Information and Quality in International Trade and the Political Economy of Trade Protection

by

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A thesis submitted in August 2007 in fulfilment of the requirements for the degree of Doctor of Philosophy in Economics at the London School of Economics and Political Science
I declare that the work presented in this thesis is my own except where the collaboration with coauthors is explicitly acknowledged.

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Abstract

This thesis examines how information costs, minimum quality standards and electoral incentives affect international trade and trade policy choice.

First, a new pairwise matching model with two-sided information asymmetry is developed to analyse the impact of information costs on endogenous network-building and matching by information intermediaries. The framework innovates by examining the role of information costs on incentives for trade intermediation, thereby endogenising the pattern of direct and indirect trade. The model is extended to analyse the strategic interaction between two information intermediaries who compete in commission rates and network size, giving rise to a fragmented duopoly market structure.

Second, unilateral minimum quality standards are endogenously determined as the outcome of a non-cooperative standard-setting game between the governments of two countries. Cross-country externalities from the implementation of minimum quality standards are shown to give rise to a Prisoners' Dilemma structure in the incentives of policy-makers leading to inefficient policy outcomes. The role of minimum quality standards as non-tariff barriers is examined and the scope for mutual gains from reciprocal adjustment in minimum standards analysed. Asymmetric externalities make a cooperative agreement at the world optimum infeasible.

Third, a new multi-jurisdictional political agency model is developed to analyse electoral incentives for trade protection in an electoral college. A
unique equilibrium is shown to exist where political incumbents build a reputation for protectionism through their policy decisions in their first term of office. A spatial dimension is introduced that shows how trade policy incentives hinge on the distribution of swing voters across decisive, swing states. The empirical analysis augments a benchmark test of the "Protection for Sale" mechanism to include a measure of how industries specialise geographically in swing and decisive states. The findings lend support to the theory.
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Acknowledgements

I would like to extend my gratitude to my thesis advisors Stephen Redding and Anthony Venables for providing outstanding guidance and support throughout my PhD. I would also like to thank Daniel Sturm, Peter Neary, Alejandro Cunat and Henry Overman for very helpful comments and suggestions. Financial support of the Economic and Social Research Council is gratefully acknowledged.

Very special thanks go to my family for their loving support and understanding.

I dedicate this thesis to my mother whose unwavering faith in me helped restore my own.
Introduction

In this thesis I theoretically examine how information costs, minimum quality standards and electoral incentives affect international trade and trade policy choice.

Chapter 1 present a new pairwise matching model with two-sided information asymmetry that is used to analyse the impact of information costs on endogenous network-building and matching by information intermediaries. The framework innovates by examining the role of information costs on incentives for trade intermediation, thereby endogenising the pattern of direct and indirect trade. The analysis delivers four key results. First, intermediation unambiguously raises expected trade volume and social welfare by expanding the set of matching technologies available to traders. Second, convexity in network-building costs is necessary for both direct and indirect trade to arise in equilibrium. Third, under assumptions of convexity in the intermediary's technology, optimal network size and hence the equilibrium pattern of trade is shown to depend on the level of information costs as well as the relative effectiveness of direct and indirect matching technologies with changing information costs. Finally, the model sheds light on the relationship between information frictions and aggregate trade volume, which may be non-monotonic as a result of conflicting effects of information costs on the incentives for direct
and indirect trade.

Chapter 2 extends the model to analyse the strategic interaction between two information intermediaries who compete in commission rates and network size, giving rise to a fragmented duopoly market structure. The analysis delivers the following results. First, the model suggests that network competition between information intermediaries has a distinctive market structure, where intermediaries are monopolist service providers to some contacts but duopolists over contacts they share in their network overlap. Second, the coordination game of traders presents the possibility of coordination failure between trade pairs, even though both traders are members of both networks and this is known to both. Third, we show that intermediaries' inability to price discriminate between the competitive and non-competitive market segments, gives rise to an undercutting game, which has no pure strategy Nash equilibrium. The incentive to randomise commission rates yields a mixed strategy Nash equilibrium. Finally, competition is affected by the technology of network development. The analysis shows that either a monopoly or a fragmented duopoly can prevail in equilibrium, depending on the network-building technology. Under convexity assumptions, both intermediaries invest in a network and compete over common matches, while randomising commission rates. In contrast, linear network development costs can only give rise to a monopoly outcome.

Chapter 3 extends a well-established vertical product differentiation model
to an international setting in order to analyse governments’ incentives for the unilateral setting of minimum quality standards, as well as the scope and effects of international cooperation on welfare and international trade. National standards are endogenous and result from a standard-setting game between governments. The analysis delivers four results. First, there exist four unregulated Nash equilibria in minimum standards, two symmetric and two asymmetric, depending on the quality ranking of firms in each market. The analysis establishes that in all four cases, unilaterally selected minimum quality standards are inefficient as a result of cross-country externalities. Second, minimum quality standards are shown to operate as non-tariff barriers to trade. Third, the world welfare maximising symmetric standard can be reached through reciprocal adjustments in national minimum standards from either of the two symmetric Nash equilibria. Finally, the scope for mutually beneficial cooperation is shown to be significantly restricted when cross-country externalities are asymmetric.

The final chapter of the thesis develops a new multi-jurisdictional political agency model for analysing electoral incentives for trade protection. A unique equilibrium is shown to exist where political incumbents build a reputation of protectionism through their policy decisions in their first term of office. The spatial dimension of the multi-jurisdictional framework shows how the incentives driving trade policy hinge on the distribution of swing voters across swing and decisive states. Finally, the theoretical hypothesis is tested
empirically and the findings provide support for the theory highlighting an important, and previously overlooked, determinant of trade protection.
1 Information Costs, Networks and Intermediation in International Trade

This chapter analyses the role of information costs on the incentives for information intermediaries to emerge as trade facilitators and addresses a broad range of issues in a tractable, unified, theoretical framework. First, the model sheds light on how barriers to information flow can affect trade patterns and the organisation of trade, either directly, or indirectly through an intermediary. Second, it explores the incentives for contact-building and intermediation with varying levels of information costs and for a broad range of parameter values reflecting different network-building technologies. The pairwise matching model developed contributes to the literature by showing how information costs affect the realisation and organisation of trade transactions, for a given set of trade opportunities, in a framework where the pattern of information intermediation is determined endogenously.

The model is particularly applicable to international trade in differentiated goods for which information about product characteristics is important. The model can also be applied more broadly to intermediated markets where contact-building and matching are key. Examples may include headhunters in the job market, real estate agents in the housing or rental market, charterers in the transportation market, matchmakers in the marriage market (in some cultures), among others.

There is a broad literature addressing the many functions of middlemen.
They have been shown to reduce search costs (Rubinstein and Wolinksy, 1987; Yavas, 1992, 1994), to offer expertise in markets with adverse selection (Biglaiser, 1993), to operate as guarantors of quality under producer moral hazard (Biglaiser and Friedman, 1994), as well as to operate as investors in quality-testing technology (Li, 1998). More recently, Shevchenko (2004) endogenises the number of intermediaries who buy and sell goods and examines the optimality of the size and composition of their inventories. Common to all of these works is the exploration of the role of middlemen as buyers and sellers of goods. In contrast, this chapter explores the role of intermediaries as brokers of information.

Information is required to identify profitable trading opportunities and locate suitable trading partners, particularly where goods are differentiated and information about product characteristics is important. Information asymmetries, coupled with costs of acquiring information, can hinder the matching of agents with opportunities and prevent prices from allocating scarce resources across countries. Portes and Rey (2005) point to a lack of information about international trading opportunities and the need to tap into ‘deep knowledge’. In such a setting, international trade can be facilitated through intermediaries who invest in information networks or contacts and match agents with suitable opportunities for a fee.

Rauch and Watson (2002) present some summary statistics from a Pilot survey of international trade intermediaries based in the US. Despite the small
number of observations, their evidence suggests that 50% of trade intermediation in differentiated products does not involve taking title of goods and reselling, as compared to only 1% for homogeneous-goods. Moreover, 36% of the revenue from trade intermediation of differentiated products is reported to come from success fees based on the value of transactions, while the figure for homogeneous-good intermediation is only 1%. This is consistent with the search based or network view of trade, pioneered by Rauch (2000), Rauch and Trindade (2000) and others, that posits that the information requirements for differentiated goods are much greater due to the need to match specific characteristics. The evidence to date supports this, pointing to a more pronounced role for information intermediaries in the trade of differentiated goods.

The facilitation of trade through information networks has only recently begun to be formally developed. Recent literature on networks in international trade (Casella and Rauch, 2002) focuses on gaining insight on how information-sharing networks among internationally dispersed ethnic minorities or business groups can overcome informal trade barriers such as inadequate information about trading opportunities and weak enforcement of international contracts (Anderson and Marcouiller, 2002).

Casella and Rauch (2002) develop a model where output is produced through a joint venture and agents cannot judge the quality of their match abroad. They show that introducing a subset of agents with social ties, who have complete information when it comes to matching with other group mem-
bers, increases aggregate trade and income, but hurts the anonymous market. More recently, Rauch and Watson (2002) model the supply of 'network intermediation' where agents endogenously choose whether to be producers or intermediaries, depending on their endowment of contacts. The emphasis of the existing literature has largely been the effects of pre-existing social ties or contacts on trade. This chapter contributes to the literature by analysing the incentives for contact-building and exploring how trade intermediation can offer a more efficient means of trade matching, without relying on any pre-existing ties between agents.

The remainder of this chapter is organised as follows. Section 1 introduces the intermediation model. Section 2 extends the network-building cost specification giving rise to a richer set of results. Section 3 concludes.

1.1 The Model

This section introduces a pairwise matching model with a continuum of importers and exporters, and a single trade intermediary. The framework captures the incentives for network-building and intermediation when there are barriers to the flow of information and sheds light on the role information costs play on the organisation of trade.

1.1.1 Model Set-up

Consider a two-sided market where importers and exporters match in pairs to exchange a single unit of output. Let there be a continuum of exporters
(X), distributed uniformly, and with unit density, over the interval [0, 1] and a continuum of importers (M), also distributed uniformly, with unit density over [0, 1]. Suppose that for each trader there is a unique partner on the other side of the market with whom they can trade. Each trade transaction generates a joint surplus $S > 0$, but if agents fail to locate their match they receive a payoff of 0. Moreover, assume all market participants are risk-neutral.

The framework best reflects trade in differentiated goods where specific characteristics have to be matched, whether these are feature of the product, timing of delivery etc. In the absence of trade frictions, importers and exporters identify each other costlessly and all trade opportunities are exploited generating a total surplus of $S$.

Suppose there is two-sided information asymmetry such that traders on both sides of the market do not know the location of their partner on the other side of the market. Within the set of infinitely many traders, the probability of each exporter (importer) locating her partner by selecting a random trader from the measure of importers (exporters) is zero. Any pair $j$ of trade partners $(X_j, M_j)$ is assumed to be able to match through a direct matching technology, however, which achieves successful matching with probability $q(i)$, where parameter $i \in [0, 1]$ reflects the level of information costs or barriers to information flow between the two sides of the market. Let $q'(i) < 0$, so a higher prevailing level of information costs implies a lower probability of matching for each pair. Parameter $i$ may be interpreted as reflecting the state
of information and communication technology (ICT). An ICT improvement reflects a decline in $i$, which in turn implies a higher probability of a direct match. Further, let $q(1) = 0$ and $q(0) = 1$, so information cost level $i = 1$ prohibits any matching, while $i = 0$ corresponds to the full information case where all trade opportunities are exploited. $q(i)$ is also the expected trade volume and $q(i)S$ the expected joint surplus from direct trade. The two-sided market is represented in figure (1).

Suppose the market has a single intermediary ($I$) with access to a technology for developing contacts with importers and exporters and finding out their trade characteristics (location, product features etc.). The intermediary incurs a set up cost, $F$, for creating a network and a marginal cost of network expansion, $c(i)$, where $c'(i) > 0$ and $c(0) = 0$. The cost of making contacts is assumed to increase monotonically with the level of information costs; but assumed to be entirely costless when $i = 0$. 

![Figure 1: The two-sided market with pairwise trade matches.](image-url)
The intermediary’s network is denoted by a measure of importers, $P_M$, and a measure of exporters, $P_X$, where $P_M \in [0,1]$ and $P_X \in [0,1]$, contacted by incurring cost $c(i)P_M$ and $c(i)P_X$, respectively. Let $C(P_X, P_M)$ denote the intermediary’s total investment cost for building a network of contacts of size $\{P_X, P_M\}$, where this is linear and described by (1):

$$C(P_X, P_M) = F + c(i)(P_X + P_M) \quad (1)$$

Once network investment costs are sunk, it is assumed costless for the intermediary to match trade pairs from within his network of contacts. The intermediary’s marginal cost of trade intermediation is zero. The proportion $P_X$ also reflects the ex ante probability that any particular exporter $X_j$ is a network member. Similarly, $P_M$ is the probability that any particular importer $M_j$ is a network member. Thus, $P_XP_M$ describes the joint probability that both trade partners in pair $(X_j, M_j)$ are contacted by the intermediary, for given network size. Once uncertainty regarding the identity of network members is resolved, the intermediary is able to match trade pairs from within his network\(^1\) with probability 1.

The intermediary raises revenue by charging a commission for matching trading partners through his network. Let $\alpha_I$ denote the share of trade surplus, or commission rate, that the intermediary demands for successful intermediation of trade. The intermediary’s power to extract trade surplus through $\alpha_I$

\(^1\)This assumption can easily be relaxed so that indirect matching takes place with a probability less than or equal to 1 but higher than the probability of a direct match.
is constrained by the traders' outside option to trade directly with probability $q(i)$. In particular, as direct matching prospects worsen with $i$, the highest commission rate consistent with trader participation increases. The commission rate is thus described as a function of information cost $i$, $\alpha_I(i)$, where $\alpha_I'(i) > 0$. The level of information costs $i$ therefore affects the intermediary's profit through two channels. First, through network-building cost, $c(i)$, and second, through commission rate $\alpha_I(i)$.

**Timing of the Game** The timing of the game between traders and intermediary $I$ is as follows:

**Stage 1 - Network investment:** The intermediary invests in a network of size $\{P_X, P_M\}$ by contacting a proportion of importers and exporters. Network investment costs, $C(P_X, P_M)$, are sunk. The intermediary offers contacts a take-it-or-leave-it contract specifying commission rate $\alpha_I$ for successful matching.

**Stage 2 - Contracting:** Traders in receipt of a contract accept or reject it.

**Stage 3 - Indirect trade:** Uncertainty over which trade matches are feasible through the network is resolved. The intermediary matches pairs of traders in his network, provided both parties accepted in stage 2.

**Stage 4 - Direct trade:** Any unmatched traders trade directly with probability $q(i)$.
Equilibrium Concept  The solution concept used is subgame perfect equilibrium (SPE) and the method used is backward induction. A strategy for intermediary $I$ is a set $\{P_X(i), P_X(i), \alpha_I(i)\}$ that describes network size and commission rate, given information costs $i$. A strategy for trader $j$ is described by a rule $R_a$ for accepting or rejecting a contract in stage 2, if such a contract is received. A set of strategies $\{P_X(i), P_X(i), \alpha_I(i), R_a\}$ can be said to form a subgame perfect equilibrium of the game if under these strategies the expected profit of the intermediary and the expected trade surplus of each trader are maximised, given the strategies of all other players.

1.1.2 Direct and Indirect Trade

The pool of unmatched traders in the final stage of the game includes three groups of traders: (a) those not contacted in stage 1, (b) those contacted but who rejected the contract in stage 2, and (c) those who were contacted and accepted, but could not be matched through the network in stage 3. Unmatched traders can expect to match directly with probability $q(i)$ in the final stage of the game. Each direct match generates $S$, so the ex ante expected surplus from the direct trade route is $q(i)S$. Let $\alpha_X$ and $\alpha_M$ denote the surplus shares of exporters and importers, respectively, where $\alpha_X + \alpha_M = 1$.

For simplicity, assume both parties have equal bargaining power so gains from any transaction are split evenly\(^2\), such that $\alpha_X = \alpha_M = \frac{1}{2}$. The expected

\(^2\)The particular values of $\alpha_X$ and $\alpha_M$ have no bearing on the intermediary’s investment decision, or choice of commission rate. Symmetry is assumed for simplicity.
payoffs from direct trade for importers and exporters, denoted by $E^{DT}(\Pi_M)$ and $E^{DT}(\Pi_X)$, respectively, can thus be expressed as:

$$E^{DT}(\Pi_X) = E^{DT}(\Pi_M) = \frac{1}{2} q(i)S$$

Intermediated trade transactions in stage 3 between network members who accept in stage 2 also generate $S$ per match. Since traders are identical in terms of their future trade prospects, they all either accept or reject the take-it-or-leave-it offer in stage 2. The intermediary maximises stage 1 expected profit subject to participation constraints, thereby ensuring that all traders contacted by the intermediary find it optimal to accept in equilibrium. Let $\alpha_j(i)$ denote the share of trade surplus captured by $j$, given information costs $i$, where $j = \{X, M, I\}$. As with direct trade, exporters and importers are assumed to split (residual) surplus equally, so $\alpha_X(i) = \alpha_M(i) \equiv \alpha_T(i)$. It follows that:

$$2\alpha_T(i) + \alpha_I(i) = 1$$

The intermediary's share, $\alpha_I(i)$, is determined endogenously and depends on $i$. It follows that $\alpha_T(i)$ also depends on the prevailing level of information costs. Traders' expected payoffs from indirect trade, denoted by $E^{IT}(\Pi_M)$ and $E^{IT}(\Pi_X)$, respectively, can thus be expressed as:
The measure of intermediated transactions, for any given network size, will vary in stage 3 depending on the degree of overlap between the two groups of contacts. Hence, the measure of intermediated trade matches, denoted by \( T_I \), is a random variable. For any network of size \((P_X, P_M)\), the largest measure of matches possible through the network is \( \min \{P_X, P_M\} \), reflecting the maximal measure of overlap between importer and exporter contacts. Similarly, the smallest measure of matches that may arise is \( \max \{P_X + P_M - 1, 0\} \), where mismatch between the two contact groups is greatest.

For any pair \((X_j, M_j)\), the ex ante probability of matching through the intermediary is given by the joint probability of both partners being contacted by the intermediary in stage 1, \( P_X P_M \). The probability of any pair \( j \) matching is integrated over the range of possible pairs to give the expected measure of intermediated matches \( E(T_I) = P_X P_M \).

In equilibrium, the intermediary builds contacts symmetrically in order to maximise \( E(T_I) \), for any given network investment. Thus, \( P_X = P_M = P \). Hence, the expected measure of intermediated matches is \( E(T_I) = P^2 \).

Proposition (1) establishes the optimality of a symmetric network, allowing the subgame perfect equilibrium strategy set to be redefined as \( \{P^*(i), \alpha^*(i), R^*_i\} \).

**Proposition 1** It is optimal for the trade intermediary to invest symmetrically in network-building on both sides of the market, such that \( P_X = P_M = P \),
where \( P \in [0,1] \).

**Proof.** Consider a network of size \((P_X, P_M)\) from which a measure of trade matches \( E(T_I) = P_X P_M \) is expected. The intermediary can maximise the return from his network investment by choosing \( P_X \) and \( P_M \) to maximise \( E(T_I) \), given \( C(P_X, P_M) = F + c(i)(P_X + P_M) \). The first order conditions of the constrained optimisation yield \( P_X = P_M \) as the trade maximising network configuration. Proposition (1) follows directly. ■

For any exporter (importer) evaluating whether to sign up with the intermediary in stage 2, the probability of her partner also being in the network is \( P \). Each exporter (or importer) can expect to receive \( E^{IT}(\Pi_X) \) (or \( E^{IT}(\Pi_M) \)) with probability \( P \) and \( E^{DT}(\Pi_X) \) (or \( E^{DT}(\Pi_M) \)) with probability \( 1 - P \). Hence, the expected payoff of exporter \( X_j \) (or importer \( M_j \)), conditional on being contacted by the intermediary in stage 1, is given by:

\[
E(\Pi_{X_j}|X_j \in P) = E(\Pi_{M_j}|M_j \in P) = \frac{1}{2} [P(1 - \alpha_I(i)) + (1 - P)q(i)] S \quad (5)
\]

Contrasting the expected payoffs described by equations (2) and (5) yields proposition (2).

**Proposition 2** In equilibrium, the intermediary offers contracts demanding commission rate \( \alpha^*_I(i) = 1 - q(i) \). All contracts are accepted.

**Proof.** To ensure trader participation in stage 2, the intermediary must set \( \alpha_I(i) \) sufficiently low so that expected payoff from signing up to the network,
described by equation (5), is at least as large as the expected payoff from an exclusively direct trade route, described by (2). The highest commission rate consistent with trader participation is thus:

$$\alpha_I(i) \leq 1 - q(i)$$  \hspace{1cm} (6)

Hence, traders' optimal acceptance rule $R^*_2$ in stage 2 is 'accept the contract if $\alpha_I(i) \leq 1 - q(i)$; reject otherwise'. Anticipating the traders' incentives in stage 2, the intermediary sets the largest participation-consistent commission rate$^3$. Hence, the intermediary selects $\alpha^*_J(i) = 1 - q(i)$ in stage 1 and all contracts offered are accepted in stage 2. ■

The intermediary is constrained by traders' outside option to trade directly, which in turn depends on the level of information costs. The worse are the traders' prospects in the market, the higher the commission rate the intermediary can charge and still ensure participation. Even though a larger network improves the chances of an indirect trade match, the option to trade directly remains available, so $\alpha^*_J(i)$ is independent of $P$. Moreover, since all surplus over and above that generated through direct trade is appropriated by the intermediary, all traders are indifferent between trading directly or the possibility of trading through the network.

**Proposition 3** *In equilibrium, importers and exporters are indifferent ex ante*

$^3$Assume that when indifferent between the two modes of trade, traders sign up with the intermediary. Alternatively, the intermediary could offer an infinitesimally small additional amount, $\varepsilon$, to ensure traders sign up to the network.
between the prospect of direct matching only and having the opportunity to trade both directly and indirectly.

**Proof.** At the outset of the game, anticipating a network of size \( P \), any pair \((X_j, M_j)\) can expect to find themselves in one of four possible positions: (i) with probability \((1 - P)^2\), both trade partners are outside the network; (ii) with probability \(P(1 - P)\), \(M_j\) is inside the network and \(X_j\) outside; (iii) with probability \(P(1 - P)\), \(X_j\) is inside the network and \(M_j\) outside, and (iv) both partners are members of the network, with probability \(P^2\). The expected payoff for each partner is \(\frac{1}{2}q(i)S\) in (i)-(iii) and \(\frac{1}{2}[1 - \alpha_T(i)]S\) in (iv). Weighing the expected payoffs with their respective probabilities yields the *ex ante* expected payoff to any trader \(j\) at the outset of the game, given \(P\). This is summarised by:

\[
E(U_{X_j} | P) = E(U_{M_j} | P) = \frac{1}{2} [q(i)(1 - P^2) + (1 - \alpha_T(i)) P^2] S
\]

Anticipating that \(\alpha_T(i) = 1 - q(i)\), (7) simplifies to give \(E(U_{X} | P) = E(U_{M} | P) = \frac{1}{2}q(i)S = E^{DT}(U_{X}) = E^{DT}(U_{M})\). Hence, traders are indifferent between having the prospect of intermediated trade, or not. ■

1.1.3 Equilibrium Network Size

The intermediary chooses \(P \in [0, 1]\) to maximise expected profits (net of network investment cost), \(E(\Pi_I)\), subject to \(\alpha_T(i) = 1 - q(i)\) and \(R^*_n\), where expected profit can be expressed by:
\[ E(\Pi_I) = [1 - q(i)] SP^2 - 2c(i)P - F \]  

(8)

The specification does not yield an interior equilibrium for \( P \), as verified by the non-negative second order condition\(^4\). Hence, the intermediary chooses to develop contacts with all traders, or none, depending on the level of information costs. When profitable at the margin, the network expands to include all traders, provided the measure of trade matches is sufficiently large to cover set up costs. Otherwise, no contacts are developed at all, and the intermediary is inactive. Which of the two corner equilibria prevails hinges on the relative magnitude of two conflicting effects of \( i \) on expected profits. The greater the prevailing barriers to information flow, the higher are the costs of network development. At the same time, traders’ direct matching prospects worsen, thereby allowing a higher commission rate to be charged. The net effect of information costs on \( E(\Pi_I) \) thus depends on the relative impact of \( i \) on \( q(i) \) and \( c(i) \). This is summarised formally in proposition (4).

**Proposition 4** Expected profit from intermediation is monotonically increasing with the level of information costs if \( c'(i) < -\frac{PS}{2}q'(i) \), for \( P > 0 \).

**Proof.** Partially differentiating (8) with respect to \( i \) yields:

\[ \frac{\partial E(\Pi_I)}{\partial i} = - [2c'(i) + PSq'(i)] P \]  

(9)

\(^4\)The second order condition is non-negative for all values of information cost \( i \) and network size \( P \). \( \frac{\partial E(\Pi_I)}{\partial P} = 2S[1 - q(i)] \geq 0. \)
It follows directly from (9) that \( E(\Pi_I) \) is monotonically increasing with \( i \), if:

\[
\frac{d'}{d} < -\frac{PS}{2q'(i)}, \text{ for } P > 0 \quad (10)
\]

Let \( P^*(i) \) describe the intermediary's optimal network investment strategy for any \( i \in [0,1] \). This defines the equilibrium network path \( P^*(i) \) in the subgame perfect equilibrium. The parameter space can be partitioned into two sets; the set of parameters for which condition (10) is satisfied, denoted by (AO), and the set for which it is not, denoted by (B0). For each set there exists a unique equilibrium pattern of intermediation.

Equilibrium path (A0) arises for parameter values that satisfy condition (10) and thus for which the intermediary's expected profit is increasing in information costs \( i \). Hence, for sufficiently small network set-up costs relative to trade surplus, there is a threshold level of information costs, \( \hat{i} \in [0,1] \), above which the intermediary finds it profitable to invest in an information network spanning the entire market and below which the intermediary is inactive. Moreover, the higher the trade surplus relative to fixed costs, the lower the threshold above which the intermediary is active.

Equilibrium path (B0) arises where expected profit fails to satisfy condition (10), so expected profit is decreasing with information costs. This describes the case where the negative effect of higher \( i \) on network investment cost outweighs the positive effect on revenue from the ability to set a higher commission rate.
Hence, for sufficiently small network set-up costs relative to trade surplus, there is a threshold level of information costs, \( \tilde{\gamma} \in [0, 1] \), below which the intermediary finds it profitable to invest in an information network that covers the entire market. The intermediary’s network investment is constrained by market size, yielding a constrained expected profit \( E(\Pi_I)|_{P=1} \), which is non-monotonic in \( \gamma \), and which yields a threshold level \( \gamma^* < \tilde{\gamma} \), below which the market size constraint is so restrictive that positive profits cannot be attained. Hence, in equilibrium (B0), a trade network is only viable for values of \( \gamma \) that lie between the two thresholds.

Propositions (5) and (6) characterise the two equilibrium patterns.

**Proposition 5** If expected profits are monotonically increasing in \( \gamma \), then equilibrium network size, \( P^* \), expected trade, \( E^*(T) \), and expected welfare, \( E^*(W) \), are:

\[
P^* = \begin{cases} 
0 & \text{if } \gamma \in \left[0, \min \left(\tilde{\gamma}, 1\right)\right] \\
1 & \text{if } \gamma \in \left[\min \left(\tilde{\gamma}, 1\right), 1\right]
\end{cases}
\]

\[
E^*(T) = \begin{cases} 
q(\gamma) & \text{if } \gamma \in \left[0, \min \left(\tilde{\gamma}, 1\right)\right] \\
1 & \text{if } \gamma \in \left[\min \left(\tilde{\gamma}, 1\right), 1\right]
\end{cases}
\]

\[
E^*(W) = \begin{cases} 
q(\gamma)S & \text{if } \gamma \in \left[0, \min \left(\tilde{\gamma}, 1\right)\right] \\
S - 2c(\gamma) - F & \text{if } \gamma \in \left[\min \left(\tilde{\gamma}, 1\right), 1\right]
\end{cases}
\]

where \( \tilde{\gamma} \) is the positive threshold level of information costs that solves \( c(\gamma) + \sqrt{c(\gamma)^2 + [1 - q(\gamma)]SF} = [1 - q(\gamma)]S \), above which \( E(\Pi_I) > 0 \).

**Proof.** Setting expected profit in equation (8) to zero, \( E(\Pi_I) = 0 \) simplifies
to give the following quadratic expression in $P$:

$$[1 - q(i)]SP^2 - 2c(i)P - F = 0 \quad (11)$$

Equation (11) describes the combinations of $i$ and $P$ for which $E(\Pi_I) = 0$. Equation (11) therefore reflects the iso-profit contour in $(i, P)$ space, along which expected profits are zero. Let $\hat{P}(i)$ denote the positive, real root of (11), in terms of $i$ and parameters of the model, where:

$$\hat{P}(i) = \frac{c(i) + \sqrt{c(i)^2 + [1 - q(i)]SF}}{[1 - q(i)]S} > 0 \quad (12)$$

$\hat{P}(i)$ gives a measure of the market size that would, given $i$, generate exactly enough revenue to cover the network set-up cost and variable costs. It can be interpreted as the minimum network size consistent with $E(\Pi_I) > 0$, given $i$. If $\hat{P}(i) \leq 1$, then the revenue generated from the unit measure of traders is sufficient to cover network costs so the intermediary invests in a trade network spanning the entire market. Conversely, for $i$ where $\hat{P}(i) > 1$, the measure of traders is not large enough for a viable network, so $P^* = 0$.

If condition (10) holds for all values of $i > 0$, then there is a unique value of $i, \hat{i}$, that solves $\hat{P}(i) = 1$ at which $E(\Pi_I) = 0$. It follows that $E(\Pi_I) \geq 0$ for $i \geq \hat{i}$ and $E(\Pi_I) < 0$ for $i < \hat{i}$. Hence, $P^* = 1$ for values of $i$ where $\hat{P}(i) \leq 1$ and 0 otherwise.

If $P^* = 0$, then there is no intermediated trade. Expected trade volume
is thus \( q(i) \) direct matches, generating an expected surplus of \( q(i)S \). If \( P^* = 1 \), then the intermediary can match all pairs. It follows that all trade is intermediated and trade volume is 1. The expected welfare is the surplus generated from trade, \( S \), less the network costs incurred by the intermediary. Hence, \( E^*(W) = S - 2c(i) - F \) when the intermediary is active.

**Proposition 6** If expected profits are non-monotonic in \( i \), then equilibrium network size, \( P^* \), expected trade, \( E^*(T) \), and expected welfare, \( E^*(W) \), are:

\[
P^* = \begin{cases} 
0 & \text{if } i \in [0, \min (\bar{i}, 1)] \\
1 & \text{if } i \in [\min (\bar{i}, 1), \min (\bar{i}, 1)] \\
0 & \text{if } i \in [\min (\bar{i}, 1), 1]
\end{cases}
\]

\[
E^*(T) = \begin{cases} 
q(i) & \text{if } i \in [0, \min (\bar{i}, 1)] \\
1 & \text{if } i \in [\min (\bar{i}, 1), \min (\bar{i}, 1)] \\
q(i) & \text{if } i \in [\min (\bar{i}, 1), 1]
\end{cases}
\]

\[
E^*(W) = \begin{cases} 
q(i)S & \text{if } i \in [0, \min (\bar{i}, 1)] \\
S - 2c(i) - F & \text{if } i \in [\min (\bar{i}, 1), \min (\bar{i}, 1)] \\
q(i)S & \text{if } i \in [\min (\bar{i}, 1), 1]
\end{cases}
\]

where \( \bar{i} \) and \( \bar{i} \) are positive roots of \( c(i) + \sqrt{c(i)^2 + [1 - q(i)]SF} = [1 - q(i)]S \), between which \( E(\Pi_I) > 0 \).

**Proof.** If \( c'(i) < -\frac{PS}{2}q'(i) \) for \( i \in [0, \bar{i}] \) and \( c'(i) \geq -\frac{PS}{2}q'(i) \) for \( i \in [\bar{i}, 1] \), given \( P > 0 \), then expected profit is non-monotonic in \( i \) and there are, in general, two positive, real roots of \( \hat{P}(i) = 1 \). Let the two roots be defined as \( \hat{i} \) and \( \bar{i} \), respectively, where \( \bar{i} > \hat{i} > 0 \) and \( \bar{i} \in [\hat{i}, 1] \). It follows that \( E(\Pi_I) \geq 0 \)
for $i \in [i, 7]$, and $E(\Pi_I) < 0$, otherwise. Hence, $P^* = 1$ for $i \in [i, 7]$, and 0 otherwise. $E^*(T)$ and $E^*(W)$ follow directly. ■

**Trade and Welfare** Since any unmatched network members in stage 3 continue to have the opportunity to trade directly in stage 4, expected trade can never be lower with an active intermediary in the market than without. This is formalised in proposition (7).

**Proposition 7** An active intermediary raises expected trade volume unambiguously compared to expected trade when only direct trade is possible.

**Proof.** Let $E(T)$ denote expected trade volume. Investment in a network of size $P$, where $P \in [0, 1]$, generates $P^2$ expected indirect matches in stage 3. A proportion $q(i)$ of the remaining $1 - P^2$ pairs trade directly in stage 4. It follows that:

$$E(T) = q(i) + P^2 [1 - q(i)]$$

$$\geq q(i) = E^{DT}(T)$$

Expected trade volume with an intermediary is thus at least as great as when only direct trade is possible, for any choice of network size $P$. Moreover, expected trade is unambiguously higher when the intermediary is active. ■

The intermediary exploits his monopoly power and sets a commission rate that leave traders as well off (in expected terms) under the intermediation.
contract as through direct trade. Hence, the intermediary's expected profit represents a pure welfare gain. The gain arises from the fact that the intermediary expands the set of possible production technologies for matching, while his exclusive appropriation of these welfare gains stems from his market power from being a monopolist provider\(^5\) of the indirect matching technology. Proposition (8) formalises this discussion.

**Proposition 8** An active intermediary raises expected welfare unambiguously compared to expected welfare when only direct trade is possible.

**Proof.** Let \(E^{DT}(W)\) denote expected welfare arising from direct trade, without an intermediary. This mirrors expected trade, and is given by:

\[
E^{DT}(W) = q(i)S
\]  

(14)

Further, let \(E(W)\) denote expected welfare with a trade network of any size \(P\), where \(P \in [0, 1]\). The total surplus generated from direct and indirect trade is \(P^2S\) and \(q(i) (1 - P^2) S\), respectively. Subtracting the intermediary's network costs gives:

\[
E(W) = (1 - P^2) q(i)S + P^2S - 2c(i)P - F
\]  

(15)

\(^5\)Chapter 2 analyses the competitive interaction of two intermediaries in the two-sided market. For equilibria where both intermediaries are active, traders with access to intermediation services are, on average, strictly better off than those with access to direct trade only.

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Rearranging (15) gives:

\[
E(W) = q(i)S + [1 - q(i)]SP^2 - 2c(i)P - F
\]  

(16)

In equilibrium, \( P^* > 0 \) if \( E(\Pi_I) > 0 \), in which case (8) implies that \([1 - q(i)]S(P^*)^2 \geq 2c(i)P^* - F\). Moreover, for all values of \( i \) where \( E(\Pi_I) < 0 \), \( P^* = 0 \). Hence:

\[
E^*(W) = q(i)S + [1 - q(i)]S(P^*)^2 - 2c(i)P^* - F
\]  

\[\geq q(i)S = E^{DT}(W)\]  

(17)

Equilibrium expected welfare with an intermediary is thus at least as large as expected welfare when only direct trade is possible. Moreover, for levels of information costs where \( P^* > 0 \), expected welfare is unambiguously higher with the intermediary. Proposition (8) follows directly. ■

**Illustrative Examples** To provide further intuition an illustrative example is provided for each of the two equilibrium patterns of intermediation. To add structure to the discussion, let marginal cost of network expansion \( c(i) \) and direct matching probability \( q(i) \) be specified as \( c(i) = i^\alpha \) and \( q(i) = 1 - i^\delta \), respectively, where \( \alpha, \delta \geq 1 \). For these specifications, equilibrium pattern (A0) arises where \( \delta \geq \alpha \), while equilibrium pattern (B0) arises for parameter values where \( \alpha > \delta \). For sufficiently large trader surplus \( S \) relative to network set-
up cost $F$, the lower threshold levels of information cost above which the intermediary is active lie within $i \in [0, 1]$. Consider the following illustrative examples for each case.

**Equilibrium Intermediation Path (A0)** Figure (2) illustrates the equilibrium path of network size with information costs, for which $\delta \geq \alpha$ and thus where condition (10) is satisfied. A map of iso-profit contours is depicted where the lowest corresponds to zero profits, and illustrates $\hat{P}(i)$, the minimum network size that allows the intermediary to break even. Threshold $\hat{i}$ corresponds to $\hat{P}(i) = 1$, below which the intermediary is inactive and above which network size is 1. The higher cost implications of higher prevailing information costs are dominated by the commission effect through condition (10), so the intermediary is active for all $i \in [\hat{i}, 1]$.

The corresponding expected trade pattern, $E(T)$, is illustrated in figure (3). $E^{DT}(T)$ depicts the declining expected trade path that would prevail without an intermediary. Despite the relatively low level of information costs that prevail when the intermediary is inactive, a proportion of trade matches $1 - q(i)$ is lost due to information frictions. As barriers to information flow worsen, an increasing measure of transactions fail to materialise, enabling the intermediary to become active beyond threshold $\hat{i}$. The network enables all trading pairs to match indirectly, raising trade volume to 1, despite the larger

---

$^6$All figures for equilibrium (A) are illustrated for $S = 9$, $q(i) = 1 - i^4$, $c(i) = i^2$ and $F = 0.001$. 

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frictions that a higher $i$ implies. Moreover, Figure (4) illustrates the positive welfare effect of the intermediary’s investment. Since profits from intermediation are monotonically increasing in $i$, the welfare gain from intermediation increases as the barriers to information flow become more severe.

**Equilibrium Intermediation Path (B0)** Consider the iso-profit map\(^7\) in figure (5) which reflects the intermediary’s incentives where $\delta < \alpha$. For relatively low levels of information costs $i$, expected profit is increasing with $i$. For this range of information costs the revenue effect of increasing information costs outweighs the cost effect. The trade-off between the two effects worsens with $i$, however, for any given network size $P > 0$, until eventually the cost

\(^7\)All figures for equilibrium (B) are illustrated for $S = 1.2$, $q(i) = 1 - i^3$, $c(i) = i^6$ and $F = 0.005$. 

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effect outweighs the revenue effect. A trade network is thus unviable when information costs are very low \((i < \bar{i})\), or very high \((i > \bar{i})\).

Figures (6) and (7) illustrate the corresponding expected trade volume and welfare effects. The trade network represents a more efficient information technology than direct matching, thereby improving welfare, but over a limited range of \(i\). The pattern of trade in equilibrium \((B_0)\) indicates that even small changes in information costs may have dramatic implications for the organisation of trade between direct and indirect as a result of pivotal thresholds that trigger network investment or, indeed, network collapse.

The model points to the possibility of a complete reorganisation of trade beyond threshold levels of information costs. The dramatic swings between direct trade and intermediated trade result from the linear network cost spec-
Figure 4: Equilibrium A0: expected welfare path.

ification. Since both direct and intermediated trade is observed in practice, it is important to examine the conditions under which an interior equilibrium exists and how it may be affected by information costs. In the next section, network size is introduced as an argument of the intermediary’s cost function and the interior equilibrium solved analytically under convexity in network-building costs. Note that core propositions (7) and (8) do not rely on any assumptions on costs and $q(i)$, so continue to hold.

1.2 Convex Network-Building Costs

This section allows the intermediary’s costs to depend on network size, $P$, in addition to information costs $i$. In particular, let marginal costs of network
Figure 5: Equilibrium B0: path of network size with information costs.

expansion be denoted by \( c(i, P) \), where:

\[
\begin{align*}
    c(0, \cdot) &= 0 ; c(\cdot, 0) = 0 \\
    c_i(i, P) &> 0 ; c_{ii}(i, P) \geq 0 \\
    c_p(i, P) &> 0 ; c_{pp}(i, P) > 0 \\
    c_{ip}(i, P) &= c_{pi}(i, P) > 0
\end{align*}
\]

(18)

As described in (18), \( c(i, P) \) is monotonically increasing in \( i \), for any given network size \( P \), and monotonically increasing in \( P \), for any given level of information costs. Convexity in network size \( P \) (but not \( i \)) is necessary in order to generate an interior equilibrium. Let \( c(i, P) \) be specified by equation (19), which satisfies the conditions in (18):
$c(i, P) = \gamma i^\alpha P^\beta$, where $\alpha \geq 1, \beta \geq 2$ and $\gamma > 0$ (19)

Parameter $\alpha$ is the elasticity of cost $c(i, P)$ with respect to information costs $i$ and $\beta$ is the elasticity of cost $c(i, P)$ with respect to network size $P$. Coefficient $\gamma$ is a shift factor, which raises (or lowers) network investment cost for given $i$ and $P$. Total network investment cost $C(P) = F + 2\gamma i^\alpha P^{\beta+1}i$ is thus convex in $P$.

Further, let $q(i)$ be described by:

$q(i) = 1 - i^\delta$, where $\delta \geq 1$ (20)

Hence, from proposition (2), the commission rate demanded by the inter-
mediary in equilibrium is $\alpha_i^*(i) = i^\delta$. Parameter $\delta$ thus denotes the elasticity of the equilibrium commission rate with respect to information cost $i$.

Substituting (19) and (20) into equation (8) yields the following expression for expected profits:

$$E(\Pi_t) = [1 - q(i)]SP^2 - 2Pc(i, P) - F$$
$$= Si^\delta P^2 - 2\gamma^\alpha P^{\delta+1} - F$$  \hspace{1cm} (21)

Maximising (21) with respect to $P$ yields equilibrium network size in terms of $\alpha, \beta, \gamma, \delta$ and $S$. Analytically, this can be expressed \(^8\) by:

\(^{8}\) A derivation of (22) is included in Appendix B.
\[ \bar{P} = \left( \frac{S^{\delta - \alpha}}{\gamma(\beta + 1)} \right)^{\frac{1}{\beta - 1}} > 0 \]  

Equilibrium network size is given by (22), provided \( E(II_I) \geq 0 \) and \( \bar{P} \leq 1 \), which shows that the equilibrium pattern of intermediation and trade depend on the relative values of \( \delta \) and \( \alpha \). The convexity of network investment costs gives rise to an interior equilibrium, subject to the constraint imposed by the size of the market and provided set-up costs \( F \) are sufficiently low relative to trade surplus \( S \).

Proposition (9) describes the necessary condition for expected profit in the interior equilibrium to be increasing in \( i \).

**Proposition 9** Unconstrained expected profit is monotonically increasing with the level of information costs if \((\beta + 1)\delta > 2\alpha\).

**Proof.** For proof see Appendix A. ■

Condition \((\beta + 1)\delta > 2\alpha\) implies that as information costs increase, the direct matching route worsens relatively more than the cost of network provision. This gives rise to higher expected profits for the intermediary by relaxing the constraint on the commission fee the intermediary can demand.

The analysis proceeds by distinguishing between four distinct equilibrium patterns of network investment. The parameter space can be split into four ranges, denoted by (A1)-(D1), each corresponding to a different set of incentives for network investment. These are discussed in turn.
(A1) $\delta > \alpha \geq 1$: For this parameter range, the elasticity of the intermediary's optimal commission rate with respect to information costs, $\delta$, exceeds the elasticity of the intermediary's marginal cost of network expansion with respect to information costs, given by $\alpha$. Hence, as information costs worsen, the increase in the commission rate the intermediary can command exceeds the increase in networking cost $c(i, P)$, making a network expansion profitable. For this parameter range, optimal network size is increasing with $i$.

(B1) $\delta = \alpha \geq 1$: If the elasticities of the commission rate and $c(i, P)$ are exactly equal, then the effects of changing information cost $i$ on the intermediary's cost and expected revenue exactly offset each other. Hence the intermediary's optimal choice of network size is unchanging with $i$. Note, however, that while the intermediary's investment decision is unaffected at the margin, it follows from proposition (9) that unconstrained profits are increasing with $i$.

(C1) $\frac{2\alpha}{\beta + 1} < \delta < \alpha$: If the elasticity of marginal networking cost $c(i, P)$ exceeds the elasticity of the commission rate with respect to $i$, then it is optimal for the intermediary to contract network size as information costs worsen. Despite the contracting network size, unconstrained expected profits are increasing with $i$. Recall that $\beta$ is the elasticity of $c(i, P)$ with respect to network size $P$. Since $(\beta + 1)\delta > 2\alpha$ holds, then within this range of parameter values, cost $c(i, P)$ is sufficiently elastic

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with respect to network size $P$, so as to offset the effects of information cost $i$ on $c(i,P)$, thereby raising equilibrium profit overall.

**(D1)** $\delta \leq \frac{2\alpha}{\beta+1}$: For this range of elasticities, the commission rate is less responsive to information cost $i$ than is networking cost $c(i,P)$ and moreover, the responsiveness of $c(i,P)$ with respect to $P$ is not sufficient so as to allow a contraction to offset the negative effect on expected profit. Hence, equilibrium (unconstrained) expected profit is decreasing with $i$.

The four equilibrium patterns of intermediation, (A1)-(D1), shed light on how information frictions affect direct and indirect matching technologies. The model thus suggests that we can learn about the relative elasticities of the costs of network provision and the probability of direct matching from an empirical examination of the impact of changing information costs on intermediation.

The rest of the section formally characterises the interior equilibrium path of network size, expected trade and expected welfare for parameter ranges, (A1)-(D1). Further intuition is provided through the discussion of illustrative examples.

### 1.2.1 Equilibrium Pattern of Intermediation (A1)

**Proposition 10** If $\delta > \alpha \geq 1$, then the interior equilibrium is characterised by the following:

(a) Network size is increasing in the level of information costs $i$ and trade surplus $S$ and decreasing in cost parameters $\beta$ and $\gamma$.  

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(b) The proportion of indirect trade to total trade is increasing in the level of information costs \( i \). The relationship between total expected trade and information costs is non-monotonic.

(c) The contribution of intermediation to social welfare is positive and increasing in the level of information costs \( i \).

