EXPECTATIONS AND EXCHANGE RATE DYNAMICS UNDER MANAGED FLOATING: AN ASSET MARKET APPROACH

by

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ABSTRACT

This thesis analyses exchange rate dynamics in a managed floating exchange rate regime, under two alternative specifications of private expectations: adaptive expectations and long-run perfect foresight. The whole analysis follows the asset market approach to exchange rate determination.

A built-in government reaction function is incorporated in a neoclassical general equilibrium portfolio balance model. The intervention rule employed, reflects both the "reference rate proposal" of Ethier and Bloomfield (1975) and the 3rd guideline of the IMF for the management of floating exchange rates. The government intervenes in the foreign exchange market in order to minimize the discrepancies of the spot exchange rate from the government reference rate (estimate of the long-run exchange rate), at every moment in time.

It is shown that the degree of success at which government intervention moderates short-run exchange rate variability depends on the degree of precision with which the government forms its estimate of the long-run exchange rate path. Any prediction errors lead to dynamic instability. A generalisation of the intervention rule leads to a stable long-run equilibrium even if the government uses the wrong estimate of the long-run exchange rate path (competitive exchange rate policies). However, the sustenance of these equilibria creates crises in the balance of payments and the system ultimately returns to the free floating long-run position.

Persuance of competitive exchange rate policies increases the degree of reserve use and could lead to intervention at cross purposes, increasing the need for international liquidity.

A re-interpretation of our intervention rule to reflect speculative behaviour leads to similar results as government intervention. The liquidation of speculative profits, however, creates additional short-run exchange rate variability, enhancing the non-profit making nature of government intervention.

ACKNOWLEDGEMENTS

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To my parents with gratitude

CONTENTS

CHAPTER 1

INTRODUCTION

The following chapters attempt to explore the nature of exchange rate dynamics under a regime of managed floating, for two alternative specifications of private expectations: adaptive expectations and long-run perfect foresight. The whole analysis follows the asset market approach to exchange rate determination.

In contrast to the traditional approach to exchange rate determination, the essence of the asset market theory is that the exchange rate is a relative asset price[1] - the relative price at which the stocks of money, bonds and other financial and real assets of a country are willingly held by domestic and foreign asset holders, at every moment in time. The asset market approach is an extension of the monetary approach to the balance of payments to the regime of flexible exchange rates. In terms of the asset market approach, dynamic exchange rate behaviour is affected by the evolution of supply and demand for the various assets together with the behaviour of expectations over time.

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1. For the asset market approach see the various articles appearing in the Scandinavian Journal of Economics, Vol. 78, No. 2, 1976. Isard (1978) provides a short survey of popular views and recent models for exchange rate determination.

During the recent period of floating exchange rates, it has been observed that exchange rates display considerable short-run fluctuations. The consensus in the literature[2] seems to be that the asset market approach provides the most theoretically satisfactory explanation of this short-run exchange rate variability. The asset market theory allows short-run exchange rate fluctuations in excess of fluctuations in underlying conditions.

However, the various models of exchange rate dynamics, e.g. Kouri (1976), Dornbusch (1976b), assume a freely flexible exchange rate regime, ignoring the active government interference in the foreign exchange markets. While in the real world central banks do intervene in the foreign exchange markets on a regular basis in order to correct disorderly market conditions, the theoretical work on exchange rate dynamics under managed floating is still in its infancy. Girton and Henderson (1976, 1977) analyse central bank operations in foreign and domestic assets in a short-run partial equilibrium portfolio balance model. More significantly, Kouri (1976) and Henderson (1978) examine the impact, long-run and dynamic effects of a once and for all foreign exchange market operation, in a general equilibrium portfolio balance model. A more realistic model should allow continuous government intervention (not just a once and for all foreign exchange market operation),

2. For a brief survey of the relevant literature see Schadler (1977).

throughout the adjustment process. In addition, an explicit analysis of private speculation could help investigate the importance of stabilizing or destabilizing speculation for short-run exchange rate variability.

In our thesis, we incorporate a built-in government intervention function in a general equilibrium portfolio balance model. Creating a shock into the system, we comparatively examine the resulting exchange rate dynamics under free and managed floating. The intervention rule we employ, reflects both the "reference rate proposal" of Ethier and Bloomfield (1975) and the 3rd guideline of the IMF for the management[3] of floating exchange rates. The government intervenes in the foreign exchange market in order to minimize the discrepancies of the spot exchange rate from the "government reference rate" (estimate of the long-run exchange rate) at every moment in time, minimizing in consequence, the welfare costs of short-run exchange rate variability.

We investigate also the effectiveness[4] of government intervention as a means of moderating short-run exchange rate variability. In addition, we examine the

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3. The different intervention rules proposed in the literature are analysed in detail in section 3.1 of chapter 3.

4. In our thesis, however, we are not trying to defend managed floating relative to either free floating or fixed exchange rates. We are simply providing a positive economics analysis of a managed floating system. For the fixed versus flexible exchange rate debate, see Ishiyama (1975), Tower and Willett (1976) and Artus and Young (1979).

determinants of reserve use and the possible repercussions of government intervention on international liquidity in general. In a world of more than two countries, managed floating may lead to intervention at cross purposes, increasing the need for international surveillance of national exchange rate policies.

A re-interpretation of our intervention rule to reflect speculative behaviour, allows us to examine the effects of stabilizing and destabilizing speculation on exchange rate dynamics relative to free and managed floating, with no change in our formal analysis. A special feature of speculative behaviour has particular importance for exchange rate dynamics. Speculators are driven by profit maximizing objectives. In order to remain in business, speculators, unlike the government, need to uncover their positions and take their profits. An action that by itself creates an endogenous source of short-run exchange rate flunctuations.

Because of their nature, exposition of our results at this stage, even in summary form, presupposes formal exposition of a framework of analysis. Instead, we will very briefly highlight the main issues dealt with in every chapter. A summary of our results is provided in the concluding sections of chapters 3 and 4.

In chapter 2 we develop a general equilibrium portfolio balance model of exchange rate determination under free floating. Its basic structure is an extention of Kouri's (1976) model to include domestic and foreign bonds, modifying the current account accordingly to allow for the service account. Our model provides an explanation of short-run exchange rate variability in excess of fluctuations in underlying conditions (overshooting), for both adaptive expectations and long-run perfect foresight.

In chapter 3, government intervention is introduced. The government forms an estimate of the long-run exchange rate path and intervenes in the foreign exchange market on the basis of the discrepancies of the spot exchange rate from its expected long-run value. The effectiveness of government intervention in moderating short-run exchange rate fluctuations is shown to depend on the degree of precision with which the government forms its estimate of the long-run exchange rate path. Any deviation of the government estimate from its true value leads to dynamic instability. A re-interpretation of our intervention rule to reflect speculative behaviour yields similar results.

In chapter 4, the intervention function is modified, on the basis of the conclusions reached in chapter 3, giving rise to a generalised government reaction function. Intervention leads now to a stable long-run stationary state, even if the government, due to prediction errors, follows competitive exchange rate policies. However, sustenance of a long-run equilibrium position other than the one that would prevail in the absence of intervention requires the government to accumulate or to decumulate foreign exchange reserves over time, creating a crisis in the balance of payments. Due to the constraints on its actions, the government is forced either to modify its estimate of the long-run exchange rate path or to give up intervention altogether. Ultimately the system returns to its free floating stationary state position. Similar results are obtained, when private speculation is considered instead. In addition, though, the analysis of chapters 3 and 4 shows that profitable speculation can only be stabilizing.

CHAPTER 2

THE ASSET MARKET APPROACH TO EXCHANGE RATE DETERMINATION

2.1 Introduction

In this chapter we develop a model of short-run and long-run exchange rate determination in terms of the asset market approach. We examine also the dynamic response of the system to exogenous changes and provide an explanation of short-run exchange rate variability.

The asset market approach focuses on the equilibrating role of the exchange rate in balancing the foreign demand for domestically issued financial and real assets and the domestic demand for foreign assets, while, at the same time, it does not neglect the role of the exchange rate in the goods market. The basic analysis is an extension of the monetary approach to the balance of payments (Frenkel and Johnson, 1976) to the regime of floating exchange rates.[1] The essence of

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1. For recent work that shares some of the present emphasis see Black (1973, 1975), Henderson (1975), Dornbusch (1976a, 1976b), Kouri (1976), Frenkel (1976), Myrhman (1976), Mussa (1976). The classics remain Fleming (1962) and Mundell (1963). For a brief survey see Schadler (1977) and Bilson (1979).

the asset market theory is that the exchange rate is a relative asset price - the relative price at which the stocks of money, bonds and other financial and real assets of a country are willingly held by domestic and foreign asset holders. The asset market approach highlights the importance of factors affecting the desired relative stocks of domestic and foreign assets in the short-run exchange rate determination. Exchange rate behaviour is affected by the evolution of supply and demand for the various assets together with the behaviour of expectations over time. This is in direct contrast to the traditional approach to flexible exchange rates which focuses attention on the behaviour of flows of exports, imports and of capital flows between countries (Robinson, 1947).

Throughout the thesis we assume continuous time and we postulate that all markets clear instantaneously. The stock formulation of our model is along the lines of the work of Foley (1975), Turnovsky and Burmeister (1977), Burmeister and Turnovsky (1977), Buiter and Woglom (1977) and Turnovsky (1977). The maintained hypothesis of the stock approach is that asset markets clear very fast compared to the goods markets. Prices, rates of return, the endogenous variables in general adjust, so as to make the wealth holders content to hold the existing stock of assets at every point in time. This is based on the empirical assumption that the

> "... speed of adjustment of portfolio imbalance is high, or equivalently, the costs of portfolio adjustments are low. Asset markets are in fact among the most organised of markets; information about prices of many (especially financial) assets is disseminated widely and rapidly, and the great bulk of the total wealth in industrialised capitalist economies is held in very large portfolios for which fixed transaction costs will be negligible in relation to portfolio shifts. These observations suggest that the vision of stock equilibrium may be a good approximation to the real situation. Empirical evidence of large transaction costs, of course, will upset this conclusion."
> (Foley, 1975, p. 319).

In terms of the asset market approach, the usual single budget constraint (Walras Law) of the end of period formulation of the asset markets gives rise to two constraints: one stock and one flow constraint.

The asset market approach is particularly relevant to countries with well developed capital and money markets, where exchange controls are free enough to allow arbitrage between domestic and foreign assets. For less developed countries, however, where these conditions are less likely to exist the exchange rate is determined by the supply and demand conditions in the goods market and by the extent of government intervention.[2]

In the short-run, the stocks of the various assets are given and along with the other exogenous variables they determine the short-run quotations of the

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2. For the relationship between exchange rate arrangements and the institutional backgrounds of domestic policy in less developed countries, see Black (1975).

endogenous variables. The accumulation or decumulation of assets together with changing expectations move the system over time. The dynamics of our model are similar to those of Foley and Sidrauski (1971). Frenkel and Rodriguez (1975), Kouri (1976) and Dornbusch (1975).

The basic model is developed in Section 2.2, the short-run equilibrium of which is analysed in 2.3 and in 2.4 we examine the long-run stationary state. Section 2.5 analyses the dynamic stability of the system both under adaptive expectations and under perfect foresight and Section 2.6 deals with the dynamic response of the model to exogenous shocks. We conclude with Section 2.7, which deals with short-run exchange variability and provides the setting for the analysis of government intervention.

2.2 The basic model

Our basic model is an extension of Kouri's (1976) model to include domestic and foreign bonds, modifying the current account accordingly to allow for the service account. We assume a one sector neoclassical small open economy, producing a fully traded output, the relative price of which is determined by the world market forces (law of one price) - transaction costs in world trade, tariffs or any other impediments to trade are assumed away. The world price level is assumed constant, normalized equal to one so that the domestic price level (P) is identical[3] to the exchange rate, defined as the domestic currency price of the unit of foreign money. Labour, our only factor of production, is inelastically supplied so that real output is constant. Domestic absorption is equal to private real consumption (C) and real government expenditures (G). Following the life-cycle hypothesis, we postulate real consumption as a positive function of real labour disposable income (Y^D) and real private wealth (W), i.e.

$$C = C(Y^D, W); \quad 0 < C_{Y^D} < 1, \ C_W > 0 \qquad (2.1)$$

The balance of trade surplus (BT) is domestic output minus domestic absorption:

$$BT = Y - C(Y^D, W) - G \qquad (2.2)$$

Our small open economy includes four assets: domestic money (M), held only by domestic residents and bearing a zero nominal yield, foreign exchange (F), domestic bonds (B) - assuming no discounting of future

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3. The law of one price assumption makes the domestic price level identical to the exchange rate. The use of this strong assumption, however, dictated by reasons of mathematical tractability does not allow independent dynamics of domestic prices and the exchange rate. See Dornbusch (1976b) for a model of independent price and exchange rate dynamics.

tax liabilities - and foreign bonds. Domestic bonds are denominated in foreign currency and are considered as perfect substitutes to foreign bonds.[4] Both are variable coupon bonds so that their value is independent of the rate of interest. The real value of bond holdings - both domestic and foreign - and of the holdings of foreign exchange are invariant to changes in the domestic price level because they are offset by identical changes in the exchange rate. Foreign exchange is assumed to bear a zero nominal rate of interest. The assumed four assets are effectively reduced to three because of the perfect substitutes assumption between domestic and foreign bonds. Our analysis, though, will remain basically invariant if domestic bonds were issued in domestic currency but still considered as perfect substitutes to foreign bonds; the mathematical complexity of the model will increase though, as the service account will have to be properly adjusted and interest rate parity should hold between domestic and foreign interest rates. F can not only represent foreign money but it might also stand for any foreign non-interest bearing asset.

Domestic real wealth can be written as

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4. Since both domestic and foreign bonds are denominated in the same currency there is no relative exchange rate risk differential. The assumption of perfect substitutes, in the language of Girton and Henderson (1977), implies that business and political risks in the domestic economy and the rest of the world are perfectly correlated.

$$W = \frac{M}{P} + F + b \tag{2.3}$$

where b is domestic real bond holdings and P is the domestic price level (exchange rate). Defining real financial wealth (v) as the sum of bond holdings and holdings of foreign exchange, i.e.

$$v \equiv F + b \tag{2.4}$$

(2.3) can also be written as

$$W = \frac{M}{P} + v \tag{2.3'}$$

Private asset demand functions (demand for stocks) are postulated as functions of real income, real wealth and the expected real rate of returns of the various assets.[5] According to the instantaneous (momentary) asset market equilibrium that we postulate, the existing stocks of assets are willingly held at every point in time. Thus, the asset market equilibrium can be described by the following conditions:[6]

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5. For the appropriate formulation of asset demand functions see, for instance, Tobin (1969), Foley and Sidrauski (1971, Ch. 3), and Tobin and Brainard (1968).

6. To simplify our analysis we do not assume any explicit banking system, so that the nominal money stock (M) is identical to high power money. Our asset demand functions, however, incorporate the behaviour of commercial and clearing banks, speculators and private asset holders. In other words, equations (2.5) - (2.7) are the reduced form equations that describe the asset market behaviour of the private sector.

$$(\frac{M}{P})^d = f_1(Y, W, i, \pi, u) = \frac{M}{P} \tag{2.5}$$

$$F^d = f_2(Y, W, i, \pi, u) = F \tag{2.6}$$

$$b^d = f_3(Y, W, i, \pi, u) = v - F \tag{2.7}$$

$$W = \frac{M}{P} + v \tag{2.3'}$$

where i is the nominal rate of interest on bonds, fixed for the domestic economy by the world market forces because of our small country assumption; π is the expected rate of inflation for the domestic economy which is identical to the expected exchange rate depreciation; and u is the subjective estimate of foreign exchange risk relative to the domestic currency.

Following Tobin (1969), we assume that all assets are strict gross substitutes with the following conditions on their partial derivatives:

$$\begin{array}{lllll} f_{1_y} > 0; & 0 < f_{1_w} < 1; & f_{1_i} < 0; & f_{1_\pi} < 0; & f_{1_u} > 0 \\ f_{2_y} < 0; & 0 < f_{2_w} < 1; & f_{2_i} < 0; & f_{2_\pi} > 0; & f_{2_u} < 0 \\ f_{3_y} < 0; & 0 < f_{3_w} < 1; & f_{3_i} > 0; & f_{3_\pi} > 0; & f_{3_u} < 0 \end{array} \tag{2.8}$$

and subject to the following adding up properties:

$$\begin{array}{ll} f_{1_w} + f_{2_w} + f_{3_w} = 1 & f_{1_y} + f_{2_y} + f_{3_y} = 0 \\ f_{1_i} + f_{2_i} + f_{3_i} = 0 & f_{1_\pi} + f_{2_\pi} + f_{3_\pi} = 0 \end{array} \tag{2.9}$$

$$f_{1_u} + f_{2_u} + f_{3_u} = 0 \tag{2.9}$$

Besides the above conventional restrictions on the partial derivatives we will need to assume the following additional ones referring to their relative magnitudes with various degrees of stringency:

$$(-C_{y^D} + C_w) > 0 \tag{2.9a}$$

$$C_w - (1 - C_{y^D})i > 0 \tag{2.9b}$$

$$-\frac{f_{1_u}}{f_{2_u}} = \frac{f_{2_w} + f_{3_w}}{f_{2_w}} \tag{2.9c}$$

The relative importance and meaning of these conditions will be developed in due course, we mention them at this stage simply to provide a systematic presentation of our maintained hypothesis.

The government is financing its expenditures by lump sux taxes (lump sum so that they do not affect asset preferences), by issuing bonds and by outside money creation. In particular we assume that the government fixes the level of its real expenditures, the level of real taxes (T) and the rate of growth of the nominal money stock (m), and varies the issue of new debt to cover the remaining budget deficit or suplus:[7]

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7. Under this formulation of the government budget constraint, B varies over time since the stock of real money balances are determined endogenously (see Section 2.3). The government by doing so avoids frequent changes in its expenditures and taxes and maintains a fixed rate of increase of the nominal money stock - type of monetary policy, while in the long-run both G and T can easily be adjusted to make the issue of new debt zero (see Section 2.4).

$$G + iB = T + \dot{B} + m\frac{M}{P} \qquad (2.10)$$

where B is the stock of outstanding government debt and $\dot{B}$ is the issue of new debt (a dot over a variable denotes a time derivative). Whenever B is different from the domestic bond holdings, b, the private sector satisfies its excess demand (supply) from the world market.

Since our model allows variable output prices, real disposable income must be appropriately defined, according to Turnovsky (1977), so that consistency of the system in real terms is preserved; i.e. real planned (actual) savings equal to the desired (actual) accumulation of wealth. Thus, real disposable income is defined to include expected capital gains ($\pi\frac{M}{P}$),[8] i.e.

$$Y^D = Y - T + i(v - F) - \pi\frac{M}{P} \qquad (2.11)$$

The current account (S) of the balance of payments is defined as the balance of trade plus the service account. It is also equal to total domestic savings, i.e. the sum of gross private savings (S_p) and government savings (S_g). Thus:

$$S = S_p + S_g = BT + i(v - F - B) \qquad (2.12)$$

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8. If expected capital gains are included in real disposable income, they affect ex ante consumption and savings alike; unexpected capital gains, however affect ex post savings only.

where S_g is equal to the government budget surplus, i.e.

$$S_g = T - (G + iB) = -(\dot{B} + m\frac{M}{P}) \qquad (2.13)$$

Substituting the balance of trade from (2.2) in (2.12) we have

$$S = Y - C(Y^D, W) - G + i(v - F - B) \quad (2.14)$$

A current account surplus (deficit) implies that the domestic economy is accumulating (decumulating) foreign assets (foreign exchange and/or bonds) from the rest of the world. Because of the assumption of flexible exchange rates, the current account surplus (deficit) is equal to the capital account deficit (surplus) so that the balance of payments is always in equilibrium. The accumulation of real financial wealth by the private sector is equal to the accumulation of foreign assets and the issue of new government bonds. The accumulation of foreign assets is given by the current account position and hence, the accumulation of real financial wealth (v) is determined by the current account balance and the issue of new domestic bonds, formally given by the time derivative of equation (2.4), i.e.

$$\dot{v} = \dot{F} + \dot{b} = S + \dot{B} \qquad (2.15)$$

2.3 The short-run equilibrium

For short-run equilibrium all markets of the system should clear, given our assumption of continuous market clearing. The equilibrium conditions for the three asset markets - money, foreign exchange and bonds - are given by equations (2.5), (2.6) and (2.7) respectively, and they are all subject to the stock constraint (2.3'). For equilibrium in the goods market, aggregate supply should equal aggregate expenditure, i.e. equation (2.2) should hold. Since, though, the current account is equal to the balance of trade plus the service account, by substitution we derive equation (2.14) which incorporates now the flow constraint of the model. Substituting real wealth from (2.3') and real disposable income from (2.11) and dropping the bonds market as redundant because of the stock constraint, the short-run equilibrium conditions of the model are the following:

$$f_1\ [Y,\ (\frac{M}{P} + v),\ i,\ \pi,\ u] = \frac{M}{P} \qquad (2.16a)$$

$$f_2\ [Y,\ (\frac{M}{P} + v),\ i,\ \pi,\ u] = F \qquad (2.16b)$$

$$S = Y - C\ [(Y - T + i(v - F) - \pi\frac{M}{P}),(\frac{M}{P} + v)] - G + i(V-F-B) \qquad (2.16$$

$$G + iB = T + \dot{B} + m\ \frac{M}{P} \qquad (2.16d)$$

where P, F, S and $\dot{B}$ are the endogenous variables, while M, v, i, π, u, Y, T, B, G and m are exogenous. M, v and B are the stocks of nominal money balances, real financial wealth and outstanding government debt respectively, which although they change over time, they are predetermined at a particular point in time, given by past accumulation, i.e.

$$M(t) = \int_0^t \dot{M}(t)dt, \quad v(t) = \int_0^t \dot{v}(t)dt, \quad B(t) = \int_0^t \dot{B}(t)dt$$

Note, though, that because bonds are internationally traded, they can be exchanged with foreign exchange or vice versa; thus the composition of real financial wealth is not fixed, but rather is endogenously determined.

Expectations about inflation and exchange rate depreciation are exogenously given for the short-run equilibrium. The precise mechanism of expectations formation will remain unspecified for the moment, but, in Section 2.5 where the dynamics of the system will be dealt with, we comparatively examine adaptive expectations and perfect foresight - the deterministic equivalent to rational expectations.

Taking total differentials of the short-run equilibrium conditions, we observe that the jacobian matrix A_1 is recursive; this is to be expected given our stock equilibrium assumption for the asset markets.

The exchange rate (price level) is determined in the money market alone, given the short-run exogenous variables, which in turn determines the short-run holdings of foreign exchange in the foreign exchange market; given the values of P and F, the current account balance is determined; and finally, given the exchange rate the issue of government debt is deduced from the government budget constraint. The short-run exchange rate is exclusively determined in the asset market so that the existing stocks of assets are willingly held; its short-run quotation reflects the demand for domestic currency relative to the demand for foreign assets (foreign exchange and bonds). The current account position affects the exchange rate only over time, through its effects on asset accumulation, unless, of course, expectations are affected such as is the case under rational expectations. It is for this property - exchange rate as a relative asset price - that the asset market approach lies in direct contrast to the traditional approach to flexible exchange rates (Robinson, 1947), which maintains that the exchange rate is mainly determined by the current account position, perceived as the relative price of domestic output.

$$A_1 = \begin{bmatrix} (1-f_{1_w})\frac{M}{P^2} & 0 & 0 & 0 \\ -f_{2_w}\frac{M}{P^2} & -1 & 0 & 0 \\ (-C_{y^D}\pi + C_w)\frac{M}{P^2} & -(1-C_{y^D})i & -1 & 0 \\ m\frac{M}{P^2} & 0 & 0 & -1 \end{bmatrix} \qquad (2.17)$$

The short-run exchange rate determination can also be illustrated geometrically. The MM_o schedule in Figure 2.1 shows the locus of combinations of exchange rates and stocks of foreign exchange for which the money market is in equilibrium; MM_o is perfectly elastic because F, as (2.17) shows, does not affect the money market equilibrium for a given stock of real financial wealth (v). Table 2.1 shows how the MM curve is shifted as the exogenous variables of the model increase.

Table 2.1

Shifts of the MM schedule as exogenous variables increase

Exchange rate (F = const.)	Exogenous variables M	v	π	u	Y
P	(+)	(-)	(+)	(-)	(-)

(+) means upwards and (-) downwards.

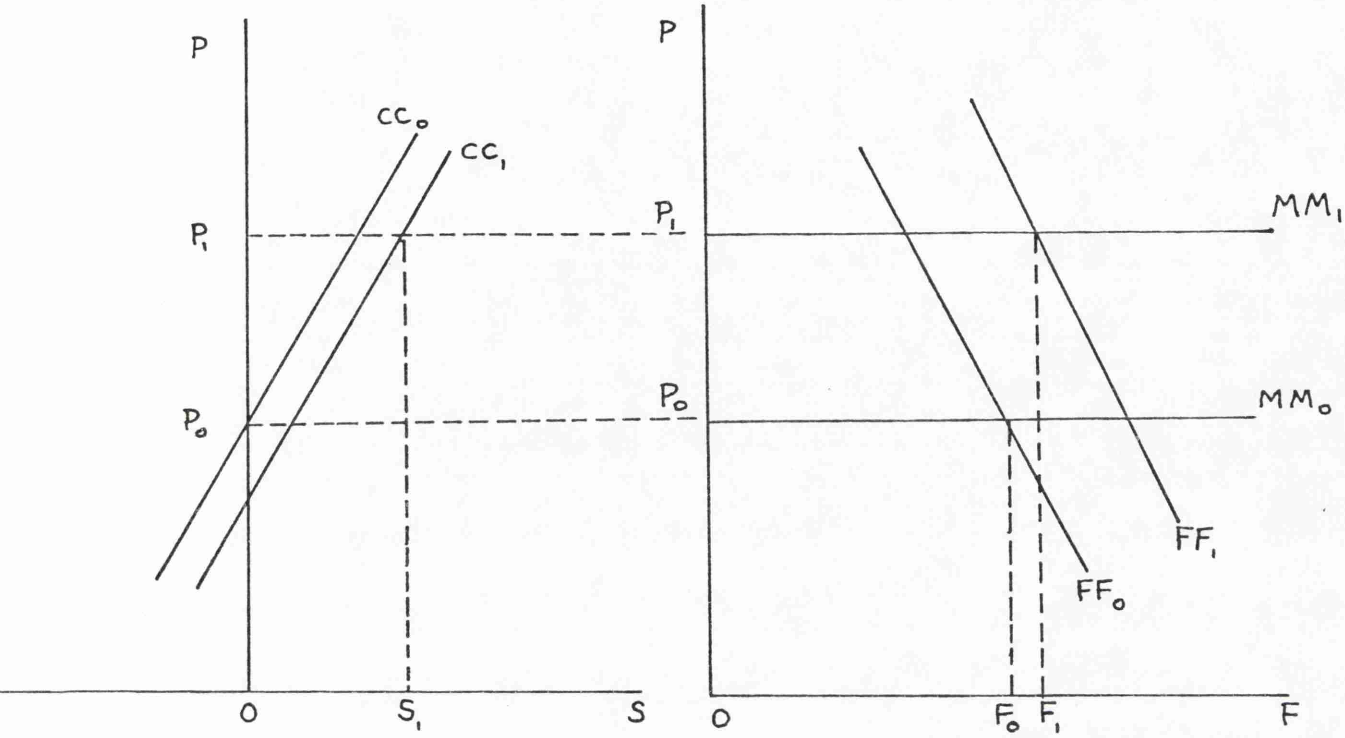

Figure 2.1. Short-run exchange rate determination;
Effects of an exogenous increase in expectations.

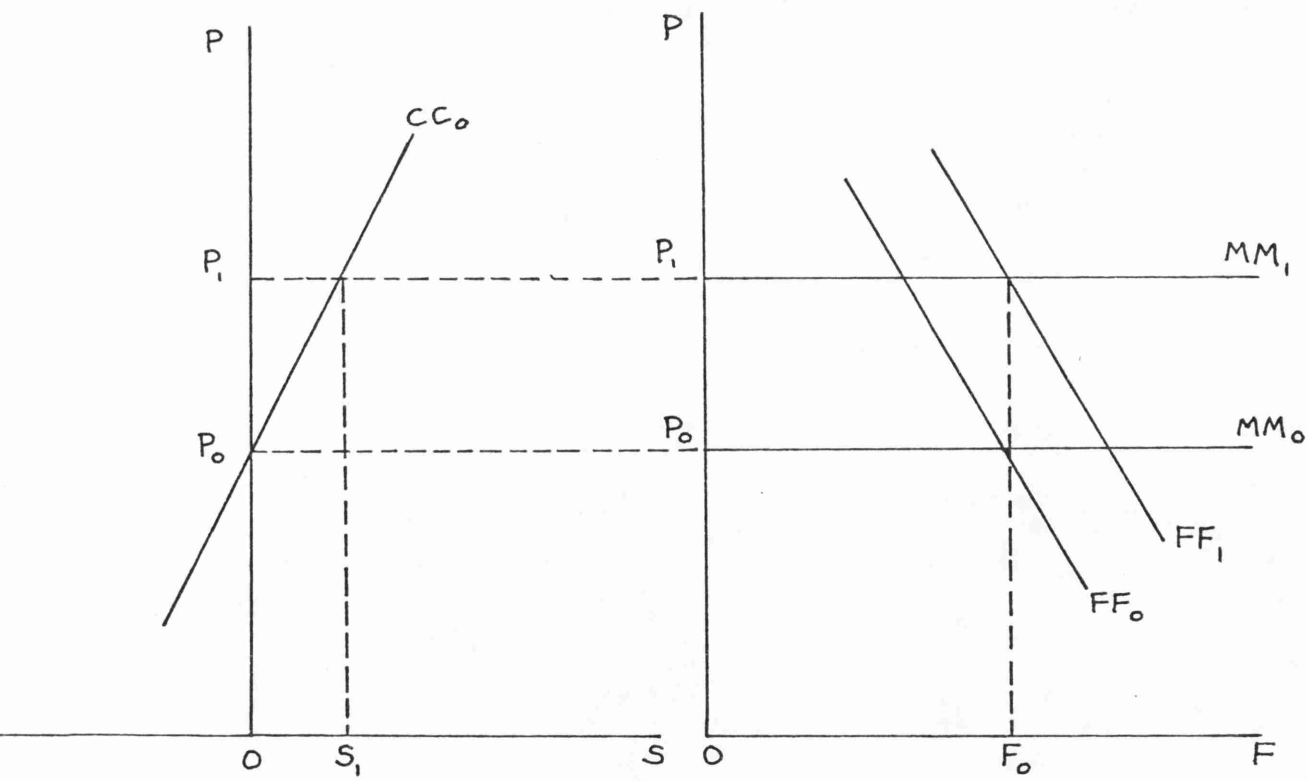

Figure 2.2. Short-run exchange rate determination;
Effects of an exogenous fall in u.

The FF_0 schedule shows combinations of exchange rates and stocks of foreign exchange for equilibrium in the foreign exchange market, and together with MM_0 they represent asset market equilibrium. It is downwards[9] sloping because an increase in the price level reduces real money balances and causes a fall in real wealth; this induces a decrease in the demand for foreign money and for equilibrium to be maintained, the short-run holdings of foreign exchange should fall as well. Table 2.2, similarly, shows how FF is shifted following increases in exogenous variables.

Table 2.2

Shifts of the FF schedule as exogenous variables increase

Exchange rate (F = const.)	Exogenous variables M	v	π	u	Y
P	(+)	(+)	(+)	(-)	(-)

(+) means upwards and (-) downwards

9. The FF schedule is downwards sloping because from (2.16b) we have

$$\left.\frac{dP}{dF}\right|_{FF} = - \frac{1}{f_{2_w} \frac{M}{P^2}} < 0; \text{ since } f_{2_w} > 0 \text{ by } (2.8).$$

The CC_0 schedule of Figure 2.1 shows the current account position against the short-run exchange rate quotations. An increase in the price level decreases real money balances and through a fall in consumption, decreases domestic absorption and improves the current account position. The increase in the price level, however, decreases the expected capital loss $(\pi\frac{M}{P})$ and hence, it causes a rise in real disposable income which tends to counteract the effects of a decrease in real money balances. Postulating that the net effect of a price rise is a decrease in domestic absorption (restriction 2.9a), the CC schedule is upwards[10] sloping. Table 2.3, in turn, shows how the CC curve moves as its shifting variables increase. Note that CC is affected by changes in G, T and B, in contrast to MM and FF which remain unchanged, and by changes in one endogenous variable (F).

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10. From (2.16c) we have

$$\left.\frac{dP}{dS}\right|_{CC} = \frac{1}{(-C_{y^D}\pi + C_W)\frac{M}{P^2}}$$

which is positive if $(-C_{y^D}\pi + C_W) > 0$. Restriction (2.9a), which suggests that the net effect of a rise in prices (exchange rate depreciation) is an improvement in the current account, implies that the positive effect of a decrease in real money balances outweights the negative effect of a rise in real disposable income, i.e. $C_W > C_{y^D}\pi$ and thus the above fraction is positive.

Table 2.3

Shifts of the CC schedule as its shifting variables increase

Exchange rate (S=const.)	Shifting variables M	v	π	u	Y	G	T	B	F	$\dot{B}$
P	(+)	(+)	(-)	0	(-)	(+)	(-)	(+)	(+)	0

(+) means upwards, (-) downwards and (0) unchanged.

Figure 2.1 depicts a particular momentary equilibrium where the exchange rate, P_0, is given by the intersection of the MM_0 schedule and the vertical axis, completely determined by the money market condition, as the recursiveness of the A_1 matrix suggests. The short-run equilibrium holdings of foreign exchange are determined by the intersection of the MM_0 and FF_0 schedules, denoting asset market equilibrium. At this particular asset market quotations there corresponds a zero current account surplus as the CC_0 schedule intersects the vertical axis at P_0. This specific short-run equilibrium was deliberately drawn to coincide with an instantaneous long-run equilibrium, where the current account is in balance and wealth is constant. As will be shown in Section 2.4, though, the current account need not be equal to zero in the stationary state if the issue of government debt, $\dot{B}$, is different from zero. To illustrate the short-run exchange rate determination we will consider the consequences of two specific disturbances in the asset market: an exogenous rise in expected inflation (expected exchange rate depreciation) and an exogenous fall in the subjective estimate of foreign exchange risk relative to domestic currency.

(a) <u>Exogenous rise in expected inflation</u>: Given our gross substitutes assumption, an increase in expected inflation increases the opportunity cost of holding money balances and induces a substitution of real money balances with real financial wealth - both foreign exchange and bonds - in private portfolios, i.e. it creates an excess supply of real money balances and an excess demand for real financial wealth. The private sector, however, cannot collectively dispose of its excess nominal money stock, and for real balances to be reduced to their new lower demand, prices have to rise inducing an exchange rate depreciation. This is shown in Figure 2.1 by the upward shift (see Table 2.1) of the MM_0 schedule to MM_1, disturbing the long-run equilibrium and depreciating the exchange rate to P_1. The excess demand for foreign exchange shifts the FF_0 schedule to FF_1 and the new short-run equilibrium holdings of foreign exchange increase to F_1; but this is not an unambiguous result, because, as is shown in the appendix and in Table 2.4, the short-run comparative static effect of an increase in inflationary expectations on the holdings of foreign exchange is uncertain. This is so because the increase in π on the one hand, leads to a substitution of domestic money with foreign assets, but on the other, the rise in prices decreases real money balances and real wealth and thus reducing the demand for all assets. The fall in real wealth reduces absorption as well, through a reduction in consumption giving rise to a current account surplus. The CC_o schedule subject to the counteracting shifting effects of rises in π and F is, without any loss of generality, shown to shift downwards to CC_1, allowing for a current account surplus, as the short-run comparative statics require (see Table 2.4).

The exogenous shock in the asset market, in the form of a rise in inflationary expectations, lets through a change in prices to an instantaneous re-evaluation of the stock of wealth and thus affects domestic absorption. Cooper (1976) questions whether changes in the market evaluation of wealth resulting from short-run exchange rate changes should be expected to affect expenditure decisions. If consumption, he argues, is a positive function of expected wealth, changes in the market evaluation of current wealth might not affect expenditure decisions. In such a formulation, though, individuals stabilize planned consumption leaving savings to absorb all the short-run variability of wealth; an assumption no less arbitrary than letting both consumption and savings to be similarly affected.[11]

(b) Exogenous fall in the subjective estimate of foreign exchange risk (u): A relatively more interesting example of exogenous disturbances is the case of a decrease in the subjective estimate of foreign exchange risk (u)[12] which, as will be argued in Section 2.7, is one of the main causes of short-run exchange rate variability. A fall in u indicates that speculators come to expect a greater risk in holding domestic currency relative to foreign assets, due say, to new expectations about the relative purchasing power of domestic money; (i.e. a change in the degree of substitutability between domestic and foreign assets). This will induce an

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11. For the formulation of the relationship between real and financial sectors in other models dealing with exchange rate dynamics see Schadler (1977).

12. The conventional method of modelling subjective arguments in asset demand functions, behavioural functions in general (such as tastes, risk aversion, etc.) is to incorporate them in the functional form; leaving as explicit arguments in the behavioural equations variables such as prices, interest rates, income or wealth. The main reason why u is presented as an explicit variable in our model, is the need to illustrate its importance in short-run exchange rate variability as one of the main factors accounting for the asset market disturbances. u, the subjective estimate of foreign exchange risk relative to domestic currency, reflects changes in wealth holders' risk aversion which might arise either from a change in their tastes and sensitiveness to risk or from a change in the uncertainty of prospective developments in the relative purchasing power of currencies.

excess supply of real money balances and an excess demand for foreign exchange and bonds, an excess demand for foreign assets in general. To clear the money market the exchange rate should depreciate (prices should rise) and hence the MM_o schedule in Figure 2.2 is shifted upwards to MM_1; the FF_o schedule, similarly, moves to FF_1 with the short-run holdings of foreign exchange remaining the same. The fall in u increases the demand for foreign exchange, while on the other hand, the fall in real money balances caused by the rise in prices decreases the demand for all assets. To facilitate our exposition without altering anything of the fundamental structure of the model, we will assume that the two counteracting effects offset each other[13] - i.e. restriction (2.9c) - so that the short-run holdings of foreign exchange remain unchanged at F_o. The reduction in real money balances, as a reduction in real wealth, decreases absorption, and given restriction (2.9a), leads to a current account surplus. The CC_o schedule remains unchanged since u is not a shifting variable (see Table 2.3) and F does not change.

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13. As is shown in the appendix, a sufficient condition for this is

$$a = -\frac{f_{1_u}}{f_{2_u}} = \frac{f_{2_w} + f_{3_w}}{f_{2_w}} .$$

Table 2.4

Short-run comparative static effects

Endogenous variables	Exogenous variables M	v	π	u	Y	T	G	B
P	(+)	(-)	(+)	(-)	(-)	0	0	0
F	0	(+)	(?)	0_a	(?)	0	0	0
S	0	$(-)_b$	(+)	$(-)_a$	(?)	(+)	(-)	(-)
$\dot{v}$	0	(-)	(+)	(-)	(?)	(-)	0	0
$\dot{B}$	0	(-)	(+)	(-)	(-)	(-)	(+)	(+)

Notes: (a) Given restriction (2.9c);
(b) given restriction (2.9b).

The short-run comparative static effects of changes in π, u and all other exogenous variables are formally derived in the appendix and summarised in Table 2.4 above. The comparative-static effects on the private sector's accumulation of real financial wealth are, given equation (2.15), the algebraic sum of the effects on the current account balance and on the issue of new debt. In deriving the effect of a change in real financial wealth (v) on the current account balance, as shown in the appendix, we make use of restriction (2.9b). Namely, we are assuming that the positive effect of a rise in real financial wealth on absorption $(C_W + C_{y^D}.i)$ that comes about through an increase in real wealth and disposable income, overoffsets the positive effect of a rise in v on the service account (i); this quantitatively very plausible restriction, i.e. $(C_W + C_{y^D} i - i) > 0$, is a sufficient condition for dS/dv to be positive. (See Ando and Modigliani, 1963)

As can be seen from Table 2.4, changes in the level of government expenditures, the outstanding stock of government debt and the level of real taxes, have no effect whatsoever on the short-run asset market equilibrium, affecting only the current account balance and the issue of government debt. Changes in G and B in particular, do not even affect the private sector's accumulation of real financial wealth and hence, we would expect them not to affect the long-run stationary state as well.

The exogenous disturbances, in general, shift the system away from its long-run equilibrium position, and through a sequence of short-run equilibria it returns to the stationary state provided the system is dynamically stable. Before examining the dynamics, though, we analyse the long-run stationary state.

2.4 The long-run stationary state

Assuming for the moment that the system is dynamically stable, the sequence of short-run equilibria will lead to the long-run stationary state where real wealth is constant, held in the desired proportions and where all nominal variables grow at the same rate; expected inflation is equal to the actual rate, equal to the rate of growth in the nominal money stock. For real wealth to be constant, net private savings (defined as the sum of gross private savings and capital gains, or equivalently, equal to the difference between real disposable income and consumption) must be zero and hence, real private consumption has to be equal to real disposable income, properly defined; thus the long-run marginal propensity to consume is equal to unity. Since, however, the rate of inflation in the long-run is positive and equal to the rate of increase in nominal balances (m), real money balances are depreciating every period by

$$m\frac{M}{P} \left(= \pi\frac{M}{P} = \frac{\dot{P}}{P} \cdot \frac{M}{P}\right) ,$$

which represents an inflation tax or a capital levy on real money balances. To make up for this capital loss, the private sector is saving at every period a part of its income equal to the expected capital loss $(\pi\frac{M}{P})$; equal to the actual capital loss $(m\frac{M}{P})$ since $m = \pi$, equal to the inflation tax levied by the government.[14]

14. Consumption in the long-run is equal to

$$C = C(Y^D, W) = Y^D = Y - T + i(v-F) - \pi\frac{M}{P}$$

Real disposable income is defined to exclude that part of private income $(Y - T + i(v - F))$ that is used to compensate capital losses, i.e. $\pi M/P$.

Hence, gross private savings are

$$S_P = Y - T + i(v - F) - C(Y^D, W)$$

$$= Y - T + i(v - F) - [Y - T + i(v-F) - \pi\frac{M}{P}]$$

$$= \pi\frac{M}{P} = m\frac{M}{P} \qquad (2.18)$$

Substituting (2.18) and government savings from (2.13) into the definition of the current account (2.14), we deduce that the current account balance in the long-run is equal to minus the issue of new government debt,[15] i.e.

$$S = m\frac{M}{P} - (\dot{B} + m\frac{M}{P})$$

$$= - \dot{B} \qquad (2.19)$$

The issue of new government debt in the long-run can be positive, negative or zero depending on the government budget constraint, i.e.

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15. This can also be seen from equation (2.15), i.e.

$$\dot{v} = \dot{S} + \dot{B}$$

Since the stock of real financial wealth in the long-run is constant, $\dot{v} = 0$, the current account balance is equal to minus the issue of new government debt.

$$G + iB = T + \dot{B} + m\overline{M}^* \qquad (2.20)$$

where $\overline{M}^*$ are the long-run real money balances, over which the government has no control. If real taxes (lump-sum) and the inflation tax revenue exactly offset the total government expenditures $(G + iB)$, the remaining budget deficit to be financed by issue of bonds is zero; if "total tax revenue" $(T + m\overline{M}^*)$ falls short or exceeds the total government expenditures, $\dot{B}$ is positive or negative and increasing or decreasing over time respectively. Of course, as will be shown below, the government can always adjust the level of its expenditures or its taxes or both to stabilize the outstanding government debt.

The equilibrium conditions for the long-run stationary state are the following:[16]

$$f_1\ [Y,\ (\overline{M} + v),\ i,\ \pi,\ u] = \overline{M} \qquad (2.21a)$$

$$f_2\ [Y,\ (\overline{M} + v),\ i,\ \pi,\ u] = F \qquad (2.21b)$$

$$Y = C[(Y - T + i(v-F) - \pi\overline{M}), (\overline{M}+v)] + G - i(v-F-B) - \dot{B} \qquad (2.21c)$$

16. The fact that asset holders in the long-run hold both foreign exchange with a zero nominal return and domestic or foreign bonds bearing a positive yield and both subject to the same exchange rate risk, implies the existence of certain differences in their inherent characteristics that make the private sector willing to diversify its portfolio in spite of their different real returns.

$$G + iB = T + \dot{B} + m\overline{M} \tag{2.21d}$$

$$\pi = \frac{\dot{P}}{P} = m \tag{2.21e}$$

Equations (2.21a) and (2.21b) represent equilibrium in the money and foreign exchange markets respectively; taken together they imply asset market equilibrium and that real wealth is held in the desired proportions. Equation (2.21c), which is derived from (2.16c) and (2.19), shows the current account balance in the long-run and reflects the fact that net private savings are zero, or equivalently, that private real wealth is constant. Equation (2.21d) repeats the government budget constraint, while equation (2.21e) shows that expectations are realised in the long-run. From the above equilibrium conditions we can solve for the long-run stocks of real money balances ($\overline{M}$), holdings of foreign exchange (F) and real financial wealth (v), the issue of new government debt ($\dot{B}$) at every point in time and for expected inflation (π); all in terms of the exogenous variables of the system, i.e. Y, T, G, m, i, u and B.

As an interesting by-product of our assumption that domestic bonds are perfect substitutes for foreign bonds, the long-run stationary state is consistent with domestic debt being constant, increasing or decreasing over time. This is similar to the result reached by McKinnon and Oates (1966) who showed that the current account deficit in the long-run is equal to the budget deficit, equal to the issue of new debt. The only difference is that in our model the issue of government debt can be increasing or decreasing over time because of the explicit incorporation of a service account, whereas the government pays interest yields on the existing bond holdings. Both in our model and in McKinnon and Oates (1966), the long-run stationary state equilibrium (when $\dot{B} \neq 0$) depends on the willingness of the government to go on increasing or decreasing its debt over time and on the ability of the rest of the world to absorb the issues of new debt without any change in its portfolio allocation decisions.[17] As can be seen from the long-run comparative static effects, derived in the appendix and shown in summary form in Table 2.5, changes in the outstanding government debt ($\dot{B} \neq 0$) have no effects whatsoever on the long-run stationary state as the asset market

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17. The fact that the rest of the world is presented in our model to be completely unaffected by the developments in the small domestic economy can be justified by either of the following three reasons:
(i) The rest of the world maintains an active policy of preserving a constant price level, P*, equal to unity a fixed nominal interest rate i and a perfectly elastic demand and supply of bonds at the given nominal yield.
(ii) The rest of the world is in a stationary state, whereas the developments in the domestic economy, being a small country, can be ignored, or
(iii) The rest of the world is in equilibrium and the repercussions of the activities of the domestic economy are offset by developments in other small countries among whom there are no economic linkages.

equilibrium and the long-run inflation rate remain unaffected. Nevertheless, the outstanding government debt cannot go on increasing or decreasing for ever and the government has ultimately to adjust its real expenditures or its taxes so that the stock of domestic bonds is stabilized; i.e. $\dot{B} = 0$. As can be seen from Table 2.5 (column 4), the government can very easily bring this adjustment about by varying analogously the level of its expenditures without deviating the system from its stationary state position. This does not hold, though, if taxes are adjusted instead since, as column (3) shows, the stationary state is disturbed.

Table 2.5

Long-run comparative static effects

Endogenous variables	Exogenous variables Y	u	T	G	B
$\overline{M}$	(+)	(+)	(-)	0	0
v	(?)	$(-)_a$	(-)	0	0
F	(?)	$(-)_a$	(-)	0	0
$\dot{B}$	(-)	(-)	(-)	(+)	(+)

Notes: (a) sufficient condition for this is restriction (2.9c).

The long-run equilibrium can be illustrated diagrammatically with the help of Figure 2.3. The MM*

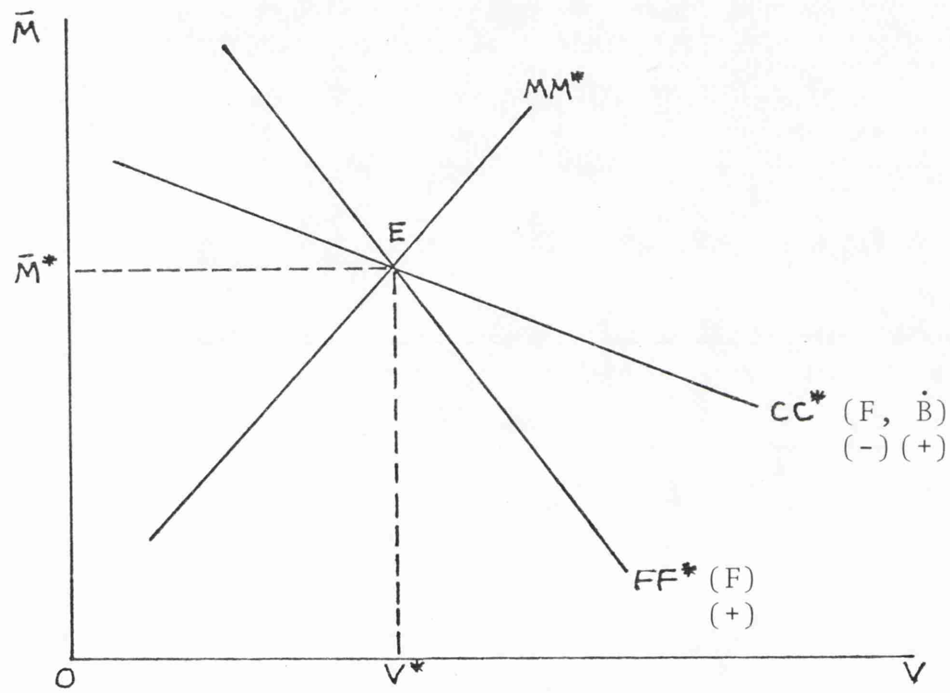

Figure 2.3. The long-run stationary state.

schedule, derived from equation (2.21a), shows combinations of real money balances and real financial wealth for which the money market is in equilibrium; it is upwards sloping[18] because an increase in real financial wealth produces an increased demand for real balances. Table 2.6 shows how the MM* curve is shifted as other variables increase.

Table 2.6

Shifts in the MM* schedule

Real money (v = const.)	Shifting variables Y	u	T	G	F	$\dot{B}$
$\bar{M}$	(+)	(+)	0	0	0	0

(+) = upwards, 0 = unchanged.

The FF* schedule, derived from (2.21b), shows combinations of real money balances and stocks of real financial wealth consistent with foreign exchange market equilibrium for a given stock of foreign exchange; it is

18. From (2.21a) we have:

$$\left.\frac{d\bar{M}}{dv}\right|_{MM*} = \frac{f_{1_w}}{1-f_{1_w}} > 0$$

downwards sloping,[19] since for a given stock of foreign exchange, an increase in real financial wealth increases wealth and leads to an excess demand for foreign exchange; to offset this real money balances have to fall by exactly the same amount (the absolute slope of FF* is unity). How FF* is shifted is shown in Table 2.7.

Table 2.7

Shifts in the FF* schedule

Real money (v = const.)	Shifting variables Y	u	T	G	F	$\dot{B}$
$\bar{M}$	(+)	(+)	0	0	(+)	0

(+) = upwards, 0 = unchanged.

Finally, the CC* schedule, derived from (2.21c), shows the locus of real money balances and real financial wealth for which private net savings are zero and hence real wealth is constant; if $\dot{B} = 0$ the current account is balanced as well (S = 0). Given restriction (2.9b), namely, that an increase in real financial wealth increases domestic absorption by more than it improves the service

19. From (2.21b) we have

$$\left.\frac{d\bar{M}}{dv}\right|_{FF^*} = - \frac{f_{2_w}}{f_{2_w}} = -1 < 0$$

account, the CC* schedule is downwards sloping,[20] since an increase in real financial wealth leads to dissaving and a fall in real money balances is needed to offset it. Table 2.8 shows how the CC* schedule is shifted; note that both the stock of government bonds and the issue of new government debt are shifting variables.

Table 2.8

Shifts in the CC* schedule

Real money (v = const.)	Shifting variables						
	Y	u	T	G	B	$\dot{B}$	F
$\bar{M}$	(+)	0	(+)	(-)	(-)	(+)	(-)

(+) = upwards, (-) = downwards and 0 = unchanged.

This does not mean, however, that CC* is shifting over time as $\dot{B}$ changes; the effect of a rise in the stock of government bonds is exactly offset by the rise in the issue of new debt that it causes.[21]

20. From (2.21c) we have

$$\left.\frac{d\bar{M}}{dv}\right|_{CC^*} = - \frac{C_W - (1 - C_{y^D})i}{(- C_{y^D}\pi + C_W)} < 0$$

since both the numerator and denominator are assumed positive by restrictions (2.9a) and (2.9b) respectively.

21. If real financial wealth is constant and the stock of government debt is increased by dB, its effects on the relative position of CC* depend on the change in the stock of real money balances. Thus linearizing (2.21c) we have:

/Continued over

The intersection of the three equilibrium schedules in Figure 2.3 determines the long-run equilibrium stocks of real money balances and real financial wealth. The long-run effects of the two exogenous disturbances examined in Section 2.3, will be analysed along with the dynamic response of the system to them in Section 2.6.

The last equilibrium condition, equation (2.21e), defines a differential equation in terms of the exchange rate (price level), i.e.

$$\frac{\dot{P}}{P} = m \tag{2.22}$$

with the following solution:

$$P(t) = P(o)e^{mt} \tag{2.23}$$

But since $\bar{M}^* = M(t)/P(t)$, $P(o)$ becomes $M(o)/\bar{M}^*$. Hence, equation (2.23) can be written as

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Footnote 21 continued from previous page.

$$(-C_{y^D}\pi + C_w)d\bar{M} - \frac{\partial \dot{B}}{\partial B} \cdot dB + (1 - C_{y^D})i\,\frac{\partial F}{\partial B} \cdot dB = -idB$$

From the long-run comparative statics (see appendix), we have: $\partial \dot{B}/\partial B = i$ and $\partial F/\partial B = 0$. Substituting above we get

$$(-C_{y^D}\pi + C_w)d\bar{M} = idB - idB = 0$$

Thus the CC* schedule remains unchanged.

$$\ln P(t) = \ln M(0) + mt - \ln \bar{M}^* \tag{2.24}$$

where M(0) is the nominal money stock at time t = 0, the initial stationary state we started from. Equation (2.24), which is exactly identical to the one derived by Kouri (1976, p. 291), defines the long-run exchange rate path; a long-run exchange rate has no meaning since the rate is steadily depreciating. Note also, that there is a one to one correspondence between the long-run exchange rate path and the stock of real money balances; the lower the stock of real money balances, the "higher" the exchange rate path. A further conclusion that can be drawn from equation (2.24), is that the long-run effect of an once and for all monetary expansion is an equiproportionate increase in the exchange rate[22] (and prices), while maintaining the same rate of depreciation (m) but from a different level; thus, long-run real money balances remain unaffected, i.e. the system is neutral to changes in the money supply. The same long-run neutrality result was also reached by Kouri (1976) and Dornbusch (1976b). Dornbusch assumes a sluggish adjustment in domestic prices and thus an increase in

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22. From (2.24) we have

$$\frac{d \ln P(t)}{d \ln M(0)} = 1$$

i.e. an increase in the nominal money stock depreciates the exchange rate in the same proportion.

money supply, while neutral in the long-run has real effects in the short-run. In our model, however, an increase in the money supply (helicopter money) is neutral even in the short-run, as can be seen from the short-run comparative statics summarized in Table 2.4; an increase in money supply increases prices (depreciates the exchange rate) immediately, leaving the rest of the system completely unaffected.

The dynamic response of the exchange rate to the various exogenous disturbances can easily be established, as in Kouri (1976), from the response of the long-run stock of real balances. An increase in $\bar{M}^*$, for a given nominal stock, implies that the exchange rate has appreciated and its path has shifted downwards; a decrease in $\bar{M}^*$, on the other hand, suggests a long-run exchange rate depreciation with an upward shift of its path. The dynamic response of the current account is no longer identified with the response of the stock of real financial wealth because of the existence of internationally traded domestic bonds. As equation (2.15) indicates, the current account balance is equal to the difference between the accumulation of real financial wealth and the issue of domestic bonds. Nevertheless, the impact effect on the current account is given from the short-run comparative statics, while the long-run effect is given by the effect of the particular exogenous shock on the long-run issue of new government debt, as the accumulation of real financial wealth in the long-run is always zero.

2.5 Dynamic Stability

What moves the system over time is the change in the stock of real money balances, the stock of real financial wealth and the change in expectations about inflation. Dynamic stability, in this context, requires the convergence of the sequence of short-run equilibria to the stationary state, where the stock of real money balances, real financial wealth, wealth in general, are constant and where expectations are both constant and fully realized.

Taking the total differential of (2.16a) in respect to real money balances, real financial wealth and expected inflation, we have:

$$-(1 - f_{1w})d\bar{M} + f_{1w}dv + f_{\pi}d\pi = 0 \qquad (2.25)$$

Since $d\bar{M}/dv > 0$ and $d\bar{M}/d\pi < 0$, (2.25) can be written as an implicit function of v and π, i.e.

$$\bar{M} = \theta(v, \pi); \quad \theta_v > 0, \quad \theta_{\pi} < 0 \qquad (2.26)$$

Defining x as the logarith of real money balances, $x = \ln \bar{M}$, we can transform (2.26) into

$$x = g(v, \pi); \quad g_v > 0, \quad g_\pi < 0 \qquad (2.27)$$

The accumulation of real financial wealth which, from equation (2.15), is made up by the accumulation of foreign exchange, domestic and foreign bonds, can also be written as a function of the stock of real financial wealth and inflationary expectations. From the short-run comparative statics, derived formally in the appendix and summarised in Table 2.4, an increase in real financial wealth appreciates the exchange rate and increases real money balances; given restrictions (2.9a) and (2.9b) the increases in v and $\overline{M}$ increase absorption, reduce the current account surplus and decrease the issue of domestic bonds, exerting thus a negative influence over the accumulation of real financial wealth. An increase in expectations, on the other hand, as it was shown in Section 2.3, leads to an improvement in the current account and to an increased issue of new debt, speeding up the accumulation of real financial wealth. Hence, we can write[23] $\dot{v}$ as

$$\dot{v} = h(v, \pi); \quad h_v < 0, \quad h_\pi > 0 \qquad (2.28)$$

To illustrate the sensitivity of the model to the way expectations are formed, we are examining two

23. For a similar formulation of differential equations out of short-run comparative statics see Branson (1976).

types of expectations generation functions - adaptive expectations and perfect foresight - the incorporation of which will produce remarkably different results both to exchange rate dynamics and to the dynamic stability of the whole system. Thus, we have the following two alternative specifications of the formation of expectations mechanism:

(a) Adaptive expectations:

$$\dot{\pi} = \beta(\frac{\dot{P}}{P} - \pi) = \beta(m - \dot{x} - \pi) \qquad (2.29a)$$

(b) Perfect foresight: $\pi = \frac{\dot{P}}{P} = m - \dot{x}$ (2.29b)

(a) Adaptive expectations: The dynamic evolution of the system in this case, is described by equations (2.29a), (2.27) and (2.28), i.e.

$$\dot{\pi} = \beta(m - \dot{x} - \pi); \quad 0 < \beta < 1 \qquad (2.30a)$$

$$x = g(v, \pi) \quad ; \quad g_v > 0, \; g_\pi < 0 \qquad (2.30b)$$

$$\dot{v} = h(v, \pi) \quad ; \quad h_v < 0, \; h_\pi > 0 \qquad (2.30c)$$

As is proved in the appendix, a sufficient condition for a locally stable solution is that the product of the absolute value of the expected inflation elasticity of the demand for money (g_π) and the speed of revision of expectations (β) is less than unity - a condition identical to that of Kouri (1976), i.e.

$$|\beta . g_\pi| < 1 \tag{2.31}$$

The convergence to the long-run stationary state is asymptotic if the characteristic roots are real and oscillating if complex. It is not possible though to prove formally the one or the other and hence, we are going to assume throughout our analysis that the dynamic adjustment is asymptotic.

