NOMINAL AND FINANCIAL FRICTIONS IN INTERNATIONAL BUSINESS CYCLE: A DSGE APPROACH

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ABSTRACT

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In this thesis I address three stylized facts widely debated in international economics. The first is the positive correlation between macroeconomic volatility and degree of international financial openness in developing countries. The second is the huge and persistent increase in U.S. trade deficit. The third is the real exchange rate volatility, persistence and the disconnection with other macroeconomic variables. I investigate the three issues by elaborating three dynamic stochastic general equilibrium models. In the first chapter I develop a small open economy model to show how higher but imperfect financial integration can induce macroeconomic instability. The imperfect nature of financial integration is captured through a constraint on international borrowing, that affects expenditure decisions of private agents; the degree of financial openness is measured by the amount of borrowing for a given value of the collateral. In the second chapter I explain the deterioration of the US trade balance in terms of the positive collateral effect of increases in U.S. house prices on private consumption: I develop a model to reproduce the positive relationship - found by estimating a structural vector autoregressive model - between real estate prices, private consumption and trade balance deficit. The key feature of the model is a borrowing constraint, in which real estate is the collateral, that affects the consumption choices of U.S. households. In the third chapter I empirically investigate the determinants of the real exchange rate dynamics by estimating three new open economy macroeconomics models. I find that shocks to the uncovered interest parity and international price discrimination, due to local currency pricing and distribution costs, are crucial features to replicate the dynamics of the real exchange rate.
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The third chapter has been presented at the Boston Fed, Bank of Chile, Bank of Finland, Bank of Italy, CEPR, European Economic Association, Federal Reserve Bank of South Africa, Harvard, University of Bologna.
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Introduction

In this thesis I analyze three stylized facts of the international business cycle. The first is the increase in volatility of consumption that systematically characterize emerging market economies when they increase the openness of their financial markets to international capital flows. The second stylized fact is the huge and persistent trade balance deficit of the U.S. economy, which is at the root of the trade and financial imbalances that currently characterize the global economy. The third is the high volatility and persistence of the real exchange rate and its disconnection with respect to the ‘fundamental’ variables (the volatility of the real exchange rate is much higher than that of the other main macroeconomic variables).

The three stylized facts are puzzling.

According to standard theory, greater access to international financial markets should reduce the volatility of consumption in a small open economy, such as the typical emerging market country. In fact, from a theoretical point of view the greater availability of financial flows should improve the capability of smoothing consumption over time and across states of nature.

Regarding the U.S. trade deficit, there is lack of consensus on its causes, its (long-run) sustainability, the mechanism of adjustment. Explanations rely on several aspects: the fiscal deficit of the U.S. government and its effect on the behavior of the U.S. households; the excess of savings in the rest of the world that finances U.S. investment; the effects of changes in the relative prices of goods and U.S. assets and liabilities.

Finally, standard models of international business cycle have troubles in replicating the stylized facts of the real exchange rate. In fact, both the alternative assumptions of complete and incomplete international financial markets create a tight link between the real exchange rate and other macroeconomic variables; as a consequence, the volatility of the former cannot be much higher than that of the latter.

I address the three issues by developing open economy dynamic stochastic general equilibrium models with financial frictions. For the analysis of the U.S. trade balance deficit and of the real exchange dynamics, I also assume nominal rigidities.

Specifically, the structure of the thesis is the following.

In the first chapter I analyze the positive relationship between the volatility of consumption and the degree of financial openness in an emerging country. I develop a small open economy real business cycle model whose main feature is the presence of a borrowing constraint. Agents can borrow from the rest of the world a constant fraction
of a collateral. This feature is consistent with empirical evidence, suggesting that access to international financial markets for emerging markets is usually in the form of short-term and conditional (to some form of guarantee) borrowing. A regime of financial openness is characterized by a higher value of $m$: for a given value of the collateral, the amount of borrowing is higher. I calibrate the model using data of Malaysia, a country that during the mid 1980s adopted measures to open its financial market to international capital flows. The main result is that, consistently with the empirical evidence, in correspondence of a higher value of $m$ the volatility of consumption is higher in absolute terms and as a ratio to the volatility of domestic output (the ratio, in disagreement with the prediction of standard business cycle models, is higher than one both in the data and in the simulations of the model).

In the second chapter I investigate the relationship between the price of the real estate and the trade balance in the U.S. economy. Unconditional evidence suggests there is a positive association between increases in the price of the real estate and external deficit. I search for (conditional) evidence of a systematic relationship between the variables by estimating a structural vector autoregressive (VAR) model and calculating the responses of real estate prices, private consumption and trade balance in the U.S. to an expansionary monetary shock and to a positive shock to house prices. I find that there is a positive (conditional) correlation between the three variables. I rationalize the evidence in terms of a two-country DSGE model, calibrated on the U.S. economy. As for the model used in the first chapter, I introduce the financial friction in terms of a borrowing constraint: some agents in the economy can borrow only a fraction $m$ of the collateral, which, consistently with the empirical evidence on the U.S. households, is the domestic real estate. In the model there are also sticky prices and incomplete exchange rate pass-through into import prices (nominal exchange rate fluctuations are not fully transmitted to import prices). The first assumption allows for a not trivial role of the monetary policy. The second is consistent with the empirical evidence on the U.S. and limits the capability of the nominal exchange rate fluctuations to modify international relative prices and hence to allow the correction of the imbalance. Differently from existing literature on the global imbalances, I emphasize the particular role of the relative price of a domestic asset, the real estate, as a possible source of the trade deficit. The main result is that the DSGE model is able to qualitatively and to some extent quantitatively replicate the conditional evidence obtained from the VAR: conditional to an expansionary monetary shock or to a positive house price shock (the latter is introduced through a shock to the preferences for housing) the higher price of the collateral stimulates the increase in borrowing, consumption and hence imports; the trade balance moves towards deficit.

Finally, in the third chapter I empirically investigate the stylized facts of the real exchange rate dynamics. Several theoretical and quantitative contributions, based on
the assumption of incomplete international financial markets (only a riskless bond is internationally traded) explain the high exchange rate volatility in terms of local currency pricing (import prices are sticky in the currency of the destination market) and a shock to the uncovered interest parity (UIP). The latter is a shortcut to capture noisy behavior, not formalized in the models, of participants in financial and currency markets. It is justified on the basis of the empirical failure of the UIP. Other contributions, instead, emphasize, almost always assuming incomplete international financial markets, the low level of the elasticity of intratemporal substitution between tradable goods as a source of the excess volatility of the real exchange rate. One way to reduce the elasticity of substitution at the producer level can be built up on the assumption of distribution services intensive in local nontradable goods.

I use the Bayesian methodology to estimate a two-country model having the features above illustrated: incomplete markets at international level, a shock to the UIP, local currency pricing, distribution services. I also estimate a version of the model without the distribution services (there is only local currency pricing) and one in which import prices are set in the currency of the producer (producer currency pricing assumption, which implies that the law of one price holds at international level). The contribution is empirical. It differs from the others in terms of richness and complexity, both justified by the existing theoretical and quantitative debate, of the estimated models. Main results are two. First, the model featuring the local currency pricing and distribution services is the most in agreement with the data, followed by that featuring producer currency pricing. The model based on the local currency pricing assumption is the most at odds with the data. Hence data express a preference for distribution services and the related low intratemporal elasticity of substitution. The second result is that the UIP shock, followed by the shock to the preferences, has a not negligible role for the real exchange rate volatility. Without these shocks, the models, when simulated, are not able to replicate it.

The three chapters of this thesis indirectly suggest that formalizing the role of assets in the decision process of the agents is a crucial step for models of international business cycle. Financial frictions such as borrowing constraints and/or UIP shocks help to reduce the distance between the standard models based on incomplete markets and the stylized facts. More work is needed to deeply analyze the open economy implications of these frictions. However, there is a whole range of theoretical options between the two opposite assumptions of complete and incomplete markets (with or without financial frictions). Hence, more work is also needed to understand the factors driving the accumulation of external asset and liabilities, the foreign asset portfolio and their relationship with the exchange rate adjustment. The analyses would be worthy not only from a theoretical point of view, but also from a policy perspective, given the recent developments in the global financial scenario.
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CHAPTER 1

Financial Openness and Macroeconomic Instability in Emerging Market Economies

1.1. Introduction

During the mid-eighties, after decades of financial repression, many emerging economies chose to pursue a financial liberalization strategy. The increase of competition among financial operators was favoured and the degree of openness of financial markets to the international capital flows was increased.

According to standard theoretical models of international economics, financial openness should lead to a decline in the volatility of consumption and other main macroeconomic variables. Thanks to the greater access to international financial markets, in fact, agents should better smooth consumption over time.

Evidence, however, suggests that in emerging markets financial openness - usually measured by gross capital flows as ratio to gross domestic product - has been systematically associated to an increase in macroeconomic volatility: during the nineties output volatility has essentially not changed relatively to the levels of previous periods, while the volatility of consumption has increased. As a result, consumption volatility has been always higher than output volatility, before and after the financial liberalization.

In this paper I explain the positive relationship between financial openness and macroeconomic instability in terms of imperfect international financial integration. My point is that emerging economies, differently from industrialized countries, have only a limited access to international liquidity: it is usually in the form of short-term debt, whose amount is conditioned to some form of guarantee, for example the value of a collateral. Given the imperfect access to international financial markets, greater financial flows from the rest of the world contribute to amplify the macroeconomic effects of a given shock, instead of allowing to smooth them over time.

To illustrate the point, I calibrate a dynamic stochastic general equilibrium model using data of Malaysia, a country that during the mid 1980s increased the degree of openness of its financial markets to foreign capital flows. The model is a standard real business cycle (RBC from now on) small open economy. Agents in the country produce an internationally traded good using capital and labor and import a good produced in the rest of the world. The two goods are used for consumption and investment purposes. The feature I add to this standard framework is a borrowing constraint: agents' borrowing from the rest of the world is a proportion $m$ (the loan-to-value ratio)
of the value of the domestic real estate. Many authors have emphasized, in fact, the role of real estate as collateral in both domestic and international financial transactions in emerging countries. That of real estate, hence, is a rather important channel to propagate shocks in the emerging economies.\footnote{See, among the others, Collyns and Senhadji (2002), Corsetti (1998), Corsetti et al. (1998), Jansen (2003), Krugman (1999), Mera and Renaud (2000), Radelet and Sachs (1998).}

The main result is that in a more financially open economy (having a higher loan-to-value ratio $m$) the volatility of consumption is higher than that of output and the ratio between the two increases as $m$ becomes bigger. The intuition is the following one. The borrowing constraint affects the intratemporal and intertemporal decisions of the agent. When $m$ is higher, a greater increase in borrowing is allowed for a given increase in the value of the collateral. Hence, there is less need to increase the demand of the collateral and more resources can be used to finance consumption of other goods. As a consequence, demand for consumption of goods different from the collateral increases more for a given shock.

Thanks to the presence of the borrowing constraint, I’m able to replicate the increase in the consumption volatility in Malaysia after the liberalization of capital inflows. Results are robust to changes in key parameters, such the elasticity of substitution between tradable goods, the share of the domestic good in the consumption and investment bundles, the persistence of the exogenous shocks.

There exists a long and growing literature that seeks to explain the volatility of consumption in emerging markets. Standard RBC models of open economies have a difficult time capturing the behavior of the consumption. With forward looking agents and transitory shocks, consumption should be “smooth” relative to income and this dampens the volatility of consumption.

One way to address the issue is to emphasize that economies are mainly subject to substantial volatility in the trend growth rate (rather than to volatility in the transitory fluctuations around a stable trend) relative to developed markets. Aguiar and Gopinath (2005) show that this feature improve the capability of a standard RBC model to match the stylized facts of an emerging markets.

A second approach in the literature relies on market imperfections. Aghion et al. (2003) find that full financial liberalization (i.e., opening the domestic market to foreign capital flows) may actually destabilize, inducing chronic phases of growth with capital inflows followed by collapse with capital flight. Tornell et al. (2004) show the existence of a positive link between financial liberalization, growth, and crises, using a disaggregate model based on the real and financial asymmetries between tradable and nontradable sectors. Also in this case, the existence of financial frictions is key for the final results. Caballero and Krishnamurthy (2001), Schneider and Tornell (2000),
Tornell and Westermann (2001, 2002, 2003) analyze the relevance of sectoral and national credit market imperfections - such as asymmetric sectoral financing constraint, currency mismatching and guarantees - in amplifying macroeconomic shocks in middle income countries.\(^2\)

Differently from them, I develop and calibrate a fully articulated small open economy dynamic equilibrium model. Hence, I can quantitatively investigate the contribution of the financial friction for replicating the stylized facts about consumption.

The paper is structured in the following way. Next section presents some stylized facts about financial openness and consumption volatility in emerging markets. Section three describes the setup: the microfoundations of the model, the optimization problems and the equilibrium relations are illustrated. Section four reports the results of the analysis, based on impulse response functions and simulations. Finally, section five contains the conclusions.

1.2. Stylized Facts

Prasad et al. (2003) show that in 1990s industrial countries have had an enormous increase in financial openness. The picture is not uniform for developing countries. Using data of actual capital flows, the authors classify developing countries in two groups. The first group comprises 22 countries. They are the developing countries that have sharply increased their degree of financial openness during the 1990s. The second group is formed by the 33 least financially open countries.\(^3\)

Table 1 reports changes in volatility for different macroeconomic aggregates over the last two decades for the three groups of countries. In the 1990s there is a significant decline in average output volatility for industrial countries and the least financially open developing countries, while for the more financially open developing countries the decline has been modest. The consumption rows show that the average volatility in the 1990s has declined in line with output volatility for both industrial and less financially open developing countries. To the contrary, for more financially open developing countries the volatility of private consumption has risen in the 1990s relative to the 1980s.\(^4\) Thus, the resulting increase in the 1990s of the ratio of volatility of consumption to

\(^2\)See also Pasche (2001). Iacoviello and Minetti (2003) analyze the effects of the effects of the financial liberalization on the business cycle of open industrialized countries. Another strand of the literature on the macroeconomics of emerging markets has studied the implications of financial frictions - such as borrowing constraint or risk premia - for the optimality of currency regimes. See among the others Cespedes, Chang and Velasco (2002a, 2002b).

\(^3\)The first group includes many countries of the South-Est Asia and South America. The second group many countries of Africa.

\(^4\)To take into account for the possible smoothing role played by public expenditure - that could be important especially in developing economies - Prasad et al. (2003) also report total consumption. This variable is generally less volatile than private consumption. However, the general pattern is confirmed: consumption volatility declined for industrial and less financially open developing economies, while it increased for the more financially open developing countries.
that of output for the more financially open developing countries suggests that finan-
cial integration has not provided better consumption-smoothing opportunities for these
economies.⁵

1.3. The Model

I consider a discrete time, infinite horizon small open economy. The economy cannot
affect the rest of the world's interest rate and demand level. The representative agent
consumes two tradable goods. One is domestically produced, the other is imported. She
also consumes a durable good, the local real estate, which is in a fixed amount. The
agent holds a riskless bond which is internationally traded, invests in physical capital,
and supplies labor. Firms produce under a perfect competition regime a good which is
sold domestically and in the rest of the world. In what follows, I denote the small
economy as Home, the rest of the world as Foreign. Starred variables are related to the
rest of the world.

1.3.1. Household

I initially illustrate the preferences, constraints and first order conditions of the Home
representative agent. In the next section, I report the problem solved by the firms.

1.3.1.1. Preferences. The representative agent's lifetime expected utility $U_0$ is de-
defined as:

$$U_0 = E_0 \sum_{t=0}^{\infty} \gamma_t [\ln C_t + j^h \ln h_t - \frac{\kappa}{\tau} L_t^{(s)}]$$

where $E_0$ is the expectational operator at time 0, $\gamma$ is the discount factor ($0 < \gamma < 1$), $C_t$
is the consumption of nondurable goods, $h_t$ represents the real estate services ($j^h > 0$),
$L_t^{(s)}$ is the labor supply ($\kappa$, $\tau > 0$).

The consumption index $C_t$ in the Home economy is given by the following constant-
elasticity-of-substitution (CES) aggregator:

$$C_t = \left[ a_H^{\frac{1}{r}} C_{H,t}^{\frac{r-1}{r}} + (1 - a_H)^{\frac{1}{r}} C_{F,t}^{\frac{r-1}{r}} \right]^{\frac{r}{r-1}}$$

where $C_{H,t}$ ($C_{F,t}$) is the consumption of Home (Foreign) produced good, $a_H$ ($0 < a_H < 1$)
is the share of the domestically produced good in the consumption expenditure,
$(1 - a_H)$ is the corresponding share of imported good, $\rho$ ($\rho > 0$) is the elasticity of
intratemporal substitution between Home and Foreign goods. A similar definition
applies to the investment in physical capital and public expenditure bundles, respectively

⁵See also Kose et al. (2003a,2003b).
equal to:

\( I_t \equiv \left[ \frac{1}{a_H} I_{H,t}^{e-1} + (1 - a_H) \frac{1}{I_{F,t}} I_{F,t}^{e-1} \right]^{\frac{1}{e-1}} \)  \\

(1.3)

\( G_t \equiv \left[ \frac{1}{a_H} G_{H,t}^{e-1} + (1 - a_H) \frac{1}{G_{F,t}} G_{F,t}^{e-1} \right]^{\frac{1}{e-1}} \)  \\

(1.4)

From the above bundles the following utility-based consumer price index can be derived: \(^6\)

\( P_t = \left[ a_H P_{H,t}^{1-\rho} + (1 - a_H) P_{F,t}^{1-\rho} \right]^{\frac{1}{1-\rho}} \)  \\

(1.5)

\( P_{H,t} \) is the price of the Home good, \( P_{F,t} \) is the price of the Foreign good. Consumption of Home and Foreign goods is a function of the relative price and total consumption, respectively:

\( C_{H,t} = a_H \left( \frac{P_{H,t}}{P_t} \right)^{-\rho} C_t, \quad C_{F,t} = (1 - a_H) \left( \frac{P_{F,t}}{P_t} \right)^{-\rho} C_t \)  \\

(1.6)

Similar definitions apply to the investment goods \( I_H, I_F \) and to the public expenditures \( G_H, G_F \):

\( I_{H,t} = a_H \left( \frac{P_{H,t}}{P_t} \right)^{-\rho} I_t, \quad I_{F,t} = (1 - a_H) \left( \frac{P_{F,t}}{P_t} \right)^{-\rho} I_t \)  \\

(1.7)

\( G_{H,t} = a_H \left( \frac{P_{H,t}}{P_t} \right)^{-\rho} G_t, \quad G_{F,t} = (1 - a_H) \left( \frac{P_{F,t}}{P_t} \right)^{-\rho} G_t \)  \\

(1.8)

Finally, let’s define the Home terms of trade:

\( T = \frac{P_{F,t}}{P_{H,t}} \)  \\

(1.9)

It is the price of the imported good relative to that of the Home good.

1.3.1.2. The constraints. The budget constraint is defined as follows:

\[ \frac{P_t B_t}{(1 + r)} - P_t B_{t-1} = W_t L_t^{(s)} + R_t K_t^{(s)} - Q_t (\bar{h}_t - \bar{h}_{t-1}) \]

\[ -P_t C_t - P_t I_t \]

\[ - \frac{\psi}{2\delta} \left( \frac{I_t}{K_{t-1}^{(s)}} - \delta \right)^2 P_t K_{t-1}^{(s)} - P_t T_t \]  \\

(1.10)

\(^6\)The utility based price index \( P \) is defined as the minimum expenditure required to buy one unit of the composite good, given the prices of the Home and Foreign goods.
where $B_t$ is a riskless bond ($B_t < 0$ is a debt) traded with the rest of the world and denominated in units of domestic consumption, $(1 + r)$ is the interest rate which is always equal to its steady state level, $W_t$ is the wage rate, $Q_t$ is the price of real estate $\bar{h}_t$ (which is a durable good), $\frac{I_t}{K_{t-1}} - \delta$ are adjustment costs of installing physical capital $K_t^{(s)}$ denominated in units of consumption ($\psi \geq 0$, $0 < \delta < 1$ is the depreciation rate of physical capital), $T_t$ are lump-sum taxes denominated in consumption units and paid to the local government.

As in Kiyotaki and Moore (1997), I assume a limit on the obligations of the agent:

$$-F_tB_t \leq mE_t(Q_{t+1}\bar{h}_t)$$

where $m$ is the loan-to-value ratio ($0 < m < 1$). The term $mE_t(Q_{t+1}\bar{h}_t)$ is the maximum amount that the agent can borrow. Hence, the term $(1 - m)E_t(Q_{t+1}\bar{h}_t)$ can be interpreted as a proportional transaction cost the lender must pay to repossess the borrowers' assets. The borrowing constraint captures the imperfect international financial integration of emerging markets: the agent cannot borrow as much as she wants; to the contrary, she faces a limit to the amount she borrows given by the expected value of the real estate. A higher value of the parameter $m$ corresponds to a higher degree of financial openness: for a given value of the collateral, the agent can borrow more from the rest of the world. In the Appendix it is shown that, under the assumption that the agent is relatively impatient (her discount factor $\gamma$ is lower than that of the rest of the world), the borrowing constraint is binding in the steady state and in a neighborhood of it.

Real estate is fixed in the aggregate. Capital is instead accumulated according to the following law:

$$K_t^{(s)} = (1 - \delta)K_{t-1}^{(s)} + I_t$$

I assume that the prices are expressed in terms of consumption $C$ ($P = 1$). The first order conditions with respect to $B_t$, $\bar{h}_t$, $I_t$, $K_t^{(s)}$, $L_t^{(s)}$ are respectively:

$$E_t\left(-\frac{1}{C_t(1 + r)} + \frac{1}{C_t} + \lambda_t\right) = 0$$

$$\frac{q_t}{C_t} = j^h\bar{h}_t + E_t\left\{\gamma\frac{q_{t+1}}{C_{t+1}} + m\lambda_{t+1}\right\}$$

$$\Psi_t = \frac{\psi}{C_t}\left[\frac{I_t}{K_t^{(s)}} - \delta\right] + \frac{1}{C_t}$$
Equations (1.15) and (1.16) are standard in models of investment with adjustment costs. Equation (1.15) defines the shadow value of one unit of investment today, $\Psi_t$, which equals the marginal cost of investment. Equation (1.16) states that this value must be equated across time periods. At the optimum, the shadow price of capital must equal the sum of the expected values of the following three components: the rental rate of capital, the shadow value of capital and the capital contribution to lower installation costs. Equation (1.17) is the first order condition for labor supply: in each period, the real wage $w_t$ is equal to the marginal rate of substitution between consumption and leisure.

Equation (1.13) is the consumption Euler equation, modified to take into account for the marginal value of additional borrowing $\lambda$. Equation (1.14) is the real estate demand equation ($q_t$ is the price of the real estate in terms of consumption units). It requires that the marginal utility of current non-durable consumption is equated to the marginal gain of the real estate. The latter depends on the following elements: (1) the direct utility gain of an additional unit of the real estate; (2) the expected utility from expanding future consumption by means of the realized resale value of the durable purchased in the previous period; (3) the marginal utility of relaxing the borrowing constraint. Both equations (1.13) and (1.14) contain $\lambda t$, the Lagrange multiplier of the borrowing constraint. $\lambda t$ equals the increase in lifetime utility that would stem from borrowing \(1+r\) today and consuming the proceeds, and reducing consumption by an appropriate amount next period. Since housing can be used as collateral, there is a distortion of the intratemporal and intertemporal allocations towards housing consumption.

Combining the equations (1.13) and (1.14) to substitute out the Lagrange multiplier and log-linearizing around the deterministic steady state (see the Appendix), we get the following equation:

$$
\dot{C}_t = \frac{1}{1-m\beta} q_t + \frac{\beta(1-m)}{1-m\beta} E_t \dot{C}_{t+1} - \frac{m(\beta-\gamma)}{1-m\beta} E_t q_{t+1}
$$

where $\beta$ is the discount factor of the rest of the world (by assumption, $\beta > \gamma$).

The equation clearly shows how, keeping constant all the expected variables, the multiplier effect on consumption of given changes in the price of real estate $q_t$ can be rather large, and is increasing with $m$, the loan-to-value ratio. Consumption is a positive
function of house prices, with a coefficient \(1/(1-m\beta)\) (the inverse of the downpayment needed to purchase one unit of the real estate).

1.3.2. Firms

Firms in the Home country operate under a perfect competition regime. They produce the Home good according to a Cobb-Douglas technology:

\[ Y_t = A_t K_{t-1}^{(d)\alpha} L_t^{(d)1-\alpha} \]

where \(0 < \alpha < 1\) and \(A\) represents a shock to the total factor productivity. Firms solve a standard static profit maximization problem. They maximize profits, expressed in terms of domestic consumption units, \(p_{H,t}Y_{H,t} - r_t K_{t-1}^{(d)} - w_t L_t^{(d)}\) with respect to \(K_{t-1}^{(d)}\) and \(L_t^{(d)}\). The first order conditions are respectively:

\[ \alpha p_{H,t} Y_{H,t} = r_t^{k} K_{t-1}^{(d)} \]
\[ (1 - \alpha) p_{H,t} Y_{H,t} = w_t L_t^{(d)} \]

1.3.3. Public Sector budget constraint

The budget constraint of the public sector is equal to:

\[ G_t = T_t \]

I introduce the public expenditure only to correctly calibrate the model. Hence both \(G\) and \(T\) are constant and equal to their steady state level.

1.3.4. Market Clearings

The market clearing condition for capital is:

\[ K_t^{(d)} = K_t^{(s)} \]

The market clearing condition for labor is:

\[ L_t^{(d)} = L_t^{(s)} \]

The market clearing condition for real estate is:

\[ \bar{h}_t = H \]

where I assume \(H\) is fixed.

\[ ^7\text{Hence, } p_H = p_R.\]
The market clearing of the domestically produced good is:

\[ Y_t = C_{H,t} + I_{H,t} + G_{H,t} + (p_{H,t})^{-\rho} X_t \]  

where \((p_{H,t})^{-\rho} X_t\) is the demand of the rest of the world for the Home good, with \(X_t\) exogenously given (see next section). For simplicity, I assume that the real price \(p_{H,t}\) of the Home good in the small open economy and in the rest of the world is the same, as well as the elasticity of intratemporal substitution \(\rho\).

The market clearing of the imported good is:

\[ Y_{F,t} = C_{F,t} + I_{F,t} + G_{F,t} \]

1.3.5. Exogenous Driving Forces

I assume that the two external sources of uncertainty that drive business cycles in the model are the amount of export \(X_t\) and technology \(A_t\). The laws of motion are:

\[ \ln(X_t) = \rho_X \ln(X_{t-1}) + \varepsilon_{X,t} \]

\[ \ln(A_t) = \rho_A \ln(A_{t-1}) + \varepsilon_{A,t} \]

where \(0 < \rho_S < 1\), with \(S = X, A\). Innovations \(\varepsilon_{S,t}\) (\(S = X, A\)) have zero mean and are normally distributed.

1.3.6. The Equilibrium

The equilibrium of the model is a set of decisions rules taken by the agent and firms such that all the optimality conditions, constraints and market clearing conditions hold, given initial conditions and the exogenous laws of motion of the shocks perturbing the economy. This equilibrium is log-linearized around a deterministic steady state. The steady state and log-linearized equilibria are reported in the Appendix. I calibrate the steady state to reproduce the main features of the economy of Malaysia, a country that during the mid '80s experienced a process of financial liberalization. After the calibration, the results of the simulations are reported, to show the relationship between macroeconomic instability and degree of financial openness.

1.3.7. Calibration

Before calibrating the model, let's consider how the degree of financial openness affects the steady state values of the main variables.

The investment-to-output ratio does not depend upon the parameter \(m\):

\[ \frac{I}{Y} = \delta \frac{\gamma \alpha}{1 - \gamma (1 - \delta)} \]
The same is true for the public expenditure $G$, which is exogenous. Given the inequality $\gamma < \beta$ ($\beta$ is the discount factor of the rest of the world), a higher value of $m$ implies a higher real estate value-to-output ratio:\(^8\)

\[
\left(1.29\right) \quad \frac{Q_h}{Y} = \frac{j^h}{1 - \gamma - m(\beta - \gamma)} \left(1 - \frac{I}{Y} - \frac{G}{Y}\right)
\]

Correspondingly, the value of the borrowing-to-output ratio is in absolute value higher:

\[
\left(1.30\right) \quad \frac{B}{Y} = -m \frac{Q_h}{Y}
\]

Higher borrowing-to-output ratio implies a lower consumption-to-output ratio, given the higher amount of interest rate payments on the existing debt:

\[
\left(1.31\right) \quad \frac{C}{Y} = -(\beta - 1) \frac{B}{Y} + 1 - \frac{I}{Y} - \frac{T}{Y}
\]

I calibrate the parameters of the model on a yearly basis. I match the yearly 'great' ratios of Malaysia over the period 1960-1996.\(^9\)

Malaysia is a good case study, given the goal of the paper. First, in the middle 1980s it liberalized capital account controls and got strong capital inflows, especially in form of short term debt.\(^10\) Second, consumption volatility during the period 1985-1996 was higher than during the period 1960-1984 (the period of financial closeness). Third, local real estate had a strong role as collateral in financial transactions during the period 1985-1996.\(^11\)

I report the chosen values in Table 2.

Following the business cycle literature, I set the discount factor of the rest of the world $\beta$ equal to 0.96 (on a quarterly basis, it corresponds to 0.99), The discount factor of the Home agent, $\gamma$, is equal to 0.92 (0.98 on a quarterly basis).

I set $j^h$ equal to 0.01. The parameter of labor disutility $\tau$ is equal to 2. The elasticity of substitution between the Home produced and imported goods, $\rho$, is equal to 1.1, while $a_H$ (the share of Home produced goods in the consumption and investment bundles) is equal to 0.44 to match the Malaysian share of imported goods over domestic output. I choose the value of the capital adjustment cost, $\psi$, to match the volatility of investment over the period 1960-1996. The share of capital in the production function

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\(^8\)See the Appendix for derivation.

\(^9\)I do not consider the years after 1996 to avoid the macroeconomic effects of the East-Asia currency and financial crises.

\(^10\)The liberalization of the capital account, that was accompanied by measures to deregulate the financial system, began in the late 1980s. See Ariyoshi et al. (2000).

is equal to 0.48 and the capital depreciation rate \( \delta \) equal to 0.12. These values allow to match the investment-to-output ratio. The loan-to-value ratio is set to 0.3.

The obtained great ratios are reported in Table 3. They closely match the actual ones.

I set the loan-to-value ratio \( m \), which is a key parameter to 0.3. In the exercises, I'll capture the regimes of low and more open financial markets setting \( m \) respectively equal to 0.15 and to 0.45. A higher \( m \) implies a lower consumption-to-output ratio. This is consistent with the empirical evidence on Malaysia: the ratio is equal to 58 percent during the period 1960-1984, while it is equal to 50 percent during the period 1985-1996.

After the steady state, I calibrate the shocks to the export volume and to the technology shock. The persistence of the shock to export is 0.38, while the standard deviation is 0.0663. The coefficient of the technology persistence is 0.67, while the standard deviation of the shock is 0.0323.12

1.4. Results

My main goal is to show that the model, thanks to the borrowing constraint (a feature that formalizes imperfect international financial integration), is able to reproduce the following stylized fact: the volatility of consumption increases when the degree of financial openness of emerging markets increases. To this purpose, I initially show results obtained under alternative values of the loan-to-value ratio; than I'll show what happens when the borrowing constraint is not in the model, and hence agents can smooth consumption over time. Finally, I'll perform a robustness analysis of the results.

1.4.1. Macroeconomic volatility and financial frictions

I compute averages of statistics from 100 simulations. Each simulation consists of 100 periods. The reported moments are averages across simulations. I discard initial observations so to work with a number of years equal to those used to construct the actual statistics. Consistently with the length of the period 1960-1984, in the case of the regime of less open financial market \( (m = 0.15) \) I have discarded the first 76 observations. In the case of a regime of more open financial markets \( (m = 0.45) \), I have discarded the first 89 observations (the period goes from 1985 to 1996). Other parameters are held constant across the two regimes.13

12I have estimated the two laws of motion using annual logged and filtered data of Malaysia. I have used the Hodrick-Prescott filter, with smoothing parameter equal to 100. The data go from 1963 to 1996. Data for export are from the International Financial Statics of the International Monetary Fund. Those for technology are calculated using data of the Penn Table.

13If the variance of the shocks is very large, the agent might not borrow up to the limit after a long series of positive productivity or export shocks. Instead, she can decide to keep a buffer stock of resources to use in bad times to avoid the possibility of hitting the borrowing constraint. By assuming
Table 4 reports the ratios of the volatility of consumption to that of output. Interesting results are two. First, both the simulated and actual ratios are greater than one in both the subperiods. Second, both theoretical and actual ratios in the second subperiod (1985-1996) are higher than their counterparts in the first subperiod.

To better understand the results in Table 4, Table 5 shows the values of consumption and output volatilities. The simulated volatility of consumption is lower than the actual one. However, it tracks the actual one and in the period 1985-1996 it is higher than in the period 1960-84. The simulated output volatility slightly increases across the two periods, while the actual one decreases.

Hence, the model is able to reproduce the increase in consumption volatility that has characterized Malaysia and other emerging markets after opening their financial markets to foreign capital flows. The key parameters is $m$, measuring the degree of collateralization of borrowing. The higher $m$, the higher the amount of borrowing from the rest of the world, the higher the volatility of consumption. This suggests that financial frictions in emerging markets can contribute to explain the positive relation between macroeconomic volatility and financial openness in emerging markets.

I also report some statistics of the other main variables. The model is able to replicate the volatility of real imports and investment, while it overstates the volatility of the trade balance. At least to some extent, it is able to replicate (see Table 6) the persistence of consumption, investment, output and imports. Actual persistence is in fact higher. This indicates that the model lacks internal mechanism of persistence (such as sticky prices, for example) or that more persistent shocks should be added as sources of the dynamics. The trade balance has a countercyclical behavior, also if the simulated moment is in absolute value lower than the actual one (Table 7). The low magnitude of the moment is due to the technology shock, which induce a shift of consumption and investment towards the domestic good, which becomes relatively cheap.

To get some intuition on the results of the simulations, I show the impulse response of the main variables (see Figure 1) to a $+1\%$ shock to the amount of export.

When $m$ is higher ($m = 0.45$), on impact the responses of real estate price, borrowing, trade balance deficit, output and labor are lower; to the contrary, on impact the responses of imports, consumption, investment are higher. The terms of trade improve more when $m$ is higher.

