Credit Frictions and the Macroeconomy

Kalin Ognianov Nikolov
PhD Thesis, Department of Economics
London School of Economics and Political Science
University of London

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Abstract

The unifying theme in my PhD thesis is the effect that credit market imperfections have on aggregate outcomes. My main interest is in the collateral amplification mechanism and on the welfare effects that economic shocks and policies have on different groups in society.

In my first chapter (which is joint with Nobu Kiyotaki and Alex Michaelides), we develop a life-cycle model of a production economy in which land and capital are used to build residential and commercial real estate. We find that, in an economy where the share of land in the value of real estates is large, housing prices react more to an exogenous change in expected productivity or the world interest rate, causing a large redistribution between net buyers and net sellers of houses. Changing financing constraints, however, has limited effects on housing prices.

My second and third chapters examine environments with credit constrained entrepreneurs similarly to the original Kiyotaki and Moore (1997) paper. My second chapter asks the question of whether tightening capital requirements may be welfare improving when firms face credit constraints. I find that the answer is 'no'. Although tightening the collateral constraint dampens business cycle fluctuations, the first order cost in terms of reduced access to credit is too great.

My third chapter examines the extent to which a borrower's reputation for repayment can serve as intangible collateral, thus explaining the movement of down-payment requirements over the business cycle. The main finding is that, under standard technology shocks, down-payments move in a pro-cyclical fashion. Introducing a pro-cyclical productivity gap between firms as well as counter-cyclical degree of idiosyncratic production risk helps to generate counter-cyclical down-payment requirements.

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Overview

The financial crisis of the last few years has given a fresh impetus to the development of macro-economic models with credit frictions. In an economy with complete markets, full information and full commitment, only the most productive firms will be in operation, consumers will enjoy smooth consumption profiles across states of nature and housing tenure will be irrelevant for consumer welfare. Once we allow for limited commitment, outcomes in the market economy depart from those predicted in the standard RBC model. This thesis examines three aspects of these departures from first best.

Chapter 1 (joint with Nobu Kiyotaki and Alex Michaelides) studies the effect of collateral constraints on housing tenure choices over the life cycle, before examining what effect housing price changes have on the welfare of different groups in society. We argue that limited commitment in the housing rental market and in the credit market are key to explaining the fact that almost a third of the US population rent. In the model we build, landlords guard against moral hazard in the housing rental market by restricting the freedom of tenants in modifying rented dwellings to their own taste. This implies that tenants get less utility from a rented house compared to the utility they would get from owning the same house. Without credit constraints, everyone would then borrow heavily and purchase a house in order to enjoy housing services to the full. However, credit constraints prevent young and poor households from buying and forces them to rent instead. Only gradually, consumers accumulate savings and purchase houses. At first they do so subject to binding borrowing constraints but increasingly over time, they accumulate their own equity as they start to save for retirement.

Chapter 1 also examines the aggregate and welfare implications of the model. We consider whether our economy is capable of delivering quantitatively realistic predictions for housing prices when hit with shocks such as a fall in the world real interest rate or an increase in the labour productivity growth rate. The chapter shows that the presence of land (which is fixed in aggregate supply) in the production of real estate services is key in delivering large movements in housing prices, following changes in fundamentals. In contrast the tightness of the collateral constraint plays almost no role because it only affects relatively poor people who account for a small fraction of the housing stock. Finally, we show that housing price movements redistribute wealth from buyers to sellers of housing.

In Chapters 2 and 3 I we focus on the collateral amplification mechanism, using a more traditional Kiyotaki and Moore (1997) environment in which entrepreneurs rather than consumers are subject to borrowing constraints. Chapter 2 studies the incentives of a benevolent government to regulate the private sector's leverage in an economy in which debt is secured by collateral. The presence of asset prices in individual agents' collateral constraints introduces a 'fire sale' externality, which can potentially make the private equilibrium constrained inefficient. Individual entrepreneurs decide how much to borrow and lend, only taking private gains and losses into account. What atomistic agents ignore is the fact that in some states of the world, they will realise gains, purchase more assets and push prices up while in other states they will realise losses, 'fire sale' assets helping to push prices down further. Usually, such pecuniary externalities are not a reason for policy to correct private outcomes. But in an environment with collateralised borrowing and lending, asset prices can affect the tightness of borrowing constraints and policy can help to stabilise access to credit and therefore economic activity. As a result, consumption variability is reduced and this has a beneficial effect on welfare. We find that regulating leverage has a substantial cost too because it denies highly productive entrepreneurs access to funds. Quantitatively, we find that the cost is too great and the government does not find it optimal to regulate leverage in our environment.

In most models of the collateral amplification mechanism, the fraction of firm tangible assets that can be used as collateral is assumed to be constant and exogenous. In contrast, during the crisis we saw big changes in the access and terms for leveraged finance. Downpayment requirements for house purchase increased and private equity firms no longer could acquire their targets with a minimal amount of own equity. It is widely believed that such fluctuations in downpayment requirements significantly amplified the credit cycle over the past few years. In Chapter 3 we extend a Kiyotaki-Moore (1997) type model in order to incorporate such fluctuations in downpayment requirements for capital goods. We do this by assuming that a lender can lend anonymously but a borrower must borrow publicly. In addition, we assume that lenders can commit to punish defaulting borrowers by permanently excluding them from future credit.

We show that, when credit constraints are binding, such permanent exclusion is costly for borrowers because they earn a higher return on their own production than the rate of return on risk free debt. Access to credit helps entrepreneurs leverage up in order to take maximum advantage of this excess return. Having to self finance or lend to others therefore leads to a substantial loss of utility. In the paper we compute how a borrower's reputation for repayment can act as intangible collateral. We find that intangible collateral can be very substantial in steady state, backing the liabilities of the private sector in addition to the more traditional tangible collateral usually considered in the literature. We introduce aggregate technology shocks and find that the value of intangible collateral is high (and downpayment requirements are low) in recessions because borrowing constraints bind more tightly and the excess return enjoyed by high productivity entrepreneurs is higher. We show that introducing a high degree of idiosyncratic production risk in recessions can help correct this implication of the model, generating counter-cyclical downpayment requirements on capital goods.

Chapter 1

Winners and Losers in Housing Markets

1.1 Introduction

Over the last few decades, we have observed considerable fluctuations in real estate values and aggregate economic activities in many economies. In Japan, both the real capital gains on real estate during the prosperous decade of the 1980s and the losses during the depressed decades of the 1990s and the early 2000s are in the order of multiple years worth of GDP. Recent fluctuations in housing prices in many countries raise concerns. To what extent are these housing price fluctuations consistent with fundamental conditions? How do the fluctuations affect the wealth and welfare of different groups of households? In this paper, we develop a life-cycle model to investigate how housing prices, aggregate production and the wealth distribution react to changes in technology and financial conditions. After confirming that the model is broadly empirically consistent with life-cycle choices of home ownership and consumption, we use the model to assess which groups of households gain and which groups lose from changes in fundamentals.

To develop a theoretical framework, we take into account the limitation on the supply of land and the limitation on the enforcement of contracts in real estate and credit markets. Land (or location) is an important input for supplying residential and commercial real estates. Because the supply of land is largely inelastic and because the real estate price includes the value of land, the real estate price is sensitive to a change in the expected productivity growth rate and the real interest rate in equilibrium. We also consider incomplete contract enforcement to be an essential feature of an economy with real estate. Often, because landlords are afraid that the tenant may modify the property against their interests, landlords restrict tenants' discretion over the use and modification of the house, and tenants enjoy lower utility from renting the house compared to owning and controlling the same house. If there were no other frictions, then the household would buy the house straight away. The household, however, may face a financing constraint, because the creditor fears that the borrowing household may default. The creditor demands the borrower to put his house as collateral for a loan and asks him to provide a downpayment. We develop an overlapping generations model of a production economy in which land and capital are used to produce residential and commercial tangible assets, taking the importance of land for production of tangible assets, the loss of utility from rented housing and the tightness of collateral constraints as exogenous parameters.[1]

The interaction between the collateral constraint and the loss of utility from renting a house turns out to generate a typical pattern of consumption and housing over a life-cycle. When the household is born without any inheritance, it cannot afford a sufficiently high downpayment for buying a house; the household rents and consumes modestly to save for a downpayment. When the household accumulates some net worth, the household buys a house subject to the collateral constraint, which is smaller than a house that would be bought without the collateral constraint. As net worth further rises, the household upgrades along the housing ladder. At some stage, the household finds it better to start repaying the debt rather than moving up the housing ladder. When the time comes for retirement possibly with idiosyncratic risk attached, the household moves to a smaller house anticipating a

[1]Here, the importance of land for the production of the tangible asset is defined as the elasticity of tangible asset supply with respect to land for a fixed level of the other input. See equation (1.2) later on.

lower income in the future.

In equilibrium, due to the limitation of land supply, the supply of tangible assets tends to grow more slowly than final output causing an upward trend in the real rental price and the purchase price of the tangible asset. The more important is land for producing tangible assets compared to capital (as in Japan or a metropolitan area), the higher is the expected growth rate of the rental price and therefore the higher is the housing price-rental ratio. In such an economy, the household needs a larger downpayment relative to wage income in order to buy a house and tends to buy a house later in life, resulting in a lower home-ownership rate.

Moreover, in an economy where land is more important for producing tangible assets, we find the housing price to be more sensitive to exogenous changes in fundamentals such as the expected growth rate of labor productivity or the world interest rate, along the perfect foresight path from one steady state to another. Consistent with these theoretical predictions, Davis and Heathcote (2007) note that housing prices are more sensitive in large U.S. metropolitan areas. Del Negro and Otrok (2007) use a dynamic factor decomposition to find that local factors are more important for the house price change in states where the share of land in the real estate value is larger in the United States.[2]

In contrast to the change in productivity growth and the world interest rate, we find that financial innovation which permanently relaxes the collateral constraint has a surprisingly small effect on housing prices, despite increasing the home-ownership rate substantially both in the transition and in the steady state. In our economy, tenants or credit-constrained home owners are relatively poor and own a small share of aggregate wealth as a group. As a result, the effect of relaxing the collateral

[2]Davis and Palumbo (2008) find that the share of land in the value of houses has risen in U.S. metropolitan areas and they argue that this contributes to faster housing price appreciation and, possibly, larger swings in housing prices. Glaeser et. al. (2005) find that land use restrictions are needed to explain recent high housing prices in Manhattan. van Nieuwerburgh and Weill (2006) also argue that the increase in the dispersion of housing prices across regions can be quantitatively generated from an increase in the dispersion of earnings in the presence of planning restrictions. We ignore the restrictions on land use and planning, even though they further increase the natural limitation of land in supplying tangible assets. Other factors that might be empirically relevant for house price determination (such as owner-occupied housing as a hedge against rent risk, the effects of inflation and money illusion) are not considered in our framework; see Sinai and Souleles (2005) and Brunnermeier and Julliard (2008).

constraint on housing prices is largely absorbed by a modest conversion from rented to owned units.

In addition to the effect on the housing price and aggregate output, the exogenous changes in the productivity growth rate and the interest rate affect the wealth and welfare of various households differently, causing winners and losers in housing markets. As a general rule of thumb, net house buyers (such as young worker-tenants) lose and net house sellers (such as retiree-home owners) gain from the house price hike, while the wealth effect of the house price change on aggregate consumption is negligible aside from the liquidity effect.[3] Since housing wealth forms the largest component of nonhuman wealth for most households, the distribution effect is substantial. The overall welfare effect depends on the underlying shocks causing house price changes. A general equilibrium framework with heterogeneous agents enables us to analyze how the shocks to fundamentals affect the distribution of wealth and welfare of different households.

Our work broadly follows two strands of the literature. One is the literature on consumption and saving of a household facing idiosyncratic and uninsurable earnings shock and a borrowing constraint, which includes Bewley (1977, 1983), Deaton (1991), Carroll (1997), Attanasio et. al. (1999) and Gourinchas and Parker (2002). Huggett (1993), Aiyagari (1994), and Krusell and Smith (1998) have examined the general equilibrium implications of such models. The second strand is the literature on the investment behavior of firms under liquidity constraints. In particular, Kiyotaki and Moore (1997) is closely related since they study the dynamic interaction between asset prices, credit limits and aggregate economic activity for an economy with credit constrained entrepreneurs. When many households borrow substantially against their housing collateral and move up and down the housing ladder, these households are more like small entrepreneurs rather than simple consumers.

[3] The household is a net house buyer if the expected present value of housing services consumption over the lifetime exceeds the value of the house currently owned. Although the present population as a whole is a net seller of the existing houses to the future population, the aggregate effect is quantitatively very small because the discounted value of selling the existing houses to the future population is negligible. Thus, unlike some popular arguments, the wealth effect of housing prices on aggregate consumption is negligible, because the positive wealth effect of the net house sellers is largely offset by the negative wealth effect of the net house buyers of present population.

Our attention to housing collateral is in line with substantial micro evidence in the UK (Campbell and Cocco (2007)) and the US (Hurst and Stafford (2004)) which suggests that dwellings are an important source of collateral for households. Given the empirical findings that connect housing prices, home equity and aggregate consumption, there has been substantial research on building models that capture these relationships, either with a representative agent (Aoki, Proudman and Vlieghe (2004), Davis and Heathcote (2005), Kahn (2007), Piazessi et. al. (2007)), or with heterogeneous agents (Chambers, Garriga and Schlagenhauf (forthcoming), Fernandez-Villaverde and Krueger (2007), Iacoviello (2005), Iacoviello and Neri (2007), Lustig and van Nieuwerburgh (2005), Nakajima (2005), Ortalo-Magne and Rady (2006), Rios-Rull and Sanchez (2005) and Silos (2007)). Distinguishing features of our analysis include an explicit account of land as a limiting factor in a production economy, an investigation of the interaction between household life-cycle choices and the aggregate economy and evaluating welfare changes across heterogeneous households stemming from shocks to fundamentals.

Section 1.2 lays out the model, Section 1.3 examines the steady state, and Section 1.4 investigates the transitions, including the impact on wealth and welfare of different households.

1.2 The Model

1.2.1 Framework

We consider an economy with homogeneous product, tangible assets, labor, reproducible capital stock, and non-reproducible land. There is a continuum of heterogeneous households of population size $\overline{N}_t$ in period t, a representative foreigner, and a representative firm.

The representative firm has a constant returns to scale technology to produce output (Y_t) from labor (N_t) and productive tangible assets (Z_{Yt}) as:

$$Y_t = F(A_tN_t, Z_{Yt}) = (A_tN_t)^{1-\eta} Z_{Yt}^{\eta}, \quad 0 < \eta < 1, \tag{1.1}$$

where A_t is aggregate labor productivity which grows at a constant rate, $A_{t+1}/A_t = G_A$. Tangible assets (Z_t) are produced according to a constant returns to scale production function using aggregate capital (K_t) and land (L):

$$Z_t = L^{1-\gamma} K_t^{\gamma}, \quad 0 < \gamma < 1. \tag{1.2}$$

The tangible assets are fully equipped or furnished and can be used as productive tangible assets (such as offices and factories) or houses interchangeably:

$$Z_t = Z_{Yt} + \int_0^{\overline{N}_t} h_t(i)\, di, \tag{1.3}$$

where $h_t(i)$ is housing used by household i in period t. With this technological specification of tangible assets, the firm can continuously adjust the way in which the entire stock of land and capital are combined and can convert between productive tangible assets and housing without any friction.[4] The parameter $(1-\gamma)$ measures the importance of land for the production of tangible assets compared to capital, which would be equal to the share of land in property income if there were separate competitive rental markets for land and capital. Thus, we often call $(1-\gamma)$ as "the share of land in the production of tangible assets" hereafter. Typically, the share of land in the production of tangible assets is higher in urban than in rural areas, because land (or location) is more important for production with the agglomeration of economic activities.[5] We assume that the aggregate supply of land L is fixed. The capital stock depreciates at a constant rate $1-\lambda \in (0,1)$ every period, but can

[4] Davis and Heathcote (2005) use a production function in which only a fixed flow of new vacant land can be used for building new houses. Because, once used, the land is no longer usable for renovation nor new construction, there would be no vibrant city older than a hundred years. Perhaps, in reality, the allocation of land and capital is not as flexible as in our model but not as inflexible as in Davis and Heathcote (2005). We also assume there is no productivity growth in the production of tangible assets, because Davis and Heathcote (2005) calculate the growth rate of productivity in the US construction sector to be close to zero (-0.27 percent per annum). We ignore labor used in this sector for simplicity.

[5] We will not attempt to explain why agglomeration arises. We should not confuse the share of land $(1-\gamma)$ with the scarcity of land (or marginal product of land), because scarcity not only depends upon the share of land, but also on labor productivity, the capital-land ratio and the capital-labor ratio. We will later discuss how the share of land in the production of structures is related to the share of land in the value of tangible assets in Section 3.4.

be accumulated through investment of goods (I_t) as:

$$K_t = \lambda K_{t-1} + I_t. \tag{1.4}$$

Tangible assets built this period can be used immediately.

The representative firm owns and controls land and capital from last period and issues equity to finance investment. As the firm increases the size of tangible assets with capital accumulation, it will be convenient in subsequent analysis to assume that the firm maintains the number of shares to be equal to the stock of tangible assets.[6] Let q_t be the price of equity before investment takes place and let p_t be the price of equity after investment takes place in this period. Let w_t be the real wage rate, and r_t be the rental price of tangible assets. The firm then faces the following flow-of-funds constraint:

$$Y_t - w_t N_t - r_t Z_{Yt} - I_t + p_t Z_t = q_t Z_{t-1} \tag{1.5}$$

The left hand side (LHS) is the sum of the net cash flow from output production, minus investment costs and the value of equities after investment. The right hand side (RHS) equals the value of equity at the beginning of the period (before investment has taken place).

The owners of equity pay p_t to acquire one unit and immediately receive r_t as a rental payment (including imputed rents). Next period, the owner earns q_{t+1} before investment takes place. Therefore, the rate of return equals

$$R_t = \frac{q_{t+1}}{p_t - r_t}. \tag{1.6}$$

There are no aggregate shocks in this economy except for unanticipated, initial shocks. As a result, we assume that agents have perfect foresight for all aggregate variables, including the rate of return.

[6] This means the firm follows a particular policy of equity issue and dividend payouts. However, alternative policies do not change allocations because the Modigliani-Miller Theorem holds in our economy under perfect foresight and would only complicate subsequent expressions.

From (1.5) and (1.6) under perfect foresight, the value of the firm (V_t^F) to the equity holders from the previous period is equal to the present value of the net cash flow from production and the rental income of tangible assets produced:

$$
\begin{aligned}
V_t^F &\equiv q_t Z_{t-1} = Y_t - w_t N_t - r_t Z_{Yt} - I_t + r_t Z_t + (p_t - r_t) Z_t \qquad (1.7)\\
&= Y_t - w_t N_t - r_t Z_{Yt} - I_t + r_t Z_t + \frac{1}{R_t} V_{t+1}^F
\end{aligned}
$$

The firm takes $\{w_t, r_t, R_t\}$ as given and chooses a production plan $\{N_t, Z_{Yt}, Y_t, I_t, K_t\}$ to maximize the value of the firm, subject to the constraints of technology (1.1),(1.2), (1.3) and (1.4).

Since the production function of output is constant returns to scale, there is no profit from output production. Therefore, the value of the firm equals the value of the tangible asset stock. Given that the number of equities are maintained to equal the stock of tangible assets by assumption, the price of equities equals the price of tangible assets. Hereafter, we refer to the shares of the firm as the shares of tangible assets.

Households are heterogeneous in labor productivity, and can have either low, medium, or high productivity, or be retired. Every period, there is a flow of new households born with low productivity without any inheritance of the asset. Each low productivity household may switch to medium productivity in the next period with a constant probability δ^l. Each medium productivity household has a constant probability δ^m to become a high productivity one in the next period. Once a household has switched to high productivity it remains at this high productivity until retirement. All the households with low, medium and high productivity are called *workers*, and all the workers have a constant probability $1-\omega \in (0,1)$ of retiring next period. Once retired, each household has a constant probability $1-\sigma \in (0,1)$ of dying before the next period. (In other words, a worker continues to work with probability ω, and a retiree survives with probability σ in the next period). The flow of new born workers is $G_N - \omega$ fraction of the workforce in the previous period, where $G_N > \omega > \delta^i$ for $i = l, m$. All the transitions are i.i.d. across a continuum of households and over time, and thus there is no aggregate uncertainty on the distribution of individual labor productivity. Let N_t^l, N_t^m and N_t^h be populations of low, medium and high productivity workers, respectively, and let N_t^r be the population size of retired households in period t. Then, we have:

$$\begin{aligned} N_t^l &= (G_N - \omega)\,(N_{t-1}^l + N_{t-1}^m + N_{t-1}^h) + (\omega - \delta^l)N_{t-1}^l, \\ N_t^m &= \delta^l N_{t-1}^l + (\omega - \delta^m)N_{t-1}^m, \\ N_t^h &= \delta^m N_{t-1}^m + \omega N_{t-1}^h, \\ N_t^r &= (1-\omega)(N_{t-1}^l + N_{t-1}^m + N_{t-1}^h) + \sigma N_{t-1}^r. \end{aligned}$$

We choose to formulate the household's life-cycle in this stylized way, following Diaz-Gimenez, Prescott, Fitzgerald and Alvarez (1992) and Gertler (1999), because we are mainly interested in the interaction between the life-cycles of households and the aggregate economy. The three levels of labor productivity give us enough flexibility to mimic a typical life-cycle of wage income for our aggregate analysis.

Each household derives utility from the consumption of output (c_t) and housing services (h_t) of rented or owned housing, and suffers disutility from supplying labor (n_t). (We suppress the index of household i when we describe a typical household). We assume that, when the household rents a house rather than owning and controlling the same house as an owner-occupier, she enjoys smaller utility by a factor $\psi \in (0,1)$. This disadvantage of rented housing reflects the tenant's limited discretion over the way the house is used and modified according to her tastes. The preference of the household is given by the expected discounted utility as:

$$E_0\left(\sum_{t=0}^{\infty}\beta^t\left[u\left(c_t,\left[1-\psi I(rent_t)\right]h_t\right) - v(n_t,\nu_t)\right]\right),\ 0<\beta<1, \tag{1.8}$$

where $I(rent_t)$ is an indicator function which takes the value of unity when the household rents the house in period t and zero when she owns it.[7] Disutility of

[7] We assume that, in order to enjoy full utility of the house, the household must own and control the entire house used. If the household rents a fraction of the house used, then she will not enjoy

labor $v(n_t, \nu_t)$ is subject to idiosyncratic shocks to its labor productivity ν_t, which consists of the persistent component ε_t and transitory component ζ_t as

$$\nu_t = \varepsilon_t \zeta_t \tag{1.9}$$

The persistent component ε_t is either high (ε^h), medium (ε^m), low (ε^l), or 0, depending on whether the household has high, medium or low productivity, or is retired, and follows the stationary Markov process described above. The transitory component ζ_t is i.i.d. across time and across households and has mean of unity.[8] $E_0(X_t)$ is the expected value of X_t conditional on survival at date t and conditional on information at date 0. For most of our computation, we choose a particular utility function with inelastic labor supply as:

$$u(c_t, h_t) = \frac{\left(\left(\frac{c_t}{\alpha}\right)^{\alpha}\left(\frac{[1-\psi I(rent_t)]h_t}{1-\alpha}\right)^{1-\alpha}\right)^{1-\rho}}{1-\rho}$$

and $v_t = 0$ if $n_t \leq \nu_t$, and v_t becomes arbitrarily large if $n_t > \nu_t$. The parameter $\rho > 0$ is the coefficient of relative risk aversion and $\alpha \in (0,1)$ reflects the share of consumption of goods (rather than housing services) in total expenditure. We normalize the labor productivity of the average worker to unity as:

$$N_t^l \varepsilon^l + N_t^m \varepsilon^m + N_t^h \varepsilon^h = N_t^l + N_t^m + N_t^h. \tag{1.10}$$

We focus on the environment in which there are problems in enforcing contracts and there are constraints on trades in markets. There is no insurance market against the idiosyncratic shock to labor productivity of each household. The only asset that households hold and trade is the equity of tangible assets (and the annuity contract upon this equity). An owner-occupier can issue equity on its own house to raise funds from the other agents. But the other agents only buy equity up to a fraction $1-\theta \in [0,1)$ of the house. Thus, to control the house and enjoy full utility of a

full utility even for the fraction of the house owned.

[8]The transitory labor productivity shock helps to generate smooth distribution of net worth of households of the same persistent labor productivity.

house of size h_t, the owner-occupier must hold sufficient equity s_t to satisfy:

$$s_t \geqslant \theta h_t. \tag{1.11}$$

We can think of this constraint as a collateral constraint for a residential mortgage — even though in our economy the mortgage is financed by equity rather than debt — and we take θ as an exogenous parameter of the collateral constraint. Because the tenant household does not have a collateral asset, we assume the tenant cannot borrow (or issue equities):

$$s_t \geq 0. \tag{1.12}$$

We restrict tradeable assets to be the homogeneous equity of tangible assets in order to abstract from the portfolio choice of heterogeneous households facing collateral constraints and uninsurable labor income risk. Because we analyze the economy under the assumption of perfect foresight about the aggregate states, this restriction on tradeable assets is not substantive (because all the tradeable assets would earn the same rate of return), except for the case of an unanticipated aggregate shock.[9]

The flow-of-funds constraint of the worker is given by:

$$c_t + r_t h_t + p_t s_t = (1 - \tau) w_t \nu_t + r_t s_t + q_t s_{t-1}, \tag{1.13}$$

where τ is a constant tax rate on wage income. The LHS is consumption, the rental cost of housing (or opportunity cost of using a house rather than renting it out), and purchases of equities. The RHS is gross receipts, which is the sum of after tax wage income, the rental income from equities purchased this period, and the

[9] Although we do not attempt to derive these restrictions on market transactions explicitly as the outcome of an optimal contract, the restrictions are broadly consistent with our environment in which agents can default on contracts, misrepresent their wage income, and can trade assets anonymously (if they wish). The outside equity holders (creditors) ask the home owners to maintain some fraction of the housing equity to prevent default. There is no separate market for equities on land and capital upon it, because people prefer to control land and capital together in order to avoid the complications. Cole and Kocherlakota (2001) show that, if agents can misrepresent their idiosyncratic income and can save privately, the optimal contract is a simple debt contract with a credit limit. See Lustig (2004) and Lustig and van Nieuwerburgh (2005b) for analysis of optimal contracts with tangible assets as collateral.

pre-investment value of equity held from the previous period.[10]

For the retiree who only survives until the next period with probability σ, there is a competitive annuity market in which the owner of a unit annuity will receive the gross returns q_{t+1}/σ if and only if the owner survives, and receive nothing if dead. The retiree also receives the benefit b_t per person from the government, which is financed by the uniform payroll tax as

$$b_t N_t^r = \tau w_t (N_t^l + N_t^m + N_t^h). \tag{1.14}$$

We assume that the retirement benefit does not exceed after-tax average wage income of the low productivity worker:

$$b_t / w_t = \tau \frac{G_N - \sigma}{1 - \omega} \leq (1 - \tau)\varepsilon^l.$$

The flow-of-funds constraint for the retiree is

$$c_t + r_t h_t + p_t s_t = b_t + r_t s_t + \frac{q_t}{\sigma} s_{t-1}. \tag{1.15}$$

Each household takes the equity from the previous period (s_{t-1}) and the joint process of prices, and idiosyncratic labor productivity shocks $\{w_t, r_t, p_t, q_t, \varepsilon_t\}$ as given, and chooses the plan of consumption of goods and housing, and the equity holding $\{c_t, h_t, s_t\}$ to maximize the expected discounted utility subject to the constraints of flow-of-funds and collateral.

The representative foreigner makes purchases of goods C_t^* and equities of tangible assets S_t^* in the home country (both C_t^* and S_t^* can be negative), subject to the

[10] When the worker is an owner-occupier of a house of size h_t and issues equity to the outside equity holders (creditors) by outstanding size of $(h_t - s_t)$ in period t, she faces the flow-of-funds constraint:

$$c_t + [p_t h_t - q_t h_{t-1}] + r_t(h_t - s_t) = (1 - \tau) w_t \nu_t + [p_t (h_t - s_t) - q_t (h_{t-1} - s_{t-1})].$$

The LHS is an outflow of funds: consumption, purchases of the owned house over the resale value of the house held from last period, and rental income paid to the outside equity holders of this period. The RHS is an inflow: after-tax wage income, and the value of new issues of outside equity above the value of outside equity from the previous period. By rearranging this, we find that both the owner-occupier and tenant face the same flow-of-funds constraint (1.13), in which only the net position of equity matters.

international flow-of-funds constraint against home agents as:

$$C_t^* + p_t S_t^* = r_t S_t^* + q_t S_{t-1}^*. \tag{1.16}$$

The LHS is gross expenditure of the foreigner on home goods and equities, and the RHS is the gross receipts. We will focus on two special cases: one is a closed economy in which $S^* = 0$, and another is a small open economy in which $R_t = R_t^*$ where R_t^* is the exogenous foreign interest rate.

Given the above choices of households, the representative firm and the foreigner, the competitive equilibrium of our economy is characterized by the prices $\{w_t, r_t, p_t\}$ which clear the markets for labor, output, equity and the use of tangible assets as:

$$N_t = \int_0^{\bar{N}_t} n_t(i)\, di = \varepsilon^l N_t^l + \varepsilon^m N_t^m + \varepsilon^h N_t^h = N_t^l + N_t^m + N_t^h, \tag{1.17}$$

$$Y_t = \int_0^{\bar{N}_t} c_t(i)\, di + I_t + C_t^*, \tag{1.18}$$

$$Z_t = \int_0^{\bar{N}_t} s_t(i)\, di + S_t^*. \tag{1.19}$$

and (1.3).[11] Because of Walras' Law, only three out of four market clearing conditions are independent.

1.2.2 Behavior of Representative Firm

The first order conditions for the value maximization of the representative firm are:

$$w_t = (1-\eta) Y_t / N_t, \tag{1.20}$$

$$r_t = \eta Y_t / Z_{Yt} = \eta \left(\frac{M_t}{f_t Z_t} \right)^{1-\eta}, \text{ where } M_t \equiv A_t N_t \text{ and } f_t \equiv Z_{Yt}/Z_t, \tag{1.21}$$

$$1 - \frac{\lambda}{R_t} = r_t \gamma \left(\frac{L}{K_t} \right)^{1-\gamma} = \gamma \eta L^{(1-\gamma)\eta} \left(\frac{M_t}{f_t} \right)^{1-\eta} K_t^{\gamma\eta - 1}. \tag{1.22}$$

[11] The name of individual household i is such that a fraction of new-born households named after the names of the deceased households and the remaining fraction of newborns are given new names for $i \in (\overline{N}_{t-1}, \overline{N}_t]$.

The first two equations are the familiar equality of price and marginal products of factors of production. The value of M_t is the labor in efficiency unit, and f_t is a fraction of tangible assets used for production. The last equation says that the opportunity cost of holding capital for one period – the cost of capital – should be equal to the marginal value product of capital. Thus we have

$$K_t = \left[\frac{\gamma\eta}{1 - \frac{\lambda}{R_t}} L^{(1-\gamma)\eta} \left(\frac{M_t}{f_t} \right)^{1-\eta} \right]^{1/(1-\gamma\eta)}, \tag{1.23}$$

$$Y_t = f_t \left[\left(\frac{\gamma\eta}{1 - \frac{\lambda}{R_t}} \right)^{\gamma\eta} L^{(1-\gamma)\eta} \left(\frac{M_t}{f_t} \right)^{1-\eta} \right]^{1/(1-\gamma\eta)}. \tag{1.24}$$

Because there is no profit associated with regular production, the value of the firm is:

$$\begin{aligned} V_t^F &= r_t Z_t - (K_t - \lambda K_{t-1}) + \frac{1}{R_t} \left[r_{t+1} Z_{t+1} - (K_{t+1} - \lambda K_t) \right] + \ldots \qquad (1.25) \\ &= \lambda K_{t-1} + \eta(1-\gamma) \left(\frac{Y_t}{f_t} + \frac{1}{R_t} \frac{Y_{t+1}}{f_{t+1}} + \frac{1}{R_t R_{t+1}} \frac{Y_{t+2}}{f_{t+2}} + \ldots \right). \end{aligned}$$

The first term of the RHS is the capital stock inherited from the previous period, and the second term is the value of land, which is proportional to the present value of the return to land which comes from output and housing service production. Thus, the equity holders as a whole receive returns from capital and land through their holdings of equities of the entire tangible asset.

1.2.3 Household Behavior

The household chooses one among three modes of housing - becoming a tenant, a credit constrained owner-occupier, and an unconstrained owner-occupier. The flow-of-funds constraint of the worker and retiree can be rewritten as

$$\begin{aligned} c_t + r_t h_t + (p_t - r_t) s_t &= (1-\tau) w_t \nu_t + q_t s_{t-1} \equiv x_t, \\ c_t + r_t h_t + (p_t - r_t) s_t &= b_t + \left[q_t / \sigma \right] s_{t-1} \equiv x_t, \end{aligned}$$

where x_t is the liquid wealth of the household. Liquid wealth is the wealth of the household, excluding illiquid human capital (the expected discounted value of future wages and pension income). We call liquid wealth "net worth" hereafter.

The tenant

The tenant chooses consumption of goods and housing services to maximize the utility, which leads to:

$$\frac{c_t}{r_t h_t} = \frac{\alpha}{1-\alpha}.$$

Using the flow-of-funds constraint we can express housing and consumption as functions of current expenditure:

$$c_t = \alpha[x_t - (p_t - r_t)s_t],$$

and

$$h_t = \frac{(1-\alpha)\left[x_t - (p_t - r_t)s_t\right]}{r_t}.$$

Substituting these into the utility function we get the following indirect utility function:

$$u^T\left(s_t, x_t; r_t, p_t\right) = \frac{1}{1-\rho}\left[\frac{x_t - (p_t - r_t)s_t}{[r_t/(1-\psi)]^{1-\alpha}}\right]^{1-\rho}.$$

Due to the lower utility from living in a rented house, the tenant effectively faces a higher rental price than the owner-occupier for the same utility, i.e., $[r_t/(1-\psi)]$ rather than r_t.

The constrained owner-occupier

The constrained owner-occupier faces a binding collateral constraint as:

$$s_t = \theta h_t.$$

Thus he consumes $h_t = s_t/\theta$ amount of housing services, and spends the remaining on goods as:

$$c_t = x_t - \left(p_t - r_t + \frac{r_t}{\theta}\right) s_t.$$

The indirect period utility of the constrained home owner is now:

$$u^C\left(s_t, x_t; r_t, p_t\right) = \frac{1}{1-\rho}\left\{\left[\frac{x_t - \left(p_t - r_t + \frac{r_t}{\theta}\right)s_t}{\alpha}\right]^{\alpha}\left[\frac{s_t/\theta}{1-\alpha}\right]^{1-\alpha}\right\}^{1-\rho}.$$

The unconstrained owner-occupier

The collateral constraint is not binding for the unconstrained owner-occupier. Her intra-temporal choice is identical to the tenant's but she does not suffer from the limited discretion associated with renting a house.

$$u^U\left(s_t, x_t; r_t, p_t\right) = \frac{1}{1-\rho}\left[\frac{x_t - (p_t - r_t)s_t}{{r_t}^{1-\alpha}}\right]^{1-\rho}$$

Value functions

Let $\overline{A}_t$ be the vector of variables and a function that characterizes the aggregate state of the economy at the beginning of period t :

$$\overline{A}_t = (A_t,\ N_t^l,\ N_t^m, N_t^h,\ N_t^r,\ K_{t-1},\ S_{t-1}^*,\ \Phi_t(\varepsilon_t(i), s_{t-1}(i)))',$$

where $\Phi_t(\varepsilon_t(i), s_{t-1}(i))$ is the date t joint distribution function of present persistent productivity and equity holdings from the previous period across households. Each household has perfect foresight about the future evolution of this aggregate state, even if each faces idiosyncratic risks on her labor productivity. The prices (w_t, r_t, p_t, q_t) would be a function of this aggregate state in equilibrium. We can express the value functions of the retiree, high, medium and the low productivity worker by $V^r(x_t, \overline{A}_t)$, $V^h(x_t, \overline{A}_t)$, $V^m(x_t, \overline{A}_t)$, and $V^l(x_t, \overline{A}_t)$ as functions of the individual net worth and the aggregate state.

The retiree chooses the mode of housing and an annuity contract on equities, s_t, subject to the flow-of-funds constraint. Then, the retiree's value function satisfies the Bellman equation:

$$V^r(x_t,\overline{A}_t)=\underset{j=T,C,U}{Max}\left(\max_{s_t}\left\{u^j\left(s_t,x_t;r_t,p_t\right)+\beta\sigma V^r\left(b_{t+1}+\left[q_{t+1}/\sigma\right]s_t,\overline{A}_{t+1}\right)\right\}\right),$$

where $u^j\left(s_t,x_t;r_t,p_t\right)$ is the indirect utility function of present consumption and housing services when the mode of housing is tenant ($j=T$), constrained owner-occupier ($j=C$), or unconstrained owner-occupier ($j=U$).

