Interactions Between Inflation, Monetary and Fiscal Policy

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Abstract

In Chapter 1 I develop a simple model, that builds upon some previous work on financial innovation processes, to account for the main stylized facts observed during extreme hyperinflations. The modeling device used here helps to reconcile some conflicting views on the causes, development and end of a hyperinflation without departing from the rational expectations assumption. Specifically, it is shown that the effectiveness of a future orthodox reform to preclude the occurrence of a hyperinflation, either speculative or fundamental, is an endogenous outcome which depends on a wide array of policy choices (fiscal and monetary) and structural features of the economy.

In Chapter 2 I examine the postulates of the Fiscal Theory of the Price Level (FTPL) under an interest rate peg. I show that the usual definition of a non-Ricardian plan involves a non-credible government policy commitment, thus confuting the interpretation of the FTPL as a policy-based equilibrium selection device. Then I identify the set of necessary conditions for the implementation of non-Ricardian fiscal plans that result in a unique equilibrium. A critical necessary condition for the credibility of a fiscalist plan is that the equilibrium level of seigniorage must be non-positive. I argue that the fiscalist stock-analogy is only meaningful, precisely, when money enters into the government constraint as a destination of funds, rather than as a source.

In Chapter 3, I extend Buiter's (2001, 2002) criticism (non-Ricardian plans are generally non-implementable whenever the monetary authority sets a non-contingent sequence of money supplies) to an infinite horizon economy. In particular, it is shown that a fiscalist deflation involves the violation of a household's optimality condition and that the notion of a fiscally-induced speculative hyperinflation cannot be rationalized invoking a symmetry between the problem of pricing a potentially fiat non-convertible asset (like money) and that of pricing the stock of a private firm as advocated by the FTPL.

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Chapter 1
A Model of Hyperinflations with Rational Agents and Orthodox Reforms

1.1 Introduction

The general objective of this paper is to provide a theoretical explanation for the main stylized facts observed during hyperinflationary processes, as those suffered by some European, Latin-American and transition countries in the 1920's, 80's and 90's, respectively. These episodes often display a common set of facts, yet much of the previous related literature has followed a partial approach, concentrating on some of these facts but, critically, neglecting others at the same time, as stressed by Bental and Eckstein (1990) and, more recently, by Marcet and Nicolini (2003). Indeed, some of the most influential "partial stories", when combined together, yield a picture that does not resemble what we actually observe in real economies. A central contribution aimed here is to present a simple rational-expectations general equilibrium model that potentially accommodates three of the most wide-spread partial approaches (the *fiscal root*, the *bubble explanation* and the *orthodox reform*, briefly described below), making them consistent with each other and with the empirical facts.

First, weak fiscal conditions that force the government to finance a significant share of the budget via money injections (seigniorage) are usually thought to be an important factor behind hyperinflations (the *fiscal root*). In a general sense, this view is not refuted by the empirical evidence since, on the one hand, explosive hyperinflations often take place in countries in which seigniorage is relatively high and, on the other, a drastic reduction in this source of funding is observed after every successful stabilization. For example, regarding the fiscal stance of some European countries in the 1920's, Sargent (1986; p. 45) observes that "*the governments of these countries (Hungary, Austria, Poland, and Germany) resorted to the printing of new unbacked money to finance government deficits.*

This was done on such a scale that it led to a depreciation of the currencies of spectacular proportions." (parentheses added)[1]. Although the exact intertemporal mechanisms linking inflation and seigniorage are likely to be rather complex, as shown by Sargent and Wallace (1981), the traditional proposition that associates high inflation to a high degree of fiscal dominance in the implementation of monetary polices remains basically uncontested[2].

Second, for some students of these episodes, the extremely high velocity taken by the inflation rate over the course of a hyperinflation, without a noticeable change in the volume of seigniorage, suggests that these processes may well be driven just by expectations that, in equilibrium, are self-confirmed (the *bubble explanation*). This approach has received a formal treatment, under the rational expectations paradigm, among others, by Sargent and Wallace (1987), and Bruno and Fischer (1990). Those authors assume a demand for real balances *à la* Cagan[3], which implies the existence of a hump-shaped inflation-tax Laffer curve with two stationary inflation rates (one "low" and one "high") associated to a unique volume of seigniorage, and interpret a hyperinflation as the economy moving from the vicinity of the low inflation rate to the high one. According to this view, seigniorage is not the sole explanation for high inflation, instead, private expectations are thought to be the prime cause. However, in explaining the occurrence of a hyperinflation as a bubble equilibrium along a standard Laffer curve, the degree of fiscal pressure does no play any significant role[4]. Still, such a possibility has also attracted much attention in the empirical literature[5]. For example, Imrohoroglu (1993), provides empirical support for Sargent and

[1]Sachs (1987) and Kiguel (1995) also point towards high levels of seigniorage as a prime cause in the hyperinflationary episodes in Bolivia and Argentina, respectively, in the 1980's. Petrovic and Bogetic (1999) stress the massive resort to seigniorage as a key determinant of the violent hyperinflation suffered by Yusgoslavia in the early 1990's. Reinhart and Savastano (2003) also share this view and provide some suggestive evidence on the size of the fiscal adjustments after the end of several recent hyperinflations.

[2]Yet there are some divergent views on this issue. For example, Loyo (2001) provides an explanation for the Brazilian several hyperinflationary bursts in the 1980's and 90's and assumes that the volume of seigniorage is negligible. Also, in the economy studied by Bental and Eckstein (1990), an *upper* bound on seigniorage is a necessary condition for the occurrence of the hyperinflation.

[3]Much of the work undertaken in the hyperinflation literature, both theoretical and empirical, has been heavily influenced by the seminal work of Cagan (1956). In particular, Cagan assumes that the only changing argument in the demand for money is the expected rate of inflation, as a measure of the cost of holding money, an artifact intended to capture the fact that variations in other arguments of that function (e.g. the level of income and consumption, the real interest rate, etc.) are overwhelmed by the huge changes in the inflation rate over a hyperinflation.

[4]More on the contrary, as the economy moves towards the decreasing arm of the Laffer curve, higher levels of seigniorage will result in lower inflation rates.

[5]Since, at least, the influential work of Flood and Garber (1980), a huge number of authors have tried to detect bubble-components using data from hyperinflationary economies. The results are mixed and the comparison across different studies is not always easy, as there is a wide heterogeneity in the specification of the money demand function, the money supply process, etc. Also the reduced length of the series and the quality of the available data pose important difficulties for the interpretation of those econometric tests. Some recent contributions on this area include those by Blackburn and Sola (1996), Engsted (2003)

Wallace's multiple-equilibria model using data from the German hyperinflation.

Third, for a wide class of rational-expectations economic models, including those in the tradition of Cagan mentioned above, Obstfeld and Rogoff (1983) and Nicolini (1996) have shown that the prospect of a fiscal-monetary reform aimed at stabilizing the real value of the currency (the *orthodox reform*) is itself a sufficient condition for precluding the occurrence of a speculative hyperinflation, thus making it difficult to reconcile the idea of a hyperinflation as a rational-bubble equilibrium. Moreover, once the *possibility* of a reform is introduced in a model with rational expectations and a Cagan-style demand for money, what one gets is that higher levels of seigniorage are uniformly associated to more severe *disinflationary* processes. As pointed out by Marcet and Nicolini (2003), if anything is unambiguously true about hyperinflations is that they are always stopped. There is little doubt either that this is possible because of the government's determination to reform the fiscal-monetary mix, i.e. to implement "*deliberate and drastic fiscal and monetary measures taken to end the hyperinflations*" (Sargent (1986; p. 44)).

Clearly, when combined together, these three *partial* views do not fit each other. Yet, there have been a relatively few number of attempts to provide a *general* theoretical explanation for hyperinflations, including their causes, dynamics and end. To the best of my knowledge, most of them consider models in which the preventive role of an eventual reform, as the one outlined above, is not explicitly considered[6]. Marcet and Nicolini (2003) consider a baseline model similar to the one studied by Sargent and Wallace (1987) but assume that individuals follow a backward-looking rule for forming expectations (*quasi-rational learning*). They show that, under some conditions on the learning rule, the model is able to match most of the stylized facts. However, as individuals do not internalize the possibility of a future change in the policy-regime when forming their expectations, the prospect of a reform is itself irrelevant for the dynamics of the model. Using the same basic Cagan-type model, Kiguel (1989)[7] argues that a volume of seigniorage above the maximum one in a steady-state inflation-tax Laffer curve will unchain an explosive hyperinflation if the money market does not clear instantaneously. Although he explicitly considers the

and Hooker (2000 and 2003).

[6]Drazen and Helpman (1990) provide an exception, for they explicitly model the effects of anticipated change of regime. However, in order for their model to generate a hyperinflationary path, an upward seigniorage-path is required, something that, as discussed below, is not a common feature in hyperinflationary episodes (see Bental and Eckstein (1990) for a discussion of this point).

[7]Romer (2001) presents a simplified version of this model.

implementation of a reform as a device to stop the hyperinflation, the expectation of a reform does not have any effect on individuals decisions before its implementation either[8].

The two models just mentioned provide a rationale for the *fiscal root* view, however at the cost of neglecting the course of the government actions executed beyond the current date. A radically different view is taken by Bental and Eckstein (1990) and Paal (2000), since in these models the anticipation of a reform is thought to be the cause of the hyperinflation. Bental and Eckstein explain the relatively low level of monetization (i.e. the stock of real balances) observed for a long time after the end of a hyperinflation as a consequence of a negative wealth effect arising from the increase in income-taxes accompanying a drastic fiscal-monetary reform. Then, they interpret a hyperinflation as the economy moving from a scenario of a high demand for real money to a stationary one in which, for any given inflation rate, the stock of real balances remains low, in line with a lower level of private wealth. Paal (2000), referring to the hyperinflationary episode in Hungary in the 1940's, argues that a reform based on the stabilization of the inflation rate without directly controlling for a monetary aggregate may actually leave the economy prone to purely expectational-driven equilibria, some of them hyperinflationary, some others exhibiting a pronounced disinflation. Anticipation of such a change in the monetary rule, rather than a weak government's fiscal condition, could have been, according to this model, the true cause of the Hungarian hyperinflation[9].

The approach followed here may be seen as a complement to the "general" explanations listed above, for it shares some basic features with them, yet takes a different route. I do not impose *ex ante* any special assumption on the current effects of a future reform. Rather, I am concerned with the conditions under which the prospect of an eventual reform exerts a preventive effect. In this sense, I build up a framework within which the effectiveness of a reform to preclude the occurrence of a hyperinflation arises as an endogenous outcome. To this aim, I formulate a rational-expectations model whose

[8] As observed by Kiguel, imposing a sufficiently tight limit to the speed of adjustment of the money market amounts to transforming the dynamics of an otherwise standard rational-expectations model into the ones of a model with adaptive expectations, an issue which is discussed later in more detail.

[9] This argument resembles the *bubble explanation* above, however with an important difference. For instance, in Sargent and Wallace (1987), the government directly sets a target for seigniorage *at every period.* Paal, on the contrary assumes that at the time of the reform the government, by committing to a non-contingent inflation target, leaves both the volume of seigniorage and the supply of money indeterminate, very much like what we would observe when a central bank switches from a rule based on the control of a monetary aggregate to one based on the control of the short-term interest rate or the foreign exchange rate. Then, she associates the occurrence of an explosive hyperinflation to a self-confirmed very high volume of seigniorage at the precise time of the reform.

cornerstone is its potential ability to display one of the most recurrent empirical facts in hyperinflationary economies, namely, a post-reform hysteresis-effect in the demand for (national currency) real balances. Referring to this phenomenon, Calvo and Végh (1992; p. 11) note that "*high inflation forces a gradual development of new financial instruments [...] Creating new financial products is costly and requires a learning process. Once this investment has taken place, the public will continue using these new financial instruments even if inflation falls*"[10]. Along this line of reasoning, the model developed later features a *gradual development* of a non-monetary financial instrument whose initial adoption is *costly* but that can be used *even if inflation falls.* There are several theoretical models in the previous literature designed to account for some of these facts. For example, Ireland (1995), Sachs (1995) and Uribe (1997) all provide examples of economies in which a form of financial innovation exhibits long-lasting effects. However, to my knowledge, no model of financial innovation or currency substitution has been previously exploited to account for the stylized effects we are after, including an explosive inflation-path and a sudden change of policy-regime. Thus, a methodological contribution of the paper consists in bringing together these two strands of the literature, a natural step after recognizing that the incentives to invest in financial products not directly subject to the inflation-tax are probably highest when inflation reaches several-digits values, and that the most pronounced episodes of rapid demonetization and slow remonetization are observed during hyperinflationary episodes[11].

Specifically, I model the demand for monetary balances as a version of Ireland's cash- and credit- goods environment. Private individuals may acquire their desired consumption basket by using either cash or credit. However, the usage of credit requires the prior investment of some valuable resources for building up long-lasting credit relationships which can be exploited for some time to avoid the inflationary cost of carrying cash[12]. On

[10] A similar observation can be found in Dornbusch *et al.* (1990; p. 23-24): "*The distinction between movements along the real money demand schedule and shifts in real money demand helps explain the actual dynamics of real balances during an inflation. The distinction may also explain why there are hysteresis effects. Once a new financial product is in use, it will continue to be used even if inflation declines. [...] Financial institutions are not ready-made to accommodate the flight from money. In the short run, households may hold more real balances than they will after alternative vehicles become available or better known. In practice, this will look like adjustment lags, but it also reflects the adjustments of financial institutions to the increased inflation.*"

[11] The possibility of employing a model of gradual financial adaptation to account for the stylized facts observed in hyperinflations was first suggested by Uribe (1997), although he limited his analysis to steady-states characterization. This paper can be viewed as following up on his observation.

[12] I stick at the cash- *versus* credit-transactions parable for simplicity and without loss of generality. The basic arguments can be extended to alternative forms of carrying out transactions that do not rely

the side of monetary supply, I follow the standard assumption of a government aiming at collecting an exogenous and constant amount of seigniorage over a *fiscal-dominance* regime which is possibly reversed at some future date by the implementation of a monetary-fiscal reform conducive to a low-inflation *monetary-dominance* regime[13]. Private agents are assumed to be aware of the eventual change of regime. Such a simple formal apparatus turns out to provide a natural interpretation on the possible links among the fiscal roots of the hyperinflations, their dynamics and termination and the relative role of expectations and fundamentals. In particular, provided some mild conditions hold, the model exhibits up to three general classes of potential equilibria over the fiscal-dominance regime. First, a unique low-inflation equilibrium that arises when the public expects a "responsible" government policy in the form of a combination of a low level of deficit-monetization and an early and restrictive reform. Second, a unique high- and increasing-inflation path that obtains in the opposite policy scenario (i.e. high seigniorage and late/insufficiently restrictive reform). Third, for some structural parameters and government policy choices lying in between the two previous extreme cases, the model displays multiple equilibria, some of them hyperinflationary, whose realization hinges crucially on private expectations, this being a reflection of the fact that a seigniorage-based monetary policy allows *per se* for individual complementarities in the decision of investment in the credit-technology.

Overall the main insights of the paper are two. First, on the theory side, the paper argues that the practical relevance of the strong arguments put forward by Obstfeld and Rogoff, concerning the effectiveness of a future reform to rule out a hyperinflation, must be qualified in view of the empirical evidence, for the presence of hysteresis suggests that the sufficient condition for the validity of those arguments (i.e. the lack of causality running from past to future states along an equilibrium in which the prospect of a reform is internalized by the households) may not hold in real economies. Indeed, in an economy in which hysteresis is a true possibility, the conditions under those arguments hold are likely to be much more complex than in the history-independent economy studied by

on the use of domestic money (e.g. dollarization, barter, etc.)

[13]As further clarified later, this is the standard assumption followed in the "general" approaches commented above, in which no attempt to explain the government behavior (beyond that of a mechanic monetization of a portion of the deficit) is made. In another branch of the literature (see e.g. Drazen (2000, chap. 10 and the references therein) and Albanesi (2002)) the degree of fiscal pressure on monetary policy is modelled explicitly as the outcome of a game between economic authorities and/or groups of interest. Within this political-economy branch, Mondino *et al.* (1996) provide a model with some similarities to the one developed here, since they allow for a mechanism of financial innovation with log-lasting effects. Yet, the questions they pose and their methodology are different from the ones pursued here.

those authors. This observation will allow us to reconcile the three influential views on hyperinflationary processes described above. Second, by examining the necessary and sufficient conditions for a hyperinflation, the model offers some insights potentially relevant for policy judgment. In particular, fiscal conditions are likely to play an important role, as stressed by Marcet and Nicolini (2003). However, using a genuine rational-expectations dynamic framework, we learn that seigniorage over the fiscal-dominance regime, while important, does not contain all the relevant fiscal-information, e.g. the horizon over which that regime is implemented and the fiscal-monetary conditions prevailing after the reform are also likely to have a direct effect on the conditions that determine whether a hyperinflation is a feasible outcome or not.

The rest of the paper is organized as follows. Section 1.2 describes the main empirical facts observed in a variety of hyperinflations. Section 1.3 contains the model. Section 1.4 characterizes the equilibrium conditions and introduces and discusses some general assumptions on the model's parameters. Section 1.5 contains the main results of the paper, dealing with the conditions under which hyperinflations (speculative or "fundamental") are possible equilibrium outcomes under two alternative reforms (contingent and fixed-date reforms). Section 1.6 summarizes the main results of the paper.

1.2 Stylized facts

Over the course of the last century and until very recent times a considerable number of countries have experienced episodes of vertiginous falls in the value of their currencies. Some of these episodes are so distant in time and/or geographical location that a detailed list of the economic factors underlying each single hyperinflationary is just an unfeasible task. Nevertheless, the literature has in most cases come to agree on a number of stylized facts that have been present in virtually all the hyperinflations occurred so far. Below, I briefly describe those facts[14].

First. Hyperinflations usually occur in countries experiencing important fiscal imbalances, in the sense that the volume of resources that the government is able to collect from regular taxes and borrowing from (national or international) financial markets is, on average, significantly below the level of its obligations, so that the resulting difference

[14]This characterization of the main facts closely follows the ones in Marcet and Nicolini (2003) and Fischer *et al.* (2002).

is funded by printing money (i.e. seigniorage). Fischer *et al.* (2002) report a strong positive relationship between seigniorage, deficits and inflation using annual observations corresponding to a sample of 24 high-inflation countries over the period 1960-1995[15].

Second. Real balances fall as inflation increases over the course of a hyperinflation.

Third. When inflation reaches its highest values within the hyperinflationary episode there is a lack of strong contemporaneous correlation between seigniorage and inflation. Indeed, seigniorage may well decrease significantly while inflation still grows at increasing rates[16].

Fourth. Stopping a hyperinflationary process usually involves a mix of fiscal and monetary reforms whose aim is to set limits to the ability of the government to monetize its deficit by e.g. setting government expenditures at levels according with its capability to cover them by taxation and debt emission, fixing the exchange rate to a foreign currency with a good reputation in terms of inflation, establishing legally independent central banks with limitations to the amount of funds to be let to the government, implementing rules to keep the growth of the money supply under limits (e.g. through currency boards), etc. The process of inflation-stabilization (from the peak of the inflation series to the, usually, stable and low rate after the monetary reform) takes a short period (two or three months at most).

Fifth. The demand for real balances remains low, but exhibiting an upward trend, for a long period after the stabilization[17]. This hysteresis-effect is behind the well documented observation that, just after the stabilization, (nominal) money supply increases at moderate-high rates without causing noticeable inflationary pressure.

Figure 1 contains some empirical evidence on several hyperinflationary episodes recorded in Germany, Argentina, Bolivia and Peru. These data generally confirm the "standard" facts above. In all cases, the rise in inflation goes parallel to the fall in real balances. The

[15]For another recent survey on this issue, see also Catao and Terrones (2005).

[16]The absence of a marked trend in seigniorage during hyperinflationary periods has led many researchers to model this variable as a constant (or with a constant mean, in stochastic environments). This is the strategy followed by e.g. Bental and Eckstein (1990), Bruno and Fischer (1990), Kiguel (1989), Marcet and Nicolini (2003), Paal (2000) and Sargent and Wallace (1987). The model presented here uses this simplifying assumption too.

[17]In recent times, this observation is closely related to the phenomenon of dollarization, i.e. the quasi-permanent substitution of a "strong" foreign currency for the local one, an issue specially relevant in countries that have experienced some episodes of high inflation. For some recent empirical treatments of this question see e.g. Kamin and Ericsson (2003) and Reinhart and Savastano (2003), and Reinhart, Rogoff and Savastano (2003).

peaks in the inflation rate uniformly go together with the lowest value of the stock of real money recorded for the entire period. At that point the stock of real balances falls bellow the 40% of its value a couple of years before. Although average seigniorage is clearly higher in the pre-reform period than after the collapse of the hyperinflationary path for the four countries, in some cases inflation reaches its highest value while seigniorage is clearly decreasing (as in Germany 10:1923 and Bolivia 12:1984-2:1985) while in others seigniorage remains high and increasing (as in Argentina) or fairly constant (as in Peru). In all cases, after the stabilization, the volume of seigniorage was drastically cut down, inflation remained low and stable, and the stock of real balances exhibited a very slow recovery, thus confirming a strong hysteresis-effect.

