The London School of Economics and Political Science

Robust Global Supply Chain Planning Under Uncertainty

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Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work.

Abstract

The New World Economy presents business organizations with some special challenges that they have never met before, when they manage their activities in the global supply chain network. Business managers find that traditional managerial approaches, techniques and principles are no longer effective in dealing with these challenges. This dissertation is a study of how to solve new problems emerging in the global supply chain network. Three main issues identified in the global supply chain network are: production loading problems for global manufacturing, logistics problems for global road transport and container loading problems for global air transport. These problems involve a higher level of uncertainty and risk. Three types of dual-response strategies have been developed to hedge the uncertainty and short lead time in the above three problems. These strategies are: a dual-response production loading strategy for global manufacturing, a dual-response logistics strategy for global road transport and a dual-response container loading strategy for global air transport. In order to implement these strategies, the two-stage stochastic recourse programming models have been formulated. The computational results show that the two-stage stochastic recourse models have an advantage in comparison to the corresponding deterministic models for the three issues. However, the two-stage stochastic recourse models lack the ability of handling risk, which is particularly important in today's highly-competitive environment. We thus develop a robust optimization framework for dealing with uncertainty and risk. The robust optimization framework consists of a robust optimization model with solution robustness, a robust optimisation model with model robustness and a robust optimization model with trade-off between solution robustness and model robustness. Each type of the robust optimization models represents a different measure of performance in terms of risk and cost. A series of experiments demonstrate that the robust optimization models can create a global supply chain planning system with more flexibility, reliability, agility, responsiveness and lower risk.

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This is the third time I changed the acknowledgement. During the past 8 years for pursuing my second PhD at LSE, many things had happened in my life. I had my viva in November 28, 2006, which was just 17 days after my mother passed away. I was unable to go back to my country to attend her funeral because of the coming viva and my heavy pregnant then. Now, the little baby is 3-year-old. Just like I promised in the 2nd revision, I brought him back to UK when he is 14-month-old, and he has been with me since then. My old son, who is already 16-year-old, has been educated in one of top independent schools in UK, Winchester College, as a scholar. On top of raising two children on my own, I am a full-time lecturer at the University of Southampton. Through the past tough 8 years, I realize that there is nothing on this planet more important than the two boys, and I will love them forever (3rd submission, Jan 2010).

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This thesis is dedicated to my mother,

Shuzhen He

who passed away on November 11, 2006 in China.

I love you very much.

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Chapter 1

Introduction

This thesis is a study of the problems emerging in the global supply chain networks under uncertainty. A brief description of the changing business environment and the competitive performances in the New World Economy explain the motivation of the thesis. After that we introduce the contribution of the thesis. Finally, we provide an overview of the thesis.

1.1 The changing business environment and competitive performances in the New World Economy

The world is a very different place than it was only a few years ago. Business organizations face complex challenges posed by advances in information technology, particularly the advent and the growing power of the Internet. These (information technology) advances constitute the very basis for several industries whose operating characteristics are substantially different from those of more traditional ones, and which, collectively, have come to be called "The New World Economy" (Hayes *et al.* 2005). The New World Economy is also called the *New Economy*, the *Internet Economy*, the *Web Economy*, the *Network Economy*, or the *Digital Economy* (Turban *et al.* 2006, Reddy and Reddy 2001). Many business organizations find that traditional managerial approaches, techniques and principles are no longer effective in dealing with the challenges in the New World

Economy - they demand more innovative strategies, tactics, and operations in order to compete and survive. Poirier (1999) thinks supply chain management has emerged as one of the most powerful business tools available today. Harrison (2003) states that supply chain management has become an important focus of competitive advantage for firms and organizations over the past ten years. Therefore, management of the supply chain under the changing business environment has become an important point of focus for business organizations.

1.1.1 The changing business landscape: driving forces

A supply chain consists of all parties involved, directly or indirectly, in producing products or delivering services, and is often represented as a network: this involves members at a variety of stages, including suppliers, manufacturers, distributors, retailers and customers. Supply chain management is a set of approaches utilized to efficiently coordinate the activities and components at different stages within the chain so that products or services are produced or distributed in the right quantities to the right locations at the right time, in order to minimize operating costs and satisfy customer requirements. Today's business has inevitably set in a global environment in which materials and products can be bought, manufactured and sold anywhere in the world. As a result, supply chain management is usually labelled as global supply chain management in the global environment (Coyle *et al.* 2003). Several forces are currently presenting challenges for business organizations in managing their supply chains in the New World Economy.

- ***Globalization:*** We are in an era when more and more companies are seeking to achieve a competitive advantage by expanding their operations to a global scale. The globalization of industry, and hence supply chains, is inevitable (Christopher 2005).

- ***Advances in information technology:*** The movement towards globalization has been mainly facilitated by the advances in information technology. During the past decade, business organizations have been irrevocably changing the way they design, purchase, process, market and support their products and services through the Internet, computerization, and a wide range of inexpensive information transmission tools. Advanced information technology has made competition truly global.

- ***e-Business:*** Inexpensive use of e-business has lead to companies, wherever or whatever size they are, being able to participate in business. As a result of e-business development, many of the core concepts or principles of supply chain have been implemented and put into practice in a much more efficient way.

- ***Service-based competition:*** In today's global marketplace, competitive advantage is driven by service-based strategies, instead of product-based strategies. As customers move at the Internet speed, they demand that companies respond at the Internet speed (Iansiti and MacCormark 1997).

- ***Time-based competition:*** Time compression has become a more critical management issue than ever before (Christopher 2005). Business success increasingly relies on speed instead of quality, which has become a minimum standard rather than a competitive advantage. Time has become the next battleground or the next strategic frontier (Tang *et al.* 2005). Customers are used to immediate availability from stock for instant gratification, which makes logistics ever so important and challenged.

- ***Powerful customers:*** Customers are empowered by the information they have from the Internet or other sources (Coyle *et al.* 2003). As they can globally compare services and prices, customers tend to have low tolerance and loyalty. They demand quick response, while expecting continuously declining product costs.

- ***Short product lifecycle and lead time:*** Product lifecycle is becoming shorter and shorter, particularly in industries like personal computers and fashion. The enlightened customers tend to delay their order commitments until they have confidence about market trends. These trends leave manufacturing companies an ever-shortening time for designing, purchasing, manufacturing, and distributing to satisfy the customers.

- ***High degree of product variety:*** There is a strong trend in industry towards increased product variety, and easy availability of information on the Internet has lead to an even higher level of customization.

- ***Global sourcing:*** Companies tend to perform those activities in the supply chain where they are able to provide a differential advantage; they outsource all other activities to partners like manufacturers and logistics providers in any part of the world that offers low cost and high quality products or services. Logistics is now considered to be one of the main sourcing functions.

- ***Third party logistics (3PL):*** Third party logistics services providers are external suppliers who provide a range of logistics services to their clients, such as transporting, warehousing, distributing, and so on. There has been a significant increase in the number of third-party logistics providers and companies that outsource their logistics activities. Outsourcing logistics provides many benefits in the supply chain, such as improving efficiency of the whole supply chain, heightening competitiveness through the use of expert staff, consolidating of different goods and bringing in new technologies like online order placing, auto-tracking and on-line inventory verification, as well as other value-added benefits, including warehouse management, packaging, labelling, etc.

- ***Distribution and centralization:*** Globalization encourages companies to rationalize production at fewer locations, which leads to a trend towards centralization of inventories (Christopher 2005). Deliveries are now increasingly being made to one or several centralized points for further onward distribution.

1.1.2 Managing the global supply chain: the competitive performances

Earlier attempts of managing the supply chain mostly centered on vertical integration, which normally implies ownership of upstream suppliers and downstream distribution channels. The major disadvantage of the vertical integration was a lack of flexibility as companies had to bear high fixed costs to perform all activities in the entire supply chain. The globalized environment requires quicker deliveries of products and services through the entire supply chain and it is beyond what any single party can provide. As a result, the vertical integration model has lost momentum since specialization has proved more powerful than integration (Lawrence *et al.* 2003). Increasingly, companies are now focusing on their core businesses - things that they do really well and where they have some differential advantages; everything else is outsourced globally (Christopher, 2005). Dornier *et al.* (1998) think the key to successful restructuring for many companies has been to focus on core competencies or strategically important activities and to withdraw from non-core functions. Magretta (1998) presents the same view: companies should focus on their core activities and outsource the rest because of the propelling changes being forced by global competition.

Some of the changes that have occurred in the New World Economy have led to more efficient methods in managing the supply chain while others have increased uncertainty and risk. To compete and survive, companies have to develop innovative strategies, tactics and operations in order to manage different functions in the global supply chain. Competitive performances have been changed over the past decade. Several dimensions are currently used to measure supply chain performances (Nair 2005, Christopher 2005) in the global supply chain management environment.

Productivity

- ***Return on assets:*** The ratio of outputs to inputs.
- ***Inventory turnover:*** The ratio of value of goods sold to total value of inventory.

Flexibility

- ***Product flexibility:*** The ability to manufacture products characterized by numerous features, options, size, colours, etc.
- ***Delivery flexibility:*** The ability to manage delivery for different customers in an effective and efficient manner.

Agility

- The ability to move quickly and to meet customer demand in time.

Responsiveness

- The ability to respond to customers' sophisticated requirements, which is critical in ever-shortening timeframes.

Reliability

- The ability to meet a delivery promise under uncertain conditions.

1.2 Motivation of the thesis

Information technology enables easy and inexpensive communication, which forces companies to provide a high degree of customization in their products and services. Companies have to meet the sophisticated requirements of the customers and respond to them at Internet speed while continuously facing the need for lowering costs. Besides, the product life cycle has become very short. For example, the life cycle of many PC products is only a couple of months and the life cycle for fashion retail is, in some cases, as low as only a few weeks. Customers tend to delay their order commitment until they are confident about future market trends and this leads to an ever shorter lead time. All these factors make it practically impossible to satisfy customer requirements, which is the ultimate aim of the supply chain management.

Hayes *et al.* (2005) state that, as the New World Economy expanded its reach, managers discovered that many of the principles, practices, and methodologies that had been proven successful in traditional industries no longer seemed effective in the new context. The main reason for this is the basic assumptions that managers and academics tend to make when thinking about managing operations in traditional industries – they are now inadequate and ineffective for information-intensive operations. For example, the traditional approach emphasises vertical integration of the supply chain, which has been proved to be difficult to maintain in the global supply chain. In addition, many traditional management approaches and techniques assume that the information that decision-making needs is available with certainty. However, this is really not the case in the current information-intensive decision-making environment. Hayne *et al.* (2005) state that as they confronte the twenty-first century, managers around the world experience mixed emotions: a sense of real accomplishment accompanied by frustration and uncertainty. Constant change, propelled by information technology, is making the job of managers increasingly difficult (Reddy and Reddy 2001). The managers are struggling to find new ways to adapt to the changing business environment to hedge against uncertainty and risk. It is not surprising that many managers have failed to fully adapt to the changing environment, resulting in performance shortcomings and lost opportunities.

In this thesis, we will study the new problems emerging in the global supply chain networks linking Asia, North America, and Europe. Different organizations in the global supply chains perform different functions aiming at satisfying North American and European markets. Typically, product sales, customer service, and market demand are centred in North America and Europe. Production facilities are most likely located in low-cost countries, such as Mainland China, Indonesia, Mauritius, Mexico, Nigeria, South Africa, South Korea, Thailand, Vietnam, and so on. Mainland China, however, is one of the favourite places for manufacturing because of its low production costs and high product quality, as well as its attractive domestic market and a highly skilled workforce. The type of the global supply chain network outlined here plays an important role in today's business world. Because of China's booming economy, more and more companies have been setting up their production bases in China. China has become a world manufacturing centre, particularly for the textile and clothing industry. It is expected that 50% of clothing of the world will be made in China by 2010. Therefore, this study will look at a global supply chain network providing fashion garments to North American and European markets. Two organizations – a garment manufacturer and a third-party logistics provider – are involved in the process of manufacturing and distributing. The garment manufacturer, headquartered in Hong Kong, distributes its production among its manufacturing factories which are located in Mainland China, Thailand, Sri Lanka, and other places. The third-party logistics provider has a global logistics network providing global transportation from manufacturing sites to demand sites. The products that are made in different manufacturing plants are first transported to and stored in China's warehouse before they are shipped to North American and European markets. The products then need to be transported from the warehouse in China by truck across the border to Hong Kong, from where the products can enter the international cargo routes. Three main issues, therefore, are identified in the global supply chain: production loading problems for global manufacturing, logistics problems for global road transport, and container loading problems for global air transport.

The motivation behind this study is to address a lack of a systematic approach in managing different activities in the global supply chain in a manner appropriate to deal with the series of changes that have occurred in the New World Economy, including global manufacturing, global logistics, import quota, uncertainty, risk, etc. Most traditional supply chain management methods assume that all information is known with certainty. However, Reddy and Reddy (2001) state that, in our era, the only constant is change, and all technology decisions have to factor this into the decision-making process. The changing business environment makes it difficult to obtain accurate demand information from the markets. However, global manufacturing processes cannot wait until accurate market information is available. Consequently, logistics managers are facing a bigger challenge and more uncertainty than ever before. Global road transportation involves many uncertain factors and must operate under a tighter time schedule in the delivery of products from one country to another country. In addition, global air transport faces an even more critical situation because moving goods by air usually involves huge capital investment and the time required is even shorter, representing more uncertain factors and higher risk. Therefore, solving these new problems in the global supply chain management network is critical to the success of the whole supply chain.

1.3 Contribution of the thesis

The contributions of this thesis can be classified into four areas.

Contribution 1: We develop a methodology for formulating a decision-making framework for tactical planning in the global supply chain management environment to deal with uncertainty and short lead time.

Practical management has discovered that the early methods and techniques that attempt to vertically integrate across the supply chain are inappropriate and difficult to put into practice in the New World Economy, characterized by the advanced information technology, especially the Internet. Over the years more and more researchers and practitioners have realized the importance of global sourcing. Vast quantities of resources were invested in implementing new strategies in basic functions, including purchasing materials, marketing products, setting up of plants and distribution centers, and distributing goods on a global scale. Leading-edge business organizations seek to achieve advantages by identifying world markets for their products and then developing a manufacturing and logistics strategy to support the marketing strategy (Christopher 2005). Much of the research in the field of global supply chain management addresses the issue of how to deal with the problems at the strategic level, focusing on globally designed supply chain infrastructure, which is the process of determining the number, location and capacity of the plants, distribution centres, markets, etc. Unfortunately, the problems at the tactical/operational level are paid less attention. Simchi-levi *et al.* (2003) state that only in the last few years, companies have recognized the importance of problems at the tactical level. Christopher (2005) states that to enable the potential of global networks to be fully realized, a wider supply chain perspective must be adopted. The global corporations' competitive advantage will increasingly depend on excellence in managing the complex web of relationships. Nowadays, globalization of the supply chain network is a reality and many companies, particularly the leading-edge companies, have already globalized their manufacturing operations, markets, distribution centres, logistics modes, etc., one example being the global supply chain system studied in this thesis. However, little research has covered global supply chain management problems at the tactical/operational level, which is the process of formulating a series of plans, including production planning, transportation planning, distribution planning, inventory planning, etc, all of which represent elements critical to the long term success of any organization. This thesis contributes to development of a methodology for decision-making for solving of the tactical planning problems in the global supply chain network. This involves multi-organizations hedging against uncertainty in short manufacturing and distribution time frames.

Contribution 2: We develop three dual-response strategies to hedge against uncertainty and short lead time: a dual-response production loading strategy for global manufacturing problems; a dual-response logistics strategy for global road transport problems; and a dual-response container loading strategy for global air transport problems, aiming at creating a more flexible, reliable, agile, responsive and less risky supply chain.

- ***The dual-response production loading strategy for global manufacturing problems***: The manufacturing company uses two types of plants: company-owned and contracted – to hedge against the short lead time and uncertainty involved in allocating production among different manufacturing plants in different countries. In the first stage, when accurate market information is not available, the company first distributes production tasks among company-owned plants with lower operating costs. In the second stage, when the uncertainty is realized, the company prepares to respond to different scenarios that have been observed and allocates production tasks among the contracted plants with higher operating costs because of the shorter lead time. By utilizing two types of plants in two different stages, the company is able to achieve a quick response to the changing market scenarios while minimizing the total cost.

- ***The dual-response logistics strategy for global road transport problems***: The logistics company has its fleet containing company-owned trucks transporting the finished products from a warehouse in one country to a warehouse in another country. However, when the capacity of the fleet is not enough, the logistics provider has to hire trucks in both the countries in order to satisfy customer shipment requirements. Because of the differences between the two countries in terms of border crossing policies, hiring cost of trucks, inventory costs in the warehouses, etc., the dual-response logistics strategy is used to hedge against the uncertain market scenarios and the short shipping notice time.

- ***The dual-response container loading strategy for global air transport problems***: In order to obtain competitive rents from air carriers, the freight forwarder first needs to book the quantities and types of containers in advance based on the incomplete customer shipment information. As the accurate shipment information can only be obtained on the shipping day, the forwarder needs to make different responses to different cargo quantities. By utilizing the dual-response container loading strategy, the forwarder is able to respond quickly to satisfy customer shipping requirements while minimizing the operating cost.

Contribution 3: We develop a robust optimization framework by using a quantitative method to measure trade-off between the cost and risk in dealing with uncertainty. The robust optimization framework includes a robust linear optimization model with solution robustness, a robust linear optimization model with model robustness, and a robust linear optimization model with trade-off between solution robustness and model robustness.

Mulvey *et al.* (1995) first develop a robust optimization technique. In this thesis, we provide three types of robust linear optimization models, which can easily be solved by mathematical programming software available today. The robust linear optimization model with solution robustness can provide a solution with low variability among different scenarios. The robust linear optimization model with model robustness permits violation of some constraints by the least amount. The robust linear optimization model with trade-off presents a quantitative method to measure trade-off between solution robustness and model robustness.

Contribution 4: We identify three main issues in managing the global supply chain, concerning a high degree of uncertainty: production loading problems for global manufacturing, logistics problems for global road transport and container loading problems for global air transport. At the same time, we formulate three types of the robust optimization models for the three problems. By comparing these with the deterministic programming and stochastic recourse programming models, we demonstrate that robust optimization models can create a global supply chain planning system with more flexibility, reliability, agility, responsiveness and less risk.

- ***Production loading problems for global manufacturing***: we are the first ones to identify uncertainties regarding production loading problems arising from import quota restraints and formulate three types of models for solving the problems: the linear programming model; the stochastic linear recourse programming model; and the robust linear optimization models.

- ***Logistics problems for global road transport***: we are the first ones to identify the logistics problems between two countries and formulate three types of models for solving the problems: the mixed 0-1 integer programming model; the stochastic mixed 0-1 integer recourse programming model; and the robust mixed 0-1 integer optimization models.

- ***Container loading problems for global air transport:*** we are the fist ones to identify the container loading problems, which are involved in the changing costs of renting a container, which depends on the cargo weight inside and the renting time. At the same time, we also consider how to load the cargo into containers when renting containers. We formulate three types of models for the problems: the mixed 0-1 integer programming model; the stochastic mixed 0-1 integer recourse programming model; and the robust mixed 0-1 integer optimization models.

In summary, this research contributes to the development of a methodology for solving uncertain supply chain planning problems. This research also develops the robust optimization framework and the dual response strategies in the global supply chain network, with a high degree of uncertainty, and a short lead time. Meanwhile, we identify the three main issues in the global supply chain and apply the robust optimization framework to find solutions for the three problems of achieving dual response strategies aiming at building a competitive advantage in the global supply chain management environment.

1.4 Overview of the thesis

This thesis is organized into seven chapters. We start this thesis with a chapter on an introduction of the thesis. This chapter emphasizes the changing business environment in the New World Economy by exploring the forces driving the changing business environment and the competition performances in the global supply chain management environment. After describing the motivation of this thesis, including the brief introduction of the company and its global supply chain network, we present the contribution of the thesis. The overview of the thesis is outlined in the final part of chapter 1.

In order to emphasize the importance of the global supply chain structure discussed in this thesis, chapter 2 first summarizes the current supply chain practice in Mainland China and Hong Kong. It emphasizes the importance of China's participation in global trade, particularly for the textile and clothing industry. By looking at the background of the companies that are involved in the global supply chain network providing garments for North American and European markets, we outline the main issues in managing the global supply chain management under uncertainty: production loading problems for global manufacturing, logistics problems for global road transport and container loading problems for global air transport. The literature review related to this study is presented at the end in terms of production loading, logistics, container loading and global supply chain management problems.

Starting with an introduction to the linear programming and stochastic linear recourse programming, chapter 3 presents the robust optimization framework, including the robust linear optimization model with solution robustness, the robust linear optimization model with model robustness, and the robust linear optimization model with trade-off between solution robustness and model robustness.

Chapter 4 discusses the production loading problems for global manufacturing. After introducing the production loading process, we identify the main problems and difficulties when the production managers in the manufacturing companies make decisions. A dual-response production loading strategy is introduced to hedge against uncertainty and short production time. After that, a linear programming model for certain production loading problems with import quota limits is presented. Then, we formulate a stochastic linear recourse programming model to deal with uncertain production loading problems associated with changing quota price. We finally formulate three types of robust optimization models to deal with uncertainty and risk. A series of experiments are designed to test the effectiveness of the proposed robust optimisation models. Compared with the results of the linear programming and the stochastic linear recourse programming model, the robust optimization models can provide a more responsive and flexible production loading plan with less risk for uncertain production loading problems associated with changing quota price.

Chapter 5 discusses the logistics problems for global road transport. After introducing the crossing border process, we identify the main problems and difficulties faced when the managers in the logistics companies make decisions. A dual-response logistics strategy is presented to hedge against uncertainty and short shipment notice. A mixed 0-1 integer programming model for certain logistics problems is presented. Then, we formulate a stochastic mixed 0-1 integer recourse programming model to deal with uncertain logistics problems for global road transport. We finally formulate three types of robust optimization models to deal with uncertainty and risk. A series of experiments are designed to test the effectiveness of the proposed robust optimization models. Compared with the results of the linear programming and stochastic mixed 0-1 integer recourse programming model, the robust optimization models can provide a more responsive and flexible global road transport plan with less risk.

Chapter 6 discusses the container loading problems for global air transport. After introducing the container loading process, we identify the main problems and difficulties faced by logistics managers in the freight forwarding company when they make decisions. A dual-response container loading strategy is presented to hedge against uncertainty and short shipment notice. After that, a mixed 0-1 integer programming model for certain container loading problems is presented. Then, we formulate a stochastic mixed 0-1 integer recourse programming model to deal with uncertain container problems. We finally formulate three types of robust optimization models to measure the trade-off between the cost and the risk. Compared with the results of the mixed 0-1 integer programming and stochastic mixed 0-1 integer recourse programming model, the robust mixed 0-1 integer optimization models can provide a more responsive and flexible container loading plan with less risk.

Chapter 7 presents the conclusions drawn by this thesis and recommendations for future research.

Chapter 2

Problem statement and literature review

2.1 Supply chain practice in China

2.1.1 Mainland China's economy and logistics

Economic development in Mainland China

China has suffered tremendous political and economic upheavals since the founding of the People's Republic of China in 1949. During the 1950s and the 1960s, economic policy was based on the philosophy of a planned economy. Economic and business activities were totally controlled by the Government. A historic change occurred in 1979 as China began its 'open door' economic policy. Economic reform regained its momentum in 1992. The Chinese Government stressed the need for establishment of a social market economic system. 11^{th} of December of 2001 makes a key date in the calendar of world trade (Jackson 2003), as on the day, China joined the World Trade Organization (WTO). China's economy registered an average growth rate of 7-8% in the 1990s. This growth is expected to persist as China's economy continues to get more and more integrated with the global economy - GDP grew 9.4% in 2004. China is the 6th largest economy in the world with a GDP of US$ 1,929.21 billion (in 2004). It became the 4th largest economy in 2005 with a GDP of approximately US$ 2.18 trillion. China is the world's largest developing economy, and its continued growth is critical to the overall world economy and to the welfare of its population of 1.3 billion.

Logistics in China

With the fast development of its economy, logistics is receiving significant attention in China. The transportation and logistics sector in China has historically been under the control of state monopolies. China's logistics industry has been growing fast since its accession to the WTO. However, despite the rapid development of the logistics industry in China, it is still not a well-defined industry in comparison with other developed countries. At present, there is little integration in logistics services throughout China. The logistics in China is seen as consisting of a number of sub-sectors for transporting, warehousing, customer brokerage, etc. Most logistics companies only participate in one or a few of parts, rather than providing the whole range of logistics services.

- ***Warehousing and distribution:*** Because of rapid economic development in China, changes have occurred within distribution systems. The older inefficient, hierarchical, vertically organized distribution networks are being replaced by a more market orientated system; however, the degree of change varies from city to city and from province to province. Distribution in China is an expensive business activity. Distribution costs are much higher in China than in North America and Europe. The main problems include poor infrastructure of the distribution network, slow delivery service, poor location and transportation links, a lack of computerized facilities, spoilage caused by poor packaging, etc.

- ***Transportation systems:*** Development of transportation systems in China has fallen behind the pace of the country's rocketing economic development. Problems include old transport technologies, limited railway services, and road systems with serious congestion, especially in fast growing areas (Yam and Tang 1996). It is still difficult to move goods around China. The country's underdeveloped transport infrastructure presents one of the biggest challenges to multinational corporations' supply chain distributions. The logistics of transportation face numerous serious problems that have major implications for the success of supply chain management.

- ***Roads:*** Inadequate road infrastructure and transport facilities remain a barrier to efficient distribution. Highway density in China is significantly less than Japan, UK, or US. The Chinese government has been aware of this problem and has planned a series of road and highway building programs using the services of many foreign investors. Actually, China has built many highway bridges and highways in recent years. However, road quality is unsuitable for heavy cargo transportation in many regions in China, and road upgrading and maintenance works fail to follow the rapid increase in demand for road transportation. These factors result in traffic jams, which can seriously impact a company's logistics and distribution strategies. Chinese vehicles are often poorly maintained and this, coupled with poor road surfaces and congestion, means that breakdowns are inevitable. In addition, many of China's highways are toll roads and this can add to a company's transportation costs. All these problems mean that companies have to tailor their logistics and distribution strategies carefully.

- ***Rail:*** Rail still plays an important role in movement of goods in China. The major problem is that there is not enough capacity, and many of the rail lines are old. Other problems in rail transport include excessive loading, spoiled and damaged goods, and unreliable delivery times. Rail shipments often need to be booked months in advance. Compared with western countries, the Chinese railway network is spread very thin. In order to rectify the situation and to cope with the fast expanding economy, China is taking up a major expansion plan for the country's rail network and is investing in rail and related projects, including new rails, rolling stock, and locomotives as well as technical renovation of the existing rail infrastructure.

- ***Air:*** The growth in China's airport sector has been extraordinary. Air freight has grown rapidly during the past ten years. China has several airlines of it's own. However, air freight accounts for only a small percentage of total freight carried in China. Transportation of cargo by air in China still suffers from routing problems, poor ground services, long cargo shipment schedules, poor cargo handling facilities, and insufficient transport infrastructure linking the airports to nearby industrial areas.

- ***Ports:*** There are still too few deepwater ports in China. Of the country's 60 major coastal ports, only 446 of a total 1,322 berths are deepwater ones. This is about the same number as in New York and less than in Antwerp. The Chinese Government has built numerous new berths and ports, including container ports and inland ports. Other ports are undergoing renovation or expansion. The country has a network of inland waterways including the Yangtze and several other large rivers. Canals link parts of the country and barges still represent an option for distribution, particularly, if time is not an issue or cost is a major consideration. Barges are a good way of transporting goods, as they can be cheap, although using canals or inland waterways can be slow, and security may be a problem.

The total logistics cost as a percentage of GDP has widely been used as an indicator of the development level of the logistics industry in many developed countries. The higher the percentage, the less efficient is the logistics industry. The total logistics cost as a percentage of GDP in China has gradually declined from 24% in 1991 to 21.3% in 2004. However, this figure is still more than double the 10% figure in some developed countries, like the US and Japan, suggesting that there is big scope to improve China's logistics industry.

2.1.2 Hong Kong's economy and logistics

Economic development in Hong Kong

Hong Kong's prosperity started with its light manufacturing industries in the 1950s. By the 1970s, it had become renowned as a manufacturing centre in the world. In the 1980s, Hong Kong industry faced a series of problems, such as global trade restrictions, rising protectionism, shortages of labour and increasing land/labour costs. Fortunately, it was at about the same time that China adopted its open door policy. Hong Kong shifted its labour-intensive production activities to China to take advantage of cheap labour and land resources. Hong Kong is characterised by its high degree of internationalization, business-friendly environment, free trade, substantial foreign exchange reserves, a fully convertible and stable currency, a simple tax system with low tax rates, well-developed financial networks, and superb transport infrastructure. Hong Kong has presence of almost all the great international names from Sony, Panasonic, HSBC, Citibank, to Toyota, YKK, and Heniz. According to the Hong Kong Trade Development Council, over the past two decades, the Hong Kong economy has more than doubled in size, with GDP growing at an average annual rate of 4.9 per cent. GDP grew by 8.2% in the third quarter of 2005, marking its ninth consecutive quarterly growth since mid-2003. The growth of GDP reached 7.3% for the first three quarters of 2005. Hong Kong is one of the richest regions in the world.

Logistics in Hong Kong

Located at the mouth of the Pearl River with a deep natural harbour, Hong Kong is geographically and strategically important as a gateway for China and trans-shipment port for intra-Asian and world trade. Hong Kong is the eighth largest trading entity in the world and the world's busiest container port. It has also been the major contact point for Mainland China, especially for Southern China, with the rest of the world for decades, and this intermediate role has been further enhanced in recent years because of China's booming economy.

- ***Road:*** Road transportation in Hong Kong, unlike in other regions in China, is currently the major mode of transport for moving goods to and from Southern China. All road freight traffic travelling between Hong Kong and Mainland China must cross one of the three border control points, which are in Sha Tau Kok, Lok Ma Chau, and Man Kan To. Well-constructed transportation networks and expressways are favourable forms of transportation between Hong Kong and Mainland China.

- ***Water:*** Endowed with a deep-water, silt-free natural harbour strategically located along a major sea route and with the Mainland China providing a huge cargo base, Hong Kong is a major sea transport hub in Asia. Hong Kong is one of the busiest container ports in the world. There are 8 container terminals with total 19 berths in Kwai Chung and Stonecutters Island.

- *Air:* Given its excellent geopolitical location, Hong Kong has grown over recent decades to become a key hub for international aviation - both for passengers and for air cargo. Almost all prominent airlines have offices in Hong Kong. Hong Kong airport has an air freight-forwarding centre, providing space for warehousing loading platforms, truck parking bays and offices. Hong Kong is also a home to a large and dynamic clustering sector with almost 300 shippers, freight forwarders and other related companies linked with customs, insurance and finance issues.

2.1.3 Economic links between Mainland China and Hong Kong

According to the Hong Kong Trade Development Council, Hong Kong is the largest source of foreign direct investment in Mainland China accounting for about 51% of the national total, with a cumulative value of US$ 157.7 billion from 1979 to 1999. Taking the first ten months of 2005, Hong Kong is the Mainland's third largest trading partner (after Japan and the US). According to China's Customs Statistics, bilateral trade between the Mainland and Hong Kong amounted to US$107.1 billion (9.3% of the Mainland China's total external trade) in Jan-Oct 2005. Exports from Mainland China to Hong Kong grew to US$ 85.6 billion, making Hong Kong the second largest export market after the US. Hong Kong has been actively participating in the re-export trade with Mainland China, particularly through outward processing activities. According to the Hong Kong Trade Development Council Statistics, in 2004, 43.5% of Hong Kong's total exports to Mainland China are related to outward processing activities.

2.1.4 China's textile and clothing exports

According to the Hong Kong Trade Council statistics, China's external trade in 2004 reached US$ 1,155 billion - the third highest in the world, with exports and imports growing at 35.4% and 36% respectively, up from the fourth place in 2003. In addition, export-processing trade continues to be the major part of external trade. In 2004, exports and imports related to export-processing trade grew 35.7% and 36.1% respectively. Export processing accounted for 55.3% of China's total exports in 2004. China is gaining an increasingly competitive position in world textile and clothing markets because of its cheap labour cost and highly skilled workforce. Today, China is the world's largest textiles and clothing exporting country. Textiles and clothing make up approximately one-quarter of China's total exports by value; and around one-quarter of China's total textiles and clothing exports go to the US and the European Union (Dickson 2005).

2.1.5 Quota limitations

Import quotas are assigned by importing countries. Quotas control the quantity or volume of certain merchandise that can be imported into North American and European countries. The importing countries allocate a certain quantity of quota to each exporting country. Any products that belong to quota restriction categories have to have the corresponding quotas for the exporting countries. Many developing countries, including China, face restraints on textile and clothing exports to their trading partners that maintain import quotas, including the US, Canada and European Union. For example, clothing and textile products are divided into 147 categories by the US and 143 categories by the European Union. However, not all the exporting countries face the same quota limitations for products. For example, China faces the US's quota limitation in 81 of 147 categories, while for India the figure is 30. At the same time, China faces quota limitation in 61 of 143 categories assigned by the EU, while for India it is 17 (Dickson 2005). Therefore, global manufacturing companies have to consider quota limitations when they distribute manufacturing tasks among different exporting countries. If the quota amount for a certain category product is used up in a country, companies have to find alternatives in other countries that own quotas for the product.

2.2 Company background

This study is concerned with problems that occurred in a global supply chain network providing garments for Northern America and Europe markets. Manufacturing factories are located in China and other low-cost countries. Two companies are involved in manufacturing and distribution: a global manufacturer, LT International Group Ltd., and a third-party logistics provider, CTSI.

Founded in 1965, LT is one of the leading apparel supply chain providers in the world, with over 17,000 employees producing over 50 million pieces of garments annually. The company operates 12 manufacturing facilities and has 14 offices around the world. Headquartered in Hong Kong, LT has its manufacturing facilities in Mainland China, the Philippines, Cambodia, Sri Lanka, Thailand, etc., and the sales and marketing offices are mainly centred in North America and Europe. Its products include sleepwear, intimate wear, pants and shorts, sports and active wear, ladies' fashion and children's wear. LT is a major supplier of some of the world's best-known and top-selling brands including ABERCROMBIE & FITCH, Dillard's, DKNY, EXPRESS, FAST RETAILING, GAP, JCPENNY, JONES NEW YORK, LIZ CLAIBORNE, NAUTICA, POLO RALPH LAUREN, STRUCTURE, THE LIMITED, TOMMY HILFIGER, UNI QLO, etc.

The vision of LT is to be recognized by their customers as the best apparel supply chain service partner in the world. The CEO of LT thinks LT is more than a manufacturing company - a leading "one-stop-shop" apparel supply chain service provider. He also states that LT does not want to be a vertical player in the whole supply chain as it would make the company lack flexibility in terms of satisfying customer needs. Additionally, LT has no intention to build it's own brands, as doing this would mean competing with its customers. Therefore, LT positions itself as only an apparel maker being able to provide the whole supply chain solution for different brands of products to different companies around the world. For this, LT has built up a design-to-store business model, which emphasizes customer relationship and develops competitive advantages by developing better products with short life cycles, shortening lead time, speeding market delivery, providing end-to-end apparel supply chain services. LT's services include:

- ***Design and development:*** LT partners with its customers at a very early stage of the supply chain - product design development stage - to provide competitive products. LT has product design facilities, including a graphics studio, fabric library, print, washing, embroidery and sample shops, technical, fabric and accessories testing centres, all of which help customers transform their concepts into a real apparel piece ready for batch production. At the same time, LT maintains professional design expertise, transforming customers' sketches into workable series of designs. The graphics studio is linked to the sales offices around the world, which allows quick response to the changing markets.

- ***Materials management:*** Increasingly closer partnership with materials suppliers is an important aspect of materials management service at LT. It can help its customers obtain high quality products, reasonable price and quick delivery.

- ***Manufacturing***: The core of the design-to-store business model is manufacturing. LT continues to expand multi-product, multi-country manufacturing services to provide customers end-to-end value propositions. The collaborative end-to-end apparel supply chain services aim at satisfying customer demand at all stages in the supply chain, including design and development, sourcing, marketing, manufacturing, warehousing, transporting and distributing on a global base.

- ***Logistics:*** In order to focus on its core business of product design, development and manufacturing, LT outsources its logistics function to CTSI Logistics. Through its affiliate, CTSI Logistics, LT provides tailored logistics services to its customers in terms of packing, transporting, tracking, warehousing, cargo forwarding and other activities related to logistics.

To further increase the competitiveness, LT partners with other vendors to create a win-win outsourcing strategy. Currently, LT has two important affiliates: IST, an information technology company, and CTSI, a third party logistics provider. Established in 1998, IST is the industry leader in providing collaborative apparel supply chain solutions. Its earliest products include GO2000, an ERP system for LT company, and an early version of ASNx/FGA, a scan and pack order fulfilment solution for finished garments. Today, IST's vision is to build solutions that enable members of the apparel supply chain to deliver the garments in the right quantities, to the right stores, at the right time and at the lowest cost. Established in 1989, CTSI is recognized by its international competitiveness, stability and an ever-expanding global network. Headquartered in Hong Kong, CTSI provides tailored logistics services for its customers, including:

- ***International freight forwarding:*** By working with the most proven consolidators and shipping lines, CTSI offers a series of logistics services, including air and sea freight import and export shipping, inland transporting, tracing shipments, in-house customs brokerage, etc.

- ***Freight management***: CTSI has an on-line global tracking and tracing system designed with a centralized database, linking shippers, consignees, carriers and other involved parties in the shipment process. Connectivity to a central databank enables CTSI and its customers to share information.

- ***Warehousing and distribution:*** CTSI can monitor and control warehouse business processes and day-to-day activities in warehouses, including receiving, packing, consolidating, shipping, etc.

- ***Inventory management:*** CTSI offers its customers convenient access to real-time information about inventory status of the consignment in warehouses. Through the Internet, this online inventory system provides customers with fast and accurate information on inventory locations and space availability, which can be utilized by CTSI's other trading partners like LT.

In relating to the business with LT, CTSI performs two major services in the whole global supply chain, which will be discussed in this study. One is global road transportation, and the other is global air cargo forwarding. To be a global road transport provider, CTSI is responsible for transporting all finished goods from the Mainland China warehouse to the Hong Kong warehouse. The logistics services include warehousing, packing, preparing all documents needed to cross the border, contacting the air transport division for further airfreight transport, etc. Meanwhile, to be a global air cargo forwarder, CTSI is engaged in transporting the shipment from the supply sites to demand sites around the world by air. The global air transport services include renting air containers from air carriers, consolidating small shipments into different types of air cargo, and loading them into the air containers. Global air cargo forwarding has some special characteristics, which differ from domestic freight forwarding. The global airfreight forwarder has to be knowledgeable in all aspects of international shipping in terms of preparing export documentation, obtaining cargo insurance, arranging cargo shipments with air carriers, packaging markings for international shipment, loading and tracing cargo, etc.

Why does the company use Hong Kong's airport to ship overseas?

Currently, moving goods in China is still difficult and expensive. Additionally, China's logistics providers have little experience in international shipping in terms of preparing all documents, obtaining cargo insurance, tracing shipments, paying freight charges, providing language translation, etc. However, logistics providers based in Hong Kong can tap into a global network of overseas branches with frequent flights, have an understanding of international practices, and can offer more customized services such as warehousing, distributing, trucking, consolidating, etc. Hong Kong is a regional air transportation hub, and tops the world in terms of international cargo handled.

Why does the company ship products by air?

Fast delivery is a main advantage offered by air transport. Being part of a global air transport network makes it possible to reduce door-to-door shipping time to 48 hours, regardless of the distance involved. The fact that the world's major cities are linked by daily air services also offers frequency and dependability. In addition, air transport offers lighter and less expensive packing costs, lower insurance premiums, elimination of transit warehousing and transfer costs, smaller inventories and related inventory costs, and faster capital turnaround. The efficiency induced by air transport is extremely critical because of heightened expectations of customers, shortened product lifecycle and fierce competition in the global environment. Supplying a market ahead of competitors provides remarkable advantages in terms of the flexibility and responsiveness to changing market demands. Time saving is particularly important for the fashion industry, which leaves global manufacturers a longer time margin to allocate production among their global factories to satisfy uncertain market demand, or to beat seasonal deadlines when sales are at their peak. Therefore, transporting the final products to North America and Europe by air is another part of the global manufacturer's strategic plan.

2.3 Problem statement

Hayes *et al.* (2005) state that as they confront the twenty-first century, managers around the world experience mixed emotions: a sense of real accomplishment accompanied by frustration and uncertainty. The garment manufacturing company under this study has its affiliate IST to provide real time data and communication between all members in the whole supply chain. However, obtaining and sharing data is one thing, and taking action on the information, particularly for the real-time information is another. In the uncertain and dynamic environment, operations managers, such as production managers and logistics managers, are losing their confidence in traditional supply chain planning approaches as these approaches are unable to deal with emerging problems in the global supply chain management environment. Particularly, these approaches suffer inability to handle the uncertainty and the risk involved, which are particularly important in the global supply chain management environment.

2.3.1 Phase I: production loading problems for global manufacturing

In this study, the manufacturing company's headquarters are in Hong Kong and its own plants are located in different countries, such as Mainland China, Thailand, the Philippines, Sri Lanka and Vietnam. When necessary, the company can outsource its production to other contracted plants. The company's sales departments, customer services and markets are centred in Northern America and Europe. The sales departments collect product information from local retailers, and send it to the Hong Kong headquarters. Based on this information, the Hong Kong headquarters need to forecast the market demand for different types of products that will be on the market in the next selling season. The products under this study are fashion garments. Like personal computers, they belong to the group of innovative products, which have a very short life cycle and lead time. The predicted demand for innovative products involves substantial uncertainty, as markets' reaction to a new, innovative product is unclear, and this increases the risk of a shortage or excess supply. The manufacturing company, however, can not wait until they are able to ascertain accurate market demand as it is impossible to globally produce and distribute the product to customers then. The manufacturing company has to determine production loading plans and start to produce products that will be on the market in the next selling season on the basis of uncertain information. However, the purpose of the production plan is to satisfy the customer. Order commitment for products become clear only when the selling season is coming. Until then, the manufacturer has to respond to the different market information that has been observed. Therefore, production managers feel challenged while allocating production because of uncertain market demand and quota prices, short lead times and other uncertain information while aiming at satisfying market demand and simultaneously trying to minimize the production costs.

Products produced in other countries are first transported to a warehouse in China, which leads to the Phase III problems. Normally all products are stored in the Mainland

China warehouse before they are transported to the Hong Kong warehouse. These products, however, need to be transported across the border to Hong Kong by truck, which leads to the logistics problems for global road transport referred in Phase II. After the products arrive in Hong Kong, they are immediately loaded into air containers for shipment to different destinations in North America and Europe: this is the container loading problem for global air transport referred to in Phase III.

2.3.2 Phase II: logistics problems for global road transport

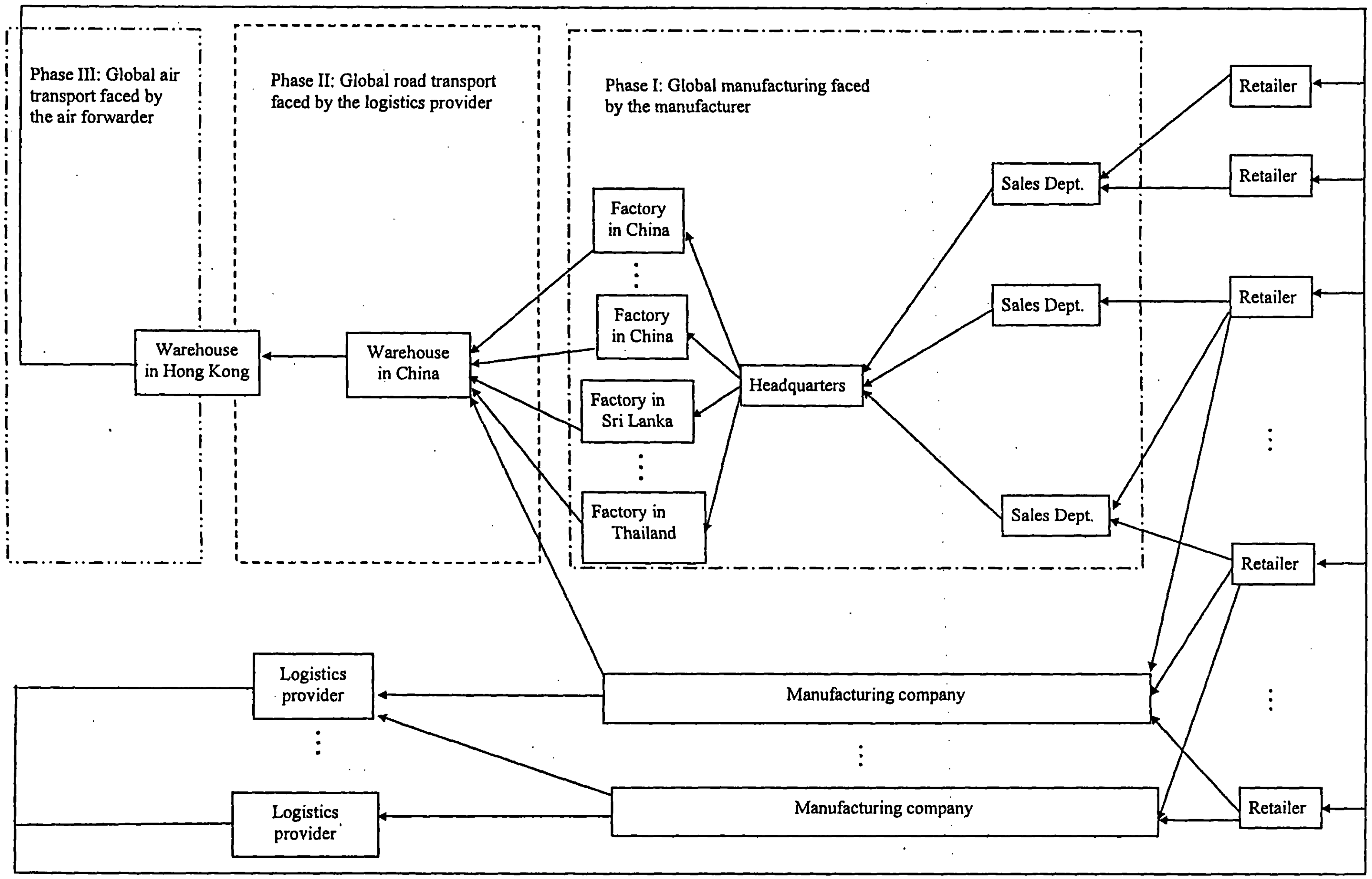
The logistics provider is responsible for the whole logistics service for the manufacturer in terms of crossing-border road transport, warehousing in two countries, packing, loading, unloading, preparing documents, contacting the air forwarders for air transport. The manufacturing company in this study, however, is only one of the logistics provider's customers. The logistics company provides the global road transport service for many customers, which need to transport their goods from Mainland China to Hong Kong. Because of the very high inventory cost and space limitation of the Hong Kong warehouse, the products are normally stored in the Mainland China warehouse, and are not moved to the Hong Kong warehouse until the onward shipment schedule is firmed up. On the shipping day, the products are transported from the Mainland China warehouse to the Hong Kong warehouse, from where they are immediately consolidated into air cargo, loaded into the air containers, and shipped to overseas markets. Therefore, the logistics managers have to determine a crossing border logistics plan in terms of the fleet composition, transportation route, inventory level, etc. Unfortunately, the logistics managers can not obtain accurate shipment information until the shipping day. Because of the capacity limitation of the fleet and changing demand of crossing-border transportation, the logistics managers have to determine the quantities and types of the trucks that will be hired from the two countries in advance for crossing border before the exact shipment information can be obtained. Therefore, the logistics managers experience the challenges of global road transport in terms of uncertain shipment information, short shipment notice, preparing for responding to different scenarios that might happen on the shipping day, aiming at satisfying customer requirements and minimizing the logistics cost.

2.3.3 Phase III: container loading problems for global air transport

The second service that the logistics provider offers is to transport their shipment to North America and Europe by air. Some functions deal with arrangement and shipping, such as renting air containers from air carriers, consolidating small shipments into different types of air cargo and loading them into the air containers. However, international air cargo forwarding has some special characteristics, which differ from domestic freight forwarding. The international airfreight forwarder has to be knowledgeable in all aspects of international shipping in terms of preparing export documentation, obtaining cargo insurance, arranging cargo shipments with air carriers, packaging markings for international shipment, loading and tracing cargo, etc. Air containers are often used in international shipping. After initial loading, the cargo is not re-handled until it is unloaded at its final destination. However, containerization changes commodities handling from a labour-intensive to a capital-intensive task. Therefore, when the international forwarder makes a decision about how to pack the cargoes into the air containers for international shipment, they have to bear in mind both the costs of renting the containers, the costs of warehousing the cargoes, and the costs of penalizing the unshipped cargoes. Therefore, the forwarder managers experience the challenge of global air transport in terms of the changing shipment information and short shipment notice, the cost of booking containers in advance, the higher penalty cost of requiring additional containers or cancelling the containers on the shipping day, and planning responses for different scenarios that might happen on the shipping day, aiming at satisfying customer shipment requirement while minimizing the operation cost.

The whole supply chain process can be summarized as follows:

Table 2.1: Global supply chain network



2.4 Literature review

2.4.1 Literature on production loading problems for global manufacturing

Analysis of production loading problems has been an active area of research for many years. See inventory carrying and set-up systems in Wagner and Within (1958); and Dillenberger *et al.* (1994); inventory carrying cost and labour cost considerations in Dzielinski and Gomory (1965), Florian and Klein (1971), and Lason and Terjung (1971); heuristic approach for multi-level lot-sizing with a bottleneck in Billington *et al.* (1986); multi-stage production and inventory systems in Goyal and Gunasekeran (1990); multi-item lot sizing systems in Pocket and Wolsey (1991), among others. Shapiro (1993), Thomas and McClain (1993), Silver *et al.* (1998) present excellent general references about production loading problems.

All the above literature present models and techniques for a deterministic environment, where all information that decision-making needs is accurately known. Sen and Higle (1999) think it is difficult to precisely estimate certain critical data elements, and it is necessary to address the impact of uncertainty during the planning process. Alonso-Ayuso *et al.* (2003) state that the treatment of the stochasticity has only recently been applied to production planning. See deterministic approximations to stochastic production systems in Bitran and Yanasse (1984); stochastic multi-item batch production systems in Zipkin (1986); a tactical planning model to evaluate capacity loading under varying demand in Graves (1986); derived demand and capacity planning under uncertainty in Modiano (1987); a scenario approach to capacity planning in Eppen *et al.* (1989); a scenario approach to characterize the uncertain demand for production planning in Escudero (1993); and models and algorithms for distribution under uncertainty in Cheung and Powell (1996).

To date there exists little research that addresses the import quota issue by modelling the production loading problems in global manufacturing, let alone considering the uncertainty involved. As a result, little research uses stochastic programming, including stochastic recourse programming and robust optimization, to model the production loading problems in the global supply chain management environment under uncertainty.

2.4.2 Literature on logistics problems for global road transport

Analysis of logistics and transportation has been an active area for researchers and practitioners since it was first carried out during the World War II. However, early work purely considered logistics problems as transportation problems without considering other factors in the logistics process such as packing, labelling, warehousing, consolidating, etc. For related work see the bi-criteria transportation problem in Aneja and Nair (1979); fleet size problem in Etezadi and Beasley (1983); multiple objectives transportation problems in Current and Min (1986) and Current and Marish (1993); interactive algorithms to solve multi-objective transportation problems in Ringuest and Rinks (1987) and Climaco *et al.*, (1993); a tabu search approach for the fixed charge transportation problem in Sun *et al.* (1998); and insertion-based savings heuristic algorithms for the fleet size and mixed vehicle routing problem with time windows in Liu and Shen (1999).

Global logistics is defined as exporting and importing products or services beyond the boundaries of a country. Global logistics present logistics managers with a more difficult challenge than domestic logistics in terms of packing, labelling, transport modes and cost, labour cost, warehousing, government policy and regulation, etc. Cohen *et al.* (1989) present international supply chain models with considerations related to global trade in terms of raw materials and production costs, the existence of duties, tariffs, different tax rates among countries, random fluctuations in currency exchange rates, and the existence of constraints not included in single-country models. Fawcett (1992) claims that limited research has been done on international logistics strategy, and that the existing literature focuses on descriptions only. Goldsborough (1992) provides an analytical report on global logistics management in which two different logistics systems – domestic and international – have been compared. Cohen and Kleindorfer (1993) present a framework for the operations of a global company to determine plant location and capacity, product categories, material and cash flow in an international scenario. However, no model formulation or experiments are provided in their paper. Vidal and Goetschalckx (1997) think global supply models are more complex and difficult to solve than domestic models, as the flow of cash and the flow of information are more important and difficult to coordinate in an international scenario than they are in a single country environment. Goetschalckx *et al.* (2002) give a review of integrated strategic and tactical models and design algorithms for global logistics systems. They point out that a great deal of research has been conducted in quantitative techniques for improvement and optimization of supply chains without global considerations; of which mixed-integer programming models are among the most widely-used techniques. They also report that most models address the problem in a regional, local, or single-country environment, where international factors do not have a significant impact on the supply chain design. Geoffrion and Powers (1995) give an evolutionary perspective to 20 years of strategic distribution system design, and think logistics has changed from a neglected activity to an essential business function. Coyle *et al.* (2003) think countries are coming closer and closer because of the success of logistics. They find many global manufacturers are using a new managerial strategy, called focused production, in which one or a few plants are designated as the worldwide supplier(s) of the given product(s). The plants are typically located in different countries, requiring a global logistics system to deliver items to the right place, in the right quantity, at the right time.

Road transport is the most important among all transport modes. Muller (1999) notes that, in the U.S, of the nearly 7.8 million tons of freight and commodities moved in 1996, an estimated 46% was moved by truck (up almost 78% since 1980), compared with 26% by rail, 13% by water, and 15% by pipeline. However, road transportation beyond the boundary of a country caught the attention of researchers and practitioners only a few years ago when globalization became an important issue in business organizations. Bergan and Bushman (1998) present the North America Trade Agreement (NAFTA) perspective on cross-border trucking transportation between the US, Canada, and Mexico, and emphasize the importance of efficient border-crossing systems. Bochner *et al.* (2001) examine the possibility of expediting current port-of-entry processing of commercial vehicles entering the US from Mexico, provide the basic prototype plan for northbound commercial border inspection stations with automated processing, and suggest bi-national links to improve cross-border system's efficiency.

Little research has been done to apply stochastic recourse programming and robust optimization techniques to solve logistics problems for global road transport under uncertainty.

2.4.3 Literature on container loading problems for global air transport

Packing cargoes into a container is an important materials-handling activity in manufacturing and distribution industries (Chen *et al.* 1995). Containers were first used in 1950s. Through the years, the cargo handled via containers has steadily increased. Containers are defined as large boxes that are used to transport goods from one destination to another (Vis and Koster 2003). The efficient stowage of goods in a means of transport can often be modelled as a container loading problem (Bortfeldt and Gehring 2001). There exists a large body of literature related to container loading problems, which is classified as the three-dimensional (3D) rectangular packing problem in the general cutting and packing problem. Cutting and packing problems involve different dimensions. Gilmore and Gomory (1965) were the first to discuss the one-dimensional stock cutting problem as a linear programming problem. Then they address the 2D and 3D problems with related algorithms. Dyckhoff (1990) presents a survey and classification of cutting and packing problems.

Bischoff and Ratcliff (1995) are critical of many publications that cover container loading, saying their material is based on pure knapsack-type formulations of the problem structure. They highlight some important shortcomings in the existing theoretical literature on container loading, and demonstrate the requirement for fundamentally new approaches to be able to tackle different situations arising in practice. Davis and Bischoff (1999) also think much of the literature considers the container purely as a storage device relying on a study of the 3D cutting-packing problems, rather than considering the problem as that of a transport medium.

In practice, a container can be classified as a road container, a sea container or an air container. Much of literature treats container loading problems as cutting-packing problems focusing on the road and sea container with the objective of minimizing the total unused space in the containers. Cattrysse *et al.* (1996) present a case study on road container transport. Their study discusses the building of a prototype decision-support system for a road container transport company in the light of constraining factors which affect scheduling of trucks and vehicle routing problems of various kinds. Vis and Koster (2003) classify the decision problems arising at sea container terminals and give an overview of relevant literature. For the sea container, much of the literature studies empty sea container allocation problems faced by shipping companies in terms of how to distribute empty containers to the shippers and how to relocate empty containers in preparation for future demand. Early work using network models for empty container allocation problems can be found in White (1972). Cheung and Chen (1998) consider the dynamic empty sea container allocation problem where they need to reposition empty containers and to determine the number of leased containers to satisfy customer demand over time. In their study, a stochastic quasi-gradient method and a stochastic hybrid approximation procedure are applied to solve the empty sea container allocation problem.

Air containers, however, have some special characteristics, which differ from road or sea containers. Delivery time is critical. Air containers usually carry low-density and high-value cargo. Air containers also have limitations on weight and volume of the cargoes inside. In addition, air transport is a capital intensive industry. It is very important to choose adequate containers to ship the cargoes at the right time with the lowest cost. For air container problems, much of work focuses on the weight distribution issue in a container or an aircraft. Martin-Vega (1995) presents a complete review of manual and computer-assisted approaches to air container loading problems, in which the centre of gravity is considered via pyramid loading. A new approach provided by Davis and Bischoff (1999) considers weight-distribution considerations in container loading, in which an even weight distribution can be attained whilst simultaneously achieving a high degree of space utilization. Mongeau and Bès (2003) address the problem of loading as much as freight while balancing the load in order to minimize fuel consumption and satisfying stability and safety requirements. A mathematical programming model is formulated to choose which containers should be loaded on the aircraft and how they should be distributed among different compartments. Ivancic *et al.* (1989) and George (1996) discuss the container packing problem with rental cost functions, but the cost of using each container is a constant related only to the container. The cost of renting the container has no relationship to the weight that the container holds. Groenewege (1996) points out the importance of air transport and reports that airfreight forwarding represents over a third of the total value of all international trade. For many countries this percentage is considerably higher, and most nations involved in international trade are seeing a steady increase in the percentage of goods being moved by air. Coyle *et al.* (2003) think containerization has gained notable acceptance in international distribution, and report that some firms containerizing shipments to foreign markets have reduced the service time and cost by 10 to 20 percent and have increased the service level they provide to these markets.

There have been few studies in the literature dealing with how to choose containers and load cargoes into them simultaneously, with the considerations of volume and weight of both the containers and cargoes - the cost of renting a container depends on the cargo weight loaded and the delivery time. To our best knowledge, there is little work in the past to model the container loading problems, as well as the consideration of uncertainty and risk.

2.4.4 Literature on global supply chain management problems

Dornier *et al.* (1998) state a vast majority of manufacturers have some form of global presence through exports, strategic alliances, joint ventures, or as part of a committed strategy to sell in foreign markets or locate production abroad. Rosenfield (1996) notes that the challenges of global manufacturing present a series of challenging management problems that are similar, but which are also very different from traditional methods. There is extensive literature on global supply chain management problems. We divide them into the following aspects:

- ***Global supply chain network design***: A great deal of research has been carried out in designing supply chain networks on a global scale. Hodder and Jucker (1985) develop models for an international plant location problem. Cohen and Lee (1989) point out how a company should structure its plants around the world to supply a global market with variations in consumer's expectations, recourse conditions, and cost structures from country to country. A survey article, presented by Verter and Dincer (1992), presents a review of modelling issues on international plant location, capacity acquisition, and technology selection. Rosenfield (1996) develops a number of deterministic and stochastic models to determine the number of plants and production levels in a global environment for a firm in order to minimize production and distribution costs for geographically dispersed markets. Taylor (1997) presents a model to integrate product choices considering global plant capacities with an assumption of known unit costs and no trade barriers. Ferdow (1997) emphasizes that a country attributes would determine whether it becomes a manufacturing hub with exports to other countries or a market for imported goods, or both. Vidal and Goetschalckx (1997) present an extensive literature review of global supply chain models, and state that there is a lack of research on mixed integer programming models for the strategic design of global supply chain systems. Goetschalckx *et al.* (2002) present the potential saving generated by the integration of the design of strategic global supply chain networks with the determination of tactical production-distribution allocations and transfer prices, which combines strategic planning and tactical planning in the global supply chain networks. Chakravarty (2005) develops a model that optimizes plant investment decisions and determines prices of products by countries. They also analyze labour costs, transportation costs, demand and import tariff on production quantities, etc.

- ***Coordination in the global supply chain:*** Supply chain coordination is increasingly viewed as a source of strategic advantage for participating members (Kulp *et al.* 2003). Cohen and Mallik (1997) emphasize that competitive advantages can be achieved through global supply chain management only if the management of the chain's geographically-dispersed activities is effectively coordinated. Coordination is, therefore, the key concept in implementing a global supply chain strategy. Kogut (1985a,1985b) was the first to describe the importance of global coordination and develop global strategies. In 1993, Dasu and Torre (1993a, 1993b) study a case covering the affiliates of a U.S. multinational firm in three Latin American countries, concentrating on the coordination problem. A single-period deterministic game theoretical model is formulated to determine the price and sale amount for each firm and this is used in two scenarios: one scenario is in the competitive environment, where affiliates compete against each other as well as with other companies; and the other scenario is in the cooperative environment where the affiliates' activities are coordinated. Different factors related with international activities are considered in the model: these include exchange rates, inflation rates, and tariff rates. Ahmadi and Yang (1995) study a parallel-import problem in a global supply chain under the assumption that a manufacturer could implement price discrimination in different markets. Thus parallel importers can buy products in low-priced markets and sell them in high-priced markets.

- ***Exchange rates:*** Co-ordination within the global supply chain provides a firm with an opportunity to respond to uncertain events such as exchange rate fluctuations, changes in government policy, competitors' decisions or new technologies. A major issue for global manufacturing is the impact of exchange rates. Lessar and Lightstone (1986) propose a qualitative study on the effect of exchange rate fluctuation in a multinational company. An extensive section of the literature (Cohen and Lee 1989, Tombak 1995, Dasu and Li 1997, Hadjinicola and Kumar 2002) discusses important factors such as tariffs, taxes, currency exchanges rates, shipping costs, domestic resources and demands, trade barriers, etc.

- ***Global purchasing:*** Some researchers focus on the perspective of global purchasing and supply functions. As Pyke and Johnson (2003) state, companies outsource an increasing amount of the value of their products, and sourcing strategies have rapidly shifted in leading companies all over the world. They also present a framework to help managers make decisions on sourcing issues in terms of strategic alliances and e-procurement. Dyer *et al.* (2001) discover that by 2001, there is an average of 60 major strategic alliances in each of the top 500 global businesses.

- ***Stochastic models for global supply chain management problems***: As Cohen and Mallik (1997) state in their analytical review of the literature, the majority of reported models lack practicality and would be difficult to implement. They also state that few of the models incorporate price and demand uncertainties in international markets. Kougut and Kulatilaka (1994) develop a stochastic dynamic programming model that explicitly treats supply chain flexibility as the equivalent of purchasing an option whose value is dependent upon the exchange rate. They consider a two-country, production switching model and derive optimal cost functions. This model does not consider detailed operational characteristics (e.g. multiple products or supply chain stages) and becomes intractable for more than one exchange rate process (such as when operating in more than two counties). Dasu and Li (1997) provide optimal strategies for a firm whose plants are located in different countries where there is exchange rate variability. A stochastic programming model is developed. The combined capacities of the plants exceed the single product deterministic demand. Thus, the firm can allocate production among the plants, depending on the exchange rate. Kouvelis and Sinha (1995) formulate a model that allows switching of production models for a firm in a foreign country. They present a profit-maximizing, multi-period, stochastic dynamic programming formulation, and conclude that a strongly depreciated home currency favours an export policy, while a strongly appreciated home currency favours a joint venture or wholly owned subsidiary. The choice between a joint venture or a wholly-owned subsidiary depends on transaction costs (including production, distribution and logistics costs), per unit demand in each production mode, as well as switching costs from these modes to the export mode. Axarloglou *et al.* (1993) address an empirical study to test the analytical results proposed by Kouvelis and Sinha (1995). Both of the studies focus on exchange rates. Huchzermeier and Cohen (1996) present a modelling framework that integrates network flows and option valuation approaches to global supply chain modelling for a multinational firm in terms of an enumerative currency. They propose a hierarchical approach to solving the problem with the discussion of the exchange rate risk for global operations.

- ***Case studies and applications for global supply chain management problems:*** Many authors report on the applications of mathematical models to global supply chain management. There are a number of interesting cases that illustrate how to develop a global supply chain strategy model. However, problems at the tactical and operational planning level are paid little attention. Breitman and Lucas (1987) were probably the first to report an application study of global supply chain management. They develop a decision support system to support multinational planning at General Motors for several years. The system applies mixed integer programming to generate a series of operational and financial reports. The system can solve multi-period location problems of significant complexity. Cohen and Lee (1989) develop a normative model of resource decisions, which is used to analyze Apple Computer's global manufacturing and distribution network. The model maximizes global after-tax profit in terms of currency. The decision include the assignment of vendors to plants and distribution centres; the assignment of supply links from plants to distribution centres, and distribution centres to customers; the assignment of products and subassemblies to plants; and production mix at each plant. Since the basic model is single period and deterministic, the model can be run for multi-period problems under alternative future scenarios to quantify the value of flexibility of global supply chains. Cohen *et al.* (1989) develop a multi-period extension of the above model, which explores the trade-offs between centralization and localization of global supply chain strategies. Lee *et al.* (1993) develop the implementation of a series of global supply chain management models at Hewlett-Packard. The models focus on a worldwide inventory network optimizer. The model is basically a network of nodes, in which each node is assumed to operate like a periodic-review inventory system. The model determines the optimal inventory in different locations and in different forms, and is used to model the Vancouver supply chain of HP Deskjet printers. Bartmess (1994) presents an analytical report from eight experts on how an American bicycle manufacturer expands its production into Mainland China. Arntzen *et al.* (1995) consider the global supply chain model at Digital Equipment Corporation. The model minimizes a weighted combination of total cost and activity days (i.e. production and transportation days) in the company's global supply chain network. The decision variables include site locations, capacity decisions, manufacturing technology at each site, product mix, shipping modes and quantities, and duty drawback locations. The model is solved using a variety of optimization tools, and it has been used by Digital to analyze new product strings and supply strategies for components.

Chapter 3

Robust optimization framework

3.1 Introduction to stochastic programming

In mathematical programming, one solves the problem of selecting one alternative, which is optimal with respect to a certain criterion, from a set of feasible solutions. An important subfield is linear programming, characterized by a linear criterion function and linear constraints that describe the set of alternatives. A general linear programming model can be formulated as follows:

$$\min c^T x \tag{3.1}$$

$$\text{s.t. } Ax=b \tag{3.2}$$

$$x \geq 0 \tag{3.3}$$

where x is an $(n \times 1)$ vector of decision variables, and c, A, b, are known data of sizes $(n \times 1)$, $(k \times n)$, and $(k \times 1)$, respectively. If needed, any less-than-or-equal-to constraint can be transformed into an equality constraint by adding a slack variable, and any greater-than-or-equal-to constraint can be transformed into an equality constraint by subtracting a surplus variable.

Linear programming is a fundamental planning tool for quantitative analysis of decision making problems. Since its introduction by Dantzig (1955), linear programming has proved to be a powerful tool in modelling and solving practical problems. The problems include marketing, finance, economics, engineering, manufacturing, transportation, facility location and layout, supply chain management, etc. However, when modelling the linear programming problems, it is assumed that the value of each parameter in the linear programming models can be accurately obtained, which means all the information for decision-making is available at the time of planning. Decision making usually involves uncertainty such as noisy, incomplete or erroneous data. Sen and Higle (1999) think it is difficult to precisely estimate certain critical data elements, and it is necessary to address the impact of uncertainty during the planning process. Explicitly considering uncertainty, in some situations, is very critical and failure to include uncertainty may lead to very expensive, even disastrous consequences if the anticipated situation is not realized (Bai *et al.* 1997). One method to handle this uncertainty is to apply sensitivity analysis, finding how changes in coefficients will influence the optimal solution. Mulvey *et al.* (1995) think sensitivity analysis is only a post-optimality study, which only discovers the impact of data uncertainties on the model's recommendations. For some applications a proactive approach may be adequate; however, in some situations, when the decisions depend heavily on the value of inaccurate data, it might be reasonable to take uncertainty into the consideration in a more fundamental way. *Stochastic programming* is a branch of mathematical programming that copes with a class of mathematical models and algorithms in which the data may be subject to uncertainty. Since its invention in the 1950s by Beale (1955), Dantzig (1955), and Charnes and Cooper (1959), stochastic programming has made significant applications in many areas, including electric power generation (Murphy *et al.* 1982), financial planning (Cariño *et al.* 1994), telecommunications network planning (Sen *et al.* 1994), transportation (Ferguson and Dantzig 1956, Powell 1988), empty container allocation (Cheung and Chen 1998), supply chain network design (Santoso *et al.* 2005), and strategic supply chain planning (Alonso-Ayuso *et al.* 2003). General references on stochastic programming are books by Vajda (1972), Kall (1976), Kall and Wallace (1994), Birge and Louvveaux (1997) and Prékopa (1995). Excellent survey articles related to stochastic programming applications and algorithms are presented by Birge (1997), Sen and Higle (1999) and Dupaćová (2002).

Consider the following model:

$$\text{"min"}\, c^T x \tag{3.4}$$

$$\text{s.t. } Ax=b \tag{3.5}$$

$$\text{“}T(\omega)x = h(\omega)\text{”} \tag{3.6}$$

$$x \geq 0 \tag{3.7}$$

where x is an $(n \times 1)$ vector of decision variables, and c, A, b, are known data of sizes $(n \times 1)$, $(k \times n)$, and $(k \times 1)$, respectively. Constraint (3.5) represents k deterministic constraints. In constraint (3.6), $\omega \in \Omega$, (Ω, F, P) is a probability space. $T(\omega)$ and $h(\omega)$ denote a random $(l \times n)$ matrix and $(l \times 1)$ vector, respectively. Constraint (3.6) represents l stochastic constraints. The above problem, expressed in (3.4)~(3.6), is not well defined since the meanings of "min" as well as of the constraints are not clear at all, and a revision of the modelling process is necessary (Kall and Wallace, 1994). In the optimization literature, there are two main approaches to construct a meaningful optimization model. One approach is called chance constrained programming, which was pioneered by Charnes and Copper (1959) and developed as a means of describing constraints in the form of probability levels of attainment (see Kall 1976, Kall and Wallace 1994, Mayer 1992, and Prékopa 1973, 1995). The other approach is called two-stage recourse programming, developed by Beale (1955) and Dantzig (1955). Recourse programs are those in which some decisions or recourse actions can be taken after uncertainty is disclosed (see Kall 1976, Vajda 1972, Birge and Louveaux 1997, and Ruszczyński and Shapiro 2003).

3.2 Chance constrained programming

Chance constrained programming is a tool used for modelling risk and risk aversion to handle uncertain problems. In the chance constrained model, infeasibility is accepted, but only if it occurs with a low probability. The model is extended by specifying a reliability coefficient $\alpha \in [0,1]$ and replacing (3.6) by

$$\Pr\{T(\omega)x = h(\omega)\} \geqslant \alpha \tag{3.8}$$

The chance constrained model can be formulated as:

$$\min c^T x \tag{3.9}$$

$$\text{s.t. } Ax=b \tag{3.10}$$

$$Q(x) \geq \alpha \tag{3.11}$$

$$x \geq 0 \tag{3.12}$$

where

$$Q(x) := \Pr\{T(\omega)x = h(\omega)\} \tag{3.13}$$

where α is some fixed constant number in [0,1], which is chosen by the decision maker, and $Q(x)$ is the *reliability* of decision x. $Q(x)$ is the probability that stochastic constraints $T(\omega)x = h(\omega)$ are satisfied; its complement 1-$Q(x)$ is the 'risk' of infeasibility concerning x. In the chance constrained model, the decision maker accepts possible violation of the stochastic constraints, but only if the risk is, at the most, 1-α.

Alternately, one may specify a reliability coefficient α_i ($0\leq\alpha_i\leq1$) for each constraint, $T_i(\omega)x = h_i(\omega)$, i=1,2,..., l, and replace (3.6) by:

$$\Pr\{T_i(\omega)x = h_i(\omega)\} \geqslant \alpha_i,\ i=1,2,\ldots,l. \tag{3.14}$$

The former model is said to have a joint chance constraint, and the latter separate chance constraints. Clearly, it is possible to combine joint and separate chance constraints in a model.

3.3 A two-stage recourse programming

3.3.1 A two-stage recourse model

Recourse models are the most important class of models in stochastic programming. This remains one of the more widely studied class of models, and most of the applications are reported in the literature (see Kall 1976, Wets 1988, Kall and Wallace 1994, Mayer 1992). This concept leads to extending the model to a so-called two-stage recourse model. At the first stage, before realization of the corresponding random variables become known, one chooses the first stage decision variables to optimize the expected value of an objective function which, in turn, is the optimal solution of the second stage optimization problem. A two-stage stochastic linear programming model can be written as follows (Ruszczyński and Shapiro 2003):

$$\min_x c^T x + E_\xi(Q(x,\xi)) \tag{3.15}$$

$$\text{s.t. } Ax=b \tag{3.16}$$

$$x\geq0 \tag{3.17}$$

where $Q(x,\xi)$ is the optimal solution of the second stage problem

$$Q(x,\xi) = \min_y\{q(\omega))^T y : W(\omega)y = h(\omega) - T(\omega)x, y \geqslant 0\} \tag{3.18}$$

We have a set of decisions to be taken without full information on some random event. These decisions are called the first stage decisions, which are represented by vector x of size ($n \times 1$). Corresponding to x are the first-stage vectors and matrices c, A and b of sizes ($n \times 1$), ($k \times n$), and ($k \times 1$), respectively. When full information is received on realization of random vector ξ, the second stage actions are taken, which are represented by vector y of size ($m \times 1$). Corresponding to y are the second-stage vectors and matrices $q(\omega), W(\omega), h(\omega), T(\omega)$ of size ($m \times 1$), ($l \times m$), ($l \times 1$) and ($l \times n$), respectively. $\omega \in \Omega$, (Ω, F, P) is a probability space. ξ is the vector formed by the components of $q(\omega), W(\omega), h(\omega), T(\omega)$. The expectation in the objective function in (3.15) is taken with respect to the probability distribution of ξ, which is known. Matrix $T(\omega)$ and $W(\omega)$ are referred as technology and recourse matrices, respectively. We assume that $W(\omega)$ is fixed (fixed resource). Often, we use the same notation ξ to represent a random vector and its particular realization. Which one of these two meanings will be used in a particular situation will be clear from the context. If in doubt, we will write $\xi = \xi(\omega)$ to emphasise that this is a random vector defined on a corresponding probability space (Ruszczyński and Shapiro 2003). Similarly, we use the same notations q, W, h, and T to represent random vectors/matrices and their particular realizations.

3.3.2 The value of stochastic solution

When the two-stage recourse model is formulated, its solution is called the stochastic solution, denoted as x^*, and its performance is called *the expected objective value of the stochastic solution*, denoted as *ESS*. Therefore, the two-stage recourse model expressed in (3.15)~(3.18) can be written as:

$$ESS := \min_x E_\xi z(x,\xi) = c^T x + \min_y\{q^T y \mid Wy = h - Tx, y \geqslant 0\} \tag{3.19}$$

$$\text{s.t. } Ax = b \tag{3.20}$$

$$x \geq 0 \tag{3.21}$$

For handling the uncertainty, a natural way is to solve a much simpler corresponding deterministic problem: the one obtained by replacing all random variables by their expected values, for all random parameters (Birge and Louveaux 1997). This problem is called *the expected value problem*, which can be expressed as:

$$EV := \min_x z(x,\bar{\xi}) = c^T x + \min_y \{\bar{q}^T y \mid \bar{W}y = \bar{h} - \bar{T}x, y \geqslant 0\} \tag{3.22}$$

$$\text{s.t. } Ax=b \tag{3.23}$$

$$x \geq 0 \tag{3.24}$$

where $\bar{\xi} = E(\xi)$ denotes the expectation of ξ, and $\bar{q}, \bar{W}, \bar{h}, \bar{T}$ represent the expectation of q, W, h, T, respectively. The solution of the expected value problem expressed in (3.22)~(3.24) is called *the expected value solution*, denoted by $\bar{x}(\bar{\xi})$. In order to measure the performance of the expected value solution $\bar{x}(\bar{\xi})$, we define *EEV* as *the expected result of the expected value solution, or the expected result of using the EV solution. EEV* measures how $\bar{x}(\bar{\xi})$ performs, allowing the second-stage decisions to be chosen optimally as functions of $\bar{x}(\bar{\xi})$ and ξ (Birge and Louveaux 1997).

$$EEV := E_\xi(z(\bar{x}(\bar{\xi}),\xi)) = \min_x z(\bar{x}(\bar{\xi}),\xi) = c^T \bar{x}(\bar{\xi}) + \min_y \{q^T y \mid Wy = h - T\bar{x}(\bar{\xi}), y \geqslant 0\} \tag{3.25}$$

$$\text{s.t. } Ax=b \tag{3.26}$$

$$x \geq 0 \tag{3.27}$$

We use *VSS* to denote the difference between *the expected objective value of the stochastic solution and expected value solution. VSS* is referred as *the value of the stochastic solution.* We have:

$$VSS=EEV\text{-}ESS \tag{3.28}$$

Madansky (1960) establishes the following relations between *EEV* and *VSS.*

Property 1: $VSS \geq 0$ *(or* $EEV \geq ESS$*)* (3.29)

Proof: Because x^* is an optimal solution for the stochastic model expressed in (3.19)~(3.21), while $\bar{x}(\bar{\xi})$ is just one solution to the stochastic model expressed in (3.19)~(3.21), we then reach the above conclusion.

Now by using a very simple production planning example shown as below, we could understand the relationship between *EEV* and *ESS*. It is assumed that a company wants to make a plan for processing a certain amount of product A. The relevant data related to product A is given in Table 3.1.

Table 3.1: An illustrative example

Scenario	1	2
Demand	10	20
Probability	40%	60%
Unit cost for underproduction	3	3
Unit cost for overproduction	1	1

Let

x production quantities in the first stage

y_1^- / y_1^+ shortage/surplus quantities for scenario 1 in the second stage

y_2^- / y_2^+ shortage/surplus quantities for scenario 2 in the second stage

A two-stage recourse model is formulated as follows:

$$\min x + 0.4(3y_1^- + 1y_1^+) + 0.6(3y_2^- + 1y_2^+) \tag{3.30}$$

$$\text{s.t. } x + y_1^- - y_1^+ = 10 \tag{3.31}$$

$$x + y_2^- - y_2^+ = 20 \tag{3.32}$$

$$x, y_1^+, y_1^-, y_2^+, y_2^+ \geq 0 \tag{3.33}$$

The above model is a linear programming model, and can be easily solved using mathematical programming software, such as Excel Solver, AIMMS, Lindo, etc. The optimal stochastic solution is: x*=20, $y_1^+ = 0$, $y_1^- = 10$, $y_2^+ = 0$, $y_2^- = 0$. It means that 20 units of the product will be manufactured before accurate demand is identified. If scenario 1 happens, there will be 10 surplus units. If scenario 2 happens, production quantities will be exactly equal to the demand. The objective function value of the stochastic solution is 24. Thus we have: *ESS* =24.

The corresponding expected value model for the above problem can be formulated in the following form, in which the stochastic demand is replaced by its expected value.

$$\min x + 3y^- + y^+ \tag{3.34}$$

$$\text{s.t } x + y^- - y^+ = 16 \tag{3.35}$$

$$x, y^-, y^+ \geq 0 \tag{3.36}$$

where x = production quantities

y^- / y^+ = shortage/ surplus

The optimal solution of the expected value model is: $\bar{x}=16$, $y^{-}=0$, $y^{+}=0$. The objective value: $EV=16$. Unfortunately, the actual demand is either 10 or 20. When we produce 16 units according to the result of the expected value model, there will be two scenarios that may happen in the future. If scenario 1 appears in the future (with the probability of 40%), there would be 6 units of surplus resulting in a surplus cost ($\overline{y_1^-}=0, \overline{y_1^+}=6$); if scenario 2 appears in the future (with the probability of 60%), there would a shortage of 4 units ($\overline{y_2^-}=4, \overline{y_1^+}=0$). Therefore, the expected result of using the EV solution is:

$$EEV=\min\{\bar{x}+0.4(3\overline{y_1^-}+\overline{y_1^+})+0.6(3\overline{y_2^-}+\overline{y_2^+})=16+0.4\times 6+0.6\times 3\times 4=25.6 \tag{3.37}$$

where $\overline{y_1^-},\overline{y_1^+}$ satisfies equation $16+\overline{y_1^-}-\overline{y_1^+}=10$, and $\overline{y_2^-},\overline{y_2^+}$ satisfies equation $16+\overline{y_2^-}-\overline{y_2^+}=20$. It can be seen that $EEV > ESS$, which indicates that the performance of the stochastic solution is better than that of the expected value model for this problem. The value of the stochastic solution for this problem is: $VSS= EEV-ESS$=25.6-24=1.6.

3.3.3 A two-stage stochastic linear recourse programming model

In the two-stage stochastic recourse model expressed in (3.15)~(3.18), it is assumed that the random data $\xi(\omega)$ has a discrete distribution with a finite number S of possible realizations $\xi_s=(q_s,W_s,h_s,T_s)$, called scenarios, with the corresponding probabilities p_s, $p_s=P(\{\omega \mid \xi(\omega)=\xi_s\}), s=1,2,...S$, $p_s>0$, and $\sum_{s=1}^{S} p_s=1$.

For example, one random variable for production planning problems could be the future state of the economy, which could be three different scenarios (or realizations) that might happen in the future: good, fair, and bad. The actual demand is dependent on the economic condition: a high demand associated with a good economy at a possibility of 30%, a medium demand associated with a fair economy at a possibility of 60%, and a low demand associated with a bad economy at a possibility of 10%.

Therefore, in the case of finite discrete distribution, the two-stage stochastic recourse programming model can be equivalently reformulated as the following algebraic equivalent linear programming form:

$$\min c^T x + \sum_{s=1}^{S} p_s (q_s)^T y_s \tag{3.38}$$

$$\text{s.t. } Ax=b \tag{3.39}$$

$$T_s x + W_s y_s = h_s,\ s=1,\dots,S \tag{3.40}$$

$$x \geq 0, y_s \geq 0, s = 1,\dots,S \tag{3.41}$$

where y_s represents the response for each realization of the random variables, s=1,…,S.

In the above model, x is referred to as a vector of the *first stage variables*, whose value is not conditional on realization of the random variable. $y_1, y_2, \dots, y_S$ is referred as the *second stage variables*, which are subject to adjustment, once the random variable is determined. The constraints, therefore, are also classified as the first stage constraints and the second stage constraints. The constraints that only involve the first stage variables are defined as the *first stage constraints*. The first stage constraints have to be satisfied before accurate information is obtained. The rest of the constraints that consist of the first stage variables and the second stage variables are referred as the *second stage constraints*, which have to be satisfied for all realizations of the stochastic variables. Equation (3.39) denotes the first stage constraints, and equation (3.40) denotes the second stage constraints. The first term in (3.38), denoted by $c^T x$, is called *the first stage cost*. The second term in (3.38), denoted by $\sum_{s=1}^{S} p_s (q_s)^T y_s$, is called *the second stage cost*. The sum of the first cost and the second stage cost in (3.38) is defined as the *expected cost* of the objective function value of the two-stage stochastic resource programming model, which is the expected total cost of making the two-stage decisions.

For example, the first stage decision variables for the production planning problems are often associated with proactive decisions, such as machine capacity, labour hours, product quantities in normal production, etc; the second stage decision variables are often associated with reactive decisions, such as quantities of surplus, quantities for outsourcing, etc. For the container loading problems, the first stage decisions include the booking information about the container types and quantities that will be needed in the following weeks; the second stage decisions are made on the shipping day, including the container types and quantities that are required or/and returned, as well as how to allocate all cargo into containers. The first stage constraints for the production planning problems include machine capacity and work force level. It means the machine and labour level can not exceed the maximum capacity available, no matter what the accurate information could be in the future. The second stage constraints have to be satisfied for each scenario that might occur in the future. For the container loading problems, for example, we have to pack all cargo in containers, which may need to require additional containers in the case of large cargo quantities or return unused containers in the case of small cargo quantities on the shipping day.

3.4 Robust optimization

3.4.1 A brief introduction to robust optimization

Stochastic recourse programming is an important approach used to handle uncertainties in the decision making process; and it has a wide range of applications. Sen and Higle (1999) think stochastic programming optimizes an expected-value criterion, and it often includes constraints on downside risk. Mulvey *et al.* (1995) point out that stochastic recourse programming optimizes only the first moment of the distribution of the objective function value, and ignores higher moment of the distribution, and the decision maker's risk attitude, which are particularly critical for asymmetric distributions, and for risk averse decision makers. In addition, the stochastic recourse programming model has no ability to handle situations in which no feasible solution exists for each scenario. Mulvey *et al.* (1995) first propose the robust optimization technique, which integrates goal programming formulations with a scenario-based description of problem data. They define two concepts: *solution robust* and *model robust*. The optimal solution of the stochastic programming model will be *solution robust* if its objective value stays 'close' to optimal for all realizations of the random variables. The solution will be *model robust* with respect to feasibility if it remains 'almost' feasible for any realization of the random variables.