The worker chooses the mode of housing and saving in equities. The value function of a high-productivity worker satisfies the Bellman equation:

$$V^h(x_t,\overline{A}_t)=\underset{j=T,C,U}{Max}\left(\max_{s_t}\left\{\begin{array}{c}u^j\left(s_t,x_t;r_t,p_t\right)+\beta\{\omega E_\zeta[V^h((1-\tau)\varepsilon^h\zeta w_{t+1}+q_{t+1}s_t,\overline{A}_{t+1})]\\+(1-\omega)V^r(b_{t+1}+q_{t+1}s_t,\overline{A}_{t+1})\}\end{array}\right\}\right).$$

The high productivity worker continues to work with probability ω and retires with probability $1-\omega$ in the next period.

The value function of a medium productivity worker satisfies:

$$V^m(x_t,\overline{A}_t)=$$
$$\underset{j=T,C,U}{Max}\left(\max_{s_t}\left\{\begin{array}{c}u^j\left(s_t,x_t;r_t,p_t\right)+\beta\{(\omega-\delta^m)E_\zeta[V^m((1-\tau)\varepsilon^m\zeta w_{t+1}+q_{t+1}s_t,\overline{A}_{t+1})]\\+\delta^m E_\zeta[V^h((1-\tau)\varepsilon^h\zeta w_{t+1}+q_{t+1}s_t,\overline{A}_{t+1})]+(1-\omega)V^r(b_{t+1}+q_{t+1}s_t,\overline{A}_{t+1})\}\end{array}\right\}\right.$$

Next period, the medium productivity worker switches to high productivity with probability δ^m, retires with probability $1-\omega$, and remains with medium productivity with probability $\omega-\delta^m$. The value function of a low productivity worker is similar to the value function of a medium productivity worker, except for m being replaced by l and h being replaced by m.

Growth in the economy with land presents a unique problem for the solution of the individual agent problem because wages grow at different rates from the rental price and the equity price even in the steady state. This means that we need to transform the non-stationary per capita variables in the model into stationary per capita units. In Appendix 4.A.2, we describe how to convert the value functions of the household into a stationary representation.

1.2.4 Steady State Growth

Before calibrating, it is useful to examine the steady state growth properties of our economy. Let $G_X = X_{t+1}/X_t$ be the steady state growth factor of variable X_t. In the following we simply call the growth factor as the "growth rate". In steady state, the growth rate of aggregate output variables should be equal:

$$\frac{Y_{t+1}}{Y_t} = \frac{I_{t+1}}{I_t} = \frac{K_{t+1}}{K_t} = G_Y.$$

The growth rate of tangible assets need not be equal the growth rate of output, but it should be equal to the growth rate of productive tangible assets:

$$\frac{Z_{t+1}}{Z_t} = \frac{Z_{Yt+1}}{Z_{Yt}} = G_Z.$$

Then, from the production functions, these growth rates depend upon the growth rates of aggregate labor productivity and population as $G_Y = (G_A G_N)^{1-\eta}\, G_Z^{\eta}$, and $G_Z = G_Y^{\gamma}$. Thus

$$G_Y = (G_A G_N)^{(1-\eta)/(1-\gamma\eta)}, \tag{1.26}$$

$$G_Z = (G_A G_N)^{\gamma(1-\eta)/(1-\gamma\eta)}.$$

Because the supply of land is fixed, to the extent that land is an important input for producing tangible assets, the growth rates of output and tangible assets are both smaller than the growth rate of labor in efficiency units. Moreover, because tangible assets are more directly affected by the limitation of land than output, the growth rate of tangible assets is lower than the growth rate of output, when labor in efficiency units is growing.

In the steady state of the competitive economy, the growth rate of the real rental price and the purchase price of tangible assets is equal to the ratio of the growth rate of output and the growth rate of tangible assets:

$$G_r \equiv \frac{r_{t+1}}{r_t} = \frac{p_{t+1}}{p_t} = \frac{G_Y}{G_Z} = G_Y^{1-\gamma}. \tag{1.27}$$

The rate of increase of the rental price and the purchase price of tangible assets is an increasing function of the growth rate of workers in efficiency units in steady state. The wage rate grows in the steady state with the same rate as the per capita output as

$$G_w = \frac{G_Y}{G_N} = \left[G_A^{1-\eta} G_N^{-\eta(1-\gamma)}\right]^{1/(1-\gamma\eta)}. \tag{1.28}$$

Because the per capita supply of land decreases with population growth, the growth rate of the wage rate is a decreasing function of the population growth rate.

Notice that the growth rates of aggregate quantities and prices only depend upon the parameters of the production function and the population and labor productivity growth rates. Because of overlapping generations and Cobb Douglas production functions, there is always a unique steady state growth in our closed or small open economy with constant population and labor productivity growth rates, even though the consumption and net worth of the individual household have different trends from the aggregate output per capita.

1.3 Observations and Steady State Implications

1.3.1 Observations

Types of Tangible Assets

Here, we gather some observations, which give us some guidance for our calibrations. Our model has clear implications about the amount of tangible assets and its split between a productive and a residential component. We use the U.S. flow of fund accounts (see Appendix 4.A.3) to compute the average quarterly tangible assets of the non-farm private sector to GDP (this includes the value of land) and this equals 3.3 for the 1952-2005 period, and is fairly stable. The fraction of productive tangible assets to total tangible assets (Z_{Yt}/Z_t) turns out to be around 0.41 (but this masks a downward trend from around 0.39 in 1991 to around 0.31 in 2005). The value of the total housing stock to GDP has an average value of around 1.94 but again this masks a marked increase from around 2.2 in 1991 to 2.6 in 2005.

Evolution of U.S. home-ownership rates and housing prices

There exists considerable variation in home ownership rates across countries and over time. Focussing on the recent U.S. experience, Figure 1.1 plots the home ownership rates (fraction of households who are owner-occupiers) across different age groups from 1991 to 2009.

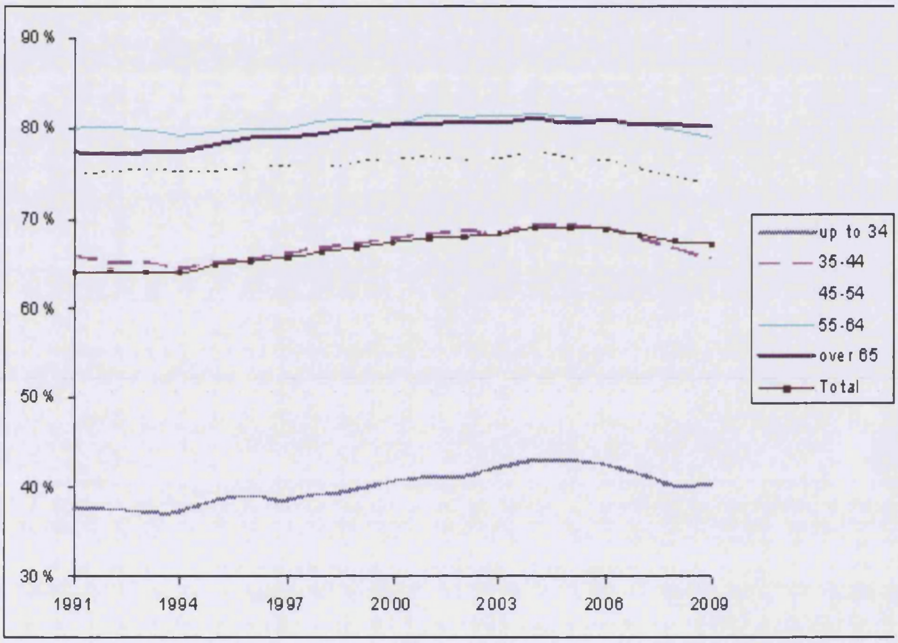


Figure 1.1: Home-ownership in the US since 1991

The figure shows a general upward trend that starts after 1995 and basically reflects the choices of younger cohorts (see Chambers, Garriga and Schlagenhauf (forthcoming) for further discussion). Variations over time across different cohorts may reflect differences in financing constraints, and utility losses from renting, factors that we analyze in the theoretical model. At the same time as homeownership goes up, real house prices also increase by a substantial amount. Figure 1.2 plots the real (deflated by the urban CPI) house price both for the value-weighted Case-Shiller index and for the equally weighted OFHEO index (for purchase-only transactions). The model we develop will have implications for these observations.

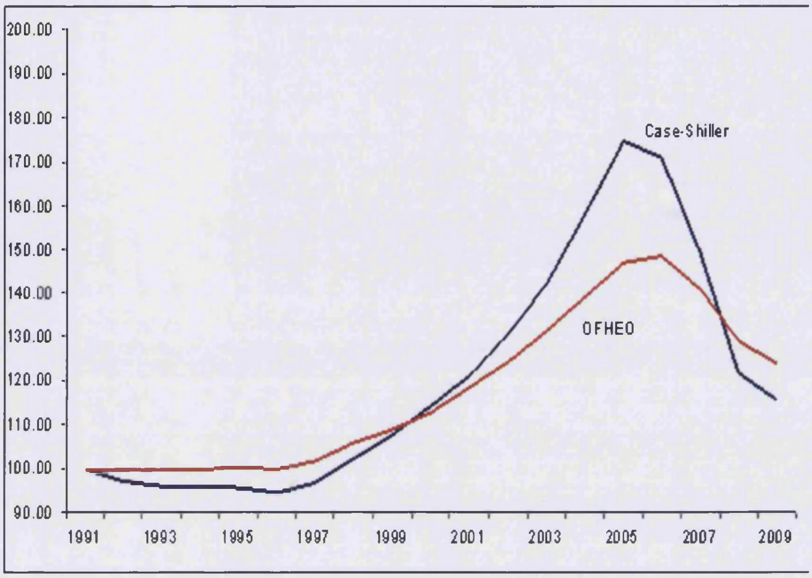


Figure 1.2: Housing prices in the US since 1991

1.3.2 Calibration

We consider one period to be one year and the baseline economy as the United States.

Labor Income Process

Our analysis will critically hinge on capturing the skewed income distribution in the data. To deal with this problem we follow Castaneda, Diaz-Gimenez and Rios-Rull (2003) and construct a simplified version of their labor income process to capture the substantial earnings inequality in U.S. data, with the aim of generating endogenously a wealth distribution close to its empirical counterpart. We pick the probabilities of switching earnings states (δ^l, δ^m) and the individual labor income productivity levels ($\varepsilon^l, \varepsilon^m, \varepsilon^h$) to match six moments. The first moment is a hump-shape in labor income; we set the ratio of mean income of 41-60 year old to the mean income of 21-40 year old to be 1.3, based on PSID evidence. The other five moments are the five quintiles of the earnings distribution. All six moments are taken from Castaneda et. al. (p.839 and table 7, p. 845) but we have independently confirmed that even though these moments change in subsequent waves of the SCF (1995, 1998, 2001 and 2004), these changes are very small. Given that we normalize the average productivity to one, this means we have 4 parameters to match 6 moments. This results in setting $\{\delta^l = 0.0338,\ \delta^m = 0.0247\}$, while the ratio of the middle to low productivity is 4.51 and the ratio of high to low productivity is 15.75. Following the buffer stock saving literature (for example, Deaton (1991) or Carroll (1997)) we assume that the transitory shock (ζ_t) is log-normally distributed with mean $-0.5*\sigma_\zeta^2$ and standard deviation $\sigma_\zeta = 0.1$.

The probability of continuing to work (ω) is set so that the expected duration of working life is 45.5 years, while the probability of the retiree to survive (σ) implies an expected retirement duration of 18.2 years. The replacement ratio (b) is chosen so that the replacement rate for the workers with low or medium productivity is 40%, consistent with the data from the PSID (very high earnings workers similar to our ε^h types will be top-coded in the PSID). We set the growth rate of labor productivity (G_A) to two percent, and the population growth rate (G_N) to one percent.

Other parameters

Using the Cooley and Prescott (1995) methodology of aligning the data to their theoretical counterparts, Appendix 4.A.3 outlines how we calculate the share of productive tangible assets in the production of non-housing final output (η) from the NIPA data for the period 1952:Q1 to 2005:Q4. This share equals 0.258 which is a bit lower than the one used in other studies (between 0.3 and 0.4), because we treat the production of housing services separately (and this is a capital intensive sector).

A key parameter in our model is the share of land in the production of tangible assets $(1 - \gamma)$. Thinking of the U.S. economy as our baseline, we set $\gamma = 0.9$ since Haughwout and Inman (2001) calculate the share of land in property income between 1987 and 2005 to be about 10.9%, while Davis and Heathcote (2005) also use $\gamma = 0.9$. Davis and Heathcote (2007) note that the share of land in residential housing values has risen recently in the U.S., and it is close to 50% in major metropolitan areas like

Boston and San Francisco. We will run some experiments for the U.K., a country where we think land restrictions are more important than in the U.S.. Absent a model with regional variation in γ (an interesting topic for further research), we will use a lower γ to match aggregate features in the U.K. with the aim of better understanding the influence of the share of land on the allocations in the steady state as well as in the transition.

The depreciation rate of the capital stock $(1-\lambda)$ is set at 10 percent per annum and the coefficient of relative risk aversion at 2. For the baseline, we consider a closed economy as the baseline. Recent papers have calibrated α (the share of non-durables in total expenditure) at around 0.8 (Diaz and Luengo-Prado (2007) use 0.83 and Li and Yao (2007) use 0.8 based on the average share of housing expenditure found in the 2001 Consumer Expenditure Survey). We use a slightly lower number (0.76) since we think of housing as inclusive of other durables, while Morris and Ortalo-Magne (2008) provide evidence supporting this choice.

The fraction of a house that needs a downpayment (θ) is set at 20%, consistent with the evidence in Chambers et. al. (forthcoming) who estimate this to be 21% for first-time buyers in the early 1990s. We perform extensive comparative statics relative to this parameter since one of our goals is to better understand the role of collateral constraints on home-ownership rates, house prices and allocations.

Model Targets

We choose the discount factor (β) to generate a reasonable tangible assets to output ratio (3.3), and the fraction of utility loss from renting a house (ψ) to generate the number of renters observed in the data (36% in 1992). This yields $\beta = 0.9469$ and $\psi = 0.0608$ for the baseline economy.

1.3.3 General Features of Household Behavior

The household chooses present consumption, saving, and mode of housing, taking into account its net worth and its expectations of future income. Figure 1.3 illustrates the consumption of goods, housing services and the mode of housing of the worker with low productivity as a function of net worth. In order to explore the stable relationship between the household choice and the state variable, we detrend all variables using their own theoretical trend as in Appendix 4.A.2.

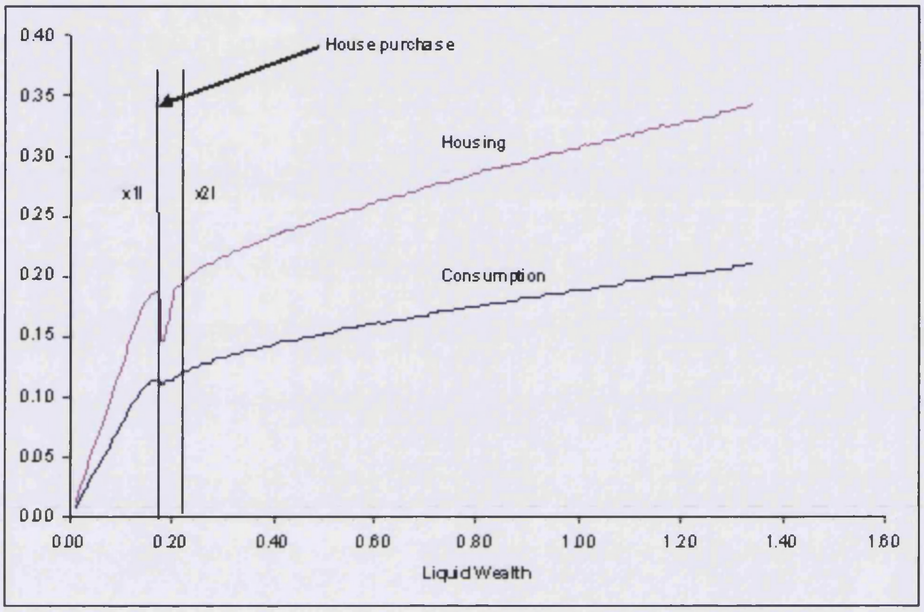


Figure 1.3: Policy functions for a low productivity household

When the worker does not have much net worth, $x < x_{1l}$, he does not have enough to pay for a downpayment of even a tiny house. He chooses to rent a modest house and consume a modest amount. In Figure 1.4, the locus $s' = s(s, q, yl)$ shows the equity-holding at the end of the present period as a function of the equity-holding at the end of the last period for the low productivity worker when the transitory income is the average ($\zeta = 1$).

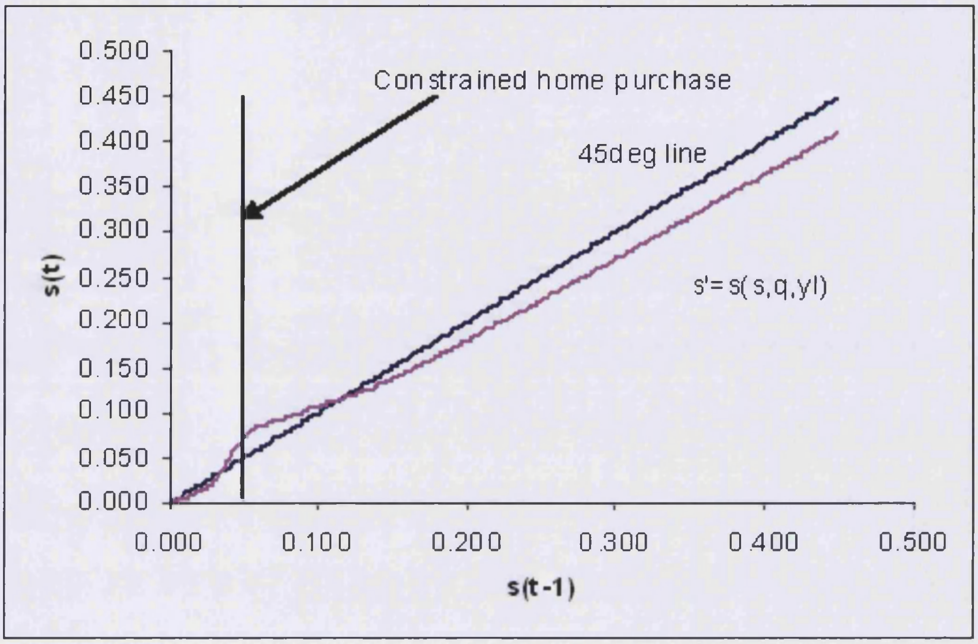


Figure 1.4: Evolution of savings for a low productivity household

Everyone enters the labor market with low productivity and no inheritance ($s_0 = 0$). Because the $s' = s(s, q, yl)$ locus lies below the 45-degree line for small enough s, as long as the worker continues to be with low productivity, he does not save - aside from small saving stemming from the transitory wage income shock - hoping to become more productive in the future. He continues to live in a rented house.[12]

Figure 1.5 shows the choice of a worker in the medium productivity state.

[12]No saving by a low productivity worker is not always true. If the income gap between low productivity and higher productivity workers is small, the transition probability from less to more productive states is small, or the pension is very limited, then the low productivity worker saves to buy a house for retirement.

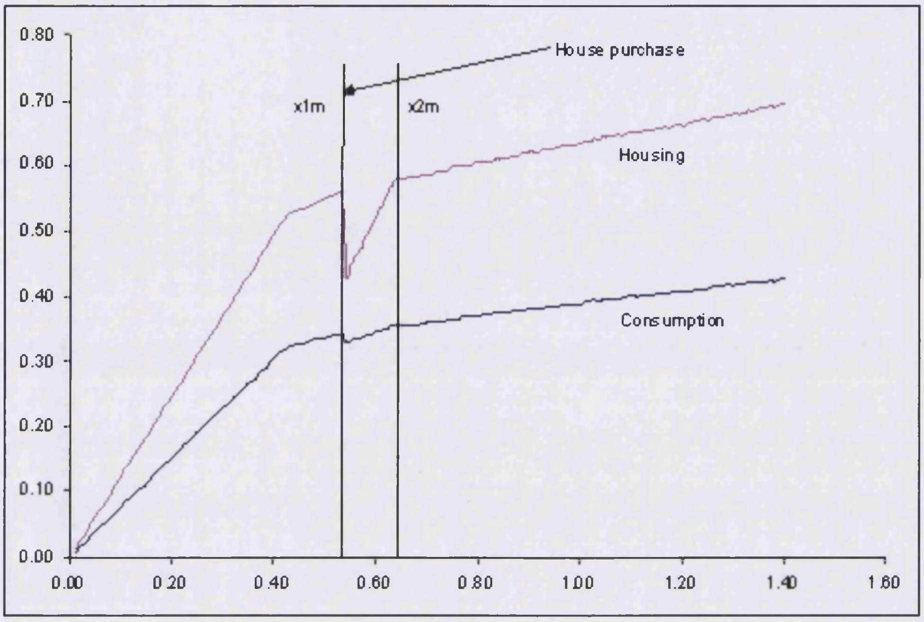


Figure 1.5: Policy functions for a medium productivity householdfigure4a

When she does not have much net worth to pay for a downpayment to buy a house, $x < x_{1m}$, she chooses to rent a place, a similar behavior with the low productivity worker. The main difference is that the medium productivity worker saves to accumulate the downpayment to buy a house in the future. In Figure 1.6, the $s' = s(s, q, ym)$ locus of the medium productivity worker lies above the 45-degree line for $s < sm^*$, so that the equity holding at the end of this period is larger than the last period.

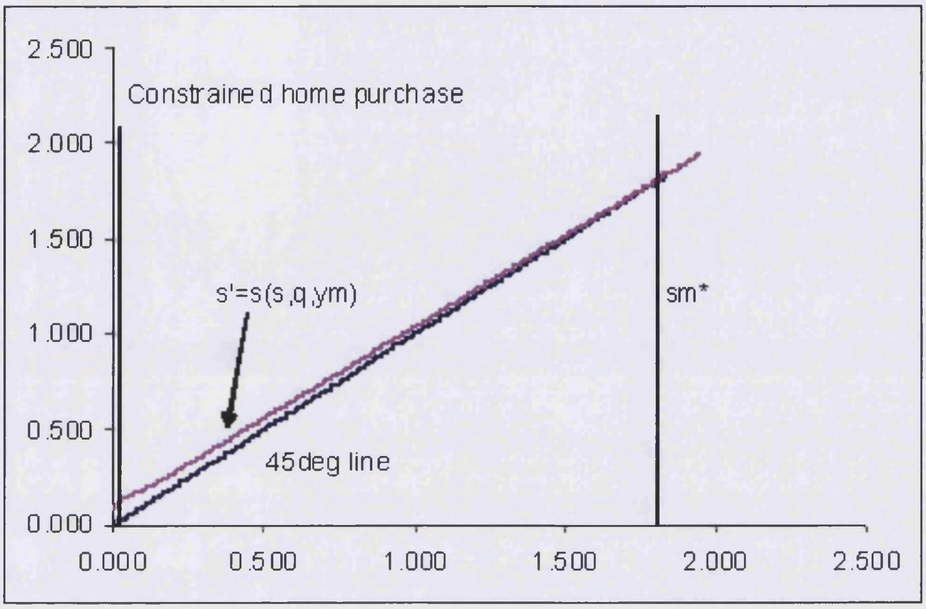


Figure 1.6: Evolution of savings for a medium productivity household

When the medium productivity worker accumulates modest net worth, $x \in [x_{1m}, x_{2m}]$ in Figure 1.5 she buys her own house subject to the binding collateral constraint. Here, the size of an owned house is a sharply increasing function of net worth, because the worker maximizes the size of the house subject to the down-payment constraint.[13] When the medium productivity worker has substantial net worth $x > x_{2m}$, she becomes an unconstrained home owner, using her saving partly to repay the debt (or increase the housing equity ownership). In Figure 1.6, the medium productivity worker continues to accumulate her equity holding until she reaches the neighborhood of equity-holding at sm^*, the intersection of $s(s, q, ym)$ and the 45-degree line.

The behavior of the high productivity worker is similar to the medium productivity one, except that she accumulates more equities: $s' = s(s, q, yh)$ lies above

[13] The size of the house at net worth $x = x_{1m}$ is smaller than the house rented at net worth slightly below x_{1m}, because she can only afford to pay downpayment on a smaller house. (Nonetheless, she is happier than before due to larger utility from an owner-occupied house). The worker moves to a bigger house every period in our model because there are no transaction costs. If there were transaction costs, the worker would move infrequently, and change housing consumption by discrete amounts, rather than continuously. She may even buy first a larger house than the house rented before, anticipating the future transaction cost. But the basic features remain the same.

$s' = s(s, q, ym)$ and her converging equity-holding sh^* is larger than that of medium productive worker sm^*. Therefore, the equity holding of all the workers is distributed between 0 and the neighborhood of sh^*, with a mass of workers in the neighborhood of $s = 0$, $s = sm^*$ and $s = sh^*$. The retiree decumulates assets very slowly as the rate of return is lower than the growth-adjusted rate of time preference.

Putting together these arguments, we can draw a picture of a typical life-cycle in Figure 1.7. The horizontal axis counts years from the beginning of work-life, and the vertical axis measures housing consumption (h) and equity-holding (s). Starting from no inheritance, he chooses to live in a rented house without saving during the young and low wage periods until the $6th$ year. When he becomes a medium productivity wage worker at the $7th$ year, he starts saving vigorously. Quickly, he buys a house subject to the collateral constraint. Then he moves up fast the housing ladder to become a unconstrained home owner. Afterwards, he starts increasing the fraction of his own equity of the house (similar to repaying the debt). By the time of retirement, he has repaid all the mortgage and has accumulated equities higher than the value of his own house. When the worker hits the wall of retirement (with the arrival of a retirement shock) at the $50th$ year, his permanent income drops, and he moves to a smaller house. He also sells all the equities to buy an annuity contract on the equities, because the annuity earns the gross rate of return which is $(1/\sigma) > 1$ times as much as straightforward equity-holding. But his effective utility discount factor shrinks by a factor σ too. Thus as the rate of return on the annuity is not sufficiently high to induce the retiree to save enough, he decumulates slowly the relative equity-holding, downsizing his consumption of goods and housing services relative to the working population as he gets older. When he dies, his assets drop to zero according to the annuity contract.

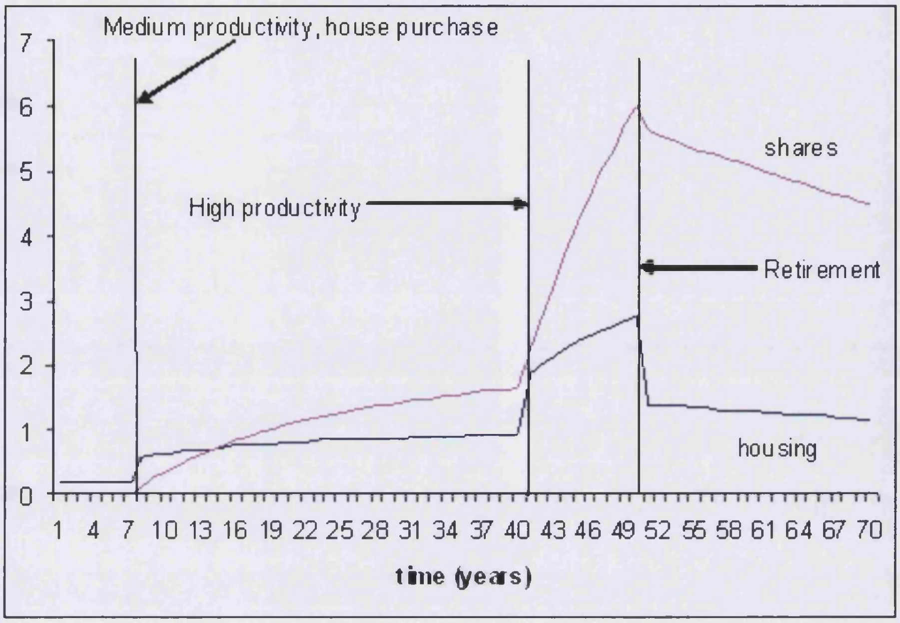


Figure 1.7: An example life time

1.3.4 Comparison of Steady States

We compare the implications of the model for the steady state economy with the data in the 1992 Survey of Consumer Finances (SCF, 1992). Table 1.1 reports the five quintiles of earnings and net worth implied by the model and their empirical counterparts. The earnings quintiles are matched exactly since the parameters of the earnings process were chosen to achieve this objective before the model is solved. Given the skewed earnings distribution, the model generates a very skewed net worth distribution as well, slightly more skewed to the right than the data. The model distribution of net worth for homeowners is even more unequal than in the data, reflecting that only very poor households remain tenants. The self-reported house value for homeowners is more evenly distributed than net worth both in the data and in the model.

Earnings quintiles (all)	1st	2nd	3rd	4th	5th
Data	0.00	0.03	0.12	0.23	0.62
Model	0.00	0.03	0.12	0.23	0.62
Net worth quintiles (all)					
Data	0.00	0.01	0.05	0.13	0.80
Model	0.00	0.00	0.01	0.11	0.88
Net Worth quintiles (Homeowners)					
Data	0.02	0.07	0.11	0.18	0.62
Model	0.00	0.01	0.05	0.16	0.78
House value quintiles (Homeowners)					
Data	0.08	0.12	0.16	0.23	0.41
Model	0.03	0.03	0.11	0.21	0.62

Table 1.1: Distribution of earnings, net worth and housing usage - SCF 1992

Table 1.2 compares mean net worth as a ratio to per capita GDP between the data and the model for different groups. The total net worth normalized by per capita GDP adds up to the calibration target of the model (3.29). Conditional on home owning, owners are wealthier than tenants, both in the model and in the data. Although the model approximately matches the average net worth of owners (4.76 in the data versus 5.52 in the model), it completely misses the net worth of tenants - tenants own very little net worth in the model while in the data they do own something. The reason is that the model abstracts from determinants of renting other than poverty. But given the richness of other moments that we match we are going to leave a more explicit calibration that captures the wealth accumulation for the tenants to future work. The average (self-reported) house value is 1.93 times as large as per capita GDP in the SCF data versus 2.34 in the model. The mean leverage ratio - the mean ratio of house value to net worth conditional on being an owner-occupier (h/s in the model) - is 1.39 in the data versus 1.49 in the model.

	Tenant NW	Total NW	Owner NW	House Value	House Value to NW
Data	0.68	3.29	4.76	1.93	1.39
Mode	0.01	3.29	5.52	2.34	1.49

Table 1.2: Aggregates - model vs SCF1992

Table 1.3 illustrates that the model captures well the rising homeownership over the lifecycle.

Age	Home-ownership	
	Data	Model
up to 34	38%	21%
35-44	65%	53%
45-54	75%	68%
55-64	80%	78%
65 or more	77%	90%

Table 1.3: Life cycle profiles of home-ownership - model vs SCF1992

Table 1.4 reports net worth and home value relative to per capita GDP for the different groups over the life cycle. Household net worth and house values increase over the life cycle in the data, which is consistent with the model.

	Net Worth (all)		Net Worth (Owners)		Home Size (Owners)	
	Data	Model	Data	Model	Data	Model
Up to 34	0.80	0.21	1.62	0.68	1.60	1.00
35-44	2.35	1.23	3.34	2.26	2.02	1.62
45-54	4.72	2.65	5.91	3.88	2.24	2.17
55-64	5.98	4.34	7.27	5.58	2.11	2.69
65 or more	3.76	3.01	4.49	3.48	1.62	1.02

Table 1.4: Life cycle profiles of net worth - model vs SCF1992

We interpret these results as suggesting that the model generates reasonable implications relative to the information in the 1992 SCF. Given this interpretation, we now would like to understand how the endogenous variables in the model (house prices and home-ownership rates) depend upon exogenous fundamentals in steady state. We restrict our attention to three main changes in the fundamentals: greater financial development, a higher productivity growth and a fall in the world real interest rate, since we view these as reasonable exogenous changes to fundamentals given the US experience in the 1990s[14].

[14]Notes to Tables 1.2-1.4: All data are from the 1992 SCF, while model refers to the baseline capturing the initial steady state for the U.S.. In Table 1.2 NW stands for net worth, and all numbers are the means relative to per capita GDP. Housing refers to the value of the home, while the house value to NW ratio is the median size of a house divided by net worth conditional on being a home-owner. Table 1.3 reports the average homeownership over the life cycle and the median house value to net worth ratio. Table 1.4 reports the average net worth over the life cycle (both for everyone and conditional on home-ownership), as well as the average home size over the life cycle (for homeowners).

Table 1.5 reports steady state comparisons for the baseline (U.S.) calibration (panel A). In the first column, the fraction of tenants in the population is 36%, which is equal to the US tenancy rate in the early 1990s (by our choice of the utility-loss from renting). The fraction of constrained home owners is 13.9%. The fraction of houses lived in by tenants and constrained home owners is smaller than the fraction of their population because they tend to live in smaller houses than the unconstrained home owners. The average house size is about 19.5% (= 7.02/35.92) of the economy average for tenants, and is about 21% for constrained home owners. The tenants and the constrained home owners live in smaller houses than the average mainly because they have lower permanent income. The distribution of equity-holding is even more unequal across the groups of households in different modes of housing. The fraction of total equities held by tenants is negligible (0.1%), the fraction of total equities held by constrained home owners is 2.97%, and the remainder is held by unconstrained home owners.

Panel A: US calibration					
	baseline	?=0.1	?=1.0	ga=1.03	R*=5.69
% of tenants	35.92	10.08	53.99	49.66	49.66
% of constrained households	13.92	26.32	4.25	2.06	1.14
% of unconstrained homeowners	50.16	63.61	41.77	48.28	49.21
% of housing used by tenants	7.02	1.82	13.20	10.82	10.15
% of housing used by constrained	2.97	5.11	2.92	0.84	0.37
% of shares owned by tenants	0.10	0.01	0.71	0.18	0.13
% of shares owned by constrained	0.26	0.23	1.29	0.17	0.06
Value of total tangible assets to GDP	3.29	3.29	3.29	3.62	3.75
Housing to total tangible assets	0.45	0.45	0.45	0.43	0.43
Value of housing to wages	2.39	2.40	2.39	2.50	2.61
Housing price to rental rate	8.58	8.58	8.58	9.56	9.87
Real return	6.69	6.69	6.69	6.69	5.69
Panel B: UK calibration					
% of tenants	31.87	7.51	54.18	49.66	49.62
% of constrained households	15.63	22.82	5.21	1.51	1.25
% of unconstrained homeowners	52.50	69.67	40.61	48.83	49.13
% of housing used by tenants	5.92	1.26	12.67	10.44	10.27
% of housing used by constrained	3.13	4.17	3.72	0.70	0.64
% of shares owned by tenants	0.09	0.02	0.79	0.19	0.02
% of shares owned by constrained	0.29	0.19	1.70	0.18	0.12
Value of total tangible assets to GDP	4.29	4.29	4.29	4.91	5.07
Housing to total tangible assets	0.46	0.46	0.46	0.44	0.44
Value of housing to wages	3.23	3.23	3.23	3.48	3.64
Housing price to rental rate	10.96	10.96	10.96	12.85	13.22
Real return	6.69	6.69	6.69	6.69	5.69

Table 1.5: Steady state comparative statics for the small open economy

Turning to prices and aggregate variables, the gross rate of return on equity-holding is 1.0669 in terms of goods, and is equal to $1.0669 \div G_r^{1-\alpha} = 1.0662$ in terms of the consumption basket. The latter is smaller than the inverse of the discount factor, which, adjusted for growth effects, equals $(1/\beta)\left(G_w/G_r^{1-\alpha}\right)^{\rho} = 1.095$. This is not because people are impatient, but because people tend to save substantially during the working period to cope with idiosyncratic shocks to wage income and to mitigate the collateral constraint. Many general equilibrium models with uninsurable idiosyncratic risk have such a feature, including Bewley (1983) and Aiyagari (1994). The ratio of average housing value to the average wage is 2.4 years, while the housing price to rental ratio is 8.6 years in the baseline economy. The share of housing in total tangible assets is 45% (compared to 41% in the post war US economy, see appendix 4.A.3).[15]

Columns 2 and 3 of Table 1.5 report the results for a different level of financial development, keeping the interest rate constant at its closed economy counterpart in column 1, by considering a corresponding small open economy. Column 2 is the case of a more advanced financial system, where the fraction of house that needs downpayment is $\theta = 0.1$ instead of $\theta = 0.2$ of the baseline. The main difference relative to the baseline economy is that now there are more constrained home owners instead of tenants. Intuitively, because borrowing becomes easier, relatively poor households buy a house with high leverage (outside equity ownership) instead of renting. Column 3, by comparison, is the case of no housing mortgage ($\theta = 1$) so that the household must buy the house from its own net worth. In this economy, more than a half of households are tenants. Financial development affects substantially the home-ownership rate. On the other hand, financial development by itself has limited effects on prices and aggregate quantities in steady state. This result arises because the share of net worth of tenants and constrained households (who are directly influenced by the financing constraint) is a small fraction of aggregate net worth, and because the required adjustment is mostly achieved through the conversion of houses from rental to owner-occupied units.

In column 4, we consider a small open economy in which the growth rate of labor productivity is three percent instead of two percent. A higher growth rate of productivity, keeping the world interest rate constant, raises the housing price-rental ratio from 8.6 to 9.6, because the real rental price is expected to rise faster as in (1.27). The value of housing to the average wage rises from 2.4 to 2.5, as does the value of tangible assets to GDP. In the new steady state, the percentage of tenants is much higher (50% from 36%) as housing prices-rental ratio is substantially higher.