**Proof.** Formally, network size, \( P^* \), expected trade volume, \( E^*(T) \), and expected welfare, \( E^*(W) \), are described by:

\[
P^* = \begin{cases} 
0 & \text{if } 0 < i \leq \min \{ \hat{i}, 1 \} \\
\left[ \frac{S}{7(\beta+1)} \right]^{\frac{1}{2}} & \text{if } \min \{ \hat{i}, 1 \} \leq i \leq \min \{ \hat{\hat{i}}, 1 \} \\
1 & \text{if } \min \{ \hat{\hat{i}}, 1 \} \leq i \leq 1
\end{cases}
\]

\[
E^*(T) = \begin{cases} 
1 - i^\delta & \text{if } 0 \leq i \leq \min \{ \hat{i}, 1 \} \\
1 - i^\delta + \left[ \frac{S}{7(\beta+1)} \right]^{\frac{1}{2}} i^\gamma A^{\gamma+1} - 2\gamma A^{\gamma+1} i^\alpha - F & \text{if } \min \{ \hat{i}, 1 \} \leq i \leq \min \{ \hat{\hat{i}}, 1 \} \\
1 & \text{if } \min \{ \hat{\hat{i}}, 1 \} \leq i \leq 1
\end{cases}
\]

\[
E^*(W) = \begin{cases} 
(1 - i^\delta) S & \text{if } 0 \leq i \leq \min \{ \hat{i}, 1 \} \\
(1 - i^\delta) S + (i^\delta S A^{\frac{2}{\beta-1}} - 2\gamma A^{\gamma+1} i^\alpha - F) & \text{if } \min \{ \hat{i}, 1 \} \leq i \leq \min \{ \hat{\hat{i}}, 1 \} \\
S - 2\gamma i^\alpha - F & \text{if } \min \{ \hat{\hat{i}}, 1 \} \leq i \leq 1
\end{cases}
\]

where \( A = \frac{S}{\gamma(\beta+1)} \hat{i} = \left[ \gamma^{\frac{2}{\beta-1}} \left( \frac{F}{\beta-1} \right) \left( \frac{\gamma+1}{\gamma} \right)^{\frac{\gamma+1}{\beta-1}} \right]^{\gamma(\beta+1)-2\alpha} > 0 \)

and \( \hat{i} = \left[ \frac{\gamma(\beta+1)}{S} \right]^{\frac{1}{\beta-\alpha}} > 0 \)

For a full proof of the above see Appendix B.
It follows from the interior equilibrium that:

\[ \frac{\partial P^*}{\partial \gamma} = \frac{(\delta - \alpha) S}{\gamma(\beta - 1)(\beta + 1)} \left( \frac{S}{\gamma(\beta + 1)} \right)^{\frac{\delta - \beta}{\beta - 1}} \frac{\beta + 1 + (\delta - \alpha)}{\beta - 1} > 0 \text{ when } \delta > \alpha \]

\[ \frac{\partial P^*}{\partial S} > 0 ; \quad \frac{\partial P^*}{\partial \gamma} < 0 ; \quad \frac{\partial P^*}{\partial \beta} < 0 \]

Moreover, \( E^*(T) \) can be decomposed into direct and indirect equilibrium trade.

Let direct\(^9\) and indirect trade in equilibrium be denoted by \( E^*_D(T) \), and \( E^*_I(T) \), respectively, where:

\[ E^*_D(T) = \left(1 - i^\delta\right) \left[ 1 - \left(\frac{S^\delta - \alpha}{\gamma(\beta + 1)}\right)^{\frac{2}{\beta - 1}} \right] \]  \hspace{1cm} (24)

\[ E^*_I(T) = \left(\frac{S^\delta - \alpha}{\gamma(\beta + 1)}\right)^{\frac{2}{\beta - 1}} \]  \hspace{1cm} (25)

Let the equilibrium direct and indirect trade shares be denoted by \( s_D \) and \( s_I \), respectively, where:

\[ s_D = \frac{E^*_D(T)}{E^*(T)} = \frac{q(i) \left[ 1 - (P^*)^2 \right]}{q(i) \left[ 1 - (P^*)^2 \right] + (P^*)^2} \]  \hspace{1cm} (26)

\[ s_I = \frac{E^*_I(T)}{E^*(T)} = \frac{(P^*)^2}{q(i) \left[ 1 - (P^*)^2 \right] + (P^*)^2} \]  \hspace{1cm} (27)

It is straightforward to show that \( \frac{\partial s_D}{\partial \gamma} < 0 \) and \( \frac{\partial s_I}{\partial \gamma} > 0 \). Higher information

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\(^9\) \( E^*_D(T) \) is not to be confused with \( E^{DT}(T) \). \( E^*_D(T) \) represents the equilibrium measure of direct trade matches, as a component of equilibrium total trade \( E^*(T) \). In contrast, \( E^{DT}(T) \) represents the measure of equilibrium total trade if there were no intermediary in the market.
costs correspond to both a larger network size and a lower probability of direct matching. Both effects drive the result that the proportion of indirect trade to total trade is increasing in the level of information costs. Moreover, for \( i \in [\hat{i}, 1] \), where \( P^* = 1 \), all trade is intermediated, so \( s_D = 0 \) and \( s_I = 1 \).

Recall that \( E^{DT}(W) \) is the expected welfare that would prevail if there were no intermediary in the market. It follows from (17) that \( E^*(W) - E^{DT}(W) = E^*(\Pi) \) is a measure of the intermediary's contribution to social welfare. Moreover, since \( \delta > \alpha \geq 1 \), it follows that \( \delta > \frac{2}{(\beta + 1)} \alpha \). Hence, from proposition (9), \( E^*(\Pi_I) \) is increasing in \( i \) in the interior equilibrium, so the contribution of intermediation to social welfare is both positive and increasing in the level of information costs, where the intermediary is active.

Illustrative Example Figure (8) - (10) illustrate equilibrium network size, expected trade and expected welfare, respectively, for \( i \in [0, 1] \), for parameter values \( \beta = 2 \), \( \gamma = 1 \), \( \delta = 4 \), \( \alpha = 2 \), \( F = 0.001 \) and \( S = \{2.5, 3, 4\} \), which satisfy \( \delta > \alpha \geq 1 \) and the convexity assumption \( \beta \geq 2 \).

Figure (8) illustrates the positive relationship between optimal network size and prevailing information costs where the elasticity of the intermediary's commission exceeds the elasticity of cost \( c(i, P) \) with respect to \( i \). The fixed set-up cost \( F \) implies that information costs must be above a threshold level for intermediation to be profitable in the two-sided market. The optimal network path is illustrated for (a) \( S = \gamma(\beta + 1) \), (b) \( S > \gamma(\beta + 1) \) and (c) \( S < \gamma(\beta + 1) \),
verifying that network size and threshold level \( \hat{S} \) are increasing in \( S \) relative to cost parameters \( \beta \) and \( \gamma \).

Figure (9) illustrates the effect of intermediation on total expected trade between the two sides of the market. The intermediary’s network investment provides access to a more efficient matching technology than direct trade, thereby raising total trade relative to access to direct matching only. The relationship between expected trade volume and information cost \( i \) is non-monotonic due to the conflicting effects of information cost \( i \) on the constituent parts of expected trade. For this range of parameters, the intermediary finds it optimal to increase network size with \( i \), thereby increasing the expected measure of intermediated trade matches. The impact on direct trade is twofold. First, higher information cost worsens the probability of a direct match, and second, the expansion in network size results in a smaller expected pool of unmatched traders in stage 4. The net effect is ambiguous, giving rise to a non-monotonic relationship between \( i \) and total expected trade \( E(T) \) in equilibrium.

Figure (10) shows that intermediation is welfare improving and that it more so when information cost is higher.

1.2.2 Equilibrium Pattern of Intermediation (B1)

**Proposition 11** If \( \delta = \alpha \geq 1 \) and \( S < \gamma(\beta + 1) \), then there exists an interior equilibrium characterised by the following:

(a) Network size is independent of the level of information costs \( i \), increasing
in trade surplus $S$ and decreasing in cost parameters $\beta$ and $\gamma$.

(b) The measure of intermediated transactions is independent of the level of information costs but represents an increasing proportion of total trade, which is unambiguously decreasing in information costs $i$.

(c) The contribution of intermediation to social welfare is positive and increasing in the level of information costs $i$.

Proof. If $\delta = \alpha \geq 1$ and $S \leq \gamma(\beta + 1)$, then equilibrium network size, $P^*$, expected trade volume, $E^*(T)$, and expected welfare, $E^*(W)$, are described by:

$$P^* = \begin{cases} 
0 & \text{if } 0 \leq i \leq \min \{i, 1\} \\
\left[ \frac{\delta}{\gamma(\beta + 1)} \right]^{\frac{1}{\beta - 1}} & \text{if } \min \{i, 1\} \leq i \leq 1
\end{cases}$$
Figure 9: Equilibrium A1: expected trade path.

\[ E^*(T) = \begin{cases} 
1 - i^\delta & \text{if } 0 \leq i \leq \min \left\{ \tilde{i}, 1 \right\} \\
1 - i^\delta \left[ 1 - B^{\frac{2}{\beta - 1}} \right] & \text{if } \min \left\{ \tilde{i}, 1 \right\} \leq i \leq 1 
\end{cases} \]

\[ E^*(W) = \begin{cases} 
(1 - i^\delta) S & \text{if } 0 \leq i \leq \min \left\{ \tilde{i}, 1 \right\} \\
(1 - i^\delta) S + i^\delta SB^{\frac{2}{\beta - 1}} - 2i^\delta \gamma B^{\frac{\delta + 1}{\beta - 1}} - F & \text{if } \min \left\{ \tilde{i}, 1 \right\} \leq i \leq 1 
\end{cases} \]

where \( B = \frac{S}{\gamma(\beta + 1)} \) and \( \tilde{i} = \left[ \gamma^{\frac{2}{\beta - 1}} \left( \frac{F}{\beta - 1} \right) \left( \frac{\beta + 1}{\beta - 1} \right) \right]^{\frac{1}{\beta - 1}} > 0 \).

For a full proof of the above see Appendix C.

If \( \delta = \alpha \geq 1 \) and \( S > \gamma(\beta + 1) \), then the unit measure of market size poses a binding constraint. The constrained optimum network size is thus \( P^* = 1 \), provided \( E(\Pi_I) \geq 0 \). The equilibrium is analogous to that of proposition (5), with cost given by \( c(i, 1) \).

Whether constrained or unconstrained, the equilibrium network size is constant over the range of values of \( i \) where \( E(\Pi_I) \geq 0 \). It follows that the
measure of intermediated trade is also constant. Where \( P^* < 1 \), Equations (24) and (25) simplify to:

\[
E_D^*(T) = \left(1 - i^d\right) \left[1 - \left(\frac{S^*}{\gamma(\beta + 1)}\right)^{\frac{2}{\beta - 1}}\right]
\]

(28)

\[
E_I^*(T) = \left(\frac{S}{\gamma(\beta + 1)}\right)^{\frac{2}{\beta - 1}}
\]

(29)

It follows immediately from (28) and (29) that indirect trade is constant and direct trade decreases with \( i \) as the probability of successful matching declines. Hence, \( \frac{\partial P^*}{\partial i} < 0 \) and \( \frac{\partial s_I}{\partial i} > 0 \). At the limit where \( P^* = 1 \), all trade is intermediated, so \( E_D^*(T) = s_D = 0 \) and \( E_I^*(T) = s_I = 1 \).

Furthermore, since \( \delta = \alpha \geq 1 \), it follows that \( \delta > \frac{2}{(\beta + 1)} \alpha \). Hence, from proposition (9), \( E^*(\Pi_I) \) is increasing in \( i \) in the interior equilibrium, so the
contribution of intermediation to social welfare is both positive and increasing in the level of information costs, where the intermediary is active.

The constrained profit path, where \( P^* = 1 \) is lower than if the intermediary could expand the trade network further, but increasing in \( i \) nonetheless, since

\[
E(\Pi_I)|_{P=1} = i^\delta (S - 2\gamma) - F. 
\]

Illustrative Example  Figures (11) and (12) illustrate\(^{10}\) the equilibrium network size where \( \delta = \alpha \geq 1 \). Figure (11) shows that optimal network size is unaffected by the level of information cost \( i \). The intermediary's optimal investment is again increasing in \( S \) relative to cost parameters \( \beta \) and \( \gamma \). Figure (12) shows that expected trade volume decreases monotonically with \( i \), but lies above the expected trade path that prevails with access to direct matching only.

1.2.3 Equilibrium Pattern of Intermediation (C1)

Proposition 12  If \( \frac{2}{\beta+1} \alpha < \delta < \alpha \), then the interior equilibrium is characterised by the following:

(a) Network size is decreasing in the level of information costs \( i \) and cost parameters \( \beta \) and \( \gamma \) and increasing in trade surplus \( S \).

(b) Indirect trade is decreasing and direct trade increasing in information costs \( i \). Total expected trade is unambiguously decreasing in information costs \( i \).

(c) The contribution of intermediation to social welfare is positive and increas-

\(^{10}\)Figures (11) and (12) are illustrated for \( \beta = 2 \), \( \gamma = 1 \), \( \alpha = \delta = 3 \), \( F = 0.001 \) and \( S = \{2,3\} \).
Figure 11: Equilibrium B1: path of network size with information costs.

\[
\begin{align*}
S &= \gamma (P + 1) \\
S/3 &= 0.75 \\
0.5 &= 0.25
\end{align*}
\]

\[
\begin{align*}
(P, i) &= (a, S = \gamma (P + 1)) \\
&= (b, S < \gamma (P + 1))
\end{align*}
\]

Proof. If \( \frac{2}{\beta + 1} \alpha < \delta < \alpha \), then equilibrium network size, \( P^* \), expected trade volume, \( E^*(T) \), and expected welfare, \( E^*(W) \), are described by:

\[
P^* = \left\{ \begin{array}{ll}
0 & \text{if } 0 \leq i \leq \min \{ \hat{i}, 1 \} \\
1 & \text{if } \min \{ \hat{i}, 1 \} \leq i \leq \min \{ \hat{\hat{i}}, 1 \} \\
\left[ \frac{S}{\gamma (\beta + 1) \alpha} \right]^{\frac{1}{\beta - 1}} & \text{if } \min \{ \hat{\hat{i}}, 1 \} \leq i \leq 1
\end{array} \right.
\]

\[
E^*(T) = \left\{ \begin{array}{ll}
1 - i^\delta & \text{if } 0 \leq i \leq \min \{ \hat{i}, 1 \} \\
1 & \text{if } \min \{ \hat{i}, 1 \} \leq i \leq \min \{ \hat{\hat{i}}, 1 \} \\
1 - i^\delta + \left[ \frac{S}{\gamma (\beta + 1) \alpha} \right]^{\frac{1}{\beta - 1}} \frac{\beta (\beta + 1) - 2 \alpha}{\beta - 1} & \text{if } \min \{ \hat{\hat{i}}, 1 \} \leq i \leq 1
\end{array} \right.
\]

\[
E^*(W) = \left\{ \begin{array}{ll}
(1 - i^\delta) S & \text{if } 0 \leq i \leq \min \{ \hat{i}, 1 \} \\
S - 2\gamma i^\alpha - F & \text{if } \min \{ \hat{i}, 1 \} \leq i \leq \min \{ \hat{\hat{i}}, 1 \} \\
(1 - i^\delta) S + i^\delta SG^{\beta - 1} - 2\gamma i^\alpha - F & \text{if } \min \{ \hat{\hat{i}}, 1 \} \leq i \leq 1
\end{array} \right.
\]

ing in the level of information costs \( i \).

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where $G = \frac{S}{\gamma(\beta+1)^{\alpha-\delta}}$ and $\hat{i}$ is the smaller positive root of

$$E(\Pi_I)|_{P=1} = Si^\delta - 2\gamma i^\alpha - F = 0 \text{ and } \hat{i} = \left[ \frac{S}{\gamma(\beta+1)} \right]^{\frac{1}{\alpha-\delta}} > 0.$$  

The proof of the above is in Appendix D.

It follows from the interior equilibrium that:

$$\frac{\partial P^*}{\partial i} = - \frac{(\alpha - \delta) S}{\gamma(\beta - 1)(\beta + 1)} \left( \frac{S}{\gamma(\beta + 1)} \right)^{\frac{\beta-\delta}{\gamma - 1}} \frac{\beta+1+(\beta-\alpha)}{\beta-1} < 0 \text{ when } \alpha > \delta$$

$$\frac{\partial P^*}{\partial S} > 0; \quad \frac{\partial P^*}{\partial \gamma} < 0; \quad \frac{\partial P^*}{\partial \beta} < 0$$  \hspace{1cm} (30)

\text{11} This is the threshold above which the intermediary can attain a positive profit. It is computed based on the constrained profit equation, where $P = 1$. If $F$ is sufficiently high, however, market size is not a binding constraint in the region where $E(\Pi_I) = 0$. so the threshold which applies is: $\hat{i}_{P=1} = \left[ \left( \frac{1}{2} \right) \frac{S^{\frac{\beta-1}{\beta+1}}}{\left( \frac{\beta+1}{\beta-1} \right)^{\frac{\beta-1}{\beta+1}}} \right]^{\frac{\gamma-1}{(\beta+1)}} > 0.$
Hence, optimal network size is decreasing in \(i\) and cost parameters, but increasing in \(S\). Moreover, \(E^*_D(T)\), and \(E^*_I(T)\), are given by:

\[
E^*_D(T) = \left(1 - i^\delta\right) \left[1 - \left(\frac{S}{\gamma(\beta + 1)i^{\alpha - \delta}}\right)^{\frac{2}{\beta - 1}}\right]
\]

(31)

\[
E^*_I(T) = \left(\frac{S}{\gamma(\beta + 1)i^{\alpha - \delta}}\right)^{\frac{2}{\beta - 1}}
\]

(32)

The decline in network size with information cost \(i\) is mirrored by \(E^*_I(T)\) when \(\alpha > \delta\). The decline in intermediated matches with \(i\) increases the measure of traders seeking a direct match in stage 4. At the same time, a higher \(i\) implies a lower probability of successful direct matching.

Furthermore, since \(\delta > \frac{2}{(\beta + 1)}\alpha\), it follows from proposition (9) that \(E^*(\Pi_I)\) is increasing in information cost \(i\) in the interior equilibrium. Hence, the contribution of intermediation to social welfare is both positive and increasing in the level of information costs, where the intermediary is active.

1.2.4 Equilibrium Pattern of Intermediation (D1)

**Proposition 13** If \(\delta \leq \frac{2}{\beta + 1}\alpha\), then the interior equilibrium is characterised by the following:

(a) Network size is decreasing in the level of information costs \(i\) and cost parameters \(\beta\) and \(\gamma\) and increasing in trade surplus \(S\).

(b) Indirect trade is decreasing and direct trade increasing in information costs.
i. Total expected trade is unambiguously decreasing in information costs i.

(c) The contribution of intermediation to social welfare is positive but decreasing in information costs i.

Proof. If $\delta \leq \frac{2}{\beta+1} \alpha$, then equilibrium network size, $P^*$, expected trade volume, $E^*(T)$, and expected welfare, $E^*(W)$, are described by:

$$P^* = \begin{cases} 
0 & \text{if } 0 \leq i < \min \{\hat{t}_{i|P=1}, 1\} \\
1 & \text{if } \min \{\hat{t}_{i|P=1}, 1\} \leq i < \min \{\hat{t}, 1\} \\
\left[\frac{8}{\gamma(\beta+1)^{n-1}}\right]^{\frac{1}{\beta-1}} & \text{if } \min \{\hat{t}, 1\} \leq i < \min \{\hat{t}_{i|P=P}, 1\} \\
0 & \text{if } \min \{\hat{t}_{i|P=P}, 1\} \leq i \leq 1 
\end{cases}$$

$$E^*(T) = \begin{cases} 
1 - t^\delta & \text{if } 0 \leq i < \min \{\hat{t}_{i|P=1}, 1\} \\
1 & \text{if } \min \{\hat{t}_{i|P=1}, 1\} \leq i < \min \{\hat{t}, 1\} \\
1 - t^\delta + \left[\frac{8}{\gamma(\beta+1)}\right]^{\frac{2}{\beta-1}} i^{\frac{\delta(\beta+1)-2\alpha}{\beta-1}} & \text{if } \min \{\hat{t}, 1\} \leq i < \min \{\hat{t}_{i|P=P}, 1\} \\
1 - t^\delta & \text{if } \min \{\hat{t}_{i|P=P}, 1\} \leq i \leq 1 
\end{cases}$$

$$E^*(W) = \begin{cases} 
(1 - t^\delta) S & \text{if } 0 \leq i < \min \{\hat{t}_{i|P=1}, 1\} \\
S - 2\gamma t^\alpha - F & \text{if } \min \{\hat{t}_{i|P=1}, 1\} \leq i < \min \{\hat{t}, 1\} \\
(1 - t^\delta) S + t^\delta SA^{\frac{2}{\beta-1}} - 2\gamma A^{\frac{\beta+1}{\beta-1}} t^\alpha - F & \text{if } \min \{\hat{t}, 1\} \leq i < \min \{\hat{t}_{i|P=P}, 1\} \\
(1 - t^\delta) S & \text{if } \min \{\hat{t}_{i|P=P}, 1\} \leq i \leq 1 
\end{cases}$$

where $A = \frac{S^{\delta-a}}{\gamma(\beta+1)}$, $\hat{t} = \left[\frac{8}{\gamma(\beta+1)}\right]^{\frac{1}{\beta-1}} > 0$,

$\hat{t}_{i|P=1}$ is the smaller positive root of $E(\Pi_{i})_{|P=1} = S t^\delta - 2\gamma t^\alpha - F = 0$

and $\hat{t}_{i|P=P} = \left[\left(\frac{1}{\gamma}\right)^{\frac{2}{\beta-1}} \left(\frac{S-1}{F}\right) \left(\frac{8}{\gamma(\beta+1)}\right)^{\frac{\beta+1}{\beta-1}}\right]^{\frac{1}{2\gamma-2\delta(\beta+1)}} > 0$.

For a proof of the above see Appendix E. The trade effects follow from the
proof of Proposition (13). Expected profit is unconstrained in the interior equilibrium. Since \( \delta \leq \frac{2^\frac{2}{\beta+1}}{\alpha} \) then it follows from proposition (9) that expected profit and thus the contribution of intermediation to social welfare is decreasing in the level of information costs \( i \).

**Illustrative Example** Figure (13) illustrates\(^{12}\) the pattern of network investment where \( \delta \leq \frac{2}{\beta+1} \alpha \). For this range of elasticities, the commission rate is less responsive to information cost \( i \) than is networking cost \( c(i,P) \), giving rise to a negative relationship between network size and information costs along the interior path. Moreover, as illustrated in figure (14), unconstrained expected profit, denoted by \( E^U(\Pi_I) \) rises without limit as \( i \to 0 \), which implies that in the absence of a binding market size constraint, the intermediary finds it profitable to invest in an ever-increasing network size as information costs tend to zero. Thus below threshold \( \hat{i} \), equilibrium network size is constrained by the size of the market. For interval \( i \in [0,\hat{i}] \) the intermediary's expected profits follow the constrained path, denoted by \( E^C(\Pi_I) |_{P=1} \) in figure (14). While unconstrained expected profit is increasing, constrained expected profit is declining as information costs tend to zero, rendering the network unviable below some threshold level \( \hat{i} \).

\(^{12}\)Illustrated for parameter values \( \alpha = 6, \delta = 3, \beta = 2, \gamma = 1, F = 0.1, \) and \( S = 2 \).
1.3 Conclusion

This chapter presents a pairwise matching model with two-sided information asymmetry between trade partners, where an intermediary has the opportunity to invest in a network of contacts and facilitate trade matching for a success fee. The framework innovates by examining the role of information costs on incentives for trade intermediation, thereby endogenising the pattern of direct and indirect trade.

The framework delivers four key results. First, intermediation unambiguously raises expected trade volume and social welfare by expanding the set of matching technologies available to traders. Second, convexity in network-building costs is necessary for both direct and indirect trade to arise in equilibrium; otherwise, the level of information costs determines whether all trade
Figure 14: Equilibrium D1: constrained and unconstrained profits.

is routed through the intermediary or takes place directly.

Third, under assumptions of convexity in the intermediary’s technology, optimal network size and hence the equilibrium pattern of trade is shown to depend on the level of information costs as well as the relative effectiveness of direct and indirect matching technologies with changing information costs. In particular, if the probability of direct matching is more responsive to changing information costs than is the cost of network expansion, then indirect trade offers a relatively more attractive matching technology than direct trade as information costs rise. Hence, the proportion of indirect trade to total trade is increasing in the level of information frictions. Conversely, if networking costs are more responsive than the probability of a direct match, then the intermediary has an incentive to contract network size with the opposite trade
implications. The model thus suggests that we can learn about the relative elasticities of direct and indirect matching technologies from an empirical examination of the impact of changing information costs on intermediation.

Finally, the model sheds light on the relationship between information frictions and aggregate trade volume, which may be non-monotonic as a result of conflicting effects of information costs on the incentives for direct and indirect trade. Higher information costs worsen direct matching prospects but can, at the same time, provide an incentive for network-building and thus indirect trade through a trade network.
References


Appendix A. Proof of Proposition 9.

Differentiating (21) partially with respect to \( i \) yields:

\[
\frac{\partial E(\Pi_f)}{\partial i} = -P \left[ 2c_i (i, P) + PSq'(i) \right]
\]  

(33)

It follows directly that expected profits are increasing with \( i \), if:

\[
c_i (i, P) < -\frac{PS}{2} q'(i)
\]

(34)

Substituting for \( c_i (i, P) \) and \( q'(i) \) simplifies the condition to:

\[
p^{\beta + 1} < \frac{S\delta i^{-\alpha}}{2\alpha\gamma}
\]

(35)

Substituting the expression for (interior) equilibrium network size, \( P^* = \left[ \frac{S\delta - \alpha}{\gamma(\beta + 1)} \right]^{\frac{1}{\beta + 1}}, \)

and rearranging, yields the necessary and sufficient condition for unconstrained equilibrium profits, \( E^*(\Pi_f) \), to be increasing in \( i \):

\[ (\beta + 1)\delta > 2\alpha \]

(36)

Appendix B. Proof of Proposition 10.

Maximising (21) with respect to \( P \) yields the first order condition:
\[ \frac{\partial E(\Pi_f)}{\partial P} = 2P \left[ S_i^\delta - \gamma (\beta + 1) P^{\beta - 1} i^{\alpha} \right] = 0 \] (37)

Solving yields the interior profit-maximising network size, \( \tilde{P} \), where:

\[ \tilde{P} = \left[ \frac{S_i^\delta - \gamma (\beta + 1) P^{\beta - 1} i^{\alpha}}{\gamma (\beta + 1)} \right]^{-\frac{1}{\beta - 1}} > 0 \] (38)

The second order condition is found to be:

\[ \frac{\partial^2 E(\Pi_f)}{\partial P^2} = 2 \left[ S_i^\delta - \gamma \beta (\beta + 1) P^{\beta - 1} i^{\alpha} \right] \] (39)

The second order condition is negative provided \( P > \left[ \frac{S_i^\delta - \gamma (\beta + 1)}{\gamma \beta (\beta + 1)} \right]^{-\frac{1}{\beta - 1}} \). Since \( \beta \geq 2, \tilde{P} > \left[ \frac{S_i^\delta - \gamma (\beta + 1)}{\gamma \beta (\beta + 1)} \right]^{-\frac{1}{\beta - 1}} \) and so corresponds to an interior maximum.

The intermediary sets \( P = \tilde{P} \) provided \( E(\Pi_f) \geq 0 \) and \( \tilde{P} \leq 1 \). Let \( \hat{i} \) denote the threshold level of information costs at which \( E(\Pi_f)_{|_{P=\tilde{P}}} = 0 \). Since \( \delta > \alpha \geq 1 \), it follows that \( (\beta + 1) \delta > 2\alpha \), so, from proposition (9), \( E(\Pi_f) \) is increasing in \( \hat{i} \) in the interior equilibrium. Hence, \( E(\Pi_f) \geq 0 \) when \( \hat{i} \geq \hat{i} \).

Solving \( E(\Pi_f)_{|_{P=\tilde{P}}} = 0 \) for \( \hat{i} \) yields:

\[ \hat{i} = \left[ \gamma \frac{2^2}{(\beta - 1)} \left( \frac{F}{\beta + 1} \right) \left( \frac{\beta + 1}{S} \right)^{\frac{\beta + 1}{\beta - 1}} \right]^{\frac{1}{\gamma (\beta + 1) - 2\alpha}} \] (40)
Equilibrium network size is thus $P^* = 0$ for $i \in \left[0, \min \left\{ \hat{i}, 1 \right\} \right]$. Furthermore, $\tilde{P}$ is increasing in $i$ since $\delta > \alpha \geq 1$, but network size is constrained by market size.

Let $\tilde{i}$ denote the threshold level of information costs, at which $\tilde{P} = 1$. Solving $\tilde{P} = 1$ for $i$ yields:

$$\tilde{i} = \frac{\gamma (\beta + 1)}{S} \frac{1}{1 - \alpha}$$

(41)

Hence, equilibrium network size is $P^* = 1$ for $i \in \left[ \min \left\{ \hat{i}, 1 \right\}, 1 \right]$. For values $i \in \left[ \min \left\{ \hat{i}, 1 \right\}, \min \left\{ \hat{i}, 1 \right\} \right]$, where $E(U_i) > 0$ and $\tilde{P} \leq 1$, network size follows the interior path $P^* = \tilde{P} = \left[ \frac{S^{\hat{i} - \alpha}}{\gamma (\beta + 1)} \right]^{\frac{1}{\beta - 1}}$. These results are summarised by:

$$P^* = \begin{cases} 0 & \text{if } 0 \leq i \leq \min \left\{ \hat{i}, 1 \right\} \\ \left[ \frac{S^{\hat{i} - \alpha}}{\gamma (\beta + 1)} \right]^{\frac{1}{\beta - 1}} & \text{if } \min \left\{ \hat{i}, 1 \right\} \leq i \leq \min \left\{ \hat{i}, 1 \right\} \\ 1 & \text{if } \min \left\{ \hat{i}, 1 \right\} \leq i \leq 1 \end{cases}$$

where $\hat{i} = \left[ \frac{\gamma \beta - 1}{\gamma \beta - 1} \left( \frac{S^{\hat{i} - \alpha}}{\gamma (\beta + 1)} \right) \right]^{\frac{1}{\beta - 1}} > 0$ and $\tilde{i} = \left[ \frac{\gamma \beta - 1}{\gamma \beta - 1} \right]^{\frac{1}{\beta - 1}} > 0$.

If $0 \leq i \leq \min \left\{ \hat{i}, 1 \right\}$, then the intermediary does not invest in a trade network and all trade takes place directly. The expected trade volume is thus $q(i) = 1 - i^\delta$. If $\min \left\{ \hat{i}, 1 \right\} \leq i \leq 1$, then the intermediary's network spans the entire market so all transactions are intermediated and trade volume is 1.

For values of $i$, $\min \left\{ \hat{i}, 1 \right\} \leq i \leq \min \left\{ \hat{i}, 1 \right\}$, both direct and indirect trade are observed in equilibrium. Substituting $P^*$ into equation (13) yields the equilibrium expected (total) trade path over this range of information costs.
These results are summarised by:

\[
E^*(T) = \begin{cases} 
1 - i^\delta & \text{if } 0 \leq i \leq \min \left\{ \tilde{i}, 1 \right\} \\
1 - i^\delta + \left[ \frac{S}{\gamma(\beta + 1)} \right]^{\frac{1}{\beta - 1}} \frac{\delta(\beta + 1) - 2\alpha}{\beta - 1} i & \text{if } \min \left\{ \tilde{i}, 1 \right\} \leq i \leq \min \left\{ \tilde{i}, 1 \right\} \\
1 & \text{if } \min \left\{ \tilde{i}, 1 \right\} \leq i \leq 1 
\end{cases}
\]

where \( \tilde{i} = \left[ \gamma^{\frac{2}{\beta - 1}} \left( \frac{E}{\beta - 1} \right) \left( \frac{\beta + 1}{S} \right)^{\frac{\beta + 1}{\beta - 1}} \right] \gamma(\beta + 1)^{-\frac{2\alpha}{\alpha}} > 0 \) and \( \tilde{i} = \left[ \frac{\gamma(\beta + 1)}{S} \right]^{\frac{1}{1 - \alpha}} > 0 \).

Finally, the piece-wise function \( E^*(W) \) follows directly from substitution of \( P^* = 0, \left[ \frac{S^{\delta - \alpha}}{\gamma(\beta + 1)} \right]^{\frac{1}{\beta - 1}} \) and 1, respectively, into equation (17). This yields:

\[
E^*(W) = \begin{cases} 
(1 - i^\delta) S & \text{if } 0 \leq i \leq \min \left\{ \tilde{i}, 1 \right\} \\
(1 - i^\delta) S + i^\delta SA^{\frac{1}{\beta - 1}} - 2\gamma A^{\frac{\beta + 1}{\beta - 1}} i^\alpha - F & \text{if } \min \left\{ \tilde{i}, 1 \right\} \leq i \leq \min \left\{ \tilde{i}, 1 \right\} \\
S - 2\gamma i^\alpha - F & \text{if } \min \left\{ \tilde{i}, 1 \right\} \leq i \leq 1 
\end{cases}
\]

where \( \tilde{i} = \left[ \gamma^{\frac{2}{\beta - 1}} \left( \frac{E}{\beta - 1} \right) \left( \frac{\beta + 1}{S} \right)^{\frac{\beta + 1}{\beta - 1}} \right] \gamma(\beta + 1)^{-\frac{2\alpha}{\alpha}} > 0 \) and \( \tilde{i} = \left[ \frac{\gamma(\beta + 1)}{S} \right]^{\frac{1}{1 - \alpha}} > 0 \) and

\[
A = \frac{S^{\delta - \alpha}}{\gamma(\beta + 1)}
\]

Appendix C. Proof of Proposition 11.

The equilibrium path if \( \delta = \alpha \geq 1 \) follows directly from equation (22). If \( \delta - \alpha = 0 \), then \( \tilde{P} \) simplifies to:

\[
\tilde{P}_{i \delta = \alpha} = \left[ \frac{S}{\gamma(\beta + 1)} \right]^{\frac{1}{\beta - 1}} > 0 \tag{42}
\]

\( \tilde{P}_{i \delta = \alpha} \) is a positive constant, that represents the profit maximising network size. From (42) it follows that equilibrium network size, trade and welfare depend on whether (i) \( S \leq \gamma(\beta + 1) \) or (ii) \( S > \gamma(\beta + 1) \):

(i) If \( S \leq \gamma(\beta + 1) \), then \( \tilde{P}_{i \delta = \alpha} = 1 \). Let \( \tilde{i} > 0 \) denote the threshold level
of $i$ at which $E^*(\Pi_f) = 0$. Since $\delta = \alpha$, it follows from proposition (9) that $(\beta + 1)\delta > 2\alpha$, so $E(\Pi_f)$ is increasing in $i$ in the interior equilibrium. Hence, $E(\Pi_f) < 0$ when $i < \hat{i}$. Solving $E(\Pi_f)|_{P=a} = 0$ for $i$ and simplifying yields:

$$E\left(\frac{n}{i}\right) < 0 \quad \text{when} \quad i < \frac{\gamma}{(\beta + 1)} \left(\frac{F}{\beta - 1}\right) \left(\frac{\beta + 1}{S}\right)^{\frac{\beta + 1}{\beta - 1}}$$

Equilibrium network size is thus $P^* = 0$ for $i \in \left[0, \min\left\{\hat{i}, 1\right\}\right]$. For all values of $i \geq \hat{i}$, expected profits are positive, so the intermediary invests in contact-building to $P_{\delta=\alpha} < 1$. Equilibrium network size is thus $P^* = P_{\delta=\alpha}$ for $i \in \left[\min\left\{\hat{i}, 1\right\}, 1\right]$. Furthermore, substituting both $P^* = 0$ and $i \in \left[\min\left\{\hat{i}, 1\right\}, 1\right]$ into equations (13) and (17), respectively, yields the piecewise functions $E^*(T)$ and $E^*(W)$.

(ii) If $S > \gamma(\beta + 1)$, then $P_{\delta=\alpha} > 1$, so the constraint imposed by market size is binding. The constrained optimum is thus $P^* = 1$, provided $E(\Pi_f) \geq 0$.

The equilibrium is analogous to that described in proposition (5). $P^* = 0$ below a threshold value $\hat{i}$, that solves $E(\Pi_f)|_{P=a} = i^\delta (S - 2\gamma) - F = 0$, and $P^* = 1$ otherwise.

**Appendix D. Proof of Proposition 12.**

The equilibrium path if $\frac{S}{\gamma(\beta + 1)} \alpha < \delta < \alpha$ follows directly from equation (22).

Since $\delta < \alpha$, then $\hat{P}$ can be rearranged to give:

$$P_{\delta<\alpha} = \left[\frac{S}{\gamma(\beta + 1)i^{\alpha-\delta}}\right]^{\frac{1}{\beta-1}} > 0$$

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From (44) it follows that $\frac{\partial \tilde{P}}{\partial i} < 0$, so the interior equilibrium path of network size is declining with information cost $i$. Moreover, the second order condition in equation (39) is negative provided $P > \left[ \frac{S}{\gamma \beta (\beta+1)i^{\alpha-1}} \right]^{\frac{1}{\alpha-1}}$. Since $\beta \geq 2$ it must be true that $\tilde{P}_{\delta < \alpha} > \left[ \frac{S}{\gamma \beta (\beta+1)i^{\alpha-1}} \right]^{\frac{1}{\alpha-1}}$. Hence (44) corresponds to an interior maximum.

Let $\tilde{i}$ denote the threshold level of information costs, at which $\tilde{P}_{\delta < \alpha} = 1$. Solving for $i$ yields:

$$\tilde{i} = \left[ \frac{S}{\gamma (\beta + 1)} \right]^{\frac{1}{\alpha-1}}$$

(45)

Let $\hat{i}$ denote the threshold level of information costs at which $E(\Pi_I) = 0$. Since the interior equilibrium path of network size is declining with information cost $i$, then for sufficiently low $F$, the threshold $\hat{i}$ corresponds to a range where $P = 1$. If so, then $\hat{i}$ solves $E(\Pi_I)|_{P=1} = St^{\delta} - 2\gamma i^{\alpha} - F = 0$ and $\hat{i} \leq \tilde{i}$, where $\tilde{i}$ is described by equation (45). If (for sufficiently high $F$) threshold $\hat{i}$ corresponds to a range where $P = P_{\delta < \alpha}$, however, then $\hat{i}$ solves $E(\Pi_I)|_{P=\tilde{P}} = 0$. This yields the threshold level in equation (40) and must exceed $\tilde{i}$, where $\tilde{i}$ is described by equation (45). If the value of $\tilde{P}_{\delta < \alpha}$ at $E(\Pi_I)|_{P=\tilde{P}} = 0$ exceeds 1, then this indicates that the constrained optimisation applies and the relevant threshold is $\hat{i}$ solves $E(\Pi_I)|_{P=1} = St^{\delta} - 2\gamma i^{\alpha} - F = 0$.

These results are summarised by:

$$P^* = \begin{cases} 
0 & \text{if } 0 \leq i \leq \min \{\hat{i}, 1\} \\
1 & \text{if } \min \{\hat{i}, 1\} \leq i \leq \min \{\tilde{i}, 1\} \\
\left[ \frac{S}{\gamma (\beta + 1)i^{\alpha-1}} \right]^{\frac{1}{\alpha-1}} & \text{if } \min \{\hat{i}, 1\} \leq i \leq 1
\end{cases}$$
where \( \hat{i} \) is as above and \( \hat{i} = \left[ \frac{8}{7(\beta+1)} \right]^\frac{1}{\beta-1} > 0. \)

The piece-wise functions \( E^*(T) \) and \( E^*(W) \) follow directly from \( P^* \) and equations (13) and (17), respectively.

**Appendix E. Proof of Proposition 13.**

If \( \delta \leq \frac{2}{\beta+1} \alpha \), then from proposition (9) it follows that \( E(\Pi_I) \) is decreasing in information cost \( i \) in the interior equilibrium. Moreover, since \( \delta < \alpha \), the interior path is described by \( P_{|\delta<\alpha} \), where \( P_{|\delta<\alpha} \) is given by equation (44).

The declining profits along the equilibrium path imply that as \( i \rightarrow 0, \overline{P} \rightarrow \infty \), hence the constraint that \( P = \min \{ \overline{P}, 1 \} \) is binding. Let \( \hat{i} \) denote the threshold level of information costs, at which \( P_{|\delta<\alpha} = 1 \). This corresponds to the threshold given by equation (45).

Further, let \( \hat{i} \) denote the threshold level of information costs at which \( E(\Pi_I) = 0 \). While unconstrained profit is decreasing with \( i \), constrained profit \( E(\Pi_I)|_{P=1} \) is increasing for low values of \( i \) (hence, expected profit is non-monotonic with information cost \( i \). Case D under convex network-building costs is analogous to Equilibrium B described in proposition (6) under the linear cost specification).

Let \( \hat{i}_{|P=1} \) solve \( E(\Pi_I)|_{P=1} = 0 \) and \( \hat{i}_{|P=\overline{P}} \) solve \( E(\Pi_I)|_{P=\overline{P}} = 0 \). It follows from \( \delta \leq \frac{2}{\beta+1} \alpha \) and the definition of \( \hat{i} \) that \( \hat{i}_{|P=1} < \hat{i} < \hat{i}_{|P=\overline{P}} \). Thus, \( E(\Pi_I) \) is non-negative between these thresholds. Hence, the intermediary is inactive.
for low levels of information cost $i \leq \hat{i}_{i|P=1}$ and also for $i \geq \hat{i}_{i|P=\bar{P}}$.

\[
P^* = \begin{cases} 
0 & \text{if } 0 \leq i \leq \min \left\{ \hat{i}_{i|P=1}, 1 \right\} \\
1 & \text{if } \min \left\{ \hat{i}_{i|P=1}, 1 \right\} \leq i \leq \min \left\{ \hat{i}, 1 \right\} \\
\left[ \frac{g}{\gamma (\beta + 1)^{2n+\xi}} \right]^{n+1} & \text{if } \min \left\{ \hat{i}, 1 \right\} \leq i \leq \min \left\{ \hat{i}_{i|P=\bar{P}}, 1 \right\} \\
0 & \text{if } \min \left\{ \hat{i}_{i|P=\bar{P}}, 1 \right\} \leq i \leq 1 
\end{cases}
\]

The piece-wise functions $E^*(T)$ and $E^*(W)$ follow directly from $P^*$ and equations (13) and (17), respectively.
2 Competing for Contacts: Network Competition, Trade Intermediation and Fragmented Duopoly

This chapter extends the two-sided, pairwise matching model with a single intermediary, developed in Chapter 1, to analyse the effects of competition between intermediaries on endogenous network-building. The simplest framework within which to undertake such an analysis is the case of two trade intermediaries competing in network size and commission rates.

The model analyses the strategic interaction between two information intermediaries with symmetric access to an information technology that allows them to develop contacts with importers and exporters seeking to form unique trade matches. The intermediation analysed takes the form of information intermediation, where the role of intermediaries is to facilitate matching in a market with information frictions. Intermediaries thus seek to match members of their network of contacts for a success fee.

The related literature on competition between information intermediaries is limited to relatively few contributions, where these focus on competing 'cybermediaries' who seek to match two sides of a market on the Internet (Caillaud and Jullien, 2001, 2003). The main role of intermediaries in this literature is to gather and process information on users that visit their website so as to assist buyers and sellers in matching through their web service. This literature focuses on the effects of competition between online intermediaries in the presence of asymmetric network externalities where the value of an in-
termediary to a buyer depends on the number of sellers or goods that can be accessed through the intermediary (e.g. access to books through Amazon versus a smaller online seller). The literature discusses different pricing rules and contractual arrangements between users and intermediaries and contrasts the effects with the findings of the traditional literature on network competition (for example, Katz and Shapiro, 1985).

This chapter examines a framework of competition in information intermediation that differs from this literature. The focus of the model is the endogenous network investment decision of competing intermediaries, which in turn affects the nature of competition between them. While the importance of network size for competition is addressed in the literature, this is explored in the context of network externalities, whereby an intermediary that offers wider access to trading partners is considered more valuable to traders. The analysis in this chapter does not consider network externalities of this kind. Moreover, there are no asymmetries built into the model (although asymmetries between intermediaries may arise in equilibrium). In fact, the model assumes that traders receive intermediation offers by intermediaries after uncertainty about matching possibilities is resolved. That is, traders who find themselves in a position to choose between the two competing intermediaries, do so in the knowledge that a match with their unique trading partner is possible.

The model focuses on the competition between intermediaries in commission rates and the coordination game played by trading partners who must
select between intermediaries. Moreover, in contrast to the analysis of Chapter 1, where the intermediary has exclusive access to traders in his network, the competition between intermediaries gives rise to a distinctive market structure as a result of network overlap and the inability to price discriminate between groups of network members. In particular, intermediaries are monopolist service providers to some contacts but duopolists over contacts they share in their network overlap. The chapter thus models competition between information intermediaries as a fragmented duopoly with a competitive and a non-competitive segment, which gives rise to an undercutting game in commission rates with no pure strategy Nash equilibrium.

To the best of my knowledge there is no literature that examines competition in endogenous network formation in this way.

A few references in the Industrial Organisation literature consider markets with similar characteristics. Baye and De Vries (1992) develop a model with brand loyal consumers and price-sensitive consumers, in a market where price discrimination is not possible. They too find no pure strategy Nash equilibrium in prices.

Beard, Ford, Hill and Saba (2005) build a model directly applicable to cable television service competition, in which cable networks overlap, but price discrimination across users is not possible. They do find a pure strategy Nash equilibrium in prices, despite the fragmented nature of the market, as a result of smoothness conditions that ensure demand is decreasing in price in both
market segments. The network overlap itself is exogenous in Beard, Ford, Hill and Saba (2005), while the distribution of brand loyal versus price sensitive consumers is also arbitrary in Baye and De Vries (1992). In contrast, intermediaries' network sizes are endogenous in the model developed in this chapter.

The rest of the chapter proceeds as follows. Section 1 describes the economic environment and describes the timing of the game between traders and intermediaries. The subgame perfect equilibrium is characterised in Section 2. Section 3 provides two illustrative examples. Section 4 concludes.