Given that the agent is relatively impatient, she prefers current to future consumption. The increase in the rental rate of capital and wage, due to the higher Foreign demand for Home good, generates a positive income effect. This stimulates current consumption. The borrowing constraint induces a wealth effect on the labor supply.

that the discount factor of the constrained agents, $\gamma$, is well below that of the agents in the rest of the world, $\beta$, I can minimize the probability that credit constraints become non-binding in some states of the world.
This effect is stronger in correspondence of lower \( m \) (tighter borrowing constraint): to get a given amount of borrowing, a higher value of the collateral is needed; to finance the purchase of the latter, the labor supply response must be stronger. To the contrary, when \( m \) is higher the link between the labor supply and the borrowing constraint is weaker. In fact, a higher \( m \) implies a "relaxed" borrowing constraint and a given amount of borrowing can be obtained with a relatively lower increase in the value of the collateral. The benefit from strongly increasing the supply of labor and the demand of the collateral is lower. The increase in borrowing is used to demand mainly the Home and Foreign nondurable goods. The increase in consumption of nondurable goods (and in investment) is hence stronger. This result is consistent with those of the simulation exercises, that positively relate the volatility of consumption to the value of \( m \). The price of the Home good (relatively to that of the Foreign good) increases to a higher extent when \( m \) is higher, given the stronger increase in Home demand and the lower increase in labor supply. The higher increase in the relative price of the Home good counteracts the higher amount of imports and the trade balance deficit is on impact lower, consistently with the lower increase in borrowing from the rest of the world.

Results in the case of a positive technology shock are similar (Figure 2). In correspondence of a higher \( m \), on impact the response of consumption and investment is higher, while the increase in labor is lower. As in the case of the shock to export, a higher value of the parameter \( m \) weakens the wealth effect of the borrowing constraint on the labor supply. Hence, the output increase is lower, contributing to a lower increase in the relative price of imports (a lower deterioration of the Home terms of trade). The decrease of imports is lower when \( m \) is higher. The trade balance deficit is lower, as well as the increase in the price of the real estate and in borrowing.

To emphasize the role of the financial friction for the obtained results, I simulate the model without the borrowing constraint.\(^{14}\) Table 8 reports the results. Consumption volatility in the model without the borrowing constraint is lower than in the model with

\(^{14}\)The model without the borrowing constraint is a standard small open economy real business cycle model. The first order conditions with respect to \( B_t \) and \( h_t \) are now:

\[
E_t \left( \frac{1}{C_t (1 + r)} \Phi(B_t) + \gamma \frac{1}{C_{t+1}} \right) = 0
\]

\[
\frac{q_t}{C_t} = \beta \frac{1}{h_t} + E_t \left( \gamma \frac{q_{t+1}}{C_{t+1}} \right)
\]

I assume the steady state of the model without borrowing constraint is similar to that of the model with \( m = 0.3 \). To make the model stationary, I assume the Home agent is as patient as the agents in the rest of the world (hence \( \gamma = \beta = \frac{1}{h_t} \)) and that there is a function \( \Phi(B_t) \) capturing the cost of undertaking positions in the international asset market. The agent does not take into this extra-term when maximizes her utility. As borrower, the agent will be charged a premium on the interest rate; as lenders, she will receive a remuneration lower than the interest rate. I introduce this additional cost to pin down a well defined steady state. The payment of this cost is rebated to agents belonging to the rest of the world. I adopt the following functional form:

\[
\Phi(B_t) = \exp \left[ \delta_t (B_t - B) \right]
\]
the borrowing constraint and lower than the actual value. It is also lower than that of the output. This result is clearly at odds with the empirical evidence on emerging markets. It is more in line with the evidence on industrialized countries, where the volatility of consumption is lower than that of the output.

To get some intuition, in Figure 3 I report the responses of the main variables to the shock to the export in the case the borrowing constraint holds \((m = 0.3)\) and in the case it does not. In the model with the borrowing constraint the increase of the real estate price is stronger. Thanks to this increase, the agent can borrow more from the rest of the world and the trade balance goes towards deficit. Instead, in the model without the borrowing constraint the agent smooths consumption over time. She uses some of the revenues from higher export to reduce her debt. The foreign asset position and the trade balance improve. Consumption and investment increase in the model without the borrowing constraint, also if to a lower extent than in the model with the borrowing constraint. On impact the increase of output is higher in the model without the borrowing constraint, given the higher increase in labor supply. The response of the labor supply is higher in the model without the borrowing constraint. In the periods after the first, the output becomes higher in the model with the borrowing constraint, given the increase in physical capital associated with the higher investments.

The responses of the main variables do not change much when the technology shock is considered. Results are reported in Figure 4. The only differences are that imports decrease and the terms of trade deteriorate. The responses of these two variables are stronger in the model without the borrowing constraint, given that the stronger response of labor induces a stronger increase in the supply of the Home good.

The reported results suggest that more open financial markets, when there are financial frictions and international financial integration is not perfect, tend to increase macroeconomic instability. In particular the volatility of consumption is higher.

1.4.2. Some sensitivity analysis

In what follows I analyze how the responses and the volatility of the main variables change by varying some key parameters of the model. Specifically, I consider the responses in correspondence of different degrees of elasticity of intratemporal substitution between tradable goods, of different shares of Home good in the consumption and investment bundles, of different persistence of the shocks.

1.4.2.1. Different elasticities of substitution. I analyze the case of a relatively high elasticity of intratemporal elasticity of substitution between Home and imported

where \(-1 \leq \delta_b \leq 0\) and \(B\) is the steady state asset position. The parameter \(\delta_b\) controls the speed of convergence to the steady state. I set it as small as possible, compatibly with the stationarity of the model. See Schmitt-Grohé and Uribe (2001) for more details.
goods. I compare the responses of the main variables when \( p \) is set to its baseline value \( (p = 1.1) \) and when it is set to 2.

Figure 5 reports the results in the case of a shock to exports. The magnitude of the responses is lower in correspondence of a higher value of \( p \). The substitution effect becomes relatively stronger. The economy becomes more stable, in the sense that both prices and quantities react less to the shock. However, qualitatively the results do not change: output and labor supply increase; the price of the real estate, borrowing and demand for consumption and investment increase as well. The trade balance shifts on impact towards deficit.

Figure 6 reports the results in the case of a technology shock. The reaction of the main variables is stronger when the elasticity of substitution is higher. In that case, in fact, the deterioration of the Home terms of trade is lower and the increase in demand for the Home good is higher. This stimulates a stronger increase in output and labor. Demand for real estate is higher, stimulating a stronger increase in the relative price of real estate. The increase in borrowing is higher, allowing higher consumption and investment. The trade balance deficit is higher.

From a qualitative point of view, the response of the main variables is similar across different values of \( p \). However, the magnitude of the effect of the elasticity of substitution on the main macroeconomic variables depends on the type of the shock affecting the economy. Some simulations I have performed show that when the economy is hit by both technology and export shocks the volatility of consumption and other main macroeconomic variables is decreasing in the value of \( p \). The reason is that, according to the chosen calibration, the shock to export is stronger than the technology shock.

1.4.2.2. Different degrees of trade openness. Figure 7 shows the responses to a shock to exports when \( a_H = 0.44 \) (the benchmark) and when \( a_H \) it is equal to 0.75. A lower degree of openness reduces the magnitude of the effects of the shock. Export constitute now a lower share of Home production. As a consequence, the related increase in labor supply and output are lower. Real estate price, borrowing, consumption and investment increase to a lower extent.

Figure 8 shows the responses in the case of a technology shock. A lower degree of openness (higher \( a_H \)) amplifies the effects of a technology shock: quantities and relative prices react to a greater extent. When the degree of openness is lower, a wider part of

\[ \frac{X}{Y} = 1 - a_H \left( \frac{C}{Y} + \frac{I}{Y} + \frac{G}{Y} \right) \]

See the Appendix for more details.
the increase in demand is direct towards the domestic good. The increase in production is higher. The increase in labor supply, real estate price, borrowing is stronger. As a consequence, also consumption and investment increase to a greater extent.

From a qualitative point of view the response of the main variables is similar across different values of $a_H$. The magnitude of the effect of the degree of openness on the main macroeconomic variables depends on the type of the shock affecting the economy. As in the case of the elasticity of substitution, I have performed some simulations in correspondence of different values of the degree of openness. They show that the volatility of consumption and other main variables is decreasing in the degree of openness.\(^\text{17}\)

Also in this case, the reason is that the shock to export is relatively stronger than the technology shock.

1.4.2.3. Different persistence of the shocks. Figure 9 shows the responses to a shock to the amount of export when its persistence is equal to its benchmark value and when it's equal to 0.92 (this value corresponds to 0.98 on a quarterly basis). When the shock is more persistent, the agent does not strongly substitute future with current labor. Hence, on impact the increase in labor is lower. There is also a stronger income effect, which induces a higher increase in demand for real estate. The price of the latter strongly increase, favouring higher borrowing, consumption, and investment.

Figure 10 shows the responses of the variables to a technology shock when its persistence is equal to 0.67 (the benchmark) and when it's equal to 0.92. Responses of borrowing, consumption and investment do not differ from those obtained under alternative values of the persistence of exports shock.

As in the cases of alternative degrees of elasticity if substitution and of trade openness, the responses of the main variables are qualitatively similar across different values of the persistence of the shocks. Simulations show that the volatility of consumption is increasing in the persistence of the shocks.\(^\text{18}\)

1.5. Conclusions

In this paper I develop a model of a small open economy to address the relation between macroeconomic volatility and financial openness in emerging markets. I find that financial liberalization *per se* does not guarantee a reduction of macroeconomic volatility: greater but, because of financial frictions, imperfect access to international liquidity can amplify the effects of exogenous shocks and cause an increase in consumption volatility. From a policy perspective, the results suggest that it is worthwhile to implement not only structural measures that increase the degree of financial openness,

\(^{17}\)When $a_H$ is equal to 0.75 (0.44 in the benchmark), the volatilities of consumption, investment and output are respectively equal to 3.43 (3.54 in the benchmark), 13.9 (14.27), 3.18 (3.15).

\(^{18}\)When the persistence of both shocks is equal to 0.92, the volatilities of consumption, investment and output are respectively equal to 4.93 (3.54 in the benchmark), 14.98 (14.27), 3.13 (3.15).
but also those that contribute to eliminate the financial frictions and hence improve financial integration. For example, incentives for prudent risk management, appropriate financial institutions for prudential supervision and regulation in the liberalizing country can contribute to reduce the asymmetric information, and hence the moral hazard problem, that are at the basis of the financial frictions; in other words, measures that contribute to develop the transparency of the financial system are important to exploit the potential benefits, in terms of macroeconomic stability, deriving from greater access to international financial markets.

According to the obtained results, more research is needed to understand how the design of financial institutions affects the macroeconomic results of financial liberalization and to quantify the effects on assets and goods prices. In particular, it would be interesting to quantitatively analyze the macroeconomic implications resulting from the interaction of the liberalization of capital account and that of the domestic financial sector. The setup illustrated in this paper could be extended to take into account for the possibility that agents in the small open economy are subject not only to an international borrowing constraint, but also to a domestic one. Finally, the model could be extended to quantitatively and empirically explore the macroeconomic effects of permanent technology shocks. This would help to measure the relative contribution of growth and financial frictions to the macroeconomic instability of emerging markets.
References


Appendix. The solution of the model

The equilibrium dynamics is characterized by solving a first-order log-linear approximation to the equilibrium conditions around the non-stochastic steady state. In what follows, the deterministic steady state and the log linearized equations are shown.

The steady state equilibrium

A steady state equilibrium is considered in which all the shocks are zero, there is no net capital accumulation \( K_t = K_{t-1} = K \), no debt change \( B_t = B_{t-1} = B \) and in which all price are constant. The following price normalization is used: \( P_H = P_F = P = 1 \). Hence, the relative prices are equal to one.

The steady state value of \( r \) satisfies:

\[
\beta = \frac{1}{1+r}
\]

where \( \beta \) is the discount factor of the rest of the world. Using the steady state version of the consumer Euler equation (1.13) and equation (1.14), the following value for \( \lambda \), the Lagrange multiplier, can be obtained:

\[
\lambda = \left( \frac{\beta - \gamma}{C} \right)
\]

By assumption, \( 0 < \gamma < \beta < 1 \); so the Lagrange multiplier is strictly greater than zero in the steady state (and in a small neighborhood of it). As a consequence, the borrowing constraint is binding.

I normalize the steady state value of the total productivity factor, \( A \), to 1. From the firms’ first order conditions, equations (1.19) and (1.20), the following two equations are respectively obtained:

\[
\frac{r^k K}{Y} = \alpha \tag{SS.3}
\]

\[
\frac{w L}{Y} = 1 - \alpha \tag{SS.4}
\]

From equations (1.15), (1.16), (SS.3) the capital-to-output ratio is:

\[
\frac{K}{Y} = \frac{\gamma \alpha}{1 - \gamma (1 - \delta)} \tag{SS.5}
\]

From the capital accumulation law (1.12), the investment-to-output ratio can be derived:

\[
\frac{I}{Y} = \delta \frac{\gamma \alpha}{1 - \gamma (1 - \delta)} \tag{SS.6}
\]
The borrowing constraint equation (1.11) becomes:

\[(SS.7)\] \[\frac{B}{Y} = -m \frac{Q_h}{Y}\]

From the budget constraint (1.10), using equations (SS.3) and (SS.4), the consumption-to-output ratio can be written as:

\[(SS.8)\] \[\frac{C}{Y} = -(\beta - 1) \frac{B}{Y} + 1 - \frac{I}{Y} - \frac{T}{Y}\]

Combining equations (SS.7), (SS.8) and the steady state version of equation (1.14) the following equation for the real estate value-to-output ratio is obtained:

\[(SS.9)\] \[\frac{Q_h}{Y} = \frac{j^h}{1 - \gamma - m(\beta - \gamma) - j^h(\beta - 1)m} \left(1 - \frac{I}{Y} - \frac{G}{Y}\right)\]

The ratio \(I/Y\) is given by equation (SS.6) and \(G/Y\) (as well as \(T/Y\)) is exogenous. Hence, the ratio \(Q_h/Y\) is function of the structural parameters of the model. Substituting back into equations (SS.7) and (SS.8), also the ratios \(B/Y\) and \(C/Y\) become function of the structural parameters.

From the intratemporal first order conditions of Home agents - equations (1.6), (1.7), (1.8) - the following steady state allocations are obtained:

\[C_H = a_H C\], \[C_F = (1 - a_H) C\]  \[(SS.10)\]
\[I_H = a_H I\], \[I_F = (1 - a_H) I\]  \[(SS.11)\]
\[G_H = a_H G\], \[G_F = (1 - a_H) G\]  \[(SS.12)\]

The demand of the rest of the world demand for the Home good in steady state is (see equation (1.24)):

\[(SS.13)\] \[\frac{X}{Y} = 1 - a_H \left(\frac{C}{Y} + \frac{I}{Y} + \frac{G}{Y}\right)\]

The Home demand for the Foreign good is (see equation (1.25) and the above steady state equations of \(C_F, I_F, G_F\)):

\[(SS.14)\] \[\frac{Y_F}{Y} = (1 - a_H) \left(\frac{C}{Y} + \frac{I}{Y} + \frac{G}{Y}\right)\]

Finally, the budget constraint of the public sector is:

\[(SS.15)\] \[G = T\]
The log-linearized equilibrium

Local dynamics is studied by linearizing the equilibrium conditions around the steady state.

Let variables with a time subscript $t$, $t + 1$ or $t - 1$ denote log-deviations from steady state and let those without a time subscript denote steady state values. The log-linearized equilibrium of the model is system of 14 equations in 14 variables: $\hat{Y}$, $\hat{Y}_F$, $\hat{C}$, $\hat{B}$, $\hat{K}$, $\hat{I}$, $\hat{L}$, $\hat{q}$, $\hat{h}$, $\hat{p}_H$, $\hat{p}_F$, $\hat{T}$, $\hat{A}$, $\hat{X}$. They are log-deviations from steady state (for example, $\hat{Y}_t \equiv \ln(Y_t/Y)$) of, respectively, output, amount of imports, consumption, net foreign asset position, stock of capital, investment, labor supply, real estate price index, real estate holdings, price of the Home good, price of the Foreign good, terms of trade, technology, amount of export.

To save on notation, I drop the expectation operator between variables dated $t + 1$. However the variables must be intended as in expected value conditional on the information available at time $t$.

The equations are the following ones:

1. Aggregate demand

$$ (AD1) \quad \hat{Y}_t = -\rho (\hat{p}_H) + a_H \hat{C}_t + a_H \hat{I}_t + \hat{X}_t $$

$$ (AD2) \quad \hat{Y}_{F,t} = -\rho (\hat{p}_F) + \frac{C}{Y} \hat{C}_t + \frac{I}{Y} \hat{I}_t $$

$$ \hat{q}_t = (1 - m\beta) \hat{C}_t $$

$$ + (\gamma + m (\beta - \gamma)) (\hat{q}_{t+1}) $$

$$ - (1 - \gamma - m\beta + m\gamma) \hat{h}_t $$

$$ - \gamma (1 - m) \hat{C}_{t+1} $$

$$ (\hat{I}_t - \hat{K}_{t-1}) = \gamma (\hat{I}_{t+1} - \hat{K}_t) + \frac{1}{\psi} (\hat{C}_t - \hat{C}_{t+1}) $$

$$ + [1 - \gamma (1 - \delta)] \left[ \hat{P}_{H,t} + \hat{Y}_{t+1} - \hat{K}_t \right] $$

2. Borrowing constraint

$$ (BC1) \quad \hat{B}_t = \hat{q}_{t+1} + \hat{h}_t $$

3. Aggregate supply
\( \dot{Y}_t = \alpha \dot{K}_{t-1} + (1 - \alpha) \dot{L}_t \)

\[(AS1)\]

\( \dot{p}_{H,t} + \dot{Y}_t = +\dot{C}_t + \tau \dot{L}_t \)

\[(AS2)\]

4. Flows of funds/ Other variables

\[(FF1)\] \( \dot{K}_t = (1 - \delta) \dot{K}_{t-1} + \delta \dot{I}_t \)

\[(FF2)\] \( \beta \frac{B}{Y} \dot{B}_t = \frac{B}{Y} \dot{B}_{t-1} + \dot{p}_{H,t} + \dot{Y}_t - \frac{C}{Y} \dot{C}_t - \frac{I}{Y} \dot{I}_t \)

\[(FF3)\] \( a_H \dot{p}_{H,t} + (1 - a_H) \dot{p}_{F,t} = 0 \)

\[(FF4)\] \( \dot{h}_t = 0 \)

\[(FF5)\] \( \dot{T}_t = \dot{p}_{F,t} - \dot{p}_{H,t} \)

5. Shock process

\[(SP1)\] \( \dot{A}_t = \rho_A \dot{A}_{t-1} + \varepsilon_{A,t} \)

\[(SP2)\] \( \dot{X}_t = \rho_X \dot{X}_{t-1} + \varepsilon_{X,t} \)

The equations are divided in 5 blocks. The first block contains the demand side of the economy.

The first equation is the demand for the Home produced good (it is the log-linearized version of the equation (1.24)).

The second equation is the demand for the imported good (it is obtained by log-linearizing the equation (1.25)).

The third equation is the derived from the agent Euler consumption equation (1.13) and real estate demand equation (1.14).

The fourth equation is obtained by combining the investment and capital first order conditions, respectively equation (1.15) and equation (1.16).

The second block is the borrowing constraint. It derives from equation (1.11).

The third block is the supply side of the economy.

The first equation derives from the technology constraint (1.18).

The second equation derives from the labor first order conditions (1.17), (1.20) and from the labor market clearing condition (1.22).
The fourth block contains equations describing flows of funds and remaining variables.

The first equation is the log-linearized version of the capital accumulation law (1.12).

The second equation derives from the budget constraint (1.10).

The third equation defines the consumption as the numeraire (see equation (1.5), with \( P = 1 \)).

The fourth equation derives from the real estate market clearing condition (1.23). The real estate is in a fixed amount.

The fifth equation is the Home terms of trade. It is the log-linearized version of equation (1.9).

Finally, block number five contains the exogenous processes: the law of motion of technology and that of foreign demand.
Table 1. Volatility of Annual Growth Rates of Consumption and Output

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<th>Full Sample</th>
<th>Decade</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1960-99</td>
<td>1960s</td>
<td>1970s</td>
<td>1980s</td>
</tr>
<tr>
<td><strong>Output</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industrial countries</td>
<td>2.18</td>
<td>1.91</td>
<td>2.46</td>
<td>2.03</td>
<td>1.61</td>
</tr>
<tr>
<td></td>
<td>(0.23)</td>
<td>(0.26)</td>
<td>(0.28)</td>
<td>(0.30)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>MFI economies</td>
<td>3.84</td>
<td>3.31</td>
<td>3.22</td>
<td>4.05</td>
<td>3.59</td>
</tr>
<tr>
<td></td>
<td>(0.20)</td>
<td>(0.42)</td>
<td>(0.37)</td>
<td>(0.44)</td>
<td>(0.62)</td>
</tr>
<tr>
<td>LFI economies</td>
<td>4.67</td>
<td>3.36</td>
<td>4.88</td>
<td>4.53</td>
<td>2.70</td>
</tr>
<tr>
<td></td>
<td>(0.35)</td>
<td>(0.61)</td>
<td>(1.01)</td>
<td>(0.69)</td>
<td>(0.38)</td>
</tr>
<tr>
<td><strong>Consumption</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industrial countries</td>
<td>2.37</td>
<td>1.47</td>
<td>2.16</td>
<td>1.98</td>
<td>1.72</td>
</tr>
<tr>
<td></td>
<td>(0.30)</td>
<td>(0.27)</td>
<td>(0.25)</td>
<td>(0.28)</td>
<td>(0.20)</td>
</tr>
<tr>
<td>MFI economies</td>
<td>5.18</td>
<td>4.57</td>
<td>4.52</td>
<td>4.09</td>
<td>4.66</td>
</tr>
<tr>
<td></td>
<td>(0.51)</td>
<td>(0.49)</td>
<td>(1.04)</td>
<td>(0.94)</td>
<td>(0.46)</td>
</tr>
<tr>
<td>LFI economies</td>
<td>6.61</td>
<td>5.36</td>
<td>7.07</td>
<td>7.25</td>
<td>5.72</td>
</tr>
<tr>
<td></td>
<td>(0.78)</td>
<td>(0.58)</td>
<td>(0.11)</td>
<td>(0.81)</td>
<td>(0.78)</td>
</tr>
</tbody>
</table>

Source: Prasad et al. (2003).

Notes: Percentage standard deviations, medians for each group of countries. Standard errors are reported in parentheses. MFI denotes more financially integrated, and LFI less financially integrated, economies.
Table 2. Model Calibration (Baseline)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.96</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.92</td>
</tr>
<tr>
<td>$j^H$</td>
<td>0.01</td>
</tr>
<tr>
<td>$\tau$</td>
<td>2</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1.1</td>
</tr>
<tr>
<td>$a_H$</td>
<td>0.44</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.17</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.48</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.12</td>
</tr>
<tr>
<td>$m$</td>
<td>0.3 (baseline), 0.15, 0.45</td>
</tr>
<tr>
<td>$\rho_X$</td>
<td>0.38</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>0.67</td>
</tr>
<tr>
<td>$\sigma_X$</td>
<td>0.0663</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>0.0323</td>
</tr>
</tbody>
</table>

Table 3. Main “Great” Ratios of Malaysia (with respect to output)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Actual</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>55.7</td>
<td>57.8</td>
</tr>
<tr>
<td>Investment</td>
<td>25.7</td>
<td>27.8</td>
</tr>
<tr>
<td>Public Expenditure</td>
<td>15.3</td>
<td>15.3</td>
</tr>
<tr>
<td>Export</td>
<td>57.8</td>
<td>56.5</td>
</tr>
<tr>
<td>Imports</td>
<td>55.8</td>
<td>55.4</td>
</tr>
<tr>
<td>Net Exports</td>
<td>2.0</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Source: Kim et al. (2003).

Notes: % values. Period 1960-1996. Actual values are taken from Kim et al. (2003).
Table 4. The collateral effect: consumption relative volatility (to output)

<table>
<thead>
<tr>
<th></th>
<th>1960-1984 Actual</th>
<th>1960-1984 Simulated (m = 0.15)</th>
<th>1985-1996 Actual</th>
<th>1985-1996 Simulated (m = 0.45)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>1.33</td>
<td>1.11</td>
<td>2.19</td>
<td>1.17</td>
</tr>
</tbody>
</table>

Notes: volatility is measured by the standard deviation of logged and filtered (HP filter, with $\lambda = 100$) variables.

Table 5. The Volatility of the Main Variables

<table>
<thead>
<tr>
<th></th>
<th>1960-96 Actual</th>
<th>1960-96 Simulated (m = 0.3)</th>
<th>1960-1984 Actual</th>
<th>1960-1984 Simulated (m = 0.15)</th>
<th>1985-1996 Actual</th>
<th>1985-1996 Simulated (m = 0.45)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>4.68</td>
<td>3.54</td>
<td>4.02</td>
<td>3.41</td>
<td>4.75</td>
<td>3.66</td>
</tr>
<tr>
<td></td>
<td>(0.30)</td>
<td>(0.33)</td>
<td>(0.33)</td>
<td></td>
<td>(0.33)</td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>3.39</td>
<td>3.15</td>
<td>3.01</td>
<td>3.07</td>
<td>2.17</td>
<td>3.14</td>
</tr>
<tr>
<td></td>
<td>(0.33)</td>
<td>(0.26)</td>
<td>(0.26)</td>
<td></td>
<td>(0.27)</td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>14.09</td>
<td>14.27</td>
<td>11.94</td>
<td>13.6</td>
<td>11.44</td>
<td>14.90</td>
</tr>
<tr>
<td></td>
<td>(1.17)</td>
<td>(1.13)</td>
<td>(1.13)</td>
<td></td>
<td>(1.34)</td>
<td></td>
</tr>
<tr>
<td>Trade Balance</td>
<td>8.29</td>
<td>36.59</td>
<td>8.75</td>
<td>43.27</td>
<td>4.59</td>
<td>30.30</td>
</tr>
<tr>
<td></td>
<td>(2.90)</td>
<td>(3.37)</td>
<td>(3.37)</td>
<td></td>
<td>(2.40)</td>
<td></td>
</tr>
<tr>
<td>Imports</td>
<td>10.29</td>
<td>7.76</td>
<td>8.68</td>
<td>7.47</td>
<td>8.30</td>
<td>7.94</td>
</tr>
<tr>
<td></td>
<td>(0.63)</td>
<td>(0.55)</td>
<td>(0.55)</td>
<td></td>
<td>(0.59)</td>
<td></td>
</tr>
</tbody>
</table>

Source: Kim et al. (2003).

Note: volatility is measured by the standard deviation of logged and filtered (HP filter, with $\lambda = 100$) variables (standard error between brackets).
Table 6. The Persistence of the Main Variables

<table>
<thead>
<tr>
<th></th>
<th>1960-96 Actual</th>
<th>1960-96 Simulated $(m = 0.3)$</th>
<th>1960-1984 Actual</th>
<th>1960-1984 Simulated $(m = 0.15)$</th>
<th>1985-1996 Actual</th>
<th>1985-1996 Simulated $(m = 0.45)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>0.52</td>
<td>0.21</td>
<td>0.30</td>
<td>0.20</td>
<td>0.44</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td></td>
<td>(0.11)</td>
<td></td>
<td>(0.11)</td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>0.62</td>
<td>0.36</td>
<td>0.48</td>
<td>0.34</td>
<td>0.30</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td></td>
<td>(0.09)</td>
<td></td>
<td>(0.09)</td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>0.73</td>
<td>0.18</td>
<td>0.61</td>
<td>0.17</td>
<td>0.38</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td></td>
<td>(0.10)</td>
<td></td>
<td>(0.11)</td>
<td></td>
</tr>
<tr>
<td>Trade Balance</td>
<td>0.33</td>
<td>-0.11</td>
<td>0.19</td>
<td>-0.09</td>
<td>0.05</td>
<td>-0.12</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td></td>
<td>(0.09)</td>
<td></td>
<td>(0.10)</td>
<td></td>
</tr>
<tr>
<td>Imports</td>
<td>0.42</td>
<td>0.07</td>
<td>0.18</td>
<td>0.04</td>
<td>0.10</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td></td>
<td>(0.12)</td>
<td></td>
<td>(0.13)</td>
<td></td>
</tr>
</tbody>
</table>

Source: Kim et al. (2003).

Note: persistence is measured by the first order autocorrelation coefficient of the logged and filtered (HP filter, with $\lambda = 100$) variables (standard error between brackets).
Table 7. Contemporaneous Comovement with Output

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual</td>
<td>Simulated $(m = 0.3)$</td>
<td>Actual</td>
<td>Simulated $(m = 0.15)$</td>
<td>Actual</td>
<td>Simulated $(m = 0.45)$</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.69</td>
<td>0.22</td>
<td>0.54</td>
<td>0.23</td>
<td>0.75</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.11)</td>
<td>(0.11)</td>
<td></td>
<td>(0.11)</td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>0.83</td>
<td>0.15</td>
<td>0.70</td>
<td>0.17</td>
<td>0.90</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.10)</td>
<td>(0.10)</td>
<td></td>
<td>(0.11)</td>
<td></td>
</tr>
<tr>
<td>Trade Balance</td>
<td>-0.37</td>
<td>-0.05</td>
<td>-0.15</td>
<td>-0.06</td>
<td>-0.23</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.09)</td>
<td>(0.09)</td>
<td></td>
<td>(0.09)</td>
<td></td>
</tr>
<tr>
<td>Imports</td>
<td>0.58</td>
<td>-0.16</td>
<td>0.47</td>
<td>-0.17</td>
<td>0.16</td>
<td>-0.18</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.11)</td>
<td>(0.11)</td>
<td></td>
<td>(0.10)</td>
<td></td>
</tr>
</tbody>
</table>

Source: Kim et al. (2003).

Note: contemporaneous comovement with output is measured by the correlation between the filtered (HP filter, with $\lambda = 100$) series and the filtered output (standard error between brackets).
Table 8. The collateral effect

<table>
<thead>
<tr>
<th></th>
<th>1960-96</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual</td>
</tr>
<tr>
<td>Consumption</td>
<td>4.68</td>
</tr>
<tr>
<td>Output</td>
<td>3.39</td>
</tr>
<tr>
<td>Investment</td>
<td>14.09</td>
</tr>
</tbody>
</table>

Notes: volatility is measured by the the standard deviation of logged and filtered (HP filter, with $\lambda = 100$) variables (standard error between brackets).
Figure 1. Alternative values of $m$: shock to the export

Notes: dashed lines $m = 0.45$; solid lines $m = 0.15$. Ordinate: time horizon in years. Coordinate: percent deviation from steady state. Size of the shock: 1%.
Figure 2. Alternative values of m: technology shock

Notes: dashed lines $m = 0.45$; solid lines $m = 0.15$. Ordinate: time horizon in years. Coordinate: percent deviation from steady state. Size of the shock: 1%. Plots of terms of trade and output are magnified to emphasize differences.
Figure 3. The role of borrowing constraint: shock to the export

Notes: dashed lines: no borrowing; solid lines \( m = 0.3 \). Ordinate: time horizon in years. Coordinate: % deviation from steady state. Size of the real estate price shock: 1%.
Figure 4. The role of borrowing constraint: technology shock

Notes: dashed lines: no borrowing; solid lines $m = 0.3$. Ordinate: time horizon in years. Coordinate: % deviation from steady state. Size of the real estate price shock: 1%. The plot of the output is magnified to emphasize differences.
Figure 5. Alternative values of the elasticity of intratemporal substitution: shock to the export

Notes: dashed lines $\rho = 2$; solid lines $\rho = 1.1$. Ordinate: time horizon in years. Coordinate: percent deviation from steady state. Size of the shock: 1%.
Figure 6. Alternative values of the elasticity of intratemporal substitution: technology shock

Notes: dashed lines $\rho = 2$; solid lines $\rho = 1.1$. Ordinate: time horizon in years. Coordinate: percent deviation from steady state. Size of the shock: 1%.
Figure 7. Alternative values of the degree of openness: shock to the export

Notes: dashed lines $a_H = 0.75$; solid lines $a_H = 0.44$. Ordinate: time horizon in years. Coordinate: percent deviation from steady state. Size of the shock: 1%.
Figure 8. Alternative values of the degree of openness: technology shock

Notes: dashed lines $a_H = 0.75$; solid lines $a_H = 0.44$. Ordinate: time horizon in years. Coordinate: percent deviation from steady state. Size of the shock: 1%.
Figure 9. Alternative values of the persistence of the shock to the export

Notes: dashed lines $\rho_X = 0.92$; solid lines $\rho_X = 0.38$. Ordinate: time horizon in years. Coordinate: percent deviation from steady state. Size of the shock: 1%.
Figure 10. Alternative values of the persistence of the technology shock

Notes: dashed lines $\rho_A = 0.92$; solid lines $\rho_A = 0.67$. Ordinate: time horizon in years. Coordinate: percent deviation from steady state. Size of the shock: 1%.
CHAPTER 2

House Prices, Consumption and Trade Balance in the U.S.

2.1. Introduction

The high level of trade and financial global imbalances is stimulating a large debate about the determinants of the U.S. trade balance and current account dynamics. The main views are two. The 'global saving glut' explain the widening of the U.S. external imbalances in terms of three factors: (1) a sharp rise in national savings in China and oil-exporting countries; (2) relatively weak investments in some key industrial countries such as Germany and Japan and in some emerging Asian countries; (3) relatively high investments in the U.S. economy.1 The other view emphasizes the reduction in domestic saving and it can be split in two strands: one highlights the influence of the widening U.S. public budget deficit on the reduction of national savings ('twin deficit' hypothesis); the other points out the negative effect of the booming house prices on household saving.2

Recent contributions tend to downplay the 'global saving glut' and the 'twin deficit' hypotheses. Regarding the first hypothesis, some authors observe that in the period 2001-2004 U.S. investments have decreased.3 Regarding the second, the Ricardian effect of public deficit on private saving seems to be not negligible: an increase in the federal budget, resulting in higher interest rates, lower private domestic consumption and investment, would have a rather low net effect on the external deficit.4

To the contrary, several observers emphasize the existence of a relationship between the recent boom in U.S. house prices and higher trade balance deficit.5 This pattern is experienced not only by the U.S. economy. The OECD, for example, reports that between 1996 and 2004 the countries whose current account have moved toward deficit

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1See Bernanke (2005).
2See Roubini and Setser (2005).
3See Roubini (2005).
4See Kim and Roubini (2004) and Ergeg et al. (2005).
5Bernanke (2005) points out "...the expansion of housing wealth, much of it easily accessible to households through cash-out refinancing and home equity lines of credit" and "...the evident link between rising household wealth and a tendency for the current account to shift toward deficit". Greenspan (2005) notes a strong correlation between the U.S. mortgage debt and the U.S. current account deficit. Rogoff (2005) says that "...low interest rates that have fuelled an increasingly speculative housing price boom, which has in turn contributed to low personal savings and a bigger current account deficit".
have also experienced substantial housing appreciation and increasing in house wealth, contrary to countries where the housing appreciation has been slow.\textsuperscript{6}

Following the latter strand of the debate, I investigate the relationship between the U.S. housing prices and trade balance.

I estimate two vectors autoregressive (VAR from now on) models to look at the empirical evidence initially, then I rationalize it in terms of a two-country dynamic stochastic general equilibrium (DSGE from now on) model.