1.3 A simple model with money-demand hysteresis

The model developed in this section is inspired in the cash-and-credit transactions economy studied by Ireland (1995), who extends the Lucas and Stokey's (1987) framework to allow for persistent effects in the use of credit. Households are assumed to exchange their consumption-goods endowments by using government-provided money or interpersonal credit. The latter form of trade requires the existence of a credit-contract between the intervening parts. Creation of such a contract involves some initial fixed and irreversible cost. However an existing contract can be used at not further cost at any subsequent date on which the contract remains operative. Transactions made using money do not carry over any explicit cost beyond that associated to the erosion of the real value of money due to inflation. The main difference with respect to Ireland's model is intended to keep the subsequent dynamic analysis as simple as possible[18]. In particular, I assume that the volume of investment in the credit technology in any period is bounded above, as the opportunities for investment in the credit technology arrive at a finite rate. This artifact allows us to focus on monetary regimes along which the government is able to extract an exogenously set amount of seigniorage over a non-trivial number of periods.

[18] Previous formal applications of models with financial innovation are either restricted to economies with exogenously determined inflation, as in Ireland (1995) and Uribe (1997), or to one-period or steady state analyses, as in Chang (1994) and Sachs (1995), respectively. The approach taken here imposes the necessity of some simplifying assumptions as we are dealing with explosive endogenously determined inflation paths and sudden changes of regime.

1.3.1 The households

Consider a discrete-time economy with N (≥ 3) types of goods and N types of households of population size $1/N$ each. The household of type j produces goods j and consumes goods $j+1$ (except the one with type N who consumes good 1). Each household consists of a continuum of members of unit mass. Half of the members within a household are producers, who can produce their goods up to a given capacity y without cost. The other half are buyers, without such a production capacity. All the members in a household share consumption equally. At each date t, a fraction n_t^j of household-j's members, producers and buyers, are linked to the credit system, while the remaining fraction $1 - n_t^j$ are not. Each linked buyer has an established bilateral relationship with a producer to buy her consumption goods $c_t^{c,j}$ through a credit-contract, subject to the capacity constraint $c_t^{c,j} \leq y$. Symmetrically, each linked producer has a bilateral relationship with a buyer to sell his product $C_t^{c,j} \leq y$ on credit. All non-linked buyers (producers) go to a centralized market to buy (sell) goods with (in exchange for) cash as anonymous members. Money received by non-linked producers cannot be passed to buyers within the same household until the beginning of the following period.

The timing of events is as follows. At the beginning of every period $t \geq 1$, households convene in the centralized financial market to settle outstanding debts and to accumulate government–issued money and bonds. Both government and private debts mature one period after issued and pay the same nominal risk-free rate. Household-j enters this market with the cash obtained from selling a part of the common endowment at $t-1$, denoted by M_{t-1}^j, with some maturing government bonds, inclusive of interest, $(1+i_{t-1}) B_{t-1}^j$, some invoices over other households, corresponding to the amount of goods sold on credit during last period, $(1+i_{t-1}) P_{t-1} C_{t-1}^{c,j}$ and, symmetrically, some debts corresponding to last period's credit-financed purchases, $(1+i_{t-1}) P_{t-1} c_{t-1}^{c,j}$. The terms B_{t-1}^j, $C_{t-1}^{c,j}$, $c_{t-1}^{c,j}$, i_{t-1} and P_{t-1} are, respectively, bond holdings, invoices over last period's sales and purchases of goods on credit (measured in real terms), the nominal rate and the general price level[19]. The government participates in this market redeeming maturing bonds, collecting seigniorage (i.e. money injections by the central bank) and taxes (assumed to be paid in

[19] I am already assuming that all goods are traded at the same relative value. Given the symmetry imposed in endowments and preferences this turns out to be true in the competitive equilibria analyzed later.

cash) and issuing new bonds.

The household's constraint in this market is

$$M_t^j + B_t^j + P_t\tau_t \leq \widetilde{M}_{t-1}^j + (1+i_{t-1})\left(B_{t-1}^j + C_{t-1}^{c,j} - c_{t-1}^{c,j}\right) \equiv W_t^j \tag{1.1}$$

where τ_t is the lump-sum tax and $\widetilde{M}_{t-1}^j$ is the household's stock of monetary balances at the end of the previous period and, hence, it satisfies $\widetilde{M}_t^j \geq \left(1 - n_t^j\right) P_t y$. Households are precluded from issuing money, i.e.

$$M_t^j \geq 0 \tag{1.2}$$

Once the households rearrange their portfolios of money and bonds, non-linked producers engage in a process of finding buyers for their goods to sign a credit-contract. A free producer finds an opportunity to establish a credit link with a buyer with a given probability $\gamma < 1$. In such an event, the producer may chose to sign a contract by paying a utility cost of θ[20]. In that case, the buyer commits to redeem the resulting debt, plus the interest rate, in exchange for money at the financial market during the following period. Clearly, imposing $\gamma < 1$ tantamounts to a limit on the speed of investment in the credit technology which, as seen later in detail, translates into an upper bond to the speed of "de-monetization". In other words, it implies that building up a certain stock of credit-links takes time[21]. In a symmetric vein, as in Ireland (1995), I assume that the households' credit capacity is subject to a form of depreciation. In particular, credit-contracts have, on average, a finite life, as they are subject to a constant probability of termination at the beginning of each period equal to δ[22]. This assumption is critical for the arguments developed later. In practical terms, it implies that current individual decisions on the mix of money and credit used to carry out transactions may have effects beyond the current

[20] A contract will generally create some "monopolistic" rents for the counterparts. As the main interest here is in the evolution of the aggregate stock of credit-contracts, real balances and, in turn, inflation, I consider a very simple (and extreme) rule governing the sharing of the benefits arising from a match. In particular, I assume that the seller pays the fixed utility-cost and leaves the buyer only with an infinitesimal share of the monopolistic pecuniary rents of the contract. As participating in a contract is costless for the shopper, she is happy to accept any strictly positive payment. For notational simplification, I omit that negligible transfer in the forthcoming expressions.

[21] The assumptions of a constant probability of arrival of a credit-link opportunity and the symmetric proportion of linked producers and buyers are compatible with a random matching framework in which each individual can only commit to maintain a single credit-link and the matching function exhibits constant returns to scale in the number of non-linked agents on both sides of the market and a linear time-invariant total-efficiency parameter γ.

[22] Similarly, Uribe (1997) assumes that the stock of "social experience" in transacting with dollars, rather than with the local currency, depreciates at an exogenous constant rate.

date and, hence, it works as a simple device aimed at capturing the presence of hysteresis in the demand for money observed after hyperinflationary episodes.

Notice that, while there is uncertainty at the individual agent level, the assumption of a continuum of agents within each household implies that at the household level, there is no uncertainty on the total measure of the flows of new and existing contracts. Thus, the law of motion of the measure of credit-linked producers within household-j can be expressed as[23]

$$n_t^j = \gamma \left[1 - (1-\delta)\, n_{t-1}^j\right] \lambda_t^j + (1-\delta)\, n_{t-1}^j \tag{1.3}$$

where the parameter λ_t^j captures the proportion of producers that do take advantage of an opportunity to create a credit link, and, hence, it satisfies $\lambda_t^j \in [0,1]$.

Then, the centralized market for goods opens. In this market, producers who do not deliver their stock of goods through a credit arrangement sell their endowments in exchange for money. Here, the following standard CIA-constraint applies,

$$c_t^{m,j} \leq \frac{M_t^j}{P_t} \tag{1.4}$$

where $c_t^{m,j}$ refers to the total measure of goods consumed by j that are purchased with cash. Finally, the members of the household join to consume the buyers' purchases. In making its choices at t, the household must respect the following flow of funds constraint

$$\frac{W_{t+1}^j + i_t \widetilde{M}_t^j}{1+i_t} + P_t \left(c_t^{m,j} + c_t^{c,j}\right) \leq P_t \left(y - \tau_t\right) + W_t^j \tag{1.5}$$

and a borrowing constraint that rules out games *à la* Ponzi

$$\lim_{T\to\infty} W_T^j / \prod_{t=1}^{T-1} (1+i_t) \geq 0 \tag{1.6}$$

All households enter period 1 with the same financial wealth, in an amount exogenously given, i.e. $W_1^j = W_1$. Also, the measure of credit-linked producers at the beginning of period 1 (just before the destruction of some of these contracts) is given by history and identical for every household, i.e. $n_0^j = n_0$.

[23]Notice that in writing (1.3), it is assumed that the set of non-linked producers who are eligible to meet a free buyer includes both those who delivered their goods in exchange for money in the previous period plus those whose contract has just expired at the beginning of this period.

Household-j chooses a path for $\left\{c_s^{m,j}, c_s^{c,j}, M_s^j, \widetilde{M}_s^j, B_s^j, n_s^j, \lambda_s^j\right\}_{s=t}^{\infty}$ in order to maximize

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} \left\{ \ln c_s^j - \theta\gamma \left[1 - (1-\delta)\, n_{s-1}^j\right] \lambda_s^j \right\} \tag{1.7}$$

where $c_s^j = c_s^{m,j} + c_s^{c,j}$, subject to the set of constraints in (1.1), (1.2), (1.3), (1.4), (1.5), and (1.6).

1.3.2 The government

The government in this economy collects taxes, issues and redeems bonds and provides the entire stock of money. When choosing a particular combination of policy-instruments, the government is restricted by the two following constraints. First, a period-by-period flow of funds constraint,

$$\frac{B_t + M_t}{P_t} + \tau_t = \frac{(1 + i_{t-1})\, B_{t-1} + M_{t-1}}{P_t} \tag{1.8}$$

where M_t and B_t represent the government supply of money and bonds at t, respectively. Government consumption is assumed to be set at zero at every date. Second, an intertemporal constraint, given by

$$(1 + i_{t-1})\, B_{t-1} = \sum_{s=0}^{\infty} \frac{P_{t+s}\tau_{t+s} + M_{t+s} - M_{t+s-1}}{\prod_{l=1}^{s} (1 + i_{t+l})} \tag{1.9}$$

I assume throughout that the government targets a constant (and finite) real-valued stock of end-of-period outstanding bonds, i.e. $\frac{B_t}{P_t} = \frac{B_0}{P_0}$ for $t \geq 1$, with $\frac{B_0}{P_0} > 0$ being exogenously given by history. Regarding monetary policy, I consider two alternative policy regimes. During the first regime, operating from period 1 to $T > 1$ (including both), the government sets an exogenous target for seigniorage. Money supply is then adjusted to satisfy a seigniorage requirement, denoted by α, according to the following rule

$$M_t = P_t\alpha + M_{t-1} \quad (1 \leq t \leq T) \tag{1.10}$$

A second monetary regime is implemented at $T+1$ (this date is henceforth referred as the date of reform). From that period on, the government targets a unique sequence of inflation rates by adjusting money supply accordingly. For notational simplicity and without loss of generality, I assume that the targeted level of inflation under this regime is constant, denoted by x^L. As, in principle, the model admits multiplicity of equilibria under a targeted-inflation monetary rule, some of them hyperinflationary, it is also assumed that the government is willing to respond to upwards deviations from the target by backing the stock of money according to the following strategy. Taking P_T and the aggregate equilibrium stock of money at that date, M_T, as given[24], the government announces a sequence of money supplies $\{M_t\}_{t=T+1}^{\infty}$. Such a sequence of money supplies will map into a unique sequence of equilibrium inflation rates if and only if the demand for real balances and, hence, the price level follow a unique path from $T+1$ on. Let's denote this sequence of non-hyperinflationary prices as $\{P_t^G\}_{t=T+1}^{\infty}$, whose exact elements are understood to depend on $\{M_t\}_{t=T+1}^{\infty}$, M_T and P_T, and satisfy $\frac{P_{T+s+1}^G}{P_{T+s}} - 1 = x^L$, for $s = 0, 1, ..., \infty$. Then, in view of an arbitrary price $P_t > P_t^G$ at any $t \geq T+1$, the government reacts by offering the households the possibility of redeeming each monetary unit in exchange for $\frac{1}{\widetilde{P}_t}$ bonds, where $P_t^G \leq \widetilde{P}_t < P_t$. Such a deal creates an arbitrage opportunity which cannot be compatible with individual optimization and, hence, any P_t above the one consistent with the government's inflation target cannot be part of an equilibrium[25]. Notice that for such a backing-scheme to be effective it must be credible, in the sense that the households expect a current and/or future fiscal adjustment consistent with the endogenous higher supply of government bonds. Specifically, credibility of this government's contingent strategy requires the commitment to increase the total discounted value of taxes by an amount equal to $\frac{M_t}{P_t}$.

As shown later, in most cases, after the reform there will be a upward adjustment in the demand for real balances. Thus, a constant targeted x^L over this regime will require money injections, i.e. $M_t > M_{t-1}$ for $t \geq T+1$, which in turn implies that the government will earn some seigniorage whose amount is (endogenously) driven by the path followed by the stock of real balances. Therefore, in this regime, for a policy of zero government consumption and constant stock of debt, taxes must adjust endogenously so as to satisfy (1.8) and (1.9). From now on, I will refer to the two policy regimes described above as *fiscal-dominance* (for $1 < t \leq T$, i.e. the *pre-reform period*) and *monetary-dominance* regimes (for $t \geq T+1$, *post-reform period*).

[24]Notice that, given the money supply rule followed up to T (inclusive), the stock of money at that date is not direclty targeted by the government.

[25]This backing scheme resembles the one considered by Obstfeld and Rogoff (1983) and Nicolini (1996).

1.4 Equilibrium: Definition and characterization

In this section I state the definition of equilibrium used in the remaining, describe the solution of the representative household's maximization problem and introduce some assumptions regarding several classes of dynamic paths followed by the endogenous variables of greatest interest (the demand for real balances and the inflation rate).

1.4.1 Equilibrium and individual behavior

Henceforth, I will focus on symmetric competitive equilibria for the economy just described[26], defined according to

Definition 1 *A perfect-foresight symmetric competitive equilibrium in this economy is a collection of sequences* $\left\{c_s^m, c_s^c, M_s, \widetilde{M}_s, B_s, n_s, \lambda_s, P_s, i_s\right\}_{s=t}^{\infty}$ *and a government policy such that the following conditions are satisfied:*

1. Households maximize their utility subject to the initial conditions, W_1 *and* n_0*, and the constraints (1.1)-(1.6), taking as given the price and interest rate sequences and the government policy.*

2. The government satisfies its budget constraints (1.8) and (1.9).

3. All markets clear at each date.

The following conditions are necessary and sufficient for characterizing the optimal individual behavior:

1) The intertemporal allocation of consumption satisfies the Euler equation

$$\frac{c_{t+1}}{c_t} = \beta\left(1+i_t\right)\frac{P_t}{P_{t+1}} \tag{1.11}$$

2) When facing an opportunity, a producer decides to establish a credit link and pay the fixed utility-cost if $\Psi_t > \theta$, it is indifferent if $\Psi_t = \theta$ and does not sign the contract otherwise. The shadow-value function Ψ_t captures the discounted sum of the expected

[26] To simplify the notation, from now on I omit the household's superscript j.

savings, in terms of the inflationary tax, delivered by a credit-link, and is given by

$$\Psi_t = \frac{\sum_{s=1}^{\infty} (1-\delta)^{s-1} \phi_{t+s-1} x_{t+s}}{\prod_{l=0}^{s-1} (1+i_{t+l}) \frac{P_{t+l}}{P_{t+l+1}}} \tag{1.12}$$

where x_{t+1} is the inflation rate between t and $t+1$, i.e. $\frac{P_{t+1}}{P_t} - 1$ and ϕ is an indicator function capturing whether the producer is using that credit-contract at each date. It takes two values: 1 if the producer uses that link and 0 otherwise. In writing (1.12), we note that the effective inflation-savings from signing a contract at t is the discounted sum of the product of the inflation rate times the amount transacted via credit. Notice that the discount factor used here takes into account the expected lifetime of the credit-link, as captured by $1-\delta$.

3) The producers of the household optimally decide to use of every existing link if $x_{t+1} > 0$ (thus setting $\phi_t = 1$), they are indifferent between transacting using the credit facility if $x_{t+1} = 0$ and decide to trade their endowments in exchange for cash if $x_{t+1} < 0$ ($\phi_t = 0$).

4) As long as[27] $i_t > 0$ bonds perform better than money as an instrument for saving, so that the household optimally decides to minimize its end-of-period money holdings, i.e. $\frac{M_t}{P_t} = c_t^m$ and $\frac{\widetilde{M}_t}{P_t} = (1 - n_{t-1})\, y$.

5) When behaving optimally, the household does not leave any intrinsically valuable resource unconsumed. Thus the flow of funds constraint (1.5) and the transversality condition (1.6) hold as equalities.

1.4.2 Inflation dynamics: Some extra assumptions

Before characterizing the main results of the paper, it is helpful to state explicitly some equilibrium results and to describe the evolution of the endogenous variables of the model under alternative kinds of (potential) equilibria. In this section I first solve for some alternative equations governing the dynamics of the aggregate demand for real balances and inflation as functions of the parameters of the model over the fiscal-dominance regime and then I introduce some assumptions concerning the volume of seigniorage collected in that regime.

[27] Henceforth, I will focus on equilibria along which this inequality holds.

Henceforth, I will mainly focus on those cases in which the households either exploit the possibility to establish a credit-link whenever possible ($\lambda_t = 1$) or refuse to invest in the credit technology at all[28] ($\lambda_t = 0$). Assuming the former case, $\lambda_t = 1$, the overall economy measure of producers with a credit-link at t, according to (1.3), is given by

$$n_t = \gamma + (1-\gamma)(1-\delta)\, n_{t-1} \tag{1.13}$$

Thus, the proportion of producers that sell their goods in exchange for money can be written as

$$m_t = (1-\gamma)\left[1-(1-\delta)(1-m_{t-1})\right] \equiv \Gamma^I(m_{t-1}) \qquad (t>1) \tag{1.14}$$

After applying the normalization $y = 1$, from the equilibrium condition $M_t = \widetilde{M_t}$, it follows that m_t above represents the end-of-period stock of real balances in the economy. As long as $i_t > 0$, m_t may take values in the unit interval.

Equation (1.14) captures the law of motion of m conditional on every producer investing in the credit technology at the highest pace. Conversely, when no new credit-links are made, i.e. $\lambda_t = 0$, it can be readily verified that the degree of monetization evolves according to

$$m_t = 1-(1-\delta)(1-m_{t-1}) \equiv \Gamma^N(m_{t-1}) \qquad (t>1) \tag{1.15}$$

Next, I introduce the following general assumptions.

(Consistency with the assumption of exogenous seigniorage) The level of seigniorage collected over the fiscal-dominance regime, α, is bounded above by

$$\alpha < \alpha^{\max} \equiv \delta(1-\gamma)\frac{1-\eta^{T-1}}{1-\eta} + \eta^{T-1} m_0 \tag{1.16}$$

where $\eta \equiv (1-\delta)(1-\gamma)$. When this inequality holds, the amount of seigniorage targeted by the government before the implementation of the reform is feasible even when every household invests in the credit technology in every period $1 \leq t \leq T-1$, so that m_{T-1} takes its minimum possible value (given by the RHS of (1.16)).

(Dynamic negative association between real balances and inflation). For $\delta < 1$, the

[28] As shown later, considering intermediate values of λ is helpful in analyzing the class of equilibria arising under the no-hysteresis assumption (to be clarified later).

targeted level of seigniorage satisfies

$$\alpha > \delta \tag{1.17}$$

This condition implies that, regardless of the evolution of the degree of monetization at every $t \in [2, T]$, i.e. whether m_t follows (1.14) or (1.15), the resulting inflation rate, x_t, is a negative function of the last-period degree of monetization, m_{t-1}. To see this, let's write (1.10) as

$$x_t = \frac{m_{t-1}}{m_t - \alpha} - 1, \qquad 2 \leq t \leq T \tag{1.18}$$

Then, substituting m_t for its two extreme values given in (1.14), $\Gamma^I(m_{t-1})$, and (1.15), $\Gamma^N(m_{t-1})$, and taking a simple derivative in (1.18), we can write

$$\begin{aligned} \left.\frac{\partial x_t}{\partial m_{t-1}}\right|_{m_t=\Gamma^I(m_{t-1})} &< 0 \Leftrightarrow \alpha > \delta(1-\gamma) \\ \left.\frac{\partial x_t}{\partial m_{t-1}}\right|_{m_t=\Gamma^N(m_{t-1})} &< 0 \Leftrightarrow \alpha > \delta \end{aligned}$$

Thus, as $\gamma < 1$, the inequality in (1.17) is a sufficient condition for the inflation rate to be negatively related to m_{t-1}, regardless of the law of motion followed by m_t. This simple observation will turn out critical in the analysis of the feasibility of hyperinflationary paths in the next section. For the moment, it is useful to state the following result, which is a direct consequence of the inflation-generation process (1.18) and the specification of the Γ-functions, (1.14) and (1.15).