2.1 Economic Environment

Consider the two-sided market of Chapter 1, where a continuum of risk-neutral importers (M) and a continuum of exporters (X), each distributed uniformly and with unit density over [0, 1], match uniquely to exchange a single unit of output generating joint surplus $S > 0$. There is two-sided information asymmetry as traders regarding the location of trading partner on the continuum. Due to the infinite number of importers (and exporters) along the continuum, the probability of any trader $j$ locating her partner through random selection is 0.

Each pair $(X_j, M_j)$ may match through a direct matching technology, which achieves successful matching with probability $q(i)$, where $i$ reflects the level of information costs or barriers to information flow between the two sides of the market and $i \in [0, 1]$. Let $q'(i) < 0$, $q(1) = 0$ and $q(0) = 1$. This direct
matching technology could reflect a search process whose success hinges on the state of information technology.

Alternatively, traders may match through a trade intermediary. Suppose there are two intermediaries, A and B, with access to the same technology for developing a network of contacts. The network of intermediary I is denoted by a measure of importer contacts, \( P_{MI} \), and exporter contacts, \( P_{XI} \), where \( P_{MI} \in [0,1] \) and \( P_{XI} \in [0,1] \), respectively, for \( I = \{A, B\} \). Given network size, the measure of feasible trade matches depends on the degree of overlap between importer and exporter contacts and is a random variable. For any given network investment, expected matches are maximised through symmetric contact-building\(^\text{13}\) in the two sides of the market. Hence intermediaries ensure \( P_{MI} = P_{XI} = P_I \) \( \forall I \), where \( P_I \in [0,1] \).

Network set-up costs are assumed to be zero, for simplicity. Let \( C(P_I) \) denote the total investment cost for a symmetric network of size \( P_I \) on either side of the market. The network investment decisions of intermediaries are analysed under two cost specifications:

(a) Linear costs: \( C(P_I) = 2P_I c \), where \( c > 0 \).

(b) Convex costs: \( C(P_I) = 2P_I c(i, P_I) \), where \( c(i, P_I) = \gamma i^\alpha P_I^2 \) and \( \alpha \geq 1, \gamma > 0 \).

For simplicity, it is assumed costless to match trade pairs from within the network of contacts. Hence, each intermediary has a marginal cost of interme-

\(^{13}\)This is proposition 1 of Chapter 1.
diation equal to zero. Intermediaries receive a success fee or commission for each intermediated trade match. Let $\alpha_A$ and $\alpha_B$ denote the commission rates of intermediaries $A$ and $B$, respectively, where $\alpha_A \in [0,1]$ and $\alpha_B \in [0,1]$.

The marginal revenue from intermediation is thus $\alpha_A S$ and $\alpha_B S$ for $A$ and $B$, respectively. Residual trade surplus is assumed to be shared equally between the importer and exporter.

The demand for each intermediary's services depends on two factors. First, the network size decisions of the two intermediaries. A larger network size gives rise to a larger measure of expected matches through the network, but also increases the expected overlap between networks. Expected overlap gives rise to an expected measure of common matches that can be intermediated through either network and for which intermediaries compete. Second, traders with access to both trade networks must choose between the intermediaries ex post and play a coordination game.

2.1.1 Timing of the Game

Intermediaries and traders interact strategically in a multi-stage game. The timing of the game is as follows.

**Stage 1 - Network investment:** Intermediaries $A$ and $B$ simultaneously and non-cooperatively choose network sizes $P_A$ and $P_B$. Network investment costs are sunk.

**Stage 2 - Commission setting:** Intermediaries simultaneously and non-cooperatively
commit to commission rates $\alpha_A$ and $\alpha_B$, respectively.

**Stage 3 - Intermediation offers:** Uncertainty over which trade matches are feasible through each network is resolved. Each intermediary makes a take-it-or-leave-it intermediation offer to traders that can be matched, specifying his commission rate. Successful matching is conditional on both trade partners accepting an offer by the same intermediary. Traders accept at most one offer.

**Stage 4 - Indirect trade:** Indirect trade takes place through $A$ and/or $B$.

**Stage 5 - Direct trade:** Any unmatched traders trade directly with probability $q(i)$.

### 2.1.2 Equilibrium Concept

The solution concept used is subgame perfect equilibrium (SPE) and the method used is backward induction. A strategy for intermediary $I$ is described by a pair $\{P_I, \alpha_I\}$. An offer acceptance strategy for trader $j$ is described by a pair $\{R_a, R_s\}$, where $R_a$ is a rule for determining whether an intermediation offer is acceptable and $R_s$ is a rule for selecting between acceptable offers. A set of strategy pairs, for intermediaries and traders, respectively, can be said to form an equilibrium of the game if these maximise the expected profit of each intermediary and the expected surplus from trade of each trader, given the strategies of all other players.

The subgame perfect equilibria of the game are characterised over the next
sections.

2.2 Traders' Incentives

Traders select their offer acceptance strategy to maximise their expected pay-off taking intermediaries strategies as given. Each trader in receipt of one or more offers must decide whether to accept one (or none) of the offers of intermediation. If all offers are rejected in stage 3 then trade can only take place directly in stage 5 of the game with probability $q(i)$. The expected pay-off from the direct trade route represents the outside option available to all traders and forms the benchmark against which all intermediation offers are assessed. Equilibrium rule $R_a$ summarises this assessment through a participation constraint.

Although uncertainty about available matching opportunities is resolved at the time of traders’ decision-making, indirect trade between matching trade partners is not guaranteed. The uncertainty in the outcome of the model arises, in part, from the coordination game played by traders in receipt of two offers of intermediation. Equilibrium rule $R_s$ summarises the incentives for selecting between available offers of intermediation, when both of these are acceptable. Since traders cannot communicate their intentions, there is a non-zero probability of coordination failure as a result of mismatch in coordination decisions.

The incentives of traders at each decision node of the game are examined in turn.
2.2.1 Stage 5 - Direct Trade

The pool of traders who attempt to match directly in stage 5 are those who either (a) receive no offers of intermediation, (b) accept no offers of intermediation, and (c) accept one offer but fail to match as a result of coordination failure. Since importers and exporters are assumed to match uniquely, any unmatched trader \( j \) can be assured that her trading partner is also a member of the pool of unmatched traders. The probability of a direct match, \( q(i) \), depends on the prevailing level of information costs, reflected in parameter \( i \). Assuming trade surplus is shared equally between trading partners, the expected payoff from direct trade of any trader \( j \) is given by:

\[
E^{DT}(\Pi_j) = \frac{1}{2} q(i) S
\]  

(46)

Recall from Chapter 1, that a monopolist intermediary sets his commission rate at \( 1 - q(i) \), thereby leaving traders indifferent between direct and indirect trade. Let \( \alpha^M \) denote the monopoly commission rate, where \( \alpha^M = 1 - q(i) \). Expressing (46) in terms of \( \alpha^M \) gives:

\[
E^{DT}(\Pi_j) = \frac{1}{2} (1 - \alpha^M) S
\]  

(47)

The expected payoff from direct trade reflects traders' outside option. All offers of intermediation must generate an expected payoff at least as good as \( E^{DT}(\Pi_j) \) in order to be acceptable. Interpreting equation (47), duopolist
intermediaries must offer traders an expected payoff from indirect trade at least as good as that which would have been received under a monopolistic market structure.

2.2.2 Stage 3 - Intermediation Offers

In stage 3, traders \( X_j \) and \( M_j \) find themselves in one of four positions:

1. Pair \( (X_j, M_j) \) cannot match through either intermediary.
2. Pair \( (X_j, M_j) \) can match through \( A \), but not \( B \); traders receive one offer from \( A \).
3. Pair \( (X_j, M_j) \) can match through \( B \), but not \( A \); traders receive one offer from \( B \).
4. Pair \( (X_j, M_j) \) can match through either \( A \) or \( B \); traders receive two intermediation offers.

If in (1), then \( X_j \) and \( M_j \) have no option but direct trade in stage 5. If in position (2)-(4), then \( X_j \) and \( M_j \) contrast the expected payoff from each offer received against the expected payoff from direct trade.

Let \( \Pi_j^A \) denote the payoff of trader \( j \) from indirect matching through \( A \) and \( \Pi_j^B \) the payoff through \( B \), where these are given by (48) and (49), respectively:

\[
\Pi_j^A = \frac{1}{2} (1 - \alpha_A) S \quad (48)
\]

\[
\Pi_j^B = \frac{1}{2} (1 - \alpha_B) S \quad (49)
\]
It follows directly from (47), (48) and (49), that if $\alpha_A \leq \alpha^M$, then intermediation through $A$ is acceptable and if $\alpha_B \leq \alpha^M$, then intermediation through $B$ is acceptable. In general, all offers must satisfy the participation constraint $\alpha_I \leq \alpha^M$ in order to be acceptable.

In the case where $X_j$ and $M_j$ receive only one offer, the optimal selection rule is thus to accept the unique acceptable offer. The optimal acceptance strategy conditional on one offer being received is thus 'accept offer $I$ if $\alpha_I \leq \alpha^M$; reject otherwise'. The next section examines the selection decision of traders in receipt of two offers.

**The Trader Coordination Subgame**  Consider a trade pair $(X_j, M_j)$ that can match through either $A$ or $B$. In stage 3, $X_j$ and $M_j$ each receive two offers of intermediation. They apply optimal rule $R_a$ to assess the acceptability of each offer. Since expected payoffs are symmetric for both traders, their assessment of offers is identical.

If both offers are deemed unacceptable, $X_j$ and $M_j$ reject both offers and can expect to receive $E^{DT}(\Pi_j)$ in stage 5. If one offer is acceptable and the other unacceptable, then the optimal decision is for $X_j$ and $M_j$ to reject the unacceptable offer and trade indirectly in stage 4. Hence, in the case of one acceptable offer, there is no possibility of coordination failure.

If both received offers prove to be acceptable for $X_j$ and $M_j$, then each trader $j$ must choose between them. This gives rise to a coordination game between $X_j$ and $M_j$. As with all games of this class, there are three equilibria,
two symmetric pure strategy Nash equilibria (both choose $A$ or both choose $B$) and one symmetric mixed strategy Nash equilibrium, where both traders choose $A$ (or $B$) with the same probability.

If both $X_j$ and $M_j$ accept $A$ in stage 3, then each receives $\Pi_j^A$. If they both accept $B$, then each receives $\Pi_j^B$. If one accepts $B$ and the other $A$, then indirect trade cannot take place due to coordination failure. Thus mismatch can arise in the model even though both traders are members of both networks and both offers are acceptable. If coordination failure takes place, traders can expect to receive expected payoff $E^{DT}(\Pi_j)$. Table (1) describes the payoff structure of the coordination game:

<table>
<thead>
<tr>
<th>$X_j$</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$\frac{S}{2} (1 - \alpha_A), \frac{S}{2} (1 - \alpha_A)$</td>
<td>$\frac{S}{2} (1 - \alpha^M), \frac{S}{2} (1 - \alpha^M)$</td>
</tr>
<tr>
<td>$B$</td>
<td>$\frac{S}{2} (1 - \alpha^M), \frac{S}{2} (1 - \alpha^M)$</td>
<td>$\frac{S}{2} (1 - \alpha_B), \frac{S}{2} (1 - \alpha_B)$</td>
</tr>
</tbody>
</table>

Table 1: Traders’ Coordination Game

For both offers to be acceptable, it must be the case that $\alpha_A \leq \alpha^M$ and $\alpha_B \leq \alpha^M$ are satisfied. This allows the payoffs in table (1) to be ranked, confirming $(A, A)$ and $(B, B)$ as the two pure strategy Nash equilibria of the coordination game. Either both traders accept $A$, or both accept $B$.

Allowing traders to randomise over $A$ and $B$, such that $X_j$ accepts $A$ with probability $\lambda$ (and $B$ with $1 - \lambda$) and $M_j$ accepts $A$ with probability $\mu$ (and
$B$ with $1 - \mu$, it is straightforward to show\(^{14}\) the coordination game has one symmetric mixed strategy Nash equilibrium $(\lambda^*, \mu^*)$ where:

$$
\lambda^* = \mu^* = \frac{\alpha^M - \alpha_B}{2\alpha^M - \alpha_B - \alpha_A}
$$

(50)

Combining (50) and the pure strategy payoffs allows the expected payoff for each trader $j$ in the mixed strategy Nash equilibrium to be computed:

$$
E(\Pi_j|\lambda^*, \mu^*) = \frac{S}{2} \left[ \frac{\alpha^M - \alpha_B}{2\alpha^M - \alpha_B - \alpha_A} (1 - \alpha_A) + \frac{\alpha^M - \alpha_A}{2\alpha^M - \alpha_B - \alpha_A} (1 - \alpha^M) \right]
$$

(51)

We can now distinguish between three cases: (a) $\alpha_A < \alpha_B$, (b) $\alpha_B < \alpha_A$ or (c) $\alpha_A = \alpha_B$. If either (a) or (b) holds, then the game between $X_j$ and $M_j$ becomes a Ranked Coordination game, which has the additional feature that the equilibria can be Pareto ranked.

Consider case (a) where $\alpha_A < \alpha_B$. If intermediary $A$ offers to match pair $j$ for a lower commission than $B$, then the payoffs received from pure strategy Nash equilibrium $(A, A)$ dominate those from $(B, B)$. By inspection of (51) it can also be observed that $E(\Pi_j|\lambda^*, \mu^*) < \Pi^A_j$. Hence pure strategy Nash equilibrium $(A, A)$ is Pareto superior to the other Nash equilibria of the coordination game. It can thus be said that although there are three equilibria,

---

\(^{14}\)A derivation of the symmetric mixed strategy Nash equilibrium, and associated expected payoff, of the coordination game is included in Appendix A.
the pure strategy Nash equilibrium \((A, A)\) offers a compelling focal point\(^{15}\) of the Ranked Coordination game when \(\alpha_A < \alpha_B\).

Similar arguments apply in case (b), which lead to the result that \((B, B)\) is Pareto superior to the other two Nash equilibria providing a focal point of the coordination game when \(\alpha_B < \alpha_A\).

In case (c), where \(\alpha_A = \alpha_B\), both pure strategy Nash equilibria yield symmetric payoffs and \(\lambda^* = \mu^* = \frac{1}{2}\). Since the two intermediaries are indistinguishable and there is no way for trade partners to indicate their action to each other, the mixed strategy Nash equilibrium provides a compelling focal point of the game when \(\alpha_A = \alpha_B\).

The multiplicity of equilibria implies there is a multiplicity of selection rules \(R_s\) that are optimal for each trader in pair \(j\), given the strategies of the other players. For example, if all traders follow the selection rule 'always accept \(A\) when two acceptable offers are received', then the outcome of their actions is \((A, A)\) and no trader \(j\) ever finds it optimal to deviate from the rule.

Let the 'focal strategy' refer to the selection rule \(R_s\), which specifies an action for each trader \(j\) to follow in each of the three cases (a) to (c), that leads to the focal point in each case. This rule gives rise to the most intuitive and likely outcome of the stage 3 coordination game. Specifically, that

---

\(^{15}\)The game theory literature points to a number of mechanisms for resolving the multiplicity of equilibria in the Ranked Coordination game so that focal pure Nash equilibria emerge as the unique solution. These include communication or signalling between coordinating parties to indicate the action to be taken. In light of the information barriers that underpin the model, such communication is prohibited by assumption. If unique pairs of traders could communicate their actions to each other then there would be no need for an intermediary. Other mechanisms include mediation where an outside party imposes a solution.
each trader in pair \( j \) simultaneously and non-cooperatively accepts the most inexpensive of two acceptable indirect trade routes when commissions differ across intermediaries, and flips a coin when commission rates are the same across intermediaries. Its conceptual appeal aside, the focal rule gives rise to strategic interactions between intermediaries that do not arise if one of the two intermediaries is always selected in stage 3, irrespective of commission rates. To explore the effects of competition in overlapping matches, we examine the subgame perfect equilibrium of the game that includes the focal strategy \( R_s \) as part of the equilibrium strategy of traders.

The focal selection strategy \( R_s \) can thus be summarised as follows for trader \( j \): If two acceptable offers are received and \( \alpha_A \neq \alpha_B \), then accept the offer with the lower commission rate with probability 1; if \( \alpha_A = \alpha_B \), then accept offer \( A \) with probability \( \frac{1}{2} \).

To confirm that randomisation when \( \alpha_A = \alpha_B \) does not give rise to an unacceptable expected payoff, consider that coordination is at \( A \) with probability \( \frac{1}{4} \), and at \( B \) with probability \( \frac{1}{4} \). Mismatch occurs with probability \( \frac{1}{2} \). Moreover, \( \alpha_A = \alpha_B \) implies that \( \Pi_j^A = \Pi_j^B = \Pi_j \). Hence the expected payoff when commissions are equal, which follows directly from (51), is given by (52):

\[
E(\Pi_j)_{|\alpha_A = \alpha_B \leq \alpha^M} = E(\Pi_j)_{|\lambda^* = \mu^* = \frac{1}{2}} = \frac{1}{2} \Pi_j + \frac{1}{2} E^{DT}(\Pi_j) \tag{52}
\]

Hence the selection rule \( R_s \) in the case of two acceptable offers is consistent
2.2.3 Traders' Offer Acceptance Strategy

The analysis of traders' optimal incentives is summarised by proposition (14).

Proposition 14 The following pair \({R_a, R_s}\) forms an optimal acceptance strategy for each trader \(j\):

\(R_a\): Any offer \(k\) is acceptable if \(\alpha_I \leq \alpha^M\) and unacceptable otherwise.

\(R_s\): If one acceptable offer is received, accept it; if two acceptable offers are received and \(\alpha_A \neq \alpha_B\), then accept the offer with the lower commission rate with probability 1; if two acceptable offers are received and \(\alpha_A = \alpha_B\), then accept offer \(A\) with probability \(\frac{1}{2}\).

Proof. The optimality of \(R_a\) follows directly from equations (47), (48) and (49). The optimality of \(R_s\) in the case of one acceptable offer also follows directly from these. The optimality of \(R_s\) given two acceptable offers and \(\alpha_A \neq \alpha_B\) follows from the payoffs in table (1). The optimality of \(R_s\) when \(\alpha_A = \alpha_B\) follows from the mixed strategy Nash equilibrium of the coordination game, described by (50) and from the expected payoff under randomisation given by (52). ■

2.3 Stage 2 - Nash Equilibrium in Commission Rates

In stage 2, intermediaries simultaneously and non-cooperatively select commission rates, \(\alpha_A\) and \(\alpha_B\), respectively, to maximise their expected profit,
taking each others' commission rate, network sizes $P_A \in [0,1]$ and $P_B \in [0,1]$ and $\{R_a, R_b\}$ as given.

The strategic interaction between intermediaries in the commission-setting game hinges on two conflicting effects. On the one hand, a lower commission rate makes it more likely that the intermediary’s offer is selected by traders in receipt of two acceptable offers. At the same time, a lower commission implies lower profit per successful match.

The measure of trade matches possible through a network of a given size is a random variable that depends on the degree of overlap between importer and exporter contacts. A crucial feature of the game is that intermediaries set commission rates prior to the realisation of this random variable and thus without knowing the identity of their future customers. This prevents intermediaries from price discriminating between trade pairs\(^\text{16}\) that are exclusive to their own network and those common to both networks.

To assess intermediaries’ incentives in the commission-setting game the structure of traders’ demand for intermediation services needs to be characterised.

\(^\text{16}\)All decisions of intermediaries are thus made on the basis of expectations. The *ex post* realisation of trade matches can differ markedly from the *ex ante* expectation. For example, if $P_A = P_B = \frac{1}{2}$, the measure of expected common matches is $P_A^2 P_B^2 = \frac{1}{16}$. The realised overlap between the two networks can range from 0 to $\frac{1}{2}$, however, depending on which specific importers and exporters are contacted in stage 1. The obvious exception is where $P_A = P_B = 1$ for which expected and realised trade matches coincide.
2.3.1 Demand for Intermediation Services

For an intermediary to be able to match pair \((X_j, M_j)\), both partners must be members of the intermediary's network. The matching probabilities for any pair \((X_j, M_j)\) are therefore given by (53) to (56):

\[
\begin{align*}
\Pr[\text{Pair } j \text{ can match via } A \text{ and } B] &= P_A^2 P_B^2 & (53) \\
\Pr[\text{Pair } j \text{ can match via } A, \text{ but not } B] &= P_A^2 (1 - P_B^2) & (54) \\
\Pr[\text{Pair } j \text{ can match via } B, \text{ but not } A] &= P_B^2 (1 - P_A^2) & (55) \\
\Pr[\text{Pair } j \text{ cannot match via } A \text{ or } B] &= (1 - P_A^2) (1 - P_B^2) & (56)
\end{align*}
\]

Suppose both intermediaries choose network sizes between 0 and 1. The market structure that results is one of fragmented duopoly. Each intermediary has a set of exclusive matches, over which there is monopoly power. At the same time, the non-zero probability of network overlap gives rise to a set of expected common matches, over which intermediaries compete.

The success of each intermediary in gaining trade matches from the competitive network overlap depends on relative commission rates. From \(R_s\) intermediaries anticipate that all common matches are won by the intermediary with the lower of the two commission rates when \(\alpha_A \neq \alpha_B\), while each expects to win \(\frac{1}{4}\) of common matches when \(\alpha_A = \alpha_B\).

Let \(E(T_A)\) denote expected indirect trade through \(A\) or, equivalently, expected demand for \(A\)'s intermediation services. Similarly, \(E(T_B)\) denotes ex-
pected indirect trade through $B$ or expected demand for $B$'s intermediation services. Combining the matching probabilities in (53) to (56) with $\{R_a, R_s\}$ yields $E(T_A)$ and $E(T_B)$, conditional on $\alpha_A$ and $\alpha_B$.

Consider the expected demand for $A$'s services. Intermediary $A$ expects a measure $P^2_A (1 - P^2_B)$ of exclusive matches while common matches between $A$ and $B$ are given by measure $P^2_A P^2_B$. If $\alpha_A < \alpha_B \leq \alpha_M$, then $A$ provides the most inexpensive trade route, so all matching traders in the competitive segment coordinate at $A$. This yields a total expected demand for $A$ of $P^2_A$.

Conversely, if $\alpha_B < \alpha_A \leq \alpha_M$ all common matching traders coordinate at $B$ giving intermediary $A$ an expected demand of $P^2_A (1 - P^2_B)$ only. If $\alpha_B = \alpha_A \leq \alpha_M$, then intermediaries' offers are acceptable but indistinguishable, so traders randomise over $A$ and $B$ in the coordination stage. $\frac{1}{4} P^2_A P^2_B$ are expected to trade through $A$, another $\frac{1}{4} P^2_A P^2_B$ through $B$, while the remaining $\frac{1}{2} P^2_A P^2_B$ fail to coordinate.

Similar arguments can be applied to $B$. The structure of $E(T_A)$ and $E(T_B)$ summarised below gives rise to the strategic incentives discussed in the rest of the section.

$$E(T_A) = \begin{cases} 
    P^2_A & \text{if } \alpha_A < \alpha_B \leq \alpha_M \\
    P^2_A (1 - P^2_B) & \text{if } \alpha_B < \alpha_A \leq \alpha_M \\
    P^2_A (1 - \frac{3}{4} P^2_B) & \text{if } \alpha_B = \alpha_A \leq \alpha_M \\
    0 & \text{if } \alpha_A > \alpha_M 
\end{cases}$$
Recall that network investment costs are sunk in stage 1. Moreover, the marginal cost of matching is assumed to be zero for simplicity. If follows that the expected operating profit of $A$ and $B$, respectively, are given by (57) and (58):

\[
E(T_A) = \alpha_A SE(T_A) \tag{57}
\]

\[
E(T_B) = \alpha_B SE(T_B) \tag{58}
\]

Intermediaries thus choose $\alpha_A$ and $\alpha_B$ to maximise $E(T_A)$ and $E(T_B)$, respectively, given $E(T_A)$ and $E(T_B)$.

### 2.3.2 Polar Cases of Market Structure

The discussion above assumes network sizes between 0 and 1. In general, the commission-setting game can be analysed for three polar cases:

1. Monopoly in intermediation services, where $\{P_B = 0; P_A \in [0,1]\}$ or $\{P_A = 0; P_B \in [0,1]\}$. Inspection of $E(T_A)$ confirms that expected indirect trade collapses to $P_A^2$ when $\alpha_A \leq \alpha_M$ and 0 otherwise when $P_B = 0$, and vice versa if $B$ is a monopolist. This corresponds to the
analysis in Chapter 1, where a monopolist intermediary chooses $\alpha^M$ in
equilibrium.

2. Bertrand duopoly in intermediation services, where $\{P_A = 1; P_B = 1\}$. If both intermediaries' networks span the entire market, then networks overlap entirely giving rise to the most competitive market outcome. Intermediaries are in direct competition for all trade pairs.

3. Fragmented duopoly in intermediation services, where $\{P_A \in (0,1); P_B \in (0,1)\}$. Each intermediary’s demand is partitioned between a competitive and non-competitive segment.

The two competitive cases are analysed in turn.

2.3.3 Competing in Commission Rates: Bertrand Duopoly

Let $P_A = P_B = 1$. All trade pairs are common to $A$ and $B$ giving rise to the following demand structure:

\[
E(T_A) = \begin{cases} 
1 & \text{if } \alpha_A < \alpha_B \leq \alpha^M \\
0 & \text{if } \alpha_B < \alpha_A \leq \alpha^M \\
\frac{1}{4} & \text{if } \alpha_B = \alpha_A \leq \alpha^M \\
0 & \text{if } \alpha_A > \alpha^M 
\end{cases}
\]

\[
E(T_B) = \begin{cases} 
1 & \text{if } \alpha_B < \alpha_A \leq \alpha^M \\
0 & \text{if } \alpha_A < \alpha_B \leq \alpha^M \\
\frac{1}{4} & \text{if } \alpha_B = \alpha_A \leq \alpha^M \\
0 & \text{if } \alpha_B > \alpha^M 
\end{cases}
\]
Expected operating profits for $A$ and $B$ when $\alpha_B = \alpha_A$ are thus:

\[
E(\Pi_A)_{\alpha_B=\alpha_A} = \frac{\alpha_A S}{4} \quad (59)
\]
\[
E(\Pi_B)_{\alpha_B=\alpha_A} = \frac{\alpha_B S}{4} \quad (60)
\]

Expected operating profits for $A$ and $B$ when $\alpha_B < \alpha_A$ are thus:

\[
E(\Pi_A)_{\alpha_B<\alpha_A} = 0 \quad (61)
\]
\[
E(\Pi_B)_{\alpha_B<\alpha_A} = \alpha_B S \quad (62)
\]

Conversely, when $\alpha_A < \alpha_B$:

\[
E(\Pi_A)_{\alpha_A<\alpha_B} = \alpha_A S \quad (63)
\]
\[
E(\Pi_B)_{\alpha_A<\alpha_B} = 0 \quad (64)
\]

Intermediaries provide a homogeneous service in a price-setting Bertrand duopoly. The pattern of $E(\Pi_A)$ and $E(\Pi_B)$ provides an incentive for intermediaries to undercut each other in order to win the entire market. In contrast to the classical Bertrand duopoly, coordination failure in traders' decisions implies that intermediaries share *half* the market when $\alpha_B = \alpha_A$, instead of sharing the entire market.

Let $\alpha^C$ denote the 'competitive' commission rate where $E(\Pi_A) = E(\Pi_B) = 0$. In the absence of a marginal cost of matching, $\alpha^C = 0$, giving rise to a
unique Nash equilibrium in commission rates at $\alpha_B = \alpha_A = \alpha^C = 0$. The Bertrand duopoly outcome is summarised by proposition (15).

**Proposition 15** If $P_A = P_B = 1$, then the commission-setting subgame has a unique, pure strategy Nash equilibrium where $\alpha_B = \alpha_A = \alpha^C = 0$.

**Proof.** To prove that $\alpha_B = \alpha_A = \alpha^C = 0$ is the unique, pure strategy Nash equilibrium of the game we show that $\alpha_B = \alpha_A = \alpha_0 > \alpha^C$ can never be an equilibrium. The proof is by contradiction. Let $(\alpha_A, \alpha_B) = (\alpha_0, \alpha_0)$, where $\alpha_0 > \alpha^C$. If $A$ deviates from $(\epsilon^*, \alpha_0)$ by undercutting $B$, then $E(\Pi_A)|_{\alpha_A = \alpha_0 - \epsilon} = S(\alpha_0 - \epsilon)$, where $E(\Pi_A)|_{\alpha_A = \alpha_0 - \epsilon} \to S\alpha_0$, as $\epsilon \to 0$. Since $E(\Pi_A)|_{\alpha_A = \alpha_0} = \frac{1}{2}S(\alpha_0)$, it follows that $E(\Pi_A)|_{\alpha_A = \alpha_0 - \epsilon} > E(\Pi_A)|_{\alpha_A = \alpha_0}$. It is thus profitable for intermediary $A$ to undercut from $\alpha_0 > \alpha^C$. Likewise, $E(\Pi_B)|_{\alpha_B = \alpha_0 - \epsilon} < E(\Pi_B)|_{\alpha_A = \alpha_A - \epsilon}$. Hence, $\alpha_B = \alpha_A = \alpha_0 > \alpha^C$ cannot be an equilibrium. ■

2.3.4 Competing in Commission Rates: Fragmented Duopoly

Let $P_A \in (0,1)$ and $P_B \in (0,1)$. This gives rise to a distinctive market structure comprised by a competitive and a non-competitive market segment between which price discrimination is not possible. Hence intermediaries are neither pure monopolists, nor pure duopolists and as a result face conflicting incentives compared to both the monopoly case of Chapter 1 and the Bertrand duopoly case.

Consider the incentives of intermediary $A$ when setting $\alpha_A$. In the frag-
mented duopoly, in contrast to the Bertrand duopoly case, A never finds it optimal to set \( \alpha_A = \alpha^C = 0 \), given \( \alpha_B < \alpha^M \). This is due to the fact that A can always relinquish the common trade matches to B and set the monopoly commission rate. The only traders that accept A's offers at \( \alpha_A = \alpha^M \), given \( \alpha_B < \alpha^M \), are traders in receipt of an A offer only. Let \( E^M(\Pi_A) \) denote the expected profit from A's monopolistic market segment corresponding to this strategy. It follows directly from \( E(T_A) \) that:

\[
E^M(\Pi_A) = \alpha^M SP_A^2 (1 - P^2_B) \tag{65}
\]

Profit level \( E^M(\Pi_A) \) is always an option for A, introducing a positive lower bound to the profits A receives in the commission-setting game.

If A sets \( \alpha_A = \alpha_0 \) then the most profit A can ever expect to earn is \( E(\Pi_A)_{|\alpha_0 < \alpha_B} = \alpha_0 SP_A^2 \), in the event that \( \alpha_0 < \alpha_B \). Contrasting \( E(\Pi_A)_{|\alpha_0 < \alpha_B} \) with \( E^M(\Pi_A) \) reveals that \( \alpha_0 \geq (1 - P^2_B) \alpha^M \) must be satisfied in order for A to find it optimal to charge \( \alpha_0 \), given \( \alpha_B \). If conversely, \( \alpha_0 < (1 - P^2_B) \alpha^M \), then the maximum expected profit that A can receive by setting \( \alpha_0 \) is lower than the profit from A's outside option, and hence \( \alpha_0 \) is never optimal.

Let \( \tilde{\alpha}_A \) denote the threshold level of \( \alpha_A \) at which \( \max E(\Pi_A)_{|\alpha_A} = E^M(\Pi_A) \). Hence:

\[
\tilde{\alpha}_A = (1 - P^2_B) \alpha^M \tag{66}
\]

It follows that if \( \alpha_A < \tilde{\alpha}_A \), then the maximum profit that A can ever
expect to generate from intermediation service provision overall is less than
that under monopolistic commission setting that yields $E^M(\Pi_A)$. Hence, $A$
ever charges a commission below $\tilde{\alpha}_A$.

Furthermore, given $R_a$, $A$ never finds it optimal to set $\alpha_A > \alpha^M$ since all
its offers are subsequently rejected. The elimination of dominated strategies
of $A$, conditional on $P_a$, yields $\alpha_A \in [\tilde{\alpha}_A, \alpha^M]$.

Similar arguments for $B$ yield the following outside option for $B$:

$$E^M(\Pi_B) = \alpha^M S P_B^2 (1 - P_A^2)$$

(67)

Let $\tilde{\alpha}_B$ denote the threshold level of $\alpha_B$ at which $\max E(\Pi_B)|_{\alpha_B} = E^M(\Pi_B)$. Hence:

$$\tilde{\alpha}_B = (1 - P_A^2) \alpha^M$$

(68)

Symmetric arguments for $B$ allow the elimination of dominated strategies,
thereby yielding $\alpha_B \in [\tilde{\alpha}_B, \alpha^M]$. Note that, in general, the threshold levels
are not symmetric since network sizes can be asymmetric in stage 1.

Consider how the threshold level of $B$ is affected by the network size of $A$.
The lower is $P_A$ then the smaller the measure of common matches between
$A$ and $B$ and thus the smaller the loss of trade matches from a deviation to
the monopolistic strategy. Thus the attractiveness of $B$'s outside option is
increasing as $P_A \rightarrow 0$ making $B$ less inclined to undercut $A$, thereby raising
the deviation threshold level $\tilde{\alpha}_B$. At the limit, when $P_A = 0$, it follows directly
from equation (68) that $\tilde{\alpha}_B = \alpha^M$. In other words, the smaller is $P_A$, *ceteris paribus*, then the weaker are competitive forces between $A$ and $B$, inducing $B$ to set $\alpha_B$ more monopolistically. At the one extreme, where $P_A = 0$, there is no competition, so $B$ sets the monopoly commission (as in Chapter 1). At the other extreme, where $P_A = 1$, there is no monopolistic segment, yielding the Bertrand pure strategy Nash equilibrium in which $B$ sets the competitive commission rate.

The partitioned market structure is thus a hybrid of the two extremes of monopoly and Bertrand duopoly that gives rise to conflicting monopolistic and competitive forces. The combined effect of these forces is to make any fixed pair of commission rates $(\alpha_A, \alpha_B)$ unstable. There is thus no pure strategy Nash equilibrium in commission rates when $P_A \in (0, 1)$ and $P_B \in (0, 1)$.

The conflicting incentives are illustrated in figure (15). The figure illustrates the case where $P_A > P_B$ and thus $\tilde{\alpha}_A > \tilde{\alpha}_B$. Consider the incentives of $A$. The optimal response to $\alpha_B > \alpha^M$ is to set $\alpha_A = \alpha^M$; but if $\alpha_A = \alpha^M$, then $B$ has an incentive to undercut $A$ for $\alpha_A \in (\tilde{\alpha}_B, \alpha^M]$. $A$ also has an incentive to undercut $B$ for $\alpha_B \in (\tilde{\alpha}_A, \alpha^M]$, driving down commission rates. $A$'s incentive to undercut is restrained, however, by the existence of $A$'s outside option. Hence, once $\alpha_B$ reaches $\tilde{\alpha}_A > \tilde{\alpha}_B$, $A$ finds it optimal to deviate to $\alpha_A = \alpha^M$; but if $\alpha_A = \alpha^M$, then $B$ has an incentive to undercut $A$...and so on.

The analysis is summarised by proposition (16).
Proposition 16 If $P_A \in (0,1)$ and $P_B \in (0,1)$, then the commission-setting subgame has no pure strategy Nash equilibrium in commission rates.

Proof. Proof is by contradiction. Let $(\alpha^*_A, \alpha^*_B)$ reflect a pair of commission rates that constitute a pure strategy Nash equilibrium, where $\alpha^*_A \in [\hat{\alpha}_A, \alpha^M]$ and $\alpha^*_B \in [\hat{\alpha}_B, \alpha^M]$. Consider the optimal response of $A$ to $\alpha^*_B$, where $\alpha^*_B$ takes the following values: (a) $\hat{\alpha}_A < \alpha^*_B \leq \alpha^M$ and (b) $\alpha^*_B = \hat{\alpha}_A < \alpha^M$.

(a) If $\hat{\alpha}_A < \alpha^*_B \leq \alpha^M$, then the optimal response of $A$ is to undercut $B$ by $\varepsilon$, where $\varepsilon \to 0$. The expected profit from $A$'s undercutting strategy is $E(\Pi_A)_{\alpha_A = \alpha^*_B - \varepsilon} = (\alpha^*_B - \varepsilon)SP^2_A - \alpha^*_BSP^2_A$, where $\alpha^*_BSP^2_A > \alpha^*_BSP^2_A (1 - \frac{3}{4}P_B^2) = \alpha^*_B E(\Pi_A)_{\alpha_A = \alpha^*_B}$. Hence, $A$ receives a higher expected profit from undercutting $B$ by $\varepsilon$, than from matching $\alpha^*_B$. It follows that $A$'s optimal response to
\( \alpha^*_B \) is always \( \alpha^*_A \equiv \alpha^*_B - \epsilon \) if \( \hat{\alpha}_A < \alpha^*_B \leq \alpha^M \).

Given \( \alpha^*_A \), consider the incentives of \( B \). Using an identical argument, it is optimal for \( B \) to deviate from \( \alpha^*_B \) by undercutting \( \alpha^*_A \) by \( \epsilon \), if \( \hat{\alpha}_B < \alpha^*_A \leq \alpha^M \).

Hence, \( B \)'s optimal response to \( \alpha^*_A \) is \( \alpha_B = \alpha^*_A - \epsilon < \alpha^*_B \) and not \( \alpha^*_B \). Thus \( \alpha^*_B \) is not an optimal response to \( \alpha^*_A \), where \( \alpha^*_A \) is an optimal response to \( \alpha^*_B \).

(b) If \( \alpha^*_B = \hat{\alpha}_A < \alpha^M \), then the optimal response of \( A \) is to deviate to \( \alpha^M \), since \( E(\Pi_A|_{\alpha_A < \hat{\alpha}_A}) < E^M(\Pi_A) \); but then it follows that intermediary \( B \) finds it optimal to deviate to \( \alpha_B = \alpha^M - \epsilon > \alpha^*_B \). Hence, the optimal reply to \( \alpha^*_B \) is \( \alpha^*_A \equiv \alpha^M \), but the optimal reply of \( B \) to \( \alpha^*_A \) not \( \alpha^*_B \).

Similar arguments apply for \( B \)'s optimal response to \( \alpha^*_A \). It follows from the above that there is no pure strategy Nash equilibrium in commission rates. ■

2.3.5 Randomising Commission Rates

This section characterises the unique, mixed strategy Nash equilibrium in intermediaries' commission rates. Intuitively, randomisation of commission rates prevents rival intermediaries from systematically undercutting each other.

Let \( H(\alpha_A) \) and \( F(\alpha_B) \) denote the cumulative distribution functions used to randomise commission rates \( \alpha_A \) and \( \alpha_B \), respectively, where \( F(\cdot) \) and \( H(\cdot) \)
are continuous and have the following features:

\[ H(\tilde{\alpha}_A) = F(\tilde{\alpha}_B) = 0 \]
\[ H(\alpha^M) = F(\alpha^M) = 1 \]  \hspace{1cm} (69)
\[ dH/d\alpha_A > 0; \quad dF/d\alpha_B > 0 \]

Since the distributions are continuous, the probability of \(A\) and \(B\) setting identical commission\(^{17}\) rates is 0. Let \(\alpha_A\) be a random draw from \(H(\cdot)\). It follows from \(F(\cdot)\) that:

\[ \Pr(\alpha_B < \alpha_A) = F(\alpha_A) \]
\[ \Pr(\alpha_B > \alpha_A) = 1 - F(\alpha_A) \]  \hspace{1cm} (70)
\[ \Pr(\alpha_A = \alpha_B) = 0 \]

Given the probabilities described in (70), we seek to find the optimal distribution \(H(\cdot)\) for intermediary \(A\), that keeps expected profits constant over the distribution, and similarly, the optimal \(F(\cdot)\) that keeps \(B\)'s expected profits constant.

The mixed strategy Nash equilibrium depends on the relative values of \(\tilde{\alpha}_A\) and \(\tilde{\alpha}_B\), which reflect the relative values of \(P_A\) and \(P_B\). There are three cases:

(i) \(\tilde{\alpha}_A > \tilde{\alpha}_B\), where \(P_A > P_B\)
(ii) \(\tilde{\alpha}_A < \tilde{\alpha}_B\), where \(P_A < P_B\) and
(iii) \(\tilde{\alpha}_A = \tilde{\alpha}_B\),

\(^{17}\)This implies that while coordination failure with probability \(\frac{1}{4}\) is expected in stage 3 in the event where traders receive two offers and commission rates are equal, the randomisation of commission rates in stage 2 ensures that this event occurs with zero probability. Thus coordination failure does not arise in equilibrium.
where $P_A = P_B$.

In (i) and (ii) intermediaries $A$ and $B$ are shown to randomise\(^ {18}\) over different distributions, while in case (iii) the symmetry in network sizes implies $H(\cdot) = F(\cdot) \equiv G(\cdot)$. Following the methodology used in Baye and De Vries (1992), but allowing for asymmetric network sizes\(^ {19}\), yields the unique, mixed strategy Nash equilibrium summarised in proposition (17).

**Proposition 17** (a) If $P_A = P_B = P \in (0, 1)$, then there exists a mixed strategy Nash equilibrium, in which intermediaries choose their commission rate randomly from the same distribution:

$$G(\alpha) = \frac{\alpha - (1 - P^2)\alpha^M}{\alpha P^2},$$

where $\alpha \in [\widehat{\alpha}, \alpha^M]$ \hspace{1cm} (71)

where $\widehat{\alpha} = (1 - P^2)\alpha^M$.

(b) If $P_A \in (0, 1)$, $P_B \in (0, 1)$ and $P_A \neq P_B$, then there is a unique, mixed strategy Nash equilibrium, in which intermediaries $A$ and $B$ choose their commission rate randomly from distributions $H(\alpha_A)$ and $F(\alpha_B)$, respectively, where:

\(^{18}\)Note that the mixed strategy Nash equilibrium does not imply randomisation of commission rates across offers made to traders. Each intermediary sets a unique commission rate that is common to all offers made, where this unique commission rate is a random draw from the relevant distribution in proposition (17) in equilibrium.

\(^{19}\)Since intermediaries’ network investment decisions in stage 1 are not necessarily symmetric, we solve for the unique, mixed strategy Nash equilibrium for general $P_A$ and $P_B$ without imposing a restriction of symmetry.
where $\alpha_A = (1 - P_B^2) \alpha^M$ and $\alpha_B = (1 - P_A^2) \alpha^M$.

**Proof.** The cases of symmetric and asymmetric network sizes are examined in turn:

(a) The optimality of $G(\cdot)$ for both intermediaries when $P_A = P_B = P$, such that $H(\cdot) = G(\cdot)$ and $F(\cdot) = G(\cdot)$ in equilibrium, can be shown by examining expected operating profit of intermediaries in the ranges $[0, \tilde{\alpha}]$, $[\tilde{\alpha}, \alpha^M]$, and greater than $\alpha^M$. From demand $E(T_A)$ and probabilities (70), it follows that:

$$E(\Pi_A) = \begin{cases} 
\alpha_A S P^2 & \text{if } \alpha_A < \tilde{\alpha} \leq \alpha^M \\
F(\alpha_A) (\alpha_A S P^2 (1 - P^2)) + (1 - F(\alpha_A)) (\alpha_A S P^2) & \text{if } \alpha_A \in [\tilde{\alpha}, \alpha^M] \\
0 & \text{if } \alpha_A > \alpha^M 
\end{cases}$$

Since it is never optimal for $A$ to set $\alpha_A$ below $\tilde{\alpha}$, the probability that $\alpha_A < \tilde{\alpha}$ is zero. Hence, randomisation over range $\alpha_A \in [\tilde{\alpha}, \alpha^M]$ need only be considered. $E(\Pi_A | \alpha_A \in [\tilde{\alpha}, \alpha^M])$ can be simplified to $\alpha_A S P^2 [1 - P^2 F(\alpha_A)]$. Hence, $A$ chooses $H(\cdot)$ to maximise $E(\Pi_A)$ over $[\tilde{\alpha}, \alpha^M]$:

$$\max_{dH} E(\Pi_A) = \int_{\tilde{\alpha}}^{\alpha^M} [\alpha_A S P^2 (1 - P^2 F(\alpha_A))] dH \quad (74)$$
Recalling that $H(\hat{\alpha}) = F(\hat{\alpha}) = 0$ and $H(\alpha^M) = F(\alpha^M) = 1$, the solution to (74) yields constant expected profit for $A$ over $[\hat{\alpha}, \alpha^M]$ equal to $E^M(\Pi_A)$. The analysis is symmetric for $B$ so:

$$\max_{dH} E(\Pi_A) = \max_{dF} E(\Pi_B) = \alpha^MSP^2(1 - P^2)$$ (75)

$A$’s expected payoff under any random draw $\alpha_A \in [\hat{\alpha}, \alpha^M]$ must be equal to (75). Similarly for random draw $\alpha_B \in [\hat{\alpha}, \alpha^M]$ by $B$. Solving from (74) and (75) yields optimal distributions $H(\cdot) = G(\cdot)$ and $F(\cdot) = G(\cdot)$, where distribution $G(\cdot)$ is described by equation (71).

(b) The optimality of $H(\cdot)$ and $F(\cdot)$ in (72) and (73), respectively, follows similarly for the case where $P_A \neq P_B$. Let $P_A < P_B$ and hence $\hat{\alpha}_A < \hat{\alpha}_B$.