Figure 2.4 depicts the phase diagrams under adaptive expectations. The $\dot{x} = o$ curve shows combinations of real money balances (in logs) and real financial wealth for which the stock of real balances is constant. Similarly, the $\dot{v} = o$ curve shows the locus of real money balances and real financial wealth consistent with a constant stock of real financial wealth. The equations of both curves are derived in the appendix from equations (2.30). The slope of the $\dot{x} = o$ locus is ambiguous, since

$$\left.\frac{dx}{dv}\right|_{\dot{x}=o} = \frac{\overset{(+)}{g_V}(\overset{(+)}{g_V}\cdot\overset{(+)}{h_\pi} - \overset{(-)}{h_V}\cdot\overset{(-)}{g_\pi}) - \beta \overset{(+)}{g_V}\cdot\overset{(-)}{g_\pi}}{\underset{(+)}{(g_V}\cdot\underset{(+)}{h_\pi} - \beta \underset{(-)}{g_\pi})} = ? \tag{2.32a}$$

When

$$g_V > \frac{h_V \cdot g_\pi}{g_\pi} \tag{2.32b}$$

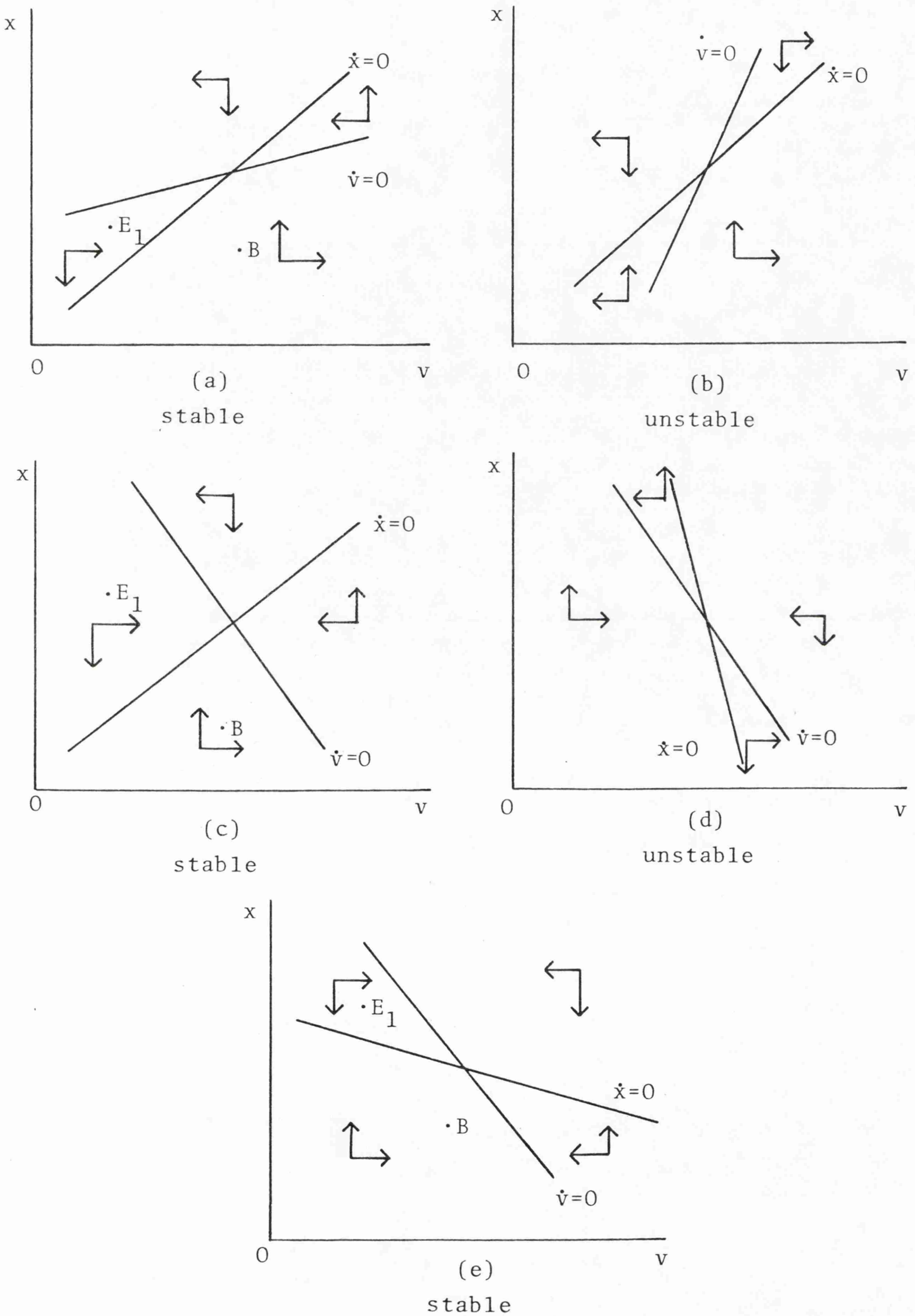

Figure 2.4. Phase diagram under adaptive expectations.

$\dot{x} = o$ is upwards sloping, but if the inequality sign is reversed, its slope can be positive or negative. Deviations of real money balances from their long-run level are always self-correcting (vertical arrows), since

$$\left.\frac{\partial \dot{x}}{\partial x}\right|_{v=\text{const.}} = \frac{\overset{(+)(+)}{(g_V \cdot h_\pi} - \overset{(-)}{\beta g_\pi)}}{\underset{(-)}{g_\pi}\underset{(+)}{(1+\beta g_\pi)}} < o \tag{2.33}$$

given stability condition (2.31). The slope of the $\dot{v} = o$ curve and the direction of the horizontal arrows depend on inequality (2.32b) as well, for

$$\left.\frac{dx}{dv}\right|_{\dot{v}=o} = \underset{(+)}{g_V} - \frac{\overset{(-)}{h_V} \cdot \overset{(-)}{g_\pi}}{\underset{(+)}{h_\pi}} \gtrless o \qquad \text{as}$$

$$g_V \gtrless \frac{h_V \cdot g_\pi}{h_\pi} \tag{2.34a}$$

$$\left.\frac{d\dot{v}}{dv}\right|_{x=\text{const.}} = \frac{\overset{(+)}{h_\pi}}{\underset{(-)}{g_\pi}} \left(\frac{h_V \cdot g_\pi}{h_\pi} - g_V\right) \gtrless o \qquad \text{as}$$

$$g_V \gtrless \frac{h_V \cdot g_\pi}{h_\pi} \tag{2.34b}$$

Fig. 2.4 illustrates the full range of possibilities. If inequality (2.32b) holds, the slopes of both

$\dot{x} = o$ and $\dot{v} = o$ are positive and deviations of real financial wealth from its long-run level are cumulative. The $\dot{v} = o$ curve can be either flatter (Fig. 2.4(a)) or steeper (Fig. 2.4(b)) than the $\dot{x} = o$ locus. The latter case, however, is unacceptable because, as the vertical and horizontal arrows show, it implies an unstable long-run equilibrium. If the inequality sign in (2.32b) is reversed, the slope of the $\dot{v} = o$ curve becomes negative and deviations of real financial wealth from its long-run level self-correcting, while the slope of the $\dot{x} = o$ locus can be positive or negative. Diagrams c, d and e of Fig. 2.4 illustrate the three possible subcases.[24] Subcase d, though, should be dismissed, for it implies an unstable long-run equilibrium.

The $\dot{x} = o$ and $\dot{v} = o$ curves do not coincide with the MM* and CC* schedules of Fig. 2.3. Along the MM* schedule real money balances are constant and expectations are fulfilled, i.e. $\pi = m$. The CC* schedule, on the other hand, shows combinations of real financial wealth consistent with constant real private wealth and expectations. Along CC*, the accumulation of real money balances is completely offset by the accumulation of real financial wealth, so that net private savings are zero. The relative positions

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24. The problems of multiple equilibria are assumed away.

of the two schedules, unlike the $\dot{x} = o$ and the $\dot{v} = o$ curves. depend on endogenous variables: MM* depends on π, while changes in π, F and B shift the CC* schedule. Nonetheless, as is shown in the next section, to enhance the intuitive interpretation of our analysis, the dynamic response of the system to exogenous changes is first formally derived in the appendix and then presented in the $\overline{M}$, v space along with the three long-run equilibrium schedules MM*, CC* and FF*.

(b) Perfect foresight: The assumption of perfect foresight appears usually in two forms: perfect myopic foresight and long-run perfect foresight. The former suggests that economic agents can successfully predict the relevant economic variable in every successive period. In our model for example, perfect myopic foresight implies that people can fully predict the rate of exchange rate depreciation (expected inflation) at every moment in time, i.e. equation (2.29b) holds. Formally, the assumption of perfect myopic foresight implies that economic agents know the short-run comparative statics with certainty. Long-run perfect foresight, on the other hand, means that people over and above the information they have under myopic foresight, also know the long-run comparative static effects; thus to the extent that a particular exogenous shock produces an impact effect different either in magnitude or in direction of change from the long-run effect, economic agents adjust their expectations accordingly, reducing thus the discrepancy between impact and long-run effects, while at the same time their expectations are fulfilled at every point in time. Long-run perfect foresight is the deterministic equivalent of the rational expectations assumption for stochastic models, where economic agents' subjective probability distribution of a particular economic variable coincides with the objective distribution; hence, the expectation of the relevant variable differs from its actual value only by an error term, orthogonal to the prediction itself.[25] In a deterministic model there are no error terms and expectations and actual values of economic variables coincide with the means of their respective probability distributions. In terms of our model, under long-run perfect foresight, equation (2.29b) still holds.

The dynamic evolution of the system under perfect foresight (both myopic and long-run perfect foresight) is described by equations (2.29b), (2.27) and (2.28). Substituting the first into the other two, we derive the following system of differential equations:

$$x = g(v, m - \dot{x}); \quad g_v > 0, \ g_\pi < 0 \tag{2.35a}$$

$$\dot{v} = h(v, m - \dot{x}); \quad h_v < 0, \ h_\pi > 0 \tag{2.35b}$$

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25. For a critical survey of the rational expectations models with an explicit analysis of the information requirements, see Schiller (1978) and Friedman (1979).

As in shown in the appendix, the characteristic roots of the above system, linearised around the stationary state, are real and of opposite signs which is a sufficient condition for the stationary state to be locally a saddle point.

The phase diagram under perfect foresight is depicted in Fig. 2.5. The $\dot{x} = o$ and $\dot{v} = o$ loci are derived from equations (2.35a) and (2.35b) respectively and have the same interpretation as in Fig. 2.4. The slope of the $\dot{v} = o$ curve and the direction of the horizontal arrows are again ambiguous, given by the same expressions as under adaptive expectations (see appendix for details). There is, however, a notable difference in the dynamic adjustment of real money balances. The $\dot{x} = o$ locus is now unambiguously upwards sloping, since

$$\left.\frac{dx}{dv}\right|_{\dot{x} = o} = g_V > 0 \tag{2.36a}$$

Moreover, under perfect foresight, deviations of real money balances from their long-run level become cumulative; initial discrepancies tend to be exacerbated rather than diminished. Formally, we have:

$$\left.\frac{\partial \dot{x}}{\partial x}\right|_{v = \text{const.}} = - \frac{1}{g_\pi} > 0 \tag{2.36b}$$

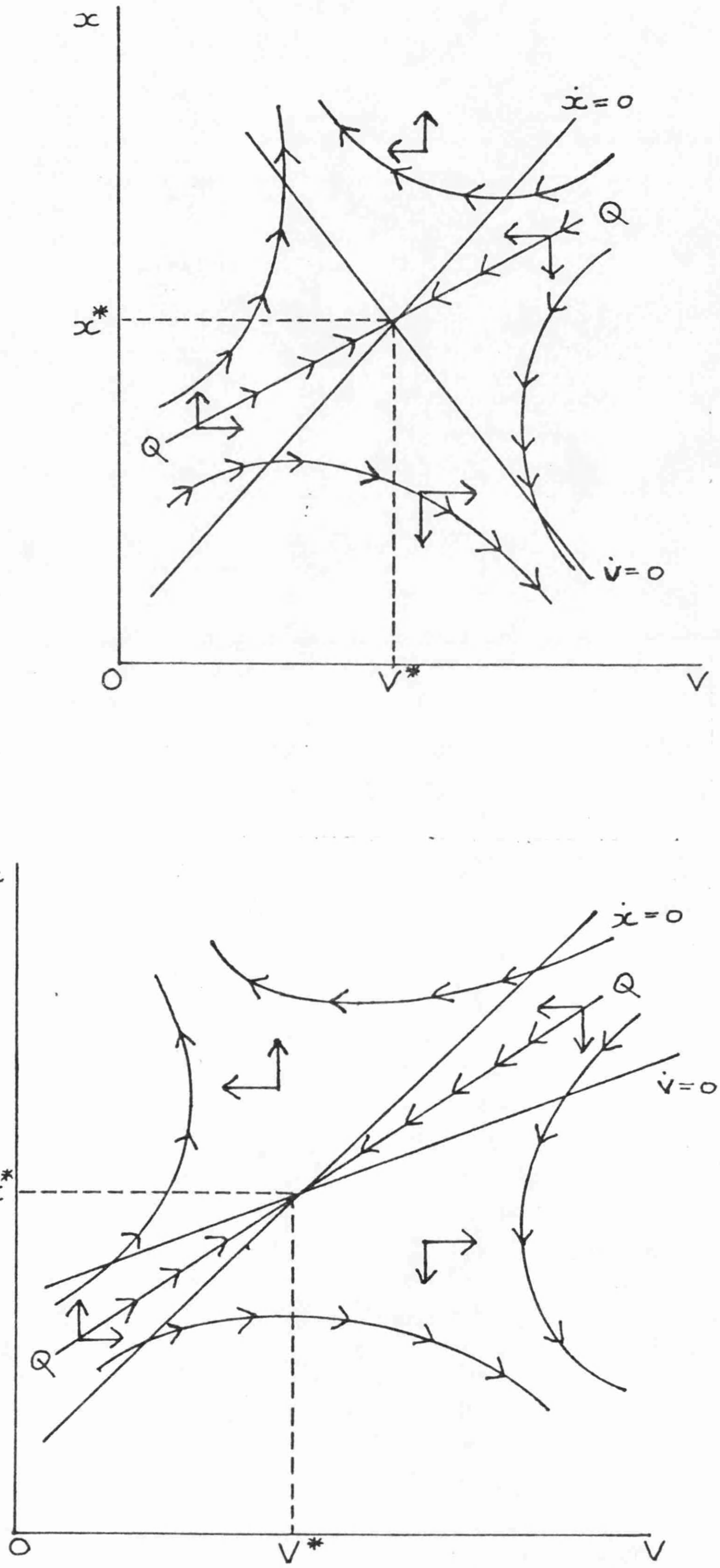

Figure 2.5. Phase diagram under perfect foresight.

The reason for this, is that to induce people to hold a larger (lower) money stock, the rate of inflation - which is the opportunity cost of holding money - has to decline (increase). Therefore, real money balances rise (fall) at a faster rate than before, deviating even further from equilibrium.[26]

The slope of the $\dot{v} = o$ curve and the direction of the horizontal arrows depend on inequality (2.32b). Fig. 2.5 illustrates the two subcases. In diagram (b), inequality (2.32b) holds and $\dot{v} = o$ is positively sloping but flatter than the $\dot{x} = o$ curve; deviations of real financial wealth from its long-run level are self - correcting. In diagram (a), the inequality sign of (2.32b) is reversed and the slope of the $\dot{v} = o$ locus becomes negative. In addition, discrepancies of real financial wealth from its long-run level are cumulative. In both cases, though, we have a saddle point equilibrium.

Under perfect foresight we have indeterminacy (the long-run stationary state is a saddle point); from any initial exchange rate there is a real money balances and real financial wealth path such that expectations are continuously fulfilled and all markets are in equilibrium. There is, however, only one path that can take the system to its long-run stationary state (the QQ trajectory). It is exactly at this juncture that perfect myopic foresight

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26. For a more detailed analysis od this phenomenon in perfect foresight models, see Stein (1970), Ch. 1.

and long-run perfect foresight really differ. If we postulate simply myopic foresight, the economy can embark on the equilibrating path only by chance. With long-run perfect foresight, however, for a given stock of real financial wealth (v_o) people on the impact of any exogenous shock, adjust their expectations in the fashion explained above so that by affecting real money balances shift the economy on the dynamic adjustment path QQ that leads to the new long-run equilibrium. The problem of indeterminacy, well known in models of money and growth,[27] became recently a familiar feature in models of exchange rate dynamics, e.g. Dornbusch (1976b) and Kouri (1976). Ethier (1979) provides an analysis of the stability implications of perfect foresight in models incorporating the asset market approach.

By assuming long-run perfect foresight (rational expectations), we are effectively ignoring the diverging dynamic paths and concentrating on the stabilizing QQ trajectory. An important property of this adjustment path, extensive use of which is going to be made in the section on the dynamic response of the system of this and the next two chapters, is that the stock of real money balances always moves monotonically and in the same direction as real financial wealth.

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27. For a discussion on these problems in models of money and growth, see Hahn (1969), Brock (1975) and Black (1974).

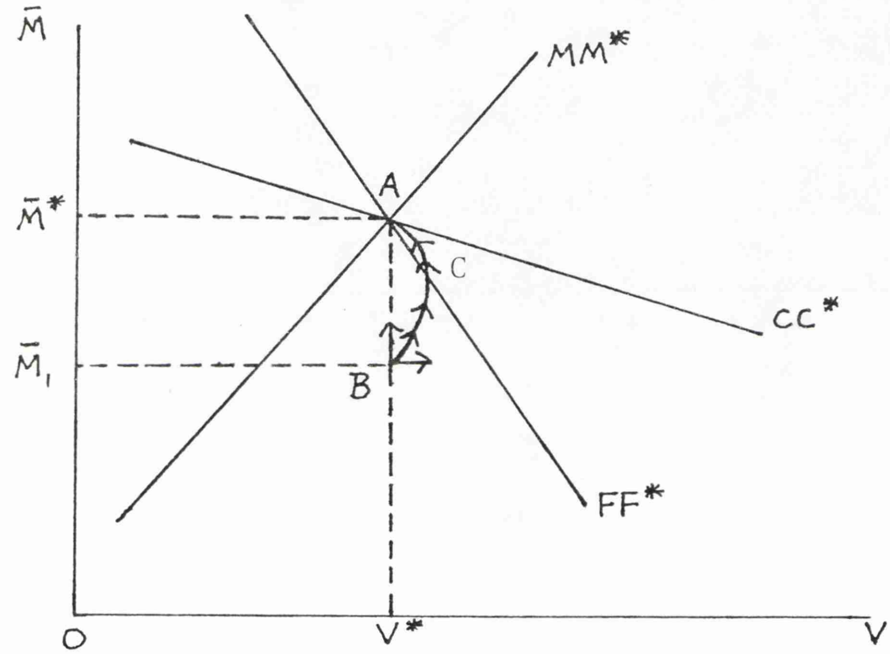

Figure 2.6. Dynamic response of the system to an exogenous rise in expectations under adaptive expectations.

2.6 Dynamic response of the system to exogenous changes

The dynamic response of the stocks of real financial wealth and real money balances to the exogenous shocks, the impact effects of which were discussed in Section 2.3, do, of course, reflect the differences in dynamic adjustment of the two expectations generation mechanisms.

Under long-run perfect foresight, an exogenous shock in expectations, unjustified by existing economic conditions, has no meaning; hence, Figure 2.6 analyses the dynamic response of the system under adaptive expectations only.

Starting from an initial long-run equilibrium position (point A), the exogenous rise in inflationary expectations produces an instantaneous exchange rate depreciation, induces an accumulation of real financial wealth, i.e. $\dot{v} > o$, and gives rise to a current account surplus, for a given nominal money stock and real financial wealth. The long-run stationary state is disturbed and the economy moves to a particular short-run equilibrium (point B), consistent with the new expectations. The exchange rate depreciation reduces instantaneously the stock of real money balances to $\overline{M}_1$, while the stock of real financial wealth remains the same. Over time, however, real money balances are to rise again to their previous stationary state level, since the change in expectations represents a speculative error that does not affect the long-run equilibrium. Hence, the exchange rate, the dynamic response of which is determined by the response of real money balances, is appreciating[28] from point B onwards till the stationary state is re-established. Trajectory BCA is the dynamic path that both real balances and real financial wealth follow to reach long-run equilibrium again. Note, that real financial wealth initially rises and from point C onwards starts falling again to its initial level.

The current account moves into surplus on the impact of the exogenous change, falling gradually under the influence of an appreciating exchange rate and a rising stock of real financial wealth; it even moves into deficit (point C onwards) returning finally to its initial position at point A. The net sum of current account surpluses and deficits added to the sum of net

---

28. The exchange rate appreciates relative to its long-run path in the sense that it is either falling in absolute terms or increasing at a lower rate than the money stock (nominal) is growing (m) - m is equal to the long-run rate of exchange rate depreciation - so that real money balances are rising. See the next section for a detailed examination of exchange rate dynamics and the appendix for the formal derivation.

changes in government bonds should be zero,[29] as the stock of real financial wealth in the stationary state returns to its previous level.

If instead of adaptive expectations, wealth holders are assumed to possess long-run perfect foresight

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29. As is shown in the appendix, the accumulation rates of real money balances and real financial wealth during the adjustment process are given by the following expressions:

$$\dot{x}(t) = \underset{(-)}{\lambda_1}\underset{(-)}{(x(t) - \bar{x})}$$

$$+ \left[\underset{(-)}{g_\pi}\underset{(?)}{(\lambda_2 - h_v)} + \underset{(+)}{h_\pi} \cdot \underset{(+)}{g_v}\right]\underset{(+)}{(\pi_o - \bar{\pi})}e^{\lambda_2 t}$$

$$\dot{v}(t) = \underset{(-)}{\lambda_1}\underset{(?)}{(v(t) - v^*)} + \underset{(+)}{h_\pi}\underset{(+)}{(\pi_o - \bar{\pi})}e^{\lambda_2 t}$$

where λ_1 and λ_2 are the two negative characteristics roots and $\bar{x} = \ln\bar{M}^*$. Given the phase diagram analysis of Figure 2.4, point B lies below the $\dot{x} = o$ curve, so that real money balances are increasing throughout the adjustment process (unless, of course, the adjustment process is oscillatory) and real financial wealth increases initially and falls afterwards.

such exogenous shocks in expectations are avoided and the exchange rate is cushioned against unreasonable speculative errors.

The dynamic response of the system to an exogenous fall in the subjective estimate of foreign exchange risk relative to domestic money (u), shown in Figure 2.7, illustrates quite clearly the sensitivity of the adjustment process to the way expectations are formed. Starting from an initial long-run equilibrium at E_0, the exogenous fall in u shifts the stationary state to E_2 where real financial wealth is increased and real money balances are reduced, reflecting the increased risk of holding domestic money relative to foreign money.

Under adaptive expectations the impact effect of a fall in u is a rearrangement of private portfolio holdings so that the proportion of real financial wealth relative to domestic real money is higher than before. In the short-run, however, wealth holders cannot increase their holdings of real financial wealth instantaneously and hence, at the new structure of their asset preferences (produced by the fall in u) they have excess real money balances; their efforts to dispose of the excess money stock drive prices up (the exchange rate depreciates instantaneously), reducing the value of real money balances and real wealth. The reduction in wealth leads to a lower consumption level and a current account

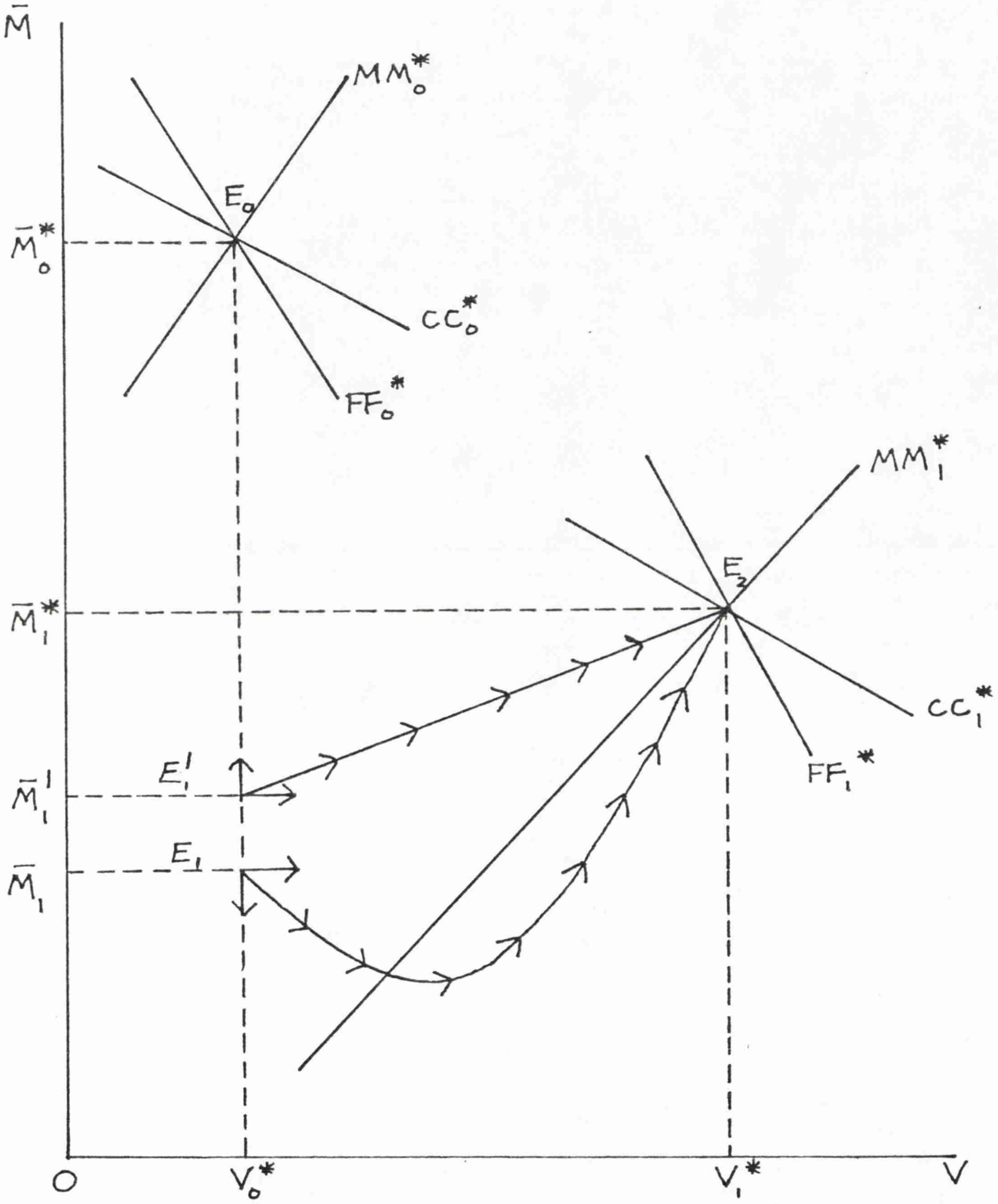

Figure 2.7. Dynamic response of the system to an exogenous fall u, both under adaptive and rational expectations.

surplus, initiating a process of accumulation of real financial wealth. In contrast to long-run perfect foresight, expectations in the short-run are given by the past history of prices, and hence, the impact effects of the exogenous fall in u are those analysed in Section 2.3. The very fact that in the short-run the private sector is constrained by the existing stock of real financial wealth, $v_o{}^*$ in Figure 2.9, implies that the holdings of real money balances have to fall considerably more on the impact of the exogenous change than in the new stationary state at E_2, where the level of real financial wealth is higher. Thus in the short-run real money balances are drastically reduced below their long-run level (point E_1), forcing the exchange rate to overshoot its long-run path. Formally, the overshooting under adaptive expectations is shown by the negative difference between the long-run and short-run comparative static effects of a fall in u on real money balances, i.e.

$$\left.\frac{d\bar{M}}{du}\right|_{LR} - \left.\frac{d\bar{M}}{du}\right|_{SR} = \left.\frac{d\bar{M}}{du}\right|_{LR} + \frac{M}{P^2} \cdot \left.\frac{dP}{du}\right|_{SR}$$

$$= \frac{1}{|A_2|} \cdot f_{1_w} \{- \frac{f_{1_u}}{(1-f_{1_w})} (-C_{y^D}\pi + C_w + m)$$

$$- (1 - C_{y^D}) i \, [f_{2_u} + \frac{f_{1_u} \cdot f_{2_w}}{(1 - f_{1_w})}]\}$$

$$= \frac{\overset{(+)}{f_{1_w}}}{\underset{(+)}{|A_2|}} \cdot \underset{(-)}{\left.\frac{\partial \dot{v}}{\partial u}\right|_{SR}} < 0 \qquad (2.37)$$

From point E_1 onwards, the economy, following the dynamic adjustment path E_1E_2, approaches the new stationary state (the adjustment process is assumed asymptotic). On the impact of the exogenous change, the current account moves into surplus, initiating a process of accumulation of foreign assets ($\dot{v} > o$). Given the phase diagram analysis of Figure 2.4, this implies that point E_1 lies below the $\dot{v} = o$ locus and that real financial wealth is monotonically increasing throughout the adjustment process. The dynamic eveolution of real money balances, however, depends on two counteracting forces: on the one hand, given the error-learning mechanism, the initial exchange rate depreciation will lead to an increase in expected inflation (π rises), which in turn tends to decrease real balances even further. The increasing stock of real financial wealth, on the other hand, increases the demand for all assets.

If the first effect prevails over the second[30],

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30. A necessary and sufficient condition for this is that point E_1 lies above the $\dot{x}$=o curve in figure 2.4. Formally, the accumulation rates of real money balances and real financial wealth during the adjustment process are given by the following expressions (see appendix for details):

$$\dot{x}(t) = \underset{(-)}{\lambda_1}\underset{(-)}{(x(t) - \bar{x})}$$
$$- \left[\frac{\overset{(-)}{g_\pi}}{\underset{(+)}{h_\pi}}\underset{(?)}{(\lambda_1 - h_v)}\underset{(?)}{(\lambda_2 - h_v)} + \underset{(+)}{g_v}\underset{(?)}{(\lambda_1 - h_v)}\right]\underset{(-)}{(v_o^* - v_1^*)}e^{\lambda_2 t}$$

$$\dot{v}(t) = \underset{(-)}{\lambda_1}(v(t) - v_1^*) - \underset{(?)}{(\lambda_1 - h_v)}\underset{(-)}{(v_o^* - v_1^*)}e^{\lambda_2 t}$$

where $\bar{x} = \ln\bar{M}_1^*$.

real money balances decrease even further than at the impact effect. Over time, however, as wealth holders catch up with their expectations, real balances start rising again, along with the increasing stock of real financial wealth.

Under rational expectations, however, the picture is quite different. Economic agents predict the long-run reduction in real money balances and the ensuing long-run exchange rate depreciation and knowing that the impact effect of a fall in u is to decrease real money balances below their long-run path (i.e. depreciating too much), they are led to expect an exchange rate appreciation. This change in expectations which occurs simultaneously with the exogenous disturbance, cushions the fall in real money balances and moderates the exchange rate overshooting (point E_1'). Thus, the reduction in real money balances and the exchange rate overshooting on the impact of the exogenous change in u are lower[31].

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31. By solving the differential equations describing the dynamic adjustment under the two expectations mechanisms, the difference between the stock of real money balances (in logs) under rational expectations (x_o') and under adaptive expectations (x_o) at time t = o (impact effect), is positive, i.e.

$$(x_o' - x_o) = - \frac{\overset{(+)}{g_v} \cdot \overset{(-)}{g_\pi} \cdot \overset{(-)}{\lambda}}{\underset{(-)(-)}{(1 + g_\pi \lambda)}} \underset{(-)}{(v_o^* - v_1^*)} > 0$$

See appendix for derivation.

Under rational expectations, however, the picture is quite different. Economic agents predict the long-run reduction in real money balances and the ensuing long-run exchange rate depreciation and knowing that the impact effect of a fall in u is to decrease real money balances below their long-run level with the exchange rate overshooting its long-run path (i.e. depreciating too much), they are led to expect an exchange rate appreciation. This change in expectations which occurs simultaneously with the exogenous disturbance, cushions the fall in real money balances and moderates the exchange rate overshooting (point E_1'). Thus, the reduction in real money balances and the exchange rate overshooting on the impact of the exogenous change in u are lower[31]

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$$(x_o' - x_o) = - \frac{\overset{(+)}{g_V} \cdot \overset{(-)}{g_\pi} \cdot \overset{(-)}{\lambda}}{\underset{(-)(-)}{(1 + g_\pi \lambda)}} (v_o^* - v_1^*) > 0$$

See appendix for derivation.

than under adaptive expectations ($E_1' > E_1$). The current account surplus though is lower, as the fall in u is accompanied by an expected exchange rate appreciation that moderates the impact effect; over time, the current account remains in surplus till the new long-run equilibrium is established.

With long-run perfect foresight the economy is shifted on the equilibrating dynamic adjustment path $E_1'E_2$ (the equivalent of the QQ trajectory of Figure 2.5) that would lead, through a gradual and monotonic increase in both real money balances and real financial wealth, to the new stationary state. Formally, both assets are monotonically rising during the adjustment process since, from the solution of the differential equations (2.35) we have (see appendix for details):

$$\dot{x}(t) = \underset{(-)}{\lambda} \; \underset{(-)}{(x(t) - \bar{x})} > 0 \tag{2.38a}$$

$$\dot{v}(t) = \underset{(-)}{\lambda} \; \underset{(-)}{(v_0^* - v_1^*)} e^{\lambda t} > 0 \tag{2.38b}$$

where λ is the negative characteristic root. Hence, during the dynamic adjustment the exchange rate will be gradually appreciating relative to its long-run path till the new stationary state is established.

Therefore, the dynamic response of the system to the exogenous fall in the subjective estimate of foreign exchange risk, irrespective of the mechanism of expectations formation, involves an overshooting of the exchange rate and the stock of real money balances; the current account during the adjustment process is always in surplus.

The major difference in adjustment between the two expectations mechanisms is that the assumption of rational expectations, with the knowledge of the short-run and long-run comparative statics that it entails, cushions the effects of exogenous changes, relative to the case of adaptive expectations, on asset holdings, the exchange rate and the economy in general, moderating the discrepancies from the long-run equilibrium position. The fundamental difference in dynamic adjustment under adaptive and rational expectations is reflected in the evolution over time of the whole system (notice the difference in the direction of the arrows at points E_1' and E_1 of Figure 2.7). While under rational expectations real balances and real financial wealth are monotonically increasing towards the stationary state with the exchange rate exhibiting an unambiguous gradual appreciation relative to its long-run path, under adaptive expectations we have initially an even further reduction in real balances and a further exchange rate depreciation before eventually, as speculators catch up with their expectations, they start rising and appreciating respectively.

It is to be expected that because of the correct expectations and the cushioning that long-run perfect foresights provides to the system, the speed of dynamic adjustment might be greater than under adaptive expectations and hence, it would lead to a shortening of the adjustment period.

2.7 Exchange rate variability and the asset market approach

During the recent period of floating exchange rates, 1973 onwards, it has notably been observed that the exchange rate is subject to excessive short-run variability, posing some troubling and still puzzling aspects of the behaviour of floating exchange rates. Friedman (1953), one of the early supporters of a flexible exchange rates regime, suggested that exchange rate variability should not surprise us since it would simply reflect the variability of the underlying economic forces. The monetary models of exchange rate determination (such as Frenkel (1976) and Myrhman (1976)), as Bilson (1979) argues, appear to be unable to account for the current short-run exchange rate fluctuations, because of their somewhat long-run nature. In a recent survey of the relevant theoretical and empirical literature, Schadler (1977, p. 225) notes:

> "... the last four years' experience with floating rates suggests that there may be more to the adjustment process than the simple monetarist argument implies. There is for example, concern that variations of floating exchange rates have been larger than those of determining factors. Also, changes in exchange rates have often exceeded differences between countries' rates of inflation."

The economic costs of these fluctuations indicate their importance and put the problem in perspective. The main costs of short-run exchange rates variability, Schadler (1977), Artus and Crockett (1978) and Artus (1978), can be summarised as follows: First, a rise in the costs of international transactions because of the risk of exchange rate changes during the contract period; these costs, however, are not so serious as was initially thought and, moreover, hedging mechanisms exist at moderate costs (Crockett and Goldstein, 1976).

Second, more prolonged deviations from the long-run equilibrium exchange rate are likely to produce misallocation of resources on a world scale, create uncertainty and weaken incentives for activities in international transactions unless proper compensation is provided.

Third, and more serious, according to Artus and Crockett (1978), are the adjustment costs of eventually correcting a disequilibrium. These involve shifts of resources from the production of traded to the production of non-traded goods or vice versa, with consequent in a world of rigidities transitional unemployment of both labour and capital which entail considerable social and political costs. Changes in the domestic structure of production will also affect foreign competitors to the extent that relative shares in world markets are affected.

Fourth, the possible incompatibilities in national exchange rate policies, in the absence of reconciling mechanisms, might produce political consequences of unquantifiable costs. Finally, the short-run exchange rate fluctuations give rise to domestic price variability; and even more seriously, to the extent that domestic monetary authorities are not capable of controlling the money supply because of economic, social and political costs, exchange rate changes may be self-validating leading to inflation or deflation respectively.

The simple monetary approach to exchange rate determination, because of its somewhat long-run nature cannot adequately explain the short-run exchange rate variability. The asset market approach, however, is currently the prevailing framework in which short-run exchange rate variability receives a more satisfactory explanation. The exchange rate, as the relative price of domestic and foreign assets, is determined in the short-run predominantly in the asset market by assumption, so that the existing stock of assets are willingly held by asset holders at every point in time. Formally, this is shown in our model by the recursiveness of matrix A_1 in equation (2.17); changes in the goods sector affect the exchange rate over time through the accumulation or decumulation of real financial wealth,[32] unless, of course, they affect expectations (e.g. perfect foresight).

Hence, exchange rate variability is simply due to changes in all those factors that affect the demand for and supply of all relevant financial and real assets, such as changes in expectations, changes in interest rates and other rates of return, changes in risk aversion of wealth holders, changes in tastes, or changes in government policies that affect asset supplies or demands or impinge on expectations. Artus and Crockett (1978) provide a thorough analysis of the importance of some of the above factors for exchange rate variability in the context of an asset market approach; Schadler (1977) provides a survey of the relevant literature.

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32. In a model that allows for traded and non-traded goods, to the extent that prices of the non-traded goods are flexible, the goods market equilibrium will affect the asset market and hence, the short-run exchange rate determination since non-traded goods prices are included in the deflator of real money balances.

More interesting, however, is the fact that the asset market approach is consistent with more than proportionate responses of the exchange rate to exogenous asset market disturbances; i.e. a "magnification" of the original shock. One such common disturbance examined in the literature, is a discrete change in money supply which, as Kouri (1976) and Dornbusch (1976b) show, produces an overshooting of the exchange rate above its long-run path without affecting the long-run equilibrium. In terms of our model, we have seen in Sections 2.3 and 2.6, how a speculative error in expectations leads to a short-run exchange rate fluctuation without affecting the long-run stationary state. On the impact of the exogenous disturbance in the asset market the exchange rate depreciates instantaneously and during the adjustment process, it gradually appreciates relative to its long-run path until long-run equilibrium is restored. This short-run exchange rate fluctuation is illustrated in Figure 2.8 which depicts the evolution of the exchange rate (in logs) over time; the PP line represents the long-run exchange rate path described by equation (2.24). Starting from an initial long-run equilibrium at A, the discrete depreciation shifts the exchange rate to B and over time, following the dynamic adjustment path BC it appreciates towards its long-run path at C.

As the exchange rate in the long-run is depreciating by a constant rate m - because of the inflation

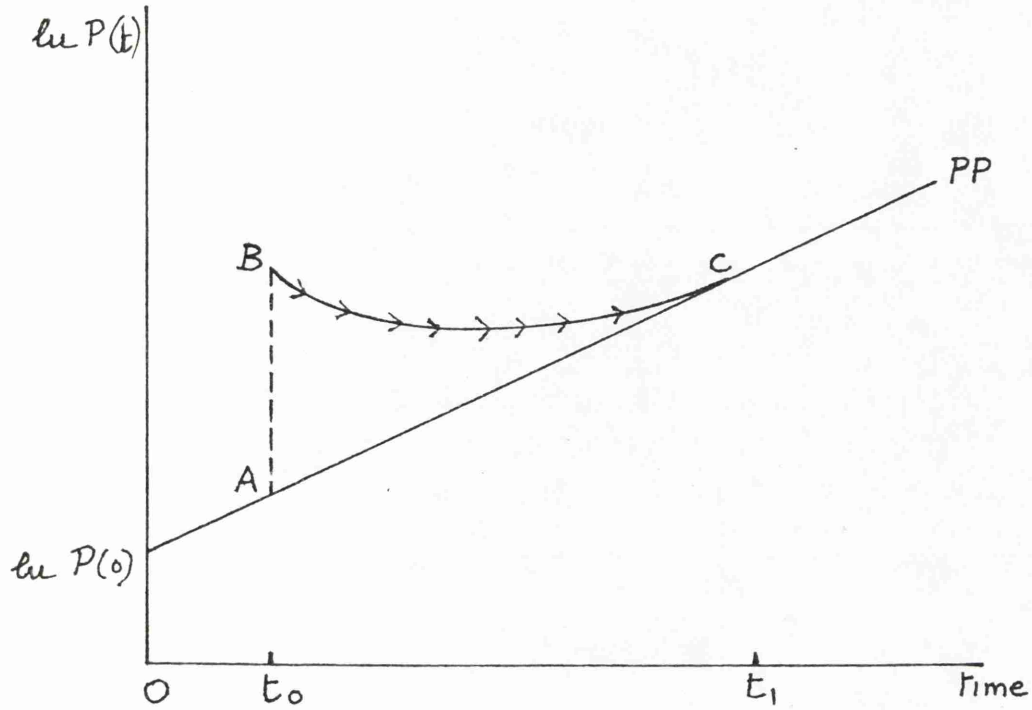

Figure 2.8. Dynamic response of the exchange rate to an exogenou rise in expectations under adaptive expectations.

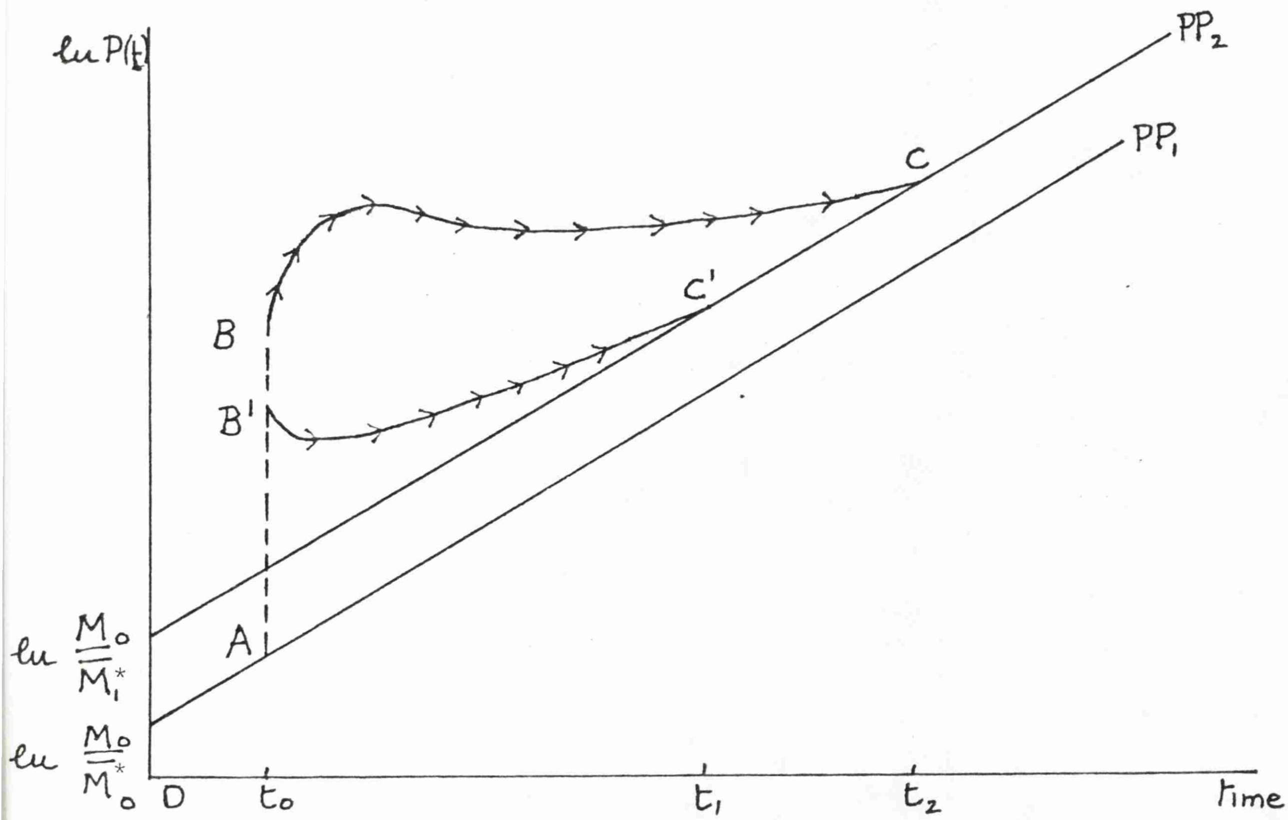

Figure 2.9. Dynamic response of the exchange rate to an exogenou fall in u, both under adaptive and rational expectations.

rates differential between the domestic economy and the rest of the world - a gradual exchange rate appreciation relative to its long-run path, implies that during the adjustment process the exchange rate is rising at a lower rate (even negative initially) than in the long - run. In the case of Figure 2.8, after the impact exchange rate depreciation at B, the rate falls initially in absolute terms and then it starts rising again at a rate lower than its long-run path.[33]

One of the predominant factors that affects the asset market equilibrium is the degree of substitutability between the various assets in private portfolios. The higher (lower) the degree of substitutability between assets the less (more) exchange rates have to adjust to

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33. Formally, the slope of the dynamic adjustment path BC (variable over time) is given by the following expression (see appendix for derivation):

$$\mathrm{DlnP}(t) = m - \underset{(-)}{\lambda_1} \underset{(-)}{(x(t) - \bar{x})}$$

$$- \left[\underset{(-)}{g_\pi} \underset{(?)}{(\lambda_2 - h_{2v})} + \underset{(+)}{h_\pi} \cdot \underset{(+)}{g_v} \right] \underset{(+)}{(\pi_o - \pi)} e^{\lambda_2 t}$$

where $(\pi_o - \bar{\pi})$ is the initial rise in expectations. Given the phase diagram analysis, during the adjustment process DlnP(t) is always less than m. At the stationary state, though, at C, DlnP(t) = m.

make asset holders content to hold the existing stock of assets, because of the ensuing capital flows that provide the necessary cushioning to exogenous disturbances. The degree of substitutability between domestic and foreign assets is related to the currency risk they involve, which in turn is related to the degree of predictability of exchange rate changes during the period a currency is held. Currency risk, as Artus and Crockett (1978) maintain, depends on the "thinness" of exchange markets, on the risk preferences of banks and other large institutional market participants and on the increasing risk of a growing exposure to a particular currency.

To capture the changes in asset substitutability and the ensuing short-run exchange rate fluctuations following changes in risk aversion, we have explicitly incorporated risk in our model by including u as an argument in the asset demand functions. u also stands for changes in the degree of uncertainty of expected relative purchasing power of currencies, which as Artus and Crockett (1978) put it, "make the foreign exchange market more like a stock market where price fluctuations are due more to changing expectations of future profitability rather than current profits" (p. 14-15). Hence, changes in the degree of substitutability among assets, or equivalently, changes in risk aversion and in uncertainty about the relative purchasing power of currencies produce sudden changes in u and lead to short-run exchange rate variability.

We may recall that from the analysis of the dynamic response of the system to a reduction in u, in Section 2.6, we have established an unambiguous exchange rate overshooting on the impact of the exogenous change, irrespective of the expectations mechanism. The exchange rate evolution over time in this case is shown in Figure 2.9 . PP_1 is the long-run exchange rate path prior to the exogenous change and PP_2 is the path consistent with the new asset market conditions; PP_2 lies above PP_1, for the reduction in u reduces real money balances in the new stationary state and shifts the long-run exchange rate path upwards, reflecting a long-run exchange rate depreciation. ABC is the dynamic exchange rate path under adaptive expectations and AB'C' the one under rational expectations. Because of the impact exchange rate overshooting, both B abd B' lie above the PP_2 line. The dynamic adjustment paths BC and B'C' reflect the differences in adjustment of the system under the two expectations mechanisms, referred to in Section 2.6. Given that the slope of the $E_1'E_2$ trajectory in Figure 2.7 is unambiguously positive, as both real money balances and real financial wealth are monotonically rising during the adjustment process (see equation (2.38)), the exchange rate is always appreciating relative to its long-run path. The extent of this appreciation is a positive function[34] of the discrepancy between actual real money balances and their long-run value; at the initial stages of the dynamic adjustment the exchange rate may be actually falling in absolute terms, over time,though,it is rising at a rate less than its long-run depreciation (m).

Under adaptive expectations the exchange rate will initially depreciate even more than the impact effect (D ln P > m) reducing further the real money stock, provided point E_1 in Figure 2.4 lies above the $\dot{x} = o$ schedule. As real balances start rising the exchange rate will be gradually appreciating, rising at a rate less than m, till the new stationary state is reached.[35]

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34. Formally, the slope of the B'C' path is given by the following equation (see appendix for derivation):

$$D \ln P(t) = m - \underset{(-)}{\lambda}(\underset{(-)}{x(t) - \bar{x}})$$

where λ is the negative characteristic root. If at the initial stages of the adjustment process $(x(t) - \bar{x})$ is high enough, it might outweigh m and cause an absolute reduction in the exchange rate. Actually this is the way that is drawn. At the long-run equilibrium at C', $D \ln P_o = m$.

35. The slope of BC over time is given by the following expression (see appendix for derivation):

$$D \ln P(t) = m - \underset{(-)}{\lambda_1}(\underset{(-)}{x(t) - \bar{x}})$$

$$+ [\overset{(-)}{\underset{(+)}{\frac{g_\pi}{h_\pi}}} \underset{(?)}{(\lambda_1 - h_v)} \underset{(?)}{(\lambda_2 - h_v)} + \underset{(+)}{g_v} \underset{(?)}{(\lambda_1 - h_v)}] \underset{(-)}{(v_o{}^* - v_1{}^*)} e^{\lambda_2 t}$$

where λ_1, λ_2 are the two negative characteristic roots. When the net sign of the square bracket is negative, the second term may be outweighted by the rest for small values of t, so that the exchange rate is rising at a rate higher than m, and this is the way that is drawn.

From Figure 2.9 we also note that the stationary state equilibrium under rational expectations is established in less time than under adaptive expectations, since even if the two paths have the same slope, the B'C' path reaches the rising long-run exchange rate path sooner, for it lies below BC. A comparison of the slopes of the two paths (footnotes 34 and 35) reveals that although no unambiguous formal proof can be derived, it seems very unlikely that BC would have a sharp negative slope at some stage during the adjustment process.

In general it is noteworthy, that although we assume instantaneous adjustment in both the goods and asset markets, we still get exchange rate overshooting, while Dornbusch (1976b) attributes this magnification of initial shocks to the sluggish adjustment of domestic prices that he assumes. Kouri (1976), however, using a model similar to ours, shows that under perfect foresight, following an increase in the rate of growth of the nominal money stock, the exchange rate overshoots as well. In general, as Schadler (1977, p. 256) argues:

> "... the explanation for such exchange rate behaviour [overshooting] lies in unresolved issues concerning expectations formation, differential speeds of adjustment among markets, the impact of uncertainty on foreign exchange transactions, and the economic policy decisions."

One word of caution, however, is in order on the use of the asset market approach as the basic explanation for short-run exchange rate variability, since it is based on somewhat strict assumptions giving disproportionate weight in its structure to the asset market with no disaggregation of the goods market and no explicit incorporation of exogenous real shocks such as changes in oil prices or prices of raw materials. The modelling of changes in risk aversion and of differential speeds of adjustment in the various markets needs to be explored even more. Schadler's concluding comment (pp. 290-291), perfectly reflects such reservations:

> "On a theoretical level, the consensus seems to be that exchange rate dynamics are best studied in an asset market equilibrium model, and that without appeal to destabilizing speculation, the model does predict exchange rate fluctuations in excess of fluctuations in underlying conditions. On a more realistic level, however, it does not seem likely that overshooting implicit in asset market equilibrium models is sufficient to account for the magnitude of exchange rate variations observed recently. There seems to be little evidence that conclusively precludes the possibility that at least over particular periods for certain currencies, destabilizing speculation has contributed to exchange rate variations."

In the analysis so far, however, nothing was said about government intervention in the foreign exchange market and we have treated exchange rate fluctuations as arising only out of exogenous shocks in the asset or good markets, implicitly excluding the possibility of moderating or exacerbating these fluctuations by government intervention - a case much more close to the real world. In the next two chapters, using the framework developed so far, we try to comparatively examine the exchange rate dynamics under free and managed floating for different specifications of the expectations mechanisms.