[15]From (1.27) we learn that the steady state annual growth rate in rents of the baseline economy will be 0.3% when $\gamma = 0.9$. Davis et. al. (2008) compute the annual rent for the U.S. economy since 1960 and the mean real growth rate is found to be 1.17% with a standard deviation of 1.5%. Another prediction of the model involves the long run growth in house prices which is predicted to be equal to the growth rate in rents (therefore 0.3%). Using the OFHEO average annual house price data from 1960 to 2007 we calculate a real (deflating using the US CPI) annual growth rate of 2.1% with a standard deviation of 3.3%.

In Column 5, we consider an open economy where the world interest rate is lower by one percentage point. A lower world interest rate increases the house price-rental ratio from 8.6 to 9.9, which leads to a higher tenancy rate, 50% instead of 36% of the baseline.

"UK calibration"

One of the key messages of our work is that the constraint imposed by land as a fixed factor of production can have important implications for the behavior of house prices and homeownership. In order to illustrate the general equilibrium effect of the different importance of land for production of tangible assets $(1-\gamma)$, we change 3 parameters from the previous calibration and argue that this can give useful insights to a country like the U.K. Specifically, $\{\beta, \gamma, \psi\}$ are chosen so that the interest rate remains at 6.69% in the closed economy, the ratio of tangible assets to GDP is equal to 4.29 (the UK average between 1987 and 2007, for which the data exist) and the homeownership rate is equal to 68% (the UK number in the early 1990s). The resulting parameter values are $\gamma = 0.783$ (a larger share of land in the production of tangible assets than in the US), $\beta = 0.9612$ and $\psi = 0.0598$.

The baseline results (column 1) in Panel B of Table 1.5 illustrate that the value of housing relative to wages rises from 2.39 in the $\gamma = 0.9$ economy (US calibration) to 3.23 in the $\gamma = 0.78$ one (UK calibration), and that the housing price to rental ratio rises from 8.58 to 10.96. Why is the value of tangible assets to GDP and the price to rental ratio much higher in the UK calibration? Since land neither depreciates nor accumulates, as land becomes more important for tangible assets relative to the capital stock, the effective depreciation of tangible capital falls and the expected growth rate of the rental price rises. Thus, the ratio of tangible asset value to GDP and the housing price to rental ratio are larger in the UK calibration.[16]

[16]From columns 2 and 3 of Table 3, we observe that changing the collateral constraint again only affects the homeownership rate and does not affect equilibrium prices. A higher productivity growth changes in column 4 substantially the house price to rental ratio (from 11.0 to 12.9). A reduction in the world interest rate in column 5 also substantially affects equilibrium prices. The main difference from the US calibration comes from the higher share of land which makes the price to rental ratio rise more in the UK calibration. In this economy the price to rent ratio rises from 11.0 to 13.2 (a 21% increase), while in the US calibration ($\gamma = 0.9$) this ratio rises from 8.6 to 9.9 (a 15% increase).

There are two ways to measure the importance of land for tangible assets. One is the share of land in the production of tangible assets $(1-\gamma)$. The other is the share of land in the value of tangible assets. In the steady state, we can compute the present value of imputed income of land and capital in order to obtain the share of land in the value of tangible assets as:

$$\frac{\frac{1-\gamma}{1-(G_Y/R)}}{\frac{\gamma}{1-(\lambda/R)}+\frac{1-\gamma}{1-(G_Y/R)}}. \tag{1.29}$$

Note that physical capital depreciates through λ, while the imputed rental income of land grows at the rate of aggregate output growth in the steady state because the ratio of land value to aggregate GDP is stable in the steady state. Thus, in the US baseline economy in which $1-\gamma = 10\%$, $R = 1.0669$ and $G_Y = 1.029$, the share of land in the value of tangible assets is equal to 33%. (Davis and Heathcote (2007) produce estimates of the share of land in U.S. residential tangible assets and the annual average between 1930 and 2000 is 24.7% with a standard deviation of 9.6%.[17]) For the UK baseline economy in which $\gamma = 0.78$, the share of land in the value of tangible assets is 55% for the same real rate of return.

1.4 Winners and Losers in Housing Markets

We now examine how the small open economy reacts to a once-for-all change in different fundamental conditions of technology and the financial environment. We change a parameter once-and-for-all unexpectedly and solve for the path of prices and quantities that lead the economy to the new steady state. Here, we assume perfect foresight except for the initial surprise. Details of the numerical procedure can be found in Appendix A, but the basic procedure is as follows. First guess a set of rental rates over the next (say) 50 years, which converges to the new steady state; then solve backwards the household problem based on these prices; and finally update this price

[17]Thus, our assumption of a Cobb-Douglas production function for structures is generally consistent with the U.S. data. Moreover, for Japan Kiyotaki and West (2006) provide evidence that the elasticity of substitution between land and capital is not significantly larger than unity for the period 1961-1995.

vector until the market for use of tangible assets clears in all periods. To highlight the importance of land, we compare the reaction of the economy with a larger share of land in the production of tangible assets ($\gamma = 0.78$, the "UK calibration") with the baseline economy ($\gamma = 0.9$, the US calibration).

1.4.1 Welfare Evaluations

We are particularly interested in how an unanticipated change in fundamentals affects the wealth and welfare of various groups of households differently. Here, using the joint distribution of current productivity and equity holdings from the previous period $\Phi\left(\varepsilon_t(i), s_{-1}(i)\right)$ in the steady state before the shock hits, we define the group as the set I_g of individual households of a particular labor productivity (low, medium, high, and retired (l, m, h, r)), and a particular range of equity holdings of the previous period which corresponds to a particular home-ownership mode (tenant, constrained owner or unconstrained owner) in the old steady state. For example, the low-wage worker tenant group is a group of agents with low labor productivity who choose to be tenants under the old steady state.

One simple measure of the distribution effect is the average rate of change of net worth. Let $j(i)$ be present labor productivity of ($j(i) = h, m, l$ and r) of individual i. Then the net worth of individual i depends upon the wage rate and equity price as:

$$x(i) = w\epsilon^{j(i)}\zeta + q\widetilde{s}_{-1}(i),$$

where $\epsilon^j = (1-\tau)\varepsilon^j$ for worker of productivity j and $\epsilon^j = (b/w)$ for $j = r$, retired, $\widetilde{s}_{-1}(i) = s_{-1}(i)$ if i was a worker and $\widetilde{s}_{-1}(i) = s_{-1}(i)/\sigma$ if i was a retiree in the previous period. Then, the average rate of change in net worth (non-human wealth) of group I_g is:

$$\text{average of } \left(\frac{[w_n\epsilon^{j(i)}\zeta + q_n\widetilde{s}_{-1}(i)]}{[w_o\epsilon^{j(i)}\zeta + q_o\widetilde{s}_{-1}(i)]} - 1\right) \text{ for all } i \in I_g \tag{1.30}$$

where (w_o, q_o) are the wage rate and equity price in the old steady state, and (w_n, q_n) are those immediately after the shock.

To calculate welfare changes we use the value functions. Given that we have solved for the prices and value functions for all the periods in the transition, we know that the value functions at the period when the change in fundamentals takes place is a sufficient statistic for the welfare effect of the shock. Let $V_o^{j(i)}(x(i))$ be the value function at the old steady state and $V_n^{j(i)}(x(i))$ be the value function in the period of the shock's arrival as a function of net worth $x(i)$ and labor productivity.[18] We compute a measure of welfare change for the group I_g as:

$$\overline{\mu}_g = \text{average of } \left[\left(\frac{V_n^{j(i)}([w_n \epsilon^{j(i)} \zeta + q_n \widetilde{s}_{-1}(i)])}{V_o^{j(i)}([w_o \epsilon^{j(i)} \zeta + q_o \widetilde{s}_{-1}(i)])} \right)^{\frac{1}{1-\rho}} - 1 \right] \text{ for all } i \in I_g. \quad (1.31)$$

We call this measure as the certainty expenditure equivalent, because we convert the change of the value into the dimension of expenditure before taking the average.[19]

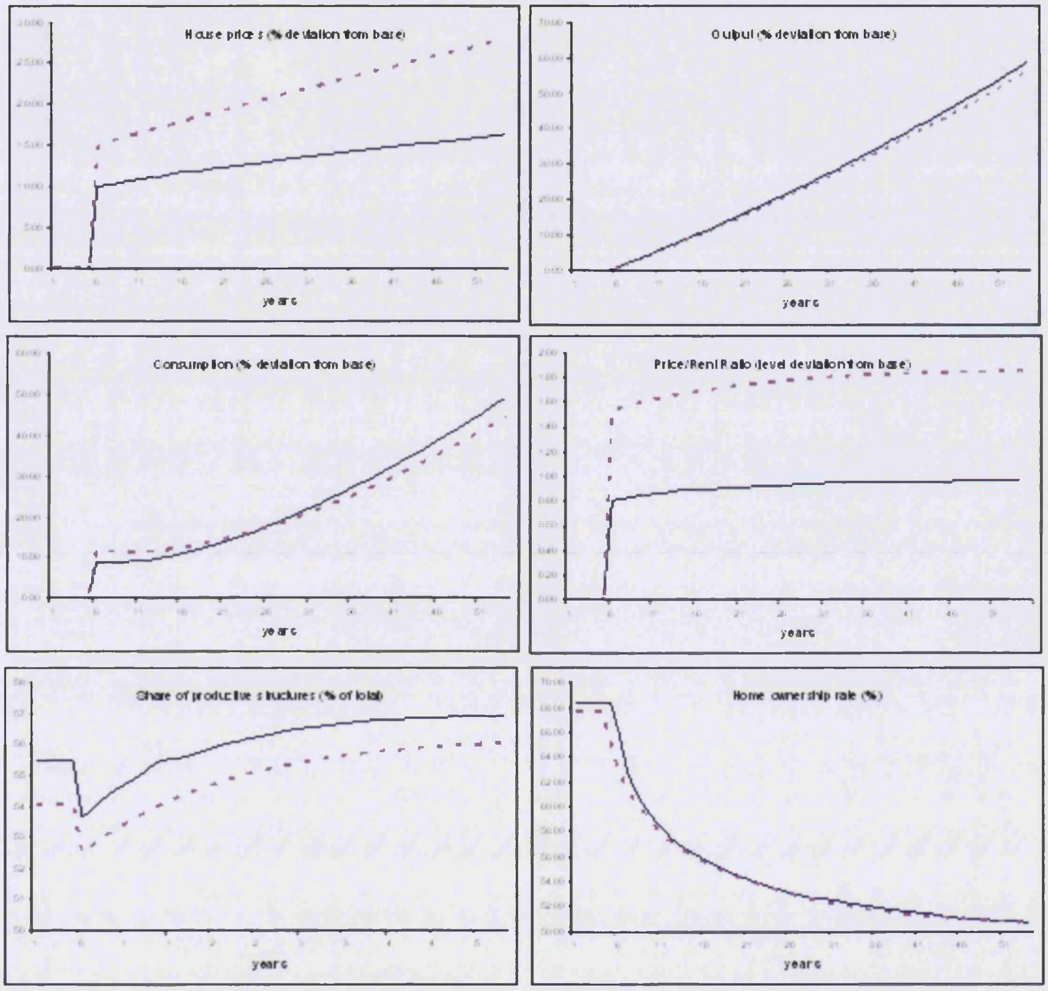
[18] Note that V_n is the value function that has been derived after the full perfect foresight transition has been solved for and therefore includes all this information about the transition to the new steady state.

[19] We also computed the net worth equivalent that would make a household indifferent between the period before and after the shock as the value of $\lambda(i)$ such that

$$V_o^{j(i)}([w_o \epsilon^{j(i)} \zeta + q_o \widetilde{s}_{-1}(i)]) = V_n^{j(i)}(\lambda(i)\,[w_n \epsilon^{j(i)} \zeta + q_n \widetilde{s}_{-1}(i)])$$

The value of $\lambda(i)$ measures how much the initial net worth must be multiplied immediately after the shock in order to maintain the same level of the expected discounted utility as the old steady state. We can find the net worth equivalent uniquely, because the value functions are monotonically increasing. We can then compute the average of individual $\lambda(i)-1$ for a particular group g of agents as $\widetilde{\mu}_g$. This welfare measure suffers from the drawback that net worth does not include the value of human capital. Thus, if two groups have different ratios of net worth (liquid wealth) to human capital, a difference in $\widetilde{\mu}_g$ may reflect the difference of the ratio of human to non-human wealth rather than the difference in the welfare effect.

1.4.2 Transition of Small Open Economy following a Change in Fundamentals



Note: dotted line ($\gamma = 0.78$), solid line ($\gamma = 0.9$)

Figure 1.8: 1 per cent increase in labor productivity growth

Figure 1.8 shows the responses to a once-for-all increase in the growth rate of labor productivity from 2% to 3%. Because the economy is growing, all the following figures show the percentage difference from the old steady state growth path of the baseline economy. In both economies the housing price increases substantially initially and continues to increase afterwards. In the economy with a larger share of land ($\gamma = 0.78$), the increase in house prices is larger, and real house price inflation afterwards is higher. The housing price-rental ratio is going to be higher, anticipating the increase in the rental price in the future. The home-ownership rate gradually declines because young workers take a longer time to accumulate a sufficient downpayment to buy a house. Consumption of goods and housing services increase initially as well as afterwards, reflecting higher permanent income. The share of productive tangible assets (Z_{Yt}/Z_t) falls initially, to accommodate a larger demand for residential tangible assets by converting productive to residential tangible assets.

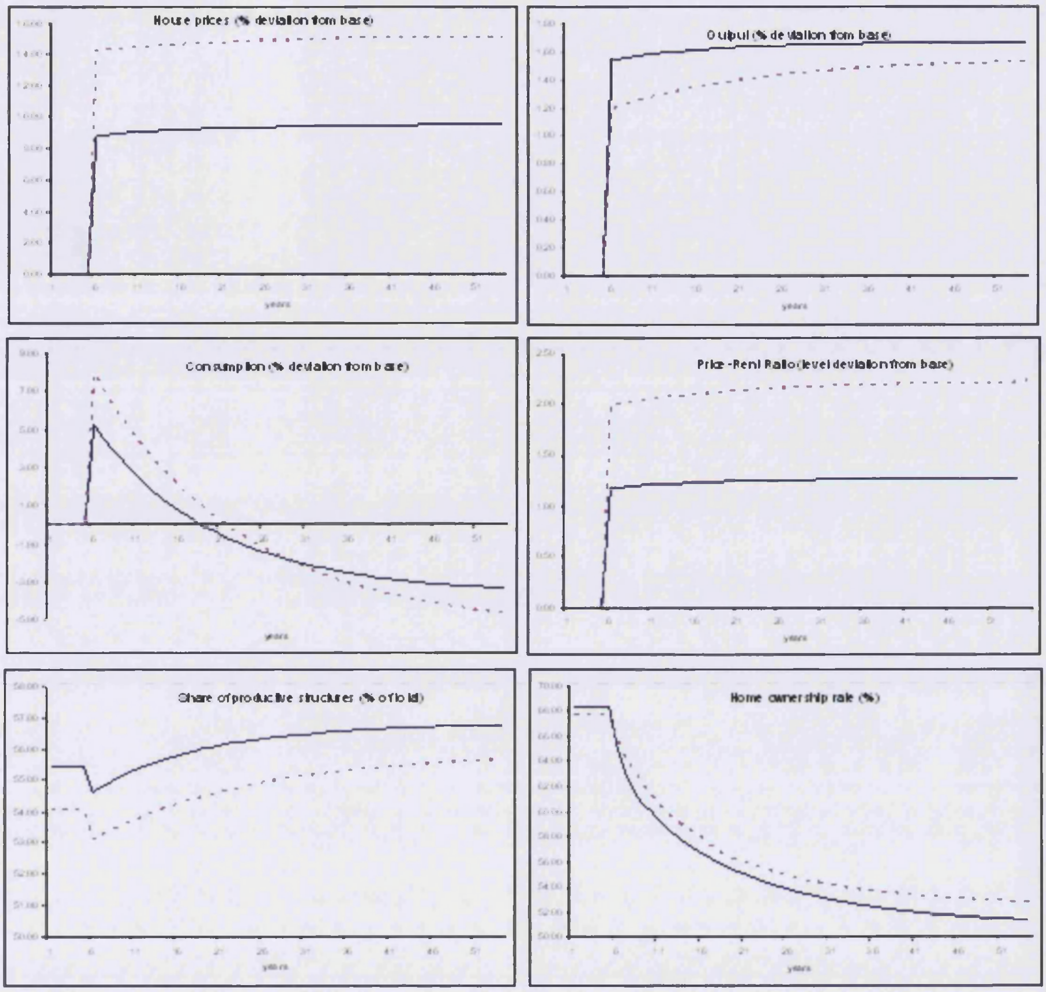
Scarcity of Land Parameter	v=0.9	v=0.78	v=0.9	v=0.78	v=0.9	v=0.78
Column	1	2	3	4	5	6
Panel A: Certainty expenditure equivalent	*ga+1%*	*ga+1%*	*R*-1%*	*R*-1%*	*all*	*all*
Workers	9.20	10.46	0.00	0.00	11.59	13.62
Tenant Workers	8.74	9.61	1.27	0.88	10.24	10.69
Constrained Homeowner Workers	9.04	9.93	1.27	1.00	11.04	11.76
Unconstrained Homeowner Workers	9.80	11.39	-0.05	0.42	12.74	15.65
Low Income Workers	8.94	9.71	1.28	0.96	10.63	11.20
Middle Income Workers	9.48	10.73	0.10	0.37	12.24	14.49
High Income Workers	10.37	12.58	0.00	0.00	13.72	18.30
Retirees	8.27	10.46	1.64	3.49	14.85	20.79
Tenant Retirees	6.86	7.07	1.24	0.78	8.59	8.75
Constrained Homeowner Retirees	7.10	7.39	1.30	1.11	9.32	9.63
Unconstrained Homeowner Retirees	10.67	11.24	3.15	4.25	16.05	21.93
Panel B: Wealth change						
Workers	4.61	7.44	4.88	8.10	15.57	24.01
Tenant Workers	0.50	0.71	0.42	0.90	1.06	2.12
Constrained Homeowner Workers	2.34	4.25	1.97	4.72	6.10	10.93
Unconstrained Homeowner Workers	8.15	12.31	8.30	13.55	25.44	36.87
Low Income Workers	1.03	1.77	1.13	2.08	3.42	5.90
Middle Income Workers	7.72	11.78	7.90	12.95	24.42	37.27
High Income Workers	10.14	14.70	10.33	16.11	31.37	46.24
Retirees	6.47	10.45	6.61	11.50	21.65	33.95
Tenant Retirees	0.81	1.74	0.84	1.80	2.16	4.74
Constrained Homeowner Retirees	3.22	4.24	3.32	4.42	6.94	10.40
Unconstrained Homeowner Retirees	10.84	11.52	11.09	12.56	23.26	36.09

Table 1.6: Welfare

Table 1.6 reports the average rate of change of welfare (1.31) in Panel A and the average rate of change of current net worth (1.30) in Panel B for each group against changes in the fundamentals, for the baseline economy ($\gamma = 0.9$) and the economy with a larger share of land ($\gamma = 0.78$). The first and second columns report the average rate of changes from an increase in the growth rate of labor productivity from 2% to 3%. Given the higher productivity growth, households are on average better off with a higher permanent income. (Remember the retiree's benefit is proportional to the wage rate of present workers). The higher housing price, however, affects the welfare of different groups of households differently. Those who buy (or expand) houses in the future gain less from the housing price hike, while those who sell houses in the future gain more. Specifically, unconstrained homeowners as a group gain more than tenants and constrained homeowners. The gap in welfare effects between unconstrained homeowners and the other groups is particularly large for the retirees. Overall, one main message from this analysis is that the redistribution effect is larger in the economy with the larger share of land since the house price hike is bigger in this economy.

We can observe the change in current net worth in Panel B. The net worth of unconstrained homeowners increases by a much larger amount than tenants' net worth because the former own much more non-human wealth. Thus, those with larger holdings of shares experience a bigger increase in net worth with the house price rise, and the increase is more pronounced where land is more important.

Figure 1.9 shows how these two economies react to a once-for-all fall in the world real interest rate by 1%. In both economies, housing prices and output increase with large inflows of capital, and the adjustment of housing prices is fast. In the economy with a larger share of land, the swing of net exports and consumption is larger, output takes a longer time to increase despite the large increase in the capital stock, because a large amount of tangible assets gets allocated to housing in the early stages of the transition. The home-ownership rate declines gradually because the lower real interest rate discourages saving, delaying the age of switching from renting to owning a house over the life cycle.



Note: dotted line ($\gamma = 0.78$), solid line ($\gamma = 0.9$)

Figure 1.9: 1per cent decrease in the world real interest rate

The third and fourth columns of Table 1.6 report the reaction of welfare to this decrease in the world real interest rate for the two economies with different shares of land. Looking at the value of net worth in Panel B, all groups have a larger net worth from a higher house price, and the net worth increase is larger group-by-group in the economy with a larger share of land ($\gamma = 0.78$). As we discussed in the Introduction (especially in footnote 3), however, the increase in housing price per se does not have an aggregate wealth effect on consumption nor welfare, but mainly redistributes wealth between net sellers and net buyers of houses. Unconstrained homeowner retirees gain most from the house price hike due to a lower interest rate. Although workers gain from a higher wage rate due to the capital inflow, workers as a whole are savers who suffer from a lower interest rate, particularly high income workers. Thus despite the capital gains on housing, the high income workers and unconstrained homeowner workers lose from a lower interest rate in our calibration, and the loss is larger when the share of land is small ($\gamma = 0.9$), that is, when the capital gains on the house is small.

These two experiments illustrate the idea that the relationship between housing price changes and welfare depends upon the underlying cause of the house price change. House prices are higher by a similar magnitude after either a higher productivity shock or a lower world interest rate, but in our calibrations workers as a whole gain from the productivity improvement but lose as a whole from the interest rate decrease.[20]

We have also done the experiment of lowering the downpayment requirement from 20% to 10% permanently. This provides extra liquidity for households, especially for constrained home owners, and encourages consumption initially. At the same time, with a less stringent collateral constraint, some low wage workers and tenants from the previous period buy houses. Overall, however, relaxing the financing constraint has a very limited effect on housing price and aggregate production in the transition, a result similar to the comparisons of the steady states, because the necessary adjustment is mostly achieved by the modest conversion of rented to owned units rather than by the housing price. This contrasts Ortalo-Magne and Rady (2006), who show that relaxing the collateral constraint increases the housing price substantially by increasing the housing demand of credit constrained households. In their model, the net worth of the home-owners with outstanding mortgage is sensitive to the housing price due to the leverage effect, which magnifies the effect of any shock to fundamentals, while there is no leverage effect in our equity

[20] Attanasio et. al. (2009) make a similar point empirically. They find that tenants' consumption is positively correlated with house price increases, contradicting the conventional wealth channel. They attribute this finding to common factors driving both consumption demand and house prices, namely better longer-run income prospects. Thus, the shock causing higher house prices can be key in determining the effect on consumption (and, therefore, welfare).

financing economy. Also the supply of houses and flats is inelastic in their model. Thus, relaxing the collateral constraint will generate a large inflow of new owners of flats and houses, which is not offset by an increase in the supply, through conversion from rented to owned units, conversion from productive to residential tangible assets and capital accumulation. A comprehensive analysis of the leverage effect and the portfolio decision in the presence of uninsurable earnings and aggregate risk is a topic for future research.

1.4.3 A Scenario for House Price Changes?

Putting together the simulation results from these experiments, we can conclude that, if we were to explain the large increase in housing prices in many developed countries in the last decades, we could look for increases in the expected growth rate of labor productivity and for decreases in the real interest rate. Moreover, to generate a positive correlation between homeownership rates and house price rises since the early 1990s, we will also need to simultaneously improve access to credit. An empirically plausible calibration will be to simultaneously increase the expected growth rate of labor productivity from 2% to 3%, decrease the world interest rate by one percent and reduce the collateral constraint from 20% to 10%.

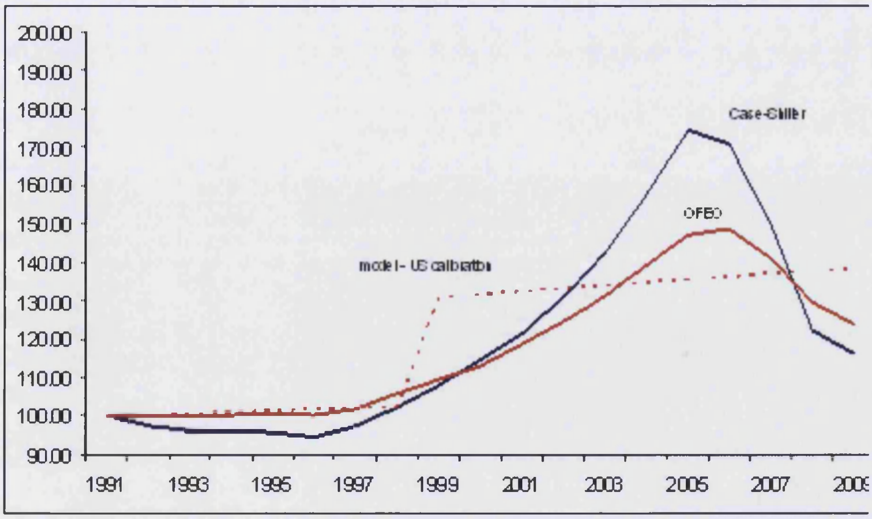


Figure 1.10: US - model versus data since 1991

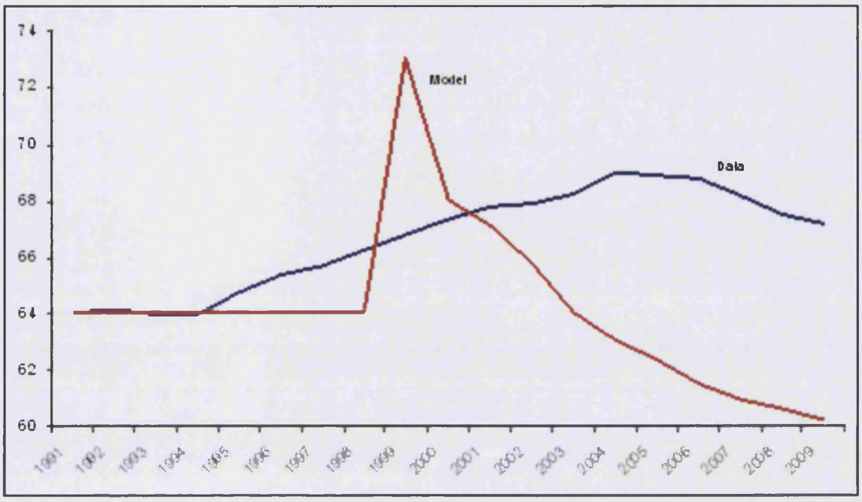


Figure 1.11: Aggregate home ownership rates since 1991: model versus data

The implications for house prices and homeownership rates are given in Figures 1.10 and 1.11 respectively for the US experience, and Figures 1.12 and 1.13 for the UK. For the US calibration Figure 1.10 illustrates that the model can explain a substantial component of the recent house price increases. Moreover, the model captures well the increase in home-ownership rates, even though this increase is much faster in the model than in the data given the perfect foresight/information assumptions of the model. Interestingly the model does predict a fall in the home-ownership rate after the initial increase as house prices begin to rise. The wealth changes and the welfare effects from this simultaneous shock for the US economy are given in column 5 of Table 1.6. Households are both richer and better off in response to this combination of shocks, with the unconstrained home owner retirees gaining the most in both wealth and welfare.

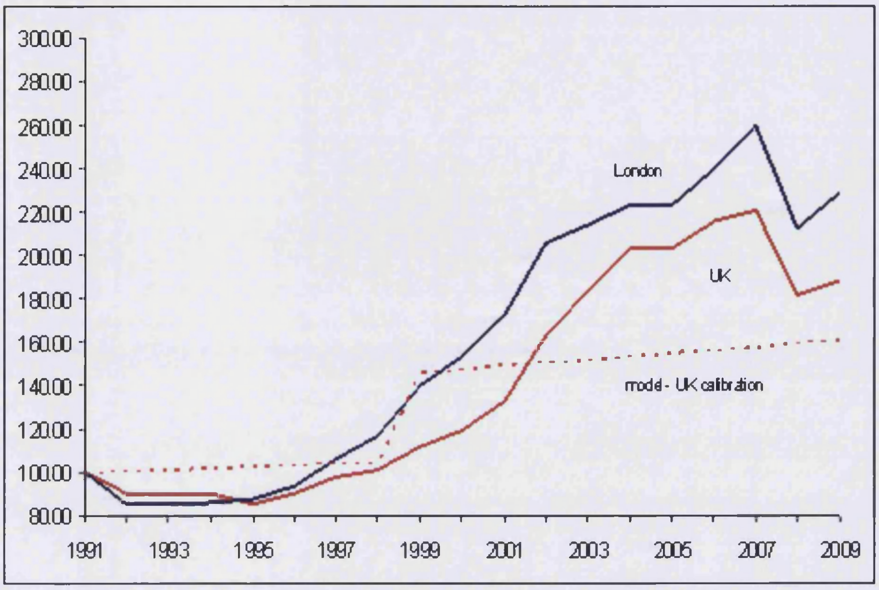


Figure 1.12: UK - model versus data since 1991

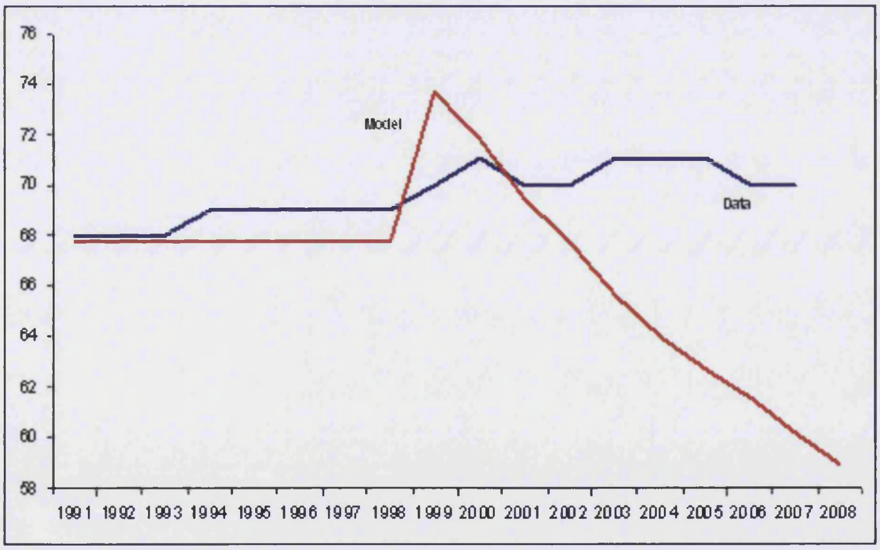


Figure 1.13: UK: Aggregate home ownership rates since 1991: model versus data

The responses of the calibration for the "UK" economy are given in Figures 1.12 and 1.13. The model captures a lower fraction of the recent runup in housing prices in the UK, but it also predicts a slight increase in homeownership rates with a decrease predicted in the future as housing prices reach a higher level. The last column of Table 1.6 illustrates that both wealth and welfare increase by more in this economy rather than in the $\gamma = 0.9$ one and that the effect is biggest for the unconstrained retirees.

1.5 Conclusions

This paper develops an aggregate life-cycle model to investigate the interaction between housing prices, aggregate production, and household behavior over a lifetime. We take into account land as a fixed factor for producing residential and commercial tangible assets in order to analyze the implications for the aggregate time series and the cross section of household choices. Comparing two small open economies with different shares of land in the production of tangible assets, the economy with a larger share of land has a higher housing price-rental ratio and a lower homeownership rate in the steady state. The transitions of the small open economy along the perfect foresight path illustrate that, where the share of land is larger, once-for-all shocks to the growth rate of labor productivity or the world interest rate generate a greater movement in housing prices.

We also find that the permanent increase in the growth rate of labor productivity and the decrease in the world real interest rate substantially redistribute wealth from the net buyers of houses (relatively poor tenants) to the net sellers (relatively rich unconstrained homeowners) with the house price hike. On average, households gain from the increase in the growth rate of labor productivity and do not gain from the decrease in the world interest rate. Because the gap in welfare effects between winners and losers in the housing market is substantial, especially where land is more important for production of tangible assets compared to capital, we think that a credible welfare evaluation should take into account household heterogeneity and contract enforcement limitations in housing and credit markets that generate realistic life-cycles of consumption and homeownership.

Chapter 2

Is Private Leverage Excessive?

2.1 Introduction

The 2007-09 financial crisis brought the world financial system to the brink of collapse, leading to calls for tighter regulation in order to prevent a repeat of the crisis. 'Excessive leverage' is thought to be one of the main culprits for the fragility of the economy in the face of shocks. This has re-opened the debate of whether private banks, corporates and households tend to take socially optimal borrowing decisions. In this paper we examine the optimality of firms' leverage decisions using a standard macroeconomic model with credit frictions. We examine whether a benevolent government can improve ex ante welfare by imposing capital requirements which are different from those chosen by the market.

A growing academic literature has shown that the prevalence of uncontingent debt has the potential of interacting with binding collateral constraints in order to magnify the effects of shocks to the economy. The mechanism is based on different versions of the the collateral amplification argument popularised by Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999). More recently, Lorenzoni (2008), Gromb and Vayanos (2002) and Korinek (2009) have shown that, in an environment of binding credit constraints, private leverage tends to be excessive from a social point of view due to the presence of a market price externality. This externality arises because private borrowers do not internalise the effects of their own financial distress on other borrowers. When collateral constraints tighten due to an adverse aggregate shock, leveraged debtors' net worth declines and they need to sell assets in order to satisfy the collateral constraint. This 'financial distress' scenario leads to private losses which are fully taken into account by firms when they decide ex ante how much debt to take on.

What private borrowers ignore, however, is the market price externality of financial distress. The larger the volume of asset sales following an adverse shock to collateral values, the bigger the eventual decline in capital prices and the wider the spectre of financial distress. Individual borrowers, however, do not take such 'general equilibrium' effects into account. They take the state contingent evolution of market prices as exogenous, treating their own leverage decisions as irrelevant for aggregate outcomes. In contrast, the government takes the market price externalities in question into account when designing the optimal state contingent capital adequacy rules.

This paper focuses on the quantitative question of whether taking the market price externality into account leads the government to choose very different capital requirements from those already required by the market. We use a business cycle model with credit constraints, which is similar to Kiyotaki (1998). In our environment borrowing and lending is motivated by a heterogeneity in the productivity of different firms. But because debt is assumed to be uncontingent and secured against collateral, aggregate shocks can damage the net worth of borrowers and reduce their access to finance. I assume that borrowing entrepreneurs in the model know that aggregate productivity shocks may hit and this gives them an incentive to hedge their net worth by borrowing less than the market determined debt limit. We nevertheless find that high productivity firms choose to take the maximum permitted leverage despite the risks to net worth this involves. The intuition for this is simple. High productivity entrepreneurs earn such a good return on their productive assets that insuring their net worth by leaving themselves with spare debt capacity is too costly. Because the owners of these fast growing firms have very good future consumption opportunities, saving at prevailing market prices is a very bad proposition for them. So they rationally choose to leverage up to the debt limit, accepting the ex post volatility in the rate of return on their portfolios.

The main result of the paper is the following. When we allow a benevolent government to choose state contingent capital requirements to maximise ex ante social welfare, we find that the government makes identical choices to the market for reasonable parameter values. In other words, the government chooses capital requirements which are equal to the incentive compatible debt limits. We find that this surprising result arises from the balance of the costs and benefits of regulation around the private optimum. Tightening capital requirements relative to the market-imposed borrowing limits has the benefit of dampening the collateral amplification mechanism and reducing the volatility of asset prices and consumption over the economic cycle. This cyclical volatility is 'excessive' from a social point of view because leveraged borrowers do not take into account the effect of their own forced asset sales on other leveraged borrowers. But the government considers the costs of regulation too. In our model, the flow of finance from low to high productivity entrepreneurs increases the economy's TFP by putting more of the economy's productive resources into the hands of those best able to make use of them. When the government regulates leverage, more production has to be undertaken by inefficient firms and this depresses average TFP and consumption over time.

How the government locates itself on this trade off between increasing the economy's average productivity and consumption and increasing its consumption volatility is a function of the costs of business cycles in the model. We find that, quantitatively, these costs are small. Because the government acts in the social interest, it allows private agents to borrow as much as can be credibly repaid without imposing tighter capital requirements than the market.

Interestingly, we find that the 'no overborrowing' result does not arise because amplification in the model is small. Contrary to the results of Cordoba and Ripoll (2004) we find that it is large, increasing the standard deviation of output by 40% higher than the first best without making any non-standard assumptions about preferences or the productive technology. The difference between our results and those of Cordoba and Ripoll (2004) arise out of our assumption of constant returns to scale to all factors, which helps to maintain productivity differences between firms even in the face of large shocks to their relative outputs. This result shows that the Kiyotaki and Moore (1997) framework is capable of generating quantitatively large amplification for reasonable calibrations. Nevertheless, despite generating a lot of amplification, the framework does not generate strong incentives to regulate financial transactions. This is because consumers care more about having a high rate of return on wealth and this dominates the welfare costs due to business cycle fluctuations.

Finally, we need to stress that the pecuniary externality our paper discusses is only one of the many reasons for capital regulation. Our framework misses out one very important reason for capital regulation - the risk shifting behaviour caused by the possibility of bankruptcy or a government bail-out. There is a large literature which has studied the incentives for banks and other private borrowers to take excessive risks when they know that losses in the worst case scenarios will be borne by lenders or the government. While such factors are undoubtedly an important cause of financial crises, we abstract from them in this paper in order to keep our framework tractable[1].

