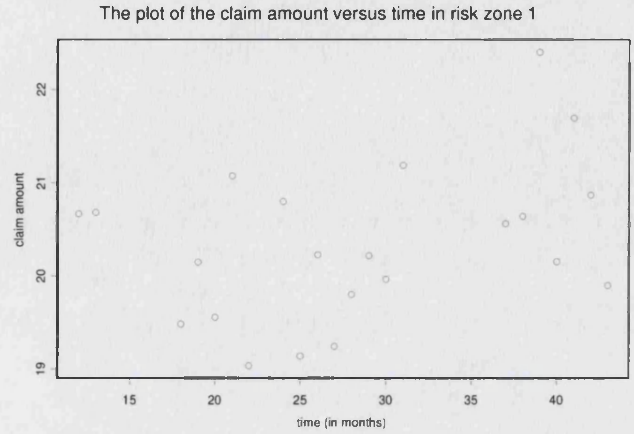

Figure 5.37: The plot of the claim amount versus time (in months) in zone 1 by the exponential kernel use

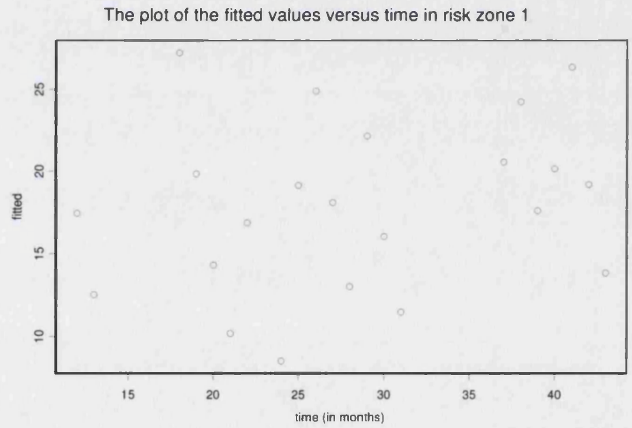

Figure 5.38: The plot of the fitted values versus time (in months) in zone 1 by the exponential kernel use

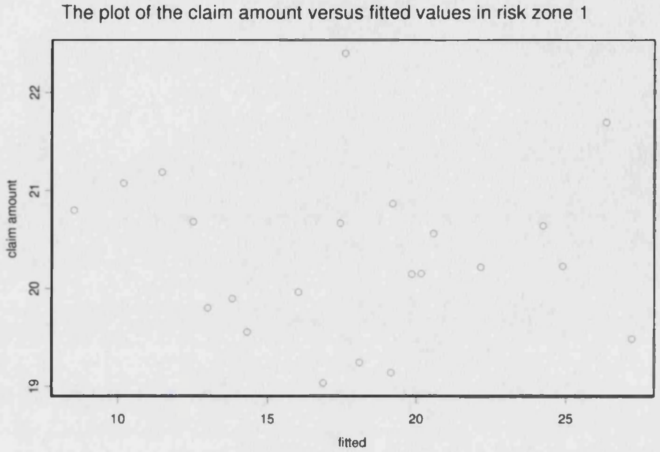

Figure 5.39: The plot of the claim amount versus fitted values in zone 1 by the exponential kernel use

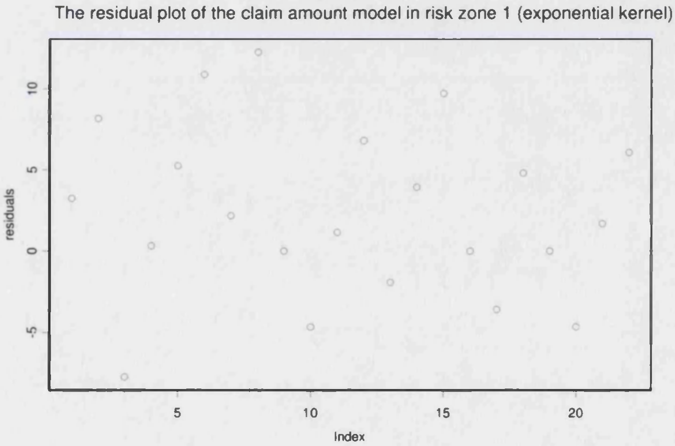

Figure 5.40: The plot of the residuals in zone 1 by the exponential kernel use

By weeks data:

The estimate of the β parameter in zone 1 is the same as the value in Table 5.3 of zone 1 (0.2) exponential kernel case and Table 5.5 of zone 1 (0.2) power kernel case in N_i models. This indicates the same fixed feature of the earthquake area, which affects the trend of the claim arrivals. This feature has the same effect on the number of claims and the paid claim amount in either kernel function use, in either time model in zone 1. On the other hand, for zone 2, the $\hat{\beta}$ value (0.7) is the highest compared to the previous models. We have higher values of the parameter estimates for zone 2 claims data. This can be linked to the awareness of the community is an important factor. An example of this is the city of 'Izmir'. Some surrounding towns of Izmir are risk zone 2 areas but a lot of claims, even for a light damage, arrive to the Turkish Catastrophe Insurance Pool if there occurs any tremor, earthquake mainly as a result of the fault movements in the Aegean Sea.

Model 6	$\hat{\beta}$	$\hat{\alpha_j}$	Residual deviance	95 % CI for $\hat{\beta}$
Zone 1	0.2	$\hat{\alpha_{60}} = 0.21$ $\hat{\alpha_{122}} = 0.18$	10.489	(0.19995,0.20004)
Zone 2	0.7	$\hat{\alpha_{36}} = 1.33$ $\hat{\alpha_{88}} = 0.26$	0.788	(0.6997,0.7003)

Table 5.7: The estimate of the selected model parameters and 95 % $\hat{\beta}$ confidence interval for risk zones 1-2 in Turkey

The use of the power kernel:

By months data:

In Table 5.8, Model 7 suggests very close β estimates for risk zones 1 and 2 for the total claim amount S_i. This means that, regardless of the zone effect, with a strike of a big earthquake, there can be huge losses in either zones and a lot of money has to be compensated to the insurance holders by using the available reserves of the Turkish Catastrophe Insurance Pool.

Model 7	$\hat{\beta}$	$\hat{\alpha_j}$	Residual deviance	95 % CI for $\hat{\beta}$
Zone 1	0.64	$\hat{\alpha_{26}} = 11.58$ $\hat{\alpha_{40}} = 0.64$	1.34	(0.610,0.669)
Zone 2	0.68	$\hat{\alpha_{26}} = 6.7$ $\hat{\alpha_{41}} = 1.7$	0.591	(0.6792,0.6808)

Table 5.8: The estimate of the selected model parameters and 95 % $\hat{\beta}$ confidence interval for risk zones 1-2 in Turkey

Higher $\hat{\beta}$ values back up the idea of the need for a large reserve pool in both risk zones 1 and 2, which also indicate that the big jumps in the claim arrivals are more likely to occur. The total money in the TCIP should be as high as possible to have a use when need arises. The significant earthquakes can still be followed by the α_j coefficients. The deviance values are reasonable for this case.

By weeks data:

Table 5.9 suggests the highest $\hat{\beta}$ estimate with the largest deviance for zone 1 among the other models of this chapter for both the number of claims N_i and the total claim amount S_i. For zone 2 claims data, a narrower width is observed, with a lower residual deviance value than that of zone 1.

Model 8	$\hat{\beta}$	$\hat{\alpha_j}$	Residual deviance	95 % CI for $\hat{\beta}$
Zone 1	0.84	$\hat{\alpha_{60}} = 0.39$ $\hat{\alpha_{122}} = 0.16$	6.734	(0.83997,0.84002)
Zone 2	0.60	$\hat{\alpha_{36}} = 0.93$ $\hat{\alpha_{88}} = 1.08$	0.1345	(0.599907,0.600093)

Table 5.9: The estimate of the selected model parameters and 95 % $\hat{\beta}$ confidence interval for risk zones 1-2 in Turkey

By using all the models of this chapter, tt can be concluded that the claims satisfy the shock kernel approach in earthquake risk zone 1 and risk zone 2 in Turkey. The confidence intervals for all cases do not cover the nice value of $\beta = 1$ and they are between $[0, 1]$ in interval.

Model Summary

The claim number and the aggregate claims (total claim amount) models of this chapter lead that the money to be paid to the insured by the Turkish Catastrophe Insurance Pool after an earthquake strike will be extremely high, since the number of claims arriving to the pool is expected to be high in zones 1 and 2. One reason for such large figure expectation is the wrong settlement very close to the fault lines. This is due to the unawareness/carelessness of the people and authorities throughout the history.

Unfortunately, another factor is the misuse or even no use of the building standards in Turkey. They are not compatible with the existing earthquake-resistant Building Code of 1998 in Turkey. It is observed of the TCIP that an increasing number of residents (especially after if any tremor/earthquake is experienced in time) are buying the mandatory earthquake insurance in higher risk zones, which leads to more number of claims and more money to pay back to the insured after an earthquake strike.

The approach of modelling the number of claims and the claim amount by the approximation of the raw event time either in months or in weeks play a crucial role in determining the capacity of the TCIP, the reserves $E(S(t))$, to compensate the possible losses. It helps to understand that a lot of claims might arrive in a short period of time, which ends in huge amount of loss payment. Therefore, in the high risk earthquake zones of Turkey, there should be a back up financial mechanism in case of an earthquake strike. In practice, it is a fact that the more the number of claims (N_i increases) especially with big earthquake shocks, the more the total amount of money paid to the insurance holder (S_i increases).

The aim of using an inhomogeneous Poisson process is to let the process to observe the significant earthquakes at any time point in a given interval. The models here, which are the claim number N_i and the aggregate claims S_i models, allow the parameters to pick the jump occurring as a result of a big earthquake. It is an interesting point to notice the following pattern: When there are no earthquakes, no claims occur, and then with a small earthquake (e.g. magnitude < 5) few claims arrive to the pool and say in next two months a big earthquake strikes. The α_j parameter, which is expected to pick the effect of this big earthquake, assigns a high importance of the few claim arrivals due to that small earthquakes that occur just before the big earthquake. The small claims absorb the effect of the large claim arrivals due to the significant earthquake. This structure best explains the use of the main shock-aftershock or initial shock aspects of the earthquake occurrence and shows the link between the geological and financial approach.

5.3.1 Simulation

The method of empirically determining probabilities by means of experimentation is called 'simulation' [Ross, 2003]. The main cornerstone of stochastic simulation is random numbers, which is used to generate independent identically distributed Uniform(0,1) random variables [Ripley, 1987, Ross, 2002b]. Most computers have built-in routines to generate such uniform random numbers. The simplest way to define a univariate random variable is the cumulative distribution function (CDF) F. [Ross, 2003, Ross, 2002b] suggest the following proposition.

Proposition: Let U be a Uniform(0,1) random variable. For any continuous distribution function F, if the random variable X is defined by

$$X = F^{-1}(U),$$

then the random variable X has distribution function F.

Proof:

$$F_X(x) = P(X \leq x) = P(F^{-1}(U) \leq x).$$

Since $F(x)$ is monotone function, it is valid that $F^{-1}(U) \leq x$ if and only if $U \leq F(x)$ $(F(F^{-1}(U)) = U)$. Then:

$$F_X(x) = P(U \leq F(x)) = F(x).$$

Therefore, the random variable X can be simulated from the continuous distribution F, by simulating a random number U and setting $X = F^{-1}(U)$. This is called the 'inverse transformation method' and is one of the methods to simulate (generate) random variables from an arbitrary distribution [Ross, 2003, Ross, 2002b].

Another method is called the 'rejection method'. Suppose there is a method available to simulate a random variable with density function $g(x)$. This is used as basis to simulate from a continuous distribution with density $f(x)$ by simulating Y from g and accepting this simulated value with a probability proportional to $\frac{f(Y)}{g(Y)}$ [Ross, 2003, Ross, 2002b]. Let c be a constant such that

$$\frac{f(Y)}{g(Y)} \leq c;\ \forall y.$$

Then the following procedure is used for simulating a random variable of density f [Ross, 2003, Ross, 2002b].

1. Simulate Y having density g and simulate a random number U.

2. If $U \leq \frac{f(Y)}{cg(Y)}$, set $X = Y$. Otherwise go to 1.

Following the procedure above, the following proposition is given in [Ross, 2003, Ross, 2002b]:

Proposition:

The random variable X, which is generated by the rejection method, has density function f.

There are other methods for simulation like the hazard rate method and special techniques for simulating specific random variables. The procedure is generally analog for discrete and continuous distributions.

Generating an inhomogeneous Poisson Process:

Counting events in a Poisson process gives a Poisson distributed random variable. An inhomogeneous Poisson process $N(t)$ is an important process in modelling purposes, which allows the possibility that the arrival rate can vary with time effect unlike a homogeneous Poisson process with stationary increments [Ross, 2002b].

The claim interoccurrence times has a very convenient use in describing the claim number process $N(t)$ [Bühlmann, 1970]. The most basic method to simulate an inhomogeneous Poisson process is to simulate the successive event times [Ross, 2003, Ross, 2002b]. Let $W_1, \ldots, W_n$ denote the successive event times of the process $N(t)$ as given in Section 3.3.1. If an event occurs at time W_1, independent of what has occurred before time W_1, the time until the occurrence of the next event has the distribution $F_i(t) = 1 - e^{-\Lambda_i(t)}$ given on Page 44 in Section 3.3.1. The event times $W_1, \ldots, W_n$ can be simulated starting with the simulation of W_1 from F_0, if $W_1 = w_1$ simulate W_2 by adding w_1 to the value generated from F_{w_1} and so on. It is possible to find the inverse of the distribution function $F^{-1}(.)$ and generate the inter-arrival times with $F^{-1}(U_i)$ where U_i is a uniform random variable on $(0, 1)$ [Ross, 2003, Ross, 2002b, Lewis and Shedler, 1979, Martinez and Martinez, 2002].

Another method to simulate an inhomogeneous Poisson process is the 'thinning' approach. Suppose the first T time units of an inhomogeneous Poisson process will be simulated. The thinning method starts by choosing a value λ, which satisfies $\lambda(t) < \lambda$ for all $t \leq T$ [Ross, 2002b]. By a random selection of the event times of a Poisson process of rate λ, an inhomogeneous Poisson process can be generated. Basically, if an event of a Poisson process of intensity λ occurs at time t is counted with probability $\frac{\lambda(t)}{\lambda}$, then the process of counted events is an inhomogeneous Poisson process with intensity function $\lambda(t)$ for $0 \leq t \leq T$. The intensity λ of the process is simply estimated $\frac{N(t)}{t}$, that is $\frac{\text{the number of claims}}{\text{time}}$.

The simulation of a Poisson process and counting its events randomly leads to the generalisation of an inhomogeneous Poisson process [Ross, 2002b]. Then, the following steps are followed in the so-called thinning algorithm for simulating an inhomogeneous Poisson process [Lewis and Shedler, 1976, Lewis, 1972, Ross, 2003, Ross, 2002b, Kuhl and Bhairgond, 2000].

1. Set $t = 0$, $I = 0$.
2. Generate a random number U (Uniform random variable).
3. Set $t = t - \frac{\log(U)}{\lambda}$. If $t > T$, stop.
4. While $t < T$, generate U.
5. If $U \leq \frac{\lambda(t)}{\lambda}$, set $I = I + 1$, $W(I) = t$.
6. Go to step 2, generate U and set $t = t - \frac{\log U}{\lambda}$.

Here, $\lambda(t)$ is the intensity function and λ is the rate of a homogeneous Poisson process such that $\lambda(t) \leq \lambda$. The final value of I denotes the number of events time T and $W(1), \ldots, W(I)$ are the event times.

Figure 5.41 is an example of the pattern of the simulated inhomogeneous Poisson process by using Model 1 of Table 5.2 parameter estimates of zone 1 with months based data, where as an approximation of λ in the bin case

$$\hat{\Lambda} = \frac{3601}{32} = \frac{\text{the number of claims in zone 1}}{\text{the number of months in given time period}} = 113.$$

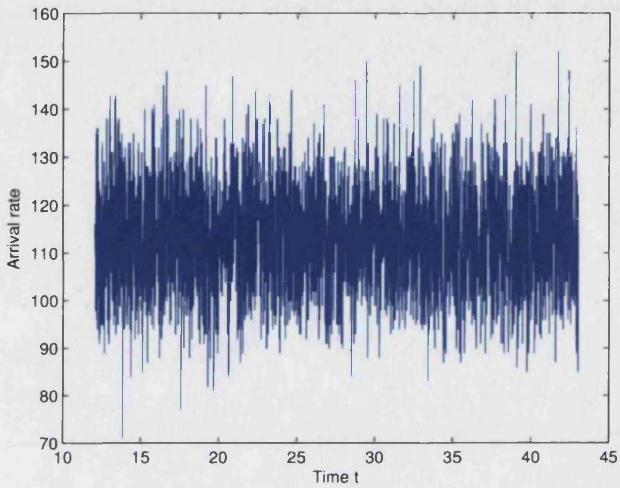

Figure 5.41: The fitted estimate of the event arrival rate $\lambda(t)$

The modelling process of the thesis can be justified by using the following steps.
Algorithm 2:

1. Fix the existing parameters $\alpha_0, \alpha_1, \ldots, \alpha_k$ and β,
2. Generate (simulate) a new random data set for the number of claims N_i by using the thinning algorithm,
3. Use the simulated data in glm modelling,
4. Get the new estimates of the parameters $\alpha_0, \alpha_1, \ldots, \alpha_k$ and β,
5. Construct the confidence interval for β,
6. Check how many times it covers the true value of β, which is set in 1.

Here we give an example on how to apply the simulation procedure explained above for the modelling purposes of this study. The simulated data for zone 1 Model 1 of Table 5.2 is generated by using the corresponding parameter estimates, that is by using the maximum likelihood estimates of the parameters in the exponential kernel form of $\hat{\Lambda}_i = e^{\hat{\alpha}_0 + \sum_{j=1}^{k} \hat{\alpha}_j e^{-\hat{\beta}(t_i - s_j)|+}}$. Hence, the mean-value function of the whole process $N(t)$ can be expressed as $\Lambda_i = \int_0^t e^{\hat{\alpha}_0 + \sum_{j=1}^{k} \hat{\alpha}_j e^{-\hat{\beta}(t_i - s_j)|+}} dt$ [Lewis, 1972].

When the generalised linear models are run with the zone claims simulated data based on the exponential kernel and months as time covariate, the $\hat{\beta}$ value is obtained close to the actual value of $\hat{\beta} = 0.37$ and the α_j parameters still pick the big earthquakes. The following table gives how many times the actual value of β is covered with 95 % confidence at the empirical knots.

5000 iterations	The number of coverage	The % coverage
$\hat{\beta}_1$=0.17	4740	94.8
$\hat{\beta}_2$=0.28	4742	94.84
$\hat{\beta}_3$=0.4	4748	94.96

Table 5.10: The simulated values of $\hat{\beta}$ and 95 % coverage for Model 1 of Table 5.2 N_i model

Table 5.10 indicates as $\hat{\beta}$ value increases, the coverage probability increases when 5000 iterations are used. Higher value of $\hat{\beta}$ means, the size of the jump is large and it was a real shock event. This is typical as shown in the Table. As a part of future research, we intend to work on bias corrected intervals as well as updating the data and fitting different types of models.

Chapter 6

Modelling with Covariates

This chapter calibrates the existing models of Chapter 5 by adding magnitude and the number of residential buildings of the earthquake area as new covariates (aka regressors or independent variables) in the selected models. The binning idea of the Poisson counts in modelling the non-normal response variable for the number of claims N_i ($N_i \sim \text{Pois}(\Lambda_i)$) and similar argument for the modelling of the aggregate claims (total claim amount) S_i is followed.

The mandatory earthquake insurance scheme in Turkey (the Turkish Catastrophe Insurance Pool) only insures the residential buildings. Therefore, it seemed to be an interesting approach to use the number of the residential buildings in the earthquake affected area as an independent variable for our models. The building number figures are obtained from the State Institute of Statistics.

6.1 Graphical analysis

The following plots are presented to show the relation between the covariates (of the raw data) and the log claim amount in all zones and in zone 1 and zone 2, respectively. Figures 6.1 and 6.2 represent the basic relation between the magnitude, the claim amount and the residential building number considering all risk zones. It seems that earthquakes of magnitude 5 or more are considered to cause serious damage. Here, it is observed from the plots that there occurs a lot of claims at smaller magnitudes as well as the magnitudes more than 5. Interestingly, when the magnitude is big (e.g 6.5), even though the number of the residential buildings is between 50000-100000, which means a small town/city, still a lot of claims are observed. This can be due to the distance of the earthquake area to the fault line or to the earthquake epicentre; or, if the building structure is poor in the earthquake region, when a big magnitude earthquake strikes heavy damage occurs.

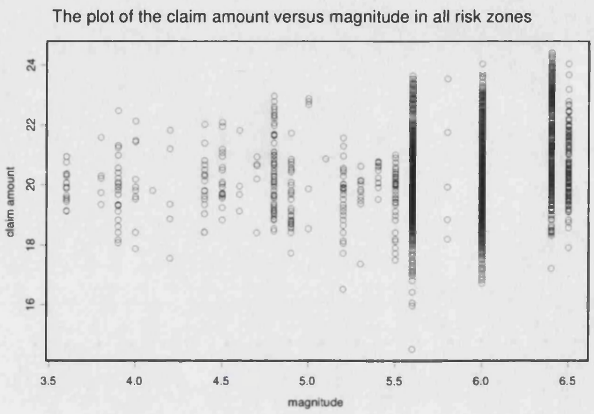

Figure 6.1: The plot of the claim amount versus magnitude in all risk zones (1-5)

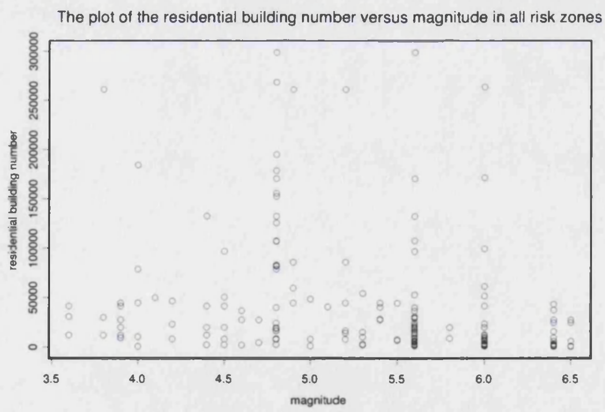

Figure 6.2: The plot of the residential building number versus magnitude in all risk zones (1-5)

Figures 6.3 and 6.4 show a similar pattern with Figures 6.1 and 6.2 because most of the all zone claims arrive from zone 1. It is also observed that the strike of an earthquake of magnitude 5.6 can result in many claims, which is the same as the effect of an earthquake of 6 or 6.5. Quiet a lot of claims occur as a result of an earthquake of magnitude less than 5, which is probably due to the geological characteristics of the town/city and the quality of the building structure. Another possible reason for the arrival of many claims is the amount of the earthquake insurance policies in the earthquake affected area. Assume there is a small magnitude earthquake and there are few dwelling units (e.g. < 100000). The number of claims and so the total paid claim amount will increase, if many of the households own the mandatory earthquake insurance of the Turkish Catastrophe Insurance Pool.

Figures 6.5 and 6.6 represent the relation between the magnitude, the residential building number and the claim amount in zone 2. The similar interpretation for zone 1 claims data can be made for zone 2 too. Ordinary small claims are also observed at earthquakes of magnitude less than 5, which probably occur due to light damage like wall cracks, sliding of walls or garden walls.

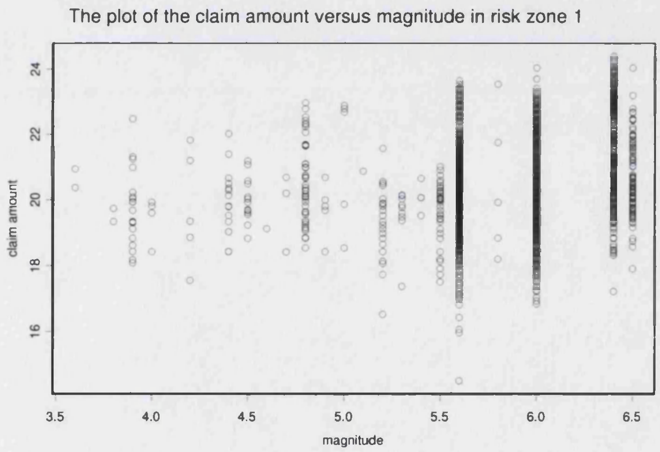

Figure 6.3: The plot of the claim amount versus magnitude in risk zone 1

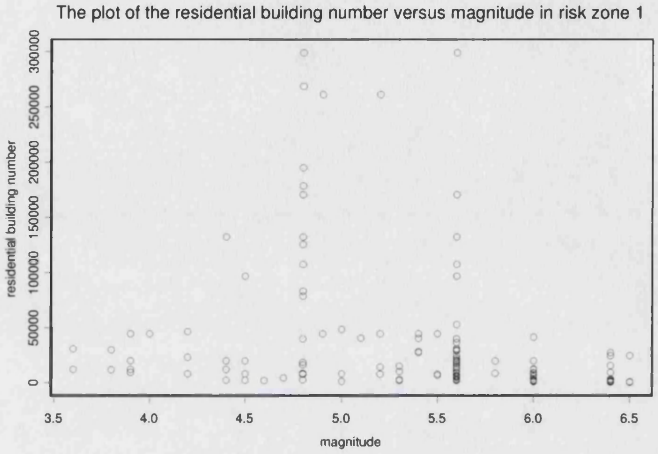

Figure 6.4: The plot of the residential building number versus magnitude in risk zone 1

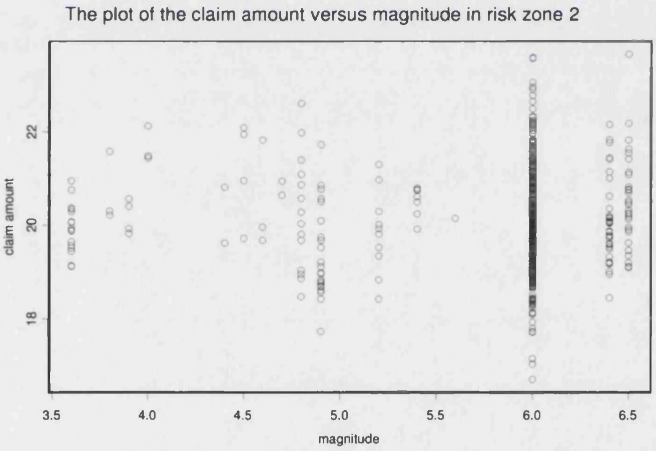

Figure 6.5: The plot of the claim amount versus magnitude in risk zone 2

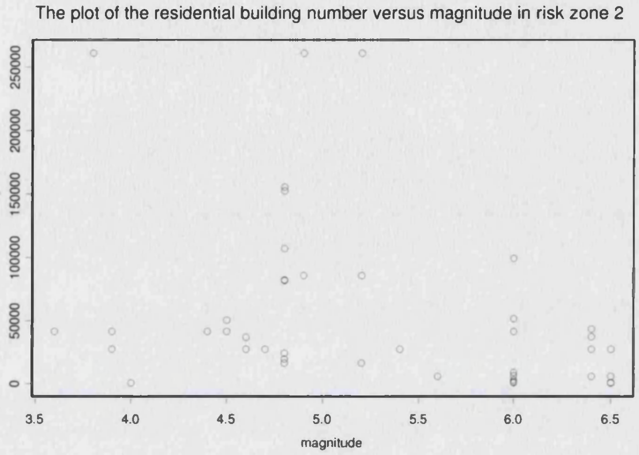

Figure 6.6: The plot of the residential building number versus magnitude in risk zone 2

Figures 6.7 and 6.8 show the relation between the number of residential buildings (the second model covariate), the magnitude and the claim amount considering all risk zones. In the data, the minimum number of the residential buildings is 506 (where many observations accumulate) and the maximum is 298841. It is observed that many claims arrive in the interval of 506 to 50000 residential buildings. This scale indicates a small to medium size town/city in Turkey. Additionally, these settlement areas are possibly very close to the fault line (either risk zone 1 or zone 2) and by the awareness of the community that they live in high risk zones, many flats/houses might have been insured against earthquake risk.

Figures 6.9 and 6.10 are drawn for zone 1 of the same variables as in Figures 6.7 and 6.8 and have a similar interpretation because most of the claims of all zone data is from risk zone 1. Figures 6.11 and 6.12 is plotted for zone 2 claims data. There are less observations in zone 2 (nzone1=3602, nzone2=676) than in zone 1. The intense pattern of the claims in Figures 6.11 and 6.12 is due to the 03/02/2002 earthquake of magnitude 6.0 (see Table 5.1) in Afyon, Sultandaği, which is a small town in the Middle-Aegean province.

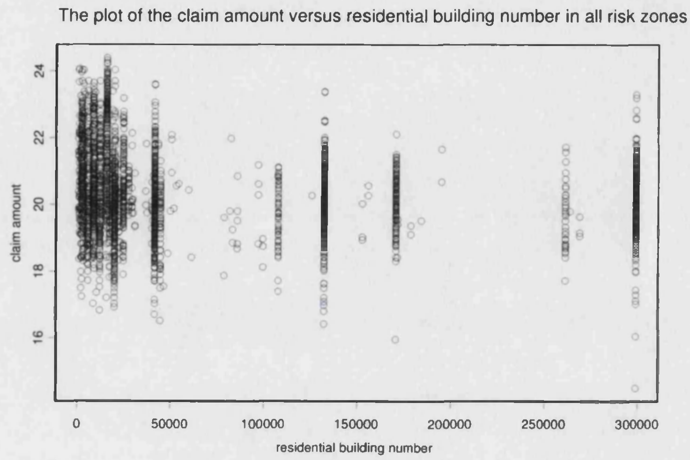

Figure 6.7: The plot of the claim amount versus residential building number in all risk zones (1-5)

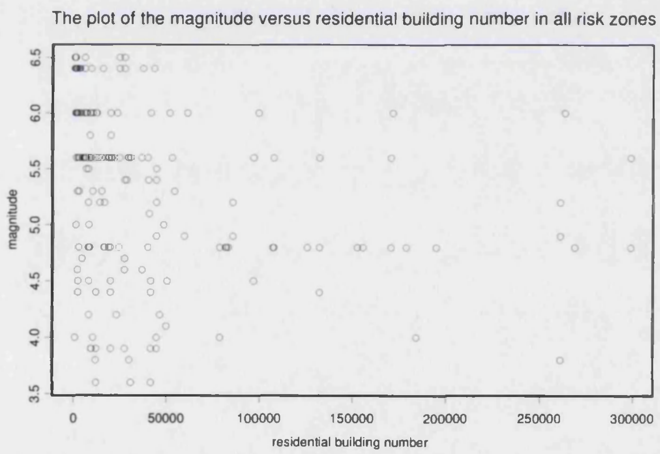

Figure 6.8: The plot of magnitude versus residential building number in all risk zones (1-5)

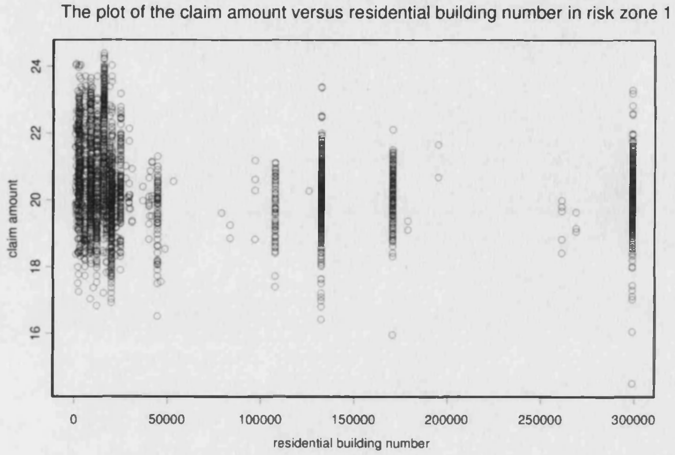

Figure 6.9: The plot of the claim amount versus residential building number in risk zone 1

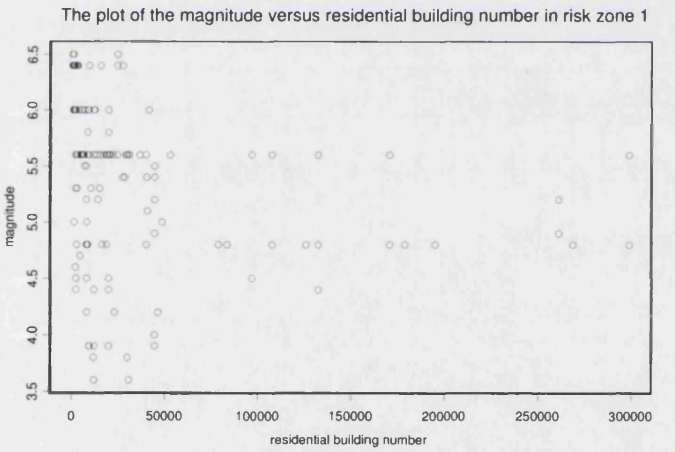

Figure 6.10: The plot of magnitude versus residential building number in risk zone 1

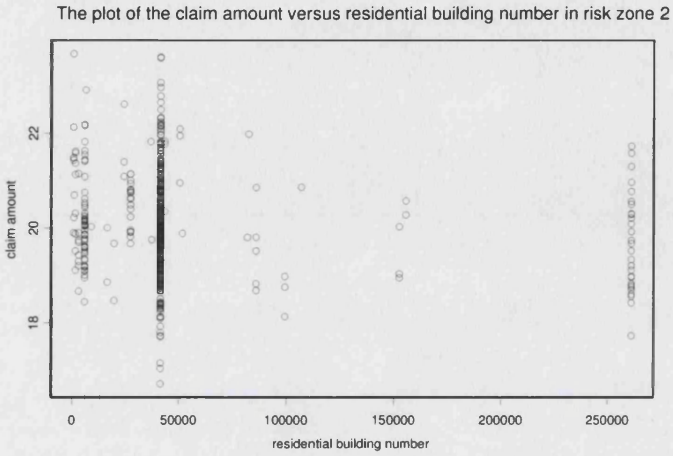

Figure 6.11: The plot of the claim amount versus residential building number in risk zone 2

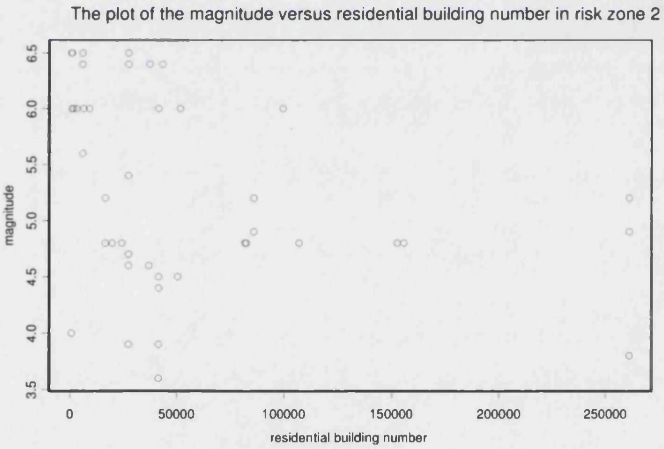

Figure 6.12: The plot of magnitude versus residential building number in risk zone 2

The jump pattern of the claims at the significant earthquakes, which is mentioned in Chapter 5, can also be recognised in the following plots. Figures 6.13, 6.14, 6.15 and 6.16 show the relation between the number of claims, the claim amount and magnitude in risk zones 1 and 2 by using weeks data.