APPENDIX 2

A2.1 Short-run Comparative Statics

Linearising the short-run equilibrium conditions, equations (2.16a) - (2.16d) of Section 2.3, we have:

$$A_1[dP\ dF\ dS\ d\dot{B}]' = B_1[dM\ dv\ d\pi\ du\ dY\ dT\ dG\ dB]' \qquad \text{(A2.1)}$$

where

$$A_1 = \begin{bmatrix} (1 - f_{1w})\frac{M}{P^2} & 0 & 0 & 0 \\ -f_{2w}\frac{M}{P^2} & -1 & 0 & 0 \\ (-C_{y^D}\pi + C_w)\frac{M}{P^2} & -(1 - C_{y^D})i & -1 & 0 \\ m\frac{M}{P^2} & 0 & 0 & -1 \end{bmatrix} \qquad \text{(A2.2)}$$

and

$$B_1 = \begin{bmatrix} (1-f_{1w})\frac{1}{P} & -f_{1w} & -f_{1\pi} & f_{1u} & -f_{1y} & 0 & 0 & 0 \\ -f_{2w}\frac{1}{P} & -f_{2w} & -f_{2\pi} & -f_{2_u} & -f_{2y} & 0 & 0 & 0 \\ (-C_{y^D}\pi+C_w)\frac{1}{P} & C_w-(1-C_{y^D})i & -C_{y^D}\frac{M}{P} & 0 & -(1-C_{y^D}) & -C_{y^D} & 1 & i \\ m\frac{1}{P} & 0 & 0 & 0 & 0 & 1 & -1 & -i \end{bmatrix}$$

(A2.3)

The determinant of the matrix of coefficients, $|A_1|$, is unambiguously negative, i.e.

$$|A_1| = -(1 - f_{1w})\frac{M}{P^2} < 0; \quad \text{since } 0 < f_{1w} < 1 \qquad \text{(A2.4)}$$

Using Cramer's rule the comparative static effects of changes in the exogenous variables are as follows:

(a) Effects of exogenous changes on the price level (exchange rate)

$$\frac{dP}{dM} = \frac{P}{M} > 0 \qquad \text{(A2.5)}$$

$$\frac{dP}{dv} = -\frac{f_{1w}}{(1-f_{1w})\frac{M}{P^2}} < 0 \qquad \text{(A2.6)}$$

$$\frac{dP}{d\pi} = -\frac{f_{1\pi}}{(1-f_{1w})\frac{M}{P^2}} > 0 \qquad \text{(A2.7)}$$

$$\frac{dP}{du} = -\frac{f_{1u}}{(1-f_{1w})\frac{M}{P^2}} < 0 \tag{A2.8}$$

$$\frac{dP}{dY} = -\frac{f_{1y}}{(1-f_{1w})\frac{M}{P^2}} < 0 \tag{A2.9}$$

$$\frac{dP}{dT} = \frac{dP}{dG} = \frac{dP}{dB} = 0 \tag{A2.10}$$

(b) Effects of exogenous changes on the stock of foreign exchange:

$$\frac{dF}{dM} = 0 \tag{A2.11}$$

$$\frac{dF}{dv} = \frac{f_{2w}}{(1-f_{1w})} > 0 \tag{A2.12}$$

$$\frac{dF}{d\pi} = \underset{(+)}{f_{2\pi}} + \frac{\overset{(-)}{f_{1\pi}} \cdot \overset{(+)}{f_{2w}}}{\underset{(+)}{(1-f_{1w})}} = ? \tag{A2.13}$$

$$\frac{dF}{du} = \underset{(-)}{f_{2u}} + \frac{\overset{(+)}{f_{1u}} \cdot \overset{(+)}{f_{2w}}}{\underset{(+)}{(1-f_{1w})}} \tag{A2.14}$$

Defining $a = -\frac{f_{1u}}{f_{2u}} > 0$ since $f_{1u} > |f_{2u}|$ and substituting in (A2.14) we deduce that

$$\frac{dF}{du} = 0 \quad \text{iff} \quad a = -\frac{f_{1u}}{f_{2u}} = \frac{f_{2w} + f_{3w}}{f_{2w}} \qquad \text{(A2.15)}$$

that is iff restriction (2.9c) holds, i.e. the effects of a change in u on foreign exchange stock are exactly offset by the consequent (because of a change in u) changes in real money balances.

$$\frac{dF}{dY} = -\underset{(-)}{\frac{1}{|A_1|}} \cdot \frac{M}{P^2}\,[\underset{(-)}{f_{2y}}\underset{(+)}{(1 - f_{1w})} + \underset{(+)}{f_{1y}}\,\underset{(+)}{f_{2w}}] = ? \qquad \text{(A2.16)}$$

$$\frac{dF}{dT} = \frac{dF}{dG} = \frac{dF}{dB} = 0 \qquad \text{(A2.17)}$$

(c) Effects of exogenous changes on the current account balance:

$$\frac{dS}{dM} = 0 \qquad \text{(A2.18)}$$

$$\frac{dS}{dv} = \frac{\partial S}{\partial v} + \frac{\partial S}{\partial P} \cdot \frac{dP}{dv} + \frac{\partial S}{\partial F} \cdot \frac{dF}{dv} + \frac{\partial S}{\partial \dot{B}} \cdot \frac{d\dot{B}}{dv}$$

$$= -\,[C_w - (1 - C_{y^D})i] \qquad (-)$$

$$-(-\,C_{y^D}\pi + C_w)\,\frac{f_{1w}}{(1-f_{1w})} \qquad (-)$$

$$-(1 - C_{y^D})i\,\frac{f_{2w}}{(1-f_{1w})} \qquad (-)$$

$$+\,0 \quad < 0 \quad ; \quad \text{given restriction (2.9b)} \qquad \text{(A2.19)}$$

$$\frac{dS}{d\pi} = \frac{\partial S}{\partial \pi} + \frac{\partial S}{\partial P} \cdot \frac{dP}{d\pi} + \frac{\partial S}{\partial F} \cdot \frac{dF}{d\pi} + \frac{\partial S}{\partial \dot{B}} \cdot \frac{d\dot{B}}{d\pi}$$

$$= C_{y^D} \frac{M}{P} \qquad (+)$$

$$- (- C_{y^D}\pi + C_w) \frac{f_{1\pi}}{(1-f_{1w})} \qquad (+)$$

$$- (1 - C_{y^D})i \; [f_{2\pi} + \frac{f_{1\pi} \cdot f_{2w}}{(1 - f_{1w})}] \qquad ?$$

$$+ 0 \qquad \text{(A2.20)}$$

The $\partial S/\partial F . dF/d\pi$ term is ambiguous because $dF/d\pi$ is ambiguous. $\partial S/\partial F = - (1 - C_{y^D})i$ is negative because the reduction in absorption that a rise in F causes through its negative effect on real disposable income (fall in interest earnings out of a given stock of real financial wealth), is less than the reduction in the service account, if the marginal propensity to consume out of disposable income is less than one. The $dF/d\pi$ term is ambiguous because a rise in expectations increases on the one hand the demand for foreign exchange ($f_{2\pi}$) due to our gross substitutes assumption, but on the other, it causes an increase in prices and hence, a reduction in real balances and wealth which decreases the demand for foreign exchange. If $dF/d\pi \leq 0$ then $dS/d\pi > 0$ unambiguously, while if $dF/d\pi > 0$ the sign of $dS/d\pi$ depends on the relatively strength of $\partial s/\partial \pi$ and

$(\partial S/\partial P)(dP/d\pi)$ on the one hand and $(\partial S/\partial f)(dF/d\pi)$ on the other. In the latter case, however, the positive effect (direct) of a rise in π on the current account and the indirect effect of a rise in prices most probably will outweigh the negative effect of a rise in F. To show this let us allow the parameters of the model to assume those values that maximize the possible negative effect of a rise in F on the current account, i.e. let $f_{2\pi} = - f_{1\pi}$ and $f_{3\pi} = 0$ so that any increase in expectations causes a shift out of domestic money into foreign exchange only, so that there is the maximum fall in the service account. Let also $C_{y^D} = 0$ so that any fall in interest earnings does not cause any reductions in absorption - the positive effect of an increase in F on the current account is minimized. Equation (A2.20) then becomes:

$$\frac{dS}{d\pi} = - \underset{(-)}{f_{1\pi}} \left[\frac{\overset{(+)}{C_w} - i \overset{(+)}{f_{3w}}}{\underset{(+)}{(1 - f_{1w})}}\right] \qquad \text{(A2.21)}$$

(A2.21) is positive if $f_{3w} < C_w/i$. Allowing some possible values for the parameters, say $C_w = .07$ from Ando and Modigliani (1963) and $i = .10$, then $f_{3w} < .7$ which is well within the acceptable range of values for f_{3w}. $(0 < f_{3w} < 1)$. Thus, the higher the marginal propensity to consume out of disposable income and the lower the foreign nominal rate of interest, the more easily the positive sign for $dS/d\pi$ prevails. In any eventuality, however, $dS/d\pi$ has to be positive for dynamic stability and so it might ultimately be considered as one of our stability conditions.

$$\frac{dS}{du} = \frac{\partial S}{\partial u} + \frac{\partial S}{\partial P}\frac{dP}{du} + \frac{\partial S}{\partial F} \cdot \frac{dF}{du} + \frac{\partial S}{\partial \dot{B}} \cdot \frac{d\dot{B}}{du}$$

$$= 0$$

$$- (- C_{y_D}\pi + C_w) \frac{f_{1u}}{(1 - f_{1w})} \qquad (-)$$

$$- (1 - C_{y_D}) i [f_{2u} + f_{1u} \frac{f_{2w}}{(1 - f_{1w})}] \qquad (?) \qquad \text{(A2.22)}$$

Restriction (2.9c), however, is sufficient for (A2.22) to be negative - though not necessary.

$$\frac{dS}{dY} = (1 - C_{y_D}) \qquad (+)$$

$$- (- C_{y_D}\pi + C_w) \frac{f_{1y}}{(1 - f_{1w})} \qquad (-)$$

$$- (1 - C_{y_D}) i [f_{2y} + f_{3w} \frac{f_{1y}}{(1 - f_{1w})}] \qquad (?)$$

$$= ? \qquad \text{(A2.23)}$$

$$\frac{dS}{dT} = C_{y_D} > 0 \qquad \text{(A2.24)}$$

$$\frac{dS}{dG} = - 1 < 0 \qquad \text{(A2.25)}$$

$$\frac{dS}{dB} = - i < 0 \qquad \text{(A2.26)}$$

(d) Effects of exogenous changes on the issue of government debt:

$$\frac{d\dot{B}}{dM} = 0 \qquad \text{(A2.27)}$$

$$\frac{d\dot{B}}{dv} = - \frac{f_{1w} \cdot m}{(1 - f_{1w})} < 0 \qquad \text{(A2.28)}$$

$$\frac{d\dot{B}}{d\pi} = - \frac{f_{1\pi} \cdot m}{(1 - f_{1w})} > 0 \qquad \text{(A2.29)}$$

$$\frac{d\dot{B}}{du} = - \frac{f_{1u} \cdot m}{(1 - f_{1w})} < 0 \qquad \text{(A2.30)}$$

$$\frac{d\dot{B}}{dY} = - \frac{f_{1y} \cdot m}{(1 - f_{1w})} < 0 \qquad \text{(A2.31)}$$

$$\frac{d\dot{B}}{dT} = - 1 < 0 \qquad \text{(A2.32)}$$

$$\frac{d\dot{B}}{dG} = 1 > 0 \qquad \text{(A2.33)}$$

$$\frac{d\dot{B}}{dB} = i > 0 \qquad \text{(A2.34)}$$

Given equation (2.15), the effects of exogenous changes on the accumulation of real financial wealth ($\dot{v}$) are given as the sum of their effects on the current account and the issue of government debt.

A2.2 Long-run comparative statics

Linearising the long-run equilibrium conditions (2.21a) - (2.21d) of Section 2.4, considering π as exogenous equal to m, given by condition (2.21e), we have:

$$A_2\ [d\bar{M}\ dv\ dF\ d\dot{B}\ ]' = B_2\ [dY\ du\ dT\ dG\ dB]' \qquad (A2.35)$$

where

$$A_2 = \begin{bmatrix} (f_{1w} - 1) & f_{1w} & 0 & 0 \\ f_{2w} & f_{2w} & -1 & 0 \\ (-C_{y^D}\pi + C_w) & C_w - (1 - C_{y^D})i & (1 - C_{y^D})i & -1 \\ -m & 0 & 0 & -1 \end{bmatrix}$$

(A2.36)

$$B_2 = \begin{bmatrix} -f_{1y} & -f_{1u} & 0 & 0 & 0 \\ -f_{2y} & -f_{2u} & 0 & 0 & 0 \\ (1 - C_{y^D}) & 0 & C_{y^D} & -1 & -i \\ 0 & 0 & 1 & -1 & -i \end{bmatrix} \qquad (A2.37)$$

The determinant of the matrix of coefficients, $|A_2|$, is unambiguously positive given restrictions (2.9b) and (2.9a):

$$|A_2| = -\underset{(-)}{(f_{1w} - 1)}\underset{(+)}{(C_w - (1-C_{y^D})i)} + \underset{(+)}{f_{1w}}\underset{(+)}{(- C_{y^D}\pi + C_w)}$$

$$-\underset{(+)}{(1-C_{y^D})}i[\underset{(+)}{f_{2w}}\underset{(-)}{(f_{1w}-1)} - \underset{(+)}{f_{1w}}\underset{(+)}{f_{2w}}] + m\,\underset{(+)}{f_{1w}} > 0$$

(A2.38)

Using Cramer's rule, the long-run comparative static results for real money balances are as follows:

$$\frac{d\overline{M}}{dY} = \underset{(+)}{\frac{1}{|A_2|}} \{\underset{(+)}{f_{1y}}\,\underset{(+)}{(C_w - (1 - C_{y^D})i)} + \underset{(+)}{f_{1w}}\underset{(+)}{(1 - C_{y^D})}$$

$$-\underset{(+)}{(1 - C_{y^D})}i[- \underset{(+)}{f_{1y}}\,\underset{(+)}{f_{2w}} + \underset{(-)}{f_{2y}}\,\underset{(+)}{f_{1w}}]\} > 0 \qquad \text{(A2.39)}$$

$$\frac{d\overline{M}}{du} = \underset{(+)}{\frac{1}{|A_2|}} \{\underset{(+)}{f_{1u}}\underset{(+)}{(C_w - (1-C_{y^D})i)} + \underset{(+)}{(1-C_{y^D})}i\,\underset{(+)}{f_{1u}}\,\underset{(+)}{f_{2w}}$$

$$-\underset{(+)}{(1 - C_{y^D})}i\,\underset{(-)}{f_{2u}}\,\underset{(+)}{f_{1w}}\} > 0 \qquad \text{(A2.40)}$$

$$\frac{d\bar{M}}{dT} = \underset{(+)}{\frac{1}{|A_2|}} \underset{(+)}{f_{1w}} (\underset{(-)}{C_{y^D} - 1}) < 0 \qquad (A2.41)$$

$$\frac{d\bar{M}}{dG} = \frac{d\bar{M}}{dB} = 0 \qquad (A2.42)$$

The comparative static effects on real financial wealth are:

$$\frac{dv}{dY} = \underset{(+)}{\frac{1}{|A_2|}} \{- (\underset{(-)}{f_{1w}} - 1)(\underset{(+)}{1 - C_{y^D}}) - \underset{(+)}{f_{1y}}(- C_{y^D}\pi + \underset{(+)}{C_w})$$

$$+(\underset{(+)}{1-C_{y^D}})i\ \underset{(-)}{f_{2y}}(\underset{(-)}{f_{1w}-1}) - (\underset{(+)}{1-C_{y^D}})i\ \underset{(+)}{f_{1y}}\ \underset{(+)}{f_{2w}} - m\ \underset{(+)}{f_{1y}}\} = ?$$

$$(A2.43)$$

$$\frac{dv}{du} = \underset{(+)}{\frac{1}{|A_2|}}\{- \underset{(+)}{f_{1u}}(- C_{y^D}\pi + \underset{(+)}{C_w}) + (\underset{(+)}{1-C_{y^D}})i\ \underset{(-)}{f_{2u}}(\underset{(-)}{f_{1w}-1})$$

$$-(\underset{(+)}{1-C_{y^D}})i\ \underset{(+)}{f_{1u}}\ \underset{(+)}{f_{2w}} - m\underset{(+)}{f_{1u}}\} = ? \qquad (A2.44)$$

Restriction (2.9c) is a sufficient condition - though not necessary - for (A2.44) to be negative. Note, that we are using (2.9c) to facilitate our short-run and long-run comparative statics, without any loss of generality.

$$\frac{dv}{dT} = \underset{(+)}{\frac{1}{|A_2|}} \underset{(-)}{(f_{1w} - 1)} \underset{(+)}{(1 - C_{y^D})} < 0 \qquad (A2.45)$$

$$\frac{dv}{dG} = \frac{dv}{dB} = 0 \qquad (A2.46)$$

The effects on the stock of foreign exchange are as follows:

$$\frac{dF}{dY} = \frac{1}{|A_2|} \{f_{1y} [f_{2w}(C_w - (1-C_{y^D})i) - f_{2w}(- C_{y^D}\pi + C_w)]$$
$$- f_{2y}[(f_{1w}-1)(C_w-(1-C_{y^D})i) - f_{1w}(- C_{y^D}\pi + C_w)]$$
$$- (1-C_{y^D})i[f_{2w}(f_{1w} - 1) - f_{1w} f_{2w}]$$
$$- m [-f_{1w} f_{2y} + f_{1y} f_{2w}]\} = ? \qquad (A2.47)$$

$$\frac{dF}{du} = \frac{1}{|A_2|}\{f_{1u} f_{2w}\left[(C_w - (1-C_{y^D})i\right] - f_{1u} f_{2w}(- C_{y^D}\pi + C_w)$$
$$- f_{2u}(f_{1w} - 1)(C_w - (1-C_{y^D})i) + f_{2u} f_{1w}(- C_{y^D}\pi + C_w)$$
$$- m(- f_{1w} f_{2u} + f_{1u} f_{2w})\} = ? \qquad (A2.48)$$

Restriction (2.9c) is again sufficient for (A2.48) to be negative.

$$\frac{dF}{dT} = \underset{(+)}{\frac{1}{|A_2|}}\{\underset{(+)}{(1 - C_{y^D})}[\underset{(+)}{f_{2w}}\underset{(-)}{(f_{1w} - 1)} - \underset{(+)}{f_{1w}} \underset{(+)}{f_{2w}}]\} < 0 \qquad (A2.49)$$

$$\frac{dF}{dG} = \frac{dF}{dB} = 0 \tag{A2.50}$$

Finally, the comparative static effects on the issue of government bonds are:

$$\frac{d\dot{B}}{dY} = \underset{(+)}{\frac{1}{|A_2|}} \{-m[\underset{(+)}{f_{1w}}(1 - \underset{(+)}{C_{y^D}}) + \underset{(+)}{f_{1y}}(C_w - (1 - \underset{(+)}{C_{y^D}})i)]$$

$$-m(1 - \underset{(+)}{C_{y^D}})i[-\underset{(+)}{f_{1w}}\ \underset{(-)}{f_{2y}} + \underset{(+)}{f_{1y}}\ \underset{(+)}{f_{2w}}]\} < 0 \tag{A2.51}$$

$$\frac{d\dot{B}}{du} = \underset{(+)}{\frac{1}{|A_2|}} \{-m[\underset{(+)}{f_{1u}}(C_w - (1 - \underset{(+)}{C_{y^D}})i)]$$

$$-m(1 - \underset{(+)}{C_{y^D}})i[-\underset{(+)}{f_{1w}}\ \underset{(-)}{f_{2u}} + \underset{(+)}{f_{1u}}\ \underset{(+)}{f_{2w}}]\} < 0 \tag{A2.52}$$

$$\frac{d\dot{B}}{dT} = \underset{(+)}{\frac{1}{|A_2|}} \{-\underset{(+)}{C_{y^D}}m \cdot \underset{(+)}{f_{1w}} + (\underset{(-)}{f_{1w} - 1})(C_w - (1 - \underset{(+)}{C_{y^D}})i)$$

$$-\underset{(+)}{f_{1w}}(-\underset{(+)}{C_{y^D}}\pi + C_w) + (1 - \underset{(+)}{C_{y^D}})i[\underset{(+)}{f_{2w}}(\underset{(-)}{f_{1w} - 1}) - \underset{(+)}{f_{1w}}\ \underset{(+)}{f_{2w}}]\} < 0$$

$$\tag{A2.53}$$

$$\frac{d\dot{B}}{dG} = \underset{(+)}{\frac{1}{|A_2|}} \{m\ \underset{(+)}{f_{1w}} - (\underset{(-)}{f_{1w} - 1})(C_w - (1 - \underset{(+)}{C_{y^D}})i) + \underset{(+)}{f_{1w}}(-\underset{(+)}{C_{y^D}}\pi + C_w)$$

$$-(1 - \underset{(+)}{C_{y^D}})i\ [\underset{(+)}{f_{2w}}(\underset{(-)}{f_{1w} - 1}) - \underset{(+)}{f_{1w}}\ \underset{(+)}{f_{2w}}]\} > 0 \tag{A2.54}$$

$$\frac{d\dot{B}}{dB} = i > 0 \tag{A2.55}$$

A2.3 Dynamic Stability

(a) Adaptive expectations: The dynamics of the system in this case, are described by the following system of differential equations (see Section 2.5):

$$\dot{\pi} = \beta(m - \dot{x} - \pi); \quad 0 < \beta < 1 \tag{A2.56a}$$

$$x = g(v, \pi) \quad ; \quad g_v > 0, \ g_\pi < 1 \tag{A2.56b}$$

$$\dot{v} = h(v, \pi) \quad ; \quad h_v < 0, \ h_\pi > 0 \tag{A2.56c}$$

Linearising around the long-run stationary state we get:

$$\dot{\pi} + \beta\dot{x} = -\beta(\pi - \pi_o) \tag{A2.57a}$$

$$0 = g_\pi(\pi - \pi_o) - (x - x_o) + g_v(v - v_o) \tag{A2.57b}$$

$$\dot{v} = h_\pi(\pi - \pi_o) + h_v(v - v_o) \tag{A2.57c}$$

or in matrix notation

$$\begin{bmatrix} 1 & \beta & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\pi} \\ \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} -\beta & 0 & 0 \\ g_\pi & -1 & g_v \\ h_\pi & 0 & h_v \end{bmatrix} \begin{bmatrix} (\pi - \pi_o) \\ (x - x_o) \\ (v - v_o) \end{bmatrix} \tag{A2.58}$$

For non-trivial solutions the characteristic polyonyum should be equal to zero, i.e.

$$\begin{vmatrix} -(\beta + \lambda) & -\beta\lambda & 0 \\ g_\pi & -1 & g_v \\ h_\pi & 0 & (h_v - \lambda) \end{vmatrix} = 0 \quad \text{(A2.59)}$$

with the following characteristic equation:

$$\underset{(-)}{(1+\beta g_\pi)}\lambda^2 + \left[\beta - \underset{(-)}{h_v}(1 + \underset{(-)}{\beta g_\pi}) + \underset{(+)}{\beta h_\pi}.\underset{(+)}{g_v}\right]\lambda - \underset{(-)}{\beta h_v} = 0 \quad \text{(A2.60)}$$

According to the Routh-Hurwitz theorem (see Samuelson, 1947), the necessary and sufficient conditions for the real parts of the characteristic roots of a second degree polyonyum to be negative - the requirement for local stability - is that the coefficients of the characteristic equation are all positive. Thus, the condition for local stability reduces to

$$(1 + \beta g_\pi) > 0 \quad \text{or} \quad |\beta . g_\pi| < 1 \quad \text{(A2.61)}$$

To derive the phase diagram analysis under adaptive expectations, totally differentiate (A2.56b) with respect to time, i.e.

$$\dot{x} = g_\pi \dot{\pi} + g_v \dot{v} \tag{A2.62}$$

Substituting $\dot{\pi}$ from (A2.57a) and $\dot{v}$ from (A2.57b) we have:

$$(1+\beta g_\pi)\dot{x} = (g_v h_\pi - \beta g_\pi)d\pi + g_v h_v dv \tag{A2.63}$$

Substituting $d\pi$ from equation (A2.57b), we derive the equation of the $\dot{x} = o$ locus, i.e.

$$(1+\beta g_\pi)\dot{x} = (g_v h_\pi - \beta g_\pi)\frac{1}{g_\pi}dx + \left[g_v h_v - \frac{g_v}{g_\pi}(g_v h_\pi - \beta g_\pi)\right] dv \tag{A2.64}$$

from which we have:

$$\left.\frac{dx}{dv}\right|_{\dot{x}=o} = \frac{\overset{(+)}{g_v}\ (\overset{(+)}{g_v}\ \overset{(+)}{h_\pi} - \overset{(-)}{h_v}\ \overset{(-)}{g_\pi}) - \beta \overset{(+)}{g_v}\ \overset{(-)}{g_\pi}}{(\underset{(+)}{g_v}\ \underset{(+)}{h_\pi} - \beta \underset{(-)}{g_\pi})} > 0 \tag{A2.65}$$

provided

$$g_v > \frac{h_v \cdot g_\pi}{h_\pi} \tag{A2.65}$$

If the inequality sign of (A2.65) is reversed, the slope of the $\dot{x} = o$ locus is ambiguous. Deviations of real money balances from their long-run level are self - correcting, since

$$\left.\frac{\partial \dot{x}}{\partial x}\right|_{v=\text{const.}} = \frac{\overset{(+)}{(g_v} \cdot \overset{(+)}{h_\pi} - \overset{(-)}{\beta g_\pi})}{\underset{(-)}{g_\pi}(1 + \underset{(+)}{\beta g_\pi})} < 0 \tag{A2.66}$$

given stability condition (A2.61).

Substituting $d\pi$ from (A2.57b) in (A2.57c), we derive the equation of the $\dot{v}$ = o locus, i.e.

$$\dot{v} = \frac{h_\pi}{g_\pi} dX + (h_v - \frac{h_\pi \cdot g_v}{g_\pi}) \frac{h_\pi}{g_\pi} dv \tag{A2.67}$$

from which we have:

$$\left.\frac{dx}{dv}\right|_{\dot{v}=o} = \underset{(+)}{g_v} - \frac{\overset{(-)}{h_v} \cdot \overset{(-)}{g_\pi}}{\underset{(+)}{h_\pi}} \gtrless 0 \quad \text{as} \quad g_v \gtrless \frac{h_v \cdot g_\pi}{h_\pi} \tag{A2.68}$$

$$\left.\frac{\partial \dot{v}}{\partial v}\right|_{x=\text{const.}} = \frac{\overset{(+)}{h_\pi}}{\underset{(-)}{g_\pi}}(\frac{h_v \cdot g_\pi}{h_\pi} - g_v) \gtrless 0 \qquad \text{as}$$

$$g_v \gtrless \frac{h_v \cdot g_\pi}{h_\pi} \tag{A2.69}$$

(b) Perfect foresight: The following system of differential equation describes the dynamics of the system under perfect foresight:

$$x = g(v, m - \dot{x}); \quad g_v > 0, \quad g_\pi < 0 \tag{A2.70a}$$

$$\dot{v} = h(v, m - \dot{x}); \quad h_v < 0, \quad h_\pi > 0 \tag{A2.70b}$$

Linearising (A2.70) around the stationary state we have:

$$g_\pi \dot{x} = -(x - x_o) + g_v(v - v_o) \tag{A2.71a}$$

$$h_\pi \dot{x} + \dot{v} = 0 + h_v(v - v_o) \tag{A2.71b}$$

or in matrix notation

$$\begin{bmatrix} g_\pi & 0 \\ h_\pi & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} -1 & g_v \\ 0 & h_v \end{bmatrix} \begin{bmatrix} (x - x_o) \\ (v - v_o) \end{bmatrix} \tag{A2.72}$$

For non-trivial solutions the characteristic equation should be equal to zero, i.e.

$$\begin{vmatrix} -(1 + g_\pi \lambda) & g_v \\ -h_\pi \lambda & (h_v - \lambda) \end{vmatrix} = 0 \tag{A2.73}$$

with the following characteristic equation

$$g_\pi \lambda^2 + (1 - h_v \cdot g_\pi + g_v \cdot h_\pi) - h_v = 0 \tag{A2.74}$$

The signs of the coefficients of (A2.74) are as follows

$$a = \underset{(-)}{g_\pi} < 0$$

$$b = 1 - \underset{(-)}{h_v} \cdot \underset{(-)}{g_\pi} + \underset{(+)}{g_v}\underset{(+)}{h_\pi} = ? \qquad \text{(A2.75)}$$

$$c = - \underset{(-)}{h_v} > 0$$

Hence, since $-4ac = 4g_\pi . h_v > 0$ and $b^2 > 0$, the characteristic roots are real and of opposite signs; a sufficient condition for a saddle point.

Equations (A2.71a) and (A2.71b) represent the $\dot{x} = 0$ and $\dot{v} = 0$ curves respectively of the phase diagram in Figure 2.5. From (A2.71a) we note that the $\dot{x} = 0$ is upwards sloping since

$$\left.\frac{dx}{dv}\right|_{\dot{x}=0} = g_v > 0 \qquad \text{(A2.76)}$$

and that deviations of real money balances from their long-run level are cumulative, i.e.

$$\left.\frac{\partial \dot{x}}{\partial x}\right|_{v=\text{const.}} = - \frac{1}{g_\pi} > 0 \qquad \text{(A2.77)}$$

Substituting $\dot{x}$ from (A2.71a) into (A2.71b) we derive an expression identical to (A2.67), which again gives an ambiguous slope for the $\dot{v} = 0$ locus and an ambiguous reaction of the accumulation of real financial wealth to discrepancies of its stock from the long-run level, i.e.

$$\left.\frac{\partial \dot{v}}{\partial v}\right|_{x=\text{const.}} = \overset{(+)}{\underset{(-)}{\frac{h_\pi}{g_\pi}}} \left(\frac{\overset{(-)(-)}{h_v \cdot g_\pi}}{\underset{(+)}{h_\pi}} - \overset{(+)}{g_v} \right) \lessgtr 0 \text{ as } \frac{h_v \cdot g_\pi}{h_\pi} \gtrless g_v \tag{A2.78}$$

and

$$\left.\frac{dx}{dv}\right|_{\dot{v}=0} = - \frac{\overset{(-)(-)}{h_v \cdot g_\pi}}{\underset{(+)}{h_\pi}} + \overset{(+)}{g_v} \lessgtr 0 \quad \text{as} \quad \frac{h_v \cdot g_\pi}{h_\pi} \gtrless g_v \tag{A2.79}$$

A2.4 Dynamic response of the system to exogenous changes

(a) Exogenous rise in expectations (adaptive expectations). The inflationary expectations, the stock of real money balances and the stock of real financial wealth can be written as a function of time, i.e.

$$\pi(t) = \bar{\pi} + A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} \tag{A2.80a}$$

$$x(t) = \bar{x} + B_1 e^{\lambda_1 t} + B_2 e^{\lambda_2 t} \tag{A2.80b}$$

$$v(t) = \bar{v} + C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \tag{A2.80c}$$

where A, B and C are constants, λ_1 and λ_2 are the two negative roots of the characteristic equation (A2.60) and $\bar{\pi}$, $\bar{x}$ and $\bar{v}$ are the stationary state values of expectations about inflation (= m), real money balances and real financial wealth respectively. This particular form of equations (A2.80) was implicitly used in A2.3(a) to check the stability of the model under adaptive expectations. The relationship between the constants A, B and C can easily be established from the characteristic matrix, i.e. from (A2.58) we have:

$$\begin{bmatrix} -(\beta + \lambda) & -\beta\lambda & 0 \\ g_\pi & -1 & g_v \\ h_\pi & 0 & (h_v - \lambda) \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = 0 \qquad \text{(A2.81)}$$

Since the three equations in (A2.81) are not independent from each other given (A2.59), using the two characteristic roots λ_1 and λ_2 we derive from (A2.81) the following relationships:

$$A_1 = \frac{(\lambda_1 - h_v)}{h_\pi} C_1 \; ; \qquad A_2 = \frac{(\lambda_2 - h_v)}{h_\pi} C_2 \qquad \text{(A2.82a)}$$

$$B_1 = g_\pi A_1 + g_v C_1 \; ; \qquad B_2 = g_\pi A_2 + g_v C_2 \qquad \text{(A2.82b)}$$

Given that at $t = 0$ we have an exogenous rise in expectations that does not affect the long-run stationary state, the following initial conditions are implied:

$$v(0) = v_o = \bar{v} \qquad \text{(A2.83a)}$$

and

$$\pi(0) = \pi_o > \bar{\pi} \qquad \text{(A2.83b)}$$

Letting $t = 0$ in (A2.80a) and (A2.80c) and using (A2.83), we have

$$A_1 = (\pi_o - \bar{\pi}) - A_2 \tag{A2.84}$$

$$C_1 = - C_2 \tag{A2.85}$$

From (A2.82a), (A2.84) and (A2.85) we get

$$A_1 = - \frac{k_1}{(k_2 - k_1)} (\pi_o - \bar{\pi}); \quad A_2 = \frac{k_2}{(k_2 - k_1)} (\pi_o - \bar{\pi}) \tag{A2.86a}$$

$$C_1 = - \frac{1}{(k_2 - k_1)} \cdot (\pi_o - \bar{\pi}); \quad C_2 = \frac{1}{(k_2 - k_1)} (\pi_o - \bar{\pi})$$

where

$$k_1 = \frac{(\lambda_1 - h_v)}{h_\pi} ; \qquad k_2 = \frac{(\lambda_2 - h_v)}{h_\pi} \tag{A2.86b}$$

Note that $\lambda_i = h_v$ is not a solution to the characteristic equation (A2.60) because for this value the characteristic polyonyum (A2.59) is different from zero.

Substituting (A2.86a) into (A2.82b) we have

$$B_2 = \frac{g_\pi k_2 + g_v}{(k_2 - k_1)} (\pi_o - \bar{\pi}) \tag{A2.87}$$

Taking the time derivatives of (A2.80b) and (A2.80c) we have:

$$\dot{x}(t) = \lambda_1 B_1 e^{\lambda_1 t} + \lambda_2 B_2 e^{\lambda_2 t} \tag{A2.88a}$$

$$\dot{v}(t) = \lambda_1 C_1 e^{\lambda_1 t} + \lambda_2 C_2 e^{\lambda_2 t} \tag{A2.88b}$$

Substituting B_1 from (A2.80b), B_2 from (A2.87), C_1 from (A2.80c) and C_2 from (A2.86a) and using (A2.86b), we have:

$$\dot{x}(t) = \lambda_1 (x(t) - \overline{x}) + \left[g_\pi(\lambda_2 - h_v) + h_\pi \cdot g_v\right](\pi_o - \overline{\pi})e^{\lambda_2 t} \tag{A2.98a}$$

$$\dot{v}(t) = \lambda_1 (v(t) - \overline{v}) + h_\pi(\pi_o - \overline{\pi})e^{\lambda_2 t} \tag{A2.98b}$$

which give the rate of change of real money balances and real financial wealth during the adjustment process (see Figure 2.6). Since $\dot{x}(t) = m - D\ln P(t)$, (A2.89a) becomes

$$D\ln P(t) = m - \lambda_1 (x(t) - \overline{X}) - \left[g_\pi(\lambda_2 - h_v) + g_v h_\pi\right](\pi_o - \overline{\pi})e^{\lambda_2 t} \tag{A2.90}$$

which gives the slope of the dynamic exchange rate path BC in Figure 2.8.

(b) <u>Exogenous fall in u</u>

In the case of adaptive expectations we have the following initial conditions:

$$\pi_o = \overline{\pi} \tag{A2.91a}$$

$$v(0) = v_o^* < v_1^* = \overline{v} \tag{A2.91b}$$

where $v_1{}^*$ is the stock of real money balances at the new long-run equilibrium. Using a similar procedure as before to estimate the constants A, B and C, we have

$$B_1 = - \frac{g_\pi k_1 \; k_2 + g_v \; k_2}{(k_1 - k_2)} (v_o{}^* - v_1{}^*) \tag{A2.92a}$$

$$B_2 = \frac{g_\pi \; k_1 \; k_2 + g_v \; k_1}{(k_1 - k_2)} (v_o{}^* - v_1{}^*) \tag{A2.92b}$$

Letting t = 0 in (A2.80b) we have:

$$x(0) = \bar{x} + B_1 + B_2 = x_o \tag{A2.93}$$

Substituting B_1 and B_2 from (A2.92) we get

$$x_o = \bar{x} + \underset{(+)}{g_v}(v_o{}^* - \underset{(-)}{v_1{}^*)} < \bar{x} \tag{A2.94}$$

Since $x_o < \bar{x}$ real money balances on the impact of the exogenous change overshoot their long-run level. Substituting B_1 from (A2.80b) and B_2 from (A2.92b), (A2.88) now becomes

$$\dot{x}(t) = \lambda_1(x(t) - \bar{x}) \tag{A2.95a}$$

$$- \left[\frac{g_\pi}{h_\pi}(\lambda_1 - h_v)(\lambda_2 - h_v) + g_v(\lambda_1 - h_v)\right](v_o{}^* - v_1{}^*)e^{\lambda_2 t}$$

which gives the rate of change of real money balances during the adjustment process under adaptive expectations (see Figure 2.7).
Similarly, following a similar procedure, the rate of change of real financial wealth during the adjustment process reduces to

$$\dot{v}(t) = \lambda_1 \ (v(t) - v_1{}^*)$$

$$-(\lambda_1 - h_v)(v_o{}^* - v_1{}^*)e^{\lambda_2 t} \qquad \text{(A2.95b)}$$

Substituting $\dot{x}(t)$ with its equal in (A2.95a), we derive the slope of the exchange rate path BC of figure 2.9, i.e.

$$D\ln P(t) = m - \lambda_1 \ (x(t) - \bar{x})$$

$$+ \left[\frac{g_\pi}{h_\pi} (\lambda_1 - h_v)(\lambda_2 - h_v) + g_v(\lambda_1 - h_v) \right] (v_o{}^* - v_1{}^*)\rho^{\lambda_2 t}$$

(A2.96)

Under rational expectations real money balances and real financial wealth are given by the following equations

$$x(t) = \bar{x} + A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} \tag{A2.97a}$$

$$v(t) = \bar{v} + B_1 e^{\lambda_1 t} + B_2 . e^{\lambda_2 t} \tag{A2.97b}$$

where λ_1 and λ_2 are now the two characteristic roots of the characteristic equation (A2.74) and A and B is a new set of constants. One of the roots is negative (say $\lambda_1 = \lambda < 0$) and the other is positive ($\lambda_2 > 0$). Hence, for a stable long-run equilibrium

$$A_2 = B_2 = 0 \tag{A2.98}$$

with (A2.97) becoming

$$x(t) = \bar{x} + Ae^{\lambda t} \tag{A2.99a}$$

$$v(t) = \bar{v} + Be^{\lambda t} \tag{A2.99b}$$

From expression (A2.72) we have the following relationship

$$\begin{bmatrix} -(1 + g_\pi \lambda) & g_v \\ - h_\pi \lambda & (h_v - \lambda) \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = 0 \tag{A2.100}$$

from the first row of which we have

$$A = \frac{g_V}{(1 + g_\pi \lambda)} B \tag{A2.101}$$

Using the initial condition (A2.91b) in (A2.99b) and solving for B we have

$$B = (v_o^* - v_1^*) \quad \text{and} \quad A = \frac{g_V}{(1 + g_\pi \lambda)} (v_o^* - v_1^*) \tag{A2.102}$$

Substituting A in (A2.99a) we get

$$x(t) = \bar{x} + \frac{g_V}{(1 + g_\pi \lambda)} (v_o^* - v_1^*) e^{\lambda t} \tag{A2.103}$$

For t = 0 we get the stock of real balances on the impact of the exogenous fall in u under long-run perfect foresight, i.e.

$$x(o) = \bar{x} + \frac{\overset{(+)}{g_V}}{\underset{(-)(-)}{(1 + g_\pi \lambda)}} \underset{(-)}{(v_o^* - v_1^*)} = x_o' < \bar{x} \tag{A2.104}$$

Since $x_o' < \bar{x}$, we have overshooting. Subtracting (A2.94) from (A2.104) we observe that the overshooting under adaptive expectations is greater since

$$(x_o' - x_o) = - \frac{\overset{(+)(-)(-)}{g_V \cdot g_\pi \cdot \lambda}}{\underset{(-)(-)}{(1 + g_\pi \lambda)}} \underset{(-)}{(v_o^* - v_1^*)} > 0 \tag{A2.105}$$

Taking the time derivatives of (A2.99) and substituting A_1 from (A2.99) and B from (A2.102) we get the rate of accumulation of real money balances and real financial wealth during the adjustment process under rational expectations (see Figure 2.7)

$$\dot{x}(t) = \lambda(x(t) - \bar{x}) \tag{A2.106a}$$

$$\dot{v}(t) = \lambda(v_o^* - v_1^*)e^{\lambda t} \tag{A2.106b}$$

From (A2.106a) the slope of the exchange rate path during the adjustment process (B'C' in Figure 2. 9) is

$$D \ln P(t) = m - \lambda(x(t) - \bar{x}) \tag{A2.107}$$

The formal analysis of this section underlies the analysis of Sections 2.6 and 2.7 of the text.

CHAPTER 3

EXCHANGE RATE DYNAMICS UNDER MANAGED FLOATING

3.1 Introduction

After the collapse of the Bretton Woods system in March 1973, the major industrialized countries, faced with substantial uncertainty concerning future balance of payments developments, allowed their currencies to float in view of the persisting marked differences among national economies with respect to levels of economic activity, inflation rates, interest rates and structural changes. The short-run exchange rate variability that resulted and the potential economic costs that it entails, led many central banks to intervene in the foreign exchange market to correct disorderly market conditions or to mitigate excessive exchange rate movements or even to drive rates aggressively in a particular direction for various economic or political reasons. The most widely used means of exchange rate management include: capital or current account restrictions, official foreign borrowing, monetary or interest rates policies, but mainly purchases or sales of foreign exchange in spot or forward markets.

In spite of this intervention, which ironically had led initially to a greater reserve usage than under fixed rates (Williamson 1976) contrary to expectations to the opposite (Williamson 1973), exchange rate changes in the major industrial countries were used as the main mechanism of adjustment in balance of payments developments (IMF 1978, 1979). A general account of the major developments during the recent floating period is given, apart from the IMF annual reports, in McKinnon (1976a), Brown (1796) and Tosini (1977).

The International Monetary Fund in June 1974, reflecting a general implicit agreement that the behaviour of governments with respect to exchange rates should be a matter for international surveillance, adopted a set of non-binding guidelines[1] which recommend that members:

(1) should prevent or moderate day to day and week to week movements in exchange rates,
(2) should moderate reversible movements in their rates from month to month or quarter to quarter,
(3) are allowed to act aggressively to lead their exchange rates towards a medium-term-norm agreed with the Fund, and
(4) should consult the Fund about their desired reserves levels, should take into account the interest of the countries in whose currencies they intervene and should refrain from the use of current account

1. These guidelines have been adopted by the executive board of the IMF in June, 13, 1974 and are included in appendix II of IMF (1974), pp. 112 - 116, and in the appendix of Mikesell and Goldstein (1975) and Fleming (1975).

restrictions.

A number of academic papers also discuss the rules that should govern intervention in the foreign exchange market. Mikesell and Goldstein (1975) suggest certain rules for intervention based on various policy motivations, which relate in general to permissible changes in official reserves between reporting periods. In contrast, two other papers refer to the mechanics of intervention rather than the presumed motivation. First, the "leaning against the wind" proposal (Tosini 1977) refers to a non-aggressive intervention to symmetrically and smoothly resist exchange rate movements without actually fixing the rate to a particular level. This proposal is reflected in guidelines (1) and (2) of the IMF which require smoothing and moderating intervention.

The second, the "reference rate proposal" put forward by Ethier and Bloomfield (1975), is of a non-mandatory nature and requires the setting of a structure of reference rates, revised periodically through some defined international procedure and that central-banks should undertake not to sell their currencies at a price below their reference rates, by more than a fixed percentage (possibly zero) or buy their currencies at a price exceeding reference rates by more than the fixed percentage. The reference rate proposal is very similar to guideline (3) of the IMF, and in fact Ethier and Bloomfield consider their proposal as an implementation of guideline (3), in which reference rates are equivalent to medium-term target zones for exchange rates, together with a suggested substitution of permissible intervention for the mandatory one recommended by the IMF. Williamson (1975), extending the reference rate proposal considers it as strengthening the IMF's guidelines and as providing the focus for stabilizing speculation. Its implementation, he argues, would permit attainment of the objectives of symmetry in the adjustment mechanism and control of global liquidity which could be achieved by the introduction of multicurrency intervention and asset settlement.

Both proposals, however, present practical and theoretical difficulties. Leaning against the wind is optimal so long as market forces are moving the exchange rate excessively in one direction; if, though, rates are moving towards a new long-run equilibrium level, this intervention rule would hinder the attainment of the equilibrium exchange rates. It might be more important, for example, to act aggressively to drive the exchange rate towards its new long-run level minimizing the discrepancies between spot and medium-term exchange rates consistent with effective balance of payments adjustment. The establishment of an internationally agreed set of reference rates for all countries, on the other hand, is far from easy, given the uncertainty about the implications for exchange rates of changes in the structure of world trade that characterized the 1970s. The attainment of such medium-term "norms" is unlikely to be politically feasible, given the persistence of marked differences in national economies as far as inflation rates, interest rates and levels of economic activity are concerned. The establishment of such a structure of reference rates presupposes some degree of stability of underlying economic conditions and some harmonization of national economic policies.

In the light of these reservations, it is perhaps not surprising that no attempt was made to implement the IMF guidelines or the reference rates proposal. The new Articles of Agreement of the IMF express the exchange rate obligations of members in more general terms; the amended article IV in particular, maintains that:[2]

> "members shall avoid manipulating exchange rates or the international monetary system in order to prevent effective balance of payments adjustment or to gain an unfair competitive advantage over other members."

In April 1977 the IMF issued some specific principles[3] for the guidance of members' exchange rate policies which avoid precise formal rules. Instead they establish procedures that would allow continuous and effective surveillance by the fund, while individual countries are left free to

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2. IMF (1976), p. 228.

3. The relevant decision of the IMF executive board is reproduced in appendix II of IMF (1977), pp. 107-109 and in the appendix of Artus and Crockett (1978)

follow any exchange rate arrangements they wish.

The literature on intervention policies and managed floating has dealt with a variety of the issues raised by the new status quo. Some papers try to justify a case for government intervention: Day (1977), using a simple model of exchange rate determination, argues that official intervention in the foreign exchange market on a quantitative basis is required to avoid any necessity for the build up of large open positions by private speculation. Lipschitz (1978), using a simple IS-LM model with domestic supply and demand shocks, makes a simple case for intervention in the exchange market in order to stabilize domestic absorption.

Girton and Henderson (1976, 1977) analyse central bank operations in foreign and domestic assets in a short-run partial equilibrium portfolio balance model. Williamson (1976), in a simple stochastic general equilibrium flow model, examines the determinants of reserve use and the trade-off between exchange rate flexibility and the extent of reserve-use with the government intervening according to the 3rd guideline of the IMF (reference rate proposal). Black (1976), in a critical comment, modifies Williamson's results for the case of rational expectations.

Kouri (1976) and Henderson (1978) examine the impact, long-run and dynamic effects of a once and for all foreign exchange market operation, in a portfolio balance general equilibrium model. Finally, in an empirical context, Artus (1976) examines the behaviour of the deutschmark-dollar exchange rate for the recent floating period with an explicit incorporation of government intervention; the reaction function used reflects both the leaning against the wind and the reference rate proposals. Branson et al. (1977), on the other hand, in an empirical study of the same exchange rate use an endogenous government reaction function derived through a process of minimizing the welfare costs of divergence from the target values of the exchange rate, the money supply and the reserve stock.

Our analysis is an attempt to incorporate a government reaction function in a general equilibrium portfolio balance model and to compare the resulting exchange rate dynamics under managed floating with those under free floating. Apart from assessing the effectiveness of the intervention policy and the determinants of the degree of reserve use involved, our analytical framework allows a rigorous examination of the effects of stabilizing and destabilizing speculation on dynamic exchange rate behaviour. First, however, government intervention needs to be justified in the world described by our model.

The neoclassical basic format of our model, analysed in detail in chapter 2, because of its highly abstract nature and its rather strict assumptions about wage and price flexibility, the law of one price and the instantaneous clearing of all markets, allows no economic costs to short-run exchange rate variability. The resulting short-run price fluctuations do not affect the allocation of resources or the government policy; they simply cause corresponding variations in domestic consumption and absorption. To justify government intervention we postulate that the government is sensitive to price variablility which affects negatively some implicit government welfare function. Thus, the monetary authorities intervene in the foreign exchange market to minimize the welfare costs of short-run price variation.[4] Formally, the government is assumed to have a loss function the minimization of which dictates a specific government reaction function. More specifically, the central bank intervenes in the foreign exchange market in order to minimize the welfare costs of deviations of the spot exchange rate (price level) from its expected long-run path. Such an intervention rule can

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4. Short-run price variability, through its effects on private wealth, causes fluctuations in domestic consumption and absorption in general. Therefore, to the extent that government intervention moderates price variation, it stabilizes domestic absorption as well. See Lipschitz (1978) for a government intervention function directly concerned with stabilizing domestic absorption. Use of a more realistic model, allowing explicit economic costs to exchange rate variability, will render our analysis mathematically intractable.

formally be presented as follows:

$$Fg(t_o) = \ell \left[P(t_o) - \overline{P}(t_o) \right]; \quad \ell > 0 \tag{3.1a}$$

$$Fg(t_o) = - \frac{Mg(t_o)}{P(t_o)} \tag{3.1b}$$

$$\dot{F}g(t) = \delta \left[P(t) - \overline{P}(t) \right]; \quad \delta > 0 \tag{3.1c}$$

$$\dot{F}g(t) = - \frac{\dot{M}g(t)}{P(t)}; \quad t > t_o \tag{3.1d}$$

where Fg is the change in the private sector's holdings of foreign exchange due to the intervention policy. P(t) is the spot exchange rate, $\overline{P}(t)$ is the government's estimate of the long-run exchange rate and ℓ and δ are positive intervention parameters. The intervention rule amounts to a government purchase or sale of foreign exchange from the private sector, depending on the discrepancy between the spot rate at a particular time t and its reference rate, given the intervention parameter. In return, the government sells or buys domestic money (Mg).

On the impact of any exogenous change (time t_o), the government changes discretely the composition of private portfolios through exchange market operations (equations (3.1a) and (3.1b). Equations (3.1c) and (3.1d) describe how the stocks of Fg and Mg vary over time.

The intervention policy can equivalently be effected by exchanging foreign bonds for domestic money, with an added complication on the service account. Because domestic bonds are denominated in foreign currency, domestic open market operations are also foreign exchange market operations. An exchange of domestic with foreign bonds, though, will have no effect because they are perfect substitutes.[5]

We are not interested in deriving the optimum reaction function that minimizes a given loss function but rather, we start with a particular intervention rule, the very existence of which implies an implicit loss function which justifies also the intervention policy. Hence, our approach is quite different from that of Fischer (1977), Henderson (1979) and Boyer (1978). Boyer in particular, dealing with a simplified stochastic model, derives an optimal exchange rate policy permitting the appropriate degree of exchange rate flexibility, given the deterministic and stochastic structure of the economy; such an analysis is conducted in terms of the techniques developed by the literature of targets, instruments and indicators under uncertainty.[6]

Our postulated government reaction function

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5. See Girton and Henderson (1977) for a thorough analysis of central bank operations in the foreign exchange market.

6. For the targets, instruments and indicators literature see Brainard (1967), Poole (1970) and Friedman (1975).

(3.1) is identical to the reference rate proposal of Ethier and Bloomfield (1975) and the 3rd guideline of the IMF, where our $\overline{P}(t)$ corresponds to the reference-rate of Ethier and Bloomfield and the IMF's exchange-rate medium - term target zone. A similar government reaction function is also used by Williamson (1976), Black (1976) and Artus (1976). We maintain the non-mandatory nature of intervention recommended by Ethier and Bloomfield, but we assume that the reference rates path ($\overline{P}(t)$) is determined by the domestic government alone and not through any consultation with the IMF or an internationally agreed process. Given the format of our model, however, variations in $\overline{P}(t)$ would not affect the world structure of exchange rates for we only have two countries with one degree of freedom (given that P* is constant). If more than two countries are introduced, things become quite different, for in the absence of any reconciling mechanism, such behaviour might lead to incompatible national policies, to possible asymmetries in intervention and it could complicate the choice of intervention currencies.[7]

To make our intervention rule operational, we need to specify the way in which the government forms its expectations about the long-run equilibrium exchange rate path, the estimate of which, as will be shown, has considerable implications for exchange rate dynamics, the long -

7. For a more thorough analysis of these issues see Fleming (1975) and Williamson (1975).

run stationary state and the stability of the whole system. In particular, we postulate that the government reference rate at every moment in time, or alternatively the government's estimate of the long-run exchange rate path, is given by the following expression:

$$\bar{P}(t) = \frac{M^S(t)}{\bar{M}^e} \tag{3.2}$$

where $\bar{M}^e$ is the stock of real money balances expected by the government to prevail in the long-run. $M^S(t)$ is the nominal money stock at every moment in time. On the impact of any exogenous change, the government works out the implications of the exogenous disturbance for the long-run stock of real money balances and adjusts its reference rate accordingly, i.e.

$$\bar{P}(t) = \frac{M^S(t)}{\bar{M}^e} = M^S(t)\ L\ (\bar{M}^e); \qquad L'(\cdot) < 0 \tag{3.3}$$

From the long-run comparative statics of chapter 2 (summarised in table 2.5), the long-run stock of real money balances, $\bar{M}^*$, can be written as a function of exogenous variables, i.e.

$$\bar{M}^* = \bar{M}^*(Y,\ u,\ T,\ G,\ B) \tag{3.4a}$$

$$\overline{M}^*_y > 0,\ \overline{M}^*_u > 0,\ \overline{M}^*_T < 0,\ \overline{M}^*_G = \overline{M}^*_B = 0 \qquad (3.4b)$$

Hence, the government reference rate can be written as

$$\overline{P}(t) = \alpha M^S(t).L(Y,\ u,\ T)\ ;\ \alpha > 0 \qquad (3.5a)$$

$$L_Y < 0,\ L_u < 0,\ L_T > 0 \qquad (3.5b)$$

where Y, u, and T represent now the levels of these variables expected by the government to hold in the long - run. Exogenous disturbances that are expected to reverse themselves, do not affect the government's estimate of the long-run exchange rate, even if the exogenous shocks affect the long-run stock of real money balances. Therefore, in cases of reversible exogenous changes or of short-run disturbances that leave the long-run equilibrium unaffected, the intervention rule (3.1) amounts effectively to a leaning against the wind policy. α is a parameter reflecting the relative precision with which the government predicts long-run real money balances. If $\alpha = 1$, the government has long-run perfect foresight and if $\alpha \neq 1$, the government's estimate of long-run real money balances deviates from its true value, indicating that the government is using the wrong estimate of the long-run exchange rate path.

The way the government is postulated to form its estimate of the reference rate path is similar to the "purchasing power parity method" suggested by Artus (1978), which, given the format of our analysis seems most suitable.[8]

The layout of this chapter is as follows: in section 3.2 we incorporate the intervention rule (3.1) in our basic model assuming that the government has long-run perfect foresight (case I). Then, in section 3.2.4, we compare the dynamic exchange rate behaviour under free and managed floating. In section 3.3 we relax the perfect foresight assumption (case II), and examine the possibilities of various asymmetries in information between the public and private sectors and their consequences for exchange rate dynamics. Under a modified interpretation, the intervention rule (3.1) is shown to reflect speculative behaviour and in section 3.4 we examine the effects of stabilizing and destabilizing speculation on the dynamic exchange rate behaviour. Finally, section 3.5 provides our concluding remarks.

3.2 Case I: Government intervention with the right estimate of the long-run exchange rate path

In this case the government's estimate of the

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8. Other methods suggested by Artus (1978), are the "underlying payments disequilibria approach" and the "asset market disturbances method". In Artus (1976), where a government intervention function similar to ours is used, the government's estimate of the long-run exchange rate is endogenously determined as a function of domestic and foreign price levels.

long-run exchange rate path is always equal to its true value, given by (3.5a) with $\alpha = 1$, because of the long-run perfect foresight assumption. The private sector, on the other hand, is assumed to form its expectations either in an error-learning process or according to long-run perfect foresight, as before.

The intervention policy will affect the stocks of real financial wealth and real money balances (in the absence of complete specialization). To reflect these repercussions of exchange market operations, we redefine our assets as follows:

$$\overline{M} \equiv \frac{M^S}{P} = \frac{M}{P} + \frac{Mg}{P}(1 - k); \quad 0 < k < 1 \tag{3.6}$$

$$V \equiv b(1 - k) + F + Fg \tag{3.7}$$

where M^S stands for the nominal money stock at every moment in time; it is made up from the stock of nominal money (M) accumulated by the normal financing of government expenditures and from Mg, which shows the accumulated changes in the nominal money supply that the intervention policy brings about. k is the sterilization parameter, showing the extent to which the government sterilizes the changes in money supply (through domestic open market operations) caused by intervention. If $k = 1$, sterilization is complete and if $k = 0$, we have no sterilization at all. In empirical studies on the DM/dollar exchange rate the sterilization parameter was found both significant and close to unity,[9] indicating that the Bundesbank was able to pursue effectively its desired domestic monetary policy. In terms of our model, however, sterilization is inconsistent with intervention. Because domestic bonds are denominated in foreign currency and are perfect substitutes to foreign bonds, domestic open market operations are effectively foreign exchange market operations. Hence, if sterilization is complete, open market operations offset the foreign exchange market operations and net intervention is zero. Thus, in our model we assume no sterilization ($k = 0$) or, equivalently, we refer to net intervention.

The accumulated changes in private holdings of foreign exchange (Fg) augment the private holdings of foreign assets. To reflect this, we introduce a new notation, V, which stands for the "augmented stock of real financial wealth", defined by equation (3.7). Hence, for $k = 0$ real home-country wealth becomes:

$$W = \frac{M^S}{P} + V = \frac{M}{P} + \frac{Mg}{P} + v + Fg \tag{3.8}$$

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9. Artus (1976, table 3, p. 326) found a sterilization coefficient equal to .745 and Branson et al. (1977, p. 318) cite a study by Herring and Marston (1977) who found a coefficient of around .9, indicating a high degree of insulation of the domestic money supply from external developments.

Given equation (3.1b), the foreign exchange market operations that the intervention policy involves, change the composition of private portfolios. Intervention amounts to a mere reallocation of the existing wealth between real money balances and real financial wealth. Any induced changes in the exchange rate (price level), however, will affect the evaluation of the existing stock of assets.

3.2.1 Short-run equilibrium

The government on the impact of any exogenous change, intervenes in the foreign exchange market, in the way described by the intervention rule (3.1), altering discretely the composition of private portfolios. Over time, both Fg and Mg vary continuously, their marginal changes following the same intervention rule, till long-run equilibrium is restored. Formally, the government behaviour can be presented as follows:

$$Fg(t_o) = \ell\left[P(t) - \alpha M^S(t_o)L(Y, u, T)\right]; \quad \ell > 0 \qquad (3.9a)$$

$$Fg(t_o) = -\frac{Mg(t_o)}{P(t_o)} \qquad (3.9b)$$

and

$$\dot{F}g(t) = \delta\left[P(t) - \alpha M^S(t)L(Y, u, T)\right]; \quad \delta > 0 \qquad (3.10a)$$

$$\dot{F}g(t) = -\frac{\dot{M}g(t)}{P(t)} \quad ; \qquad \text{for } t \geqq t_o \qquad (3.10b)$$

Equation (3.9) shows the discrete change in Fg and Mg on the impact of the exogenous change at time $t = t_o$ that the intervention policy brings about. (3.10), similarly, shows how Fg and Mg evolve over time. If the foreign exchange market operation that the intervention policy involves, does not affect the composition of private portfolios on the impact of the exogenous change but simply affects the dynamic evolution of the system (i.e. if only equation (3.10) is used), the impact effects of the exogenous shock under free and managed floating will be the same.

The short-run equilibrium conditions of chapter 2 have to be modified to take account of the discrete government reaction, i.e. equation (3.9). Assuming, with no loss of generality, that we start from an initial stationary state position where the initial stocks of both Mg and Fg are zero, the short-run equilibrium condition (impact effect) under managed floating are as follows:

$$f_1\left[Y, \left(\frac{M}{P} + v\right), i, \pi, u\right] = \frac{M}{P} - Fg \qquad (3.11a)$$

$$f_2\left[Y, \left(\frac{M}{P} + v\right), i, \pi, u\right] = F + Fg = \overline{F} \qquad (3.11b)$$

$$Fg = \ell\left[P - \alpha(M - FgP)\ L\ (Y,\ u,\ T)\right] \tag{3.11c}$$

$$S = Y - C\left[(Y - T + i(v-F) - \pi\frac{M}{P} + \pi Fg),\ (\frac{M}{P} + v)\right]$$

$$- G + i(v - F - B) \tag{3.11d}$$

$$G + iB = T + \dot{B} + m\frac{M}{P} - mFg \tag{3.11e}$$

Since the initial stocks of Fg and Mg are zero, real private wealth reduces to (M/P + v). Fg is the stock of foreign exchange by which the intervention policy will augment the private holdings of foreign assets, on the impact of any exogenous change. The intervention policy affects the composition of private portfolios, but given (3.9b), total wealth remains (M/P + v). Using (3.9b), the supply of real money balances becomes ((M/P - Fg) and the short-run holdings of foreign exchange, ($\overline{F}$ = F + Fg). Real disposable income, as before, includes the expected capital losses on total money supply M^S, which is now affected by the intervention policy. The government is assumed to maintain its monetary policy of increasing the money stock M by a fixed proportion m (= $\dot{M}/M^S$) of the total money supply M^S, at every moment in time. Under perfect foresight, $\alpha = 1$.

The short-run (impact effect) endogenous variables are P, F, Fg, S and $\dot{B}$ while M, v, i, π, u, Y, T, G, B, α and ℓ are exogenous. The intervention policy involves exchanges of domestic for foreign money, with no direct involvement in the goods market equilibrium. Therefore, the system maintains its asset-market-approach recursiveness in that the exchange rate, the short-run holdings of foreign exchange and the intervention policy are determined in the asset market exclusively, unless, of course, the goods market influences expectations, as is the case under long-run perfect foresight. Given the asset market equilibrium, the current account balance and the issue of new government debt are determined by the flow market. This is formally shown by the matrix of coefficients of the endogenous variables, A_3, of the linearised form of the short-run equilibrium conditions:

$$A_3 = \left[\begin{array}{ccc|cc} (1-f_{1w})\frac{M}{P^2} & 0 & 1 & 0 & 0 \\ -f_{2w}\frac{M}{P^2} & -1 & -1 & 0 & 0 \\ -\ell & 0 & (1-\frac{P^2}{M}\cdot\ell) & 0 & 0 \\ \hline (-C_y^D\cdot\pi + C_w)\frac{M}{P^2} & -(1-C_y^D)i & -C_y^D\,\pi & -1 & 0 \\ m\frac{M}{P^2} & 0 & m & 0 & -1 \end{array}\right] \tag{3.12}$$

As an illustration of the short-run exchange rate determination under managed floating we will examine the effects of an exogenous fall in u, the subjective estimate of foreign exchange risk relative to domestic money, stressing, at the same time, the differences in the impact effects under free and managed floating.

