On the Coexistence of Money and Credit

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Abstract

This thesis explores the idea of money and credit as complementary media of exchange. Complementarity has interesting implications for the effects of monetary policy on macroeconomic variables. The impact of inflation is markedly different in a world in which money and credit are complementary or substitute, i.e. cooperate or compete. In the first chapter, I review some recent literature on the coexistence of money and credit in matching models of money à la Kiyotaki and Wright and in models with spatial separation à la Townsend. I argue that the literature, virtually without exceptions, has seen money and credit as competing media of exchange and concentrated on the role of record keeping technologies in supporting credit as a medium of exchange. Moreover, money doesn't normally serve to clear debts. In chapter 2, I construct an economy with microfoundations for the use of money and bilateral credit as media of exchange. The model features spatial separation,

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interesting applications of the model. First a model in which agents in equilibrium endogenously decide to become debtors or creditors. Second the issue of seigniorage and finally the question of circulation of promises. In chapter 5, I consider the question of coexistence and social benefits of having a zero rate of return asset -fiat money- and an illiquid nominal, risk-free, interest bearing bond. I consider the model by Kocherlakota (2003) where illiquid bonds coexist with money because they serve to insure against liquidity shocks. I introduce a commitment technology giving agents the ability to issue promises fully backed by bonds. I show that illiquidity is not sufficient to guarantee a role for bonds. For illiquid bonds to be essential some legal restrictions on the issue of promises backed by bonds should be introduced. Finally I present a model in which changes in the liquidity of assets generate interesting predictions about the riskiness of projects undertaken in the economy, output and welfare. In the last chapter, coauthored with Raoul Minetti, we develop a theory of the interaction between the entry of lenders and the real sector. The high liquidation skills of incumbent lenders render them too tough in terminating high-risk/return projects. Being "foreign" to the market, newcomers have lower ability to liquidate than incumbents. This makes them softer in liquidating high-risk/return projects but renders their funding more costly. We show that the entry of lenders and the share of high-risk/return projects can reinforce each other through firms' liquidation values. This interaction dampens the output impact of liquidity shocks. Hence, financial liberalization can enhance stability.

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Contents

Introduction

"This, as I see it, is really the central issue in the pure theory of money. Either we have to give an explanation of the fact that people do hold money when rates of interest are positive or we have to evade the difficulty somehow" (Hicks (1935). John Hicks suggests that the answer to the central issue in the pure theory of money is frictions.

Frictions are necessary to generate a role for money as a medium of exchange. Absence of double coincidence of wants, spatial separation, absence of a record keeping technology and absence of commitment have been advocated to explain the use of money to lubricate exchange. Unfortunately some of these frictions, while making room for money, prevent the use of alternative -interest bearing- media of exchange like credit. A recent theoretical literature studies credit as a medium of exchange introducing a record keeping technology. Trade can be monitored and agents can be punished for not keeping their promises. Money and credit are typically seen as substitute, competing means of exchange. Money never plays the role of the means of payment, i.e. the means to repay promises. The literature has largely ignored the fact that money and credit can be complementary, with money being the means of payment as well as a medium of exchange. This thesis explores the idea of credit as a bilateral promise of future money and money and credit as complementary or cooperating media of exchange. This turns out to have interesting implications for the effects of monetary policy on macroeconomic variables. The impact of inflation is markedly different in a world in which money and credit cooperate instead of competing.

The first four chapters of the thesis deal with the issue of the coexistence of money and credit as media of exchange. The fifth chapter concerns the role of the liquidity of alternative interest-bearing assets -e.g. bonds- in insuring against liquidity shocks. The last chapter -which is based on a paper coauthored with Raoul Minetti- examines a more applied issue relating to the liquidity of assets. Differences in the liquidity of assets can have an impact on the riskiness of projects undertaken, on output and welfare in the economy.

In the first chapter, I review some recent literature on the coexistence of money and credit in matching models of money à la Kiyotaki and Wright and in models with spatial separation à la Townsend. I argue that the literature, while stressing the relationship of substitutability between money and credit, has overlooked the fact that money and credit might in some cases be complementary. This may have important implications for monetary policy.

In chapter 2, I construct an economy with microfoundations for the use of money and bilateral credit as media of exchange. The model features spatial separation, absence of double coincidence of wants and competitive markets. Money is the means of payment: in equilibrium, bilateral credit is paid back with money. Money and credit are complementary. Complementarity generates a reverse Mundell-Tobin effect. The nominal interest rate is more than unit elastic in the inflation rate and therefore the real interest rate increases with inflation. The credit to money and credit to output ratios, output and welfare all decrease with inflation. A model where the two media of exchange are complementary generates opposite predictions on the effect of inflation on credit with respect to a model where money and credit are substitute.

In chapter 3, I consider a modification of the model presented in the previous chapter where the elasticity of the interest rate is less than one. This shows that what is really crucial is that the elasticity is different from one. While there is in the literature a consensus that empirically the elasticity is not unitary, it is less clear whether the elasticity is greater or smaller than one. I then use macroeconomic data for 59 countries over the period 1993-2003 to estimate the elasticity of the nominal interest rate with respect to the inflation rate. I find an elasticity significantly greater than one. I also test the prediction on Credit/GDP.

The model presented in the previous chapters has the potential to address a number of issues relating to the role of money and credit and the interaction with monetary policy. In chapter 4, I discuss two applications. First, I analyze the decision to become a borrower or a lender in more details, proposing a model in which in equilibrium agents endogenously partition between borrowers and lenders. Second I consider the possibility that the government, instead of making transfers to agents, prints money and buys commodities on the market . Finally, I speculate on the possibility of circulating promises.

In chapter 5, I consider the question of coexistence and social benefits of having two assets: a zero rate of return asset -fiat money- and an illiquid nominal, risk-free, interest bearing bond. In Kocherlakota (2003) illiquid bonds are socially beneficial since they allow agents to insure against liquidity shock. Here I introduce a commitment technology allowing agents to issue promises fully backed by bonds. When the liquidity shock is sufficiently high, legal restrictions can be advocated to prevent agents from issuing promises.

Finally I present a model in which changes in the liquidity of assets generate interesting predictions about the riskiness of projects undertaken in the economy, output and welfare. In the last chapter, coauthored with Raoul Minetti, we develop a theory of the interaction between the entry of lenders and the real sector. The high liquidation skills of incumbent lenders render them too tough in terminating high-risk/return projects. Being "foreign" to the market, newcomers have lower ability to liquidate than incumbents. This makes them softer in liquidating high-risk/return projects but renders their funding more costly. We show that the entry of lenders and the share of high-risk/return projects can reinforce each other through firms' liquidation values. This interaction dampens the output impact of liquidity shocks. Hence, financial liberalization can enhance stability.

Chapter 1

Money and Credit in Monetary Economies

1.1 Introduction

The fundamental issue in the pure theory of money is to explain why people are willing to hold worthless, non interest-bearing pieces of paper and use them in the process of exchange. The basic idea -as old as monetary theory- is that the use of money is motivated by absence of double coincidence of wants: if an economist and a hairdresser meet, it may happen that the economist wants a haircut but the hairdresser doesn't want an economics lecture, in which case trade cannot take place without a medium of exchange of some sort. This is however not enough to explain why people hold money, since in principle the economist could buy on credit. Kiyotaki and Wright (1989) explicitly formalize the former situation using a search-matching model, in which absence of double coincidence of wants is the main ingredient and several other frictions make credit systems difficult to implement. Alternatively when markets are spatially separated as in models a' la Townsend (1980), money serves as a carrier of value between markets, provided there are impediments to the use of credit. Therefore, it is crucial to explain why agents use simultaneously money and credit.

The present chapter is a review of the literature that has recently dealt -in the matching and in the spatial separation traditions- with the latter issue: the coexistence of money and credit as media of exchange. This is an important issue not only from a theoretical point of view but also for monetary policy reasons. As Kocherlakota (2003) put it: "real life monetary policy is primarily about interest rates -that is the relative price of money and claims to future money-. The basic literature generally abstracts from other assets besides money, and offers no compelling reason why societies need risk-free claims to future money as well as money itself." A first step toward such real life monetary policy analysis involves constructing models were money coexists with alternative assets. With few exceptions, the literature has followed a paper by Kocherlakota (1998) -where money is seen as a form of memory- in seeing the absence of a record keeping technology as the crucial feature that prevents credit from working. Both in the matching and the spatial separation literature the emphasis has been on the possibility to monitor agents in various ways and to different degrees. Credit thus has been seen as a sort of centralized contract among agents, implemented through a monitoring technology, while money as a useful device for agents that cannot be monitored or drop out of the contract. Implicitly the two instruments are always seen as substitute. Money is never used as a means of payment -i.e. as a means to repay promises. An idea of complementarity between money and credit is absent from the literature. I will argue that to view money and credit as substitute rather than complementary is not indifferent when monetary policy issues are addressed.

1.1.1 Overview

A very recent literature has explained -within the theoretical framework of a random matching model- the coexistence of means of exchange with different rates of return, relaxing some of the assumptions that were made in the original work by Kiyotaki and Wright (1989). The literature concentrated on the absence of a monitoring technology and on the impossibility for individuals to commit to future actions. If all transactions were recorded on a computer money would completely disappear. The imperfections in the record keeping technology could thus explain the use of both money and credit in our economies. This approach was followed by Kocherlakota and Wallace (1998) and Cavalcanti and Wallace (1999).

Shi (1996) focused instead on the limitation of commitment. If agents are unable to commit to future actions, credit will not emerge. If commitment is unlimited, on the other hand, no one will use money and trade will be organized exclusively on a credit basis. The idea in Shi (1996) is to secure credit with the use of collateral: agents can commit up to the limit of the value of the collateral, otherwise they use money to trade. I pursue this approach further in chapter 2.

Other papers focus on the use of credit in enduring relationships. In the models I will examine -by Jin and Temzelides (2001) and Corbae and Ritter (2004)- the idea of substitution between money and credit is even more explicit: money may actually reduce the incentives to enter in credit relationships.

A related literature looks at the question of private versus public money. Interest in the question, dating back at least to Hayek and Friedman, has been revived by new developments in the use of stored-value cards and advances in the payment systems. In Williamson (1999) banks are able to issue notes backed by productive investments funded by agents when they visit the bank. Abstracting from asymmetric information and lemons problems, the introduction of private money, since it stimulates investment, seems to unambiguously enhance welfare. When possible inefficiencies of private money are taken into account however, it may be beneficial to ban private issue and stick to public money. Again the two instruments are seen as alternative. I also present three very recent papers exploring the combination of money, credit and banking in search economies, namely Berentsen, Camera and Waller (2004), Faig (2004) and He,Huang and Wright (2003).

I then consider the literature in the spatial separation tradition, where I examine the recent efforts to introduce credit alongside money in the turnpike model and in the Cass-Yaari model. Again -following an idea developed by Townsend (1989)- credit has been seen as a useful medium of exchange when there is the possibility to monitor trading histories or in enduring relationships. I finally look at a literature on payment systems, where the issue of substitution versus complementarity becomes relevant for monetary policy issues. What is the correct behavior of a Central Bank in the presence of both money and a payment system? Is inflation bad -as in Freeman (1996)- or good -as in Williamson (2002)- for the working of the payment system? As emphasized by Green (2003) inflation may destroy the acceptability of money in a model featuring substitutability but not in a model featuring complementarity -in a model where money serves as the means to settle debt-. Inflation may favour credit or it may discourage it. I will discuss these points further in the following chapters.

The chapter is structured as follows. Section 2 introduces the basic matching model of money and discusses coexistence in that framework. Section 3 deals with the question of coexistence of money and credit in models with spatial separation. Section 4 concludes.

1.2 The Matching Model

1.2.1 The basic Model

I will follow Kiyotaki and Wright (1989), Kiyotaki and Wright (1993) and Trejos and Wright (1995) in describing the random matching model of money. The aim of the literature on the search-theoretic approach to monetary economics is to generate a role for money in lubricating the process of exchange. The first essential feature to generate a role for money is some form of absence of double coincidence of wants. This has been considered - at least since the time of Menger and Wicksell- one of the crucial prerequisite to generate a role for money. Otherwise all trades would involve just barter and no monetary exchange. This is embedded into the model as follows.

In the economy there are $N \geq 3$ different production opportunities. Opportunity i is used to produce good i. Goods are perishable, in the sense that they have to be consumed immediately after trade otherwise they are lost. The economy is inhabited by N types of agents with a $[0, 1]$ continuum of each type. Agents of type i produce good $i + 1$ (modulo N) and consume i. Agents derive utility from consumption. The utility function $u(q)$ -where q is the quantity of the good- is twice continuously differentiable, strictly increasing and strictly concave, with $u(0) = 0, u'(0) = \infty$ and $u'(\infty) = 0$. Each agent incurs a production cost in terms of utils given by $c(q) = q$. The rate of time preference $r > 0$ is the same for all agents. Time continues for ever. This is necessary to induce people to voluntarily hold intrinsically useless pieces of paper (fiat money). If the time horizon were finite, no agent would be willing to accept money in the last period and working backward no agent would be willing to accept money at all. This feature also suggest that money will be held and exchanged because agents expect it to be accepted in the future by someone else.

Each agent maximizes the expected discounted utility of consuming minus the cost of producing

$$E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t (u(q_t) - q_t)$$

At time t an agent meets randomly (according to a process specified below) another agent and produces for him. In the following period the same agent will start searching for his own consumption good. Production and consumption cannot happen in the same period.

There exists in the economy one intrinsically useless and durable[1] object called fiat money. Money is indivisible and agents can hold one unit of money or nothing. This zero/one restriction on the portfolios of money that agents can hold is due to the technical difficulty of keeping track of the holdings of money of a continuum population that -as it is explained below- meets at random. Among each type there is a mass M of agents that hold one unit of fiat money and $1-M$ that hold nothing. There is no market place. People are randomly assigned to meet bilaterally. In each period the arrival rate of trading partners is a Poisson process with constant rate $\sigma > 0$. The randomness assumption together with the fact that there is a continuum of agents of each type rules out the possibility of the circulation of bilateral promises since the probability of a promise to get back to the issuer and be repaid is zero. This implies that there wouldn't be anyone willing to accept a promise in the first place. There is no record keeping technology other than money. This assumption is motivated by the fact that money wouldn't be essential in the presence of a centralized record keeping technology monitoring agents. Trade is observable only by the two agents involved. Agents cannot commit to perform future actions. This assumption is also crucial to obtain a role for money. If agents could costlessly commit to keep their promises there would be no need for money.

Due to the randomness of the matching process there are many meetings in

[1]The fact that fiat money is durable while goods are perishable is to avoid the complication of having to consider commodity money.

which nothing happens and only some meetings in which exchange takes place. If a type i and a type $j \neq i+1, i-1$ meet nothing happens. These are no-coincidence meetings, since type i doesn't want to consume what type j produces and viceversa. An interesting meeting is on the contrary a meeting between type i and a type $j = i+1$ or $j = i-1$. In this case there is single coincidence of wants, namely one of the two partners wanting what the other produces but having nothing -in terms of consumption goods- to offer. If the potential consumer has money, trade can take place; otherwise nothing happens. When a money holder meets a producer of his consumption good who doesn't hold money, he offers one unit of money for q_m units of the good. If exchange takes place he consumes and returns home. To determine the quantity produced q_m -which gives also implicitly the relative price of money and goods $\frac{1}{q_m}$- it is assumed that when the two partners meet they bargain over the terms of trade using a Nash Bargaining procedure. In particular they will split the surplus generated by trade according to their relative bargaining power:

$$\underset{q_m}{Max}\,(V_m - V_p - q_m)^{\theta}\,(u(q_m) + V_p - V_m)^{1-\theta}$$

$$\begin{aligned} s.t. \quad V_m - V_p - q_m &\geq 0 \\ u(q_m) + V_p - V_m &\geq 0 \end{aligned}$$

where V_m is the value function of a money holder, V_p is the value function of a producer and $\theta \in [0,1]$ is the bargaining power of a producer. In most of the discussion below for simplicity take $\theta = 0$, thus giving all the bargaining power to money holders, who in fact will make take-it-or-leave-it offers to producers. This assumption -without affecting in any substantial way the results- will reduce the former maximization problem to a situation in which the producer will get zero surplus and consumers will get all the positive surplus (if surplus were zero or negative the outcome would be no-trade):

$$\begin{aligned} V_m - V_p - q_m &= 0 \\ u(q_m) + V_p - V_m &> 0 \end{aligned}$$

The second inequality is an Incentive Condition insuring that money holders are willing to give up money to consume.

In a stationary equilibrium the value functions are given by the following system of equations:

$$\begin{aligned} rV_m &= \sigma(1-M)x\,(u(q_m) + V_p - V_m) \qquad (1.1) \\ rV_p &= \sigma M x\,(-q_m + V_m - V_p) = 0 \end{aligned}$$

where $x = 1/N$ is the probability of a single coincidence meeting, σ is the probability of meeting someone and $(1-M)$ is the probability of meeting an agent without money and M is the probability of meeting a money holder.

A money holder when meeting the "right" producer -which happens with probability $\sigma(1-M)x$- consumes, looses money and becomes a producer. A producer with probability $\sigma M x$ meets the "right" money holder, produces for him and gets money, obtaining zero surplus from the deal.

Solving system (1):

$$\begin{aligned} V_m &= \frac{\sigma(1-M)x}{\sigma(1-M)x + r} u(q_m) \\ V_p &= 0 \end{aligned}$$

and using the take-it-or-leave-it condition:

$$q_m = V_m = \frac{\sigma(1-M)x}{\sigma(1-M)x + r} u(q_m)$$

which gives the equilibrium value $\overline{q}_m$. Moreover $u(\overline{q}_m) > \overline{q}_m$ is clearly verified for any $M \in (0,1)$. This proves that a monetary equilibrium exists for any $M \in (0,1)$. It is worth emphasizing that there always exists also a non-monetary equilibrium. This phenomenon, which is also present in other models of monetary economies like the overlapping generations model, is due to the fact that money in the matching model is used as a medium of exchange because agents expect it to have value in the future for other agents. If however agents expect money to be valueless, the expectation will be self-fulfilling and money will not be used at all[2]. Agents use money because they expect other agents to accept it in the future.

1.2.2 Restrictions on Money Holdings

One of the features making the matching model of money tractable -i.e. the $\{0,1\}$ restriction on money holdings- forecloses the possibility to address monetary policy questions. One of the main efforts in the recent literature on the pure theory of money has been to get rid of the restriction while preserving the nice microfoundation for the use of money. Lagos and Wright (2002) modify the previous setting by introducing two types of commodities: general and special goods. There is absence of double coincidence of wants for special goods, while general goods are produced and consumed by everyone. The utility $U(X)$of consuming the general good is increasing and concave and the cost to produce it is $C(X) = X$. Special goods are exchanged during the day on a matching basis, while general goods are exchanged during the night in a centralized market. Terms of trade are determined by bargaining in the matching market, while in the centralized market one dollar buys ϕ units of the general good. Let F be the distribution of money holdings in the decentralized market: $F(\widetilde{m})$ is the measure of agents holding $m \leq \widetilde{m}$ units of money. If M is the

[2] I only mentioned pure strategy equilibria. There exist however also a stationary mixed strategy equilibrium and non-stationary sunspot equilibria where the probability that money is used varies over time.

total money stock, F will satisfy $\int mdF(m) = M$ at every date. Let $V(m, \phi, F)$ be the value function for an agent with m units of money entering the decentralized market and $W(m, \phi, F)$ the value function of the agent when he enters the centralized market and $q(m, \widetilde{m}, \phi, F)$ and $d(m, \widetilde{m}, \phi, F)$ respectively the quantity of goods and money changing hands in a single coincidence meeting between a buyer with m units of money and a seller with $\widetilde{m}$ units of money. Agents discount the future at a rate $\beta = \frac{1}{1+r}$. In recursive form the problem for an agent entering the decentralized market can be written as

$$\begin{aligned} V(m, \phi, F) &= \sigma x \int \left[u\left(q(m, \widetilde{m}, \phi, F)\right) + W(m - d(m, \widetilde{m}, \phi, F))\right] dF(\widetilde{m}) + \\ &\quad + \sigma x \int \left[-q(\widetilde{m}, m, \phi, F) + W(m + d(\widetilde{m}, m, \phi, F))\right] dF(\widetilde{m}) + \\ &\quad + (1 - 2\sigma x)\, W(m, \phi, F) \end{aligned}$$

where the first integral represents the buyer's side, the second integral the seller's side and the third part a no-coincidence meeting. When the agent enters the centralized market, he faces the following maximization problem:

$$\begin{aligned} W(m, \phi, F) &= \max_{X, Y, m'} U(X) - Y + \beta V(m', \phi', F') \\ s.t. X &= Y + \phi m - \phi m' \end{aligned}$$

By solving backwards, starting from the centralized market, an expression is obtained for $W(m, \phi, F)$ which is linear in m :

$$W(m, \phi, F) = U(X^*) - X^* + \phi m + \max_{m'} - \phi m' + \beta V(m', \phi', F') = W(0, \phi, F) + \phi m$$

Quantities are determined using bargaining. Assuming that consumers have all the bargaining power:

$$-q + W(m + d, \phi, F) - W(m, \phi, F) = 0$$

using the fact that $W(m + d, \phi, F) = W(m, \phi, F) + \phi d$, the previous expression becomes

$$-q + \phi d = 0$$

The value function for the decentralized market -which can be shown to exist, to be unique, differentiable and strictly concave- becomes then

$$V(m, \phi, F) = \max_{m'} \sigma x \left[u\left(q(m)\right) - \phi d(m)\right] + U(X^*) - X^* + \phi m - \phi m' + \beta V(m', \phi', F')$$

with the first order condition

$$\beta V'(m', \phi', F') = \phi$$

and the envelope

$$V'(m, \phi, F) = \sigma x \left[u'\left(q(m)\right) q'(m) - \phi\right] + \phi$$

Lagos and Wright (2002) show that in any monetary equilibrium the distribution F is degenerate and every agent chooses the same $m' = m = M$. This also implies $d = m$. In a steady state $\phi' = \phi = \frac{q}{M}$.

Combining the last two equations in a steady state I get

$$u'(q) = 1 + \frac{1 - \beta}{\sigma x \beta}$$

which gives the unique solution $\widehat{q}$. The incentive condition $u(q) > q$ is also verified at $\widehat{q}$ since $u'(\widehat{q}) > 1$.

To summarise the discussion. Any monetary equilibrium implies $m = M$ with probability 1, which gives $d = m$. A steady state q exists and is unique, it has $\widehat{q} < q^*$ -where q^* is the efficient level of production $(u'(q^*) = 1)$- and increasing in β and σx. Nominal variables are proportional to M and real variables are independent of it. It can be easily shown that the optimal monetary policy is the Friedman rule.

Intuitively, the centralized market serves the purpose of making the distribution of money holdings degenerate. The main difficulty with a matching model with general money holdings is the ability to keep track of each agent's holdings: the distribution may become so complicated as to make the analysis impossible. Quasi-linearity eliminates the wealth effects: the value function on the central market is linear in m and m' is independent of m, agents adjust their money holdings on the central market in such a way that everyone at the end of the round of trade holds the same amount. This makes the distribution degenerate and the model tractable. A different idea involving households with a continuum of members is used by Shi (1997) to achieve the same result.

1.2.3 Credit in the Matching Model

In recent years there have been several attempts to modify the matching model described above to allow for the use of credit as a medium of exchange alongside money. I will focus on some papers that have obtained the result, modifying two crucial assumptions of the matching model. I will first review the papers by Kocherlakota and Wallace (1998) and Cavalcanti and Wallace (1999) and then Shi (1996). The first two introduce an imperfect record keeping technology while the third introduces the possibility for people to costly commit to future actions. I will then move to models where credit arises in enduring relationships, like in Jin and Temzelides (2001) and Corbae and Ritter (2004).

On the absence of a record-keeping Technology

Suppose society has a public record of individual histories, in the sense that every trade is recorded in a computer costlessly accessible by everybody[3]. If the record is updated for sure and continuously the whole process of exchange could be organized using simply credit. No agent would have the incentive to deviate from the credit arrangement, since everybody would immediately discover the defector and punish him refusing to trade with him for ever. The purely monetary economy is at the other extreme, without any public record of past trade.

Assume that the public record of histories is updated probabilistically[4]. Every period there is a probability π that the record is updated. In this economy there are (potentially) several groups of agents behaving differently: money holders, agents that conduct trade using credit, defectors i.e. agents that used credit to consume but didn't repay and producers.

$$
\begin{aligned}
rV_m &= \sigma(1-M)x\left(u(q_m)+V_p-V_m\right) \\
rV_c &= \sigma(1-M)x\left(u(q_c)+V_p-V_c\right) \\
rV_c' &= (1-\pi)\sigma(1-M)x\left(u(q_c')+V_p'-V_c'\right) \\
rV_p &= rV_p'=0
\end{aligned}
$$

The first value function describes money holders and is interpreted as before. The second concerns people using the credit system. With probability $\sigma(1-M)x$ they meet the right producer and exchange happens. In the next period they are supposed to produce for someone else to repay. The third equation describes agents defecting from the credit arrangement. With probability π the record is updated and they are discovered and punished by all the others: they will never consume

[3] One could interpret the record keeping technology as a credit card system.

[4] In Kocherlakota and Wallace (1998) a different approach based on mechanism design is used. In this section I adapt those results to a standard matching model.

again. With probability $(1-\pi)$ they continue undiscovered to consume. Producers -that have not and have defected in the past (V_p and V_p' respectively)- are down to zero because of the take-it-or-leave-it offers.

To sustain an equilibrium with both money and credit it has to be that the following incentive conditions are verified:

Money holders want to give up money for consumption

$$u(q_m) + V_p - V_m > 0$$

Agents consuming on credit have a positive surplus

$$u(q_c) + V_p - V_c > 0$$

Agents don't want to defect from the credit arrangement

$$V_c > V_c'$$

The last incentive condition is the crucial one to get an equilibrium with money and credit. Solving the system of value functions:

$$\begin{aligned}
V_m &= \frac{\sigma(1-M)x}{\sigma(1-M)x+r}u(q_m) \\
V_c &= \frac{\sigma(1-M)x}{\sigma(1-M)x+r}u(q_c) \\
V_c' &= \frac{(1-\pi)\sigma(1-M)x}{(1-\pi)\sigma(1-M)x+r}u(q_c') \\
V_p &= V_p' = 0
\end{aligned}$$

and from the take-it-or-leave-it offers condition

$$\begin{aligned} q_m &= \frac{\sigma(1-M)x}{\sigma(1-M)x+r}u(q_m) \\ q_c &= \frac{\sigma(1-M)x}{\sigma(1-M)x+r}u(q_c) \\ q_c' &= \frac{(1-\pi)\sigma(1-M)x}{(1-\pi)\sigma(1-M)x+r}u(q_c') \end{aligned}$$

which immediately imply that $q_m = q_c$ and that the first two incentive conditions are verified. Moreover $V_m = V_c > V_c' \Leftrightarrow$

$$r\pi\sigma(1-M)x > 0$$

which is verified for $\pi > 0$. Finally observe that when $\pi \to 1$ the last incentive condition becomes less stringent until $\pi = 1$ where the condition is equivalent to the second incentive condition. At this point people are just indifferent between using money or credit. Notice that when there is no record keeping technology ($\pi = 0$) credit doesn't work.

Money and credit from this perspective are purely substitute. Welfare increases when π increases, i.e. when the record keeping technology becomes more and more efficient in detecting defectors. The welfare problem is defined as follows

$$\underset{\pi}{Max}\ W = MV_m + CV_c \quad s.t. V_c > V_c'$$

observe that the welfare function is independent of π. Only the constraint depends on π through V_c': for agents to obtain some positive surplus it has to be that defection is not profitable. C is the proportions of agents that consume on credit. To characterize the solution to the problem in π, it is thus enough to compute the derivative of V_c' with respect to π :

$$\frac{\partial V_c'}{\partial \pi} = -\frac{2(1-\pi)\left[\sigma(1-M)x\right]^2 + r\sigma(1-M)x}{\left[(1-\pi)\left[\sigma(1-M)x\right]+r\right]^2}u(q_c') < 0$$

To quote Kocherlakota and Wallace (1998): "Technological advances improve welfare through their effect on the way transactions are made". In Cavalcanti and Wallace (1999), the population is divided in two subgroups: in one group agents' histories cannot be monitored while in the second group histories are completely monitored. The latter group is meant to represent stylised bankers, people that can issue private -or inside- money. Private money is accepted because, should bankers refuse to redeem it, they would be punished by being excluded from trade for ever. The focus of the paper is to prove that inside money is better than outside money, being able to implement a wider range of allocations. I will come back to this idea later when I will describe the model by Williamson (1999).