Robust optimization has a number of applications. Vassiadou-Zeniou and Zenios (1996) integrate traditional simulation models for bond pricing with robust optimization technique to develop tools for management of portfolios of callable bonds. They present two models: a single period model that imposes robustness by penalizing downside tracking error, and a multi-stage stochastic program with recourse. Gutiérrez *et al.* (1996) use the robust optimization technique to solve incapacitated network design problems under uncertainty. They present a formal definition of "robustness" for incapacitated network design problems, and develop algorithms aimed at finding robust network designs. The computational experiments show that robust solutions are able to handle incapacitated network design problems and the proposed algorithm performance is satisfactory in terms of cost and number of robust network designs obtained. Yu (1997) discuss the classical economic order quantity (EOQ) model under significant uncertainties. A robust optimization approach is proposed to find an inventory policy that performs well under all realizations of stochastic parameters. An efficient linear time algorithm is designed to find the robust decisions. By comparing the results of the stochastic decisions, the paper demonstrates the advantages of the robust approach. Laguna (1998) formulate a robust optimization model to solve the problem regarding capacity expansion at one location in telecommunications, with demand uncertainty. In their paper, the graphical display of the trade-off between expected shortage reduction and the total cost has been found to be a particularly appealing analysis tool by actual network planners. Sen and Higle (1999) give an introductory tutorial on stochastic linear models, in which the robust optimization approach is discussed. A mean-variance robust optimization is presented. An example is provided to illustrate the first-stage decisions, the second stage decisions, expected cost, and variance of the robust model. Darlington *et al.* (1999) discuss robust formulation for controlling the constraints of systems under uncertainty. They present a nonlinear and stochastic model, and a mean-variance robustness framework is proposed. In their paper, they also discuss the feasibility via a penalty framework. Yu and Li (2000) propose a robust optimization model for solving logistics problems. Two examples from a wine company and an airline company are presented to demonstrate the computational efficiency of the proposed model. List *et al.* (2003) formulate a robust optimization model for a fleet planning problem. An example illustrates the importance of including uncertainty in the fleet sizing problem formulation, and the nature of the fundamental trade-off between acquiring more vehicles and accepting the risk of potentially high costs of outsourcing resources. Takriti and Ahmed (2004) examine the robust optimization approach in the context of two-stage planning systems. They study the impact of different measures for variability on two-stage planning problems.

In 2006, the *Mathematical Programming* journal published a special issue on robust optimization, which carried 10 articles exploring different topics in this field. For example, Adida and Perakis (2006) discuss a robust approach to dynamic pricing and inventory control with no backorders problem. In the introduction part of this issue, Ben-Tal *et al.* (2006) state that robust optimization is a relatively recent technique, which has been successfully applied in a number of areas. They also think robust optimization is a challenging field with many real-world applications; it also has strong connection with other fields, such as statistics and control. Chen *et al.* (2007) use new deviation measures for random variables, namely, the forward and backward deviations, to construct uncertainty sets for robust optimization. They also propose a tractable approximation approach to solve a class of multistage chance-constrained stochastic optimization problems. A project management problem is presented to demonstrate the framework of the approach.

3.4.2 A brief introduction about different types of risk measurement

Markowitz (1952) is probably the first one to propose a measure of the risk associated with the return of each investment, where the variance of random returns or losses is used as a measure of risk. Markowitz (1952) suggestes that investors consider expected return a desirable objective to maximize, but only while also considering risk an undesirable element that needs to be minimized. Scegö (2005) states that the Markowitz model goes in hand with appropriate utility functions, allowing a subjective ordering of preferences of assets and their combinations. In the case of non-normal, albeit symmetric distributions, utility functions must be quadratic, which, in practice, restricts the use of this model to portfolios characterized by normal joint return distribution, i.e. to the case in which returns of all assets, as well as their dependence structure, is normal.

Value at Risk (VaR) is a popular measure of risk, which is extensively used in analysis of portfolio optimization. VaR is defined as a threshold value; the probability of a loss function exceeding this value is limited to a special level (Jorion 1997, Basak and Shapiro, 2001). Although VaR is a very popular measure of risk, it has undesirable mathematical characteristics such as a lack of subadditivity and convexity (Rockafellar and Ursasev, 2000). Additionally, VaR is coherent only when it is based on standard deviation of normal distribution. Furthermore, VaR is difficult to optimize when it is calculated from different scenarios. A very serious shortcoming of VaR is that it provides no handle on the extent of the losses that might be suffered beyond the threshold amount indicated by this measure (Rockafellar and Ursasev, 2002). It is incapable of distinguishing between situations where losses that are worse may be deemed only a little worse, and those which could well be overwhelming. Scegö (2005) points out that VaR, if applied to most (not elliptical) return distributions, is not an acceptable risk measure because:

- it does not measure losses exceeding VaR;
- a reduction of VaR may lead to stretch the tail exceeding VaR;
- it may provide conflicting results at different levels;
- non-subadditivity implies that portfolio diversification may lead to an increase of risk and prevents adding up VaR of different risk sources;
- non-convexity makes it impossible to use VaR in optimization problems;
- VaR has many local extremes leading to unstable VaR rankings.

Rockafellar and Uryasev (2000) propose a new approach as an alternative measure of risk, called Conditional Value-at-Risk (CVaR). CVaR, also called Mean Excess, Mean Loss, Mean Shortfall, or Tail VaR, is defined as the expected value of tail distributions of returns or losses. CVaR is known to have better properties than VaR (Rockafellar and Uryasev, 2000). Pflug (2000) proves that CVaR is a coherent risk measure having the following prosperities: transition-equivariant, positively homogeneous, convex, etc. Krokhmal *et al.* (2002) investigate CVaR models, and reformulated them as convex optimization problems in a portfolio problem. Szego (2005) gives a review on the main recently proposed risk measures, in which the mean, linear correlation coefficient, VaR and CVaR approaches are discussed. Alexander *et al.* (2006) develop a CVaR model for a portfolio problem and solve it by using a Monte Carlo method. Despite the interest in coherent risk measures, CVaR in returns has received criticism because its size grows linearly with the size of positions, thereby ruling out many of the inherently nonlinear, certainty equivalent-type risk measures suggested by the traditional utility theory (Brown, 2007). Andersson *et al.* (2001) think CVaR is a currency-denominated measure of significant undesirable changes in the value of the portfolio. To the best of our knowledge, VaR and CVaR are mainly applied in finance or the insurance industry, and there have been few studies in the literature dealing with risk issues in supply chain, using VaR or CVaR.

The modelling paradigm called "robust optimization" emerged from dissatisfaction with limitations of Markowitz's mean-risk model, and the stochastic recourse model (Bertsimas and Thiele, 2004). In their paper, they state that unlike the mean-risk model, the robust models need not have variances available. Unlike the chance-constrained model, VaR or CVaR, robust models need not know any probability distribution. The key elements of robust optimization are volatility and flexibility: the former asks for a solution that is relatively insensitive to data variations and hedges against catastrophic outcomes, and the latter is concerned with keeping options open in a sequential decision process having recourses for the effects of earlier decisions (Bertsimas and Thiele, 2004). The chance-constrained model, mean-risk model, VaR or CVaR has less ability to make a decision first and correct it when the stochasticity is realized. This property is important in the supply chain planning process as it is difficult to precisely forecast customers demand when production starts. Additionally, the lack of information about the probability of random events makes it impractical to use the chance-constrained model, mean-risk model, VaR or CVaR. For example, companies have less historic data to forecast market demand for a new product. Furthermore, robust optimization does not need full knowledge about the probability of random parameters. Additionally, robust optimization provides a quantitative method to measure the trade-off between cost and risk. Robust optimization provides decision makers accurate information in terms of what actions need to be taken for different realizations (scenarios), and what levels of risk the decision-makers would like to take. These properties are important in making decisions during the supply chain planning process under uncertainty; other risk measures have less ability to do so. Robust optimization, however, has some drawbacks, for example, specifying effective procedures for selecting scenarios, specifying multi-objective programming weights, and requiring high performance computers for solving robust optimization models (Mulvey *et al.*, 1995).

3.4.3 A robust linear optimization model with solution robustness

A robust optimization model with solution robustness means the solution will not differ substantially under different scenarios and there is less variability in the objective function across different scenarios of the stochastic variables. This kind of model is a suitable framework for quantitative analysis of decision problems involving trade off between the risk and the cost, which represent a less aggressive management style. The decision makers would like to pay more to reduce the risk of variability among different scenarios. A robust optimization model with solution robustness can be formulated as:

$$\min c^T x + \sum_{s=1}^{S} p_s (q_s)^T y_s + \lambda \sum_{s=1}^{S} p_s \left| (q_s)^T y_s - \sum_{s=1}^{S} p_s (q_s)^T y_s \right| \tag{3.42}$$

$$\text{s.t. } Ax = b \tag{3.43}$$

$$T_s x + W_s y_s = h_s,\ s = 1,2,\ldots,S \tag{3.44}$$

$$x \geq 0, y_s \geq 0,\ s = 1,2,\ldots,S \tag{3.45}$$

In the objective function (3.42), the third term $\lambda \sum_{s=1}^{S} p_s \left| (q_s)^T y_s - \sum_{s=1}^{S} p_s (q_s)^T y_s \right|$ is defined as the *expected variability cost*, where λ is a goal programming parameter representing the measurement of the variability of the objective function in the two-stage stochastic program. $\sum_{s=1}^{S} p_s \left| (q_s)^T y_s - \sum_{s=1}^{S} p_s (q_s)^T y_s \right|$ is defined as the *expected variability*, which measures the variability among all realizations of the stochastic variables. Clearly, in objective function (3.42), λ=0 means the variability is not considered in the decision-making process. Then the above model becomes a two-stage stochastic recourse programming model, which is the same model as is expressed in (3.38) ~ (3.41).

From the robust model with solution robustness described in (3.42)~(3.45), and the recourse model described in (3.38)~(3.41), we could observe that the optimal objective function value of the robust optimization model with solution robustness is not less than that of the corresponding stochastic recourse programming model. However, the solution of the robust optimization model with solution robustness is less sensitive, particularly for random data with asymmetric distribution, than that of the corresponding stochastic recourse programming model.

Yu and Li (2000) propose a robust model with the absolute term for a logistic management problem, and present an effective method to transform the model into a linear programming model by introducing additional deviation variables. In this study, we use the method proposed by Yu and Li (2000) to convert the model with the absolute term into a linear programming one. The model above can be formulated as a linear programming model by introducing a deviation variable $\theta_s \geq 0$.

$$\min c^T x + \sum_{s=1}^{S} p_s (q_s)^T y_s + \lambda \sum_{s=1}^{S} p_s ((q_s)^T y_s - \sum_{s=1}^{S} p_s (q_s)^T y_s + 2\theta_s) \tag{3.46}$$

$$\text{s.t. } Ax = b \tag{3.47}$$

$$T_s x + W_s y_s = h_s,\ s = 1,2,...,S \tag{3.48}$$

$$-(q_s)^T y_s + \sum_{s=1}^{S} p_s (q_s)^T y_s - \theta_s \leq 0, s = 1,2,...,S \tag{3.49}$$

$$x \geq 0, y_s \geq 0,\ \theta_s \geq 0\ s = 1,2,...,S \tag{3.50}$$

Proof: If $(q_s)^T y_s \geq \sum_{s=1}^{S} p_s (q_s)^T y_s$, we have $\theta_s = 0$. Then the objective function is equal to $c^T x + \sum_{s=1}^{S} p_s (q_s)^T y_s + \lambda \sum_{s=1}^{S} p_s ((q_s)^T y_s - \sum_{s=1}^{S} p_s (q_s)^T y_s)$; If $(q_s)^T y_s \leq \sum_{s=1}^{S} p_s (q_s)^T y_s$, we have: $\theta_s = -(q_s)^T y_s + \sum_{s=1}^{S} p_s (q_s)^T y_s$. The objective function is equal to: $c^T x + \sum_{s=1}^{S} p_s (q_s)^T y_s + \lambda \sum_{s=1}^{S} p_s ((q_s)^T y_s - \sum_{s=1}^{S} p_s (q_s)^T y_s)$.

3.4.4 A robust linear optimization model with model robustness

A robust optimization model with model robustness means violation of the second stage constraint is permitted, but this is done by the least amount by introducing a penalty function. A robust optimization model with model robustness can be formulated as:

$$\min c^T x + \sum_{s=1}^{S} p_s (q_s)^T y_s + \omega \sum_{s=1}^{S} p_s |e_s| \tag{3.51}$$

$$\text{s.t. } Ax = b \tag{3.52}$$

$$e_s = -T_s x - W_s y_s + h_s,\ s = 1,2,\dots,S \tag{3.53}$$

$$x \geq 0, y_s \geq 0,\ \ s = 1,2,\dots,S \tag{3.54}$$

Equation (3.53) denotes that some (or all) of the constraints in the second stage can be violated by the amount e_s, which is penalized in the objective function (3.51). In (3.51), $\sum_{s=1}^{S} p_s |e_s|$ is defined as the *expected infeasibility*, which is used to measure the infeasibility of the second stage constraints. In (3.51), ω is a parameter as a measurement of the infeasibility of the second stage constraints, and $\omega \sum_{s=1}^{S} p_s |e_s|$ *is* defined as the *expected infeasibility cost*. ω=0 means there is no penalty for not satisfying the second stage constraints. In this case, the second stage constraints can be violated as much as possible. On the other hand, $\omega \to +\infty$ means that any amount of violation of the second stage constraints is hardly accepted. As a result, any constraints at the second stage have to be satisfied because of the large penalty value of ω. Therefore, when ω is set up large enough, the robust optimization model with model robustness is converted into a two-stage recourse programming model, which is the same model shown in (3.38)~(3.41).

From the robust model with model robustness described in (3.51)~(3.54), and the recourse model described in (3.38)~(3.41), we could observe that the optimal objective function value of the robust optimization model with model robustness is not more than that of the corresponding stochastic recourse model. If the penalty for not satisfying the stochastic constraints in the robust optimization model with model robustness is not too large, some (or all) of the stochastic constraints in the robust optimization model with model robustness will be violated. When the penalty is large enough, the robust optimization model with model robustness becomes the stochastic recourse programming model, in which all constraints have to be satisfied.

By introducing a deviation variable $\delta_s \geq 0$, the robust optimization model with model robustness can be formulated as the following linear programming model:

$$\min c^T x + \sum_{s=1}^{S} p_s (q_s)^T y_s + \omega \sum_{s=1}^{S} p_s (e_s + 2\delta_s) \tag{3.55}$$

$$\text{s.t. } Ax = b \tag{3.56}$$

$$e_s = -T_s x - W_s y_s + h_s,\ s = 1,2,...,S \tag{3.57}$$

$$-e_s - \delta_s \le 0,\ s = 1,2,...,S \tag{3.58}$$

$$x \ge 0, y_s \ge 0,\ \delta_s \ge 0\ s = 1,2,...,S \tag{3.59}$$

3.4.4 A robust linear optimization model with trade-off between solution robustness and model robustness

When we consider the variability and infeasibility simultaneously, a robust optimization model featuring trade-off between the solution and model robustness:

$$\min c^T x + \sum_{s=1}^{S} p_s (q_s)^T y_s + \lambda \sum_{s=1}^{S} p_s \left| (q_s)^T y_s - \sum_{s=1}^{S} p_s (q_s)^T y_s \right| + \omega \sum_{s=1}^{S} p_s |e_s| \tag{3.60}$$

$$\text{s.t. } Ax = b \tag{3.61}$$

$$e_s = -T_s x - W_s y_s + h_s,\ s = 1,2,...,S \tag{3.62}$$

$$x \ge 0, y_s \ge 0,\ s = 1,2,...,S \tag{3.63}$$

In objective function (3.60), the first term is the first stage cost, and the second term is the second stage cost. The sum of the first stage cost and the second stage cost is the expected cost. The third term is the variability cost, and the fourth term is the infeasibility cost. Meanwhile, constraint (3.61) is the first stage constraint, and constraint (3.62) is the second stage constraint. The above robust optimization model can be further formulated as the following linear programming model by introducing two additional variables $\theta_s \ge 0$ and $\delta_s \ge 0$.

$$\min c^T x + \sum_{s=1}^{S} p_s (q_s)^T y_s + \lambda \sum_{s=1}^{S} p_s ((q_s)^T y_s - \sum_{s=1}^{S} p_s (q_s)^T y_s + 2\theta_s) + \omega \sum_{s=1}^{S} p_s (e_s + 2\delta_s) \tag{3.64}$$

$$\text{s.t. } Ax = b \tag{3.65}$$

$$e_s = -T_s x - W_s y_s + h_s,\ s = 1,2,...,S \tag{3.66}$$

$$-(q_s)^T y_s + \sum_{s=1}^{S} p_s (q_s)^T y_s - \theta_s \le 0, s = 1,2,...,S \tag{3.67}$$

$$-e_s - \delta_s \le 0,\ s = 1,2,...,S \tag{3.68}$$

$$x \ge 0, y_s \ge 0,\ \theta_s \ge 0, \delta_s \ge 0\ s = 1,2,...,S \tag{3.69}$$

Chapter 4

Production loading problems for global manufacturing

4.1 Introduction

4.1.1 Production loading process

Production loading has a fundamental role in any manufacturing operation. It is the process of determining what type of, and how many, products should be produced in future time periods. Manufacturing companies operating today, however, face a very different environment from that which was prevalent only a few years ago. With the substantial differentials in labour salary and raw material supply, continuously improving global logistics networks and dramatically decreased transportation costs, products can be manufactured anywhere in the world where it is feasible. In today's fiercely competitive global markets, companies are forced to compete on price and delivery performance to their customers in the face of rapidly changing conditions. Under the global manufacturing environment, effective production loading strategies can provide a critical competitive advantage for manufacturing companies in terms of the lower cost of production operations, the responsiveness and flexibility to changing market conditions and reducing risk. This is particularly true for industries, whose products have short life cycles and lead times, and market demand fluctuates over time.

Production loading problems under the global manufacturing environment are identified in global manufacturing companies, which are involved in global supply chain networks linking Asia, North America and Europe. Typically, product sales, R&D, customer service and market demand are centred in North America and Europe. Production facilities are most likely located in low-cost countries, such as Indonesia, Mauritius, Mexico, Nigeria, South Africa, South Korea, Thailand, Tunisia, Vietnam, and so on. However, China is one of the favourite places for manufacturing because of its low labour and production costs, its large supply of skilled workers, well-equipped facilities, high quality products, as well as its lucrative consumer market. This study considers a garment manufacturing company, which provides fashion garments to the North American and European markets. Products are manufactured in company-owned and contracted plants. The main products under this study are clothes, which are seasonal and timely. Decision makers need to determine the quantity of each product manufactured by different plants to fulfil market demand. The decision makers also need to determine the machine processing time, workforce level, inventory level, and quota utility etc.

Loading production is affected by some production constraints. To produce products, machine and labour are necessary recourses. However, in some production situations, the company can change the capacity of the sources by increasing the machine capacity (using additional machine capacity through leasing) and changing of workforce (through hiring, firing and overtime). Decisions include how much these recourses are needed.

Production is used to satisfy market demand. The ideal situation is production equals to market demand. Costs, however, are induced when the production is either less or greater than the demand, namely shortage cost or surplus cost, respectively. When the production exceeds the demand, the surplus products have to be stored, which incurs the surplus cost. On the other hand, when the production is not enough to satisfy the demand, the company has to purchase products from its contracted plants at a higher cost, which incur the shortage cost.

Loading production tasks globally is a more complicated process than domestic production plans. Not only do decision makers need to consider the factors in domestic production plans discussed above, but also some international issues; for example, the import quota limitations being considered in this study. Import quotas are assigned by importing countries and can be legally traded on the markets of exporting countries. Import quotas control the quantity or volume of certain merchandise that can be imported into North American and European counties. The importing countries allocate a certain quantity of quota to each exporting country. Any companies that want to export their products to North America and Europe have to buy the corresponding quotas for the products from local markets in exporting countries. At the beginning of the planning horizon, the company allocates a certain amount of quota for each type of products for each period. If the initial quota amount allocated in a period is less than market demand, the company has to buy additional quotas at market prices. On the other hand, if the initial quota allocated is not used up, the company suffers because of purchasing unused quota. The unused quota can be passed to the next period.

In Section 4.3.1, a linear programming model is formulated to determine production plans with import quota limits under the global manufacturing environment. The model assumes that all data that decision-making needs is known with certainty.

4.1.2 A dual-response production loading strategy for global manufacturing under uncertainty

Under the current global manufacturing environment, the production planning process involves many uncertain factors, such as market demand and quota price. One of the uncertain factors is quota purchasing price, which fluctuates frequently and depends on politics, economy, market supply and demand either from the exporting countries or from the importing countries, and so on. Before accurate market information is available, the company initially allocates a certain amount of quota for products to each period. After the stochastic variables are realized, the quota amounts that are initially allocated may not be equal to the actual demand in that period. The decision makers, therefore, need to make responses related to quota for different situations.

Additionally, demand uncertainty is also an important factor affecting production loading decisions. Under the global manufacturing environment, accurate market information becomes more and more difficult to obtain. Market demand usually come from different retailers mainly located in the North American and Europe markets, and these retailers tend to delay their commitments for their actual demand, which leaves manufacturers even less time to produce the products. The products under this study are fashion garments with short lead times. The manufacturing company, however, has to start production among the company-owned plants before accurate market demand is observed. When the sales season is nearing, the commitment for products will be clear. The company then has to take corresponding actions to satisfy the demand that has been realized.

In this study, we propose a dual-response production loading strategy, which consists of two-stage decisions. In the first stage, when accurate market information is not available, the company distributes initial quotas and production tasks among the company-owned plants. The first stage decisions include the production quantities for products, machine capacity, changes of workforce level (including the number of workers hired and fired), worker overtime and the allocated quota. In the second stage, once the stochasticity is realized, the company has to make responses for different scenarios that have been observed, such as how many additional products need to be outsourced to its contracted plants for urgent production to satisfy the high demand scenario, how many products have a surplus in the case of low demand, how many quotas need to be purchased from local markets when there is not enough quota, or how many quotas are left in the case of low demand.

In Section 4.3.2, a stochastic linear recourse programming model is formulated to structure the dual-response production loading strategy.

4.1.3 Risk

Despite its significant applications in many areas, stochastic recourse programming still has limitations owing to its inability to deal with risk and infeasibility of real-world applications under uncertainty. Today's customers have more power than ever before. They have more opportunity to compare price, quality, service, and delivery speed due to the massive amount of information captured from the Internet and other sources. Therefore, providing fast, responsive and flexible production while keeping risk and costs low in response to changing market demand gives a competitive advantage for manufacturing companies. In section 4.3.3, three types of robust optimization models, the robust optimization model with solution robustness, the robust optimization model with model robustness, the robust optimization model with trade-off between solution robustness and model robustness are presented for the production loading problems, which proposes a straightforward way to measure risk and cost.

4.1.4 Overview of chapter 4

The rest of the chapter is organized as follows. Section 4.2 presents the notation and definitions. Section 4.3 formulates a series of models, including the linear programming model under the assumption that all parameters are known with certainty, the two-stage stochastic recourse programming model under uncertainty, and the robust optimization models, which present a direct way to measure trade-off between risk and cost. Section 4.4 gives the computational results and analysis for the models. The final section gives the summary of the production planning problems under the global manufacturing environment.

4.2 Notation and definitions

In formulating the production loading models, the following notation and definitions are used.

4.2.1 Indices

i	for products (i=1,...,m);
j	for plants (j=1,...,n);
t	for time periods (t=1,...,T);

4.2.2 Parameters

Raw material and machine

r_{ij}	raw material cost of production for a unit of product i in plant j
a_j^1 / a_j^2	machine regular/additional cost of production per hour in plant j
g_{ij}^1 / g_{ij}^2	machine time for production of a unit of product i by skilled/non-skilled workers in plant j
C_{jt} / A_{jt}	maximum regular/additional machine capacity of plant j in period t
V_{jt}	minimum work time in plant j in period t

Labour

k_{ij}^1 / k_{ij}^2	labour cost of skilled/non-skilled workers making a unit of product i in plant j
o_j^1 / o_j^2	labour overtime cost of skilled/non-skilled workers per hour in plant j

h_{jt}^1 / h_{jt}^2 labour cost for hiring skill/non-skilled workers per hour in plant j at the beginning of period t

f_{jt}^1 / f_{jt}^2 labour cost for firing skill/non-skilled workers per hour in plant j at the beginning of period t

v_{j0}^1 / v_{j0}^2 initial labour level of skilled/non-skilled workers in plant j

α_j limit ratio between skilled and non-skilled workers for production in plant j

l_{ij}^1 / l_{ij}^2 labour time for production of a unit of product i in plant j by skilled/non-skilled workers

L_{jt}^1 / L_{jt}^2 maximum capacity of hiring skilled/non-skilled workers in plant j in period t

W_{jt}^1 / W_{jt}^2 maximum overtime for skilled/non-skilled workers in plant j in period t

Demand

D_{it} demand for product i in period t

Surplus/ shortage production

b_{it}^- / b_{it}^+ under-/over-production cost of a unit of product i in period t

I_i maximum inventory capacity for product i

B_i maximum purchasing capacity for product i

d_{i0}^+ initial inventory of product i at the beginning of the planning horizon

Quota

c_i initial quota purchasing cost of a unit of product i

c_{it}^- / c_{it}^+ under-/over-quota cost of a unit quota of product i in period t

Q_i initial quota quantity of product i at the beginning of the planning horizon

4.2.3 Decision variables

x_{ijt}^1 / x_{ijt}^2	production quantities of product *i* by skilled/non-skilled workers in plant *j* in period *t*
y_{jt}^1 / y_{jt}^2	planned labour time of hiring skilled/non-skilled workers in plant *j* in period *t*
z_{jt}^1 / z_{jt}^2	planned labour time of firing skilled/non-skilled workers in plant *j* in period *t*
u_{jt}^1 / u_{jt}^2	used regular/additional machine capacities in plant *j* in period *t*
v_{jt}^1 / v_{jt}^2	used labour time of skilled/non-skilled workers in plant *j* in period *t*, including overtime
w_{jt}^1 / w_{jt}^2	used overtime of skilled/non-skilled workers in plant *j* in period *t*
q_{it}	initially allocated quota quantity of product *t* in period *i*
d_{it}^- / d_{it}^+	shortage/surplus production for product *i* in period *t*
q_{it}^- / q_{it}^+	under-/over-quota quantities of product *i* in period *t*

4.2.4 Constraints

Demand constraints

In each period, for each product, market demand has to be met by a combination of production in that period, inventory from the previous period, purchasing from the contracted plants and inventory in that period.

$$\sum_{j=1}^{n}(x_{ijt}^1 + x_{ijt}^2) + d_{i,t-1}^+ + d_{it}^- - d_{it}^+ = D_{it},\ i=1,\ldots,m,\ t=1,\ldots,T \tag{4.1}$$

Quota constraints

In each period, each product needs to have its own quota. The ideal situation is that in each period the demand is equal to the initial allocated quota. However, when the quota amount is insufficient, the company needs to purchase quota from local markets at the market price. On the other hand, when the quota is not used fully, the company incurs the penalty.

$$q_{it} + q^{+}_{i,t-1} + q^{-}_{it} - q^{+}_{it} = D_{it},\ i=1,\ldots,m,\ t=1,\ldots,T \tag{4.2}$$

Machine capacity constraints

Machine regular and additional capacity must be sufficient to produce the required number of products.

$$\sum_{i=1}^{m} (g^{1}_{ij} x^{1}_{ijt} + g^{2}_{ij} x^{2}_{ijt}) = u^{1}_{jt} + u^{2}_{jt},\ j=1,\ldots,\ n,\ t=1,\ldots,T \tag{4.3}$$

Workforce capacity constraints

Constraints (4.4) and (4.5) are the capacity requirements of skilled and non-skilled workers.

$$\sum_{i=1}^{m} l^{1}_{ij} x^{1}_{ijt} = v^{1}_{jt},\ j=1,\ldots,\ n,\ t=1,\ldots,T \tag{4.4}$$

$$\sum_{i=1}^{m} l^{2}_{ij} x^{2}_{ijt} = v^{2}_{jt},\ j=1,\ldots,\ n,\ t=1,\ldots,T \tag{4.5}$$

Workforce level constraints

The available workforce in any period equals the workforce in the previous period plus the change of workforce level in the current period. The change in workforce may be due to hiring extra workers, firing redundant workers or overtime.

$$v^{1}_{jt} = v^{1}_{jt-1} + y^{1}_{jt} - z^{1}_{jt} + w^{1}_{jt}\ \ ,j=1,\ldots,n,\ t=1,\ldots,T \tag{4.6}$$

$$v^{2}_{jt} = v^{2}_{jt-1} + y^{2}_{jt} - z^{2}_{jt} + w^{2}_{jt},j=1,\ldots,n,\ t=1,\ldots,T \tag{4.7}$$

Production quality constraints

The ratio between work time of skilled workers and non-skilled workers should not be less than a given constant so as to guarantee product quality.

$$\sum_{t=1}^{T} v_{jt}^{1} \geq \alpha_j \sum_{t=1}^{T} v_{jt}^{2}, j=1,\ldots, n, \tag{4.8}$$

Initial quota allocation constraints

At the beginning, the initial quota is allocated in each time period.

$$\sum_{t=1}^{T} q_{it} = Q_i, i=1,\ldots, m \tag{4.9}$$

Minimum work time constraints

Each plant has a minimum work time in each period.

$$v_{jt}^{1} + v_{jt}^{2} \geq V_{jt}, j=1,\ldots,n, t=1,\ldots,T \tag{4.10}$$

Upper bound constraints

The capacity has the upper bound limits in terms of purchasing capacity, inventory capacity, machine regular and additional capacity, and available labour time and overtime for skilled/non-skilled workers.

$$d_{it}^{-} \leq B_{it}, i=1,\ldots, n, t=1,\ldots,T \tag{4.11}$$

$$d_{it}^{+} \leq I_{it}, i=1,\ldots, n, t=0,\ldots,T \tag{4.12}$$

$$u_{jt}^{1} \leq C_{jt}, j=1,\ldots,n, \quad t=1,\ldots,T \tag{4.13}$$

$$u_{jt}^{2} \leq A_{jt}, j=1,\ldots,n, \quad t=1,\ldots,T \tag{4.14}$$

$$y_{jt}^{1} - z_{jt}^{1} \leq L_{jt}^{1}, j=1,\ldots,n, t=1,\ldots,T \tag{4.15}$$

$$y_{jt}^{2} - z_{jt}^{2} \leq L_{jt}^{2}, j=1,\ldots,n, \quad t=1,\ldots,T \tag{4.16}$$

$$w_{jt}^{1} \leq W_{jt}^{1}, j=1,\ldots,n, t=1,\ldots,T \tag{4.17}$$

$$w_{jt}^{2} \leq W_{jt}^{2}, j=1,\ldots,n, t=1,\ldots,T \tag{4.18}$$

Variable type contraints

$$x_{ijt}^{1}, x_{ijt}^{2}, y_{jt}^{1}, y_{jt}^{2}, z_{jt}^{1}, z_{jt}^{2}, u_{jt}^{1}, u_{jt}^{2}, v_{jt}^{1}, v_{jt}^{2}, w_{jt}^{1}, w_{jt}^{2}, q_{it} \geq 0, i=1,\ldots, m, j=1,\ldots,n, t=1,\ldots,T \tag{4.19}$$

$$d_{it}^{-}, d_{it}^{+}, q_{it}^{-}, q_{it}^{+} \geq 0, i=1,\ldots, m, t=1,\ldots,T \tag{4.20}$$

4.2.5 Costs

The objective is to distribute the production task so that market demand can be fulfilled at a minimum cost. To achieve the optimal plan, this study takes the following cost factors into account.

Raw material cost

Products are manufactured by the skilled workers and non-skilled workers

$$RC=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T} r_{ij}\,(x_{ijt}^{1}+x_{ijt}^{2}) \tag{4.21}$$

Machine cost

Machine capacity includes regular and additional machine capacity. To satisfy demand, additional machine capacity may be used at an extra cost.

$$MC=\sum_{j=1}^{n}\sum_{t=1}^{T}(a_{j}^{1}u_{jt}^{1}+a_{j}^{2}u_{jt}^{2}) \tag{4.22}$$

Labour cost

Plant j will pay the skilled workers k_{ij}^{1} for processing each product i, and the non-skilled workers k_{ij}^{2}.

$$LC=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T}(k_{ij}^{1}x_{ijt}^{1}+k_{ij}^{2}x_{ijt}^{2}) \tag{4.23}$$

Overtime cost

To satisfy demand, overtime can be used. The expression (4) gives the labour cost for overtime production for skilled and non-skilled workers.

$$OC=\sum_{j=1}^{n}\sum_{t=1}^{T}(o_{j}^{1}z_{jt}^{1}+o_{j}^{2}z_{jt}^{2}) \tag{4.24}$$

Hiring/firing worker cost

It costs the company to hire or fire skilled/non-skilled workers.

$$HC=\sum_{j=1}^{n}\sum_{t=1}^{T}(h_{jt}^{1}y_{jt}^{1}+h_{jt}^{2}y_{jt}^{2}+f_{jt}^{1}z_{jt}^{1}+f_{jt}^{2}z_{jt}^{2}) \tag{4.25}$$

Initial quota purchasing cost

c_i is the original quota cost of purchasing a unit of product i at the beginning of the planning horizon.

$$IC=\sum_{i=1}^{m}\sum_{t=1}^{T}c_i\, q_{it} \tag{4.26}$$

Surplus/shortage cost

When market demand is not satisfied, the company will purchase products from its subcontracted plants at the unit cost b_{it}^{-}. On the other hand, when production exceeds market demand in each period, the surplus products have to be stored at the unit cost b_{it}^{+}.

$$SC=\sum_{i=1}^{m}\sum_{t=1}^{T}(b_{it}^{-}d_{it}^{-}+b_{it}^{+}d_{it}^{+}) \tag{4.27}$$

Under-/over- quota cost

When the demand D_{it} is less than the initial allocated quota q_{it} in period t, some quotas, called over-quota q_{it}^{+}, are left. The unit penalty cost is b_{it}^{+} On the other hand, when the initial allocated quota quantities q_{it} is not enough to satisfy the demand D_{it} in period t, the company has to buy under-quota quantities q_{it}^{-} at the market price b_{it}^{-}. Therefore, under-/over-quota cost can be formulated as follows.

$$UC=\sum_{i=1}^{m}\sum_{t=1}^{T}(c_{it}^{-}q_{it}^{-}+c_{it}^{+}q_{it}^{+}) \tag{4.28}$$

4.3 Model formulations

4.3.1 A linear programming model for the deterministic production loading problems

When all parameters in Section 4.2.2 are known and certain, a linear programming model is formulated as follows:

$$\min RC+MC+LC+OC+HC+IC+SC+UC \tag{4.29}$$

s.t.

(4.1) ~ (4.20)

4.3.2 A stochastic recourse programming model for the uncertain production loading problems

The following parameters in Section 4.2.2 are defined as random parameters.

Random parameters

D_{it} demand for product i in period t

b_{it}^{-}/b_{it}^{+} shortage/surplus cost of a unit of product i in period t

c_{it}^{-}/c_{it}^{+} under-/over-quota cost of a unit quota of product i in period t

It is assumed that the uncertainties are represented by a set of possible realizations, called *scenarios*. Each scenario provides one possible course of future events. The recourse production policy allows compensating for discrepancies in the second-stage in each scenario s by incurring a cost of b_{it}^- / b_{it}^+ per unit of production deviation from market demand, and by incurring a cost of c_{it}^- / c_{it}^+ per unit of market demand deviation from the initial allocated quota. When the recourse actions are taken for the realization D_{its} of the demand D_{it}, the realization b_{its}^- of the unit shortage cost b_{it}^- for purchasing product i, the realization b_{its}^+ of the unit surplus cost b_{it}^+ for storing product i, the realization c_{its}^- of the unit under-quota cost c_{it}^- for purchasing quota, and the realization c_{its}^+ of unit over-quota c_{it}^+ for penalizing unused quota, the random parameters D_{it} , b_{it}^- , b_{it}^+, c_{it}^-, and c_{it}^+, are independent random variables, and have the same finite discrete distribution specified by:

$$\begin{bmatrix} p_1 & p_2 & \cdots & p_S \\ D_{it1} & D_{it2} & \cdots & D_{itS} \\ b_{it1}^- & b_{it2}^- & \cdots & b_{itS}^- \\ b_{it1}^+ & b_{it}^+ & \cdots & b_{itS}^+ \\ c_{it1}^- & c_{it2}^- & \cdots & c_{itS}^- \\ c_{it1}^+ & c_{it2}^- & \cdots & c_{itS}^+ \end{bmatrix} \tag{4.30}$$

Decision variables

Decision variables are divided into the first stage decision variables and the second stage decision variables. The first stage decision variables have to be determined before accurate information are obtained, including production quantity x_{ijt}^1 / x_{ijt}^2, hiring workers quantities y_{jt}^1 / y_{jt}^1, firing workers quantities z_{jt}^1 / z_{jt}^2, used machine capacity u_{jt}^1 / u_{jt}^2, used labour time v_{jt}^1 / v_{jt}^2, overtime w_{jt}^1 / w_{jt}^2, and initially allocated quota quantities q_{it}. After the realization of the stochastic variables is observed, we have to decide the values of the second stage decision variables, including shortage/surplus production d_{its}^- / d_{its}^+, and under-/over-quota q_{its}^- / q_{its}^+.

Constraints

The constraints are divided into the first stage constraints and the second stage constraints. The first stage constraints are the constraints that only involve the first stage decision variables, including (4.3) ~ (4.10), and (4.13) ~ (4.19). The constraints that involve the first stage decision variables and the second stage decision variables are the second stage constraints, including demand constraints, quota constraints, upper bound and variable type constraints. In each scenario s, the following constraints have to be satisfied.

- ***Random demand constraints***

$$\sum_{j=1}^{n}(x_{ijt}^{1}+x_{ijt}^{2})+d_{i,t-1,s}^{+}+d_{its}^{-}-d_{its}^{+}=D_{its},\ i=1,\ldots,m,\ t=1,\ldots,T,\ s=1,\ldots,S \tag{4.31}$$

- ***Random quota constraints***

$$q_{it}+q_{i,t-1,s}^{+}+q_{its}^{-}-q_{its}^{+}=D_{its},\ i=1,\ldots,m,\ t=1,\ldots,T,\ s=1,\ldots,S \tag{4.32}$$

- ***Random upper bound constraints***

$$d_{its}^{-}\leq B_{it},\ i=1,\ldots,n,\ t=1,\ldots,T,\ s=1,\ldots,S \tag{4.33}$$

$$d_{its}^{+}\leq I_{it},\ i=1,\ldots,n,\ t=0,\ldots,T,\ s=1,\ldots,S \tag{4.34}$$

- ***Variable type constraints***

$$d_{its}^{-},d_{its}^{+},q_{its}^{-},q_{its}^{+}\geq 0,\ i=1,\ldots,m,\ t=1,\ldots,T,\ s=1,\ldots,S \tag{4.35}$$

Objective function

The objective is to minimize the total cost, which equals the first stage cost plus the second stage cost. The first stage cost, denoted by *FirstCost*, is the cost that we need to pay for the first stage production loading decisions among the company-owned plants, including the raw material cost, the used machine cost, the used labour cost, the overtime cost, the cost of hiring/firing workers and the initial quota purchasing cost.

$$FirstCost=RC+MC+LC+OC+HC+IC \tag{4.36}$$

The second stage cost, denoted by *SecondCost*, is the cost that we need to pay for the second stage production loading decisions. After realization of the random variable has been observed, the decision makers have to make the second stage decisions, such as the quantity of purchasing products from contracted plants, inventory, purchasing quota and the quota unused. Therefore, the second stage cost is the sum of the cost of shortage/surplus production and the cost of under-/over-quota, which is shown as follows.

$$SecondCost=\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}p_s(b_{its}^{-}d_{its}^{-}+b_{its}^{+}d_{its}^{+}+c_{its}^{-}q_{its}^{-}+c_{its}^{+}q_{its}^{+}) \tag{4.37}$$

A stochastic recourse programming model for the uncertain production loading problems is formulated as follows:

$$\min FirstCost+SecondCost \tag{4.38}$$

s.t.

The first stage constraints: (4.3)~(4.10), and (4.13)~(4.19)

The second stage constraints: (4.31)~(4.35)

4.3.3 Robust optimization models for the uncertain production loading problems

4.3.3.1 A robust linear optimization model with solution robustness

Based on the analysis in Section 3.4.2, a robust optimization model with solution robustness for the production loading problems with the importing quota limits under global supply chain environments can be formulated as:

min *FirstCost+SecondCost*

$$+\lambda\sum_{s=1}^{S}p_s\left|\sum_{i=1}^{m}\sum_{t=1}^{T}(b_{its}^{-}d_{its}^{-}+b_{its}^{+}d_{its}^{+}+c_{its}^{-}q_{its}^{-}+c_{its}^{+}q_{its}^{+})-\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}p_s(b_{its}^{-}d_{its}^{-}+b_{its}^{+}d_{its}^{+}+c_{its}^{-}q_{its}^{-}+c_{its}^{+}q_{its}^{+})\right| \quad (4.39)$$

s.t.

The first stage constraints: (4.3)~(4.10), and (4.13)~(4.19)

The second stage constraints: (4.31)~(4.35)

The final term in objective function (4.39) is the variability cost for shortage/surplus production and under-/over- quota. The model above can be converted into a linear programming model by introducing a deviational variable $\theta_s \geq 0$ as follows:

min *FirstCost+SecondCost*

$$+\lambda\sum_{s=1}^{S}p_s\left[\sum_{i=1}^{m}\sum_{t=1}^{T}(b_{its}^{-}d_{its}^{-}+b_{its}^{+}d_{its}^{+}+c_{its}^{-}q_{its}^{-}+c_{its}^{+}q_{its}^{+})-\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}p_s(b_{its}^{-}d_{its}^{-}+b_{its}^{+}d_{its}^{+}+c_{its}^{-}q_{its}^{-}+c_{its}^{+}q_{its}^{+})+2\theta_s\right] \quad (4.40)$$

s.t.

The first stage constraints: (4.3)~(4.10), and (4.13)~(4.19)

The second stage constraints: (4.31)~(4.35), and

$$-\sum_{i=1}^{m}\sum_{t=1}^{T}(b_{its}^{-}d_{its}^{-}+b_{its}^{+}d_{its}^{+}+c_{its}^{-}q_{its}^{-}+c_{its}^{+}q_{its}^{+})+\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}p_s(b_{its}^{-}d_{its}^{-}+b_{its}^{+}d_{its}^{+}+c_{its}^{-}q_{its}^{-}+c_{its}^{+}q_{its}^{+})-\theta_s \leq 0,$$

$s=1,\ldots,S$ (4.41)

$\theta_s \geq 0,\ s=1,\ldots,S$ (4.42)

4.3.3.2 A robust linear optimization model with model robustness

Stochastic recourse programming models determine the first stage decision variables such that for each realized scenario the second stage decision variables can satisfy all the constraints. For systems with some redundancy, the stochastic recourse programming model solution might be feasible. However, the stochastic recourse programming model is infeasible when feasible decision variables do not exist either in the first stage or second stage. The robust optimization model with model robustness can handle this kind of situation. By introducing the penalty function, the model will generate a solution with the least amount of violation of the stochastic constraints. Based on the analysis in Section 3.3.2, a robust optimization model with model robustness for production loading problems with the importing quota limits under global supply chain environments can be formulated as:

$$\min FirstCost+SecondCost+\omega\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}p_s(e_{its}^1+e_{its}^2) \tag{4.43}$$

s.t.

The first stage constraints: (4.3)~(4.10), and (4.13)~(4.19),

The second stage constraints: (4.33)~(4.35), and

$$e_{its}^1 = D_{its}-\sum_{j=1}^{n}(x_{ijt}^1+x_{ijt}^2)-d_{i,t-1,s}^- - d_{its}^+ + d_{its}^-,\ i=1,\ldots,m,\ t=1,\ldots,T,\ s=1,\ldots,S \tag{4.44}$$

$$e_{its}^2 = D_{its}-q_{it}-q_{i,t-1,s}^- - q_{its}^+ + q_{its}^- \le 0,\ i=1,\ldots,m,\ t=1,\ldots,T,\ s=1,\ldots,S \tag{4.45}$$

$$e_{its}^1 = e_{its}^2,\ i=1,\ldots,m,\ t=1,\ldots,T,\ s=1,\ldots,S \tag{4.46}$$

$$e_{its}^1, e_{its}^2 \ge 0,\ i=1,\ldots,m,\ t=1,\ldots,T,\ s=1,\ldots,S \tag{4.47}$$

Constraint (4.44) denotes the random demand constraints in (4.31) that can be violated to the extent of e_{its}^1. In other words, there is an unsatisfied demand e_{its}^1 in scenario s. Constraint (4.45) denotes the random quota constraints in (4.32) that can be violated to the extent of e_{its}^2. Constraint (4.46) ensures that we only buy quotas for goods that we are going to deliver to overseas markets. Constraint (4.47) is a variable type of requirement, which ensures that we do not produce what we are not going to deliver, and we also do not buy quotas that we are not going to use.

In the objective function (4.43), ω represents the unit weighting penalty for the infeasibility of the random demand and quota constraints. When the unit weighting parameter ω increases, the unit penalty cost for the infeasibility of the random constraints increases. We have to pay more for the violation of the random constraints. If the value of

ω is increased by enough, the value of e^1_{its} and e^2_{its} will be forced to become zero simultaneously, which means all random constraints have to be satisfied for each scenario.

4.3.3.3 A robust linear optimization model with trade-off between robustness solution and model robustness

When the variability and infeasibility are considered simultaneously, a robust optimization model with robustness solution and model robustness is formulated to solve uncertain production loading problems with import quota.

$$\min \mathit{FirstCost}+\mathit{SecondCost}$$
$$+\lambda\sum_{s=1}^{S} p_s\left|\sum_{i=1}^{m}\sum_{t=1}^{T}(b^-_{its}d^-_{its}+b^+_{its}d^+_{its}+c^-_{its}q^-_{its}+c^+_{its}q^+_{its})-\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}p_s(b^-_{its}d^-_{its}+b^+_{its}d^+_{its}+c^-_{its}q^-_{its}+c^+_{its}q^+_{its})\right|$$
$$+\omega\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}p_s(e^1_{its}+e^2_{its}) \tag{4.48}$$

s.t.

The first stage constraints: (4.3)~(4.10), and (4.13)~(4.19)

The second stage constraints: (4.33)~(4.35), and (4.44)~(4.47)

Furthermore, the above model can be expressed as the following linear programming model by introducing a deviational variables $\theta_s \geq 0$:

$$\min \mathit{FirstCost}+\mathit{SecondCost}$$
$$\lambda\sum_{s=1}^{S} p_s\left[\sum_{i=1}^{m}\sum_{t=1}^{T}(b^-_{its}d^-_{its}+b^+_{its}d^+_{its}+c^-_{its}q^-_{its}+c^+_{its}q^+_{its})-\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}p_s(b^-_{its}d^-_{its}+b^+_{its}d^+_{its}+c^-_{its}q^-_{its}+c^+_{its}q^+_{its})+2\theta_s\right]$$
$$+\omega\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}p_s(e^1_{its}+e^2_{its}) \tag{4.49}$$

s.t.

The first stage constraints: (4.3)~(4.10), and (4.13)~(4.19)

The second stage constraints: (4.33)~(4.35), (4.41)~(4.42), and (4.44)~(4.47).

4.4 Computational results and analysis

4.4.1 Known and fixed parameters

In order to illustrate the effectiveness of the proposed models for the production loading problems with importing quota limits, we use the data provided by a garment manufacturing company. Based on the information from its retailers in North American and European markets, the company decides to produce three types of products for new season's fashions in the three plants in China. The company will look at a 4-week planning horizon. The data given in Tables 4.1~4.4 are known and fixed parameters in all decision-making processes. Table 4.1 gives the unit raw material cost, labour cost, labour time and machine time. Table 4.2 gives the unit machine cost for regular and additional production, and the unit overtime cost for skilled and non-skilled workers. Table 4.3 gives the maximum machine regular/additional capacity, maximum labour capacity, maximum overtime capacity and minimum work time. The unit initial quota purchasing cost is shown in Table 4.4. Currently, there is no cost in hiring/firing workers because there is a large supply of skilled and non-skilled workers in China's market and there is no union contract limitation in China. Thus we assume that the initial workforce level is zero. The work time of skilled workers is not less than that of non-skilled workers. There is no initial inventory. Additionally, it is assumed that the contracted plants have enough capacity to satisfy the company's demand, and there is no limitation of inventory as long as it is profitable to hold it.

Table 4.1: Unit raw material cost, labour cost, labour time and machine time

Product	Plant	Raw material cost ($)	Labour cost of skilled workers ($)	Labour cost of non-skilled workers ($)	Labour time for skilled workers (hrs)	Labour time for non-skilled workers (hrs)	Machine time for skilled workers (hrs)	Machine time for non-skilled workers (hrs)
1	1	4	4.5	4	2	2.25	1.75	2.25
	2	4.2	4	3.5	2.25	2.5	2	2.5
	3	4.3	3.5	3	2.5	2.75	2.25	2.75
2	1	3	4	3.5	1.5	1.75	1.25	1.75
	2	3.2	3.5	3	1.75	2	1.5	2
	3	3.3	3	2.5	2	2.25	1.75	2.25
3	1	2	3	2.5	1	1.25	0.75	1.25
	2	2.2	2.5	2	1.25	1.5	1	1.5
	3	2.3	2	1.5	1.5	1.75	1.25	1.75

Table 4.2: Unit machine cost and overtime cost

Plant	Regular machine cost for production ($)	Additional machine cost for production ($)	Overtime cost for skilled worker ($)	Overtime cost for non-skilled worker ($)
1	0.05	0.055	6	5
2	0.06	0.065	5	4
3	0.07	0.75	4	3

Table 4.3: Maximum capacity for machine, labour and overtime and minimum labour work time

Plant	Period	Maximum machine regular capacity (hrs)	Maximum machine additional capacity (hrs)	Maximum capacity of skilled workers (hrs)	Maximum capacity of non-skilled workers (hrs)	Maximum overtime by skilled workers (hrs)	Maximum overtime by non-skilled workers (hrs)	Minimum labour work time (hrs)
1	1	5500	250	4800	2400	2400	1200	2400
	2	5500	250	4800	2400	2400	1200	2400
	3	5500	250	4800	2400	2400	1200	2400
	4	5500	250	4800	2400	2400	1200	2400
2	1	5000	250	3840	1920	1920	960	1800
	2	5000	250	3840	1920	1920	960	1800
	3	5000	250	3840	1920	1920	960	1800
	4	5000	250	3840	1920	1920	960	1800
3	1	5000	200	2400	1200	1200	600	1500
	2	5000	200	2400	1200	1200	600	1500
	3	5000	200	2400	1200	1200	600	1500
	4	5000	200	2400	1200	1200	600	1500

Table 4.4: Unit initial quota cost

Product	1	2	3
Initial quota cost	20.5	13	6.55

4.4.2 Computational results of the linear programming model

4.4.2.1 Deterministic parameters

It is assumed that at the beginning of the planning horizon, there are quotas for 7,700 units of product 1, 6,800 units of product 2 and 5,200 units of product 3. The market demand, unit shortage/surplus cost and unit under-/over-quota cost are shown in Table 4.5 (All data presented here are the expect values of stochastic variables in Section 4.4.3.1 for Test III, See Table 4.12 and Table 4.26).

Table 4.5: Unit shortage/surplus cost, unit under/over- quota cost and demand

Product	Period	Shortage cost ($)	Surplus cost ($)	Under-quota cost ($)	Over-quota cost ($)	Demand (units)
1	1	86	1.89	22.6	2.7	1730
	2	86	1.89	22.6	2.7	1830
	3	86	1.89	22.6	2.7	1930
	4	86	1.89	22.6	2.7	2030
2	1	51.6	0.89	14.4	1.7	1330
	2	51.6	0.89	14.4	1.7	1530
	3	51.6	0.89	14.4	1.7	1730
	4	51.6	0.89	14.4	1.7	1930
3	1	34.4	0.39	7.4	0.7	1030
	2	34.4	0.39	7.4	0.7	1130
	3	34.4	0.39	7.4	0.7	1230
	4	34.4	0.39	7.4	0.7	1330

4.4.2.2 Computational results

Using the input data shown in Tables 4.1~4.5, the linear programming model can be solved using AIMMS, and the optimal solution can be obtained. The total cost is $402,471. Additionally, we can obtain other results such as production amount in Table 4.6, machine work time in Table 4.7, labour work time in Table 4.8, hiring/firing worker time in Tables 4.9 and 4.10, and initial quota allocated in Table 4.11

Table 4.6: Production quantity for the deterministic problems

Plant	Product	Skilled workers (hrs)				Non-skilled workers (hrs)			
		Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	1	1200			1200		1067	1067	
	2								
	3								
2	1	207	47	863	794				
	2								
	3					1030	1130	222	485
3	1	323	717		36				
	2	797	463	914	1408	533	1067	816	522
	3							1008	845

Table 4.7: Machine work time for the deterministic problems

Product	Regular capacity used (hrs)				Additional capacity used (hrs)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	2100	2400	2400	2100				
2	1960	1788	2059	2314				
3	3320	4823	5000	5000			200	200

Table 4.8: Labour work time for the deterministic problems

Plant	Skilled workers (hrs)				Non-skilled workers (hrs)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	2400			2400		2400	2400	
2	467	105	1943	1785	1545	1695	332	727
3	2400	2718	1829	2907	1200	2400	3600	2654

Table 4.9: Hiring workers for the deterministic problems

Plant	Skilled workers (hrs)				Non-skilled workers (hrs)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	2400			2400		2400		
2	467		1838		1545	150		395
3	2400	318		1079	1200	1200	1200	

Table 4.10: Firing workers for the deterministic problems

Plant	Skilled workers (hrs)				Non-skilled workers (hrs)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1		2400						2400
2		362		157			1363	
3			890					946

Table 4.11: Quotas allocated for the deterministic problems

Plant	Period 1	Period 2	Period 3	Period 4
1	1730	1830	1930	2210
2	1330	1530	1730	2210
3	1030	1130	1230	1810

Additionally, there is no need to work overtime, and there is no inventory for any products. No contracted plants need to be used for urgent production. There is also no need to purchase any additional quotas for any product in any period. Additionally, there is a certain amount of unused quotas in period 4 (180 for product 1, 280 product 2, and 480 for product 3.

4.4.3 Computational results of the stochastic linear recourse programming model

4.4.3.1 Random parameters

It is assumed that the uncertainty is represented by the possible states of the economy, in terms of the scenarios, i.e. good, fair, or bad. Let s_1 represent a good economy scenario with probability p_1, p_1=Pr{s_1}; s_2 represent a fair economy scenario with probability p_2, p_2=Pr{s_2}; and s_3 represent a bad economy scenario with probability p_3, p_3=Pr{s_3}. The probability of a good economy in the new season is 10%, fair economy is 10%, and bad economy is 80%. Table 4.12 gives the realizations of random parameters in each scenario, including the unit shortage cost for purchasing products from the contacted plants, the unit surplus cost for storing left products, the unit under-quota cost for purchasing quota from the market, and the over-quota cost for penalizing unused quota. Additionally, market demand in each scenario is also shown in Table 4.12.

Table 4.12: Unit shortage/surplus cost, unit under/over- quota cost and demand

Scenario	Product	Period	Shortage cost ($)	Surplus cost ($)	Under-quota cost ($)	Over-quota cost ($)	Demand (units)
s_1	1	1	120	2.5	26	4	1900
		2	120	2.5	26	4	2000
		3	120	2.5	26	4	2100
		4	120	2.5	26	4	2200
	2	1	72	1.5	17	3	1500
		2	72	1.5	17	3	1700
		3	72	1.5	17	3	1900
		4	72	1.5	17	3	2100
	3	1	48	1	10	2	1200
		2	48	1	10	2	1300
		3	48	1	10	2	1400
		4	48	1	10	2	1500
s_2	1	1	100	2	24	3	1800
		2	100	2	24	3	1900
		3	100	2	24	3	2000
		4	100	2	24	3	2100
	2	1	60	1	15	2	1400
		2	60	1	15	2	1600
		3	60	1	15	2	1800
		4	60	1	15	2	2000
	3	1	40	0.5	8	1	1100
		2	40	0.5	8	1	1200
		3	40	0.5	8	1	1300
		4	40	0.5	8	1	1400
s_3	1	1	80	1.8	22	2.5	1700
		2	80	1.8	22	2.5	1800
		3	80	1.8	22	2.5	1900
		4	80	1.8	22	2.5	2000
	2	1	48	0.8	14	1.5	1300
		2	48	0.8	14	1.5	1500
		3	48	0.8	14	1.5	1700
		4	48	0.8	14	1.5	1900
	3	1	32	0.3	7	0.5	1000
		2	32	0.3	7	0.5	1100
		3	32	0.3	7	0.5	1200
		4	32	0.3	7	0.5	1300

4.4.3.2 Computational results

The stochastic recourse programming presented in section 4.3.2 is solved using AIMMS.

The first stage decisions

Before the accurate market and quota price data are available, the company has to start production among its company-owned plants. The first stage decisions are shown in Tables 4.13 ~ 4.18. Table 4.13 shows the production quantities. Tables 4.14 and 4.15 show the machine work time and labour work time. Tables 4.16 and 4.17 show the hiring and firing worker time. The initial quota allocated in each period is shown in Table 4.18. There is no need to work overtime.

Table 4.13: Production quantity for the uncertain problems using the stochastic recourse model

Plant	Product	Skilled workers (hrs)				Non-skilled workers (hrs)			
		Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	1			1200	1200	1067	1067		
	2								
	3								
2	1	467	40	855	1000				
	2								
	3					1100	1140	396	906
3	1	267	793	45					
	2	867	580	1070	1128	533	1020	730	972
	3						60	904	594

Table 4.14: Machine work time for the uncertain problems using the stochastic recourse model

Plant	Regular capacity used (hrs)				Additional capacity used (hrs)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	2400	2400	2100	2100				
2	2583	1790	2303	3359				
3	3317	5000	5000	5000		200	200	200

Table 4.15: Labour work time for the uncertain problems using the stochastic recourse model

Plant	Skilled workers (hrs)				Non-skilled workers (hrs)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1			2400	2400	2400	2400		
2	1050	90	1923	2250	1650	1710	593	1359
3	2400	3143	2253	2256	1200	2400	3226	3226

Table 4.16: Hiring workers for the uncertain problems using the stochastic recourse model

Plant	Skilled workers (hrs)				Non-skilled workers (hrs)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1			2400		2400			
2	1050		1833	327	1650	60		766
3	2400	743		3	1200	1200	826	

Table 4.17: Firing workers for the uncertain problems using the stochastic recourse model

Plant	Skilled workers (hrs)				Non-skilled workers (hrs)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1							2400	
2		960					1117	
3			891					

Table 4.18: Quotas allocated for the uncertain problems using the stochastic recourse model

Plant	Period 1	Period 2	Period 3	Period 4
1	1700	1800	2000	2200
2	1300	1600	1800	2100
3	1100	1200	1400	1500

The second stage decisions

When the uncertainty is realized, the company can make the second stage production loading decisions. The results are shown in Tables 4.19~4.24.

Scenario 1: Good economy

The probability of a good economy is 10%. If this scenario happens, the company will take the second-stage decisions, shown in Tables 4.19 and 4.20. If the unexpected situation (high demand) happens (the possibility is 10%), there will exist the option of outsourcing a certain amount of production (Table 4.19), while additional quotas will also be required (Table 4.20). In this situation, there will be no leftover inventory or unused quota.

Table 4.19: Shortage/surplus in scenario 1 for the uncertain problems using the stochastic recourse model

Product	Purchased products from contractors (units)				Inventory (units)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	100	100						
2	100	100	100					
3	100	100	100					

Table 4.20: Under-/over-quota in scenario 1 for the uncertain problems using the stochastic recourse model

Product	Purchased quota (units)				Unused quota (units)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	200	200	100					
2	200	100	100					
3	100	100						

Scenario 2: Fair economy

The probability of a fair economy is 10%. If the fair demand is realized, the company will take the corresponding second-stage production loading decisions, shown in Tables 4.21 and 4.22. If the unexpected situation (fair economy) happens (the possibility is 10%), there will be a small amount of leftover inventory (Table 4.21) and unused quota (Table 4.22). Additionally, a small amount of additional quota will be required in periods 1 and 2 (Table 4.22).

Table 4.21: Shortage/surplus in scenario 2 for the uncertain problems using the stochastic recourse model

Product	Purchased products from contractors (units)				Inventory (units)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1							100	200
2								100
3								100

Table 4.22: Under-/-over quota in scenario 2 for the uncertain problems using the stochastic recourse model

Product	Purchased quota (units)				Unused quota (units)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	100	100						100
2	100							100
3							100	200

Scenario 3: Bad economy

The probability of a bad economy is 80%. If the demand is low, the company will take the second-stage production loading decisions shown in Tables 4.23 and 4.24. If this situation (bad economy) happens (the possibility is 80%), there will be a large amount of leftover inventory (Table 4.23) and unused quota (Table 4.24).

Table 4.23: Shortage/surplus in scenario 3 for the uncertain problems using the stochastic recourse model

Product	Purchased products from contractors (units)				Inventory (units)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1					100	200	400	600
2					100	200	300	500
3					100	200	300	500

Table 4.24: Under/-over quota in scenario 3 for the uncertain problems using the stochastic recourse model

Product	Purchased quota (units)				Unused quota (units)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1							100	300
2						100	200	400
3					100	200	400	600

4.4.3.3 Comparison between the expected value model and stochastic recourse model

Comparison between the expected value model and recourse model in Test III

Computational results of the expected value model in Test III are shown in Section 4.4.2.2, and computational results of the recourse model in Test III are shown in Section 4.4.3.2. Table 4.25 summarizes the related cost for the two models in Test III.

Table 4.25: Comparison between the expected value model and recourse model in Test III

Model	Material cost	Machine cost	Labour cost	Overtime cost	Initial quota cost	Production Shortage cost	Production surplus cost	Quota shortage cost	Quota surplus cost	Total cost
Recourse model	64741	2366	60755		280310	7200	2587	2810	2240	423010
Expected value model	61280	2237	57346	0	280310	0	0		1298	402471

The recourse model considers three scenarios for future demand and their corresponding probabilities and makes two-stage decisions. The total cost of the recourse model is \$423,010 (See Table 4.25). The expected value model assumes that the future demand will be the expected value of stochastic demand (See Table 4.5). Therefore, the decision is made on the basis of the expected value of stochastic demand. The total cost of the expected value model is \$402,471 (See Table 4.25). 1. Unfortunately, the situation that the expected value describes will not happen in the future. The real demand will be one of three scenarios, i.e. either Scenario 1, or Scenario 2, or Scenario 3 (See Table 4.12). Based on the solution of the expected value model, the company has to take an action when the real situation (either Scenario 1, or Scenario 2, or Scenario 3) unfolds. This can be done by outsourcing production and/or purchasing additional quotas in the case of high demand or storing the products and/or unused quotas in the case of low demand. The total cost of this action will be \$38,001. Therefore, the total cost of the expected value model in Test III is \$440,472(=\$402,471+\$38,001). It means that the company will save \$17,462 (=\$440,472-\$423,010) from using the recourse model rather than the expected value model.

Comparison between the expected value model and recourse model for Tests I, II and III

Let EV represent the objective function value of the expected value model. When the uncertainty is realized, the actual situation may be: scenario 1 happens; or scenario 2 happens; or scenario 3 happens (see Table 4.12). At this stage, the company has to make a decision to respond to the realized situation, in order to satisfy the demand. Let *EEV* represent the expected results of using the solution of the expected value problem. The quantity, *EEV*, measures how the solution of the expected value problem performs, allowing the second-stage decisions to be chosen optimally (Birge and Louveaux 1997). *EEV* can be obtained by solving the stochastic recourse model, in which the first stage decisions are made by the expected value model. Let *ESS* represent the optimal solution of the stochastic recourse model. From the stochastic recourse model in Section 4.3.2, we know that *EEV* is only one of the solutions for the stochastic recourse model, but *ESS* is the best solution. Letting *VSS* represent the value of the stochastic solution (*VSS=EEV-ESS*), we have the following inequality: $VSS \geq 0$. The comparative results for the stochastic recourse model and the expected value model are shown in Table 4.27.

In order to further demonstrate the effectiveness of the recourse model, we perform three different tests under different probabilities. Other than the change in probability of occurrences of the different future economic scenarios, other conditions in the three tests are the same. The test data are shown in Table 4.26. Test I represents the situation where it is most likely that the economy will perform well, Test II represents the situation where it is most likely that the economic performance will be fair, and Test III represents the situation where it will be poor. The problem, which is described in 4.3.3.1, is the case in Test III. Table 4.27 shows the computational results for the expected value model and stochastic recourse model for the three tests.

Table 4.26: Three tests for the uncertain problems

Test	p_1=Pr{s_1}	p_2=Pr{s_2}	p_3=Pr{s_3}
I	0.8	0.1	0.1
II	0.1	0.8	0.1
III	0.1	0.1	0.8

Table 4.27: Comparison between the expected value model and stochastic recourse model

Test	*EV*	*EEV*	*ESS*	*VSS (=EEV-ESS)*
I	426643	444205	432865	11340
II	408974	437078	420705	16373
III	402471	440472	423010	17462

From Table 4.27, it can be seen that in the three tests, all values of *EEV* are greater than the values of *ESS*. The expected value solution, therefore, can have unfavourable consequences because of the higher level of costs incurred, compared to those incurred when using the stochastic recourse model. In Test I, the total cost difference between the stochastic and expected value models (see the value of *VSS* in Table 4.27) is \$11,340, which is the possible gain from solving the stochastic model. The total cost in Test I decreases by \$11,340, from \$444,205 to \$432,865, if we choose the stochastic recourse model, rather than the expected value model. The total cost in Test II will decrease by \$16,373, from \$437,078 to \$420,705. The total cost in Test III will decrease by \$17,462, from \$440,472 to \$423,010. Compared with the expected value model, it is more beneficial to use the stochastic recourse model in Tests II and III, than in Test I. Test I represents the situation where it is most likely that demand will be high. If the anticipated situation does not happen, there will be a certain amount of surplus inventory of products and quotas. In Tests II and III, if the unanticipated situation (high demand) happens (with the possibility of 10%), there will be a certain amount of shortage of products and quotas. The unit surplus cost of products/quotas is lower than the unit shortage cost of products/quotas. The expected value model has limited ability to handle unanticipated situations, which may result in a very high cost. This is particularly true in Tests II and III, when the unanticipated situation (high demand) is realized. We can conclude that it is more beneficial to use the recourse model in Tests II and III than in Test I. These results show that explicitly considering uncertainty is a critical aspect of decision making and failure to include uncertainty may lead to very expensive, even disastrous consequences, if the anticipated situation is not realized (Bai *et al.* 1997).

4.4.4 Computational results of the robust linear optimization model with trade-off between solution robustness and model robustness

4.4.4.1 Computational results

The following content shows the computational results of the robust optimization model with trade-off between solution robustness and model robustness for Test III (p_1=10%, p_2=10%, p_3=80%) by setting up $\lambda = 0.1$, $\omega = 50$. The total cost is \$421,948.

The first stage decisions

Before the accurate market and quota price data are available, the company has to start production among its company-owned plants. The first stage decisions among the company-owned plants are shown in Tables 4.28~4.33. Table 4.28 shows the production quantities. Tables 4.29 and 4.30 show the machine work time and labour work time. Tables 4.31 and 4.32 show hiring and firing worker time. The initial quota allocated in each period is shown in Table 4.33. There is no need to work overtime. The first stage cost is $409,367.

Table 4.28: Production quantity for the uncertain problems using the robust optimization model

Plant	Product	Skilled workers (hrs)				Non-skilled workers (hrs)			
		Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	1	1200			1200		1067	1067	
	2								
	3								
2	1	333	226	933	900				
	2								
	3					1200	932	452	1005
3	1	267	607						
	2	867	819	1469	783	533	781	431	1317
	3						368	948	495

Table 4.29: Machine work time for the uncertain problems using the robust optimization model

Plant	Regular capacity used (hrs)				Additional capacity used (hrs)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	2400	2400	2400	2100				
2	2467	1851	2544	3307				
3	3317	5000	5000	5000		200	200	200

Table 4.30: Labour work time for the uncertain problems using the robust optimization model

Plant	Skilled workers (hrs)				Non-skilled workers (hrs)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	2400			2400		2400	2400	
2	750	509	2100	2025	1800	1399	678	1507
3	2400	3157	2937	1566	1200	1200	2630	3830

Table 4.31: Hiring workers for the uncertain problems using the robust optimization model

Plant	Skilled workers (hrs)				Non-skilled workers (hrs)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	2400			2400		2400		
2	750		1590		1800			830
3	2400	757			1200	1200	230	1200

Table 4.32: Firing workers for the uncertain problems using the robust optimization model

Plant	Skilled workers (hrs)				Non-skilled workers (hrs)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1		2400						2400
2		241		75		401	721	
3			219	1371				

Table 4.33: Quotas allocated for the uncertain problems using the robust optimization model

Plant	Period 1	Period 2	Period 3	Period 4
1	1700	1800	2000	2200
2	1300	1600	1800	2100
3	1100	1200	1400	1500

The second stage decisions

When the uncertainty is observed, the company can make the second stage production loading decisions, which are shown in Tables 4.34 ~ 4.40. The second stage cost is \$6,552.

Scenario 1: Good economy

The probability of a good economy is 10%. If this scenario is realized, the company will take the second stage decisions of purchasing certain quantities of products from its contractors, as well as purchasing additional quotas, to satisfy the high market demand. These results are shown in Tables 4.34 and 4.35. In the good economy scenario, 100 units of product 1 are leftover in periods 2 and 3, respectively. The inventory cost is \$500. There is no need to purchase products from contracted plants. As the initial quota is not enough to satisfy the higher demand in the good economy, the company needs to buy additional quotas, as shown in Table 4.35. The cost of purchasing quota in Scenario 1 is $8,000. However, in the good economy, a small amount of demand is not satisfied (see Tables 4.36). Table 4.36 also shows that the amount of unsatisfied demand is equal to the amount of unused quota in each period, which means that we only purchase quotas for products that will be actually shipped to overseas markets. The total penalty cost for violating the random constraints is $6,000.

Table 4.34: Shortage/surplus in scenario 1 for the uncertain problems using the robust optimization model

Product	Purchased products from contractors (units)				Inventory (units)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1						100	100	
2								
3								

Table 4.35: Under/-over quota in scenario 1 for the uncertain problems using the robust optimization model

	Purchased quota (units)				Unused quota (units)			
Product	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	100							
2	100							
3	100	100	100					

Table 4.36: Unsatisfied demand and unsatisfied quota

	Unsatisfied demand (units)				Unsatisfied quota (units)			
Product	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	100	200	100		100	200	100	
2	100	100			100	100		
3								

Scenario 2: Fair economy

The probability of a fair economy is 10%. If the fair economy is realized, the company will take the corresponding second-stage production loading decisions as follows.

Table 4.37: Shortage/surplus in scenario 2 for the uncertain problems using the robust optimization model

Product	Purchased products from contractors (units)				Inventory (units)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1								
2							100	200
3					100	200	300	400

Table 4.38: Under/-over quota in scenario 2 for the uncertain problems using the robust optimization model

Product	Purchased quota (units)				Unused quota (units)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	100	100						100
2	100							100
3							100	200

In the fair economy scenario, all random constraints are satisfied. Therefore, no infeasibility cost is incurred. At the same time, the first stage production is able to satisfy the demand in the fair economy. Thus there is no purchasing cost involved, for urgent production. However, some products produced in the first stage are leftover, as shown in Table 4.37, resulting in an inventory cost of \$800. The initial quota available in periods 1 and 2 for products 1 and 2 cannot satisfy the demand in the fair economy; so the company needs to buy a certain amount of quotas for Products 1 and 2 - these are shown in Table 4.38. The cost of purchasing quota is \$6,300 in Scenario 2. However, the initial quota in periods 3 and 4 exceeds the demand in those two periods, as shown in Table 4.38. The penalty cost for unused quotas is \$800 in Scenario 2.

Scenario 3: Bad economy

The probability of a bad economy is 80%. If the bad economy is realized, the company will take the second stage production loading decisions as follows:

Table 4.39: Shortage/surplus in scenario 3 for the uncertain problems using the robust optimization model

Product	Purchased products from contractors (units)				Inventory (units)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1					100	200	300	400
2					100	200	400	600
3					200	400	600	800

Table 4.40: Under/-over quota in scenario 3 for the uncertain problems using the robust optimization model

Product	Purchased quota (units)				Unused quota (units)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1							100	300
2						100	200	400
3					100	200	400	600

In the bad economy scenario, all random constraints are satisfied. There is no infeasibility cost. Meanwhile, as the first stage production is able to satisfy demand in the bad economy, there is no cost involved for purchasing urgent production. However, some products produced in the first stage are leftover, as shown in Table 4.39. This results in an inventory cost of $3,440 in Scenario 3. The initial quota available in each period is also too much for the demand in the bad economy, as shown in Table 4.40. The penalty cost for not fully using the initial quota is $2,700. There is no cost for purchasing additional quotas.

4.4.4.2 Comparison between the recourse and robust models

Table 4.41 gives the computational results of the robust optimization and the stochastic recourse model, for Test III. The expected cost under the stochastic recourse model is $423,010, and the expected cost under the robust model is $415,919. Using the robust optimization model, the expect cost decreases by $7,091, and the expected variability decreases by $12,908, which means the robust model presents a less sensitive production loading strategy. However, the robust model involves the infeasibility of 120 for not satisfying all market demand. If we increase the penalty of ω to 100 (see the last row in Table 4.41), no random constraint is violated. Compared with the recourse model, in which the expected variability decreases by \$7,569, the expected cost in the robust model increases by only \$419. It means that the production loading plan proposed by the robust model is not expensive, and it reduces the risk.

Table 4.41: Comparison between the recourse model and robust model

	Expected variability	Expected variability cost	Expected infeasibility	Expected infeasibility cost	Expected cost	Total cost
Recourse model	13567				423010	423010
Robust model ($\lambda = 0.1$, ω= 50)	659	66	120	6000	415919	421984
Robust model ($\lambda = 0.1$, ω= 100)	5998	600			423429	424028

4.4.5 Further tests for the robust optimization models

We perform three different tests, described in Table 4.26 in Section 4.4.3.3.

4.4.5.1 Tests for the robust linear optimization model with solution robustness

Table 4.42 shows the computational results of the robust optimization with solution robustness for the three tests, in which λ is assigned different values.

Table 4.42: Results for the robust optimization model with solution robustness

Test	λ	Expected variability	First stage cost	Second stage cost	Expected cost	Expected variability cost	Total cost
I	0*	4472	413860	19005	432865	0	432865
	0.1	4472	413860	19005	432865	447	433312
	0.5	4472	413860	19005	432865	2236	435101
	0.9	0	413860	21800	435660	0	435660
II	0*	14028	409045	11660	420705	0	420705
	0.1	8659	411354	9705	421059	866	421925
	0.5	3016	413860	8085	421945	1508	423453
	0.9	3016	413860	8085	421945	2714	424659
III	0*	13567	423010	13965	423010	0	423010
	0.1	5998	412421	11008	423429	600	424028
	0.5	2347	413860	10066	423926	1173	425099
	0.9	2347	413860	10066	423926	2112	426038

Note: * represents where the robust optimization model becomes the stochastic recourse programming model.

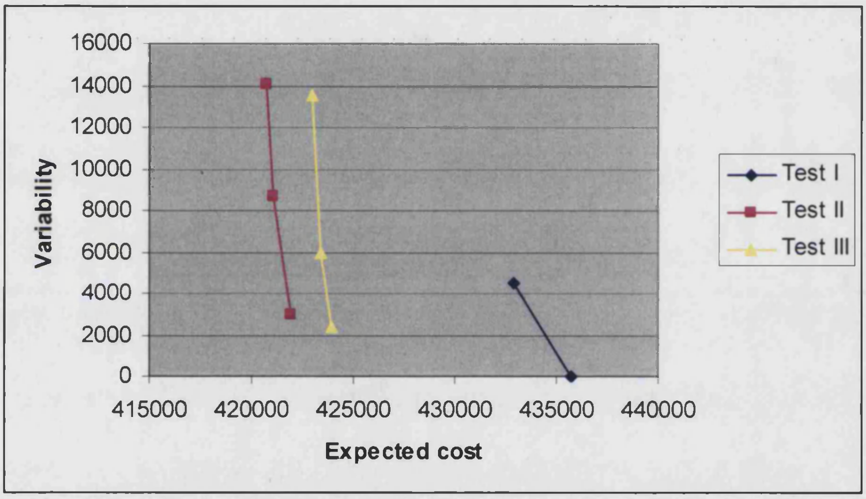
We first analyze the whole trend of the three tests in Table 4.42. When λ=0, the robust optimization model becomes a two-stage stochastic recourse model in which the variability is not considered. In Table 4.42, for each test, the expected variability for the two-stage recourse model is greater-than-or-equal-to that of the robust optimization model. This means that the two-stage stochastic recourse model is riskier than the robust optimization model with solution robustness. The total cost of the robust-optimization model is greater than that of the two-stage stochastic recourse model. Compared with the recourse model, the total cost of the robust model increases by 0.62% in Test I, 0.94% in Test II, and 0.72% in Test III. However, the variability decreases by 100% in Test I, 80.58% in Test II, and 89.02% in Test III. Compared with the recourse model, the expected cost of the robust model increases by 0.62% in Test I, 0.29% in Test II, and 0.22% in Test III. However, the variability decreases by 100% in Test I, 80.58% in Test II, and 89.02% in Test III.

Additionally, from Table 4.42, we could find that the first-stage cost is the same when λ=0.5, and λ=0.9, but the second-stage cost is different. It means the value of 0.9 is used to reduce the variability, but at a cost: the variability is reduced by 4472 at a cost of an increase in the expected cost of 2795. If a decision-maker is risk averse, she/he may want to choose a solution with a larger value of λ, such as λ=0.9. On the other hand, if the decision-maker has an active management style, she/he may want to adopt a solution with a smaller value of λ, such as λ=0.5.

Figure 4.1 presents the trade off of the expected cost against the expected variability for the three tests, when λ=0.9. Test I shows that the expected variability is reduced by $4,472 at a cost of an increase in the expected cost of $2,795, when we use the robust model with solution robustness (λ=0.9), rather than the recourse model. Test II shows that the expected variability is reduced by $11,012 at a cost of an increase in the expected cost of $1,240, when we use the robust model with solution robustness (λ=0.9), rather than the recourse model. Test III shows that the expected variability is reduced by $11,220 at a cost of an increase in the expected cost of $916, if we use the robust model with solution robustness (λ=0.9), rather than the recourse model. Tests II and III show a better improvement from the use of the robust model than Test I. The reason for this is that Test I represents a situation where it is most likely that Scenario 1 will happen (with the probability of 80%). If the unexpected situation (Scenarios 2 or 3) happens (with the probability of 10% and

10%, respectively), there will be a certain amount of surplus products and unused quotas. However, the cost of storing the surplus products and the cost of unused quotas is lower than the cost of purchasing the products from contracted plants and the cost of purchasing additional quotas from the markets. In Tests II and III, the possibility of purchasing a large amount of products and quotas to deal with the situation of high demand is 10%. If the unexpected situation (Scenario 1) happens, the cost of purchasing a large amount of products and quotas is high. The variability of Test I is less than the variability of Tests II and III.

Figure 4.1: Trade off of the expected cost against variability



Figures 4.2~4.4 further demonstrate the trade off between the expected cost and expected variability in the three tests when λ=0, 0.1, 0.5, and 0.9. Figure 4.2 shows the trade off between the expected cost and expected variability in Test I. The production plan based on the robust model with λ=0.9 is suitable for risk averse decision makers. However, the decision makers may choose the recourse model if the variability of \$4,472 is acceptable for them. Figure 4.3 shows the trade off between the expected cost and expected variability in Test II. The decision makers may choose the production plan based on the robust model with λ=0.5, as this will decrease the variability by \$11,012, against an increase in the expected cost of \$1,240. Figure 4.4 shows the trade off between the expected cost and expected variability in Test III. The decision makers may choose the production plan based on the robust model with λ=0.5, as this will decrease the variability

by 11,220, against an increase in the expected cost of \$916. Figures 4.2~4.4 show that the decrease in the expected variability is substantially lower than the increase in the expected cost, when λ increases. This is particularly true for Tests II and III. Compared with the recourse model, it is more beneficial to use the robust model with solution robustness in Tests II and III, than in Test I, as Tests II and III involve a high level of variability.

Figure 4.2: Trade off between the expected cost and variability in Test I

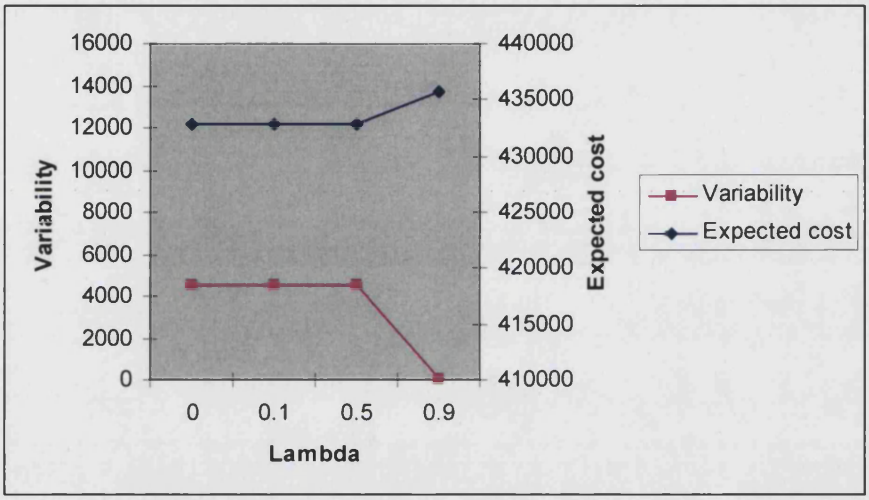


Figure 4.3: Trade off between the expected cost and variability in Test II

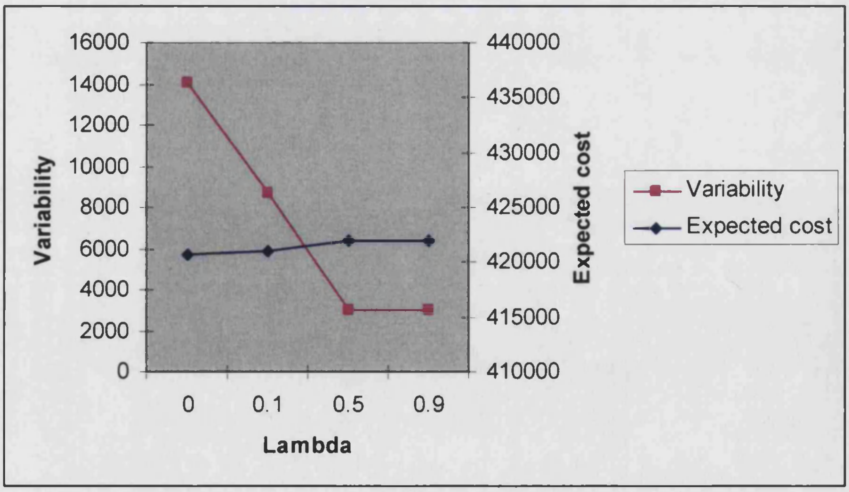
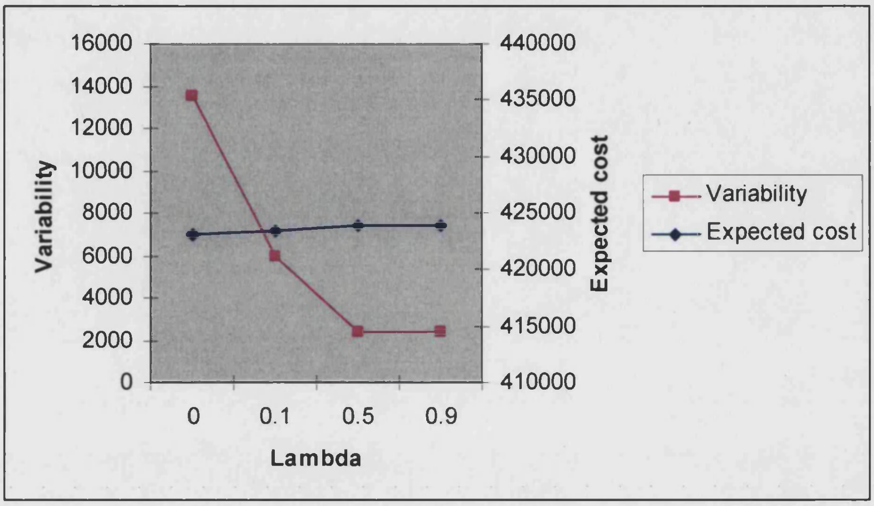


Figure 4.4: Trade off between the expected cost and variability in Test III



4.4.5.2 Tests for the robust linear optimization model with model robustness

Table 4.43 shows the computational results of the robust optimization with model robustness for the three tests.