The rest of the paper is organised as follows. Section 2.2 discusses the related literature in a little more detail. Section 2.3 outlines the model environment. Section 2.4 outlines the competitive equilibrium for our model economy. Section 2.6 outlines the government's objective function and policy instrument. Section 2.5 compares private and government leverage choices and uses numerical simulation of the economy to illustrate the costs and benefits of tighter collateral requirements. Finally, Section 2.8 concludes.

[1]We study borrowing contracts which feature no bankruptcy in equilibrium. Also we assume that the government cannot make transfers. This rules out two of the most widely studied mechanism which generate overborrowing by private agents.

2.2 Related Literature

2.2.1 The collateral amplification mechanism

This model is related to a large and rapidly growing literature on the credit amplification mechanism and on the pecuniary externalities this generates. The collateral amplification transmission channel was first popularised by the work of Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Kiyotaki (1998) Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999). All these models examine the effect of financing frictions on aggregate allocations. In them, the net worth of agents who have productive opportunities is key in determining the cost and availability of external finance. Adrian and Shin (2009) have explored this mechanism in the context of multiple leveraged traders in financial markets.

2.2.2 Pecuniary externalities and the efficiency of private leverage

The central question of this paper is related to an older literature which has examined the constrained efficiency of the competitive equilibrium in an economy with moral hazard and adverse selection. Arnott and Stiglitz (1986) showed using a simple insurance moral hazard example that the competitive equilibrium is constrained inefficient when prices affect insurees' incentives to take care. Kehoe and Levine (1993) show that the competitive equilibrium in their 'debt constrained' economy is only efficient in a single good world. Multi-good economies are not necessarily constrained efficient because relative prices affect the value of default and this introduces a market price externality which is not taken into account by atomistic private agents. What these papers show is that when relative prices determine the tightness of incentive compatibility constraints, this drives a wedge between the decisions of private agents and the decisions of the social planner. Private individuals take prices as given while the social planner recognises that manipulating prices can relax some of the constraints it is facing.[2]

[2] Prescott and Townsend (1984) showed that introducing man-made lotteries into the economy can remove the externality in question and restore the constrained efficiency of the competitive equilibrium.

Even more closely related to the topic of this paper, work by Lorenzoni (2008), Korinek (2009) and Gromb and Vayanos (2002) have shown rigorously that the presence of asset prices in the collateral constraint can generate a pecuniary asset price externality between leveraged borrowers. Distressed sales by one set of borrowers can push down asset prices, damaging the net worth and credit access of other borrowers. Private agents ignore this externality, generating incentives for government intervention in order to bring the social costs and benefits of leverage into line with one another. These papers provide the theoretical motivation in a simple three period framework for the quantitative investigation we undertake here in an infinite horizon macro model.

Korinek (2008) and Bianchi (2009) have also examined the possibility of excessive external debt in the an emerging market context. In Korinek (2008), borrowing in foreign currency is cheaper for individual firms because of the risk premium on domestic currency debt. However, foreign currency debt leaves domestic entrepreneurs vulnerable to a sharp appreciation of the domestic real exchange rate. In Bianchi (2009), fluctuations in the price of non-traded goods work in the same way to introduce sudden sharp changes in real debt values. In both of these models, just like in the model of this paper, the externality works through pecuniary externalities that affect the tightness of borrowing constraints.

2.2.3 The welfare costs of business cycles

How the government trades off average consumption against the volatility of consumption is an important reason behind the results of this paper. This issue connects with the literature on the welfare costs of business cycles, which was started by Lucas (1987)'s seminal contribution. Lucas (1987) found that the cost of aggregate consumption volatility was of the order of 0.08% of annual consumption, implying that business cycle volatility is not an important determinant of social welfare. Lucas (1987), of course, recognised that imperfections in risk sharing had the potential of increasing the cost of business cycles at least for some groups in society.

This finding spurred a lot of research on the effect of risk sharing and consumer

δ heterogeneity on the welfare costs of business cycles. Krussell and Smith (1998) examine this question in an infinitely lived economy with aggregate uncertainty in which individuals are subject to unsinsurable idiosyncratic shocks. Storsletten et al. (2001) extended Krussell and Smith's analysis to an economy with finitely lived overlapping generations. They found that the welfare costs of the business cycle vary substantially across different groups in society and are larger than Lucas' orginal numbers but still far from enormous. We find that the small costs of business cycles play a substantial role in determining the costs and benefits of regulation in our framework too.

2.3 The Model

2.3.1 The Economic Environment

Population and Production Technology

The economy is populated with a continuum of infinitely lived entrepreneurs and a continuum of infinitely lived workers - both of measure 1. Each entrepreneur is endowed with a constant returns to scale production function which uses capital k, labour h and intermediate inputs x to produce gross output y.

$$y_t = a_t A_t \left(\frac{k_{t-1}}{\alpha}\right)^{\alpha} \left(\frac{x_{t-1}}{\eta}\right)^{\eta} \left(\frac{h_{t-1}}{1-\alpha-\eta}\right)^{1-\alpha-\eta}$$

where a is the idiosyncratic component of productivity which is revealed to the entrepreneur one period in advance and can be high a^H or low a^L. The idiosyncratic state evolves according to a Markov process. Following Kiyotaki (1998) let $n\delta$ be the probability that a currently unproductive firm becomes productive and let be the probability that a currently productive firm becomes unproductive. This implies that the steady state ratio of productive to unproductive firms is n. The aggregate state also evolves according to a persistent Markov process.

A_t is the aggregate component of productivity which also evolves according to a Markov process and alternates between high and low values. The realisation of the aggregate state A_t occurs at the beginning of time t.

Intermediate inputs x are produced one for one from consumption goods and fully depreciate between periods. Capital is in fixed aggregate supply and does not depreciate. The only financial asset is simple debt.

Commitment technology and private information

Agents suffer from limited commitment. They cannot make binding promises unless it is in their interests to do so. In addition, idiosyncratic productivity realisations and individual asset holdings are private information.

2.3.2 Entrepreneurs

Preferences

Entrepreneurs are ex-ante identical and have logarithmic utility over consumption streams

$$U^E = E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t$$

Flow of Funds

Entrepreneurs purchase consumption (c), working intermediate inputs (x), capital (k) at price q and labour (h) at wage w. All inputs are chosen a period in advance. Entrepreneurs borrow using debt securities b_t at price $1/R_t$.

$$c_t + w_t h_t + x_t + q_t k_t - \frac{b_t}{R_t} = y_t + q_t k_{t-1} - b_{t-1}$$

Because we assume that idiosyncratic shocks and individual asset holdings are private information, securities contingent on the realisation of the idiosyncratic state will not trade in equilibrium.

Collateral constraints

Due to moral hazard in the credit market, agents will only honour their promises if it is in their interests to do so. We assume that only a fraction θ of capital holdings can be seized by creditors. We also assume that entrepreneurs only have the opportunity to default before the aggregate shock has been realised. Hence the collateral constraint limits the entrepreneur's debt to the expected value of collateralisable capital[3]:

$$b_t \leqslant \theta E_t q_{t+1} k_t \tag{2.1}$$

Note that θ here is assumed to be exogenously given by the underlying limited commitment problem in this economy. It therefore cannot be affected by the government. When we come to analyse the government's choice of capital requirements, we will allow it to choose the capital requirement $\widetilde{\theta_t} \leqslant \theta$. This will then place a limit on private leverage over and above the limit imposed by the incentive compatibility constraint (2.1).

2.3.3 Workers

Preferences

Workers have the following preferences:

$$U^W = E_0 \sum_{t=0}^{\infty} \beta^t \ln \left(c_t - \varkappa \frac{h_t^{1+\omega}}{1+\omega} \right)$$

[3]We also consider an alternative collateral constraint which limits borrowing by the realisation of the land price in the worst case scenario. In our case there are only two aggregate productivity states so lenders look at the value of collateral in the low aggregate state.

$$b_{t+1} \leqslant \theta q_{t+1}^L k_{t+1}$$

Such a collateral constraint would obtain if borrowers were allowed to default after the realisation of the aggregate productivity shock. Lenders would then want to insure themselves against losses by only lending up to the value at which entrepreneurs would never default.

We found that using such a form of the collateral constraint did not significantly affect the results we get.

Flow of Funds

Workers do not have the opportunity to produce. They purchase consumption (c) and save using debt securities b_t at price $1/R_t$. Their net worth consists of labour income ($w_t h_t$) and bonds b_{t-1}.

$$c_t + \frac{b_t}{R_t} = w_t h_t + b_{t-1}$$

Collateral constraints

Due to moral hazard in the credit market, workers cannot borrow:

$$b_t \geqslant 0 \tag{2.2}$$

2.4 Competitive Equilibrium

2.4.1 Entrepreneurial behaviour

Entrepreneurs make decisions based on three key margins. First of all they decide how much to consume today and how much to save for future consumption. Secondly, they need to decide how to divide their savings between safe bonds and risky production - the portfolio problem. Thirdly, within the amount they invest in production, they need to decide on the input mix between capital, intermediate inputs and labour - the production problem.

Let $V(z_t, a_t, X_t)$ denote the value of an entrepreneur with wealth z_t, idiosyncratic productivity level a_t (determined and revealed to the entrepreneur at time $t-1$) when the aggregate state is $X_t \equiv [A_t, Z_t, d_t]$. For now we simply assume that the aggregate state consists of the aggregate technology realisation A_t, total wealth in the economy Z_t as well as the share of wealth held by high productivity entrepreneurs d_t. We will prove subsequently that this is the case.

The value function is defined recursively as follows:

$$V(z_t, a_t, X_t) = \max_{x_t, k_t, b_t, h_t, c_t} \left\{ \ln c_t + \beta E_t V(z_{t+1}, a_{t+1}, X_{t+1}) \right\} \tag{2.3}$$

where the maximisation is performed subject to the current resource constraint,

$$c_t + w_t h_t + x_t + q_t k_t - \frac{b_t}{R_t} \leqslant z_t$$

the transition law for individual wealth,

$$z_{t+1} = a_{t+1} A_{t+1} \left(\frac{k_t}{\alpha}\right)^{\alpha} \left(\frac{x_t}{\eta}\right)^{\eta} \left(\frac{h_t}{1-\alpha-\eta}\right)^{1-\alpha-\eta} + q_{t+1} k_t - b_t$$

the collateral constraint

$$b_t \leqslant \theta E_t q_{t+1} k_t$$

the Markov process for the idiosyncratic productivity shock and the transition law for the aggregate state. The aggregate technology shock evolves according to a Markov process. The share of wealth held by high productivity entrepreneurs is an endogenous variable and we will describe its evolution as part of our characterisation of the competitive equilibrium of our model economy.

Optimal consumption

In Appendix 4.B.1 we prove that the log utility assumption ensures that consumption is always a fixed fraction of wealth that depends upon the discount factor.

$$c_t = (1 - \beta)\, z_t$$

Optimal production

When borrowing constraints bind, high and low productivity entrepreneurs will make different production decisions. This is why we examine the optimal production decisions of the two groups separately.

High productivity entrepreneurs In equilibrium, the high productivity entrepreneurs will turn out to be the borrowers in this economy. Optimal production implies that the input mix between capital, labour and intermediate inputs is given by the following expressions:

$$x_t = \eta u_t^H k_t / \alpha \tag{2.4}$$

and

$$h_t = \frac{1 - \alpha - \eta}{\alpha} \frac{u_t^H}{w_t} k_t \tag{2.5}$$

where u_t^H is the user cost of capital faced by high productivity entrepreneurs.

When the borrowing constraint is binding, this means that the entrepreneur derives additional value from purchasing capital because this relaxes the collateral constraint. This value (in terms of goods) can be easily derived from the first order condition with respect to borrowing:

$$\begin{aligned} \frac{\mu_t}{\lambda_t} &= \frac{1}{R_t} - \beta E_t \left(\frac{c_t}{c_{t+1}} \right) \\ &= \frac{1}{R_t} - E_t \left(\frac{1}{R_{t+1}^H} \right) \end{aligned}$$

where R_{t+1}^H is the rate of return on wealth for high productivity entrepreneurs (to be pinned down later in the paper) and μ_t and λ_t are the Lagrange multipliers on the borrowing and resource constraints. The value of relaxing the borrowing constraint by a unit is equal to the difference between the market price of future consumption (the price of debt) and the private valuation of future consumption. Credit constrained borrowers are those who value future consumption less than the market because their wealth and consumption are growing fast. They would like to borrow unlimited amounts at prevailing market prices but are prevented from doing so by binding collateral constraints.

In general the user cost expression is given by:

$$u_t^H = q_t - E_t \left(\frac{q_{t+1}}{R_{t+1}^H} \right) - \theta E_t q_{t+1} \frac{\mu_t}{\lambda_t}$$

When credit constraints bind, the user cost expression is give by:

$$u_t^H = q_t - E_t \left(\frac{q_{t+1}}{R_{t+1}^H} \right) - \theta E_t q_{t+1} \left(\frac{1}{R_t} - E_t \left(\frac{1}{R_{t+1}^H} \right) \right)$$

while when they do not bind, the shadow price on the borrowing constraint $\mu_t = 0$ and the user cost is given by:

$$u_t^H = q_t - E_t\left(\frac{q_{t+1}}{R_{t+1}^H}\right)$$

Low productivity entrepreneurs In equilibrium, low productivity entrepreneurs are always unconstrained savers. When borrowing constraints bind sufficiently tightly, they also end up producing using their inefficient technology. Suppose that we are in such an environment where efficient and inefficient technologies are both used due to the borrowing constraint. Then the first order condition for optimal capital input by the low productivity producers is as follows:

$$u_t^L = q_t - E_t\left(\frac{q_{t+1}}{R_{t+1}^L}\right)$$

where $R_{t+1}^L \equiv \frac{z_{t+1}}{\beta z_t}$ is the rate of return on wealth for a low productivity entrepreneur (to be specified later on in the paper). This is a standard user cost expression. Because our economy has two aggregate states and two assets (debt and productive projects), markets for aggregate risk are complete and $\pi(s)/R_{t+1}^L(s)$ is the price of an Arrow security that pays a unit of consumption if state s is realised in the next period. The $E_t\left(\frac{q_{t+1}}{R_{t+1}^L}\right)$ term is the present value of the capital unit tomorrow evaluated at Arrow security prices.

Conditional upon the user cost of capital, low productivity entrepreneurs have the same input mix as high productivity types. However, high productivity entrepreneurs will use less capital intensive production strategies because they face a higher cost of capital compared to low productivity ones. We will return to the link between downpayment requirements and the user cost of capital later because it is key to the policy conclusions of the paper.

The portfolio problem

In the previous two subsections we characterised the solution of two of the consumer's three decision margins: the consumption function and the optimal input mix into production. Now what remains is to solve for the optimal mix between productive projects and loans to other entrepreneurs. For the high productivity entrepreneurs who are the borrowers in our economy this problem boils down to choosing optimal leverage. For the low productivity savers, it will be a choice of whether to produce or lend at the margin.

High productivity entrepreneurs In equilibrium, high productivity entrepreneurs have investment opportunities in excess of the rates of return available on market securities (in this model, simple debt). Consequently they will want to leverage up in order to take advantage of this (temporary) investment opportunity. Let $l_t \equiv b_t / E_t q_{t+1} k_t$ denote the fraction of the entrepreneur's capital purchase which is financed by debt. This fraction is bounded from above by the collateral constraint, which states that, in the laissez faire economy, at most θ fraction can be borrowed. In the regulated economy l_t will be bounded by the capital requirement chosen by the government, $\widetilde{\theta}_t$.

In Appendix 4.B.2 we show that a high productivity entrepreneur who borrows a fraction $l_t \leqslant \theta$ to fund his capital purchases will earn the following rate of return:

$$R^H_{t+1} = \frac{\left(A_{t+1} a^H / \alpha\right) w_t^{\alpha+\eta-1} \left(u_t^H\right)^{1-\alpha} + q_{t+1} - l_t E_t q_{t+1}}{q_t + (1-\alpha) u_t^H / \alpha - \left(l_t / R_t\right) E_t q_{t+1}} \tag{2.6}$$

The numerator of the above expression denotes project revenues consisting of output per unit of capital ($\left(A_{t+1} a^H / \alpha\right) w_t^{\alpha+\eta-1} \left(u_t^H\right)^{1-\alpha}$) and the value of capital (q_{t+1}) net of debt repayments $l_t E_t q_{t+1}$. The denominator denotes the total cost of undertaking the project. It consists of the total cost of capital (q_t) and other inputs ($(1-\alpha) u_t^H / \alpha$) minus the amount of financing the entrepreneur chose to undertake via debt markets $\left(l_t / R_t\right) E_t q_{t+1}$. So in other words, R^H_{t+1} is the leveraged rate of return on production.

In Appendix 4.B.3 we show that the entrepreneur chooses l_t in order to maximise the expected log rate of return on wealth.

$$\ln R^{H*} = \max_{l_t} E_t \ln \left[\frac{\left(A_{t+1} a^H / \alpha\right) w_t^{\alpha+\eta-1} \left(u_t^H\right)^{1-\alpha} + q_{t+1} - l_t E_t q_{t+1}}{q_t + (1-\alpha) u_t^H / \alpha - \left(l_t / R_t\right) E_t q_{t+1}} \right] \tag{2.7}$$

subject to the constraint:

$$l_t \leqslant \widetilde{\theta}_t \tag{2.8}$$

To get a more intuitive understanding of the leverage decision, we can think of the entrepreneur's leverage decision as a standard portfolio problem in which the entrepreneur chooses how much of his savings to put into a risky and a safe asset. We define the return on the risky asset as the return on a productive project together with the returns from the capital holding that goes with it:

$$R^k_{t+1} = \frac{\left(A_{t+1}a^H/\alpha\right) w_t^{\alpha+\eta-1}\left(u_t^H\right)^{1-\alpha} + q_{t+1}}{q_t + (1-\alpha)\, u_t^H/\alpha}$$

Then we can write the rate of return on the entrepreneur's total portfolio as the weighted average between the risky and the safe rate of return:

$$R^H_{t+1} = \varpi_t^H R^k_{t+1} + \left(1 - \varpi_t^H\right) R_t$$

where

$$\varpi_t^H \equiv \frac{q_t + (1-\alpha)\, u_t^H/\alpha}{q_t + (1-\alpha)\, u_t^H/\alpha - (l_t/R_t)\, E_t q_{t+1}} > 1 \tag{2.9}$$

is the share of the risky asset in the high productivity entrepreneur's portfolio. Entrepreneurs are free to choose a value of l_t below θ if they are unconstrained. However, the maximum share of the risky asset is determined by the borrowing constraint and is given by:[4]

$$\varpi_{\max}^H \equiv \frac{q_t + (1-\alpha)\, u_t^H/\alpha}{q_t + (1-\alpha)\, u_t^H/\alpha - (\theta/R_t)\, E_t q_{t+1}} > 1 \tag{2.10}$$

[4] The larger l_t the higher the share of risky assets in the entrepreneur's portfolio. As (2.9) shows, when $l_t > 0$, the share of the risky asset ϖ_t^H is greater than unity. But even when the entrepreneur borrows the full value of her capital purchases, this does not mean that she is unconstrained in her borrowing. As long as the expected return on the risky asset R^k_{t+1} is sufficiently greater than the interest rate on safe debt R_t to compensate for risk, the entrepreneur will remain credit constrained and would like to borrow against the value of her future output as well.

Reducing the value of l_t below the market determined θ is tantamount to the entrepreneur choosing to reduce his holdings of the risky asset. As the entrepreneur borrows less and less, l_t falls and with it ϖ_t^H falls too. If the entrepreneur decides to become a net saver, l_t falls below zero. In the limit, as l_t becomes large and negative, ϖ_t^H tends to zero and the portfolio of the entrepreneur consists of only the safe asset.

In Appendix 4.B.5 we show that we can take a second order approximation to the portfolio problem as follows:

$$\ln R^{H*} \approx \max_{\varpi_t^H} \left[\ln R_t + \varpi_t^H \left(E_t \rho_{t+1}^H - 1 \right) - \frac{\left(\varpi_t^H\right)^2}{2} \sigma_{Rt+1}^2 \right]$$

where the expected excess return on production for high productivity agents is defined as follows:

$$E_t \rho_{t+1}^H = \frac{E_t R_{t+1}^k}{R_t} = E_t \left(\frac{\left(A_{t+1} a^H / \alpha\right) w_t^{\alpha+\eta-1} \left(u_t^H\right)^{1-\alpha} + q_{t+1}}{q_t + (1-\alpha) u_t^H / \alpha} \right) / R_t \tag{2.11}$$

The conditional variance of the log rate of return of the risky asset σ_{Rt+1}^2 is dominated by the variance of the capital price as well as the covariance of the capital price with the technology shock (for more details see Appendix 4.B.5). Both of these terms increase strongly as the collateral amplification mechanism becomes stronger. The first order condition is:

$$\frac{\partial \ln R^{H*}}{\partial \varpi_t^H} \approx E_t \rho_{t+1}^H - 1 - \varpi_t^H \sigma_{Rt+1}^2 \geqslant 0 \tag{2.12}$$

It holds with equality if the collateral constraint does not bind. Re-arranging we get:

$$\varpi_t^H \approx \frac{S_{t+1}^H}{\sigma_{Rt+1}}$$

where $S_{t+1}^H \equiv \frac{E_t \rho_{t+1}^H - 1}{\sigma_{Rt+1}}$ is the conditional Sharpe ratio on the risky asset for the high productivity entrepreneur. σ_{Rt+1} is determined by the volatility of the technology shock σ_A^2 as well as the volatility of the capital price σ_{qt+1}^2. The higher these are, the smaller the share of the risky asset chosen by the entrepreneur. Equally a higher premium $E_t \rho_{t+1}^H - 1$ leads to a larger share invested in the risky asset.

This means that, in general, the share of the risky asset in the high productivity entrepreneur's portfolio is given by:

$$\varpi_t^H = \min \left[\frac{E_t \rho_{t+1}^H - 1}{\sigma_{Rt+1}^2}, \frac{q_t + (1-\alpha) u_t^H / \alpha}{q_t + (1-\alpha) u_t^H / \alpha - (\theta / R_t) E_t q_{t+1}} \right]$$

where $\frac{q_t+(1-\alpha)u_t^H/\alpha}{q_t+(1-\alpha)u_t^H/\alpha-(\theta/R_t)E_tq_{t+1}}$ is the share of the risky asset when the constraint is binding.

Low productivity entrepreneurs Low productivity entrepreneurs may or may not produce in equilibrium, depending on the tightness of the collateral constraint. When the constraint binds very tightly, high productivity firms will be constrained in their ability to purchase the productive assets in the economy and some of them will have to be bought by low productivity firms. Consistent with the large variance of plant level productivity, we focus on a level of θ such that low productivity firms do end up producing in equilibrium, financing themselves using their own net worth. In Appendix 4.B.4 we show that the rate of return on their net worth is given by:

$$R_{t+1}^L = \frac{\left[(A_{t+1}/\alpha)\, w_t^{\alpha+\eta-1} \left(u_t^L\right)^{1-\alpha} + q_{t+1}\right] k_t + b_t}{\left[q_t + (1-\alpha)\, u_t^L/\alpha\right] k_t + b_t/R_t}$$

where the numerator consists of the revenues from production as well as debt repayments received from other entrepreneurs, while the denominator is the cost of purchasing the portfolio. Unlike, high productivity entrepreneurs who leverage up in order to invest in production, low productivity entrepreneurs have more balanced portfolios, consisting of loans to other entrepreneurs as well as own productive projects.

The portfolios of high and low productivity entrepreneurs are linked by the market clearing conditions in the capital and debt markets. This means that once we have solved for the optimal portfolio of the high productivity entrepreneurs, this also gives us the investment choices of low productivity ones. In Appendix 4.B.4 we show that the equilibrium rate of return on wealth for the low types is given below:

$$R_{t+1}^L = \varpi_t^L \left[\frac{(A_{t+1}/\alpha)\, w_t^{\alpha+\eta-1} \left(u_t^L\right)^{1-\alpha} + q_{t+1}}{q_t + (1-\alpha)\, u_t^L/\alpha}\right] + \left(1-\varpi_t^L\right) R_t$$

where

$$\varpi_t^L \equiv \frac{\left[q_t + (1-\alpha)\, u_t^L/\alpha\right](1-K_t)}{\left[q_t + (1-\alpha)\, u_t^L/\alpha\right](1-K_t) + l_t E_t q_{t+1}/R_t} < 1$$

is the share of the risky asset in the low productivity entrepreneur's portfolio. Note that this is always less than one because this entrepreneur invests part of his savings into risk free loans to other entrepreneurs. The risky asset available to the low productivity entrepreneur earns a lower rate of return compared to the one held by high productivity ones. The excess return for the 'low' type is given by:

$$E_t\rho^L_{t+1} = E_t\left(\frac{(A_{t+1}/\alpha)\, w_t^{\alpha+\eta-1}\left(u_t^L\right)^{1-\alpha} + q_{t+1}}{q_t + (1-\alpha)\, u_t^L/\alpha}\right)/R_t \tag{2.13}$$

The conditions for the optimal portfolio composition of the low productivity type are similar to those in the previous subsection:

$$\varpi_t^L \approx \frac{S^L_{t+1}}{\sigma_{rt+1}}$$

where $S^L_{t+1} \equiv \frac{E_t\rho^L_{t+1}-1}{\sigma_{rt+1}}$ is the conditional Sharpe ratio on the risky asset for the low productivity entrepreneur and σ_{rt+1} is the standard deviation of the log return on the risky asset. Analogously with σ_{Rt+1}, σ_{rt+1} is determined by the volatility of the technology shock σ_A^2 as well as the volatility of the capital price σ^2_{qt+1}.

2.4.2 Behaviour of Workers

Let $V^W(b_{t-1}, X_t)$ denote the value function of a worker with individual financial wealth b_t when the aggregate state is X_t. The value function is given by:

$$V^W(b_{t-1}, X_t) = \max_{c_t, h_t, b_{t+1}}\left\{\ln\left(c_t - \varkappa\frac{h_t^{1+\omega}}{1+\omega}\right) + \beta E_t V^W(b_t, X_{t+1})\right\}$$

subject to the flow of funds constraint and the borrowing constraint. The first order conditions are given by:

$$w_t = \varkappa h_t^{\omega} \tag{2.14}$$

$$\frac{1}{c_t - \varkappa\frac{h_t^{1+\omega}}{1+\omega}} = \beta R_t E_t\left(\frac{1}{c_{t+1} - \varkappa\frac{h_{t+1}^{1+\omega}}{1+\omega}}\right)$$

In equilibrium, workers will not save as long as the volatility of the aggregate wage is not too great. This is because the risk free interest rate is below the workers' rate of time preference. This means that workers will consume their entire wage income in equilibrium and their welfare will be dominated by the stochastic process for the aggregate wage rate[5].

The result that workers consume their entire labour income allows us to drop the financial wealth state variable and simplify their value function considerably. Using the optimal labour supply condition (2.14) we get to the following simple expression:

$$V^W(X_t) = \Theta + \frac{\omega}{1+\omega}\ln w_t + \beta E_t V^W(X_{t+1})$$

where Θ is a constant that depends on parameter values.

2.4.3 Aggregation and Market Clearing

We complete the characterisation of the competitive equilibrium of our model economy by specifying the evolution equations for the endogenous state variables well as the market clearing conditions.

There are three market clearing conditions. The bond,

$$\int b_{t+1}(i)\,di = 0 \tag{2.15}$$

capital

$$\int k_{t+1}(i)\,di = 1 \tag{2.16}$$

and goods markets

$$C_t^H + C_t^L + C_t^W + X_{t+1}^H + X_{t+1}^L = Y_t^H + Y_t^L \tag{2.17}$$

all clear.

Finally the economy's endogenous state variables evolve according to the follow-

[5] In solving the model we verify at each point in time that the condition for no saving holds

$$\frac{1}{c_t - \varkappa\frac{h_t^{1+\omega}}{1+\omega}} > \beta R_t E_t\left(\frac{1}{c_{t+1} - \varkappa\frac{h_{t+1}^{1+\omega}}{1+\omega}}\right)$$

ing transition law.

$$\begin{aligned} Z_{t+1} &= R^H_{t+1}\beta Z^H_t + R^L_{t+1}\beta Z^L_t \\ &= \left[d_t R^H_{t+1} + (1-d_t) R^L_{t+1}\right]\beta Z_t \end{aligned} \tag{2.18}$$

$$\begin{aligned} d_{t+1} &= \frac{Z^H_{t+1}}{Z_{t+1}} \\ &= \frac{(1-\delta) d_t R^H_{t+1} + n\delta (1-d_t) R^L_{t+1}}{d_t R^H_{t+1} + (1-d_t) R^L_{t+1}} \end{aligned} \tag{2.19}$$

2.4.4 Equilibrium Definition

Recursive competitive equilibrium of our model economy is a price system w_t, u^H_t, u^L_t, q_t, R_t, value functions V^E_t and V^W_t, entrepreneur decision rules k_t, x_t, b^e_t, h^e_t and c^e_t, worker decision rules b^w_{t+1}, h^w_{t+1} and c^w_t, and equilibrium laws of motion for the endogenous state variables (2.18) and (2.19) such that

(i) The value function V^E_t and the decision rules k_t, x_t, h^e_t, b^e_t and c^e_t solve the entrepreneur's decision problem conditional upon the price system w_t, u^H_t, u^L_t, q_t, R_t, the value function V^W_t and the decision rules b^w_t, h^w_t and c^w_t solve the worker's decision problem conditional upon the price system w_t, u^H_t, u^L_t, q_t, R_t.

(ii) The process governing the transition of the aggregate productivity and the household decision rules k_t, x_t, b^e_t, h^e_t, c^e_t, b^w_t, h^w_t and c^w_t induce a transition process for the aggregate state given by (2.18) and (2.19).

(iii) All markets clear

2.5 The Economic Impact of Capital Requirements

Capital requirements are the main policy instrument for the government in our framework. In this section we examine using numerical solutions of our model economy what their effect is on economic outcomes. We focus on the ways in which tighter borrowing limits affects the different distortions in the credit constrained economy in order to see how the government trades them off against one another.

Section 2.6 will derive the optimal capital requirement.

2.5.1 Baseline Calibration

In this section we outline the basic features of the baseline calibration. More details can be found in Appendix 4.B.7.

We calibrate η, the share of intermediate inputs in gross output to 0.45 using data from the 2007 BEA Industrial Accounts. Using the Cooley and Prescott (1995) methodology we calibrate α (the share of capital in gross output) to 0.2 which gives a share of 0.36 in value added. We set θ (the share of capital which can be collateralised for loans) to 1.0 in line with the value used in Kiyotaki (1998) and Aoki et al (2009). However, since there is very little information on the collateralisability of capital goods we conduct extensive sensitivity analysis due to the highly uncertain value of this parameter.

The technology process at the firm level consists of an aggregate and an idiosyncratic component. Because TFP is endogenous in the Kiyotaki-Moore framework we pick the process for the aggregate exogenous technology shock to match the standard deviation of HP-filtered real GDP. The high (low) realisations of the aggregate TFP shock are 0.6% above (below) the steady state TFP level. The probability that the economy remains in the same aggregate state it is today is equal to 0.8.

Calibrating the cross-sectional dispersion of TFP is important because the quantitative importance of the pecuniary externality studied in our paper is related to the productivity gap between high and low productivity firms. Bernard et al. (2003) report an enormous cross-sectional variance of plant level value added per worker using data from the 1992 US Census of Manufactures. The standard deviation of the log of value added per worker is 0.75 in the data while their model is able to account for only around half this number. The authors argue that imperfect competition and data measurement issues can account for much of this discrepancy between model and data. In addition, the study assumes fixed labour share across plants so any departures from this assumption would lead to more variations in the measured dispersion of labour productivity.

In a comprehensive review article on the literature on cross-sectional productivity differences, Syverson (2009) documents that the top decile of firms has a level of TFP which is almost twice as high as the bottom decile. He finds that unobserved inputs such as the human capital of the labour force, the quality of management and plant level 'learning by doing' can account for much of the observed cross-sectional variation in TFP.

This model does not have intangible assets of the sort discussed in Syverson (2009) and consequently calibrating the model using the enormous productivity differentials identified in the productivity literature would overestimate the true degree of TFP differences. In addition, the Kiyotaki-Moore model would need very tight borrowing constraints or a very small number of high productivity entrepreneurs in order for credit constraints to be binding if some firms are so much more productive than others. And within the framework we have, binding credit constraints are the only mechanism for generating cross-sectional differences in productivity. Aoki et al. (2009) also consider these issues in their calibration of a small open economy version of Kiyotaki and Moore (1997). They argue that a ratio of the productivities of the two groups of 1.15 is broadly consistent with the empirical evidence and I choose this number for the baseline case. However I conduct extensive sensitivity analysis on this hard to pin down parameter because there is very little strong evidence for how to calibrate the productivity dispersion across firms.

Moving on to the parameters governing labour supply we set ω^{-1} (the Frisch elasticity of labour supply) to 3. This is higher than micro-data estimates (references) but is consistent with choices made in the macro literature. We then pick $\varkappa$, a parameter governing the disutility of labour to get a value of labour supply as a fraction of workers' time endowment which is equal to 0.33.

The discount factor β, the probability that a highly productive entrepreneur switches to low productivity δ, and the ratio of high to low productivity entrepreneurs n are parameters I pick in order to match three calibration targets - the ratio of tangible assets to GDP, aggregate leverage and the leverage of the most indebted decile of firms.

I use data on tangible assets and GDP from the BEA National Accounts in the 1952-2008 period. The concept of tangible assets includes Business and Household Equipment and Software, Inventories, Business and Household Structures and Consumer Durables. GDP excludes government value added so it is a private sector output measure.

Aggregate leverage is defined as the average ratio of the value of the debt liabilities of the non-financial corporate sector to the total value of assets. Leverage measures can be obtained from a number of sources. In the US Flow of Funds, aggregate leverage is approximately equal to 0.5 for the 1948-2008 period. This is broadly consistent with the findings of den Haan and Covas (2007) who calculate an average leverage ratio of 0.587 in Compustat data from 1971 to 2004. Den Haan and Covas (2007a) also examine the leverage of large firms and find that it is slightly higher than the average in the Compustat data set. Firms in the top 5% in terms of size have leverage of around 0.6. Den Haan and Covas (2007b) have similar findings in a panel of Canadian firms. There the top 5% of firms have leverage of 0.7-0.75 compared to an average of 0.66 for the whole sample. High productivity entrepreneurs in our economy run larger firms so differences in productivity and therefore leverage could be one reason for the findings of Den Haan and Covas (2007a and 2007b). But the perfect correlation of firm size and leverage that holds in our model will not hold in the data. So if we are interested in the distribution of firm leverage, the numbers in Den Haan and Covas will be an underestimate. This is why we pick a target for the average leverage of the top 10% most indebted firms to be equal to 0.75. This number is broadly consistent with the findings in Den Haan and Covas.

Table 2.1 below summarises the calibration targets we match while Table 2.2 summarises the baseline parameter values used in the paper.

Target	Value	Source
Tangible Assets to GDP $= q/\left(Y^H + Y^L - X^H - X^L\right)$	3.49	BEA National Accounts
Aggregate Leverage $=L^A = B/\left(q + Y^H + Y^L\right)$	0.50	Flow of Funds
Leverage of indebted firms $=L^H = B/\left(qK + Y^H\right)$	0.75	Den Haan-Covas (2007a)
Share of intermediate inputs in gross output $= \eta$	0.45	BEA National Accounts
Share of capital in GDP $= \alpha/(1-\eta)$	0.36	BEA National Accounts
Cross sectional productivity dispersion $= a^H/a^L$	1.15	Aoki et al. (2009)
Collateralisability of capital $= \theta$	1.00	Aoki et al. (2009)
Standard deviation of annual real GDP	2.01	BEA National Accounts

Table 2.1: Calibration targets

Parameter Name	Parameter Value
β	0.896
δ	0.145
n	0.084
α	0.20
η	0.45
ω	0.33
$\varkappa$	2.29
p_{gg}	0.80
p_{bb}	0.80
A^h	1.006
A^l	0.994
a^H/a^L	1.15
θ	1.00

Table 2.2: Summary of baseline model calibration

2.5.2 Model evaluation

Having chosen parameter values to match the first moments of the model to those in the data and to match the volatility of real GDP, in this section we evaluate the model by analysing how key moments of the model compare to those in the data. All variables have been detrended using the HP filter (for more details see Appendix 4.B.7) Table 2.3 below compares the second moments of the model relative to the data[6]. The numbers we focus on is the standard deviation of annual aggregate non-durable consumption, aggregate labour hours and the stock market

	Data	Model
σ_c	1.55	2.01
σ_h	1.32	1.25
σ_v	6.06	2.55

Note: σ_c is the standard deviation of the logarithm of aggregate consumption, σ_h is the standard deviation of the logarithm of aggregate labour hours, σ_v is the standard deviation of the logarithm of stock prices

Table 2.3: Model second moments

The standard deviation of aggregate labour hours in the model are broadly in line with those in the data. The model does less well in the other two key dimensions we use in our evaluation. Aggregate consumption is too volatile relative to the data. This is a feature of the model that can be improved upon in future work by adding a better means of aggregate saving. Capital is fixed and the only means of aggregate saving for agents in the model is to purchase intermediate inputs. In addition, due to the low risk free interest rate, workers do not save and their consumption is as volatile as labour income. In future work I intend to extend the model by adding capital which does not depreciate fully and which can, therefore, be accumulated in the aggregate, allowing households to smooth consumption better. The volatility

[6] More details on how the data moments were computed are in Appendix 4.B.7.

of the real value of the S&P 500 in the data is also considerably higher than the volatility of asset prices in the model.

