Essays in Incomplete Insurance and Frictional Labour Markets

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Abstract

This thesis consists of three chapters that investigate the importance of frictions in insurance and labour markets and their effects on macroeconomic outcomes. It asks how the behavior of aggregate employment and unemployment are affected, or the behavior of a planner who sets benefits to maximize welfare, when agents possess a number of risk sharing opportunities and luck in the labour market is the principal component of idiosyncratic risks.

Chapter one deals with the technical aspects of this question. I introduce wealth accumulation in a battery of familiar search models and explore the implications for wages, allocations and the amount of risk sharing that firms can provide to their workforce.

The second chapter investigates how the government should optimally set unemployment benefits depending on the range of private insurance opportunities in the economy. I consider a class of models that feature heterogeneous agents and wealth accumulation and contrast their properties with another where firms can provide additional insurance to their workforce. I show that the role of public policy is substantially different between the two economies.

The third chapter is joint work with Jochen Mankart. We consider another margin of insurance, namely family self insurance, whereby household members can adjust jointly their labour supply to insure against income losses. We investigate how this feature can affect the cyclical behavior of key labour market statistics. In the US data we find that insurance within the family is important in explaining why the labour force is acyclical and not volatile but when we turn to the model we get the converse prediction. We then evaluate what important additions need to be made to our framework to make the model consistent with the data.

Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it). The copyright of this thesis rests with the author. Quotation from it is permitted, provided that full acknowledgement is made. This thesis may not be reproduced without the prior written consent of the author. I warrant that this authorization does not, to the best of my belief, infringe the rights of any third party.

Declaration

I certify that chapter three of this thesis was coauthored with Jochen Mankart. Rigas Oikonomou contributed 50 percent to the genesis of the project, 50 percent to the computational and the empirical implementation, and 50 percent to the writing of the text.

Chris Pissarides (Supervisor)

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Date

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There were also others; I had the opportunity to test these ideas in the seminars at LSE but also in private, talking with faculty and research students and to them I owe a lot. Their suggestions improved significantly the quality of this thesis. I also owe a lot to my friends Christian, Jochen, Luca, Nuno and Ramin for making my time as a PhD student an unforgettable experience.

There are two people to which I am particularly grateful, perhaps more so than to anyone else; My father has been my inspiration, one of the reasons that I embarked upon this journey. At times he was even more emotionally involved in this adventure than me. Rea has been there for me in every difficulty. Her love, patience and support made all of this possible. This thesis belongs partly to them.

Preface

Throughout their working lives, economic agents face a considerable amount of idiosyncratic earnings uncertainty. They dislike these risks and they are willing to pay high premia to avoid them, but the insurance opportunities that are available through markets are far from perfect. Understanding how uncertainty and incomplete insurance markets shape economic outcomes is the main goal of this thesis. It assigns a very precise interpretation to the background economic risks and presents agents with a well defined array of opportunities to insure against them.

In the three chapters that compose this work, unemployment is the principal component of income losses; agents enjoy a higher income when they are working but they are constantly faced with a probability that they will loose their job. When they become unemployed they have to confront the frictions in the labour market that make the length of their spell uncertain. Against these risks they possess a number of insurance margins; they can accumulate assets to buffer shocks in labour income or they can rely on the government and their employers for transfers, but they can also be part of a family whose members adjust their labour supplies jointly. How these insurance arrangements affect the behavior of agents in frictional labour markets and how they translate into aggregate outcomes is focal point of this work.

Chapter 1 deals with some of the technical aspects of this venture. It builds on the observation that many influential models of search in the labour market assign a secondary role to risks; they rely on environments that are populated by risk neutral agents, and I introduce wealth accumulation and risk aversion to these models. I characterize allocations under two important arrangements; in one firms can sign long term contracts with their workforce subject to limited commitment and in another allocations have to be re-bargained each period according to a Nash sharing rule. I also present two general equilibrium frameworks to close these models. Depending on the scope of commitment, of the firm and the worker, allocations can entail much more risk sharing in some economies than others (in particular more risk sharing when commitment is abundant). I then ask whether the role of public policy differs along this dimension, and I find that differences in the range of private insurance opportunities present the planner with substantially different tradeoffs.

Chapter 2 takes a closer look to this last implication. It investigates how the government should devise its UI scheme to minimize the interference with private markets. I consider two economies: In the first one agents can do no better than to accumulate wealth during employment and in the second firms can provide additional insurance to their workforce. This is again comes in the form of contracts with limited commitment although in this context I reinterpret the arrangement to show that it can summarize other realistic insurance margins such as severance payments. Public policy can crowd out private markets in both economies but I find that the optimal level of benefits is much smaller in the second case than in the first. Further on, with firm insurance the optimal UI scheme doesn't have the typical shape; optimal payments in this economy should increase in the duration of an unemployment spell. To the extent that models of heterogeneous agents have been used to evaluate the welfare implications of public policy, the results of this chapter call for a more detailed account of the risk sharing opportunities in private markets.

Chapter 3 is the product of joint work with Jochen Mankart. We contrast the implications for the aggregate labour market of economies with realistic frictions, heterogenous agents and wealth accumulation and pay particular attention to the structure of the household unit. In one case we use the standard incomplete market model of bachelors households and in another we introduce couples of two ex ante identical agents that form search, labour supply and consumption decisions jointly. We use the model to investigate whether joint insurance within the family can explain the low procyclicality of the US labour force simultaneously with the suggestive business cycle correlations of other labour market statistics. Using samples of married couples from the CPS we show that joint insurance is an important feature of the US data, but our models are unable to capture it. We then go on to investigate what important additions need to be made to the baseline framework to reconcile the model with the data.

Contents

1 On the Joint Modeling of Incomplete Asset and Labour Markets

1.1 Introduction

Modern economic theory has become increasingly assertive of the fact that economic agents face a considerable amount of earnings uncertainty throughout their working lives, and that the insurance opportunities against these risks available to them are limited. A large body of work has tried to make sense of the large cross sectional dispersion of wage outcomes experienced by the economy's workforce, predominantly by viewing as central the notion that search and luck components in the labour market are important [1], and another voluminous literature has relied on estimations of the idiosyncratic earnings processes to assess the welfare implications of the lack of insurance markets and those of redistributive policies. [2] It seems however that there are very few formal connections between these two attempts. For instance modern micro theories of the labour market have had an enormous amount of success in matching the cross sectional distribution of wages (see Postel-Vinay and Robin (2002, 2005), Postel Vinay and Turon (2009)), by developing complicated economic environments only to colonize them with risk neutral agents, and quantitative models with heterogeneous agents and wealth accumulation have remained largely agnostic about the sources of risk that economic agents face over their lifetimes.

This chapter embraces the idea that the two frictions, in asset and labour markets should not be viewed in isolation but rather modeled jointly and presents an exhaustive account of their interactions. The aim is to develop a theory of wages in environments where active matches entail the existence of rents, and workers and firms can transfer resources intertemporally through the accumulation of assets. With very few exceptions up to date, search theoretic models that allow for these ingredients (see Lise (2007), Alvarez and Veracierto (2001)) have assumed that wage profiles remain constant throughout the life of the match. In contrast one of the focal points of this chapter is that the sharing of rents that accrue to active matches must be optimal in some sense, and that 'optimal wages' need not be fixed. Firms can rearrange the timing of payments in such a way so as to encourage the accumulation of assets and thus provide insurance against unemployment even in the absence of any other formal instruments. Another possibility (see Krusell et al (2007), Bils et al (2009(a), 2009(b)) is to assume that match rents are bargained for period by period say through a Nash protocol. Here Nash solutions are shown to be a special case of the firm's general contracting

[1] See Eckstein and Van Den Berg (2005) for a survey.
[2] See Heathcote et al (2009) for a survey.

problem, one that requires that optimal policies be time consistent.

Section 1.2 explores these ideas in depth. I set up a simple matching model where job opportunities arrive to unemployed job seekers at a constant rate and matches are heterogeneous in productivity. These ingredients form a theory of search, matching and bargaining with assets that derives from the work Mortensen and Pissarides (1994) and off course the relevance of the latter framework for quantitative macro cannot be overstated. To develop the concept of 'optimal wages' recursive representations of the firm's Pareto program are written whereby in the constraint set, the ability of the firm and the worker to commit to policy rules that dictate allocations at various horizons is included. There are three possibilities; first, in section 1.2.2, I consider an ex ante Pareto optimal program with enforceable contracts. Then I refine this concept to require that allocations satisfy participation by the firm and the worker at all future dates in section 1.2.3. Finally, section 1.2.4, describes a model with lack of commitment and per period bargaining.

All of these arrangements appear to be important in the relevant literature. For instance Rudanko (2008, 2009) uses commitment contracts to investigate how risk sharing between workers and firms affects the business cycle properties of aggregate wages, vacancies and unemployment (her model however doesn't have self insurance) and similarly Nash bargaining has been a primitive assumption for labour market models since the seminal work of Pissarides (1985). But with very few exceptions (namely the recent of work of Krusell et al (2007) and Bils et al (2009(a), (2009(b))) these models that explain the sources of idiosyncratic risks, feature too few private insurance opportunities for the economy's workforce and this is the gap that this chapter aims to bridge.

More substantively section 1.3 attempts to incorporate the analysis into a general equilibrium framework whereby distributions of wealth and wages and the contact rates between vacant jobs and job seekers are endogenously determined. It does so by relying on two equilibrium concepts: The first in section 1.3.1 builds on the directed search model of Moen (1997) and Acemoglu and Shimer (1999) to develop a notion of the equilibrium whereby firms post contracts and workers channel their search to the most profitable direction. The main task here is to characterize the equilibrium set of contracts and show that its a manageable object, thus making the model suitable for quantitative macro work. The second (section 1.3.2) is an undirected search equilibrium similar to the models of Krusell et al (2007) and Bils et al (2009(a), 2009(b)). In this case my work extends previous attempts by adding the notion that firms can commit to long term allocations with their workforce.

To put this theory at work I setup a simple optimal policy problem: A benevolent social planner chooses the level of non-employment income and levies taxes on the firm's output subject to budget balance each period. The finding here is that optimal policy prescriptions differ markedly depending on the contract offered to employed workers in the economy. I argue that many of the conclusions for optimal policy drawn from models with search and self insurance (see Alvarez and Veracierto (2001)) may have been misguided by the fact that the impact of wages on risk sharing opportunities has not been properly accounted for. Section 1.4 concludes. The Appendix (in section 1.5) contains a number of derivations extensions and numerical algorithms for the models of this chapter.

The theory presented in this chapter is in itself a contribution, in that it brings together incomplete insurance markets and a battery of familiar models of search. It is aimed to help researchers setup models with realistic heterogeneity where search frictions play a central role in labour market outcomes and which can be used in evaluations of optimal policy (this is a task that I take up seriously in chapter 2 of this thesis), or more generally to explore the aggregate implications of heterogeneity in individual labour supply rules. Both this and the next chapters can be viewed as complementary to this attempt.

1.2 The Model

I consider a labour market populated by a continuum of infinitely lived workers and entrepreneurs of equal but irrelevant measure. Workers are strictly risk averse, derive utility from the consumption of a general multipurpose good and discount the future at rate β; entrepreneurs on the other hand are risk neutral and discount future cash flows at rate $\frac{1}{R}$.

At any point in time a fraction e of the economy's workforce will be employed, matched with entrepreneurs in a joint production project, and the remaining u workers are unemployed and waiting for a suitable matching opportunity to arrive. In employment a worker-entrepreneur pair produce $z\epsilon$ units of output per unit of labour, where z is the aggregate component of labour productivity and ϵ is a match specific (idiosyncratic) component that derives from a general probability distribution F_ϵ. The latter is assumed to remain constant throughout the life of the match.

Unemployed workers produce a flow value of income b per unit of time and meet a potential trading partner (entrepreneur) at a constant rate p each period. I assume that workers have access to incomplete financial markets and can only borrow up to exogenous (ad hoc) limit $\overline{a}$ to finance consumption. Let r denote the rate of return on savings and assume that: $r \leq R \leq \frac{1}{\beta}$ and $r < \frac{1}{\beta}$[3].

Applying standard arguments we can represent the unemployed worker's dynamic programming problem as:

$$U(a_t) = \max_{a_{t+1}} u(c_t) + \beta\,(1-p)\;U(a_{t+1}) + \beta\,p \int \max\{U(a_{t+1}), \mathcal{W}(a_{t+1}, \epsilon)\}\;\; d\,F_\epsilon \tag{1.2.1}$$

Subject to the constraint set:

$$a_{t+1} \geq \overline{a} \qquad\qquad a_{t+1} = r(a_t + b - c_t) \tag{1.2.2}$$

In the notation $U(a_t)$ is the lifetime utility of an unemployed worker with wealth a_t in the current period and $\mathcal{W}(a_{t+1}, \epsilon)$ denotes her expected utility conditional on the event of meeting an entrepreneur next period in the market. The pair, upon the arrival of the job opportunity, draw a match specific value ϵ and then decide whether or not to give up search and form a productive match. To make matters simpler assume that the joint surplus of the match is strictly positive for every ϵ in the support of F, that is to say in what follows I always assume that $\max\{U(a_{t+1}), \mathcal{W}(a_{t+1}, \epsilon)\} = \mathcal{W}(a_{t+1}, \epsilon) \quad \forall\epsilon$. Furtheron, to introduce an outflow from employment into unemployment, assume that existing partnerships terminate exogenously at rate s each period. These properties (no reservation wage and exogenous separations) are not at all restrictive for the analysis that follows.

1.2.1 The General Contracting Problem

When they meet, the worker entrepreneur pair draw a value for the match specific productivity ϵ, and an allocation rule of the form $\mathcal{W}(a, \epsilon)$ (that is an object that

[3]Assumption $r < \frac{1}{\beta}$ is necessary to have a well defined equilibrium in this class of models (that is to guarantee that savings do not diverge).

I scrutinize below) gives the share of the surplus that accrues to each party. What happens after this initial assignment is the focal point of this section. I argue that firms and workers will rearrange payments over the life of the match in a manner that is Pareto optimal, and generally this requires to fix the value $\mathcal{W}(a, \epsilon)$ and choose a sequence of allocations that maximize the firm's profit stream. With this notion of equilibrium the analysis writes recursive representations of the firm's program where in the constraint set, the ability of the worker and the firm to commit to policy rules that dictate allocations at various horizons is included.

I distinguish between the following cases: In section 1.2.2 I assume that contracts in this economy are enforceable and that both the principal (firm) and the worker have sufficient commitment to adhere to date zero optimal policies. I refer to this program as the full commitment first best solution and generally I treat it as a benchmark relative to which more realistic alternatives are compared. Section 1.2.3 refines the equilibrium concept, to require that efficient outcome paths (the ones that solve the firm's dynamic program) satisfy certain sustainability conditions; namely that anywhere on the optimal contract both parties should be weakly better off than in autarky (unemployment). Finally section 1.2.4 introduces the notion that optimal contracts must be time consistent, and relies on a Markov perfect structure with Nash Bargaining to characterize optimal allocations.

The scope of the firm's and the worker's commitment is shown to have profound impact on the shape of the optimal compensation scheme. Generally with commitment introducing wealth as a state variable makes possible (and optimal) to transfer resources, in the first period, between the firm and the worker in a manner that is most cases is shown to be time inconsistent. Markov perfect contracts on the other hand induce a time invariant wage schedule as a function of the agent's wealth endowment.

There is a battery of results that I highlight. First an important issue is to determine the extent to which the firm can rearrange payments in such as way, so that allocations provide insurance against unemployment risks. The finding here is that with commitment there are cases where the optimal contract features complete insurance (and also wealth is a perfect substitute for any other form of insurance) but without commitment this is no longer possible. Further on another substantive theme that I pursue is whether asset contractibility (that is whether the worker or the firm dictates optimal savings decisions) matters for the optimal policies. This turns out to be the case for time consistent (Markov Perfect) contracts, but with commitment I show either theoretically or numerically that this is not the case. Very few of these results can be established analytically and instead I have to rely on numerical methods to discern something about the optimal decision rules. The details of the algorithms are delegated to the Appendix.

1.2.2 Full Commitment

Commitment programs are nothing but ex ante Pareto optimal allocations. They involve maximizing the expected utility of one party, subject to the other party getting at least the payoff that is prescribed when the contract is signed. For instance here if W_0 is the agreed level of utility for the worker (under any some initial allocation rule) then the planner (entrepreneur) must choose a sequence of transfers and wealth to deliver W_0 to the worker in the most efficient (profit maximizing) way. Further on these optimal policies define a sequence of payoffs for the worker, and assume that at some generic point in time t her expected continuation utility under the allocation is given by W_t and her outside option (unemployment) by $U(a_t)$. There are two important points:

First the time paths for W_t and $U(a_t)$ are inessential for the complete (full commitment) contract analyzed in this section (but not for the other arrangements studied in this chapter). That is to say the allocation here permits to have $W_t < U(a_t)$ for some t, since contracts are enforceable and participation need not be satisfied and the same holds for the stream of profits that accrue to the entrepreneur.

Second the economy studied in this section (and more generally all of the models of this chapter) admits to a recursive representation; it allows us to take any point in time t and summarize the optimal allocation by the state variable W_t, the level of expected utility that the entrepreneur must deliver to the worker from that point onwards. There are other inputs in the state vector (current assets a_t and the firm specific productivity ϵ) but recursive representations mean that optimal choices for the next period are time invariant functions of these arguments.

Consider a firm that maximizes the present value of its profits Π in a complete contract. The per period payoff is the difference between the wage paid w_t and the labour productivity $z\epsilon$, and the match terminates at a rate s per period so the effective discount factor for the firm equals $\frac{1-s}{R}$. The firm must choose current transfers w_t (wages), next period's wealth a_{t+1} and a continuation payoff W_{t+1} that will be taken as given in the next period. These solve the following functional equation:

$$\Pi(W_t, a_t, \epsilon) = \max_{W_{t+1}, a_{t+1}, w_t} \; z\,\epsilon - w_t + \frac{1-s}{R}\,\Pi(W_{t+1}, a_{t+1}, \epsilon) \tag{1.2.3}$$

Subject to the constraint set:

$$(\lambda_t) \quad u(-a_{t+1}/r + a_t + w_t) + \beta(1-s)\,W_{t+1} + \beta s\,U(a_{t+1}) \geq W_t \tag{1.2.4}$$

$$(\chi_t) \quad a_{t+1} \geq \overline{a} \tag{1.2.5}$$

Added in parentheses are the multipliers on the constraints. Equation (1.2.4) is the so called promise keeping constraint stating that the expected level of utility delivered to the worker from the optimal contract must be weakly greater than the promised value W_t. Note that by varying the value of W_t it is possible to trace the entire frontier of utilities for the firm and the worker in the current context. Equation 1.2.5 is the borrowing constraint on assets.

The policy rules for w_t W_{t+1} and a_{t+1} define an implicit consumption sequence that adds up to the payoff W_t in expectation. Notice that, in the absence of separations, this sequence could be financed solely by wages and indeed in this case the risk sharing role of wealth becomes meaningless. But if $s > 0$ then wealth is an important variable because it allows to (partially) control the agent's consumption when she becomes unemployed. Since the entrepreneur can control this variable, the above program corresponds to a full commitment allocation with contractible wealth. That is to say that the implications of this arrangement could in principle be different to those of contract where the firm can only set wages, and the worker makes optimal savings and investment decisions.

Optimality. Taking first order conditions with respect to W_{t+1} , a_{t+1} and w_t we get:

$$\lambda_t u'(c_t) = 1 \tag{1.2.6}$$

$$\frac{1-s}{R}\Pi_{W_{t+1}} + \lambda_t \beta(1-s) = 0 \tag{1.2.7}$$

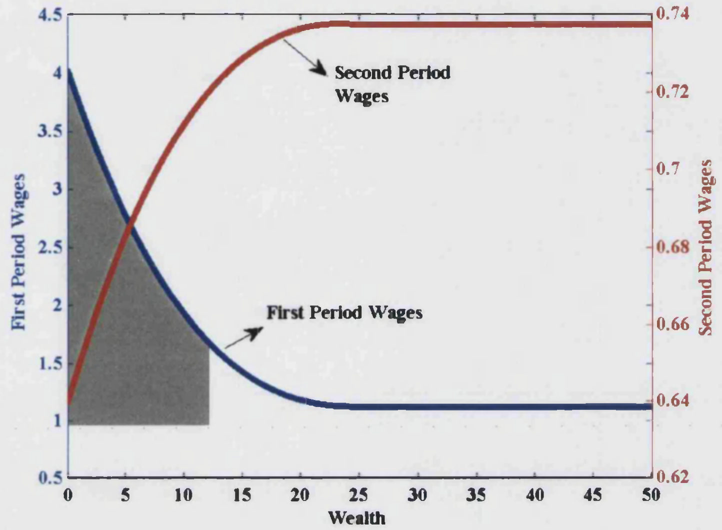

FIGURE 1.1: FULL COMMITMENT $R = r$ WAGE PROFILES

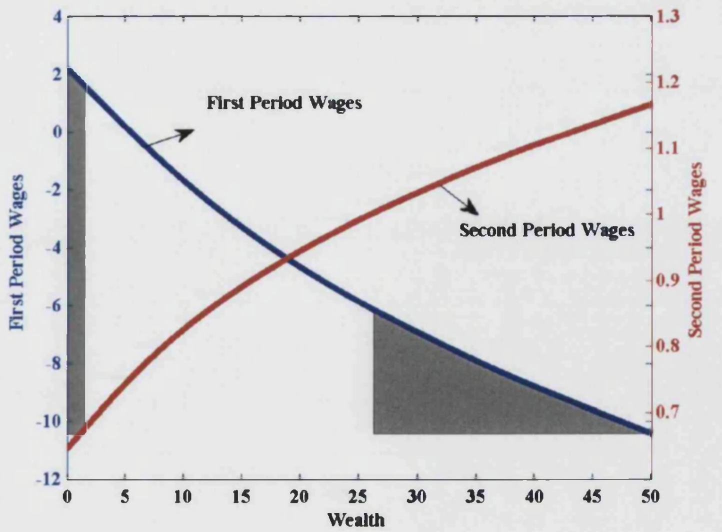

FIGURE 1.2: FULL COMMITMENT $R = \frac{1}{\beta}$ WAGE PROFILES

$$\frac{1-s}{R}\Pi_{a_{t+1}} - \frac{\lambda_t u'(c_t)}{r} + \beta s \lambda_t U_{a_{t+1}} - \chi_t = 0 \tag{1.2.8}$$

Along with the Envelope conditions: $\Pi_{W_t} = -\lambda_t$ and $\Pi_{a_t} = \lambda_t u'(c_t) = 1$

These objects have a straightforward interpretation. For instance in equation (1.2.6) the multiplier λ_t is the (relative) Pareto weight assigned to the worker on the optimal program. Lowering wages by one unit entails a unitary marginal utility benefit for the entrepreneur (due to risk neutrality) whilst the local cost for the worker is given by $u'(c_t)$. Moreover equation (1.2.7) gives the law of motion of this weight over time. To see this make use of the envelope condition for W_t to derive a general recursion of the form:

$$\lambda_{t+1} = \beta R \lambda_t \tag{1.2.9}$$

Finally equation (1.2.8) determines the optimal policy for asset accumulation in the next period. Off corners (when $\chi_t = 0$) the firm equates the net marginal cost of supplying an extra unit of savings ($\frac{1-s}{R} - \frac{1}{r}$) [4] to the marginal benefit of insuring the worker against unemployment next period whereby the allocation takes into account the weight λ_t and the relative discounting of the firm and the worker.

These conditions can go a long way towards characterizing some of the salient features of the optimal allocation. For instance consider equation (1.2.9). It is clear that differences in discounting (in the sense $R = r < \frac{1}{\beta}$) make the sequence of Pareto weights strictly decreasing over time and in the limit the optimal allocation implies that the marginal utility of the worker will tend to infinity (her consumption will tend to zero). Since wealth and wages are the two instruments used to finance consumption in the current context this result suggests that over time the values for these objects are decreasing.

Further on equation (1.2.8) determines the extent to which the optimal contract provides sufficient insurance against the event of a job loss. To see this rearrange (1.2.8) making use of the envelope conditions and the law of motion of marginal

[4]Note that by the envelope condition for a_t the firms profit function is linear homogeneous in wealth and thus $\lambda_t \frac{u'(c_t)}{r} = \frac{1}{r}$.

utility in (1.2.9) to get:

$$u'(c_u, t+1) = U_{a_{t+1}} = u'(c_e, t+1) + (\frac{R}{r} - 1)\frac{u'(c_e, t+1)}{s} + \tilde{\chi}_t \qquad (1.2.10)$$

Where $\tilde{\chi}_t = \frac{\chi_t}{\beta\lambda_t s}$ [5] and subscripts e and u denote the relevant quantities for employed and unemployed workers respectively.

The following proposition summarizes the optimal provision of insurance in first best full commitment contracts.

PROPOSITION 1.1 *Consider the special case* $r = R$*: Then if* $a_{t+1} > \overline{a}$ *the worker is perfectly insured against unemployment (in the sense that* $u'(c_u, t+1) = u'(c_e, t+1)$ *). If* $a_{t+1} = \overline{a}$ *consumption rises when the agent becomes unemployed. On the other hand with sufficient discounting* $r < R = \frac{1}{\beta}$ *the agent is underinsured almost everywhere on the optimal contract unless* $a_{t+1} = \overline{a}$ *in which case it is impossible to sign the difference in marginal utilities.*

The result follows readily from equation (1.2.10) . It suggests that if the firm's discount rate is equal to the market interest rate (i.e. $R = r$) assets are sufficient to insure the agent against unemployment spells and indeed wealth in this case can be shown to be a perfect substitute for any form of severance compensation. With the same discount factor though (i.e. when $R = \frac{1}{\beta}$ or more generally when $R > r$), this result no longer holds, because in this case the entrepreneur has access to a technology that transfers resources intertemporally earning a higher rate of return ($\frac{1}{\beta} > r$) than the riskless savings in the market. Under such return dominance it is not uncommon for optimal allocations to feature an extraction of the agent's wealth endowment in the initial period and in general for underinsurance to carry over in the entire optimal path.

Optimal Compensation. The implications of these results for optimal compensation are shown in Figures 1.1 and 1.2; they trace wages in the first (left axis) and second (right axis) period of the contract as a function of the initial wealth endowment of a newly employed worker. The value of the firm productivity is normalized to unity and the worker's discount factor and the

[5] Note that if the constraint binds $\chi_t < 0$. This derivation makes use of the fact that $\lambda_{t+1}u'(c_{e,t+1}) = 1$ from the first order conditions in the next period.

market interest rate equal $\beta = .995$ and $r = 1.0041$. In both of these Figures function $\mathcal{W}(a, \epsilon)$ (that determines the initial placement of workers on the Pareto frontier) is a solution to the following Nash Bargaining program:

$$\mathcal{W}(a, \epsilon) \in \arg\max_{W} (W - U(a))^{\eta} \Pi(W, a, \epsilon)^{1-\eta} \tag{1.2.11}$$

where I set $\eta = 1/2$.

Consider the case $\mathcal{W}(a, \epsilon) >> U(a)$ and $R = r$ (Figure 1.1). If the worker enjoys a capital gain from search (say if $\eta > 0$) then her consumption will rise when she becomes employed and if βR is close to unity it will remain high for several periods. The complete insurance result states that wages in the first period must be high enough to finance the accumulation of assets, and in the next period wages will fall below productivity and continue to fall as the expected utility path is downward sloping in the optimal contract. [6]

If on the other hand $r < R = \frac{1}{\beta}$ (Figure 1.2) and capital gains are not sufficiently high, then part of the agent's wealth endowment is extracted in the initial period, which could make wages negative, and this extraction is used to finance higher wages permanently from period two onwards. As capital gains increase the difference between period one and period two wages decreases.

These implications highlight the importance of the relative discounting assumptions and of the timing of wages for the notion that active matches provide insurance against unemployment. They seem to hold over a range of reasonable calibrations for the model's parameters. For instance in the limit when $s \to 0$, whereby the risk sharing role of assets becomes meaningless, it can be established that whenever $R > r$ wealth is zero everywhere on the optimal contract (it is extracted in the initial period), whereas it becomes redundant whenever $R = r$. In the latter case any form of contract including flat wage contracts can be optimal [7]. More generally higher values of s imply that insurance in the

[6] It is easy to argue that even if $\mathcal{W}(a, \epsilon) = U(a)$ and $R = r$ the optimal path of wages is front-loaded. To see this consider a constrained worker i.e. one that finds a job with an initial endowment of wealth equal to $\overline{a}$ and let without loss of generality $\overline{a} = 0$. For this agent unemployment is equivalent to consuming b units each period. But when employed the optimal allocation implies that eventually her consumption path will be driven below b since marginal utility is increasing over time. To compensate for this the entrepreneur must offer an initial level of consumption that is strictly greater than b, and coupled with complete insurance it must be that $a_{t+1} > \overline{a}$. Such an accumulation of assets can only be financed if wages are sufficiently high in the first period of the contract.

[7] From equation 1.2.8 it follows that when $s \to 0$ then $\chi_t = \frac{1}{R} - \frac{1}{r}$ where χ_t is the multiplier on the borrowing constraint. When $r < R$ the optimal policy is for wealth to be zero anywhere on the optimal contract and hence it is extracted in the initial period.

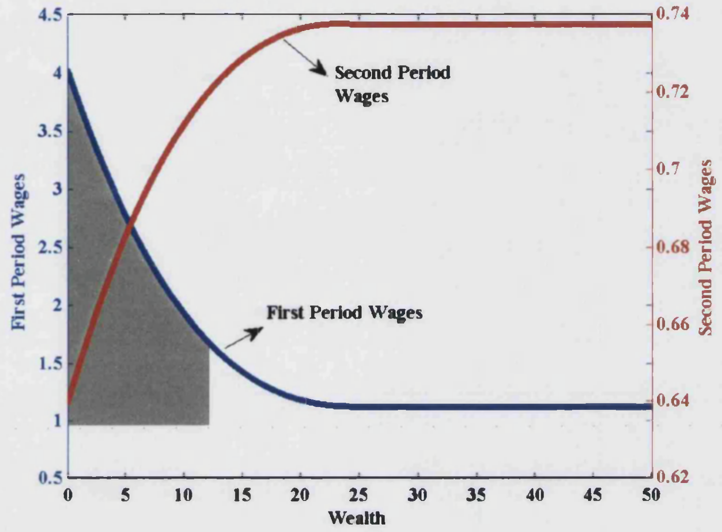

FIGURE 1.1: FULL COMMITMENT $R = r$ WAGE PROFILES

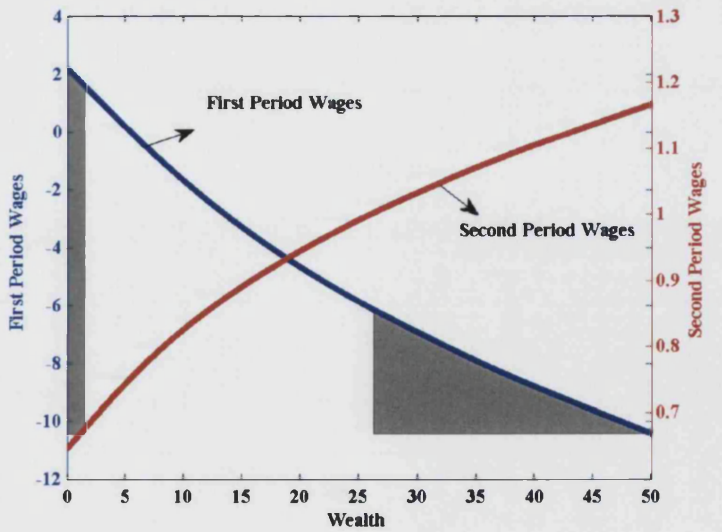

FIGURE 1.2: FULL COMMITMENT $R = \frac{1}{\beta}$ WAGE PROFILES

in Figure 1.1 can be reinterpreted as a simple wage and severance payment scheme. This means that wages don't have to be frontloaded if this feature seems rather (empirically) unappealing. [10].

1.2.3 Limited Commitment

In the previous section I defined the nature of the interaction between the firm and the worker as one where both parties can commit to a long term plan, independent of the path of utility that it entails. In other words contracts were fully enforceable once signed at date zero. Here I refine this concept by requiring that efficient outcome paths (the ones that solve the firm's program) satisfy certain sustainability conditions; namely that neither the principal nor the agent must be strictly better off anywhere on the optimal contract, reneging on their commitments and reverting to autarky (unemployment).

This requirement can a priori rule out certain aspects of the optimal allocation under full commitment that were discussed in the previous section. For instance when $r = R < \frac{1}{\beta}$ it was shown that marginal utility tends to infinity in the long run, but clearly such solutions are not admissible here since the worker can always quit to become unemployed, in which case her consumption is bounded by b. Or even in the case $R = \frac{1}{\beta}$ I showed that under-investment in wealth could mean that the firm's profits become negative after the first period and in this case, entrepreneurs would better off unmatched.

[10] In Chapter 2 I provide a general proof for a similar search model. Here consider the following argument based on the first order conditions of the firm's program when an additional control variable ζ_t (severance payment) is included. It can be shown that off corners (when $a_{t+1} > \overline{a}$), optimality for wealth and severance compensation is determined by the following equations:

$$\frac{1-s}{R} - \frac{1}{r} + \lambda_t \beta s U_{a_{t+1}}(a_{t+1} + \zeta_t) = 0 \quad \text{wrt } a_{t+1}$$
$$\frac{s}{R} + \lambda_t \beta s U_{\zeta_t}(a_{t+1} + \zeta_t) = 0 \quad \text{wrt } \zeta_t$$

Clearly when $R > r$ the first condition is not satisfied and wealth is equal to zero (insurance is provided only through severance payments). Severance payments in this case are equivalent to the firm extracting the agents wealth endowment and investing it, earning a higher rate of return (or otherwise allowing access to a superior storage technology). When $R = r$ the two equations are the same and since a_{t+1} and ζ_t enter additively in the utility the optimal allocation is indeterminate.

match is more valuable, and hence initial investments in wealth increase with this parameter[8]. Similarly higher ϵ (and higher η) means that the worker commands more gains from search under the initial allocation rule, and hence given the insurance properties of the models, asset accumulation should be higher [9]. The qualitative patterns of the Figures are preserved.

Finally note that some of the features of the optimal allocation in both cases are not sustainable if either party can unilaterally revert to autarky at some point in the future. The shaded regions in Figures 1.1 and 1.2 identify the locations where the optimal contract makes either the entrepreneur or the worker better off in unemployment than in the match. In Figure 1.1 all of this violation comes from the side of the worker, since by front-loading wages in the first period, the firm makes unemployment a less unattractive state (remember that in this case utility for the worker falls over time). A similar intuition applies to the highlighted part at the low end of the asset grid in Figure 1.2 but when wealth is high it is the entrepreneur that must pay subsequent wages greater than productivity and hence would rather renege on this commitment. The next section makes an explicit reference to these ideas by introducing the notion that optimal contracts must be self enforcing in the sense that at no point should they violate participation by either the firm or the worker.

Noncontractible Assets. How are these results affected when the agent's control over her assets is restored? The answer is that for the first best contract of this section the standard Euler equations are satisfied, and given her control over transfers, the principal can implement the same allocations as an equilibrium outcome in an environment with non contractible wealth. To see this formally summarize the optimal allocation in the sequence $\{c_t^*,\ c_{e,t+1}^*,\ c_{u,t+1}^*\ ,a_{t+1}^*\}$ and in the following conditions:

$$u'(c_t^*) = \beta R u'(c_{e,t+1}^*) \tag{1.2.12}$$

$$u'(c_{u,t+1}^*) = u'(c_{e,t+1}^*) + (\frac{R}{r} - 1)\frac{u'(c_{e,t+1}^*)}{s} + \tilde{\chi}_t \tag{1.2.13}$$

Then note that off corners ($\tilde{\chi}_t = 0$) if the agent were to depart from the optimal savings schedule the marginal cost of doing so is $u(c_t^*)$ and the marginal benefit

[8] This is visible from equation (1.2.10) whereby a higher separation rate means that the wedge between the unemployed and the employed worker's consumption shrinks given r and R.
[9] Again this can be read off equation (1.2.10).

for next period is given by:

$$\beta(1-s)ru'(c^*_{e,t+1}) + \beta sru'(c^*_{u,t+1})$$

But notice that the later quantity equals:

$$\frac{r}{R}u(c^*_t)(1-s) + \frac{r}{R}(su'(c^*_t) + (\frac{R}{r} - 1)u(c^*_t)) = u'(c^*_t)$$

(a similar condition applies when the optimal contract features $a_{t+1} = \overline{a}$.)

This result is very intuitive. It suggests that by making payments as erratic as those shown Figures 1.1 and 1.2 the firm can induce the worker to save (or dissave) to finance a smooth consumption path. For instance when $R = r$ the agent expects his income to fall between the first and the second period and her optimal response is to accumulate precautionary savings to buffer consumption against this drop. In the first best this motive turns out to yield the same optimal investment for the worker, as the one that would be chosen by the firm if the latter dictated allocations in the match. The following proposition summarizes the result.

PROPOSITION 1.2 *Optimal first best contracts are equivalent under contractible and non contractible savings.*

To conclude this section there are a few points that need to be clarified: First complete insurance in the current context (whenever that obtains) should not be misconstrued to imply that the worker's consumption path is unaffected by the risk of job separations. On the contrary when the job terminates, the worker is beyond the reach of the firm and her consumption falls as an unemployment spell progresses through standard wealth effects. It is only in the initial period that sufficient wealth is stored to alleviate the risk of a drop in consumption.