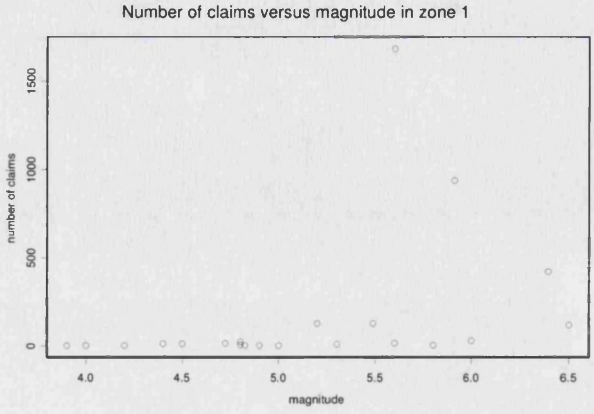

Figure 6.13: The scatter plot of number of claims versus magnitude in risk zone 1 by weeks data

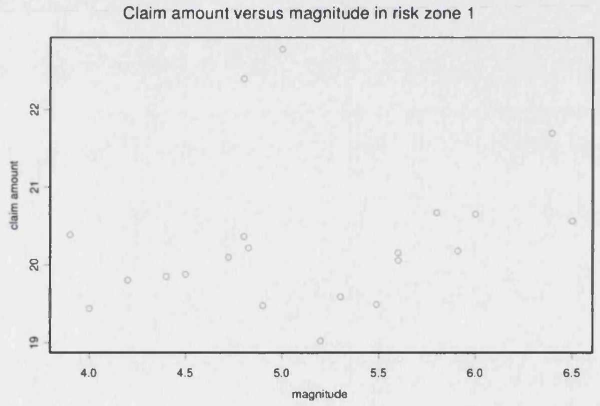

Figure 6.14: The scatter plot of claim amount versus magnitude in risk zone 1 by weeks data

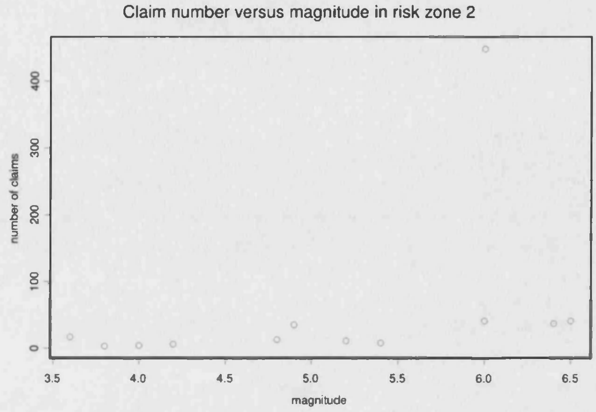

Figure 6.15: The scatter plot of number of claims versus magnitude in risk zone 2 by weeks data

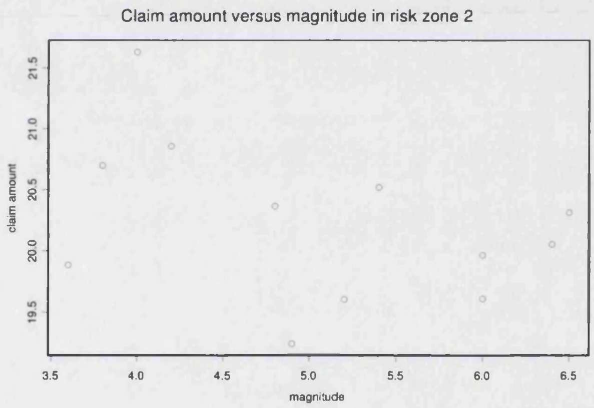

Figure 6.16: The scatter plot of claim amount versus magnitude in risk zone 2 by weeks data

The biggest earthquake in the data is of magnitude 6.5. However, the jump of the number of claims N_i and a large amount of claim payments might occur at lower magnitude earthquakes. This can be linked with the number of people owning the compulsory earthquake insurance, the income, education level and the awareness of the community in the earthquake area, or, the large number of claim payments is due to poor building construction in the earthquake town/city, where small magnitude earthquakes end up in heavy building damage.

6.2 Modelling

In the modelling process, the similar analysis of Chapter 5 is followed with the addition of covariates to the existing models. It is observed that the magnitude is a significant variable, which reduces the deviance and calibrates the models. Here, the causality can be explained as, at a certain time point, there is a magnitude, which causes a lot of claims. When the magnitude and the residential building number are included together in the models, the best results in terms of the diagnostic plots of the suggested models and the residual deviance are observed. The following algorithm is used to obtain the required models.

Algorithm 3:

1. Choose on the empirical event times (where large earthquakes occur resulting in large claims) and the corresponding kernel knots (one-sided kernel to represent the jump effect ahead in the time period),
2. Use the exponential kernel to model claim number N_i with rate $\Lambda_i = e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|_+}}$ and to model the total claim amount S_i with rate $\mu_i = \alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|_+}$ and use the power kernel correspondingly with rates $\Lambda_i = e^{\alpha_0+\sum_{j=1}^{k}\alpha_j (t_i-s_j)|_+^{-\beta}}$ and $\mu_i = \alpha_0 + \sum_{j=1}^{k}\alpha_j(t_i - s_j)|_+^{-\beta}$,
3. Use additive generalised linear models,
4. Model the number of claims N_i with a Poisson log-link function as counts ($N_i \sim \text{Pois}(\Lambda_i)$) and model the total claim amount, S_i, as Gaussian by using the selected empirical kernel knots,
5. Add magnitude to the model as a linear explanatory variable,
6. Add residential building number to the existing model as the other linear explanatory variable,

7. Use the non-linear parameter β as a representative of the characteristics of earthquake risk zones 1 and 2 and use the α_j parameters as the coefficients picking the significant earthquakes with big number of claims,

8. Set an initial value for β (random),

9. Run step-wise regression,

10. Choose the β value, which corresponds to the minimum deviance since the non-linear models are mainly based on deviance (see the Appendix for deviance). Note that, the formulae in Chapter 4 for the estimation of $\hat{\beta}$ is used as a check as well,

11. Run the model again with the fixed β,

12. Obtain the values of α_j estimates,

13. Construct the Hessian matrix H by using the second partial derivatives of the log-likelihood, which are derived in Chapter 4 and the parameter estimates and invert the Hessian,

14. Construct the confidence interval of β parameter by using the corresponding diagonal value of the inverse observed information matrix I,

15. If interested, by the similar argument in the previous step, construct the confidence interval of α_j,

16. By using the parameter estimates of the models, calculate the expected total claim amount $E(S(t))$ and other statistical measures.

The claim number (N_i) Models:

When the magnitude of an earthquake is added as a linear explanatory variable to the models, which are generated in Chapter 5, the new suggested models for the intensity of the claim number is

$$N_i \sim \text{Pois}(\Lambda_i),$$

where the log-linear Poisson count rates is modelled with $\log \Lambda_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+}$ for the exponential kernel choice and $\log \Lambda_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j((t_i - s_j)|_+)^{-\beta}$ for the power kernel choice with $i = 1, \ldots, n$, $j = 1, \ldots, k$.

When the magnitude is added as a covariate, the models can be rewritten as

$$\log \Lambda_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+} + m_l,$$

and

$$\log \Lambda_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j((t_i - s_j)|_+)^{-\beta} + m_l,$$

where $j = 1, \ldots, k$, $l = 1, \ldots, k$ refer to the knots that the empirical kernels sit.

The addition of the residential building number of the earthquake are changes the models above into:

$$\log \Lambda_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+} + m_l + r_l,$$

and

$$\log \Lambda_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j((t_i - s_j)|_+)^{-\beta} + m_l + r_l,$$

where m_l stands for the magnitude and r_l denotes the number of residential buildings.

The S-Plus analysis suggests the following covariate models as suitable models to explain the claim number N_i with the replacement of the empirical kernel knots at the time of the significant earthquakes. The values of the non-linear parameter β is kept fixed since it is a universal constant, which denotes the exponential decay in the claim arrival process of the risk zone 1 and zone 2. The addition of the magnitude and the residential building number affect the value of the α_j coefficients and this leads to a change in the deviance value and also slight variation in the confidence limits. The significance level to construct the confidence interval for β is used as 0.05 (95 % confidence).

The use of the exponential kernel:

By months data:

Model 9 of Table 6.1 suggests the following figures by using the same estimate value of the non-linear parameter β in Table 5.2 ($\hat{\beta}_{zone1} = 0.37$, $\hat{\beta}_{zone2} = 0.325$), when the covariates are added to the models of previous Chapter. Then, we observe that the deviance values drop when compared to Model 1 of Table 5.2. The α coefficients to represent the big earthquakes still pick the significant earthquake event occurrence.

Model 9	$\hat{\beta}$	$\hat{\alpha_j}$	Residual deviance	95 % CI for $\hat{\beta}$
Zone 1	0.370	$\hat{\alpha_{26}} = 8.99$ $\hat{\alpha_{40}} = 8.15$	56.71	(0.26962,0.47328)
Zone 2	0.325	$\hat{\alpha_{26}} = 3.95$ $\hat{\alpha_{41}} = 1.63$	17.09	(0.31505,0.33495)

Table 6.1: The estimate of the selected model parameters and 95 % $\hat{\beta}$ confidence interval for risk zones 1-2 in Turkey

Figure 6.17 and Figure 6.18 show the efficiency of the model fit. Similar diagnostic plot analysis back up the same argument for zone 2 claims data.

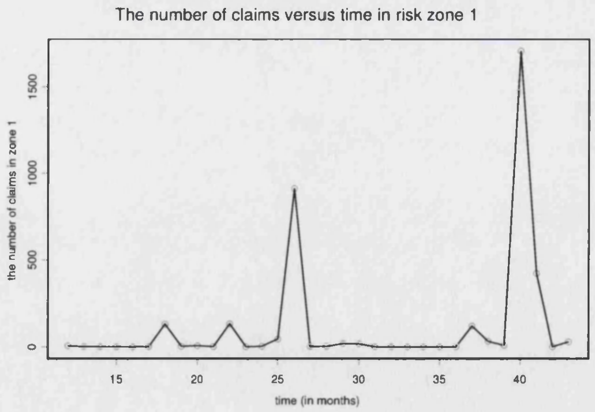

Figure 6.17: The plot of the claim number versus time (in months) in risk zone 1 by the exponential kernel use

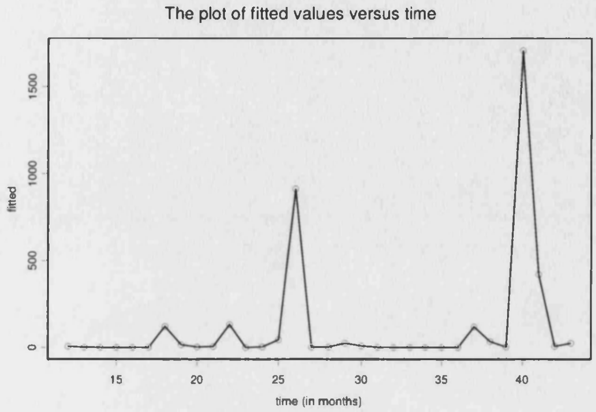

Figure 6.18: The plot of the fitted values versus time (in months) in risk zone 1 by the exponential kernel use

By weeks data:

Table 6.2 suggests a good model in terms of deviance values and diagnostic plots in both zones (see Figures 6.19 and 6.20 below for zone 1 data plots). When the model is run with covariates, the significant earthquakes are obviously picked among others by their corresponding α_j coefficients like $\hat{\alpha}_{\text{week}122} = 4.93$ with $n_{\text{week }122} = 1682$ claims, $\hat{\alpha}_{\text{week}60} = 3.87$ with $n_{\text{week}60} = 939$ claims. More consistent confidence intervals are obtained, especially for zone 1 when compared to the values of Table 5.3 (the same model without covariate case).

Model 10	$\hat{\beta}$	$\hat{\alpha_j}$	Residual deviance	95 % CI for $\hat{\beta}$
Zone 1	0.2	$\hat{\alpha_{60}} = 3.87$ $\hat{\alpha_{122}} = 4.93$	156.403	(0.1264,0.2736)
Zone 2	0.3	$\hat{\alpha_{36}} = 3.83$ $\hat{\alpha_{88}} = 4.08$	21.624	(0.2241,0.3759)

Table 6.2: The estimate of the selected model parameters and 95 % $\hat{\beta}$ confidence interval for risk zones 1-2 in Turkey

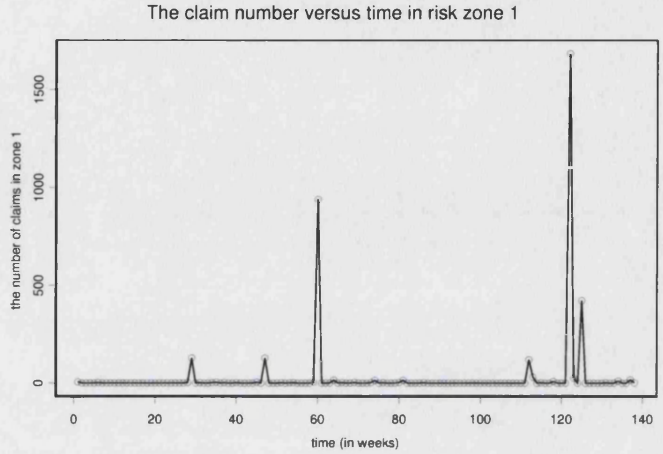

Figure 6.19: The plot of the claim number versus time (in weeks) in risk zone 1 by the exponential kernel use

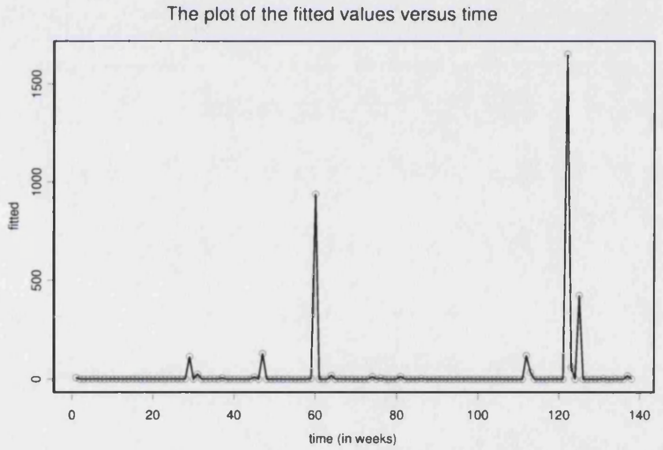

Figure 6.20: The plot of the fitted values versus time (in weeks) in risk zone 1 by the exponential kernel use

The use of the power kernel:

By months data:

Larger confidence limits for zone 1 claims data in Table 6.3 can be interpreted as the fix region characteristic of zone 1, there are more factors, like demographic, economical or geological indicators, to play a role on the earthquake occurrence and resulting analysis.

Model 11	$\hat{\beta}$	$\hat{\alpha_j}$	Residual deviance	95 % CI for $\hat{\beta}$
Zone 1	0.43	$\hat{\alpha_{26}} = 6.75$ $\hat{\alpha_{40}} = 1.85$	174.805	(0.3865,0.4734)
Zone 2	0.52	$\hat{\alpha_{26}} = 7.91$ $\hat{\alpha_{41}} = 4.11$	84.69	(0.51998,0.52001)

Table 6.3: The estimate of the selected model parameters and 95 % $\hat{\beta}$ confidence interval for risk zones 1-2 in Turkey

The selected presentation of the α_j values for months 26, 40 and 41 are good examples to denote the big magnitude earthquake, more claims combination. The following plots give the diagnostic check of the zone 1 claims model suggested in Table 6.3 to show that our models provide consistent and sufficient results. Zone 2 claims data also support the model consistency, when diagnostic plot check is done.

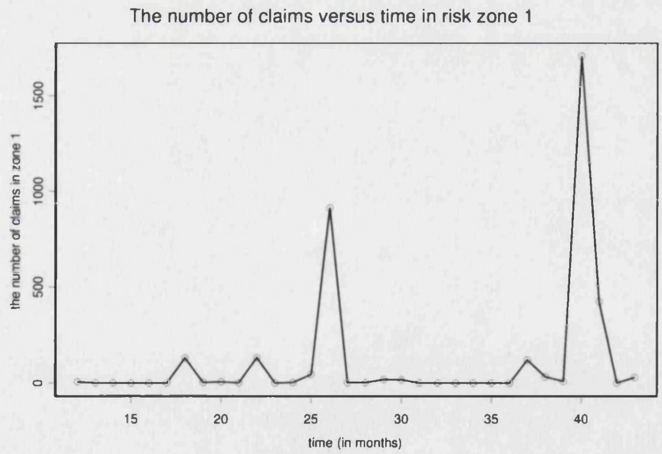

Figure 6.21: The plot of the claim number versus time (in months) in risk zone 1 by the power kernel use

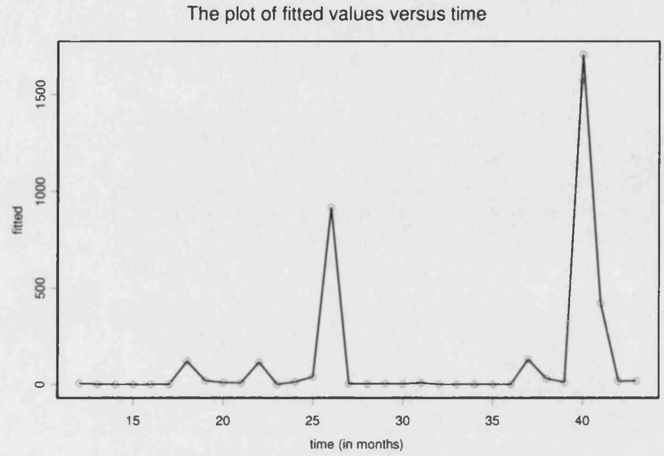

Figure 6.22: The plot of the fitted values versus time (in months) in risk zone 1 by the power kernel use

By weeks data:

In Model 12, the variance of the non-linear β parameter is obtained to be very low in risk zone 2 with a narrower confidence interval than of zone 1. An interpretation of this can be the number of claims in zone 2 by the use of weeks data does not show a large variability. That is, the distance to a fault line, the population of the region or the density of the number of buildings in the area have more dominant effect on the number of claims arriving in zone 2.

Model 12	$\hat{\beta}$	$\hat{\alpha_j}$	Residual deviance	95 % CI for $\hat{\beta}$
Zone 1	0.2	$\hat{\alpha_{60}} = 1.50$ $\hat{\alpha_{122}} = 2.37$	434.36	(0.074,0.326)
Zone 2	0.4	$\hat{\alpha_{36}} = 7.45$ $\hat{\alpha_{88}} = 6.34$	95.531	(0.391,0.408)

Table 6.4: The estimate of the selected model parameters and 95 % $\hat{\beta}$ confidence interval for risk zones 1-2 in Turkey

Figures 6.23 and 6.24 are presented to show the jump in the actual earthquake claim events and the fitted values of the model.

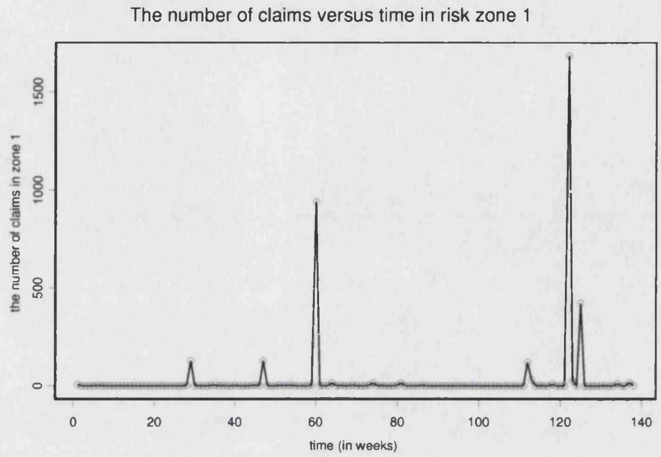

Figure 6.23: The plot of the claim number versus time (in weeks) in risk zone 1 by the power kernel use

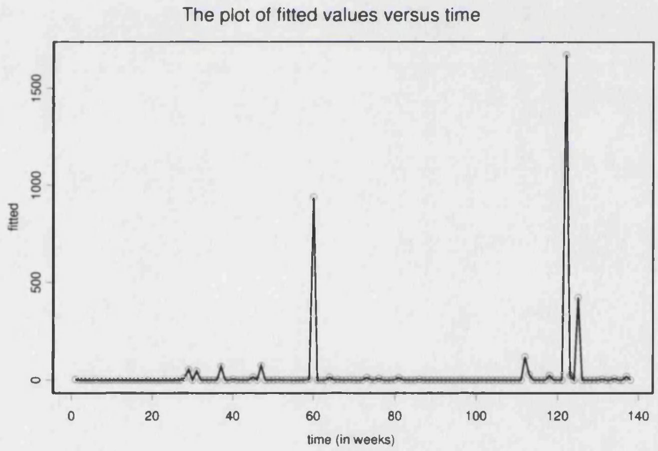

Figure 6.24: The plot of the fitted values versus time (in weeks) in risk zone 1 by the power kernel use

It is observed that the magnitude and the residential building number are crucial actors for the modelling of the number of claims by using the special kernel functions. These variables decrease the deviance significantly. In Turkey, if an earthquake of a big magnitude hits an area in risk zones 1 or 2, where there are many buildings, both the life and socio-economical losses are expected to be high due to the lack of organisation and coordination of the emergency response personnel, lack of building code applications and lack of community awareness.

In such scenario, the number of claims will increase and they will arrive in a short period of time depending how many households have an earthquake insurance. The collected premiums are the main source of the Turkish Catastrophe Insurance Pool. Therefore, during the TCIP premium calculations, the magnitude and the building number should be carefully considered. Currently, it is known that the premium tariffs of the TCIP (see Section 7.2.1, Table 7.1) are calculated by using the type of the building (e.g masonry, reinforced concrete) and the risk zone effect (in terms of magnitude). It might be more useful to improve the methodology of the tariff calculations by adding the effects of the residential building number (as the TCIP only insures the residential buildings) and the age of the building, because our suggested models show the significance of the residential building number of the earthquake area, which reduces the residual deviance of the model.

The total claim amount (S_i) Models:

In the covariate models, the total claim amount S_i uses the same notation for non-linear parameter β as in Chapter 5. By using the same argument to use the same kernel function both for the claim number N_i and aggregate claims S_i, the proposed model is again

$$\log S_i \sim \mathrm{N}(\mu_i, \sigma_i^2),$$

where $\mu_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+} + \log N_i + m_l + r_l$ for the use of the exponential kernel and $\mu_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j((t_i - s_j)|_+)^{-\beta} + \log N_i + m_l + r_l$, where the power kernel replaces the exponential kernel. Here m_l stands for the magnitude, r_l denotes the number of residential buildings, $i = 1, \ldots, n$, $j = 1, \ldots, k$ and $l = 1, \ldots, k$.

The use of the exponential kernel:

By months data:

The total claim amount to be paid by the insurer naturally increases if there are a lot of earthquake-damaged buildings in the earthquake area and if many of the households have an earthquake insurance. The level of the damage (slight-moderate-heavy or total collapse) is an important factor in the claim payment process. Table 6.5 expresses the chosen models for the total claim amount S_i in earthquake risk zones 1 and 2 by using the exponential kernel function.

Model 13	$\hat{\beta}$	$\hat{\alpha_j}$	Residual deviance	95 % CI for $\hat{\beta}$
Zone 1	0.29	$\hat{\alpha_{26}} = 3.34$ $\hat{\alpha_{40}} = 5.97$	1.35	(0.289994,0.29006)
Zone 2	0.40	$\hat{\alpha_{26}} = 3.15$ $\hat{\alpha_{41}} = 1.24$	0.06	(0.399963,0.400036)

Table 6.5: The estimate of the selected model parameters and 95 % $\hat{\beta}$ confidence interval for risk zones 1-2 in Turkey

For this case, the confidence intervals of the non-linear parameter β estimates in both zones are very narrow due to a very low variance estimates obtained from the computations. The deviance values are reasonable. The diagnostic plots (e.g. claim amount versus time, fitted values versus claim amount, residual plot) support the validity of the model.

By weeks data:

Model 14	$\hat{\beta}$	$\hat{\alpha_j}$	Residual deviance	95 % CI for $\hat{\beta}$
Zone 1	0.2	$\hat{\alpha_{60}} = 1.61$ $\hat{\alpha_{122}} = 1.99$	8.609	(0.19941,0.20059)
Zone 2	0.7	$\hat{\alpha_{36}} = 1.76$ $\hat{\alpha_{88}} = 2.14$	0.143	(0.699951,0.700049)

Table 6.6: The estimate of the selected model parameters and 95 % $\hat{\beta}$ confidence interval for risk zones 1-2 in Turkey

The time dependency of the total claim amount S_i observations is obvious regarding Model 6 of Table 5.7. The covariates lessen the effect of time, so the deviance values are much lower than that of Table 5.7 for weeks data. In this case, the reasonability of the model is also backed up with the corresponding diagnostic plots.

The use of the power kernel:

By months data:

In this case, the models in Table 5.8 of Chapter 5 are well calibrated with the addition of the magnitude and the residential building number. The new values are presented below in Table 6.7. The confidence intervals of the β parameter are consistent to explain the characteristics of the parameter in both zones.

Model 15	$\hat{\beta}$	$\hat{\alpha_j}$	Residual deviance	95 % CI for $\hat{\beta}$
Zone 1	0.64	$\hat{\alpha_{26}} = 3.98$ $\hat{\alpha_{40}} = 1.38$	0.46	(0.562,0.718)
Zone 2	0.68	$\hat{\alpha_{26}} = 1.15$ $\hat{\alpha_{41}} = 1.22$	0.165	(0.673,0.687)

Table 6.7: The estimate of the selected model parameters and 95 % $\hat{\beta}$ confidence interval for risk zones 1-2 in Turkey

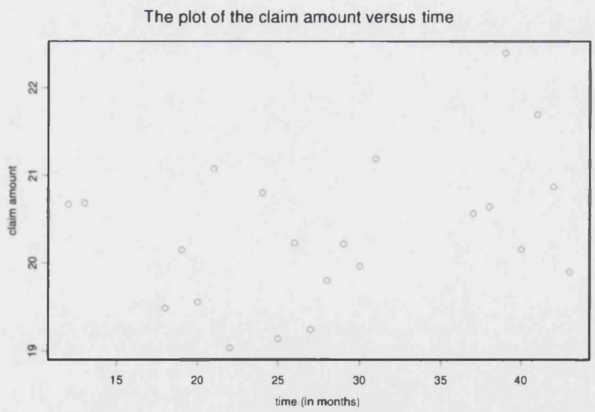

Figure 6.25: The plot of the claim number versus time (in months) in risk zone 1 by the power kernel use

Figures 6.26 and 6.27 are some examples of the diagnostic check plots to show the usage of the model in representing the behaviour of the claim amount in risk zone 1, when the power kernel function is used.

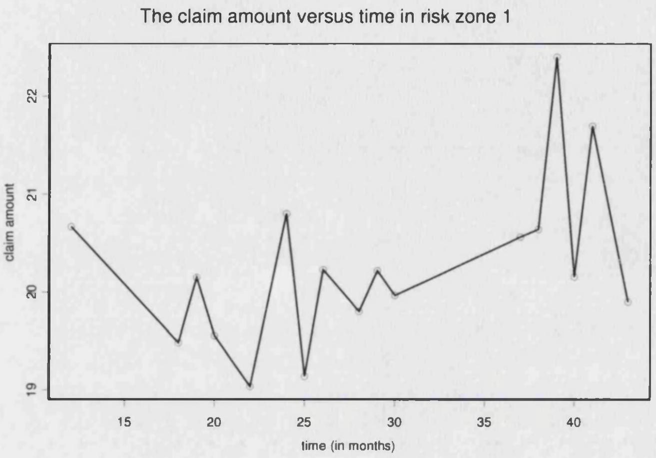

Figure 6.26: The plot of the claim amount versus time (in months) in risk zone 1 by the power kernel use in covariate models

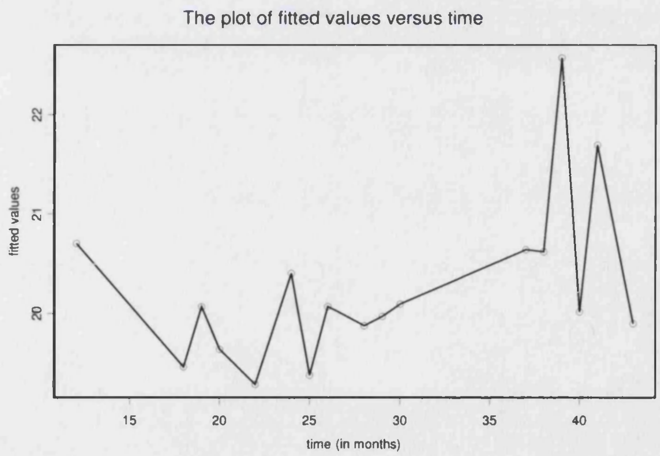

Figure 6.27: The plot of the fitted values versus time (in months) in risk zone 1 by the power kernel use in covariate models

By weeks data:

When $\beta = 1$, the power kernel function approximates the pure log-linear case for the modelling of the counts. This idea is presented when explaining the Gutenberg-

Richter and Omori's Law in Section 4.2. In Table 6.8, the high $\hat{\beta}$ values indicates a good approximation of the Gutenberg-Richter formula as a result of the use of the power kernel function. The big earthquakes are obvious to observe by their high coefficient values.

Model 16	$\hat{\beta}$	$\hat{\alpha_j}$	Residual deviance	95 % CI for $\hat{\beta}$
Zone 1	0.84	$\hat{\alpha_{60}} = 2.48$ $\hat{\alpha_{122}} = 5.45$	3.097	(0.8390,0.8416)
Zone 2	0.60	$\hat{\alpha_{36}} = 2.38$ $\hat{\alpha_{88}} = 3.50$	0.03	(0.5995,0.6005)

Table 6.8: The estimate of the selected model parameters and 95 % $\hat{\beta}$ confidence interval for risk zones 1-2 in Turkey

When the number of claims increases, the number of payments for individual claims increases and so the total paid claim amount S_i increases. This increase affects the value of the $E(S(t))$, which determines the necessary amount of reserves to keep the Turkish Catastrophe Insurance Pool safe.

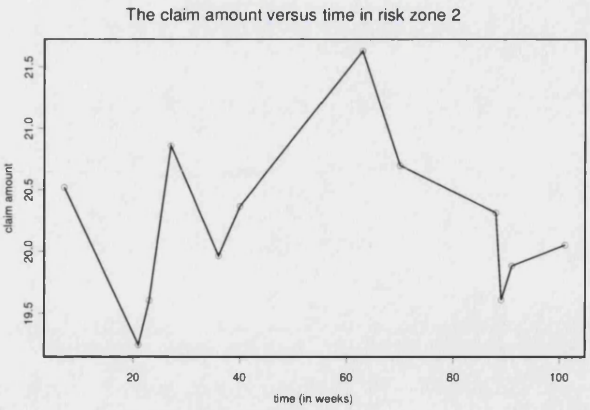

Figure 6.28: The plot of the claim amount versus time (in weeks) in risk zone 2 by the power kernel use in covariate models

Figures 6.28 and 6.29 are presented to show the diagnostic check of zone 2 claims data in this case. The corresponding plots for zone 1 claims data also support the same idea of the reasonability of the suggested models.

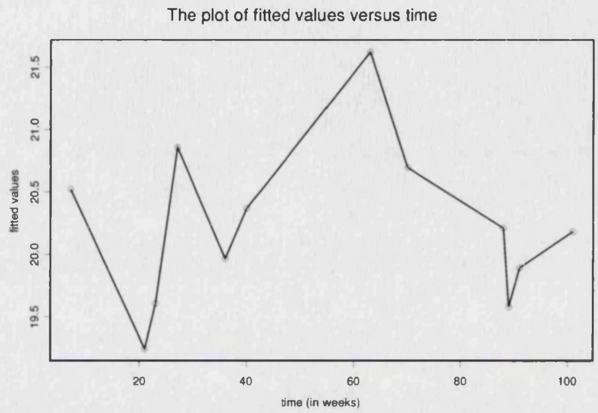

Figure 6.29: The plot of the fitted values versus time (in weeks) in risk zone 2 by the power kernel use in covariate models

Model Summary

In non-linear regression, the residual deviance is the main criteria to check the reasonability of the suggested models. This argument seems to be enough to validate the statistical applications in the non-linear and the generalised linear model analysis. The use of the penalised likelihood is suitable for more complex models and also depending on the interest of the research. Therefore, in the following tables, the residual deviance values, which are obtained from all of the models of Chapter 5 and Chapter 6 is gathered together to give an idea on how the use of the exponential and the power kernel functions affect the claim number N_i and the aggregate claims S_i models by risk zones and by time effect, either in months or weeks.