Finally, Aiyagari and Williamson (1998) have a similar model in which consumers enter into long-term contracts with a financial intermediary. Each period a fraction of the people cannot enter the long-term contract. Agents can defect from the long-term contract and trade every period on the money market. Monetary policy affects the decision to stick to the contract. Inflation reduces the incentives to defect inducing people to economise on real money balances and increases welfare dispersion and consumption variability across the population.

On the Commitment assumption

Shi (1996) explores a different idea to generate a role for credit. A crucial assumption in the matching model is that people cannot commit to future actions. Assume that there is a way of partially committing to future action in the form of collateral. In particular each agent has a personalized consumption tool -think of a spoon- which is necessary to consume. Without their spoons agents are unable to consume. In this case an agent without money can write a promise to his trading partner and give him his spoon as a guarantee of repayment. For the potential creditor to accept the promise, credit should yield a higher rate of return than money, or he would be better off waiting for a money holder to come. Creditors stay out of exchange and wait for the debtor[5]. There is no record keeping technology and the structure is maintained as before.

Among each type there are M money holders, P producers, C creditors and D debtors. In a stationary equilibrium, the proportions of new debtors have to be equal to the proportion of agents redeeming their promises -i.e. the total stock of promises has to be constant in a stationary equilibrium-. Assume that agents can hold either one unit of money or one promise -but not both simultaneously- or nothing and a promise -like money- is indivisible . The assumption that agents cannot hold simultaneously money and promises and the fact that creditors stay out of exchange implies that the number of creditor is the same as the number of debtors: $C = D$. The stationarity condition is

$$\sigma(1 - M - C)xP = \sigma MxD$$

The LHS is the inflow of debtors: $\sigma(1-M-C)x$ is the probability for a producer to meet a producer that is not going to issue a promise and P is the number of producers. The RHS is the outflow of debtors: a promise gets repaid when a debtor acquires money, which happens with probability σMx. He then visits his creditor and redeems his promise.

[5]This is due to avoid the problem that in a random matching model two individuals may not meet again.

The value functions in a stationary monetary equilibrium with credit will be:

$$\begin{aligned}
rV_m &= \sigma(1-M-D)x\,(u(q_m)+V_p-V_m)+\sigma Dx\,(u(q_d)+V_p-V_m)\\
rV_p' &= \sigma(1-M-C)x\,(u(q_c)+V_d-V_p')\\
rV_c &= \sigma Mx\,(V_m-V_c)\\
rV_p &= 0\\
rV_d &= 0
\end{aligned}$$

The first equation describes money holders and is interpreted as follows. Money holders can consume if they meet a producer, which happens with probability $\sigma(1-M-D)x$, or a debtor with probability σDx. In the two cases the quantity exchanged is different and is given by two different take-it-or-leave-it conditions. The second one concerns the issuers of promises. With probability $\sigma(1-M-C)x$ they meet the producer of their consumption good, they issue a promise and become debtors. The third equation gives the value for creditors. They stay out of exchange and wait for their debtor to acquire money -which happens with probability σMx-, come back and swap money for the promise. The fourth equation is the value to a pure producer which is zero by the take-it-or-leave-it assumption. The fifth equation gives the value for a debtor. A debtor is a producer who wants to acquire money as soon as possible to repay his debt and get back his consumption tool to start consuming again. The value is zero by the take-it-or-leave-it assumption.

An equilibrium with money and credit has money holders wanting to give up money for consumption

$$u(q_m)+V_p-V_m>0$$

promise issuers gaining a positive surplus

$$u(q_c)+V_d-V_p'>0$$

and creditors accepting the repayment if

$$V_m > V_c$$

Define $R \equiv r/\sigma x$. The main result shows that there exist values $(\widehat{R}, \widehat{M})$ such that for $R \leq \widehat{R}$ and $1 > M \geq \widehat{M}$ an equilibrium with money and promises exists. The conditions for existence of the equilibrium can be interpreted as follows. The repayment of a promise is a time consuming activity and the time spent on that activity depends on the proportions of money holders, on σ and on x. The higher is the number of money holders in the economy, the more efficient is the matching technology (σ higher) and the higher is the probability of meeting the right producer (x higher) the easier is for a debtor to find money and repay the promise. Finally a smaller r (agents are more patient), reduces the time cost that a debtor experiences until he regains the consumption tool. All these features make the life of a debtor easier and thus promote the issue of promises. The reason why people use promises even though it is an "inferior"[6] means of exchange is that not everyone has money $(1 > M)$. Promises sell at a discount compared to fiat money. This can be interpreted as a difference in rates of return. The nominal price of a promise is $\frac{q_c}{q_m}$ since a promise costs q_c units of goods and sells at a price $\frac{1}{q_m}$. Define the interest rate ρ as

$$1 = \left(\frac{q_c}{q_m}\right) e^{\rho t_r}$$

where t_r is the random time to maturity of the promise. Another equivalent way of writing this expression is

$$\rho = \frac{1}{t_r} \log\left(\frac{q_m}{q_c}\right)$$

since $q_m > q_c$ in equilibrium, for any finite maturity t_r, the interest rate will be

[6]In the sense that credit is costly in terms of the time needed to repay it. In a monetary trade, on the contrary, debt is discharged immediately.

positive in equilibrium. The rate of return on money, on the contrary, is zero in equilibrium since money costs q_m and sells at $\frac{1}{q_m}$.

Finally it can be easily shown that for R sufficiently small welfare is higher in the economy with promises than in the economy with only money. Some agents buy with money, other agents buy with credit and repay with money. This is the paper that comes closer to set out an idea of complementarity between money and credit.

Local Interaction of Money and Credit

Jin and Temzelides (2001) explore a different reason why money and credit may coexist. Their model connects the use of credit to frequent exchange and money to infrequent exchange. They modify a standard random matching model to allow for local matching in the sense that neighboring agents are more likely to meet than far away agents. The unit circle is divided into a large number of arcs of equal length which can be interpreted as villages. Two assumptions play a crucial role. The probability of meeting someone from the same village is high and the probability of meeting someone from different villages decreases with the distance between the villages. Suppose there are K villages, ordered according to the increasing distance from village 0. The probability to meet agents from each village is decreasing in the distance $\sigma_0 > \sigma_1 > \sigma_2 > ... > \sigma_K$. Matching probabilities are perfectly correlated across agents in the same village. Inside each village there is a record keeping technology to monitor the trading history of agents living in the village but not agents living in different villages. For simplicity, assume that agents exchange one unit of the good for one unit of money and quantities need not to be determined. Let also σ_M be the sum of the matching probabilities of the villages whose agents use money in meetings with agents from the same village, while -with an abuse of notation- σ_0 denotes the sum of the matching probabilities of the villages whose agents use credit in meetings with agents from the same village. Define u the utility of consuming one unit of a good and c the cost of producing it. The value functions for credit trade for a producer and a consumer are respectively

$$V_p = (\sigma_0 + \sigma_k)\left[x(u-c) + \beta V_p\right] + \sigma_M\left[xM(-c+\beta V_c) + (1-xM)\beta V_p\right]$$

$$V_c = (\sigma_0 + \sigma_k)\left[x(u-c) + \beta V_c\right] + \sigma_M\left[x(1-M)(u+\beta V_p) + (1-x(1-M))\beta V_c\right]$$

where the first is the value function for a producer meeting people from his own village and exchanging with credit, meeting agents from village k and exchanging with credit and producing for money with agents from his own village. The second value function represents an agent consuming with money when meeting agents from his own village.

The value functions in autarky, i.e. without exchange in meetings with agents living outside the village, for a producer and a consumer are respectively:

$$\widetilde{V}_p = (\sigma_0)\left[x(u-c) + \beta V_p\right] + \sigma_M\left[xM(-c+\beta V_c) + (1-xM)\beta V_p\right] + \sigma_k\beta\widetilde{V}_p$$

$$\widetilde{V}_c = (\sigma_0)\left[x(u-c) + \beta V_c\right] + \sigma_M\left[x(1-M)(u+\beta V_p) + (1-x(1-M))\beta V_c\right] + \sigma_k\beta\widetilde{V}_c$$

In order to have credit transactions between two distinct villages at a distance k, the following incentive conditions must be satisfied

$$-c + \beta V_p > \beta\widetilde{V}_p$$

$$-c + \beta V_c > \beta\widetilde{V}_c$$

i.e. it has to be that both a producer and a consumer are better off exchanging with someone from village k than exchanging only with agents from their own village.

Taking the difference

$$V_p - \widetilde{V}_p = V_c - \widetilde{V}_c = \frac{\sigma_k x (u-c)}{(1-\beta)}$$

it can be seen that the incentive conditions are satisfied if

$$\sigma_k > \frac{c(1-\beta)}{\beta x (u-c)} = \overline{\sigma}_k$$

Consider now the set of villages matched with the generic village j with probability less than $\overline{\sigma}_k$ and call it B. Define $\sigma_B = \sum_k \sigma_k$ for $\sigma_k < \overline{\sigma}_k$. The value functions for producers and consumers trading with money are respectively

$$\widehat{V}_p = (1-\sigma_B)\left[x(u-c) + \beta\widehat{V}_p\right] + \sigma_B\left[xM\left(-c+\beta\widehat{V}_c\right) + (1-xM)\beta\widehat{V}_p\right]$$

$$\widehat{V}_c = (1-\sigma_B)\left[x(u-c) + \beta\widehat{V}_c\right] + \sigma_B\left[x(1-M)\left(u+\beta\widehat{V}_p\right) + (1-x(1-M))\beta\widehat{V}_c\right]$$

where the first is the value function for a producer using credit in a meeting with an agent from a sufficiently close village and using money with others. The second is the value function for a consumer using credit with an agent from a sufficiently close village and using money with others.

For the monetary equilibrium to exist it has to be that a money holder wants to use money

$$-c + \beta\widehat{V}_c > \beta\widehat{V}_p$$

Solving the value functions and plugging them in the incentive condition, it gives

$$\sigma_B > \frac{c(1-\beta)}{\beta x(1-M)(u-c)} = \overline{\sigma}_B$$

To summarise the discussion. Trade between villages that are matched with probability higher than $\overline{\sigma}_k$ use credit and if $\sigma_B > \overline{\sigma}_B$ the remaining transactions are monetary. The higher is the probability to meet frequently a trading partner the higher is the incentive to trade on credit. Money serves to trade with strangers.

Enduring Relationships

A related recent model emphasizes further the role of enduring relationships. Suppose -as in Corbae and Ritter (2004)- that some double coincidence is possible: conditional on the event that i wants to consume what j produces, y is the probability that j wants to consume what i produces. Agents can choose to stay together and produce for each other until a breakdown of their technology happens and they become unable to produce unless they abandon the partnership. The breakdown occurs with probability $\zeta/2\sigma$. Suppose first that money is not available. In this case agents are either in search or in a double coincidence meeting, call them potential producers (P) and creditors (C). They exhaust the entire population: $P + C = 1$. The outflows and inflow are stationary if

$$\zeta C - xyP = 0$$

The value function for an agent in a double coincidence meeting and an agent in search are respectively

$$rV_c = \sigma (u - c) + \zeta (V_p - V_c)$$

$$rV_p = xy (V_c - V_p)$$

In a double coincidence meeting the agent consumes and produces until the partnership breaks down in which case the agent starts searching and with probability xy he finds a new partner.

The following incentive condition must be satisfied in an equilibrium with credit and no money:

$$-c + \frac{\zeta}{2\sigma} V_p + \left(1 - \frac{\zeta}{2\sigma}\right) V_c \geq V_p \tag{1.2}$$

i.e. staying in the partnership must be better than walking away.

Solving for the equilibrium:

$$P = \frac{1}{1 + \frac{xy}{\zeta}}$$

$$C = \frac{\frac{xy}{\zeta}}{1 + \frac{xy}{\zeta}}$$

$$V_c = \frac{\sigma\,(u-c)\,(r+xy)}{r\,(r+\zeta+xy)}$$

$$V_p = \frac{\sigma\,(u-c)\,(xy)}{r\,(r+\zeta+xy)}$$

From (2), provided $\frac{\sigma(u-c)(1-\frac{\zeta}{2\sigma})}{(r+\zeta+xy)} \geq c$ a symmetric steady state credit equilibrium exists. The following step involves showing that an equilibrium with money and credit exists. The logic to show the existence of a coexistence equilibrium is very similar and I will not go into the details here. The striking feature is that the introduction of money might be bad for the functioning of the credit economy since it reduces the incentives to stick to an enduring relationship. On one hand, money is useful since it allows agents to trade when there is only single coincidence; on the other it is armful because it dilutes the incentive to remain in a double coincidence meeting. This feature stresses once more the relationship of substitution between money and credit.

1.2.4 Private Money and Banks in a Matching Model

There is a classic question in monetary theory -dating back at least to Hayek and Friedman-. Should the government have the monopoly over the provision of money or should private banks be allowed to issue banknotes? The Free Banking Era in the US before the Civil War, Canada prior to 1935 and Scotland in the beginning of the nineteenth century are historical examples of private provision of money. The renewed interest is connected to the recent developments in payment system technologies, like stored-value cards, providing close substitutes for currency. Williamson (1999) analyses the former question in a search model with a bank.

A search model with a bank

At the start of the period each agent can be in the search sector -where goods are exchanged- with probability π or in the banking sector with probability $1-\pi$. When in the banking sector, the agent can fund an investment project, which is indivisible and needs an injection of q units of goods to begin. After funding an investment project, the agent is given an indivisible and portable banknote, which is a claim to the investment made. The banknote can be redeem at any time. When the banknote is returned to the bank the investment project backing it is interrupted. The agent then receives ρ units of consumption good and consumes them. Agents can hold one unit of money, one banknote or nothing but they cannot hold simultaneously money and banknotes. Finally and crucially, when in the banking sector agents cannot contact each other and trade. This prevents to organize a centralized market place, which would destroy the possibility of using money. The structure of the search sector is the same as before. The only change is in the source of randomness. In this model matching is directed instead of random, in the sense that an agent wanting to consume always meets an agent who produces his consumption good. Randomness is however still present, being embedded in preferences. Every period, each agent wants to consume with probability 1/2. Agents maximize

$$E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t (\theta_t u(q_t) - q_t)$$

where $\theta_t \in \{0,1\}$ and $\Pr[\theta_t = 1] = \Pr[\theta_t = 0] = 1/2$.

Among each type at any point in time there will be P agents not holding any asset, B agents holding banknotes and M money holders. Terms of trade are determined through bargaining. Consumers have all the bargaining power.

The Bellman equations determining the value functions will be

$$\begin{aligned} rV_m &= \frac{\pi P}{2}\left(u(q_m) + V_p - V_m\right) \\ rV_b &= \frac{\pi P}{2}\left(u(q_b) + V_p - V_b\right) + \frac{1-\pi}{2}\left(u(\rho) + V_p - V_b\right) \\ rV_p &= 0 \end{aligned}$$

The first equation describes a money holder. When he is in the search sector (with probability π) and meets a producer willing to produce (with probability $\frac{P}{2}$) he consumes and becomes a producer. In the banking sector he gets zero since he already has money and he cannot simultaneously hold a banknote.

The second equation gives the value for a banknote holder. In the search sector he can consume -with probability $\frac{\pi P}{2}$- using the banknote and then become a producer. In the banking sector if he is willing to consume he turns the banknote in, he is given ρ to consume and becomes a producer. Finally the producer gets zero by the take-it-or-leave-it conditions.

For an equilibrium with money and banknotes to exist, the following incentive conditions have to be verified.

Money holders want to give up money for consumption

$$u(q_m) + V_p - V_m > 0$$

Banknote holders want to give up their banknote for consumption

$$u(q_b) + V_p - V_b > 0$$

A banknote holder should get a positive surplus from redeeming a banknote

$$u(\rho) + V_p - V_b > 0$$

A banknote holder is indifferent -after redeeming his note- between funding a new project and having a new banknote or holding no assets.

$$V_b - q = V_p$$

Solving the system of value functions:

$$\begin{aligned} V_m &= \frac{\frac{\pi P}{2}}{\frac{\pi P}{2} + r} u(q_m) \\ V_b &= \frac{\frac{\pi P}{2} u(q_b) + \frac{1-\pi}{2} u(\rho)}{\frac{\pi P}{2} + \frac{1-\pi}{2} + r} \\ V_p &= 0 \end{aligned}$$

and from the take-it-or-leave-it conditions:

$$\begin{aligned} q_m &= \frac{\frac{\pi P}{2}}{\frac{\pi P}{2} + r} u(q_m) \\ q_b &= \frac{\frac{\pi P}{2} u(q_b) + \frac{1-\pi}{2} u(\rho)}{\frac{\pi P}{2} + \frac{1-\pi}{2} + r} \end{aligned}$$

By inspection the first incentive condition is immediately verified. The fourth implies that $q = q_b$. A solution is found by solving the following equation

$$q = \frac{\frac{\pi P}{2} u(q) + \frac{1-\pi}{2} u(\rho)}{\frac{\pi P}{2} + \frac{1-\pi}{2} + r}$$

subject to the second and the third incentive condition. These constraints can in turn be rewritten as

$$\frac{\left(\frac{\pi P}{2} + r\right)}{\frac{1-\pi}{2}} u(q) > u(\rho) > \frac{\frac{\pi P}{2}}{\left(\frac{\pi P}{2} + r\right)} u(q)$$

thus providing upper and lower bounds for ρ. A solution can be easily shown to exist and to imply $q = q_b > q_m$. To summarise there exist $\left(\underline{\rho}, \overline{\rho}\right) \gg 0$ such that for

$\underline{\rho}< \rho < \overline{\rho}$, a monetary equilibrium with private banknotes exists.

Two aspects are worth emphasizing. The first is that the upper and lower bounds on ρ are easily understood as a consequence of the way the investment takes place and yields returns. If ρ is too small the investment offers a low return and agents will never want to hold banknotes. If ρ is too high the returns of the investment are so high that agents will never want to give a banknote away for consumption in the search sector. In equilibrium $q = q_b > q_m$. This implies that banknotes sell at a premium with respect to money since they have a redemption value -being backed by a productive investment- while money doesn't have any.

To give an answer to the question whether money should be public or private, a maximization of social welfare is solved. In the present framework the result is unambiguously in favor of private money. Defining welfare as the sum

$$W = PV_p + BV_b + MV_m$$

and using the equilibrium value functions, since $B = 1 - P - M$ and $V_b > V_m$, it's easily seen that welfare is decreasing in M.

This shouldn't be surprising since private money has the advantage over public money of being backed by productive investment activities and no other disadvantages. In this model private money can accomplish the same role as fiat money as a medium of exchange and unlike money can promote productive investment.

Private Money, Lemons and Counterfeiting

In more elaborate models some of the inefficiencies of private money can be analyzed. In particular the literature has focused on the lemons problem and on the possibility of counterfeits -as in Williamson (2001)-. The first approach is very similar to the previous one except that investment projects are assumed to be of two ex-ante unknown qualities: good and bad. A good project requires an investment q_g

and yields a return R_g, while a bad project requires an investment q_b and yields a return R_b with $q_g > q_b$ and $R_g > R_b$. There are two types of banks, one specialized in good projects and one in bad projects. The same structure as in the case without private information is preserved. There may exist multiple equilibria: there is one equilibrium with only good projects backing notes, one with only bad projects and one with a mix of good and bad projects. For some parameter values, the only equilibrium is the one with bad projects. From a welfare point of view, the equilibrium with both money and private notes is never superior to an equilibrium with only notes or only money. When the letter is superior, a ban on the circulation of private banknotes can be justified.

When the possibility of counterfeiting, in the sense of creating a banknote not backed by a productive investment, is taken into account and there is a cost of counterfeiting a banknote, it is shown that if counterfeiting is sufficiently easy it may be better to ban the private issue of notes, since the equilibrium may involve the circulation of counterfeits only, which have no welfare improving role.

The Suffolk Banking System, in place in New England between 1824 and the Civil War had a clearing arrangement to redeem notes. Temzelides and Williamson (2001) analyse this case. There is no public money. The main objective of the paper is to compare the behavior of different banks when different circulating private notes coexist. There are two banks and two groups of people living in two separate regions. The model can be thought of as a decoupling of previous models in which banknotes originating in one region circulate in the other region. It is shown that in the presence of a clearing arrangement, notes circulate at par in both regions. Without clearing arrangement, notes from one region circulate at a discount in the other region.

Cavalcanti, Erosa and Temzelides (1999) construct a model with banks to address the question of the instability of a private money system. The presumption is that such a system would be plagued by the overissue problem. In the model some agents

-called bankers- have the right to be members of a clearinghouse. When a banker produces for someone, the clearinghouse credits his account with one unit of money. When a banker buys goods with money his account is debited by one unit of money. A banker can create banknotes when meeting both a non-banker and a banker. In both cases the creation of a note is recorded as a debt in the account of the banker. The account of each banker cannot go below zero -i.e. reserves of fiat money and banknotes should always at least balance- or the membership in the clearinghouse is terminated and the banker loses the possibility to issue notes. The question is whether the clearinghouse mechanism is sufficient to prevent overissue of banknotes and a private banknote system is feasible. If agents are patient enough, the private banknote system is stable, not being subject to the overissue problem. If bankers are patient enough their incentive to issue a note when their account balance is down to zero -which implies immediate consumption but also exclusion from the note issuing arrangement- is lower than the incentive to stick to the arrangement. When fiat money is sufficiently scarce, welfare is higher with private banknotes. The most serious obstacle to the feasibility of a private money system is the "lemons" problem and not the overissue problem.

1.2.5 Money, Credit and Banking

In a recent paper, Berentsen, Camera and Waller (2004) introduce a banking sector in the Lagos and Wright (2000) model with divisible money. A period is divided into day and night. During the day a random half of the population can consume and the other half can produce. During the night everyone consumes and produces the general good. Both markets are competitive. There are also perfectly competitive banks offering the possibility to deposit money and take out a loan of money. Through a record keeping technology, banks are able to monitor financial transactions. Trade on the goods market cannot be monitored. Money is essential to trade on the goods market during the day. Agents can commit to fully repay their loans with banks but not with other agents[7]. Before trading, agents can decide to take a loan or deposit money at a bank. The sequence of events is as follows. First an agent observes whether he will be a consumer or a producer in the morning. If he is a consumer he takes out a loan of money, if he is a producer he deposits money at the bank. Then the goods market opens and agents trade. In the night all agents trade and debtors repay their loans with cash. There is a government injecting money through lump-sum transfers. The growth rate of money is given by $M_t = (1 + z)\, M_{t-1}$ and the inflation rate $\pi = z$. Due to the fact that agents have quasi-linear utility during the night, every agent will end each period with the same amount of money. The following period during the day, some agents (consumers) will need money while others (producers) won't. Banks provide a costly way to redistribute money from producers to consumers. They perform a role similar to illiquid bonds in Kocherlakota (2003), to be discussed below. During the day, consumption is given by the Euler equation

$$u'(x) = (1 + i)c'(x)$$

where i is the nominal interest rate and the interest factor by

$$1 + i = \frac{1 + \pi}{\beta}$$

The equilibrium with money and banks is sub-optimal but it improves upon the equilibrium with only money. Consumption is decreasing in the inflation rate and the Friedman Rule implements the first best outcome.

Credit -implemented through a limited record keeping technology- allows money to be redistributed from agents not needing it to agents in need of it.

Other papers introducing banks in the matching models include Faig (2004) and

[7] The paper deals also with the case without commitment and the possibility of default.

He, Huang and Wright (2003). In the former, agents can deposit money and take loans from banks and use them when trading with trading partners whose history can be monitored. When trading with strangers who cannot be easily monitored, deposits can be made transferrable only at a cost. This creates a role for money. Money and credit compete as media of exchange in trades among strangers. In equilibrium, money specializes in small ticket items, credit in large ones. In the latter paper, banks have the role of accepting deposit to keep money safe from theft. Money and deposits compete as media of exchange.

1.2.6 Bonds in the Matching Model

A related question -dating back to 1935, when Hicks wrote his "Suggestion for simplifying the theory of money"- concerns the coexistence of money and bonds. Aiyagari, Wallace and Wright (1996) construct a matching model of money where agents use indivisible bonds. Agents can use fiat money and interest-bearing securities as alternative media of exchange. These nominal, bearer and safe securities are introduced in the following way. There is a proportion G of government agents that take part into the exchange process in an exogenously specified way. When an agent with a unit of money meets an agent willing to produce for him, trade happens. A government agent, with probability π, offers a two period pure discount bond and destroys his own money. At maturity a private agent holding a bond can exchange it for one unit of money. Before maturity government agents can however refuse to redeem the bond with probability r. The cumbersome way chosen to introduce bonds is meant to guarantee that the total amount of asset is unchanged in the economy. The probability of rejection is crucial to obtain a discount on securities. There are two types of stationary equilibria with money and circulating bonds. In the first, matured securities exchange at par and not-yet-matured securities exchange at a discount if and only if $r > 0$; in the second, matured securities exchange at a discount and not–yet-matured securities exchange at less than matured securities.

Coexistence of money and bonds is obtained since government agents discriminate against not-yet-matured bonds (the probability r with which they are refused is positive) and bonds are indivisible. These features can be interpreted as forms of legal restrictions on the circulation of bonds.

1.3 Spatial Separation

In this section I will examine two different models with spatial separation giving rise to the use of money and to some form of coexistence of money and credit. I will use the terminology adopted by Townsend (1980) and call the two models the Turnpike Model and the Cass-Yaari Model respectively. While in the matching model of money markets are absent and agents meet at random, here there are competitive markets for the exchange of commodities but they are spatially separated or incomplete and money serves to transfer value from one market to the next. The assumptions on the limitations of commitment and the absence of a record keeping technology are again crucial to obtain a role for money. Overall the central idea in this literature is that money serves to facilitate trade among strangers -agents knowing little about each other- while credit mediates exchange between agents that can be monitored. In the last part of the Section, I will analyse models of payment systems and finally the coexistence of money and illiquid bonds.

1.3.1 Money in the Turnpike Model

The economy is populated by a countably infinite number of agents, each living forever. There is an infinite turnpike with two lanes running in opposite direction: east and west. Agents are split between those living on one side of the turnpike and those living on the other side traveling eastward and westward respectively. Each period they travel exactly one mile, stop, trade with the agent traveling on the opposite lane and then resume their itinerary. It is a pure exchange economy and each agent is alternatively endowed with one unit or zero units of the only good in the economy. When two agents meet, one and only one has an endowment of the good. Agents have preferences given by a strictly increasing, strictly concave and differentiable utility function $u(x)$, with $u'(0) = \infty$ and discount the future at a rate β.

There is no role for credit, since two agents meet only once in their lifetime and there isn't any record keeping technology. There is an amount M of money in the economy. Define the price of the good in terms of money as p. In recursive form the problem for a representative agent of type A starting with no endowment and m units of money will be

$$\begin{aligned} V^A(m) &= \max_{x,m'} \quad u(x_1^A) + \beta u(x_2^A) + \beta^2 V^A(m') \\ s.t. \quad m' &= m + p' - px_1^A - p'x_2^A \\ m' &\geq 0 \end{aligned}$$

The Euler equation for an agent of type A is

$$\beta u'(x_1^A) = u'(x_2^A)$$

Analogously for a representative agent of type B starting with one unit of the good and no money

$$u'(x_1^B) = \beta u'(x_2^B)$$

Market clearing requires

$$x_1^A + x_1^B = 1$$

$$x_2^A + x_2^B = 1$$

The solution of the four equations gives consumption in the monetary equilibrium. Clearly the equilibrium is inefficient, since efficiency requires equality of the marginal rates of substitution. First best can be however attained by an activist monetary policy in which the gross rate of deflation is equal to the discount factor: $z = \beta$, the Friedman rule. Money is thus used to transfer value from one market to the next and it has a role in mediating intertemporal trade.