From demand $E(T_A)$ and probabilities (70), it follows that:

$$E(\Pi_A) = \begin{cases} 
\alpha_A SP_A^2 & \text{if } \alpha_A < \hat{\alpha}_A \\
\alpha_A SP_A^2 & \text{if } \alpha_A \in [\hat{\alpha}_A, \hat{\alpha}_B] \\
F(\alpha_A)(\alpha_A SP_A^2(1 - P_B^2)) + (1 - F(\alpha_A)) (\alpha_A SP_A^2) & \text{if } \alpha_A \in [\hat{\alpha}_B, \alpha^M] \\
0 & \text{if } \alpha_A > \alpha^M
\end{cases}$$

The probability that $\alpha_A < \hat{\alpha}_A$ is zero, but there now exists a range of commission rates $[\hat{\alpha}_A, \hat{\alpha}_B]$, where $A$ finds it optimal to follow an undercutting strategy but $B$ finds the monopolistic strategy optimal. This gives rise to a positive probability that $\alpha_A < \hat{\alpha}_B \leq \alpha^M$. Hence $A$ chooses $H(\cdot)$ to maximise
the following:

\[
\max_{dH} E(\Pi_A) = \int_{\alpha_B}^{\alpha_M} [\alpha_A S P^2_A (1 - P^2_B F(\alpha_A))] dH + \int_{\alpha_A}^{\alpha_B} (\alpha_A S P^2_A) dH \quad (76)
\]

Solving (76) yields constant expected profit for A:

\[
\max_{dH} E(\Pi_A) = E^M(\Pi_A) = \alpha_M S P^2_A (1 - P^2_B) \quad (77)
\]

B chooses \( F(\cdot) \) to maximise:

\[
\max_{dF} E(\Pi_B) = \int_{\alpha_B}^{\alpha_M} [\alpha_B S P^2_B (1 - P^2_A H(\alpha_B))] dF \quad (78)
\]

Solving (78) yields constant expected profit for B:

\[
\max_{dF} E(\Pi_B) = E^M(\Pi_B) = \alpha_M S P^2_B (1 - P^2_A) \quad (79)
\]

From (76), (77), (78) and (79) it follows that the optimal strategy of A and B is to randomise commission rates according to distributions (72) and (73), respectively. ■

Intuitively, randomisation of commission rates prevents intermediaries from systematically undercutting each other, thereby allowing the expected operating profits \( E^M(\Pi_A) \) and \( E^M(\Pi_B) \) to be attained in equilibrium. That is, randomisation allows each intermediary to attain expected operating profit equal to that which would arise from the monopolistic segment of their net-
work under the monopoly commission rate.

This is summarised by Corollary (1).

**Corollary 1** In the unique, mixed strategy Nash equilibrium in commission rates:

(a) If \( P_A = P_B = P \), then each intermediary expects to receive constant operating profit equal to \( \alpha^M SP^2 (1 - P^2) \).

(b) If \( P_A \neq P_B \), then intermediaries A and B expect to receive constant operating profit equal to \( E^M (\Pi_A) = \alpha^M SP_A^2 (1 - P_A^2) \) and \( E^M (\Pi_B) = \alpha^M SP_B^2 (1 - P_B^2) \), respectively.

**Proof.** This follows directly from the proof of proposition (17). ■

An implication of randomised commission rates is that traders' payoffs from indirect trade through A and B, respectively, are random variables that mirror \( H(\alpha_A) \) and \( F(\alpha_B) \) when \( P_A \neq P_B \) and \( G(\alpha) \), when \( P_A = P_B \). The conflicting monopolistic and competitive forces for commission-setting in the fragmented duopoly give rise to a unique, non-cooperative mixed strategy Nash equilibrium where intermediaries randomise their commission rates. The range of commission rates over which randomisation takes place has an upper bound of \( \alpha^M \), imposed by \( R_a \), and a positive lower bound due to intermediaries' monopolistic outside option. While the upper bound for both commission rates is exogenously determined by traders' direct trade option, that in turn hinges on the level of information costs in the market, the lower bounds hinge on network sizes. In particular, an intermediary with a larger network, enjoys a
relatively larger set of exclusive trade matches, and thus behaves more monop-
olistically when randomising commission rates than does an intermediary with
a smaller network. Each intermediary’s expected operating profit is constant
and corresponds to expected monopoly profit from the monopolistic market
segment. Intermediaries invest in network development in stage 1, anticipat-
ing the implications of their decisions. It is the Nash equilibrium in network
sizes to which we now turn.

2.4 Stage 1 - Nash Equilibrium in Network Sizes

In this section we seek a Nash equilibrium, or Nash equilibria, in network sizes,
where these are set simultaneously and non-cooperatively by competing inter-
mediaries, each taking the network size of his rival, and the offer acceptance
strategy of traders, as given.

2.4.1 Network Investment Cost

Intermediaries are assumed to have access to the same technology for develop-
ing a network of contacts, where the total investment cost for a network of size
$P_I$ is denoted by $C(P_I)$. The network investment decisions of intermediaries
are analysed under two cost specifications:

(a) Linear cost: $C(P_I) = 2P_Ic$, where $c > 0$. Hence:

$$\frac{\partial C_I}{\partial P_I} = 2c > 0$$

(80)
The marginal cost of network expansion is constant.

(b) Convex cost: \( C(P_I) = 2P_Ic(i, P_I) \), where \( c(i, P_I) = \gamma i^\alpha P_I^2 \) and \( \alpha \geq 1 \), \( \gamma > 0 \). Hence:

\[
C(P_I) = 2\gamma i^\alpha P_I^3 \quad (81)
\]

\[
\frac{\partial C_I}{\partial P_I} = 6\gamma i^\alpha P_I^2 > 0, \quad \frac{\partial^2 C_I}{\partial P_I^2} = 12\gamma i^\alpha P_I > 0 \quad (82)
\]

\[
\frac{\partial C_I}{\partial i} = 2\alpha \gamma i^{\alpha-1} P_I^3 > 0 \quad (83)
\]

The marginal cost of network expansion is thus increasing monotonically in the level of information costs and network size, while convexity in network size is assumed. Cost parameter \( \gamma \) is a scale factor. In addition, let the probability of direct matching \( q(i) \) take the functional form \( q(i) = 1-i^\delta \), where \( \delta > \alpha \geq 1 \).

2.4.2 Stage 1 Expected Profit

In stage 1, intermediary \( A \) chooses \( P_A \) to maximise stage 1 expected profit, denoted by \( E^1(\Pi_A) \), taking \( P_B \) and \( \{R_a, R_s\} \) as given. Similarly, intermediary \( B \) chooses \( P_B \) to maximise \( E^3(\Pi_B) \), taking \( P_A \) and \( \{R_a, R_s\} \) as given. Intermediaries anticipate expected operating profit levels \( E^M(\Pi_A) \) and \( E^M(\Pi_B) \), respectively, to arise from the stage 2 commission-setting subgame. Hence,
$E^1(\Pi_A)$ and $E^1(\Pi_B)$ are given by:

$$
E^1(\Pi_A) = E^M(\Pi_A) - C(P_A) 
= \alpha^M S P_A^2 (1 - P_B^2) - C(P_A) 
$$

and:

$$
E^1(\Pi_B) = E^M(\Pi_B) - C(P_B) 
= \alpha^M S P_B^2 (1 - P_A^2) - C(P_B) 
$$

where $\alpha^M = 1 - q(i)$.

Equations (84) and (85) offer a general description of expected stage 1 profit, where the three polar market structures correspond to different configurations of $P_A$ and $P_B$:

1. Monopoly: if $P_B = 0$ and $P_A \in [0,1]$ then (84) and (85) yield $E^1(\Pi_A) = \alpha^M S P_A^2 - C(P_A)$ and $E^1(\Pi_B) = 0$. Thus $B$ is inactive and $A$ is a monopolist and vice versa if $P_A = 0$ and $P_B \in [0,1]$.

2. Bertrand duopoly: if $P_A = P_B = 1$ then (84) and (85) yield $E^1(\Pi_A) = E^1(\Pi_B) = -C(1) < 0$. Hence, $P_A = P_B = 1$ can never constitute a Nash equilibrium in network sizes.

3. Fragmented duopoly in intermediation services, where $P_A \in (0,1)$ and $P_B \in (0,1)$. Equations (84) and (85) allow for both symmetric and
asymmetric network size selection.

It follows that Bertrand duopoly can never arise in a subgame perfect equilibrium (SPE) of the game, since it does not constitute a Nash equilibrium in stage 1. Only monopoly and fragmented duopoly are thus consistent with SPE.

2.4.3 Linear Network-Building Costs

Substituting \( C (P_t) = 2P_t c \) into (84) and (85) yields:

\[
\begin{align*}
E^1(\Pi_A) &= \alpha^M S P_A^2 (1 - P_B^2) - 2c P_A \\
E^1(\Pi_B) &= \alpha^M S P_B^2 (1 - P_A^2) - 2c P_B
\end{align*}
\] (86) (87)

Equations (86) and (87) show that the expected profit of an intermediary is decreasing in the network size of the rival intermediary and increasing in his own network size. Intuitively, the larger the network size of the rival, the greater the measure of common matches as a result of network overlap; and hence the lower the expected operating profit arising from the mixed strategy Nash equilibrium in commission rates.

Examination of (86) and (87) shows there is no pair of network sizes \((P_A^*, P_B^*)\) that are best responses to each other and simultaneously satisfy \(P_A^* \in (0, 1]\) and \(P_B^* \in (0, 1]\) when network-building costs are linear.

The results are summarised by propositions (18) and (19).
Proposition 18 If network-building cost is linear, then there is no pure strategy Nash equilibrium in which both intermediaries are active.

Proof. Let \( \overline{P}_B (P_A) \) and \( \overline{P}_A (P_B) \) describe the locus of network size pairs along which \( E^1(\Pi_A) = 0 \) and \( E^1(\Pi_B) = 0 \), respectively. Hence, for a given network size \( P_A \), \( \overline{P}_B (P_A) \) gives the threshold level of \( P_B \) above which \( E^1(\Pi_A)|_{P_A} < 0 \). Similarly, for given network size \( P_B \), \( \overline{P}_A (P_B) \) gives the threshold level of \( P_A \) above which \( E^1(\Pi_B)|_{P_B} < 0 \). Rearranging \( E^1(\Pi_A) = 0 \) and \( E^1(\Pi_B) = 0 \) from equations (86) and (87) yields:

\[
\overline{P}_B (P_A) = \left( 1 - \frac{2c}{SP_A\alpha_M} \right)^{\frac{1}{2}}
\]

(88)

\[
\overline{P}_A (P_B) = \left( 1 - \frac{2c}{SP_B\alpha_M} \right)^{\frac{1}{2}}
\]

(89)

Proof by contradiction. Let \( (P^*_A, P^*_B) \) reflect a pure strategy Nash equilibrium in network sizes where \( P^*_A \in (0,1) \) and \( P^*_B \in (0,1) \). Consider the incentives of intermediary A. If \( \overline{P}_B (P^*_A) < P^*_B \), then it follows that \( E^1(\Pi_A)|_{P^*_A} < 0 \); but if A is making losses, \( P^*_A \) cannot be an optimal reply to \( P^*_B \). Recall that \( E^1(\Pi_A) \) is increasing in \( P_A \). It follows that the maximum expected profit that A can attain, given \( P^*_B \), is that which corresponds to \( P_A = 1 \). If \( E^1(\Pi_A)|_{P^*_B,P_A=1} > 0 \), then A's optimal reply to \( P^*_B \) is \( P_A (P^*_B) = 1 \). If \( E^1(\Pi_A)|_{P^*_B,P_A=1} < 0 \), then A's optimal reply is \( P_A (P^*_B) = 0 \).

Suppose \( P_A (P^*_B) = 1 \). Then B is making losses under \( P^*_B > 0 \). Thus B's optimal reply to \( P_A (P^*_B) = 1 \) is \( P^*_B (1) = 0 \).
Suppose instead that $P_A(P_B) = 0$. Then $P_B^* \in (0,1)$ is not an optimal reply to $P_A(P_B) = 0$ since $B$ can raise $E^1(\Pi_B)$ by increasing $P_B^*$ to 1. Thus $B$’s optimal reply to $P_A(P_B^*) = 0$ is $P_B(0) = 1$.

If instead $P_B^* < \bar{P}_B(P_A^*)$, then it follows that $E^1(\Pi_A)|P_A^* > 0$; but then interior network size $P_A^* \in (0,1)$ is not an optimal reply to $P_B^*$. $A$’s optimal reply is thus $P_A(P_B^*) = 1$ and arguments apply as above.

We can thus conclude that neither is $P_A^* \in (0,1)$ an optimal reply to $P_B^* \in (0,1)$, nor is $P_B^* \in (0,1)$ an optimal reply to $P_A^* \in (0,1)$. Moreover, since $P_B^*(1) = P_A^*(1) = 0$, $(P_A^*, P_B^*) = (1,1)$ cannot be a Nash equilibrium either. Thus $(P_A^*, P_B^*)$ cannot constitute a Nash equilibrium where both network sizes are non-zero. It follows that there is no Nash equilibrium in which both intermediaries are active.

**Proposition 19** If network-building cost is linear and provided $c < \frac{1}{2} S \alpha^M$, then there are two pure strategy Nash equilibria where $(P_A^*, P_B^*) = (1,0)$ and $(P_A^*, P_B^*) = (0,1)$.

**Proof.** It follows from proposition (18) that $(P_A^*, P_B^*)$ cannot constitute a Nash equilibrium where both network sizes are non-zero. The only remaining candidate Nash equilibria are $(P_A^*, P_B^*) = (1,0)$ and $(P_A^*, P_B^*) = (0,1)$. If $P_A^* = 1$, then there is no scope for $B$ to gain exclusive trade matches from investment in $P_B$. All resulting trade matches are common and thus no profit can be attained. The optimal reply of $B$ is thus $P_B^*(1) = 0$. A symmetric argument applies where $P_B^* = 1$. Hence, $(P_A^*, P_B^*) = (1,0)$ and $(P_A^*, P_B^*) = (0,1)$.
constitute the two pure strategy Nash equilibria in network sizes. Moreover, it follows directly from equations (86) and (87) that $c < \frac{1}{2}S\alpha^M$ must be satisfied for expected profit to be positive in equilibrium for the monopolist intermediary. ■

The analysis has shown that under linear costs of network expansion the equilibrium outcome of the game is monopolisation of the market by either $A$ or $B$. The non-convexity in network-building costs provides incentives for intermediaries to increase network size without bound, other than the constraint imposed by market size. Each intermediary provides complete coverage of the market, when active, thereby preventing the rival intermediary from gaining any exclusive trade matches.

Substituting $P_A = 1$ and $P_B = 1$ into (88) and (89), respectively, yields:

$$P = P_B(1) = P_A(1) = \left(1 - \frac{2c}{S\alpha^M}\right)^{\frac{1}{2}}$$

(90)

$P$ denotes the threshold level of $P_A$ below which it is optimal for $B$ to invest in a network size that covers the whole market and above which $B$ is inactive. By symmetry, $P$ is also the threshold for $P_B$. Figure (16) illustrates the reaction functions\(^{20}\) of $A$ and $B$. The $E^1(\Pi_A) = 0$ and $E^1(\Pi_B) = 0$ loci pin down threshold level $P$ for the two intermediaries and confirm the monopolisation of the market by either $A$ or $B$ (at NE$_2$ and NE$_1$, respectively) is the only market outcome consistent with profit maximisation under linear

\(^{20}\)The figure is drawn for parameter values $S = 10$, $c = 1$ and $\alpha^M = 0.7$. 

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costs of network expansion.

Figure 16: Monopoly Nash equilibria in network sizes.

2.4.4 Convex Network-Building Costs

This section shows how convexity in the costs of developing a network provides sufficient incentives for 'restraint' in network investment, so as to allow both intermediaries to survive in a fragmented duopoly with incomplete network overlap.

Note that in the analysis that follows we assume \( \gamma \) is sufficiently low relative to \( S \) so that \( E^1(\Pi_A) \) and \( E^1(\Pi_B) \) are positive in equilibrium. The results are summarised by proposition (20).

**Proposition 20** If network-building cost is convex, then there exists a unique,
pure strategy Nash equilibrium in network sizes where:

\[ P_A^* = P_B^* = \frac{3\gamma}{2S^{\delta - \alpha}} \left[ \left( 4 \left( \frac{S^{\delta - \alpha}}{3\gamma} \right)^2 + 1 \right)^{\frac{1}{2}} - 1 \right] \in (0, 1) \]

**Proof.** Substituting \( C(P_I) = 2P_I c(i, P_I) = 2\gamma^2 P^3_I \) and \( q(i) = 1 - i^\delta \) into (84) and (85), where \( \delta > \alpha \geq 1 \) and \( \gamma > 0 \), yields:

\[
\begin{align*}
E^1(\Pi_A) &= i^\delta S P_A^2 (1 - P_B^2) - 2\gamma i^\alpha P_A^3 \\
E^1(\Pi_B) &= i^\delta S P_B^2 (1 - P_A^2) - 2\gamma i^\alpha P_B^3
\end{align*}
\]

(91) (92)

The network size reaction functions of \( A \) and \( B \), denoted by \( P_A (P_B) \) and \( P_B (P_A) \), respectively, are derived from the first order conditions:

\[
\begin{align*}
\frac{\partial E^1(\Pi_A)}{\partial P_A} \bigg|_{P_B} &= 2i^\delta S P_A (1 - P_B^2) - 6\gamma i^\alpha P_A^2 = 0 \quad (93) \\
\frac{\partial E^1(\Pi_B)}{\partial P_B} \bigg|_{P_A} &= 2i^\delta S P_B (1 - P_A^2) - 6\gamma i^\alpha P_B^2 = 0 \quad (94)
\end{align*}
\]

The first order conditions (93) and (94) simplify to give:

\[
\begin{align*}
P_A (P_B) &= \frac{S^{\delta - \alpha}}{3\gamma} (1 - P_B^2) \quad (95) \\
P_B (P_A) &= \frac{S^{\delta - \alpha}}{3\gamma} (1 - P_A^2) \quad (96)
\end{align*}
\]

Solving the reaction functions simultaneously, and confirming that we have a maximum\(^\text{21}\), yields Nash equilibrium network sizes \((P_A^*, P_B^*)\) in terms of

\(^{21}\text{The second derivatives and confirmation that } P_A^* \text{ and } P_B^* \text{ correspond to a maximum can }\)

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information costs $i$, and parameters $\gamma$, $S$, $\delta$ and $\alpha$:

\[
P^* = P_A^* = P_B^* = \frac{3\gamma}{2S^2\delta - \alpha} \left[ \left(4 \left(\frac{S^2\delta - \alpha}{3\gamma}\right)^2 + 1\right)^{\frac{1}{3}} - 1 \right]
\]

(97)

Discarding complex and negative solutions to (93) and (94) confirms that (97) describes the unique Nash equilibrium in network sizes. ■

It follows directly from the reaction functions, (95) and (96), that network sizes are strategic substitutes. An increase in the network size of $A$ gives rise to a strategic contraction in the network investment of $B$, and vice versa. Intuitively, when intermediary $A$ invests in a larger network, the expected overlap between the two networks is larger, thereby lowering $E_M(\Pi_B)$. Hence, for given investment cost $C(P_B)$, $B$ can expect a lower revenue than before, thereby inducing a network contraction.

Moreover, network sizes are increasing in trade surplus $S$, declining in cost parameter $\gamma$, and increasing in information cost $i$ (since $\delta > \alpha \geq 1$).

Figure (17) illustrates\textsuperscript{22} $P_A(P_B)$ and $P_B(P_A)$ and depicts a unique, pure strategy Nash equilibrium in network sizes, in which both intermediaries invest symmetrically in network development. This arises from the symmetry in the costs incurred by $A$ and $B$. It is straightforward to show that when cost parameter $\gamma$ varies across intermediaries, the intermediary with the lower cost has a larger network size in equilibrium.

\textsuperscript{22}The figure is drawn for parameter values $\gamma = 1$, $\delta = 4$, $\alpha = 2$, $S = 4$ and $i = 0.8$. 

be found in Appendix B.
Figure 17: Fragmented duopoly Nash equilibrium in network sizes.

Nash equilibrium network sizes \((P_A^*, P_B^*)\) in terms of information costs \(i\), and parameters \(\gamma, S, \delta\) and \(\alpha\) are:

\[
P^* = P_A^* = P_B^* = \frac{3\gamma}{2Si\delta - \alpha} \left[ \left( 4 \left( \frac{Si\delta - \alpha}{3\gamma} \right)^2 + 1 \right)^{\frac{1}{2}} - 1 \right]
\]  

(98)

Let \(P^M\) denote the monopoly network size that prevails under the same cost specification. It follows directly from proposition (10) in Chapter 1 that:

\[
P^M = \frac{Si\delta - \alpha}{3\gamma} > P^*
\]  

(99)

Equations (98) and (99) describe the equilibrium monopoly and duopoly network sizes for network cost specification (81). Figure (18) illustrates the
path of network size with information costs in the two cases for parameter values $\gamma = 1$, $\delta = 4$, $\alpha = 2$, $S = 4$.

![Figure 18: Network size and information cost.](image)

To summarise, the analysis shows that either a monopoly or a fragmented duopoly can prevail in equilibrium, depending on the network-building technology. Under convexity assumptions, both intermediaries invest in a network and compete over common matches, while randomising commission rates. In contrast, linear network development costs can only give rise to a monopoly outcome.
2.5 Conclusion

This chapter presents a new theoretical framework to analyse the strategic interaction between two information intermediaries who compete in commission rates and network size. The intermediaries are assumed to have symmetric access to an information technology that allows them to develop contacts with importers and exporters who match uniquely in pairs. Intermediaries have the opportunity to invest in a network of contacts and subsequently compete in commission rates before making offers of intermediation to members in their network. Traders select between intermediaries ex post, when uncertainty in the realisation of a match is resolved.

The analysis delivers the following results. First, the model suggests that network competition between information intermediaries has a distinctive market structure, where intermediaries are monopolist service providers to some contacts but duopolists over contacts they share in their network overlap. Traders in the network overlap receive two intermediation offers, while other members are exclusive to one intermediary and thus receive only one offer of intermediation. Traders in receipt of two intermediation offers play a coordination game when deciding which offer to select. The information frictions in the model make it impossible for traders to signal their decisions to each other, so there are multiple equilibria to the game. The model thus emphasises the role of ‘beliefs’ in determining market outcomes when there are information frictions and traders gain from making coordinated decisions.
Second, the coordination game of traders presents the possibility of coordination failure between trade pairs, even though both traders are members of both networks and this is known to both.

Third, we show that if traders choose to accept the offer from the intermediary with the lower commission and randomise when commissions are the same, then intermediaries have an incentive to undercut each other. Moreover, intermediaries’ inability to price discriminate between the competitive and non-competitive market segments, gives rise to an undercutting game, which has no pure strategy Nash equilibrium due to the option to charge the monopoly commission to exclusive contacts, and relinquish the overlap to the rival. Randomising over the strategy space of commission rates results in a mixed strategy Nash equilibrium yielding expected profit equal to that which would have been earned in the monopolistic outside option. In this mixed strategy Nash equilibrium, an intermediary with a larger network sets a higher commission rate, on average, than an intermediary with a smaller network. Moreover, average commission rates lie below the monopoly commission rate. Hence, compared to the monopoly case in Chapter 1, traders who match indirectly enjoy a trade surplus over and above their outside option.

The multiplicity of equilibria of the coordination game and the randomisation of commission rates that results shows how information problems can give rise to endogenous uncertainty in market outcomes.

Finally, competition is affected by the technology of network development.
The analysis shows that either a monopoly or a fragmented duopoly can prevail in equilibrium, depending on the network-building technology. Under convexity assumptions, both intermediaries invest in a network and compete over common matches, while randomising commission rates. In contrast, linear network development costs can only give rise to a monopoly outcome.

References


Appendix A. The Coordination Game

Recall the payoff matrix summarising the payoff structure of traders’ simultaneous and non-cooperative coordination game in stage 3:

\[
\begin{array}{ccc}
 & \text{A} & \text{B} \\
\text{A} & \frac{s}{2} (1 - \alpha_A), \frac{s}{2} (1 - \alpha_A) & \frac{s}{2} (1 - \alpha_M), \frac{s}{2} (1 - \alpha_M) \\
\text{B} & \frac{s}{2} (1 - \alpha_M), \frac{s}{2} (1 - \alpha_M) & \frac{s}{2} (1 - \alpha_B), \frac{s}{2} (1 - \alpha_B) \\
\end{array}
\]

There are two pure strategy Nash Equilibria, (A, A) and (B, B), and one symmetric, mixed strategy Nash equilibrium. Suppose \(X_j\) selects A with probability \(\lambda\) (and B with \(1 - \lambda\)) and \(M_j\) selects A with probability \(\mu\) (and B with \(1 - \mu\)). For probabilities \((\lambda^*, \mu^*)\) to form a mixed strategy Nash equilibrium the expected payoffs from mixing between A and B must be equalised for each trader.

Equalising the expected payoff from the mixed strategy of \(X_j\) yields:

\[
\lambda (1 - \alpha_A) + (1 - \lambda) (1 - \alpha_M) = (1 - \lambda) (1 - \alpha_B) + \lambda (1 - \alpha_M)
\]  \hspace{1cm} (100)

Rearranging (100) yields:

\[
\lambda^* = \frac{\alpha_M - \alpha_B}{2\alpha_M - \alpha_B - \alpha_A}
\]  \hspace{1cm} (101)

Equalising the expected payoff from the mixed strategy of \(M_j\) yields:
\[
\mu (1 - \alpha_A) + (1 - \mu) (1 - \alpha^M) = (1 - \mu) (1 - \alpha_B) + \mu (1 - \alpha^M) \quad (102)
\]

Rearranging (102) yields:

\[
\mu^* = \frac{\alpha^M - \alpha_B}{2\alpha^M - \alpha_B - \alpha_A} \quad (103)
\]

Since \(\lambda^* = \mu^*\), the unique mixed strategy Nash equilibrium is symmetric.

The expected payoffs of \(X_j\) and \(M_j\) in the mixed strategy Nash equilibrium are found by substituting \(\lambda^*\) and \(\mu^*\) into each side of (100) and (102) yields:

\[
E(\Pi_j)\vert \lambda^*,\mu^* = \frac{S}{2} \left[ \frac{\alpha^M - \alpha_B}{2\alpha^M - \alpha_B - \alpha_A} (1 - \alpha_A) + \frac{\alpha^M - \alpha_A}{2\alpha^M - \alpha_B - \alpha_A} (1 - \alpha^M) \right]
\]

\[
= \frac{S}{2} \left[ \frac{\alpha^M - \alpha_A}{2\alpha^M - \alpha_B - \alpha_A} (1 - \alpha_B) + \frac{\alpha^M - \alpha_B}{2\alpha^M - \alpha_B - \alpha_A} (1 - \alpha^M) \right]
\]

**Appendix B. Second Derivatives**

The second derivatives that follow from (93) and (94) are:

\[
\frac{\partial^2 E^1(\Pi_A)}{\partial P_A^2} \bigg|_{P_B} = 2i^\delta S \left( 1 - P_B^2 \right) - 12i^\alpha P_A \quad (104)
\]

\[
\frac{\partial^2 E^1(\Pi_B)}{\partial P_B^2} \bigg|_{P_A} = 2i^\delta S \left( 1 - P_A^2 \right) - 12i^\alpha P_B \quad (105)
\]
To have a maximum, (104) and (105) must be negative. Hence, the following must hold in equilibrium:

\[
P_A > \frac{S_i^{\delta - \alpha}}{6\gamma} (1 - P_B^2) \tag{106}
\]

\[
P_B > \frac{S_i^{\delta - \alpha}}{6\gamma} (1 - P_A^2) \tag{107}
\]

Comparing (106) and (107) with (95) and (96) confirms the constraints are satisfied in equilibrium and thus that \( P_A^* \) and \( P_B^* \) correspond to a maximum.
3 International Trade, Minimum Quality Standards and the Prisoners’ Dilemma

This chapter extends a well-established vertical product differentiation model to a two-country framework in which international duopolists compete in quality and price in each market. Unilateral minimum quality standards are endogenously determined as the outcome of a non-cooperative standard-setting game between the governments of the two countries. The international context highlights the effects of cross-country externalities from the implementation of minimum quality standards that can be both positive, both negative, or asymmetric, depending on the quality of traded goods. These externalities are shown to give rise to a Prisoners’ Dilemma structure in the incentives of the two policy-makers that leads to inefficient policy outcomes. The role of minimum quality standards as non-tariff barriers is examined and the incentives and scope for international cooperation analysed.

The chapter contributes to both the international trade and industrial organisation literature in a number of ways.

First, the chapter extends the literature that examines the effects of minimum quality standards in markets where firms offer vertically differentiated products by analysing national incentives to regulate quality in an open-economy setting. The cross-country externalities generated when countries are linked through international trade are not present in the literature that studies quality standards in the context of a single economy.
Second, the chapter endogenously determines national minimum quality standards through the strategic interaction between policy-makers. To the best of my knowledge, this is the first analysis that endogenises national decisions to regulate quality in an international context. The industrial organisation literature has widely analysed the effects of minimum standards in a single country, but has done so by introducing minimum standards as exogenous constraints. Only recently has the issue of endogenous determination of quality standards begun to be addressed. Ecchia and Lambertini (1997) endogenously determine the minimum quality standard in the context of one country where a social planner sets the standard to maximise national welfare. This chapter extends to two policy-makers, each of which unilaterally selects their national minimum quality standard to maximise national welfare. The individually optimal standard are shown to be jointly suboptimal as a result of the cross-country externalities.

Third, the analysis contributes to the literature on international cooperation by examining whether bargaining from a non-cooperative Nash equilibrium in minimum quality standards can lead to an efficient outcome. The analysis follows the approach of the literature on cooperation in tariffs (e.g. Bagwell and Staiger, 1999, 2002; Staiger and Tabellini, 1987) but shows that endogenous country asymmetries arising from specialisation in goods of different quality levels introduce constraints to cooperation that do not arise in the literature on cooperation in tariffs.
The related literature on minimum quality standards originates in the industrial organisation literature, with the development of vertical quality differentiation models (e.g. Gabszewicz and Thisse, 1979; Shaked and Sutton, 1982). In his renowned paper, Ronnen (1991) uses Shaked and Sutton's framework to demonstrate that mild minimum quality standards are welfare improving in a duopoly where firms compete in prices and incur fixed quality-development costs. Similar results are obtained by Crampes and Hollander (1995) assuming that quality improvements increase variable rather than fixed costs.

The literature has more recently turned to open economy versions of the vertically differentiated duopoly framework. Motta and Thisse (1993) analyses the effects of environmental quality standards in autarky and free trade, while Boom (1995) analyses the effects of differing national standards for firms who cannot tailor quality to different markets. She finds that exit from the market with the more stringent standard may occur if the difference in national minimum standards is beyond a certain threshold, but does not determine national standards endogenously through a standard-setting game.

The model in this chapter differs from Boom (1995) and other works assuming up-front quality development costs (Ronnen 1991, Zhou et al., 2000, Herguera et al., 2002, among others), by assuming firms incur quality dependent variable costs (as in Motta, 1993, Crampes and Hollander, 1994, and Lutz, 2005). Hence the model applies more closely to industries where quality
improvements stem from higher quality of materials or ingredients or other factors embedded in the production process (e.g. textiles) rather than from innovative characteristics or design that arise from up-front investment in research and development (e.g. pharmaceuticals). The advantage of this cost specification is it gives firms the flexibility to tailor quality levels to different markets and to thus respond endogenously and asymmetrically to different quality standards.

The rest of the chapter proceeds as follows. Section 1 introduces the model and characterises the unregulated equilibria. Section 2 examines national incentives for standard-setting and solves for the non-cooperative Nash equilibria in minimum quality standards. The properties of these are examined and contrasted to world welfare-maximising international standards. International cooperation in setting quality standards is analysed in Section 3. Section 4 concludes.

3.1 The Model

This section describes the economic environment of the two-country model of vertical product differentiation and characterises the unregulated equilibria. This lays the groundwork for the rest of the chapter that analyses the non-cooperative standard-setting game between policy-makers and examines the scope and effects of international cooperation in standard-setting. The underlying quality differentiation model is closest to Motta (1993), Crampes and Hollander (1995) and Ecchia and Lambertini (1997) for a single economy.
3.1.1 Economic Environment

Consider two segmented markets, A and B, with a single firm located in each (firms A and B). The firms compete in quality and prices in each market, producing a vertically differentiated good. The firms can supply goods of a single quality level in each market but are able to differentiate the quality of their exports from the quality of their domestic sales. Let $q_{ij}$ be the quality level of the good produced in country $i$ (by firm $i$) and consumed in country $j$, where $i, j \in \{A, B\}$. We assume no upper bound to quality level so $q_{ij} \in [0, \infty)$. There is no potential entry of additional firms, but the duopolists may choose not to supply goods to either or both markets. Finally, we assume no transport costs.

The firms interact in a two-stage game. In the first stage, firms non-cooperatively select quality levels $q_{AA}, q_{AB}$ and $q_{BA}, q_{AB}$, respectively. Perfect and costless commitment to these quality levels is assumed. Firms compete in prices in the second stage, given first stage quality levels. Firms have access to the same production technology, which involves variable costs that are convex in quality and linear in quantity. No sunk costs of quality development are assumed. Let $V(S_{ij}, q_{ij})$ denote variable costs of production as a function of quality, $q_{ij}$, and sales, $S_{ij}$, of firm $i$ in market $j$, where these are as in (108):

\[ V(S_{ij}, q_{ij}) \]

Note that the absence of fixed sunk costs of quality development implies the equilibrium choice of qualities and prices would not change if firms chose these simultaneously and non-cooperatively, rather than sequentially and non-cooperatively. The sequential structure is preserved for purposes of comparability with the related literature.
\[ V(S_{ij}, q_{ij}) = bq_{ij}^2 S_{ij}, \text{ where } b > 0 \]  

(108)

Convexity in quality is both necessary and sufficient for the existence of an interior solution\(^{24}\) for quality choice in stage 1, and can thus be accompanied by an unlimited range for quality level \(q_{ij}\).

The demand side is assumed to consist of a continuum of consumers in each market with a varying taste parameter\(^{25}\), \(\theta\). Consumers are uniformly distributed, with unit density, over the interval \([0, \bar{\theta}]\) and derive utility from the first unit of purchase only. The indirect utility function of a consumer with taste parameter \(\theta\), purchasing a unit of a good with quality \(q\) and price \(p\) is described by (109):

\[ U = \theta q - p \]  

(109)

Given firms' decisions \((q_{ij}, p_{ij})\), consumers in each market choose between (i) purchasing one unit of the good from firm \(A\), (ii) purchasing one unit of the good from firm \(B\), or (iii) making no purchase. Consumers receive zero utility

\(^{24}\)There is no interior solution for firm quality levels if the variable cost function is non-convex. For example, if variable costs are linear in quality, e.g. \(V(q) = cq\), the low quality firm's best response is a quality level \(q\) that of the high quality firm, while the high quality firm adds to profit by raising quality without limit (e.g. Choi and Shin, 1992). A solution can only be pinned down by assuming quality has a finite upper limit, \(\bar{q}\), where the high quality firm locates.

\(^{25}\)Parameter \(\theta\) may also be interpreted as the marginal rate of substitution between income and quality such that a consumer with a higher \(\theta\) has a lower marginal rate of substitution between income and quality and thus a higher income. With this interpretation the framework presented is analogous to models where consumers vary in their income level rather than preference over quality e.g Gabszewicz and Thisse (1979), Shaked and Sutton (1982).
if they do not buy the good. Note that since the minimum value for $\theta$ is zero, there is always a measure of consumers who prefer not to buy the good when prices are positive, implying incomplete market coverage. Parameter $\bar{\theta}$ measures market size, which is symmetric in both countries.

Further suppose the governments of countries $A$ and $B$ have the opportunity to regulate quality in their market by unilaterally setting minimum quality standards in a stage 0, prior to the strategic interaction of firms. Governments choose standards to maximise national welfare, anticipating firms' optimal price and quality responses.

The solution concept employed to solve the multi-stage game is subgame perfect equilibrium (SPE), found by backward induction. First, the unique second-stage equilibrium in prices is analysed and firms' payoffs described in terms of first-stage qualities. Second, optimal first stage quality decisions of firms are examined. This allows the unregulated equilibria to be characterised. Then, the effects of minimum quality standards are examined, yielding the regulated optimal quality responses of firms. Finally, the incentives for standard setting are analysed and the non-cooperative Nash equilibria in minimum standards characterised. The issue of international cooperation in standard setting is analysed in the next section of the chapter.

3.1.2 The Price-Setting Subgame

Firms $A$ and $B$ compete in prices in each market in the final stage of the game, given stage 1 quality levels. It is common practice in the industrial organisation
literature with one market to arbitrarily assign one firm as high quality, solving for equilibrium prices and qualities assuming an exogenous quality ranking between firms. Rather than assigning a particular quality ordering between firms in each market, we suppose that in each market \( j \), there is a 'high' quality supplier (\( H \)), with quality level \( q_{Hj} \), and a 'low' quality supplier (\( L \)), with quality level \( q_{Lj} \), where \( q_{Hj} \geq q_{Lj} \). This allows for the possibility that firms choose identical quality levels in stage 1. The associated prices levels of these goods are denoted by \( p_{Hj} \) and \( p_{Lj} \), respectively. We proceed to characterise equilibrium prices and qualities for the \( H \) and \( L \) quality goods supplied in each market and then examine the multiple equilibria that correspond to different configurations of quality rankings of firms \( A \) and \( B \) in the two markets.

Hence, in the final stage of the game, firm quality levels are fixed in market \( j \), such that \( q_{Hj} \geq q_{Lj} \). Let \( x_{Hj} \) and \( x_{Lj} \) denote quality-deflated prices for the \( H \) and \( L \) goods, respectively, and let \( r_j \) denote the quality ratio, such that:

\[
x_{Hj} = \frac{p_{Hj}}{q_{Hj}} \quad \text{and} \quad x_{Lj} = \frac{p_{Lj}}{q_{Lj}}
\]

(110)

\[
r_j = \frac{q_{Hj}}{q_{Lj}}
\]

(111)

Consider the structure of demand for the two goods. Let \( z_j \) denote the preference parameter of the marginal consumer in market \( j \) who is indifferent between purchasing one unit of the good of quality \( q_{Hj} \) at price \( p_{Hj} \) and one unit of the good of quality \( q_{Lj} \) at price \( p_{Lj} \). The marginal consumer \( z_j \) follows
directly from (109) and is given by:

\[
z_j = \frac{p_{Hj} - p_{Lj}}{q_{Hj} - q_{Lj}} = \frac{r_j x_{Hj} - x_{Lj}}{r_j - 1}
\]  

(112)

Moreover, let \( k_j \) denote the preference level of the consumer who is indifferent between buying the differentiated good and not making a purchase. Consumption of one unit of the good of quality \( q_{Lj} \) at price \( p_{Lj} \) yields zero utility for this consumer, so from (109) it follows that \( k_j \equiv x_{Lj} = \frac{p_{Lj}}{q_{Lj}} \). Hence, consumers with preference parameter \( z_j < \theta \leq \bar{\theta} \) purchase good \( H \) and consumers for whom \( x_{Lj} \leq \theta < z_j \) purchase good \( L \). Consumers with \( 0 \leq \theta < x_{Lj} \) make no purchase.

The quantity demand for \( H \) and \( L \) goods in market \( j \), and thus firm sales, are denoted by \( S_{Hj} \) and \( S_{Lj} \), respectively, and given by:

\[
S_{Hj} = \bar{\theta} - z_j = \bar{\theta} - \frac{p_{Hj} - p_{Lj}}{q_{Hj} - q_{Lj}}
\]  

(113)

\[
S_{Lj} = z_j - x_{Lj} = \frac{p_{Hj} - p_{Lj}}{q_{Hj} - q_{Lj}} - \frac{p_{Lj}}{q_{Lj}}
\]  

(114)

The corresponding profits, \( \Pi_{Hj} \) and \( \Pi_{Lj} \), from \( H \) and \( L \) sales in market \( j \), are thus:
Substituting for $z_j$ and rearranging yields:

$$\Pi_{Hj} = \frac{1}{r_j - 1} q_{Hj} (x_{Hj} - bq_{Hj}) \left( \bar{\theta} (r_j - 1) - r_j x_{Hj} + x_{Lj} \right)$$  \hspace{1cm} (117)$$

$$\Pi_{Lj} = \frac{r_j}{r_j - 1} q_{Hj} (x_{Lj} - bq_{Lj}) (x_{Hj} - x_{Lj})$$  \hspace{1cm} (118)$$

Maximising $\Pi_{Hj}$ with respect to $x_{Hj}$, given $x_{Lj}$ and $r_j$, yields the quality-deflated price reaction function ($R_H$) of firm $H$ in $j$:

$$x_{Hj} (x_{Lj}) = \frac{1}{2r_j} \left[ \bar{\theta} (r_j - 1) + br_j q_{Hj} + x_{Lj} \right]$$ \hspace{1cm} (119)$$

Maximising $\Pi_{Lj}$ with respect to $x_{Lj}$, given $x_{Hj}$ and $r_j$, yields the reaction function ($R_L$) of firm $L$ in $j$:

$$x_{Lj} (x_{Hj}) = \frac{1}{2} (bq_{Lj} + x_{Hj})$$ \hspace{1cm} (120)$$

The slopes of the price reaction functions, $\frac{dx_{Hj}}{dx_{Lj}} |_{R_H} = \frac{1}{2r_j}$ and $\frac{dx_{Hj}}{dx_{Lj}} |_{R_L} = 2$, respectively, confirm prices are strategic complements. Moreover, if $q_{Hj} > q_{Lj}$ in stage 1, then $r_j > 1$ and $x_{Hj} > x_{Lj}$, while if $q_{Hj} = q_{Lj}$ in stage 1, then
\( r_j = 1 \) and (119) and (120) imply that \( x_{Hj} = x_{Lj} \).

Solving (119) and (120) simultaneously yields the unique Nash equilibrium in quality-deflated prices, in terms of parameters \( b, \bar{\theta}, \) and firm quality levels:

\[
x_{Hj} = \frac{1}{4q_{Hj} - q_{Lj}} \left( 2q_{Hj}\bar{\theta} - 2q_{Lj}\bar{\theta} + 2bq_{Hj}^2 + bq_{Lj}^2 \right)
\]

\[
x_{Lj} = \frac{1}{4q_{Hj} - q_{Lj}} \left( q_{Hj}\bar{\theta} - q_{Lj}\bar{\theta} + bq_{Hj}^2 + 2bq_{Hj}q_{Lj} \right)
\]

Substituting (121) and (122) into (112) yields the equilibrium marginal consumer \( z_j \):

\[
z_j = \frac{1}{4q_{Hj} - q_{Lj}} \left( 2q_{Hj}\bar{\theta} - q_{Lj}\bar{\theta} + 2bq_{Hj}^2 + bq_{Hj}q_{Lj} \right)
\]

Equilibrium prices follow directly from (121) and (122):

\[
p_{Hj} = \frac{q_{Hj}}{4q_{Hj} - q_{Lj}} \left( 2\bar{\theta} (q_{Hj} - q_{Lj}) + 2bq_{Hj}^2 + bq_{Lj}^2 \right)
\]

\[
p_{Lj} = \frac{q_{Lj}}{4q_{Hj} - q_{Lj}} \left( \bar{\theta} (q_{Hj} - q_{Lj}) + bq_{Hj}^2 + 2bq_{Hj}q_{Lj} \right)
\]

Inspection of (124) and (125) reveals the quality gap, \( q_{Hj} - q_{Lj} \), to be a key determinant of prices. If firms choose identical quality levels \( q_{Hj} = q_{Lj} = q_j \), then the quality gap is zero and prices collapse to marginal cost, \( p_{Hj} = p_{Lj} = bq_j^2 \). The Bertrand outcome for homogeneous goods where price is equal to marginal cost and firm profits are zero results. This drives the
quality differentiation result of the literature, confirmed in the next section.

3.1.3 Nash Equilibrium in Firm Qualities

Anticipating the price implications of their quality decisions, firms set quality levels non-cooperatively in stage 1. Substituting (124) and (125) into (113) and (114) gives demands in terms of stage 1 qualities:

\[ S_{Hj} = \bar{\theta} - z_j = \frac{q_{Hj}}{4q_{Hj} - q_{Lj}} \left(2\bar{\theta} - 2bq_{Hj} - bq_{Lj}\right) \]  
\[ S_{Lj} = z_j - x_{Lj} = \frac{q_{Hj}}{4q_{Hj} - q_{Lj}} \left(\bar{\theta} + bq_{Hj} - bq_{Lj}\right) \]  

It is now straightforward to express firms' first stage profits as a function of quality levels, market size and the cost parameter:

\[ \Pi_{Hj}(q_{Hj}, q_{Lj}) = (q_{Hj} - q_{Lj}) \frac{q_{Hj}^2 \left(2\bar{\theta} - 2bq_{Hj} - bq_{Lj}\right)^2}{(4q_{Hj} - q_{Lj})^2} \]  
\[ \Pi_{Lj}(q_{Hj}, q_{Lj}) = (q_{Hj} - q_{Lj}) \frac{q_{Hj}q_{Lj} \left(\bar{\theta} + bq_{Hj} - bq_{Lj}\right)^2}{(4q_{Hj} - q_{Lj})^2} \]  

The profit equations confirm that firms can only earn positive profits when a quality gap is established in stage 1. Profits are affected by quality choice in two ways. Firms trade off the cost of producing a higher quality good with the higher price made possible by the higher quality level, but also consider the disparity in quality levels, which affects the intensity of price competition. The quality levels in the unregulated equilibrium are found by solving the
following system of first-order conditions, that captures these two effects:

\[
\frac{\partial \Pi_{Hj}(q_{Hj}, q_{Lj})}{\partial q_{Hj}} = \frac{q_{Hj}(2bq_{Hj} + bq_{Lj} - 2\theta)}{(4q_{Hj} - q_{Lj})^3} (24bq_{Hj}^3 - 22bq_{Hj}^2 q_{Lj} + 5bq_{Hj}q_{Lj}^2 + 2bq_{Lj}^3 - 4q_{Hj}^2 \bar{\theta} + 6q_{Hj}q_{Lj} \bar{\theta} - 8q_{Lj}^2 \bar{\theta}) = 0
\]

(130)

\[
\frac{\partial \Pi_{Lj}(q_{Hj}, q_{Lj})}{\partial q_{Lj}} = \frac{q_{Lj}(2bq_{Hj} + bq_{Lj} - 2\theta)}{(4q_{Hj} - q_{Lj})^3} (4bq_{Hj}^3 - 19bq_{Hj}^2 q_{Lj} + 17bq_{Hj}q_{Lj}^2 - 2bq_{Lj}^3 + 4q_{Hj}^2 \bar{\theta} - 7q_{Hj}q_{Lj} \bar{\theta} = 0
\]

(131)

The first order conditions simplify to (132) and (133), implicitly define the quality reaction functions of the two firms, \(q_{Hj}(q_{Lj})\) and \(q_{Lj}(q_{Hj})\):

\[
24bq_{Hj}^3 - 22bq_{Hj}^2 q_{Lj} + 5bq_{Hj}q_{Lj}^2 + 2bq_{Lj}^3 - 4q_{Hj}^2 \bar{\theta} + 6q_{Hj}q_{Lj} \bar{\theta} - 8q_{Lj}^2 \bar{\theta} = 0
\]

(132)

\[
4bq_{Hj}^3 - 19bq_{Hj}^2 q_{Lj} + 17bq_{Hj}q_{Lj}^2 - 2bq_{Lj}^3 + 4q_{Hj}^2 \bar{\theta} - 7q_{Hj}q_{Lj} \bar{\theta} = 0
\]

(133)

Confirming a maximum, the analytical expressions for \(q_{Hj}(q_{Lj})\) and \(q_{Lj}(q_{Hj})\) can be found. These are not included in the main text due to their length, but can be found in Appendix A. The reaction functions are illustrated for parameter values \(\bar{\theta} = 5\) and \(b = \frac{1}{2}\) in figure (19), and shown to be positively sloped,
indicating that firm quality levels are strategic complements. The intuition behind the upward sloping $q_{Hj}(q_{Lj})$ reaction function is straightforward. A rise in $q_{Lj}$ narrows the quality gap, thereby intensifying stage 2 price competition. The high quality firm thus has an incentive to increase its quality in order to widen the quality gap and alleviate competition. The convexity of costs with respect to quality ensures it is not optimal for the high quality firm to fully offset the impact of higher $q_{Lj}$ and thus $q_{Hj}(q_{Lj})$ has a slope less than 1.