N_i	Months	Weeks
Exponential kernel	247.905 (Z1)-176.069 (Z2)	1048.398 (Z1)-691. 175 (Z2)
Power kernel	1237.057 (Z1)-239.165 (Z2)	4518.727 (Z1)-1286.833 (Z2)

Table 6.9: Comparison of the deviance values of **Chapter 5** N_i models for zone 1 (Z1) and zone 2 (Z2) and months and weeks based data

N_i	Months	Weeks
Exponential kernel	56.70 (Z1)-17.092 (Z2)	156.403 (Z1)-21.624 (Z2)
Power kernel	174.81 (Z1)-84.69 (Z2)	434.36 (Z1)-95.531 (Z2)

Table 6.10: Comparison of the deviance values of **Chapter 6** N_i models for zone 1 (Z1) and zone 2 (Z2) and months and weeks based data

Based on the deviance values, for the claim number (N_i) models, the power kernel function is especially effective for zone 2 analysis in both with and without covariate cases of Table 6.9 and Table 6.10. Although there are extreme event claims in zone 2, generally less number of observations are expected from this zone and the power kernel function use fits well to represent the idea behind the models. This use of the power kernel function is a kind of generalisation of the Gutenberg-Richter model of Section 4.2.

The aggregate claims (S_i) models express the aim of the modelling process well, when the exponential kernel is used in no- covariate case, Table 6.11. For the covariate case, it is indifferent to use the exponential or the power kernel functions in the required analysis.

S_i	Months	Weeks
Exponential kernel	5.35 (Z1)- 0.05 (Z2)	10.489 (Z1)- 0.788 (Z2)
Power kernel	1.34 (Z1)- 0.591 (Z2)	6.734 (Z1)- 0.1345 (Z2)

Table 6.11: Comparison of the deviance values of **Chapter 5** S_i models for zone 1 (Z1) and zone 2 (Z2) and months and weeks based data

S_i	Months	Weeks
Exponential kernel	1.35 (Z1)- 0.06 (Z2)	8.609 (Z1)- 0.143 (Z2)
Power kernel	0.46 (Z1)- 0.165 (Z2)	3.097 (Z1)- 0.03 (Z2)

Table 6.12: Comparison of the deviance values of **Chapter 6** S_i models for zone 1 (Z1) and zone 2 (Z2) and months and weeks based data

The comparison of using months or weeks based data indicates more or less equal preference. It depends on the main interest of research. If there are many claims arriving one after each other in a short period of time, e.g one year, then, it might be suggested to use the weeks based data. For our analysis here, months data seems like a slightly better choice to work with. The similar pattern of the jumps in claim arrivals and other variables are observed in both months and weeks cases.

Parameters	Zone 1	Zone 2	no. of claims	95 % CI
$\hat{\beta}$	0.37	0.325	Z1: 3602 / Z2: 676	Z1: (0.369, 0.372) Z2: (0.323, 0.327)
$\hat{\alpha_{18}}$	7.85	NA	Z1: 130 / Z2: 0	Z1: (7.821,7.939) Z2: NA
$\hat{\alpha_{19}}$	NA	5.79	Z1: 3 / Z2: 8	Z1: NA Z2: (5.73999,5.83001)
$\hat{\alpha_{22}}$	6.5	5.62	Z1: 132 / Z2: 46	Z1: (6.5199,6.5311) Z2: (5.609,5.631)
$\hat{\alpha_{25}}$	4.61	NA	Z1: 45 / Z2: 1	Z1: (4.599,4.6401) Z2: NA
$\hat{\alpha_{26}}$	5.268	7.99	Z1: 912 / Z2: 461	Z1: (5.2599,5.2701) Z2: (7.9899,8.0019)
$\hat{\alpha_{29}}$	3.51	NA	Z1: 19 / Z2: 2	Z1: (3.483,3.533) Z2: NA
$\hat{\alpha_{37}}$	8.01	7.47	Z1: 120 / Z2: 41	Z1: (7.999,8.0311) Z2: (7.4599,7.4901)
$\hat{\alpha_{38}}$	1.35	2.42	Z1: 32 / Z2: 58	Z1: (1.3289,1.37001) Z2: (2.3991,2.43001)
$\hat{\alpha_{40}}$	7.62	0.55	Z1: 1708 / Z2: 2	Z1: (7.58,7.6501) Z2: (0.5309,0.5601)
$\hat{\alpha_{41}}$	2.01	4.13	Z1: 423 / Z2: 37	Z1: (2.0099,2.0199) Z2: (4.1299,4.1401)

Table 6.13: Parameter summary of the selected Model 1

In Table 6.13, we present an example of the parameter values and corresponding confidence intervals for a selected model. Same procedure applies for all models. This table is compiled from the results of Model 1 in Table 5.2 for the claim number N_i. In Table 6.13, it is possible to follow the strike effect of the big earthquakes by α_j coefficients as aimed, where more claims arrive in the given time period. Same argument apply for all the other models.

NA values in Table 6.13 are used for the very low and no claim cases either in zone 1 or zone 2. Since the variance-covariance matrix has low values, the confidence interval for the parameters are observed to be narrow. Some α_j coefficients absorb the before and after event claims (or no claims), where one-sided kernel sits. For instance, if there are no claims until time 19 for zone 2 and then an earthquake occurs, the coefficient for time 19 is larger than the coefficient of time 22, although there occurs less claim at time 19 than time 22.

The magnitude M_i models

In this part, the magnitude M (in Richter Scale), is modelled with time as a covariate by using generalised linear models. The number of claims N_i is basically replaced with the magnitude just of an interest. Since the magnitude is already in logarithmic form by Gutenberg-Richter Law (See Section 4.2), the models can be expressed for the exponential and power kernel functions, respectively

$$M_{ij} = \alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+} + \epsilon_{ij},$$

and

$$M_{ij} = \alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta} + \epsilon_{ij},$$

where M_{ij} stands for the magnitude of observations in each corresponding bin and $i = 1, \ldots, n$, $j = 1, \ldots, k$. The error term has the same assumptions as in Page 183.

By months data:

The following table summarises the results of the magnitude models for zones 1 and 2 and by using the exponential and the power kernel functions with the use of months data.

Residual deviance	Exponential kernel	Power kernel
a- Zone 1	45.554 (df 21)	61.392 (df 21)
b- Zone 2	79.004 (df 23)	77.363 (df 24)

Table 6.14: The residual deviance values of the magnitude models for earthquake risk zones 1 and 2 in Turkey by months

By weeks data:

Table 6.15 shows the results of the analysis of similar argument of Table 6.14 for weeks based data.

Residual deviance	Exponential kernel	Power kernel
a- Zone 1	353.380 (df 124)	420.797 (df 123)
b- Zone 2	254.651 (df 91)	297.447 (df 92)

Table 6.15: The residual deviance values of the magnitude models for earthquake risk zones 1 and 2 in Turkey by weeks

The residual deviance values are lower for the months data in Table 6.14 than that of the weeks data in Table 6.15 when modelling the magnitude as a dependent variable on time. The pre-shock ordinary time period is observed to have some effect in the magnitude models and the models still pick the big earthquakes, which occur at the knots, in both cases. Generally, earthquakes of big magnitude are expected to occur in risk zone 1 classified areas. In Table 6.14, the lower deviance values are observed for zone 1 both with the use of the exponential and the power kernel functions. Conversely, in Table 6.15, in terms of weeks data, the higher deviance values come from zone 1. This might suggest that the choice of months is rather preferable time domain for the magnitude analysis. In a way, it can be interpreted as the magnitude overwrites the effect of time, when time itself is used as a covariate. At certain times, magnitude is the factor to cause a lot of claims. It is a well-known fact that magnitude is one of the key factors in most of the earthquake studies. The risk zoning map and the earthquake insurance premium calculations consider its effect in practice.

When the magnitude of the data is increased one degree, for instance magnitude of 5.4 is changed to magnitude 6.4 with 10 times more destructive effect ($\frac{10^{6.4}}{10^{5.4}} = 10^{6.4-5.4} = 10$), the following tables are observed for months and weeks data, respectively. The deviance values increase in both risk zones and for the use of the exponential and the power kernel functions. The significant earthquakes, which are parametrised by α_j coefficients, are still selected with their slightly higher values than the actual data magnitude. The lower deviance values are still observed with the use of the exponential kernel by months-based time. The use of the exponential kernel for zone 2 claims gives the minimum deviance among the other cases.

Residual deviance	Exponential kernel	Power kernel
a- Zone 1	56.563	73.894
b- Zone 2	95.304	93.207

Table 6.16: The residual deviance values of the magnitude models for earthquake risk zones 1 and 2 in Turkey by months

Residual deviance	Exponential kernel	Power kernel
a- Zone 1	428.785	505.049
b- Zone 2	308.375	358.761

Table 6.17: The residual deviance values of the magnitude models for earthquake risk zones 1 and 2 in Turkey by weeks

As a general conclusion on the claims modelling, the most reasonable way to express the models for the claim number N_i and the aggregate claims (total claim amount) S_i of the Turkish Catastrophe Insurance Pool data is to model the response variable by considering the effect of the magnitude and the number of residential buildings of the earthquake place. The suggested models of the thesis data are given below first with the use of the exponential kernel function

$$N_i \sim \text{Pois}(\Lambda_i),$$

where $\log \Lambda_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+} + m_l + r_l$. For the aggregate claims S_i

$$\log S_i \sim \text{N}(\mu_i, \sigma_i^2),$$

where $\mu_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+} + \log N_i + m_l + r_l$ for the use of the exponential kernel. Here m_l stands for the magnitude, r_l denotes the number of residential buildings, $i = 1, \ldots, n$, $j = 1, \ldots, k$ and $l = 1, \ldots, k$. These formulas change into the following with the use of the power kernel function as

$$\log \Lambda_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta} + m_l + r_l,$$

and

$$\log S_i \sim \text{N}(\mu_i, \sigma_i^2),$$

where $\mu_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta} + \log N_i + m_l + r_l$.

The following Tables 6.18 and 6.19 give the selection of the most reasonable models as a summary of the modelling chapters. Table 6.18 is for the number of claims N_i and Table 6.19 presents the total claim amount S_i of the same data.

Models	Chapter 5 (without covariate)	Chapter 6 (with covariate)
i) Exponential case-Zone 1	Model 1 - Table 5.2	Model 9 - Table 6.1
ii) Power case-Zone 2	Model 3 - Table 5.4	Model 11 - Table 6.3

Table 6.18: The final N_i models of the modelling process

Models	Chapter 5 (without covariate)	Chapter 6 (with covariate)
i) Exponential case-Zone 1	Model 5 - Table 5.6	Model 13 - Table 6.5
ii) Power case-Zone 2	Model 7 - Table 5.8	Model 15 - Table 6.7

Table 6.19: The final S_i models of the modelling process

It is a fact that the magnitude of an earthquake is the key factor to cause life losses and damage to the structures/infrastructures. However, if the building code applications (Turkish Building Code 1998) are strong enough, especially in disaster-prone countries, whatever the level of the magnitude is, the damage can be quite small. For instance, an earthquake of magnitude 7.0 causes few casualties and less damage in San Francisco; however, it ends up with thousands of deaths and a huge amount of monetary losses in any part of Turkey due to the lack of building code applications, unawareness of the public and authorities and lack of ex-ante and post-ante (mitigation/rescue/rehabilitation) facilities. Another point to notice is that the data of the Turkish Catastrophe Insurance Pool is just the claims of the residential buildings within the municipality borders. If the TCIP (the regulations are given in the next chapter) offers the mandatory earthquake insurance for the commercial facilities, the number of claims and the claim amount will change after an earthquake. The industrial plants buy insurance from the international sources. If the business premises buy earthquake insurance via the TCIP system, and since the pool reinsures itself, the reserves might increase by the profit. The suggested models can be calibrated with the updated data and the effect of the high number and high amount of the claims of industry can be observed on the parameters.

Chapter 7

Disaster Risk Management

An important instrument in the education of actuarial science is the 'Risk Theory' [Kaas et al., 2001]. Therefore, it is a good start to this chapter with some details of 'Risk', which is simply defined as Risk = Hazard $\times$ Vulnerability (see the Glossary). Risk management is a process of identifying and selecting the most appropriate techniques for treating loss exposures. The following are the basic steps of this process [Rejda, 2003]:

1. Identification of the potential losses,
2. Evaluation of the potential losses,
3. Selection of the appropriate techniques for treating loss exposures,

a) Risk control

i) Avoidance (e.g. Not marrying for not getting divorce.)

ii) Loss prevention (e.g. Take safe-driving courses and drive defensively.)

iii) Loss control

b) Risk financing

i) Retention (e.g. A risk manager can fail to identify all company assets, which could be damaged in an earthquake.)

ii) Noninsurance transfers (e.g. bonds, hedging)

iii) Commercial insurance

4. Implementation and administration of the program.

The suggested risk management techniques according to the loss severity and loss frequency (see the Glossary) are given below. Since the earthquakes are low frequency and high severity events, the best way to deal with their consequences is re/insurance applications.

Type of loss	Loss frequency	Loss severity	Risk Management Technique
1	Low	Low	Retention
2	High	Low	Loss prevention and retention
3	Low	High	Insurance
4	High	High	Avoidance

In terms of disaster risk management, the following chart can be suggested [Ibarra and Mechler, 2006]:

Risk identification (e.g. GIS systems, hazard maps)

↓

Risk reduction (e.g. building code application, early warning systems)

↓

Risk transfer (e.g insurance, cat bonds, micro-insurance)

7.1 Natural Hazard Insurance

The number of the catastrophic events has dramatically increased in the past decades. This increase can be named to result from the rise in world's population (demographic changes), the unbalance of nature, the Greenhouse effect (changes in the level of Chlorofluorocarbon-CFC- in the atmosphere), the climate change, the widening hole in the ozone layer, the loss of rain forests and the nuclear tests conducted by developed countries.

Countries suffer from different types of losses depending on the severity of the disaster. The loss figures can reach to huge amounts both in life and economical terms. The amount of the number of deaths is almost four times more in developing countries than that of the developed ones [Ibarra and Mechler, 2006]. The countries try to reduce the effect of disasters by some precautions. One of the suggested ways to deal with the possible economical losses due to natural disasters is to have a well-developed and effective insurance system (contingent credit, reserve funds, bonds are some other methods) within the context of the national disaster risk management program. Some devastating disasters made the countries pay more attention and develop disaster emergency plans for mainly by the help of insurance sector. For instance, the Great fire of London in 1666 made a significant improvement in the fire insurance sector of United Kingdom.

The economic impacts of disasters at the macro level are basically categorised as follows:

1. Direct effects (e.g. life losses, building damage, energy loss, communication lacks),
2. Indirect effects (e.g. wage losses due to life losses, tax changes),
3. Secondary (e.g. the change in GDP, export/import rates) effects.

Mitigation and risk financing are the main two mechanisms to reduce the net economic losses caused by the disasters. Physical vulnerability can be reduced by using mitigation and financial vulnerability can be reduced by risk financing. If abundant amount of capital can be kept in the reserved by the government during the ex-ante period, most of the direct losses can be covered with it and this helps to lessen the indirect and secondary effects of the disasters [Linnerooth-Bayer and Mechler, 2005].

Micro-finance (e.g. savings, investment, credit, insurance) is another useful tool in the speed up process of the post-disaster recovery. It is one of the methods that began to be used in the recent years as a part of the whole disaster risk reduction/mitigation process of the countries. Micro-finance also helps to reduce vulnerability and increase the capacity to deal with the disaster socio-economic shocks.

Micro insurance is a branch mechanism of micro-finance. Micro insurance does not need the involvement of the national governments and can be handled by the local governments and other institutions; whereas, macro insurance requires the step in of the national and local governments and sometimes the private insurers depending on the size of the damage.

EM-DAT

The Emergency Disaster Database (EM-DAT), which is directed by the Centre for Research on the Epidemiology of Disasters (CRED), is established in 1988 at the Catholic University of Louvain, Belgium. EM-DAT is the most complete publicly accessible international database with the core data on the occurrence and effects of 15,000 disasters, which occurred across the world from 1900 to present. The data is obtained from NGOs, UN agencies, insurance companies, media-press and research centres. It is possible to obtain the human and economic losses via EM-DAT [IFRC, 2005].

The following conditions are required to enter disaster data into EM-DAT system [IFRC, 2005]:

1. Ten or more people recorded killed (i.e. confirmed dead and missing people),
2. 100 people reported affected (i.e. homeless, people with urgent survival needs),
3. Declaration of a state of emergency,
4. Call for international assistance.

The following tables are prepared from the EM-DAT, CRED, University of Louvain sources and give information on the number of natural disasters, the number of people killed and the estimated damage by continent and by year due to disasters [IFRC, 2005].

	2000	2001	2002	2003	2004	Total
Africa	196	186	201	163	161	907
Americas	147	128	153	119	131	678
Asia	289	295	301	264	313	1,462
Europe	122	92	111	85	93	503
Oceania	13	18	18	20	21	90
Total	767	719	784	651	719	3,640

Table 7.1: Total number of reported disasters from 2000 until 2004. *Source:* EM-DAT, CRED, University of Louvain, Belgium.

The high figures in the following Tables 7.2 and 7.3 are due to the Indonesia tsunami disaster, which occurred in December 2004. Generally, most of the life (60 % of the world's life losses) and economical losses (50 % of the world's total economical disaster losses) happen in the Asia continent as a result of the densely populated (40 % of the world's population), developing or undeveloped, disaster-prone countries of the region.

	2000	2001	2002	2003	2004	Total
Africa	5,756	4,462	8,272	5,810	4,308	28,608
Americas	1,820	3,460	2,285	2,026	8,269	17,860
Asia	11,608	29,255	13,358	37,860	236,102	328,183
Europe	1,627	2,196	1,699	31,046	1,182	37,750
Oceania	205	9	91	64	35	404
Total	21,016	39,382	25,705	76,806	249,896	413,572

Table 7.2: Total number of people killed due to disasters from 2000 until 2004. *Source:* EM-DAT, CRED, University of Louvain, Belgium.

	2000	2001	2002	2003	2004	Total
Africa	164	339	144	5,684	1480	7,811
Americas	3,588	11,090	3,547	14,272	27,891	60,388
Asia	18,074	17,160	7,853	17,766	67,395	128,248
Europe	9,070	895	17,081	19,108	2,028	48,182
Oceania	553	364	410	617	516	2,460
Total	31,449	29,848	29,035	57,447	99,310	247,089

Table 7.3: Total amount of disaster estimated damage in millions of USD (2004 prices) from 2000 until 2004. *Source:* EM-DAT, CRED, University of Louvain, Belgium.

Figure 7.1 shows the economical consequences of the different types of natural disasters between 1975-2005.

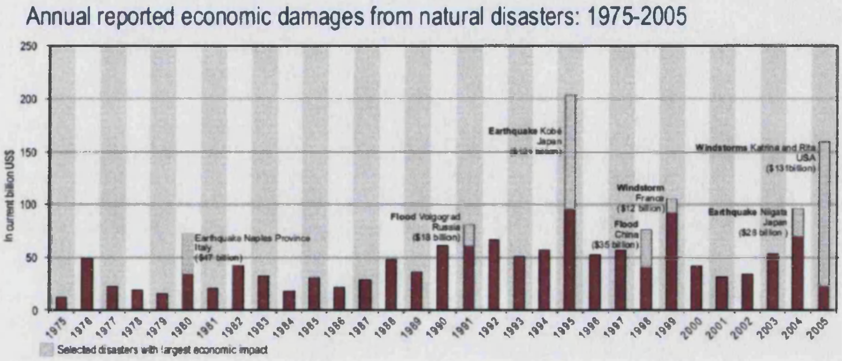

Figure 7.1: Annual reported economic damages from natural disasters: 1975-2005. Source: EMDAT

The effect of natural disasters on the whole economy and disaster insurance sector between 1950 and 2002 is given in Figure 7.2.

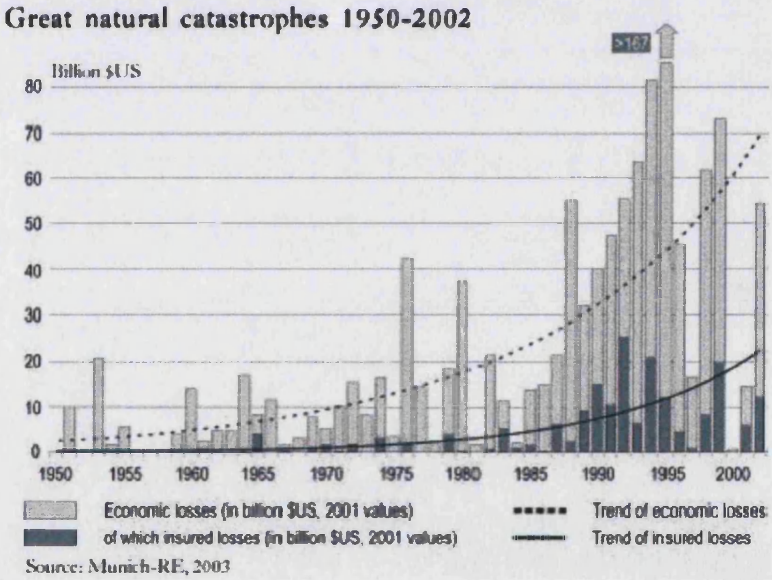

Figure 7.2: Great natural catastrophes 1950-2002.

The Application of Natural Disaster Management Programs in Some Countries

Sudden and widespread events like earthquakes, windstorms (hurricanes, cyclones and typhoons), floods, droughts, famines, fires, avalanche, chemical and nuclear accidents, epidemics, volcanic eruptions cause loss of human life and damage to the social and economical systems. Earthquakes capture the largest portion among those disasters by 19 %. The most disaster vulnerable countries in the world, considering the yearly total number of disasters are ranked as Colombia, China and the Philippines.

There are different disaster management programmes in application in some countries, which face different types of disasters. For instance [ulk, 2000]:

1. Japan is prone to earthquakes and reinsurance (Japanese Earthquake Reinsurance Company - JER) is an important tool to response to the effects of quakes. JER was established in 1966 against earthquake, tsunami and volcanic damage risks [Gurenko et al., 2006],

2. California (California Earthquake Authority - CEA- established in 1996 /earthquake risk coverage only) and New Zealand (Earthquake and War Damage Commission - founded after 1944) also face with earthquakes frequently due to the active faults. After the 17/August/1999 Marmara earthquake, Turkey has developed a programme called the 'Turkish Catastrophe Insurance Pool - TCIP', in which the systems of the CEA and New Zealand Commission are combined as basis,

3. Hurricanes are the most frequent disasters in Hawaii (Hurricane Relief Fund) and Florida (Joint Underwriting Association - JUA, Hurricane Catastrophe Fund),

4. France has a system in use, Caisse Central de Reassurance, to mitigate the effects of the disasters, especially floods (It is estimated that there is a severe flood in Paris in every 100 years.). Also, there is Catastrophe Naturalles (CatNat) system established in 1982 to cover all disasters except windstorm, ice and snow [Gurenko et al., 2006],

5. In Spain, there is Consorcio de Compensacion de Segures, which is established in 1954, to cope with the effects of earthquakes, tidal waves, floods, volcanic eruptions and cyclonic storms [Gurenko et al., 2006],

6. In Northern Ireland, governmental insurance is implied against terrorist acts,

7. There were two big explosions in London in 1993. After these, a program to cover terrorism attacks is in operation in England. This program is reinsured by the Pool Re, which can borrow from the Bank of England when it is out of its resources. Moreover, the 'Thames Barrier' in London is the world's largest movable flood barrier that was designed to protect the capital from floods until at least year 2030,

8. Taiwan has an earthquake risk coverage system, which is established in 2002 and called Taiwan Residential Earthquake Insurance Pool (TREIP).

7.1.1 Earthquake Insurance

The 1906 San Francisco earthquake initiated the important implications of earthquake insurance. Since the occurrence, frequency and damage of the earthquakes are not predictable, it is not in the same category with life, automobile or health insurance for the insurance companies. Some countries have direct earthquake insurance schemes, whereas some offers the earthquake coverage in addition to the fire insurance policies.

Today, it is possible to estimate losses of earthquakes up to a degree. The magnitude of an earthquake, the depth of the epicentre, duration of the earthquake, the distance of the settlement area to the focus of the earthquake and some ground structure help scientists to calculate the loss ratio (the ratio of incurred claims to the earned premiums). Beside all of these factors, the earthquake zone maps, the location, the height and the age of the buildings, the historical earthquake records help insurance companies to determine the premium, the amount that the insured pays to the insurer for the earthquake risk it carries, of the earthquake insurance. The premium also changes whether the insured object is a household or commercial building.

Applications of Earthquake Insurance in Some Countries

The United States of America

In U.S.A, private companies provide policies for earthquakes, tsunamis, landslides and other kinds of natural hazards. California is in a very active seismic zone. Therefore, in 1984, by a state law, it was required that insurance companies add earthquake coverage to the policies. However, by that time 80 % of the homeowners did not have such insurance probably due to the substantial deductible amount. Later in 1995, the California Earthquake Authority (CEA) was established to spread and operate earthquake insurance in California State. The CEA depends on the insurance market financially but it is managed by the public. It was launched in December 1996 under the partnership of thirteen Californian insurance companies. The revenue of CEA investment is exempted from taxes. It has a reserve of USD 7.2b. The insurance coverage is up to USD 100,000 for rebuilding and repair of a house, USD 5,000 for its contents and USD 1,500 for other living expenses [Bolt, 1988].

There is also a National Flood Insurance Program of the U.S.A. managed by Federal Emergency Management Agency (FEMA), Federal Insurance Administration and Mitigation Directorate. The program includes the standard flood insurance policy with buildings and contents on separate policies. It also covers the increased cost of compliance coverage and preferred risk policy premiums.

Japan

The insurance sector is highly developed in Japan and the country is ranked the first for life insurance applications all over the world. The seismic risk is also high in Japan, like in California. Earthquake insurance became very important after the 1966 Niigata earthquake and has been revised after the 1995 Kobe earthquake. The islands of Japan are divided into twelve zones according to the degree of the earthquake hazard. Zone 5, including Tokyo, Kanagawa, Chiba and Yokohama, is classified as one of the most risky areas.

Buildings and their contents are included in the earthquake insurance. The implications are divided into two groups: the personal and the industrial risks. The private insurance companies cover a big amount of the personal risks. The rest of those risks are under the responsibility of Japan Earthquake Reinsurance Company (JER). On the other hand, the private sector offers a full coverage of the industrial risks by transferring some amount to the international reinsurance companies [Bolt, 1988].

New Zealand

Following the 1944 earthquake, the companies added the compulsory earthquake coverage to the fire policies in 1946. Later on, all insurance policies included this earthquake coverage. Today, there is a Natural Disaster Fund in New Zealand with a reserve of NZD 3.2b under the administration of Earthquake Commission. The policies cover up to the damage of NZD 100,000 for homes and NZD 20,000 for the contents [Bolt, 1988].

France

Caisse Centrale de Reassurance (CCR) is the main state-owned authority and reassurer. All of the natural disasters were included in policies by a law of 1982, after the flooding of the River Seine in Paris in 1981. The natural disasters only have a cover under the declaration of the Inter-Ministrial Commission. The coverage is available by the original fire policy.

Other Countries

United Kingdom experienced small earthquakes in some areas throughout its history (see the Appendix). The earthquake coverage is available under some policies within the overall rate. Spain and Switzerland have compulsory earthquake coverage in the policies. In Italy and Portugal, it is included in the fire policies, whereas in Greece it is separate from fire insurance. In the Philippines, there is a very wide and effective use of the Crop Insurance. The country is also in the process of developing a similar system to the Turkish Catastrophe Insurance Pool (TCIP), which is planned to be names as the Philippines Catastrophe Insurance Pool (PCIP).

Colombia, Ethiopia, India, Mexico, Sri-Lanka, Nepal, El Salvador and Honduras are some countries with applications of different types of mitigation/recovery financial strategies (e.g. micro finance, weather derivatives, contingent credit, cat bonds, index-based weather insurance) for different types of disasters striking those. In Canada, people have earthquake insurance under the fire policies, which differs from zone to zone. Some regions of Canada are highly prone to earthquakes (West Coast), some have moderate risk (Eastern Canada), some have little risk (Southern Prairies) and some virtually none (Canadian Shield). In countries without seismic risk, like Belgium, Germany, the Netherlands, Denmark, Sweden and Norway, the earthquake coverage is not available [Bolt, 1988].

7.2 The Insurance System in Turkey

The insurance business was managed by the international agencies, especially after the 1870 Beyoğlu fire. The first national insurance company 'Ottoman General Insurance Company' was founded in 1916 during the Ottoman Empire and the foundation of the others followed. Türkiye İş Bankasi was established in 1924. Then in 1925 Anadolu Sigorta A.Ş (Anadolu Insurance), which is the insurance group of Türkiye İş Bankasi, was launched. In the following years, there was a decline in the number of insurance companies, which caused a stop-period in the sector as a result of the implication of the development programmes of the government. Some improvements started in the insurance market by the law dated 1987. The life and non-life insurance separation of the policies started after 1991. Today, there are about sixty seven insurance and reinsurance companies in total in Turkey, most of which are privately owned. Those companies have to be a member of the 'Insurance and Reinsurance Companies' Association', which links the government and insurance industry for the development of insurance and solidarity between companies. Insurance Supervisory Office as a part of the State Ministry supervises insurance in Turkey. The Prime Ministry Undersecretariat of the Treasury has a general insurance directorate, which regulates insurance system of the country.

The legal system in Turkey for disasters consists of the Law of Disasters No.7269, the Development Law No.3194 and the Tender Law No.2886. The Law of Disasters mainly deals with post-ante events and has a little part on disaster mitigation. The Development Law defines the plans to follow in disaster preparedness. The Tender Law states the conditions for the implementation of the public construction projects. These laws are being reviewed to establish a strong National Disaster Risk Management Program and to make the existing Building Code of 1998 applied more efficiently. Decree Law No.587 officialized the compulsory earthquake insurance on 27/December/1999. This law explains the roles of the General Directorate of Insurance at the Treasury, the TCIP board and the operational manager [Gurenko et al., 2006].

The earthquake coverage was added to the fire policies with the demand of the insured between 1904 and 1929. After 1939 Erzincan earthquake, the earthquake risk was put out of that coverage and replaced back during 60's. More than 50 % (665,000) of the fire policies (1,240,000), which were sold in 1998, contained an earthquake coverage. This coverage requires the insured to pay 20 % of the loss and 5 % deductible amount over the price of the insured-value, the insurer pays the 80 % of the loss as coinsurance.

7.2.1 The Turkish Catastrophe Insurance Pool (TCIP)

As mentioned earlier, before the Marmara earthquake, fire insurance policies used to cover the earthquake risk in Turkey. The Prime Ministry Undersecretariat of the Treasury started the construction of the insurance pool titled the 'Turkish Catastrophe Insurance Pool (TCIP)', which is established with the basis of the systems of California (California Earthquake Authority - CEA) and New Zealand (Earthquake and War Damage Commission - founded after 1944), as a conclusion of studies for the necessity of an earthquake insurance. The Turkish Catastrophe Insurance Pool started operation on 27/September/2000.

The aim of the TCIP is to transfer the national risk to world-wide risk sharing pools under the management of the international reinsurance companies. The substantial capital resources support the TCIP. The plan is to have an earthquake coverage of USD 30.000 per housing unit. In the TCIP contracts, the deductible amount is 2 % of the losses. The losses, which exceed this threshold, is covered by the TCIP.

As well as the idea of the importance of an earthquake insurance, the TCIP will also play a significant role in the control of the use of the necessary building codes during the construction, since it is required by the reinsurers [Erdik, 2000, Bibbee et al., 2000]. The use and the control of the current Building Code 1998 is one of the main problems in terms of disaster management in Turkey.

The damage and losses due to the following hazards are in the coverage of the Turkish Catastrophe Insurance Pool:

1. Earthquakes,

2. Fires due to an earthquake,

3. Explosions as a result of an earthquake,

4. Landslides following an earthquake.

There are five different earthquake risk zones in Turkey according to the classification by the General Directorate of the Disaster Affairs. The aim is to provide minimum amount standard insurance for residents living in those areas. The mandatory earthquake insurance only covers the losses of the residential buildings within the municipality borders. It does not offer any coverage for the rural areas or for the building contents. House-aimed built buildings can be used as offices afterwards and can be covered with the earthquake insurance but office-built buildings are excluded from the coverage. This urban coverage is one of the obstacles of the TCIP since the ideal prototype insurance should be able to reach all parts of the community, that is including the poor and the vulnerable areas.

The TCIP is directed by a board of members, who represent government, academia and insurance companies. The administrative power is the General Directorate of Insurance, Prime Ministry Undersecretariat of Turkey. The operational management of the Pool is contracted out for five year period. The national reinsurance company 'Milli Reassurance (Milli Re.Ltd.)' was the Pool Manager from the establishment until August 2005 and since then Garanti Insurance took it over. There are 32 authorised insurance companies to sell TCIP policies by March 2005 figures [Gurenko et al., 2006]. The following chart shows the marketing process of the mandatory earthquake insurance scheme [Gurenko et al., 2006]:

TCIP
↓
Insurance companies
↓
Agents
↓
Homeowners

There are fifteen tariffs in the TCIP, which are calculated according to five earthquake risk zones and three types of buildings. Table 7.4 gives the yearly premium rates decided by the Turkish Catastrophe Insurance Pool. These values are decided by international advisors during the foundation period of the TCIP. Then it is revised by the administration of the TCIP according to the changes in the cost-value of the dwellings each year. The methodology to obtain these tariff prices in Table 7.4 is not given by the data providers for confidential reasons.

Building type	Zone 1 (%)	Zone 2 (%)	Zone 3 (%)	Zone 4 (%)	Zone 5 (%)
Steel, reinforced concrete	2.20	1.55	0.83	0.55	0.44
Masonry	3.85	2.75	1.43	0.60	0.50
Other	5.50	3.53	1.76	0.78	0.98

Table 7.4: The yearly premium rates of the TCIP by earthquake risk zone and building type [TCIP, 2006].

The insured value of a property is decided every year by $\Big((\text{cost per square meter} \times \text{gross area of the house/flat}) \times \text{tariff price}\Big)$. This value changes every year according to the increase in the construction cost of a property. The maximum coverage of the TCIP is 100,000 YTL for each house/flat.

Place	Risk Zone	Yearly premium (YTL)
Ankara-centre	4	20.90
Artvin-centre	3	31.54
Bingöl-centre	1	83.60
Erzurum-centre	2	58.90
Karaman-centre	5	16.72

Table 7.5: The yearly premium rates of the compulsory earthquake insurance for a $100m^2$ reinforced concrete flat in Turkey in five risk zones obtained from [TCIP, 2006].

Table 7.5 gives an example of a yearly mandatory earthquake insurance premium amount to be paid by the insured for $100m^2$ reinforced concrete flat, which has an insured value of 38,000 YTL. These values in Table 7.5 are calculated by using the rates for reinforced concrete building type in Table 7.4 (e.g. for risk zone 1, Bingöl-centre, the premium to be paid in 1-year is: $38,000(2.20\%) = 83.60$).