Main feature of the model is the presence of a domestic borrowing constraint: some agents, I define them ‘constrained’, in the country have only access to domestic financial markets and can borrow using the domestic real estate holdings as collateral.\textsuperscript{7} Unconstrained agents, instead, can also borrow from the rest of the world. The trade balance is determined by the difference between the value of the output and that of the total demand. Other features are rather standard to get a model as simple as possible: in each country two tradable non-durable goods are consumed, one domestically produced, the other imported from the other country; monetary policy is introduced in form of a Taylor rule; there are nominal rigidities in form of staggered prices; to capture the observed low degree of import price pass-through at consumer level, I assume import prices are set in local currency terms. As a first step, I develop a simplified version of the model. It is a one-country open economy model, with the rest of the world taken as exogenous. Thanks to this basic setup, I can focus on the relationship between house prices, consumption and trade balance in the U.S. economy.\textsuperscript{8} I subsequently extend the basic setup and develop a two-country model to take into account general equilibrium considerations resulting from the United States’ large size in the global economy.

The main result of the model, roughly in line with the empirical evidence suggested by the estimated VARs, is that changes in house prices contribute to determine the trade balance dynamics through a collateral effect on agents subject to the borrowing constraint. Higher housing prices allow higher borrowing and hence higher consumption; as a consequence, imports increase contributing to shift the trade balance towards deficit.\textsuperscript{9}

Other results are the following ones. First, the effect of a house price shock on the trade balance is higher in correspondence of higher values of the loan-to-value ratio

\textsuperscript{6}OECD Economic Outlook (2004).
\textsuperscript{8}Obstfeld and Rogoff (2000) also develop a partial equilibrium model of U.S. that negleects its large size.
\textsuperscript{9}Several empirical papers find a positive relationship between consumption and house prices. Case et al. (2001) find long-run elasticities of consumption to housing prices of around 0.06 for a panel of U.S. states. Davis and Palumbo (2001) estimate a long-run elasticity of consumption to housing wealth of 0.08. These positive elasticities are hard to reconcile with the traditional life-cycle model. For more detailed discussion, see Iacoviello (2005).
(the amount of borrowing for unit of collateral) faced by the U.S. constrained agents. Second, lower values of the elasticity of substitution between domestically produced and imported goods limit the size of the trade balance deficit in the aftermath of a shock. Third, the deterioration of the trade balance after a U.S. expansionary monetary shock is higher under a flexible exchange rate regime than under a regime in which the other country pegs the dollar. Finally, I analyze how the nominal exchange rate depreciation affects the trade balance dynamics when the assumption of low pass-through of nominal exchange rate into import prices is relaxed and it is assumed that pass-through is complete (so that the depreciation of, let's say, the U.S. nominal exchange rate has a strong impact on international relative prices and shifts world demand towards U.S. produced goods). I show that the depreciation of the exchange rate contributes to some extent to limit the increase of the trade deficit under this counterfactual assumption.

This paper is related to recent studies that have formalized the 'credit view' in open economy general equilibrium models.

Iacoviello and Minetti (2005) develop an international real business cycle model based on a borrowing constraint a la Kiyotaki-Moore, having real estate as collateral. Also Gilchrist et al. (2002) and Faia (2005) study the open economy implications of financial frictions, which are introduced via a risk premium on borrowing proportional to the level of leverage. The performed analysis focuses on how cross-country financial heterogeneity affects and amplifies the propagation of international spillovers under alternative monetary policies and exchange rate regimes.

My paper differs in three main aspects. First, I perform a systematic evaluation of the implications of the household borrowing constraint for the trade balance dynamics. Second, I focus on the real estate market and consumer's expenditure. Third, I focus on the U.S. economy by estimating the VARs and appropriately calibrating the DSGE model.

The plan of the paper is as follows. Next section illustrates the VAR evidence on the relationship between trade deficit and real estate price. Section three rationalizes the evidence in terms of a single open economy model with a real estate credit channel (the rest of the world is taken as exogenous). Section four develops a two-country version of the model used in section three. Conclusions are in section five.

2.2. Empirical evidence

I use the VAR methodology to find evidence of a relationship between house prices, private consumption and external deficit in the U.S. economy.

I estimate two VARs. The two models share the variables needed to identify a monetary policy and a house price shock; they differ in terms of the variable representing the U.S. trade. The shared variables are the detrended log of real gross domestic product (Y), the change in the log of consumer price index (π), the detrended log of real house prices (q), the Fed Funds rate (R), the detrended log of real private consumption (C).

The model-specific variables are:
1) the ratio of detrended balance of goods and services (divided by the consumer price level) to the average (over the estimation period) real gross domestic product (TB); 2) detrended log of real export (X) and imports (M).\(^{12}\)

Data are from 1974Q1 to 2005Q2. The Fed Funds rate is the average value in the first month of each quarter. The house price series (deflated with the consumer price deflator) is the Conventional Mortgage Home Price Index from Freddie Mac.\(^{13}\) The logs of gross domestic product, housing prices, consumption, exports and imports are detrended with a band-pass filter that removes frequencies above 32 quarters. All the considered variables are expressed in percentages and in quarterly rates.

The VARs have lag length equal to three (chosen according to the Hannah-Quinn criterion) and include a constant, a time trend, a shift dummy from 1979Q4 (to take into account the change in the U.S. monetary policy) and one lag of the log of the CRB (Commodity Research Bureau) commodity spot price index. To make the VAR and the DSGE models more comparable, the shocks are orthogonalized in the order R, π, q, Y, C, TB; in the other VAR X and M, taken in that order, substitute TB, while the order of the remaining variables does not change.\(^{14}\)

Figure 1 presents impulse responses to a negative interest rate shock.\(^{15}\) There is a positive response of real housing prices, consumption, while inflation increases with some lag. The trade balance moves towards deficit.

Figure 2 presents impulse responses to a positive house price shock. Also in this case, there is a positive comovement between house prices and consumption and the trade balance moves towards deficit.

To better understand the dynamics of the trade balance, I run the VAR substituting the trade balance variable with U.S. export and imports.

Figure 3 presents impulse responses to a negative interest rate shock. Consumption and real estate prices increase, as well as imports.

A similar pattern - an increase in consumption and imports - is observed when a positive house price shock is considered, as shown by Figure 4.

\(^{12}\)The variable TB has the property of being consistent with the trade balance variable used in the DSGE model.

\(^{13}\)Data on gross domestic product, private consumption, consumer prices, export, imports, trade balance are taken from the database FREDII of the St. Louis Federal Reserve and from the Bureau of Economic Analysis.

\(^{14}\)However changing the order does not greatly affect the results.

\(^{15}\)All the VAR impulses have 90-percent bootstrapped confidence bands. Shocks have a one standard deviation size.
TB

in the

date
The reported evidence suggests two points. First, following an exogenous increase in housing prices or decrease in interest rate, there is a positive comovement between housing real prices and consumption; this evidence, which is in line with the results found by Iacoviello (2005), suggests the existence of a real estate credit channel; through it, the asset “real estate” affects domestic demand for consumption. Second, trade balance deteriorates when real estate prices and consumption increase, while the amount of imports increases.

In what follows, I rationalize this evidence through a microfounded open economy DSGE model having a real estate credit channel. This channel, in fact, is able to create, following an increase in house prices, a collateral effect on consumption and hence a positive comovement between the two variables, in line with the found empirical evidence.

2.3. The Basic Setup

In this section I illustrate the basic model. It is a one-country open economy model, with exogenous rest of the world. In the next section, I illustrate the complete two-country model.

The country, denominated Home, can be interpreted as a stylized representation of the U.S. economy. The rest of the world is denominated Foreign. In the Home country there is a continuum of economic agents. Some agents are financially unconstrained (they lie on the interval \((0, 1]\)) and the others are constrained (they lie on \((1, 2]\)).

Constrained agents are subject to a borrowing constraint: they can borrow a certain fraction of the expected value of the collateral, which is the real estate. Thanks to this assumption, I formalize in a simple way the real estate credit channel. Both types of households belonging to the Home country consume, work, demand money and real estate services. I assume real estate is fixed in aggregate and not tradable at international level. Agents supply labor to a wholesale sector which, according to a Cobb-Douglas technology, produces a homogenous good. The good is differentiated by the retailer sector, and sold domestically and in the rest of the world. Firms in the retail sector, owned by the unconstrained agents, are the source of nominal rigidities in the model. In the country there is a central bank that adjusts money supply and transfers to support an interest rate rule.

In what follows, starred variables are referred to variables belonging to rest of the world.

\(^{16}\)See also Aoki et al. (2004) and Iacoviello (2002) for other VAR models on the real estate credit channel.
2.3.1. Bundle and price indexes

The consumption index $C$ of agents belonging to the Home country is given by the following constant-elasticity-of-substitution (CES) aggregator:

\[ C_t \equiv \left[ \frac{1}{a_H} C_{H,t}^{\frac{\rho-1}{\rho}} + (1-a_H) \frac{1}{\rho} C_{F,t}^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \quad \rho > 0 \]

where $C_H$ ($C_F$) is the consumption of Home (Foreign) produced good, $a_H$ ($0.5 < a_H < 1$) measures the home bias in consumption, $\rho$ is the intratemporal elasticity of substitution between Home and imported goods. In the rest of the world, the CES aggregator is (I assume mirror symmetric home bias):

\[ C_t^* \equiv \left[ (1-a_H)^{\frac{1}{\rho}} C_{H,t}^* \rho^{\frac{\rho-1}{\rho}} + a_H C_{F,t}^* \rho^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \]

The consumption bundles $C_H$ and $C_F$ are defined as:

\[ C_{H,t} = \left( \int_0^1 C_t(h)^\frac{\rho-1}{\rho} \, dh \right)^{\frac{\rho}{\rho-1}}, \quad C_{F,t} = \left( \int_0^1 C_t(f)^\frac{\rho-1}{\rho} \, df \right)^{\frac{\rho}{\rho-1}}, \quad \theta > 1 \]

where $C(h)$ and $C(f)$ are respectively the consumption of the generic Home and imported brand. $C^*(h)$ and $C^*(f)$ are similarly defined. For each type of good, I assume that one brand is an imperfect substitute for all the other brands produced within a country, with a constant elasticity of substitution equal to $\theta$.

The utility-based consumer price index in the Home country is:

\[ P_t = \left[ a_H P_{H,t}^{1-\rho} + (1-a_H) P_{F,t}^{1-\rho} \right]^{\frac{1}{1-\rho}} \]

where $P_H$ is the price of the domestically produced composite good and $P_F$ is that of the imported one. Prices are in units of the Home currency.

A similar price index of tradable goods holds in the rest of the world:

\[ P_t^* = \left[ (1-a_H) P_{H,t}^{*1-\rho} + a_H P_{F,t}^{*1-\rho} \right]^{\frac{1}{1-\rho}} \]

where prices are in units of the Foreign currency. Given that I assume that firms set their prices in the buyer's currency (local currency pricing policy), prices of the same goods should not be necessarily the same when measured in the same currency (see below for the description of the pricing policy of the firms). Hence: $P_H^* \neq P_H/S$ and $P_F \neq SP_F^*$, where $S$ is the nominal exchange rate of the Home economy, defined as number of Home currency units per unit of Foreign currency.

\[ ^{17}\text{The consumer price index } P \text{ is defined as the minimum expenditure necessary to buy one unit of } C, \text{ given the prices of the Home and Foreign good.} \]
The utility-based price index of the good produced in the Home country is:

\[ P_{H,t} = \left( \int_0^1 p_t(h)^{1-\theta} \, dh \right)^{\frac{1}{1-\theta}} \]

The indexes \( P_F, P_H^* \) and \( P_F^* \) are analogously defined.

Given the structure of preferences of the Home agents, the demands of the Home and Foreign bundles are respectively:

\[ C_{H,t} = a_H \left( \frac{P_{H,t}}{P_t} \right)^{-\rho} \]

\[ C_{F,t} = (1 - a_H) \left( \frac{P_{F,t}}{P_t} \right)^{-\rho} \]

They are functions of the relative prices and the total consumption.

In the rest of the world, the demands of the Home and Foreign bundles are respectively equal to:

\[ C_{H,t}^* = (1 - a_H) \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-\rho} \]

\[ C_{F,t}^* = a_H \left( \frac{P_{F,t}^*}{P_t^*} \right)^{-\rho} \]

In the Home country, the demands of the brands \( h \) and \( f \) are respectively:

\[ C_t(h) = \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\theta} C_{H,t}, \quad C_t(f) = \left( \frac{p_t(f)}{P_{F,t}} \right)^{-\theta} C_{F,t} \]

They are function of the relative price and the total consumption of the Home and imported goods. Demands for brands \( h \) and \( f \) in the rest of the world are similarly defined. I assume that investment goods and public expenditure have the same composition as that of consumption. Here I define the following relative prices:

\[ T_t \equiv \frac{P_{F,t}}{P_{H,t}}, \quad T_t^* \equiv \frac{P_{F,t}^*}{P_{H,t}^*} \]

They represent the relative prices of the Foreign good in terms of the Home good, expressed in local currency, respectively in the Home country and in the rest of the world. Finally, I define the real exchange rate of the Home country:

\[ RS_t \equiv \frac{S_t P_t^*}{P_t} \]
2.3.2. Households

The Home country is populated by unconstrained and constrained households, each infinitely lived. Unconstrained agents have access to international financial markets. In what follows, unconstrained agents are indexed by \( j \), constrained agents by \( j' \).

**Unconstrained Agents**

Each unconstrained household maximizes a lifetime utility function \( U_0 \) given by:

\[
U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln C_t (j) + \gamma x_0 \ln \tilde{h}_t (j) + \chi \ln \frac{M_t (j)}{P_t} - \frac{\kappa}{\tau} L_t^R (j) \right] \quad \chi, \kappa, \tau > 0
\]

where \( E_0 \) is the expectation operator at time 0, \( \beta \in (0,1) \) is the discount factor, \( C \) is consumption of tradable goods, \( \tilde{h} \) denotes the holdings of housing, \( L \) are hours of work and \( M/P \) are money balances \( M_t \) divided by the price level. The subscript \( t \) under \( j^h \) allows for random disturbances to the weight of housing in the utility function; it constitutes a parsimonious way to formalize an exogenous disturbance on housing prices. The budget constraint of the agent \( j \) is defined as follows:

\[
B_t (j) = B_{t-1} (j) + \frac{B_{H,t} (j)}{1 + i_{H,t}} - B_{H,t-1} (j) + M_t (j) - M_{t-1} (j)
\]

\[
= \int_0^1 \Pi_t (h, j) dh + R^K K(j) + W_t L_t (j) - P_t C_t (j) - Q_t (\tilde{h}_t (j) - \tilde{h}_{t-1} (j))
- TAX_t (j) - P_t I_t (j)
\]

\( B \) and \( B_H \) are respectively household's holding of an internationally traded and of a domestically traded risk-free one-period nominal bond, both denominated in units of the Home currency.\(^{18}\) This assumption is consistent with empirical evidence, which suggests that a large fraction of U.S. foreign debt is denominated in U.S. dollars. The nominal interest rate on \( B \) is \( i \), while the nominal interest on \( B_H \) is \( i_H \). I introduce a financial friction, \( \Phi (\frac{B}{B_t}) \), that multiplies the nominal interest rate \( 1+i \) on internationally traded bonds. The function \( \Phi (.) \) captures the costs of undertaking the financial position. It depends on the real holdings of internationally traded bonds by the entire class of unconstrained agents. Hence unconstrained agents take the function \( \Phi (.) \) as given when deciding the optimal holding of the bond. As borrowers, they will charged a premium on the interest rate; as lenders, they will receive a remuneration lower than the interest rate. Unconstrained agents are the only agents in the model subject to this cost; neither Home constrained agents nor Foreign agents are subject to it.\(^{19}\)

\(^{18}\)The international financial structure of the model is similar to that of Benigno (2001).

\(^{19}\)I introduce the friction to make the model stationary and well define the steady state.
in the next section) are $\int_0^1 \Pi(h, j) dh$. $W$ is the nominal wage. $K$ is the capital stock. I assume capital is equal to its steady state level (see the Appendix A). I introduce it only to correctly calibrate the model. I also assume that in each period firms demand of capital is constant. Hence also $R^k$, the rental rate of capital, is constant and equal to its steady state level.\(^{20}\) In the complete model, illustrated later, I allow the physical capital to change over time. Here, I prefer to have the model as simple as possible, so to focus on the relationship between real estate, consumption and trade balance. The real estate $\tilde{h}$ is a durable good and $Q$ is its nominal price. $I$ is the investment in physical capital, which I assume to be constant at its steady state level. $TAX$ are non-distortionary (lump-sum) net taxes to unconstrained households. I assume that: (a) preferences and initial wealth conditions are the same for all agents; (b) retailer profits are equally shared across agents. As a consequence, all unconstrained agents take the same optimal consumption and labor paths; I can drop the index $j$ and consider a representative unconstrained agent. The first order conditions for the choice of the internationally traded bond, domestically traded bond, real estate demand, labor supply are respectively:

\[
\frac{1}{P_tC_t (1 + i_t) \Phi \left( \frac{B_t}{P_t} \right)} = E_t \left( \beta \frac{1}{P_{t+1}C_{t+1}} \right)
\]

(2.16)

\[
(1 + i_t) \Phi \left( \frac{B_t}{P_t} \right) = (1 + i_{H,t})
\]

(2.17)

\[
\frac{Q_t}{P_tC_t} = j_t \frac{1}{h_t} + \beta E_t \left( \frac{Q_{t+1}}{P_{t+1}C_{t+1}} \right)
\]

(2.18)

\[
\frac{W_t}{P_tC_t} = \kappa L_t^{-1}
\]

(2.19)

First order conditions are rather standard. Equation (2.17) is the no-arbitrage condition, which derives from the fact that at the margin the agent equates the bonds revenues. Since I assume that in each country the local monetary authority follows an interest rate rule (as illustrated later in the text), money supply will always respond to meet money demand at the desired equilibrium nominal interest rate; as utility is separable in money balances and the assumption of equality between money injections

\(^{1}\) I require that $\Phi(0) = 1$ and that $\Phi(.) = 1$ only if $B_t = 0$. $\Phi(.)$ is a, differentiable, (at least) decreasing function in the neighborhood of zero. Revenues from the financial friction (which are always positive given the shape of the function $\Phi(.)$) are assumed to be distributed to Foreign agents: $FF = \frac{B}{P(1+i)} \left[ \frac{1}{\Phi(0)} - 1 \right]$. Benigno (2001) and Schmitt-Grohé and Uribe (2001) use a similar feature to make the model stationary.

\(^{20}\) Campbell and Hercowitz (2004) also make the same assumption.
and net taxes for each group of agents holds (see later), the actual quantity of money has no implications for the rest of the model; therefore the money first order condition can be ignored.

**Constrained Agents**

Constrained households (denoted with a \( j' \)) have the same preferences as the unconstrained ones. The only difference is that their discount factor, \( \gamma \), is lower than that of the unconstrained agents, \( \beta \). This assumption guarantees that the constrained agents face a binding borrowing constraint in equilibrium (see the Appendix A). The budget constraint of the constrained agents is defined as:

\[
B_{H,t}(j') - B_{H,t-1}(j') + M_t(j') - M_{t-1}(j') = W_t^j L_t(j') - P_t C_t(j') - Q_t(\bar{h}_t(j') - \bar{h}_{t-1}(j')) - TAX_t(j')
\]

Constrained agents do not have access to international financial markets. This assumption allows to uniquely determine in a simple way the asset position of the constrained agents.

Main aspect of the allocation problem is the presence of the borrowing constraint. Following Kiyotaki and Moore (1997), it is assumed the existence of a limit on the debt of the agent \( j' \). In each period \( t \), the amount she can borrow from other domestic agents, comprehensive of interest payment, cannot exceed a fraction \( m \leq 1 \) of next period’s expected value of real estate holdings. Hence, the constrained household debt is limited by:

\[
-B_{H,t}(j') \leq mE_t(Q_{t+1}\bar{h}_t(j'))
\]

One way to rationalize this constraint is by thinking to the existence of liquidation costs: in case of default, costs amount to a fraction \( 1 - m \) of the real estate value. As in the case of unconstrained agents, I assume that preferences and initial wealth conditions are the same across constrained agents. Given that they face the same prices, they choose the same optimal path for borrowing, consumption and labor supply. I hence drop the index \( j' \) and consider a constrained representative agent.

The first order conditions with respect to \( B_{H,t}^j, \bar{h}_t^j \) and \( L_t^j \) are:

\[
E_t \left( -\frac{1}{P_t C_t^j (1 + i_{H,t}^j)} + \gamma \frac{1}{P_{t+1} C_{t+1}^j} + \frac{1}{P_t} \lambda_t^j \right) = 0
\]

\[
\frac{Q_t}{P_t C_t^j} = j^h \frac{1}{\bar{h}_t^j} + \gamma E_t \left( -\frac{Q_{t+1}}{P_{t+1} C_{t+1}^j} \right) + \lambda_t^j m E_t \left( \frac{Q_{t+1}}{P_t} \right)
\]
Both the Euler and the housing demand equations differ from the usual formulations because of the presence of $\lambda_t$, the Lagrange multiplier on the borrowing constraint, which distorts both intratemporal and intertemporal allocation between houses and non-durable consumption.

As in the case of the unconstrained agents, the assumptions of (1) monetary authority following an interest rate rule, (2) equality between money injections and taxes for each group of agents, (3) separable utility in money balances imply that the actual quantity of money is residual with respect to the rest of the model; the money first order condition of constrained agents can therefore be ignored.

I assume that the following uncovered interest parity (expressed in log-linear terms) holds for Foreign agents:

$$
(2.25) \quad (1 + i_t) - (1 + \hat{i}_{t+1}) = E_t(\Delta S_{t+1})
$$

where $E_t(\Delta S_{t+1})$ is the expected variation of the nominal exchange rate (see the Appendix A for more details).

I take the rest of the world as exogenous. The above condition can be rationalized by assuming that Foreign agents can allocate their wealth among domestic money and two bonds. Both bonds are risk-free with one-period maturity. One, $B^*_F$, is denominated in Foreign currency, pays a nominal interest rate equal to $i^*_t$ and is not internationally traded; the other is the bond exchanged with Home unconstrained agents. Optimal behavior requires that at the margin revenues from the two bonds must be equal.

### 2.3.3. The Production Sector

The supply side of the Home economy is constituted by a wholesale and a retailer sector. The former produces a homogeneous good and sells it to the latter. The retailer sector differentiates the good and sells it both domestically and abroad. In this section, I illustrate the problem solved in each sector.

### 2.3.4. The wholesale sector

The wholesale sector produces a homogenous good under a perfect competition regime. It assembles labor supplied by the two agents to operate a constant return to scale production function:

$$
(2.26) \quad Y_t = K^\alpha L_t^{(1-\alpha)\mu} L_t^{\mu(1-\alpha)(1-\mu)} \quad 0 < \alpha < 1 \quad 0 < \mu < 1
$$

Capital is assumed to be constant at the steady state level (I relax this assumption in the complete model). Labor demand results from a static problem of cost minimization.
First order conditions for $L_t$ and $L'_t$ are:

\begin{align}
W_t &= (1 - \alpha) \mu P_W Y_t / L_t \\
W'_t &= (1 - \alpha) (1 - \mu) P_W Y_t / L'_t
\end{align}

where $P_W$ is the price of the wholesale good.

### 2.3.5. Retailers

There is a continuum of retailers having mass one which buy the domestic homogeneous good at the given price $P_W$ from the domestic wholesale sector, differentiate at no cost and then sell it both domestically and internationally. Each retailer is the only producer of a single differentiated good. The retail sector is subject to a monopolistic competition regime. In this way I introduce price stickiness as in Calvo (1983): when a Home retailer $h$ has the opportunity to set a new price in the Home or in the Foreign market in period $t$, it does so to maximize the expected valued of its profits taking into account that there is a nonzero probability of not adjusting prices in each future period.

I assume there is international good market segmentation: retailers engage in third degree price discrimination across countries and set prices in each country in terms of local currency, taking the demand curve as given (hence the international law of one price does not hold). Home retailers choose Home and Foreign currency prices, respectively $p^*_t(h)$ and $p^*_{t^*}(h)$, to maximize the expected discounted value of profits. Hence, they solve the following problem:\textsuperscript{21}

\begin{equation}
\max_{p^*_t(h), p^*_{t^*}(h)} \sum_{k=0}^{\infty} \vartheta^k \Lambda_{t,t+k} E_t \left( \frac{p^*_t(h) Y_{t+k}(h) + S p^*_{t^*}(h) Y^*_{t+k}(h)}{p^*_{t+t+k}(h) + Y^*_{t+k}(h)} \right)
\end{equation}

where $Y_{t+k}(h)$ and $Y^*_{t+k}(h)$ are respectively the Home and Foreign demand for Home produced good:

\begin{equation}
Y_{t+k}(h) = \left( \frac{p^*_t(h)}{P_{H,t+k}} \right)^{-\theta} \left[ a_H \left( \frac{P_{H,t+k}}{P_{t+k}} \right)^{-\rho} \left( C_{t+k} + C^*_{t+k} + I \right) \right]
\end{equation}

and

\begin{equation}
Y^*_{t+k}(h) = \left( \frac{p^*_{t^*}(h)}{P^*_{H,t+k}} \right)^{-\theta} \left[ (1 - a_H) \left( \frac{P^*_{H,t+k}}{P^*_{t+k}} \right)^{-\rho} \left( C^*_{t+k} \right) \right]
\end{equation}

$\Lambda_{t,t+k} = \beta (C_t/C_{t+k})$ is the unconstrained household discount factor (retailers are agents for the unconstrained household, to whom transfer profits in a lump-sum fashion). The parameter $\vartheta$ is the probability of not adjusting the price of the brand $h$ in the Home

\textsuperscript{21}Benigno (2004) solves a similar problem.
and in the Foreign market. Since retailers simply repackage wholesale goods, \( P^W_t \) is the marginal cost of producing a unit of output. The optimal choices of \( p_t^* (h) \) and \( p_t^* (h) \) satisfy the following first order conditions, respectively:

\[
\sum_{k=0}^{\infty} \vartheta^k E_t \left( \Lambda_{t, t+k} \left( \frac{(\theta - 1) p_t^* (h) - \theta P_{W, t+k} Y_{t+k} (h)}{P_{t+k}} \right) \right) = 0 \tag{2.32}
\]

\[
\sum_{k=0}^{\infty} \vartheta^k E_t \left( \Lambda_{t, t+k} \left( \frac{(\theta - 1) S_t p_t^* (h) - \theta P_{W, t+k} Y_{t+k}^* (h)}{P_{t+k}} \right) \right) = 0 \tag{2.33}
\]

Since there are no firm-specific state variables, all retailers setting price at \( t \) will choose the same optimal prices \( p_t^* (h) \) and \( p_t^* (h) \). It can be shown that the price indexes of the Home good evolve in the Home and in rest of the world respectively according to:

\[
P_{H, t} = \left( \vartheta P_{H, t-1}^{1-\vartheta} + (1 - \vartheta) (p_t^* (h))^{1-\vartheta} \right)^{1/(1-\vartheta)} \tag{2.34}
\]

\[
P_{H, t}^* = \left( \vartheta P_{H, t-1}^{1-\vartheta} + (1 - \vartheta) (p_t^* (h))^{1-\vartheta} \right)^{1/(1-\vartheta)} \tag{2.35}
\]

Equations (2.32) and (2.34), once log-linearized around the steady state, yield a Phillips curve of the Home good holding in the Home country; it positively relates Home good inflation rate in the Home country, \( P_{H, t} / P_{H, t-1} \), to its expected value \( E_t (P_{H, t+1} / P_{H, t}) \) and to the marginal cost \( P_{W, t} \). Equations (2.33) and (2.35) yield a similar curve holding in the rest of the world. I assume similar equations hold for the Foreign retailers that export to the Home country:

\[
\sum_{k=0}^{\infty} \vartheta^k E_t \left( \Lambda_{t, t+k} \left( \frac{(\theta - 1) p_t^* (f) - \theta P_{W, t+k} Y_{t+k}^* (f)}{P_{t+k}} \right) \right) = 0 \tag{2.36}
\]

\[
P_{F, t} = \left( \vartheta P_{F, t-1}^{1-\vartheta} + (1 - \vartheta) (p_t^* (f))^{1-\vartheta} \right)^{1/(1-\vartheta)} \tag{2.37}
\]

Profits are rebated lump-sum to the agents in the rest of the world. Thanks to the assumption of local currency pricing, I introduce a low degree of nominal exchange rate pass-through into import prices and reduce the size of substitution effect (between domestic and imported goods) induced by changes in the nominal exchange rate. This assumption has two advantages: first, it is consistent with empirical evidence on the U.S. nominal exchange rate pass-through into import prices, which is extremely low in the short run; second, it allows to better emphasize the real estate credit channel and
the wealth effect of changes in the housing prices on the trade balance and imports. However, in the two-country model I also consider the case of complete pass-through.\textsuperscript{22}

In agreement with the preferences of Home and Foreign agents, the aggregate final goods are defined as the CES composite of individual retail goods:

\begin{equation}
Y_{H,t} \equiv \left( \int_0^1 Y_t(h) \frac{h^{1-\rho_y}}{\rho_y} dh \right)^{\frac{1}{1-\rho_y}}
\end{equation}

\begin{equation}
Y^*_{H,t} \equiv \left( \int_0^1 Y^*_t(h) \frac{h^{1-\rho_y}}{\rho_y} dh \right)^{\frac{1}{1-\rho_y}}
\end{equation}

Similar aggregators hold for goods produced in the rest of the world.\textsuperscript{23}

2.3.6. Monetary Policy Rule

In the Home country, the central bank makes lump sum transfers of money to the unconstrained and constrained agents to implement a Taylor-type interest rate rule. The rule has the form:

\begin{equation}
\left(1 + \delta_{H,t}\right) = \left(1 + \delta_{H,t-1}\right)^{\rho_R} \left(\frac{p_{t-1}}{p_{t-2}}\right)^{(1-\rho_R)\rho_y} \left(\frac{Y_{t-1}}{Y}\right)^{(1-\rho_y)\rho_Y} \epsilon_{R,t}
\end{equation}

with $0 < \rho_R < 1$, $\rho_x, \rho_Y > 0$. Monetary policy responds systematically to the deviations of past inflation $P_{t-1}/P_{t-2}$ and total output from their correspondent steady state levels, $1 + \delta_H$, $\pi$ and $Y$. The lagged value of the nominal interest rate is introduced to allow for interest rate inertia. $\epsilon_{R,t}$ is a shock to monetary policy.

2.3.7. Market Clearing Conditions, Public Sector Budget Constraints and the trade balance

The model is closed by the market clearing conditions and the public sector budget constraints. Equation (2.41) is the housing market clearing condition:

\begin{equation}
\bar{h}_t + \bar{h}'_t = H
\end{equation}

where $H$ is in a fixed amount. Equations (2.42) and (2.43) are the Home good market clearing conditions respectively in the Home country and in the rest of the world:

\textsuperscript{22}The assumptions of international price discrimination, local currency pricing, incomplete pass-through, their implications for the monetary policy and the empirical evidence are widely discussed in Campa and Goldberg (2002), Devereux and Engel (2001, 2002), Engel (2002).

\textsuperscript{23}CES makes aggregation difficult. However, it is possible to show that within a small neighborhood of the steady-state a linear aggregator $Y'_{H,t} = \int_0^1 Y_t(h) dh$ is equal to $Y_{H,t}$ ($Y^*_{H,t}$), with the sum of $Y_{H,t}$ and $Y^*_{H,t}$ equal to the aggregate wholesale output $Y_t$. In what follows I'll consider $Y_t$ as home aggregate output.
Equation (2.42)

\[ Y_{H,t} = C_{H,t} + C'_{H,t} + I_H \]

Equation (2.43)

\[ Y^*_{H,t} = C^*_{H,t} \]

Equation (2.44) is the Home demand for the good produced in the rest of the world:

Equation (2.45) is the Home country resource constraint:

In the Home country, government rebates seigniorage revenues in a lump-sum fashion to domestic agents. Hence the public sector budget constraint implies:

Equation (2.46) is the market clearing of the bond traded in the Home country:

In the Home country, government rebates seigniorage revenues in a lump-sum fashion to domestic agents. Hence the public sector budget constraint implies:

The trade balance of the Home country is obtained by consolidating the budget constraints of the Home agents, equations (2.15) and (2.20), and of the public sector; it is equal to:

Hence the trade balance is expressed as the change in the international asset position of the Home unconstrained agent, comprehensive of interest payments. It corresponds to the difference between total production and the total expenditure in the Home country.

2.3.8. The Equilibrium

The equilibrium is defined as a sequence of allocations and prices such that, given the initial conditions of the state variables and the stochastic processes of the exogenous shocks, (a) households, firms in the wholesale sectors and retailers solve their respective maximization problems, (b) the market clearing conditions, the government budget constraint and the monetary policy rules hold. For simplicity, and consistently with the focus of the paper (which is on the trade balance effects of the U.S. real estate credit channel) I assume that in the rest of the world the aggregate consumption, interest rate, marginal costs of production, inflation rate of the Foreign good do not change after a
shock in the U.S. economy. The equilibrium dynamics is characterized by solving a first-order log-linear approximation to the equilibrium conditions around the non-stochastic steady state. They are shown in the Appendix A. I relax the assumption of exogenous rest of the world in the two-country model.

2.3.9. Solution of the model

I calibrate the model as similarly as possible to Iacoviello (2005), who studies the real estate credit channel of monetary policy in the U.S. economy using a closed economy DSGE model. The time period is one quarter. Values are reported in Table 1. For the standard parameters, values which are within the range considered in the monetary business cycle literature are chosen. The discount factor of the unconstrained agents, $\beta$, is set equal to 0.99; the value of the parameter $\mu$ - determining the unconstrained agent's wage share - is set equal to 0.67; the share of capital is 0.33. The depreciation rate of capital, $\delta$, is 0.025. Following Lawrance (1991), which estimates discount factors for poor households (which are more likely to be debtors) in the range of 0.97 - 0.98, the discount factor of constrained agents, $\gamma$, is set equal to 0.98. The parameter measuring the weight of housing in the utility function, $j^h$, is set to 0.1. The parameter governing disutility of labor effort is set to 1.01. This value rationalizes the weak observed response of real wages to macroeconomic disturbances. For the Taylor rules the parameters are set as follows: $\rho_R$ is set equal to 0.79, $\rho_\pi$ is set equal to 1.29, $\rho_y$ is set equal to 0.16. The parameter $m$, which measures the loan-to-value ratio of the constrained agents, is equal to 0.4. The parameter $\vartheta$, which measures the retailers’ probability of not adjusting prices, is set equal to 0.75. The parameter $a_H$, which measures the degree of home bias, is set equal to 0.95. The parameter $\rho$, which measures the elasticity of substitution between Home and Foreign goods, is equal to 1.5. The parameter $\theta$, which measures the elasticity of substitution between the various types of brands, is set equal to 21, so that the steady state markup is equal to 1.05. The autocorrelation parameter, $\rho_j$, of the stochastic process of the preference shock, is set equal to 0.85, while its standard deviation $\sigma_j$ is set equal to 28.5. The Home monetary shock has a standard deviation $\sigma_R$ equal to 0.29, in line with the standard error of the interest rate equation in the VARs. The steady state net foreign asset position of the unconstrained agents is assumed to be zero. Unconstrained agents hold a positive asset position against constrained agents. Finally, the parameter $\delta_b$ in the financial friction, that guarantees the stationarity of the net foreign asset position, is set equal to 0.00001.
2.3.10. Results

To explain how the real estate credit channel affects the trade balance dynamics, I report the main predictions of the DSGE model and compare them with those of the VAR.