![Diagram](image_url)

Figure 19: Nash equilibrium in qualities.

The upward sloping reaction function of the low quality firm, $q_{Lj}(q_{Hj})$, is less straightforward, since an increase in $q_{Hj}$ widens the quality gap, relaxing price competition. The intuition behind the relationship is that the low quality
firm’s revenue is increasing in its own quality level, but its ability to raise $q_{Lj}$ is constrained by the stronger price competition that ensues. The alleviation of price competition through a higher $q_{Hj}$ thus permits an increase in $q_{Lj}$, that would otherwise not be optimal.

Since product differentiation relaxes \textit{ex post} price competition\footnote{This result is reminiscent of the well-known result of Kreps and Scheinkman (1983), where duopolists choose capacity constraints and then compete in prices. The incentive to constrain output in order to alleviate price competition gives rise to the Cournot outcome. Commitment to quality differentiation serves a similar purpose in the vertical product differentiation literature, but there are key differences; quality-differentiated goods command different prices, so firms are asymmetric in equilibrium. In contrast, capacity constraints serve to uniformly raise price above marginal cost preserving the symmetry between firms.}, firms find it optimal to offer distinct quality levels in equilibrium. Solving (132) and (133) simultaneously yields the unregulated equilibrium quality levels $q^{UR}_{Hj} = 0.40976\theta^\theta_k$ and $q^{UR}_{Lj} = 0.19936\theta^\theta_k$. They are increasing in market size, but decreasing in the cost parameter of the model.

In the context of the two-country model, either high or low quality goods are imported by $j$. The larger the market size of country $j$, or the higher is the highest income level (depending on the interpretation of $\theta$), then the higher the quality of traded goods of a given type. The results of the model are thus consistent with recent empirical studies that find a positive relationship between the quality of traded goods and country size and income (Hallak, 2006; Hummels and Klenow, 2005).

Substituting equilibrium qualities $q^{UR}_{Hj} = 0.40976\theta^\theta_k$ and $q^{UR}_{Lj} = 0.19936\theta^\theta_k$ into the prices, sales, and profit equations fully characterises the unregulated equilibrium. The results are reported in table (1).
The consumer surplus in \( j \), \( C_j \), is comprised by the surplus of consumers of high quality goods \( H \), denoted by \( C_{Hj} \), and of consumers of low quality goods \( L \), denoted by \( C_{Lj} \). Integrating utility over the relevant range of consumers gives the expressions (134), (135) and (136), expressed in terms of firm quality levels and model parameters \( \overline{\theta} \) and \( b \). Substituting for equilibrium quality levels yields consumer surplus in the unregulated equilibrium, reported in table (2).

\[
C_{Hj} = \int_{\overline{q}_H}^{\overline{q}_H} (\theta q_{Hj} - p_{Hj}) \ d\theta \\
= \frac{q_{Hj}^2}{2(4q_{Hj} - q_{Lj})^2} \left( b q_{Hj} \overline{\theta} + b q_{Lj} \overline{\theta} - 2b q_{Lj}^2 - 2b q_{Hj}^2 + b q_{Lj} q_{Hj} \right) \tag{134}
\]

\[
C_{Lj} = \int_{\overline{q}_L}^{\overline{q}_L} (\theta q_{Lj} - p_{Lj}) \ d\theta \\
= \frac{q_{Lj}^2}{2(4q_{Hj} - q_{Lj})^2} \left( \overline{\theta} + b q_{Hj} - b q_{Lj} \right)^2 \tag{135}
\]

\[
C_j = C_{Hj} + C_{Lj} \\
= \frac{q_{Hj}^2}{2(4q_{Hj} - q_{Lj})^2} \left( 3b^2 q_{Lj}^3 - 2b q_{Lj} q_{Hj} \overline{\theta} + b^2 q_{Hj}^3 + b^2 q_{Lj} q_{Hj}^2 \right) + b^2 q_{Lj} q_{Hj} + 5q_{Lj} \overline{\theta}^2 - 8b q_{Lj} q_{Hj} \overline{\theta}^2 - 8b q_{Hj} q_{Lj} \overline{\theta} \right) \tag{136}
\]

### 3.1.4 Firm Quality Rankings and Multiple Equilibria

Sections 1.2 and 1.3. solve for unregulated equilibrium prices and qualities in each market, without specifying which of the two firms, \( A \) or \( B \), is the high or low quality supplier in each market. The assumption that firms can freely choose quality levels for domestic and export sales and the absence of
Table 2: The Unregulated Market

<table>
<thead>
<tr>
<th>The Unregulated Market $j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality levels</td>
</tr>
<tr>
<td>$q_{UR}^j = 0.40976^f$</td>
</tr>
<tr>
<td>$q_{LR}^j = 0.19936^f$</td>
</tr>
<tr>
<td>Quality ratio</td>
</tr>
<tr>
<td>$r_j = 2.0554$</td>
</tr>
<tr>
<td>Quality gap</td>
</tr>
<tr>
<td>$q_{Hj} - q_{Lj} = 0.2104^f$</td>
</tr>
<tr>
<td>Prices</td>
</tr>
<tr>
<td>$p_{Hj} = 0.22666^f$</td>
</tr>
<tr>
<td>$p_{Lj} = 0.075010^f$</td>
</tr>
<tr>
<td>Quality-deflated prices</td>
</tr>
<tr>
<td>$x_{Hj} = 0.53314^f$</td>
</tr>
<tr>
<td>$x_{Lj} = 0.37625^f$</td>
</tr>
<tr>
<td>Marginal consumer</td>
</tr>
<tr>
<td>$z_j = 0.72075^f$</td>
</tr>
<tr>
<td>Sales</td>
</tr>
<tr>
<td>$S_{Hj} = 0.27925^f$</td>
</tr>
<tr>
<td>$S_{Lj} = 0.3445^f$</td>
</tr>
<tr>
<td>Profits</td>
</tr>
<tr>
<td>$\Pi_{Hj} = 0.016407^f$</td>
</tr>
<tr>
<td>$\Pi_{Lj} = 0.012149^f$</td>
</tr>
<tr>
<td>Consumer surplus</td>
</tr>
<tr>
<td>$C_j = 0.046985^f$</td>
</tr>
</tbody>
</table>

1. Each firm supplies its home market with low quality goods and exports high quality goods, so $q_{AA} < q_{BA}$ and $q_{AB} > q_{BB}$. Countries trade in high quality goods.

2. Each firm supplies its home market with high quality goods and exports low quality goods, so $q_{AA} > q_{BA}$ and $q_{AB} < q_{BB}$. Countries trade in low quality goods.

3. Firm $A$ is the world high quality supplier, so $q_{AA} > q_{BA}$ and $q_{AB} > q_{BB}$. Country $B$ imports high quality goods from $A$ while $A$ imports low quality goods from $B$.

4. Firm $B$ is the world high quality supplier, so $q_{AA} < q_{BA}$ and $q_{AB} < q_{BB}$. Country $A$ imports high quality goods from $B$ while $B$ imports low quality goods from $A$. 
Hence the model gives rise to three possible trade patterns. Countries may trade in high quality goods, low quality goods, or bilateral trade may be in goods of different quality levels (equilibria 3 and 4 are symmetric).

The welfare of a country is measured as the sum of consumer surplus and profits of the domestic firm from domestic sales and exports. The welfare of each country thus depends on the quality rankings of firms in each market, and thus the pattern of trade in equilibrium. While world welfare, denoted by $W$, is unchanged between equilibria, the distribution of welfare between countries varies. Consumer surplus is symmetric between countries at $C_j = 0.046985\frac{p^3}{r}$ but the higher profits earned from $H$ sales in the unregulated equilibrium imply higher welfare from a higher quality ranking of firm $j$ relative to the foreign firm.

Equations (137) to (140) give the welfare equations for country $A$ under the four equilibrium configurations:

\begin{align*}
W_{\text{AA}<q_{BA},q_{AB}<q_{BB}}^A &= C_A + \Pi_{LA} + \Pi_{HB} \quad (137) \\
W_{\text{AA}>q_{BA},q_{AB}<q_{BB}}^A &= C_A + \Pi_{HA} + \Pi_{LB} \quad (138) \\
W_{\text{AA}>q_{BA},q_{AB}>q_{BB}}^A &= C_A + \Pi_{HA} + \Pi_{HB} \quad (139) \\
W_{\text{AA}<q_{BA},q_{AB}>q_{BB}}^A &= C_A + \Pi_{LA} + \Pi_{LB} \quad (140)
\end{align*}
Correspondingly, equations (141) to (144) describe welfare for country B under the four equilibrium configurations:

\[
\begin{align*}
W^B_{qAA < qBA, qAB > qBB} & = C_B + \Pi_{HA} + \Pi_{LB} \\
W^B_{qAA > qBA, qAB < qBB} & = C_B + \Pi_{LA} + \Pi_{HB} \\
W^B_{qAA > qBA, qAB > qBB} & = C_B + \Pi_{LA} + \Pi_{LB} \\
W^B_{qAA < qBA, qAB < qBB} & = C_B + \Pi_{HA} + \Pi_{HB}
\end{align*}
\]  

Combining the unregulated market outcome reported in table (2) with welfare equations (137) to (140) and (141) to (144) yields the unregulated welfare levels in table (3) for the four equilibrium configurations.

<table>
<thead>
<tr>
<th>Firm Rankings</th>
<th>( W^A )</th>
<th>( W^U )</th>
<th>( W )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) ( qAA &lt; qBA, qAB &gt; qBB )</td>
<td>0.075541</td>
<td>0.075541</td>
<td>0.151082</td>
</tr>
<tr>
<td>(2) ( qAA &gt; qBA, qAB &lt; qBB )</td>
<td>0.075541</td>
<td>0.075541</td>
<td>0.151082</td>
</tr>
<tr>
<td>(3) ( qAA &gt; qBA, qAB &gt; qBB )</td>
<td>0.079799</td>
<td>0.071283</td>
<td>0.151082</td>
</tr>
<tr>
<td>(4) ( qAA &lt; qBA, qAB &lt; qBB )</td>
<td>0.071283</td>
<td>0.079799</td>
<td>0.151082</td>
</tr>
</tbody>
</table>

Table 3: Welfare distribution in the unregulated equilibria.

Table (3) shows the asymmetric distribution of welfare in equilibria (3) and (4) where national firms are either world quality leaders or world low quality suppliers. Equilibria (1) and (2) give rise to symmetric welfare effects in the unregulated equilibrium. This symmetry is not preserved, however,
in the regulated equilibrium where governments set minimum standards non-cooperatively, as is made clear in the next section.

3.2 Non-Cooperative Minimum Quality Standards

This section endogenises the choice of minimum quality standards in the two countries, where these are the result of a non-cooperative standard-setting game between the governments of $A$ and $B$. The objective function of each policy-maker is to maximise national welfare, taking as given the minimum standard of the other country, while anticipating the optimal quality response of the high quality firm and ensuing duopolistic price competition.

The industrial organisation literature on minimum quality standards has only recently endogenised the choice of national minimum quality standard (Ecchia and Lambertini, 1997), prior to which standards were modelled as exogenous constraints. This chapter extends the analysis to examine the incentives for standard-setting in an international context, thereby showing the effects of cross-country externalities and the role of trade patterns in shaping national incentives. Section 1.4 establishes the three possible trade patterns that arise as equilibria of the two-country model. These in turn correspond to three non-cooperative Nash equilibria in minimum standards.

The section first examines the effects of minimum quality standards on key market variables that influence national decisions. The government reaction functions in minimum standards are then examined and the Nash equilibria in minimum quality standards characterised. These are contrasted to the world
optimum pair of quality standards and the role of unilateral standards as non-tariff barriers to trade is analysed.

3.2.1 The Effects of Minimum Quality Standards

The welfare improving effects of minimum quality standards in a single market with two price-competing duopolists was first found in Ronnen (1991) and is a feature of the subsequent literature, such as Crampes and Hollander (1995) and more recently Ecchia and Lambertini (1997). It is common to the literature that the intensity of price competition induces a greater degree of quality differentiation than is optimal from a social welfare perspective, which a minimum standard can correct by narrowing the quality gap between the two goods and raising both quality levels.

As the next few sections show, the incentive to regulate is also a feature of the two-country model, but the open economy characteristics of the framework distort policy-makers’ incentives to correct the inefficiency in quality-differentiation. This section analyses the effects of a minimum quality standard, $s^j$, in country $j$, such that $s^j > q_{Lj}^R$. The related industrial organisation literature usually examines the effects of ‘mild’ minimum standards as exogenous constraints, defined as a standard in between the two unregulated qualities, $q_{Hj}^U > s^j > q_{Lj}^U$. Since $s^j$ is determined endogenously in this model as a result of a strategic game between policy-makers, we prefer not to restrict policy-makers’ strategy space through a priori assumptions about whether unilaterally selected standard are ‘mild’ or ‘severe’, i.e. $s^j \geq q_{Hj}^U > q_{Lj}^U$. The
effects of $s^j$ on key market variables, for the range of values consistent with both firms remaining in market $j$, are summarised by the following:

(i) The quality levels of both firms increase and the degree of quality differentiation decreases. $q_{Lj} > q_{Lj}^{UR}$ as a result of the binding standard, and $q_{Hj}(s^j) > q_{Hj}^{UR}$ as a result of the strategic complementarity between quality levels. The standard has the effect of raising the quality of the low quality firm closer to that of its rival. As discussed in section 1.3, the optimal response for the high quality firm to raise its own quality level to alleviate the price competition induced by the implementation of a minimum quality standard. The convexity of costs ensures that the high quality firm’s quality rises less than proportionally, as a result of the trade-off between the intensified price competition that a smaller quality gap implies and the convex costs of quality improvement. The high quality firm’s quality response to minimum standard standard $s^j$ found by substituting $q_{Lj} = s^j$ into the high quality firm’s reaction function, implicitly defined by (132), which yields $q_{Hj}(s^j)$, the optimal response of the high quality firm to standard $s^j$. Figure (20) illustrates the path of quality levels and the quality ratio $r^j$ with minimum standard $s^j$ for $\tilde{\theta} = 5$ and $b = \frac{1}{2}$. As the severity of the minimum standard increases, the quality ratio converges to 1, while the quality gap converges to zero. Let $s^{Pj}$ denote the ‘prohibitive’ minimum standard in country $j$.  

\footnote{The analytical expression for $q_{Hj}(s^j)$ can be found in Appendix A.}
\( j \), at which quality levels are equal. The ensuing price competition is
at its strongest and both firms earn zero profit. \( s^{Pj} \) is thus the highest standard consistent with the survival of both firms in market \( j \). For
standards \( s > s^{Pj} \), firms would make losses giving rise to exit. Solving
\( q_{Hj}(s^{Pj}) = q_{Lj} = s^{Pj} \) yields the general expression for \( s^{Pj} \):

\[
s^{Pj} = \frac{2 \bar{\theta}}{3b}
\]

The larger the market, or the lower is firm cost, then the higher the maximum standard consistent with a duopolistic outcome. For \( \bar{b} = 5 \)
and \( b = \frac{1}{2} \), \( s^{Pj} = \frac{20}{3} \) as illustrated in figure (20).

(ii) Prices are increasing and converging over \( s^j \in \left[ q_{Lj}^{UR}, \frac{2\bar{\theta}}{3b} \right] \). There are two conflicting effects on price levels. First, a higher \( s^j \) implies higher quality levels for both firms and thus higher variable costs, that are increasing at an increasing rate due to the convexity assumption. Second, the convergence of quality levels intensifies price competition, moderating the impact of costs of price levels. The cost effects dominate under the assumptions of the model, in contrast to other contributions, where prices fall with standards (e.g. Ronnen, 1991, Boom, 1995). Substituting \( q_{Lj} = s^j \) and \( q_{Hj}(s^j) \) into price equations (124) and (125) yields
\( p_{Hj}(s^j, b, \bar{\theta}) \) and \( p_{Lj}(s^j, b, \bar{\theta}) \), which are confirmed to be increasing in \( s^j \).

Note that if firms incur a fixed costs of production in addition to the variable cost specification assumed, the threshold standard above which exit occurs is lower than \( s^{Pj} \). For simplicity, fixed costs are set at zero.
while the price gap is declining in $s^j$. Figure (21) illustrates $p_{HJ}(s^j, b, \bar{\theta})$
and $p_{LJ}(s^j, b, \bar{\theta})$ for $\bar{\theta} = 5$ and $b = \frac{1}{2}$.

(iii) Profits of the high quality supplier decrease with standard $s^j$ and profits of the low quality increase with standards up to a certain threshold level, $\bar{s}$, above which they also decline. Substituting $q_{LJ} = s^j$ and $q_{HJ}(s^j)$ into the profit equations (128) and (129) yield $\Pi_{HJ}(s^j, b, \bar{\theta})$ and $\Pi_{LJ}(s^j, b, \bar{\theta})$, from which $\frac{\partial \Pi_{HJ}(s^j, b, \bar{\theta})}{\partial s^j} < 0$ is confirmed. Solving $\frac{\partial \Pi_{LJ}(s^j, b, \bar{\theta})}{\partial s^j} = 0$ and confirming the maximum yields threshold level:

$$\bar{s} = 0.27763 \frac{\bar{\theta}}{b}$$ (146)

Hence, $\frac{\partial \Pi_{LJ}(s^j, b, \bar{\theta})}{\partial s^j} > 0$ for $s^j \in [\frac{UR}{q_{LJ}}, \bar{s}]$ and $\frac{\partial \Pi_{LJ}(s^j, b, \bar{\theta})}{\partial s^j} < 0$ for $s^j \in [\bar{s}, s^{Pj}]$. The path of firm profits earned from high and low quality sales in $j$ is illustrated for $\bar{\theta} = 5$ and $b = \frac{1}{2}$ in figure (22). The economic mechanism for these contrasting effects operates through the narrowing quality gap and price implications of $s^j$. As the quality gap narrows consumers switch from consuming the high quality good to consuming the low quality good. At the same time, the increasing prices imply some consumers switch from consuming low quality goods to not making any purchase. From (123), (122) and (132), $\frac{\partial x^j}{\partial s^j} > 0$ and $\frac{\partial x_{LJ}}{\partial s^j} > 0$. The implications of the increase in the preference parameter of the marginal consumers, is an unambiguous decline in demand for high quality goods with $s^j$, lowering $\Pi_{HJ}$. This negative relationship is common feature
Figure 20: Regulated equilibrium qualities.

Figure 21: Regulated equilibrium prices.
of the related literature. The low quality firm enjoys a larger overall market share for sufficiently low \( s^j \) but beyond a threshold \( \hat{s} \), the stronger price competition dominates the market share effect and profits decline. Expressing low quality sales in terms of \( s^j \), \( b \) and \( \bar{\vartheta} \) by substituting \( q_{Lj} = s^j \) and \( q_{Hj}(s^j) \) into (127) allows threshold level \( \hat{s} \) to be computed from \( \frac{\partial S_{Lj}(s^j,b,\bar{\vartheta})}{\partial s^j} = 0 \) in terms of model parameters:

\[
\hat{s} = \frac{0.34104}{b}
\]  

(147)

It thus follows that \( \frac{\partial S_{Lj}(s^j,b,\bar{\vartheta})}{\partial s^j} > 0 \) for \( s^j \in [q_{Lj}^{JR}, \hat{s}] \) and \( \frac{\partial S_{Lj}(s^j,b,\bar{\vartheta})}{\partial s^j} < 0 \) for \( s^j \in [\hat{s}, s^{P_j}] \). Ronnen (1991) and Crampes and Hollander (1995) find a similar pattern for profits, while Boom (1995) finds losses for both firms as a result of assumptions that keep market shares constant. The effects of \( s^j \) on firm profits and sales are illustrated in figures (22)-(24) for \( \bar{\vartheta} = 5 \) and \( b = \frac{1}{2} \).

(iv) There are two conflicting effects of minimum quality standards on consumer surplus under variable quality costs. First, the convexity of unit costs implies higher prices as a result of higher quality levels, an effect which is exacerbated by firms' strategic responses to each others' price increases. Second, stronger price competition associated with diminished quality disparity has a positive effect on total consumer surplus. The pro-competitive effect of \( s^j \) dominates the cost effect for standards up to a threshold level \( \hat{s} \), above which the converse is true. Equation 152
Figure 22: Regulated equilibrium profit levels.

Figure 23: Regulated equilibrium high quality sales.
(136) describes total consumer surplus as a function of quality levels and parameters $b$ and $\bar{\theta}$. Substituting $q_{Lj} = s^j$ and $q_{Uj}(s^j)$ into $C_j$ yields $C_j(s^j, b, \bar{\theta})$. Solving $\frac{\partial C_j(s^j, b, \bar{\theta})}{\partial s^j} = 0$ and confirming the maximum yields threshold level:

$$\tilde{s} = 3.668 \frac{\bar{\theta}}{b}$$  \hspace{1cm} (148)

It follows that $\frac{\partial C_j(s^j, b, \bar{\theta})}{\partial s^j} > 0$ for $s^j \in [q_{Lj}^U, \tilde{s}]$ and $\frac{\partial C_j(s^j, b, \bar{\theta})}{\partial s^j} < 0$ for $s^j \in [\tilde{s}, q_{Uj}^R]$. The path of consumer surplus is illustrated for $\bar{\theta} = 5$ and $b = \frac{1}{2}$ in figure (25). Note the effect of $s^j$ on consumers is not uniform.

Crampes and Hollander (1995) obtain the result that all consumers gain for sufficiently low minimum quality standards, under similar assumptions. For a higher standard, however, the higher costs and prices induces losses in high quality consumers, relative to low quality consumers, who increase in number as an increasing measure switches to purchasing the low quality good. Ecchia and Lambertini (1997) show that all consumers lose for a sufficiently high standard. Hence, as $s^j$ rises over $s^j \in [q_{Lj}^R, \tilde{s}]$, welfare is being redistributed from the supplier of high quality goods to the supplier of low quality goods and to consumers in aggregate, but also between consumers of high quality goods to consumers of low quality goods.

The effects of $s^j$ on $C_j$ and $\Pi_{Uj}$ and $\Pi_{Lj}$ described in this section shape the incentives of policy-makers in the standard-setting game. Section 2.2 examines these incentives more closely.
Figure 24: Regulated equilibrium low quality sales.

Figure 25: Regulated equilibrium consumer surplus.
3.2.2 National Incentives

In stage 0, the governments of A and B are assumed to set minimum quality standards $s^A$ and $s^B$ simultaneously and non-cooperatively, under the assumption that both firms remain in the market, taking the strategic interaction of the firms and the resulting effects on market variables as given. In section 2.1 the 'prohibitive' standard, $s^{Pj}$, which defines the highest minimum quality standard consistent with the survival of both firms in market $j$, is found to be $s^{Pj} = \frac{\bar{\theta}}{3\epsilon}$. Hence, the strategy space of each policy-maker of country $j$ is $s^j \in \left[ q^{UR}_{Lj}, s^{Pj} \right]$. The policy-maker of each country $j$ chooses $s^j$ from within this strategy space to maximise its national welfare, $W^j$, taking the standard of the other country as given.

Recall that policy decision $s^j$, gives rise to a strategic response of the high quality firm in market $j$, $q_{Hj}(s^j)$. Substituting the reaction function $q_{Hj}(s^j)$ implicit\(^{29}\) in (132) and $q_{Lj} = s^j$ into equations (136), (128) and (129), yields consumer surplus and profits from the sale of high and low quality goods in market $j$, as functions of policy variable $s^j$ and market parameters $\bar{\theta}$ and $b$. These are defined generally as $C_j(s^j, \bar{\theta}, b)$, $\Pi_{Hj}(s^j, \bar{\theta}, b)$ and $\Pi_{Lj}(s^j, \bar{\theta}, b)$.

Consider welfare equations (149) and (150) that describe the objective functions of governments A and B in terms of $s^A$, $s^B$ and parameters $\bar{\theta}$ and $b$:

\(^{29}\)The analytical expression for $q_{Hj}(s^j)$ can be found in Appendix A.
\[ W^A(s^A, s^B, \bar{q}, b) = C_A(s^A, \bar{q}, b) + \Pi_{AA}(s^A, \bar{q}, b) + \Pi_{AB}(s^B, \bar{q}, b) \]  
\[ W^B(s^A, s^B, \bar{q}, b) = C_B(s^A, \bar{q}, b) + \Pi_{BB}(s^B, \bar{q}, b) + \Pi_{BA}(s^A, \bar{q}, b) \]

Suppressing market and cost parameters \( b \) and \( \bar{q} \) for convenience, the governments' reaction functions are implicitly defined by (151) and (152):

\[ \frac{\partial W^A(s^A, s^B)}{\partial s^A} \bigg|_{s^B} = \frac{\partial C_A(s^A)}{\partial s^A} + \frac{\partial \Pi_{AA}(s^A)}{\partial s^A} = 0 \]  
\[ \frac{\partial W^B(s^A, s^B)}{\partial s^B} \bigg|_{s^A} = \frac{\partial C_A(s^B)}{\partial s^B} + \frac{\partial \Pi_{BB}(s^B)}{\partial s^B} = 0 \]

For a pair of standards \((s^A, s^B)\) to constitute a Nash equilibrium in minimum standards, the pair must solve both (151) and (152). Since the distribution of consumers is symmetric in the two markets, the choice of the minimum standard is influenced symmetrically by its effect \( C_j(s^j) \) in the two countries. Recall from section 2.1 that \( \frac{\partial C_j(s^j)}{\partial s^j} > 0 \) holds for a range of values \( s^j \in [q^U_R, \bar{s}] \), where \( \bar{s} = 3.668 \bar{q} \), providing incentives to regulate. Asymmetries in incentives for setting minimum quality standards hinge on the quality of its domestic sales. If firm A supplies its home market with high quality goods, then domestic profit is unambiguously lowered by the implementation of \( s^A > q^U_R \), since \( \frac{\partial \Pi_{AA}(s^A)}{\partial s^A} = \frac{\partial \Pi_{BA}(s^A)}{\partial s^A} < 0 \). In contrast, if firm A is a supplier of low quality goods then profit from domestic sales is increasing with \( s^A \) for sufficiently low minimum standards, \( s^A \in [q^U_L, \bar{s}] \), where \( \bar{s} = 0.27763 \bar{q} \).
Similar arguments apply for country $B$.

Moreover, inspection of the implicit reaction functions of $A$ and $B$, given by (151) and (152), respectively, reveals that while the level of $W^A$ and $W^B$ depend on both $s^A$ and $s^B$, the optimal responses of the two policy-makers depend only on the quality ranking of the national firm in the domestic market and parameters $b$ and $\bar{b}$. The policy-makers' optimal choice of minimum standard thus involves a trade-off between the gains to domestic consumers and the effects on domestic profit, which is unaffected by the foreign standard but depends on the quality of domestically produced goods. The discussion is summarised by proposition (21).

**Proposition 21** A country's optimal unilateral minimum quality standard is higher when the domestic firm is the low quality supplier in the domestic market, than if the domestic firm is the high quality supplier.

**Proof.** Let $W^j_H$ denote the welfare of a country $j$ when the domestic firm is the high quality supplier in $j$ and $W^j_L$ denote welfare when the domestic firm is the low quality supplier in $j$. Further, let $\Pi_{ji}$ denote the profit of firm $j$ from exports (of unspecified quality) to $i$, where $i \neq j$. Hence (suppressing market and cost parameters $b$ and $\bar{b}$):

\[
W^j_H (s^i, s^j) = C_j (s^j) + \Pi_{Hj} (s^j) + \Pi_{ji} (s^i) \tag{153}
\]

\[
W^j_L (s^i, s^j) = C_j (s^j) + \Pi_{Lj} (s^j) + \Pi_{ji} (s^i) \tag{154}
\]
Let $s^j_H$ and $s^j_L$ denote the optimal unilateral standard when $W^j = W^j_H$ and when $W^j = W^j_L$, respectively, given $s^i$. Since $\Pi_{ji}(s^i)$ is not a function of the domestic standard $s^j$, optimal standard $s^j_H = \arg \max_{s^j} W^j_H (s^i, s^j) = \arg \max_{s^j} \hat{W}^j_H (s^j)$ and $s^j_L = \arg \max_{s^j} W^j_L (s^i, s^j) = \arg \max_{s^j} \hat{W}^j_L (s^j)$, where $\hat{W}^j_H (s^j) = C_j (s^j) + \Pi_{Hj} (s^j)$ and $\hat{W}^j_L (s^j) = C_j (s^j) + \Pi_{Lj} (s^j)$. Recall that $\frac{\partial \Pi_{Hj}(s^j)}{\partial s^j} < 0$ for all $s^j$ and $\frac{\partial \Pi_{Lj}(s^j)}{\partial s^j} > 0$ for $s^j \in [\underline{q}^{UR}, \bar{s}]$, while $\frac{\partial C_j(s^j)}{\partial s^j} > 0$ for $s^j \in [\underline{q}^{UR}, \bar{s}]$. It follows that $\arg \max_{s^j} \hat{W}^j_H (s^j) < \arg \max_{s^j} \hat{W}^j_L (s^j)$ and hence $s^j_H < s^j_L$. 

In general, the policy-maker finds it optimal to set $s^j_H$ that solves (155), or $s^j_L$ that solves (156), depending on whether the domestic firm is a high or low quality supplier in $j$:

$$\frac{\partial C_j(s^j, \bar{v}, b)}{\partial s^j} + \frac{\partial \Pi_{Hj}(s^j, \bar{v}, b)}{\partial s^j} = 0 \quad (155)$$

$$\frac{\partial C_j(s^j, \bar{v}, b)}{\partial s^j} + \frac{\partial \Pi_{Lj}(s^j, \bar{v}, b)}{\partial s^j} = 0 \quad (156)$$

Solving (155) and (156) yields non-cooperative standards and $s_L^j = 0.34691 \bar{b}$, respectively. In the former case, the policy-maker sets a relatively ‘soft’ unilateral standard, while in the latter case a relatively ‘tough’ standard is set. Symmetry across countries implies that $s_H^j = s_H^j = s^S = 0.23995 \bar{b}$ and $s_L^j = s_L^j = s^T = 0.34691 \bar{b}$, where $s^S$ denotes the ‘soft’ unilateral quality standard and $s^T$ ‘tough’ unilateral standard. Both $s^S$ and $s^T$ lie

\footnote{Tables (3) and (4) in Appendix B report the solutions to all market variables under the two unilateral optimal minimum standards.}
between the unregulated high and low quality levels, so unilaterally selected minimum standards are, indeed, mild.

**Proposition 22** Unilateral minimum quality standards are always mild.

**Proof.** Since \( \frac{\partial C_1(s')}{\partial s} = 0 \) at \( s = 3.668 \frac{\bar{b}}{b} \) and \( \frac{\partial \Pi_H(s')}{\partial s} = 0 \) at \( s = 0.27763 \frac{\bar{b}}{b} \), then \( s^*_L \) that solves a convex combination of these in (156) must satisfy \( \bar{s} < s^*_L < \hat{s} \). Moreover, since \( \bar{s} > q^U_L \) and \( \hat{s} < q^U_H \) it is also true \( q^U_L < s^*_L < q^U_H \).

Hence, \( s^*_L \) is mild. Following a similar line of argument for \( s^*_H \) and noting that \( s^*_H > 0 \), it is straightforward to show that \( s^*_H \) is also mild. ■

Tables (4) and (5) report the regulated market outcome under \( s^S \) and \( s^T \), respectively. They show that consumer surplus is higher and profit from high quality sales lower under \( s^T \) than under \( s^S \). Furthermore, the tougher standard corresponds to a smaller quality gap, indicating the lower degree of product differentiation. Prices for both high and low quality goods are higher with the tougher standard, as the convex variable costs effect from the higher quality levels outweighs the pro-competitive effect of stronger price competition.

### 3.3 Nash Equilibria in Minimum Quality Standards

Section 2.2 establishes minimum quality standards \( s^S = 0.23995 \frac{\bar{b}}{b} \) and \( s^T = 0.34691 \frac{\bar{b}}{b} \) as the optimal unilateral policy decisions of national welfare-maximising policy-makers in countries whose national firm supplies the domestic market.
with high or low quality goods, respectively. Examination of the four configurations of firm quality rankings show that there exist four non-cooperative Nash equilibria in minimum standards: two symmetric Nash equilibria, where national minimum standards are either both tough \((s^A = s^B = s^T)\) or both ‘soft’ \((s^A = s^B = s^S)\), and two asymmetric Nash equilibria where one country sets the soft standard and the other the ‘tough’ standard \((s^A = s^S\) and \(s^B = s^T\), or \(s^A = s^T\) and \(s^B = s^S)\).

The multiplicity of non-cooperative Nash equilibria in minimum standards

<table>
<thead>
<tr>
<th>Regulated Equilibrium with High Quality Domestic Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality levels: (q_{Hj} = 0.42856) (\bar{q}_H)</td>
</tr>
<tr>
<td>Quality ratio: (r_j = 1.8318)</td>
</tr>
<tr>
<td>Quality gap: (q_{Hj} - q_{Lj} = 0.18861) (\bar{q}_H)</td>
</tr>
<tr>
<td>Prices: (p_{Hj} = 0.23487) (\bar{p}_H)</td>
</tr>
<tr>
<td>Quality-deflated prices: (x_{Hj} = 0.54805) (\bar{q}_H)</td>
</tr>
<tr>
<td>Marginal consumer: (z_j = 0.73685) (\bar{z}_H)</td>
</tr>
<tr>
<td>Sales: (S_{Hj} = 0.26315) (\bar{S}_H)</td>
</tr>
<tr>
<td>Profits: (\Pi_{Hj} = 0.013476) (\bar{\Pi}_H)</td>
</tr>
<tr>
<td>Consumer surplus: (C_j = 0.050122) (\bar{C}_H)</td>
</tr>
</tbody>
</table>

Table 4: Regulated Equilibrium with High Quality Domestic Sales.

<table>
<thead>
<tr>
<th>Regulated Equilibrium with Low Quality Domestic Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality levels: (q_{Hj} = 0.49272) (\bar{q}_H)</td>
</tr>
<tr>
<td>Quality ratio: (r_j = 1.4203)</td>
</tr>
<tr>
<td>Quality gap: (q_{Hj} - q_{Lj} = 0.14581) (\bar{q}_H)</td>
</tr>
<tr>
<td>Prices: (p_{Hj} = 0.27231) (\bar{p}_H)</td>
</tr>
<tr>
<td>Quality-deflated prices: (x_{Hj} = 0.55267) (\bar{q}_H)</td>
</tr>
<tr>
<td>Marginal consumer: (z_j = 0.79743) (\bar{z}_H)</td>
</tr>
<tr>
<td>Sales: (S_{Hj} = 0.20257) (\bar{S}_H) (imports)</td>
</tr>
<tr>
<td>Profits: (\Pi_{Hj} = 0.005983) (\bar{\Pi}_H)</td>
</tr>
<tr>
<td>Consumer surplus: (C_j = 0.055502) (\bar{C}_H)</td>
</tr>
</tbody>
</table>

Table 5: Regulated Equilibrium with Low Quality Domestic Sales.
arises from the multiplicity of equilibria of the firms’ strategic interaction in prices and quality levels. It is interesting to note that asymmetric national standards can arise endogenously even though markets are symmetric and the duopolists have access to the same technology.

Consider the four configurations of firm quality rankings in $A$ and $B$:

1. If $q_{AA} < q_{BA}$ and $q_{AB} > q_{BB}$, then each firm supplies its home market with low quality goods and trade is in high quality goods. Policy-makers’ incentives are symmetric and give rise to a symmetric Nash equilibrium pair of standards $(s^{A*}, s^{B*}) = (s^T, s^T)$.

2. If $q_{AA} > q_{BA}$ and $q_{AB} < q_{BB}$, then each firm supplies its home market with high quality goods and trade is in low quality goods. Policy-makers’ incentives are again symmetric, but standards are less stringent due to the negative effect on the profits of the domestic firm. The Nash equilibrium pair of standards is thus $(s^{A*}, s^{B*}) = (s^S, s^S)$.

3. If $q_{AA} > q_{BA}$ and $q_{AB} > q_{BB}$, then firm $A$ is the world high quality supplier. Country $B$ imports high quality goods from $A$ while $A$ imports low quality goods from $B$. Policy-makers’ incentives are asymmetric such that the policy maker in $A$ has an incentives to set the ‘soft’ standard to protect the interests of the high quality producing $A$ firm, while policy-maker $B$ sets the ‘tough’ standard. The Nash equilibrium pair of standards is thus $(s^{A*}, s^{B*}) = (s^S, s^T)$. 

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4. If \( q_{AA} < q_{BA} \) and \( q_{AB} < q_{BB} \), then firm \( B \) is the world high quality supplier and \( A \) exports low quality goods to country \( B \). The Nash equilibrium is again asymmetric, where \((s^{A\ast}, s^{B\ast}) = (s^{T}, s^{S})\).

These results are summarised in proposition (23).

**Proposition 23** Non-cooperative Nash equilibrium minimum quality standards are higher when countries trade in high quality goods than if they trade in low quality goods. If trade flows vary in quality, then a higher minimum standard is set by the country importing high quality goods than by the country importing low quality goods.

**Proof.** If countries trade in high quality goods then national firms are low quality suppliers in their home markets. Conversely, if trade is in low quality goods, then national firms are high quality suppliers in their home market. Moreover, if one country imports high quality goods and the other low quality goods, then the national firm of the high quality importing country is the world low quality supplier, while the other firm is the world high quality supplier. The proposition then follows directly from proposition (21).

The key difference between the international duopoly and having both duopolists in a single country is that only the profits of the national firm are incorporated into each policy-maker's objective function. At the same time, the trade links between countries give rise to cross-country externalities from standard-setting as each standard affects the profits of the foreign firm.
In contrast to the widely explored negative terms-of-trade externalities of the strategic tariff-formation literature (e.g. Bagwell and Staiger, 1999, 2002; Staiger and Tabellini, 1987), the cross-country externalities arising from mild quality standards can be either positive or negative, depending on the quality of traded goods. Profits from low quality exports are increasing in foreign minimum standards, provided these are not too severe, yielding a positive cross-country externality. Profits from high quality exports unambiguously decrease with foreign minimum standards, giving rise to a negative cross-country externality. The four Nash equilibria thus correspond to the four different combinations of externalities that may arise between the two countries: symmetric positive externalities, symmetric negative externalities, or asymmetric positive and negative externalities.

More formally, the externalities between countries $A$ and $B$ are reflected in (157) and (158), which describe the impact that $s^B$ and $s^A$ have on $W^A$ and $W^B$, respectively, through their effect on profit flowing abroad.

\[
\begin{align*}
\frac{\partial W^A}{\partial s^B} &= \frac{\partial \Pi_{AB}}{\partial s^B} < 0 \quad (157) \\
\frac{\partial W^B}{\partial s^A} &= \frac{\partial \Pi_{BA}}{\partial s^A} > 0 \quad (158)
\end{align*}
\]

The sign of $\frac{\partial \Pi_{AB}}{\partial s^B}$ and $\frac{\partial \Pi_{BA}}{\partial s^A}$, respectively, depends on the quality of traded goods, and thus on the pattern of trade. For example, if $A$ exports high quality goods to $B$, then even a very mild minimum standard $s^B$ imposes
a negative externality on $A$. Conversely, if low quality goods are exported, then for sufficiently low $s^B$, the externality on country $A$ is positive. Since these positive or negative external effects do not factor into unilateral decision-making, the Nash equilibrium standards are inefficiently high or inefficiently low relative to the world optimum pair of standards that internalises these effects.

To examine the efficiency characteristics of the Nash equilibria, we derive the efficiency condition for pairs of minimum quality standards. Then the world-welfare maximising pair of standards is calculated and contrasted with the non-cooperative policy outcome. This paves the way for the analysis of international cooperation in quality standards that forms the rest of the chapter.

### 3.3.1 Efficiency

Consider the welfare level of $A$ from a given pair $(s^A, s^B)$.

\[
W^A = W^A(s^A, s^B) \quad (159)
\]

Consider an iso-welfare contour for $A$, that describes all the combinations of national standards that yield the welfare level described in (159). Along the iso-welfare contour, it must hold that:

\[
\frac{\partial W^A}{\partial s^A} ds^A + \frac{\partial W^A}{\partial s^B} ds^B = 0 \quad (160)
\]
Hence, the slope of the iso-welfare contour of \( A \) is given by:

\[
\frac{ds^A}{ds^B} \bigg|_{dW^A=0} = -\frac{\partial W^A/\partial s^B}{\partial W^A/\partial s^A}
\]  

(161)

Similarly along an iso-welfare contour for \( B \), for constant welfare \( W^B = W^B(s^A, s^B) \), it must hold that:

\[
\frac{\partial W^B}{\partial s^B} ds^B + \frac{\partial W^B}{\partial s^A} ds^A = 0
\]

(162)

Hence, the slope of the iso-welfare contour of \( B \) is given by:

\[
\frac{ds^A}{ds^B} \bigg|_{dW^B=0} = -\frac{\partial W^B/\partial s^B}{\partial W^B/\partial s^A}
\]

(163)

Substituting (151) and (152) into (161) and (163) allows the slopes of the country \( A \) and \( B \) iso-welfare contours to be expressed in terms marginal effects of national standards on consumer surplus and profit flows:

\[
\frac{ds^A}{ds^B} \bigg|_{dW^A=0} = -\frac{\partial \Pi_{AB}/\partial s^B}{\partial C_A/\partial s^A + \partial \Pi_{AA}/\partial s^A}
\]

(164)

\[
\frac{ds^A}{ds^B} \bigg|_{dW^B=0} = -\frac{\partial C_B/\partial s^B + \partial \Pi_{BB}/\partial s^B}{\partial \Pi_{BA}/\partial s^A}
\]

(165)

For a pair of minimum quality standards \((s^A, s^B)\) to be efficient, there must be no possibility Pareto improvement from \((s^A, s^B)\), so the iso-welfare contours must be tangential to each other at \((s^A, s^B)\). The efficiency requirement is
thus:

\[
\begin{bmatrix}
\frac{ds^A}{ds^B}
\end{bmatrix}_{|dW^A=0} = \begin{bmatrix}
\frac{ds^A}{ds^B}
\end{bmatrix}_{|dW^B=0}
\]  

(166)

Results from the examination of the Nash equilibria against efficiency condition (166) are summarised in Proposition (24).

Proposition 24 The Nash equilibria in minimum quality standards are inefficient.

Proof. Recall the condition for efficiency \(\frac{ds^A}{ds^B}|_{dW^A=0} = \frac{ds^A}{ds^B}|_{dW^B=0}\) that implies (164) and (165) must be equal for Nash equilibrium pair \((s^*_A, s^*_B)\) to be efficient. Moreover, in order for the pair to constitute a Nash equilibrium, it must be true that it solves the government reaction functions, (151) and (152). However, if \((s^*_A, s^*_B)\) satisfies both (151) and (152), then \(\frac{ds^A}{ds^B}|_{dW^A=0} = \infty > \frac{ds^A}{ds^B}|_{dW^B=0} = 0\), thus violating the condition for efficiency. ■

3.3.2 World Optimum Minimum Standards

Let pair \((s^A, s^B) = (s^{WO}_A, s^{WO}_B)\) denote the pair of minimum standards that maximise world welfare \(W(s^A, s^B)\), given by (167), where:

\[
W(s^A, s^B) = W_A(s^A, s^B) + W_B(s^A, s^B) = C_A(s^A) + \Pi_{AA}(s^A) + \Pi_{BA}(s^A) + C_B(s^B) + \Pi_{BB}(s^B) + \Pi_{AB}(s^B)
\]

(167)

Since each market has a low and high quality supplier with symmetric costs and consumer preferences are identical across markets of equal size \(\bar{\theta}\),
then by symmetry $s^A_W = s^B_W = s^W_O$, where $s^W_O = \arg\max_s W(s) = 2[C_j(s) + \Pi_{H_j}(s) + \Pi_{L_j}(s)]$. Thus $s^W_O$ solves:

$$\frac{\partial W}{\partial s} = \frac{\partial C_j(s)}{\partial s} + \frac{\partial \Pi_{H_j}(s)}{\partial s} + \frac{\partial \Pi_{L_j}(s)}{\partial s} = 0$$  \hspace{1cm} (168)$$

Rearranging (168) yields the efficiency condition:

$$-\frac{\partial \Pi_{H_j}(s)}{\partial s} / \partial C_j(s) / \partial s + \partial \Pi_{L_j}(s) / \partial s = -\frac{\partial C_j(s)}{\partial s} + \partial \Pi_{L_j}(s) / \partial s = 1$$  \hspace{1cm} (169)$$

It follows directly from (169) that efficiency is satisfied at the world optimum. Hence, the world welfare-maximising pair of minimum standards is shown to be both symmetric and efficient. The implication is that world welfare is maximised at a unique point, where both countries harmonise their standards at $s^W_O$. Since the world optimum does not constitute a non-cooperative Nash equilibrium, it may only be reached through cooperative agreement. The feasibility of international cooperation at $(s^A, s^B) = (s^W_O, s^W_O)$ is analysed in the section 3.

Solving (168) yields the world optimum common standard $s^W_O = 0.25241 \frac{\bar{y}}{\bar{k}}$.