The TCIP aims to pay back the claims latest in one month time after the claim is made. The experts of the TCIP arrive to the earthquake area just after an occurrence of an earthquake and calculate the initial expected losses of the earthquake insurance holders. The experts pay the insurance holders some money in advance without any limitation (level of damage, type of the building etc.), so that the affected people can provide some of their urgent needs. The payouts to the policyholders in the TCIP scheme is given in the Marmara Earthquake Emergency Construction Project (MEER) as

$$\text{Paid claims} = \left(\frac{\text{Maximum sum insured}}{\text{Property Value at time of disaster}}\right) \times \text{Losses}.$$

Next, Table 7.6 gives the number of the earthquake claims reported to the TCIP and the payments made for these claims in terms of years by the figures of 26/October/2006.

Year	The number of earthquakes	The number of claims	The total payment in YTL
2000	1	6	23,022
2001	17	338	127,497
2002	21	1558	2,284,835
2003	20	2504	5,203,990
2004	31	586	768,692
2005	39	3448	7,970,223
2006	13	368	1,089,633
Total	142	8808	17,467,892

Table 7.6: The claim and payment information of the TCIP [TCIP, 2006].

There are tremors (earthquake of magnitude < 4) everyday in different parts of Turkey. Even slight damages, which are caused by these tremors, are reported to the TCIP and the claim payment data is updated regularly. The TCIP reassured USD 540m of its risk to A-level and above rated international reassurers in 2001, that of USD 840m in 2002 and USD 740m in 2003. In 2006, the reinsured amount of the pool is EUR 920m ($\sim$ USD 1.1b). By the 2005 Figures, the payment capacity of the TCIP is approximately EUR 1.1b ($\sim$ USD 1.33b). In case of a disaster strike, the payment sources of the TCIP are the collected premiums in the pool, the reinsurance back-up and a special fund from the World Bank. This fund will cover the 100 % of the claims, which the Pool and the international reinsurance will not be able to cover.

The culture of insurance is still far from many parts of the country. The authorities are trying to change this situation by education, seminars, use of media. It will be a continuous process for years until the importance of a disaster insurance settles down. It is interesting to observe that every car owner has the compulsory traffic insurance in Turkey. They care about their cars more than they care about their homes.

The contract period for the compulsory earthquake insurance is 1-year and many householders do not renew their contracts when it is terminated, if they did not experience an earthquake during the contract year. When they do not renew their contracts, although no earthquake occurred during the contract period, they can not get any money back as they have received the service by the TCIP. It is interesting to observe that if an earthquake occurs, the next day there is an increase in the number of applications to buy the mandatory earthquake insurance policies from the agencies.

Next, Table 7.7 gives the number of buildings and the number of policies in 2004, 2005 and 2006 according to the administrative regions of Turkey by the figures of 24/May/2006. Unfortunately, there is still not enough number of the compulsory earthquake insurance holders as originally planned at the beginning of the establishment of the TCIP. The penetration rate in Turkey as of 01/February/2005 is 16.51 %, where most of the policies are sold in the Marmara region [Gurenko et al., 2006]. By the figures of 24/May/2006:

Region	The no. of residential buildings	The no. of policies in 2004	The no. of policies in 2005	The no. of policies in 2006
Marmara	4.143.469	1.085.630	1.074.603	1.146.142
Mid Anatolia	2.227.056	363.529	366.519	445.546
Aegean	2.318.262	347.676	362.134	460.823
Mediterranean	1.663.126	136.558	147.944	202.543
Black Sea	1.282.098	104.435	111.653	167.343
East Anatolia	611.788	42.105	47.181	59.320
South East Anatolia	742.866	29.329	31.596	50.317
Total	12.988.665	2.109.262	2.141.630	2.532.034

Table 7.7: The comparison of the number of buildings and the number of policies by 2004-2005-2006 according to the TCIP [TCIP, 2006].

The worst fear in the country is to experience the famous severe Istanbul earthquake, which is expected at any time in the future. It can hit in one hour, tomorrow, next week or next year. Some people neglect to buy the earthquake insurance although they are aware of the risk. The regulators of the TCIP is planning to start an automatic insurance system, where the premiums will be collected by adding some amount in the utility bills, as a solution suggestion for this negligence. Moreover, some residents of Istanbul can not afford even low premium rates and again the TCIP regulators are trying to improve the system by finding sources to provide insurance for the poor by the help of the government.

7.2.2 Financial estimation

The calculation of the expected total claim amount (aggregate mean), $E(S(t))$, in the portfolio of risks (the Turkish Catastrophe Insurance Pool in our case) due to the catastrophic events is a high interest of the actuaries in the recent years. The aggregate mean, $E(S(t))$, is both used in determining the sufficient amount of the reserves to keep the surplus (free reserves) of the company in positive (see (3.8)) and in the calculation of the net premium (aka pure premium). The number of claims (aka claim frequency) N_i (N_i notation is used for binning case) and the claim amount (aka claim severity) X_i are the key elements in the computation of the aggregate claims (total claim amount) S_i of the portfolio. The relation between the response variables, which are used in the modelling chapters, and the aggregate mean can be summarised with the following chart:

$$\boxed{\text{The number of claims } (N_i) \text{ \& The claim amount } (X_i)}$$
$$\downarrow$$
$$\boxed{\text{The total claim amount } (S(t))}$$
$$\downarrow$$
$$\boxed{\text{Net (Pure) premium } (E(S(t)))}$$

The results of the mathematical models of Chapter 4 and the parameter estimates of β and α_j's from the claim models of Chapters 5 and 6 are used in the scenarios of this Chapter to estimate the aggregate mean $E(S(t))$, the aggregate variance $Var(S(t))$, the standard deviation, the skewness and the relative variance of the Turkish Catastrophe Insurance Pool claims data. Here one should be reminded of the notation difference between $S(t)$ and S_i, where S_i refers to the aggregate claims in the corresponding bin i of the bin case. Also, X_i refers to the raw claim amount of the bin case.

The rate of an inhomogeneous claim number process $N(t)$ (see **Remark 2**) is $\Lambda(t)$ (in non-binning case) and the intensity of the aggregate claims S_i is the mean μ_i (binning case). The two main assumptions, which are used throughout the thesis work, are the independency of the claim number process $N(t)$ and the raw claim amount X_i and that the raw X_i's being independent identically distributed (iid). The dependency of the claim number and the aggregate claims (or the individual claim amount) can be studied as a part of future work by using the idea of copula functions (see the Appendix for basic definition of copula).

After using the binning approach, the estimates of the two intensities (Λ_i, μ_i) are needed to compute the aggregate mean $E(S(t))$, which is same as the necessary free reserve amount of the pool or the net (pure) premium to charge for the portfolio. It should be noted that the external effects like the inflation, interest rate, unemployment rate, other economical factors (seasonal trends etc.) and the management expenses (office equipment, cost of personnel) are excluded for the calculations of the required premium amounts [Booth et al., 1999].

The special kernel functions are chosen as the exponential and the power kernel functions to represent the exponential decay of the claim arrivals following an earthquake strike in a given time period. The necessary Λ_i and μ_i values are estimated by using the maximum likelihood estimates ($\hat{\alpha_0}$, $\hat{\beta}$, $\hat{\alpha_j}$) of the suggested generalised linear models. The substitution of the related estimates in the exponential kernel function suggests the following by using the fact in (5.3) and also (5.6)

$$\hat{\Lambda_i} = e^{\hat{\alpha}_0 + \sum_{j=1}^{k} \hat{\alpha}_j e^{-\hat{\beta}(t_i - s_j)|_+}},$$

and

$$\hat{\mu_i} = \hat{\alpha}_0 + \sum_{j=1}^{k} \hat{\alpha}_j e^{-\hat{\beta}(t_i - s_j)|_+} + \log(N_i),$$

and the use of the power kernel function results in

$$\hat{\Lambda_i} = e^{\hat{\alpha}_0 + \sum_{j=1}^{k} \hat{\alpha}_j (t_i - s_j)|_+^{-\hat{\beta}}},$$

and

$$\hat{\mu_i} = \hat{\alpha}_0 + \sum_{j=1}^{k} \hat{\alpha}_j (t_i - s_j)|_+^{-\hat{\beta}} + \log(N_i).$$

The choice of the exponential and the power kernel functions are compatible with the idea of the decreasing function since they are aimed to represent the claim decays as a result of the high jumps of the earthquake claims.

In Section 3.2, the total claim amount process (the aggregate claims) is given as $S(t) = \sum_{i=1}^{N(t)} X_i$, which is also called the 'collective risk model' in the actuarial context since the process is the sum of the claim amount from each policy in a certain time period. The claim number process $N(t)$ (an inhomogeneous Poisson process) and the claim amount process $X(t)$ are stochastic variables changing over time. As mentioned earlier, the necessary assumptions for the aggregate claims model are the independency of the $N(t)$ and raw claim amount X_i (mean is η_i and variance is τ_i) and the independent identically distributed X_i's. Then, the aggregate mean $E(S(t))$ and the aggregate variance $Var(S(t))$ are obtained by using

$$E(S(t)) = E(N(t))E(X(t)) = \Lambda(t)\eta_i,$$

and

$$Var(S(t)) = E(N(t))Var(X(t)) + Var(N(t))E(X(t)) = \Lambda(t)(\eta_i^2 + \tau_i),$$

where η_i and τ_i are the parameters of the chosen X distribution and the mean μ_i of the aggregate claims is estimated by using the related maximum likelihood estimates of the modelling chapters as given in the equations above. The raw variance is used in the further calculations, which is obtained from the data itself.

The rate of the whole process is $\Lambda_i = \Lambda_1(t) + \ldots + \Lambda_n$ because the claim number process $N(t)$ is independently Poisson distributed over each bin so $N_i \sim$ Pois(Λ_i). The integrals should be evaluated over the fixed time interval of $[0, 43]$ when analysing the months case and $[0, 138]$ or $[0, 101]$ for the weeks case of zone 1 and zone 2 claims data respectively according to the coding of the time variable in this study. The rough estimate of the mean intensity $\Lambda(t)$ can be calculated as

$$\hat{\Lambda(t)} = \frac{\text{The total number of claims up to time}}{\text{total time}} = \frac{4297}{32 \text{ (in terms of months)}} = 134,$$

which can be interpreted that each month approximately 134 claims arrived to the Turkish Catastrophe Insurance Pool between December 2000 and July 2003. Or in terms of weeks

$$\hat{\Lambda(t)} = \frac{\text{The total number of claims up to time}}{\text{total time}} = \frac{4297}{138 \text{ (in terms of weeks)}} = 31.14.$$

The same estimation procedure above, except the exponentiation of the log-linear rate for the claim number counts, applies for the computation of the mean μ_i of the aggregate claims S_i with the use of the exponential kernel as

$$\hat{\mu_i} = \hat{\alpha}_0 + \sum_{j=1}^{k} \hat{\alpha}_j e^{-\hat{\beta}(t_i - s_j)|_+} + \log(N_i), \tag{7.1}$$

and for the power kernel function

$$\hat{\mu_i} = \hat{\alpha}_0 + \sum_{j=1}^{k} \hat{\alpha}_j (t_i - s_j)|_+^{-\hat{\beta}} + \log(N_i). \tag{7.2}$$

Since the distribution of the aggregate claims S_i is chosen as lognormal in our case, the mean and the variance of the lognormal S_i are as in (5.1) and (5.2), respectively

$$E(S) = e^{\mu_i + \frac{\sigma_i^2}{2}}, \tag{7.3}$$

and

$$Var(S) = e^{2\mu_i + \sigma_i^2}(e^{\sigma_i^2} - 1), \tag{7.4}$$

where the value of the intensity μ_i is estimated in (7.3) and (7.4) with the use of the exponential and the power kernel functions by the binning approach. The total claim amount model S_i is based on $\log S_i$ values, where $\log S_i \sim$ Normal (see Figure 5.1), and the size of the claim due to the intensity of the magnitude is a main factor to affect the claim amount modelling.

The claim number process $N(t)$ and the claim amount process $X(t)$ are both stochastic processes along the given time interval $(0, t]$. If the observation time t is fixed, the state $X(t)$ of the claim amount process $X(t)$ is a random variable in the given time interval with a distribution function $F_t(X)$. At each time point t, the stochastic process X is uniquely determined with the distribution function

F. The values of the states $X(t)$, meaning each claim amount corresponding to each time point, at different times t $(t_1, \dots, t_n)$ are time-dependent (correlated) [Daykin et al., 1994b].

When $N(t)$ and $X(t)$ are both stochastic processes, the aggregate claim amount $S(t)$ is considered as a doubly-stochastic (Cox) process. However, in our case, the rate $\Lambda(t)$ (as an approximation of $\lambda(t)$) is not random itself. That means the claims only occur as a result of one type of event, in this case an earthquake. If the claims would come from different types of disasters then the intensity $\Lambda(t)$ would be random and the whole process would be doubly-stochastic [Albrecher and Asmussen, 2005]. With this argument, although each claim amount is time-dependent itself, the computation of the Compound Poisson aggregate claim process $S(t)$ uses the calculation of the $\mu_i(t)$'s as if the X_i's are independent of the distribution of time.

The relation between the aggregate mean and variance of the total claim amount $S(t)$ and its cumulant values are given in Section 3.3.1 in Chapter 3, which states that

$$E(S(t)) = \kappa_1^{(s)} = \Lambda(t)\mu_1 = \Lambda(t)\eta, \tag{7.5}$$

$$Var(S(t)) = \kappa_2^{(s)} = \Lambda(t)\mu_2 = \Lambda(t)(\eta^2 + \tau). \tag{7.6}$$

In Statistics, the mean and the variance are the measures for central location and dispersion, respectively. There is another measure of the data, 'skewness', which is an indicator of deviation from a fully symmetric shape [Daykin et al., 1994b]. The skewness is calculated as

$$\gamma(S(t)) = \frac{\kappa_3^{(s)}}{\sigma^3} = \frac{\Lambda(t)\mu_3}{(\Lambda(t)\mu_2)^{\frac{3}{2}}}.$$

If the skewness of a random variable is greater than 0, large values of the difference of the random variable and its mean, say $X - \mu$, are likely to occur, so the right tail of the distribution function is heavy. A negative skewness indicates a heavy left tail and if the random variable X is symmetric then skewness is 0, but 0 value of skewness does not indicate symmetry [Kaas et al., 2001].

The lognormal distribution choice for the aggregate claims S_i in our case has all moments in order [Asmussen, 2000]. As stated in (7.3), the first moment is the mean (here we drop the the sub-index i to express the binning case not to cause confusion, originally μ_i when binned)

$$E(S) = e^{\mu + \frac{\sigma^2}{2}},$$

and the second moment is [Asmussen, 2000]

$$E(S^2) = e^{2\mu + 2\sigma^2}.$$

Then the variance (see (7.4)) of the lognormal S_i is

$$Var(S) = E(S^2) - (E(S))^2 = e^{2\mu+2\sigma^2} - (e^{\mu+\frac{\sigma^2}{2}})^2 = e^{2\mu+\sigma^2}(e^{\sigma^2} - 1).$$

The σ^2 here is the raw variance of the log aggregate claims data and μ is obtained from the kernel functions, where the model estimates are substituted.

The use of the moment generating function of the Normal distribution is another way to obtain the mean and the variance of the lognormal distribution since it is known that if $\log S \sim$ Normal then $S \sim$ lognormal. The moment generating function of the Normal distribution with parameters μ and σ^2 is given in Section 3.3.1 as in the form of $e^{\mu\vartheta + \frac{\sigma^2\vartheta^2}{2}}$. By using this moment generating function, the mgf of the lognormal distribution is obtained as

$$Y = \log S \rightarrow M_Y(\vartheta) = E(e^{\vartheta y}) = e^{\mu\vartheta + \frac{\sigma^2\vartheta^2}{2}},$$

then by exponentiating

$$M_Y(\vartheta) = E(e^{\vartheta \ln s}) = E(e^{\ln s^{\vartheta}}) = E(S^{\vartheta}) = e^{\mu\vartheta + \frac{\sigma^2\vartheta^2}{2}}.$$

Substituting $\vartheta = 1$ gives the mean of the lognormal distribution as

$$E(S) = e^{\mu + \frac{\sigma^2}{2}},$$

and the second moment is equal to $\vartheta = 2$, that is

$$E(S^2) = e^{2\mu + \frac{4\sigma^2}{2}},$$

then the variance of the lognormal distribution is

$$Var(S) = E(S^2) - (E(S))^2 = e^{2\mu+\frac{\sigma^2 4}{2}} - (e^{\mu+\frac{\sigma^2}{2}})^2 = e^{2\mu+\sigma^2}(e^{\sigma^2} - 1).$$

Another measure of the data, the skewness of the lognormal distribution is given in [Bühlmann, 1970] as

$$\gamma_S = \frac{E(S-E[S])^3}{\sigma^3} = (e^{\sigma^2}+2)\sqrt{e^{\sigma^2}-1}.$$

The third central moment of the lognormal distribution can be obtained by using the following skewness formula

$$\begin{aligned}\gamma_S &= \frac{E(S-E[S])^3}{\sigma^3} = \frac{E(S^3)-3\mu E(S^2)}{\sigma^3}\\ &= (e^{\sigma^2}+2)\sqrt{e^{\sigma^2}-1},\end{aligned} \tag{7.7}$$

and previously given mean and variance values. If the necessary simplification is done in (7.7) and it is solved for $E(S^3)$, the third central moment of the lognormal distribution is derived as

$$E(S^3) = 3(e^{\sigma^2}+2)\sqrt{e^{\sigma^2}-1} + (3\mu e^{2\mu+2\sigma^2}).$$

Then the skewness of the Compound Poisson $S(t)$ is computed with

$$\gamma(S(t)) = \frac{\Lambda(t)\mu_3}{(\Lambda(t)\mu_2)^{\frac{3}{2}}} = \frac{\Lambda(t)\Big(3(e^{\sigma^2}+2)\sqrt{e^{\sigma^2}-1} + (3\mu e^{2\mu+2\sigma^2})\Big)}{(\Lambda(t)e^{2\mu+2\sigma^2})^{\frac{3}{2}}}. \tag{7.8}$$

The presentation of the first three moments of the total claim amount $S(t)$ seems to be enough for the purposes of this study (See also Pages 60-61). As a further research interest, the kurtosis and higher moments of $S(t)$ can be studied.

Another statistical measure is the relative variance (coefficient of variation) of the total claim amount process $S(t)$, that is

$$c.o.v(S(t)) = \frac{\sqrt{Var(S(t))}}{E(S(t))}. \tag{7.9}$$

Generally, as the number of the policyholders increases, less variation is expected in the portfolio [Booth et al., 1999].

The following scenarios are prepared to give the reader the idea of how the suggested models of the previous chapters can be used in actual life. The best reasonable models of Chapters 5 and 6 are summarised in Tables 6.18 and 6.19 in terms of the residual deviance values of the claims data by months classification. However, here we want to make the estimation of the expected total claim amount $E(S(t))$ by weeks based claims data to observe the effect of the even small earthquakes, although the residual deviance values are slightly higher in weeks case.

As mentioned before, the exponential kernel function seems to have a better use with the zone 1 claims data; whereas zone 2 claims data fits well with the use of the power kernel function. The following calculations are presented with the use of the exponential kernel because the chosen form of the power kernel function seems to behave like the exponential kernel so that it seems to be a better choice to use the exponential kernel. As an example, the covariate (magnitude and the residential building number) and non-covariate models are used to explore the required reserves of the Turkish Catastrophe Insurance Pool. The empirical selection of the significant earthquakes at selected kernel knots and corresponding parameter estimates of β and α_j's are used in the estimation process. The financial estimations (e.g. $E(S(t))$, premiums) are based on Turkey New Lira (YTL).

Scenario 1: No more earthquake occurs

Estimation of the necessary TCIP reserves for risk zone 1 by using the exponential kernel

Covariate Models

In this case, we would like to estimate the required reserves and calculate some statistical measures of the zone 1 claims data, when the exponential kernel function is used for the modelling purposes. The time is based on weeks and the models include the time, magnitude and the residential building number effects. The time covariate, which we use here for estimation, is the same as the thesis data period, that is from December 2000 (starting from week 1 for the first claim) until July 2003 (ends in week 138). The aim is to estimate the losses of this period. Therefore, the estimates of the model parameters β and α's are used from the results of Model 10 of Table 6.2 (N_i model) and Model 14 of Table 6.6 (S_i model) in Chapter 6. Figure

7.3 shows the pattern of the claims in earthquake risk zone 1.

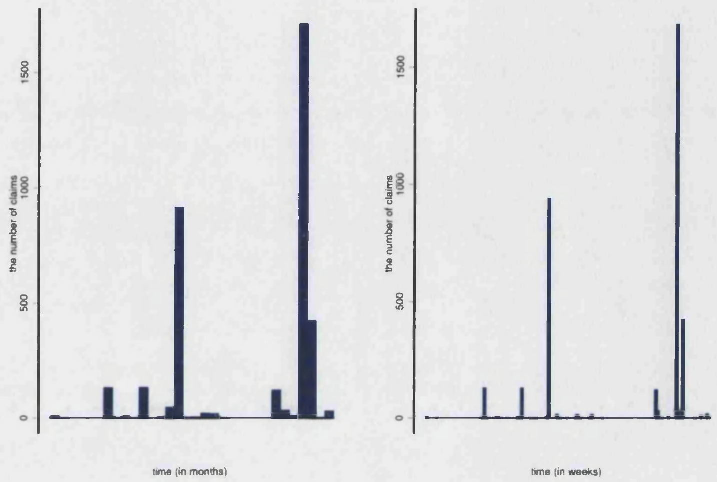

Figure 7.3: The bar plot of the number of claims in risk zone 1. Left: in terms of months, right: in terms of weeks.

By using the binning approach, Model 10 provides the parameter estimates for the number of claims model with intensity $\hat{\Lambda}_i = e^{\hat{\alpha}_0 + \sum_{j=1}^{k} \hat{\alpha}_j e^{-\hat{\beta}(t_i - s_j))_+}}$. The results of Model 10 of Table 6.2 suggests the parameter estimates for zone 1 claims as $\hat{\beta}_{\text{zone 1}} = 0.2$, $\hat{\alpha}_0 = -0.09$ and some α_j estimates for big earthquakes like $\hat{\alpha}_{\text{week 122}} = 4.93$ ($n_{\text{week 122}} = 1682$ claims), $\hat{\alpha}_{\text{week 60}} = 3.87$ ($n_{\text{week 60}} = 939$ claims). The diagnostic check of this model is presented in Figures 6.19 and 6.20 in Chapter 6 on the actual observations and fitted values.

Next, Figures 7.4 and 7.5, respectively, show the pattern of the intensity of the claim number process and the fitted $\hat{\Lambda}_i$ in risk zone 1 when the exponential kernel is used for the number of claims N_i model for weeks data. We can observe the claim jumps, for instance at week 60 and week 122 of Figure 7.4 in the curves of Figure 7.5. The jump pattern of these claims are also observed in Figure 5.7 and Figure 5.8 in Chapter 5 both for months and weeks data.

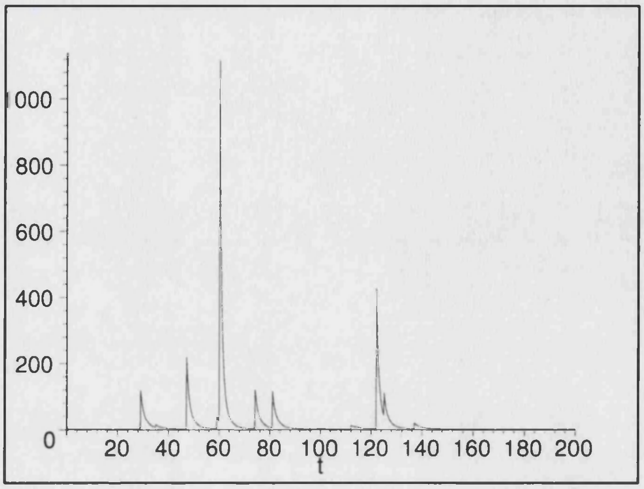

Figure 7.4: The plot of the exponential kernel function in risk zone 1. x-axis: time (in weeks), y-axis: the exponential kernel

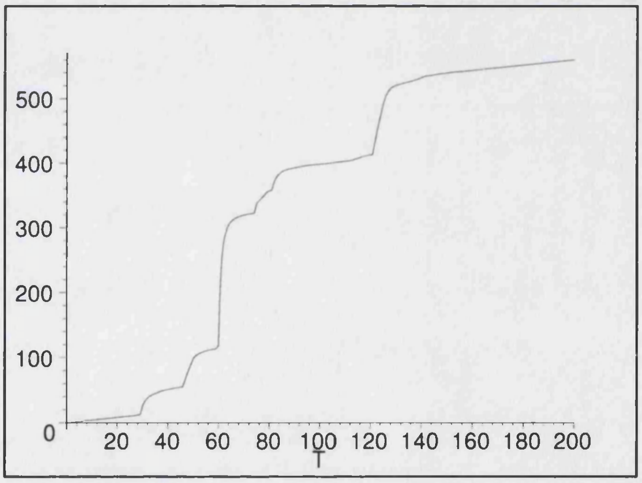

Figure 7.5: The plot of the fitted $\hat{\Lambda}_i$ in risk zone 1 by exponential kernel. x-axis: time (in weeks), y-axis: $\hat{\Lambda}_i$

By using (7.1) for our case, the mean intensity function $\Lambda(t)$ is estimated for instance as

$$\hat{\Lambda}_i = \int_0^{138} e^{-0.09+3.91e^{-0.2(t_i-29)}+4.37e^{-0.2(t_i-47)}+...+1.71e^{-0.2(t_i-137)}} dr = 529.$$

It should be noted that the claim number $N_i \sim \text{Pois}(\Lambda_i)$, so the estimate here seems reasonable on this basis. By a similar argument to the computation of Λ_i, the estimate of the mean η_i of the claims X is calculated by the estimates of Model 14 of Table 6.6 ($\hat{\beta} = 0.2$ and the α_j coefficients from S models in S-Plus). Also, the sample variance of the raw claim amount is used to compute the aggregate variance. Figure 7.6 shows the decreasing jump pattern of the exponential kernel function for some significant earthquake kernels in risk zone 1.

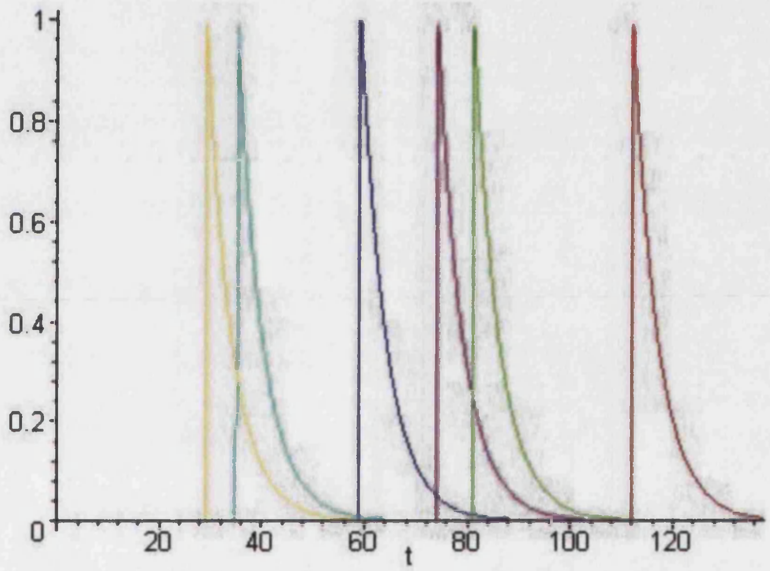

Figure 7.6: The exponential kernel in risk zone 1 at weeks 29, 35, 59, 74, 81 and 112

We calculate the mean of the total claim amount S by using the estimate of $\hat{\Lambda}_i = 529$ for the claim number and the estimate of the mean $\hat{\eta}_i$ for the claim amount as given in the basic theory in Sections 3.2 and 3.3, where we have the i.i.d claims X's. Then, the aggregate mean $E(S(t))$, is obtained for this scenario as

$$E(S(t)) = \hat{\mu}_i = 529(0.17) = 1,563.77,$$

and the corresponding aggregate variance is computed as

$$Var(S(t)) = \hat{\sigma_i^2} = 529(258.89 + 0.17^2) = 28,677.06,$$

by using (7.5) and (7.6), respectively. The greater variance suggests the greater the dispersion (heterogeneity) of the data. This high variance value can be modified by computing the standard deviation (square root of the variance) as

$$sd(S(t)) = \sqrt{Var(S(t))} = 169.343.$$

The corresponding coefficient of variation is obtained by (7.11) and equals to 0.108. (7.8) suggests that the skewness of this case is 0.001. These results are based on the portfolio of the Turkish Catastrophe Insurance Pool in zone 1 risk areas. Similar methodology can be applied based on individual cities in the chosen risk zone as part of future research.

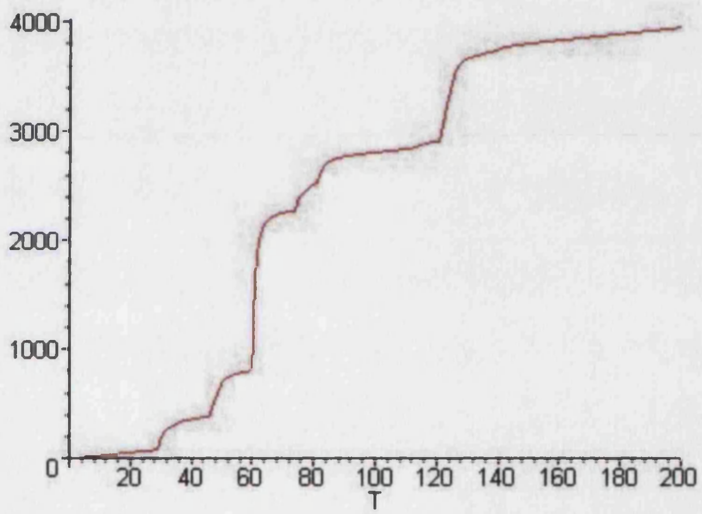

Figure 7.7: The plot of the aggregate mean in risk zone 1 by the use of the exponential kernel. x-axis: time (in weeks), y-axis: the aggregate mean $E(S(t))$

Figures 7.7 and 7.8 are the plots for the future prediction of the aggregate mean $E(S(t))$ and the aggregate variance $Var(S(t))$. The increasing trend is observed in both plots and one can conclude that the required reserves of the Turkish Catastrophe Pool is expected to increase in time.

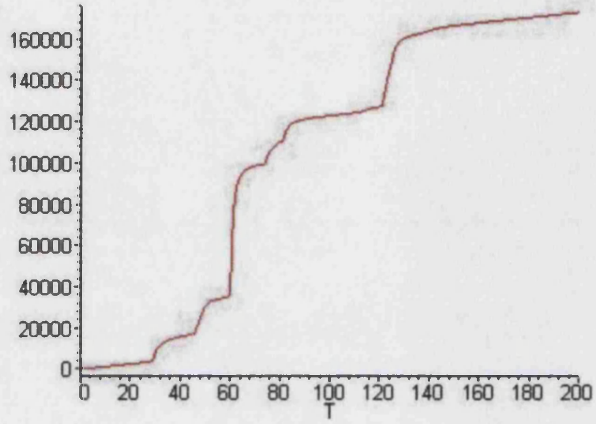

Figure 7.8: The plot of the aggregate variance in risk zone 1 by the use of the exponential kernel. x-axis: time (in weeks), y-axis: the aggregate variance $Var(S(t))$

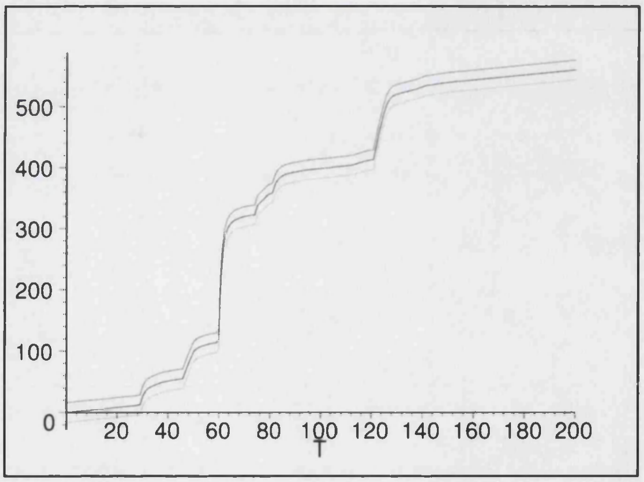

Figure 7.9: The plot of the $\hat{\Lambda}_i$ deviation versus time. x-axis: time (in weeks), y-axis: the deviation of $\hat{\Lambda}_i$

Above, Figure 7.9 shows the combination of the estimates $\hat{\Lambda}_i$, $\hat{\Lambda}_i + \text{Std}(S)$ and $\hat{\Lambda}_i - \text{Std}(S)$ in terms of weeks data, where the total claim amount S is lognormal.

Another use of the aggregate mean is to calculate the net premium amount as mentioned before. By using the Normal approximation (assuming a large number of claims will arrive) referring to the example in Page 50, and the Expected Value

Principle (EVP), the minimum premium to charge the insured is obtained by the following security loading values, that is

1. the probability of a loss in the TCIP is less than 10 % is

$$Pr(S < (1+c)E(S)) = 0.10,$$

after standardising

$$Pr(Z < \frac{cE(S)}{\sqrt{Var(S)}}) = 0.90.$$

The corresponding test-statistic value from the Standard Normal Table is 1.282, so the security loading is

$$\frac{cE(S)}{\sqrt{Var(S)}} = 1.282 \rightarrow c = \frac{(1.282)(1,563.77)}{169.343} = 0.138,$$

which means 0.138 % security in the portfolio. Then the minimum amount of premium with no deductible to charge to ensure that the probability of loss is less than 10 % is $cE(S) = (0.138)(1,563.77) = 217.098$.

2. the probability of a loss in the TCIP is less than 5 % gives a security loading of

$$\frac{cE(S)}{\sqrt{Var(S)}} = 1.645 \rightarrow c = 0.178,$$

with a required premium amount of $cE(S) = (0.178)(1,563.77) = 278.569$ with no deductible.

3. the probability of a loss in the TCIP is less than 1 % concludes

$$\frac{cE(S)}{\sqrt{Var(S)}} = 2.33 \rightarrow c = 0.252,$$

which requires a premium amount of $cE(S) = (0.252)(1,563.77) = 394.569$ to be collected in the TCIP to be able to compensate the possible earthquake losses.