2.5.3 Borrowing Constraints and Steady State Productive Efficiency

In this subsection we consider what would happen in the steady state (i.e. in the absence of aggregate shocks) if the government chooses to impose tighter capital requirements (a lower value of $\widetilde{\theta}$). Perhaps the biggest welfare cost of tighter borrowing constraints arises because borrowing constraints reduce the efficiency of the economy. This happens for two reasons. Firstly, the downpayment requirements on capital acts as a tax on the capital purchases of high productivity entrepreneurs and distorts their production mix relative to the first best. Secondly, borrowing constraints increase the share of low productivity firms in economic activity, reducing aggregate TFP. Below we explain both of these sources of inefficiency.

Capital requirements and the 'downpayment tax' on high productivity entrepreneurs

In Appendix 4.B.8 we show that we can write the steady state user cost of capital for high productivity entrepreneurs in the tax wedge form popularised by Chari, Kehoe and McGrattan (2007):

$$\begin{aligned} u_t^H &= q_t - \left[\frac{\widetilde{\theta}_t}{R_t} + \frac{1-\widetilde{\theta}_t}{R_{t+1}^H}\right] q_{t+1} \\ &= u_t^L \left(1 + \tau_t\left(\widetilde{\theta}\right)\right) \end{aligned}$$

where the tax is given by the following expression

$$\tau_t\left(\widetilde{\theta}_t\right) = \left(1 - \widetilde{\theta}_t\right)\left(\frac{q_t}{u_t^L} - 1\right)\left(1 - \frac{R_t}{R_{t+1}^H}\right) \tag{2.20}$$

The collateral requirement acts like a tax on the capital purchases of constrained producers. The size of the tax is determined by the following factors. First of all, the tax is increasing in the required downpayment on capital goods $1 - \widetilde{\theta}_t$. This fraction determines how much of the capital purchase needs to be financed by expensive own savings as opposed to cheap external funds. The difference between the valuation of internal funds and the market price of loans is given by the $1 - \frac{R_t}{R^H_{t+1}}$ term in (2.20). It arises when the borrowing constraint leads to a deterioration in consumption smoothing. High productivity entrepreneurs experience faster consumption growth making them less willing to save. And because the collateral constraint forces them to save, this acts to increase their user cost relative to unconstrained low productivity agents. Secondly, the tax is increasing in the price to rent ratio of capital. This is because a high price to rent ratio increases the internal funds required by a constrained borrower (who needs to have a fraction of the cost of capital as downpayment) relative to an unconstrained borrower (who effectively faces only the user cost). The first row in Table 2.4 below shows how the 'downpayment tax' varies with the value of downpayment requirement. As $\widetilde{\theta}_t$ - the collateralisability of capital - declines from 1 and 0.8, the 'tax' increases from 0 to more than 20%.

Interestingly the impact of capital requirements on the real wage and nonmonotonic. At high levels of $\widetilde{\theta}$, the wage is increasing in the downpayment of capital goods, while at low levels of $\widetilde{\theta}$, it is decreasing. Capital requirements have two opposing effects on the real wage. Lower $\widetilde{\theta}_t$ allows high productivity entrepreneurs to expand production which boosts TFP and increases wages. But there is another effect. Lower $\widetilde{\theta}_t$ increases the user cost of capital and skews the input mix by high productivity entrepreneurs towards intermediate inputs and labour. The higher labour demand increases the wage. At high levels of $\widetilde{\theta}_t$, the share of production done by the efficient producers is high and the input mix effect dominates, increasing the real wage.

Capital Requirements	$\widetilde{\theta}=0.8$	$\widetilde{\theta}=0.9$	$\widetilde{\theta}=1.0$	1st best
τ	0.21	0.12	0.00	0.00
w	1.590	1.592	1.586	1.788
K^H	0.30	0.42	0.69	1.00
TFP	1.050	1.066	1.104	1.15

Notes: τ is the 'downpayment tax' rate, w is the wage rate, K^H is the share of the capital stock held by high productivity entrepreneurs, TFP is aggregate total factor productivity.

Table 2.4: Selected first moments under different capital requirements

Capital requirements and the level of TFP

The aggregate level of TFP in this economy is given by the ratio of aggregate output in the economy to the inputs that are used in production.

$$TFP_t = A_t \frac{a^H (K)^{\alpha} (X^H)^{\eta} (H^H)^{1-\alpha-\eta} + (1-K)^{\alpha} (X^L)^{\eta} (H^L)^{1-\alpha-\eta}}{(X^H + X^L)^{\eta} (H^H + H^L)^{1-\alpha-\eta}}$$

In Appendix 4.B.9 we show that aggregate TFP in the economy is given by the following expression:

$$TFP_t = \frac{1 + K_t \left[a^H (1+\tau(\theta))^{1-\alpha} - 1\right]}{1 + \tau(\theta) K_t}$$

The downpayment tax and the existence of inefficient production under binding borrowing constraints endogenously reduces the economy's level of TFP. This can be seen in the last row of Table 3.4 above. As $\widetilde{\theta}$ declines from unity to 0.8, the share of capital held by high productivity entrepreneurs declines from 0.69 to 0.30, bringing about a decline in aggregate TFP of more than 5%. This is a crucial feature of the Kiyotaki (1998) framework. When borrowing constraints bind tightly, not enough funds get into the hands of the high productivity firms. As a result, the economy operates within the production possibility frontier because some of the scarce capital input is held by low productivity firms.

2.5.4 Borrowing Constraints and Aggregate Volatility in the Stochastic Economy

In this subsection we consider how the imposition of capital requirements affect the equilibrium of the economy with aggregate uncertainty. Here we focus on the ways in which capital requirements affect the volatility of aggregate consumption as well as the consumption of different groups and link it to the endogenous fluctuations in TFP which arise due to the amplification mechanism.

Leverage leads to a reallocation of capital between high and low productivity entrepreneurs over the business cycle. This happens through the standard collateral amplification mechanism of Kiyotaki and Moore (1997), which can cause substantial endogenous fluctuations in TFP amplifying the normal shocks to technology over the business cycle. The mechanism which generates this amplification is the following. When the aggregate productivity state A_t changes (say, it falls), this reduces the capital price in both the borrowing constrained and in the 'first best' economy. But whereas in the 'first best' world, there is very little additional propagation, in the credit constrained (leverage financed) economy, the fall in asset prices impacts the wealth of high productivity and low productivity agents differently. Because they are leveraged, high productivity entrepreneurs are badly affected and have to scale down their capital investments because they can no longer afford the required downpayment as well as the cost of the capital input needed to operate productive projects with a large capital input. The purchasers of capital are the low productivity entrepreneurs and consequently the economy's aggregate TFP declines as inefficient production expands. The additional fall in TFP puts further downward pressure on capital prices and on the wealth and borrowing capacity of high productivity entrepreneurs. This is the amplification channel of Kiyotaki and Moore (1997): small declines in the economy's aggregate technology can set off a self-reinforcing spiral of falling TFP and asset prices, magnifying the effect of the original technology shock. The amplification mechanism is very important because its quantitative strength will be a crucial determinant of whether capital requirements can be welfare improving or not.

Capital Requirements	$\widetilde{\theta} = 0.80$	$\widetilde{\theta} = 0.90$	$\widetilde{\theta} = 1.00$	1st best
σ_y	1.48	1.57	2.01	1.39
σ_q	1.52	1.68	2.55	1.39
σ_c	1.48	1.57	2.01	1.39
σ_w	0.57	0.59	0.65	0.34
σ_{TFP}	0.53	0.66	0.85	0.61

Note: σ_y is the standard deviation of the log of output, σ_q is the standard deviation of the log of the capital price, σ_c is the standard deviation of the log of aggregate consumption, σ_{TFP} is the standard deviation of the log of aggregate total factor productivity, σ_w is the standard deviation of the log of the real wage rate.

Table 2.5: Selected second moments under different capital requirements

Cordoba and Ripoll (2004) have argued that the amount of amplification in the Kiyotaki and Moore (1997) framework is very small when one assumes concave utility and decreasing returns to scale in production. They show that large amplification needs a large productivity gap, a large share of constrained agents in production and substantial reallocation of collateral in response to shocks. Cordoba and Ripoll (2004) find that, in particular, there is a trade off between having a large productivity gap and having a lot of production in the hands of constrained entrepreneurs. This is because they assume decreasing returns to scale at the plant level. When constrained firms are very small and their output is low they are much more productive than the larger unconstrained firms. But the downside is that their share in total output is low. At the other extreme, when constrained firms are large, their productivity advantage relative to unconstrained ones is small. In both cases, at least one condition for large amplification is not satisfied and so the additional volatility from the model is negligible.

As Table 2.5 shows, the amplification we obtain from out calibrated version of the Kiyotaki (1998) model is very substantial. In the baseline case, the standard deviations of TFP and output are, respectively, 38% and 45% higher compared to the first best while the standard deviation of the capital price is 84% higher. So contrary to the results in Cordoba and Ripoll (2004) we get quantitatively large amplification from the framework. Our differences from Cordoba and Ripoll (2004) arise from one main source - our assumption of constant returns to scale to all factors at the plant level. Even though we have decreasing returns to the collateral factor (capital), the production function is constant returns in all the three factors. This means that in our calibration we do not face the trade off between the size of the productivity gap and the share of constrained producers in economic activity. The productivity gap is largely driven by the value of a^H as well as the 'downpayment tax' $\tau(\theta)$. It is independent of the level of output at any individual firm. When we add the effects of leverage (again realistically calibrated to match US data), we get substantial reallocation of collateral between high and low productivity entrepreneurs as asset prices fluctuate. So Cordoba and Ripoll's conditions for amplification are satisfied and this explains why our constrained economy is so much more volatile relative to the 'first best'. Our results are similar to those in Vlieghe (2005) who found something very similar in a version of Kiyotaki (1998) with nominal rigidities. In his model (which also featured constant returns to all factors) amplification was very substantial showing the potential of the framework to propagate shocks.

In addition to the amplification of aggregate fluctuations, leverage concentrates the aggregate risk in the hands of only a small subset of agents in the economy. When capital is largely held by high productivity entrepreneurs who finance their capital holdings using simple debt, risk sharing between the two groups deteriorates. We can see this in Table 2.6 below which shows the variance of the aggregate consumption of the two groups. This difference grows as credit constraints are relaxed due to the increasing collateralisability of capital.

Capital Requirements	$\widetilde{\theta}=0.80$	$\widetilde{\theta}=0.90$	$\widetilde{\theta}=1.00$	FB
σ_{cH}	2.48	3.12	5.57	1.39
σ_{cL}	1.33	1.34	1.52	1.39
σ_{cW}	1.45	1.50	1.65	1.39

Note: σ_{cH} is the unconditional standard deviation of the log of the consumption of high productivity entrepreneurs, σ_{cL} is the unconditional standard deviation of the log of the consumption of low productivity entrepreneurs, σ_{cL} is the unconditional standard deviation of the log of the consumption of workers.

Table 2.6: Consumption volatility for the two groups

This result is not surprising. The low productivity entrepreneurs hold largely riskless debt and small positions in risky capital. In contrast, high productivity entrepreneurs hold leveraged positions in risky capital. This asymmetry in the asset holdings of the two groups leads to a concentration of the aggregate risk in the economy into the hands of very few (high productivity) individuals whose consumption fluctuates very substantially. Our results are in line with the findings of Vissing-Jorgensen and Parker (2009) who find that the aggregate risk is borne by a small fraction of high consumption/high income households. Tightening firms' access to borrowing reduces this asymmetry in the riskiness of different individuals' portfolios and consequently reduces the volatility in their relative consumption levels over the business cycle.[7]

2.5.5 Discussion

In this section we examined the quantitative significance of four ways in which the credit constrained economy is distorted relative to the first best. These distortions, however, do not necessarily imply that the economy is constrained inefficient. As long as the government cannot do anything directly about borrowing constraints, many of these distortions will be an unavoidable consequence of credit market im-

[7]In the limit, when no borrowing is allowed and all production is entirely net worth financed, both types of agents hold identical portfolios (only productive projects) and risk sharing is perfect.

perfections.

For example, any deviations of the economy's steady state from first best would be constrained efficient. The trade off between productive efficiency and consumption smoothing is identical for private individuals and for the government. Private borrowers with good productive opportunities choose to borrow up to the limit and experience a steeply sloped consumption path because the rates of return they can earn on productive projects are much better compared to the cost of debt. The government will make an identical decision because it can redistribute capital holdings between the two groups and compensate the low productivity firms for their lost output while still making the high productivity borrowers better off. The only constraint on this redistribution is the collateral constraint, which binds for the government in the same way as it binds for the laissez faire economy.

In a stochastic environment, the efficiency properties of the competitive equilibrium change. The collateral amplification mechanism of Kiyotaki and Moore (1997) introduces feedback effects between asset prices, the net worth of leveraged borrowers and the tightness of borrowing constraints. When aggregate productivity switches from high to low, asset prices fall and this has a disproportionately negative effect on the net worth of leveraged high productivity borrowers. Because part of the capital purchase and the whole of the intermediate input purchase is non-collateralisable, borrowers need their own net worth in order to produce on a large scale. Therefore the fall in the net worth of high productivity borrowers reduces the amount of capital they can invest in production and forces them to scale down their capital holdings. The low productivity agents absorb the capital sold by the high productivity ones but only at lower prices. But this fall in the price of capital further damages the net worth of leveraged firms and forces them to cut their capital holdings even further. This completes the 'credit cycle', amplifying and propagating small shocks into larger fluctuations in output, TFP and asset prices.

Where does the inefficiency of private leverage come from? As identified in Lorenzoni (2008) and Korinek (2009), when collateral constraints bind, the pecuniary externalities we usually consider harmless from an economic efficiency point of view, begin to interfere with the allocative efficiency of the economy. The forced sales of leveraged borrowers depress asset prices and tighten the credit constraints of all other constrained borrowers, forcing them to sell assets themselves[8].

2.6 The Model Economy under Capital Requirements

In this section we turn to the main question of this research: are private leverage decisions optimal from a social point of view? From the work of Lorenzoni (2008) and Korinek (2009) we know that, qualitatively, the answer is 'no'. Here we examine whether, quantitatively, the inefficiency is large or small.

We assume that capital requirements are chosen by a benevolent government who maximises a social welfare function which weights the values of all agents in the economy. The government is subject to the same collateral and budget constraints facing private agents. So any differences in private and social leverage choices are due to the market price externality discussed above.

2.6.1 The Government's Problem

The government optimises the coefficient on a simple state contingent capital requirement rule

$$\widetilde{\theta}_t = \min\left[\exp\left(\chi_0^i + \chi_1^i \ln d_t + \chi_2^i \ln Z_t\right), \theta\right] \tag{2.21}$$

in order to maximise the following social welfare function

$$\Omega_0 = \max_{\{\chi^i\}} E_0 \left[\sum_i \varsigma_E^i \sum_{t=0}^{\infty} \beta^t \ln c_t^i\right] + \varsigma_W \sum_{t=0}^{\infty} \beta^t \ln\left(C_t^W - \varkappa \frac{(H_t)^{1+\omega}}{1+\omega}\right) \tag{2.22}$$

[8]But although such pecuniary externalities exist they are not always quantitatively significant. For example, Guerrieri (2007) examines the constrained efficiency of a competitive labour market search model with private information and limited commitment. In her model, workers take the value of the outside unemployment option as given while the planner recognises that it is endogenous because the expected value of job matches affects the continuation value of the unemployed. Although Guerrieri (2007) identifies this very interesting source of inefficiency of the competitive equilibrium, she finds that, quantitatively, the externality in question is very small.

where ς_E^i is the Pareto-Negishi weight on entrepreneur i while ς_W is the Pareto-Negishi weight on the workers. We do not consider any other policy instruments.[9] Note that the capital requirements $\widetilde{\theta}_t$ is constrained by the exogenously given limit θ.

$$\widetilde{\theta}_t \leqslant \theta$$

In other words the government has no advantage in enforcing debt repayment over the private sector and therefore it cannot choose looser capital requirements than the market. The policy rule (2.21) allows the capital requirement to undergo mean shifts as the aggregate productivity state changes. Capital requirements also can respond to changes in the other aggregate state variables - total wealth w_t and the share of wealth held by high productivity people d_t. Once the government has chosen capital requirements, the collateral constraint in the regulated economy becomes:

$$b_t \leqslant \widetilde{\theta}_t E_t q_{t+1} k_t \tag{2.23}$$

Private agents then perform exactly the same maximisation problem as in the unregulated economy, but the collateral constraint they now face may be tighter if $\widetilde{\theta}_t < \theta$ in some states of the world.

In Appendix 4.B.3 we showed that the value function of entrepreneurs depend on the net present value of future expected log rates of return on wealth as well as the logarithm of current financial wealth. We assume a particular initial wealth distribution in which all high and all low productivity entrepreneurs have an initial level of wealth equal to the group average in the 'no regulation' steady state. This

[9] We do not solve a social planning problem because the collateral constraints in our economy depend on prices and these do not admit to a simple closed form solution in the same way as in Lorenzoni (2008) and Korinek (2009).

In future work, we intend to solve for the full Ramsay problem. We do not do this here because it complicates the solution of the model. At the same time the policy we consider does capture a lot of intuitive features about the way capital requirement policy may be implemented. It is fully state contingent and it is conducted under commitment because the government chooses the χ^i coefficients at the beginning of time and sticks to them for ever.

Our policy rule is, therefore, similar to the 'Optimal non-inertial plan' popularised by Woodford (2003) because it is conducted under commitment (the central bank opimises its coefficients in a once and for all fashion) but without responding to lagged variables (which is what the optimal Ramsay commitment policy does).

allows us to consider the following social welfare function which weights the utilities of the three groups by the inverse of their marginal utility of consumption evaluated at the initial wealth distribution (more details in Appendix 4.B.11):

$$\Omega_0 = \max_{\{\chi^i\}} E_0 \left[\varsigma^H \left(\varphi^H \left(X_0 | \chi^i \right) + \frac{\ln Z_0^H \left(\chi^i \right)}{1-\beta} \right) + \varsigma^L \left(\varphi^L \left(X_0 | \chi^i \right) + \frac{\ln Z_0^L \left(\chi^i \right)}{1-\beta} \right) + \varsigma^W V^W \left(X_0 | \chi^i \right) \right] \tag{2.24}$$

where Z_0^H and Z_0^L are the initial wealth levels of the high and low productivity entrepreneurs. Of course future switches in the idiosyncratic as well as aggregate state will cause ex post wealth heterogeneity, which the government cares about. the welfare costs of this ex post wealth heterogeneity is fully captured by the $\varphi^H \left(X_0 | \chi^i \right)$ and $\varphi^L \left(X_0 | \chi^i \right)$ terms and the government fully takes these welfare costs into account when setting $\widetilde{\theta}_t$.

2.6.2 When is private leverage excessive?

The benevolent government chooses and commits to a time invariant capital requirement function $\widetilde{\theta}_t$ which maximises social welfare (2.24). The government cares about three things in (2.24). It wants to maximise the Pareto weighted average of the net present expected value of log returns on wealth for the two types of entrepreneurs. These are the the $\varphi_0^H$and φ_0^L terms in the social welfare function. But it also wants to maximise the welfare of workers which depends on the average level and volatility of real wages. Finally, the government cares about the current financial wealth of entrepreneurs too. It knows that any policy announcement will immediately be reflected in the capital price, impacting on the wealth of the two groups and it takes this into account when designing the optimal policy. In the next section we will compute numerically how these determinants of the welfare of the three groups change as we vary capital requirements. Then we will see whether the government can increase welfare relative to the market.

Here however we try to add a little more intuition by considering how the capital requirement choices of the government differ from those of private individuals in more detail. We do this by looking at what choices the government would make if allowed to choose $\widetilde{\theta}_t$ in order to maximise the log expected portfolio return of the two groups of entrepreneurs as well as the log wage rate of workers. We compute the government's first order condition for each group's portfolio problem and evaluating them at private leverage choices l_t^m. This exercise will be useful for two reasons. First of all it identifies any sources of re-distribution between the two groups as capital requirements are tightened. But secondly, it pinpoints where the externalities discussed by Lorenzoni (2008) and Korinek (2009) might occur in our framework.

High productivity entrepreneurs

Starting with the portfolio problem of high productivity entrepreneurs we find how R_{t+1}^{H*} is affected by tightening collateral requirements around the private optimum l_t^m

$$\frac{\partial R_{t+1}^{H*}\left(\widetilde{\theta}_t = l_t^m\right)}{\partial \widetilde{\theta}_t} \approx \frac{\partial \varpi_t^H}{\partial \widetilde{\theta}_t}\left(E_t\rho_{t+1}^H - 1 - \varpi_t^H \sigma_{Rt+1}^2\right) - \frac{\left(\varpi_t^H\right)^2}{2}\frac{\partial\left(\sigma_{Rt+1}^2\right)}{\partial \widetilde{\theta}_t} + \left(\frac{\partial E_t \rho_{t+1}^H}{\partial \widetilde{\theta}_t} + \frac{\partial \ln R_t}{\partial \widetilde{\theta}_t}\right) \tag{2.25}$$

Here ρ_{t+1}^H is the excess return on leveraged production for high productivity entrepreneurs, which was defined in equation (2.11). The value of (2.25) depends strongly on whether borrowing constraints bind or not in the current period. When borrowing constraints bind, the entrepreneur's portfolio hits the constraint and the private first order condition with respect to the share of the risky asset (equation (2.12)) holds with inequality:

$$E_t\rho_{t+1}^H - 1 - \varpi_t^H \sigma_{Rt+1}^2 > 0$$

But the government takes an additional amplification effect into account. This is the $\frac{\left(\varpi_t^H\right)^2}{2}\frac{\partial\left(\sigma_{Rt+1}^2\right)}{\partial \widetilde{\theta}_t}$ term in equation (2.25). It takes into account the endogeneity of the variance of the portfolio rate of return for high productivity entrepreneurs. The more they borrow to invest into risky assets, the larger the impact of capital price shocks on their rates of return on wealth. And this is where the amplification mechanism generates the externality identified in Lorenzoni (2008) and Korinek (2009). When capital prices fall, leveraged entrepreneurs make low returns on wealth and this forces them to sell capital because they no longer have the net worth to purchase the non-collateralised inputs needed to support a large capital input into production. The capital sales can only be absorbed by low productivity firms at lower prices, leading to another round of forced capital sales by credit constrained entrepreneurs.

But the government also recognises the fact that its policy instrument has its costs. Raising the downpayment requirement on capital acts like a tax on high productivity entrepreneurs, which reduces their excess return on production: $\frac{\partial E_t \rho_{t+1}^H}{\partial \widetilde{\theta}_t} > 0$. So when capital requirements are tightened, the excess return on high productivity projects is reduced due to their distorted input mix. Partially offsetting that, the risk free rate increases when the government tightens credit limits: $\frac{\partial \ln R_t}{\partial \widetilde{\theta}_t} < 0$. But overall, tighter capital requirements leads to a lower rate of return on wealth for high productivity entrepreneurs. Finally, high productivity entrepreneurs have substantial capital positions which depreciate in value when regulation is introduced. This has a negative effect on their welfare.

Low productivity entrepreneurs

Moving on to the portfolio of low productivity entrepreneurs we have the following first order condition, which determine the way the capital requirements for high productivity entrepreneurs impact on the log rate of return on wealth for the low types:

$$\frac{\partial \ln R_{t+1}^{L*}\left(\widetilde{\theta}_t = l_t^m\right)}{\partial \widetilde{\theta}_t} \approx \frac{\partial \varpi_t^L}{\partial \widetilde{\theta}_t}\left(E_t \rho_{t+1}^L - 1 - \varpi_t^L \sigma_{rt+1}^2\right) - \frac{\left(\varpi_t^L\right)^2}{2}\frac{\partial\left(\sigma_{rt+1}^2\right)}{\partial \widetilde{\theta}_t} + \frac{\partial E_t \rho_{t+1}^L}{\partial \widetilde{\theta}_t} + \frac{\partial \ln R_t}{\partial \widetilde{\theta}_t} \tag{2.26}$$

Capital requirements will affect low productivity types indirectly because they will reduce the available supply of the risk free asset and force them to invest more of their net worth in production. This is the first term in (2.26). But in addition, the volatility of the aggregate economy will decline and this will reduce the variance of the returns on the risky asset $\left(\frac{(\varpi_t^L)^2}{2}\frac{\partial(\sigma_{rt+1}^2)}{\partial \widetilde{\theta}_t}\right)$. The excess return on the risky asset for low productivity types will also change $\left(\frac{\partial E_t \rho_{t+1}^L}{\partial \widetilde{\theta}_t}\right)$ depending on whether the overall portfolio has become riskier or safer as a result of the policy change. Finally risk free rates will change as the economy becomes more regulated.

For unconstrained low productivity entrepreneurs most of the terms in (2.26) are zero. $E_t\rho^L_{t+1} - 1 - \varpi^L_t \sigma^2_{rt+1} = 0$ from optimal portfolio choice. Because the low productivity type prices assets in our economy, any change in the volatility of returns will be reflected in the excess returns demanded in equilibrium. This means that $-\frac{(\varpi^L_t)^2}{2}\frac{\partial(\sigma^2_{rt+1})}{\partial\tilde{\theta}_t} + \frac{\partial\rho^L_{t+1}}{\partial\tilde{\theta}_t} = 0$: more volatile returns will be accompanied by a higher excess return leaving the welfare of low productivity entrepreneurs unaffected.

There is an interesting difference between the way the government treats the portfolios problems of the two groups. In the case of the high productivity agents, the government was concerned with the welfare consequences of the market price externality which increased the value of σ^2_{Rt+1}- the variance of the log rate of return on the risky asset for high productivity entrepreneurs. But in this case changes in σ^2_{rt+1} - the variability of the log excess return on the risky asset for the low types - did not represent any allocative inefficiency.

This difference arises because low productivity entrepreneurs are always unconstrained in their portfolio choice so, on the margin, any increase in the volatility of capital prices due to the excessive leverage of other entrepreneurs is compensated in equilibrium by higher excess returns. For the low productivity types the behaviour of the productive types represents a pure pecuniary externality with no consequences for allocative efficiency. In contrast, high productivity entrepreneurs are borrowing constrained (at least in some states of the world) and the tightness of the borrowing constraint depends on the level of asset prices. So the pecuniary externalities caused by the forced capital sales by leveraged entrepreneurs in downturns do have consequences for the allocative efficiency of the economy. By tightening borrowing constraints for everyone else, forced sales exert an externality the benevolent government should be concerned with correcting.

Because most of the terms in (2.26) drop out, the expected net present value of future returns for low productivity types is driven largely by what is happening to the log of risk free rates.

$$\frac{\partial \ln R_{t+1}^{L*}\left(\widetilde{\theta}_t = l_t^m\right)}{\partial \widetilde{\theta}_t} \approx \frac{\partial \ln R_t}{\partial \widetilde{\theta}_t}$$

Because tightening collateral requirements in the Kiyotaki (1998) model reduces aggregate TFP and pushes down on capital prices, the lower user cost of capital increases the rate of return on production for low productivity types and, by arbitrage, increases the risk free rate. This effect raises the welfare of low productivity entrepreneurs.

But there are other factors which reduce the welfare of low productivity entrepreneurs. First of all, the continuation value of low productivity agents φ_t^L partly depends on the value of a possible future high productivity opportunity φ_t^H and as we have seen in the previous subsection, this can be reduced by regulation. But secondly, as capital regulation is tightened, this depresses capital prices which form a part of all entrepreneurs' portfolios. So the wealth terms of (2.24) will fall. Overall, the welfare of the unproductive will rise if they do not hold much capital (hence the loss of wealth from lower prices is small) and if they are not very likely to transit to the high productivity state (hence the fall in the value of productive opportunities does not affect them much).

Workers

Workers' period welfare is determined by the log of the real wage.

$$\frac{\partial \ln w_t\left(\widetilde{\theta}_t = l_t^m\right)}{\partial \widetilde{\theta}_t}$$

As the results in Table 3.5 above showed, tightening capital requirements in relatively well developed financial systems (with a high value of $\widetilde{\theta}_t$) resulted in slightly higher real wages and higher welfare for workers. However, tightening collateral requirements in a less well developed financial system resulted in lower wages for workers.

To summarise. We can see that introducing capital requirements may improve the welfare entrepreneurs and workers although this is by no means guaranteed. When collateral requirements are already binding at the time of capital requirement reform, such a reform may not be welfare increasing despite the existence of externalities. This is because the binding collateral constraint makes the policy instrument (tightening collateral constraints even further in some states of the world) a very distortionary one. In order for the government to distort an already distorted economy even further, two things have to be true: the collateral amplification mechanism must be very powerful and/or private individuals must care very much about consumption volatility. We now proceed to check whether numerically this is is the case or not in our economy.

2.7 Optimal Collateral Requirements and Welfare

2.7.1 Numerical Results

In this section we use numerical simulations to compare the market and the government's choices of the collateral requirements on capital. We do this under different states of the financial system as measured by θ - the fraction of capital which is collateralisable[10]. This is done in Table 2.7 below. The first row of the table shows that firms always choose to invest up to the debt limit in the competitive equilibrium. The second row shows the government's choice of capital requirement as it tries to maximise the social welfare function (2.24). The capital requirement turns out to be invariably equal to the privately permissible maximum leverage and, unsurprisingly, private agents borrow the same amount as they do in the unregulated economy (shown in the third row of the table). This is the main result of this paper - when credit constraints bind tightly due to a substantial productivity differential between the two types of entrepreneurs in our economy, the government wants to encourage investment all the way to the incentive-compatibility determined borrowing limit θ.

[10] In each we recalibrated the model to match the target discussed in the calibration section. These are (1) aggregate leverage, (2) leverage of the most indebted decile of firms, (3) the ratio of tangible assets to GDP, (4) the fraction of time spent working and (5) the standard deviation of real GDP.

	$\theta = 0.80$	$\theta = 0.90$	$\theta = 1.00$
$E\left(l_t^m\right)$	0.80	0.90	1.00
$E\left(\widetilde{\theta}_t\right)$	0.80	0.90	1.00
$E\left(l_t^g\right)$	0.80	0.90	1.00

Note: $E\left(l_t^m\right)$ is the average private choice of debt as a fraction of tangible assets in the Laissez Faire economy, $E\left(\widetilde{\theta}_t\right)$ is the average capital requirement in the regulated economy and $E\left(l_t^g\right)$ is the average private choice of debt as a fraction of tangible assets in the regulated economy.

Table 2.7: The government's collateral requirement choices

Table 2.8 below tries to delve a little deeper into the determinants of welfare for individual groups as well as the aggregate economy in order to see how they they are affected by changes in capital requirements. The table looks at the change in a number of measures of welfare from the imposition of a capital requirement $\widetilde{\theta}_t = \theta - 0.01$ in all states of the world. Because we are interested in how the initial state of the financial system affects the incentives of the government to regulate leverage, we repeat our exercise for several financial systems, represented by different values of the maximum collateral limit θ.

So for example, the first column of the table takes an economy where the state of the financial system can collateralise up to a 0.8 fraction of capital values. To see the local incentives for the government to regulate we consider the welfare effects of the imposition of a capital requirement $\widetilde{\theta}_t = 0.79$.

	$\theta = 0.80$	$\theta = 0.90$	$\theta = 1.00$
Welfare of high productivity entrepreneurs			
$100\triangle \ln \varphi_0^H$	-0.33	-0.32	-0.23
$100\triangle \ln Z_0^H$	-1.06	-1.91	-4.15
$100\triangle \ln V_0^H$	-0.33	-0.58	-1.15
Welfare of high productivity entrepreneurs			
$100\triangle \ln \varphi_0^L$	0.37	0.58	1.11
$100\triangle \ln Z_0^L$	-0.14	-0.32	-0.71
$100\triangle \ln V_0^L$	0.05	0.03	0.04
Workers' welfare			
$100\triangle \ln V_0^W$	-0.09	-0.02	0.14
Aggregate welfare			
$100\triangle \ln V_0$	-0.18	-0.23	-0.33

Note: All variables in the table measure the percentage change in the relevant component of welfare from a tightening of collateral requirements by 0.01 (or 1% of the value of tangible assets). $\triangle \ln \varphi_0^H$ is the change in the net present value of future expected log rates of return on wealth for high productivity entrepreneurs, $\triangle \ln \varphi_0^L$ is the change in the net present value of future expected log rates of return on wealth for low productivity entrepreneurs, $\triangle \ln w_0^H$ is the wealth change for high productivity entrepreneurs, $\triangle \ln w_0^L$ is the wealth change for low productivity entrepreneurs, $\triangle \ln V_0^H$ is the welfare change for high productivity entrepreneurs, $\triangle \ln V_0^L$ is the welfare change for low productivity entrepreneurs, $\triangle \ln V_0$ is the aggregate welfare change.

Table 2.8: Capital requirements and welfare

Starting with the baseline case of $\theta = 1$ we can see that changing capital requirements a little in the neighbourhood of the competitive equilibrium reduces aggregate welfare by 0.3%. But this masks a number of different competing effects on welfare. Starting with the high productivity entrepreneurs, the second row of the table shows that the expected net present value of future log returns on wealth decreases by just under 0.2%. There is also a 4% decline in wealth (second row) and causes a 1% drop in the welfare of high productivity entrepreneurs. Further down the $\theta = 1$ column we have the components of welfare for low productivity entrepreneurs. The expected net present value of future rates of return increases by around 1% driven by the higher safe rate of return. Lower asset prices depress the wealth of this group which falls by 0.7%. The effect of higher rates of return on wealth dominates, leading to a 0.04% increase in welfare. Workers' welfare also rises by a small amount driven by a small rise in the real wage and a decline in the volatility of real wages.

The cases of $\theta = 0.9$ and $\theta = 0.8$ (the first and second column of the Table) are qualitatively similar to the baseline case though all the magnitudes get progressively smaller in absolute value as the economy gets more and more distorted at lower levels of financial development. Appendix 4.B.12 contains a number of other sensitivity checks we performed in order to be sure of the robustness of the 'no regulation' result. We found that our results were robust to different values of the productivity differential a^H as well as to the form of the borrowing constraint.

2.7.2 Discussion

Our numerical results show that the capital requirement is a very blunt instrument, which is best left unused in the context of our model and calibration. The main losers from tighter regulation of private leverage are the high productivity entrepreneurs who find that their access to borrowing is reduced with detrimental effects on their steady state consumption and welfare. On the positive side, the volatility in their consumption declines very sharply. This is because reduced leverage improves both consumption smoothing over the idiosyncratic productivity cycle as well as risk sharing over the business cycle. But the beneficial impact of greater consumption stability are insufficient to generate a welfare improvement.

Low productivity entrepreneurs also lose out though by a smaller margin. For them, capital regulation represents a finer balance. On the one hand they gain because the reduced access to credit reduces capital prices and boosts the rate of return they earn on their own production. The consumption is also smoother due to the reduced volatility of consumption over the productivity cycle as well as the business cycle. But these gains are relatively small because low productivity entrepreneurs are not leveraged and their consumption is already smooth. On the other hand, lower wealth due to poorer borrowing opportunities and lower asset prices affects them too.

Taken as a whole, the economy is made worse off by capital requirements. This is because the productivity reducing effect of regulation turns out to have a larger impact on welfare compared to its impact in terms of greater macro-economic stability. This suggests that one reason for the surprising result of this paper is that private agents value average consumption a lot more than they value consumption stability. One simple way to test this hypothesis is to examine the premium on risky assets in our economy. This is done in Table 2.9 below, which shows the difference between the expected return on the risky asset for low productivity entrepreneurs and the risk free rate. We focus on low productivity entrepreneurs because they are unconstrained and therefore they price assets in our economy.

	$\theta = 0.80$	$\theta = 0.90$	$\theta = 1.00$
$100\left(E_t r_{t+1}^k - R_t\right)$	0.0010	0.0015	0.0020

Note: 0.01 denotes 1 basis point.

Table 2.9: The risk premium under different financial systems

The table shows that the risk premium is very small - less than 1bp for the calibration we consider. Put another way, low productivity entrepreneurs strongly prefer excess returns to smooth returns. It is therefore clear why the government finds that it cannot improve on the competitive allocation. The pecuniary externality results in excessive volatility of consumption and asset prices while the policy response we consider has its own costs in terms of the level of output and consumption. Consumers in this model do not find such a trade off advantageous.

Again, note that an absence of amplification in the Kiyotaki (1998) model is not the reason for this result. Contrary to the findings of Cordoba and Ripoll (2004) we find that there is substantial amplification with the standard deviation of output and TFP around 40% higher than the 'first best' and the standard deviation of consumption and asset prices more than 80% higher than the 'first best'. This shows that the model can magnify the effects of shocks but consumers do not care sufficient about this to be willing to pay the costs of the regulation.

There are at least three reasons for this. First of all, the assumption of log utility limits entrepreneurs' risk aversion and the amount of steady state consumption they are willing to give up in order to have a smooth consumption profile over time. This reduces the costs of weak risk sharing and consumption smoothing in our economy and therefore makes regulation (which improves both risk sharing and consumption smoothing) less desirable. Secondly, aggregate shocks are small. The high productivity state alternates between values 0.6% above or below steady state. This is consistent with aggregate fluctuations in developed economies during the recent 'Great Moderation' period. It remains to be seen whether the volatility of technology shocks picks up following the 2008 Lehmans Crisis.