Given $\delta < 1$, if (1.17) holds, then the following equalities are satisfied

$$\begin{aligned} x_t^I &> x_t^N \quad \text{for } 2 \leq t \leq T \\ x_{t+1}^I &\geq x_t^I \text{ and } x_{t+1}^N \leq x_t^N \quad \text{for } 2 \leq t \leq T-1 \end{aligned}$$

where x_t^I is the inflation rate satisfying (1.18), conditional on m_s satisfying $m_s = \Gamma^I(n_{s-1})$, for $s = 2, ..., t$. Analogously, x_t^N is defined as the inflation rate at t conditional on no credit-link being established at any date from 1 up to t, both inclusive, i.e. $m_s = \Gamma^N(m_{s-1})$, for $s = 2, ..., t$. Thus, it follows that, for a sufficiently high level of seigniorage, the inflation rate associated with a sequence of degrees of monetization satisfying $m_s = \Gamma^I(m_{s-1})$ is always higher than the corresponding to a sequence obeying $m_s = \Gamma^N(m_{s-1})$. Also, from the above inequalities, we learn that, for a sufficiently high level of seigniorage, a decreasing m-sequence is associated with an increasing sequence of inflation rates and vice versa, i.e. increasing m over time leads to lower inflation rates[29].

Thus, the basic economic message contained in the above result is a very simple one: by restricting the volume of seigniorage to satisfy (1.17), we focus on fiscal-dominance regimes along which a higher degree of monetization is invariably associated with a lower inflation rate[30]. This idea resembles the one underlying the analyses based on the interpretation of seigniorage as a tax paid by money holders, along the lines of a standard hump-shaped inflation-tax Laffer curve, in the sense that a higher tax base (i.e. real balances or degree of monetization) needs a lower tax rate (i.e. inflation) to yield the same total revenue (seigniorage). In view of this reflection, (1.17) seems a natural ingredient in analyzing the dynamics of this economy under the fiscal-dominance regime. Also, as (1.17) imposes a lower bound on seigniorage or, alternatively, a lower bound on the degree of persistence of the credit technology, $1-\delta$, it is not at odds with the common view of high levels of seigniorage being, at least in part, responsible for hyperinflationary processes and with the empirical evidence presented before regarding the slow recovery in the degree of monetization observed after the end of the hyperinflations.

1.5 Results

In this section I check the ability of the model developed before to match the empirical facts listed earlier and to provide an answer to two of the most recurrent questions in the previous literature on hyperinflationary economies:

First. *When is a credible fiscal-monetary reform effective to prevent a hyperinflation?*

Second. *What are the factors that may explain the occurrence of a hyperinflation as a self-fulfilled prophecy?*

I order to make the above qualitative queries explicit within the formal apparatus of the model, it is convenient to give a meaning to the terms "hyperinflation" and "self-fulfilled hyperinflation" using the own language of the model. For this purpose, an impor-

[29]This statement depicts situations in which the degree of monetization is not constant over the fiscal-dominance regime, i.e. it either increases or decreases monotonically over time. In the special case in which the initial condition, m_0, happens to be equal to the stationary value of m under the corresponding law of motion, the degree of monetization remains constant over this regime and, hence, the inflation rate is constant too. The stationary values for m under the two law of motions (1.14) and (1.15) are, respectively, $\frac{\delta(1-\gamma)}{\delta+(1-\delta)\gamma}$ and 1.

[30]Althoug, in the next section I also examine an economy in which that condition is violated.

tant part of the following discussion is referred to two possible equilibrium inflation paths: the *highest inflationary path* and the *lowest inflationary path* (HIP and LIP, henceforth). Using these two concepts (whose formal definition is given below), I will henceforth use the term "hyperinflation" as the occurrence of the HIP and "self-fulfilled" or "speculative" hyperinflation as the occurrence of the HIP whenever the LIP is, simultaneously, a possible equilibrium outcome. On the one hand, this particular convention sacrifices some generality. First, because, for a class of reforms (*non-contingent* or fixed-T reforms), on purely empirical grounds, it might be difficult to distinguish the HIP from other possible equilibria that yield inflation paths almost as high as the HIP. Second, because several inflationary paths may coexist as potential equilibria with the LIP and/or the HIP or other intermediate paths, so that could equally be labelled as self-fulfilled inflationary prophecies. On the other hand, by focusing on the conditions under which the HIP may occur we will draw an important and interesting conceptual line, namely that one separating an "effective" fiscal-monetary reform (to be clarified later) from an irrelevant one. This distinction turns out to be critical when analyzing *contingent reforms* (e.g. reforms whose implementation hinges on the value taken by the inflation rate). Also, restricting the analysis of the existence of self-fulfilled hyperinflations to situations in which these two extreme kinds of inflation paths may arise facilitates the comparison of the results yielded by this model with some more traditional approaches on the existence of multiple equilibrium inflation paths, along which it is thought that the same level of seigniorage can be obtained at either a low and stable average inflation path or alongside an explosive, ever-increasing, one. Nonetheless, the formal discussion that follows also devotes some attention to other non-extreme inflationary paths, in part because it is shown that under some conditions, non-extreme paths are the only possible equilibrium outcome.

In the remaining of this section, I first explore the conditions under which the HIP may occur, assuming that the government credibly commits to reform its fiscal-monetary plan at some future date $T + 1$, regardless of the state of the economy at that date[31]. Maintaining the assumption of non-contingent reform, I then investigate analytically the conditions that tend to favor the existence of speculative hyperinflationary paths (i.e. situations in which both the HIP and LIP can be simultaneously supported as equilibrium

[31] The assumption that the public knows with certainty the date of the reform is followed by Bental and Eckstein (1990) and Paal (2000).

outcomes) and then perform some numerical simulations of the model. Finally, I explore the determinants of the (in)effectiveness of an alternative state-contingent reform.

1.5.1 Hyperinflationary paths: The case of an ineffective reform

Let's suppose that the government credibly commits to implement a reform as the one described in section 3.2 at some date $T+1$ and regardless of the state of the economy at that time. Also, to make life interesting, let's assume that, independently of the history of this economy up to T, the inflation rate targeted in the post-reform monetary-dominance regime, x^L, is bounded above by[32]

$$x^L \equiv x_{T+s} < \min\left\{x_T, \ \frac{\theta}{\sum_{s=1}^{\infty} \beta^s (1-\delta)^{s-1}}\right\}, \quad s \geq 1 \tag{1.19}$$

This inequality implies, first, that the change of regime always leads to a fall in inflation at the time of the reform, regardless of the history up to that date, and, second, that the reform always eliminates the individual incentives to reduce the usage of money, i.e. the reform is successful in stopping an "eventual flight from money" from the time of its implementation on, i.e., in equilibrium $\lambda_s = 0$, $s \geq T$.

Assuming (1.17) holds, I define the two extreme paths (HIP and LIP) using the laws of motion for the degree of monetization under the assumption that λ_t takes one of the extreme values, 1 or 0, obtained before, (1.14) and (1.15), respectively.

Definition 2 *The highest inflationary path (HIP) is the sequence* $\{x_t^I\}_{t=2}^{T}$ *that satisfies (1.18) given that the sequence* $\{m_t\}_{t=2}^{T-1}$ *satisfies* $m_t = \Gamma^I(m_{t-1})$. *The lowest inflationary path (LIP) is the sequence* $\{x_t^N\}_{t=2}^{T}$ *that satisfies (1.18) given that the sequence* $\{m_t\}_{t=2}^{T-1}$ *satisfies* $m_t = \Gamma^N(m_{t-1})$.

Using the shadow-value function (1.12) together with the equilibrium conditions $\frac{1+i_t}{1+x_{t+1}} = \beta^{-1}$ and $c_t = 1$, we can define the time-t (unique) shadow-value function associated to the HIP as

$$\Psi_t^I = \sum_{s=1}^{T-t} \beta^s (1-\delta)^{s-1} x_{t+s}^I + \frac{\beta^{T-t+1}(1-\delta)^{T-t}}{1-\beta(1-\delta)} \phi^L x^L \tag{1.20}$$

[32]In writing this expression, we note that the real return on debts is constant over time, $1+r \equiv (1+i_t)\frac{P_t}{P_{t+1}} = \beta^{-1}$, a result that follows from the goods-market clearing condition and the Euler equation (1.11).

where $\phi^L \in \{0,1\}$ denotes whether pre-existing credit-contracts are used after the reform[33]. According to Definition 2, x_t^I is given by

$$x_t^I = \frac{\delta(1-\gamma)\frac{1-\eta^{t-1}}{1-\eta} + \eta^{t-1}m_0}{\delta(1-\gamma)\frac{1-\eta^t}{1-\eta} + \eta^t m_0 - \alpha} \tag{1.21}$$

Thus, equation (1.20) captures the (gross) profit given by a credit contract when every household exploits every opportunity for investment in the credit technology at each single period within the pre-reform period ($t = 1, ..., T-1$). As the households follow this pattern, provided α satisfies (1.17), the resulting equilibrium sequence of inflation rates is the highest possible one. Then, denoting inf $\{\Psi_t^I\}_{t=1}^{T-1}$ by $\underline{\Psi}_t^I$, and according to (1.12), the following condition is necessary and sufficient for the feasibility of the HIP as an equilibrium path[34]

$$\underline{\Psi}_t^I \geq \theta \tag{1.22}$$

By simple inspection of (1.20) and (1.21), the qualitative effect of most parameters and policy variables on condition (1.22) becomes apparent. Higher values of β increase the total return of the investment in a credit-link, thus raising Ψ_t^I, and, hence, $\underline{\Psi}_t^I$, making the HIP a more likely equilibrium outcome. Assuming (1.17) holds, a similar effect comes from δ, as lower δ means a longer average life for a contract. Also, from (1.21), higher values γ contribute to a more rapid demonetization and, thus, to a higher sequence of future inflation rates and a higher value for the credit-technology at any t. This latter effect is also caused by a lower initial condition, m_0. From (1.20) and (1.22), it is also clear that a lower value for the cost of establishing a credit relationship, θ, tends to favor the occurrence of the HIP.

Regarding the effects of the policy instruments, α, T and x^L, their influence on condition (1.22) is also very intuitive. A high value of seigniorage goes in hand with higher inflation rates, given a path for m, and with a higher $\underline{\Psi}_t^I$. A restrictive monetary supply over the post-reform period, contributes, given every thing else, to keep inflation low also in the pre-reform period, since low values for x^L reduce the expected return gained

[33]Notice that, since x^L is time-invariant, the equilibrium value of ϕ^L will be constant too. As I will focus on situations in which inflation is striclty positive in the pre-reform regime, ϕ will be optimally be 1 duirng that regime.

[34]For the ease of the exposition, it is assumed that the vector of parameters and policy-variables are such that $\underline{\Psi}_t^I$ is unique.

from each credit-link. Also, assuming (1.19) holds, an early reform (low T) reduces the incentives to invest in the credit technology, since it shortens the period over which that investment yields high inflation-savings (the pre-reform period). Finally, the effect of time on Ψ_t^I is likely to be non-monotonic, due to the discount-effect. Indeed, all the numerical simulations discussed later yield a hump-shaped Ψ_t^I function, i.e. the lowest values for Ψ_t^I are located at the beginning and at the end of the pre-reform period.

Thus, an equilibrium along which the prospect of an orthodox fiscal-monetary reform does not have any positive effect on the demand for real balances and, hence, on the inflation rate, except just at the moment of its implementation[35], cannot always be ruled out. This result is in clear contrast with the analysis of similar orthodox reforms carried out by Obstfeld and Rogoff (1983) and Nicolini (1996), within the context of purely forward-looking economies. In an economy like this one, in which past decisions of the households have long-lasting effects, time matters. When this is case, the effectiveness of a future reform to rule out explosive hyperinflationary paths hinges on the timing of the reform, the monetary policy implemented in the post-reform regime (as captured by x^L) and also on the monetary stance before the reform (as captured by α) and on some structural parameters of the economy governing the *history-dependent* processes of demonetization and remonetization (as m_0, δ, γ and θ). Also, when time plays a non-trivial role, the concept of a static inflation-tax Laffer curve is no longer a useful analytical tool, since whether a given level of seigniorage can be collected at a relatively stable inflation rate or at an explosive one does not only depend on the particular volume of seigniorage but, critically, also on the horizon over which that fiscal-dominance regime is to prevail. For example, a given amount of seigniorage may unchain an explosive hyperinflation if it is maintained over a long period, but could equally be financed at a moderate and stable inflation rate if the public understands that such a regime is being reversed at an earlier date. In this sense, the total discounted value of the tax paid by money holders is more informative than the current value of that tax for understanding individual current money-holdings decisions, an observation that helps to understand the lack of contemporaneous correlation between seigniorage and the inflation rate detected in the data.

It is worth noticing the different nature of the reform-ineffectiveness result in this

[35] Recall that the first exogenously targeted inflation rate is $x_{T+1} = x^L$.

environment and in those studied in some earlier works. On the one hand, in Bental and Eckstein (1990) and Paal (2000) the prospect of a particular reform is the underlying determinant of an increasing inflation path and, hence, in both cases, the absence of such a reform would preclude such an outcome. Equivalently, the fiscal-monetary conditions prevailing over the pre-reform period do not play any meaningful role in unchaining an increasing inflationary path, notwithstanding the fact that a higher amount of seigniorage collected during that period implies, given everything else, more inflation[36]. In contrast, in the economy studied here, the prospect of a reform may or not preclude the occurrence of a hyperinflation, but its sole expectation never causes a hyperinflation. Also, the fiscal-monetary stance observed over the pre-reform period now has a first-order importance in determining whether the economy enters into a hyperinflationary spiral, that is, *what happens before the reform matters.* The fact that the reform considered here may exert an anti-inflationary effect distinguishes this economy from the ones studied by Kiguel (1989) and Marcet and Nicolini (2003), since in both cases the possibility of a reform is never acknowledged by the public. However, in this economy, when the reform takes place and how it is implemented, i.e. the time of the reform T and the targeted inflation rate x^L, matters.

1.5.2 The relative role of expectations and fundamentals

Next, I extend the previous analysis to account for one of the most recurrent themes in the previous theoretical and empirical studies on hyperinflations: the occurrence of speculative or self-fulfilled hyperinflations. As pointed out above, for the sake of the clarity, I concentrate the discussion on the conditions under which the coexistence of the two extreme inflationary paths in Definition 2 is possible. For the moment, I retain the exogenous reform-date assumption. Let's define the time-t shadow-value function associated to the LIP as

$$\Psi_t^N = \sum_{s=1}^{T-t} \beta^s (1-\delta)^{s-1} x_{t+s}^N + \frac{\beta^{T-t+1} (1-\delta)^{T-t}}{1-\beta(1-\delta)} \phi^L x^L \tag{1.23}$$

[36] That is, in those models seigniorage matters for determining the location of the resulting equilibrium inflation path, but not its shape, with the qualification observed before: a "too high" volume of seigniorage may result in a disinflationary process in Bental and Eckstein's model.

where x_t^N is given by

$$x_t^N = \frac{1-(1-\delta)^{t-1}(1-m_0)}{1-(1-\delta)^t(1-m_0)-\alpha} \tag{1.24}$$

The function Ψ_t^N captures the potential benefits for a single household from investing in a credit-link conditional on that link being the only one made from that period on, i.e. it captures the individual incentives to invest in the credit technology when it is understood that nobody else will ever invest. As $x_T < x^L$ (see (1.19)) and (1.17) holds, we learn that Ψ_t^N is a decreasing function of time satisfying $\Psi_t^N < \Psi_t^I$ for $t = 1, ..., T-1$. Hence, the following sign condition is necessary and sufficient for the feasibility of the LIP

$$\Psi_1^N \leq \theta \tag{1.25}$$

Clearly, the influence of the parameters and policy variables regarding the satisfaction of condition (1.25) is the opposite one with respect to condition (1.22), that is, low values of β, α, x^L and T, and high values of δ, m_0 and θ run in favor of the feasibility of the LIP.

Then, combining (1.22) and (1.25), we learn that an extreme speculative hyperinflation is possible in this environment if the following condition holds

$$\Psi_1^N \leq \theta \leq \underline{\Psi}_t^I \tag{1.26}$$

Having stated the condition that guarantees the coexistence of the two extreme equilibrium inflation paths, the next step is to analyze the role of parameters of the model regarding the satisfaction of condition (1.26). However, given the large number of relevant parameters and the algebraic complexity of the differential equations involved in the definition of the shadow-value functions, performing a complete analytical study of the influence of each parameter on condition (1.26) is an unfeasible task. Given this limitation, I restrict most of the analysis to the following three parameters: δ, γ and α. Devoting special attention to δ and γ is obliged here, since these parameters are the critical ingredients in this economy. Also, by focusing on seigniorage, we get a net picture of the main differences between this model and previous approaches based on relating the observed inflation rate to the amount of seigniorage through an inflation-tax Laffer curve.

A dynamic Laffer-curve. Let's characterize the threshold conditions (1.22) and (1.25) as functions of α, taking everything else as given. To this aim, let's define two threshold-values for seigniorage, α^I and α^N, both falling within the interval $(\delta, \alpha^{\max})$ and satisfying $\underline{\Psi}_t^I(\alpha^I) = \theta$ and $\Psi_1^N(\alpha^N) = \theta$, respectively. The conditions that guarantee the existence of these two threshold-values for α can be stated in terms of the following bounds for θ:

$$\begin{aligned} \theta &\leq \overline{\theta} \equiv \lim_{\alpha \uparrow \alpha^{\max}} \beta x_T^I + \frac{\beta^2(1-\delta)}{1-\beta(1-\delta)} \phi^L x^L \\ \theta &\geq \underline{\theta} \equiv \lim_{\alpha \downarrow \delta} \sum_{s=1}^{T-1} \beta^s (1-\delta)^{s-1} x_{s+1}^N + \frac{\beta^T (1-\delta)^{T-1}}{1-\beta(1-\delta)} \phi^L x^L \end{aligned}$$

The upper bound $\overline{\theta}$ implies that the unit cost of a credit-contract is not sufficiently high so as to discourage investments in the credit technology just before the reform (at $T-1$) when the government sets α arbitrarily close to its maximum sustainable level[37], $\alpha^{\max}$. Symmetrically, the lower bound $\underline{\theta}$ implies that θ is always sufficiently high so as to allow individuals to optimally give up their opportunities to expand their portfolio of credit-links whenever α is set close to its minimum possible value, i.e. δ, consistently with the sign condition (1.17). Then, if $\underline{\theta} < \overline{\theta}$ and θ falls within the interval $[\underline{\theta}, \overline{\theta}]$, both α^I and α^N exist[38].

The fact that $\underline{\Psi}_t^I$ and Ψ_1^N are monotonically increasing functions of α implies that, provided α^I and α^N exist, they are unique. Then, using these threshold-values, we can rewrite condition (1.26) as

$$\alpha \in [\alpha^I, \alpha^N]$$

Thus, the following is a necessary condition for an extreme speculative hyperinflation

$$\alpha^I \leq \alpha^N \tag{1.27}$$

Figure 2 depicts x_T (i.e. the last endogenously determined inflation rate) as a function of α, under the assumption that (1.27) holds as a strict inequality, for the two extreme paths (HIP and LIP). This figure shares an important feature with a standard hump-shaped inflation-tax Laffer-curve, namely that some levels of seigniorage can be financed at either

[37]Notice that $\lim_{\alpha \uparrow \alpha^{\max}} x_{T-1}^I \to \infty$, so for finite T and θ, $\lim_{\alpha \uparrow \alpha^{\max}} \Psi_t^I \to \infty > \theta$ for $1 \leq t \leq T-2$, and, hence, feasibility of the HIP hinges exclusively on the sign of $\lim_{\alpha \uparrow \alpha^{\max}} \Psi_{T-1}^I - \theta$.

[38]Notice that the existence of α^I and α^N amounts to impose a rather mild condition, namely, that we can find a sufficiently high (low) α, within the limits imposed in (1.16) and (1.17), such that the HIP (LIP) is a possible equilibrium outcome.

a stable and low (or moderate) inflation rate or at an unstable rising one. However, in this economy a speculative hyperinflation will not take place for low values of seigniorage ($\alpha < \alpha^I$) since the prospect of a future reform renders the investment in the credit technology no profitable enough. Another important difference is that in this economy, along a speculative hyperinflationary path, inflation is always positively associated with the level of seigniorage, just the opposite result found in the analyses performed by Sargent and Wallace (1987) and Bruno and Fischer (1990), since, in those models, the fact that a speculative hyperinflation is understood as the economy slipping into the decreasing arm of the Laffer-curve implies that one should expect higher rates of inflation when seigniorage is low. Figure 2 also makes clear that the model is compatible with the main result obtained in the *money-market partial-adjustment* framework employed by Kiguel (1989) and Romer (2001), namely that "too high" seigniorage, i.e. in this model values of $\alpha > \alpha^N$, will invariably result in a hyperinflationary path. However, while admitting the potential importance of seigniorage in causing a hyperinflation, this model still allows for equilibrium hyperinflationary paths even when the quantitative measure of fiscal dominance before the reform is thought to be only "moderate", i.e. values of α falling within the interval $\left[\alpha^I, \alpha^N\right]$.