The premium requirements calculated above is much higher than the full payment of the TCIP to the earthquake affected policy holders in the past six years as mentioned in Table 7.6. Moreover, if the no-covariate models are used as in the next case, the values of $E(S(t))$, $Var(S(t))$ and the rest increase significantly.

The $E(S(t))$ and the premium values here one more time shows how indeed the current premium ratings of the TCIP is very low even in the highest risk zones if the figures in Table 7.5 is considered. One way to increase the amount of the reserves of the TCIP is to reinsure the risk to the international companies, as the TCIP does within its system. The reinsurance companies invest the collected premiums of disaster free time in tax-free banks, or invest for the construction of shopping centres, hotels etc., where money can be earned. In case of a big disaster strike, the financial sources would be available to compensate the affected people. As given previously, the TCIP reassured USD 540m of its risk to A-level and above rated international reassurers in 2001, that of USD 840m in 2002 and USD 740m in 2003. In 2006, the reinsured amount is given as EUR 920m ($\sim$USD 1.1b) with a full payment capacity of EUR 1.1b ($\sim$ USD 1.33b 2005 estimate).

Scenario 2: A hypothetical earthquake of magnitude 7.6 hits Istanbul

In Turkey, most of the earthquake scenarios are based on the city of Istanbul. Therefore, in Scenario 2, lets assume that an earthquake of magnitude 7.6 in Richter Scale, as suggested in [Erdik, 2003], occurs in Istanbul, which is in the category of earthquake risk zone 1. Assume that 100000 claims arrive to the Turkish Catastrophe Insurance Pool as a result of this earthquake, where there are approximately 500000 residential buildings in Istanbul (not every flat/apartment/house have a compulsory earthquake insurance). The magnitude is the first concept to come to people's mind when an earthquake strikes. Therefore, in this scenario we will present the case of a covariate model and try to see what changes in the previous Scenario 1, if a shock earthquake hits Istanbul.

In Figures 7.10 and 7.11, it is obvious that the ordinary and any other small, moderate or big earthquake claims are all absorbed with one big earthquake strike in Istanbul. The total claims arriving in risk zone 1 is more than 400000 in this scenario, which will probably be much higher if the real event occurs.

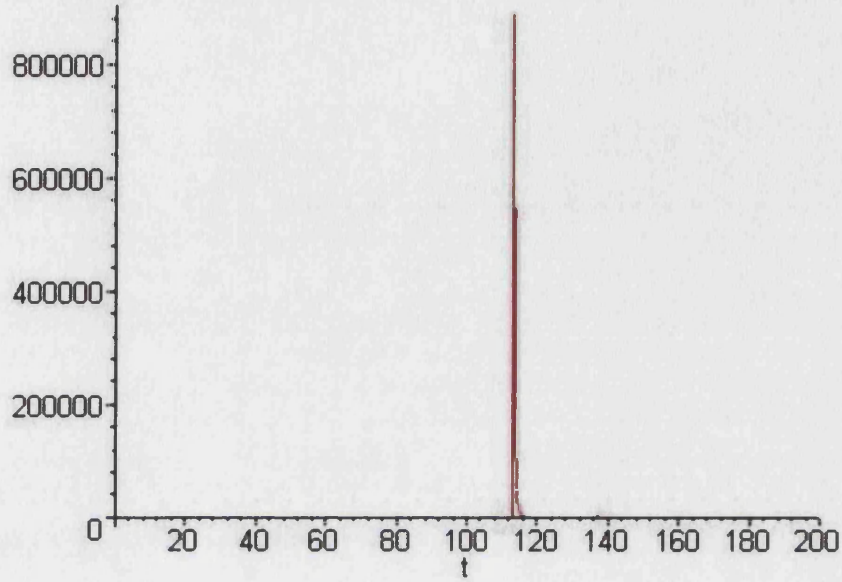

Figure 7.10: The plot of the exponential kernel function in risk zone 1. x-axis: time (in weeks), y-axis: the exponential kernel

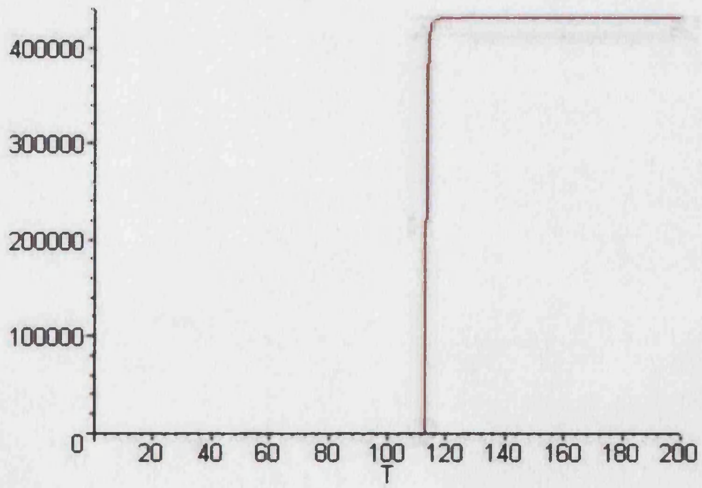

Figure 7.11: The plot of the fitted $\hat{\Lambda}_i$ in risk zone 1 by exponential kernel. x-axis: time (in weeks), y-axis: $\hat{\Lambda}_i$

The aggregate mean and variance figures on Scenario 1 change as follows with the strike of a 7.6 magnitude earthquake in Istanbul

$$E(S(t)) = 1,275,417.99,$$

and the aggregate variance is

$$Var(S(t)) = 23,389,190,$$

with a standard deviation of

$$sd(S(t)) = \sqrt{Var(S(t))} = 4,836.2.$$

The coefficient of variation is 0.0037. By (7.8), the skewness is computed as 0.000029. The similar argument of the earlier calculation suggests the security loadings as 0.0048, 0.0062 and 0.0088 with less than 10 %, 5 % and 1 % probability of loss, respectively. Then, the corresponding premium with no deductible for these security loading values are 6,200.06, 7,955.61 and 11,243.68.

The needed reserve estimate $E(S(t))$ is much higher than the value obtained in Scenario 1, where the covariate model estimates are used. The estimated TCIP reserve need s 1,275,417.99 YTL for a possible earthquake in Istanbul, which is more than the so far 2006 total payment of the TCIP (1,089,633 YTL). This is an optimistic approach because if there is an earthquake in Istanbul, the damage will be extremely high and the capacity of the TCIP itself will not be enough to cover the possible losses. Then the government will use the other financial possible coverage methods like contingent credit, external borrowing.

Our estimate here is still just shows a small portion of the 'loss-cake' since including the losses of the industry, transportation (roads, the two big bridges connecting the continents), communication facilities, public/state buildings, the loss figures in Istanbul will hit the ceiling. Istanbul is the financial centre of the country and all micro/macro economical indicators will change significantly with the earthquake strike. One suggestion has been discussed to build another financial centre in some other part of the country, which is earthquake-safer.

This shows that the premium ratings of the mandatory earthquake insurance contracts should be revised so carefully for Istanbul and surrounding areas (this are is the heart of the industry in Turkey), because one big earthquake can open a gigantic hole in the economical structure of the country and it takes a long time to recover and go back to pre-disaster period.

The current mitigation strategies in Turkey

Before the 1999 Marmara earthquake, there was no valid disaster risk and mitigation plan in Turkey. This sad experience showed the vital importance of the need for such a plan. As mentioned before, scientists expect another severe earthquake to hit Istanbul area in near future. They forecast that the fault line in the base of the Marmara Sea is accumulating some energy now, which will be released as an earthquake in time. The Republic of Turkey Prime Ministry Project Implementation Unit is mainly completed the following projects: Disaster Rehabilitation and Reconstruction (DRR) Project, Turkey Emergency Flood and Earthquake Recovery (TEFER) Project and Marmara Emergency Earthquake Recovery (MEER) Project. What seems fearful is most of the earthquake studies are focused on the Marmara region especially in Istanbul area. The vital question is: 'What happens if as a surprise, the fault line breaks in the opposite direction of Istanbul and a severe earthquake hits somewhere in any other part of the country?'

The main body in the case of a disaster strike is the central government in Turkey with top-down approach. If an earthquake occurs, the government immediately starts a crisis centre, which is run by chosen ministers of the cabinet. This crisis centre coordinates all the rescue and relief operations in cooperation with other government, military and voluntary organisations. Before the disaster, as a part of mitigation plan, some authority should be given to the local administration so that in case of need, no time will be lost to respond the disaster.

Financially, there were no spare reserves to respond to the direct, indirect and secondary effects of the disasters in Turkey before 1999. The 'father government' approach was the only way to cover the losses, that is the government was the only hope of the disaster-affected people. The Turkish Catastrophe Insurance Pool is an example of the first public-private insurance system in a developing country. However, there are many things to revise in the system to make it function more efficiently. One of the most important lacks of the TCIP is that it only insures the residential buildings within the municipality borders. There is no insurance offered in rural areas or business premises or for the contents of the buildings. The poorer part of the population has no insurance coverage, although they will be the first to be affected in case of a disaster. Currently, the TCIP is the only source to respond the financial effects of a disaster with a capacity of more than USD 1b.

On the 26/May/2005, the World Bank's (WB) Board of Executive Directors has approved a four year project titled 'Istanbul Seismic Risk Mitigation and Emergency Preparedness Project-ISMEP' with a loan amount of USD 400m. The project aims to reduce life losses, social and economic losses in Istanbul in case of a severe earthquake. The key steps of the process are: Enhancing emergency preparedness, seismic risk mitigation for public facilities (retrofitting of hospitals, schools etc.), enforcement of building codes and land use plans, implementation of the project in the most efficient way and to build institutional capacity. The involvement of the community is one of the main aims of the Istanbul Seismic Risk Mitigation and Emergency Preparedness Project. It is a well-known fact that the elderly, the poor, the children and the women are the worst affected parts of the community in case of a disaster strike. The education program, which is supported by seminars, workshops, media advertisement, leaflets and brochures, is organised by the authorities to increase the public awareness. This will also help to remind the people, who easily forget about their disaster experience and go back to their routine lives. A National Mitigation Program can not work thoroughly without the community involvement.

There is also a microzonation pilot project in process in a province of Istanbul, which is called 'Zeytinburnu'. A large database is under construction with the use of Geographical Information Systems (GIS-like Arcview, Hazus- and HazTurk is in process) for Turkey, which will give all the necessary information like building number, the age and the type of the buildings, Cresta zone codes (risk zones), disaster history and the number of earthquake insurance policies to be able to calculate possible losses of a future disaster. The use of the GIS will help to determine the vital resources after a disaster hit like the distance of the nearest hospitals to the earthquake epicentre, the bed capacity of these hospitals, the number of health personnel or the nearest fire stations with the number of the personnel and the fire equipment available.

The earthquakes are the main disasters in Turkey. However, because the country suffers from other types of natural disasters, the General Directorate of Disaster Affairs is in the process of the development of a multi hazard (including floods, landslides, earthquakes, avalanches etc.) map, which shows all the possible disasters in one map.

7.2.3 Estimation of losses and financial vulnerability of Turkey in a hypothetical earthquake

In this section, different than the previous estimation depending on our suggested models of Chapters 5 and 6, the main aim of the study is to decide if the government should make insurance investment for its public infrastructure assets and other post-disaster liabilities and if it needs additional funding in post-disaster recovery period. In such case, what need to be measured to cover the potential financial gap are: the governments's exposure to catastrophic risks, the government's liabilities in case of a disaster strike and the post-disaster financial availability. The following earthquake scenario is submitted as a course project in February 2005 to the World Bank Institute Natural Disaster Risk Management Program: Financial Strategies module, to assess the financial gap situation for the case of Turkey. The calculations are based on the ECLAC (Economic Commission of Latin America) methodology and the IIASA CATSIM (CATastrophe SImulation Model) model, of which the computation details are confidential and so not provided to the participants.

Ex-ante mitigation measures are vital, especially in developing countries to reduce the social and financial losses of a disaster. It is also important to have a disaster response plan in advance not to lose much time waiting for the international assistance by institutions and non govermental organisations (NGOs) to arrive.

An efficient National Disaster Mitigation and Recovery Program should include the possible catastrophe risks of country, the post-disaster government liabilities, the estimation of the possible losses due to a disaster, the level of preparedness of the government to be able to cope with its liabilities and the possible solutions to decrease the government's financial vulnerability in such case. The main responsibilities of the governments are to start a crisis centre immediately, to keep the coordination-cooperation of the bodies, to provide temporary accommodation for the disaster-affected area, to provide urgent assistance to the poor, elderly, women and children, and to reconstruct the public assets like roads, bridges, hospitals, communication.

Let's assume the following figures are estimated by the World Bank and the State Planning Organisation of Turkey (SPO). The total capital stock of Turkey is USD 40b in 2005 and the value of public assets is USD 6b (15 % of the total capital stock). The Turkish government's post-disaster liabilities for reconstruction of infrastructure (railways, highways, telecommunication etc.) and disaster relief is decided to be 20 % of total capital stock, which is USD 8b. This means in total, 35 % of the losses will be financed by the government, which is USD 14b. The following probability distribution is based on information from Swiss Re. They obtain this information by the analysis of their database on past events and losses and future catastrophe models [Freemand et al., 2002, Pollner et al., 2001]. By the IIASA CATSIM tool, the losses to be financed by the government given the probability and the amount of the capital stock destroyed in percentage is

$$\text{Losses}(10\text{year}) = 14 \times \frac{0.1}{100} = 0.014$$

$$\text{Losses}(50\text{year}) = 14 \times \frac{3.8}{100} = 0.532$$

$$\text{Losses}(100\text{year}) = 14 \times \frac{7.5}{100} = 1.05$$

$$\text{Losses}(500\text{year}) = 14 \times \frac{17.4}{100} = 2.436$$

Event (Year)	Probability (%)	Capital Stock Destroyed (%)	Losses to be financed by the government (35 % of the losses) USDb
10	10	0.1	0.014
50	2	3.8	0.532
100	1	7.5	1.05
500	0.2	17.4	2.436

Table 7.8: The probability distribution of losses to infrastructures caused by the earthquakes in Turkey in 10 to 500 years time

Therefore, Table 7.8 indicates, for instance, in a 50-year event, a possible earthquake will cause losses of approximately 3.8 % of the total capital stock. In the national plan, the government should consider these values in their budget arrangements and determine the alternative resources to cover a possible financial gap.

If the government can not provide the financial needs of reconstruction and relief after a disaster via domestic and international sources, this implies the financial vulnerability of the country. It is important to know the availability of the post-disaster financing sources of the Turkish government to be able to make a more reliable analysis of the financial vulnerability for an earthquake scenario. Generally, the sources are the diversions from other expenditures of the budget, special disaster tax arrangements, borrowing from the domestic reserves (Central Bank, bonds, credits), international aid and external borrowing from international institutions like the World Bank, International Monetary Fund (IMF), European Bank. There is a need of experts in the key positions of the financial institutions, who can prepare the pre-disaster and post-disaster financial plans for the government with a good knowledge of disaster economy.

Table 7.9 denotes the financial needs and availability for the government in case of an earthquake risk in Turkey in 10-,50-,100- and 500-year period. The direct loss ratio is the amount of the losses to be financed by the government. The diversion, international aid, domestic borrowing and foreign financing are the key sources in this analysis and it is assumed that the maximum available amounts are given as

USD 0.400b from the diversion, USD 0.100b from domestic credit, USD 0.200b from foreign financing and 10 % of the total losses from international aid.

Since the total resources needed is USD 0.014b in 10-year period and the maximum amount of diversion is USD 0.400b, it can be covered by the current resources of the government that is within the capacity of the Turkish Catastrophe Insurance Pool (TCIP), which is currently more than USD 1b. In 50-, 100- and 500- year period, the losses to be financed by the government is more than the maximum diversion available, so the rest will be covered by other sources like international aid or domestic credit. The calculations continue with the same argument, using the maximum amount available for domestic credit, foreign financing and the maximum coverage of the losses by the aid.

Earthquake risk	10 years	50 years	100 years	500 years
Direct loss ratio(%)	0.1 %	3.8 %	7.5 %	17.4 %
Losses Government (USD b)(Total resources needed)	0.014	0.532	1.05	2.436
Diversion (USD b) (Max. available USD 0.400b)	0.014	0.400	0.400	0.400
Aid (USD b) (10 % of total losses)	0	0.053	0.105	0.244
Domestic credit (USD b) (Max. available USD 0.100b)	0	0.079	0.100	0.100
Foreign financing (USD b) (Max. available USD 0.200b)	0	0	0.200	0.200
Sum financing (USD b) (Total resources available)	0.014	0.532	0.805	0.944
Financing gap (USD b)	0	0	-0.245	-1.492

Table 7.9: Financial vulnerability analysis

When the sum financing is obtained, according to Table 7.9, the losses caused by an earthquake, which is expected to occur every 10 or 50 years, can be covered by government liabilities and do not cause a gap in economy. However, the losses due to the 100- and 500-year events cause financial gap of USD 0.245b and 1.492b respectively

$$\begin{aligned}\text{Financial gap(100year)} &= \text{Total resources available} - \text{Total resources needed}\\ &= 0.805 - 1.05 = -0.245,\end{aligned}$$

and

$$\begin{aligned}\text{Financial gap(500year)} &= \text{Total resources available} - \text{Total resources needed}\\ &= 0.944 - 2.436 = -1.492.\end{aligned}$$

How can the Turkish government cover the financial gap?

The methodology and the earthquake scenario in this chapter can be linked to the studies on the total claim amount (the aggregate claims), $S(t)$. The $S(t)$ is used as an estimate of the reserves of the country to measure the risk level of the government in case of an occurrence of a disaster. The expectation of the aggregate claims, $E(S(t))$, will be a deterministic value for the improvements in government's disaster finance strategy because it gives the approximate amount of the money to keep enough reserves in the Turkish Catastrophe Insurance Pool.

Financial disaster risk management is needed to prepare the country against huge financial losses, which government is not likely to cope with by its own sources. In pre-disaster time, the government should invest more in risk transfer mechanisms and disaster management projects as part of mitigation activities. The Turkish Catastrophe Insurance Pool (TCIP) is the only risk transfer mechanism in Turkey. The pool will be the first source to try to cover the possible losses of an earthquake. Therefore, the compulsory earthquake insurance payments is very important to develop disaster economy strategies in Turkey. The capacity of the TCIP will add a large amount to the diversions from government in case of an earthquake. Then if there is still more money needed, next source should be the international aid. In Table 7.9, 10- and 50-year events seem like the losses could be covered by diversion and TCIP, international aid and some amount from domestic borrowing, probably the Central Bank reserves in Turkey.

The 100- and 500-year events cause big amount of gaps in the budget (USD 0.245b and USD 1.492b). The country will definitely need a long-term, low-interest loans from international institutions, in which case, the government will be in debt to external sources. What looks like a possible solution is to increase the contribution of diversion as much as possible. Since the Turkish Catastrophe Insurance Pool will be the backbone in terms of diversion, compulsory earthquake insurance should be spreaded in every part of the country including the business premises and rural/urban residential areas. The government should provide the same insurance for the poor with low premium rates. Also, the wealthy business associations might contribute a little more on behalf of the insurance provided to the poor by government. Some special tax arrangements can be applied in pre-disaster period rather than increasing all the taxes in post-disaster time.

However, the government should not fully invest in the TCIP. Catastrophe bonds and other types of insurance might be other possible sources. The TCIP has some expenses on its education, advertisement and application period itself. The construction work (of highways, ports, commercial and residential buildings etc.) should be strictly under control that if it is done according to the valid Building Code 1998. This will help to reduce the possible losses with a big amount. The emergency services should be well-trained and be ready at any time to respond. This will help to save man-power, so to reduce the economic impact even with a little percentage. All the reserve funds and strategies should be revised very carefully during the preparation of a time and cost efficient financial disaster risk management plan in Turkey. The elements to affect the stability of the economy of the country should be avoided in such a plan as much as possible.

Figure 7.12 gives the impact of the 1999 Marmara earthquake on major macroeconomic indicators. The under base figures of the year 1999 is a good example of a financial gap of the economy due to a shock earthquake hit. 1999 figures will be a comparison base for a highly probable earthquake strike in Turkey.

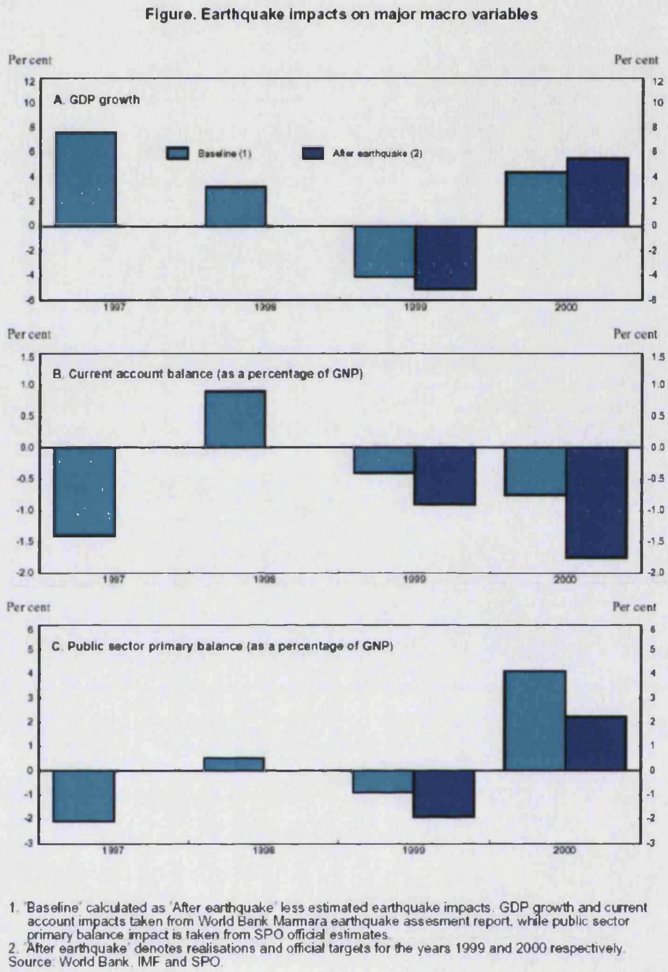

Figure 7.12: The effects of the Marmara earthquake on major macroeconomic variables [Bibbee et al., 2000].

Chapter 8

Conclusion

The Turkish Catastrophe Insurance Pool data is used for the first time in such statistical analysis in this thesis. The aim is to establish and present some links between the claim number N_i, the claim amount X_i, and the aggregate claims (total claim amount) S_i by using the binning approach, the available literature and the statistical concepts of likelihood, parameter estimation and modelling to obtain a nice interpretation for this claims data.

An inhomogeneous Poisson process allows a stochastic process to pick the time-varying events in the given time period. The log-linear rate function is a good choice to work with in analysing an inhomogeneous Poisson process. By the argument in **Remark 2**, Λ_i is used as an approximation of the process rate λ_i. Therefore, an inhomogeneous Poisson process, $N(t) \sim \text{Poisson}(\Lambda(t))$, is chosen to model the number of claims $N_i \sim \text{Pois}(\Lambda_i)$ with the use of the special kernel functions, the exponential kernel and the power kernel representing the intensity of the process. The exponential and the power kernel functions are respectively given in the following forms

$$\log \Lambda_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+},$$

and

$$\log \Lambda_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta},$$

where α_0 is the effect of the claims in the ordinary time period, β (the universal constant) is the non-linear parameter to denote the exponential decay (trend) of the claim arrivals, α_j are the coefficients for the effect of the big earthquakes, t_i is the start of the i^{th} bin and s_j are the kernel knots at which the significant earthquake jumps occur.

The reasons why the log-linear rate is preferred to linear rate are given as [Cox and Lewis, 1966, Lewis and Shedler, 1979]:

1. The log-linear rate is positive for all values of α_0 and α_j,

2. The log-linear model is suitable to use basic statistical procedures, like the use of the residual deviance criteria rather than the use of penalised likelihood, which need more complex models.

The extreme value analysis in Chapter 2 suggests that the mandatory earthquake insurance claims data of the TCIP can be modelled by using another type of distribution rather than the widely used generalised Pareto distribution for claims in the extreme value context, because the generalised Pareto distribution does not seem to provide a very reasonable fit for this data with the idea of heavy-tailedness. The data rather behaves like a short-tailed distribution.

The non-linear parameter β (exponential decay or trend) is estimated in Chapter 5 for the claim number N_i models and the total claim amount S_i models to represent the different characteristics of earthquake risk zones 1 and 2 of Turkey. The β parameter shows how steep the jump of the claims is, when an earthquake hits either risk zone 1 or risk zone 2. The value of the $\hat{\beta}$ changes from zone to zone and also with the use of either the exponential or the power kernel function to represent the claim decay. The estimates of the model parameters, these are $\hat{\beta}$, $\hat{\alpha_0}$ and $\hat{\alpha_j}$, lead to the calculation of the reserves, $E(S(t))$ (aka the aggregate mean), of the Turkish Catastrophe Insurance Pool (TCIP).

The aim of the relatively new insurance scheme in Turkey is to keep the pool reserves as enough as possible to be able to cover the losses of a future disaster. This can be done by appropriate premium arrangements, which mainly requires a careful study of the underwriters. The TCIP reinsures its risk to worldwide reinsurance companies to keep the reserves capable of payment in case of a disaster strike. The government will then use first the TCIP resources, then the agreed World Bank loan will be transferred (it is an agreement between the TCIP and the

WB authorities) and then reinsurance money will be used. Perhaps, even this will not be enough and budget diversions (e.g. tax changes), international aid and any other financial support will try to be provided. However, people believe that with a big earthquake hit, the system might not respond as efficient as it is expected to do so. It is unfortunate that scientists, economists and bureaucrats are waiting for the next significant earthquake, especially the soon Istanbul earthquake scenario, so that the insurance scheme/regulations of the TCIP can be improved.

In a developing country like Turkey, having such insurance system like the Turkish Catastrophe Insurance Pool is a big step ahead in the process of developing a National Disaster Risk Management Program. One leg of this program will be the financial aspects of the disaster losses and will include the strategies to lessen the economical impacts of disasters.

In Chapter 6, magnitude variable is observed to be a very significant to affect the number of claims N_i, and the total claim amount S_i. It is realised that magnitude reduces the residual deviance and calibrates the suggested models. It also plays a crucial role in the premium calculations when considering the earthquake risk zones. The premium rates in Table 7.4 already takes into account the effect of the magnitude so magnitude is an unchangeable actor in the mandatory earthquake insurance scheme in Turkey. The earthquake catalogue records, which include the level of magnitude as well as the location, epicentre, date etc., should be kept with maximum expertise to have an efficient use for the disaster studies of the country.

The addition of the number of residential building variable to the existing models even gives better results in terms of graphical explanatory analysis, the deviance and in observing the effect of the significant earthquakes. Therefore, when calculating the premiums of the compulsory earthquake insurance for different risk zones of Turkey, the actuaries/the underwriters should definitely consider the number (density) of residential buildings in the related disaster town/city. Until now, the Turkish Catastrophe Insurance Pool only uses the type of the building (e.g. masonry, reinforced concrete) and the risk zone (includes the magnitude effect) in the tariff calculations. It is strongly recommended that the number of residential buildings of the earthquake area/town/city should be revised for the improvement of the regulations of the TCIP. The effect of the population of an earthquake area and if possible the age of the building will be studied to create more complex models for further research.

The following models are suggested for the number of claims when covariates (time, magnitude, building number) are added in a fixed time interval. The first two equations represent the case, when the exponential kernel function is used

$$N_i \sim \text{Pois}(\Lambda_i),$$

where

$$\log \Lambda_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+} + m_l,$$

and

$$N_i \sim \text{Pois}(\Lambda_i),$$

where

$$\log \Lambda_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+} + m_l + r_l,$$

where m_l and r_l are used for the magnitude and the residential building number with $i = 1, \ldots, n$, $j = l = 1, \ldots, k$.

If the power kernel function replaces the exponential kernel function, the claim number model is presented as

$$N_i \sim \text{Pois}(\Lambda_i),$$

where

$$\log \Lambda_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta} + m_l,$$

and

$$N_i \sim \text{Pois}(\Lambda_i),$$

where

$$\log \Lambda_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j ((t_i - s_j)|_+)^{-\beta} + m_l + r_l,$$

The models above can be rewritten by the same argument of using the special kernel functions in the aggregate claims S_i with mean function μ_i

$$\log S_i \sim \mathrm{N}(\mu_i, \sigma_i^2),$$

where $\mu_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|_+} + \log N_i + m_l + r_l$ stands for the exponential kernel use. For the power kernel, the rate is $\mu_i = \alpha_0 + \sum_{j=1}^{k} \alpha_j((t_i - s_j)|_+)^{-\beta} + \log N_i + m_l + r_l$, where m_l stands for the magnitude, r_l denotes the number of residential buildings, $i = 1, \dots, n$, $j = 1, \dots, k$ and $l = 1, \dots, k$.

The methodology, which is developed in this research, can be summarised as:

1. The data of the Turkish Catastrophe Insurance Pool, which includes the earthquake claims that arrived to the pool between December 2000 and July 2003, is obtained from the TCIP authorities,

2. The time (months/weeks), the number of claims N_i, the corresponding total claim amount S_i, magnitude and the residential building number of the earthquake place are organised,

3. The data is split into zones. The analysis are done on the basis of risk zone 1 and risk zone 2 only due to the scarcity (< 30) of the data from risk zones 3, 4 and 5,

4. The exponential and the power kernel functions are used to represent the ordinary claim (α_0) arrival times of small earthquakes/tremors, big jumps (α_j, knots) with a significant earthquake occurrence in risk zones 1-2 and the non-linear parameter β of the exponential decay of the claims due to some fixed characteristics of the region,

5. The models are produced on the basis of with and without covariate cases,

6. The number of claims (N_i) are used as counts and modelled with generalised linear models of Poisson regression by using the suggested kernel functional forms in S-Plus,

7. The total claim amount (in the binning case $S_i = \sum_{i=1}^{N_i} X_i$) is modelled as lognormal (note that if $S_i \sim$ lognormal then $\log S_i \sim$ Normal) by using the same form of the suggested kernel functions in generalised linear models,

8. In Chapter 5, only the time effect is used to model the response variables, which are mentioned before. The approximation of the event times by using the empirical kernel knots are used as a feature of an inhomogeneous Poisson process,

9. By using the mean of the claim amount in related zone and the estimates of the number of events in that zone, the required reserves, $E(S(t)) = \eta\Lambda(t)$ (the aggregate mean), the aggregate variance $E(S(t))$ and the skewness $\gamma(S(t))$ are calculated for the selected models in Turkey. The result is used as an indication for the capacity of the reserves of the Turkish Catastrophe Insurance Pool and suggested that $E(S(t))$ is the minimum amount to be kept in the pool to cover possible losses of a future earthquake.

In summary:

$\boxed{\text{Mathematical model}} \longrightarrow \boxed{\text{Fit/test/verify the models}} \longrightarrow \boxed{\text{Use the models}}$

The statistical analysis of the thesis are mainly based on these three steps. The first big earthquake to hit Istanbul will affect all the country and will cause huge social and economical losses. Some mitigation effort is going on in Istanbul and surrounding cities like the retrofitting of the hospitals, schools, public buildings, roads, bridges and houses. The main aim of the current mitigation studies is to reduce the life losses initially. The findings in the financial estimation of Chapter 7 are very scaring high figures and gives an idea of an approximate amount that the insurance holders in Istanbul will need from the TCIP reserves in case of a strike of an earthquake of magnitude 7.6. The losses are estimated only for the ordinary households, who own compulsory earthquake insurance offered by the TCIP agencies. The industrial facilities have their own earthquake insurance via the private (re)insurers.

Our estimate of losses for Istanbul is just a small portion of the 'loss-cake' as including the losses of the industry, transportation (roads, the two big bridges (The Bosphorus Bridge and Fatih Sultan Mehmet Bridge) connecting the continents), communication facilities, public/state buildings, the loss figures in Istanbul will hit the ceiling. The two main bridges are told to be earthquake resistant but the best way to see if it is true is experiencing the earthquake. The Turkish Catastrophe Insurance Pool should at least reach to its 10^{th} year establishment anniversary to be able to cover possible losses of a future earthquake and even of regulations are improved, effects of any type of disaster.

Istanbul is the financial centre of the country and all micro/macro economical indicators will change significantly with an earthquake strike. One discussion among the scientists is to build another financial centre in some other part of the country, which is earthquake-safer. An interesting point is, although people are aware of the earthquake risk of Istanbul and the Marmara region, the migration to Istanbul and surrounding places with job hopes is still increasing.

The total losses due to a highly possible earthquake strike will affect the whole structure of the Turkish Catastrophe Insurance Pool. The authorities should strongly advertise the compulsory earthquake insurance and encourage public to buy insurance. Although the advertisement procedure is continuing, because the insurance culture is not common among the Turkish people, there is still not enough number of policies all around the country compared to the population and the residential number of buildings of the regions (remember that the TCIP only insures the residential buildings). One other improvement the TCIP urgently needs is the update of the system for all other types of the disasters to occur in Turkey. The natural disaster policies should be valid for earthquakes, floods, landslides, avalanches and other types. The TCIP should also provide coverage for all types of buildings (like residential, commercial/business, public/state) and partly to their contents.

As mentioned before, the stochastic processes $N(t)$ and the total claim amount $S(t)$, have a wide use in insurance and financial context. Credibility theory (see the Glossary), martingales (sub-martingale and super-martingale), arbitrage, famous Brownian Motion, the Black-Scholes model are some topics, where further research for the use of the exponential and power kernel functions in the claim number process rate $\lambda(t)$ (approximated by $\Lambda(t)$ as in **Remark 2** in the non-binning case) is suggested.

The World Bank Institute is in the process of initialising a sustainable online Disaster Risk Management program for Turkey, in which there will be global and country-based (local) modules including readings, case studies, audios and power point presentations. The program consists of five core courses, which will train the participants (e.g local governments, health personnel, rescue operators, academics) in general framework of disasters, financial strategies (here the TCIP is one of the most important example), safe cities (implementation of the building codes, construction quality), damage assessment and the community-based disaster risk management. Some findings of this thesis are aimed to be partly used in the 'Financial Strategies' module of the course.

This thesis one more time emphasis the need for an obligatory earthquake insurance system in Turkey with its methodology, ideas and findings, and hopes for the success of a continuous National Disaster Mitigation Plan, where the involvement of the community is crucial.