Table 4.43: Computational results for the robust optimization model with model robustness

Test	ω	Expected infeasibility	First stage cost	Second stage cost	Expected cost	Expected Infeasibility cost	Total cost
I	$0^{\dagger}$	17980	357866	21986	379852	0	379852
	10	1480	407297	2293	409590	14600	424190
	50*	0	413860	19005	432865	0	432865
II	$0^{\dagger}$	17394	358750	15137	373887	0	373887
	10	420	405556	778	406334	4200	410534
	50	80	407430	7686	415116	4000	419116
	100*	0	409045	11660	420705	0	420705
III	$0^{\dagger}$	16620	359862	10825	370687	0	370687
	10	720	398950	1910	400860	7200	408060
	50	80	407430	10083	417513	4000	421513
	100*	0	409045	13965	423010	0	423010

Note: † represents the robust optimization model without considering the random demand and quota constraints, and * represents when the robust optimization model becomes the stochastic recourse programming model.

In the three tests, when ω=0 there is no penalty for violating the second stage constraints consisting of the random demand constraints and random quota constraints. The second stage cost arises mainly from the over-quota cost of penalizing the unused quota in the three tests (see the first row in each test). Only a small amount of demand is satisfied because of the requirement of minimum work time. The expected infeasibility is very high: \$17,980 in Test I, \$17,394 in Test II and \$16,620 in Test III (see the third column), which means the higher violation of the random constraints. When ω increases, the expected infeasibility decreases, the expected cost increases, and the total cost increases When ω increases by enough, the expected infeasibility becomes zero, which means that all random constraints in the second stage are satisfied because of the higher penalty for the infeasibility. The robust optimization model then becomes the two-stage stochastic recourse model (see the final row in each test). From Tables 4.42 and 4.43, we know that the first row of each test in Table 4.42 (when λ=0) has the same result as that shown in the final row in Table 4.43 (when ω is large enough), as both of them represent the result of the two-stage stochastic recourse programming model.

4.4.5.3 Tests for the robust linear optimization model with trade-off between solution robustness and model robustness

Parameters λ and ω are used to measure trade-off between solution robustness and model robustness. When ω=0, there is no penalty for the infeasibility of random constraints in the objective function. The infeasibility representing un-fulfilment is a higher value. Clearly, decision-makers would not like this kind of production loading plan. However, a large weight ω means the penalty function dominates the total objective function value and would result in a higher variability and a higher total cost. Therefore, there is always a trade-off between the risk and the cost. During the production loading process, it is necessary to test the proposed robust optimization model with difference λ and ω in order to measure trade-off between the risk and cost. The computational results for Test III are provided in this section.

When λ is a constant

Figures 4.5~4.8 show the computational results for Test III in terms of variability, infeasibility, expected cost and total cost, when λ keeps constant.

Figure 4.5 gives the trends in variability when ω increases for λ=0.1, 0.5 and 0.9. For λ=0.1, when ω increases, variability sharply increases from 416 to 5,998. However, variability keeps steady at 5,998 after ω increases to 100. When λ=0.5 and 0.9, the value of ω has a relatively small impact on variability. The reason for this is that when λ is given a large value, variability cost dominates the objective function value. ω has a small impact on the objective value and variability.

Figure 4.5: Variability when λ is a constant

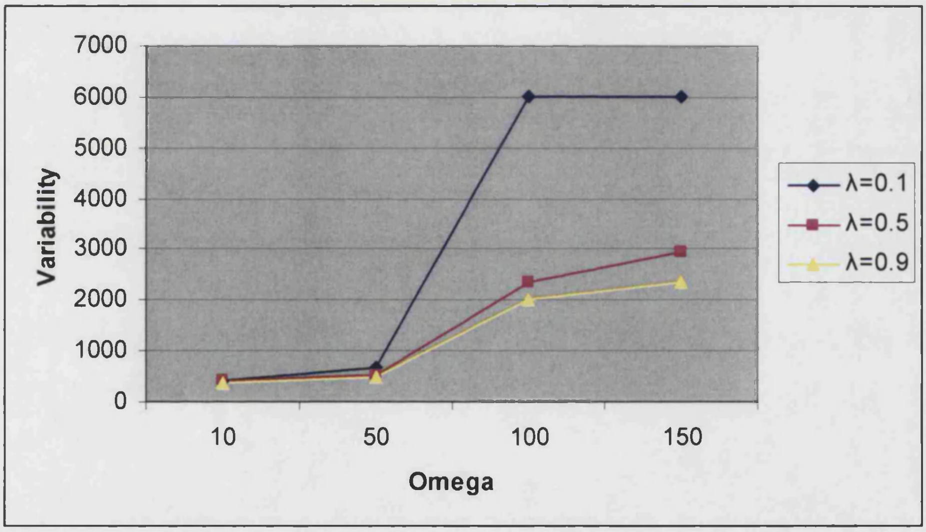
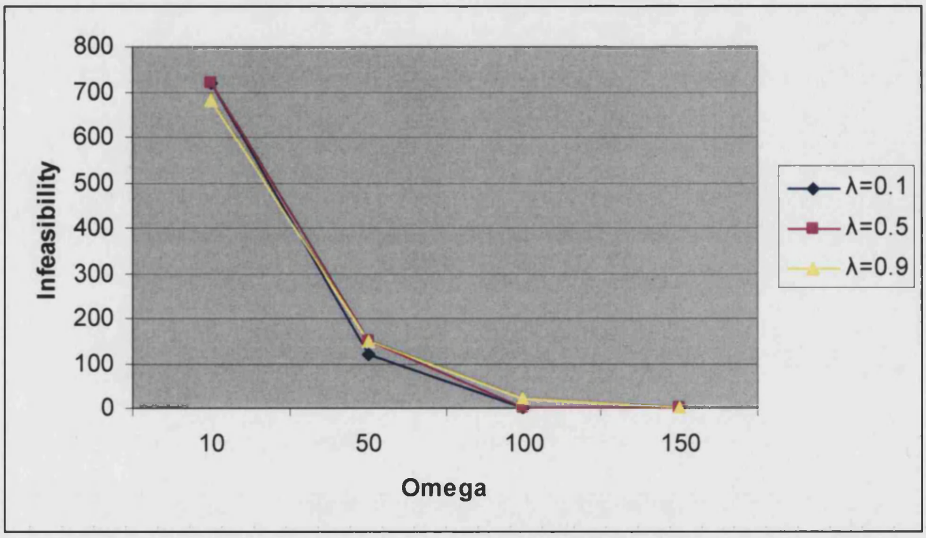


Figure 4.6 gives the trends in infeasibility when ω increases for λ=0.1, 0.5 and 0.9. Clearly, the value of ω has a big influence on the system's infeasibility.

Figure 4.6: Infeasibility when λ is a constant



In Figures 4.7 and 4.8, when ω increases, the expected and total costs increase accordingly. The value of ω has more impact on the system's cost.

Figure 4.7: Expected cost when λ is a constant

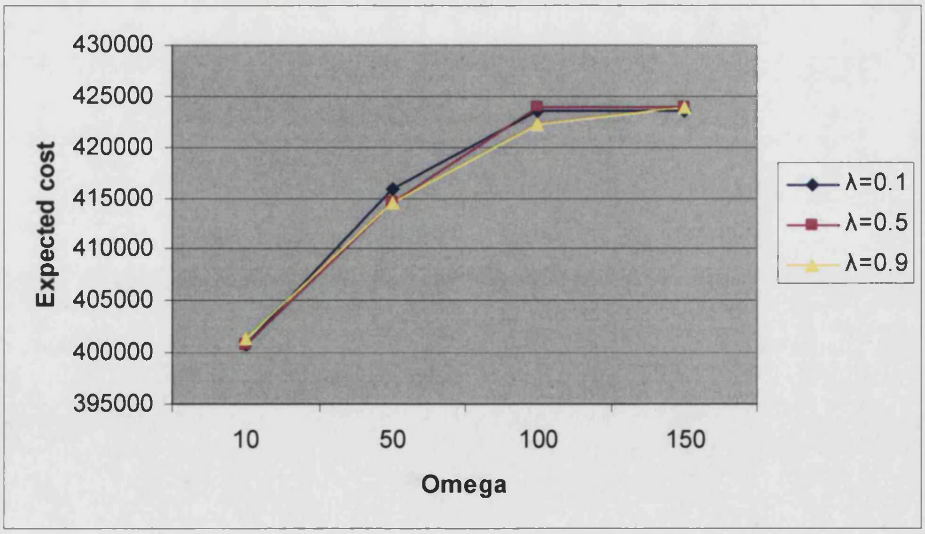
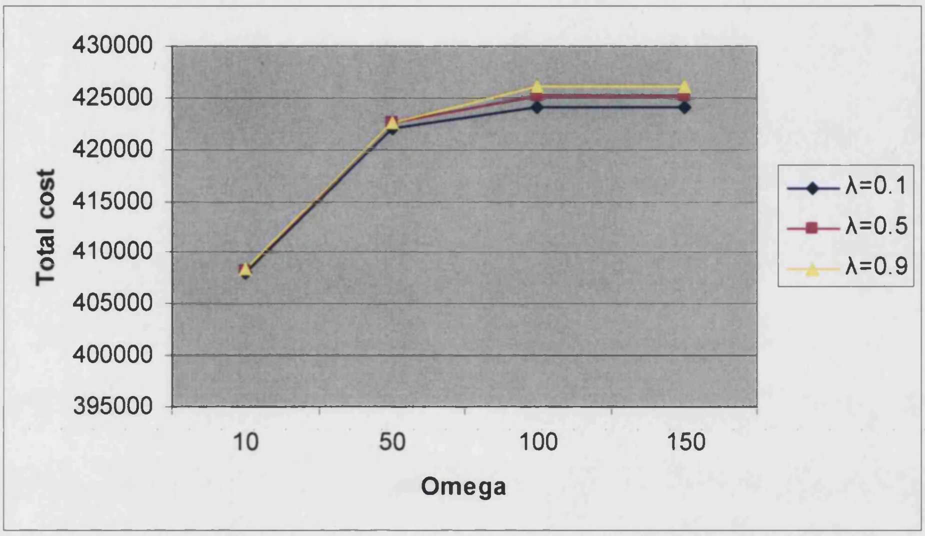


Figure 4.8: Total cost when λ is a constant



When ω is a constant

Figures 4.9~4.12 show the computational results of Test III in terms of variability, infeasibility, expected cost and total cost, when ω keeps constant.

Figure 4.9 shows the trends in variability when λ increases for ω=10, 50, 100 and 150. If λ increases from 0.1 to 0.9, for ω=10, variability decreases by 58; for ω=50, variability decreases by 179, for ω=100, variability decreases by 3,989; for ω=150, variability decreases by 3,651. When ω is given a large number, infeasibility becomes small. The variability cost dominates the objective function value. The value of λ has more impact on variability.

Figure 4.9: Variability when ω is a constant

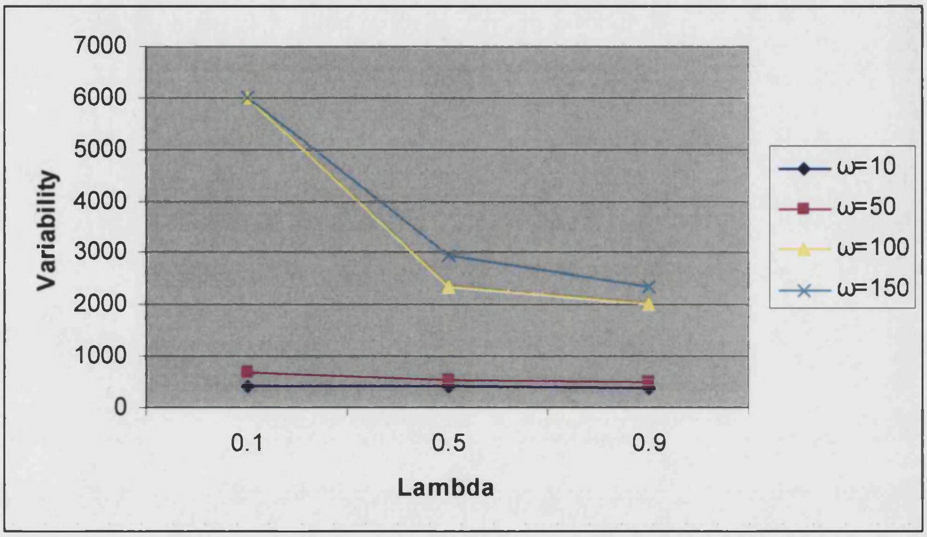
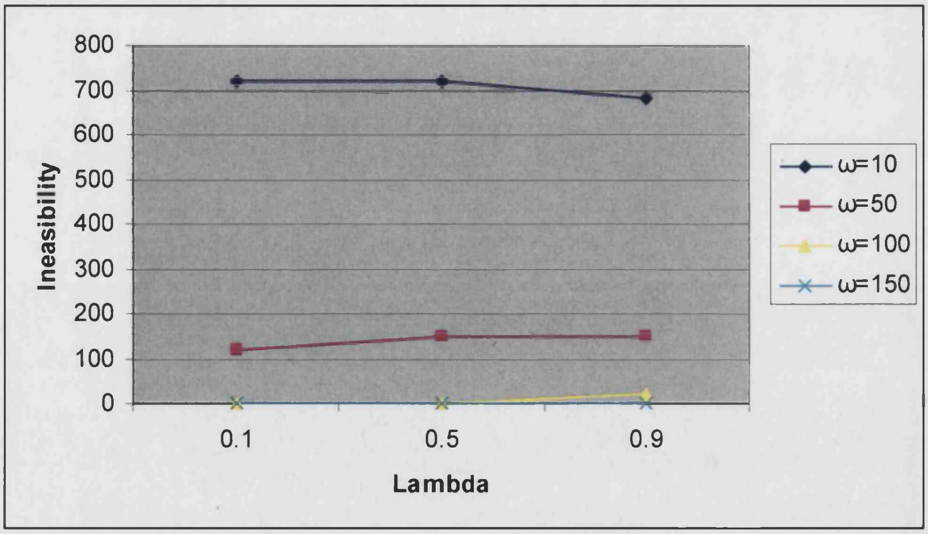


Figure 4.10 shows the trends in infeasibility when λ increases for ω=10, 50, 100 and 150. We can see that λ has less impact on infeasibility.

Figure 4.10: Infeasibility when ω is a constant



Figures 4.11 and 4.12 show the trends in expected and total costs. The value of λ has less impact on expected and total costs.

Figure 4.11: Expected cost when ω is a constant

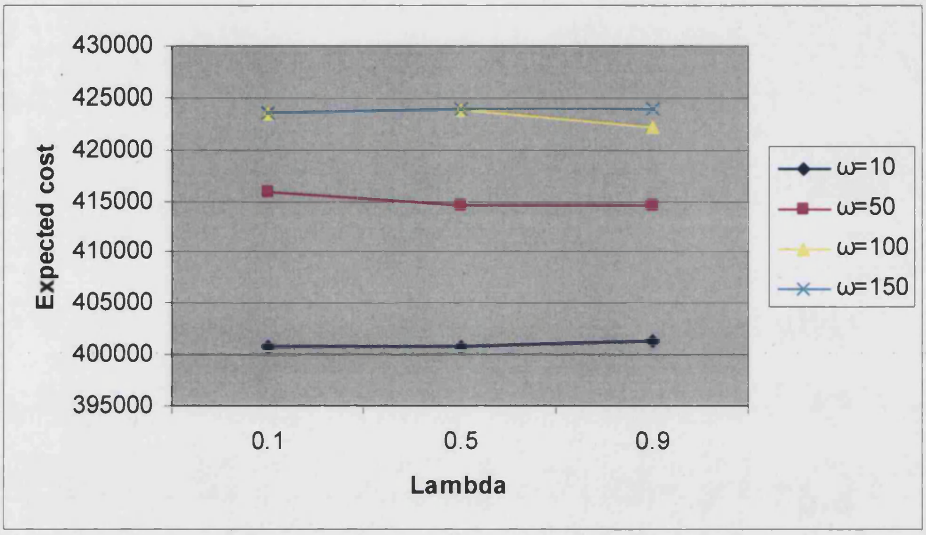
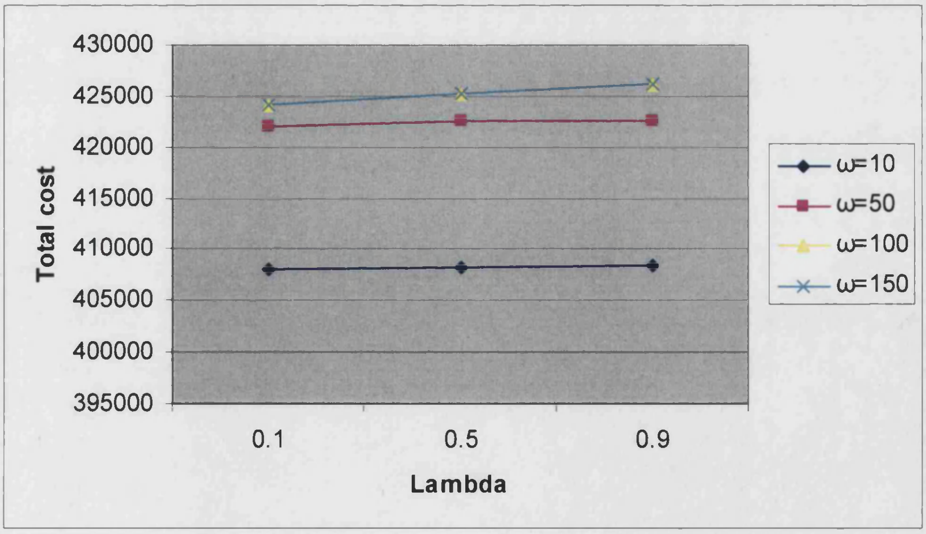


Figure 4.12: Total cost when ω is a constant



Model validation

To validate the efficiency of the models, a series of tests were carried out, using the data provided by the company for 12 months. Based on the company's strategies, all customer orders have to be fulfilled, which leads to using the robust optimization model with solution robustness, proposed in this study. Figures 4.13, 4.14 and 4.15 show the variability, expected cost, and total cost for 12 months. We can see that the robust model has less risk than the two-stage stochastic recourse model, and the cost of reducing the risk is not high.

Figure 4.13: Expected variability for 12 months

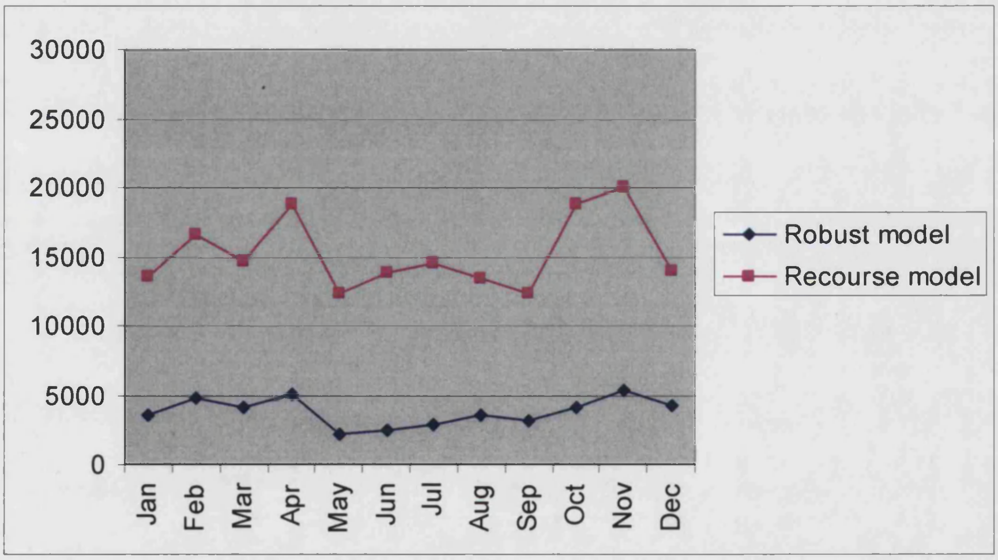


Figure 4.14: Expected cost for 12 months

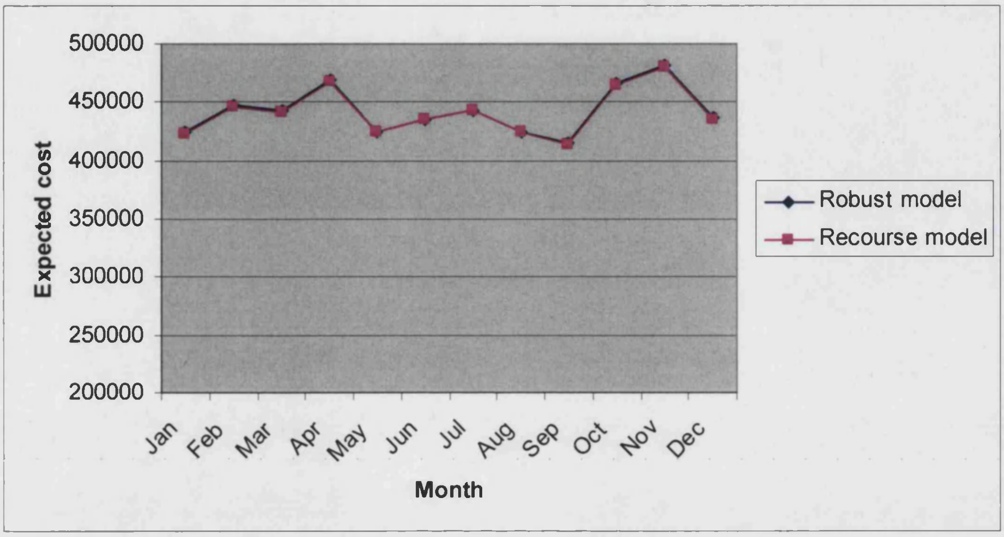
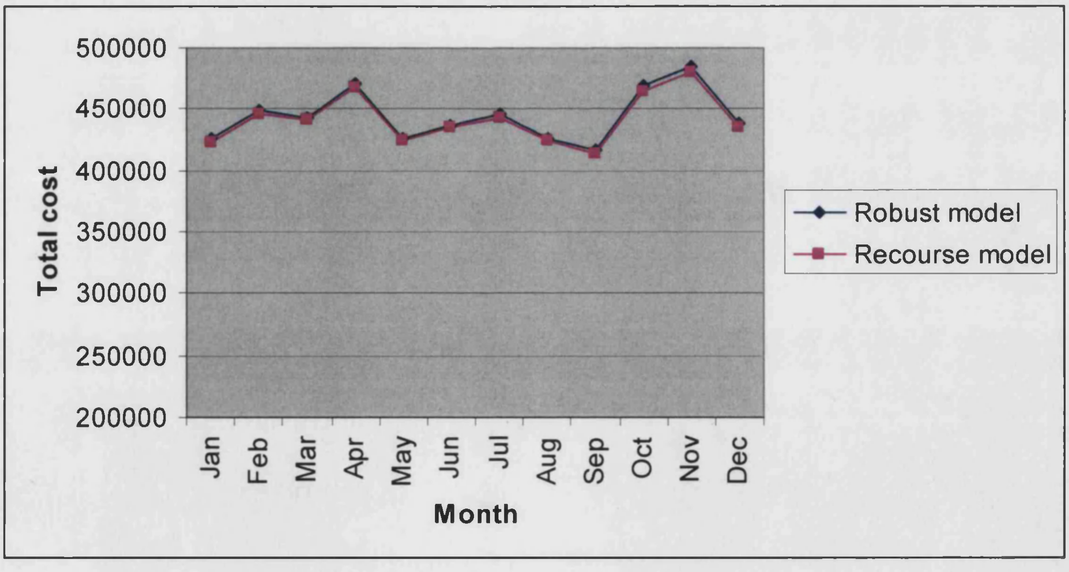


Figure 4.15: Total cost for 12 months



4.5 Summary

Global supply chain management presents some special challenges and issues for manufacturing companies in production planning; these challenges are different from those discussed in domestic production plans. Production managers find that they have to develop competitive production strategies in order to survive. This chapter examines production loading problems with import quota limitations in a global supply chain network. We first develop a linear programming model to determine the optimal production plan, which assumes that all information is available at the time of decision making. The computational results, based on data from a garment company, present the production loading strategy in terms of quantities of used resources, including machine, labour and initial quotas, as well as inventory levels, outsourcing levels, quotas purchased from local markets and unused quotas, production volumes, etc. However, globally, production loading problems involve substantial uncertainty because of uncertain market demand, and fluctuating quota prices. In addition, the lead time of products under this study is very short. The company has to start manufacture of products before accurate information is available. We propose a dual-response production loading strategy to hedge against uncertainty involved in loading production among different manufacturing plants in different countries. In the first stage, when accurate market information is not available, the company distributes production tasks among the company-owned plants. The decisions in this stage include production quantity, machine capacity, work force level and initially available quotas. In the second stage, when the uncertainty is realized, the company allocates production tasks among contracted plants. The decisions in this stage include the quantities of products to be outsourced from contracted plants, inventory levels, quantities of additional quotas required, and the quantities of quotas that are unused. In order to achieve the dual-response production loading strategy, a two stage stochastic recourse model is developed. Computational results demonstrate how the recourse model can provide the dual-response production loading strategy to handle uncertainty during the decision-making process. A series of experiments are also designed to show that the recourse model has favourable consequences because of the lower level of costs, compared to costs incurred when using the corresponding expected value model, in which all stochastic parameters are replaced by their expected values. The computational results from the data provided by the company also show that it is more beneficial to use the stochastic recourse model in some production scenarios, than others.

As the stochastic recourse model has less capability to handle the risk, three types of robust optimization models are proposed: the robust optimization model with solution robustness, the robust optimization model with model robustness, and the robust optimization model with trade-off between solution robustness and model robustness. The robust model with solution robustness provides a direct way to measure trade-off between cost and risk, which is characterised by variability. The solution from the robust model with solution robustness has low variability among different scenarios. The computational results demonstrate that robust model with solution robustness has lower risk than the two-stage stochastic recourse model, and the cost of reducing the risk is low. The computational results also show that it is more profitable to use the robust model with solution robustness in production loading problems when the level of risk is high. Furthermore, we propose a robust model with model robustness to handle infeasibility during the decision-making process. A series of experiments give results of comparison between the recourse model and the robust model with model robustness in terms of expected infeasibility, expected cost and total cost. Computational results show that the robust model with model robustness is able to handle infeasibility in production loading problems under uncertainty. Finally, a general robust model with solution robustness and model robustness is presented, which provides a direct way to measure the trade off between solution robustness and model robustness. A series of experiments, and the figures that are based on computational results, show the impact of λ and ω on the production system's performance in terms of variability, infeasibility, expected cost and total cost. Decision-makers can choose their favourite production loading strategy, based on their attitude toward the risk by adjusting the value of λ and ω.

Products that are discussed in this chapter are fashion products, which belong to innovative products category. Compared with functional products with stable demand and long life cycle, demand for innovative products is uncertain and their life cycle is short.

Production managers have to quickly forecast the demand and the corresponding probability of the forecast demand being realized. The demand data is mainly based on production managers' experience and judgment, as introduction of new products suffers from the absence of historical data that could be useful in forecasting; a quick forecast is required during the decision making process. Forecasting future demand for innovative products is a challenging task for researchers. It, however, goes beyond the scope of this research. In addition, computation and analysis of the models may lead to different outcomes if the values of model parameters change.

Chapter 5

Logistics problems for global road transport

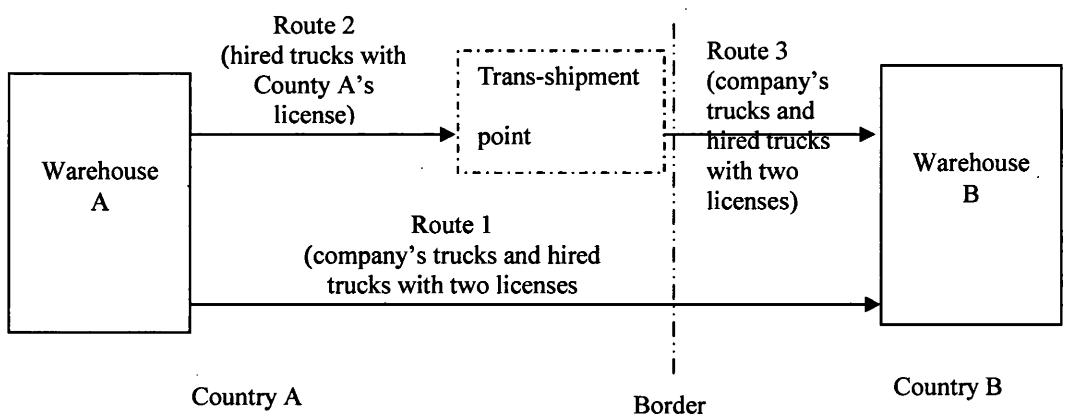
5.1 Introduction

5.1.1 Global road transport process

Over the past ten years supply chain management has become an important focus of competitive advantage for business organizations. Logistics, as a critical part of the supply chain management, controls capital, materials, services and information to anticipate customer requirements. Logistics has never played such an important role in the global supply chain management environment, because the movement of shipments from supply site to demand site tends to be more frequent than ever before. In this study, we consider the global logistic problems for road transportation, which involves transporting goods from country A across the border to country B. There are some differentials between two countries in terms of truck operation cost, truck capacity, labour cost, warehousing cost, etc. Compared with country B, country A is a low-cost country in terms of production, transportation, warehouse, labour, etc. Two centralized warehouses 1 and 2, are located in the two countries A and B, respectively. It is assumed that both of the warehouses have enough capacity for storing goods. The unit inventory cost in warehouse B is much higher than in warehouse A. As a result, the goods are normally stored in warehouse A in country

A, and need to be transported to warehouse B in country B, where there is a demand for the good. The logistics company has its own trucks with two licenses and which can operate in both countries. However, when the company fleet does not have enough capacity to satisfy demand in country B, the company has to hire additional trucks. There are two types of trucks available for rental: the first type of truck only has a license for country A and can only operate in that country; the second type of truck has two licenses and can operate in both countries. The company has two strategies for delivering goods. The first strategy is to use company-owned trucks or/and hire trucks with two licenses to directly transport goods from warehouse A to warehouse B. The second strategy is first to load the goods into hired trucks with a country A license only. Then the goods are trans-shipped into the company-owned trucks or the hired trucks with two licenses at the border in order to get across to country B. The goods cannot stay overnight on the border, as there is no warehouse there. Although the transhipment process involves a certain cost associated with unloading and loading, the company may adopt this strategy as the cost of hiring a truck with a country A license only is very low. Therefore, the road network consists of three routes: Route 1, connecting warehouse A in country A and warehouse B in country B; Route 2, connecting warehouse A and the trans-shipment point on the border in country A; and Route 3, connecting the trans-shipment point on the border in country A and warehouse B in country B. As shown in Figure 5.1, Routes 1 and 3 include a border-crossing process.

Figure 5.1: Truck routes



It is assumed the cost of hiring a truck either with one license or two licenses only covers one trip each day between two places. If the truck makes two trips, the hiring cost will double so the company does not adopt this strategy. If necessary, the company could hire more trucks, as this ensures faster delivery for the same cost. Thus, only company-owned trucks will make a round-trip journey every day on Routes 1 and 3.

In section 5.3.1, a mixed 0-1 linear programming model is formulated to determine an optimal global logistics transportation strategy including optimal composition of the company's fleet and route plans to minimize total cost. The model assumes that all data that decision-making needs is known with certainty.

5.1.2 A dual-response logistics strategy for global road transport under uncertainty

The goods will be transported to warehouse B located in country B. Unfortunately, the accurate shipment information can only be obtained on the shipping day from the freight forwarders, who are responsible for the global air transport. However, the logistics company has a limited capacity of fleet transportation, and has to determine the numbers and types of trucks that will be hired from the two countries in advance. Therefore, a dual-response logistics strategy for global road transport is developed in dealing with the uncertain information and short shipment notice. In the first stage, when accurate information is not available, we determine the fleet composition and route. In the second stage on the shipping day, when accurate shipment information is obtained, we need to make responses for different scenarios that might happen on the shipping day.

Section 5.3.2 presents a two-stage stochastic mixed 0-1 integer recourse programming model to determine optimal delivery routes and the optimal truck fleet composition for a weekly planning horizon under uncertainty.

5.1.3 Risk

As the stochastic recourse programming model is unable to handle infeasibility and risk, section 5.3.3 formulates three types of robust optimization model for the logistics road transport problems between two countries. The first type of model is called the robust optimization model with solution robustness, which provides a solution that is less sensitive to the realizations of the stochastic parameters. The two-stage stochastic mixed 0-1 integer recourse programming model is infeasible if a feasible solution, including the first stage and the second stage decision variables, does not exist. In section 5.3.3, we formulate the robust optimization model with model robustness for the logistics road transport problems: this model can be used to find a solution that violates the stochastic constraints by the least amount through the penalty function. The third type of model, called the robust optimization model with trade-off between solution robustness and model robustness, provides a direct way to measure the trade-off between the risk and cost during the global transportation process.

5.1.4 Overview of chapter 5

The rest of the chapter is organized as follows. Section 5.2 presents the notation and definitions. Section 5.3 formulates a series of models, including the mixed 0-1 integer programming model under the assumption that all parameters are known with certainty, the two-stage stochastic mixed 0-1 integer recourse programming model under uncertainty, and the robust optimization models to handle uncertainty and risk. Section 5.4 gives the computational results and analysis for all the models. The final section gives the summary on the logistics problems for global road transport.

5.2 Notation and definitions

In formulating the logistics models for global road transport, the following notation and definitions are used.

5.2.1 Indices

I^0 = set of company-owned trucks with licences to operate in both countries

I^1 = set of trucks for hire with a country A licence only

I^2 = set of trucks for hire with licenses for both countries

J = set of routes. J={1, 2,3}

T= set of time periods

K = set of round-trips

i=index of trucks, $i \in I^0 \cup I^1 \cup I^2$

j= index of routes, $j \in J$

t= index of time periods, $t \in T$

k=index of round-trips, $k \in K$

5.2.2 Parameters

Supply/demand

s_t = volume of products arriving in country A's warehouse on day t, $t \in T$

d_t = volume of products demanded in country B on day t, $t \in T$

Truck capacity

L_i = maximum loading capacity of truck i, $i \in I^0 \cup I^1 \cup I^2$

Company-owned trucks

c_{ij}^{0} = unit trip cost of company-owned truck i operating on Route j, $i \in I^{0}$, j={1,3}

r_j = a round-trip time using Route j, j={1,3}

H= maximum working hours for drivers of company-owned trucks per day

Hired trucks

h_{i}^{1} = unit hiring cost of truck i operating in country A on Route 2, $i \in I^{1}$

h_{ij}^{2} = unit hiring cost of truck i operating in countries A and B on Routes j, $i \in I^{2}$, j={1,3}

Warehousing/trans-shipping

w_{0}^{1} = initial volume of products stored in warehouse A in country A

w_{0}^{2} = initial volume of products stored in warehouse B in country B

c^{1} = unit inventory cost in warehouse A

c^{2} =unit inventory cost in warehouse B

c^{T} =unit cost of trans-shipment on the border

Penalty cost

c^{3} =unit penalty cost for not satisfying the demand in country B

5.2.3 Decision variables

Trucks used

$$x_{ijkt}^{0} = \begin{cases} 1 & \text{if company-owned truck } i \text{ operates the } k^{\text{th}} \text{ round trip on Route } j \text{ on day } t \\ 0 & \text{otherwise} \end{cases},$$

$i \in I^{0}$, j={1,3}, $k \in K$, $t \in T$

$$x_{it}^{1} = \begin{cases} 1 & \text{if hired truck } i \text{ operates from country A to border on Route 2 on day } t \\ 0 & \text{otherwise} \end{cases}, \; i \in I^1,$$

$t \in T$

$$x_{ijt}^{2} = \begin{cases} 1 & \text{if hired truck } i \text{ operates from country A to country B on Route } j \text{ on day } t \\ 0 & \text{otherwise} \end{cases}, \; i \in I^2,$$

j={1,3}, $t \in T$

Volume loaded

X_{ijkt}^{0} = volume of goods loaded by company-owned truck i on Route j on k^{th} round on day t, $i \in I^0$, j={1,3}, $k \in K$, $t \in T$

X_{it}^{1} = volume of goods loaded by hired truck i with one license on Route 2 on day t, $i \in I^1$, $t \in T$

X_{ijt}^{2} = volume of goods loaded by hired truck i with two licenses on Route j on day t, $i \in I^2$, j={1,3}, $t \in T$

Surplus/shortage

w_t^1 = surplus in warehouse A on day t, $t \in T$

w_t^2 = surplus in warehouse B on day t, $t \in T$

w_t^3 = shortage in country B on day t, $t \in T$

5.2.4 Constraints

Destination constraints

Demand in country B has to be satisfied by the sum of the initial inventory in warehouse B, the total volume of the products that arrive in warehouse B on day t and any shortage, minus surplus goods at the end of day.

$$d_t = w_{t-1}^2 + \sum_{k \in K} \sum_{j=\{1,3\}} \sum_{i \in I^0} X_{ijkt}^0 + \sum_{j=\{1,3\}} \sum_{i \in I^2} X_{ijt}^2 - w_t^2 + w_t^3, \; t \in T \tag{5.1}$$

Supply constraints

On day t, the total volume of the products that arrive in warehouse A plus its initial inventory is equal to the sum of the products leaving warehouse A on day t and the products left at the end of day.

$$s_t + w_{t-1}^1 = w_t^1 + \sum_{k\in K}\sum_{i\in I^0} X_{i1kt}^0 + \sum_{i\in I^1} X_{it}^1 + \sum_{i\in I^2} X_{i1t}^2,\ t\in T \tag{5.2}$$

Trans-shipment constraints

Constraint (5.3) ensures that, on day t, the total products arriving at the transhipment point on the border is equal to the total products leaving the trans-shipment point to go to warehouse B. This constraint is needed since the goods cannot be kept at the trans-shipment point overnight.

$$\sum_{i\in I^1} X_{it}^1 = \sum_{k\in K}\sum_{i\in I^0} X_{i3kt}^0 + \sum_{i\in I^1} X_{i3t}^2,\ t\in T \tag{5.3}$$

Work time constraints

Constraint (5.4) ensures that the working hours for drivers of the company-owned trucks cannot exceed their maximum working hours.

$$\sum_{j=\{1,3\}}\sum_{k\in K} r_j x_{ijkt}^0 \le H,\ i\in I^0, j=\{1,3\},\ t\in T \tag{5.4}$$

Round-trip constraints

Each company-owned truck could make the next round trip only after the previous round trip has been completed.

$$x_{ijkt} \ge x_{ij,k+1,t},\ i\in I^0, j=\{1,3\},\ k\in K,\ t\in T \tag{5.5}$$

Capacity constraints

Constraints (5.6)~(5.7) ensure that, for every truck, the loading volume of products cannot exceed its capacity.

$$X_{ijkt}^0 \le L_i x_{ijkt}^0,\ i\in I^0, j=\{1,3\},\ k\in K,\ t\in T \tag{5.6}$$

$$X_{it}^1 \le L_i x_{it}^1,\ i \in I^1,\ t \in T \tag{5.7}$$

$$X_{ijt}^2 \le L_i x_{ijt}^2, i \in I^2, j=\{1,3\},\ t \in T \tag{5.8}$$

Variable type constraints

$$x_{ijkt}^0 \in \{0,1\}, X_{ijkt}^0 \ge 0, i \in I^0, j = \{1,3\}, k \in K, t \in T \tag{5.9}$$

$$x_{it}^1 \in \{0,1\}, X_{it}^1 \ge 0, i \in I^1, t \in T \tag{5.10}$$

$$x_{ijt}^2 \in \{0,1\}, X_{ijt}^2 \ge 0, i \in I^2, j = \{1,3), t \in T \tag{5.11}$$

$$w_t^1 \ge 0,\ t \in T \tag{5.12}$$

$$w_t^2, w_t^3 \ge 0,\ t \in T \tag{5.13}$$

5.2.5 Costs

Transportation cost

This cost is associated with fuel, maintenance, loading cost, labour cost, etc for the company-owned trucks. The company-own trucks can operate on Route 1 connecting warehouse A and warehouse B and on Route 3 connecting the trans-shipment point at the border and warehouse B.

$$TC = \sum_{t \in T} \sum_{k \in K} \sum_{j=\{1,3\}} \sum_{i \in I^0} c_{ij}^0 x_{ijkt}^0 \tag{5.14}$$

Hiring cost

The hired trucks with a country A licence only operate on Route 2, while the hired tucks with licenses for both countries operate on Routes 1 and 3, which includes the cost of crossing the border.

$$HC = \sum_{t \in T} \sum_{i \in I^1} h_i^1 x_{it}^1 + \sum_{t \in T} \sum_{j=\{1,3\}} \sum_{i \in I^2} h_i^2 x_{ijt}^2 \tag{5.15}$$

Trans-shipment cost

When products are transported from warehouse A in country A to the trans-shipment point on the border using Route 2, products need to be unloaded from the trucks, and are loaded into the truck with two licenses on order to cross the border. The trans-shipment cost involves the unloading and loading cost.

$$RC = \sum_{t \in T} \sum_{i \in I^1} c^T X_{it}^1 \tag{5.16}$$

Inventory cost in warehouse A

An inventory cost is incurred at warehouse A when the goods are not fully transported to country B on day t and have to be stored in warehouse A on day t.

$$IC^1 = \sum_{t \in T} c^1 w_t^1 \tag{5.17}$$

Inventory cost in warehouse B

An inventory cost is also incurred in warehouse B when the total goods being stored and arriving in warehouse B exceed the demand from country B on day t.

$$IC^2 = \sum_{t \in T} c^2 w_t^2 \tag{5.18}$$

Shortage cost in country B

The company will incur a penalty when demand is not satisfied.

$$SC = \sum_{t \in T} c^3 w_t^3 \tag{5.19}$$

5.3 Model formulations

5.3.1 A mixed 0-1 integer programming model for the deterministic logistics problems

The objective is to minimize the sum of all costs listed in Section 5.2.5, and satisfy all constraints described in Section 5.2.4. The deterministic global logistics problem can be formulated as a mixed 0-1 integer programming model as follows:

Min $TC+HC+RC+IC^1+IC^2+SC$ (5.20)

s.t.

(5.1)~(5.13)

5.3.2 A stochastic mixed 0-1 integer recourse programming model for the uncertain logistics problems

Random parameters

In this study, demand d_t, unit inventory cost c^2 in warehouse B, and unit shortage cost c^3 in country B are defined as stochastic parameters. It is assumed that uncertainties are represented by a set of possible realizations, called *scenarios*. Each scenario s with probability p_s, where $s \in S$ and $\sum_{s\in S} p_s = 1$, provides one possible course of future events. When recourse actions are taken after realization d_{ts} of random parameter d_t, realization c_s^2 of random parameter c^2, and realization of c_s^3 of random parameter c^3, d_t, c^2 and c^3 are independent random parameters, and have the same finite discrete distribution specified by:

$$\begin{bmatrix} p_1 & p_2 & \cdots & p_S \\ d_{t1} & d_{t2} & \cdots & d_{tS} \\ c_1^2 & c_2^2 & \cdots & c_S^2 \\ c_1^3 & c_2^3 & \cdots & c_S^3 \end{bmatrix} \tag{5.21}$$

Decision variables

The second stage decision variable includes the volume of product surplus stored in warehouse B, denoted by w_{ts}^2, and the volume of product shortage in country B denoted by w_{ts}^3 for each scenario. Other decision variables, which are defined in Section 5.2.3, belong to the first stage decision variables.

Constraints

- **The first stage constraints:**
  Constraints (5.2)~(5.12)

- **The second stage constraints:**

$$d_{ts} = w_{t-1,s}^2 + \sum_{k \in K} \sum_{j=\{1,3\}} \sum_{i \in I^0} X_{ijkt}^0 + \sum_{j=\{1,3\}} \sum_{i \in I^2} X_{ijt}^2 - w_{ts}^2 + w_{ts}^3,\ t \in T,\ s \in S \tag{5.22}$$

$$w_{ts}^2, w_{ts}^3 \geq 0, t \in T,\ s \in S \tag{5.23}$$

Constraint (5.22) ensures that, on day t and in each scenario s, the total volume of the products transported received from warehouse A plus the products currently stored in warehouse A is equal to the total volume of products required in warehouse B plus the products stored or product shortage in warehouse B. Constraint (5.23) is a variable type constraint.

Objective function

The objective is to minimize the total cost, which is the sum of the first stage cost and the second stage cost. The first stage cost is the sum of the company-owned trucks cost, hiring cost for all trucks, trans-shipment cost on the border, and inventory cost at warehouse A. The first stage cost, denoted by *FirstCost*, can be formulated as:

$$FirstCost=TC+HC+RC+IC^1 \tag{5.24}$$

The second stage cost, denoted by *SecondCost*, is the cost of the second stage decisions, and can be expressed as:

$$SecondCost=\sum_{s\in S}\sum_{t\in T} p_s\left(c_s^2 w_{ts}^2 + c_s^3 w_{ts}^3\right) \tag{5.25}$$

A two-stage stochastic recourse programming model for the global logistics problem under uncertainty is summarized as follows:

$$\text{Min } FirstCost+SecondCost \tag{5.26}$$

s.t.

The first stage constraints: (5.2)~(5.12)

The second stage constraints: (5.22)~(5.23)

5.3.3 Robust optimization models for the uncertain logistics problems

5.3.3.1 A robust mixed 0-1 integer optimization model with solution robustness

Based on the analysis in section 3.3.1, a robust optimization model with solution robustness for the uncertain logistics problems for global road transport can be formulated as:

$$\text{Min } FirstCost+SecondCost+\lambda\sum_{s\in S} p_s\left|(c_s^2 w_{ts}^2 + c_s^3 w_{ts}^3) - \sum_{s\in S} p_s(c_s^2 w_{ts}^2 + c_s^3 w_{ts}^3)\right| \tag{5.27}$$

s.t.

The first stage constraints: (5.2)~(5.12)

The second stage constraints: (5.22)~(5.23)

The sum of the first term and the second term in objective function (5.27) is the objective function of the stochastic recourse programming model expressed in equation

(5.26). The final term in (5.27) is the cost of the variability, in which the parameter λ is intended as a measure of decision-maker's aversion to the variability. Clearly, when $\lambda = 0$, the above model becomes a two-stage stochastic recourse programming model, and this is precisely formulated in section 5.3.2. The above model can be converted into a linear programming model by introducing a deviational variable $\theta_s \geq 0$ as follows:

$$\text{Min } FirstCost+SecondCost+\lambda\sum_{s\in S} p_s\left[\sum_{t\in T}(c_s^2 w_{ts}^2 + c_s^3 w_{ts}^3) - \sum_{s\in S}\sum_{t\in T} p_s(c_s^2 w_{ts}^2 + c_s^3 w_{ts}^3) + 2\theta_s\right] \quad (5.28)$$

s.t.

The first stage constraints: (5.2)~(5.12)

The second stage constraints: (5.22)~(5.23)

$$-\sum_{t\in T}(c_s^2 w_{ts}^2 + c_s^3 w_{ts}^3) + \sum_{s\in S}\sum_{t\in T} p_s(c_s^2 w_{ts}^2 + c_s^3 w_{ts}^3) - \theta_s \leq 0 \quad (5.29)$$

$$\theta_s \geq 0 \quad (5.30)$$

5.3.3.2 A robust mixed 0-1 integer optimization model with model robustness

A robust optimization model with model robustness for the uncertain logistics problems for global road transport can be formulated as:

$$\text{Min } FirstCost+SecondCost+\omega\sum_{s\in S}\sum_{t\in T} p_s\left|e_{ts}\right| \quad (5.31)$$

s.t.

The first stage constraints: (5.2)~(5.12)

The second stage constraints: (5.23), and

$$e_{ts} = d_{ts} + w_{ts}^2 - w_t^3 - w_{t-1,s}^2 - \sum_{k\in K}\sum_{j=\{1,3\}}\sum_{i\in I^0} X_{ijkt}^0 - \sum_{j=\{1,3\}}\sum_{i\in I^2} X_{ijt}^2,\ t\in T \quad (5.32)$$

The sum of the first term and the second term in objective function (5.31) is the objective function of the stochastic recourse programming model expressed in the equation

(5.26). The final term in (5.31) penalizes a norm of the infeasibility, weighted by parameter ω. The infeasibility of the initial second stage constraints is formulated in constraint (5.32). Based on the analysis in section 3.3.2, the above model can be expressed as a linear programming model by using the absolute term to denote the norm in (5.31) and introducing a deviational variable $\delta_{ts} \geq 0$. The robust optimization model with model robustness for the uncertain logistics problems can be formulated as:

$$\text{Min } FirstCost + SecondCost + \omega \sum_{s \in S} \sum_{t \in T} p_s [e_{ts} + 2\delta_{ts}] \tag{5.33}$$

s.t.

The first stage constraints: (5.2)~(5.12)

The second stage constraints: (5.23), (5.32) and

$$-e_{ts} - \delta_{ts} \leq 0 \tag{5.34}$$

$$\delta_{ts} \geq 0 \tag{5.35}$$

5.3.3.3 A robust mixed 0-1 integer optimization model with trade-off between solution robustness and model robustness

A robust optimization model with solution robustness and model robustness for the logistics problems under uncertainty can be formulated as follows:

$$\text{Min } FirstCost + SecondCost + \lambda \sum_{s \in S} p_s \left| (c_s^2 w_{ts}^2 + c_s^3 w_{ts}^3) - \sum_{s \in S} p_s (c_s^2 w_{ts}^2 + c_s^3 w_{ts}^3) \right| + \omega \sum_{s \in S} \sum_{t \in T} p_s |e_{ts}| \tag{5.36}$$

s.t.

The first stage constraints: (5.2)~(5.12)

The second stage constraints: (5.23) and (5.32)

Furthermore, the above model can be expressed as the following linear programming model by introducing the deviational variables $\theta_s \geq 0$ and $\delta_{ts} \geq 0$.

Min *FirstCost+SecondCost*

$$+\lambda\sum_{s\in S}p_s\left(\sum_{t\in T}(c_s^2w_{ts}^2+c_s^3w_{ts}^3)-\sum_{s\in S}\sum_{t\in T}p_s(c_s^2w_{ts}^2+c_s^3w_{ts}^3)+2\theta_s\right)+\omega\sum_{s\in S}\sum_{t\in T}p_s(e_{ts}+2\delta_{ts}) \quad (5.37)$$

s.t.

The first stage constraints: (5.2)~(5.12)

The second stage constraints: (5.23), (5.29), (5.30), (5.32), (5.34) and (5.35)

5.4 Computational results and analysis

5.4.1 Known and fixed parameters

All data that is used in this study is provided by a third-party logistics company. The company has two warehouses: one is located in Southern China, while the other is in Hong Kong's port terminal. Goods are usually stored at the Mainland China's warehouse. The logistics company is responsible for transporting these goods from the Mainland China's warehouse to the Hong Kong's warehouse from where the goods can be shipped to overseas markets. The logistics company under this study has three trucks (V1, V2 and V3). Each truck has a capacity of 250 units. The costs of a trip on Routes 1 and 3 are \$300 and \$200, respectively. There are 4 trucks (V4, V5, V6 and V7) with a China license that the company can rent. The capacity of each truck is 250, and the cost of hiring each truck is \$500. In addition, there are 2 trucks (V8 and V9) with China and Hong Kong licenses available for rental. The capacity of each of these trucks is 450. In addition, the cost of hiring the truck bears no relationship to its transportation route. The hiring cost for each truck for each round trip is \$1,500. Computational results for all following tests show that the hired trucks with two licenses will not operate on Route 3 between the border and the warehouse B. The round trip time for Routes 1, 2 and 3 are 10 hours, 3 hours and 5 hours, respectively. However, the drivers' maximum working time is 10 hours every working day. The unit inventory cost in the China warehouse is $1, and the unit inventory cost in the Hong Kong warehouse is $5. The unit trans-shipment cost is $0.5. The unit penalty cost for not satisfying demand is $12. We also assume that the two warehouses have enough space to store any goods left.

5.4.2 Computational result of the mixed 0-1 integer recourse programming model

5.4.2.1 Deterministic parameters

Three tests with various levels of required demand are analysed and shown in Table 5.1. Test I shows the situation when supply is equal to demand daily; Test II when supply is more than or equal to demand daily; and Test II when supply is less than or equal to demand daily. Table 5.2 summarizes the costs incurred for the three tests.

Table 5.1: Three test data of supply and demand

Test	Supply/Demand	Mon	Tue	Wed	Thu	Fri	Sat	Total
I	Supply	1600	2100	1800	2000	1500	1700	10700
	Demand	1600	2100	1800	2000	1500	1700	10700
II	Supply	2000	1800	2300	1600	2100	1500	11300
	Demand	1800	1700	2200	1500	1900	1400	10500
III	Supply	1700	1900	2000	1900	1800	2000	11300
	Demand	1700	2000	2050	1900	1900	2050	11600

Table 5.2: Summary of costs incurred in the three tests

Test	Transportation cost	Hiring cost	Trans-shipment cost	Surplus cost	Shortage cost	Total cost
I	5800	22000	925	500	1200	30425
II	5500	22000	1000	3500	0	32000
III	6000	24000	1450	450	3600	35500

5.4.2.2 Computational results

Table 5.3 gives the optimal solution of six days for Test I. From Table 5.3, we can know that although the demand from country B is equal to the supply in country A, the company does not have to transport all goods from country A to B to satisfy the demand in country B. For example, on Thursday, country B has a shortage of 100, but warehouse A in country A has an inventory of 100. The optimal solutions suggest that it is not necessary to hire additional trucks to deliver small amounts (only 100 units). The company would like to wait one or more days when more goods need to be transported from country A to B, even the inventory and shortage cost incur simultaneously.

Table 5.3: Test I results for the deterministic problems

Day	Company-owned trucks on Route 1	Hired trucks with two licenses on Route 1	Hired trucks with one license on Route 2	Company-owned trucks on Route 3	Surplus in warehouse A	Surplus in warehouse B	Shortage in country B
Mon	T1 (200) T2 (250) T3 (250)	T8 (450) T9 (450)					
Tue	T5 (250)	T8 (450) T9 (450)	T4 (200) T5 (250) T6 (250) T7 (250)	T1 (200) T1 (250) T3 (250) T3 (250)			
Wed	T2(250) T3 (250)	T8 (450) T9 (450)	T4 (150) T6 (250)	T1 (150) T1 (250)			
Thu	T1 (250) T2 (250)	T8 (450) T9 (450)	T4 (250) T7 (250)	T3 (250) T3 (250)	100		100
Fri	T1 (150) T2 (250) T3 (250)	T8 (450) T9 (450)			50	50	
Sat	T1 (250) T2 (250) T3 (250)	T8 (450) T9 (450)			100		

Test II represents the situation when the supply in country A is greater than the demand in country B. From Table 5.4, we can see that, on Monday and Tuesday, there are some goods are left in warehouse B in country B. This method fully utilizes the truck load, although the inventory cost in warehouse B is much higher than that in warehouse A. We can see that all trucks reach their maximum capacity during the whole week in Test II, except on Thursday.

Table 5.4: Test II results for the deterministic problems

Day	Company-owned trucks on Route 1	Hired trucks with two licenses on Route 1	Hired trucks with one license on Route 2	Company-owned trucks on Route 3	Surplus in warehouse A	Surplus in warehouse B	Shortage in country B
Mon	T2 (250) T3 (250)	T8 (450) T9 (450)	T4 (250) T7 (250)		100	100	
Tue	T1 (250) T2 (250) T3 (250)	T8 (450) T9 (450)			250	50	
Wed	T3 (250)	T8 (450) T9 (450)	T4 (200) T5 (250) T6 (250) T7 (250)		400		
Thu	T1 (100) T2 (250) T3 (250)	T8 (450) T9 (450)			500		
Fri	T1 (250) T2 (250)	T8 (450) T9 (450)	T5 (250) T7 (250)		700		
Sat	T2 (250) T3 (250)	T8 (450) T9 (450)			800		

Table 5.5 gives the optimal solution when the supply in country A is less than the demand in country B. However, there are still some goods left in the country A warehouse, even when there is a shortage from country B (See the results on Monday, Tuesday and Wednesday in Table 5.4). There is always a trade-off between transportation cost, inventory cost and shortage cost.

Table 5.5: Test III results for the deterministic problems

Day	Company-owned trucks on Route 1	Hired trucks with two licenses on Route 1	Hired trucks with one license on Route 2	Company-owned trucks on Route 3	Surplus in warehouse A	Surplus in warehouse B	Shortage in country B
Mon	T1 (250) T2 (250) T3 (250)	T8 (450) T9 (450)			50		50
Tue	T1 (250) T2 (250)	T8 (450) T9 (450)	T4 (250) T5 (250)	T3 (250) T3 (250)	50		100
Wed	T1 (250) T2 (250)	T8 (450) T9 (450)	T6 (250) T7 (250)	T3 (250) T3 (250)	150		150
Thu	T1 (250) T3 (250)	T8 (450) T9 (450)	T5 (250) T6 (250)	T2 (250) T2 (250)	150		
Fri	T1 (250) T3 (250)	T8 (450) T9 (450)	T4 (250) T5 (250)	T2(250) T2(250)	50		
Sat	T2 (250)	T8 (450) T9 (450)	T4 (200) T5 (250) T6 (250) T7 (250)	T1 (200) T1 (250) T3 (250) T3 (250)			

5.4.3 Computational results of the stochastic mixed 0-1 integer recourse programming model

5.4.3.1 Random parameters

This paper considers the shipment demand, unit surplus cost and shortage cost at the Hong Kong warehouse as random parameters, whose values depend on the future economic situation. As economic conditions are uncertain decision makers can only capture the realizations of future economic conditions. It is assumed that the future economic situation will fit into one of three possible situations – good, fair and bad – with associated probabilities. Let s_1 represent a good economy with probability p_1, p_1=Pr$\{s_1\}$; s_2 represents a fair economy with probability p_2, p_2=Pr$\{s_2\}$; and s_3 represents a bad economy with probability p_3, p_3=Pr$\{s_3\}$. In the Table 5.6 shows the unit surplus cost, unit shortage cost and demand for each scenario. Supply is known: 1000 on Monday, 1300 on Tuesday, 2000 on Wednesday, 1700 on Thursday, 1400 on Friday and 1500 on Saturday.

Table 5.6: The unit surplus cost, unit shortage cost and demand

Scenario	Unit surplus cost	Unit shortage cost	Demand					
			Mon	Tue	Wed	Thu	Fri	Sat
s_1	6	15	1100	1300	2100	1800	1500	1600
s_2	5	12	1000	1200	2000	1700	1400	1500
s_3	4	10	900	1100	1900	1600	1300	1400

5.4.3.2 Computational results

In this paper, we perform three different tests under different probabilities. Apart from the change in probability of occurrences of the future economic situation, other conditions in the three tests are the same. The test data are shown in Table 5.7. Test I represents the situation where it is most likely that the economy will perform well, Test II the situation where it is most likely that the economic performance will be fair, and Test III represents where it will be poor. The optimal solution and related costs are showed in Tables 5.8 and 5.9.

Table 5.7: Three tests

Test	p_1=Pr{s_1}	p_2=Pr{s_2}	p_3=Pr{s_3}
I	0.8	0.1	0.1
II	0.1	0.8	0.1
III	0.1	0.1	0.8

Table 5.8: The dual-response logistics plan

		The first stage decision					The second stage decision					
		Route 1		Route 2	Route 3		Surplus in HK			Shortage in HK		
Test	Day	Company -owned trucks	Hired trucks with two licenses	Hired trucks with one license	Company -owned trucks	Surplus in China	s_1	s_2	s_3	s_1	s_2	s_3
I	Mon	T1 (250) T3 (250)		T4 (250) T5 (250)	T2 (250) T2 (250)				100	100		
I	Tue	T1 (250) T2 (250) T3 (250)	T8 (450)						200	100		
I	Wed	T1 (250) T2 (250)	T8 (450) T9 (450)	T4 (250) T6 (250)	T3 (250) T3 (250)	100			200	200	100	
I	Thu	T1 (250) T2 (250) T3 (250)	T8 (450) T9 (450)			150			250	150	50	
I	Fri	T2 (200) T3 (250)	T9 (450)	T4 (250) T5 (250)	T1 (250) T1 (250)	100		50	400	50		
I	Sat	T1 (100) T2 (250) T3 (250)	T8 (450) T9 (450)					150	600			
II	Mon	T2 (250) T3 (250)		T4 (250) T6 (250)	T1 (250) T1 (250)				100	100		
II	Tue	T1 (150) T2 (250) T3 (250)	T8 (450)						200	100		
II	Wed	T2 (250)	T8 (450) T9 (450)	T4 (250) T5 (100) T6 (250) T7 (250)	T1 (100) T1 (250) T3 (250) T3 (250)				300	100		
II	Thu	T1 (250) T2 (250) T3 (250)	T8 (450) T9 (450)			50			350	150	50	
II	Fri	T2 (200) T3 (250)	T8 (450) T9 (450)			50			450	100		
II	Sat	T1 (100) T2 (250) T3 (250)	T8 (450) T9 (450)			50			550	100		
III	Mon	T1 (200) T2 (250)	T9 (450)			100				200		
III	Tue	T1 (200) T2 (250) T3 (250)	T9 (450)			150			50	150	100	
III	Wed	T1 (200) T2 (250)	T8 (450) T9 (450)	T4 (250) T7 (250)	T3 (250) T3 (250)	250			50	200	50	
III	Thu	T1 (250) T2 (250) T3 (250)	T8 (450) T9 (450)			300			100	150	100	
III	Fri	T1 (250) T2 (250) T3 (250)	T9 (450)			500				300	50	
III	Sat	T2 (250) T3 (250)	T8 (450) T9 (450)			600				200	200	

Table 5.9: Summary of costs incurred in the dual-response logistics planning process

Test	The first stage cost				The second stage cost		Total cost
	Transportation cost	Hiring cost	Trans-shipment cost	Surplus cost in China	Surplus cost in HK	Shortage cost in HK	
I	5700	15000	750	350	800	7380	29980
II	5400	16500	675	150	780	1455	24960
III	4900	14500	250	1900	640	2520	24710

5.4.3.3 Comparison of the expected value model and stochastic recourse model

The stochastic problem has a related problem, namely, the expected value problem. This arises when all uncertain parameters are replaced by their expected values. Table 5.10 shows the expected value of uncertain demand, unit surplus cost and unit shortage cost.

Table 5.10: The expected value of unit surplus cost, unit shortage cost and demand

Test	Unit surplus cost	Unit shortage cost	Demand					
			Mon	Tue	Wed	Thu	Fri	Sat
I	5.7	14.2	1070	1170	2070	1770	1470	1570
II	5	12.1	1000	1200	2000	1700	1400	1500
III	4.3	10.7	930	1130	1930	1630	1330	1430

The expected value model is a mixed 0-1 integer programming model for deterministic logistics problems presented in Section 5.3.1. Using the input data shown in Table 5.10, the above model can be solved, and logistics plan obtained. The results are shown in Table 5.11.

Table 5.11: The logistic plan for the expected value problem

Test	Day	Route 1		Route 2	Route 3	Surplus in China	Surplus in HK	Shortage in HK
		Company-owned trucks	Hired trucks with two licenses	Hired trucks with one license	Company-owned trucks			
I	Mon	T2 (250) T3 (250)		T4 (250) T5 (250)	T3 (250) T3 (250)			70
	Tue	T1 (250) T2 (250) T3 (250)	T9 (450)				30	

	Wed	T2 (250) T3 (250)	T8 (450) T9 (450)	T6 (250) T7 (250)	T1 (250) T1 (250)	100		140
	Thu	T1 (250) T2 (250) T3 (250)	T8 (450) T9 (450)			150		120
	Fri	T1 (70) T2 (200) T3 (250)	T8 (450) T9 (450)			80		
	Sat	T1 (170) T2 (250) T3 (250)	T8 (450) T9 (450)			10		
II	Mon	T2 (250) T3 (250)		T4 (250) T5 (250)	T1 (250) T1 (250)			
	Tue	T1 (150) T2 (250) T3 (250)	T8 (450)					
	Wed	T2 (100)	T8 (450) T9 (450)	T4 (250) T5 (250) T6 (250) T7 (250)	T1 (100) T1 (250) T3 (250) T3 (250)			
	Thu	T1 (250) T2 (250) T3 (250)	T8 (450) T9 (450)			50		50
	Fri	T2 (200) T3 (250)	T8 (450) T9 (450)			50		
	Sat	T1 (100) T2 (250) T3 (250)	T8 (450) T9 (450)			50		
III	Mon	T1 (230) T2 (250)	T9 (450)			70		
	Tue	T1 (210) T2 (250) T3 (250)	T9 (450)			110	30	
	Wed	T1 (250) T2 (250)	T8 (450) T9 (450)	T5 (250) T7 (250)	T3 (250) T3 (250)	210		
	Thu	T1 (230) T2 (250) T3 (250)	T8 (450) T9 (450)			280		
	Fri	T1 (160) T2 (250) T3 (250)	T9 (450)			120	230	
	Sat	T1 (250) T2 (250) T3 (250)	T8 (450) T9 (450)			420		

When the uncertainty is realized, the actual situation is that either Scenario 1 happens; or Scenario 2 happens; or Scenario 3 happens (see Table 5.6). Based on the solution of the expected problem, the company has to determine a response for each scenario. Let *EV* represent the objective function value of the expected value model. Therefore, the total cost will not be *EV*. Let *EEV* represent the expected results of using the solution of the expected value problem. *EEV* can be obtained by solving the stochastic recourse model, in which the first stage decisions are made by the expected value model. Let *ESS* represent the optimal solution of the stochastic recourse model, which is presented in Section 5.3.2. We know that *EEV* is only one of the solutions for the stochastic recourse model, but *ESS* is the best solution. Letting *VSS* represent the value of the stochastic solution (*VSS=EEV-ESS*), we have the following inequality: *VSS* ≥ 0 (see Property 1 in Chapter 3). The comparative results for the stochastic recourse model and expected value model are shown in Table 5.12.

Table 5.12: Comparison between the expected value model and stochastic recourse model

Test	*EV*	*EEV*	*ESS*	*VSS*
I	28046	32065	29980	2085
II	23330	27423	24960	2463
III	22460	27062	24710	2352

From Table 5.12, it can be seen that the expected value model solution can have unfavourable consequences because of the higher costs incurred, compared to costs incurred when using the stochastic recourse model. In Test I, decision makers will pay $2,085 more in terms of the logistic plan determined by the expected value model, than the stochastic recourse model (see the *VSS* values in the last column in Table 5.12). The total cost of Test I will decrease by $2,085, from $32,065 to $29,980, if we choose the stochastic recourse model, rather than the expected value model. It means the company could save $2,085 by using the stochastic recourse model. In Test II, the total cost will decrease by $2,463, from $27,423 to $24,960. In Test III, the total cost will decrease by $2,352, from $28,062 to $24,710.

The three tests show that the stochastic recourse model improves the performance in Tests II and III more significantly than in Test I. Test I represents the situation where it is most likely that demand will be high. If the anticipated situation does not happen, there will be a certain amount of surplus inventory and quotas. In Tests II and III, if the unanticipated situation (high demand) happens (with the possibility of 10%), there will be a certain amount of shortage (of products and quotas). The unit surplus cost is lower than the unit shortage cost. The expected value model has limited ability to handle unanticipated situations, which may result in very high costs. This is particularly true in Tests II and III, when the unanticipated situation (high demand) is realized. We can conclude that it is more beneficial to use the recourse model in Tests II and III than in Test I.

5.4.4 Computational results of the robust optimization model with trade-off between solution robustness and model robustness

5.4.4.1 Computational results

All input is the same as the data given in section 5.4.3. Tables 5.13 and 5.14 give computational results of the robust optimization model with trade-off between solution robustness and model robustness for Test II by setting up different values of λ and ω.

Table 5.13: The logistics plan for Test II under different λ and ω

(λ, ω)	Day	The first stage decision					The second stage decision					
		Route 1		Route 2	Route 3		Surplus in HK			Shortage in HK		
		Company -owned trucks	Hired trucks with two licenses	Hired trucks with one license	Company -owned trucks	Surplus in China	s_1	s_2	s_3	s_1	s_2	s_3
(0.1,20)	Mon	T2 (250) T3 (250)		T4 (250) T6 (250)	T1 (250) T1 (250)					100		
	Tue	T1 (250) T2 (250) T3 (250)	T8 (450)							100		
	Wed	T2 (250)	T8 (450) T9 (450)	T4 (100) T5 (250) T6 (250) T7 (250)	T1 (100) T1 (250) T3 (250) T3 (250)				100	100		
	Thu	T1 (250) T2 (250) T3 (250)	T8 (450) T9 (450)			50			150	150	50	
	Fri	T2 (200) T3 (250)	T8 (450) T9 (450)			50			250	100		
	Sat	T1 (100) T2 (250) T3 (250)	T8 (450) T9 (450)			50			350	100		
(0.5, 10)	Mon	T2 (250) T3 (250)		T4 (250) T6 (100)	T1 (250) T1 (250)							
	Tue	T1 (250) T2 (250) T3 (250)	T8 (450)									
	Wed	T1 (200) T2 (250)	T8 (450) T9 (450)	T4 (250) T5 (250)	T3 (250) T3 (250)	100					5	
	Thu	T1 (250) T2 (250) T3 (250)	T8 (450) T9 (450)			150						
	Fri	T2 (200) T3 (250)	T8 (450) T9 (450)			150						

	Sat	T1 (100) T2 (250) T3 (250)	T8 (450) T9 (450)			150			100			
	Mon	T1 (200) T2 (250)	T9 (450)	T4 (250) T6 (250)	T3 (250) T3 (250)				100	100		
	Tue	T1 (250) T2 (250) T3 (250)	T9 (450)						200	100		
	Wed	T2 (250) T3 (250)	T8 (450) T9 (450)	T5 (250) T6 (250)	T1 (250) T1 (250)	100			200	200	100	
(0.9,45)	Thu	T1 (250) T2 (250) T3 (250)	T8 (450) T9 (450)			150			250	150	50	
	Fri	T2 (250) T3 (250)	T9 (450)			150			350	100		
	Sat	T1 (100) T2 (250) T3 (250)	T8 (450) T9 (450)			150			450	100		

Table 5.14: Summary of costs for Test II

(λ, ω)	Expected variability	Expected infeasibility	Expected cost	Expected variability cost	Expected infeasibility cost	Total cost
(0.1, 20)	2121	20	24520	212	400	25132
(0.5, 10)	18	226	21942	9	2257	24208
(0.9, 45)	2083	0	25035	1875	0	26910

5.4.4.2 Comparison between the stochastic recourse model and robust optimization model

Table 5.15 gives the computational results of the robust optimization model and the two-stage recourse programming model for Test II. The total cost under the recourse model is $24,960 (See the second row in Table 5.15) and the total cost under the robust model is $24,208 (See the third row in Table 5.15 when λ=0.5 and ω=10). Using the robust optimization model by setting λ=0.1 and ω=10, the total cost decreases by 3.01% and the expected variability of the robust model decreases 99.26%, which means the robust model presents a less sensitive logistic plan. However, the robust model involves the infeasibility cost of $2,257 for not satisfying all shipment requirements. If we increase the weighting penalty of ω to 45 (See the last row in Table 5.15), no constraint is violated. Compare this with the stochastic recourse model, the variability decreases 14.53%, the expected cost (the fourth column in Table 5.15) and total cost (the last column in Table 5.15) of the robust model only increases by 0.30% and 4.48%, respectively. It means that the logistics plan proposed by the robust model is not expensive, and it reduces the risk.

Table 5.15: Comparison between the stochastic recourse model and robust optimization model

Model	Expected variability	Expected infeasibility	Expected cost	Expected variability cost	Expected infeasibility cost	Total cost
Recourse model	2437	0	24960	0	0	24960
Robust model (λ=0.5, ω=10)	18	226	21942	9	2257	24208
Robust model (λ=0.5, ω=20)	115	81	23608	57	2020	25685
Robust model (λ=0.5, ω=45)	2083	0	25035	1042	0	26077

5.4.5 Further tests for the robust optimization models

We perform three different tests, described in Table 5.16 in section 5.4.3

5.4.5.1 Tests for the robust mixed 0-1 integer optimization model with solution robustness

We perform four tests for a weekly plan when λ=0, 0.1, 0.5 and 0.9 for Test I, II and III. Table 5.16 gives the related costs. In a weekly logistics plan, when the value of λ increases from 0 to 0.9, the variability decreases by 60.55% in Test I, 15.78% in Test II and 6.44% in Test III, respectively. The total cost increases by 12.23% in Test I, 7.81% in Test II and 19.53% in Test III, respectively.

Table 5.16: Costs incurred in the robust model with solution robustness under different λ in three tests

Test	λ	Expected variability	First-stage cost	Second-stage cost	Expected cost	Expected variability cost	Total cost
I	0	4917	21800	8180	29980	0	29980
	0.1	4917	21800	8180	29980	492	30472
	0.5	4917	21800	8180	29980	2459	32439
	0.9	2155	21800	9906	31706	1940	33646
II	0	2473	22725	2235	24960	0	24960
	0.1	2473	22725	2235	24960	247	25207
	0.5	2083	21850	3185	25035	1042	26077
	0.9	2083	21850	3185	25035	1875	26910

III	0	5436	21550	3160	24710	0	24710
	0.1	5436	21550	3160	24710	544	25254
	0.5	5436	21550	3160	24710	2718	27428
	0.9	5086	21750	3210	24960	4577	29537

5.4.5.2 Tests for the robust mixed 0-1 integer optimization model with model robustness

Tables 5.17 show the related costs when ω=0, 5, 10 and 15 in the three tests. When ω=0, there is no delivery in the whole planning horizon because there is no penalty for not satisfying the demand.

Table 5.17: Costs incurred in the robust model with model robustness under different ω

Test	ω	Expected infeasibility	First-stage cost	Second-stage cost	Expected cost	Expected infeasibility cost	Total cost
I	0	3910	17400	0	17400	0	17400
	5	540	21800	130	21930	2700	24630
	10	520	21800	300	22100	5200	27300
	15	500	21800	540	22340	7500	29840
	20	10	21800	7940	29740	200	29940
	25	0	21800	8180	29980	0	29980
II	0	3490	17400	0	17400	0	17400
	5	230	21850	40	21890	1150	23040
	10	220	21850	120	21970	2200	24170
	15	95	22725	660	23385	1425	24810
	20	10	22725	1995	24720	200	24920
	25	0	22725	2235	24960	0	24960
III	0	3350	17400	0	17400	0	17400
	5	520	20000	160	20160	2600	22760
	10	180	21550	640	22190	1800	23990
	15	120	21550	1360	22910	1800	24710
	20	0	21550	3160	24710	0	24710

5.4.5.3 Tests for the robust mixed 0-1 integer optimization model with trade-off between solution robustness and model robustness

Table 5.18 shows the summary of costs incurred of the robust optimization model with a trade-off between solution robustness and model robustness.

Table 5.18: Trade-off between solution robustness and model robustness under different λ and ω

Test	λ	ω	Expected variability	Expected infeasibility	Expected cost	Expected variability cost	Expected infeasibility cost	Total cost
I	0.1	0	0	3910	1700	0	0	17400

		5	80	550	21880	8	550	24638
		10	250	525	22050	25	5250	27325
		15	255	452	23060	26	6780	29866
		20	5061	10	29740	506	200	30446
		25	4917	0	29980	492	0	30472
	0.5	0	0	3910	17400	0	0	17400
		5	0	570	21800	0	2850	24650
		10	126	539	21940	63	5392	27395
		15	101	471	22806	50	7062	29918
		20	413	375	24350	207	7507	32063
		25	4917	0	29980	2459	0	32439
	0.9	0	0	3910	17400	0	0	17400
		5	0	570	21800	0	2850	24650
		10	36	524	22163	33	5240	27463
		15	101	471	22806	91	7062	29958
		20	0	389	24350	0	7776	32126
		25	2155	0	31707	1940	0	33646
II	0.1	0	0	3490	17400	0	0	17400
		5	40	230	21890	4	1150	23044
		10	120	220	21970	12	2200	24182
		15	85	83	23575	9	1243	24827
		20	2121	20	24520	212	400	25132
		25	2281	10	24720	228	250	25198
		30	2473	0	24960	247	0	25207
	0.5	0	0	3490	17400	0	0	17400
		5	0	240	2185	0	1200	23050
		10	18	226	21942	9	2257	24208
		15	85	83	23575	43	1243	24861
		20	85	83	23575	43	1657	25275
		25	115	81	23608	57	2020	25685
		30	1022	36	24400	511	1090	26001
		35	2083	0	25035	1042	0	26077
	0.9	0	0	3490	17400	0	0	17400
		5	0	240	21850	0	1200	23050
		10	11	224	21960	10	2243	24213
		15	85	83	23575	77	1243	24895
		20	85	83	23575	77	1657	25309
		25	85	83	23575	77	2072	25723
		30	115	81	23608	103	2424	26135
		35	686	42	24400	617	1482	26499
		40	686	42	24400	617	1693	26711
		45	2083	0	25035	1875	0	26910
III	0.1	0	0	3350	17400	0	0	17400
		5	160	520	20160	16	2600	22776
		10	640	180	22190	64	1800	24054
		15	2045	113	23015	205	11695	24915
		20	5436	0	24710	534	0	25254
	0.5	0	0	3350	17400	0	0	17400
		5	160	520	20160	80	2600	22840
		10	61	185	22382	31	1852	24264
		15	597	172	22296	298	2581	25175
		20	597	172	22286	298	3441	26035
		25	2045	113	23015	1023	2826	26863
		30	2045	113	23015	1023	3391	27428
		35	5436	0	24710	2718	0	27428
	0.9	0	0	3350	17400	0	0	17400
		5	160	520	20160	144	2600	22904
		10	0	182	22443	0	1824	24267
		15	0	182	22443	0	2736	25179
		20	0	182	22443	0	3648	26091
		25	0	182	22443	0	4560	27003
		30	0	182	22443	0	5472	27915
		35	1555	118	23243	1400	4134	28777
		40	1535	107	23658	1382	4284	29325
		45	5086	0	24860	4577	0	29537

From Table 5.15, we arrive at the following conclusion: there is always a trade-off between the variability and infeasibility. The role of weight ω and λ in the robust optimization model objective function is used to measure the trade-off between model robustness ("almost" feasible for any realization of all scenarios) and solution robustness ("close" to optimal for any realization of all scenarios). Robust optimization allows for the infeasibility in the random constraints by means of penalties. When $\omega = 0$, there is no penalty for the infeasibility of random constraints in the objective function. The infeasibility that represents under-fulfilment attains a higher value. Clearly, decision makers do not adopt this kind of production plan. However, a large weight ω shows that the infeasibility penalty dominates the total objective function value and results in a higher variability and a higher total cost. This is an inappropriate approach for those decision makers who are risky and prefer to pay less. Therefore, there is always a trade-off between the risk and cost. For the decision makers, it is necessary to test the proposed robust optimization with various ω and λ on the global logistics problems.

When λ is a constant

Figures 5.2~5.4 denote the computational results for Test II (see Table 5.7) in terms of the expected variability, expected infeasibility and total cost, when λ=0.1, 0.5 and 0.9. Figure 5.2 shows the trend in variability when ω increases. As weight ω increases, variability increases. In particular, when weight ω is more than 15, variability increases dramatically. After weight ω reaches 45, variability does not change. On the other hand, as weight ω increases, the total under-fulfilment denoted by infeasibility drops dramatically (see Figure 5.3). When weight ω is greater than or equal to 45, infeasibility is equal to zero. This means there is no under-fulfilment; all constraints can be satisfied for any scenario. Figure 5.4 shows the trend in total cost.

Figure 5.2: Variability when λ keeps constant

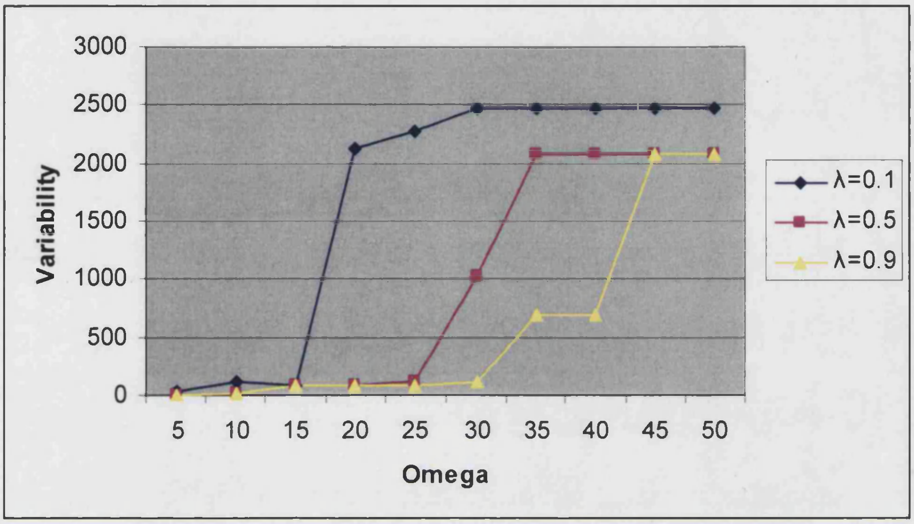


Figure 5.3: Infeasibility when λ keeps constant

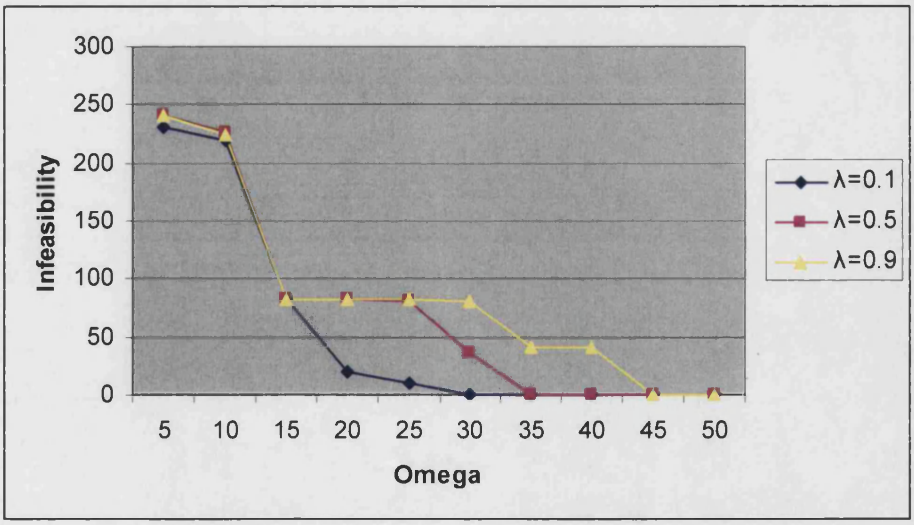
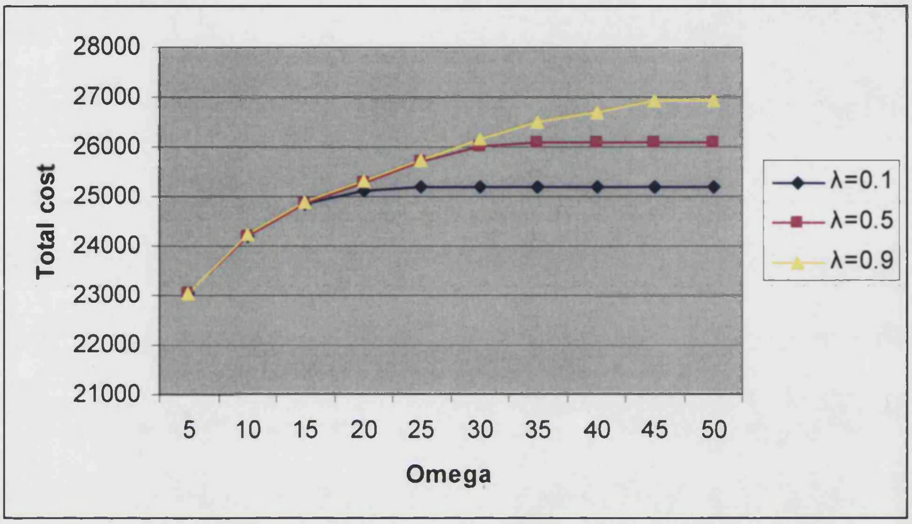


Figure 5.4: Total cost λ when keeps constant



When ω is a constant

Figures 5.5~5.7 denote the computational results for Test II in terms of expected variability, expected infeasibility and total cost, when ω increases for λ=0.1, 0.5 and 0.9.

Figure 5.5 shows the trend of variability when λ increases for ω=5, 15, 25, 35 and 45. If λ increases from 0.1 to 0.9, for ω=5, variability decreases from \$40 to \$0; for ω=25, variability decreases from \$2281 to \$85; for ω=35, variability decreases from \$2,437 to \$2,083.