Coexistence of Money and Credit

Townsend (1980) shows that by converting the turnpike into a circle opens the possibility for agents to exchange promises, since they can now meet again in the future and their promises can be repaid. The model, however, pointed to some difficulties in deriving an equilibrium with promises whose existence is guaranteed only when some exogenous bound on the amount of promises issued is imposed. The model moreover left open the question of the coexistence of money and credit. In a similar setting Townsend and Wallace (1982) have shown that private securities may emerge and circulate in order to facilitate intertemporal exchange. They show that circulating private debt often generates a coordination problem, in the sense that the amount of debt issued has to satisfy restrictions not implied by individual maximization and market clearing alone.

A different idea was explored by Ireland (1994) and Manuelli and Sargent (1992). They modified slightly the previous setting to introduce credit and obtain coexistence between money and credit. Suppose that agents, instead of moving every period to the next location, are allowed to remain in the same location for two periods. Agents will use private securities to trade during the period in which they stay in the same location and money to connect one trading session to the next. Lengthening the trading session improves welfare and in the limit -i.e. when agents stay always in the same location- first best is reached: if agents are allowed to stay in the same location for ever, the model effectively becomes an Arrow-Debreu model with complete markets.

Chatterjee and Corbae (1996) add costly commitment to the model. Each of an infinite number of locations is inhabited by $N+1$ types of infinitely lived households. There are two members in each household a producer and an accountant. During a period three sub-periods can be distinguished. In the first sub-period the producer is moved to a new randomly chosen location, while the accountant stays at the old location. In the second sub-period, the producer can produce goods, enter into contracts -i.e. issue promises- with other agents and ship goods back to his old location where the accountant still resides. In the third sub-period the accountant joins the producer in the new location. Promises can now be issued since the producer can costlessly ship goods back to the accountant. The problem is that the accountant doesn't have any incentive to keep the promise since he will never meet the creditor again. With a record keeping technology an equilibrium with both money and credit exists. Eliminating money may improve welfare.

1.3.2 Money in the Cass-Yaari Model

In the turnpike model money acts as a store of value rather than a medium of exchange. In the Cass-Yaari model, money acts as a medium of exchange. There are a countably infinite number of households and a countably infinite number of perishable commodities. Each household, living forever, consists of a pair of agents and is located on a line, one household per integer. Household i cares about consumption of commodity i and $i+1$ only. Each period an household receives an endowment of one unit of good i. Each member is able to travel one half of the distance to the adjacent integers where there are the markets for the exchange of good i and $i+1$ (to the right) and good i and $i-1$ (to the left). The construction generates absence of double coincidence of wants and spatial separation which make money essential as a medium of exchange. Members of Household i have preferences given by a strictly increasing, strictly concave and differentiable utility function $u(x_i, x_{i+1})$, with indifference curves asymptotic to the axes and discount the future at a rate β. Promises in this economy will not be used since agent i has nothing to offer agent $i+1$ in order to repay the promise and there is no record keeping technology. Suppose there is a certain amount M of money in the economy and define the price of the good in terms of money as p, then in recursive form the problem for a representative agent will be

$$\begin{aligned} V(m) &= \max_{x,m'} \quad u(x_1, x_2) + \beta V(m') \\ s.t. \quad m' &= m + p - px_1 - px_2 \\ m &\geq px_2 \end{aligned}$$

The Euler equation is

$$\beta \frac{\partial u(x_1, x_2)}{\partial x_2} = \frac{\partial u(x_1, x_2)}{\partial x_1}$$

Market clearing requires

$$x_1 + x_2 = 1$$

The equilibrium is inefficient, since efficiency requires equalizing the marginal rates of substitution. First best can however be reached by the Friedman rule: $z = \beta$. Money here has the role of an intratemporal medium of exchange.

Coexistence of Money and Credit

Imagine to convert the line into a circle, preserving the remaining part of the model. Suppose also -along the lines of Bernhardt (1989)-that each period a shock hits the economy and households are relocated to different points on the circle. The new setting permits the exchange of promises since two trading partners will meet at some point in the future with their role reversed: agent i meets $i+1$, makes a purchase on credit -issuing a promise-. He will repay in the future when he will be of type k and his creditor will be of type $k-1$. The arrangement is sustained by bilateral punishments for reneging on a promise -i.e. if i reneges, $i+1$ will never produce for him again-. If the economy is not too big -i.e. the number of types is not too high- and the probability of meeting again is sufficiently high, promises will circulate in the economy. The model creates room for a promise to be used as a medium of exchange, but doesn't generate coexistence of money and credit unless consumption shocks are added. In a sufficiently large economy and for a sufficiently large consumption shock, agents first use money to purchase and then issue promises. If agents urgently need to consume, they try to issue large promises. This increases the incentive to renege, making the equilibrium with promises collapse. The use of money reduces the incentives to renege and it allows trade to happen. The result point to the fact that in small communities trade is mainly organised on a credit basis, while in large communities of strangers -who will probably never meet again- money is instead the medium of exchange. Once again the idea that credit is associated to frequent relationships, while money to infrequent trades is exploited. In the following chapter, I will propose a model of coexistence of money and credit that resembles a Cass-Yaari economy but it is based on the availability of collateral and explores the idea of the complementarity between money and credit.

1.3.3 Payment systems

Consider a model with spatial separation -as in Temzelides and Williamson (2000)- in which there is a countable infinity of locations. Each location is inhabited by a representative household with two members a producer and a shopper. Household i consumes commodity i and produces $i+1$. Production of y units of a commodity cost

$c(y)$ and the cost function is increasing convex and twice continuously differentiable. First, shoppers leave their home locations and travel to the next location, then production takes place, agents trade and shoppers return home. There are three types of households: each type is unproductive in one period, type 1 is unproductive in periods 0,3,6..., type 2 is unproductive in periods 2,5,8..., and type 3 in periods 1,4,7... The structure makes sure that people cannot barter or exchange promises. Trade can take place using money or taking part in a centralized payment system. When agents are not taking part in a payment system the problem for an agent unproductive in period one will be

$$V_1(m_1) = \max \quad u(x_1) + \beta V_2(m_2)$$

$$s.t. \quad p_1 x_1 \leq m_1$$

$$m_2 = m_1 - p_1 x_1$$

In period two the agent can consume and produce

$$V_2(m_2) = \max \quad u(x_2) - c(y_2) + \beta V_3(m_3)$$

$$s.t. \quad p_2 x_2 \leq m_2$$

$$m_3 = m_2 - p_2 x_2 + p_2 y_2$$

In period three he can only produce

$$V_3(m_3) = \max \quad -c(y_3) + \beta V_1(m_1)$$

$$s.t. \quad m_1 = m_3 + p_3 y_3$$

The equilibrium will exhibit price dispersion and the allocation will not be efficient, since two cash in advance constraints are involved. When a centralized payment system is in place with settlement on net at the end of each period, the cash in advance constraint in the second sub-period is relaxed, average money holdings are reduced and there is a welfare improvement. When an agent is simultaneously producing and consuming, he can use current receipts to finance current expenditures and settle on net at the end. The way to achieve efficiency is to relax also the first cash in advance constraint. This can be done if the payment system operates account balances -without settlement at the end of each period- at an interest rate equal to β. Alternatively the Central Bank could operate a system of daylight overdrafts at zero nominal interest rate financed by the issue of money. In a similar spatial separation model of money and credit by Williamson (2002), the use of private money replaces completely outside money as a medium of exchange.

Green (2003) asks the following question: is inflation going to make money inessential, in the sense that people will increasingly make use of alternative payment arrangements not involving the use of money? In a model in which money and credit are substitute, inflation may indeed spoil the acceptability of money. In a model, however, in which money serves as a means of payment -to settle debts- inflation doesn't spoil the acceptability of money. Consider for instance the model by Freeman (1996). In an OLG economy he considers the role of monetary policy when a payment arrangement is in place. Overlapping generations of agents are spatially separated and divided between debtors and creditors. Private debts are redeemed with money at a central clearing house and there is a market for second hand promises. Liquidity problems might arise due to the fact that trips to the clearing house are not synchronized and creditors and debtors may not be simultaneously at the clearinghouse for redemption. Freeman shows that equilibria with constrained liquidity are inefficient and the Central Bank should issue currency temporarily to overcome the distortion and attain efficiency. In such a model inflation doesn't spoil the acceptability of money since money is necessary as a means of payment for debt. Mills (2004) shows however that in Freeman's world such a payment system is not essential in the sense that there is an alternative payment system not using money as a means of payment that implements the same outcome. Too much commitment on the part of agents is assumed. To obtain essentiality Mills (2004) introduces in Freeman's model collateralized lending.

1.3.4 Money and Illiquid Bonds

A recent paper by Kocherlakota (2003) deals with the coexistence of money and illiquid bonds. The model economy is built in such a way as to give money an essential role in performing intratemporal exchanges. Agents have different intertemporal marginal rates of substitution, being subject to liquidity shocks, in the form of multiplicative shocks to preferences. This creates a need for further intertemporal trades of money that are accomplished using nominal risk-free bonds. A liquid bond -a bond that is portable and can be exchanged for goods- wouldn't serve the purpose since it would have the same rate of return as money. Illiquid non-portable bonds command a positive rate of return and are essential to insure against liquidity shocks. The analogy is with registered bonds issued by the US Treasury. In Kocherlakota's words "if an individual tries to buy apples with a registered bond, transferring ownership of that bond requires a lot of resources". In chapter 4 I introduce a commitment technology in the model by Kocherlakota (2003) and show that illiquidity may not be enough to guarantee the essentiality of bonds.

1.4 Conclusion

The matching model and spatial separation model of money have contributed greatly to improve our understanding of why people use money to lubricate exchange, providing a framework in which money arises endogenously as a genuine medium of exchange. An important limitation in both models was the absence of alternative media of exchange. The literature has generated a number of models were coexistence of money and credit is possible. Money and credit or money and private notes compete as media of exchange and only an imperfection in the functioning of credit creates room for the use of money. In these models, money doesn't serve as a means of payment, to settle debts. We do however observe in reality instances in which money and credit are complementary and debts are settled with money. To see money and credit as substitute or complementary can have implications for monetary policy. In the presence of steady inflation if money and credit are substitute, we should expect people to switch to credit in order to avoid the inflation tax. In an economy where they are complementary and money is the means of payment, the reverse may happen. The following chapter further explores this issue.

Chapter 2

On the Complementarity of Money and Credit

2.1 Introduction

Money performs four main functions. It is a unit of account, a store of value, a medium of exchange and a means of payment. The most neglected of these in the literature has been the means of payment, i.e. the means to settle debt. Other objects can play the role of unit of account, store of value or medium of exchange. Only money however acts as the means of payment[1].

The aim of the present chapter is to construct a model where money serves as the means of payment. To address this issue, an environment is needed where agents use both money and credit as media of exchange. In particular, I look at bilateral credit -in the form of promissory notes- and show that money is used to settle bilateral promises. Money and credit are complementary.

Complementarity between money and credit turns out to have interesting impli-

[1] "A medium of exchange includes those assets, or claims, whose transfer to the seller will commonly allow a sale to proceed. Payment is in some sense final. The most important general function of money is to serve as a means of payment."(Goodhart (1989))

cations for the effects of monetary policy on macroeconomic variables.

The model features lack of double coincidence of wants and spatial separation. When purchasing goods, agents can use fiat money to overcome the absence of double coincidence of wants and they can buy on credit issuing bilateral promises.

The lack of double coincidence of wants has been crucial to explain why money is useful in lubricating exchange, ever since the works of Menger and Wicksell. As seen in the previous chapter, Kiyotaki and Wright (1989) constructed a search model around the lack of double coincidence of wants to explain the use of money as a medium of exchange. In order to make money essential however they had to introduce limitations on enforcement and commitment, thus ruling out alternative means of exchange as multilateral and bilateral credit. Specifically, they assumed that no public memory technology was in place and agents couldn't commit to future actions. Kocherlakota and Wallace (1998) studied multilateral credit as an alternative medium of exchange, introducing in the Kiyotaki and Wright model a public memory technology. In their model multilateral credit -which resembles a credit card system- is a claim to future commodities, not to money. In their model money and credit are substitute.

In this chapter, I rule out any public memory technology and thus multilateral credit. I assume that agents -being endowed with collateral- can commit to keep their bilateral promises. Repayment is decentralized and it requires time. Markets are walrasian and money is durable and perfectly divisible. I drop the assumption, typical of the search theory of money, that agents meet at random and assume that agents can choose their itineraries: this allows me to avoid restrictions on money holdings and study an economy with general portfolios of money.

I assume that a day is divided in three periods of eight hours. Each period can be used to either consume, produce or rest. After producing, agents need to rest for eight hours during which they will not be able to consume or produce. This

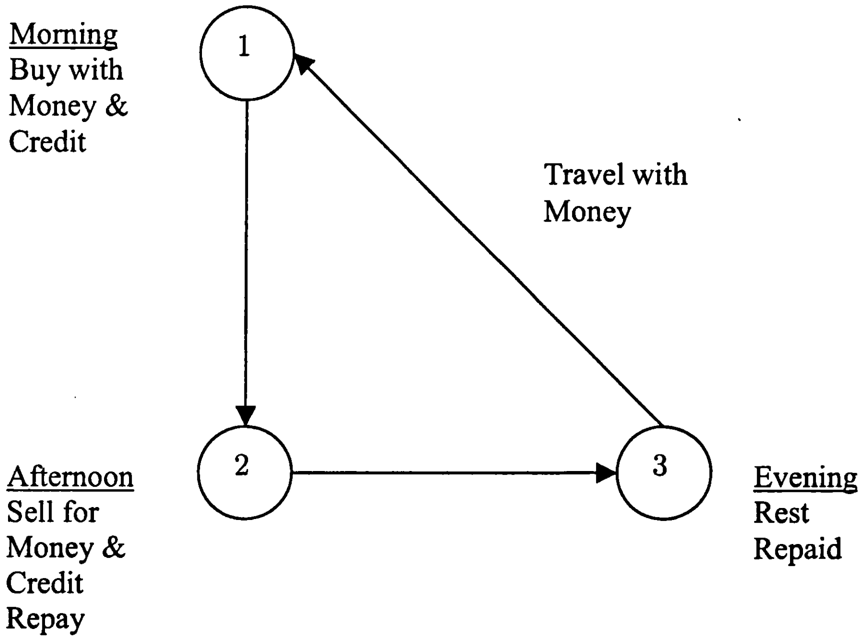

Figure 2.1: Life of an Agent

assumption captures in a simple, deterministic way a time mismatch between the arrival of liquidity and of consumption or production opportunities, inducing agents to hold money for some time before being able to spend it.

In equilibrium each agent uses both money and bilateral credit to exchange commodities and bilateral credit is repaid with money. The typical life of an agent in the economy is summarized in Figure 1. In the morning, agent 1 is on his island, where he purchases commodities with money and promises. Then he travels to island 2 together with his creditor. In the afternoon he produces for money, repays his debt and produces for promises becoming a creditor himself. Then he travels with his debtor to island 3 and in the evening -his period of rest- he waits for his debtor to repay. Finally he travels to island 1 with money and the next morning the cycle restarts. Output and welfare are higher than in the corresponding economy without bilateral credit.

The model generates interesting predictions on the effect of anticipated inflation on macroeconomic variables. Anticipated inflation drives up the nominal interest rate, which makes costly to hold promises of future money. The elasticity of the nominal interest rate to inflation is greater than one and thus the real interest rate increases with inflation. The credit to money ratio and the credit to output ratio decrease with inflation. An agent holding a promise will get final payment with money in the future. He will then have to hold money overnight before spending it to purchase goods. With inflation the opportunity cost of holding money is higher: the agent will reduce his holdings of promises of money. In a recent paper, Green (2003) argues that inflation may destroy the acceptability of money in a model where debt is not settled with money, but not in a model where money serves as the means of payment. Here I go one step further, arguing that inflation may harm credit. The nominal interest rate is more than unit elastic with respect to inflation and thus the real interest rate increases with inflation. The failure of the Fisher effect is due to the time mismatch between the arrival of liquidity and consumption opportunities: in an environment with inflation some agents hold money for one period. Output and welfare decrease with inflation. Inflation, harming credit, reduces the amount of transactions performed and thus reduces output. Interestingly, the negative effect of inflation on output emerges for high enough inflation rates. In Tobin (1965) anticipated inflation induces agents to substitute in their portfolios capital for money thus increasing the capital stock and output and decreasing the real interest rate. In the literature this is known as the Mundell-Tobin effect. My model generates a reverse Mundell-Tobin effect[2]: credit and output decrease with inflation, the nominal interest rate is more than unit elastic in inflation and therefore the real interest rate increases with inflation.

Recent empirical analyses highlight three facts consistent with the model:

[2]In a different model Stockman (1981) derived similar predictions.

1. there is negative correlation between inflation and private credit (Boyd, Levine and Smith (2001))

2. there is negative correlation between inflation and output for high enough inflation (Bullard and Keating (1995); Fischer, Sahay and Vegh (2002))

3. the nominal interest rate is not unit elastic in the inflation rate (Koustas and Serletis (1999))

Boyd, Levine and Smith (2001), in particular, find evidence of a negative relationship between financial activity and inflation, using both cross-country and panel data. Their measure of financial activity is private credit to GDP. As for the second fact: "There is now a substantial body of evidence indicating that sustained -and, therefore, likely predictable- high rates of inflation can have adverse consequences either for an economy's long-run rate of real growth or for its long-run level of real activity." (Boyd, Levine and Smith (2001)). Bullard and Keating (1995) show that output is decreasing in the inflation rate when inflation is high enough. In a recent paper, Stanley Fischer highlights the fact that in high inflation countries credit and output have been severely harmed by inflation.

Baumol-Tobin cash management theory is the basis for recent estimates of the demand for cash with microeconomic data (Attanasio, Guiso and Jappelli (2002)). Households choose their money holdings for transaction purposes taking into account the cost of transaction time and forgone interest. My model suggests that alongside interest rate, time and consumption, debt might have a role in determining agents' demand for cash.

In Freeman (1996), unlike in the present paper, the clearing system is centralized: agents need to move to the central island to clear debts. Here the repayment process is completely decentralized.

The chapter is structured as follows. Section 2 describes the model. Section

3 derives the equilibrium and analyses the impact of anticipated inflation on the interest rate, output, the credit to money ratio and welfare. Section 4 discusses the results. Finally, section 5 concludes.

2.2 The Model

Time is discrete and continues for ever. Agents are infinitely lived. They can produce, consume and rest. There are $N > 4^3$ islands arranged on a circle, indexed by $j = 1, ..., N$. Each island is inhabited by a continuum of mass three of agents. On each island j one and only one type of perishable commodity (j) can be produced: agents j are the producers of commodity $j + 1$ (modulo N). After producing individuals need to rest for one period: they will not be able to consume or produce for one period[4]. In order to clear the markets, some of agents j will be consuming in the morning, producing in the afternoon and resting in the evening (type 1), some others producing in the afternoon, resting in the evening and consuming in the morning (type 2) and finally some resting in the evening, consuming in the morning and producing in the afternoon (type 3). There will therefore be eight hours periods in a day and agents will do either the morning, afternoon or evening shift. (See Figure 2).

To induce absence of double coincidence of wants, I assume that agents j enjoys consuming commodity j only. Every ordered pair of islands is connected by ships. Agents are free to choose their itineraries. On each island competitive markets for the exchange of the local commodity open each period, closing at the end of the period. When agents arrive on the island they are randomly matched to a trading partner. They can however be randomly re-matched without cost to another trading

[3] $N > 4$ guarantees that promises cannot be simply swapped instead of being repayed.

[4] The cost of consuming or producing without resting is infinite. This assumption can be relaxed by having a finite disutility of not resting. Temzelides and Williamson (2001) use a similar assumption.

Day

	Evening	Morning	Afternoon	Evening	Morning
Type 1	Rest	**Consume**	**Produce**	**Rest**	Consume
Type 2	Consume	**Produce**	**Rest**	**Consume**	Produce
Type 3	Produce	**Rest**	**Consume**	**Produce**	Rest

Figure 2.2: A Day: Height Hours Shifts

partner if they want to. This assumption -together with the fact that there is a continuum of agents- keeps the price competitive. At the beginning of time, on each island, there is an amount M of fiat money, in the form of durable and worthless pieces of paper. Agents have also the option of issuing their own promises. There is no central record-keeping technology to monitor and enforce promises. Each agent has an amount of durable collateral specific to him without which he is unable to consume[5]. When issuing a promise, each agent surrenders his collateral to his trading partner in exchange for the commodity he purchases. Repayment takes time: the issuer of a promise needs to go back to his island, produce and sell in order to gain the money needed to make final payment. There is no clearing-house for promises. The economy has limited commitment: the commitment power of agents is confined to the use of collateral. At all times agents can simply decide not to participate in the exchange process. On each island there are competitive markets for the exchange of goods for money (the money market) and for the exchange of goods for promises (the credit market).

Agents are characterized by a utility function which I assume to be linear in

[5]The utility of consuming without the collateral is zero. Shi (1996) uses a similar assumption.

consumption

$$u(x_{t,j}) = x_{t,j}$$

where $x_{t,j}$ is the quantity bought at time t on island j and by a cost function -in terms of utils- which I assume to be quadratic in the productive effort

$$c(y_{t,j}) = \frac{1}{2}(y_{t,j})^2$$

where $y_{t,j}$ is the quantity produced by producer j.[6] The objective of an agent of type j is to maximize

$$\sum_{t \in T} \beta^t \left[x_{t,j}^M + x_{t,j}^C - \beta \frac{1}{2} \left(y_{t+1,j+1}^M + y_{t+1,j+1}^C \right)^2 \right]$$

where $T = \{1, 4, 7, ...\}$ in order to take into account the resting period and $0 < \beta < 1$ is the time discount rate. $x_{t,j}^M$ and $x_{t,j}^C$ are the quantities of good j bought at time t by agent j on the money market and on the credit market respectively, while $y_{t+1,j+1}^M$ and $y_{t+1,j+1}^C$ the quantities of good $j+1$ sold for money and for credit. Define the price on the money market for good j at time t as $p_{t,j}$ and the price on the credit market as $q_{t,j}$.

2.2.1 Sequence of Events

Let me first describe the sequence of events informally. For simplicity, suppose there are 8 islands. In the Morning agent 1 is on island 1. He spends money to buy his consumption good from agent 8. He then increases his purchases issuing bilateral promises secured by collateral. At the end of the morning agent 1 and 8 travel together to island 2[7]. Meanwhile agent 2 is also coming to island 2. Agent 1 sells

[6] The specific functional forms are not crucial. Quasi-linearity is however crucial to compute the solution explicitly.

[7] Agents are randomly matched once they arrive on an island. Since there is a continuum of agents, a debtor and a creditor may not find each other again. To overcome this problem I make them stick together.

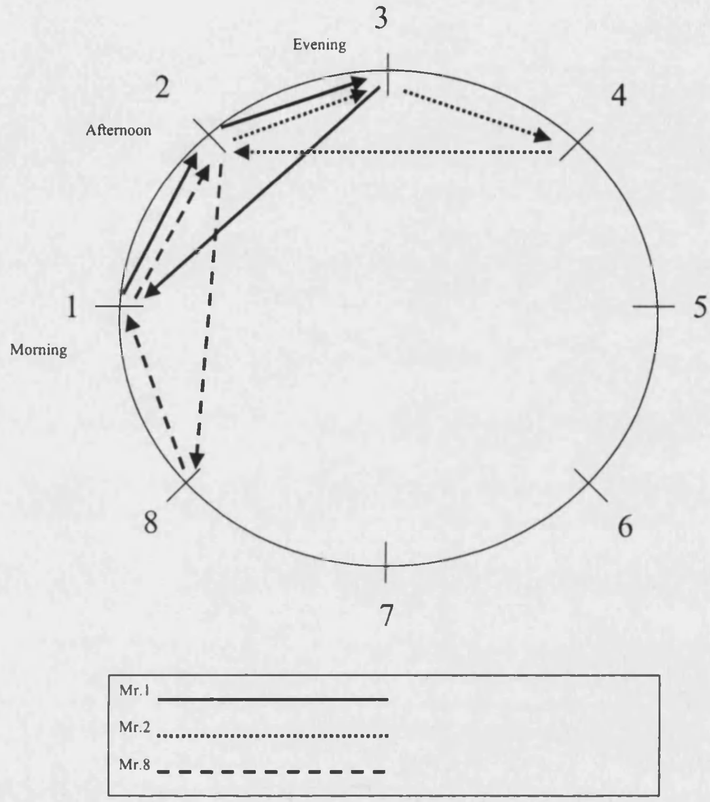

Figure 2.3: Sequence of Events

goods to agent 2 for money. With the money he got, he repays agent 8 and gets his collateral back. He then sells goods to agent 2 for promises, becoming a creditor. At the end of the afternoon he travels with agent 2 to island 3. Meanwhile agent 8 travels to his consumption island. In the evening agent 1 will be resting and waiting for repayment. The next morning he will restart the cycle. (See Figure 3).

Formally the sequence of events is as follows. Agent j at time t uses part or possibly all of the money he accumulated in the past (m_{t-1}^j) to buy goods on the money market

$$p_{t,j}x_{t,j}^M \leq m_{t-1}^j$$

then proceeds to the credit market where he sells a promise d_t^j to be repaid in the future:

$$q_{t,j}x^C_{t,j} = d^j_t$$

In the following period he produces for money in order to pay off his debt:

$$p_{t+1,j+1}y^M_{t+1,j+1} = m^j_{t+1} \geq d^j_t$$

He is then free to produce for a promise that will be repaid the next day when he will follow his debtor on island $j+2$ in order to get his money back:

$$q_{t+1,j+1}y^C_{t+1,j+1} = b^j_{t+1} = m^j_{t+2}$$

At time $t+2$ he receives a lump-sum money transfer from the government. The previous sequence of exchanges gives rise to the following budget constraint

$$m^j_{t+2} = p_{t+1,j+1}y^M_{t+1,j+1} + q_{t+1,j+1}y^C_{t+1,j+1} + m^j_{t-1} - p_{t,j}x^M_{t,j} - q_{t,j}x^C_{t,j} + \tau_{t+2,j}$$

the cash-in-advance constraint

$$p_{t,j}x^M_{t,j} \leq m^j_{t-1}$$

and the repayment constraint

$$p_{t+1,j+1}y^M_{t+1,j+1} \geq q_{t,j}x^C_{t,j}$$

which states that the amount of money obtained producing has to be at least enough to repay the debt. The repayment constraint embeds the relationship of complementarity between money and credit in an otherwise fairly standard cash-in-advance framework and is going to play a major role in the analysis.

The economy features limited enforcement. Agents -as in Kocherlakota (1998)-

always have the option to stay home and do nothing. The relevant participation constraint states that producing tomorrow for promises and consuming in three periods must give non-negative utility:

$$\beta^{t+3}\left(x_{t+3,j}^{M}+x_{t+3,j}^{C}\right)-\beta^{t+1}\frac{1}{2}\left(y_{t+1,j+1}^{M}+y_{t+1,j+1}^{C}\right)^{2}\geq 0$$

2.2.2 Key Assumptions

The model features five key assumptions:

1. absence of double coincidence of wants and spatial separation are necessary to give money a role as a medium of exchange;
2. absence of a record keeping technology rules out multilateral credit contracts;
3. limited enforcement;
4. collateral makes the exchange of promises possible, giving agents a limited form of bilateral commitment power;
5. agents do not always participate in the exchange process: they have to rest after producing. Rest is crucial to generate an essential role for credit as a medium of exchange.

The model differs from a standard search model in that agents can choose their itineraries. This allows me to lift the $\{0,1\}$ restriction on money holdings, typical of the search theory of money, and analyse a model where general portfolios of perfectly divisible money can be held. The absence of double coincidence of wants alone wouldn't be enough in my model to induce a role for money as a medium of exchange if agents could meet in the same market place and strike multilateral deals. Spatial separation of markets prevents multilateral deals.

The paper focuses on the interaction of money and bilateral credit rather than multilateral credit. I therefore rule out all devices that could allow agents to exchange multilateral credit contracts. In fact I assume that there isn't any technology allowing agents to keep track of each other. I also rule out any technology that could allow agents to commit to future actions other than collateral.

Collateral in the model is individual specific, has no public value and is necessary to consume. The reader can think of a consumption tool as in Shi (1996) or alternatively of a blueprint for future production. In the model the borrower leaves the collateral with the lender until the debt is repaid. Such form of collateralized borrowing is known as a repurchase agreement (repo). The fact that collateral is necessary to consume induces agents to repay their promises for sure and rules out default .