It is observed to lie between the two optimal unilateral minimum standards $s^S = 0.23995 \frac{\bar{y}}{\bar{k}}$ and $s^T = 0.34691 \frac{\bar{y}}{\bar{k}}$. This leads to Proposition (25).

Intuitively, the cross-country externalities imply that when a government of a country imposes a minimum quality standard, part of the costs (or benefits) of the standard are borne by the trading partners of that country. As a result,
each government faces less than the full costs (or benefits) of the standard and hence over (or under) provides regulation of quality relative to the world optimum minimum standard that internalises the cross-country effects.

**Proposition 25** The world optimum standard \((s^{WO})\) lies between the unilateral 'soft' standard \((s^s)\) and 'tough' standard \((s^T)\).

**Proof.** Since \(s^{WO} = \arg \max_s \hat{W}(s)\), \(s^j_H = \arg \max_s \hat{W}^j_H\) and \(s^j_L = \arg \max_s \hat{W}^j_L\), where \(\hat{W}(s) = C_j(s) + \Pi_{Hj}(s) + \Pi_{Lj}(s)\), \(\hat{W}^j_H = C_j(s^j) + \Pi_{Hj}(s^j)\) and \(\hat{W}^j_L = C_j(s^j) + \Pi_{Lj}(s^j)\), then from the properties of \(\Pi_{Lj}\), \(\Pi_{Hj}\) and \(C_j\) it follows that \(s^j_H < s^{WO} < s^j_L\). ■

Table (6) reports the solutions to all market variables under \(s^{WO}\), the world welfare maximising (common) minimum quality standard.

<table>
<thead>
<tr>
<th>Market Outcome</th>
<th>Quality levels</th>
<th>Quality ratio</th>
<th>Prices</th>
<th>Quality-deflated prices</th>
<th>Marginal consumer</th>
<th>Sales</th>
<th>Profits</th>
<th>Consumer surplus</th>
</tr>
</thead>
<tbody>
<tr>
<td>(q_{Hj})</td>
<td>(0.43885)</td>
<td>(r_j = 1.7386)</td>
<td>(p_{Hj} = \frac{q_{Hj}}{r_j})</td>
<td>(x_{Hj} = 0.54676)</td>
<td>(z_j = 0.74601)</td>
<td>(S_{Hj} = 0.25399)</td>
<td>(\Pi_{Hj} = 0.012028)</td>
<td>(C_j = 0.05151)</td>
</tr>
<tr>
<td>(q_{Lj})</td>
<td>(0.25241)</td>
<td>(p_{Lj} = \frac{q_{Lj}}{r_j})</td>
<td>(x_{Lj} = 0.39958)</td>
<td>(S_{Lj} = 0.34642)</td>
<td>(\Pi_{Lj} = 0.012869)</td>
<td>(\frac{r_j}{b})</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6: Market Outcomes Under the World Optimum Standard.

### 3.3.3 Non-Cooperative Standard-Setting and Trade

Consider the implications of the over- or under-regulation of quality on the trade flows between \(A\) and \(B\). When countries trade in high quality goods,
the Nash equilibrium is symmetric and characterised by 'tough' standards in both countries. These are more stringent than is optimal from a world-welfare perspective, however, as a result of the bilateral negative externalities. Since demand for high quality goods is decreasing in the minimum quality standard, over-regulation implies lower demand for high quality goods in each country and thus lower bilateral trade in these goods than is efficient. The tougher standards also imply the quality of traded goods is higher than is efficient.

Conversely, trade in low quality goods gives rise to a symmetric Nash equilibrium in which both countries set 'soft' standards, which are laxer than is optimal from a world-welfare maximising perspective. For low quality goods, however, laxer standards imply less demand in each country, and thus lower bilateral trade in low quality goods than is efficient. Moreover, the quality of traded goods is lower than is efficient.

Similar arguments apply for the asymmetric Nash equilibria for the flows of high and low quality exports, respectively. It follows that irrespective of the quality level of traded goods, and thus for all patterns of trade, the inefficiency in unilateral decision-making in quality standards operates as a non-tariff barrier to trade. Moreover, the result follows without any of the usual assumptions that generate non-tariff barrier effects of minimum standards, such as certification or labelling costs for firms to meet different national standards or quality modification costs that prohibit the customisation of quality to different markets.
The discussion is summarised by proposition (26).

**Proposition 26**  Trade flows are lower under Nash equilibrium quality standards than under world optimum standards.

**Proof.** Recall that \( \frac{\partial S_{H}}{\partial s} < 0 \) and \( \frac{\partial S_{L}}{\partial s} > 0 \) for \( s \in \left( q_{U}^{UR}, \hat{s} \right) \) where \( \hat{s} = 3.4104 \). If both countries export high quality goods, then Nash equilibrium standards are \( s^{A*} = s^{B*} = s^{T} \). From Proposition (25) it follows that \( s^{T} > s^{WO} \), so exports must be lower than under world optimum standards. If countries trade in low quality goods, then Nash equilibrium standards are \( s^{A*} = s^{B*} = s^{S} \). Proposition (25) implies \( s^{S} < s^{WO} \). Moreover, since \( s^{WO} < \hat{s} \) then \( \frac{\partial S_{H}}{\partial s} > 0 \) holds in the region of the Nash equilibrium and world optimum. Hence low quality exports are lower in the Nash equilibrium than under world optimum standards. Finally, if \( A \) exports high quality goods and \( B \) exports low quality goods, then \( s^{A*} = s^{S} < s^{WO} \) and \( s^{B*} = s^{T} > s^{WO} \).

Thus high quality exports of country \( A \) are lower under \( s^{B*} = s^{T} \) than under \( s^{B*} = s^{WO} \). Furthermore, country \( B \)'s low quality exports are lower under \( s^{A*} = s^{S} \) than under \( s^{A*} = s^{WO} \).


3.4 International Cooperation in Quality Standards

Section 2 establishes the inefficiency of the non-cooperative Nash equilibria in minimum quality standards, as well as the efficiency of harmonisation of quality standards at the world optimum. This section analyses the potential gains from international cooperation between governments under the three distinct
trade patterns: trade in high quality goods, trade in low quality goods, and bilateral trade in goods of different qualities. A common feature of all three cases is the Prisoners' Dilemma structure in the incentives of the two policymakers. While countries stand to gain through a cooperative agreement, this does not constitute a Nash equilibrium. Taking as given the minimum quality standard of the other country, each policy-maker has an incentive to defect from the cooperative agreement. The analysis follows the general approach used in the analysis of cooperative agreements in tariffs (Bagwell and Staiger, 1999, 2002) by examining Pareto-improving reciprocal adjustment of minimum quality standards as a means of establishing the scope for cooperative agreement.

3.4.1 Bargaining from Symmetric Nash Equilibria

The two distinct trade patterns that give rise to a symmetric Nash equilibrium are analysed in turn.

Trade in High Quality Goods

Proposition 27 If countries trade in high quality goods, then a cooperative agreement in quality standards must involve a reciprocal lowering of minimum quality standards from the non-cooperative Nash equilibrium.

Proof. If countries trade in high quality goods, then the national firms are low quality suppliers in their home market. It follows from propositions (21) and (25) that A and B set ‘tough’ standards unilaterally, denoted by $s^* =$
\[ s^{B^*} = s^T > s^{WO}. \]

For a pair of standards \((s_0^A, s_0^B)\) to be welfare improving for both \(A\) and \(B\) relative to the Nash standards \((s^T, s^T)\), it is necessary that \(s_0^A < s^T\) and \(s_0^B < s^T\). This is shown to be true by considering the effect of \(s^B\) on \(W^A\). The minimum standard in \(B\) affects the welfare of \(A\) through its effect on profit from high quality exports. From the properties of \(\Pi_{Hj}\) it follows that \(W^A\) is decreasing in \(s^B\). A symmetric argument applies for \(B\), so \(W^B\) is decreasing in \(s^A\). The cross-country negative externalities are summarised by:

\[
\begin{align*}
\frac{\partial W^A}{\partial s^B} &= \frac{\partial \Pi_{AB}}{\partial s^B} < 0 \quad (170) \\
\frac{\partial W^B}{\partial s^A} &= \frac{\partial \Pi_{BA}}{\partial s^A} < 0 \quad (171)
\end{align*}
\]

While each country's welfare is decreasing in the quality standard of its trading partner, the implicit reaction functions (151) and (152) imply that \(\frac{\partial W^A}{\partial s^A} \vert_{s^B}\) and \(\frac{\partial W^B}{\partial s^B} \vert_{s^A}\) are independent of \(s^B\) and \(s^A\), respectively. The best responses of governments \(A\) and \(B\) are thus

\[ s^{A*}(s^B) = s^T \forall s^B \quad \text{and} \quad s^{B*}(s^A) = s^T \forall s^A, \]

respectively. Hence, if \(s_0^B > s^T\), (170) implies the best attainable welfare for \(A\) given \(s_0^B\) is:

\[
W^A(s^{A*}(s_0^B), s_0^B) = W^A(s^T, s_0^B) < W^A(s^T, s^T) \quad (172)
\]
Similarly, if $s_0^A > s^T$, (171) implies the best attainable welfare for $B$ given $s_0^A$ is:

$$W^B\left(s_0^A, s_0^B (s_0^A)\right) = W^B\left(s_0^A, s^T\right) < W^B\left(s^T, s^T\right)$$

Inequalities (172) and (173) imply that if $s_0^A > s^T$ or $s_0^B > s^T$ then $(s_0^A, s_0^B)$ cannot be Pareto improving relative to $(s^T, s^T)$ for both $A$ and $B$ and thus cannot be the outcome of a cooperative agreement in standards. Hence for $(s_0^A, s_0^B)$ to be a cooperative agreement both $s_0^A < s^T$ and $s_0^B < s^T$ must hold. ■

Figure (26) illustrates the incentives of the two policy-makers under the configuration of firm qualities that generates trade in high quality goods. The curves illustrated are plotted for parameter values $b = \bar{b}$ and $d = 5$ using the profit equations for high and low quality goods, consumer surplus and the quality reaction function of the high quality firm given in Appendix A.

The origin of the figure corresponds to no regulation, where $s^A = s^B = q_0^{UR}$. Iso-welfare contours $W^{A*}$ and $W^{B*}$ are drawn for the welfare level attained in the symmetric Nash equilibrium, denoted by NE, where $(s^{A*}, s^{B*}) = (s^T, s^T)$, while contours $\tilde{W}^A$ and $\tilde{W}^B$ correspond to the welfare of $A$ and $B$ from a cooperative agreement at the world optimum (WO), where $s^A = s^B = s^{WO} < s^T$. Moreover, the dotted contour $W^*$ reflects iso-world-welfare contour at the Nash equilibrium level of welfare $W^* = W^{A*} + W^{B*}$.

Consider the iso-welfare contours for each country. Higher welfare levels correspond to contours closer to the axes, reflecting the cross-country nega-
tive externalities that apply. For each country, the unique, optimal unilateral standard is \( s^T \), which reflects the optimal reply of each country to any standard set by the other. The welfare level of each is decreasing in the standard of the other, so the highest welfare point is \( (s^A, s^B) = (s^T, q^{UR}_L) \) for \( A \) and \( (s^A, s^B) = (q^{UR}_L, s^T) \) for \( B \).

The efficiency locus is denoted by EE, which plots all pairs of minimum standards at which the iso-welfare contours of \( A \) and \( B \) are tangent. WO is the efficient, symmetric pair, which maximises world welfare. The inefficiency of NE is confirmed since the iso-welfare contours of the two countries are not tangential at NE, indicating scope for Pareto improvement. The core, enclosed by \( W^A^* \) and \( W^B^* \), gives the set of all Pareto-improving points relative to NE, within which the darker segment reflects the contract curve of \( A \) and \( B \), along which cooperative pairs \( (s^A, s^B) \) are both efficient and Pareto-improving relative to NE.

Consider the opportunities for cooperation reflected in the figure. While \( (s^T, s^T) \) is a Nash equilibrium, mutual gains can be reaped through a reciprocal adjustment of minimum quality standards downwards, as follows from proposition (27). Each reciprocal adjustment places countries on a lower, symmetric contour (reflecting higher welfare), through which all gains from bargaining are exhausted at WO.

The analysis confirms the potential welfare gains from cooperation, but also highlights the Prisoners' Dilemma structure of incentives. Points \( D^A \)
and $D^B$ illustrated optimal defection points from cooperation for $A$ and $B$, respectively. If $s^B = s^{WO}$, then country $A$’s optimal reply is to defect to $D^A$, by setting $s^A = s^T$ and thereby attaining a higher welfare level. While the scope for Pareto-improving cooperation is established, the analysis raises concerns over the enforceability of such cooperation.

In practice, national quality standards are developed by National Standards Bodies. 153 of these national bodies are members of the International Organization for Standardization (ISO), the world’s largest developer of standards. While the ISO has greatly contributed to the development of international standards alongside national standards, it has no legal authority to enforce the implementation of its standards. Despite the ISO’s large membership, and the impetus it creates for the implementation of international standards, there are still widespread differences in national standards.

Trade in Low Quality Goods

**Proposition 28** If countries trade in low quality goods, then a cooperative agreement in quality standards must involve a reciprocal raising of minimum quality standards from the non-cooperative Nash equilibrium.

**Proof.** If countries trade in low quality goods, it follows from propositions (21) and (25) that $A$ and $B$ set ‘soft’ standards unilaterally, denoted by $s^{A^\ast} = s^{B^\ast} = s^S < s^{WO}$. Moreover, the implicit reaction functions (151) and (152) yield $s^{A^\ast}(s^B) = s^S \forall s^B$ and $s^{B^\ast}(s^A) = s^S \forall s^A$, respectively.

For a pair of standards $(s^A_0, s^B_0)$ to be welfare improving for both $A$ and $B$
relative to the Nash standards \((s^S, s^S)\), it is necessary that \(s_0^A > s^S\) and \(s_0^B > s^S\). If these conditions do not both hold, then at least one country has lower welfare under \((s_0^A, s_0^B)\) than under the Nash equilibrium standards. From the properties of \(\Pi_{ij}\) and since \(\bar{q} > s^S\) it follows that \(W^A\) is increasing in \(s^B\) and \(W^B\) increasing in \(s^A\) in the region of the Nash equilibrium. The cross-country externalities are thus positive and summarised by:

\[
\begin{align*}
\frac{\partial W^A}{\partial s^B} &= \frac{\partial \Pi_{AB}}{\partial s^B} > 0 \text{ for } s^B \in (q_{UR}^L, \bar{q}) \\
\frac{\partial W^B}{\partial s^A} &= \frac{\partial \Pi_{BA}}{\partial s^A} > 0 \text{ for } s^A \in (q_{UR}^L, \bar{q})
\end{align*}
\]
Hence, if \( s^B_0 < s^S \) the best attainable welfare for \( A \) given \( s^B_0 \) is:

\[
W^A(s^A^*, (s^B_0), (s^B_0)) = W^A(s^S, s^B_0) < W^A(s^S, s^S) (176)
\]

Similarly, if \( s^A_0 < s^S \) the best attainable welfare for \( B \) given \( s^A_0 \) is:

\[
W^B(s^A_0, (s^B_0), (s^A_0)) = W^B(s^A_0, s^S) < W^B(s^S, s^S) (177)
\]

From (176) and (177) it follows that if \( s^A_0 < s^S \) or \( s^B_0 < s^S \), then \((s^A_0, s^B_0)\) cannot be Pareto improving relative to \((s^S, s^S)\) for both \( A \) and \( B \) and thus cannot be the outcome of a cooperative agreement in standards. Hence for \((s^A_0, s^B_0)\) to be a cooperative agreement both \( s^A_0 > s^S \) and \( s^B_0 > s^S \) must hold.

**Proposition 29** From a symmetric Nash equilibrium in minimum standards, mutually beneficial reciprocal adjustment of national standards increases national welfare monotonically for both countries until the world optimum is reached.

**Proof.** Consider reciprocal changes in standards \( ds^A \) and \( ds^B \) from an initial bargaining position at Nash equilibrium standards, \((s^A^*, s^B^*)\). If trade is in high quality goods, then \( s^A^* = s^B^* = s^T > s^{WO} \) and from Proposition (27) adjustments \( ds^A < 0 \) and \( ds^B < 0 \) from the Nash equilibrium are mutually beneficial. Similarly, if low quality goods are traded, then \( s^A^* = s^B^* = s^S < s^{WO} \) and from Proposition (28) adjustments \( ds^A > 0 \) and \( ds^B > 0 \) from
the Nash equilibrium are mutually beneficial. Since $s^A^* = s^B^*$, reciprocal adjustments give rise to symmetric Pareto improvements until $s^A^* + \sum ds = s^B^* + \sum ds = s^{WO}$, where standards $(s^{WO}, s^{WO})$ are efficient. ■

Figure (27) illustrates the strategic incentives of the two policy-makers where trade is in low quality goods. As before, the curves illustrated are plotted for parameter values $b = \frac{1}{2}$ and $\theta = 5$ using the profit equations for high and low quality goods, consumer surplus and the quality reaction function of the high quality firm given in Appendix A. All notation is as in figure (26).

The iso-welfare contours for countries that trade in low quality goods are elliptical, as illustrated in the figure. In particular, the iso-welfare contours for $A$ and $B$ form concentric ellipses that correspond to higher welfare as they converge to the unique, preferred point of each country.

Consider the welfare of country $A$. Welfare level $W^{A*}$ is attained at the Nash equilibrium (NE), where both countries set the ‘soft’ standard $s^S < s^{WO}$. Welfare $W^A$ from a cooperative agreement at the world optimum (WO) corresponds to an elliptical iso-welfare contour that lies within the contour corresponding to the Nash equilibrium $(s^A^*, s^B^*) = (s^S, s^S)$. The unique, preferred point of country $A$ is denoted by $\bar{W}^A$, corresponding to the pair of minimum standards $(s^A, s^B) = (s^S, \bar{s})$. The intuition behind the shape of the iso-welfare contours and preferred point $\bar{W}^A$ lies in the positive cross-country externalities between countries for minimum standards $J^i \in (q^{UR}_J, \bar{s})$, which become negative for $J^i \in (\bar{s}, s^P)$, where $\bar{s} = 0.27763 \frac{\theta}{b} = 2.7763$ and $s^P = \frac{20}{3}$. 

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Hence, at $s^B = \bar{s} = 2.7763$, the profit from firm A's low quality exports is maximised, yielding the highest attainable welfare for A at $(s^A, s^B) = (s^S, \bar{s})$.

Similar arguments apply for country $B$.

The dotted contour reflects the iso-world-welfare contour through the Nash equilibrium point, corresponding to welfare level $W^* = W^{A*} + W^{B*}$, and centred around the world welfare maximising WO point. The core, enclosed by the intersection contours $W^{A*}$ and $W^{B*}$, gives the set of Pareto-improving cooperative agreements, the efficient of which lie on the contract curve, that forms a subset of the efficiency locus EE.

Propositions (28) and (29) are reflected by reciprocal increases in standards $s^A$ and $s^B$, which increase welfare by 'internalising' the positive externalities drive firms to under-regulate at the NE, relative to WO. These Pareto-improving adjustments shift countries onto smaller and smaller concentric circles from NE, which correspond to higher welfare levels, until efficiency is achieved at WO where both iso-welfare contours are tangent. As with trade in high quality goods, the Prisoners' Dilemma structure exists, since countries $A$ and $B$ have an incentive to defect to $D^A$ and $D^B$, respectively, from a cooperative agreement at WO.

### 3.4.2 Bargaining from Asymmetric Nash Equilibria

This section examines the incentives and scope for cooperative agreement between the two countries from an initial asymmetric Nash equilibrium in minimum quality standards. This corresponds to the trade pattern where one
firm ranks as the high quality leader in both markets, and the other firm is the world low quality supplier. The resulting trade pattern is where one country exports high quality goods, and finds it optimal to set minimum quality standard $s^S < s^{WO}$, while the other exports low quality goods, and sets $s^T > s^{WO}$.

The results point to scope for mutual gains from reciprocal adjustment in minimum standards, but show that the asymmetric welfare measures and externalities (positive for one country and negative for the other) make a cooperative agreement at the world optimum infeasible. The asymmetries
between $A$ and $B$ are such that the world optimum does not offer a Pareto gain to both countries and hence does not lie on the contract curve. In the absence of lump-sum transfers that can correct for the asymmetries, the models shows that harmonisation of minimum quality standards cannot form a cooperative agreement. This result is similar to Mayer (1981) and Kennan and Riezman (1988), who show in the context of tariff negotiations that free trade may be unattainable if countries are sufficiently asymmetric\textsuperscript{31} in size.

The results are summarised by general propositions (30) to (34) and then illustrated for particular parameter values.

**Proposition 30** If trade flows vary in quality, then a cooperative agreement reached from the non-cooperative Nash equilibrium must involve a higher standard in the high quality exporting country and a lower standard in the low quality exporting country.

**Proof.** Suppose $A$ exports high quality goods and $B$ exports low quality goods (or vice versa). It follows from propositions (21) and (25) that $A$ sets a ‘soft’ standard and $B$ a ‘tough’ standard unilaterally, where $s_A^* = s^S < s^{WO}$ and $s_B^* = s^T > s^{WO}$. The implicit reaction functions (151) and (152) yield $s_A^*(s_B) = s^S \forall s_B$ and $s_B^*(s_A) = s^T \forall s_A$, respectively. Equations (170) and (175) imply a negative externality on $A$ from $s_B^* = s^T$ and a positive externality on $B$ from $s_A^* = s^S$. Hence for standards $(s_A^*, s_B^*)$ to be welfare

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\textsuperscript{31}An important difference is that asymmetric Nash equilibria arise endogenously as a feature of the vertical product differentiation in this chapter, and not as a result of asymmetric assumptions in country or firm characteristics.
improving for both A and B relative to the Nash standards it is necessary that $s^A_0 > s^B$ and $s^B_0 < s^T$. If these conditions are not both satisfied, then at least one country has lower welfare under $(s^A_0, s^B_0)$ than under $(s^S, s^T)$, as shown by (176) and (173), and $(s^A_0, s^B_0)$ cannot be the result of a cooperative agreement. ■

**Lemma 31** The world optimum does not lie on the contract curve of governments whose initial bargaining position is an asymmetric Nash equilibrium in minimum standards.

**Proof.** Consider the asymmetric Nash equilibrium where A exports high quality goods and B exports low quality goods, where Nash equilibrium standards are $s^A* = s^S < s^{WO}$ and $s^B* = s^T > s^{WO}$. For the world optimum to lie on the contract curve it must be both (i) efficient and (ii) Pareto improving. The world optimum is shown to be efficient in section 2.3.2. Agreement at the world optimum is shown not to be Pareto improving, however, by examination of country welfare levels at the Nash equilibrium and at the world optimum:

Substituting $s^S = 0.23395\frac{g}{b}$ and $s^T = 0.34691\frac{g}{b}$ into equation (132) yields Nash equilibrium high qualities $q_{HA}(s^S) = 0.42856\frac{b}{f}$ and $q_{HB}(s^T) = 0.49272\frac{b}{f}$. Firm quality rankings imply $q_{AA}^* = q_{HA}(s^S) = 0.42856\frac{b}{f}$, $q_{AB}^* = q_{HB}(s^T) = 0.49272\frac{b}{f}$, $q_{BB}^* = s^T = 0.34691\frac{b}{f}$ and $q_{BA}^* = s^S = 0.23395\frac{b}{f}$. Substitution of the equilibrium values in (149) and (150) yields Nash welfare levels $W^A* = (6.9585 \times 10^{-2}) \frac{fi}{b}$ and $W^B* = (8.0616 \times 10^{-2}) \frac{gi}{b}$, respectively.

At the world optimum low qualities are $q_{LA} = q_{LB} = s^{WO} = 0.25241\frac{g}{b}$. From
(132) the high quality best responses are \( q_{HA}(s^{WO}) = q_{HB}(s^{WO}) = 0.43885 \frac{\sigma}{5} \).

It follows that \( q_{AA}^{WO} = q_{AB}^{WO} = 0.43885 \frac{\sigma}{5} \) and \( q_{BB}^{WO} = q_{BA}^{WO} = 0.25241 \frac{\sigma}{5} \).

Substituting into (??) and (??) yields welfare levels at the world optimum \( \tilde{W}^A = (7.5567 \times 10^{-2}) \frac{\sigma^3}{5} \) and \( \tilde{W}^B = (7.7249 \times 10^{-2}) \frac{\sigma^3}{5} \), respectively. \( \tilde{W}^A > W^A^* \) and \( \tilde{W}^B < W^B^* \) imply the world optimum is not Pareto improving relative to the asymmetric Nash equilibrium and thus cannot lie on the contract curve.

**Proposition 32** From an initial asymmetric Nash equilibrium in minimum standards, an agreement at the world optimum cannot be reached through Pareto improving reciprocal adjustment of national standards.

**Proof.** This follows directly from lemma (31). ■

**Lemma 33** From initial asymmetric Nash equilibrium minimum standards a lump-sum transfer from the high quality to the low quality exporting country can ensure mutual gains from a cooperative agreement at the world optimum.

**Proof.** Consider the asymmetric Nash equilibrium where \( A \) exports high quality goods and \( B \) exports low quality goods. The proof of lemma (31) confirms that \( \tilde{W}^A - W^A^* > W^B^* - \tilde{W}^B \). Thus any lump-sum transfer \( L^{AB} \) from \( A \) to \( B \) where \( L^{AB} \in (W^B^* - \tilde{W}^B, \tilde{W}^A - W^A^*) \) ensures mutual gains from an agreement at the world optimum. ■

**Proposition 34** If lump sum transfers between countries are possible, then the world optimum can be reached through international cooperation for all
configurations of firm quality rankings.

**Proof.** This follows directly from propositions (29) and (33). ■

Figure (28) illustrates the strategic incentives and opportunities for cooperative bargaining from the asymmetric Nash equilibrium where \( s^A = s^S \) and \( s^B = s^T \), corresponding to a quality ordering in each market where firm \( A \) is the high quality supplier. As before, the curves illustrated are plotted for parameter values \( b = \frac{1}{2} \) and \( \overline{\theta} = 5 \) using the profit equations for high and low quality goods, consumer surplus and the quality reaction function of the high quality firm given in Appendix A.

The asymmetric Nash equilibrium depicted contains elements of figures (26) and (27) of the previous sections. Country \( A \) exports high quality goods, and thus always loses welfare from the implementation of a binding standard in country \( B \), through the negative effect on export profit. The iso-welfare contours for \( A \) are thus increasing in welfare for lower \( s^B \), and centred around \( s^A = s^S \), the optimal minimum standard for \( A \). Welfare level \( W^A^* \) is attained at the asymmetric Nash equilibrium.

Country \( B \) is an exporter of low quality goods to country \( A \) and thus experiences a positive welfare effect from the implementation of \( s^A \in (q^U^L, \overline{s}) \), and a negative welfare effect for \( s^P \in (\overline{s}, s^P) \), giving rise to elliptical iso-welfare contours. Welfare \( W^B^* \) corresponds to the welfare level at the Nash equilibrium.

The concentric dotted iso-world-welfare contours are centred around \( W^O \).
and the efficiency locus EE passes through WO and into the core formed by the two (reservation) iso-welfare contours at NE. The main observation is that WO is not on the countries’ contract curve, so in the absence of lump sum transfers, this point cannot be reached through reciprocal adjustment of $s^A$ and $s^B$. While country A gains from WO (relative to NE), country B experiences a loss in welfare, since $\tilde{W}^B < W^B$. Mutual gains from a cooperative agreement are possible, for example at C, but the resulting agreement does not entail harmonisation of standards and corresponds to a world welfare level lower than at WO. Note that the Prisoners’ Dilemma structure in incentives continues to apply, with $D^A$ and $D^B$ reflecting the defection points of A and B, respectively.

If lump-sum transfers are possible between countries, then a lump sum transfer $L^{AB} = W^B - \tilde{W}^B$ is the smallest transfer consistent with cooperation of B at WO. Under $L^{AB}$, country B is indifferent between NE and WO, while A gains welfare relative to the non-cooperative Nash equilibrium.

Meza and Tombak (2007) introduce asymmetries in marginal costs and Jinji and Toshimitsu (2004) assume asymmetric fixed quality-development costs to obtain an endogenously determined quality ranking. For sufficiently small cost differentials, they each find a unique duopoly equilibrium in which the low cost firm offers high quality. Their results suggest that even small cost asymmetries can eliminate symmetric Nash equilibria. If the quality ranking of firms is preserved across markets through a cost advantage of one firm over
another, then only the asymmetric equilibria discussed survive.

3.4.3 Cooperative Standard-Setting and Trade

Proposition 35 International cooperation in standard-setting raises trade volume relative to the non-cooperative Nash equilibrium for all configurations of firm quality rankings.

Proof. Proposition (26) establishes that trade flows are lower under the Nash equilibrium than under world optimum standards. Under symmetric initial
conditions, Proposition (29) establishes that reciprocal mutual adjustments in standards allow the world optimum to be reached. Hence international cooperation from a symmetric Nash equilibrium raises or lowers standards towards the world optimum, thereby raising trade flows. Moreover, if lump sum transfers can be made between countries, then Proposition (34) establishes that the world optimum can be reached even under asymmetric initial conditions. Pareto improving cooperation is possible even in the absence of lump-sum transfers, however. Proposition (30) establishes that such cooperation must raise the standard of the high quality exporting country and lower the standard of the low quality exporting country, albeit not to the world optimum level. Since \( \frac{\partial S_L}{\partial s} < 0 \) and \( \frac{\partial S_U}{\partial s} > 0 \) for \( s \in \left( q^U_R, \hat{s} \right) \), any cooperative agreement from an asymmetric Nash equilibrium is trade enhancing. 

\[ \textbf{3.5 Conclusion} \]

This chapter extends a well-established vertical product differentiation model to an international setting where international duopolists compete in two segmented markets. The framework is used to analyse governments' incentives for the unilateral setting of minimum quality standards, as well as the scope and effects of international cooperation on welfare and international trade. Firms compete in qualities and prices internationally and incur variable costs of quality improvement, allowing quality of domestic sales and exports to be differentiated. National standards are endogenous and result from a standard-setting game between governments whose objective function is to maximise
Multiple equilibria arise as a feature of the underlying vertical product differentiation model. Four unregulated Nash equilibria in minimum standards are shown to exist, two symmetric and two asymmetric, depending on the quality ranking of firms in each market. The framework delivers several new propositions. First, the analysis establishes that in all four cases, unilaterally selected minimum quality standards are inefficient as a result of cross-country externalities. Second, trade flows are shown to be lower under non-cooperative Nash equilibrium standards than under a mutually beneficial cooperative agreement, suggesting higher trade flows between countries that cooperate in standard-setting than between countries that set minimum standards unilaterally. Unilateral minimum standards are thus shown to operate as non-tariff barriers to trade.

In contrast to the widely explored negative terms-of-trade externalities of the strategic tariff-setting literature, the cross-country externalities arising from mild quality standards can be either positive or negative, depending on the quality of traded goods. Profits from low quality exports are increasing in foreign minimum standards, provided these are not too severe, yielding a positive cross-country externality. Profits from high quality exports unambiguously decrease with foreign minimum standards, giving rise to a negative cross-country externality. The four Nash equilibria thus correspond to the four different combinations of externalities that may arise between the two
countries: symmetric positive externalities, symmetric negative externalities, or asymmetric positive and negative externalities. Hence unilateral minimum standards may be inefficiently high or inefficiently low relative to the efficient world optimum symmetric standards.

Third, the existence of Pareto improving cooperative agreements from an initial bargaining position at any of the four Nash equilibria, is established. Moreover, the world welfare maximising symmetric standard can be reached through reciprocal adjustments in national minimum standards from either of the two symmetric Nash equilibria. These correspond to firm rankings that give rise to trade in high quality goods only, or trade in low quality goods only. While the underlying Prisoners' Dilemma structure of the standard-setting game raises concerns about enforcement of cooperative agreements, the theoretical results show that an efficient cooperative agreement to harmonise minimum quality standards is feasible and mutually beneficial for countries that trade in goods of similar quality levels.

Finally, the potential scope for mutually beneficial cooperation is shown to be significantly restricted when cross-country externalities are asymmetric. New propositions establish that although asymmetric countries can mutually gain from cooperation, the resulting cooperative standards are asymmetric and do not maximise world welfare. Cross-country asymmetries that arise endogenously in equilibrium, and not by assumption, correspond to the setting where trade is between a country who is a high quality leader and a country
that supplies both markets with low quality. The resulting contract curve does not include the symmetric world optimum. While lump-sum transfers can correct for this asymmetry, a mutually beneficial cooperative agreement at the world optimum cannot be reached in their absence. The results suggest that successful cooperation in the setting of minimum standards between high quality and low quality exporting countries is less likely, particularly if the agenda for cooperation is to harmonise minimum standards.

The chapter provides a motivation for international cooperation in minimum standards that stems from the inefficiencies of national decisions in an international context where countries are linked through trade, but also points to potential difficulties in the realisation of successful cooperation as international asymmetries hinder the incentives to implement jointly, but not individually, beneficial harmonised standards.

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Appendix A. Quality Reaction Functions

The full expression for the high quality firm's optimal quality response to an unregulated low quality, $q_{Hj}(ql_j)$, or response $q_{Hj}(s^j)$ to a minimum standard $s^j \in \left[q_{ij}^U, s^{pj}\right]$, is given below. Due to its length the country subscript $j$ is dropped and $q_{Hj}(\cdot) \Delta q_H(s)$. It is expressed in terms of the standard, $s$, and the ratio of market size to cost, $\frac{\theta}{\delta}$, denoted by $m$

$$q_H(s) = \frac{1}{9}m + \frac{11}{36}s + A - \frac{1}{A} \left( \frac{5}{324}ms - \frac{1}{81}m^2 - \frac{31}{1296}s^2 \right)$$

where:


\[ A = \left( \frac{1}{729}m^3 - \frac{1049}{23328}m^4 + \frac{503}{7776}m^5s - \frac{5}{1944}m^2s + B \right)^{\frac{1}{3}} \]

and:

\[ B = \left( \frac{1999}{995328}s^6 - \frac{1441}{248832}m^2s^5 + \frac{9803}{2239488}m^3s^4 - \frac{119}{279936}m^4s^3 + \frac{23}{139968}m^5s^2 \right)^{\frac{1}{3}} \]

The low quality firm’s reaction function with \( q_{Lj} (q_{Hj}) \) can be expressed in terms of high quality \( q_{Hj} \) and the ratio of market size to cost denoted \( m \).

For expositional convenience the country subscript \( j \) is dropped.

\[ q_L (q_H) = \frac{17}{6}q_H + D - \frac{1}{D} \left( \frac{7}{6}mq_H - \frac{175}{36}q_H^2 \right) \]

where:

\[ D = \left( \frac{1111}{108}q_H^2 - \frac{95}{24}mq_H^2 + \left( \frac{343}{216}m^3q_H^3 - \frac{7225}{1728}m^2q_H^4 + \frac{365}{288}m^4q_H^5 - \frac{579}{64}q_H^8 \right) \right)^{\frac{1}{3}} \]

The unregulated Nash equilibrium quality levels reported in the text can be found by solving \( q_{Hj} (q_{Lj}) \) and \( q_{Lj} (q_{Hj}) \) simultaneously. The polynomial expressions yield a number of solutions. All negative and complex solutions are discarded, as well as those for which \( q_{Hj} < q_{Lj} \). There is a unique real solution for which \( q_{Hj} > q_{Lj} > 0 \).

The minimum quality standards \( s^*, s^T, s^{WO} \) and threshold standards \( \bar{s}, \)
\( \hat{s} \) and \( \tilde{s} \) referred to in the main text are found by substituting \( q_{Hj}(s^j) \) into the relevant equations for welfare, profit, sales and consumer surplus, which are then expressed in terms of \( s^j \) and market parameters \( b \) and \( \bar{b} \) only. Applying optimisation techniques to these expressions, with respect to \( s^j \), yields the solutions. All solutions described in the text have been computed using Scientific Workplace.

Furthermore, all figures in the chapter are plotted by substituting \( q_{Hj}(s^j) \) into the relevant equations and setting parameter values \( b = \frac{1}{2} \) and \( \bar{b} = 5 \).
4 A Swing-State Theory of Trade Protection in the Electoral College

In this chapter we develop a multi-jurisdictional, infinite horizon, elections model characterised by asymmetric information between politicians and voters and an absence of policy commitment with regards to trade policy. The political districts of the model, or states, form an electoral college that elects the president from two candidates from rival parties. The model is used to investigate how the distribution of voters with heterogeneous preferences across swing states gives rise to incentives for strategic trade protection by incumbent politicians who wish to maximise their chance of re-election.

The chapter contributes to the literature in three ways. First, the model presented extends the trade policy literature by using a political agency methodology that has never been used to address trade policy issues. The approach examines the electoral incentives for the strategic choice of secondary policy issues in a framework characterised by asymmetric information between politicians and voters regarding politicians' preferences over trade policy and lack of pre-commitment to a particular trade policy prior to election. Electoral incentives can cause political incumbents to alter their policy choice in early years in power in order to influence voter beliefs about the nature of future trade policy. By building a reputation as a protectionist or free-trader, the incumbent attracts swing voters to his platform.

The type of policy modelled in this type of framework is characterised by
the inability to tailor it to satisfy the preference of voters at the state level, making it a national policy. Trade policy is thus an excellent candidate for a policy with this feature. Hence, it is the ability to garner electoral college votes nationally that drives results, rather than 'pork-barrel' state level politics. Moreover, it is assumed that the political incumbent has discretion over the selection of trade policy. While this is a reduced form of a more general notion of a cohesive government whose policy decisions are influenced by the desire to retain control of power, it is also the case that over the past few decades there have been periods where the US President was granted trade promotion authority (formerly fast-track authority) to determine trade policy. When granted such authority, the President is able to negotiate trade agreements faster, and while Congress retains power to reject proposed legislation, it has no power of amendment and limited room for debate. While discretion of certain policy instruments is constrained by multilateral agreements, there is still considerable scope for erecting Non-Tariff Barriers, or implementing safeguards, granting relevance to the assumptions of our framework.

Second, we contribute to the political agency literature by developing a tractable multi-jurisdictional framework that extends the single-district political agency framework of recent contributions to the literature by List and Sturm (2006) and Besley and Burgess (2002). We model the electoral system as an electoral college, where electoral votes are attached to political states. This innovation adds a spatial dimension that delivers additional results on
how the distribution of single-issue voters across swing states can influence trade policy decisions. The framework delivers three new propositions that relate the location of swing voters across swing states to the likelihood that incumbents engage in strategic trade protection.

The third contribution of the chapter is that we provide empirical evidence using data for the United States that lends support for the type of mechanisms present in the theoretical model. By augmenting the benchmark empirical specification used by Gawande and Bandyopadhyay (2000) we find evidence in the data to support the theoretical hypothesis that the concentration of a sector across states that are both swing and decisive for election outcomes is a significant determinant of the level of trade protection of that sector. This provides formal support for the claims made in the popular press about the politics behind the recent United States - European Union steel tariffs dispute, “that steel tariffs were introduced for short-term political advantage ... in order to gain votes in key states like West Virginia, Ohio, Pennsylvania and Michigan where the steel industry is a major employer” (The Guardian, November 17th, 2003).

The literature with regards to the role of concentration on endogenous protection is, in general, very different to the framework employed in this chapter. The first strand of the literature is the long-standing tradition that addresses the role of concentration for collective action. The effect of geographical concentration on facilitating lobby formation and therefore positively affecting
trade policy, was first put forward in Olson (1971). The relationship between
the location of industry and import barriers has been debated at length in
this literature. The "close group" hypothesis that the concentration of firms
allows them to overcome free-rider problems and organise lobbying efficiently
is widely accepted and Hansen (1990), among others, provides supporting em­
pirical evidence. This contrasts with the "dispersed group" argument which
posits that geographically dispersed industries enjoy broader political repre­
sentation (depending on the electoral rules) as empirically supported by Pin­
between geographical concentration, and ‘political concentration’, defined as
the spread of industry across political districts, in order to reconcile the two
hypotheses. Their finding that geographically concentrated but politically dis­
persed industries in the US are more likely to be protected, suggests that the
mechanisms linking location, concentration and protection are more complex
than simply those that can be captured through standard measures of con­
centration. This chapter is not related to the collective action literature on
concentration, focusing instead on the effects of concentration for electoral
outcomes and thus electoral incentives to protect. Our framework suggests
concentration might not always matter as such, but rather it is the presence
of industrial concentrations in pivotal locations that has an impact on trade
protection.

The second strand of the literature stems from the seminal contribution
of Grossman and Helpman (1994, 1996) on "Protection for Sale" that analyses the effects of campaign contributions for policy decision-making. Mitra (1999) considers endogenous lobby formation in a theoretical extension of the Grossman and Helpman framework. A multitude of papers have followed in this strand to explain the determinants of trade policy and are surveyed in Helpman (1997) and Grossman and Helpman (2002). Recent contributions to the lobbying literature for trade include Bombardini (2005) who introduces the decisions of individual firms and hence the role of size distributions within industries in determining protection. The relevance of lobbies has been widely tested, for example by Goldberg and Maggi (1997), Gawande and Bandyopadhyay (2000), Eicher and Osang (2002). While geographical concentration measures have also been included in empirical tests of the lobby model, such as Gawande and Bandyopadhyay (2000), they have not been linked to location in swing states. We augment their specification in the empirical section of this chapter to show that political decisions also react to electoral incentives.

The most common electoral approach to the political economy of trade and secondary policy issues is that of median voter models, such as Mayer (1984) and probabilistic voting frameworks such as Yang (1995). These have been used, for example, to explain differences in protectionism based on countries' constitutional set-up (Roelfsema, 2004) or to consider how trade retaliation and liberalisation is affected by the ideological distribution of voters in trading partners (Wiberg, 2005). Our framework is distinct from these approaches
since we examine the effects of swing voters in a model of the electoral college without policy commitment. We show that a redistribution of voters between states in the electoral college, holding the population of each voter type constant, can make trade protection more or less likely. Such redistributions have no impact in frameworks in the spirit of Mayer (1984).

Willmann (2005) employs a median voter model to offer an explanation for the empirical relationship between geographical concentration and protection by introducing regional voters who anticipate that their representatives will internalise the costs of protection, once at the national level. The model cannot offer an explanation, however, as to why industries with the same degree of geographical concentration, that are located in different political states, may be systematically awarded different levels of protection.

Finally, a growing political agency literature has more recently addressed the issue of electoral incentives for policy choices in secondary policy issues, such as trade policy or environmental policy, about which smaller groups of voters have very strong views. Recent contributions to this literature include Coate and Morris (1998), Besley and Case (1995), Besley and Burgess (2002) and List and Sturm (2006). Our basic modelling approach is closest to Besley and Burgess (2002) and List and Sturm (2006), while extending to a multi-jurisdictional framework.

The remainder of the chapter proceeds as follows. Section 1 develops the theoretical model of the electoral college and discuss the testable empirical
implications of the model. The theoretical predictions of the model are tested empirically with US data in section 2. Section 3 concludes.

4.1 The Model

In this section we develop a multi-jurisdictional, infinite horizon, elections model characterised by asymmetric information between politicians and voters and an absence of policy commitment with regards to trade policy. Political incumbents with private preferences over trade policy may have an incentive to build a reputation through the strategic selection of trade policy, in order to swing single-issue voters to their platform in forthcoming elections.

The model contributes to the political agency literature by extending the single-district political agency framework of List and Sturm (2006) and Besley and Burgess (2002) to include a continuum of political districts that form an electoral college. This innovation adds a spatial dimension to the political agency framework that delivers results on how the distribution of single-issue voters across swing states can influence trade policy decisions. Moreover, the model extends the trade policy literature by using a methodology from the political agency literature that has not been used before to examine the strategic incentives for trade policy choice. The empirical implications that arise from the theoretical framework are then tested in section 2.
4.1.1 Economic Environment

Consider a country with a continuum of political districts\textsuperscript{32}, or states, \( s \), over the interval \([0,1]\), each with a unit mass of voters. These states form an electoral college, through which electoral outcomes are determined. In particular, let each state contribute to the electoral outcome through a single electoral college vote, so the aggregate measure of electoral college votes over the continuum of unit interval is also 1.

Further suppose that in any presidential election in the infinite-horizon game there are two candidates from rival parties, Democrat (\(D\)) and Republican (\(R\)), competing for votes. An election may be between two newcomers, or alternatively, between an incumbent politician and a challenger. If a candidate wins a majority of votes in a state, then the electoral college vote of that state is won by that candidate. The election is won by the candidate with the majority of electoral college votes, which corresponds to gaining a majority in a measure of states greater than \(\frac{1}{2}\).

Politicians are assumed to face a binding term limit of two periods. After two terms of holding office an incumbent leaves the political arena and a new candidate from within the party competes with the rival candidate in the presidential elections.

\textsuperscript{32}The assumption of a continuum of political districts allows us to appeal to the law of large number in the calculation of electoral college votes won by each candidate. This facilitates the analysis greatly by making the framework tractable. The role of this assumption is discussed in more detail in section 4.1.4.
Incumbents Policy Preferences  During each term of office the incumbent politician must choose the level of public spending, or 'ideology', denoted by $g$, and a secondary policy, such as trade policy for a particular sector, denoted by $r$. Politicians of either party whose personal views are in favour of free trade are referred to as 'free-traders' ($F$), while those in favour of trade protection are referred to as 'protectionists' ($P$). Suppose that a randomly selected candidate, of either party, is a protectionist with probability $\pi$. While politicians' preferences over public spending are assumed to be public knowledge, their preferences over $r$ are private. Moreover, electoral candidates are unable to commit to a particular trade policy prior to election.

The level of public spending is assumed to be continuous, or, equivalently, ideology is selected from a continuous spectrum. In contrast, trade policy takes the form of a binary choice, to be made by the incumbent politician, between trade protection ($r = 1$) and free trade ($r = 0$). The trade policy is assumed to have negligible financial impact on government revenue, and so the model abstracts from any possible revenue-raising incentives for trade protection.

Suppose politicians earn an 'ego-rent', $\zeta$, from holding a term in office and receive zero payoff when out of office. In addition, a politician faces a utility cost $c = \{c_L, c_H\}$ from deviating from his own preferred trade policy, where $c_H > c_L$. Let the probability of any politician having a low utility cost be $\Pr(c = c_L) = p$. Cost $c$ can be interpreted as a psychological cost of setting
a policy in conflict with personal views. Moreover, let $\beta$ denote the common discount factor, where $\beta$ is assumed to satisfy the following restriction:

$$c_H > \beta \zeta > c_L > 0$$  \hspace{1cm} (178)

Inequality (178) states that the ego-rent from holding one more term in office lies between the high and low utility costs.