Starting from an initial long-run equilibrium position where the government's estimate of the long-run exchange rate path is equal to the actual one, Fg = 0 and $M^S = M$, an exogenous fall in u occurs which is expected not to reverse itself. Under free floating, this leads to an excess supply of domestic money and an excess demand for foreign assets, shifting the MM_o and FF_o schedules of fig. 3.1 upwards to MM'_1 and FF'_1 respectively. In the absence of intervention this leads to an exchange rate depreciation (P'_1) and a current account surplus (S'_1), leaving the short-run holdings of foreign exchange unchanged ($\overline{F}$ = Fo, since Fg = 0), given restriction (2.9c). Under managed floating, however, the reduction in u, because it affects the long-run stock of real money balances and is expected to be permanent, shifts the government's estimate of the long-run exchange rate path upwards, for it leads to a long-run reduction in real balances, $L_u < 0$ by (3.5b). Prior to the exogenous change, the government reference rate was equal to P_o; the fall in u shifts it upwards to $\overline{P}$, which lies below P'_1 as the spot rate under free floating overshoots its long-run value (P*), on the impact of an

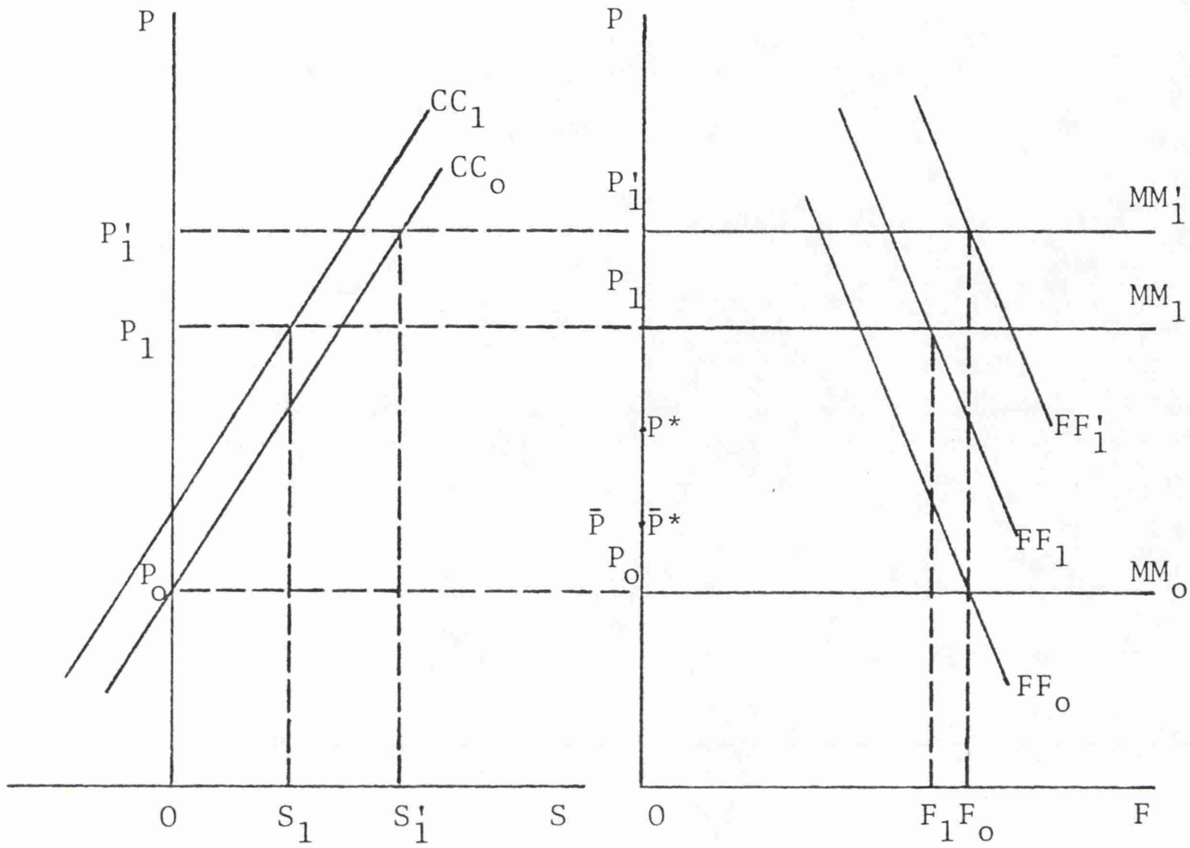

Figure 3.1. Short-run exchange rate determination under managed floating: effects of an exogenous fall in u.

exogenous fall in u. Under managed floating, given the intervention rule (3.9), the trend of the spot rate to overshoot its long-run level, activates the intervention policy. As $P(t_o) > \overline{P}(t_o)$, the government through exchange market operations sells foreign exchange to the private sector, buying in return domestic money. Effectively, the intervention policy moderates both the excess demand for real financial wealth and the excess supply of domestic money. For a given intervention parameter, the greater the difference between the spot rate and the government reference rate, the greater the magnitude of the exchange market operation. But the greater the government purchase of domestic money the less the excess supply of real money balances and hence, the less the exchange rate has to depreciate (the price level has to rise) to restore equilibrium in the money market. This, however, reduces the difference between the spot rate and the reference rate $\overline{P}$ and it thus reduces the volume of government sales of foreign money, i.e. Fg falls. Formally, this is shown by the fact that both Fg and P are endogenous variables.

Given the structure of the government reaction function, the government is implicitly assumed not to be interested in a once and for all exchange, that would take the system to the new stationary state. In the particular example of a fall in u, any attempt to do so, and hence to supply all the foreign exchange needed to restore long-run equilibrium, will also mean a great reduction in the excess supply of real money balances, bringing the spot exchange rate closer to the reference rate $\overline{P}$. Given the intervention rule, however, this will reduce the magnitude of government sales of foreign exchange, counter-acting the initial objective. Thus, the government does not completely eliminate the discrepancies of the spot rate from its expected long-run path; it simply allows the "optimum" exchange rate flexibility that minimizes the welfare costs.

From the short-run comparative statics, derived in the appendix and summarised in table 3.1 below, Fg unambiguously rises on the impact of the exogenous fall in u, implying that the spot rate has to lie somewhere between P'_1 and $\overline{P}$; the discrete exchange rate depreciation on the impact of the exogenous change is moderated relative to free floating, but we still have overshooting.[10]

10. Formally, government intervention moderates the discrete exchange rate depreciation, because:

$$\left.\frac{dP}{du}\right|^{M} - \left.\frac{dP}{du}\right|^{F} = -\underset{(-)}{\frac{\ell}{|A_3|}}\underset{(+)}{\left[\alpha(M-FgP)L_u + \frac{f_{1u}}{(1-f_{1W})\frac{M}{P^2}}\right]} > 0$$

However, we still have overshooting, confirmed by the analysis of the dynamic response of the system to the exogenous fall in u in section 3.2.4. The government reference rate $\overline{P}$, lies below P* in fig.3.1, since

$$P^*(t_o) = \frac{M(t_o)}{\overline{M}^*} > \frac{M^S(t_o)}{\overline{M}^*} = \frac{\left[M(t_o) - Fg(t_o)P_o\right]}{\overline{M}^e} = \overline{P}(t_o)$$

as $Fg(t_o) > 0$ and $\overline{M}^* = \overline{M}^e$ by the assumption of long-run perfect foresight.

In terms of fig. 3.1, the MM_o and FF_o schedules shift upwards to MM_1 and FF_1 respectively under the combined influence of the fall in u and the rise in Fg, which are both shifting variables. The fall in u alone shifts the system to the free floating position but the rise in F_g shifts both schedules downwards[11] so that the exchange rate overshooting is mitigated.

The short-run holdings of foreign exchange, as can be seen from (3.11b) are equal to (F + Fg). From equation (3.7), dFg is the increase in the stock of augmented real financial wealth resulting from the impact of the exogenous disturbance since real financial wealth (v), given by past accumulation, is constant in the short-run. Differentiating (3.7) we deduce that the short-run comparative static effects on bond holdings are identical to - dF, since

$$dV = db + dF + dFg = dFg$$

i.e.

$$db = - dF \qquad (3.13)$$

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11. From (3.11a) and (3.11b) we have

$$\left.\frac{dP}{dFg}\right|_{F=\text{const.}}^{MM} = - \frac{1}{(1-f_{1W})\frac{M}{P^2}} < 0$$

$$\left.\frac{dP}{dFg}\right|_{F=\text{Const.}}^{FF} = - \frac{1}{f_{2W}\frac{M}{P^2}} < 0$$

Hence the increase in Fg shifts both schedules downwards from what they would otherwise have been (free floating position).

The increase in augmented real financial wealth that the intervention policy brings about avails the private sector of the opportunity of increasing its short-run holdings of both foreign exchange and bonds simultaneously with the exogenous change,unlike the case of free floating where that is possible only over time, as current account surpluses allow accumulation of foreign assets. The short-run comparative statics show that foreign exchange[12] and bond holdings both increase (F falls in fig. 3.1).

The impact effect on the current account balance will depend on the relative shift of the CC_o schedule. The rise in Fg shifts CC_o upwards, while the fall in F tends

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12. From the short-run comparative statics - see appendix - by adding (dF/du) and (dFg/du) and using restriction (2.9c) we have:

$$\frac{d\bar{F}}{du} = \frac{dF}{du} + \frac{dFg}{du}$$

$$= \underset{(-)}{\frac{\ell}{|A_3|}} \underset{(+)}{\left[\alpha(M-FgP)L_u + \frac{f_{1u}}{(1-f_{1W})\frac{M}{p^2}} \right]} \underset{(+)}{f_{2W} \frac{M}{p^2}} < 0$$

since, from footnote 10, the numerator is positive.

TABLE 3.1

Short-run comparative statics under managed floating (impact effects)

Endogenous Variables	Exogenous variables							
	M	V	π	u	Y	T	G	B
P	(+)	(-)	(+)	(-)	(-)	(+)	0	0
F	0	(+)	(?)	(+)	(?)	(+)	0	0
F_g	0	(-)	(+)	(-)	(?)	(-)	0	0
S	0	(?)	(?)	(?)	(?)	(?)	(-)	(-)
$\dot{B}$	0	(-)	(+)	(-)	(-)	(?)	(+)	(+)

to shift it downwards.[13] Fig. 3.1 has been drawn showing as a net effect an upward shift to CC_1. The fall in private real balances, because of the exchange rate depreciation, and the rise in bond holdings tend to drive the current account into surplus, while the reduction in nominal balances that the intervention policy causes,decreases expected capital losses and tends to counteract the above two effects through a rise in real disposable income. The latter effect, though, might not be strong enough to push the current account into deficit and hence,in fig. 3.1 the current account is shown to be in surplus (S_1).

To the extent that the CC_o schedule shifts upwards, the current account surplus under managed floating is unambiguously reduced relative to free floating. This reduction in the current account imbalance is due to the cushioning that the intervention policy provides to private wealth: the foreign exchange market operations moderate the exchange rate overshooting and thus real balances are reduced by less than is the case under free floating;

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13. Table 2.3 shows that a fall F shifts the CC_o schedule downwards. From equation (3.11d) on the other hand, an increase in Fg shifts CC_o upwards since

$$\left.\frac{dP}{dFg}\right|_{\substack{CC \\ S=\text{const.}}} = \frac{C_y{}^D.\pi}{(-C_y{}^D.\pi + C_W)\frac{M}{P^2}} > 0$$

hence, consumption is higher than in the absence of intervention. The increase in short-run bond holdings (under managed floating), though, tends to offset the effect of the moderated fall in wealth[14] on the current account surplus, by improving the service account. However, assuming that the balance of trade effects prevail, the current account imbalance is unambiguously moderated.[15]

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14. The exchange rate depreciation reduces real wealth, for from equation (3.8) we have:

$$W = \frac{M}{P} + \frac{M_g}{P} + v + Fg = \frac{M}{P} + v$$

given equation (3.9b). Thus, the rise in prices reduces real money balances and wealth. Since, however, the exchange rate depreciation, under managed floating, is moderated relative to free floating, the reduction in wealth is moderated as well.

15. Formally, the reduction in the current account imbalance under managed floating is always positive, i.e.

$$\frac{dS}{du}\Bigg|^{M} - \frac{dS}{du}\Bigg|^{F} = \underset{(+)}{(-C_y{}^{D}\pi + C_w)} \frac{M}{P^2} \underset{(+)}{\left[\frac{dP}{du}\Bigg|^{M} - \frac{dP}{du}\Bigg|^{F}\right]}$$

$$- i \underset{(+)}{\frac{dF}{du}}\Bigg|^{M} + \underset{(+)}{C_y{}^{D}i} \underset{(+)}{\frac{dF}{du}}\Bigg|^{M} - \underset{(+)}{C_y{}^{D}\pi} . \underset{(-)}{\frac{dFg}{du}}\Bigg|^{M} > 0$$

provided the effects of the increased bond holdings on the current account (-i(dF/du)) under managed floating do not offset the balance of trade effects. The inclusion of the service account in our model, apart from complicating the analysis, suggests the possibility that the service account effects of a particular disturbance might be strong enough to more than offset the balance of trade effects. That said, though, we assume that the balance of trade effects always dominate.

The net effects of the intervention policy intuitively seem quite clear: the exogenous fall in u creates an excess demand for foreign assets and an excess supply of domestic money and leads to a current account surplus, providing over time the foreign assets needed to establish a higher proportion of real financial wealth in private portfolios at the new stationary state. The intervention policy simply alleviates the excess demand for real financial wealth and the excess supply of real balances by increasing the supply of foreign exchange and decreasing the nominal money supply simultaneously with the exogenous change. As a consequence, the exchange rate's discrete depreciation and overshooting are moderated and the current account imbalance is reduced. In addition, the intervention policy cushions the reduction in private wealth and consumption, stabilising domestic absorption. Government intervention in the foreign-exchange market continues so long as the spot rate deviates from its reference rate, speeding up the dynamic adjustment, a fact confirmed by the analysis of the dynamic response of the system to the exogenous change in section 3.2.4.

The short-run comparative static effects of a change in u and all other exogenous variables are formally derived in the appendix and summarised in table 3.1 above. Note that an increase in the nominal money stock (M) maintains its short-run neutrality, for it increases the price level (the exchange rate) in the same proportion, leaving all other variables unaffected. Similarly, changes in government expenditure and in the outstanding stock of domestic bonds (B) leave the asset market equilibrium unchanged, affecting only the issue of new debt and the current account balance, as under free floating.

The short-run equilibrium conditions (3.11) describe the impact effects of any exogenous change when the government, based on the reaction function (3.9), changes discretely the composition of private portfolios contemporaneously with the exogenous disturbance . Subsequently, the exchange rate, determined by the system (3.11), determines recursively the evolution of both Fg and Mg over time through the reaction functions (3.10). The short-run equilibrium conditions, that describe the behaviour of the system at all points in time (not just only on the impact of any shocks as conditions (3.11) do), are the same as under free floating plus the intervention rule (3.10). There is one modification, however: real wealth, real money balances and augmented real financial wealth are now given by equations (3.8), (3.6) and (3.7) respectively. Thus, the short run equilibrium conditions become:

$$f_1 \left[Y, \left(\frac{M^S}{P} + V\right), i, \pi, u \right] = \frac{M^S}{P} \tag{3.14a}$$

$$f_2 \left[Y, \left(\frac{M^S}{P} + V\right), i, \pi, u \right] = \overline{F} = F + Fg \tag{3.14b}$$

$$\dot{F}g = \delta \left[P - \alpha M^S . L(Y, u, T) \right] \tag{3.14c}$$

$$S = Y - C\left[(Y - T + i(V - \bar{F}) - \pi \frac{M^S}{P}), \ (\frac{M^S}{P} + V) \right]$$

$$- G + i(V - \bar{F} - B) \tag{3.14d}$$

$$G + iB = T + \dot{B} + m \frac{M^S}{P} \tag{3.14e}$$

where $\bar{F}$ are the short-run holdings of foreign exchange. Apart from equation (3.14c) and the redefinition of wealth variables, conditions (3.14) are the same as under free floating. Government intervention, based on (3.14c), affects the stocks of Fg and Mg only over time and for a particular short-run equilibrium all stocks of assets are given by past accumulation. The endogenous variables are P, $\bar{F}$, $\dot{F}g$, S and $\dot{B}$, while M^S, V, i, π, u, Y, T, G, B, m, α and δ are exogenous. Moreover, (3.14) is perfectly consistent with (3.11) because the government policy simply alters, on the impact of any exogenous change, the initial conditions of (3.14) by changing the existing stocks of assets. As is shown in the appendix, the short-run comparative statics of (3.14) are identical to those under free floating, for $\dot{F}g$ is determined recursively by other endogenous variables. Table 3.2 below, provides a summary of the short-run comparative statics. The last row shows the effects of exogenous changes on the rate of accumulation of augmented

TABLE 3.2

Short-run comparative static effects under managed floating

Endogenous Variables	Exogenous variables							
	M	V	π	u	Y	T	G	B
P	(+)	(-)	(+)	(-)	(-)	0	0	0
$\bar{F}$	0	(+)	(?)	0	(?)	0	0	0
$\dot{F}g$	0	(-)	(+)	(-)	(?)	(-)	0	0
S	0	(-)	(+)	(-)	(?)	(+)	(-)	(-)
$\dot{B}$	0	(-)	(+)	(-)	(-)	(-)	(+)	(+)
$\dot{V}$	0	(-)	(+)	(-)	(?)	(-)	0	0

real financial wealth, given as the algebraic sum of the comparative static effects on S, $\dot{B}$ and $\dot{F}g$, since by differentiating[16] (3.7), we have (k = o):

$$\dot{V} = \dot{v} + \dot{F}g = S + \dot{B} + \dot{F}g \qquad (3.15)$$

3.2.2 The long-run stationary state

Provided the system is dynamically stable, the sequence of short-run equilibria will lead to the long-run

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16. More rigorously, $\dot{V}$ is derived from the equality between ex post accumulation of real wealth ($\dot{W}$) and ex post net private savings (Sp^n, defined as gross private savings plus capital gains), i.e.

$$\dot{W} = Sp^n \quad \text{or} \quad \dot{V} + \dot{\bar{M}} = S - Sg - \frac{M^S}{P} \cdot \frac{\dot{P}}{P}$$

where Sg are government savings and $(-\frac{M^S}{P} \cdot \frac{\dot{P}}{P})$ are the ex post capital gains. Substituting Sg from the government budget constraint, equation (3.14e), we have:

$$\dot{V} + \frac{\dot{M}^S}{P} - \frac{M^S}{P} \cdot \frac{\dot{P}}{P} = S + \dot{B} + m\frac{M^S}{P} - \frac{M^S}{P} \cdot \frac{\dot{P}}{P}$$

$$\text{i.e. } \dot{V} = S + \dot{B} + \frac{\dot{M}}{P} - \frac{M^S}{P} \cdot \frac{\dot{P}}{P} - (\frac{\dot{M}}{P} + \frac{\dot{M}g}{P} - \frac{M^S}{P} \cdot \frac{\dot{P}}{P})$$

$$= S + \dot{B} - \frac{\dot{M}g}{P} = S + \dot{B} + \dot{F}g$$

given equation (3.10b)

stationary state where real wealth is constant and held in the desired proportions, expectations are constant and fully realised and where all nominal variables grow at the same rate. The exchange rate follows its long-run path, which, by definition, is identical to the government's estimate, and given intervention rule (3.10), there is no reason for the government to intervene.

Hence, under case I where the government uses the right estimate of the long-run exchange rate path, the long-run equilibrium conditions are as follows:

$$f_1 \left[Y, (\bar{M} + V), i, \pi, u\right] = \bar{M} \tag{3.16a}$$

$$f_2 \left[Y, (\bar{M} + V), i, \pi, u\right] = \bar{F} = F + Fg \tag{3.16b}$$

$$\begin{aligned} Y = C\left[(Y - T + i(V-\bar{F}) - \pi\bar{M}), (\bar{M} + V)\right] \\ + G - i(V-\bar{F}-B) - \dot{B} \end{aligned} \tag{3.16c}$$

$$G + iB = T + \dot{B} + m\bar{M} \tag{3.16d}$$

$$\pi = m \tag{3.16e}$$

which are exactly identical to those under free floating, given the redefinition of assets. The endogenous variables are $\bar{M}$, $\bar{F}$, V, $\dot{B}$ and π, while Y, i, u, T, G, B and m are exogenous. Table 2.5 of chapter 2 summarizes the long-run comparative static results. The long-run stationary state is illustrated diagramatically in fig. 3.2 below. The equilibrium schedules have the same interpretation as under free floating. On the horizontal axis we now have augmented real financial wealth (V) which is equal in value to the long-run stock of real financial wealth (v) under free floating. Because we restrict the presentation of the long-run equilibrium in a two-dimensional space, the CC* and FF* schedules depend on endogenous variables as well as on exogenous. Tables 2.6 - 2.8 of chapter 2 summarize the shifting variables of the three equilibrium schedules.

3.2.3 Dynamic stability

The accumulation of real money balances and augmented real financial wealth together with changing expectations move the system over time. Dynamic stability requires convergence of the sequence of short-run equilibria to the stationary state, where wealth is constant and expectations are constant and fully realised. Since under case I $\dot{Fg} = o$ in the long-run, the accumulations of augmented real financial wealth ($\dot{V}$) and real financial wealth ($\dot{v}$) will both be zero, given equation (3.15); this though, does not hold if $\dot{Fg} \neq o$, as can be the case under the generalised government reaction function discussed in chapter 4. As under free floating, the dynamic adjustment is considered under two alternative expectations mechanisms:

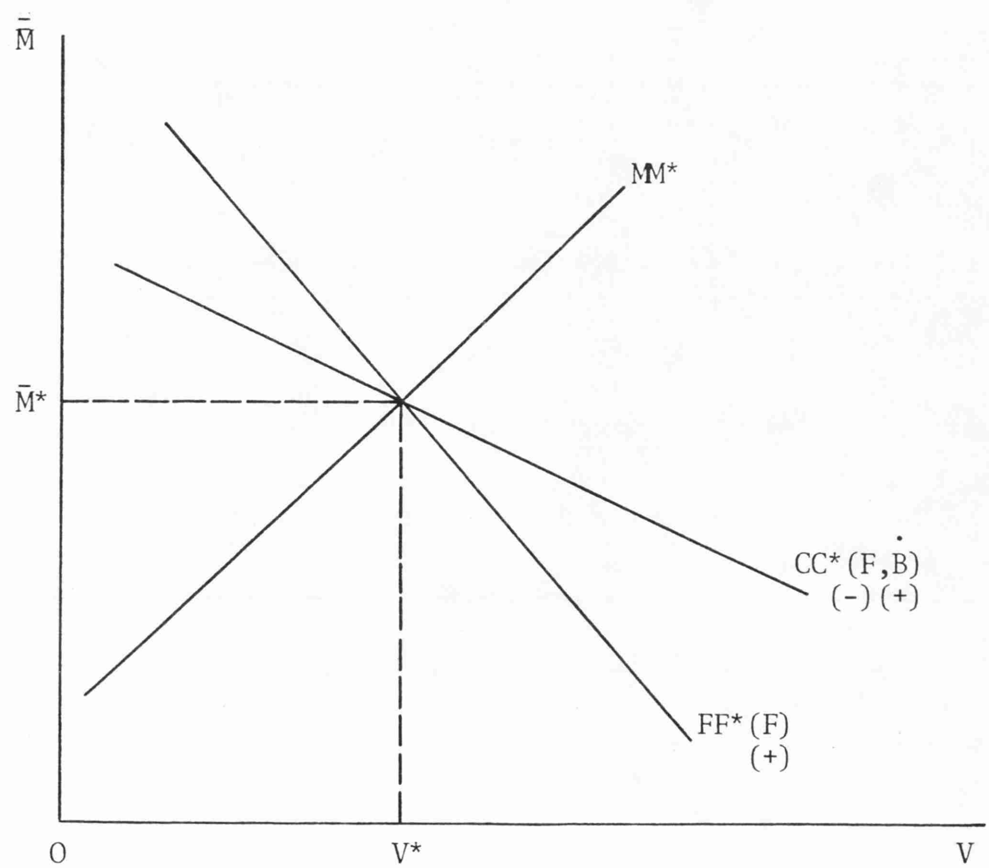

Figure 3.2. The long-run stationary state

adaptive expectations and perfect foresight.

(a) Adaptive expectations: The dynamic behaviour of the system under adaptive expectations is described by the following system of differential equations, derived formally in section A3.2 of the appendix:

$$\dot{\pi} = \beta\left[m - \frac{F\dot{g}}{\overline{M}} - \dot{x} - \pi\right]; \quad o<\beta<1,\ E_V>o,\ E_\pi<o \tag{3.17a}$$

$$x = g_1\ (V,\ \pi); \qquad g_{1V}>o,\ g_{1\pi}<o \tag{3.17b}$$

$$\dot{V} = h_1(V,\ \pi); \qquad h_{1V}<o,\ h_{1\pi}>o \tag{3.17c}$$

The first two terms in the square bracket of (3.17a) show the rate of growth of the nominal money stock, $\dot{M}^S/M^S$; in the absence of complete sterilization the rate of increase of the nominal money supply is no longer equal to m, but is affected by the intervention policy. As is proved in the appendix, the following conditions are sufficient for a locally stable stationary state:

$$|\beta.g_{1\pi}| < 1 \tag{3.18a}$$

$$\underset{(-)}{E_\pi}\ .\ \underset{(-)}{h_{1V}}\ >\ \underset{(+)}{E_V}\ .\ \underset{(+)}{h_{1\pi}} \tag{3.18b}$$

The first condition is identical in form to the stability condition under free floating. The presence of of the second condition makes the conditions for dynamic stability under managed floating more stringent than under free floating. This, however, is not due to the intervention policy *per se*, but it arises because the rate of growth of the money supply, in the absence of complete sterilization, is no longer constant over time.

Fig. 3.3 illustrates the phase diagram under adaptive expectations. The $\dot{x} = 0$ and the $\dot{V} = 0$ curves show the loci of real money balances (in logs) and augmented real financial wealth for which the stocks of real balances and augmented real financial wealth, respectively, are constant. The equations of both schedules are formally derived in the appendix from equations (3.17). The slope of the $\dot{x} = 0$ locus is ambiguous, since

$$\left.\frac{dx}{dV}\right|_{\dot{x}=0} = \frac{\overset{(+)}{g_{1V}}(\overset{(+)}{g_{1V}}\ \overset{(+)}{h_{1\pi}} - \overset{(-)}{g_{1\pi}}\ \overset{(-)}{h_{1V}}) - \beta \overset{(-)}{g_{1\pi}}\overset{(+)}{g_{1V}}}{(\underset{(-)}{g_{1\pi}}\ \underset{(-)}{E_{\pi}} + \underset{(+)}{g_{1V}}\ \underset{(+)}{h_{1\pi}})} = ? \tag{3.19a}$$

When

$$g_{1V} > \frac{h_{1V}\ g_{1\pi}}{h_{1\pi}} \tag{3.19b}$$

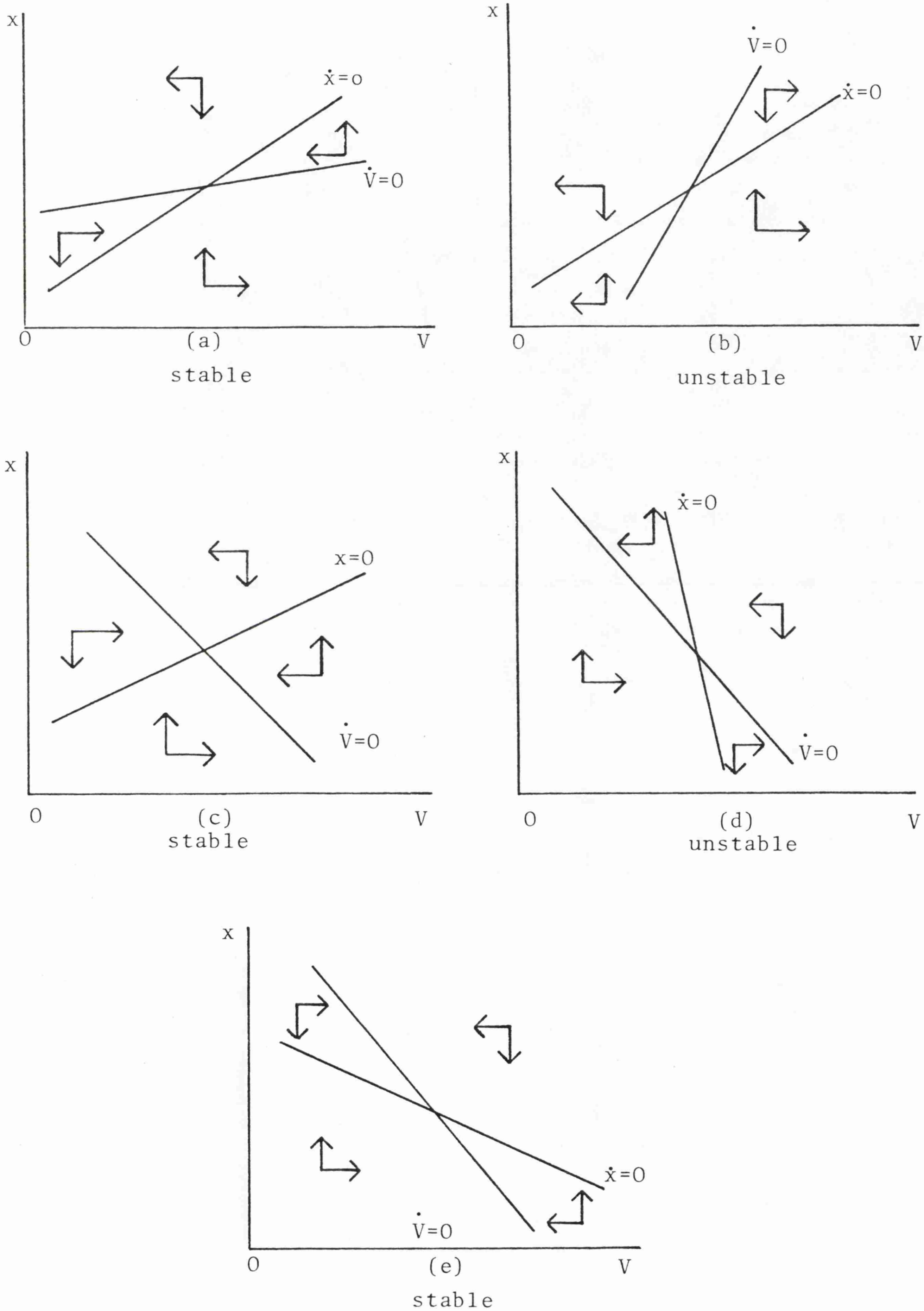

Figure 3.3. Phase diagram under adaptive expectations.

$\dot{x} = 0$ is unambiguously upwards sloping, but if the inequality sign of (3.19b) is reversed, its slope can be positive or negative. However, deviations of real money balances from their long-run level are always self correcting (vertical arrows), since

$$\left.\frac{\partial \dot{x}}{\partial x}\right|_{V=\text{const.}} = \frac{(\overset{(-)}{g_{1\pi}}\overset{(-)}{E_{\pi}} + \overset{(+)}{g_{1V}}\overset{(+)}{h_{1\pi}})}{\underset{(-)}{g_{1\pi}}\underset{(+)}{(1+\beta g_{1\pi})}} < 0 \qquad (3.19c)$$

given stability condition (3.18a).

The slope of the $\dot{V} = 0$ curve and the direction of the horizontal arrows depend on inequality (3.19b) as well:

$$\left.\frac{dx}{dV}\right|_{\dot{V}=0} = \underset{(+)}{g_{1V}} - \frac{\overset{(-)}{h_{1V}}\cdot \overset{(-)}{g_{1\pi}}}{\underset{(+)}{h_{1\pi}}} \gtrless 0 \quad \text{as} \quad g_{1V} \gtrless \frac{h_{1V}\, g_{1\pi}}{h_{1\pi}} \qquad (3.20a)$$

$$\left.\frac{\partial \dot{V}}{\partial V}\right|_{x=\text{const.}} = \frac{\overset{(+)}{h_{1\pi}}}{\underset{(-)}{g_{1\pi}}}\left(\frac{h_{1V}\, g_{1\pi}}{h_{1\pi}} - g_{1V}\right) \gtrless 0$$

$$\text{as } g_{1V} \gtrless \frac{h_{1V}\, g_{1\pi}}{h_{1\pi}} \qquad (3.20b)$$

Figure 3.3 illustrates the full range of possibilities. If inequality (3.19b) holds, both $\dot{x} = 0$ and $\dot{V} = 0$ are positively sloping and deviations of augmented real financial wealth from its long-run level are cumulative. The $\dot{V} = 0$ curve can be either flatter (fig. 3.3(a)) or steeper (fig. 3.3(b)) than the $\dot{x} = 0$ locus. The latter case, however, is inconsistent with dynamic stability. If the inequality sign in (3.19b) is reversed, $\dot{V} = 0$ becomes downwards sloping and deviations of augmented real financial wealth from its long-run level are self-correcting, while the slope of the $\dot{x} = 0$ locus can be positive or negative. Diagrams c, d and e of figure 3.3 illustrate the three possible subcases. Subcase d should be dismissed, for it implies an unstable long-run equilibrium.

As in chapter 2, to enhance intuitive interpretation, the dynamic response of the system to the exogenous fall in u, derived formally in the appendix, will be presented in the ($\overline{M}$, V) space, along with the three long-run equilibrium schedules MM*, CC* and FF* (section 3.2.4).

(β) <u>Perfect foresight</u>: With perfect foresight, expected inflation or equivalently expected exchange rate depreciation is always equal to the actual rate (see appendix for detail derivation), i.e.

$$\pi = \frac{\dot{M}^S}{M^S} - \dot{x} = \omega(V, \dot{x}); \qquad \omega_V > 0,\ \omega_{\dot{x}} < 0 \tag{3.21}$$

Substituting (3.21) in (3.17b) and (3.17c) we get a system of differential equations that describe the dynamic behaviour of the system under perfect foresight, i.e.

$$x = g_1\left[V,\ \omega(V,\ \dot{x})\right];\quad g^*_{1V} > 0,\ g_{1\pi} < 0 \tag{3.22a}$$

$$\dot{V} = h_1\left[V,\ \omega(V,\ \dot{x})\right];\quad h^*_{1V} = ?,\ h_{1\pi} > 0 \tag{3.22b}$$

where $g^*_{1V} = (\partial x/\partial V) + (\partial x/\partial \pi)\ \omega_V$ and $h^*_{1V} = (\partial \dot{V}/\partial V) + (\partial \dot{V}/\partial \pi)\omega_V$.

As is shown in the appendix, the stationary state is a saddle point equilibrium, provided:

$$h^*_{1V} < 0 \tag{3.23}$$

Fig. 3.4 below, depicts the phase diagram under perfect foresight. The $\dot{x} = 0$ and $\dot{V} = 0$ curves are derived from (3.22a) and (3.22b) respectively (see appendix). The $\dot{x} = 0$ locus is positively sloping, but deviations of real money balances from their long-run level are cumulative, since

$$\left.\frac{dx}{dV}\right|_{\dot{x} = 0} = g^*_{1V} > 0 \tag{3.24a}$$

$$\left.\frac{d\dot{x}}{dx}\right|_{V=\text{const.}} = \frac{1}{\underset{(-)}{g_{1\pi}} \cdot \underset{(-)}{\omega_{\dot{x}}}} > 0 \tag{3.24b}$$

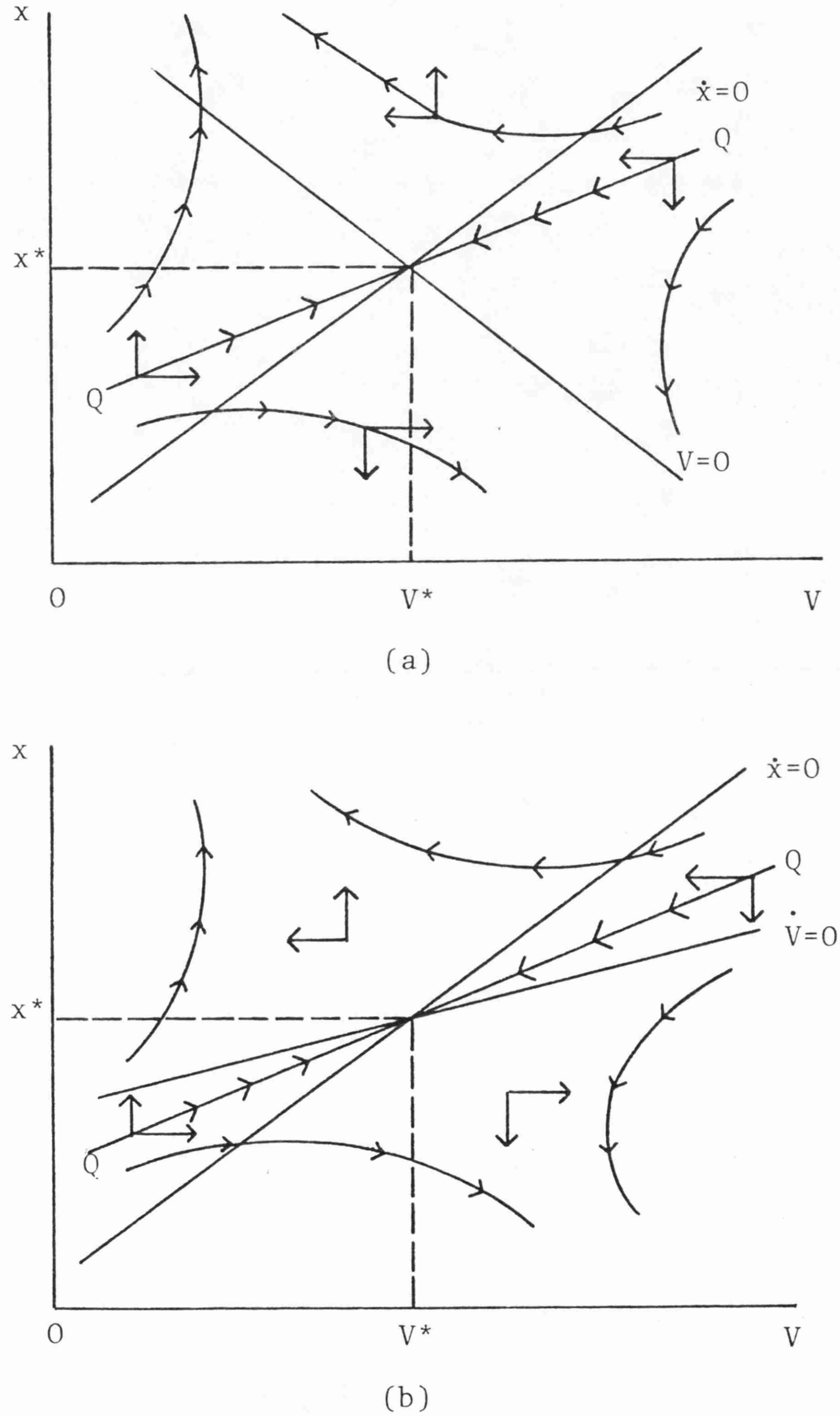

Figure 3.4. Phase diagram analysis under perfect foresight.

The slope of the $\dot{V} = 0$ curve and the reaction of the accumulation rate of augmented real financial wealth to deviations of its stock from the long-run level are again ambiguous:

$$\left.\frac{dx}{dV}\right|_{\dot{V}=0} = -\frac{h^*_{1V} \cdot g_{1\pi}}{h_{1\pi}} + g^*_{1V} \lessgtr 0 \qquad (3.25a)$$

$$\left.\frac{d\dot{V}}{dV}\right|_{x=\text{const.}} = \underset{(-)}{\overset{(+)}{\frac{h_{1\pi}}{g_{1\pi}}}}\left[\frac{h^*_{1V} \cdot g_{1\pi}}{h_{1\pi}} - g^*_{1V}\right] \lessgtr 0 \qquad (3.25b)$$

as
$$\frac{\overset{(-)}{h^*_{1V}} \cdot \overset{(-)}{g_{1\pi}}}{\underset{(+)}{h_{1\pi}}} \gtrless \underset{(+)}{g^*_{1V}} \qquad (3.25c)$$

Fig. 3.4(a) illustrates the case where the $\dot{V} = 0$ locus is downwards sloping and fig. 3.4(b) when it is positively sloping but flatter than the $\dot{x} = 0$ curve.

The assumption of long-run perfect foresight eliminates the diverging paths and ensures that the economy lies always on the equilibrating trajectory QQ (see section 2.4 of chapter 2 for details); during the adjustment process real money balances and augmented real financial wealth monotonically move in the same direction.

3.2.4 Dynamic response of the system to an exogenous change

The real test of the intervention policy depends crucially on its consequences for the dynamic adjustment of the system, as the long run stationary state is the same for both free and managed floating, given our assumption that the government uses the right estimate of the long-run exchange rate path. Fig. 3.5 partly repeats fig. 2.9 of chapter 2 and illustrates the dynamic response of the system to the exogenous fall in u, both under free and managed floating. In the absence of intervention and for both expectations mechanisms, real balances, on the impact of the exogenous change, overshoot their new long-run level, the overshooting being greater under adaptive expectations. Over time, the system follows the BE_1 path under adaptive and the AE_1 path under rational expectations till the new long-run equilibrium is reached.

The overshooting is required for short-run equilibrium in the money market, in view of the fall of u, which, at the given stock of real financial wealth (v_0*) requires a lower proportion of real money balances in private portfolios. Under managed floating, however, as we have seen, the government, through exchange market operations, increases the stock of real financial wealth by Fg(o) on the impact of the asset market disturbance. This by itself allows a higher level of real money balances at the same value of u, which effectively reduces overshooting.

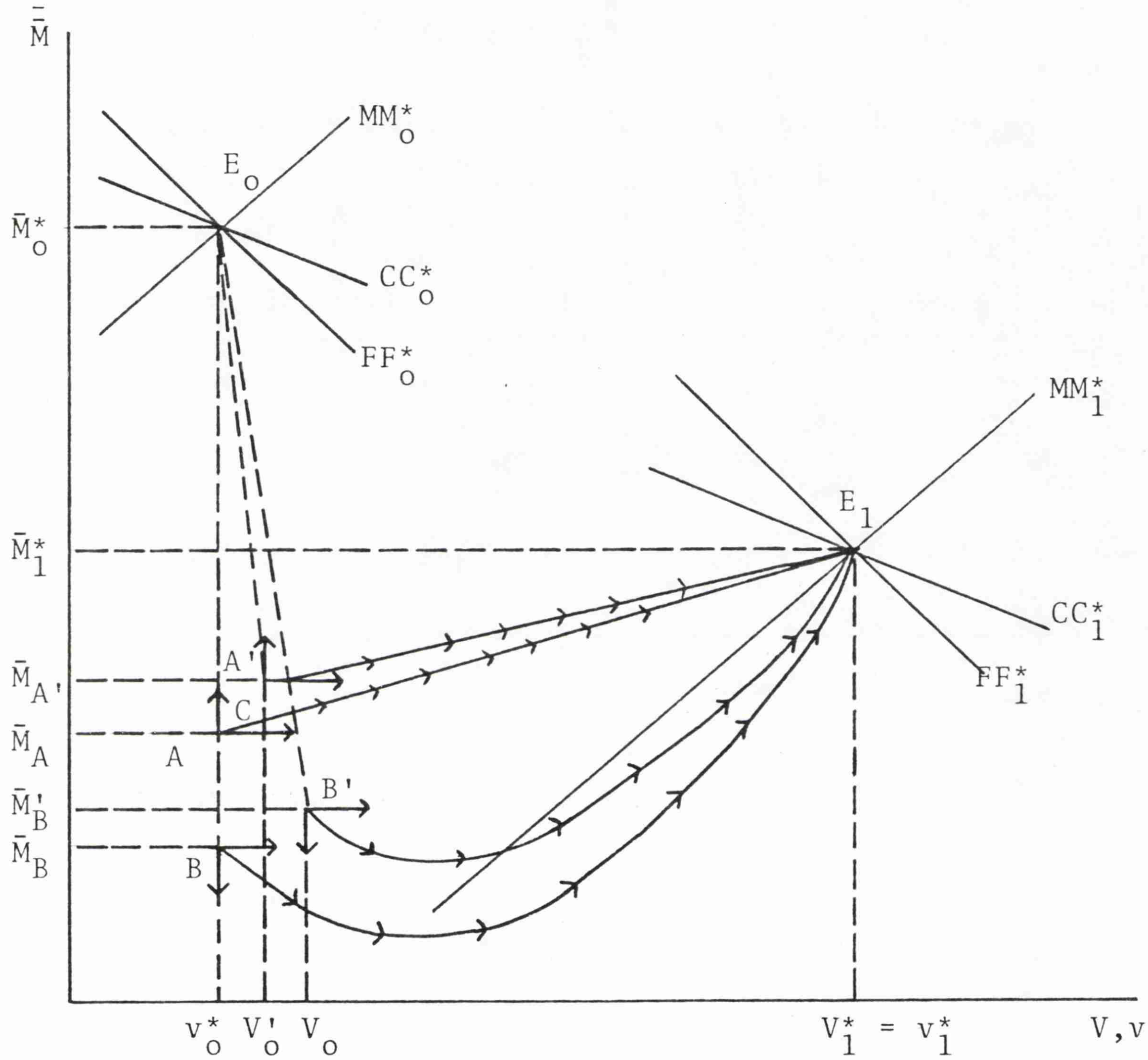

Figure 3.5. Dynamic response of the system to an exogenous fall in u.

Under adaptive expectations, on the impact of the fall in u the system shifts to B' which shows both a higher stock of real money balances ($\bar{M}_{B'}$) and augmented real financial wealth (V_o) relative to free floating.[17] The intervention policy succeeds in cushioning the reduction in real wealth and hence it makes possible a higher level of consumption and a lower current account surplus. From point B' onwards the economy follows the $B'E_1$ adjustment path to long-run equilibrium. After the impact effect real balances fall even further[18] as wealth holders, because

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17. The stock of real money balances on the impact of the exogenous change still overshoots its long-run level, since (see section A3.3 of the appendix for derivation)

$$x_o(B') = \bar{x} + \underset{(+)}{g_{1V}} \underset{(-)}{(V_o - V^*_1)} < \bar{x}$$

where $\bar{x}$ is the log of real money balances at the new stationary state, i.e. $\bar{x} = \ln\bar{M}^*_1$. $\bar{M}_{B'}$ is greater than $\bar{M}_B$ since

$$x_o(B') - x_o(B) = \underset{(+)}{g_{1V}} \cdot \underset{(+)}{Fg(o)} > 0$$

18. Under free floating, real money balances at the initial stages of the adjustment process, fall even further below $\bar{M}_B$. This is so, because in chapter 2 B was assumed to lie below the $\dot{x} = o$ and above the $\dot{v} = o$ schedules, where the vertical and horizontal arrows move the system in a south-east direction. Assuming the same for B', real money balances under managed floating follow a similar pattern of movement.

of their error-learning mechanism, come to expect an exchange rate depreciation, unaware of the fact that it has already depreciated too much. Over time, however, as wealth holders catch up with their expectations real balances start rising again along with the monotonically increasing stock of augmented real financial wealth. Formally, the accumulation rates of both assets during the adjustment process are given by the following expressions (see section A3.3 of the appendix for derivation):

$$\dot{x}\,(t) = \underset{(-)}{\lambda_1}\,\underset{(-)}{(x(t) - \bar{x})}$$

$$\overset{(-)}{-}\left[\frac{g_{1\pi}}{h_{1\pi}}\underset{(+)}{}\underset{(?)}{(\lambda_1 - h_{1V})}\underset{(?)}{(\lambda_2 - h_{1V})} + \underset{(+)}{g_{1V}}\underset{(?)}{(\lambda_1 - h_{1V})}\right]\underset{(-)}{(Vo - V^*_1)}e^{\lambda_2 t} \tag{3.26a}$$

$$\dot{V}(t) = \underset{(-)}{\lambda_1}\underset{(-)}{(V(t) - V^*_1)} - \underset{(?)}{(\lambda_1 - h_{1V})}\underset{(-)}{(V_o - V^*_1)}e^{\lambda_2 t} \tag{3.26b}$$

where λ_1 and λ_2 are the two negative characteristic roots. If B' lies above the $\dot{x} = o$ and below the $\dot{V} = o$ schedules, the phase diagram analysis implies that at the initial stages of the adjustment process (small values for t) the square bracket is negative and that the second term in (3.26a) outweighs the first, so that real money balances fall even further than at the impact of the exogenous change. If B' lies below the $\dot{x} = o$ schedule, however, real money balances will be increasing monotonically throughout the adjustment process (notice the difference in the direction of the vertical arrows in fig. 3.3 in the areas above and below $\dot{x} = o$). Given the direction of the horizontal arrows, augmented real financial wealth is monotonically increasing throughout the adjustment process.

Under long-run perfect foresight, economic agents are assumed to know the short-run and long-run comparative statics; thus, on the impact of the exogenous change they come to expect an exchange rate appreciation which shifts the system on the equilibrating path (the QQ trajectory in fig. 3.4), moderating the overshooting of real money balances below their long-run level relative to the case of adaptive expectations[19] (point A' in fig. 3.5). From point

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19. As shown in the appendix, under long-run perfect foresight real money balances on the impact of the exogenous change are equal to

$$x_o(A') = \bar{x} + \frac{\overset{(+)}{g_{1V}}}{(1-\underset{(-)}{g_{1\pi}} \cdot \underset{(-)}{\omega_{\dot{x}}} \cdot \underset{(-)}{\lambda})} (V'_o - V^*_1) < \bar{x}$$

where λ is the negative characteristic root. The moderation in the overshooting of real money balances relative to adaptive expectations depends on how much smaller Fg'(o) is relative to Fg(o), since $V_o' - V_o = Fg'(o) - Fg(o)$.

A' onwards real money balances and augmented real financial wealth are both increasing monotonically until the new stationary state is reached, along the adjustment path $A'E_1$. Formally, the accumulation rates of the two assets during the adjustment process are given by the following expressions (see appendix for derivation):

$$\dot{x}(t) = \underset{(-)}{\lambda}(\underset{(-)}{x(t) - \overline{x}}) \tag{3.27a}$$

$$\dot{V}(t) = \underset{(-)}{\lambda}(\underset{(-)}{V'_o - V^*_1})e^{\lambda t} \tag{3.27b}$$

which are both unabiguously positive (λ is the negative characteristic root). V'_o is less than V_o, because the impact government sales of foreign exchange under long-run perfect foresight, Fg'(o), are less than those under adaptive expectations, Fg(o). On the impact of the exogenous change, the moderated overshooting of real money balances relative to adaptive expectations ($\overline{M}_{A'} > \overline{M}_{B'}$), decreases the discrepancy between the spot exchange rate and its reference rate and hence, F'g(o) < Fg(o). The dynamic adjustment path $A'E_1$ in fig. 3.5 is drawn to be different from the corresponding path under free floating. This can be justified only if the QQ trajectories of fig. 2.5 and fig 3.4 are not the same, as in fact is the case since equations (2.35) and (3.22), that describe the dynamic behaviour of the system under free and managed floating respectively, are not the same functions (they have different partial derivatives).

Given the assumption of long-run perfect foresight for both the private sector and the government, there is no asymmetry of information and private economic agents can predict the consequences of any exogenous changes just as accurately as the government can; their actual behaviour though might be quantitively different, because of their wealth constraint. It would thus be interesting to examine whether intervention is beneficial to the economy, if we assume that the government policy has no qualitative effect on the dynamic adjustment process (i.e. AE_1 and $A'E_1$ are identical), although this is ruled out in our model by the apparent differences between equations (2.35) and (3.22). If that is the case and AE describes the dynamic adjustment path under free and managed floating, we still get different impact effects: at C, the impact effect under managed floating, real money balances and augmented real financial wealth are both greater relative to the impact effect under free floating (A). Government intervention is still beneficial, for it changes the private stock of foreign assets simultaneously with the exogenous change, altering the wealth constraint that restrains the behaviour of private wealth holders under free floating - quantitative effect of intervention.

But the dynamic adjustment paths are not invariant to exchange rate arrangements, as equations (2.35) and (3.22) suggest. The intervention policy, apart from its quantitative effects, has qualitative effects on the adjustment process as well; the dynamic adjustment can be speeded up or delayed depending on whether the equilibrating trajectory $A'E_1$ lies above or below AE_1 respectively. If $A'E_1$ lies below AE_1, real money balances on the impact of the exogenous change might be less than in the absence of intervention, exacerbating the overshooting of real money balances. More rigorously, this is only possible, if the speed of dynamic adjustment under managed floating is so much less relative to free floating, so that the increase in augmented real financial wealth that the intervention policy brings about is not enough to offset it, i.e. the negative qualitative effect exceeds the positive quantitative effect. Formally, this can be illustrated by estimating the difference in real money balances at the two impact effects; i.e.

$$x_o(A') - x_o(A) = \left[\frac{\overset{(+)}{g^*_{1V}}}{\underset{(+)}{(1-g_{1\pi} \cdot \omega_{\dot{x}} \cdot \lambda^M)}} - \frac{\overset{(+)}{g_V}}{\underset{(+)}{(1+g_{\pi} \cdot \lambda^F)}} \right] \underset{(-)}{(v^*_o - v^*_1)}$$

$$+ \frac{\overset{(+)}{g^*_{1V}}}{\underset{(+)}{(1-g_{1\pi} \cdot \omega_{\dot{x}} \cdot \lambda^M)}} \cdot \underset{(+)}{F'g(o)} = ? \qquad (3.28)$$

where λ^M and λ^F are the negative characteristic roots under managed and free floating respectively. The first term shows the ambiguous qualitative effect and the second the positive quantitative effect of an increase in augmented real financial wealth. If the square bracket is positive, the first term might offset the positive quantitative effect, so that government intervention exacerbates the overshooting of real money balances. This is, of course, an outcome in direct contrast to the objectives of the intervention policy. Since, however, the government has long-run perfect foresight, it can anticipate such complications. Intervention under such circumstances is irrational because it would exacerbate rather than diminish the deviations of the spot rate from its reference rate. Therefore, if the private sector has long-run perfect foresight, the government intervenes in the foreign exchange market, only if it anticipates a reduction in the overshooting of real money balances.

In summary, under both expectations' mechanisms, the intervention policy moderates the overshooting of real money balances and increases the stocks of both assets on the impact of the exogenous change, relative to free floating. It also affects the dynamic adjustment process both qualitatively and quantitavely, speeding up the adjustment process (see the analysis of fig. 3.6 below). Fluctuations in real wealth are cushioned since real money balances and augmented real financial wealth are higher than their respective values under free floating at every point in time during the adjustment process. As a result, domestic consumption and absorption in general are stabilized and current account imbalances are moderated. The differences in dynamic adjustment under the two expectations' mechanisms are identical to those under free floating: while under long-run perfect foresight real money balances and augmented real financial wealth are monotonically increasing until long-run equilibrium is restored, under adaptive expectations real money balances initially fall even further before they start rising again as speculators catch up with their expectations (i.e. they come to expect an exchange rate appreciation).

The dynamic exchange rate response to exogenous changes is completely determined by the response of the nominal and real money balances ($P(t) = M^S(t)/\overline{M}$). Figures 3.6 and 3.7 below, illustrate the dynamic response of the exchange rate to an exogenous fall in u, both under free and managed floating and for adaptive and rational expectations respectively. PP^*_o is the original long-run exchange rate path under free floating and PP^*_1 is the new one consistent with the lower value for u. In both diagrams, ABC shows the dynamic adjustment path of the exchange rate under free floating.

Under managed floating, the government throughout the adjustment process sells foreign exchange in return for domestic money. At the new stationary state, however, real

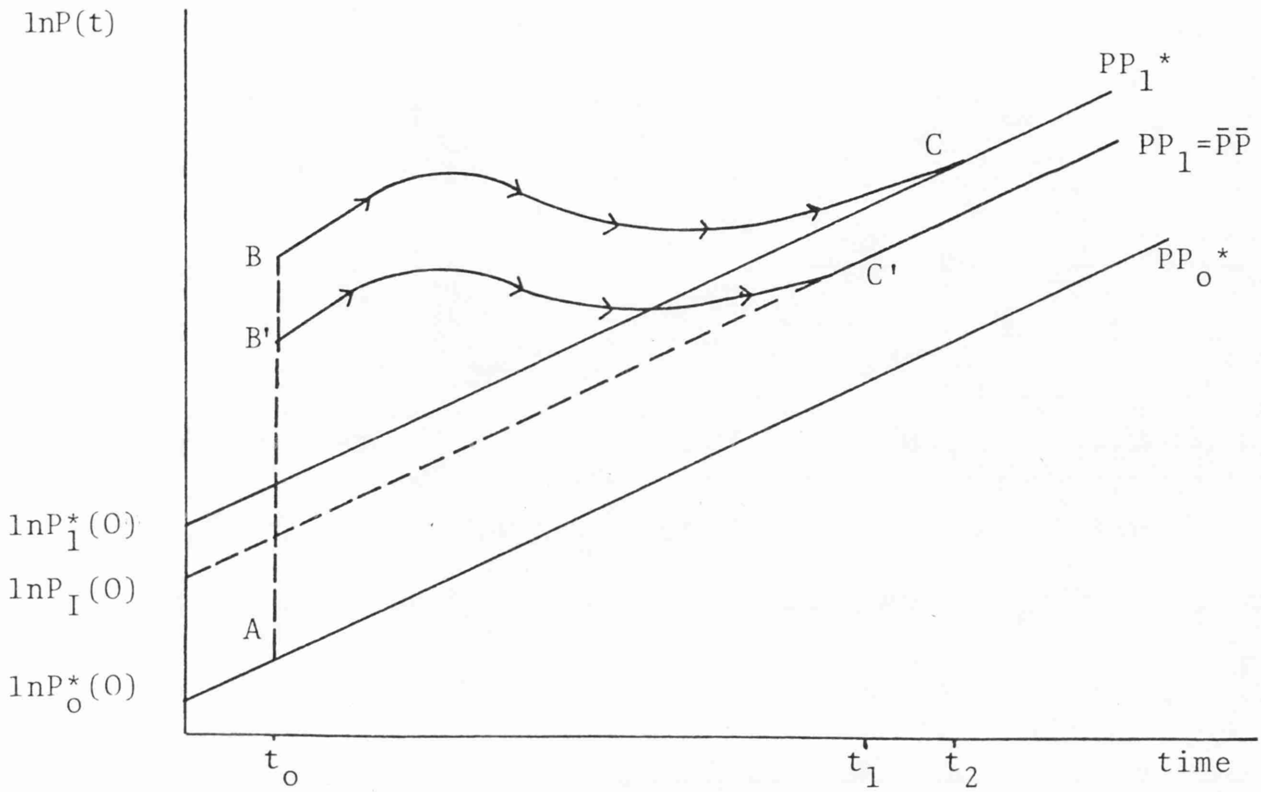

Figure 3.6. Dynamic response of the exchange rate to an exogenous fall in u, under adaptive expectations.