In the model agents need to rest for one period after producing. This assumption captures in a simple way the fact that consumption, investment or production opportunities may arise at a different time with respect to the time at which contracts come due and agents may thus have to hold money for some time before being able to spend it. Temzelides and Williamson (2001) have a similar assumption.

2.2.3 Individuals

In a symmetric equilibrium individuals choose $\left(x_t^M, x_t^C, y_{t+1}^M, y_{t+1}^C, m_{t+2}\right)$ to solve

$$\max \quad \sum_{t=1,4,\dots} \beta^t \left[x_t^M + x_t^C - \beta\frac{1}{2}\left(y_{t+1}^M + y_{t+1}^C\right)^2\right]$$

$$s.t. \quad p_t x_t^M \leq m_{t-1} \qquad [\lambda_t]$$

$$p_{t+1} y_{t+1}^M \geq q_t x_t^C \qquad [\mu_t]$$

$$m_{t+2} = p_{t+1} y_{t+1}^M + q_{t+1} y_{t+1}^C + m_{t-1} - p_t x_t^M - q_t x_t^C + \tau_{t+2} \qquad [\gamma_t]$$

$$\beta^{t+3}\left(x_{t+3}^M + x_{t+3}^C\right) \geq \beta^{t+1}\frac{1}{2}\left(y_{t+1}^M + y_{t+1}^C\right)^2 \qquad [\phi_t]$$

together with the transversality condition on money holdings.

In the interior solution the Euler equations are

$$\frac{1}{q_t} = \frac{(1+\phi_t)\,\beta\left(y_{t+1}^M + y_{t+1}^C\right)}{p_{t+1}} \tag{2.1}$$

which equates the marginal benefit of consuming on credit to the marginal cost of producing tomorrow for money in order to repay and

$$\frac{\beta^3}{p_{t+3}} = \frac{(1+\phi_t)\,\beta\left(y_{t+1}^M + y_{t+1}^C\right)}{q_{t+1}} \tag{2.2}$$

which equates the marginal benefit of consuming with money in three periods time to the marginal cost of producing for a promise tomorrow. Define the interest factor $(1+i_t) = \frac{q_t}{p_t}$ for all t. Observe that $\lambda_t > 0$ and $\mu_t > 0$ when $(1+i_t) > 1$ for all t.

The binding cash-in-advance constraint is

$$x_t^M = \frac{m_{t-1}}{p_t} \tag{2.3}$$

the repayment constraint is

$$y_{t+1}^M = (1+i_t)\,\frac{p_t}{p_{t+1}}x_t^C \tag{2.4}$$

the budget constraint

$$m_{t+2} = p_{t+1}y_{t+1}^M + q_{t+1}y_{t+1}^C + m_{t-1} - p_t x_t^M - q_t x_t^C + \tau_{t+2} \tag{2.5}$$

and the complementary slackness condition for the participation constraint is

$$\phi_t\left[\beta^3\left(x_{t+3}^M + x_{t+3}^C\right) - \beta\frac{1}{2}\left(y_{t+1}^M + y_{t+1}^C\right)^2\right] = 0 \tag{2.6}$$

with $\phi_t \geq 0$.

Observe that the Euler equations can be solved together to give the two period interest factor

$$(1 + i_t)(1 + i_{t+1}) = \frac{1}{\beta^3} \frac{p_{t+3}}{p_t}$$

Finally as a benchmark consider an economy where credit cannot be used (for instance because there is no collateral). In this case the model turns into a cash-in-advance model where the repayment constraint and the participation constraint are not binding and the solution is

$$\widehat{y}_{t+1}^M = \beta^2 \frac{p_{t+1}}{p_{t+3}}$$

2.2.4 The Government

On each island, the government issues money every period and gives lump-sum transfers to agents in their period of rest[8]. The government budget constraint equates the increase in the money supply on each island to the total transfers to agents:

$$M_t - M_{t-1} = T_t$$

where I dropped the index for the islands since I solve for a symmetric equilibrium. The money supply grows at a rate z_t :

$$M_t = (1 + z_t) M_{t-1}$$

[8] I assume that agents receive transfers only when they are resting in order to rule out redistributive effects of monetary policy.

2.3 Equilibrium

2.3.1 Equilibrium without Inflation

I solve first for a stationary symmetric competitive equilibrium without inflation. In such an equilibrium

1. agents maximize utility subject to the budget constraint, the cash-in-advance constraint, the repayment constraint, the participation constraint and the transversality condition;

2. all markets clear at all times: the goods-for-money market $x_t^M = y_t^M$, the goods-for-credit market $x_t^C = y_t^C$, the market for money $m_t = M_t$;

3. the Government fulfills its budget constraint and sets $z_t = 0$ ($T_t = 0$) at all times;

The system (1)-(6) can be solved in a stationary equilibrium for the one period nominal interest factor, for the credit to money ratio, for total output and for the multiplier ϕ. The interest factor is

$$(1+i) = \frac{1}{\beta^{\frac{3}{2}}}$$

the credit to money ratio is

$$\frac{y^C}{y^M} = \beta^{\frac{3}{2}}$$

and equation (1) and (6) determine together output and the multiplier

$$1 = (1+\phi)(1+i)\beta\left(y^M + y^C\right)$$

$$\phi\left[2\beta^2 - \left(y^M + y^C\right)\right] = 0$$

When $\beta < \left(\frac{1}{2}\right)^{\frac{2}{3}}$, the Participation Constraint is binding and total output and consumption in the economy are given by

$$y^M + y^C = 2\beta^2 = x^M + x^C$$

When $\beta \geq \left(\frac{1}{2}\right)^{\frac{2}{3}}$ the Participation Constraint is not binding and total output and consumption are

$$y^M + y^C = \beta^{\frac{1}{2}} = x^M + x^C$$

In the monetary equilibrium without credit, total output and consumption would be

$$\widehat{y}^M = \beta^2 = \widehat{x}^M$$

In equilibrium, each period one third of the population is consuming using both money and credit, one third is producing for money and credit and one third is resting while repayment takes place. One third of agents j in the morning - period t- are on island j and trade money for goods. They also issue promises secured by the collateral to increase their consumption of good j. Then they travel to island $j+1$. In the afternoon - period $t+1$- they produce for money and for promises. They repay their promise in the afternoon, handing fiat money to their creditors (agents $j-1$) who have travelled with them and are waiting on island $j+1$. The collateral is returned and promises destroyed. In the evening they leave to island $j+2$ where they rest and wait for repayment to take place. Finally they travel to island j to restart the cycle. Money and credit are both media of exchange and money is the means of payment.

To show that this is indeed an equilibrium I have to check all the relevant incentive and participation constraint. When an agent is consuming, it is fairly intuitive that he does want to use both money and credit since they allow him to transact and consume more. When an agent is producing though, several deviations have to be considered. First, he may refuse to repay his debt. In this case he will loose his collateral, will be unable to consume in the future and will get zero for ever. This deviation is thus captured by the participation constraint. Second, he could repay his debt, but then produce more for money instead of becoming a creditor. Here is where the resting period is playing its technical role. This deviation would be particularly attractive if the agent could run away immediately, spend his cash and consume. The fact that he is stuck for some time, unable to spend money, induces him to become a creditor in order to gain an interest over the resting period. The equilibrium interest rate will make him willing to lend.

Consider then the possibility that an agent repays his debt with someone else's promise instead of money. In this case the original issuer of the promise may not meet the holder of his collateral again since agents, once they are on an island, are matched randomly and there is a continuum of them[9]. Agents wouldn't issue promises in the first place and this is enough to rule out the deviation. Notice finally that the lack of double coincidence of wants excludes repayment with commodities.

A feature of the equilibrium is that the price on the credit market is higher than the one on the money market: $(1+i) \equiv \frac{q}{p} = \beta^{-\frac{3}{2}} > 1$, i.e. there is rate of return dominance. This is necessary for a promise of money-later to be accepted instead of money-now. Total production in the economy with credit is higher than total production in the economy with only money: $\beta^{\frac{1}{2}} > \beta^2$and $2\beta^2 > \beta^2$. Credit is a valuable medium of exchange that increases the number of transactions. More transactions induce agents to consume and produce more compared to an economy without credit. The two media of exchange coexist in a fundamental sense: each agent uses both at every stage to exchange.

To summarise, credit is repaid with money, there is an endogenous interest rate and output is higher than in the corresponding economy without credit.

[9]Technically it is a zero measure event.

2.3.2 Arrow-Debreu Economy and First Best

Suppose for a moment to drop the assumption on the spatial separation of market, so that Arrow-Debreu markets can be organized. The maximization problem for an agent consuming at time t (type 1) becomes

$$\max \sum_{t=1,4,\ldots} \beta^t \left[x_t^1 - \beta \frac{1}{2} \left(y_{t+1}^1 \right)^2 \right]$$

$$s.t. \sum_{t=1,4,\ldots} \rho_t x_t^1 \leq \sum_{t=1,4,\ldots} \rho_{t+1} y_{t+1}^1$$

for an agent resting at time t (type 2) it will be

$$\max \sum_{t=1,4,\ldots} \beta^t \left[\beta x_{t+1}^2 - \beta^2 \frac{1}{2} \left(y_{t+2}^2 \right)^2 \right]$$

$$s.t. \sum_{t=1,4,\ldots} \rho_{t+1} x_{t+1}^2 \leq \sum_{t=1,4,\ldots} \rho_{t+2} y_{t+2}^2$$

and for an agent producing at time t (type 3)

$$\max \sum_{t=1,4,\ldots} \beta^t \left[\beta^2 x_t^3 - \frac{1}{2} \left(y_{t+1}^3 \right)^2 \right]$$

$$s.t. \sum_{t=1,4,\ldots} \rho_{t+2} x_{t+2}^3 \leq \sum_{t=1,4,\ldots} \rho_t y_t^3$$

where ρ are Arrow-Debreu prices. Call the proportions of agents of each type $\alpha^1, \alpha^2, \alpha^3$ with $\alpha^1 + \alpha^2 + \alpha^3 = 3$.

In a stationary symmetric Arrow-Debreu equilibrium:

1. agents maximize utility subject to the life-time budget constraint;

2. markets clear: $\alpha^1 x^1 = \alpha^3 y^3$, $\alpha^2 x^2 = \alpha^1 y^1$ and $\alpha^3 x^3 = \alpha^2 y^2$.

To solve for a stationary symmetric equilibrium I will guess that Arrow-Debreu prices are given by $\rho_{t+1} = \beta \rho_t$. The budget constraint for agents of type 1 is then

$x^1 = \beta y^1$. The solution gives $y^1 = 1$ and $x^1 = \beta$. For an agent of type 2 the budget constraint becomes $x^2 = \beta y^2$. The solution gives $y^2 = 1$ and $x^2 = \beta$. For an agent of type 3, the budget constraint is $x^3 = \frac{1}{\beta^2} y^3$. The solution gives $y^3 = 1$ and $x^3 = \frac{1}{\beta^2}$.

The proportions of agents can be computed from the equilibrium conditions. These are respectively $\alpha^1 = 3\beta / \left(1 + \beta + \beta^2\right)$, $\alpha^2 = 3 / \left(1 + \beta + \beta^2\right)$ and $\alpha^3 = 3\beta^2 / \left(1 + \beta + \beta^2\right)$. Finally, welfare in the Arrow-Debreu equilibrium is equal to $9\beta^2 / 2 \left(1 + \beta + \beta^2\right)$.

To compute the first best allocation I solve the following problem

$$\max \quad \alpha^1 \left[x^1 - \beta \frac{1}{2} \left(y^1\right)^2\right] + \alpha^2 \left[\beta x^2 - \beta^2 \frac{1}{2} \left(y^2\right)^2\right] + \alpha^3 \left[\beta^2 x^2 - \frac{1}{2} \left(y^2\right)^2\right]$$

$$\begin{aligned} s.t.\ \alpha^1 x^1 &= \alpha^3 y^3 \\ \alpha^2 x^2 &= \alpha^1 y^1 \\ \alpha^3 x^3 &= \alpha^2 y^2 \end{aligned}$$

which gives the allocation $x^1 = \alpha^3/\alpha^1$, $x^2 = \alpha^1/\alpha^2$, $x^3 = \alpha^2/\alpha^3$ and $y^1 = y^2 = y^3 = 1$. The first best frontier is given by

$$W^{FB} = \alpha^1 \frac{\beta}{2} + \alpha^2 \frac{\beta^2}{2} + \alpha^3 \frac{1}{2}$$

As expected welfare at the Arrow-Debreu equilibrium is a point on the first best frontier, as it can be easily checked substituting the values for the proportions of agents found above.

2.3.3 Welfare

To show that money and credit together can achieve an outcome that is socially preferred to the outcome that can be achieved with money alone, I will compare welfare in the equilibrium with credit and in the equilibrium without it, taking as a measure of welfare the sum of the value functions of the three groups of individuals -i.e. consumers, producers and resting agents-.

$$W = V_c + V_p + V_r = \frac{x^M + x^C - \frac{1}{2}\left(y^M + y^C\right)^2}{(1-\beta)}$$

is the sum of the value functions of the three groups of individuals in the money and credit equilibrium and

$$\widehat{W} = \widehat{V}_c + \widehat{V}_p + \widehat{V}_r = \frac{\widehat{x}^M - \frac{1}{2}\left(\widehat{y}^M\right)^2}{(1-\beta)}$$

is the sum of the value functions of the three groups of individuals in the money only equilibrium.

The difference $W - \widehat{W}$ -when $\beta > \left(\frac{1}{2}\right)^{\frac{2}{3}}$- is

$$W - \widehat{W} = \frac{2\beta^{\frac{1}{2}} - \beta}{2(1-\beta)} - \frac{2\beta^2 - \beta^4}{2(1-\beta)}$$

which is positive for all $0 < \beta < 1$.

The difference $W - \widehat{W}$ -when $\beta < \left(\frac{1}{2}\right)^{\frac{2}{3}}$- is

$$W - \widehat{W} = \frac{4\beta^2 - 4\beta^4}{2(1-\beta)} - \frac{2\beta^2 - \beta^4}{2(1-\beta)}$$

which is positive for $\beta < \left(\frac{2}{3}\right)^{\frac{1}{2}}$. Notice that $\left(\frac{2}{3}\right)^{\frac{1}{2}} > \left(\frac{1}{2}\right)^{\frac{2}{3}}$.

Money and credit thus achieve a socially preferred outcome. The combination of the two means of exchange allows the economy to move closer to the first best frontier, which is however never attained except for $\beta = 1$. Money helps credit to work providing the means of payment and the use of credit improves output and welfare compared to an economy without credit since it allows agents to perform more transactions.

2.3.4 Equilibrium with Anticipated Inflation

Next, I solve for a stationary symmetric competitive equilibrium with steady inflation. In such an equilibrium

1. agents maximize utility subject to the budget constraint, the cash-in-advance constraint, the repayment constraint, the participation constraint and the transversality condition;

2. all markets clear at all times: the goods-for-money market $x_t^M = y_t^M$, the goods-for-credit market $x_t^C = y_t^C$, the market for money $m_t = M_t$;

3. the Government fulfills its budget constraint and sets $z_t = z > 0$ at all times;

Observe that by money market clearing, the inflation rate π is equal to the growth rate of money z: $\pi = \frac{p_{t+1}-p_t}{p_t} = \frac{M_{t+1}-M_t}{M_t} = z$.

Consider first the case in which the Participation Constraint is not binding, which happens when $1+\pi \geq \frac{1}{4\beta^3}$.

The Euler equations will be

$$1 = \beta \frac{1+i}{1+\pi} \left(y^M + y^C\right) \tag{2.7}$$

$$y^M + y^C = \beta^2 \frac{1+i}{(1+\pi)^2} \tag{2.8}$$

where the first equation describes the behavior of a debtor who increases his utility in the afternoon borrowing an extra unit (LHS) and will have to repay his debt with interest producing an extra unit in the evening (RHS). The second equation describes the behavior of a creditor who is lending an extra unit in the afternoon (LHS) and will get repayment with interest in the evening but will be able to spend it only the next morning since he has to rest during the night (RHS). There is a time wedge between the borrower -who cares about one period ahead- and the lender

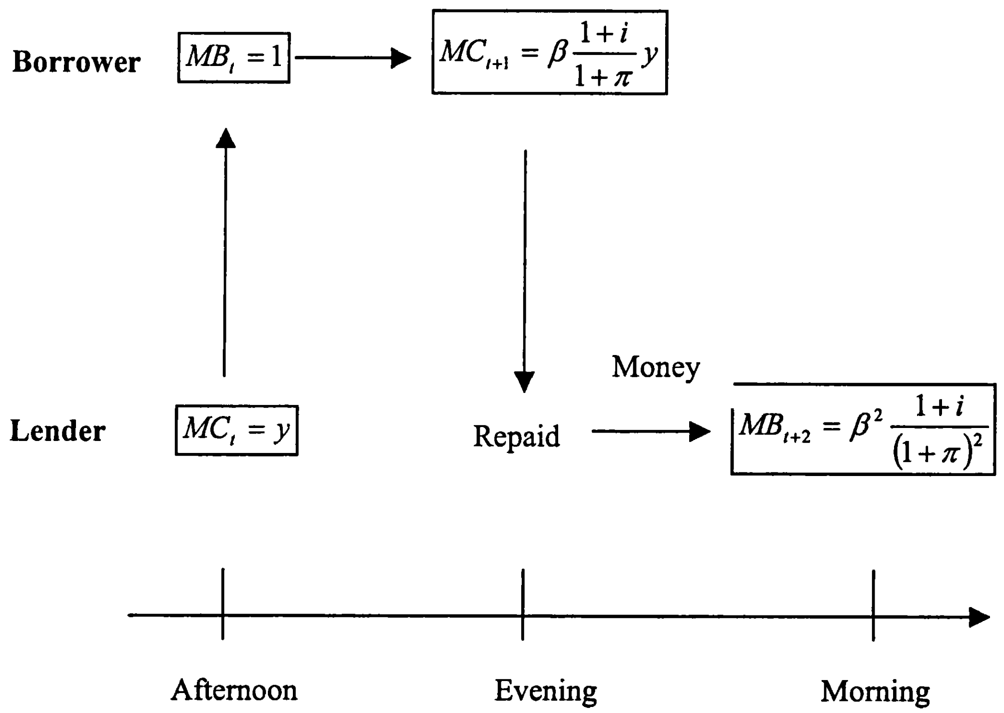

Figure 2.4: Euler Equations

-who cares about two periods ahead, because of the resting period-. The creditor is holding money overnight, before being able to spend it. (See Figure 4, where $y = y^M + y^C$ and MB stands for marginal benefit, MC for marginal cost).

The repayment in real terms must be equal to the amount borrowed plus the interest discounted by the inflation factor, since debt is repaid one period later:

$$y^M = \left(\frac{1+i}{1+\pi}\right) x^C \tag{2.9}$$

which, using the equilibrium condition on the credit market, gives the ratio of the real demand for promises to the real demand for money.

Solving (1),(2) and (3) together the nominal interest rate, output and the credit to money ratio are derived. The nominal interest factor is

$$1 + i = \left(\frac{1+\pi}{\beta}\right)^{\frac{3}{2}}$$

total output will be given by

$$y^M + y^C = \frac{1}{\beta}\left(\frac{1+\pi}{1+i}\right) = \frac{\beta^{\frac{1}{2}}}{(1+\pi)^{\frac{1}{2}}}$$

and the credit to money ratio is

$$\frac{y^C}{y^M} = \left(\frac{1+\pi}{1+i}\right) = \frac{\beta^{\frac{3}{2}}}{(1+\pi)^{\frac{1}{2}}}$$

When $1 + \pi < \frac{1}{4\beta^3}$, the participation constraint is binding and total output is given by:

$$y^M + y^C = 2\beta^2$$

2.3.5 Credit, Output and Real Interest Rate

The credit to money ratio in the economy goes down with inflation since lenders have to hold money overnight after repayment takes place

$$\frac{y^C}{y^M} = \frac{\beta^{\frac{3}{2}}}{(1+\pi)^{\frac{1}{2}}}$$

In turn, the amount of goods sold for credit as a share of output is decreasing in the inflation rate

$$\frac{y^C}{y^C + y^M} = \frac{\beta^{\frac{3}{2}}}{\beta^{\frac{3}{2}} + (1+\pi)^{\frac{1}{2}}}$$

At time t a producer in equilibrium holds a claim to future money which will be paid off at $t+1$. He will however have to hold money overnight, from $t+1$ until $t+2$

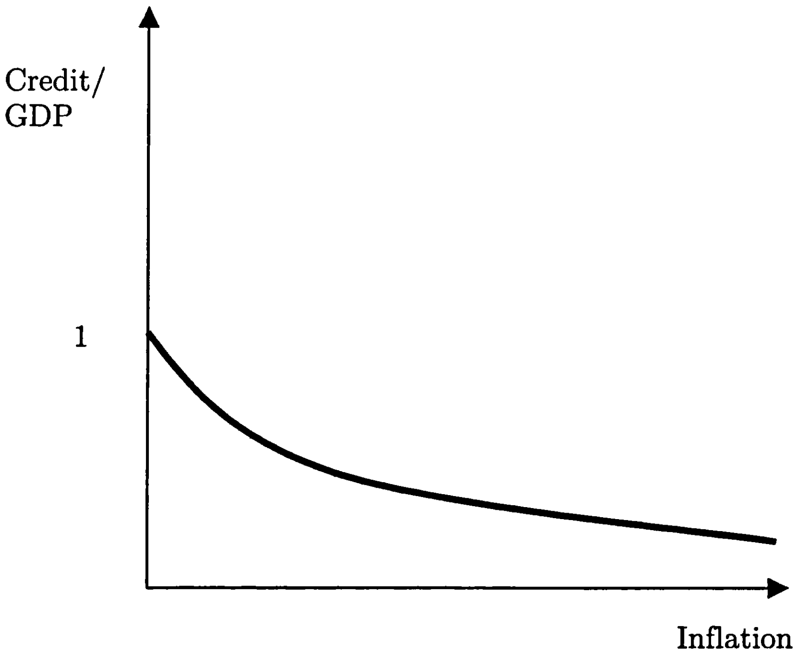

Figure 2.5: Inflation harms Credit

when he will spend it to buy consumption goods. Inflation drives up the nominal interest rate and makes it costly to hold claims to future money: agents reduce their credit holdings. (See Figure 5).

Output is decreasing in the inflation rate. Credit increases the amount of transactions which increases output. This can be seen from the Euler equation

$$y^M + y^C = \frac{1}{\beta}\left(\frac{1+\pi}{1+i}\right) = \frac{1}{\beta}\left(\frac{y^C}{y^M}\right)$$

where the second equality is obtained using the binding repayment constraint. Higher inflation reduces the credit to money ratio and this reduces transactions and the incentive to produce. Output and consumption when the participation constraint is

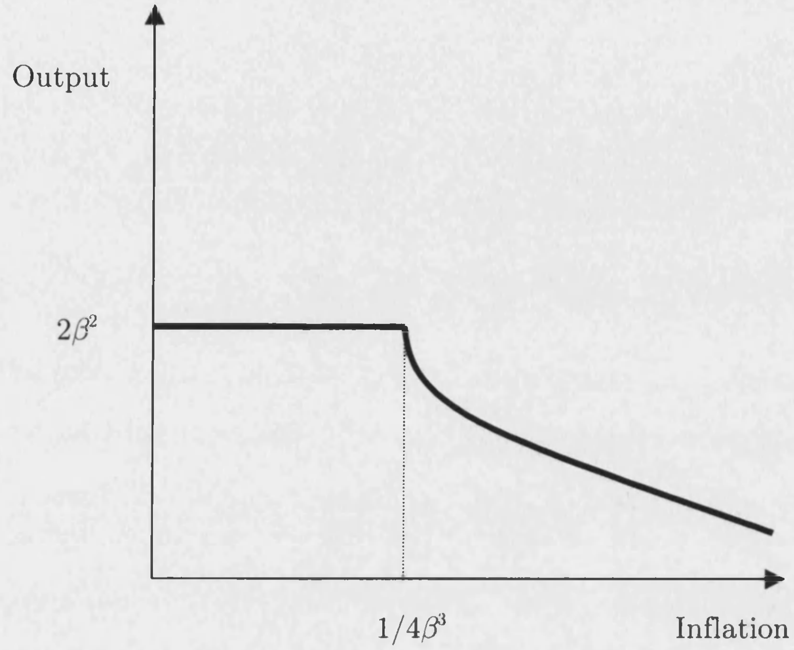

Figure 2.6: High Inflation harms Output

not binding are decreasing in the inflation rate

$$y^M + y^C = \frac{\beta^{\frac{1}{2}}}{(1+\pi)^{\frac{1}{2}}} = x^M + x^C$$

while, when the participation constraint is binding, output is insensitive to the inflation rate

$$y^M + y^C = 2\beta^2 = x^M + x^C$$

Total output increases with the credit to money ratio: more credit allows agents to trade more driving up production. Inflation in turn decreases the credit to money mix, reducing the incentive to hold future money. Thus inflation reduces output in the economy. This however happens only when the inflation rate is sufficiently high -i.e. for $1 + \pi \geq \frac{1}{4\beta^3}$. For lower inflation factors output is constant (see Figure 6).

Observe that the real interest factor increases with the inflation rate

$$(1+r)=\frac{1+i}{1+\pi}=\frac{(1+\pi)^{\frac{1}{2}}}{\beta^{\frac{3}{2}}}$$

The nominal interest factor is more than unit elastic in the inflation factor, with elasticity equal to 3/2, and the real interest factor increases with inflation. In the model the creditor is bearing the cost of inflation, since he is holding money for one period while he is resting. The higher interest rate compensate him for the cost of holding money in the presence of inflation. Changing slightly the model, it would be possible to make the debtor hold the money for one period. The analysis would go through unchanged except for the real interest rate which would be decreasing in inflation. In this case it would be the debtor who would have to be compensated for holding money in the presence of inflation. What is really crucial is therefore that the real interest rate changes with inflation -and thus the nominal interest rate is not unit elastic in inflation- rather than the direction of the change. (See Figure 7). The mismatch between the arrival of liquidity and of consumption opportunities is responsible for the break-down of the Fisher effect.

To summarise, total output decreases, the credit to money ratio decreases and the real interest rate increases with fully anticipated inflation.

2.3.6 Welfare

A steady inflation generates a welfare loss, since agents transact, produce and consume less. Welfare, when the participation constraint is not binding, is given by

$$W(\pi)=\frac{2\beta^{\frac{1}{2}}(1+\pi)^{\frac{1}{2}}-\beta}{2(1-\beta)(1+\pi)} \tag{2.10}$$

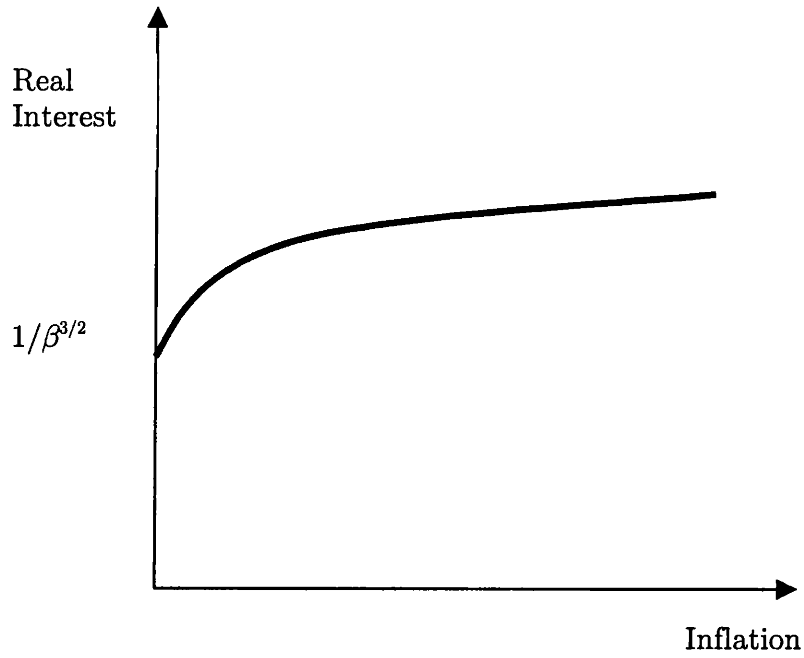

Figure 2.7: Inflation increases the Real Interest Factor

and when the Participation Constraint is binding, by

$$W(\pi) = \frac{4\beta^2 - 4\beta^4}{2(1-\beta)} \tag{2.11}$$

As a measure of the welfare cost of inflation consider the difference between welfare in an economy with inflation and in an economy without inflation: $W(\pi) - W(0)$. Observe that (10) is decreasing in the inflation rate for any $\pi \geq 0$

$$W'(\pi) = \frac{\beta^{\frac{1}{2}}\left[\beta^{\frac{1}{2}} - (1+\pi)^{\frac{1}{2}}\right]}{2(1-\beta)(1+\pi)^2}$$

Therefore, welfare with inflation is lower than welfare without inflation. (11) is constant and equal to welfare without inflation.