Persistency of the credit-contracts (δ). The role of the assumption about the persistent effects of the credit technology ($\delta < 1$) is critical for the existence of speculative equilibria. Below I show that the presence of a mechanism allowing for the possibility of hysteresis in the demand for real balances is a necessary condition for the existence of multiple equilibrium inflation-paths when individuals rationally expect the future implementation of a monetary-fiscal reform as the one considered here. The proof of this claim involves a kind of negation argument since the absence of an underlying mechanism that makes hysteresis possible (i.e. $\delta = 1$) renders a reform an *effective* equilibrium selection device, where the term *effective* refers to the fact that such a reform is always compatible with only one equilibrium inflation path, regardless of the set of fundamentals (e.g. level of seigniorage, speed of demonetization and remonetization, endowments, timing and dimension of the reform, etc.) being observed[39].

As $\delta = 1$ the investment in the credit technology only lasts for one period, that is, each

[39] Of course, the value of the endogenous variables of the model in the (*unique*) equilibrium will depend on the particular value of the vector of fundamentals.

credit-contract is only effective at the period of its creation. By imposing this particular condition, we get a version of the model that replicates one of the salient features of the canonical Cagan's demand for real balances function, namely, that the only inflation rate relevant for the individual's decision of how much money to hold is the next period's one. This feature is critical in removing multiple equilibria. To see this, let's first notice that, as $\delta = 1$, the relevant savings-function (1.12), in equilibrium, simplifies to

$$\Psi_t = \beta \phi_t x_{t+1}$$

where the function ϕ_t now simply indicates whether the credit-link is established ($\phi_t = 1$) or not ($\phi_t = 0$). Since the reform, by assumption (see (1.19)), precludes the creation of new links, i.e. $x^L < \frac{\theta}{\beta}$, we learn that in equilibrium $\phi_T = 0$ and, according to (1.15), $m_T = 1$ regardless of the value of m_{T-1}. Let's consider the following two extreme values of the degree of monetization at $T-1$, each associated with an extreme λ_{T-1}: 0 and 1, and the corresponding (according to (1.18)) inflation rates at T:

$$n_{T-1} = \begin{cases} m_{T-1}^I = 1 - \gamma \\ m_{T-1}^N = 1 \end{cases} \qquad x_T = \begin{cases} x_T^I = \frac{1-\gamma}{1-\alpha} - 1 \\ x_T^N = \frac{1}{1-\alpha} - 1 \end{cases} \tag{1.28}$$

Thus, as $\gamma < 1$, we learn that $x_T^I < x_T^N$, i.e. just before the implementation of the reform, there is a negative relationship between the last period's degree of monetization, m_{T-1}, and the current period's inflation rate[40], x_T. Next, let's consider three alternative cases, depending on the sign of the following differences: $x_T^I - \widehat{\theta}$ and $x_T^N - \widehat{\theta}$, where $\widehat{\theta} \equiv \frac{\theta}{\beta}$.

Case 1. $x_T^N - \widehat{\theta} \leq 0$. As $x_T^I < x_T^N$, it follows that $x_T^I - \widehat{\theta} < 0$. Then, the optimal households' decision at $T-1$ is not to invest in credit-contracts, i.e. there is no $\lambda_{T-1} \in (0, 1)$ such that $\frac{1-\lambda_{T-1}\gamma}{1-\alpha} - 1 - \widehat{\theta} > 0$. Thus, in equilibrium, $m_{T-1} = m_T = 1$. The same argument clearly applies to any period $t \in [1, T-1]$, so, from (1.18), we learn that the equilibrium inflation rate is also uniquely determined (and constant) over the pre-reform regime[41], i.e.

$$x_t^{(1)} = \frac{1}{1-\alpha} - 1 \ \text{ for } \ 2 \leq t \leq T.$$

[40]Notice that this result is directly driven by the assumption $\delta = 1$. Under this assumption, the sign condition in (1.17) cannot be satisfied and, thus, the chains of implications following that condition do not hold.

[41]In every case, after the reform the equilibrium inflation rate is equal to the government target, i.e. $x_t = x^L$ for $t \geq T+1$.

Case 2. $x_T^I - \widehat{\theta} \geq 0$. In this case $x_T^N - \widehat{\theta} > 0$ must hold and the households optimally decide to exhaust their opportunities to invest in the credit technology at $T-1$, so that $\lambda_{T-1} = 1$ and $m_{T-1} = 1-\gamma$. Then, the minimum inflation rate attainable at $T-1$ is equal to $\frac{1-\gamma}{1-\gamma-\alpha} - 1 > x_T^I > \widehat{\theta}$, i.e. the rate that would prevail in a situation in which the households set $\lambda_{T-2} = 1$. Thus, the households optimally set $\lambda_{T-2} = 1$, and, applying this argument backwards, we find that in any equilibrium $\lambda_t = 1$ and $m_t = 1 - q$ for $t = 1, ..., T-1$. Then, given a unique equilibrium sequence of m's, it follows that there is also a unique equilibrium sequence of inflation rates satisfying

$$x_t^{(2)} = \begin{cases} \frac{1-\gamma}{1-\gamma-\alpha} - 1, & 2 \leq t \leq T-1 \\ \frac{1-\gamma}{1-\alpha} - 1 & t = T \end{cases} \tag{1.29}$$

Case 3. $x_T^N - \widehat{\theta} > 0$ and $x_T^I - \widehat{\theta} < 0$. This case is clearly incompatible with the limiting values of λ_{T-1}, 0 or 1. First, $\lambda_{T-1} = 0$ cannot be optimal, since the inflation rate resulting from the highest level of monetization, $m_{T-1}^N = 1$, is higher than the unit cost of a credit-contract. Symmetrically, $\lambda_{T-1} = 1$ cannot be optimal either, for the inflation rate consistent with $m_{T-1}^I = 1-\gamma$ falls below the cost of a contract. Thus, in a symmetric equilibrium, the household chooses an intermediate $\lambda_{T-1} \in (0,1)$, i.e. some producers within the household invest and some others, while having the opportunity, do not. But this implies that, at the margin, the household must be indifferent between establishing a link and giving up that opportunity, so that the (unique) equilibrium inflation rate at T satisfies $x_T = \widehat{\theta}$ and $x_T \in \left(x_T^I, x_T^N\right)$. Then, we can use (1.18) to solve for the unique equilibrium λ_{T-1}, i.e.

$$\widehat{\theta} = \frac{1 - \lambda_{T-1}\gamma}{1-\alpha} - 1$$

Given a unique $m_{T-1} \left(= 1 - \lambda_{T-1}\gamma\right)$, we learn that $x_{T-1}^N \equiv \frac{1}{1-\lambda_{T-1}\gamma-\alpha} - 1 > x_T^N > \widehat{\theta}$, so that the relevant condition governing the households' choice for λ_{T-2} is given by the sign of $x_{T-1}^I - \widehat{\theta}$, where $x_{T-1}^I \equiv \frac{1-\gamma}{1-\lambda_{T-1}\gamma-\alpha} - 1$. If $x_{T-1}^I - \widehat{\theta} < 0$, following the same argument above, the equilibrium inflation rate at $T-1$ is equal to $\widehat{\theta}$ and (1.18) can be solved for the unique equilibrium λ_{T-2}. This iterative process continues up to a period t^* at which the following inequality holds,

$$x_{t^*+1}^I \equiv \frac{1-\gamma}{1-\lambda_{t^*+1}\gamma - \alpha} - 1 \geq \widehat{\theta} \tag{1.30}$$

with $\lambda_{t^*+1} < 1$, in which case, we learn that individual optimality calls for $\lambda_{t^*} = \lambda_{t^*-1} = ... = \lambda_1 = 1$. Combining (1.30) for two consecutive dates, $s-1$ and s, we can write the following dynamic equation

$$\lambda_{s-1} = \frac{1-\widehat{\theta}\,(1-\alpha)}{\gamma} + \widehat{\theta}\lambda_s$$

which, for a terminal condition λ_{T-1} given in (1.30), can be solved for a unique equilibrium sequence $\{\lambda_s\}_{s=T-1}^{t^*+1}$. Further, since $\left(\widehat{\theta}, \gamma, \lambda_s\right) \ll 1$, the elements in this sequence satisfy $\lambda_{s-1} > \lambda_s$ and, hence, $m_{s-1} < m_s$. The degree of monetization for all $t = 1,..,t^*$ is constant and equal to $\frac{1-\gamma}{1-\gamma-\alpha}$. Then, given a unique equilibrium sequence $\{m_t\}_{t=1}^{T}$, we can solve for the unique equilibrium sequence of inflation rates $\{x_t\}_{t=2}^{T}$,

$$x_t^{(3)} = \begin{cases} \frac{1-\gamma}{1-\gamma-\alpha} - 1, & 2 \leq t \leq t^* \\ \frac{1-\gamma}{1-\lambda_{t^*+1}\gamma-\alpha} - 1 & t = t^*+1 \\ \widehat{\theta} & t = t^*+2, ..., T \end{cases}$$

Using the terminology introduced above, Case 1 corresponds to the LIP and Case 2 to the HIP. We can then solve for the two critical α-thresholds, such that both extreme cases are feasible, to find the following step-function relating seigniorage to the equilibrium inflation sequence over the pre-reform period

$$\underset{2\leq t\leq T}{x_t} = \begin{cases} x_t^{(1)} & \text{for } \alpha^{(1)} \leq \alpha^N \equiv \frac{\widehat{\theta}}{1+\widehat{\theta}} \\ x_t^{(2)} & \text{for } \alpha^{(2)} \geq \alpha^I \equiv \frac{\gamma+\widehat{\theta}}{1+\widehat{\theta}} \\ x_t^{(3)} & \text{for } \alpha^{(3)} \in \left(\alpha^L, \alpha^H\right) \end{cases} \tag{1.31}$$

The three possible equilibrium inflation sequences, $x_t^{(1)}$, $x_t^{(2)}$ and $x_t^{(3)}$ are plotted in figure 3.

Thus, in the absence of a mechanism that allows for the possibility of hysteresis in the degree of monetization, the expectation of a future reform precludes the possibility of multiple equilibria. The reason for this result is very intuitive. The $\delta = 1$ condition, when coupled with a post-reform monetary policy that results in a unique optimal transaction-instruments portfolio decision over that regime, implies that the households' optimal actions from T on are completely independent of their actions at any earlier date. In plain words, "the future is independent of past". Then, the ability of the government to implement a monetary rule consistent with an exogenously fixed level of seigniorage implies, in view of (1.18), that, for the unique "possible future" there is a unique "possible past". In this sense, one policy choice, the fiscal-monetary reform, gives every piece of relevant information about the future, while another policy choice, the level of seigniorage, gives all the sufficient information regarding the past, as from the date of reform. In sum, in this economy, as in the one examined Obstfeld and Rogoff (1983) and Nicolini (1996), there is only space for one set of beliefs. On the other hand, when $\delta < 1$, the previous conclusion does not necessarily hold. A successful post-reform policy aimed at stabilizing the rate of inflation at a low level induces a unique optimal private behavior from the time of the reform on but it is not necessarily sufficient to always induce a unique optimal behavior before its implementation and, hence, private expectations may play a role in choosing the observed equilibrium.

Obviously, if the government chooses a post-reform inflation target, x^L, below zero then hysteresis in the demand for money after the reform is ruled out, as well. However, this policy-choice may not induce a unique equilibrium path over the pre-reform period whenever $\delta < 1$, in spite of inducing a unique m_T (equal to 1, i.e. the full-monetization level). At any period t before the reform, the characterization of the households' optimal portfolio decision differs from the three cases above, since now, for $\delta < 1$, past individuals' decisions affect the current optimal portfolio choice, and vice versa, thus, giving rise to the possibility of potential multiple equilibria in the same way as described in the above discussion about figure 2. In terms of the terminology of the previous paragraph, when $x^L < 0$ and $\delta < 1$, even if there is a unique "possible future" from the perspective of time T, there is not necessarily a unique "possible past" before that date. Of course, a sufficiently restrictive monetary policy supporting a negative inflation rate after the reform will always tend to exert a strong counter-inflationary effect, due to the discontinuity in the optimal portfolio rule at $x^L = 0$, and such a reform-policy is likely to have stabilizing effects before its implementation except, e.g., under very extreme fiscal conditions (high values for α). Nevertheless, the empirical evidence commented in section 1.2 reveals that an instantaneous and complete recovery in the degree of monetization at the end of the hyperinflationary period is never observed in real economies, thus supporting the view that reforms that are unsuccessful to preclude a hyperinflation are a real possibility.

The speed of demonetization (γ). In some previous models of hyperinflations that incorporate a mechanism for sluggish adjustment in the demand for real balances the existence of an upper bound in the velocity of demonetization (i.e. the fall in m between two consecutive periods) is a critical necessary condition to generate a hyperinflationary path when seigniorage is high, such that violations of that threshold lead to a reversal of the dynamics of the model. For example, in the Kiguel-Romer framework with partial adjustment in the money market, a sufficiently high speed of adjustment may invert the sign of the correlation between seigniorage and inflation, yielding disinflationary (hyperinflationary) paths when seigniorage is above (below) the maximum level dictated by the stationary inflation-tax Laffer-curve. A similar effect is also found in some models with backward looking expectations (crude adaptive or quasi-rational expectations as in the modern literature on learning mechanisms[42]). To put it in simple terms, such a feature implies that should individuals be able to "fly from money" at a sufficiently fast rate, say because there are not important obstacles to access to alternative means of payment (i.e. high γ in the present framework), we would observe very severe hyperinflations when seigniorage is low and pronounced disinflations with high volumes of seigniorage.

Such an "unpleasant" feature is also shared by the restricted version of the model analyzed above with $\delta = 1$. The following example illustrates this point. Let's assume an initial situation with a level of seigniorage satisfying $\alpha > \frac{\gamma^0+\widehat{\theta}}{1+\widehat{\theta}}$. According to (1.31), optimal individual behavior implies that equilibrium inflation in the pre-reform regime is given by (1.29), with $\gamma = \gamma^0$. Now, let's suppose that γ rises up to $\gamma' > \gamma^0$, such that, for the same α, now $\alpha < \frac{\gamma'+\widehat{\theta}}{1+\widehat{\theta}}$. This upward adjustment in γ moves the economy into the intermediate region in (1.31), thus lowering the inflation rate at T (since $\frac{1-\gamma^0}{1-\alpha} - 1 > \widehat{\theta}$) and at every date sufficiently close to the reform, at which $x = \widehat{\theta}$ for $\gamma = \gamma'$. Further, for sufficiently high γ (or low T) the equilibrium inflation sequence is uniformly lower under γ' than under γ^0, that is, a more effective mechanism for bringing together free shoppers and sellers (higher γ) results in a lower degree of usage of credit and, thus, at some dates, in lower inflation.

[42]For a discussion of this point in the context of a mechanism of crude adaptive expectations see, e.g. Bruno and Fischer (1990). Lettau and van Zandt (2003) and Adam, Evans and Honkapojha (2004) investigate how the usage of current versus past inflation rates in forming expectations affect the dynamic properties of a standard model of seigniorage under alternative learning rules, showing that under some widely used specifications for the learning rule, the use of updated, rather than lagged, information tends to make an explosive hyperinflation a more unlikely result.

The above, rather counterintuitive, result vanishes as soon as the parameters allow for the possibility of hysteresis after the reform (i.e. $\delta < 1$ and $x^L > 0$[43]) and (1.17) holds. First, as stressed before, a higher velocity of demonetization, as captured by high values of γ, given everything else, always tends to favor the occurrence of the HIP (by decreasing α^I) and to push-up the resulting hyperinflationary path (by decreasing m). Second, higher values of γ in this model, also tend to increase the range of pre-reform seigniorage choices such that extreme speculative hyperinflations are possible equilibrium outcomes. To see this, notice from (1.20), (1.21), (1.23) and (1.24) that α^I is a negative function of γ, while α^N is independent of that parameter. Thus, when the private sector can easily substitute credit for money as an alternative means of carrying out transactions *and* individuals internalize the persistence of their decisions, a simple and intuitive proposition applies: extreme speculative *hyperinflations* are more likely outcomes[44].

Finally, when the velocity of demonetization or its degree of persistence are sufficiently low, given everything else, the necessary condition for the existence of an extreme speculative hyperinflation (1.27) is violated, so that there is never an overlap between the HIP and LIP, even if (1.17) holds. When this is the case, it can be shown that for intermediate levels of seigniorage $\alpha \in \left(\alpha^N, \alpha^I\right)$ there is always, at least, one equilibrium in which every household exhausts its opportunities to invest in the credit technology up to some period t, for $2 \leq t \leq T-2$ and stop investing from $t+1$ on, so that the equilibrium path for m exhibits a $\cup$-shape over time. Further, over this range of α's, this class of m-paths are the only admissible ones, thus implying that the reform only exerts a positive effect on the demand for real balances when the date of the reform is perceived as sufficiently close[45] (see the Appendix).

1.5.3 Some numerical illustrations

Figures N1 to N7 contain the results of some numerical simulations of the model in which the values of some parameters are set at some arbitrary levels. In all cases considered

[43] If, on the contrary, $x^L < 0$ holds, then the "perverse" effect of a high velocity of demonetization arises.

[44] In his classic study on inflationary finance, Nichols (1974) formulates an intuitive principle to avoid hyperinflations, or at least to keep inflation under some control, in face of a given target for seigniorage: restrict the public's access to those assets that may be seen as close substitutes for money. Chang (1994) also points out that policies tending to decrease the usage of foreign currency should reduce inflation. Those reflections are plainly compatible within our model when $\delta < 1$.

[45] In general, this class of paths may also exist when (1.27) is satisfied.

here, the minimum value of the shadow-value function Ψ_t^I over the period running from $t = 1$ to $T - 1$ was found at $T - 1$, this reflecting, among other things, that the prospect of a reform supporting a low inflation rate (x^L was set in most numerical simulations at 0.1) tends to reduce the incentives for investing in the credit technology specially in the period preceding its implementation. Figures N1 and N2 depict the size of the overlap between the two extreme inflation paths (i.e. the the difference $\alpha^N - \alpha^I$) as a function of 1-δ and q, respectively, for several pre-reform time-horizons. For both parameters, the size of the α-set such that extreme speculative hyperinflations is positively related to the value of the relevant parameter regardless of the length of the pre-reform period. For low values of these parameters the overlap disappears (i.e. $\alpha^N - \alpha^I$ takes negative values).

Figure N3 is the numerical counterpart of the time-T Laffer curve in figure 2. The parameters are chosen so that the overlap is positive, thus resulting in a non-empty α-set for which both the LIP and the HIP are possible equilibria. In addition to these two extreme paths, this figure plots the time-T inflation rates arising under non-monotonic paths along which the equilibrium level of monetization, m, decreases up to some period $n - 1$ and then grows up uniformly from n on. The discontinuous effect of seigniorage on the time-T inflation rate (and, hence, on the entire inflation-sequence) and the scope for multiple, and potentially very different in quantitative terms, equilibrium paths become apparent. Figure N4, on the other hand, depicts a situation in which δ, while below 1, is so high that $\alpha^N < \alpha^I$. The emerging picture is then very different: the effect of seigniorage on the optimal portfolio decision and the resulting inflation sequence exhibits a high degree of "smoothness", very much as the suggested by the conventional arguments based on the upward sloped branch in a static inflation-tax Laffer curve, in the sense that high volumes of seigniorage tend to uniformly cause high inflation (but here without associating values of seigniorage above the peak of the static Laffer curve with disinflationary paths).

Figures N5 to N7 represent the two extreme (LIP and HIP) shadow-value functions together with those associated to $\cup$-shaped m-paths[46], the time-invariant cost function (left column) and the corresponding sequences of the inflation rate and the degree of monetization (right column) for $T = 10$. As in every case considered here (1.17) holds, the shadow value function is uniformly higher as individuals postpone the time for stopping

[46] The function $\widetilde{\Psi}_t$ is also depicted (see the Appendix for its definition) to facilitate the graphic inspection of the fulfillment of the necessary conditions for the occurence of $\cup$-shaped n-paths.

their investments in credit-contracts. In the three cases depicted in figures N5 to N7, x^L is equal to 0.1, i.e. hysteresis in m after the reform is possible, and $\alpha^N > \alpha^I$, so that there is an overlap between the LIP and the HIP. The case depicted in the first row (figures N5A and N5B) corresponds to a level of seigniorage sufficiently low so that the only feasible equilibrium corresponds to the LIP. Hence, for a given initial condition for m_0 below 1, m grows and x decreases over the pre-reform period. Figures N6A and N6B represent a situation with an intermediate level of seigniorage falling within the interval (α^I, α^N). Thus, consistently with the above discussion, both the LIP and the HIP are possible equilibrium outcomes. Also, a path along which individuals coordinate their actions to invest in credit-contracts up to $t = 8$ and to stop investing from $t = 9$ on is possible (at the same time, no other ∪-shaped m-path can be an equilibrium). In the case depicted in Figures N7A and N7B, seigniorage is above α^I and it is high enough so as to preclude any path different from the HIP: the prospect of a reform does not help to prevent an all-the-way flight from money. It is worth noticing that the required increase in α to move the economy from the situation in figure 6 to the one in figure 7 may be only a marginal one. The consequences, however, can be dramatic in terms of the potential differences in the observed inflation paths.

Also notice that the inflation rate in the HIP just before the reform, x_T is in all cases below the one observed in the preceding period, so that the inflation rate sequence is not a monotonic function of time along that path. This attenuating effect is driven by the positive (negative) effect of the reform on m_T (x_T). However this feature of the model cannot be understood as being incompatible with the empirical evidence, as a short period of downward adjustment in the inflation rate following its peak and just before the implementation of the reformed were observed in some of the most explosive hyperinflations, as those experienced by Argentina and Germany (see figure 1).