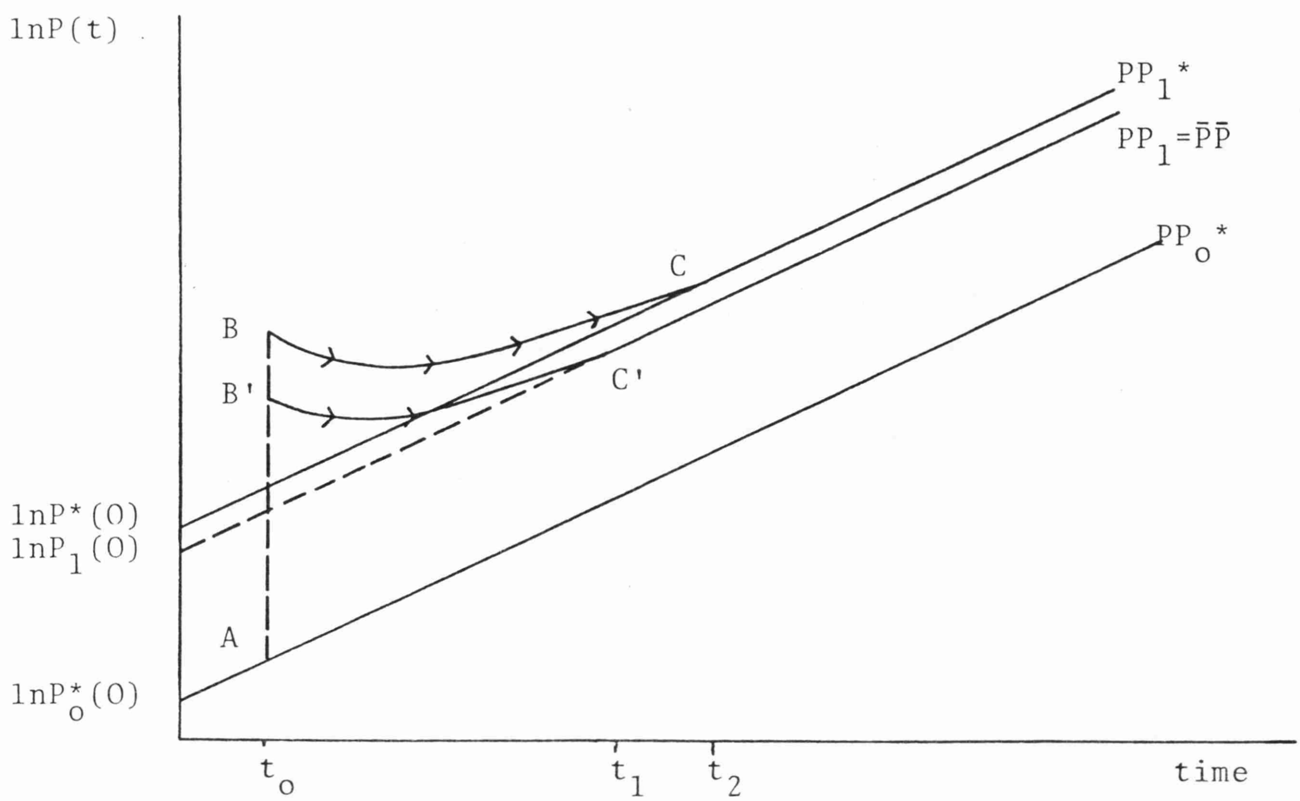

Figure 3.7. Dynamic response of the exchange rate to an exogenous fall in u, under long-run perfect foresight.

money balances are the same as in the absence of intervention, since intervention with the right estimate of the long-run exchange rate path has no long-run effects. Consequently, the long-run equilibrium exchange rate path, PP_1, lies below PP^*_1 for both expectations' mechanisms, because the nominal money stock under managed floating is less[20] than in the absence of intervention.

Considering initially the case of adaptive expectations, we have shown above that the overshooting in real money balances is reduced, implying effectively that the exchange rate overshooting is moderated as well (point B' in figure 3.6). Given the dynamic response of real money balances to the exogenous fall in u, over time, the

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20. Formally, by solving the differential equation defined by the long-run equilibrium condition (3.16e), the equation of the PP_1 path is as follows:

$$\ln P(t) = \ln M^S(o) + mt - \ln\bar{M}^*$$

Real money balances are the same for both free and managed floating. What is different is the initial nominal money stock, affected by the impact government intervention that changes discretely the initial stocks of assets. Thus, $M^S(o)$ is highest under free floating and lowest under managed floating (adaptive expectations), since

$$M(o) > \left[M(o) - Fg'(o)P_o\right] > \left[M(o) - Fg(o).P_o\right]$$

as

$$F'g(o) < Fg(o)$$

exchange rate follows the same pattern of movements as under free floating, depreciating initially more and appreciating afterwards, but at a lower path; during the adjustment process, the exchange rate is always lower than its equivalent value under free floating. Formally, the slope of the dynamic exchange rate path B'C' at every moment in time is given by the following expression (see appendix for derivation):

$$D\ln P(t) = D\ln M^S(t) - \underset{(-)}{\lambda_1}\underset{(-)}{(x(t) - \bar{x})}$$

$$+ \left[\underset{(+)}{\overset{(-)}{\frac{g_{1\pi}}{h_{1\pi}}}} \underset{(?)}{(\lambda_1 - h_{1V})} \underset{(?)}{(\lambda_2 - h_{1V})} + \underset{(+)}{g_{1V}} \underset{(?)}{(\lambda_1 - h_{1V})}\right] \underset{(-)}{(V_o - V^*_1)} e^{\lambda_2 t} \tag{3.29}$$

The first term shows the rate of increase of the nominal money stock, which is now less than m and variable over time, as the intervention policy affects the nominal money supply. So long as point B' of figure 3.5 lies above the $\dot{x} = o$ and below the $\dot{V} = o$ loci, the phase diagram analysis implies that at the initial stages of the adjustment process, the first and third terms outweigh the second, forcing the exchange rate to depreciate even more relative to its long-run path. During the adjustment process, the government reference rate $\bar{P}(t)$ is growing at a variable rate, reflecting the movements of the nominal money stock.

From point C' onwards, however, it coincides with the actual exchange rate, growing both at the same rate m. PP_1 and PP_1* are parallel to each other, since the inflation rate under free and managed floating is the same.

Similarly, under long-run perfect foresight the exchange rate overshooting is reduced, since on the impact of the exogenous change, real money balances under managed floating are higher relative to free floating (pont B' in figure 3.7). Over time, following the dynamic adjustment path B'C', the exchange rate is always appreciating relative to its long-run path, being always lower than its respective value in the absence of intervention (B'C' lies below BC). Formally, the slope of the B'C' path is given by the following expression:

$$D\ln P(t) = DmM^S(t) - \underset{(-)}{\lambda}(\underset{(-)}{x(t) - \bar{x}}) \tag{3.30}$$

indicating that throughout the adjustment process the exchange rate grows at a rate lower than the rate of growth of the nominal money stock. As under adaptive expectations, from point C' onwards the spot exchange rate coincides with the government reference rate. PP_1 is again parallel to PP_1*.

The analysis of the dynamic response of the system to the exogenous fall in u shows, that for both expectations mechanisms, the exchange rate is closer to its expected long-run path. Real money balances and augmented real financial wealth are higher, throughout the adjustment process, than their respective values in the absence of intervention. Government intervention succeeds in moderating short-run exchange rate variability (price variability) and consequently, fluctuations in real private wealth and consumption are cushioned. In addition, the dynamic adjustment process is speeded up, as for both expectations mechanisms the stationary state equilibrium is established in less time than under free floating.

However, we note that exchange rate overshooting and short-run exchange rate fluctuations are both consistent with a regime of managed floating; a result that conforms with the empirical observation of both substantial short-run exchange rate variability and active government interference in the foreign exchange market.

3.3 Case II: Government intervention with the wrong estimate of the long-run exchange rate path

In the analysis so far the government was assumed to be able to anticipate the long-run real money stock correctly. Given the uncertainties, though, that in the real world are involved in forming such an expectation, it would be interesting to relax the assumption of long-run perfect foresight and examine the consequences of any possible deviations between the expected ($\overline{M}^e$) and the actual ($\overline{M}^*$) long run real money balances, i.e. $\alpha \neq 1$. In other words, we are considering situations where the government reference rate $\overline{P}(t)$ deviates from its true value $\overline{P}^*(t)$, where $\overline{P}^*(t) = M^S(t)\ L\ (Y, u, T)$. More rigorously, the government reference rates (both right or wrong) are implicitly assumed to be point estimates. They should be interpreted though, as the centre of a zone and whenever the actual spot rate falls within this zone, government intervention ceases. Thus, we are effectively referring to deviations between $\overline{P}(t)$ and $\overline{P}^*(t)$ that are greater than the span of the neutral zone around $\overline{P}(t)$.

These discrepancies are particularly relevant in an uncertain world, where the long-run effects of exogenous disturbances are difficult to infer with certainty; more realistically, economic agents and/or the government would predict a certain range inside which the true value will lie with some probability. Discrepancies are even more likely to arise in cases where exogenous changes involve subjective variables, such as is the case with a fall in the subjective estimate of foreign risk relative to domestic money (u). Estimation of the true magnitude of such an exogenous shock is extremely difficult in itself.

In such a context, the government in practice uses as an estimate of the long-run exchange rate path either an overestimate or an underestimate of the true path. Any deviations between $\overline{P}^*(t)$ and $\overline{P}(t)$ are considered as the result of prediction errors. Deliberate competitive exchange rate policies are irrational. The law of one price and the small country assumptions that we employ, do not allow to the government any leverage to affect the foreign exchange price of domestic output.

3.3.1 Short-run equilibrium

In terms of fig. 3.1, any mistake by the government in predicting the long-run equilibrium exchange rate path would shift $\overline{P}$ either below or above $\overline{P}^*$ on the impact of exogenous fall in u, reflecting respectively an underestimate or an overestimate of the true reference rate at time t = o. To the extent that the spot rate, in the absence of intervention, lies above both $\overline{P}$ and $\overline{P}^*$ - in the same direction from both in general - the short-run equilibrium analysis will remain qualitatively the same as under Case I, the only difference being the magnitude of government intervention. In subsequent short-run equilibria that the dynamic evolution of the system will bring about, the exchange rate will appreciate, moving towards both $\overline{P}$ and $\overline{P}^*$ until it falls in the region between them ($\overline{P}$, $\overline{P}^*$), which has to be greater than the neutral zone around the government's reference rate, for otherwise intervention will stop. The behaviour of the system within the ($\overline{P}$, $\overline{P}^*$) range merits particular analysis. To illustrate this case, we assume without any loss of generality, that on the impact of the exogenous fall in u, the spot rate in the absence of intervention lies between $\overline{P}$ and $\overline{P}^*$; the government, in other words, overadjusts its estimate of the long - run exchange rate path so that the prediction error (deviation of $\overline{P}$ from $\overline{P}^*$) more than exceeds the extent of impact exchange rate overshooting[21] under free floating, i.e. $\overline{P} > P_1'$ in fig. 3.8 below. The short-run equilibrium conditions (impact effect) are equations (3.11a) - (3.11e), the same as under Case I.

Given the intervention rule (3.11c) and that $\overline{P}(t) > P'(t)$, the government will purchase foreign exchange from private wealth holders in return for domestic money; this exacerbates both the excess demand for real financial wealth and the excess supply of real money balances, caused by the fall in u. The decreased stock of augmented real

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21. Formally, this implies that α is large enough ($\alpha>1$) so that

$$\alpha(M-FgP)L_u < -\frac{f_{1u}}{(1-f_{1W})\frac{M}{P^2}} = \left.\frac{dP}{du}\right|^F$$

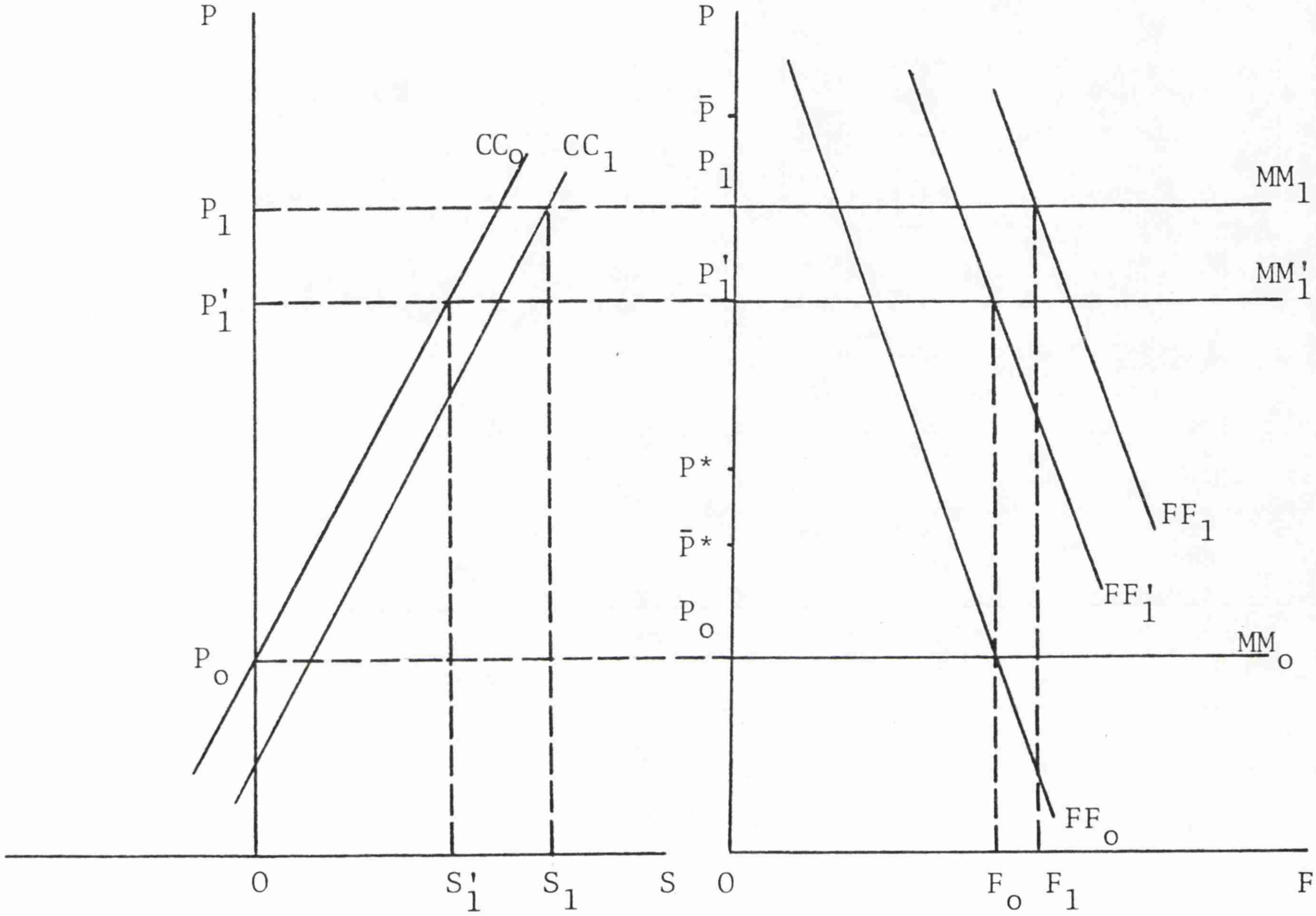

Figure 3.8. Short-run exchange rate determination under managed floating (Case II): effects of an exogenous fall in u.

financial wealth[22] leads to a reduction of the short-run private holdings of both foreign exchange and bonds.[23] (F in fig.3.8 rises). The increased excess supply of real money balances requires a greater exchange rate depreciation[24] (a larger increase in prices) relative to both free and managed floating (Case I) to restore equilibrium in the money market. In terms of fig. 3.8 the combined influences of the falls in u and Fg shift the MM_o and FF_o schedules upwards to MM_1 and FF_1 respectively, exacerbating the exchange rate overshooting, i.e. $P_1 > P'_1$.

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22. From equation (3.13), the change in the stock of augmented real financial wealth, on the impact of the exogenous change, is equal to the change in Fg. From the short-run comparative statics (section A3.1 of the appendix), contrary to Case I, we now have:

$$\left.\frac{dFg}{du}\right|_{W}^{M} = \underset{(-)}{\frac{1}{|A_3|}}\left[\alpha(M-FgP)L_u + \frac{f_{1u}}{(1-f_{1W})\frac{M}{P^2}}\right] \underset{(+)}{\ell(1-f_{1W})\frac{M}{P^2}} > o$$

since, by assumption, the square bracket is negative (see footnote 21). W indicates wrong estimate of $\overline{P}^*(t)$

23. Using restriction (2.9c) and the relationship in footnote 21, we have:

$$\left.\frac{dF}{du}\right|_{W}^{M} = -\underset{(-)}{\frac{1}{|A_3|}}\underset{(-)}{\left[\alpha(M-FgP)L_u + \frac{f_{1u}}{(1-f_{1W})\frac{M}{P^2}}\right]} \underset{(+)}{\ell . f_{3W}\cdot\frac{M}{P^2}} < 0$$

24. The exchange rate overshooting is now exacerbated, since

$$\left.\frac{dP}{du}\right|^{F} - \left.\frac{dP}{du}\right|_{W}^{M} = \underset{(-)}{\frac{1}{|A_3|}}\,\ell \underset{(-)}{\left[\alpha(M-FgP)L_u + \frac{f_{1W}}{(1-f_{1W})\frac{M}{P^2}}\right]} > 0$$

The fall in Fg shifts the CC_0 schedule downwards, while the rise in F (fall in bond holdings) pushes it upwards, fig. 3.8 showing a net downward shift. The increased exchange rate overshooting reduces real wealth and consumption by more than in the absence of intervention, improving the balance of trade. Assuming that the balance of trade effects prevail over the service account effects (negative since bond holdings fall), the current account surplus under Case II, on the impact of the exogenous change, is unambiguously greater[25] than under free floating ($S_1 > S_1'$).

Therefore, whenever the spot exchange rate falls within the region between $\overline{P}$ and $\overline{P}^*$, the intervention policy works against market forces, exacerbating the effects of exogenous shocks. Exchange rate variability is magnified

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25. Assuming again that the balance of trade effects dominate the service account effects, the current account surplus under Case II is greater than in the absence of intervention, since

$$\left.\frac{dS}{du}\right|_{W}^{M} - \left.\frac{dS}{du}\right|^{F} = \underset{(+)}{(-C_y{}^{D}\pi + C_W)} \frac{M}{P^2} \underset{(-)}{\left[\left.\frac{dP}{du}\right|_{W}^{M} - \left.\frac{dP}{du}\right|^{F} \right]}$$

$$- i \underset{(-)}{\left.\frac{dF}{du}\right|_{W}^{F}} + \underset{(+)}{C_y{}^{D}}.i \underset{(-)}{\left.\frac{dF}{du}\right|_{W}^{M}} - \underset{(+)}{C_y{}^{D}}.\pi \underset{(+)}{\left.\frac{dFg}{du}\right|_{W}^{M}} < 0$$

and fluctuations in real wealth, consumption and the current account balance are exacerbated rather than diminished; exactly the opposite of the effects of managed floating under Case I.

3.3.2 Long-run equilibrium and dynamic stability

Under Case II where $\overline{P}(t) \neq \overline{P}^*(t)$, a long-run stationary state equilibrium, if it exists, leads to a long-run exchange rate that lies somewhere between $\overline{P}(t)$ and $\overline{P}^*(t)$. The reason is quite simple: if the spot exchange rate lies in the same direction from both $\overline{P}$ and P^*, we are back to the analysis of managed floating under Case I, but with a different intervention parameter δ. Given the stability conditions for Case I, for both expectations mechanisms, the exchange rate through a sequence of short-run equilibria will be driven closer to both $\overline{P}$ and $\overline{P}^*$ till it coincides with one of the two, whichever is closer. Let us assume that we have a situation where $P = \overline{P}$, would that be a long-run equilibrium ? No; although intervention is zero, we are back to free floating with a depreciated or appreciated exchange rate relative to its long-run path in the absence of intervention $P^*(t)$, depending on whether $\overline{P}$ is greater or smaller than $P^*(t)$; thus the normal interplay of market forces would drive the exchange rate closer to $\overline{P}^*(t)$. If, alternatively, $P(t) = \overline{P}^*(t) \neq \overline{P}(t)$ the intervention policy would shift the spot rate closer to the government reference rate. Therefore, irrespective of the initial position, the exchange rate is finally driven in the region between $\overline{P}(t)$ and $\overline{P}^*(t)$, where market forces are acting at cross purposes with the intervention policy.

For long-run equilibrium to exist within this region, the effects of the intervention policy have to exactly offset the counteracting effects of market forces, i.e. $\dot{F}g \neq 0$. At the stationary state real wealth should be constant and held in the desired proportions and all nominal variables should grow at the same rate; expected inflation should be constant, equal to the actual rate and equal to the rate of growth of the nominal money stock. The last equilibrium condition[26] requires that

$$\pi = \frac{\dot{P}}{P} = \frac{\dot{M}^S}{M^S} = m + \frac{\dot{M}g}{M^S}$$

$$= m - \frac{\dot{F}g}{\overline{M}}$$

$$= m + \gamma \tag{3.31}$$

where $\gamma = -\dot{F}g/\overline{M}$. In the absence of complete sterilization, intervention affects the nominal money supply. Real money balances, $\overline{M}$, are constant since in the stationary state real wealth and its composition are constant. Hence, for

26. See chapter 4 for a full analysis of the long-run equilibrium conditions in the case of continued government intervention in the long-run.

a constant long-run inflation rate, $\dot{F}g$ has to be constant; the government has to purchase from or sell to the private sector a fixed amount of foreign exchange per period.

In the long-run stationary state, intervention rule (3.10a) becomes:

$$\dot{F}g(t) = \delta\left[P(t) - \alpha M^S(t)L(Y, u, T)\right]$$

$$= \delta\left[\frac{M^S(t)}{\overline{M}^*} - \alpha M^S(t)L(Y, u, T)\right]$$

$$= \delta\left[\frac{1}{\overline{M}^*} - \alpha L(Y, u, T)\right]M^S(t) \qquad (3.32)$$

since $P(t) = M^S(t)/\overline{M}^*$. $\overline{M}^*$ is the long-run stock of real money balances. Given that $L(Y, u, T) = 1/\overline{M}^*$, equation (3.32) becomes:

$$\dot{F}g(t) = \delta\left[\frac{1}{\overline{M}^*} - \alpha\frac{1}{\overline{M}^*}\right]M^S(t) \qquad (3.33)$$

Hence, in the presence of prediction errors ($\alpha \neq 1$), government intervention continues in the long-run ($\dot{F}g \neq 0$). The existence or not of a stationary state equilibrium under Case II, amounts finally, to whether $\dot{F}g$ is constant or not. Taking the time derivative of (3.33), we have:

$$\frac{d\dot{F}g(t)}{dt} = \delta(1-\alpha)\frac{1}{\overline{M}^*} \cdot \dot{M}^S(t) \neq 0 \qquad (3.34)$$

which shows that $\dot{F}g$, far from being constant, is increasing or decreasing over time, depending on whether α is smaller or greater than unity. Thus, the conditions for long-run equilibrium are not satisfied and government intervention under Case II leads to instability.

This is a counter-intuitive result in that any deviation of the government's estimate of the long-run exchange rate path from its true value leads to dynamic instability. The intervention policy poises the economy on a knife-edge position. Any deviation between $\overline{P}(t)$ and $\overline{P}^*(t)$ leads to instability. The reasons for this result might be the formulation of our intervention rule, the way the government forms its estimate of the long-run exchange rate path or even a possible inconsistency in government policies. On closer examination, though, instability is attributable to the particular structure of our intervention rule. The behaviour of a real variable, $\dot{F}g$, over time is determined by the difference between two nominal variables, P(t) and $\overline{P}(t)$. Building on these results, the generalised government reaction function introduced in chapter 4, modifies the intervention and allows a stable long-run equilibrium even if $\overline{P}(t) \neq P^*(t)$.

The ability of the private sector to predict these long-run effects under Case II, and its probable reaction to them will be of particular importance to exchange rate dynamics and the continuation of the intervention policy itself.

3.3.3 Dynamic exchange rate response

Exchange rate dynamics under Case II depend critically on the way the private sector forms its expectations and rest finally on the symmetry or asymmetry of information available to the private sector and the government. The analyses of Cases I and II provides the full range of possibilities.

Under Case I we have assumed that the government has long-run perfect foresight while the private sector has either adaptive expectations or long-run perfect foresight. In the former case, the government unambiguously has more information than private wealth holders, who have to rely on their error learning process. As was shown in section 3.2.4 above, the government succeeds in stabilizing exchange rate fluctuations and in moderating the variability of real wealth and private consumption. In the latter case we have no asymmetry; both the private sector and the monetary authorities have the same information set, as they are both equipped with long-run perfect foresight (the deterministic equivalent of rational expectations). Nevertheless, intervention is still beneficial because of its qualitative and quantitative effects on the adjustment process that result in moderated exchange rate variability and stabilized real wealth.

The government information set is reduced considerably when we relax the assumption of long-run perfect foresight. The crucial factor in the dynamic exchange rate response is the ability of the private sector to anticipate or not the dynamic effects of intervention under Case II and to respond to them accordingly. Under adaptive expectations the private sector is in no better position than the government whose relative superiority in information depends on the margin of error in predicting the long-run exchange rate path. If the deviations between $\overline{P}(t)$ and $\overline{P}^*(t)$ are small, the dynamic exchange rate response will initially be similar to that under Case I till the spot rate falls in the region between $\overline{P}(t)$ and $\overline{P}^*(t)$ where we have instability. The greater the deviations between the estimated and the true exchange rate paths the greater the instability region within which the government, far from stabilizing exchange rate fluctuations, exacerbates initial disturbances.

Such dynamic exchange rate behaviour is illustrated in figure 3.9 below, for an exogenous fall in u. PP^*_0 and PP^*_1 are the original and the new long-run exchange rate paths respectively and ABC is the dynamic exchange rate path under free floating. PP_1 is the long-run equilibrium exchange rate path that prevails under managed floating if the government uses the right estimate of the long-run exchange rate (i.e. when $\overline{P}(t) = \overline{P}^*(t)$). In the particular example of figure 3.9, the government overestimates the true reference rate, $\overline{P}(t) > \overline{P}^*(t)$, i.e. $\alpha > 1$, forcing the exchange rate to follow the AB'DE path. Between t_0 and

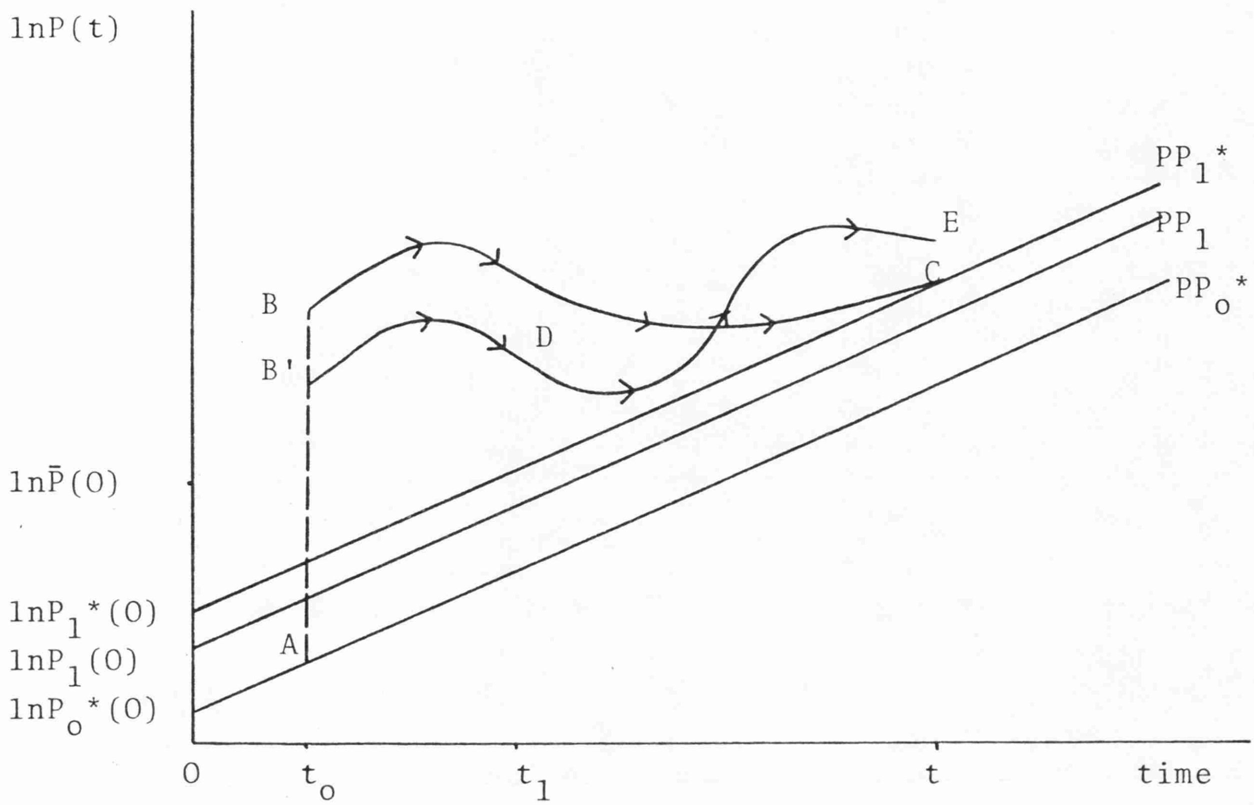

Figure 3.9. Dynamic exchange rate response under adaptive expectations: Case II; $\bar{P}(t) > \bar{P}^*(t)$

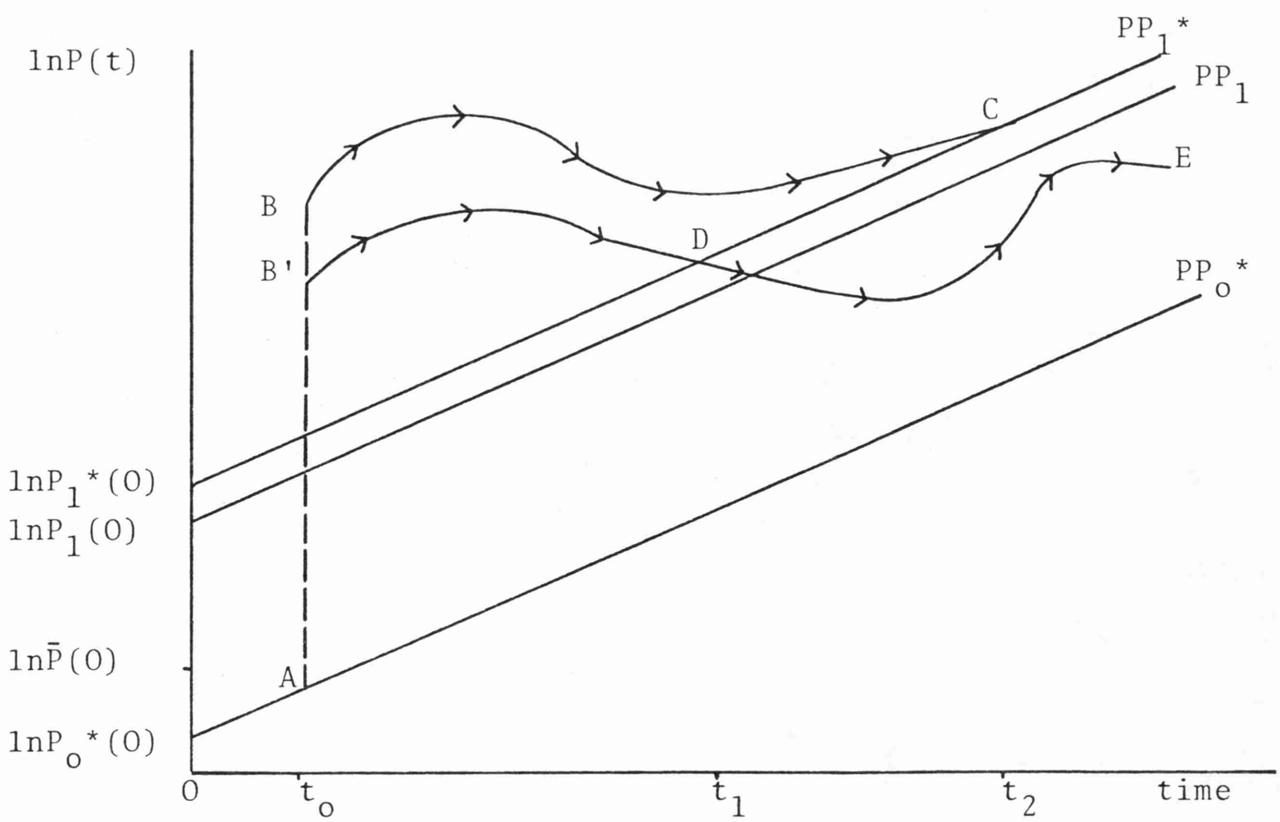

Figure 3.10. Dynamic exchange rate response under adaptive expectations: Case II; $\bar{P}(t) < \bar{P}^*(t)$

t_1, the spot rate under Case II lies above both $\overline{P}(t)$ and $\overline{P}^*(t)$ and the intervention policy stabilizes exchange rate variability in a way similar to that of Case I (B'D lies below BC). From point D (at which $P(t) = \overline{P}(t)$) onwards, however, the spot rate falls within the range $(\overline{P}, \overline{P}^*)$ where we have instability; depending on the strength of market forces relative to the intervention policy, the exchange rate can even move below PP^*_1 (though not shown in figure 3.9).

During the adjustment process, the exchange rate is predominantly above PP^*_1, its long-run path under free floating, being relatively depreciated (higher domestic inflation). If, instead, the government were to underestimate the true reference rate, i.e. $\alpha < 1$ and $\overline{P}(t) < \overline{P}^*(t)$, the spot rate in the instability region would be appreciated relative to its free floating long-run path (lower domestic inflation). This case is illustrated in fig. 3.10.

Given the adaptive expectations mechanism, private wealth holders are neither aware of the apparent instability, nor able to anticipate the possible government reaction to it; as a result, their behaviour remains invariant. The government, on the other hand, cannot afford to remain idle for long. Given the intervention rule, in the case of figure 3.9 the government will have to accumulate foreign exchange reserves for ever. Similarly, in the case of fig. 3.10 the government will have to tolerate a gradual decumulation of its foreign reserves. In the basic structure of our model we have not incorporated a stock constraint for the government intervention policy. This does not mean, however, that the government can sustain the intervention policy for ever in the face of these developments. Although it is easier to accumulate reserves - ignoring a possible reaction from the rest of the world - than to decumulate, and in spite of the extra leverage that sales of domestic bonds offer (being perfect substitutes to foreign bonds and denominated in foreign currency), the government has ultimately either to give up the intervention policy, or to adjust accordingly its estimate of the long-run real money stock (i.e. adjust α). The effects of intervention on the stocks of reserves can be used as an indicator of the way the government should adjust its estimate of the long-run path.[27] Finally the system returns to the full long-run equilibrium position of Case I. The government cannot sustain its support for a particular exchange rate path for ever against market forces because of the constraints on its actions. In the language of Krugman (1979), we have a "crisis" in the balance of payments; the way and the timing out of the crisis rest entirely on the government, given the inability of the private sector to anticipate the long-run instability.

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27. This is consistent with the 4th guideline of the IMF which calls for member countries (in consultation with the Fund) to take into account their desired reserves level, in setting their exchange rates medium term target zones.

If instead of adaptive expectations the private sector is assumed to possess long-run perfect foresight, it would have more information than the government and it would be able to recognise the deviation between $\overline{P}(t)$ and $\overline{P}^*(t)$ and the consequent instability of the dynamic adjustment. Fig. 3.11 and 3.12 below, illustrate the dynamic response of the exchange rate to a fall in u, when the government either overestimates or underestimates the true reference rate respectively. In both diagrams ABC shows the dynamic exchange rate path in the absence of intervention, while AB'DE is the respective path under Case II, when speculators react passively to the apparent exchange rate instability. Unlike adaptive expectations, speculators are now in a position to anticipate the government reaction to a continuous accumulation (fig. 3.11) or decumulation (fig. 3.12) of foreign exchange reserves, adjusting their behaviour in accordance with some maximising process. The resulting exchange rate dynamics will be quite different from those shown in fig. 3.11 and 3.12 and will depend on the anticipated specific government reaction and the respective speculative response to it. As in the case of adaptive expectations, the system finally returns to the full long-run equilibrium position of Case I.

The situation is identical to that of Krugman's (1979) model of balance of payments crises where the government is unable to defend a particular fixed parity; and, running out of reserves, gives up intervention causing a

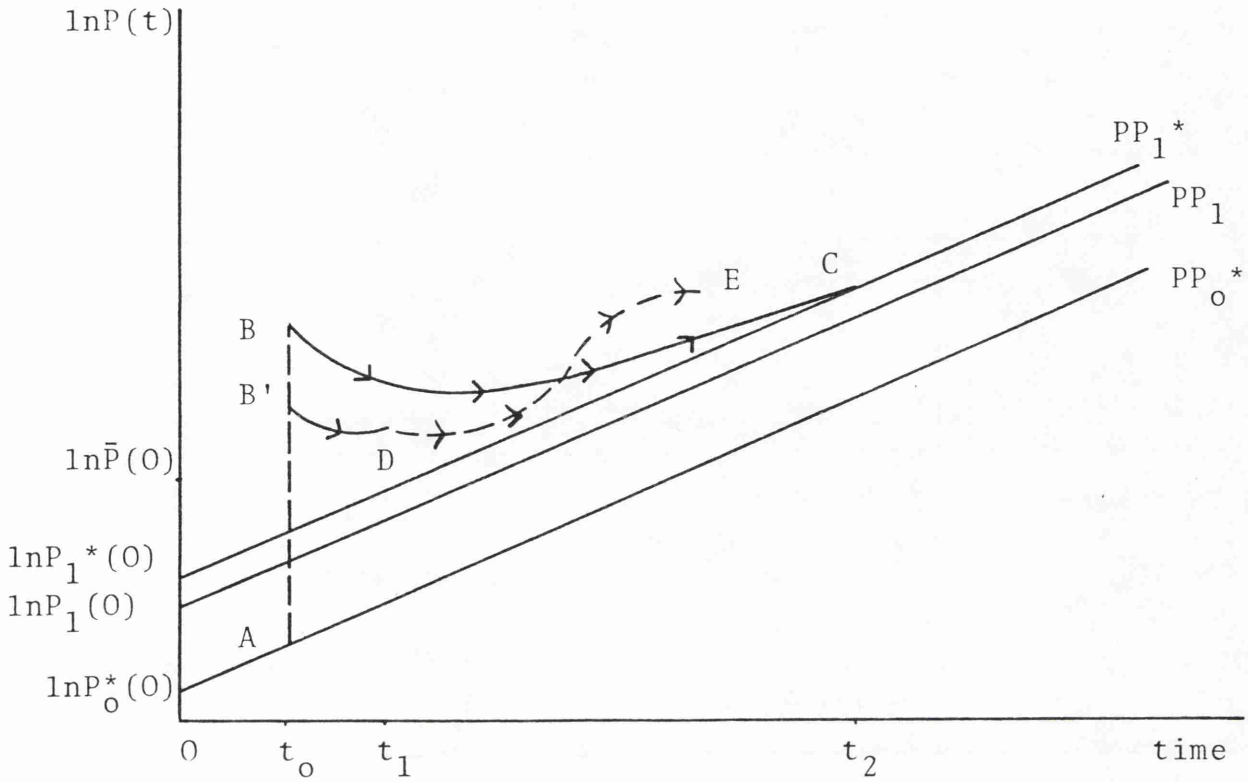

Figure 3.11. Dynamic exchange rate response under long-run perfect foreright: Case II; $\bar{P}(t) > \bar{P}^*(t)$

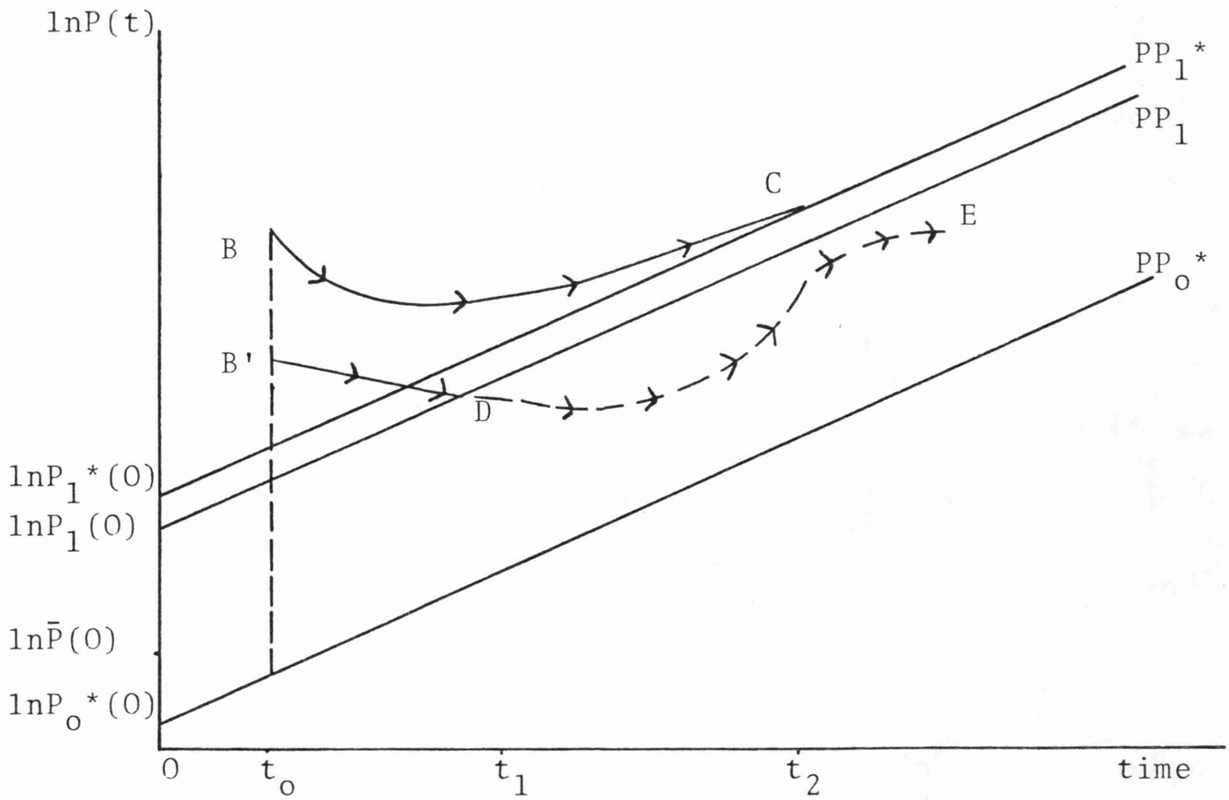

Figure 3.12. Dynamic exchange rate response under long-run perfect foreright: Case II; $\bar{P}(t) < \bar{P}^*(t)$

discrete jump in the exchange rate (depreciation) from its dynamic adjustment path under fixed to that under flexible rates. Speculators equipped with long-run perfect foresight, anticipate the forthcoming capital loss and through a speculative attack, force the government to run out of reserves earlier than otherwise; an attack that helps avoid the discrete exchange rate depreciation and is consistent with maximising behaviour by wealth holders.

In terms of our model, speculators anticipate the constraints that exchange rate variability imposes on the government stock of foreign reserves and they adjust their expectations accordingly. Unlike Krugman's model, however, the government might react in a variety of ways, e.g. give up intervention, revise the estimate of the long-run real money stock, or borrow more reserves. Private speculators have to identify which possible reaction the government is going to use and adjust their behaviour accordingly. We will not pursue the issue further at this stage, however, for in the analysis of exchange rate dynamics under a generalised government reaction function in chapter 4, we will return to this issue in more detail. Suffice it to say that the degree of symmetry of information available to the government and the private sector plays a crucial role in exchange rate dynamics under managed floating. The government is unable to impose its own exchange rate path against a better informed private sector.[28] But even under

28. Such a possibility is considered in both the IMF guidelines (commentary) and the IMF specific principles for the management of floating exchange rates.

adaptive expectations where private wealth holders have less information, intervention with the wrong estimate of the long-run exchange rate path would force the government to modify its policy in the face of its repercussions on the stock of foreign exchange reserves.

3.4 Speculation and exchange rate dynamics

Exchange rate behaviour and speculation has always been a controversial issue; as long ago as 1944 Nurske was arguing that exchange rate fluctuations in the 1920s were due to "destabilizing" speculation, whereas Friedman (1953) was suggesting that profitable speculation for the market as a whole can only be stabilizing as destabilizing speculators would lose money and be driven out of business. Commenting on the current floating experience, Kindleberger (1976) attributes the observed variability to "destabilizing and profitable speculation".[29]

A re-interpretation of our intervention rule - equations (3.9) and (3.10) - to reflect speculative behaviour allows us to examine the effects of speculation on dynamic exchange rate adjustment with no change at all in our formal analysis. To start with, assume that the government does not intervene in the foreign exchange market at all and maintains the same behaviour as under free floating, described

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29. See Schadler (1977, Section 1) for a brief survey of the relevant literature.

by the government budget restraint (2.10). The asset demand functions reflect the asset preferences of the private sector, excluding speculators. The latter are assumed to have no preferred monetary habitat, but hold non-interest bearing working balances in domestic and foreign currencies. Their only function in the system is to provide foreign exchange cover to wealth holders wishing to reallocate their portfolios.[30] They do this, by taking open positions in domestic and foreign currency at a premium. To facilitate our analysis and to keep it symmetrical to that under managed floating, we would not impose any stock constraint on speculators, although we will consider the possibility of their running out of domestic or foreign money reserves. Speculators are assumed to form expectations about the long-run exchange rate path ($\overline{P}(t)$) in the same way as the government does under managed floating. They are also assumed to take open positionsin foreign exchange depending on the discrepancies between the spot exchange rate and their "reference rate" for the respective period ($-\dot{F}g$ is the increment in speculative holdings of foreign exchange at every moment in time). In return they sell domestic money, allowing themselves a reasonable profit (exchange rate premium).

By buying domestic money when they expect an exchange rate appreciation ($P(t) > \overline{P}(t)$), speculators

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30. In this respect our model is directly analagous to that of McKinnon (1976), as cited by Schadler (1977, section III).

increase the expected real value of their money holdings at the new stationary state. ($\left[\bar{P}(t) - P(t)\right]/P(t)$ = exchange rate premium). Given the law-of-one-price assumption, the real value of their foreign exchange holdings is invariant to exchange rate changes. Similarly, when speculators anticipate a long-run exchange rate depreciation, they get out of domestic money acquiring more foreign exchange, minimizing the expected long-run reduction in their real money holdings. To liquidate their potential profits, speculators uncover their positions at the new stationary state, exchanging domestic money with foreign exchange or vice versa. Ignoring, for the moment, the effects of liquidation of potential profits, speculative activity produces exactly the same effects on exchange rate dynamics as government intervention. To illustrate this, consider again the case of an exogenous fall in u.

Allowing speculators initially to possess long - run perfect foresight, the analysis of section 3.2 illustrates (fig. 3.6 and 3.7) that, for either expectation mechanism for the private sector, speculative activity succeeds both in moderating exchange rate fluctuations and in stabilizing real wealth and consumption, in the same way as government intervention does. We assume implicitly, of course, that speculators have enough reserves to carry out their covering activities. Speculators, on the other hand, maximise their profits: by selling (buying) domestic money when they expect a long-run exchange rate depreciation (appreciation) they maximize (minimize) their expected capital gains (losses).

Relaxing the assumption of long-run perfect foresight and allowing speculative errors in predicting the long-run exchange rate path, we have an analysis similar to that of section 3.3. Fig. 3.9 and 3.10 describe now the dynamic exchange rate response to an exogenous fall in u when the private sector forms its expectations in an adaptive process and speculators use either an overestimate or an underestimate of the true reference rate respectively, at every moment in time. In fig. 3.9, speculators are accumulating foreign exchange reserves over time paying in return domestic money. Given the error learning mechanism, private wealth holders are unable to anticipate the likely outcome of this instability. As we have no government intervention, the balance of payments is always in equilibrium and instead of the "crisis" in the balance of payments we had before, we now have a "speculative crisis". Unlike the government who is able to print money at no cost, speculators are unable to defend their particular estimate of the long-run exchange rate path, as they are going to run out of reserves of domestic money. This will force them to give up their covering activities or adjust accordingly their estimate of the long-run real money stock, (i.e. change α). In either case, the system returns finally to the same long-run position (PP_1) as under Case I and speculation ceases. In terms of fig. 3.9, up to time t_1 speculators are accumulating domestic money, anticipating a capital gain, given their expectations for a long-run exchange rate appreciation. In the instability region, however, speculators are decumulating domestic money anticipating a capital loss, since they expect now a long-run exchange rate depreciation. The ultimate exchange rate appreciation, as speculators running out of stocks of domestic money either adjust their expectations or are driven out of business, increases the capital value of domestic money and speculators incur a financial loss.

In the case of fig. 3.10, speculators in the instability region are now decumulating foreign exchange and accumulating domestic money, expecting a long-run exchange rate appreciation. Over time, however, they are forced either to give up speculation or to adjust their expectations as they are running out of foreign exchange reserves. The final exchange rate depreciation causes capital losses. In both cases, the sooner speculators adjust their expectations in the face of exchange rate instability, the less their capital losses. Prolonged exchange rate instability, however, would drive speculators out of business suffering financial losses.

Under adaptive expectations, private wealth holders are unable to anticipate these developments and hence their behaviour remains invariant. Under long-run perfect foresight, however, private asset holders are able to anticipate the speculative errors and their likely outcome, adjusting their expectations accordingly. Ultimately, the system returns to the full long-run equilibrium position of Case I, at which $\overline{P}(t) = P(t)$, i.e. $\alpha = 1$ and speculation ceases. The dynamic exchange rate behaviour, however, depends, as in section 3.3.3, on the particular reaction (and its timing) of private wealth holders to speculative errors.[31]

The above analysis leads to the interesting conclusion that profitable speculation is always stabilizing and that destabilizing speculation leads to financial losses and ultimately drives speculators out of business; a conclusion that supports Friedman's (1953) argument and casts doubts on the possibility of profitable and destabilizing speculation.

The uncovering of speculative positions, however, that allows speculators to take their profits and to sustain themselves in business, disturbs the long-run equilibrium position. The cashing in of speculative profits represents an endogenous source of asset market disturbances that forces the exchange rate to deviate in the short-run from its long-run path, wiping out at the same time some of the speculative profits. Unlike speculators, the government is not governed by any profit maximization objectives in its intervention policy. In consequence, the choice between government intervention

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31. If the government, in the face of the exchange market developments, resumes intervention or alters in general its policies, the resulting exchange rate dynamics are complicated even further.

and private speculation as a means of moderating short-run exchange rate variability depends on three factors. First, on the ability of the government relative to private speculators to predict correctly the long-run exchange rate path ($\overline{P}^*(t)$). Second, on the relative significance of the additional exchange rate fluctuations that the liquidation of speculative profits causes; and finally, on whether sufficient speculative capital is available.[32]

3.5 Concluding remarks

The extension of our basic model to the regime of managed floating gives rise to some very interesting conclusions about the effectiveness of intervention and the resulting exchange rate dynamics. The success of an intervention policy geared towards minimizing the discrepancies of the spot exchange rate from its long-run value, depends crucially on the way the government forms its expectations about the long-run exchange rate path. If the government possesses long-run perfect foresight, intervention moderates exchange rate variability, stabilizes real wealth and consumption and reduces the extent of current account imbalances relative to free floating, irrespective of the way the private sector forms its expectations about exchange rate

32. McKinnon (1976a) argues that during the current floating period, the increased foreign exchange and banking risks, the absence of one "safe" currency and the introduction of new government regulations have reduced the availability of speculative capital.

depreciation (inflation). Managed floating increases the speed of dynamic adjustment but still allows exchange rate dynamics consistent with overshooting. Intervention is still beneficial even if there is no asymmetry of information between the private sector and the government. To the extent that the government's welfare function expresses social preferences, intervention improves social welfare even in a country with no market imperfections.

If, however, the government is unable to predict successfully the long-run exchange rate path, intervention exacerbates the effects of exogenous disturbances whenever the spot rate falls in the region bounded by the government's estimate of the long-run path and its true value. The resulting exchange rate variability and current account imbalances are now greater than in the absence of intervention. More crucially, though, the system becomes dynamically unstable and a "crisis" develops in the balance of payments as the government is unable to defend a particular exchange rate path against market forces, running out of or accumulating foreign exchange reserves over time. The economy, under managed floating, is poised on a knife-edge equilibrium: any deviation between the government's estimate of the long-run exchange rate path and its true value makes the system dynamically unstable.

Under an alternative interpretation, the intervention function can be seen as describing speculative behaviour providing foreign exchange cover to private wealth holders wishing to change the allocation of their portfolios. Private speculation, prior to the cashing in of speculative profits, exercises the same influence on exchange rate dynamics as government intervention. The uncovering of speculative positions, however, creates additional short - run exchange rate variation that enhances the importance of the non-profit nature of government intervention in moderating short-run exchange rate fluctuations. Nonetheless, the main issue for this end is, which of the two, the government or speculators, is better equipped to predict successfully the long-run exchange rate path.

Throughout the analysis of managed floating under Case I, i.e. when the government has long-run perfect foresight, we have implicitly assumed that the government has sufficient foreign exchange reserves to perform uninhibitedly its intervention policy. For any exogenous shock the government has either to buy or sell foreign exchange during the adjustment process. Hence, for the government stock of foreign exchange reserves to be relatively stable over a particular time horizon, the distribution of exogenous exchange rate shifts has to be symmetric with a zero mean. If the exogenous changes force the government more in the direction of selling (buying) foreign exchange, the government stock of reserves will fall (rise). If the welfare gains of moderated exchange rate variability are greater than the costs of borrowing foreign exchange, it pays for the government to borrow from abroad to finance its intervention policy. Under Case II, in contrast, the change in the government stock of reserves serves as an indicator for the proper adjustment of the government reference rates.

The degree of reserve use under managed floating can be greater or smaller than under fixed exchange rates, depending on the frequency of exogenous disturbances under the two regimes. The degree of reserve use over a particular period is usually defined as the sum of the absolute changes in the government stock of reserves. To the extent that managed floating is more prone to exogenous disturbances than fixed rates are, the degree of reserve use can very easily be higher. Moreover, whenever $\overline{P}(t)$ deviates from its true value, intervention continues as long as the government is willing to deplete its foreign exchange reserves (before the government either adjusts its estimate or gives up intervening altogether) which may involve a substantial reserve usage. Williamson (1976) provides empirical evidence that shows greater reserve use for the initial stages of the current floating period relative to the previous fixed rates period.[33]

The moderation of exchange rate fluctuation and the stabilization of real wealth and consumption with the consequent improvement in social welfare, depends on the ability of the government to limit prediction errors of the

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33. For an analysis of the expected world liquidity effects of free and managed floating relative to fixed exchange rates, see Williamson (1973), Fleming (1975) and Crockett and Goldstein (1976).

long-run exchange rate path so that its true value lies within the neutral zone around $\overline{P}(t)$ in which the government abstains from intervention. This avoids the possibility of dynamic instability. Information gathering activities and periodic revisions of the estimate of the long-run path in the light of experience will help to promote such an objective. Any failure would not only exacerbate exchange rate variability, but, in the presence of more than two countries, would also produce an inconsistent world structure of reference rates with unnecessary intervention at cross purposes that "exports" exchange rate variability from one country to another.

In the next chapter, an improvement of our intervention rule gives rise to a generalised government reaction function allowing a stable long-run equilibrium, even if the government uses the wrong estimate of the long-run exchange rate path.

APPENDIX 3

A3.1 Short-run comparative statics under managed floating.