Natural disasters are inevitable and it is mainly the developing countries, who suffer the most and the worst from the effects of these disaster. We hope that the findings of this thesis might lead to some useful solutions for the disaster-prone countries mainly in financial aspects and also suggest some general mitigation strategies against the disasters. The history should not let to repeat itself.

Chapter 9

Glossary

Some parts of the following glossary is prepared from the online Natural Disaster Risk Management Program, which is offered by the World Bank Institute. The online courses of this program are: Comprehensive Disaster Risk Management Framework, Financial Strategies for Managing the Economic Impacts of Natural Disasters, Safe Cities, Community-based Disaster Risk Management and Damage Assessment.

Adverse selection: The tendency of a person with a higher-than-average chance of loss to seek insurance at standard rates.

Aftershock: An earthquake, which occurs after the main shock with a smaller magnitude. It normally originates close to the focus of the main earthquake.

Aggregate Claim Amount: Let n be the number of claims received by the insurance company in a certain period of time. The aggregate claim amount S is defined as:

$$S = \sum_{i=1}^{n} X_i$$

where X_i is the claim size of the i^{th} claim in that time period.

Building Code: It is the regulations as a combination of technical and functional standards, which controls the design, construction, materials, alteration and occupancy of any structure for human safety and welfare. Each country has a different building code depending on the risk type and amount of a natural disaster.

Capacity (in finance): The largest amount of insurance/reinsurance available from a company.

Capacity-Capability: A threshold level of the physical, social, economical and institutional resources of a community, society or organisation to reduce the effects of a disaster.

Catastrophe Bonds: Securitization of disaster risk and its transfer via bond issue to capital markets.

Catastrophe Model: A risk-analytic technique, which uses simulation modelling to supplement or replace historical data for the purposes of estimating probabilities and outcomes.

Catastrophe Reinsurance: A form of excess of loss reinsurance, which, subject to a specific limit, indemnifies the ceding company in excess of a specified retention with respect to an accumulation of losses resulting from a catastrophic event or series of events arising from one occurrence.

Cede: To transfer all or part of the insurance or reinsurance risk, which is written by a ceding company, to a reinsurer.

Cession: The amount of insurance risk transferred to the reinsurer by the ceding company.

Civil Defense: It is normally a government body to response to disasters and emergencies and to protect and mitigate the civilians in both wartime and peacetime.

Claim: There are two types of claims, **incurred** and **paid**. Paid claims are the ones, which are compensated by the insurer on time. Incurred claims are defined as the total amount of claims in the accounting year t, arising from the events which have occurred in the year irrespective of when the final payment is made.

Claim Delay: The time difference between the claim payment after the event and the late payment of that claim.

Credibility Theory: It assumes that the risk quality is a drawing from a certain structure distribution, and that conditionally given the risk quality, the actual claims experience is a sample from a distribution having the risk quality as its mean value [Kaas et al., 2001].

Damage: Any economical loss or destruction, which is caused by earthquakes, windstorms, and other perils.

Damage ratio: The repair cost of a location represented as a percentage of the value at that location.

Deductible: The amount d, decided between the insurer and the insured that the insurer pays only the part of the claim which exceeds d. There are two reasons to determine a deductible amount. First, to reduce the claim handling costs by excluding coverage for the often numerous small claims; second, to provide some motivation to the insured to prevent claims.

Design Earthquake: Selected earthquake parameters to design earthquake resistant structures compiling with the building code requirements.

Direct damage: Negative consequences of disasters in terms of assets lost, damaged or affected. First perceived in physical terms, i.e. miles of roads, hectares affected either in agricultural land, forests or environmental reserves, production already completed but lost as tons of agricultural products, numbers of industrial production units; or infrastructure affected as number of health services facilities, number of hospital beds, schools or number of classrooms destroyed, etc. Part of the direct damage, although not quantified specifically in terms of monetary value, is life losses, injuries and the primary, secondary or tertiary affected population.

Disaster: Natural or manmade disruption of the functioning society which causes human, material and environmental losses that the society can not cope with by its own resources. It is also a function of the risk process. It results from the combination of hazards, conditions of vulnerability and insufficient capacity or measures to reduce the consequences of risk.

Disaster Insurance: The insurance policies, which are provided by the government or private insurance companies to protect the insured from the economical losses caused by a disaster.

Disaster Risk Management-DRM: DRM is a combination of the administrative and operational strategies, policies and capacities of the society and communities to reduce the impact of natural disasters including structural and non-structural measures.

Earthquake: Earthquakes are generally defined as the shaking of the ground resulting from the reshaping of the Earth. An earthquake occurs when the vibrations, which are caused by the release of the accumulated energy as a result of the movements of the tectonic plates, reach to the Earth's surface.

Economic Impacts of Disasters: Damages to physical assets and losses in an economic activity. These can be classified into direct, indirect, and macroeconomic (also called secondary) effects. Direct losses occur from physical damage to assets or stocks, including public infrastructure, homes and commercial buildings, building contents and agricultural assets. Indirect losses consist of damages to in the flow of goods and services. Macroeconomic losses consist of changes to gross domestic product (GDP), consumption, inflation and employment and other macroeconomic indicators. These macroeconomic effects are due to the disaster as well as to the reallocation of government resources to relief and reconstruction.

Elements at Risk: Persons, building structures, machinery, infrastructure (e.g. water facilities, roads and bridges) or agricultural and other assets in harm's way.

Epicentre: The point on the ground surface just directly above the focus.

Exposure: The total value or replacement cost of assets (such as structures), which is at risk from a loss-causing event such as a catastrophe.

Exposure data: Information describing the exposures, which is used as an input for risk modelling. For insured property exposure, this information includes: geographic location (e.g., state, county, postal code), physical characteristics (e.g., occupancy type, construction class, year built, height of the structure, building/contents/time element contributions), replacement cost value (building/contents/time element), and financial structure (limits, deductibles, % insured, insurance-to-value).

Foreshock: It is an earthquake that is often part of a distinctive sequence, which precedes and originates close to the focus of a main shock.

Focus: The origin, or the source of the earthquake's energy is called 'the focus of the earthquake'. In natural earthquakes, the focus is located below the ground; whereas, in artificial ones, such as caused by nuclear explosions, it is near the earth's surface. Earthquakes with a depth of 70 km (43.5 miles) from the surface are called 'shallow-focus earthquakes', the ones with that of from 70 to 300 km (43.5 to 186 miles) are called 'intermediate-focus earthquakes' and those deeper than 300 km are called 'deep-focus earthquakes'. The depth may reach more than 700 km (435 miles) in deep-focus ones.

Geographic Information Systems (GIS): Computer programs developed to capture, store, check and analyse the data about the Earth. These systems are mainly used for hazard and vulnerability mapping and analysis and for disaster risk management.

Hazard: It is a phenomenon caused by nature or human, which causes social, economical and environmental losses as well as life losses and injuries. Each hazard is characterised by its location, probability, intensity and frequency.

IBNR: An important statistical problem for the practicing actuary is the forecasting of the total of the claims that are 'Incurred But Not Reported (IBNR)' or not fully settled ('RBNS (Reported But Not Settled)') [Kaas et al., 2001].

Indirect effects: Consequences, either positive or negative, for flows related to the production, provision, distribution or performance of goods and services, i.e. additional costs of transport, reduced income of enterprises, increased expenses of government, reduced tax revenues, insurance payments received, increased imports or reduced exports, etc.

Insurance: A way of spreading risk within a collective.

Insured loss: The portion of total economic loss from a catastrophe that is paid by insurance policies, including payments made by insurance carriers based on recoveries from reinsurance contracts or other financial guarantees. This excludes deductibles paid by the policy holder as well as losses that are not covered by insurance (such as losses above insurance limits or losses for perils that are not insured).

Intensity: Intensity is a scale number which is determined by the effects of an earthquake on people, structures and earth materials. The most commonly used scales are Modified Mercalli (MM) and Medvedev, Sponheuer and Karnik (MSK), both having twelve degrees indicating the maximum damage level of an earthquake.

The Law of Large Numbers: The greater the number of exposures, the more closer the actual result to the probable results, which are expected from an infinite number of exposures.

Lifelines: The public facilities and systems, which provide basic life support services such as water, energy, sanitation, communications and transportation.

Loss frequency: The probable number of losses, which may occur during some given time period.

Loss severity: The probable size of losses, which may occur.

Magnitude: It is a measure of the size of an earthquake.

Man-made Disaster: Loss event caused by human actions.

Megacity: A type of urban region, containing more than 10 million inhabitants. It is a recent phenomenon that the average size of the world's largest 100 cities increased from 2.1 million in 1950 to 5.1 million in 1990.

Mitigation: The precautions to reduce the impact of natural disasters.

Natural Disaster: Loss event, which is caused by nature that results in damage, disruption and casualties mostly in vulnerable communities.

Natural Hazard: It is a geographical, atmospheric or hydrological event, which has potential for causing loss or harm.

Peril: The cause of loss.

Preparedness: Being ready to be able to response as quick as possible after a natural disaster occurs.

Prevention: All kinds of precautions, which are taken to reduce the amount of losses as a result of a natural disaster.

Reinsurance: The insurance of an insurer.

Relief/Response: To provide all the facilities (e.g food, energy) to the disaster-affected region until all the systems (transportation, communication etc.) are rebuilt.

Recovery/Rehabilitation: The application of a suitable risk management system in the disaster-prone area to reduce the amount of risk.

Reserve: An amount, which is set aside to provide for payment of a future obligation.

Retrocession: Insurance of the reinsurers.

Retrofitting: A complex parameter of mitigation, which involves reinforcement of the existing built environment in order to be more resistant to the forces of natural hazards. Retrofitting involves consideration of changes in the mass, stiffness, damping, load path and ductility of materials, as well as radical changes such as the introduction of energy absorbing dampers and base isolation systems. Examples of retrofitting includes the consideration of wind loading to strengthen and minimize the wind force, or in earthquake prone areas, the strengthening of structures. Typically, retrofit starts with essential and critical facilities such as health care facilities, emergency shelters, and emergency response facilities. It also includes critical infrastructure that is needed for post-emergency operations. Retrofitting is costly, disturbing and often poses legal, operational and financial constraints.

Richter Scale: The Richter Scale is an index of the seismic energy, which is released by an earthquake. It is developed by the scientist C.F. Richter in 1935.

Risk: The probability of expected losses (deaths, injuries, property, livelihoods, economical activity disruption or environmental damage) resulting from interactions between natural or human caused hazards and vulnerable conditions. Mathematically: Risk = Hazards × Vulnerability.

Risk Transfer: An arrangement, where the liability for the costs of a risk is transferred from one party to the other.

Shock Model: The model, which explains the effects of a disaster in terms of mathematical and statistical analysis.

The maximum possible loss: The worst loss that could possibly happen to the firm during its lifetime.

The maximum probable loss: The worst loss that is likely to happen.

Uncertainty: A situation in which there is insufficient data to estimate 'risk' in terms of mathematical probability.

Underwriting: The process of selection and classification of the applicants for insurance.

Vulnerability: Susceptibility of a community to be severely affected by the consequences of a natural disaster. This includes social, economical, environmental and physical aspects.

Chapter 10

Appendix

Figures 10.1 to 10.4 are sourced from the EM-DAT and gives some information on the number of disasters, corresponding life losses and economical losses in 2004.

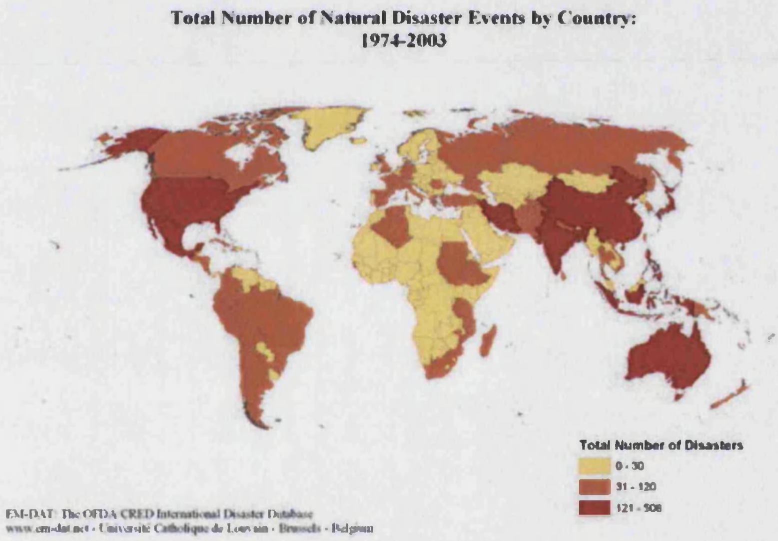

Figure 10.1: The total number of natural disaster events by country: 1974-2003

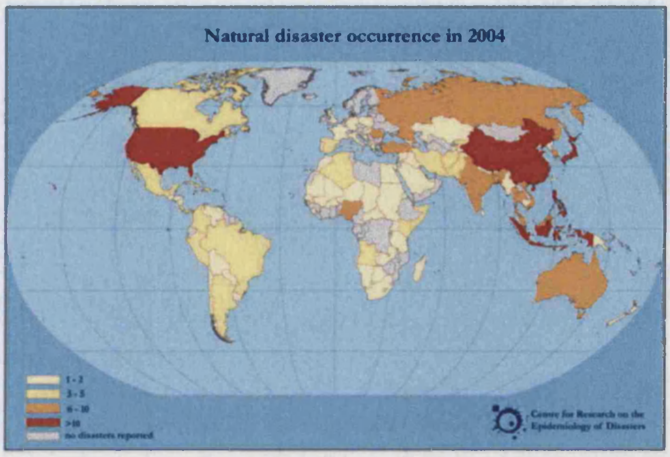

Figure 10.2: Natural disaster occurrence in 2004

Figure 10.3: Number of people reported killed by natural disasters in 2004

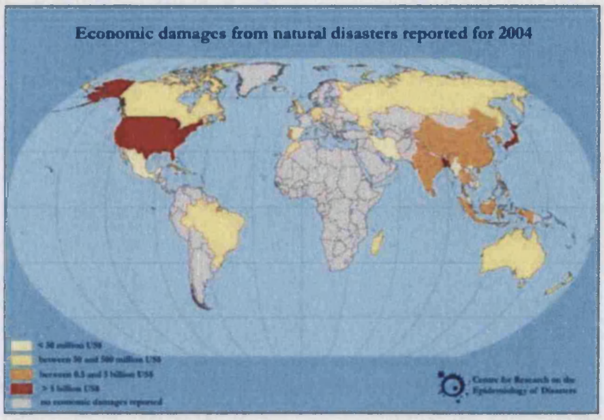

Figure 10.4: Economic damage from natural disasters reported for 2004

Figures 10.5 to 10.10 are obtained from the Kandilli Observatory and Earthquake Research Centre in Istanbul, the MEER Project by the Project Implementation Unit of the Prime Ministry of Turkey and the General Directorate of the Disaster Affairs (GDDA).

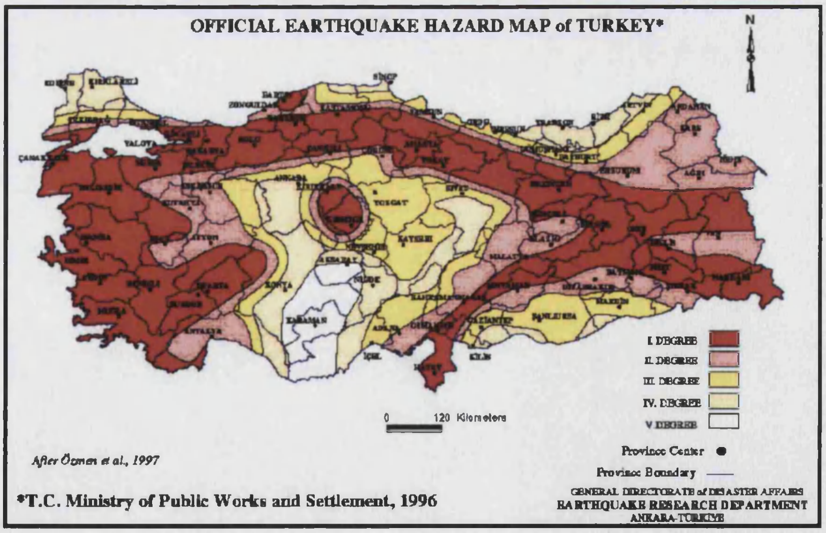

Figure 10.5: The earthquake zone map of Turkey. Source: GDDA

Figure 10.6: The fault map of Turkey

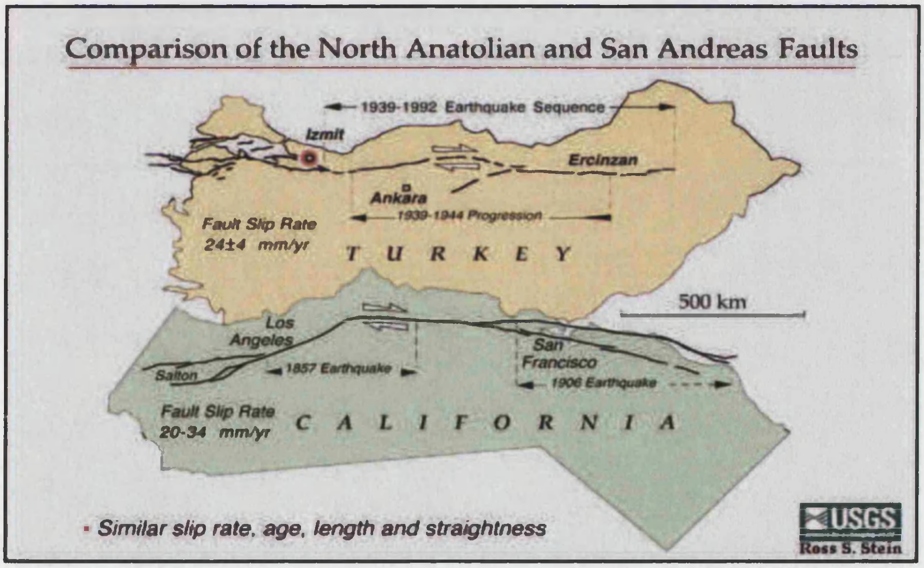

Figure 10.7: Comparison of the North Anatolian Fault and San Andreas Fault

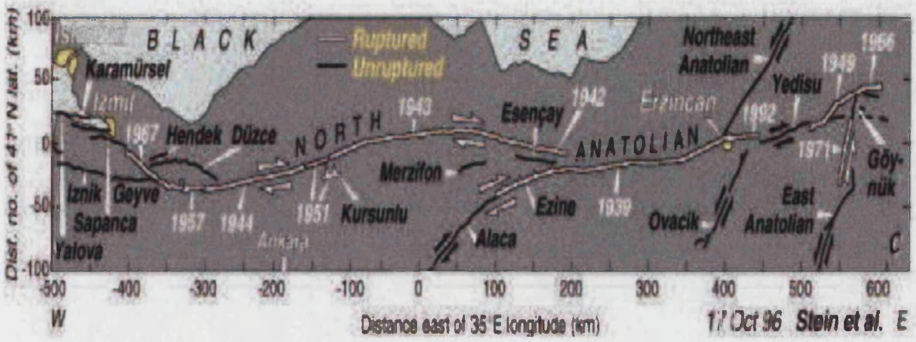

Figure 10.8: The ruptures in the NAF by years

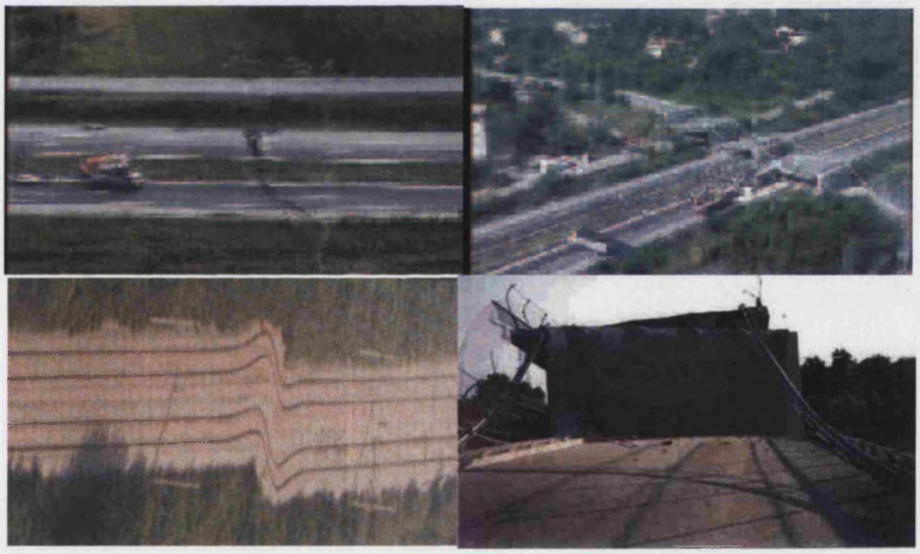

Figure 10.9: Some images after the Marmara earthquake

Figure 10.10: Some images of the house damage after the Marmara earthquake

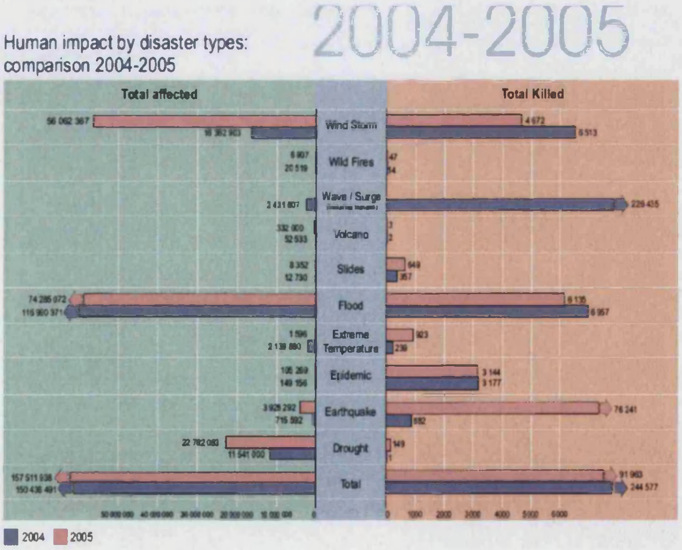

Figure 10.11: Human impact by disaster types: comparison 2004-2005. Source: EMDAT

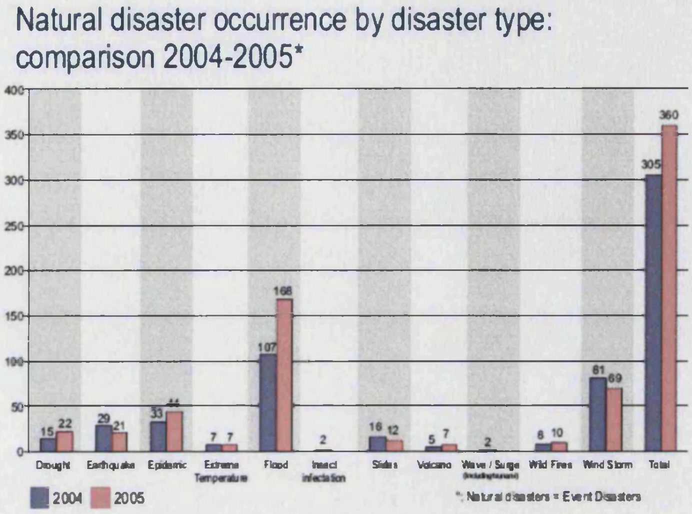

Figure 10.12: Natural disaster occurrence by disaster type: comparison 2004-2005. Source: EMDAT

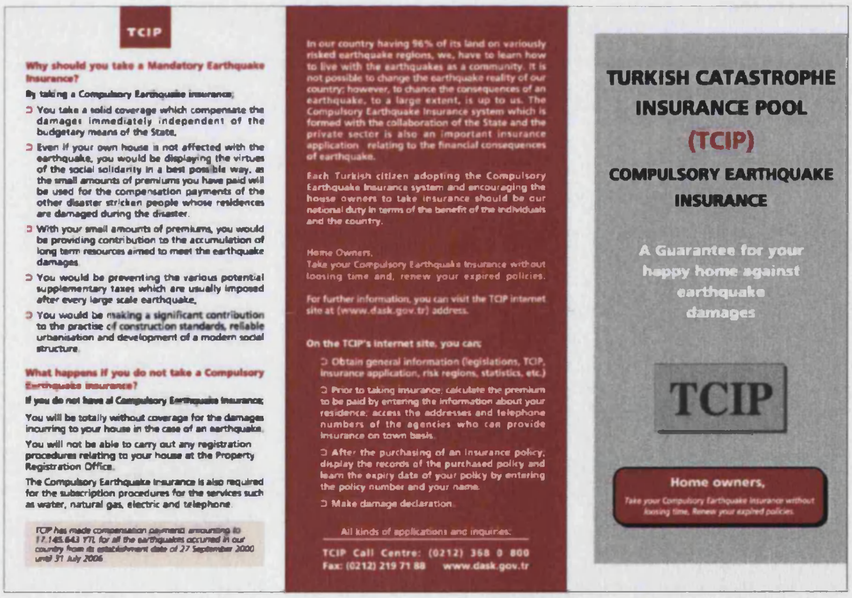

Figure 10.13: The Turkish Catastrophe Insurance Pool

Tables on different natural disaster types in Turkey

Rank	Province	Annual frequency	Population at risk
1	İzmir	3.484	450,000
2	Rize	1.841	55,000
3	Kahramanmaraş	1.608	35,000
4	Denizli	0.596	20,000
5	Trabzon	0.508	32,000
6	Antalya	0.408	400,0000
7	Kirikkale	0.396	10,000
8	Balikesir	0.172	15,000
9	Bartin	0.132	60,000
10	Bitlis	0.132	10,000
11	Sivas	0.132	10,000
12	Van	0.132	70,000
13	Batman	0.044	5,000
14	Zonguldak	0.024	25,000
15	Ankara	0.024	100,000
Total			1,297,000

Table 10.1: Most Vulnerable Provinces for Flood Risk in Turkey according to the data between 1955 and 2002. *Source:* The General Directorate of State Hydrolic Works, [JICA, 2004]

Rank	Province	Number of events	Population at risk
1	Trabzon	272	16,500
2	Kastamonu	229	13,800
3	Zonguldak	204	12,250
4	Kahramanmaraş	201	12,100
5	Erzurum	155	9,300
6	Rize	151	9,100
7	Malatya	141	8,500
8	Sivas	137	8,200
9	Ankara	131	7,900
10	Erzincan	125	7,500
11	Sinop	120	7,300
12	Çorum	117	7,200
13	Bingöl	115	6,900
14	Artvin	114	6,850
15	İçel	108	6,500
Total		2,320	139,900

Table 10.2: Most Vulnerable Provinces for Landslide Risk in Turkey according to the data between 1958 and 2003. *Source:* The General Directorate of Disaster Affairs (GDDA), [JICA, 2004]

Rank	Province	Number of events	Population at risk
1	Kayseri	34	10,000
2	Niğde	28	8,400
3	Erzincan	20	6,000
4	Aksaray	18	5,400
5	Karaman	17	5,100
6	Kahramanmaraş	16	4,800
7	Adiyaman	16	4,800
8	Sivas	14	4,200
9	Bitlis	13	3,900
10	Diyarbakir	12	3,600
11	Nevşehir	12	3,600
12	Mardin	10	3,000
13	Malatya	9	2,700
14	Hakkari	9	2,700
15	Kars	7	2,100
Total		235	70,300

Table 10.3: Most Vulnerable Provinces for Rock-fall Risk in Turkey according to the data since 1955. *Source:* GDDA, [JICA, 2004]

Year	Number of events	Deaths	Injuries	Households relocated
1981	2	14	-	52
1982	10	15	-	117
1983	14	6	-	400
1984	6	-	-	94
1985	2	7	-	29
1986	2	1	4	16
1987	10	18	-	146
1988	13	27	8	365
1989	7	4	-	77
1990	4	4	1	47
1991	12	7	-	267
1992	112	328	53	1,656
1993	31	135	95	146
1994	6	26	7	-
1995	3	7	2	68
1996	5	8	1	67
1997	8	16	3	88
1998	13	6	5	178
1999	5	10	3	31
2000	9	12	14	-
Total	344	974	258	5,154

Table 10.4: Avalanches in Turkey between 1980-2000. *Source:* GDDA, [JICA, 2004]

Convolution

Lets assume there are two independent random variables, X and Y. The operation 'convolution' calculates the distribution of $X+Y$ as [Kaas et al., 2001]

$$\begin{aligned} F_{X+Y}(s) &= Pr[X+Y \leq s] = \int_{-\infty}^{+\infty} Pr[X+Y \leq s \mid X = x] dF_X(x) \\ &= \int_{-\infty}^{+\infty} Pr[Y \leq s - x \mid X = x] dF_X(x) = \int_{-\infty}^{+\infty} Pr[Y \leq s - x] dF_X(x) \\ &= \int_{-\infty}^{+\infty} F_Y(s-x) dF_X(x) = F_X * F_Y(s). \end{aligned}$$

The cumulative distribution function (cdf) $F_X * F_Y(.)$ is called the 'convolution' of the cdf's $F_X(.)$ and $F_Y(.)$. The same notation is used for the density function. If X and Y are discrete random variables [Kaas et al., 2001]

$$F_X * F_Y(s) = \sum_x F_Y(s-x) f_X(x),$$

and

$$f_X * f_Y(s) = \sum_x f_Y(s-x) f_X(x),$$

where the sum is over x with $f_X(x) > 0$. If X and Y are continuous random variables

$$F_X * F_Y(s) = \int_{-\infty}^{+\infty} F_Y(s-x) f_X(x) dx,$$

and by derivation under the integral sign

$$f_X * f_Y(s) = \int_{-\infty}^{+\infty} f_Y(s-x) f_X(x) dx.$$

The convolution operation can be applied for more than two random variables. For instance, if there are three random variables, X, Y, Z, then

$$(F_X * F_Y) * F_Z \equiv F_X * (F_Y * F_Z) \equiv F_X * F_Y * F_Z.$$

Generally, for the sum of n independent and identically distributed random variables with marginal cdf F, the cdf is called the n-fold convolution power of F, that is [Kaas et al., 2001]

$$F * F * \ldots * F = F^{*n}.$$

The moment generating functions of some distributions

1- If the claim amount X_i is Exponential (β_e), the moment generating function for the special case of Gamma, where $\alpha_g = 1$, is

$$M_S(\theta) = \sum_{k=0}^{\infty} \frac{e^{-\Lambda(t)}\Lambda(t)^k}{k!}(\frac{1}{1-\beta_e\theta})^k = e^{-\Lambda(t)} \sum_{k=0}^{\infty} \frac{(\Lambda(t)\frac{1}{1-\beta_e\theta})^k}{k!}$$
$$= e^{-\Lambda(t)} e^{\Lambda(t)(\frac{1}{1-\beta_e\theta})} = e^{\Lambda(t)\Big((\frac{1}{1-\beta_e\theta})-1\Big)},$$

and the corresponding cumulant function is

$$\kappa_S(\theta) = \Lambda(t)\Big((\frac{1}{1-\beta_e\theta}) - 1\Big).$$

If the exponential kernel function is replaced in the intensity $\Lambda(t)$ for the moment generating and cumulant functions, the mgf and the cumulant functions are respectively

$$M_S(\theta) = e^{\int_{w_i}^{t+w_i} e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)|+}} dr\Big((\frac{1}{1-\beta_e\theta})-1\Big)},$$

and

$$\kappa_S(\theta) = \int_{w_i}^{t+w_i} e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j e^{-\beta(t_i - s_j)+}} dr\Big((\frac{1}{1-\beta_e\theta}) - 1\Big).$$

If the power kernel replaces the exponential kernel above

$$M_S(\theta) = e^{\int_{w_i}^{t+w_i} e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j (t_i - s_j)|_+^{-\beta}} dr\Big((\frac{1}{1-\beta_e\theta})-1\Big)},$$

and the cumulant function is

$$\kappa_S(\theta) = \int_{w_i}^{t+w_i} e^{\alpha_0 + \sum_{j=1}^{k} \alpha_j (t_i - s_j)|_+^{-\beta}} dr\Big((\frac{1}{1-\beta_e\theta}) - 1\Big).$$

2- If X is distributed as Logarithmic (p)

$$M_S(\theta) = \sum_{k=0}^{\infty} \frac{e^{-\Lambda(t)}\Lambda(t)^k}{k!}\Big(\frac{\log(1-pe^\theta)}{\log(1-p)}\Big)^k$$
$$= e^{-\Lambda(t)} \sum_{k=0}^{\infty} \frac{\Big(\frac{\Lambda(t)\log(1-pe^\theta)}{\log(1-p)}\Big)^k}{k!}$$
$$= e^{-\Lambda(t)} e^{\Lambda(t)\frac{\log(1-pe^\theta)}{\log(1-p)}} = e^{\Lambda(t)\Big(\frac{\log(1-pe^\theta)}{\log(1-p)}-1\Big)},$$

and

$$\kappa_S(\theta) = e^{\Lambda(t)\left(\frac{\log(1-pe^{\theta})}{\log(1-p)}-1\right)}.$$

Here, the use of the exponential kernel will give the following

$$\begin{aligned}
M_S(\theta) &= \sum_{k=0}^{\infty} \frac{e^{-\Lambda(t)}\Lambda(t)^k}{k!}\Big(\frac{\log(1-pe^{\theta})}{\log(1-p)}\Big)^k \\
&= e^{-\Lambda(t)} \sum_{k=0}^{\infty} \frac{\left(\frac{\Lambda(t)\log(1-pe^{\theta})}{\log(1-p)}\right)^k}{k!} \\
&= e^{-\Lambda(t)} e^{\Lambda(t)\frac{\log(1-pe^{\theta})}{\log(1-p)}} = e^{\int_{w_i}^{t+w_i} e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|+}} dr\left(\frac{\log(1-pe^{\theta})}{\log(1-p)}-1\right)},
\end{aligned}$$

and

$$\kappa_S(\theta) = \int_{w_i}^{t+w_i} e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|+}} dr\Big(\frac{\log(1-pe^{\theta})}{\log(1-p)} - 1\Big).$$