Figure 5.5: Variability when ω keeps constant

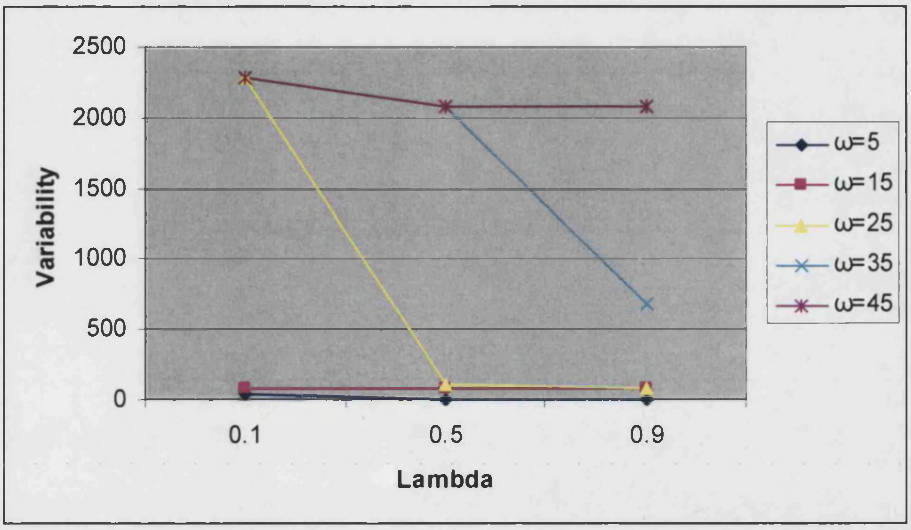


Figure 5.6 shows the trend of infeasibility when λ increases for ω =5, 15, 25 and 35. The greater the value of ω, the less the value of λ impacts the infeasibility.

Figure 5.6: Infeasibility when ω keeps constant

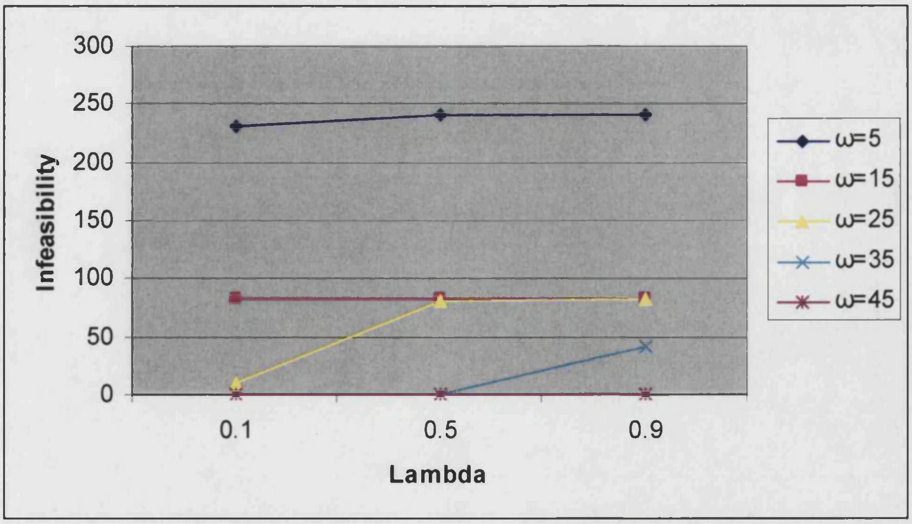
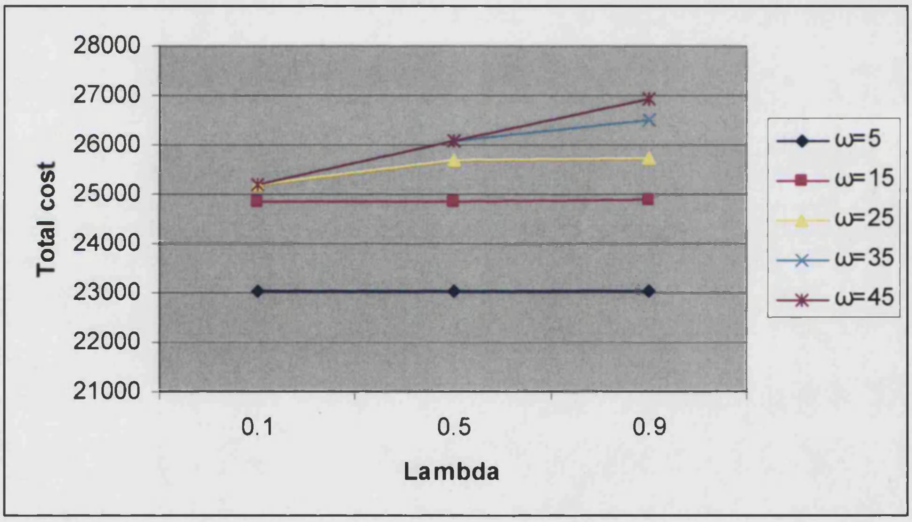


Figure 5.7 shows the trend of the total cost when λ increases for ω =5, 15, 25, 35 and 45. If λ increases from 0.1 to 0.9, for ω=5, the total cost increases by 0.026%; for ω =15, the total cost increases by 0.274%; for ω =25, the total cost increases by 2.083%; for ω=35, the total cost increases by 5.163%; and for ω=45, the total cost increases by 6.834%. Compared with changes in variability and infeasibility, the total cost increases by only a small amount when λ increases. This means that the robust model proposed in this study is not expensive for a low risk dual-response logistics system.

Figure 5.7: Total cost when ω keeps constant



5.5 Summary

Today's business has inevitably been set in the global supply chain management environment. Global logistics, therefore, have never played such an important role in the global supply chain network, because movement of goods, particularly from one country to another, tends to be more frequent than ever before. This chapter examines the global logistics problems experienced by a logistics company that is responsible for transporting goods from one country to another by road, as well as warehousing them in two countries. We first develop a mixed 0-1 integer programming model to determine the optimal logistics strategy, which assumes that shipment information is available by the time of decision making. The computational results, which are based on data from the company, present the logistics strategy in terms of fleet composition (the number of company-owned trucks, hired trucks with one country's license and hired trucks with two countries' licenses), cross-border routes, and inventory levels in two warehouses located in two different countries.

In practice, accurate shipment information can not be obtained until the shipping day. Logistics managers have to book trucks they intend to hire in advance. The decision making process about global logistics strategies involves uncertainty. We propose a dual-response logistics strategy to copy with the short shipment notice time, and the uncertainty involved. In the first stage, when accurate shipment information is not available, logistics managers need to determine cross-border transportation plans for company-owned and hired trucks. In the second stage, when the uncertainty is realized, the company needs to determine the inventory level and shortage level in the demand country. In order to achieve the dual-response logistics strategy, a two stage stochastic mixed 0-1 integer recourse model is developed; the computational results demonstrate how the recourse model can provide the dual-response logistics strategy to handle uncertainty during the decision-making process. A series of experiments show that the recourse model has favourable consequences because of the lower level of costs incurred, compared to those incurred when using the corresponding expected value model, in which all stochastic parameters are replaced by their expected values. Computational results from the data provided by the company also show that it is more beneficial to use the stochastic recourse model in some logistics scenarios than others.

As the stochastic recourse model is unable to handle the risk, we propose three types of robust optimization models for global road transportation problems. The first type of model is the robust mixed 0-1 integer optimization model with solution robustness, which can provide a solution that is less sensitive to realization of uncertainty. The second type of model is the robust mixed 0-1 integer optimization model with model robustness, which provides an approach to handle infeasibility arising from stochastic constraints. The third type of model is the robust mixed 0-1 optimization model with trade-off between solution robustness and model robustness, which provides a direct way to measure the trade-off between risk and cost during the global transportation process. The three models can be applied to different decision-making scenarios of global road transport problems under uncertainty, to deal with the risk issues. If decision-makers prefer a logistics plan less sensitive to realization of uncertainty, they can choose the robust mixed 0-1 integer optimization model with solution robustness. By adjusting the value of λ, they could obtain the trade-off between cost and risk, which is characterised by variability. Additionally, the computational results show that the robust model with solution robustness carries less risk than the two-stage stochastic recourse model, and the cost of reducing the risk is low. Furthermore, it is more beneficial to use the robust model with solution robustness in global logistics problems with high levels of risk.

However, if the decision-makers prefer a trade-off between cost and infeasibility, they can use the robust model with model robustness. By adjusting the value of ω, they could obtain the trade-off between cost and risk, which is characterised by infeasibility. Finally, the robust mixed 0-1 integer optimization model with solution robustness and model robustness provides a direct way to measure the trade off between solution robustness and model robustness. A series of experiments show the impact of λ and ω on the logistics system's performance in terms of variability, infeasibility and total cost. The decision-makers can choose their preferred logistics strategy, based on their attitude toward risk, by adjusting the value of λ and ω.

Finally, it should be noted that computation and analysis of the models may lead to different outcomes if the values of model parameters change.

Chapter 6

Container loading problems for global air transport

6.1 Introduction

6.1.1 Container loading process

Today's air transport is exerting an ever increasing impact on transportation, particularly global air transport, as compared with only a few years ago. Although the average shipment size is still limited by today's aircraft, the nature of air cargoes, mostly high-value and low-density items, has caused the total value of air carges to comprise a greater portion of total global cargoes. The tremendous speed of aircraft and high frequency of scheduled flights to the majority of cities in the world has reduced transit time from as many as 50 days to one or two days. In today's global competitive environment, business success increasingly replies on speed. With easy and instant access to the Internet, the inexpensive launch of B2B or B2C businesses, and advancements in information technology, products and services can be manufactured and sold anywhere in the world where feasible. Because product and service information is available on a real-time basis and comparisons can quickly be made, customers are increasingly empowered to have more complicated requirements and tend to have a low tolerance to poor quality either in products or in services. They demand a quick response and speedy delivery while continuously lowering costs. Supplying a market ahead of competitors can provide a competitive advantage by offering remarkable flexibility to the dynamic and changing demand. Time is extremely important for certain industries, like the PC and apparel industries. The time saved by using air freight can leave manufacturers and transporters a margin to beat product variety, short lead time and life cycles, and uncertain demand. Additionally, air transport offers substantial savings for its customers through low insurance, cheap labour costs for packing, loading and unloading, dramatically decreasing the costs of warehousing and inventory, less capital needing to be invested in large shipments by sea and faster capital turnover.

Containerization is an approach to cost-effectively and efficiently organize shipments. It changes shipment handling from a labour-intensive to a capital- and time- intensive operation, which is particularly true for containerizing air cargoes for global transport because of their higher freight rates. In this study, airlines offer different types of containers for rent. Each type of container has its weight and volume limits for holding cargoes, and each type of cargo has its own weight and volume. Each cargo must be packed into a single container. Breaking a cargo into different containers is not allowed. Typically, the forwarders book containers from the airline one week before shipment. The airlines give different rental prices when booking different types of containers. The cost of renting a container is based on a fixed cost and a variable cost that depends on the weight that the container holds. Therefore, the cost of renting a container is not a linear function but a piece-wise function.

If cargo shipping information is accurately obtained when booking, the forwarder can book containers that will be used next week aiming at minimizing the total rental cost. The decisions about booking include what quantities and types of containers are needed and how cargoes are loaded into containers. In this situation, a deterministic program can be applied to solve the container loading problems under certain cargo shipping information. Section 6.3.1 presents a mixed 0-1 integer programming model for the deterministic container loading problems.

6.1.2 A dual-response container loading strategy for global air transport under uncertainty

If accurate cargo information is not available when booking, the forwarders have to book containers in advance in order to get a low rental price. As airlines discourage urgent requirements for containers, they impose a heavy penalty for renting/returning containers on the shipping day. If all cargoes have to be loaded on the shipping day, the booked containers may not meet all container needs on the shipping day. In this situation, additional containers are required: but this comes at a high penalty cost. On the other hand, if too many containers are booked, the unused containers have to be returned to the airlines: in this case a penalty is incurred because of the forwarders breaking a contract.

In this study, we develop a dual-response container loading strategy. In the first stage, the forwarders have to make a response based on the inaccurate information by determining the booking quantities and types of containers. In the second stage, the forwarders have to make responses for different situations that might happen on the shipping day by determining the required or returned containers and loading all cargoes into containers. Under uncertain information and a no-delay policy, a two-stage stochastic mixed 0-1 integer programming technique can be applied to solve the uncertain cargo forwarding problems. Section 6.3.2 presents a two-stage stochastic mixed 0-1 integer programming model for the uncertain container loading problems.

6.1.3 Risk

The deterministic model and stochastic recourse model above share a common assumption: that all cargoes available on the shipping day have to be loaded into containers without delay. This assumption means that the forwarder has to change the types or/and quantity of booked containers on the shipping day at a high price if more or fewer cargoes appear. If a container only holds a small weight, the container is not fully utilized. This means the container is rented at a relatively high cost. In general, the larger the weight, the lower the unit rate charged by the airline. In particular, urgently renting a container on the shipping day will result in a high penalty. It is assumed that not all cargoes have to be shipped on the shipping day. However, the unshipped cargoes will incur a penalty. If the penalty for the delay is not too high, the decision makers could choose to deliver some cargoes on the following days. In this situation, a robust optimization with model robustness can be applied to solve the uncertain cargo forwarding problem, which provides a way of measuring the trade-off between risk and cost. Section 6.3.3 presents the robust optimization model with model robustness, which allows the violation of the random constraint by the least amount. In the container loading problems under uncertainty, we also present the robust optimization model with solution robustness and the robust optimization model with trade-off between solution robustness and model robustness, which proposes a straightway to measure risk and cost.

6.1.4 Overview of chapter 6

The rest of the chapter is organized as follows. Section 6.2 presents the notation and definitions. Section 6.3 presents three types of models, including a mixed 0-1 integer programming model under the assumption that all parameters are known with certainty, a two-stage stochastic mixed 0-1 integer programming model under uncertainty, and robust optimization models to handle uncertainty and risk. Section 6.4 gives the computational results and analysis for all the models. The final section gives the conclusions on the container loading problems for global air transport.

6.2 Notation and definitions

In formulating container loading problems for global air transport, the following notation and definitions are used.

6.2.1 Indices

i for container types (i=1,...,m);

l the l^{th} container (l=1,...,L_i);

j for cargo types (j=1,...,n);

t for time periods (t=1,...,T);

6.2.2 Parameters

Containers

V_i volume limit of container type i

W_j weight limit of container type i

c_i^0 fixed cost of renting a container of type i

Cargoes

v_j volume of cargo type j

w_j weight of cargo type j

q_j quantity of cargo type j

6.2.3 Decision variables

$$x_{il} = \begin{cases} 1 & \text{if the } l^{\text{th}} \text{ container of type } i \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$$

y_{ilj} = quantities of cargo type j loaded into the l^{th} container of type i

6.2.4 Constraints

Container volume constraints

Constraint (6.1) ensures that the volume of all cargoes allocated to a container cannot exceed the container's volume limits.

$$\sum_{j=1}^{n} v_j y_{ilj} \le V_i x_{il}, \; i=1,\ldots,m, \; l=1,\ldots,L_i \tag{6.1}$$

Container weight constraints

Constraint (6.2) ensures that the weight of all cargoes allocated to a container cannot exceed the container's weight limits.

$$\sum_{j=1}^{n} w_j y_{ilj} \le W_i x_{il}, \; i=1,\ldots,m, \; l=1,\ldots,L_i \tag{6.2}$$

Cargo quantity constraints

Constraint (6.3) requires all cargoes to be loaded into the containers without any delay.

$$\sum_{i=1}^{m} \sum_{l=1}^{L_i} y_{ilj} = q_j, \; j=1,\ldots,n \tag{6.3}$$

Variable type contraints

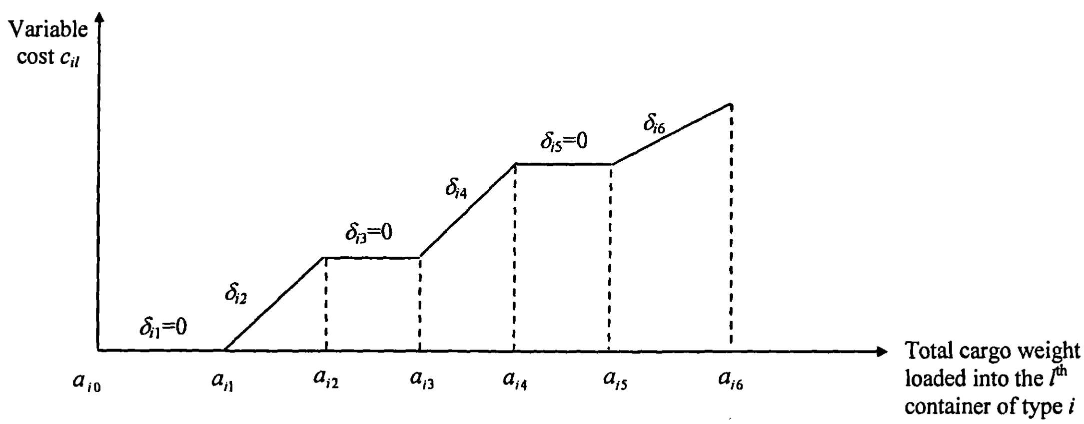
$$x_{il} = \{0,1\} \; i=1,\ldots,m, \; l=1,\ldots,L_i \tag{6.4}$$

y_{ilj} is an non-negative integer, $i=1,\ldots, m$; $l=1,\ldots,L_i, j=1,\ldots,n$ (6.5)

6.2.5 Cost

Whenever a container is rented, it causes a fixed cost c_i^0. Once the cargo loaded into the container exceeds a permitted weight limit, a variable cost c_{il} will be incurred, and this is associated with the weight of cargoes loaded into the container. Figure 6.1 shows the relationship between the weight and the variable cost.

Figure 6.1: Variable cost of renting the l^{th} container of type i



In Figure 6.1, a_{ik} represents the break point for container type i, where i=1,..,m, k=1,..., K_i, where K_i is the maximum number of break points. In this study, the air carriers provide six cost break points: $a_{i1}, a_{i2}, a_{i3}, a_{i4}, a_{i5}$, and a_{i6}. Let a_{i0} be the initial point, i.e. a_{i0} =0. Thus, a_{i1} is the first cost break point for the variable cost, and a_{i6} is the maximum weight limit of container type i. The definition of the variable cost c_{il} can be formulated as follows:

$$c_{il} = \begin{cases} 0 & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i0}, a_{i1}] \\ \delta_{i2}(\sum_{j=1}^{n} w_j y_{ilj} - a_{i1}) & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i1}, a_{i2}] \\ \delta_{i2}(a_{i2} - a_{i1}) & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i2}, a_{i3}] \\ \delta_{i2}(a_{i2} - a_{i1}) + \delta_{i4}(\sum_{j=1}^{n} w_j y_{ilj} - a_{i3}) & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i3}, a_{i4}] \\ \delta_{i2}(a_{i2} - a_{i1}) + \delta_{i4}(a_{i4} - a_{i3}) & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i4}, a_{i5}] \\ \delta_{i2}(a_{i2} - a_{i1}) + \delta_{i4}(a_{i4} - a_{i3}) + \delta_{i6}(\sum_{j=1}^{n} w_j y_{ilj} - a_{i5}) & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i5}, a_{i6}] \end{cases} \tag{6.6}$$

where $i = 1,2,\ldots,m$, $l=1,2,\ldots, L_i$

6.3 Model formulations

6.3.1 A mixed 0-1 integer programming model for the deterministic container loading problems

The objective is to satisfying all constraints described in Section 6.2.4 while minimizing the total renting cost consisting of the fixed cost and variable cost. Thus, the deterministic container loading problems for global air transport can be formulated as follow:

$$\text{Min} \ \sum_{i=1}^{m}\sum_{l=1}^{L_i} c_i^0 x_{il} + \sum_{i=1}^{m}\sum_{l=1}^{L_i} c_{il} \tag{6.7}$$

s.t.

Constraints: (6.1)~(6.5)

The objective function expressed in (6.7) is a piecewise function, and it is difficult to solve this kind of model by employing optimization software packages. Thus, two variables are introduced to transform the model into a mixed 0-1 integer programming model. One variable g_{ilk} is a continuous variable representing the cargo weight lying in the range $(a_{i,k-1},a_{ik})$ inside the l^{th} container of type i. The other variable z_{ilk} is a binary variable indicating whether the cargo weight is distributed in the range $(a_{i,k-1},a_{ik})$ inside the l^{th} container of type i. Therefore, the above model can be formulated as the following mixed 0-1 integer programming model:

$$\text{Min } \sum_{i=1}^{m}\sum_{l=1}^{L_i} c_i^0 x_{il} + \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{k=1}^{K_i} \delta_{ik} g_{ilk} \tag{6.8}$$

s.t.

$$\sum_{k=1}^{K_i} g_{ilk} = \sum_{j=1}^{n} w_j y_{ilj}\text{, } i=1,\ldots, m, l=1,\ldots, L_i \tag{6.9}$$

$$g_{ilk} \le z_{ilk}(a_{i,k} - a_{i,k-1}),\ i=1,\ldots, m;\ l=1,\ldots, L_i,\ k=1,\ldots,K_i \tag{6.10}$$

$$g_{ilk} \ge z_{il,k+1}(a_{i,k} - a_{i,k-1}),\ i=1,\ldots, m;\ l=1,\ldots,L_i,\ k=1,\ldots,K_i-1 \tag{6.11}$$

$$z_{ilk}=\{0,1\},\ i=1,\ldots, m;\ l=1,\ldots,L_i,\ k=1,\ldots,K_i \tag{6.12}$$

$$g_{ilk} \ge 0,\ i=1,\ldots, m,\ l=1,\ldots,L_i,\ k=1,\ldots,K_i \tag{6.13}$$

Constraints: (6.1)~(6.5)

There are two items in the objective function (6.8). The first component is the fixed cost, which is as the same as in the objective function (6.7). The second component in (6.8) represents the sum of the variable costs for all containers. The variable cost for each container is the sum of the variable costs distributed in all ranges, described in Figure 6.1. The variable cost of the l^{th} container of type i in the range $(a_{i,k-1},a_{ik})$ is the unit charge rate of container i in the range $(a_{i,k-1},a_{ik})$, represented by δ_{ik}, multiplied by the cargo weight distributed in the range $(a_{i,k-1},a_{ik})$ inside the l^{th} container of type i, represented by g_{ilk}.

Constraint (6.9) ensures that the sum of the cargo weight distributed in all areas inside a container is equal to the total weight of the cargoes loaded into the container. Constraint (6.10) ensures z_{ilk} is equal to 1 if the total cargo weight inside the l^{th} container of type i reaches the range $(a_{i,k-1},a_{ik})$. In addition, the cargo weight g_{ilk} in the range $(a_{i,k-1},a_{ik})$ is less-than-or-equal-to the maximum weight value in the range $(a_{i,k-1},a_{ik})$, which is $a_{ik}-a_{i,k-1}$.

Constraint (6.11) ensures that once the total cargo weight inside the l^{th} container of type i reaches the range $(a_{i,k},a_{ik+1})$, the cargo weight in the range $(a_{i,k-1},a_{ik})$, which is g_{ilk}, is not less than the difference between a_{ik} and $a_{i,k-1}$. Constraints (6.10) and (6.11) ensure that the weight ranges are reached by priority: g_{ilk} cannot be positive unless the range $(a_{i,k-1},a_{ik})$ is fully occupied by the cargo weight. In other words, constraints (6.10) and (6.11) ensure that g_{ilk} cannot have a positive value unless all g_{ilt} are at their maximum value, which is a_{it}-$a_{i,t-1}$, $1 \le t \le k$. Constraints (6.12) and (6.13) are the variable type requirements.

6.3.2 A stochastic mixed 0-1 integer programming model for the uncertain container loading problems

This section is concerned with the stochastic recourse model of the container loading problems, in which the cargo quantity q_j is a random parameter. It is assumed that q_j has a discrete distribution with a finite number S of possible realizations, q_{js}, s=1,2,…, S, with the corresponding probabilities p_s, $\sum_{s=1}^{S} p_s = 1$. Two types of response are made in different stages: the first-stage response is the decisions regarding booking under uncertain information; the second-stage response is the decisions that are made on the shipping day when the stochasticity is realized. Two types of decision variables are defined as follows:

The first-stage decision variables

n_i= number of containers of type i to be booked

The second-stage decision variables

n_{is}^{+}= number of type i containers returned on the shipping day in scenario s

n_{is}^{-}= extra number of containers of type i rented on the shipping day in scenario s

$$x_{ils} = \begin{cases} 1 & \text{if the } l^{\text{th}} \text{ container of type } i \text{ is selected in scenario } s \\ 0 & \text{otherwise} \end{cases}$$

y_{iljs} = quantities of cargo of type j loaded into the l^{th} container of type i in scenario s

Based on the analysis above, we know that the total cost for shipping cargoes consists of two parts: cost of usage and penalty cost. Penalty costs arise from urgent needs or the cancellation of containers on the shipping day. For each scenario, the cost of usage includes a fixed cost c_i^0 and a variable cost c_{ils}. The variable cost under uncertainty can be formulated as follows:

$$c_{ils} = \begin{cases} 0 & \sum_{j=1}^{n} w_j y_{iljs} \in (a_{i0}, a_{i1}] \\ \delta_{i2}(\sum_{j=1}^{n} w_j y_{iljs} - a_{i1}) & \sum_{j=1}^{n} w_j y_{iljs} \in (a_{i1}, a_{i2}] \\ \delta_{i2}(a_{i2} - a_{i1}) & \sum_{j=1}^{n} w_j y_{iljs} \in (a_{i2}, a_{i3}] \\ \delta_{i2}(a_{i2} - a_{i1}) + \delta_{i4}(\sum_{j=1}^{n} w_j y_{iljs} - a_{i3}) & \sum_{j=1}^{n} w_j y_{iljs} \in (a_{i3}, a_{i4}] \\ \delta_{i2}(a_{i2} - a_{i1}) + \delta_{i4}(a_{i4} - a_{i3}) & \sum_{j=1}^{n} w_j y_{iljs} \in (a_{i4}, a_{i5}] \\ \delta_{i2}(a_{i2} - a_{i1}) + \delta_{i4}(a_{i4} - a_{i3}) + \delta_{i6}(\sum_{j=1}^{n} w_j y_{iljs} - a_{i5}) & \sum_{j=1}^{n} w_j y_{iljs} \in (a_{i5}, a_{i6}] \end{cases} \tag{6.14}$$

where i= 1,2,…,m, l=1,2,…,L_i, s=1,2,…,S

The objective is to load all cargoes into the containers on the shipping day, where the containers are either booked containers or urgent requirement or cancellation made on the shipping day, while minimizing the total cost charged by the airlines. Uncertain air cargo forwarding problems can be formulated as a two-stage stochastic recourse programming model:

$$\text{Min } \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{s=1}^{S} p_s c_i^0 x_{ils} + \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{s=1}^{S} p_s c_{ils} + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^- n_{is}^- + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^+ n_{is}^+ \tag{6.15}$$

s.t.

$$\sum_{j=1}^{n} v_j y_{iljs} \le V_i x_{ils},\ i=1,\dots,m,\ l=1,\dots,L_i,\ s=1,\dots,S \tag{6.16}$$

$$\sum_{j=1}^{n} w_j y_{iljs} \le W_i x_{ils},\ i=1,\dots,m,\ l=1,\dots,L_i,\ s=1,\dots,S \tag{6.17}$$

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i} y_{iljs} = q_{js},\ j=1,\dots,n,\ s=1,\dots,S \tag{6.18}$$

$$n_i = \sum_{l=1}^{Li} x_{ils} + n_{is}^+ - n_{is}^-,\ i=1,\dots,m,\ s=1,\dots,S \tag{6.19}$$

$$x_{ils} = \{0,1\},\ i=1,\dots,m;\ l=1,\dots,h,\ s=1,\dots,S \tag{6.20}$$

$$y_{iljs}, n_i, n_{is}^-, n_{is}^+ \text{ are non-negative integers, } i=1,\dots,m;\ l=1,\dots,L_i, j=1,\dots,n,\ s=1,\dots,S \tag{6.21}$$

The objective function in (6.15) is the total cost of renting the containers, and it includes four parts. The first part is the expected value of the total fixed costs. The second part is the expected value of the total variable costs. The definition of the variable cost $c_{ils,}$ can be seen by referring to equation (6.14). The third part is the expected value of the total penalty cost for renting additional containers on the shipping day. The fourth part is the expected value of total penalty cost for returning unused containers on the shipping day. Each scenario has to satisfy the container volume constraints in (6.16), container weight constraints in (6.17), cargo quantity constraints in (6.18), and container quantity constraints in (6.19). Constraints (6.20) and (6.21) are the variable type requirements.

The objective function expressed in (6.15) is a piecewise function. We use the same method that is described in the deterministic model, in which two new variables are introduced to transform the model into a mixed 0-1 integer programming model. One variable g_{ilks} is a continuous variable representing the cargo weight distributed in the range $(a_{i,k-1},a_{ik})$ inside the l^{th} container of type i in scenario s. The other variable z_{ilks} is a binary variable indicating whether the cargo weight is distributed in the range $(a_{i,k-1},a_{ik})$ inside the l^{th} container of type i in scenario s. Thus the above model can be formulated as a two-stage stochastic mixed 0-1 integer programming model:

$$\text{Min} \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{s=1}^{S} p_s c_i^0 x_{ils} + \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{k=1}^{K_i}\sum_{s=1}^{S} p_s \delta_{ik} g_{ilks} + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^- n_{is}^- + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^+ n_{is}^+ \tag{6.22}$$

s.t.

$$\sum_{k=1}^{K_i} g_{ilks} = \sum_{j=1}^{n} w_j y_{iljs},\ i=1,\ldots,m,\ l=1,\ldots,L_i,\ s=1,\ldots,S \tag{6.23}$$

$$g_{ilks} \le z_{ilks}\left(a_{i,k} - a_{i,k-1}\right),\ i=1,\ldots,m,\ l=1,\ldots,L_i,\ k=1,\ldots,K_i,\ s=1,\ldots,S \tag{6.24}$$

$$g_{ilks} \ge z_{il,k+1,s}\left(a_{i,k} - a_{i,k-1}\right),\ i=1,\ldots,m,\ l=1,\ldots,L_i,\ k=1,\ldots,K_i-1,\ s=1,\ldots,S \tag{6.25}$$

$$z_{ilks}=\{0,1\},\ i=1,\ldots,m,\ l=1,\ldots,L_i,\ k=1,\ldots,K_i,\ s=1,\ldots,S \tag{6.26}$$

$$g_{ilks} \ge 0,\ i=1,\ldots,m,\ l=1,\ldots,L_i,\ k=1,\ldots,K_i,\ s=1,\ldots,S \tag{6.27}$$

Constraints: (6.16)~(6.21)

6.3.3 Robust optimization models for the uncertain container loading problems

6.3.3.1 A robust mixed 0-1 integer optimization model with solution robustness

Based on the analysis in Section 3.3.1, a robust optimization model with solution robustness for the uncertain container loading problems for global air transport can be formulated as:

$$\begin{aligned}\text{Min} & \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{s=1}^{S} p_s c_i^0 x_{ils} + \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{s=1}^{S} p_s c_{ils} + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^- n_{is}^- + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^+ n_{is}^+ \\ & + \lambda \sum_{s=1}^{S} p_s \left| \sum_{i=1}^{m}\sum_{l=1}^{L_i}(c_i^0 x_{ils} + c_{il}) - \sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{l=1}^{L_i} p_s (c_i^0 x_{ils} + c_{ils}) + \sum_{i=1}^{m}(c_i^- n_{is}^- + c_i^- n_{is}^-) - \sum_{s=1}^{S}\sum_{i=1}^{m} p_s (c_i^- n_{is}^- + c_i^- n_{is}^-) \right|\end{aligned} \tag{6.28}$$

s.t.

Constraints: (6.16)~(6.21)

Using the same method introduced in Section 6.3.1 by defining two new variables: $g_{ilks} \geq 0$ and $z_{ilks} = 0$ or 1, the above model can be formulated as the following form:

$$\text{Min} \ \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{s=1}^{S} p_s c_i^0 x_{ils} + \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{k=1}^{K_i}\sum_{s=1}^{S} p_s \delta_{ik} g_{ilks} + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^- n_{is}^- + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^+ n_{is}^+$$
$$+ \lambda \sum_{s=1}^{S} p_s \left| \sum_{i=1}^{m}\sum_{l=1}^{L_i}(c_i^0 x_{ils} + \sum_{k=1}^{K_i}\delta_{ik} g_{ilks}) - \sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{l=1}^{L_i} p_s(c_i^0 x_{ils} + \sum_{k=1}^{K_i}\delta_{ik} g_{ilks}) + \sum_{i=1}^{m}(c_i^- n_{is}^- + c_i^- n_{is}^-) - \sum_{s=1}^{S}\sum_{i=1}^{m} p_s(c_i^- n_{is}^- + c_i^- n_{is}^-) \right| \tag{6.29}$$

s.t.

Constraints: (6.16)~(6.21), and (6.23)~(6.27)

The above model can be further converted to a mixed 0-1 integer programming model by introducing a derivation variable $\theta_s \geq 0$.

$$\text{Min} \ \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{s=1}^{S} p_s c_i^0 x_{ils} + \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{k=1}^{K_i}\sum_{s=1}^{S} p_s \delta_{ik} g_{ilks} + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^- n_{is}^- + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^+ n_{is}^+$$
$$+ \lambda \sum_{s=1}^{S} p_s \left[\sum_{i=1}^{m}\sum_{l=1}^{L_i}(c_i^0 x_{ils} + \sum_{k=1}^{K_i}\delta_{ik} g_{ilks}) - \sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{l=1}^{L_i} p_s(c_i^0 x_{ils} + \sum_{k=1}^{K_i}\delta_{ik} g_{ilks}) + \sum_{i=1}^{m}(c_i^- n_{is}^- + c_i^- n_{is}^-) - \sum_{s=1}^{S}\sum_{i=1}^{m} p_s(c_i^- n_{is}^- + c_i^- n_{is}^-) + 2\theta_s \right] \tag{6.30}$$

s.t.

$$-\sum_{i=1}^{m}\sum_{l=1}^{L_i}(c_i^0 x_{ils} + \sum_{k=1}^{K_i}\delta_{ik} g_{ilks}) + \sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{l=1}^{L_i} p_s(c_i^0 x_{ils} + \sum_{k=1}^{K_i}\delta_{ik} g_{ilks}) - \sum_{i=1}^{m}(c_i^- n_{is}^- + c_i^- n_{is}^-) + \sum_{s=1}^{S}\sum_{i=1}^{m} p_s(c_i^- n_{is}^- + c_i^- n_{is}^-) - \theta_s \leq 0,$$

$s=1,\ldots,S$ (6.31)

$\theta_s \geq 0$, $s=1,\ldots,S$ (6.32)

Constraints: (6.16)~(6.21), and (6.23)~(6.27)

6.3.3.2 A robust mixed 0-1 integer optimization model with model robustness

Robust optimization allows the violation of the random constraints. Let e_{js} denote cargo quantities of type j not shipped on the shipping day under scenario s. A robust optimization model with model robustness for the uncertain container loading can be formulated as:

$$\text{Min} \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{s=1}^{S} p_s c_i^0 x_{ils} + \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{s=1}^{S} p_s c_{ils} + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^- n_{is}^- + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^+ n_{is}^+ + \sum_{j=1}^{n}\sum_{s=1}^{S} p_s \omega_j e_{js} \quad (6.33)$$

s.t.

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i} y_{iljs} = q_{js} - e_{js}, j=1,\ldots,n,\ s=1,\ldots,S \quad (6.34)$$

e_{js} is an non-negative integer, $j=1,\ldots,n$, $s=1,\ldots,S$ (6.35)

Constraints: (6.16)~(6.17), and (6.19)~(6.21)

Compared with the objective function of the stochastic recourse model in (6.15), the objective function in (6.33) includes one fifth additional part, which is the expected value of the penalty cost for not shipping cargoes on the shipping day, where ω_j is the penalty cost for not shipping one cargo of type j. All constraints in the above robust optimization model are the same as the constraints in the two-stage stochastic model, except for the cargo quantity constraint expressed in (6.18). Constraint (6.34) allows e_{js} cargoes of type j not to be shipped under scenario s. However, the cargo quantity constraint in (6.18) for the two-stage stochastic programming model requires all cargoes to be loaded into containers.

Using the same method as in the deterministic and stochastic model, the above model can be converted into the following mixed 0-1 integer programming model by introducing two variables: g_{ilks} and z_{ilks}.

$$\text{Min} \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{s=1}^{S} p_s c_i^0 x_{ils} + \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{k=1}^{K_l}\sum_{s=1}^{S} p_s \delta_{ik} g_{ilks} + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^- n_{is}^- + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^+ n_{is}^+ + \sum_{j=1}^{n}\sum_{s=1}^{S} p_s \omega_j e_{js}$$

(6.36)

s.t. Constraints: (6.16)~(6.17), (6.19)~(6.21), (6.23)~(6.27), and (6.34)~(6.35)

6.3.3.3 A robust mixed 0-1 inter optimization model with trade-off between solution robustness and model robustness

When the variability and infeasibility are considered simultaneously, a robust optimization model with trade-off between solution robustness and model robustness is formulated to solve the uncertain container loading problems.

$$\begin{aligned}
\text{Min } & \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{s=1}^{S} p_s c_i^0 x_{ils} + \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{s=1}^{S} p_s c_{ils} + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^- n_{is}^- + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^+ n_{is}^+ \\
& + \lambda \sum_{s=1}^{S} p_s \left| \sum_{i=1}^{m}\sum_{l=1}^{L_i} (c_i^0 x_{ils} + c_{il}) - \sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{l=1}^{L_i} p_s (c_i^0 x_{ils} + c_{ils}) + \sum_{i=1}^{m} (c_i^- n_{is}^- + c_i^- n_{is}^-) - \sum_{s=1}^{S}\sum_{i=1}^{m} p_s (c_i^- n_{is}^- + c_i^- n_{is}^-) \right| \\
& + \sum_{j=1}^{n}\sum_{s=1}^{S} p_s \omega_j e_{js}
\end{aligned} \tag{6.37}$$

s.t.
Constraints: (6.16)~(6.21), and (6.34)~(6.35)

By introducing a new variable $\theta_s \geq 0$, the above model can be converted into the following mixed 0-1 integer programming model:

$$\begin{aligned}
\text{Min } & \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{s=1}^{S} p_s c_i^0 x_{ils} + \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{k=1}^{K_i}\sum_{s=1}^{S} p_s \delta_{ik} g_{ilks} + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^- n_{is}^- + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^+ n_{is}^+ \\
& + \lambda \sum_{s=1}^{S} p_s \left[\sum_{i=1}^{m}\sum_{l=1}^{L_i} (c_i^0 x_{ils} + \sum_{k=1}^{K_i} \delta_{ik} g_{ilks}) - \sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{l=1}^{L_i} p_s (c_i^0 x_{ils} + \sum_{k=1}^{K_i} \delta_{ik} g_{ilks}) + \sum_{i=1}^{m} (c_i^- n_{is}^- + c_i^- n_{is}^-) - \sum_{s=1}^{S}\sum_{i=1}^{m} p_s (c_i^- n_{is}^- + c_i^- n_{is}^-) + 2\theta_s \right] \\
& + \sum_{j=1}^{n}\sum_{s=1}^{S} p_s \omega_j e_{js}
\end{aligned} \tag{6.38}$$

s.t.
Constraints (6.16)~(6.21), (6.23)~(6.27), (6.31)~(6.32), and (6.34)~(6.35)

6.4 Computational results and analysis

6.4.1 Known and fixed data

A forwarding company in Hong Kong provides air transport services worldwide. The company collects shipping information from its customers in terms of the weight, volume and shape of shipments, delivery time and destinations. Based on this information, the company consolidates the small shipments into three types of cargo: large, medium and small. The volume and weight of each type of cargo are given in Table 6.1.

Table 6.1: Air cargo characteristics

Cargo types	Cargo volume	Cargo weight
Large	1500	750
Medium	1200	600
Small	1000	500

The forwarder then contacts the airline to arrange rental of air containers. The air carrier can provide 7 types of containers for renting, and currently there are 2 of each type of container available. The airline provides the following information shown in Table 6.2, including the types and quantities of the containers, the volume and weight limits of the containers, the fixed cost, the break points and the unit charge rate in the different ranges.

Table 6.2: Air container characteristics

Container type	Container quantity	Fixed cost	Volume limit	Weight limit	Break point						Charged rate					
					a_{i1}	a_{i2}	a_{i3}	a_{i4}	a_{i5}	a_{i6}	δ_{i1}	δ_{i2}	δ_{i3}	δ_{i4}	δ_{i5}	δ_{i6}
1	2	161617	6489	6800	3968	4722	5290	5976	6273	6800	0	32	0	29	0	25
2	2	105898	6300	5400	2600	3050	3467	3954	4111	5400	0	32	0	29	0	25
3	2	85207	5008	4200	2092	2490	2789	3140	3307	4200	0	32	0	29	0	25
4	2	74373	4882	4000	1826	2173	2434	2741	2886	4000	0	32	0	29	0	25
5	2	48713	3700	3900	1196	1423	1594	1825	1917	3900	0	32	0	29	0	25
6	2	46553	3150	3500	1643	1747	2000	2500	2591	3500	0	32	0	29	0	25
7	2	20695	1400	1200	505	602	674	758	799	1200	0	32	0	29	0	25

6.4.2 Computational results of the deterministic mixed 0-1 integer programming model

6.4.2.1 Deterministic parameters

It is assumed that there are 7 large cargoes, 6 medium cargoes and 5 small cargoes, which need to be shipped one week later. Based on the deterministic information, decision makers need to make decisions on what types and how many containers to book for the next week's shipping and how to pack these cargoes into containers. The mixed 0-1 integer programming model presented in Section 6.2.1 is used to solve the cargo forwarding problem under certainty.

6.4.2.2 Computational results

Table 6.3 gives the computational results. The solution includes which containers to select and which cargoes to be loaded into them. The total rental cost for shipping 7 large cargoes, 6 medium cargoes and 5 small cargoes is 387,237. Additionally, Table 6.3 provides other related results including the loaded volume and weight for each container, the fixed cost, variable cost and total cost for each container. Table 6.4 gives the cargo weight at all ranges in each container.

Table 6.3: Optimal plan for container rental and cargo loading

Selected Containers	Loaded cargoes	Loaded volume	Loaded weight	Fixed cost	Variable cost	Total cost
Container 4 (1st)	1 large, 1 medium, 2 small	4700	2350	74373	11104	85477
Container 4 (2nd)	4 medium	4800	2400	74373	11104	85477
Container 5 (1st)	1 large, 2 small	3500	1750	48713	11788	60501
Container 5 (2nd)	1 large, 1 medium, 1 small	3700	1850	48713	13963	62676
Container 6 (1st)	2 large	3000	1500	46553	0	46553
Container 6 (2nd)	2 large	3000	1500	46553	0	46553

Table 6.4: The cargo weight at all ranges for each container

Container		Cargo weight in the different ranges						Total cargo weight in the container
		$(a_{i0},a_{i1}]$	$(a_{i1},a_{i2}]$	$(a_{i2},a_{i3}]$	$(a_{i3},a_{i4}]$	$(a_{i4},a_{i5}]$	(a_{i5},a_{i6})	
Container type 1	1st							
	2nd							
Container type 2	1st							
	2nd							
Container type 3	1st							
	2nd							
Container type 4	1st	1826	347	177				2350
	2nd	1826	347	227				2400
Container type 5	1st	1196	227	171	156			1750
	2nd	1196	227	171	231	25		1850
Container type 6	1st	1500						1500
	2nd	1500						1500
Container type 7	1st							
	2nd							

From Table 6.3, we know that the rental cost for the two containers of type 4 is the same, although the total weight and volume of cargoes loaded into the second type 4 container is greater than for the first one. The reason is the two containers reach the same range $(a_{i2},a_{i3}]$ (see Table 6.4), which results in the same variable cost for renting the two containers. The rental cost of the second type 5 container is more than for the first one, as the first container only reaches the range $(a_{i3},a_{i4}]$; while the second container reaches the range $(a_{i4},a_{i5}]$. At the same time, Table 6.4 shows the two type 6 containers do not exceed the first cost breaking-point, so no variable cost is incurred.

6.4.2.3 Container loading strategy analysis

Table 6.5 gives four scenarios for the shipping cargo process that the forward company may face in the future. Scenario 1 is the optimal solution using the existing data above. Scenarios 2, 3 and 4 are drawn up on the assumption that the cargo quantities are lowered by one for every type of cargo, representing the different situations that the forwarder might experience.

Table 6.5: Scenario assumptions

Scenario	Description of changes	Cargo quantities
1	Using exist data	7 large, 6 medium and 5 small cargoes
2	Quantity of large cargoes decreases by 1	6 large, 6 medium and 5 small cargoes
3	Quantity of medium cargoes decreases by 1	7 large, 5 medium and 5 small cargoes
4	Quantity of small cargoes decrease by 1	7 large, 6 medium and 4 small cargoes

The optimal solutions for four scenarios are shown in Tables 6.6 and 6.7. From Tables 6.6 and 6.7, we see that types and quantities of cargoes have a dramatic impact on the decisions of how to select containers and how to load cargoes, as well as on the total rental cost. The reason is that the total cost for renting the container not only depends on a fixed cost, but also includes a variable element, which is associated with the cargo weight that the container holds.

Table 6.6: Optimal plan for container rental and cargo loading under different scenarios

Scenario	Loaded cargoes	Container type 1		Container type 2		Container type 3		Container type 4		Container type 5		Container type 6		Container type 7	
		1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd
1	7 large cargo							1		1	1	2	2		
	6 medium cargo							1	4	1					
	5 small cargo							2		1	2				
2	6 large cargo							1		1		2	2		
	6 medium cargo							1	4	1					
	5 small cargo							2		1				1	1
3	7 large cargo							1	1	1		2	2		
	5 medium cargo							1	1	1				1	1
	5 small cargo							2	2	1					
4	7 large cargo							1		2		2	2		
	6 medium cargo							1	4		1				
	4 small cargo							2			2				

Table 6.7: Related costs for container rental and cargo loading under different scenarios

Scenario	Related cost	Container type 1		Container type 2		Containe r type 3		Container type 4		Container type 5		Container type 6		Container type 7	
		1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd
1	Fixed cost							74373	74373	48713	48713	46553	46553		
	Variable cost							11104	11104	11788	13963				
	Total cost							85477	85477	60501	62676	46553	46553		
2	Fixed cost							74373	74373	48713		46553	46553	20695	20695
	Variable cost							11104	11104	13963					
	Total cost							85477	85477	62676		46553	46553	20695	20695
3	Fixed cost							74373	74373	48713		46553	46553	20695	20695
	Variable cost							11104	11104	13963				3040	3040
	Total cost							85477	85477	62676		46553	46553	23735	23735
4	Fixed cost							74373	74373	48713	48713	46553	46553		
	Variable cost							11104	11104	7264	7438				
	Total cost							85477	85477	55977	55977	46553	46553		

In Scenario 2 there is the same amount of medium and small cargoes as in Scenario 1. However, there is less large cargo in Scenario 2 than in Scenario 1 (see Table 6.5). Comparing the computational results of Scenario 1 and 2 in Table 6.6, the company only needs to choose two type 7 containers in Scenario 2 instead of one type 5 container in Scenario 1. Two type 7 containers are enough to hold 2 small cargoes in Scenario 2. However, in Scenario 1, a larger and more expensive type 5 container is needed to carry two small cargoes as well as one large cargo. The cost of renting two type 7 containers to carry two small cargoes is 41,390 in Scenario 2, but the cost of renting one type 5 container to carry two small cargoes plus one large cargo is 62,676 in Scenario 1.

In Scenarios 3 and 2 the same containers are selected: one type 5 container and two type 4, 6 and 7 containers, respectively. The cargo loading plans into the first type 4 and 5 container, as well as the two type 6 containers, are exactly same in both scenarios. Therefore, the costs for renting them are also the same in both scenarios. However, the cargo loading plans are different for the second type 4 container and two type 7 containers. In Scenario 2, the second type container holds 4 medium cargoes, and the two type 7 containers hold two small cargoes. In Scenario 3, however, the second type 4 container carries 1 large, 1 medium and 2 small cargoes, and the two type 7 containers carry two medium cargoes. Therefore, the variable cost for each type 7 container is 23,735 in Scenario 3, compared with 20,695 in Scenario 2. In Scenario 2, the type 7 container only carries one small cargo with a weight of 500, which is less than the first cost breaking-point of 505 for type 7 container. Thus there is no variable cost in renting the two type 7 containers in Scenario 2. In Scenario 3, however, each type 7 container carries one medium cargo, which incurs a variable cost of 3040, because the weight of the medium cargo of 600 exceeds the first cost-breaking-point of 505 for a type 7 container. The related data can be found in Tables 6.6 and 6.7.

Scenarios 4 and 1 select the same containers: 2 type 4, 2 type 5 and 2 type 6. The cargo loading plans are the same in both scenarios, except for the type 5 containers. In Scenario 1, the first type 5 container holds 1 large, 1 medium and 1 small cargo, while the container needs to hold 2 large cargoes in Scenario 4. At the same time, the second type 5 container holds 1 large and 2 small cargoes in Scenario 1, while it carries 1 medium and 2 small cargoes in Scenario 4.

From the computational results and analysis conducted under different scenarios, we conclude that container selecting and cargo loading plans have a dramatic impact on the company's profit.

6.4.3 Computational results for the stochastic mixed 0-1 integer recourse programming model

6.4.3.1 Random parameters

If the cargo quantities are uncertain when booking, decision makers have to make decisions before accurate information is obtained. In the following tests, there is only 1 container of each type available for rental. The unit penalty cost for returning unused containers and renting additional containers is shown in Table 6.8.

Table 6.8: The unit penalty cost for returning unused containers and renting additional containers

Container type	Unit penalty cost for returning unused containers	Unit penalty cost for renting additional containers
Container type 1	100000	200000
Container type 2	70000	150000
Container type 3	60000	120000
Container type 4	50000	100000
Container type 5	40000	80000
Container type 6	35000	70000
Container type 7	30000	60000

The uncertainty of cargo quantities of each type can be captured by three scenarios, as shown in Table 6.9. Scenario 1 denotes that on the shipping day there are 3 of each type of cargo to be shipped; Scenario 2 denotes 2 of each type of cargo and Scenario 3 denotes 1 of each type of cargo.

Table 6.9: Cargo quantities under different scenarios

Cargo type	Scenario 1	Scenario 2	Scenario 3
Large Cargo	3	2	1
Medium Cargo	3	2	1
Small Cargo	3	2	1

In the following tests, we perform three different tests under different probability for the realization of stochastic cargo quantities. Other than the probability of occurrence of cargo quantities, the other conditions in the three tests are kept constant. The test data are shown in Table 6.10.

Table 6.10: Three tests

Test	$p_1=\Pr\{s_1\}$	$p_2=\Pr\{s_2\}$	$p_3=\Pr\{s_3\}$
I	0.8	0.1	0.1
II	0.1	0.8	0.1
III	0.1	0.1	0.8

Test I represents the situation where there are most likely 3 of each type of cargo; Test II the situation where there are most likely 2 of each type of cargo; and Test III where there are most likely 1 of each type of cargo.

6.4.3.2 Computational results

The optimal selection and loading plan of the proposed model in this study can be obtained using mathematical programming software. The first stage response for booking containers is shown in Table 6.11. Tables 6.12 and 6.13 gives the second stage response for renting/returning containers and the cargo loading plan on the shipping day. The related cost is shown in Table 6.14.

Table 6.11: The first stage response for booking

Test	Container type						
	1	2	3	4	5	6	7
I				1	1	1	
II					1	1	1
III					1		

Table 6.12: The second stage response for urgent container requirements on the shipping day

Test	Container type	Scenario 1		Scenario 2		Scenario 3	
		Containers rented	Containers returned	Containers rented	Containers returned	Containers rented	Containers returned
	1						
	2						
	3						
I	4						1
	5						
	6						
	7						
	1						
	2						
	3						
II	4	1					
	5						1
	6						
	7						
	1						
	2						
	3						
III	4	1		1			
	5						
	6	1					
	7						

Table 6.13: The second stage response for loading cargo on the shipping day

Test	Container type	Scenario 1			Scenario 2			Scenario 3		
		Large cargo	Medium cargo	Small cargo	Large cargo	Medium cargo	Small cargo	Large cargo	Medium cargo	Small cargo
	1									
	2									
	3									
I	4	1	1	2	2					
	5		2	1			2	1		
	6	2				2			1	1
	7									
II	1									
	2									
	3									
	4	1	1	1						

	5		2	1		2	1			
	6	2			2		1	1	1	1
	7			1						
III	1									
	2									
	3									
	4	1	1	2	1	1	1			
	5		2	1	1	1	1	1	1	1
	6	2								
	7									

Table 6.14: Related cost

Test	Fixed cost of renting containers	Variable cost of renting containers	Penalty cost for urgent rental	Penalty cost for urgent return	Total cost
I	162202	17154	0	5000	184355
II	118527	9381	10000	4000	141908
III	68243	14788	27000	0	110031

Test I represents the situation where the possibility that there are 3 cargoes of each type is 80%. In Test I, the first stage response is to book 1 container each of type 4, 5 and 6 (see Table 6.11). In the second stage, if Scenario 1 (probability=80%) occurs on the shipping day, there is no need to rent additional containers or return any redundant containers (see Table 6.12). If Scenario 2 (probability=10%) occurs on the shipping day, then also there is no need to rent additional containers or return any redundant containers. If Scenario 3 (probability 10%) occurs on the shipping day, a container of type 4 is cancelled. Any cancellation will incur a penalty. The total expected penalty cost is $5,000 (see Table 6.14). However, the probability that Scenarios 2 and 3 occur is only 20%. Therefore, in Test I, decision makers would like to book more containers in advance to ship a most likely large quantity of cargoes. If the unexpected situation (Scenario 3) occurs, container 4 may need to be returned because of the low shipment requirement; this is shown in Table 6.12. Table 6.13 shows the cargo loading plan on the shipping day for Test I, for each scenario.

In Test II, the most likely cargo quantity for each type is 2 (possibility is 80%). Based on the results of Test II, as shown in Table 6.11, decision makers make the first stage decisions by booking 1 container each of type 5, 6 and 7, a week in advance. Compared with the container selection plan in Test I, the decision makers do not choose a container type 4, which has a comparatively high capacity and cost, as the cargo quantities in Test II would most probably be less than in Test I. In Test II, if Scenario 1 (probability 10%) occurs on the shipping day, which is an unexpected situation where there are 3 cargoes of each type to be shipped, a container of type 4 is required (see Table 6.12), in order to ship all cargoes. If Scenario 2 (probability 80%) occurs on the shipping day, there is no further renting or returning of containers. If Scenario 3 (probability 10%) occurs, the second stage response for this situation is to return a container of type 5 to account for a small quantity of cargoes on the shipping day. The corresponding cargo loading plan for each scenario is shown in Table 6.13, for Test II. The penalty cost for urgent rental of containers in Test II is $10,000, and the penalty for cancellation is $4,000 (see Table 6.14).

Test III shows that the cargo quantity for each type is most likely to be 1. Based on the results of Table 6.11, decision makers will book only 1 container of type 5, a week in advance. Quantities and types of booked containers in Test III are different from those in Tests I and II. In contrast with Tests I and II, Test III selects a container with a comparatively small capacity and cost, since cargo quantities in Test III are most likely to be less than in Tests I and II. In Test III, if the unexpected situation of Scenario 1 (probability 10%) occurs on the shipping day (which means there are 3 cargoes of each type for shipping), the decision maker makes the second stage response by renting a container of type 4 and a container of type 6 on the shipping day, to meet urgent requirements. If another unexpected situation, Scenario 2 (probability 10%), occurs on the shipping day, when there are 2 cargoes for each type waiting for shipping, the decision makers respond by renting a container of type 4. If Scenario 3 (probability 80%) occurs on the shipping day, containers booked in advance are able to meet the requirements on the shipping day. Therefore, there is no need for additional containers or returning redundant containers. The cargoes can be loaded according to the cargo loading plans under different scenarios provided in Table 6.13 (see Test III). The penalty cost for urgent rental of containers in Test III is $27,000, but there is no penalty for cancellation (see Table 6.14).

6.4.3.3 Comparison between the expected value model and stochastic recourse model

The stochastic recourse problem has a related problem, namely, the expected value problem. This arises when all random variables are replaced by their expected values. Table 6.15 shows the expected value of stochastic cargo quantities of each type, for the above three tests.

Table 6.15: Expected value of stochastic cargo quantities in the three tests

Test	Cargo quantities		
	Large	Medium	Small
I	3	3	3
II	2	2	2
III	1	1	1

The expected value model is a mixed 0-1 integer programming model for deterministic container loading problems presented in Section 6.3.1. The model can be solved, and the container solution be obtained, as shown in Table 6.16. Let *EV* represent the objective function value of the expected value model.

Table 6.16: The container renting plan for the expected value model

Test	Container type						
	1	2	3	4	5	6	7
I			1	1	1		
II			1	1			
III						1	1

When stochastic cargo quantities are obtained on the shipping day, the actual situation can be: either scenario 1 happens; or scenario 2 happens; or scenario 3 happens (see Table 6.9). Based on the container renting plan in Table 6.16, decision makers need to make a response for each realization. Table 6.17 shows the results for the three tests. Therefore, the total cost will not be *EV*. Let *EEV* represent the expected results of using the solution of the expected value problem. The comparative results for the stochastic recourse model and expected value model are shown in Table 6.18.

Table 6.17: The quantity of renting and returning on the shipping day

Test	Container type	Scenario 1		Scenario 2		Scenario 3	
		Containers rented	Containers returned	Containers rented	Containers returned	Containers rented	Containers returned
I	1						
	2						
	3				1		1
	4						1
	5						
	6						
	7						
II	1						
	2						
	3						1
	4						
	5	1					
	6						
	7						
III	1						
	2						
	3						
	4	1					
	5	1		1			
	6						
	7		1				

Table 6.18: Comparison between the expected value model and stochastic recourse model

Test	*EV*	*EEV*	*ESS*	*VSS*
I	191081	210448	184335	26093
II	126299	170854	141908	28946
III	62676	130536	110031	20505

In Test I, the expected value model assumes there will be 3 cargoes of each type of cargo for shipping. The plan, which is based on the expected value model, is to book 1 container each of types 3, 4 and 5 (see Table 6.16). If Scenario 1 happens (probability=80%), there are 3 cargoes of each type on the shipping day. The booked containers can meet the shipment requirements. The total renting cost in Scenario 1 is \$216,755. If the unexpected situation of Scenario 2 (probability=10%) occurs on the shipping day, there are 2 cargoes of each type on the shipping day. A container of type 3 is cancelled, which incurs a penalty cost of \$60,000 (see Table 6.8). The total renting cost in Scenario 2 is \$197,817. If the unexpected situation of Scenario 3 (probability=10%) occurs on the shipping day, there will be 1 cargo of each type to ship. A container of type 3 and a container of type 4 will be cancelled, incurring a penalty cost of \$110,000 (see Table 6.8). The total renting cost in Scenario 3 is \$172,676. Therefore, the expected result of using the solution of the expected value problem, denoted by *EEV* is 80%*216,755+10%*197,817+10%*172,626=\$210,453. The total renting cost of using the stochastic recourse model, denoted by *ESS*, is \$184,355 (see Table 6.14). Therefore, the potential gain form using the stochastic model, denoted by *VSS*, is 210,453-184,355=\$26,098 (see Table 6.18).

In Test II, the expected value model assumes there will be 2 cargoes of each type for shipping (see Table 6.15). The plan, which is based on the expected value model, is to book 1 container each of types 3 and 4. If the unexpected situation of Scenario 1 happens (probability=10%), there are 3 cargoes of each type on the shipping day. The booked containers can not carry all the cargoes. A container of type 5 is needed on the shipping day, which incurs a penalty cost of \$80,000 for urgent requirement (see Table 6.8). The total renting cost in Scenario 1 is \$296,755. If Scenario 2 (probability=80%) occurs on the shipping day, there is no need to rent additional containers or return any redundant containers. The total renting cost in Scenario 2 is \$159,580. If the unexpected situation of Scenario 3 (probability=10%) occurs on the shipping day, there is 1 cargo of each type to ship. A container of type 3 is cancelled, which incurs a penalty cost of \$60,000 (see Table 6.8). The total renting cost in Scenario 3 is \$135,141. Therefore, the expected result of using the solution of the expected value problem, denoted by *EEV*, is: *EEV*=10%*296,755+80%*159,580+10%*135,141= \$171,070. The total renting cost of using the stochastic recourse model, denoted by *ESS*, is \$141,908 (see Table 6.14). Therefore, the potential gain form using the stochastic model denoted by *VSS* is 171,070-141,908= \$29,162 (see Table 6.18).

In Test III, the expected value model assumes there will be 1 cargo of each type to ship (see Table 6.15). The plan, which is based on the expected value model, is to book 1 container each of types 6 and 7. If the unexpected situation of Scenario 1 happens (probability=10%), there are 3 cargoes of each type on the shipping day. The booked containers can not hold all the cargoes. A container of type 4 and a container of type 5 are required, and a container of type 7 is cancelled. The penalty cost for urgent rental is $180,000, and the penalty cost for cancellation is $30,000. The total renting cost in Scenario 1 is $401,081. If the unexpected situation of Scenario 2 (probability=10%) occurs on the shipping day, a container of type 5 is required. The penalty cost for urgent rental is $80,000. The total renting cost in Scenario 2 is $206,299. If Scenario 3 (probability=80%) occurs on the shipping day, there is no need to rent additional containers or return any redundant containers. The total renting cost in Scenario 3 is $87,248. Therefore, the expected result of using the solution of the expected value problem, denoted by *EEV*, is: *EEV*=10%*401,081+10%*206,299+80%*87,248=$130,536. The total renting cost of using the stochastic recourse model, denoted by *ESS*, is $110,031 (see Table 6.14). Therefore, the potential gain form using the stochastic model denoted by *VSS* is 130,536-110,031= $20,505 (see Table 6.18).

Based on the above computational results, we can conclude that the optimal solution of the stochastic model is cheaper than that of the corresponding expected value model.

6.4.4 Computational results of the robust models

6.4.4.1 Computational results of the robust model with solution robustness

We perform four tests when λ=0, 0.1, 0.5 and 0.9 for Tests I, II and III (See Table 6.10). The first-stage response for booking containers is shown in Table 6.19. Table 6.20 gives the second-stage response for renting and returning containers. Table 6.21 shows cargo loading plan on the shipping day. Table 6.22 shows the computational results regarding the variability, variability cost, expected cost, penalty cost, total cost, etc. From Tables 6.19 and 6.20, we can see that λ has impact on the first stage and second stage decisions in terms of the containers booked in advance, and the containers returned and required on the shipping day. Table 6.21 shows that λ has also impact on the cargo loading on the shipping day. From Table 6.22, we can see that the expected variability in Test I is reduced by \$9,073 at a cost of an increase in the expected cost of \$5,671, when we use the robust model with solution robustness (λ=0.9), rather than the recourse model (λ=0). The expected variability in Test II is reduced by \$6,442 at a cost of an increase in the expected cost of \$1,838, when we use the robust model with solution robustness (λ=0.9), rather than the recourse model (λ=0). The expected variability in Test III is reduced by \$46,514 at a cost of an increase in the expected cost of \$18,393, if we use the robust model with solution robustness (λ=0.9), rather than the recourse model (λ=0). Therefore, we can conclude that the container loading plan proposed by the robust optimization model with solution robustness is not expensive, and it reduces the risk.

Table 6.19: The first stage response under different λ in three tests

Test	λ	Container type						
		1	2	3	4	5	6	7
I	0				1	1	1	
	0.1				1	1	1	
	0.5				1	1	1	
	0.9				1	1	1	
II	0					1	1	1
	0.1					1	1	1
	0.5				1		1	
	0.9				1		1	
III	0					1		
	0.1					1		
	0.5				1		1	
	0.9				1		1	

Table 6.20: The second stage response under different λ in three tests

Test	λ	Container type	Scenario 1		Scenario 2		Scenario 3	
			Containers rented	Containers returned	Containers rented	Containers returned	Containers rented	Containers returned
I	0	1						
		2						
		3						
		4						1
		5						
		6						
		7						
	0.1	1						
		2						
		3						
		4						1
		5						
		6						
		7						
	0.5	1						
		2						
		3						
		4						1
		5						
		6						
		7						
	0.9	1						
		2						
		3						
		4						
		5						
		6						
		7						
II	0	1						
		2						
		3						
		4	1					
		5						1
		6						
		7						
	0.1	1						
		2						
		3						
		4	1					
		5						1
		6						
		7						
	0.5	1						
		2						
		3						
		4						

		5	1					
		6						1
		7						
		1						
		2						
		3						
	0.9	4						
		5	1					
		6						1
		7						
		1						
		2						
		3						
	0	4	1		1			
		5						
		6	1					
		7						
		1						
		2						
		3						
	0.1	4	1		1			
		5						
		6	1					
III		7						
		1						
		2						
		3						
	0.5	4						
		5	1					
		6						1
		7						
		1						
		2						
		3						
	0.9	4						
		5	1					
		6						1
		7						

Table 6.21: The second stage response for loading cargo under different λ in three tests

Test	λ	Container type	Scenario 1			Scenario 2			Scenario 3		
			Large cargo	Medium cargo	Small cargo	Large cargo	Medium cargo	Small cargo	Large cargo	Medium cargo	Small cargo
		1									
		2									
		3									
	0	4	1	1	2		1	2			
		5		2	1	1			1		
		6	2			1	1			1	1
		7									
		1									
		2									
		3									
	0.1	4	1	1	2		2	1			
		5		2	1	1				1	
		6	2			1		1	1		1
I		7									
		1									
		2									
		3									
	0.5	4	1	1	2		1	2			
		5		2	1		1			1	
		6	2			2			1		1
		7									
		1									
		2									
		3									
	0.9	4	1	1	2		2	2			
		5		2	1	2			1	1	1
		6	2								
		7									
II		1									
		2									
		3									
	0	4	1	1	1						
		5		2	1		2	1			
		6	2			2			1	1	
		7			1			1			1
		1									
		2									
		3									
	0.1	4	1	1	1						
		5		2	1		2	1			
		6	2			2			1	1	
		7			1			1			1
	0.5	1									
		2									
		3									
		4	1	1	2	1	1	2	1	1	1

		5		2	1						
		6	2			1	1				
		7									
		1									
		2									
		3									
	0.9	4	1	1	2		2	2	1	1	1
		5		2	1						
		6	2			2					
		7									
III		1									
		2									
		3									
	0	4	1	1	2	1	1	1			
		5		2	1	1	1	1	1	1	1
		6	2								
		7									
		1									
		2									
		3									
	0.1	4	1	1	2	1	1	1			
		5		2	1	1	1	1	1	1	1
		6	2								
		7									
		1									
		2									
		3									
	0.5	4	1	1	2		2	2	1	1	1
		5		2	1						
		6	2			2					
		7									
		1									
		2									
		3									
	0.9	4	1	1	2		2	2	1	1	1
		5		2	1						
		6	2			2					
		7									

Table 6.22: Summary of costs under different λ in three tests

Test	λ	Fixed cost of renting containers	Variable cost of renting container	Penalty cost for urgent rental	Penalty cost for urgent return	Expected cost	Variability	Variability cost	Total cost
I	0	162202	17154	0	5000	184355	10761	0	184355
	0.1	162202	17154	0	5000	184355	10761	1076	185431
	0.5	162202	17154	0	5000	184355	10761	5381	189736
	0.9	169639	20387	0	0	190026	1688	1520	191545

II	0	118527	9381	10000	4000	141908	31906	0	141908
	0.1	118527	9381	10000	4000	141908	31906	3191	145099
	0.5	121142	11104	8000	3500	143746	25464	12733	156480
	0.9	121142	11104	8000	3500	143746	25464	22920	166666
III	0	68243	14788	27000	0	110031	75767	0	110031
	0.1	68243	14788	27000	0	110031	75767	7577	117607
	0.5	88555	3869	8000	28000	128424	29253	14626	143050
	0.9	88555	3869	8000	28000	128424	29253	26327	154751

6.4.4.2 Computational results of the robust model with model robustness

If the decision makers would like to consider a trade-off between the shipping cost, delivery time and penalty cost for late delivery, they need to consider the mixed 0-1 integer programming model with model robustness. Therefore, in the following tests, we discuss the mixed 0-1 integer programming model with model robustness. For each type of cargo, it is assumed that there is a fixed penalty if the cargo cannot be shipped on the shipping day. Table 6.23 shows the unit penalty cost.

Table 6.23: Unit penalty cost for unshipped cargoes

Container type	Large	Medium	Small
Unit penalty cost	20000	18000	16000

The optimal selection and loading plan of the robust mixed 0-1 integer optimization model with model robustness can be obtained using AIMMS. The first-stage response for booking containers is shown in Table 6.24. Table 6.25 gives the second-stage response for renting and returning containers. Table 6.26 shows the second-stage decision about the unshipped cargo quantities for each type. Table 6.27 shows cargo loading plan on the shipping day. The related cost is shown in Table 6.28.

Table 6.24: The first-stage response for booking

Test	Container type						
	1	2	3	4	5	6	7
I					1	1	
II					1	1	
III					1		

Table 6.25: The second-stage response for urgent container requirements on the shipping day

Test	Container type	Scenario 1		Scenario 2		Scenario 3	
		Containers rented	Containers returned	Containers rented	Containers returned	Containers rented	Containers returned
I	1						
	2						
	3						
	4						
	5						
	6						1
	7			1			
II	1						
	2						
	3						
	4	1					
	5						1
	6						
	7			1			
III	1						
	2						
	3						
	4	1		1			
	5						
	6	1					
	7						

Table 6.26: Cargo quantities for unshipped cargoes under different scenarios in the three tests

Test	Cargo Type	Scenario 1	Scenario 2	Scenario 3
I	Large	3		
	Medium			
	Small			
II	Large			
	Medium		1	
	Small			
III	Large			
	Medium			
	Small			

Table 6.27: Optimal cargo loading plans in the three tests

Test	Container Type	Scenario 1			Scenario 2			Scenario 3		
		Large	Medium	Small	Large	Medium	Small	Large	Medium	Small
I	1									
	2									
	3									
	4									
	5		3			2	1	1	1	1
	6			3	2					
	7						1			
II	1									
	2									
	3									
	4	1	1	2						
	5		2	1		2	2	1	1	1
	6	2			2					
	7									
III	1									
	2									
	3									
	4	1	1	2	1	1	1			
	5		2	1	1	1	1	1	1	1
	6	2								
	7									

Table 6.28: Related cost for container selection and cargo loading problems in the three tests

Test	Fixed cost of renting containers	Variable cost of renting containers	Renting penalty cost	Returning penalty cost	Late delivery penalty cost	Total cost
I	92680	13021	1000	1000	60000	92680
II	98048	9491	3000	1000	18000	129539
III	68243	14788	27000	0	0	110031

In Test I, the most likely cargo quantities for each type of cargo are 3. Table 6.24 provides booking information by ordering 1 container each of types 5 and 6 a week in advance. If Scenario 1 (probability=80%) occurs on the shipping day, this means there are 3 cargoes of each type. In this situation, there is no change in containers needed on the shipping day (see Table 6.25). However, three large cargoes are not shipped (see Table

6.26). Table 6.27 shows that 3 medium cargoes are loaded into container 5 and 3 small cargoes are placed into container 6. If Scenario 2 (probability=10%) occurs on the shipping day in Test I, this means there are 2 cargoes of each type waiting for shipping. From Table 6.25, we know that a container of type 7 is rented on the shipping day. All cargoes are shipped without delay (see Table 6.26). Table 6.27 shows that container 6 holds 2 large cargoes; container 5 holds 2 medium cargoes and 1 small cargo; and container 7 (which is rented on the shipping day) holds 1 small cargo. If Scenario 3 (probability 10%) occurs on the shipping day in Test I, this means there is only 1 cargo of each type for shipping. Therefore, a container type 6 is cancelled on the shipping day (see Table 6.27), and all cargoes can be loaded into container 5 without delay (see Table 6.26).

In Test II, the most likely cargo quantities for each type of cargo are 2. Table 6.24 shows that 1 container of type 5 and 1 container of type 6 are booked a week before. If Scenario 1 (probability 10%) occurs on the shipping day, this means that there are 3 cargoes of each type waiting for shipping. Based on the results of Test II shown in Table 6.26, a container of type 4 is required on the shipping day. Additionally, all cargoes are shipped without delay (see Table 6.26). Therefore, container 4 holds 1 large cargo, 1 medium cargo, and 2 small cargoes; container 5 holds 2 medium cargoes and 1 small cargo; and container 6 holds 2 large cargoes (see Table 6.27). If Scenario 2 (probability 80%) occurs on the shipping day in Test II, there are 2 cargoes of each type waiting for shipping. No additional containers are required on the shipping day, but there is one medium cargo left (See Table 6.26). Thus container 5 holds 2 medium cargoes and 2 small cargoes, and container 6 holds 2 large cargoes (see Table 6.27). If Scenario 3 (probability 10%) occurs in Test II, it means a cargo of each type is waiting for shipping. In this situation, a container of type 6 is cancelled on the shipping day (see Table 6.25). All cargoes can be loaded into container 5 for shipping with out any delay (see Table 6.26). The cargo loading plan is shown in Table 6.27.

In Test III, the most likely cargo quantity for each type is 1. The containers booked in Test III differ from those in Tests I and II. In Test III, only one container is booked (see Table 6.24), because the cargo quantities in Test III are most likely less than those in Tests I or II. In Test III, if the unexpected Scenario 1 (probability=10%) occurs on the shipping day, it means that 3 cargoes of each type are waiting for shipping. On the shipping day, a container of type 4 and a container of type 6 are required to deal with this unexpected large cargo situation (see Table 6.25). Container 4 holds 1 large cargo, 1 medium cargo and 2 small cargoes; container 5 holds 2 medium cargoes and 1 small cargo; container 6 holds 2 large cargoes (see Table 6.27). No cargoes are left under scenario 1 (see Table 6.26). If Scenario 2 occurs (probability=10%) in Test III, it means there are 2 of each type of cargo quantities waiting for shipping. In this situation, a container of type 4 is rented on the shipping day (see Table 6.25). Container 4 holds 1 large cargo, 1 medium cargo and 1 small cargo; container 5 holds 1 largo cargo, 1 medium cargo and 1 small cargo. No cargoes are left (see Table 6.26). If Scenario 3 (probability 80%) occurs in Test III, there is only 1 of each type of cargo for shipping. There is no need to rent or return any containers on the shipping day (see Table 6.25). All cargoes can be loaded into a container of type 5, which has been ordered a week in advance.

In the above three tests, the cargo quantities for each type of cargo under the different scenarios are 3, 2 and 1 respectively. However, the probability of each scenario occurring is different in each of the three tests, which results in different container loading plans in the first stage (when booking) and the second stage (on the shipping day). Additionally, the plans are dependent on the penalty cost associated with unshipped cargoes.

Further tests for the robust optimization model with model robustness

The following tests assume that the uncertainty of the random variable can be captured by three scenarios: Scenario 1 (or s_1) denotes 3 cargoes of each type with probability 25%; Scenario 2 (or s_2) denotes 2 cargoes of each type with probability 50%; Scenario 3 (or s_3) denotes 1 cargo of each type with probability 25%.

1) The unit penalty for not shipping large, medium and small cargoes increases or decreases by the same amount

Table 6.29 shows the optimal solution of the stochastic recourse model. Table 6.30 shows the computational results of the robust optimization model under different unshipped penalty costs ω. As the stochastic recourse model does not permit the violation of stochastic constraints, all cargoes have to be shipped on the shipping day. Table 6.29 shows that the total cost is 138,982. From Table 6.30, when the unit penalty cost for not shipping cargo is more than 16000 for large cargo, 14000 for medium cargo and 12000 for small cargo, no cargoes are left on the shipping day because of the high penalty charge. In this situation, the total cost of the robust optimization model is equal to the total cost of the stochastic recourse model. When the unit penalty cost is less-than-or-equal-to 16000 for large cargo, 14000 for medium cargo and 12000 for small cargo, some cargoes are left on the shipping day. Because of the low unit penalty cost for not shipping cargoes, the decision makers would like to leave some cargoes for future shipment. Therefore, the total costs decrease as the unit penalty cost for not shipping cargo decreases.

When the unit penalty cost is lower than 11000 for large cargo, 9000 for medium cargo and 7000 for small cargo, more cargoes are not shipped on the shipping day because of this lower unit penalty cost. As soon as the unit penalty cost falls to 7000 for large cargo, 5000 for medium cargo and 3000 for small cargo, no cargoes need to be shipped on the shipping day. The total costs equal the penalty cost for the unshipped cargoes.

Table 6.29: Optimal solution of the stochastic recourse model

Fixed cost of renting containers	Variable cost of renting containers	Rent penalty cost	Return penalty cost	Total cost
117202	10530	7500	3750	138982

Table 6.30: Optimal solution of robust optimization model under different ω

Unit penalty cost ω	Unshipped cargo quantities			Unshipped penalty cost	Fixed cost of renting containers	Variable cost of renting containers	Rent penalty cost	Return penalty cost	Total cost
	s_1	s_2	s_3						
(20000,18000,16000)	0	0	0	0	117202	10530	7500	3750	138972
(19000,17000,15000)	0	0	0	0	117202	10530	7500	3750	138972
(18000,16000,14000)	0	0	0	0	117202	10530	7500	3750	138972
(17000,15000,13000)	0	0	0	0	117202	10530	7500	3750	138972
(16000,14000,12000)	0	1	0	12000	102221	14020	7500	2500	138241
(15000,13000,11000)	0	1	0	11000	102221	14020	7500	2500	137241
(14000,12000,10000)	0	1	0	10000	102221	14020	7500	2500	136241
(13000,11000,9000)	0	1	0	9000	102221	14020	7500	2500	135241
(12000,10000,8000)	0	1	0	8000	102221	14020	7500	2500	134241
(11000,9000,7000)	0	1	0	7000	102221	14020	7500	2500	133241

(10000,8000,6000)	4	6	0	76000	35995	6800	5000	5000	128795
(9000,7000,5000)	4	6	0	66000	35995	6800	5000	5000	118795
(8000,6000,4000)	7	6	1	78000	23277	0	0	5000	106277
(7000,5000,3000)	6	4	3	90000	0	0	0	0	90000
(6000,4000,2000)	6	4	3	72000	0	0	0	0	72000

2) The unit penalties for not shipping large, medium and small cargoes change by different amounts

We first set the unit penalty cost for not shipping cargo ω at 13000 for large cargoes, 1100 for medium cargoes and 9000 for small cargoes (see Row 2, Table 6.31). The difference in the unit penalty between large and medium cargoes is the same as between medium and small cargoes. Now, let the unit penalty for not shipping small cargo increase by 2000 (see Row 3, Table 6.31). From Table 6.31, we know that unshipping cargo from a small cargo becomes a medium one. When the unit penalty for not shipping all types of cargo rises to 13000, one medium cargo is left over. However, when the unit penalty for not shipping all types of cargo falls to 11000, a large cargo is left over, When the unit penalty for not shipping cargo falls to 9000 for all types of cargoes, 3 large cargoes are left in Scenario 1, and 2 large cargoes and 1 medium cargo are left in Scenario 2.

Based on the above tests, we can reach the following conclusion: the cargo forwarding strategy is heavily dependent on the unit penalty cost for not shipping cargoes. When the unit penalty cost is large enough, no cargoes are left unshipped on the shipping day under all scenarios. However, when the unit penalty cost is small enough, no cargoes need to be shipped on the shipping day.

Table 6.31: Unit penalty for not shipping cargo by different amounts

Unit penalty cost of not shipping cargo (ω)	Unshipped cargoes			Non-shipped penalty cost	Total cost
	s_1	s_2	s_3		
(13000,11000,9000)	0	1 small	0	9000	135241
(13000,11000,11000)	0	1 medium	0	11000	135791
(13000,13000,13000)	0	1 medium	0	30000	134015
(11000,11000,11000)	0	1 large	0	11000	135791
(9000,9000,9000)	3 large	2 large, 1 medium	0	54000	129871

6.4.4.4 Computational results of the robust model with trade-off between solution robustness and model robustness

Table 6.32 shows the summary of costs incurred of the robust optimization model with trade-off between solution robustness and model robustness.

Table 6.32: Trade-off between solution robustness and model robustness under different λ and ω

Test	λ	ω	Expected variability	Expected infeasibility	Expected cost	Expected variability cost	Expected infeasibility cost	Total cost
I	0.1	0	0	8.1	0	0	0	0
		10000	0	8.1	0	0	81000	81000
		20000	0	5.3	46553	0	106000	152553
		30000	3046	2.5	106600	305	75000	181905
		40000	10761	0	184355	1076	0	185431
	0.5	0	0	8.1	0	0	0	0
		10000	0	8.1	0	0	81000	81000
		20000	0	5.3	46553	0	106000	152553
		30000	0	5.3	46553	0	159000	205553
		40000	10761	0	184355	5381	0	189736
	0.9	0	0	8.1	0	0	0	0
		10000	0	8.1	0	0	81000	81000
		20000	0	5.3	46553	0	106000	152553
		30000	0	5.3	46553	0	159000	205553
		40000	1688	0	190026	1520	0	191545
II	0.1	0	0	6	0	0	0	0
		10000	0	6	0	0	60000	60000
		20000	1178	3	56786	118	60000	116904
		30000	4195	0.3	123677	420	90000	133096
		40000	4195	0.3	123677	420	12000	136096
		50000	4195	0.3	123677	420	15000	139096
		60000	4195	0.3	123677	420	18000	142096
		70000	4195	0.3	123677	420	21000	145096
		80000	31906	0	141908	3191	0	145099
	0.5	0	0	6	0	0	0	0
		10000	0	6	0	0	60000	60000
		20000	1178	3	56786	589	60000	117375
		30000	1178	3	56786	589	90000	147375
		40000	4195	0.3	123677	2098	12000	137775
		50000	4195	0.3	123677	2095	15000	140775
		60000	4195	0.3	123677	2098	18000	143775
		70000	4195	0.3	123677	2098	21000	146775
		80000	4195	0.3	123677	2098	24000	149775
		90000	4195	0.3	123677	2098	27000	152775
		100000	4195	0.3	123677	2098	30000	155775
		110000	25467	0	143746	12733	0	156480
	0.9	0	0	6	0	0	0	0
		10000	0	6	0	0	60000	60000
		20000	1178	3	56786	1060	60000	117846
		30000	185	0.3	126192	167	9000	135359
		40000	185	0.3	126192	167	12000	138359
		50000	185	0.3	126192	167	15000	141359
		60000	185	0.3	126192	167	18000	144359
		70000	185	0.3	126192	167	21000	147359
		80000	185	0.3	126192	167	24000	150359
		90000	185	0.3	126192	167	27000	153359
		100000	185	0.3	126192	167	30000	156359
		110000	185	0.3	126192	167	33000	159359
		120000	185	0.3	126192	167	36000	162359

		130000	1899	0.2	137508	1709	26000	165217
		140000	25467	0	143746	22920	0	166666
		0	0	3.9	0	0	0	0
		10000	0	3.9	0	0	39000	39000
		20000	0	3.9	0	0	78000	78000
		30000	2116	0.9	61354	212	27000	88565
	0.1	40000	2116	0.9	61354	212	36000	97565
		50000	20980	0.6	73606	2098	30000	105704
		60000	20980	0.6	73606	2098	36000	111704
		70000	36137	0.4	85262	3614	28000	116875
		80000	75767	0	110031	7577	0	11607
		0	0	3.9	0	0	0	0
		10000	0	3.9	0	0	39000	39000
		20000	0	3.9	0	0	78000	78000
		30000	2116	0.9	61354	1058	27000	89412
		40000	2116	0.9	61354	1058	36000	98412
		50000	2116	0.9	61354	1058	45000	107412
		60000	547	0.8	67552	274	48000	115826
	0.5	70000	547	0.8	67552	274	56000	123826
		80000	3308	0.4	97334	1654	32000	130988
		90000	3308	0.4	97334	1654	36000	134988
III		100000	3308	0.4	97334	1654	40000	138988
		110000	51174	0.1	106160	25587	11000	142746
		120000	51174	0.1	106160	25587	12000	143746
		130000	51174	0.1	106160	25587	13000	144746
		140000	29253	0	128424	14626	0	143050
		0	0	3.9	0	0	0	0
		10000	0	3.9	0	0	39000	39000
		20000	0	3.9	0	0	78000	78000
		30000	0	0.9	62676	0	27000	89676
		40000	0	0.9	62676	0	26000	98676
		50000	0	0.9	62676	0	45000	107676
		60000	547	0.8	67552	492	48000	116044
	0.9	70000	547	0.8	67552	492	56000	124044
		80000	547	0.8	67552	492	64000	132044
		90000	3308	0.4	97334	2977	36000	136311
		100000	3308	0.4	97334	2977	40000	140311
		110000	3308	0.4	97334	2977	44000	144311
		120000	3308	0.4	97334	2977	48000	148311
		130000	3308	0.4	97334	2977	52000	152311
		140000	9119	0.2	118375	8207	28000	154582
		150000	29253	0	128424	26327	0	154751

From Table 6.32, we have the following conclusion: there is always a trade-off between the variability and infeasibility. ω and λ in the robust optimization model objective function is used to measure the trade-off between model robustness and solution robustness. Robust optimization allows for the infeasibility in the random constraints by means of penalties. When $\omega = 0$, there is no penalty for the infeasibility of random constraints in the objective function. The infeasibility that represents under-fulfilment attains a higher value. Clearly, decision makers do not adopt this kind of production plan. However, a large weight ω shows that the infeasibility penalty dominates the total objective function value and results in a higher variability and a higher total cost. This is an inappropriate approach for those decision makers who are risky and prefer to pay less. Therefore, there is always a trade-off between the risk and cost. For the decision makers, it is necessary to test the proposed robust optimization with various ω and λ on the container loading problems.