Thirdly, the nature of borrowing in this model is entirely constrained efficient. The flow of funds between borrowers and lenders serves to boost productivity and benefit everyone. There is no misalocation of resources such as might arise if lenders or borrowers make mistakes in allocating credit; there are no defaults and no bankruptcy costs associated with default. So perhaps it is unsurprising that regulation cannot help in this environment: we have made its task relatively difficult.

These considerations introduce many possible avenues for future work. Examining the robustness of the 'no regulation' result to different preferences is one obvious extension I am already working on. But examining other economic environments is also a promising avenue in studying the question of whether and how capital requirements can improve social welfare.

2.8 Conclusions

This paper aims to assess quantitatively the extent to which private leverage choices are inefficient from a social point of view. We found that, to a very close approximation, these choices are efficient. In the Kiyotaki-Moore framework credit constraints bind because limited commitment makes the financing of productive opportunities more difficult. Thus although leverage introduces a certain degree of financial fragility into the economy, it also allows the funding of high value added activities which, on average, allow society to enjoy a higher level of output and consumption.

So we find that regulation has a number of costs and benefits for economic agents. The main benefits involve reducing the inefficient volatility of output and consumption which arises from the workings of the collateral amplification mechanism. In the laissez faire equilibrium individual borrowers decide to borrow up to the debt limit in order to take advantage of attractive productive opportunities. They know that when aggregate shocks hit, leverage will magnify the effect of asset prices on balance sheets and force them to sell productive assets at a time when the price is already low. But atomistic agents take the low price in downturns as given even though the amount of asset sales and the size of the price fall are closely linked. The more assets are sold by leveraged high productivity entrepreneurs the more the price falls because the only buyers are the unleveraged low productivity types. This exerts downward pressure on the aggregate efficiency of the economy and depresses asset prices even further tightening credit constraints even more. It is binding borrowing constraints that make the usually harmless pecuniary externalities between different agents important for allocative efficiency.

But regulation has substantial costs too. When borrowing constraints bind, not enough funds flow from low to high productivity entrepreneurs and this reduces average TFP and consumption over time. Imposing tighter collateral requirements further squeezes the flow of credit and further reduces its average productive efficiency even though it makes it more stable as a result.

The social choice between the level and the volatility of consumption is largely driven by the preferences of economic agents as well as the marginal rate of transformation between the level and volatility of consumption. In our calibration we find that economic agents do not care about volatility as much as they care about the level of consumption. This is clearly demonstrated by the low premium on risky assets (below 1bp). In addition, capital requirements reduce volatility at too high a cost in terms of average efficiency. Consequently, the benevolent government chooses not to regulate finance in our model economy.

In future research I want to explore the robustness of this result. One obvious extension is to change the structure of the model in order to generate a more realistic equity risk premium, for example by incorporating the Epstein-Zin-Weil preferences and an environment of long run consumption risk. A high equity premium indicates that private investors are very concerned about risk. So an environment with a high equity premium is more likely to be one in which the imposition of capital requirements is optimal.

Chapter 3

Borrower Reputation as Intangible Collateral

3.1 Introduction

The financial boom and bust cycle of 2005-2009 was characterised by a substantial increase and subsequent fall in the permissible leverage for all sectors of the economy. Downpayment requirements on housing, capital and financial asset purchases fell during the boom and then increased sharply as the financial crisis unfolded during 2008. At the same time, asset prices and output fell sharply across the world, raising questions about the linkages between financial conditions, asset prices and real quantities during the financial crisis. And while we have a good theoretical understanding of how credit constraints affect the interaction between output and asset prices, there has been comparatively less work on downpayment requirements and other aspects of the financial conditions facing private borrowers.

In this paper, we build a framework which can generate fluctuations in downpayment requirements by appealing to changes in the value of borrower's reputation for repayment. We extend the framework of Kiyotaki and Moore (1997) and Kiyotaki (1998) by considering an environment, in which savers can keep their anonymity but borrowers cannot. This allows lenders to punish defaulting borrowers by excluding them from future borrowing. They cannot however stop them from saving in the anonymous financial market or from engaging in self-financed production. We show how the possibility of such market exclusion can lead to the emergence of intangible collateral in equilibrium alongside the tangible collateral which is usually studied in the literature.

The intangible collateral is essentially the value of a borrower's reputation for debt repayment. We find that this collateral form can back a very significant part of the liabilities of the private sector. One of the key contributions of this paper is to show how the financial contract in a model with tangible and intangible collateral can still be represented as a linear borrowing constraint, where a fall in the value of intangible collateral manifests itself in a higher 'haircut' (or downpayment) while a rise in the value of intangible collateral can manifest itself as a lower haircut.

In our numerical experiments we find that intangible collateral is large (and haircuts are low) when the cost to an entrepreneur of being excluded from borrowing is substantial. Intangible (or reputational) collateral has a non-linear relationship with the more conventional tangible collateral. While inefficient production remains, higher tangible collateral boosts the excess return of high productivity entrepreneurs and therefore increases the value of intangible collateral. This is because increasing the borrowing capacity of high productivity entrepreneurs increases the price of capital and reduces the real rate of interest (equal to the rate of return on the production of low productivity entrepreneurs). Once, the availability of tangible collateral becomes high enough, inefficient production disappears and further increases in tangible collateral starts to push up real interest rates, depressing the excess rate of return for high productivity entrepreneurs. Once the excess return starts to fall so does the value of intangible collateral.

Finally we solve our model economy with aggregate uncertainty in order to study how intangible collateral interacts with the business cycle. Assuming that entrepreneurs borrow using empirically realistic simple debt contracts, the model does generate some endogenous fluctuations in 'haircuts'. For conventional technology shocks, these fluctuations are small and pro-cyclical. In other words, the model generates low haircuts in recessions and high haircuts in booms. The reason for this counterintuitive finding is the following. In recessions, asset prices are low, financial constraints bind strongly and the excess return for leveraged high-productivity firms over the unleveraged low-productivity firms increases. Since recessions are expected to be persistent, this increase in excess returns leads to a rise in the value of debt repayment, reducing lenders' required haircuts. In contrast, in booms, asset prices are high, financial frictions are reduced and the leveraged high productivity entrepreneurs enjoy a smaller excess return relative to unleveraged low productivity firms. Hence the value of intangible assets declines, increasing lenders' required haircuts. In order to replicate the counter-cyclical behaviour of downpayment requirements in the data, we augment the model by allowing pro-cyclical fluctuations in the technological gap between 'high' and 'low' productivity firms and also by allowing counter-cyclical fluctuations in the degree of uninsurable idiosyncratic production risk. This introduces a pro-cyclical component in the value of being a leveraged high productivity producer, helping to motivate counter-cyclical haircut movements.

3.2 Related Literature

This paper studies the nature of dynamic borrowing contracts in an environment with permanent exclusion from credit markets. There is a large literature on dynamic optimal contracts starting with the seminal contributions of Kehoe and Levine (1993) and Kocherlakota (1996) who developed the first general equilibrium models with endogenous borrowing constraints. Subsequently, work by Alvarez and and Jermann (2001) showed how the allocation of Kehoe and Levine (1993) can be decentralised by a set of state contingent borrowing limits in a general economy with permanent exclusion from risk sharing arrangements.

Our paper is also related to the literature on the collateral amplification literature started by Kiyotaki and Moore (1997) and Kiyotaki (1998). These papers have shown that when debts are collateralised, leverage magnifies the impact of small shocks on the net worth of producers, thus amplifying and propagating impulses over time. This mechanism is central in our paper too. In the standard Kiyotaki and Moore (1997) set up, borrowers can commit to repay an exogenous fraction of project revenues or tangible asset values. In contrast, this paper explicitly models the fluctuations in such 'haircuts' as a function of the value to a borrower of being able to access credit markets in future. We find that the value of credit access is counter-cyclical. It is low in booms because entrepreneurial net worth is already high and excess returns from production are relatively low. It is high in recessions because entrepreneurial net worth is low and excess returns are high. So the value of intangible collateral acts as a dampening mechanism in our model.

There has been relatively little work on the importance of intangible collateral. Hellwig and Lorenzoni (2007) is a notable exception. They study an endowment economy with limited commitment in which there is no collateral to secure borrowing. Because the autrarkic equilibrium is dynamically inefficient and stationary bubbles on intrinsically useless assets can exist as in the classic Samuelson (1958) model. Hellwig and Lorenzoni show that when private borrowers can be permanently excluded from future credit market access, an equilibrium with bubbles on inside liquidity (private debt) can achieve an identical allocation as the equilibrium with bubbles on outside liquidity. Here private agents capture the seigniorage benefits of debt issue and these seigniorage benefits serve as the intangible collateral needed to back borrowing.

Gertler and Karadi (2009) is closer to this paper in the sense that they motivate intangible collateral by appealing to the value of excess returns in an equilibrium with no bubbles. They develop a model of a bank which can pledge the net present value (NPV) of future profits as collateral. Their mechanism is very similar to the intangible collateral studied in this paper. In Gertler and Karadi (2009) the entrepreneur is threatened with the loss of the NPV of future profits (which come from excess returns). Here the entrepreneur is threatened with the loss of the NPV of future utility flows from excess productive returns. Because of risk aversion, the value of intangible collateral is lower in this paper but not significantly so because log utility does not lead to high risk aversion.

We find that counter-cyclical variation in idiosyncratic production risk is one mechanism that is capable of causing counter-cyclical movements in haircuts in a way that amplifies the business cycle. Angeletos and Calvet (2006) and Perez (2006) are two papers that examine the importance of idiosyncratic production risk for the business cycle. They both show that the presence of uninsurable idiosyncratic production risk can have a profound impact on risk-taking and capital accumulation. And if the degree of idiosyncratic production risk varies in a counter-cyclical fashion (i.e. it is higher in recessions), Angeletos and Calvet (2006) show that this can amplify the business cycle by affecting entrepreneurs' investment into risky but high yielding projects. In this paper, our focus is mainly on the impact of idiosyncratic production uncertainty on haircuts. High ex post productivity variability causes the expected return from production (in utility terms) to decline and this reduces the value of borrowing. So to the extent that production uncertainty is high in recessions, this channel is capable of producing counter-cyclical downpayment requirements.

3.3 Motivating Observations

There is a lot of evidence that permissible leverage fluctuates very substantially for many private borrowers. Figure 3.1 below (reprinted from Geannakoplos (2009)) shows how the haircuts on securities purchases have fluctuated for a hedge fund named Ellington. The chart clearly shows that haircuts average around 20% of the purchase price although they rose to 40% during the Russian default in 1998 and during the 2008-2009 financial crisis. During the 2006-2007 credit boom haircuts were unusually low at levels just above 10%.

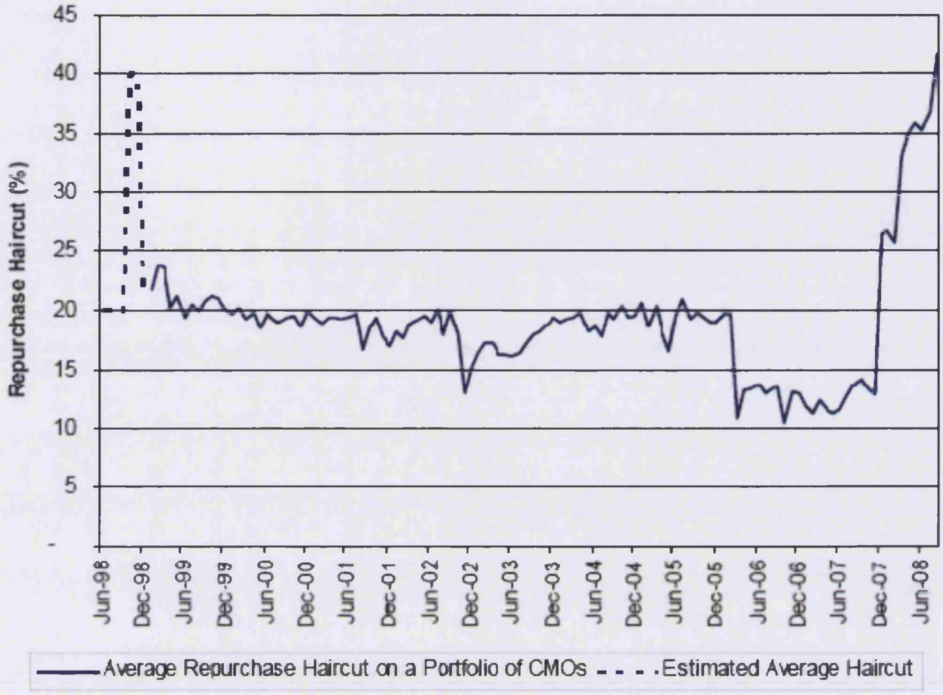


Figure 3.1: CMO margins at Ellington (reprinted from Geannakoplos (2009))

In housing markets, leverage fluctuations have also received a lot of recent attention. Figure 3.2 below shows the movement of the monthly LTV ratio for new home buyers. The chart shows that the ratio varies in a pro-cyclical fashion, with local peaks in booms (1984, 1988, 1995-1999 and 2007) and troughs in recessions (1975, 1982, 1991, 2003 and 2008). The contemporaneous correlation between HP-filtered GDP and the HP-filtered LTV ratio is 0.4.

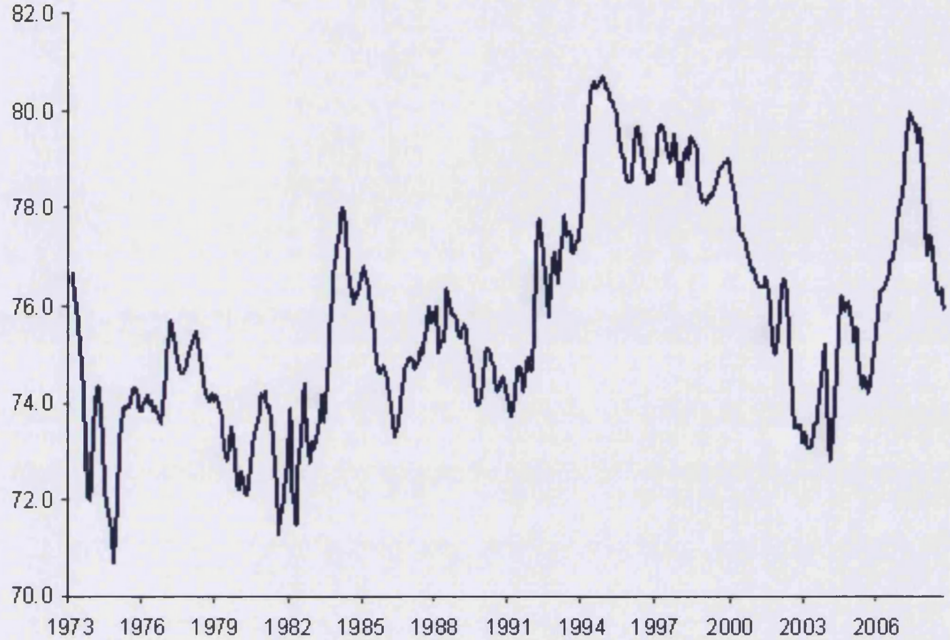


Figure 3.2: Loan to Value Ratios in the US: 1973-2008 (Source: FHFA)

These data show that downpayments in financial markets move in a counter-cyclical fashion and the leverage used in security or housing purchase varies in a pro-cyclical fashion. This is the feature of the data our model aims to explain.

3.4 The Model

3.4.1 The Economic Environment

Production Technology

- Population and productive technology

The economy is populated with a continuum of infinitely lived entrepreneurs of measure 1. Each entrepreneur is endowed with a constant returns to scale production function which uses land and capital to produce gross output y.

$$y_t = a_t A_t \left(\frac{k_t}{\alpha}\right)^{\alpha} \left(\frac{x_t}{1-\alpha}\right)^{1-\alpha}$$

k is fixed capital (which does not depreciate), x is working capital (which fully depreciates) and a is the idiosyncratic component of productivity (which can be high or low) and A is the aggregate component of productivity (which also can be high or low). The idiosyncratic state evolves according to a Markov process. Following Kiyotaki (1998) let $n\delta$ be the probability that a currently unproductive firm becomes productive and let be the probability that a currently productive firm becomes unproductive. This implies that the steady state ratio of productive to unproductive firms is n. The aggregate state also evolves according to a persistent Markov process.

Finally we assume that agents suffer from limited commitment. They cannot make binding promises to repay their debts unless it is in their interests to do so.

3.4.2 Entrepreneurs

Preferences

Entrepreneurs are ex-ante identical and have logarithmic utility over consumption streams

$$U^e = \sum_{t=0}^{\infty} \beta^t \ln c_t$$

Flow of Funds

Agents purchase consumption (c), investment goods (x), capital (k) at price q_t and borrow using debt securities b_{t+1} at price R_t^{-1} where $X_t \equiv (A_t, \Gamma_t)$ is a vector describing the aggregate state of the economy. A_t is aggregate TFP and Γ_t denotes the wealth distribution.

$$c_t + q_t k_{t+1} - \frac{b_{t+1}}{R_t} = y_t + q_t k_t - b_t$$

Because the idiosyncratic shocks and individual asset holdings are private information, this implies that Arrow securities contingent on the idiosyncratic state will not trade in equilibrium. Hence, we have an asset market which is complete over aggregate but not idiosyncratic states.

Collateral constraints

Due to moral hazard in the credit market, agents will only honour their promises if it is in their interests to do so. This is why we need to carefully describe the precise nature of contract enforcement in the event of default. We assume that entrepreneurs have the ability to 'run away' with a (possibly state contingent) fraction $1-\phi_t$ of the firm's revenues - y_{t+1}. However we assume that lenders can seize the firm's capital $q_{t+1}k_{t+1}$. Upon default, entrepreneurs can anonymously lend to other entrepreneurs or produce without any leverage.

The collateral constraint then takes the form of a value function comparison at each value of the aggregate and individual state. Let $V(s_t, X_t)$ denote the value of an entrepreneur who has never defaulted and let $V^d(s_t, X_t)$ denote the value of an entrepreneur who has defaulted in the past. $s_t \equiv (w_t, a_t)$ is the idiosyncratic state where w_t is individual wealth and α_t is the idiosyncratic level of TFP. When an entrepreneur borrows using state contingent debt, her lender would have to assess whether such an entrepreneur would find it optimal to repay the promised amount in each state of the world. This is tantamount to knowing that the value of repaying is larger than the value of defaulting in each state of the world next period.

$$V(s_{t+1}, X_{t+1}|s_t, X_t) \geqslant V^d(s_{t+1}, X_{t+1}|s_t, X_t)$$

For the time being we will conjecture that this value function comparison can be reduced to a state contingent collateral constraint of the following form.

$$b_{t+1} \leqslant E_t[\theta_t y_{t+1} + q_{t+1}k_{t+1}]$$

We verify subsequently that this is indeed the case.

3.4.3 Entrepreneurial behaviour

The entrepreneurs in our economy have to make two types of decisions. They have to choose consumption over time optimally (the consumption problem) and they have to choose the assets they invest in (the portfolio problem). Fortunately, the budget constraint is linear in all the assets at the entrepreneur's disposal and as a result we can utilise the result due to Samuelson (1968), which states that we can solve separate the consumption and portfolio decisions.

The consumption problem

Due to logarithmic utility, consumption is a fixed fraction of wealth at each point in time for all entrepreneurs regardless of their level of idiosyncratic productivity. This general result is proved in Appendix 4.B.1 and it greatly simplifies the aggregation of consumption decisions.

$$c_t = (1-\beta)\, w_t$$

The portfolio problem

The portfolio problem is more complex because we have three assets and a collateral constraint. The first order condition for capital use is:

$$-\lambda_t q_t + \beta E_t \left[\frac{\alpha y_{t+1}}{k_{t+1}} + q_{t+1} \right] \lambda_{t+1} + \mu_t \theta_t E_t \left[\frac{\alpha y_{t+1}}{k_{t+1}} + q_{t+1} \right] = 0 \qquad (3.1)$$

where $\lambda_t = 1/c_t$ is the shadow price on the flow of funds constraint while μ_t is the shadow price the collateral constraint. The first order condition for capital investment is:

$$-\lambda_t + E_t \left[\beta \frac{(1-\alpha)\, y_{t+1}}{x_{t+1}} \lambda_{t+1} + \theta_t \frac{(1-\alpha)\, y_{t+1}}{x_{t+1}} \mu_t \right] = 0 \qquad (3.2)$$

Finally the first order condition for the bond holdings is:

$$-\frac{\lambda_t}{R_t} + \beta E_t \lambda_{t+1} + \mu_t = 0$$

3.4.4 Borrowing limit determination

Our economy is a limited commitment one. Borrowers repay their debts only if it is in their interests to do so. Upon default, a borrower loses his tangible assets but also he loses his reputation for repayment and therefore the ability to borrow in the future. As we now show, entrepreneurs will be allowed to borrow up to the value of the tangible and intangible assets they can lose when they default.

The value of a non-defaulting entrepreneur

We now combine the optimal consumption and portfolio choices of entrepreneurs to derive the value function that characterises their maximum lifetime utility. Let $V(s_t, X_t)$ be the value of a non-defaulting entrepreneur with idiosyncratic state s_t when the aggregate state is X_t.

$$V(s_t, X_t) = \max_{c_t, k_{t+1}, x_{t+1}, b_{t+1}} \{\ln c_t + \beta E_t V(s_{t+1}, X_{t+1})\}$$

In Appendix 4.C.1 we show that the value function takes the following form

$$V(s_t, X_t) = \varphi(s_t, X_t) + \frac{\ln w_t}{1-\beta}$$

where the intercept $\varphi(s_t, X_t)$ satisfies a functional equation:

$$\varphi(s_t, X_t) = \ln(1-\beta) + \max_{k_{t+1}, x_{t+1}, b_{t+1}} \beta E_t \left[\frac{\ln \beta}{1-\beta} + \frac{\ln r^i_{t+1}}{1-\beta} + \varphi(s_{t+1}, X_{t+1})\right] \quad (3.3)$$

Intuitively, the value of an entrepreneur depends on his current wealth (this is the term in $\ln a_t$) as well as the rate of return the entrepreneur can earn on his wealth (this is the intercept term). Looking at (3.3) we can see that, if the rate of return on wealth is equal to the inverse of the rate of time preference at all times ($r^i = 1/\beta$), the intercept $\varphi(s_t, X_t)$ will be equal to zero and the value of an entrepreneur will be solely determined by his current wealth. In contrast, values of r^i above $1/\beta$ would generate a positive value of φ reflecting the net present value of 'excess returns' to the entrepreneur.

The value of a defaulting entrepreneur

An entrepreneur who defaults makes a very high return on her investments for one period because she avoids paying some of her debt. The cost of this is that she then loses her right to borrow in future and she loses her right to use the high productivity technology. We guess that the value of an entrepreneur who has defaulted in the past is given as follows:

$$V^d(s_t, X_t) = \varphi^d(s_t, X_t) + \frac{\ln w_t}{1-\beta}$$

where the intercept of the value function satisfies the now familiar functional equation:

$$\varphi^d(s_t, X_t) = \ln(1-\beta) + \max_{k_{t+1}, x_{t+1}, b_{t+1}} \beta E_t \left[\frac{\ln\beta + \ln r^{di}_{t+1}}{1-\beta} + \varphi^d(s_{t+1}, X_{t+1}) \right]$$

This guess is verified in Appendix 4.C.1. Intuitively, once the entrepreneur defaults he can only lend to others or produce without leverage. This is reflected in the above value function which depends on r^{di}_{t+1} - the rate of return on the portfolio of entrepreneur who have defaulted in the past.

The value of an entrepreneur defaulting in state X_{t+1} next period is higher than this, however, because of the large one-off return due to the avoidance of debt repayment:

$$\begin{aligned} V^d(s_{t+1}, X_{t+1}) &= \varphi^d(s_{t+1}, X_{t+1}) + \frac{\ln w^d_{t+1}}{1-\beta} \\ &= \varphi^d(s_{t+1}, X_{t+1}) + \max_{k_{t+1}, x_{t+1}, b_{t+1}} \frac{(1-\phi) E_t \ln[y_{t+1}]}{1-\beta} \end{aligned}$$

Solving for the borrowing limits

Alvarez and Jermann (2001) solve for borrowing limits which are 'not too tight' as the highest possible borrowing limit consistent with repayment. In our setting this is given by the Incentive Compatibility Constraint which equates the expected value of repayment with the expected value of defaulting.

$$E_t V\left(s_{t+1}, X_{t+1}|\theta\right) = E_t V^d\left(s^d_{t+1}, X_{t+1}\right)$$

This implies that the loss of reputation due to default (LHS of the expression below) exactly offsets the one-off gain from having one's debt written off (the RHS of the expression below).

$$\begin{aligned}
& (1-\beta) E_t \left[\sum_{\alpha_{t+1}} \pi\left(s_{t+1}|s_t\right) \varphi\left(s_{t+1}, X_{t+1}|\theta\right) - \varphi^d\left(X_{t+1}\right)\right] \\
\geqslant\ & E_t \ln\left[(1-\phi)\, y_{t+1}\right] - E_t \ln\left[y_{t+1} + q_{t+1}k_{t+1} - b_{t+1}\right]
\end{aligned}$$

Using the approximation:

$$E \ln x \approx \ln Ex - \frac{1}{2} var(\ln x)$$

we get:

$$\begin{aligned}
& (1-\beta) E_t \left[\sum_{\alpha_{t+1}} \pi\left(s_{t+1}|s_t\right) \varphi\left(s_{t+1}, X_{t+1}|\theta\right) - \varphi^d\left(X_{t+1}\right)\right] - \Omega_t \\
\geqslant\ & \ln E_t \left[(1-\phi)\, y_{t+1}\right] - \ln E_t \left[y_{t+1} + q_{t+1}k_{t+1} - b_{t+1}\right]
\end{aligned}$$

where

$$\Omega_t = \frac{1}{2}\left\{var_t(\ln\left[y_{t+1} + q_{t+1}k_{t+1} - b_{t+1}\right]) - var_t(\ln\left[(1-\phi)\, y_{t+1}\right])\right\}$$

is an approximate risk premium term which reflects the greater ex post wealth variability for repaying entrepreneurs. Re-arranging we have:

$$\begin{aligned}
\frac{(1-\phi)\, y_{t+1}}{y_{t+1} + q_{t+1}k_{t+1} - b_{t+1}} &\leqslant \exp\left\{(1-\beta)\left[\sum_{\alpha_{t+1}} \pi\left(s_{t+1}|s_t\right) \varphi\left(s_{t+1}, X_{t+1}|\phi\right) - \varphi^d\left(X_{t+1}\right)\right] - \Omega_t\right\} \\
&\equiv \Delta\left(s_{t+1}, X_{t+1}|\phi\right)
\end{aligned}$$

So the borrowing constraint is:

$$b_{t+1} \leqslant \left\{ \frac{\Delta(s_{t+1}, X_{t+1}|\phi) + \phi - 1}{\Delta(s_{t+1}, X_{t+1}|\phi)} \right\} y_{t+1} + q_{t+1}k_{t+1}$$

Solving for the borrowing constraints requires us to solve for the value function and for the borrowing constraints until both have converged.

Discussion

The entrepreneur's borrowing limit is determined by the trade off between the benefits of gaining some current wealth by defaulting against the costs of permanently losing the ability to borrow. The benefit from defaulting is determined by the size of unsecured borrowing - $(\theta_t - \phi_t)\, y_t$. The costs are dominated by the gap between the expected value of being a non-defaulting entrepreneur $(\sum_{a_{t+1}} \pi(s_{t+1}|s_t)\, \varphi(s_{t+1}, X_{t+1}|\phi))$ and the value of defaulting $(\varphi^d(X_{t+1}))$. This gap is driven by the utility value of the entrepreneur's stream of excess returns relative to current financial wealth.

Because most of these excess returns are in the future, the discount factor is one of the main determinants of the value of repayment. A discount factor of 0.95 implies that the entrepreneur is indifferent between a 1pp increase in his rate of return on wealth in perpetuity and a 19% increase in his current financial wealth. With a discount factor of 0.9, the consumer is only willing to accept a 9.5% increase in current wealth in exchange for a 1pp increase in returns. Furthermore, since a defaulting entrepreneur only suffers a rate of return disadvantage during high productivity spells, the value of her reputation for repayment is largely determined by the frequency of high productivity spells

3.4.5 Market clearing

There are four market clearing conditions in our model economy - two Arrow security markets (one for each aggregate state), the land market and the goods market.

$$\int b_{t+1}^i di = 0 \tag{3.4}$$

$$\int k_{t+1}^{i} di = 1 \tag{3.5}$$

The total quantity of land in the economy is normalised to unity.

$$\int c_t^i di + \int x_{t+1}^i di = \int y_t^i di \tag{3.6}$$

3.4.6 Behaviour of the aggregate economy

Due to the presence of binding borrowing constraints, high and low productivity entrepreneurs have different demands for assets at a given level of wealth. High productivity agents prefer to invest in production in order to take advantage of high productivity. Low productivity agents have a more balanced portfolio - they invest in production too but also lend funds to the high productivity entrepreneurs through the Arrow security market. This implies that the wealth distribution does matter for equilibrium. But even though the individual decision rules differ according to idiosyncratic productivity, these decision rules remain linear in wealth which means that a within-groups aggregation result obtains. The economy behaves as if it is populated by two agents (a high productivity and a low productivity one). In determining the equilibrium of our model economy we can concentrate on just two moments of the wealth distribution - the mean of the wealth distribution W_t and the share of wealth owned by high-productivity agents d_t.

At any given date, the state of the aggregate economy can be summarised by the state vector

$$X_t = \{A_t, W_t, d_t\}$$

consisting of the level of aggregate productivity, the level of aggregate wealth and the share of aggregate wealth held by productive agents. A_t evolves according to an exogenous two state Markov process while the evolution of the two state variables W_t and d_t is governed by the following relations.

$$W_{t+1} = \beta \left[d_t R_{t+1}^H + (1 - d_t) R_{t+1}^L \right] W_t \tag{3.7}$$

$$d_{t+1} = \frac{(1-\delta)d_t R^H_{t+1} + n\delta (1-d_t) R^L_{t+1}}{d_t R^H_{t+1} + (1-d_t) R^L_{t+1}} \tag{3.8}$$

where R_{t+1} and r_{t+1} are the rates of return on wealth of, respectively, high productivity and low productivity agents.

In equilibrium, productive agents' wealth grows at state contingent rate which depends on their leverage choices

$$R^H_{t+1} = \frac{\left[a_{t+1}A_{t+1} - \theta_t a^H E_t A_{t+1}\right] \left(u^H_t\right)^{1-\alpha} + q_{t+1} - E_t q_{t+1}}{q_t + \frac{(1-\alpha)}{\alpha} u^H_t - E_t \left(q_{t+1} + \theta_t a^H A_{t+1} \left(u^H_t\right)^{1-\alpha}\right) / R_t} \tag{3.9}$$

where u^H_t is the user cost of capital for high productivity agents.

Aggregating the individual capital demands yields an expression for the aggregate land investment by productive entrepreneurs as a function of the state of the economy:

$$K_{t+1} = \frac{\beta d_t W_t}{q_t + \frac{(1-\alpha)}{\alpha} u^H_t - E_t \left(q_{t+1} + \theta_t Y^H_{t+1}\right) / R_t} \tag{3.10}$$

Unproductive agents are unconstrained and invest in their own projects as well as in the loans they make to the productive agents. This means that the wealth of low-productivity entrepreneurs grows at the following rate:

$$\begin{aligned} R^L_{t+1} &= \frac{W^L_{t+1}}{W^L_t} \\ &= \frac{Y^L_{t+1} + q_{t+1}(1-K_{t+1}) + B_{t+1}}{q_t(1-K_{t+1}) + B_{t+1}} \end{aligned} \tag{3.11}$$

where W^L_t and Y^L_t, and $1 - K_t$ are, respectively, the aggregate wealth, output, and capital investments of low productivity workers. Also since we know that high productivity entrepreneurs are constrained, we know that

$$B_{t+1} = E_t \left(\theta_t Y^H_{t+1} + q_{t+1} K_{t+1}\right) \tag{3.12}$$

Substituting (3.10) and (3.12) into (3.11) we get an expression for the growth rate of the wealth of low productivity workers as a function of expected land prices and land investments by the productive entrepreneurs:

$$R_{t+1}^{L}=\frac{\left[a_{t+1}A_{t+1}\frac{\left(u_t^L\right)^{1-\alpha}}{\alpha}+q_{t+1}\right](1-K_{t+1})+B_{t+1}}{\left[q_{t+1}+\frac{1-\alpha}{\alpha}u_t^L\right](1-K_{t+1})+B_{t+1}/R_t} \tag{3.13}$$

Due to log utility, individual and aggregate consumption are linear in individual and aggregate wealth. Hence goods market clearing implies:

$$(1-\beta)\,W_t+X_{t+1}^{H}+X_{t+1}^{L}=Y_t^H+Y_t^L$$

3.4.7 Competitive equilibrium

Recursive competitive equilibrium of our model economy is a price system u_t^H, u_t^L, q_t, R_t, household decision rules k_{t+1}^i, x_{t+1}, b_{t+1}^i and c_t^i, $i=H,L$ and equilibrium laws of motion for the endogenous state variables (3.7) and (3.7) such that

(i) The decision rules k_{t+1}^i, x_{t+1}^i, b_{t+1}^i and c_t^i, $i=H,L$ solve the household decision problem conditional upon the price system u_t^H, u_t^L, q_t, R_t.

(ii) The process governing the transition of the aggregate productivity and the household decision rules k_{t+1}^i, x_{t+1}^i and c_t^i, $i=H,L$ induce a transition process for the aggregate state s given by (3.7) and (3.8).

(iii) All markets clear

3.5 Calibration

We calibrate our model economy as follows. We set α, the share of fixed capital in output, equal to 0.2 in line with the calibration in Davis and Heathcote (2004) of the share of land in GDP. For the baseline calibration, I set ϕ, the percentage of output that can be seized in the event of default, to zero. So any collateralisability of output in the steady state is due to the value of intangible collateral. I also set $\triangle^I$, the standard deviation of 'ex post' idiosyncratic productivity shocks, equal to zero in the baseline calibration. Following the arguments in Chapter 2 I set the ratio of the productivities of the two groups to 1.15.

The discount factor β, the probability that a highly productive entrepreneur switches to low productivity δ, and the ratio of high to low productivity entrepreneurs n are parameters I pick in order to match three calibration targets - the ratio of tangible assets to GDP, aggregate leverage and the leverage of the most indebted decile of firms. The data is constructed in the same way as in Chapter 2 and the data sources are discussed in more detail in Appendix 4.B.7.

Finally, the high (low) realisations of the aggregate TFP shock are 0.5% above (below) the steady state TFP level. The probability that the economy remains in the same aggregate state it is today is equal to 0.8. Table 3.1 below displays a summary of the baseline calibration.

Parameter Name	Parameter Value
β	0.921
δ	0.344
n	0.066
α	0.20
p_{gg}	0.80
p_{bb}	0.80
$\triangle^A$	0.005
$\triangle^I$	0.00
ϕ	0.00
a^H	1.15

Table 3.1: Baseline calibration

3.6 Numerical Results for the Baseline Economy

3.6.1 Steady state comparative statics

In this section we consider how the steady state collateralisability of output θ varies with different features of the economy's production technology and nature of contract enforcement. Figure 3.3 shows the value of intangible collateral as a percentage of output. We compute the value of intangible collateral as the difference between the amount the market would lend against the firm's future output and the fraction of the firm's output which can be seized from defaulting entrepreneurs. The three lines on the chart correspond to three different values of a^H/a^L - the productivity differential between high and low productivity entrepreneurs. In the absence of any long term punishments for defaulters, all three lines on the figure should be zero - the downpayment should be exactly pinned down by the collateralisability of the firm's capital and output. But in our framework borrowing capacity is determined by the values of a borrower's reputation for repayment as well as by the value of tangible assets. We can see from the figure that intangible collateral first increases with ϕ before declining.