Further on in this simple model insurance against unemployment is accomplished through investments in assets and the timing of payments is central to the notion that active matches provide insurance. It can be established though that assets are a perfect substitute for other (more popular) forms of insurance such as severance payments. I mentioned this earlier but it is important to repeat here because it has the following implication for the timing of payments: If the firm is given both margins of insurance then the optimality conditions cannot uniquely determine the ratio investments, and in this case the wage profile shown in Figure 1.1 can be reinterpreted as a simple wage and severance payment scheme. This means that wages don't have to be frontloaded if this feature seems rather (empirically) unappealing. [10].

1.2.3 Limited Commitment

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This requirement can a priori rule out certain aspects of the optimal allocation under full commitment that were discussed in the previous section. For instance when $r = R < \frac{1}{\beta}$ it was shown that marginal utility tends to infinity in the long run, but clearly such solutions are not admissible here since the worker can always quit to become unemployed, in which case her consumption is bounded by b. Or even in the case $R = \frac{1}{\beta}$ I showed that under-investment in wealth could mean that the firm's profits become negative after the first period and in this case, entrepreneurs would better off unmatched.

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This type of arrangement also entails many more appealing features relative to the first best contracts. For one thing wage profiles are not so extreme in the sense that the differences between first and subsequent period wages are not as large and this is consistent with the notion that empirically wages are very persistent. On the other hand I argued previously that the optimal transfers shown in Figure 1.1 are the result of the particular insurance arrangement considered here and that under alternative mechanisms one can get more realistic wage profiles out of the model. Private risk sharing can be reinterpreted as a simple wage and severance payment scheme and even for the limited commitment model of this section it is possible to construct examples where this equivalence holds.

The self-enforcing contracts described in this section are rather common in the related literature. In fact Thomas and Worall (1988) were the first to explore their implications in a labour market context and Rudanko (2008, 2009) uses this model to investigate how risk sharing between workers and firms affects the business cycle behavior of aggregate wages, vacancies and unemployment. This work however doesn't give to agents any self insurance opportunities through assets and in a labour market with unemployment such an inclusion is important since wealth can buffer the risk of a job loss. This at least was one of the central implications of the model of the previous section.

To characterize policy rules in an environment where date zero contracts are self enforcing, a dynamic programming procedure can be followed here similar to the one defined in equations (1.2.3) to (1.2.5), the main difference being that a set of forward looking sustainability constraints must be added to the firm's Pareto program (see Ligon et al (2000, 2002)). These are:

$$(\gamma_{t+1}) \quad W_{t+1} \geq U(a_{t+1}) \tag{1.2.14}$$

$$(\phi_{t+1}) \quad \Pi(W_{t+1}, a_{t+1}, \epsilon) \geq 0 \tag{1.2.15}$$

Equation (1.2.14) imposes that investment in wealth must not be too high (or future continuation utility too low) to make the worker better off in unemployment, and equation (1.2.15) is the analogous participation constraint for the firm. Added in the parentheses are the multipliers on these constraints.

The nature of these objects (that they are forward looking) entails a theoretical difficulty in that they may not define a convex set, and establishing convergence to a concave value function becomes a formidable task. The analysis that follows

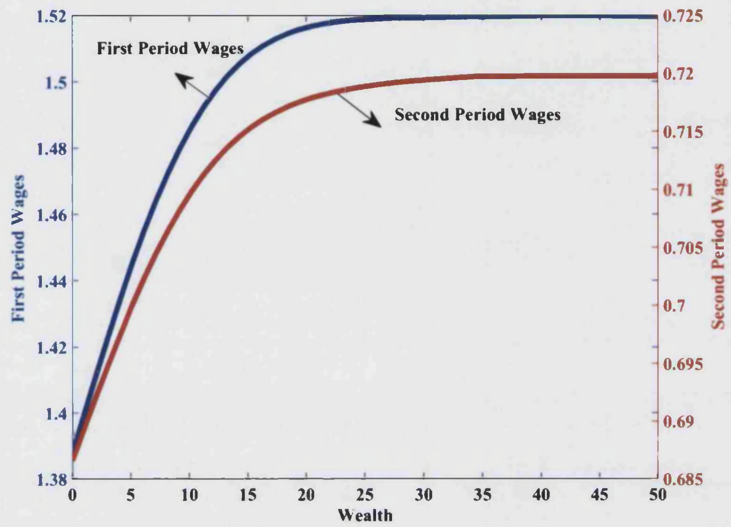


FIGURE 1.3: LIMITED COMMITMENT $R = r$ WAGE PROFILES

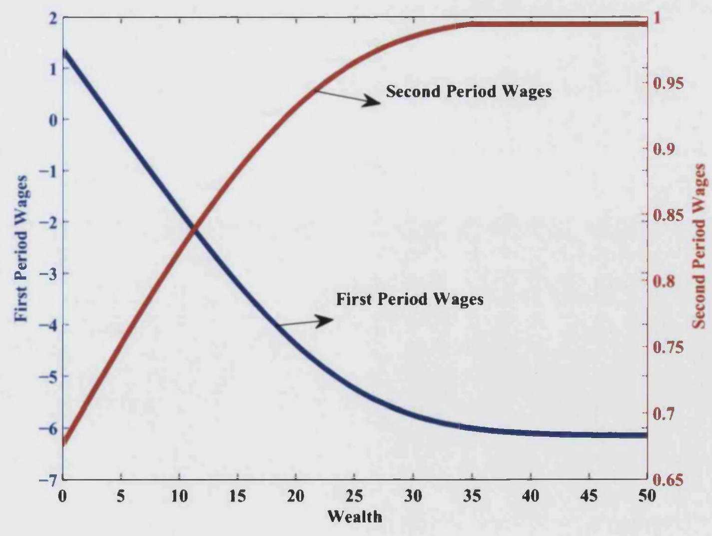


FIGURE 1.4: LIMITED COMMITMENT $R = \frac{1}{\beta}$ WAGE PROFILES

uses the optimality conditions to characterize some features of the optimal plan but these require sufficiency which is impossible to prove. Nonetheless, in all the simulations carried out in this and the next chapter I found that Π is strictly decreasing and concave in W_t. The Appendix outlines the numerical procedure I used.

Optimal Policies. The first order conditions for the optimum can be written as:

$$\frac{1-s}{R} - \frac{\lambda_t u(c_t)}{r} + \lambda_t \beta U_{a_{t+1}} s + \gamma_{t+1} U_{a_{t+1}} - \phi_{t+1} = 0 \tag{1.2.16}$$

$$\frac{1-s}{R} \Pi_{W_{t+1}} + \lambda_t \beta (1-s) - \gamma_{t+1} - \phi_{t+1} \Pi_{W_{t+1}} = 0 \tag{1.2.17}$$

Along with the Envelope conditions: $\Pi_{W_t} = -\lambda_t$ And $\Pi_{a_t} = \lambda_t u'(c_t) = 1$

These equations have a similar interpretation to the analogous conditions derived for the full commitment program the novel elements being the multipliers on the constraints (1.2.14) and (1.2.15). This addition gives rise to the following two interesting implications for the optimal contract:

First forwarding the envelope condition one period we can restate equation (1.2.17) as:

$$\lambda_{t+1} = \frac{\lambda_t \beta (1-s) - \gamma_{t+1}}{\frac{1-s}{R} - \phi_{t+1}} \tag{1.2.18}$$

Remember that λ_t is the inverse of the marginal utility of consumption, the Pareto weight attached to the worker's utility from the planning problem. Condition (1.2.18) then states that this weight is time varying, even if the firm and the worker share the same discount factor. When the constraint on the worker's outside option binds, so that $\gamma_{t+1} < 0$, the weight assigned to her jumps, and consumption increases within the period, whilst if the principal needs to be made better off in the future date, resources will shift from the worker to the firm (since in this case $\phi_{t+1} < 0$).

It is clear that the limiting behavior of the sequence of multipliers defined in (1.2.18) is substantially different from the analogous sequence in the first best contract. There the value of λ_t shrunk to zero whenever the worker discounted the future more heavily than the entrepreneur but here the multipliers will adjust to keep this stationary value strictly greater than zero. A similar argument can be applied to the case $R = \frac{1}{\beta}$.

The second interesting object is a modified Euler equation (whose derivation is delegated to the Appendix). It can be shown that optimal savings in the current context are governed by the following condition:

$$u'(c_t) \geq \beta(1-s)ru'(c_{t+1,e}) + r\beta su'(c_{t+1,u}) + ru'(c_t)\gamma_{t+1}(u'(c_{t+1,u}) - u'(c_{t+1,e})) \tag{1.2.19}$$

(With equality if $a_{t+1} > 0$)

The leading terms pertain to the familiar marginal benefit of an extra unit of savings adjusted (through the inequality) for debt limit facing the agent. The last term in (1.2.19) reveals the effect of the sustainability constraints on optimal inter-temporal consumption choices whereby an extra unit of savings has a direct effect on the allocation by affecting the magnitude of the multiplier γ_{t+1}.

For instance consider the case $\gamma_{t+1} < 0$ and $u'(c_{t+1,u}) - u'(c_{t+1,e}) < 0$. In this case the optimal allocation dictates a rise in the weight λ_{t+1} next period and hence a rise to the worker's consumption. Should this rise be financed by an increase in current savings then this would relax the constraint. To see this note that, from the envelope condition of the unemployed worker's program, the marginal utility of consumption in unemployment in period t+1 ($u'(c_{t+1,u})$) equals the marginal valuation of an extra unit of savings in this state (i.e $U_{a_{t+1}}$). Increasing consumption next period financed through current savings implies that the value of autarky (unemployment) rises less than the value of employment. Hence the constraint is relaxed. The converse holds if $u'(c_{t+1,u}) - u'(c_{t+1,e}) > 0$.

Further on it should also be clear from equation (1.2.19) that in an environment of limited commitment the assumptions about asset contractibility are no longer innocuous to the shape of the optimal paths. Generally these last terms in equation (1.2.19) drive a wedge between marginal costs and benefits to an extra unit of savings, though establishing the direction of the difference between desired savings for the worker and optimal savings with contractible assets for the firm, would require to sign the differences in marginal utilities and to know where exactly the enforcement constraints bind. [11] Instead one can rely on numerical

[11] It substantially harder to solve the firm's program in recursive form when wealth is non-

analysis to see whether departures from full commitment signal violations of the standard Euler equation. I tend to find that such violations are minimal, undistinguishable from numerical errors of an acceptable order of magnitude, which suggests that including the Euler equation as an additional constraint to the firm's program would not affect the shape of the optimal compensation scheme (see section 1.5.1 of the Appendix for further details).

Implications for wages. Figures 1.3 and 1.4 display the wage profiles in the first and the second period of the optimal contract as a function of the wealth endowment of the worker. As in Figure 1.1 when $R = r$ wages are front-loaded to provide insurance against unemployment through the accumulation of assets, but the extent to which these initial transfers are possible is limited here by the requirement that optimal contracts satisfy participation for both parties. Full and limited commitment profiles differ most, precisely in the region where violations of the worker's participation constraint occur (i.e. the highlighted part of Figure 1.1).

On the other hand the bulk of the difference, between full and limited commitment profiles when $R = \frac{1}{\beta}$, comes from the higher end of the grid of assets. There period one wages were extremely low to extract wealth initially in Figure 1.2, and for all subsequent periods they exceeded the marginal productivity of the job. This path is off course no longer admissible.

Insurance properties with limited commitment. It is possible to show that the optimal investment in wealth (when the borrowing constraint doesn't bind) is governed by the following optimality condition:

$$\frac{u'(c_u, t+1)}{u'(c_e, t+1)} = 1 + \omega(\phi_{t+1}, \gamma_{t+1}, r, R, s) \tag{1.2.20}$$

contractible. The relevant state variable is no longer a promised level of utility but rather a promised utility function of the form $W_t(a)$ which has to be endogenously determined along with the optimal policies. Although it is theoretically feasible to characterize solutions in this context an application of this theory would involve dealing with curse of dimensionality issues. It is beyond the scope my analysis to attempt this. Alternatively one could use the methodology developed by Marcet and Marimon (1997) that can handle Euler equations as a additional constraint in programs with limited commitment. There are two reasons why I don't pursue this here. First in all the model simulations I tend to find that violations of the Euler equation are minimal and indistinguishable from numerical errors of acceptable tolerance. Second possible non-convexities introduced by the enforcement constraints may actually throw off these calculations.

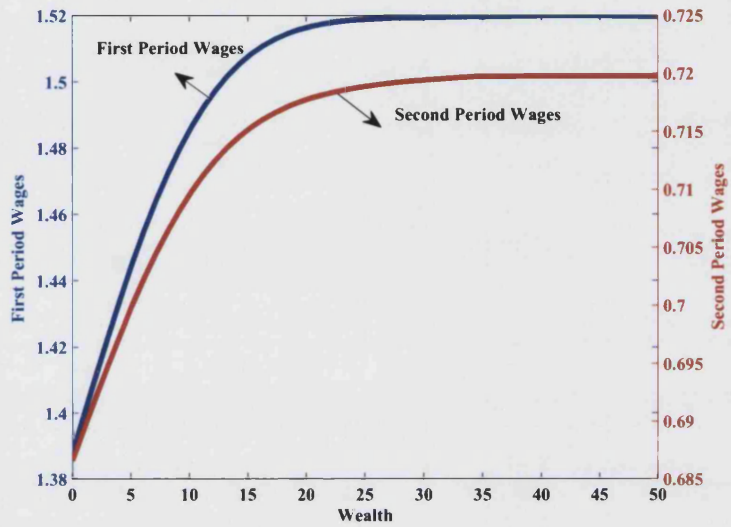


FIGURE 1.3: LIMITED COMMITMENT $R = r$ WAGE PROFILES

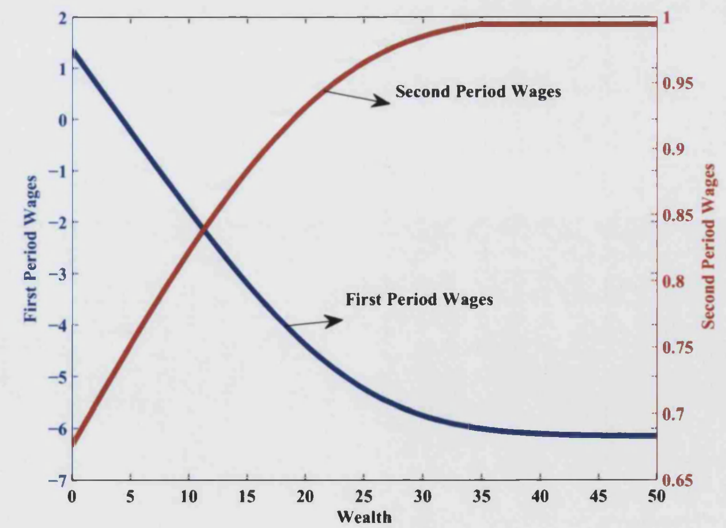


FIGURE 1.4: LIMITED COMMITMENT $R = \frac{1}{\beta}$ WAGE PROFILES

Where:

$$\Omega(a_t, \epsilon) \in \arg\max_W (W - U(a_t))^\eta (\Pi(W, a_t, \epsilon))^{1-\eta} \tag{1.2.24}$$

$$\Phi(a_t, \epsilon) = \Pi(\Omega(a_t, \epsilon), a_t, \epsilon) \tag{1.2.25}$$

Equations (1.2.21) to (1.2.25) set the basis for an efficient algorithm of computing Markov equilibria in this context whereby the functional equation (1.2.21) along with the unemployed worker's program in (1.2.1) can be iterated to convergence. Equation (1.2.23) is the analogous object to the promise keeping constraint of the firm's program under commitment. It requires that at least a level of lifetime utility W_t be delivered to the worker although in this case continuation utility must be consistent with the equilibrium payoff $\Omega(a_{t+1}, \epsilon)$. [13] Further on the firm's profit is defined in (1.2.25) after the requirement that $W = \Omega(a_t, \epsilon)$ is imposed and the reason that the program is written this way (as opposed to working with a functional equation for $\Phi(a_t, \epsilon)$) is that this formulation makes clear how choices of wages and wealth need to be consistent with the Nash sharing rule in (1.2.24).

This type of contract does not appear to be new in the literature. In fact Krusell et al (2007) and Bils et al (2009(a) , 2009(b)) among others, build models with search frictions in the labour market and incomplete insurance and assume that rents are bargained for each period with a Nash protocol, but their approach is very different from mine; they approximate the Nash sharing rule with an invariant function $w(a)$ and solve the worker's optimal control program. Instead I treat Markov perfect solutions as part of a more general contracting problem and what this approach offers, is the possibility to incorporate additional features in the analysis, such as other insurance margins, a choice of effort etc and hence this formulation may prove useful in other contexts as well.

Optimal Policies. It can be shown that optimal choices of w_t and a_{t+1} satisfy the following first order conditions :

$$\lambda_t u'(c_t) = 1 \tag{1.2.26}$$

$$\lambda_t \beta (sU_{a_{t+1}} + (1-s)\Omega_{a_{t+1}}) - \frac{1}{r} + \frac{1-s}{R}(1 - \lambda_{t+1}\Omega_{a_{t+1}}) \leq 0 \tag{1.2.27}$$

[13] Notice that participation constraints need not be added to the program since solutions to Nash sharing rule will always induce positive capital gains for both parties (so long as they are defined).

$$\omega(\phi_{t+1}, \gamma_{t+1}, r, R, s) = (\frac{1}{r} - \frac{1}{R} + \frac{\phi_{t+1}}{1-s} - \frac{u'(c_u, t+1)}{1-s}\gamma_{t+1})\frac{1}{\frac{s}{R} - \frac{s}{1-s}\phi_{t+1}}$$

Notice that when $\gamma_{t+1} = \phi_{t+1} = 0$ and $r = R$ the term $\omega(\phi_{t+1}, \gamma_{t+1}, r, R, s)$ equals zero, and in this case perfect insurance obtains, as in a complete (full commitment) contract. But when the worker's participation constraint binds (in which case $\gamma_{t+1} < 0$) then the worker is underinsured (her consumption would fall if she becomes unemployed) and the converse holds when the firm must be made better off (hence when $\phi_{t+1} < 0$).

To understand how these results relate to the optimal profiles shown in this and the previous section notice that when $R = r$ allocations entail an initial loan from the firm to the worker and hence it seems that $\phi_{t+1} = 0$ always holds in this case. Then the only relevant constraint is on the worker's participation and the optimal contract could provide at most as much insurance against unemployment as the full commitment allocation. In this case the ratio of marginal utilities reduces to

$$\frac{u'(c_u, t+1)}{u'(c_e, t+1)} = 1 - \frac{u'(c_u, t+1)}{1-s}\gamma_{t+1}\frac{R}{s} \geq 1$$

On the other hand when $R > r$ both constraints could bind (although not simultaneously) and when the firm needs to be made better off, the optimal investment is higher than under full commitment (or the initial extraction of the agent's endowment is smaller). In this case the allocations provide more insurance against unemployment than what is implied by the full commitment model.

$$\frac{u'(c_u, t+1)}{u'(c_e, t+1)} = 1 + (\frac{1}{r} - \frac{1}{R} + \frac{\phi_{t+1}}{1-s})\frac{1}{\frac{s}{R} - \frac{s}{1-s}\phi_{t+1}} \leq 1 + (\frac{R}{r} - 1)\frac{1}{s}$$

1.2.4 Equilibria with Nash Bargaining

The maintained assumption in the versions of the model studied so far has been that both parties (the worker and the firm) have sufficient commitment to adhere to date zero optimal policies without ever renegotiating the optimal contract no matter if such renegotiations are from the perspective of one of the parties profitable. These commitments, were sustained by the threat of mutual reversion to autarky if ever the terms of trade were to be violated; for instance if the

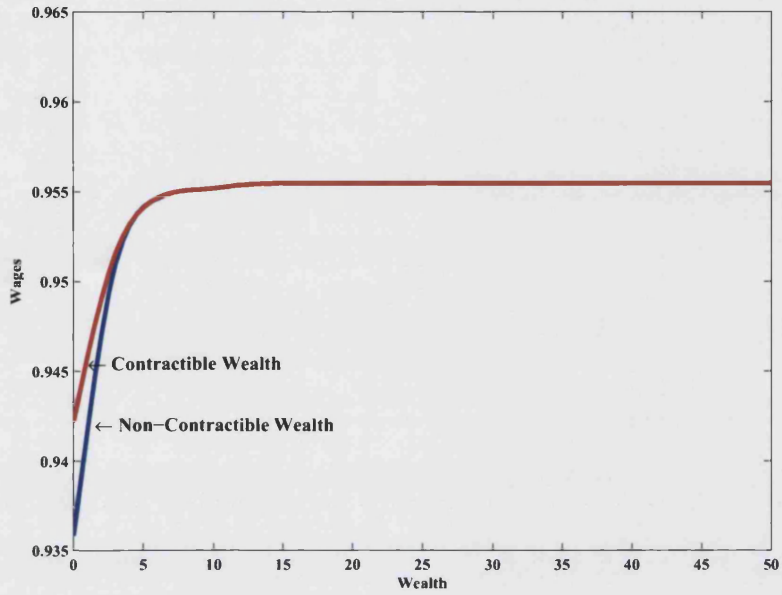


FIGURE 1.5: MARKOV PERFECT WAGE PROFILES: CONTRACTIBLE AND NONCONTRACTIBLE WEALTH

Further on consider the case $\Phi_{a_{t+1}} \leq 0$. From the Nash rule it is easy to show that $\Omega_{a_{t+1}} - U_{a_{t+1}} < 0$ (i.e. that the marginal increment from an extra unit of wealth is higher for an unemployed that for an employed worker). Then off corners rearranging equation (1.2.27) we reach the following expression:

$$\lambda_{t+1}\Omega_{a_{t+1}} = 1 + \frac{\lambda_t R}{r(1-s)}(\beta r s U_{a_{t+1}} + (1-s)r\Omega_{a_{t+1}} - u'(c_t)) < \lambda_{t+1}U_{a_{t+1}} \quad (1.2.29)$$

Equation (1.2.29) generalizes the underinsurance result for markov perfect contracts. It states that whenever $\Phi_{a_{t+1}} \leq 0$ the term in the parenthesis is positive and consumption falls as the agent becomes unemployed. [15] Whenever $\Phi_{a_{t+1}} > 0$ underinsurance is impossible to prove.

Finally from equation (1.2.27) it is possible to argue that differences in discounting with markov perfect contracts have a minimal impact on the shape of the optimal compensation scheme. To see this notice that even if $R = \frac{1}{\beta}$ and $s = 0$ equation (1.2.27) could be consistent with a positive level of a_{t+1} whereas

[15]This follows from the fact that: $\lambda_{t+1}U_{a_{t+1}} = \frac{U_{a_{t+1}}}{u'(c_{e,t+1})} > 1$.

firm was to reset the agreement at some future date the implicit assumption of the previous sections was that the worker would break the match to become unemployed. There was such an opportunity in the case $R = \frac{1}{\beta}$, where the firm optimally extracted part of the agent's wealth endowment in the initial period and made unemployment a strictly inferior state to be in.

To throw off any profitable deviation of this form optimal contracts must be signed every period or otherwise date zero policies must be time consistent and this section introduces this notion by focusing on Markov perfect equilibria where no history matters for the optimal policies, other than what is summarized in the endowment of wealth that the agent holds. I assume that at any point in time the optimal contract can be renegotiated and that the shares of the match surplus that accrue to the firm and the worker are determined as the solution to the familiar Nash sharing rule with respective weights $1-\eta$ and η.

Contrary to the commitment models of sections 1.2.2 and 1.2.3 this process induces a time invariant wage schedule and equilibrium payoffs of the form $\Omega(a_t, \epsilon)$ and $\Phi(a_t, \epsilon)$ for workers and firms respectively. Optimal continuation policies must conform with these objects in the sense that any expected utility path that is inconsistent with $\Omega(a_t, \epsilon)$ is not an equilibrium outcome. In turn for the firm the payoff function $\Omega(a_t, \epsilon)$ gives pairs of current wages and wealth accumulation that are consistent with Nash bargaining.

Value Functions. To see how the equilibrium in this economy can be computed consider an auxiliary program of a firm that must deliver a level of lifetime utility W to the worker (not necessarily on the equilibrium path), and let $\Pi(W, a_t, \epsilon)$ be the associated profit function borne out of the optimal policies. In the standard notation the firm's program can be represented recursively as:

$$\Pi(W_t, \epsilon, a_t) = \max_{w_t, a_{t+1} \geq \bar{a}} z\epsilon - w_t + \frac{1-s}{R} \Phi_{a_{t+1}, \epsilon} \tag{1.2.21}$$

Subject to [12]:

$$W_t \leq u(-a_{t+1}/r + a_t + w_t) + \beta s U(a_{t+1}) + \beta(1-s)\Omega(a_{t+1}, \epsilon) \tag{1.2.23}$$

[12]If savings decisions are non contractible, the firm's dynamic programming problem must be solved subject to the Euler equation as an additional constraint:

$$u'(c_t) \geq \beta r (sU_{a_{t+1}} + (1-s)\Omega_{a_{t+1}}) \tag{1.2.22}$$

Where:

$$\Omega(a_t, \epsilon) \in \arg\max_W (W - U(a_t))^\eta (\Pi(W, a_t, \epsilon))^{1-\eta} \tag{1.2.24}$$

$$\Phi(a_t, \epsilon) = \Pi(\Omega(a_t, \epsilon), a_t, \epsilon) \tag{1.2.25}$$

Equations (1.2.21) to (1.2.25) set the basis for an efficient algorithm of computing Markov equilibria in this context whereby the functional equation (1.2.21) along with the unemployed worker's program in (1.2.1) can be iterated to convergence. Equation (1.2.23) is the analogous object to the promise keeping constraint of the firm's program under commitment. It requires that at least a level of lifetime utility W_t be delivered to the worker although in this case continuation utility must be consistent with the equilibrium payoff $\Omega(a_{t+1}, \epsilon)$. [13] Further on the firm's profit is defined in (1.2.25) after the requirement that $W = \Omega(a_t, \epsilon)$ is imposed and the reason that the program is written this way (as opposed to working with a functional equation for $\Phi(a_t, \epsilon)$) is that this formulation makes clear how choices of wages and wealth need to be consistent with the Nash sharing rule in (1.2.24).

This type of contract does not appear to be new in the literature. In fact Krusell et al (2007) and Bils et al (2009(a) , 2009(b)) among others, build models with search frictions in the labour market and incomplete insurance and assume that rents are bargained for each period with a Nash protocol, but their approach is very different from mine; they approximate the Nash sharing rule with an invariant function $w(a)$ and solve the worker's optimal control program. Instead I treat Markov perfect solutions as part of a more general contracting problem and what this approach offers, is the possibility to incorporate additional features in the analysis, such as other insurance margins, a choice of effort etc and hence this formulation may prove useful in other contexts as well.

Optimal Policies. It can be shown that optimal choices of w_t and a_{t+1} satisfy the following first order conditions :

$$\lambda_t u'(c_t) = 1 \tag{1.2.26}$$

$$\lambda_t \beta(sU_{a_{t+1}} + (1-s)\Omega_{a_{t+1}}) - \frac{1}{r} + \frac{1-s}{R}(1 - \lambda_{t+1}\Omega_{a_{t+1}}) \leq 0 \tag{1.2.27}$$

[13] Notice that participation constraints need not be added to the program since solutions to Nash sharing rule will always induce positive capital gains for both parties (so long as they are defined).

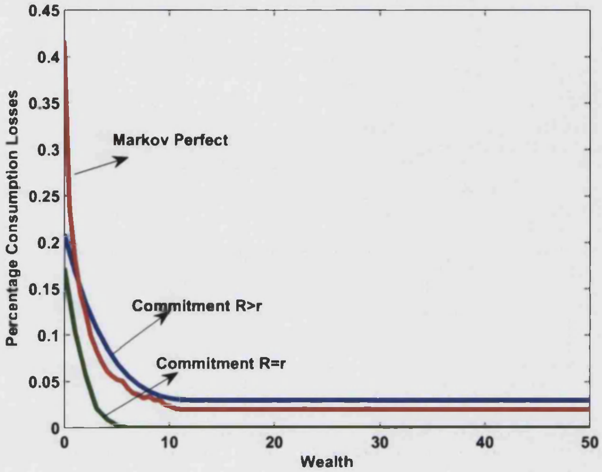


FIGURE 1.6: CONSUMPTION LOSSES IN UNEMPLOYMENT

1.3 General Equilibrium

The analysis of the previous section carried out a general description of optimal compensation schemes in various environments but they had nothing to say about observable equilibrium outcomes whereby distributions of wealth and wages in the economy are endogenously determined. Optimal wage patterns where shown to vary with the agent's endowment of wealth and can be shown to vary with the productivity of the match, and thus the distribution of job seekers across the relevant state space is important for drawing implications from the various schemes. Further more and perhaps more substantively, contact rates between firms and workers have been purposefully held fixed so far, but it is precisely these objects that make the framework amenable to provide answers to an array of economically interesting questions, be they related to optimal policy experiments or to changes in aggregate conditions.

To address these concerns Sections 1.3.1 and 1.3.2 attempt to close the model by making these contact rates a function of the willingness of firms to create jobs and of the workers to look for them by relying on two alternative equilibrium concepts: The first (in section 1.3.1) builds on the directed search model of Moen (1997) and Acemoglu and Shimer (1999) to develop a notion of equilibrium (with strict equality if $a_{t+1} > 0$).

$$\text{Envelope:} \quad \Phi_{a_t} = 1 - \lambda_t \Omega_{a_t}$$

These equations have a similar interpretation to the analogous objects under commitment. An increment is wealth in equation (1.2.27) has two distinct effects on the firm's profits: it lowers required wages to finance a given consumption stream, but also it increases the level of promised utility that the firm must deliver to the worker (according to the derivative $\Omega_{a_{t+1}}$). The latter effect would tend to dominate the closer the wealth is to the bound $\overline{a}$ since it is precisely there that an increment in assets encounters the higher marginal utility gains.

To see how that is important rearrange equation (1.2.27) making use of the envelope conditions to get the following Euler condition for the model with Markov perfect contracts:

$$u'(c_t) \geq \beta r(sU_{a_{t+1}} + (1-s)\Omega_{a_{t+1}}) + \frac{1-s}{R}\frac{r}{\lambda_t}\Phi_{a_{t+1}} \tag{1.2.28}$$

Equation (1.2.28) sets the marginal cost of saving an extra unit today ($u'(c_t)$ equal to the standard life cycle theory marginal benefit ($\beta r(sU_{a_{t+1}} + (1-s)\Omega_{a_{t+1}})$) and an extra term that pertains to the shape of the profit function. Should $\Phi_{a_{t+1}}$ be less than zero the marginal cost would be less than the marginal benefit and the agent would be savings constrained. The converse holds if $\Phi_{a_{t+1}} > 0$.

The difference between optimal (for the firm) and desired savings (for the worker) turns out to have a significant impact on the shape of the wage rule in the current context. Figure 1.5 traces the optimal (time invariant) wage rules for the cases of contractible and non-contractible wealth when the share of the firm in the Nash protocol is set at $\eta = 1/2$. Violations of the Euler equation occur when these policy rules diverge since the two programs are not equivalent in this case, and a higher wage signals that the entrepreneur can control the agents wealth since combinations of wages and assets must be consistent with the Nash rule in all cases. When assets are near the borrowing constraint an increment in wealth encounters the highest returns and it is precisely there that $\Phi_{a_{t+1}} < 0$ holds. Larger values of η make this distinction less and less relevant [14].

[14] In fact it is possible to show that the model with $\eta = 1$ (where the worker extracts all the surplus from the job) features a flat wage contract equal to $z\epsilon$ each period and that coupled with incomplete markets and the unemployment risks the worker will have the standard precautionary savings behavior. Notice that in that case $\Phi_{a_{t+1}} = 0$

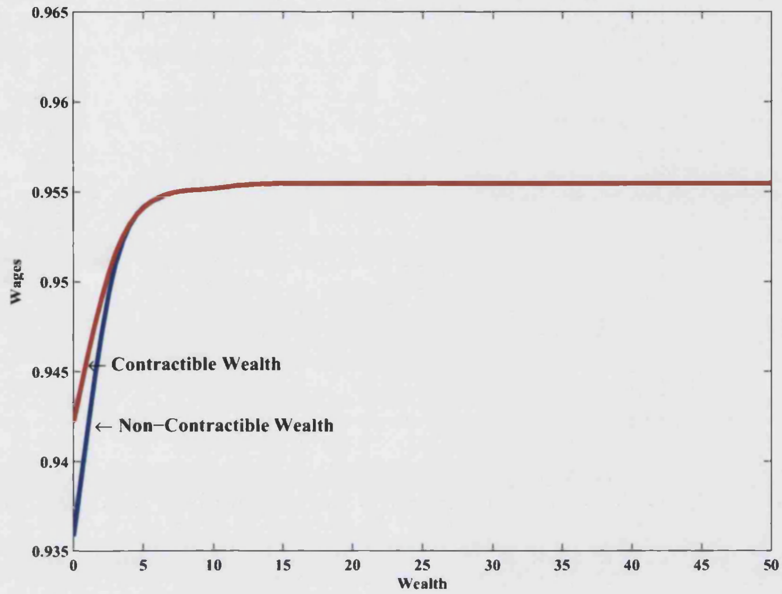


FIGURE 1.5: MARKOV PERFECT WAGE PROFILES: CONTRACTIBLE AND NONCONTRACTIBLE WEALTH

Further on consider the case $\Phi_{a_{t+1}} \leq 0$. From the Nash rule it is easy to show that $\Omega_{a_{t+1}} - U_{a_{t+1}} < 0$ (i.e. that the marginal increment from an extra unit of wealth is higher for an unemployed that for an employed worker). Then off corners rearranging equation (1.2.27) we reach the following expression:

$$\lambda_{t+1}\Omega_{a_{t+1}} = 1 + \frac{\lambda_t R}{r(1-s)}(\beta rsU_{a_{t+1}} + (1-s)r\Omega_{a_{t+1}} - u'(c_t)) < \lambda_{t+1}U_{a_{t+1}} \quad (1.2.29)$$

Equation (1.2.29) generalizes the underinsurance result for markov perfect contracts. It states that whenever $\Phi_{a_{t+1}} \leq 0$ the term in the parenthesis is positive and consumption falls as the agent becomes unemployed. [15] Whenever $\Phi_{a_{t+1}} > 0$ underinsurance is impossible to prove.

Finally from equation (1.2.27) it is possible to argue that differences in discounting with markov perfect contracts have a minimal impact on the shape of the optimal compensation scheme. To see this notice that even if $R = \frac{1}{\beta}$ and $s = 0$ equation (1.2.27) could be consistent with a positive level of a_{t+1} whereas

[15] This follows from the fact that: $\lambda_{t+1}U_{a_{t+1}} = \frac{U_{a_{t+1}}}{u'(c_{e,t+1})} > 1$.

where firms post contracts and workers channel their search to the most profitable direction. These contracts consist of a sequence of transfers from the firm to the worker (a wage tenure profile essentially) and the implied rules for asset accumulation. I establish that the equilibrium set of contracts can be summarized in a sufficient statistic, the present discounted value of profits that accrue to the entrepreneur, which I denote by J, and that they always place the firm worker pair on the Pareto frontier of utility. Along this J dimension markets are (possibly) segmented, and each segment attracts a group of workers that have the same wealth endowment. With this, the equilibrium set becomes manageable and computations can be extremely efficient. The second framework (section 1.3.2) is the standard search and matching model where a centralized undirected algorithm brings together vacant jobs and job seekers. Although this approach doesn't appear to be new in the literature [16], the main contribution here is to extend previous attempts by applying the notion that firms can commit to long term payment plans with their workforce.

Rather than pursuing just a simple description of the recursive equilibria in these economies I use the tools laid out to address a very relevant question. In section 1.3.3 I set up a simple framework for optimal policy. I ask how would a social planner choose the level of unemployment insurance b to maximize the economy's welfare when there are different contracting schemes at work. This experiment is motivated by the finding of section 1.2 that commitment contracts have considerably different implications for risk sharing than Markov Perfect contracts under certain conditions. The results suggest that optimal policy should account for the extent of these private insurance arrangements between workers and firms.

1.3.1 Directed Search With Commitment

Consider the following generalization of the environment studied in section 1.2.3 of this chapter. To simplify assume that all firms have the same constant level of productivity and that the population of searchers consists exclusively of unemployed agents. The discount rate of workers is denoted by β (assume that there is a unit mass of these agents) and the analogous object for entrepreneurs

[16] See Krusell et al (2007), Bils et al (2009(a),2009(b))

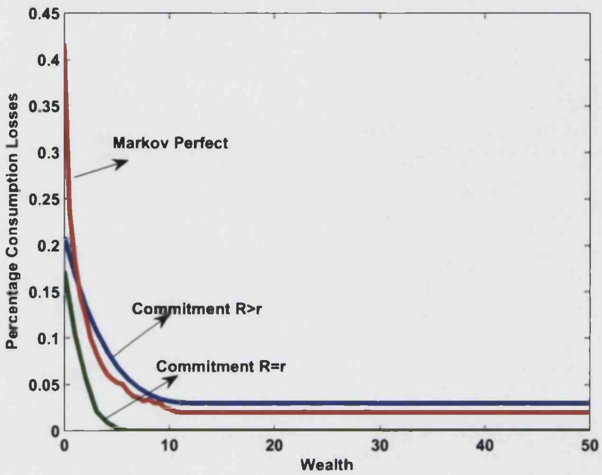
with commitment that was never the case). This is so because initial extractions of the agent's wealth holdings are not time consistent in the sense that after they take place, the level of promised utility cannot exceed the one implied by the bargaining problem (i.e. $\Omega(0,\epsilon)$).