We know that first best output is given by $y^* = 1$. The optimal monetary policy would be the Friedman Rule. The government should tax money balances, deflating at a rate $\pi = \beta - 1$, until the rate of return on money and credit are equalised, allowing the distortion due to market incompleteness to be overcome and the first best allocation to be reached. However, the Friedman Rule cannot be implemented through taxes since enforcement is limited in the economy and agents cannot be forced to surrender money to the government.

Output and consumption in the monetary equilibrium without credit would be

$$\widehat{y}^M = \frac{\beta^2}{(1+\pi)^2} = \widehat{x}^M$$

Welfare in the money economy without credit would then be

$$\widehat{W}(\pi) = \frac{\beta^2 \left[2(1+\pi)^2 - \beta^2\right]}{2(1-\beta)(1+\pi)^4}$$

The economy with credit always performs better than the corresponding economy without credit. Taking the difference between welfare with credit -when the participation constraint is slack- and welfare without credit, I obtain

$$W(\pi) - \widehat{W}(\pi) = \frac{2\beta^{\frac{1}{2}}(1+\pi)^{\frac{7}{2}} - \beta(1+\pi)^3 - 2\beta^2(1+\pi)^2 + \beta^4}{2(1-\beta)(1+\pi)^4}$$

which is strictly positive for any $\pi \geq 0$. When the participation constraint is binding, the difference in welfare is

$$W(\pi) - \widehat{W}(\pi) = \frac{4\beta^2(1+\pi)^4 - 4\beta^4(1+\pi)^4 - 2\beta^2(1+\pi)^2 + \beta^4}{2(1-\beta)(1+\pi)^4}$$

which is positive for any $\pi \geq 0$ when $\beta < \left(\frac{2}{3}\right)^{\frac{1}{2}}$.

The model highlights a neglected component of the welfare costs of inflation: inflation drives down the credit to money mix in the economy. Production suffers from the decrease in credit: welfare is lower.

2.4 Discussion

Complementarity between money and credit in the model is represented by the repayment constraint which is responsible for the effects of inflation on the interest rate, on output and on credit. Anticipated inflation gives rise to a reverse Mundell-Tobin effect. The nominal interest rate is more than unit elastic in the inflation rate and thus the real interest rate increases with inflation, the credit to money and credit to GDP ratios decrease with inflation and finally output decreases with inflation, when the inflation rate is sufficiently high.

Boyd, Levine and Smith (2001) use macroeconomic data to show that anticipated inflation has a negative impact on private credit to GDP and interestingly, Bullard and Keating (1995) provide evidence of the fact that output is relatively insensitive to inflation when inflation is low and decreasing for higher inflation rates. There is also evidence that the nominal interest rate is not unit elastic in the inflation rate (Koustas and Serletis (1999)). I explore these issues further in the next chapter.

The predictions differ from those found by Gillman (1993) and in similar models by Aiyagari, Braun and Eckstein (1998) and English (1999). These models modify a cash in advance economy à la Lucas and Stokey to allow the representative agent to decide which goods to buy with cash and which goods with a costly transaction technology that produces credit services. Gillman (1993) shows that "the consumer chooses between a foregone-interest cost of cash and a time cost of credit when purchasing any one good. Avoiding the inflation tax means switching from fiat that uses no resources to exchange credit that uses up societal resources.[...]the consumer substitutes away from cash until the marginal cost of avoiding inflation, through credit use, equal the marginal inflation tax on cash use". The increase in the nominal interest rate makes more costly to hold money. Agents thus switch to an alternative -inflation free- medium of exchange. The credit to money ratio and credit to output ratio increase with inflation. The nominal interest rate elasticity to inflation is equal to one. These papers see money and credit as substitute.

The literature typically identifies the welfare costs of inflation with the resource cost of producing transaction services alternative to cash. Resources are diverted from production to credit services and welfare is lower. The point is neatly summarized in a recent paper by Lucas (2000): "In a monetary economy, it is in everyone's private interest to try to get someone else to hold non-interest-bearing cash and reserves. [...] All of us spend several hours per year in this effort and we employ thousands of talented and highly-trained people to help us. These person-hours are simply thrown away, wasted on a task that should not have to be performed at all."

In my model, inflation, driving up the nominal interest rate, makes costly to hold a promise of money, thus reducing the credit to money ratio which in turn reduces transactions, output, consumption and welfare.

The repayment constraint is the driving force of the model and it embeds complementarity. Broadly speaking it says that agents with debt will have to hold cash in the future in order to repay. Money demand functions have been estimated on the basis of theoretical models of cash management of the Baumol-Tobin type, in which consumers -when deciding their money holdings for transaction purposes- take into account the cost of transaction time and the forgone interest of other assets. To test the Baumol-Tobin model Attanasio, Guiso and Jappelli (2002) estimate a money demand function with time, interest on checking accounts and consumption as explanatory variables. In my model, debt influences cash management decisions by households, since debt is paid off with money. Specifically higher debt would induce agents to hold more cash, suggesting that debt might have a role in determining agents' demand for cash.

2.5 Conclusion

The aim of the chapter was to address the issue of the role of money as a means to repay debt. I constructed a model with microfoundations for the use of money and bilateral credit as media of exchange, where money serves also as the means of payment. In equilibrium money and bilateral credit coexist, credit is repaid with money and it dominates money in the rate of return: the two assets are complementary. Complementarity generates a reverse Mundell-Tobin effect: the real interest rate increases with inflation, the credit to money ratio and output both decrease with inflation.

Chapter 3

On the Elasticity of the Nominal Interest Rate

3.1 Introduction

In the previous chapter, I constructed a model in which the nominal interest rate was more than unit elastic in the inflation rate and thus the real interest rate was increasing in inflation. In this chapter, I show that the model can be slightly modified to give a less than unit elastic nominal interest rate and thus capture a decrease -rather than an increase- in the real interest rate. If the bilateral credit contract induces the debtor -instead of the creditor- to hold money overnight, then the interest rate will have to compensate him for bearing that cost and the real interest rate will decrease with inflation. What is really crucial in the model is therefore that the nominal interest rate is not unit elastic and the real interest rate is not independent of inflation and not whether it increases or decreases. The mismatch between the arrival of liquidity and of consumption opportunities is responsible for the break-down of the Fisher effect. The paper by Koustas and Serletis (1999) provides empirical evidence that the nominal interest rate is not unit elastic in the inflation rate and thus the

Fisher effect is not at work. Here I explore the issue of the direction of the change in the interest rate.

I then use macroeconomic data on interest rates and inflation between 1993 and 2003 for 59 countries to estimate the elasticity of the nominal interest rate with respect to inflation. To account for the difference in the size of countries and thus the informational content of data, I use weighted least square with GDP in 2002 as a weight. I use GDP in order to have a sound measure of both the size and the efficiency of the market. I find that the elasticity of the nominal interest rate with respect to inflation is close to 1.4. I also test the prediction on credit/GDP.

The chapter proceeds as follows. Section 2 presents the modification of the model and solves for the equilibrium. Section 3 contains the evidence. Section 4 concludes.

3.2 The Model

Consider the same model as in chapter 2 with one modification. Assume that there exist a technology allowing a creditor and a debtor to keep track of each other -and of no one else in the economy-. In this case the following sequence of events can be an equilibrium[1]. In the Morning agent 1 is on island 1. He spends money to buy his consumption good from agent 8. He then increases his purchases issuing bilateral promises secured by collateral. At the end of the morning agent 1 travels to island 2. Meanwhile agent 2 is also coming to island 1. Agent 1 sells goods to agent 2 for money. He then sells goods to agent 2 for promises, becoming a creditor. At the end of the afternoon he travels to island 8 to meet his creditor. With the money he has got, he repays agent 8 and gets his collateral back. In the evening agent 1 is resting. The next morning he meets his debtor on island 1 who repays him and he restarts the cycle. (See Figure 1).

The crucial difference is that now the debtor -instead of the creditor- holds money

[1] To illustrate the point, I take the same -8 islands- example as before.

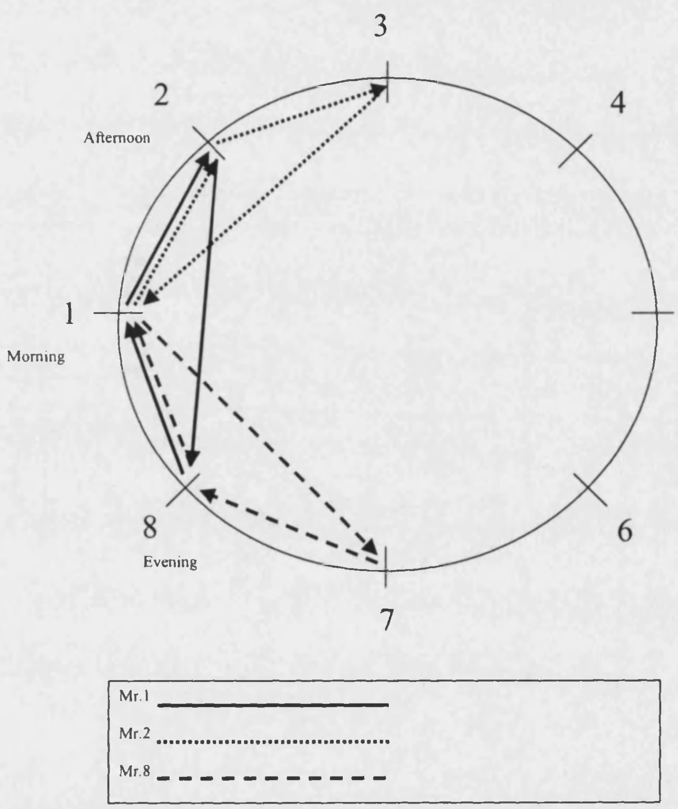

Figure 3.1: Sequence of Events

for eight hours.

3.2.1 Equilibrium

The Euler equations[2] are

$$1 = \beta \frac{(1+i)^2}{1+\pi} \left(y^M + y^C\right) \tag{3.1}$$

$$y^M + y^C = \beta^2 \frac{(1+i)^2}{(1+\pi)^2} \tag{3.2}$$

where the first equation describes the behavior of a debtor who is increasing his utility in the afternoon by borrowing an extra unit (LHS) and will have to repay his debt with interest for two periods producing in the evening (RHS). The second equation describes the behavior of a creditor who is lending an extra unit in the afternoon (LHS) and will get repayment with interest the next morning -i.e. two periods later- (RHS). The debtor is holding money for one period. (See Figure 2, where $y = y^M + y^C$ and MB stands for marginal benefit, MC for marginal cost).

The repayment in real terms must be equal to the amount borrowed plus the two periods interest discounted by the inflation factor:

$$y^M = \frac{(1+i)^2}{1+\pi} x^C \tag{3.3}$$

which, using the equilibrium condition on the credit market, gives the ratio of the real demand for promises to the real demand for money.

Solving (1),(2) and (3) together the nominal interest rate, output and the credit to money ratio are derived. The nominal interest factor is

$$1 + i = \left(\frac{1+\pi}{\beta}\right)^{\frac{3}{4}}$$

[2]For simplicity, I ignore the participation constraint. The solution when the constraint is binding mirrors the one in the previous chapter.

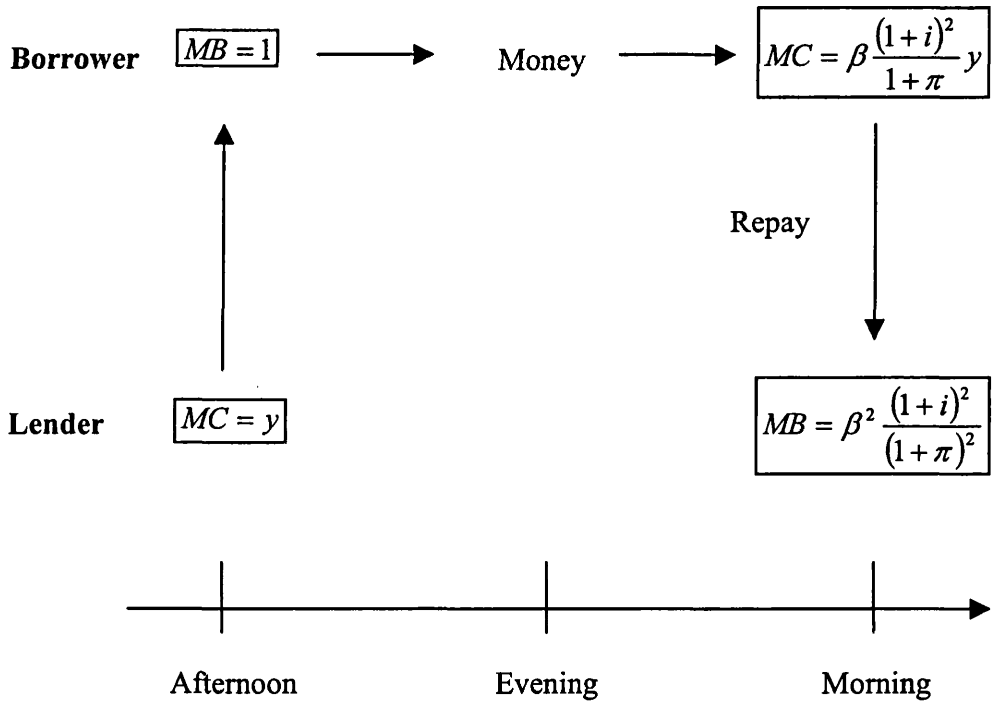

Figure 3.2: Euler Equations

total output is given by

$$y^M + y^C = \frac{1}{\beta}\frac{1+\pi}{(1+i)^2} = \frac{\beta^{\frac{1}{2}}}{(1+\pi)^{\frac{1}{2}}}$$

and the credit to money ratio is

$$\frac{y^C}{y^M} = \frac{1+\pi}{(1+i)^2} = \frac{\beta^{\frac{3}{2}}}{(1+\pi)^{\frac{1}{2}}}$$

3.2.2 Nominal and Real Interest Rate

The credit to money ratio and output are the same as before and thus decreasing in the inflation rate.

The nominal interest factor is however now less than unit elastic with respect to inflation, with elasticity equal to 3/4, and the real interest factor decreases with the inflation rate

$$(1+r) = \frac{1+i}{1+\pi} = \frac{1}{\beta^{\frac{3}{4}}(1+\pi)^{\frac{1}{4}}}$$

In the model the debtor is bearing the cost of inflation, since he is holding money for one period while he is resting. The lower interest rate compensate him for the cost of inflation. (See Figure 3).

To summarise, total output decreases, the credit to money ratio decreases and the real interest rate decreases with the inflation rate.

3.3 Elasticity of the nominal interest rate: evidence

To distinguish between the two possible cases -i.e. an elasticity greater or smaller than one- I tested the elasticity of the nominal interest rate to the inflation rate with macroeconomic data. I use data on 59 countries between 1993 and 2003 on

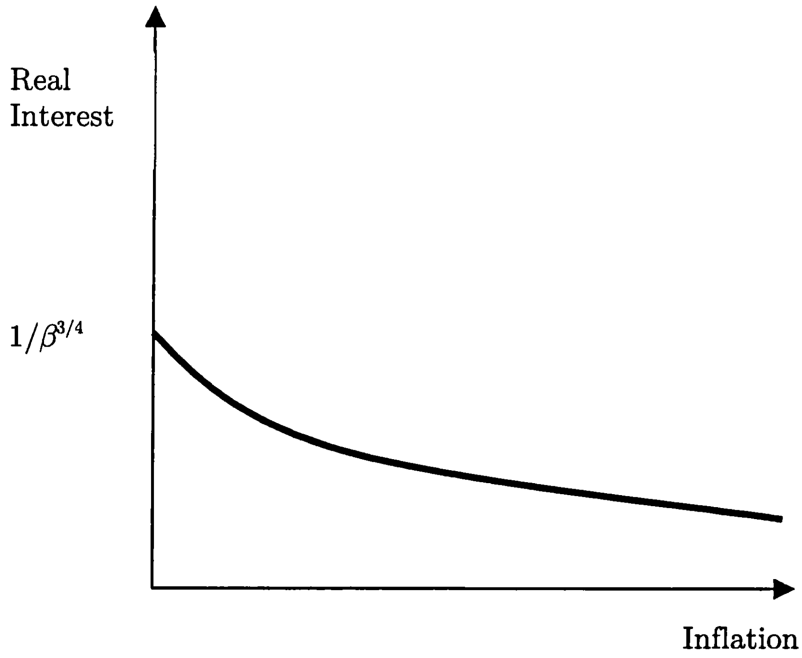

Figure 3.3: Inflation decreases the Real Interest Factor

the nominal interest rate and the inflation rate[3]. I use the Lending Rate -the bank rate that meets the short and medium-term financing needs of the private sector (International Financial Statistics, line 60p)- as the nominal interest rate and the Consumer Price Index - (International Financial Statistics, line 64)- to compute inflation rates. I take averages over the period 1993-2003 to concentrate on steady state inflation. Some countries experienced, over the last ten years, one year of sudden inflation or deflation returning to previous levels afterwards. In those cases I regressed the inflation rate against a constant and a dummy for the specific year and used the coefficient of the constant as a measure of average inflation. I included

[3]The 59 countries are: Argentina, Australia, Bangladesh, Barbados, Belgium, Bolivia, Botswana, Cameroon, Canada, Chile, Colombia, CostaRica, Cyprus, Czech Republic, Denmark, Dominican Republic, Ecuador, Finland, France, Germany, Greece, Grenada, Guatemala, Honduras, Hungary, Iceland, India, Indonesia, Ireland, Israel, Italy, Jamaica, Japan, Kenya, Korea, Malaysia, Mexico, Netherlands, New Zealand, Norway, Panama, Paraguay, Peru, Philippines, Poland, Singapore, South Africa, Spain, SriLanka, Suriname, Switzerland, Syrian Arab Republic, Tanzania, Thailand, Trinidad and Tobago, United Kingdom, United States, Venezuela, Zimbabwe.

only countries that have a C- or better in the Summers and Heston's ranking on the quality of data and countries that have data for the entire period considered. To account for the difference in size among countries I use weighted least squares with GDP in 2002 for each country as a weight. I use GDP to capture simultaneously the dimension and efficiency of the market. The model generates an interest factor of the form

$$1+i=\left(\frac{1+\pi}{\beta}\right)^{a}$$

where a is the elasticity. Taking logs I get

$$\log(1+i)=-a\log\beta+a\log(1+\pi)$$

I therefore estimate the following equation:

$$\begin{aligned}\log(1+i) \quad = \quad & \underset{(0.002)^{***}}{0.035} \quad + \quad \underset{(0.104)^{***}}{1.381}\log(1+\pi)\end{aligned}$$

The coefficients are statistically significant (standard errors are in parenthesis) and the R^2 is 0.982. Using the Wald test, the hypothesis of the elasticity being equal to one is rejected (F-statistic 13.234, p-value 0.0006). The implied value for the discount factor is close to 0.975. The elasticity turns out to be close to 1.4.

3.4 Credit/GDP

Following the approach by Boyd, Levine and Smith (2001), I also tested the prediction of my model on the credit to GDP ratio. I use data for the period 1960-1995 for 63 countries[4]. The dependent variable is Private Credit to GDP (PC), which

[4]Argentina, Australia, Austria, Belgium, Bolivia, Brazil, Canada, Chile, Colombia, Congo Dem. Rep., Costa Rica, Cyprus, Denmark, Dominican Rep., Ecuador, El Salvador, Fiji, Finland, France, Germany, Ghana, Greece, Guatemala, Guyana, Honduras, Haiti, India, Iceland, Ireland, Israel,

measures claims on the private sector by bank and non-bank financial institution, excluding credit to the government and public enterprises. I compute the inflation rate from the consumer price index. I take annual averages over the 35 years period, in order to focus on the long-run (steady state) relationship of variables. I include three main controls: the average annual growth rate of real per capita GDP (Growth), the (log of the) average years of schooling in total population (School), and the average share of government expenditure on GDP (GovExp)[5]. These are meant to control for alternative factors that may affect inflation and lending activities. Higher growth may imply lower inflation and a more rapidly developing financial system. More developed countries have more developed financial systems. Also the government may combine inflation with restrictions on the financial sector to fund expenditures.

The regression shows a (statistically and economically) significant negative coefficient for Inflation thus supporting the conclusion that inflation has a negative impact on credit. The adjusted R^2 is 0.404. White heteroskedasticity consistent t-statistics are in parenthesis.

$$\begin{aligned} \text{PC} &= 2.325 - 0.608\text{Inflation} + 2.337\text{Growth} + 26.594\text{School} + 0.189\text{GovExp} \\ & \quad (0.211) \qquad (-2.911)^{**} \qquad (1.154) \qquad (4.058)^{**} \qquad (0.384) \end{aligned}$$

3.5 Conclusion

The model presented in chapter 2 is able to generate different behaviors for the interest rate. The nominal interest rate might be more or less than unit elastic

Italy, Jamaica, Japan, Korea, Malaysia, Malta, Mauritius, Mexico, Netherlands, Nepal, New Zeland, Niger, Norway, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Portugal, Senegal, Sierra Leone, South Africa, Spain, Syria, Switzerland, Thailand, Trinidad and Tobago, Uruguay, United Kingdom, USA, Venezuela, Zimbabwe.

[5] Here are the sources for the variables: Private Credit, International Financial Statistics (IFS), lines 22D and 42D. GDP, IFS, line 99B. CPI, IFS, line 64. Government, Share of government expenditures in GDP, World Development Indicators (WDI). School, WDI, Per capita GDP, WDI.

depending on whether the creditor or the debtor are holding money for one period. In order to distinguish between the two cases I estimated the elasticity of the nominal interest rate with respect to inflation with data for 59 countries over the years 1993-2003 and found that the nominal interest rate is more than unit elastic with respect to the inflation rate.

Chapter 4

Money and Credit: Related Issues

4.1 Introduction

The model developed in the previous chapters can be used to explore a number of further questions relating to the interaction of money and credit. In this chapter I investigate three such issues. First, I analyze in more details the decision to become a creditor or a debtor. Second, I consider the question of seigniorage assuming that the government buys goods on the money market printing new money. Third, I speculate on the possibility of having circulating private credit in the model. To address the first issue I assume agents cannot visit both the money market and the credit market. The two markets are separated -suppose they are at two extreme ends of the island and it takes too long to travel from one side to the other- and agents have to decide whether they will visit one or the other. The remaining part of the model is unchanged.

In equilibrium agents endogenously partition between debtors and creditors. Debtors will travel to their consumption island. They will visit the credit market and issue promises. Then they will travel back to their own island together with their creditors. They will produce for money and repay their debt. The following day they will rest. A creditor produces for a promise, then follows his debtor and waits for repayment with money. The following day he consumes using money. Both credit and money are used in exchange although not simultaneously by the same agent. Money serves as the means of payment. Credit bears interest.

It is interesting to compare my model with a model of private debt developed by Townsend (1980). It is a version of the turnpike model I described in chapter 1. The economy is populated by a countably infinite number of agents, each living forever. Imagine now an infinite turnpike with two lanes running in opposite direction: east and west. Agents are split between those living on one side of the turnpike and those living on the other traveling eastward and westward respectively. Each period they travel exactly one mile , they stop, trade with the agent traveling on the opposite lane and then resume their itinerary. It is a pure exchange economy and endowments are distributed in such a way that each agent is alternatively endowed with one unit or zero units of the only good in the economy. When two agents meet one and only one has an endowment of the good. Two agents never meet twice in their lifetime. This prevents credit to arise, since a promise cannot be repaid. Convert now the turnpike into a circle. This opens the possibility for agents to exchange promises, since they will now meet for sure in the future and they will thus be able to repay their promises. In the model some exogenous bound on the amount of promises to be issued has to be imposed in order to obtain a non trivial equilibrium with credit. The model moreover left open the question of the coexistence of money and credit.

In my setting the itineraries of agents and their very identity as debtors or creditors are not fixed in advance: in equilibrium the population endogenously splits in two, debtors and creditors. The issue of the bound on the amount of promises is resolved using collateral and the fact that money is used to repay promises. By providing an answer to the coexistence of money and credit I obtain an endogenous bound on the amount of promises which insures the existence of equilibrium. I then discuss seigniorage when the government uses revenue to buy goods on the market instead of making transfers to agents. The last part of the chapter deals with circulating promises.

The chapter is structured as follows. In Section 2 I present the model and I solve for the equilibrium. Section 3 compares welfare and output in the economy with credit and in the one without. Section 4 analyzes circulating promises. Section 5 concludes.

4.2 The Model

Consider the same structure as before, except that the money market and the credit market are now separate and agents can visit only one of them each period. Individuals are characterized by a utility function which I assume to be linear in consumption

$$u(x_{t,j}) = x_{t,j}$$

where $x_{t,j}$ is the quantity bought at time t on island j and by a quadratic cost function

$$c(y_{t,j}) = \frac{1}{2}\left(y_{t,j}\right)^2$$

where $y_{t,j}$ is the quantity produced by producer j. The objective of an individual of type j will be to maximize

$$\sum_{t\in T}\beta^t \left[x_{t,j} - \beta\frac{1}{2}\left(y_{t+1,j+1}\right)^2\right]$$

where $T = \{1, 4, 7, ...\}$ in order to take into account the period in which he is going to rest and $0 < \beta < 1$ is the rate at which he discounts the future[1].

In what follows, I will analyse two arrangements. In the first one, agents visit only the money market and don't issue promises. A purely monetary economy will arise, in which agents will use fiat money to exchange in order to overcome the absence of double coincidence of wants. I will then move to an economy in which agents visit only the credit market and issue their own promises. In this equilibrium money acts as a means of final payment for credit, but not as a medium of exchange. The only media of exchange are promises.

Let me summarise here what the typical life of an individual in this economy will be in each case. In the pure money equilibrium, in the morning an agent j will be on his island ready to trade with agents $j-1$. In the afternoon he will travel to island $j+1$, in order to be able to produce good $j+1$ and sell his production for money. In the evening he will rest. In the equilibrium with promises, agents have to decide whether to become debtors or creditors. A debtor j issues promises secured by the collateral in the morning. In the afternoon he produces for money. He repays his own promises in the afternoon, by handing fiat money to his creditors (agents $j-1$) who have travelled with him and are waiting on island $j+1$. The collateral is returned and promises destroyed. In the evening he rests. A creditor j produces for promises and leaves to island $j-1$ where the next period he waits for repayment with money to take place. In the evening he travels to island j in order to consume using money.

[1]Assuming those specific functional forms for the utility and the cost function allows me to simplify the algebra. Similar results can be obtained by putting the curvature on the utility function (e.g. logarithmic utility and linear cost would equally do).

4.2.1 Money Holders

I will first consider the case in which all agents visit only the money market. In this case, individual j at time t will use the amount of money accumulated in the past (m^j_{t-1}) to buy goods on the money market, $p_{t,j}x^M_{t,j} \leq m^j_{t-1}$, where he will consume. He will then go on to produce for money in order to carry it into the future, after resting for one period: $p_{t+1,j+1}y^M_{t+1,j+1} = m^j_{t+1}$. Then he will rest for one period. I will use $x^M_{t,j}$ to indicate the quantity bought at time t of good j with money, $y^M_{t,j+1}$ the quantity sold at time t of good $j+1$ for money and $p_{t,j+1}$ the (walrasian) price at time t of good $j+1$. The sequence of exchanges gives rise to the following budget constraint

$$m^j_{t+2} = p_{t+1,j+1}y^M_{t+1,j+1} + m^j_{t-1} - p_{t,j}x^M_{t,j}$$

and the cash-in-advance constraint

$$p_{t,j}x^M_{t,j} \leq m^j_{t-1}$$

Subject to the previous constraints he will maximize

$$\sum_{t\in T}\beta^t\left[x^M_{t,j} - \beta\frac{1}{2}\left(y^M_{t+1,j+1}\right)^2\right]$$

where $T = \{1, 4, 7, ...\}$ in order to take into account the period in which he is going to rest and $0 < \beta < 1$ is the rate at which he discounts the future. In recursive form the problem becomes[2]

$$\widehat{V}(m) = \max \quad x^M - \beta\frac{1}{2}\left(y^M\right)^2 + \beta^3\widehat{V}(m')$$

$$s.t. \quad px^M \leq m$$

[2] Since I will compute a symmetric equilibrium I omit indices referring to individuals.

$$m' = p'y^M + m - px^M$$

where the term $\widehat{V}(m')$ appears discounted by β^3 to take into account the resting period.