1.5.4 State-contingent reforms

Although the discussion up to here has referred to the case in which the government commits to implement a fiscal-monetary reform at some time T regardless of the history of the economy up to that date, most of the previous key results regarding the (in)effectiveness of an anticipated reform can be easily extended to account for the possibility that the government will only react if things get sufficiently bad, i.e. if the inflation rate threatens to go out of hands. The case of a state-contingent reform is interesting in its own as, indeed, this is the assumption followed by Obstfeld and Rogoff (1983), Nicolini (1996) and Marcet and Nicolini (2003). Those authors consider a simple rule followed by the government for deciding whether to reform or not: to implement a real-backing mechanism at some date t if, absent the threat of such a reform, the inflation rate would violate some finite threshold value $\overline{x}$[47]. Also, as the interest here is to analyze the effects of such a commitment on the occurrence of purely expectations-driven hyperinflations, the natural context to frame this question is one in which, for a given fiscal-monetary pre-reform regime, there is a non-hyperinflationary path (the *no-crisis* or *status quo* equilibrium). This general picture may be embedded in our framework in a simple way by imposing the following conditions

$$\begin{array}{l} \text{Reform at } t \text{ if } x_t^{NR} \geq \overline{x} \text{ and } \left\{x_t^{NR}\right\}_{s=2}^{t-1} < \overline{x} \\ \text{Do not reform at } t \text{ if } \left\{x_t^{NR}\right\}_{s=2}^{t} < \overline{x} \end{array}$$

$$\sum_{s=1}^{\infty} \beta^s p^{s-1} \frac{1-(1-\delta)^s(1-m_0)}{1-(1-\delta)^{s+1}(1-m_0)-\alpha} \leq \theta \tag{1.32}$$

$$\overline{x} < \left[1 - \frac{\delta(1-\gamma)}{1-\eta}\alpha\right]^{-1} \tag{1.33}$$

The first line describes the rule followed by the government to switch from a regime of exogenously targeted seigniorage to one of exogenously targeted (low) inflation rate, i.e. the government abandons the former regime at the first period at which the inflation rate within that regime, x_t^{NR}, would be equal or above some limit $\overline{x}$, exogenously chosen. The inequality in (1.32) imposes an implicit upper bond to the volume of seigniorage targeted in the first regime, such that the LIP is a possible equilibrium[48]. The left side of that inequality corresponds to Ψ_1^N in (1.23) when this function is defined over an infinite horizon, i.e. here we are associating the LIP to the no-crisis (long-run sustainable)

[47] Although they differ in the particular mechanism through which the real-backing scheme is implemented. While Obstfeld and Rogoff assume that the government prevents an excessive depreciation of its currency by offering the possibility of redeeming money giving some productive capital in exchange, in Nicolini and Marcet and Nicolini it is assumed that the government sets the relative price of local money and a foreign currency.

[48] This inequality implies that there is, at least, one no-crisis equilibrium (i.e. the LIP) that is consistent with the seigniorage target, α, being implemented forever, i.e. a scenario in which the threat of the reform is never executed.

equilibrium. Notice that, provided (1.17) holds, this is the relevant measure of the long-run sustainability of the LIP as a potential equilibrium since inflation and the time-t corresponding shadow-value function are both decaying over time along this path. The right side in the inequality in (1.33) is the (limiting) constant inflation rate attained when the degree of monetization is at its minimum possible value, i.e. the stationary state with zero net flow of new credit contracts. Thus, that inequality rules out the possibility that the threshold value $\overline{x}$ is set at a sufficiently high level so as to always preclude the implementation of the reform. Notice that this last sign condition is only relevant if α is low enough to be maintained in the stationary state in which m reaches its minimum possible value, i.e. $\frac{\delta(1-\gamma)}{1-\eta}$. But nothing, in principle, forces the government to choose a α that is sustainable in every contingency, e.g. a particular α may not be implementable by the government over a long period *if* individuals find optimal to fly from money massively[49]. All we are requiring is that α is consistent with the LIP being a possible equilibrium (i.e. (1.32) holds).

Now, in deriving a necessary and sufficient condition for the feasibility of the HIP similar to (1.22), we must take into account that the time of the eventual reform is determined endogenously. Apart from this observation, the problem is similar in its basic aspects to the case with exogenous T. In deriving the counterpart of (1.22), it is helpful to consider the following two steps. First, compute the inflation rate sequence associated to the HIP using (1.21), from $t = 2$ up to the first date, $T^* + 1$, at which the inflation threshold would be violated in the absence of a reform. The solution for T^* is implicitly given by

$$\frac{m_{T^*-1}}{m_{T^*} - \alpha} - 1 < \overline{x} \leq \frac{m_{T^*}}{m_{T^*+1} - \alpha} - 1, \quad \left[m_t = \Gamma^I\left(m_{t-1}\right) \text{ for } t = 2, ..., T^* + 1\right]$$

Second, use T^* to compute $\inf \left\{\Psi_t^I\right\}_{t=1}^{T^*-1}$ using (1.20) and check whether it is above θ.

[49]This illustrates an important feature of this model. For some parameter-configuration, we can find a non-empty set of α's for which the *status quo* equilibrium is a possible equilibrium while the HIP necessarily triggers a reform since, otherwise, the seigniorage-target at some point becomes unsustainable, which in turn means that inflation can reach arbitrarily high values just before the change of regime. Such a set is $\left[\alpha^N, \frac{\delta(1-\gamma)}{1-\eta}\right]$. For some authors, see e.g. Gutiérrez and Vázquez (2004) and Vázquez (1998), this is a desirable property for a model of hyperinflations, since the empirical evidence suggests that these processes may be better understood as being inherently unstable along which inflation explodes without bound at an ever increasing rate, a reasoning which is in clear contrast to what one would observe if a hyperinflation is interpreted as a movement between the low and the high inflation (stationary) equilibrium along an standard hump-shaped Laffer curve.

If this is the case, then the threat of a reform is not sufficient to preclude a speculative hyperinflationary path that is brought to its end, indeed, by implementing the reform.

Clearly, the sign of the influence of the parameters entering in Ψ_t^I on the condition ruling the existence of speculative paths is the same as in the exogenous-T scenario described before. In particular, if $\delta = 1$, given the assumption on the amount of seigniorage in (1.32), then the threat of a contingent-reform will always restrict the set of possible equilibria to one: the constant-inflation LIP. Now, if the government credibly commits to a negative post-reform steady inflation rate x^L, so that $\phi_s = 0$ for $s \geq T^*$ (i.e. no hysteresis after the reform), the reform need not to be implemented in equilibrium, for its sole expectation removes the individuals' incentives to invest in the credit technology, keeping inflation away from the shadow-threshold $\overline{x}$. The formal argument here parallels the one given in Case 1 above, with $\delta = 1$. Feasibility of the LIP, consistently with (1.32), implies that $\Psi_{T^*-1}^I < \theta$ and, hence, $\inf \{\Psi_t^I\}_{t=1}^{T^*-1} - \theta < 0$, i.e. there is no an equilibrium inflation-path that will ever trigger the reform. Notice that (1.32) is a sufficient condition for this result. As $x^L < 0$, then $\Psi_{T^*-1}^I = \beta x_{T^*}^I$. At the same date, the shadow-value function associated to the status quo LIP, $\Psi_{T^*-1}^N$ (computed over an infinite period), satisfies $\Psi_{T^*-1}^N \leq \Psi_1^N \leq \theta$, with $x_s^N > 0$ for $s \geq T^* + 1$ and $x_{T^*}^N \left[= \frac{m_{T^*-1}^{LIP}}{m_{T^*}^{LIP} - \alpha} - 1 \right] > x_{T^*}^I \left[= \frac{m_{T^*-1}^{HIP}}{1 - \alpha} - 1 \right]$, where $m_{T^*-1}^{LIP}$ and $m_{T^*-1}^{HIP}$ are the time-$(T^* - 1)$ stocks of monetization associated to the respective extreme path. From this last equality, it follows that $\Psi_{T^*-1}^I < \Psi_{T^*-1}^N \leq \theta$ and the reform is never executed.

The role of the new parameter, $\overline{x}$, is clear, as well. Notice that in the problem above, the solution for T^* will depend positively on $\overline{x}$, since, given everything else, it takes longer to reach a higher inflation rate. Then, substituting T^* for the exogenous T in (1.20), we learn that a contingent reform will only be effective in removing the expectation of a hyperinflation (provided $x^L > 0$ holds) and, hence, will not need to be implemented *in equilibrium*, only if the public understands that the government will not let the inflation rate to reach high values before intervening.

Thus, putting altogether, what we find here is that a credible commitment by the government to switch from a fiscal-dominance to an orthodox monetary-dominance one when the inflation rate threatens to reach a sufficiently high value will only prevent a speculative hyperinflation and, hence, the regime switch will not take place, if the triggering inflation rate, $\overline{x}$, is not very high or if the government is able to implement a sufficiently restrictive monetary policy so that the economy jumps to the full-level of monetization at the time of the reform, that is, two requirements that are widely violated in real hyperinflationary episodes.

Figure 4 depicts three alternative scenarios. In the first one, with $x^L > 0$, the threshold $\overline{x}$ is set at a sufficiently low level, x^a, so that the reform is never implemented, i.e. $\inf \left\{ \Psi_t^I \right\}_{t=1}^{T^*(x^a)-1} < \theta$. In the second one, maintaining $x^L > 0$, the government is assumed to react only when inflation is sufficiently high, x^b ($x^b > x^a$) so that the eventual reform is effective in stopping the hyperinflation but not in preventing it, i.e. $\inf \left\{ \Psi_t^I \right\}_{t=1}^{T^*(x^b)-1} > \theta$. In the last one, the government commits to a negative post-reform rate, $x^L > 0$, so that regardless the particular $\overline{x}$ (x^c in the figure), the reform is never implemented.

One can also follow a similar strategy of treating a certain parameter of interest as exogenously given, like $\overline{x}$ above, to shed light on a different channel through which fiscal conditions may affect monetary movements beyond that of an exogenous level of seigniorage collected over the pre-reform period. Regarding the different nature in the fiscal-monetary regime before and after the reforms implemented in several European countries in the interwar period, Sargent (1986; p. 45-46) writes the following:

> The hyperinflations were each ended by restoring or virtually restoring covertibility to the dollar or equivalently to gold. For this reason it is good to keep in mind the nature of the restrictions that the adherence to the gold standard imposed on a government. Under the gold standard, a government issued demand notes and longer-term debt it promised to convert into gold under certain specified conditions, that is, on demand, for notes [...]. More important in practice, since usually a government did not hold 100 percent reserves of gold, a government's notes and debts were backed by the commitment of the government to levy taxes in sufficient amounts, given its expenditures, to make good on its debt [...]. According to this view, what mattered was not the current government deficit but the present value of current and prospective future government deficits.

The money-backing mechanism described in section 1.3.2 clearly resembles the one highlighted by Sargent and, hence, its success in stopping a continuos fall in the real value of money rests on the same factor: the degree of government's fiscal solvency over the post-reform period. Thus a similar argument to the one exploited before would follow: the government threat to implement a backing-scheme conditional on a speculative flight from money will be effective in precluding it if the public understands that the volume of resources that the government can seize to back its currency is not too low[50].

1.6 Concluding remarks

The model presented in this paper aims to reconcile several long-standing views on the causes and dynamics of extreme hyperinflationary processes with the objective of providing a simple theoretical explanation of the main stylized facts observed during those episodes.

A central theme in the paper is how to make three of the most influential stories in the literature compatible with each other and, of course, with the empirical facts we are after. First, a high level of fiscal pressure leading to relatively high levels of deficit monetization is usually identified as a leading cause of hyperinflations. Indeed, stopping a hyperinflation usually involves, among other things, a drastic reduction in seigniorage. Second, it is well understood that a seigniorage-based monetary rule may be compatible with a multiplicity of inflation equilibrium-paths, as this rule leaves the monetary aggregate indeterminate. Private expectations are frequently thought as of being an important mechanism behind hyperinflationary processes, an argument that seems specially attractive to account for one of the most robust stylized facts: the lack of a strong correlation between seigniorage (i.e. the fiscal "fundamental") and inflation over the course of a hyperinflation. Third, for a large class of models widely studied in the previous theoretical and empirical literature (namely, models incorporating a demand for real balances *à la* Cagan), it is difficult to accept an explanation for a hyperinflation based on rational self-fulfilled prophecies, for the government commitment to eventually implement a regime-reform, abandoning the fiscal-dominance regime, and to back its currency with taxes implies, first, that rational speculative paths are not possible, as shown by Obstfeld and Rogoff (1983) and Nicolini (1996), and, second, that higher volumes of seigniorage over that regime would be associated with more severe *disinflationary* processes. The mutual incompatibility of the above

[50] Marcet and Nicolini (2003) also devote some attention to this point, as they argue that the drastic fall in the stock of real balances at the end of the hyperinflationary episodes in the late 80's and early 90's, indeed, could have enabled the Argentinian government to peg its currency to the dollar by reducing the required volume of reserves to do so.

arguments with the hypothesis of rational expectations and with the empirical evidence is not a new result. The solutions given to this problem in the previous literature vary to a great extent. In some cases the rational expectations assumption is abandoned while in others the possibility of a fiscal-monetary reform is left out of the picture or, even, the anticipation of such a reform is blamed for being the cause of a hyperinflation.

The approach taken here follows a different route. Based on some previous works on *dynamic* processes of financial innovation, I study how the persistence in the use of alternative means of transactions affects the effectiveness of an equilibrium selection device similar to the one studied by Obstfeld and Rogoff (1983) and Nicolini (1996). This is a natural question in this context, for the experience of those countries which suffered extreme hyperinflations reveals a high degree of hysteresis in the demand for real money. Using a simple cash&credit model that allows for persistence in the usage of credit, it is argued that the robustness, and the rather extreme implications, of an argument *à la* Obstfeld and Rogoff's follow directly from the assumption of instantaneous adjustment in the private demand for real money (i.e. no hysteresis). Once this assumption is relaxed, the effectiveness of the prospect of a future orthodox reform for ruling out hyperinflationary paths, speculative or "fundamental", is an endogenous outcome. In particular, whether the government's commitment to reform exerts any anti-inflationary effect before the time of its implementation, hinges on a wide array of structural factors and policy choices. For example, a high volume of seigniorage collected over the pre-reform period, a long period of government inactivity and a weak fiscal position after the reform are likely to set down the necessary conditions for a hyperinflation. Further, when those variables reach "too high" values the promise of a low-inflation future may well be totally disregarded by the public: no credible threat to reform will preclude a continuous flight from money and a hyperinflation will be the unique possible outcome. When, in addition, individuals do not face important barriers to access to the credit-technology and the effects of the investments in this technology extend over a longer horizon, an extreme speculative hyperinflation happens to be a true possibility, even if individuals rationally expect a future drastic reform.

While the model offers a theoretical basis compatible with the view of a hyperinflation as a bubble phenomenon, it also provides some useful guidance in identifying the economic-policy conditions that may lead to such a painful experience, for the conditions under which such paths are possible are not arbitrary, as in some of the previous literature. Namely, only countries that, for some reason, are advocated to rely on seigniorage in a significant amount and/or for a sufficiently prolonged period are likely to put themselves on the knife-edge. On the theoretical side, the model offers a simple resolution to the incompatibility problem among the three popular approaches aforementioned in a way that renders it a useful tool to understand the empirical evidence. And it does so by including an ingredient which can hardly be labelled as unrealistic or empirically irrelevant: the existence of hysteresis in the degree of monetization following the end of the hyperinflation.

Appendix

Proof of the existence of ∪-shaped n-paths when $\alpha \in \left(\alpha^N, \alpha^I\right)$

Let's define $\widetilde{\Psi}_t$ as the time-t shadow-value function associated with the following n-sequence

$$m_s = \begin{cases} \Gamma^I\left(m_{s-1}\right) & \text{for } s = 2, ..., t-1 \\ \Gamma^N\left(m_{s-1}\right) & \text{for } s = t, ..., \infty \end{cases}$$

It can be readily verified that (1.17) is sufficient for $\widetilde{\Psi}_t$ to be bounded by Ψ_t^N and Ψ_t^I, i.e. $\Psi_t^N < \widetilde{\Psi}_t < \Psi_t^I$, for $t = 2, ..., T-1$[51]. Let's first consider the case in which $\widetilde{\Psi}_2 \leq \theta$. As $\alpha \in \left(\alpha^N, \alpha^I\right)$, we learn that $\Psi_1^I > \Psi_1^N > \theta$ must hold, where the first inequality follows from (1.17) and the second one holds by construction. Consider a n-path such that $m_t = \Gamma^N\left(m_{t-1}\right)$ for $t \geq 2$. Clearly such a path is an equilibrium one. First, $\widetilde{\Psi}_2 \leq \theta$ implies, by definition, that having invested in the credit technology at $t = 1$,no investing from period 2 on is optimal. Second, the actual shadow-value function at $t = 1$ conditional on investing in that period and no investing from period, Ψ_1, must satisfy $\Psi_1^I > \Psi_1 > \Psi_1^N > \theta$, so it is indeed optimal to invest in the first period. Then, consider the case in

[51] To see this notice that at any t within this interval, $\widetilde{\Psi}_t$ and Ψ_t^N look similarly, as both functions are defined under the assumption that no household is investing in the credit technology from date (inclusive) on. However, as n_{t-1} under $\widetilde{\Psi}_t$ is lower than under Ψ_t^N, (1.17) implies that every element in the equilibrium inflation sequence $\{x_s\}_{s=t+1}^T$ is higher for $\widetilde{\Psi}_t$. When comparing $\widetilde{\Psi}_t$ and Ψ_t^I, notice that n_{t-1} is common for both functions, but n_s for $s \geq t$ is lower under Ψ_t^I, so the corresponding equilibrium inflation sequence $\{x_s\}_{s=t+1}^T$ is higher for Ψ_t^I than for $\widetilde{\Psi}_t$ and, hence, $\widetilde{\Psi}_t < \Psi_t^I$.

which $\widetilde{\Psi}_2 > \theta$, so that $\Psi_2^I > \theta$, as well. As $\underline{\Psi}_t^I < \theta$, there must exist, at least, one date $t_1 \geq 3$ such that $\Psi_{t_1=1}^I \geq \theta > \Psi_{t_1}^I$. Let's denote the lowest possible t_1 as $t_1^{\min}$. As $\widetilde{\Psi}_t < \Psi_t^I$, there exists also, at least, one date t_2, with $3 \leq t_2 \leq t_1^{\min}$, such that $\Psi_{t_1=1}^I \geq \theta > \Psi_{t_1}^I$. Let's denote the lowest possible t_2 as $t_2^{\min}$. Then, by looking forward, not investing from period $t_2^{\min}$ on is optimal, conditional on having invested in every period $t = 1, ..., t_2^{\min} - 1$. Looking backwards, as $\widetilde{\Psi}_t > \theta$ for $t = 1, ..., t_2^{\min} - 1$, we learn that the actual Ψ_t satisfies $\Psi_t > \widetilde{\Psi}_t > \theta$ for $t = 1, ..., t_2^{\min} - 1$, so it is optimal to invest over that interval when it is anticipated that no new investments will be made from $t_2^{\min}$ on. Thus, there is always, at least, one equilibrium path for any $\alpha \in \left(\alpha^N, \alpha^I\right)$ along which the time-path of m has a $\cup$-shape and, hence, the equilibrium inflation path depicts a $\cap$-shape over the pre-reform period.