(a) Impact effects: Linearising the short-run equilibrium conditions (3.11a) - (3.11e) of section 3.2.1, starting from an initial long-run equilibrium at which Fg = Mg = 0, we have:

$$A_3 \begin{bmatrix} dP & dF & dFg & dS & d\dot{B} \end{bmatrix}' = B_3 \begin{bmatrix} dM & dv & d\pi & du & dY & dT & dG & dB \end{bmatrix}' \tag{A3.1}$$

where

$$A_3 = \begin{bmatrix} (1-f_{1W})\frac{M}{p^2} & 0 & 1 & 0 & 0 \\ -f_{2W}\frac{M}{p^2} & -1 & -1 & 0 & 0 \\ -\ell & 0 & (1-\frac{p^2}{M}.\ \ell) & 0 & 0 \\ (-C_y{}^D\pi+C_W)\frac{M}{p^2} & -(1-C_y{}^D)i & -C_y{}^D\pi & -1 & 0 \\ m\frac{M}{p^2} & 0 & m & 0 & -1 \end{bmatrix} \tag{A3.2}$$

with

$$|A_3| = -(1 - \frac{p^2}{M}\ .\ \ell)\underset{(+)}{(1-f_{1W})}\ \underset{(+)}{\frac{M}{p^2}} - \ell < 0 \tag{A3.3}$$

provided

$$\left(1 - \frac{p^2}{M} \cdot \ell\right) > 0 \quad \text{or} \quad \ell < \frac{M}{p^2} \tag{A3.3a}$$

which we assume to be so.

$$B_3 = \begin{bmatrix} (1-f_{1W})\frac{1}{P} & -f_{1W} & -f_{1\pi} & -f_{1u} & -f_{1y} & 0 & 0 & 0 \\ -f_{2W}\frac{1}{P} & -f_{2W} & -f_{2\pi} & -f_{2u} & -f_{2y} & 0 & 0 & 0 \\ -\ell\frac{P}{M} & 0 & 0 & -\alpha\ell(M-FgP)L_u & -\alpha\ell(M-FgP)L_y & -\alpha\ell(M-FgP)L_T & 0 & 0 \\ (-C_y^D\pi + C_W)\frac{1}{P} & \left[C_W-(1-C_y^D)i\right] & -C_y^D(\frac{M}{P} - Fg) & 0 & -(1-C_y^D) & -C_y^D & 1 & i \\ m\frac{1}{P} & 0 & 0 & 0 & 0 & 1 & -1 & -i \end{bmatrix}$$

(A3.4)

Using Cramer's rule, the comparative statics effects on the exchange rate are as follows:

$$\frac{dP}{dM} = \frac{P}{M} > 0 \tag{A3.5}$$

$$\frac{dP}{dv} = \underset{(-)}{\frac{1}{|A_3|}}\ \underset{(+)}{f_{1W}}\ \Big(1 - \underset{(+)}{\frac{P^2}{M}}\ \ell\Big) < 0 \tag{A3.6}$$

$$\frac{dP}{d\pi} = \underset{(-)}{\frac{1}{|A_3|}}\ \underset{(-)}{f_{1\pi}}\ \Big(1 - \underset{(+)}{\frac{P^2}{M}}\ \ell\Big) > 0 \tag{A3.7}$$

$$\frac{dP}{du} = \underset{(-)}{\frac{1}{|A_3|}} \left[\underset{(+)}{\Big(1-\frac{P^2}{M}\ell\Big)} \underset{(+)}{f_{1u}} - \underset{(+)}{\alpha\ell(M-FgP)} \underset{(-)}{L_u} \right] < 0 \tag{A3.8}$$

$$\frac{dP}{dY} = \underset{(-)}{\frac{1}{|A_3|}} \left[\underset{(+)}{\Big(1-\frac{P^2}{M}\ell\Big)} \underset{(+)}{f_{1y}} - \underset{(+)}{\alpha\ell(M-FgP)} \underset{(-)}{L_y} \right] < 0 \tag{A3.9}$$

$$\frac{dP}{dT} = - \underset{(-)}{\frac{1}{|A_3|}}\ \underset{(+)}{\alpha\ell(M-FgP)} \underset{(+)}{L_T} > 0 \tag{A3.10}$$

$$\frac{dP}{dG} = \frac{dP}{dB} = 0 \tag{A3.11}$$

Comparative statics effects on F:

$$\frac{dF}{dM} = 0 \tag{A3.12}$$

$$\frac{dF}{dv} = -\underset{(-)}{\frac{1}{|A_3|}}\left[\underset{(+)}{f_{2W}}\underset{(+)}{(1 - \frac{p^2}{M}\ell)} + \ell\underset{(+)}{(1 - f_{1W})}\right] > 0 \tag{A3.13}$$

$$\frac{dF}{d\pi} = \underset{(+)}{\frac{\partial F}{\partial \pi}} + \underset{(-)}{\frac{\partial F}{\partial P}} \cdot \underset{(+)}{\frac{dP}{d\pi}} + \underset{(-)}{\frac{\partial F}{\partial Fg}} \cdot \underset{(+)}{\frac{dFg}{d\pi}}$$

$$= f_{2\pi} - f_{2W}\frac{M}{p^2}\frac{1}{|A_3|}(1 - \frac{p^2}{M}.\ell)f_{1\pi} - \frac{1}{|A_3|}\ell f_{1\pi} = ? \tag{A3.14}$$

$$\frac{dF}{du} = \underset{(-)}{\frac{\partial F}{\partial u}} + \underset{(-)}{\frac{\partial F}{\partial P}} \cdot \underset{(-)}{\frac{dP}{du}} + \underset{(-)}{\frac{\partial F}{\partial Fg}} \cdot \underset{(-)}{\frac{dFg}{du}}$$

$$= -\underset{(-)}{\frac{\ell}{|A_3|}}\left[\alpha(M-FgP)L_u + \frac{f_{1u}}{(1-f_{1W})\frac{M}{p^2}}\right]\underset{(+)}{f_{3W}}\frac{M}{p^2} > 0 \tag{A3.15}$$

using restriction (2.9c); the square bracket is positive since

$$\alpha(M - FgP)L_u > -\frac{f_{1u}}{(1-f_{1W})\frac{M}{p^2}} = \left.\frac{dP}{du}\right|^{F} \tag{A3.15a}$$

as the exchange rate under free floating overshoots its long-run value (i.e. $\overline{P} < P^* < P'$ in figure 3.1).

$$\frac{dF}{dY} = \frac{1}{|A_3|} \{ - \alpha\ell(M-FgP)L_y f_{3W} \frac{M}{P^2} + \ell f_{3y} - (1-\frac{P^2}{M}\ell)\left[f_{2y}(1-f_{1W}) + f_{1y}f_{2W}\right] \frac{M}{P^2}\} = ? \tag{A3.16}$$

$$\frac{dF}{dT} = - \underset{(-)}{\frac{1}{|A_3|}} \alpha\ell\underset{(+)}{(M-FgP)}L_T \underset{(+)}{f_{3W}} \frac{M}{P^2} > 0 \tag{A3.17}$$

$$\frac{dF}{dG} = \frac{dF}{dB} = 0 \tag{A3.18}$$

Comparative statics effects on Fg:

$$\frac{dFg}{dM} = 0 \tag{A3.19}$$

$$\frac{dFg}{dv} = \underset{(-)}{\frac{1}{|A_3|}} \ell \underset{(+)}{f_{1W}} < 0 \tag{A3.20}$$

$$\frac{dFg}{d\pi} = \underset{(-)}{\frac{1}{|A_3|}} \ell \underset{(-)}{f_{1\pi}} > 0 \tag{A3.21}$$

$$\frac{dFg}{du} = \frac{1}{|A_3|}\left[\alpha(M-FgP)L_u + \frac{f_{1u}}{(1-f_{1W})\frac{M}{p^2}}\right]\ell(1-f_{1W})\frac{M}{p^2} < 0$$

$$\begin{matrix} (-) & (+) & (+) \end{matrix}$$

(A3.22)

given (A3.15a)

$$\frac{dFg}{dY} = \frac{\ell}{|A_3|}\left[\alpha(M - FgP)L_y(1 - f_{1W})\frac{M}{p^2} + f_{1y}\right] = ? \qquad (A3.23)$$

$$\frac{dFg}{dT} = \frac{1}{|A_3|}\alpha\ell(M - FgP)L_T(1 - f_{1W})\frac{M}{p^2} < 0 \qquad (A3.24)$$

$$\begin{matrix} (-) & (+) & (+) & (+) \end{matrix}$$

$$\frac{dFg}{dG} = \frac{dFg}{dB} = 0 \qquad (A3.25)$$

Comparative statics effects on the current account balance:

$$\frac{dS}{dM} = 0 \qquad (A3.26)$$

$$\frac{dS}{dv} = \frac{\partial S}{\partial v} + \frac{\partial S}{\partial P} \cdot \frac{dP}{dv} + \frac{\partial S}{\partial F} \cdot \frac{dF}{dv} + \frac{\partial S}{\partial Fg} \cdot \frac{dFg}{dv}$$

$$(-) \quad (+) \quad (-) \quad (-) \quad (+) \quad (-) \quad (-)$$

$$= - \left[C_W - (1 - C_y{}^D) i\right] \qquad (-)$$

$$+ (-C_y{}^D\pi + C_W)\frac{M}{p^2} \cdot \frac{1}{|A_3|} (1 - \frac{p^2}{M} \cdot \ell) f_{1W} \qquad (-)$$

$$+ (1 - C_y{}^D) i \left[f_{2W}(\frac{M}{p^2} - \ell) + \ell(1-f_{1W})\right] \qquad (-)$$

$$- C_y{}^D\pi \frac{1}{|A_3|} \ell f_{1W} \qquad (+)$$

$$= ? \qquad \text{(A3.27)}$$

$$\frac{dS}{d\pi} = \frac{\partial S}{\partial \pi} + \frac{\partial S}{\partial P} \cdot \frac{dP}{d\pi} + \frac{\partial S}{\partial F} \cdot \frac{dF}{d\pi} + \frac{\partial S}{\partial Fg} \cdot \frac{dFg}{d\pi}$$

$$(+) \quad (+) \quad (+) \quad (-) \quad (?) \quad (-) \quad (+)$$

$$= C_y{}^D (\frac{M}{P} - Fg) \qquad (+)$$

$$+ \left(- C_y{}^{D}{}_{\pi} + C_W\right) \frac{M}{p^2} \cdot \frac{1}{|A_3|} f_{1\pi}\left(1 - \frac{p^2}{M}\ell\right) \qquad (+)$$

$$- \left(1 - C_y{}^{D}\right) i \left[f_{2\pi} - f_{2W}\frac{M}{p^2} \frac{1}{|A_3|} \left(1 - \frac{p^2}{M}\ell\right) f_{1\pi} - \frac{1}{|A_3|} \ell f_{1\pi} \right] \qquad (?)$$

$$- C_y{}^{D}{}_{\pi} \frac{1}{|A_3|} \ell f_{1\pi} \qquad (-)$$

$$= ? \qquad (A3.28)$$

$$\frac{dS}{du} = \underset{(0)}{\frac{\partial S}{\partial u}} + \underset{(+)}{\frac{\partial S}{\partial P}} \circ \underset{(-)}{\frac{dP}{du}} + \underset{(-)}{\frac{\partial S}{\partial F}} \circ \underset{(+)}{\frac{dF}{du}} + \underset{(-)}{\frac{\partial S}{\partial Fg}} \cdot \underset{(-)}{\frac{dFg}{du}}$$

$$= \left(-C_y{}^{D}{}_{\pi} + C_W\right)\frac{M}{p^2} \frac{1}{|A_3|} \left[\left(1-\frac{p^2}{M}\ell\right) f_{1u} - \alpha \ell (M-FgP) L_u \right] \qquad (-)$$

$$+ \left(1-C_y{}^{D}\right) i \frac{\ell}{|A_3|} \cdot \left[\alpha (M-FgP) L_u + \frac{f_{1u}}{(1-f_{1W})\frac{M}{p^2}} \right] f_{3W}\frac{M}{p^2} \qquad (-)$$

$$- C_y{}^D\pi \frac{1}{|A_3|} \left[\alpha(M-FgP)L_u + \frac{f_{1u}}{(1-f_{1W})\frac{M}{p^2}} \right] \ell(1-f_{1W})\frac{M}{p^2} \quad (+)$$

$$= \; ? \tag{A3.29}$$

$$\frac{dS}{dY} = \underset{(-)}{\frac{\partial S}{\partial Y}} + \underset{(+)}{\frac{\partial S}{\partial P}} \cdot \underset{(-)}{\frac{dP}{dY}} + \underset{(-)}{\frac{\partial S}{\partial F}} \cdot \underset{(?)}{\frac{dF}{dY}} + \underset{(-)}{\frac{\partial S}{\partial Fg}} \cdot \underset{(?)}{\frac{dFg}{dY}} = \; ? \tag{A3.30}$$

$$\frac{dS}{dT} = \underset{(+)}{\frac{\partial S}{\partial T}} + \underset{(+)}{\frac{\partial S}{\partial P}} \cdot \underset{(+)}{\frac{dP}{dT}} + \underset{(-)}{\frac{\partial S}{\partial F}} \cdot \underset{(+)}{\frac{dF}{dT}} + \underset{(-)}{\frac{\partial S}{\partial Fg}} \cdot \underset{(+)}{\frac{dFg}{dT}}$$

$$= C_y{}^D + \frac{1}{|A_3|} \left[- (-C_y{}^D\pi + C_W) + (1-C_y{}^D)i \; f_{3W} \right.$$

$$\left. - C_y{}^D\pi(1 - f_{1W}) \right] \frac{M}{p^2} \cdot \alpha\ell(M - FgP)L_T = \; ? \tag{A3.31}$$

$$\frac{dS}{dG} = - 1 < 0 \tag{A3.32}$$

$$\frac{dS}{dB} = - i < 0 \tag{A3.33}$$

Comparative statics effects on the issue of new debt:

$$\frac{d\dot{B}}{dM} = 0 \tag{A3.34}$$

$$\frac{d\dot{B}}{dv} = \underset{(-)}{\frac{1}{|A_3|}} \underset{(+)}{f_{1W}} \cdot m \frac{M}{p^2} < 0 \tag{A3.35}$$

$$\frac{d\dot{B}}{d\pi} = \underset{(-)}{\frac{1}{|A_3|}} \underset{(-)}{f_{1\pi}} \cdot m \frac{M}{p^2} > 0 \tag{A3.36}$$

$$\frac{d\dot{B}}{du} = \underset{(-)}{\frac{1}{|A_3|}} \left[\underset{(+)}{f_{1u}} \cdot m \frac{M}{p^2} - \alpha \ell \underset{(+)}{(M - FgP)} \underset{(-)}{L_u} \cdot m \frac{M}{p^2} \underset{(+)}{f_{1W}} \right] < 0 \tag{A3.37}$$

$$\frac{d\dot{B}}{dY} = \underset{(-)}{\frac{1}{|A_3|}} \left[\underset{(+)}{f_{1y}} \cdot m \frac{M}{p^2} - \alpha\ell \underset{(+)}{(M - FgP)} \underset{(-)}{L_y} \cdot m \cdot \underset{(+)}{f_{1W}} \frac{M}{p^2} \right] < 0$$

(A3.38)

$$\frac{d\dot{B}}{dT} = - \underset{(-)}{\frac{1}{|A_3|}} \alpha\ell \underset{(+)}{(M - FgP)} \underset{(+)}{L_T} \underset{(+)}{f_{1W}} \ m \frac{M}{p^2} - 1 = ? \quad \text{(A3.39)}$$

$$\frac{d\dot{B}}{dG} = 1 > 0 \quad \text{(A3.40)}$$

$$\frac{d\dot{B}}{dB} = i > 0 \quad \text{(A3.41)}$$

(β) <u>Short-run comparative statics (general case)</u>: Linearising the equilibrium conditions (3.14a) - (3.14e) we have:

$$A_4 \left[dP \ d\overline{F} \ dS \ d\dot{B} \ d\dot{F}g \right]' = B_4 \left[dM^S \ dV \ d\pi \ du \ dY \ dT \ dG \ dB \right]'$$

(A3.42)

where

$$A_4 = \left[\begin{array}{cccc|c} (1-f_{1W})\,\dfrac{M^S}{p^2} & 0 & 0 & 0 & 0 \\ -f_{2W}\,\dfrac{M^S}{p^2} & -1 & 0 & 0 & 0 \\ (-C_y{}^D\pi + C_w)\,\dfrac{M^S}{p^2} & -(1-C_y{}^D)\,i & -1 & 0 & 0 \\ m\,\dfrac{M^S}{p^2} & 0 & 0 & -1 & 0 \\ \hline -\delta & 0 & 0 & 0 & 1 \end{array}\right] \tag{A3.43}$$

with

$$|A_4| = -(1 - f_{1W})\,\frac{M^S}{p^2} < 0 \tag{A3.44}$$

$$
B_4 = \begin{bmatrix}
(1-f_{1W})\frac{1}{P} & -f_{1W} & -f_{1\pi} & -f_{1u} & -f_{1y} & 0 & 0 & 0 \\
-f_{2W}\,\frac{1}{P} & -f_{1W} & -f_{2\pi} & -f_{2u} & -f_{2y} & 0 & 0 & 0 \\
(-C_y^{D}\pi+C_W)\frac{1}{P} & C_W-(1-C_y^{D})i & -C_y^{D}\frac{M^S}{P} & 0 & -(1-C_y^{D}) & -C_y^{D} & 1 & i \\
m\,\frac{1}{P} & 0 & 0 & 0 & 0 & 1 & -1 & -i \\
-\delta\,\frac{P}{M^S} & 0 & 0 & -\alpha\delta M^S L_u & -\alpha\delta M^S L_y & -\alpha\delta M^S L_T & 0 & 0
\end{bmatrix}
\qquad (A3.45)
$$

Matrix A_4 is recursive and has the same determinant as matrix A_1 of chapter 2. The matrix of coefficients of the exogenous variables B_4 is exactly the same as B_1, except for the last row entries for u, Y and T. In spite of that, however, because matrix A_1 is recursive, for P, $\overline{F}$, S and $\dot{B}$ we get exactly the same comparative statics results as in chapter 2.

The comparative statics effects on $\dot{F}g$ are as follows:

$$\frac{d\dot{F}g}{dM^S} = 0 \tag{A3.46}$$

$$\frac{d\dot{F}g}{dV} = \delta \frac{dP}{dV} = \underset{(-)}{\frac{1}{|A_4|}} \delta \underset{(+)}{f_{1W}} < 0 \tag{A3.47}$$

$$\frac{d\dot{F}g}{d\pi} = \delta \frac{dP}{d\pi} = \underset{(-)}{\frac{1}{|A_4|}} \cdot \delta \underset{(-)}{f_{1\pi}} > 0 \tag{A3.48}$$

$$\frac{d\dot{F}g}{du} = \frac{\partial \dot{F}g}{\partial u} + \delta \frac{dP}{du} = - \alpha\delta M^S \underset{(-)}{L_u} + \underset{(-)}{\frac{1}{|A_4|}} \delta \underset{(+)}{f_{1u}} < 0 \tag{A3.49}$$

since, in the absence of intervention, the exchange rate overshoots its long-run path.

$$\frac{d\dot{F}g}{dY} = \frac{\partial \dot{F}g}{\partial Y} + \delta \frac{dP}{dY} = -\underset{(-)}{\alpha\delta M^S L_y} + \underset{(-)}{\frac{1}{|A_4|}} \underset{(+)}{\delta f_{1y}} = ? \qquad \text{(A3.50)}$$

$$\frac{d\dot{F}g}{dT} = \frac{\partial \dot{F}g}{\partial T} + \delta \frac{dP}{dT} = -\underset{(+)}{\alpha\delta M^S L_T} + 0 < 0 \qquad \text{(A3.51)}$$

$$\frac{d\dot{F}g}{dG} = \frac{d\dot{F}g}{dB} = 0 \qquad \text{(A3.52)}$$

The short-run comparative static effects on $\dot{V}$ are given as the algebraic sum of the effects on S, $\dot{B}$ and $\dot{F}g$.

A3.2 Dynamic stability

From equation (3.6) of the text, real money balances in the absence of sterilization are equal to:

$$\bar{M} = \frac{M}{P} + \frac{Mg}{P} = \frac{M^S}{P} \qquad \text{(3.6) repeated}$$

Hence,

$$\frac{d\bar{M}}{dV} = -\frac{M^S}{p^2} \cdot \frac{dP}{dV} = \frac{f_{1W}}{(1-f_{1W})} > 0 \qquad \text{(A3.53a)}$$

$$\frac{d\bar{M}}{d\pi} = -\frac{M^S}{p^2} \cdot \frac{dP}{d\pi} = \frac{f_{1\pi}}{(1-f_{1W})} < 0 \qquad \text{(A3.53b)}$$

Thus, we can write real money balances as a function of augmented real financial wealth and expectations, i.e.

$$\bar{M} = \theta_1(V, \pi) \qquad \text{(A3.54)}$$

with

$$\theta_{1V} = \frac{d\bar{M}}{dV} = \frac{f_{1W}}{(1-f_{1W})} > 0;$$

$$\theta_{1\pi} = \frac{d\bar{M}}{d\pi} = \frac{f_{1\pi}}{(1-f_{1W})} < 0 \qquad \text{(A3.54a)}$$

Taking the logarithm of (A3.54) we have:

$$x = \ln \bar{M} = \ln \theta_1 (V, \pi) \tag{A3.55a}$$

or

$$x = g_1(V, \pi) \tag{A3.55b}$$

with

$$g_{1V} = \frac{\theta_{1V}}{\theta_1(V, \pi)} > 0; \quad g_{1\pi} = \frac{\theta_{1\pi}}{\theta_1(V, \pi)} < 0 \tag{A3.55c}$$

Equations (A3.54) and (A3.55) are exactly identical to equations (2.26) and (2.27) respectively, of chapter 2.

From the short-run comparative statics the accumulation of augmented real financial wealth can also be written as a function of augmented real financial wealth and expectations (Table 3.2), i.e.

$$\dot{V} = h_1(V, \pi); \quad h_{1V} < 0; \quad h_{1\pi} > 0 \tag{A3.56}$$

(a) Adaptive expectations: the expectations generation mechanism under adaptive expectations can be written as:

$$\dot{\pi} = \beta(\frac{\dot{M}^S}{M^S} - \dot{x} - \pi); \quad 0 < \beta < 1 \tag{A3.57}$$

The rate of growth of the national money stock is equal to

$$\frac{\dot{M}^S}{M^S} = \frac{\dot{M}+\dot{M}g}{M^S} = m + \frac{\dot{M}g}{M^S} = m - \frac{\dot{F}g.P}{M^S} = m - \frac{\dot{F}g}{\overline{M}} \qquad (A3.58)$$

given equation (3.10b).

From the short-run comparative statics (Table 3.2) we can write $\dot{F}g$ as a function of augmented real financial wealth and expectations, i.e.

$$\dot{F}g = \dot{F}g(V, \pi) \qquad (A3.59)$$

with

$$\dot{F}g_V = \frac{d\dot{F}g}{dV} = - \frac{\delta f_{1W}}{(1-f_{1W})\frac{M^S}{P^2}} < 0 \qquad (A3.59a)$$

$$\dot{F}g_\pi = \frac{d\dot{F}g}{d\pi} = - \frac{\delta f_{1\pi}}{(1-f_{1W})\frac{M^S}{P^2}} > 0 \qquad (A3.59b)$$

Substituting (A3.59) and (A3.58) in (A3.57) we have:

$$\dot{\pi} = \beta\left[m - \frac{\dot{F}g(V, \pi)}{\theta_1(V, \pi)} - \dot{x} - \pi \right] \qquad (A3.57a)$$

with

$$E_V = \frac{d\dot{\pi}}{dV} = - \beta \frac{\dot{F}g_V}{\overline{M}} > 0 \qquad (A3.57b)$$

$$E_\pi = \frac{d\dot{\pi}}{d\pi} = - \beta \left(\frac{\dot{F}g_\pi}{\overline{M}} + 1 \right) < 0 \qquad (A3.57c)$$

since, under Case I, in the long-run stationary state $\dot{F}g = 0$.

The dynamics of the system under adaptive expectations are described by equations (A3.57a), (A3.55b) and (A3.56) which form the following system of differential equations:

$$\dot{\pi} = \beta\left[m - \frac{\dot{F}g(V, \pi)}{\theta_1(V, \pi)} - \dot{x} - \pi\right]; \quad 0 < \beta < 1,\ E_V > 0,\ E_\pi < 0 \tag{A3.60a}$$

$$x = g_1(V, \pi); \quad g_{1V} > 0, \quad g_{1\pi} < 0 \tag{A3.60b}$$

$$\dot{V} = h_1(V, \pi); \quad h_{1V} < 0, \quad h_{1\pi} > 0 \tag{A3.60c}$$

Linearising around the stationary state we have:

$$\dot{\pi} + \beta\dot{x} = E_\pi d\pi + E_V dV \tag{A3.61a}$$

$$0 = g_{1\pi}d\pi - dx + g_{1V}dV \tag{A3.61b}$$

$$\dot{V} = h_{1\pi}d\pi + h_{1V}dV \tag{A3.61c}$$

or in matrix notation

$$\begin{bmatrix} 1 & \beta & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\pi} \\ \dot{x} \\ \dot{V} \end{bmatrix} = \begin{bmatrix} E_\pi & 0 & E_V \\ g_{1\pi} & -1 & g_{1V} \\ h_{1\pi} & 0 & h_{1V} \end{bmatrix} \begin{bmatrix} d\pi \\ dx \\ dV \end{bmatrix} \tag{A3.62}$$

The characterisitc polyonyum is:

$$\begin{vmatrix} (E_\pi - \lambda) & -\beta\lambda & E_V \\ g_{1\pi} & -1 & g_{1V} \\ h_{1\pi} & 0 & (h_{1V} - \lambda) \end{vmatrix} = 0 \quad \text{(A3.63)}$$

with the following characteristic equation:

$$\underset{(-)}{(1 + \beta g_{1\pi})}\lambda^2 + \left[\underset{(-)}{-E_\pi} - \underset{(-)}{h_{1V}}\underset{(-)}{(1+\beta g_{1\pi})} + \underset{(+)}{\beta g_{1V}} \cdot \underset{(+)}{h_{1\pi}}\right] \lambda$$

$$+ \underset{(-)}{E_\pi} \cdot \underset{(-)}{h_{1V}} - \underset{(+)}{E_V}\underset{(+)}{h_{1\pi}} = 0 \quad \text{(A3.64}$$

Sufficient condition for local stability is that all coefficients are positive. This reduces to:

$$1 + \beta g_{1\pi} > 0 \quad \text{or} \quad |\beta \cdot g_{1\pi}| < 1 \quad \text{(A3.65a)}$$

$$E_\pi \cdot h_{1V} - E_V \cdot h_{1\pi} > 0 \quad \text{(A3.65b)}$$

To derive the $\dot{x} = 0$ and $\dot{V} = 0$ loci, we totally differentiate (A3.60b) with respect to time, i.e.

$$\dot{x} = g_{1\pi}\,\dot{\pi} + g_{1V}\,\dot{V} \tag{A3.66}$$

Substituting $\dot{\pi}$ and $\dot{V}$ from (A3.61a) and (A3.61c) respectively, we have:

$$(1 + \beta g_{1\pi})\dot{x} = (g_{1\pi}\,E_\pi + g_{1V}\,h_{1\pi})\,d\pi + (g_{1\pi}\,E_V + g_{1V}\cdot h_{1V})\,dV \tag{A3.67}$$

Substituting $d\pi$ from (A3.61b), we get the equation of the $\dot{x} = 0$ locus, i.e.

$$(1 + \beta g_{1\pi})\dot{x} = (g_{1\pi}\cdot E_\pi + g_{1V}\,h_{1\pi})\,\frac{1}{g_{1\pi}}\,dx + \left[(g_{1\pi}\,E_V + g_{1V}\,h_{1V}) - \frac{g_{1V}}{g_{1\pi}}(g_{1\pi}\,E_\pi + g_{1V}\,h_{1\pi})\right] dV \tag{A3.68}$$

from which we have:

$$\left.\frac{d\dot{x}}{dx}\right|_{V=\text{const.}} = \frac{\overset{(-)}{(g_{1\pi}}\cdot \overset{(-)}{E_\pi} + \overset{(+)}{g_{1V}}\cdot \overset{(+)}{h_{1\pi})}}{\underset{(-)}{g_{1\pi}}\ \underset{(+)}{(1 + \beta g_{1\pi})}} < 0 \tag{A3.69}$$

given stability condition (A3.65a). The slope of the $\dot{x} = 0$ locus is ambiguous since

$$\left.\frac{dx}{dV}\right|_{\dot{x}=0} = \frac{\overset{(+)}{g_{1V}}(\overset{(+)}{g_{1V}}\ \overset{(+)}{h_{1\pi}} - \overset{(-)}{g_{1\pi}}\ \overset{(-)}{h_{1V}}) + \overset{(-)}{g_{1\pi}}(\overset{(+)}{g_{1V}}\ \overset{(-)}{E_{\pi}} - \overset{(-)}{g_{1\pi}}\ \overset{(+)}{E_{V}})}{(\underset{(-)}{g_{1\pi}}\ \underset{(-)}{E_{\pi}} + \underset{(+)}{g_{1V}}\ \underset{(+)}{h_{1\pi}})} = ? \tag{A3.70}$$

However, using equations (A3.53), (A3.54a), (A3.59a), (A3.59b and (A3.59c), the second bracket on the numerator of (A3.70) is equal to $-\beta g_{1V} < 0$.

Hence, when

$$g_{1V} \geqslant \frac{h_{1V} \cdot g_{1\pi}}{h_{1\pi}} \tag{A3.70a}$$

the $\dot{x} = 0$ curve is upwards sloping, but if the inequality sign is reversed, its slope can be positive or negative.

Substituting $d\pi$ from (A3.61b) in (A3.61c), we get the $\dot{V} = 0$ locus, i.e.

$$\dot{V} = \frac{h_{1\pi}}{g_{1\pi}}\, dx + \frac{h_{1\pi}}{g_{1\pi}} \left(\frac{h_{1V} \cdot g_{1\pi}}{h_{1\pi}} - g_{1V}\right) dV \tag{A3.71}$$

from which we have:

$$\left.\frac{dx}{dV}\right|_{\dot{V}=0} = \underset{(+)}{g_{1V}} - \frac{\overset{(-)}{h_{1V}} \cdot \overset{(-)}{g_{1\pi}}}{\underset{(+)}{h_{1\pi}}} \gtrless 0 \quad \text{as} \quad g_{1V} \gtrless \frac{h_{1V} \cdot g_{1\pi}}{h_{1\pi}} \tag{A3.72a}$$

$$\left.\frac{d\dot{V}}{dV}\right|_{x=\text{const.}} = \frac{h_{1\pi}}{g_{1\pi}}\left(\frac{h_{1V}\cdot g_{1\pi}}{h_{1\pi}} - g_{1V}\right) \gtrless 0 \quad \text{as}$$

(+)

(-)

$$g_{1V} \gtrless \frac{h_{1V}\cdot g_{1\pi}}{h_{1\pi}} \tag{A3.72b}$$

(β) <u>Perfect foresight</u>: Under perfect foresight expectations are always fulfilled, i.e.

$$\pi = \frac{\dot{M}^S}{M^S} - \dot{x} = m - \frac{\dot{F}g(V,\ \pi)}{\theta_1(V,\ \pi)} - \dot{x} \tag{A3.73}$$

Linearising around the stationary state, we have:

$$\left[1 + \frac{\dot{F}g}{\overline{M}}\right] d\pi = -\frac{\dot{F}g_V}{\overline{M}}\, dV - \dot{x} \tag{A3.74}$$

since, under Case I, $\dot{F}g = 0$ in the long-run. From (A3.74) we can write expectations as a function of V and $\dot{x}$, i.e.

$$\pi = \omega(V,\ \dot{x}) \tag{A3.75}$$

with

$$\omega_V = \frac{d\pi}{dV} = - \frac{\overset{(-)}{\dot{F}g_V}}{\bar{M} + \underset{(+)}{\dot{F}g_\pi}} > 0 \qquad \text{(A3.75a}$$

$$\omega_{\dot{x}} = \frac{d_\pi}{d\dot{x}} = - \frac{\bar{M}}{\bar{M} + \underset{(+)}{\dot{F}g_\pi}} < 0 \qquad \text{(A3.75b)}$$

Substituting (A3.75) in (A3.55b) and (A3.56) we have:

$$x = g_1\left[V,\ \omega(V,\ \dot{x})\right]; \quad g^*_{1V} > 0,\ g_{1\pi} < 0 \qquad \text{(A3.76a)}$$

$$\dot{V} = h_1\left[V,\ \omega(V,\ \dot{x})\right]; \quad h^*_{1V} = ?.\ \ h_{1\pi} > 0 \qquad \text{(A3.76b)}$$

where, using (A3.55c), (A3.54a) and (A3.75a)

$$g^*_{1V} = \underset{(+)}{g_{1V}} + \underset{(-)}{g_{1\pi}} \cdot \underset{(+)}{\omega_V} = \frac{\overset{(+)}{\theta_{1V}}}{\bar{M} + \underset{(+)}{\dot{F}g_{\pi}}} > 0 \qquad \text{(A3.77a)}$$

$$h^*_{1V} = \underset{(-)}{h_{1V}} + \underset{(+)}{h_{1\pi}} \cdot \underset{(+)}{\omega_V} = ? \qquad \text{(A3.77b)}$$

Linearising (A3.76) around the stationary state, we have:

$$\begin{bmatrix} -g_{1\pi} \cdot \omega_{\dot{x}} & 0 \\ -h_{1\pi} \cdot \omega_{\dot{x}} & 1 \end{bmatrix} \begin{bmatrix} x \\ \dot{V} \end{bmatrix} = \begin{bmatrix} -1 & g^*_{1V} \\ 0 & h^*_{1V} \end{bmatrix} \begin{bmatrix} dx \\ dV \end{bmatrix} \qquad \text{(A3.78)}$$

with the following characteristic polyonyum

$$\begin{vmatrix} (-1 + g_{1\pi} \cdot \omega_{\dot{x}} \ \lambda) & g^*_{1V} \\ h_{1\pi} \cdot \omega_{\dot{x}} \ \lambda & (h^*_{1V} - \lambda) \end{vmatrix} = 0 \qquad \text{(A3.79)}$$

and characteristic equation

$$\underset{(-)}{g_{1\pi}} \cdot \underset{(-)}{\omega_{\dot{x}}} \ \lambda^2 + \left[\underset{(+)}{g^*_{1V}} \cdot \underset{(+)}{h_{1\pi}} \cdot \underset{(-)}{\omega_{\dot{x}}} - \underset{(?)}{h^*_{1V}} \cdot \underset{(-)}{g_{1\pi}} \cdot \underset{(-)}{\omega_{\dot{x}}} - 1\right] \lambda$$

$$+ \underset{(?)}{h^*_{1V}} = 0 \qquad \text{(A3.80)}$$

Thus a necessary and sufficient condition for saddle point equilibrium is $(-4a\gamma > 0)$:

$$\gamma = h^*_{1V} < 0 \qquad \text{(A3.81)}$$

Equations (A3.76a) and (A3.76b) describe the $\dot{x} = 0$ and $\dot{V} = 0$ loci respectively. From (A3.76a) the slope of the $\dot{x} = 0$ curve is positive and discrepancies of real money balances from their long-run level are cumulative, since

$$\left.\frac{dx}{dV}\right|_{\dot{x}=0} = g^*_{1V} > 0 \qquad \text{(A3.82a)}$$

$$\left.\frac{d\dot{x}}{dx}\right|_{V=\text{const.}} = \frac{1}{\underset{(-)}{g_{1\pi}} \cdot \underset{(-)}{\omega_{\dot{x}}}} \quad 0 \qquad \text{(A3.82b)}$$

Substituting (A3.76a) in (A3.76b) we have:

$$\dot{V} = \frac{h_{1\pi}}{g_{1\pi}} \cdot dx + \frac{h_{1\pi}}{g_{1\pi}} \left(\frac{h^*_{1V} \cdot g_{1\pi}}{h_{1\pi}} - g^*_{1V} \right) dV \qquad \text{(A3.83)}$$

from which we have:

$$\left.\frac{dx}{dV}\right|_{\dot{V}=0} = - \frac{h^*_{1V} \cdot g_{1\pi}}{h_{1\pi}} + g^*_{1V} \lesseqgtr 0 \qquad \text{as}$$

$$\frac{\overset{(-)}{h^*_{1V}} \cdot \overset{(-)}{g_{1\pi}}}{\underset{(+)}{h_{1\pi}}} \gtrless \overset{(+)}{g^*_{1V}} \tag{A3.84a}$$

$$\left.\frac{d\dot{V}}{dV}\right|_{X=\text{const.}} = \frac{h_{1\pi}}{g_{1\pi}} \left(\frac{h^*_{1V} \cdot g_{1\pi}}{h_{1\pi}} - g^*_{1V} \right) \lesseqgtr 0 \quad \text{as}$$

$$\frac{h^*_{1V} \cdot g_{1\pi}}{h_{1\pi}} \gtrless g^*_{1V} \tag{A3.84b}$$

A3.3 Dynamic response of the system to an exogenous fall fall in u

(a) Adaptive expectations: Inflationary expectations, real money balances and augmented real financial wealth can all be written as functions of time, i.e.

$$\pi(t) = \bar{\pi} + A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} \tag{A3.85a}$$

$$x(t) = \bar{x} + B_1 e^{\lambda_1 t} + B_2 e^{\lambda_2 t} \tag{A3.85b}$$

$$V(t) = \bar{V} + C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \tag{A3.85c}$$

where A, B and C are constants, λ_1 and λ_2 are the characteristic roots of (A3.64) and π, $\bar{x}$ and $\bar{V}$ are the stationary state values of π, x and V respectively.

For an exogenous fall in u at time t = 0 we have the following initial conditions:

$$\pi(0) = \pi_o = \bar{\pi} \tag{A3.86a}$$

$$V(0) = V_o = v^*_o + Fg(o) \tag{A3.86b}$$

where v^*o is stock of real financial wealth at the initial stationary state. From (A3.85) and (A3.86) and using (A3.63) we can solve for the six constants in a fashion similar to that of section A3.4 of chapter 2, i.e.

$$A_1 = - \frac{k_1 k_2}{(k_1 - k_2)} (V_o - \bar{V}); \quad A_2 = \frac{k_1 k_2}{(k_1 - k_2)} (V_o - \bar{V}) \tag{A3.87a}$$

$$B_1 = - \frac{g_{1\pi} \cdot k_1 k_2 + g_{1V} \cdot k_2}{(k_1 - k_2)} (V_o - \bar{V});$$

$$B_2 = \frac{g_{1\pi} k_1 k_2 + g_{1V} k_1}{(k_1 - k_2)} (V_o - \bar{V}) \tag{A3.87b}$$

$$C_1 = - \frac{k_2}{(k_1 - k_2)} (V_o - \bar{V}); \quad C_2 = \frac{k_1}{(k_1 - k_2)} (V_o - \bar{V}) \tag{A3.87c}$$

where

$$k_1 = \frac{\lambda_1 - h_{1V}}{h_{1\pi}}, \qquad k_2 = \frac{\lambda_2 - h_{1V}}{h_{1\pi}} \tag{A3.87d}$$

k_1 and k_2 are different from zero because $\lambda = h_{1V}$ is not a solution for the characteristic equation (A3.64). Letting $t = 0$ in (A3.85b) and substituting B_1 and B_2 from (A3.87b) we get

$$x(0) = x_0 = \bar{x} + \underset{(+)}{g_{1V}} (V_0 - \bar{V})$$

which gives the stock of real balances on the impact of the exogenous change. From the analysis of fig. 3.1, for a positive $Fg(0)$ the spot rate lies between P'_1 and $\bar{P}$ implying an exchange rate overshooting, i.e. $x_o < \bar{x}$, which requires $(V_o - \bar{V}) < 0$. If on the other hand $(V_o - \bar{V}) > 0$, $Fg(o)$ has to increase by more than $(\bar{V} - v_o^*) > o$ making $x_o > \bar{x}$. But $x_o > \bar{x}$ implies that $P_1 < \bar{P}$, which, given the intervention rule, requires a decrease rather than an increase in $Fg(o)$ which is inconsistent. Therefore, $(V_o - \bar{V})$ is unambiguously negative.

Using equation (A2.94) of chapter 2 and equation (A3.88), the difference between the stocks of real money balances under managed (x_o^M) and free (x_o^F) floating at time $t = o$ is as follows:

$$x_o^M - x_o^F = g_{1V}(V_o - \bar{V}) - g_V(v_o^* - v_1^*)$$

$$= (g_{1V} - g_V)(v_o^* - v_1^*) + g_{1V}Fg(o)$$

$$= \underset{(+)}{g_{1V}} \cdot \underset{(+)}{Fg(o)} > 0 \qquad \text{(A3.89)}$$

since, at the stationary state, the stock of real financial wealth under free floating (v^*_1) is equal to the stock of augmented real financial wealth ($\overline{V}$) under managed floating and that $g_{1V} = g_V$. (A3.89) indicates that the overshooting under managed floating is reduced.

Taking the time derivatives of (A3.85b) and (A3.85c) we have:

$$\dot{x}(t) = \lambda_1 B_1 e^{\lambda_1 t} + \lambda_2 B_2 e^{\lambda_2 t} \tag{A3.90a}$$

$$\dot{V}(t) = \lambda_1 C_1 e^{\lambda_1 t} + \lambda_2 B_2 e^{\lambda_2 t} \tag{A3.90b}$$

Substituting B_1 from (A3.85b), B_2 from (A3.87b), C_1 from (A3.85c), C_2 from (A3.87c) and using (A3.87d) we have:

$$\dot{x}(t) = \lambda_1 (x(t) - \overline{x}) - \left[\frac{g_{1\pi}}{h_{1\pi}} (\lambda_1 - h_{1V})(\lambda_2 - h_{1V}) + g_{1V}(\lambda_1 - h_{1V})\right](V_o - \overline{V}) e^{\lambda_2 t} \tag{A3.91a}$$

$$\dot{V}(t) = \lambda_1 (V(t) - \overline{V}) - (\lambda_1 - h_{1V})(V_o - \overline{V}) e^{\lambda_2 t} \tag{A3.91b}$$

which describe the accumulation of real money balances and augmented real financial wealth during the adjustment process.

Differentiating (A3.55a) with respect to time we have

$$\dot{x}(t) = \frac{d(\ln\bar{M})}{dt} = D\ln M^S(t) - D\ln P(t) \tag{A3.92}$$

Substituting in (A3.91a) we have:

$$D\ln P(t) = D\ln M^S(t) - \lambda_1(x(t) - \bar{x})$$

$$+ \left[\frac{g_{1\pi}}{h_{1\pi}}(\lambda_1 - h_{1V})(\lambda_2 - h_{1V}) + g_{1V}(\lambda_1 - h_{1V})\right](V_o - \bar{V})e^{\lambda_2 t} \tag{A3.93}$$

which gives the slope of the dynamic exchange rate path BC' in fig. 3.6.

(β) <u>Perfect foresight</u>: The stocks of real money balances and augmented real financial wealth, can be written as:

$$x(t) = \bar{x} + A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} \tag{A3.94a}$$

$$V(t) = \bar{V} + B_1 e^{\lambda_1 t} + B_2 e^{\lambda_2 t} \tag{A3.94b}$$

where λ_1 and λ_2 are the characteristic roots of (A3.80) and A and B is a new set of constants. The roots are real and of opposite signs - say $\lambda_1 = \lambda < o$ and $\lambda_2 > 0$ - so that for stability

$$A_2 = B_2 = 0 \tag{A3.95}$$

with (A3.94) becoming

$$x(t) = \bar{x} + Ae^{\lambda t} \tag{A3.96a}$$

$$V(t) = \bar{V} + Be^{\lambda t} \tag{A3.96b}$$

Under long-run perfect foresight, at the impact of the exogenous fall in u, initial condition (A3.86b) becomes:

$$V'(o) = V'_o = v^*_o + F'g(o) < V_o \tag{A3.86b'}$$

since $F'g(o) < Fg(o)$.

From (A3.96) and the initial condition (A3.86b') and using (A3.79) we can solve for the two constants, i.e.

$$A = \frac{g^*_{1V}}{(1-g_{1\pi} \cdot \omega_{\dot{x}}\lambda)} (V'_o - \bar{V}); \qquad B = (V'_o - \bar{V})$$

(A3.97)

Letting t = o in (A3.96a) and substituting A from above, we get the stock of real balances on the impact of the exogenous fall in u, i.e.

$$x(o) = x_o' = \bar{x} + \frac{g^*_{1V}}{(1-g_{1\pi} \cdot \omega_{\dot{x}}\lambda)} (V'_o - \bar{V}) < \bar{x} \tag{A3.98}$$

$$(-)(-)(-)$$

Since $x'_o < \bar{x}$, we still have overshooting.

The difference between the stocks of real money balances under managed $(x'_o{}^M)$ and free $(x'_o{}^F)$ floating is as follows (using equation (A2.104) of chapter 2):

$$x'_o{}^M - x'_o{}^F = \left[\underset{(+)}{\frac{g^*{}_{1V}}{(1-g_{1\pi} \cdot \omega_{\dot{x}}\lambda^M)}} - \underset{(+)}{\frac{g_V}{(1+g_\pi \lambda^F)}} \right] \underset{(-)}{(v_o{}^* - v_1{}^*)}$$

$$+ \underset{(+)}{\frac{g^*{}_{1V}}{(1-g_{1\pi} \cdot \omega_{\dot{x}}\lambda^M)}} \underset{(+)}{F'g(o)} = ? \qquad (A3.99)$$

which has an ambiguous sign.

Taking the time derivative of (A3.96) and substituting A and B from (A3.97) we derive the accumulation rates of real money balances and augmented real financial wealth during the adjustment process, i.e.

$$\dot{x}(t) = \lambda(x(t) - \overline{x}) \qquad (A3.100a)$$

$$\dot{V}(t) = \lambda(V'_o - \overline{V})e^{\lambda t} \qquad (A3.100b)$$

Substituting $\dot{x}(t)$ from (A3.92) we get the slope of the adjustment path B'C' in fig. 3.7 i.e.

$$D\ln P(t) = D\ln M^S(t) - \lambda(x(t) - \overline{x}) \qquad (A3.101)$$

CHAPTER 4

MANAGED FLOATING WITH A GENERALISED GOVERNMENT REACTION FUNCTION

4.1 Introduction

In chapter 3 we have seen that any discrepancies between the government's estimate of the long-run exchange rate path, $\bar{P}(t)$, and its true value, $\bar{P}^*(t)$, lead to dynamic instability. The root-cause of these deviations is the use by the government of the wrong estimate of the long-run stock of real money balances in estimating its reference rate at every moment in time (i.e. $\alpha \neq 1$). Given these prediction errors and the specific formulation of intervention rule (3.1), no long-run equilibrium exists, because one of the long-run equilibrium conditions is violated. Government intervention continues in the long-run involving variable rates of purchasing or selling foreign exchange ($\dot{F}g$) and domestic money ($\dot{M}g$). As a result, the rate of growth of the nominal money stock varies over time, violating the long-run equilibrium condition that requires that the inflation rate in the long-run is constant and equal to the rate of growth of the nominal money stock.

On the basis of intervention rule (3.1), the government purchases or sales of foreign exchange ($\dot{F}g$) at every moment in time are determined by the difference between the spot exchange rate and the government reference rate. In the long-run under Case II, these two rates still differ from each other and as a result, a real variable of the system, $\dot{F}g$, is given as the difference between two nominal variables that grow over time. In consequence, no long-run equilibrium exists and short-run exchange rate fluctuations are prolonged. In spite of this deficiency of the intervention rule, government intervention under Case I leads to a stable long-run equilibrium, because in the long-run, the spot rate coincides with the government reference rate and $\dot{F}g$ is always zero.

Hence, for stable long-run stationary state irrespective of whether α equals one or not, the intervention rule needs to be modified. The discrepancies between the spot exchange and the government reference rate should be deflated by another nominal variable, so that $\dot{F}g$ can be constant in the long-run at a non-zero level. Choosing the nominal money supply as our deflator, intervention rule (3.1) becomes:

$$Fg(t_o) = \ell\left[\frac{P(t_o) - \overline{P}(t_o)}{M^S(t_o)}\right]; \quad \ell > o \qquad (4.1a)$$

$$Fg(t_o) = -\frac{Mg(t_o)}{P(t_o)} \qquad (4.1b)$$

$$\dot{F}g(t) = \delta \left[\frac{P(t) - \overline{P}(t)}{M^S(t)} \right] ; \ \delta > o \qquad (4.1c)$$

$$\dot{F}g(t) = - \frac{\dot{M}g(t)}{P(t)} ; \quad t \geqslant t_o \qquad (4.1d)$$

Equation (4.1) constitutes now our new "generalised government reaction function". (4.1a) and (4.1b) describe the foreign exchange market operation through which, the government, on the impact of any exogenous change (time t_o), alters discretely the composition of private portfolios. Equations (4.1c) and (4.1d) show how the stocks of Fg and Mg are adjusted over time.

The government reference rate at every moment in time, or alternatively the government's estimate of the long-run exchange rate path, is given by the same expression as in chapter 3, i.e.

$$\overline{P}(t) = \frac{M^S(t)}{\overline{M}^e} = \alpha M^S(t) \frac{1}{\overline{M}^*} = \alpha M^S(t) L(Y, u, T) \qquad (4.2a)$$

$$L_Y < 0, \ L_u < 0, \ L_t > 0 \qquad (4.2b)$$

The intervention policy, as in the previous chapter, is geared towards minimising the discrepancies of the spot exchange rate from the government reference rate $\overline{P}(t)$ at every moment in time. Substituting equation (4.2a) however, in intervention rule (4.1), the generalised government reaction function resumes an alternative intuitive interpretation:

$$Fg(t_o) = \ell \left[\frac{1}{\bar{M}(t_o)} - \frac{1}{\bar{M}^e} \right] \tag{4.3a}$$

$$Fg(t_o) = - \frac{Mg(t_o)}{P(t_o)} \tag{4.3b}$$

$$\dot{F}g(t) = \delta \left[\frac{1}{\bar{M}(t)} - \frac{1}{\bar{M}^e} \right] \tag{4.3c}$$

$$\dot{F}g(t) = - \frac{\dot{M}g(t)}{P(t)} \; ; \qquad t \geq t_o \tag{4.3d}$$

since $P(t)/M^S(t) = \bar{M}(t)$. The government intervenes in the foreign exchange market on the basis of the discrepancies of the inverse of actual real money balances from the inverse of the government's estimate of the long-run money stock ($\bar{M}^e$). Whenever α differs from unity, $\bar{M}^e$ deviates from its true value $\bar{M}^*$.

In this chapter we incorporate the generalised government reaction function in our basic model and we compare the resulting exchange rate dynamics to those under free floating. Use of the wrong estimate of the long-run exchange rate path ($\alpha \neq 1$) constitutes an unintentional competitive exchange rate policy with consequent real effects. The format of our model does not allow any deliberate competitive exchange rate policies, for the foreign currency price of domestic output is fixed by the world market forces (law of one price and small country assumptions). Formally, though, competitive policies, deliberate or not, have the same long-run effects. Where they differ, as will be shown, is on the specific way the government would decide to solve the resulting balance of payments crises. As in the previous chapter, the government is either having long-run perfect foresight in predicting the long-run real money stock ($\alpha = 1$ and $\overline{P}(t) = \overline{P}^*(t)$), or using the wrong estimate of the long-run exchange rate path ($\alpha \neq 1$ and $\overline{P}(t) \neq \overline{P}^*(t)$).

As for the layout of this chapter, in section 4.2 we present the short-run equilibrium conditions under managed floating with the generalised government reaction function. In section 4.3 we analyse the long-run stationary state, whereas in section 4.4 we investigate the dynamic stability of the system. In section 4.5 we examine the dynamic response of the system to an exogenous disturbance, whose implications for exchange rate dynamics are dealt with in section 4.6. In section 4.7, a re-interpretation of our intervention rule allows us to consider the effects of speculation on exchange rate dynamics.

Finally, section 4.8 provides our concluding remarks.

4.2 Short-run equilibrium

The incorporation of the generalised government reaction function into our basic model yields in the short-run, basically the same results as the analysis of section 3.2.1 in chapter 3. In this section we will briefly re - state the short-run equilibrium conditions when the new intervention rule (4.1) replaces equations (3.9) and (3.10) of chapter 3. An illustration of the short-run effects of a particular exogenous shock is provided in section 4.5, where we examine the dynamic response of the system to an exogenous fall in u, the subjective estimate of foreign exchange risk relative to domestic money.

Assuming that we start from an initial long - run equilibrium where there is no government intervention and where Fg and Mg are both zero, the short-run equilibrium conditions on the impact of any exogenous change are as follows:

$$f_1\left[Y,\ (\frac{M}{P} + v),\ i,\ \pi,\ u\right] = \frac{M}{P} - Fg \qquad (4.4a)$$

$$f_2\left[Y,\ (\frac{M}{P} + v),\ i,\ \pi,\ u\right] = F + Fg \qquad (4.4b)$$

$$Fg = \ell\left[\frac{P}{(M-FgP)} - \alpha L(Y,\ u,\ T)\right] \tag{4.4c}$$

$$S = Y - C\left[(Y - T + i(v-F) - \pi\frac{M}{P} + \pi Fg),\ (\frac{M}{P} + v)\right]$$

$$- G + i(v-F-B) \tag{4.4d}$$

$$G + iB = T + \dot{B} + m\frac{M}{P} - mFg \tag{4.4e}$$

Equation (4.4c) replaces intervention rule (3.9a) and is derived by substituting equation (4.2a) in (4.1a). It describes the discrete change in the stocks of augmented real financial wealth and real money balances that the intervention policy brings about on the impact of any exogenous change. (M - FgP) is equal to the nominal money supply (M^S) at time t_o (impact effect), which is an endogenous variable affected by the intervention policy (-FgP = Mg). The interpretation of the other equilibrium conditions is the same as in the previous chapter. The short-run endogenous variables (impact effect) are P, F, Fg, S and $\dot{B}$, while M, v, i, π, u, Y, T, G, B, m and ℓ are exogenous.

As is shown in the appendix, system (4.4) qualitatively yields the same short-run comparative statics results (impact effects) as its respective system in chapter 3 (summarised in table 3.1). However, whenever the government uses the wrong estimate of the long-run exchange rate path (i.e. $\alpha \neq 1$), the sign of the comparative statics effects of changes in Y, u and T (variables that affect the government reference rate) becomes ambiguous, depending on the relative value of α.

The short-run equilibrium conditions (4.4) describe the behaviour of the system on the impact of any exogenous shocks, when the government, based on intervention rules (4.1a) and (4.1b), changes discretely the composition of private portfolios contemporaneously with the exogenous disturbances. Subsequently, the exchange rate determines recursively the evolution of both Fg and Mg over time, through reaction functions (4.1c) and (4.1d). The short-run equilibrium conditions that describe the behaviour of the system at all points in time are the following:

$$f_1\left[Y,\ (\frac{M^S}{P} + V),\ i,\ u,\ \pi\right] = \frac{M^S}{P} \tag{4.5a}$$

$$f_2\left[Y,\ (\frac{M^S}{P} + V),\ i,\ u,\ \pi\right] = \overline{F} = F + Fg \tag{4.5b}$$

$$\dot{F}g = \delta\left[\frac{p}{M^S} - \alpha L(Y,\ u,\ T)\right] \tag{4.5c}$$

$$S = Y - C\left[(Y - T + i(V-\overline{F}) - \pi\frac{M^S}{P}),\ (\frac{M^S}{P} + V)\right]$$

$$- G + i(V - \overline{F} - B) \tag{4.5d}$$

$$G + iB = T + \dot{B} + m\frac{M^S}{P} \tag{4.5e}$$

As in chapter 3, real money balances and real financial wealth are redefined to include the stocks of Mg and Fg respectively (see equations (3.6) and (3.7)). Government intervention, based on (4.5c), affects the stocks of Fg and Mg only over time and for a particular short-run equilibrium, the stocks of all assets are given by past accumulation. The endogenous variables are P, $\overline{F}$, S, $\dot{B}$ and $\dot{F}g$, while M^S, V, i, u, π, Y, T, G, B, M, m and δ are exogenous. As is shown in the appendix, the short-run comparative statics of system (4.5) yield exactly the same results, both qualitatively and quantitatively, for all endogenous variables except $\dot{F}g$ as in the previous chapters (summarised in table 3.2). The introduction of the new intervention rule changes the absolute value of the comparative statics effects on $\dot{F}g$ but not their signs. Again, whenever $\alpha \neq 1$, the effects on $\dot{F}g$ of changes in Y, u and T become ambiguous.

4.3 Long-run stationary state

If the system is stable, the sequence of short - run equilibria will lead to the long-run stationary state, where real wealth is constant, held in the desired proportions and where all nominal variables grow at the same rate.

For real private wealth to be constant, net private savings (defined as gross private savings plus capital gains) have to be zero. In the long-run, if inflation is different from zero, real money balances are depreciating or appreciating over time, depending on whether inflation is positive or negative respectively. Hence, for constant real wealth, gross private savings must compensate capital losses or gains, i.e.

$$
\begin{aligned}
Sp &= Y - T + i(V-\overline{F}) - C(Y^D, W) \\
&= Y - T + i(V-\overline{F}) - \left[Y - T + i(V-\overline{F}) - \pi\overline{M}\right] \\
&= \pi\overline{M} \qquad (4.6)
\end{aligned}
$$

since consumption in the long-run is equal to real disposable income. In the long-run, expected inflation is equal to the actual rate, equal to the rate of increase of the nominal money stock, i.e.

$$\pi = \frac{\dot{P}}{P} = \frac{\dot{M}^S}{M^S} = m + \frac{\dot{M}g}{M^S} = m - \frac{\dot{F}g}{\overline{M}} \tag{4.7}$$

given equation (4.1d).

Given the format of our model, the current account balance is given by the sum of gross private and government savings. Adding gross private savings, equation (4.6), to the budget deficit and using (4.7), we derive the current account balance in the long-run, i.e.

$$S = Sp + Sg = \pi\overline{M} + (-\dot{B} - m\overline{M})$$

$$= (m - \frac{\dot{F}g}{\overline{M}})\ \overline{M} - \dot{B} - m\overline{M}$$

$$= -\dot{B} - \dot{F}g \tag{4.8}$$

Given equation (4.8), the long-run equilibrium conditions un er managed floating with the generalised government reaction function are as follows:

$$f_1\left[Y,\ (\overline{M} + V),\ i,\ u,\ \pi\right] = \overline{M} \tag{4.9a}$$

$$f_2\left[Y,\ (\overline{M} + V),\ i,\ u,\ \pi\right] = \overline{F} \tag{4.9b}$$

$$\dot{F}g = \delta\left[\frac{1}{\overline{M}} - \alpha L(Y,\ u,\ T)\right] \tag{4.9c}$$

$$Y = C\left[(Y - T + i(V-\overline{F}) - \pi\overline{M}),\ (\overline{M} + V)\right] + G$$

$$- i(V - \overline{F} - B) - \dot{B} - \dot{F}g \qquad (4.9d)$$

$$G + iB = T + \dot{B} + m\overline{M} \qquad (4.9e)$$

$$\pi = m - \frac{\dot{F}g}{\overline{M}} \qquad (4.9f)$$

Equation (4.9c) is derived from (4.1c), given that $P(t) = M^S(t)/\overline{M}$. Using equation (4.2a), (4.9c) becomes:

$$\dot{F}g = \delta\left[\frac{1}{\overline{M}} - \frac{1}{\overline{M}^e}\right] = \delta(1 - \alpha)\,\frac{1}{\overline{M}} \qquad (4.10)$$

When the government uses the right estimate of the long-run stock of real money balances ($\overline{M}^e = \overline{M}$), α equals unity and government intervention ceases in the long-run ($\dot{F}g = 0$). In this case the long-run equilibrium conditions (4.9) reduce to the long-run equilibrium conditions under free floating. When α is greater (smaller) than unity, the government underestimates (overestimates) the true long-run real money stock and hence, intervention continues in the long-run. $\dot{F}g$ is different from zero, but constant over time. In other words, whenever the government uses the wrong estimate of the long-run exchange rate path ($\overline{P}(t) \neq \overline{P}^*(t)$).

government intervention continues in the long-run. Unlike Case II of chapter 3, though, now long-run equilibrium is possible.

For a non-zero $\dot{Fg}$ ($\alpha \neq 1$), the continued foreign exchange market operations that the intervention policy involves, tend to cause a continuous re-allocation of private portfolios. Long-run equilibrium, however, requires that private real wealth is constant, held in the desired proportions. For this to hold, private wealth holders have to offset the effects of intervention on the composition of their portfolios. In particular, for real money balances to be constant, private gross savings have to offset the capital losses (gains) that a positive (negative) rate of inflation causes. Equation (4.6) shows exactly just that. In addition, for a constant stock of augmented real financial wealth ($\dot{V} = 0$), private wealth holders have to vary their stock of real financial wealth (i.e. $\dot{v} \neq 0$) to offset the changes in their holdings of foreign exchange that the intervention policy brings about at every moment in time (since $\dot{Fg} \neq 0$). Substituting (4.8) in equation (3.15) of chapter 3, it turns out that the accumulation of real financial wealth is exactly the opposite of government sales in the long-run ($\dot{Fg}$), maintaining augmented real financial wealth constant, i.e.

$$\dot{V} = \dot{v} + \dot{Fg} = S + \dot{B} + \dot{Fg} = (-\dot{B} - \dot{Fg}) + \dot{B} + \dot{Fg} = 0 \qquad (4.11a)$$

and $$v = - \dot{Fg} \qquad (4.11b)$$

Intuitively, what is happening is quite clear. In the long-run, the stock of augmented real financial wealth (V) is constant, held in the desired proportions. This implies that $\bar{F}$ and b are both constant. If $\dot{Fg}$ is positive (negative), the government sells (buys) foreign exchange to (from) the private sector which immediately adjusts its consumption accordingly, so that its holdings of foreign exchange remain constant. This is possible because the lower (higher) inflation rate allows gross private savings to be lower (higher) and consumption higher (lower Hence, for a positive (negative) $\dot{F}g$, the current account balance falls (rises) by $\dot{F}g$ and the balance of payments is in deficit (surplus), i.e. B of P = $-\dot{F}g$.

The endogenous variables in the long-run are $\bar{M}$, V, $\bar{F}$, $\dot{F}g$, π and $\dot{B}$ while Y, T, G, i, u, M^S, B', δ and m are exogenous. The long-run equilibrium under managed floating with the generalised government reaction function can be illustrated diagramatically. Figure 4.1 presents, in the two dimensional space, the three equilibrium schedules of the system. Since, though, we have six endogenous variables, the relative positions of the three schedules depend, apart from the exogenous variables, on endogenous variables as well. Substituting π from equation (4.9f) into the other long-run equilibrium conditions, leaves $\bar{F}$, $\dot{F}g$ and $\dot{B}$ as the shifting endogenous variables.