The equilibrium condition on the goods market will simply be $x^M = y^M$, on the money market $m = M$ and I will require that consumers, producers and individuals currently resting in equilibrium want to participate in the exchange process.

The first order conditions for the problem are

$$\begin{aligned} 1 - (\delta + \theta)p &= 0 \\ -\beta y^M + \theta p' &= 0 \\ \beta^3 \widehat{V}'(m') &= \theta \end{aligned}$$

where δ is the multiplier of the cash in advance constraint and θ of the budget constraint. The envelope condition is

$$\widehat{V}'(m) = (\delta + \theta)$$

observe that $\delta > 0$ when $\beta < 1$. Stationarity requires $p = p'$ and $m = m'$. Using the first order conditions and the envelope condition I can solve for the quantity consumed and produced:

$$x^M = y^M = \beta^2$$

the price level can be computed from the binding cash in advance constraint

$$p = \frac{1}{\beta^2} M_0$$

and the participation constraints for consumers, producers and individuals currently resting are all satisfied.

4.2.2 Debtors

Now I will turn to the economy where agents exchange promises, secured by the collateral. Agents will issue promises secured by the collateral and they will repay their promises with money. The objective of a debtor of type j will be to maximize

$$\sum_{t\in T}\beta^t\left[x^D_{t,j}-\beta\frac{1}{2}\left(y^D_{t+1,j+1}\right)^2\right]$$

where $T=\{1,4,7,...\}$ in order to take into account the period in which he is going to rest and travel to the island for repayment and $0<\beta<1$ is the rate at which he discounts the future. $x^D_{t,j}$ is the quantity of good j bought at time t by individual j on the credit market, while $y^D_{t+1,j+1}$ the quantity sold for money. Let me define the price on the money market for good $j+1$ at time t as $p_{t,j+1}$ and the price on the credit market as $q_{t,j}$. Individual j at time t will visit to the credit market where he will sell a promise d^j_t that will need to be repaid in the future:

$$q_{t,j}x^D_{t,j}=d^j_t$$

In the following period he will then produce for money in order to pay off his debt:

$$p_{t+1,j+1}y^D_{t+1,j+1}=m^j_{t+1}\geq d^j_t$$

He will then rest for one period.

The previous sequence of exchanges gives rise to the following budget constraint

$$m^j_{t+2}=p_{t+1,j+1}y^M_{t+1,j+1}+q_{t+1,j+1}y^C_{t+1,j+1}+m^j_{t-1}-p_{t,j}x^M_{t,j}-q_{t,j}x^C_{t,j}$$

and the repayment constraint

$$p_{t+1,j+1}y^M_{t+1,j+1}\geq q_{t,j}x^C_{t,j}$$

In recursive form the problem becomes

$$V(m) = \max \quad x^D - \beta\frac{1}{2}\left(y^D\right)^2 + \beta^3 V(m')$$

$$s.t. p'y^D \geq qx^D$$

$$m' = p'y^D + m - qx^C$$

where $V(m')$ is discounted by β^3 to take into account the resting period.

The first order conditions for x^D, y^D are respectively

$$1 - (\lambda + \gamma)q = 0$$

$$-\beta\left(y^D\right) + (\lambda + \gamma)p' = 0$$

$$\beta^3 V'(m') = \gamma$$

where λ is the multiplier associated to the repayment constraint and γ to the budget constraint. The envelope condition is

$$V'(m) = \gamma$$

Notice that necessarily $\lambda > 0$ when $\beta < 1$. Stationarity requires $p = p'$, $q = q'$ and $m = m'$. Define $1 + i = \frac{q}{p}$. The system of equations given by the first order conditions and the envelope condition can be simplified to give

$$y^D = \frac{1}{(1+i)\beta}$$

$$x^D = \frac{y^D}{(1+i)} = \frac{1}{(1+i)^2\beta}$$

The participation constraints for the individuals currently consuming, producing and resting are respectively

$$
\begin{aligned}
V_c &= \frac{1}{2\left(1+i\right)^2 \beta \left(1-\beta^3\right)} > 0 \\
V_p &= \frac{2\beta^3 - 1}{2\left(1+i\right)^2 \beta^2 \left(1-\beta^3\right)} \\
V_r &= \frac{\beta}{2\left(1+i\right)^2 \beta \left(1-\beta^3\right)} > 0
\end{aligned} \tag{4.1}
$$

(1) is the only condition that is not automatically verified. To make sure that producers want indeed to participate the following condition is therefore needed:

$$\beta > \left(\frac{1}{2}\right)^{\frac{1}{3}}$$

4.2.3 Creditors

The objective of a creditor of type j will be to maximize

$$\sum_{t \in T} \beta^t \left[-\frac{1}{2} \left(y^C_{t+1,j+1}\right)^2 + \beta^2 x^C_{t,j} \right]$$

where $T = \{1, 4, 7, ...\}$ in order to take into account the period in which he is going to rest and travel to the island for repayment and $0 < \beta < 1$ is the rate at which he discounts the future. $x^C_{t,j}$ is the quantity of good j bought at time t by individual j on the money market, while $y^C_{t+1,j+1}$ the quantity sold for credit. A creditor will first produce for a promise that will be repaid the next day when he will follow his debtor on island $j+1$ in order to get his money back:

$$q_{t,j+1} y^C_{t,j+1} = d^j_t = m^j_{t+1}$$

then proceed to the money market where he will use the money just acquired to purchase commodities:

$$p_{t+2,j}x^C_{t+2,j} \leq m^j_{t+1}$$

The previous sequence of exchanges gives rise to the following budget constraint

$$m^j_{t+2} = q_{t,j+1}y^C_{t,j+1} + m^j_{t+1} - p_{t+2,j}x^C_{t+2,j}$$

and the cash in advance constraint

$$q_{t,j+1}y^C_{t,j+1} \geq p_{t+2,j}x^C_{t+2,j}$$

In recursive form the problem is

$$V(m) = \max \quad -\frac{1}{2}\left(y^C\right)^2 + \beta^2 x^C + \beta^3 V(m')$$

$$s.t. qy^C \geq p'x^C$$

$$m' = qy^C + m - p'x^C$$

where $V(m')$ is discounted by β^3 to take into account the resting period.

The first order conditions for x^C, y^C are respectively

$$\beta^2 - (\lambda + \gamma)p' = 0$$

$$-\left(y^C\right) + (\lambda + \gamma)q = 0$$

$$\beta^3 V'(m') = \gamma$$

where λ is the multiplier associated to the repayment constraint and γ to the budget constraint. The envelope condition is

$$V'(m) = \gamma$$

Notice that necessarily $\lambda > 0$ and $\mu > 0$ when $\beta < 1$. Stationarity requires $p = p'$, $q = q'$ and $m = m'$. The system of equations given by the first order conditions and the envelope conditions can be simplified to give

$$y^C = \beta^2 (1+i)$$

$$x^C = (1+i)\, y^C = \beta^2 (1+i)^2$$

The participation constraints for the individuals currently consuming, producing and resting are respectively

$$\widehat{V}_c = \frac{\left(2\beta^2 - \beta^5\right)(1+i)^2}{2\left(1-\beta^3\right)} > 0$$

$$\widehat{V}_p = \frac{\beta^4 (1+i)^2}{2\left(1-\beta^3\right)} > 0$$

$$\widehat{V}_r = \frac{\left(2\beta^3 - \beta^6\right)(1+i)^2}{2\left(1-\beta^3\right)} > 0$$

4.2.4 Indifference and incentive conditions

Each agent can choose whether he wants to become a debtor or a creditor. The choice involves the decision to be a consumer or a producer in the first period. In order to have both debtors and creditors -i.e. for exchange with credit to be possible- a new debtor and a new creditor must be indifferent. This is captured by the following indifference condition

$$\frac{1}{2(1+i)^2 \beta \left(1-\beta^3\right)} = \frac{\beta (1+i)^2}{2\left(1-\beta^3\right)}$$

which can be solved to give the interest rate

$$(1+i) = \beta^{-\frac{5}{4}} > 1$$

I also need to check that a new creditor doesn't have the incentive to become a money holder. This is verified since

$$\frac{\beta^4 (1+i)^2}{2(1-\beta^3)} > \frac{\beta^4}{2(1-\beta^3)}$$

4.2.5 Equilibrium

In equilibrium the population living on each island partitions between debtors and creditors. Let μ^D be the proportion of debtors and μ^C the proportions of creditors. Market clearing requires

$$\mu^D \left(\frac{1}{(1+i)^2 \beta}\right) = \mu^C \left(\beta^2 (1+i)\right)$$

Moreover

$$\mu^D + \mu^C = 1$$

and

$$(1+i) = \beta^{-\frac{5}{4}}$$

I can solve the system to obtain the exact proportions of debtors and creditors

$$\mu^D = \frac{\beta^{-\frac{3}{4}}}{1+\beta^{-\frac{3}{4}}}$$

$$\mu^C = \frac{1}{1+\beta^{-\frac{3}{4}}}$$

Let me briefly summarise and comment on the main features of the equilibrium with credit. In equilibrium agents of every type partition between debtors and creditors. A Debtor visits the credit market and issues promises. Then he travels together with his creditor. He produces for money and repays his debt. The following period he rests. A creditor produces for a promise, then follows his debtor and waits for repayment with money. The following period he consumes using money. Both credit and money are used in exchange although not simultaneously by the same agent. Money serves as the means of payment. Credit bears a higher rate of return.

4.3 Welfare

I compare welfare in the economy with only money and the economy with credit. My welfare measure is given by the sum of the value functions of each group of agents in the two equilibria, with credit and with money:

$$W = \frac{1}{3}\left[\frac{\beta^{-\frac{1}{4}}(2\beta-1)+\beta^{-\frac{5}{2}}\left(2-\beta^{2}\right)}{2(1-\beta)\left(1+\beta^{-\frac{3}{4}}\right)}\right]$$

$$\widehat{W} = \frac{1}{3}\frac{\beta^{2}\left(2-\beta^{2}\right)}{2(1-\beta)}$$

Taking the difference between the two, I obtain

$$W-\widehat{W} = \frac{1}{3}\frac{\beta^{-\frac{5}{2}}\left(2-\beta^{2}-2\beta^{\frac{9}{2}}-\beta^{\frac{9}{4}}+\beta^{\frac{13}{2}}+2\beta^{\frac{13}{4}}-2\beta^{\frac{15}{4}}+\beta^{\frac{23}{4}}\right)}{2(1-\beta)\left(1+\beta^{-\frac{3}{4}}\right)}$$

which is positive for any $\beta < 1$.

This confirms that the use of credit improves welfare compared to an economy with only money.

4.4 Seigniorage

Suppose that the government, instead of giving lump-sum transfers to agents, intervenes issuing money every period and buying goods on the money market on each island. I will label $G_{t,j}$ the quantity bought by the government on island j at time t. This modifies the equilibrium condition on the money market to:

$$x_{t,j}^M + G_{t,j} = y_{t,j}^M$$

The remaining structure and notation is left unchanged. The government budget constraint equates the increase in the money supply on each island to the value of the commodities bought on the money market:

$$M_t - M_{t-1} = p_t G$$

which can be rewritten as

$$\frac{M_t}{p_t} = \frac{M_{t-1}}{p_{t-1}} \frac{p_{t-1}}{p_t} + G$$

In a stationary equilibrium the real quantity of money is constant from period to period

$$\frac{M_t}{p_t} = \frac{M_{t-1}}{p_{t-1}} = \mu$$

Therefore the government budget constraint becomes

$$\mu \left(\frac{\pi}{1+\pi} \right) = G$$

The problem in recursive form for the equilibrium with money and credit will be

$$V(m) = \max \quad x^M + x^C - \beta \frac{1}{2} \left(y^M + y^C \right)^2 + \beta^3 V(m')$$

$$s.t. \quad px^M \leq m$$

$$p'y^M \geq qx^C$$

$$m' = p'y^M + q'y^C + m - px^M - qx^C$$

The equilibrium conditions are

$$\begin{aligned} x^M + G &= y^M \\ x^C &= y^C \\ m &= M \end{aligned}$$

The solution of the problem gives

$$R = \left(\frac{1+\pi}{\beta}\right)^{\frac{3}{2}}$$

total output is

$$y^M + y^C = \frac{\beta^{\frac{1}{2}}}{(1+\pi)^{\frac{1}{2}}}$$

The real demand for money

$$\mu = \frac{\beta^{\frac{1}{2}}}{\alpha\left(\beta^{\frac{3}{2}} + z^{\frac{1}{2}}\right)}$$

which gives, using the government budget constraint, the government expenditures

$$G = \frac{\beta^{\frac{1}{2}}\pi}{\left(\beta^{\frac{3}{2}} + (1+\pi)^{\frac{1}{2}}\right)(1+\pi)}$$

and using the equilibrium condition on the money market

$$x^M = \frac{\beta^{\frac{1}{2}}}{\left(\beta^{\frac{3}{2}} + (1+\pi)^{\frac{1}{2}}\right)(1+\pi)}$$

Total consumption will then be

$$x^M + x^C = \frac{\beta^{\frac{1}{2}} + \beta^2 (1+\pi)^{\frac{1}{2}}}{\left(\beta^{\frac{3}{2}} + (1+\pi)^{\frac{1}{2}}\right)(1+\pi)}$$

The interesting aspect to notice is that the revenue collected by the government in the form of seigniorage

$$\mu \frac{\pi}{1+\pi} = \frac{\beta^{\frac{1}{2}} \pi}{\left(\beta^{\frac{3}{2}} + (1+\pi)^{\frac{1}{2}}\right)(1+\pi)}$$

is an inverted U shape function of the inflation rate, similar to a Laffer curve.

4.5 On the Circulation of Promises

An interesting question to address in the framework presented so far is whether credit can circulate in the economy, in the sense that the same promise is held successively by two different agents and then redeemed by the original issuer from a third party. Credit becomes inside money. "Inside money can be defined very broadly as any privately-issued long-term paper that is held by a number of agents in succession. Whenever paper circulates as a means of short-term savings (liquidity), it can properly be considered as money, or a medium of exchange, because agents hold it not for its maturity value but for its exchange value." Kiyotaki and Moore (2000).

The model features absence of double coincidence of wants, spatial separation and competitive markets. Agents move to different islands to consume and produce. Moreover they can issue promises secured by collateral and repaid with money. Money and credit coexist thanks to a friction in the exchange technology that prevents agents from spending money immediately -the resting period-. In the equilibrium with money and circulating promises, a typical agent j in the morning exchanges money, old promises and newly issued promises to purchase and consume good j. In the afternoon he meets agents $j+1$ and produces for money, second hand promises and newly issued promises. Agent j embarks on the ship leaving to island $j-1$, where he arrives in the evening. There he meets the holders of his own promises. With the money acquired in the previous period on the money market he repays the promises thus getting back his collateral. Promises are then destroyed and he is ready to travel to island $j+1$ and restart the cycle. Money and credit coexist, credit circulates and money serves also to repay credit. There is rate of return dominance: the gross return on money is one, the return on second hand promises is $1+i_2 = \beta^{-1} > 1$ and the return on newly issued promises is $1+i_1 = \beta^{-2} > \beta^{-1}$. Moreover total production is higher than in an economy with only money: $\frac{\beta}{\alpha} > \frac{\beta^2}{\alpha}$. The higher return that producers are getting induces them to increase their production compared to an economy with only money. Finally, welfare in the economy with circulating credit is higher than in the economy with only money. With steady inflation, the nominal interest rate increases making costly to hold money. Agents holding a second hand promise will have to hold money overnight before repaying their promises: they will reduce their holdings of second hand promises more than their holdings of newly issued promises. The ratio of second hand promises to newly issued promises in real terms decreases with inflation. Also the ratio of newly issued promises to money decreases with inflation. Since less transactions are performed also output, consumption and welfare are lower. Concerning the question of the circulation of credit there are few contributions in the literature. The paper by Townsend and Wallace (1982) features spatial separation and circulating debt. The focus in that paper is however on the coordination problems arising in a decentralised economy where agents use privately issued means of exchange rather than the issue of the coexistence with fiat money. Kiyotaki and Moore (2000) construct a model of inside money "to ask: when and why is the circulation of inside money essential to the smooth running of the economy?". The model -which addresses also a number of other issues not directly connected to the present inquiry- is built around an absence of double coincidence of wants in dated goods rather than in physical goods. Markets are competitive but there is no walrasian auctioneer. Agents can issue their own promises by mortgaging only a fraction of the output they are going to produce in the future. They find parameter values such that promises are essential and circulate.

Freeman (1996) has a paper where purchases are made with debt, debt is settled with a final payment of fiat money and there is a market for resale of debt. The model itself is however quite different, being cast in an overlapping generation structure to which he added spatial separation. Moreover in Freeman (1996) the clearing system is centralized: agents need to move to a central island to clear debts, while in my paper the repayment process is completely decentralised. A related paper in the OLG tradition is Azariadis, Bullard and Smith (2001). In their framework, the question of the coexistence of the two instruments can be meaningfully posed and some answers are provided about the social benefits of a mixed system. Their environment features heterogeneous agents, spatial separation and limited communication. Borrowers and lenders can meet only once during their three period lifetime. In such a setting private liabilities can be issued by borrowers to lenders who in turn use them to exchange with trading partners that will meet the original issuers and will thus be able to present the liabilities for redemption. These liabilities therefore circulate in the economy and constitute indeed a form of inside money. In an economy without outside money they find indeterminacy of equilibrium and high volatility of consumption and interest rates. In an economy with both outside and inside money, the equilibrium turns out to be determinate and -strikingly- efficient.

4.6 Conclusion

In this Chapter, I derived an equilibrium in which money and credit coexist, credit has a higher rate of return and improves welfare. Agents decide whether they want to become creditors or debtors and partition accordingly in equilibrium. Some agents -debtors- will buy on credit and sell on money (and use money to repay their debt), some others -creditors- will sell for credit and buy with money. This arrangement dominates the one in which only money is used. I then moved on to derive the seigniorage revenue of a government which is buying goods instead of making transfers. I also discussed the interesting possibility of having promises circulate in the economy. Promises can be issued to purchase commodities and then used by the creditor to purchase commodities from a third party and then redeemed with money by the issuer from the third party. Privately issued promises would thus constitute a form of inside money in the model. Further interesting questions to be addressed in this framework include studying the interaction of cooperating and competing media of exchange and understanding the redistributive effects of monetary policy. These are left for future research.

Chapter 5

Illiquid Bonds and Promises

5.1 Introduction

According to Hicks -in his famous "Suggestion for simplifying the theory of money"- the main challenge of monetary theory is to explain the coexistence of money and interest bearing assets. Why do people hold simultaneously assets having different rates of return? Kocherlakota (2003) gives a clever explanation of why nominal risk-free bonds yielding a positive rate of return may coexist with money -the zero rate of return asset- and be essential in monetary economies. The intuition is as follows. Money has an essential role in performing intratemporal exchanges being the medium of exchange. Agents have different intertemporal marginal rates of substitution: half of the agents experience a liquidity shock. This creates a need for further intertemporal trades of money that are accomplished using nominal risk-free bonds with a positive rate of return. A liquid bond -i.e. a bond that is portable and can be exchanged for goods- wouldn't serve the purpose since it would have the same rate of return as money. An illiquid bond -i.e. a bond that is non-portable and cannot be exchanged for commodities- commands a positive rate of return and is essential in insuring against liquidity shocks. Agents with urgent need of liquidity, sell their bonds for money to agents with less need for immediate liquidity. An example of an illiquid bond is a registered US Treasury bond.

In this paper, I argue that illiquidity alone may not be enough to guarantee that bonds are essential. If agents cannot be prevented from using a commitment technology allowing them to issue promises secured by bonds, bonds themselves cannot be used to insure against liquidity shocks, since their rate of return is driven to zero. Consider the following scenario. Agents, instead of trading bonds for money on the asset market, have the option to store them in a vault and issue promises backed by them, using the key of the vault as a guarantee. Agents not experiencing the liquidity shock are willing to issue promises. This drives down the rate of return on bonds to zero. For low values of the liquidity shock, agents experiencing it are better off and promises improve welfare. For higher values of the liquidity shock however agents experiencing the shock are worse off and welfare is lower with promises. In this case legal restrictions on the intermediation of bonds would improve welfare. In the 19th century in the US banks used to be able to issue private money fully backed by bonds. One way to interpret the present result is to argue that such an activity was often welfare reducing. Section 2 presents the result. Section 3 concludes.

5.2 The Model

Time is discrete and continues for ever. There are three islands. Island 3 is inhabited by a unit measure of households, where each household has two members, a consumer and a producer. There are two types of perishable goods and two types of households: households of type 1 consume good 1 and produce good 2, while households of type 2 consume good 2 and produce good 1. A type $i = 1, 2$ household seeks to maximize

$$\theta_j \log(c_0) - y_0 + \sum_{t=1}^{\infty} \beta^t (\log(c_t) - y_t)$$

where c_t is consumption of good i, y_t is production of good $i+1$ and $\theta_j \in \{\theta_H, \theta_L\}$, with $\theta_L = 1$ and $\theta_H > 1 > \beta$. θ_j is a shock to preferences, capturing the liquidity shock and is privately observed. The shock hits half of the agents in period zero only.

At the beginning of each period all households are on island 3. Good 1 can be produced on island 2 and good 2 on island 1 only. Then, producers of good 1 move to island 2 and producers of good 2 move to island 1. Consumers of good 1 visit island 1 and consumers of good 2 visit island 2. Exchange takes place in competitive markets. There is no record keeping technology and enforcement is limited. There are also two durable and divisible objects: money and nominal bonds. A nominal bond lasts only one period, it is withdrawn and exchanged for money by the government in period one. On island 3 in period zero there is a competitive asset market on which households can exchange money and bonds. Agents are endowed with an amount M of money and an amount B of bonds and they trade them at a price q in terms of money. The relative price of goods in terms of money is p. Households choose $(c_t, y_t, M_t, M_0', B_0) \geq 0$ to maximize utility subject to the constraint on trade in the asset market in period zero $M_0' + qB_0 \leq M + qB$, where B_0 is the amount of bonds agents buy; the cash-in-advance constraint in period zero $p_0 c_0 \leq M_0'$; the budget constraint in period one $M_1 = M_0' - p_0 c_0 + p_0 y_0 + B_0$; from period one on, the economy is purely monetary and the constraints are the cash-in-advance and budget constraint $p_t c_t \leq M_t$ and $M_{t+1} \leq M_t - p_t c_t + p_t y_t$ respectively. Finally, there is a capacity constraint on the amount agents can produce $0 \leq y_t \leq 1$. An Equilibrium for the economy with money and bonds is a sequence of consumption, production, money holdings, bond holdings and prices such that at all times $(c_t, y_t, M_t, M_0', B_0)$ maximize utility subject to the budget constraint, the cash-in-advance constraint and the constraint on the exchange of money and bonds; (p_t, q) clear the goods market and the asset market. Surprisingly the solution in Kocherlakota (2003) doesn't account for the participation constraints of agents. In an economy without enforcement agents should be willing to participate at every stage including the asset market stage. In this economy, agents have always the option to secure $c = 1$ for themselves by not trading on the asset market. Once the participation constraints are taken into account, the following is an equilibrium for sufficiently high β and θ. The price in period zero is $p_0 = M$ and from period one onwards is $p_t = M + B$. The price on the bonds market is $q = \frac{\beta M}{M+\beta B+2B}$. Production is always at capacity: $y_t = 1$. Consumption for agents with a high shock is higher in period zero than in period one: $c_0(H) = \frac{M+qB}{M}$, $c_0(L) = \frac{M-qB}{M}$. Consumption for agents with a low shock is lower in period zero than in period one: $c_1(H) = \frac{M}{M+B}$, $c_1(L) = \frac{M+2B}{M+B}$. Consumption thereafter is $c_t = 1$ for all agents. Agents with a high shock sell all their bonds in period zero $B_0(H) = 0$ and agents with a low shock buy all bonds $B_0(L) = 2B$. In equilibrium agents experiencing a high liquidity shock (H) sell their bonds to agents with a low (L) shock for money -at a relative price less than one-. Agents with a more urgent need to consume can increase their purchases of goods. The following period the L agents receive their payoff from matured bonds and consume more than H agents. From then on the economy reverts to the stationary equilibrium without bonds. Illiquid bonds insure agents against preference shocks. All agents are better off and illiquid bonds are essential. When β and θ are lower, agents choose $B_0(H) = B_0(L) = B$, i.e. they don't trade on the asset market, and consume $c_0(j) = 1$ for all j.

Assume on island 3 vaults are available. The key of the vault can be taken away only if bonds are stored in the vault. Vaults are secure against theft. Bonds and keys cannot be counterfeited. Bond holders can now use bonds to issue their own promises. Before moving to the next island to trade, they can store the bond in a vault and take the key with them. When they arrive on their consumption island they can issue a promise, giving the key to the producer as a guarantee. The vault is a safe keeping technology and the key a commitment technology. The holder of the key has the right to go to island 3 and take the bond from the vault. Agents cannot commit not to use the vault. Historically, banks used to provide safe keeping and issue private money backed by bonds.

The equilibrium with money and promises backed by bonds has $p_t = M + B$, $q = 1$ and $c_t = y_t = 1$ at all times. In particular $q = 1$ by no arbitrage. Suppose agents with a low shock get the same utility in the two equilibria:

$$\log\left(\frac{M - qB}{M}\right) + \beta \log\left(\frac{M + 2B}{M + B}\right) = \log 1 + \beta \log 1 = 0 \tag{5.1}$$

which requires agents to be impatient enough:

$$\beta = \frac{\log\left(\frac{M}{M-qB}\right)}{\log\left(\frac{M+2B}{M+B}\right)} = \overline{\beta}$$

Agents with a high shock are better off in the equilibrium with promises if the liquidity shock is not too high:

$$\theta_H \log\left(\frac{M + qB}{M}\right) + \beta \log\left(\frac{M}{M + B}\right) \leq \theta_H \log 1 + \beta \log 1 = 0 \tag{5.2}$$

and substituting $\overline{\beta}$ in (2):

$$\theta_H \leq \frac{\log\left(\frac{M+B}{M}\right) \log\left(\frac{M}{M-qB}\right)}{\log\left(\frac{M+2B}{M+B}\right) \log\left(\frac{M+qB}{M}\right)} = \overline{\theta}$$

Since $\frac{M+B}{M} > \frac{M+2B}{M+B}$ and $\frac{M}{M-qB} > \frac{M+qB}{M}$, $\overline{\theta} > 1$.

Therefore, there exist a $\overline{\theta} > 1$, such that for $1 < \theta_H < \overline{\theta}$, low shock agents are indifferent and high shock agents are better off with promises. Not surprisingly, this happens exactly in the parameters region where illiquid bonds were not traded. When $\theta_H > \overline{\theta}$ high shock agents are worse off. For higher values of the liquidity shock, legal restrictions on the intermediation of bonds can be advocated.

5.3 Conclusion

The point is reminiscent of the literature on legal restrictions. "There are no natural barriers that limit substitution between privately issued inside money on the one hand and outside or government-issued money on the other hand" (Wallace (1983)). In an economy with money and nominal risk-free interest bearing securities some barrier has to be present to explain the difference in the rate of return. It could be that securities come in large denominations. Indivisibility was the subject of Aiyagari, Wallace and Wright (1996). It could also be that bonds are registered, not bearer assets. Illiquidity is the subject of Kocherlakota (2003). Both reasons are however not enough to explain the difference in the rate of returns, since -as pointed out by Wallace (1983)- some agents could create promises fully backed by bonds and drive the rate of return to zero. Historically the best example is the National Banking System in place in the US between 1865 and 1913, where each bank could issue its own banknotes secured by government bonds. At the time, nominal interest rates were exceptionally low. Legal restrictions is the subject of the present paper. When some agents have a very urgent need of liquidity, legal restrictions against intermediation can be advocated to improve welfare. A way to interpret the result is to argue that in 1913 banks were prevented to issue private money backed by bonds since the activity was welfare decreasing.