An ordering of insurance across the various schemes

As a general matter it is really difficult to go far in characterizing the optimal allocations from the first order conditions for the models analyzed in the chapter. It is also hard to discern an ordering between the limited commitment and the markov perfect contracts in terms of the overall risk sharing that they entail. In the absence of a general proof this can be done numerically though.

Figure 1.6 presents such an ordering for the baseline calibration of the economy (see section 1.3.3 for details). It plots consumption losses suffered by an agent whose job is destroyed as a function of her endowment of assets and this corresponds to an imperfect measure of insurance provided by the two models. By far more losses accumulate at high levels of wealth, for the limited commitment contract with $R = \frac{1}{\beta}$ since the optimal allocation was shown to feature an initial extraction of the agent's endowment and under-investment carried over in the entire path. Markov perfect contracts on the other hand imply much lower risk sharing when wealth is low. In that region both commitment solutions featured some insurance. Finally from all these models the limited commitment contract with $R = r$ features the smallest consumption losses.

These results are more general for all the versions of the model in this and the next chapter and they suggest that when firms and workers can commit, and firms discount the future at the market interest rate r, near complete insurance against (short term) unemployment can obtain. A worker in this environment doesn't have to worry about her consumption dropping on impact during an unemployment spell and faces considerably less risks than a worker that must rely on precautionary savings (as in a Markov perfect context) to finance insurance. In chapter 2 I reinterpret the limited commitment model as an arrangement that summarizes other important forms of insurance that firms provide to their workforce, such as severance payments. I exploit measures of consumption losses for unemployed workers to argue that public policy (in particular unemployment benefits) has a smaller importance in this economy.



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Further on an unemployed worker's optimal strategy is a choice σ^* such that:

$$\sigma^* \in \arg\max_{\sigma \in \Sigma^*} f(\theta_\sigma) W_\sigma + (1 - f(\theta_\sigma))U' \tag{1.3.2}$$

where W_σ is the lifetime utility that the generic contract promises and U' is (an abbreviation of) the next period lifetime utility for the unemployed worker if she fails to find a job.

This is a standard definition of the directed search equilibrium that appears in many different contexts in related theoretical work. For instance Acemoglu and Shimer (1999) consider an economy where firms make ex ante investments in capital, and workers are risk averse and hold wealth, to show that the output maximizing level of unemployment insurance is greater than zero. Their theoretical results derive from a static version of this model, and in a dynamic context they only allow for flat wage contracts. Further on Rudanko (2008, 2009) uses the directed search equilibrium in an environment with commitment contracts, but no assets, to describe the business cycle implications for the aggregate labour market. My treatment here is much more general and shows how this model can be used for quantitative macro work.

Computation. The above (informal) definition describes an equilibrium that is computationally unmanageable (because the equilibrium set Σ^* is infinite dimensional) but there are ways to make it much more tractable by illustrating that the firm's expected profits J is a sufficient statistic for market clearing and optimization in this economy. The following three steps summarize this argument:

STEP 1. Notice that given risk neutrality of entrepreneurs, from the zero profit condition (1.3.1) it follows that for any two contracts $\sigma, \sigma' \in \Sigma^*$ such that $J_\sigma = J_{\sigma'}$ potential entrants are indifferent and equation (1.3.1) traces a locus of points that determines the tightness ratio as a function of the expected profits J that accrue to the firm.

STEP 2. All contracts offered in equilibrium will place workers and firms on the Pareto frontier of utility. To see this consider two contracts $\sigma_P, \sigma_{NP} \in \Sigma^*$ such that $J_{\sigma_P} = J_{\sigma_{NP}}$. Contract σ_P corresponds to a Pareto efficient allocation where by definition the value of the worker is maximized subject to the firm's where firms post contracts and workers channel their search to the most profitable direction. These contracts consist of a sequence of transfers from the firm to the worker (a wage tenure profile essentially) and the implied rules for asset accumulation. I establish that the equilibrium set of contracts can be summarized in a sufficient statistic, the present discounted value of profits that accrue to the entrepreneur, which I denote by J, and that they always place the firm worker pair on the Pareto frontier of utility. Along this J dimension markets are (possibly) segmented, and each segment attracts a group of workers that have the same wealth endowment. With this, the equilibrium set becomes manageable and computations can be extremely efficient. The second framework (section 1.3.2) is the standard search and matching model where a centralized undirected algorithm brings together vacant jobs and job seekers. Although this approach doesn't appear to be new in the literature [16], the main contribution here is to extend previous attempts by applying the notion that firms can commit to long term payment plans with their workforce.

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[16] See Krusell et al (2007), Bils et al (2009(a),2009(b))

by $\frac{1}{R}$. Financial markets are incomplete and workers can only borrow up to an ad hoc limit $\overline{a}$.

Firms and workers are brought together via search. At the beginning of each period entrepreneurs open up vacant positions, and each vacancy advertises a contract that consists of a sequence of payments from the firm to the worker. Let σ denote a generic contract and define Σ to be the set of feasible contracts in this economy. More precisely the latter is the collection of all contracts that satisfy participation by firms and workers in the current context.

Unemployed workers observe the set of contracts offered by entrepreneurs and channel their search to the most profitable direction. Each worker can only fill one application per period but depending on the decisions of the population of searchers there may be more competition for some jobs than others. Let θ_σ be the ratio of vacant jobs (offering contract σ) to unemployed applicants, which is an index of the extent of this competition. The 2-tuple $\{\sigma\ ,\ \theta_\sigma\}$ defines a submarket for this contract. I assume that the total number of matches that occur in this submarket is governed by a technology of the form $m(v_\sigma, u_\sigma)$. By the standard properties of the matching function (CRTS and concave), any firm in the sub-market for contract σ has a vacancy filling rate equal to $q(\theta_\sigma)$ and every unemployed worker meets a firm at a rate $f(\theta_\sigma)$.

What does an equilibrium look like in this economy? In a decentralized equilibrium with undirected search the set of contracts Σ^* is such that three conditions are met [17]: First firms choose the allocation σ to maximize their profits. Second unemployed agents choose the contract to which they apply given complete information about the structure of payoffs in the economy. Third there is free entry (perfect competition among potential entrants) in the market, that drives the expected profits off all entrepreneurs from the creation of vacancies to zero.

More formally let ξ be the per period cost associated with keeping a vacancy open and denote by V_σ and J_σ the value of a vacant and a filled job respectively for a generic contract in the equilibrium set. Then free entry requires:

$$V_\sigma = -\xi + \frac{1}{R}q(\theta_\sigma)(J_\sigma - V_\sigma) = 0 \rightarrow \xi = \frac{1}{R}q(\theta_\sigma)J_\sigma \qquad (1.3.1)$$

[17] See Moen (1997), Acemoglu and Shimer (1999) and Rudanko (2009) for further details.

Further on an unemployed worker's optimal strategy is a choice σ^* such that:

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STEP 2. All contracts offered in equilibrium will place workers and firms on the Pareto frontier of utility. To see this consider two contracts $\sigma_P, \sigma_{NP} \in \Sigma^*$ such that $J_{\sigma_P} = J_{\sigma_{NP}}$. Contract σ_P corresponds to a Pareto efficient allocation where by definition the value of the worker is maximized subject to the firm's profit being equal to J_{σ_P}. Contract σ_{NP} does not. Letting W_{σ_P} and $W_{\sigma_{NP}}$ denote the expected lifetime utilities of employed workers at date zero for these two contracts it should be clear that $W_{\sigma_P} \geq W_{\sigma_{NP}}$. If this condition holds with equality then allocation σ_{NP} is Pareto optimal. On the other hand if $W_{\sigma_P} > W_{\sigma_{NP}}$ and both contracts are part of the equilibrium set then by the zero profit condition (1.3.1) it must be that they encounter the same ratio of vacancies to unemployed workers. But firms that offer σ_{NP} can actually do much better, since by increasing the worker's utility (or offering an efficient allocation for that matter), they can convince more workers to join the pool of searchers for that particular contract. Thus equation (1.3.1) would fail to hold.

In turn efficient allocations are a solution to the following program:

$$W(a_t, J_t) = \max_{J_{t+1}, a_{t+1}, w_t} u(c_t) + \beta U(a_{t+1})s + \beta(1-s)W(a_{t+1}, J_{t+1}) \tag{1.3.3}$$

Subject to the constraint set:

$$a_{t+1} = r(a_t + w_t - c_t) \tag{1.3.4}$$

$$J_t = z - w_t + \frac{1-s}{R} J_{t+1} \tag{1.3.5}$$

$$J_{t+1} \geq 0 \tag{1.3.6}$$

$$W(a_{t+1}, J_{t+1}) \geq U(a_{t+1}) \tag{1.3.7}$$

STEP 3. For an unemployed agent given the iso-profit line for vacant jobs and the payoff $W(a_t, J_t)$ optimal policies are a solution to the following functional equation:

$$U(a_t) = \max_{a_{t+1} \geq \bar{a}, J} u(-\frac{a_{t+1}}{r} + a_t + b) + \beta(1 - f(\theta_J))U(a_{t+1}) + \beta f(\theta_J) W(a_{t+1}, J) \tag{1.3.8}$$

A few comments are in order here. First notice how these equations pose the Pareto program by having the worker choose prices as opposed to the firm choosing allocations as studied in section 1.2 of this chapter. This is a convenient representation for the current context and doesn't affect the optimal policies. The worker can rearrange payments between the present and future to maximize her utility while adhering to a promised value J_t to be delivered to the firm, and equations (1.3.6) and (1.3.7) require that participation for both firms and workers be satisfied on the optimal path.

Further on the solution to equation (1.3.8) is a time invariant policy rule J_a^* that defines a mapping from assets to the optimal choice of J. Under general conditions this map can be shown to be monotonically decreasing in wealth and in this case the equilibrium gives rise to an interesting form of market segmentation whereby wealthier workers choose a lower J (higher wages) because they can afford to wait in unemployment for better opportunities to come along. However, this property is not necessary for the equilibrium to be well defined in the current context.

Finally notice that since the equilibrium here is based on the assumption that firms advertise a wage tenure profile to attract job applicants it is important that there is sufficient commitment in the economy to adhere to the announced path. With Markov perfect contracts say, either the firm or the worker would have the incentive to re-bargain after all investments in search are made, and the only way whereby payoffs are well defined in this economy is for firms to advertise different values of the weight η.

The Appendix generalizes this analysis to a model that features uncertainty about match quality and describes a numerical procedure that makes computations in the economy a near trivial task. The next paragraph defines the competitive equilibrium in this economy.

Recursive Competitive Equilibrium

The stationary competitive equilibrium consists of a set of value functions $\{U_{(a)}, W_{(a,J)}, V\}$for workers and firms, a set of decision rules on asset holdings and continuation values $\{a'_{w,(a,J)}, a'_{u,(a)}, J'_{w,(a,J)}\}$ for employed and unemployed agents and a decision rule J_a^* defining optimal search policies of unemployed workers as a function of assets. It also consists of a set of prices $\{r, w_{(a,J)}\}$ and an invariant measure μ of agents across assets and employment states such that:

1) Free entry condition (2) $V = 0 \rightarrow \xi = \beta_e q_\theta J$ holds $\forall J$

2) Given R and $f(\theta_J)$ asset accumulation and optimal search rules are the solution to the agent's program.

3) Given optimal search rules J_a^* and the stationary distribution of workers across states, flows in and out of each segment are balanced. Define A to be a set on the asset grid with the property, $J^*_{a\in A} = \overline{J}$ for some $\overline{J}$. Also define the steady state measure of unemployed in segment $\overline{J}$: $u_{\overline{J}}$. The balance of flows requires:

$$s\sum_{J}\sum_{a:\, a'=a'_{w,(a,J)}\in A}\mu_{w,(a,J)} + \sum_{a:\, a'=a'_{u,(a)}\in A,\ a\notin A}\mu_{u,(a)}(1-f(\theta_{J^*_{(a)}}))$$

(inflow) be equal to:

$$u_{\overline{J}}f(\theta_{\overline{J}}) + (1-f(\theta_{\overline{J}}))\sum_{a:\, a'=a'_{u,(a)}\notin A, a\in A}\mu_{u,(a)}$$

(outflow).

4) Consistency: μ is the invariant distribution generated by optimal decision rules of households.

1.3.2 Undirected Search

This section uses a concept of equilibrium that is entirely different from the one defined in section 1.3.1. There the set of contracts offered was such that the equilibrium gave rise to an interesting form of market segmentation whereby workers with a different endowment of wealth made different choices of J and hence encountered job opportunities in the market at different rates each period. Here job creation is the outcome of a centralized undirected algorithm, where the locations of vacant jobs and the identities of job seekers are submitted each period, and a common unemployment to employment transition rate (which I denote by $f(\theta)$) applies to all workers in the economy.

I already said that this setup is not new in the related literature; Krusell et al (2007) and Bils et al (2009(a), 2009(b)) build similar models where job creation encounters matching frictions and workers can self insure against unemployment, and use these to address how changes in aggregate productivity affect unemployment and vacancies in the labour market. My contribution here, besides of the different focal point of my analysis, is to extend this previous work to a more general framework that distinguishes between alternative contracting schemes.

I again assume that there is a unit mass of risk averse agents in the economy and a continuum of firms (entrepreneurs) of irrelevant measure. The standard assumptions, parameters (discount rates and borrowing constraints) and technologies are maintained here. At any point in time a fraction e of the economy's workforce are employed workers matched with firms in joint production, and the remaining u workers are active job seekers in the labour market. There is also a number v of available job opportunities.

Firms and workers are brought together via search which is a time consuming activity. In particular the frictions that impede instantaneous transitions from unemployment to employment are summarized in a technology of the form $m_{v,u}$ that gives the total number of matches as a function of these inputs. I assume that the standard properties of the matching function (constant returns to scale, increasing in both arguments and concave) apply. By these properties we can denote the job finding rate for unemployed workers and the filling rate for vacant jobs by $f(\theta) = \frac{m(u,v)}{u}$ and $q(\theta) = \frac{m(u,v)}{v}$ respectively. In turn $\theta = \frac{v}{u}$ is an index of competition (tightness) in the labour market.

Finally to describe the equilibrium in this economy I consider an environment whereby match rents must be rebargained each period. The analysis can be easily extended to include limited commitment contracts and in the Appendix I provide a thorough description of the equilibrium in that case.

Value Functions In section 1.2.4 I established that in an economy with time consistent contracts, payoffs for firms and workers are of the form $\Phi(a_t, \epsilon)$ and $\Omega(a_t, \epsilon)$ respectively. These are a solution to the following functional equations:

$$\Pi(W_t, a_t, \epsilon) = \max_{a_{t+1} \geq \overline{a}, w_t} z\epsilon - w_t + \frac{1-s}{R}\Phi(a_{t+1}, \epsilon) \tag{1.3.9}$$

Subject to:

$$W_t \leq u(-a_{t+1}./R + a_t + w_t) + \beta(1-s)\Omega(a_{t+1}, \epsilon) + \beta s U(a_{t+1}) \tag{1.3.10}$$

$$\Omega(a_t, \epsilon) \in \arg\max_{W} (W - U(a_t))^{\eta}(\Pi(W, a_t, \epsilon))^{1-\eta} \quad \textbf{Nash Bargaining} \tag{1.3.11}$$

$$\Phi(a_t, \epsilon) = \Pi(\Omega(a_t, \epsilon), a_t, \epsilon) \quad \textbf{Consistency} \tag{1.3.12}$$

Equation 1.3.9 represents the firm's dynamic programming problem where expected utility W_t is a state variable along with wealth and productivity. In equilibrium it can only be that $W_t = \Omega(a_t, \epsilon)$ (since no promise can be sustained that is inconsistent with the Nash bargaining rule) and this is precisely the requirement imposed by condition (1.3.12).

Further on the lifetime utility for an unemployed worker with a stock of wealth a_t in the current period solves:

$$U(a_t) = \max_{a_{t+1} \geq \bar{a}} u(-fraca_{t+1}r + a_t + b) + \beta f(\theta) \int \Omega(a_{t+1}, \epsilon)\, dF_\epsilon + \beta(1 - f(\theta))U(a_{t+1}) \tag{1.3.13}$$

The solutions to the value function programs (1.3.9) and (1.3.13) give rise to a set of optimal policy rules $\{a'_{w,(a,\epsilon)}, a'_{u,(a)}\}$ for employed and unemployed workers respectively. In turn these policies induce a invariant measure μ of agents across assets, productivity and employment status which evolves according to the following law of motion:

$$\mu'_{A,u} = s \int_{a'_{w,(a,\epsilon)} \in A} \int d\, \mu_e(a, \epsilon) + (1 - f_\theta) \int_{a'_{u,(a)} \in A} d\, \mu_u(a) \tag{1.3.14}$$

$$\mu'_e(A, E) = (1 - s) \int_{\epsilon \in E} \int_{a'_{w,(a,\epsilon)} \in A} d\mu_e(a, \epsilon) \; + f_\theta \int_{\epsilon \in E} \int_{a'_u(a) \in A} d\mu_u(a)\, dF_\epsilon \tag{1.3.15}$$

For all $\quad A \subset \mathcal{A}, \quad E \subset \mathcal{E}$

Where are $\mu_e(a, \epsilon)$ and $\mu_u(a)$ are the marginal cdfs for employed and unemployed workers and $\mathcal{A}$ and $\mathcal{E}$ denote the relevant state space of wealth and productivity respectively.

Free Entry Condition. Just like in the equilibrium of section 1.3.1 job creation in this economy needs to be consistent with the notion that prospective entrants in the market make zero profits from vacant jobs. Letting ξ be the per period cost of posting a vacancy and given the steady state measure μ it must be that:

$$-\xi + \frac{1}{R} q_\theta \int \int_{a'_u(a)} \Phi(a, \epsilon) \frac{d\mu_u(a)}{\int d\mu_u(a)}\, dF_\epsilon = 0 \tag{1.3.16}$$

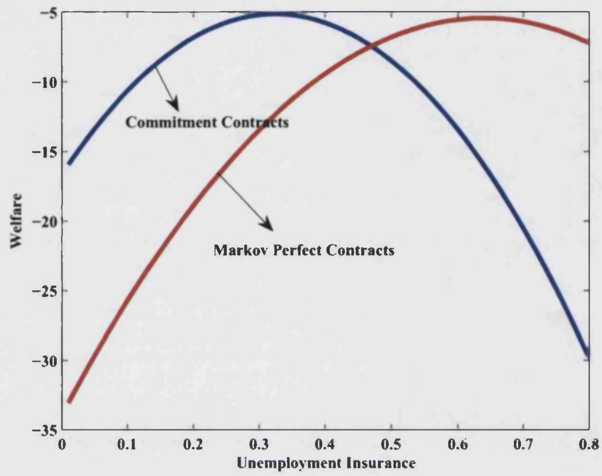

FIGURE 1.7: WELFARE CRITERION MARKOV PERFECT AND COMMITMENT CONTRACTS

Where do these choices come from? There is a simple interpretation of these results: First notice that in all economies higher values of b increase utility for unemployed workers, but also imply higher taxes (and hence lower output) on existing matches. This is so because given the number of unemployed workers an economy that satisfies budget balance requires that: $\tau = \frac{ub}{e}$ and hence an increase in b engineers a rise in required revenues for the government. Further on given the level of search costs in the economy, a loss in the match surplus associated with higher taxes, translates into lower desired job creation for firms, and with fewer active jobs the tax burden on existing matches increases. For all of the economies considered the response of the aggregate labour market to changes in the level of benefits is similar; I find that a rise in b from .4 to .5 increases aggregate unemployment from the baseline value to .0788 and .0791 in the undirected model with markov perfect and commitment contracts respectively.

But where these models seem to differ, is in the amount of risk sharing that private partnerships between workers and firms entail. I previously summarized this by looking at the consumption losses that the workers suffer when they become unemployed. In this case, in the baseline calibration this statistic is 21% for the economy with markov perfect contracts and only 8% with optimal

Recursive Competitive Equilibrium

The stationary competitive equilibrium consists of a set of value functions $\{U(a), \Omega(a,\epsilon), \Phi(a,\epsilon)\}$, and a set of decision rules on asset holdings $\{a'_{w,(a,\epsilon)}, a'_{u,(a)}\}$ for employed and unemployed agents and an invariant wage rule of the form $w_{a,\epsilon}$. It also consists of an index of labour market tightness θ and an invariant measure μ of agents across assets, productivity and employment status such that:

1) Equilibrium payoffs solve the functional equations (1.3.13) and (1.3.9) and optimal policies derive.

2)The invariant measure μ is consistent: In particular the law of motion of μ must be consistent with(1.3.14) and (1.3.15)

3) Equilibrium tightness θ is consistent with the free entry condition (1.3.16)

1.3.3 Numerical Analysis: An Optimal Policy Experiment

Section 1.2 of this chapter highlighted the important role of the timing of wages in providing insurance against unemployment risks by encouraging the accumulation of assets. In this context commitment contracts were shown to feature considerably more insurance than markov perfect contracts and in some cases (with appropriate discounting assumptions and when the enforcement constraints were slack) the worker's consumption was unaffected in the event of a job loss.

This section makes a point that prescriptions of optimal policy in quantitative macro models with heterogeneous agents and idiosyncratic risk cannot be accurate unless they account for the extent of private risk sharing arrangements between economic agents (here the choice of wage setting scheme between firms and workers). It envisages that a benevolent social planner would choose the level of unemployment income b optimally and use lump sum taxes levied on entrepreneurial profits to finance the expenditures subject to budget balance (BB) each period. The choice of the location of taxes is almost inconsequential in the current context in the sense that both commitment and markov perfect models feature a sharing rule (the former as an initial allocation rule) that implies that a reduction in one party's expected payoffs translate into a loss of utility for the other party.

My findings suggest that conclusions for optimal policy drawn from models with search and self insurance (see Alvarez and Veracierto (2001), Hansen and

Imhrohoroglu (1992) and Wang and Williamson (2002)) may have been misguided by the fact that the impact of wages on risk sharing opportunities has not been properly accounted for. Indeed a result that comes out of the analysis is that when commitment is abundant in the economy there is much less scope for publicly provided insurance against unemployment, since increases in the level of benefits can disturb private insurance arrangements. This is a result that I establish using both the concept of the undirected and the directed search equilibrium although in the latter case lack of commitment cannot be made consistent with the notion that workers direct their search to a market with well defined ex ante payoffs (unless each contract consists of a different bargaining share η). Instead I assume that workers and firms trade with flat wage contracts in one case and with optimal limited commitment contracts in another.

The Environment. To simplify matters assume that all firms have the same level of productivity and a discount factor $R = r$. These choices imply that insurance against unemployment can be perfect in some regions of the limited commitment contract (I established this numerically before) so the results that follow can be interpreted as an upper bound on the difference in optimal policies between the two schemes. Let τ denote the tax levi on the match output u be the fraction of unemployed agents in the economy (analogously $e = 1 - u$ is the fraction of employed workers).

I what follows I restrict attention to steady state outcomes and I assume that optimal policies derive from a welfare function whereby the social planner assigns an equal weight to all agents in the economy. Further on notice that, since both models used in this section feature zero profits for the population of unmatched entrepreneurs, there are three inputs in the welfare criterion: The average utility of unemployed and employed workers and the profits of existing jobs (matched entrepreneurs). Given the invariant measures of agents across the relevant state space the welfare criterion (in an economy with time consistent contracts say) is given by:

$$\int U(a)\, d\, \mu_u(a) + \int (\Omega(a) + \Phi(a))\, d\mu_e(a) \qquad (1.3.17)$$

Calibration. I briefly explain my choice of parameters and functional forms: Given that the model period is set to one month the target interest rate is

$r = 1.0041$ [18] and the rate of time preference β is set at .995. Following Shimer (2005 (a)) I set the separation rate s to 3 % per month and the contact rate between unemployed workers and firms to .4 in steady state. In this case in the undirected search equilibrium the unemployment rate equals $\frac{s}{s+p} = .0698$ and with a value $b = .4$ taxes burden the match output by an amount equal to .0279. I also calibrate the search cost ξ to be consistent with these targets. With directed search, matters are more complicated because there the equilibrium unemployment rate is partly determined by the workforce's optimal choices, and I need to adjust ξ to make the average job finding and unemployment rates as above [19].

The matching algorithm is standard and is guided by the relevant literature: $m_{v,u} = \chi v^{\eta} u^{1-\eta}$. The value of η is set to .5 and $\chi = .4$ is calibrated to make compatible the steady state job finding rate for unemployed workers equal to .4 with a tightness ratio $\theta = 1$. Aggregate productivity z is normalized to unity. Finally the utility function of the workers is of the form:

$$u(c_t) = \log(c_t)$$

Results: Directed and Undirected Search Models

Figure (1.7) plots the value of the Welfare criterion (for the undirected search equilibrium) as a function of the level of benefits b [20]. Optimal policies differ markedly between the two contracting schemes and with commitment the social planner is required to set $b = .3$ to maximize welfare while her preferred value is around .65 in the economy with Nash Bargaining.

A similar result emerges in the directed search equilibrium; I find that with optimal contracts the government sets $b = .28$ but without them (this economy has flat wages instead of markov perfect contracts) the optimal level of benefits equals .55. The differences between the two models stem from the fact that the directed search equilibrium features considerably less risks for unemployed agents since they can determine the probability of finding jobs by choosing the market to which they apply.

[18] This value yields and yearly analogue of 5 %.

[19] Notice that the steady state numbers differ depending on the contracting scheme. I calibrate all economies to be consistent with the targets by choosing different values for the parameters.

[20] These values are normalized to fit the same scale

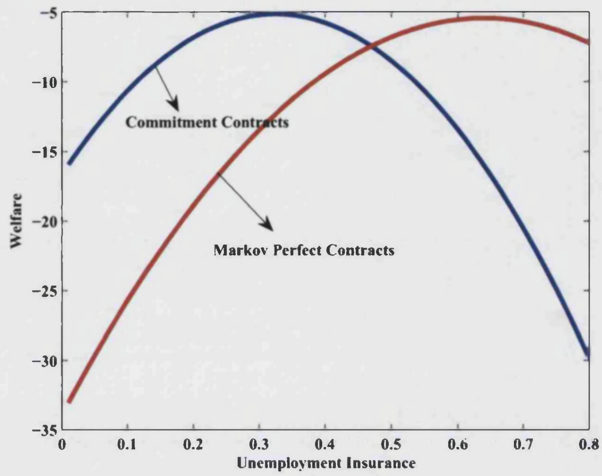

FIGURE 1.7: WELFARE CRITERION MARKOV PERFECT AND COMMITMENT CONTRACTS

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But where these models seem to differ, is in the amount of risk sharing that private partnerships between workers and firms entail. I previously summarized this by looking at the consumption losses that the workers suffer when they become unemployed. In this case, in the baseline calibration this statistic is 21% for the economy with markov perfect contracts and only 8% with optimal contribution of my work here is to present a flexible framework that can be used to deal with the two frictions jointly.

commitment contracts given the distribution of agents over the relevant state space (I obtain similar numbers for the directed search model). By this metric there is considerably more insurance in one case than in the other and hence a different role is assigned to public policy to complete the market.

These results mask considerable heterogeneity in outcomes. For one thing these economies may not be comparable due to differences in the steady state distributions and to the number of periods that the typical agent spends in unemployment. However important these features maybe, it is not my intention here to go far in analyzing the different tradeoffs with which the planner is confronted. Rather the point was to put the theories laid out in this chapter at work and to explore some of their aggregate implications. What these results suggest is that aggregate effects could be important.

In Chapter 2 of this thesis I scrutinize the implication that the range of private risk sharing sets the scope of public insurance by calibrating my artificial economy to be consistent with a large range of stylized facts for unemployment and the current UI scheme in the United States. As here I contrast the properties of two models; in one firms can allocate risks efficiently with their workforce and in another only flat wage contracts are permitted and I find that the predictions of this section survive the more detailed analysis that I attempt. Public policy is different in the two models. I then go on to evaluate the extent to which the government can devise more complicated insurance schemes to avoid or minimize the interference with private markets. In the context of UI payments more complicated mechanisms have an obvious interpretation; they are related to the timing of benefits and one of the findings is that unemployment insurance should be back-loaded in some cases (in the sense that transfers should increase over time).

1.4 Conclusions

Quantitative macro models of heterogeneous agents and wealth accumulation have had a tremendous impact in shaping and economic policy propositions over the past decade. Yet these models are agnostic about the sources of risk that give rise to a meaningful role of insurance markets. On the other hand search models of the labour market present a convincing foundation for the uncertainty that agents face over their working lives but build complicated environments only to colonize them with risk neutral agents whereby the role of insurance is meaningless.

This chapter sets out to build a theory that accounts for the interactions of these two frictions in asset and labour markets. It uses a standard search matching and bargaining framework where job availability in the economy is limited and agents can self insure against unemployment risks. It goes far in exploring modeling strategies for wages and characterizes the implications of alternative wage setting schemes. More substantively, by extending the existing literature, I develop two equilibrium concepts whereby the contact rates between workers and firms in these economies are endogenously determined.

This theory can help researchers setup models with realistic heterogeneity where search frictions play a central role in labour market outcomes, and my calculations illustrate that there is scope to reconsider many of the messages and implications of the literature of welfare effects of policies under limited insurance. I setup a simple experiment whereby the government chooses the level of unemployment insurance optimally along with taxes to finance the scheme and I find that depending on the range of insurance opportunities in worker employer relationships, the welfare benefits of public policy can be small or large. This important dimension seems to be missing from the literature of heterogeneous agents and labour market frictions (see Wang and Williamson (2002), Hansen and Imhrohoroglu (1992) and Alvarez and Veracierto (2001)) in spite of the fact that insurance provided by firms to their workforce is not uncommon (Pissarides (2004), Chetty (2007)).

A lot of other important implications are left unexplored. For one thing the work of Krusell et al (2007) and Bils et al (2009(a), 2009(b)) explains how models that feature both frictions in asset and labour markets, fare in matching the suggestive business cycle correlations in key labour market statistics (they both use an undirected search model). We yet don't know much about how heterogeneity in individual labour supply rules aggregates over the business cycle but this appears to be an important question for future work. For instance Chang and Kim (2006) use a model with heterogeneous agents and an extensive margin of labour supply (their economy doesn't feature search frictions) to show that the elasticity of labour supply is much larger at the aggregate than at the individual level, and Chang and Kim (2007) use a similar framework to point out that these ingredients explain the cyclical behavior of labour market wedges. One important

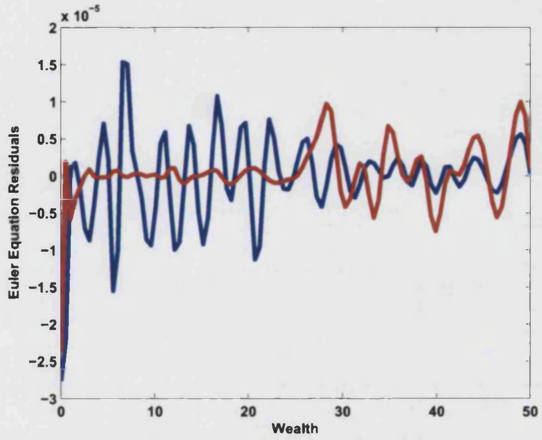


FIGURE 1.8: EULER RESIDUALS LIMITED COMMITMENT MODEL

value (around -.15) for low levels of wealth. This is shown in Figure 1.9 that plots the violations of the Euler equation along with the profit function. The partial derivative of the latter is negative (profits fall in wealth) suggesting that the term $\frac{1-s}{R}\frac{r}{\lambda_t}\Phi_{a_{t+1}}$ is negative. Clearly the agent is savings constrained in this case.

1.5.2 Algorithms for Computing Equilibria in the Undirected Search Model

An Equilibrium with Commitment and Undirected Search

Consider the model of section 1.2.3 where firms and workers have the ability to commit to long term date zero policies, insofar as they satisfy the sustainability condition, that the partnership is weakly preferable to autarky (unemployment). Central to the notion of equilibrium here is a job finding rate $f(\theta)$ for unemployed job seekers, an analogous rate $q(\theta)$ for vacant jobs, and the payoff functions $\mathcal{W}(a_t, \epsilon)$ (initial allocation for employed workers) and $U(a_t)$ (for unemployed agents). The Algorithm consists of the following steps:

STEP 1. Form initial guesses for tightness θ_0 and payoff functions $\mathcal{W}_0(a_t, \epsilon)$ and $U_0(a_t)$. Given these objects solve the firm's problem:

contribution of my work here is to present a flexible framework that can be used to deal with the two frictions jointly.

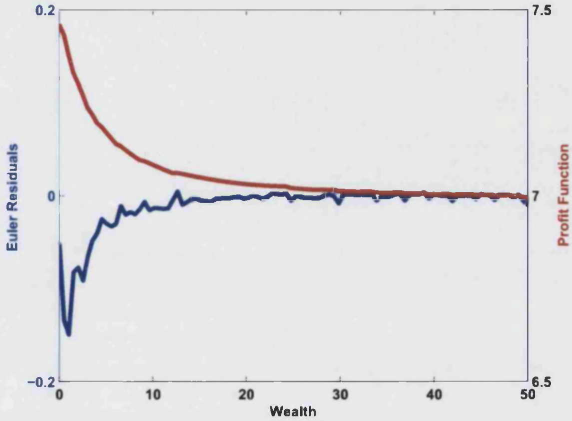


FIGURE 1.9: EULER RESIDUALS MARKOV PERFECT MODEL

$$\Pi(W_t, a_t, \epsilon) = \max_{W_{t+1}, a_{t+1}, w_t} \quad z\,\epsilon - w_t + \quad \frac{1-s}{R}\,\Pi(W_{t+1}, a_{t+1}, \epsilon) \tag{1.5.8}$$

Subject to the constraint set:

$$u(-a_{t+1}/r + a_t + w_t) + \beta(1-s)\,W_{t+1} + \beta s\,U(a_{t+1}) \geq W_t \tag{1.5.9}$$

$$a_{t+1} \geq \overline{a}$$

$$W_{t+1} \geq U(a_{t+1})$$

$$\Pi(W_{t+1}, a_{t+1}, \epsilon) \geq 0$$

Standard techniques (eg. value function iteration) can be applied here and optimal policies can be derived by iterating convergence on the functional equation (1.5.8). [21]

[21] A few comments are in order here: First to reduce the number of control variables we can get rid of w_t by making use of the fact that the promise keeping constraint will always bind, given a period utility that satisfies the Inada conditions. It follows from the worker's promise keeping constraint that:

$$w_t = a_{t+1}/r - a_t + u^{-1}(W_t - \beta(1-s)\,W_{t+1} - \beta s\,U(a_{t+1}))$$

With this addition we can write the firm's program as:

1.5 Appendix

1.5.1 Proofs and Derivations

Derivation of equation (1.2.19) in Text

Consider the first order conditions (1.2.16) and (1.2.17) that solve the firm's optimal program.

$$\frac{1-s}{R} - \frac{\lambda_t u(c_t)}{r} + \lambda_t \beta U_{a_{t+1}} s + \gamma_{t+1} U_{a_{t+1}} - \phi_{t+1} = 0 \tag{1.5.1}$$

$$\frac{1-s}{R} \Pi_{W_{t+1}} + \lambda_t \beta (1-s) - \gamma_{t+1} - \phi_{t+1} \Pi_{W_{t+1}} = 0 \tag{1.5.2}$$

Note that by the envelope conditions: $-\Pi_{W_{t+1}} = \lambda_{t+1} = \frac{1}{u'(c_{e,t+1})}$. Then clearly:

$$\phi_{t+1} = \frac{1-s}{R} - \frac{\lambda_t}{\lambda_{t+1}} \beta (1-s) - \frac{\gamma_{t+1}}{\lambda_{t+1}} \tag{1.5.3}$$

$$\frac{1}{r\lambda_t} = \beta s U_{a_{t+1}} + \beta (1-s) \frac{1}{\lambda_{t+1}} + \frac{\gamma_{t+1}}{\lambda_t} (U_{a_{t+1}} - \frac{1}{\lambda_{t+1}}) \tag{1.5.4}$$

Which is equation (1.2.19) in text.