Also, notice that if $\alpha^I > \alpha^N$ holds, then there cannot exist any equilibrium with a $\cap$-shaped path for m over the pre-reform period, i.e. situations in which the households coordinate to give up every opportunity for investing in the credit technology up to some date between 2 and $T-1$ and to invest from that particular date on. To see this, notice that a necessary condition for not investing in the first period is that $\Psi_1^N \leq \theta$, and, hence, an equilibrium $\cap$-shaped path for m requires $\alpha \leq \alpha^N$. Also, for investing from some date $\widehat{t}$ on, such that $2 \leq \widehat{t} \leq T-1$, not having invested over the period running from 1 to $\widehat{t}-1$, the following condition must hold

$$\widehat{\Psi}_t \equiv \sum_{s=1}^{T-t} \beta^s (1-\delta)^{s-1} \widehat{x}_{t+s} + \frac{\beta^{T-t+1}(1-\delta)^{T-t}}{1-\beta(1-\delta)} x^L \geq \theta, \;\; \forall t = \widehat{t}, ..., T-1 \tag{A1}$$

where

$$\widehat{x}_{t+s} = \frac{\delta(1-\gamma)\frac{1-\eta^{s-1}}{1-\eta} + \eta^{s-1} m_{\widehat{t}}}{\delta(1-\gamma)\frac{1-\eta^s}{1-\eta} + \eta^s m_{\widehat{t}} - \alpha}, \tag{A2}$$

$$\text{where } \; s = 1, ..., T-\widehat{t} \; \text{ for some } \widehat{t} = 2, ..., T-1$$

$$\text{and } \; m_{\widehat{t}} = 1 - p^{\widehat{t}}(1-m_0)$$

By comparing (1.21) and (A2), and exploiting the fact that the shadow-value functions at any date t are inversely related to m_t, regardless of the evolution of m from that date on, we learn that

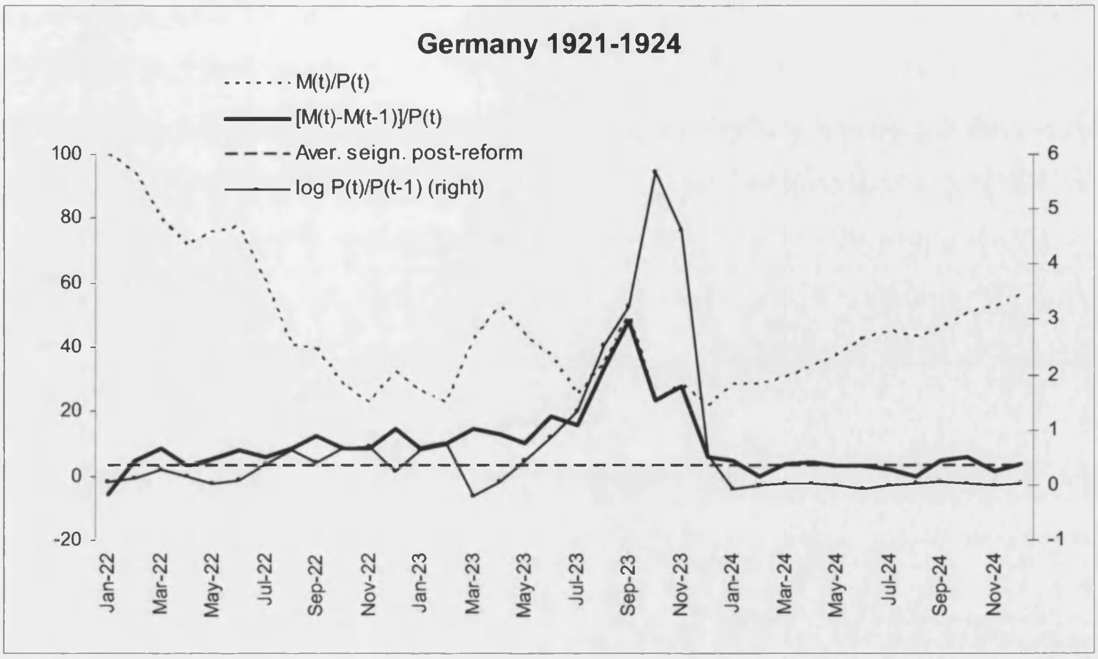
$$\Psi_t^I > \widehat{\Psi}_t, \;\; \forall t = \widehat{t}, ..., T-1$$

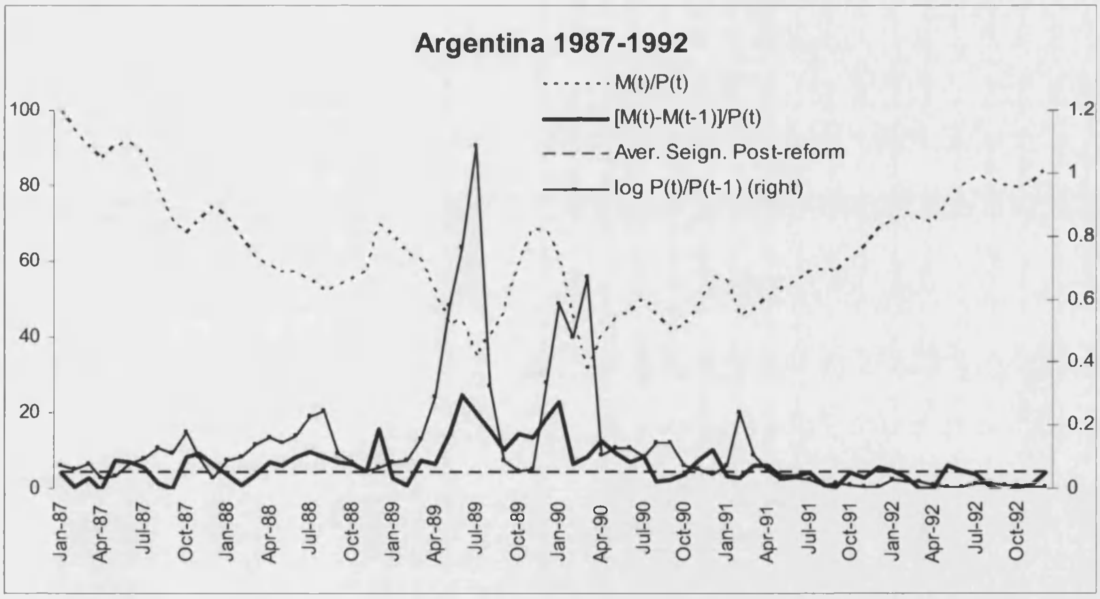
Thus, if the necessary condition for a ∩-shaped m-path (A1) holds and $\underline{\Psi}_t^I$ is located within the time-interval $\widehat{t}, ..., T-1$, then it must be the case that $\underline{\Psi}_t^I > \theta$. A stronger result can be obtained for the alternative case in which $\underline{\Psi}_t^I$ is located before $\widehat{t}$, since, in this case, the positive effect of a lower m on the shadow-value function is reinforced by a positive effect coming from a longer horizon over which the investment in the credit technology is expected to yield positive returns, i.e. strictly positive savings, so that the following inequality must hold

$$\inf \left\{\Psi_t^I\right\}_{t=1}^{\widehat{t}-1} > \sup \left\{\widehat{\Psi}_t\right\}_{t=\widehat{t}}^{T=1} \geq \theta$$

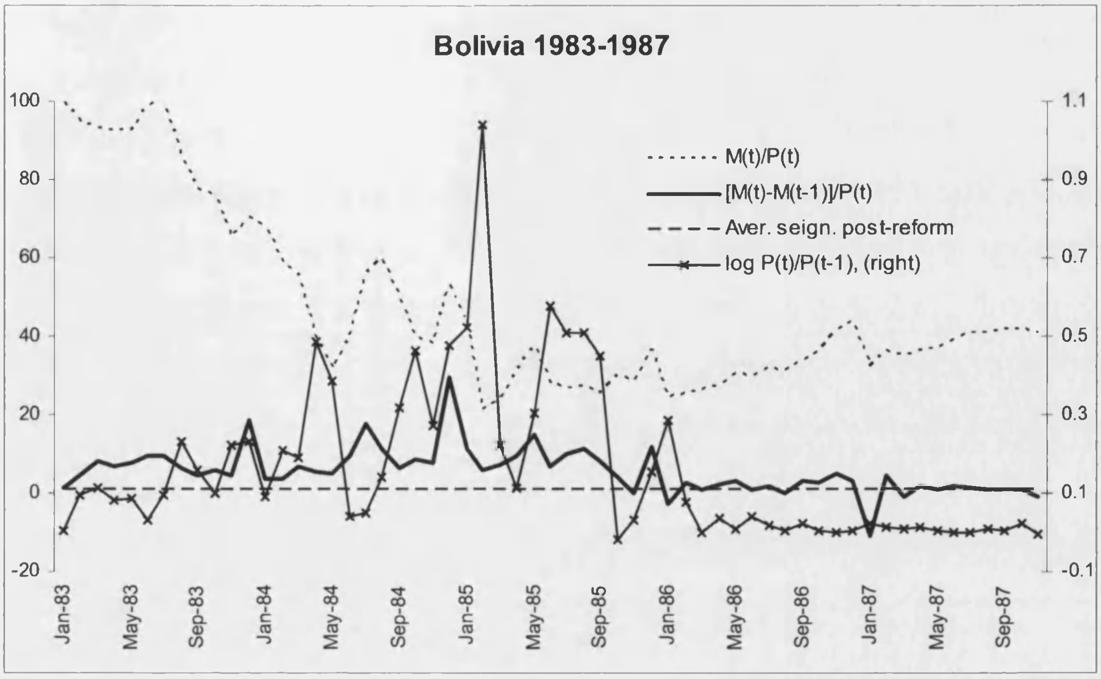
Therefore, regardless of the time-location of $\underline{\Psi}_t^I$, the existence of a ∩-shaped m-path requires a level of seigniorage α, such that $\alpha^I < \alpha \leq \alpha^N$, and, as a result, when $\alpha^I > \alpha^N$ holds, the only class of non-monotonic equilibrium m-paths are ∪-shaped, i.e., as the time of the reform comes closer, its prospect exerts a positive effect on the demand for real balances. *Q.E.D.*

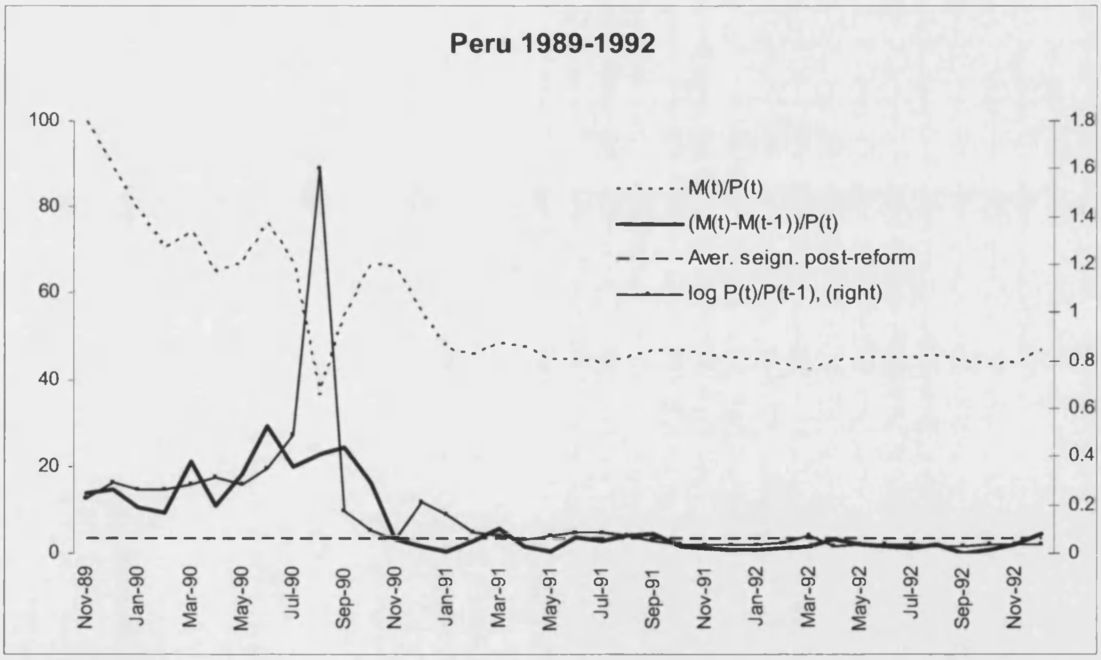
Figure 1: Monetary base, seigniorage and inflation[1]





[1] Left scale: real balances and seigniorage computed using base money (M_0) as percentage of the initial date value for real balances. Average seigniorage in the post-reform regime is computed as the simple mean of seigniorage for the following periods: Germany (Jan 1924 – Dec 1924), Argentina (Apr 1990 - Dec 1992), Bolivia (Mar 1986 – Nov 1987), Peru (Sep 1990 – Dec 1992).





Sources: Germany: Holtfrerich (1986) and Sargent (1986). Argentina: Boletín Estadístico, Banco Central de la República Argentina (several issues). Bolivia: Memoria, Banco Central de Bolivia (several issues). Perú: Boletín Mensual, Banco Central de Reserva del Perú (several issues).

Figure 2. Dynamic Laffer-curve ($\alpha^I < \alpha^N$)

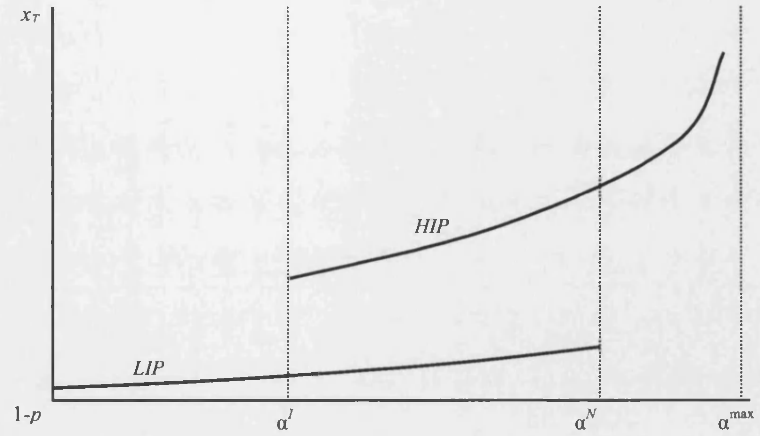
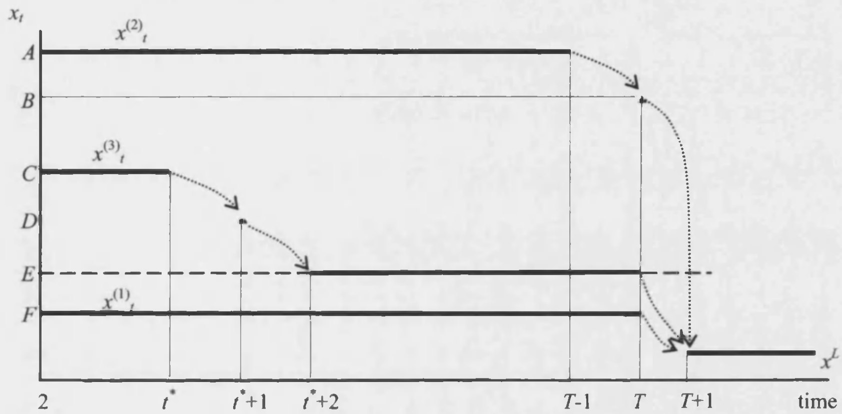
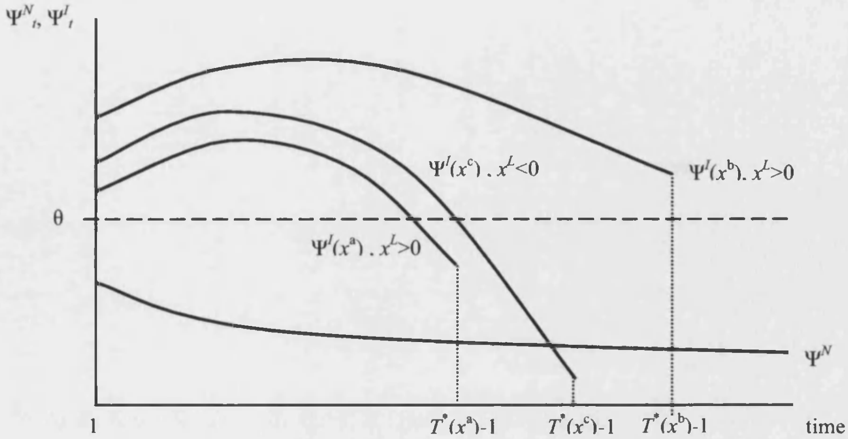


Figure 3. Alternative inflation-paths for $\delta=0$



$A=(1-\gamma)/(1-\gamma-\alpha^{(2)})$, $B=(1-\gamma)/(1-\alpha^{(2)})$, $C=(1-\gamma)/(1-\gamma-\alpha^{(3)})$, $D=(1-\gamma)/(1-\lambda_{t^*-1}\gamma-\alpha^{(3)})$, $E=\theta$, $A=1/(1-\alpha^{(1)})$

Figure 4. Contingent reforms



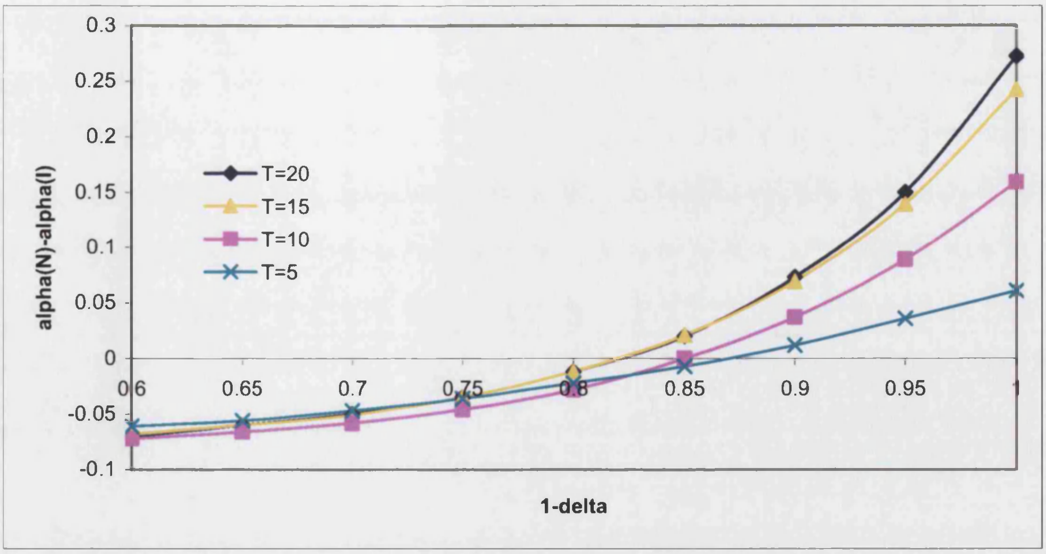


Figure N1: α^N- α^I as a function of credit persistence (1-δ)
Parameter values: $y = 1$, $m_0 = 1$, $\theta = 1.35$, $\beta = 0.8$, $x^L = 0.1$, $\gamma = 0.15$

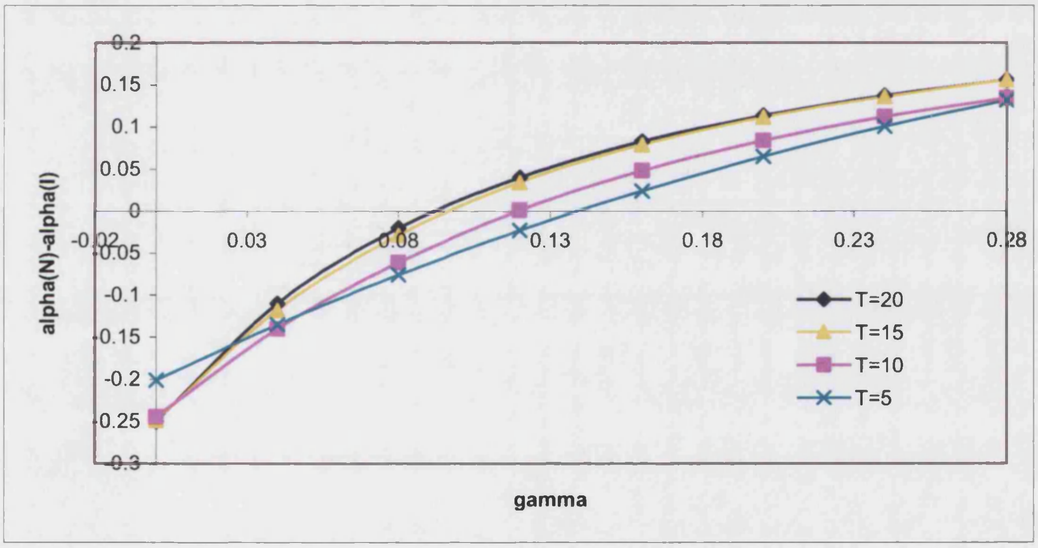


Figure N2: α^N- α^I as a function of γ.
Parameter values: same as in Figure N1 with $\delta = 0.1$

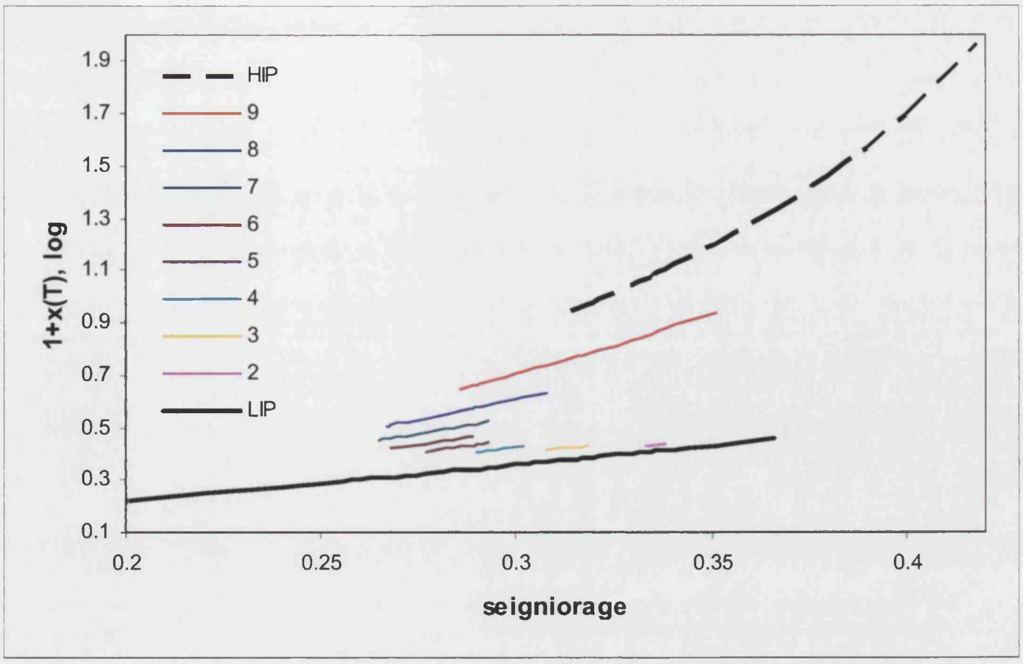


Figure N3: Dynamic Laffer-curve with extreme overlap (x_T as a function of a).
Parameter values: $y = 1$, $m_0 = 1$, $\theta = 1.35$, $\beta = 0.8$, $x^L = 0.1$, $\delta = 0.1$, $\gamma = 0.15$, $T = 10$. Computed (extreme) threshold seigniorage-values: $\alpha^N = 0.366$ and $\alpha^I = 0.314$. *LIP* = lowest inflationary path, *HIP* = highest inflationary path, lines labeled $n = 2$ to 9 represent x_T conditional on $\lambda_t = 1$ for $t = 1,..., n$-1 and $\lambda_t = 0$ for $t \geq n$ (see main text).