Chapter 6

Entry of Lenders and Liquidity of Assets

In[1] the last two decades several financially liberalized countries, such as the United States, the United Kingdom, the Nordic countries, the East Asian countries, have experienced an aggressive entry of new lenders into their credit markets. Cross-border lending has played an important role in this process. Japanese banks increased their presence in the United States during the 1980's and in East Asia during the 1990's (Peek and Rosengren (2000a)).[2] Within the United States, after the abolition of interstate branching restrictions by the 1994 Riegle-Neal Act, banks have expanded their business beyond state borders. The impact of financial liberalization has had a functional dimension besides a geographic one, with lenders spreading their loan portfolios beyond their traditional area of activity.[3]

The mechanisms through which new (foreign) lenders interact with incumbent economic actors are non-obvious. The entry of lenders into liberalized economies

[1] This chapter is coauthored with Raoul Minetti.

[2] Peek and Rosengren (2000a) report that in the late 1980's Japanese banks accounted for up to 18% of Consumer and Investment loans in the United States.

[3] For example, in the 1980's, Nordic and UK banks increased their involvement in the real estate sector, funding speculative builders.

and sectors has allegedly had dramatic effects on firms' behavior and the nature of their projects. According to *The Economist* (1999), the effect of entry has been to "increase the riskiness of traditional behavior or introduce new and inexperienced players".[4] Is this view justified? Do changes in project riskiness have a feedback effect on the entry of lenders? What is the impact on economic stability? Recent research on financial crises (Kaminsky and Reinhart (1999)) has identified a pattern: crises tend to be preceded by episodes of financial liberalization. Concerns have mounted on the possible destabilizing behavior of foreign financial institutions (Dages, Goldberg and Kinney (2000)). "Arguments against allowing the entry of foreign banks into domestic markets usually include concerns (...) that foreign banks will not serve as a stabilizing influence by providing additional credit during a crisis in the host country"(Peek and Rosengren (2000b, p. 147)). Yet, the few existing empirical studies reveal that lending by foreign banks exhibits a less procyclical pattern than that of domestic ones and that a stronger presence of foreign banks is associated with greater output stability.

In this chapter, we address these issues. We put forward a theory of the interaction between the entry of lenders into a market and the nature of projects based on firms' liquidation values. We then analyze the implications that this interaction has for output stability. We call "incumbents" (respectively "newcomers") lenders with a consolidated (lack of) experience of the market. The crucial feature of our economy is that newcomers have lower ability than incumbents to extract value from the assets of borrowers. The disadvantage of newcomers could materialize at the bankruptcy stage because newcomers could be less aware of local insolvency practices. Hermalin and Rose (1999, p. 373), argue that "an alien legal system means that a foreign lender's domestic expertise on enforcement is of lower value; the foreign lender may, therefore, need to make expensive investments in acquiring the neces-

[4] The Economist. "Finance on the loose." May 15th 1999.

sary expertise or become reliant on expensive local expertise". The disadvantage of newcomers could also materialize at the redeployment stage. When second-hand users feature heterogeneous efficiency in employing assets, the disadvantage of newcomers in liquidating assets could be their lower ability to identify the most efficient users in the secondary market. As this ability is at least partly a by-product of the information gathered by a lender in her credit relationships, a newcomer with no history of lending to local firms will be likely to lack this ability (see section 6.1.5 for an example along these lines).

We show that the entry of newcomers and the riskiness of projects can reinforce each other through firms' liquidation values. The intuition is as follows. In our economy, entrepreneurs can choose between safe projects and risky projects that offer higher returns.[5] With some probability, both types of projects need to be refinanced at an intermediate stage. Because of their high liquidation skills, incumbents have strong incentives to terminate projects with high probability of failure and liquidate their assets rather than refinancing. Because of their lower ability to liquidate, instead, newcomers are reluctant to liquidate a project prematurely and always refinance, even if it becomes clear that the project has high probability of failure. Their softer budget constraint in refinancing high-risk/return projects renders newcomers more appealing lenders than incumbents. However, the lower liquidation skills of newcomers make their funding more costly because newcomers expect a lower return from asset liquidation if a project fails. In equilibrium, entrepreneurs who derive high returns from risky projects borrow from newcomers, sustaining the associated extra cost of funding, and choose high-risk/return projects; entrepreneurs who derive low returns from risky projects borrow from incumbents and choose safe projects, saving on funding costs.

Now, consider an exogenous shock to asset liquidity that affects liquidation val-

[5]The assumption that on average risky projects are more productive than safe projects is common in the literature. New riskier technologies can be more productive.

ues. Suppose a negative shock is realized depressing liquidation values. The fall in the liquidation price of assets erodes the cost advantage of incumbents that stems from their higher liquidation skills. By eroding the cost advantage of incumbents, the price fall pushes some entrepreneurs to switch to newcomers and choose risky projects, spurring the expected default rate and supply of liquidated assets. In turn, this induces a further fall of the liquidation price of assets. In fact, as the number of liquidated assets increases, lenders have to sell assets to less productive second-hand users who are willing to pay a lower price. This induces further entrepreneurs to switch to newcomers and high-risk/return projects and so forth.

The mutually reinforcing interaction between the entry of newcomers and the share of high-risk/return projects stabilizes output following liquidity shocks, such as those occurring in booms and recessions. In our economy, the share of high-risk/return projects is suboptimally low. The high liquidation skills of incumbents, coupled with contractual incompleteness, render incumbents too conservative towards high-risk/return projects. Entrepreneurs can escape incumbents' too hard budget constraint by borrowing from newcomers who, however, are only an imperfect substitute for incumbents. In fact, newcomers impose on entrepreneurs the cost of their low liquidation skills, discouraging some entrepreneurs from borrowing from them and implementing risky projects. Following, for example, a negative shock to asset liquidity, the cost of the low liquidation skills of newcomers declines and more entrepreneurs choose newcomers and risky projects, approaching the optimal share of high-risk/return projects. This dampens the negative impact on output of the decline in asset liquidity. The model can therefore rationalize the empirical findings on the *stabilizing* role of foreign financial institutions (see section 6.2.4).

The closest papers in the literature are Diamond and Rajan (2001a, b), Dewatripont and Maskin (1995) and Shleifer and Vishny (1992). In analyzing the rationale for the short-term liability structure of banks, Diamond and Rajan (2001a, b) argue that banks have better liquidation skills than dispersed investors. Diamond and Rajan (2001b) also qualitatively discuss applications of their theory. By interpreting dispersed investors as foreign investors, they argue that foreign, short-term lending intermediated by domestic banks allows domestic banks to commit their superior liquidation skills and fund illiquid projects. When we interpret newcomers as foreign lenders, there are at least three differences between our analysis and this application of their theory. First, we analyze the implications of direct foreign lending, in substitution of domestic lending, rather than of foreign lending intermediated by domestic banks. Secondly, we describe a two way interaction between type of projects undertaken and direct foreign lending. Diamond and Rajan (2001b) analyze how short term debt spurs the number of illiquid projects but do not analyze possible feedbacks, even less so through firms' liquidation values (exogenous in their context). Finally, the type of projects that foreign lending allows financing differs: while they focus on illiquid projects, we focus on high-risk/return ones. In the analysis (section 6.2.1), we show that our model can offer an explanation for the evolution of projects in East Asia before the 1997-98 crisis different from that of Diamond and Rajan (2001b).

In Dewatripont and Maskin (1995), some firms choose between long term, very profitable projects and short term profitable ones while other firms choose between long term poor projects and short term profitable ones.[6] Dewatripont and Maskin (1995) show that in a decentralized economy, meant as one in which the ownership of capital is diffuse and firms borrow from multiple sources, lenders can have a too hard budget constraint towards long term very profitable projects. This leads firms to prefer short term projects (short-termism). In their context, lenders have all the same skills so that there is no room for analyzing the entry of lenders "foreign" to

[6] Dewatripont and Maskin (1995) introduce long term very profitable projects in a second stage of their analysis. In a first stage, they focus on the choice between short term, profitable projects and long term, poor ones.

the market. Furthermore, Dewatripont and Maskin (1995) focus on "investment horizons" and project length while we focus on the riskiness of projects and their endogenous liquidation values. Shleifer and Vishny (1992) analyze the interaction between the debt leverage of firms and liquidation values and the role of the liquidity of assets in this interaction. Despite our different focus, we share with them the emphasis on endogenous firms' liquidation values. If one believes that the entry of lenders has important effects on firms' average riskiness and default rate, treating liquidation values as endogenous and analyzing their possible feedback on the entry of lenders appears to be necessary.

The chapter also relates to the literature on the entry of new lenders into a credit market (see, for example, Broecker (1990); Dell'Ariccia, Friedman, and Marquez (1999)). This literature generally analyzes the impact of this entry on the efficiency and on the structure of the credit sector. In analyzing the limits to perfect competition in the credit market, Dell'Ariccia et al. (1999) assume that existing lenders have more information than new lenders on the riskiness of borrowers. This allows them to fund safe borrowers, leaving potential new lenders exposed to adverse selection and blocking their entry. In their context, risk deters the entry of new lenders while in ours the riskiness of projects and the entry of newcomers reinforce each other. Furthermore, in their model, as in the other studies in this literature, asset liquidity and liquidation values play no role in the entry of new lenders. Finally, our model departs from this literature in its emphasis on the real sector. In particular, the paper analyzes how the interaction between the entry of lenders and the nature of projects affects output stability.

The chapter is organized as follows. Section 1 presents the model. Section 2 discusses its applications and limitations. In this section, we also assess the empirical predictions of the model, comparing them with the existing evidence. Section 3 concludes. The proofs are in the Appendix.

6.1 The Model

6.1.1 Setup

Environment and Technology The economy lasts for three dates, 0, 1 and 2. There is a continuum of entrepreneurs of mass 1 and two continua of lenders ("incumbents" and "newcomers") each of mass greater than 1. There is initially only the final good, while assets can be produced. All agents are risk neutral and consume final good at date 2.

Entrepreneurs have no endowment. At date 0, each entrepreneur can start a safe project or a risky one. In both projects at date 0 the entrepreneur can invest an amount I_0 of final good and at date 1 transform it into A indivisible, productive assets.[7] With probability $1 - \alpha$ production is "fast" and at date 1 the assets yield Y, whether the project is safe or risky. With probability α production is "slow" and at date 1 the entrepreneur must inject an additional amount I_1 of final good. If the refinancing occurs and the project is safe, at date 2 the assets yield Y/p_s with probability p_s $(0 < p_s < 1)$ or the project fails, the assets yield 0 and one non-depreciated asset can be redeployed outside the firm. If the refinancing occurs and the project is risky, at date 2 the assets yield $(Y + y)/p_r$ with probability p_r $(0 < p_r < p_s)$ or the project fails, the assets yield 0 and one non-depreciated asset can be redeployed. Finally, if the refinancing does not occur and the assets are not used in production, at date 2 the project fails with certainty and the A non-depreciated assets can be redeployed. y/p_r constitutes the entrepreneur-specific return of a risky project, with y uniformly distributed on the support $[0, 1]$.

If at date 2 an asset is redeployed, it can be used by other entrepreneurs. For simplicity, we assume that each entrepreneur can use zero or one liquidated asset. Let $\alpha\ell \in \alpha\,[\underline{\ell},\ 1]$ $(\underline{\ell} > 1-p_r)$ be the share of entrepreneurs who can use one liquidated

[7] The assumption that assets are indivisible simplifies the analysis of their secondary market later on.

asset and λ the amount of final good that each of them can produce with it, with λ uniformly distributed on the support $[0, \ell]$.[8]

We impose a lower bound on the output of completed projects:

$$Y > I_0/(1-\alpha) + I_1 + A. \tag{6.1}$$

Assumption (1) guarantees that at date 0 the net expected return from a project is strictly positive and that at date 1 the expected return from refinancing a project exceeds the return from liquidating its assets.

In Figure 1 we summarize the timing of events.

Financing Each lender has at least an amount $I_0 + I_1$ of final good at date 0 that she can store or lend. At date 0 and date 1 lending takes place in one to one relationships, i.e. one lender funds one entrepreneur and one entrepreneur is funded by one lender only. Therefore, at date 1 the project of an entrepreneur can be refinanced only by her date 0 lender.[9]

We assume that the date 1 refinancing decision of a lender is non-contractible. We restrict the date 0 (non-renegotiable) contract between a lender and an entrepreneur to a standard debt contract that specifies a loan of I_0 at date 0 and a repayment R at date 2. If at date 2 the project fails, the lender can recover proceeds from the redeployment of the non-depreciated assets up to the agreed repayment R.[10]

The two types of lenders differ only in their ability to recover and/or redeploy the assets of their borrower. Possibly because of lack of earlier experience of the local insolvency practices or secondary market, each newcomer faces a liquidation

[8] The lower bound on ℓ will guarantee that the demand and the supply of assets always cross at a price greater than zero.

[9] We are assuming that at date 1 an entrepreneur cannot use funds saved from the first period. For example, we can think that in the first period the entrepreneur cannot run the project and simultaneously store funds in excess of I_0 borrowed at date 0.

[10] We are implicitly assuming that a lender cannot be repaid with the proceeds that her borrower can obtain from using a liquidated asset, possibly because these proceeds accrue to the entrepreneur too late.

cost proportional to the value of liquidated assets.[11] Let $1-\theta$ be the fraction of liquidated value that is lost by a newcomer with

$$\theta < (1-p_s)/(1-p_r). \tag{6.2}$$

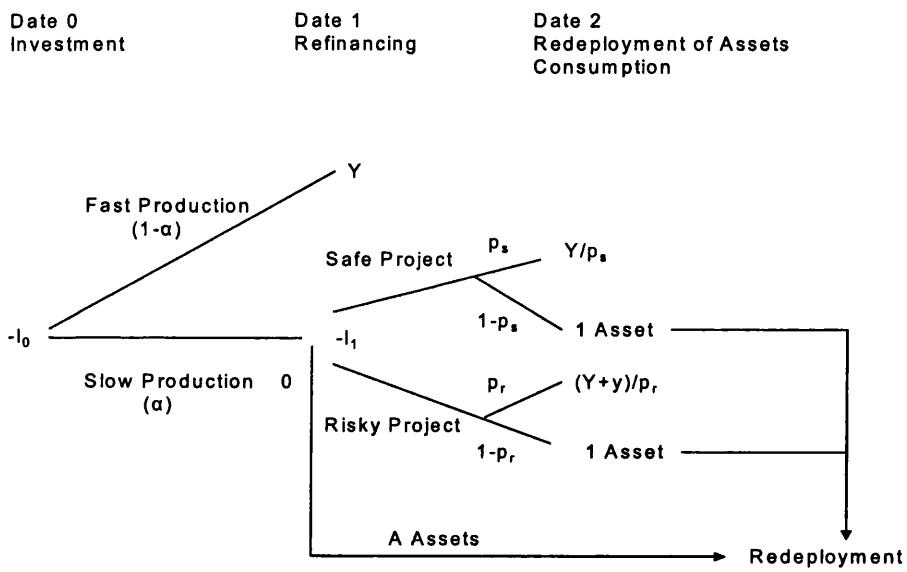

Figure 1: Timing of Events.

We motivate assumption (2) later in the analysis (section 6.1.3). We also assume that an entrepreneur faces a liquidation cost higher than that of a newcomer. This assumption captures the idea that concentrated lenders, such as banks, develop superior skills for reorganizing funded firms and liquidating their assets (Sheard (1989); Habib and Johnsen (1999)).[12] For the results it does not matter whether the liquidation cost is real or a transfer (see also section 6.1.4).

[11] In a similar way, Bencivenga, Smith, and Starr (1995) assume that transaction costs exist in transacting capital in a secondary capital market and that these costs are proportional to the value of traded capital. Liquidation costs in selling assets include commissions, fees or also the time required to arrange a sale or purchase of an asset.

[12] According to Habib and Johnsen (1999) it is reasonable to assume that the entrepreneur lacks the skill even to identify the asset's next best use or to recognize the occurence of the bad states, in which case he risks mantaining it in a suboptimal use.

Secondary Market Lenders can sell the assets recovered from borrowers in a spot secondary market that opens at date 2. We denote with $q \geq 0$ the resale price of an asset in the secondary market. Entrepreneurs can finance the purchase of liquidated assets borrowing in the credit market. We assume that the mass of lenders is such that at date 2 the available funds are more than enough to finance these purchases.

Discussion of the Setup

Our specification of technology and financing is close to that of several studies. We now discuss this specification in detail, relating it to the literature.

Technology The timing of production decisions resembles that proposed by Dewatripont and Maskin (1995). In their economy, long term, profitable projects and long term poor ones populate entrepreneurs' opportunity set and both types of projects must be refinanced at an intermediate stage. Our specification of the technology departs from Dewatripont and Maskin (1995) in two main dimensions: the nature of the projects and the endogenous, general equilibrium determination of liquidation values. The way we generate a downward sloping demand for liquidated assets, i.e. allowing for heterogeneous ability of second-hand users, is also standard. A similar assumption can be found in Gorton and Huang (2004); furthermore, Ramey and Shapiro (2001), Shleifer and Vishny (1992) and Habib and Johnsen (1999) provide several foundations for this assumption.[13] Later in the paper (section 6.1.5), we also show that the heterogeneous ability of second-hand users can be used to endogenize the different liquidation skills of incumbents and newcomers.

[13] According to Ramey and Shapiro (2001, p. 961), "Most capital is specialized by industry, so that used capital typically has greater value inside than outside the industry. Even within an industry, though, capital from one firm may not be a perfect match for another firm". In Shleifer and Vishny (1992), the best alternative users of assets are other entrepreneurs active in the same sector.

Financing On the financing side, we have to discuss two interrelated features: the control exerted by the lender through the interim refinancing and the incompleteness of contracts. A word of caution is due here. This paper has an aggregate focus. Hence, we have chosen a parsimonious specification of these features that effectively conveys our message. In what follows, we show that this specification can be motivated in different ways and that we could obtain the same results borrowing richer, though more complex, specifications from the literature on financial contracts.

The Control of the Lender. Starting by the lender's control, we borrow from a vast literature the idea that firms are frequently locked into their original lenders and cannot address new financiers for the refinancing of projects (Rajan (1992)). In some papers, this feature is justified with firms' informational opaqueness. In other words, new lenders are unwilling to refinance ongoing projects because, unlike the original lenders, they lack information on these projects. We could slightly enrich the model and endogenize such a problem of adverse selection. For example, some of the slow projects could be "rotten" (in the spirit of Dewatripont and Maskin (1995)) and at the refinancing stage only the original lender could be able to discern a rotten project from one with a positive net expected return.

It is also important to stress that in our economy the lender's "refinancing" could be replaced by *any* costly action that the lender must implement to allow continuation of the project. Aghion and Bolton (1992) introduce the analogous assumption that the lender takes an interim, non-contractible action which is critical for the success of the project. This action may consist of voting for the adoption of a production plan if the lender has representatives on the board of the firm, providing strategic advice to the firm, and so forth. For many types of actions, the original lender may be irreplaceable because she has achieved experience and soft knowledge which cannot be transferred to a new financier.

Contractual Incompleteness. It is well known in the literature that the lender's control potentially generates inefficiency if the lender cannot contractually commit to the interim action and it is costly or, in the limit infeasible, to provide her with pecuniary incentives to take this action (Aghion and Bolton (1992); Rajan (1992); Diamond and Rajan (2001a); Dewatripont and Maskin (1995)). In fact, in our economy the interim action (refinancing) is non-contractible. Furthermore, the use of standard debt contracts prevents from specifying a repayment to the lender contingent on the realized output. The use of debt contracts can be motivated by assuming, for example, that only an amount Y of output is verifiable in courts, regardless of the state of nature and of the type of project. Alternatively, there are several institutional rationales for our focus on debt contracts, which we share with a vast literature (e.g. Rajan, (1992)). For example, if we interpret lenders as banks, this focus matches the regulatory restrictions that in several countries prevent banks from holding equity participations in firms. Furthermore, Tornell and Westermann (2003) document firms' widespread use of standard debt contracts in a large number of countries.

Finally, the non-renegotiability of contracts is worth further discussion. In the literature, this is often exogenously motivated with the presence of high renegotiation costs. Even if we allowed contracts to be perfectly renegotiable at the refinancing stage, under the assumption that output is partly non-verifiable, our results would be unaltered. Indeed, in a previous version of this paper we allowed for renegotiability but we imposed a restriction on the amount of verifiable output Y. Alternatively, following Rajan (1992), there is a straightforward, endogenous way to motivate the lack of renegotiability in our economy. Since an entrepreneur is locked into her original lender at the refinancing stage, a problem of hold-up will arise in the ex-post renegotiation of the contract. Precisely, the original lender may exploit her monopoly and extract surplus beyond what is strictly necessary to compensate her for the refinancing of the project. As shown by Rajan (1992), if the lender has sufficiently high bargaining power, expecting this rent-extraction, an entrepreneur may prefer entering a non-renegotiable contract to prevent the hold-up.[14] Clearly, an appealing feature of this approach is that it bundles together the lock-in problem and the non-renegotiability of contracts.

6.1.2 Equilibrium

Entrepreneurs First, we characterize the choice of entrepreneurs at date 0. Each entrepreneur chooses whether to borrow from an incumbent (henceforth denoted by superscript i) or from a newcomer (denoted by n) and whether to implement a safe project (henceforth denoted by subscript s) or a risky one (denoted by r). Let R^i (R^n) stand for the repayment due to an incumbent (a newcomer) at date 2. Let also d_j^i (d_j^n) be an indicator variable $(0, 1)$ for the date 1 refinancing decision of an incumbent (a newcomer) if the slow state is realized and the project is of type j, with $j = s, r$ (when d_j^i (d_j^n) $= 1$ the incumbent (newcomer) refinances). In (3) ((4)), we report the expected return of an entrepreneur from choosing a lender of type t

[14] Ex-ante the lender may be unable to compensate the entrepreneur for the ex-post rent extraction. For example, the expected return from the fast state may fall short of the expected rents extracted in the slow state. Furthermore, without any change in the results, we can think that the entrepreneur must exert an effort which is critical for the continuation of the project in the slow state (as in Rajan, 1992, for example). If the entrepreneur expects the rent extration, she will not exert the effort. Hence, the only way to ensure the continuation of a project will be to enter a non-renegotiable contract.

$(t = i, n)$ and a safe (risky) project[15]

$$(1-\alpha)(Y - R^t) + \alpha d_s^t(Y - p_s R^t), \tag{6.3}$$

$$(1-\alpha)(Y - R^t) + \alpha d_r^t(Y + y - p_r R^t) \qquad t = i, n. \tag{6.4}$$

Lenders Suppose that at date 1 the slow state is realized and the lender has to choose whether to refinance the project. An incumbent will refinance ($d_j^i = 1$) if and only if

$$qA < p_j R^i + (1 - p_j)q - I_1 \qquad j = s, r, \tag{6.5}$$

i.e. if and only if the return from the redeployment of the A non-depreciated assets is lower than the net expected return from refinancing (we assume that if indifferent a lender will not refinance). Note that, all else equal, in equilibrium the agreed repayment to a lender will always exceed q. Hence, the lower the probability of success p_j of a project, the weaker is the incentive of an incumbent to refinance.

Analogously, a newcomer will refinance ($d_j^n = 1$) if and only if

$$\theta qA < p_j R^n + (1 - p_j)\theta q - I_1 \qquad j = s, r, \tag{6.6}$$

where the only difference from the corresponding condition of an incumbent is the lower return that the newcomer obtains from recovering and redeploying the assets of her borrower. It is easy to see that, because of this lower return from the liquidation of assets, for a given type of project, liquidation price q and repayment R, newcomers have a stronger incentive to refinance than incumbents.

[15] Throughout the analysis we focus on the case in which, in case of success, output always exceeds the agreed repayment. It is straightforward to verify that we can always choose values of Y that are consistent with the parameter restrictions and such that the limited liability constraint of the entrepreneur does not bind. Moreover, we focus on the case in which the agreed repayment always exceeds the liquidation value of assets, i.e. $qA < R$. In fact, the maximum value of q is p_s (see below in the analysis) and in Lemma 1 we will introduce restrictions on I_0 such that $p_s A < I_0$. Therefore, necessarily in equilibrium $R > qA$ otherwise the lender would receive a repayment lower than I_0 and would not break even in equilibrium.

At date 0, each lender must expect zero profits at least. The repayment R^i (R^n) to an incumbent (a newcomer) has to satisfy the non-negative profit condition of the incumbent (newcomer) (7) ((8)):

$$\begin{aligned} (1-\alpha)R^i + \alpha\left\{d_j^i\left[p_j R^i + (1-p_j)q - I_1\right] + (1-d_j^i)qA\right\} &\geq I_0, \quad (6.7) \\ (1-\alpha)R^n + \alpha\left\{d_j^n\left[p_j R^n + (1-p_j)\theta q - I_1\right] + (1-d_j^n)\theta qA\right\} &\geq I_0. \quad (6.8) \end{aligned}$$

The only difference between the non-negative profit condition of an incumbent and that of a newcomer is the lower return that a newcomer obtains from the redeployment of assets. It is easy to see that, for a given type of project, liquidation price q, lender's expected return and refinancing decision, borrowing from a newcomer is more costly ($R^n > R^i$) because a newcomer expects a lower return from liquidation if a project fails.

Secondary Market In the secondary market, all the entrepreneurs who can produce more than q with it will demand one liquidated asset. Therefore, the demand for assets is

$$D = \alpha \Pr(\lambda \geq q) = \alpha(\ell - q). \quad (6.9)$$

Assumption (1) guarantees that in equilibrium necessarily all entrepreneurs are funded and implement projects. Then, let y_s be the share of entrepreneurs who choose safe projects.[16] Let also δ_s (δ_r) stand for the share of safe (risky) projects activated at date 0 and in the slow state at date 1 that are refinanced. The supply of assets is

$$S = \alpha\left\{(1-p_s)y_s\delta_s + (1-p_r)(1-y_s)\delta_r + \left[1-\delta_s y_s - \delta_r(1-y_s)\right]A\right\}. \quad (6.10)$$

[16] Since contracts are not contingent on entrepreneurs' choice of projects, y_s defines a threshold specific return such that entrepreneurs with $y < y_s$ choose safe projects.

The supply is given by the share of projects that are in the slow state at date 1 and fail at date 2 times the number of assets liquidated for each project that fails. In turn, using the law of large numbers, the share of projects that fail is given by the share of projects not refinanced $1 - \delta_s y_s - \delta_r(1 - y_s)$ plus the fraction that are refinanced times the share of refinanced projects that fail. Finally, the fraction of safe refinanced projects that fail is $1 - p_s$ while the fraction of risky refinanced projects that fail is $1 - p_r$.

Equilibrium Characterization Let y^i $(1 - y^i)$ be the fraction of entrepreneurs who borrow from incumbents (newcomers). The equilibrium is defined by a vector $(d_s^i, d_s^n, d_r^i, d_r^n, \delta_s, \delta_r, R^i, R^n, q, y_s, y^i)$ such that entrepreneurs and lenders maximize their utility, the non-negative profit conditions of lenders hold and the credit and the secondary market clear. Lemma 1 characterizes the conditions under which in equilibrium incumbents and newcomers coexist in the credit market, making the analysis meaningful.

LEMMA 1: *Suppose that $\underline{\theta} < \theta < \overline{\theta}$ and $I_0 > \underline{I_0}$ (see the Appendix for the values of $\underline{\theta}$, $\overline{\theta}$ and $\underline{I_0}$ as functions of the parameters of the model). In equilibrium a share $y_s = y^i$ of entrepreneurs (with $0 < y_s < 1$) borrow from incumbents and choose safe projects and a share $1 - y_s = 1 - y^i$ borrow from newcomers and choose risky projects. Incumbent and newcomers are always expected to refinance projects ($\delta_s = \delta_r = d_s^i = d_r^n = 1$).*

The intuition behind lemma 1 is as follows. All else equal, the lower liquidation skills of newcomers make their funding more costly because they expect a lower return from asset liquidation if a project fails. However, their lower liquidation skills render newcomers softer in liquidating projects. In fact, because of their strong ability to redeploy the assets of their borrowers, incumbents have high incentive to liquidate projects prematurely rather than refinancing them. In particular, since risky projects have high probability of failure, incumbents must be offered a particularly high repayment R^i to refinance them. Instead, if their disadvantage in liquidating assets is not too small, newcomers will always have the incentive to refinance risky projects, even at the repayment that guarantees them expected zero profits at date 0.