The MM* schedule, as in the previous chapters, shows the compinations of real money balances and augmented real financial wealth for which the money market is in equilibrium and expectations are fulfilled. Assuming that we start from an initial long-run equilibrium where there is no government intervention ($\dot{F}g$ = 0), the MM* schedule is upwards sloping[1] and has the same slope as under free

---

1. Substituting π from equation (4.9f) into equation (4.9a), we have:

$$\left.\frac{d\bar{M}}{dV}\right|^{MM^*} = \frac{f_{1W}}{(1-f_{1W}-f_{1\pi}\frac{\dot{F}g}{\bar{M}^2})} = \frac{f_{1W}}{(1-f_{1W})} > 0$$

given our assumption that we start from an initial stationary state with no government intervention (i.e. $\dot{F}g$ = 0).

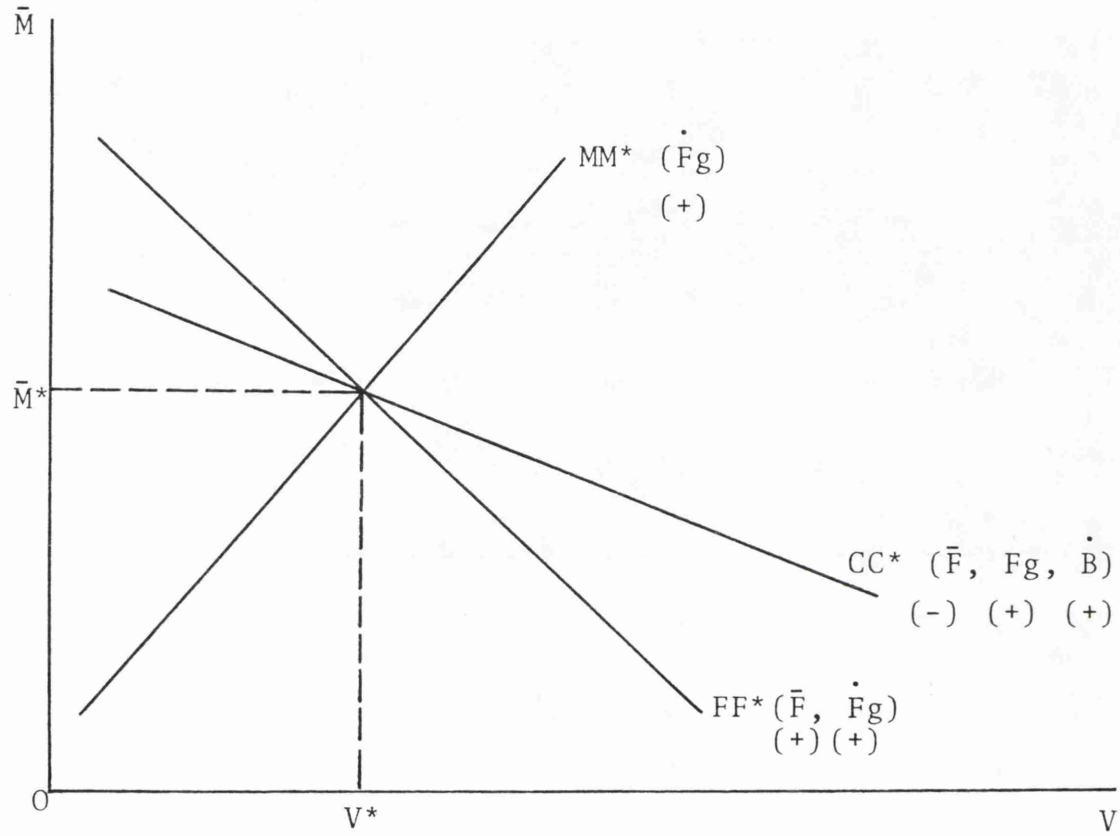

Figure 4.1. Long-run stationary state.

floating. An increase in augmented real financial wealth increases the demand for money and real money balances have to rise to restore equilibrium in the money market. The FF* schedule shows the locus of real money balances and augmented real financial wealth for which the foreign exchange market is in equilibrium and expectations are fulfilled. It is downwards sloping[2] (the same slope as under free floating), for an increase in augmented real financial wealth increases the demand for foreign exchange. For equilibrium in the foreign exchange market to be preserved, real money balances have to fall by the same amount. Similarly, the CC* schedule shows combinations of real money balances and augmented real financial wealth for which private wealth is constant (net private savings are zero) when expectations are realised. If $\dot{B}$ and $\dot{Fg}$ are both equal to zero, along CC* the current account is also in

---

2. Substituting equation (4.9f) in (4.9b), we have:

$$\left.\frac{d\overline{M}}{dV}\right|_{FF^*} = - \frac{f_{2W}}{f_{2W} + f_{2\pi}\dfrac{\dot{Fg}}{\overline{M}^2}} = - \frac{f_{2W}}{f_{2W}} = - 1$$

since at the initial stationary state $\dot{Fg}$ is assumed to be zero.

equilibrium. The CC* schedule is downwards sloping[3], with the same slope as under free floating. An increase in augmented real financial wealth causes net private dissavings and for a constant stock of wealth, real money balances have to fall to offset it. In fig. 4.1, for each of the equilibrium schedules the shifting endogenous variables are shown in brackets. The signs below the variables indicate the direction of shift of the relevant schedule.[4]

---

3. Substituting (4.9f) in equation (4.9d), we have:

$$\left.\frac{d\bar{M}}{dV}\right|_{CC^*} = - \frac{(-C_y^D . m + C_W)}{C_W - (1 - C_y^D)i} > 0$$

The numerator is positive because under free floating $\pi = m$ in the long-run and by restriction (2.9a), $(- C_y^D \pi + C_W)$ is positive.

4. Substituting (4.9f) in equations (4.9a), (4.9b) and (4.9d), the following table shows how the three equilibrium schedules are shifted when the shifting endogenous variables increase.

V=const.	Equilibrium schedules		
	MM*	FF*	CC*
$\frac{d\bar{M}}{d\dot{F}g}$	$-\frac{f_{1\pi}}{(1-f_{1W})}\frac{1}{\bar{M}}>0$	$\frac{f_{2\pi}}{f_{2W}}.\frac{1}{\bar{M}}>0$	$\frac{(1-C_y^D)}{\left[C_W-(1-C_y^D)i\right]}>0$
$\frac{d\bar{M}}{d\bar{F}}$	0	$\frac{1}{f_{2W}}>0$	$-\frac{(1-C_y^D)\,i}{\left[C_W-(1-C_y^D)i\right]}<0$
$\frac{d\bar{M}}{d\dot{B}}$	0	0	$\frac{1}{\left[C_W-(1-C_y^D)i\right]}>0$

The fact that $\dot{F}g$ is a shifting variable, suggests that unintentional competitive exchange rate policies (being the result of prediction errors), apart from their consequences on the dynamic adjustment process, can cause real effects in the long-run. These effects come about through the influence that continued government intervention has on the long-run inflation rate. A change in the inflation rate relative to free floating affects the opportunity cost of holding money and modifies the long-run equilibrium configuration of the asset market.

To illustrate, we will consider an exogenous fall in the subjective estimate of foreign exchange risk relative to domestic money (u). Starting from an initial long-run equilibrium with no government intervention ($\dot{F}g = 0$), E_0 in figure 4.2, the exogenous fall in u induces private wealth holders to increase in the new long-run equilibrium the proportion of augmented real financial wealth relative to real money balances in their portfolios. Under free floating, the new long-run equilibrium is shown by point E_1 of figure 1.2. Under managed floating, the long-run equilibrium position depends on whether the government uses the right estimate of the long-run exchange rate path or not. Or, alternatively, on whether the government predicts correctly the long-run stock of real money balances, i.e. it depends on α. When $\alpha = 1$, the long-run equilibrium is identical to that under free floating, since at the new stationary state $\dot{F}g = 0$. Whenever α deviates from unity, the

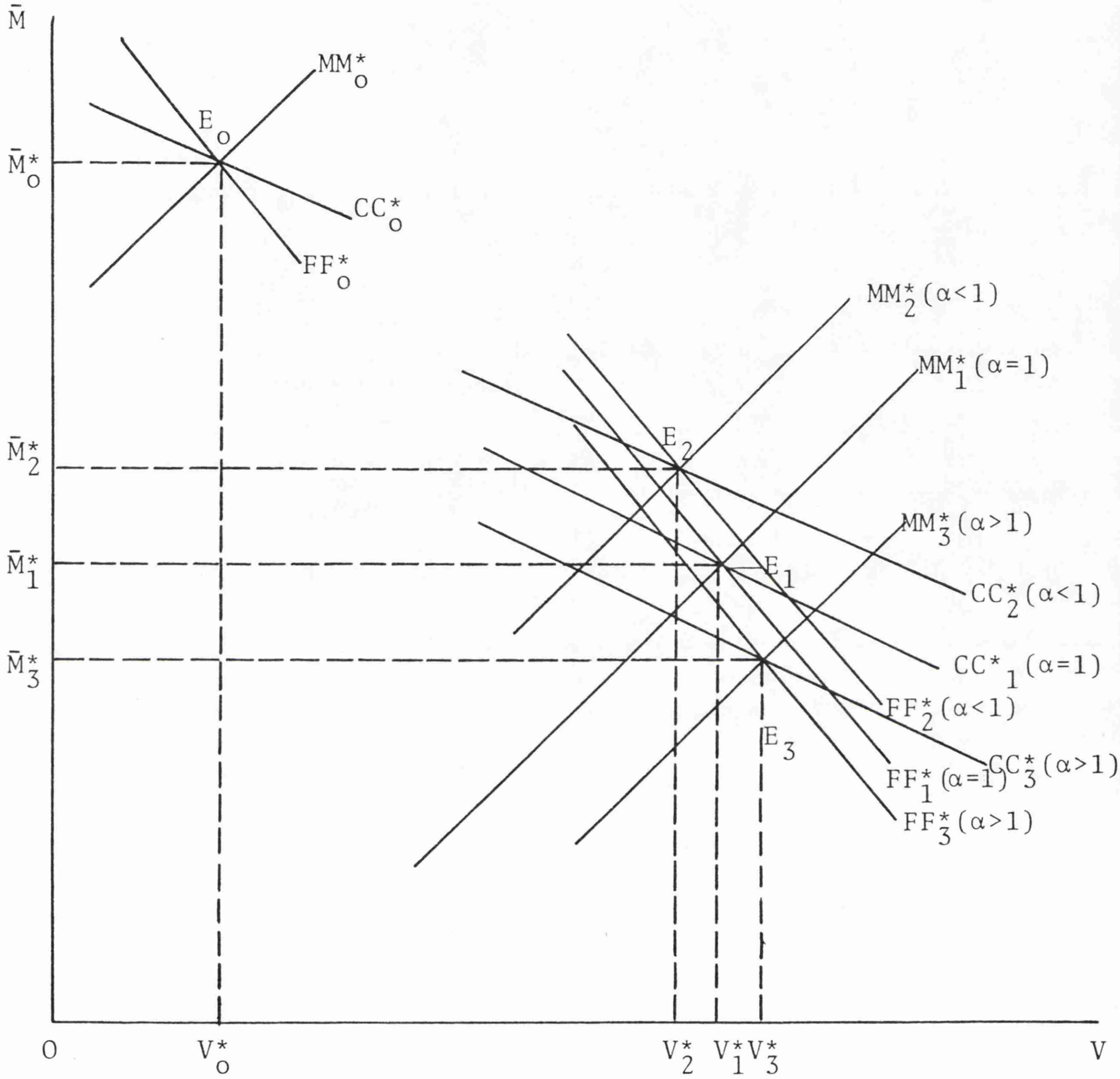

Figure 4.2. Long-run stationary state: effects of an exogenous fall in u.

government unintentionally follows a competitive exchange rate policy.

When α is less than one, the government underestimates the true long-run exchange rate path. The continued government intervention in the long-run ($\dot{F}g > 0$), reduces long-run inflation relative to free floating (see appendix for details, section A4.2). As the opportunity cost of holding money falls, the demand for real money balances relative to free floating rises and the demand for foreign exchange and bonds falls. The long-run stock of real wealth will also be affected; if it rises (falls) the demand for all assets will rise (fall). As is shown in the appendix, the fall in α (since we start from the free floating position at which $\alpha = 1$) unambiguously increases real money balances and decreases the issue of government debt ($\dot{B}$). Its effects, though, on augmented real financial wealth and foreign exchange holdings are ambiguous. A sufficient condition for them to be negative is that the fall in α does not increase real wealth. Similarly, when α rises (i.e. $\alpha > 1$), inflation rises and real money balances fall relative to free floating. A sufficient condition for augmented real financial wealth and foreign exchange holdings to rise, is that real wealth does not fall.

Diagramatically, for $\alpha < 1$ the MM^*_1 schedule shifts upwards, while the net shifts of the other two schedules depend on the relative shifting strengths of changes in $\overline{F}$, $\dot{F}g$ and $\dot{B}$. Figure 4.2, however, shows a net upward shift in all schedules that results in a long-run equilibrium position with higher real money balances and lower augmented real financial wealth relative to the free floating position (point E_2). Similarly, for $\alpha > 1$ the three schedules shift downwards, indicating at the new stationary state at E_3, a lower stock of real money balances and a higher stock of augmented real financial wealth.

Thus, under managed floating, the long-run comparative statics effects of the fall in u depend critically on the value of α (i.e. the degree of precision of the government's estimate of the long-run exchange rate path). As is shown in the appendix, while real money balances fall and the issue of government bonds rises unambiguously, the effects on the other endogenous variables are ambiguous.

However, whenever $\dot{Fg}$ is different from zero ($\alpha \neq 1$), the government is either accumulating (balance of payments surplus) or decumulating (balance of payments deficit) foreign exchange reserves over time, depending on whether $\dot{Fg}$ is negative or positive respectively. Evidently, the government cannot pursue this policy for ever, for it would either accumulate too much or run out of foreign exchange reserves. The government will be forced either to modify its estimate of the long-run exchange rate path (adjust α) or even to give up intervention. The long-run equilibria at E_2 and E_3 cannot be sustained for ever and, ultimately, the system returns to the free floating long-run position at E_1. We will return to this issue in section 4.6, where we examine in more detail the dynamic exchange rate response to the exogenous fall in u.

4.4 Dynamic stability

Changes in real money balances, augmented real financial wealth and private expectations move the system over time. Dynamic stability requires convergence of the sequence of short-run equilibria to the long-run stationary state, where real wealth is stabilized and expectations are constant and fully realised. The accumulation of real money balances and augmented real financial wealth ($\dot{V}$) are both zero. As equation (4.11b) shows, however, the accumulation of real financial wealth ($\dot{v}$) is not zero whenever government intervention continues in the long-run ($\dot{F}g \neq 0$).

The dynamic behaviour of the system is considered under two alternative specifications of private expectations. Moreover, the conditions for dynamic stability are investigated irrespective of whether the government is using the right estimate of the long-run exchange rate path or not, i.e. $\dot{F}g$ can be different from zero in the long-run.

(a) Adaptive expectations: As is shown in the appendix (section A4.3(a)), the dynamic behaviour of the system in this case is described by the following system of differential equations:

$$\dot{\pi} = \beta\left[m - \frac{\dot{F}g_2(V, \pi)}{\theta_2(V, \pi)} - \dot{x} - \pi\right];$$

$$0 < \beta < 1, \; E_{2V} = ?, \; E_{2\pi} = ? \tag{4.12a}$$

$$x = g_2(V, \pi); \quad g_{2V} > 0, \quad g_{2\pi} < 0 \tag{4.12b}$$

$$\dot{V} = h_2(V, \pi); \quad h_{2V} < 0, \quad h_{2\pi} > 0 \tag{4.12c}$$

The signs of both E_{2V} and $E_{2\pi}$ are now ambiguous because $\dot{F}g$ in the long-run may be different from zero. As is proved in the appendix, for a locally stable stationary state, the following stability conditions are required:

$$\underset{(-)}{|\beta g_{2\pi}|} < 1 \tag{4.13a}$$

$$\underset{(?)}{E_{2\pi}} < \beta \underset{(+)}{h_{2\pi}} \underset{(+)}{g_{2V}} - \underset{(-)}{h_{2V}}(1 + \underset{(+)}{\beta g_{2\pi})} \tag{4.13b}$$

$$\underset{(?)}{E_{2\pi}} \underset{(-)}{h_{2V}} - \underset{(?)}{E_{2V}} \underset{(+)}{h_{2\pi}} > 0 \tag{4.13c}$$

When $\dot{F}g \geqslant 0$ (i.e. when $\alpha \leqslant 1$), the signs of $E_{2\pi}$ and E_{2V} become unambiguously negative and positive respectively. Thus, condition (4.13b) is satisfied and the other two are exactly the same as in Case I of chapter 3.

The sensivity of the rate of change of expectations ($\dot{\pi}$) to changes in expectations and in augmented real financial wealth, is strengthened when $\dot{F}g$ is positive and weakened when $\dot{F}g$ is negative.[5] This is so, because, given equation (4.9f), in the former case long-run inflation is lower and in the latter is higher, relative to free floating.

The stability conditions under managed floating are more stringent relative to those under free floating, because the intervention policy affects the rate of growth of the nominal money stock. Consequently, through the expectations mechanism, the revision of private expectations ($\dot{\pi}$) is affected as well.

Figure 4.3 illustrates the phase diagram under

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5. From section A4.3(a) of the appendix we have:

$$E_{2V} = -\beta\frac{\overset{(-)}{\dot{F}}_{g2V}.\bar{M}-\overset{(?)}{\dot{F}g}.\overset{(+)}{\theta_{2V}}}{\bar{M}^2} \text{ and } E_{2\pi} = -\beta\left[\frac{\overset{(+)}{\dot{F}g_{2\pi}}.\bar{M}-\overset{(?)}{\dot{F}g}.\overset{(-)}{\theta_{2\pi}}}{\bar{M}^2} + 1\right]$$

When $\dot{F}g$ is positive (negative) both E_{2V} and $E_{2\pi}$ increase (fall) in absolute terms.

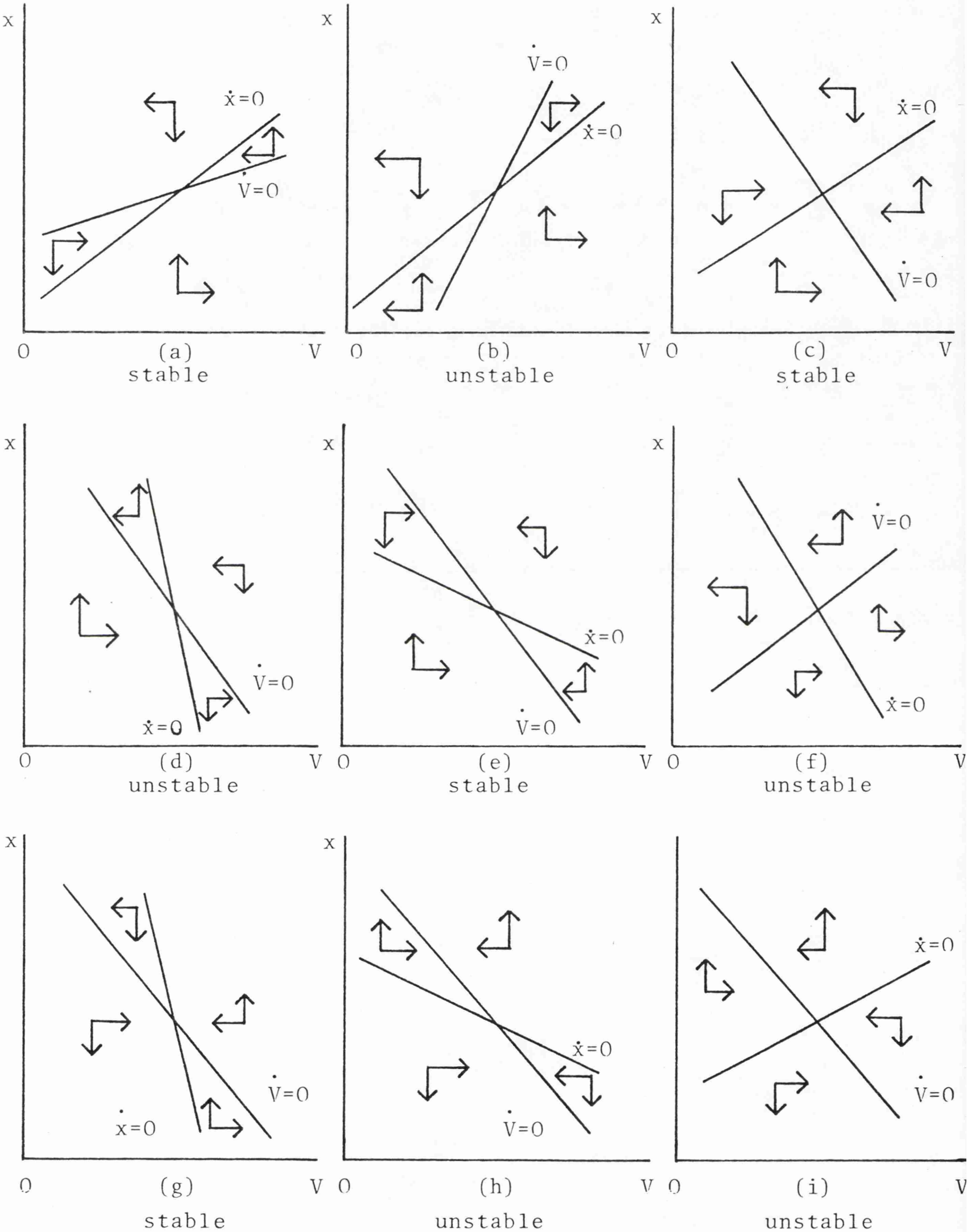

Figure 4.3. Phase diagram under adaptive expectation.

adaptive expectations. The interpretation of the $\dot{x} = 0$ and $\dot{V} = 0$ loci is the same as before. The slopes of both curves and the direction of the horizontal and vertical arrows is ambiguous (see appendix for derivation):

$$\left.\frac{dx}{dV}\right|_{\dot{x}=0} = \frac{\overset{(+)}{g_{2V}}(\overset{(+)}{g_{2V}}\overset{(+)}{h_{2\pi}} - \overset{(-)}{g_{2\pi}}\overset{(-)}{h_{2V}}) - \beta \overset{(-)}{g_{2\pi}}\overset{(+)}{g_{2V}}}{(\underset{(-)}{g_{2\pi}}\underset{(?)}{E_{2\pi}} + \underset{(+)}{g_{2V}}\underset{(+)}{h_{2\pi}})} = ? \qquad (4.14a)$$

$$\left.\frac{d\dot{x}}{dx}\right|_{V=\text{const.}} = \frac{(\overset{(-)}{g_{2\pi}}\overset{(?)}{E_{2\pi}} + \overset{(+)}{g_{2V}}\overset{(+)}{h_{2\pi}})}{\underset{(-)}{g_{2\pi}}\underset{(+)}{(1+\beta g_{2\pi})}} = ? \qquad (4.14b)$$

and

$$\left.\frac{dx}{dV}\right|_{\dot{V}=0} = - \frac{\overset{(-)}{h_{2V}}\overset{(-)}{g_{2\pi}}}{\underset{(+)}{h_{2\pi}}} + \underset{(+)}{g_{2V}} \lessgtr 0 \text{ as } g_{2V} \lessgtr \frac{h_{2V}g_{2\pi}}{h_{2\pi}} \qquad (4.15a)$$

$$\left.\frac{d\dot{V}}{dV}\right|_{x=\text{const.}} = \frac{\overset{(+)}{h_{2\pi}}}{\underset{(-)}{g_{2\pi}}}\left(\frac{h_{2V}g_{2\pi}}{h_{2\pi}} - g_{2V}\right) \lessgtr 0 \text{ as } g_{2V} \lessgtr \frac{h_{2V}g_{2\pi}}{h_{2\pi}}$$

(4.15b)

When $E_{2\pi} < 0$, i.e. when $\alpha \leqslant 1$ so that $\dot{F}g \geqslant 0$, the phase diagram analysis is identical to that of figure 3.3 in chapter 3 (shown by diagrams (a) - (e) of figure 4.3). When $\alpha > 1$, $\dot{F}g$ is negative and $E_{2\pi}$ might be positive. If $E_{2\pi}$ is positive enough to make the numerator of (4.14b) negative, deviations of real money balances from their long-run level become cumulative. Diagrams (f) - (i) of figure 4.3 show the full range of possibilities under this case. The sub-cases of diagrams (f), (h) and (i), however, should be dismissed because they are inconsistent with a stable long-run stationary state.

(β) Perfect foresight: Under perfect foresight expectations are always fulfilled and hence

$$\pi = \frac{\dot{M}^S}{M^S} - \dot{x} = \omega_2(V, \dot{x}); \qquad \omega_{2V} = ?, \qquad \omega_{2\dot{x}} = ? \qquad (4.16)$$

The signs of ω_{2V} and $\omega_{2\dot{x}}$ are now ambiguous because $\dot{F}g$ may be different from zero in the long-run. Substituting expectations from (4.16) into equations (4.12b) and (4.12c), we derive a system of differential equations that describe the dynamic behaviour of the system under perfect foresight, i.e.

$$x = g_2\left[V, \omega_2(V, \dot{x})\right]; \quad g_2{}^*{}_V = ?, \quad g_{2\pi} < 0 \qquad (4.17a)$$

$$\dot{V} = h_2\left[V, \omega_2(V, \dot{x})\right]; \quad h_2{}^*{}_V = ?, \quad h_{2\pi} > 0 \qquad (4.17b)$$

where $g_2{}^*{}_V = (\partial x/\partial V) + (\partial x/\partial \pi)\omega_{2V}$ and $h_2{}^*{}_V = (\partial \dot{V}/\partial V) + (\partial V/\partial \pi)\omega_{2V}$. As is shown in the appendix, the stationary state is a saddle point equilibrium, provided:

$$h_2{}^*{}_V < 0 \tag{4.18a}$$

$$\omega_{2\dot{x}} < 0 \tag{4.18b}$$

When $\alpha \leqslant 1$, in the long-run $\dot{F}g \geqslant 0$ and ω_{2V} and $\omega_{2\dot{x}}$ become positive and negative respectively,[6] making the second condition redundant. Condition (4.18a) is identical to the saddle point equilibrium requirement, equation (3.23), of chapter 3. Use by the government of the wrong estimate of the long-run exchange rate path ($\alpha \neq 1$) affects the dynamic adjustment process through the influence of a non-zero $\dot{F}g$ on expectations (the absolute value of ω_{2V} and $\omega_{2\dot{x}}$ changes). As is shown in the appendix, the requirement

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6. From section A4.3(b) of the appendix we have:

$$\omega_{2V} = -\frac{\overset{(-)}{\dot{F}g_{2V}}\,\bar{M}-\overset{(?)}{\dot{F}g}.\overset{(+)}{\theta}_{2V}}{\underset{(+)}{\bar{M}^2}+\dot{F}g_{2\pi}.\bar{M}.\underset{(?)}{\dot{F}g}.\underset{(-)}{\theta}_{2\pi}} \text{ and } \omega_{2\dot{x}} = -\frac{\bar{M}^2}{\underset{(+)}{\bar{M}^2}+\dot{F}g_{2\pi}.\bar{M}-\underset{(?)}{\dot{F}g}.\underset{(-)}{\theta}_{2\pi}}$$

when $\dot{F}g$ is positive, ω_{2V} and $\omega_{2\dot{x}}$ become positive and negative respectively.

that $\omega_{2\dot{x}}$ is negative implies that g^*_{2V} is unambiguously positive (g^*_{2V} is the slope of the $\dot{x} = 0$ locus).

The phase diagram analysis under perfect foresight is identical to that of section 3.2.3(b) of chapter 3. The only difference is that the subscripts of the ω, h and g functions change from 1 to 2.

The analysis of the dynamic behaviour of the system shows that, for both expectations mechanisms, the introduction of the generalised government reaction function corrects the main shortcoming of the simple intervention rule employed in chapter 3. The economy is no longer poised on a knife-edge equilibrium position, that causes dynamic instability whenever α deviates from unity. Provided the stability conditions hold, the system is now dynamically stable, irrespective of whether the government uses the right estimate of the long-run exchange rate path or not.

4.5 Dynamic response of the system to an exogenous change

The dynamic response of the system, like the stability analysis, is considered under two alternative formulations of the private expectations mechanism.

Figure 4.4 illustrates the dynamic response of the system to an exogenous fall in u, both under free and managed floating, when the private sector has adaptive expectations. In the absence of intervention, the exogenous change leads to a new long-run stationary state (E_1)

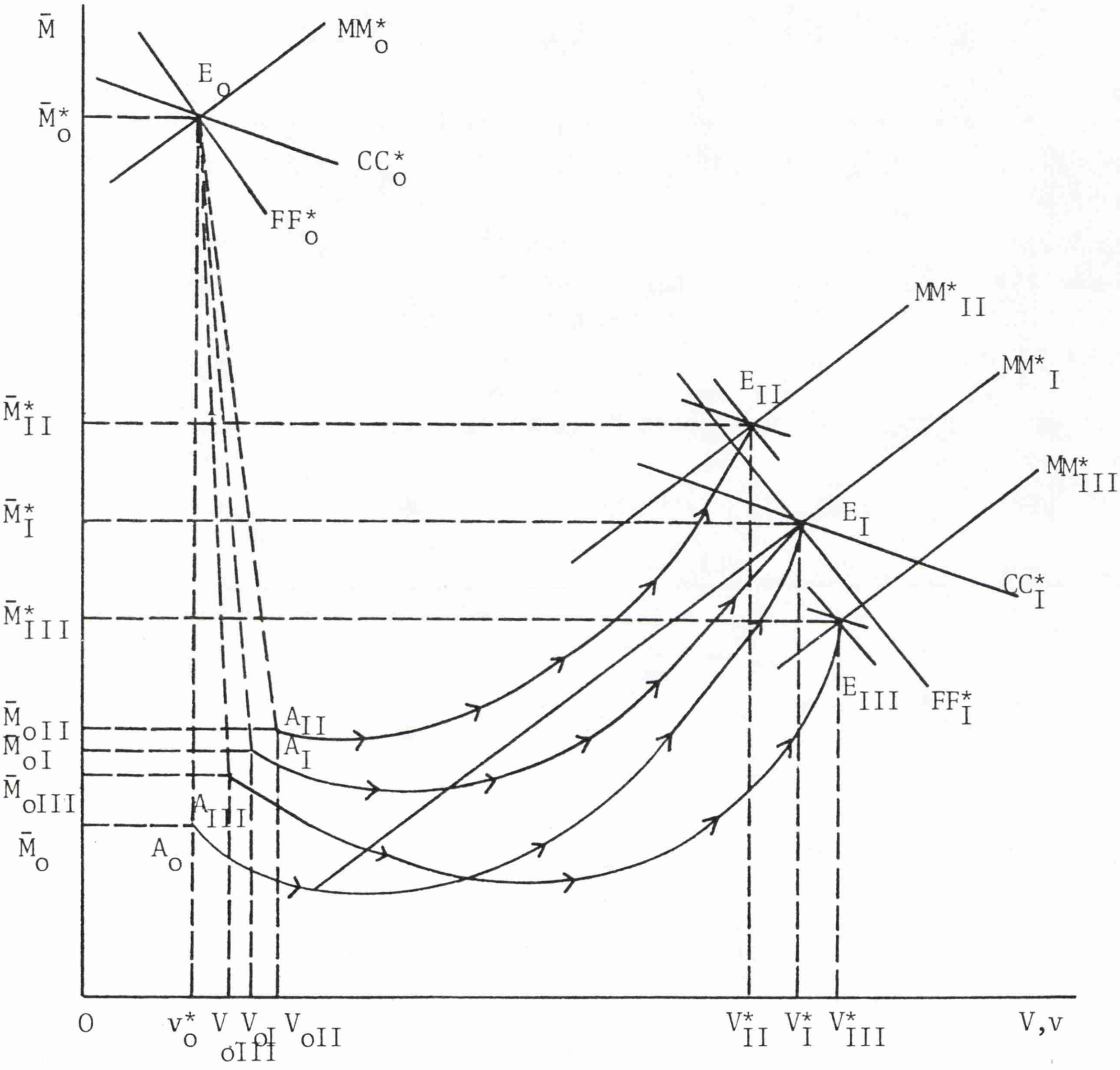

Figure 4.4. Dynamic response of the system to an exogenous fall in u (adaptive expectations).

consistent with a reduced stock of real money balances and an increased stock of real financial wealth. On the impact of the fall in u, real balances overshoot their long-run value (A_o) and over time, following the A_oE_1 adjustment path, fall initially even further and increase afterwards until the new stationary state is established. Throughout the adjustment process, augmented real financial wealth is monotonically increasing towards its long-run value (in chapter 3, point A_o was assumed to lie below the $\dot{V} = 0$ and above the $\dot{x} = 0$ loci).

Under managed floating, the dynamic response of the system is directly related to the degree of precision with which the government estimates the long-run exchange rate path (or alternatively, the degree of precision in estimating the long-run stock of real money balances), i.e. it depends on α. We can distinguish three different subcases, depending on whether α is equal, smaller or greater than one. In subcase I ($\alpha = 1$), the government is using the right estimate of the long-run exchange rate path and the long-run equilibrium under managed floating coincides with the free floating stationary state position (E_I). In subcase II ($\alpha < 1$), the government underestimates the long-run exchange rate path, $\overline{P}(t) < \overline{P}^*(t)$, and in the long-run real money balances are higher and augmented real financial wealth lower[7] than in the absence of intervention (E_{II}),

7. As we have seen in section 4.3, the effect of a fall in α on augmented real financial wealth is ambiguous. A sufficient condition for it to be positive is that private real wealth at E_{II} is lower or equal to that at E_I.

In subcase III ($\alpha < 1$), the government overestimates the long-run exchange rate path, $\overline{P}(t) > \overline{P}^*(t)$, and the long-run equilibrium shifts to E_{III} where real balances are lower and augmented real financial wealth higher.[8]

The impact effects of an exogenous fall in u depend on the value of α as well. In subcase I, given the short-run comparative statics (impact effects), the government steps in the foreign exchange market selling foreign exchange (Fg_I (o) > 0) in return for domestic money. This alleviates the private sector's excess demand for real financial wealth and excess supply of real money balances. The exchange rate overshooting is moderated and hence, real money balances and augmented real financial wealth are both higher than in the absence of intervention (point A_I).

When α deviates from unity, the magnitude and nature (i.e. whether purchases or sales) of the exchange market operations on the impact of the exogenous change, vary accordingly. In subcase II, given the intervention rule, the deviation of the spot exchange rate from its reference rate $\overline{P}_{II}(o)$ increases relative to subcase I (as the government underestimates the true reference rate) and hence, the impact government sales of foreign exchange are

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8. A sufficient condition for $V^*_{III} > V^*_I$, is that private real wealth at E_{III} is either greater or equal to that at E_I.

greater[9] ($Fg_{II}(o) > Fg_I(o)$). Real money balances at A_{II} are greater that $\overline{M}_{oI}$, because the increased stock of augmented real financial wealth ($V_{oII} > V_{oI}$) increases the demand for money.[10]

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9. Formally, given the short-run comparative statics (impact effects), when $\alpha = 1$ the exogenous fall in u increases Fg, since:

$$Fg_I(o) = -\frac{dFg}{du} = -\underset{(-)}{\frac{1}{|A_5|}}\left[\alpha L_u + \underset{(+)}{\frac{f_{1u}}{(1-f_{1W})\frac{M}{p^2}}}\right] \underset{(+)}{\ell(1-f_{1W})\frac{M}{P^2}} > 0$$

In subcase II, $\alpha < 1$ and hence

$$\frac{d}{d\alpha}\left(-\frac{dFg}{du}\right) = -\frac{\overset{(-)}{\ell L_u}\overset{(+)}{(1-f_{1W})\frac{M}{P^2}}}{\underset{(-)}{|A_5|}} < 0$$

Thus

$$Fg_{II}(o) = \underset{(+)}{Fg_I(o)} - \underset{(-)}{\frac{d}{d\alpha}\left(-\frac{dFg}{du}\right)} > Fg_I(o)$$

10. Formally,

$$x_o(A_{II}) - x_o(A_I) = \underset{(+)}{(\overline{x}_{II} - \overline{x}_I)} + \underset{(+)}{g_{2V}}\left[\underset{(+)}{(Fg_{II}(o)-Fg_I(o))} + \underset{(+)}{(V_I{}^* - V_{II}{}^*)}\right] > 0$$

where $\overline{x}_{II} = \ln\overline{M}^*{}_{II}$ and $\overline{x}_I = \ln\overline{M}^*{}_I$

In subcase III, the impact deviation of the spot exchange rate from its reference rate $\overline{P}_{III}(o)$ decreases relative to subcase I and it can even be negative, as the government overestimates the true reference rate. Assuming, with no loss of generality, that the deviation remains positive, on the impact of the exogenous change the government sales of foreign exchange $Fg_{III}(o)$ and the stock of real money balances are both lower[11] than their respective levels in subcase I. Real money balances at A_{III}, however, can even be less than their respective value under free floating, depending on the magnitude of $Fg_{III}(o)$ relative to the difference of the long-run assets at E_I and E_{III}.[12] Nevertheless,

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11. Formally,

$$Fg_{III}(o) = \underset{(+)}{Fg_I(o)} + \underset{(-)}{\frac{d}{d\alpha}}\left(-\frac{dFg}{du}\right) > 0$$

provided the deviation between the spot rate and $\overline{P}_{III}(o)$ is positive. Real money balances at A_{III} are less than $\overline{M}_{oI}$ since

$$x_o(A_I) - x_o(A_{III}) = \underset{(+)}{(\overline{x}_I - \overline{x}_{III})} + \underset{(+)}{g_{2V}}\Big[\underset{(+)}{(Fg_I(o) - Fg_{III}(o)}$$
$$+ \underset{(+)}{(V^*_{III} - V^*_I)}\Big] > 0$$

12. Formally, we have

$$x_o(A_{III}) - x_o(A_o) = \underset{(-)}{(\overline{x}_{III} - \overline{x}_I)} + \underset{(+)}{g_{2V}}\underset{(+)}{Fg_{III}(o)} + \underset{(+)}{g_{2V}}\underset{(-)}{\left[V^*_I - V^*_{III}\right]} =$$

Figure 4.4, though, shows that $x_o(A_{III}) - x_o(A_o) > 0$

in all subcases, real money balances on the impact of the exogenous change, still overshoot their respective long-run values.[13] Table 4.1 below, summarises the impact and long-run effects of the exogenous fall in u for the three subcases.

In subcase I, government intervention with the generalised government reaction function unambiguously reduces the overshooting of real money balances on the impact of the exogenous disturbance. In all three subcases (provided $\bar{M}_{oIII} > \bar{M}_o$), the stock of private real wealth and consumption are higher than in the absence of intervention and the current account imbalance is reduced. However, the moderating effects of intervention on the impact of any shocks in the system depend on the relative degree of precision with which the government can predict the long-run stock of real money balances in forming its estimate of the long-run exchange rate path (α). Excessive prediction errors could exacerbate the impact effects of exogenous disturbances. If, in the particular example of figure 4.4, α is too high ($\alpha > 1$) in subcase III so that

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13. We still have overshooting, for

$$x_o(A_j) = \bar{x}_j + \underset{(+)}{g_{2V}}(\underset{(-)}{V_{oj} - V^*_j}) < \bar{x}_j$$

where j = I, II, III.

TABLE 4.1

Impact and long-run effects of an exogenous fall in u

Subcases	Impact effects			Long-run effects			
	$Fg(o)$	V_o	$\bar{M}_o$	$\dot{Fg}$	$\dot{P}/P$	V^*	$\bar{M}^*$
0: free floating	-	v^*_o	$\bar{M}_o$	-	m	V^*_I	$\bar{M}^*_I$
I: $\alpha = 1$	$Fg_I(o) > 0$	$V_{oI}=v^*_o+Fg_I(o)>v^*_o$	$\bar{M}_{oI} > \bar{M}_o$	$\dot{Fg}_I = 0$	$(\frac{\dot{P}}{P})_I = m$	V^*_I	$\bar{M}^*_I$
II: $0 < \alpha < 1$	$Fg_{II}(o)>Fg_I(o)$	$V_{oII}=v^*_o+Fg_{II}(o)>V_{oI}$	$\bar{M}_{oII}>\bar{M}_{oI}$	$\dot{F}_{gII}>0$	$(\frac{\dot{P}}{P})_{II}<m$	$V^*_{II}<V^*_I$	$\bar{M}^*_{II}>\bar{M}^*_I$
III: $\alpha > 1$	$0<Fg_{III}(o)<Fg_I(o)$	$V_{oIII}=v^*_o+Fg_{III}(o)<V_{oI}$	$\bar{M}_o \lessgtr \bar{M}_{oIII}<\bar{M}_{oI}$	$\dot{Fg}_{III}<0$	$(\frac{\dot{P}}{P})_{III}>m$	$V^*_{III}>V^*_1$	$\bar{M}^*_{III}<\bar{M}^*_I$

$Fg_{III}(o) < 0$, the intervention policy augments the impact effects of the fall in u. The government, by purchasing rather than selling foreign exchange, exacerbates the excess demand for real financial wealth and the excess supply of real money balances. In consequence, the overshooting of real money balances is increased relative to free floating.

During the adjustment process, the system moves along the A_jE_j (j = I, II, III) paths, till the new stationary state is established. As in chapters 2 and 3, the nature of these paths depends on two counteracting forces: First, on the basis of the error-learning mechanism, the initial exchange rate depreciation induces expectations for further price rises (π increases), which, given the short-run comparative statics, tend to reduce real money balances even further (the exchange rate depreciates). On the other hand, if on the exogenous fall in u the current account moves into surplus (dS/du = ?), the stock of augmented real financial wealth will increase, appreciating the exchange rate and increasing real money balances. The A_jE_j trajectories in figure 4.4 assume that the first effect prevails over the second, or alternatively, that points A_j lie below the $\dot{V} = 0$ and above the $\dot{x} = 0$ loci of figure 4.3. Formally, the accumulation rates of real money balances and augmented real financial wealth during the adjustment process are given by the following expressions:

$$\dot{x}_j(t) = \underset{(-)}{\lambda_{1j}}\underset{(-)}{(x_j(t) - \overline{x}_j)}$$

$$- \overset{(-)}{\left[\underset{(+)}{\frac{g_{2\pi}}{h_{2\pi}}} \underset{(?)}{(\lambda_{1j} - h_{2V})} \underset{(?)}{(\lambda_{2j} - h_{2V})} + \underset{(+)}{g_{2V}} \underset{(?)}{(\lambda_{1j} - h_{2V})} \right]} \cdot$$

$$\underset{(-)}{(V_{oj} - V^*_j)} e^{\lambda_{2j} t} \tag{4.19a}$$

$$\dot{V}_j(t) = \underset{(-)}{\lambda_{1j}} \underset{(-)}{(V_j(t) - V_j^*)} - \underset{(?)}{(\lambda_{1j} - h_{2V})} \underset{(-)}{(V_{oj} - V_j^*)} e^{\lambda_{2j} t} \tag{4.19b}$$

where j = I, II, III. For each path, the initial and long-run equilibrium conditions are given by table 4.1. The characteristic roots (λ's) vary from case to case because the characteristic equations are different.

Provided A_j lie below the $\dot{V} = 0$ and above the $\dot{x} = 0$ schedules, the phase diagram analysis implies that, at the initial stages of the adjustment process (small values for t) the square bracket in (4.19a) is negative and that the second term more than offsets the first. As a result, real money balances fall even further than at the impact effect. Over time, as private wealth holders catch up with their expectations, real balances start rising. Since A_j are assumed below the $\dot{V} = 0$ locus, augmented real financial wealth is increasing monotonically throughout the adjustment process.

The nature of the dynamic adjustment process, however, changes considerably when, instead of adaptive expectations, the private sector has long-run perfect foresight. On the impact of the exogenous disturbance, private wealth holders adjust their expectations accordingly, so that the system is shifted to the equilibrating trajectory that leads to the stationary state. Figure 4.5 below, illustrates the dynamic response of the system to the exogenous fall in u, both under free and managed floating when the private sector is equipped with long-run perfect foresight. E_I, E_{II} and E_{III} show the long-run equilibrium positions in the respective subcases. A_o and A_oE_I show the impact effect and the dynamic adjustment path under free floating.

Under managed floating, the intervention policy exerts the same qualitative (but not quantitative) effects on the system, on the impact of the exogenous change, as in the analysis of figure 4.4 above. The stocks of real money balances and augmented real financial wealth are higher, relative to free floating. Intervention reduces the overshooting of real money balances and moderates the fluctuations in real wealth, private consumption and current account balance. As before, the effects of the intervention

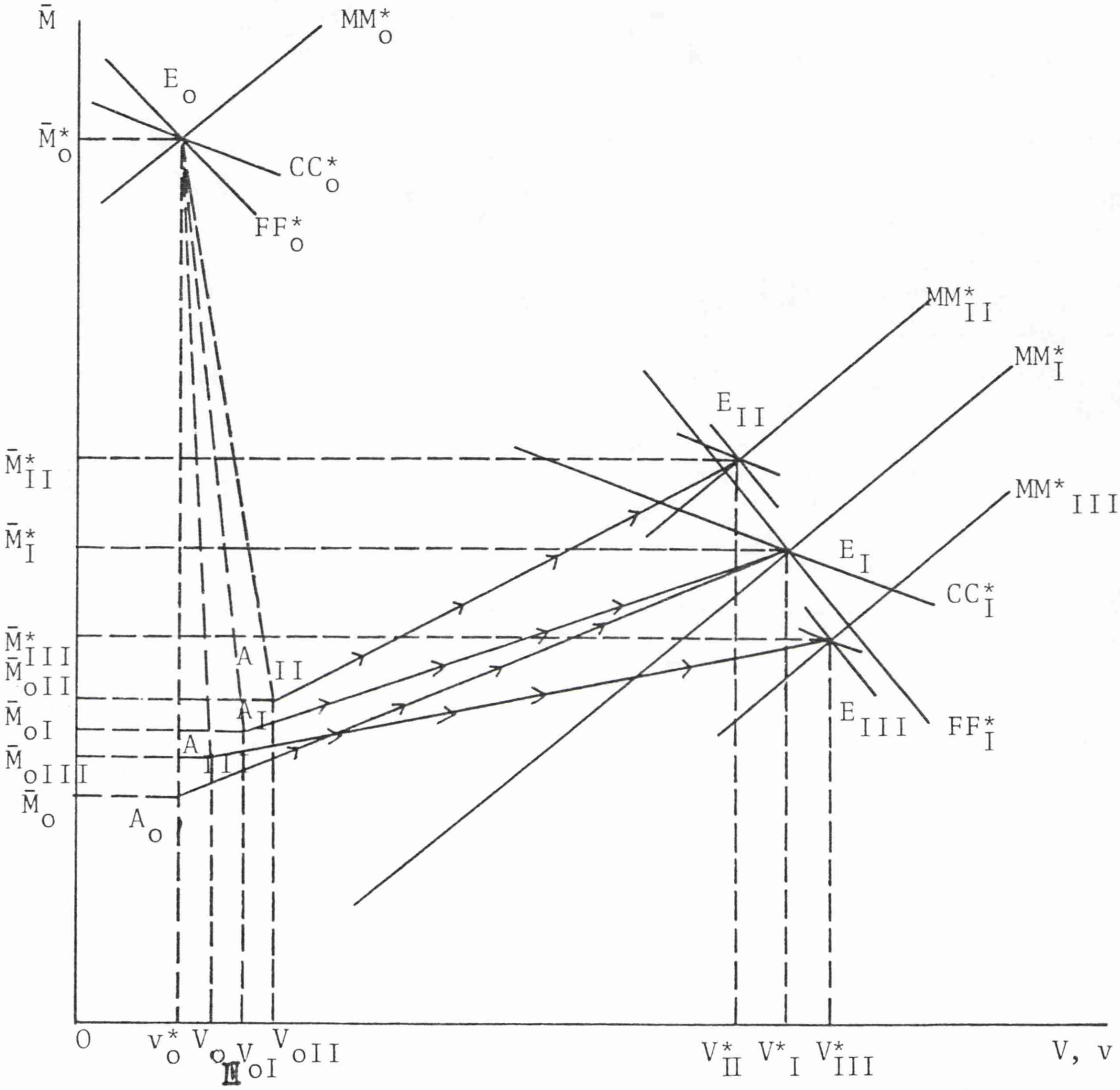

Figure 4.5. Dynamic response of the system to an exogenous fall in u (long-run perfect foresight).

policy depend on the value of α, the degree of precision with which the government predicts the long-run exchange rate path. In subcase III where $\alpha > 1$, real money balances on the impact of the exogenous change (at A_{III}) may even be lower than their respective level under free floating, depending on the slope of the A_{III} E_{III} trajectory. Table 4.1 summarises the impact and long-run effects of the exogenous fall in u for the three different subcases. Unlike the long-run effects, the impact effects under long-run perfect foresight are quantitatively different from those under adaptive expectations. In all subcases, over time real money balances and augmented real financial wealth are monotonically increasing along the equilibrating adjustment paths A_j E_j (j = I, II, III), till long-run equilibrium is established. Formally, the accumulation rates of the two assets during the adjustment process are given by the following expressions:

$$\dot{x}_j(t) = \underset{(-)}{\lambda_j}\,\underset{(-)}{(x_j(t) - \bar{x}_j)} \tag{4.20a}$$

$$\dot{V}_j(t) = \underset{(-)}{\lambda_j}\,\underset{(-)}{(V_{oj} - V_j{}^*)}\,e^{\lambda jt} \tag{4.20b}$$

(j = I, II, III), which are both unambiguously positive.

4.6 Exchange rate dynamics

The generalised government reaction function, as an improvement over the intervention rule employed in chapter 3, allows a stable long-run equilibrium even if the government uses the wrong estimate of the long-run exchange rate path. In consequence, the analysis of exchange rate dynamics when α deviates from unity, assumes particular importance.

Figure 4.6 illustrates the dynamic response of the exchange rate to an exogenous fall in u, both under free and managed floating, when the private sector has adaptive expectations. PP_o^* is the initial long-run exchange rate path under free floating and PP_1^* is the new one, consistent with the lower value for u. ABC is the familiar dynamic adjustment path of the exchange rate, in the absence of intervention. Under managed floating the dynamic behaviour of the exchange rate depends on the value of α; in figure 4.6, the three subcases are considered separately.

In subcase I, government intervention reduces the overshooting of real money balances on the impact of the exogenous change. As a result, the impact exchange rate overshooting is moderated. Given the dynamic response of real money balances, over time, the exchange rate depreciates even further and appreciates afterwards, till the long-run equilibrium is reached. During the adjustment process, the

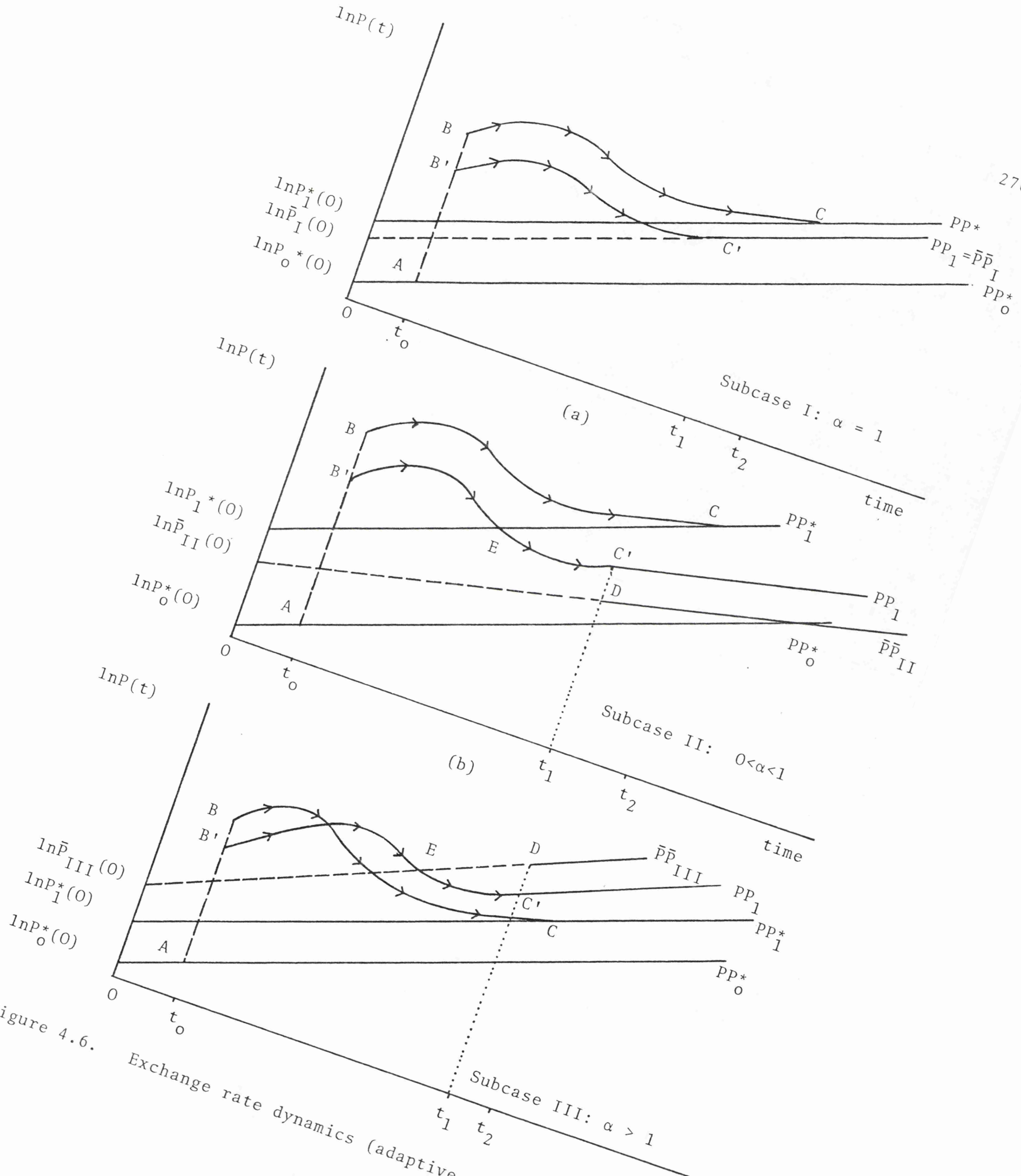

Figure 4.6. Exchange rate dynamics (adaptive expectations)

exchange rate is always lower than its corresponding value under free floating (the adjustment path $A_I E_I$ in figure 4.4 lies above $A_o E_I$). PP_1 is the long-run exchange rate path under managed floating. It lies below $PP_1{}^*$, because the foreign exchange market operations on the impact of the exogenous change, reduce the initial nominal money stock. During the adjustment process, the government reference rate grows at a variable rate, dominated by the evolution of nominal money balances. From point C' onwards, however, $\overline{P}(t)$ coincides with the actual exchange rate, growing both at the same rate. PP_1 and $PP_1{}^*$ are parallel to each other, for long-run inflation under managed floating is the same as under free floating.

The dynamic exchange rate behaviour in this subcase is very similar to that of case I (adaptive expectations) of chapter 3. Short-run exchange rate fluctuations are moderated and the process of dynamic adjustment is speeded up, as the intervention policy augments the adjustment in private portfolios.

In subcase II, the moderation in the impact exchange rate overshooting is greater than in any other subcase, because real money balances at point A_{II} in figure 4.4 are higher. Over time, the exchange rate is initially depreciating even further and appreciating afterwards, till the long-run equilibrium is reached at C'. The nature of the dynamic path B'C' depends on the dynamic response of real money balances. From some point E (at which $P(t) = \overline{P}^*(t)$)

onwards, the spot rate falls in the region between $\overline{P}^*(t)$ - which shows the government reference rate when $\alpha = 1$ - and $\overline{P}(t)$, in which the intervention policy and the market forces work at cross purposes. Under the generalised government reaction function, however, and contrary to the results of the analysis of section 3.3.3 of chapter 3, the intervention policy just offsets the effects of market forces, so that a stable long-run equilibrium is established. During the adjustment process, as in subcase I, the government reference rate $\overline{P}_{II}(t)$ grows at a variable rate. From point D onwards, though, it increases at the same constant rate as the actual exchange rate. The long-run exchange rate path under managed floating, PP_1, lies between $\overline{PP}_{II}$ and the free floating long-run exchange rate path $PP_1{}^*$. Both PP_1 and $\overline{PP}_{II}$ are flatter than $PP_1{}^*$, for long-run inflation in subcase II is less than under free floating.

The reduction in the impact exchange rate overshooting in subcase III is much less than in the other two subcases. If α is high enough, so that real money balances at point A_{III} of figure 4.4 are less than $\overline{M}_o$, government intervention in subcase III exacerbates the impact exchange rate overshooting. During the adjustment process, the exchange rate follows a similar pattern of movements as in the other two subcases.[14] As in subcase II, from some

14. Formally, the slopes of the B'C' paths in figure 4.6 are given by the following expressions:

$$DlnP_j(t) = DlnM^S{}_j(t) - \underset{(-)}{\lambda_{1j}}\underset{(-)}{(x_j(t) - \overline{x}_j)}$$

(continued on next page)

point E (at which now $P(t) = \overline{P}(t)$) onwards, the spot exchange rate falls in the region where the intervention policy counteracts the market forces. Along PP_1, the long-run exchange rate path under managed floating, market forces exactly offset the effects of intervention and the system is stabilised. At the stationary state, $P(t)$ and $\overline{P}_{III}(t)$ grow at the same rate, equal to the long-run inflation. Their paths are steeper than $PP_1{}^*$, because the long-run inflation rate is greater than m. In neither of subcases II and III, though, is the government's estimate of the long-run exchange rate path equal to the actual one.

Figure 4.7 below, illustrates the dynamic exchange rate response when private wealth holders are equipped with long-run perfect foresight.ABC is the dynamic exchange rate path under free floating. On the impact of the exogenous change, the exchange rate overshoots its long-run value and over time, it appreciates monotonically towards its long-run path.

Footnote 14 continued

$$+\left[\underset{(+)}{\overset{(-)}{\frac{g_{2\pi}}{h_{2\pi}}}}\underset{(?)}{(\lambda_{1j} - h_{2V})}\underset{(?)}{(\lambda_{2j} - h_{2V})} + \underset{(+)}{g_{2V}}\underset{(?)}{(\lambda_{1j} - h_{2V})}\right]\underset{(-)}{(V_{oj} - V_j{}^*)}e^{\lambda_{2j}t}$$

where j = I, II, III. Given the dynamic response of real money balances, at the initial stages of the adjustment process, the first and third terms outweigh the second, forcing the exchange rate to depreciate even further.