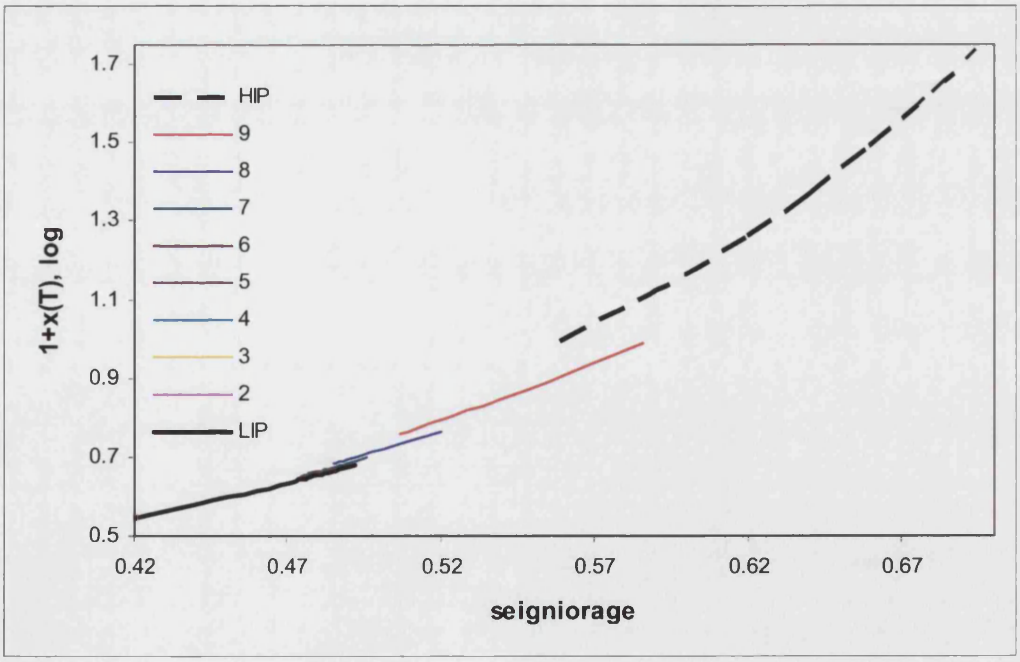
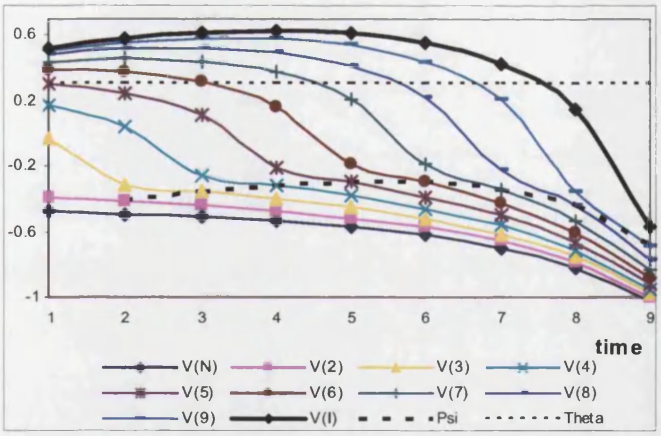


Figure N4: Dynamic Laffer-curve without extreme overlap.
Parameter values: $y = 1$, $m_0 = 1$, $\theta = 1.35$, $\beta = 0.8$, $x^L = 0.1$, $\delta = 0.4$, $\gamma = 0.15$, $T = 10$. Computed (extreme) threshold seigniorage-values: $\alpha^N = 0.492$ and $\alpha^I = 0.552$. Legend as in figure N3.



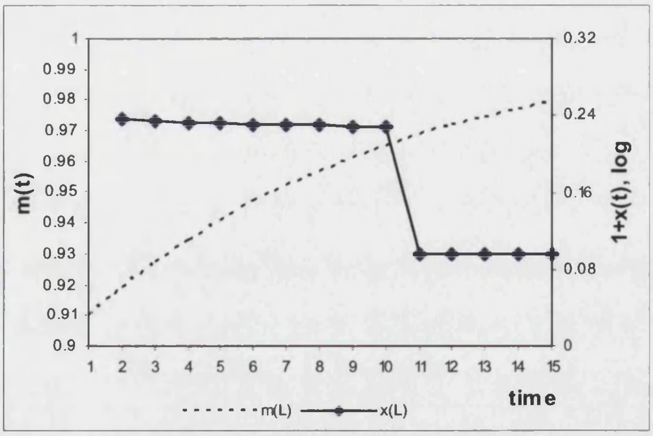
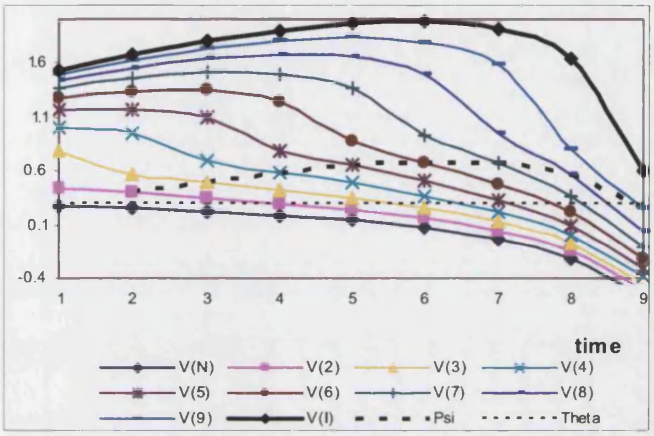


Fig. N5: Shadow-value functions (α=0.20)

Fig. N5A: LIP-Inflation and real balances



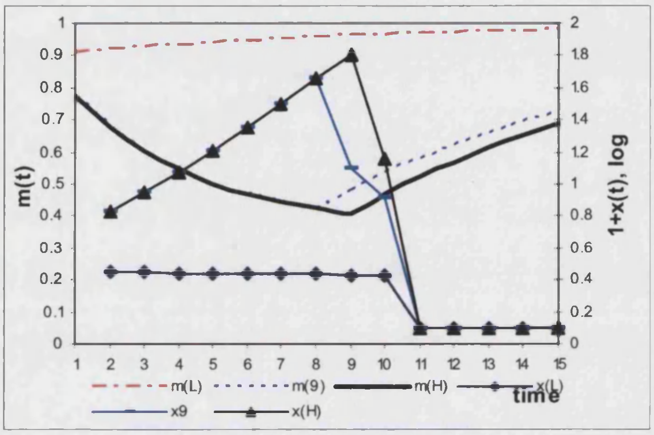
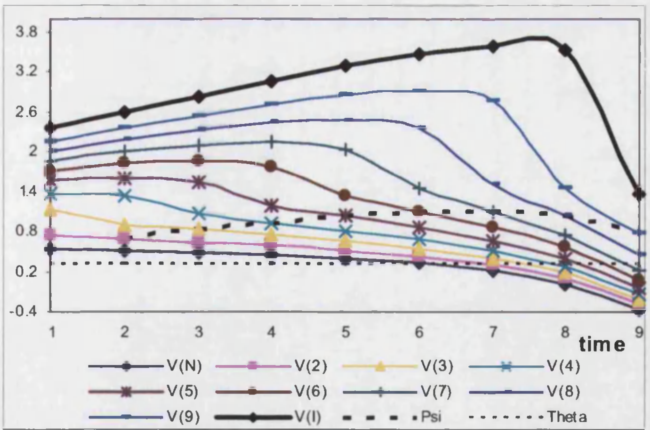


Fig. N6: Shadow-value functions (α=0.34)

Fig. N6A: Multiple inflation and balances paths



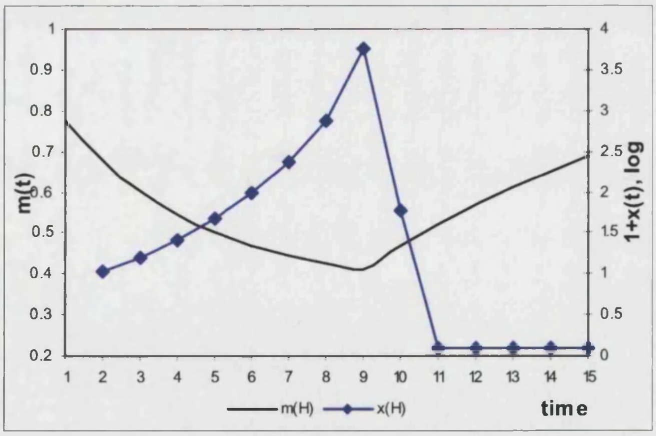


Fig. N7: Shadow-value functions (α=0.40)

Fig. N7A: HIP-Inflation and real balances

Parameter values (Figs. N5 to N7A): $T = 10$, $y = 1$, $m_0 = 0.9$, $\theta = 1.35$, $\beta = 0.8$, $x^L = 0.1$, $\gamma = 0.15$, $\delta = 0.1$. $V(i)$ denotes the shadow-value function associated to the m-path computed as $m_t = \Gamma^I(m_{t-1})$ for $t \leq i-1$ and $m_t = \Gamma^N(m_{t-1})$ for $t \geq i$. $V(N)$ and $V(I)$ denote, respectively, the shadow-value function associated to the LIP and HIP. *Psi* corresponds to the function Ψ_t defined in the appendix.

Chapter 2

The Fiscal Theory of the Price Level: A Narrow Theory for non-Fiat Money

2.1 Introduction

Sargent and Wallace (1981), in a pathbreaking article, showed a natural way in which fiscal and monetary policies are related to each other by highlighting the role played by the government's intertemporal budget constraint. One of the most important messages from the "Unpleasant Monetarist Arithmetic" is that, regardless of the policy regime at work, a certain degree of coordination between fiscal and monetary decisions is always needed, provided the government is willing to honor its policy announcements. Another critical observation drawn from Sargent and Wallace's story is that fiscal variables, such as taxes or public debt, can only affect nominal variables, such as prices, money supply or nominal interest rates, as long as the central bank accommodates its policy to satisfy some fiscal requirements, printing as much money (i.e. collecting seigniorage) as needed to cover a portion of the government's outlays, i.e., when the fiscal-monetary plan is conducted under a fiscal-dominance regime. Overall, in their economy the monetarist dictum goes on: the price level is always a monetary phenomenon, in spite of the eventual fiscal roots of the observed monetary stance.

Over the last decade, a number of economists[1] have challenged the above arguments concerning both the necessity of some degree of monetary and fiscal coordination and the

[1]Some of the seminal works on the FTPL include the following: Leeper (1991), Sims (1994) and Woodford (1995). Other contributions supporting its theoretical foundations are contained in Bassetto (2002), Cochrane (1998, 2003, 2004), Daniel (2003, 2004a), Davig, Leeper and Chung (2004), Davig and Leeper (2005), Gordon and Leeper (2002), Sims (1999, 2002) and Woodford (1998, 2001). Christiano and Fitzgerald (2000), and Kocherlakota and Phelan (1999) contain clear explanations of the basic FTPL's arguments.

monetary nature of the price level, developing the so-called Fiscal Theory of the Price Level (FTPL, henceforth). The cornerstone of this novel theory is the assumption that the government can commit to implement non-Ricardian policies (to be defined later). Taking this assumption as valid forces an important reconsideration of the main results of Sargent and Wallace's. First, under a non-Ricardian policy, the policy coordination problem vanishes: fiscal and monetary policies can be designed in a completely uncoordinated fashion without it meaning that the government violates any budget constraint in equilibrium. Second, the previous monetarist dictum breaks down: under the fiscalist postulates the price level and inflation are, fundamentally, fiscal phenomena, with money playing a secondary role.

The fact that under a non-Ricardian policy the price level is directly influenced by fiscal variables (e.g. the stock of government debt and the sequence of primary surpluses/deficits) can be exploited to design policies which are supposed to remove the classic nominal indeterminacy problem associated with a pure nominal interest rate peg and the multiplicity of equilibria under an exogenous money supply rule. Over the last few years, a growing number of papers have proliferated in which the intellectual framework of the FTPL is employed to provide answers to some practical macroeconomic questions such as liquidity traps, hyperinflations, currency crisis, international monetary policy coordination problems, questions related to the fiscal policy design in monetary unions, the effectiveness of the independent central bank paradigm, etc.[2].

Given the long-reaching implications of the FTPL, the validity of its basic postulates has attracted much attention. Among the advocates of the FTPL, it is usually argued that the government is not limited in its actions by any intertemporal budget constraint, that the real value of fiat government-issued money can be determined in a fiscalist world as the price of a private firm's equity (this is the so-called stock-analogy, defended by Cochrane (2003, 2004), Sims (1999) and Woodford (2001), among others), and, in general, that a

[2]Benhabib *et al.* (2002) study alternative policies, including non-Ricardian ones, to avoid liquidity traps. Loyo (2001) describes some hyperinflationary episodes in Brazil using the postulates of the FTPL. Daniel (2001b, 2004b), Mackowiak (2003) and Corsetti and Mackowiak (2003) explain currency crisis along the fiscalist arguments. Daniel (2001a), Dupor (2000) and Sims (1997) explore the implications of the FTPL in an open economy. Bergin (2000) and Andrés *et al.* (2002) discuss the fiscal-monetary coordination problem in a monetary union in light of the FTPL's arguments. Afonso (2002), Canzoneri *et al.* (2001) and Sala (2002) discuss the empirical relevance of the FTPL.

non-Ricardian policy is theoretically as valid as it is a Ricardian one, with the assumptions underlying the former being neither unfeasible nor unrealistic (e.g. Cochrane (2005), Daniel (2003, 2004a), Woodford (1998, 2001)). Some critiques of the FTPL point towards the implausibility of non-Ricardian policies, since these policies imply that the government is allowed to violate its intertemporal budget constraint at off-equilibrium prices (Buiter (1999, 2002)), the incompatibility of non-Ricardian policies with the requirements imposed by the Walrasian competitive equilibrium concept (Marimón (2001)), the possibility of government default under a non-Ricardian fiscal policy (Cushing (1999)) or the unrealistic assumption on the existence of a non-zero stock of nominal government-issued assets at the "beginning of the world" (Niepelt (2004)).

Bassetto (2002) suggests that much of the current confusion about the FTPL is mainly due to two particular reasons. First, he argues that the Walrasian framework employed in all the previous papers in this area is not the appropriate one to deal with a theory which critically depends on some assumptions about the off-equilibrium behavior of the economic agents. Second, he makes the point that the usual way in which non-Ricardian policies are defined is not a "correct" one. Bassetto frames a simple non-monetary model, exploiting the so-called cash-less economy assumption, into a non-Walrasian economy. His main results are summarized in the following passage: "*I show that there exist government strategies that lead to a version of the fiscal theory, in which the price level is determined by fiscal variables alone. However, these strategies are more complex than the simple budgetary rules usually associated with the fiscal theory* [...]".

In this paper, I identify a minimum set of conditions that guarantee the internal consistency of the FTPL's arguments, under the assumption that the government commits to maintain a nominal interest rate peg, supplying as much money as demanded by the private sector, a policy that has received much attention in earlier works in this area. Some of the arguments developed here resemble one of the observations made by Bassetto: the design of a fiscal-monetary program in the spirit of the FTPL's non-Ricardian policies is not as straightforward as it usually assumed. However, this paper departs form Bassetto's one in two main respects. First, I allow individuals to hold money, by introducing real balances as an additional argument in the utility function and, second, I cast my arguments in a standard Walrasian framework widely employed by an important number of previous authors (both proponents and opponents of the FTPL). The first departure is rooted on an obvious observation: in an economy in which money does not exist, one cannot get any meaningful result on how the real value of money is determined. The second departure is of technical nature. Firstly, I argue that the analysis of the fundamental "weak points" of the FTPL can be carried out within a Walrasian framework, for some critical questions in this analysis, such as the fulfilment of flow of funds constraints and individual rationality, are not specific of the equilibrium concept employed or of the particular assumptions about how prices are formed. Secondly, I show that, under some conditions, the government can credibly commit to maintain some exogenously targeted fiscal-monetary policy which is consistent with a unique equilibrium price sequence, in the spirit of the FTPL, by implementing a price-contingent fiscal plan in a standard Walrasian economy.

I first argue that the validity of the FTPL's argument needs a mechanism linking government policies, individual's optimal decisions and the price level and that the standard definition of a non-Ricardian policy *assumes* that such a mechanism exists without *proving* it. Moreover, I show that, according to the most basic arguments of the standard FTPL, such a theory can never perform as a policy-based equilibrium selection device in the context of the economy and the policy rule studied here. In doing so, I add a number of new arguments to the existing critiques against the FTPL. Then I identify a minimum set of conditions that allow the government to credibly commit to a fiscal-monetary plan which results in a unique equilibrium by creating a link between *prices*, *policies* and *individual optimal behavior*. A critical necessary condition for a credible government policy, that includes a monetary plan not involving a direct control of a monetary aggregate, to yield a unique equilibrium price sequence is that the equilibrium level of seigniorage cannot be positive, i.e., except in the particular case in which seigniorage is equal to zero, the government must redeem a fraction of the existing stock of money in exchange for consumption goods. This observation implies that the fiscalist arguments under a nominal interest rate peg, as they are usually presented in the literature, are not generally consistent with the notion of fiat (i.e. non-convertible) money, or, equivalently, a properly defined non-Ricardian plan requires dollars (rather than bonds) to be net wealth. This is, perhaps, the main result of this paper. Finally, I also provide a critical revision of the general validity of the comparison between the FTPL's arguments and the standard financial asset-pricing theory. In particular, I argue that there is no analogy between the standard FTPL and the asset-pricing theory, although there is a similarity between the mechanism underlying the determination of the real value of a firm's stock and the one underlying the determination of the equilibrium price sequence under a credible (i.e. with convertible money) non-Ricardian fiscal-monetary plan.

The paper is structured as follows. Section 2.2 describes the general set-up of the monetary economy studied here and presents a basic "canonical" example of a non-Ricardian fiscal-monetary program. Section 2.3 explores in detail the problems associated with the standard definition of a non-Ricardian policy using a simple one-period model. Section 2.4 describes a set-up in which the government can credibly commit to implement non-Ricardian policies. Section 2.5 deals with the fiscalist stock-analogy. Section 2.6 contains the extension of the main results to a multi-period economy. Section 2.7 summarizes the main conclusions of the paper.

2.2 The canonical non-Ricardian argument

In this paper I consider a simple representative agent model with exogenous and constant endowments of consumption goods. Time is discrete and there is no uncertainty. Money is introduced as an argument in the household's utility function. Thus, in its basic aspects, the economy studied here is similar to the ones considered by some advocates (e.g. Daniel (2003, 2004a), Leeper (1991) and Woodford (1995, 1998, 2001)), and opponents (e.g. Buiter (2001, 2002), Cushing (1999), Niepelt (2002) and McCallum (2001)) of the FTPL.

2.2.1 The model

Households form their expectations rationally and try to maximize the following objective function[3]

$$\max_{\{c,M\}} \sum_{t=1}^{\infty} \beta^{t-1} \left[u\left(c_t\right) + v\left(M_t/P_t\right) \right] \tag{2.1}$$

where u and v are increasing and concave functions in their respective arguments and satisfy the Inada conditions. The function u captures the utility enjoyed by the household from consuming a certain amount of goods, c_t, and v stands for the liquidity services provided by real money holdings, i.e. nominal balances, M_t, deflated by the general price level, P_t. The parameter β is the subjective discount factor and satisfies $0 < \beta < 1$.