Derivation of equation (1.2.20) in Text

For the first order conditions of the limited commitment program (equations (1.2.16) and (1.2.17)) it follows that off the borrowing constraint:

$$\begin{aligned} \lambda_t \beta (1-s) &= \gamma_{t+1} - \phi_{t+1} \lambda_{t+1} + \frac{1-s}{R} \lambda_{t+1} \rightarrow \\ \rightarrow \lambda_t \beta &= \frac{1}{1-s} (\gamma_{t+1} - \phi_{t+1} \lambda_{t+1}) + \frac{1}{R} \lambda_{t+1} \end{aligned} \tag{1.5.5}$$

$$\begin{aligned} s \lambda_t \beta U_{a_{t+1}} &= \frac{1}{r} - \frac{1}{R} + \frac{s}{R} + \phi_{t+1} - \gamma_{t+1} U_{a_{t+1}} \\ &= \frac{1}{r} - \frac{1}{R} + s(\frac{1}{R} - \phi_{t+1} \frac{1}{1-s}) + \phi_{t+1} \frac{1}{1-s} - \gamma_{t+1} U_{a_{t+1}} \end{aligned} \tag{1.5.6}$$

Substituting out $\lambda_t\beta$ in equation (1.5.6) and rearranging we get:

$$sU_{a_{t+1}}\lambda_{t+1}\left(\frac{1}{R}-\frac{\phi_{t+1}}{1-s}\right) = s\left(\frac{1}{R}-\frac{\phi_{t+1}}{1-s}\right)+\frac{1}{r}-\frac{1}{R}+ + \frac{\phi_{t+1}}{1-s}-\gamma_{t+1}U_{a_{t+1}}\frac{1}{1-s} \tag{1.5.7}$$

Which is equation (1.2.20) in text.

Discussion of the Importance of Contractible Assets

In the text it was shown that optimal investments in limited commitment and markov perfect contracts was governed by the two following Euler equations:

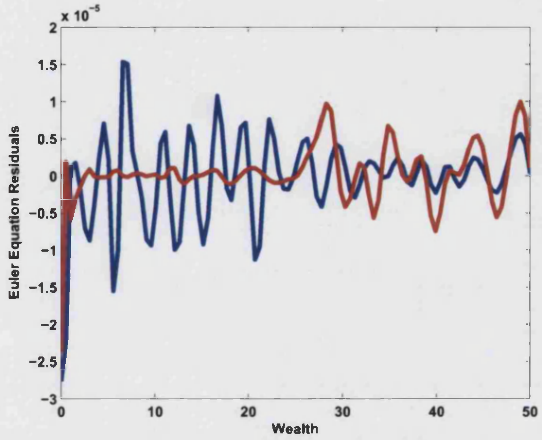
$$u'(c_t) \geq \beta(1-s)ru'(c_{t+1,e}) + r\beta su'(c_{t+1,u}) + ru'(c_t)\gamma_{t+1}(u'(c_{t+1,u}) - u'(c_{t+1,e}))$$

$$u'(c_t) \geq \beta r(sU_{a_{t+1}} + (1-s)\Omega_{a_{t+1}}) + \frac{1-s}{R}\frac{r}{\lambda_t}\Phi_{a_{t+1}}$$

The discussion emphasized that the last terms on the right hand side of these equations represent terms to the standard life cycle theory Euler equations that result from the fact that wages and investment are determined on the same side of the market (in other words assets are contractible by the firm). To the extent that these terms are important optimal contracts need to incorporate an additional state variable, the agent's marginal utility as in Abraham and Pavoni (2005) or Werning (2002) (or otherwise the standard life cycle theory Euler equation as an additional constraint to the firm's program).

Although it is not possible to provide a general proof the simulations of the model suggest that such violations are minimal in the context of a contract with limited commitment. This is visible from Figure 1.8 that plots the violations of the Euler equation (essentially the term $u'(c_t) - \beta(1-s)ru'(c_{t+1,e}) - r\beta su'(c_{t+1,u})$) in the baseline calibration of the model. These residuals are really small in absolute value to discern a pattern and in fact they seem consistent with numerical errors of acceptable order of magnitude.

In contrast a model with per period bargaining and contractible assets has a discernible pattern whereby the residuals are negative and large in absolute



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An Equilibrium with Commitment and Undirected Search

Consider the model of section 1.2.3 where firms and workers have the ability to commit to long term date zero policies, insofar as they satisfy the sustainability condition, that the partnership is weakly preferable to autarky (unemployment). Central to the notion of equilibrium here is a job finding rate $f(\theta)$ for unemployed job seekers, an analogous rate $q(\theta)$ for vacant jobs, and the payoff functions $\mathcal{W}(a_t,\epsilon)$ (initial allocation for employed workers) and $U(a_t)$ (for unemployed agents). The Algorithm consists of the following steps:

STEP 1. Form initial guesses for tightness θ_0 and payoff functions $\mathcal{W}_0(a_t,\epsilon)$ and $U_0(a_t)$. Given these objects solve the firm's problem:

IF the update θ_1 is close enough to the initial guess θ_0 **EXIT**. **ELSE** repeat STEP 1.

Markov Perfect Equilibria with undirected search

STEP 1. Form an initial guess for the index of tightness θ_0 and equilibrium payoff functions $U_0(a_t)$ and $\Omega_0(a_t, \epsilon)$ for unemployed and employed workers respectively. Choose a large grid of expected utility W to solve the firm's auxiliary program which can be represented recursively as follows:

$$\Pi(W, a_t, \epsilon) = \max_{w_t, a_{t+1} \geq \bar{a}} z\epsilon - w_t + \frac{1-s}{R}\Pi(\Omega_0(a_{t+1}), a_{t+1}, \epsilon) \qquad (1.5.15)$$

Subject to the constraint set:

$$W \leq u(-a_{t+1}/r + a_t + w_t) \quad + \quad \beta(1-s)\Omega(a_{t+1}, \epsilon) + \beta s U(a_{t+1})$$

$$u'(c_t) \geq \beta(1-s)\Omega_{a_{t+1}} + \beta s U_{a_{t+1}} \qquad \text{With Equality If } a_{t+1} \geq 0$$

Notice that continuation utility promises must conform with the function $\Omega_0(a_t, \epsilon)$ to be time consistent. Further on if assets are non-contractible (as in Krusell et al (2007), Bils et al (2009(a), 2009(b)), the Euler equation is an additional constraint to the firm's program [22] Consistent with the equilibrium is the notion that the payoff function of the firm satisfies the following condition:

$$\Phi(a_t, \epsilon) = \Pi(\Omega(a_t, \epsilon), a_t, \epsilon)$$

Standard Methods (say Value function iteration) say can be applied here to obtain numerical solutions to the optimal policies.

[22] The algorithm for non-contractible savings in these two papers doesn't use the firm's program as I do here. It approximates the function $w(a, \epsilon)$ (wages as a function of wealth) and solves for assets by maximizing the worker's utility who takes as given the wage schedule. The wages have to be consistent with Nash bargaining. For a detailed description see Bils et al (2009(a), 2009(b)).

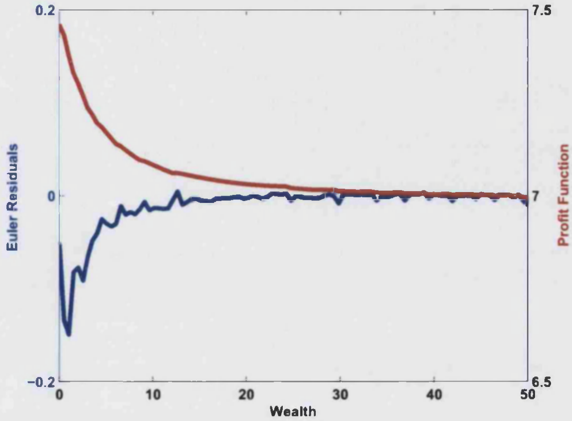

FIGURE 1.9: EULER RESIDUALS MARKOV PERFECT MODEL

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Subject to the constraint set:

$$u(-a_{t+1}/r + a_t + w_t) + \beta(1-s)\,W_{t+1} + \beta s\,U(a_{t+1}) \geq W_t \tag{1.5.9}$$

$$a_{t+1} \geq \overline{a}$$

$$W_{t+1} \geq U(a_{t+1})$$

$$\Pi(W_{t+1}, a_{t+1}, \epsilon) \geq 0$$

Standard techniques (eg. value function iteration) can be applied here and optimal policies can be derived by iterating convergence on the functional equation (1.5.8). [21]

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$$w_t = a_{t+1}/r - a_t + u^{-1}(W_t - \beta(1-s)\,W_{t+1} - \beta s\,U(a_{t+1}))$$

With this addition we can write the firm's program as:

STEP 2. Given the optimal value function, update the employed worker's payoff $\Omega(a_t, \epsilon)$ by solving the following Nash Bargaining Program:

$$\Omega_1(a_t, \epsilon) \in \arg\max_W (W - U(a_t))^{1-\eta}(\Pi(W, a_t, \epsilon))^\eta \tag{1.5.16}$$

IF function $\Omega_1(a_t, \epsilon)$ is close to the initial guess $\Omega_0(a_t, \epsilon)$ proceed to STEP 3 . ELSE repeat STEP 1.

STEP 3. Update the unemployed worker's value function. This can be accomplished by iterating convergence on the following functional equation:

$$U(a_t) = \max_{a_{t+1} \geq \bar{a}} u(-a_{t+1}/r + a_t + b) + \beta\,(1 - f_\theta)\,U(a_{t+1}) + \beta\, f_\theta \int \Omega(a_{t+1}, \epsilon)\; d\, F_\epsilon$$

IF the update is close enough to the initial guess proceed to STEP 4 . **ELSE** repeat STEP 1.

STEP 4. Update the value of θ. Given the steady state measures $\mu_{e,a}$ and $\mu_{u,a}$ for employed and unemployed job seekers over the relevant state space (wealth) the zero profit condition for vacant jobs requires:

$$-\xi \;+\; \frac{1}{R} q_\theta (\int \int_{a \in \mathcal{A}} \Pi(\Omega(a, v), a) d\, \mu_{u,a}\; d\, F_v = 0$$

1.5.3 Equilibria With Directed Search

Numerical Algorithm

STEP 1. Choose a grid of profits J for firms and assets a. Given the parametrization of search costs and the matching technology equilibrium tightness solves the zero profit condition (1.3.1). Form an initial guess for the unemployed worker's value function $U_0(a_t)$ and solve the employed worker's program by iterating convergence on the following functional equation:

STEP 2. Update objects $\mathcal{W}_0(a_t, \epsilon)$ and $U_0(a_t)$. In particular solve the following Generalized Nash Bargaining problem:

$$\mathcal{W}_1(a_t, \epsilon) \in \arg\max_W (W - U(a))^{1-\eta}\Pi(W, a, \epsilon)^\eta \tag{1.5.13}$$

The update for $U_1(a_t)$ can be obtained by solving the unemployed worker's value function, equation (1.2.1) in text.

IF the updates $U_1(a_t)$ and $\mathcal{W}_1(a_t, \epsilon)$ are close enough to $U_0(a_t)$ $\mathcal{W}_0(a_t, \epsilon)$ respectively proceed to STEP 3. **ELSE** repeat STEP 1.

STEP 3. Update the tightness index θ. Given a stationary measure of unemployed workers over assets $\mu_{u,a}$, the free entry condition for vacant jobs requires:

$$-\xi + \frac{1}{R} q_{\theta_1} \int \int_{a \in \mathcal{A}} \Pi(\mathcal{W}(a, \epsilon), a, \epsilon) d\, \mu_{u,a} \;\; d\, F_\epsilon = 0 \tag{1.5.14}$$

$$\begin{aligned} \Pi(W_t, a_t, \epsilon) &= \max_{W_{t+1}, a_{t+1}} \;\; z\,\epsilon - a_{t+1}/r + a_t - u^{-1}(W_t - \beta(1-s)\, W_{t+1} - \beta s\, U(a_{t+1})) \\ &+ \;\; \frac{1-s}{R}\, \Pi(W_{t+1}, a_{t+1}, \epsilon) \end{aligned} \tag{1.5.10}$$

Further on note that from (1.5.10) it follows readily that the firm's profit function is linear homogeneous in a_t. By this property, $\Pi(W_t, a_t, \epsilon) = \Pi(W_t, 0, \epsilon) + a_t$, we can drop current wealth as a state variable and rewrite equation (1.5.10) as:

$$\begin{aligned} \Pi(W_t, 0, \epsilon) &= \max_{W_{t+1}, a_{t+1}} \;\; z\,\epsilon - a_{t+1}/r - u^{-1}(W_t - \beta(1-s)\, W_{t+1} - \beta s\, U(a_{t+1})) \\ &+ \;\; \frac{1-s}{R}\, (a_{t+1} + \Pi(W_{t+1}, 0, \epsilon)) \end{aligned} \tag{1.5.11}$$

This problem is considerably easier to deal with but still it contains two control variables and dimensionality may be an issue even in this case. However by constraining pairs of W_{t+1}, a_{t+1} to those that satisfy the relevant Euler equations we can go a long way towards alleviating this burden. For instance in section 1.2.3 we saw that optimal insurance against unemployment (the choice of assets to be more precise) obeys the following first order condition:

$$\begin{aligned} u'(c_{u,t+1}) &= u'(c_{e,t+1})(1 + \frac{1}{s}(\frac{R}{r} - 1)) = \frac{u'(c_{e,t})}{\beta R}(1 + \frac{1}{s}(\frac{R}{r} - 1)) \\ &= \frac{u'(u^{-1}(W_t - \beta(1-s)\, W_{t+1} - \beta s\, U(a_{t+1})))}{\beta R}(1 + \frac{1}{s}(\frac{R}{r} - 1)) \end{aligned} \tag{1.5.12}$$

In turn since $u'(c_{u,t+1})$ depends on wealth and is independent of the firm's optimal policies, equation (1.5.13) can be used to determine off corners solutions to the value function under limited commitment. When the enforcement constraints bind we know that either $W_{t+1} = U(a_{t+1})$ or $\Pi(W_{t+1}, a_{t+1}, \epsilon) = 0$ and in this case it is again much simpler to characterize optimal policies.

IF the update θ_1 is close enough to the initial guess θ_0 **EXIT**. **ELSE** repeat STEP 1.

Markov Perfect Equilibria with undirected search

STEP 1. Form an initial guess for the index of tightness θ_0 and equilibrium payoff functions $U_0(a_t)$ and $\Omega_0(a_t, \epsilon)$ for unemployed and employed workers respectively. Choose a large grid of expected utility W to solve the firm's auxiliary program which can be represented recursively as follows:

$$\Pi(W, a_t, \epsilon) = \max_{w_t, a_{t+1} \geq \overline{a}} z\epsilon - w_t + \frac{1-s}{R}\Pi(\Omega_0(a_{t+1}), a_{t+1}, \epsilon) \tag{1.5.15}$$

Subject to the constraint set:

$$W \leq u(-a_{t+1}/r + a_t + w_t) \quad + \quad \beta(1-s)\Omega(a_{t+1}, \epsilon) + \beta s U(a_{t+1})$$

$$u'(c_t) \geq \beta(1-s)\Omega_{a_{t+1}} + \beta s U_{a_{t+1}} \quad \text{With Equality If } a_{t+1} \geq 0$$

Notice that continuation utility promises must conform with the function $\Omega_0(a_t, \epsilon)$ to be time consistent. Further on if assets are non-contractible (as in Krusell et al (2007), Bils et al (2009(a), 2009(b)), the Euler equation is an additional constraint to the firm's program [22] Consistent with the equilibrium is the notion that the payoff function of the firm satisfies the following condition:

$$\Phi(a_t, \epsilon) = \Pi(\Omega(a_t, \epsilon), a_t, \epsilon)$$

Standard Methods (say Value function iteration) say can be applied here to obtain numerical solutions to the optimal policies.

[22] The algorithm for non-contractible savings in these two papers doesn't use the firm's program as I do here. It approximates the function $w(a, \epsilon)$ (wages as a function of wealth) and solves for assets by maximizing the worker's utility who takes as given the wage schedule. The wages have to be consistent with Nash bargaining. For a detailed description see Bils et al (2009(a), 2009(b)).

STEP 2. Given the optimal value function, update the employed worker's payoff $\Omega(a_t, \epsilon)$ by solving the following Nash Bargaining Program:

$$\Omega_1(a_t, \epsilon) \in \arg\max_{W}(W - U(a_t))^{1-\eta}(\Pi(W, a_t, \epsilon))^{\eta} \tag{1.5.16}$$

IF function $\Omega_1(a_t, \epsilon)$ is close to the initial guess $\Omega_0(a_t, \epsilon)$ proceed to STEP 3 . ELSE repeat STEP 1.

STEP 3. Update the unemployed worker's value function. This can be accomplished by iterating convergence on the following functional equation:

$$U(a_t) = \max_{a_{t+1} \geq \bar{a}} u(-a_{t+1}/r + a_t + b) + \beta\,(1 - f_\theta)\,U(a_{t+1}) + \beta\, f_\theta \int \Omega(a_{t+1}, \epsilon)\, d\, F_\epsilon$$

IF the update is close enough to the initial guess proceed to STEP 4 . **ELSE** repeat STEP 1.

STEP 4. Update the value of θ. Given the steady state measures $\mu_{e,a}$ and $\mu_{u,a}$ for employed and unemployed job seekers over the relevant state space (wealth) the zero profit condition for vacant jobs requires:

$$-\xi \;+\; \frac{1}{R} q_\theta (\int \int_{a \in \mathcal{A}} \Pi(\Omega(a, v), a) d\, \mu_{u,a}\, d\, F_v = 0$$

1.5.3 Equilibria With Directed Search

Numerical Algorithm

STEP 1. Choose a grid of profits J for firms and assets a. Given the parametrization of search costs and the matching technology equilibrium tightness solves the zero profit condition (1.3.1). Form an initial guess for the unemployed worker's value function $U_0(a_t)$ and solve the employed worker's program by iterating convergence on the following functional equation:

$$\begin{aligned} W(a_t, J_t) &= \max_{a_{t+1}\geq \overline{a}, J_{t+1}} u(-\frac{a_{t+1}}{r} + a_t + z - J_t + \frac{1-s}{r} J_{t+1}) \quad (1.5.17) \\ &+ \beta s U(a_{t+1}) + \beta(1-s) W(a_{t+1}, J_{t+1}) \end{aligned}$$

Subject to:

$$J_{t+1} \geq 0 \qquad W(a_{t+1}, J_{t+1}) \geq U(a_{t+1})$$

Notice that this formulation makes use of the fact that the promise keeping and resource constraints are satisfied with equality. In turn equation (1.5.18) can be solved using standard techniques (i.e value function iteration).

STEP 2. Update the value function of the unemployed worker by iterating convergence on the following functional equation:

$$U(a_t) = \max_{a_{t+1}\geq \overline{a}, J} u(-\frac{a_{t+1}}{r} + a_t + b) + \beta(1 - f(\theta_J))U(a_{t+1}) + \beta f(\theta_J) W(a_{t+1}, J) \quad (1.5.18)$$

IF the update $U_1(a_t)$ is close enough to the initial guess $U_0(a_t)$ **EXIT**. **ELSE** repeat STEP 1.

Notice that once objects $U(a_t)$ $W(a_t, J_t)$ converge the equilibrium has been computed in the sense that tightness ratios are determined from equation (1.3.1) and there is no need to iterate on labour market conditions as in the undirected search equilibrium model. On the other hand because optimal allocations are computed by maximizing the worker's utility (as opposed to the firm's) it is much harder to get rid off state variables and make computations more efficient. In the next paragraph I outline a way around dimensionality problems that makes the value of the firm's profits J a permanent state that is relevant on job creation. The idea is to combine the worker's wealth endowment and the promised utility to the firm into a composite state variable which I define as cash in hand. Then the agent's program consists of choosing a level of assets and a level of dept obligations to the firm and I find that this formulation increases the efficiency of computations.

A modified Program For Directed Search Equilibria

Consider the resource constraint of an employed with a current stock of wealth a_t and a promised present discounted value of profits to the firm equal to J:

$$a_{t+1}/r = a_t - J + z + \frac{(1-s)}{R} J_{t+1} - c_t \tag{1.5.19}$$

This section shows that it is possible to make J a permanent state variable in the worker's optimal control program which facilitates computing numerical solutions in the directed search model.

Define $\overline{w}_J = z - J(1 - \frac{1-s}{R})$ to be the constant per period wage that delivers the prescribed value J to the firm (over the infinite horizon) and note that it follows from (1.5.19) and the definition of $\overline{w}_J$ that:

$$a_{t+1}/r = a_t - J + \overline{w}_J + \frac{1-s}{R}(J_{t+1} - J) - c_t \tag{1.5.20}$$

Define $d_{f,t} = -\frac{1-s}{R}(J_{t+1} - J)$ to be the agent's debt to the firm (issued in the current period). Then clearly if $J_{t+1} > J$ the agent front-loads part of the compensation and next period enters a state with a lower present discounted value of wages. Rearranging we can obtain the budget constraint of the agent for this period as:

$$a_{t+1}/r + c_t + d_{f,t} = a_t + \overline{w}_J \equiv x_t \tag{1.5.21}$$

In the next period conditional on the survival of the job and given her optimal choices, the agent will be in state $\{J_{t+1}, a_{t+1}\}$, and in the analogy of equation (1.5.19) forwarded one period, total available resources to finance consumption and asset accumulation are: $a_{t+1} + z - J_{t+1} + \frac{1-s}{R} J_{t+2}$ Rearranging we can write:

$$\begin{aligned} &a_{t+1} + z - J_{t+1} + \frac{1-s}{R} J_{t+2} = a_{t+1} + \overline{w}_J - (J_{t+1} - J) + \frac{1-s}{R}(J_{t+2} - J) = \\ &a_{t+1} + \overline{w}_J + \frac{d_{f,t} R}{1-s} + \frac{1-s}{r}(J_{t+2} - J) \end{aligned}$$

The last two terms in equation (1.5.22) represent interest payments on firm debt outstanding and new debt issued in period 2. Applying the same reasoning we can derive the constraint set facing the agent having spend an arbitrary amount of time on the job as:

$$a_{t+1}/R + c_t + d_{f,t} = x_t \qquad x_{t+1} = a_{t+1} + \overline{w}_J + \frac{R d_{f,t}}{(1-s)} = a_{t+1} + \overline{w}_J + R_f d_{f,t}$$

Value Functions Given the above derivations the employed worker's program can be cast in the following form:

$$W(x_t, J) = \max_{a_{t+1}, d_{f,t}} \; u(x_t - a_{t+1}/r - d_{f,t}) + \beta(1-s)W(x_{t+1}, J) + \beta s U(a_{t+1}) \tag{1.5.22}$$

Subject to the constraint set:

$$x_{t+1} = \overline{w}_J + a_{t+1} + d_{f,t}R_f \tag{1.5.23}$$

$$W(x_{t+1}, J) \geq U(a_{t+1}) \qquad -\frac{R}{1-s}d_{f,t} + J \geq 0 \tag{1.5.24}$$

Similarly the unemployed worker's program can be written as:

$$U(a_t) = \max_{a_{t+1} \geq \overline{a}, J} \; u(-\frac{a_{t+1}}{r} + a_t + b) + \beta(1 - f(\theta_J))W(a_{t+1} + \overline{w}_J, J) + \beta f(\theta_J)U(a_{t+1}) \tag{1.5.25}$$

To conclude this section it is important to make the following remark: by writing the program as in equation (1.5.22) it is possible that the agent will embark upon a path of increasing wealth and indebtness to the firm that would violate the transversality condition. In the simulations carried out for this chapter this was never the case; It is possible to verify ex post that no such violations occur, although this may be unnecessary in the presence of participation constraints.

The Treatment of Uncertainty

Consider an extension of the directed search equilibrium of section 1.3.1 that allows for different productivities ϵ at the firm level. Assume that productivity becomes known after the worker and the entrepreneur meet and that it remains constant throughout the life of the match. The representation of the optimal contracting problem for a matched worker is analogous to equation (1.3.3), the only difference being that that the lifetime utility is of the form $W(J_t, a_t, \epsilon)$ (i.e. ϵ is added to the list of state variables in the value function).

Unemployed workers also face a similar program in this case. Given that firms post contracts summarized in an expected present discount value of profits J and the equilibrium tightness ratio θ_J will determine the extent of competition in this market, then optimal choices of a market segments are made to maximize gains from search. Off course since productivity is not known a priori but rather revealed when a match is formed and the investments in search are sunk, an initial allocation $\mathcal{J}(a, \epsilon, J)$ (i.e. a map from initial wealth, productivity and J to expected utility) needs to be defined. For that assume that date zero allocations solve the following program:

$$\max_{\mathcal{J}(a_0,\epsilon,J)} EW(a_0, \mathcal{J}(a_0, \epsilon, J), \epsilon)$$

Subject to:

$$\int \mathcal{J}(a_0, \epsilon, J)\; d\, F_\epsilon = J$$

$$W(a_0, \mathcal{J}(a_0, \epsilon, J), \epsilon) \geq U(a_0) \;\; \nabla\epsilon$$

$$\mathcal{J}(a_0, \epsilon, J) \geq 0$$

In words initial allocations in the current context must adhere to the same principle of optimality whereby the workers utility is maximized subject to the firm making in expectation J profits and participation being satisfied for both parties. Given the above initial period optimization problem the equilibrium is similar to the one described in section 1.3.1 of the text.

2 Optimal Unemployment Insurance In The Presence of Private Insurance

2.1 Introduction

The optimal provision of unemployment insurance by governments has generated a voluminous literature since the work of Shavell and Weiss (1978) highlighting a central theme: In environments that are fraught with moral hazard allocations must be distorted away from optimal risk sharing to provide private incentives. Further on in these economies the ability of agents to insure privately against fluctuations in labour income is assigned an important role for two reasons; first because the range of these insurance opportunities determines the overall welfare gains from the UI scheme, and second because, in some cases, private insurance makes the implementation of public policy harder by taking away from the planner her control over allocations.

These ideas are well understood and yet in most of the relevant theoretical work the risk sharing role of private markets is largely understated. For instance there are numerous attempts in the literature that build on the baseline incomplete market model of heterogenous agents, where private insurance is simply wealth accumulation (as in Hansen and Imhrohoroglu (1992) and Wang and Williamson (2002)), and these models are often used to quantify the welfare gains from one UI scheme or the other. The problem is that in reality private risk sharing goes far beyond that; there is a wealth of instruments in partnerships between workers and firms such as severance payments and dismissal delays that can be readily used, along with assets, to buffer the risks of unemployment and minimize it's impact on consumption.

To the best of my knowledge no previous effort was made to put these realistic margins of insurance in a dynamic model, and understand their important interactions, and this is precisely the gap I attempt to bridge with this work. I do so by comparing the outcomes of two economies. In the first one agents can do no better than to trade non-contingent claims in financial markets subject to borrowing constraints and to rely on benefits provided by the government to insure against unemployment; In the second firms can provide additional insurance by signing contracts with their workforce subject to limited commitment. In both economies financial markets are incomplete but the range of insurance opportunities is not the same; I argue that in the second environment contracts can in some cases provide complete insurance in a way that other formal instruments such as severance payments or dismissal delays become redundant (in fact I establish this equivalence).

In line with the literature of heterogenous agents and wealth accumulation I focus on a restricted class of policies. In one experiment I keep constant the duration of benefits and allow the planner to choose the replacement ratio. In another I let the government choose the level of benefits for two classes of unemployed agents; those with duration less than two quarters and those with longer durations. In both cases employed workers are burdened with a constant tax levi.

I find that the optimal provision of unemployment benefits varies with the range of private markets and in particular when firms can remove part of the overall uncertainty, the optimal level of benefits is much lower than when no private risk sharing (other than simple wealth accumulation) exists. Put differently the provision of benefits by governments can crowd out private insurance by the same token that in the standard model with incomplete markets, UI payments crowd out the precautionary role of assets. Even more important is the result I get for the timing of payments; With firm insurance optimal payments are back-loaded in the sense that the government sets benefits equal to zero for the first two periods of an unemployment spell and positive afterwards, but without firm insurance, UI payments have the typical downward sloping path.

None of these implications seems to be qualitatively surprising. For one thing the possibility that public insurance can crowd out other arrangements in the economy has already been addressed in different contexts, and insofar as the timing of payments is concerned, the fact that workers can save means that benefits don't have to be decreasing in the duration of an unemployment spell for consumption to be decreasing (Werning (2002), Shimer and Werning (2005)). Then the important contribution of this work is first that it introduces a tractable framework that can be used to quantify the role of unemployment insurance provided by governments in the two economies, and second to establish quantitatively that optimal policies are indeed different. I argue that the latter is not a trivial task since in models where the distribution of agents across the state space is endogenously determined, steady state comparisons confound genuine differences in the scope of public policy with differences in the distributions and the analysis contains an exhaustive account of these issues.

Related Literature.

There is a large literature that investigates the role of labour market policies such as firing costs severance payments and unemployment benefits in economies with labour market frictions. Usually these models build on the tradition of the search and matching framework and rarely include risk averse consumers that can self insure with assets. [1] Alvarez and Veracierto (2001) is the only exception where the role of severance payments in a dynamic model with wealth accumulation and unemployment benefits is considered but in their economy all forms of insurance (other than wealth) are controlled by the government. In contrast my intention here is to investigate how the planner's program changes when all other insurance (except benefits) is provided by optimizing firms and the planner takes these private policies as given.

Moreover the idea that public policy can crowd out private insurance doesn't appear to be new in the literature. In a different context (with redistributive taxation) Krueger and Perri (2006) illustrate how public policy can throw off private risk sharing arrangements and reduce welfare. In their model allocations are also summarized in contracts with limited commitment but there the role of wealth accumulation is secondary since partnerships never break up. When there are unemployment risks or to put differently states where agents are beyond the reach of their insurance agencies wealth accumulation is important. Furtheron Attanasio and Rios Rull (2002) consider an economy where private risk sharing occurs between two ex ante identical agents that form a family and risks derive from a stochastic labour income process. They show that public insurance that takes the form of a reduction in the aggregate (family) component of risk has a disruptive role on the realm of private risk sharing.

Closer to mine is the work of Pissarides (2004) who considers an environment where workers have access to savings and can search for job opportunities whilst employed, and uses it to investigate the optimal response of firms when they are granted two margins of insurance; one is that they can make payments contingent on the event of job loss (severance compensation), and another is that they can retain their workforce for a given period of time even when jobs become unproductive (dismissal delay). The worker employer relationship is contaminated with moral hazard in that the firm doesn't observe the optimal search policy of the worker. In some cases, he argues, benefits can crowd out

[1] See for instance Bentolila and Betrola (1990), Hopenhayn and Rogerson (1993), Ljungqvist (2002).

private insurance (in particular they can eliminate the incentive to postpone layoffs) and this result introduces an array of economically meaningful interactions that he discusses. My intention here is different however. I 'm mostly interested in aggregate effects and instead of characterizing the optimal response of private policies taking public insurance as given, I explore how alternative UI schemes affect welfare in economies where some aspects of private insurance are important and in economies where they are not.

In the model the arrangement that firms and workers use to tradeoff risk is made under the following assumptions; Assets are observable and investment lies in the same side of the market as wages. There are three reasons for this: First it turns out that with limited commitment and exogenous separations asset contractibility is a rather innocuous assumption to make [2]. Indeed in the simple benchmark that delivers the model's main result standard Euler equations seem to hold approximately in the relevant state space, although a modified Euler equation applies more generally. Second in macro contexts these assumptions are rather common. [3] For instance macroeconomists are not concerned with unobservable storage, in the same ways that the literature on optimal allocations with incomplete information or moral hazard condemns private savings (Cole and Kocherlakota (2001(a), 2001(b)), Werning (2002), Kocherlakota (2004), Abraham and Pavoni (2003)). The focal point of this analysis is not to put limits of private risk sharing possibilities but rather to take them as given and investigate how they vary when public policy regimes change.

Finally chapter one of this thesis is also part of the related literature; Many of the features of the comparison between these two economies, are analogous to the contrast between commitment and markov perfect allocations attempted there. For one thing there is a similar ordering of the set of insurance opportunities but as before it is impossible to put bounds on risk sharing just by working out optimality conditions. This is more of an applied issue and in the analysis that follows I look closely at measures that summarize the consumption costs of unemployment in the two economies.

The rest of this chapter is organized as follows: Section 2.2 presents the baseline model. The framework is a variant of the work Wang and Williamson (2002) that features risk sharing opportunities between firms and workers. In the economy of section 2.2.1 wage contracts are flat and agents have to rely on self insurance

[2]See the first chapter of this thesis.

[3]See Krusell et al (2008) for a model with observable savings and Nash bargaining.

to alleviate unemployment risks. In section 2.2.2 private partnerships between workers and firms provide additional insurance by trading risks optimally. In the benchmark separations occur at a constant rate each period, and unemployed workers choose the level of effort that determines their reemployment probability (standard moral hazard problem). Section 2.3 contains the main results. I compare various UI schemes and rank their welfare properties and I find that the differences in optimal policy between the two environments are stark. Finally in section 2.3.3 I add endogenous separations to the model (as in the original work of Wang and Williamson (2002)) by endowing employed workers with a search technology that maps effort into job retention. I explain how this addition limits the range of insurance opportunities by introducing moral hazard in the worker employer relationship, but again I find that the implications of the comparison of the two economies are the same.

2.2 The Model.

There is a continuum of infinitely lived risk averse households with the preferences of the following form:

$$E\sum_{0}^{\infty}\beta^t(\log(c_t) - v(s_t)) \tag{2.2.1}$$

where c_t denotes the consumption of a general multipurpose good and $v(s_t)$ denotes the disutility of search in the labour market.[4]

Each period a fraction $e = 1 - u$ of the economy's workforce are employed matched with firms in joint production and the remaining u workers are unemployed waiting for suitable employment opportunities to arrive. Such opportunities entail the investment of resources in the form of labour market search and each unemployed worker possesses a technology that maps search effort s_t in a job finding probability $\gamma_j(s_t)$. I use the subscript $j = 0, 1, 2, ...$ to denote the number of periods that this worker has spent in unemployment (a worker with an index j is running her $j + 1$ period) and this construction permits to match the decreasing hazard rate (escape rate from unemployment) in labour market data (Wang and Williamson (2002)).

[4]Notice that with separable utility the first best allocation would feature the same level of consumption in employment and unemployment.

In production an agent is entitled to y units of output per period. Production presupposes the collaboration of a worker with a firm that is an entity whose contribution in output is irrelevant. Since search costs are borne exclusively by workers here, it is assumed that firms earn zero profits in expectation when the job starts, but there is nothing that precludes active matches from generating positive payoffs for firms at longer horizons. Further on employed workers don't search (hence $s_t = 0$ for them) but their matches terminate at an exogenous rate λ per unit of time. When this occurs they become unemployed and an index $j = 0$ applies for each newly unemployed worker.

In this economy the government provides insurance against unemployment in the form of replacement income which is denoted by b_j. It depends on the index j to make clear that not all workers are eligible for the scheme. In particular there is a maximum horizon m (duration of non-employment spell) beyond which the unemployed worker's income is normalized to zero (i.e. $b_j = 0 \ \nabla j \geq m$) and for all $j < m$ the level of income received by the worker is a constant b. This construction is meant to capture the current UI scheme that operates in most the US whereby workers are eligible for benefits for a period up to 26 weeks (Wang and Williamson (2002)).

Total resources available to firms and workers are taxed each period and τ denotes the amount that is levied from the match product. The proceeds are used to finance benefits subject to budget balance. If there are e employed workers in the economy then it must be that $e\tau = \sum_j u_j b_j$ where u_j denotes the total number of unemployed workers who are running their $j+1$ period of joblessness. Finally financial markets are incomplete and agents can trade non-contingent securities subject to an ad hoc limit $\overline{a}$. The interest rate on savings is denoted by R. The equilibrium in this economy mandates that $\beta R < 1$ for the agent's program to be well defined.

Insurance Opportunities. I consider two environments. In the first one (in section 2.2.1) agents can do not better than to accumulate assets to buffer shocks in labour income and they can also find comfort in the replacement income b provided by the government. Wage contracts are flat in the sense that active jobs pay out y (gross of taxes) each period. In the second (section 2.2.2) firms sign contracts with their workforce subject to limited commitment and any timing of payments is possible, so long as the equivalent in present discounted value of income is delivered to the worker over the life of the match.

Many of the features of the comparison between these two economies, are analogous to the contrast between commitment and markov perfect allocations made in the previous chapter. For one thing there is a similar ordering of the set of insurance opportunities with which agents are presented; the economy of section 2.2.2 features complete insurance in some of the regions of the state space (this depends on whether participation binds) and section 2.2.1 has a standard incomplete markets model with ad hoc borrowing constraints and uncertain labour income and under general conditions these ingredients grant the agents with a precautionary savings behavior. [5] Since the technical aspects were laid out in the previous chapter, I only summarize some of these features here informally. There is one implication that I exploit here much more than previously though; that optimal allocations under commitment are attainable with a simple wage and severance payment scheme. In fact I establish this equivalence.

In a slight abuse of the term I refer to the economy of section 2.2.1 as the no private insurance case. By that I mean that besides wealth accumulation (and the precautionary effort that unemployed workers exert) there is no risk sharing between workers and their employers in this economy.

2.2.1 No Private Insurance

Consider an environment where insurance opportunities are limited and wage contracts are restricted to pay y each period. Let $W(a)$ and $U(a,j)$ denote the lifetime utilities for an employed and an unemployed worker respectively (the latter running her $j\text{th}+1$ period in unemployment) when their current stock of wealth is a. Their optimal choices consist of current consumption and, for the unemployed agent, a level of search intensity that will determine her probability of finding a job in the next period. Applying standard arguments we can represent these lifetime utilities recursively as:

Employed Workers.

$$W(a) = \max_{a' \geq \bar{a}} \log(c) + \beta\ (\ \lambda\ U(a',0) + (1-\lambda)\ W(a')) \qquad (2.2.2)$$

[5] It is possible to show that the latter economy corresponds to a markov perfect contract where the share of the worker equals one.