In the analysis, we restrict ourselves to the region of the parameter space identified in lemma 1. The restriction $I_0 > \underline{I_0}$ guarantees that incumbents are willing to refinance safe projects at the repayment that guarantees them expected zero profits at date 0. The restriction $\underline{\theta} < \theta < \overline{\theta}$ guarantees that newcomers are willing to refinance both safe and risky projects at the repayment that guarantees them expected zero profits and that the repayment that induces an incumbent to refinance a risky project exceeds the one that guarantees expected zero profits to a newcomer. In equilibrium, entrepreneurs who derive a high return y from a risky project borrow from newcomers, sustaining the associated extra cost of funding, and choose high-risk/return projects; entrepreneurs who derive a low return from a risky project borrow from incumbents and choose safe projects, saving on funding costs. Henceforth, we define equivalently with y_s $(1 - y_s)$ the share of entrepreneurs who choose safe (risky) projects and who borrow from incumbents (newcomers). In the proof of lemma 1 we report the equilibrium values of y_s, q, R^i and R^n.

6.1.3 Impact of a Shock

We now show that shocks to asset liquidity can originate waves of entry of newcomers that mutually interact with increases in project riskiness through liquidation values. In the next subsection, we analyze output implications of this interaction. We consider an exogenous shock to asset liquidity in the form of a shock to the number of potential second-hand users of assets. We assume that the shock occurs at the beginning of time, before contracts are written. Henceforth, when we refer to the effects of this shock and to changes in the variables, we implicitly compare the equilibrium that is realized in our economy with the one that would be realized in the absence of the shock.

The shock to the number of potential second-hand users proxies for any aggregate shock that, by modifying firms' possibility to buy assets in the secondary market, affects the liquidation value of assets exogenously. It could be a change of the cash flow of potential users due to a boom or recession or a change in government regulation that affects the number of potential second-hand users directly, like a change in antitrust policies and in limitations of foreign investment (Shleifer and Vishny (1992)).

We consider a negative shock and we assume that a fraction of the most efficient users of liquidated assets exit the secondary market. Formally, we assume that ℓ falls so that the demand for assets shifts inward in a parallel way. Proposition 1 presents the first result of the paper.

PROPOSITION 1: *A fall in the demand for firms' assets increases the share of entrepreneurs borrowing from newcomers and implementing risky projects and decreases the liquidation value of assets. These effects are bigger the stronger is the disadvantage of newcomers in liquidating assets (i.e. the lower is θ).*

The intuition behind proposition 1 is as follows. Following a shrink in the demand for firms' assets, in the secondary market supply exceeds demand and the liquidation price of assets falls. Newcomers face a liquidation cost proportional to the value of the assets they liquidate so that the price fall erodes their expected return from liquidation less than that of incumbents and the gap $R^n - R^i$ narrows. By eroding the cost advantage of incumbents, the price fall pushes some entrepreneurs to switch to newcomers. Since entrepreneurs choose risky projects when borrowing from newcomers, the share of risky projects, and hence the expected default rate and supply of assets, rise. In turn, this induces a further fall of liquidation values and erosion of the cost advantage of incumbents and so forth, until a new equilibrium is reached. The magnitude of these effects is inversely related to θ. The erosion in the cost advantage of incumbents due to falls in liquidation values is bigger the lower is θ. Therefore, the overall increase in the share of entrepreneurs who choose newcomers and risky projects and decrease in the liquidation value of assets is inversely related to θ. In sum, the result in proposition 1 incorporates a mutually reinforcing interaction between the entry of lenders foreign to the market and the share of risky projects, with the link being firms' liquidation values.[17]

Assumption (2) on θ guarantees that declines in liquidation values resulting from increases in the share of risky projects encourage further entrepreneurs to switch to newcomers and risky projects. In fact, a standard, opposite force makes risky projects less appealing as liquidation values fall. Since a risky project fails and leads to the liquidation of the residual asset with higher probability than a safe one, its expected return is eroded by a fall in the liquidation price more than that of a safe project. This conventional effect is unrelated to differences across lenders and would also operate in an economy populated only by incumbents. The condition $\theta < (1 - p_s)/(1 - p_r)$ ensures that the erosion of the cost advantage of incumbents deriving from their higher liquidation skills overwhelms this standard effect and declines in the liquidation price of assets spur the share of risky projects.

6.1.4 Output

We now analyze the output implications of the model. Consistent with proposition 1, we consider a negative shock to asset liquidity. The decline in the return from liquidated assets due to the fall in the average productivity of second-hand users has

[17]Note that in the model the entry of new lenders is associated with a rise in loan rates. This could appear at odds with the popular view that the entry of new lenders tends to reduce loan rates, mainly by favoring competition. However, what is crucial for our results is that the spread between the loan rates charged by new lenders and those charged by incumbents narrows and *not* that loan rates increase. Possibly, one could think of a richer framework in which the entry of new lenders leads to a decline in loan rates but the mechanism in the model is fully operational.

a *direct* negative impact on output. In this subsection, we investigate the *indirect* impact of the shock on output due to the change in the share y_s of safe projects and how this impact depends on the disadvantage of newcomers in liquidating assets. We also analyze how the share of safe projects changes relative to the share of safe projects that would be chosen by a social planner maximizing total output.

In comparing the composition of projects in the decentralized equilibrium with the one that would be chosen by a social planner we do not need to take into account the nature of the liquidation cost. The liquidation cost can be a real resource loss, like a dead-weight output loss due to the sale of the asset to an inefficient second-hand user. Alternatively, the liquidation cost can be a transfer, like a fee a newcomer pays to an efficient liquidator of the asset, i.e. someone facing no liquidation cost.[18] The social planner's optimal y_s will be independent of whether the liquidation cost is real or a transfer. In fact, if the liquidation cost is real, the social planner will be able to induce incumbents to fund risky projects engaging in transfers among agents.[19] Therefore, newcomers will not participate in the credit market at date 0 and their real liquidation cost will not affect output to be maximized.

In proposition 2 we compare the share of risky projects in our economy with the one that would be chosen by a social planner.

PROPOSITION 2: *Regardless of whether the liquidation cost of newcomers is a real cost or a transfer, the share of risky projects is lower than the optimal share (equal to 1) that maximizes total output.*

[18]The fees collected could be rebated to agents as a lump sum, hence not affecting their decisions. In a model with transaction costs in trading capital in the secondary market, Bencivenga, Smith and Starr (1996) distinguish between transaction costs that are real resource costs and transaction costs that are pure transfers (such as fees or rents to brokers or market makers). In a different context, Diamond and Rajan (2001a) distinguish between the cases in which a new lender can and cannot address an old lender familiar with the assets of the firm for liquidation. In their context, the entrepreneur herself is an efficient liquidator, so that the lower liquidation ability of new lenders never results in a social loss.

[19]For example, the planner could tax all the lenders who liquidate more than one asset in the secondary market discouraging the premature liquidation of projects.

The intuition behind proposition 2 is as follows. The combination of contractual incompleteness and high liquidation skills of incumbents render the latter too conservative towards high-risk/return projects. Entrepreneurs can implement these projects by borrowing from newcomers but newcomers are only an imperfect substitute for incumbents. In fact, newcomers impose on entrepreneurs the cost of their low liquidation skills, discouraging some entrepreneurs from borrowing from them and implementing risky projects.[20]Therefore, the liquidation cost of newcomers distorts the share of risky projects downward.

Proposition 2 implies that, regardless of the nature of the liquidation cost, the increase in the share of risky projects that follows a negative liquidity shock increases output and that the stronger is the described mechanism (the lower is θ) the bigger the output rise is. In fact, when a negative shock to asset liquidity occurs and the price of assets falls, the liquidation cost of newcomers falls with it and more entrepreneurs choose newcomers and risky projects. Therefore, the share of risky projects approaches the social optimum.

COROLLARY 1: *The increase in the share of risky projects $1-y_s$ induced by a fall in the demand for firms' assets increases output, whether the liquidation cost of newcomers is a real cost or a transfer. This output rise is inversely related to θ. After the shock the share of risky projects is closer to the optimal one.*

Generalizing it to a positive shock, corollary 1 shows that the interaction between

[20]The liquidation cost has also an indirect distortionary effect. Consider the following thought experiment. Assume that the cost is real and that necessarily incumbents (newcomers) fund safe (risky) projects even in the centralized equilibrium. It can be shown that the share of risky projects in the decentralized equilibrium is lower than the optimal choice of the social planner. Two externalities are at work. On the one hand, entrepreneurs tend to choose risky projects too often because they take the average productivity of liquidated assets as given. Therefore, they do not internalize the bigger reduction in the average productivity of liquidated assets that occurs when they choose risky projects. On the other hand, entrepreneurs do not fully internalize the social return of a liquidated asset since they care only about its resale price q. This makes them choose safe projects too often. When θ is equal to one the two externalities offset each other and the share of risky projects is the optimal one. When θ is below one the latter externality is stronger and the share of risky projects is too low.

the entry of newcomers and the share of high-risk/return projects generates a change in output opposite in sign to the direct effect of a liquidity shock. Put differently, this interaction acts as a stabilizer when shocks to asset liquidity are realized. An immediate policy implication is that not only binding regulatory restrictions to the entry of new (foreign) lenders reduce output by further depressing the share of high-risk/return projects (proposition 2),[21] but also they increase output volatility.

6.1.5 Endogenous Liquidation Skills

The key assumption of the model is the lower ability of newcomers to extract value from the assets of bankrupt firms. The advantage of incumbents in liquidating assets can be motivated in several ways. However, the model offers a straightforward way to endogenize this advantage. Since second-hand users differ in their efficiency in producing with the assets of bankrupt firms, an incumbent could find easier to identify the most efficient users of these assets. Ramey and Shapiro (2001, p. 961) stress the importance of search costs in the redeployment of assets and argue that "Thin markets and costly search complicate the process of finding buyers whose needs best match the capital's characteristics. The cost of search includes not only monetary costs, but also the time it takes to find good matches within the industry". We now analyze a simple way to endogenize the disadvantage of newcomers in liquidating assets along this intuition.

Suppose that the secondary market for firms' assets is segmented in two islands: one island is populated by highly productive users ($\lambda > \widehat{\lambda}$) while the other is populated by low productive users ($\lambda \leq \widehat{\lambda}$). At date 2, lenders have to decide in which island to sell the recovered assets. The choice of island is reversible but, once an

[21]The inefficiency in our economy can be compared to that in Dewatripont and Maskin (1995). In a context in which lenders cannot distinguish between long term, very profitable projects and long term poor ones, borrowing from multiple lenders dilutes the incentive of lenders to gather information for the continuation of projects. This leads to the liquidation of very profitable projects and, therefore, deters their implementation ex-ante. Both the rationale for the inefficiency in our economy and the type of the projects deterred differ from Dewatripont and Maskin (1995).

island has been chosen, switching to the other has a cost of $2(1-\theta)$ times the value of the assets to be sold. For example, we can think that a "shipper" offers transportation between the two islands and the switching cost is the fee charged by the shipper.[22] Assume also that in the two islands the sale of assets takes place after any switching has occurred. Finally, while ex-ante incumbents know the type of users who populate each island, newcomers cannot observe it until they have chosen island. Therefore, with probability 1/2 newcomers will initially select the island populated by low productive users. In particular, assume that newcomers have chosen the island populated by low productive users. Provided the difference between the liquidation price in the two islands exceeds the cost of switching island, newcomers will always switch and sell the recovered assets in the island populated by highly productive users. Denoting with q the liquidation price in the island with highly productive users, a sufficient condition for the switching to occur is $q - \widehat{\lambda} > 2(1-\theta)q$.[23] At date 0 the expected date 2 liquidation cost faced by a newcomer in redeploying the residual asset of her borrower will be $(1-\theta)q$.

6.2 Applications and Empirical Assessment

In section 6.2.1 we discuss an alternative application of the model in which, rather than focussing on cyclical changes in asset liquidity, we reinterpret our results in the light of cross-sectional differences in asset liquidity. In sections 6.2.2 and 6.2.3 we discuss limitations of the model. Finally, in 6.2.4 we assess the empirical predictions of the model, comparing them with the existing evidence.

[22] The shipper could be a metaphor for any middle-man who helps inexperienced newcomers to find the best users of assets in the secondary market.

[23] Clearly, this condition is meaningful only if $\theta > 1/2$.

6.2.1 Liquid and Illiquid Markets

In sections 6.1.3 and 6.1.4 we have considered cyclical changes in the liquidity of assets. Besides changing over time, asset liquidity differs across sectors/economies (Shleifer and Vishny (1992)). Therefore, our results could be reinterpreted in terms of cross-sectional differences in the presence of new (foreign) lenders and in the riskiness of projects. In particular, proposition 1 predicts that, if the liquidation skills of newcomers are not too high, environments with low asset liquidity will feature a higher share of newcomers and high-risk/return projects and that cross-sectional differences will be stronger the bigger is the liquidation disadvantage of newcomers. This result could explain why, from the mid-1980's to the 1997-98 crisis, East Asian countries experienced both an increasing illiquidity of projects and a strong penetration of foreign lending. Focussing on short term, foreign lending intermediated by domestic banks, Diamond and Rajan (2001b) argue that this allowed domestic banks to commit their superior liquidation skills and fund highly productive but illiquid projects. Focussing on direct foreign lending, our model offers a different explanation based on the increasing productivity and riskiness of these projects. This would have endogenously depressed expected liquidation values, increasing the incentive of firms to forego the liquidation skills of domestic lenders for the willingness of foreign lenders to fund high-risk/return projects. Interestingly, our explanation can complement that put forward by Diamond and Rajan (2001b). In fact, while their argument appears especially relevant for those East Asian countries in which foreign lending was mainly intermediated by domestic banks, our argument would apply to those countries, such as Indonesia, in which the inflow of foreign funds was more direct than intermediated by domestic lenders.

6.2.2 Firm and Sector Specificity

The core assumption that drives our results is the higher ability of incumbents to recover and/or liquidate the assets of borrowers. As shown in section 6.1.5, when the second-hand users of assets have heterogeneous efficiency in using liquidated assets, this higher ability can be interpreted as incumbents' higher ability to identify the best users in the secondary market. This interpretation will be reasonable if the specificity of assets is at the firm level, i.e. if the firms *within* the sector/region familiar to incumbents feature heterogenous efficiency in using liquidated assets. Conversely, this interpretation will be less relevant if the heterogeneous ability to use assets is at the sectorial level, i.e. firms within the sector/region familiar to incumbents have the same ability but firms *outside* the sector have lower ability to use liquidated assets. In this case, incumbents' knowledge of sector-insiders will probably give them little advantage over newcomers in redeploying assets.

6.2.3 Emerging and Industrialized Economies

The model can be applied to different contexts. For example, we have shown that it can help to explain the evolution of projects in East Asian countries during the second half of the 1980's and the 1990's. Obviously, other mechanisms of interaction between foreign lenders and incumbent economic actors can be at work. Thus, the question becomes: in what contexts do we expect the described mechanism to be especially important relative to other possible mechanisms of interaction? If we interpret newcomers as foreign lenders, we believe that the described mechanism can be relatively more important for cross-border lending across industrialized economies. Lenders of an industrialized country that operate in an emerging economy may have higher efficiency and monitoring ability than local lenders. For example, Dages, Goldberg and Kinney (2000) find that, in the second half of the nineties, by exploiting their high monitoring ability, in Mexico and in Argentina foreign banks engaged in cherry-picking activities, leaving less lucrative, riskier customers to domestic lenders.[24] Furthermore, it is sometimes argued that entrant banks from industrialized economies can rely on cheaper sources of funds than local banks, typically because of a reputational advantage in the international financial markets (Dages, Goldberg and Kinney (2000)). This will be especially likely if entrant banks are subsidiaries of well-established foreign banks. In the model, instead, we have ruled out any heterogeneity in lenders' monitoring ability or funding opportunities, restricting lenders' heterogeneity to a different knowledge of the local market. For this reason, we believe that cross-border lending across industrialized countries constitutes the best application of our setup. In fact, in this case there is no reason to expect that foreign lenders have an intrinsic advantage or disadvantage relative to incumbents.

6.2.4 Empirical Assessment

We now assess the empirical predictions of the model, comparing them with those of related studies and with the existing evidence. We classify predictions according to whether they refer to the impact of the share of foreign lending on the fundamentals of the economy (riskiness of projects, firms' liquidation values and output) or on the feedback effect.

i) *Impact of the share of foreign lending on fundamentals.* The model predicts that the diffusion of new (foreign) lenders increases the riskiness of projects and leads to a decline in firms' liquidation values. While the first implication can be obtained in a model of adverse selection (Dell'Ariccia et al. (1999)), existing models do not offer predictions on the pattern of liquidation values at the time of new lenders' entry.

[24] In general, the authors argue that the overall effect of the higher efficiency of foreign lenders is ambiguous. On the one hand, the efficiency of foreign banks can have a positive spill-over effect on domestic lenders. On the other hand, by cherry picking the most lucrative customers, foreign banks can harm domestic institutions.

ii) *Impact of fundamentals on the share of foreign lending.* The model predicts that increases in the riskiness of projects foster the diffusion of new lenders. As argued in the Introduction, assuming that new lenders are exposed to more severe adverse selection than incumbent ones would imply that increases in the riskiness of projects deter the entry new lenders (Dell'Ariccia et al. (1999)). The model also predicts that declines in the liquidity and liquidation values of assets foster the diffusion of new lenders. To the extent that asset liquidity is procyclical,[25] the model implies therefore a countercyclical pattern of the relative share of foreign lending. Finally, the model predicts that, at least if we limit ourselves to shocks to asset liquidity, economies with less restrictions to the entry of foreign lenders experience lower output volatility.

The empirical literature on the pattern of foreign and domestic lending and, more generally, on the entry into a credit market is at its early stages. Dages, Goldberg and Kinney (2000) argue that, while a sizable body of literature has explored the potential effects of financial liberalization broadly meant, few studies have analyzed the effects of an increased foreign participation in banking and finance. Furthermore, these studies have generally focused on the impact of entry on the efficiency and competitiveness of the local credit market, neglecting the lending behavior of foreign and domestic financiers, its (possibly different) response to cyclical changes in fundamentals and its feedback on fundamentals. Dages, Goldberg and Kinney (2000), Goldberg (2002) and Morgan, Rime and Strahan (2003) are among the exceptions we are aware of. Dages, Goldberg and Kinney (2000) find that in Mexico and in Argentina in the late nineties the relative share of foreign lending exhibited an anticyclical pattern. Goldberg (2002) finds that the lending of US banks to industrialized countries does not respond significantly to cyclical changes in the fundamentals of the host country, less so than the lending of domestic banks. Fi-

[25]For example, during a recession the cash flow of potential users can be lower, leading to a contraction in the demand for assets.

nally, Morgan, Rime and Strahan (2003) focus on interstate branching in the United States and find a negative correlation between out-of-state bank share and within state business volatility. All these results are consistent with the predictions of the model.

The existing evidence is therefore broadly supportive of the model findings. However, this evidence is still too scarce to draw conclusions. In particular, the above studies do not allow to disentangle the specific contribution of different mechanisms of interaction between foreign lenders and incumbent economic actors.[26] In a study in progress based on data on US banks, we aim at assessing the contribution of our mechanism and of alternative ones to the observed pattern of international banking flows.

6.3 Conclusion

In this chapter, we have investigated the interaction between the entry of lenders "foreign" to a market, the nature of projects and firms' liquidation values. In our economy, the combination of contractual incompleteness and high liquidation skills of incumbent lenders render the latter conservative towards high-risk/return projects. Possibly because of lack of knowledge of the local insolvency practices or secondary market, new (foreign) lenders exhibit a disadvantage in recovering and liquidating the assets of borrowers relative to incumbents. This implies that foreign lenders have a softer budget constraint towards high-risk/return projects and specialize in their financing, leaving low return, safe borrowers to incumbents.

When a negative shock to asset liquidity is realized, the fall in firms' liquidation values erodes the comparative cost advantage of incumbents deriving from their higher liquidation skills, pushing more firms to choose foreign lenders and high-

[26] Dages, Goldberg and Kinney (2000) conceptually discuss possible explanations for the pattern of the share of foreign lending based on the different efficiency and funding opportunities of foreign and domestic lenders.

risk/return projects. The fall in liquidation values due to the increased riskiness of projects further erodes the cost advantage of incumbents and so forth. This interaction acts as a stabilizer, dampening the output impact of liquidity shocks, such as those occurring in booms and recessions.

In the wake of the financial crises that have recently hit liberalized countries, such as the Nordic countries and the East Asian countries, widespread concerns have developed on the possibly destabilizing effects of financial liberalization. The model suggests instead a channel through which an economy more open to the direct presence of (new) foreign financial institutions enjoys greater stability.

6.4 Appendix

Proof of Lemma 1: The values of $\underline{I_0}$, $\underline{\theta}$, and $\overline{\theta}$ are respectively

$$\underline{I_0} = \frac{(1-\alpha)I_1}{p_s} + (1-\alpha+\alpha p_s)(A-1) + p_s; \tag{A1}$$

$$\underline{\theta} = \frac{I_0 - I_1(1+\alpha p_r) - (\underline{\ell} - 1 + p_r)(A - 1 + p_r)(1-\alpha+\alpha p_r)}{\alpha(1-p_r)(\underline{\ell} - 1 + p_r)}; \tag{A2}$$

$$\overline{\theta} = \frac{p_r I_0 - (1-\alpha)I_1}{(1-\alpha+\alpha p_r)(p_s A - 1 + p_r) + p_r \alpha(1-p_r)p_s}. \tag{A3}$$

It is easily verified that there exist regions of the parameter space such that the interval $[\underline{\theta}, \overline{\theta}]$ is non-empty and the restriction $I_0 > \underline{I_0}$ and assumption (2) hold. We show first that it is possible to write a contract with an incumbent that guarantees her zero profits and gives her incentives to continue a safe project at date 1 ($d_s^i = 1$). Using (5) and (7), for this to be true it has to be that $[I_0 + \alpha I_1 - \alpha(1-p_s)q]/(1-\alpha+\alpha p_s) > [qA + I_1 - (1-p_s)q]/p_s$. The highest possible value of q is p_s. The lower bound on I_0 guarantees that the inequality is always verified. Observe also that, all else equal, the gross repayment that guarantees zero profits to an incumbent is always lower than the one that guarantees zero profits to a newcomer. This implies that for a borrower the choice of a safe project financed by a newcomer is necessarily dominated. Moreover, it will never be preferable to write a contract with a lender that leads her to liquidate a project ($\delta_s = \delta_r = 1$). In fact, it is straightforward that this would be dominated by a contract with an incumbent that leads her to continue a safe project and guarantees her zero profits. We now show that, if $\theta < \overline{\theta}$, at the repayment that guarantees her expected zero profits, a newcomer has always the incentive to continue a project, whether safe or risky ($d_s^n = d_r^n = 1$). In fact, using (6), to have $d_j^n = 1$, it has to be that $\theta q A + I_1 < p_j R^n + (1-p_j)\theta q$. Using (8), in an equilibrium in which a newcomer continues a project and realizes zero profits, $R^n = [I_0 + \alpha I_1 - \alpha(1-p_j)\theta q]/(1-\alpha+\alpha p_j)$. Moreover, the maximum value of q is p_s. Plugging these values into the inequality above, we have that if $\theta < \overline{\theta}$ it will be satisfied. We now show that if, in addition, $\theta > \underline{\theta}$, an entrepreneur will never choose an incumbent to finance a risky project. In fact, using (5), the minimum repayment that would give an incumbent the incentive to refinance a risky project is $[qA + I_1 - (1-p_r)q]/p_r$ that, from the restriction $\theta > \underline{\theta}$, certainly exceeds the repayment that guarantees zero profits to a newcomer under a risky project for any feasible value of q. All the above implies $y_s = y^i$. We now show that $0 < y_s < 1$. Considering the zero profit conditions of a newcomer funding a risky project and of an incumbent funding a safe project, we obtain respectively:

$$R^n = \frac{I_0 + \alpha I_1 - \alpha(1-p_r)\theta q}{1 - \alpha + \alpha p_r} \tag{A4}$$

$$R^i = \frac{I_0 + \alpha I_1 - \alpha(1-p_s)q}{1 - \alpha + \alpha p_s} \tag{A5}$$

Substituting $\delta_s = \delta_r = 1$ into the right hand side of (10) and using (9) and (10), we obtain the price q that equates demand and supply in the secondary market as a function of the share of safe projects y_s. Analogously, equalizing (3) and (4), we obtain the share of safe projects y_s as a function of q, R^n and R^i. The values of q and y_s are respectively:

$$q = (p_s - p_r)y_s + \ell - 1 + p_r \tag{A6}$$

$$y_s = \frac{1}{\alpha}\left\{[(1-\alpha) + \alpha p_r]\, R^n - [(1-\alpha) + \alpha p_s]\, R^i\right\}. \tag{A7}$$

Substituting the values of R^n and R^i from (A4) and (A5) into the right hand side of (A7) and solving the system (A6) and (A7), we obtain:

$$y_s = \frac{[1 - p_s - (1-p_r)\theta]\,(\ell - 1 + p_r)}{1 - [(p_s - p_r)\,[1 - p_s - (1-p_r)\theta]]}. \tag{A8}$$

It is straightforward that the right hand side of (A8) is always positive and smaller than 1.

Proof of Proposition 1: To prove the proposition it is sufficient to show that y_s decreases when ℓ decreases (negative demand shock) and the decrease is stronger the lower is θ. Computing the derivative of y_s with respect to ℓ, we obtain

$$\frac{\partial y_s}{\partial \ell} = \frac{[(1-p_s)-(1-p_r)\theta]}{1-(p_s-p_r)\left[(1-p_s)-(1-p_r)\theta\right]}, \tag{A9}$$

which is positive if $\theta < (1-p_s)/(1-p_r)$. In turn, differentiating it with respect to θ we obtain

$$\frac{\partial}{\partial \theta}\left(\frac{\partial y_s}{\partial \ell}\right) = -\frac{(1-p_r)\left\{1-2(p_s-p_r)\left[(1-p_s)-(1-p_r)\theta\right]\right\}}{\left\{1-(p_s-p_r)\left[(1-p_s)-(1-p_r)\theta\right]\right\}^2}. \tag{A10}$$

Observe that $1-2(p_s-p_r)\left((1-p_s)-(1-p_r)\theta\right) > 0$ since the maximum of $2(p_s - p_r)\left((1-p_s)-(1-p_r)\theta\right)$ is reached at $\theta = 0,\ p_r = 0,\ p_s = 1/2$ where the expression is equal to $1/2$. This implies that $\partial\left(\partial y_s/\partial \ell\right)/\partial\theta$ is negative.

Proof of Proposition 2: A social planner would choose the optimal share of safe projects in order to maximize

$$\widehat{Y}(y_s) = Y - I_0 + \alpha \left\{ \begin{array}{c} -I_1 + y_s\left[\frac{(1-p_s)(\ell+q)}{2}\right] + \\ +(1-y_s)\left[\frac{(1-p_r)(\ell+q)}{2}\right] + \frac{(1-y_s)(1+y_s)}{2} \end{array} \right\}. \tag{A11}$$

On the right hand side of (A11), the terms in the two square brackets are the average productivity of a liquidated asset $(\ell+q)/2$ times the probability that respectively a safe and risky project fails. The last term on the right hand side is the total specific returns enjoyed by entrepreneurs who choose risky projects. Let us plug $q = (p_s - p_r)y_s + \ell - 1 + p_r$ into $\widehat{Y}(y_s)$. Define the share of safe projects that maximizes $\widehat{Y}(y_s)$ as $y_{s,\widehat{Y}}$. We need to show that $y_{s,\widehat{Y}} < y_s$. Solving for $y_{s,\widehat{Y}}$, we obtain

$$y_{s,\widehat{Y}} = \frac{(p_r - p_s)(\ell - 1 + p_r)}{1 + (p_s - p_r)^2} < 0 \tag{A12}$$

This implies that the share of safe projects chosen by the social planner is $y_{s,\widehat{Y}} = 0 < y_s$.

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