The use of the power kernel will change those above into

$$\begin{aligned}
M_S(\theta) &= \sum_{k=0}^{\infty} \frac{e^{-\Lambda(t)}\Lambda(t)^k}{k!}\Big(\frac{\log(1-pe^{\theta})}{\log(1-p)}\Big)^k \\
&= e^{-\Lambda(t)} \sum_{k=0}^{\infty} \frac{\left(\frac{\Lambda(t)\log(1-pe^{\theta})}{\log(1-p)}\right)^k}{k!} \\
&= e^{-\Lambda(t)} e^{\Lambda(t)\frac{\log(1-pe^{\theta})}{\log(1-p)}} = e^{\int_{w_i}^{t+w_i} e^{\alpha_0+\sum_{j=1}^{k}\alpha_j (t_i-s_j)|_+^{-\beta}} dr\left(\frac{\log(1-pe^{\theta})}{\log(1-p)}-1\right)},
\end{aligned}$$

and the cumulant function is

$$\kappa_S(\theta) = \int_{w_i}^{t+w_i} e^{\alpha_0+\sum_{j=1}^{k}\alpha_j (t_i-s_j)|_+^{-\beta}} dr\Big(\frac{\log(1-pe^{\theta})}{\log(1-p)} - 1\Big).$$

3- If X is Uniform (a, b), the moment generating and the cumulant functions are, respectively

$$\begin{aligned}
M_S(\theta) &= \sum_{k=0}^{\infty} \frac{e^{-\Lambda(t)}\Lambda(t)^k}{k!}\Big(\frac{e^{b\theta}-e^{a\theta}}{(b-a)\theta}\Big)^k \\
&= e^{-\Lambda(t)} \sum_{k=0}^{\infty} \frac{(\Lambda(t)\frac{e^{b\theta}-e^{a\theta}}{(b-a)\theta})^k}{k!} \\
&= e^{-\Lambda(t)} e^{\Lambda(t)\frac{e^{b\theta}-e^{a\theta}}{(b-a)\theta}} = e^{\Lambda(t)\left(\frac{e^{b\theta}-e^{a\theta}}{(b-a)\theta}-1\right)},
\end{aligned}$$

and

$$\kappa_S(\theta) = \Lambda(t)\Big(\frac{e^{b\theta} - e^{a\theta}}{(b-a)\theta} - 1\Big).$$

The moment generating and cumulant functions take the following forms, if the exponential kernel is used

$$\begin{aligned}
M_S(\theta) &= \sum_{k=0}^{\infty} \frac{e^{-\Lambda(t)}\Lambda(t)^k}{k!}\Big(\frac{e^{b\theta} - e^{a\theta}}{(b-a)\theta}\Big)^k \\
&= e^{-\Lambda(t)} \sum_{k=0}^{\infty} \frac{(\Lambda(t)\frac{e^{b\theta}-e^{a\theta}}{(b-a)\theta})^k}{k!} \\
&= e^{-\Lambda(t)} e^{\Lambda(t)\frac{e^{b\theta}-e^{a\theta}}{(b-a)\theta}} = e^{\int_{w_i}^{t+w_i} e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|_+}} dr \left(\frac{e^{b\theta}-e^{a\theta}}{(b-a)\theta}-1\right)},
\end{aligned}$$

and the cumulant function is

$$\kappa_S(\theta) = \int_{w_i}^{t+w_i} e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|_+}} dr\Big(\frac{e^{b\theta} - e^{a\theta}}{(b-a)\theta} - 1\Big),$$

and the power kernel will change these to

$$\begin{aligned}
M_S(\theta) &= \sum_{k=0}^{\infty} \frac{e^{-\Lambda(t)}\Lambda(t)^k}{k!}\Big(\frac{e^{b\theta} - e^{a\theta}}{(b-a)\theta}\Big)^k \\
&= e^{-\Lambda(t)} \sum_{k=0}^{\infty} \frac{\Big(\Lambda(t)\frac{e^{b\theta}-e^{a\theta}}{(b-a)\theta}\Big)^k}{k!} \\
&= e^{-\Lambda(t)} e^{\Lambda(t)\frac{e^{b\theta}-e^{a\theta}}{(b-a)\theta}} = e^{\int_{w_i}^{t+w_i} e^{\alpha_0+\sum_{j=1}^{k}\alpha_j (t_i-s_j)|_+^{-\beta}} dt \left(\frac{e^{b\theta}-e^{a\theta}}{(b-a)\theta}-1\right)},
\end{aligned}$$

and the cumulant function is

$$\kappa_S(\theta) = \int_{w_i}^{t+w_i} e^{\alpha_0+\sum_{j=1}^{k}\alpha_j (t_i-s_j)|_+^{-\beta}} dr\Big(\frac{e^{b\theta} - e^{a\theta}}{(b-a)\theta} - 1\Big).$$

4- If X is distributed with Logistic (μ, β_l)

$$\begin{aligned}
M_S(\theta) &= \sum_{k=0}^{\infty} \frac{e^{-\Lambda(t)}\Lambda(t)^k}{k!}(e^{\mu\theta}\Gamma(1-\beta_l\theta)\Gamma(1+\beta_l\theta))^k \\
&= e^{-\Lambda(t)} \sum_{k=0}^{\infty} \frac{\Big(\Lambda(t)e^{\mu\theta}\Gamma(1-\beta_l\theta)\Gamma(1+\beta_l\theta)\Big)^k}{k!} \\
&= e^{-\Lambda(t)} e^{\Lambda(t)e^{\mu\theta}\Gamma(1-\beta_l\theta)\Gamma(1+\beta_l\theta)} = e^{\Lambda(t)\Big((e^{\mu\theta}\Gamma(1-\beta_l\theta)\Gamma(1+\beta_l\theta))-1\Big)},
\end{aligned}$$

and the cumulant function is

$$\kappa_S(\theta) = \Lambda(t)\Big((e^{\mu\theta\Gamma(1-\beta_l\theta)\Gamma(1+\beta_l\theta)}) - 1\Big).$$

The moment generating and the cumulant functions for this distribution will be as follows with the use of the exponential kernel

$$\begin{aligned}
M_S(\theta) &= \sum_{k=0}^{\infty} \frac{e^{-\Lambda(t)}\Lambda(t)^k}{k!}(e^{\mu\theta}\Gamma(1-\beta_l\theta)\Gamma(1+\beta_l\theta))^k \\
&= e^{-\Lambda(t)} \sum_{k=0}^{\infty} \frac{\Big(\Lambda(t)e^{\mu\theta}\Gamma(1-\beta_l\theta)\Gamma(1+\beta_l\theta)\Big)^k}{k!} \\
&= e^{-\Lambda(t)} e^{\Lambda(t)e^{\mu\theta}\Gamma(1-\beta_l\theta)\Gamma(1+\beta_l\theta)} = e^{\int_{w_i}^{t+w_i} e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|+}} dr \Big((e^{\mu\theta\Gamma(1-\beta_l\theta)\Gamma(1+\beta_l\theta)})-1\Big)},
\end{aligned}$$

and the cumulant function is

$$\kappa_S(\theta) = \int_{w_i}^{t+w_i} e^{\alpha_0+\sum_{j=1}^{k}\alpha_j e^{-\beta(t_i-s_j)|+}} dr\Big((e^{\mu\theta\Gamma(1-\beta_l\theta)\Gamma(1+\beta_l\theta)}) - 1\Big),$$

and when $\Lambda(t)$ is replaced with the power kernel

$$\begin{aligned}
M_S(\theta) &= \sum_{k=0}^{\infty} \frac{e^{-\Lambda(t)}\Lambda(t)^k}{k!}(e^{\mu\theta}\Gamma(1-\beta_l\theta)\Gamma(1+\beta_l\theta))^k \\
&= e^{-\Lambda(t)} \sum_{k=0}^{\infty} \frac{\Big(\Lambda(t)e^{\mu\theta}\Gamma(1-\beta_l\theta)\Gamma(1+\beta_l\theta)\Big)^k}{k!} \\
&= e^{-\Lambda(t)} e^{\Lambda(t)e^{\mu\theta}\Gamma(1-\beta_l\theta)\Gamma(1+\beta_l\theta)} = e^{\int_{w_i}^{t+w_i} e^{\alpha_0+\sum_{j=1}^{k}\alpha_j (t_i-s_j)|_+^{-\beta}} dr \Big((e^{\mu\theta\Gamma(1-\beta_l\theta)\Gamma(1+\beta_l\theta)})-1\Big)},
\end{aligned}$$

and the cumulant function is

$$\kappa_S(\theta) = \int_{w_i}^{t+w_i} e^{\alpha_0+\sum_{j=1}^{k}\alpha_j (t_i-s_j)|_+^{-\beta}} dr\Big((e^{\mu\theta\Gamma(1-\beta_l\theta)\Gamma(1+\beta_l\theta)}) - 1\Big).$$

Remark 7:

If a random variable $X \sim$ lognormal, then the probability density function of X is

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{\left(\frac{-(\log x-\mu)^2}{2\sigma^2}\right)},$$

where $-\infty < x < \infty$, with mean

$$E(X) = e^{\mu+\frac{\sigma^2}{2}},$$

and variance

$$Var(X) = (e^{\sigma^2-1})e^{2\mu+\sigma^2}.$$

Copula

In the actuarial context, a copula is a mathematical function, which is used to estimate the dependence structure of the claims in the form of the total claim amount (aggregate claims). The name 'copula' is first introduced by Abe Skler in 1959 as a function, which couples (connects) a joint distribution function with its marginal distributions [Pfeifer and Neslehová, 2003, Walhin, 2002, Emanuele, 2004, Cebrián et al., 2003]. Copulas provide a means to construct random vectors with a wide range of possible joint distributions [Kaas et al., 2001].

Let us assume the random variable $X = (X_1, \ldots, X_n)$ has the following distribution function

$$F(x_1, \ldots, x_n) = Pr(X_1 < x_1, \ldots, X_n < x_n).$$

The copula is based on the transformation using the marginal distribution of the random variable X and can be denoted as

$$F(x_1, \ldots, x_n) = C(F_1(x_1), \ldots, F_n(x_n)),$$

or

$$C(c_1, \ldots, c_n) = F(F_1^{-1}(x_1), \ldots, F_n^{-1}(x_n)),$$

where the $F_1^{-1}(x_1), \ldots, F_n^{-1}(x_n)$ denotes the quantiles of the univariate marginal $F_1, \ldots, F_n$. In practice, marginal distribution is usually provided.

The uniform continuity and the existence of all partial derivatives provide an easy and wide use of copulas in financial and actuarial analysis in the past decade. In [Pfeifer and Neslehová, 2003], some basic and useful properties of copulas are given as:

1. The range of a copula C is the $[0, 1]$ unit interval,
2. $C(c)$ is zero for all c in $[0, 1]^n$ for which at least one coordinate is zero,
3. $C(c) = c_k$ if all coordinates of c are 1 except the k^{th} one,

4. C is n-increasing in the sense that every $a \leq b$ in $[0,1]^n$ the value assigned by C to the n-box $[a,b] = [a_1, b_1] \times \ldots \times [a_n, b_n]$,

5. If all the margins are continuous, then the copula is unique.

Regression Methods

Various methods have been developed to build regression models by either adding or deleting regressors (explanatory variables) of the model one at a time. There are three main categories of the selection procedure [Montgomery et al., 2001b]:

1. Forward selection

   The method starts with the assumption that there are no regressors in the model except the intercept term. The regressors are inserted to model one by one, generally starting with the one, which has the highest simple correlation with the response variable y.

2. Backward elimination

   This works in the opposite direction of the forward selection. All the possible variables are used in the model and the ones with smaller correlation and F-values are eliminated until a suitable model is obtained.

3. Stepwise regression

   Stepwise regression is a modification of the forward and backward selection methods. This type of selection is generally preferred as it is fast, available in almost all computer packages and easy to implement. The analyst should consider all possible subset models before deciding on the optimal model.

Basic properties of a kernel function

A kernel function $K(t)$ mainly have the following properties:

1. $K(t) \geq 0$ for all t,

2. $\int_{-\infty}^{+\infty} K(t)dt = 1$,

3. $K(-t) = K(t)$.

Deviance and Akaike Criterion Information (AIC)

Deviance

The statistical softwares generally calculates deviance values for the generalised linear models based on twice the difference of the log-likelihood from that for a saturated model [Lindsey, 1997]. That is

$$D = 2\Big(L(\text{saturated}) - L(\text{the model under consideration with the parameter})\Big),$$

and the deviance is distributed as χ^2_p, where p is the number of parameters

$$D \sim \chi^2_p.$$

The use of deviance is convenient in modelling since it provides the comparison of the models additively instead more complex models (e.g. multiplicative, interaction models) [Lindsey, 1997].

In this thesis, since we are using the additive models to explain our research, the minimum deviance is used as a criteria to pick the required non-linear parameter β estimate. The automatically generated deviance for our choices of the models is given with:

$$D = 2\Big(L(\text{saturated}) - L(\text{the model under consideration with the parameter } \beta)\Big)$$

AIC

During the model selection process, AIC is used as one of the main tools to help the researchers to decide on a better fit model. The larger models provide a better fit with smaller residual sum of squares but use more parameters. A preferable model is the one, which explains a lot of the response variable with a good model size, not too many parameters [Faraway, 2005]. Akaike Criterion Information is calculated as

$$\text{AIC} = -2\text{max log-likelihood} + 2p,$$

where p is the number of parameters used in the model. An alternative to the AIC is the Bayesian Information Criterion (BIC), which is

$$\text{BIC} = -2\text{max log-likelihood} + p \log n,$$

where n is the number of observations.

Both the AIC and BIC are used to provide a 'penalty' for the likelihood, which is called the 'penalised likelihood', of more complex models rather than the additive case [Lindsey, 1997]. The BIC penalises larger models more heavily than the AIC. This causes a tendency to prefer smaller models compared to the AIC. The models of this thesis are not based on the idea of the penalised likelihood as the additive models with the deviance criteria seems to be enough to explain the research conducted here.

Example: A 2×2 Contingency Table

In this example, we wanted to check the independency of the number of claims and the risk zone effect in Turkey. There are 4297 claims in the data of this study. The number of claims are divided by zones as: Zone 1=3602, Zone 2=676 and Zone 3 & 4=19. The following table is prepared from the data to check the independency of the number of claims more than 100 and less than 100 in zones 1 and 2. Also, the log-odds for the Poisson count data can be estimated and the confidence interval for this estimate can be constructed.

The number of claims N_i	$N_i < 100$	$N_i > 100$	Total
Zone 1	177	3425	3602
Zone 2	215	461	676
Total	392	3886	4278

from the table above, the expected values of the corresponding observations are calculated as

$$e_{11} = \frac{3602 \times 392}{4278} = 330.057$$
$$e_{12} = \frac{3602 \times 3886}{4278} = 3271.943$$
$$e_{21} = \frac{676 \times 392}{4278} = 61.943$$
$$e_{22} = \frac{676 \times 3886}{4278} = 614.057$$

The following expected values table is obtained by using the expected values above.

The number of claims N_i	$N_i < 100$	$N_i > 100$
Zone 1	330.057	3271.943
Zone 2	61.943	614.057

The following hypothesis is constructed to check the independency of the claim number and the zone effect. Let

$$H_0 : \text{Claims from different zones are independent}$$

$$H_1 : \text{Claims from different zones are not independent}$$

In this case, the test-statistic is χ^2

$$\chi^2 = \sum_n \frac{(o_i - e_i)^2}{e_i} = \frac{(177 - 330.057)^2}{330.057} + \ldots + \frac{(461 - 614.057)^2}{614.057} = 494.48$$

At $\alpha = 0.05$ significance level, the table value of χ^2 is $\chi^2_{(2-1)(2-1);0.05} = 3.841$. Since the calculated χ^2 is much greater than the table value, that is $\chi^2 = 494.48 > 3.841$, the null hypothesis H_0 is rejected at this significance level. Also, at $\alpha = 0.10$ significance level, the calculated $\chi^2 = 494.48 > \chi^2_{1;0.10} = 2.706$ so H_0 is rejected again. This indicates that there is a significant relation in the number of claims received in earthquake risk zones 1 and 2 in Turkey. Moreover, the estimate of the odds-ratio is $\frac{34256 \times 215}{177 \times 461} = 9.023$ which shows the probability of the claims occurring to they are not occurring (the range of odds is $(0, +\infty)$).

The proportion of the claims can be also tested, for the case when N_i is less than or greater than 100. The related hypothesis is constructed as

$$H_0 : p_1 = p_2$$

$$H_1 : p_1 \neq p_2$$

where $\hat{p_1} = \frac{3886}{4278} = 0.908$ and the log-odds (logit) is

$$\hat{l} = \text{logit}(\hat{p_1}) = \ln(\frac{3886}{392}) = 2.29.$$

The deviance for the estimate $\hat{l}$ is

$$sd(\hat{l}) = \sqrt{\frac{1}{4278(\frac{392}{4278})(\frac{3886}{4278})}} = 0.053.$$

The 95 % confidence interval for the logit will be in the form

$$\hat{l} \pm z_{\frac{\alpha}{2}} sd(\hat{l}) = 2.29 \pm (1.96)(0.053) = (2.186, 2.394),$$

and for 90 % confidence interval

$$\hat{l} \pm z_{\frac{\alpha}{2}} sd(\hat{l}) = 2.29 \pm (1.645)(0.053) = (2.203, 2.377),$$

which gives a narrower confidence interval for the estimate $\hat{l}$ for $N_i > 100$ case.

The tables of the number of claims and the calendar time

There are 3602 earthquake insurance claims arriving from risk zone 1 with the following frequency table:

Months	Frequency	Months	Frequency
12	6	28	3
13	1	29	19
14	0	30	17
15	0	31	1
16	0	32	0
17	0	33	0
18	130	34	0
19	3	35	0
20	6	36	0
21	1	37	120
22	132	38	32
23	0	39	8
24	3	40	1708
25	45	41	423
26	912	42	1
27	2	43	29

For zone 2676 claims have been received by the Turkish Catastrophe Insurance Pool.

Months	Frequency	Months	Frequency
12	0	28	0
13	0	29	2
14	0	30	2
15	0	31	0
16	0	32	6
17	2	33	3
18	0	34	0
19	8	35	1
20	0	36	0
21	0	37	41
22	46	38	58
23	3	39	0
24	3	40	2
25	1	41	37
26	461	42	0
27	0	43	0

There are 19 observations from zone 3 (9) and zone 4 (10), which are observed in the following tables, respectively.

Months	Frequency	Months	Frequency
12	0	28	0
13	0	29	1
14	0	30	0
15	0	31	0
16	0	32	0
17	0	33	0
18	0	34	0
19	0	35	0
20	0	36	0
21	0	37	0
22	1	38	2
23	0	39	0
24	0	40	1
25	0	41	0
26	3	42	0
27	0	43	1

Months	Frequency	Months	Frequency
12	0	28	0
13	0	29	0
14	0	30	0
15	0	31	0
16	0	32	0
17	0	33	0
18	0	34	0
19	0	35	0
20	0	36	0
21	0	37	0
22	0	38	2
23	0	39	0
24	0	40	0
25	0	41	0
26	0	42	0
27	8	43	0

In the next page, the number of claims arriving to zone 1 is given in terms of weeks data:

Weeks	Frequency	Weeks	Frequency
1	6	70	0
2	0	71	0
3	0	72	0
4	0	73	3
5	0	74	14
6	2	75	0
7	0	76	2
8	0	77	0
9	0	78	0
10	0	79	0
11	0	80	3
12	0	81	14
13	0	82	0
14	0	83	0
15	0	84	0
16	0	85	0
17	0	86	2
18	0	87	0
19	0	88	0
20	0	89	0
21	0	90	0
22	0	91	0
23	0	92	0
24	0	93	0
25	0	94	0
26	0	95	0
27	0	96	0
28	2	97	0
29	128	98	0
30	0	99	0
31	2	100	0
32	0	101	0
33	0	102	0
34	2	103	0
35	4	104	0
36	0	105	0
37	2	106	0
38	0	107	0
39	0	108	0
40	2	109	0
41	0	110	0
42	0	111	0
43	0	112	120
44	0	113	31
45	4	114	0
46	0	115	2
47	128	116	0
48	0	117	0
49	0	118	8
50	0	119	0
51	2	120	0
52	0	121	0
53	2	122	1682
54	2	123	26
55	0	124	0
56	0	125	423
57	0	126	0
58	0	127	0
59	4	128	0
60	939	129	0
61	0	130	0
62	0	131	0
63	0	132	2
64	15	133	0
65	0	134	11
66	2	135	0
67	0	136	0
68	0	137	17
69	3	138	2

For zone 2:

Weeks	Frequency	Weeks	Frequency
1	2	52	0
2	0	53	0
3	0	54	0
4	0	55	0
5	0	56	2
6	0	57	2
7	8	58	0
8	0	59	0
9	0	60	0
10	0	61	0
11	0	62	0
12	0	63	4
13	0	64	2
14	0	65	0
15	0	66	0
16	0	67	0
17	0	68	0
18	0	69	0
19	0	70	3
20	0	71	0
21	35	72	0
22	0	73	0
23	11	74	0
24	0	75	0
25	0	76	0
26	0	77	0
27	6	78	2
28	0	79	0
29	0	80	0
30	0	81	0
31	0	82	0
32	0	83	0
33	0	84	0
34	0	85	0
35	2	86	0
36	448	87	0
37	0	88	41
38	0	89	41
39	0	90	0
40	13	91	17
41	0	92	0
42	0	93	0
43	0	94	0
44	0	95	0
45	0	96	0
46	0	97	0
47	0	98	2
48	0	99	0
49	0	100	0
50	2	101	37
51	0	102	0

Table 10.6 gives the information on the significant British earthquakes of the twentieth century and compiled from the British Geological Survey (BGS-http://www.earthquakes.bgs.ac.uk/earthquakes/historical/historical_search_date.htm).

Date	Magnitude	Location
18/09/1901	5.0	Inverness
24/03/1903	4.6	Derby
19/06/1903	4.9	Caernarvon
27/06/1906	5.2	Swansea
14/10/1916	4.6	Stafford
30/07/1926	5.5	Channel Islands
15/08/1926	4.8	Ludlow
24/01/1927	5.7	North Sea
17/02/1927	5.4	Channel Islands
19/11/1927	4.9	Normandy
07/06/1931	6.1	Dogger Bank
12/04/1933	5.2	Normandy
12/12/1940	4.7	North Wales
30/12/1944	4.8	Skipton
11/02/1957	5.3	Derby
12/02/1957	4.2	Derby
09/02/1958	5.1	North Sea
02/01/1959	5.4	Brittany
03/11/1976	4.5	Widnes
26/12/1979	4.7	Carlisle
19/07/1984	5.4	Lleyn Peninsula
02/04/1990	5.1	Bishop's Castle
13/04/1992	5.9	Roermond

Table 10.5: Significant British earthquakes in the twentieth century.

Bibliography

[ulk, 2000] (2000). The Management of Catastrophic Risks Using Pooled Insurance Structures and Alternative Financing & Risk Transfer Mechanisms, Discussion Draft, The Reinsurance Market and the Case of Caribbean Region.

[Adler et al., 1998] Adler, R. J., Feldman, R. E., and Taqqu, M. S. (1998). *A Practical Guide to Heavy Tails.* Birkhäuser.

[Aitchison and Lauder, 1985] Aitchison, J. and Lauder, I. J. (1985). Kernel density estimation for compositional data. *Applied Statistics*, 34(2):129–137.

[Akbank, 2001a] Akbank (2001a). Turkish economy August 2001. Technical report, The library of the Central Bank of Turkey.

[Akbank, 2001b] Akbank (2001b). Turkish economy June-July 2001. Technical report, The library of the Central Bank of Turkey.

[Akbank, 2001c] Akbank (2001c). Turkish economy October-November 2001. Technical report, The library of the Central Bank of Turkey.

[Akgönenç and Verivakis, 1999] Akgönenç, O. and Verivakis, E. (13/12/1999). Economic consequences of the recent earthquakes in Turkey and Greece. Technical report, Council of Europe.

[Albala-Bertrand, 1993a] Albala-Bertrand, J. M. (1993a). Natural disaster situations and growth: A macroeconomic model for sudden disaster impacts. *World Development*, 21(9):1417–1434.

[Albala-Bertrand, 1993b] Albala-Bertrand, J. M. (1993b). *Political Economy of Large Natural Disasters.* Clarendon Press.

[Albrecher and Asmussen, 2005] Albrecher, H. and Asmussen, S. (2005). Ruin probabilities and aggregate claims distributions for shot noise Cox processes. Technical report, Centre for Analytical Finance University of Aarhus Aarhus School of Business. Working Paper Series No.211.

[Alesch and Petak, 2001] Alesch, D. J. and Petak, W. J. (2001). Overcoming obstacles to implementing earthquake hazard mitigation policies. Technical report, IIASA, www.iiasa.ac.at/Research/RMS.

[Alexander, 1997] Alexander, D. (1997). The study of natural disasters, 1977-1997: Some reflections on a changing field of knowledge. *Disasters*, 21(4):284–304.

[Amendola et al., 2000] Amendola, A., Ermoliev, Y., and Ermolieva, T. (2000). Earthquake risk management: a case study for an Italian region. Technical report, IIASA, www.iiasa.ac.at/Research/RMS/july2000/papers.html.

[Andersen et al., 1993] Andersen, P. K., Borgan, Ø., Gill, R. D., and Keiding, N. (1993). *Statistical Models Based on Counting Processes.* Springer-Verlag.

[Anderson, 1974] Anderson, D. R. (1974). The national flood insurance program problems and potential. *The Journal of Risk and Insurance*, 41(4):579–599.

[Anderson and Weinrobe, 1986] Anderson, D. R. and Weinrobe, M. (1986). Insurance issues related to mortgage default risks associated with natural disasters. *The Journal of Risk and Insurance*, 53(3):501–513.

[Andrews et al., 1999] Andrews, G. E., Askey, R., and Roy, R. (1999). *Special Functions.* Cambridge University Press.

[Ashimbaev and Itskov, 2000] Ashimbaev, M. and Itskov, I. (2000). The lessons of the destructive earthquakes occurred in Turkey and Armenia. Technical report, IIASA, www.iiasa.ac.at/Research/RMS/july2000/papers.html.

[Asmussen, 2000] Asmussen, S. (2000). *Ruin Probabilities.* World Scientific.

[Baranov et al., 2002] Baranov, S., Digas, B., Ermolieva, T., and Rozenberg, V. (2002). Earthquake risk management: A scenario generator. Technical report, IIASA, www.iiasa.ac.at/Research/RMS.

[Barndorff-Nielsen and Cox, 1989] Barndorff-Nielsen, O. E. and Cox, D. R. (1989). *Asymptotic Techniques for Use in Statistics.* Chapman and Hall.

[Basu and Dassios, 2002] Basu, S. and Dassios, A. (2002). A Cox Process with Log-Normal Intensity. Technical report, Department of Statistics, London School of Economics.

[Bates and Watts, 1988] Bates, D. M. and Watts, D. G. (1988). *Nonlinear Regression Analysis and Its Applications.* John Wiley & Sons.

[BBC, 2001] BBC (2001). Hurricane Michelle batters Cuba.

[Beattie, 2001] Beattie, A. (2001). Economic Disaster for Developing Countries-www.ft.com.

[Beirlant and Teugels, 1992] Beirlant, J. and Teugels, J. L. (1992). Modelling large claims in life insurance. *Insurance: Mathematics and Economics*, 11:17–29.

[Benckert and Jung, 1974] Benckert, L.-G. and Jung, J. (1974). Statistical models of claim distributions in fire insurance. *ASTIN Bulletin*, 8:1–25.

[Bendimerad, 2001] Bendimerad, F. (2001). Modelling the macroeconomic impacts of natural disasters and megacities-megarisks.

[Benktander, 1978] Benktander, G. (1978). Largest claims reinsurance - A quick method to calculate LCR rates from the excess of loss risk rates. *ASTIN Bulletin*, 10:54–58.

[Bensallah, 2000] Bensallah, Y. (2000). Steps in applying Extreme Value Theory to Finance: A Review. Working paper, Bank of Canada, www.bank-banque-canada.ca/en/res/wp00-20.htm.

[Benson and Clay, 2004] Benson, C. and Clay, E. J. (2004). *Understanding the Economic and Financial Impacts of Natural Disasters.* The World Bank.

[Bertoin, 1996] Bertoin, J. (1996). *Lévy Processes.* Cambridge University Press.

[Bibbee, 2001] Bibbee, A. (2001). Economic Survey of Turkey, 2000-01. Technical report, OECD Economics Department, Paris.

[Bibbee et al., 2000] Bibbee, A., Gönenç, R., Jacobs, S., Konvitz, J., and Price, R. (2000). Economic effects of the 1999 Turkish earthquakes: An interim report. Technical report, OECD Economics Department Working Papers No. 247, Paris.

[Bierens and Gallant, 1997] Bierens, H. J. and Gallant, A. R. (1997). *Nonlinear Models*, volume 2. Edward Elgar Publishing Company.

[Bingham, 2002] Bingham, N. H. (2002). Probability Models Based on Processes with Independent Increments, Ten-Lecture Postgraduate Course, Royal Statistical Society, Nuffield College, Oxford, 23-27 September 2002.

[Bingham et al., 1987] Bingham, N. H., Goldie, C. M., and Teugels, J. L. (1987). *Regular Variation.* Cambridge University Press.

[Blaikie et al., 1994] Blaikie, P., Cannon, T., Davis, I., and Wisner, B. (1994). *At Risk: natural hazards, people's vulnerability, and disasters.* Routledge.

[Bolt, 1988] Bolt, A. B. (1988). *Earthquakes.* W. H. Freeman and Company.

[Booth et al., 1999] Booth, P., Chadburn, R., Cooper, D., Haberman, S., and James, D. (1999). *Modern Actuarial Theory and Practice.* CRC Press.

[Borch, 1978] Borch, K. (1978). Problems in the economic theory of insurance. *ASTIN Bulletin*, 10:1–11.

[Bouye, 2002] Bouye, E. (2002). Multivariate extremes at work for portfolio risk measurement. http://gloriamundi.org/var/pub/ebo.pdf.

[Bowman and Azzalini, 1997] Bowman, A. W. and Azzalini, A. (1997). *Applied Smoothing Techniques for Data Analysis: The Kernel Approach with S-Plus Illustrations.* Oxford Science Publications.

[Bresch et al., 2000] Bresch, D. N., Bisping, M., Lemcke, G., and Perils, C. (2000). Storm over europe an underestimated risk. Technical report, Swiss Re Reinsurance and Risk Division.

[Briane, 1999] Briane, J. (21/12/1999). Economic consequences of the recent earthquakes in Turkey and Greece. Technical report, Council of Europe.

[Brooks and Marron, 1991] Brooks, M. M. and Marron, J. S. (1991). Asymptotic optimality of the least-squares cross validation bandwidth for kernel estimates of intensity functions. *Stochastic Processes and Their Applications*, 38:157–165.

[Brookshire et al., 1985] Brookshire, D. S., Thayer, M. A., Tschirhart, J., and Schulze, W. D. (1985). A test of the expected utility model: evidence from earthquake risks. *The Political Economy*, 3(2):369–389.

[Bühlmann, 1970] Bühlmann, H. (1970). *Mathematical Methods in Risk Theory.* Springer-Verlag.

[Burby, 1991] Burby, R. J. (1991). *Sharing Environmental Risks: How to Control Government's Losses in Natural Disasters.* Westview Press.

[Cameron and Trivedi, 1998] Cameron, A. C. and Trivedi, P. K. (1998). *Regression analysis of count data.* Cambridge University Press.

[Carlson and Doyle, 1999] Carlson, J. M. and Doyle, J. (1999). Highly optimised tolerance: A mechanism for power laws in designed systems. Technical report, http://www.aip.org/enews/physnews/2000/split/pnu474-1.htm.

[Castillo, 1988] Castillo, E. (1988). *Extreme Value Theory in Engineering.* Academic Press.

[Cebrián et al., 2003] Cebrián, A. C., Denuit, M., and Lambert, P. (2003). Analysis of bivariate tail dependence using extreme value copulas: An application to the SOA medical large claims database. *Belgian Actuarial Bulletin*, 3(1).

[Celasun et al., 1999] Celasun, O., Denizer, C., and He, D. (1999). Capital flows, macroeconomic management and the financial system: The Turkish case, 1989-97. Technical report, World Bank.

[Chang, 2000] Chang, S. E. (2000). Disaster and Transport Systems: Loss, Recovery and Competition at the Port of Kobe after the 1995 earthquake. *Journal of Transport Geography*, 8:53–65.

[Chang et al., 1996] Chang, S. E., Seligson, H. A., and Eguchi, R. T. (1996). Estimation of the economic impact of multiple lifeline disruption. Technical report, NCEER Bulletin.

[Chen et al., 2004] Chen, T.-I., Huang, C.-S., and Hong, C.-H. (2004). Statistical Assesment for the Hazard of Large Aftershocks Post to the Chi-Chi Mainshock. *TAO*, 15(3):493–502.

[Cho et al., 2001] Cho, S., Gordon, P., II, J. E. M., Richardson, H. W., Shinozuka, M., and Chang, S. (2001). Integrating transportation network and regional economic models to estimate the costs of a large urban earthquake. *Journal of Regional Science*, 41(1):39–45.

[Cinlar, 1975] Cinlar, E. (1975). *Introduction to Stochastic Process*. Prentice-Hall Inc.

[Coburn and Spence, 1992] Coburn, A. and Spence, R. (1992). *Earthquake Protection*. John Wiley and Sons.

[Cohen et al., 1973] Cohen, A., Cutts, J. F., Fielder, R., Jones, D. E., Ribbans, J., and Stuart, E. (1973). *Numerical Analysis*. John Wiley & Sons.

[Coles, 2001] Coles, S. (2001). *An Introduction to Statistical Modeling of Extreme Values*. Springer.

[Cox and Isham, 1980] Cox, D. R. and Isham, V. (1980). *Point processes.* Chapman & Hall.

[Cox and Lewis, 1966] Cox, D. R. and Lewis, P. A. W. (1966). *The Statistical Analysis of Series of Events.* Methuen & Co Ltd.

[Cox and Miller, 1965] Cox, D. R. and Miller, H. D. (1965). *The Theory of Stochastic Processes.* Methuen and Co Ltd.

[Cuny, 1983] Cuny, F. C. (1983). *Disasters and Development.* Oxford University Press.

[Dacy and Kunreuther, 1969] Dacy, D. C. and Kunreuther, H. (1969). *The Economics of Natural Disasters: Implications for Federal Policy.* The Free Press.

[Daley and Vere-Jones, 2003] Daley, D. J. and Vere-Jones, D. (2003). *An Introduction to the Theory of Point Processes.* Springer.