Monetary Policy Shock. In Figure 5 I consider a one standard deviation negative shock to the nominal interest rate (expansionary monetary shock). The trade balance moves towards a deficit position. The mechanism is the following one. The lower nominal and real interest rates induce constrained agents to substitute future consumption with current one. Their demand for real estate increases, inducing an increase in the correspondent price. Given that the real estate is used as collateral for financial transactions, its higher price stimulates borrowing by the constrained agent. The extra-borrowing, the 'symptom' of the collateral effect, further stimulates consumption.

Unconstrained households, instead, reduce to a limited extent present consumption: the higher inflation rate and the lower interest rate reduce the real value of domestic outstanding credit and the revenue from the credit service; these effects more than compensate the positive substitution effect of lower interest rate on current consumption.

The total consumption, however, increases; the higher consumption is partly satisfied by the domestic output, which increases, and partly by the higher amount of imported goods.

Housing Price shock. In Figure 6 I consider a one standard deviation shock to the weight of housing in the utility function of the unconstrained households. This shock can be interpreted as a temporary tax advantage to housing, which shifts the demand. Also in this case, there is a deterioration of the trade balance.

The increase in house pricing induces a positive collateral effect on constrained agents, who increase their borrowing, consumption of the nondurable goods and consumption of the real estate.

Given the rise in the inflation rate and output, the monetary authority increases the nominal interest rate.

Unconstrained agents reduce consumption of both housing and nondurable good: the increase in the interest rate discourage current consumption by increasing its price in terms of future consumption.

However, the aggregate consumption increases and induces a higher domestic output and higher imports.

Comparison with the VAR. I compare the DSGE impulse responses with those of the estimated VARs (the one including the trade balance, the other including the export and imports separately). To make this comparison is not strictly correct, given
that the identification schemes of the DSGE and the VAR are different. I perform this exercise to assess the empirical properties of the model.

Figures 7 and 8 report the responses estimated to a one standard deviation decrease in the nominal interest rate.

As it can be seen, the sign of the trade balance and imports responses in the model is the same as that in the data. The same is true for the remaining variables. However, the initial increase in trade deficit and in the imports is higher in the model than in the VAR.

The Figures suggest that the model is able to reproduce qualitatively and to some extent quantitatively the empirical evidence. The model reproduces the observed positive conditional correlation between private consumption, real estate price and trade balance deficit. It also reproduces the positive conditional correlation between real estate, consumption and amount of imports. The model overstates the impact response of imports. Adding an adjustment cost on the amount of imports, or habit in consumption, would improve the capability of fitting the empirical responses. However, the main message of the Figure, which is to illustrate the effects of the U.S. real estate credit channel on the trade balance and imports, is not affected by the lack of these features.

Figures 9 and 10 report the responses to an increase in the housing preference shock. Also in this case, qualitatively and to some extent quantitatively the model is able to reproduce the main features of the data. In particular, the model reproduces the observed correlation between housing prices, consumption, imports and trade balance deficit.

2.4. The two-country model

I extend the basic framework illustrated in the previous section by elaborating a two-country model. In this way I can take into account general equilibrium considerations resulting from the United States' large size in the global economy. I focus on the effects of the shocks on the trade balance under alternative monetary regimes and alternative degrees of exchange rate pass-through into import prices. I also perform a robustness analysis of the main results.

2.4.1. The setup

In the model there are two countries symmetric and of equal size. U.S. is the Home country. Equations characterizing the Home country are similar to those of the basic model, with one exception: I allow the physical capital to vary, instead of setting it at the steady state level. In what follows, I'll report only equations that change with respect to the basic setup.
2.4.1.1. Home unconstrained agents. The budget constraint of the Home unconstrained agent $j$ becomes:

$$\frac{B_t(j)}{1 + i_t} - B_{t-1}(j) + \frac{B_{H,t}(j)}{1 + i_{H,t}} - B_{H,t-1}(j) + M_t(j) - M_{t-1}(j)$$

$$= \int_0^1 \Pi_t(h,j)dh + R^K_t K_{t-1}(j) + W_t L_t(j) - P_t C_t(j) - Q_t \left( \bar{h}_t(j) - \bar{h}_{t-1}(j) \right) - T A X_t(j) - P_t I(j) - \frac{\psi}{2 \delta} \left( \frac{I_t}{K_{t-1}} - \delta \right)^2 P_t K_{t-1}(j)$$

where the term $\frac{\psi}{2 \delta} \left( \frac{I_t}{K_{t-1}} - \delta \right)^2 P_t K_{t-1}$ represents quadratic adjustment costs of capital ($\psi > 0, \ 0 < \delta < 1$ is the depreciation rate of capital). Physical capital is accumulated according to the following law:

$$K_t(j) = (1 - \delta) K_{t-1}(j) + I_t(j)$$

Hence the agent maximizes the utility function also with respect to $I_t$ and to $K_t$. In a symmetric equilibrium where all unconstrained agents are equal, the following two first-order conditions hold, respectively:

$$\Psi_t = \frac{\psi}{C_t} \left[ \left( \frac{I_t}{K_{t-1}} \right) - \delta \right] + \frac{1}{C_t}$$

$$\Psi_t = \beta E_t \left( R^K_{t+1} / P_{t+1} \right) + (1 - \delta) \beta E_t \Psi_{t+1} + \left\{ \frac{1}{C_{t+1}} \left[ \frac{\psi}{2} \left( \frac{I_{t+1}}{K_t} - \delta \right)^2 - \psi \frac{I_{t+1}}{K_t} \left( \frac{I_{t+1}}{K_t} - \delta \right) \right] \right\}$$

Equations (2.52) and (2.53) are standard in models of investment with adjustment costs. Equation (2.52) defines the shadow value of one unit of investment today, which equals the marginal cost of investment. Equation (2.53) states that this value must be equated across time periods. At the optimum, the shadow price of capital must equal the sum of the expected values of the following three components: the rental rate of capital, the shadow value of capital and the capital contribution to lower installation costs.

2.4.1.2. Firms. The supply side is similar to that illustrated in the basic setup. The only difference is that firms belonging to the wholesale sector now maximize profits out of the steady state choosing not only the optimal quantity of labor but also that of physical capital $K_{t-1}$. Hence, the following (standard) first order condition with respect to physical capital $K_{t-1}$ should be added to those reported in section illustrating the basic setup:

$$R^K_t = \alpha P_{W,t} Y_t / K_{t-1}$$
As in the basic model, the wholesale sector produces a homogenous good which is
differentiated and distributed by the retailers both domestically and in the Foreign
country. The assumption of local currency pricing continues to hold: Home retailers
choose a price \( p_t(h) \) for the Home market and a price \( p_t^*(h) \) for the Foreign market. The
prices are denominated in the currency of the destination market. The supply-side of
the Foreign economy is similar to that of the Home country. Foreign retailers buy a
homogenous good produced by the local wholesale sector, differentiate and sell it both
domestically and in the Home country. They set a price \( p_t^*(f) \) for the Foreign market
and a price \( p_t(f) \) for the Home market.

2.4.1.3. Foreign unconstrained agents. The Foreign country is symmetric to the
Home country. The consumption and investment bundles \( C^* \) and \( I^* \) are mirror-symmetric
(because of the home bias) to those in the Home country. Other bundles \( C_H^*, C_F^*, I_H^* \)
and \( I_F^* \) are symmetric to their Home counterparts. There are unconstrained and con­
strained agents. Foreign unconstrained agents trade the riskless bond denominated in
the Home currency with the Home unconstrained agents. They also trade, but only do­
mentically, a riskless bond \( B_{F,t} \) denominated in Foreign currency and paying an interest
rate equal to \( i_t^* \). The budget constraint of the Foreign unconstrained agent \( j^* \) is:

\[
\frac{B_t(j^*)}{S_t(1 + i_t)} - \frac{B_{t-1}(j^*)}{S_t} + \frac{B_{F,t}(j^*)}{1 + i_t^*} - B_{F,t-1}(j^*) + M_t^* - M_{t-1}(j^*)
\]

\[
= \int_0^1 \Pi_t(f, j^*) df + R_t^K K_t-1(j^*) + W_t^* L_t(j^*) + FF^* - P_t^* C_t(j^*) - Q_t^* (\tilde{h}_t(j^*) - \tilde{h}_{t-1}(j^*))
\]

\[- TAX_t(j^*) - P_t I_t(j^*) - \frac{\psi}{2\delta} \left( \frac{I_t(j^*)}{K_{t-1}(j^*)} - \delta \right)^2 P_t^* K_{t-1}^*(j^*)
\]

The internationally traded bond is divided by the nominal exchange rate (\( S_t \) is the
number of Home currency units per unit of Foreign currency) to convert it in Foreign
currency terms. The terms \( \int_0^1 \Pi(f, j^*) df \) represents profits from holding local firms.
The term \( \tilde{h} \) represents local real estate, which is not internationally traded. \( FF^* \)
are revenues from the financial friction \( \Phi(.) \) paid by the Home unconstrained agents.
First order conditions are similar to those of the Home unconstrained agents. There
is only one difference: given that the Foreign unconstrained agent trades two bonds
denominated in different currencies, she will maximize her utility with respect to \( B_F \)
and \( B \). The following first order conditions hold, respectively:

\[
1 = (1 + i_t^*) \beta E_t \left( \frac{C_{t+1}(j^*)^{-1} P_t^*}{C_t(j^*)^{-1} P_{t+1}^*} \right)
\]

\[
= (1 + i_t) \beta E_t \left( \frac{C_{t+1}(j^*)^{-1} P_t^*}{C_t(j^*)^{-1} P_{t+1}^*} \frac{S_t}{S_{t+1}} \right)
\]
Combining the log-linearized version of these equations we get the uncovered interest rate parity equation (2.25). Other first order conditions are similar to those of the Home unconstrained agents.

2.4.1.4. Foreign constrained agents. The Foreign constrained agent $j^*$ is subject to a borrowing constraint in which local real estate is the collateral:

$-B_{F,t}(j^*) \leq m^* E_t(Q_{t+1} h^*_t (j^*))$

with $0 < m < 1$. Foreign constrained agents are symmetric to their Home counterparts. They do not have access to international financial markets and trade only the bond $B_F$. First order conditions are similar to those of Home agents.

As in the basic model, I assume that within each class of agents (Home unconstrained, Home constrained, Foreign unconstrained, Foreign constrained), initial wealth conditions are the same. This assumption and that of symmetric preferences imply that there is a representative agent for each class. I can hence drop the indexes $j, j^*, j', j''$.

2.4.1.5. The Foreign monetary authority. The Foreign monetary authority sets the interest rate according to the following rule:

$\left(\frac{1 + i^*_t}{1 + \bar{r}^*}\right) = \left(\frac{1 + i_{H,t-1}}{1 + \bar{r}^*}\right)^{\rho_R^*} \left(\frac{P_{t-1}^*}{P_{t-2}^*}\right)^{(1-\rho_R^*)} \left(\frac{Y_{t-1}^*}{Y^*}\right)^{(1-\rho_Y^*)} \left(\frac{S_t}{S_{t-1}/\Delta S}\right)^{(1-\rho_{\Delta S})}$

where $0 < \rho_R^* < 1$, $\rho_Y^*, \rho_{\Delta S} > 0$. I allow monetary policy to react not only to deviations of past inflation $P_{t-1}^*/P_{t-2}^*$ and past output from their correspondent steady state levels, but also to the deviations of the nominal exchange rate variation from its steady state value $\Delta S$: when the U.S. dollar appreciates ($S_t/S_{t-1}$ decreases) the interest rate $i_t^*$ increases. I'll consider alternative exchange rate regimes by opportunely varying the parameter $\rho_{\Delta S}$. When $\rho_{\Delta S}$ is set equal to zero, the monetary authority reacts only to domestic inflation and output. To introduce a fixed exchange rate regime, $\rho_{\Delta S}$ is set to an extremely high value: the monetary authority changes the nominal interest rate to exclusively stabilize the nominal exchange rate.

2.4.1.6. The market clearing conditions. The model is closed by the market clearing conditions and the public sector budget constraints. The Home country house market clearing condition is:

$\bar{h}_t + \bar{h}'_t = H$

where $H$ is in a fixed amount. A similar equation holds in the Foreign country.

The market clearing condition of the good produced in the Home country is:

$Y_t = C_{H,t} + C'_{H,t} + I_{H,t}$

$+ C^*_{H,t} + C'^*_{H,t} + I^*_{H,t}$
The market clearing condition of the Foreign good is:

\[ Y_t^* = C_{F,t} + C'_{F,t} + I_F \]

(2.60)

\[ + C_{F,t}^{*} + C'_{F,t}^{*} + I_{F,t}^{*} \]

The market clearing of the bond traded in the Home country is:

(2.61)

\[ B_{H,t} + B'_{H,t} = 0 \]

The market clearing of the bond traded in the Foreign country is:

(2.62)

\[ B_{F,t}^{*} + B'_{F,t} = 0 \]

The market clearing of the internationally traded bond is:

(2.63)

\[ B_t + B_t^{*} = 0 \]

In the Home country, government rebate seigniorage revenues in a lump-sum fashion to domestic agents. Hence the public sector budget constraint implies:

(2.64)

\[ M_t - M_{t-1} = -TAX_t \]

(2.65)

\[ M'_t - M'_{t-1} = -TAX'_t \]

Similar equations hold in the Foreign country.

Home country trade balance is obtained by consolidating the budget constraints of the Home agents, equations (2.50) and (2.20), and of the public sector budget constraint. The following equation is obtained:

(2.66)

\[ TB_t = \frac{B_t}{(1 + \tilde{i}_t)} - B_{t-1} = P_{H,t}Y^*_{H,t} + S_{t}P^*_{H,t}Y'_{H,t} - C_t - C'_t - I_t \]

2.4.1.7. Equilibrium. The equilibrium is defined as a sequence of allocations and prices such that, given the initial conditions of the state variables and the stochastic processes of the exogenous shocks, in each country (a) households, firms in the wholesale sectors and retailers solve their respective maximization problems, (b) the market clearing conditions, the government budget constraint and the monetary policy rules hold. The equilibrium dynamics is characterized by solving a first-order log-linear approximation to the equilibrium conditions around the non-stochastic steady state. The steady state and the log-linearized system are in the Appendix B. Given the assumption of symmetric countries, every parameter is set to the same value in each country. The values of those parameters that are present also in the basic model do not change (see Table 1). The physical capital adjustment cost, not present in the basic model, is set equal to 1. The depreciation rate of capital, \( \delta \), is set equal to 0.025.
2.4.2. Results

I analyze the implications for the U.S. trade balance of the two U.S.-based shocks introduced in the basic model: an expansionary monetary shock and a positive shock to the preferences for real estate of the unconstrained agents, initially. Then, I analyze the change in results when some assumptions are modified. In particular, I consider different exchange rate regimes, different values of the loan-to-value ratio and of the intratemporal elasticity of substitution between Home and Foreign good. I also consider different assumptions about the degree of pass-through of nominal exchange rate into import prices.

2.4.2.1. Response Analysis. Monetary Policy Shock. In Figure 11 I consider a negative shock to the U.S. nominal interest rate (expansionary monetary shock). The responses of the main U.S. variables are not much different from those in the basic model (see also Figure 5). The amount of imports increases and the trade balance moves towards deficit. In fact, the lower nominal (and real) interest rate makes current consumption more convenient than future consumption. The demand for housing increases. The related price increase induces a positive collateral effect of constrained household through the borrowing constraint. Private consumption is hence further stimulated. The Home inflation increases. The nominal and real exchange rates (not reported in the Figure) depreciate, given the lower Home interest rate. However, the assumption of local currency pricing implies that the pass-through is low: the depreciation of the nominal exchange rate is not fully transmitted to the import prices faced by the agents; hence, the increase in the relative price of the Foreign good is relatively low and the agents do not strongly substitute the Foreign good with the Home good.24

Housing Price shock. In Figure 12 I consider a shock to the weight of housing in the utility function of the U.S. unconstrained households. Also in this case the trade balance and the net foreign asset position of the Home country deteriorate. The increase in house pricing induces a positive collateral effect on the constrained agent. As a consequence, borrowing and aggregate consumption increase, inducing a higher domestic output and higher imports from the Foreign country.25

2.4.2.2. Robustness Analysis. I analyze the effects of the monetary and house price shocks under alternative assumptions. The goal is to investigate the robustness of the positive effects of the real estate credit channel on the trade balance deficit and on

24U.S. investment in physical capital (not reported to save on space) increases, given the stimulating effect of the lower real interest rate.
25U.S. investment in physical capital (not reported to save space) decreases. The higher interest rate has a depressing effect on the physical capital accumulation. The decrease in investment, however, only partially compensate the increase in private consumption due to the collateral effect of higher real estate prices. Hence, the trade balance moves towards deficit.
imports. I consider alternative exchange rate regimes, different values of the loan-to-value ratio \( m \), of the elasticity of substitution between tradable, alternative pricing policy of the firms.

**Alternative exchange rate regimes.** I compare the effects of the shocks on the trade balance in the benchmark case, in which the exchange rate is flexible, to the case of a pegging regime. In the benchmark case the Taylor rule of the Foreign monetary authority is similar to that of the Home authority; under the pegging regime, the Foreign authority moves its nominal interest rate to peg the U.S. dollar. In the latter case, the nominal exchange rate does not change (\( S_t/S_{t-1} = 1 \)).

Figure 13 reports the responses to a U.S. expansionary monetary shock. Under the pegging regime, there is no deterioration of the U.S. trade balance nor change in international relative prices, given that both monetary authorities de facto adopt the same expansionary policy. Home consumption increases in both cases. Foreign consumption (not reported to save space) increases in the fixed exchange rate regime, given the reduction of the Foreign interest rate.

In Figure 14 I consider the effects of a U.S. positive house price shock. In this case there is always a deterioration of the U.S. trade balance. The size of the deficit is not much different across the regimes. What matters is the relative price of the housing, that increases and stimulates consumption in the U.S. economy. The Home monetary authority augments the nominal interest rate to contrast the higher inflation, caused by the increase in consumption and investment. Under the pegging regime, the Foreign monetary authority increases the interest rate to peg the U.S. dollar. Foreign consumption and house prices decrease; hence, the U.S. export decrease to a greater amount.26

**Different values of \( m \).** I consider a positive U.S. housing price shock under two alternative values of \( m \), the loan-to-value ratio: when \( m \) is equal to its benchmark value, 0.4, and when it assumes a lower value, 0.2. Results are in Figure 15.

In correspondence of the lower value of \( m \), the collateral effect is lower and the model predicts a lower deterioration of the trade balance, because the increase of the constrained agent’s consumption, and hence imports, is lower.

These responses show that the conditional correlation between housing prices and consumption is rather low when the collateral effect, associated to the real estate credit channel, is close to zero (\( m \) extremely low). Housing prices and the consumption of non durable goods would have a one to one relationship (the multiplier effect of higher

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26 The relative price of U.S. imports is not greatly different across the two regimes, given the assumption of imperfect exchange rate pass-through: in the case of the flexible exchange rate regime, nominal exchange rate fluctuations are not fully transmitted to the import prices. From this point of view, there is not a great difference between a fixed and a flexible exchange rate regime.
housing prices on consumption would be lower) and the depressing effect of the higher interest rate would dominate. However, a negative correlation between housing prices and consumption of non durable goods is counterfactual, as shown by the VARs.

Different values of the elasticity of substitution $\rho$. I consider the effects on trade balance of a one standard deviation expansionary monetary policy shock in the U.S. under two alternative values of the elasticity of substitution between Home and Foreign goods, $\rho$: when it is equal to its benchmark value, 1.5, and when it assumes a lower value, 1.1.

In correspondence of the lower value of $\rho$ (see Figure 16), the model predicts a lower deterioration of the trade balance. Given the decrease (favoured by the home bias) in the price of imports relative to that of the U.S. domestic good (see the definition of $T$ in equation (2.12)), U.S. households are less willing to substitute domestic for imported goods; hence, while the increase in aggregate consumption is the roughly the same in the two cases, the increase of the imports is lower in the case of lower elasticity. The amount of U.S. export decreases less in correspondence of lower $\rho$ for a given increase in their Foreign relative price (equal to $1/T^*$, see equation (2.12)). Results are similar in the case of a house price shock (Figure 17).

The effects of the nominal exchange rate fluctuations. One strand of the debate on the U.S. current account deficit has focused on the possibility that the U.S. nominal exchange rate fluctuations could help in reducing the U.S. external imbalance. A depreciating U.S. dollar, in fact, would increase the relative price of Foreign goods and shift world demand towards U.S. goods (substitution effect). The implicit assumption behind this statement is that nominal exchange rate fluctuations are, at least to some extent, passed-through to the import and export prices. Until now I have assumed that the prices are set in local currency terms, so that the degree of pass-through is relatively low.

To understand if the substitution effect of the nominal exchange rate can counteracts the (expansionary) collateral effect of the housing prices increase on the trade balance, I make the assumption of producer currency pricing: producers set the price of the good they produce in their own currency (hence Home producers set $p_t(h)$ and Foreign producers set $p_t^*(f)$ given the respective world demands) so that the international law of one price holds ($p_t(h) = S p_t^*(h)$ and $p_t(f) = S p_t^*(f)$). I consider an expansionary monetary shock in the U.S. economy in Figure 18. The depreciation of the U.S. dollar (not reported in the Figure) modifies international relative prices. Home goods are cheaper than Foreign ones. The substitution effect counteracts the collateral effect and contributes to reduce the trade balance deficit on impact, by increasing the relative
price of Foreign goods (the U.S. terms of trade deteriorate), reducing the imports and increasing U.S. exports.\textsuperscript{27}

The determination of the degree of pass-through, and hence of the relative size of the substitution effect with respect to the collateral effect, is an empirical matter. Evidence for the U.S. suggests that the degree of pass-through is rather low at the border and at consumer level. Hence a flexible exchange rate hardly would be able to limit the effects of the higher housing prices on the trade balance dynamics.

\textbf{2.5. Conclusions}

Recent contributions emphasize the role of technology and fiscal shocks to explain the U.S. trade balance deficit. I add to the existing literature a new dimension, by investigating whether changes in real estate prices influence the U.S. trade balance dynamics.

The model, calibrated on the U.S. economy, shows that the domestic monetary and housing price shocks can also contribute to explain the movement of the U.S. trade balance through the real estate credit channel. Because of households' imperfect access to financial markets, real estate price changes generate a collateral effect which induces consumption to positively commove with house price; as a consequence, the trade balance moves towards deficit. Impulse responses obtained from the model are roughly in line with those obtained from an estimated VAR using U.S. data. I also show that a depreciation of the U.S. dollar limits the shift of the trade balance towards deficit when the pass-through is, counterfactually, complete.

The model can be extended to allow the constrained agents to borrow internationally, so to better understand the role of international financial flows over the U.S. business cycle. Fiscal policy could also be added, to consider the implications of the financial frictions for the 'twin-deficit' story. These are the next steps in the research agenda. More theoretical and empirical work is needed.

\textsuperscript{27}The U.S. terms of trade is defined as $P_{F,t}/SP_{H,t}$. In the case of complete pass-through, the law of one price holds and the previous definition can be rewritten as $S_t P_{F,t}/P_{H,t}$.
References


Appendix A. The basic model

The steady state equilibrium

I assume a symmetric steady state flexible price equilibrium in which all the shocks are zero, there is no change in asset positions \( (B_t = B_{t-1} = B, B_{H,t} = B_{H,t-1} = B_H) \), in house holdings \( (h_t = h_{t-1} = h) \), no net accumulation of physical capital \( (K_{t-1} = K_t = K) \) all price inflation rates as well as exchange rate depreciation are zero \( (P_t/P_{t-1} = 1, Q_t/Q_{t-1} = 1, S_t/S_{t-1} = 1) \), the net foreign asset position is zero (hence also the trade balance is equal to zero). I assume all tradable goods have the same price \( (P_H = P_F = P = SP^* = SP_F^* = P_H^*/S) \) and I normalize them to one. Let’s consider the Home country equations.

The pricing equation (2.32) implies:

\[
P_W = \frac{\theta - 1}{\theta}
\]

Normalizing all the variable by \( Y \) (the total national production), the steady state values become function of the deep parameters (technology and preferences) and hence are uniquely determined. I derive all the quantities that enter as coefficients in the log-linearized equilibrium system (see next part of this Appendix).

Labor demand first order conditions, equations (2.27) and (2.28), become respectively:

\[
\frac{W_L}{Y} = (1 - \alpha) \mu P_W
\]

\[
\frac{W'L'}{Y} = (1 - \alpha) (1 - \mu) P_W
\]

The value of real estate held by the unconstrained agent, obtained from equation (2.18), is equal to:

\[
\frac{Q_H}{Y} = \frac{j^h C}{1 - \beta Y}
\]

The consumer Euler equations of the Home unconstrained agent, equation (2.16) and (2.17), imply:

\[
\beta = \frac{1}{1 + \bar{v}} = \frac{1}{1 + \bar{v}_H}
\]
I assume that agents invest to replace the depreciated capital ($\delta$ is the depreciation rate):

$$\frac{I}{Y} = \delta \frac{K}{Y}$$

and the steady state capital stock satisfies the traditional first order condition (hence I assume that in steady state standard first order conditions for capital hold for agents and firms):

$$\frac{K}{Y} = \frac{\alpha \beta}{1 - \beta (1 - \delta)}$$

Combining the steady state versions of the constrained and unconstrained agent Euler equations, (2.16) and (2.23), the following value for $\lambda$, the Lagrange multiplier of the borrowing constraint, can be obtained:

$$\lambda' = \left(\frac{\beta - \gamma}{C}\right)$$

By assumption, $0 < \gamma < \beta < 1$; so the Lagrange multiplier is strictly greater than zero in the steady state (and in a small neighborhood of it). As a consequence, the borrowing constraint is binding.

From the first order equations, (2.22) and (2.23), the real estate held by the constrained agent is:

$$\frac{Q^h}{Y} = \frac{j^h}{1 - \gamma - m (\beta - \gamma) Y} \cdot C'$$

The borrowing constraint equation (2.21) becomes:

$$\frac{B'_H}{Y} = -m \frac{j^h}{1 - \gamma - m (\beta - \gamma) Y} \cdot C'$$

Using the budget constraint of the constrained agent and the above two equations it is possible to derive the equation of the constrained agent’s consumption, $C'$:

$$\frac{C'}{Y} = (1 - \alpha) (1 - \mu) \left[ \frac{1 - \gamma - m (\beta - \gamma)}{1 - \gamma - m (\beta - \gamma) + mj^h (1 - \beta)} \right] P_W$$

I assume that the net foreign asset position is zero. From the trade balance equation (2.49), the consumption of the unconstrained agent, $C$, can be derived:

$$\frac{C}{Y} = 1 - \frac{C'}{Y} - \frac{I}{Y}$$
Using the market clearing condition for the bonds traded in the Home country, equation (2.46), the asset position of the unconstrained agent can be determined:

\[ \frac{B_H}{Y} = -\frac{B'_H}{Y} \]  

The Home demand function for the Home produced good, using equations (2.7) and (2.42), is:

\[ \frac{Y_H}{Y} = a_H \left( \frac{C}{Y} + \frac{C'}{Y} + \frac{I}{Y} \right) \]  

Total demand for the Home produced good, equation (2.45), is:

\[ 1 = \frac{Y_H}{Y} + \frac{Y_H}{Y} \]  

From it, the share of Home export over the total output is obtained.
The log-linearized equilibrium

Let hatted variables denote percent changes from the steady state, and those without subscript denote steady state values. The trade balance and the net foreign asset position of the Home country are not log-linearized, given that they are equal to zero in steady state; I hence exploit the following definition: \( Y(tb_t) = (TB_t - TB) \) and \( Yb_t = B_t - B \), with \( TB = B = 0 \). I apply a similar definition to the asset position of the constrained agents, also if its steady state value is not zero (but negative). Note that \( \pi_{.,t} \equiv \ln(1 + \pi_{.,t}) \equiv \ln(P_{.,t}/P_{.,t-1}) \) and that \( \Delta S_t \equiv \ln(S_t/S_{t-1}) \). To save on notation, I drop the expectation operator before variables dated \( t - 1 \), which must be intended in expected value conditional on the information available at time \( t \). The model can be reduced to the following linearized system.

I. Relative Price and Inflation Rates.

\[ \hat{T}_t = \hat{T}_{t-1} + \hat{\pi}_{F,t} - \hat{\pi}_{H,t} \]  
\[ \hat{T}^*_t = \hat{T}^*_{t-1} - \hat{\pi}^*_{H,t} \]  
\[ \hat{\pi}_t = a_H \hat{\pi}_{H,t} + (1 - a_H) \hat{\pi}_{F,t} \]  
\[ \hat{\pi}^*_t = (1 - a_H) \hat{\pi}^*_{H,t} \]

II. Aggregate supply

\[ \hat{Y}_t = (1 - \alpha) \mu \hat{L}_t + (1 - \alpha) (1 - \mu) \hat{L}'_t \]  
\[ \hat{P}_{W,t} + \hat{Y}_t = \hat{C}_t + \tau \hat{L}_t \]  
\[ \hat{P}_{W,t} + \hat{Y}_t = \hat{C}_t + \tau \hat{L}'_t \]  
\[ \hat{\pi}_{H,t} = \beta \hat{\pi}_{H,t+1} + \frac{(1 - \vartheta) (1 - \beta \vartheta)}{\vartheta} \left( \hat{P}_{W,t} + (1 - a_H) \hat{T}_t \right) \]  
\[ \hat{\pi}_{F,t} = \beta \hat{\pi}_{F,t+1} + \frac{(1 - \vartheta) (1 - \beta \vartheta)}{\vartheta} \left( -a_H \hat{T}_t + \hat{R} \right) \]  
\[ \hat{\pi}^*_{H,t} = \beta \hat{\pi}^*_{H,t+1} + \frac{(1 - \vartheta) (1 - \beta \vartheta)}{\vartheta} \left( \hat{P}_{W,t} + a_H \hat{T}^*_t - \hat{R} \right) \]
III. Aggregate Demand

(AD1) \[-\dot{C}_t = -\dot{C}_{t+1} + (1 + \dot{i}_t) - \delta_b \frac{B}{Y} \hat{b}_t - \hat{\pi}_{t+1}\]

(AD2) \[(1 + i_{H,t}) = (1 + \dot{i}_t) - \delta_b \frac{B}{Y} \hat{b}_t\]

(AD3) \[\dot{q}_t = \dot{C}_t + \beta \dot{q}_{t+1} - \beta \dot{C}_{t+1} + (1 - \beta) \frac{q\hat{h}'/Y}{q\hat{h}/Y} \hat{h}_t + (1 - \beta) \hat{y}_t\]

(AD4) \[\dot{q}_t = (1 - m\beta) \dot{C}_t + \dot{q}_{t+1} (\gamma + m(\beta - \gamma)) - \dot{C}_{t+1} \gamma (1 - m) - \hat{h}_t (1 - \gamma - m(\beta - \gamma)) - m\beta (1 + \dot{i}_t) + m\beta \hat{\pi}_t\]

(AD5) \[(1 + \dot{i}_t) = \Delta S_t\]

(AD6) \[\dot{Y}_{H,t} = (1 - a_H) \rho \hat{T}_t + \frac{C}{Y} \dot{C}_t + \frac{C'}{Y} \dot{C}'_t\]

(AD7) \[\dot{Y}_{H,t}^* = a_H \rho \hat{T}_t^*\]

(AD8) \[\dot{Y}_{F,t} = -a_H \rho \hat{T}_t + \frac{C}{Y} \dot{C}_t + \frac{C'}{Y} \dot{C}'_t\]

(AD9) \[Y_t = \frac{Y_H}{Y} \dot{Y}_{H,t} + \frac{Y_H^*}{Y} \dot{Y}_{H,t}^*\]

IV. Borrowing constraint and flows of funds

(BC1) \[\beta b_t - b_{t-1} = \frac{Y_H}{Y} \left[ (1 - a_H) \hat{T}_t + \dot{Y}_{H,t} \right] + \frac{Y_H^*}{Y} \left[ (1 - a_H) \hat{T}_t^* + \dot{R}_S + \dot{Y}_{H,t}^* \right] - \frac{C}{Y} \dot{C}_t - \frac{C'}{Y} \dot{C}'_t\]

(BC2) \[\beta \frac{B'_{H,t}}{Y} \left[ b'_{H,t} - (1 + \dot{i}_t) \right] - \frac{B_{H,t-1}}{Y} (b'_{H,t-1} - \hat{\pi}_{t-1}) = (1 - \alpha) (1 - \mu) p \dot{W}_t + \hat{Y}_t - \frac{q\hat{h}'/Y}{q\hat{h}_t} \hat{h}_t - \frac{C}{Y} \dot{C}_t\]

(BC3) \[-b'_{H,t} = m \frac{q\hat{h}'/Y}{\hat{h}_t} (\dot{q}_{t+1} + \hat{\pi}_t + \hat{h}_t)\]

(BC4) \[t b_t = \beta b_t - b_{t-1}\]
V. Monetary Policy rules and shock processes

\[(MP1)\quad (1 + \hat{i}_{H,t}) = \rho_R(1 + \hat{i}_{H,t-1}) + (1 - \rho_R)\rho_H\hat{x}_{t-1} + (1 - \rho_R)\rho_Y\hat{Y}_{t-1} + \hat{e}_{R,t}\]

\[(PRE1)\quad j = \rho_j\hat{j} + \hat{e}_{j,t}\]

The first block contains the definitions of international relative prices and of inflation rates: equations (RP1) and (RP2) represent the log-linearized versions of relative prices (2.12) (note that the price of Foreign good in the Foreign country is assumed to be constant); equation (RP3) is the log-linearized Home country real exchange rate (2.13); equations (RP4) and (RP5) are the Home and Foreign country inflation rates, respectively obtained from equations (2.4) and (2.5).

The second block is formed by the equations describing the supply side of the economy: (AS1) is the log-linearized version of the production function (2.26); (AS2) is obtained from log-linearizing labor demand (2.27) and supply (2.19); (AS3) is the analogous of equation (AS2) for the constrained agent (obtained from equations (2.24) and (2.28)); the short run Phillips curves, equations (AS4), (AS5), (AS6) are obtained from the solution of the retailers' problem.