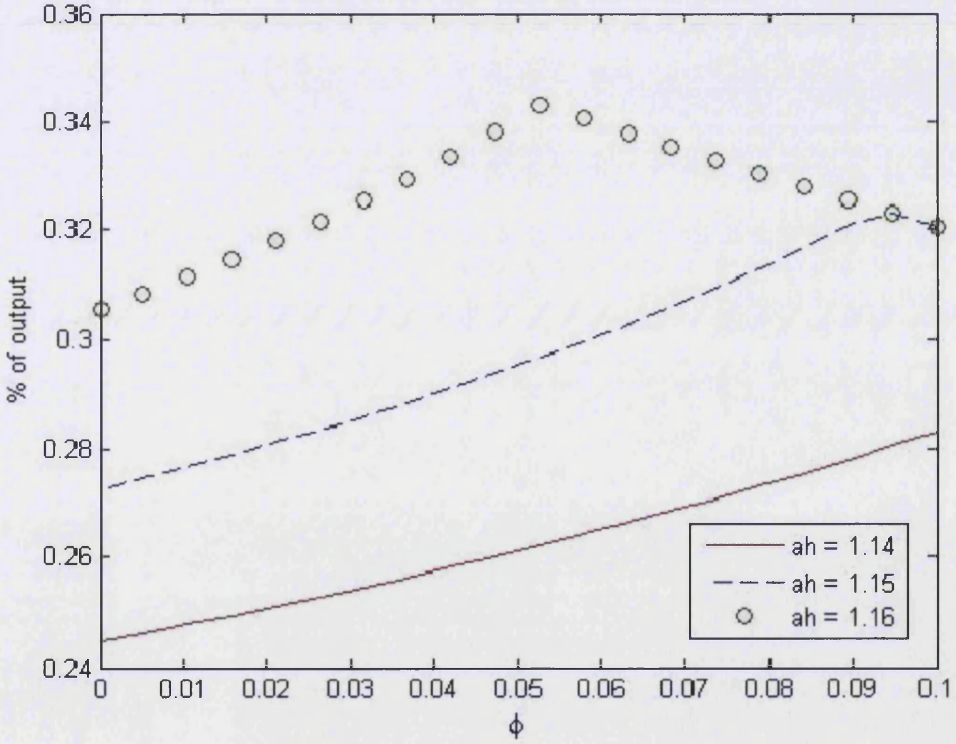


Figure 3.3: Collateralisability of output

Figure 3.4 below examines the determinants of the value of a borrower's reputation. The evolution of reputational collateral in response to changes in ϕ is governed by the interplay of the impact of rising capital prices and falling real interest rates on the excess return for high productivity entrepreneurs. While the economy is productively inefficient ($K < 1$), rising ϕ reduces real interest rates by depressing the rate of return to the production of low productivity entrepreneurs. Rising capital prices and falling real interest rates increase the leverage available to high productivity entrepreneurs, boosting the excess rate of return during high productivity spells. This in turn makes access to borrowing more attractive, driving up intangible collateral values higher and helping to increase leverage and capital prices even more. Once the economy achieves productive efficiency, productive entrepreneurs start to bid up the real interest rate, collectively reducing their excess return in the process. Lower excess returns, in turn, erode the value of reputational collateral.

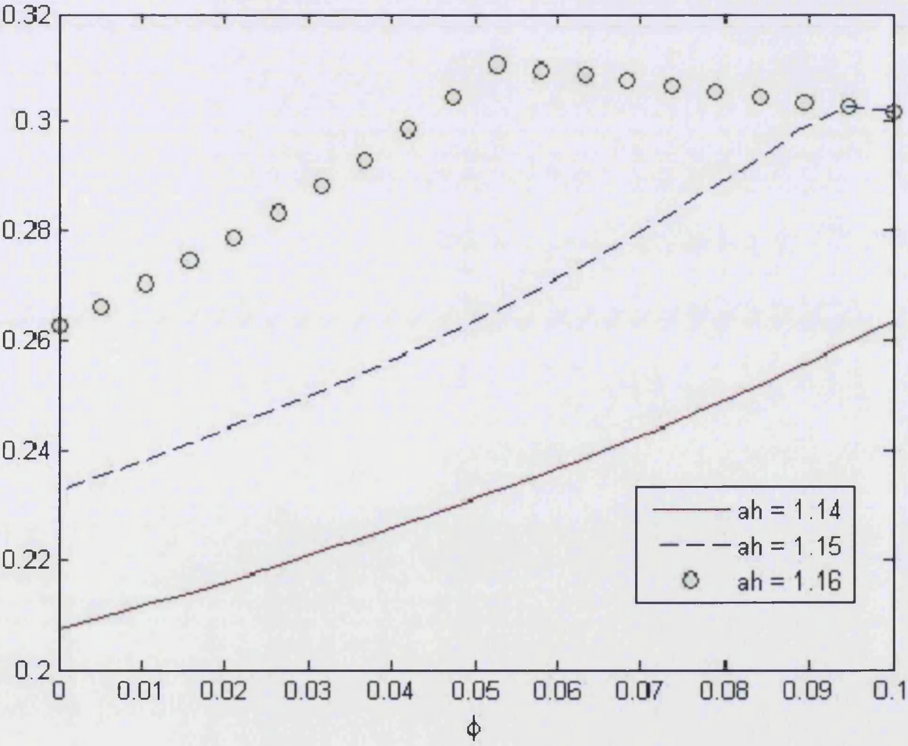


Figure 3.4: Excess return for high productivity entrepreneurs

Unsurprisingly, Figures 3.3 and 3.4 show, the value of repayment also increases as the productivity differential a^H rises. The bigger the productivity advantage the greater the benefit of leverage and therefore the greater the leverage a borrower can obtain by mortgaging his tangible assets and reputation for repayment.

3.6.2 Numerical results for the stochastic economy

Table 3.2 below shows how debt limits evolve over the cycle for different parameterisations of the economy. In the baseline case (the first column of the table) we can see that debt limits are slightly counter-cyclical. They tend to be lower in booms than in recessions though the difference is very small. The reason for this is that the net worth of high productivity entrepreneurs is low in recessions and this lowers asset prices. Low current asset prices implies higher rates of return on investment, and this is magnified by the availability of leverage. Access to debt markets is more beneficial in recessions in our economy because asset prices are low and the potential profits from leveraged investments is high.

In columns 2 and 3 we consider a situation in which firms face a high degree of idiosyncratic investment risk in recessions but not in booms. Counter-cyclical idiosyncratic production risk is capable of rationalising the pro-cyclical borrowing limits faced by firms. This is because uninsurable idiosyncratic risk reduces the value of leveraged investments in productive projects. As the value of an entrepreneur's reputation for repayment falls, this makes entrepreneurs less able to pledge it as collateral and borrow against its value in the capital market. As idiosyncratic uncertainty in the recession increases, the average value of reputational collateral falls and the gap between haircuts in the boom and the recession increases.

	Baseline	$\Delta^I = 0.10$	$\Delta^I = 0.20$	$\Delta^E = 0.0025$	$\Delta^A = 0.01$
$\theta(h)$	0.267	0.262	0.243	0.265	0.257
$\theta(l)$	0.268	0.259	0.230	0.264	0.260

Note: $\theta(i)$ is the average downpayment from a 2000 periods long simulation

Table 3.2: Borrowing limits over the economic cycle in the Debt Economy

3.7 Conclusions

This paper extends the collateral amplification framework of Kiyotaki and Moore (1997) and Kiyotaki (1998) by adding intangible collateral. Intangible collateral arises due an assumption that although lending can be done anonymously in this economy, borrowing cannot. Consequently, a defaulting entrepreneur not only loses a fraction of her tangible assets but also permanently loses her ability to borrow. When credit constraints bind, leveraged high productivity entrepreneurs have a rate of return on investments which exceeds the market interest rate. Leveraged production can boost low productivity firms' rate of return on wealth and consequently exclusion from debt markets is costly to borrowers. This generates a value for intangible collateral - in our model this is a borrower's reputation for repayment.

We study the way such intangible collateral varies with the nature of technology and contract enforcement in the economy both in steady state and over the business cycle. Steady state intangible collateral is higher the larger the excess return of leveraged production relative to saving. This is the case when the productivity differential between the high and low efficiency technology is large and when the collateralisability of tangible assets is high.

When we introduce aggregate uncertainty we found that the baseline model predicted, counterfactually, that downpayment requirements are mildly pro-cyclical. This is because credit constraints are tighter in recessions and the excess return of leveraged high productivity entrepreneurs is higher, increasing the value of intangible collateral. However, introducing counter-cyclical variability of idiosyncratic productivity shocks helped to generate pro-cyclical movements in the value of intangible collateral.

Chapter 4

Appendices

4.A Chapter 1

4.A.1 Solving the model

Solving the household's decision problem

We discretize net worth (x_t^i) using 400 grid points, with denser grids closer to zero to take into account the higher curvature of the value function in this region. The grid range for the continuous state variable is verified ex-post by comparing it with the values obtained in the simulations. For points which do not lie on the state space grid, we evaluate the value function using cubic spline interpolation along net worth. We simulate the idiosyncratic exogenous productivity shock from its three-point distribution. The realizations of these exogenous random variables are held constant when searching for the market clearing prices (p and r). We use the policy functions to simulate the behavior of 10000 agents over 600 (the exact number depends on the probability of exiting working life and the survival probability) periods and aggregate the individual housing and equity demands to determine the market clearing rental and housing price and the equilibrium household allocations.

Solving the perfect foresight model

We guess a sequence of tangible asset rental rates $\{r_t\}_{t=1}^{T}$ such that the rental rate has converged to the new steady state. For an exogenous real interest rate R in the small open economy, use (1.22) to calculate a sequence of capital stocks $\{K_t\}_{t=1}^{T}$ and then use (1.2) to compute the sequence of $\{Z_t\}$. Then we get tangible asset prices $\{q_t, p_t\}_{t=1}^{T}$ from (1.25) and $V_t^F = q_t Z_{t-1} = p_t Z_t - I_t$ (which follows from the firm flow-of-funds and the zero profit condition). Given these guessed prices, we solve the household's problem backwards from period T when the economy is assumed to have converged to the new steady state. Households are assumed to know the realization of the entire path of tangible asset prices and rental rates. The value function in period T is the value function for the new steady state. Then the value function in period T-1 is computed as follows:

$$V_{T-1}\left(x_{T-1}|r_{T-1}, p_{T-1}\right) = \max_{c_T, h_T}\left[u\left(c_{T-1}, h_{T-1}\right) + \beta V_T\left(x_T|r_T, p_T\right)\right]$$

We simulate the model forward, starting from the capital stock and the joint distribution of labor productivity and equity of the original steady state. In each period, we simulate a cross-section of 10000 agents over 600 periods and aggregate their individual housing choices, computing the excess demand for tangible assets in each period. We increase the rental rate in periods with an excess demand in the market for tangible assets use, and decrease the rental rate in periods with an excess supply, generating a new path $\{r_t\}_{t=1}^{T}$ of the rental rate. We repeat this until successive paths of the rental rate are less than 0.0001% from each other.

4.A.2 Stationary Representation of Value Functions

The stationary representation of the household's problem

Using the property of the steady state equilibrium of Section 2.4, we normalize the quantities and prices using the power function of labor in efficiency units $M_t \equiv A_t N_t$ and population N_t. Both variables are exogenous state variables, and there can be a jump or a kink in the trend if labor productivity experiences a once-for-all change in its level or growth rate. Let us denote the normalized variable X_t as $\widetilde{X}_t$. Then we have:

$$\begin{aligned}
\widetilde{K}_t &= K_t/M_t^{\frac{1-\eta}{1-\gamma\eta}}, \quad \widetilde{S}_t^* = S_t^*/M_t^{\gamma\frac{1-\eta}{1-\gamma\eta}} \\
(\widetilde{w}_t, \widetilde{x}_t) &= (w_t, x_t)/(M_t^{\frac{1-\eta}{1-\gamma\eta}}/N_t) \\
(\widetilde{h}_t, \widetilde{s}_t) &= (h_t, s_t)/(M_t^{\gamma\frac{1-\eta}{1-\gamma\eta}}/N_t) \\
(\widetilde{r}_t, \widetilde{p}_t, \widetilde{q}_t) &= (r_t, p_t, q_t)/M_t^{(1-\gamma)\frac{1-\eta}{1-\gamma\eta}} \\
\widetilde{V}_t^i &= V_t^i/\left[\frac{M_t^{\frac{1-\eta}{1-\gamma\eta}}/N_t}{M_t^{(1-\alpha)(1-\gamma)\frac{1-\eta}{1-\gamma\eta}}}\right]^{1-\rho}, \text{ for } i = l, m, h, \text{ or } r
\end{aligned}$$

We also define the normalized discount factor as:

$$\widetilde{\beta} = \beta\left(\frac{G_w}{G_r^{1-\alpha}}\right)^{1-\rho}.$$

Let us assume population grows along the steady state path. Let $\widetilde{A}_t$ be deviation of labor productivity from the trend. Then the vector of normalized state variables adjusted by the productivity change are:

$$\overline{\widetilde{A}}_t = \left(\widetilde{A}_t, \widetilde{K}_{t-1}, \widetilde{S}_{t-1}^*, \widetilde{\Phi}_t\left(\varepsilon_t, \widetilde{s}_{t-1}(i)\right)\right)'.$$

Using these normalized variables, we can define the normalized value function. For an example, the stationary representation of the retiree's problem is (noting that prices and quantities grow at different rates, explaining the use of (1.28) in the normalizations:

$$\widetilde{V}^r\left(\widetilde{x}_t, \overline{\widetilde{A}}_t\right) = Max($$

$$\max_{\widetilde{s}}\left\{\begin{array}{c} \frac{1}{1-\rho}\left[\frac{\widetilde{x}_t-(\widetilde{p}_t-\widetilde{r}_t)\widetilde{s}_t}{[\widetilde{r}_t/(1-\psi)]^{1-\alpha}}\right]^{1-\rho} \\ +\widetilde{\beta}\sigma\widetilde{V}^r\left(\widetilde{b}_{t+1}+\frac{\widetilde{q}_{t+1}}{\sigma}\frac{\widetilde{s}_t}{G_w}, \overline{\widetilde{A}}_{t+1}\right) \end{array}\right\},$$

$$\max_{\widetilde{s}}\left\{\begin{array}{c} \left\{\left[\frac{\widetilde{x}_t-\left(\widetilde{p}_t-\widetilde{r}_t+\frac{\widetilde{r}_t}{\theta}\right)\widetilde{s}_t}{\alpha}\right]^{\alpha}\left[\frac{\widetilde{s}_t\widetilde{r}_t/\theta}{1-\alpha}\right]^{1-\alpha}\right\}^{1-\rho}/(1-\rho) \\ +\widetilde{\beta}\sigma\widetilde{V}^r\left(\widetilde{b}_{t+1}+\frac{\widetilde{q}_{t+1}}{\sigma}\frac{\widetilde{s}_t}{G_w}, \overline{\widetilde{A}}_{t+1}\right) \end{array}\right\}, .$$

$$\max_{\widetilde{s}}\left\{\begin{array}{c} \frac{1}{1-\rho}\left[\frac{\widetilde{x}_t-(\widetilde{p}_t-\widetilde{r}_t)\widetilde{s}_t}{\widetilde{r}_t{}^{1-\alpha}}\right]^{1-\rho} \\ +\widetilde{\beta}\sigma\widetilde{V}^r\left(\widetilde{b}_{t+1}+\frac{\widetilde{q}_{t+1}}{\sigma}\frac{\widetilde{s}_t}{G_w}, \overline{\widetilde{A}}_{t+1}\right) \end{array}\right\})$$

4.A.3 Data sources and definitions

To compute the share of income of productive tangible assets (η), we use quarterly data from the US Flow of Funds accounts and from the NIPA for the period of 1952 Q1 - 2005Q4. We follow Cooley and Prescott (1995). We define unambiguous capital income as the sum of corporate profits (π), net interest (i), non-housing rental income (r) from the NIPA (table 1.12)[1]. We also measure the depreciation of capital (DEP) by the consumption of fixed capital (NIPA, table 1.14). We allocate η fraction of proprietors' income (Y_P, NIPA, Table 1.12) to the income from productive tangible assets. Then, the income from productive tangible assets, Y_{ZP}, can be computed as the sum of unambiguous capital income, depreciation, and η fraction of proprietors' income:

$$Y_{ZP} = \pi + i + r + DEP + \eta Y_P = \eta Y$$

where Y is GDP excluding explicit and implicit rents from housing. Solving this for η, we have

$$\eta = \frac{\pi + i + r + DEP}{Y - Y_P}$$

[1]We use the average share of residential to total structures to compute non-housing rental income from the total rental payments of all persons reported in NIPA table 1.12.

This is a similar expression for the share of capital in output found in Cooley and Prescott (1995, p.19).

Averaging the quarterly data for the U.S. from 1952 to 2005, we obtain a value of η equal to 0.26. This is lower than the share of capital in output in the real business cycle literature (estimates there range between 0.3 and 0.4) because our η excludes the capital intensive production of housing services. We can decompose economy-wide tangible assets between the household and the firm. The exact definitions in the data and their counterparts in the theoretical model are given in the following table:

Economic concept	**Flow of Funds concept**
pZ_y	Non-farm, non-financial tangible assets (Non-residential tangible assets+Equipment+software+Inventories) Flow of funds, Tables B.102 and B.103 FL102010005.Q+FL112010005.Q-FL115035023.Q
$p\int h(i)di = pH$	Household tangible assets (Residential tangible assets+Equipment+software+Consumer durables) Flow of funds, Table B.100 FL152010005.Q+FL115035023.Q

Non-corporate tangible assets include residential properties occupied by renters. Therefore, this series (FL115035023.Q) is subtracted from pZ_y and added to household tangible assets. Using these definitions, we compute the average numbers of $Z_Y/\left(Z_Y+H\right) = 0.59$ between 1952:Q1 and 2005:Q4. The ratio of total tangible assets to GDP $\left(p\left(Z_y+H\right)/Y\right)$ is 3.3, giving an average value of residential tangible assets to GDP of around 1.94. If farm corporate and non-corporate tangible assets (FL132010005.Q in the Flow of Funds)[2] are added to the non-farm tangible assets, then the ratio of household tangible assets to total tangible assets falls from 0.59 to 0.55, while the ratio of total tangible assets to GDP rises from 3.3 to 3.6.

[2]Thanks to Michael Palumbo (Board of Governors) of kindly sending us this series in private correspondence.

4.A.4 Survey of Consumer Finances

We use primarily the 1992 SCF to calibrate our parameters. The labor income process is intended to use entrepreneurial income on top of wages and salaries. Following Castaneda et. al. (2003) we add to wages and salaries and proportion of proprietors' income that can be attributed to self-employment. Thus, total labor income is wages and salaries plus 0.93 of business income where the 0.93 comes from the average ratio of (wages_sal/(wages_sal+bus_inc)). Net worth is total assets minus total debt for each household, corresponding to variable s in the model. The house value is the self-reported value of the primary residence conditional on owning a house. The SCF homeownership rate matches the Census one in 1992 exactly (64%).

4.B Chapter 2

4.B.1 Solving for the consumption function

Suppose the entrepreneur has optimally chosen her capital, labour and intermediate inputs and purchases/short sales of the risk free security. This means that she can earn a state contingent rate of return on invested wealth of R_{t+1}. The first order condition for optimal consumption then becomes:

$$\frac{1}{c_t} = \beta E_t \left(R_{t+1} \frac{1}{c_{t+1}} \right)$$

We guess that the entrepreneur consumes a fixed fraction of her available resources:

$$c_t = (1-\beta) z_t$$

where z_t is the entrepreneur's wealth. This means that

$$z_{t+1} = \beta R_{t+1} z_t$$

Substituting into the consumption Euler equation we have:

$$\begin{aligned} \frac{1}{(1-\beta) z_t} &= \beta E_t \left(R_{t+1} \frac{1}{(1-\beta) z_{t+1}} \right) \\ &= \beta E_t \left(R_{t+1} \frac{1}{(1-\beta) \beta R_{t+1} z_t} \right) \\ &= \frac{1}{(1-\beta) z_t} \end{aligned}$$

This confirms our initial guessed consumption function.

4.B.2 Solving for the rate of return on wealth of a high productivity entrepreneur

We start with the flow of funds constraint of the agent.

$$c_t + w_t h_t + x_t + q_t k_t - \frac{b_t}{R_t} = y_t + q_t k_{t-1} - b_{t-1}$$

From the conditions for optimal production (2.4) and (2.5) we know that

$$x_t = (1-\alpha)\, u_t^H k_t/\alpha$$

and

$$w_t h_t = (1-\alpha-\eta)\, u_t^H k_t/\alpha$$

Then if entrepreneurs borrow $l_t \leqslant \theta$ of the expected value of collateral, this allows us to solve for their debt choice:

$$b_t \leqslant l_t E_t q_{t+1} k_t$$

The entrepreneur's total saving is given by:

$$w_t h_t + x_t + q_t k_t - \frac{b_t}{R_t} = \left(q_t + (1-\alpha)\, u_t^H/\alpha - \frac{l_t E_t q_{t+1}}{R_t}\right) k_t$$

This will deliver the following level of wealth in the following period:

$$\begin{aligned} w_{t+1} &= y_{t+1} + q_{t+1} k_t - b_t \\ &= \left[\left(A_{t+1} a^H/\alpha\right) w_t^{\alpha+\eta-1} \left(u_t^H\right)^{1-\alpha} + q_{t+1} - l_t E_t q_{t+1}\right] k_t \end{aligned}$$

The entrepreneur's rate of return on total wealth invested is given by:

$$\begin{aligned} R_{t+1}^H &= \frac{y_{t+1} + q_{t+1} k_t - b_t}{x_{t+1} + q_t k_t - \frac{b_t}{R_t}} \\ &= \frac{\left(A_{t+1} a^H/\alpha\right) w_t^{\alpha+\eta-1} \left(u_t^H\right)^{1-\alpha} + q_{t+1} - l_t E_t q_{t+1}}{q_t + (1-\alpha)\, u_t^H/\alpha - (l_t/R_t)\, E_t q_{t+1}} \end{aligned}$$

4.B.3 Solving for the value function

The value function of an entrepreneur is:

$$V(z_t, a_t, X_t) = \max_{x_t,k_t,h_t,b_t,c_t} \{\ln c_t + \beta E_t V(z_{t+1}, a_{t+1}, X_{t+1})\} \tag{4.1}$$
$$= \max_{x_t,k_t,h_t,b_t} \{\ln(1-\beta) + \ln w_t + \beta E_t V(z_{t+1}, a_{t+1}, X_{t+1})\} \tag{4.2}$$

Guess that the solution is of the form

$$V(z_t, a_t, X_t) = \varphi(a_t, X_t) + \frac{\ln z_t}{1-\beta}$$

This implies that:

$$\begin{aligned}
&\varphi(a_t, X_t) + \frac{\ln z_t}{1-\beta} \\
= &\max_{x_t,k_t,h_t,b_t} \left\{\ln(1-\beta) + \ln z_t + \beta E_t \left[\varphi(a_{t+1}, X_{t+1}) + \frac{\ln z_{t+1}}{1-\beta}\right]\right\} \\
= &\max_{x_t,k_t,h_t,b_t} \left\{\ln(1-\beta) + \ln z_t + \frac{\beta}{1-\beta} E_t \left[\varphi(a_{t+1}, X_{t+1}) + \ln(\beta R_{t+1} z_t)\right]\right\} \\
= &\left\{\ln(1-\beta) + \frac{\beta \ln \beta}{1-\beta} + \frac{\beta}{1-\beta} E_t \left[\max_{x_t,k_t,h_t,b_t} [\ln(R_{t+1})] + \varphi(a_{t+1}, X_{t+1})\right] + \frac{\ln z_t}{1-\beta}\right\}
\end{aligned}$$

Equating coefficients we get the expression for the intercept of the value function:

$$\varphi(a_t, X_t) = \ln(1-\beta) + \frac{\beta \ln \beta}{1-\beta} + \frac{\beta}{1-\beta} \max_{x_t,k_t,h_t,b_t} E_t \left[[\ln(R_{t+1})] + \varphi(a_{t+1}, X_{t+1})\right]$$

The above expression shows that the agent has to choose productive inputs and borrowing so as to maximise the expected log rate of return on wealth in each period.

So the value of an entrepreneur in our economy depends on the net present value of expected log returns on the optimal portfolio as well as the log of current financial wealth.

4.B.4 Solving for the rate of return on wealth of a low productivity entrepreneur

We start with the flow of funds constraint of the agent.

$$c_t + w_t k_t + x_t + q_t k_t + \frac{b_t}{R_t} = y_t + q_t k_{t-1} + b_{t-1}$$

From the condition for optimal production (2.4) and (2.5) we know that

$$x_t = (1-\alpha)\, u_t^L k_t/\alpha$$

and

$$w_t h_t = (1-\alpha-\eta)\, u_t^L k_t/\alpha$$

The entrepreneur's total saving is given by:

$$w_t h_t + x_t + q_t k_t + \frac{b_t}{R_t} = \left(q_t + (1-\alpha)\, u_t^L/\alpha\right) k_t + \frac{b_t}{R_t}$$

This will deliver the following level of wealth in the following period:

$$\begin{aligned} w_{t+1} &= y_{t+1} + q_{t+1} k_t + b_t \\ &= \left[(A_{t+1}/\alpha)\, w_t^{\alpha+\eta-1} \left(u_t^L\right)^{1-\alpha} + q_{t+1}\right] k_t + b_t \end{aligned}$$

The entrepreneur's rate of return on total wealth invested is given by:

$$\begin{aligned} R_{t+1}^L &= \frac{y_{t+1} + q_{t+1} k_t + b_t}{w_t h_t + x_t + q_t k_t + \frac{b_t}{R_t}} \\ &= \frac{\left[(A_{t+1}/\alpha)\, w_t^{\alpha+\eta-1} \left(u_t^L\right)^{1-\alpha} + q_{t+1}\right] k_t + b_t}{\left(q_t + (1-\alpha)\, u_t^L/\alpha\right) k_t + b_t} \end{aligned}$$

Imposing market clearing in the capital and debt markets and recognising that all low productivity entrepreneurs chose the same portfolio, we get the following equilibrium rate of return on wealth for the low type:

$$R_{t+1}^{L} = \frac{\left[(A_{t+1}/\alpha)\, w_t^{\alpha+\eta-1} \left(u_t^L\right)^{1-\alpha} + q_{t+1} \right] (1-K_t) + l_t E_t q_{t+1} K_t}{\left(q_t + (1-\alpha)\, u_t^L/\alpha\right)(1-K_t) + l_t E_t q_{t+1} K_t / R_t}$$

where K_t is the aggregate capital-holding of the high productivity entrepreneurs.

4.B.5 Approximating the optimal portfolio problem as a mean variance utility problem

The entrepreneur's portfolio problem involves maximising the log return on his portfolio of assets. The portfolio can be written as the weighted sum of the return on the risky asset and the rate of return on the safe asset

$$R_{t+1}^{i} = \varpi_t^i \left[\frac{(A_{t+1} a^i/\alpha)\, w_t^{\alpha+\eta-1} \left(u_t^i\right)^{1-\alpha} + q_{t+1}}{q_t + (1-\alpha)\, u_t^i/\alpha} \right] + \left(1 - \varpi_t^i\right) R_t \tag{4.3}$$

Let

$$R^{i*} = \max_{\varpi_t^i} E_t \ln R_{t+1}^i$$

denote the maximum value of the expected log portfolio return. Using the approximation

$$E_t \ln x \approx \ln E_t x - \frac{1}{2} var(\ln x) \tag{4.4}$$

we can write the portfolio problem as a mean-variance utility maximisation problem.

High Productivity Entrepreneurs

For high productivity entrepreneurs the (4.3) expression above can be written as follows:

$$\begin{aligned}
R^H_{t+1} &= \varpi^H_t \left[\frac{\left(A_{t+1}a^H/\alpha\right) w_t^{\alpha+\eta-1}\left(u^H_t\right)^{1-\alpha} + q_{t+1}}{q_t + (1-\alpha)\, u^H_t/\alpha}\right] + \left(1-\varpi^H_t\right) R_t \\
&= R_t + \varpi^H_t \left[\frac{\left(A_{t+1}a^H/\alpha\right) w_t^{\alpha+\eta-1}\left(u^H_t\right)^{1-\alpha} + q_{t+1}}{q_t + (1-\alpha)\, u^H_t/\alpha} - R_t\right] \\
&= R_t \left\{1 + \varpi^H_t \left[\frac{\left(A_{t+1}a^H/\alpha\right) w_t^{\alpha+\eta-1}\left(u^H_t\right)^{1-\alpha} + q_{t+1}}{q_t + (1-\alpha)\, u^H_t/\alpha} / R_t - 1\right]\right\} \\
&\equiv R_t \left\{1 + \varpi^H_t \left[\rho^H_{t+1} - 1\right]\right\}
\end{aligned}$$

where

$$\rho^H_{t+1} = \frac{\left(A_{t+1}a^H/\alpha\right) w_t^{\alpha+\eta-1}\left(u^H_t\right)^{1-\alpha} + q_{t+1}}{q_t + (1-\alpha)\, u^H_t/\alpha} / R_t$$

Taking logs and using the approximation $\ln(1+x) \approx x$ for small x we have

$$\ln R^H_{t+1} \approx \ln R_t + \varpi^H_t \left[\rho^H_{t+1} - 1\right]$$

Applying the approximation (4.4) we have:

$$\begin{aligned}
R^{H*} &\approx \max_{\varpi^H_t} \left[\ln E_t R^H_{t+1} - \frac{1}{2} var\left(\ln R^H_{t+1}\right)\right] \\
&= \max_{\varpi^H_t} \left[\ln R_t + \ln\left(1 + \varpi^H_t \left(E_t \rho^H_{t+1} - 1\right)\right) - \frac{1}{2} var\left(\ln R_t + \ln\left(1 + \varpi^H_t \left(\rho^H_{t+1} - 1\right)\right)\right)\right] \\
&\approx \max_{\varpi^H_t} \left[\ln R_t + \varpi^H_t \left(E_t \rho^H_{t+1} - 1\right) - \frac{1}{2} var\left(\ln R_t + \varpi^H_t \left(\rho^H_{t+1} - 1\right)\right)\right] \\
&\approx \max_{\varpi^H_t} \left[\ln R_t + \varpi^H_t \left(E_t \rho^H_{t+1} - 1\right) - \frac{\left(\varpi^H_t\right)^2}{2} \sigma^2_{Rt+1}\right]
\end{aligned}$$

Define

$$\psi^H_{t+1} = \left(a^H/\alpha\right)\left(u^H_t\right)^{1-\alpha}$$

as output per efficiency unit of capital at time $t+1$ for high productivity entrepreneurs. Then the variance of the risky asset's rate of return is given by:

$$\begin{aligned}
\sigma_{Rt+1}^{2} &= \left(\frac{\psi_{t+1}^{H}}{q_t+(1-\alpha)\,u_t^{H}/\alpha}\right)^{2}\sigma_A^2+\left(\frac{\psi_{t+1}^{H}}{q_t+(1-\alpha)\,u_t^{H}/\alpha}\right)\sigma_{Aqt+1} \\
&\quad +\left(\frac{1}{q_t+(1-\alpha)\,u_t^{H}/\alpha}\right)^{2}\sigma_{qt+1}^{2} \\
&= \frac{\left(\psi_{t+1}^{H}\right)^{2}\sigma_A^2+\psi_{t+1}^{H}\left(q_t+(1-\alpha)\,u_t^{H}/\alpha\right)\sigma_{Aqt+1}+\sigma_{qt+1}^{2}}{\left(q_t+(1-\alpha)\,u_t^{H}/\alpha\right)^{2}}
\end{aligned}$$

where σ_A^2 is the variance of the technology shock, σ_{Aqt+1} is the conditional covariance of the technology shock and the capital price and σ_{qt+1}^2 is the conditional variance of the capital price.

Low Productivity Entrepreneurs

Analogously with the previous subsection we learn that the log rate of return on wealth for low productivity agents can be approximated by:

$$\ln R_{t+1}^{L}\approx\ln R_t+\varpi_t^{L}\left[\rho_{t+1}^{L}-1\right]$$

where

$$\rho_{t+1}^{L}=\frac{\left(A_{t+1}/\alpha\right)\left(u_t^{L}\right)^{1-\alpha}+q_{t+1}}{q_t+(1-\alpha)\,u_t^{L}/\alpha}/R_t$$

Then we can approximate the expected log rate of return on wealth of low productivity agents by the following expression:

$$\begin{aligned}
R^{L*} &\approx \max_{\varpi_t^{L}}\left[\ln E_tR_{t+1}^{L}-\frac{1}{2}var\left(\ln R_{t+1}^{L}\right)\right] \\
&\approx \max_{\varpi_t^{L}}\left[\ln R_t+\varpi_t^{L}\left(E_t\rho_{t+1}^{L}-1\right)-\frac{\left(\varpi_t^{L}\right)^{2}}{2}\sigma_{rt+1}^{2}\right]
\end{aligned}$$

Define

$$\psi_{t+1}^{L}=(1/\alpha)\left(u_t^{L}\right)^{1-\alpha}$$

as output per efficiency unit of capital at time $t+1$ for low productivity entrepreneurs. Then the variance of the risky asset's rate of return is given by:

$$\begin{aligned}\sigma_{rt+1}^2 &= \left(\frac{\psi_{t+1}^L}{q_t + (1-\alpha)\, u_t^L/\alpha}\right)^2 \sigma_A^2 + \left(\frac{\psi_{t+1}^L}{q_t + (1-\alpha)\, u_t^L/\alpha}\right) \sigma_{Aqt+1} \\ &+ \left(\frac{1}{q_t + (1-\alpha)\, u_t^L/\alpha}\right)^2 \sigma_{qt+1}^2 \\ &= \frac{\left(\psi_{t+1}^L\right)^2 \sigma_A^2 + \psi_{t+1}^L \left(q_t + (1-\alpha)\, u_t^L/\alpha\right) \sigma_{Aqt+1} + \sigma_{qt+1}^2}{\left(q_t + (1-\alpha)\, u_t^L/\alpha\right)^2}\end{aligned}$$

Again, just like in the previous subsection, the variance of the risky rate of return for the low productivity is driven by σ_A^2 - the variance of the technology shock, σ_{Aqt+1} - the conditional covariance of the technology shock and the capital price and σ_{qt+1}^2 - the conditional variance of the capital price.

4.B.6 The Frictionless Benchmark

The Problem of Entrepreneurs

Entrepreneurs solve the following problem

$$\max_{c_t, x_t} E_t \sum_{t=0}^{\infty} \beta^t \ln c_{t+s}^e$$

subject to the resource constraint:

$$c_t^e + x_t + u_t + w_t h_t^d + \sum_s \frac{b_t^s}{R_{t+1}^s} = \frac{a^H A_t}{\alpha^\alpha} \left(\frac{x_{t-1}}{\eta}\right)^\eta \left(\frac{h_{t-1}}{1-\alpha-\eta}\right)^{1-\alpha-\eta} + b_{t-1}$$

Here we have already taken into account the fact that only high productivity entrepreneurs will produce in equilibrium and the entire capital supply will be used in production. $\sum_s \frac{b_t^s}{R_{t+1}^s}$ are the entrepreneurs' net purchases (or sales) of Arrow securities at price $1/R_{t+1}^s$ from workers. The first order conditions are as follows:

(1) Investment

$$x_t = \frac{\eta}{\alpha} u_t$$

(2) Labour demand

$$w_t h_t^d = \frac{1-\alpha-\eta}{\alpha} u_t$$

(3) Arbitrage between production and Arrow securities

$$R^s_{t+1} = \frac{a^H A^s_{t+1}}{\alpha^\alpha}\left(\frac{x_{t-1}}{\eta}\right)^\eta \left(\frac{h_{t-1}}{1-\alpha-\eta}\right)^{1-\alpha-\eta}$$

(4) Entrepreneurs' consumption function

$$c^e_t = (1-\beta)\left[\frac{a^H A_t}{\alpha^\alpha}\left(\frac{x_{t-1}}{\eta}\right)^\eta \left(\frac{h_{t-1}}{1-\alpha-\eta}\right)^{1-\alpha-\eta} + b_{t-1}\right]$$

The Problem of Workers

Workers have the following preferences

$$\max_{c^w_t, h_t} E_t \sum_{s=0}^{\infty} \beta^t \ln\left(c^w_t - \varkappa \frac{h_t^{1+\omega}}{1+\omega}\right)$$

subject to the resource constraint:

$$c^w_t + \sum_s \frac{b^s_t}{R^s_{t+1}} = b_{t-1} + w_t h_t$$

First order conditions are given by:

$$\frac{1}{c^w_t - \varkappa \frac{h_t^{1+\omega}}{1+\omega}} = \beta \pi^s \frac{R^s_{t+1}}{c^{ws}_{t+1} - \varkappa \frac{\left(h^s_{t+1}\right)^{1+\omega}}{1+\omega}}$$

and

$$w_t = \varkappa h^\omega_t \tag{4.5}$$

We can derive the consumption function of the workers as follows. Define:

$$\begin{aligned} \widetilde{c}_t &= c^w_t - \varkappa \frac{h_t^{1+\omega}}{1+\omega} \\ &= c^w_t - \varkappa \frac{(w_t/\varkappa)^{\frac{1+\omega}{\omega}}}{1+\omega} \end{aligned}$$

and

$$\begin{aligned}
\widetilde{w_t} &= w_t h_t - \varkappa \frac{h_t^{1+\omega}}{1+\omega} \\
&= w_t \left(w_t/\varkappa\right)^{\frac{1}{\omega}} - \varkappa \frac{\left(w_t/\varkappa\right)^{\frac{1+\omega}{\omega}}}{1+\omega}
\end{aligned}$$

Then redefine the inter-temporal budget constraint using $\widetilde{c_t}$ and $\widetilde{w_t}$:

$$\widetilde{c_t} + \sum_s \frac{b_t^s}{R_{t+1}^s} = b_{t-1} + \widetilde{w_t}$$

and the Euler equation:

$$\frac{1}{\widetilde{c_t}} = \beta \pi^s \frac{R_{t+1}^s}{\widetilde{c}_{t+1}}$$

This problem now looks like the standard consumption-savings problem with log utility. The consumption function is:

$$\widetilde{c_t} = (1-\beta)\left(H_t + b_{t-1}\right)$$

where

$$H_t = \widetilde{w_t} + E_t\left(\frac{H_{t+1}}{R_{t+1}}\right)$$

is the human wealth of the worker. The workers' aggregate consumption function is therefore given by:

$$c_t^w = (1-\beta)\left(H_t + b_{t-1}\right) + \varkappa \frac{\left(w_t/\varkappa\right)^{\frac{1+\omega}{\omega}}}{1+\omega}$$

Aggregate consumption in the economy is given by:

$$c_t^w + c_t^e = (1-\beta)\left(H_t + q_t + Y_t\right) + \varkappa \frac{\left(w_t/\varkappa\right)^{\frac{1+\omega}{\omega}}}{1+\omega}$$

The full set of aggregate equilibrium conditions

Aggregate output

$$Y_t = \frac{a^H A_t}{\alpha} u_t^{1-\alpha} w_t^{\alpha+\eta-1}$$

Market clearing

$$(1-\beta)(H_t + q_t + Y_t) + \varkappa \frac{(w_t/\varkappa)^{\frac{1+\omega}{\omega}}}{1+\omega} + \frac{\eta}{\alpha} u_t = \frac{a^H A_t}{\alpha} u_t^{1-\alpha} w_t^{\alpha+\eta-1}$$

Human wealth

$$H_t = w_t (w_t/\varkappa)^{\frac{1}{\omega}} - \varkappa \frac{(w_t/\varkappa)^{\frac{1+\omega}{\omega}}}{1+\omega} + E_t\left(\frac{H_{t+1}}{R_{t+1}}\right)$$

Price of capital

$$q_t = u_t + E_t\left(\frac{q_{t+1}}{R_{t+1}}\right)$$

Arrow security price

$$R_{t+1} = a^H A_t u_t^{-\alpha} w_t^{\alpha+\eta-1}$$

Labour demand

$$w_t (w_t/\varkappa)^{\frac{1}{\omega}} = \frac{1-\alpha-\eta}{\alpha} u_t$$

4.B.7 Data Definitions and Sources

Computing the share of capital in private value added

We compute the share of capital in private value

$$\widetilde{\alpha} \equiv \frac{\alpha}{1-\eta}$$

added following the method in Cooley and Prescott (1995). We define unambiguous capital income (Y^U) as the sum of [] and ambiguous capital income (Y^A) as Proprietors income. We assume that the share of capital in ambiguous capital income is equal to its share in total national income. All series are obtained from the BEA national accounts. Then the share of capital in total income (Y) is defined as the sum of unambiguous capital income and the capital share of ambiguous capital income:

$$\widetilde{\alpha}Y = Y^U + \widetilde{\alpha}Y^A$$

Hence

$$\widetilde{\alpha} = \frac{Y^U}{Y - Y^A}$$

Computing η the share of intermediate inputs in gross output

We use the BEA Industrial Accounts to compute this parameter. The Industrial Accounts produces sector by sector input output tables, showing the value added and gross output of each sector. This allows us to compute the share of intermediate inputs for each sector. The aggregate share of intermediate inputs can be obtained by averaging across all the sectors. Weighting different sectors by their weight in aggregate gross output gave almost identical results.