Subject to the constraint set:

$$a' = Ra + y - \tau - c \tag{2.2.3}$$

Unemployed Workers.

$$U(a,j) = \max_{a' \geq \overline{a}, s} \log(c) - v(s) + \beta\, \gamma_j(s)\, W(a') + \beta(1 - \gamma_j(s))\, U(a', j+1)) \tag{2.2.4}$$

Subject to the constraint set:

$$a' = Ra + b_j - c \tag{2.2.5}$$

In the standard notation primes denote next period variables. Notice that in (2.2.2) whenever the job is destroyed, which occurs at rate λ each period, the worker becomes unemployed and an index $j = 0$ applies. Further on for the unemployed worker if search in the labour market is unsuccessful, then next period the state j is updated deterministically to $j + 1$ since in this case the worker accumulates an additional period of nonemployment duration.

The implications of this environment for the agent's optimal policies are well understood in the literature. Under standard assumptions uncertainty about income coupled with incomplete markets result in a precautionary savings behavior whereby the agent will accumulate assets in employment and run them down whilst unemployed to minimize the impact of income fluctuations on her private consumption path. Further on search in the labour market is an additional margin of insurance here and wealthier workers will search less intensively [6] cause they can finance current consumption out of wealth. The following paragraph defines the competitive equilibrium in this economy.

Recursive Competitive Equilibrium: No Private Insurance Economy

The stationary competitive equilibrium, in the economy without private insurance consists of a set of value functions $\{U(a,j), W(a)\}$, and a set of decision rules on asset holdings $\{a'_{w,(a)}, a'_{u,(a,j)}\}$ and an optimal search intensity rule of the form $s(a,j)$. It also consists of a level of taxes τ and an invariant measure μ of agents over the relevant state space (employment status and wealth) such that:

1) Agents optimize: Lifetime utilities solve the functional equations (2.2.2) and

[6] See Lentz and Tranaes (2005) for discussion of wealth effects on optimal search intensity.

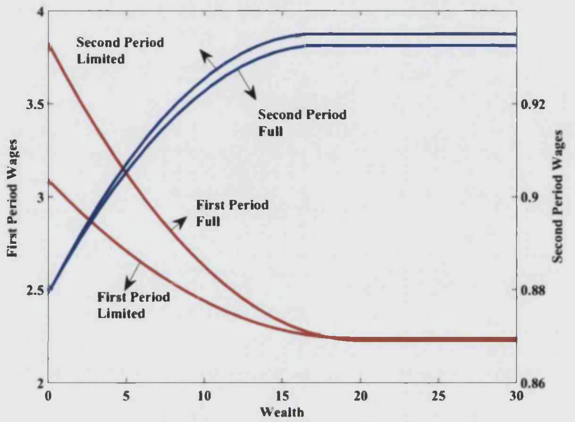

FIGURE 2.1: COMPLETE AND INCOMPLETE CONTRACTS: FIRST AND SECOND PERIOD WAGES.

following functional equation: [7]:

$$\begin{aligned} W(a,J) &= \max_{a',J',\xi} \log(-a' + Ra - \tau - J + y + \frac{1-\lambda}{R}J' - \frac{\lambda\xi}{R}) \quad (2.2.13) \\ &+ \beta(\lambda U(a' + \frac{\xi}{R}, 0) + (1-\lambda)W(a', J')) \end{aligned}$$

Subject to the constraint set:

$$a' \geq \overline{a} \qquad a' + \frac{\xi}{R} \geq \overline{a}$$

The following simple Ricardian equivalence argument can be used to establish that the worker's program under (2.2.14) is the same as that in equation (2.2.6): Increase wealth for the employed worker by $\frac{\xi}{R}$ and let the new level of assets be $\tilde{a}' = a' + \frac{\xi}{R}$. Also decrease the continuation utility J' by ξ and define $\tilde{J}' = J' - \xi$. Then clearly $W(\tilde{a}', \tilde{J}') = W(a', J')$ since the amount of resources available to finance consumption for the worker next period is unchanged. Thus a program that sets $\xi = 0$ and uses next period's wealth as a single control variable as in equation (2.2.6) is payoff equivalent to one where both investment in wealth and

[7]The promise keeping constraint in this case is given by $J = y - w + \frac{1-\lambda}{R}J' - \frac{\lambda}{R}\xi$. The formulation of the value function in (2.2.14) makes use of the fact that with log utility the promise keeping constraint binds with equality.

(2.2.4) and optimal policies derive.

2) The invariant measure μ is consistent: In particular the law of motion of μ can be represented in the following equations:

$$\mu_{e,\mathcal{A}} = (1-\lambda)\int_{a'_{(e,a)}\in\mathcal{A}} d\,\mu_{e,a} + \sum_j \int_{a'_{(u,a,j)}\in\mathcal{A}} \gamma_j(s(a,j))d\,\mu_{u,a,j}$$

$$\mu_{u,\mathcal{A},j} = \lambda\mathcal{I}_{(j=0)}\int_{a'_{(e,a)}\in\mathcal{A}} d\,\mu_{e,a} + \mathcal{I}_{j>0}\int_{a'_{(u,a,j-1)}\in\mathcal{A}} (1-\gamma_j(s(a,j-1)))d\,\mu_{u,a,j-1}$$

where $\mu_{e,a}$ ($\mu_{u,a}$) is the marginal cdf for employed (unemployed) workers and $\mathcal{I}$ is an indicator function.

3) Taxes and benefits are consistent with Budget Balance: $e\tau = \sum_j u_j b_j$

2.2.2 Private Insurance

Consider now an environment whereby instead of constraining active jobs to pay out y each period to workers, wage tenure profiles are optimal. When they meet, the firm and the worker enter in a long term agreement that specifies the entire path of consumption, wealth and wages (assume that there is sufficient commitment to adhere to such date zero allocations) and let J and $W(a,J)$ be the associated payoffs (lifetime utilities) for the two parties. Allocations in this economy are optimal in the sense that they maximize $W(a,J)$ subject to delivering an expected present discounted value of profits equal to J to the firm, and in line with the notion that there are limits to the scope of commitment, efficient outcome paths must satisfy participation by both parties at all future dates. This program admits a recursive representation (see Ligon et al (2002)) that can be written as follows:

$$W(a,J) = \max_{a'\geq\bar{a},J'} \log(c) + \beta(\lambda U(a',0) + (1-\lambda)W(a',J')) \tag{2.2.6}$$

Subject to the constraint set:

$$a' = Ra + w - \tau - c \tag{2.2.7}$$

$$J \leq y - w + \frac{1-\lambda}{R}J' \tag{2.2.8}$$

$$J' \geq 0 \tag{2.2.9}$$

$$W(a', J') \geq U(a', 0) \tag{2.2.10}$$

In equation (2.2.6) the worker solves for a pair of optimal policies for next period wealth a' and a continuation utility J' for the firm. Equation (2.2.8) is the so called promise keeping constraint that imposes the requirement that the firm's expected profit is at least J over the life of the match. Further on optimal allocations must never violate participation by either firms or workers a requirement that is imposed by equations (2.2.9) and (2.2.10). In particular in (2.2.10) the worker must be weakly better off than quitting her job and becoming unemployed (notice that her outside option is the value function $U(a', 0)$). In (2.2.9) the expected profits of the firm next period must be non-negative.

Equilibrium payoffs for unemployed workers solve the following functional equation:

$$U(a, j) = \max_{a' \geq \bar{a}, s} \log(c) - v(s) + \beta\, \gamma_j(s)\, W(a', 0) + \beta(1 - \gamma_{j+1}(s))\, U(a', j+1)) \tag{2.2.11}$$

Subject to the constraint set:

$$a' = Ra + b_j - c \tag{2.2.12}$$

Notice that when the job starts it must always be that $J = 0$ so that firms make zero profits in expectation as in the model of section 2.2.1. But here it is obvious that allocations that solve equation (2.2.6) can accomplish a lot more than in the case of the constrained contracts. Optimal choices of J' define an implicit sequence of transfers (wages) that could in principle be different from y at any point in time and in general there is a whole range of outcomes that is admissible here, and that was not available when contracts were restricted to be flat. Without loss of generality we can write $W_I(a, 0) \geq W_{NI}(a)$ where subscripts I and NI denote models of insurance and no insurance respectively.

This type of arrangement whereby a number of agents pool their resources to alleviate risks subject to limited commitment is rather common in the literature (see Attanasio and Rios Rull (2002), Krueger and Perri (2006)) but what is different here is that wealth is an important state variable along with expected utility J. Most of the relevant literature analyzes the implications of limited risk sharing in economies where there are two (or more) ex ante identical agents and private partnerships never dissolve. In contrast in the current context, at rate λ each period the worker and the firm separate and wealth is the only way to contract upon unemployment.

Implications.

Chapter one of this thesis examined the implications of a risk sharing arrangement between workers and firms that was, at least technically, very similar the one I use here. I showed that incomplete (limited commitment contracts) provide some insurance against unemployment but there were also regions of the state space where complete insurance obtained (that depended critically on whether the participation constraints were slack). These conditions continue to hold in this context.

The timing of payments was shown to be central to the notion that firms could alleviate the risks of unemployment and this feature is summarized in Figure 2.1. It shows wages in the first two periods of the optimal contract (borne out of the baseline calibration of this chapter) along with the analogous objects of an arrangement that ignores the constraints in equations (2.2.9) and (2.2.10) altogether (more precisely the full commitment solution).

The gap between the two first period wage functions indicates the range of investment possibilities in these two cases. Complete (full commitment) contracts offer higher compensation against unemployment, by allowing the worker to accumulate more savings but they violate participation and the two paths differ precisely where the desired provision of insurance makes the worker better off in unemployment (the marginal gains from investment are higher when the wealth endowment is low).

There were also other implications but one that is particularly important here is that in this simple environment it is possible to implement allocations that solve the worker's program in (2.2.6) using severance payments as an alternative margin of insurance. I establish this in this chapter (I only mentioned this property preciously) by showing that whenever wealth and severance payments (the latter is denoted by ξ) are part of the worker's optimal control variables, they are perfect substitutes and that there always exists a payoff equivalent allocation that sets $\xi = 0$ as in (2.2.6).

To see this consider first a model that features complete contracts and both margins of insurance are available to the firm. Optimal allocations then solve the

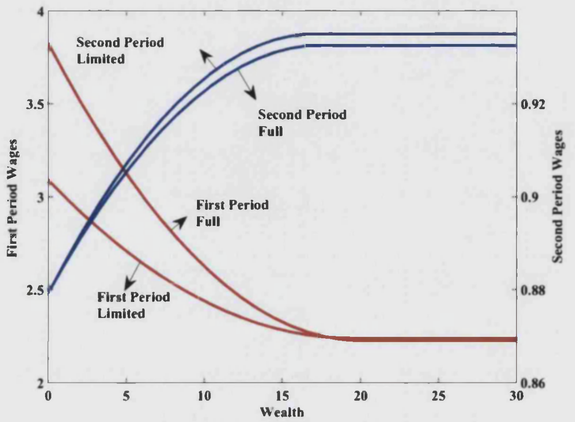

FIGURE 2.1: COMPLETE AND INCOMPLETE CONTRACTS: FIRST AND SECOND PERIOD WAGES.

following functional equation: [7]:

$$\begin{aligned} W(a,J) &= \max_{a',J',\xi} \log(-a' + Ra - \tau - J + y + \frac{1-\lambda}{R}J' - \frac{\lambda\xi}{R}) \quad (2.2.13) \\ &+ \beta(\lambda U(a' + \frac{\xi}{R}, 0) + (1-\lambda)W(a', J')) \end{aligned}$$

Subject to the constraint set:

$$a' \geq \overline{a} \qquad a' + \frac{\xi}{R} \geq \overline{a}$$

The following simple Ricardian equivalence argument can be used to establish that the worker's program under (2.2.14) is the same as that in equation (2.2.6): Increase wealth for the employed worker by $\frac{\xi}{R}$ and let the new level of assets be $\tilde{a}' = a' + \frac{\xi}{R}$. Also decrease the continuation utility J' by ξ and define $\tilde{J}' = J' - \xi$. Then clearly $W(\tilde{a}', \tilde{J}') = W(a', J')$ since the amount of resources available to finance consumption for the worker next period is unchanged. Thus a program that sets $\xi = 0$ and uses next period's wealth as a single control variable as in equation (2.2.6) is payoff equivalent to one where both investment in wealth and

[7]The promise keeping constraint in this case is given by $J = y - w + \frac{1-\lambda}{R}J' - \frac{\lambda}{R}\xi$. The formulation of the value function in (2.2.14) makes use of the fact that with log utility the promise keeping constraint binds with equality.

on following Wang and Williamson the real rate of return on savings is set to zero so that agents in the economy have access to a simple storage technology.

I assume that the search functions are of the following standard form: $\gamma_j = 1 - e^{(-\gamma_j s)}$ and the cost function is quadratic (i.e $v(s) = s^\delta$ where $\delta = 2$). The parameters γ_j for $j = 0, 1, 2, ...$ are calibrated to match certain observations in the US. In particular Wang and Williamson (2002) use a sample from the CPS spanning the years 1977-87 and find an unemployment rate of 7.5 % and of these workers approximately 70 % have had a duration in unemployment of a quarter or less and 15 % between one and two quarters. For these observations it suffices to let the reemployment probability drift between the first and second quarters of an unemployment spell and then remain constant; I obtain $\gamma_0 = 1.81$ (1.855) and $\gamma_{\geq 1} = .62$ (.68) for the economy of section 2.2.1 (2.2.2) respectively. With these choices there are three relevant value functions for the unemployed worker to consider, and one for employed agents and this reduces the computational burden considerably. Finally separations in the model are pinned down by the requirement that 70 % of unemployed workers are in state $j = 0$. The implied value is $\lambda = .0568$.

As the baseline UI scheme I adopt the calibration of Wang and Williamson (2002) whereby the worker earns 50 % of gross after tax income in the first two quarters of her spell and zero afterwards. Hence the replacement income is of the following form: $b_j = .5$ For $j = 0, 1$ and $b_j = 0$ For $j \geq 2$. Table (1) summarizes these choices.

TABLE 2.1: THE MODEL PARAMETERS (QUARTERLY VALUES)

Parameter	Symbol	Baseline
Time Preference	β	.99
Real Rate	$R - 1$	0
Replacement Rate	b	.5
Separation Rate	λ	0.0568
Search Cost Function	δ	2
Search Technology γ	γ_0, $\gamma_{\geq 1}$	(see text)

There is a number of important features that are missing from this analysis. The first is that all newly unemployed workers in the economy are eligible for the UI scheme, whilst in reality only a fraction of them qualify for benefits [11]. This

[11] Wang and Williamson (2002) calibrate the fraction of eligible workers to 30% which is the

severance payments are allowed.

Under limited commitment matters are more complicated but we can always construct examples where the constraint set is such that allowing for severance payments makes no difference for optimal allocations. For instance two such participation constraints are given by the following equations:

$$J' - \xi \geq 0 \qquad W(a', J') \geq U(a' + \frac{\xi}{R}, 0)$$

These objects mandate that next period profits are such that the employer would never find it profitable to dissolve the match and pay ξ to the worker, and that the worker would not want to quit her job and become unemployed when her resources in that state are given by $a' + \frac{\xi}{R}$. Under these conditions the equivalence between assets and severance payments as vehicles of insurance is straightforward to establish. [8]

A corollary from this is that the optimal transfers shown in Figure 2.1 are the result of the particular insurance arrangement considered here and that under alternative mechanisms one can get more realistic wage profiles out of the model. Private risk sharing can be reinterpreted as a simple wage and severance payment scheme and off course the empirical relevance of this arrangement is obvious. [9] The caveat from this analysis is that since severance compensation is not uniquely defined in the model it is also difficult to have an empirical evaluation of the range of private insurance opportunities in the economy (although imperfect measures can be constructed).

Finally note that wherever through wealth or severance payments (or any mixture between the two), limited commitment contracts feature complete insurance, that doesn't mean that the worker's entire consumption path is unaffected by the risk of job separations. On the contrary since in unemployment the worker is beyond the reach of the firm, her consumption will fall (it just won't fall in the first period of the spell if insurance is perfect) through standard wealth effects as she runs down her stock of assets. That is to say that in this economy there is still

[8] Note however that an outside option $W(a', J') \geq U(a' + \xi, 0)$ may not be realistic here in the sense that when the worker quits she may not be entitled to severance compensation. A constraint of the form $W(a', J') \geq U(a', 0)$ however would make an allocation between assets and severance payments non neutral since by increasing ξ and lowering a' the worker's participation constraint is relaxed.

[9] For instance Chetty and Saez (2009) use a sample from a survey conducted by Mathematica on behalf of the department of Labor and find that 15 % of newly unemployed workers receive severance compensation. See also Pissarides (2004).

ample room for public policy to alleviate the welfare costs of prolonged periods of joblessness. This is an implication that will prove to be particularly important for the optimal timing of UI payments considered in section 2.3.2 that contains the baseline results. The next paragraph defines the competitive equilibrium.

Recursive Competitive Equilibrium

The stationary competitive equilibrium consists of a set of value functions $\{U_{(a,j)}, W_{(a,J)}\}$ for employed and unemployed workers respectively, and a set of decision rules on asset holdings $\{a'_{w,(a,J)}, a'_{u,(a,j)}\}$ and continuation values ($J'_{(a,J)}$) and search intensity ($s(a,j)$). It also consists of a level of taxes τ and an invariant measure μ of agents across assets, employment status and (promised value to firms) such that:

1) Agents optimize: $\{U_{(a,j)}, W_{(a,J)}\}$ solve functional equations 2.2.6 and 2.2.11 above and optimal policies derive.

2) Taxes and benefits are consistent with Budget Balance: $e\tau = \sum_j u_j b_j$

3) The measure μ is consistent: In particular the law of motion of μ can be represented as: [10]

$$\mu_{(e,\mathcal{A},\mathcal{J})} = (1-\lambda)\int_{a'_{(e,a,J)}\in\mathcal{A},J'(a,J)\in\mathcal{J}} d\,\mu_{(e,a,J)} + \mathcal{I}_{0\in\mathcal{J}}\sum_j \int_{a'_{(u,a,j)}\in\mathcal{A}} \gamma_j(s(a,j))d\,\mu_{(u,a,j)}$$

$$\mu_{(u,\mathcal{A},j)} = \mathcal{I}_{j=0}(\lambda)\int_{a'_{(e,a,J)}\in\mathcal{A}} d\,\mu_{(e,a,J)} + \mathcal{I}_{j>0}\int_{a'_{(u,a,j-1)}\in\mathcal{A}} (1-\gamma_j(s(a,j-1)))d\,\mu_{(u,a,j-1)}$$

2.3 Numerical Analysis

2.3.1 Calibration

I briefly explain the choice of parameters and functional forms: Since one period in the model corresponds to one quarter, the time preference parameter β is set to .99. Choosing a long horizon serves to make computations more manageable and for the case in hand it doesn't seem to matter for the conclusions. Further

[10] Notice that expected utility J is part of the relevant state space here.

on following Wang and Williamson the real rate of return on savings is set to zero so that agents in the economy have access to a simple storage technology.

I assume that the search functions are of the following standard form: $\gamma_j = 1 - e^{(-\gamma_j s)}$ and the cost function is quadratic (i.e $v(s) = s^{\delta}$ where $\delta = 2$). The parameters γ_j for $j = 0, 1, 2, ...$ are calibrated to match certain observations in the US. In particular Wang and Williamson (2002) use a sample from the CPS spanning the years 1977-87 and find an unemployment rate of 7.5 % and of these workers approximately 70 % have had a duration in unemployment of a quarter or less and 15 % between one and two quarters. For these observations it suffices to let the reemployment probability drift between the first and second quarters of an unemployment spell and then remain constant; I obtain $\gamma_0 = 1.81$ (1.855) and $\gamma_{\geq 1} = .62$ (.68) for the economy of section 2.2.1 (2.2.2) respectively. With these choices there are three relevant value functions for the unemployed worker to consider, and one for employed agents and this reduces the computational burden considerably. Finally separations in the model are pinned down by the requirement that 70 % of unemployed workers are in state $j = 0$. The implied value is $\lambda = .0568$.

As the baseline UI scheme I adopt the calibration of Wang and Williamson (2002) whereby the worker earns 50 % of gross after tax income in the first two quarters of her spell and zero afterwards. Hence the replacement income is of the following form: $b_j = .5$ For $j = 0, 1$ and $b_j = 0$ For $j \geq 2$. Table (1) summarizes these choices.

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Search Technology γ	γ_0, $\gamma_{\geq 1}$	(see text)

There is a number of important features that are missing from this analysis. The first is that all newly unemployed workers in the economy are eligible for the UI scheme, whilst in reality only a fraction of them qualify for benefits [11]. This

[11] Wang and Williamson (2002) calibrate the fraction of eligible workers to 30% which is the

would be an important addition if the focal point here was to assess the welfare implications of currently active policies in the US, but rather the purpose is to compare the outcomes of two economies that offer a different menu of insurance opportunities. Further on making this consideration part of the analysis would raise additional concerns for how eligibility and non eligibility can be handled in an economy with contracts, because there optimal policies could in principle be contingent on the outcome of this lottery. Such additions would complicate the analysis unnecessarily without being clear how they would affect the ordering of policies.

The second important omission is capital and there is a good reason why market clearing conditions for private savings are not part of my analysis here. The work of Young (2004) shows that capital overwhelms the welfare calculations, and always the optimal UI scheme features zero benefits. This in my case would throw off any meaningful comparison between the two economies. The intuition for this result is simple; when b increases precautionary savings fall and lower equilibrium capital labour ratios result in lower productivity for active jobs. On the other hand search theory is filled with examples that produce the converse implication (just think of a model with a distribution of productivity and reservation wage policies) and there is no reason to draw the line by including one feature and leaving out the rest. I leave all these possibilities to future work.

Welfare Criterion. To evaluate optimal policies I assume that the social planner assigns equal weight to all agents in the economy. The welfare criterion is of the form:

$$\Theta = \int W(a)\, d\,\mu_{e,a} + \sum_j \int U(a,j) d\,\mu_{u,a,j}$$

and denotes the ex ante utility of the typical agent in the economy. [12] Further on to rank the various policies I convert the welfare numbers in terms of percentage consumption using the following calculation:

$$\Theta_1 = \Theta_0 + \frac{1}{1-\beta}\log(1+\epsilon)$$

corresponding observation for the US economy

[12]With optimal contracts the relevant state space includes expected utility of the firm so that:

$$\Theta = \int W(a,J)\, d\,\mu_{e,a,J} + \sum_j \int U(a,j) d\,\mu_{u,a,j}$$

Where Θ_0 is the expected utility in the baseline case and Θ_1 is the analogous object under the new policy regime. The fraction ϵ is a standard measure of compensated variation. After each policy change I compute the value of ϵ to give sense of the magnitude of the associated gains or losses.

2.3.2 Results

There are three dimensions for optimal policy considered in this section. First I determine the level of optimal benefits in the two economies when duration is kept constant at two quarters. Then I let the duration of payments be indefinite and I determine the optimal permanent UI scheme. Finally I consider a less constrained optimal policy, one that keeps b constant for two periods and sets a (potentially different) level of benefits for higher durations.

The result that comes out of the analysis is that the range of private insurance sets the scope of optimal public policy. I find that the level of benefits is lower when firms can give additional insurance than in the simple incomplete market economy with wealth accumulation. Further on the shape of optimal payments is different in the two economies with the economy of section 2.2.1 having the typical front-loaded payment scheme whilst in that of section 2.2.2 (with contracts) payments are typically zero for the first two quarters of an unemployment spell and positive afterwards.

Section 2.3.2 assigns an interpretation to these differences by looking at the risk sharing role of private markets and the shape of the worker's policy functions for consumption and search, that determine the wealth distribution. The latter object is central to any meaningful comparison, for in a steady state the asset holdings of unemployed workers determine the gains for public insurance. There is much less dispersion in wealth in the baseline incomplete market economy and to understand whether the differences are important for the results I perform the following simple experiment: I let the government choose the optimal permanent level of benefits and a constant tax levi for each worker as a function of her employment status and wealth, subject to the scheme being self financing. This is a different requirement than having to rely on the distribution to finance unemployment compensation and in fact it is similar (dual) to the more standard program considered in the literature. The finding is that preferred benefits are

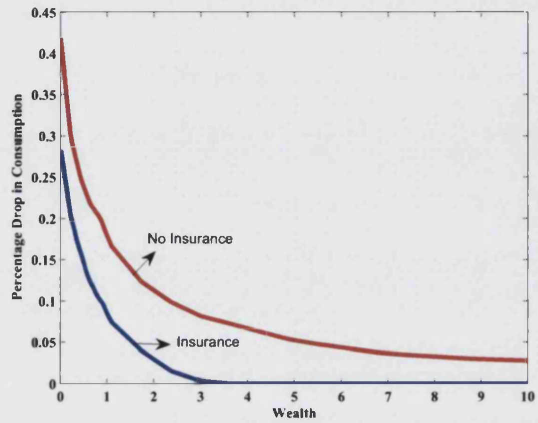

FIGURE 2.2: PERCENTAGE DROP IN CONSUMPTION: 2 ECONOMIES.

Figure 2.2 makes this point by plotting the drop in consumption as a function of wealth associated with the event of a job loss (these values are borne out of the steady state calibration). When wages are flat (no risk sharing with firms) agents have to rely on their savings to insure against unemployment and in this case consumption drops significantly even for those who have stored sufficient wealth. On the other hand with optimal contracts there are regions of the state space where complete insurance obtains and the drop in consumption in the first period of unemployment is zero. In the first case the steady state allocation is such that consumption falls on average by roughly 17% for newly unemployed workers, whilst in the later this statistic is in the order of 7%.

Table (2.3) summarizes this feature and indicates how changes in the level of benefits impact on private consumption for unemployed workers at various horizons. The first three columns refer to the percentage drop from one period to the next for the typical worker holding her wealth constant. For instance the column labeled 1st is the average loss for workers who are running their first quarter in unemployment, and fail to find a job next period, but have the same level of assets.

There are three points that merit attention; First when benefits decrease to zero newly unemployed workers (column labeled EU) suffer substantially more losses in one economy than in the other. Without private insurance opportunities much lower in the economy with private risk sharing which I interpret as an indication that the impact of the wealth distribution on the ranking of optimal policies is of second order.

Optimal Policy.

Simple Policies. The baseline results for optimal policy are summarized in table 2.2. The first row presents the optimal level of UI in the two economies when payments are constrained to be constant for two quarters and then zero forever and the associated welfare gains in terms of the value of ϵ. With flat wage contracts (no insurance) the value of b that maximizes ex ante welfare is equal to 1.81 but when insurance opportunities include risk sharing between workers and firms (as in the model of section 2.2.2) the optimal replacement ratio is 1.32. With permanent benefits (second row) these numbers are .47 and .39 respectively.

TABLE 2.2: OPTIMAL POLICY

	Insurance		No Insurance	
	$b,\ b_{\geq 1}$	**Gains** ϵ	$b,\ b_{\geq 1}$	**Gains** ϵ
2 Quarters	(1.32 , 0.00)	.40%	(1.81 , 0.00)	.40%
Permanent	(0.39 , 0.39)	.59%	(0.47 , 0.47)	.50%
Unconstrained	(0.00 , 0.46)	.63%	(0.95 , 0.38)	.63%
No UI	(0.00 , 0.00)	-.03%	(0.00 , 0.00)	-.21%

To give sense of the magnitude of welfare gains that accrue to the economy's workforce in steady state, the second and fourth columns of Table 2.2 report the value of ϵ. For both regimes a move to the optimal policy with benefits that last for two quarters, is associated with a .40 % gain in welfare whereas with permanent benefits the gains are .59 % and 50% with and without private insurance respectively. These gains are not too large but they are still an order of magnitude larger than what Wang and Williamson (2002) get out of their model and there are two reasons for this; first in their calibration roughly 70% of all newly unemployed workers are not eligible for benefits and second workers can exert search effort on the job to avoid unemployment. With separations being exogenous here, and eligibility not being an issue, it is no wonder that changes in b have a larger impact on welfare, since workers face more risks when employed, and the scope of the gains from public insurance is wider to include all newly unemployed workers.

Finally the last row of Table 2.2 presents the percentage losses from a policy that sets benefits equal to zero in the two economies. Relative to the benchmark these losses are minuscule with private risk sharing and they are substantially larger without, which seems to suggest that private and public insurance are substitutes and that when the government steps down private employers step in to insure their workforce.

To understand these results notice that both economies present the planner with the following tradeoff: First to minimize the cost of the scheme optimal policy must invoke a penalty against long term unemployment (this is a standard moral hazard interpretation). Second since search becomes much less effective after a quarter spent in unemployment, part of the planner's objective is to minimize the loss of utility associated with long durations. For a policy that lasts for two quarters this means that benefits must be high enough to allow the agent to build a buffer against these risks, but then again such a policy cannot be optimal since the cost of financing insurance unconditionally for everyone is much larger than financing it conditionally for those who fail to find a job. This is why permanent benefits can perform so much better.

To address these possibilities I turn to a more flexible policy, one that allows the planner to treat differently the short and long term unemployed, and investigate whether it can engineer even larger gains by virtue of optimizing along the important timing dimension; There is an additional reason why such a comparison is important between the two economies; since contracts can go a long way towards insuring the worker against short term unemployment (I give a quantitative assessment of how far later on) the main concern becomes to manage utility losses for the long term unemployed and in this environment there could be large differences in the optimal timing of payments stemming from the insurance role of private markets.

Flexible Policy. I consider the following experiment: I introduce replacement income for agents with durations that exceed two periods which I denote by $b_{\geq 1}$, and I let the planner choose b and $b_{\geq 1}$ to maximize welfare subject to budget balance. The results are stacked in the third row of Table 2.2 . Optimal policies in the absence of private insurance do indeed have the typical shape. They are front-loaded since part of the planner's concern is to minimize the costs of the scheme by penalizing long durations of unemployment. With private contracts however they are zero for the first two quarters and positive (equal to .46) indefinitely after.

I give the following interpretation to these results; First, when workers can save, UI benefits need not be decreasing for consumption to be decreasing and indeed under both arrangements consumption falls through standard wealth effects when agents run down their assets. In principle in this environment any timing of payments could be optimal (Werning (2002), Shimer and Werning (2004)); Second the difference between the two economies is again that with private contracts the role of insurance against unemployment is partly assigned to firms and thus benefits need not be large for consumption smoothing at least in the first period of an unemployment spell. Although the cost of insurance per worker increases with the duration of her spell (since search is more ineffective along this dimension) the cost to a society of financing the scheme out of the distribution of workers is not. In the calibration of both economies there are substantially more workers unemployed up to than more than 2 quarters and by setting $b = 0$ in an environment with private contracts the planner saves a lot in the tax levi.

How robust are these predictions to changes in the institutional environment, or to more complicated UI schemes (say a scheme that sets a potentially different level of benefits in every quarter) is a point that merits considerable attention from future work. For instance I find that there is a large range of payment schedules that perform almost as well as the policy that sets $(b, b_{\geq 1}) = (0, 46)$ but their allocations and the prediction on who gains who doesn't amongst the population differ substantially. It is important however to realize that the central message of this section is that in the context of an economy where private and public insurance margins coexist the government would like to devise more complicated mechanisms to minimize its interference with private markets. Here more complicated mechanisms have an obvious form; they are related to the timing of benefits and by postponing benefits the government leaves ample scope for private insurance and minimizes the costs of financing the UI scheme.

Understanding The Result.

Allocations and Consumption Losses. The range of private insurance opportunities lies at the heart of the differences in the scope of public policy here.

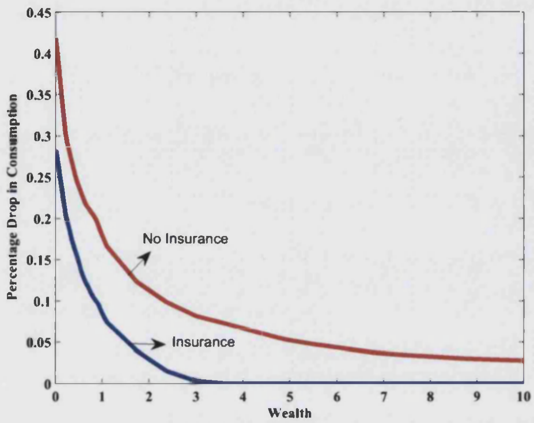

Figure 2.2: Percentage Drop in Consumption: 2 Economies.

Figure 2.2 makes this point by plotting the drop in consumption as a function of wealth associated with the event of a job loss (these values are borne out of the steady state calibration). When wages are flat (no risk sharing with firms) agents have to rely on their savings to insure against unemployment and in this case consumption drops significantly even for those who have stored sufficient wealth. On the other hand with optimal contracts there are regions of the state space where complete insurance obtains and the drop in consumption in the first period of unemployment is zero. In the first case the steady state allocation is such that consumption falls on average by roughly 17% for newly unemployed workers, whilst in the later this statistic is in the order of 7%.

Table (2.3) summarizes this feature and indicates how changes in the level of benefits impact on private consumption for unemployed workers at various horizons. The first three columns refer to the percentage drop from one period to the next for the typical worker holding her wealth constant. For instance the column labeled 1st is the average loss for workers who are running their first quarter in unemployment, and fail to find a job next period, but have the same level of assets.

There are three points that merit attention; First when benefits decrease to zero newly unemployed workers (column labeled EU) suffer substantially more losses in one economy than in the other. Without private insurance opportunities agents in the two economies. When private contracts are present firms encourage the accumulation of wealth by front-loading part of the compensation to newly employed workers whereas without private risk sharing wealth accumulation can only be financed out of current consumption for these workers. Smaller differences in the ratios $\frac{W_e}{W_u}$ also reflect this timing but these don't have any allocational implication whatsoever; if say insurance was given through severance payments which is an equivalent margin of insurance here, then unemployed workers would hold more assets but all other statistics (unemployment rates, durations etc.) would be unaffected.

I conclude the following from this analysis; Although the model of section 2.2.2 doesn't permit to construct empirical measures of the extent of private risk sharing in the economy (such as severance payments relative to income say), imperfect measures can be constructed (i.e. consumption losses in unemployment) and these indicate that the insurance role of private markets is important. Further on equilibrium allocations are also different in a way that raises some concern of whether the differences in optimal policy are partly attributed to differences in the steady state distribution. This is a possibility that I scrutinize in the next paragraph.

The Wealth Distribution. Figure 2.3 describes the distribution of assets in the two economies. It shows fractions of agents as a function of the ratio of current wealth to the steady state level of wealth in the economy without private contracts (which is equal to 1.05). [13] There are large differences in the distribution of agents over the relevant state space. For instance the standard incomplete market model (red bars) predicts that most agents cluster in the region where wealth is between .75 and 1.25 relative to the average. With private insurance (blue bars) there is a lot more dispersion in outcomes; more agents near the borrowing constraint but also a lot more holding higher levels of wealth.

To understand these differences it is important to look at some of the features of the calibrated solution in the two economies. Figure 2.4 plots the consumption for unemployed workers for the economy with private insurance. Under the baseline calibration agents decrease consumption between periods one and two because the search technology deteriorates, and a further cut back takes place after the second quarter since government benefits drop to zero thereafter. Further on the

[13] This serves to make the two distributions more comparable since mean assets differ in the two economies.

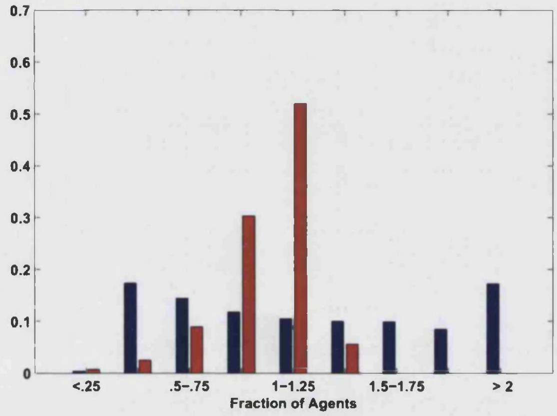

FIGURE 2.3: STEADY STATE DISTRIBUTION OF WEALTH.

analogous functions for unemployed agents in the incomplete market economy of section 2.2.1 turned out to be very similar (since the environments present unemployed agents with a similar program).

What explains the differences in allocations is the behavior of employed agents. In Figure 2.5 I graph these policies showing that there is a large gap in terms of the overall level of consumption for the generic worker between the two economies. Consumption is substantially higher at the start of the job for an agent in the insurance economy because the optimal contract prescribes a path that is decreasing over time and wealth can be financed through higher initial wages. In the incomplete market of section 2.2.1 the optimal behavior is to consume little at the start of the job, accumulate assets and then after a sufficient buffer is build, consumption and wealth stay constant.

These features explain the shape of the wealth distributions shown in Figure 2.3. In effect in the private insurance model of section 2.2.2 agents that are 'lucky' enough to experience frequent transitions across labour market states (employment and unemployment) will also hold higher levels of wealth. This implication is not counterfactual and in fact it is that is not at odds even with conventional incomplete market models, since there those agents that face higher risks of unemployment will also have a stronger precautionary savings motive. It is inconsistent however with an economy where all agents face the same separation the percentage drop given the wealth distribution is 23% when $b = 0$ and 17% in the baseline economy. With private insurance these numbers are 7.9% and 7.7 % respectively. Second the drop in consumption from the first to the second quarter comes from the change in the search technology and the deterioration of reemployment prospects for the typical worker. Then the possibility of long term unemployment is much more imminent (or the drop in expected permanent income larger) and the worker needs to account for this in her optimal path. Finally consumption between the second and third quarters drops when benefits are positive but not when they are zero because in the latter case the worker's conditions in terms of income or search remain the same.