[Danielsson and de Vries, 1997] Danielsson, J. and de Vries, C. G. (1997). Tail index and quantile estimation with very high frequency data. *Journal of Empirical Finance*, 4:241–257.

[Dassios and Jang, 2000] Dassios, A. and Jang, J.-W. (2000). Pricing of catastrophe reinsurance and derivatives using the Cox process with shot noise intensity. Technical report, Department of Statistics, London School of Economics.

[Datta, 1992] Datta, S. (1992). Some asymptotic bounds for Li density estimation using kernels. *The Annals of Statistics*, 20:1658–1667.

[Daykin et al., 1994a] Daykin, C. D., Pentikäinen, T., and Pesonen, M. (1994a). *Practical Risk Theory for Actuaries.* Chapman & Hall.

[Daykin et al., 1994b] Daykin, C. D., Pentikäinen, T., and Pesonen, M. (1994b). *Practical Risk Theory for Actuaries.* Chapman and Hall.

[de Boor, 1978] de Boor, C. (1978). *A Practical Guide to Splines.* Springer-Verlag.

[Demirtaş, 2000a] Demirtaş, R. (2000a). 17 Ağustos 1999 Izmit Körfezi Deprem Raporu (in Turkish). Technical report.

[Demirtaş, 2000b] Demirtaş, R. (2000b). 6 Haziran 2000 Orta çankiri depremi raporu (in Turkish). Technical report, http://sismo.deprem.gov.tr/orta/orta1.htm.

[Devarajan et al., 1993] Devarajan, S., Lewis, J. D., and Robinson, S. (1993). External shock, purchasing power parity, and the equilibrium real exchange rate. *The World Bank Economic Review*, 7(1):45–63.

[Diggle, 1985] Diggle, P. (1985). A kernel method for smoothing point process data. *Applied Statistics*, 34(2):138–147.

[Dobson, 1990] Dobson, A. J. (1990). *An Introduction to Statistical Modelling.* Chapman and Hall.

[Doherty and Clifford W. Smith, 1999] Doherty, N. and Clifford W. Smith, J. (1999). Corporate Insurance Strategy: The Case of British Petroleum. *Journal of Applied Corporate Finance*, 6(3):4–15.

[Doherty, 1998] Doherty, N. A. (1998). Financial innovation in the management of catastrophe risk. Technical report, Wharton Business School.

[Domaç and Ferri, 1998] Domaç, I. and Ferri, G. (1998). The Real Impact of Financial Shocks: Evidence from Korea. Technical Report 2010, The World Bank Working Paper.

[Drees et al., 2000] Drees, H., de Haan, L., and Resnick, S. (2000). How to make a Hill plot. *The Annals of Statistics*, (28):254–274.

[Edwards, 1972] Edwards, A. W. F. (1972). *Likelihood.* Cambridge University Press.

[Ellson et al., 1984] Ellson, R. W., Milliman, J. W., and Roberts, R. B. (1984). Measuring the regional economic effects of earthquakes and earthquake predictions. *Journal of Regional Science*, 24(4):559–579.

[Emanuele, 2004] Emanuele, V. (2004). Risk sharing between cedent and reinsurer when dependence is described by copulae. Technical report, Dipartimento di Statistica e Mathematica applicata all'Economia, Università di Pisa.

[Embrechts et al., 1997] Embrechts, P., Klüppelberg, C., and Mikosch, T. (1997). *Modelling Extremal Events for Insurance and Finance.* Springer, Germany.

[Engel and Meller, 1993] Engel, E. and Meller, P. (1993). *External Shocks and Stabilisation Mechanisms.* Inter-American Development Bank.

[EQE, 1999a] EQE (1999a). August 17, 1999, Izmit, Turkey earthquake. Technical report, EQE International, http://www.eqe.com/revamp/turkey2.htm.

[EQE, 1999b] EQE (1999b). August, 17 Marmara earthquake. Technical report, EQE International, http://www.eqe.com/revamp/izmitreport/izmit-1.pdf.

[Erdik, 2000] Erdik, M. (2000). Report on 1999 Kocaeli and Düzce (Turkey) earthquakes. Technical report, Department of Earthquake Engineering, Boğaziçi University, İstanbul, Turkey, http://koeri.boun.edu.tr/earthqk/Kocaelireport.pdf.

[Erdik, 2003] Erdik, M. (2003). Earthquake vulnerability of buildings and a mitigation strategy: Case of Istanbul. WBI Workshop.

[Ermolieva et al., 2001] Ermolieva, T., Rozenberg, V., and Blizorukova, M. (2001). Modelling earthquakes via computer programs. Technical report, IIASA, www.iiasa.ac.at/Research/RMS.

[Fahrmeir and Tutz, 1994] Fahrmeir, L. and Tutz, G. (1994). *Multivariate Statistical Modelling Based on Generalized Linear Models.* Springer-Verlag.

[Fang and Wu, 2005] Fang, Y. and Wu, R. (2005). On Gerber-Shiu discounted penalty function of the Sparre Andersen risk model with constant interest and constant divided barrier. Department of Mathematics and LPMC, Nankai University, Tianjin, P. R. China.

[Faraway, 2005] Faraway, J. J. (2005). *Linear Models with R.* Chapman & Hall/CRC.

[Finance et al., 2000] Finance, P. S., infrastructure department Latin America, and the Caribbean Region the World Bank (2000). The management of catastrophic risks using pooled insurance structures and alternative financing and risk transfer mechanisms / the reinsurance market and the case of the caribbean region.

[Freeman, 2001] Freeman, P. K. (2001). Estimating chronic risk from natural disasters in developing countries: A case study on Honduras. Technical report, World Bank.

[Freemand et al., 2002] Freemand, P. K., Martin, L., Mechler, R., Warner, K., and Hausman, P. (2002). Catastrophes and Development, Integrating Natural Catastrophes into Development Planning, Disaster Risk Management Working Paper Series No.4, World Bank, Washington DC.

[Froot, 1997] Froot, K. A. (1997). The limited financing of catastrophe risk: an overview. Technical report, National Bureau of Economic Research.

[Froot and O'Connell, 1997] Froot, K. A. and O'Connell, P. G. J. (1997). On the pricing of intermediate risks: theory and application to catastrophe insurance. Technical report, National Bureau of Economic Research(NBER).

[Froot and O'Connell, 1997] Froot, K. A. and O'Connell, P. G. J. (1997). On the pricing of intermediated risks: theory and application to catastrophe insurance. Technical report, National Bureau of Economic Research.

[Galambos et al., 1994] Galambos, J., Lechner, J., and Simiu, E. (1994). *Extreme Value Theory and Applications.* Kluwer Academic Publishers.

[Gençay et al., 2001] Gençay, R., Selçuk, F., and Ulugülyağci, A. (2001). Evim: A Software Package for Extreme Value Analysis in MATLAB. *Studies in Nonlinear Dynamics and Econometrics*, 5:213–239.

[Gentle, 2003] Gentle, J. E. (2003). *Random Number Generation and Monte Carlo Methods.* Springer.

[Gerber and Shiu, 2005] Gerber, H. U. and Shiu, E. S. W. (2005). The Time Value of Ruin in a Sparre Andersen Model: Ruin Theory by Divided Differences.

[Gilchrist, 2000] Gilchrist, W. G. (2000). *Statistical Modelling with Quantile Functions.* Chapman and Hall/CRC.

[Goovaerts et al., 1986] Goovaerts, M., de Vylder, F., and Haezendonck, J. (1986). *Insurance and Risk Theory.* Reidel Pub. Co.

[Gospodinov and Rotondi, 2001] Gospodinov, D. and Rotondi, R. (2001). Exploratory analysis of marked poisson process applied to Balkan earthquake sequence. *Journal of the Balkan Geophysical Society*, 4(3):61–68.

[Granot, 2002] Granot, H. (2002). The choreography of death: observations from Turkey and Greece. *Disaster Prevention and Management*, 11(5):389–394.

[Grossi, 1999] Grossi, P. (1999). Assessing the benefits and costs of earthquake mitigation. Technical report, Wharton Business School.

[Grossi et al., 1999] Grossi, P., Kleindorfer, P., and Kunreuther, H. (1999). The impact of uncertainty in managing seismic risk: the case of earthquake frequency and structural vulnerability. Technical report, Wharton Business School.

[Guin and Saxena, 2000] Guin, J. and Saxena, V. (2000). Extreme losses from natural disasters - earthquakes, tropical cyclones and extratropical cyclones. Technical report, IIASA, www.iiasa.ac.at/Research/RMS/july2000/.

[Gülkan, 2000] Gülkan, P. (2000). Rebuilding the Sea of Marmara Region: Recent Structural Revisions in Turkey to Mitigate Disasters.

[Gumbel, 1958] Gumbel, E. J. (1958). *Statistics of Extremes.* Columbia University Press.

[Gurenko et al., 2006] Gurenko, E., Lester, R., Mahul, O., and Gonulal, S. O. (2006). *Earthquake Insurance in Turkey.* The World Bank.

[Haindl, 2001] Haindl, D. A. (2001). 11/September/2001. Technical report, Munich Re.

[Hauksson et al., 2001] Hauksson, H. A., Dacorogna, M., Domenig, T., Muller, U., and Samorodnitsky, G. (2001). Multivariate extremes, aggregation and risk estimation. *Quantative Finance*, 1:79–95.

[Hausmann and Perils, 1998] Hausmann, P. and Perils, C. (1998). Floods: an insurable risk? Technical report, Swiss Re Reinsurance and Risk Division.

[Healy, 1988] Healy, M. J. R. (1988). *Glim An Introduction.* Oxford University Press.

[Hogg and Klugman, 1984] Hogg, R. and Klugman, S. (1984). *Loss Distributions.* John Wiley & Sons, Inc., USA.

[Horwich, 1999] Horwich, G. (1999). Economic Lessons of the Kobe Earthquake. *Economic Development and Cultural Change*, 48(1):521–542.

[Howard, 2002] Howard, J. V. (2002). Regression and Generalised Linear Models Lecture Notes.

[Huet et al., 1996] Huet, S., Bouvier, A., Gruet, M.-A., and Jolivet, E. (1996). *Statistical Tools for Nonlinear Regression: A Practical Guide with S-Plus Examples.* Springer.

[Husler and Reiss, 1987] Husler, J. and Reiss, R.-D. (1987). *Extreme Value Theory: Proceedings of a Conference held in Oberwolfach.* Springer-Verlag.

[Ibarra and Mechler, 2006] Ibarra, H. and Mechler, R. (2006). Disasters and Development: Natural Disaster Framework for Developing Countries. Financial Strategies for Managing the Economic Impact of Disasters.

[IFRC, 2005] IFRC (2005). *World Disaster Report 2005.* IFRC.

[III, 1975] III, J. P. (1975). Statistical inference using extreme order statistics. *Annals of Statistics*, 3(1):119–131.

[JERC, 1998] JERC (1998). Aftershock Probability Evaluation Methods. Technical report, The Headquarters for Earthquake Research Promotion Earthquake Research Committee Prime Minister's Office Government of Japan, http://www.jishin.go.jp/main/yoshin2-e/yoshin2.htm.

[JICA, 2004] JICA (2004). Country Strategy Paper for Natural Disasters in Turkey. Technical report, Japan International Cooperation Agency.

[Johnson, 2000] Johnson, L. A. (2000). Earthquake loss modelling applications for disaster risk management: lessons from the 1999 Turkey, Greece and Taiwan earthquakes. Technical report, IIASA, www.iiasa.ac.at/Research/RMS/july2000/papers.html.

[Johnson et al., 1999] Johnson, R. A., Evans, J. W., and Green, D. W. (1999). Some bivariate distributions for modelling the strength properties of lumber. http://www.fpl.fs.fed.us/documents/FPLRP/fplrp575.pdf.

[Jones and Casti, 2000] Jones, R. D. and Casti, J. (2000). Insurance world: a process for understanding risk flows due to catastrophes in the insurance/reinsurance industry. Technical report, IIASA, www.iiasa.ac.at/Research/RMS/july2000/.

[Jorion, 1997] Jorion, P. (1997). *Value At Risk: The New Benchmark for Controlling Market Risk.* McGraw-Hill.

[Jr., 1988] Jr., W. A. T. (1988). *Point process models with applications to safety and reliability.* Chapman & Hall.

[Kaas et al., 2001] Kaas, R., Goovaerts, M., Dhaene, J., and Denuit, M. (2001). *Modern Actuarial Risk Theory.* Kluwer Academic Publishers.

[Kaas and Tang, 2005] Kaas, R. and Tang, Q. (2005). A large deviation result for aggregate claims with dependent claim occurrences. *Insurance: Mathematics and Economics*, (36):251–259.

[Kajitani et al., 2000] Kajitani, Y., Tatano, H., and Okada, N. (2000). Long-term Effects of the Hyogo Nanbu earthquake on the economic activities at the port of Kobe: Time Series Analysis. Technical report, IIASA, www.iiasa.ac.at/Research/RMS/july2000/.

[Karadağ and Westaway, 1999a] Karadağ, M. and Westaway, T. (1999a). A sam-based computable general equilibrium model of the Turkish economy. Technical report, University of Lougbough.

[Karadağ and Westaway, 1999b] Karadağ, M. and Westaway, T. (1999b). The impact on consumer and producer prices in Turkey of VAT changes designed to meet EU membership criteria: A computable general equilibrium model. Technical report, University of Lougbough.

[Karlin and Taylor, 1994] Karlin, S. and Taylor, M. H. (1994). *An Introduction to Stochastic Modelling.* Academic Press Inc.

[Karr, 1986] Karr, A. F. (1986). *Point Processes and Their Statistical Inference.* Marcel Dekker, Inc.

[Kauermann and Ortlieb, 2004] Kauermann, G. and Ortlieb, R. (2004). Temporal pattern in number of staff on sick leave: the effect of downsizing. *Applied Statistics*, 53(2):355–367.

[Keiding et al., 1998] Keiding, N., Andersen, C., and Fledelius, P. (1998). The Cox Regression Models for Claims Insurance Data in Non-life Insurance. *ASTIN Bulletin*, 28(1):95–118.

[Kharab and Guenther, 2002] Kharab, A. and Guenther, R. B. (2002). *An introduction to numerical methods: A Matlab Approach.* CRC Press.

[Klein and Moeschberger, 1997] Klein, J. P. and Moeschberger, M. L. (1997). *Survival Analysis Techniques for Censored and Truncated Data.* Springer.

[Kleindorfer and Kunreuther, 1998] Kleindorfer, P. and Kunreuther, H. (1998). The complementary roles of mitigation and insurance in managing catastrophic risks. Technical report, Wharton Business School.

[Kleindorfer and Sertel, 2001] Kleindorfer, P. and Sertel, M. (2001). *Mitigation and Financing of Seismic Risks: Turkish and International Perspectives*, volume 3 of *NATO Science Series-IV. Earth and Environmental Sciences.* Kluwer Academic Publishers, the Netherlands.

[Klugman et al., 1998] Klugman, S. A., Panjer, H. H., and Willmot, G. E. (1998). *Loss Models from Data to Decisions.* John Wiley & Sons, Inc.

[KOERI, 1999] KOERI (1999). Technical report, Kandilli Observatory and Earthquake Research Centre, İstanbul, Turkey, http://www.koeri.boun.edu.tr/earthqk.

[Kuhl and Bhairgond, 2000] Kuhl, M. E. and Bhairgond, P. S. (2000). Nonparametric Estimation of Nonhomogeneous Poisson Processes Using Wavelets. Proceedings of the 2000 Winter Simulation Conference.

[Kulkarni, 1995] Kulkarni, V. G. (1995). *Modelling and analysis of stochastic systems.* Chapman & Hall.

[Kunreuther, 1998] Kunreuther, H. (1998). Managing catastrophic risks through insurance and mitigation. Technical report, Wharton Business School.

[Kupper, 1971] Kupper, J. (1971). Contributions to the theory of the largest claim cover. *ASTIN Bulletin*, 6:134–146.

[Lawless, 1987] Lawless, J. F. (1987). Regression Methods for Poisson Process Data. *Journal of the American Statistical Association*, 82(399):808–815.

[Leadbetter et al., 1983] Leadbetter, M. R., Lindgren, G., and Rootzen, H. (1983). *Extremes and related properties of random sequences and processes.* Springer-Verlag.

[Lewis, 1972] Lewis, P. A. W. (1972). *Stochastic Point Processes: Statistical Analysis, Theory, and Applications.* Wiley Interscience.

[Lewis and Shedler, 1976] Lewis, P. A. W. and Shedler, G. S. (1976). Simulation of Nonhomogenous Poisson Processes with Log Linear Rate Function. *Biometrika*, 63(3):501–505.

[Lewis and Shedler, 1979] Lewis, P. A. W. and Shedler, G. S. (1979). Simulation of nonhomogenous Poisson processes by thinning. *IBM J. Research Development*, (32):403–413.

[Lindsey, 1995] Lindsey, J. K. (1995). Fitting Parametric Counting Processes by Using Log-Linear Models. *Applied Statistics*, 44(2):201–212.

[Lindsey, 1997] Lindsey, J. K. (1997). *Applying Generalised Linear Models.* Springer.

[Linnerooth-Bayer and Mechler, 2005] Linnerooth-Bayer, J. and Mechler, R. (2005). Financing Natural Disaster Risks in Developing and Emerging-Economy Countries. IIASA.

[Lustig, 1999] Lustig, N. (1999). First to suffer, last to recover. Technical report, IADB.

[MacNeill and Umphrey, 1987] MacNeill, I. B. and Umphrey, G. J. (1987). *Actuarial Science.* D. Reidel Publishing Company.

[Marron, 1988] Marron, J. S. (1988). Automatic smoothing parameter selection. *Empirical Economics*, 13:187–208.

[Martinez and Martinez, 2002] Martinez, W. L. and Martinez, A. R. (2002). *Computational Statistics Handbook with MATLAB.* Chapman & Hall/CRC.

[Matthes et al., 2000] Matthes, K., Kerstan, J., and Mecke, J. (2000). *Infinite divisible point processes.* Kluman.

[McNeil, 1997] McNeil, A. (1997). Estimating the tails of loss severity distributions using extreme value theory. *ASTIN Bulletin*, 27(1):117–137.

[McNeil, 2001] McNeil, A. (2001). EVIS 2001 Software for extreme values in S-Plus.

[Michels, 1992] Michels, P. (1992). Asymmetric kernel functions in non-parametric regression analysis and prediction. *The Statistician*, 41:439–454.

[Moat et al., 2000] Moat, A. M., Morrison, A. J. T., and Wong, S. (2000). Performance of industrial facilities during 1999 earthquakes:implications for risk managers. Technical report, IIASA, www.iiasa.ac.at/Research/RMS/july2000/papers.html.

[Montgomery et al., 2001a] Montgomery, D. C., Peck, E. A., and Vining, G. G. (2001a). *Introduction to Linear Regression Analysis.* John Wiley & Sons Inc.

[Montgomery et al., 2001b] Montgomery, D. C., Peck, E. A., and Vining, G. G. (2001b). *Introduction to Linear Regression Analysis.* John Wiley & Sons.

[Munasinghe and Clarke, 1995] Munasinghe, M. and Clarke, C. (1995). *Disaster Prevention and Sustainable Development.* World Bank.

[Nelder and Wedderburn, 1972] Nelder, J. A. and Wedderburn, R. W. M. (1972). Generalised Linear Models. *Journal of the Royal Statistical Society, Series A (General)*, 135(3):370–384.

[Neter et al., 1996] Neter, J., Kutner, M. H., Nachtsheim, C. J., and Wasserman, W. (1996). *Applied Linear Statistical Models.* WCB/McGraw-Hill.

[Newton L. Bowers et al., 1997] Newton L. Bowers, J., Gerber, H. U., Hickman, J. C., Jones, D. A., and Nesbitt, C. J. (1997). *Actuarial Mathematics.* The Society of Actuaries.

[Nomer, 2002] Nomer, C. (2002). An evaluation of the Turkish economy and insurance industry in 2000 and the projections for the year 2001 by the general director and manager.

[Nomer and Yunak, 1998] Nomer, C. and Yunak, H. (1998). *Reasürans (in Turkish).* Ceyma Matbaacilik.

[Northop, 2002] Northop, P. (2002). Statistical aspects of flood frequency estimation.

[O. E. Barndorff-Nielsen and D. R. Cox, 1989] O. E. Barndorff-Nielsen and D. R. Cox (1989). *Asymptotic Techniques for Use in Statistics.* Chapman and Hall.

[Ogata, 1988] Ogata, Y. (1988). Statistical Models for Earthquake Occurrences and Residual Analysis for Point Processes. *Journal of the American Statistical Association*, 83(401):9–27.

[Ozaki, 1979] Ozaki, T. (1979). Maximum Likelihood Estimation of Hawkes' Self-Exciting Point Processes. *Annals of the Institute of Statistical Mathematics*, 31:145–155.

[Özerdem and Barakat, 2000] Özerdem, A. and Barakat, S. (2000). After the Marmara earthquake: lessons for avoiding short cuts to disasters. *Third World Quarterly*, 21(3):425–439.

[Özmen and Bağci, 2000] Özmen, B. and Bağci, G. (2000). 12 Kasim 1999 Düzce depremi raporu (in Turkish). Technical report.

[Özmen et al., 2000] Özmen, B., Nurlu, M., and Güler, H. (2000). 17 Ağustos 1999 Izmit körfezi deprem raporu (in Turkish). Technical report, Bayindirlik ve Iskan Bakanliği Afet Işleri Genel Müdürlüğü.

[Part-Enander et al., 1996] Part-Enander, E., Sjoberg, A., Melin, B., and Isaksson, P. (1996). *The Matlab Handbook*. Prentice Hall.

[Petak and Elahi, 2000] Petak, W. J. and Elahi, S. (2000). The Northridge earthquake, USA and its economic and social impacts. Technical report, IIASA, www.iiasa.ac.at/Research/RMS/july2000/.

[Pfeifer and Neslehová, 2003] Pfeifer, D. and Neslehová, J. (2003). Modelling and generating dependent risk processes for Integrated Risk Management (IRM) and Dynamic Financial Analysis (DFA). Technical report, ASTIN Colloquim, Topic 3: Risk Control, Berlin.

[Pollner et al., 2001] Pollner, J., Camara, M., and Martin, L. (2001). Honduras. Catastrophe risk exposure of public assets. An analysis of financing instruments for smoothing fiscal volatility, World Bank, Washington DC.

[Presura and Yetkiner, 2002] Presura, C. and Yetkiner, H. (2002). Earthquake and Growth II. University of Groningen SOM.

[Proceedings, 1992] Proceedings (1992). *The Economic Consequences of a Catastrophic Earthquake: Proceedings of a Forum August 1 & 2, 1990.* National Academy Press.

[Ramlau-Hansen, 1983] Ramlau-Hansen, H. (1983). Smoothing counting process intensities by means of kernel functions. *The Annals of Statistics*, 11(2):453–466.

[Re, 2001a] Re, M. (2001a). Earthquake seattle. Technical report, Munich Reinsurance Company.

[Re, 2001b] Re, M. (2001b). World map of natural catastrophes 2000. Technical report, Munich Reinsurance Company.

[Reasenberg and Jones, 1989] Reasenberg, P. A. and Jones, L. M. (1989). Earthquake Hazard After a Mainshock in California. *Science*, 243:1173–1176.

[Reiss, 1993] Reiss, R.-D. (1993). *A course on point processes.* Springer-Verlag.

[Reiss and Thomas, 2001] Reiss, R.-D. and Thomas, M. (2001). *Statistical Analysis of Extreme Values with Applications to Insurance, Finance, Hydrology and Other Fields.* Birkhäuser Verlag, Germany.

[Rejda, 2003] Rejda, G. E. (2003). *Principles of Risk Management and Insurance.* Addison Wesley.

[Repenning, 2001] Repenning, N. P. (2001). Disaster dynamics: Understanding the role of quantity in organisational collapse. Technical report, Jenny W. Rudolph Organisation Studies Department, Carroll School of Management, Boston College.

[Research and Consulting, 2001] Research, S. R. E. and Consulting (2001). Economic risk scenarios survey results second half of 2001. Technical report, Swiss Reinsurance Company.

[Resnick, 1997] Resnick, S. I. (1997). Discussion of the Danish data on large fire insurance losses. *ASTIN Bulletin*, 27(1):139–151.

[Ripley, 1987] Ripley, B. D. (1987). *Stochastic Simulation.* John Wiley and Sons.

[Rolski et al., 1999] Rolski, T., Schmidli, H., Schmidt, V., and Teugels, J. (1999). *Stochastic Processes for Insurance and Finance.* John Wiley & Sons.

[Ross, 2002a] Ross, S. (2002a). *A First Course in Probability.* Pearson International Edition.

[Ross, 2002b] Ross, S. M. (2002b). *Simulation.* Academic Press.

[Ross, 2003] Ross, S. M. (2003). *Probability Models.* International Edition.

[Ruggeri and Pievatolo, 2002] Ruggeri, F. and Pievatolo, A. (2002). Bayesian reliability analysis of complex repairable systems. Technical report, CNR-IAMI.

[Ruggeri et al., 2000] Ruggeri, F., Pievatolo, A., and Argiento, R. (2000). Bayesian analysis and prediction of failures in underground trains. Technical report, CNR-IAMI.

[Ruggeri and Sivaganesan, 2002] Ruggeri, F. and Sivaganesan, S. (2002). Change points in nonhomogenous poisson processes. Technical report, CNR-IAMI.

[Rundle et al., 1995] Rundle, J. B., Turcotte, D. L., and Klein, W. (1995). *Complex Systems in the Earth Sciences: An Introduction Predictability, Mitigation, and Reduction of Natural Hazards.* http://www.santafe.edu/sfi/publications/Bookinforev/nhrpreface.html.

[Sato, 1999] Sato, K.-I. (1999). *Lévy Processes and Infinitely Divisible Distributions.* Cambridge University Press.

[Scawthorn, 2001] Scawthorn, C. (2001). National programs for natural hazards insurance. Technical report, IIASA, www.iiasa.ac.at/Research/RMS/july2000/.

[Scawthorn and Yanev, 1995] Scawthorn, C. and Yanev, P. I. (1995). Preliminary Report 17 January 1995, Hyogo-ken Nambu Japanese Earthquake. *Engineering Structures*, 17(3):146–157.

[Schoutens, 2000] Schoutens, W. (2000). *Stochastic Processes and Orthogonal Polynomials.* Springer.

[Seal, 1969] Seal, H. (1969). *Stochastic Theory of a Risk Business.* John Wiley & Sons, Inc., USA.

[Seal, 978a] Seal, H. L. (1978a). From aggregate claims distribution to probability of ruin. *Astin Bulletin*, 10:47–53.

[Seber and Wild, 1989] Seber, G. A. F. and Wild, C. J. (1989). *Nonlinear Regression.* John Wiley & Sons, Inc.

[Selçuk, 1999] Selçuk, A. S. (1999). The Turkish insurance market: Land of opportunity. *Journal of International Insurance*, 1(1).

[Selçuk and Yeldan, 2001] Selçuk, F. and Yeldan, E. (2001). On the macroeconomic impact of the August 1999 earthquake in Turkey: a first assessment. *Applied Econometric Letters*, (2):483–488.

[Shinozuka et al., 2000] Shinozuka, M., Feng, M. Q., Lee, J., and Naganuma, T. (2000). Statistical analysis of fragility curves. *Journal of Engineering Mechanics*, 126(12).

[Sigma, 2001] Sigma (2001). Large man-made insured losses 1979-2000 and Izmit earthquake. Technical report, Swiss Re Sigma Publications.

[Silverman, 1986] Silverman, B. W. (1986). *Density Estimation for Statistics and Data Analysis.* Chapman & Hall.

[Smith, 1989] Smith, R. L. (1989). Extreme value analysis of environmental time series: An application to trend detection in ground-level ozone. *Statistical Science*, 4(4):367–377.

[Smith and Shively, 1995] Smith, R. L. and Shively, T. S. (1995). Point process approach to modelling trends in tropospheric ozone based on exceedances of a high threshold. *Atmospheric Environment*, 29(23):3489–3499.

[Sorkin, 1982] Sorkin, A. L. (1982). *Economic Aspects of Natural Hazards.* Lexington Books.

[SPO, 1999] SPO (1999). DPT Yönetici Özeti. Technical report, The State Planning Organisation, http://ekutup.dpt.gov.tr/deprem.

[SPO, 2001] SPO (2001). The State Planning Organisation: Main Economic Indicators of Turkey.

[Taylor, 2000] Taylor, G. (2000). *Loss reserving.* Kluwer Academic Publishers.

[TCIP, 2006] TCIP (2006). http://www.dask.gov.tr/bilgi.htm.

[TCMB, 1999] TCMB (1999). Quarterly bulletin. Technical report, http://www.tcmb.gov.tr.

[TCMB, 2000] TCMB (2000). The domestic economy. Technical report, The Central Bank of Turkey Library.

[Team et al., 1999] Team, J. R., BU, ITU, and METU (1999). Report on damage investigation after Kocaeli earthquake.

[The State Institute of Statistics, 1996] The State Institute of Statistics (1996). The Input-Output Structure of the Turkish Economy. Technical report.

[The State Institute of Statistics, 2001a] The State Institute of Statistics (2001a). 17 Ağustos 1999 depreminin yaşandiği bölgede 11-19 Eylül 1999 tarihleri arasinda yapilan geçici iskan tercih eğilimlerini belirleme raporu (in Turkish). Technical report.

[The State Institute of Statistics, 2001b] The State Institute of Statistics (2001b). The report on the survey conducted between the dates of 4-11 October 1999 to determine the impact of the earthquake zone that a severe earthquake occurred on 17 August 1999. Technical report.

[Townsend, 1995] Townsend, R. M. (1995). Consumption insurance: An evaluation of risk bearing systems in low-income economies. *The Journal of Economic Perspectives*, 9(3):83–102.

[(TUSIAD), 1999a] (TUSIAD), T. B. A. (1999a). Economic Impact of the Turkish Earthquake Final Draft (in Turkish). Technical report.

[(TUSIAD), 1999b] (TUSIAD), T. B. A. (1999b). Economic Outlook of the Turkish Economy After the Earthquake (in Turkish). Technical report.

[(TUSIAD), 1999c] (TUSIAD), T. B. A. (1999c). Izmit depreminin ekonomik etkileri (in Turkish). Technical report.

[Utsu, 1961] Utsu, T. (1961). Statistical study on the occurrence of aftershocks. *Geophysical Magazine*, 30:521–605.

[Venables and Ripley, 2002] Venables, W. and Ripley, B. (2002). *Modern Applied Statistics with S-PLUS.* Springer, USA.

[Vere-Jones, 1970] Vere-Jones, D. (1970). Stochastic models for earthquake occurrence. *Journal of Royal Statistical Society. Series B (Methodological)*, 32(1):1–62.

[Vere-Jones and Ozaki, 1982] Vere-Jones, D. and Ozaki, T. (1982). Some Examples of Statistical Estimation Applied to Earthquake Data. *Annals of the Institute of Statistical Mathematics*, 34:189–207.

[Vermeiren, 1989] Vermeiren, J. C. (1989). Natural disasters: Linking economics and the environment with a vengeance. Technical report, OAS.

[Walhin, 2002] Walhin, J. H. (2002). On the Optimality of Multiline Excess of Loss Covers. Technical report, Institut des Sciences Actuarielles, Université Catholique de Louvain; Secura Belgian Re.

[Wallis and Hosking, 1987] Wallis, J. and Hosking, J. (1987). Parameter and quantile estimation for the generalised pareto distribution. *Technometrics*, 29:339–349.

[Wand and Jones, 1995] Wand, M. P. and Jones, M. C. (1995). *Kernel Smoothing.* Chapman & Hall.

[West and Lenze, 1994] West, C. T. and Lenze, D. G. (1994). Modelling the regional impact of natural disaster and recovery: A general framework and an application to hurricane andrew. *International Regional Science Review*, 17(2):121–150.

[Woo, 1999] Woo, G. (1999). *The Mathematics of Natural Catastrophes.* Imperial College Press, London.

[World Bank, 2000] World Bank (2000). Selected world development indicators.

[World Bank, 2001] World Bank (2001). World development report.

[Wright et al., 1979] Wright, J. D., Rossi, P. H., Wright, S. H., and Weber-Burdin, E. (1979). *After the Clean-Up: Long-Range Effects of Natural Disasters.* Sage Publications.

[Wu, 1997] Wu, C. O. (1997). The effects of kernel choices in density estimation with biased data. *Statistics and Probability Letters*, 34:373–383.

[Zanetti et al., 2004a] Zanetti, A., Enz, R., Heck, P., Green, J., and Suter, S. (2004a). Natural catastrophes and man-made disasters in 2003: many fatalities, comparatively moderate insured losses. Technical Report 1/2004, Swiss Reinsurance Company Economic Research and Consulting.

[Zanetti et al., 2004b] Zanetti, A., Enz, R., Heck, P., Green, J., and Suter, S. (2004b). World insurance in 2003: insurance industry on the road to recovery. Technical Report 3/2004, Swiss Reinsurance Company Economic Research and Consulting.

[Zanetti et al., 2000a] Zanetti, A., Enz, R., Hitz, L., and Hess, T. (2000a). Natural catastrophes and man-made disasters in 1999: Storms and earthquakes lead to the second highest-losses in insurance industry. Technical Report 2/2000, Swiss Re, Switzerland.

[Zanetti et al., 2000b] Zanetti, A., Enz, R., Hitz, L., and Hess, T. (2000b). Natural catastrophes and man-made disasters in 1999: Storms and earthquakes lead to the second-highest losses in the insurance history. Technical Report 2/2000, Swiss Re Sigma Publications.

[Zanetti et al., 2003] Zanetti, A., Enz, R., Menzinger, I., Mehlhorn, J., and Suter, S. (2003). Natural catastrophes and man-made disasters in 2002: high flood loss burden. Technical Report 2/2003, Swiss Re Sigma Publications.

[Zanetti et al., 2002] Zanetti, A., Enz, R., and Schaad, W. (2002). Natural catastrophes and man-made disasters in 2001: man-made losses take on a new dimension. Technical Report 1/2002, Swiss Re Sigma Publications.

[Zanetti et al., 2001] Zanetti, A., Enz, R., Schweizer, U., and Hess, T. (2001). Natural catastrophes and man-made disasters in 2000: fewer insured losses despite huge floods. Technical Report 2/2001, Swiss Re Sigma Publications.

[Zanetti and McNamara, 2002] Zanetti, A. and McNamara, M. (2002). Catastrophes in 2001: man-made losses take on a new dimension. Technical report, Swiss Re Sigma Publications.