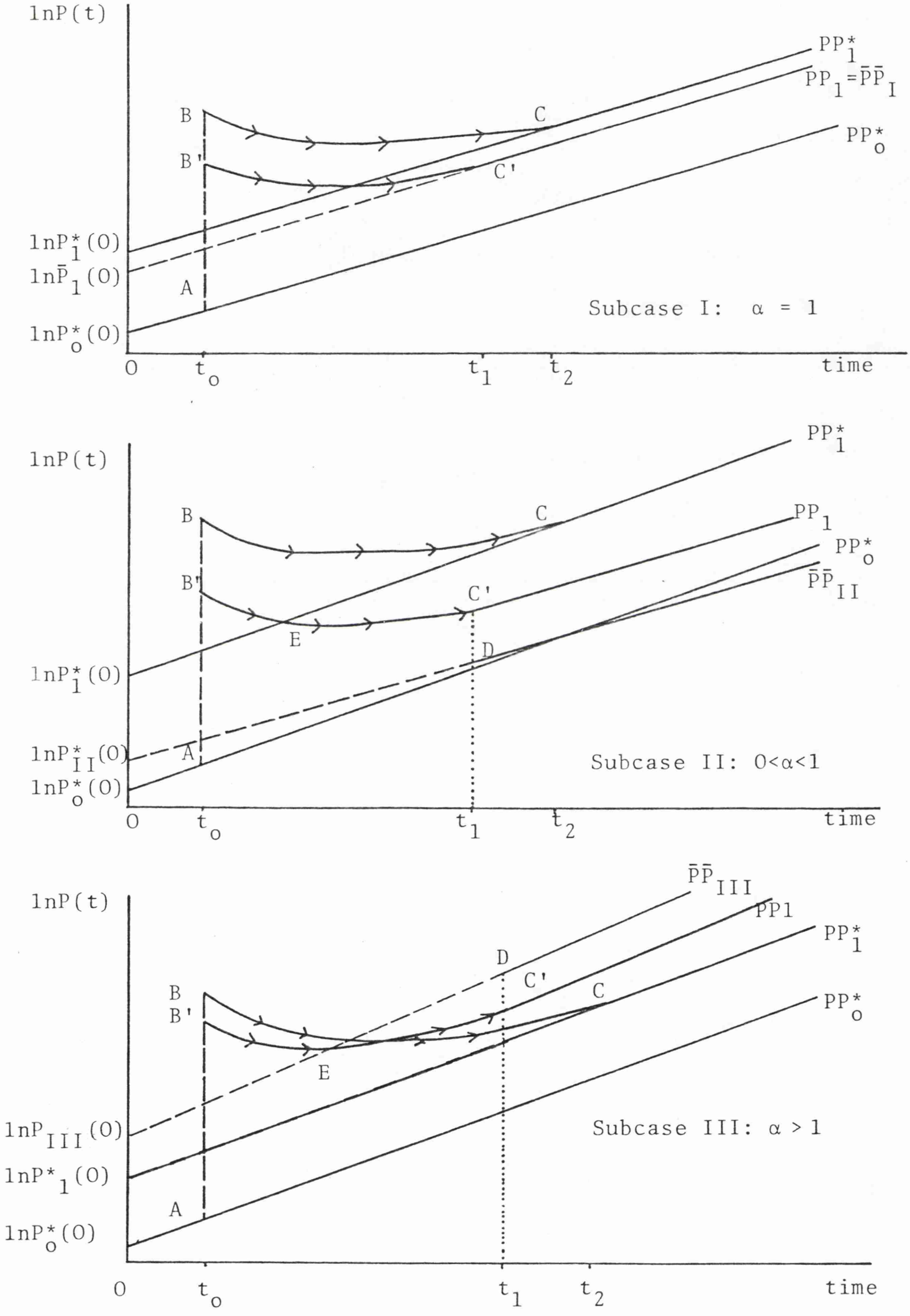

Figure 4.7. Exchange rate dynamics (long-run perfect foresight)

Under managed floating, for all subcases the long-run effects are the same as under adaptive expectations. On the impact of the exogenous change, the exchange rate over-shooting is reduced relative to free floating. This reduction is higher in subcase II, less in subcase I and in subcase III the exchange rate overshooting may even be exacerbated if real money balances at point A_{III} of figure 4.5 are less than $\overline{M}_o$. Over time, contrary to the case of adaptive expectations, the exchange rate is monotonically appreciating[15] (growing at a rate less than long-run inflation) till the new stationary state is established.

Tha analysis so far, however, tacitly assumes that the continued government intervention in the long-run (subcases II and III) does not affect in any way wealth holders' expectations or induce the government to modify its policy. In subcase II, both under adaptive and rational expectations, the long-run exchange rate path under managed floating, PP_1, lies above the government's estimate of the long-run exchange rate path $\overline{PP}_{II}$. Given the intervention

---

15. Formally, the slopes of the B'C' paths in figure 4.7 are given by the following expressions:

$$D\ln P_j(t) = D\ln \dot{n} M^S{}_j(t) - \underset{(-)}{\lambda_j}\underset{(-)}{(x_j(t) - \overline{x}_j)}$$

where j = I, II, III. Throughout the adjustment process the exchange rate grows at a lower rate than the nominal money stock, so that real money balances rise.

rule, this implies that the government is decumulating foreign exchange reserves over time. Similarly in subcase III, the continued government intervention in the long-run involves a continuous accumulation of foreign exchange reserves. In either case, the government cannot sustain its intervention policy for ever. Because of the constraints on its actions, the government is unable to defend a particular long-run exchange rate path and a "crisis" develops in the balance of payments.

As was already assumed, the government has no interest in deliberate exchange rate policies. Hence, the long-run implications of intervention in subcases II and III are unwelcomed by the government, for they involve continuous accumulation or decumulation of foreign exchange reserves. In addition, domestic inflation is lower and higher respectively relative to free floating and the asset market equilibrium is affected analagously.

The government will be forced, therefore, either to modify its estimate of the long-run real money stock (i.e. adjust α), revising its estimate of the long-run exchange rate path, or give up intervention altogether. A stabilized domestic inflation with a constant and non-zero $\dot{F}g$ indicates that the government is using the wrong estimate of the long-run exchange rate path (i.e. $\alpha \neq 1$). Moreover, the rate of change of the government stock of foreign exchange reserves suggests the direction in which α should be adjusted so that prediction errors are minimized. Consequently, the long-run equilibria of subcases II and III are intermediary stages of a more general adjustment process, which ultimately leads to a full long-run equilibrium where $\alpha = 1$ (subcase I), the free floating long-run position.

Of particular importance to the resulting exchange rate dynamics is the possible ability of private wealth holders to anticipate the long-run effects of government policy and to adjust their behaviour accordingly. Under adaptive expectations the private sector is unable to predict the long-run implications of government intervention. Long-run perfect foresight, however, provides this information and allows private wealth holders to adjust their expectations accordingly, even well before the long-run positions in subcases II and III are reached. The resulting exchange rate dynamics will, of course, be very different from those under adaptive expectations.

4.7 Speculation and exchange rate dynamics

A re-interpretation of our intervention rule (4.7), along the lines of the analysis of section 3.4 of chapter 3, allows us to examine the effects of speculation on the dynamic exchange rate adjustment with no changes in our formal analysis.

Speculators are assumed to have no preferred monetary habitat, but hold non-interest bearing working balances in domestic and foreign currencies. Their speculative activities provide foreign exchange cover to wealth holders wishing to re-allocate their portfolios. They form expectations about the long-run exchange rate path $\overline{P}(t)$ in the same way as government does under managed floating. Speculators take open positions in foreign currency depending on the discrepancies between the spot exchange rate and their reference rate, at every moment in time. Through their covering activities, speculators maximize the expected long-run value of their real wealth: they buy (sell) domestic money when they expect a long-run exchange rate depreciation (appreciation), in return for foreign exchange. At the new stationary state, speculators take their profits by uncovering their positions.

As in chapter 3, speculative activity, prior to the liquidation of potential profits, produces exactly the same effects on exchange rate dynamics as government intervention. To illustrate, consider again the case of an exogenous fall in u: the effects of speculation on the resulting exchange rate dynamics depend, as under managed floating, on α, the degree of precision with which speculators predict the long-run stock of real money balances. In subcase I ($\alpha = 1$), irrespective of the private expectations mechanisms, speculation leads to an unambiguous moderation in short-run exchange rate variability and to stabilization of private real wealth and consumption and of the current account imbalances (figures 4.6(a) and 4.7(a)). In return, speculators maximize their potential profits.

When α deviates from unity, speculators make mistakes in predicting the long-run exchange rate path. Figures 4.6 and 4.7 illustrate the exchange rate dynamics under the two expectations mechanisms. In subcase II ($\alpha < 1$), speculators underestimate the true long-run exchange rate path. This leads to a long-run equilibrium position at which speculators are decumulating foreign exchange reserves over time. In return, they buy domestic money, anticipating an exchange rate appreciation ($P(t) > \overline{P}_{II}(t)$). The near depletion of their foreign exchange holdings will force them either to modify their estimate of long-run real money balances (i.e. adjust α), revising their estimate of the long-run exchange rate path, or to give up their covering activities, driven out of business. In either case, the system returns to the long-run equilibrium position of subcase I, where $\alpha = 1$ (free floating stationary state). The exchange rate depreciates and speculators incur a financial loss on their holdings of domestic money.

A similar "speculative crisis" occurs when speculators overestimate the long-run exchange rate path (subcase III, $\alpha > 1$). Speculators are now accumulating foreign exchange selling domestic money (in the long-run), as they expect an exchange rate depreciation ($P(t) < \overline{P}_{III}(t)$). The running down of their stock of domestic money will force them again to revise their estimate of the long-run exchange rate path (i.e. adjust α) or to give up speculation. The system will return finally to the free floating long-run equilibrium position (subcase I). The exchange rate appreciates relative to $\overline{PP}_{III}$ and speculators suffer financial losses. The present value of the domestic money they have used in their covering activities supporting $\overline{PP}_{III}$, is greater than the value of foreign exchange they have acquired in return. In both subcases, the resulting exchange rate dynamics are further influenced by a possible reaction of private wealth holders. Under adaptive expectations, the private sector is unable to predict the long-run implications of speculative activities and hence, its behaviour remains invariant. Under long-run perfect foresight, however, private wealth holders are able to anticipate the speculative errors and their likely outcome, adjusting their expectations accordingly.

Therefore, to the extent that the speculators' estimate of the long-run exchange rate path is likely to deviate significantly from its true value, speculation will exacerbate short-run exchange rate fluctuations. In addition, the variability of private wealth and consumption and the current account imbalances will be increased as well.

Contrary to the analysis of section 3.4 of chapter 3, use of the wrong estimate of the long-run exchange rate path, leads to a temporarily stable long-run equilibrium (subcases II and III). To sustain this equilibrium, however, speculators have to be willing to tolerate a gradual decumulation of their reserves of domestic money or foreign exchange. In this respect, destabilizing speculation leads, for at least some time, to a long-run equilibrium position other than the one that would prevail in the absence of any speculative activity. Nevertheless, as in chapter 3, destabilizing speculation is unprofitable and will either drive speculators out of business, or will force them to modify their reference rates.

Tacitly implied in our analysis is that there is enough speculative capital available; a prerequisite condition for any speculation, stabilizing or otherwise.

The uncovering of speculative positions, however, that allows speculators to take their profits and to sustain themselves in business, disturbs the long-run equilibrium position. The cashing in of speculative profits represents an endogenous source of asset market disturbances, creating additional short-run exchange rate variability that wipes out some of the speculative profits.

4.8 Concluding remarks

The introduction of the generalised government reaction function corrects the main shortcomings of the simple intervention rule of chapter 3. The economy is no longer poised on a knife-edge equilibrium position, that causes dynamic instability whenever the government's estimate of the long-run real money stock deviates from its true value. Managed floating with the generalised intervention rule leads to a stable long-run equilibrium, even if the government, due to prediction errors, follows competitive exchange rate policies.

To sustain long-run equilibrium in these cases, the government is required to accumulate or to decumulate foreign exchange reserves for ever. In the face of these balance of payments crises, the government is forced either to revise its estimate of the long-run exchange rate path or to give up intervention altogether. In either case, the system returns to the free floating long-run equilibrium position. Of particular importance to the dynamic response of the system and the dynamic exchange rate behaviour in particular is the private wealth holders' ability to predict the effects of government policies and to adjust their behaviour accordingly.

Government intervention is an efficient policy of moderating short-run exchange rate variability so long as the government uses an estimate of the long-run exchange rate path close enough to its true value. Excessive prediction errors exacerbate rather than diminish short-run exchange rate fluctuations. In addition, they magnify the variability of private wealth and consumption and of the current account imbalances.

A re-interpretation of our intervention rule to reflect speculative behaviour yields results similar to those of government intervention. One feature, though, of speculation represents an endogenous source of asset market disturbances. Unlike the government, speculators, driven by their profit maximising behaviour, need to uncover their positions and take their speculative profits, if they are to remain in business. An action that disturbs the long-run equilibrium position, causing additional short-run exchange rate variability. If speculators were to have enough speculative capital and exactly the same information as the government to predict the long-run exchange rate path, the government would seem to be in a better position to moderate short-run exchange rate variability. This is so, because government actions are not dominated by any profit maximising objectives. In general, however, the choice between government intervention and private speculation depends on the following factors: the relative ability of speculators to predict the long-run exchange rate path, the availability of speculative capital and the relative significance of the additional exchange rate fluctuations that the liquidation of speculative profits causes.

Pursuance of competitive exchange rate policies leads to a considerable increase in the degree of reserve use. More interestingly, however, these policies increase the demand for international liquidity and in the presence of more than two countries, produce an inconsistent world structure of reference rates that causes intervention at cross purposes. The possibility of such problems arising increases the need for international surveillance of national exchange rate policies.

APPENDIX 4

A4.1 Short-run comparative statics

(a) Impact effects: Linearising the short-run equilibrium conditions (4.4a) - (4.4c) of section 4.2 we have:

$$A_5\left[dP\ dF\ dFg\ dS\ d\dot{B}\right]' = B_5\left[dM\ dv\ d\pi\ du\ dY\ dT\ dG\ dB\right]' \tag{A4.1}$$

where

$$A_5 = \begin{bmatrix} (1-f_{1W})\frac{M}{P^2} & 0 & 1 & 0 & 0 \\ -f_{2W}\frac{M}{P^2} & -1 & -1 & 0 & 0 \\ -\frac{\ell}{M} & 0 & (1-\ell\frac{P^2}{M^2}) & 0 & 0 \\ (-C_y{}^D\pi+C_W)\frac{M}{P^2} & -(1-C_y{}^D)i & -C_y{}^D\pi & -1 & 0 \\ m\,\frac{M}{P^2} & 0 & m & 0 & -1 \end{bmatrix} \tag{A4.2}$$

with

$$|A_5| = -(1 - \ell\frac{P^2}{M^2})(1-f_{1W})\frac{M}{P^2} - \frac{\ell}{M} < 0 \tag{A4.3}$$

since $(1 - \ell\frac{P^2}{M^2}) >$, given equation (A3.3a) of chapter 3.

$$B_5 = \begin{bmatrix} (1-f_{1W})\frac{1}{P} & -f_{1W} & -f_{1\pi} & -f_{1u} & -f_{1y} & 0 & 0 & 0 \\ -f_{2W}\frac{1}{P} & -f_{2W} & -f_{2\pi} & -f_{2u} & -f_{2y} & 0 & 0 & 0 \\ -\ell\frac{P}{M^2} & 0 & 0 & -\alpha\ell L_u & -\alpha\ell L_y & -\alpha\ell L_T & 0 & 0 \\ (-C_y{}^D\pi + C_W)\frac{1}{P} & \left[C_W - (1-C_y{}^D)i\right] & -C_y{}^D(\frac{M}{P}-Fg) & 0 & -(1-C_y{}^D) & -C_y{}^D & 1 & i \\ m\frac{1}{P} & 0 & 0 & 0 & 0 & 1 & -1 & -i \end{bmatrix} \quad \text{(A4.4)}$$

Matrices A_5 and B_5 are very similar to matrices A_3 and B_3 of chapter 3. The only difference is that some coefficients change their numerical value but they retain their signs. In particular, $(1 - \ell \frac{P^2}{M})$ becomes $(1 - \ell\frac{P^2}{M^2})$ and $-\alpha\ell(M - FgP)L_i$ becomes $-\alpha\ell L_i$, for i = u, Y and T. Given these changes, the short-run comparative statics results (impact effects) are qualitatively the same as in the previous chapter. Whenever $\alpha \neq 1$, the sign of the effects of changes in u, Y and T become ambiguous, depending on the relative value of α.

(β) <u>Short-run comparative statics (general case)</u>:
Linearising the short-run equilibrium conditions (4.5a) - (4.5e), we have:

$$A_6 \begin{bmatrix} dP & d\bar{F} & dS & d\dot{B} & d\dot{F}g \end{bmatrix}' = B_6 \begin{bmatrix} dM^S & dV & d\pi & du & dY & dT & dG & dB \end{bmatrix}' \qquad (A4.5)$$

where

$$A_6 = \begin{bmatrix} (1 - f_{1W})\frac{M^S}{P^2} & 0 & 0 & 0 & 0 \\ -f_{2W}\frac{M^S}{P^2} & -1 & 0 & 0 & 0 \\ (-C_y{}^D\pi + C_W)\frac{M^S}{P^2} & -(1 - C_y{}^D)i & -1 & 0 & 0 \\ m\frac{M^S}{P^2} & 0 & 0 & -1 & 0 \\ -\delta\frac{1}{M^S} & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (A4.6)$$

$$B_6 = \begin{bmatrix} (1-f_{1W})\frac{1}{P} & -f_{1W} & -f_{1\pi} & -f_{1u} & -f_{1y} & 0 & 0 & 0 \\ -f_{2W}\frac{1}{P} & -f_{2W} & -f_{2\pi} & -f_{2u} & -f_{2y} & 0 & 0 & 0 \\ (-C_y{}^D{}_\pi + C_W)\frac{1}{P} & \left[C_W - (1-C_y{}^D)i\right] & -C_y{}^D\frac{M^S}{P} & 0 & -(1-C_y{}^D) & -C_y{}^D & 1 & i \\ m\frac{1}{P} & 0 & 0 & 0 & 0 & 1 & -1 & -i \\ -\delta\frac{P}{(M^S)^2} & 0 & 0 & -\alpha\delta L_u & -\alpha\delta L_y & -\alpha\delta L_T & 0 & 0 \end{bmatrix} \qquad \text{(A4.7)}$$

with

$$|A_6| = -(1 - f_{1W}) \frac{M^S}{P^2} < 0 \qquad \text{(A4.8}$$

which is exactly the same as $|A_4|$.

The comparative statics results for P, $\overline{F}$, S and $\dot{B}$ are exactly the same as in chapters 2 and 3. The effects on $\dot{F}g$, though qualitatively the same as in the previous chapter, differ in absolute terms. Because of their importance for the stability analysis, are repeated again below:

$$\frac{d\dot{F}g}{dM^S} = 0 \qquad \text{(A4.9)}$$

$$\frac{d\dot{F}g}{dV} = \underset{(-)}{\frac{1}{|A_6|}} \cdot \underset{(+)}{\delta f_{1W}} \frac{1}{M^S} < 0 \qquad \text{(A4.10)}$$

$$\frac{d\dot{F}g}{d\pi} = \underset{(-)}{\frac{1}{|A_6|}} \cdot \underset{(-)}{\delta f_{1\pi}} \frac{1}{M^S} > 0 \qquad \text{(A4.11)}$$

$$\frac{d\dot{F}g}{du} = \underset{(+)}{\frac{\partial \dot{F}g}{\partial u}} + \delta \underset{(-)}{\frac{dP}{du}} = -\underset{(-)}{\alpha\delta L_u} + \underset{(-)}{\frac{1}{|A_6|}} \underset{(+)}{\delta f_{1u}} \frac{1}{M^S} = ? \qquad \text{(A4.12)}$$

depending on the relative value of α ($\alpha > 0$).

$$\frac{d\dot{F}g}{dY} = \underset{(+)}{\frac{\partial \dot{F}g}{\partial Y}} + \delta \underset{(-)}{\frac{dP}{dY}} = - \underset{(-)}{\alpha\delta L_y} + \underset{(-)}{\frac{1}{|A_6|}} \underset{(+)}{\delta f_{1y}} \frac{1}{M^S} = ? \qquad \text{(A4.13)}$$

$$\frac{d\dot{F}g}{dT} = \frac{\partial \dot{F}g}{\partial T} + \delta \frac{dP}{dT} = - \underset{(+)}{\alpha\delta L_T} + 0 < 0 \qquad \text{(A4.14)}$$

$$\frac{d\dot{F}g}{dG} = \frac{d\dot{F}g}{dB} = 0 \qquad \text{(A4.15)}$$

A4.2 Long-run comparative statics

Linearising the long-run equilibrium conditions (4.9a) - (4.9f), we have:

$$A_7 \begin{bmatrix} d\bar{M} \\ dV \\ d\bar{F} \\ d\dot{F}g \\ d\pi \\ d\dot{B} \end{bmatrix} = \begin{bmatrix} -f_{1u} & 0 \\ -f_{2u} & 0 \\ -\alpha\delta L_u & -\delta L \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} du \\ d\alpha \end{bmatrix} \qquad \text{(A4.16)}$$

where

$$A_7 = \begin{bmatrix} (f_{1W}-1) & f_{1W} & 0 & 0 & f_{1\pi} & 0 \\ f_{2W} & f_{2W} & -1 & 0 & f_{2\pi} & 0 \\ \delta \dfrac{1}{\overline{M}^2} & 0 & 0 & 1 & 0 & 0 \\ (-C_y{}^D\pi + C_W) & \left[C_W - (1 - C_y{}^D)i\right] & (1 - C_y{}^D)i & -1 & -C_y{}^D\,\overline{M} & -1 \\ -m & 0 & 0 & 0 & 0 & -1 \\ -\dfrac{\dot{F}g}{\overline{M}^2} & 0 & 0 & \dfrac{1}{\overline{M}} & 1 & 0 \end{bmatrix} \qquad (A4.17)$$

Assuming that we start from an initial long-run stationary state where $\dot{F}g = 0$, the determinant of A_7 is positive, since

$$
\begin{aligned}
|A_7| = {} & \underset{(+)}{(-C_y{}^D\pi + C_W + m)}\ \underset{(+)}{f_{1W}} + \underset{(+)}{(1 - f_{1W})}\underset{(+)}{\left[C_W - (1 - C_y{}^D)i\right]} \\
& + \underset{(+)}{(1 - C_y{}^D)i}\ \underset{(+)}{f_{2W}} + \delta\,\frac{1}{\overline{M}^2}\Bigg[\underset{(+)}{f_{1W}}\underset{(+)}{(1 - C_y{}^D)} \\
& - \frac{1}{\overline{M}}\ \underset{(-)}{f_{1\pi}}\underset{(+)}{(C_W - (1 - C_y{}^D)i)} \\
& + \frac{1}{\overline{M}}\underset{(+)}{(1 - C_y{}^D)}i(\underset{(+)(+)}{f_{1W}f_{2\pi}} - \underset{(+)(-)}{f_{2W}f_{1\pi}})\Bigg] > 0
\end{aligned}
\tag{A4.18}
$$

Thus, the comparative statics effects of a change in u are as follows:

$$
\begin{aligned}
\frac{d\overline{M}}{du} = {} & \underset{(+)}{\frac{1}{|A_7|}}\ \underset{(-)}{\alpha\delta L_u}\{\, - \underset{(+)}{f_{1W}}\underset{(+)}{(1 - C_y{}^D)} + \frac{1}{\overline{M}}\ \underset{(-)}{f_{1\pi}}\underset{(+)}{\left[C_W - (1 - C_y{}^D)i\right]} \\
& - \frac{1}{\overline{M}}\ \underset{(+)}{(1 - C_y{}^D)}i\ (\underset{(+)(+)}{f_{1W}f_{2\pi}} - \underset{(+)(-)}{f_{2W}f_{1\pi}})\}
\end{aligned}
$$

$$+ \frac{1}{|A_7|} \{f_{1u}\left[C_W - (1 - C_y{}^D)i\right]$$

$$\quad (+) \quad (+) \qquad (+)$$

$$+ (1 - C_y{}^D)i(f_{1u}f_{2W} - f_{2u}f_{1W})\} > 0$$

$$\quad (+) \qquad (+)(+) \quad (-)(+)$$

(A4.19)

$$\frac{dV}{du} = - \frac{1}{|A_7|} m(f_{1u} + \alpha\delta L_u \cdot f_{1\pi} \frac{1}{\overline{M}})$$

$$- \frac{1}{|A_7|} f_{1u}\left[\delta\frac{1}{\overline{M}^2} (1 - C_y{}^D) + (-C_y{}^D\pi + C_W)\right]$$

$$+ \frac{1}{|A_7|} \alpha\delta L_u\left[(1 - f_{1W})C_y{}^D - f_{1\pi}\frac{1}{\overline{M}}(-C_y{}^D\pi + C_W)\right.$$

$$\left. + (f_{1W} - 1)\right]$$

$$+ \frac{1}{|A_7|} (1 - C_y{}^D)\frac{1}{\overline{M}} \left[\delta\frac{1}{\overline{M}^2} (f_{2u}f_{1\pi} - f_{iu}f_{2\pi})\right.$$

$$\left. + \alpha\delta L_u(f_{2\pi}(f_{1W} - 1) - f_{1\pi}f_{2W})\right] = ? \qquad \text{(A4.20)}$$

$$\frac{d\overline{F}}{du} = \frac{1}{|A_7|} \cdot m(f_{1W}f_{2u} - f_{1u}f_{2W})$$

$$+ \frac{1}{|A_7|} \alpha\delta L_u\{-f_{2W}(1 - C_y^D) + m\frac{1}{\overline{M}}(f_{1W}f_{2\pi} - f_{1\pi}f_{2W})$$

$$+ \frac{1}{\overline{M}} (-C_y^D\pi + C_W)(f_{1W}f_{2\pi} - f_{1\pi}f_{2W})$$

$$+ \frac{1}{\overline{M}}\left[C_W - (1 - C_y^D)i\right]\left[f_{1\pi}f_{2W} - f_{2\pi}(f_{1W} - 1)\right]\}$$

$$+ \frac{1}{|A_7|} \delta \frac{1}{\overline{M}^2}\{(1 - C_y^D)(f_{1W}f_{2u} - f_{1u}f_{2W})$$

$$+ \frac{1}{\overline{M}}\left[C_W - (1 - C_y^D)i\right](f_{1u}f_{2\pi} - f_{1\pi}f_{2u})\} = ?$$

(A4.21)

$$\frac{d\dot{F}g}{du} = \frac{\partial \dot{F}g}{\partial u} + \frac{\partial \dot{F}g}{\partial \overline{M}} \cdot \frac{d\overline{M}}{du}$$

(+) (-) (+)

$$= \frac{1}{|A_7|} \alpha\delta L_u\{-f_{1W}(-C_y^D\pi + m + C_W)$$

$$+ \ (f_{1W} - 1)\left[C_W - (1 - C_y{}^D)i\right] - f_{2W}(1 - C_y{}^D)i\}$$

$$+ \frac{1}{|A_7|}\ \delta\ \frac{1}{\overline{M}^2}\ \{\ (1 - C_y{}^D)i(f_{1W}f_{2u} - f_{1u}f_{2W})$$

$$- \ f_{1u}\left[C_W - (1 - C_y{}^D)i\right]\ \} \ = \ ? \qquad \text{(A4.22)}$$

$$\frac{d\pi}{du} = -\frac{1}{\overline{M}}\frac{d\dot{F}g}{du} \gtrless 0 \qquad \text{as} \qquad \frac{d\dot{F}g}{du} \lessgtr 0 \qquad \text{(A4.23)}$$

$$\frac{d\dot{B}}{du} = -\ m\ \underset{(+)}{\frac{d\overline{M}}{du}} < 0 \qquad \text{(A4.24)}$$

Comparative statics effects of changes in α:

$$\frac{d\overline{M}}{d\alpha} = \underset{(+)}{\frac{1}{|A_7|}}\ \delta L\ \frac{1}{\overline{M}}\ \{\ -\ \underset{(+)}{(1 - C_y{}^D)}\underset{(+)}{f_{1W}}\ \overline{M} + \underset{(-)}{f_{1\pi}}\underset{(+)}{\left[C_W - (1 - C_y{}^D)i\right]}$$

$$- \ \underset{(+)}{(1 - C_y{}^D)}i(\underset{(+)}{f_{1W}}\underset{(+)}{f_{2\pi}} - \underset{(-)}{f_{1\pi}}\underset{(+)}{f_{2W}})\ \} \ < 0 \qquad \text{(A4.25)}$$

$$\frac{dV}{d\alpha} = \underset{(+)}{\frac{1}{|A_7|}} \delta L \frac{1}{\overline{M}} \{ - \underset{(-)}{f_{1\pi}} \underset{(+)}{(-C_y{}^D\pi + m + C_W)} + \underset{(+)}{(1 - C_y{}^D)}\overline{M}\underset{(+)}{(f_{1W} - 1}$$

$$+ \underset{(+)}{(1 - C_y{}^D)} i \left[\underset{(+)}{f_{2\pi}} \underset{(-)}{(f_{1W} - 1)} - \underset{(-)}{f_{1\pi}} \underset{(+)}{f_{2W}} \right] \} = ? \qquad (A4.26)$$

From the definition of real wealth, however, we have:

$$\frac{dW}{d\alpha} = \underset{(-)}{\frac{d\overline{M}}{d\alpha}} + \underset{(?)}{\frac{dV}{d\alpha}} = ? \qquad (A4.27)$$

If real wealth rises or remains the same after a rise in α, augmented real financial wealth unambiguously rises. If real wealth falls, dV/dFg remains ambiguous.

$$\frac{d\overline{F}}{d\alpha} = \underset{(+)}{\frac{1}{|A_7|}} \delta L \frac{1}{\overline{M}} \{ -\underset{(+)}{f_{2W}} \underset{(+)}{(1 - C_y{}^D)}\overline{M}$$

$$+ \underset{(+)}{(-C_y{}^D\pi + m + C_W)} (\underset{(+)}{f_{1W}}\underset{(+)}{f_{2\pi}} - \underset{(+)}{f_{2W}}\underset{(-)}{f_{1\pi}})$$

$$- \left[C_W - (1 - C_y{}^D) i \right] \left[f_{2\pi} (f_{1W} - 1) - f_{2W} f_{1\pi} \right] \} = ?$$

$$\quad (+) \qquad\qquad (+) \quad (-) \qquad\qquad (+)(-)$$

(A4.28)

Alternatively, from equation (4.9b) we have:

$$\frac{d\overline{F}}{d\alpha} = f_{2W} \frac{dW}{d\alpha} + f_{2\pi} \frac{d\pi}{d\alpha} \qquad \text{(A4.29)}$$

$$(+)\ (?)\quad (+)\ (+)$$

If real wealth rises or remains the same, $\overline{F}$ rises ($d\overline{F}/d\alpha > 0$); if real wealth falls, $d\overline{F}/d\alpha$ is ambiguous.

$$\frac{d\dot{F}g}{d\alpha} = \frac{1}{|A_7|} \delta L \left\{ (f_{1W} - 1) \left[C_W - (1 - C_y{}^D) i \right] - f_{2W} (1 - C_y{}^D) i \right.$$

$$(+) \qquad\qquad (-) \qquad\qquad (+) \qquad\qquad (+) \quad (+)$$

$$\left. - f_{1W} (- C_y{}^D \pi + m + C_W) \right\} < 0 \qquad \text{(A4.30)}$$

$$(+) \qquad\qquad (+)$$

$$\frac{d\pi}{d\alpha} = - \frac{1}{\overline{M}} \frac{d\dot{F}g}{d\alpha} > 0 \qquad \text{(A4.31)}$$

$$(-)$$

$$\frac{d\dot{B}}{d\alpha} = - m \underset{(-)}{\frac{d\overline{M}}{d\alpha}} = > 0 \tag{A4.32}$$

A4.3 Dynamic stability

From the short-run comparative statics (general case), we can write $\overline{M}$ and $\dot{V}$ as functions of expectations and augmented real financial wealth, i.e.

$$\overline{M} = \theta_2(V, \pi) \tag{A4.33}$$

with

$$\theta_{2V} = \frac{d\overline{M}}{dV} = \frac{f_{1W}}{(1 - f_{1W})} > 0 \tag{A4.33a}$$

$$\theta_{2\pi} = \frac{d\overline{M}}{d\pi} = \frac{f_{1\pi}}{(1 - f_{1W})} < 0 \tag{A4.33b}$$

$$\dot{V} = h_2(V, \pi); \qquad h_{2V} < 0, \qquad h_{2\pi} > 0 \tag{A4.34}$$

Taking the logs of (A4.33), we have:

$$x = \ln \overline{M} = \ln\theta_2 \; (V, \pi) \tag{A4.36}$$

or $$x = g_2\ (V,\ \pi) \qquad \text{(A4.36)}$$

$$g_{2V} = \frac{\theta_{2V}}{\theta_2(V,\ \pi)} > 0; \qquad g_{2\pi} = \frac{\theta_{2\pi}}{\theta_2(V,\ \pi)} < 0 \qquad \text{(A4.36a)}$$

Equations (A4.33) and (A4.34) are identical to the respective expressions of chapters 2 and 3 because the comparative statics effects on P are identical.

The dynamic process, as before, is considered under two alternative specifications of private expectations.

(a) Adaptive expectations: From the short-run comparative statics $\dot{F}g$ can be written as a function of augmented real financial wealth and expectations:

$$\dot{F}g = \dot{F}g_2\ (V,\ \pi) \qquad \text{(A4.37)}$$

$$\dot{F}g_{2V} = \frac{d\dot{F}g}{dV} = -\frac{\delta f_{1W}}{(1 - f_{1W})\frac{M^S}{P^2}} \cdot \frac{1}{M^S} < 0 \qquad \text{(A4.37a)}$$

$$Fg_{2\pi} = \frac{d\dot{F}g}{d\pi} = -\frac{\delta f_{1\pi}}{(1 - f_{1W})\frac{M^S}{P^2}} \cdot \frac{1}{M^S} > 0 \qquad \text{(A4.37b)}$$

Hence, the adaptive expectations mechanism becomes:

$$\dot{\pi} = \beta\left[m - \frac{\dot{F}g_2(V, \pi)}{\theta_2(V, \pi)} - \dot{x} - \pi\right]; \quad 0 < \beta < 1 \qquad \text{(A4.38)}$$

with

$$E_{2V} = \frac{d\dot{\pi}}{dV} = -\beta\,\frac{\overset{(-)}{\dot{F}g_{2V}} \cdot \bar{M} - \overset{(?)}{\dot{F}g} \cdot \overset{(+)}{\theta_{2V}}}{\bar{M}^2} = ? \qquad \text{(A4.38a)}$$

$$E_{2\pi} = \frac{d\dot{\pi}}{d\pi} = -\beta\left[\frac{\overset{(+)}{\dot{F}g_{2\pi}} \cdot \bar{M} - \overset{(?)}{\dot{F}g} \cdot \overset{(-)}{\theta_{2\pi}}}{\bar{M}^2} + 1\right] = ? \qquad \text{(A4.38b)}$$

In the long-run stationary state, around which equation (A4.38) is linearised, $\dot{F}g$ can be positive, negative or zero, depending on whether α is smaller, greater or zero, respectively. When $\dot{F}g \geqslant 0$, $E_{2V} > 0$ and $E_{2\pi} < 0$ and when $\dot{F}g < 0$, E_{2V} and $E_{2\pi}$ are both ambiguous.

The dynamic behaviour of the system under adaptive expectations is described by equations (A4.38), (A4.35) and (A4.34), i.e.

$$\dot{\pi} = \beta\left[m - \frac{\dot{F}g_2(V, \pi)}{\theta_2(V, \pi)} - \dot{x} - \pi\right];$$

$$0 < \beta < 1, \qquad E_{2v} = ?, \qquad E_{2\pi} = ? \qquad \text{(A4.39a)}$$

$$x = g_2\ (V, \pi); \qquad g_{2V} > 0, \qquad g_{2\pi} < 0 \qquad \text{(A4.39b)}$$

$$\dot{V} = h_2\ (V, \pi); \qquad h_{2V} < 0, \qquad h_{2\pi} > 0 \qquad \text{(A4.39c)}$$

The characteristic equation of (A4.39) is as follows:

$$\underset{(-)}{(1 + \beta g_{2\pi})}\lambda^2 + \left[-\underset{(?)}{E_{2\pi}} - \underset{(-)}{h_{2V}}(1 + \underset{(-)}{\beta g_{2\pi}}) + \beta \underset{(+)}{h_{2\pi}}\underset{(+)}{g_{2V}}\right]\lambda$$

$$+ \underset{(?)}{E_{2\pi}}\underset{(-)}{h_{2V}} - \underset{(?)}{E_{2V}}\underset{(+)}{h_{2\pi}} = 0 \qquad \text{(A4.40)}$$

Necessary and sufficient condition for local stability is that all coefficients are positive. This reduces to

$$(1 + \beta g_{2\pi}) > 0 \quad \text{or} \quad |\beta \cdot g_{2\pi}| < 1 \qquad \text{(A4.41a)}$$

$$E_{2\pi} < \beta h_{2\pi} g_{2V} - h_{2V}(1 + \beta g_{2\pi}) \qquad \text{(A4.41b)}$$

$$E_{2\pi} h_{2V} - E_{2V} h_{2\pi} > 0 \qquad \text{(A4.41c)}$$

When $\dot{F}g \geqslant 0$, $E_{2\pi}$ and E_{2V} become negative and positive respectively, and the stability conditions are identical to those under case I of chapter 3.

To derive the $\dot{x} = 0$ and $\dot{V} = 0$ schedules, we totally differentiate equation (A4.39b) with respect to time, i.e.

$$\dot{x} = g_{2\pi}\dot{\pi} + g_{2V}\dot{V} \tag{A4.42}$$

Substituting $d\pi$ from the linearised form of (A4.39b) into the linearised forms of (A4.39a) and (A4.39c), and then substituting $\dot{\pi}$ and $\dot{V}$ in (A4.42), we derive the equation of the $\dot{x} = 0$ curve:

$$(1 + \beta g_{2\pi})\dot{x} = (g_{2\pi}E_{2\pi} + g_{2V}h_{2\pi})\frac{1}{g_{2\pi}}dx + \left[(g_{2\pi}E_{2V} + g_{2V}h_{2V}) - \frac{g_{2V}}{g_{2\pi}}(g_{2\pi}E_{2\pi} + g_{2V}h_{2\pi})\right]dV \tag{A4.43}$$

from which we have:

$$\left.\frac{d\dot{x}}{dx}\right|_{V=\text{const.}} = \frac{(\overset{(-)}{g_{2\pi}} \cdot \overset{(?)}{E_{2\pi}} + \overset{(+)}{g_{2V}} \cdot \overset{(+)}{h_{2\pi}})}{\underset{(-)}{g_{2\pi}}\underset{(+)}{(1 - \beta g_{2\pi})}} \tag{A4.44}$$

$$\left.\frac{dx}{dV}\right|_{\dot{x}=0} = \frac{\overset{(+)}{g_{2V}}(\overset{(+)}{g_{2V}}\,\overset{(+)}{h_{2\pi}} - \overset{(-)}{g_{2\pi}} \cdot \overset{(-)}{h_{2V}}) + \overset{(-)}{g_{2\pi}}(\overset{(+)}{g_{2V}} \cdot \overset{(?)}{E_{2\pi}} - \overset{(-)}{g_{2\pi}}\,\overset{(?)}{E_{2V}})}{(\underset{(-)}{g_{2\pi}}\,\underset{(?)}{E_{2\pi}} + \underset{(+)}{g_{2V}}\,\underset{(+)}{h_{2\pi}})} \tag{A4.45}$$

Using equations (A4.33a), (A4.33b), (A4.36a), (A4.37a), (A4.37b), (A4.38a) and (A4.38b) the second bracket on the numerator of (A4.45) becomes

$$(\underset{(+)}{g_{2V}} \cdot \underset{(?)}{E_{2\pi}} - \underset{(-)}{g_{2\pi}} \cdot \underset{(?)}{E_{2V}}) = -\beta \underset{(+)}{g_{2V}} < 0 \tag{A4.46}$$

Hence, (A4.45) becomes:

$$\left.\frac{dx}{dV}\right|_{\dot{x}=0} = \frac{\overset{(+)}{g_{2V}}(\overset{(+)}{g_{2V}} \cdot \overset{(+)}{h_{2\pi}} - \overset{(-)}{g_{2\pi}}\,\overset{(-)}{h_{2V}}) - \beta \overset{(-)}{g_{2\pi}} \cdot \overset{(+)}{g_{2V}}}{(\underset{(-)}{g_{2\pi}}\,\underset{(?)}{E_{2\pi}} + \underset{(+)}{g_{2V}}\,\underset{(+)}{h_{2\pi}})} = \, ? \tag{A4.45a}$$

The numerator of (A4.44), which is equal to the denominator of (A4.45), is ambiguous, depending on the relative value of $E_{2\pi}$. If $E_{2\pi}$ is negative, the numerator is positive so that deviations of real money balances from their long-run level are self-correcting. However, if $E_{2\pi}$ is positive, deviations of real money balances might be cumulative.

Similarly, by substituting $d\pi$ from the linearised form of (A4.39b) into the linearised form of (A4.39c), we derive the equation of the $\dot{V} = 0$ locus, i.e.

$$\dot{V} = \frac{h_{2\pi}}{g_{2\pi}} dx + \frac{h_{2\pi}}{g_{2\pi}} \left[\frac{h_{2V} \cdot g_{2\pi}}{h_{2\pi}} - g_{2V} \right] dV \qquad \text{(A4.47)}$$

from which we have:

$$\left.\frac{dx}{dV}\right|_{\dot{V}=0} = - \frac{\overset{(-)}{h_{2V}} \cdot \overset{(-)}{g_{2\pi}}}{\underset{(+)}{h_{2\pi}}} + \underset{(+)}{g_{2V}} \lesseqgtr 0$$

as

$$g_{2V} \lesseqgtr \frac{h_{2V} \cdot g_{2\pi}}{h_{2\pi}} \qquad \text{(A4.48a)}$$

$$\left.\frac{d\dot{V}}{dV}\right|_{x=\text{const.}} = \frac{\overset{(+)}{h_{2\pi}}}{\underset{(-)}{g_{2\pi}}} \left[\frac{h_{2V} \cdot g_{2\pi}}{h_{2\pi}} - g_{2V} \right] \lesseqgtr 0$$

as

$$g_{2V} \lesseqgtr \frac{h_{2v} \cdot g_{2\pi}}{h_{2\pi}} \qquad \text{(A4.48b)}$$

(β) <u>Perfect foresight</u>: Under perfect foresight expectations are always fulfilled, i.e.

$$\pi = \frac{\dot{M}^S}{M^S} - \dot{x} = m - \frac{\dot{Fg}_2(V, \pi)}{\theta_2(V, \pi)} - \dot{x} \tag{A4.49}$$

Linearising around the long-run stationary state, we have:

$$\left[1 + \frac{\dot{Fg}_{2\pi} \cdot \bar{M} - \dot{Fg} \cdot \theta_{2\pi}}{\bar{M}^2}\right] d\pi = - \frac{\dot{Fg}_{2V} \cdot \bar{M} - \dot{Fg} \cdot \theta_{2V}}{\bar{M}^2} dV - \dot{x} \tag{A4.50}$$

From (A4.50), expectations can be written as a function of V and $\dot{x}$, i.e.

$$\pi = \omega_2(V, \dot{x}) \tag{A4.51}$$

with

$$\omega_{2V} = \frac{d\pi}{dV} = - \frac{\overset{(-)}{\dot{Fg}_{2V}} \, \bar{M} - \overset{(?)}{\dot{Fg}} \, \overset{(+)}{\theta_{2V}}}{\bar{M}^2 + \underset{(+)}{\dot{Fg}_{2\pi}} \, \bar{M} - \underset{(?)}{\dot{Fg}} \, \underset{(-)}{\theta_{2\pi}}} = ? \tag{A4.51a}$$

$$\omega_{2\dot{x}} = \frac{d\pi}{d\dot{x}} = - \frac{\bar{M}^2}{\bar{M}^2 + \underset{(+)}{\dot{Fg}_{2\pi}} \cdot \bar{M} - \underset{(?)}{\dot{Fg}} \cdot \underset{(-)}{\theta_{2\pi}}} = ? \tag{A4.51b}$$

The signs of both partial derivatives is ambiguous because $\dot{F}g$ is ambiguous. When $\dot{F}g \geqslant 0$, though, $\omega_{2V} > 0$ and $\omega_{2\pi} < 0$. Substituting (A4.51) in equations (A4.34) and (A4.36), we derive a system of differential equations that describes the dynamics of the system under perfect foresight, i.e.

$$x = g_2\left[V, \omega_2(V, \dot{x})\right]; \qquad g^*_{2V} = ?, \qquad g_{2\pi} < 0 \qquad \text{(A4.52a)}$$

$$\dot{V} = h_2\left[V, \omega_2(V, \dot{x})\right]; \qquad h^*_{2V} = ?, \qquad h_{2\pi} > 0 \qquad \text{(A4.52b)}$$

where

$$g^*_{2V} = \underset{(+)}{g_{2V}} + \underset{(-)}{g_{2\pi}} \cdot \underset{(?)}{\omega_{2V}} = \frac{\overset{(+)}{\theta_{2V}}}{\underset{(+)}{\overline{M}^2 + \dot{F}g_{2\pi}} \cdot \overline{M} - \underset{(?)}{\dot{F}g} \cdot \underset{(-)}{\theta_{2\pi}}} = ? \qquad \text{(A4.53a)}$$

$$h^*_{2V} = \underset{(-)}{h_{2V}} + \underset{(+)}{h_{2\pi}} \cdot \underset{(?)}{\omega_{2V}} = ? \qquad \text{(A4.53b)}$$

given equations (A4.33a), (A4.33b), (A4.36a) and (A4.51a). The characteristic equation of the linearised form of (A4.52) is as follows:

$$\underset{(-)}{g_{2\pi}} \cdot \underset{(?)}{\omega_{2\dot{x}}}\, \lambda^2 + \left[\underset{(?)}{g^*_{2V}}\, \underset{(+)}{h_{2\pi}}\, \underset{(?)}{\omega_{2\dot{x}}} - \underset{(?)}{h^*_{2V}}\, \underset{(-)}{g_{2\pi}}\, \underset{(?)}{\omega_{2\dot{x}}} - 1\right]\lambda + \underset{(?)}{h^*_{2V}} = 0 \qquad \text{(A4.54)}$$

The stationary state is a saddle point equilibrium provided:

$$\omega_{2\dot{x}} < 0 \tag{A4.55a}$$

$$h^*_{2V} < 0 \tag{A4.55b}$$

For $\dot{F}g \geqslant 0$, $\omega_{2\dot{x}}$ is unambiguously negative. Stability condition (A4.55b) is the same as in chapter 3. Given equation (A4.51b), condition (A4.55a) implies that

$$\underset{(+)}{\overline{M}^2 + Fg_{2\pi}} \cdot \overline{M} - \underset{(?)}{\dot{F}g} \cdot \underset{(-)}{\theta_{2\pi}} > 0 \tag{A4.56}$$

and given equation (A4.53a) makes g^*_{2V} unambiguously positive (g^*_{2V} is the slope of the $\dot{x} = 0$ locus).

The phase diagram analysis under perfect foresight is identical to that of section A3.2(b) of chapter 3. The only difference is that the subscripts of the h, ω and g functions change from 1 to 2.

For both expectations mechanisms, the dynamic response of the system to an exogenous fall in u is very similar to that of section 3 (section A3.3) and need not be repeated here.

REFERENCES

Ando, A. and Modigliani, F. (1963): "The Life Cycle Hypothesis of Saving: Aggregate Implications and Tests" American Economic Review, March.

Artus, J.R. (1976): "Exchange Rate Stability and Managed Floating: The Experience of the Federal Republic of Germany". IMF Staff Papers, vol. 23, July, pp. 312 - 33.

---------- (1978): "Models for Assessing the Long-run Equilibrium Value of an Exchange Rate". Journal of International Economics, vol. 8, June, pp. 277 - 99.

---------- and Crockett, A. (1978): "Floating Exchange Rates and the Need for Surveillance". Essays in International Finance, no. 127, Princeton University, May 1978.

---------- and Young, J.H. (1979); "Fixed and Flexible Exchange Rates: A Renewal of the Debate" IMF Staff Papers, vol. 26, December, pp. 654 - 699.

Bilson, J.F. (1979): "Recent Developments in Monetary Models of Exchange Rate Determination" IMF Staff Papers, vol. 26, June, pp. 201 - 223.

Black, F. (1974): "Uniqueness of the Price Level in Monetary Growth Models with Rational Expectations" Journal of Economic Theory, vol. 7, no. 1, pp. 53 - 65.

Black, S.W. (1973): "International Money Markets and Flexible Exchange Rates", Princeton Studies in International Finance, no. 32, Princeton University.

---------- (1975): "Exchange Rate Policies for Less Developed Countries in a World of Floating Exchange Rates", Essays in International Finance, no. 119, December, Princeton University.

---------- (1976): "Comment on J. Williamson 'Exchange Rate Flexibility and Reserve Use", Scandinavian Journal of Economics, vol. 78, no. 2, May, pp. 340 - 45.

Boyer, R.S. (1978): "Optimal Foreign Exchange Market Intervention", Journal of Political Economy, vol. 86, no. 6, pp. 1045 - 55.

Brainard, W. (1967): "Uncertainty and the Effectiveness of Policy", American Economic Review, May, pp. 411 - 33.

Branson, W.H. (1976): "The Dual Roles of the Government Budget and the Balance of Payments in the Movement from SR to LR Equilibrium". Quarterly Journal of Economics, August, pp. 345 - 367.

----------, Halttunen, H. and Mason, P. (1977): "Exchange Rates in the Short-run: The Dollar Deutschmark Rate", European Economic Review, vol. 10, December, pp. 303 - 24.

Brock, W.A. (1974): "Money and Growth: The Case of Long-run Perfect Foresight", International Economic Review, vol. 15, no. 3, October, pp. 750 - 777.

Brown, W.H. (1976): "World Afloat: National Policies Ruling the Waves", Essays in International Finance, no. 116, May, Princeton University.

Buiter, W. and Woglom, G. (1977): "On the Two Specifications of Asset Market Equilibrium in Macroeconomic Models: A Note", Journal of Political Economy, vol. 85, no. 2, April, pp. 395 - 400.

Burmeister, E. and Turnovsky, S.J. (1976): "The Specifications of Adaptive Expectations in Continuous Time Dynamic Economic Models", Econometrica, vol. 44, No. 5, September, pp. 879 - 905.

Crockett, A. and Goldstein, M. (1976): "Inflation under Fixed and Flexible Exchange Rates", IMF Staff Papers, vol. 23, November, pp. 509 - 44.

Cooper, R.N. (1976): "Monetary Theory and Policy in an Open Economy", Scandinavian Journal of Economics, vol. 78, no. 2, May, pp. 146 - 164.

Day, W.H.L. (1977): "Flexible Exchange Rates: A Case for Official Intervention", IMF Staff Papers, vol. 24, July, pp. 330 - 43.

Dornbusch, R. (1975): " A Portfolio Balance Model of the Open Economy", Journal of Monetary Economics, vol. I, January, pp. 3 - 19.

---------- (1976a): "The Theory of Flexible Exchange Rates and Macroeconomic Policy", Scandinavian Journal of Economics, vol. 78, no. 2, May, pp. 255 - 76.

Dornbusch, R. (1976b): "Expectations and Exchange Rate Dynamics", Journal of Political Economy, vol. 84, no. 6, December, pp. 1161 - 76.

Ethier, W. (1979): "Expectations and the Asset Market Approach to the Exchange Rate", Journal of Monetary Economics, vol. 5, pp. 259 - 282.

---------- and Bloomfield, A. (1975): "Managing the Managed Float", Essays in International Finance, no. 112, October, Princeton University.

Fischer, S. (1977): "Stability and Exchange Rate Systems in a Monetarist Model of the Balance of Payments", chapter 5 in R.Z. Aliber (ed.): "The Political Economy of Monetary Reform", Chicago 1977, pp. 59 -73.

Fleming, M.J. (1962): "Domestic Financial Policies under Fixed and Floating Exchange Rates", IMF Staff Papers, vol. 9, November, pp. 369.- 80

---------- (1975): "Floating Exchange Rates, Asymmetrical Intervention and the Management of International Liquidity", IMF Staff Papers, vol. 22, July, pp. 263 - 83.

Foley, D.K. (1975): "On Two Specifications of Asset Equilibrium in Macroeconomic Models". Journal of Political Economy, vol. 83, no. 2, pp. 303 - 24.

---------- and Sidranski, M. (1971): "Fiscal and Monetary Policy in a Growing Economy", New York, MacMillan, 1971.

Frenkel, J.A. (1976): "A Monetary Approach to the Exchange Rate: Doctrinal Aspects and Empirical Evidence", Scandinavian Journal of Economics, vol. 78, no. 2, May, pp. 200 - 25.

---------- and Johnson, H.G. (eds.) (1976): "The Monetary Approach to the Balance of Payments", George Allen and Unwin, London.

---------- and Rodriguez, C.A. (1975): "Portfolio Equilibrium and the Balance of Payments: A Monetary Approach", American Economic Review, vol. 65, no. 4, September, pp. 674 - 688.

Friedman, B.M. (1975): "Targets, Instruments and Indicators of Monetary Policy", Journal of Monetary Economics, vol. 1, October, pp. 443 - 73.

---------- (1979): "Optimal Expectations and the Extreme Information Assumptions of Rational Expectations Macromodels", Journal of Monetary Economics, vol. 5, pp. 23 - 41.

Friedman, M. (1953): "The Case for Flexible Exchange Rates", in his "Essays in Positive Economics", University of Chicago Press, 1953, pp. 157 - 203.

Girton, L. and Henderson, D. (1976): "Financial Capital Movements and Central Bank Behaviour in a Two Country Short-run Portfolio Balance Model", Journal of Monetary Economics, vol. 2, pp. 33 - 61.

---------- and --------- (1977): "Central Bank Operations in Foreign and Domestic Assets under Fixed and

Flexible Exchange Rates", in P.B. Clark et al. (eds.): "The Effects of Exchange Rate Adjustments", U.S. Government Printing Office, Washington, D.C., 1977.

Hahn, F. (1969): "On Money and Growth", Journal of Money, Credit and Banking, vol. 1, pp. 172 - 188.

Henderson, D. (1975): "Monetary, Fiscal and Exchange Rate Policy in a Two Country Short-run Macroeconomic Model", mimeographed, Board of Governors, Federal Reserve 1975.

---------- (1978): "The Dynamic Effects of Exchange Market Intervention Policy: Two Extreme Views and a Synthesis", unpublished manuscript, 1978.

---------- (1979): "Financial Policies in Open Economies", American Economic Review, May, pp. 232 - 39.

Herring, R.J. and Marston, R.C. (1977): "National Monetary Policies and International Financial Markets", North-Holland, Amsterdam.

IMF (1974): "Annual Report", Washington, D.C., September.

--- (1976): "Proposed Second Amendment to the Articles of Agreement of the IMF", Washington D.C., March.

--- (1977): "Annual Report", Washington D.C., September.

--- (1978): "Annual Report", Washington D.C., September.

--- (1979): "Annual Report", Washington D.C., September.

Isard, P. (1978): "Exchange Rate Determination: A Survey of Popular Views and Recent Models", Princeton Studies in International Finance, no. 42, May, Princeton University.

Ishiyama, Y. (1975): "The Theory of Optimum Currencies Areas: A Survey", IMF Staff Papers, vol. 22, July, pp. 344 - 383.

Kindleberger, C.P. (1976): "Lessons of Floating Exchange Rates", in K. Brunner and A. Meltzer (eds.): "Institutional Arrangements and the Inflation Problem", North-Holland, Amsterdam, 1976, pp. 51 - 77.

Kouri, P. (1976): "The Exchange Rate and the Balance of Payments in the Short-run and in the Long-run: A Monetary Approach", Scandinavian Journal of Economics, vol. 78, no. 2, May, pp. 280 - 305.

Krugman, P. (1979): "A Model of Balance of Payments Crises", Journal of Money Credit and Banking, vol. 11, August, pp. 311 - 28.

Lipschitz, L. (1978): "Exchange Rate Policies for Developing Countries: Some Simple Arguments for Intervention", IMF Staff Papers, vol. 25, December, pp. 650 - 75.

McKinnon, R.I. (1976a): "Floating Foreign Exchange Rates 1973-1974: The Emperor's New Clothes", in Brunner, K. and Meltzer, A. (eds.): "The Institutional Arrangements and the Inflation Problem", North - Holland, Amsterdam, pp. 79 - 114.

---------- (1976b): "Instability in Floating Foreign Exchange Rates: A Qualified Monetary Interpretation", unpublished manuscript.

---------- and Oates, W. (1966): "The Implications for International Economic Integration for Monetary, Fiscal and Exchange Rate Policy", Princeton Studies in International Finance, no. 16, Princeton University.

Mikesell, R. and Goldstein, H. (1975): "Rules for a Floating Regime", Essays in International Finance, no. 109, April, Princeton University.

Mundell, R. (1963): "Capital Mobility and Stabilization Policy under Fixed and Flexible Exchange Rates", Canadian Journal of Economics and Political Science, November 1963, reprinted in his "International Economics", McMillan, New York, 1968.

Mussa, M. (1976): "The Exchange Rate, the Balance of Payments and Monetary and Fiscal Policy under a Regime of Controlled Floating", Scandinavian Journal of Economics, vol. 78, no. 2, May, pp. 229 - 249.

Myrhman, J. (1976): "Experiences of Flexible Exchange Rates in Earlier Periods: Theories, Evidence and a New View". Scandinavian Journal of Economics, vol. 78, no. 2, May, pp. 169 - 197.

Poole, W. (1970): "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model", Quarterly Journal of Economics, vol. 84, May, pp. 197 - 216.

Robinson, J. (1974): "The Foreign Exchanges", chapter 4 in Ellis, H.S. and Metzler, L.A. (eds.): "Readings in the Theory of International Trade", Blakiston Company, Philadelphia and Toronto, 1947, pp. 83 - 104.

Samuelson, P.A. (1974): "Foundations of Economic Analysis", Harvard Economic Studies, no. 80, Harvard University.

Schadler, S. (1977): "Sources of Exchange Rate Variability: Theory and Empirical Evidence", IMF Staff Papers, vol. 24, July, pp. 253 - 96.

Shiller, R. (1978): "Rational Expectations and the Dynamic Structure of Macroeconomic Models: A Critical Review", Journal of Monetary Economics, vol. 4, January, pp. 1 - 44.

Stein, J. (1971): "Money and Capacity Growth", Columbia University Press, New York, 1971.

Tobin, J. (1969): "A General Equilibrium Approach to Monetary Theory", Journal of Money, Credit and Banking, vol. 1, February, pp. 15 - 29.

---------- and Brainard, W. (1968): "Pitfalls in Financial Model Building", American Economic Review, vol. 58, May.

Tosini, P. (1977): "Leaning Against the Wind: A Standard for Managed Floating", Essays in International Finance, no. 126, December, Princeton University.

Turnovsky, S.J. (1977): "On the Formulation of Continuous Time Macroeconomic Models with Asset Accumulation", International Economic Review, vol. 18, no. 1, February, pp. 1 - 28.

Williamson, J. (1973): "Surveys in Applied Economics: International Liquidity", Economic Journal, September, pp. 685 - 746.

Williamson, J. (1975): "The Future Exchange Rate Regime", Banca Nationale del Lavorno Quarterly Review, no. 113, June, pp. 127 - 144.

---------- (1976): "Exchange Rate Flexibility and Reserve Use", Scandinavian Journal of Economics, vol. 78, no. 2, pp. 327 - 40.