Aggregating these statistics over all periods would seem to suggest that changes in UI payments send consumption losses to the opposite direction. However this implication should not be misconstrued to contradict the findings of Gruber (1997), that UI alleviates unemployment risks by minimizing impact on consumption, because these calculations don't not account for the drop in wealth from one period to the other and the differences in the reemployment probabilities of workers who hold different levels of assets. This is performed in the last column of Table 2.3 that looks at the percentage drop in consumption of workers who 'report' being employed in one year and unemployed in the next (clearly there are many transitions consistent with this censored spell).

This is precisely the measure constructed by Gruber (1997) and he finds that a fall in benefits to zero (relative to the current UI scheme in the US) increases annual consumption losses from 6.8% to 22 % in his sample (equivalently a rise in the replacement income of 10% reduces the consumption loss by 2.68%). By this metric both economies seem to underestimate the importance of publicly provided insurance precisely because the ability of agents to insure privately (either by accumulating wealth of through contracts with firms) mitigates the effects of an unemployment spell on consumption in the steady state. Put differently Engen and Gruber (2001) estimate that a reduction in the level of benefits by 50% increase households financial wealth by 14% giving an indication of the extent to which UI payments crowd out the precautionary role of assets. Both models fail to match this elasticity; in the no insurance economy a similar reduction in UI engineers an increase of roughly 35% in the households net worth and 25% in the insurance model. It is no wonder therefore that the two models tend to underestimate the impact of the UI scheme on risk sharing.

Table 2.4 summarizes some important features of equilibrium allocations in the

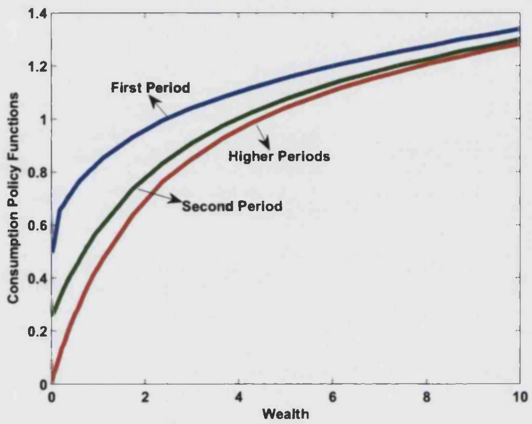

FIGURE 2.4: CONSUMPTION POLICY FUNCTIONS: UNEMPLOYED AGENTS.

risk since in this case more frequent transitions mean that agents don't have enough time to accumulate wealth.

Preferred Benefits. It is clear that these asymmetries are far too important for the central message of this chapter to be left unexplored. To investigate their impact I perform the following simple experiment: I let the government choose the optimal permanent level of benefits (denoted by $\bar{b}$) and constant taxes separately for each worker as a function of her employment status and wealth, subject to the scheme being self financing (that is the overall discounted cost has to equal zero). This is a different requirement than having to rely on the distribution to finance the UI scheme, and closer to the standard program considered in the literature whereby an insurance agency sets the level of benefits and taxes to minimize the costs of insurance, subject to delivering to the worker a prescribed level of utility. [14]. Off course since the qualitative features of optimal policies and transitions are still present it is only an imperfect measure of controlling for the different steady allocations in the two economies.

More specifically consider a planner that solves the agent's program in the no insurance economy (equations (2.2.2) and (2.2.4) in section 2.2.1) subject to the following laws of motions of costs and taxes:

[14]See Shimer and Werning (2004) for a similar experiment

TABLE 2.3: PERCENTAGE DROP IN CONSUMPTION DUE TO UNEMPLOYMENT

Model	b	Period EU	(1st)	(2nd)	Total
	0.0	.23	.17	.00	.30
No Insurance	0.5	.17	.28	.22	.28
	0.0	.079	.18	.00	.21
Insurance	0.5	.077	.28	.31	.20

two environments across alternative policy regimes. Benefits are assumed to last for two quarters and hence $b = .5$ corresponds to the benchmark steady state calibration of the two economies. Columns 1 through 3 give the economy's rate of unemployment, the fraction of unemployed workers with duration greater than 2 quarters and the level of taxes needed to finance the scheme respectively.

In both cases (with and without private contracts) associated with increases in benefits is a fall in search effort and a rise in the number of unemployed workers and in the incident of long term unemployment. For instance in the economy without private insurance a value of b equal to unity means that 16.1% of unemployed workers have spent at least six months looking for work and a similar value (16.6%) obtains when private risk sharing is present. The reason is that a large upfront provision of insurance encourages savings for unemployed workers that affect their search behavior even when benefits stop. Further on notice that insofar as the aggregate labour market is concerned the two economies share more or less the same response to changes in the policy regime.

TABLE 2.4: ALLOCATIONS

Model	b	u (1)	τ (2)	$u_{\geq 2}$ (3)	Wealth (4)	$\frac{W_e}{W_u}$ (5)
	0.0	.0731	0.0	.142	1.97	1.14
No Insurance	0.5	.0750	.033	.150	1.05	1.10
	1.0	.0771	.065	.161	0.26	0.82
	0.0	.0735	0.0	.145	2.24	1.17
Insurance	0.5	.0750	.033	.150	1.34	1.11
	1.0	.0773	.065	.166	0.69	.95

Columns 4 and 5 give the average level of assets in the economy and the ratio of wealth ($\frac{W_e}{W_u}$) held by employed to unemployed workers respectively. The standard result that government insurance crowds out the precautionary role of assets holds in both cases but there are large differences in average wealth held by

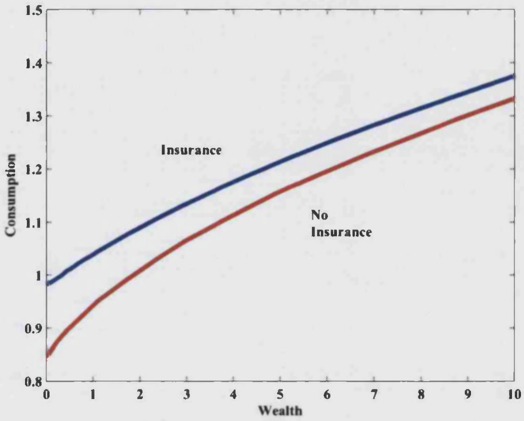

FIGURE 2.5: CONSUMPTION POLICY FUNCTIONS: EMPLOYED AGENTS

$$\mathcal{C}_0(a,\bar{b},\tau) = \bar{b} - \frac{\gamma_0(s_{a,\bar{b},\tau})}{R}\mathcal{T}(a'(a),\bar{b},\tau) + \frac{1-\gamma_0(s_{a'(a),\bar{b},\tau})}{R}\mathcal{C}_1(a,\bar{b},\tau) \qquad (2.3.1)$$

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$$\mathcal{T}(a,\bar{b},\tau) = \tau + \frac{1-\lambda}{R}\mathcal{T}(a'(a),\bar{b},\tau) - \frac{\lambda}{R}\mathcal{C}_0(a'(a),\bar{b},\tau) \qquad (2.3.3)$$

Equations (2.3.1) to (2.3.3) represent the laws of motion of the present value costs (benefits) and taxes (notice that since benefits are permanent only the first two quarters of an unemployment spell are different). To conserve notation I let $a'(a)$ and $s(a,\bar{b},\tau)$ be the optimal choice of assets and search intensity for the generic agent that has a current level of wealth a. Further on these objects generalize in the insurance model of section 2.2.2 the only difference being that the present discounted value of profits J must be added to the list of state variables along with assets for the employed agent.

To ensure convergence I assume that the interest rate R equals 1.005 so that the compound return is 2.02% at an annual horizon. With the same discount factor an increase in the interest rate implies that the employed agent's consumption in the optimal contract economy is not as front-loaded and thus if anything the optimal agents in the two economies. When private contracts are present firms encourage the accumulation of wealth by front-loading part of the compensation to newly employed workers whereas without private risk sharing wealth accumulation can only be financed out of current consumption for these workers. Smaller differences in the ratios $\frac{W_e}{W_u}$ also reflect this timing but these don't have any allocational implication whatsoever; if say insurance was given through severance payments which is an equivalent margin of insurance here, then unemployed workers would hold more assets but all other statistics (unemployment rates, durations etc.) would be unaffected.

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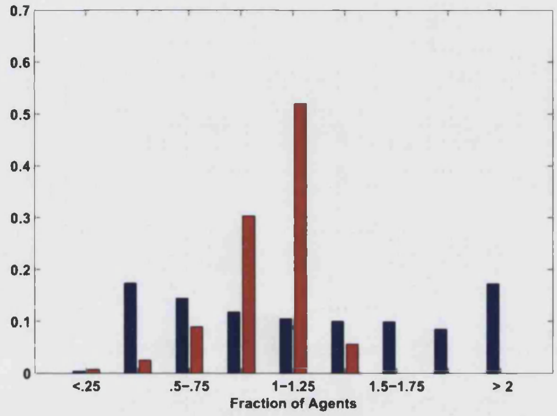

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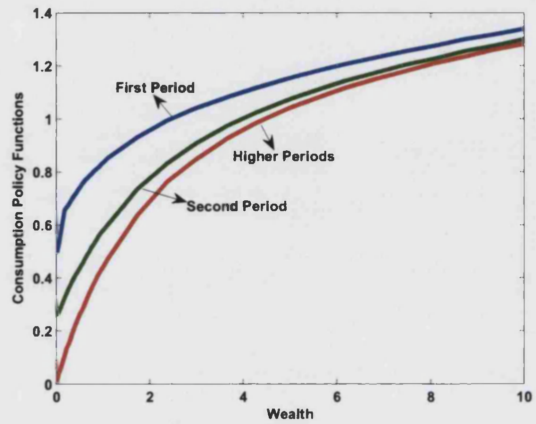

FIGURE 2.4: CONSUMPTION POLICY FUNCTIONS: UNEMPLOYED AGENTS.

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More specifically consider a planner that solves the agent's program in the no insurance economy (equations (2.2.2) and (2.2.4) in section 2.2.1) subject to the following laws of motions of costs and taxes:

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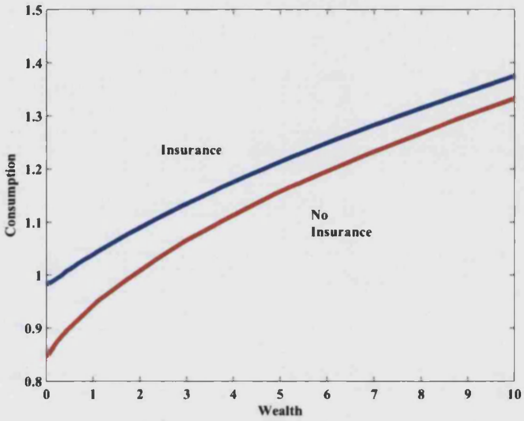
2.3.3 Other Models

How robust are the results of the previous sections to changes in the economic environment? In the first chapter of this thesis I take stock of the vast literature of search and matching models of the labour market to develop a similar policy experiment and I reach similar conclusions. I rely on a two alternative equilibrium concepts; the first one is the undirected search model of Krusell et al (2007) and Bils et al (2009(a), (2009(b)) (there the no insurance case is represented with a contract that is rebargained each period) and the second is a variant of the directed search equilibrium with assets of Acemoglu and Shimer (1999) (there no insurance corresponds to a flat wage contract). Both models have the same implication that depending on the range of private risk sharing in the economy the insurance role of public policy is different.

It is not important to summarize this work here; rather what I seek to address in this section is whether some important additions to the model that limit the insurance role of private markets, can throw off the main predictions of the previous analysis. For this I turn to a model where separations are endogenous as in the work of Wang and Williamson (2002) and as I explain this feature alone implies that the optimal contract of section 2.2.2 is fraught with moral hazard which, as is well known, trades off risk sharing for incentives to maximize the duration of the job.

The technical treatment of these issues attempted in this chapter is rather primitive. It appears that there is a growing literature that investigates the effects of self insurance in partnerships that are fraught with moral hazard or that feature incomplete information (see Karaivanov and Martin (2009) and the references therein). For one thing the assumptions about who controls investments in this context are much less innocuous than before. The previous chapter argued that fundamental violations of the Euler equation were tiny and in this sense adding marginal utility as an additional state variable (as in Abraham and Pavoni (2005) or Werning (2002)) would have no impact on optimal policies. This need not be the case here however. Further on neither the equivalence between assets and severance compensation holds, since the latter is a margin of insurance that is contingent on the observable outcome (separation) whilst the former is not.

These considerations may be important for a more thorough look at optimal policies by firms and their response to the government's scheme but this possibility is left for future work. Further on there appears to be only one paper in the



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$$\mathcal{C}_1(a, \bar{b}, \tau) = \bar{b} - \frac{\gamma_1(s_{a,\bar{b},\tau})}{R}\mathcal{T}(a'(a), \bar{b}, \tau) + \frac{1 - \gamma_1(s_{a'(a),\bar{b},\tau})}{R}\mathcal{C}_1(a, \bar{b}, \tau) \tag{2.3.2}$$

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Equations (2.3.1) to (2.3.3) represent the laws of motion of the present value costs (benefits) and taxes (notice that since benefits are permanent only the first two quarters of an unemployment spell are different). To conserve notation I let $a'(a)$ and $s(a, \bar{b}, \tau)$ be the optimal choice of assets and search intensity for the generic agent that has a current level of wealth a. Further on these objects generalize in the insurance model of section 2.2.2 the only difference being that the present discounted value of profits J must be added to the list of state variables along with assets for the employed agent.

To ensure convergence I assume that the interest rate R equals 1.005 so that the compound return is 2.02% at an annual horizon. With the same discount factor an increase in the interest rate implies that the employed agent's consumption in the optimal contract economy is not as front-loaded and thus if anything the optimal literature that attempts to provide an answer to a similar question (Pissarides (2004)) but again in that analysis the principal (firm) can control all investment decisions and on top of this contracts are complete. I do think however that the model of presented in this section can be a good starting point for this research agenda.

There are two types of arrangements to consider. One derives from an environment where commitment is abundant and utility promises summarize the history of the contract and allocations are determined at date zero and specify a path for the all possible future states. This is similar to the insurance model studied in section 2.2.2. The other is an environment where commitment is scarce and allocations have to be re-bargained each period to make the optimal plan time consistent. This latter case corresponds to a Markov perfect contract whereby no variable other than the endowment of wealth of the agent can summarize the histories (see Karaivanov and Martin (2009)). I establish that under some conditions the optimal policy under this contract is a flat wage similar to the model of section 2.2.1.

The optimal public policy in these two worlds differs in the same way that it differed so far. The scope of public policy gains is reduced when private risk sharing permits agents to mitigate consumption losses in the first period of a unemployment spell. Further on with less risks (agents choose the separation rate) the importance of public insurance is dwarfed by the concern to minimize the outflow rate from employment and indeed in this case I find that in both regimes the optimal policy features near zero benefits, even when the UI scheme is constrained to last for two quarters. Finally the conclusion that the optimal timing of payments is different is preserved here.

Commitment.

Consider the program with commitment as in section 2.2.2 with the addition that employed workers can determine the separation rate $\gamma_e(s)$ as a function of the choice variable s (search intensity). Further on let $v(s)$ be the per period level of disutility suffered by the worker associated with this choice. The rest of the setup (interest rates, discounting, taxes etc) is the same as before.

Each period an employed worker's contract with her firm is summarized in the promised value J (expected value of cash flows that accrue to the firm) which is

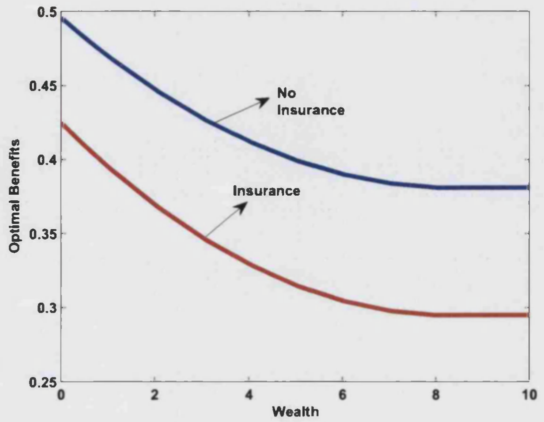

FIGURE 2.6: PREFERRED BENEFITS: FIRST PERIOD UNEMPLOYED AGENTS.

policies in the two economies are closer than in the baseline model. Further on I use the same search technology parameters in both economies setting $\gamma_0 = 1.81$ and $\gamma_{\geq 1} = .62$. To calculate the preferred level of benefits, I solve the worker's value functions using standard methods (value function iteration) and then I use the optimal policies to iterate convergence on equations (2.3.1) to (2.3.3). For a worker with assets a running her first quarter in unemployment it must be $\mathcal{C}_0(a, \bar{b}, \tau) = 0$ for the scheme to be self financing and similarly $\mathcal{C}_1(a, \bar{b}, \tau) = 0$ for an agent that has accumulated more than one quarter in unemployment. For any given level of $\bar{b}$ I use a simple bisection algorithm to bracket the required the tax levi and then I use a simple grid search method for the optimum over all levels of benefits.

Figure 2.6 displays the optimal levels of benefits as a function of wealth for workers in their first quarter in unemployment. The analogous object for agents with higher durations is almost the same. It is clear that the two economies deliver largely different levels of preferred government insurance for their workforce with standard incomplete markets economy having systematically higher levels of benefits for all unemployed workers. Over the state space the preferred taxes are lower by 20% to 25% in the economy with private insurance. From this I conclude that the impact of the wealth distribution is of second order insofar as the main results are concerned.

an argument in her value function, and optimal policies consist of a level of wealth for next period, a continuation value J' and a level of effort s that determines the probability of unemployment. Let $W(a,J)$ be the expected lifetime utility of the worker that solves the following functional equation:

$$W(a,J) = \max_{a' \geq \bar{a}, J', s} \log(c) - v(s) + \beta((1-\gamma_e(s))U(a',0) + \gamma_e(s)W(a',J')) \quad (2.3.4)$$

Subject to the constraint set:

$$a' = Ra + w - \tau - c \qquad J \leq y - w + \frac{\gamma(s)}{R} J'$$

$$J' \geq 0 \qquad W(a',J') \geq U(a',0)$$

This type of arrangement has wealth and search effort chosen on the same side as wages and hence it corresponds to an environment where these choices are both observable (no moral hazard) and contractible. The optimal allocation would feature (can be characterized by) a set of first order conditions for wealth and continuation payoffs to the firm (similar to those of the model of section 2.2.2) with an additional object the optimality condition for the level of s. The latter solves the following equation:

$$v'(s) = \gamma'(s)\beta(W(a',J') - U(a',0) + \frac{u'(c)}{R\beta} J') \quad (2.3.5)$$

where $u'(c)$ denotes the marginal utility of current consumption. Thus the optimal allocation (with complete information) sets search effort such that the marginal cost to the worker equals the weighted benefit to her and the firm. On the other hand when the worker's action is not observed by the firm, the optimal choice of search intensity that is incentive compatible sets this last term equal to zero and incentive compatible allocations rather satisfy the following first order condition:

$$v'(s) = \gamma'(s)\beta(W(a',J') - U(a',0)) \quad (2.3.6)$$

This distinction is crucial for the ordering of optimal policies in the two environments. For instance a planner that seeks to maximize the duration of existing jobs will always set a lower level of benefits in the economy where the agent's program includes equation (2.3.6) in the constraint set, since the worker doesn't internalize the payoff to the firm in her optimal choice. On the other hand since usually such an addition trades off insurance for incentives in private policies it is

2.3.3 Other Models

How robust are the results of the previous sections to changes in the economic environment? In the first chapter of this thesis I take stock of the vast literature of search and matching models of the labour market to develop a similar policy experiment and I reach similar conclusions. I rely on a two alternative equilibrium concepts; the first one is the undirected search model of Krusell et al (2007) and Bils et al (2009(a), (2009(b)) (there the no insurance case is represented with a contract that is rebargained each period) and the second is a variant of the directed search equilibrium with assets of Acemoglu and Shimer (1999) (there no insurance corresponds to a flat wage contract). Both models have the same implication that depending on the range of private risk sharing in the economy the insurance role of public policy is different.

It is not important to summarize this work here; rather what I seek to address in this section is whether some important additions to the model that limit the insurance role of private markets, can throw off the main predictions of the previous analysis. For this I turn to a model where separations are endogenous as in the work of Wang and Williamson (2002) and as I explain this feature alone implies that the optimal contract of section 2.2.2 is fraught with moral hazard which, as is well known, trades off risk sharing for incentives to maximize the duration of the job.

The technical treatment of these issues attempted in this chapter is rather primitive. It appears that there is a growing literature that investigates the effects of self insurance in partnerships that are fraught with moral hazard or that feature incomplete information (see Karaivanov and Martin (2009) and the references therein). For one thing the assumptions about who controls investments in this context are much less innocuous than before. The previous chapter argued that fundamental violations of the Euler equation were tiny and in this sense adding marginal utility as an additional state variable (as in Abraham and Pavoni (2005) or Werning (2002)) would have no impact on optimal policies. This need not be the case here however. Further on neither the equivalence between assets and severance compensation holds, since the latter is a margin of insurance that is contingent on the observable outcome (separation) whilst the former is not.

These considerations may be important for a more thorough look at optimal policies by firms and their response to the government's scheme but this possibility is left for future work. Further on there appears to be only one paper in the literature that attempts to provide an answer to a similar question (Pissarides (2004)) but again in that analysis the principal (firm) can control all investment decisions and on top of this contracts are complete. I do think however that the model of presented in this section can be a good starting point for this research agenda.

There are two types of arrangements to consider. One derives from an environment where commitment is abundant and utility promises summarize the history of the contract and allocations are determined at date zero and specify a path for the all possible future states. This is similar to the insurance model studied in section 2.2.2. The other is an environment where commitment is scarce and allocations have to be re-bargained each period to make the optimal plan time consistent. This latter case corresponds to a Markov perfect contract whereby no variable other than the endowment of wealth of the agent can summarize the histories (see Karaivanov and Martin (2009)). I establish that under some conditions the optimal policy under this contract is a flat wage similar to the model of section 2.2.1.

The optimal public policy in these two worlds differs in the same way that it differed so far. The scope of public policy gains is reduced when private risk sharing permits agents to mitigate consumption losses in the first period of a unemployment spell. Further on with less risks (agents choose the separation rate) the importance of public insurance is dwarfed by the concern to minimize the outflow rate from employment and indeed in this case I find that in both regimes the optimal policy features near zero benefits, even when the UI scheme is constrained to last for two quarters. Finally the conclusion that the optimal timing of payments is different is preserved here.

Commitment.

Consider the program with commitment as in section 2.2.2 with the addition that employed workers can determine the separation rate $\gamma_e(s)$ as a function of the choice variable s (search intensity). Further on let $v(s)$ be the per period level of disutility suffered by the worker associated with this choice. The rest of the setup (interest rates, discounting, taxes etc) is the same as before.

Each period an employed worker's contract with her firm is summarized in the promised value J (expected value of cash flows that accrue to the firm) which is an argument in her value function, and optimal policies consist of a level of wealth for next period, a continuation value J' and a level of effort s that determines the probability of unemployment. Let $W(a,J)$ be the expected lifetime utility of the worker that solves the following functional equation:

$$W(a,J) = \max_{a' \geq \bar{a}, J', s} \log(c) - v(s) + \beta((1-\gamma_e(s))U(a',0) + \gamma_e(s)W(a',J')) \quad (2.3.4)$$

Subject to the constraint set:

$$a' = Ra + w - \tau - c \qquad J \leq y - w + \frac{\gamma(s)}{R}J'$$

$$J' \geq 0 \qquad W(a',J') \geq U(a',0)$$

This type of arrangement has wealth and search effort chosen on the same side as wages and hence it corresponds to an environment where these choices are both observable (no moral hazard) and contractible. The optimal allocation would feature (can be characterized by) a set of first order conditions for wealth and continuation payoffs to the firm (similar to those of the model of section 2.2.2) with an additional object the optimality condition for the level of s. The latter solves the following equation:

$$v'(s) = \gamma'(s)\beta(W(a',J') - U(a',0) + \frac{u'(c)}{R\beta}J') \quad (2.3.5)$$

where $u'(c)$ denotes the marginal utility of current consumption. Thus the optimal allocation (with complete information) sets search effort such that the marginal cost to the worker equals the weighted benefit to her and the firm. On the other hand when the worker's action is not observed by the firm, the optimal choice of search intensity that is incentive compatible sets this last term equal to zero and incentive compatible allocations rather satisfy the following first order condition:

$$v'(s) = \gamma'(s)\beta(W(a',J') - U(a',0)) \quad (2.3.6)$$

This distinction is crucial for the ordering of optimal policies in the two environments. For instance a planner that seeks to maximize the duration of existing jobs will always set a lower level of benefits in the economy where the agent's program includes equation (2.3.6) in the constraint set, since the worker doesn't internalize the payoff to the firm in her optimal choice. On the other hand since usually such an addition trades off insurance for incentives in private policies it is well understood that the optimal allocation under moral hazard would feature less risk sharing between workers and firms thus inducing higher gains from public policy. The overall effect is ambiguous.

These features are not central to my analysis, since the purpose here is really to get some confirmation that an alternative model yields more or less the same predictions for optimal policy as the baseline economy. Rather I report the optimal UI scheme in both cases (with and without moral hazard) and I use this result to infer the impact of unobserved actions on optimal allocations. I find only a small effect.

Markov Perfect Contracts.

I next turn to a model that features no private private risk sharing as the one laid out in section 2.2.1. There is a very straightforward way to generalize the worker's value function to include search intensity as a control variable here, but instead I attempt to connect this model with the markov perfect contracts that were discussed in the previous chapter. The reason is that this construction is instructive and permits to generalize the analysis in other equally important models of labour market frictions.

In the analogy of the commitment programs the general contracting problem here will also feature a state variable (firm's profits) that summarizes the allocation. But in markov perfect contract this object is a function of the wealth endowment of the agent and continuation policies must always conform with this object. This was the significant difference between commitment and time consistent allocations before, in particular that equilibrium payoffs are of the form $\mathcal{J}(a)$ and $W(a, \mathcal{J}(a))$, rather than simply J and $W(a, J)$ for firms and workers respectively. Then optimal allocations solve the following functional equation:

$$W(a, \mathcal{J}(a)) = \max_{a' \geq \overline{a}, J', s} \log(c) - v(s) + \beta((1 - \gamma_e(s))U(a', 0) + \gamma_e(s)W(a', \mathcal{J}(a'))) \tag{2.3.7}$$

Subject to the constraint set:

$$a' = Ra + w - \tau - c \qquad \mathcal{J}(a) \leq y - w + \frac{\gamma(s)}{R}\mathcal{J}(a')$$

Further on what determines the shares of the surplus that accrue to each party is a generalized Nash bargaining procedure where η and $1 - \eta$ are the weights attached to the firm and the worker respectively. Then the solution to $\mathcal{J}(a)$ satisfies the following requirement:

$$\mathcal{J}(a) \in \arg\max_{J} \left(W(a,J) - U(a,0)\right)^{1-\eta} J^{\eta} \tag{2.3.8}$$

where J is a generic value of the firm's payoffs (not necessarily on the equilibrium path). Clearly it must always be (at least for matches that generate positive surplus as in this model) that $\mathcal{J}(a) \geq 0$ everywhere on the state space. When η equals zero the allocation corresponds to a particular case where all the gains from search accrue to the worker and the following proposition argues that in this case the solution to equation (2.3.7) sets wages equal to productivity each period.

PROPOSITION 2.1 *In a Markov Perfect Equilibrium with* $\eta = 0$ *the only incentive compatible allocation has wages equal to productivity each period (flat wage contract).*

To see this suppose that there is a point a in the state space where payments are frontloaded (i.e. $y < w$) in the sense that the worker receives a loan from the firm that helps her build a stock of wealth. Assume without loss of generality that the worker's choice of assets for next period is a' and her current choice of search intensity s. The firm's payoff is given $\mathcal{J}(a')$ and must conform with the notion that firms break even in equilibrium (hence $\mathcal{J}(a') = 0$). But then it must be that $\mathcal{J}(a) = y - w + \frac{\gamma_e(s)}{R}\mathcal{J}(a') < 0$ which is a contradiction. [15]

Hence flat wage contracts in the current context are nothing but a time consistent allocation where the worker's bargaining power is the highest possible. When I compute the equilibrium for this model I make no use of the value functions above, but instead I add a choice of effort in the worker's value function that is similar otherwise to that in section 2.2.1. That is I simply approximate

[15] More generally the solution to the workers value function in equation (2.3.7) is characterized by the following two first order conditions:

$$v'(s) = \gamma_e'(s)(\beta(W(a',\mathcal{J}(a')) - U(a',0)) + \frac{1}{Ru'(c_t)}\mathcal{J}(a))$$

$$u'(c_t) \geq \beta\gamma_e(s)\frac{dW(a',\mathcal{J}(a'))}{da} + \beta(1-\gamma_e(s))\frac{dU(a',0)}{da} + \frac{1}{Ru'(c_t)}\mathcal{J}'(a)$$

The first equation governs the choice of search intensity and the second determines the optimal investment for the worker. Clearly under $\eta = 0$ whether these choices are observable or not by the firm makes no difference for allocations since in the case $\mathcal{J}(a) = 0$ every where on the state space.

the solution to the following functional equation:

$$W(a) = \max_{a' \geq \bar{a}, s} \log(-a' + Ra + y - \tau) + \beta(\gamma_e(s)W(a') + (1 - \gamma_e(s))U(a', 0))$$

Results.

To solve the model I use the same calibration procedure outlined in section 2.3.1. There is parameter γ_w governs the rate at which in equilibrium workers exit from jobs (the search function is $1 - e^{-\gamma_w s}$) and I find that $\gamma_w = 7.45$ for the no insurance model and $\gamma_w = 7.48$ for the commitment cases make the steady state consistent with the targets. Search costs derive from a quadratic function. The baseline UI scheme is again one that sets $b = .5$ for two quarters and no benefits afterwards.

Table 2.5 presents the levels of optimal policy in the various environments. For all models I consider the optimal value of the insurance scheme that runs for two periods, the optimal permanent policy and a flexible policy that treats differently short and long term unemployed (i.e. workers with more than six months in unemployment). The first two columns refer to the optimal policies in economies with flat (time consistent) wage contracts and the associated welfare gains, whilst columns three to six present the analogous objects for the commitment models with and without moral hazard respectively.

What is different here relative to the models of sections 2.2.1 and 2.2.2 is that part of the planner's objective is to maximize the duration of existing jobs by encouraging higher search effort by employed workers and off course this is accomplished by lowering benefits at all horizons. But the central implications of the previous section (that the two economies present the planner with a different tradeoff) seem to hold in this case as well.

Allocations under commitment feature much more risk sharing between workers and firms and optimally the planner sets lower benefits in these economies. With benefits that run for two quarters the planner optimally sets $b = .26$ for a markov perfect contract (the first row of Table 2.5) and .1 and .11 in the two commitment economies. A similar result obtains for a scheme that features permanent benefits. Further on the shape of the optimal UI scheme has again the typical shape with flat contracts and an inverted shape with private insurance.

Finally notice that between the two commitment economies, moral hazard doesn't seem to have such a big impact on the optimal policy. Despite the fact that allocations with moral hazard feature much less insurance against unemployment they also lead to inefficient separations since the worker doesn't account for the firm's payoff on her choice of effort. These effects seem to balance each other out here.

TABLE 2.5: OPTIMAL POLICY WITH ENDOGENOUS SEPARATIONS

	Markov Perfect		Commitment Moral Hazard		Commitment No Moral Hazard	
	$b,\ b_{\geq 1}$	ϵ	$b,\ b_{\geq 1}$	ϵ	$b,\ b_{\geq 1}$	ϵ
2 Quarters	(0.26 , 0.00)	.17%	(0.10 , 0.00)	.19%	(0.11 , 0.00)	.19%
Permanent	(0.19 , 0.19)	.53%	(0.12 , 0.12)	.55%	(0.13 , 0.13)	.50%
Unconstrained	(0.20 , 0.13)	.62%	(0.00 , 0.22)	.65%	(0.00 , 0.28)	.60%

It would generally take an extensive discussion to understand what private insurance does or doesn't accomplish in this simple model. I do find for instance that this model features transfers between workers and firms that are similar to the ones I showed in the previous section but here there is a choice of another variable (precautionary effort) that brings a different perspective; generally agents with lower wealth will also exert higher effort so that insurance comes from two margins. Instead the results of this section can be interpreted more broadly as indicating that different risk sharing arrangements imply different choices of optimal benefits for the planner. This is precisely the confirmation that I was looking for from this model; that an optimal policy prescription should account closely for the range of private insurance opportunities.

There is a number of other important interactions here that are beyond the scope of this analysis but still are worth mentioning. An interpretation of the search technology possessed by employed workers in the current context is that separations are both quits and layoffs, and to the extent that the government cannot distinguish between the two it is not so clear what the optimal mechanism (UI scheme) is. Do firms have superior information so that governments should minimize the interference with private markets? For instance Hopenhayn and Nicolini (2009) use a model that features both quits and layoffs to conclude the optimal UI must take into account the worker's employment history. But they don't consider how the government could optimally contract with both firms and workers when it can gain information from both sources regarding the nature of separations. This could have important implications for the optimal UI scheme and the payroll taxes paid by firms (experience rating). Providing answers to these questions is an important task for future work but this entails a more exhaustive account of the interactions between firms workers and government than this chapter explores.

2.4 Conclusions

In this chapter I illustrate that alternative assumptions about the scope of risk sharing opportunities that are available through private markets affect the prescriptions for optimal public policy. This conclusion comes out of a comparison of two economies; the first one is a standard incomplete market model with wealth accumulation, the second is a model where firms can provide additional insurance by signing contracts with their workforce subject to limited commitment.

I find that the optimal UI is considerably different between these two economies; In the presence of private risk sharing optimal benefits are lower and the optimal payment schedule is backloaded. I argue that these predictions are symptomatic of the incentive of the planner to minimize his interference with private markets.

These implications don't lack empirical relevance. Chetty (2008) finds that severance payments increase unemployment durations through a liquidity effect on the worker's search intensity and Chetty and Saez (2009) use variation in the UI benefit laws across states to estimate that a 10% increase in UI reduces severance pay by around 7%. Since empirical estimates cannot go much further than characterizing simple policy rules one important contribution of this chapter is to provide a tractable framework that can be used to evaluate more complicated policies and the associated welfare gains from government provided insurance.

2.5 Computational Appendix

No insurance program: To calibrate the model of section 2.2.1 the search functions must be recovered. In equilibrium the model has to be consistent with a target unemployment rate of 7.5% and a division of the workforce between workers that are running their first and second periods in unemployment of 70% and 15% respectively. Further on in this economy prices are fixed and taxes must be consistent with these targets and budget balance. To estimate the search parameters γ_0 and $\gamma_{\geq 1}$ I use a bisection algorithm that consists of the following steps:

1 Form an initial guess for the parameters γ_0 and $\gamma_{\geq 1}$.

2. Solve the worker's program in equations 2.2.2 and 2.2.4 in text. Standard methods can be applied here to recover the optimal policy functions. The procedure is to approximate the lifetime utilities using a grid of wealth with $n_a = 200$ unevenly spaced nodes (more nodes near the borrowing constraint). Between the nodes value functions are interpolated using cubic splines.

Given γ_0 and $\gamma_{\geq 1}$ form an initial guess for the value functions $W_0(a)$ and $U_0(a, j)$. Solve for the optimal policy rules in equations 2.2.2 and 2.2.4 and recover the new (updated) lifetime utilities $W_1(a)$ and $U_1(a, j)$. For consumption a fixed grid of equally spaced nodes is used and for search policies the grid is endogenously determined after each iteration. When the updates are close enough to the initial functions optimal policies are found.

Use a finer grid of $n_{aB} = 10000$ nodes on assets to approximate the optimal policy of the form $\{a'_{w,(a)}, a'_{u,(a,j)}\}$ and $s(a, j)$.

The final step is to compute the invariant distribution μ over the relevant state space. This can be accomplished by iterating convergence on the following equations:

3 Joint Search and Aggregate Fluctuations

3.1 Introduction

The idea that economic agents lack sufficient access to markets to insure against misfortune has been one of the founding blocks of modern macroeconomics. By now the literature that assigns a central role to heterogeneity and postulates that risk sharing is far from perfect is voluminous and has addressed most interesting aspects of macroeconomic theory (see Heathcote et al (2008) for a survey). It is not entirely clear though how far from complete markets, the actual risk sharing opportunities available to economic agents are. For instance the baseline incomplete markets paradigm builds on the assumption that households are formed by bachelor agents who, by trading claims on the aggregate capital stock, can self-insure against shocks in labour income. Over time alternative sources of insurance (either private or government provided) have been introduced to this framework, but much less common is the idea that within the family a considerable amount of employment and productivity risk diversification can be provided in the form of adjustments of the family members' labour supplies.

In this chapter we set out to achieve two ambitious and closely related goals; first we perform an accounting exercise of the differences between economies where risk sharing is limited because agents stand alone against uncertain contingencies, and those where households are formed by unions of two ex ante identical (ex post heterogeneous) members that can mutually insure against economic risks. We do so by taking stock from the vast literature of search models of the labour market and the kind of risks that arise in this environment are uncertainty about the job quality and the possibility of rationing of employment opportunities. On both these margins joint labour supply decisions present households with an array of economically meaningful opportunities that we explore.