6.5 Summary

Globalization is forcing companies to compete on price and delivery speed and these factors highlight the importance of air transport. Effective transport strategies can provide a competitive advantage in terms of quick delivery, responsiveness and flexibility to changing and uncertain market information, while continuously lowering transportation costs. In this paper, we first formulate a mixed 0-1 integer programming model under the assumption that all cargo shipping information can be obtained when booking and there will not be any changes in future. Even if the cargo shipping information is known and fixed, the container loading process is still complicated because the cost of hiring a container is not a fixed value, which depends on the container type and cargo weight inside. In addition, each container has its own special shape with its limitations of weight and volume, and the cargoes inside must not exceed these limitations. Decisions based on the mixed 0-1 integer programming model include the types and numbers of containers that are required, and decisions about which cargo should be loaded into which containers on the shipping day. However, in reality, accurate cargo shipping information can only be obtained on the shipping day. The company also wants to book containers in advance in order to get lower rents, as the penalty cost for urgent renting on the shipping day is very high. At the same time, there is a risk that there may not be enough containers available for rental on the shipping day. On the other hand, it is also very expensive to return booked containers. Therefore, we develop a dual-response container loading strategy to deal with uncertain cargo quantity and short shipping notice. In the first stage, usually a week before the shipping day, when accurate information is not available, the company has to book containers that will be needed on the shipping day, in terms of types and quantities. In the second stage, on the shipping day, when accurate shipping information is realized, the company has to respond to the scenario that has occurred. Decisions in the second stage include the types and quantities of additional containers required in case of large quantities of cargo, and the types and quantities of containers to be returned in case of small quantities of cargo. By adopting the dual-response container loading strategy, the company can make a quick response to different probable scenarios on the shipping day while minimizing the total renting cost. In order to implement the dual-response production loading strategy, we develop a two-stage stochastic mixed 0-1 integer recourse programming model. Computational results show that the container loading plan based on the two-stage stochastic recourse model is cheaper than the corresponding deterministic model for uncertain container loading problems, in global air transport.

Furthermore, we develop three types of robust optimization models for dealing with risk and uncertainty: the robust optimization model with solution robustness, the robust optimization model with model robustness, and the robust optimization model with trade-off between solution robustness and model robustness. Computational results show that the robust model with solution robustness has lower risk than the two-stage stochastic recourse model, and the cost of reducing the risk is low. In addition, a series of experiments are presented to demonstrate the effectiveness of the robust optimization model with model robustness, in which late shipping is permitted with a penalty. In comparison to the stochastic recourse model, the robust optimization model with model robustness shows flexibility in dealing with risk and cost. Finally, a general robust model with solution robustness and model robustness is presented, which provides a direct way to measure the trade off between solution robustness and model robustness. A series of experiments show the impact of λ and ω on the container loading problems in terms of variability, infeasibility, expected cost, variability cost, infeasibility cost and total cost. Decision-makers can choose their favourite container loading strategy, based on their attitude toward the risk by adjusting the value of λ and ω.

In conclusion, the robust models can provide a more flexible, reliable, agile and responsive container loading system with lower risk. Finally, it should be noted that computation and analysis of the models may lead to different outcomes if the model parameters change.

Chapter 7

Conclusions and recommendations

7.1 Conclusions

Today's business has been set in the so-called New World Economy, which has been fuelled by the advances in information technology, particularly accelerated by the Internet. Over the past decade, supply chain management has proved to be a major source of gaining competitive advantages for business companies. More and more companies now realize the importance of global supply chain management by seeking suitable locations and facilities anywhere in the world for manufacturing, marketing and distributing. The infrastructure for global supply chain networks have already been formed, although they will have to be changed over time, as only those companies providing innovative products and services can survive in this highly competitive environment. Today's environment is so rich in information that communication brings real-time data to all participants in the supply chain networks. While business managers are overjoyed with so much quick and rich information, they also find that the traditional managerial approaches, techniques and principles are no longer effective in dealing with these challenges.

This study is motivated by the frustration and uncertainty that many operations managers experience in managing global supply chain networks, characterized by continuously changing information, increasingly shortening product lifecycle and lead time and higher level of customization. Business mangers are struggling to seek innovative ways of dealing with these challenges occurring in the global supply chain management environment in order to compete and survive. This thesis studies the problems emerging in the global supply chain network under uncertainty. By looking at a global supply chain network providing garments to North American and European markets, we outline three main operations in the global supply chain network: globally loading production among different plants located in different countries, globally transporting goods by road from one country to another country, and globally transporting cargos by air from supply sites to demand sites. In these three operations processes, three issues have been identified: the production loading problems for global manufacturing, the logistics problems for global road transport and the container loading problems for global air transport. Through analysis of the operations processes and problems, we find that there is a higher level of uncertainty and risk involved in the global supply chain network.

In order to solve these problems, we first develop a robust optimization framework for decision-making under uncertainty, which provides a quantitative method to obtain a trade-off between cost and risk. The robust optimization framework consists of a robust linear optimization model with solution robustness, a robust linear optimization model with model robustness and a robust linear optimization model with trade-off between solution robustness and model robustness, all of which can be easily solved by mathematical software available. We conclude that the robust linear optimization model with solution robustness is more effective for the uncertainty problems with more variability among different scenarios for the stochastic variables, such as the uncertain production loading and logistics problems discussed in the study. However, we find that there is no need to formulate the robust optimization model with solution robustness for solving the uncertain container loading problems. The robust linear optimization model with solution robustness provides a solution with less variability for the sensitive data in comparison to the solution of the stochastic linear recourse programming model. The robust linear optimization model with model robustness is used to deal with the infeasibility of the stochastic constraint by introducing a penalty function in the objective function. The robust linear optimization model with trade-off between solution robustness and model robustness considers the variability and feasibility simultaneously and provides a trade-off between cost and risk. Comparing the solutions of the deterministic model and two-stage stochastic recourse model, we prove that the robust models have the ability to handle either the variability for uncertain data or handle the infeasibility of uncertain constraints, or both.

For the production loading problems in global manufacturing, we formulate a linear programming model under the assumption that all data is known and fixed during the whole planning process. Globally loading production not only includes the factors in domestic production plans, such as plant capacity, workforce level, etc., but also some global trading factors, such as importing quota limitations. The computational results present the production loading plans, which are based on the deterministic model. After that, we develop a dual-response production loading strategy to deal with the short lead time and uncertain information, such as random demand for products, random unit cost of surplus/shortage and the cost of under-/over-quota. In the first stage, when accurate market information is not available, the production managers first distribute production tasks among company-owned plants. Decisions at this stage include production quantities, machine capacity, changes in workforce level (including the number of workers hired and fired), worker overtime and the appropriate quotas. In the second stage, once the stochasticity is realized, the production managers have to prepare for possible responses to different scenarios that have been observed, such as what additional quantities need to be outsourced to its contracted plants for urgent production to satisfy the high demand scenario, which products have a surplus because of low demand, the quantity of quotas that need to be purchased from local markets etc. By utilizing two types of plants in two different stages, the manufacturing company is able to achieve quick responses to the changing market scenarios while minimizing the total operating cost. In order to implement the dual-response production loading strategy, we develop a two-stage stochastic linear recourse programming model. Computational results show that the linear programming model has less advantage than the two-stage stochastic recourse model when the uncertain factors are addressed in the production loading process in global manufacturing problems. Furthermore, we develop three types of robust optimization models for dealing with risk. Computational results show that the robust optimization models have more advantages over the two-stage stochastic model in terms of less sensitivity to the realization of stochastic variables and the ability to handle the infeasibility. In conclusion, the production loading plans based on the robust optimization models can provide a more flexible, reliable, agile and responsive production loading system with lower risk.

For the logistics problems in global road transport, we formulate a mixed 0-1 integer programming model under the assumption that all data is known and fixed during the whole planning process. The logistics plans, which are based on the deterministic model, are presented, including the fleet composition, transporting routes and warehousing plans in the two countries. After that, we develop a dual-response logistics strategy to deal with the uncertain information and short shipment notice. In the first stage, when accurate crossing-border shipment information is not available, the logistics managers have to develop a logistics plan because of the limited capacity of the fleet. Decisions at this stage include the numbers and types of the hired vehicles operating in one country with low rentals and the hired vehicles operating in two countries with high rentals. In the second stage, typically the shipping day, on which the accurate shipping information is confirmed, the logistics managers have to respond to different scenarios that have been observed. By adopting the dual-response logistics strategy, the company is able to achieve quick response to the changing market demand while minimizing the total logistics cost. In order to implement the dual-response logistics strategy, we develop a two-stage stochastic mixed 0-1 integer recourse programming model. Computational results show that the logistics plan based on the stochastic recourse model is less expensive than the logistics plan based on the the expected value model. Furthermore, we develop three types of robust optimization models for dealing with risk. Computational results show that the robust optimization models for the logistics problems in global road transport have more advantage over the two-stage stochastic recourse model in terms of less sensitivity to the realization of stochastic variables and the ability to handle the infeasibility. In conclusion, the robust optimization models can provide a more flexible, reliable, agile and responsive logistics system with lower risk.

For the container loading problems in global air transport, we first formulate a mixed 0-1 integer programming model under the assumption that all cargo shipping information can be obtained when booking and there will not be any changes in future. Even if the cargo shipping information is known and fixed, the container loading process is still complicated for the freight forwarder because the cost of hiring a container is not a constant, and it depends on the container type and the cargo weight inside. In addition, each container has its own special shape with its limitations of weight and volume, and the cargos inside must not exceed these limitations. In this study, we not only consider how to rent suitable containers with the lowest renting cost, but also consider loading cargos into the containers simultaneously, on the shipping day. Therefore, decisions based on the mixed 0-1 integer programming model for the certain container loading problems include the types and numbers of containers that are booked, and decisions about which cargos should be loaded into which containers on the shipping day. Unfortunately, cargo shipping information is always changing anyway and it can not be confirmed until the shipping day. Therefore, we develop a dual-response container loading strategy to deal with the uncertain cargo quantity and short shipping notice. In the first stage, usually a week before the shipping day when the accurate information is not available, the logistics managers have to book the container types and quantities that will be used on the shipping day. In the second stage, on the shipping day, when accurate shipping information is confirmed, the logistics managers have to respond to different scenarios that have been observed, such as the types and quantities of additional containers required in the case of large quantities of cargo, and the types and quantities of containers returned in case of small quantities of cargo. By adopting the dual-response container loading strategy, the freight forwarder is able to achieve quick response to different cargo shipping scenarios on the shipping day while minimizing the total renting cost. In order to implement the dual-response production loading strategy, we develop a two-stage stochastic mixed 0-1 integer recourse programming model. Computational results show that the container loading plan based on the two-stage stochastic recourse model is cheaper than the corresponding deterministic model for the uncertain container loading problems in global air transport. Furthermore, we develop three types of robust optimization models in dealing with risk and uncertainty. Computational results, however, show that the robust model with solution robustness has less ability to reduce the variability for container loading problems discussed in this study. A series of experiments is presented to demonstrate the effectiveness of the robust optimization model with model robustness, in which late shipping is permitted with a penalty. In conclusion, the robust model with model robustness can provide a more flexible, reliable, agile and responsive container loading system with lower risk.

From the above analysis, we can conclude that when uncertainty is a significant factor in the decision-making process, it has to be addressed accordingly. Failure to consider the uncertainty may lead to very expensive, even disastrous consequences if the unanticipated situation happens. The two-stage stochastic recourse programming model is a suitable tool for dealing with uncertainty by adopting two-stage decisions. Comparing with the deterministic model, the global supply chain plans based on the two-stage stochastic recourse models are less expensive and have the ability to respond to different scenarios of stochastic variables. Despite its success of handling uncertainty, the two-stage stochastic recourse model optimizes only the first moment of the distribution of the objective value, and ignores higher moments of the distribution and the decision maker's risk attitude, which are particularly critical for asymmetric distribution, and for risk averse decision makers. The computational results show that the robust optimization models with solution robustness are very effective for uncertain production loading and logistics problems. However, they have less ability to handle risks in container loading problems with uncertainty. In addition, comparing with the two-stage stochastic recourse programming models, the robust optimization models with model solution robustness are able to provide a more flexible system in dealing with infeasibility arising in the stochastic constraints.

7.2 Recommendations for future research

Our work in this thesis has provided important insights into the global supply chain planning problems under uncertainty. It represents a building block for extended research. There are several paths we can take for future research. These are:

- The models developed in this thesis need input data. The quality of this data, including the deterministic and stochastic parameters, clearly affects the solutions offered by the models. Particularly, the development of forecasting models of stochastic demand in global manufacturing and distributing is an important area for further investigation.

- The robust optimization models provided in this thesis still belong to goal programming, which means that there is no priori mechanism for specifying a "correct" choice of the parameters in the models. This issue is prevalent in multi-criteria programming. Further research might consider how to determine the model parameters for different types of problems as the value of the parameters would affect the solution performance.

- The robust optimization models do not provide means of specifying a scenario, which also occurs when formulating the two-stage stochastic recourse programming models. Development of means of determining the scenarios for different types of global supply chain planning problems is a potential area for further research.

- In the two-stage stochastic recourse models and robust optimization models, we make only two-stage decisions. However, every piece of information is continuously changing over time. Development of multiple-stage stochastic recourse models and robust optimization models could well represent the problems occurring in the global supply chain management environment. If the problems belong to linear programming genre, multi-stage problems should not cause problems of computation time. Most software available are able to handle large linear programming problems. However, if the problems involve integer solutions, computation time would increase substantially. Artificial intelligence algorithms like genetic algorithm, tabu search, simulated annealing, etc., could be considered to solve large integer programming problems.

- For the production loading problems in global manufacturing, other international trading factors can be considered further: for example, the changing exchange rates.

- For the logistics problems in global road transportation, we discuss the crossing-border transportation from country A to country B. Simultaneously transporting goods from country B back to country A is a potential research area to examine, which should substantially improve the fleet's efficiency, while reducing the logistics cost.

- For the container loading problems in global air transport, it is assumed that there is no competition among air carriers in terms of container rentals and container type and number of containers available. Future research could consider how to select containers provided by different air carriers.

- There exist many potential areas for future research in the global supply chain network, which involve uncertainty and risk. For example, global purchasing problems of upstream suppliers, replenishing inventory problems of downstream retailers, vehicle routing problems for delivering goods to different retailers' stores, etc. The robust optimization framework can be applied in these areas also.

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Appendix A

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Robust optimization applied to uncertain production loading problems with import quota limits under the global supply chain management environment

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Abstract Global supply chain management presents some special challenges and issues for manufacturing companies in planning production: these challenges are different from those discussed in domestic production plans. Globally loading production among different plants usually involves substantial uncertainty and great risk because of uncertain market demand, fluctuating quota costs incurred in the global manufacturing process, and shortening lead times. This study proposes a dual-response production loading strategy for two types of plants – company-owned and contracted – to hedge against the short lead time and uncertainty, and to be as responsive and flexible as possible to cope with the uncertainty and risk involved. Three types of robust optimization models are presented: the robust optimization model with solution robustness, the robust optimization model with model robustness, and the robust optimization model with the trade-off between solution robustness and model robustness. A series of experiments are designed to test the effectiveness of the proposed robust optimization models. Compared with the results of the two-stage stochastic recourse programming model, the robust optimization models provide a more responsive and flexible system with less risk, which is particularly important in the current context of global competitiveness.

Key Words: Dual-response production loading; Global supply chain management; Linear programming; Model robustness; Production loading; Robust optimization; Stochastic programming; Solution robustness; Two-stage stochastic resource programming.

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1. Introduction

Production loading has a fundamental role in any manufacturing operation. It is the process of determining what type of, and how much, products should be produced in future time periods. Manufacturing companies operating today, however, face a very different environment from that which was prevalent only a few years ago. With the substantial differentials in labour salary and raw material supply, continuously improving global logistics networks, and dramatically decreased transportation costs, products can be manufactured anywhere in the world where it is feasible. Business has been set in a global environment, where global corporations and brands dominate most markets in the world. Manufacturing companies have discovered that they either develop competitive strategies, tactics, and operations for the global market or be beaten by other manufacturers who have embraced more innovative approaches. Several forces are currently driving changes in the global supply chain environment:

- Global outsourcing of different activities.
- Empowered customers, who demand quick responses and speedy delivery while continuously lowering costs.
- Increasingly shortening products lifecycles, which leaves shorter time for manufacturers to produce.
- Increased product variety, which makes it more difficult to accurately forecast market demand.
- Advancement of information technology and easy access to the Internet.
- Development of e-business, which can lead to global visibility for purchasing, production and distribution.

As customers move at the Internet speed, they need companies to respond at the Internet speed (Iansiti and MacCormark 1997). This puts companies in a tighter squeeze trying to meet more and more sophisticated customer demand with shorter time to develop, produce, and distribute products. Time has become a new and powerful dimension of performance (Stalk, 1988). However, increasing competitive pressures dictate that costs must also continually decrease (Tang *et al.*, 2005). As a result, companies need to work on new manufacturing strategies in order to cope with the current trends under the global supply chain environment.

The analysis of production loading problems has been an active area of research for many years. See inventory carrying and set-up systems in Wagner and Within (1958); and Dillenberger *et al.* (1994); inventory carrying cost and labour cost consideration in Dazelinski and Glmory (1965), Florian and Klein (1971), and Lason and Terjung (1971); heuristic approach for multi-level lot-sizing with a bottleneck in Billington *et al.* (1986); multi-stage production and inventory systems in Goyal and Gunasekeran (1990); multi-item lot sizing systems in Pocket and Wolsey (1991), among others. Shapiro (1993), Thomas and McClain (1993), Silver *et al.* (1998) present excellent general references about production loading problems.

All the above literature presents models and techniques for the deterministic environment, where all information that decision-making needs is accurately known. Sen and Higle (1999) think it is difficult to precisely estimate certain critical data elements, and it is necessary to address the impact of uncertainty during the planning process. Explicitly considering uncertainty, in some situations, is a very critical and failure to include uncertainty may lead to very expensive, even disastrous consequences if the anticipated situation is not realized (Bai *et al.*, 1997). Stochastic programming is a branch of mathematical programming that copes with a class of mathematical models and algorithms in which of the data may be subject to significant uncertainty. Since its invention in the 1950s by Beale (1955), Dantzig (1955) and Charnes and Cooper (1959), stochastic programming has made significant applications in many areas including electric power generation (Murphy *et al.*, 1982), financial planning (Cariño *et al.*, 1994), telecommunications network planning (Sen *et al.*, 1994), transportation (Ferguson and Dantizig 1956, Powell 1988) and supply chain management (Fisher *et al.*, 1997). Excellent survey articles related to stochastic programming application and algorithms are presented by Birge (1997), Sen and Higle(1999), and Dupaćová (2002).

Alonso-Ayuso et al. (2003) state that the treatment of the stochasticity has only relatively recently been applied to production planning. See deterministic approximations to stochastic production system in Britran and Yanasse (1984); stochastic multi-item batch production systems in Zipkin (1986); a tactical planning model to evaluate capacity loading under varying demand in Graves (1986); derived demand and capacity planning under uncertainty in Modiano (1987); a scenario approach to capacity planning in Eppen *et al.* (1989); a scenario approach to characterize the uncertain demand for production planning in Escudero (1993); and models and algorithms for distribution under uncertainty in Cheung and Powell (1996).

Despite its significant applications in many areas, including production loading problems, stochastic programming still has limitations owing to its inability to deal with risk and infeasibility of real-world applications under uncertainty. Mulvey *et al.* (1995) first develop robust optimization that integrates goal programming formulations with a scenarios-based description of problem data. The solutions of robust optimization models are progressively less sensitive, and/or more flexible to the realizations of stochastic variables. They characterize the desirable properties of solutions to models by defining solution robust and model robust. A solution to an optimal model is defined as *solution robust* if it remains "close" to optimal for all input data scenarios, and *model robust* if it remains "almost" feasible for all data scenarios. They also use the robust optimization to solve several real-world problems, including diet problems, power capacity, matrix balance, airline scheduling, scenario immunization for financial planning, and minimum weight structural design.

Robust optimization has a number of applications in areas dealing with uncertainty and risk. Vassiadou-Zeniou and Zenios (1996) investigate traditional simulation models for bond pricing with robust optimization techniques, and develop tools for the management of portfolios of callable bonds. Two models are formulated for single-period and multi-stage problems by using robust optimization. Gutierrez *et al.* (1996) use a robustness approach to solve an incapacitated network design problem considering of a variety of likely future scenarios rather than a fixed future scenario. Vladimirou and Zenios (1997) introduce the notion of restricted resource, which incorporates parameterized satisfying constraints in stochastic programs to directly enforce robustness in recourse decisions. They formulate three alternative models of stochastic program with restricted recourse and compare their performance on several test problems. In their paper, they investigate the trade-off between the stability of recourse decisions and the expected cost of a solution in a robust optimization model. Yu (1997) develops a robust optimization model for stochastic logistics problems. Two logistics examples from a wine and airline company are presented to demonstrate the computational efficiency of the proposed robust model. Darlington *et al.* (1999) propose robust formulations for the constrained control of systems under uncertainty. A mean-variance robustness framework is adopted for formulating a nonlinear and stochastic model. They discuss the flexibility of the formulation via a penalty framework, and a chemical engineering optimization problem is presented to test the robust strategies. Yu and Li (2000) develop a robust optimization model for stochastic logistic problems, for which they propose an efficient method to reduce the computational burden in practice.

In this study, we propose a dual response production loading strategy, in which a company utilizes two types of plant: company-owned and contracted, to satisfy uncertain information and the short lead time. The company first makes the production loading response among the company-owned plants based on the incomplete information. After the uncertainty is realized, the company makes the different production loading responses among the contracted plants. To our best knowledge, there exists little research to use quantitative techniques to model uncertain production loading problems with the concept of dual-response production loading. In this study, robust optimization is used to solve uncertain production loading in order to structure a dual-response production loading system that is as responsive, flexible, and less risky as possible to adequate changing market information and the shorter lead time under the global supply chain management environment.

The rest of the paper is organized as follows. Section 2 describe the dual-response production loading process, and illustrates the uncertainty and risk involved. Section 3 presents a robust optimization framework. Three types of robust optimization models are presented: (1) a *robust optimization model with solution robustness*, (2) a *robust optimization model with model robustness*, and (3) a *robust optimization model with a trade-off between solution robustness and model robustness*. Section 4 introduces the notations and definitions of formulating robust optimization models in terms of parameters, variables, constraints and cost. Section 5 formulates three types of robust optimization models used to structure the dual response production loading strategies. Section 6 presents the computational results and analysis. The final section gives the conclusions of the paper and the recommendations for future research.

2. Dual-response manufacturing process

In today's fiercely competitive global markets, companies are forced to compete on price and delivery performance to their customers in the face of rapidly changing conditions. Under the global supply chain management environment, effective production loading strategies can provide a critical competitive advantage for manufacturing companies in terms of the lower cost of production operations, the responsiveness and flexibility to changing market conditions, and reducing risk. This is particularly true for industries, whose products have short life cycles and lead times, and market demand fluctuates over time. This study is motivated by the problem experienced by global manufacturing companies involved in global supply chain networks linking Asia, North America and Europe. Typically, product sales, R&D, customer service, and market demand are centred in North America and Europe. Production facilities are most likely located in low-cost countries, such as Indonesia, Mauritius, Mexico, Nigeria, South Africa, South Korea, Thailand, Tunisia, Vietnam, and so on. However, China is one of the favourite places for manufacturing because of its low labour and production costs, its large supply of skilled workers, well-equipped facilities, high quality products, as well as its lucrative consumer market. This study considers a garment manufacturing company, which provides fashion garments to the North American and European markets. Products are manufactured in company-owned and contracted the plants in China.

Loading manufacturing tasks globally is a more complicated process than domestic production plans. Not only do decision makers need to consider the factors in domestic loading plans, such as plant capacity, customer requirements, workers skill and cost, inventory cost, and raw material supply, but also some international issues; for example, the import quota limitations being considered in this study. Import quotas are assigned by importing countries and can be legally traded on the markets of exporting countries. Import quotas control the quantity or volume of certain merchandise that can be imported into North American and European counties. The importing countries allocate a certain quantity of quota to each exporting country. Any companies that want to export their products to North America and Europe have to buy the corresponding quotas for the products from local markets. Quota purchasing prices, therefore, fluctuate frequently, depending on many factors such as politics, economy, and market supply and demand either from the exporting countries or from the importing countries. Before accurate market information is available, the company allocates a certain amount of quotas for products in each period. After the stochastic variables are realized, the quota amounts that are initially allocated may not be equal to the actual demand in that period. If the amount of the allocated quota is less than the product demand in that period, the company has to buy additional quotas from local markets at market prices. On the other hand, if the allocated quota is not used up, the company suffers because of buying the unused quota.

Production is used to satisfy market demand. Demand uncertainty is another important factor affecting production loading. Under the global supply chain management environment, accurate market information becomes more and more difficult to obtain. Market demand usually come from different retailers mainly located in the North American and Europe markets, and these retailers tend to delay their commitments for their actual demand, which leaves manufacturers even less time in which to produce the goods. Today's retailers have more power than ever before. They have more opportunity to compare price, quality, service, and delivery speed due to the massive amount of information captured from the Internet and other sources. A high quality product has become a minimum standard rather than a point of differentiation for many industries. Therefore, providing fast, responsive, and flexible production while keeping costs low in response to changing market demand becomes a competitive advantage for manufacturing companies.

The products under this study are fashion garments with short lead times. The manufacturing company, however, has to start production among its plants located in China before accurate market demand is observed. When the sales season is nearing, the commitment for products will be clear. The company then has to take corresponding actions in its manufacturing plans. The dual-response production loading process, therefore, consists of two stages. In the first stage, when accurate market information is not available, the company distributes production tasks among the company-owned plants. The first stage decisions include the production quantities for products, machine capacity, changes in the workforce (including the number of workers hired and fired), worker overtime, and the allocated quota. In the second stage, once the stochasticity is realized, the company has to make responses for different scenarios that have been observed, such as how many additional products need to be outsourced to its contracted plants for urgent production to satisfy the high demand scenario, how many products have a surplus in the case of low demand, how many quotas need to be purchased from local markets when there is not enough quota, or how many quotas are left in the case of low demand.

Loading production involves a great risk of a shortage and surplus both for manufacturing products and for purchasing quotas. Adopting a dual-response production loading strategy, the company is able to quickly respond the changing market information at a low cost while hedging against the risk. Robust optimization is an adequate technique to deal with the risk and uncertainty in the dual-response production loading process. Not only does the company make two stages decisions, but also provides a trade-off between the risk and cost in a direct way. The following section gives a framework of robust optimization.

3. Robust Optimization Framework

3.1. A linear programming model

A general linear programming model can be formulated as follows:

$$\min c^{\mathrm{T}} x \tag{1}$$

s.t.

$$Ax = b \tag{2}$$

$$x \geq 0 \tag{3}$$

where A is a fixed matrix, b is a fixed vector, and x is the vector of decision variables.

3.2. A two-stage stochastic recourse programming model

It is assumed that x consists of two subvectors: x_1 and x_2. x_1 represents a vector of the decision variables that has to be determined before accurate information can be observed. x_1 is referred to as a vector of the *first stage variables*. x_2 represents a vector of the decision variables that can be postponed until the realization of stochastic variables is identified. x_2 is referred as the vector of the *second stage variables*. The constraints, therefore, are classified as the first stage constraints and the second stage constraints. The constraints that only involve the first stage variables are defined as the *first stage constraints*. The rest of the constraints that consist of the first stage variables and the second stage variables are referred as the *second stage constraints*. Therefore, the above linear programming model can be rewritten as:

$$\min c_1^{\mathrm{T}} x_1 + \tilde{c}_2^{\mathrm{T}} x_2 \tag{4}$$

s.t.

$$A_1 x_1 = b_1 \tag{5}$$

$$\tilde{T} x_1 + \tilde{W} x_2 = \tilde{b}_2 \tag{6}$$

$$x_1, x_2 \geq 0 \tag{7}$$

Equation (5) denotes the first stage constraints, and equation (6) denotes the second stage constraints. The stochastic variables that are indicated by $\sim$ represent a stochastic entity. Let S represent all realizations of the stochastic variables. For each $s \in S$, let $p_s = P\{(\widetilde{c}_2, \widetilde{T}, \widetilde{W}, \widetilde{b}_2) = (c_{2s}, T_s, W_s, b_{2s})\}$. A two-stage stochastic recourse programming model is formulated as follows:

$$\min c_1^{\mathrm{T}} x_1 + \sum_{s \in S} p_s c_{2s} x_{2s} \tag{8}$$

s.t.

$$A_1 x_1 = b_1 \tag{9}$$

$$T_s x_1 + W_s x_{2s} = b_{2s}, s \in S \tag{10}$$

$$x_1, x_2 \geq 0 \ , \tag{11}$$

In objective function (8), the first term $c_1^T x_1$ is as the first stage cost, and $\sum_{s \in S} p_s c_{2s} x_{2s}$ is the second stage cost. The sum of the first stage cost and the second stage cost in (8) is defined as the *expected cost* of the objective function of the two-stage stochastic resource programming model.

3.3 Robust optimization

3.3.1 A robust optimization model with solution robustness

A robust optimization model with solution robustness means the solution will not differ substantially among different scenarios and there is less variability in the objective function across scenarios, which presumes a less aggressive management style. A robust optimization model with solution robustness can be formulated as:

$$\min c_1^{\mathrm{T}} x_1 + \sum_{s \in S} p_s c_{2s} x_{2s} + \lambda \sum_{s \in S} p_s \left\| c_{2s} x_{2s} - \sum_{s \in S} p_s c_{2s} x_{2s} \right\| \tag{12}$$

s.t.

$$A_1 x_1 = b_1 \tag{13}$$

$$T_s x_1 + W_s x_{2s} = b_{2s}, \ s \in S \tag{14}$$

$$x_1, x_2, \lambda \geq 0 \ , \tag{15}$$

In the objective function (12), the third term $\lambda \sum_{s \in S} p_s \left\| c_{2s} x_{2s} - \sum_{s \in S} p_s c_{2s} x_{2s} \right\|$ is defined as the *expected variability cost*, where λ is a goal programming parameter. λ

is intended as a measurement of the variability of the objective function in the two-stage stochastic program. $\sum_{s\in S} p_s \left\| c_{2s}x_{2s} - \sum_{s\in S} p_s c_{2s} x_{2s} \right\|$ is defined as the *expected variability* of the objective function of the two-stage stochastic recourse program. Clearly, in objective function (12), λ=0 means the variability is not considered in decision-making process. Then the above model becomes a two-stage stochastic recourse programming model, which is the same model as is expressed in (8) ~ (11).

In objective function (12), $\|o\|$ denotes the norm of *o*, which can be chosen in an arbitrary way. However, its choice influences solution performance. If the norm is denoted by the variance, the quadratic terms contain numerous cross products among variables, which contribute a large computational burden. Yu and Li (2000) propose a robust model with absolute term for a logistic management problem, and present an effective method to transform the model into a linear programming model by introducing additional deviation variables. In this study, we use the absolute term $|o|$ of *o* to denote norm $\|o\|$, and use the method proposed by Yu and Li (2000) to convert the model with the absolute term into a linear programming one. The robust model with solution robustness can be formulated as:

$$\min c_1^{\mathrm{T}} x_1 + \sum_{s\in S} p_s c_{2s} x_{2s} + \lambda \sum_{s\in S} p_s \left| c_{2s}x_{2s} - \sum_{s\in S} p_s c_{2s} x_{2s} \right| \tag{16}$$

s.t.

$$A_1 x_1 = b_1 \tag{17}$$

$$T_s x_1 + W_s x_{2s} = b_{2s}, s \in S \tag{18}$$

$$x_1, x_2, \lambda \geq 0 \tag{19}$$

The model above can be formulated as a linear programming model by introducing a deviation variable $\theta_s \geq 0$.

$$\min c_1^{\mathrm{T}} x_1 + \sum_{s\in S} p_s c_{2s} x_{2s} + \lambda \sum_{s\in S} (p_s c_{2s} x_{2s} - \sum_{s\in S} p_s c_{2s} x_{2s} + 2\theta_s) \tag{20}$$

s.t.

$$A_1 x_1 = b_1 \tag{21}$$

$$T_s x_1 + W_s x_{2s} = b_{2s},\ s \in S \tag{22}$$

$$-\theta_s - c_{2s} x_{2s} + \sum_{s\in S} p_s c_{2s} x_{2s} \leq 0,\ s \in S \tag{23}$$

$$x_1, x_2, \lambda, \theta_s \geq 0, \ s \in S \tag{24}$$

This can be proved as follows:

If $c_{2s}x_{2s} \geq \sum_{s \in S} p_s c_{2s} x_{2s}$, then $\theta_s = 0$.

Thus, the objective function = $c_1^{\mathrm{T}} x_1 + \sum_{s \in S} p_s c_{2s} x_{2s} + \lambda \sum_{s \in S} (p_s c_{2s} x_{2s} - \sum_{s \in S} p_s c_{2s} x_{2s})$;

If $c_{2s}x_{2s} \leq \sum_{s \in S} p_s c_{2s} x_{2s}$, then $\theta_s = -c_{2s}x_{2s} + \sum_{s \in S} p_s c_{2s} x_{2s}$.

The objective function $= c_1^{\mathrm{T}} x_1 + \sum_{s \in S} p_s c_{2s} x_{2s} + \lambda \sum_{s \in S} (p_s c_{2s} x_{2s} - \sum_{s \in S} p_s c_{2s} x_{2s})$.

3.3.2. A robust linear optimization model with model robustness

A robust optimization model with model robustness means the violation of the second stage constraint is permitted, but this is done by the least amount by introducing a penalty function. A robust optimization model with model robustness can be formulated as:

$$\min c_1^{\mathrm{T}} x_1 + \sum_{s \in S} p_s c_{2s} x_{2s} + \omega \sum_{s \in S} p_s \|y_s\| \tag{25}$$

s.t.

$$A_1 x_1 = b_1 \tag{26}$$

$$T_s x_1 + W_s x_{2s} + y_s = b_{2s} \ \ s \in S \tag{27}$$

$$x_1, x_2, \omega \geq 0 \tag{28}$$

$\sum_{s \in S} p_s \|y_s\|$ is defined as the *expected infeasibility*, which is used to measure the violation of the second stage constraints. In (25), the final term $\omega \sum_{s \in S} p_c \|y_s\|$ *is* defined as the *expected infeasibility cost*, where ω is a parameter as a measurement of the infeasibility of the second stage constraints. ω =0 means there is no penalty for not satisfying the second stage constraints. In this case, the second stage constraints can be violated as much as possible. On the other hand, $\omega \to +\infty$ means that any amount of violation for the second stage constraints is hardly accepted. As a result, any constraints at the second stage have to be satisfied because of the large penalty ω. Therefore, when ω is set up large enough, the robust optimization model with model robustness is converted to a two-stage recourse programming model.

By using the absolute term $|o|$ of o to denote norm $\|o\|$ and introducing a deviation variable $\delta_s \geq 0$, the robust optimization model with model robustness can be formulated as the following linear programming model:

$$\min c_1^{\mathrm{T}} x_1 + \sum_{s \in S} p_s c_{2s} x_{2s} + \omega \sum_{s \in S} (p_s y_s + 2\delta_s) \tag{29}$$

s.t.

$$A_1 x_1 = b_1 \tag{30}$$

$$T_s x_1 + W_s x_{2s} + y_s = b_{2s}, s \in S \tag{31}$$

$$-\delta_s - y_s \leq 0, s \in S \tag{32}$$

$$x_1, x_2, \omega, \delta_s \geq 0,\ s \in S \tag{33}$$

3.3.3. A robust optimization model with the trade-off between solution robustness and model robustness

When we consider the variability and infeasibility simultaneously, a robust optimization model featuring a trade-off between solution and model robustness can be formulated as:

$$\min c_1^{\mathrm{T}} x_1 + \sum_{s \in S} p_s c_{2s} x_{2s} + \lambda \sum_{s \in S} p_s \left\| c_{2s} x_{2s} - \sum_{s \in S} p_s c_{2s} x_{2s} \right\| + \omega \sum_{s \in S} p_s \|y_s\| \tag{34}$$

s.t.

$$A_1 x_1 = b_1 \tag{35}$$

$$T_s x_1 + W_s x_{2s} + y_s = b_{2s},\ s \in S \tag{36}$$

$$x_1, x_2, \lambda, \omega \geq 0,\ s \in S \tag{37}$$

In objective function (34), the first term is the first stage cost, and the second term is the second stage cost. The sum of the first stage cost and the second stage cost is the expected cost. The third term is the variability cost, and the fourth term is the infeasibility cost. Meanwhile, constraint (35) is the first stage constraint, and constraint (36) is the second first stage constraint. The robust optimization model above can be further formulated as the following linear programming model by using the absolute term $|o|$ of o to denote norm $\|o\|$ and introducing a deviation variable $\theta_s \geq 0$, and $\delta_s \geq 0$, then:

$$\min c_1^{\mathrm{T}} x_1 + \sum_{s \in S} p_s c_{2s} x_{2s} + \lambda \sum_{s \in S} p_s \left(c_{2s} x_{2s} - \sum_{s \in S} p_s c_{2s} x_{2s} + 2\theta_s \right) + \omega \sum_{s \in S} p_s (y_s + 2\delta_s) \tag{38}$$

s.t.

$$A_1 x_1 = b_1 \tag{39}$$

$$T_s x_1 + W_s x_{2s} + y_s = b_{2s}, s \in S \tag{40}$$

$$-\theta_s - c_{2s}x_{2s} + \sum_{s\in S} p_s c_{2s} x_{2s} \leq 0, s \in S \tag{41}$$

$$-\delta_s - y_s \leq 0, \ s \in S \tag{42}$$

$$x_1, x_2, \lambda, \omega, \theta_s, \delta_s \geq 0, \ s \in S \tag{43}$$

4. Notations and Definitions

4.1. Notations

In formulating the production loading models, the following notations are used.

4.1.1. Indices

i for products (i=1,…,m);

j for plants (j=1,…,n);

t for time periods (t=1,…,T);

s for scenarios (s=1,..,S)

4.1.2. Deterministic parameters

Raw material and machine

c_{ij}^{1} raw material cost of production for a unit of product i in plant j;

c_j^{21}/c_j^{22} machine regular/additional cost of production per hour in plant j;

h_{ij}^1/h_{ij}^2 machine time for production of a unit of product i by skilled/non-skilled workers in plant j;

C_{jt}/A_{jt} maximum regular/additional machine capacity of plant j in period t;

V_{jt} minimum work time in plant j in period t;

Labour

c_{ij}^{31}/c_{ij}^{32} labour cost of skilled/non-skilled workers making a unit of product i in plant j;

c_j^{41}/c_j^{42} labour overtime cost of skilled/non-skilled workers per hour in plant j;

$c^{51+}_{jt-1,t} / c^{52+}_{jt-1,t}$ labour cost for hiring skill/non-skilled workers per hour in plant j between periods t-1, t;

$c^{51-}_{jt-1,t} / c^{52-}_{jt-1,t}$ labour cost for firing skill/non-skilled workers per hour in plant j between periods t-1, t;

v^1_{j0} / v^2_{j0} initial labour level of skilled/non-skilled workers in plant j;

α_j limit ratio between skilled and non-skilled workers for production in plant j;

l^1_{ij} / l^2_{ij} labour time for production of a unit of product i in plant j by skilled/non-skilled workers;

L^1_{jt} / L^2_{jt} maximum capacity of hiring skilled/non-skilled workers in plant j in period t;

W^1_{jt} / W^2_{jt} maximum overtime for skilled/non-skilled workers in plant j in period t;

Quota

c^7_i original quota purchasing cost of a unit of product i;

Q_i initial quota quantity of product i at the beginning of the planning horizon;

Surplus/ Shortage

I_i maximum inventory capacity for product i;

B_i maximum purchasing capacity for product i;

d^+_{i0} initial inventory of product i at the beginning of the planning horizon;

Probability

p_s probability of scenario s occurrence;

4.1.3. Random parameters

D_{it} demand for product i in period t;

$c^{6-}_{it} / c^{6+}_{it}$ shortage/surplus cost of a unit of product i in period t;

$c^{7-}_{it} / c^{7+}_{it}$ under-/over-quota cost of a unit quota of product i in period t;

It is assumed that the uncertainties are represented by a set of possible realizations called *scenarios*. Each scenario provides one possible course of future events. The recourse production policy allows compensating for discrepancies in the second-stage in each scenario s by incurring a cost of $c^{6-}_{its} / c^{6+}_{its}$ per unit of production deviation from market demand, and by incurring a cost of $c^{7-}_{its} / c^{7+}_{its}$ per unit of market demand deviation from the initial allocated quota. When the recourse actions are taken for the realization D_{its} of the demand D_{it}, the realization c_{its}^{6-} of the unit shortage cost c_{it}^{6-} for purchasing product i, the realization c_{its}^{6+} of the unit surplus cost c_{it}^{6+} for storing product i, the realization c_{its}^{7-} of the unit under-quota cost c_{it}^{7-} for purchasing quota, and the realization c_{its}^{7+} of unit over-quota c_{it}^{7+} for penalizing unused quota, the random parameters D_{it} , c_{it}^{6-} , c_{it}^{6+} , c_{it}^{7-} , and c_{it}^{7+} , are independent random variables, and have the same finite discrete distribution specified by:

$$\begin{bmatrix} p_1 & p_2 & \cdots & p_S \\ D_{it1} & D_{it2} & \cdots & D_{itS} \\ c_{it1}^{6-} & c_{it2}^{6-} & \cdots & c_{itS}^{6-} \\ c_{it1}^{6+} & c_{it}^{6+} & \cdots & c_{itS}^{6+} \\ c_{it1}^{7-} & c_{it2}^{7-} & \cdots & c_{itS}^{7-} \\ c_{it1}^{7+} & c_{it2}^{7-} & \cdots & c_{itS}^{7+} \end{bmatrix} \tag{44}$$

4.1.4. The first stage decision variables

x_{ijt}^1 / x_{ijt}^2 production quantities of product i by skilled/non-skilled workers in plant j in period t;

$y_{jt-1,t}^{1+} / y_{jt-1,t}^{2+}$ planned labour time of hiring skilled/non-skilled workers in plant j between periods t-1, t;

$y_{jt-1,t}^{1-} / y_{jt-1,t}^{2-}$ planned labour time of firing skilled/non-skilled workers in plant j between period t-1 and t;

v_{jt}^1 / v_{jt}^2 used labour time of skilled/non-skilled workers in plant j in period t;

z_{jt}^1 / z_{jt}^2 used overtime of skilled/non-skilled workers in plant j in period t;

u_{jt}^1 / u_{jt}^2 used regular/additional machine capacities in plant j in period t;

q_{it} initially allocated quota quantity of product t in period i.

4.1.5. The second stage decision variables:

d_{its}^- / d_{its}^+ shortage/surplus production for product i in period t in each scenario s;

q_{its}^- / q_{its}^+ under-/over-quota quantities of product i in period t in each scenario s;

4.2. Constraints

4.2.1. The first stage constraints

Machine capacity constraints

Machine regular and additional capacity must be sufficient to produce the required number of products.

$$\sum_{i=1}^{m} h_{ij}^1 x_{ijt}^1 + h_{ij}^2 x_{ijt}^2 = u_{jt}^1 + u_{jt}^2, j=1,\ldots, n, t=1,\ldots,T. \qquad (45)$$

Workforce capacity constraints

Constraint (46) and (47) are the capacity requirements of skilled and non-skilled workers.

$$\sum_{i=1}^{m} l_{ij}^1 x_{ijt}^1 = v_{jt}^1, j=1,\ldots, n, t=1,\ldots,T. \qquad (46)$$

$$\sum_{i=1}^{m} l_{ij}^2 x_{ijt}^2 = v_{jt}^2, j=1,\ldots, n, t=1,\ldots,T. \qquad (47)$$

Workforce level constraints

The available workforce in any period equals the workforce in the previous period plus the change of workforce level in the current period. The change in workforce may be due to hiring extra workers, firing redundant workers or overtime.

$$z_{jt}^1 + y_{jt-1,t}^{1+} - y_{jt-1,t}^{1-} = v_{jt}^1 - v_{jt-1}^1, j=1,\ldots,n;\ t=1,\ldots,T. \qquad (48)$$

$$z_{jt}^2 + y_{jt-1,t}^{2+} - y_{jt-1,t}^{2-} = v_{jt}^2 - v_{jt-1}^2, j=1,\ldots,n;\ t=1,\ldots,T. \qquad (49)$$

Initial quota allocation constraints

In the first-stage, the initial quota is allocated in each time period.

$$\sum_{t=1}^{T} q_{it} = Q_i, i=1,\ldots, m \qquad (50)$$

Production quality constraints

The ratio between work time of skilled workers and non-skilled workers should not be less than a given constant so as to guarantee product quality.

$$\sum_{t=1}^{T} v_{jt}^1 \geq \alpha_j \sum_{t=1}^{T} v_{jt}^2, j=1,\ldots, n, t=1,\ldots,T. \qquad (51)$$

Minimum work time constraints

Constraint (52) ensures each plant has a minimum work time in each period.

$$v_{jt}^1 + v_{jt}^2 \geq V_{jt}, j=1,\ldots,n, t=1,\ldots,T. \tag{52}$$

Upper bound constraints

The capacity has the upper bound limits in terms of machine regular/additional capacity, and available labour time and overtime for skilled/non-skilled workers.

$$u_{jt}^1 \leq C_{jt}, j=1,\ldots,n,\ t=1,\ldots,T. \tag{53}$$

$$u_{jt}^2 \leq A_{jt}, j=1,\ldots,n,\ t=1,\ldots,T. \tag{54}$$

$$y_{jt-1,t}^{1+} - y_{jt-1,t}^{1-} \leq L_{jt}^1, j=1,\ldots,n, t=1,\ldots,T. \tag{55}$$

$$y_{jt-1,t}^{2+} - y_{jt-1,t}^{2-} \leq L_{jt}^2,\ j=1,\ldots,n,\ t=1,\ldots,T. \tag{56}$$

$$z_{jt}^1 \leq W_{jt}^1,\ j=1,\ldots,n, t=1,\ldots,T. \tag{57}$$

$$z_{jt}^2 \leq W_{jt}^2,\ j=1,\ldots,n, t=1,\ldots,T. \tag{58}$$

Variable type constraints

$$x_{ijt}^1, x_{ijt}^2, y_{jt-1,t}^{1+}, y_{jt-1,t}^{1-}, y_{jt-1,t}^{2+}, y_{jt-1,t}^{2-}, z_{it}^1, z_{it}^2, u_{jt}^1, u_{jt}^2, v_{jt}^1, v_{jt}^2, q_{it} \geq 0,$$

$$i=1,\ldots, m, j=1,\ldots,n, t=1,\ldots,T. \tag{59}$$

4.2.2. The second stage constraints

Random demand constraints

In each scenario, in each period, for each product, market demand has to be met by a combination of production in that period, inventory from the previous period, purchasing from the contracted plants, and inventory in that period.

$$\sum_{j=1}^{n}(x_{ijt}^1 + x_{ijt}^2) + d_{i,t-1,s}^+ + d_{its}^- - d_{its}^+ = D_{its},\ i=1,\ldots,m,\ t=1,\ldots,T, s=1,\ldots,S. \tag{60}$$

Random quota constraints

In each scenario, in each period, each product needs to have its own quota. The ideal situation is that in each period the demand is equal to the initial allocated quota. However, when the quota amount is insufficient, the company needs to purchase quota from local markets at the market price. On the other hand, when the quota is not used fully, the company incurs the penalty.

$$q_{it} + q_{i,t-1,s}^+ + q_{its}^- - q_{its}^+ = D_{its},\ i=1,\ldots,m,\ j=1,\ldots,n, s=1,\ldots,S. \tag{61}$$

Random upper bound constraints

Shortage/surplus production has capacity limits.

$$d_{its}^- \leq B_{it},\ i=1,\ldots, n, t=1,\ldots,T, s=1,\ldots,S. \tag{62}$$

$$d_{its}^{+} \le I_{it},\ i=1,\ldots,n,\ t=0,\ldots,T,\ s=1,\ldots,S. \tag{63}$$

Variable type constraints

$$d_{its}^{-}, d_{its}^{+}, q_{its}^{-}, q_{its}^{+} \ge 0,\ i=1,\ldots,m,\ t=1,\ldots,T,\ s=1,\ldots,S. \tag{64}$$

4.3. Cost

4.3.1. The first stage cost

The first stage cost, denoted by *FC*, is the cost that we need to pay for the first stage production loading decisions among the company-owned plants. This is the sum of the raw material cost, the used machine cost, the used labour cost, the overtime cost, the cost of hiring/firing workers, and the initial quota purchasing cost.

$$FC=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T}c_{ij}^{1}(x_{ijt}^{1}+x_{ijt}^{2})+\sum_{j=1}^{n}\sum_{t=1}^{T}(c_{j}^{21}u_{jt}^{1}+c_{j}^{22}u_{jt}^{2})+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T}(c_{ij}^{31}x_{ijt}^{1}+c_{ij}^{32}x_{ijt}^{2})$$
$$+\sum_{j=1}^{n}\sum_{t=1}^{T}(c_{j}^{41}z_{jt}^{1}+c_{j}^{42}z_{jt}^{2})+\sum_{j=1}^{n}\sum_{t=1}^{T}(c_{jt-1,t}^{51+}y_{jt-1,t}^{1+}+c_{jt-1,t}^{52+}y_{jt-1,t}^{2+}+c_{jt-1,t}^{51-}y_{jt-1,t}^{1-}+c_{jt-1,t}^{52-}y_{jt-1,t}^{1+})$$
$$+\sum_{i=1}^{m}\sum_{t=1}^{T}c_{i}^{7}q_{it} \tag{65}$$

4.3.2. The second stage cost

The second stage cost, denoted by *SC*, is the cost that we need to pay for the second stage production loading decisions. After realization of the random variable has been observed, the decision makers have to make the second stage decisions, such as the quantity of purchasing product from contracted plants, inventory, purchasing quota, and the quota unused. Therefore, the second stage cost is the sum of the cost of shortage/surplus production and the cost of under-/over-quota, which is shown as follows.

$$SC=\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}p_{s}(c_{its}^{6-}d_{its}^{-}+c_{its}^{6+}d_{its}^{+})+\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}p_{s}(c_{its}^{7-}q_{its}^{-}+c_{its}^{7+}q_{its}^{+}) \tag{66}$$

5. Model formulation

5.1. A two-stage stochastic recourse programming model for uncertain production loading problems with import quota limits

Based on the analysis in Section 3.2, the production loading problem with importing quota limits can be formulated as a two-stage stochastic recourse programming model:

$$\text{Min } FC+SC \tag{67}$$

s.t.

The first stage constraints: (45) ~ (59).

The second stage constraints: (60) ~ (64).

5.2. A robust optimization model with solution robustness for uncertain production loading problems with import quota limits

Based on the analysis in Section 3.3.1, a robust optimization model with solution robustness for the production loading problems with the importing quota limits under global supply chain environments can be formulated as:

$$\text{Min } FC+SC$$

$$+\lambda\sum_{s=1}^{S}p_s\left\|\sum_{i=1}^{m}\sum_{t=1}^{T}(c_{its}^{6-}d_{its}^{-}+c_{its}^{6+}d_{its}^{+}+c_{its}^{7-}q_{its}^{-}+c_{its}^{7+}q_{its}^{+})-\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}p_s(c_{its}^{6-}d_{its}^{-}+c_{its}^{6+}d_{its}^{+}+c_{its}^{7-}q_{its}^{-}+c_{its}^{7+}q_{its}^{+})\right\| \tag{68}$$

s.t.

The first stage constraints: (45) ~ (59).

The second stage constraints: (60) ~ (64).

The final term in objective function (68) is the variability cost for shortage/surplus production and under-/over- quota. The model above can be converted into a linear programming model by using the absolute term to denote the norm in (68), and introducing a deviational variable $\theta_s \geq 0$ as follows:

$$\text{Min } FC+SC$$

$$+\lambda\sum_{s=1}^{S}p_s\left[\sum_{i=1}^{m}\sum_{t=1}^{T}(c_{its}^{6-}d_{its}^{-}+c_{its}^{6+}d_{its}^{+}+c_{its}^{7-}q_{its}^{-}+c_{its}^{7+}q_{its}^{+})-\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}p_s(c_{its}^{6-}d_{its}^{-}+c_{its}^{6+}d_{its}^{+}+c_{its}^{7-}q_{its}^{-}+c_{its}^{7+}q_{its}^{+})+2\theta_s\right] \tag{69}$$

s.t.

The first stage constraints: (45) ~ (59).

The second stage constraints: (60) ~ (64).

$$-\sum_{i=1}^{m}\sum_{t=1}^{T}(c_{its}^{6-}d_{its}^{-}+c_{its}^{6+}d_{its}^{+}+c_{its}^{7-}q_{its}^{-}+c_{its}^{7+}q_{its}^{+})+\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}p_s(c_{its}^{6-}d_{its}^{-}+c_{its}^{6+}d_{its}^{+}+c_{its}^{7-}q_{its}^{-}+c_{its}^{7+}q_{its}^{+})-\theta_s\le 0$$

$$s=1,\ldots,S. \quad (70)$$

$$\theta_s \ge 0,\ s=1,\ldots,S. \quad (71)$$

5.3. A robust optimization model with model robustness for uncertain production loading problems with importing quota limits

Based on the analysis in Section 3.3.2, a robust optimization model with solution robustness for production loading problems with the importing quota limits under global supply chain environments can be formulated as:

$$\text{Min } FC+SC+\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}\omega^1\left\|e_{its}^1\right\|+\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}\omega^2\left\|e_{its}^2\right\| \quad (72)$$

s.t.

The first stage constraints: (45) ~ (59).

The second stage constraints: (60) ~ (64).

$$e_{its}^1 = D_{its}-\sum_{j=1}^{n}(x_{ijt}^1+x_{ijt}^2)-d_{i,t-1,s}^{-}-d_{its}^{+}+d_{its}^{-},\ i=1,\ldots,m,\ t=1,\ldots,T,\ s=1,\ldots,S. \quad (73)$$

$$e_{its}^2 = D_{its}-q_{it}-q_{i,t-1,s}^{-}-q_{its}^{+}+q_{its}^{-},\ i=1,\ldots,m,\ t=1,\ldots,T,\ s=1,\ldots,S. \quad (74)$$

In objective function (72), ω^1 represents the unit weighting penalty for the infeasibility of the random demand constraints, and ω^2 represents the unit weighting penalty for the infeasibility of the random quota constraints. Constraint (73) denotes the random demand constraints in (60) can be violated at an amount e_{its}^1. e_{its}^1 is a deviation variable, and it is the difference between the demand, production and shortage/surplus. Constraint (74) denotes the random quota constraints in (61) can be violated at an amount e_{its}^2. e_{its}^2 is a deviation variable, and it is the difference between demand, initial quota allocated and under-/over- quota. In the objective function (72), when the unit weighting parameter ω^1 increases, the unit penalty cost for the infeasibility of the random demand constraints increase. We have to pay more for the violation of the random demand constraint. If the value of ω^1 is increased by enough, the value of e_{its}^1 will be forced to become zero, which means all random demand constraints have to be satisfied for each scenario. The same phenomenon occurs at the unit weighting penalty ω^2 and the corresponding random quota constraint (74).

Based on the analysis in Section 3.3.2, the above model can be expressed as a linear programming model by using the absolute term to denote the norm in (72), and introducing two deviational variables $\delta_{its} \geq 0$ and $\gamma_{its} \geq 0$. The robust optimization model with model robustness for the production loading problems with the importing quota limits under global supply chain environments can be formulated as:

$$\text{Min } FC+SC+\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}\omega^1(e_{its}^1+2\delta_{its})+\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}\omega^2(e_{its}^2+2\gamma_{its}) \tag{75}$$

s.t.

The first stage constraints: (45) ~ (59).

The second stage constraints: (60) ~ (64).

$$-D_{its}+\sum_{j=1}^{n}(x_{ijt}^1+x_{ijt}^2)+d_{i,t-1,s}^{-}+d_{its}^{+}-d_{its}^{-}-\delta_{its}\leq 0,\ i=1,\ldots,m,\ t=1,\ldots,T,\ s=1,\ldots,S. \tag{76}$$

$$-D_{its}+q_{it}+q_{i,t-1,s}^{-}+q_{its}^{+}-q_{its}^{-}-\gamma_{its},\ i=1,\ldots,m,\ t=1,\ldots,T,\ s=1,\ldots,S. \tag{77}$$

$$\delta_{its},\gamma_{its}\geq 0,\ i=1,\ldots,m,\ t=1,\ldots,T,\ s=1,\ldots,S. \tag{78}$$

5.4. A robust optimization model with the trade-off between robustness solution and model robustness for uncertain production loading problems with import quota limits

When the variability and infeasibility are considered simultaneously, a robust optimization model with robustness solution and model robustness is formulated to solve uncertain production loading problems with import quota limits under the global supply chain environment.

$$\text{Min } FC+SC$$
$$+\lambda\sum_{s=1}^{S}p_s\left\|\sum_{i=1}^{m}\sum_{t=1}^{T}(c_{its}^{6-}d_{its}^{-}+c_{its}^{6+}d_{its}^{+}c_{its}^{7-}q_{its}^{-}+c_{its}^{7+}q_{its}^{+})-\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}p_s(c_{its}^{6-}d_{its}^{-}+c_{its}^{6+}d_{its}^{+}c_{its}^{7-}q_{its}^{-}+c_{its}^{7+}q_{its}^{+})\right\|$$
$$+\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}\omega^1\left\|e_{its}^1\right\|+\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}\omega^2\left\|e_{its}^2\right\| \tag{79}$$

s.t.

The first stage constraints: (45) ~ (59),

The second stage constraints: (60) ~ (64),

and (73), (74).

Furthermore, the above model can be expressed as the following linear programming model by using the absolute term to denote the norm in (79), and introducing three deviational variables $\theta_s \geq 0$, $\delta_{its} \geq 0$ and $\gamma_{its} \geq 0$:

Min $FC+SC$

$$+\lambda\sum_{s=1}^{S}p_s\left[\sum_{i=1}^{m}\sum_{t=1}^{T}(c_{its}^{6-}d_{its}^{-}+c_{its}^{6+}d_{its}^{+}+c_{its}^{7-}q_{its}^{-}+c_{its}^{7+}q_{its}^{+})-\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}p_s(c_{its}^{6-}d_{its}^{-}+c_{its}^{6+}d_{its}^{+}+c_{its}^{7-}q_{its}^{-}+c_{its}^{7+}q_{its}^{+})+2\theta_s\right]$$

$$+\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}\omega^1(e_{its}^1+2\delta_{its})+\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T}\omega^2(e_{its}^2+2\gamma_{its}) \tag{80}$$

s.t.

The first stage constraints: (45) ~ (59),

The second stage constraints: (60) ~ (64),

and (70), (71), (73), (74), (76), (77), (78).

6. Computational Results and Analysis

6.1 Data and implementation

In order to illustrate the effectiveness of the proposed three recourse models for the uncertain production loading problems with importing quota limits, we use the data provided by the garment manufacturing company. Based on the information from its retailers in North American and European markets, the company decides to produce three types of products for new season's fashions in the three plants in China. The company will look at a 4-week planning horizon. Table 1 gives the unit raw material cost, labour cost, labour and machine time. Table 2 gives the unit machine cost for regular and additional production, and the unit overtime cost for skilled and non-skilled workers. Table 3 gives the maximum machine regular/additional capacity, maximum labour capacity, maximum overtime capacity and minimum work time. Table 4 shows maximum inventory capacity and purchasing capacity. Currently, there is no cost in hiring/firing workers because there is a large supply of skilled and non-skilled workers in China's market and there is no union contract limitation in China. Thus we assume that the initial workforce level is zero. Additionally, there is no inventory for the new products.

Table 1. Unit raw material cost, labour cost, labour time and machine time.

Product	Plant	Raw material cost	Labour cost of skilled workers	Labour cost of non-skilled workers	Labour time for skilled workers	Labour time for non-skilled workers	Machine time for skilled workers	Machine time for non-skilled workers
1	1	4	4.5	4	2	2.25	1.75	2.25
	2	4.2	4	3.5	2.25	2.5	2	2.5
	3	4.3	3.5	3	2.5	2.75	2.25	2.75
2	1	3	4	3.5	1.5	1.75	1.25	1.75
	2	3.2	3.5	3	1.75	2	1.5	2
	3	3.3	3	2.5	2	2.25	1.75	2.25
3	1	2	3	2.5	1	1.25	0.75	1.25
	2	2.2	2.5	2	1.25	1.5	1	1.5
	3	2.3	2	1.5	1.5	1.75	1.25	1.75

Table 2: Unit machine cost and overtime cost.

Plant	Regular machine cost for production	Additional machine cost for production	Overtime cost for skilled worker	Overtime cost for non-skilled worker
1	0.05	0.055	6	5
2	0.06	0.065	5	4
3	0.07	0.75	4	3

Table 3. Maximum machine capacity, labour capacity, overtime, as well as minimum labour work time.

Plant	Period	Maximum machine regular capacity	Maximum machine additional capacity	Maximum capacity of skilled workers	Maximum capacity of non-skilled workers	Maximum overtime by skilled workers	Maximum overtime by non-skilled workers	Minimum labour work time
1	1	5500	250	4800	2400	2400	1200	2400
	2	5500	250	4800	2400	2400	1200	2400
	3	5500	250	4800	2400	2400	1200	2400
	4	5500	250	4800	2400	2400	1200	2400
2	1	5000	250	3840	1920	1920	960	1800
	2	5000	250	3840	1920	1920	960	1800
	3	5000	250	3840	1920	1920	960	1800
	4	5000	250	3840	1920	1920	960	1800
3	1	5000	200	2400	1200	1200	600	1500
	2	5000	200	2400	1200	1200	600	1500
	3	5000	200	2400	1200	1200	600	1500
	4	5000	200	2400	1200	1200	600	1500

Table 4. Maximum inventory and purchasing capacity.

Product	Period	Maximum inventory	Maximum purchasing
1	1	1500	500
	2	1500	500
	3	1500	500
	4	1500	500
2	1	1500	500
	2	1500	500
	3	1500	500
	4	1500	500
3	1	1500	500
	2	1500	500
	3	1500	500
	4	1500	500

It is assumed that the uncertainty can be represented by a set of possible economic situations, namely good, fair, and bad, for the new season. Let s_1 represent a good economy with probability p_1, p_1=Pr{s_1}; s_2 represents a fair economy with probability p_2, p_2=Pr{s_2}; and s_3 represents a bad economy with probability p_3, p_3=Pr{s_3}. Let

p_1=0.1, p_2=0.1, and p_3=0.8, representing that the probability of a good economy in the new season as 10%, fair economy as 10%, and bad economy as 80%. Table 5 gives the unit shortage cost for purchasing products from the contacted plants, the unit surplus cost for storing left products, the unit under-quota cost for purchasing quota from the market, and the over-quota cost for penalizing unused quota. Additionally, market demand in each scenario is also shown in Table 5.

Table 5. Unit shortage/surplus cost, unit under/over- quota cost, demand.

Scenario	Product	Period	Shortage cost	Surplus cost	Under-quota cost	Over-quota cost	Demand
s_1	1	1	120	2.5	26	4	2000
		2	120	2.5	26	4	2100
		3	120	2.5	26	4	2200
		4	120	2.5	26	4	2300
	2	1	72	1.5	17	3	1500
		2	72	1.5	17	3	1700
		3	72	1.5	17	3	1900
		4	72	1.5	17	3	2100
	3	1	48	1	10	2	1200
		2	48	1	10	2	1300
		3	48	1	10	2	1400
		4	48	1	10	2	1500
s_2	1	1	100	2	24	3	1800
		2	100	2	24	3	1900
		3	100	2	24	3	2000
		4	100	2	24	3	2100
	2	1	60	1	15	2	1400
		2	60	1	15	2	1600
		3	60	1	15	2	1800
		4	60	1	15	2	2000
	3	1	40	0.5	8	1	1100
		2	40	0.5	8	1	1200
		3	40	0.5	8	1	1300
		4	40	0.5	8	1	1400
s_3	1	1	80	1.8	22	2.5	1700
		2	80	1.8	22	2.5	1800
		3	80	1.8	22	2.5	1900
		4	80	1.8	22	2.5	2000
	2	1	48	0.8	14	1.5	1300
		2	48	0.8	14	1.5	1500
		3	48	0.8	14	1.5	1700
		4	48	0.8	14	1.5	1900
	3	1	32	0.3	7	0.5	1000
		2	32	0.3	7	0.5	1100
		3	32	0.3	7	0.5	1200
		4	32	0.3	7	0.5	1300

6.2. Computational results

The models are solved using AIMMS 3.4 and the problems are executed on a Pentium IV 2.60GHz PC. The following content shows the computational results of the robust optimization model with the trade-off between solution robustness and model robustness by setting up $\lambda = 0.1$, $\omega^1 = \omega^2 = 100$.

6.2.1. The first-stage decisions

Before the accurate market and quota price data are available, the company has to start production among its company-owned plants. The first-stage decisions among the company-owned plants are shown in Tables 6 ~ 11. Table 6 shows the production quantities. Tables 7 and 8 show the machine work time and labour work time. Tables 9 and 10 show hiring and firing worker time. The initial quota allocated in each period is shown in Table 11. There is no need to work overtime.

Table 6. Production quantity.

Plant	Product	Skilled workers				Non-skilled workers			
		Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	1	1200		600	600		1067	533	533
	2								
	3								
2	1	332	172	867	1167				
	2								
	3					1200	1151	452	1005
3	1	267	662						
	2	867	749	1469	783	5 33	951	431	1317
	3						149	948	495

Table 7. Machine work time.

Plant	Regular capacity used				Additional capacity used			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	2100	2400	2250	2250				
2	2467	2070	2411	3840				
3	3317	5000	5000	5000		200	200	200

Table 8. Labour work time.

Plant	Skilled workers				Non-skilled workers			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	2400		1200	1200		2400	1200	1200
2	750	386	1950	2625	1800	1727	678	1507
3	2400	3153	2938	1567	1200	2400	2629	3829

Table 9. Hiring workers.

Plant	Skilled workers				non-skilled workers			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	2400		1200			2400		
2	750		1564	675	1800			830
3	2400	753			1200	1200	1229	1200

Table 10. Firing workers.

Plant	Skilled workers				non-skilled workers			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1		2400					1200	
2		364				73	1049	
3			214	1371				

Table 11. Quotas allocated.

Plant	Period 1	Period 2	Period 3	Period 4
1	1700	1900	2100	2300
2	1400	1600	1900	2100
3	1000	1200	1300	1500

6.2.2. The second-stage decisions

When the uncertainty is observed, the company can make the second-stage production loading decisions, which are shown in Tables 12 ~ 17.

Scenario 1: Good economy

The probability of a good economy is 10%. If this scenario is observed, the company will take the second-stage decisions by purchasing certain products from its contractors for urgent production, as well as purchasing quotas to satisfy the high market demand.

Table 12. Shortage/surplus production.

Product	Purchased products from contractors				Inventory			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1								
2	100							
3								

Table 13. Under-/over- quota.

Product	Purchased quota				Unused quota			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	300	200	100					
2	100	100						
3	200	100	100					

In the good economy scenario, there are 200 product 1 unfinished in periods 1, 2 and 3, respectively. The penalty cost for unfulfilled production is 6000. The company also has to buy 100 product 12 in period from its contractors for urgent production, as shown in Table 12. The purchasing cost is 7200. Obviously, there is no inventory cost. In addition, as the initial quota amount is not enough to satisfy the higher demand in the good economy, the company needs to buy additional quotas from market, shown in Table 13, and the cost for purchasing quota is 23000.

Scenario 2: Fair economy

The probability of a fair economy is 10%. If the fair economy is realized, the company will take the corresponding second-stage production loading decisions as follows.

Table 14. Shortage/surplus production

Product	Purchased products from contractors				Inventory			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1								200
2						100	200	300
3					100	200	300	400

Table 15. Under/-over quota

Product	Purchased quota				Unused quota			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	100						100	300
2							100	200
3	100							100

In the fair economy scenario, all random constraints are satisfied. Therefore, no infeasibility cost is incurred. At the same time, the first stage production is able to satisfy the demand in the fair economy. Thus there is no purchasing cost involved for urgent production. However, some products produced in the first stage are left: as shown in Table 14, this results in an inventory cost of 1500. The initial quota amount allocated in period 1 for products 1 and 3 cannot satisfy the demand in period 1 in the fair economy, so the company needs to buy a certain amount of quotas for products 1 and 3 - these are shown in Table 15. The cost of purchasing quota is 3200. However, the initial quota amount allocated in period 3 and 4 exceed the demand in those two periods, as shown in Table 15. The penalty cost for the unused quotas is 1900.

Scenario 3: Bad economy

The probability of a bad economy is 80%. If the future economy is bad, the company will take the second-stage production loading decisions as follows:

Table 16. Shortage/surplus production

Product	Purchased products from contractors				Inventory			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1					100	200	300	600
2					100	300	500	700
3					200	400	600	800

Table 17. Under/-over quota

Product	Purchased quota				Unused quota			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1						100	300	600
2					100	200	400	600
3						100	200	400

In the bad economy scenario, all random constraints are satisfied. There is no infeasibility cost. Meanwhile, as the first-stage production is able to satisfy demand in the fair economy, there is no purchasing cost involved for urgent production. However, some products produced in the first-stage are left, as shown in Table 16. This results in an inventory cost of 4040. The initial quota amount allocated in each period is also too much for the demand in the bad economy, as shown in Table 17, and the penalty cost for not fully using the initial quota allocated is 4800.There is no cost for purchasing quotas.

6.3. Comparison with the two-stage recourse programming model

Table 18 gives the computational results of the robust optimization model and the two-stage recourse programming model. The total cost under the recourse model is 436557 (See the second row in Table 18), and the total cost under the robust model is 436194 (See the third row in Table 18),. Using the robust optimization model, the total cost decreases by 0.083%, and the expected variability of the robust model decreases 78.03%, which means the robust model presents a less sensitive production loading strategy. However, the robust model involves the infeasibility cost of 6000 for not satisfying all market demand. If we increase the weighting penalty of ω^1 and ω^2 to 150 (See the last row in Table 18), no random constraint is violated. Compare this with the recourse model, in which the expected variability decreases 55.49%, and the total cost of the robust model only increases by 0.30%. It means that the dual response production loading plan proposed by the robust model is not expensive, and it reduces the risk.

Table 18. Comparing the robust model and the recourse model.

	Expected variability	Expected infeasibility	First stage cost	Second stage cost	Expected cost	Expected variability cost	Expected infeasibility cost	Total cost
Recourse model	17709	0	418100	18457	436557	0	0	436557
Robust optimization model ($\lambda = 0.1$, $\omega^1=\omega^2= 100$)	3890	60	419053	10752	429805	389	6000	436194
Robust optimization model ($\lambda = 0.1$, $\omega^1=\omega^2= 150$)	7882	0	42284	14792	437076	788	0	437864

6.4. Tests

We perform three different tests under different probabilities for the production loading problems. Except for the probability of occurrences of future economic situations, all other conditions in the three tests are the same. From Table 19, we can see that Test I represents the situation where it is most likely that economy will be good, Test II where it is most likely that economy will be fair and Test II represents where it will be bad.

Table 19. Three tests.

Test	p_1=Pr{s_1}	p_2=Pr{s_2}	p_3=Pr{s_3}
Test I	0.8	0.1	0.1
Test II	0.1	0.8	0.1
Test III	0.1	0.1	0.8

6.4.1. Computational results for robust optimization model with solution robustness

Table 20 shows the computational results of the robust optimization with solution robustness for the three tests, in which λ is assigned different values.

Table 20. Computational results for robust optimization model with solution robustness.

Test	λ	Expected variability	First stage cost	Second stage cost	Expected cost	Expected variability cost	Total cost
Test I	0*	4200	424531	20375	444906	0	444906
	0.1	4200	424531	20795	445326	4202	445326
	0.5	4200	424531	22475	447006	2100	447006
	0.9	0	424531	23000	447531	0	447531
	0*	18661	418100	13694	431784	0	431794
Test II	0.1	11025	421217	12176	43393	1103	43393
	0.5	3624	424531	10777	435308	1812	435308
	0.9	3624	424531	12227	436757	3262	436757
Test III	0*	17709	418100	18457	436557	0	436557
	0.1	7882	422284	15580	437864	788	437864
	0.5	1944	424531	14252	438783	972	438783
	0.9	1944	424531	15030	439560	1750	439560

Note: * represents where the robust optimization model becomes the two-stage stochastic recourse programming model

We first analyze the whole trend of the three tests. When λ=0, the robust optimization model becomes a two-stage stochastic recourse model in which the variability is not considered. In Table 20, for each rest, the expected variability for the two-stage recourse model is greater-than-or-equal-to that of the robust optimization model. This means that the two-stage stochastic recourse model is riskier than the robust optimization model with solution robustness. The total cost of the robust-optimization model is greater than that of the two-stage stochastic recourse model. Compared with the recourse model, the total cost of robust model increases by 0.59% in Test I, 1.15% in Test II, and 0.69% in Test III. However, the variability decreases by 100% in Test I, 80.58% in Test II, and 89.02% in Test III.

In Test I, the first stage cost of the recourse model is equal to the first stage cost of the robust model. However, the second stage cost increases when λ increases, which means the different decisions in the second stage are made based on the decision makers' risk attitude (different λ values). The expected variability in Test I keeps constant (4200) until λ increases to 0.9. In Test II and Test III, the expected variability decreases when λ decreases, which means the risk in Test I is less than in Test II and III. Test I represents the situation where the future economy is most likely to be good. Once the unexpected situation (fair or bad) happens, the second stage cost arises mainly from the surplus cost for inventory and over-quota cost for penalizing the unused quota. This cost, however, is less than the second stage cost in Tests II and III, which mainly arises from purchasing products and buying quotas. In Table 20, the expected variability of the recourse model in Test I is 4200. However, the expected variability of the recourse model for Tests II and III is 18661 and 17709, respectively. Compared with the recourse model, it is more important to use the robust model with solution robustness in Tests II and III than in Test I, as the risk is higher in Tests II and III.

6.4.2. Computational results for the robust optimization model with model robustness

Table 21 shows the computational results of the robust optimization with model robustness for the three tests. In the tests, ω is used to represent ω^1 and ω^2. Thus we have: $\omega = \omega^1 = \omega^2$.

Table 21. Computational results for robust optimization model with model robustness.

Test	ω	Expected infeasibility	First stage cost	Second stage cost	Expected cost	Expected Infeasibility cost	Total cost
Test I	$0^{\dagger}$	12364	364534	0	364534	0	4534
	20	1290	418069	4315	422384	12900	435284
	50*	0	424531	20375	444906	0	444906
Test II	$0^{\dagger}$	10221	354534	0	364534	0	364534
	10	300	413864	1550	415354	3000	418354
	50	120	414239	5578	419817	6000	425817
	100	60	416485	8182	424667	6000	430667
	150*	0	418100	13694	431794	0	431794

Test III	0†	11046	364534	0	364534	0	364534
	10	590	406390	3170	409506	5900	415460
	50	180	411850	9143	420993	9000	429993
	100	60	416485	13001	429486	6000	435486
	150*	0	418100	18457	436557	0	436557

Note: † represents the robust optimization model without considering the random demand and quota constraints, and * represents when the robust optimization model becomes the two-stage stochastic recourse programming model.

In the three tests, when ω=0 there is no penalty for violating the second stage constraints consisting of the random demand constraints and random quota constraints. The second stage cost is equal to 0 in the three tests (see the first row in each test) because no decision is made in the second stage. However, the expected infeasibility is very high: 12364 in Test I, 10221 in Test II, and 11046 in Test III (see the third column), which means the higher violation of the random constraints. When ω increases, the expected infeasibility decreases, and the total cost increases. When ω increases by enough, the expected infeasibility becomes zero, which means that all random constraints in the second stage are satisfied because of the higher penalty for the infeasibility. The robust optimization model then becomes the two-stage stochastic recourse model (see the final row in each test). From Tables 20 and 21, we know that the first row of each test in Table 20 (when λ=0) has the same result as that shown in the final row in Table 21 (when ω is large enough), as both of them represents the result of the two-stage stochastic recourse programming model.

6.4.3. Computational results for the robust optimization model with the trade-off between solution robustness and model robustness

Parameter λ and ω are used to measure the trade-off between solution robustness and model robustness. When $\omega = 0$, there is no penalty for the infeasibility of random constraints in the objective function. The infeasibility representing un-fulfilment is a higher value. Clearly, decision-makers would not like this kind of production loading plan. However, a large weight ω^1 and ω^2 means the penalty function dominates the total objective function value and would result in a higher variability and a higher total cost. Therefore, there is always a trade-off between the risk and the cost. During the production loading process, it is necessary to check the proposed robust optimization model with difference λ in order to measure the trade-off between the risk and cost.

When λ keeps constant

Figures 1~3 show the computational results for Test II in terms of the variability, infeasibility, and total cost, when λ keeps constant.

Figure 1 gives the trend of the variability when ω increases for λ =0.1, 0.5, and 0.9, respectively. For λ =0.1, when ω increases, the variability sharply increases from 1102 to 11025. However, the variability keeps steady at 11025 after ω increases to 150. When λ =0.5, and 0.9, the value of ω has a small impact on the variability. The reason for this is that when λ is given a large value, the variability cost dominates the objective function value, and the infeasibility cost measured by ω has less impact on the total cost.

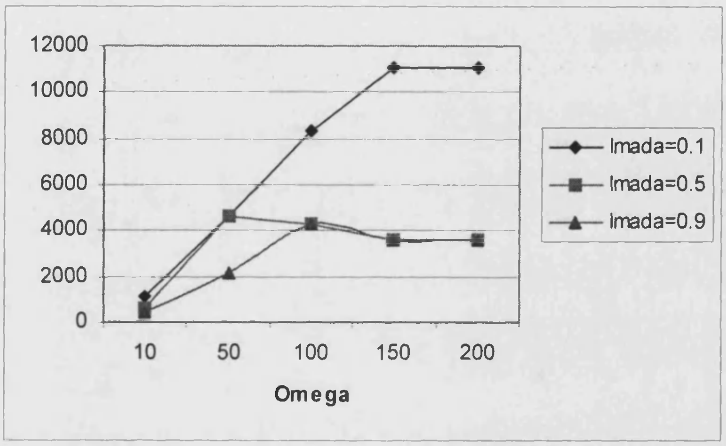

Figure 1. Variability.

Figure 2 gives the trend of the infeasibility when ω increases for λ =0.1, 0.5, and 0.9, respectively. Clearly, the value of ω has a big influence on the system's infeasibility.

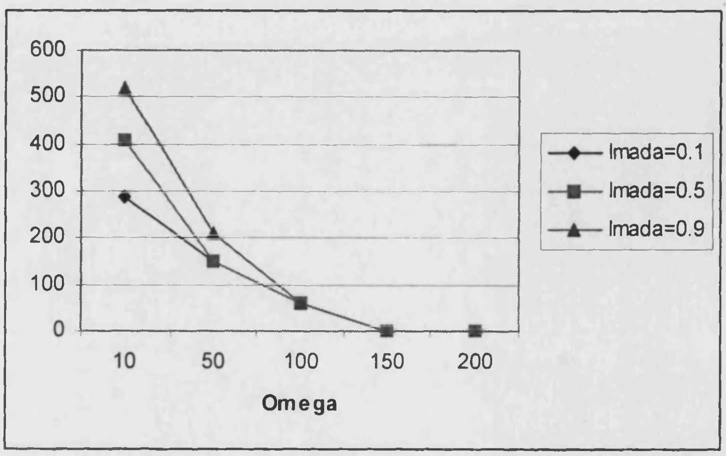

Figure 2. Infeasibility.

In Figure 3, when ω increases, the total cost increases accordingly. The value of ω has more impact on the system when the value of λ is small.

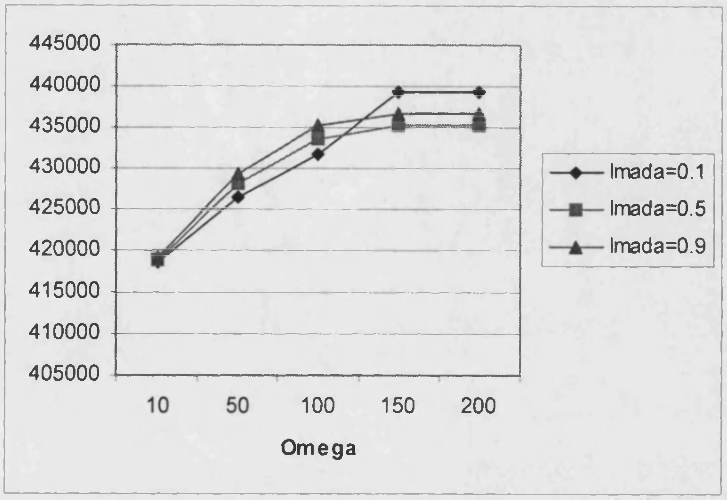

Figure 3. Total cost.

When ω keeps a constant

Figures 4~6 show the computational results of Test II in terms of the variability, infeasibility, and total cost, when ω keeps constant.

Figure 4 shows the trend in the variability when λ increases for ω =10, 50, 100, and 150, respectively. If λ increases from 0.1 to 0.9, for ω =10, the variability decreases by 63.25%; for ω =50, the variability decreases by 54.01%; for ω =100, the variability decreases by 48.34%; for ω =150, the variability decreases by 67.13%. The value of λ has a great impact to the variability.

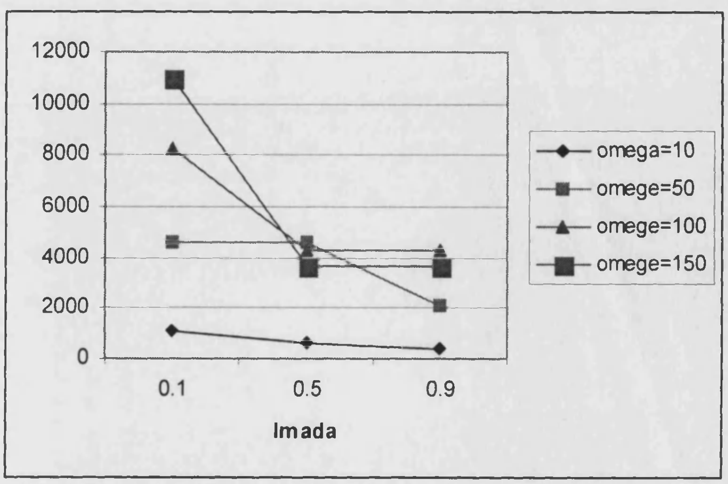

Figure 4. Variability.

Figure 5 shows the trend of the infeasibility when λ increases for ω =10, 50, 100, and 150, respectively. The greater the value of ω, the less the value of λ has an impact on the variability. If λ increases from 0.1 to 0.9, for ω =10, the variability increases by 81.28%; for ω =50, the variability increases by 40%; for ω =100, and 150, the value of λ has no impact on the infeasibility. The reason for this is that when ω is given a large value, the infeasibility cost dominates the objective function value, and the variability cost measured by λ has less impact on the total cost.

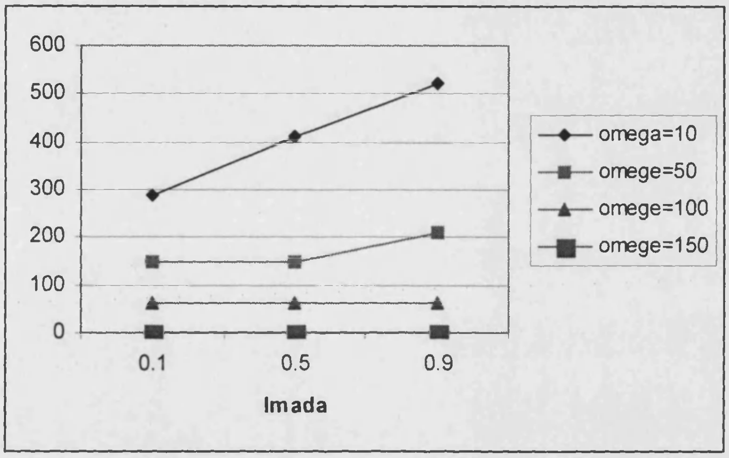

Figure 5. Infeasibility.

Figure 6 shows the trend of the total cost when λ increases for ω =10, 50, 100, and 150, respectively. If λ increases from 0.1 to 0.9, for ω =10, the total cost increases by 0.13%; for ω =50, the total cost increases by 0.68%; for ω =100, the total cost increases by 0.85%; for ω =150, the variability increases by 0.78%. Compared with the changes in variability and infeasibility, the total cost only increases by a small amount when λ increases. This means that the robust model proposed in this study is not expensive for a low risk dual-response production loading system.