The third block is the demand block, composed by the Home unconstrained agent and interest parity, (AD1) and (AD2), respectively obtained by log-linearizing equations (2.16) and (2.17); the unconstrained and constrained agent real estate Euler equations, (AD3) and (AD4), respectively obtained by log-linearizing (2.18) and (2.23) (using the house market clearing condition (2.41)); the modified uncovered interest parity (AD5), in which the Foreign interest rate is assumed to be constant; the market clearing conditions (AD6), (AD7), (AD8), (AD9), respectively obtained from equations (2.42), (2.43), (2.44), (2.45) (note that \(C^*\) is assumed to be constant).

The fourth block is formed by the equations describing the borrowing constraint, the flows of funds and the trade balance: equation (BC1) describes the Home country's net foreign asset position and it is obtained by consolidating equations (2.15), (2.20), (2.41), (2.47), (2.48); equations (BC2) and (BC3) are the constrained agent's budget and borrowing constraints (obtained respectively from equations (2.20) and (2.21)); equation (BC4) is the Home country's trade balance, obtained from the equation (2.49).

The last block is formed by equations describing the monetary policy and the exogenous shocks: equations (MP1) is the log-linearized version of equation (2.40). Equation (PRE1) is the exogenous law of motion of the housing preference shock.
Appendix B. The two-country model

The two country are symmetric (the only exception is that there is symmetric home bias). The steady state equilibrium is the same as that of the basic model. In what follows, I describe the log-linearized equations of the dynamic equilibrium. Starred variables are referred to the Foreign country. \( \hat{I}, \hat{K}, \hat{r}^k \) are respectively the investment in physical capital, the stock of physical capital, the rental rate of capital (log-deviation from the steady state)

I. Relative prices and inflation rates.

\[
\text{(RP1)} \quad \hat{T}_t = \hat{T}_{t-1} + \hat{\pi}_{F,t} - \hat{\pi}_{H,t}
\]

\[
\text{(RP2)} \quad \hat{T}^*_t = \hat{T}^*_{t-1} + \hat{\pi}^*_{F,t} - \hat{\pi}^*_{H,t}
\]

\[
\text{(RP3)} \quad \hat{R}S_t = \hat{R}S_t + \Delta \hat{S}_t + \hat{\pi}_t - \hat{\pi}_t
\]

\[
\text{(RP4)} \quad \hat{\pi}_t = a_H \hat{\pi}_{H,t} + (1 - a_H) \hat{\pi}_{F,t}
\]

\[
\text{(RP5)} \quad \hat{\pi}^*_t = (1 - a_H) \hat{\pi}^*_{H,t} + a_H \hat{\pi}^*_{F,t}
\]

II. Aggregate supply

\[
\text{(AS1)} \quad \hat{Y}_t = \alpha \hat{K}_{t-1} + (1 - \alpha) \mu \hat{L}_t + (1 - \alpha) (1 - \mu) \hat{L}^t_t
\]

\[
\text{(AS2)} \quad \hat{Y}^*_t = \alpha \hat{K}^*_t + (1 - \alpha) \mu \hat{L}^*_t + (1 - \alpha) (1 - \mu) \hat{L}^*_t
\]

\[
\text{(AS3)} \quad \hat{p}_{W,t} + \hat{Y}_t = \hat{C}_t + \tau \hat{L}_t
\]

\[
\text{(AS4)} \quad \hat{p}_{W,t} + \hat{Y}_t = \hat{C}_t + \tau \hat{L}^*_t
\]

\[
\text{(AS5)} \quad \hat{p}_{W,t} + \hat{Y}^*_t = \hat{C}^*_t + \tau \hat{L}^*_t
\]

\[
\text{(AS6)} \quad \hat{p}_{W,t} + \hat{Y}^*_t = \hat{C}^{**}_t + \tau \hat{L}^{**}_t
\]

\[
\text{(AS7)} \quad \hat{p}_{W,t} + \hat{Y}_t = \hat{r}^k_t + \hat{k}_{t-1}
\]

\[
\text{(AS8)} \quad \hat{p}_{W,t} + \hat{Y}^*_t = \hat{r}^{k*}_t + \hat{k}^*_t
\]

\[
\text{(AS9)} \quad \hat{\pi}_{H,t} = \beta \hat{\pi}_{H,t+1} + \frac{(1 - \vartheta)(1 - \beta \vartheta)}{\vartheta} \left( \frac{\hat{p}_{W,t} + (1 - a_H) \hat{T}_t}{\hat{p}_{W,t}} \right)
\]
\( i \frac{A}{I} + i \frac{\varphi}{\varphi_0} + i \frac{\lambda}{\lambda_0} + i \varphi (H_0 - 1) = \psi H \)  

(AD0)

\( \frac{i}{\varphi} \left( (\varphi - 1) \varphi - 1 \right) + 
\left( \psi + \frac{i}{\varphi} \right) \frac{\varphi}{1} + \left( 1 + \varphi i_4 \right) = \left( 1 - i \varphi - i_4 \right) \)

(AD9)

\( \frac{i}{\varphi} \left( (\varphi - 1) \varphi - 1 \right) + 
\left( \psi + \frac{i}{\varphi} \right) \frac{\varphi}{1} + \left( 1 + \varphi i_4 \right) = \left( 1 - i \varphi - i_4 \right) \)

(AD9)

\( \psi g w u + (\psi + 1) g w u - (\varphi - \varphi) w - \psi - 
\frac{w - 1}{\psi} \left( \psi + i \psi - 1 \right) \frac{\psi}{1} \) = \( \psi \)  

(AD8)

(AD7)

\( \psi g w u + (\psi + 1) g w u - (\varphi - \varphi) w - \psi - 
\frac{w - 1}{\psi} \left( \psi + i \psi - 1 \right) \frac{\psi}{1} \) = \( \psi \)  

(AD6)

(AD5)

\( \psi (\varphi - 1) + \frac{\psi}{\varphi} \frac{\lambda}{\lambda} \frac{\psi}{\psi} (\varphi - 1) + 1 \psi \varphi + i \psi + \frac{\psi}{\psi} = \psi \)  

(AD4)

\( \psi \frac{\lambda}{\psi} - (\varphi + 1) = (\psi \frac{\lambda}{\psi} + 1) \)  

(AD3)

\( \frac{1}{\varphi} \psi - (\varphi + 1) + 1 \psi \varphi - \frac{\psi}{\psi} \) = \( \frac{1}{\varphi} \psi \)  

(AD2)

\( \frac{1}{\varphi} \psi - (\varphi + 1) + 1 \psi \varphi - \frac{\psi}{\psi} \) = \( \frac{1}{\varphi} \psi \)  

(AD1)

\( \frac{1}{\varphi} \psi - (\varphi + 1) + 1 \psi \varphi - \frac{\psi}{\psi} \) = \( \frac{1}{\varphi} \psi \)  

\( \frac{1}{\varphi} \psi - (\varphi + 1) + 1 \psi \varphi - \frac{\psi}{\psi} \) = \( \frac{1}{\varphi} \psi \)  

(AD1)

\( \psi (\varphi - 1) + \frac{\psi}{\varphi} \frac{\lambda}{\lambda} \frac{\psi}{\psi} (\varphi - 1) + 1 \psi \varphi + i \psi + \frac{\psi}{\psi} = \psi \)  

(AD5)

\( \psi \frac{\lambda}{\psi} - (\varphi + 1) = (\psi \frac{\lambda}{\psi} + 1) \)  

(AD3)

\( \frac{1}{\varphi} \psi - (\varphi + 1) + 1 \psi \varphi - \frac{\psi}{\psi} \) = \( \frac{1}{\varphi} \psi \)  

(AD2)

\( \frac{1}{\varphi} \psi - (\varphi + 1) + 1 \psi \varphi - \frac{\psi}{\psi} \) = \( \frac{1}{\varphi} \psi \)  

(AD1)

\( \psi (\varphi - 1) + \frac{\psi}{\varphi} \frac{\lambda}{\lambda} \frac{\psi}{\psi} (\varphi - 1) + 1 \psi \varphi + i \psi + \frac{\psi}{\psi} = \psi \)  

(AD5)

\( \psi \frac{\lambda}{\psi} - (\varphi + 1) = (\psi \frac{\lambda}{\psi} + 1) \)  

(AD3)

\( \frac{1}{\varphi} \psi - (\varphi + 1) + 1 \psi \varphi - \frac{\psi}{\psi} \) = \( \frac{1}{\varphi} \psi \)  

(AD2)

\( \frac{1}{\varphi} \psi - (\varphi + 1) + 1 \psi \varphi - \frac{\psi}{\psi} \) = \( \frac{1}{\varphi} \psi \)  

(AD1)

\( \psi (\varphi - 1) + \frac{\psi}{\varphi} \frac{\lambda}{\lambda} \frac{\psi}{\psi} (\varphi - 1) + 1 \psi \varphi + i \psi + \frac{\psi}{\psi} = \psi \)  

(AD5)

\( \psi \frac{\lambda}{\psi} - (\varphi + 1) = (\psi \frac{\lambda}{\psi} + 1) \)  

(AD3)

\( \frac{1}{\varphi} \psi - (\varphi + 1) + 1 \psi \varphi - \frac{\psi}{\psi} \) = \( \frac{1}{\varphi} \psi \)  

(AD2)

\( \frac{1}{\varphi} \psi - (\varphi + 1) + 1 \psi \varphi - \frac{\psi}{\psi} \) = \( \frac{1}{\varphi} \psi \)  

(AD1)
\[(AD12)\quad \dot{Y}_{H,t} = a_H \rho \hat{\pi}_t + \frac{C}{Y} \hat{C}_t + \frac{C'}{Y} \hat{C}_t' + \frac{I}{Y} \hat{I}_t
\]

\[(AD13)\quad \dot{Y}_{F,t} = -a_H \rho \hat{\pi}_t + \frac{C}{Y} \hat{C}_t + \frac{C'}{Y} \hat{C}_t' + \frac{I}{Y} \hat{I}_t
\]

\[(AD14)\quad \dot{Y}_{F_t}^* = -(1 - a_H) \rho \hat{\pi}_t + \frac{C}{Y} \hat{C}_t + \frac{C'}{Y} \hat{C}_t' + \frac{I}{Y} \hat{I}_t
\]

\[(AD15)\quad \dot{Y}_t = \frac{Y_H}{Y} \dot{Y}_{H,t} + \frac{Y_H^*}{Y} \dot{Y}_{H,t}^*
\]

\[(AD16)\quad Y_t^* = \frac{Y_F}{Y} \dot{Y}_{F,t} + \frac{Y_F^*}{Y} \dot{Y}_{F,t}^*
\]

**IV. Borrowing constraint and flows of funds**

\[(BC1)\quad \beta b_t - b_{t-1} = \frac{Y_H}{Y} \left( -(1 - a_H) \hat{T}_t + \dot{Y}_{H,t} \right) + \frac{Y_H^*}{Y} \left( -(1 - a_H) \hat{T}_t + \dot{Y}_{H,t}^* \right) - \frac{C}{Y} \hat{C}_t - \frac{C}{Y} \hat{C}_t' - \frac{I}{Y} \hat{I}_t
\]

\[(BC2)\quad \beta B_{H}^t \left[ b_{H,t} - (1 + \hat{\pi}_t) \right] - B_{H}^t (b_{H,t-1} - \hat{\pi}_{t-1}) = (1 - \mu) p_w \left( \hat{p}_w + \hat{Y}_t \right) - \frac{q \hat{h}'}{Y} \left( \hat{h}_t - \hat{h}_{t-1} \right) - \frac{C}{Y} \hat{C}_t
\]

\[(BC3)\quad -b_{H,t} = m \frac{q \hat{h}'}{Y} \left( \hat{q}_{t+1} + \hat{\pi}_t + \hat{h}_t \right)
\]

\[(BC4)\quad \beta B_{H}^t \left[ b_{H,t}^* - (1 + \hat{\pi}_t^*) \right] - B_{H}^t (b_{H,t-1}^* - \hat{\pi}_{t-1}^*) = (1 - \alpha) (1 - \mu) p_w \left( \hat{p}_w + \hat{Y}_t^* \right) - \frac{q \hat{h}'}{Y} \left( \hat{h}_t^* - \hat{h}_{t-1}^* \right) - \frac{C}{Y} \hat{C}_t^*
\]

\[(BC5)\quad -b_{F,t}^* = m \frac{q \hat{h}'}{Y} \left( \hat{q}_{t+1} + \hat{\pi}_t^* + \hat{h}_t^* \right)
\]

\[(BC6)\quad \dot{K}_t = (1 - \delta) \dot{K}_{t-1} + \delta \dot{I}_t
\]

\[(BC7)\quad \dot{K}_t^* = (1 - \delta) \dot{K}_{t-1}^* + \delta \dot{I}_t^*
\]

\[(BC8)\quad th_t = \beta b_t - b_{t-1}
\]
V. Monetary policy rules and shock processes

\[(MP1) \quad (1 + i_{H,t}) = \rho_R(1 + i_{H,t-1}) + (1 - \rho_R)\rho_s \hat{\pi}_{t-1} + (1 - \rho_R)\rho_y \hat{Y}_{t-1} + \hat{\varepsilon}_{R,t}\]

\[(MP2) \quad (1 + i^*_t) = \rho_R^* (1 + i^*_{t-1}) + (1 - \rho_R^*)\rho_s^* \hat{\pi}^*_{t-1} + (1 - \rho_R^*)\rho_y^* \hat{Y}^*_{t-1} - (1 - \rho_R^*)\rho_{AS} \Delta S_t\]

\[(MP3) \quad \hat{j} = \rho_j \hat{j} + \hat{\varepsilon}_{j,t}\]

The first two equations in the first block contain define the relative price of the Foreign good in terms of the Home good respectively in the Home and in the Foreign country. The third equation defines the real exchange rate of the Home country. The last two equations define the consumer price inflation rate respectively in the Home and in the Foreign country.

In the second block (the 'supply side') the first two equations are the Home and Foreign production functions. The third and the fourth equations are the labor market equilibrium conditions of the Home unconstrained and constrained agents. The fifth and sixth equations are the correspondent Foreign equations. Equations (AS7) and (AS8) are the Home and Foreign demand of capital. The last four equations of the block are the Phillips curves of the Home and Foreign good in the Home country and their counterparts in the Foreign country.

In the third block (the 'aggregate demand') there are the Euler equation of the Home unconstrained agent, the Euler equation of the Foreign unconstrained agent, the no-arbitrage condition holding for the Home unconstrained agent, the uncovered interest parity holding for the Foreign unconstrained agent, the real estate first order conditions of the Home unconstrained agent, Home constrained agent, Foreign unconstrained agent and Foreign constrained agent. Note that in the first order conditions of the unconstrained agents I have used the relevant log-linearized real estate market clearing conditions \((\frac{\partial}{\partial t} \hat{h}_t + \frac{\partial}{\partial \pi^*} \hat{h}^*_t = 0 \text{ for the Home agent, } \frac{\partial}{\partial \pi^*} \hat{h}^*_t + \frac{\partial}{\partial \pi^*} \hat{h}''^*_t = 0 \text{ for the Foreign agent). Equations (AD8) and (AD9) are the Home and Foreign investment schedules, respectively. The last six equations are the related to the goods markets: the Home good market clearing conditions in the Home and in the Foreign country, the correspondent equations for the Foreign good, the resource constraints of the Home and Foreign good.

In the fourth block ('borrowing constraint and flows of fund') there are the following equations (taken in that order): the Home country net foreign asset position, the budget constraint and the borrowing constraint equations of the Home constrained agent, the budget constraint and the borrowing constraint equations of the Foreign constrained agent, the Home and Foreign accumulation laws of physical capital, the Home country trade balance.
The last block ('monetary policy rules and shock processes') contains the Home and Foreign monetary policy rules and the shock to the weight of housing in the utility function of the Home unconstrained agent.
Table 1. Model Calibration

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Figure 1. VAR responses to a U.S. expansionary monetary shock.

The U.S. trade balance

Notes: The dashed lines indicate 90-percent confidence bands. Ordinate: time horizon in quarters. Coordinate: % deviation from steady state. Size of the shock: one standard deviation.
Figure 2. VAR responses to a U.S. positive housing price shock.

The U.S. trade balance

Notes: The dashed lines indicate 90-percent confidence bands. Ordinate: time horizon in quarters. Coordinate: % deviation from steady state. Size of the shock: one standard deviation.
Figure 3. VAR responses to a U.S. expansionary monetary shock.

The U.S. imports

Notes: The dashed lines indicate 90-percent confidence bands. Ordinate: time horizon in quarters. Coordinate: % deviation from steady state. Size of the shock: one standard deviation.
Figure 4. VAR responses to a U.S. positive housing prices shock. The U.S. imports

Notes: The dashed lines indicate 90-percent confidence bands. Ordinate: time horizon in quarters. Coordinate: % deviation from steady state. Size of the shock: one standard deviation.
Figure 5. DSGE responses to a U.S. expansionary monetary shock

Notes: Ordinate: time horizon in quarters. Coordinate: % deviation from steady state. Asset position and trade balance are ‘deviation from steady state-to-steady state output’ ratio.
Figure 6. DSGE responses to a U.S. positive housing prices shock

Notes: Ordinate: time horizon in quarters. Coordinate: % deviation from steady state. Asset position and trade balance are 'deviation from steady state-to-steady state output' ratio.
Figure 7. DSGE and VAR responses to a U.S. expansionary monetary shock.

The U.S. trade balance

Figure 8. DSGE and VAR responses to a U.S. expansionary monetary shock.

The U.S. imports

Figure 9. DSGE and VAR responses to a U.S. positive housing prices shock.

The U.S. trade balance

Notes: Ordinate: time horizon in quarters. Coordinate: % deviation from steady state. Asset position and trade balance are 'deviation from steady state-to-steady state output' ratio.
Figure 10. DSGE and VAR responses to a U.S. positive housing prices shock.

The U.S. imports

Notes: Ordinate: time horizon in quarters. Coordinate: % deviation from steady state. Asset position and trade balance are ‘deviation from steady state-to-steady state output’ ratio.
Figure 11. DSGE responses of U.S. variables to a U.S. expansionary monetary shock.

The two-country model

Notes: Ordinate: time horizon in quarters. Coordinate: % deviation from steady state. Asset position and trade balance are 'deviation from steady state-to-steady state output' ratio.
Figure 12. DSGE responses of U.S. variables to a U.S. positive housing price shock.

The two-country model

Notes: Ordinate: time horizon in quarters. Coordinate: % deviation from steady state. Asset position and trade balance are ‘deviation from steady state-to-steady state output’ ratio.
Figure 13. DSGE responses under alternative exchange rate regimes.
U.S. expansionary monetary shock

Notes: Ordinate: time horizon in quarters. Coordinate: % deviation from steady state.
Solid line: flexible exchange rate regime. Dashed line: pegging regime.
Figure 14. DSGE responses under alternative exchange rate regimes.

U.S. positive housing price shock

Notes: Ordinate: time horizon in quarters. Coordinate: % deviation from steady state. Solid line: flexible exchange rate regime. Dashed line: pegging regime.
Figure 15. DSGE responses of U.S. variables to a positive U.S. housing price shock. Different values of the loan-to-value ratio $m$

Notes: Ordinate: time horizon in quarters. Coordinate: % deviation from steady state. Solid line: $m = 0.4$. Dashed line: $m = 0.2$. 
Figure 16. Responses of U.S. variables to an expansionary U.S. monetary shock.
Alternative elasticities of substitution

Notes: Ordinate: time horizon in quarters. Coordinate: % deviation from steady state. Asset position and trade balance are ‘deviation from steady state-to-steady state output’ ratio. Solid line: $\rho = 1.5$. Dashed line: $\rho = 1.1$. 
Figure 17. Responses of U.S. variables to a positive U.S. house price shock.
Alternative elasticities of substitution

Notes: Ordinate: time horizon in quarters. Coordinate: % deviation from steady state. Solid line: $\rho = 1.5$. Dashed line: $\rho = 1.1$. 
Figure 18. DSGE responses of U.S. variables to an expansionary U.S. monetary shock. Different degrees of exchange rate pass-through

Notes: Ordinate: time horizon in quarters. Coordinate: % deviation from steady state. Solid line: incomplete pass-through. Dashed line: complete pass-through.
CHAPTER 3

The Dynamics of the Real Exchange Rate: a Bayesian DSGE Approach

3.1. Introduction

Three main stylized facts in international economics are related to the real exchange rate dynamics. Its fluctuations are extremely volatile, persistent and do not matter for the other variables ('disconnect' from the real economy).

Several contributions in the new open economy macroeconomics (NOEM from now on) literature have extended the Obstfeld and Rogoff sticky-price general equilibrium model, in which the real exchange rate is constant and the purchasing power parity (PPP from now on) holds, to explain the quoted stylized facts.¹

The goal of this paper is to empirically investigate these extensions. To that purpose, I estimate three two-country NOEM models using Bayesian techniques and data of euro area and U.S. economy.

In one model (let’s call it the 'complete' model) I relax the three assumptions at the basis of the PPP condition: international law of one price, symmetric preferences, tradeability of all goods.

I make two assumptions to remove the international law of one price and introduce international price discrimination. One is the local currency pricing (LCP from now on) assumption. Contrary to the producer currency pricing (PCP from now on) case, exporters face costs of adjusting prices not only in their own currency, but also in the currency of the importing country. Hence, import prices are sticky in the currency of the destination market. Chari et al. (2002) show that a high degree of sticky prices is necessary to reproduce the volatility of the real exchange rate when there are monetary shocks.² The other source is that of distribution services intensive in local nontradable goods. Distribution services induce differences in demand elasticity across countries. Thus, with monopolistic producers the law of one price does not hold in general, even in a flexible price equilibrium. Firms in the tradable sector would optimally charge different wholesale prices in the two countries. The advantage of

¹Obstfeld and Rogoff (1995).
²Kollman (2001) studies a small open economy model in which nominal prices and wages are set two or four periods in advance, and in which monetary shocks are the dominant source of exchange rate fluctuations. His model generates variability in the real and nominal exchange rate not much different from that in the data.
introducing distribution services is that, when combined with standard preferences, they contribute to generate a low price elasticity of tradables. Thanks to the price discrimination, fluctuations in the nominal exchange rate are not fully transmitted to the price of imports (assumption of incomplete exchange rate pass-through into import prices). Hence, large exchange-rate swings do not translate into large consumer prices movements.\(^3\) I also relax the assumption of symmetric preferences, imposing home bias in consumption. Finally I assume that some goods are nontradable.\(^4\) Also these features can contribute to increase the volatility of the real exchange rate.

I assume that financial markets are incomplete and there is a shock to the uncovered interest parity (UIP from now on). In fact, incomplete exchange rate pass-through and nominal rigidities \textit{per se} are not sufficient to replicate the high volatility of the real exchange rate. When international financial markets are complete, there is a risk sharing condition linking the real exchange rate to the cross-country ratio of consumption marginal utilities. According to this condition, the volatility of the real exchange rate is proportional to that of the fundamentals. The risk sharing condition can be removed by assuming that only a riskless bond is internationally traded. Incomplete financial market are not able to reproduce the high volatility of the exchange rate. In fact, the uncovered interest rate parity links real exchange rate fluctuations to the real interest rate differential; the related tilting of the consumption path and current account adjustment limit the volatility of the real exchange rate. One possible solution is to assume that the coefficient of risk aversion is high.\(^5\) Another is to introduce a low elasticity of intratemporal substitution between domestic tradable and imported goods using, as I do, distribution services. Alternatively, it is possible to add an uncovered interest parity shock that drives a wedge between the real interest rates. The result is the weakening of the link between exchange rate and fundamentals. The introduction of the UIP shock is justified by the empirical evidence on the failure of the UIP condition as well as by its long-standing use in the theoretical literature.\(^6\)

\(^3\)See Corsetti et al. (2004). Campa and Goldberg (2004) provide evidence that the presence of distribution services helps to explain a lower exchange rate pass-through at the consumer level than at the producer level.

\(^4\)The role of non tradable goods in the explanation of the observed behaviour of prices and exchange rates is still debated. Chari et al. (2002) observe that empirically the real exchange rate dynamics is mainly determined by rigidities in tradable goods prices so that the inclusion of a non tradable sector in a model is unnecessary. Burnstein et al. (2005) reach opposite conclusions. Given this lack of consensus, and the presence of distribution services intensive in local nontradable goods in our model, I include them.

\(^5\)Chari et al. (2002) find that a high coefficient of risk aversion is necessary to reproduce real exchange rate volatility. The result is robust to the structure, complete or incomplete, of the financial markets.

\(^6\)Devereux and Engel (2002), Jeanne and Rose (2002), Duarte and Stockman (2005) show how deviation from the UIP condition, as well as from the complete pass-through assumption, are required to reproduce the high real exchange rate volatility in a monetary model.
Finally, I exploit a combination of nominal price and wage rigidities and ‘inertial’ monetary policy to reproduce the volatility and persistence of the real exchange rate. Nominal rigidities increase the correlation between the nominal and the real exchange rates and allow the overshooting of the correspondent long-run responses. Inertial monetary policy means that the adjustment towards the monetary target is smoothed over time. Benigno (2004) shows that with staggered prices and inertial monetary policy the real exchange rate is persistent because, through the interest rate differential, also its adjustment is smoothed over time.\(^7\)

The other two models are down-sized versions of the complete one. In the first, incomplete pass-through is due only to LCP (there are no distribution services, I call it the ‘LCP’ model). In the second the pass-through is complete because I assume that prices are set in the currency of the producer (I call it the ‘PCP’ model).

The main results of the estimation are the following.

First, incomplete pass-through and the UIP shock are the features most supported by the data. The model with distribution services produces the best fitting.

Second, the real exchange rate variance breakdown into its main economic determinants along with parameters estimates prove that international price discrimination - and hence incomplete pass-through - and home bias are the main determinants of real exchange rate deviations from PPP, at least for the complete and LCP models, where the incomplete pass-through limits the transmission of exchange rate fluctuations to other variables. In the case of the PCP model this is obtained through the home bias, whose estimated value becomes extremely high so as to limit the effects of import prices, which fully react to exchange rate movements, on consumer prices.

Third, in the three models the relative prices of nontradable goods (the internal real exchange rate) play a limited role in generating real exchange rate fluctuations. However nontradable goods cannot be dismissed: in the complete model (the one that better fits the data, as said before) the estimate of the distribution margin is around 50\%, suggesting that distribution services are an important component in the final sale price of tradables and hence an important source of international price discrimination.

Fourth, median estimates of nominal rigidities say that at the border import prices change once a quarter for the euro area and once every two quarters in the U.S. economy. In the complete model, the pass-through of nominal exchange rate into euro area import prices at the border is equal to 40 percent (10 percent for U.S. import prices), at the consumer level 20 percent (5 percent for U.S. imports). The estimates are lower than those usually found in the literature on empirical NOEM models. They

\(^7\)The benchmark model of Chari et al. (2002) with prices fixed for 1 year is able to reproduce the observed volatility in the real exchange rate but generates persistence below the observed one. Bergin and Feenstra (2001) use a model with translog preferences to amplify the effect of staggered price contracts and pricing-to-market in generating persistence. To generate the high persistence observed in the data, they have to impose contracts of 2–3 years length.
are also lower than the estimated nominal rigidities of nontradable goods and wages. These results suggest that models that disregard nontradability, distribution services and price discrimination may severely distort the importance of nominal frictions for import prices.

Fifth, all the models are able to reproduce the persistence of the real exchange rate, thanks to presence of nominal rigidities and inertial monetary policy.

Finally, a forecast error variance decomposition shows that in the complete model about 75% of the real exchange rate variability is explained by the shock to the UIP, and the rest by shocks to preferences. Consumptions, inflation rates and short-term interest rates are, on the contrary, mostly explained by technology and preference shocks.

I'm not the first to estimate NOEM models using Bayesian techniques. Adolfson et al. (2004), Batini et al. (2005), Lubik and Schorfeide (2005) and Rabanal and Tuesta (2006) also estimate open economy model with nominal rigidities and incomplete pass-through. Differently from them, I consider a wider array of sources of real exchange rate volatility and persistence. In fact, I have nontradable goods, distribution services and shocks to the UIP, so to minimize the risk that model mis-specification would affect estimates.8

The paper is organized as follows. Section 2 illustrates the setup. Section 3 describes the main determinants of the real exchange rate dynamics. Section 4 reports the results of the empirical analysis. Section 5 concludes.

3.2. The Setup

The world economy is composed by two large countries of equal size (Home and Foreign country). The two countries are symmetric in terms of technology and tastes, with the notable exception of home bias in preferences. In each of them consumers maximize their utility with respect to leisure and a composite good resulting from the aggregation of non tradable and tradable commodities. The latter can be either imported or domestically produced. Monopolistic firms in the two sectors produce a differentiated variety of either tradable or nontradable goods using labor.

Nominal rigidities should improve the capability of the model to match the volatility and persistence of the real exchange rate. Standard theory predicts that monetary shocks have no effect on the real exchange rate under price flexibility, but induce real exchange rate movements that closely track the nominal exchange rate when prices are (sufficiently) sticky.

I consider not only nominal price rigidities, but also nominal wage rigidities. Sticky wages, in fact, make the nominal marginal cost less responsive to a given shock; as a consequence, also nominal prices should be less responsive.

In what follows, I illustrate the complete model. I report only the equations of the Home country. Those of the Foreign country are similar.

3.2.1. Household optimization

In this section I illustrate households' preferences and budget constraint.

3.2.1.1. Preferences. In each country there is a continuum \([0,1]\) of symmetric households. Home households are indexed by \(j\), Foreign households by \(j^*\). The expected value of agent \(j\)'s lifetime utility \(U_0\) is given by:

\[
U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t Z_{PR,t} \left[ \frac{C_t(j)^{1-\sigma_C}}{1-\sigma_C} - \frac{\kappa}{\tau} L_t(j)^{\tau} \right] \right\} \quad \sigma_C, \kappa, \tau > 0
\]

where \(E_0\) denotes the expectation conditional on information set at date 0 and \(\beta\) is the intertemporal discount factor \((0 < \beta < 1)\). The household obtains utility from consumption \(C(j)\) \((1/\sigma_C\) is the elasticity of intertemporal substitution) and receives disutility from labor supply \(L(j)\). I assume that households are monopolistic competitive and supply a differentiated labor service to firms. \(Z_{PR}\) is a preference shifter, common to all Home households, that scales the overall period utility. It is a country specific exogenous process that evolves as:

\[
\ln Z_{PR,t} = \rho_{Z_{PR}} \ln Z_{PR,t-1} + \epsilon_{Z_{PR,t}}
\]

where \(0 < \rho_{Z_{PR}} < 1\) and \(\epsilon_{Z_{PR,t}}\) is an independently and identically distributed shock with mean and variance respectively equal to 0 and \(\sigma^2_{Z_{PR}}\). A similar preference shifter scales the Foreign agent's utility (hence there is no cross-country correlation between the shifters nor between the shocks). The index \(C_t(j)\) is defined as follows:

\[
C_t(j) \equiv \left[ a_T C_{T,t}(j)^{\frac{1-\phi}{\phi}} + (1-a_T) C_{N,t}(j)^{\frac{1-\phi}{\phi}} \right]^{\frac{\phi}{1-\phi}} \quad \phi > 0
\]

\(C_{T,t}(j)\) is the bundle of tradable goods, while \(C_{N,t}(j)\) is the that of nontradable goods. The parameter \(\phi\) is the elasticity of substitution between the two bundles. The parameters \(a_T\) and \((1-a_T)\) are the weights on the consumption of traded and nontraded goods, respectively. I assume a similar index holds in the Foreign country. The index of traded goods \(C_T(j)\) is given by the following constant elasticity of substitution aggregator:

\[
C_{T,t}(j) \equiv \left[ a_H C_{H,t}(j)^{\frac{1-\rho}{\rho}} + (1-a_H) C_{F,t}(j)^{\frac{1-\rho}{\rho}} \right]^{\frac{\rho}{1-\rho}} \quad \rho > 0
\]

where \(\rho\) is the elasticity of substitution between the consumption of the Home good \(C_H(j)\) and the Foreign good \(C_F(j)\).

---

9I do not model money explicitly, but I interpret this model as a cash-less limiting economy, in the spirit of Woodford (1998), in which the role of money balances in facilitating transactions is negligible.
The parameter $a_H$ ($a_H > 1/2$) and $(1 - a_H)$ are the weights on the consumption of Home and Foreign traded goods. I assume that a parallel bundle of tradable goods holds in the Foreign country, but with a weight $a_H$ ($a_H > 1/2$) on consumption of the domestically produced good $F$.\textsuperscript{10} This assumption of “mirror symmetric” baskets generates a home bias in tradable goods. $C_{H,t}(j)$, $C_{F,t}(j)$, $C_{N,t}(j)$ are indexes of consumption across the continuum of differentiated goods produced respectively in the Home tradable, Foreign tradable and Home nontradable sector:

\begin{equation}
C_{H,t}(j) \equiv \left[ \int_0^1 C_t(h,j)^{\frac{\theta_T-1}{\theta_T}} dh \right]^{\frac{\theta_T}{\theta_T-1}} \quad \theta_T > 1
\end{equation}

\begin{equation}
C_{F,t}(j) \equiv \left[ \int_0^1 C_t(f,j)^{\frac{\theta_T-1}{\theta_T}} df \right]^{\frac{\theta_T}{\theta_T-1}} \quad \theta_T > 1
\end{equation}

\begin{equation}
C_{N,t}(j) \equiv \left[ \int_0^1 C_t(n,j)^{\frac{\theta_N-1}{\theta_N}} dn \right]^{\frac{\theta_N}{\theta_N-1}} \quad \theta_N > 1
\end{equation}

$C_t(h,j)$ and $C_t(f,j)$ are respectively agent $j$’s consumption of Home brand $h$ and Foreign brand $f$. $C_t(n,j)$ is the same agent’s consumption of nontradable brand $n$. $\theta_T$ is the elasticity of substitution between brands belonging to the Home tradable or to the Foreign tradable sector. $\theta_N$ is the elasticity of substitution between brands belonging to the nontradable sector. The Home brand demand function resulting from cost minimization is:

\[ C_t(h,j) = \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\theta_T} C_{H,t}(j) \]

Similar demands can be derived for the other brands produced in the Home and in the Foreign country.\textsuperscript{11} $p_t(h)$ is the price of the generic Home brand $h$ in the Home country.