Second (and this is our main contribution) we use the model to understand whether granting joint insurance and labour supply to couples, can help match the suggestive business cycle correlations of aggregate employment, unemployment and labour force participation. Our substantive theme here is that if recessions are periods of high incidences of unemployment or low opportunities to find work then this induces household members to search jointly and intensively to insure against potential earnings losses. By contrast in bachelor household frameworks inactive workers are either those who have experienced a sequence of bad shocks, or those who have accumulated sufficient wealth to finance leisure or both. We do not believe that either is realistic is but rather view inactivity as a state that entails the presence of a main provider at home.

In section 3.2 we use the data from the CPS to illustrate that joint insurance though adjustments of labour supplies of household members can indeed explain the low procyclicality of the US labour force. We show that if it weren't for fluctuations in the employment status of the main earner in the family (husbands in our sample), secondary earners (wives) would have a considerably more procyclical labour force participation. Further on there is a small literature on the added worker effect which, at least when it asked a similar question to ours, we found it to be conducive to our hypothesis.

Then we turn to the theory in section 3.3. We build a general equilibrium framework that features realistic frictions in the labour market, and is flexible enough to allow for a comparison between the bachelor and couples household economies. The question we ask is whether family self insurance in the model economy can match the empirical facts, or to put differently we would like to use the model as a laboratory to see how far joint labour supply can go towards matching the qualitative patterns that we find in the data.

When we introduce aggregate fluctuations however, we find that the model fails miserably on both margins. We only get some improvement in the cyclicality of unemployment (in the bachelor household model it's more procyclical) but this comes at a cost of a more volatile labour force. Further on there are virtually no gains in the correlation of the labour force with aggregate output at business cycle frequencies.

We explain that these predictions are consistent with two important failings of our model; first the benchmark economy features too few risks and too many choices to assign an important role to intra-family insurance. Second our simulations suggest that with two ex ante identical agents in the model, it is extremely difficult to match the patterns of specialization of household members in home and market work and simultaneously match the average monthly flows of the labour flows from one state to the other (the latter is a crucial target for our calibration). We argue that if a model is to fare well against the data in terms of the aggregate labour market, it must also match the persistence of the identity of the main and the secondary earners in the family. Both of these possibilities however are left to future work.

3.1.1 Related literature

This chapter is related to several strands in the literature: First a central motivation of our work is that traditional theories that rely on realistic frictions in the labour market have had a hard time to match the cyclical patterns of the labour force (see for example Veracierto (2008)). The reason is that the strong inter-temporal substitution motive that grants to these theories ample fluctuations in aggregate employment, convinces agents to flow into the labour force in good times and abandon it in bad. Further on when agents have to confront the frictions that impede instantaneous transitions between employment and unemployment these theories contain the counterfactual implication that aggregate unemployment is procyclical.

It is this implication along with the apparent a-cyclicality of the US labour force that has guided theoretical research in the field to restrict attention to models that feature only two labour market states (employment and unemployment), or equivalently feature large fixed costs of moving in and out of the labour force. In turn we argue that this runs into the difficulty of explaining why labour flows between activity and inactivity are large even at a monthly horizon, and we show that rather it is joint insurance within the household that explains the patterns that we see in the data.

Further on there has been an enormous interest on the implications of heterogeneity and incomplete insurance markets for the aggregate labour fluctuations (for example Gomes et al (2001) and Chang and Kim (2007)). All of these attempts however build on the bachelor household paradigm which is precisely our point of departure here. Interestingly Chang and Kim (2006) develop a framework where families consist of two members (a male and female) and use it to address how individual supply rules affect the value of the aggregate elasticity of labour supply. As far as incomplete markets models go this work is admittedly the closest to our intentions but many of the ingredients are different. First we emphasize the role of family in circumventing frictions in the labour market (such as the limited availability of job opportunities) whilst in CK (2006) the role assigned to frictions is secondary. Second contrasting the properties of two economies (the one with bachelor households and the ones with couples) in various environments is one of the main themes that we pursue. Most importantly none of the models of incomplete insurance markets from this literature takes up seriously on the task of matching the patterns of worker reallocation between employment unemployment and inactivity, but we do. For this reason we introduce a wealth of shocks to make our model consistent with the relevant empirical labour market flows.

There appears to be a sizeable literature that highlights the role of family labour supply as a mean of insuring against idiosyncratic labour income risks. In Attanasio et al (2005, 2008) and Heathcote et al (2008) an additional margin of insurance provided by female labour market participation becomes a valuable instrument to buffer shocks in labour income, and these papers go on to analyze the effects of various changes in the economic environment on the historical trends of female labour supply. This is not the interpretation we want to give to our story however. For all we are concerned our model is one of complete markets within the household unit and incomplete outside. More akin to our attempt is the recent work by Guler et al (2008) that characterizes the effects of joint search to optimal reservation wage policies. Relative to them, they use a stylized search model, we build a general equilibrium framework with realistic heterogeneity that accounts for the observed labour market flows as well as the effects of shocks in aggregate productivity.

The rest of this chapter is organized as follows: Section 3.2 uses the estimated flows from the CPS to provide evidence that joint insurance and labour supply are key factors that explain the low procyclicality of the US LF participation. In section 3.3, we develop the bachelor household model and the couple household model. In section 3.4, we show and discuss the basic results and implications of our theory. Section 3.5 concludes and the computational details are delegated to the appendix.

3.2 Labour Market Flows in the US

Table (3.1) summarizes the US labour market business cycle statistics. The data are constructed from the CPS and they correspond to observations spanning the years 1976 to 2005. They are logged and HP filtered and all quantities refer to quarterly aggregates and are expressed relative to a de-trended measure of GDP. Unemployment is extremely counter-cyclical and more than 6 times as volatile as aggregate output. Aggregate employment has two thirds of the volatility of output at business cycle frequencies and is very procyclical. The LF is not volatile and its contemporaneous correlation with GDP is low (.22).

TABLE 3.1: US BUSINESS CYCLE: LABOUR MARKET STATISTICS

	Employment	Unemployment	LF	LF Couples	LF Wives
	Aged 16 and Above			Aged 22 to 55	
$\frac{\sigma_x}{\sigma_y}$	0.66	6.68	0.34	0.35	0.47
$\rho_{x,y}$	.81	-.88	.22	.05	.2

The last columns of Table (3.1) present a breakdown of the relevant quantities into demographic groups that are of particular interest to us. For married couples aged 22 to 55 in our sample, aggregate statistics are no different than those of the full population (aged 16 and above). The labour force for this demographic is somewhat less procyclical (and hence even more puzzling from the point of view of theory) owning to the strong acyclical attachment of males in the sample, but also to the low contemporaneous correlation with GDP of female labour force participation. The volatility of both males (not shown) and females are higher than the aggregate volatility for this demographic group (column 4). In turn this might suggest that there is negative correlation of labour force participations of wives and husbands in our sample.

We note that this break down corresponds to an imperfect measure of our notion of couples in the model. Ideally we would like to have duads of agents that are linked with near perfect insurance opportunities and make labour supply decisions jointly, but the data preclude us from doing so. In what follows we treat household units that comprise of two spouses as an ideal ground to provide evidence for our theory.

Implications for models: Fixed participation? Are these observations consistent with the tendency of macro labour market theory to restrict attention to environments where economic agents can be either employed or unemployed at any point in time? We provide an answer to this question by looking at the monthly transitions of the US workforce across adjacent labour market states.

In Table (3.2) we summarize the relevant flows estimated from the CPS. Each month roughly 7 % of OLF (out of labour force) workers join the labour force, and 3 % of employed workers quit and become inactive. Further on to dilute the

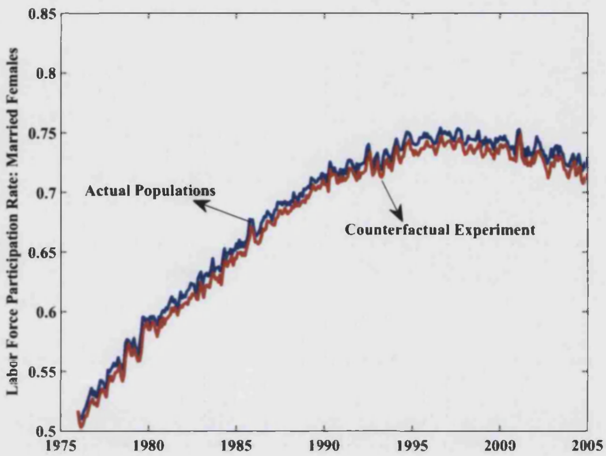

FIGURE 3.1: ACTUAL AND COUNTERFACTUAL LABOUR FORCE PARTICIPATION RATES OF MARRIED WOMEN

since for this demographic flow rates are remarkably stable over time (there is no secular trend in employment say).

TABLE 3.4: EXPERIMENTS

	Actual n_t	Actual $\tilde{n}_t$	CounterFactual $\overline{n}_t$	Actual $\tilde{n}_t$	CounterFactual $\overline{n}_t$
		One Month Horizon		Three Month Horizon	
$\frac{\sigma_x}{\sigma_y}$	.3604	.3770	.3805	.4294	.4362
$\rho_{x,y}$	.2963	.2988	.3703	.2570	.3216

In table (3.4) we summarize the results from this experiment. We compare the relative standard deviations and contemporaneous correlation of our constructed measures with a de-trended measure of GDP. The first column refers to the cyclical properties of the labour force participation rate of married wives based on the actual population measure n_t (the one we get from the data). [1] Columns 2

[1] The differences in the quantities $\frac{\sigma_n}{\sigma_y}$ and $\rho_{n,y}$ relative to Tables (2) and (3) stem from the fact that the population is normalized to unity.

suspicion that these results are driven by demographics Table (3.3) presents the analogous matrix for the sub-sample of workers aged 22 to 55.

TABLE 3.2: MATRIX FOR FLOW RATES OF AGENTS AGED ABOVE 16

	E	U	I
E	.9543	.0146	.0311
U	.2743	.4983	.2274
I	.0466	.0245	.9289

TABLE 3.3: MATRIX FOR FLOW RATES OF MARRIED COUPLES AGED 22 TO 55

	E	U	I
E	.9662	.0112	.0226
U	.2891	.5159	.195
I	.0623	.0282	.9095

A point that merits some attention is the fact that roughly 5 % of OLF workers find a job and become employed in the following month. There are two relevant possibilities: The first is that this is an immediate consequence of time aggregation since monthly horizons are more than enough for a worker to make a transition between inactivity and employment without having a recorded unemployment spell. The second pertains to the search behavior of passive searchers and marginally attached and discouraged workers. For these groups the work Jones and Riddell (1998,1999) demonstrates that they have transition probabilities into employment that are half as large as those of unemployed workers, and this implies that some of the flows between states U and I can be broadly interpreted simply as time variation in optimal search intensity for these groups. These implications have already been explored in the literature and it appears that adjusting the transition probabilities to embrace the idea that marginally attached workers should be treated as unemployed rather than inactive doesn't make a big difference in the matrices of Tables (3.2) and (3.3) (see Krusell et al (2009)).

Hence we draw two conclusions from these calculations. First that the line between economically active and inactive workers is somewhat arbitrarily drawn by the theoretical models of the labour market and second that our model, calibrated at monthly frequencies should allow all agents (independent of their labour market status) to receive job offers and experience transitions between nonemployment and employment.

How can we use the data to demonstrate our point?. One possibility would be to run limited dependent variable models (such as linear probability or probit models) and estimate the effect of the husbands employment status, on the wife's labour force transitions, and this would allow us to control for some relevant aspects of heterogeneity. Such attempts however, to determine the magnitude of the added worker effect (AWE) are numerous in the literature and we can summarize these estimates without relying on our own empirical work (we do so in the following paragraphs). Further on this kind of analysis would have very little to say about the contribution of the joint labour supply on the low procyclicality of the LF which is precisely our focal point here.

Contrasting the (cyclical) behavior singles vs couples, even after controlling for demographic characteristics, would fare no better as an alternative, since our notion of singles is a very different one from what the data could potentially suggest. In our framework singles are those agents who have an own idiosyncratic productivity and more importantly don't possess ties with any other agent in the economy that could alleviate the risk from this process. In the data unmarried agents or even those who form a household unit on their own, could have joint insurance with other agents in the economy (a broad interpretation of family) and this consideration would cloud the conclusions we could potentially draw.

Rather we treat the two spouses (husband and wife) in the household unit as the closest data analogue to our notion of partnerships with joint labour supply and insurance. Using data on individual transitions we want to test the following prediction: If it weren't for employment fluctuations over the business cycle of primary household earners, the labour force participation of secondary earners would be considerably more procyclical. We focus on individuals aged 22 to 55 and for this demographic group married agents account for roughly 60 % of the population (for the entire sample of agents aged above 16 they form 36% of all individuals). In our sample we treat husbands as primary and wives as secondary earners.

For each period t we estimate the transition probabilities of a wife from state i to state j conditional on her spouse making a transition from state k to l. We denote this object by $p_t^f(i,j,k,l)$ and analogously we let by $p_t^m(k,l)$ be the unconditional probability that the husband (and household head in our sample) makes the transition from state k to state l over the course of a month. Due to data limitations we cannot define conditional transition probabilities for all relevant labour market states. For this reason we restrict our attention to $i, j \in \{LF, OLF\}$ (that is wives can either be in the labour force or inactive) and $k, l \in \{E, N\}$ (husbands can either be employed or not). Finally we let $n_t(i, k)$ be the share of the population of couples with a secondary earner is state i and a primary earner in state k.

The evolution of these measures is central to our experiment. With the estimates $p_t^f(i, j, k, l)$, $p_t^m(k, l)$ and $n_t(i, k)$ we construct counterfactual Markov transition matrices for couples over the relevant state space $\{LF, OLF\} \times \{E, N\}$ and counterfactual populations over time. The typical element of the matrices is given $p_t^f(i, j, k, l)\overline{p}^m(k, l)$ where $\overline{p}^m(k, l)$ denotes the transition probability of the husband averaged over all periods. What we mean to accomplish by that is to have data on household transitions whereby the probability distribution of primary earners across labour market states is independent of time or, in other words, to shut down business cycle variation in the labour market flows between non-employment and employment for husbands.

First we use these matrices to construct population measures at one and three month ahead horizons. The way we do that is by feeding the actual populations n_t once and track the measures over the relevant horizon using our constructed matrices. We denote by $\overline{n}_t$ the constructed measure based on the time averaged probabilities for husbands.

Second to make our comparison meaningful we also compute populations based on the actual transition probabilities (that is without averaging) so that the typical element of the transition matrix is $p_t^f(i, j, k, l)p_t^m(k, l)$, and we let $\tilde{n}_t$ be the analogous measure under this calculation. The reason is that since there are small errors that compile over time, the comparison between $\overline{n}_t$ and $\tilde{n}_t$ is much more meaningful than between $\overline{n}_t$ and n_t.

Figure (3.1) plots the labour force participation rate (based on measure n_t that we draw from the data) for wives over the sample period with the three month ahead counterfactual time series (based on $\overline{n}_t$). Reassuringly the correlations between actual and counterfactual measures is high above .99 at our longest horizon. The correlation between n_t and $\tilde{n}_t$ is even higher. Notice that this high correlation is a direct consequence of the fact that averaging out the transition probabilities of husbands over the years 1976 and 2005 (as opposed to using any other method of eliminating business cycles) involves no loss of generality

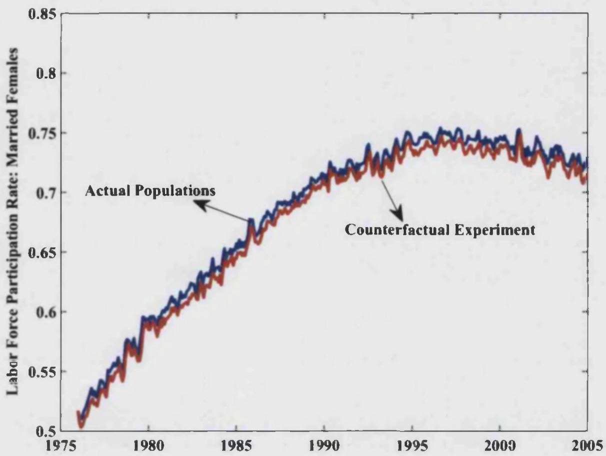

FIGURE 3.1: ACTUAL AND COUNTERFACTUAL LABOUR FORCE PARTICIPATION RATES OF MARRIED WOMEN

since for this demographic flow rates are remarkably stable over time (there is no secular trend in employment say).

TABLE 3.4: EXPERIMENTS

	Actual n_t	Actual $\tilde{n}_t$	CounterFactual $\overline{n}_t$	Actual $\tilde{n}_t$	CounterFactual $\overline{n}_t$
		One Month Horizon		Three Month Horizon	
$\frac{\sigma_x}{\sigma_y}$	.3604	.3770	.3805	.4294	.4362
$\rho_{x,y}$	.2963	.2988	.3703	.2570	.3216

In table (3.4) we summarize the results from this experiment. We compare the relative standard deviations and contemporaneous correlation of our constructed measures with a de-trended measure of GDP. The first column refers to the cyclical properties of the labour force participation rate of married wives based on the actual population measure n_t (the one we get from the data). [1] Columns 2

[1]The differences in the quantities $\frac{\sigma_n}{\sigma_y}$ and $\rho_{n,y}$ relative to Tables (2) and (3) stem from the fact that the population is normalized to unity.

loss of income due to unemployment translates into a drop in consumption), when the wife's contribution to household resources is significant (so that the insurance role of labour supply adjustments of secondary earners is important) and when the household production (or leisure) technology allows for substitutability between the time inputs of the household members. Below we explain why our framework bodes well with these requirements.

3.3 The model

We develop two related models in which households face uninsurable idiosyncratic labour income risk. In the first model a household consists of one agent, a bachelor. In the second model, and this is our key contribution, a household consists of two agents, a couple, who share their income risk.

3.3.1 Bachelor economy

We consider an economy populated by a unit mass of strictly risk averse bachelor households that are identical in preferences and value the consumption of a general multipurpose market good c. We denote the discount factor for these agents by β_S and the period utility deriving from consumption by $u(c)$.

At any point in time a household member can be either employed, unemployed or not part of the labour force and we assume that labour supply decisions are formed at the extensive margin and are subject to the frictions that impede instantaneous transitions across these adjacent labour market states. In particular employed agents spend a fraction $\overline{h}$ of their unitary time endowment each period in market activities associated with a utility cost which we denote by $\Phi(\overline{h})$. For non employed agents we assume that job availability in the economy is limited: We endow them with a technology that transforms units of search effort s into arrival rates of job opportunities $p(s)$ at a cost $k(s)$ per unit of time. As we elaborate below on the basis of these optimal choices, we classify household members as either unemployed (active searchers) or out of labour force workers.

Further on we assume that households face idiosyncratic labour productivity risks and we summarize this in two independent stochastic processes ϵ and to 3 and 4 to 5 compare the analogous objects based on the measures $\tilde{n}_t$ and $\overline{n}_t$, for one and three months horizons respectively. As the horizon expands the errors that compile over time make the processes display considerably more volatility. The result however is both qualitatively and quantitatively encouraging. The cyclical correlation of labour force participation for wives jumps from .2988 to .3703 in columns 2 and 3 and from .257 to .3216 in columns 4 and 5 (which roughly corresponds to a 25% increase in cyclicality). Further on in light of this higher correlation with GDP we can argue that the increase in volatility from $\tilde{n}_t$ to $\overline{n}_t$ is mostly due to the business cycle.

We give the following interpretation for this result; If the US economy was populated by bachelor households then the labour force would be substantially more volatile and procyclical. Off course this conclusion is reached rather prematurely we are unable to control for observed heterogeneity and our notion joint insurance cannot be perfectly captured by couples. We can only do so much as to summarize a related literature below that has estimated the magnitude of the added worker effect and when it asked a similar question to ours we found it to be conducive to our hypothesis.

The literature on added worker effects. We give a brief summary of a related literature that uses panel data to investigate the effect of income shocks experienced by the husband on the spousal supply of labour. Our reading suggests that at least with respect to data and methodology there are three strands in this literature.

First there are models that use variation in annual hours of work to identify how the husband's recorded unemployment spells affect the wife's labour supply. There doesn't appear to be a consensus in this empirical work for the magnitude of the AWE. For instance Heckman and MaCurdy (1980) find a small but significant AWE but the work of Pencavel (1982) doesn't. The reason for this is twofold. First there are other forms of insurance that minimize the loss of income due to an unemployment spell, and the work of Culen and Gruber (2000) shows that unemployment benefits do indeed have a massive crowding out effect on family self insurance. Second more recently Stevens (2001) argues that the empirical literature fails to identify unemployment spells that result in substantial earnings losses (essentially the distinction of job leavers and job losers) and he shows that for displaced workers family insurance does have an important role.

There is a recent subset of studies that focus on the responses of spousal labour supply to shocks other than unemployment (health shocks in particular) such as Gallipoli and Turner (2008(a), 2008(b)) for Canada and Coyle (2004) for the US. This work documents the complete lack of AWEs although in the context of health shocks this lack of mutual insurance has an obvious interpretation; since disability and health shocks entail an intra-household transfer of time (that allows wives to 'care' for the their ill spouse) they are unable to increase hours in the market to make up for the lost income.

What is more related to our story is the subset of studies that use short run transitions across labour market states (employment, unemployment and inactivity). These studies tend to find significant added worker effects even when controlling for observed heterogeneity (which is missing from our experiment). Lundberg (1985) uses monthly employment histories from a sample of the Seattle and Denver Income Maintenance experiments to conclude that if a husband is unemployed then the probability that the wife enters the LF increases by 25 % and the probability of leaving the LF is 33 % lower. The wives are also 28 % less likely to leave employment for unemployment. Furtheron Speltzer (1997) uses a sample from the CPS monthly data and estimates limited dependent variable models of the probability that wives enter the LF on demographics and the husbands employment transitions. His estimates show that there is an important AWE even when observable heterogeneity is taken into account [2].

We conclude this section by noting that when the relevant literature (on the AWE) has asked the 'right' question the answer has been conducive to our intuition. It is clear that insofar as monthly transitions between labour market states are concerned joint insurance is important and further on our own empirical work illustrates that it's important in explaining the cyclical patterns of the US labour force. Further on if there is anything to be taken from the literature on spousal labour supply that can guide us in building the right theory its the following; the AWE is more pronounced when markets are incomplete (so that the

[2] The sample used by Speltzer (1997) spans the months June to December in the years 1988 - 1989 and 1990 -1991 which is a much smaller range than what we use, and off course it is an entirely different empirical perspective. Our approach is much more similar to Lundberg (1985) who after estimating the transition probabilities plots the impulse response functions of a spousal hours and labour force participation when the husband's unemployment rate falls by 5 percentage points. She gets a similar result to us. Further on Speltzer (1997) shows that there are two variables that drive the AWE to be near insignificant; these are the previous year unemployment spells of the husband and the previous year LF participation of the wife. He interprets this as evidence of a spurious AWE due to assortative mating, but it is also consistent with the AWE in the data being driven by couples that can use more readily the family self insurance margin (and this shows as higher propensity to experience transitions between labour market states).

loss of income due to unemployment translates into a drop in consumption), when the wife's contribution to household resources is significant (so that the insurance role of labour supply adjustments of secondary earners is important) and when the household production (or leisure) technology allows for substitutability between the time inputs of the household members. Below we explain why our framework bodes well with these requirements.

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Further on we assume that households face idiosyncratic labour productivity risks and we summarize this in two independent stochastic processes ϵ and

x. The former (ϵ) is an agent specific process (an own labour productivity component) that is a persistent state variable in the agents value function independent of her labour market status. The latter x is a job specific component that pertains to the quality of active jobs and available job opportunities in the economy. These objects evolve stochastically over time according to the transition cumulative distribution functions $\pi_{\epsilon',\epsilon} = Pr(\epsilon_{t+1} < \epsilon', \epsilon_t = \epsilon)$ and $\pi_{x',x} = Pr(x_{t+1} < x', x_t = x)$ respectively. Further on we assume that the initial assignment of job quality x derives from a general density $H(x)$.

Financial markets are incomplete and agents can self insure by trading non contingent claims on the aggregate capital stock, earning a return R_t each period, subject to an ad hoc borrowing limit $a_t \geq \overline{a}\ \forall t$. Wages per efficiency units of labour w_t as well as rental rates R_t are determined in competitive markets where it is assumed that a representative firm aggregates all inputs into a multipurpose final good. The technology is of the standard form $Y_t = K_t^{\alpha}(L_t\lambda_t)^{1-\alpha}$ where capital K_t depreciates at rate δ each period and $L_t = \int\int\int \epsilon x h_{a,\epsilon,x} I_{h_{a,\epsilon,x}=\overline{h}} d\Gamma_t$ denotes the aggregate efficiency units of the labour input. Finally Γ_t is the density over the relevant state space (of employment status, productivity and wealth) and λ_t is the TFP process which evolves according to the non-stochastic transition cdf $\pi_{\lambda'|\lambda'} = Prob(\lambda_{t+1} < \lambda'|\lambda_t = \lambda')$. The law of motion for the distribution of workers is defined as: $\Gamma_{t+1} = \mathcal{T}(\Gamma_t, \lambda_t)$ where $\mathcal{T}$ is the relevant transition operator.

The timing of events. Each period t (and after the resolution of all relevant uncertainty) a non-employed agent chooses optimally the number of search units s_t to exert and finances her consumption out of the current stock of savings. Her choice of s_t maps into a probability $p(s_t)$ of receiving a job offer in the next period. When this opportunity arrives the new value ϵ_{t+1} and the value x_{t+1} are sampled and the aggregate state vector $\{\Gamma_{t+1}, \lambda_{t+1}\}$ is revealed and the agent will decide whether she wants to give up search and become employed. Notice that given that all jobs entail a fixed cost $\Phi(\overline{h})$ the realization of the relevant state vector might not be such that the prospective match (job) generates a positive surplus for the worker. In that case the agent continues to search in the labour market.

Similarly for an employed agent the sampling of the new values for x_{t+1} and ϵ_{t+1} generates the risk of separation. In this case the worker may decide that it is not worthwhile to spend $\overline{h}$ of her time working and would rather search for new opportunities next period. For this worker optimal consumption and savings decisions are borne out of the stock of wealth and labour earnings, conditional on her keeping her current employment status.

Value functions. Consider the problem of an agent with a stock of wealth a_t and a productivity endowment ϵ_t who is currently non employed. She must optimally allocate resources between current consumption and savings and choose the number of units of search effort to exert to maximize her well-being. In the notation we let V^n be the lifetime utility for this worker. We also define an auxiliary object $Q^e = \max\{V^n, V^e\}$ which is the outer envelope over the relevant menu of choices for this worker conditional on her receiving a job offer next period. Applying standard arguments we can represent her program recursively as:

$$\begin{aligned} V^n(a,\epsilon,\Gamma,\lambda) &= \max_{a'\geq\bar{a},s} u(c) - k(s) + \beta_S(\int_{\epsilon',\lambda'} p(s) \int x' Q^e(a',\epsilon',x',\Gamma',\lambda'))\, d\, H(x') \\ &+ (1-p(s))\, V^n(a',\epsilon',\Gamma',\lambda')\ \ d\pi_{\epsilon'|\epsilon} d\pi_{\lambda'|\lambda} \qquad (3.3.1) \end{aligned}$$

Subject to the constraint set:

$$a' = R_{\lambda,\Gamma} a - c \qquad (3.3.2)$$

Notice that the distribution Γ becomes a state variable in the worker's value function. In order to forecast prices in the current context and to make optimal savings and labour market search decisions knowledge of Γ' is necessary since this object determines the economy's aggregate capital stock and effective labour in the next period. [3]

In a similar fashion we can represent the employed worker's lifetime utility as a solution to the following functional equation:

[3]We use primes to denote next period variables. Furtheron we chose to use integrals instead of the conventional expectation operators to clarify that the relevant uncertainties faced by employed and non employed workers differ in the current context. The initial draws of x derive from the general distribution $H(x)$ and the continuation match qualities are determined by $\pi_{x'|x}$ so that in general:

$$(\int_{x'} Q^e(a',\epsilon',x',\Gamma',\lambda'))d\pi_{x'|x} d\, x') \neq \int_{x'} Q^e(a',\epsilon',x',\Gamma',\lambda'))\, d\, H(x')$$

.

$$V^e(a,\epsilon,x,\Gamma,\lambda) = \max_{a'\geq\overline{a}} u(c) - \Phi(\overline{h})$$
$$+ \quad \beta_S\Big(\int_{\epsilon',\lambda',x'} Q^e(a',\epsilon',x',\Gamma'|\lambda')\Big)\ d\pi_{\epsilon'|\epsilon} d\pi_{\lambda'|\lambda} d\pi_{x'|x}\Big) \tag{3.3.3}$$

$$a' \quad = \quad R_{\lambda,\Gamma}a + w_{\lambda,\Gamma}\overline{h}\epsilon x - c \tag{3.3.4}$$

.

A few comments are in order here: First our classification criterion for nonemployed workers is of the following form:

$$\text{IF} \quad s^* \begin{cases} < s_{min} & \text{Worker is OLF} \\ \geq s_{min} & \text{Worker is Unemployed} \end{cases}$$

That is to say that we classify a worker as unemployed if she chooses effort above a given threshold s_{min}, and as out of the labour force otherwise. This mapping is consistent with the notion that inactive agents search less intensively in the labour market and as coarse as it may seem it is very close to the analogous criterion used by the CPS. [4]

Further on we normalize the value of income for both unemployed and OLF workers to zero so that their consumption is financed exclusively out of the stock of savings. This assumption is made mainly to avoid the complications of having to talk about eligibility in government insurance schemes as it is not clear how benefits would be distributed across the population. For instance inactive workers in principle should not receive any sort of replacement income but in our model there is a considerable amount of mobility between the two non employment states. In turn keeping track of benefit histories would add to the computational burden of our exercise without being clear how it would affect the main results. [5]

[4] More specifically the CPS classifies non employed workers on the basis of the following algorithm. First a non-employed respondent is asked whether he would like to have a job. Those who reply 'no' are automatically considered as OLF workers. Those who reply 'yes' are then asked to indicate what steps they have taken towards finding employment in the previous month, and in particular they are asked to outline their methods of search (there are twelve such methods). Those that have not searched but also those who have not exerted sufficiently active search effort are classified as OLF workers. Further on active search effort consists of using any of the proposed methods of search other than or beyond reading newspaper adds. See Shimer (2003) for further details.

[5] Arguably the unemployment insurance in the current context would crowd out family self-

Competitive Equilibrium

The equilibrium consists of a set of value functions $\{V^n, V^e\}$, and a set of decision rules for consumption, asset holdings ($a'_e(a, x, \epsilon, \lambda, \Gamma)$ and $a'_n(a, \epsilon, \lambda, \Gamma)$), search ($s(a, \epsilon, \lambda, \Gamma)$) and labour supply ($h(a, x, \epsilon, \lambda, \Gamma)$). It also consists of a collection of quantities $\{K_t, L_t\}$ and prices $\{w_t, R_t\}$ and a law of motion of the distribution $\Gamma_{t+1} = T(\Gamma_t, \lambda_t)$ such that:

- Given prices households solve the maximization program in 3.3.1 and 3.3.3 and optimal policies derive.
- The final goods firm maximizes its profits:

$$w_t = K_t^{\alpha} \lambda_t^{1-\alpha} L_t^{-\alpha} \quad \textbf{And} \quad r_t = K_t^{-\alpha} \lambda_t^{1-\alpha} L_t^{1-\alpha}$$

- Goods and factor markets clear:

$$Y_t + (1-\delta)K_t = \int \mathcal{I}_{h=\bar{h}}(a'_w(a_t, \epsilon_t, x_t, \Gamma_t, \lambda_t) + c_w(a_t, \epsilon_t, x_t, \Gamma_t, \lambda_t)) \; d\Gamma_t$$
$$+ \int \mathcal{I}_{h=0}(a'_n(a_t, \epsilon_t, \Gamma_t, \lambda_t) + c_n(a_t, \epsilon_t, \Gamma_t, \lambda_t)) \; d\Gamma_t \quad \textbf{Resource Constraint}$$

$$L_t = \int \epsilon x h_{a,\epsilon,x,\lambda,\Gamma} \mathcal{I}_{(h_{a,\epsilon,x,\lambda,\Gamma}=\bar{h})} \; d\Gamma_t \quad \textbf{Labour Market}$$

$$K_t = \int a_t \; d\Gamma_t \quad \textbf{Savings Market}$$

- Individual behavior is consistent with the aggregate behavior.[6]

---

insurance (see Cullen and Gruber (2000)) but it would also crowd out the precautionary role of assets (see Engen and Gruber (2001)) . Further more although empirically one effect may not make up for the other it seems to be the case for the incomplete market model that we use here. For instance Young (2004) finds that the optimal level of UI in an economy with search frictions is always zero and part of the explanation is that in general equilibrium wealth accumulation minimizes the utility costs from a lack of buffer provided by the government. In the context of our model it seems likely that introducing UI would only shift the regions in the state space where all the action takes place without any significant impact on the main conclusions. On the other hand eliminating UI can be interpreted as a necessary feature of our analysis because we want to get the maximum AWE we can out of the model.

[6]The law of motion of the measure Γ can be represented as follows:

3.3.2 Couples economy

We introduce households that consist of two members in the economy retaining as many elements from the singles environment as possible. In particular we have a measure one of agents (so a total mass a half of households) and each one of them is endowed with a unit of time. Household members derive utility from consumption and the felicity function is given again by the general form $u(c_t)$. We denote the time preference parameters for households in this case by β_C.

As far as intra-household allocations are concerned we adopt the unitary model whereby the household as a whole is treated as a decision unit and the members share the same common utility function; income and wealth are pooled and consumption and labour supply or search decisions are formed jointly to maximize the households well being. Each agent in the economy has her own idiosyncratic productivity and consequently household members differ in their productive endowments and we denote by ϵ_t and x_t the vector of productivities of the members of a generic household. Further on to conserve on the notation we let $\Pi_{\epsilon'|\epsilon}$ be the joint cdf for the household members own productivities.

Having labour supply decisions formed jointly within households that comprise of two members gives rise to opportunities of specialization in market and non-market work that were absent in a world of bachelor agents. Ideally a household would like to have at any point in time, the most productive agent in the market but it cannot do so without confronting the frictions that impede instantaneous transitions across labour market states. In what follows we adopt the convention that the array (k, l) $k, l \in \{E, N\}$ denotes a household whose first and second

---

$$
\begin{aligned}
\Gamma'_e(\mathcal{A}, \mathcal{E}, \mathcal{X}) &= \int_{a'_e \in \mathcal{A}, \epsilon' \in \mathcal{E}, x' \in \mathcal{X}} \mathcal{I}_{h(a', \epsilon', x', \Gamma', \lambda') = \bar{h}} \; d\pi_{\epsilon'|\epsilon} d\pi_{x'|x} d\Gamma_e \\
&+ \int_{a'_n \in \mathcal{A}, \epsilon' \in \mathcal{E}, x' \in \mathcal{X}} \mathcal{I}_{h(a', \epsilon', x', \Gamma', \lambda') = \bar{h}} \; p(s(a, \epsilon, x, \Gamma, \lambda)) \, d\pi_{\epsilon'|\epsilon} \, dH(x') \, d\Gamma_n
\end{aligned}
$$

$$
\begin{aligned}
\Gamma'_n(\mathcal{A}, \mathcal{E}) &= \int_{a'_e \in \mathcal{A}, \epsilon' \in \mathcal{E}} \mathcal{I}_{h(a', \epsilon', x', \Gamma', \lambda') = 0} \; d\pi_{\epsilon'|\epsilon} d\pi_{x'|x} d\Gamma_e \\
&+ \int_{a'_n \in \mathcal{A}, \epsilon' \in \mathcal{E}} \mathcal{I}_{h(a', \epsilon', x', \Gamma', \lambda') = 0} \; p(s(a, \epsilon, x, \Gamma, \lambda)) d\pi_{\epsilon'|\epsilon} dH(x') d\Gamma_n \\
&+ \int_{a'_n \in \mathcal{A}, \epsilon' \in \mathcal{E}} \mathcal{I}_{h(a', \epsilon', x', \Gamma', \lambda') = 0} \; (1 - p(s(a, \epsilon, x, \Gamma, \lambda))) d\pi_{\epsilon'|\epsilon} dH(x') d\Gamma_n
\end{aligned}
$$

Where Γ_n and Γ_e denote the marginal cdfs for non-employed and employed workers respectively and $\mathcal{A}$ $\mathcal{E}$ $\mathcal{X}$ are subsets of the relevant state space.

members are in state k and l respectively. Also it will prove useful to define the following objects beforehand:

$$
\begin{aligned}
Q^{en} &= \max\{V^{nn}, V^{en}\} && (3.3.5)\\
Q^{ne} &= \max\{V^{nn}, V^{ne}\} && (3.3.6)\\
Q^{ee} &= = \max\{Q^{en}, Q^{ne}, V^{ee}\} && (3.3.7)
\end{aligned}
$$

These objects define the relevant menu of choices for our households. For instance a household with one employed member can in any given period decide to withdraw her from the labour market and allocate both agents to search. This option is described in equation (3.3.5). Analogously in (3.3.7) a household with both members employed, can withdraw them to non-employment, or keep one working or both. With these definitions we can represent the dynamic programming problem of a household with two non-employed members as:

$$
\begin{aligned}
V^{nn}_{a,\epsilon,\lambda,\Gamma} &= \max_{a'\geq\bar{a},s_1,s_2} \quad u(c_t) - \sum_i k(s_i)\\
&+ \beta_C\Big(\int_{\epsilon',\lambda'} p(s_1)p(s_2)\int_{x_1',x_2'} Q^{ee}(a',\epsilon',x_1',x_2',\lambda',\Gamma')dH(x_1'), dH(x_2')\\
&+ p(s_1)(1-p(s_2))\int_{x_1'} Q^{en}(a',\epsilon',x_1',\lambda',\Gamma')dH(x_1')\\
&+ p(s_2)(1-p(s_1))\int_{x_2'} Q^{ne}(a',\epsilon',x_2',\lambda',\Gamma')dH(x_2')\\
&+ (1-p(s_2))(1-p(s_1))Q^{nn}(a',\epsilon',\lambda',\Gamma')d\pi_{\epsilon'|\epsilon}d\pi_{\lambda'|\lambda}\Big) && (3.3.8)
\end{aligned}
$$

subject to:

$$
a' = R_{\lambda,\Gamma}a - c \qquad (3.3.9)
$$

Optimal choices for these agents consist of current consumption and a pair of search intensity levels. Note that nothing precludes household members from setting $s_i \neq s_j$ although with standard convexity assumptions this can only be the case if the productivity endowments ϵ_i and ϵ_j are unequal. Further on with probability $p(s_1)p(s_2)$ both members receive an offer and the sampling from the distribution of qualities $H(x)$ is independent. Both joint search coupled with the limited availability of job opportunities, and the independent sampling introduce risk sharing possibilities to households (through adjustments of labour supply) that were non-existent in the singles economy.