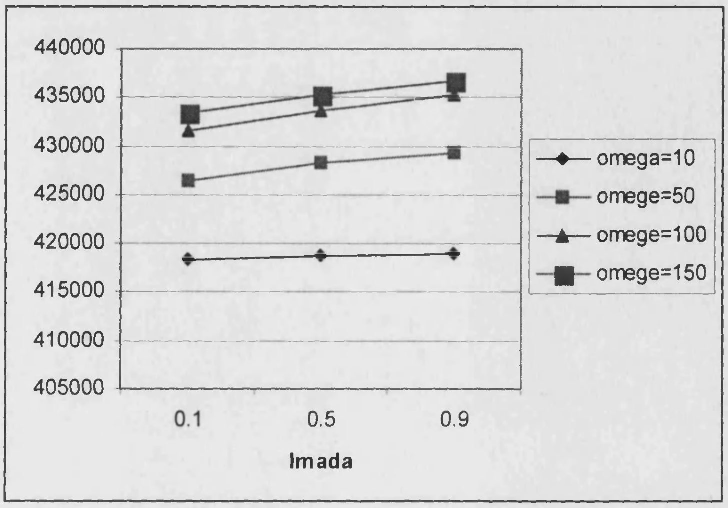

Figure 6. Total cost

6.5. Model validation

To validate the efficiency of the models, a series of computational experiments are carried out using the data provided by the company for 12 months. Based on the company's strategies, all customer orders have to be fulfilled, which leads to using the robust optimization model with solution robustness proposed in this study. Figure 7 and 8 shows the variability and total cost for 12 months. We can see that robust model has less risk than the two-stage stochastic recourse model, but the cost for reducing the risk is not high.

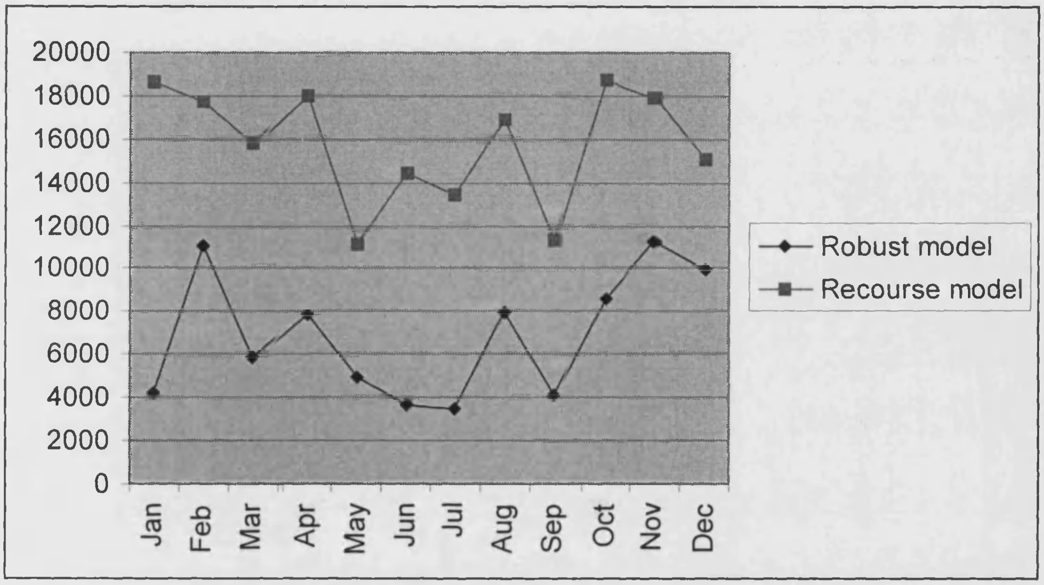

Figure 7. Variability.

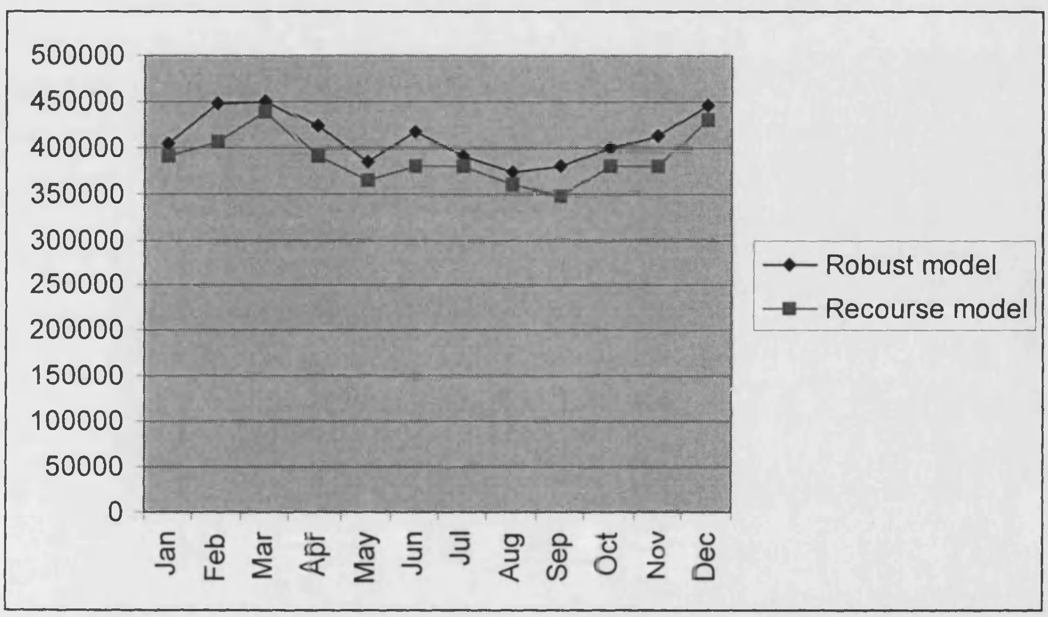

Figure 8. Total cost.

7. Conclusions

The global supply chain management environment is forcing manufacturing companies to provide competitive manufacturing strategies. This study provides a quantitative approach to forming a dual-response production loading strategy in dealing with uncertain market information, increasingly shortening lead times, as well as the greater risks involved. Three different types of robust optimization models are proposed: the robust optimization model with solution robustness, the robust optimization model with model robustness, and the robust optimization model with the trade-off between solution robustness and model robustness. A global manufacturing garment company is selected to be an example for these three types of robust optimization models. By analyzing the different weights in the robust models, the dual-response production loading strategies are determined in terms of the cost and risk. From a series of computational tests, we can conclude that the robust optimization models have advantages over the two-stage stochastic recourse model in dealing with the uncertainty and risk. The robust model solutions are progressively less sensitive to the realizations of the stochastic variables, and are able to handle the infeasibility that occurs in the two-stage recourse programming model. However, as the robust optimization still belongs to goal programming, there is no *a priori* mechanism for specifying a "correct" choice of the parameters, as is prevalent in multi-criteria programming. In addition, robust optimization does not provide a means of specifying a scenario set, which also occurs, when formulating a two-stage stochastic recourse programming model. Further research will consider designing a robust global supply chain system that integrates different activities in the global supply chain networks, such as integrating production, warehouse, road transport, sea transport, air transport, etc.

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Appendix B

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A Mixed-integer programming model for global logistics transportation problems

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Abstract: In today's highly competitive global environment, companies are forced to compete on price and delivery speed. Global logistics transportation presents some special challenges and issues for business organizations, and these issues differ from those posed by domestic logistics transportation. This study considers road transportation problems between two countries. A mixed programming model is formulated to determine the optimal fleet components, route plans, and warehouse control in two countries. A series of experiments is designed to test the effectiveness of the proposed model. To enhance the practical implications of the model, different logistics plans are evaluated according to future changes.

Keywords: Global Logistics; Global Supply Chain Management; Globalization; Road Transportation; Mixed Integer Programming.

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1. Introduction

Over the past ten years supply chain management has become an important focus of competitive advantage for companies and organizations (Harrison, 2003). Logistics, as a critical part of the supply chain management, controls capital, materials, services, and information to anticipate customer requirements. Today's business is set in a global environment in which materials and products can be bought, manufactured and sold anywhere in the world wherever possible. Logistics has never played such an important role in global supply chain networks, because the movement of shipments from supply site to demand site tends to be more frequent than ever before. Several forces are currently highlighting the importance of logistics in the global supply chain management environment:

1. *Globalization*: Global companies seek to achieve competitive advantage by identifying world markets for its products, and then to develop a manufacturing and logistics strategy to support its marketing strategy (Christopher 2005).
2. *Time-based competition*: Time compression has become a more critical management issue than ever before (Christopher 2005). Business success increasingly replies on speed instead of quality. Quality has become a minimum standard rather than a competitive advantage in many industries. Time has become the next battleground or the next strategic frontier (Tang *et al.* 2005).
3. *Serviced-based competition*: In today's global marketplace, competitive advantage is driven by serviced-based strategies instead of product-based strategies. Customers are used to immediate availability from stock for instant gratification.
4. *Customers taking control:* Customers are empowered by the information they have from the Internet or other sources (Coyle *et al.* 2003). They tend to have low tolerance to poor products and services, and demand quick response and delivery speed, while expecting continuously lowering costs.
5. *Products lifecycles:* Products lifecycles are increasingly shortening (Yang *et al.* 2005), which leaves companies an ever-shortening time to produce, transport, and distribute products.

6. *Focused business/global sourcing*: Companies tend to focus on their core business and outsource other activities to any part of the world that offers low cost and high-quality products or services (Coyle *et al.* 2003). Logistics is considered to be the main sourcing function.
7. *Centralized inventory*: Globalization has encouraged companies to rationalize production into fewer locations, and so it has led to a trend towards the centralization of inventories (Christopher, 2005).
8. *e-Business*: Inexpensive development and use of e-business has lead companies, even small ones, to gain global visibility for their purchasing, production, transportation and distribution.
9. *Information technology:* During the past decade, business organizations have been irrevocably affected by the Internet, computerization, and other advancements.
10. *Growing third-party logistics:* Coyle *et al.* (2003) define a third-party logistics company as an external supplier that performs all or part of a company's logistics functions, such as transportation, warehousing, distribution, financial services, and so on. They also note that there have been significant increases in the number of firms offering such services, and that this trend is expected to continue.

The analysis of logistics and transportation has been an active area for researchers and practitioners since it was invented in the World War II. However, early work purely considered logistics problems as transportation problems without considering other factors in the logistics process such as packing, labelling, warehousing, consolidating, etc. For related work see the bi-criteria transportation problem in Aneja and Nair (1979); fleet size problem in Etezadi and Beasley (1983); multiple objectives transportation problems in Current & Min (1986) and Current and Marish (1993); interactive algorithms to solve multi-objective transportation problems in Ringuest and Rinks (1987) and Climaco *et al.*, (1993); a tabu search approach for the fixed charge transportation problem in Sun *et al.* (1998); and insertion-based savings heuristic algorithms for the fleet size and mixed vehicle routing problem with time window in Liu and Shen (1999).

Global logistics is defined as exporting and importing products or services beyond the boundaries of a country. Global logistics presents logistics managers with a more difficult challenge than domestic logistics in terms of packing, labelling, transport modes and cost, labour cost, warehousing, government policy and regulation etc. Cohen *et al.* (1989) present international supply chain models with the considerations related to global trade in terms of raw materials and production cost, the existence of duties, tariffs, different tax rates among countries, random fluctuations in currency exchange rates, and the existence of constraints not included in single-country models. Fawcett (1992) claims that limited research has been done on international logistics strategy, and that the existing literature focuses on descriptions only. Goldsborough (1992) provides an analytical report on global logistics management in which two different logistics systems – domestic and international – are compared. Cohen and Kleindorfer (1993) present a framework for the operations of a global company to determine plant location and capacity, product categories, material and cash flow in an international scenario. However, no model formulation or experiments are provided in their paper. Vidal and Goetschalckx (1997) think global supply models are more complex and difficult to solve than domestic models, as the flow of cash and the flow of information are more important and difficult to coordinate in an international scenario than they are in a single country environment Goetschalckx *et al.* (2002) give an excellent review of integrated strategic and tactical models and design algorithms for global logistics systems. They point out a great deal of research has been conducted in quantitative techniques for the improvement and optimization of supply chains without global considerations, and mixed-integer programming models are among the most widely-used techniques. They also report that most models address the problem in a regional, local, or single-country environment, where international factors do not have a significant impact on the supply chain design. Geoffrion and Powers (1995) give an evolutionary perspective to 20 years of strategic distribution system design, and think logistics has changed from a neglected activity to an essential business function. Coyle *et al.* (2003) think countries are becoming closer and closer because of the success of logistics. They find many global manufacturers are using a new managerial strategy, called focused production, in which one or a few plants are designated as the worldwide supplier(s) of the given product(s). The plants are typically located in different countries, requiring a global logistics system to deliver items to the right place, in the right quantity, at the right time anywhere in the global marketplace.

Road transport is the most important among all transport modes. Muller (1999) notes that, in the U.S, of the nearly 7.8 million tons of freight and commodities moved in 1996, an estimated 46% was moved by truck (up almost 78% since 1980), compared with 26% by rail, 13% by water, and 15% by pipeline. However, road transportation beyond the boundary of a country only caught the attention of researchers and practitioners a few years ago when globalization became an important issue in business organizations. Bergan and Bushman (1998) present the North America Trade Agreement (NAFTA) perspective on cross-border trucking transportation between the US, Canada, and Mexico, and emphasize the importance of efficient border-crossing systems. Bochner *et al.* (2001) examine the possibility of expediting current port-of-entry processing of commercial vehicles entering the US from Mexico, provide the basic prototype plan for northbound commercial border inspection stations with automated processing, and suggest bi-national links to improve cross-border system's efficiency.

In this study, we consider a global logistic problem for road transportation, which involves transporting goods from one country across the border to another country. There are some differentials between two countries in terms of vehicle operation cost, vehicle capacity, labour cost, warehousing cost, etc. The aim of this study is to present a modelling framework for global logistics transportation problems in order to determine the fleet components of trucks from two countries, as well as transportation route from supply site in one country to demand site in the other country. This paper is organized as follows. After this introduction and literature review in this section, the detailed global logistics process is presented in Section 2. Section 3 presents a mixed-integer programming model for the global logistics transportation problems. In Section 4, a series of experiments are designed to test the effectiveness of the proposed models. Different logistics strategies are provided so that logistics managers can handle complicated future changes for the global logistics problems.

2. Dual response logistic process

Compared with country B, country A is a low-cost country in terms of production, transportation, warehouse, labour, and so on. Two centralized warehouses, 1 and 2, are located in the two countries A and B respectively. It is assumed that both of the warehouses have enough capacity for storing goods. The unit inventory cost in warehouse 2 is much higher than in warehouse 1. The goods are manufactured in country A and are stored in warehouse 1 in country A. However, country B has a certainty amount of demand for the goods. The goods, therefore, need to be transported from warehouse 1 in country A to warehouse 2 in country B. The logistics company has its own trucks with two licenses and which can operate in both countries. However, when the company fleet does not have the capacity to satisfy demand in country B, the company has to hire additional trucks. There are two types of trucks available for rental: the first type of truck only has a license for country A and can only operate in that country; the second type of trucks has two licenses and can operate in both countries. The company has two strategies for delivering goods. The first strategy is to use company-owned trucks or/and hired trucks with two licenses to directly transport goods from warehouse 1 to warehouse 2. The second strategy is first to load the goods into hired trucks with a country 1 license only. Then the goods are trans-shipped into the company-owned trucks or to hired trucks with two licenses at the border in order to get across to country B. The goods cannot stay overnight on the border, as there is no warehouse there. Although the transhipment involves a certain cost associated with unloading and loading, the company may adopt this strategy as the cost of a hiring truck with a country A license only is very low. Therefore, the road network consists of three routes: Route 1, connecting warehouse 1 in country A and warehouse 2 in country B; Route 2, connecting warehouse 1 and the trans-shipment point on the border in country

A; and Route 3, connecting the trans-shipment point on the border in country A and warehouse 2 in country B. As shown in Figure 1, Routes 1 and 3 include a border-crossing process.

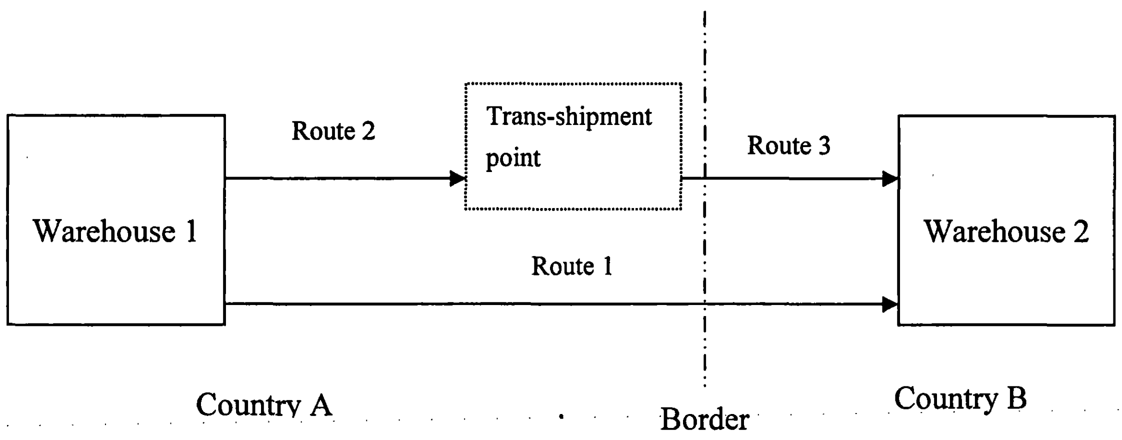

Figure 1: Truck routes

It is assumed the cost of hiring a truck either with one license or two licenses only covers one trip each day between two places. If the truck makes two trips, the hiring cost will double so the company does not adopt this strategy. If necessary, the company could hire more trucks, as this ensures faster delivery for the same cost. Thus, only company-owned trucks will make a round-trip journey every day. In addition, trucks with two licenses will not be allowed to operate Route 2, which connects warehouse 1 and the trans-shipment point within country A, because this is a waste of fleet resources.

The purpose of this study is to find an optimal global logistics transportation strategy including optimal composition of the company's fleet and route plans to minimize total cost.

3. Model formulation

3.1. Indices

I^0 = set of company-owned trucks with licences to operate in both countries.

I^1 = set of hired trucks with a country A licence only.

I^2 = set of hired trucks with licenses for both countries.

J = set of routes. J={1, 2,3}.

T= set of time periods.

K = set of round-trips.

i=index of trucks, $i \in I^0 \cup I^1 \cup I^2$

j= index of routes, $j \in J$.

t= index of time periods, $t \in T$.

k=index of round-trips, $k \in K$.

3.2 Parameters

s_t = amount of products arriving in country A's warehouse on day t, $t \in T$;

d_t = amount of products demanded in country B on day t, $t \in T$.

Truck capacity

L_i = maximum loading capacity of truck i, $i \in I^0 \cup I^1 \cup I^2$.

Company-owned trucks

c_{ij}^0 = unit trip cost of company-owned truck i operating on Route j, $i \in I^0$, j={1,3};

r_j = a round-trip time using Route j, j={1,3};

H= maximum working hours for drivers of company-owned trucks per day.

Hired trucks

c_i^1 = unit hiring cost of truck i operating in country A on Route 2, $i \in I^1$;

c_i^2 = unit hiring cost of truck i operating in countries A and B on Routes 1 and 3, $i \in I^2$.

Warehousing/trans-shipping

w_0^1 = initial amount of products stored in warehouse 1 in country A;

w_0^2 = initial amount of products stored in warehouse 2 in country B;

c^1 = unit inventory cost in warehouse 1;

c^2 =unit inventory cost in warehouse 2;

c^T=unit cost of trans-shipment on the border.

Penalty cost

c^3 =unit penalty cost for not satisfying the demand in country B.

3.3 Decision variables

Trucks used

$$x^0_{ijkt} = \begin{cases} 1 & \text{if company-owned truck } i \text{ operates on Route } j \text{ on the } k^{\text{th}} \text{ round on day } t \\ 0 & \text{otherwise} \end{cases},$$

$i \in I^0$, $j=\{1,3\}$, $k \in K$, $t \in T$;

$$x^1_{it} = \begin{cases} 1 & \text{if hired truck } i \text{ operates from country A to border on day } t \\ 0 & \text{otherwise} \end{cases}, \ i \in I^1,\ t \in T;$$

$$x^2_{ijt} = \begin{cases} 1 & \text{if hired truck } i \text{ operates on Route } j \text{ on day } t \\ 0 & \text{otherwise} \end{cases}, \ i \in I^2, j=\{1,3\},\ t \in T.$$

Amount loaded

X^0_{ijkt} = amount of goods loaded by company-owned truck i on Route j on k^{th} round on day t, $i \in I^0$, $j=\{1,3\}$, $k \in K$, $t \in T$;

X^1_{it} =amount of goods loaded by hired truck i on Route 2 on day t, $i \in I^1$, $t \in T$;

X^2_{ijt} =amount of goods loaded by hired truck i on Route j on day t, $i \in I^2$, $j=\{1,3\}$, $t \in T$.

Surplus/shortage

w^1_t =surplus in warehouse 1 on day $t, t \in T$;

w_t^2 = surplus in warehouse 2 on day $t, t \in T$;

w_t^3 =shortage in country B on day t, $t \in T$.

3.4 Constraints

Warehouse 1 constraint

Constraint (1) ensures that, on day *t*, the total volume of the products that arrive in warehouse 1 plus the products already stored in the warehouse is equal to the sum of the products left at the end of day, the products transported to the trans-shipment point on Route 2 by the hired trucks using a country A license, and the products transported to warehouse 2 on Route 1 by the company-owned trucks or hired trucks with two licenses.

$$s_t + w_{t-1}^1 = w_t^1 + \sum_{i \in I^1} X_{it}^1 + \sum_{k \in K} \sum_{i \in I^0} X_{i1kt}^0 + \sum_{i \in I^2} X_{i1t}^2, t \in T. \quad (1)$$

Warehouse 2 constraint

Constraint (2) ensures that, on day *t*, the total amount of the products that arrive in warehouse 2 plus the products already stored in warehouse 2 is equal to the total products needed by the country B's markets, plus surplus products in warehouse 2, minus any shortage of products in warehouse 2.

$$w_{t-1}^2 + \sum_{k \in K} \sum_{j=\{1,3\}} \sum_{i \in I^0} X_{ijkt}^0 + \sum_{j=\{1,3\}} \sum_{i \in I^2} X_{ijt}^2 = d_t + w_t^2 - w_t^3, t \in T. \quad (2)$$

Trans-shipment constraint

Constraint (3) ensures that, on day *t*, the total products arriving at the transhipment point on the border is equal to the total products leaving the trans-shipment point to go to warehouse 2. This constraint is needed since the goods cannot be kept at the trans-shipment point overnight.

$$\sum_{i \in I^1} X_{it}^1 = \sum_{k \in K} \sum_{i \in I^0} X_{i3kt}^0 + \sum_{i \in I^2} X_{i3t}^2. \quad (3)$$

Work time constraint

Constraint (4) ensures that the working hours for drivers of the company-owned trucks cannot exceed their maximum working hours.

$$\sum_{j=\{1,3\}} \sum_{k \in K} r_j x^0_{ijkt} \le H \text{, } i \in I^0 \text{,} j=\{1,3\} \text{, } t \in T \text{.} \tag{4}$$

Round-trip constraints

Each company-owned truck could make the next round trip only after the previous round trip has been completed.

$$x_{ijkt} \ge x_{ij,k+1,t} \text{, } i \in I^0 \text{,} j=\{1,3\} \text{, } k \in K \text{, } \quad t \in T \text{.} \tag{5}$$

Capacity constraints

Constraints (6) ~ (7) ensure that, for every truck, the loading amount of products cannot exceed its capacity.

$$X^0_{ijkt} \le L_i x^0_{ijkt} \text{, } i \in I^0 \text{,} j=\{1,3\} \text{, } k \in K \text{, } \quad t \in T \text{.} \tag{6}$$

$$X^1_{it} \le L_i x^1_{it} \text{, } i \in I^1 \text{, } t \in T \text{.} \tag{7}$$

$$X^2_{ijt} \le L_i x^2_{ijt} \text{,} i \in I^2 \text{,} j=\{1,3\} \text{, } t \in T \text{.} \tag{8}$$

Variable type constraints

$$x^0_{ijkt} \in \{0,1\}, X^0_{ijkt} \ge 0, i \in I^0, j = \{1,3\}, k \in K, t \in T \text{.} \tag{9}$$

$$x^1_{it} \in \{0,1\}, X^1_{it} \ge 0, i \in I^1, t \in T \tag{10}$$

$$x^2_{ijt} \in \{0,1\}, X^2_{ijt} \ge 0, i \in I^2, j = \{1,3\}, t \in T \text{.} \tag{11}$$

$$w^1_t, w^2_t, w^3_t \ge 0, t \in T \text{.} \tag{12}$$

3.5 Costs

Company-owned trucks cost

This cost is associated with fuel, maintenance, loading cost, labour cost, etc. The company-own trucks only operate on Route 1 connecting warehouse 1 and warehouse 2 and on Route 3 connecting the trans-shipment point at the border and warehouse 2.

$$CC = \sum_{t\in T}\sum_{k\in K}\sum_{j=\{1,3\}}\sum_{i\in I^0} c_{ij}^0 x_{ijkt}^0 \tag{13}$$

Hiring cost

The hired trucks with a country A licence only operate on Route 2, while the hired tucks with licenses for both countries operate on Routes 1 and 3, which includes the cost of crossing the border.

$$HC = \sum_{t\in T}\sum_{i\in I^1} c_i^1 x_{it}^1 + \sum_{t\in T}\sum_{j=\{1,3\}}\sum_{i\in I^2} c_i^2 x_{ijt}^2 \tag{14}$$

Trans-shipment cost

When products are transported from Warehouse 1 in country A to the trans-shipment point on the border using Route 2, products need to be unloaded from the trucks, and are loaded into the truck with two licenses on order to cross the border. The change cost involves the unloading and loading cost.

$$TC = \sum_{t\in T}\sum_{i\in I^1} c^T X_{it}^1 \tag{15}$$

Warehouse cost

An inventory cost is incurred at warehouse 1 when the goods are not fully transported to country B on day t and have to be stored in warehouse 1 on day t. An inventory cost is also incurred in warehouse 2 when the total goods being stored and arriving in warehouse 2 exceed the demand from country B on day t.

$$WC = \sum_{t\in T} c^1 w_t^1 + \sum_{t\in T} c^2 w_t^2 \tag{16}$$

Penalty cost

The company will incur a penalty when demand is not satisfied.

$$PC = \sum_{t \in T} c^3 w_t^3 \tag{17}$$

3.6 A mixed-integer programming model

The objective is to minimize the sum of all costs listed in Section 3.5, and satisfy all constraints described in Section 3.4. The global logistics problem can be formulated as a mixed-integer programming model as follows:

$$\text{Min } CC+HC+TC+WC+PC \tag{18}$$

s.t. (1)~(12)

4. Experiments

4.1 A practical global logistics problem between mainland China and Hong Kong

All data that is used in this study is provided by a third-party logistics company. The company has two warehouses: one is located in Guangzhou, Southern China, while the other is in Hong Kong's port terminal. Goods manufactured in mainland China arrive at the mainland Chinese warehouse. The logistics company is responsible for transporting these goods from the Guangzhou warehouse to the Hong Kong warehouse from where the goods can be shipped to overseas markets.

Because of China's booming economy and its low manufacturing cost, more and more global companies have been establishing their production facilities in mainland China. Currently, consumer markets are mainly centered in North America and Europe. However, it is still difficult to move goods around China and many global companies prefer to ship their goods from Hong Kong. China's transportation and logistics sector has historically been under government control until only a few years ago. Logistics is not yet a well-defined industry in China and there is still little integration in the provision of logistics services throughout China. In China, logistics is seen as consisting of a number of sub-sectors, such as (air, sea, road and rail) transportation, warehousing, consolidation, freight forwarding and customer brokerage, etc. Most companies participate in one or a few of the parts of this service rather than providing an integrated and whole logistic service. Located at the mouth of the Pearl River with a deep natural harbour, Hong Kong is geographically and strategically important as a gateway for China and is an important trans-shipment port for intra-Asian and world trade. Hong Kong is the eighth largest trading entity in the world and hosts the world's busiest container port. It has also been the major contact point for China (especially Southern China) with the rest of the world for decades, and this intermediary role has been further enhanced in recent years because of its global supply chain management environment.

The logistics company under this study has three trucks (V1, V2 and V3). Each truck has a capacity of 250 units. The costs of a trip on Routes 1 and 3 are 300 and 200, respectively. There are 4 trucks (V4, V5, V6 and V7) with a China license that the company can rent. The capacity of each truck is 250, and the cost of hiring each truck is 500. In addition, there are 2 trucks (V8 and V9) with China and Hong Kong licenses available for rental. The capacity of each of these trucks is 450, and the hiring cost for each truck for each round trip is 1500. The round trip time for Routes 1, 2 and 3 are 6 hours, 3 hours, and 5 hours respectively. However, the drivers' maximum working time is 10 hours every working day. The unit inventory cost in the China warehouse is 1, and the unit inventory cost in the Hong Kong warehouse is 5. The unit trans-shipment cost is 0.5. The unit penalty cost for not satisfying demand is 12. We also assume that the two warehouses have enough space to store any goods left.

The model is solved using AIMMS 3.4, which is a new type of mathematical modelling software, and is provided by Bisschop and Boelofs in 1999. All problems are executed on a Pentium IV 2.60 GHz PC.

4.2 Computational result and analysis from a single day

Three tests are presented to illustrate the proposed model on a fixed day (see Table 1). Test I represents the case when the demand from Hong Kong is equal to supply arriving in the Guangzhou warehouse in mainland China. Test II represents the case when supply exceeds demand. Test III represents the case when supply is less than demand.

Table 1: The data on three tests for a fixed day

Test	Supply	Demand
Test I	2000	2000
Test II	1600	1400
Test III	1800	1900

After solving the model, the optimal routes and fleet composition can be obtained. The results are shown in Table 2, and the related costs incurred are given in Table 3. From Table 2, we know that when the supply is equal to the demand, there is no surplus or shortage of products in both the warehouses (see Test I). When supply exceeds demand (see Test II), the surplus products are usually stored in the mainland China warehouse because of its cheap inventory cost. When supply is less than demand (see Test III in Table 2), the shortage penalty incur, which is very 1200 (See Table 3), representing a penalty for underachievement.

The current unit shortage cost is 12. When the unit shortage cost is decreased by 1 to 11, and other conditions are not changed, the results for Test II and III are unchanged. However, the result of Test I is changed. The result, represented by Test I`, is shown in Tables 2 and 3. In Test Ĩ, warehouse 1 in Guangzhou has a surplus of 100. However, there is also a shortage of 100 in country B. Because of the smaller unit penalty cost for shortage in Test Ĩ , the company would prefer to pay a penalty cost of 1000 and a surplus cost of 100 instead of transporting only 100 goods using a truck. The total cost in Test Ĩ is 6350. However, when the unit shortage cost is 12, the company has to change its plans (see test I in Table 2) by sending all goods to country B because of higher penalty cost associated with the shortage.

Table 2: The optimal route plan with vehicle composition for a fixed day

Test	Route 1		Route 2	Route 3		Surplus In warehouse 1	Surplus In warehouse 2	Shortage in country 2
	Company-owned trucks	Hired trucks with two licenses	Hired trucks with one license	Company -owned trucks	Hired trucks with two licenses			
Test I	V2 (250)	V8 (450) V9 (450)	V4 (100) V5 (250) V6 (250) V7 (250)	V1(100) V1(250) V3 (250) V3 (250)				
Test II	V1 (200) V2(250) V3 (250)	V8 (450) V9 (450)				100		
Test III	V1 (250) V3 (250)	V8 (450) V9 (450)	V5 (150) V6 (250)	V2 (150) V2 (250)				100
Test Ĩ	V1(250) V2 (250)	V8 (450) V9 (450)	V4 (250) V5 (250)	V3 (250) V3 (250)		100		100

Table 3: Summary of costs incurred for a fixed day

Test	Trip cost	Hiring cost	Surplus cost	Change cost	Shortage cost	Total cost
Test I	1100	5000	0	425	0	6525
Test II	900	3000	200	0	0	4100
Test III	1000	4000	0	200	1200	6400
Test Ĩ	1000	4000	100	250	1000	6350

4.3 Computational result and analysis from six days

In the following, the managerial plan for six working days is considered. Three sets of six-day tests with various levels of required demand are analysed and shown in Table 4. Test IV shows the situation when supply is equal to demand daily; Test V when supply is more than or equal to demand daily; and Test VI when supply is less than or equal to demand daily. Table 5 summarizes the costs incurred for the three tests.

Table 4: Three test data of supply and demand for six working days

		Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Total
Test IV	Supply	1600	2100	1800	2000	1500	1700	10700
	Demand	1600	2100	1800	2000	1500	1700	10700
Test V	Supply	2000	1800	2300	1600	2100	1500	11300
	Demand	1800	1700	2200	1500	1900	1400	10500
Test VI	Supply	1700	1900	2000	1900	1800	2000	11300
	Demand	1700	2000	2050	1900	1900	2050	11600

Table 5: Summary of costs incurred in the three tests

Test	Trip cost	Hiring cost	Surplus cost	Shortage cost	Trans-shipment cost	Total cost
Test IV	5800	22000	500	1200	925	30425
Test V	5500	22000	3500	0	1000	32000
Test VI	6000	24000	450	3600	1450	35500

Table 6 gives the optimal solution of six days for Test IV. From Table 6, we can know that although the demand from country B is equal to the supply in country A, the company does not have to transport all goods from country A to B to satisfy the demand. For example, on day 4, Country B has a shortage of 100, but warehouse 1 in country A has an inventory of 100. The optimal solutions suggest that it is not necessary to hire additional trucks to deliver small amounts (only 100 units). The company would like to wait one or more days when more goods need to be transported from country A to B, even the inventory and shortage cost incur simultaneously.

Table 6: Test IV results for six days

Day	Company-owned trucks	Hired trucks with two licenses	Hired trucks with one license	Company -owned trucks	Hired trucks with two licenses	Surplus in warehouse 1	Surplus in warehouse 2	Shortage in country B
Day 1	V1 (200) V2 (250) V3 (250)	V8 (450) V9 (450)						
Day 2	V5 (250)	V8 (450) V9 (450)	V4 (200) V5 (250) V6 (250) V7 (250)	V1 (200) V1 (250) V3 (250) V3 (250)				
Day 3	V2(250) V3 (250)	V8 (450) V9 (450)	V4 (150) V6 (250)	V1 (150) V1 (250)				
Day 4	V1 (250) V2 (250)	V8 (450) V9 (450)	V4 (250) V7 (250)	V3 (250) V3 (250)		100		100
Day 5	V1 (150) V2 (250) V3 (250)	V8 (450) V9 (450)				50	50	
Day 6	V1 (250) V2 (250) V3 (250)	V8 (450) V9 (450)				100		

Test V represents the situation when the supply in country A is greater than the demand in country B. From Table 7, we can see that, on days 1 and 2, there are some goods are left in warehouse 2 in country B. This method fully utilizes the truck load, although the inventory cost in warehouse 2 is much higher than that in warehouse 1. We can see that all trucks reach their maximum capacity during the whole week in Test V.

Table 7: Test V results for six days

Day	Route 1		Route 2	Route 3		Surplus in warehouse 1	Surplus in warehouse 2	Shortage in country B
	Company-owned trucks	Hired trucks with two licenses	Hired trucks with one license	Company -owned trucks	Hired trucks with two licenses			
Day 1	V2 (250) V3 (250)	V8 (450) V9 (450)	V4 (250) V7 (250)	V1 (250) V1 (250)		100	100	
Day 2	V1 (250) V2 (250) V3 (250)	V8 (450) V9 (450)				250	50	
Day 3	V3 (250)	V8 (450) V9 (450)	V4 (200) V5 (250) V6 (250) V7 (250)	V1 (200) V1 (250) V2 (250) V2 (250)		400		
Day 4	V1 (100) V2 (250) V3 (250)	V8 (450) V9 (450)				500		
Day 5	V1 (250) V2 (250)	V8 (450) V9 (450)	V5 (250) V7 (250)	V3(250) V3(250)		700		
Day 6	V2 (250) V3 (250)	V8 (450) V9 (450)				800		

Table 8 gives the optimal solution when supply in country A is less than demand in country B. However, there are still some goods left in the country A warehouse, even when there is a shortage from country B, such as on days 1, 2, and 3. There is always a trade-off between transportation cost, inventory cost, and shortage cost.

Table 8: Test VI results for six days

Day	Route 1		Route 2	Route 3		Surplus in warehouse 1	Surplus in warehouse 2	Shortage in country B
	Company-owned trucks	Hired trucks with two licenses	Hired trucks with one license	Company -owned trucks	Hired trucks with two licenses			
Day 1	V1 (250) V2 (250) V3 (250)	V8 (450) V9 (450)				50		50
Day 2	V1 (250) V2 (250)	V8 (450) V9 (450)	V4 (250) V5 (250)	V3 (250) V3 (250)		50		100
Day 3	V1 (250) V2 (250)	V8 (450) V9 (450)	V6 (250) V7 (250)	V3 (250) V3 (250)		150		150
Day 4	V1 (250) V3 (250)	V8 (450) V9 (450)	V5 (250) V6 (250)	V2 (250) V2 (250)		150		
Day 5	V1 (250) V3 (250)	V8 (450) V9 (450)	V4 (250) V5 (250)	V2(250) V2(250)		50		
Day 6	V2 (250)	V8 (450) V9 (450)	V4 (200) V5 (250) V6 (250) V7 (250)	V1 (200) V1 (250) V3 (250) V3 (250)				

4.4 Further analysis

This study not only offers an optimal solution to the present logistics management problem, but also gives insights into alternative logistics strategies that can help the company meet future rapid changes in terms of hiring costs, inventory costs and shortage costs. Seven scenarios are presented in Table 9. Scenario 1 is to find the optimal solution using the existing data. Scenarios 2 to 3 assume that the unit hiring cost will increase. Scenarios 4 and 5 consider the situation if the unit inventory cost is increased. Scenarios 7 and 8 consider a change of the unit shortage cost in country B.

Table 9: Scenario Description

Scenario	Description
Scenario 1	Use existing data
Scenario 2	Increase unit cost for trucks with two licenses from 1500 to 1600.
Scenario 3	Increase unit cost for truck with one license from 500 to 600.
Scenario 4	Increase unit surplus cost in warehouse 1 from 1 to 2, and then 2 to3.
Scenario 5	Increase unit surplus cost in warehouse 2 from 5 to 6, and then 6 to 7.
Scenario 6	Increase unit shortage cost in country B from 12 to 13, and then 13 to 14.
Scenario 7	Decrease unit shortage cost in country B from 12 to 11, and then 11 to 10.

Scenario 1: Using existing data

The optimal solution for existing six-day data has been obtained, and the summary of the optimal route plan and composition of the fleet are given in Table 10.

Table 10: Summary of the optimal route plan with fleet composition in scenario 1

	Route 1		Route 2	Route 3	
	Company-owned trucks	Hired trucks with two licenses	Hired trucks with 1 license	Company-owned trucks	Hired trucks with two licenses
Test IV	15	12	8	8	0
Test V	13	12	8	8	0
Test VII	12	12	12	12	0

Scenario 2: Increase the unit cost for hiring trucks with two licenses

Due to the increase in the unit hiring cost of trucks with two licences from 1500 to 1600 in Scenario 2, less Hong Kong trucks will be hired (Table 11). In particular, the company-owned trucks choose to make more trips on Route 2 than on Route 1. Results show that Route 2-Route 3 is chosen as the main delivery route because of the increase in cost in hiring trucks with two licenses.

Table 11: Summary of the optimal route plan with fleet composition in scenario 2

	Route 1		Route 2	Route 3	
	Company-owned trucks	Hired trucks with two licenses	Hired trucks with 1 license	Company-owned trucks	Hired trucks with two licenses
Test IV	14	11	10	10	0
Test V	12	11	12	12	0
Test VII	10	10	14	14	0

Scenario 3: Increased the unit cost for hiring trucks with a country A license

As well as considering an increase in the unit cost of hiring trucks with two licenses, it is also important to take into account changes in strategy when the unit cost of hiring a truck with a country A license increases from 500 to 600. Table 12 shows the optimal fleet composition and routes. From Table 12, it can be seen that the increase in the unit hiring cost for a truck with a country A license will directly affect the routes chosen.

Table 12: Summary of the optimal route plan with fleet composition in scenario 3

	Route 1		Route 2	Route 3	
	Company-owned trucks	Hired trucks with two licenses	Hired trucks with 1 license	Company-owned trucks	Hired trucks with two licenses
Test IV	16	12	8	8	0
Test V	14	12	6	6	0
Test VII	15	12	5	5	0

Scenario 4: Increase the unit surplus cost in warehouse 2

When considering the result of increasing the unit surplus cost at warehouse 1. The computation results of three tests show that the optimal fleet component and route plan is identical to that given in Scenario 1. Decision makers do not need to change the optimal solution in Scenario 4, even if the unit surplus cost of warehouse 1 increases 1 from 1 to 2, and then 2 to3.

Scenario 5: Increase the unit surplus cost in warehouse 1

The routing results and the flow of products in Scenario 5 are the same as in Scenario 1 for all three tests. Thus decision makers do not need to change the optimal solution in Scenario 5 even if the unit surplus cost at the mainland China warehouse increases from 5 to 6, and then 6 to 7.

Based on the analysis of Scenarios 4 and 5, we reach the following conclusion: the unit surplus cost in the two warehouses is not a significant factor in determining the vehicle route plan and fleet composition.

Scenario 6: Increase the unit shortage cost in country B

In order to test the results of increasing the unit shortage cost from 12 to 13, and then 13 to 14, we select Test IV as an example. Table 13 shows that when the unit shortage cost is greater than 13, there is no surplus/shortage cost involved. Table 14 gives the optimal route plans, which shows that the route plans do no change when the unit shortage cost is greater than 13.

Table 13: Summary of costs incurred in scenario 6 for Test IV

Unit shortage cost	Trip cost	Hiring cost	Surplus cost	Shortage cost	Trans-shipment cost	Total cost
12	5800	22000	500	1200	925	30425
13	5675	24000	0	0	1450	31125
14	5675	24000	0	0	1450	31125

Table 14: Summary of the optimal route plan with fleet composition in scenario 6 for Test IV

Unit shortage cost in country B	Route 1		Route 2	Route 3	
	Company-owned trucks	Hired trucks with two licenses	Hired trucks with 1 license	Company-owned trucks	Hired trucks with two licenses
12	13	12	8	8	0
13	12	12	10	10	0
14	12	12	10	10	0

Scenario 7: Decrease the unit shortage cost in country B

After understanding the results of increasing the unit shortage in country B, we want to know what happens if the cost falls. Test IV is chosen as an example when the unit shortage cost in country B falls from 12 to 11, and then 11 to 10. Table 15 presents the results and shows that there is an increase trend of the surplus product in warehouse 1 when the unit shortage cost decreases. When the unit shortage cost falls, the total shortage becomes an insignificant component of the total cost, and the trip cost and the hiring cost become the important factors. The plant would like to store more products in warehouse 1 so that the trucks can approach their maximum capacity for every trip. The optimal route plan is shown in Table 16.

Table 15: Summary of costs incurred in scenario 7 for Test IV

Unit shortage cost	Trip cost	Hiring cost	Surplus cost	Shortage cost	Trans-shipment cost	Total cost
12	5800	22000	500	1200	925	30425
11	5675	19000	850	1850	750	28125
10	5500	18500	1200	2300	900	28400

Table 16: Summary of the optimal route plan with fleet composition in scenario 7 for Test IV

Unit shortage cost in country B	Route 1		Route 2	Route 3	
	Company-owned trucks	Hired trucks with two licenses	Hired trucks with 1 license	Company-owned trucks	Hired trucks with two licenses
12	13	12	8	8	0
11	14	12	6	6	0
10	12	12	10	10	0

Finally, in this practical problem, the hired trucks with licenses for both countries are not assigned to Route 3, which connects the trans-shipment point on the border in mainland China with warehouse 2 in Hong Kong. The reason is that the cross-border cost for each hired truck is the same when the truck operates either Route 1 or Route 3. Therefore, all hired trucks with two licenses operate on only Route 1, which directly connects the two warehouses.

5. Conclusions

In this study, a mixed-integer programming model is proposed to deal with the global road transportation problem between two countries. The model can effectively find an optimal transportation strategy in terms of optimal delivery routes and optimal vehicle fleet composition. A real case in a logistics company is studied under this research. Some useful findings are observed. In order to meet future demand, a variety of logistics strategies are provided for different global logistics environments. It is believed that global logistics problems have increased with the implementation of global supply chain management environment. Heuristic algorithms might need to be considered when the number of trucks increases. Further research will consider uncertainty in the decision-making process, such as changing the product supply from country A, or changing the hiring cost: stochastic programming or fuzzy programming techniques can be applied to these new problems.

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Appendix C

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Modeling Containerization of Air Cargo Forwarding Problems

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Key words air container, cargo loading, containerization, globalization, just-in-time (JIT), mixed integer programming (MIP), global supply chain management (GSCM).

Abstract. This study presents a decision-making framework for modelling containerization of air cargo forwarding problems. The objective is to help logistics managers make decisions about how to rent air containers from air carriers and how to load air cargos into these containers. The air carriers can provide different types of air containers with differing weight and volume limits. The problem is further complicated by the cost of renting a container charged by the air carriers: this is based on a fixed cost for using the container and a variable cost that depends on the weight that the container holds. A mathematic programming model is formulated to minimise the total rental cost while satisfying the customer's shipping requirements. The objective function in the model, however, is a non-decreasing piece-wise linear one. We change the model into a mixed integer linear programming model by introducing two new variables, and the new model can be solved by employing many mathematical programming software packages available today. The model is illustrated with practical problems faced by a logistics company, with an analysis of different scenarios.

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1. Introduction

Logistics managers face a very different environment today from that of only a few years ago. Today's business is set in a global supply chain management environment. Global brands and companies dominate most markets in the world. With substantial differentials in production costs, advanced information technology, and improved logistics networks worldwide, materials and products can be bought, manufactured and sold anywhere in the world where it is feasible. The distance factor, therefore, becomes critical with shipments moving thousands of miles from supply sites to demand sites. In addition, increased product varieties with much shorter life cycles and lead times, and highlighted customer expectations for products and delivery speed, present today's logistics managers with special challenges in moving shipments in the right quantities, to the right destination, at the right time, and at the minimum cost.

Globalisation is heightening the importance of air transport, which provides geographical spread and fast delivery. Supplying a market ahead of competitors yields competitive advantages in terms of flexibility and responsiveness to dynamic and customized market demand. Time-saving is particularly important for certain industries with shorter product life-cycles and lead-times, such as the personal computer and fashion industries. Moving goods quickly by air can leave the manufacturing process with a margin to set up production to satisfy changing market demand, or to beat seasonal deadlines when sales are at their peak.

Containerization is an approach of organizing shipments effectively, and efficiently. Containerization changes shipment handling from a labour-intensive to a capital and time intensive operation. The objective of this study is to provide a decision-making framework for modelling the containerization of air cargo forwarding problems in order to help logistics managers to determine what types and numbers of air containers they need to rent from air carriers and how customer's shipments will be allocated, with the aim of simultaneously satisfying customer shipping requirements while minimizing the rental cost. The paper is organized into 6 sections. Section 1 is an introduction to the background to this study. The related literature is reviewed in Section 2. Section 3 describes the air container selecting and cargo loading process. Model formulation and problem analysis are presented in Section 4. Section 5 demonstrates how the proposed model can be used to solve practical container selecting and cargo loading problems with the experiments under different scenarios. The final section gives the conclusions to this study.

2. Literature review

Containers are defined as large boxes that are used to transport goods from one destination to another (Vis and Koster, 2003). The efficient stowage of goods in means of transport can often be modelled as a container loading problem (Bortfeldt and Gehring, 2001). There exists a large body of literature related to the container loading problems, which is usually classified as the three-dimensional (3D) rectangular packing problem in the general cutting and packing problem. Cutting and packing problems involve different dimensions. Gilmore and Gormory (1961) is the first researchers to discuss the one-dimensional stock cutting problem as a linear programming problem. In 1965, they extend this work to two-, and three-dimensional problems with related algorithms (Gilmore and Gormory, 1965). A survey and classification of cutting and packing problems is presented by Dyckhoff (1990). Bischoff and Ratcliff (1995) criticize the fact that a great deal of research on container loading is based on pure knapsack-type formulations of the problem structure, and they highlight some important shortcomings in the existing theoretical literature on container loading problems.

To date, much of the literature focuses on sea containers of a standardized unit, the TEU (twenty-foot equivalent), which leads to a discussion of cutting and packing problems (Bischoff and Marriott 1990, George *et al.* 1993, George 1996, Han *et al.* 1989, and Ivancic *et al.* 1989). Vis and Koster (2003) classify the decision problems arising at sea container terminals and give an overview of the relevant literature. Some research discusses empty sea container allocation problems faced by shipping companies in terms of how to distribute empty containers to shippers and how to relocate empty containers in preparation for future demand. Early work using network models for empty container allocations problems can be found in White (1972). Cheung and Chen (1998) consider the dynamic empty container allocation problem where they need to reposition empty containers and determine the number of leased containers to satisfy customer demand over time. In their study, a stochastic quasi-gradient method and a stochastic hybrid approximation procedure are applied to solve the empty container allocation problem.

The air container has some special characteristics, and these mean it cannot be treated as a sea container or a general rectangular box waiting for loading. There are different types of air containers with differing limitations on weight and volume, and these containers usually carry low-density and high-value items. Cost and time are particularly sensitive and important in the air transport business, as air container charges can be very expensive and late delivery may cause loss of goodwill and customer dissatisfaction. Therefore, it is vital that logistics companies make the best decisions about the issues of renting air containers and loading cargos. Little research has been conducted on the cost issues related to selecting air containers, let alone considering allocating air cargos simultaneously. Martin-Vega (1995) presents a complete review of the manual and the computer-assisted approaches to air container loading problems, considering the centre of gravity via pyramid loading. A new approach provided by Davies and Bischoff (1999) considers weight distribution considerations in container loading, in which an even weight distribution can be attained in a container whilst simultaneously achieving a high degree of space utilization. Mongeau and BÈS (2003) address the problem of loading as much as freight as possible in an aircraft while balancing the load in order to minimize fuel consumption and satisfy stability and safety requirements. A mathematical programming model is formulated to choose which containers should be loaded on the aircraft, and how they should be distributed among different compartments.

3. Containerizing air cargos

Logistics companies perform many functions in delivering customer's shipments by air, such as preparing all documents for air shipment, obtaining cargo insurance, collecting items from their customers, warehousing, packing, tracing, etc. However, there are several other tasks that the logistics companies provide. Typically customers' items to be shipped by air have a relatively higher value than sea shipments and they require quick and accurate shipping. Logistics companies incur a heavy penalty if delivery is late or missed. Therefore, just-in-time (JIT) shipping has become a standard of service provided by the logistics companies. Based on the shape, weight, and volume of the shipments, and on the shipping time and destinations, the logistics companies consolidate small shipments and form different types of cargos ready to be loaded into air containers. Typically, the air carriers offer several types of air cargo containers for rental. The air containers vary in shape and in their limits on the total volume and weight to be carried. The problem is further complicated by the nature of the rental cost for a container charged by the air carriers. This cost is based on a fixed charge for using the container plus a variable charge that depends on the total cargo weight that the container holds.

Air containers come in different irregular shapes to enable them to fit into the aircraft's hold. The volume limitation provided by the air carriers is only approximate, and is smaller than the actual space that the container has. This is because the air carriers cannot assume the irregular space will be fully occupied by the cargo, and the weight issue is more important than the volume issue for the air transport. However, when the forwarders consolidate the shipments, they usually form a relatively small volume cargo, which can be easily loaded into the air containers. Therefore, this study does not consider the shape issue, and focuses on how to containerize the cargos. In this study, each type of cargo has its own weight and volume. Each cargo must be packed into a single container. Breaking a cargo into different containers is not allowed. In addition, all cargos have to be allocated to containers without delay. The decisions that the logistics companies have to make include how to select adequate containers from different types of containers available, and how to allocate the different types of cargos into these containers while satisfying customer JIT delivery requirements and minimizing the rental cost.

4. Model formulation

Let $\{1,2,\ldots,n\}$ be n types of air cargos, and let v_j and w_j denote the volume and the weight of cargo type j. There are q_j cargos of type j available for shipping. All cargos have to be loaded into the air containers provided by the air carriers. There be m types of containers, number $\{1,2,\ldots,m\}$, for rental. Each type of container i has L_i cargos available, i.e. number $\{1,2,\ldots,L_i\}$. For container type i, V_i and W_i represent the volume and weight limits, respectively. The cost of renting the l^{th} container of type i includes a fixed cost c_i^0

and a variable cost c_{il}. Whenever you rent one container, you have to pay a fixed cost for using it. Once the cargo loaded into the container exceeds a permitted weight limit, a variable cost will be incurred, and this is associated with the weight of cargo loaded into the container. Figure 1 shows the variable cost c_{il}.

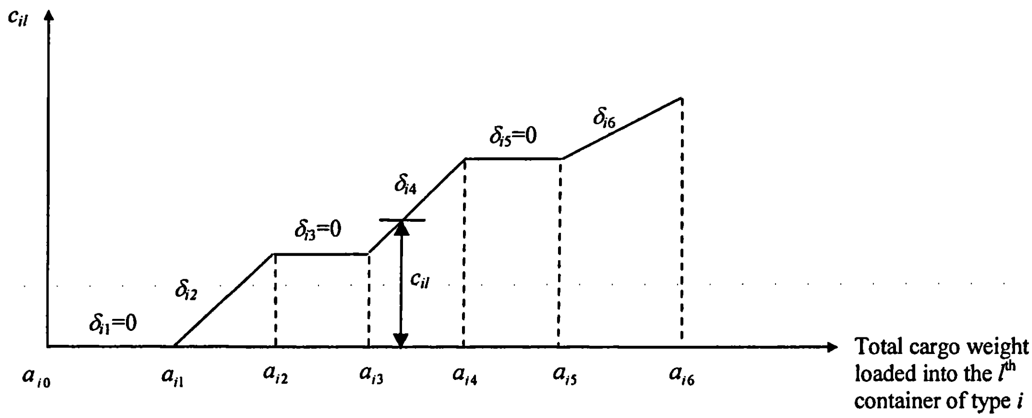

Figure 1: The variable cost of renting the l^{th} container of type i

In Figure 1, a_{ik} represents the breaking-point for container type i, where i=1,..,m, k=1,..., K_i, where K_i is the maximum number of breaking-points. In this study, the air carriers provide six cost breaking-points: a_{i1}, a_{i2}, a_{i3}, a_{i4}, a_{i5}, and a_{i6}. Let a_{i0} be the initial point, i.e. a_{i0} =0. Thus, a_{i1} is the first cost breaking point for causing the variable cost, and a_{i6} is the maximum weight limits of container type i.

When the l^{th} container of type i is selected for rental, a fixed cost c_i^0 will be incurred immediately. However, a variable cost c_{il} will be incurred only if the total cargo weight that is loaded into the l^{th} container of type i exceeds the first cost breaking-point a_{i1}. The unit variable cost is charged at a rate, denoted by δ_{ik}, which is the slope of the piece-wise linear cost function in the range $(a_{i,k-1}, a_{ik}]$. Clearly if the total cargo weight in the l^{th} container of type i does not exceed the first breaking-point a_{i1}, there is no variable cost. The unit rate δ_{i1} in the range $(a_{i0}, a_{i1}]$ is zero.

When the total weight of cargo loaded into the container i exceeds the first cost breaking-point a_{i1} and reaches the range $(a_{i1}, a_{i2}]$, the variable cost of renting the container equals the unit rate δ_{i2} multiplied by the difference between the total cargo weight and the first cost breaking-point a_{i1}. When the weight of the loaded cargo exceeds the second cost breaking-point a_{i2} and reaches the range $(a_{i2}, a_{i3}]$, the variable cost will keep a constant value of $\delta_{i2}(a_{i2}-a_{i1})$ because the unit rate δ_{i3} is zero in the range $(a_{i2}, a_{i3}]$. When the cargo weight loaded into the l^{th} container of type i exceeds the third cost breaking-point a_{i3} and reaches the range $(a_{i3}, a_{i4}]$, the variable cost c_{il} will increase by a unit rate δ_{i4} multiplied the difference between the total cargo weight in the container and the cost breaking-point a_{i3}. The definition of the variable cost c_{il} can be formulated as follows:

$$c_{il} = \begin{cases} 0 & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i0}, a_{i1}] \\ \delta_{i2}(\sum_{j=1}^{n} w_j y_{ilj} - a_{i1}) & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i1}, a_{i2}] \\ \delta_{i2}(a_{i2} - a_{i1}) & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i2}, a_{i3}] \\ \delta_{i2}(a_{i2} - a_{i1}) + \delta_{i4}(\sum_{j=1}^{n} w_j y_{ilj} - a_{i3}) & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i3}, a_{i4}] \\ \delta_{i2}(a_{i2} - a_{i1}) + \delta_{i4}(a_{i4} - a_{i3}) & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i4}, a_{i5}] \\ \delta_{i2}(a_{i2} - a_{i1}) + \delta_{i4}(a_{i4} - a_{i3}) + \delta_{i6}(\sum_{j=1}^{n} w_j y_{ilj} - a_{i5}) & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i5}, a_{i6}] \end{cases} \quad (1)$$

where, i= 1,2,...,m; l=1,2,..., L_i.

Therefore, the containerization of the air cargo forwarding problem can be formulated as follows:

$$\text{Minimize} \quad \sum_{i=1}^{m}\sum_{l=1}^{L_i} c_i^0 x_{il} + \sum_{i=1}^{m}\sum_{l=1}^{L_i} c_{il} \quad (2)$$

subject to

$$\sum_{j=1}^{n} v_j y_{ilj} \le V_i x_{il}, \; i=1,\ldots,m; \; l=1,\ldots, L_i; \quad (3)$$

$$\sum_{j=1}^{n} w_j y_{ilj} \le W_i x_{il}, \; i=1,\ldots,m; \; l=1,\ldots, L_i; \quad (4)$$

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i} y_{ilj} = q_j, \; j=1,2,\ldots,n; \quad (5)$$

$$x_{il} = \{0,1\}, \; i=1,\ldots,m; \; l=1,\ldots,h_i; \quad (6)$$

y_{ilj} is an non-negative integer, i=1,..., m; l=1,..., L_i; j=1,2,..., n; (7)

The objective function in (2) is the total cost of renting the containers, which includes two elements. The first element is the total fixed cost for using the containers, and the second is the total variable cost. The value of x_{il} will be equal to 1 if the l^{th} container of container type i is selected for rental; otherwise, the value of x_{il} will be zero, representing the fact that the l^{th} container of type i has not been selected. The definition of the variable cost $c_{il,}$ can be referred to Equation (1) and Figure 1.

y_{ilj} represents the quantities of cargo type j loaded into the l^{th} container of container type i. Constraint (3) is the container volume constraint, which ensures that the volume of all cargos allocated to a container cannot exceed the container's volume limits. Constraint (4) is the container weight constraint, which ensures that the weight of all cargos allocated into a container cannot exceed the container's weight limits. Constraint (5) is the cargo quantity constraint, which requires all cargos to be loaded into the containers without any delay. Constraints (6) and (7) are the variable type requirements.

The objective function expressed in (2) is a piecewise function, and it is difficult to solve this kind of model by employing optimal software packages. Thus, we introduce two new variables in order to transform the model into a mixed integer programming model. One variable g_{ilk} is a continuous variable representing the cargo weight distributed in the range $(a_{i,k\text{-}1},a_{ik}]$ inside the l^{th} container of type i. The other variable z_{ilk} is a binary variable indicating whether the cargo weight is distributed in the range $(a_{i,k\text{-}1},a_{ik}]$ inside the l^{th} container of type i. Thus the proposed model can be formulated as the following mixed integer programming model:

Minimise $$\sum_{i=1}^{m}\sum_{l=1}^{L_i} c_i^0 x_{il} + \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{k=1}^{K_i} \delta_{ik} g_{ilk} \tag{8}$$

subject to

$$\sum_{j=1}^{n} v_j y_{ilj} \le V_i x_{il},\ i=1,\ldots,m;\ l=1,\ldots,L_i; \tag{9}$$

$$\sum_{j=1}^{n} w_j y_{ilj} \le W_i x_{il},\ i=1,\ldots,m;\ l=1,\ldots,L_i; \tag{10}$$

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i} y_{ilj} = q_j,\ j=1,2,\ldots,n; \tag{11}$$

$$\sum_{k=1}^{K_i} g_{ilk} = \sum_{j=1}^{n} w_j y_{ilj}\ ,\ i=1,\ldots,m;\ l=1,\ldots,L_i; \tag{12}$$

$$g_{ilk} \le z_{ilk}\left(a_{i,k} - a_{i,k-1}\right),\ i=1,\ldots,m;\ l=1,\ldots,L_i;\ k=1,\ldots,K_i; \tag{13}$$

$$g_{ilk} \ge z_{il,k+1}\left(a_{i,k} - a_{i,k-1}\right),\ i=1,\ldots,m;\ l=1,\ldots,L_i;\ k=1,\ldots,K_i-1; \tag{14}$$

$$x_{il} = \{0,1\},\ i=1,\ldots,m;\ l=1,\ldots,L_i; \tag{15}$$

y_{ilj} is an non-negative integer variable, $i=1,\ldots,m;\ l=1,\ldots,L_i; j=1,2,\ldots,n;$ (16)

$$z_{ilk} = \{0,1\},\ i=1,\ldots,m;\ l=1,\ldots,L_i,\ k=1,\ldots,K_i; \tag{17}$$

There are two items in the objective function (8). The first component is the fixed cost, which is as the same as in the objective function (2). The second component in (8) represents the sum of the variable costs for all containers. The variable cost for each container is the sum of the variable cost distributed in all ranges. The variable cost of the l^{th} container of type i in the range $(a_{i,k-1},a_{ik}]$ is the unit charge rate of container i in the range $(a_{i,k-1},a_{ik}]$, represented by δ_{ik}, multiplied by the cargo weight distributed in the range $(a_{i,k-1},a_{ik}]$ inside the l^{th} container of type i, represented by g_{ilk}.

Constraints (9), (10) and (11) are the container volume constraint, container weight constraint and cargo quantity constraint, respectively. Constraint (12) ensures that the sum of the cargo weight distributed in all districts inside a container is equal to the total weight of the cargos loaded into the container. Constraint (13) ensures z_{ilk} is equal to 1 if the total cargo weight inside the l^{th} container of types i reaches the range $(a_{i,k-1},a_{ik}]$. In addition, the cargo weight g_{ilk} in the range $(a_{i,k-1},a_{ik}]$ is less-than-or-equal-to the maximum weight value in the range $(a_{i,k-1},a_{ik}]$, which is a_{ik}-$a_{i,k-1}$. Constraint (14) ensures that once the total cargo weight inside the l^{th} container of type i reaches the range $(a_{i,k},a_{ik+1}]$, the cargo weight in the range $(a_{i,k-1},a_{ik}]$, which is g_{ilk}, is not less than the difference between a_{ik} and $a_{i,k-1}$. Constraints (13) and (14) ensure that the weight ranges are reached by priority: g_{ilk} cannot be positive unless the range $(a_{i,k-1},a_{ik}]$ is fully occupied by the cargo weight. In other words, constraints (13) and (14) ensure that g_{ilk} cannot have a positive value unless all g_{ilt} are at their maximum value, which is a_{it}-$a_{i,t-1}$, $1 \le t \le k$. Constraints (16), (17) and (18) are the variable type requirements.

5. Computational result analysis

A logistics company in Hong Kong provides air transport services worldwide. The company collects shipping information from its customers in terms of the weight, volume and shape of shipments, delivery time and destinations. Based on this information, the company consolidates the small shipments into three types of cargo: large, medium and small. Currently there are 7 large cargos, 6 medium cargos and 5 small cargos that need shipping by air from Hong Kong to London at the same time. The volume and weight of each type of cargo are given as follows:

Table 1: Air cargo characteristics

Cargo Types	Cargo Volume	Cargo Weight
Large	1500	750
Medium	1200	600
Small	1000	500

The logistics company then contacts the air carrier to arrange rental of air containers. The air carrier can provide 7 types of containers for renting, and currently there are 2 of each type of container available. The air carrier provides the following information shown in Table 2, relating to the 7 types of container, including the types and quantities of the containers, the volume and weight limits of the containers, the fixed cost, the breaking-points, and the unit charge rate in the different ranges. Based on the information presented in Table 2, the logistics company needs to decide what types and how many of each air container it needs to rent, and how to pack 7 large, 6 medium, and 5 small cargos into the containers.

Table 2: Air container characteristics

Container Type	Container Quantity	Fixed Cost	Volume Limit	Weight Limit	Breaking-Point						Charged Rate					
					a_{i1}	a_{i2}	a_{i3}	a_{i4}	a_{i5}	a_{i6}	δ_{i1}	δ_{i2}	δ_{i3}	δ_{i4}	δ_{i5}	δ_{i6}
1	2	161617	6489	6800	3968	4722	5290	5976	6273	6800	0	32	0	29	0	25
2	2	105898	6300	5400	2600	3050	3467	3954	4111	5400	0	32	0	29	0	25

3	2	85207	5008	4200	2092	2490	2789	3140	3307	4200	0	32	0	29	0	25
4	2	74373	4882	4000	1826	2173	2434	2741	2886	4000	0	32	0	29	0	25
5	2	48713	3700	3900	1196	1423	1594	1825	1917	2900	0	32	0	29	0	25
6	2	46553	3150	3500	1643	1747	2000	2500	2591	3500	0	32	0	29	0	25
7	2	20695	1400	1200	505	602	674	758	799	1200	0	32	0	29	0	25

Table 3 gives the computational results obtained using mathematical programming software AIMMS to solve the problem. The solution includes which containers to select and which cargos to load into them. The total rental cost for shipping 7 large cargos, 6 medium cargos and 5 small cargos is 387237. Additionally, Table 3 provides other related results including the loaded volume and weight for each container, the fixed cost, variable cost and total cost for each container. Table 4 shows the cargo weight at all ranges in each container.

Table 3: Optimal plan for container rental and cargo loading

Selected Containers	Loaded Cargos	Loaded Volume	Loaded Weight	Fixed Cost	Variable Cost	Total Cost
Container 4 (1st)	1 large, 1 medium, 2 small	4700	2350	74373	11104	85477
Container 4 (2nd)	4 medium	4800	2400	74373	11104	85477
Container 5 (1st)	1 large, 2 small	3500	1750	48713	11788	60501
Container 5 (2nd)	1 large, 1 medium, 1 small	3700	1850	48713	13963	62676
Container 6 (1st)	2 large	3000	1500	46553	0	46553
Container 6 (2nd)	2 large	3000	1500	46553	0	46553

Table 4: The cargo weight at all ranges for each container

Container		Cargo Weight in the different ranges						Total Cargo Weight in the Container
		$(a_{i0},a_{i1}]$	$(a_{i1},a_{i2}]$	$(a_{i2},a_{i3}]$	$(a_{i3},a_{i4}]$	$(a_{i4},a_{i5}]$	$(a_{i5},a_{i6}]$	
Container Type 1	1st							
	2nd							
Container Type 2	1st							
	2nd							
Container Type 3	1st							
	2nd							
Container Type 4	1st	1826	347	177				2350
	2nd	1826	347	227				2400
Container Type 5	1st	1196	227	171	156			1750
	2nd	1196	227	171	231	25		1850
Container Type 6	1st	1500						1500
	2nd	1500						1500
Container Type 7	1st							
	2nd							

From Table 3, we know that the rental cost for the two containers of type 4 is the same, although the total weight and volume of cargos loaded into the second type 4 container is greater than for the first one. The reason is the two containers reach the same range $(a_{i2},a_{i3}]$ (see Table 4), which results in the same variable cost for renting the two containers. The rental cost of the second type 5 container is more than for the first one, as the first container only reaches the range $(a_{i3},a_{i4}]$; while the second container reaches the range $(a_{i4},a_{i5}]$. At the same time, Table 4 shows the two type 6 containers do not exceed the first cost breaking-point, so no variable cost is incurred.

Table 5 gives four scenarios for the shipping cargo process that the logistics company may face in the future. Scenario I is the optimal solution using the existing data above. Scenarios II, III and IV are drawn up on the assumption that the cargo quantities are lowered by one for every type of cargo, representing the different situations that the forwarder might experience because of customers supplying inaccurate shipping information.

Table 5: Scenario assumptions

Scenario	Description of Changes	Cargo Quantities
I	Using exist data	7 large, 6 medium and 5 small cargos.
II	Quantity of large cargos decreases by 1.	6 large, 6 medium and 5 small cargos.
III	Quantity of medium cargos decreases by 1.	7 large, 5 medium and 5 small cargos.
IV	Quantity of small cargos decrease by 1.	7 large, 6 medium and 4 small cargos.

The optimal solutions for four scenarios are shown in Table 6. The related results are presented in Table 7. From Tables 6 and 7, we see that types and quantities of cargos have a dramatic impact on the decisions of how to select containers and how to load cargos, as well as on the total rental cost. The reason is that the total cost for renting the container not only depends on a fixed cost, but also includes a variable element which is associated with the cargo weight that the container holds.

Table 6: Optimal plan for container rental and cargo loading under different scenarios

Scenario		Container Type 1		Container Type 2		Container Type 3		Container Type 4		Container Type 5		Container Type 6		Container Type 7	
		1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd
Scenario I	7 large cargo							1		1	1	2	2		
	6 medium cargo							1	4	1					
	5 small cargo							2		1	2				
Scenario II	6 large cargo							1		1		2	2		
	6 medium cargo							1	4	1					
	5 small cargo							2		1				1	1
Scenario III	7 large cargo							1	1	1		2	2		
	5 medium cargo							1	1	1				1	1
	5 small cargo							2	2	1					
Scenario IV	7 large cargo							1		2		2	2		
	6 medium cargo							1	4		1				
	4 small cargo							2			2				

Table 7: Related costs for container rental and cargo loading under different scenarios

Scenario		Container Type 1		Container Type 2		Container Type 3		Container Type 4		Container Type 5		Container Type 6		Container Type 7	
		1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd
Scenario I	Fixed cost							74373	74373	48713	48713	46553	46553		
	Variable cost							11104	11104	11788	13963				
	Total cost							85477	85477	60501	62676	46553	46553		
Scenario II	Fixed cost							74373	74373	48713		46553	46553	20695	20695
	Variable cost							11104	11104	13963					
	Total cost							85477	85477	62676		46553	46553	20695	20695
Scenario III	Fixed cost							74373	74373	48713		46553	46553	20695	20695
	Variable cost							11104	11104	13963				3040	3040
	Total cost							85477	85477	62676		46553	46553	23735	23735
Scenario IV	Fixed cost							74373	74373	48713	48713	46553	46553		
	Variable cost							11104	11104	7264	7438				
	Total cost							85477	85477	55977	55977	46553	46553		

In Scenario II there is the same amount of medium and small cargos as in Scenario I. However, there is less large cargo in Scenario II than in Scenario I (see Table 5).

Comparing the computational results of Scenario I and II in Table 6, the company only needs to choose two type 7 containers in Scenario II instead of one type 5 container in Scenario I. Two type 7 containers are enough to hold 2 small cargos in Scenario II. However, in Scenario I, a larger and more expensive type 5 container is needed to carry two small cargos as well as one large cargo. The cost of renting two type 7 containers to carry two small cargos is 41390 in Scenario II, but the cost of renting one type 5 container to carry two small cargos plus one large cargo is 62676 in Scenario I.

In Scenarios III and II the same containers are selected: one type 5 container and two type 4, 6 and 7 containers, respectively. The cargo loading plans into the first type 4 and 5 container, as well as the two type 6 containers, are exactly same in both scenarios. Therefore, the costs for renting them are also the same in both scenarios. However, the cargo loading plans are different for the second type 4 container and two type 7 containers. In Scenario II, the second type container holds 4 medium cargos, and the two type 7 containers hold two small cargos. In Scenario III, however, the second type 4 container carries 1 large, 1 medium, and 2 small cargos, and the two type 7 containers carry two medium cargos. Therefore, the variable cost for each type 7 container is 23735 in Scenarios III, compared with 20695 in Scenario II. In Scenario II, the type 7 container only carries one small cargo with a weight of 500, which is less than the first cost breaking-point of 505 for type 7 container. Thus there is no variable cost in renting the two type 7 containers in Scenario II. In Scenario III, however, each type 7 container carries one medium cargo, which incurs a variable cost of 3040, because the weight of the medium cargo of 600 exceeds the first cost-breaking-point of 505 for a type 7 container. The related data can be found in Table 7.

Scenarios IV and I select the same containers: 2 type 4, 2 type 5, and 2 type 6. The cargo loading plans are the same in both scenarios, except for the type 5 containers. In Scenario I, the first type 5 container holds 1 large, 1 medium, and 1 small cargo, while the container needs to hold 2 large cargos in Scenario IV. At the same time, the second type 5 container holds 1 large, and 2 small cargos in Scenario I, while it carries 1 medium, and 2 small cargos in Scenario IV.

From the computational results and analysis conducted under different scenarios, we conclude that container selecting and cargo loading plans have a dramatic impact on the company's profit.

5. Conclusions

This study presents a decision-making framework for modelling containerization of air cargo forwarding problems experienced by logistics companies when they use aircrafts for transportation. The decisions they face include how to select the air containers provided by the air carriers, and how to load the cargos into them. The decision-making process is complex because of the air containers' volume and weight limits and the fact that the container rentals costs consist of a fixed and a variable element, with the latter associated with the total cargo weight that each container holds. The companies have to satisfy their customers' shipping requirements while minimizing container rental costs. A major contribution of this study is that not only to consider the containerizing air cargo problems relating to the cost charged by the air carriers, but also in considering cargo loading problems simultaneously. We first formulate a mathematical programming model, whose objective function is a non-decreasing piece-wise linear function. By introducing two new variables, we then change the model into a mixed integer linear programming model, which can be solved by many mathematical programming software packages available today. Finally, the application of the proposed model is illustrated using the examples from a Hong Kong logistics company with analysis conducted under different scenarios. The heuristics algorithms might need to be considered when the container and cargo quantity increase. However, this issue is not addressed in this research as the containers' quantity is limited by the number of aircrafts flying from one city to another. Further studies will consider the uncertainty of customer shipment requirements, as well as dynamic nature of airline rental costs, in which heuristic approaches might be used to solve these problems because of the large computational burden caused by the uncertain and dynamic factors.

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Appendix D

A paper accepted by *Production Planning and Control*

A stochastic model for production loading in a global apparel manufacturing company under uncertainty

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This paper studies production loading problems with uncertainties of demand and import quotas experienced by a global apparel manufacturing company, whose markets are located in Northern America and Europe, manufacturing factories are in Asia (Mainland China, Thailand, the Philippines, Sri Lanka and Vietnam) and headquarters in Hong Kong. Loading production among different factories in different countries involves many uncertain factors, such as market information and quota premium. The paper presents a two-stage stochastic programming model for production loading problems with uncertainties where the first stage decisions are made before accurate information is available, and the second stage decisions are made when the stochasticity is realized. By using the two-stage production planning, the company is able to achieve a quick response to changing market information while minimizing the total production cost. A series of experiments, based on data from the apparel company, are designed to test the effectiveness of the proposed model. Compared with results of the deterministic model, the stochastic recourse model can provide a more flexible, responsive and cheaper production loading system.

Keywords: production loading; stochastic programming; deterministic programming; global supply chain;

1. Introduction

Supply chain management is a fundamental issue for organizations to improve their efficiency and effectiveness in today's highly competitive environment. The concept of the "global marketplace" has resulted in new all organizations (Large, medium, and small) planning their production in a new and different manner. Many senior level managers in numerous organizations now recognize the potential of a supply chain approach to making organizations more competitive globally, and to increase market shares. More and more companies are increasingly devoting themselves to international expansion and integration

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of functions such as production, marketing, and R&D. Companies are also focusing on international collaborations and networking with other firms in order to gain competitive advantages. Therefore, the concept of production management has evolved beyond the scope of a single manufacturing location.

This paper is motivated by production loading problems experienced by an apparel manufacturing company. Its headquarters are in Hong Kong and its manufacturing plants are located in several Asian countries, such as Mainland China, Thailand, the Philippines, Sri Lanka and Vietnam. When necessary, the company can outsource its production to other contracted plants, which are also located in these Asian countries. Planning production for different global plants is a critical management task for the company. While planning production, not only does the company need to consider factors normally relevant in production plans, such as raw material cost, labour cost, inventory cost, plant capacity, and warehouse capacity, etc., but also some international issues; for example, quota limitations, which are considered in this study. Import quotas are initially assigned by the importing country's government. Import quotas control the quantity or volume of certain imported merchandise that can be imported into Northern-American and European countries. China's exports of textiles and clothing products to four major trading partners that maintain import quotas, namely, the United States, European Union, Canada and Norway, are severely restrained (Cass *et al.* 2003). For example, the United States divides textiles and clothing products into 147 categories for quota administration purposes. Each exporting country selects major exporters and allocates them a certain amount of quota. Any other exporters who want to export their products to Northern American and European markets need to buy quotas from companies that own quota for exports from their country. Therefore, the quota cost is dependent on several factors, including government policies, market conditions and demand for quotas in exporting countries, or consumer demand in importing markets.

In addition, accurate market demand for products is usually unknown during the decision-making process. The apparel company's sales and marketing offices are located in Northern America and Europe. The sales departments collect product and market demand information from local retailers, and send it to the Hong Kong headquarters. Based on this information, the Hong Kong headquarters need to estimate market demand for different types of products that will be on sale in the next selling season. The products covered under this study are fashion garments, which, like personal computers, are innovative products, having a short life cycle and lead time. The demand estimates for such products involve substantial uncertainty, as markets' reaction to new and innovative products is generally unclear; this increases the risk of a shortage or an excess supply scenario. The manufacturing company, however, can not wait until it is able to ascertain accurate market demand, as it may be too late to produce the products by then. The company has to determine production loading plans and commence manufacturing of products that will be on the market in the next selling season on the basis of uncertain information. Order commitments for products become clear only when the selling season begins. Until then, the company has to react to the market information, because the purpose of the production plan is to satisfy customer demand. Therefore, the apparel manufacturing company feels challenged while allocating production to its different manufacturing facilities, because of uncertain market demand and quota prices, and short lead times.

The stochastic recourse programming model is one of the most important models in stochastic programming. The recourse model is derived from reformulations of decision-making problems, to address stochasticity by subsequent corrections. In this paper, we formulate a two-stage stochastic linear recourse model to solve uncertain production planning problems in the apparel manufacturing company. Decisions in the first-stage include production quantities, workforce level, machine capacities, and worker overtime, all of which are determined before accurate information is available. Decisions in the second-stage include surplus/shortage productionand under-/over-quota quantities, which are made when the stochasticity is realized. These decisions represent responses to actual realizations of the stochastic parameters. Suppose the recourse production planning policy allows one to compensate for demand-supply imbalances at the second-stage by incurring two penalty costs: surplus/shortage cost, and under-/over-quota cost. When demand exceeds actual production quantity, the policy may dictate that part of production needs to be outsourced at a higher cost; on the other hand, when the volume of production exceeds demand, an inventory cost will be incurred. The production planning policy in this study also covers recourse options to different quota availability situations. When the quotas available for a product are not sufficient to satisfy demand for the product, the company has to buy quotas from the market, at the prevailing market price. This will incur an under-quota cost; on the other hand, when available quota quantities for a product are more than the realized demand for the product, some quotas are left unused. This will incur an over-quota cost.

The rest of the paper is organized as follows. The related literature is reviewed in Section 2. Model formulation and problem analysis are presented in Section 3. Section 4 demonstrates how the stochastic model can be used to solve practical production loading problems in the apparel manufacturing company. The final section gives the conclusion of the paper and the recommendations for future research.

2. Literature review

Linear programming is a fundamental planning tool. It is a suitable framework for analysis of many decision-making problems. Linear programming models assume that all information necessary for decision-making is available at the time of planning. However, in practical situations, it is often the case that decision makers are not sure about the accuracy of values of some (or all) coefficients. Stochastic programming has attracted researchers' attention in the area of optimization methods since the early stages of the development of the field. Dantzig (1955) point out that most practical applications are stochastic, and uncertainty problems could be formulated as a linear program with very special structure and, typically in practice, of huge dimensionality. This is the first paper on stochastic programming. General references on stochastic programming are books by Vajda (1972), Kall and Wallace (1994), Birge and Louvveaux (1997) and Prékopa (1995). Excellent publications related to stochastic programming applications and algorithms include Birge (1997), Sen and Higle (1999), Wallace (2000), Dupaćová (2002), and Higle and Wallace (2003). There are many areas where stochastic programming tools have found significant applications. These include electric power generation (Murphy *et al.* 1982), financial planning (Cariño *et al.* 1994), telecommunications network planning (Sen *et al.* 1994), supply chain management (Fisher *et al.* 1997), and portfolio problems (Høyland *et al.* 2002).

Production loading problems have been cast in the form of deterministic mathematical optimization models and many real instances have been computationally solved. Li *et al.*

(2000) propose a genetic algorithm to multi-objective production planning problems for manufacturing systems. Lee *et al.* (2006) develop an integrated mathematical model for the semiconductor industry supply chain consisting of production and distribution chains. However, Wiendahl and Breithaupt (1999) state that companies have to adapt their production structures rapidly in a fast-changing production environment, and new methods for production planning and control are required that consider these dynamic changes. Therefore, designing and implementing a model that captures the time phasing and the uncertain elements in production planning problems remains a challenging task in the changing business environment. Williams (1984) develop a two-location system consisting of a manufacturing facility pulled by a finished goods storage facility under stochastic demand. Lee and Billington (1993) formulate a heuristic stochastic model for managing material flows. They develop a pull-type, periodic, order-up-to inventory system, and determine the review period and the order-up-to quantity as model outputs. Pyke and Cohen (1993) provide a Markov chain model for a single product; a three-level supply chain with a factory, a finished goods storage facility, and a retailer. Near-optimal algorithms are provided to determine the expedited batch size, the normal replenishment batch size, the normal reorder point, the expedited reorder point, and the order-up-to level at the retailer. Escudero *et al.* (1993) work on modelling supply chain management optimization under uncertainty, based on a scenario approach, using the non-anticipativity principle. Lee *et al.* (2002) develop a mathematical model for a multi-period, multi-product, multi-shop production and distribution problem, in which the machine capacity and distribution capacity are considered as stochastic factors. Rappold and Yoho (2008) examine a multi-item integrated production-inventory system, in which customer demand is highly uncertain. To date, there exists little research that addresses the import quota issue by modelling production loading problems in global manufacturing under uncertainty. As a result, few researchers have used stochastic programming to model production loading problems in the global supply chain management environment under uncertainty.