**Voter Preferences** Voters are assumed to have heterogeneous preferences over the two policy issues. Suppose four types of voters comprise the measure of voters in each state. A voter of type $k$ in state $s$, can be either a Democrat ($D$), a Republican ($R$), a free-trader ($F$), or a protectionist ($P$). Let $\gamma_k^s$ denote the proportion of voter type $k$ in the unit measure of voters in state $s$, such that:

$$\sum_k \gamma_k^s = 1, \text{ where } k \in \{D, R, F, P\} \text{ and } \gamma_k^s \in [0, 1]$$  \hspace{1cm} (179)

The $D$ and $R$ voters are indifferent about the trade policy issue and vote purely on the basis of their preferences over public policy. Politicians’ choice of $g$ may also be interpreted as reflecting their ideological position, so $D$ and $R$ voters cast their vote according to their ideological preferences. Even though trade protection, e.g. a tariff, raises the relative domestic price of the protected good, we assume this negative effect is negligible compared to the intensity of their ideological preferences. That is, although a price increase in one good in the consumption basket lowers consumer surplus, it is not a sufficient cost
to cause voters to shift their support to another platform. Hence, measure $\gamma_D^*$ of voters always vote Democrat, while $\gamma_R^*$ always vote Republican, in any presidential election.

$P$ and $F$ voters are 'single-issue voters' or 'swing voters' with strong preferences over the secondary policy issue, trade policy. Protectionists may be voters employed in import-competing sectors, whose jobs may be at risk from foreign competition under free trade e.g. Steel industry workers whose employment may be secured through a steel tariff. In contrast, free-traders reflect any voters with strong preferences against trade protection, such as, perhaps, students of economics.

The intensity of swing voters’ preferences is assumed to be such that the payoff received from the implementation of their preferred trade policy dominates any ideological considerations. Suppose protectionists receive a payoff of $x > 0$ if $r = 1$ and 0 otherwise, while supporters of free trade receive $x$ if $r = 0$ and 0 otherwise. Swing voters thus vote for the candidate they believe has the highest probability of implementing their preferred policy. Where candidates are perceived to be identical in this respect, swing voters are assumed to cast their vote by flipping a coin.

Note that $r$, referred to as trade policy in this chapter, can be interpreted as any secondary policy about which a subset of voters have strong views and which has two key characteristics. The first is that $r$ represents a national policy decision that cannot be tailored to satisfy the preferences of voters at the
state level. While some voters may have strong preferences regarding, say, the introduction or abolition of the death penalty, it is possible for a policy decision to be made at the state-level, as is observed in the US. In contrast, a tariff on steel imports, or any other trade policy, can only apply at the national level. Other national policies include immigration policy, foreign policy, participation in a regional trade agreement (e.g. European Union membership), membership in international organisation (e.g. WTO), to mention a few.

The second key characteristic of policy r is that the political incumbent is assumed to have discretion over its selection. Whilst we model the decision-maker as an incumbent politician, the model is consistent with a broader interpretation, where decisions are made by a group of government agents operating as a cohesive entity, whose decisions may be influenced by their desire to perpetuate their control of power.

**Electoral Uncertainty** Uncertainty in the outcome of the election stems from uncertainty at both the state level and the national level. Each state is assumed to be subject to an idiosyncratic pro-D shock, $\nu^s$, that can be interpreted as a shock to voter turnout. Since a vote gained by the D candidate, is also a vote lost by the R candidate, a positive (or negative) $\nu^s$ gives the D candidate an advantage (or disadvantage) of $2\nu^s$. For convenience, we redefine $2\nu^s$ as $\varepsilon^s$. Assume $\varepsilon^s$ is distributed identically and independently according to a symmetric, single-peaked probability density function $h(\varepsilon^s)$, with support $[-\psi, \psi]$, and a continuous cumulative distribution function $H(\varepsilon^s)$. The value
of $\psi$ is important to the extent that it affects the degree of uncertainty over the outcome of elections in each state. We assume a sufficiently wide support so that all states are 'swing states'. That is, no candidate can be certain of winning a majority in any state, but the probability of each candidate winning a majority can be computed for any state with a distribution of voter types, $\gamma^*_k$, where $k \in \{D, R, F, P\}$, given the incumbent’s policy choice $r$ and the cumulative distribution function $H(e^*)$.

In addition to uncertainty at the state level, we introduce aggregate uncertainty in the form of a 'pro-incumbent shock', $u$, in electoral college votes. In an election between two untested politicians, the shock can be in favour of either. Shock $u$ widens (or narrows) the difference in electoral college votes between candidates by $2u$. For convenience, we redefine $2u$ as $\eta$, where $\eta$ is distributed according to a symmetric, single-peaked probability density function, $f(\eta)$ and a continuous cumulative distribution function $F(\eta)$. Again, we assume a sufficiently wide support so that no candidate can secure a majority of electoral college votes. In combination, the state-level and national shocks ensure that no candidate can guarantee to win any state $s$, or the electoral college overall.

In the US, the president is elected indirectly through the Electoral College. Voters vote for state electors who pledge to vote for a particular candidate.

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\[22\text{The uncertainty reflected in the state-specific shocks is insufficient to give rise to aggregate uncertainty, as a result of the infinite nature of states along the continuum. We thus introduce aggregate uncertainty in the form of a shock to electoral college votes at the national level. The importance of this assumption is made clear in section 4.1.4.}\]
These electors cast their electoral vote and the candidate with a majority of electoral votes wins the presidency. In our model, voters are assumed to vote for the candidates directly, while the electoral college system is embodied by the fact that candidates need to win a majority in a majority of states to win the election, rather than a direct majority. The assumptions we make are equivalent to assuming that state-level elections are between two honest electors that have pledged to vote for the $D$ or $R$ candidate, respectively, if elected. A state-level majority won by a $D$ elector corresponds to an electoral college vote won by the $D$ presidential candidate, and similarly for states where the $R$ elector wins a majority. Interpreting our model in this way allows shock $\eta$ to be interpreted as mistakes made by electors when voting, or the presence of a random measure of ‘faithless electors’ who vote for a candidate other than the candidate pledged. Assuming $f(\eta)$ is symmetric around 0 and single-peaked implies that large measures of mistakes in electoral votes cast or large measures of faithless electors are increasingly unlikely.

**Timing of the Elections Game**  Events in the infinitely repeated elections model with infinitely-lived voters occur in the following order.

1. The incumbent politician draws a period one utility cost $c = \{c_L, c_H\}$, observed only by the incumbent.

2. The incumbent makes policy decisions $g$ and $r$.

3. Policy choices are observed by voters and the election for the presidency
in period two takes place.

(a) If the term limit is non-binding, then the election is between the incumbent and a randomly selected rival from the other party.

(b) If the term limit is binding, the election is between two randomly selected candidates from either party.

4. The winner of the presidential election is in office in the next period.

The game is then repeated infinitely through stages (1) to (4). In the next few sections we solve the game by backwards induction and characterise the unique equilibrium strategies of voters and politicians, for a given distribution of voters. The strategic incentives for trade policy choice are examined and the role that the distribution of swing voters plays in shaping these incentives is analysed.

4.1.2 Political Equilibrium

The Markov Perfect equilibria of the game between politicians and voters can be characterised by restricting attention to strategies that depend only on payoff-relevant past events, rather than the entire history of the game. Markov strategies for the incumbent politician, $C_{ij}$, where $i \in \{D, R\}$ and $j \in \{F, P\}$ and for type $k^*$ voters, where $k^* \in \{D, R, F, P\}$, can be said to form an equilibrium if they maximise the value functions of voters and the incumbent politician, given the strategies of the other players.
For the incumbent politician choosing trade policy, the payoff-relevant history of the game is fully described by (a) his utility cost draw, and (b) the number of terms he has already spent in office. Hence, we define a strategy for an incumbent politician as a rule that describes the probability with which he implements trade protection as a function of parameters describing the distribution of voters\textsuperscript{34} across the electoral college, his realised utility cost c and whether he is in his first or second term of office.

For type $k^*$ voters, the payoff-relevant history of the game is, where applicable, the first term trade policy decision of an incumbent who is up for reelection against a randomly selected challenger. In elections between two new candidates, there is no payoff-relevant history on which voters can condition their behaviour. For voter types $k^* = \{D, R\}$ a strategy is a rule that specifies the probability with which they vote for the Democrat or Republican candidate. For voter types $k^* = \{P, F\}$, a strategy is a re-election rule that specifies the probability with which they vote for the incumbent in elections between an incumbent and a challenger, where this probability depends on the updated beliefs regarding the incumbent’s private preferences regarding $r$, conditional on the incumbent’s trade policy decision in his first term of office.

Let $g^*(D)$ and $g^*(R)$ be the unique preferred levels of public spending for $D$ and $R$ voters, respectively, where $g^*(D) > g^*(R)$. It follows directly that $D$ and

\textsuperscript{34}These are defined fully in the next sections.
$R$ candidates always find it optimal to select public spending accordingly\textsuperscript{35} and measure $\gamma_D$ of voters always vote Democrat, while $\gamma_R$ always vote Republican, in any presidential election.

The game between incumbents and swing voters\textsuperscript{36} has two symmetric reputation-building equilibria, where incumbents choose $r$ strategically in order to swing either $P$ or $F$ voters to their platform. Which of the two applies depends on the distribution of swing voters in the electoral college, as is discussed in more detail in section 4.1.5. If the incumbent stands to gain from choosing free trade relative to trade protection, then a protectionist incumbent may have an incentive to deviate from his preferred policy choice and choose free trade. The focus of our analysis is the converse case where the distribution of swing voters is such that the Free-trader incumbent may find it optimal to build a reputation as a protectionist. Note that the incentives for Republican and Democrat incumbents are symmetric, since the incentives for trade policy choice hinge on the extent to which free-trader incumbents of either party can improve their re-election probability through trade protection. Since ideology plays no part in the voting decisions of swing voters, the effects are symmetric for $D$ and $R$ incumbents.

The trade policy game is solved by backward induction, starting from the incentives of any politician facing a binding term limit. For any distribution

\textsuperscript{35}For simplicity, we abstract from strategic incentives in public spending
\textsuperscript{36}The focus of the chapter is the strategic interaction between incumbents and swing voters. For completeness, a discussion of elections between two untested politicians is included in Appendix C.
of ideologists and single-issue voters across the electoral college, an incumbent politician in his second term of office has no incentive to choose a trade policy that conflicts with his personal views, since he can never be re-elected. Hence, incumbents always find it optimal to implement their preferred trade policy in their final term of office.

Over the next sections we derive the conditions under which the following strategies constitute an equilibrium of the trade policy game in incumbents' first term of office: free-trader incumbents deviate from their preferred policy and implement trade protection in the first term of office following a low utility cost draw; protectionist incumbents always implement their preferred policy in the first term of office. Furthermore, protectionist voters vote for the incumbent if trade protection has been implemented in the first term of office, and for the challenger otherwise, while free-trader voters vote for the incumbent if trade protection has not been implemented, and for the challenger otherwise. Moreover, this 'reputation-building' equilibrium is unique for distributions of swing voters under which incumbents can expect to improve their re-election chances through trade protection.

The strategy of a protectionist incumbent is clearly optimal since by implementing trade protection he improves his reelection probability while simultaneously setting his preferred policy. Moreover, if a free-trader incumbent

37 Appendix B shows this reputation-building equilibrium to be unique for distributions of swing voters where the measure of protectionists versus free-trader voters, and their distribution across the electoral college is such that incumbents stand to gain from implementing trade protection in the first term. A symmetric unique equilibrium exists in the case where incumbents stand to gain through free trade.
draws a high utility cost $c = c_H$, then he always follows his preferred policy choice, since $c_H > \beta \zeta$. The benefits in re-election probability can never outweigh the costs of a policy change.

In contrast, a draw of $c_L$ may induce a free-trader to set $r = 1$ if protectionism sufficiently increases the proportion of electoral college votes won so as to alter the election outcome. Since the incumbent's personal preference over $r$ is hidden from voters, a free trade incumbent in his first term may have an incentive to build a reputation as a protectionist in order to attract protectionist voters to his platform in the next election. The lack of a credible commitment to a choice of $r$ implies that pre-election promises carry no weight with single-issue voters, who recognise that politicians can deviate ex post. The only opportunity for candidates to convey information to voters regarding their preferences over trade policy, is through policy decisions made when in power. Voters can update their beliefs on the basis of the incumbent's historical trade policy decisions and thus condition their vote on the history of the elections game. It is this feature of the political agency model that can give rise to strategic behaviour by political incumbents.

Consider the incentives of swing voters in the election for the period two presidency, given the policy change strategy of free-trader incumbents described above. Protectionist and free trade voters maximise their expected

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38Besley and Case (1995) as well as List and Sturm (2006) examine how term limits change the incentives of politicians to build a reputation, with significant effects on policy choice. In this chapter, the optimality of a reputation building strategy depends on both the measure and distribution of $P$ voters relative to $F$ voters across states in the electoral college.
payoff by supporting the candidate with the highest probability of implementing \( r = 1 \) and \( r = 0 \), respectively, in their second term. Consider a free-trader incumbent who can improve the probability of winning a majority of electoral college votes if protectionists support his platform (and free traders support the challenger). If nature draws \( c_H \), the incumbent sets \( r = 0 \), thus revealing himself as a free trader and gaining the support of \( \gamma_p^n \) voters in all states over the continuum. Protectionists support the challenger who is a free-trader with probability \( 1 - \pi \). If \( c_L \) is drawn, the \( D \) free-trader incumbent strategically sets \( r = 1 \) to build a reputation as a protectionist.

The observed first-term trade policy choice provides voters with information with which they update their beliefs about the preferences of the incumbent. Let \( \tilde{\pi} \) denote the updated probability, derived from Bayes' rule, where:

\[
\tilde{\pi} = \frac{\Pr(r = 1 \text{ in 2nd term} \mid r = 1 \text{ in 1st term})}{\Pr(r = 1 \text{ in 2nd term} | r = 1 \text{ in 1st term})} = \frac{\Pr(r = 1 \text{ in 2nd term}) \Pr(r = 1 \text{ in 1st term} \mid r = 1 \text{ in 2nd term})}{\Pr(r = 1 \text{ in 1st term})} = \frac{\pi}{\pi + (1 - \pi)p} \tag{180}
\]

Since politicians set their preferred trade policy when the term limit is binding, the probability that trade protection is set in the second term is the probability that any randomly selected politician is a protectionist, i.e. \( \pi \). Moreover, if the incumbent protects in his second term, he is revealed to be a protectionist and thus protects in the first term with probability 1.
probability that the industry in question is protected in the incumbent’s first term in office is the composite probability of being a protectionist, \( \pi \), or being a free trader who had low cost draw, \((1 - \pi)p\).

Swing voters contrast \( \pi \), the updated probability of the incumbent being a protectionist, with the probability that a randomly selected challenger sets \( r = 1 \) in his first term of office. For a sufficiently small value\(^{39}\) for \( p \), first term protectionism is a sufficiently strong signal of protectionist preferences, so that:

\[
\bar{\pi} > \pi + (1 - \pi)p
\]

(181)

For the rest of the chapter we assume \( p \) is sufficiently small to satisfy condition (181) so as to ensure that \( \gamma^p_s \) support the incumbent government if trade protection is implemented in the first term, while \( \gamma^p_n \) voters support the challenger, given politicians’ strategies in equilibrium. The optimality of swing voters’ re-election strategies is confirmed in Appendix A, where these are shown to maximise voters’ value functions, given politicians’ strategies.

The next section examines how a shift from free trade in the first term of office affects the incumbent’s probability of winning a majority in any state \( s \), given its characteristics. State level probability changes are translated into

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\(^{39}\)List and Sturm (2006) identify two conflicting effects. Applied to our trade policy game, these are: first, an incentive effect that follows from the term limit assumption that lowers the probability of \( r = 1 \) in the second term, since a free-trader will set \( r = 0 \) with certainty; and second, a selection effect that raises the likelihood of \( r = 1 \), since re-elected politicians in their second term of office are more likely to be protectionist. The size of \( p \) determines which of the two effects dominates.
electoral college votes that in turn allow the change in probability of re-election to be derived. We examine incentives for trade protection and confirm that politicians' and voters' strategies constitute a Markov Perfect equilibrium of the game.

4.1.3 Trade Policy and State-Level Majority

Recall that in each state $s$, $\sum_k \gamma_k^s = 1$. Let $\omega_p^s = (\gamma_D^s - \gamma_R^s)$ represent the lead of the $D$ candidate in state $s$, referred to as the 'political lead', and $\omega_P^s = (\gamma_P^s - \gamma_F^s)$ represent the excess of $P$ voters relative to $F$ voters, referred to as the 'trade policy lead'. A state with a larger proportion of Republican voters than Democrat voters has a negative political lead, while a state with a larger proportion of free trade supporters relative to protectionists has a negative trade policy lead.

Let $\rho_{r=0}^s$ denote the probability that the incumbent wins a majority in state $s$ given free trade in the first term, and $\rho_{r=1}^s$ if trade protection is implemented. Given voters' strategies, protectionists vote for the incumbent if trade protection is implemented in the first term of office and for the challenger otherwise, and vice versa for free-trader voters.

Consider a Democrat incumbent in his first term of office. Consider the implications of switching from free-trade to trade protection in his first term of office on the probability of winning a majority in state $s$. The $D$ incumbent gains $\gamma_D^s + \gamma_P^s + \nu^s$ by setting $r = 0$ in his first term, while the $R$ challenger gains the remaining votes. The incumbent wins a majority of votes in state
s, given \( r = 0 \), if \( \gamma_D + \gamma_F + \nu^s > \gamma_R + \gamma_F - \nu^s \), that implies \( \varepsilon^s \) must exceed \( \omega_i^s - \omega_p^s \). If the \( D \) incumbent sets \( r = 1 \), he gains \( \gamma_D + \gamma_F + \nu^s \) and the remaining \( \gamma_R + \gamma_F - \nu^s \) are gained by the \( R \) challenger. Hence, a majority in state \( s \) is won if \( \varepsilon^s \) exceeds \(-\omega_i^s - \omega_p^s \). It follows from the distribution\(^{40}\) of \( \varepsilon^s \) that:

\[
\rho^s_{r=0} = \Pr (\varepsilon^s > \omega_i^s - \omega_p^s) = H (\omega_p^s - \omega_i^s) \tag{182}
\]
\[
\rho^s_{r=1} = \Pr (\varepsilon^s > -\omega_i^s - \omega_p^s) = H (\omega_p^s + \omega_i^s) \tag{183}
\]

Now consider the probabilities \( \rho^s_{r=0} \) and \( \rho^s_{r=1} \) for a Republican incumbent. The \( R \) incumbent gains \( \gamma_R + \gamma_F - \nu^s \) by setting \( r = 0 \) in his first term, while the \( D \) challenger gains the remaining votes. A majority is won by \( R \) in state \( s \) if \( \gamma_R + \gamma_F - \nu^s > \gamma_D + \gamma_F + \nu^s \), that is, if \( \varepsilon^s < -(\omega_i^s + \omega_p^s) \). If the Republican sets \( r = 1 \) in his first term, he gains \( \gamma_R + \gamma_F - \nu^s \) and the remaining \( \gamma_D + \gamma_F + \nu^s \) are gained by the \( D \) challenger. A majority in state \( s \) is won if \( \varepsilon^s < \omega_i^s - \omega_p^s \).

An \( R \) incumbent's probability of majority can thus be expressed by as:

\(^{40}\)Voter turnout across US states has been repeatedly found to be positively correlated with the closeness of electoral competition (Geys, 2006, Matsusaka, 1993, Cox and Munger, 1989). This suggests that the state-specific turnout shock may plausibly depend on \( \omega^{*} \). For simplicity and so as to be able to characterise the political equilibrium, we abstract from this and maintain the assumption of independently and identically distributed state-specific shocks.
Let $\Delta \rho^s = \rho^s_{|r=1} - \rho^s_{|r=0}$ denote the change in the probability of winning a majority in $s$ through trade protection. Combining (182) and (183), as well as (184) and (185), yields that $\Delta \rho^s = H (\omega^s_p + \omega^s_t) - H (\omega^s_p - \omega^s_t)$ for both a Democrat incumbent and a Republican incumbent. The incentives for trade policy implementation are thus symmetric for incumbents of either party. Furthermore, symmetry of $h(\varepsilon^s)$ allows $\Delta \rho^s$ to be summarised by:

$$\Delta \rho^s = H (|\omega^s_p| + \omega^s_t) - H (|\omega^s_p| - \omega^s_t)$$ (186)

Equation (186) shows that the impact of the implementation of first term trade protection by an incumbent, of either party, on the probability of that incumbent winning a majority in state $s$ depends on two factors. First, the absolute value of the political lead, $|\omega^s_p|$, that reflects the degree of electoral competition in state $s$, and second, the trade policy lead, $\omega^s_t$, the reflects the 'swingness' of state $s$, as measured by the difference between protectionist voters and free-trader voters.

For any given level of electoral competition, the magnitude and sign of $\omega^s_t$ determine the extent to which trade policy can 'swing' the state in the incum-
bent's favour. If \( \gamma_P > \gamma_F \), then deviating from free-trade to trade protection improves the incumbent's probability of a majority, so \( \Delta P > 0 \). Conversely, if \( \gamma_P < \gamma_F \) then an incumbent of either party worsens the probability of winning a majority of votes in \( s \), so \( \Delta P < 0 \). Finally, if \( P \) and \( F \) voters have equal measure in state \( s \), then \( \omega^*_s = 0 \) and trade policy has no power in altering electoral outcomes for state \( s \). Moreover, the greater the trade policy lead (lag), the greater the impact on the probability of a majority in \( s \).

For a given trade policy lead, \( \omega^*_s \), the closer is electoral competition between the candidates, the larger the impact of the existing swing voters on \( \Delta P \). To see why this is the case, consider that distribution \( h(\varepsilon^s) \) is symmetric around 0 and single-peaked. For a given \( \omega^*_s \), as \( |\omega^*_s| \to 0 \), the probability gain is from the centre of the distribution, implying a larger \( \Delta P \).

The pair of leads, \( (\omega^*_P, \omega^*_F) \), therefore provides a complete description of state \( s \), in terms of assessing the probability of it being won by either candidate. The discussion has shown that in states where \( \gamma_P > \gamma_F \) the incumbent stands to improve the probability of winning a majority, while chances are worsened in states where \( \gamma_P < \gamma_F \). States where \( \gamma_P = \gamma_F \) are neutral to the trade policy decision. In a multi-jurisdictional setting, the implications of the trade policy decision for incumbents' overall re-election probability depends crucially on the distribution of trade policy and political leads across states in the electoral college. If some states have more \( P \) than \( F \) voters, and others the converse, the incumbent stands to worsen his chances of winning certain electoral college
votes and improve the probability of winning others. The next section turns to the question of aggregation of these effects and characterises the probability of the incumbent winning the election overall.

4.1.4 Trade Policy in the Electoral College

Section 4.1.3 establishes how the trade policy lead and degree of electoral competition in a state determine how the incumbent's first term policy decision alters his subsequent probability of winning the electoral college vote of that state. This section examines how the distribution of state probability changes, $\Delta p^s$, arising from pairs of leads $(\omega_p^s, \omega_f^s)$, can be translated into a measure of electoral college votes. The conditions under which reputation-building occurs in the political equilibrium are then characterised.

The law of large numbers implies that if each state along a continuum is subject to an identically distributed and independent shock $\varepsilon^s$ described by a particular distribution, $h(\varepsilon^s)$, then the distribution of realised shocks over the infinite number of states along the continuum is exactly described by $h(\varepsilon^s)$. This implies that if all states over a continuum have identical $|\omega_p^s|$ and $\omega_f^s$, then $\Delta p^s = H (|\omega_p^s| + \omega_f^s) - H (|\omega_p^s| - \omega_f^s)$ not only describes the change in the incumbent's probability of winning the electoral college vote of each state $s$, but also describes the change in electoral college votes actually won over the continuum of unit length.

There is no aggregate uncertainty, despite the individual uncertainty reflected in the state-specific shocks, as a result of the infinite nature of states.
along the continuum. It follows that in the absence of an additional national shock, there is no aggregate uncertainty over the continuum and election outcomes can be predicted deterministically for different policy choices. To add smoothness to our results, and capture the uncertainty of election outcomes, we introduce aggregate uncertainty in the model through the national pre-incumbent shock $\eta$, distributed by $f(\eta)$. The distribution of shock $\eta$ is assumed to be symmetric around 0 and single-peaked, and distributed over a sufficiently wide support so that no candidate can be certain of a majority of electoral college votes.

To apply the law of large numbers and be able to convert changes in probability into changes in electoral college votes won, it must be the case that $|\omega_p|$ and $\omega_f$ are identical for all states over the continuum. Assuming all states are identical, however, removes all interesting effects that can arise from having a non-uniform distribution of $|\omega_p|$ and $\omega_f$. We thus choose to 'discretise' the continuum into $N$ state 'types', each forming a sub-continuum of the overall continuum of states. States of a given type have identical $|\omega_p|$ and $\omega_f$, but states from different types may differ in their characteristics. Since there are infinitely many states in a continuum of small measure and a continuum of large measure, it follows that we can apply the law of large numbers on a type-by-type basis. Hence the analysis is facilitated greatly through the assumption of a continuum of states, while the discretization of the continuum into types allows us to investigate the role of voter distribution
in a tractable way.

Let there be \( N \) state types, denoted by \( n \), where \( n = \{1, 2, ..., N\} \). All states of a given type are assumed to be identical in terms of their degree of electoral competition \(|\omega_p^n|\) and the trade policy lead \( \omega_p^2 \). Let \( \phi_n \geq 0 \) denote the proportion of states \( s \) that are of type \( n \), such that \( \sum_{n=1}^{N} \phi_n = 1 \). Moreover, suppose state types are ranked in declining \(|\omega_p^n|\) such that \(|\omega_p^k| \geq |\omega_p^j|\), where \( k > j \) and \( k, j \in \{1, 2, ..., N\} \). Further assume \(|\omega_p^1| \leq 1 \) and \(|\omega_p^N| \geq 0 \).

The ranking of discrete state types over the continuum implies that the distribution of \(|\omega_p^s|\) across the electoral college is a step function, as illustrated in figure (29). The distribution of states across the electoral college can be changed through (i) the relative weight of state types in the electoral college through \( \phi_n \), (ii) the finite number of types \( N \), and (iii) the distribution of \(|\omega_p^n|\).

![Figure 29: Representation of \( N \) state types in the continuum.](image-url)
Let $\Delta v^n$ denote the change in electoral college votes of type $n$ won by the incumbent as a result of implementing trade protection in his first term. Moreover, let $\Delta v = \sum_{n=1}^{N} \Delta v^n$ denote the total change in electoral college votes over the whole continuum of states from a deviation from preferred trade policy in the first term. For any state of type $n$, the change in the incumbent’s probability of winning a majority by deviating from free trade is $\Delta \rho^n$, where $\Delta \rho^n = H \left( |\omega_p^n| + \omega_t^n \right) - H \left( |\omega_p^n| - \omega_t^n \right)$. It follows from the law of large numbers that $\phi_n \Delta \rho^n$ gives the change in electoral college votes of type $n$ won by the incumbent. Aggregating over all state types yields:

$$\Delta v = \sum_{n=1}^{N} \Delta v^n = \sum_{n=1}^{N} \phi_n \Delta \rho^n \quad (187)$$

It follows from (187) that $\Delta v$ is a weighted sum of the state type probability changes. The incumbent may gain or lose electoral college votes from setting $r = 1$ depending on sign and magnitude of $\Delta \rho^n$ for each state type, and the weight of that state type in the electoral college, given by $\phi_n$. If the characteristics and distribution of state types are such that $\Delta v < 0$ overall, then the free-trader incumbent cannot improve his chances of re-election through the implementation of trade policy and always selects $r = 0$ in his first term.

The reputation building equilibrium described in section 4.1.2 requires that $\Delta v > 0$, so that free-trader incumbents gain from the shift from free trade. As discussed, there are two symmetric reputation-building equilibria, where
\( \Delta v > 0 \) and where \( \Delta v < 0 \), respectively. We focus on the former, where free-trader incumbents may have an incentive to implement trade protection. In the latter, a protectionist incumbent may choose to build a reputation as a free-trader by abstaining from trade protection in his first term. We return to this issue in the next section where we examine how a redistribution of swing voters gives results in a shift from one equilibrium to another.

It is appealing to interpret \( \Delta v \) in (187) as the change in electoral college votes when there are \( N \) states (rather than \( N \) measures of states), each with \( \phi_n \) electoral college votes, where \( \Delta \rho^n \) represents the change in the probability of winning the electoral college votes of state \( n \). This interpretation is intuitive but important conceptual differences exist between the discrete state interpretation and the continuous measures of states assumed in the model. Under a discrete state interpretation, the electoral votes of a state \( n \), are won or lost as a block \( \phi_n \), while in the continuous measures of state types imply that proportions of votes \( \phi_n \) are won or lost. Hence, with a continuum of states, \( \Delta v \) reflects the actual change in electoral college votes won by the incumbent, not the expected change in electoral college votes.

Recall that \( u \) is the pro-incumbent shock in electoral college votes won. Moreover, let \( v_i^r \) denote the electoral college votes won by the incumbent when he sets trade policy \( r \) in his first term of office. Similarly, \( v_C^r \) denote those won by the challenger, given \( r \). Let \( \omega_u^r = (v_i^r - v_C^r) \) denote the incumbent’s lead over the challenger in the electoral college, given \( r \), where \( \omega_u^r \) can take
values between $-1$ and 1 and reflects the degree of electoral competition at the national level.

For the incumbent to be re-elected, given $r$, it must be the case that $v^r_j + u > v^C_j - u$. Hence, $2u = \eta$ must exceed $v^C_j - v^r_j$. Finally, let $\theta^r$ denote the incumbent’s probability of re-election, given trade policy selection $r$ in the first term of office. Given distribution $F(\eta)$ probabilities $\theta^0$ and $\theta^1$ can be expressed as:

\begin{align*}
\theta^0 &= \Pr(\eta > v^0_C - v^0_j) = 1 - F(v^0_C - v^0_j) = F(\omega^0_v) \\
\theta^1 &= \Pr(\eta > v^1_C - v^1_j) = 1 - F(v^1_C - v^1_j) = F(\omega^1_v)
\end{align*}

(188) (189)

Since $\Delta v$ reflects the change in electoral college votes won by the incumbent from a policy shift, it follows that $v^1_j = v^0_j + \Delta v$ and $v^1_C = v^0_C - \Delta v$. Hence, $\omega^1_v = v^1_j - v^1_C = v^0_j - v^0_C + 2\Delta v = \omega^0_v + 2\Delta v$. The re-election probabilities can thus be re-written as:

\begin{align*}
\theta^0 &= F(\omega^0_v) \\
\theta^1 &= F(\omega^0_v + 2\Delta v)
\end{align*}

(190) (191)

Defining $\Delta\theta$ as the change in re-election probability from a policy shift, it follows directly from (190) and (191) that $\Delta\theta = \theta^1 - \theta^0 = F(\omega^0_v + 2\Delta v) -$
Furthermore, symmetry of $f(\eta)$ allows $\Delta \theta$ to be summarised by:

$$\Delta \theta = F(|\omega_v^0| + 2\Delta v) - F(|\omega_v^0|) \quad (192)$$

It follows from (192) that the incumbent enjoys an improvement in re-election probability ($\Delta \theta > 0$) from the implementation of trade protection provided there is an overall gain in electoral college votes from the policy ($\Delta v > 0$). If $\Delta v > 0$, then the expected payoff from implementing trade protection in the first term is $(\Delta \theta) \beta \zeta$ for a free-trader incumbent of either party. For $r = 1$ to be an optimal strategy, the expected payoff must exceed the incumbent’s utility cost draw. Since $(\Delta \theta) \beta \zeta < \beta \zeta$ and $c_H > \beta \zeta$, the analysis confirms that a free-trader incumbent with a high utility cost draw never finds it optimal to deviate from free trade. If a low utility cost $c_L$ is drawn, then $(\Delta \theta) \beta \zeta$ must be larger than $c_L$ for the reputation-building strategy to be optimal.

In the symmetric equilibrium where $\Delta v < 0$, a protectionist incumbent improves his re-election probability by setting $r = 0$ in his first term. Since $\Delta \theta$ is defined as the change in re-election probability from a policy shift, then $\Delta \theta = \theta^0 - \theta^1 > 0$. If the expected payoff exceeds $c_L$ then his reputation-building strategy is optimal.

**Proposition 36** If $(\Delta \theta) \beta \zeta > c_L$, then there is a unique equilibrium in which incumbent politicians with a low utility cost draw ($c_L$) deviate from their preferred trade policy in their first term of office if this increases their re-election
probability and follow their private preferences otherwise.

Proof. It follows from (186) that 
\[ \Delta \rho^n = H \left( \left| \omega_p^n \right| + \omega_i^n \right) - H \left( \left| \omega_p^n \right| - \omega_i^n \right) \]
is the change in the probability of winning the electoral college vote of a state of type \( n \). The resulting change in type \( n \) electoral college votes won is \( \phi_n \Delta \rho^n \). Aggregating over state types gives the total change in electoral college votes from a policy shift, 
\[ \Delta v = \sum_{n=1}^{N} \phi_n \Delta \rho^n. \]
If \( \Delta v > 0 \), then a free-trader incumbent of either party enjoys a gain in re-election probability \( \Delta \theta \) from setting \( r = 1 \) in his first term of office. Provided a low cost is drawn and \( (\Delta \theta) \beta \zeta > c_L \), the incumbent enjoys a positive net expected payoff from setting \( r = 1 \), so finds it optimal to deviate from his preferred private policy. If a high utility cost \( c_H \) is drawn by an incumbent or the gain in re-election probability \( \Delta \theta \) is not sufficiently large for \( (\Delta \theta) \beta \zeta > c_L \) to be satisfied, then the incumbent sets his preferred policy, free trade. In this equilibrium, a protectionist incumbent cannot increase his re-election probability through a policy shift, so always finds it optimal to follow his private preferences and set \( r = 1 \).

Conversely, if \( \Delta v < 0 \), then a protectionist incumbent of either party enjoys a gain in re-election probability \( \Delta \theta \) from setting \( r = 0 \) in his first term of office. Provided a low cost is drawn and \( (\Delta \theta) \beta \zeta > c_L \), the incumbent enjoys a positive net expected payoff from setting \( r = 0 \), so finds it optimal to deviate from his preferred private policy. If a high utility cost \( c_H \) is drawn by an incumbent or the gain in re-election probability \( \Delta \theta \) is not sufficiently large.
for \((\Delta \theta) \beta \zeta > c_L\) to be satisfied, then the incumbent sets his preferred policy, trade protection. In this equilibrium, a free-trader incumbent cannot increase his re-election probability through a policy shift, so always finds it optimal to follow his private preferences and set \(r = 1\).

For a given distribution of voters in the electoral college, and thus given \(\Delta v\), the equilibrium in which reputation-building forms part of incumbent’s optimal strategies is the unique equilibrium. A proof of uniqueness can be found in Appendix B.

Inspection of (192) reveals that the reputation-building equilibrium depends on two key national-level parameters of the model. First, the closeness of electoral competition at the national level, as measured by \(|\omega_0|\) and second, the gain in electoral college votes \(\Delta v\) from a policy shift. The characteristics of \(f(\eta)\) imply that a closer degree of electoral competition between candidates at the national level, the greater the probability gain from an increase in electoral college votes from a policy shift.

Intuitively, the closer the competition between the two candidates, that is the smaller is \(|\omega_0|\), then the more likely it is that the pro-incumbent shock perturbs the election outcome. Since the pro-incumbent shock is more likely to be near 0, a given gain in electoral college votes through a strategic trade policy decision is more beneficial the closer \(|\omega_0|\) is to 0.

Conversely, relatively weak electoral competition, reflected by high \(|\omega_0|\), implies that one of the candidates has a large lead in electoral college votes.
over the other. The probability that a sufficiently large shock is realised to change the election outcome is relatively low. A gain $\Delta v$ implies a smaller shock is sufficient to change the election result, but the further from 0 is the initial difference in electoral college votes, the smaller the associated gain in probability.

Furthermore, for any given degree of national electoral competition, the greater the increase in electoral college votes $\Delta v$ that can be won through a policy shift, the greater is the incumbent's gain in re-election probability. Intuitively, the more votes that can be 'swung' at the national level from trade policy, the larger the impact of the trade policy decision on re-election probability.

A change in either $|\omega_v^0|$ or $\Delta v$ has an impact on re-election probability $\Delta \theta$ and thus on the likelihood that condition\(^{41}\) $(\Delta \theta) \beta \zeta > c_L$ is satisfied. These results are summarised in proposition (37).

**Proposition 37** An increase in the number of electoral college votes that can be won by deviating from preferred trade policy ($\Delta v$) or an increase in electoral competition at the national level (lower $|\omega_v^0|$) make reputation-building through the strategic selection of trade policy more likely.

**Proof.** Consider a distribution of voters such that $\Delta v > 0$. It follows directly from $\Delta \theta = F (|\omega_v^0| + 2\Delta v) - F (|\omega_v^0|)$ that an increase in $\Delta v$, *ceteris paribus*,

\(^{41}\) An increase in the discounted ego-rent, $\beta \zeta$, or decrease in $c_L$ also increase the likelihood of there being a reputation-building equilibrium.
increases the change in the incumbent's re-election probability from the implementation of trade protection. Moreover, since \( f(\eta) \) is symmetric around 0 and single-peaked, \( \Delta \theta \) increases as \( |\omega^0_v| \to 0 \). A higher \( \Delta \theta \) from either increase makes it more likely that condition \( (\Delta \theta) \beta \zeta > c_L \) is satisfied, and thus that reputation building takes place. ■

Propositions (36) and (37) confirm the same properties apply in the multi-jurisdictional framework as in the related literature with one jurisdiction. Namely, that there exists a unique reputation-building equilibrium that is more likely the larger the number of votes that can be swung through a policy decision, and the closer is electoral competition between candidates.

The multi-jurisdictional framework extends the literature in two ways. First, the electoral college structure provides new insights into how state-level characteristics in the electoral college combine to influence the incentives for strategic trade protection at the national level. This provides for a more nuanced analysis of how swing-voters affect policy decisions. Second, the framework adds a spatial dimension that allows distributional effects to be examined in a highly tractable way. The analysis delivers three new propositions that describe how the distribution of voters in the electoral college influence trade policy decisions. These effects are analysed in the next section.

4.1.5 Distribution of Voters and Electoral Incentives

Section 4.1.4 establishes that the reputation building equilibrium depends on parameters, \( |\omega^0_v| \) and \( \Delta v \), that contribute to the change in the incum-
bent's re-election probability arising from a first term policy shift. While these national-level parameters confirm the importance of electoral competition and the change in electoral college votes won as key determinants, they represent summary statistics of the underlying state-level characteristics in the electoral college. Expressing $\Delta \theta$ in terms of state-level parameters gives rise to proposition (38).

**Proposition 38** The likelihood of strategic trade policy implementation depends on the distribution of swing voters and ideologists within states of a given type $([\omega_p^n, \omega_t^n])$, the distribution of state types in the electoral college ($\phi_n$) and the probability distributions of state-level ($H(e^*)$) and national shocks ($F(\eta)$).

**Proof.** Consider the change in re-election probability summarised by (192). Recall that $\Delta v = \sum_{n=1}^{N} \phi_n \Delta \rho^n$. This can be expressed in terms of state-level characteristics by substituting for $\Delta \rho^n$. This yields:

$$
\Delta v = \sum_{n=1}^{N} \phi_n \Delta \rho^n = \sum_{n=1}^{N} \phi_n \left( H(|\omega_p^n| + \omega_t^n) - H(|\omega_p^n| - \omega_t^n) \right)
$$

(193)

Moreover, electoral competition at the national level $|\omega_v^0| = |v_f^0 - v_C^0|$, where $v_f^0$ and $v_C^0$ are the electoral college votes won by the incumbent and challenger, respectively, under free trade in the first term. $v_f^0$ is the weighted sum of electoral college votes won by state type, when $r = 0$. Thus $v_f^0 = \sum_{n=1}^{N} \phi_n \rho^n_{r=0}$. Moreover, since $v_f^0 + v_C^0 = 1$, it is straightforward to express the challenger's electoral college votes as $v_C^0 = 1 - \sum_{n=1}^{N} \phi_n \rho^n_{r=0}$. Combining
these allows national-level electoral competition to be expressed in terms of state-level characteristics:

\[
\omega_v^0 = 2 \sum_{n=1}^{N} \phi_n p^n_{r=0} - 1 = 2 \sum_{n=1}^{N} \phi_n H (|\omega_p^n| - \omega_r^n) - 1
\]  \hspace{1cm} (194)

Substituting (193) and (194) into (192) allows the incumbents re-election probability to be expressed in terms of state-level variables and distributional parameters:

\[
\Delta \theta = F (|\omega_1^v|) - F (|\omega_v^0|) \\
= F \left( 2 \sum_{n=1}^{N} \phi_n H (|\omega_p^n| + \omega_r^n) - 1 \right) - F \left( 2 \sum_{n=1}^{N} \phi_n H (|\omega_p^n| - \omega_r^n) - 1 \right)
\]  \hspace{1cm} (195)

Inspection of (195) shows that the change in re-election probability, and thus the likelihood of strategic trade policy implementation, hinges on (i) the distribution of swing voters and ideologists within states of a given type, summarised by \((|\omega_p^n|, \omega_r^n)\), (ii) the distribution of state types in the electoral college, reflected by proportions \(\phi_n\), and (iii) the distributions of state-level and national-level shocks, \(H(\varepsilon^s)\) and \(F(\eta)\). ■

To show how the spatial position of swing voters can influence policy decisions, we consider two redistribution experiments that satisfy the following conditions:

1. The aggregate population of each voter type in the electoral college is
kept constant. In particular, if we let $\Gamma_k$ denote the total measure of $k$ voters in the electoral college, then the distribution of $k$ voters across $n$ state types, as reflected by $\gamma^n_k$, must satisfy the following condition:

$$\Gamma_k = \sum_{n=1}^{N} \phi_n \gamma^n_k , \text{ where } k \in \{D, R, F, P\}$$  \hspace{1cm} (196)$$

2. All states always have a unit measure of voters, so $\sum_k \gamma^n_k = 1$. This implies that an increase in the measure of voters of a particular type in a state, must be accompanied by a decrease in voters of some other type. Denoting the total measure of voters by $\Gamma$, conditions 1 and 2 imply that the total measure of voters in the electoral college must be 1:

$$\Gamma = \sum_k \Gamma_k = \sum_k \sum_n \phi_n \gamma^n_k = \sum_n \phi_n \sum_k \gamma^n_k = 1$$  \hspace{1cm} (197)$$

3. Feasibility constraints regarding pairs of values $(\omega^n_p, \omega^n_t)$ for all state types $n$ are adhered to. To see how these apply, consider pair $(|\omega^n_p|, \omega^n_t)$ that describes states of type $n$. Since the sum of all voter types is 1 in each state, there is a finite range of values that leads $\omega^n_p$ and $\omega^n_t$ may feasibly take. In particular, the larger is the lead in any one dimension, the smaller the scope for variability in the lead in the other dimension. For example, if $\omega^n_p = 1$ (or $-1$), then a state of type $n$ is made up entirely of $D$ voters (or $R$ voters) so $\omega^n_t = 0$. At the other extreme, $\omega^n_t = 1$ (or $-1$) implies $|\omega^n_p| = 0$. Figure (30) illustrates the set of all
feasible combinations of \((\omega_p^n, \omega_p^R)\), given \(\sum_k \gamma_k^n = 1\). Consider \(|\omega_p^n| = \alpha\).

This implies that \(D\) voters exceed \(R\) voters by \(\alpha\), or vice versa. For example, suppose \(\gamma_D^n = 0.4\) and \(\gamma_R^n = 0.2\), in states of type \(n\), implying a Democratic lead \(\omega_p^n = 0.2\). The sum of ideologists is 0.6, so the swing voters represent 0.4 of each state. If all swing voters are protectionist, then \(\omega_p^n = 0.4\), while if all are free-traders, then \(\omega_p^n = -0.4\). Suppose instead that \(\omega_p^n = 0.2\) arises from \(\gamma_D^n = 0.3\) and \(\gamma_R^n = 0.1\). In this case, \(\omega_p^n\) ranges from \(-0.5\) to \(0.5\). It is straightforward to see that if there are no \(R\) voters at all, then \(\omega_p^n\) ranges from \(-0.8\) to \(0.8\). This gives the largest possible range consistent with \(\omega_p^n = \gamma_D^n = 0.2\). Similar reasoning applies for a state where \(\omega_p^n = -0.2\).

In general, the maximum measure of single-issue voters consistent with \(|\omega_p^n| = \alpha\) is \(1 - \alpha\). Hence, the maximum trade policy lead is \(\omega_p^n = 1 - \alpha\), where all swing voters are protectionists. Conversely, the minimum trade policy lead consistent with \(|\omega_p^n| = \alpha\) is \(\omega_p^n = \alpha - 1\), where all single-issue voters are free-traders. These maximum and minimum leads form the rhombus in figure (30). States with positive measures of all voter types are described by \((\omega_p^n, \omega_p^R)\) that lie inside the rhombus. The discussion can be summarised by the following range for \(\omega_p^n\), given \(|\omega_p^n|\):

\[
\omega_p^n \in [\alpha - 1, 1 - \alpha], \text{ if } |\omega_p^n| = \alpha, \text{ where } \alpha \in [0, 1] \quad (198)
\]

Any redistribution of voters across state types must be consistent with
The analysis in the chapter up to this point has been concerned with politicians' optimal strategies for a given distribution of voters. The two redistribution experiments in this section address a different set of questions. In particular, how a change in the spatial location of a measure of swing voters can alter the electoral incentives for trade protection of a given industry, whether through variation in the degree of state-level competition across the electoral college, or through institutional parameters, such as variation in the contribution of electoral votes of different state types in the electoral college. While we model the redistribution as a physical migration of voters with fixed preferences, this need not be the case. Preferences of voters may change in a given location, without migration, through changes in the pattern of industrial concentration and employment. The experiments reveal two key distributional determinants of electoral incentives. First, state 'swingness', as measured by the closeness of state-level electoral competition, and second, state 'decisiveness', as measured by the proportion of electoral college votes represented by states of a given type.