The lifetime utility for a household with the first member employed solves the following functional equation:

$$
\begin{aligned}
V^{en}_{a,\epsilon,x_1,\lambda,\Gamma} &= \max_{a'\geq\bar{a},s_2} \quad u(c_t) - k(s_2) - \Phi(\bar{h}) \\
&+ \quad \beta_C\Big(\int_{\epsilon',\lambda'}(p(s_2)\int_{x_1',x_2'} Q^{ee}(a',\epsilon',x_1',x_2',\lambda',\Gamma')d\pi_{x_1'|x_1}dH(x_2') \\
&+ \quad (1-p(s_2))\int_{x_1'} Q^{en}(a',\epsilon',x_1',\lambda',\Gamma')d\pi_{x_1'|x_1})d\pi_{\epsilon'|\epsilon}d\pi_{\lambda'|\lambda}\Big)
\end{aligned}
\tag{3.3.10}
$$

$$
a' = R_{\lambda,\Gamma}a + w_{\lambda,\Gamma}\bar{h}x_1\epsilon_1 - c \tag{3.3.11}
$$

For the sake of brevity we omit object V^{ne} since the recursive representation is similar to that of equation (3.3.10). Finally for a household with both members employed we can write:

$$
V^{ee}_{a,\epsilon,x_1,x_2,\lambda,\Gamma} = \max_{a'\geq\bar{a}} \quad u(c_t) - \sum_i \Phi(\bar{h}) \tag{3.3.12}
$$

$$
+ \quad \beta_C\Big(\int_{\epsilon',\lambda'}\Big(\int_{x_1',x_2'} Q^{ee}(a',\epsilon',x_1',x_2',\lambda',\Gamma')d\pi_{x_1'|x_1}d\pi_{x_2'|x_2}\Big)d\pi_{\epsilon'|\epsilon}d\pi_{\lambda'|\lambda}\Big)
$$

$$
a' = R_{\lambda,\Gamma}a + w_{\lambda,\Gamma}\bar{h}\sum_i x_i\epsilon_i - c \tag{3.3.13}
$$

Competitive Equilibrium

The definition is similar to the one in section 3.3.1 and for the sake of brevity is omitted.

3.3.3 Discussion

Our story is similar to Chang and Kim (2006, 2007) and Gomes Greenwood and Rebelo (2001) who use models of heterogeneous agents with aggregate uncertainty and assess their labour market implications. There as well as in our case the distribution of match (job) rents is governed by the idiosyncratic productivity endowments and according to their realizations, each period agents adjust their labour market status. To this framework we add the following ingredients: We introduce both own productivity shocks ϵ and match quality shocks x and we assume that search in the labour market is subject to a technology that maps search effort s into arrival rates of job offers $p(s)$. We devote a few paragraphs to discuss why we think these additions are crucial.

Why do we need a rich structure of shocks? The answer here is simple. Without them we wouldn't be able to match the worker flows which we summarize in Tables 3.2 and 3.3. Since our model has to disassociate the behavior of agents who make frequent transitions between employment and unemployment from those who move in and out of the labour force it is imperative that we introduce both own productivity and match quality shock. For instance in our calibration we choose the moments of the two processes in such as way so that the transitions between unemployment and employment are governed by the x type shocks and those between unemployment an inactivity by the ϵ. Further on decomposing the overall labour market risk in these two processes seems to be empirically relevant since in the data firm effects as well as individual effects both account for substantial fractions of the individual earnings uncertainty (see Abowd et al (1999)).

The search technology. We adopt a very parsimonious representation of the search technology. In particular we assume that there two levels of search intensity that a worker can exert $s \in \{s_I, s_U\}$ where the subscripts I and U stand for inactive (out of labour force) and unemployment (active searchers) respectively. Associated with these choices are the following probabilities of receiving a job offer next period:

$$p(s) = \begin{cases} p_I & \text{if } s = s_I \\ p_U & \text{if } s = s_U \end{cases}$$

Further on the search costs are assumed to be of the form: $k(s) = 0$ if $s = s_I$ and $k(s) = k$ if $s = s_U$. These discrete choices are enough to capture our division between workers that search actively, and hence are counted as unemployed, and those whose optimal choice of search does not translate into a large enough contact rate with potential employers and hence are considered out of the labour force workers. Adding more thresholds would in general complicate things for us by requiring that the model be consistent with a larger set of targets. For instance if we were to include two thresholds of search for inactive workers we would have to make the model match the populations of agents who don't search at all (and this is a large fraction of respondents in the CPS) and those who do search albeit in a passive way. We don't believe that these considerations are important and that they would impact the results. Notice that there is in general nothing that precludes us from setting ($p_I = 0$) but in principle to match the observed flows from inactivity to employment in our model's horizon it must be that $p_I > 0$.

We give the following interpretation to our technology: p_U and p_I are treated as technological upper bounds to the number of matches that are possible each period from states U and I respectively. When we increase the values of these parameters we also need to increase the variance of the x shocks to keep the transition rates close to the data, since by the standard intuition a mean preserving spread in a match quality distribution would make searchers more selective. Generally we set $p_U << 1$ for our main result the reason being that with limited job availability we want give couples meaningful insurance opportunities against unemployment spells. Further on these bounds must not be too tight since in our model these probabilities are constant over the business cycle. If say we were to set $p_U = .28$ (the steady state UE rate in the data) there would be no room for an increase job finding rates when the expansion comes, and unemployment in the economy would be counter-factually procyclical.

This last point merits some attention. If in our model the flows between labour market states were governed by the firms' willingness to create jobs over the business cycle (as in Mortensen and Pissarides (1994)) then the probabilities p_I and p_U would change over time. However the implications of such a model would be no different from ours, since search and matching models generate procyclical search intensity (so agents would flow from inactivity to unemployment) which is precisely what we want to avoid by introducing couples. Further on our model generates endogenous separations and job finding by virtue of the processes x and ϵ and the fixed cost of participation in the labour market. Whether firms bear the costs of investment in search (as in the Mortensen and Pissarides framework) or workers as we assume here is completely irrelevant. The only thing that matters is how these investments change over the business cycle.

3.4 Calibration and Baseline Results

3.4.1 Parametrization

We briefly discuss our choice of parameters and functional forms: We adopt a period utility function of the form:

$$u(c_t) = \log(c_t)$$

Following Chang and Kim (2007) we set the disutility from working equal to $B\frac{\bar{h}^{1+\gamma}}{1+\gamma}$ and we normalize γ to unity (this is unimportant in the current context). Parameter B is chosen to target the average employment population ratio of 60 % in the data. Since we draw no distinction between male and female population in the economy we don't have to worry about matching the division of employment between these two demographics and we set the disutility of labour for a household that comprises of two employed members equal to $2B\frac{\bar{h}^{1+\gamma}}{1+\gamma}$. We do however show how the model fares in terms of the specialization of home vs market activity in primary and secondary earners against the data.

For the search technology we set $p_U = .5$ and $p_I = .1$ in our benchmark which given the empirical labour market flows seem like reasonable values. The cost of search for unemployed workers k is chosen to target the fraction of the population of nonemployed workers that are unemployed (i.e. those that set $s = s_U$). In the US data the unemployment rate is on average 5.5 % over our sample period.

Given that the model's horizon is one month we fix the time preference parameter for couples β_C to .995 and the depreciation rate δ to .0083. These values turn out to be roughly consistent with an (average steady state) interest rate $R = 1 + r - \delta$ of 1.0041 (a yearly analogue of 5 %). The discount factor for singles β_S is chosen so that the produced capital labour ratios (and hence the interest rates) in the two economies are equal.

Further on the share of capital to value added α is calibrated to .33 and we assume that the employed agents spend roughly a third of their time endowment in market work (hence we set $\overline{h} = 0.33$). Following Chang and Kim (2007) the aggregate TFP process is calibrated such that the quarterly first order autocorrelation is $\rho_\lambda = 0.95$ and the conditional standard deviation $\sigma_\lambda = 0.007$. Table 3.5 summarizes these choices.

Finally our idiosyncratic labour productivity processes are of the following form:

$$\log(x_t) = \rho_x \log(x_{t-1}) + v_{x,t}$$
$$\log(\epsilon_t) = \rho_\epsilon \log(\epsilon_{t-1}) + v_{\epsilon,t}$$

These choices is guided by the relevant literature that uses similar representations of the stochastic process of labour income (see Heathcote et al (2008)). Further on we assume that the innovations are mean zero processes (i.e. $v_{x,t} \sim \mathcal{N}(0, \sigma_x)$ and $v_{\epsilon,t} \sim \mathcal{N}(0, \sigma_\epsilon)$).

TABLE 3.5: THE MODEL PARAMETERS (QUARTERLY VALUES)

Parameter	Symbol	Baseline
std of TFP shock	σ_λ	0.007
AR1 of TFP shock	ρ_λ	0.95
Share of capital	α	0.33
Depreciation rate	δ	0.025
Discount Factor Couples	β_C	0.995
Fraction of time working	$\overline{h}$	0.33
Offer Rate: OLF	p_I	.1
Offer Rate: Unemployed	p_U	policy paratmeters
Labor Disutility	B	
Discount Factor Singles	β_S	
Search cost	k	jointly calibrated
Moments of x	σ_x, ρ_x	(see text)
Moments of ϵ	σ_ϵ, ρ_ϵ	

Our calibration procedure is as follows: For each one of the models (singles, couples) we choose the moments of the idiosyncratic productivity processes ρ and σ along with B and k to match the observed labour market flows. We have six parameters for six targets but this doesn't mean that we can match the worker flows perfectly. It turns out that there is a range of the ρ and σ parameters where the models perform well in some dimensions and bad in some others. In turn we set our targets so that fit is good and the calibration matches aspects of the data that are really important for our exercise. As we explain our couples economy is able to match the flow rate from unemployment to employment and the flows in and out of the labour force. What it cannot match is the division between the EU and EI flows, given a total outflow rate from employment.

Solution method

We solve the model with aggregate uncertainty using the bounded rationality approach whereby agents forecast future prices using a finite set of moments of the distribution Γ_t. As in Krusell and Smith (1998) we find that first moments (means) are sufficient for very accurate forecasts in our context (approximate aggregation holds) . A detailed description of the algorithm is delegated to the appendix.

3.4.2 Steady State Findings

We use this section to provide information on the models' performances in a number of relevant dimensions. In Table 3.6 we summarize the estimated worker

TABLE 3.6: ESTIMATED LABOUR MARKET FLOWS: SINGLES VS COUPLES

	Bachelor Households			Couples Households		
	E	U	I	E	U	I
E	.9507	.00432	.0450	.9515	.00567	.0428
U	.2801	0.5831	.1368	.2830	.5051	.2119
I	.0503	.0322	.9175	.0507	.0381	.9112

flows from the bachelor and the couple economy. We use Table 3.2 for our targets (so the flows for all agents that are aged 16 and above independent of marital status) because when we compare the business cycle properties of our economies relative to the data we don't have aggregate statistics (say output) for different demographic groups. In both cases the decomposition between movements in and movements out of labour market states is such that the model output is consistent with an employment population ratio to 60 % rate an unemployment rate to 5.5 % and a outflow rate from unemployment to employment of 28 % which is what we find in the data.

Both models can match the total outflow from employment to non-employment but the composition between the number of workers who leave their jobs to search intensively (unemployed) and those who leave their jobs but don't is off targets. In particular in the data the EU rate is around 1.4 % on average and the EI is 3.11 % but even the couples economy produces values for these objects of .567 % and 4.28 % respectively.

Further on a striking difference in terms of the performance of the two models is the resulting UI flows. We find that in the data the couple household economy can easily attain a target of 21 % (which is the data counterpart for this quantity) whilst with bachelor households the best we can do is a value of 14 %. This discrepancy is at the center of our notion of joint insurance here. In the steady state there is a large fraction of families where one member is employed and another not and also a large number of families were both members are unemployed. In the first case the choice of search intensity for the non-employed member is affected by the own productivity state ϵ and the composite productivity of their partner's (ϵ and x). Changes in household income in this case (a change in productivity of the employed agent) entail a wealth effect on the labour supply of non employed agents which could induce them to drop out of the labour force. Similarly when both members of a household are unemployed and one of them receives a job offer and becomes employed, there is an analogous wealth effect to the labour supply of the other family member. We show below that this happens a lot in the equilibrium with couples. In contrast in a bachelor household economy this channel is absent and the two factors that determine the choice of labour market status are wealth and productivity. Since wealth is run down in nonemployment, productivity must be less persistent to match the data.

To give sense of the magnitudes of these differences we report the values for the implied stochastic processes in the two models. First given the above intuition we estimate $\rho_{\epsilon}^{C} = .7$ and $\rho_{\epsilon}^{C} = .5$ (so that persistence is larger in the couples economy). Second it turns out that the required conditional standard deviation of the shocks in the singles economy is twice as large as the analogous object in the couples model, so we get that $\sigma_{x}^{S} = .095$ and $\sigma_{\epsilon}^{S} = .42$ and $\sigma_{x}^{C} = .043$ and $\sigma_{\epsilon}^{C} = .21$ although the overall household risk in both cases may be similar. Finally in both models match quality shocks need to be equally and very persistent so that we set $\rho_{x}^{C} = \rho_{x}^{S} = .99$.

With these numbers in our baseline calibration we get that employed agents care more about match quality than own productivity shocks. That is to say an agent in a high x job can let her own productivity ϵ component drift to a very low level before she considers quitting her job (since it is likely to drift back again due to low persistence) but when the match quality deteriorates she is almost certain to become non-employed. On the other hand the transition between unemployment and OLF is governed by the ϵ shocks and in this case they have to be less permanent to give us the UI and IU flow rates that we see in the data.

These numbers don't have a particular interpretation since our model confounds risks from many sources and below we provide more relevant statistics by estimating wage processes from a sample of agents out of our steady state calibration. For the moment suffice is to say although our model features two independent stochastic processes for labour income risk it is yet too parsimonious to match some aspects of the data and the rest of this section is devoted to analyzing that.

Other calibrations. We briefly discuss how the model performs when different values for the stochastic processes are chosen. First when we increase the persistence we always get a smaller UI flow rate and a larger EU flow. For instance in the couples model with $\rho_\epsilon^C = .88$ and $\sigma_\epsilon^C = .13$ we get UI equal to .14 and the EU flow rate equals .008 (much closer to the data). The reason is that the assignment of household members to market work and leisure is much more persistent in this case so when the employed member looses her job the family assigns her to become unemployed rather than withdraw her from labour force. In contrast in our baseline calibration with $\rho_\epsilon^C = .7$ the pool of non employed agents is more or less equally productive, in terms of ϵ, as the pool of employed agents and there are frequent changes in the identity of the main earner within the family (this is something that we scrutinize below).

Further on changes in the other parameters present us with much worse tradeoffs. For instance decreasing the value of ρ_x increases the UE flow rate to above .4 (a similar result applies if we decrease the value of σ_x) since now match quality shocks become less important and there are virtually no gains in the other flows.

Overall our criterion in choosing the best model is the following. First we demand that the equilibrium output is such that the UE flow rate is .28 (as in the data). The reason is that, as we said before we want expansions to increase the job finding rate in the economy without necessarily hitting the upper bound on the number of matches (which is $p_U = 1/2$). When this target is met we adjust the relevant parameters to match as close as possible the flows between unemployment and inactivity and the total outflow from employment. Why this order? Because we found that models that match all the relevant flow rates between employment and unemployment usually feature too few transitions between inactivity and the labour force; and too few transitions mean that these models could potentially have the labour force close to being fixed. In section 3.4.3 which contains our main results we also report the cyclical properties for the economy that sets $\rho_\epsilon^C = .88$ and $\sigma_\epsilon^C = .13$ as an alternative calibration.

How readily can household members substitute in terms of their labour income? To answer this question we look at the persistence of employment status over time in a cohort of agents (a sample of 5000 families) simulated from the steady state distribution. For each period we assume that a family's primary earner is the agent that had the highest recorded annual labour income. Annual horizons serve to mitigate the effect of frictions on recorded employment histories.

To uncover the persistence we simply estimate the Markov transition matrix of primary and secondary earners (that is to say the probability that the identity of the household head changes from one year to the next). By this metric we find that roughly 30 % of our families alternate roles as primary and secondary earners in the labour market each year. Further on when we use the number of hours as our index, and drop productivity from the calculation we find that this rate decreases to 20%.

Arguably the employment status of agents is a much more persistent state, and the reason that our theory cannot match this aspect of the data is precisely that we put two ex ante identical agents within each household. In reality agents differ in fixed productivity and command different rewards in the labour market based on age, sex experience among other things. We can only do so much as to summarize some of these features in our two stochastic processes but our model requires low persistence in the ϵ risk to match the flows between inactivity and unemployment.

To see how specialization in market work vs leisure is determined within the household consider the decomposition inactivity and unemployment in the steady state summarized in Table 3.7. Roughly a 35 % of all OLF agents in the economy live in households where both members are inactive and the remaining 65 %

percent are in families where one member is either unemployed or employed. In the data the analogous fractions are 24 % and 76 % respectively, for a population aged between 16 and 65, and 50 % for ages 16 and above. Clearly demographics play a significant role here but we think that our model strikes a good balance between the two samples in the data.

Further on insofar as the cross section of unemployed agents is concerned we observe that our model overestimates the fraction of agents that are part of households where both members are non-employed. In the data for instance conditional on unemployment the probability that an agent is part of a family where the second member is also unemployed is 7% when the couple occupies the age bracket 16-65. Analogously the probability that the other member is out of the labour force is 19%. The model produced values of 20% and 27% respectively and again this probably is symptomatic of the fact that independent shocks and identical agents exacerbate the role of insurance in the couples economy.

TABLE 3.7: DECOMPOSITIONS OF UNEMPLOYMENT AND INACTIVITY

Unemployed	**UU**	**UI**	**UE**
Bechmark Model	.2	.27	.53
US Data: Ages 16-65	.07	.19	.74
US Data: Ages >16	.1	.22	.68
OLF	**II**	**UI**	**IE**
Bechmark Model	.354	.026	.62
US Data: Ages 16-65	.24	.026	.734
US Data: Ages >16	.5	.02	.48

The implied process of wages. Since in our model idiosyncratic labour incomes confound risks from various sources (search frictions and the joint stochastic processes of productivity) to evaluate how realistic our choices are we need to estimate the realized profiles of wages for individuals in our economy. We use a simple representation of the logarithm of annual (time aggregated) wages: $\ln w_t = \phi \ln w_{t-1} + v_t$ and use a sample of 10000 individuals over 20 years to estimate the implied values for ϕ and the variance of the shock σ_v. Further on since in our model the distinction between household heads and secondary earners seems to be virtually irrelevant (with two ex ante identical agents) we pool the estimates from all household members in the simulated population.

Both of these values seem to be far away from the data. Our estimates are $\phi_S = .1$ for the singles economy and $\phi_C = .4$ for couples (notice that couples is much closer to a high persistence process that is empirically relevant). Furtheron there is a wealth of estimates for the data analogues for these statistics (see Heathcote et al (2008)) and all of them yield a value for ϕ in the neighborhood of .9. [7] Given that both of our models imply that labour income is less persistent than in the data we conclude that only temporary components of shocks are important in matching the labour market flows.

3.4.3 Cyclical properties

Table 3.8 presents the results from our benchmark calibration with $p_U = 1/2$ $p_I = .1$ for both the couples and the bachelor household economies. We restrict attention to key labour market statistics and all quantities are expressed relative to a de-trended measure of GDP (They are logged and HP filtered with a parameter $\lambda = 1600$). The data are quarterly aggregates of the simulated aggregate paths.

In the singles economy unemployment is extremely procyclical (contemporaneous correlation with GDP is .65) and so is the labour force. The model produces a contemporaneous correlation with GDP equal to .65 and .97 for these quantities whilst in the data the analogous statistics are -.81 and .2 respectively. Further on aggregate unemployment is not nearly as volatile as in the the data (1.78 vs 6.68) and the LF is nearly 50% more volatile (.32 vs .22 in the data).

The benchmark couples model (columns 3-4) produces a slightly different set of statistics. Unemployment now becomes more acyclical (the contemporaneous correlation with GDP is .22) and more volatile than with bachelor households. It is closer to the data. Aggregate employment is more volatile and equally procyclical and the LF is nearly twice as volatile (.62) and only marginally less procyclical (.95) than in the previous case. Finally columns 5-6 contain the results of the economy that sets $\rho_\epsilon^C = .88$ and $\sigma_\epsilon^C = .13$. There aggregate unemployment is slightly more countercyclical (contemporaneous correlation with GDP is -.05) and more volatile ($\frac{\sigma_u}{\sigma_Y}= 3.5$). Aggregate employment is still more volatile than in the bachelor household model and equally procyclical, and the LF is again more

[7] In Chang and Kim (2007) a model that accounts for selection effects yields a value for the persistence component of .73

volatile (although less than the baseline couples model) and again somewhat less procyclical [8].

TABLE 3.8: RESULTS: CYCLICAL PROPERTIES OF LABOUR MARKETS

	Bachelors Benchmark		**Couples Benchmark**		**Couples High ρ_ϵ**	
	$\frac{\sigma_x}{\sigma_y}$	$\rho_{x,y}$	$\frac{\sigma_x}{\sigma_y}$	$\rho_{x,y}$	$\frac{\sigma_x}{\sigma_y}$	$\rho_{x,y}$
Unemployment	1.78	.65	2.7	.22	3.5	-.05
Employment	0.54	0.96	.85	0.97	.77	.96
Labor Force	0.32	0.97	.62	0.95	.41	.95

These results are extremely disappointing from the point of view of our theory. How so? Well in section 3.2 we showed that the labour force participation of females in our sample was substantially more procyclical and volatile when the influence of the husband's employment status was removed. We interpreted this result as indicating that if the US economy was populated by bachelor agents (and joint insurance opportunities didn't exist) then the LF would be considerably more procyclical. By this metric the model fails miserably in replicating this feature of the data. When we move from the singles to the couples economy (so more insurance) we see that the volatility of the LF increases and there are virtually no gains in the cyclical correlation of this statistic [9]. The comparison of the two economies therefore sends the qualitative patterns to the opposite direction and this comes out of a model that features considerable insurance as we showed in section 3.4.2.

It is clear that this failure of all the models to generate statistics close to the US data is due to the overwhelming motive in these economies to allocate agents to activity (employment and unemployment) during economic expansions. In the upturn jobs become more attractive and the typical agent increases her optimal level of search intensity. However before these agents can be allocated to employment they have to spend time in the pool of unemployed workers due to the

[8] Note that although the differences are small they are not the result of sampling variation; in the Appendix we outline an algorithm due to Young (2009) that computes the equilibrium in the economy by working with the histogram instead of simulating panels of a finite number of agents. There is no sampling variation due to the Law of Large Numbers.

[9] Notice that repeating the analysis of section 2 here would be meaningless since our economy is not inhabited by males and females but by identical agents. Further on we argued that households change very frequently the identity of the main earner.

existence of frictions. This is why aggregate unemployment becomes procyclical (or nearly procyclical) in the models.

Further on the larger volatility that we get in the couples model is possibly a result of three features all of which relate to the identity of the marginal worker in the economy. First our singles model has considerably more uncertainty in the idiosyncratic process and thus individual decisions are guided less by the aggregate state (expansion vs recession) and more by the labour income. Second the distribution of agents over the relevant state space is different in the two economies and bachelors OLF agents have sufficient wealth to finance leisure whilst in the couples model inactivity usually entails the presence of a main earner at home. In the latter case households use more readily labour supply as a margin of insurance and business cycles move more workers between labour market states. Finally another reason we get larger volatilities is precisely that households have secondary earners and in this case the aggregate elasticity of labour supply is considerably larger (as in Chang and Kim (2006)) [10].

We find that all of these possibilities are relevant here; for instance when we increase the volatility of idiosyncratic endowments in the couples economy we do get some fall in volatility in the aggregate labour market (we don't report this because this model produces wrong labour market flows). Uncertainty however cannot be the only reason since our alternative calibration of the couples economy, features a similar level of unconditional uncertainty and yet produces a slightly different set of statistics.

Other models.

We take stock from the results of this section to discern whether there are important features that our model misses out on and that could potentially change its implications. For one thing with two ex ante identical agents we argued that our model is unable to match patterns of specialization within the household

[10] Chang and Kim (2006) build a model similar to ours that features husbands and wives in the household, incomplete markets and an extensive margin of labour supply and they get a much larger amplification of business cycle shocks to aggregate employment than what the values of elasticity of labour supply they assume would otherwise give. In their model as well as in ours the aggregate elasticity of labour supply is borne out of the reservation wage distributions and not the willingness to substitute leisure inter-temporally. The problem is that their analysis doesn't go as far as to discern which one of the ingredients is responsible for the results; They don't compare with a bachelor household economy (so as to single out secondary earners) vs the extensive margin of labour supply. Further on husbands and wives in their model are not ex ante identical.

in terms of market work and leisure. And yet this appears to be important since in the US data over our sample period we find that the LF participation of husbands is considerably higher and acyclical (it has a zero contemporaneous correlation with GDP).

But matching these aspects would probably have something to say about volatility but very few for the cyclical correlation. This is precisely what happens in our calibration of the model with $\rho_\epsilon^C = .88$ and $\sigma_\epsilon^C = .13$. There the assignment of roles within the family is much more persistent (note that this is why the model produces a higher EU flow) and whilst the labour force becomes less volatile the cyclical correlation doesn't budge, because there is always a marginal worker that flows in economic activity in expansions. Analogously if we were to assign a gender to each member of our families and we assumed that secondary earners are less productive, as in the data, then we could hypothetically go a lot further towards matching volatility of the labour force. The problem is that this addition would kill off the insurance margin since the contribution to household resources of a wife when she increases her labour supply would also be considerably smaller than in our model, and in this sense matching aspects of intra-household correlation of incomes would also be important.

For the same reason what doesn't seem to hold promise is to incorporate some departure from complete insurance within the family in our framework. If our modeling of household consumption and employment decisions was based on the collective approach as in Chiappori (1988) then, it is well understood, that household members would behave much more like bachelor agents. In contrast the unitary model that we adopt maximizes the added worker effect which we found to be responsible for the cyclical behavior of the labour force.

Then when should we expect for family self insurance to be most important? We answer; when there are unemployment risks (that entail large losses of income) and when there are incomplete markets so that consumption is affected. Our model builds on these assumptions but here risks are partly choices since the decision to move out of employment depends on the reservation wage that increases in wealth. For a constrained worker a fall in match quality doesn't necessarily mean unemployment since the match surplus becomes negative only when wealth is sufficiently high. But on the other hand wealthier workers are nearly permanent income agents (Krusell and Smith (1998)).

Not even search and matching models of the labour market as in Mortensen and Pissarides (1994) escape this critique since there the job surplus falls with wealth (see Bils et al 2008). Moreover if we were to add firms in the background that make hiring and firing decisions in our framework and we endowed agents with a search technology as we did here, then the search intensity of the economy's workforce would still be too procyclical (see Shimer (2003), Merz (1995)). Further on the problem is that such as model would also have to deal with the low equilibrium volatility of unemployment and vacancies and hence it would create an additional concern.

3.5 Conclusion

In this chapter we contrast the properties of economies where lack of insurance opportunities means that agents stand alone against uncertain contingencies with those where risk sharing exists in households that comprise of two ex ante identical members. We ask how the implications for the labour market are affected in an otherwise standard incomplete market model with search frictions and endogenous labour force participation, depending on the structure of the household, and especially how the two economies respond to fluctuations in aggregate productivity.

What we get is that the model is completely unable to match the empirical patterns that we see in the data. The labour force in the artificial economy is too pro-cyclical and too volatile relative to the data and it is also too volatile relative to a model that populates the economy with bachelor agents. Using data from the CPS we were led to the converse implication. We found that joint insurance is an important factor that explains why the participation of secondary earners (wives in our sample) is not correlated with aggregate output.

We explain why although out theory is incomplete in some respects, we build a model that we anticipated to give us a very large effect (possibly the maximum) from joint search and insurance. Instead it produces disappointing results. In the outset we explore what other relevant additions need to be made to our baseline framework to be able to match the data. We single out the following; first matching better the cross sectional aspects of intrahousehold division of time in home vs market work and second matching better the cross sectional aspects of risks. These are possibilities that we explore in future work.

It is important to note that our contribution goes far beyond analyzing the cyclicality of the key labour market statistics in a search model with imperfect insurance. A generation of macro-economists believed that the key of explaining fluctuations in aggregate employment is the elasticity of the labour supply of secondary household earners (females). And whilst we find some theoretical work that is conducive to this intuition our conclusions point to the fact that it is misleading to draw implications for the aggregate elasticity of labour supply from models that circumvent the effort of matching the cross sectional aspects of labour supply.

Our attempt can be viewed as a necessary step to a more ambitious research agenda. We yet don't have a clear understanding of how allocations are affected in economies where insurance is abundant in the family. There must be a wealth of policy or welfare related questions where these alternative environments produce different answers. For instance in incomplete market models with bachelor households wealth encodes the history of productivity and those agents who build up a stock of wealth can finance leisure and drop out of work. In contrast in social planning economies most productive agents are always send to work. We suspect that allocations in couples economies must be somewhat in between, and the interest lies in determining how much.

3.6 Appendices

3.6.1 Computational strategy for steady-state equilibrium

In steady state, factor prices are constant and the distribution of agents over the relevant state space Γ is time invariant. The calibration consists of three nested loops. The outer loop is the estimation loop where we set the endogenous parameters $\{B, k, \rho_\epsilon, \sigma_\epsilon, \rho_x, \sigma_x\}$ is chosen, We solve the model and check whether the generated moments (labour market flows) are close enough to their empirical counterparts. If not, we try a new set of parameters.

The middle loop is the market clearing loop. We guessed an interest rate r which implies a wage rate w and then solve for the value functions and the steady state distribution Γ). The steady sate distribution yields an aggregate savings supply. If the implied marginal product of capital is equal to the guessed interest rate, we found the equilibrium. If not, we update our interest rate guess. For the singles version of our model instead of changing interest rates to clear the market of savings we adjust the discount factor β_S and keep constant the aggregate rate of return R.

The inner loop is the value function iteration. Details are as follows:

1. We choose an unevenly spaced grid for asset holdings (a) (with more nodes near the borrowing constraint) and a grid for individual productivities ϵ and x. We experiment with different number of nodes for the asset grid, usually between $N_a = 101$ and $N_a = 161$.The number nodes for the idiosyncratic labour market risks are $N_\epsilon = 5$ and $N_x = 2$. These are equally spaced and the transition matrix of idiosyncratic shocks is obtained by the discretization procedure described by Adda and Cooper (2003).
2. Given our guess for the interest rate r, we solve for the individual value functions, V^n, V^e in the bachelor model and V^{nn}, V^{en}, V^{ee} in the couples model. This is done by finding the optimal savings and search intensity choice at each node. Values that fall outside the grid are interpolated with cubic splines. Once the value functions have converged we recover the optimal policy functions of the form $a'(a, \epsilon)$, $s(a, \epsilon)$ and $h(a, \epsilon)$.
3. The final step is to obtain the invariant measure Γ over the relevant state space (asset productivities and employment status).

   a) We first approximate the optimal policy rules on a finer grid which

$N_{a_{BIG}} = 2000$ nodes and we initialize our measure Γ_0.

b) We update it and obtain a new measure Γ_1

c) The invariant measure is found when the maximum difference between Γ_0 and Γ_1 is smaller than a pre-specified tolerance level.

d) By using the invariant measure, we compute aggregate labour supply and asset supply. This implies a new marginal product of capital which we then compare to our initial guess.

3.6.2 Computational strategy for equilibrium with aggregate fluctuations

Aggregate shocks imply that factor prices are time varying. When solving their optimization program agents have to predict future factor prices. Therefore they have to predict all the individual policy decisions in all possible future states. This requires agents to keep track of every other agent. Thus in order to approximate the equilibrium in the presence of aggregate shocks, one has to keep track of the measure of all groups of agents over time. Since Γ is an infinite dimensional object it is impossible to do this directly. We therefore follow Krusell and Smith (1998) and assume that agents are boundedly rational and use only the mean of wealth and aggregate productivity to forecast future capital K and factor prices w and R.

Compared to the steady-state algorithm we now have two additional state variables that we must add in the list of the existing state variables in the inner loop: aggregate productivity λ and aggregate capital K. As the outer loop, we iterate on the forecasting equations for aggregate capital and factor prices.[11] The details are as follows:

a) We approximate the aggregate productivity process with 2 nodes and use again the methodology of Adda and Cooper (2003) to obtain the values and transition probabilities. We choose a capital grid around the steady-state level of capital $K^s s$, particularly we $N_k = 6$ equally spaced nodes to form a grid with range $[0.95 * K^{ss}; 1.05K^{ss}]$.

b) As already mentioned, we choose the means of aggregate capital and

[11] In the steady state algorithm, there were three loops. Since we use the steady state values for the endogenous parameters, we do not have an estimation loop here.

aggregate productivity as explanatory variables in the forecasting equations. We use a log-linear form

$$\begin{aligned} ln\, K_{t+1} &= \kappa_0^0 + \kappa_1^0 ln\, K_t + \kappa_2^0 ln\, \lambda_t & (3.6.1) \\ ln\, w_t &= \omega_0^0 + \omega_1^0 ln\, K_t + \omega_2^0 ln\, \lambda_t & (3.6.2) \\ ln\, R_t &= \varrho_0^0 + \varrho_1^0 ln\, K_t + \varrho_2^0 ln\, \lambda_t & (3.6.3) \end{aligned}$$

c) We initialize the coefficients so that K_{t+1}, w, R are equal to their steady state values.

d) Given equations 3.6.1 to 3.6.3, we solve the value function problems as before, just that now the state vector is four-dimensional. Values that are not on the asset grid are interpolated using cubic splines. Values that are not on the aggregate capital grid are interpolated linearly.

e) Instead of simulating the economy with a large finite number of agents we use the procedure of Young (2009) and simulate a continuum of agents. This procedure has the advantage of avoiding cross-sectional sampling variation. We simulate the economy for 10,000 periods and discard the first 2,000. In each period we get an observation for K, w and R. We use the simulated data to run OLS regressions on equations 3.6.1 to 3.6.3 which yield new coefficient estimates κ^1's, ω^1's, ϱ^1's. If these coefficients are close to the previous ones we stop, otherwise we update equations 3.6.1 to 3.6.3 with the new coefficients and solve the problem again.

The convergent solutions for the forecasting equations of our models are as follows:

TABLE 3.9: COUPLES ECONOMY BASELINE.

Equation	**Constant**	$\ln(K_t)$	$\ln(\lambda_t)$	R^2
$\ln(K_{t+1})$	.05427	.98317	.04203	. 99996
$\ln(w_t)$	-.16841	.39621	.55531	.99627
$\ln(R_t)$	.04858	-.01355	.01546	.99108

TABLE 3.10: SINGLES ECONOMY.

Equation	**Constant**	$\ln(K_t)$	$\ln(\lambda_t)$	R^2
$\ln(K_{t+1})$	.07115	.98175	.02802	.99997
$\ln(w_t)$	-.32154	.39338	.60769	.99636
$\ln(R_t)$	.04485	-.01026	.01023	.98717

TABLE 3.11: COUPLES ECONOMY HIGH ρ_ϵ CALIBRATION.

Equation	**Constant**	$\ln(K_t)$	$\ln(\lambda_t)$	R^2
$\ln(K_{t+1})$	.06117	.98221	.03501	. 99997
$\ln(w_t)$	-.2184	.39011	.56834	.99598
$\ln(R_t)$	.04766	-.01145	.01342	.99101

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