\textsuperscript{10}Hence:

\begin{equation}
C_t(j^*) \equiv a_T C_{T,t}(j^*)^{\frac{1}{\theta_T}} + (1 - a_T) C_{N,t}(j^*)^{\frac{1}{\theta_N}}
\end{equation}

and:

\begin{equation}
C_{T,t}(j^*) \equiv \left( 1 - a_H \right) C_{H,t}(j^*)^{\frac{1}{\theta_T}} + a_H C_{F,t}(j^*)^{\frac{1}{\theta_T}}
\end{equation}

\textsuperscript{11}Using the above indexes, the demand for the bundles $C_H$, $C_F$, $C_T$ and $C_N$ can be derived:

\begin{equation}
C_{H,t}(j) = a_H \left( \frac{P_{H,t}}{P_{T,t}} \right)^{-\phi} C_{T,t}(j), \quad C_{F,t}(j) = (1 - a_H) \left( \frac{P_{F,t}}{P_{T,t}} \right)^{-\phi} C_{T,t}(j)
\end{equation}

\begin{equation}
C_{T,t}(j) = a_T \left( \frac{P_{T,t}}{P_t} \right)^{-\phi} C_t(j), \quad C_{N,t}(j) = (1 - a_T) \left( \frac{P_{N,t}}{P_t} \right)^{-\phi} C_t(j)
\end{equation}
$P_{H,t}$ is a utility-based price index:

$$
(3.11) \quad P_{H,t} = \left( \int_0^1 \rho_t (h)^{1-\theta T} \, dh \right)^{\frac{1}{1-\theta T}}
$$

It is the minimum expenditure necessary to buy one unit of the good (3.6). A similar definition applies to $P_{F,t}$ and $P_{N,t}$ and their Foreign counterparts. Similarly, the consumer price index and the price index of tradable goods can be derived respectively from the bundles (3.2) and (3.3):

$$
(3.12) \quad P_t = \left[ a_T (1-\rho_T) + (1-a_T) P_{N,t}^{1-\rho} \right]^{\frac{1}{1-\rho}}
$$

$$
(3.13) \quad P_T = \left[ a_H P_{H,t}^{1-\rho} + (1-a_H) P_{F,t}^{1-\rho} \right]^{\frac{1}{1-\rho}}
$$

Similar indexes hold in the Foreign country.\(^{12}\)

3.2.1.2. Budget Constraint. I assume that households belonging to the Home country can allocate their wealth among two bonds. Both bonds are risk-free with one-period maturity. One is denominated in the Home currency and the other in the Foreign currency. In contrast, households that belong to the Foreign country can allocate their wealth only in one risk-free nominal bond denominated in the Foreign currency.

The budget constraint of household $j$ in the Home country is:\(^{13}\)

$$
(3.16) \quad \frac{B_{H,t}(j)}{(1+i_t)} + \frac{S_t B_{F,t}(j)}{(1+i_t)^*} \Phi \left( \frac{S_t B_{F,t}}{P_t} \right) Z_{U1F,t} = B_{H,t-1}(j) - S_t B_{F,t-1}(j)
$$

$$
= \int_0^1 \Pi_t(h,j) \, dh + \int_0^1 \Pi_t(n,j) \, dn
$$

$$
+ W_t(j) L_t(j) - P_t C_t(j) - A C_t^{NW} W_t(j) L_t(j)
$$

$B_H(j)$ is household $j$'s holding of the risk-free one-period nominal bond denominated in units of Home currency.\(^{14}\) This bond pays a nominal interest rate $i_t$. $S_t$ is the nominal exchange (number of Home currency units per unit of Foreign currency). $B_F(j)$ is

\(^{12}\)Hence:

$$
(3.14) \quad P_t^* = \left[ a_T (1-\rho_T) + (1-a_T) P_{N,t}^{1-\rho} \right]^{\frac{1}{1-\rho}}
$$

and:

$$
(3.15) \quad P_T^* = \left[ (1-a_H) P_{H,t}^{1-\rho} + a_H P_{F,t}^{1-\rho} \right]^{\frac{1}{1-\rho}}
$$

\(^{13}\)See Benigno (2001) for a similar financial structure.

\(^{14}\)Our time convention has $B_{H,t-1}(j)$ and $B_{F,t-1}(j)$ as agent $j$’s nominal bonds accumulated during period $t-1$ and carried over $t$. 


the holding of the risk-free one-period nominal bond denominated in units of Foreign
currency. It pays a nominal interest $i^*_t$. The nominal rates $i_t$ and $i^*_t$ are paid at the
beginning of period $t + 1$ and are known at time $t$. They are directly controlled by
the national monetary authorities. The function $\Phi(S_{t}B_{F,t}^{*}/P_t)$ captures the costs, for the
households belonging to Home country, of undertaking positions in the international
asset market. As borrowers, they will be charged a premium on the Foreign interest
rate; as lenders, they will receive a remuneration lower than the Foreign interest rate. I
introduce this additional cost to pin down a well defined steady state. The payment
of this cost is rebated to agents belonging to the Foreign country. I adopt the following
functional form:

$$\Phi(S_{t}B_{F,t}^{*}/P_t) = \exp(\delta_b S_{t}B_{F,t}^{*}/P_t)$$

where $0 \leq \delta_b \leq 1$. The parameter $\delta_b$ controls the speed of convergence to the steady
state.

I add a risk premium shock $Z_{UIP,t}$ to allow for exogenous variations in international
financial markets conditions. It evolves as:

$$\ln Z_{UIP,t} = \rho_{UIP} \ln Z_{UIP,t-1} + \epsilon_{UIP,t}$$

where $0 < \rho_{UIP} < 1$ and $\epsilon_{UIP,t}$ is an independently and identically distributed shock
with mean and variance respectively equal to 0 and $\sigma^2_{UIP}$. Households derives income
from two sources: nominal wages $W_t(j)$; profits of domestic tradable and nontradable
firms, respectively $\int_0^1 \Pi_t(h,j) \, dh$ and $\int_0^1 \Pi_t(n,j) \, dn$. Each household is the monopolis-
tic supplier of a labor input $j$. When setting her wage $W(j)$, each Home agent takes
into account firms’ demand for her type of labor (see later):

$$L_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\theta_{L,t}} L_t, \quad \theta_{L,t} > 1$$

where $L_t$ is the total amount of labor supplied in the economy and $W_t$ is a wage index
defined later. The elasticity of substitution among labor inputs $j$s, $\theta_{L,t}$, is subject to
independently and identically distributed shocks with mean and variance respectively
equal to 0 and $\sigma^2_{\theta_{L,t}}$. I assume there is a wage-adjustment cost, which is given by the

---

15The function $\Phi(S_{t}B_{F,t}^{*}/P_t)$ depends on real holdings of the foreign assets in the entire Home economy.
Hence, domestic households take it as given when deciding on the optimal holding of the foreign bond.
We require that $\Phi(0) = 1$ and that $\Phi(.) = 1$ only if $B_{F,t} = 0$. $\Phi(.)$ is a, differentiable, (at least)
decreasing function in the neighborhood of zero. See also Schmitt-Grohé and Uribe (2003).


17Hence, in the budget constraint of household $j^*$ in the Foreign country the following term appears:

$$K = \frac{B_{F,t}}{(1 + i^*_t)} \left[\Phi(S_{t}B_{F,t}^{*}/P_t)Z_{UIP,t} - 1\right]$$
following quadratic function:\(^{18}\)

\[
(3.17) \quad AC_t^W (j) = \frac{\kappa^W}{2} \left( \frac{W_t(j)}{W_{t-1}(j)} - 1 \right)^2 \quad \kappa^W \geq 0
\]

I assume that all the households belonging to the same country have the same initial level of wealth and share the profits of domestic firms in equal proportion. Hence, within a country all the households face the same budget constraint. In their optimal decisions, they will choose the same path of bonds, consumption and wages. First order conditions with respect to \(B_{H,t}(j), B_{F,t}(j)\) are rather standard:

\[
(3.18) \quad C_t(j)^{-\sigma_{C}} Z_{PR,t} = (1 + \iota_t) \beta E_t \left( C_{t+1}(j)^{-\sigma_{C}} Z_{PR,t+1} \frac{P_t}{P_{t+1}} \right)
\]

\[
(3.19) \quad C_t(j)^{-\sigma_{C}} Z_{PR,t} = (1 + \iota_t^*) \Phi \left( \frac{S_B F_{C,t}}{P_t} \right) Z_{UIP,t} E_t \left( C_{t+1}(j)^{-\sigma_{C}} Z_{PR,t+1} \frac{P_t}{P_{t+1}} \frac{S_{t+1}}{S_t} \right)
\]

In a symmetric equilibrium \((L_t(j) = L_t, W_t(j) = W, C_t(j) = C_t)\), the first order condition with respect to \(W_t(j)\) can be written as:

\[
\frac{W}{P} = \left\{ \frac{1 + \frac{\kappa^w}{(\theta_{W,t-1}) W_{t-1}} \left( \frac{W_t}{W_{t-1}} - \pi_W \right)}{\frac{\theta_{w,t}}{(\theta_{W,t} - 1) L_{t-1}} \kappa^{\sigma_{C}} Z_{PR,t+1} Z^{\sigma_{C}} Z_{PR,t} \left( \frac{L_{t+1}}{L_t} \right)} \right\}^{-1}
\]

\[
\times \frac{\theta_{L,t}}{(\theta_{L,t} - 1)} \kappa^{\sigma_{C}} Z_{PR,t}
\]

The real wage is equal to a the marginal rate of substitution between consumption and leisure times the term \(\theta_{L,t}/(\theta_{L,t} - 1)\) and the reciprocal of a term including the wage adjustment costs. In the case of no wage rigidities, the real wage would be equal to the marginal rate of substitution times the markup, \(\theta_{L,t}/(\theta_{L,t} - 1)\).

### 3.2.2. Firms

In what follows I illustrate the problems solved by the Home firms. Foreign firms solve similar problems.

#### 3.2.2.1. Production

There is a continuum of firms. Firms belonging to the tradable sector are indexed by \(h \in [0,1]\); those belonging to the nontradable sector are indexed by \(n \in [0,1]\). Foreign firms belonging to the tradable sector are indexed by \(f \in [0,1]\); those belonging to the nontradable sector are indexed by \(n^* \in [0,1]\).

The technology used by firms is the following one:

\[
(3.21) \quad y_t(h) + y_t^*(h) = Z_{T,t} L_t(h), \quad y_t(n) = Z_{N,t} L_t(n)
\]

\(^{18}\)See Kim (2000).
$Z_{X,t} \ (X = T, N)$ is a technology shock common to all the firms belonging to the same sector ($T$ is for Home tradable sector, $N$ for Home nontradable sector). It follows a stationary AR(1) process:

\begin{equation}
\ln Z_{X,t} = \rho_Z \ln Z_{X,t-1} + \epsilon_{Z_{X,t}} \quad X = T, N
\end{equation}

where $0 < \rho_Z < 1$ and $\epsilon_{Z_{X}}$ is an independently and identically distributed shock with mean and variance respectively equal to 0 and $\sigma_{Z_{X}}^2$. The only input used in the production is a basket $L_t$ of labor. It is a combination of differentiated labor inputs defined over a continuum of mass one ($j \in [0, 1]$):

\[
L_t(x) = \frac{\int_0^1 L_t(x,j) dj}{\int_0^1 1 - \theta_{L,t} dj}
\]

Firms take the prices of labor inputs as given. Cost minimization implies that the demand for labor input $j$ by firm $x$ is a function of the relative wage:

\[
L_t(x,j) = \left( \frac{W_t(j)}{W_t} \right)^{-\theta_{L,t}} L_t(x)
\]

where $W_t(j)$ is the nominal wage paid to labor input $j$ and the wage index $W$ is defined as:

\[
W_t = \left[ \int_0^1 W_t(j)^{1-\theta_{L,t}} dj \right]^{\frac{1}{1-\theta_{L,t}}}
\]

Cost minimization also yields an expression for the marginal cost $MC_t(x)$:

\begin{equation}
MC_t(x) = \frac{W_t}{Z_{X,t}}
\end{equation}

Hence the marginal cost is the same for all firms belonging to the same sector.

### 3.2.2.2. Price setting.

A feature common to all firms is that there is sluggish price adjustment due to resource costs measured in terms of total profit. The adjustment cost is quadratic.\(^{19}\) In the case of a firm belonging to the Home nontradable sector, it is:

\begin{equation}
AC^p_{N,t}(n) = \frac{\kappa^p_N}{2} \left( \frac{p_t(n)}{p_{t-1}(n)} - 1 \right)
\end{equation}

where $\kappa^p_N \geq 0$. In the case of a firm belonging to the Home tradable sector, we have:

\begin{align*}
AC^p_{H,t}(h) &= \frac{\kappa^p_H}{2} \left( \frac{p_t(h)}{p_{t-1}(h)} - 1 \right), \\
AC^p_{H,t}(h) &= \frac{\kappa^p_H}{2} \left( \frac{p_t(h)}{p_{t-1}(h)} - 1 \right)
\end{align*}

where $\kappa^p_H, \kappa^p_H \geq 0$. Firms producing tradables pay a cost according to the destination market. Let’s consider the profit maximization problem, starting with firms belonging to the nontradable sector.

\(^{19}\)See Dedola and Leduc (2001), Rotemberg (1982).
Nontradable sector. Each firm $n$ takes into account the demand for its product and sets the nominal price $p_t(n)$ by maximizing the present discounted value of real profits. Demand comes not only from consumers but also from the firms belonging to the local distribution sector. These firms purchase Home and Foreign tradable goods and distribute them domestically using a Leontief technology: they combine one unit of the tradable good with $\eta > 0$ units of the nontradable goods basket $N:$

$$
\eta = \left[ \int_0^1 \eta(n)^{\theta_N - 1} \frac{dn}{\theta_N - 1} \right]^{\frac{\theta_N}{\theta_N - 1}} \theta_N > 1
$$

Firms in the distribution sector are perfectly competitive. The distribution costs introduce a wedge between wholesale and consumer prices. Let's denote with $\hat{p}$ the consumer price and with $\bar{p}$ the wholesale price. It follows that in the case of the Home good (similar relations hold for the Foreign traded good), I can write:

$$
p_t(h) = \hat{p}_t(h) + \eta P_{N,t}, \quad p^*_t(h) = \bar{p}_t(h) + \eta P^*_{N,t}
$$

Given that there is market segmentation, the law of one price for tradable goods does not hold; hence at the wholesale level I have $\bar{p}_t \neq \hat{p}_t(h)/S_t$. Similar inequalities hold at the consumer level.

Corsetti et al. (2004) show that, under incomplete markets, high home bias and low price elasticity of tradable goods are key to make the real exchange rate volatile and to obtain a negative correlation between relative consumption and real exchange rate. Given the elasticity of substitution between tradable goods at the consumer level, $\rho$, distribution services reduce the price elasticity of tradable goods of the producers, which is equal to:

$$
\rho \left( 1 - \eta \frac{P_N}{\bar{p}(h)} \right)
$$

where $P_N$ and $\bar{p}(h)$ are set at their steady state values. All these features, which are present in the complete model I estimate, contribute to increase the real exchange rate volatility. There are no distribution costs for nontradable goods ($p_t(n) = \hat{p}_t(n)$). Hence, the demand for Home nontradable goods is:

$$
y_t(n) = \left( \frac{p_t(n)}{P_{N,t}} \right)^{-\theta_N} \left[ C_{N,t} + \eta \left( \int_0^1 C_t(h,j) \, dj + \int_0^1 C_t(f,j) \, dj \right) \right]
$$

---

20 Hence consumers cannot purchase the tradable goods directly from producers but from firms belonging to the distribution sector.

21 The term $\left( \frac{p_t(n)}{P_{N,t}} \right)^{-\theta_N} C_{N,t}$ is obtained by maximizing, for every agent $j$, equation (3.8) subject to the constraint $\int_0^1 p_t(n) d\gamma(n) = Z$ ($Z$ is any fixed total nominal expenditure on goods). Demands for all other brands are similarly derived.
The price-setting problem is characterized as:

\[
\max_{\{p_t(n)\}} E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{P_t C_{t-\sigma C}}{P_t C_t^{-\sigma C}} (p(n) - MC_t(n)) \left( \frac{P_t(n)}{P_{N,t}} \right)^{-\theta_N} \\
\left[ C_{N,t} + \eta \left( \int_0^1 C_t(h,j) \, dj + \int_0^1 C_t(f,j) \, dj \right) \right] (1 - AC_{N,t}^p (n))
\]

The term \((\beta P_t C_{t-\sigma C} / P_t C_t^{-\sigma C})\) is the household discount factor. Firms are agents for the local households, to whom transfer profits in a lump-sum fashion. In a symmetric equilibrium \((p(n) = P_N, C_t(h) = C_{H,t}, C_t(f) = C_{F,t}, C_t(n) = C_{N,t})\) the first order condition is:

\[(3.29) \quad P_{N,t} = A_N^{-1} \left[ \theta_N \frac{W_t}{Z_{N,t}} \right] \]

where:

\[
A_N = (\theta_N - 1) + \frac{P_{N,t}}{P_{N,t-1}} \kappa_N \left( \frac{P_{N,t}}{P_{N,t-1}} - 1 \right)
\]

\[
- \beta C_{t+1}^{-\sigma C} \frac{P_t}{P_{t+1}} \frac{P_{N,t+1}^2}{C_{t}^{-\sigma C}} \left[ C_{N,t+1} + \eta (C_{H,t+1} + C_{F,t+1}) \right] \kappa_N \left( \frac{P_{N,t+1}}{P_{N,t}} - 1 \right)
\]

In absence of nominal rigidities, the above condition says that the price is a constant markup over the marginal cost.

Tradeable sector. Firm producing the generic brand \(h\) sets a price \(\bar{p}_t(h)\) (in Home currency) in the Home market subject to the demand constraint

\[
y_t(h) = \left( \frac{\bar{p}_t(h) + \eta P_{N,t}}{P_{H,t}} \right)^{-\theta_T} C_{H,t}
\]

and a price \(\bar{p}^*_t(h)\) (in Foreign currency) in the Foreign market, subject to the demand constraint:

\[
y^*_t(h) = \left( \frac{\bar{p}^*_t(h) + \eta P^*_{N,t}}{P^*_{H,t}} \right)^{-\theta_T} C^*_{H,t}
\]

Hence, the maximization problem can be written as follows:

\[
\max_{\{\bar{p}_t(h), \bar{p}^*_t(h)\}} E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{P_t C_{t-\sigma C}}{P_t C_t^{-\sigma C}} \left\{ (\bar{p}_t(h) - MC_T(h)) y_t(h) (1 - AC_{H,t}^p(h)) \right\}
\]

\[
+ (S_T \bar{p}^*_t(h) - MC_T(h)) y^*_t(h) (1 - AC_{H,t}^{*p}(h)) \}
\]

In a symmetric equilibrium \((\bar{p}(h) = \bar{P}_{H,t}, \bar{p}^*_t(h) = \bar{P}^*_{H,t}, C_t(h) = C_{H,t}, C_t(f) = C_{F,t})\), the first order condition with respect to \(\bar{p}(h)\) and to \(\bar{p}^*_t(h)\) can be written respectively as:

\[(3.30) \quad \bar{P}_{H,t} = A^{-1} \left( \theta_T \frac{W_t}{Z_{T,t}} + \eta P_{N,t} \right) \]
\[ (3.31) \quad \bar{P}_{H,t} = A^{* \smallfrown 1} \left( \frac{W_t}{Z_{T,t}} + \eta P_{N,t}^* \right) \]

where

\[
A_H = \begin{bmatrix}
\theta_T - 1 + \kappa_P^P \frac{P_{H,t}}{P_{H,t-1}} \left( \frac{P_{H,t}}{P_{H,t-1}} - 1 \right) \\
-\kappa_H^P \frac{C_{t+1}^c + \frac{P_t}{C_{t+1}^c}}{C_{C}^c} \left( \frac{P_{H,t+1}}{P_{H,t}} \right)^2 \left( \frac{P_{H,t+1}}{P_{H,t}} - 1 \right) \frac{C_{H,t+1}}{C_{H,t}}
\end{bmatrix}
\]

\[
A_H^* = \begin{bmatrix}
\theta_T - 1 + \kappa_P^P \frac{P_{H,t}}{P_{H,t-1}} \left( \frac{P_{H,t}}{P_{H,t-1}} - 1 \right) \\
-\kappa_H^P \frac{C_{t+1}^c + \frac{P_t}{C_{t+1}^c}}{C_{C}^c} \left( \frac{P_{H,t+1}}{P_{H,t}} \right)^2 \left( \frac{P_{H,t+1}}{P_{H,t}} - 1 \right) \frac{C_{H,t+1}}{C_{H,t}}
\end{bmatrix}
\]

The two above equations are the result of the international price discrimination. Its determinants are two: the local currency pricing policy, e.g. the prices are sticky in the currency of the buyer; the distribution services intensive in local nontradable goods, which imply that the elasticity of demand for any brand is not necessarily the same across markets.

In presence of international price discrimination the law of one price does not hold (that is, \( \bar{p}_t(h) \neq S_t \bar{p}_t^* (h) \)) and exchange rate pass-through into import prices is not complete (\( \partial \log P_t^* (h) / \partial \log S_t < 1 \)). It is possible to derive a structural coefficient measuring the degree of the pass-through of nominal exchange rate into import prices. From equation (3.31), after some algebra, a structural coefficient measuring the pass-through at the border can be obtained:

\[ (3.32) \quad \bar{P}_{H,t} = ... - \theta_T w \frac{1}{\bar{p}_H + \eta P_N} \left\{ \left[ \kappa_P^P (1 + \beta) \right] + \theta_T \bar{p}_H \left[ \frac{(\eta P_N + w)}{(\bar{p}_H + \eta P_N)^2} \right] \right\}^{-1} S_t \]

where prices and wages are expressed in terms of consumption. Structural exchange rate pass-through is the percentage change in import prices - denominated in local currency - resulting from a one percent change in the bilateral exchange rate, other things equal. A higher pass-through implies that fluctuations of the nominal exchange rate are transmitted to a greater extent to import prices; in the limiting case, when the pass-through is complete (i.e. equal to 1) the fluctuations are transmitted one for one into import prices. In my case, the pass-through is complete when there are no distribution services (\( \eta = 0 \)) and no nominal rigidities (\( \kappa_P^P = 0 \)). If at least one of the two features is in the model, then the pass-through is incomplete. Hence, the fluctuations of the nominal exchange rate can be not fully transmitted to the import prices even if prices are flexible, given the presence of distribution services.

International price discrimination and the related pass-through incompleteness help to increase the exchange rate volatility and disconnect: given that the exchange rate change has little effect on the behavior of final purchasers of goods, than it may take large changes in exchange rates to achieve equilibrium after some shock to fundamentals.
3.2.3. Monetary Rules

In the Home country, the monetary authority sets the nominal interest according to the following rule:

\[
(1 + i_t) = \left(1 + i_{t-1}\right)^{\rho_R} \left(\frac{P_t}{P_{t-1}}\right)^{(1-\rho_R)\rho_Y} \left(\frac{Y_t}{\bar{Y}}\right)^{(1-\rho_R)\rho_Y(1-\rho_R)\rho_s} \left(\frac{S_t}{S_{t-1}}\right)^{(1-\rho_R)\rho_s} Z_{R,t}
\]

with \(0 < \rho_R < 1, \rho_Y, \rho_s > 0\). The interest rate \(1 + i_t\) is set by the central bank, \(P_t/P_{t-1}\) is the consumer price inflation, \(Y_t\) is the total output and the coefficient, \(S_t/S_{t-1}\) is depreciation rate of the nominal exchange rate, \(1 + \bar{i}\) and \(\bar{Y}\) are the steady state levels of interest rate and output, respectively. The parameter \(\rho_R\), which assumes values between zero and one, captures inertia in conducting monetary policy: the higher the coefficient, the more inertial is the monetary policy. The parameters \(\rho_Y\) and \(\rho_s\) measure the reaction of monetary authority respectively to consumer price inflation, output and nominal exchange rate depreciation. \(Z_{R,t}\) is a shock having the following law of motion:

\[
\ln Z_{R,t} = \epsilon_{Z_{R,t}}
\]

where \(\epsilon_{Z_{R,t}}\) is an identically and independently distributed shock to the monetary policy function having mean zero and variance \(\sigma_{Z_R}^2\). A similar monetary policy function holds for the Foreign country:

\[
\left(1 + \bar{i}_t\right) = \left(1 + \bar{i}_{t-1}\right)^{\rho_R^*} \left(\frac{P_t^*}{P_{t-1}^*}\right)^{(1-\rho_R^*)\rho_Y^*} \left(\frac{Y_t^*}{\bar{Y}}\right)^{(1-\rho_R^*)\rho_Y^*(1-\rho_R^*)\rho_s^*} \left(\frac{S_t}{S_{t-1}}\right)^{(1-\rho_R^*)\rho_s^*} Z_{R,t}^*
\]

The law of motion of \(Z_{R,t}^*\) is similar to that of \(Z_{R,t}\).

Monetary policy contributes to match the high persistence of the real exchange rate. When monetary policy is conducted in an “inertial” way, so that the adjustment of the instrument toward its target is smoothed over time, the real exchange rate exhibits persistence. Hence, higher values of the real exchange rate persistence should be associated to higher values of the parameters \(\rho_R\) and \(\rho_R^*\) in equations (3.33) and (3.34), respectively.\(^{22}\)

3.2.4. Market Clearing Conditions

I assume that the bond denominated in the Home currency is traded only among Home agents. Hence the market clearing is:

\[
\int_0^1 B_{H,t}(j) \, dj = 0
\]

\(^{22}\)See G.Benigno (2004) and Chari et al. (2002).
The market clearing condition of the internationally traded bond is:

\[(3.36) \quad \int_0^1 B_{F_t}(j) \, dj + \int_0^1 B_{F_t}(j^*) \, dj^* = 0\]

The market clearing conditions of the generic brand \( h \) are:

\[(3.37) \quad y_t(h) = \int_0^1 C_t(h, j) \, dj\]

\[(3.38) \quad y^*_t(h) = \int_0^1 C_t(h, j^*) \, dj^*\]

The market clearing condition of the generic brand \( n \) in the Home country is:

\[(3.39) \quad y_t(n) = \int_0^1 C_t(n, j) \, dj + \left( \frac{p_t(n)}{P_{N,t}} \right)^{-\theta_N} \eta \left[ \int_0^1 C_t(h, j) \, dj + \int_0^1 C_t(f, j) \, dj \right]\]

The labor market clearing of the generic labor type \( j \) is:

\[(3.40) \quad L_t(j) = \int_0^1 L_t(h, j) \, dh + \int_0^1 L_t(n, j) \, dn\]

Similar equations hold in the Foreign country. In agreement with the preferences of Home and Foreign agents, the aggregate final goods are defined as the constant-elasticity-of-substitution composite of individual retail goods:

\[(3.41) \quad Y_{H,t} \equiv \left( \int_0^1 Y_t(h)^{\frac{\theta_T-1}{\theta_T}} \, dh \right)^{\frac{\theta_T}{\theta_T-1}}\]

\[(3.42) \quad Y_{H,t}^* \equiv \left( \int_0^1 Y^*_t(h)^{\frac{\theta_T-1}{\theta_T}} \, dh \right)^{\frac{\theta_T}{\theta_T-1}}\]

\[(3.43) \quad Y_{N,t} \equiv \left( \int_0^1 Y_t(n)^{\frac{\theta_N-1}{\theta_N}} \, dn \right)^{\frac{\theta_N}{\theta_N-1}}\]

Home total tradable output can be defined as:

\[Y_{T,t} = Y_{H,t} + Y_{H,t}^*\]

Home total output as:

\[Y_t = Y_T + Y_{N,t}\]

Similar aggregators hold for goods produced in the Foreign country.

In a symmetric equilibrium (households within the same country are similar, as well as firms within the same sector), the trade balance \( TB \) of the Home country can be
obtained by combining the equation (3.35) with the integral (over the Home population) of the equation (3.16)

\[
TB_t = \frac{S_t B_{F,t}}{(1 + i_t^*)\phi\left(\frac{S_t B_{F,t}}{P_t}\right)Z_{UIP,t}}
- S_t B_{F,t-1}
= \bar{P}_{H,t} Y_{H,t} + S_t \bar{P}_{H,t} Y_{H,t} + P_{N,t} Y_{N,t}
- P_t C_t - AC_t W_t L_t
\]

(3.44)

3.2.5. The equilibrium

Let's consider an initial allocation of \(B_{H,t_0}, B_{F,t_0}, B^*_{F,t_0}\) and initial prices \(W_{t_0}, W_{t_0}, \bar{p}_{t_0}(x)\) \((x = h, h^*, n, n*)\), \(\bar{p}_{t_0}(x)\) \((x = f, f^*, n^*)\), \(i_{t_0}, \bar{i}_{t_0}\) and the processes \(Z_{FR,t}, Z^*_{FR,t}, \theta_{L,t}, \theta^*_{L,t}, Z_X (X = T, N, T^*, N^*), Z_{R,t}, Z^*_{R,t}, Z_{UIP,t}\). Given these initial conditions and processes, for \(t \geq t_0\) I define an equilibrium as a sequence of allocations and prices such that the households and firms satisfy their respective first order conditions, the monetary authorities follow their respective monetary rules and the market clearing conditions hold. The rest of the paper considers an equilibrium in which agents and firms are symmetric respectively in each country and in each sector, dropping the indexes \(j, j^*, h, h^*, f, f^*, n, n^*\). Note that in an equilibrium where \(L_t(j) = L_t\) and \(L_t(j^*) = L_t^*\) output is a linear function of labor: \(Y_N = Z_N L_N\) and \(Y_H + Y_{H} = Z_T L_T\). The LCP model is obtained from the complete model by setting \(\eta = 0\) (hence, distribution services are eliminated as well as the distinction between wholesale and consumer prices of the tradable goods). The PCP model is obtained by setting \(\eta = 0\) and substituting the assumption of local currency pricing with that of producer currency pricing. A firm producing the Home tradable good sets only one price \(p_t(h)\), in Home currency, given the world demand for the good and subject to a single price adjustment cost. The Foreign currency price of the Home good follows the law of one price \((p_t^*(h) = p_t(h)/S_t\), hence the pass-through of nominal exchange rate is complete). Similarly, a firm producing the Foreign tradable good set only one price in its own currency, \(p_t^*(f)\). The price of exports to the Home country follows the law of one price \((p_t^*(f) = S_t p_t(f))\).

3.3. The determinants of the real exchange rate dynamics

The real exchange rate of the Home country is:

\[
RS = \frac{S_t P_t^*}{P_t}
\]

(3.45)
The price indexes can be used to decompose the real exchange rate into its main determinants (international price discrimination, relative price of nontradable goods, home bias). Jointly with the assumption of incomplete international financial markets, the decomposition is at basis of the performed empirical analysis of the real exchange rate dynamics.

After log-linearizing the price indexes around the steady state of the model (see details of the steady state in the Appendix) and some algebraic manipulation I obtain:

\[
\Delta R S_t = (1 - a_T)(\hat{\pi}_{NT,t}^* - \hat{\pi}_{TT,t}^*) - (1 - a_T)(\hat{\pi}_{NT,t} - \hat{\pi}_{TT,t}) + \\
+ (2a_H - 1)(\hat{\pi}_{FT,t} - \hat{\pi}_{HT,t}) \\
+ a_H(\Delta S_t + \hat{\pi}_{FT,t}^* - \hat{\pi}_{HT,t}) + (1 - a_H) (\Delta S_t + \hat{\pi}_{HT,t}^* - \hat{\pi}_{HT,t})
\]

where \(\Delta R S_t\) and \(\Delta S_t\) are the percentage change in the real and nominal exchange rate between period \(t\) and \(t - 1\), \(\hat{\pi}_{TT,t}\), \(\hat{\pi}_{HT,t}\), \(\hat{\pi}_{FT,t}\), \(\hat{\pi}_{NT,t}\) represent, respectively, the consumer price inflation rates in the Home country of tradable, Home tradable, Foreign tradable and nontradable goods. The variables with a star represent the corresponding inflation rates in the Foreign country. The first two terms in equation (3.46) are the Home and Foreign internal real exchange rate, respectively; the second row is the home bias; finally, the last two terms show the deviations from the international law of one price for the Foreign and Home tradable good, respectively. I now consider each of them in turn.

The two terms in the first row of equation (3.46) represent the part of deviation from the PPP due to nontradable goods. As for international price discrimination, the presence of nontradable goods into the bundle of the agents limits the transmission of nominal exchange rate fluctuations to the relative prices faced by the consumers. Hence larger real exchange rate fluctuations are needed for an economy with a high share of nontradable goods \((1 - a_T)\) in our model) in order to achieve the same degree of relative price adjustment which accommodates asymmetric shocks. Note that in our model, the prices of nontradable goods affect the internal real exchange rate not only directly, but also indirectly through changes in the prices of traded goods, induced by distribution costs intensive in local nontraded goods.

The second term in equation (3.46), \((2a_H - 1)(\hat{\pi}_{FT,t} - \hat{\pi}_{HT,t})\), measures the home bias component of the real exchange rate. When the parameter \(a_H\) equals 0.5 (no home bias) this term vanishes; when \(a_H\) is greater than 0.5 then the higher the home bias,$^{24}$

---

$^{24}$See Benigno and Thoenissen (2006).

$^{25}$I use the following definition: \(\hat{\pi}_{.,t} \equiv \ln (1 + \pi_{.,t}) \equiv \ln (P_{.,t}/P_{.,t-1})\).

the wider are the changes in the real exchange rate induced by changes in the relative price of the imported good.\textsuperscript{27}

The term $a_H \left( \Delta \hat{S}_t + \hat{\pi}_{F,t}^* - \pi_{F,t} \right) + (1 - a_H) \left( \Delta \hat{S}_t + \hat{\pi}_{H,t}^* - \hat{\pi}_{H,t} \right)$ in equation (3.46) is a measure of international price discrimination: if the law of one price holds (so that the price of a tradable good, when expressed in a common currency, is the same in each country), each of the two terms would be equal to zero.

These terms are crucial for the dynamics of the real exchange rate. Their estimates constitute one of the results of the empirical analysis.

A crucial assumption is also that of incomplete international financial markets.\textsuperscript{28} When a complete set of state contingent nominal assets is traded, hence international financial markets are complete, the following risk-sharing condition holds in every state of nature:

$$RS_t = \Gamma_0 \left( \frac{C_t}{C_t^*} \right)^{\sigma_C}$$

where $\Gamma_0$ is a constant, depending on initial conditions. Log-linearizing around a given steady state, I get:

$$\hat{RS}_t = \sigma_C (\hat{C}_t - \hat{C}_t^*)$$

where the hat "·" is for log-deviation from the steady state value. This condition says that the real exchange rate is proportional to the relative marginal utility of consumption, and hence, given the specification of preferences, to the relative consumption. The drawback is that it is hard to replicate the exchange rate volatility without assuming a sufficiently high level of the coefficient of risk aversion $\sigma_C$ (i.e., a relatively low intertemporal elasticity of substitution). Hence, I weak the risk-sharing condition by assuming that international financial market are incomplete, so that the relation between the real exchange rate and marginal utilities of consumption holds only in expected values. In fact, the combination of the Home and Foreign agent's (log-linearized) first order conditions with respect to the internationally traded bond $B_{F,t}$ gives:\textsuperscript{29}

$$E_t \left[ R S_{t+1} - \hat{R S}_t \right] = E_t \left[ \sigma_C (\hat{C}_{t+1} - \hat{C}_t) - \left( \hat{Z}_{PR,t+1} - \hat{Z}_{PR,t} \right) \right] - E_t \left[ \sigma_C (\hat{C}_{t+1}^* - \hat{C}_t^*) - \left( \hat{Z}_{PR,t+1}^* - \hat{Z}_{PR,t}^* \right) \right] + \delta b_{F,t} - \hat{Z}_{UIP,t}$$

\textsuperscript{27}See Warnock (2003).
\textsuperscript{28}See Chari et al. (2002) and Devereux and Engel (2002).
\textsuperscript{29}The first-order conditions with respect to $B_{F,t}$ in the Foreign country is equal to:

$$C_t (j^*)^{-\sigma_C} = \beta (1 + i_t^*) E_t \left( C_{t+1} (j^*)^{-\sigma_C} \frac{P_t^*}{P_{t+1}^*} \right)$$

Combining the log-linearized versions of the above equation, the correspondent equation holding for the Home agents and the real exchange rate definition I get the equation in the text.
where $Y_{b,F,t} \equiv S_t B_{F,t}/P_t$ (see the Appendix). The assumption of incomplete markets has other two advantages. First, shocks lead to wealth redistribution across countries and hence favour the increase in the persistence of the real exchange rate. Second, under risk sharing, the correlation between real exchange rate and relative consumption is, counterfactually, positive (Backus-Smith puzzle).\(^3\)0 This is not necessarily the case under incomplete markets: since the bond is traded only after a given shock is realized, in the impact period the above equation does not necessarily hold. As a consequence, the correlation between real exchange rate and relative consumption can be, consistently with empirical evidence, negative. I also add preference shocks $Z_{PR}$ and $Z_{PR}^*$, that contribute to weak the link between real exchange rate and relative marginal utility.\(^3\)1 Finally, the assumed financial structure affects the behavior of the nominal and real exchange rates also through the shock to the modified UIP condition. The shock can be justified on the basis of the well known weak empirical evidence in favour of the UIP condition. From a theoretical point of view, the shock can be seen as a shortcut, among the others, for "noise traders" having biased expectations on the exchange rate, or of "information shocks" affect the risk premia required by foreign-exchange markets.\(^3\)2 This shock contributes to create the disconnection between the real exchange rate and the fundamentals.