Let us define the initial distribution of swing voters prior to any redistribution. This is referred to as the 'benchmark distribution' in the rest of the section. Suppose the $N$ state types are ranked such that $1 > |\omega_p^1| > \ldots > |\omega_p^n| > \ldots > |\omega_p^N| > 0$. Condition (198) implies that the maximum measure of single-issue voters in states of type $n$ consistent with $|\omega_p^n|$ is $1 - |\omega_p^n|$. Assume
the maximum feasible measure of single-issue voters is present in all states of
types $n$. It follows that the measure of swing voters is increasing with $n$ since
$|\omega^p|_n$ is decreasing with $n$. Further assume that in the benchmark distribution,
the swing voters of each state of type $n$ are split evenly between $P$ and $F$
voters, such that $\gamma^p_p = \gamma^p_F = \frac{1}{2} [1 - |\omega^p|_n]$. This implies that for each state
of type $n$, $\omega^p_n = 0$, thereby placing the distribution of state types along the
$\omega_p$ axis in Figure (30). Hence, by construction, the benchmark distribution is
characterised by $\Delta \rho^a = \Delta v^a = 0$, $\forall n$, and thus $\Delta v = \Delta \theta = 0$, so trade policy
has no impact on re-election probability. The conditions for a reputation-
building equilibrium are not satisfied under the benchmark distribution so all
incumbents set their preferred trade policy in their first term of office.
Redistribution A - 'Swingness’ From the benchmark distribution, consider a redistribution of $P$ and $F$ voters that increases the concentration of protectionist voters in states with relatively low $|\omega_p^n|$, and vice versa for free-traders. The additional assumption is made that all state types contribute equally to the electoral college, such that $\phi_n = \phi$, $\forall n$. Under these assumptions and provided the redistribution satisfies conditions (1) to (3), the following proposition holds.

Proposition 39 A redistribution of protectionist voters from states with weaker electoral competition (higher $|\omega_p^n|$) to states with stronger electoral competition (lower $|\omega_p^n|$) makes it more likely that incumbents engage in strategic trade protection.

Proof. Starting from the initial distribution where there are $\frac{1}{2} [1 - |\omega_p^n|]$ protectionists and free traders in each state of type $n$, and state types are ranked in decreasing $|\omega_p^n|$, it follows by construction that if a positive measure $k \leq \frac{1}{2} [1 - |\omega_p^n|]$ of protectionist voters is redistributed from each state of type $i$ to each state of type $j$, where $i < j$, then:

(i) there are sufficient free-trader voters in each state of type $j$ to replace the $k$ voters redistributed to $j$ from state $i$, since $\frac{1}{2} [1 - |\omega_p^i|] < \frac{1}{2} [1 - |\omega_p^j|]$.

(ii) this exchange of swing voters redistributes $P$ voters towards a measure of states with a closer electoral competition and $F$ voters towards states with weaker electoral competition.

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42 This simplifying assumption controls for the effects on reputation-building incentives arising from different state-type contributions of electoral college votes.
In each $j$ state, the measure of protectionists rises by $k$ and the measure of free traders falls by $k$, hence $\omega^j_p = \gamma^j_p - \gamma^j_p = 2k > 0$. Conversely, in each state $i$, $\omega^i_p = \gamma^i_p - \gamma^i_p = -2k < 0$. For all states of type $n$, where $n \neq \{i, j\}$, $\omega^n_p = 0$.

Consider the effects of a deviation from free trade by an incumbent in his first term of office post-redistribution. For states of type $i$ and $j$, the change in a free-trader incumbent's probability of winning a majority from setting $r = 1$ in his first term are:

$$\Delta \rho^j = H (|\omega^j_p| + 2k) - H (|\omega^j_p| - 2k) > 0$$  \hspace{1cm} (199)

$$\Delta \rho^i = H (|\omega^i_p| - 2k) - H (|\omega^i_p| + 2k) < 0$$  \hspace{1cm} (200)

It follows from (199) and (200) that setting $r = 1$ improves the incumbent's probability of winning $j$ state electoral college votes, where $P$ voters exceed $F$, but worsens his chances of winning $i$ state electoral votes where the opposite is the case. The overall change in electoral college votes is given by:

$$\Delta v = \phi \Delta \rho^j + \phi \Delta \rho^i + \phi \sum_{n \neq i,j} \Delta \rho^n = \phi (\Delta \rho^j + \Delta \rho^i)$$  \hspace{1cm} (201)

$$= \phi [H (|\omega^j_p| + 2k) - H (|\omega^j_p| - 2k)] - \phi [H (|\omega^i_p| + 2k) - H (|\omega^i_p| - 2k)] > 0$$

Since $|\omega^i_p| > |\omega^j_p|$, it follows from the characteristics of $h(\varepsilon^n)$ that the change in electoral college votes won by the incumbent increases, from 0 in the benchmark distribution, to $\Delta v > 0$. It follows that from having no effect on re-election probability under the benchmark distribution, the redistribution of
protectionists to states with closer electoral competition increases their relative importance in the electoral college, giving rises to an improvement in re-election probability through first term trade protection. Thus an increase in the concentration of protectionists in states with closer electoral competition makes strategic trade protection by incumbents more likely. ■

The redistribution considered has the dual effect of giving protectionists a lead in one group of states, and free-traders a lead in another group of states, where both groups have equal measure. It is the closeness of electoral competition in the former group of states that gives protectionists a greater weight in the overall assessment of the change in electoral college votes and thus in re-election probability. If the degree of electoral competition were the same in the two state types, then these probability changes would entirely offset each other. It is the difference in the 'swingness' of states across which redistribution takes place that drives the electoral incentives to implement trade protection after the redistribution.

A symmetric redistribution that gives free-traders a lead in groups of states that are more competitive has the opposite effect, such that \( \Delta v < 0 \) holds post-redistribution. This corresponds to the symmetric reputation-building equilibrium where protectionist incumbents override their protectionist views and choose free-trade in their first term following a low cost draw. Thus a population-preserving redistribution of swing voters can generate either of the two symmetric reputation-building equilibria.
Intuitively, the preferences of concentrations of swing voters that contribute most in probability terms to election outcomes are given more weight by incumbents when making policy decisions. Moreover, the concentrations that contribute most are those in swing states whose electoral outcome is most uncertain.

**Redistribution B - ‘Decisiveness’** From the benchmark distribution, consider a redistribution of protectionists from states of type $i$ to states of type $j$, where both states types are characterised by the same degree of electoral competition, but where $j$ states represent a larger proportion of electoral college votes than do $i$ states. The assumption that $|\omega^i_p| = |\omega^j_p| = |\omega_p|$ controls for the ‘swingness’ effect, while $\phi_j > \phi_i$ isolates the effect of distributing swing voters across larger or smaller measures of swing states. Suppose that all states of type $n$, where $n \neq \{i,j\}$ remain unchanged.

Starting from $\omega^i = \omega^j = 0$, the redistribution described has the effect of concentrating a measure of $F$ voters over a smaller measure of swing states, $i$, while the same volume of $P$ voters is spread evenly over a larger measure of states, $j$, with an identical degree of electoral competitiveness. This gives rise to two conflicting effects on the electoral incentives for trade protection. On the one hand, the relatively large concentration of free-traders in $i$ states implies that a first term protectionist policy reduces the incumbent’s probability of winning a majority in each state $i$ by more than the probability gain in winning a majority in each state $j$, where protectionists are less concentrated.
On the other hand, $j$ states represent a larger measure of electoral college votes than $i$ states.

Whether the former 'concentration effect' or the latter 'decisiveness effect' dominates determines whether the redistribution increases or decreases the electoral college votes won overall by setting $r = 1$ in the first term of office. If $\Delta v > 0$ overall, then trade protection is more likely than under the benchmark distribution of swing voters. Otherwise, $\Delta v < 0$ and the symmetric reputation-building equilibrium is more likely.

The decisiveness effect dominates the concentration effect when the degree of electoral competition is strong in states $i$ and $j$. Intuitively, the greater the swingness of states, the greater the impact in probability terms of even a small lead in protectionist swing voters. Thus the gain in electoral college votes from trade protection is larger, ceteris paribus, when a given measure of protectionist voters is spread over a large measure of highly swing states, than when concentrated over a smaller measure of identical states. Conversely, a small protectionist lead has less potency when electoral competition is weak than when electoral competition is strong, causing the concentration effect to outweigh the decisiveness effect such that the more concentrated $F$ voters in states of type $i$ have a larger impact on electoral college votes won than the less concentrated $P$ voters in type $j$ states, under first term strategic trade protection.

**Proposition 40** A redistribution of protectionist voters from swing states that
constitute a smaller proportion of electoral college votes (lower $\phi$) to swing states that constitute a larger proportion of electoral college votes (higher $\phi$) makes it more likely that incumbents engage in strategic trade protection.

**Proof.** Consider state types $i$ and $j$ where $|\omega_p^i| = |\omega_p^j| = |\omega_p|$ and $\phi_j > \phi_i$. The total population of swing voters over states of type $i$ is $\phi_i [1 - |\omega_p|]$, which is less than the total population of swing voters over $j$ states, given by $\phi_j [1 - |\omega_p|]$. Recall that $P$ and $F$ voters are assumed to have equal measure in the benchmark distribution, such that $\Gamma_P^i = \Gamma_P^j = \frac{\phi_i}{2} [1 - |\omega_p|]$ and $\Gamma_P^j = \frac{\phi_i}{2} [1 - |\omega_p|]$. Since, by construction, $\Gamma_P^i < \Gamma_P^j$, any redistribution of protectionist voters from $i$ to $j$ states is feasible up to $\Gamma_P^i$. Suppose $k$ protectionist voters from each state $i$ are redistributed evenly across states $j$. It follows that $\phi_i k$ voters are distributed evenly over $\phi_j$ states. Let $\lambda$ denote the additional protectionist voters in each state $j$, where $\lambda = \frac{\phi_i}{\phi_j} k$. Moreover, $\phi_j \lambda$ free-traders are redistributed evenly across $i$ states. Thus $\phi_i k = \phi_j \lambda$. Since $\phi_j > \phi_i$, it follows that $\lambda < k$.

In each $j$ state, the measure of protectionist rises and free traders falls by $\frac{\phi_i}{\phi_j} k$. Hence, $\omega_t^j = \gamma_p^j - \gamma_F^j = 2k \frac{\phi_i}{\phi_j} > 0$. Conversely, in each state $i$, $\omega_t^i = \gamma_p^i - \gamma_F^i = -2k < 0$. For all states of type $n$, where $n \neq \{i, j\}$, $\omega_t^n = 0$.

Consider the effects of a deviation from free trade by an incumbent in his first term of office post-redistribution. For states of type $i$ and $j$, the change in a free-trader incumbent’s probability of winning a majority from setting $r = 1$
in his first term are:

\[ \Delta \rho^i = H \left( |\omega_p| + 2k \frac{\phi_i}{\phi_j} \right) - H \left( |\omega_p| - 2k \frac{\phi_i}{\phi_j} \right) > 0 \quad (202) \]

\[ \Delta \rho^j = H \left( |\omega_p| - 2k \right) - H \left( |\omega_p| + 2k \right) < 0 \quad (203) \]

Since \( \phi_j > \phi_i \), it follows that \( 2k \frac{\phi_i}{\phi_j} < 2k \) so the protectionist lead in \( j \) states is smaller than the free-trader lead in \( i \) states. Inspection of (202) and (203) reveal that setting \( r = 1 \) improves the incumbent’s probability of winning each \( j \) state electoral college vote but worsens his chances of winning each \( i \) state electoral college vote. Moreover, since the degree of electoral competition is the same across the two state types, it follows that \( \Delta \rho^j < -\Delta \rho^i \). This reflects the ‘concentration effect’ of the redistribution of swing voters across state types of different measure. However, \( \phi_j > \phi_i \), so there is also a ‘decisiveness effect’ since there are more \( j \) state than \( i \) state electoral college votes. Using \( \lambda = k \frac{\phi_i}{\phi_j} < k \) and \( \phi_j = \phi_i \lambda^k > \phi_i \) for simplification allows the overall change in electoral college votes won by the incumbent as a result of first term protectionist to be expressed as:

\[
\Delta v = \phi_i \Delta \rho^i + \phi_j \Delta \rho^j + \sum_{n \neq i,j} \phi_n \Delta \rho^n = \phi_i \Delta \rho^i + \phi_j \Delta \rho^j
\]

(204)

\[
= \phi_i \frac{k}{\lambda} \left[ H \left( |\omega_p| + 2\lambda \right) - H \left( |\omega_p| - 2\lambda \right) \right] - \phi_i \left[ H \left( |\omega_p| + 2k \right) - H \left( |\omega_p| - 2k \right) \right]
\]

Inspection of (204) reveals the trade-off between the two conflicting effects.

The first term shows a smaller probability change per \( j \) state, with weight \( \phi_i \).
magnified by $\frac{k}{\lambda}$ as a result of the larger scale of electoral college votes. The second term shows the larger probability change for $i$ states weighted only by $\phi_i$. The characteristics of $H(\cdot)$ imply that $\Delta v > 0$ when electoral competition is sufficiently close. Hence, when states $i$ and $j$ are characterised by low $|\omega_p|$ and thus a high degree of swingness, the redistribution of protectionist voters across a measure of more decisive states makes strategic trade protection more likely.

Propositions (39) and (40) provide new insights concerning how the distribution of voters can influence the decisions of policy makers driven by electoral incentives. The model emphasizes the differences between direct and indirect voting for a presidential candidate by showing how the electoral college system places different weights on the preferences of swing voters, depending on their location. The propositions show analytically that incremental distributional changes between states that alter the distribution of leads within states can have a significant effect on the incentives for policy implementation.

The propositions show that concentrations of swing voters with a particular trade policy stance have a larger impact on electoral outcomes when located in swing states. Moreover, their overall impact on the re-election probability of incumbents increases if their influence is spread over swing states that constitute a larger proportion of electoral college votes and are thus more decisive for the election.

The propositions thus combine to give the overall prediction that the trade
policy preferences of a measure of swing voters are more likely to be satisfied if these swing voters are concentrated in states that are both swing and decisive for the election outcome. Since voters with strong views over the protection of a particular industry are likely to be stakeholders in that industry, whether employees, entrepreneurs, shareholders etc., the main testable empirical implication of the model is that industries that are concentrated in swing and decisive states are more likely to be protected. The next section describes the results of our empirical investigation using US data that tests for the empirical implication of the model.

4.2 Empirical Analysis

This section provides evidence supporting the theoretical prediction that industries with large concentrations in swing and decisive states are more likely to be protected. The empirical analysis employs a benchmark test of the “Protection for Sale” mechanism of Grossman and Helpman (1994) using the empirical model and data of Gawande and Bandyopadhyay (2000). This baseline constitutes the “state-of-the-art” in empirical political economy of trade. We augment it with the data necessary to test our hypothesis that industrial concentration in key political districts is a significant determinant of trade policy. While the empirical specification does not form a direct test of our model, we present reduced form evidence that suggests previous empirical studies of the Grossman and Helpman (1994) model have omitted variables from their analysis that our theoretical analysis puts forward as being relevant.
The rest of the section proceeds as follows. First, we outline the model and data of Gawande and Bandyopadhyay (2000). Second, we present the data and method of construction for the measure used to capture the swingness and decisiveness elements of the model. Finally, our results are described.

4.2.1 Data and Empirical Specification

The theoretical model developed in section 1 considers how electoral incentives influence a binary trade policy decision that reflects either free trade or trade protection. The precise nature of this trade protection instrument is unspecified in the model, but is distinguished by the discretion the political incumbent is assumed to have over it.

In practice, unilateral political discretion over trade policy, in particular import tariffs, is constrained by multilateral agreements. Import tariffs are thus jointly determined through multilateral trade negotiations rather than the sole result of a government's political agenda. Moreover, tariff levels for manufacturing products are very low since they have been greatly reduced over last few decades under the GATT and WTO. In contrast, Non-Tariff Barriers (NTBs) allow governments to exercise more discretion in trade protection since these are not regulated to the extent of tariffs. For this reason, the literature has mainly employed coverage ratios for non-tariff barriers as a measure of trade protection, where these represent the share of products within an industry that benefit from one or more quantitative or qualitative trade restrictions: quantity-oriented barriers such as voluntary export restraints and
quotas, price-oriented measures such as antidumping and countervailing duties, and threats of quantity and quality monitoring. We therefore adopt the same approach as in the related literature\textsuperscript{43} in considering NTB coverage ratios as our measure of trade protection. Data on Non-Tariff Barriers for 1983\textsuperscript{44} has been collected by the UNCTAD\textsuperscript{45} and combined with data from World Bank tapes\textsuperscript{46}.

The benchmark specification by Gawande and Bandyopadhyay (2000) tests the original "Protection for Sale" equation of Grossman and Helpman (1994), reproduced in (205), where $t_i$ denotes the protection of industry $i$, $z_i$ is the inverse of the import penetration ratio, $e_i$ is the price elasticity of imports and $I_i$ describes whether sector $i$ is politically organised and represented by a lobby. Further, $\alpha_L$ represents the proportion of the population that is organised and $\alpha$ denotes the weight of contributions to the linear welfare function of the government.

$$\frac{t_i}{1 + t_i} = \frac{I_i - \alpha_L z_i}{\alpha + \alpha_L e_i}$$

(205)

Gawande and Bandyopadhyay (2000) demonstrate that lobbying competition and lobbying spending have an influence on protection in the US by estimating a system of three equations, of which only one is relevant to this analysis. This

\textsuperscript{43}Leamer (1990) details the construction of NTB coverage ratios. These have been widely used, for example, in Learner (1990), Trefler (1993), Gawande (1998), Lee and Swagel (1997), Goldberg and Maggi (1997), Gawande and Bandyopadhyay (2000) and Bombardini (2005).

\textsuperscript{44}Since 1983 is the only year for which NTB data is available, it is not possible to test the term limit effects predicted by the model.

\textsuperscript{45}UNCTAD: United Nations Conference on Trade and Development.

\textsuperscript{46}This dataset has been kindly provided by Kishore Gawande.
equation is reproduced in (206), where \( t_i \) is the coverage ratio for industry \( i \), \( z_i \) is the inverse of the import penetration ratio, the share of imports to total production in sector \( i \), \( e_i \) is the price elasticity of imports and \( I_i \) is a dummy variable that describes whether the sector is politically organised and represented by a lobby. Moreover, \( Z_{1i} \) includes tariffs on intermediate goods and \( Z_{2i} \) includes NTBs on intermediate goods as controls. The error term is denoted by \( s_i \).

\[
\frac{t_i}{1 + t_i} = \gamma_0 + \gamma_1 I_i \frac{z_i}{e_i} + \gamma_2 z_i + Z_{1i} + Z_{2i} + s_i \tag{206}
\]

A simultaneity problem was raised by Trefler (1993). Higher trade protection is likely to reduce import penetration, as reflected in the following equation, in which \( e_i \) is the error term\(^{47}\).

\[
\frac{1}{z_i} = \phi \frac{t_i}{1 + t_i} + e_i \tag{207}
\]

Import penetration and trade protection are therefore determined simultaneously. In order to correct for the simultaneity bias implied by the system of equations (206) and (207), an instrumental variables approach is adopted. The capital-labour ratio interacted with industry dummies and comparative advantage variables (fractions of managers, scientists and unskilled labour per industry) are used as instruments, as in Trefler (1993). A complete list of the

\(^{47}\)Note that coefficient \( \phi \) is not the same character employed in the theoretical section.
instruments used is reported in Appendix D. As in Gawande and Bandyopadhyay (2000), we use a two-stage least-squares estimator, and include for each of the instruments a linear term, a squared term, and the interactions of the linear term with, $e_i$, the price elasticity of imports.

The data used for import penetration ratios for the US are identical to those used by Trefler (1993). Considered as the most accurate estimate of sector-level price elasticity of imports, the data was taken originally from Shiells et al. (1986). The dummy variable, $I_i$, indicates whether a sector is politically organised and is constructed by Gawande and Bandyopadhyay (2000) based on US data from the Federal Election Commission$^{48}$.

4.2.2 Measuring Concentration

To test the hypothesis that sectors whose activity is concentrated in US states with strong electoral competition ('swingness') and with the electoral votes to influence electoral outcomes ('decisiveness') are more likely to be protected, we require a measure to capture this form of geopolitical concentration. We therefore construct a measure of this concentration by combining two datasets. The first dataset allows us to construct the geographical concentration of industries across US states, based on employment. We use the 1987 Standard Industrial Classification (SIC) data from the Bureau of Labor Statistics (BLS) Quarterly Census of Employment and Wages (QCEW) for the year 1983, which gives us

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$^{48}$Gawande and Bandyopadhyay (2000) give a detailed description of the derivation of this dummy.
state-level employment at the four digit SIC.

The second dataset measures the swingness and decisiveness of electoral states in the presidential election\(^{49}\) of 1984. Strömberg (2005) develops a probabilistic voting approach to presidential election campaigns and estimates an approximate measure \(Q_s\) of the joint probability of a state \(s\) being both decisive in the Electoral College and a swing state with a very close state-level election. It therefore encompasses the two factors put forward by Propositions (39) and (40) as being important in determining trade policy. He shows how measure \(Q_s\) depends on several factors, such as the variance of national popularity-swings or the variance of electoral vote distribution, which could be interpreted as the state level and aggregate level uncertainties in the model of Section 1.

The \(Q\)-values are estimated for each presidential election using national and state-level measures. We use measure \(Q_s\), estimated by David Strömberg for the 1984 presidential election for each state, whose mean is 0.02 and that ranges between a value close to zero and 0.07. The probability of being swing and decisive is never 0 or 1, reflecting, as in our model, that no state is expected to be won with certainty. The NTBs in place in 1983 would, according to our model, be related to the expected swingness and decisiveness for the forthcoming election. This is exactly what the \(Q_s^{1984}\) measure. At the national level, the Democrat proportion of the two-party vote share in trial-heat polls,

\(^{49}\)This data was kindly provided by David Strömberg.
economic growth, incumbency and incumbent president running for re-election are used. Moreover, at the state level, the difference from the national mean of the Democrat proportion of the two-party vote share in the 1980 election, the average ADA-scores\(^{50}\) of each state's Congress members the year prior to the election and the difference between state and national polls are included.

The well-established \(Q_s\) measure of Strömberg (2005) constitutes a convenient measure for the reduced form specification as it combines the 'swingness' of states, reflecting the electoral competitiveness, with 'decisiveness', reflecting the size of states and the necessity of winning a certain number of states to win the overall election. To check the suitability of this measure, we calculate the correlation between the \(Q_s\) for the 1984 presidential election and a state-industry Herfindahl index in 1983. This is found to be -0.4 (significant at the 1\% level), showing that industrial concentration is not directly correlated with the probability of being swing and decisive.

Since the political data, encapsulated by measure \(Q_s\), is constructed at the state level, while trade protection is measured at the industry level, we use the BLS dataset to link the two dimensions by creating an industry-specific measure of swingness and decisiveness, \(q_i\). Besides being necessary for the empirical analysis, it also corresponds to the assumption of our model that employees of a sector in a state are protectionist swing voters in that state. In order to abstract from any size effects, we measure the state specialisation

\(^{50}\) ADA (Americans for Democratic Action) scores, ranging from 0 to 100, are used as a measure of legislator ideology.
of each industry as the deviation in each state from its mean share of national employment. We can then compute a 4-digit SIC ‘Q\_s’ measure, denoted by q\_i using:

\[ q_i = 1000 \times \sum_{s=1}^{S} Q_s \times \left( \frac{L_{1s}}{L_s} - \frac{L_i}{L} \right) \]  

(208)

where \( i \in I \) denotes each of the 242 4-digit SIC industries used by Gawande and Bandyopadhyay (2000) and \( s \in S \) denotes each of the 48 continental states\(^1\). Total US employment is represented by \( L \), while aggregate industry and state employment are respectively \( L_i \) and \( L_s \). Industries that constitute a higher proportion of a state’s employment than their proportion of national employment, for a given \( Q_s \), have a higher \( q_i \). Conversely, if an industry constitutes a lower proportion of a state’s employment than it does of national employment, then \( q_i \) is lower. Moreover, for a given proportion of a state’s employment, if the state has a low joint probability of being both swing and decisive, then \( q_i \) is low. Taking the deviation from the mean rather than a pure state level measure of concentration allows to abstract from the possibility that nationally important industries will be important in all states. The sum is multiplied by 1000 as multiplying the probability \( Q_s \) by a share yields very small numbers.

Table (7) presents the descriptive statistics of this constructed measure, which show that \( q_i \) varies widely across industries. This confirms that industrial concentration through space and in specific swing and decisive states is

\(^{11}\)Excluding the District of Columbia, Alaska, Hawaii, Puerto Rico and the Virgin Islands.
Table 7: Descriptive statistics of qi

<table>
<thead>
<tr>
<th>Descriptive statistics of qi</th>
<th>Correlation of qi with</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>Min</td>
</tr>
<tr>
<td>0.07</td>
<td>-0.49</td>
</tr>
<tr>
<td>Median</td>
<td>Max</td>
</tr>
<tr>
<td>0.03</td>
<td>1.45</td>
</tr>
<tr>
<td>sd</td>
<td>Range</td>
</tr>
<tr>
<td>0.17</td>
<td>1.94</td>
</tr>
</tbody>
</table>


Not uniform. We check that our results are robust to excluding outlying observations of qi. The correlations with other industry characteristics are reported in the third column of the table. Total employment, labour and skill intensity are not correlated with qi, demonstrating that larger, or more skill or labour intensive industries do not systematically concentrate more in states that are more likely to be swing and decisive.

Augmenting the specification of Gawande and Bandyopadhyay (2000) to include the constructed industry level swingness and decisiveness variable, qi, gives the following specification:

\[
\frac{t_i}{1 + t_i} = \gamma_0 + \gamma_1 I_i \frac{z_i}{e_i} + \gamma_2 \frac{z_i}{e_i} + \gamma_3 q_i + Z_1 t_i + Z_2 t_i + s_i
\]  

(209)
which is also corrected for the simultaneity bias by using IV. The campaign contributions literature does not suggest the concentration of industries in swing and decisive states as a determinant for trade policy decision-making, implying that $\gamma_3$ is zero. The next section provides evidence that $q_i$ is a significant determinant of NTB protection of an industry, thus lending support to our theoretical results.

4.2.3 Empirical Results

Our findings are reported in table (8). The first column reports the results of the benchmark specification given by (206). It is consistent with the coefficients reported in Gawande and Bandyopadhyay (2000) and qualitatively close to those obtained by Goldberg and Maggi (1999). As predicted by Grossman and Helpman (1995), in politically organised sectors, higher industry output relative to imports and a lower price elasticity of imports increases the level of protection ($\gamma_1 > 0$). In politically disorganised sectors, the coefficient has the opposite sign ($\gamma_2 < 0$).

The results from specification (209) appear in column (2). Our measure of "industry swingness and decisiveness" does not affect the sign, magnitude of the coefficients on $I_i(z_i/e_i)$ and $z_i/e_i$. Their significance is only slightly reduced, indicating a relative robustness of the Grossman Helpman model. The point estimate of $\gamma_3$ is 0.192 (significant at the 1%, with a robust stan-

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52 The significance levels of the coefficients are smaller than those reported in their paper due to our use of robust standard errors.
Table 8: Reduced form regression results

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>NTB (_i) / (1 + NTB)</th>
<th>Beta</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td></td>
</tr>
<tr>
<td>(\epsilon_i)</td>
<td>0.192** (0.038)</td>
<td>0.233</td>
<td></td>
</tr>
<tr>
<td>(I_i (\epsilon_i / e_i))</td>
<td>0.734** (0.319)</td>
<td>0.809** (0.312)</td>
<td></td>
</tr>
<tr>
<td>(\text{Intermediates' tariffs})</td>
<td>0.378** (0.090)</td>
<td>0.337** (0.086)</td>
<td></td>
</tr>
<tr>
<td>(\text{Intermediates' NTBs})</td>
<td>0.734** (0.319)</td>
<td>0.809** (0.312)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: IV-2SLS regressions, instruments reported in appendix D. Robust standard errors in parentheses; + denotes statistical significance at the 10% level; * denotes statistical significance at the 5% level; ** denotes statistical significance at the 1% level. Includes constant not reported. The dependent variable is the Non Tariff Barriers coverage ratio. In both specifications, \((\epsilon_i / e_i)\) is the ratio of inverse import penetration to import elasticity. \(I_i (\epsilon_i / e_i)\) is the same ratio multiplied by a dummy \(I\), that indicates whether a sector is politically organized or not. Intermediates tariff is computed as the average tariff on intermediate goods used by industry \(i\) and Intermediates NTBs the average Non Tariff Barriers coverage of these intermediates. In the second specification, an additional explanatory variable is added. Industry-specific measure of swingness and decisiveness, \(\epsilon_i\), computed from data for 242 four-digit SIC industries using Strömberg (2005) measure of the probability of being swing and decisive and the Bureau of Labor Statistics employment dataset. The beta coefficients are reported for both specifications. The p-values of the F-test model and J-test overidentification are reported. Data source: Gawande and Bandyopadhyay (2000), Strömberg (2005), BLS (1983) and authors’ own calculations.
standard error of 0.038). Thus sectors that concentrate more than their national average in swing and decisive states receive more protection. This estimate translates into a normalised beta coefficient of 0.233, such that a one standard deviation increase in the industry’s swingness and decisiveness will increase the US NTB coverage ratio for that sector by approximately 0.233 standard deviations. Although this beta is smaller than that of the Grossman-Helpman variables, it is more significant, and as important as the trade protection measures on intermediates. Moreover, including our measure of swingness and decisiveness explains a larger proportion of the variation of protection levels across sectors, as it increases the centered \( R^2 \) by 30% relative to the Gawande and Bandyopadhyay (2000) benchmark specification.

These findings provide supporting evidence for the hypothesis that industrial concentration in swing and decisive states is an important determinant of trade protection of that industry, highlighting geographical concentration of industries in politically key states an important, and previously overlooked, determinant of trade protection in the US Electoral College.

4.3 Conclusion

The political agency model developed in this chapter offers a multi-jurisdictional framework for analysing electoral incentives for trade protection. For distributions of voters where support by swing voters increases re-election probability, a unique equilibrium is shown to exist where political incumbents build a reputation of protectionism through their policy decisions in their first term of
office. The extension to a multi-state framework modelled as an electoral college introduces a spatial dimension that shows how the incentives driving trade policy hinge on the distribution of swing voters across swing states. We show that strategic trade protection is more likely when protectionist swing voters have a lead over free-trade supporters in states with relatively strong electoral competition, swing states, that also represent a larger proportion of electoral votes, thus being more decisive in the overall election. The analytical results offer a theoretical explanation for why governments may sometimes push for the protection of industries with concentrations in pivotal locations, such as the US steel production industry. Moreover, our empirical strategy augments the benchmark test of the lobbying political economy of trade literature to include a measure of how industries specialise geographically in these swing and decisive states. The reduced form evidence is that the concentration of industries in politically important states is a significant element in explaining trade policy. These findings provide support for the theory highlighting an important, and previously overlooked, determinant of trade protection.

References


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Appendix A. Voter Value Functions

Section 1 establishes the optimality of the incumbents' strategies, given voters' strategies, for the equilibrium where $\Delta v > 0$. This appendix shows that the re-election rule of the infinitely-lived $F$ and $P$ swing voters is also optimal, given politicians' strategies. This confirms that the politicians' and voters' strategies constitute a Markov Perfect equilibrium of the game.

Let $V_P$ denote the value function for a protectionist voter. Further, let $\sigma_P^r$ denote the probability that a $P$ voter votes for the incumbent, given policy $r$ in his first term of office. $\sigma_P^r$ contributes to the incumbent's re-election probability by a tiny amount, thus marginally affecting his prospective payoffs. $\sigma_P^r$ is thus introduced in $V_P$ as an argument of the incumbent's re-election probability, $f(\cdot)$, which is smooth and continuous from the assumptions of the model. Further, let $u^1_p(\pi)$ denote the utility of $P$ voters in the incumbent's first term of office, where $\pi$ is the probability of the incumbent having protectionist views. Similarly, denote $P$ voters' second term utility as $u^2_p(\bar{\pi}^r)$, where this is a function of update beliefs after observing $r$ in the first term. Finally, $\beta$ is the common discount factor. Combining these allows the value function, $V_P$, to be expressed as follows:

$$V_P = u^1_p(\pi) + \beta \sum_r \left[ f(\sigma_P^r) (u^2_p(\bar{\pi}^r) + \beta V_P) + (1 - f(\sigma_P^r)) V_P \right]$$

(210)

The following proof uses (210) to show that given incumbents' strategies, $\sigma_P^0 =$

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0 and \( \sigma_P^1 = 1 \) are optimal responses. That is, protectionists vote for the incumbent if he chooses trade protection in his first term and for the challenger if free trade is chosen. In order for \( \sigma_P^1 = 1 \) to be an optimal response, it must be true from (210) that \( u_P^2(\pi^r) + \beta V_P \geq V_P \). This can be rearranged to the following condition:

\[
u_P^2(\pi^r) \geq (1 - \beta)V_P \quad (211)\]

To see this, consider that \( f(\sigma_P^r) \) and \( 1 - f(\sigma_P^r) \) are weights for \( u_P^2(\pi^r) + \beta V_P \) and \( V_P \), respectively, in the value function. Voter \( P \) maximises his effect on \( f(\sigma_P^r) \) through \( \sigma_P^r = 1 \), and thus places the largest possible weight on \( u_P^2(\pi^r) + \beta V_P \) relative to \( V_P \). Hence, \( \sigma_P^r = 1 \) can only be optimal if (211) holds.

Recall that \( P \) voters receive a payoff \( x \) if \( r = 1 \) and 0 otherwise. Since \( \Pr(r = 1 \text{ in } 1^{st} \text{ term}) = \pi + (1 - \pi)p \), it follows that \( u_P^1(\pi) = [\pi + (1 - \pi)p] x > 0 \). Moreover, since \( \Pr(r = 1 \text{ in } 2^{nd} \text{ term} | r = 0 \text{ in } 1^{st} \text{ term}) = 0 \), it follows that \( u_P^2(\pi^0) = 0 \). That is, the incumbent reveals himself to be a free-trader if he chooses \( r = 0 \) in his first term, given \( \Delta v > 0 \). Since the incumbent follows his preferences in his final term in office, so \( P \) voters can be certain of a 0 payoff. If the incumbent sets \( r = 1 \) in his first term, then voters can update their beliefs regarding the probability of \( r = 1 \) being chosen in his second term, if re-elected. Applying Bayes' rule for \( \pi^1 \), \( P \) voters can expect

\[
u_P^2(\pi^1) = \frac{\pi x}{\pi + (1 - \pi)p}.
\]
It must be true that $V_P \geq \frac{1}{1-\beta}u^1_P(\pi)$, where $\frac{1}{1-\beta}u^1_P(\pi)$ is the discounted stream of period 1 utilities, if the incumbent is never re-elected. Substituting into (211) yields:

$$u^2_P(\pi^r) \geq u^1_P(\pi)$$

(212)

This must hold for $\sigma^r_P = 1$ to be optimal, for all $r$, but leads to a contradiction. It cannot be true that $u^2_P(\pi^0) \geq u^1_P(\pi)$ since $u^2_P(\pi^0) = 0$ and $u^1_P(\pi) > 0$. Hence, $\sigma^r_P = 1$ (for all $r$) cannot be an optimal response. Since $u^2_P(\pi^0) < u^1_P(\pi)$, a new politician is always a better bet than an incumbent who set $r = 0$ in his first term. Hence, $\sigma^0_P = 0$ is optimal. Moreover, continuation payoff $\frac{1}{1-\beta}u^1_P(\pi)$ must be smaller than $\frac{1}{1-\beta}u^2_P(\pi^1)$ under the equilibrium strategies of incumbents', so $\sigma^1_P = 1$ is an optimal response.

The value function of free-traders, $V_F$, is symmetric to $V_P$ and the optimality strategies $\sigma^r_F = 1$ and $\sigma^1_F = 0$ follows with arguments symmetric to those used above. We can thus conclude that the politicians' and voters' strategies constitute a Markov Perfect equilibrium of the game.

**Appendix B. Equilibrium Uniqueness**

There are two symmetric cases, $\Delta v > 0$ and $\Delta v < 0$, where reputation-building through strategic policy implementation forms part of incumbents' optimal strategies. In each of these symmetric cases, there is a unique equilibrium. To show that the equilibrium found in the chapter is unique, consider
a distribution of swing voters under which $\Delta v > 0$ from the implementation of trade protection in the first term.

Recall that when a high cost $c_H$ is drawn, it is a dominant strategy for free-trader politicians to set $r = 1$. Moreover, let $\sigma^r_P$ denote the probability that a $P$ voter votes for the incumbent, given policy $r$ in his first term of office. Under a sufficiently low cost draw, $c_L$, it must be the case that $\sigma^1_P > \sigma^0_P$ for a free-trader to deviate from $r = 0$. Similarly, for a protectionist to deviate from $r = 1$ in his first term of office, it must be true that $\sigma^1_P < \sigma^0_P$. Hence, in any equilibrium at most one type of politician deviates from his preferred policy in the first term.

Moreover, to show that mixing between $r = 0$ and $r = 1$ cannot be an equilibrium, consider a strategy where a free-trader incumbent sets $r = 1$ with a probability less than 1 when $c = c_L$. For this to be an equilibrium, it must be the case that $\sigma^1_P \beta \zeta - c_L = \sigma^0_P \beta \zeta$ and hence that $c_L = (\sigma^1_P - \sigma^0_P) \beta \zeta$. Inspection of $V_P$ in Appendix A shows that $\sigma^1_P = 1$ and $\sigma^0_P = 0$ remain optimal. This, however, implies that $c_L = \beta \zeta$ that contradicts the assumption that $\beta \zeta > c_L$.

It can similarly be shown that a strategy in which a protectionist sets $r = 1$ with less than certainty can never form part of an equilibrium. Such a strategy requires that $\sigma^1_P \beta \zeta = \sigma^0_P \beta \zeta - c_L$, that implies $c_L = (\sigma^0_P - \sigma^1_P) \beta \zeta$. This is impossible, however, since voters’ optimal strategy in this case is to set $\sigma^1_L = 1$ and $\sigma^0_L = 0$. It follows that the unique equilibrium outcome is for
an $F$ incumbent to set $r = 1$ when $c = c_L$ and for a $P$ incumbent to also set $r = 1$ under a low cost draw.

We can conclude that the equilibrium discussed in the chapter is unique for distributions of swing voters that satisfy the conditions for this case, and sufficiently low $p$ and $c_L$. Symmetric arguments apply for the alternative case where $\Delta v < 0$.

Appendix C. Untested Candidates

Consider an election taking place between two randomly selected candidates, each with a probability $\pi$ of being protectionist. Since neither candidate has a history of a trade policy decision on which swing voters can condition their voting decision, the swing voters cast their vote on the basis of a coin toss. Each candidate can thus expect to gain $\frac{1}{2} (\gamma_D^s + \gamma_F^s)$. Hence, the Democrat candidate gains $\gamma_D^s + \frac{1}{2} (\gamma_P^s + \gamma_F^s) + \nu^s$ and the Republican candidate gains $\gamma_R^s + \frac{1}{2} (\gamma_P^s + \gamma_F^s) - \nu^s$. For the $D$ candidate to win a majority in state $s$, $2\nu^s = \varepsilon^s$ must exceed $\gamma_R^s - \gamma_D^s = -\omega_p^s$. Let $\rho^s$ denote the probability that the $D$ candidate wins a majority in state $s$. It follows from the distribution of $\varepsilon^s$ that:

$$\rho^s = \Pr (\varepsilon^s > -\omega_p^s) = 1 - H(-\omega_p^s)$$  \hspace{1cm} (213)

$$= H(\omega_p^s)$$  \hspace{1cm} (214)
Hence, \(1 - H(\omega_p^s)\) is the probability that \(R\) wins majority in state \(s\). Hence state-level outcomes depend only on the political lead in \(s\) and \(H(\varepsilon^s)\). This stems from the assumption that single-issue voters randomly select between the two candidates, so each candidate can expect to gain support by half. An alternative voting strategy could allocate swing voters in a different proportion. For example, when candidates are not distinguishable with regards to trade policy, voters may cast a vote on the basis of underlying ideological position, that is otherwise dominated by trade policy considerations.
Appendix D. Variables and Instruments

The following table provides descriptions of all the variables and instruments used in the empirical analysis of section 4.2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>NTB_i</td>
<td>Aggregate US Non Tariff Barriers coverage ratio across all partners for industry i</td>
</tr>
<tr>
<td>9</td>
<td>Constructed measure of the concentration of 4-digit SIC industry i in swing and decisive political states</td>
</tr>
<tr>
<td>I_i</td>
<td>Dummy variable, value 1 when sector i is politically organized</td>
</tr>
<tr>
<td>z</td>
<td>Inverse of import penetration ratio divided by 10000 (= (US consumption in 1983/ US total imports)/10000) in sector i</td>
</tr>
<tr>
<td>e^*</td>
<td>Price elasticity of imports in sector i, corrected for errors-in-variables (GB, 2000)</td>
</tr>
<tr>
<td>Interm. tariffs</td>
<td>Average tariff on intermediate goods used in industry i</td>
</tr>
<tr>
<td>Interm. NTBs</td>
<td>Average NTB coverage ratio on intermediate goods used in industry i</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Average tariff on intermediate goods used in industry i</td>
</tr>
<tr>
<td>2</td>
<td>Average NTB coverage ratio on intermediate goods used in industry i</td>
</tr>
<tr>
<td>3</td>
<td>Price elasticity of imports (1986)</td>
</tr>
<tr>
<td>4</td>
<td>Logarithm of the price elasticity of imports e^*</td>
</tr>
<tr>
<td>5</td>
<td>Measure of the size of firms in an industry: Value added per firm, 1982, ($Bn/firm)</td>
</tr>
<tr>
<td>6</td>
<td>Share of output in a sector produced by the four largest producers. concentration ratio, 1982</td>
</tr>
<tr>
<td>7</td>
<td>Share of employees in the industry defined as scientists and engineers, 1982</td>
</tr>
<tr>
<td>8</td>
<td>Share of employees in the industry defined as managerial, 1982</td>
</tr>
<tr>
<td>9</td>
<td>Share of employees in the industry defined as unskilled, 1982</td>
</tr>
<tr>
<td>10</td>
<td>Real Exchange Rate elasticity of imports</td>
</tr>
<tr>
<td>11</td>
<td>Cross price elasticity of imports with respect to domestic prices, corrected for errors-in-variables (GB, 2000)</td>
</tr>
<tr>
<td>12</td>
<td>Log percentage of an industry's output used as intermediate good in other sectors</td>
</tr>
<tr>
<td>13</td>
<td>Logarithm of the intermediate goods buyer concentration</td>
</tr>
<tr>
<td>14</td>
<td>Herfindahl index of the industry</td>
</tr>
<tr>
<td>15</td>
<td>Ad valorem tariff</td>
</tr>
<tr>
<td>16</td>
<td>Capital-Labor ratio of the industry x Dummy for food processing industry</td>
</tr>
<tr>
<td>17</td>
<td>Capital-Labor ratio of the industry x Dummy for resource-intensive industry</td>
</tr>
<tr>
<td>18</td>
<td>Capital-Labor ratio of the industry x Dummy for general manufacturing industry</td>
</tr>
<tr>
<td>19</td>
<td>Capital-Labor ratio of the industry x Dummy for capital intensive industry</td>
</tr>
<tr>
<td>20-36</td>
<td>Instruments 3 to 19 squared</td>
</tr>
<tr>
<td>37-52</td>
<td>Instruments 4 to 19 x price elasticity of imports e^*</td>
</tr>
</tbody>
</table>
Conclusion

This thesis theoretically examined how information costs, minimum quality standards and electoral incentives affect international trade and trade policy choice.

A new pairwise matching model with two-sided information asymmetry was presented and used to analyse the impact of information costs on endogenous network-building and matching by information intermediaries. Intermediation was found to raise expected trade volume and social welfare by expanding the set of matching technologies available to traders. Network building incentives were analysed under both linear and convex costs of network expansion and convexity proved necessary for both direct and indirect trade to arise in equilibrium. Moreover, optimal network size and the pattern of direct and intermediated trade was shown to depend on the level of information costs and the relative effectiveness of direct and indirect matching technologies, shedding light on the relationship between information frictions and trade.

The model was extended to analyse the strategic interaction between two information intermediaries. Competition in commission rates and network size was shown to give rise to a fragmented duopoly market structure in equilibrium. The coordination game of traders was shown to present the possibility of coordination failure between trade pairs. Intermediaries' inability to price discriminate between the competitive and non-competitive market segments,
was shown to give rise to an undercutting game with no pure strategy Nash equilibrium. A mixed strategy Nash equilibrium was shown to exist. The analysis concluded that either a monopoly or a fragmented duopoly can prevail in equilibrium, depending on the network-building technology. Under convexity assumptions, both intermediaries invest in a network and compete over common matches, while randomising commission rates. In contrast, linear network development costs can only give rise to a monopoly outcome.

Furthermore, the thesis developed an open economy model of vertical product differentiation to analyse governments' incentives for the unilateral setting of minimum quality standards. National minimum quality standards were endogenously determined in a standard-setting game between governments, which yielded four unregulated Nash equilibria, two symmetric and two asymmetric, depending on the quality ranking of firms in each market. The analysis established that in all four cases, unilaterally selected minimum quality standards are inefficient as a result of cross-country externalities. Furthermore, minimum quality standards were shown to operate as non-tariff barriers to trade. The analysis of international cooperation in minimum quality standards established that the world welfare maximising symmetric standard can be reached through reciprocal adjustments in national minimum standards from either of the two symmetric Nash equilibria. In the case of asymmetric cross-country externalities, the scope for mutually beneficial cooperation was shown to be significantly restricted.
The final chapter of the thesis developed a new multi-jurisdictional political agency model for analysing electoral incentives for trade protection. A unique equilibrium was shown to exist where political incumbents build a reputation of protectionism through their policy decisions in their first term of office. The incentives driving trade policy were shown to depend on the distribution of voters across swing and decisive states. Finally, the results of the empirical analysis testing the theoretical hypothesis were reported and shown to provide supporting evidence.