Computing the ratio of tangible assets to GDP

We compute the economy's stock of tangible assets by adding the nominal value of tangible assets of the Household (Table B.100, FL152010005), Corporate Non-Financial sector (Table B.102, FL102010005) and Non-corporate Non-Financial sector (Table B.103, FL112010005) from the September 2009 release of the US Flow of Funds. GDP is nominal GDP excluding the value added of the Government sector (Table 1.1.5, Line 1-Line 21). Data is for the period 1952-2008. The model counterparts to the ratio of tangible assets to GDP is defined as follows:

$$\frac{q}{Y^H + Y^L - X^H - X^L}$$

Computing aggregate corporate leverage

We use corporate (Table B.102, FL102000005) and non-corporate (Table B.103, FL112000005) total assets. This includes both tangible and financial assets on firms' books. For corporate net worth we use the market value of corporate equity (Table

B.102, FL103164003). For non-corporate net worth we use the net worth data in Table B.103, FL112090205. Leverage is computed as (Assets-Net Worth)/Assets.

The model counterpart to aggregate corporate leverage is defined as follows:

$$L^A = \frac{\theta q K}{q + (Y^H + Y^L)/R}$$

Computing the second moments in the data

Our measure of GDP is private sector value added (Table 1.1.5, Line 1-Line 21). Consumption is the sum of non-durable goods and services consumption. The value of the firm is proxied by the S&P 500. All series have been deflated by the non-durable goods deflator to convert them them into real terms (non-durables consumption goods). All data is annual and the data sample is 1929-2008. Total employment in hours is obtained from the Bureau of Labour Statistics. The sample is 1964 - 2008. We convert the monthly data into annual averages. All data is detrended using the HP filter. Following Uhlig and Ravn (2001) we use a smoothing parameter of 2.06 for annual data

4.B.8 Deriving the tax wedge formulation for steady state u_t^H

The user cost of capital for low productivity entrepreneurs is

$$u_t^L = q_t - \frac{q_{t+1}}{R_t}$$

while that for high productivity entrepreneurs is

$$u_t^H = q_t - \left[\frac{\theta}{R_t} + \frac{1-\theta}{R_{t+1}^H}\right] q_{t+1}$$

We can re-write the u_t^H expression in terms of the excess return on wealth for high productivity entrepreneurs

$$\begin{aligned} u_t^H &= q_t - \frac{q_{t+1}}{R_t} + (1-\theta)\, q_{t+1} \left(\frac{1}{R_t} - \frac{1}{R_{t+1}^H} \right) \\ &= u_t^L + (1-\theta) \frac{q_{t+1}}{R_t} \left(1 - \frac{1}{\rho_{t+1}} \right) \end{aligned}$$

where

$$\rho_{t+1} = \frac{R_{t+1}^H}{R_t}$$

is the excess return. We can use the user cost expression to substitute out the expected future price in terms of 'ex-dividend' value of capital:

$$\begin{aligned} u_t^H &= u_t^L + (1-\theta)\left(q_t - u_t^L\right)\left(1 - \frac{1}{\rho_{t+1}}\right) \\ &= u_t^L \left[1 + (1-\theta)\left(\frac{q_t}{u_t^L} - 1\right)\left(1 - \frac{1}{\rho_{t+1}}\right) \right] \\ &= u_t^L \left[1 + \tau_t\right] \end{aligned}$$

This completes our derivation of the downpayment 'tax wedge':

$$\tau_t = (1-\theta)\left(\frac{q_t}{u_t^L} - 1\right)\left(1 - \frac{1}{\rho_{t+1}}\right)$$

4.B.9 Deriving the level of TFP in steady state

The level of TFP is given by the following expression:

$$TFP_t = \frac{Y_t^H + Y_t^L}{\left(\frac{1}{\alpha}\right)^{\alpha} \left(\frac{H_{t-1}^H + H_{t-1}^L}{\eta}\right)^{\eta} \left(\frac{X_{t-1}^H + X_{t-1}^L}{1-\alpha-\eta}\right)^{1-\alpha-\eta}}$$

We know that:

$$Y_t^L = \frac{1}{\alpha} w_{t-1}^{\alpha+\eta-1} \left(u_{t-1}^L\right)^{1-\alpha} (1 - K_{t-1})$$

$$\begin{aligned} Y_t^H &= \frac{a^H}{\alpha} w_{t-1}^{\alpha+\eta-1} \left(u_{t-1}^H\right)^{1-\alpha} K_{t-1} \\ &= \frac{a^H \left(1 + \tau(\theta)\right)^{1-\alpha}}{\alpha} w_{t-1}^{\alpha+\eta-1} \left(u_{t-1}^L\right)^{1-\alpha} K_{t-1} \end{aligned}$$

where we have used the fact that $u_t^H = (1+\tau(\theta))\,u_t^L$. Aggregate intermediate input investment in given by:

$$\begin{aligned} X_{t-1}^H + X_{t-1}^L &= \frac{\eta}{\alpha} u_{t-1}^L \left(1 - K_{t-1} + (1+\tau(\theta))\,K_{t-1}\right) \\ &= \frac{\eta}{\alpha} u_{t-1}^L \left(1 + \tau(\theta)\,K_{t-1}\right) \end{aligned}$$

$$\begin{aligned} H_{t-1}^H + H_{t-1}^L &= \frac{1-\alpha-\eta}{\alpha} \frac{u_{t-1}^L}{w_{t-1}} \left(1 - K_{t-1} + (1+\tau(\theta))\,K_{t-1}\right) \\ &= \frac{1-\alpha-\eta}{\alpha} \frac{u_{t-1}^L}{w_{t-1}} \left(1 + \tau(\theta)\,K_{t-1}\right) \end{aligned}$$

Aggregate TFP for our economy is therefore given by:

$$TFP_t = \frac{1 + K_{t-1}\left[a^H (1+\tau(\theta))^{1-\alpha} - 1\right]}{1 + \tau(\theta)\,K_{t-1}}$$

4.B.10 Deriving the aggregate state

In setting up the individual maximisation problem, we had assumed that aggregate wealth Z_t and the share of wealth that belongs to productive individuals d_t are the key endogenous state variables. Following the derivation of the conditions for optimal consumption and investment by entrepreneurs, we can see why this is indeed the case. We do this by showing that our market clearing conditions are functions of current and expected future market prices as well as the state variables in question.

Starting with the bond market clearing condition (2.15) we can see straight away from the collateral constraint that the gross amount of debt in any given period is given by the condition:

$$B_t = \theta E_t q_{t+1} K_t$$

The aggregate capital holding of high productivity entrepreneurs is given by:

$$K_t = \beta \frac{d_t Z_t}{q_t + (1-\alpha)\,u_t^H/\alpha - \theta E_t q_{t+1}/R_t}$$

which implies that debt is a function of market prices and W_t and d_t.

Moving on to the capital market clearing condition (2.16) we already know that capital demand by high productivity agents is recursive in the aggregate state. The capital demand of low productivity entrepreneurs is:

$$\begin{aligned}(1-K_t) &= \frac{\beta(1-d_t)Z_t - B_t/R_t}{\left(q_t + \frac{1-\alpha}{\alpha}u_t^L\right)} \\ &= \frac{\beta(1-d_t)Z_t - \theta E_t q_{t+1} K_t/R_t}{\left(q_t + \frac{1-\alpha}{\alpha}u_t^L\right)}\end{aligned}$$

This implies that the capital market clearing condition is a function of market prices as well as W_t and d_t.

Finally looking at the goods market clearing condition (2.17) we can see that because of log utility, consumption is proportional to individual wealth and, consequently, aggregate consumption by entrepreneurs is proportional to aggregate wealth:

$$C_t^E \equiv C_t^H + C_t^L = (1-\beta) Z_t$$

The consumption of workers is very simple because they do not save in equilibrium:

$$C_t^W = w_t H_t = w_t \left(\frac{w_t}{\chi}\right)^{\frac{1}{v}}$$

Due to the Cobb-Douglas production function, the aggregate expenditure on intermediate input in the economy is given by the following expression:

$$X_t^H + X_t^L = \frac{1-\alpha}{\alpha}\left(u_t^H K_t + u_t^L (1-K_t)\right)$$

where we already know that the capital demands of the two groups are recursive in the state. The definition of total wealth implies that:

$$Y_t^H + Y_t^L = W_t - q_t$$

So goods market clearing depends on market prices as well as W_t and d_t.

4.B.11 Deriving the Social Welfare Function

The government solves the following policy problem.

$$\Omega_0 = \max_{\{\chi^i\}} E_0 \left[\sum_i \varsigma_E^i \sum_{t=0}^{\infty} \beta^t \ln c_t^i \right] + \varsigma_W \sum_{t=0}^{\infty} \beta^t \ln \left(C_t^W - \varkappa \frac{(H_t)^{1+\omega}}{1+\omega} \right) \tag{4.6}$$

We can represent the net present value of period utilities of the two groups as the sum of Pareto weighted value functions:

$$\begin{aligned} \Omega_0 &= \max_{\{\chi^i\}} E_0 \left[\sum_i \varsigma_E^i V^E \left(z_0^i, a_0^i, X_0 | \chi^i \right) + \varsigma^W V^W \left(X_0 | \chi^i \right) \right] && (4.7) \\ &= \max_{\{\chi^i\}} E_0 \left[\sum_i \varsigma^i \left(\varphi \left(a_0^i, X_0 | \chi^i \right) + \frac{\ln z_0^i \left(\chi^i \right)}{1-\beta} \right) + \varsigma^W V^W \left(X_0 | \chi^i \right) \right] && (4.8) \end{aligned}$$

Under the assumption that all entrepreneurs hold their group average level of initial wealth and all workers hold zero wealth allows us to re-write the value function (4.8) as follows:

$$\Omega_0 = \max_{\{\chi^i\}} E_0 \left[\begin{array}{c} d_0 Z_0 \left(\varphi^H \left(X_0 | \chi^i \right) + \frac{\ln Z_0^H (\chi^i)}{1-\beta} \right) + \\ (1-d_0) Z_0 \left(\varphi^H \left(X_0 | \chi^i \right) + \frac{\ln Z_0^H (\chi^i)}{1-\beta} \right) + \left(\frac{1+\omega}{\omega} w_t \right) V^W \left(X_0 | \chi^i \right) \end{array} \right]$$

4.B.12 Sensitivity Analysis

Sensitivity to a^H

We performed extensive sensitivity analysis to check whether the value of a^H affected the results. We found that it did not and the result from the exercise are shown in Table 4.1 below. Again, at each value of a^H, the model is recalibrated for each parameter value in order to match our five targets from the data).

The value of a^H has two offsetting effects on the incentives to regulate. A higher value of a^H increases amplification because fluctuations in the share of wealth of high productivity entrepreneurs leads to bigger endogenous fluctuations in TFP and land prices. This would increase the incentive of the government to impose capital requirements in order to dampen the amplification mechanism. But a higher value of a^H also increases the benefits of getting more funds into productive hands so the welfare costs of capital requirements in terms of lower average productivity and consumption also increase. We examined a number of different values of a^H and found that at all of them, the government chose not to regulate.

	$a^H = 1.05$	*Baseline*	$a^H = 1.25$
$100\Delta \ln \varphi_0^H$	-0.25	-0.23	0.02
$100\Delta \ln Z_0^H$	-1.07	-4.15	-7.60
$100\Delta \ln V_0^H$	-0.44	-1.15	-2.01
$100\Delta \ln \varphi_0^L$	0.26	1.11	1.97
$100\Delta \ln Z_0^L$	-0.16	-0.71	-1.55
$100\Delta \ln V_0^L$	0.03	0.04	-0.11
$100\Delta \ln V_0^W$	0.06	0.14	0.17
$100\Delta \ln V_0$	-0.10	-0.33	-0.66
$100\Delta\sigma_c$	-0.02	-0.12	-0.21
$100\Delta\sigma_{cW}$	-0.01	-0.03	-0.08
$100\Delta\sigma_{cH}$	-0.22	-0.60	-0.75
$100\Delta\sigma_{cL}$	0.02	-0.06	-0.18

Table 4.1: Capital requirements and welfare under different values of ah

Sensitivity to the form of the borrowing constraint

'Worst case' borrowing limit We also experimented with an alternative borrowing constraint of the form:

$$b_t \leqslant \theta q_{t+1}^l k_t$$

Such a constraint focuses on the value of collateral in the low aggregate productivity state. So it would be equivalent to a 'worst case' scenario value of collateral. Such a borrowing constraint also introduces two opposing incentives for the government. The case for higher regulation arises because the externality is much more severe under this constraint. This is because volatility of asset prices now has a first order effect on borrowing constraints. The more volatile land prices are, the more constrained entrepreneurs become because lenders become worried by large falls in the land price. This externality means that capital requirements might be beneficial because they reduce volatility and may even relax borrowing constraints.

But there is another offsetting effect. Suppose entrepreneurs attempt to leverage up and this leads to an increase in land price volatility. This would lead to tighter borrowing limits, stopping the rise in leverage in the first place. So the 'worst case' borrowing constraints exhibit a lot self-regulation which is missing in the standard 'expected value' borrowing constraints we consider in the main paper. This self-regulation effect makes government regulation unnecessary in equilibrium.

	$\theta = 0.80$	$\theta = 0.90$	$\theta = 1.00$
$100\Delta \ln \varphi_0^H$	-0.33	-0.31	-0.25
$100\Delta \ln Z_0^H$	-1.06	-1.79	-3.68
$100\Delta \ln V_0^H$	-0.33	-0.55	-1.04
$100\Delta \ln \varphi_0^L$	0.37	0.54	0.96
$100\Delta \ln Z_0^L$	-0.14	-0.30	-0.61
$100\Delta \ln V_0^L$	0.05	0.04	0.05
$100\Delta \ln V_0^W$	-0.09	-0.02	0.11
$100\Delta \ln V_0$	-0.18	-0.34	-0.29
$100\Delta\sigma_c$	-0.02	-0.03	-0.04
$100\Delta\sigma_{cW}$	-0.01	-0.02	-0.01
$100\Delta\sigma_{cH}$	-0.08	-0.13	-0.20
$100\Delta\sigma_{cL}$	0.00	0.01	-0.01

Table 4.2: Capital requirements and welfare under 'worst case' borrowing contracts

Collateralisable output In this case the borrowing constraint is of the form:

$$b_t \leqslant E_t \left(q_{t+1}k_t + \theta_y y_{t+1}\right)$$

Entrepreneurs can now borrow up to the full value of their capital holdings and also up to a fraction θ_y of their future output. The results are shown in Table 4.3 below. Again, looking at the effects of this parameter did not change the basic result that aggregate welfare declined as the result of the imposing tighter capital requirements.

	Baseline	$\theta_y = 0.1$	$\theta_y = 0.2$
$100\triangle \ln \varphi_0^H$	-0.23	-0.34	-0.39
$100\triangle \ln Z_0^H$	-4.15	-1.27	-1.78
$100\triangle \ln V_0^H$	-1.15	-0.44	-0.55
$100\triangle \ln \varphi_0^L$	1.11	0.25	0.35
$100\triangle \ln Z_0^L$	-0.71	-0.21	-0.28
$100\triangle \ln V_0^L$	0.04	-0.01	-0.01
$100\triangle \ln V_0^W$	0.14	-0.05	-0.07
$100\triangle \ln V_0$	-0.33	-0.13	-0.14
$100\triangle\sigma_c$	-0.12	-0.06	-0.10
$100\triangle\sigma_{cW}$	-0.03	-0.03	-0.04
$100\triangle\sigma_{cH}$	-0.60	-0.26	-0.42
$100\triangle\sigma_{cL}$	-0.06	-0.04	-0.07

Table 4.3: Capital requirements and welfare under collateralisable output

4.B.13 Solution method

The Laissez Faire economy

We use the following 'parameterised expectations' algorithm in order to solve for the recursive competitive equilibrium of our model economy.

1. Start by guessing parameter values for current and future expected price functions. All equilibrium pricing functions are log linear in the state variables d_t and Z_t.

$$\ln q\left(X_{t+1}|X_t\right) = \omega_c\left(X_{t+1}|X_t\right) + \omega_d\left(X_{t+1}|X_t\right)\ln d_t + \omega_w\left(X_{t+1}|X_t\right)\ln Z_t \qquad (4.9)$$

$$\ln q(X_t) = \varphi_c(X_t) + \varphi_d(X_t)\ln d_t + \varphi_w(X_t)\ln Z_t \tag{4.10}$$

$$\ln r(X_{t+1}|X_t) = \kappa_c(X_{t+1}|X_t) + \kappa_d(X_{t+1}|X_t)\ln d_t + \kappa_w(X_{t+1}|X_t)\ln Z_t \tag{4.11}$$

where X_t is the aggregate state of the economy.

2. Static portfolio maximisation

Next we find optimal leverage levels. Due to the non-convex choice set we need to compute and compare the value function when the constraint is binding and when it is non-binding. We pick the leverage choices associated with the largest of the two value functions.

(a) The value of the constraint binding is

$$\begin{aligned} R^{H*}(l_{t+1} = \theta) &= E_t \ln R^H_{t+1} \\ &= E_t \ln \left[\frac{\left(A_{t+1}a^H/\alpha\right) w_t^{\alpha+\eta-1} \left(u_t^H\right)^{1-\alpha} + q_{t+1} - \theta E_t q_{t+1}}{q_t + (1-\alpha) u_t^H/\alpha - (\theta/R_t) E_t q_{t+1}} \right] \end{aligned}$$

where

$$u_t^H = q_t - E_t\left(\frac{q_{t+1}}{R^H_{t+1}}\right) - \theta E_t q_{t+1} E_t\left(\frac{1}{R^L_{t+1}} - \frac{1}{R^H_{t+1}}\right) \tag{4.12}$$

is the user cost of capital under the binding constraint.

(b) The value of the constraint not binding

$$\begin{aligned} R^{H*}(l_{t+1} < \theta) &= \max_{0<l_{t+1}<\theta} E_t \ln R^H_{t+1} \\ &= \max_{0<l_{t+1}<\theta} E_t \ln \left[\frac{\left(A_{t+1}a^H/\alpha\right) w_t^{\alpha+\eta-1} \left(u_t^H\right)^{1-\alpha} + q_{t+1} - l_t E_t q_{t+1}}{q_t + (1-\alpha) u_t^H/\alpha - (l_t/R_t) E_t q_{t+1}} \right] \end{aligned}$$

where

$$u_t^H = q_t - E_t\left(\frac{q_{t+1}}{R^H_{t+1}}\right)$$

is the user cost when the constraint does not bind. We solve this maximisation problem using the inbuilt Matlab function fmincon.m

3. Compute the equilibrium at time t:

We use the latest guess of the q_{t+1} pricing function, the portfolio policy function

l_{t+1} as well as the current realisations of the state variables A_t, d_t and W_t.

$$R_{t+1}^L = \frac{\left[A_{t+1}\frac{w_t^{\alpha+\eta-1}\left(u_t^L\right)^{1-\alpha}}{\alpha} + q_{t+1}\right](1-K_t) + l_{t+1}q\left(X_{t+1}\right)K_t}{\left[q_t + \frac{1-\alpha}{\alpha}u_t^L\right](1-K_t) + \left(l_{t+1}/R_t\right)E_t q_{t+1}K_t} \tag{4.13}$$

where

$$u_t^L = q_t - E_t\left(\frac{q_{t+1}}{R_{t+1}^L}\right)$$

High productivity entrepreneurs invest the following fraction of their wealth in capital.

$$K_t = \frac{\beta d_t W_t}{q_t + \frac{1-\alpha}{\alpha}u_t^H - \left(l_{t+1}/R_t\right)E_t q_{t+1}} \tag{4.14}$$

Their rate of return is given by:

$$R_{t+1}^H = \frac{\left(A_{t+1}a^H/\alpha\right)w_t^{\alpha+\eta-1}\left(u_t^H\right)^{1-\alpha} + q_{t+1} - l_t E_t q_{t+1}}{q_t + (1-\alpha)u_t^H/\alpha - \left(l_t/R_t\right)E_t q_{t+1}} \tag{4.15}$$

when the collateral constraint is slack and

$$u_t^H = q_t - E_t\left(\frac{q_{t+1}}{R_{t+1}^H}\right) - \theta E_t q_{t+1}\left(\frac{1}{R_t} - E_t\left(\frac{1}{R_{t+1}^H}\right)\right)$$

Finally, goods market clearing is:

$$(1-\beta)W_t + w_t H_t + \frac{1-\alpha}{\alpha}\left[u_t^L(1-K_t) + u_t^H K_t\right] = W_t - q_t$$

Using the inbuilt Matlab zero-finding routine fsolve.m, solve for the values of $\left\{R_t, R_{t+1}^L, K_t, q_t, R_{t+1}^H, u_t^H, u_t^L\right\}$ at which these conditions are satisfied up to an error tolerance level.

4. Use the state evolution equations to compute next period's state vector:

$$W_{t+1} = \left[d_t R_{t+1}^H + (1-d_t)R_{t+1}^L\right]\beta W_t \tag{4.16}$$

$$d_{t+1} = \frac{(1-\delta)d_t R_{t+1}^H + n\delta(1-d_t)R_{t+1}^L}{d_t R_{t+1}^H + (1-d_t)R_{t+1}^L} \tag{4.17}$$

5. Repeat steps (1)-(4) for 2000 periods. Using the simulated data (minus a 200

period 'burn in' period), update the price and forecasting function coefficients using linear regression.

6. Re-compute a simulated time series of the endogenous variables in our model economy under the new forecasting rule. Repeat steps (1)-(5) until the coefficients on the forecasting rule have converged up to an error tolerance level.

The economy with capital requirements

In our government economy, the government chooses state contingent leverage functions $\widetilde{\theta}_t$ in order to maximise social welfare

$$\begin{aligned}\Omega &= \max_{\{\chi^i\}} \left[\varsigma^H n V^H \left(z_0^H/n, X_0\right) + \varsigma^L V^L \left(z_0^L, X_0\right) + \varsigma^W V^W \left(X_0\right)\right] \\ &= \max_{\{\chi^i\}} \left[\varsigma^H n \left(\varphi^H \left(X_0\right) + \frac{\ln\left(z_0^H/n\right)}{1-\beta}\right) + \varsigma^L \left(\varphi^L \left(X_0\right) + \frac{\ln z_0^L}{1-\beta}\right) + \varsigma^W V^W \left(X_0\right)\right]\end{aligned} \tag{4.18}$$

(1.) Pose a candidate leverage function and make a starting guess on its parameters. In this paper we guess a first order log-linear formulation for each aggregate state $i = h, l$.

$$\ln \widetilde{\theta}_t = \chi_0^i + \chi_1^i \ln d_t + \chi_2^i \ln Z_t$$

(2.) Compute the equilibrium quantities of our model economy using steps (1)-(6) in the previous subsection

(3.) Compute the entrepreneurs' value function

$$\varphi\left(a_t, d_t, Z_t, A_t\right) = \ln\left(1-\beta\right) + \frac{\beta \ln \beta + \beta E_t \left(\ln R_{t+1}^i\right)}{1-\beta} + \beta E_t \varphi\left(a_{t+1}, d_{t+1}, Z_{t+1}, A_{t+1}\right)$$

and the workers' value function

$$V^W\left(d_t, Z_t, A_t\right) = \Theta + \frac{\omega}{1+\omega} \ln w_t + \beta E_t V^W\left(d_{t+1}, Z_{t+1}, A_{t+1}\right)$$

(3.1.) Discretise the space of the continuous state variables d_t and Z_t. We use 10 grid points on each state variable. The value function is almost linear in the direction of both state variables so using more grid points makes very little difference to the results while slowing down the computations considerably.

(3.2.) Use value function iterations to compute the value function at each grid point. When state variables fall in between grid points, we use bi-linear interpolation to approximate the value function.

(4.) Compute social welfare for the candidate leverage function $\widetilde{\theta}_t$. This consists of two steps:

(4.1.) Compute the realisation of the capital price in the initial period when the private sector is surprised by the policy change. This allows us to compute the realisations of the aggregate state variables (the vector X_0) when the policy is announced. It also allows us to compute the realisations of the wealth of each group when the policy is announced.

(4.2) Evaluate the the social welfare function (4.18) at the post regulation reform aggregate state X_0 and individual wealth positions - z_0^H and z_0^L.

(5.) Place steps (1)-(4) above in a function which outputs the value of social welfare for a candidate leverage function and maximise it with respect to the parameters of the leverage function. Because function evaluations are very time consuming we use the inbuilt Matlab routine fminsearch.m which uses a Nelder-Meade algorithm.

4.C Chapter 3

4.C.1 The value function of a non-defaulting entrepreneur

We now combine the optimal consumption and portfolio choices of entrepreneurs to derive the value function that characterises their maximum lifetime utility. Let $V(s_t, X_t)$ be the value of a non-defaulting entrepreneur with idiosyncratic state s_t when the aggregate state is X_t.

$$V(s_t, X_t) = \max_{c_t, k_{t+1}, x_{t+1}, b_{t+1}} \left\{ \ln c_t + \beta \sum_{X_{t+1}} \sum_{s_{t+1}} \pi(X_{t+1}|X_t)\, \pi(s_{t+1}|s_t)\, V(s_{t+1}, X_{t+1}) \right\}$$

We guess a solution of the form:

$$V(s_t, X_t) = \varphi(s_t, X_t) + \varsigma(s_t, X_t) \ln w_t$$

Hence the value function equals:

$$\varphi(s_t, X_t) + \varsigma(s_t, X_t) \ln w_t \tag{4.21}$$

$$= \max_{k_{t+1}, x_{t+1}, b_{t+1}} \left\{ \begin{array}{c} \ln(1-\beta) + \ln w_t + \\ \beta \sum_{X_{t+1}} \sum_{s_{t+1}} \pi(X_{t+1}|X_t)\, \pi(s_{t+1}|s_t) \\ \left[\varphi(s_{t+1}, X_{t+1}) + \varsigma(s_{t+1}, X_{t+1}) \ln w_{t+1}\right] \end{array} \right\} \tag{4.22}$$

$$= \max_{k_{t+1}, x_{t+1}, b_{t+1}} \left\{ \begin{array}{c} \ln(1-\beta) + \ln w_t + \\ \beta \sum_{X_{t+1}} \sum_{s_{t+1}} \pi(X_{t+1}|X_t)\, \pi(s_{t+1}|s_t) \\ \left[\varphi(s_{t+1}, X_{t+1}) + \varsigma(s_{t+1}, X_{t+1}) \left(\ln \beta + \ln R(X_{t+1}|s_t, X_t) + \ln w_t\right)\right] \end{array} \right\}$$

Equating coefficients we have:

$$\varsigma(s_t, X_t) = 1 + \beta \sum_{X_{t+1}} \sum_{s_{t+1}} \pi(X_{t+1}|X_t)\, \pi(s_{t+1}|s_t)\, \varsigma(s_{t+1}, X_{t+1}) \tag{4.23}$$

and

$$\varphi(s_t, X_t) = \ln(1-\beta) \tag{4.24}$$

$$+ \max_{k_{t+1}, x_{t+1}, b_{t+1}} \beta \sum_{X_{t+1}} \sum_{s_{t+1}} \pi(X_{t+1}|X_t)\, \pi(s_{t+1}|s_t) \begin{bmatrix} \varsigma(s_{t+1}, X_{t+1}) (\ln\beta + \ln R(X_{t+1}|s_t, X_t)) \\ +\varphi(s_{t+1}, X_{t+1}) \end{bmatrix} \tag{4.25}$$

Equation (4.23) implies that

$$\varsigma(s_t, X_t) = \frac{1}{1-\beta}$$

Plugging this into (4.24) we have

$$\varphi(s_t, X_t) = \ln(1-\beta) \tag{4.26}$$

$$+ \max_{k_{t+1}, x_{t+1}, b_{t+1}} \frac{\beta}{1-\beta} \sum_{X_{t+1}} \sum_{s_{t+1}} \pi(X_{t+1}|X_t)\, \pi(s_{t+1}|s_t) \begin{bmatrix} \ln\beta + \ln R(X_{t+1}|s_t, X_t) \\ +(1-\beta)\varphi(s_{t+1}, X_{t+1}) \end{bmatrix} \tag{4.27}$$

Solving for the optimal portfolio allocation of the high productivity entrepreneurs (a) The value of the constraint binding

$$\Omega(l_t = \theta_t) = E_t \ln R^H_{t+1}$$

where

$$R^H_{t+1} = \frac{\left[(A_{t+1} - \theta_t E_t A_{t+1}) \frac{a^H \left(u^H_t\right)^{1-\alpha}}{\alpha} + q_{t+1} - E_t q_{t+1}\right]}{q_t + \frac{1-\alpha}{\alpha} u^H_t - E_t(q_{t+1} + \theta_t y_{t+1}/k_{t+1})} \tag{4.28}$$

and

$$u^H_t = q_t - E_t \left\{ \frac{q_{t+1}}{R^H_{t+1}} \right\}$$

(b) The value of the constraint not binding

$$\Omega\left(l_t < \theta_t\right) = \max_{k_{t+1}, l_t} E_t \ln R_{t+1}^H$$

where

$$R_{t+1}^H = \frac{\left[\left(A_{t+1} - l_t E_t A_{t+1}\right) \frac{a^H A^i \left(u_t^H\right)^{1-\alpha}}{\alpha} + q_{t+1} - E_t q_{t+1}\right]}{q_t + \frac{1-\alpha}{\alpha} u_t^H - E_t\left(q_{t+1} + l_t y_{t+1}/k_{t+1}\right)} \tag{4.29}$$

and

$$u_t^H = q_t - E_t\left\{\frac{q_{t+1}}{R_{t+1}^H}\right\}$$

Value function iterations Let $\widetilde{R}_{t+1}\left(X_{t+1}|s^h, X_t\right)$ denote the rate of return on entrepreneurial wealth under optimal leverage. We are now ready to compute the value functions by iterating on the functional equation below.

$$\varphi\left(s^h, X_t\right) = \ln(1-\beta) + \frac{\beta}{1-\beta} \sum \pi\left(X_{t+1}|X_t\right) \pi\left(s_{t+1}|s^h\right) \left[\begin{array}{c} \ln\beta + \ln\widetilde{R}\left(X_{t+1}|s^h, X_t\right) \\ +(1-\beta)\varphi\left(s_{t+1}, X_{t+1}\right) \end{array}\right] \tag{4.30}$$

$$\varphi\left(s^l, X_t\right) = \ln(1-\beta) + \frac{\beta}{1-\beta} \sum \pi\left(X_{t+1}|X_t\right) \pi\left(s_{t+1}|s^l\right) \left[\begin{array}{c} \ln\beta + \ln\widetilde{R}\left(X_{t+1}|s^l, X_t\right) \\ +(1-\beta)\varphi\left(s_{t+1}, X_{t+1}\right) \end{array}\right] \tag{4.31}$$

$$\varphi^d\left(X_t\right) = \ln(1-\beta) + \frac{\beta}{1-\beta} \sum \pi\left(X_{t+1}|X_t\right) \left[\begin{array}{c} \ln\beta + \ln\widetilde{R}\left(X_{t+1}|s^l, X_t\right) \\ +(1-\beta)\varphi^d\left(X_{t+1}\right) \end{array}\right] \tag{4.32}$$

Computing aggregate equilibrium Armed with the optimal leverage functions l_t obtained in the solution of the maximisation step above, we are ready to compute equilibrium for a simulated time series of shocks.

From market clearing in the capital and the debt markets we can pin down the state contingent growth rate of the low productivity household without solving an explicit portfolio problem:

$$R_{t+1}^L = \frac{\left[A_{t+1}\frac{\left(u_t^L\right)^{1-\alpha}}{\alpha} + q_{t+1}\right]\left(1 - K_{t+1}\right) + E_t\left[l_t Y_{t+1} + q_{t+1} K_{t+1}\right]}{\left[q_{t+1} + \frac{1-\alpha}{\alpha} u_t^L\right]\left(1 - K_{t+1}\right) + E_t\left[l_t Y_{t+1} + q_{t+1} K_{t+1}\right]/R_t} \tag{4.33}$$

where

$$u_t^L = q_t - E_t\left(\frac{q_{t+1}}{R_{t+1}^L}\right)$$

High productivity entrepreneurs invest the following fraction of their wealth in land.

$$K_{t+1} = \frac{\beta d_t W_t}{q_t + \frac{1-\alpha}{\alpha}u_t^H - E_t\left[l_t Y_{t+1}/K_{t+1} + q\left(X_{t+1}\right)\right]} \tag{4.34}$$

Their rate of return is given by:

$$R_{t+1}^H = \frac{\left(A_{t+1} - l_t E_t A_{t+1}\right)\frac{a^H\left(u_t^H\right)^{1-\alpha}}{\alpha} + q_{t+1} - E_t q_{t+1}}{q_t + \frac{1-\alpha}{\alpha}u_t^H - E_t\left[l_t Y_{t+1}/K_{t+1} + q_{t+1}\right]} \tag{4.35}$$

where the user cost of land is given by

$$u_t^H = q_t - E_t\left(\frac{q_{t+1}}{R_{t+1}^H}\right)$$

The real interest rate on debt securities is given by the consumption euler equation:

$$R_t = \beta E_t\left(\frac{1}{R_{t+1}^L}\right)$$

Finally, goods market clearing implies that:

$$\left(1-\beta\right)W_t + \frac{1-\alpha}{\alpha}\left[u_t^L\left(1-K_{t+1}\right) + u_t^H K_{t+1}\right] = W_t - q_t$$

Using a zero-finding routine, solve for the values of $\left\{R_t, K_{t+1}, q_t, R_{t+1}^H, R_{t+1}^L, u_t^H, u_t^L, K_{t+1}\right\}$ at which these conditions are satisfied up to an error tolerance level. I use Matlab's own fsolve.m routine.

3. Use the state evolution equations to compute next period's state vector:

$$W_{t+1} = \left[d_t R_{t+1}^H + \left(1-d_t\right)R_{t+1}^L\right]\beta W_t \tag{4.36}$$

$$d_{t+1} = \frac{(1-\delta)d_t R_{t+1}^H + n\delta\left(1-d_t\right)R_{t+1}^L}{d_t R_{t+1}^H + \left(1-d_t\right)R_{t+1}^L} \tag{4.37}$$

4. Repeat steps (1)-(3) for a large number of periods. Using the simulated data,

update the price and forecasting function coefficients using linear regression.

5. Re-compute a simulated time series of the endogenous variables in our model economy under the new forecasting rule. Repeat steps (1)-(4) until the coefficients on the forecasting rule have converged up to an error tolerance level.

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University of London

London, 1 June 2010

I declare that the work presented in this thesis is my own except where the collaboration with coauthors is explicitly acknowledged.

Kalin Ognianov Nikolov

University of London

London, 1 June 2010

I declare that the work presented in the first chapter of my thesis was co-authored with Professor Nobu Kiyotaki and Dr Alex Michaelides.

Kalin Ognianov Nikolov.