3.4. The empirical analysis

The analysis is based on the estimation of three models. I compare the complete model - whose setup has been described in previous sections - with two downsized versions: one without distribution services (international price discrimination is due only to LCP); the other in which the PCP holds and pass-through is complete. The Home country is the euro area; the Foreign country is the U.S. economy. I report the posterior estimates of the parameters and the overall fit of the alternative model specifications, initially. Then, on the basis of the results of the estimation, I analyze the capability of the models to reproduce the main stylized facts of the real exchange rate and investigate its main sources of fluctuations.

3.4.1. Model solution

Since a closed form solution is not possible, the behavior of the economy is studied by looking at a log-linear approximation to the model equations in the neighborhood of a deterministic steady state. In this steady state the shocks are set to their mean values, price inflation, wage inflation and exchange rate depreciation are set to zero, interest rates are equal to the agents' discount factor, consumption is equalized across countries,

\(^{30}\)See Backus and Smith (1993).

\(^{31}\)For a similar use of preference shocks, see Stockman and Tesar (1995).

\(^{32}\)See Devereux and Engel (2002) and Duarte and Stockman (2005).
the trade balance and the net foreign asset position are zero. Given the presence of distribution costs, price of nontradable goods is different from that of traded goods; however, prices are symmetric between countries and the real exchange rate is one. The elasticities of substitution between tradable brands ($\theta_T$) and between nontradables ($\theta_N$) are calibrated so that the steady state mark-ups are equal across sectors. More details can be found in the Appendix.

3.4.2. The Bayesian estimation

The estimation procedure consists of various steps: the transformation of the data into a form suitable for the computation of the likelihood function using the stationary state-space representation of the model; the choice of appropriate prior distributions; the estimation of the posterior distribution with Monte Carlo methods. These steps are discussed in turn in this section.

The Bayesian approach starts from the assertion that both the data $Y$ and of the parameters $\Theta$ are random variables. From their joint probability distribution $P(Y, \Theta)$ one can derive the fundamental relationship between their marginal and conditional distributions known as Bayes theorem:

$$P(\Theta|Y) \propto P(Y|\Theta) \ast P(\Theta)$$

Reinterpreting these distributions, the Bayesian approach reduces to a procedure for combining the a priori information I have on the model, as summarized in the prior distributions for the parameters $P(\Theta)$, with the information that comes from the data, as summarized in the likelihood function for the observed time series $P(Y|\Theta)$. The resulting posterior density of the parameters $P(\Theta|Y)$ can then be used to draw statistical inference either on the parameters themselves or on any function of them or of the original data.

The computation of the posterior distribution of the estimated parameters cannot be done analytically and thus I resort to Monte Carlo simulations in order to obtain a sample of draws from this distribution that can be used to compute all moments and quantities of interest. I use a Metropolis-Hastings algorithm to explore the parameter space starting from a neighborhood of the posterior mode (found by maximizing the kernel of the posterior using a numerical routine) and then moving around using a random walk “jump distribution” whose covariance matrix is chosen so as to achieve an efficient exploration of the posterior. The algorithm defines a Markov Chain which eventually generates draws coming from the posterior distribution, although the sequence of

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33See An and Schorfheide (2005) for a review of Bayesian methods for estimation of DSGE models.
draws will be correlated; keeping one every n-th draws results in a sub-sample of almost uncorrelated draws which can be used to approximate the posterior distribution.34

3.4.2.1. The data. Estimations is based on nine quarterly key macroeconomic variables sampled over the period 1983:1-2005:2: real consumption, CPI inflation, nontradable inflation and nominal short-term interest rates for both countries (the euro area and the U.S.) and the euro—dollar real exchange rate. Figure 1 shows all the used data. The model has implications for the log deviations from steady state of all these variables, and thus I transformed them before estimating the model. All series are demeaned and real consumption is detrended by fitting a linear trend to the original series. Seasonality has been removed from those series that were available only in unadjusted form regressing them on a set of seasonal dummies. The euro area is the Home country.

3.4.2.2. Prior distributions and calibrated parameters. A very small number of parameters are calibrated. The discount factor $\beta$ is calibrated at 0.99, implying an annual steady state real interest of 4%; the elasticity of substitution between nontradable varieties, $\theta_N$, is set equal to 6, while the elasticity of substitution between tradable varieties, $\theta_T$, is endogenously determined so that $\theta_T = \theta_N (1 + \eta)$, which assures that markups are equal across sectors; the parameter of labour disutility, $\tau$, is set equal to 2; the steady state elasticity of substitution between labour varieties, $\theta_L$, to 4.3.

For all the other I have set the means and variances of the prior distributions (see Table 1). The choice of the priors functional forms is rather standard. I assume all distributions to be a priori independent. The share of tradables in the consumption basket, $a_T$, is set equal to 0.45 (standard deviation 0.1).35 The share of the Home produced goods in the Home tradable composite good, $a_H$, is set equal to 0.8 (standard deviation 0.1). The mean of intratemporal elasticity of substitution between Home and Foreign tradable goods, $\rho$, is set equal to 1.14, while the mean of intratemporal elasticity of substitution between tradable and non tradable goods $\phi$ is equal to 0.74. The elasticity of marginal utility with respect to consumption $\sigma_G$ has a mean value equal to 2 (standard deviation 0.2).

The priors on the coefficients in the monetary policy reaction functions are standard: the persistence coefficient mean is set to 0.8 (standard deviation equal to 0.1); a relatively high prior mean on the inflation coefficient (1.5, with standard deviation equal to 0.1) helps to guarantee a unique solution path when solving the model.


35Stockman and Tesar (1995) suggest that the share of nontradables in the consumption basket of the seven largest OECD countries is roughly 50 percent.
Parameters measuring the degree of price stickiness have the same mean value, equal to 5.6, with standard deviation equal to 10. Parameters measuring the degree of wage stickiness have a mean value equal to 63, with standard deviation equal to 40.

The autoregressive parameters of the shocks follow a beta distribution with mean values set to 0.9. The standard errors have a common value, equal to 0.05. The variances of all shocks have non informative distributions.

3.4.2.3. Estimates. Tables 2-5 report the results. Tables 2-4 reports the posterior median, the 2.5th and 97.5th percentile, the mean and the standard deviation of the estimated parameters. Table 5 reports a comparison of the median estimates.

Nominal rigidities.

Sticky prices and wages are crucial for matching the volatility and persistence of the real exchange rate and the disconnect. Table 6 reports the frequency of adjustment obtained from the estimated parameters of prices and wage adjustment costs.

In each country nontradable goods are stickier than tradable goods. The ranking is common to the three estimated models. It is consistent with the evidence for the euro area: according to Fabiani et al. (2005), in the euro area prices are stickier in the services sector (the frequency of price changes per year for the majority of the firms is not greater than one) than in the trade sector (the frequency for the majority of the firms is between one and three). The ranking is also consistent with the evidence for the U.S. economy. According to Bils and Klenow (2002), in fact, in the U.S. price changes are more frequent for goods than for services.

The degree of nominal rigidity for the import prices is crucial in our models. In fact, it is one of the determinants of the incompleteness of the nominal exchange rate pass-through and, hence, of the behavior of the real exchange replicated by the estimated models. There are some differences across the estimates (for example, in the LCP model the U.S. import prices are more flexible than in the complete model). However, the overall results suggest that import prices adjust once a quarter and a half or two-quarters. These values are lower than those usually found in the literature. Rabanal and Tuesta (2006) estimate a LCP model using U.S. and euro area data. They find an average price duration equal to one year for the euro area and five quarters for the U.S. economy. Lubik and Schorfheide (2005) estimate an average price duration of U.S. import prices equal to two quarters and of the euro area import prices between three and four quarters. Batini et al. (2005) find a degree of price stickiness for the euro area and the U.S. import prices greater than 20 quarters.

The estimates of nominal rigidities for import prices I obtain are hence lower than those commonly found in the literature. The reason can be that the other authors do not have in their models nontradable goods, nor distribution services nor, with the exception of Batini, staggered wages. All the stickiness that is in the data on prices used for estimation is hence reproduced by increasing the import price stickiness to
values that are puzzling, at least according to conventional wisdom on the degree of competition in the import price sector. The lack of nontradable goods probably cause a serious problem of model mis-specification, with the implication that the estimation may severely distort the importance of nominal frictions for import prices.

Wages are stickier than prices. The ranking is in line with the estimates of Altig et al. (2005). Their results suggest that in the U.S. households re-optimize wages on average once every 3.6 quarters, while firms change price on average once every 1.5 quarters. Our estimates indicate that in the U.S. wages are stickier than in the euro area. This result is in line with the estimates of Batini et al. (2005) and Smets and Weulers (2004). Batini finds that the average duration of euro area wage contracts is 5.8 quarters, while that of the U.S. contracts is 9 quarters. Smets and Weulers find that the degree of wage rigidity is three quarters for the euro area, five quarters for the U.S. economy.

On the basis of the obtained estimates, I compute the exchange rate pass-through into import prices. In Table 7 I report the values of the coefficients (see equation 3.32). In the complete model the pass-through at the border is 42 per cent for euro area and 10 per cent for the U.S. economy; the presence of distribution costs halves the pass-through at the consumer level in both countries. According to the LCP model, pass-through at the consumer level is equal to 32 per cent in the euro area and to 44 per cent in the U.S. economy.

Elasticities of substitution and shares

Corsetti et al. (2006) show that the combination of incomplete markets, high home bias and low elasticity of substitution between tradable goods, the latter obtained also thanks to the presence of distribution services intensive in local nontradable goods, increases the capability of a model to reproduce the main stylized facts of the real exchange rate. This approach, also shared by Backus et al. (1995), is opposite to that of Chari et al. (2002), which relies on a low value of the intertemporal elasticity of substitution.

The results I obtain do not seem to support the low intertemporal elasticity approach (see Table 5). The estimate of the inverse of intertemporal elasticity of substitution, $\sigma_C$, is around 2.2, while Chari et al. (2002) set it equal to 5. Our result is not different from that of Smets and Weulers (2004), whose estimate of the inverse of the elasticity of intertemporal substitution is around 1.5 and 2. Lubik and Schorfheide (2005) have an estimate that is between mine and the value of Chari et al. (2002), equal to 3.8.

To some extent, instead, the estimates are in line with the approach based on the low intratemporal elasticity. The estimate of the intratemporal elasticity of substitution between Home and Foreign tradable goods, $\rho$, is equal to 1.2. Rabanal and Tuesta (2005) and Lubik and Schorfheide (2005) find a value, around 0.5, which is lower than the one I find. However, note that the model with distribution services is the most
preferred by the data (see later). In this model, the distribution services introduce a wedge between the elasticity of substitution of tradable goods at producer level and at the consumer level, with the first lower than the second. 95% of the probability mass for the parameter \( \eta \) lies between 0.92 and 1.27 (see Table 2), with a median value of 1.08.\(^{36}\) Given these estimates, the producer price elasticity of tradables, \( \rho \left(1 - \frac{\eta}{\Pi_{P}}\right) \), is more or less equal to 0.58.

Data are also informative about the degree of substitutability between tradable and nontradable goods. \( \phi \) is pushed below the prior mean (1.2) to 0.91 in the complete model and 0.75 in the LCP and PCP models. This result is in line with the 0.74 estimated by Mendoza (1991) for a sample of industrialized countries.

The estimates of home bias are high, and range from 0.8 (complete model) to 0.97 (PCP model). The high estimate is expected, given that, as said before, the volatility of the real exchange rate is higher for higher values of the home bias. The extremely high value in the case of the PCP model could be interpreted as a signal of mis-specification: the high home bias substitutes exchange rate pass-through incompleteness as feature for increasing the volatility of the real exchange rate without augmenting that of the fundamentals. Many previous attempts to estimate or calibrate this parameter ended up in the same ballpark of our numbers: both Rabanal and Tuesta (2006) and Lubik and Schorfheide (2005) find a value of 0.87, while Chari et al. (2002) choose 0.984 in their calibration.

The share of nontradable goods has a non-negligible median weight, \((1 - \alpha_T)\), that ranges from 0.37 (PCP model) to 0.46 (LCP model).

Monetary policy rules.

The ability of the model to reproduce the persistence of the real exchange rate and of other variables hinges, among other things, on the interplay between the degree of monetary policy inertia, the degree of nominal rigidities and the persistence provided exogenously by the shocks. In both countries the parameter regulating nominal interest rate inertia is pushed up by the data. The high persistence of the monetary policy is transmitted to the real exchange rate through the uncovered interest parity. Instead, data are not informative on the response of U.S. monetary policy to inflation and output.

Exogenous shocks.

The estimated structural shocks are rather persistent. The standard deviations are in line with the findings of Rabanal and Tuesta (2005) and Lubik and Schorfeide (2005). Since the importance of shocks cannot directly be assessed from the magnitude of the

\(^{36}\)Corsetti et al. (2006), following Burstein et al. (2003), set \( \eta \) equal to 1.22, to match the share of the retail price of traded goods accounted for by local distribution services in the U.S. (approximately equal to 50%).
associated standard deviation, I decompose the variance of exchange rate fluctuations (see below).

3.4.2.4. Goodness-of-fit. I compare the three estimated model in terms of their fitting of the data.\textsuperscript{37} Table 8 reports the results. The complete model has the highest Bayes factor, followed by the PCP model and LCP model. Rabanal and Tuesta (2006) also find evidence in favour of the PCP model over the LCP model, while Lubik and Schorfeide (2005) find the opposite result.

3.4.3. What determines the real exchange rate dynamics?

In this section I investigate the implications of the estimated models for the dynamics of the real exchange rate. The issue is to understand if and under which conditions the empirical models I have are able to replicate the dynamics of the real exchange rate, in particular its high volatility, persistence and the disconnection from the other variables. I initially investigate which - among the home bias, the international price discrimination and the internal exchange rate - is the main economic determinant of the variance of the real exchange rate. Subsequently, I perform a forecast error variance decomposition, an analysis of the impulse responses and of the moments.

3.4.3.1. The role of international price discrimination. To quantify the relevance of the price discrimination for the real exchange rate variance, I compute equation (3.46) using the obtained median estimates.\textsuperscript{38} The main result, reported in Table 9, is that the variance term attributable to the international price discrimination explains around 56 per cent of the whole real exchange rate variance in the complete model, 25 per cent in the LCP model. Home bias is also not negligible, given that it weighs 7.5 per cent in the complete model, 36 per cent in the LCP model. The covariance between home bias and international price discrimination terms plays a significant role in the first two models. The contribution of the internal real exchange rate is small, as shown by the variance and covariance terms in which they are involved. These results

\textsuperscript{37}In Bayesian analysis the comparison among alternative models is typically carried out in terms of posterior odds ratio and we follow this approach. Let’s assume that we are given two models $M_1$ and $M_2$ and that we assign prior probabilities $\pi_{1,0}$ and $\pi_{2,0}$ to the each of them, then the posterior odds ratio is defined as the product of the prior odds and the Bayes factor:

$$\frac{\pi_{1,0}}{\pi_{2,0}} \times \frac{p(Y^T/M_1)}{p(Y^T/M_2)} = \text{Bayes factor}$$

where $p(Y^T/M_i)$ is the marginal data density, obtained integrating the posterior kernel over all possible parameters values. We assume the models are a priori equally weighed and adopt Geweke’s (1999) harmonic mean approximation to evaluate $p(Y^T/M_i)$.

\textsuperscript{38}Variances and covariances, divided by the variance of the real exchange rate changes, are obtained simulating the models as in the section ‘Analysis of the moments’ (see later).
are in line with Chari et al. (2002). In our case, however, the importance of nontradable goods cannot be entirely dismissed, given that nontradable goods are a source of international price discrimination through the distribution sector.

3.4.3.2. Forecast error variance decomposition. To quantify the contribution of each shock to the variance of the real exchange rate, I compute the asymptotic forecast error variance decomposition of the three models. Tables 10-12 report the results. The UIP shock accounts for slightly more than three quarters of the real exchange rate variance. The remaining part is explained by preference shocks (around 20 per cent). Technology and interest rate shocks are not relevant for the fluctuations of the real exchange rate. To the contrary, consumption and nominal interest rate fluctuations are mainly driven by domestic preference shocks, while inflation rates by domestic technology shocks (in particular by the domestic tradable technology shock). Wage shocks are not important, while monetary policy shocks are, to some extent, only relevant for their country interest rate fluctuations. They are not relevant for the fluctuations of the real exchange rate, a result also found by Rabanal and Tuesta (2006).

3.4.3.3. Impulse Response Analysis. I show the qualitative implications of the main shocks determining the real exchange rate fluctuations by computing some impulse responses. I use the complete model, given that it has the better fit. Figure 2 reports the responses of the main variables of the model to a positive shock to the UIP. The size of the shock is one standard deviation. I report the mean response, the 5th and the 95 percentiles.

Following the shock, there is a depreciation of the nominal exchange rate of the Home country (the euro area). Given the price stickiness, the correlation between nominal and real exchange rate is high. Hence, there is also a depreciation of the real exchange rate. The Home terms of trade at the border slightly improve on impact, due to the incompleteness pass-through and local currency pricing. At the consumer level, on impact the terms of trade deteriorate: the increase in the price of the Home distribution services, intensive in local nontradables, induce an increase in the relative price in the Home consumer price of the Foreign good. The Home output, relative to the Foreign one, increases, while the Home relative consumption decreases (the Home monetary authority increases the interest rate as the consumer price inflation increases). Home

\[ T_t = \frac{P_{F,t}}{S_t P_{H,t}} \]

At the consumer level, as:

\[ T_t = \frac{P_{F,t}}{S_t P_{H,t}} \]
agents accumulate assets against the Foreign country, and the Home trade balance improves.

Figure 3 considers a shock to the Home preferences. Home agents increase consumption relative to Foreign agents. The nominal and real exchange rate appreciate. The reason is that the Home monetary authority increases the interest rate as the Home consumer price inflation increases. The Home terms of trade improve both at the border and at the consumer level. Home output also increases relatively to Foreign output, given the higher demand. The net foreign asset position and the trade balance of the Home country deteriorate.

Finally, Figure 4 considers a shock to the technology of the Home tradable sector. The Home nominal and real exchange rate depreciate, given that the Home monetary authority reduces the Home interest rate. The terms of trade deteriorate both at the border and at consumer level. Home consumption and output increase relative to Foreign correspondent quantities. As a result, there is an impact a slight improvement of the Home trade balance and net foreign asset position. These results do not support the idea of a strong wealth effect of technology shock (similar results are obtained when a shock to the U.S. tradable sector is considered). There is not an appreciation of the real exchange rate nor an improvement of the terms of trade after a technology shock to the Home tradable sector. However, if all the results I get are considered, the wealth effect à la Corsetti et al. (2004) cannot be entirely dismissed. First, the model with the distribution services, and hence with the lower intratemporal elasticity of substitution between tradable goods, has the best fitting. Second, all the estimates suggest that the degree of home bias is relatively high. The two elements are crucial for a positive and strong wealth effect of the technology shock to the tradable sector. Finally, I do not have investment in physical capital in the models I estimate. Capital accumulation is also an important element for producing an appreciation of the real exchange rate (conditional to a shock to the technology of the tradable sector). I leave this issue for future research.

3.4.3.4. Analysis of the moments. Tables 13-15 report the results of the analysis of moments. I compute the moments drawing the parameters of the model from the posterior distribution. I performs 100 draws. For each draw, I simulate the model 20 times. The length of each simulated time series is 89, the length of the data used in the estimation. For each moment, I compute the 2.5th and 97.5th percentile. The estimated models are able to replicate the main stylized facts of the real exchange. The relatively high volatility of the real exchange rate is always almost matched as well as its persistence. The models are also able to replicate the volatility and persistence of the other variables. Hence, they are able to replicate the exchange rate disconnect. There are some troubles in replicating the cross correlations of the observable variables. This is expected, given that the shocks are by assumption not cross-correlated. However,
the models are able to replicate the negative correlation between the real exchange rate and the relative consumption (the Backus-Smith puzzle).

Given the results of the variance decomposition analysis, it is interesting to analyze how the moments change when some shocks are switched off. I perform the exercises using the complete model, given that its goodness-of-fit of the data is better than that of the other two models.

Table 16 reports the results obtained when the UIP shock, the main determinant of the real exchange rate fluctuations, is absent. Consistently with the result of the variance decomposition, the absence of the UIP shock strongly limits the capability of the model to reproduce the volatility of real exchange rate. In fact, the simulated volatility is now much lower than the actual one and closer to the volatility of the other variables.

Table 16 also reports the results under the assumption that there is no UIP shock nor preference shocks. According to the forecast error variance decomposition, almost all the variance of the real exchange rate is explained by these three shocks. The simulated variance of the real exchange rate further decreases without them. The variances of the both consumption variables also decrease, given that the preference shock are their main determinants. The lack of consumption preferences has also another implication. In Table 17 show the cross-correlation between relative consumption and the real exchange rate. In absence of preference shocks, the capability of the model to replicate the negative sign of the cross-correlation decreases.

A clear message appear from all the reported results. Incomplete pass-through and the shocks to the UIP and preferences are essential ingredients to make empirical open economy models consistent with the data. The lack of at least one of these two features induces a worsening of the capability of the model to reproduce the real exchange rate dynamics.

3.5. Concluding remarks

I have estimated three two-country NOEM models using euro area and U.S. data to replicate the high volatility and persistence of the real exchange rate. The models differ for assumption on degree of exchange rate pass-through into import prices. I find that the model that better fits the data is the one based on the local currency pricing and distribution services. The relevance of the UIP shock for the real exchange rate fluctuations is confirmed by the forecast error variance decomposition. Hence, incomplete pass-through and UIP shock can be thought as crucial features to reproduce the high exchange rate volatility without any implications for the volatility of other macroeconomic aggregates.
The empirical relevance of the two estimated features suggests that the switching effect of changes in the nominal exchange rate is relatively low for consumer prices. However, the size of the effect can be higher at the border.

These results stimulate further work. In this paper, I have focused on the role of pass-through for the real exchange rate volatility and persistence. However, the capability of a model to replicate the persistence of real exchange rate could be improved by introducing physical capital. This feature should also improve the matching of the negative correlation between relative consumption and real exchange rate (the Backus Smith puzzle). More general preferences could be introduced to increase persistence. For example, translog preferences, or habit in consumption and in labor.

Finally, the empirical estimates could be used as a starting point for a microfounded two-country welfare analysis. Incomplete pass-through, in fact, modify the relative strength of substitution and wealth effect of a given change in the nominal exchange rate. The spillovers and the related welfare-improving policy measures could be not obvious.
References


Appendix

I firstly report the steady state of the model. Later, I illustrate the dynamic equilibrium.

The Steady state equilibrium

I assume a symmetric steady state equilibrium, in which nominal wages are equal cross countries and where markups are equal cross country and cross sectors. Prices and wages are flexible, there is no inflation nor nominal exchange rate depreciation, the net foreign asset position is zero. In each country the consumer price index is the numeraire ($P = P^* = 1$). The real exchange rate is one. There are no shocks.

In what follows, I report the equations of the complete model. The steady states of LCP and of the PCP model can be obtained by setting $\eta = 0$.

In the Home country, the steady state version of the optimal pricing conditions (3.29) and (3.30) imply (wage and all prices are expressed as a ratio to the consumer price index):

\[(SS1)\]

$$p_N = \frac{\theta_N}{\theta_N - 1}w$$

\[(SS2)\]

$$\bar{p}_H = \frac{\theta_T}{\theta_T - 1}w + \frac{\eta}{\theta_T - 1}p_N$$

$$= \left(\frac{\theta_T}{\theta_T - 1} + \frac{\eta}{\theta_T - 1} \frac{\theta_N}{\theta_N - 1}\right)w$$

The equality between sectoral markups implies that:

$$\frac{\theta_N}{\theta_N - 1} = \frac{\theta_T}{\theta_T - 1} + \frac{\eta}{\theta_T - 1} \frac{\theta_N}{\theta_N - 1}$$

After some algebra, the following markup equality condition can be obtained:

$$\theta_T = \theta_N (1 + \eta)$$

The common wage and markup imply that:

$$\bar{p}_H = p_N$$

Give that $p_H = \bar{p}_H + \eta p_N$, the following equation is obtained:

$$p_H = (1 + \eta)p_N$$

Given the assumption of cross-country symmetry, $\bar{p}_H = \bar{p}_F$, $p_N = p^*_N$ and hence, using the price index of the tradable goods, equation (3.13), $p_H = p_F = p_T$. Using the consumer price index (3.12) I get:

$$1 = [a_T p_N^{1-\phi} (1 + \eta)^{1-\phi} + (1 - a_T) p_N^{1-\phi}]$$
which can be rewritten as:

\[ p_N = \left[ a_T (1 + \eta)^{1-\phi} + (1 - a_T) \right]^{-\frac{\phi}{1-\phi}} \]

Hence the relative price of the nontradable good is function of the structural parameters of the model. Using equations (SS1) and (SS2) the real wage and the relative price of the tradable goods can be found.

Given the relative prices, the relative amounts of output can be found. The market clearing conditions (3.37) and (3.39) and the demand equations (3.9) and (3.10) imply that:

\[ Y_T = a_H a_T \left( \frac{P_H}{P_T} \right)^{-\rho} p_T^{-\phi} C \\
+ (1 - a_H) a_T \left( \frac{P_H}{P_T} \right)^{-\rho} p_T^{-\phi} C \\
= a_T \left( \frac{P_H}{P_T} \right)^{-\rho} p_T^{-\phi} C = a_T p_T^{-\phi} C \]

\[ Y_N = (1 - a_T) (p_N)^{-\phi} C + \eta a_T p_T^{-\phi} C \]

Hence:

\[ \frac{Y_H}{Y_T + Y_N} = \frac{a_T p_T^{-\phi} C}{a_T p_T^{-\phi} C + (1 - a_T) (p_N)^{-\phi} C + \eta a_T p_T^{-\phi} C} \]

\[ = \frac{a_T p_T^{-\phi}}{a_T p_T^{-\phi} + (1 - a_T) (p_N)^{-\phi} + \eta a_T p_T^{-\phi}} \]

and:

\[ \frac{Y_N}{Y_T + Y_N} = \frac{(1 - a_T) (p_N)^{-\phi} C + \eta a_T p_T^{-\phi} C}{a_T p_T^{-\phi} C + (1 - a_T) (p_N)^{-\phi} C + \eta a_T p_T^{-\phi} C} \]

\[ = \frac{(1 - a_T) (p_N)^{-\phi}}{a_T p_T^{-\phi} + (1 - a_T) (p_N)^{-\phi} + \eta a_T p_T^{-\phi}} \]

I define total output in the Home country as:

\[ Y = Y_T + Y_N \]

Given the assumption of linearity in labor for the production function, we also have:

\[ Y_T = L_T, \quad Y_N = L_N \]

The total amount of labor can be written as:

\[ 1 = \frac{L_T}{L} + \frac{L_N}{L} \]
where:

\[ \frac{L_T}{L} = \frac{Y_T}{Y}, \quad \frac{L_N}{L} = \frac{Y_N}{Y} \]

Using the Euler equations, in steady state the following condition holds:

\[ \frac{1}{1 + \hat{\iota}} = \frac{1}{1 + \hat{\iota}^*} = \beta \]

From equation (3.44), using the assumption of zero net foreign asset position, we derive the steady state consumption (as a ratio of \(Y_T + Y_N\)):

\[ \frac{C}{Y} = \bar{P}_H \frac{Y_H}{Y} + p_N \frac{Y_N}{Y} \]

Given the assumption of country symmetry, Foreign relative prices and quantities are equal to their Home counterparts.
The dynamic equilibrium

1. Price indexes

(PR1) \[ \hat{p}_{H,t} = \tilde{p}_{H,t} + \hat{\pi}_{H,t} - \hat{\pi}_t \]

(PR2) \[ \hat{p}_{F,t} = \tilde{p}_{F,t} + \hat{\pi}_{F,t} - \hat{\pi}_t \]

(PR3) \[ \hat{p}_{H,t}^{\ast} = \tilde{p}_{H,t}^{\ast} + \hat{\pi}_{H,t}^{\ast} - \hat{\pi}_t^{\ast} \]

(PR4) \[ \hat{p}_{F,t}^{\ast} = \tilde{p}_{F,t}^{\ast} + \hat{\pi}_{F,t}^{\ast} - \hat{\pi}_t^{\ast} \]

(PR5) \[ \hat{p}_{N,t} = \tilde{p}_{N,t} + \hat{\pi}_{N,t} - \hat{\pi}_t \]

(PR6) \[ \hat{p}_{N,t}^{\ast} = \tilde{p}_{N,t}^{\ast} + \hat{\pi}_{N,t}^{\ast} - \hat{\pi}_t^{\ast} \]

(PR7) \[ \hat{p}_{H,t} = \frac{\tilde{p}_{H,t}}{p_H} + \frac{\eta}{p_H} \tilde{p}_{N,t} \]

(PR8) \[ \hat{p}_{H,t}^{\ast} = \frac{\tilde{p}_{H,t}^{\ast}}{p_H} + \frac{\eta}{p_H} \tilde{p}_{N,t}^{\ast} \]

(PR9) \[ \hat{p}_{F,t} = \frac{\tilde{p}_{F,t}}{p_H} + \frac{\eta}{p_H} \tilde{p}_{N,t} \]

(PR10) \[ \hat{p}_{F,t}^{\ast} = \frac{\tilde{p}_{F,t}^{\ast}}{p_H} + \frac{\eta}{p_H} \tilde{p}_{N,t}^{\ast} \]

(PR11) \[ \hat{p}_{T,t} = a_h \tilde{p}_{H,t} + (1 - a_h) \tilde{p}_{F,t} \]

(PR12) \[ \hat{p}_{T,t}^{\ast} = a_h \tilde{p}_{H,t}^{\ast} + (1 - a_h) \tilde{p}_{F,t}^{\ast} \]

(PR13) \[ 0 = a_T \tilde{p}_T^{1 - \rho} \tilde{p}_{T,t} + (1 - a_T) \tilde{p}_N^{1 - \rho} \tilde{p}_{N,t} \]

(PR14) \[ 0 = a_T \tilde{p}_T^{1 - \rho} \tilde{p}_{T,t}^{\ast} + (1 - a_T) \tilde{p}_N^{1 - \rho} \tilde{p}_{N,t}^{\ast} \]

(PR15) \[ \tilde{w}_t = \tilde{w}_{t-1} + \hat{\pi}_{W,t} - \hat{\pi}_t \]

(PR16) \[ \tilde{w}_t^{\ast} = \tilde{w}_{t-1}^{\ast} + \hat{\pi}_{W,t}^{\ast} - \hat{\pi}_t^{\ast} \]
2. Inflation Rates

\( \hat{\pi}_{H,t} = \frac{\bar{p}_H}{\bar{p}_H} \hat{\pi}_{H,t} + \eta \frac{p_N}{\bar{p}_H} \hat{\pi}_{N,t} \) (INF1)

\( \hat{\pi}_{F,t} = \frac{\bar{p}_H}{\bar{p}_H} \hat{\pi}_{F,t} + \eta \frac{p_N}{\bar{p}_H} \hat{\pi}_{N,t} \) (INF2)

\( \hat{\pi}^{\ast}_{H,t} = \frac{\bar{p}_H}{\bar{p}_H} \hat{\pi}^{\ast}_{H,t} + \eta \frac{p_N}{\bar{p}_H} \hat{\pi}^{\ast}_{N,t} \) (INF3)

\( \hat{\pi}^{\ast}_{F,t} = \frac{\bar{p}_H}{\bar{p}_H} \hat{\pi}^{\ast}_{F,t} + \eta \frac{p_N}{\bar{p}_H} \hat{\pi}^{\ast}_{N,t} \) (INF4)

\( \hat{\pi}_{T,t} = a_H \hat{\pi}_{H,t} + (1 - a_H) \hat{\pi}_{F,t} \) (INF5)

\( \hat{\pi}^{\ast}_{T,t} = (1 - a_H) \hat{\pi}^{\ast}_{H,t} + a_H \hat{\pi}^{\ast}_{F,t} \) (INF6)

\( \hat{\pi}_{t} = a_H \hat{\pi}_{T,t} + (1 - a_H) \hat{\pi}_{N,t} \) (INF7)

\( \hat{\pi}^{\ast}_{t} = a_T \hat{\pi}^{\ast}_{T,t} + (1 - a_T) \hat{\pi}^{\ast}_{N,t} \) (INF8)

3. Aggregate supply

\( \dot{Y}_{T,t} = \dot{Z}_{T,t} + \dot{L}_{T,t} \) (AS1)

\( \dot{Y}^{\ast}_{T,t} = \dot{Z}^{\ast}_{T,t} + \dot{L}^{\ast}_{T,t} \) (AS2)

\( \dot{Y}_{N,t} = \dot{Z}_{N,t} + \dot{L}_{N,t} \) (AS3)

\( \dot{Y}^{\ast}_{N,t} = \dot{Z}^{\ast}_{N,t} + \dot{L}^{\ast}_{N,t} \) (AS4)