3. Model formulation

3.1. *Notations*

Indices

i =for products (i=1,...,m);

j =for plants (j=1,...,n);

s= for scenarios (s=1,...,S);

Deterministic parameters

k_{ij}^1 / k_{ij}^2 cost of skilled/non-skilled workers making a unit of product i in plant j

o_j^1 / o_j^2 overtime cost of skilled/non-skilled workers per hour in plant j

h_{jt}^1 / h_{jt}^2 cost of hiring skilled/non-skilled workers per hour in plant j at the beginning of period t

f_{jt}^1 / f_{jt}^2 cost of firing skilled/non-skilled workers per hour in plant j at the beginning of period t

v_{j0}^1 / v_{j0}^2 initial number of skilled/non-skilled workers in plant j

α_j limit for the ratio between skilled and non-skilled workers for production in plant j

l_{ij}^1 / l_{ij}^2 labour time for production of a unit of product i in plant j by skilled/non-skilled workers

r_{ij} raw material cost of production per unit of product i in plant j

a_j^1 / a_j^2 regular/additional machine cost of production per hour in plant j

g_{ij}^1 / g_{ij}^2 machine time for production of a unit of product i by skilled/non-skilled workers in plant j

b_{it}^- / b_{it}^+ under-/over-production cost of a unit of product i in period t

d_{i0}^+ initial inventory of product i at the beginning of the planning horizon

c_i initial quota purchasing cost per unit of product i

c_{it}^- / c_{it}^+ under-/over-quota cost per unit of product i in period t

Q_i initial quota quantity of product i at the beginning of the planning horizon

p_s probability of scenario s occurrence

L_{jt}^1 / L_{jt}^2 maximum capacity of hiring skilled/non-skilled workers in plant j in period t

W_{jt}^1 / W_{jt}^2 maximum overtime for skilled/non-skilled workers in plant j in period t

C_{jt} / A_{jt} maximum regular/additional machine capacity of plant j in period t

V_{jt} minimum work time in plant j in period t

I_i maximum inventory capacity for product i

B_i maximum purchasing capacity for product i

<u>*Random parameters*</u>

D_{it} demand for product i in period t

b_{it}^- / b_{it}^+ shortage/surplus cost of a unit of product i in period t

c_{it}^- / c_{it}^+ under-/over-quota cost per unit of product i in period t

It is assumed that the uncertainties are represented by a set of possible realizations, called *scenarios*. Each scenario refers to one possible course of future events. The recourse production policy allows compensating for demand-supply imbalances in the second-stage, in each scenario s, by incurring cost b_{it}^- / b_{it}^+ per unit of production deviations from market demand, and by incurring cost c_{it}^- / c_{it}^+ per unit product for quota required for meeting deviations in market demand, compared to the initial allocated quota. When recourse actions are taken for realization D_{its} of expected demand D_{it}, realization b_{its}^- of the unit shortage cost b_{it}^-, realization b_{its}^+ of the unit surplus cost b_{it}^+, realization c_{its}^- of the unit under-quota cost c_{it}^-, and realization c_{its}^+ of unit over-quota c_{it}^+, random parameters D_{it}, b_{it}^-, b_{it}^+, c_{it}^-, and c_{it}^+, are independent random variables, and have the same finite discrete distribution specified by:

$$\begin{bmatrix} p_1 & p_2 & \cdots & p_S \\ D_{it1} & D_{it2} & \cdots & D_{itS} \\ b_{it1}^- & b_{it2}^- & \cdots & b_{itS}^- \\ b_{it1}^+ & b_{it}^+ & \cdots & b_{itS}^+ \\ c_{it1}^- & c_{it2}^- & \cdots & c_{itS}^- \\ c_{it1}^+ & c_{it2}^- & \cdots & c_{itS}^+ \end{bmatrix} \quad (1)$$

3.2. *A two-stage stochastic linear recourse programming model*

3.2.1. *The first-stage decisions*

Decision variables in the first-stage

x_{ijt}^1 / x_{ijt}^2 production quantities of product i by skilled/non-skilled workers in plant j in period t

y_{jt}^1 / y_{jt}^2 planned labour time of hiring skilled/non-skilled workers in plant j in period t

z_{jt}^1 / z_{jt}^2 planned labour time of firing skilled/non-skilled workers in plant j in period t

u_{jt}^1 / u_{jt}^2 used regular/additional machine capacities in plant j in period t

v_{jt}^1 / v_{jt}^2 used labour time of skilled/non-skilled workers in plant j in period t

w_{jt}^1 / w_{jt}^2 used overtime of skilled/non-skilled workers in plant j in period t

q_{it} initially allocated quota quantity of product t in period i

Constraints in the first-stage

$$\sum_{i=1}^{m} (g_{ij}^1 x_{ijt}^1 + g_{ij}^2 x_{ijt}^2) = u_{jt}^1 + u_{jt}^2, j=1,\ldots, n, t=1,\ldots,T \quad (2)$$

$$\sum_{i=1}^{m} l_{ij}^1 x_{ijt}^1 = v_{jt}^1, j=1,\ldots, n, t=1,\ldots,T \quad (3)$$

$$\sum_{i=1}^{m} l_{ij}^2 x_{ijt}^2 = v_{jt}^2, j=1,\ldots, n, t=1,\ldots,T \quad (4)$$

$$v_{jt}^1 = v_{jt-1}^1 + y_{jt}^1 - z_{jt}^1 + w_{jt}^1, j=1,\ldots,n, t=1,\ldots,T \quad (5)$$

$$v_{jt}^2 = v_{jt-1}^2 + y_{jt}^2 - z_{jt}^2 + w_{jt}^2, j=1,\dots,n, t=1,\dots,T \tag{6}$$

$$\sum_{t=1}^{T} v_{jt}^1 \ge \alpha_j \sum_{t=1}^{T} v_{jt}^2, j=1,\dots, n, \tag{7}$$

$$\sum_{t=1}^{T} q_{it} = Q_i, i=1,\dots, m \tag{8}$$

$$v_{jt}^1 + v_{jt}^2 \ge V_{jt}, j=1,\dots,n, t=1,\dots,T \tag{9}$$

$$d_{it}^- \le B_{it}, i=1,\dots, n, t=1,\dots,T \tag{10}$$

$$d_{it}^+ \le I_{it}, i=1,\dots, n, t=0,\dots,T \tag{11}$$

$$u_{jt}^1 \le C_{jt}, j=1,\dots,n, t=1,\dots,T \tag{12}$$

$$u_{jt}^2 \le A_{jt}, j=1,\dots,n, t=1,\dots,T \tag{13}$$

$$y_{jt}^1 - z_{jt}^1 \le L_{jt}^1, j=1,\dots,n, t=1,\dots,T \tag{14}$$

$$y_{jt}^2 - z_{jt}^2 \le L_{jt}^2, j=1,\dots,n, t=1,\dots,T \tag{15}$$

$$w_{jt}^1 \le W_{jt}^1, j=1,\dots,n, t=1,\dots,T \tag{16}$$

$$w_{jt}^2 \le W_{jt}^2, j=1,\dots,n, t=1,\dots,T \tag{17}$$

$$x_{ijt}^1, x_{ijt}^2, y_{jt}^1, y_{jt}^2, z_{jt}^1, z_{jt}^2, u_{jt}^1, u_{jt}^2, v_{jt}^1, v_{jt}^2, w_{jt}^1, w_{jt}^2, q_{it} \ge 0, i=1,\dots, m, j=1,\dots,n, t=1,\dots,T, \tag{18}$$

Inequality (2) denotes regular and additional machine capacity must be sufficient to produce the required quantity of products. Constraints (3) and (4) are the requirements of skilled and non-skilled workers. Constraints (5) and (6) ensure that the available workforce in any period equals the workforce in the previous period plus the change of workforce level in the current period. The change in workforce may be due to hiring extra workers, firing redundant workers, or payment of overtime. Constraint (7) ensures that the ratio between work time of skilled and non-skilled workers should not be less than a given constant, so as to guarantee product quality. Constraint (8) ensures that the initial quota is available in each time period at the beginning of planning horizon. Constraint (9) ensures each plant has a minimum work time in each period. Constraints (10)~(17) are the upper bound constraints. Constraint (18) is the variable type requirements.

Cost at the first-stage

$$Firstcost=\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T}r_{ij}\,(x^1_{ijt}+x^2_{ijt})+\sum_{j=1}^{n}\sum_{t=1}^{T}(a^1_j u^1_{jt}+a^2_j u^2_{jt})+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T}(k^1_{ij}x^1_{ijt}+k^2_{ij}x^2_{ijt})$$

$$+\sum_{j=1}^{n}\sum_{t=1}^{T}(o^1_j z^1_{jt}+o^2_j z^2_{jt})+\sum_{j=1}^{n}\sum_{t=1}^{T}(h^1_{jt}y^1_{jt}+h^2_{jt}y^2_{jt}+f^1_{jt}z^1_{jt}+f^2_{jt}z^2_{jt})+\sum_{i=1}^{m}\sum_{t=1}^{T}c_i\,q_{it} \quad (19)$$

In production loading problems under uncertainty, the cost at the first stage is the sum of raw materials cost, machines cost, labour cost, overtime cost, workers hiring/firing cost, and initial quota purchasing cost.

3.2.2. *The second-stage decisions*

Decision variables in the second-stage

$d^-_{its}\,/\,d^+_{its}$ shortage/surplus of product i in period t in scenario s;

$q^-_{its}\,/\,q^+_{its}$ under-/over-quota quantities of product i in period t in scenario s;

Constraints in the second-stage

$$\sum_{j=1}^{n}(x^1_{ijt}+x^2_{ijt})+d^+_{i,t-1,s}+d^-_{its}-d^+_{its}=D_{its},\ i=1,\ldots,m,\ t=1,\ldots,T,\ s=1,\ldots,S \quad (20)$$

$$q_{it}+q^+_{i,t-1,s}+q^-_{its}-q^+_{its}=D_{its},\ i=1,\ldots,m,\ t=1,\ldots,T,\ s=1,\ldots,S \quad (21)$$

$$d^-_{its}\le B_{it},\ i=1,\ldots,n,\ t=1,\ldots,T,\ s=1,\ldots,S \quad (22)$$

$$d^+_{its}\le I_{it},\ i=1,\ldots,n,\ t=0,\ldots,T,\ s=1,\ldots,S \quad (23)$$

$$d^-_{its},d^+_{its},q^-_{its},q^+_{its}\ge 0,\ \mathbf{i=1,\ldots,m,\ t=1,\ldots,T,\ s=1,\ldots,S} \quad (24)$$

Constraint (20) denotes demand constraints, which means, in each scenario, in each period, for each product, market demand has to be met by a combination of production in that period, inventory from the previous period, purchasing from the contracted plants and inventory in that period. Constraint (21) denotes quota constraints, which means, in each scenario, in each period, each product needs to have its own quota. The ideal situation is that in each period the demand is equal to the initially available quota. However, when the quota amount is insufficient, the company needs to purchase quota from local markets at market price, which is usually higher than the cost of initial quota. On the other hand, when the quota is not used fully, the company incurs the penalty cost. Constraints (22) and (23) are the upper bounds of surplus/shortage production. Constraint (24) is the variable type requirements.

<u>*Cost at the second-stage*</u>

$$Secondcost=\sum_{s=1}^{S}\sum_{i=1}^{m}\sum_{t=1}^{T} p_s(b_{its}^{-}d_{its}^{-}+b_{its}^{+}d_{its}^{+}+c_{its}^{-}q_{its}^{-}+c_{its}^{+}q_{its}^{+}) \tag{25}$$

The recourse production policy allows one to compensate for imbalance between actual production and realized demand in the second-stage by incurring a penalty cost of b_{its}^{-}/b_{its}^{+} per unit of production deviation from market demand, and for imbalance between available and required quantity of quota by incurring a penalty cost of c_{its}^{-}/c_{its}^{+} per unit of quota deviation from the market demand.

3.2.3. *A two-stage stochastic recourse model*

A two-stage stochastic recourse programming model for uncertain production loading problems in the apparel manufacturing company can be formulated as follows:

$$\min\ Firstcost+Secondcost \tag{26}$$

s.t. (2) ~ (25)

4. Computational results analysis

4.1. *A practical problem*

In order to illustrate the effectiveness of the proposed recourse model for uncertain production planning problems under global supply chain environments, we use data provided by a global apparel company. The company headquarters determine that three types of products will be manufactured by three main factories, located in Dongguan, Huidong, and Zhongshan, in China. The decision-maker in headquarters will consider a four-period planning. There is no inventory for any product at the beginning of the planning period. Table 1 gives the unit raw material cost, labour cost, labour time and machine time. Table 2 gives machine cost for regular and additional production, and overtime cost for skilled and non-skilled workers, per unit of the product. Table 3 gives the maximum regular/additional machine capacity, maximum labour capacity, maximum overtime capacity and the minimum work time. The initial quota purchasing cost per unit product is shown in Table 4. Currently, there is no cost involved in hiring/firing workers because there is a large supply of skilled and non-skilled workers in China and there is no union contract limitation either; therefore, workers can be hired or fired without incurring any extra costs. The work time of skilled workers is not less than that of non-skilled workers. There is no initial inventory. Additionally, it is assumed that the plants have enough capacity to satisfy the company's demand, and there is no limitation of inventory, as long as it is profitable to hold it.

Table 1. Raw material cost, labour cost, labour time and machine time per unit of product.

Product	Plant	Raw material cost ($)	Labour cost of skilled workers ($)	Labour cost of non-skilled workers ($)	Labour time for skilled workers (hrs)	Labour time for non-skilled workers (hrs)	Machine time for skilled workers (hrs)	Machine time for non-skilled workers (hrs)
1	1	4	4.5	4	2	2.25	1.75	2.25
	2	4.2	4	3.5	2.25	2.5	2	2.5
	3	4.3	3.5	3	2.5	2.75	2.25	2.75
2	1	3	4	3.5	1.5	1.75	1.25	1.75
	2	3.2	3.5	3	1.75	2	1.5	2
	3	3.3	3	2.5	2	2.25	1.75	2.25
3	1	2	3	2.5	1	1.25	0.75	1.25
	2	2.2	2.5	2	1.25	1.5	1	1.5
	3	2.3	2	1.5	1.5	1.75	1.25	1.75

Table 2. Unit machine cost and overtime cost.

Plant	Regular machine cost for production ($)	Additional machine cost for production ($)	Overtime cost for skilled worker ($)	Overtime cost for non-skilled worker ($)
1	0.05	0.055	6	5
2	0.06	0.065	5	4
3	0.07	0.75	4	3

Table 3. Maximum capacity for machine, labour and overtime, and minimum labour work time.

Plant	Period	Maximum machine regular capacity (hrs)	Maximum machine additional capacity (hrs)	Maximum capacity of skilled workers (hrs)	Maximum capacity of non-skilled workers (hrs)	Maximum overtime by skilled workers (hrs)	Maximum overtime by non-skilled workers (hrs)	Minimum labour work time (hrs)
1	1	5500	250	4800	2400	2400	1200	2400
	2	5500	250	4800	2400	2400	1200	2400
	3	5500	250	4800	2400	2400	1200	2400
	4	5500	250	4800	2400	2400	1200	2400
2	1	5000	250	3840	1920	1920	960	1800
	2	5000	250	3840	1920	1920	960	1800
	3	5000	250	3840	1920	1920	960	1800
	4	5000	250	3840	1920	1920	960	1800
3	1	5000	200	2400	1200	1200	600	1500
	2	5000	200	2400	1200	1200	600	1500
	3	5000	200	2400	1200	1200	600	1500
	4	5000	200	2400	1200	1200	600	1500

Table 4. Unit initial quota cost.

Product	1	2	3
Initial quota cost ($)	20.5	13	6.55

It is assumed that the uncertainty is represented by the possible states of the economy, in terms of the scenarios, i.e. good, fair, or bad. Let s_1 represent a good economy scenario with probability p_1, $p_1=\Pr\{s_1\}$; s_2 represent a fair economy scenario with probability p_2, $p_2=\Pr\{s_2\}$; and s_3 represent a bad economy scenario with probability p_3, $p_3=\Pr\{s_3\}$. The probability of a good economy in the new season is 10%, probability of a fair economy is 10%, and probability of a bad economy is 80%. Table 5 gives the realizations of random parameters, including the per product unit shortage cost (of purchasing products from contracted plants), surplus cost per unit product (storing unsold products), under-quota cost per unit product (for purchasing quota from the market), and the over-quota cost per unit (for unused quota). Additionally, market demand in each scenario is also shown in Table 5.

Table 5. Shortage/surplus cost per unit, under/over- quota cost per unit, and demand.

Scenario	Product	Period	Shortage cost ($)	Surplus cost ($)	Under-quota cost ($)	Over-quota cost ($)	Demand (units)
	1	1	120	2.5	26	4	1900
		2	120	2.5	26	4	2000
		3	120	2.5	26	4	2100
		4	120	2.5	26	4	2200
	2	1	72	1.5	17	3	1500
		2	72	1.5	17	3	1700
		3	72	1.5	17	3	1900
		4	72	1.5	17	3	2100

	3	1	48	1	10	2	1200
		2	48	1	10	2	1300
		3	48	1	10	2	1400
		4	48	1	10	2	1500
s_2	1	1	100	2	24	3	1800
		2	100	2	24	3	1900
		3	100	2	24	3	2000
		4	100	2	24	3	2100
	2	1	60	1	15	2	1400
		2	60	1	15	2	1600
		3	60	1	15	2	1800
		4	60	1	15	2	2000
	3	1	40	0.5	8	1	1100
		2	40	0.5	8	1	1200
		3	40	0.5	8	1	1300
		4	40	0.5	8	1	1400
s_3	1	1	80	1.8	22	2.5	1700
		2	80	1.8	22	2.5	1800
		3	80	1.8	22	2.5	1900
		4	80	1.8	22	2.5	2000
	2	1	48	0.8	14	1.5	1300
		2	48	0.8	14	1.5	1500
		3	48	0.8	14	1.5	1700
		4	48	0.8	14	1.5	1900
	3	1	32	0.3	7	0.5	1000
		2	32	0.3	7	0.5	1100
		3	32	0.3	7	0.5	1200
		4	32	0.3	7	0.5	1300

4.2. *Computational results and analysis*

The optimal container booking and cargo loading plan of the stochastic linear recourse programming model can be obtained using mathematical programming software, called AIMMS 3.8 (with CPLX 11.1 Solver), which is initially provided by Bisschop and Roelofs (1999). All the programs are executed on an Intel (R) Core (TM) 2 Duo CPU 2.39GHz laptop.

The first stage decisions

Before accurate market and quota price data are available, the company starts production in its own plants. The first stage decisions are shown in Tables 6 ~ 11. Table 6 shows the production quantities. Tables 7 and 8 show the machine work time and labour work time. Tables 9 and 10 show the time of hiring and firing workers. The initial quota allocated in each period is shown in Table 11. There is no need to work overtime.

Table 6. Production quantity.

Plant	Product	Skilled workers (hrs)				Non-skilled workers (hrs)			
		Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	1			1200	1200	1067	1067		
	2								
	3								
2	1	467	40	855	1000				
	2								
	3					1100	1140	396	906
3	1	267	793	45	1128				
	2	867	580	1070		533	1020	730	972
	3						60	904	594

Table 7. Machine work time.

Plant	Regular capacity used (hrs)				Additional capacity used (hrs)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	2400	2400	2100	2100				
2	2583	1790	2303	3359				
3	3317	5000	5000	5000		200	200	200

Table 8. Labour work time.

Plant	Skilled workers (hrs)				Non-skilled workers (hrs)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1			2400	2400	2400	2400		
2	1050	90	1923	2250	1650	1710	593	1359
3	2400	3143	2253	2256	1200	2400	3226	3226

Table 9. Hiring workers.

Plant	Skilled workers (hrs)				Non-skilled workers (hrs)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1			2400		2400			
2	1050		1833	327	1650	60		766
3	2400	743		3	1200	1200	826	

Table 10. Firing workers.

Plant	Skilled workers (hrs)				Non-skilled workers (hrs)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1							2400	
2		960					1117	
3			891					

Table 11. Quotas allocated.

Plant	Period 1	Period 2	Period 3	Period 4
1	1700	1800	2000	2200
2	1300	1600	1800	2100
3	1100	1200	1400	1500

The second stage decisions

When the uncertainty is realized, the company can make the second stage production loading decisions. The results are shown in Tables 12~17.

Scenario 1: Good economy

The probability of a good economy is 10%. If this scenario occurs, the company will take the second-stage decisions as shown in Tables 12 and 13. If the unexpected situation (high demand) happens (the possibility is 10%), there will exist the option of outsourcing a certain amount of production (Table 12), while additional quotas will also be required (Table 13). In this situation, there will be no leftover inventory or unused quota.

Table 12. Shortage/surplus in Scenario 1.

Product	Purchased products from contractors (units)				Inventory (units)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	100	100						
2	100	100	100					
3	100	100	100					

Table 13. Under-/over-quota in Scenario 1.

Product	Purchased products from contractors (units)				Inventory (units)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	200	200	100					
2	200	100	100					
3	100	100						

Scenario 2: Fair economy

The probability of a fair economy is 10%. If fair demand is realized, the company will take the corresponding second-stage production loading decisions, as shown in Tables 14 and 15. If the unexpected situation (fair economy) happens (the possibility is 10%), there will be a small amount of leftover inventory (Table 14), and of unused quota (Table 15). Additionally, a small amount of additional quota will be required in periods 1 and 2 (Table 15).

Table 14. Shortage/surplus in Scenario 2.

Product	Purchased products from contractors (units)				Inventory(units)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1							100	200
2								100
3								100

Table 15. Under-/over- quota in scenario 2.

Product	Purchased products from contractors (units)				Inventory(units)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	100	100						100
2	100							100
3							100	200

Scenario 3: Bad economy

The probability of a bad economy is 80%. If demand is low, the company will take the second-stage production loading decisions as shown in Tables 16 and 17. If this situation (bad economy) happens (the possibility is 80%), there will be a large amount of leftover inventory (Table 16), and unused quota (Table 17).

Table 16. Shortage/surplus in Scenario 3.

Product	Purchased products from contractors (units)				Inventory (units)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1					100	200	400	600
2					100	200	300	500
3					100	200	300	500

Table 17. Under/-over quota in Scenario 3.

Product	Purchased products from contractors (units)				Inventory (units)			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1							100	300
2						100	200	400
3					100	200	400	600

4.3. Comparing the deterministic model and the stochastic model

4.3.1. *Definitions*

The reason for conducting research on stochastic optimization is that the traditional deterministic optimization is not suitable for capturing the truly dynamic behaviours of most real-world markets, which usually involve uncertainties. Information that will be needed in subsequent stages of decision-making is not available when decisions need to be made. The production loading problems in this study exhibit some of uncertain parameters, such as product demand and import quotas for clothing. Long-term product demand and import quota forecasts would be helpful for making production plans but unfortunately, product demand and import quotas cannot be accurately predicted ahead of production. Production plans must be made without perfect information regarding demand and import quotas. Therefore, if the quantities of products manufactured are less than demand, it means that extra quantities need to be bought from elsewhere, at a higher price, in order to satisfy the demand. On the other hand, if the quantities of products manufactured are more than demand, an inventory cost will be incurred for the surplus quantities. As the decision-makers realize that they are unable to make a perfect decision that would be best in all circumstances, they would like to assess the benefits and losses of decisions they make. In this paper, we have established the two-stage stochastic programming model with recourse for production planning problems in the garment manufacturing industry, which can be expressed in the following general form:

$$\min z(x,\xi) = c^T x + \min\{q(\xi)^T y \mid Wy = h(\xi) - T(\xi), y \geq 0\} \tag{27}$$

$$\text{s.t. } Ax = b, x \geq 0, \tag{28}$$

where ξ is a random vector whose realizations correspond to the various scenarios. When a two-stage recourse model is developed, its solution is called the stochastic solution, denoted as x^*, and its performance is called the expected objective values of the stochastic solution, denoted as *ESS*. Therefore, the two-stage recourse model can be written as: $ESS = \min_x E_\xi z(x,\xi)$. A natural temptation is to solve a much simpler problem: the one obtained by replacing all random variables by substituting their expected values of the stochastic parameters (Birge and Louveaux 1997). This is called *the expected value problem or mean value problem*, which is $EV = \min_x z(x,\bar{\xi})$, where $\bar{\xi} = E(\xi)$ denotes the expectation of stochastic variable ξ, and its solution is called *the expect value solution*, denoted as $\bar{x}(\bar{\xi})$. We define the so-called *expected result of using the EV solution*, denoted by *EVV*, $EEV = E_\xi(z(\bar{x}(\bar{\xi}),\xi))$. The quantity EEV measures how $\bar{x}(\bar{\xi})$ performs. *The value of the stochastic solution*, denoted as *VSS*, is then defined as *VSS=EEV-ESS*. *VSS* represents the potential gain if we use the stochastic model, rather than the expected value model. In other words, *VSS* represents the cost of ignoring the uncertainty during the decision-making process.

4.3.2. *Computational results*

One approach for handling uncertainty is to solve the expected value problem, using the linear programming model, in which all random parameters are replaced by their expected values. Table 18 gives the expected values of unit under-/over- production cost, unit under-/over- quota cost, and demand.

Table 18. Expected value of unit shortage/surplus cost, unit under-/over- quota cost, and demand.

Product	Period	Shortage cost ($)	Surplus cost ($)	Under-quota cost ($)	Over-quota cost ($)	Demand (units)
1	1	86	1.89	22.6	2.7	1730
	2	86	1.89	22.6	2.7	1830
	3	86	1.89	22.6	2.7	1930
	4	86	1.89	22.6	2.7	2030
2	1	51.6	0.89	14.4	1.7	1330
	2	51.6	0.89	14.4	1.7	1530
	3	51.6	0.89	14.4	1.7	1730
	4	51.6	0.89	14.4	1.7	1930
3	1	34.4	0.39	7.4	0.7	1030
	2	34.4	0.39	7.4	0.7	1130
	3	34.4	0.39	7.4	0.7	1230
	4	34.4	0.39	7.4	0.7	1330

In order to demonstrate the effectiveness of the recourse model, we perform three different tests under different probabilities. Other than the change in probability of occurrence of the different future economic scenarios, other conditions in the three tests are the same. The test data are shown in Table 19. Test I represents the situation where it is most likely that the economy will perform well, Test II represents the situation where it is most likely that the economic performance will be fair, and Test III represents the situation where it will be poor. The problem, which is described in Section 3.2, is the case in Test III. Table 20 shows computational results for the expected value model, and stochastic recourse model, for the three tests.

Table 19. Three tests for uncertain problems.

Test	p_1=Pr{s_1}	p_2=Pr{s_2}	p_3=Pr{s_3}
I	0.8	0.1	0.1
II	0.1	0.8	0.1
III	0.1	0.1	0.8

Table 20. Comparison between the expected value model and stochastic recourse model.

Test	*EV*	*EEV*	*ESS*	*VSS (=EEV-ESS)*
I	426643	444205	432865	11340
II	408974	437078	420705	16373
III	402471	440472	423010	17462

From Table 20, it can be seen that in the three tests, all values of *EEV* are greater than the values of *ESS*. The expected value solution, therefore, can have unfavourable consequences because of the higher costs incurred, compared to those incurred when using the stochastic recourse model. In Test I, the total cost difference between the stochastic and expected value models (see the value of *VSS* in Table 20) is \$11,340, which is the possible gain from solving the stochastic model. The total cost in Test I decreases by \$11,340, from \$444,205 to \$432,865, if we choose the stochastic recourse model, rather than the expected value model. The total cost in Test II will decrease by \$16,373, from \$437,078 to \$420,705. The total cost in Test III will decrease by \$17,462, from \$440,472 to \$423,010. Compared with the expected value model, it is more beneficial to use the stochastic recourse model in Tests II and III, than in Test I. Test I represents the situation where it is most likely that demand will be high. If the anticipated scenario does not occur, there will be a certain amount of surplus inventory of products and quotas. In Tests II and III, if the unanticipated situation (high demand) happens (with the possibility of 10%), there will be a certain amount of shortage of products and quotas. The unit surplus cost of products/quotas is lower than the unit shortage cost of products/quotas. The expected value model has limited ability to handle unanticipated situations, which may result in a higher cost. This is particularly true in Tests II and III, when the unanticipated situation (high demand) is realized. We can conclude that it is more beneficial to use the recourse model in Tests II and III than in Test I. These results show that explicitly considering uncertainty is a critical aspect of decision-making and failure to include uncertainty may lead to very expensive, even disastrous consequences, if the anticipated situation is not realized.

5. Conclusions

Global supply chain management presents some special challenges and issues for manufacturing companies in production planning; these challenges are different from those discussed in domestic production plans. Production managers find that they have to develop competitive production strategies in order to survive. This paper examines production loading problems with import quota limitations in a global apparel manufacturing company. Globally loading production involves substantial uncertainty because of uncertain market demand and fluctuating quota prices. In addition, the lead time of products under this study is very short. The company has to start manufacture of products before accurate information is available. We present a two-stage stochastic recourse model to hedge against uncertainty involved in loading production among different manufacturing plants in different countries. In the first stage, when accurate market information is not available, the company distributes production tasks among company-owned plants. The decisions in this stage include production quantity, machine capacity, work force level and initially available quotas. In the second stage, when the uncertainty is realized, the company allocates production tasks among contracted plants also. The decisions in this stage include the quantities of products to be outsourced from contracted plants, inventory levels, quantities of additional quotas required, and the quantities of quotas that are unused. Computational results demonstrate how the stochastic recourse model can provide an effective production loading strategy to handle uncertainty during the decision-making process. A series of experiments are also designed to show that the stochastic recourse model has favourable consequences because of the lower level of costs, compared to costs incurred when using the corresponding expected value model, in which all stochastic parameters are replaced by their expected values. Computational results from the data provided by the company also show that it is more beneficial to use the stochastic recourse model in some production scenarios, than in others. In addition, it should be noted that computation and analysis of the models may lead to different outcomes if the model parameters change; for example, change of the probability of occurrence of a future economic scenario. Therefore, it is important to determine the probability of occurrence of different possible future economic scenarios during the decision-making process. Determination of the probability can be based on experts' judgement. For example, forecasting techniques, together with information from select websites or other companies, can be used to determine the probability. Finally, it should be realized that we only make two-stage decisions in this paper. However, every piece of information is continuously changing over time. Development of a multi-stage stochastic recourse model could well represent problems occurring in the global supply chain management environment. Additionally, consideration of other international trading factors, for example, the changing exchange rates, can also be one of directions for future research.

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Appendix E

A paper accepted by *Computer & Industrial Engineering*

A time staged linear programming model for production loading problems with import quota limit in a global supply chain

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Abstract

Globalization has ushered in a new era when more and more companies are expanding their manufacturing operations on a global scale. This poses some special challenges and raises certain issues. This paper examines production loading problems that involve import quota limits in the global supply chain network. Import quota, which is imposed by importing countries (mostly in North America and Europe), requires that any products imported into these countries are against valid quotas held by the exporters. Globally loading of production, therefore, requires new methods and techniques, which are different from those used in domestic loading of production. This paper presents a time staged linear programming model for production loading problems with import limits to minimize the total cost, consisting of raw materials cost, machine cost, labour cost, overtime cost, inventory cost, outsourcing cost and quota related costs. To enhance the practical implications of the proposed model, different managerial production loading plans are evaluated according to expected changes in future production policies and situations. A series of computational results demonstrate the effectiveness of the proposed model.

Keywords: Production loading; Import quota; Globalization; Global supply chain management; Time staged linear programming.

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1. Introduction

Today's business has inevitably set in a global environment in which materials and products can be bought, manufactured and sold anywhere in the world. Managing supply chains in such a globalized environment has become an important factor for gaining competitive advantages for business organizations. A vast majority of manufacturers have some form of global presence through exports, strategic alliances, joint ventures, or as part of a committed strategy to sell in foreign markets or locate production abroad (Dornier *et al.* 1998). Although global supply chains have many of the same fundamental functions and concepts as domestic supply chains, the differences are quite substantial and require different managerial approaches and techniques.

This study is motivated by the production loading problems faced by multinational manufacturing companies that participate in global supply chain activities. In the global supply chain, the multinational companies have their headquarters at one place, somewhere in the world. Product sales, R&D and customer service are typically centred in different markets, mainly North America and Europe. However, companies would like to establish production facilities in low-cost countries. Investment destinations have been diverse with production networks now extending to practically all over the world. China (mainland) is so far one of the favourite places for companies because of its low production and labour costs. This kind of global supply chain network plays an important role in today's business.

In the global supply chain systems, one of the most important decisions is loading production among plants, which are typically located in different regions and/or countries. While loading production, companies not only consider cost and capacity in terms of raw materials, machine, workforce, inventory and market demand, but also the import quota limits allowed to the country of manufacture. Import quotas are assigned by importing countries. Quotas control the quantity or volume of certain merchandise that can be imported into North American and European countries. The importing countries allocate a certain quantity of quota to each exporting country. Any products that belong to quota restriction categories have to have the corresponding quotas for the exporting countries. Many developing countries, including China, face restraints on textile and clothing exports to their trading partners that maintain import quotas, including the US, Canada, and European Union. For example, clothing and textile products are divided into 147 categories by the US and 143 categories by the European Union. Dickson (2005) states that not all the exporting countries face the same quota limitations for products. For example, China faces the US's quota limitation in 81 of 147 categories, while for India the figure is 30. At the same time, China faces quota limitation in 61 of 143 categories assigned by the EU, while for India it is 17. Therefore, global manufacturing companies have to consider quota limitations when they distribute manufacturing tasks among different plants, which are typically located in different cities and countries. If the quota for a certain category or product is used up in a country or quota price for that product/category is very high, companies may need to find alternatives in other countries that own quotas with reasonable price for the same product. Quota prices fluctuate because of many factors, like changing market demand and government policies.

In this study, we will look at a multinational garment manufacturing company, whose headquarters is in Hong Kong, and product sales, R&D, customer service and consumer markets are spread across North America and Europe. The Hong Kong headquarters collects customer information through its American and European branch offices. Then the headquarters commissions the plants, which are located in Mainland China, Sri Lanka, the Philippines, etc., to undertake the processing work. The finished products are then shipped to Hong Kong for onward shipping to overseas markets. Thus loading production among different plants is a critical managerial task for the company. The aim of this paper is to present a decision-making framework for modelling the production loading problems involving import quota limitation in the global supply chain. The rest of the paper is organized as follows. Section 2 is literature review part. Section 3 describes production loading process in the global supply chain. Section 4 presents a time staged linear programming model for the production loading problems with import quota limits. In Section 5, a set of data from the company is used to test the effectiveness of the proposed model. Different production loading strategies are provided to match different production requirements so that decision-makers can handle complicated changes under the global supply chain management environment. The final section gives the conclusions of the paper and the recommendations for future research.

2. Literature review

In recent years, researchers and practitioners have devoted a great deal of attention to global supply chain management. When configuring global supply chains, additional complicating factors arise such as duties, taxes, exchange rates and trade blocks. Effective management of supply chain activities dispersed throughout the global supply chain results in lower production and distribution costs. There is extensive literature on global supply chain management problems. A great deal of research has been carried out for designing supply chain networks on a global scale. Hodder and Jucker (1985) develop a series of models for an international plant location problem. Hodder and Diner (1986) further develop a model for analyzing international plant location and financing decisions with the considerations of uncertain taxes and currencies in different countries. Cohen and Lee (1989) point out how a company should structure its plants around the world to supply a global market with variations, from country to country, in consumers' expectations, recourse conditions, and cost structures. A survey article, presented by Verter and Dincer (1992), presents a review of modelling issues of international plant location, capacity acquisition, and technology selection. Rosenfield (1996) develops a number of deterministic and stochastic models to determine the number of plants and production levels in a global environment for a firm in order to minimize production and distribution costs for geographically dispersed markets. Arntzen *et al.* (1995) present a global supply chain model at Digital Equipment Corporation to minimize the cost, including fixed and variable production charges, taxes, duties and duty drawback. This model recommends a production, distribution and vendor network and has saved the company over $100 million. Taylor (1997) presents a model to integrate product choices, considering global plant capacities with an assumption of known unit costs and no trade barriers. Ferdow (1997) emphasizes that country attributes would determine whether a country becomes a manufacturing hub with exports to other countries or a market for imported goods, or both. Vidal and Goetschalckx (1997) present an extensive literature review on global supply chain models, and state that there is a lack of research on mixed integer programming models for the strategic design of global supply chain systems. Goetschalckx *et al.* (2002) present the potential savings generated by the integration of the design of strategic global supply chain networks with the determination of tactical production-distribution allocations and transfer prices, which combines strategic planning and tactical planning in global supply chain networks. Chakravarty (2005) develops a model that optimizes plant investment decisions and determines prices of products by countries. The model also analyses labour costs, transportation costs, demand and import tariff on production quantities, *etc.*

Supply chain coordination is increasingly viewed as a source of strategic advantage for participating members (Kulp *et al.* 2003). Cohen and Mallik (1997) emphasize that competitive advantages can be achieved through global supply chain management only if the management of the chain's geographically-dispersed activities is effectively coordinated. Coordination is, therefore, the key concept in implementing a global supply chain strategy. Kogut (1985a, b) first describe the importance of global coordination and develope global strategies. Dasu and Torre (1993a, b) study a case covering the affiliates of a U.S. multinational firm in three Latin American countries, concentrating on the coordination problem. A single-period deterministic game theoretical model is formulated to determine the price and sale amount for each firm and this is used in two scenarios: one scenario is in the competitive environment, where affiliates compete against each other as well as with other companies; and the other scenario is in the cooperative environment where the affiliates' activities are coordinated. Different factors related with international activities are considered in the model: these include exchange rates, inflation rates and tariff rates. Ahmadi and Yang (1995) study a parallel-import problem in a global supply chain under the assumption that a manufacturer could implement price discrimination in different markets. Thus parallel importers can buy products in low-priced markets and sell them in higher-priced markets. A major issue for global manufacturing companies is the impact of exchange rates. Lessar and Lightstone (1986) propose a qualitative study on the effect of exchange rate fluctuation in a multinational company. An extensive section of the literature (Cohen and Lee 1989; Tombak 1995; Dasu and Li 1997; Hadjinicola and Kumar 2002) discusses important factors such as tariffs, taxes, currency exchange rates, shipping costs, domestic resources and demand, and trade barriers. Some interesting works include global manufacturing strategy planning problem (Dyment 1987 and Noori 1994); global outsourcing problems (Flaherty 1989 and McMillan (1990); and global services operations problems (Lawrence 1993 and McLaughlin 1993). A wide variety of production loading techniques have been developed since the early 1950s. An important review about models and methodologies for production loading problems can be found in Nam and Logendran (1992), in which 140 journal articles and 14 books are categorized into optimal and near-optimal classifications. Lee *et al.* (2002) use a hybrid approach to solve production-distribution planning problems in the global supply chain environment. Dejonckheere *et al.* (2003) study the relationship and analogues between the dynamic responses of factory aggregate planning systems and those of production ordering systems used at the individual SKU level. Techawiboonwong and Yenradee (2003) discuss aggregate production planning with workforce transferring plan for multiple product types. Park (2005) presents solutions for integrated production and distribution planning and investigates the effectiveness of their integration through a computational study, in a multi-plant, multi-retailer, multi-item, and multi-period logistic environment where the objective is to maximize the total net profit.

3. Problem analysis

In this study, the headquarters in Hong Kong distributes production tasks among the different plants in Mainland China. The products under this study are fashion garments, which have a very short life cycle and lead time. For cost effectiveness, decision makers need to determine the quantity of each product manufactured by different plants to fulfil market demand in the next selling season. The decision makers also need to determine the machine processing time, workforce level, inventory level and quota utilisation, etc. Loading production is affected by some production constraints. To produce and import products overseas, machine, labour and quota are necessary resources. However, in some production situations, the company can change the capacity of these sources by increasing the machine capacity (using additional machine capacity through leasing), changing of workforce (through hiring, firing and overtime) and purchasing quota from local market in case of higher demand. Decisions include the quantity of each resource needed.

Labour consists of skilled and non-skilled workers. In order to guarantee product quality, skilled workers need to occupy a certain ratio among the workforce. For every product, it is known how many machine-hours and labour-hours (skilled and non-skilled workers) are necessary for processing each type of product. Labour cost depends on product quantities. Unit labour cost of skilled workers is greater than that of non-skilled workers. However, skilled workers need less time to do the work than non-skilled workers. Production planning needs to determine the workforce level (skilled and non-skilled workers) in each period, including how many workers are to be hired or fired. When a large number of orders are received, the company may require workers to work overtime, or hire additional workers (either skilled or non-skilled workers) within the constraints of machine capacity. On the other hand, if production task is not enough, redundant workers will be laid-off to reduce overheads.

For each period, market demand has to be met. If demand is high, additional labour (hiring and/or overtime) and machine capacity (leasing) can be used by incurring extra costs, although limitations apply to this recourse. Costs are also affected when production is either too much or too less to match demand. When production exceeds demand, a surplus cost will be incurred for storing excess products. On the other hand, when production is not enough to satisfy demand, a shortage cost will have to be incurred for purchasing products at a higher cost from the contracted plants.

Globally, loading production becomes more complex because the finished products need to be shipped to overseas markets and they need a certain amount of quotas for each type of product. Quota prices fluctuate frequently. At the beginning of the planning horizon, the company holds certain quantities of quota for each type of product at the original purchasing price. The ideal quota quantities for every product are equal to the expected demand. When quota quantities are not enough, the company has to purchase quotas at market price from local markets. This will mean incurring under-quota costs. On the other hand, when the quotas are not used up, an over-quota cost will be incurred because generally the unused quota either goes waste or has to be sold at low prices.

4. Model formulation

4.1. Notations

- *Subscripts*

i for products (i=1,…,m);

j for plants (j=1,…,n);

t for time periods (t=1,…,T);

- *Parameters*

c_{ij}^{1}: raw material cost of production for a unit of product *i* in plant *j*;

c_{j}^{21} / c_{j}^{22} machine regular/additional cost of production per hour in plant *j*;

c_{ij}^{31} / c_{ij}^{32} labour cost of skilled/non-skilled workers producing a unit of product *i* in plant *j*;

c_{jt}^{41} / c_{jt}^{42} labour overtime cost of skilled/non-skilled workers per hour in plant *j* in period *t*;

$c_{jt-1,t}^{51+}$ / $c_{jt-1,t}^{52+}$ labour cost for hiring skilled/non-skilled workers per hour in plant *j* between periods *t*-1, *t*;

$c_{jt-1,t}^{51-}$ / $c_{jt-1,t}^{52-}$ labour cost for firing skilled/non-skilled workers per hour in plant *j* between periods *t*-1, *t*;

c_{it}^{6-} / c_{it}^{6+} shortage/surplus cost of purchasing/storing a unit of product *i* in period *t*;

c_{i}^{7} initial quota purchasing cost of a unit of product *i*;

c_{it}^{7-} / c_{it}^{7-} under-/over-quota cost of product *i* in period *t*;

l_{ij}^{1} / l_{ij}^{2} labour time for production of a unit of product *i* in plant *j* by skilled/non-skilled workers;

h_{ij}^{1} / h_{ij}^{2} machine time for production of a unit of product *i* by skilled/non-skilled workers in plant *j*;

C_{jt} / A_{jt} maximum regular/additional machine capacity of plant *j* in period *t*;

I_{i} / B_{i} maximum inventory/purchasing capacity for product *i*;

V_{jt} minimum work time in plant *j* in period *t*;

L_{jt}^{1} / L_{jt}^{2} maximum labour capacity of hiring skilled/non-skilled workers in plant *j* in period *t*;

W_{jt}^{1} / W_{jt}^{2} maximum overtime for skilled/non-skilled workers in plant *j* in period *t*;

d_{i0}^{+} initial inventory of product *i* in plant *j*;

v_{j0}^{1} / v_{j0}^{2} initial labour time of skilled/non-skilled workers in plant *j* at the beginning of the planning horizon;

α_{j} limit ratio of labour work time between skilled and non-skilled workers in plant j;

D_{it} demand for product i in period t during the whole planning horizon;

Q_i initial quota quantities of product i at the beginning of the planning horizon;

- *Decision Variables*

x_{ijt}^1 / x_{ijt}^2 production quantities of product i by skilled/non-skilled workers in plant in period t;

$y_{jt-1,t}^{1+} / y_{jt-1,t}^{2+}$ planned labour time of hiring skilled/non-skilled workers in plant j between periods t-1, t;

$y_{jt-1,t}^{1-} / y_{jt-1,t}^{2-}$ planned labour time of firing skilled/non-skilled workers in plant j between periods t-1, t;

z_{jt}^1 / z_{jt}^2 overtime of skilled/non-skilled workers in plant j in period t;

u_{jt}^1 / u_{jt}^2 planned regular/additional machine capacities in plant j in period t;

v_{jt}^1 / v_{jt}^2 planned labour time of skilled/non-skilled workers in plant j in period t;

d_{it}^- / d_{it}^+ shortage/surplus production for product i at the end of period t;

q_{it} initially allocated quota quantity of product i in period t;

q_{it}^- / q_{it}^+ under-/over-quota quantities of product i in period t;

4.2. Objective function

The aim of this study is to load production task so that market demand can be fulfilled at a minimum total cost. To achieve the optimal plan, our study takes several cost factors into account.

- *Raw material cost*: As each plant has its raw material suppliers, raw material cost is different for each plant.

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T} c_{ij}^1 (x_{ijt}^1 + x_{ijt}^2) \tag{1}$$

- *Machine cost*: Machine capacity includes regular and additional machine capacity.

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T} (c_j^{21} u_{jt}^1 + c_j^{22} u_{jt}^2) \tag{2}$$

- *Labour cost:* Plant j will pay the skilled workers c_{ij}^{31} for processing each product i, and the non-skilled workers c_{ij}^{32}.

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T}(c_{ij}^{31}x_{ijt}^{1}+c_{ij}^{32}x_{ijt}^{2}) \tag{3}$$

- *Overtime cost:* To satisfy demand, overtime can be used.

$$\sum_{j=1}^{n}\sum_{t=1}^{T}(c_{jt}^{41}z_{jt}^{1}+c_{jt}^{42}z_{jt}^{2}) \tag{4}$$

- *Hiring/firing worker cost:* It costs the company to hire or fire skilled/non-skilled workers.

$$\sum_{j=1}^{n}\sum_{t=1}^{T}(c_{jt-1,t}^{51+}y_{jt-1,t}^{1+}+c_{jt-1,t}^{52+}y_{jt-1,t}^{2+}+c_{jt-1,t}^{51-}y_{jt-1,t}^{1-}+c_{jt-1,t}^{52-}y_{jt-1,t}^{1+}) \tag{5}$$

- *Shortage/surplus cost*: When market demand is not satisfied, the company will purchase products from its contracted plants at the cost c_{it}^{6-}. On the other hand, when production exceeds market demand in each period, the surplus products have to be stored at the cost c_{it}^{6+}.

$$\sum_{i=1}^{m}\sum_{t=1}^{T}(c_{it}^{6-}d_{it}^{-}+c_{it}^{6+}d_{it}^{+}) \tag{6}$$

- *Quota related cost:* c_i^7 is the original quota cost of purchasing a unit of product i. When initially allocated quota amount is not enough, the company will purchase additional quotas from markets at the unit cost c_{it}^{7-}. On the other hand, when quota amount on hand exceeds market demand, a penalty cost c_{it}^{7+} per unit will be incurred. The initial quota purchasing costs and under-/over-quota cost can be formulated as:

$$\sum_{i=1}^{m}\sum_{t=1}^{T}(c_i^{7}q_{it}+c_{it}^{7-}d_{it}^{-}+c_{it}^{7+}d_{it}^{+}) \tag{7}$$

The objective is to minimize the total cost, which is the sum of all the above costs. The objective function of the time staged linear programming model can be formulated as follows:

$$\text{Min } \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T}c_{ij}^{1}(x_{ijt}^{1}+x_{ijt}^{2})+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T}(c_{j}^{21}u_{jt}^{1}+c_{j}^{22}u_{jt}^{2})+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{t=1}^{T}(c_{ij}^{31}x_{ijt}^{1}+c_{ij}^{32}x_{ijt}^{2})$$

$$+\sum_{j=1}^{n}\sum_{t=1}^{T}(c_{jt}^{41}z_{jt}^{1}+c_{jt}^{42}z_{jt}^{2})+\sum_{j=1}^{n}\sum_{t=1}^{T}(c_{jt-1,t}^{51+}y_{jt-1,t}^{1+}+c_{jt-1,t}^{52+}y_{jt-1,t}^{2+}+c_{jt-1,t}^{51-}y_{jt-1,t}^{1-}+c_{jt-1,t}^{52-}y_{jt-1,t}^{1+})$$

$$+\sum_{i=1}^{m}\sum_{t=1}^{T}(c_{it}^{6-}d_{it}^{-} + c_{it}^{6+}d_{it}^{+}) + \sum_{i=1}^{m}\sum_{t=1}^{T}(c_{i}^{7}q_{it} + c_{it}^{7-}d_{it}^{-} + c_{it}^{7+}d_{it}^{+}) \tag{8}$$

4.3. Constraints

In this study, we aim to minimize the total cost described in section 4.2. At the same time, production loading is restricted by a series of constraints, including demand constraints, quota constraints, machine capacity constraints, workforce level constraints, quality constraints, upper bound and lower constraints and non-negative constraints.

- *Demand constraints:* In each period and for each product, market demand D_{it} has to be met by a combination of production $\sum_{j=1}^{n}(x_{ijt}^{1} + x_{ijt}^{2})$ in all plants, purchases d_{it}^{-} from their contracted plants in this period and inventory from previous periods d_{it-1}^{+}. Surpluses d_{it}^{+} in this period have to be stored.

$$\sum_{j=1}^{n}(x_{ijt}^{1} + x_{ijt}^{2}) + d_{it-1}^{+} + d_{it}^{-} - d_{it}^{+} = D_{it},\ i=1,..,m,\ t=1,\dots,T. \tag{9}$$

- *Quota constraints:* In each period, each product needs to have its own quota. The ideal situation is that in each period the demand is equal to the initially allocated quota. However, when the quota amount is insufficient, the company needs to purchase additional quota from local markets. On the other hand, when the quota is not used fully, there are some quotas left.

$$q_{it} + q_{i,t-1}^{+} + q_{it}^{-} - q_{it}^{+} = D_{it},\ i=1,\dots,m,\ t=1,\dots,T. \tag{10}$$

- *Machine capacity constraints*: In each period and for each plant, regular machine and additional capacity must be sufficient to produce the desired number of products.

$$\sum_{i=1}^{m} h_{ij}^{1}x_{ijt}^{1} + h_{ij}^{2}x_{ijt}^{2} = u_{jt}^{1} + u_{jt}^{2}. \tag{11}$$

- *Labour processing constraints:* v_{jt}^{1} denotes the processing time of the skilled workers, and v_{jt}^{2} denotes the processing time of the non-skilled workers.

$$\sum_{i=1}^{m} l_{ij}^{1}x_{ijt}^{1} = v_{jt}^{1},\ j=1,\dots,n,\ t=1,\dots,T. \tag{12}$$

$$\sum_{i=1}^{m} l_{ij}^{2}x_{ijt}^{2} = v_{jt}^{2},\ j=1,\dots,n,\ t=1,\dots,T. \tag{13}$$

- *Workforce level constraints:* The available workforce in any period equals the workforce in the previous period plus the change of workforce level in the current period. The change in workforce may be due to hiring extra workers, firing redundant workers or overtime.

$$v_{jt}^{1} = v_{jt-1}^{1} + y_{jt-1,t}^{1+} - y_{jt-1,t}^{1-} + z_{jt}^{1}\ , j=1,\dots,n;\ t=1,\dots,T. \tag{14}$$

$$v_{jt}^{2} = v_{jt-1}^{2} + y_{jt-1,t}^{2+} - y_{jt-1,t}^{2-} + z_{jt}^{2}\ , j=1,\dots,n;\ t=1,\dots,T. \tag{15}$$

- *Quality constraints:* Constraint (16) ensures that the ratio between labour processing time for each product processed by skilled workers and non-skilled workers should not be less than a given constant in each period so as to guarantee production quality.

$$v_{jt}^{1} \geq \alpha_j v_{jt}^{2}\ , j=1,\dots,n,\ t=1,\dots,T. \tag{16}$$

- *Initial quota allocation constraints:* At the beginning, the initial quota is allocated in each time period.

$$\sum_{t=1}^{T} q_{it} = Q_i\ , i=1,\dots,m. \tag{17}$$

- *Minimum work time constraints:* Each plant has a minimum work time in each period.

$$v_{jt}^{1} + v_{jt}^{2} \geq V_{jt}\ j=1,\dots,n,\ t=1,\dots,T. \tag{18}$$

- *Upper bound constraints:* The capacity has the upper bound limits in terms of purchasing products from contracted plants, inventory, machine regular/additional capacity, and available labour time and overtime for skilled/non-skilled workers.

$$d_{it}^{-} \leq B_{it}\ , i=1,\dots,n,\ t=1,\dots,T, \tag{19}$$

$$d_{it}^{+} \leq I_{it}\ , i=1,\dots,n,\ t=0,\dots,T, \tag{20}$$

$$u_{jt}^{1} \leq C_{jt}\ , j=1,\dots,n,\ t=1,\dots,T. \tag{21}$$

$$u_{jt}^{2} \leq A_{jt}\ , j=1,\dots,n,\ t=1,\dots,T. \tag{22}$$

$$y_{jt-1,t}^{1+} - y_{jt-1,t}^{1-} \leq L_{jt}^{1}\ , j=1,\dots,n,\ t=1,\dots,T. \tag{23}$$

$$y_{jt-1,t}^{2+} - y_{jt-1,t}^{2-} \leq L_{jt}^{2}\ , j=1,\dots,n,\ t=1,\dots,T. \tag{24}$$

$$z_{jt}^{1} \leq W_{jt}^{1}\ , j=1,\dots,n,\ t=1,\dots,T. \tag{25}$$

$$z_{jt}^{2} \leq W_{jt}^{2}\ , j=1,\dots,n,\ t=1,\dots,T. \tag{26}$$

- *Variable type constraints:* All decision variables are required to be non-negative.

$$x_{ijt}^{1}, x_{ijt}^{2}, y_{jt-1,t}^{1+}, y_{jt-1,t}^{1-}, y_{jt-1,t}^{2+}, y_{jt-1,t}^{2-}, z_{it}^{1}, z_{it}^{2}, u_{jt}^{1}, u_{jt}^{2}, v_{jt}^{1}, v_{jt}^{2}, d_{it}^{-}, d_{it}^{+}, q_{it}, q_{it}^{-}, q_{ii}^{+} \geq 0,$$

$$i=1,\ldots, m, j=1,\ldots,n, t=1,\ldots,T. \tag{27}$$

5. Computational results

5.1. A practical problem

In order to illustrate the effectiveness of the proposed model for production loading, we use data provided by a garment manufacturing company involving a global supply chain network. Based on customer information from North American and European markets, the manufacturing company will load production tasks for three types of new products. Based on the strategic plan, the company has decided to process the products in their Chinese plants located in Dongguan, Huidong and Zhongshan. The company will look at a four-period planning. Table 1 gives the unit raw material cost, labour cost, and labour and machine time. Table 2 gives the unit machine cost for regular and additional production, and the unit overtime cost for skilled and non-skilled workers. Table 3 gives the maximum machine regular/additional capacity, maximum labour capacity, maximum overtime capacity and minimum work time. Table 4 gives maximum inventory capacity and purchasing capacity. Table 5 shows unit shortage/surplus cost, unit under-/over-quota cost and market demand. Currently, there is no cost involved in hiring/firing workers because there is a large supply of skilled and non-skilled workers in China and there is no union contract limitation either. In addition, there is no existing inventory of the new products, which will on sale in the next season. At the beginning of the planning horizon, there are quotas for 9,500 units of product 1, 8,000 units of product 2 and 6,600 units of product 3. The initial quota purchasing cost is 20.5 for product 1, 13 for product 2, and 6.55 for product 3, respectively.

Table 1

Unit raw material cost, labour cost, labour time and machine time

Product	Plant	Raw material cost	Labour cost of skilled workers	Labour cost of non-skilled workers	Labour time of skilled workers	Labour time of non-skilled workers	Machine time for skilled workers	Machine time for non-skilled workers
1	1	4	4.5	4	2	2.25	1.75	2.25
	2	4.2	4	3.5	2.25	2.5	2	2.5
	3	4.3	3.5	3	2.5	2.75	2.25	2.75
2	1	3	4	3.5	1.5	1.75	1.25	1.75
	2	3.2	3.5	3	1.75	2	1.5	2
	3	3.3	3	2.5	2	2.25	1.75	2.25
3	1	2	3	2.5	1	1.25	0.75	1.25
	2	2.2	2.5	2	1.25	1.5	1	1.5
	3	2.3	2	1.5	1.5	1.75	1.25	1.75

Table 2

Unit machine cost and overtime cost

Plant	Regular machine cost for production	Additional machine cost for production	Overtime cost for skilled worker	Overtime cost for non- skilled worker
1	0.05	0.088	11	8
2	0.08	0.010	9	6
3	0.10	0.120	8	5

Table 3

Maximum capacity for machine, labour and overtime and minimum labour work time

Plant	Period	Maximum machine regular capacity	Maximum machine additional capacity	Maximum capacity of hiring skilled workers	Maximum capacity of hiring non-skilled workers	Maximum overtime by skilled workers	Maximum overtime by non-skilled workers	Minimum labour work time
1	1	5500	250	4800	2400	2400	1200	2400
	2	5500	250	4800	2400	2400	1200	2400
	3	5500	250	4800	2400	2400	1200	2400
	4	5500	250	4800	2400	2400	1200	2400
2	1	5000	250	3840	1920	1920	960	1800
	2	5000	250	3840	1920	1920	960	1800
	3	5000	250	3840	1920	1920	960	1800
	4	5000	250	3840	1920	1920	960	1800
3	1	5000	200	2400	1200	1200	600	1500
	2	5000	200	2400	1200	1200	600	1500
	3	5000	200	2400	1200	1200	600	1500
	4	5000	200	2400	1200	1200	600	1500

Table 4

Maximum inventory and purchasing capacity

Product	Period	Maximum inventory capacity	Maximum purchasing
1	1	1500	500
	2	1500	500
	3	1500	500
	4	1500	500
2	1	1500	500
	2	1500	500
	3	1500	500
	4	1500	500
3	1	1500	500
	2	1500	500
	3	1500	500
	4	1500	500

Table 5

Unit shortage/surplus cost, unit under-/over-quota cost and market demand

Product	Period	Shortage cost	Surplus cost	Under-quota cost	Over-quota cost	Demand
1	1	100	2	24	3	1800
	2	110	2	26	3	1900
	3	120	2	28	3	2000
	4	110	2	26	3	2100
2	1	60	1	15	2	1400
	2	65	1	17	2	1600
	3	70	1	17	2	1800
	4	70	1	16	2	2000
3	1	40	0.5	8	1	1100
	2	50	0.5	10	1	1200
	3	50	0.5	11	1	1300
	4	45	0.5	9	1	1400

5.2. Computational result

Using the input data shown in Tables 1~5, the time staged linear programming model presented in Section 4 can be solved. The optimal production loading plans can be obtained, and the total cost is 514,168. Additionally, we can obtain other results such as production amount, work force level, worker overtime, use of regular machine capacity and additional machine capacity, use of skilled workers and non-skilled workers, inventory/purchase production, and purchase and surplus quantities of quota.

- *Production quantities:* Table 6 shows each plant's production output for each product during the planning horizon by skilled and non-skilled workers. All products of types 2 and 3 are loaded in plant 3. All products of type 1 are distributed in plants 1 and 2 for manufacturing.

Table 6

Production quantity of products in different plants

Plant	Product	Skilled workers				Non-skilled workers			
		Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	1	600	600	804	600	533	533	715	533
	2								
	3								
2	1	740	985	1176	1005	126	381	105	362
	2								
	3								
3	1								
	2	1200	1600	1387	1387	300	400	913	613
	3					300	857	410	796

- *Machine capacity:* Table 7 shows the used regular machine capacity as well as additional machines. Except period 3 in plant 2, plant 1 and 2 have some unused capacity in other periods. As plant 3 has a relatively low cost in terms of raw materials, labour and machine, it has used up all it's capacity of 5,000 in periods 2, 3 and 4, and additional capacity of 200 in these periods is also needed in order to satisfy market demand while keeping the total cost low.

Table 7
Machine work time

Plant	Regular capacity used				Additional capacity used			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	2250	2250	3015	2250				
2	3146	4188	5000	4270				
3	3300	5000	5000	5000		200	200	200

- *Workforce level:* Table 8 shows planned labour work time of skilled and non-skilled workers in each plant for each period. The table shows that the labour hours for skilled-workers are not less than those of non-skilled workers in each period, which guarantees product quality in each period. Tables 9 and 10 give the planned results of hiring or firing labour hours of skilled and non-skilled workers in each period.

Table 8
Labour work time

Plant	Skilled workers				Non-skilled workers			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	1200	1200	1608	1200	1200	1200	1608	1200
2	1666	2217	2647	2261	1666	2217	2647	2261
3	2400	3200	2773	2773	1200	2400	2773	2773

Table 9
Hiring workers

Plant	Skilled workers				Non-skilled workers			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	1200		408		1200		408	
2	1666	552	430		1666	552	430	
3	2400	800			1200	1200	373	

Table 10
Firing workers

Plant	Skilled workers				Non-skilled workers			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	1200		408		1200		408	
2	1666	552	430		1666	552	430	
3	2400	800			1200	1200	373	

- *Overtime:* No workers need to work overtime for each period in the whole planning horizon because of high unit overtime cost. It is cheaper to use part-time workers rather than overtime.
- *Shortage/surplus production:* There is no inventory for any products in each period. At the same time, no contracted plants need to be used for urgent production in the case of high demand since production resources of machine and labour are enough to satisfy market demand. In fact, there is unused machine capacity in plants 1 and 2 (see Table 7).
- *Quota:* At the beginning of the horizon, the company has a certain amount of quotas on hand for each type of product: 9,500 for product 1, 8,000 for product 2, and 6,600 for product 3. Table 11 shows the allocated quotas for each product in each period. As the initial quota quantity for product 1 is not enough, additional quotas for product 1 are needed. Table 12 gives the quota amount that needs to be bought from the market for each type of product. For product 2, there are some quotas left (see Table 12), which cause a penalty cost for purchasing excess quotas. As the quota amount of product 3 matches the total demand during the whole planning horizon, there are no unused or additional quotas required (See Tables 12).

Table 11
Quotas allocated

Product	Period 1	Period 2	Period 3	Period 4
1	1700	2500	2800	2500
2	1500	2000	2300	2200
3	1200	1700	2000	1700

Table 12
Under-/over-quotas

Product	Under-quota amount				Over-quotas amount			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	300							
2								200
3								

5.3. Production loading strategy analysis

The optimal production loading plan can be obtained by solving the time staged linear programming model proposed in Section 4. In order to understand fully the production loading strategies, we look into alternative production planning strategies that can help production managers make better decisions. Let Scenario 0 represent the production loading scenario, described in Section 5.1 and 5.2. In this paper, we discuss another six scenarios, which are described in Table 13.

Table 13
Scenario Assumptions

Scenario	Description
0	Using existing data
1	Demand in Scenario 0 is increased by 10%
2	Demand in Scenario 0 is decreased by 10%
3	Quota in Scenario 0 is increased by 10%
4	Quota in Scenario 0 is decreased by 10%
5	Demand and quota in Scenario 0 are simultaneously increased by 10%
6	Demand and quota in Scenario 0 are simultaneously decreased by 10%

The computational results are shown in Table 14. In the following seven scenarios, the cost of overtime, shortage and surplus cost are equal to zero. It means that the company would not adapt overtime production strategy in any scenario. Also, there is no need to produce early or to outsource.

Table 14
Computational results in different scenarios

Scenario	Raw material cost	Machine cost	Labour cost	Overtime cost	Shortage cost	Surplus cost	Initial quota cost	Under -quota cost	Over -quota cost	Total cost
0	79113	11156	74319	0	0	0	341980	7200	400	514168
1	86787	11999	82586	0	0	0	341980	44700	0	568043
2	71304	10186	66404	0	0	0	341980	0	4660	494535
3	79113	11156	74319	0	0	0	376178	0	4610	545376
4	79113	11156	74319	0	0	0	307782	44280	0	516650
5	86787	11999	82586	0	0	0	376178	7920	440	565892
6	71304	10186	66404	0	0	0	307782	6480	360	462517

- *Production loading strategy I – only demand is increased (scenario 1):* It is important to take into account the changes in production loading strategy when market demand increases or decreases. Table 14 shows that the total cost increases by 10.48% when demand increases by 10% (see row 2, Table 13). The machine cost and the labour cost increase by 7.53% and 11.12%, respectively. As initial quota amount remains unchanged, the initial quota purchasing cost remains unchanged in scenario 1. However, the company has to buy additional quotas from the market in order to satisfy high demand, which costs the company 44,700. There is no penalty cost involved for unused quotas in scenario 1.

- *Production loading strategy II –only demand is decreased (scenario 2):* In scenario 2 (see row 3, Table 14), the total cost decreases by 3.82% when demand decreases by 10%. The machine cost and the labour cost decrease by 8.69% and 10.65%, respectively. As initial quota amount is not changed, some quotas are left, incurring the over-quota penalty cost of 4,660. The quota purchasing cost is very high (44,700) in scenario 1 in comparison to quota purchasing cost of 7,200 in scenario 0 and the over-quota penalty cost of 4,660 in scenario 2.
- *Production strategy III – only initial quota is increased (scenario 3):* Quota is an important factor that influences production loading strategy. It is assumed that the company would like to change the initially allocated quota quantities. In scenario 3, if quota amount is increased by 10%, the total cost increases by 5.44%. The reason is that the company spends more money on purchasing the quotas at the initial price. As a result, there is no need to purchase additional quota during the whole planning horizon, and some quotas are left, incurring the over-quota penalty cost of 4,610 in scenario 3. The quota penalty cost is 400 in scenario 1.
- *Production strategy IV – only initial quota is decreased (scenario 4)*: In scenario 4, when the quota quantity is decreased by 10%, the total cost increases by 0.48% because the initial quotas are not enough to meet demand. In scenario 4, the company has to buy additional quotas at the market price (see Table 5), which varies in different periods for each types of products and is much higher than their initial purchasing cost. Therefore, the total cost increases by 0.48% in scenario 4, although the initial quota quantity is decreased.
- *Production strategy V – demand and quota are increased simultaneously (scenario 5):* Scenario 1 considers only the market demand increases, and other parameters remain unchanged. In Scenario 2, we consider only the import quotas increases, and other parameters remain unchanged. Scenario 5 would consider what the impact would be on the total production cost if both market demand and import quotas are

simultaneously increased by 10%. Comparing with the total costs in scenario 0, the total cost in Scenario 5 increases by 9.94%, because of the increase in cost of purchasing raw material, using machine and labour, and purchasing initial quota. The total cost of adopting production strategy V (565,892) is less than the total cost of production strategy I (568,043), in which case only market demand is increased by 10% and the import quota is not changed. Thus we can conclude that when market demand increases, the initial quota allocation amount should be increased accordingly.

- *Production strategy VI – demand and quota are decreased simultaneously (scenario 6).* In scenario 6, we consider a situation where both market demand and import quota are simultaneously decreased by 10%. Comparing with the total cost in scenario 0, the total cost in scenario 6 decreases by 10.05%, but the total cost in scenario 1 decreases by 3.82% only, in which case only demand is decreased by 10%. Based on the above production loading analysis, we can conclude that import quota is a very important factor that affects cost, and it should match market demand in order to keep production cost low.

5.4. Model validation

To validate the effectiveness of the models, a series of computational experiments are carried out using the data provided by the company for 12 months. Fig. 1 shows the total demand and initially allocated quota in the 12 months. Initially, the company has 270,000 quotas on hand for the whole year. The company believes that demand will increase over time during the year. Therefore, the company allocates different amount of quotas to different quarters during the year: 21,000 per month in the first quarter, 22,000 per month in the second quarter, 23,000 per month in the third quarter and 24,000 per month in the fourth quarter.

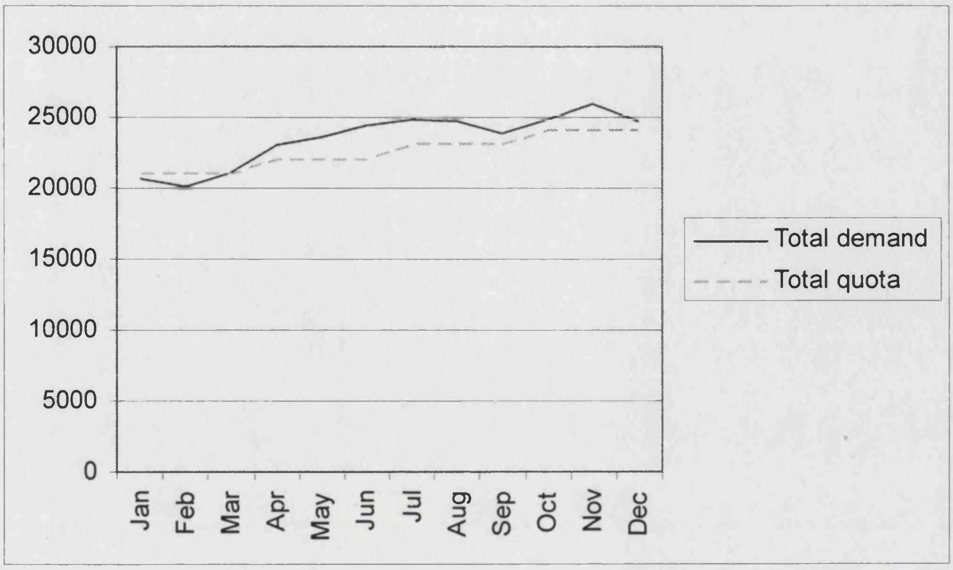

Fig. 1. Total demand and initial quota amount during the whole year

Fig. 2 shows the computational results regarding material cost, machine cost, labour cost, quota cost and the total cost during the whole year. In the whole year, there is no overtime cost or shortage/surplus cost. Fig. 2 shows that the company spends a large amount of money on import quotas, including the cost of purchasing initial quota, the cost of purchasing additional quota at a high market cost, as well as the penal cost of unused quota. The quota cost becomes the most significant factor in the total cost for the multinational company, which works out to about 64.52% of the total cost. The material cost, machine cost and labour cost are 18.13%, 2.57% and 14.76% of the total cost, respectively. Labour cost is usually very high in other countries, particularly in the developed countries. However, labour cost in China is very low, which is one of the main reasons for multinational companies to locate their production facilities in China. In this study, three plants are located in three cities in southern China, which belongs to one of the highest labour cost areas in comparison to other cities in northern and western China. In the past few years, there has been an increased trend for the multinational companies in locating production bases towards lower labour cost cities in China.

Fig. 1 shows the quota amount in the first quarter is greater than demand. As a result, the quota cost is very low in the first quarter (see Fig 2), as there is no need to buy additional quotas at market price. In the first quarter, the quota cost mainly includes the cost for purchasing initial quota and the penalty cost for not using up the allocated quotas. From the second quarter, demand increases significantly. The initial allocated quota amount also increases (see Fig. 1). However, the total quota amount is still not enough to satisfy demand in the second quarter. Thus additional quotas are purchased from the markets, which results in higher total cost. This situation becomes even worse when the summer arrives. In September, demand decreases sufficiently to come close to the quota on hand. Therefore, the company does not need to purchase any quotas from markets in September, which significantly reduces the total cost in September. From October, demand increases sharply because Christmas is approaching, which requires purchase of additional quotas. Finally, the total cost decreases in December when Christmas actually arrives.

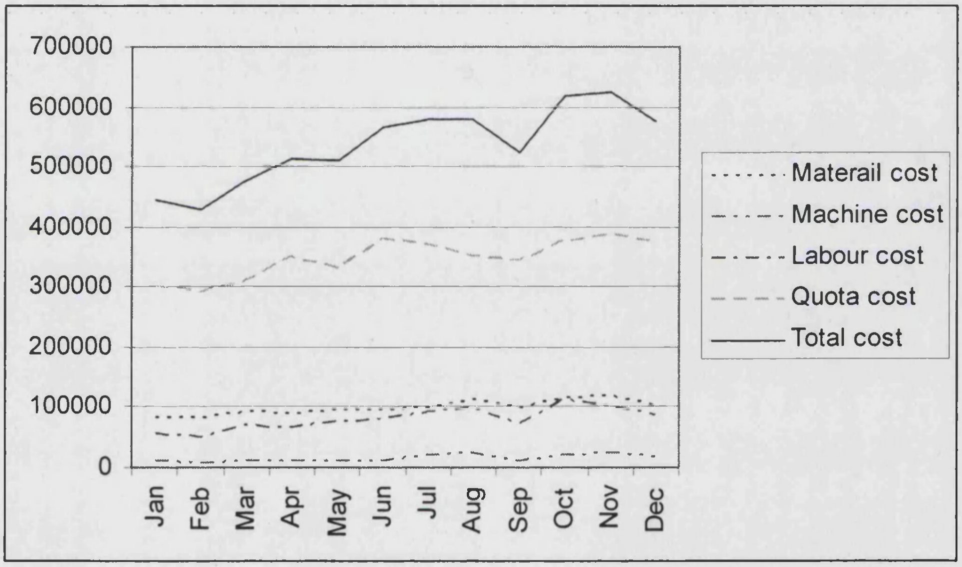

Fig. 2. Different costs during the whole year

6. Conclusions

Today's business has been set in the global supply chain management environment. More and more companies have realized the importance of global supply chain management by seeking suitable locations and facilities anywhere in the world for manufacturing, marketing and distributing. This paper studies the production loading problems in the global supply chain network, in which the import quota limit is applied for companies, which distribute production task among different plants in China aiming at satisfying North American and European market demand while attempting to minimize the total production cost. This type of production loading problem becomes more and more important in today's highly competitive global markets. Therefore, effective production loading strategies can provide a competitive advantage by reducing production cost. In this paper, a time-stage linear programming model is presented for modelling production loading problems with import quota limits. Decisions include the quantities of used resources, including machine, labour and initial quotas, as well as inventory levels, outsourcing levels, purchased quotas from local markets and unused quotas. A series of experiments, whose data is from a multinational garment company, are designed to test the effectiveness of the proposed model in solving practical production loading problems. Different production loading strategies are provided so that production managers can handle complicated future changes for the production loading problems in the global manufacturing environment. The computational results also show that import quota is a significant factor in loading production in terms of availability and cost in importing-exporting trade. Production managers have to adopt new approaches and techniques to handle production loading problems with import quota limit in the global supply chain environment. The methods used in global manufacturing are different from those in the domestic production loading process. Failure to consider the international factors, such as quota limits discussed in this paper, may lead to higher production costs and even disastrous consequences. For example, the finished products are not permitted to be exported to demand locations because of the lack of the corresponding quotas for the specific products. Future research will consider uncertainties in production loading process, such as dynamic and changing market demand and quota price; dynamic programming, stochastic programming and fuzzy programming techniques can be applied to these problems. As these models would substantially increase the computational time, artificial intelligence algorithms like genetic algorithm, tabu search and simulated annealing may be considered to solve the large scale of problems.

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Appendix F

A paper accepted by *European Journal of Operational Research*, subject to revision

A Dual-Response Forwarding Approach for Containerizing Air Cargo under Uncertainty Based on Stochastic 0-1 Programming

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Abstract

This study proposes a new approach, namely dual-response forwarding, to help airfreight forwarders to make decisions about renting air containers and loading cargos under uncertain information. The main uncertain parameter is cargo quantities. The first response that the airfreight forwarders make is to book air containers in advance from the airlines without full information. The second response is to take corresponding action for different scenarios that might happen on the shipping day after complete information becomes available. The airlines provide different types of air containers with differing weight and volume limits for renting. A discount rental rate is offered by the airlines to encourage the forwarders to book in advance. The cost of renting a container is based on a fixed cost plus a variable cost that depends on the weight inside the container. At the same time, the airlines impose a heavy penalty for any changes to the booked containers on the shipping day. In this study, we first build a 0-1 model for the deterministic version. Then a two-stage stochastic 0-1 programming is formulated to model the dual-response forwarding approach, whose goal is to minimize the total costs charged by airlines. A series of experiments are designed to test the effectiveness of the two-stage stochastic 0-1 programming model. Compared with the results of the deterministic model, the stochastic model provides a more cost-efficient, flexible, and responsive approach for air cargo forwarding.

Key words: Air container; Cargo loading; Containerization; Globalization; Global supply chain; Stochastic programming;

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1. Introduction

Today's business is set in a global environment, where companies are forced to compete on price and delivery performance in the face of continuously changing conditions. Logistics has never before played such an important role in the supply chain. This is particularly true for the global supply chain network, where products are typically manufactured in low-cost regions, such as Asia, and are mainly marketed in Northern America and Europe. The distance factor, therefore, becomes critical with shipments moving thousands of miles from one site to another site, particularly for shipments with a very short delivery time. Because the majority of the world is separated by water, air and ocean become the major modes for global transport (Coyle, *et al.*, 2003). Ocean transport is the most popular mode of global transport because of its low rates, and its ability to transport either very heavy or large cargos. However, ocean transport takes a long time. The low transit times for air transport are having a dramatic effect on transportation, particularly for the global market. The tremendous speed of airplanes combined with a high frequency of scheduled flights to the majority of cities in the world has reduced cargo transit time from as many as 50 days to one or two days. Although air cargo presently accounts for a small percentage of global freight by weight, the nature of air cargo – mostly high-value and low density – causes the total value of airfreight cargo to account for an ever-increasing proportion of total world cargo (Muller, 1999). According to the *1996/1997 World Air Cargo Forecast*, published by the Boeing Commercial Airline Group, the 6.6% annual increase of air cargo is less than the 7.8% growth between 1970 and 1992. By 2010, world air cargo is expected to triple, and the international market will account for about 80% of total revenue ton kilometres (RTKs) (Muler, 1999). In addition to speed, dependability, frequency, air transport also offers substantial savings for its customers in low insurance, cheap labour costs for packing, loading and unloading, dramatically decreasing cost of warehousing and inventory, having less capital invested in large shipments by sea, and faster capital turnover. Particularly, fast delivery by air cargo provides a competitive advantage of improving customer service by offering a flexible response in a dynamic and changing market. Nowadays, business success increasingly replies on the speed instead of quality, which has become a minimum standard rather than a competitive advantage in many industries. In addition, globalization, an increased variety of products and customerized products, shorter life cycles and lead times of products, empowered customers with high expectations of quick responses, speedy delivery, and low costs, are heightening the importance of air transport, and are forcing logistics managers to develop competitive strategies, tactics, and operations in order to survive in this highly competitive, dynamic and uncertain environment.

As the air transport is cost-, and time- sensitive, it is crucial for logistics managers to choose adequate containers for shipping. However, uncertain and changing cargo information which customers provide, and price discounts and penalty policies that airlines offer, makes the decision-making process very complicated. In this study, we propose a dual-response forwarding approach for selecting containers and loading cargos, in which two-stage actions are taken to respond to uncertain and changing market information. The first stage action is made before the accurate information is available. The second stage action is made after the uncertainty is realized. A two-stage stochastic 0-1 programming model is formulated to structure a dual-response forwarding system that is as responsive and flexible as possible to satisfy changing market information.

The rest of the paper is organized as follows. The related literature is reviewed in Section 2. Section 3 describes the dual-response strategy and air cargo forwarding problems. A deterministic version model is formulated in Section 4 when all information is known and certain. Section 5 presents a 0-1 stochastic programming for the uncertain air cargo forwarding problem. Section 6 demonstrates how the proposed models can be used to solve practical container selecting and cargo loading problems with experiments under different scenarios. The final section gives the conclusions to this study.

2. Literature Review

Containers first started to be used in the 1950s and the proportion of cargo handled has been steadily increasing since then. Containers are defined as large boxes that are used to transport goods from one destination to another (Vis and Koster, 2003). The efficient stowage of goods in means of transport can often be modelled as a container loading problem (Bortfeldt and Gehring, 2001). The analysis of containers loading problems has been an active research area for many years. In the literature, the container loading problems are differentiated in several ways (Dyckhoff and Finke, 1992). The first type of differentiation is to classify container loading as a three-dimensional (3D) rectangular packing problem, which belongs to the general cutting and packing problem (Bischoff and Marriott 1990, George *et al.* 1993, George 1996, Han *et al.* 1989, and Ivancic *et al.* 1989). An excellent survey and classification of cutting and packing problems is given in Dyckhoff (1990).

The second way of differentiation is by transport modes: sea or air. Bischoff and Ratcliff (1995) highlight some important shortcomings in the existing theoretical literature on sea container loading problems, and outline a series of considerations in loading sea containers, such as orientation constraints, handling constraints, load stability in a vessel, multi-drop situation, separation of items within a sea container, weight distribution in a sea container, etc. In their study, they also criticize the fact that much of the work published only considers container loading as a pure knapsack type problem. Davies and Bischoff (1999) consider weight distribution in sea container loading problem, and provide a new approach to obtain an even weight distribution in a container whilst simultaneously achieving a high degree of space utilization. Bortfeldt and Gehring (2001) present a hybrid genetic algorithm for the sea container loading problem with boxes of different sizes and a single container for loading. Vis and Koster (2003) give an overview of the literature related to the trans-shipment of the sea container at a container terminal, including the loading/unloading process, facilities and vehicles for container movement, intermodel transportation, and related decision problems.

The current literature on the container loading problems, however, mainly focuses on sea container loading. During the past decade, there has been a continuous increase in publications discussing air container loading problems. However, the majority of the literature is concerned with the gravity issue of container loading in an aircraft. Martin-Vega (1985) presents a complete review of the manual and the computer-assisted approaches to air container loading problems, considering the centre of gravity via pyramid loading. Mathur (1998) further extends Martin-Vega's work in 1985 by providing an algorithm with a better worst-case performance. Amiouny et al. (1992) present a simple greedy heuristics for balancing when loading a container with the assumption that all given air containers must be loaded and containers are positioned on a one-dimensional hold. Ng (1992) considers a military application, in which air cargo must be fully loaded with a priority sequence. Mongeau and BÈS (2003) address the problem of maximizing freight loading in an aircraft while balancing the weight in order to minimize fuel consumption and satisfy stability and safety requirements. A mathematical programming model is formulated in their work to choose which containers should be loaded on the aircraft, and how they should be distributed among different compartments.

To our best knowledge, little research has been conducted on the cost issue of air container loading, as well as uncertainty involved. Billington and Johnson (2003) present a dual-response manufacturing concept, in which a firm utilizes two types of capacity to balance lead times against cost: one resource with lead times but lower cost, and the other resource with short lead times and a higher cost. Their paper uses Hewlett Packard as an example of using the dual-response manufacturing concept to supply inkjet printers in the Northern American market. However, there is little work on the use of quantitative techniques to model the dual-response concept in solving uncertain air cargo forwarding problems. Stochastic programming is a branch of mathematical programming that copes with a class of mathematical models and algorithms in which the data may be subject to significant uncertainty. Since its invention in the 1950s by Beale (1955), Dantzig (1955) and Charnes and Cooper (1959), stochastic programming has made significant applications in many areas including production planning (Escudero et al., 1993), financial planning (Cariño *et al.*, 1994), telecommunications network planning (Sen *et al.*, 1999), electric power generation (Murphy *et al.*, 1982, Takriti *et al.* 1994), bank portfolio (Kusy and Ziemba 1986), transportation (Ferguson and Dantizig 1956, Powell 1988), Hydropower system control (Infanger, 1994), and supply chain management (Fisher *et al.*, 1997, Santoso *et. al.* 2005). Excellent survey books and articles are in Kall (1976), Kall and Wallace (1994), Prépkopa (1995) Birge (1997), Sen and Higle (1999), and Dupaćová (2002).

3. Problem Statement

Containerization is an approach of effectively organizing shipments. It changes shipment handling from a labour-intensive to a capital- and time-intensive operation, which is particularly true for containerizing air cargos because of its higher freight rates. This study is motivated by the problems faced by airfreight forwarders, who perform many functions in delivering cargos by air, such as consolidation, booking, documentation, insurance, picking up, delivering, warehousing, tracing, etc. Among the above, the most important function is consolidation, as the airfreight forwarders profit by consolidating small customer shipments to obtain discounts offered by the airline. Airfreight forwarders account for a large percent of users for air cargos.

In this study, each type of cargo has its own weight and volume. Each cargo must be packed into a single container. Breaking a cargo into different containers is not allowed. All cargos have to be allocated to containers without delay. Typically, the airlines publish different booking rates to its customers, and these prices depend on the container types and the cargo weight that the container holds. In general, the larger the weight, the lower the unit rate charged. Airfreight forwarders usually book the air containers one week before the actually shipping day in order to get a cheap rental price from the airlines. Cargo information provided by the customer is usually uncertain and changing. The airfreight forwarders, however, can not wait until the shipping day, when the actual cargo amount is eventually identified, as any urgent changes in the details of booked containers will incur a high penalty.

Therefore, in the first stage, the airfreight forwarders have to make a response based on the inaccurate information by determining the booking quantities and types of containers. Clearly, containers that are booked in advance may not meet the actual requirements on the shipping day, because of continuously changing customer information. If the containers that have been ordered cannot hold all cargos, additional containers are required. On the other hand, if too many containers have been ordered, redundant containers have to be returned to the airlines: the forwarders incur a penalty because they are breaking a contract. The rental cost consists of two parts: the cost of using containers and the penalty cost for changing urgent requirements on the shipping day. The cost of using containers is based on a fixed charge plus a variable charge that depends on the total cargo weight that the container holds. The penalty cost includes the cost of renting more containers or the cost of returning unused containers on the shipping day.

The difficult and challenging tasks faced by the airfreight forwarders are to determine the quantities and types of air containers for booking and actual shipping, as well as loading cargos with the aim of minimizing the total fee charged by the airlines under the uncertain environment. Under this study, it is assumed that the cargo quantity is a random parameter. In formulating a stochastic recourse programming model for this problem, the first stage decision variables are types and quantities of booking containers, and these are determined before the accurate cargo quantities are obtained. At the same time, all decisions taken on the shipping day belong to the second stage response: these include types and the quantities of containers rented or returned and loading cargos into containers for each scenario. The second stage decision variables are determined after the values of random cargo quantities are observed.

4. A Deterministic Model for Air Cargo Forwarding Under Certainty

This section is devoted to the deterministic version of the containerized air cargo problems, in which the cargo quantity information is known with certainty. Because accurate cargo shipment information has been obtained in advance, it will not change on the shipping day. Therefore, we can book containers in advance without incurring any penalty from the airlines for urgent requirements. It is assumed there are q_j air cargos of type j that will be shipped one week later, j=1,2,..., n. Let v_j and w_j denote the volume and the weight of cargo type j. All cargos have to be loaded into the air containers provided by the airlines on the shipping day. There are m types of containers, numbered $\{1,2,\ldots,m\}$, for rental. Each type of container i has L_i cargos available, i.e. number $\{1,2,\ldots,L_i\}$. For container type i, V_i and W_i represent the volume and weight limits respectively. The total cost of renting the l^{th} container of type i only includes a fixed cost c_i^0 plus a variable cost c_{il}. Whenever one container is rented, the forwarder has to pay a fixed cost. Once the cargo loaded into the container exceeds a permitted weight limit, a variable cost will be incurred, and this is associated with the weight of cargo loaded into the container. Figure 1 shows the variable cost.