\( \hat{\pi}_{H,t} = \beta \hat{\pi}_{H,t+1} - \frac{\theta_T \bar{p}_H (\eta p_N + w)}{k_H (\bar{p}_H + \eta p_N)^2} \dot{P}_{H,t} + \frac{\theta_T \eta p_N (\bar{p}_H - w)}{k_H (\bar{p}_H + \eta p_N)^2} \dot{P}_{N,t} + \frac{\theta_T w}{k_H (\bar{p}_H + \eta p_N)} \dot{\omega}_t \) (AS5)
(AS6) \[ \hat{\pi}_{F,t} = \beta \hat{\pi}_{F,t+1} - \frac{\theta_T \bar{p}_H (\eta_P + w)}{k_F (\bar{p}_H + \eta_P)} \hat{P}_{F,t} + \frac{\theta_T \eta_P (\bar{p}_H - w)}{k_F (\bar{p}_H + \eta_P)} \hat{P}_{N,t} + \frac{\theta_T w}{k_F (\bar{p}_H + \eta_P)} (\hat{w}_t + \hat{R}_S_t) \]

(AS7) \[ \hat{\pi}_{H,t} = \beta \hat{\pi}_{H,t+1} - \frac{\theta_T \bar{p}_H (\eta_P + w)}{k_H (\bar{p}_H + \eta_P)} \hat{P}_{H,t} + \frac{\theta_T \eta_P (\bar{p}_H - w)}{k_H (\bar{p}_H + \eta_P)} \hat{P}_{N,t} + \frac{\theta_T w}{k_H (\bar{p}_H + \eta_P)} (\hat{w}_t - \hat{R}_S_t) \]

(AS8) \[ \hat{\pi}_{F,t}^* = \beta \hat{\pi}_{F,t+1} - \frac{\theta_T \bar{p}_H (\eta_P + w)}{k_F^* (\bar{p}_H + \eta_P)} \hat{P}_{F,t} + \frac{\theta_T \eta_P (\bar{p}_H - w)}{k_F^* (\bar{p}_H + \eta_P)} \hat{P}_{N,t} + \frac{\theta_T w}{k_F^* (\bar{p}_H + \eta_P)} \hat{w}_t^* \]

(AS9) \[ \hat{\pi}_{N,t} = \beta \hat{\pi}_{N,t+1} - \frac{\theta_P - 1}{k_N} \hat{P}_{N,t} + \frac{\theta_P - 1}{k_N} \hat{w}_t \]

(AS10) \[ \hat{\pi}_{N,t}^* = \beta \hat{\pi}_{N,t+1} - \frac{\theta_P - 1}{k_N^*} \hat{P}_{N,t} + \frac{\theta_P - 1}{k_N^*} \hat{w}_t^* \]

(AS11) \[ \hat{\pi}_{W,t} = \beta \hat{\pi}_{W,t+1} + \frac{\sigma_C (\theta_W - 1)}{k_W} \hat{C}_t - \frac{(\theta_W - 1)}{k_W} \hat{w}_t + \frac{(\tau - 1)(\theta_W - 1)}{k_W} \hat{L}_t - \frac{1}{k_W} \hat{\theta}_{L,t} \]

(AS12) \[ \hat{\pi}_{W,t}^* = \beta \hat{\pi}_{W,t+1} + \frac{\sigma_C (\theta_W - 1)}{k_W^*} \hat{C}_t^* - \frac{(\theta_W - 1)}{k_W^*} \hat{w}_t^* + \frac{(\tau - 1)(\theta_W - 1)}{k_W^*} \hat{L}_t^* - \frac{1}{k_W^*} \hat{\theta}_{L,t}^* \]

(AS13) \[ \hat{L}_t = \frac{L_T}{L} \hat{L}_{T,t} + \frac{L_N}{L} \hat{L}_{N,t} \]

(AS14) \[ \hat{L}_t^* = \frac{L_T}{L^*} \hat{L}_{T,t}^* + \frac{L_N^*}{L^*} \hat{L}_{N,t}^* \]

4. Aggregate Demand

(AD1) \[ -\sigma_C \hat{C}_t + \hat{Z}_{PR,t} = -\sigma_C \hat{C}_{t+1} + \hat{Z}_{PR,t+1} + (1 + i_t) - \hat{\pi}_{t+1} \]

(AD2) \[ (1 + i_t) - (1 + i_t^*) + k_0 \hat{b}_{F,t} - \hat{Z}_{UIP,t} = \Delta S_{t+1} \]

(AD3) \[ -\sigma_C \hat{C}_t^* + \hat{Z}_{PR,t}^* = -\sigma_C \hat{C}_{t+1}^* + \hat{Z}_{PR,t+1}^* + (1 + i_t^*) - \hat{\pi}_{t+1}^* \]

(AD4) \[ \hat{Y}_{H,t} = -\rho \hat{p}_{H,t} + (\rho - \phi) \hat{p}_{T,t} + \frac{C}{Y} \hat{C}_t \]

(AD5) \[ \hat{Y}_{H,t}^* = -\rho \hat{p}_{H,t} + (\rho - \phi) \hat{p}_{T,t} + \frac{C}{Y} \hat{C}_t^* \]

(AD6) \[ \hat{Y}_{F,t} = -\rho \hat{p}_{F,t} + (\rho - \phi) \hat{p}_{T,t} + \frac{C}{Y} \hat{C}_t \]
(AD7) \[ Y_{F,t} = -\rho \hat{p}_{F,t} + (\rho - \phi) \hat{p}_{T,t} + \frac{C}{Y} \hat{C}_t \]

(AD8) \[ \frac{Y_N}{Y} \hat{Y}_{N,t} = -\phi \hat{p}_{N,t} + \frac{C}{Y} \hat{C}_t + \eta \frac{Y_H}{Y} \hat{Y}_H + \eta \frac{Y_F}{Y} \hat{Y}_F \]

(AD9) \[ \frac{Y_N}{Y} \hat{Y}_{N,t} = -\phi \hat{p}_{N,t} + \frac{C}{Y} \hat{C}_t + \eta \frac{Y_H}{Y} \hat{Y}_H + \eta \frac{Y_F}{Y} \hat{Y}_F \]

(AD10) \[ \frac{Y_T}{Y} \hat{Y}_T = \frac{Y_H}{Y} \hat{Y}_{H,t} + \frac{Y_F}{Y} \hat{Y}_{F,t} \]

(AD11) \[ \frac{Y_T}{Y} \hat{Y}_{T,t} = \frac{Y_F}{Y} \hat{Y}_{F,t} + \frac{Y_F}{Y} \hat{Y}_{F,t} \]

5. Flow of Funds

(FF1) \[ \beta b_{F,t} - b_{F,t-1} = \beta \frac{Y_H}{Y} \left( \hat{p}_{H,t} + \hat{Y}_{H,t} \right) + \beta \frac{Y_N}{Y} \left( \hat{R}_S + \hat{p}_{H,t} + \hat{Y}_{H,t} \right) + \beta \frac{Y_N}{Y} \left( \hat{p}_{N,t} + \hat{Y}_{N,t} \right) - \hat{C}_t \]

(FF2) \[ t_b = \beta b_{F,t} - b_{F,t-1} \]

6. Monetary policies

\[ (1 + \epsilon t) = \rho_R (1 + \epsilon_{t-1}) + (1 - \rho_R) \rho_y \left( \frac{Y_H}{Y} \hat{Y}_{H,t} + \frac{Y_F}{Y} \hat{Y}_{F,t} + \frac{Y_N}{Y} \hat{Y}_{N,t} \right) \]
\[ + (1 - \rho_R) \rho_{\Delta e} \Delta S_t + Z_{R,t} \]

\[ (1 + \epsilon t^*) = \rho_R^* (1 + \epsilon_{t-1}^*) + (1 - \rho_R^*) \rho_y^* \left( \frac{Y_F^*}{Y} \hat{Y}_{F,t}^* + \frac{Y_F^*}{Y} \hat{Y}_{F,t}^* + \frac{Y_N^*}{Y} \hat{Y}_{N,t}^* \right) \]
\[ - (1 - \rho_R^*) \rho_{\Delta e} \Delta S_t + Z_{R,t}^* \]

7. Shock processes

(SH1) \[ \hat{Z}_{T,t} = \rho_T \hat{Z}_{T,t-1} + \epsilon \hat{Z}_{T,t} \]

(SH2) \[ \hat{Z}_{T,t}^* = \rho_T \hat{Z}_{T,t-1}^* + \epsilon \hat{Z}_{T,t} \]

(SH3) \[ \hat{Z}_{N,t} = \rho_N \hat{Z}_{N,t-1} + \epsilon \hat{Z}_{N,t} \]

(SH4) \[ \hat{Z}_{N,t}^* = \rho_N \hat{Z}_{N,t-1}^* + \epsilon \hat{Z}_{N,t} \]

(SH5) \[ \hat{Z}_{PR} = \rho_{PR} \hat{Z}_{PR,t-1} + \epsilon \hat{Z}_{PR,t} \]

(SH6) \[ \hat{Z}_{PR}^* = \rho_{PR} \hat{Z}_{PR,t-1}^* + \epsilon \hat{Z}_{PR,t} \]
(SH7) \[ \dot{\hat{Z}}_{UIP} = \rho_{UIP} \hat{Z}_{UIP,t-1} + \varepsilon_{UIP,t} \]

(Sh8) \[ \dot{\hat{\theta}}_{L,t} = \varepsilon_{ZL,t} \]

(Sh9) \[ \dot{\hat{\theta}}^{*}_{L,t} = \varepsilon_{ZL,t} \]

(Sh10) \[ \dot{\hat{Z}}_{R,t} = \varepsilon_{ZR,t} \]

(Sh11) \[ \dot{Z}^{*}_{R,t} = \varepsilon_{ZR,t} \]

The first block contains the price indexes. The first two equations are respectively the laws of motion of the Home and Foreign good producer price in the Home country. The third and fourth equations are the correspondent equations in the Foreign country. The fifth and the sixth equations are the laws of motion of the nontradable goods. Equations (PR7)-(PR10) are the consumer prices of the Home and Foreign good in the Home and Foreign country. Equations (PR11) and (PR12) are the consumer prices of the composite tradable good in the Home and in the Foreign country, respectively. Equations (PR13) and (PR14) define the numeraire (the consumption unit). Equations (PR15) and (PR16) are the laws of motion of the Home real wage and its Foreign counterpart. The last three equations are respectively the laws of motion of the Home country real exchange rate, terms of trade at the border and at the consumer level.

The second block contains the inflation rates. The first two equations are those of the Home and Foreign good in the Home country at the consumer level, respectively. The third and the fourth equations are the correspondent equations in the Foreign country. The fifth equation is the inflation rate of the composite tradable good. The sixth equation is its Foreign analogue. Finally, the last two equations of the block are the Home and Foreign consumer price Index inflation rates.

The third block contains the equations describing the supply-side of the economy. The first four equations are the technology constraints for the Home tradable, Foreign tradable, Home nontradable, Foreign nontradable. Equations (AS5)-(AS10) are the Phillips curves of the Home and Foreign tradable good in the Home country, of the Home and Foreign good in the Foreign country, of the Home and Foreign nontradable goods. Equations (AS11) and (AS12) are the Phillips curves of Home and Foreign labor. The last two equations are the Home and Foreign labor market clearing conditions, respectively.

The fourth block contains the equations of the demand side of the economy. The first equation is the Euler equation of the Home agent. The second equation is the modified uncovered interest parity, holding for the Home agent. The third equation is the Euler equation of the Foreign agent. Equations (AD4) and (AD5) are the market clearing
conditions of the Home tradable good in the Home and Foreign market. Equations (AD6) and (AD7) are the market clearing conditions of the Foreign tradable good in the Home and Foreign markets. Equations (AD8) and (AD9) are the market clearing conditions of the Home and Foreign nontradable goods. The last two equations define the Home and Foreign tradable total outputs, respectively.

In the fifth block there are the net foreign asset position of the Home agent and the Home trade balance. Given that in steady state both the net foreign asset position and the trade balance are zero, I cannot log-linearize the variables $B_p$ and $TB$. Hence, I define: $b_{F,t}Y \equiv B_{F,t}$ and $tb_{t}Y \equiv TB_{t}$.

The sixth block contains the Home and Foreign monetary policy rules.

Finally, the last block contains the equations of the shock processes. The first two equations are respectively the processes of the technology shock in the Home and Foreign tradable sectors. The third and the fourth equations are the processes of the technology shock in the Home and Foreign nontradable sectors, respectively. Equations (SH5) and (SH6) are the process of the Home and Foreign preference shocks. Equation (SH7) is the process of the UIP shock. Finally, the last four equations are the processes of the Home and Foreign markup shocks and of the Home and Foreign monetary policy shocks, respectively.

The LCP model is obtained by the above log-linear equations by setting $\eta = 0$ (hence there is not anymore the distinction between wholesale and consumer prices). The PCP model is obtained by setting $\eta = 0$ in the above system and imposing the law of one price ($p_t(h) = S p_t^*(h)$ and $p_t(f) = p_t^*(f) / S_t$). Equations (AS6) and (AS7) are replaced respectively by:

\[
\hat{\pi}_{F,t} = \Delta S_t + \hat{\pi}_{F,t}^*
\]

\[
\hat{\pi}_{H,t} = \hat{\pi}_{H,t} - \Delta S_t
\]

Equations (PR2) and (PR3) are replaced respectively by:

\[
\hat{p}_{F,t} = \hat{p}_{F,t}^* + \hat{R}S
\]

\[
\hat{p}_{H,t} = \hat{p}_{H,t}^* - \hat{R}S
\]
Table 1. Estimated parameters (prior distribution)

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<th>st. dev</th>
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<td>Gamma</td>
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<td>0.10</td>
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<td>Gamma</td>
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Table 2. Posterior distribution. Complete Model

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Table 3. Posterior distribution. LCP Model

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Table 4. Posterior distribution. PCP Model

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<td>0.001</td>
<td>0.001</td>
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<tr>
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<td>0.225</td>
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Table 5. Posterior median estimates. Models comparison

<table>
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<th>PCP</th>
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<td>-</td>
<td>-</td>
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<tr>
<td>$\alpha_H$</td>
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<td>-0.01</td>
<td>-0.02</td>
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<td>0.87</td>
<td>0.91</td>
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<td>0.93</td>
<td>0.91</td>
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<tr>
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<td>0.95</td>
<td>0.95</td>
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<tr>
<td>$\rho_{ST}$</td>
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<td>0.96</td>
<td>0.96</td>
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<tr>
<td>$\rho_{SN}$</td>
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<td>0.95</td>
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<td>0.02</td>
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<td>0.00</td>
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<td>0.00</td>
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<tr>
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<td>0.01</td>
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<tr>
<td>$\sigma_{SN}$</td>
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<td>0.01</td>
<td>0.01</td>
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<tr>
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<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
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<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
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<tr>
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<td>$\sigma_{Z^*_W}$</td>
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### Table 6. Nominal rigidities in the complete model

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<th>Complete</th>
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<th>PCP</th>
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<td><strong>Euro area</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Import (border)</td>
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<td>1.6</td>
<td></td>
</tr>
<tr>
<td>Domestic tradable (wholesale)</td>
<td>1.6</td>
<td>1.9</td>
<td>1.5</td>
</tr>
<tr>
<td>Nontradable</td>
<td>3.8</td>
<td>4.3</td>
<td>2.6</td>
</tr>
<tr>
<td>Wages</td>
<td>10.0</td>
<td>5.2</td>
<td>11.1</td>
</tr>
<tr>
<td><strong>U.S.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Import (border)</td>
<td>2.0</td>
<td></td>
<td>1.4</td>
</tr>
<tr>
<td>Domestic tradable (wholesale)</td>
<td>1.2</td>
<td>1.6</td>
<td>1.4</td>
</tr>
<tr>
<td>Nontradable</td>
<td>2.9</td>
<td>4.2</td>
<td>1.9</td>
</tr>
<tr>
<td>Wages</td>
<td>11.1</td>
<td>4.8</td>
<td>11.1</td>
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</table>

### Table 7. Nominal exchange rate pass-through into import prices (%)

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<tr>
<th>at the:</th>
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<th>LCP Model</th>
</tr>
</thead>
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<tr>
<td>Border euro area</td>
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<td></td>
</tr>
<tr>
<td>Consumer euro area</td>
<td>20</td>
<td>32</td>
</tr>
<tr>
<td>Border U.S.</td>
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<tr>
<td>Consumer U.S.</td>
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<td>44</td>
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</table>
### Table 8. Overall goodness of fit

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<th>Marginal density</th>
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<td>Complete</td>
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<td>LCP</td>
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<tr>
<td>PCP</td>
<td>2974</td>
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</table>

*Notes:* The marginal density is computed using the harmonic mean estimator proposed by Geweke (1999).

### Table 9. Real exchange rate fluctuations: economic decomposition

(\% of the real exchange rate variance)

<table>
<thead>
<tr>
<th>Component</th>
<th>Complete</th>
<th>LCP</th>
<th>PCP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Var(Internal real exchange rate)</td>
<td>0.2</td>
<td>0.7</td>
<td>0.2</td>
</tr>
<tr>
<td>Var(Home bias)</td>
<td>7.5</td>
<td>35.7</td>
<td>96.6</td>
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<tr>
<td>Var(International price discrimination)</td>
<td>55.9</td>
<td>24.9</td>
<td>0.0</td>
</tr>
<tr>
<td>cov(Internal real exchange rate, home bias)</td>
<td>-0.1</td>
<td>1.7</td>
<td>3.1</td>
</tr>
<tr>
<td>cov(Internal real exchange rate, international price discrimination)</td>
<td>2.2</td>
<td>1.9</td>
<td>0.0</td>
</tr>
<tr>
<td>cov(Home bias,international price discrimination)</td>
<td>34.0</td>
<td>34.6</td>
<td>0.0</td>
</tr>
<tr>
<td>Total</td>
<td>99.8</td>
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Table 10. Variance decomposition: the complete model

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<tr>
<th>Variable</th>
<th>$z_T$</th>
<th>$z_T^*$</th>
<th>$z_N$</th>
<th>$z_N^*$</th>
<th>$z_R$</th>
<th>$z_R^*$</th>
<th>$z_{PR}$</th>
<th>$z_{PR}^*$</th>
<th>$z_{U1P}$</th>
<th>$\theta_L$</th>
<th>$\theta_L^*$</th>
<th>tot</th>
</tr>
</thead>
<tbody>
<tr>
<td>UEM $C$</td>
<td>10.2</td>
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<td>13.3</td>
<td>0</td>
<td>5.4</td>
<td>0</td>
<td>63.9</td>
<td>0.7</td>
<td>5.9</td>
<td>0.3</td>
<td>0</td>
<td>100</td>
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<td>4.8</td>
<td>1.1</td>
<td>0</td>
<td>100</td>
</tr>
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<td>1.7</td>
<td>1.2</td>
<td>0</td>
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<tr>
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<td>0</td>
<td>56.2</td>
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<td>0.5</td>
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<td>0.3</td>
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Table 11. Variance decomposition: the LCP model

<table>
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<th>Variable</th>
<th>$z_T$</th>
<th>$z_T^*$</th>
<th>$z_N$</th>
<th>$z_N^*$</th>
<th>$z_R$</th>
<th>$z_R^*$</th>
<th>$z_{PR}$</th>
<th>$z_{PR}^*$</th>
<th>$z_{U1P}$</th>
<th>$\theta_L$</th>
<th>$\theta_L^*$</th>
<th>tot</th>
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<td>0.0</td>
<td>7.3</td>
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</tr>
</tbody>
</table>

Table 12. Variance decomposition: the PCP model

<table>
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<tr>
<th>Variable</th>
<th>$z_T$</th>
<th>$z_T^*$</th>
<th>$z_N$</th>
<th>$z_N^*$</th>
<th>$z_R$</th>
<th>$z_R^*$</th>
<th>$z_{PR}$</th>
<th>$z_{PR}^*$</th>
<th>$z_{U1P}$</th>
<th>$\theta_L$</th>
<th>$\theta_L^*$</th>
<th>tot</th>
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<td>0.9</td>
<td>0.0</td>
<td>21.7</td>
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<td>3.9</td>
<td>0.7</td>
<td>0.0</td>
<td>100</td>
</tr>
<tr>
<td>UEM $(1+i)$</td>
<td>18.5</td>
<td>0.0</td>
<td>4.9</td>
<td>0.0</td>
<td>10.0</td>
<td>0.0</td>
<td>62.7</td>
<td>0.1</td>
<td>3.5</td>
<td>0.2</td>
<td>0.0</td>
<td>100</td>
</tr>
<tr>
<td>UEM $RS$</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.0</td>
<td>0.2</td>
<td>0.3</td>
<td>4.5</td>
<td>3.1</td>
<td>91.6</td>
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<td>0.0</td>
<td>100</td>
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<tr>
<td>U. S. $C^*$</td>
<td>0.1</td>
<td>4.9</td>
<td>0.0</td>
<td>1.4</td>
<td>0.0</td>
<td>15.8</td>
<td>0.6</td>
<td>65.5</td>
<td>11.6</td>
<td>0.0</td>
<td>0.1</td>
<td>100</td>
</tr>
<tr>
<td>U. S. $\pi^*$</td>
<td>0.0</td>
<td>66.6</td>
<td>0.0</td>
<td>10.0</td>
<td>0.0</td>
<td>2.7</td>
<td>0.5</td>
<td>7.1</td>
<td>12.2</td>
<td>0.0</td>
<td>0.9</td>
<td>100</td>
</tr>
<tr>
<td>U. S. $\pi_N$</td>
<td>0.0</td>
<td>12.3</td>
<td>0.0</td>
<td>53.3</td>
<td>0.0</td>
<td>4.2</td>
<td>0.7</td>
<td>13.6</td>
<td>14.7</td>
<td>0.0</td>
<td>1.3</td>
<td>100</td>
</tr>
<tr>
<td>U. S. $(1+i^*)$</td>
<td>0.0</td>
<td>8.2</td>
<td>0.0</td>
<td>5.6</td>
<td>0.0</td>
<td>21.9</td>
<td>0.8</td>
<td>49.4</td>
<td>14.0</td>
<td>0.0</td>
<td>0.1</td>
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</table>
Table 13. Moments (Complete Model)

### Table 13a. Volatility

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data 2.5</th>
<th>Median</th>
<th>97.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$RS$</td>
<td>20.7</td>
<td>8.02</td>
<td>14.71</td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.41</td>
<td>0.38</td>
<td>0.50</td>
</tr>
<tr>
<td>$\pi^*$</td>
<td>0.36</td>
<td>0.36</td>
<td>0.46</td>
</tr>
<tr>
<td>$\pi_N$</td>
<td>0.55</td>
<td>0.29</td>
<td>0.42</td>
</tr>
<tr>
<td>$\pi_N^*$</td>
<td>0.36</td>
<td>0.24</td>
<td>0.35</td>
</tr>
<tr>
<td>$C$</td>
<td>1.94</td>
<td>0.91</td>
<td>1.37</td>
</tr>
<tr>
<td>$C^*$</td>
<td>1.78</td>
<td>0.81</td>
<td>1.17</td>
</tr>
<tr>
<td>$(1+i)$</td>
<td>0.77</td>
<td>0.27</td>
<td>0.48</td>
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<tr>
<td>$(1+i^*)$</td>
<td>0.55</td>
<td>0.22</td>
<td>0.36</td>
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</table>

### Table 13b. Persistence

<table>
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<tr>
<th>Variable</th>
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<th>Median</th>
<th>97.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$RS$</td>
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<td>0.79</td>
<td>0.93</td>
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<tr>
<td>$\pi$</td>
<td>0.75</td>
<td>0.39</td>
<td>0.64</td>
</tr>
<tr>
<td>$\pi^*$</td>
<td>0.40</td>
<td>0.22</td>
<td>0.49</td>
</tr>
<tr>
<td>$\pi_N$</td>
<td>0.85</td>
<td>0.58</td>
<td>0.78</td>
</tr>
<tr>
<td>$\pi_N^*$</td>
<td>0.57</td>
<td>0.52</td>
<td>0.75</td>
</tr>
<tr>
<td>$C$</td>
<td>0.49</td>
<td>0.68</td>
<td>0.84</td>
</tr>
<tr>
<td>$C^*$</td>
<td>0.96</td>
<td>0.68</td>
<td>0.84</td>
</tr>
<tr>
<td>$(1+i)$</td>
<td>0.98</td>
<td>0.86</td>
<td>0.95</td>
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<tr>
<td>$(1+i^*)$</td>
<td>0.97</td>
<td>0.82</td>
<td>0.93</td>
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</tbody>
</table>

### Table 13c. Cross-correlations

<table>
<thead>
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<th>Variable</th>
<th>Data 2.5</th>
<th>Median</th>
<th>97.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$RS,C/C^*$</td>
<td>-0.48</td>
<td>-0.80</td>
<td>-0.39</td>
</tr>
<tr>
<td>$RS,\pi$</td>
<td>0.54</td>
<td>-0.59</td>
<td>-0.10</td>
</tr>
<tr>
<td>$RS,\pi^*$</td>
<td>0.03</td>
<td>-0.34</td>
<td>0.10</td>
</tr>
<tr>
<td>$RS,\pi_N$</td>
<td>0.52</td>
<td>-0.68</td>
<td>-0.18</td>
</tr>
<tr>
<td>$RS,\pi_N^*$</td>
<td>0.38</td>
<td>-0.39</td>
<td>0.19</td>
</tr>
<tr>
<td>$RS,\pi_F$</td>
<td>-0.24</td>
<td>0.02</td>
<td>0.22</td>
</tr>
<tr>
<td>$RS,\pi_H^*$</td>
<td>-0.11</td>
<td>-0.57</td>
<td>-0.29</td>
</tr>
<tr>
<td>$C,C^*$</td>
<td>-0.10</td>
<td>-0.58</td>
<td>-0.08</td>
</tr>
<tr>
<td>$tb,y$</td>
<td>-0.70</td>
<td>-0.37</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Notes: 2.5 and 97.5 are percentiles.
### Table 14. Moments (LCP model)

#### Table 14a. Volatility

<table>
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<th>2.5</th>
<th>Median</th>
<th>97.5</th>
</tr>
</thead>
<tbody>
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<td>$RS$</td>
<td></td>
<td>20.7</td>
<td>8.0</td>
<td>14.2</td>
</tr>
<tr>
<td>$\pi$</td>
<td></td>
<td>0.41</td>
<td>0.30</td>
<td>0.42</td>
</tr>
<tr>
<td>$\pi^*$</td>
<td></td>
<td>0.36</td>
<td>0.35</td>
<td>0.47</td>
</tr>
<tr>
<td>$\pi_N$</td>
<td></td>
<td>0.55</td>
<td>0.28</td>
<td>0.41</td>
</tr>
<tr>
<td>$\pi^*_N$</td>
<td></td>
<td>0.36</td>
<td>0.29</td>
<td>0.41</td>
</tr>
<tr>
<td>$C$</td>
<td></td>
<td>1.94</td>
<td>1.00</td>
<td>1.70</td>
</tr>
<tr>
<td>$C^*$</td>
<td></td>
<td>1.78</td>
<td>0.83</td>
<td>1.33</td>
</tr>
<tr>
<td>$(1+i)$</td>
<td></td>
<td>0.77</td>
<td>0.26</td>
<td>0.47</td>
</tr>
<tr>
<td>$(1+i^*)$</td>
<td></td>
<td>0.55</td>
<td>0.24</td>
<td>0.41</td>
</tr>
</tbody>
</table>

#### Table 14b. Persistence

<table>
<thead>
<tr>
<th>Variable</th>
<th>Data</th>
<th>2.5</th>
<th>Median</th>
<th>97.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$RS$</td>
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<td>0.97</td>
<td>0.80</td>
<td>0.93</td>
</tr>
<tr>
<td>$\pi$</td>
<td></td>
<td>0.75</td>
<td>0.44</td>
<td>0.69</td>
</tr>
<tr>
<td>$\pi^*$</td>
<td></td>
<td>0.40</td>
<td>0.39</td>
<td>0.61</td>
</tr>
<tr>
<td>$\pi_N$</td>
<td></td>
<td>0.85</td>
<td>0.63</td>
<td>0.81</td>
</tr>
<tr>
<td>$\pi^*_N$</td>
<td></td>
<td>0.57</td>
<td>0.64</td>
<td>0.80</td>
</tr>
<tr>
<td>$C$</td>
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<td>0.49</td>
<td>0.72</td>
<td>0.89</td>
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<tr>
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<td>0.96</td>
<td>0.69</td>
<td>0.87</td>
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<tr>
<td>$(1+i)$</td>
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<td>0.98</td>
<td>0.85</td>
<td>0.95</td>
</tr>
<tr>
<td>$(1+i^*)$</td>
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<td>0.97</td>
<td>0.84</td>
<td>0.94</td>
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</table>

#### Table 14c. Cross-Correlations

<table>
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<th>Variable</th>
<th>Data</th>
<th>2.5</th>
<th>Median</th>
<th>97.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$RS, C/C^*$</td>
<td></td>
<td>-0.48</td>
<td>-0.82</td>
<td>-0.36</td>
</tr>
<tr>
<td>$RS, \pi$</td>
<td></td>
<td>0.54</td>
<td>-0.56</td>
<td>-0.03</td>
</tr>
<tr>
<td>$RS, \pi^*$</td>
<td></td>
<td>0.03</td>
<td>-0.52</td>
<td>-0.00</td>
</tr>
<tr>
<td>$RS, \pi_N$</td>
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<td>-0.08</td>
</tr>
<tr>
<td>$RS, \pi^*_N$</td>
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<td>0.38</td>
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</tr>
<tr>
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<td>0.25</td>
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<tr>
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<td>-0.44</td>
<td>-0.24</td>
</tr>
<tr>
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<td>-0.10</td>
<td>-0.68</td>
<td>-0.09</td>
</tr>
<tr>
<td>$tb, y$</td>
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</table>

Notes: 2.5 and 97.5 are percentiles.
### Table 15a. Volatility

<table>
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<th>Variable</th>
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<th>97.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS</td>
<td>20.7</td>
<td>8.44</td>
<td>14.5</td>
</tr>
<tr>
<td>π</td>
<td>0.41</td>
<td>0.29</td>
<td>0.39</td>
</tr>
<tr>
<td>π*</td>
<td>0.36</td>
<td>0.31</td>
<td>0.40</td>
</tr>
<tr>
<td>πₙ</td>
<td>0.55</td>
<td>0.26</td>
<td>0.37</td>
</tr>
<tr>
<td>πₙ*</td>
<td>0.36</td>
<td>0.21</td>
<td>0.30</td>
</tr>
<tr>
<td>C</td>
<td>1.94</td>
<td>0.91</td>
<td>1.31</td>
</tr>
<tr>
<td>C*</td>
<td>1.78</td>
<td>0.80</td>
<td>1.14</td>
</tr>
<tr>
<td>(1 + i)</td>
<td>0.77</td>
<td>0.24</td>
<td>0.42</td>
</tr>
<tr>
<td>(1 + i*)</td>
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<td>0.19</td>
<td>0.32</td>
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</table>

### Table 15b. Persistence

<table>
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<tr>
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<th>Data 2.5</th>
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<th>97.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS</td>
<td>0.97</td>
<td>0.80</td>
<td>0.93</td>
</tr>
<tr>
<td>π</td>
<td>0.75</td>
<td>0.31</td>
<td>0.57</td>
</tr>
<tr>
<td>π*</td>
<td>0.40</td>
<td>0.17</td>
<td>0.43</td>
</tr>
<tr>
<td>πₙ</td>
<td>0.85</td>
<td>0.53</td>
<td>0.74</td>
</tr>
<tr>
<td>πₙ*</td>
<td>0.57</td>
<td>0.37</td>
<td>0.66</td>
</tr>
<tr>
<td>C</td>
<td>0.49</td>
<td>0.66</td>
<td>0.83</td>
</tr>
<tr>
<td>C*</td>
<td>0.96</td>
<td>0.65</td>
<td>0.81</td>
</tr>
<tr>
<td>(1 + i)</td>
<td>0.98</td>
<td>0.85</td>
<td>0.94</td>
</tr>
<tr>
<td>(1 + i*)</td>
<td>0.97</td>
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</table>

### Table 15c. Cross-Correlations

<table>
<thead>
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<th>Median</th>
<th>97.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>RS, C/C*</td>
<td>-0.48</td>
<td>-0.82</td>
<td>-0.51</td>
</tr>
<tr>
<td>RS, π</td>
<td>0.54</td>
<td>-0.61</td>
<td>-0.14</td>
</tr>
<tr>
<td>RS, π*</td>
<td>0.03</td>
<td>-0.29</td>
<td>0.10</td>
</tr>
<tr>
<td>RS, πₙ</td>
<td>0.52</td>
<td>-0.66</td>
<td>-0.19</td>
</tr>
<tr>
<td>RS, πₙ*</td>
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<td>-0.35</td>
<td>0.19</td>
</tr>
<tr>
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<td>0.17</td>
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<tr>
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<td>-0.31</td>
<td>-0.17</td>
</tr>
<tr>
<td>C, C*</td>
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</table>

Notes: 2.5 and 97.5 are percentiles.
Table 16. Volatility in the complete model without UIP and preference shocks

<table>
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<th>NO UIP and preference shock</th>
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<td>Data</td>
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</tr>
<tr>
<td>$\pi$</td>
<td>0.41</td>
<td>0.39</td>
</tr>
<tr>
<td>$\pi^*$</td>
<td>0.36</td>
<td>0.35</td>
</tr>
<tr>
<td>$\pi^*_n$</td>
<td>0.55</td>
<td>0.30</td>
</tr>
<tr>
<td>$\pi^*_n$</td>
<td>0.36</td>
<td>0.25</td>
</tr>
<tr>
<td>$C$</td>
<td>1.94</td>
<td>0.91</td>
</tr>
<tr>
<td>$C^*$</td>
<td>1.78</td>
<td>0.74</td>
</tr>
<tr>
<td>$(1 + i)$</td>
<td>0.77</td>
<td>0.29</td>
</tr>
<tr>
<td>$(1 + i^*)$</td>
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<td>0.19</td>
</tr>
</tbody>
</table>

Table 17. Cross-correlation between real exchange rate and relative consumption

<table>
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<tr>
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<th>NO UIP shock, NO Preference Shocks</th>
</tr>
</thead>
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<tr>
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</tr>
<tr>
<td></td>
<td>-0.48</td>
<td>-0.7400</td>
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</tbody>
</table>

Notes: 2.5 and 97.5 are percentiles.
Figure 1. Data

Range: 1983:1-2005:2 (90 observations, quarterly data). The real exchange rate is $SP^*/P$. $S$ is the euro price of a U.S. dollar, $P^*$ is the U.S. CPI, $P$ is the euro area CPI.
Figure 2. UIP shock

Notes: Solid line: median response; dashed lines: 5th - 95th percentiles. Ordinate: quarters. Coordinate: percent deviation from steady state. Trade balance and net foreign asset position are % of steady state output. Size of the shock: one standard deviation.
Figure 3. Shock to the Home preferences

Notes: Solid line: median response; dashed lines: 5th - 95th percentiles. Ordinate: quarters. Coordinate: percent deviation from steady state. Trade balance and net foreign asset position are % of steady state output. Size of the shock: one standard deviation.
Figure 4. Shock to the Home tradable technology

Notes: Solid line: median response; dashed lines: 5th - 95th percentiles. Ordinate: quarters. Coordinate: percent deviation from steady state. Trade balance and net foreign asset position are % of steady state output. Size of the shock: one standard deviation.