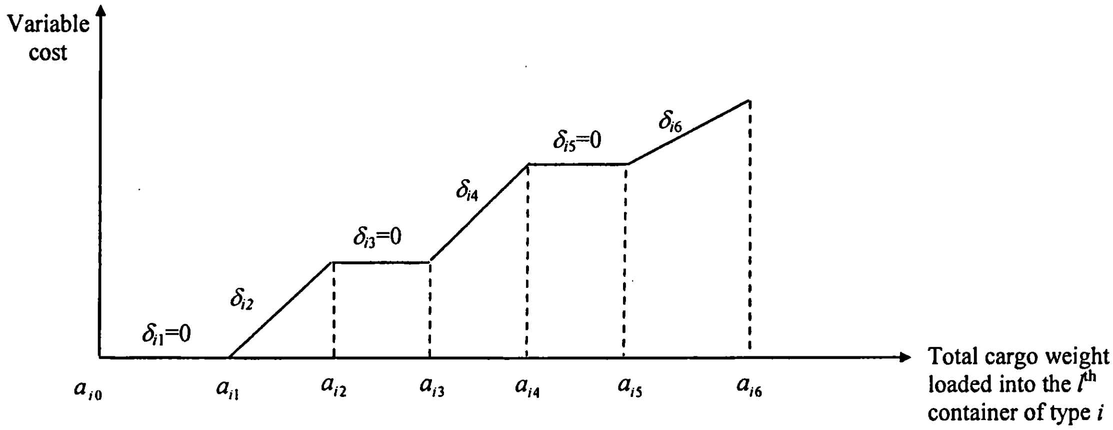

Figure 1: Variable cost of renting the l^{th} container of type i

In Figure 1, a_{ik} represents the break point for container type i, where i=1,..,m, k=1,..., K_i, where K_i is the maximum number of break points. In this study, the air carriers provide six cost break points: a_{i1}, a_{i2}, a_{i3}, a_{i4}, a_{i5}, and a_{i6}. Let a_{i0} be the initial point, i.e. a_{i0} =0. Thus, a_{i1} is the first cost break point for the variable cost, and a_{i6} is the maximum weight limit of container type i. The definition of the variable cost c_{il} in the deterministic version model can be formulated as follows:

$$c_{il} = \begin{cases} 0 & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i0}, a_{i1}] \\ \delta_{i2}(\sum_{j=1}^{n} w_j y_{ilj} - a_{i1}) & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i1}, a_{i2}] \\ \delta_{i2}(a_{i2} - a_{i1}) & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i2}, a_{i3}] \\ \delta_{i2}(a_{i2} - a_{i1}) + \delta_{i4}(\sum_{j=1}^{n} w_j y_{ilj} - a_{i3}) & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i3}, a_{i4}] \\ \delta_{i2}(a_{i2} - a_{i1}) + \delta_{i4}(a_{i4} - a_{i3}) & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i4}, a_{i5}] \\ \delta_{i2}(a_{i2} - a_{i1}) + \delta_{i4}(a_{i4} - a_{i3}) + \delta_{i6}(\sum_{j=1}^{n} w_j y_{ilj} - a_{i5}) & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i5}, a_{i6}] \end{cases} \tag{1}$$

where, i= 1,2,...,m; l=1,2,..., L_i.

For the deterministic environment, decisions include what types of and how many containers to book, and how to load the cargos into the containers, while simultaneously minimizing the rental cost. The decision variables for the deterministic models are defined as follows:

$$x_{il} = \begin{cases} 1 & \text{if the } l^{\text{th}} \text{ container of type } i \text{ is selected} \\ 0 & \text{otherwise} \end{cases};$$

y_{ilj} = quantities of cargo type j loaded into the l^{th} container of type i;

Therefore, the containerization of the air cargo forwarding problem can be formulated the following integer programming model:

$$\text{Minimize } \sum_{i=1}^{m}\sum_{l=1}^{L_i} c_i^0 x_{il} + \sum_{i=1}^{m}\sum_{l=1}^{L_i} c_{il} \tag{2}$$

subject to

$$\sum_{j=1}^{n} v_j y_{ilj} \le V_i x_{il},\ i=1,\ldots,m;\ l=1,\ldots,L_i; \tag{3}$$

$$\sum_{j=1}^{n} w_j y_{ilj} \le W_i x_{il}, \; i=1,\ldots,m;\; l=1,\ldots,L_i; \tag{4}$$

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i} y_{ilj} = q_j, \; j=1,2,\ldots,n;$$

(5)

$$x_{il} = \{0,1\}, \; i=1,\ldots,m;\; l=1,\ldots,h_i; \tag{6}$$

y_{ilj} is an non-negative integer, i=1,..., m; l=1,..., L_i; j=1,..., n; (7)

The objective function in (2) is the total cost of renting the containers, which includes two elements. The first element is the total fixed cost for using the containers, and the second is the total variable cost. The definition of the variable cost $c_{il,}$ can be seen by referring to Figure 1 and equation (1). Constraint (3) is the container volume constraint, which ensures that the volume of all cargos allocated to a container cannot exceed the container's volume limits. Constraint (4) is the container weight constraint, which ensures that the weight of all cargos allocated into a container cannot exceed the container's weight limits. Constraint (5) is the cargo quantity constraint, which requires all cargos to be loaded into the containers without any delay. Constraints (6) and (7) are the variable type requirements.

The objective function expressed in (2) is a piecewise function, and it is difficult to solve this kind of model by employing optimal software packages. Two variables are introduced to transform the model into a mixed-integer programming model. One variable g_{ilk} is a continuous variable representing the cargo weight distributed in the range $(a_{i,k-1},a_{ik})$ inside the l^{th} container of type i. The other variable z_{ilk} is a binary variable indicating whether the cargo weight is distributed in the range $(a_{i,k-1},a_{ik})$ inside the l^{th} container of type i. Therefore, the above model can be formulated as the following mixed-integer programming model:

$$\text{Min} \sum_{i=1}^{m}\sum_{l=1}^{L_i} c_i^0 x_{il} + \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{k=1}^{K_i} \delta_{ik} g_{ilk} \tag{8}$$

subject to

$$\sum_{j=1}^{n} v_j y_{ilj} \le V_i x_{il}, \; i=1,\ldots,m;\; l=1,\ldots,L_i; \tag{9}$$

$$\sum_{j=1}^{n} w_j y_{ilj} \le W_i x_{il}, \; i=1,\ldots, m;\; l=1,\ldots, L_i; \tag{10}$$

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i} y_{ilj} = q_j, j=1, 2,\ldots, n; \tag{11}$$

$$\sum_{k=1}^{K_i} g_{ilk} = \sum_{j=1}^{n} w_j y_{ilj}, i=1,\ldots, m; l=1,\ldots, L_i; \tag{12}$$

$$g_{ilk} \le z_{ilk}\left(a_{i,k} - a_{i,k-1}\right), i=1,\ldots, m; l=1,\ldots, L_i; k=1,\ldots, K_i; \tag{13}$$

$$g_{ilk} \ge z_{il,k+1}\left(a_{i,k} - a_{i,k-1}\right), i=1,\ldots, m; l=1,\ldots,L_i; k=1,\ldots, K_i-1; \tag{14}$$

x_{ils}, z_{ilks} are binary integers, $i=1,\ldots, m$; $l=1,\ldots,L_i$; $k=1,\ldots,K_i$; $s=1,\ldots, S$; (15)

y_{ilj} is an non-negative integer, $i=1,\ldots, m$; $l=1,\ldots, L_i$; $j=1,\ldots, n$; (16)

$g_{ilk} \ge 0$, $i=1,\ldots, m$; $l=1,\ldots,L_i$, $k=1,\ldots,K_i$; (17)

There are two items in the objective function (8). The first component is the fixed cost, which is as the same as in the objective function (2). The second component in (8) represents the sum of the variable costs for all containers. The variable cost for each container is the sum of the variable costs distributed in all ranges, described in Figure 1. The variable cost of the l^{th} container of type i in the range $(a_{i,k-1},a_{ik}]$ is the unit charge rate of container i in the range $(a_{i,k-1},a_{ik}]$, represented by δ_{ik}, multiplied by the cargo weight distributed in the range $(a_{i,k-1},a_{ik})$ inside the l^{th} container of type i, represented by g_{ilk}.

Constraints (9), (10), and (11) are the container volume constraint, container weight constraint and cargo quantity constraint respectively. Constraint (12) ensures that the sum of the cargo weight distributed in all areas inside a container is equal to the total weight of the cargos loaded into the container. Constraint (13) ensures z_{ilk} is equal to 1 if the total cargo weight inside the l^{th} container of type i reaches the range $(a_{i,k-1},a_{ik})$. In addition, the cargo weight g_{ilk} in the range $(a_{i,k-1},a_{ik})$ is less-than-or-equal-to the maximum weight value in the range $(a_{i,k-1},a_{ik})$, which is $a_{ik}-a_{i,k-1}$. Constraint (14) ensures that once the total cargo weight inside the l^{th} container of type i reaches the range $(a_{i,k},a_{ik+1})$, the cargo weight in the range $(a_{i,k-1},a_{ik})$, which is g_{ilk}, is not less than the difference between a_{ik} and $a_{i,k-1}$. Constraints (13) and (14) ensure that the weight ranges are reached by priority: g_{ilk} cannot be positive unless the range $(a_{i,k-1},a_{ik})$ is fully occupied by the cargo weight. In other words, constraints (13) and (14) ensure that g_{ilk} cannot have a positive value unless all g_{ilt} are at their maximum value, which is $a_{it}-a_{i,t-1}$, $1 \le t \le k$. Constraints (15), (16), and (17) are the variable type requirements.

5. A two-stage stochastic programming models for the air cargo forwarding problems under uncertainty

This section is concerned with the stochastic version of the air cargo forwarding model, in which the cargo quantity q_j is a random parameter. It is assumed that q_j has a discrete distribution with a finite number S of possible realizations, q_{js}, s=1,2,..., S, with the corresponding probabilities p_s, $\sum_{s=1}^{S} p_s = 1$. Twp types of response are made in different stages: the first-stage response is the decision regarding booking with uncertain information; the second-stage response is the decision that is made on the shipping day when the stochasticity is realized. Two types of decision variables are defined as follows:

The first-stage decision variables

n_i= number of containers of type i to be booked.

The second-stage decision variable

n_{is}^{+}= number of type i containers returned on the shipping day in scenario s;

n_{is}^{-}= number of containers of type i rented on the shipping day in scenario s;

$$x_{ils} = \begin{cases} 1 & \text{if the } l^{\text{th}} \text{ container of type } i \text{ is selected in scenario } s \\ 0 & \text{otherwise} \end{cases};$$

y_{iljs} = quantities of cargo of type j loaded into the l^{th} container of type i in scenario s.

Based on the analysis in Section 2, we know that the total cost for shipping cargos consists of two parts: cost of usage and penalty cost. Penalty costs arise from urgent needs or the cancellation of containers on the shipping day. For each scenario, the cost of usage includes a fixed cost c_i^0 and a variable cost c_{ils}. The variable cost under uncertainty can be formulated as follows:

$$c_{ils} = \begin{cases} 0 & \sum_{j=1}^{n} w_j y_{iljs} \in (a_{i0}, a_{i1}] \\ \delta_{i2}(\sum_{j=1}^{n} w_j y_{iljs} - a_{i1}) & \sum_{j=1}^{n} w_j y_{iljs} \in (a_{i1}, a_{i2}] \\ \delta_{i2}(a_{i2} - a_{i1}) & \sum_{j=1}^{n} w_j y_{iljs} \in (a_{i2}, a_{i3}] \\ \delta_{i2}(a_{i2} - a_{i1}) + \delta_{i4}(\sum_{j=1}^{n} w_j y_{iljs} - a_{i3}) & \sum_{j=1}^{n} w_j y_{iljs} \in (a_{i3}, a_{i4}] \\ \delta_{i2}(a_{i2} - a_{i1}) + \delta_{i4}(a_{i4} - a_{i3}) & \sum_{j=1}^{n} w_j y_{iljs} \in (a_{i4}, a_{i5}] \\ \delta_{i2}(a_{i2} - a_{i1}) + \delta_{i4}(a_{i4} - a_{i3}) + \delta_{i6}(\sum_{j=1}^{n} w_j y_{iljs} - a_{i5}) & \sum_{j=1}^{n} w_j y_{iljs} \in (a_{i5}, a_{i6}] \end{cases} \quad (18)$$

where, i= 1,2,...,m; l=1,2,..., L_i., s=1,2,...S.

The objective is to load all cargos into the containers on the shipping day, where the containers are either booked containers or urgent requirements made on the shipping day, while minimizing the total cost charged by the airlines. Uncertain air cargo forwarding problems can be formulated as the following stochastic integer programming model:

$$\text{Min} \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{s=1}^{S} p_s c_i^0 x_{ils} + \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{s=1}^{S} p_s c_{ils} + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^- n_{is}^- + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^+ n_{is}^+ \quad (19)$$

subject to

$$\sum_{j=1}^{n} v_j y_{iljs} \le V_i x_{ils}, \; i=1,\dots,m;\; l=1,\dots,L_i,\; s=1,\dots,S; \quad (20)$$

$$\sum_{j=1}^{n} w_j y_{iljs} \le W_i x_{ils}, \; i=1,\dots,m;\; l=1,\dots,L_i,\; s=1,\dots,S; \quad (21)$$

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i} y_{iljs} = q_{js}, \; j=1,\dots,n;\; s=1,\dots,S; \quad (22)$$

$$n_i = \sum_{l=1}^{Li} x_{ils} + n_{is}^+ - n_{is}^-, \; i=1,\dots,m;\; s=1,\dots,S; \quad (23)$$

$$x_{ils} = \{0,1\}, \; i=1,\dots,m;\; l=1,\dots,h;\; s=1,\dots,S; \quad (24)$$

$y_{iljs}, n_i, n_{is}^-, n_{is}^+$ are non-negative integers, i=1,..., m; l=1,..., L_i; j=1,..., n, s=1,..., S. (25)

The objective function in (19) is the total cost of renting the containers, and includes four parts. The first part is the expected value of the total fixed costs. The second part is the expected value of the total variable costs. The definition of the variable cost c_{ils}, can be seen by referring to Figure 1 and equation (18). The third part is the expected value of the total penalty cost for renting additional containers on the shipping day. The fourth part is the expected value of total penalty cost for returning unused containers on the shipping day. Each scenario has to satisfy the container volume constraints in (20), container weight constraints in (21), cargo quantity constraints in (22), and container quantity constraints in (23). Constraints (24) and (25) are the variable type requirements.

The objective function expressed in (19) is a piecewise function. We use the same method that is described in the deterministic model, in which two new variables are introduced to transform the model into a mixed-integer programming model. One variable g_{ilks} is a continuous variable representing the cargo weight distributed in the range $(a_{i,k-1}, a_{ik})$ inside the l^{th} container of type i in scenario s. The other variable z_{ilks} is a binary variable indicating whether the cargo weight is distributed in the range $(a_{i,k-1}, a_{ik})$ inside the l^{th} container of type i in scenario s. Thus the model can be formulated as the following mixed-integer programming model:

$$\text{Min} \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{s=1}^{S} p_s c_i^0 x_{ils} + \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{k=1}^{K_i}\sum_{s=1}^{S} p_s \delta_{ik} g_{ilks} + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^- n_{is}^- + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^+ n_{is}^+ \quad (26)$$

subject to

$$\sum_{j=1}^{n} v_j y_{iljs} \le V_i x_{ils},\ i=1,\dots,m;\ l=1,\dots,L_i;\ s=1,\dots,S; \quad (27)$$

$$\sum_{j=1}^{n} w_j y_{iljs} \le W_i x_{ils},\ i=1,\dots,m;\ l=1,\dots,L_i;\ s=1,\dots,S; \quad (28)$$

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i} y_{iljs} = q_{js},\ j=1,\dots,n;\ s=1,\dots,S; \quad (29)$$

$$n_i = \sum_{l=1}^{Li} x_{ils} + n_{is}^+ - n_{is}^-,\ i=1,\dots,m;\ s=1,\dots,S; \quad (30)$$

$$\sum_{k=1}^{K_i} g_{ilks} = \sum_{j=1}^{n} w_j y_{iljs},\ i=1,\dots,m;\ l=1,\dots,L_i;\ s=1,\dots,S; \quad (31)$$

$$g_{ilks} \le z_{ilks}\left(a_{i,k} - a_{i,k-1}\right),\ i=1,\dots,m;\ l=1,\dots,L_i;\ k=1,\dots,K_i;\ s=1,\dots,S; \quad (32)$$

$$g_{ilks} \ge z_{il,k+1,s}\left(a_{i,k} - a_{i,k-1}\right),\ i=1,\dots,m;\ l=1,\dots,L_i;\ k=1,\dots,K_i-1;\ s=1,\dots,S; \quad (33)$$

x_{ils}, z_{ilks} are binary integer, $i=1,\dots,m$; $l=1,\dots,L_i$; $k=1,\dots,K_i$; $s=1,\dots,S$; (34)

$y_{iljs}, n_i, n_{is}^-, n_{is}^+, e_{js}$ are non-negative integers, $i=1,\dots,m$; $l=1,\dots,L_i$; $j=1,\dots,n$; $s=1,\dots,S$; (35)

$g_{ilks} \ge 0$, $i=1,\dots,m$; $l=1,\dots,L_i$, $k=1,\dots,K_i$, $s=1,\dots,S$. (36)

6. Computational Results and Analysis

6.1 Known and fixed data

A forwarding company in Hong Kong provides air transport services worldwide. The company collects shipping information from its customers in terms of the weight, volume and shape of shipments, delivery time and destinations. Based on this information, the company consolidates the small shipments into three types of cargo: large, medium and small. The volume and weight of each type of cargo are given in Table 1.

Table 1: Air cargo characteristics

Cargo Types	Cargo Volume	Cargo Weight
Large	1500	750
Medium	1200	600
Small	1000	500

The forwarder then contacts the airline to arrange rental of air containers. The air carrier can provide 7 types of containers for renting, and currently there are 2 of each type of container available. The airline provides the following information shown in Table 2, including the types and quantities of the containers, the volume and weight limits of the containers, the fixed cost, the break points, and the unit charge rate in the different ranges.

Table 2: Air container characteristics

Container Type	Container Quantity	Fixed Cost	Volume Limit	Weight Limit	Break Point						Charged Rate					
					a_{i1}	a_{i2}	a_{i3}	a_{i4}	a_{i5}	a_{i6}	δ_{i1}	δ_{i2}	δ_{i3}	δ_{i4}	δ_{i5}	δ_{i6}
1	2	161617	6489	6800	3968	4722	5290	5976	6273	6800	0	32	0	29	0	25
2	2	105898	6300	5400	2600	3050	3467	3954	4111	5400	0	32	0	29	0	25
3	2	85207	5008	4200	2092	2490	2789	3140	3307	4200	0	32	0	29	0	25
4	2	74373	4882	4000	1826	2173	2434	2741	2886	4000	0	32	0	29	0	25
5	2	48713	3700	3900	1196	1423	1594	1825	1917	2900	0	32	0	29	0	25
6	2	46553	3150	3500	1643	1747	2000	2500	2591	3500	0	32	0	29	0	25
7	2	20695	1400	1200	505	602	674	758	799	1200	0	32	0	29	0	25

6.2 Computational results for the deterministic model

It is assumed that there are 3 large cargos, 5 medium cargos and 7 small cargos, which need to be shipped one week later. Based on the certain information, the decision maker needs to make decisions on what types and how many containers to book for the next week's shipping and how to pack these cargos into containers. The mixed-integer programming model presented in Section 5 is used to solve the cargo forwarding problem under certainty. The optimal container booking and cargo loading plan can be obtained by using AIMMS. Table 3 shows the computational results, including which containers will be booked for shipping and which cargo will be loaded into them. Table 3 also provides the other related results including the loaded volume and weight for each container, the fixed cost, variable cost and total rental cost. The total rental cost for shipping these cargos is 295235.

Table 3: Optimal plan for container selection and cargo loading under certainty

Selected Containers	Loaded Cargos	Loaded Volume	Loaded Weight	Fixed Cost	Variable Cost	Total Cost
Container 4 (1st)	4 medium	4800	2400	74373	1104	85477
Container 5 (1st)	1 medium, 2 small	3200	1600	48713	7438	56151
Container 5 (2nd)	1 large, 2 small	3500	1750	48713	11788	60501
Container 6 (1st)	3 small	3000	1500	46553	0	46553
Container 6 (2nd)	2 large	3000	1500	46553	0	46553

From Table 3, we know that two containers of type 5 are booked. Because the cargo weight inside is different, the variable cost is different, which results in different rental costs for the same type container. Additionally, the cargo weight in each type 6 container is only 1500, which is less than the first break point (1643) for container type 6. Therefore, no variable cost is incurred.

6.3 Computation results for the stochastic integer programming model

If the cargo quantities are uncertain when booking, the decision maker has to make decisions before accurate information is obtained. It is assumed that there is only 1 container of each type available for rental. The uncertainty of cargo quantities of each type can be captured by three scenarios, as shown in Table 4. Scenario 1 denotes that on the shipping day there are 4 of each type of cargo to be shipped; Scenario 2 denotes 3 of each type of cargo and Scenario 3 denotes 2 of each type of cargo.

Table 4: Cargo quantities under different scenarios

Cargo type	Scenario 1	Scenario 2	Scenario 3
Large Cargo	4	3	2
Medium Cargo	4	3	2
Small Cargo	4	3	2

In the following tests, we perform three different tests under different probability for the realization of stochastic cargo quantities. Other than the probability of occurrence of cargo quantities, the other conditions in the three tests are kept constant. The test data are shown in Table 5.

Table 5: Three tests

Test	$p_1=\Pr\{s_1\}$	$p_2=\Pr\{s_2\}$	$p_3=\Pr\{s_3\}$
Test I	0.8	0.1	0.1
Test II	0.1	0.8	0.1
Test III	0.1	0.1	0.8

Test I represents the situation where there are most likely 4 of each type of cargo; Test II the situation where there are most likely 3 of each type of cargo; and Test III where there are most likely 2 of each type of cargo. The optimal selection and loading plan of the proposed model in this study can be obtained using AIMMS. All the problems are executed on a Pentium IV 2.60GHz PC. The first stage response for booking containers is shown in Table 6. Tables 7 and 8 gives the second stage response for renting/returning containers and the cargo loading plan on the shipping day. The related cost is shown in Table 9.

Table 6: The first stage response for booking

Test	Container type						
	1	2	3	4	5	6	7
Test I			1	1	1	1	
Test II				1	1	1	
Test III					1	1	1

Table 7: The second stage response for urgent container requirements on the shipping day

Test	Container type	Scenario 1		Scenario 2		Scenario 3	
		Containers rented	Containers returned	Containers rented	Containers returned	Containers rented	Containers returned
Test I	1						
	2						
	3				1		

	4						1
	5						1
	6						
	7						
Test II	1						
	2						
	3	1					
	4						
	5						1
	6						
	7						
Test III	1						
	2						
	3	1					
	4	1		1			
	5						
	6						
	7		1		1		

Table 8: The second stage response for loading cargo on the shipping day

Test	Container type	Scenario 1			Scenario 2			Scenario 3		
		Large cargo	Medium cargo	Small cargo	Large cargo	Medium cargo	Small cargo	Large cargo	Medium cargo	Small cargo
Test I	1									
	2									
	3	3							2	2
	4		4		1	1	2			
	5	1		1		2	1			
	6			3	2			2		
	7									
Test II	1									
	2									
	3	3								
	4		4		1	1	2		2	2
	5	1		1		2	1			
	6			3	2			2		
	7									
Test III	1									
	2									
	3	1		3						
	4		4		1	1	2			
	5	1		1		2	1		2	1
	6	2			2			2		
	7									1

Table 9: Related cost

Test	Fixed cost	Variable cost	Penalty t cost for urgent rental	Penalty t cost for urgent return	Total cost
Test I	234017	16800	0	4500	255317
Test II	173288	20053	4000	1000	198341
Test III	135217	12203	10000	1000	158421

Test I represents the situation where the possibility that there are 4 cargos of each type is 80%. In Test I, the first stage response is to book 1 container of type 3, 4, 5 and 6 (see Table 5). In the second stage, if Scenario 1 (probability=80%) occurs on the shipping day, there is no need to rent additional containers or return any redundant containers (see Table 5). If Scenario 2 (probability=10%) occurs on the shipping day, a container of type 3 is cancelled (see Table 7). If Scenario 3 (probability 10%) occurs on the shipping day, a container of type 4 and a container of type 5 are cancelled. Any cancellation will incur a penalty. The total expected penalty cost is 4500. However, the probability that Scenarios 1 and 2 occur is only 20%. Therefore, in Test I, decision makers would like to book more containers in advance to satisfy a most likely large quantity of cargo. If unexpected situations occur, some containers may need to be returned, and this is shown in Table 5. Table 6 shows the cargo loading plan on the shipping day for each scenario.

In Test II, the most likely cargo quantity for each type is 3 (possibility is 80%). Based on the results of Test II as shown in Table 5, the decision maker makes the first stage response by booking 1 container each of type 4, 5 and 6 a week in advance. When compared with the container selection plan in Test I, the decision makers does not choose a container type 3 with a comparably high capacity and cost, as the cargo quantities in Test II are most probably less than Test I. In Test II, if Scenario 1 (probability 10%) occurs on the shipping day, which is an unexpected situation where there are 4 cargos of each type to be shipped, a container of type 3 is required (see Table 5) in order to ship all cargos. If Scenario 2 (probability 80%) occurs on the shipping day, there is no further renting or returning of containers. If Scenario 3 (probability 10%) occurs, the second stage response for this situation is to return a container of type 5 to satisfy a small quantity of cargos on the shipping day. The corresponding cargo loading plan for each scenario is shown in Table 7 for Test II. The penalty cost for urgent rental of containers in Test II is 5000, and the penalty for cancellation is 1000.

Test III shows that the cargo quantities for each type are most likely 2. Based on the results of Table 5, the decision maker will book 1 each container of type 5, 6 and 7 a week in advance. The quantities and types of booked containers in Test III are different from those in Tests I and II. In contrast with Tests I and II, Test III selects containers with a comparably small capacity and cost, since the cargo quantities in Test III are most likely less than Tests I and II. In Test III, if the unexpected situation of Scenario 1 (probability 10%) occurs on the shipping day (which means there are 4 cargos of each type for shipping) the decision maker makes the second stage responses by requiring two containers of type 3 and 4 with large capacity and cost and cancelling a container of type 7 on the shipping day to satisfy urgent requirements. If another unexpected situation Scenario 2 (probability 10%) occurs on the shipping day when there are 3 cargos for each type waiting for shipping, the decision maker takes responses by renting a container of type 4 and cancelling a container of type 7 on the shipping day. If Scenario 3 (probability 80%) occurs on the shipping day, all containers booked in advance are able to satisfy the actual cargo quantities on the shipping day. Therefore, there is no need for additional containers or returning redundant containers. The cargos can be loaded according the cargo loading plans under different scenarios provided in Table 7 (see Test III). The penalty cost for the urgent rental of containers in Test III is 10000, and the penalty for cancellation is 1000.

6.4 Comparing the deterministic and stochastic models

A natural temptation when solving uncertainty problems is to solve a much simpler deterministic problem: the one obtained by replacing all random variables by substituting their expected value of the stochastic parameters. Let *EV* represent the objective function value of the expected value problem. EEV is the expected results of the

Assume the deterministic model can be represented as $\min(z(x) = c^{\mathrm{T}} x : Ax = b, x \geq 0\}$. The stochastic programming can also be formulated as follows: $\min\{z(x,\xi) = c^T x + \min\{q(\xi)^T y : Ax = b, Wy = h(\xi) - T(\xi), x, y \geq 0\}$, where ξ is a random parameter vector, whose realizations correspond to the various scenarios.

The solution of two-stage recourse model is called the *stochastic solution*, denoted as x^*, and its performance is called the *expected objective value of the stochastic solution*, denoted as *ESS*. Thus we have: $ESS = \min_x E_\xi z(x,\xi)$. A natural temptation for solving the uncertainty problem is to solve a much simpler deterministic problem: the one obtained by replacing all random variables by substituting their expected value of the stochastic parameters. Let *EV* represent the *objective function value of the expected* *value problem.* Thus, we have: $EV = \min_x z(x, \bar{\xi})$, where $\bar{\xi} = E(\xi)$ denotes the expectation of stochastic variable ξ, and its solution is called *the mean value solution*, denoted as $\bar{x}(\bar{\xi})$. *EVV* is defined as the *expected result of using the EV solution*, denoted by $EEV = E_\xi(z(\bar{x}(\bar{\xi}), \xi))$, whose value measures how solution $\bar{x}(\bar{\xi})$ performs. The difference between the EEV and ESS is called *the value of the stochastic solution,* denoted as *VSS,* and is then defined as *VSS=EEV-ESS*. From the above definition, it can be easily seen that $VSS \geq 0$. (This is because x^* is an optimal solution of the recourse model, i.e. $ESS = \min_x E_\xi z(x, \xi)$, while $\bar{x}(\bar{\xi})$ is just one solution to $ESS = \min_x E_\xi z(x, \xi)$).

Now we introduce another concept of *the expected value of perfect information* (*EVPI*). For a given ξ, let $\hat{x}(\xi)$ denote an optimal solution to the deterministic model. Thus we can find all solutions $\hat{x}(\xi)$ and the corresponding objective values $z(\hat{x}(\xi), \xi)$ for all scenarios. *The expected value of the wait-and-see solution* (*EWS*) is calculated by $EWS = E_\xi z(\hat{x}(\xi), \xi)$. The *expected value of perfect information* (*EVPI*) is the difference between the expected objective value of the wait-and-see solution and the stochastic solution, i.e. *EVPI=ESS-EWS*. It can be noted that $EVPI \geq 0$. In fact, from the above definition, for each realization of ξ, we have the inequality $z(\hat{x}(\xi), \xi) \leq z(x^*, \xi)$, where x^* denotes an optimal solution to $ESS = \min_x E_\xi z(x, \xi)$. Taking the expectations of both sides and combining them with the above definition of *ESS* and *EWS* yields the following inequality: $z(\hat{x}(\xi), \xi) \leq z(x^*, \xi)$.

Table 10 shows the expected value of uncertain cargo quantities of each type for the above three tests. Based on the model and approach provided in Section 4, the corresponding deterministic model of the stochastic model can be solved using AIMMS. The optimal container selection and cargo loading plans for the deterministic model are shown in Table 11. Table 11 gives the value of *EV*, *EWS*, *EEV*, *ESS*, *VSS*, and *EVPI*.

Table 10: Expected value of stochastic variables in the three tests

Test	Cargo quantities		
	Large	Medium	Small
Test I	4	4	4
Test II	3	3	3
Test III	2	2	2

Table 11: Optimal cargo forwarding plan of the deterministic model in the three tests

Container type	1	2	3	4	5	6	7
Test I		4 large		4 medium		3 small	1 small
Test II				1 large, 1 medium, 1 small	2 medium, 1 small	2 large	
Test III					2 medium, 1 small	2 large	1 small

Table 12: Comparing the deterministic model and the stochastic model

Test	*EV*	*EWS*	*EEV*	*ESS*	*VSS*	*EVPI*
Test I	271423	248948	303162	255317	48845	6369
Test II	191081	192639	230853	198341	32512	5702
Test III	126299	147290	172549	158421	14128	11131

From Table 12, it can be seen that all the *VSS* values in the three tests are non-negative, which means that the objective values of the proposed stochastic recourse model are less than the objective values in the corresponding deterministic model in the three tests above. For example, in Test I, *VSS* is equal to 48845. This is the possible gain from solving the stochastic model rather than the deterministic model. In other words, we have to pay 48845 more if the deterministic model is used to determine cargo forwarding plans instead of the stochastic model. Therefore, adopting the deterministic model solution can have unfavourable consequences because the company will incur a higher level of costs compared with those incurred when using the stochastic model. From all the tests above, we also know that the mean value of the wait-and-see problem (*EWS*) is less than or equal to the value of stochastic problem (*ESS*). *EVPI*, equal to the difference between *EWS* and *ESS*, measures the maximum amount the decision maker would be ready to pay in return for complete information about the future.

7. Conclusions

In today's fiercely competitive global markets, companies are forced to compete on price and delivery performance to their customers in the face of rapidly changing conditions. Under the global supply chain management environment, effective logistic strategies can provide a critical competitive advantage for companies in terms of the lower cost, responsiveness and flexibility to changing market conditions. This study presents a dual-response approach to modelling the air cargo forwarding problems experienced by logistics companies when they use aircrafts for cargo transportation. The decisions they face include how to book the air containers provided by the airlines under uncertain customer shipment information, and how to load the cargos into the containers on the shipping day. The decision-making process is complex because of the air containers' volume and weight limits and the fact that container rental costs include a fixed cost and a variable cost for booking, as well as the penalty cost for renting or returning containers on the shipping day. The companies have to satisfy their customers' shipping requirements while minimizing the total container rental costs.

A major contribution of this study is to present a dual-response strategy and use quantitative techniques to model cargo forwarding problems under uncertainty. We first formulate a deterministic version mode, and change the model into a mixed-integer linear programming model, which can be solved by many mathematical programming software packages available today. Then we present a two-stage 0-1 stochastic programming model to solve cargo loading problems under the uncertain and changing customer information. Different experiments are designed to demonstrate the effectiveness of the proposed models. From the computational results we can conclude that the stochastic model has advantages over the deterministic model in dealing with the uncertainty. Further research will consider intermodal freight transportation problems, which involves the transfer of cargo between vehicles of different modes. Practically, all air transport is intermodal, because either pick-up or delivery services normally rely on the other mode, which is generally by truck. Thus, the challenging task for the logistics managers is how to integrate different transport modes in the logistics process in order that shipments can be picked up and delivered at the right time, in the right quantities, to the right destination with the minimum operational cost. In addition, this research does not consider risk and flexibility factors involved in cargo forwarding. The following factors can be considered in the future:

- Allowing shipment of cargos on subsequent days with or without penalty.
- The cost of renting containers subject to change with time.
- The different price policies offered by different airlines.
- Dealing with risk situations, such as not enough containers being available

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Appendix G

A paper accepted by *Journal of the Operational Research Society*, subject to revision

Robust Optimization Applied to Containerzing Air Cargo Forwarding Problems under Uncertainty

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Abstract

This study considers how to containerize air cargo into containers under uncertain information. Airlines offer different types of containers, with different weight and volume limits, for rental in advance. The rental price for each container differs, and is based on a fixed cost plus a variable cost that depends on the cargo weight that the container holds. However, a penalty cost will be incurred if additional containers are required on the shipping day. At the same time, containers that have been booked and which are returned will also incur a penalty. A deterministic model is formulated for the cargo loading problem under certainty. If the cargo information is uncertain when booking, a two-stage stochastic programming model is presented. The first-stage decisions are to determine what quantities and types of containers should be booked under the incomplete information. The second-stage decisions are to make different responses on the shipping day in order to load all the cargo into containers. The decisions include the quantities and types of containers that are required or/and returned on the shipping day and how the cargos are loaded into containers. When delayed shipping is permitted, a robust optimization model is presented to handle the infeasibility and risk involved. A series of experiments are designed to test the effectiveness of the proposed robust optimization models. Compared with the results of the two-stage stochastic programming model, the robust optimisation models provide a more responsive and flexible system with less risk, which is particularly important in the current context of global competitiveness.

Keywords: Air transport; Container loading; Deterministic programming; Globalization; Logistics; Robust optimization; Stochastic programming; Supply chain management.

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1. Introduction

Today's air transport is exerting an ever increasing impact on transportation, particularly global transport, as compared with only a few years ago. Although the average shipment size is still limited by today's aircraft, the nature of air cargo, mostly high-value and low-density items, has caused the total value of air cargo to comprise a greater portion of total global cargoes (Muller, 1999). The tremendous speed of aircrafts and high frequency of scheduled flights to the majority of cities in the world has reduced transit time from as many as 50 days to one or two days. Today's business success increasingly replies on speed instead of quality: this has become a minimum standard rather than a competitive advantage in many industries. There are several factors that are currently driving changes in business, for example, shorter product lead times and life cycles, increased product variety, instant customization, highlighted retailers, etc. Globalisation is among the most important driving forces changing the business landscape (Coyle *et al.* 2003). During the past few years, globalisation has relabelled many terminologies in the business world. It is now common to talk about the global market, global economy, global sourcing, global manufacturing, global logistics, global purchasing, global supply chain management, etc. With easy and instant access to the Internet, the inexpensive launch of B2B or B2C business, and advancements in information technology, products and service can be manufactured and sold anywhere in the world where feasible. Because product and service information is available on a real-time basis and comparisons can quickly be made, customers are increasingly empowered to have more complicated requirements and tend to have a low tolerance to poor quality either in products or in services. They demand a quick response and speedy delivery while continuously lowering costs. Supplying a market ahead of competitors can provide a competitive advantage by offering remarkable flexibility to the dynamic and changing demand. Time is extremely important for certain industries, like the PC and apparel industries. The time saved by using air freight can leave manufacturers and transporters a margin to beat product variety, short lead time and life cycles, and uncertain demand. Additionally, air transport offers substantial savings for its customers through low insurance, cheap labour costs for packing, loading and unloading, dramatically decreasing the costs of warehousing and inventory, less capital needing to be invested in large shipments by sea, and faster capital turnover.

Containers are large boxes that are used to transport goods from one destination to another (Vis and Koster, 2003). Containers first started to be used in the 1950s. In this initial stage, however, they were usually sea containers. The analysis of sea cargo loading problems has been an active area of research for many years. See three-dimensional (3D) rectangular packing in George and Robinson (1980), Bischoff, E.E., and Marriott, M.D. (1990) and Chen et al. (1995); empty container allocation among different ports in White (1972), Crainic *et. al.* (1993) and Cheung and Chen (1998); weight distribution consideration in Davies and Bischoff (1999) and Bortfeldt and Gehring (2001); transshipment of containers at a container terminal in Vis and Koster (2003). Also excellent survey articles related to sea container loading are presented by Bischoff and Ratcliff (1995) and Vis and Koster (2003).

During the past decade, there has been a continuous increase in the number of publications discussing air container loading problems. However, the majority of the literature is concerned with the gravity issue of container loading in an aircraft. See a complete review of the manual and the computer-assisted approaches to the centre of gravity via pyramid loading for air containers in Martin-Vega (1985); a further extension work in Mathur (1998) providing an algorithm with a better worst-case performance; a simple greedy heuristics for balancing in Amiouny *et al.* (1992); a military application for loading air cargo in Ng (1992); and maximizing air freight loading while balancing the weight to minimize fuel consumption and satisfy stability and safety requirements in Mongeau and BÈS (2003).

All the above literature presents models and techniques for the deterministic environment, where all information that the decision maker needs is accurately known. Sen and Higle (1999) think it is difficult to precisely estimate certain critical data elements, and it is necessary to address the impact of uncertainty during the planning process. Explicitly considering uncertainty, in some situations, is highly critical and failure to include uncertainty may lead to very expensive, even disastrous consequences if the anticipated situation is not realized (Bai *et al.*, 1997).

Stochastic programming is first presented in the 1950s by Beale (1955), Dantzig (1955) and Charnes and Cooper (1959). It is a branch of mathematical programming that copes with a class of mathematical models and algorithms in which of the data may be subject to significant uncertainty. Crainic *et. al.* (1993) propose a stochastic network model for the inland transportation of empty sea containers. Cheung and Chen (1998) formulate a dynamic empty allocation problem as a two-stage stochastic network model, as well as discussing how to reposition empty sea containers and where and how many leased containers are needed at ports. Mulvey *et al.* (1995) first develop robust optimization that integrates goal programming formulations with a scenario-based description of problem data. They think robust optimization, while not without limitations, has some advantages over stochastic programming and is more generally applicable to the problem.

To our best knowledge, little research has been conducted on the rental cost issue related to air container loading, let alone dealing with uncertain information. In this study, we first present a deterministic model for certain environment. Then, a stochastic programming model is formulated to determine two-stage decisions under uncertain information: the first-stage decision is to determine what type of, and how many, containers are booked; the second-stage decision is made on the shipping day, and includes what type of, and how, many additional containers are required, as well as how to load all the cargo into containers. We finally formulate a robust optimization model, which allows un-fulfilment of shipping by assigning a penalty function. A series of experiments are designed to demonstrate the effectiveness of the robust model in dealing with cost, risk, and flexibility under uncertainty.

The rest of the paper is organized as follows. Section 2 describes selecting container and loading cargo process, and illustrates the uncertainty and risk involved. Section 3 presents a robust optimization framework. Section 4 formulates three types of models: 1) an integer programming model; 2) a two-stage stochastic integer programming model; and 3) A robust optimization model. Section 5 gives the computational results and analysis for the models proposed. The final section gives our conclusions and recommendations for future research.

2. Problem Statement

Containerization is an approach to cost-effectively and efficiently organize shipments. It changes shipment handling from a labour-intensive to a capital- and time-intensive operation, which is particularly true for containerizing air cargos because of their higher freight rates. This study is motivated by the problems faced by airfreight forwarders, who perform many functions in delivering cargos by air, such as consolidation, booking, documentation, insurance, picking up, delivering, warehousing, tracing, etc. Among the above, the most important function is consolidation, as the airfreight forwarders profit by consolidating customer small shipments to obtain discounts offered by airlines. In this study, airlines offer different types of containers for rent. Each type of container has its weight and volume limits for holding cargos, and each type of cargo has its own weight and volume. Each cargo must be packed into a single container. Breaking a cargo into different containers is not allowed. Typically, the forwarders book containers from the airline one week before shipment. The airlines give different rental prices when booking different types of containers. The cost of renting a container is based on a fixed cost and a variable cost that depends on the weight that the container holds. Therefore, the cost of renting a container is not a linear function but a piece-wise function.

If cargo shipping information is accurately obtained when booking, the forwarder can book containers that will be used next week aiming at minimizing the total rental cost. The decisions about booking include what quantities and types of containers are needed for next week's shipping and how cargos are loaded into containers. In this situation, a deterministic program can be applied to solve the cargo forwarding problems under certain cargo shipping information.

If accurate cargo information is not available when booking, the forwarders have to book containers in advance in order to get a low rental price. As airlines discourage urgent requirements for containers, they impose a heavy penalty for renting containers on the shipping day. If all cargos have to be loaded on the shipping day, the booked containers may not meet all container needs on the shipping day. In this situation, additional containers are required: but these come at a high penalty cost. On the other hand, if too many containers are booked, the unused containers have to be returned to the airlines: in this case a penalty is incurred because of the forwarder breaking a contract. Therefore, in the first stage, the forwarders have to make a response based on the inaccurate information by determining the booking quantities and types of containers. In the second stage, the forwarders have to make responses for different situations that might happen on the shipping day by determining the required or retuned containers and loading all cargos into containers. Under uncertain information and a no-delay policy, a two-stage stochastic programming technique can be applied to solve the uncertain cargo forwarding problems.

The deterministic model and stochastic model above share a common assumption: that all cargos available on the shipping day have to be loaded into containers without delay. This assumption means that the forwarder has to change the quantity of booked containers on the shipping day at a high price if more or less cargos appear. If a container only holds a small weight, the container is not fully utilized. This means the container is rented at a relatively high cost. In general, the larger the weight, the lower the unit rate charged by the airline. In particular, urgently renting a container on the shipping day results in a high penalty. It is assumed that not all cargos have to be shipped on the shipping day. If the penalty for the delay is not too high, the decision makers could choose to deliver certain of the cargos on the following days. In this situation, a robust optimisation can be applied to solve the uncertain cargo forwarding problem, which provides a way of measuring the trade-off between risk and cost. The following section provides a framework for this robust optimization.

3. Robust optimisation framework

A general linear programming model can be formulated as follows:

$$\min c^{\mathrm{T}} x \tag{1}$$

s.t.

$$Ax = b \tag{2}$$

$$x \in \Re_{+}^{n_1} \tag{3}$$

where A is a fixed matrix, b is a fixed vector, and x is the vector of decision variables.

When some data elements in a linear program are represented by stochastic variables, the result is a stochastic linear program. Assume that a linear programming problem has been completely specified, apart from some coefficients that are random variables with a joint known distribution. A two-stage linear recourse model can be formulated as follows:

$$\min c^{T} x + E[Q(x,\xi)] \tag{4}$$

s.t.

$$Ax=b \tag{5}$$

$$x \in \Re_{+}^{n_1} \tag{6}$$

where

$$Q(x,\xi) = \min q^{T}(\xi) y \tag{7}$$

$$Wy = h(\xi) - T(\xi)x \tag{8}$$

$$y \in \Re_{+}^{n_2} \tag{9}$$

and A is an $m_1 \times n_1$ whereas W is an $m_2 \times n_2$ matrix. Thus the dimensions of all of the other arrays in the above model are fixed accordingly. x denotes the vector of the first-stage variables, whose optimal value is not conditioned on the realization of the random variable $\xi(\omega)$, and y denotes the vector of the second-stage variables, which are subject to adjustment once the random variable $\xi(\omega)$ is observed. Thus, equation (5) is called the deterministic constraint or the first-stage constraint, and equation (6) is called the random constraint or the second-stage constraint.

Suppose that the random variable ξ has a discrete distribution with a finite number of S possible realizations $\xi_s = (q_s, h_s, T_s), s = 1,...,S$, each with the corresponding probabilities p_k. The above two-stage stochastic programming model can be formulated as the following linear program form:

$$\min c^{\mathrm{T}} x + \sum_{s=1}^{S} p_s (q^s)^{\mathrm{T}} y^s \tag{10}$$

s.t.

$$Ax=b \tag{11}$$

$$T^s x + W^s y^s = h^s,\ s=1,\ldots,S \tag{12}$$

$$x \in \Re_{+}^{n_1}, y^s \in \Re_{+}^{n_2}, s = 1,...,S \tag{13}$$

Let z^s represent the error variable that measures the infeasibility allowed in the second-stage constraint under scenario s. The robust optimization model can be formulated as follows:

$$\min c^{\mathrm{T}} x + \sum_{s=1}^{S} p_s (q^s)^{\mathrm{T}} y^s + + \omega \rho(z^1,...,z^S) \tag{14}$$

s.t.

$$Ax=b \tag{15}$$

$$T^s x + W^s y^s + z^s = h^s,\ s=1,\ldots,S \tag{16}$$

$$x \in \Re_{+}^{n_1}, y^s \in \Re_{+}^{n_2},\ z^s \in \Re_{+}^{n_2}\ s = 1,...,S \tag{17}$$

In (14), $\rho(\bullet)$ is an infeasibility penalty function, which is used to penalize the violations of the second-stage constraints under scenario s. Parameter $\omega \geq 0$ is used to measure the trade-off between the cost and risk for violating the random constraints. Clearly, for $\omega \to +\infty$, the above model becomes a two-stage stochastic programming model, because the large value of ω forces all the second-stage constraints to be satisfied.

From the modelling point of view, the choice of the penalty function $\rho(\bullet)$ depends on the nature of the real-life problem to be solved, computational times, input data characteristics, etc. However, its choice influences solution performance. Here we introduce three types of penalty functions:

- Mean absolute deviation: $g(z_1^1,...,z_1^S) = \sum_{s=1}^{S} p_s |z_1^s|$
- Quadratic penalty function: $g(z_1^1,...,z_1^S) = \sum_{s=1}^{S} p_s (z_1^s)^{\mathrm{T}} z_1^s$.
- Consider only positive violations: $g(z_1^1,...,z_1^S) = \sum_{s=1}^{S} p_s \max\{0, z_z^s\}$.

4. Model Formulation

4.1 A deterministic model for the air cargo forwarding problems under certainty

This section is devoted to the deterministic version of the containerized air cargo problems, in which the cargo quantity information is known with certainty. Because accurate cargo shipment information has been obtained in advance, it will not change on the shipping day. Therefore, we can book containers in advance without incurring any penalty from the airlines for urgent requirements. It is assumed there are q_j air cargos of type j that will be shipped one week later, j=1,2,..., n. Let v_j and w_j denote the volume and the weight of cargo type j. All cargos have to be loaded into the air containers provided by the airlines on the shipping day. There are m types of containers, numbered $\{1,2,...,m\}$, for rental. Each type of container i has L_i cargos available, i.e. number $\{1,2,...,L_i\}$. For container type i, V_i and W_i represent the volume and weight limits respectively. The total cost of renting the l^{th} container of type i only includes a fixed cost c_i^0 plus a variable cost c_{il}. Whenever one container is rented, the forwarder has to pay a fixed cost. Once the cargo loaded into the container exceeds a permitted weight limit, a variable cost will be incurred, and this is associated with the weight of cargo loaded into the container. Figure 1 shows the variable cost.

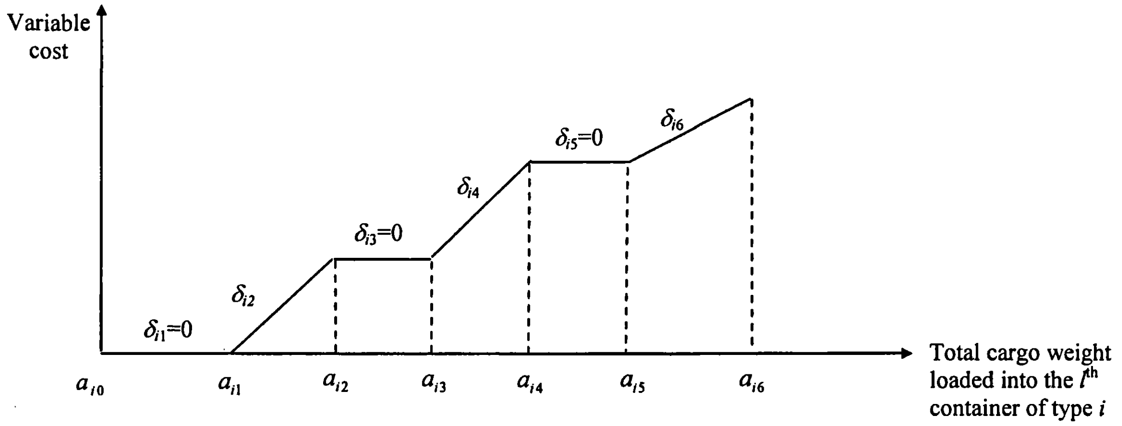

Figure 1: Variable cost of renting the l^{th} container of type i

In Figure 1, a_{ik} represents the break point for container type i, where i=1,..,m, k=1,..., K_i, where K_i is the maximum number of break points. In this study, the air carriers provide six cost break points: a_{i1}, a_{i2}, a_{i3}, a_{i4}, a_{i5}, and a_{i6}. Let a_{i0} be the initial point, i.e. a_{i0} =0. Thus, a_{i1} is the first cost break point for the variable cost, and a_{i6} is the maximum weight limit of container type i. The definition of the variable cost c_{il} in the deterministic version model can be formulated as follows:

$$c_{il} = \begin{cases} 0 & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i0}, a_{i1}] \\ \delta_{i2}(\sum_{j=1}^{n} w_j y_{ilj} - a_{i1}) & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i1}, a_{i2}] \\ \delta_{i2}(a_{i2} - a_{i1}) & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i2}, a_{i3}] \\ \delta_{i2}(a_{i2} - a_{i1}) + \delta_{i4}(\sum_{j=1}^{n} w_j y_{ilj} - a_{i3}) & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i3}, a_{i4}] \\ \delta_{i2}(a_{i2} - a_{i1}) + \delta_{i4}(a_{i4} - a_{i3}) & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i4}, a_{i5}] \\ \delta_{i2}(a_{i2} - a_{i1}) + \delta_{i4}(a_{i4} - a_{i3}) + \delta_{i6}(\sum_{j=1}^{n} w_j y_{ilj} - a_{i5}) & \sum_{j=1}^{n} w_j y_{ilj} \in (a_{i5}, a_{i6}] \end{cases} \tag{18}$$

where, i= 1,2,...,m; l=1,2,..., L_i.

For the deterministic environment, decisions include what types of and how many containers to book, and how to load the cargos into the containers, while simultaneously minimizing the rental cost. The decision variables for the deterministic models are defined as follows:

$$x_{il} = \begin{cases} 1 & \text{if the } l^{\text{th}} \text{ container of type } i \text{ is selected} \\ 0 & \text{otherwise} \end{cases};$$

y_{ilj} = quantities of cargo type j loaded into the l^{th} container of type i;

Therefore, the containerization of the air cargo forwarding problem can be formulated the following 0-1 integer programming model:

$$\text{Minimize } \sum_{i=1}^{m}\sum_{l=1}^{L_i} c_i^0 x_{il} + \sum_{i=1}^{m}\sum_{l=1}^{L_i} c_{il} \tag{19}$$

subject to

$$\sum_{j=1}^{n} v_j y_{ilj} \le V_i x_{il},\ i=1,\ldots,m;\ l=1,\ldots,L_i; \tag{20}$$

$$\sum_{j=1}^{n} w_j y_{ilj} \le W_i x_{il},\ i=1,\ldots,m;\ l=1,\ldots,L_i; \tag{21}$$

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i} y_{ilj} = q_j,\ j=1,2,\ldots,n; \tag{22}$$

$$x_{il} = \{0,1\},\ i=1,\ldots,m;\ l=1,\ldots,h_i; \tag{23}$$

$$y_{ilj} \text{ is an non-negative integer, } i=1,\ldots,m;\ l=1,\ldots,L_i; j=1,\ldots,n; \tag{24}$$

The objective function in (19) is the total cost of renting the containers, which includes two elements. The first element is the total fixed cost for using the containers, and the second is the total variable cost. The definition of the variable cost $c_{il,}$ can be seen by referring to Figure 1 and equation (18). Constraint (20) is the container volume constraint, which ensures that the volume of all cargos allocated to a container cannot exceed the container's volume limits. Constraint (21) is the container weight constraint, which ensures that the weight of all cargos allocated into a container cannot exceed the container's weight limits. Constraint (22) is the cargo quantity constraint, which requires all cargos to be loaded into the containers without any delay. Constraints (23) and (24) are the variable type requirements.

The objective function expressed in (19) is a piecewise function, and it is difficult to solve this kind of model by employing optimal software packages. Two variables are introduced to transform the model into a mixed-integer programming model. One variable g_{ilk} is a continuous variable representing the cargo weight distributed in the range $(a_{i,k-1},a_{ik})$ inside the l^{th} container of type i. The other variable z_{ilk} is a binary variable indicating whether the cargo weight is distributed in the range $(a_{i,k-1},a_{ik})$ inside the l^{th} container of type i. Therefore, the above model can be formulated as the following 0-1 integer programming model:

$$\text{Min}\sum_{i=1}^{m}\sum_{l=1}^{L_i} c_i^0 x_{il} + \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{k=1}^{K_i} \delta_{ik} g_{ilk} \tag{25}$$

subject to

$$\sum_{j=1}^{n} v_j y_{ilj} \le V_i x_{il}, \; i=1,\dots,m;\; l=1,\dots,L_i; \tag{26}$$

$$\sum_{j=1}^{n} w_j y_{ilj} \le W_i x_{il}, \; i=1,\dots, m;\; l=1,\dots, L_i; \tag{27}$$

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i} y_{ilj} = q_j, \; j=1, 2,\dots, n; \tag{28}$$

$$\sum_{k=1}^{K_i} g_{ilk} = \sum_{j=1}^{n} w_j y_{ilj}, \; i=1,\dots, m;\; l=1,\dots, L_i; \tag{29}$$

$$g_{ilk} \le z_{ilk}\left(a_{i,k} - a_{i,k-1}\right), \; i=1,\dots, m;\; l=1,\dots, L_i;\; k=1,\dots, K_i; \tag{30}$$

$$g_{ilk} \ge z_{il,k+1}\left(a_{i,k} - a_{i,k-1}\right), \; i=1,\dots, m;\; l=1,\dots,L_i;\; k=1,\dots, K_i\text{-}1; \tag{31}$$

x_{ils}, z_{ilks} are binary integers, $i=1,\dots, m$; $l=1,\dots,L_i$; $k=1,\dots,K_i$; $s=1,\dots, S$; (32)

y_{ilj} is an non-negative integer, $i=1,\dots, m$; $l=1,\dots, L_i$; $j=1,\dots, n$; (33)

$g_{ilk} \ge 0$, $i=1,\dots, m$; $l=1,\dots,L_i$, $k=1,\dots,K_i$; (34)

There are two items in the objective function (25). The first component is the fixed cost, which is as the same as in the objective function (19). The second component in (25) represents the sum of the variable costs for all containers. The variable cost for each container is the sum of the variable costs distributed in all ranges, described in Figure 1. The variable cost of the l^{th} container of type i in the range $(a_{i,k-1},a_{ik}]$ is the unit charge rate of container i in the range $(a_{i,k-1},a_{ik}]$, represented by δ_{ik}, multiplied by the cargo weight distributed in the range $(a_{i,k-1},a_{ik})$ inside the l^{th} container of type i, represented by g_{ilk}.

Constraints (26), (27), and (28) are the container volume constraint, container weight constraint and cargo quantity constraint respectively. Constraint (29) ensures that the sum of the cargo weight distributed in all areas inside a container is equal to the total weight of the cargos loaded into the container. Constraint (30) ensures z_{ilk} is equal to 1 if the total cargo weight inside the l^{th} container of type i reaches the range $(a_{i,k-1},a_{ik})$. In addition, the cargo weight g_{ilk} in the range $(a_{i,k-1},a_{ik})$ is less-than-or-equal-to the maximum weight value in the range $(a_{i,k-1},a_{ik})$, which is a_{ik}-$a_{i,k-1}$. Constraint (31) ensures that once the total cargo weight inside the l^{th} container of type i reaches the range $(a_{i,k},a_{ik+1})$, the cargo weight in the range $(a_{i,k-1},a_{ik})$, which is g_{ilk}, is not less than the difference between a_{ik} and $a_{i,k-1}$. Constraints (30) and (31) ensure that the weight ranges are reached by priority: g_{ilk} cannot be positive unless the range $(a_{i,k-1},a_{ik})$ is fully occupied by the cargo weight. In other words, constraints (30) and (31) ensure that g_{ilk} cannot have a positive value unless all g_{ilt} are at their maximum value, which is a_{it}-$a_{i,t-1}$, $1 \le t \le k$. Constraints (32), (33), and (34) are the variable type requirements.

4.2 A two-stage stochastic programming models for air cargo forwarding problems under uncertainty

This section is concerned with the stochastic version of the air cargo forwarding model, in which the cargo quantity q_j is a random parameter. It is assumed that q_j has a discrete distribution with a finite number S of possible realizations (sometimes called scenarios), q_{js}, s=1,2,..., S, with the corresponding probabilities p_s, $\sum_{s=1}^{S} p_s = 1$. Twp types of response are made in different stages: the first-stage response is the decision regarding booking with uncertain information; the second-stage response is the decision that is made on the shipping day when the stochasticity is realized. Two types of decision variables are defined as follows:

The first-stage decision variables

n_i= number of containers of type i to be booked.

The second-stage decision variable

n_{is}^{+}= number of type i containers returned on the shipping day in scenario s;

n_{is}^{-}= number of containers of type i rented on the shipping day in scenario s;

$$x_{ils} = \begin{cases} 1 & \text{if the } l^{\text{th}} \text{ container of type } i \text{ is selected in scenario } s \\ 0 & \text{otherwise} \end{cases};$$

y_{iljs} = quantities of cargo of type j loaded into the l^{th} container of type i in scenario s.

Based on the analysis in Section 2, we know that the total cost for shipping cargos consists of two parts: cost of usage and penalty cost. Penalty costs arise from urgent needs or the cancellation of containers on the shipping day. For each scenario, the cost of usage includes a fixed cost c_i^0 and a variable cost c_{ils}. The variable cost under uncertainty can be formulated as follows:

$$c_{ils} = \begin{cases} 0 & \sum_{j=1}^{n} w_j y_{iljs} \in (a_{i0}, a_{i1}] \\ \delta_{i2}(\sum_{j=1}^{n} w_j y_{iljs} - a_{i1}) & \sum_{j=1}^{n} w_j y_{iljs} \in (a_{i1}, a_{i2}] \\ \delta_{i2}(a_{i2} - a_{i1}) & \sum_{j=1}^{n} w_j y_{iljs} \in (a_{i2}, a_{i3}] \\ \delta_{i2}(a_{i2} - a_{i1}) + \delta_{i4}(\sum_{j=1}^{n} w_j y_{iljs} - a_{i3}) & \sum_{j=1}^{n} w_j y_{iljs} \in (a_{i3}, a_{i4}] \\ \delta_{i2}(a_{i2} - a_{i1}) + \delta_{i4}(a_{i4} - a_{i3}) & \sum_{j=1}^{n} w_j y_{iljs} \in (a_{i4}, a_{i5}] \\ \delta_{i2}(a_{i2} - a_{i1}) + \delta_{i4}(a_{i4} - a_{i3}) + \delta_{i6}(\sum_{j=1}^{n} w_j y_{iljs} - a_{i5}) & \sum_{j=1}^{n} w_j y_{iljs} \in (a_{i5}, a_{i6}] \end{cases} \tag{35}$$

where, i= 1,2,…,m; l=1,2,…, L_i., s=1,2,…S.

The objective is to load all cargos into the containers on the shipping day, where the containers are either booked containers or urgent requirements made on the shipping day, while minimizing the total cost charged by the airlines. Uncertain air cargo forwarding problems can be formulated as the following 0-1 stochastic programming model:

$$\text{Min } \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{s=1}^{S} p_s c_i^0 x_{ils} + \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{s=1}^{S} p_s c_{ils} + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^- n_{is}^- + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^+ n_{is}^+ \tag{36}$$

subject to

$$\sum_{j=1}^{n} v_j y_{iljs} \le V_i x_{ils}, \ i=1,\dots,m;\ l=1,\dots,L_i,\ s=1,\dots,S; \tag{37}$$

$$\sum_{j=1}^{n} w_j y_{iljs} \le W_i x_{ils}, \ i=1,\dots,m;\ l=1,\dots,L_i,\ s=1,\dots,S; \tag{38}$$

$$\sum_{i=1}^{m}\sum_{l=1}^{L_l} y_{iljs} = q_{js}, \ j=1,\dots,n;\ s=1,\dots,S; \tag{39}$$

$$n_i = \sum_{l=1}^{Li} x_{ils} + n_{is}^+ - n_{is}^-, \ i=1,\dots,m;\ s=1,\dots,S; \tag{40}$$

$$x_{ils} = \{0,1\}, \ i=1,\dots,m;\ l=1,\dots,h;\ s=1,\dots,S; \tag{41}$$

$y_{iljs}, n_i, n_{is}^-, n_{is}^+$ are non-negative integers, i=1,…, m; l=1,…, L_i; j=1,…, n, s=1,…, S. (42)

The objective function in (36) is the total cost of renting the containers, and includes four parts. The first part is the expected value of the total fixed costs. The second part is the expected value of the total variable costs. The definition of the variable cost $c_{ils,}$ can be be seen by referring to Figure 1 and equation (35). The third part is the expected value of the total penalty cost for renting additional containers on the shipping day. The fourth part is the expected value of total penalty cost for returning unused containers on the shipping day. Each scenario has to satisfy the container volume constraints in (37), container weight constraints in (38), cargo quantity constraints in (39), and container quantity constraints in (40). Constraints (41) and (42) are the variable type requirements.

The objective function expressed in (36) is a piecewise function. We use the same method that is described in the deterministic model, in which two new variables are introduced to transform the model into a 0-1 integer programming model. One variable g_{ilks} is a continuous variable representing the cargo weight distributed in the range $(a_{i,k-1},a_{ik}]$ inside the l^{th} container of type i in scenario s. The other variable z_{ilks} is a binary variable indicating whether the cargo weight is distributed in the range $(a_{i,k-1},a_{ik}]$ inside the l^{th} container of type i in scenario s. Thus the model can be formulated as the following 0-1 integer programming model:

$$\text{Min } \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{s=1}^{S} p_s c_i^0 x_{ils} + \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{k=1}^{K_i}\sum_{s=1}^{S} p_s \delta_{ik} g_{ilks} + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^- n_{is}^- + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^+ n_{is}^+ \tag{43}$$

subject to

$$\sum_{j=1}^{n} v_j y_{iljs} \le V_i x_{ils},\ i=1,\ldots,m;\ l=1,\ldots,L_i;\ s=1,\ldots,S; \tag{44}$$

$$\sum_{j=1}^{n} w_j y_{iljs} \le W_i x_{ils},\ i=1,\ldots,m;\ l=1,\ldots,L_i;\ s=1,\ldots,S; \tag{45}$$

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i} y_{iljs} = q_{js},\ j=1,\ldots,n;\ s=1,\ldots,S; \tag{46}$$

$$n_i = \sum_{l=1}^{Li} x_{ils} + n_{is}^+ - n_{is}^-,\ i=1,\ldots,m;\ s=1,\ldots,S; \tag{47}$$

$$\sum_{k=1}^{K_i} g_{ilks} = \sum_{j=1}^{n} w_j y_{iljs},\ i=1,\ldots,m;\ l=1,\ldots,L_i;\ s=1,\ldots,S; \tag{48}$$

$$g_{ilks} \le z_{ilks}(a_{i,k} - a_{i,k-1}),\ i=1,\ldots,m;\ l=1,\ldots,L_i;\ k=1,\ldots,K_i;\ s=1,\ldots,S; \tag{49}$$

$$g_{ilks} \ge z_{il,k+1,s}(a_{i,k} - a_{i,k-1}),\ i=1,\ldots,m;\ l=1,\ldots,L_i;\ k=1,\ldots,K_i-1;\ s=1,\ldots,S; \tag{50}$$

x_{ils}, z_{ilks} are binary integer, $i=1,\ldots,m$; $l=1,\ldots,L_i$; $k=1,\ldots,K_i$; $s=1,\ldots,S$; (51)

$y_{iljs}, n_i, n_{is}^-, n_{is}^+, e_{js}$ are non-negative integers, i=1,…, m;l=1,…, L_i;j=1,…, n; s=1,…, S; (52)

$g_{ilks} \geq 0$, i=1,…, m; l=1,…,L_i, k=1,…,K_i, s=1,…, S. (53)

4.3 A robust optimization model for the air cargo forwarding problems under uncertainty

Robust optimization allows the violation of the random constraints. Let e_{js} denote cargo quantities of type j not shipped on the shipping day under scenario s. Clearly, e_{js} is a second-stage variable, which is determined after the random parameter value q_j is observed. A robust optimization model can be formulated as:

$$\text{Min} \sum_{i=1}^{m}\sum_{l=1}^{L_l}\sum_{s=1}^{S} p_s c_i^0 x_{ils} + \sum_{i=1}^{m}\sum_{l=1}^{L_l}\sum_{s=1}^{S} p_s c_{ils} + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^- n_{is}^- + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^+ n_{is}^+ + \sum_{j=1}^{n}\sum_{s=1}^{S} p_s \omega_j e_{js} \quad (54)$$

subject to

$$\sum_{j=1}^{n} v_j y_{iljs} \leq V_i x_{ils},\ i=1,\ldots,m;\ l=1,\ldots,L_i;\ s=1,\ldots,S; \quad (55)$$

$$\sum_{j=1}^{n} w_j y_{iljs} \leq W_i x_{ils},\ i=1,\ldots,m;\ l=1,\ldots,L_i;\ s=1,\ldots,S; \quad (56)$$

$$\sum_{i=1}^{m}\sum_{l=1}^{L_l} y_{iljs} = q_{js} - e_{js},\ j=1,\ldots,n;\ s=1,\ldots,S; \quad (57)$$

$$n_i = \sum_{l=1}^{Li} x_{ils} + n_{is}^+ - n_{is}^-,\ i=1,\ldots,m;\ s=1,\ldots,S; \quad (58)$$

$$x_{ils} = \{0,1\},\ i=1,\ldots,m;\ l=1,\ldots,h;\ s=1,\ldots,S; \quad (59)$$

$y_{iljs}, n_i, n_{is}^-, n_{is}^+, e_{js}$ are non-negative integers, i=1,…, m;l=1,…, L_i;j=1,…, n; s=1,…, S; (60)

Compared with the objective function of the two-stage stochastic model, the objective function in (54) includes one additional part, which is repressed in the final part in (54). The final part is the expected value of the penalty cost for not shipping cargos on the shipping day, where ω_j is the penalty cost for not shipping one cargo of type j. All constraints in the above robust optimization model are the same as the constraints in the two-stage stochastic model, except for the cargo quantity constraint expressed in (57). Constraint (57) allows e_{js} cargos of type j not to be shipped under scenario s. However, the cargo quantity constraint in (39) for the two-stage stochastic programming model requires all cargos to be loaded into containers without considering the high rental cost.

Using the same method as in the deterministic and stochastic model, the above model can be changed into a 0-1 integer programming model as follows:

$$\text{Min } \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{s=1}^{S} p_s c_i^0 x_{ils} + \sum_{i=1}^{m}\sum_{l=1}^{L_i}\sum_{k=1}^{K_i}\sum_{s=1}^{S} p_s \delta_{ik} g_{ilks} + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^- n_{is}^- + \sum_{i=1}^{m}\sum_{s=1}^{S} p_s c_i^+ n_{is}^+$$

$$+\sum_{j=1}^{n}\sum_{s=1}^{S} p_s \omega_j e_{js} \tag{61}$$

subject to

$$\sum_{j=1}^{n} v_j y_{iljs} \le V_i x_{ils},\ i=1,\ldots,m;\ l=1,\ldots,L_i;\ s=1,\ldots,S; \tag{62}$$

$$\sum_{j=1}^{n} w_j y_{iljs} \le W_i x_{ils},\ i=1,\ldots,m;\ l=1,\ldots,L_i;\ s=1,\ldots,S; \tag{63}$$

$$\sum_{i=1}^{m}\sum_{l=1}^{L_i} y_{iljs} = q_{js} - e_{js},\ j=1,2,\ldots,n;\ s=1,\ldots,S; \tag{64}$$

$$n_i = \sum_{l=1}^{Li} x_{ils} + n_{is}^+ - n_{is}^-,\ i=1,\ldots,m;\ s=1,\ldots,S; \tag{65}$$

$$\sum_{k=1}^{K_i} g_{ilks} = \sum_{j=1}^{n} w_j y_{iljs},\ i=1,\ldots,m;\ l=1,\ldots,L_i;\ s=1,\ldots,S; \tag{66}$$

$$g_{ilks} \le z_{ilks}\left(a_{i,k} - a_{i,k-1}\right),\ i=1,\ldots,m;\ l=1,\ldots,L_i;\ k=1,\ldots,K_i;\ s=1,\ldots,S; \tag{67}$$

$$g_{ilks} \ge z_{il,k+1,s}\left(a_{i,k} - a_{i,k-1}\right),\ i=1,\ldots,m;\ l=1,\ldots,L_i;\ k=1,\ldots,K_i\text{-}1;\ s=1,\ldots,S; \tag{68}$$

x_{ils}, z_{ilks} are binary integer, $i=1,\ldots,m$; $l=1,\ldots,L_i$; $k=1,\ldots,K_i$; $s=1,\ldots,S$; (69)

$y_{iljs}, n_i, n_{is}^-, n_{is}^+, e_{js}$ are non-negative integer, $i=1,\ldots,m$; $l=1,\ldots,L_i$; $j=1,\ldots,n$; $s=1,\ldots,S$;

(70)

$g_{ilks} \ge 0$, $i=1,\ldots,m$; $l=1,\ldots,L_i$, $k=1,\ldots,K_i$, $s=1,\ldots,S$. (71)

5. Computational Results

For each type of cargo, there is a fixed penalty if the cargo cannot be shipped on the shipping day. Table 1 shows the unit penalty cost.

Table 1: Penalty cost for unshipped cargos

Container type	Large	Medium	Small
Unit penalty cost	20000	18000	16000

It is assumed that quantities for each type of cargo can be captured by the three scenarios. Scenario 1 denotes that 3 cargos of each cargo type are expected to be shipped one week later; Scenario 2 denotes 2 cargos of each cargo type will be shipped; and Scenario 3 denotes only 1 cargo of each cargo type will be shipped (see Table 2).

Table 2: Cargo quantities under different scenarios

Scenario	Large Cargo	Medium Cargo	Small Cargo
Scenario 1	3	3	3
Scenario 2	2	2	2
Scenario 3	1	1	1

In the following, we perform three different tests under different probabilities for the realization of the stochastic variable: cargo quantities. Other than the probability of occurrence of cargo quantities, the other conditions in the three tests are kept the same. The test data are shown in Table 3.

Table 3: Three tests

Test	p_1=Pr{S_1}	p_2=Pr{S_2}	p_3=Pr{S_3}
Test I	0.8	0.1	0.1
Test II	0.1	0.8	0.1
Test III	0.1	0.1	0.8

Test I represents the situation where the cargo quantity is most likely 3 for each type of cargo; Test II the situation where the cargo quantity is most likely 2 for each type of cargo; and Test III the situation where the cargo quantity is most likely 1 for each type of cargo. The optimal selection and loading plan of the proposed model in this study can be obtained using AIMMS. The first-stage response for booking containers is shown in Table 4. Table 5 shows the second-stage decision about the unshipped cargo quantities for each type. Table 6 gives the second-stage response for renting and returning containers. Table 7 shows cargo loading plan on the shipping day. The related cost is shown in Table 8.

Table 4: The first-stage response for booking

Test	Container type						
	1	2	3	4	5	6	7
Test I					1	1	
Test II					1	1	
Test III					1		

Table 5: The second-stage response for urgent container requirements on the shipping day

Test	Container type	Scenario 1		Scenario 2		Scenario 3	
		Containers rented	Containers returned	Containers rented	Containers returned	Containers rented	Containers returned
Test I	1						
	2						
	3						
	4						
	5						
	6						1
	7			1			
Test II	1						
	2						
	3						
	4	1					
	5						1
	6						
	7			1			
Test III	1						
	2						
	3						
	4	1					
	5						
	6	1		1			
	7			1			

Table 6: Cargo quantities for unshipped cargos under different scenarios in the three tests

Test	Cargo Type	Scenario 1	Scenario 2	Scenario 3
Test I	Large	3	0	0
	Medium	0	0	0
	Small	0	0	0
Test II	Large	0	0	0
	Medium	0	1	0
	Small	0	0	0
Test III	Large	0	0	0
	Medium	0	0	0
	Small	0	0	0

Table 7: Optimal cargo loading plans in the three tests

Test	Container Type	Scenario 1			Scenario 2			Scenario 3		
		Large	Medium	Small	Large	Medium	Small	Large	Medium	Small
Test I	1									
	2									
	3									
	4									
	5		3			2	1			
	6			3	2					
	7						1			
Test II	1									
	2									
	3									
	4	1	1	2						
	5		2	1		1	2	1	1	1
	6	2			2					
	7									
Test III	1									
	2									
	3									
	4	1	1	2						
	5		2	1		2	1	1	1	1
	6	2			2					
	7						1			

Table 8: Related cost for container selection and cargo loading problems in the three tests

Test	Fixed cost	Variable cost	Renting penalty cost	Returning penalty cost	Late delivery penalty cost	Total cost
Test I	92680	13021	1000	1000	60000	92680
Test II	98048	9491	3000	1000	18000	129539
Test III	67530	14348	8000	0	0	89879

In Test I, the most likely cargo quantities for each type of cargo are 3. Table 4 provides booking information by ordering 1 container each of types 5 and 6 a week in advance. If Scenario 1 (probability=80%) occurs on the shipping day, this means there are 3 cargos of each type. In this situation, there is no change in containers needed on the shipping day (see Table 5). However, three large cargos are not shipped (see Table 6). Table 7 shows that 3 medium cargos are loaded into container 5 and 3 small cargos are placed into container 6. If Scenario 2 (probability=10%) occurs on the shipping day in Test I, this means there are 2 cargos of each type waiting for shipping. From Table 5, we know that a container of type 7 is rented on the shipping day. All cargos are shipped without delay (see Table 6). Table 7 show that container 6 holds 2 large cargos; container 5 holds 2 medium cargos and 1 small cargo; and container 7 (which is rented on the shipping day) holds 1 small cargo. If Scenario 3 (probability 10%) occurs on the shipping day in Test I, this means there is only 1 cargo of each type for shipping. Therefore, a container type 6 is cancelled on the shipping day, and all cargos can be loaded into container 5 without delay.

In Test II, the most likely cargo quantities for each type of cargo are 2. Table 4 shows that 1 container of type 5 and 1 container of type 6 are booked a week before. If Scenario 1 (probability 10%) occurs on the shipping day, this means that there are 3 cargos of each type waiting for shipping. Based on the results of Test II shown in Table 5, a container of type 4 is required on the shipping day. Additionally, all cargos are shipped without delay (see Table 6). Therefore, container 4 holds 1 large cargo, 1 medium cargo, and 2 small cargos; container 5 holds 2 medium cargos and 1 small cargo; and container 6 holds 2 large cargos. If Scenario 2 (probability 80%) occurs on the shipping day in Test II, there are 2 cargos of each type waiting for shipping. No additional containers are required on the shipping day, but there is one medium cargo left over. Thus container 5 holds 2 medium cargos and 2 small cargos, and container 6 holds 2 large cargos. If Scenario 3 (probability 10%) occurs in Test II, it means a cargo of each type is waiting for shipping. In this situation, a container of type 6 is cancelled on the shipping day (see table 5). All cargos can be loaded into container 5 for shipping with out any delay.

In Test III, the most likely cargo quantity for each type is 1. The containers booked in Test III differ from those in Tests I and II. In Test III, only one container is booked (see Table 4), because the cargo quantities in Test III are most likely less than those in Tests I or II. In Test III, if the unexpected Scenario 1 (probability=10%) occurs on the shipping day, it means that 3 cargos of each type are waiting for shipping. On the shipping day, a container of type 4 and a container of type 6 are required to deal with this unexpected large cargo situation (see Table 5). Container 4 holds 1 large cargo, 1 medium cargo and 2 small cargos; container 5 holds 2 medium cargos and 1 small cargo; container 6 holds 2 large cargos. No cargos are left. If Scenario 2 occurs (probability=10%) in Test III, it means there are 2 of each type of cargo quantities waiting for shipping. In this situation, a container of type 6 and a container of type 7 are rented on the shipping day (see Table 3). Container 5 holds 2 medium cargos and 1 small cargo; container 6 holds 2 largo cargos; container 7 holds 1 small cargo. No cargos are left. If Scenario 3 (probability 80%) occurs in Test III, there is only 1 of each type of cargo for shipping. There is no need to rent or return any containers on the shipping day (see Table 5). All cargos can be loaded into a container of type 5, which has been ordered a week in advance.

In the above three tests, the cargo quantities for each type of cargo under the different scenarios are 3, 2 and 1 respectively. However, the probability of each scenario occurring is different in each of the three tests, which results in different container selection and cargo loading plans in the first stage (when booking) and the second stage (on the shipping day). Additionally, the plans are dependent on the penalty cost associated with unshipped cargos.

Further Discussion

The following tests assume that the uncertainty of the random variable can be captured by three scenarios: Scenario 1 (or S1) denotes 3 cargos of each type with probability 25%; Scenario 2 (or S2) denotes 2 cargos of each type with probability 50%; Scenario 3 (or S3) denotes 1 cargo of each type with probability 25%.

Test IV: The unit penalty for not shipping large, medium and small cargos increases or decreases at the same amount.

Table 9 shows the computational results of the robust optimization model solved using AIMMS under different unshipped penalty costs ω. Table 10 shows the optimal solution of the two-stage stochastic programming model solved using AIMMS. As the two-stage stochastic recourse programming model does not permit violation of stochastic constraints, all cargos have to be shipped on the shipping day. The total cost is 138982, which is shown in Table 10. From Table 9, when the unit penalty cost for not shipping cargo is more than 16000 for large cargo, 14000 for medium cargo and 12000 for small cargo, no cargos are left on the shipping day because of the high penalty charge. In this situation, the total cost of the robust optimization is equal to the total cost of the stochastic model. When the unit penalty cost is less-than-or-equal-to 16000 for large cargo, 14000 for medium cargo and 12000 for small cargo, some cargos are left on the shipping day. Because of the low unit penalty cost for not shipping cargos, the decision makers would like to leave some cargos for future shipment. Therefore, the total costs decrease as the unit penalty cost for not shipping cargo decreases. When the unit penalty cost is lower than 11000 for large cargo, 9000 for medium cargo and 7000 for small cargo, more cargos are not shipped on the shipping day because of this lower unit penalty cost. As soon as the unit penalty cost falls to 7000 for large cargo, 5000 for medium cargo and 3000 for small cargo, no cargos need to be shipped on the shipping day. The total costs equal the penalty cost for the unshipped cargos.

Table 9: Optimal solution of robust optimization model under different ω (Test IV)

Unit penalty cost ω	Unshipped cargo quantities			Unshipped penalty cost	Fixed cost	Variable cost	Rent penalty cost	Return penalty cost	Total cost
	S1	S2	S3						
(20000,18000,16000)	0	0	0	0	117202	10530	7500	3750	138972
(19000,17000,15000)	0	0	0	0	117202	10530	7500	3750	138972
(18000,16000,14000)	0	0	0	0	117202	10530	7500	3750	138972
(17000,15000,13000)	0	0	0	0	117202	10530	7500	3750	138972
(16000,14000,12000)	0	1	0	12000	102221	14020	7500	2500	138241
(15000,13000,11000)	0	1	0	11000	102221	14020	7500	2500	137241
(14000,12000,10000)	0	1	0	10000	102221	14020	7500	2500	136241
(13000,11000,9000)	0	1	0	9000	102221	14020	7500	2500	135241
(12000,10000,8000)	0	1	0	8000	102221	14020	7500	2500	134241
(11000,9000,7000)	0	1	0	7000	102221	14020	7500	2500	133241
(10000,8000,6000)	4	6	0	76000	35995	6800	5000	5000	128795
(9000,7000,5000)	4	6	0	66000	35995	6800	5000	5000	118795
(8000,6000,4000)	7	6	1	78000	23277	0	0	5000	106277
(7000,5000,3000)	6	4	3	90000	0	0	0	0	90000
(6000,4000,2000)	6	4	3	72000	0	0	0	0	72000

Table 10: The optimal solution of the stochastic programming model

Fixed cost	Variable cost	Rent penalty cost	Return penalty cost	Total cost
117202	10530	7500	3750	138982

Test V: The unit penalties for not shipping large, medium and small cargos change by different amounts.

Table 11: Unit penalty for not shipping large, medium and small cargo changes by different amounts

Unit penalty cost of not shipping cargo (ω)	Unshipped cargos			Non-shipped penalty cost	Total cost
	S1	S2	S3		
(13000,11000,9000)	0	1 small	0	9000	135241
(13000,11000,11000)	0	1 medium	0	11000	135791
(13000,13000,13000)	0	1 medium	0	30000	134015

(11000,11000,11000)	0	1 large	0	11000	135791
(9000,9000,9000)	3 large	2 large 1 medium	0	54000	129871

In Test V, we first set the unit penalty cost for not shipping cargo ω at 13000 for large cargos, 1100 for medium cargos and 9000 for small cargos (see Row 2, Table 11). The difference in the unit penalty between large and medium cargos is the same as between medium and small cargos. Now, let the unit penalty for not shipping small cargo increase by 2000 (see Row 3, Table 11). From Table 11, in Scenario 2, we know that the unshipping cargo is a medium cargo. When the unit penalty for not shipping all types of cargo rises to 13000, one medium cargo is left over. However, when the unit penalty for not shipping all types of cargo falls to 11000, a large cargo is left over, as shown in Scenario I. When the unit penalty for not shipping cargo falls to 9000 for all types of cargos, 3 large cargos are left in Scenario 1, and 2 large cargos and 1 medium cargo are left in Scenario 2.

Based on the above tests, we can reach the following conclusion: the cargo forwarding strategy is heavily dependent on the unit penalty cost for not shipping cargos. When the unit penalty cost is large enough, no cargos are left unshipped on the shipping day under all scenarios. However, when unit penalty cost of is small enough, no cargos need to be shipped on the shipping day.

6. Conclusions

Globalisation is forcing companies to compete on price and delivery to their customers, and these factors highlight the importance of air transport. Effective transport strategies can provide a competitive advantage in terms of quick delivery, responsiveness and flexibility to changing and uncertain market information, while continuously lowering transportation costs. This study is concerned with the air cargo forwarding problems experienced by the air forwarders when they rent containers from airlines. Usually, the cargo information is changing and uncertain, but air forwarders have to book containers in advance in order to obtain a price discount from the airlines, as any urgent requirements for containers made on the shipping day will incur a penalty charge. We first formulate a deterministic model, in which the cargo information is known when booking. Therefore, the forwarders can book containers in advance in order to load all cargos into the containers on the shipping day without any needing any containers urgently on the say. The total cost thus only includes a fixed cost plus a variable cost depending on the weight that the container holds. We then formulate a two-stage stochastic model for uncertain cargo information. The first-stage decision is to determine the container booking information in terms of container quantities and types. The second-stage decision includes determining the quantities and types of required or/and returned containers on the shipping day, as well as loading all cargos into the containers. We finally present a robust optimization model for uncertain cargo information, in which late shipping is permitted with at a penalty. A series of experiments is presented to demonstrate the effectiveness of the robust optimization model. In comparison with the two-stage stochastic model, the robust optimization model shows its flexibility in dealing with the risk and cost. Further research will consider designing a robust global supply chain system that integrates different activities in the global supply chain network, such as integrating production, warehousing, road transport, sea transport, etc.

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Appendix H

List of papers submitted for publication

1. A Dual-response strategy for global logistics under uncertainty based on stochastic mixed 0-1 integer programming.

2. Linear Robust Models for International Logistics Problems under Uncertainty.

(Copies available on request)