The representative household's flow budget constraints are

$$\frac{B_t + M_t}{P_t} \leq y - \tau_t - c_t + \frac{\left(1 + i_{t-1}\right) B_{t-1} + M_{t-1}}{P_t} \quad (t \geq 1) \tag{2.2}$$

where y is the (positive) exogenous household's endowment of consumption goods, τ_t is a lump-sum tax payable to the government; B_t is the amount of dollar-denominated government bonds which are assumed to mature one period after issued. Each bond issued at $t-1$ is sold at a price of one dollar and redeemed at a value equivalent to $1+i_{t-1}$ dollars at t. The household may also be endowed with an initial stock of financial wealth: some dollar-denominated government bonds, inclusive of interest, $\left(1 + i_0\right) B_0$, and some money balances, M_0. The value of each of these two terms is given exogenously and assumed to be non-negative. Also, the households are precluded from issuing money, i.e. $M_t \geq 0$. For the ease of the exposition, it is assumed that the households do not issue debt either[4], i.e. $B_t \geq 0$. In choosing a path for lifetime consumption and money holdings, the household is restricted by the following transversality condition

$$\lim_{T\to\infty} \left(M_{T-1} + \left(1 + i_{T-1}\right) B_{T-1}\right) / \prod_{s=1}^{T-1} \left(1 + i_s\right) \geq 0 \tag{2.3}$$

[3] In some of the following sections I use some restricted versions of the infinite horizon model described in this section (e.g. one-period and two-periods economies). Those restricted versions can be readily derived as special cases of this more general framework.

[4] This assumption does not affect the generality of the arguments in this paper. See Bassetto (2002) and McCallum (2001) for a similar constraint.

I define the household's utility maximization problem as his choice of a lifetime consumption and portfolio paths[5] $\{c_t, M_t, B_t\}$ satisfying (2.2), the non-negativity constraints on assets holdings, given an initial stock of financial wealth, the price and interest rate sequences $\{P_t, i_t\}$, and the income and tax sequences $\{y, \tau_t\}$. The following conditions are necessary and sufficient for an optimal choice of consumption, money and bonds holdings plans.

When the household behaves optimally, i.e. not leaving any intrinsically valuable resources unconsumed, both the set of budget constraints (2.2) and the limiting condition (2.3) hold as equalities, in which case we learn that the following present-value constraint necessarily holds

$$(1+i_0)\,B_0 + \sum_{t=1}^{\infty} \frac{P_t\,(y-\tau_t)}{\prod_{s=1}^{t-1}(1+i_s)} = \sum_{t=1}^{\infty} \frac{P_t c_t + M_t - M_{t-1}}{\prod_{s=1}^{t-1}(1+i_s)} \tag{2.4}$$

This optimality condition states that the total value of the household's wealth at any period $t \geq 1$, inclusive of maturing government bonds plus current and future endowments net of taxes be equal to the total value of the household's expenditure which includes consumption and seigniorage (i.e. purchases of government-issued money).

The following standard first order conditions complete the characterization of the solution for this maximization problem

$$\frac{v'(m_t) - u'(c_t)}{P_t} + \beta\frac{u'(c_{t+1})}{P_{t+1}} = 0 \tag{2.5}$$

$$\beta\,(1+i_t)\,\frac{u'(c_{t+1})}{P_{t+1}} - \frac{u'(c_t)}{P_t} = 0 \tag{2.6}$$

where $m_t \equiv \frac{M_t}{P_t}$ is the household's demand for real balances. Unless otherwise stated, it is also assumed that the felicity function v satisfies the following inequality

$$\lim_{m\to 0} m v'(m) > 0 \tag{2.7}$$

[5]Notice that there is another choice faced by the household at every period: whether to redeem his entire stock of maturing bonds in exchange for the goods and assets (money and/or newly issued bonds) given by the government. Since maturing bonds are assumed to expire after their maturity date, it will be assumed throughout that the household always redeems his entire stock of bonds.

In view of the above inequality, we learn that speculative hyperinflations are not possible equilibrium outcomes[6].

The government in this economy sets the level of taxes and public consumption and manages the public debt, issuing and redeeming bonds, fixing the nominal interest rate and supplying money endogenously in the amount required by the private sector so as to accommodate the "needs of trade". The consolidated government's sequence of flow of funds constraints is given by[7]

$$\frac{B_t^s + M_t^s}{P_t} = g_t - \tau_t + \frac{M_{t-1}^s + (1 + i_{t-1}) B_{t-1}^s}{P_t} \qquad (t \geq 1) \tag{2.8}$$

where B_t^s and M_t^s are, respectively, the government supply of bonds and money and g_t stands for government consumption and satisfies $g_t < y$. Iterating forward the constraint (2.8) and imposing the following transversality condition

$$\lim_{T \to \infty} \left(M_{T-1}^s + (1 + i_{T-1}) B_{T-1}^s \right) / \prod_{s=1}^{T-1} (1 + i_s) = 0 \tag{2.9}$$

gives the government intertemporal budget constraint

$$(1 + i_0) B_0 + \sum_{t=1}^{\infty} \frac{P_t g_t}{\prod_{s=1}^{t-1} (1 + i_s)} = \sum_{t=1}^{\infty} \frac{P_t \tau_t + M_t^s - M_{t-1}^s}{\prod_{s=1}^{t-1} (1 + i_s)} \tag{2.10}$$

which holds if and only if (2.8) and (2.9) are simultaneously satisfied.

2.2.2 Ricardian and Non-Ricardian policies

Bellow I introduce a standard definition of Ricardian and non-Ricardian policies and examine the implications of alternative fiscal-monetary programs for the equilibrium of the model, using the paradigm of the Walrasian competitive equilibrium. The aim here is to make a description of the implications of these two alternative classes of policies

[6]See Obstfeld and Rogoff (1983). This assumption is made here for simplicity as the main arguments in this paper do not depend on the possibility of rational speculative paths. This class of paths are analyzed in detail in Chapter 3.

[7]In writing this constraint as an equality, I stick to the convention followed by Bassetto (2002) and McCallum (2001), in that the government does not waste resources. The same argument applies to the transversality condition listed below.

in terms of the (in)determinacy of the equilibrium price sequence, as they are usually presented in the previous literature. I use the term "canonical" here to refer to the class of solutions of the model in which it is explicitly assumed that the government always honors its policy-announcements, (i.e. redeeming its eventual outstanding obligations at their contractual value and meeting its targeted sequences of taxes and government consumption). Whether this a sensible assumption to be held under any circumstance or not is at the heart of the current discussion on the validity of the FTPL's postulates, an issue which is analyzed in detail in the following sections.

Definition 1 (non-Ricardian vs Ricardian distinction) *A policy is Ricardian if it is formulated in such a way that the government intertemporal constraint (2.10) is satisfied for any price sequence* $\{P_t\}_{t=1}^{\infty}$. *It is non-Ricardian if it is only satisfied in equilibrium. Equivalently, a policy is Ricardian if the transversality condition (2.9) is satisfied for any price sequence and non-Ricardian if it is only satisfied in equilibrium*[8].

In examining the cases presented below, I focus on the following particular, although widely used in the literature, Ricardian and non-Ricardian policies[9]:

1. Ricardian interest rate peg (RP): the government sets a price-invariant level of public consumption for each period, $\{g_t\}$, and commits to a sequence of non-negative interest rates $\{i_t\}$, which is also assumed to be independent of the observed price-level.

2. Non-Ricardian interest rate peg (NRP): the government chooses the same policy-instruments and money supply rule as in RP and also commits to a price-invariant sequence of taxes, $\{\tau_t\}$.

The definition of a competitive equilibrium for this economy is the following:

Definition 2 *A perfect foresight competitive equilibrium in this economy is a set of allocations* $\{c_t\}, \{g_t\}, \{B_t\}$, *and* $\{M_t\}$, *price and interest rate sequences* $\{P_t, i_t\}$, *and a sequence of taxes* $\{\tau_t\}$ *such that the following conditions are satisfied:*

[8] This definition corresponds to the one given by Woodford (1995). Although Woodford explicitly refers to equation (2.10), the fact that the flow of funds constraint (2.8) holds as an identity allows for an equivalent definition for a non-Ricardian policy based on the transversality condition (2.9).

[9] The main arguments of this paper do not hinge on the particular design of Ricardian and non-Ricardian policies, as it will become clear later. Focusing on the simple policy rules described here, however, adds to the clarity of the exposition. All the sequences are understood to be defined for $t \geq 1$.

1. *The government satisfies its budget constraint (2.8) as an equality in every single period and the transversality condition (2.9).*

2. *Households maximize their utility subject to constraints (2.2) and (2.3), and the non-negativity constraints $M_t \geq 0$ and $B_t \geq 0$, given the sequences $\{P_t, i_t\}$ and the government policy.*

3. *All markets clear in every period, i.e., $y = c_t + g_t$, $M_t = M_t^s$, and $B_t = B_t^s$.*

The Ricardian solution Here I study how equilibrium prices are determined under the hypothesis that the government intertemporal budget constraint (2.10) is always binding, regardless of the value taken by the endogenous variables of the model. The following proposition contains the main results with respect to the (in)determinacy of the equilibrium price sequence.

Proposition 1 *Under the Ricardian policy RP, the are a continuum of equilibrium price sequences.*

Proof. Given the set of government's commitments, a competitive equilibrium is characterized by the following conditions:

(i) The first order conditions (2.5) and (2.6).

(ii) The household's flow of funds constraint (2.2) and the transversality condition (2.3) hold as equalities.

(iii) The government implements price-invariant sequences of public consumption and nominal interest rates, according to the definition of the policy RP given above, adjusting the sequence of taxes so as to satisfy (2.10) and (2.8) for any price sequence.

(iv) All markets (consumption goods, money and bonds) clear.

Let's consider an arbitrary positive and finite initial price level P_1^*. It can be verified that there exists a competitive equilibrium associated with that initial price level. As the government sets the sequence of public consumption without any feedback from the price level, we learn that in equilibrium private consumption satisfies $c_t = y - g_t$ for all $t \geq 1$. Then, the government's commitment to an exogenous sequence of nominal rates implies that, according to the first order conditions (2.5) and (2.6), the optimal household's demand for nominal balances in the initial period $t = 1$, in equilibrium, satisfies

$$M_1\left(P_1^*\right) = v'^{-1}\left[\frac{i_1}{1+i_1}u'\left(y-g_1\right)\right]P_1^* \tag{2.11}$$

Given that particular P_1^*, the government sets a sequence of taxes satisfying (2.10), i.e. the whole tax-sequence satisfies

$$\frac{(1+i_0)\,B_0}{P_1^*} + \sum_{t=1}^{\infty}\frac{g_t}{\prod_{s=1}^{t-1}(1+r_s)} = \sum_{t=1}^{\infty}\frac{\tau_t + \frac{M_t^s - M_{t-1}^s}{P_t}}{\prod_{s=1}^{t-1}(1+r_s)} \tag{2.12}$$

where $r_s = (1+i_s)\left(p_s/p_{s+1}\right) - 1$ is the real rate of return on bonds. Then, for a particular first element of the tax sequence, call it $\tau_1\left(P_1^*\right)$, there is a unique end-of-period stock of government bonds satisfying the budget constraints (2.2) and (2.8) and the market clearing conditions at $t = 1$, given by

$$B_1\left(P_1^*\right) = P_1^*\left(g_1 - \tau_1\left(P_1^*\right)\right) - M_1\left(P_1^*\right) + M_0 + (1+i_0)\,B_0$$

The equilibrium price level for the second period can then be solved uniquely from (2.6), and applying the same steps above, the entire collection of the equilibrium sequences for the endogenous variables (i.e. private consumption, stock of nominal money, stock of government bonds, taxes and prices) can be solved recursively for an arbitrary P_1^*. ■

The main message from this Ricardian fiscal-monetary program is that, provided we assume that the present-value constraint (2.10) must hold regardless of the particular values of the endogenous variables, the government must react to the real value of the initial stock of debt by adjusting the sequence of taxes as required (by (2.12)), so as to avoid default or supersolvency premium on the initial stock of debt. In other words, according to Sargent and Wallace's "game of chicken" parable, as the fully elastic money supply rule at work leaves the equilibrium price sequence undetermined, fiscal policy must "blink".

The non-Ricardian canonical solution The following proposition contains the main implications arising from the assumption that the government implements a plan without any degree of fiscal-monetary coordination, as indicated by the non-Ricardian program defined above.

Proposition 2 *Under the non-Ricardian policy NRP, the equilibrium price sequence is unique provided a sign condition holds.*

Proof. Combining the market clearing and the household's optimality conditions, including the transversality condition (2.3) holding as an equality, we learn that in equilibrium the following equation must hold in the initial period

$$\frac{M_0 + (1 + i_0) B_0}{P_1} = \sum_{t=1}^{\infty} \frac{\tau_t - g_t + \frac{i_t}{1+i_t} m_t}{\prod_{s=1}^{t-1} (1 + r_s)} \tag{2.13}$$

Notice that every term in the RHS of (2.13) is independent of the price level, P_1, under this policy. Some elements are directly fixed by the government (τ_t and g_t), r_s can be derived by combining the exogenously set fiscal instruments together with the market clearing and individual optimization conditions, while M_0 and $(1 + i_0) B_0$ in the LHS are given at the beginning of period 1. It follows that under this set of policy-choices the price level in the initial period, P_1, is uniquely determined by equation (2.13). For this price to be positive and finite, the following sign condition must hold

$$0 < \sum_{t=1}^{\infty} \frac{\tau_t - g_t + \frac{i_t}{1+i_t} m_t}{\prod_{s=1}^{t-1} (1 + r_s)} < \infty \tag{2.14}$$

The above sign-condition is the only restriction placed upon the sequence of taxes and government consumption[10]. Once the price level for the initial period is pinned down, the entire set of sequences of endogenous variables is uniquely determined too. The equilibrium money supply in the initial period, M_1^S, can be obtained from (2.5), then P_2 can be solved uniquely from (2.6), with the supply of government debt in real terms, $\frac{B_1^s}{P_1}$, obeying the sequence of budget constraints (2.8) and so on. ■

[10] For the remaining of the paper, I will assume that this sign condition holds unless otherwise stated.

Clearly, assuming that the economic authorities can commit to a non-Ricardian policy like the one outlined in the previous proposition results in a considerable "gain" in terms of the (lower) number of potential equilibria. Treating the intertemporal constraint (2.10) (or the transversality condition (2.9)) as an independent equilibrium condition rather than as an identity, which must hold always, provides an "extra" equation that can be exploited through the implementation of a non-Ricardian policy so as to solve the "classical" nominal indeterminacy problem described in Proposition 1. The main practical implications, from the perspective of the policy-mix design, arising from Proposition 2 is that fiscal-monetary coordination is not longer needed: if neither player is blinking the price level will adjust.

In providing supportive arguments for the fiscalist (unique) equilibrium constructed before, the proponents of the FTPL justify the interpretation of (2.13) as an additional separate equilibrium condition either by accepting that the government is limited on its actions by an intertemporal budget constraint, but with the extra qualification that the government is a "big player", so that it need not to take prices as given (see Woodford (1998, 2001)), or even by denying that such a constraint exists at all, arguing that (2.13) must be interpreted as a "government valuation equation" rather than as a constraint on its actions. Under this approach, the government can be viewed as a private firm whose equity is priced according to the future stream of profits, where an analogy is drawn between a firm's equity and the nominal-denominated government's obligations (the term $(1 + i_0) B_0 + M_0$), and between the firm's profit stream and the government's total surpluses (the RHS of (2.13)). From this perspective, the plausibility of non-Ricardian policies is accepted as it is the general idea that the future profits of a firm need not to be influenced by the current price of its shares and, therefore, the transversality condition is only satisfied when the stock of the firm is "correctly" valued (i.e. at equilibrium prices). Further, if money and nominal debt can be understood as the firm's stock and, hence, as residual claims to government total surpluses, the issue of default on government-issued assets is irrelevant, simply because it makes no sense to talk about default on shares. Thus, the non-default assumption embedded in the "canonical" case presented here would be plainly justified[11].

2.3 The role of transversality conditions

This section explores the internal consistency of the assumptions underlying the non-Ricardian canonical case discussed in Proposition 2. Some other authors have shown their concern about the validity of the FTPL under an interest rate peg, as well. Buiter argues that, as the government commits to run a completely exogenous fiscal policy, there is no reason why the sign condition (2.14) will hold with generality: "*The most the fiscal theory of the price level therefore could aspire to, when the arbitrary (sign condition) is satisfied, is to be a way of removing the price level indeterminacy characteristic of equilibria under a Ricardian nominal interest rule, when nominal prices are flexible*" (Buiter (1999), parentheses added). Niepelt (2004) shows his concern about the assumption of a non-zero initial stock of nominal assets: "*In this paper, I offer a resolution to this debate. The fundamental problem of the FTPL is that the feasibility of non-Ricardian policy hinges on the assumption of non-zero initial nominal government liabilities. This assumption is not well founded* [...]". Cushing (1999) argues that, in face of an initial stock of government debt, the possibility of default in the first period breaks down the fiscalist uniqueness result: "*In this section, I point out that admitting the possibility of fiscal default shows the price level to be indeterminate under an interest rate rule*".

Notwithstanding the relevance of the above criticisms, the arguments given in this section point towards a different direction. Kocherlakota and Phelan (1999) argue that the FTPL, at its core, contains prescriptions about the government behavior at off-equilibrium prices, an issue which is not testable, thus rendering the question on the validity of the FTPL as an equilibrium selection device "*a religious, not a scientific issue*". Below, I argue that it is indeed possible to analyze the arguments given by the FTPL regarding the behavior of the economic agents in those situations that this theory claims to be off-equilibrium outcomes, even after recognizing that this question is not testable. In doing so, I exploit a feature of a critical element in the model: flow of funds constraints must always hold (in- and off-equilibrium) regardless of the identity, size and behavior of the

[11] A clear exposition of this stock-analogy argument can be found in Cochrane (2005).

corresponding agent. Such a status for the flow of funds constraints, to the best of my knowledge, has not been challenged by any proponent of the FTPL[12]. Interestingly, the analysis pursued here does not need to give such a status to the transversality conditions and conforms the fiscalist view that those conditions are, indeed, only equilibrium conditions. The role of the transversality conditions have been placed at the center of the debate. Treating them as "equilibrium" conditions is at the core of non-Ricardian policies, as stated in Definition 2. For some critics of this theory (e.g. Buiter), this is not an admissible step. By keeping on the interpretation given by the proponents of the FTPL, I aim to provide a clearer and less controversial exposition of the weak points of this theory.

In particular, I show that the equilibrium uniqueness result in Proposition 2 is not an outcome driven by government's policies, but rather by some special assumptions about the households' decisions. In this sense, it can be argued that the FTPL is not a policy-based equilibrium selection device. And this proposition goes on even when the economist does not observe the off-equilibrium implementation of the particular equilibrium selection device at work, something which is necessarily the case when in face of a successful device, like e.g. the one designed by Obstfeld and Rogoff (1983) to rule out speculative hyperinflations.

To keep the exposition as simple as possible, I first consider a one-period model[13], in which the representative household holds an initial stock of financial wealth consisting on some dollars and bonds. As in the general model presented before, the household receives an endowment, y, and tries to maximize his utility by choosing how much to consume, how many dollars to hold at the end of the period and how many bonds, maturing after death, to buy. The government sets the level of taxes, τ_1 and public consumption, g_1. Notice, that within this narrow one-period framework there is no room for the government to commit to a non-zero effective redemption-value for bonds issued this period. However, for the arguments defended here, the critical fact concerning monetary policy is the commitment

[12] An explicit treatment of the government flow of funds constraint (2.8) as an *identity* can be found, for instance, in Bassetto (2002), Cochrane (2005), Davig *et al.* (2004) and Sims (2002).

[13] This simplification can also be found in Christiano and Fiztgerald (2000). Cochrane (2005) also provides some examples in one-period economies.

to elastically supply as much money as demanded, rather than the eventual announcement of a particular next period's redemption value payable by government's bonds[14]. The extension of the main results obtained within this one-period framework to a multi-period setting, where the government announces a particular interest rate is straightforward, as shown later.

The household is constrained in his choices by the following flow of funds and no-Ponzi games constraints

$$c_1 \leq y - \tau_1 + \frac{M_0 + (1 + i_0) B_0 - M_1 - B_1}{P_1} \tag{2.15}$$

$$\frac{B_1}{P_1} \geq 0 \tag{2.16}$$

The optimal behavior of the household is characterized by the following conditions:

$$u'(c_1) = v'\left(\frac{M_1}{P_1}\right) \tag{2.17}$$

$$\frac{B_1}{P_1} = 0 \tag{2.18}$$

Equation (2.17) implies that the individual optimally chooses consumption and real balances equating their respective marginal utilities. Condition (2.18) follows from the "end of the world" assumption: as bonds issued in period 1 mature one period after, no rational agent would be willing to give up some valuable resources for that debt. An additional necessary condition for optimization is that the budget constraint (2.18) holds as an equality.

The consolidated government's budget constraint is

$$\frac{B_1^s + M_1^s}{P_1} = g_1 - \tau_1 + \frac{M_0 + (1 + i_0) B_0}{P_1} \tag{2.19}$$

Regarding the monetary rule at work, it is assumed that the government commits to supply as many dollars as demanded at the ongoing price level.

[14] At this point, it is worth noticing that what drives the multiplicity of equilibria result in the Ricardian case discussed in Proposition 1 is the mathematical property of homogeneity of degree zero in the equilibrium demand for real balances. The one-period framework preserves that feature and, not surprisingly, yields similar "canonical" results as in Propositions 1 and 2.

According to Proposition 2, if the government commits to set τ_1 and g_1 without any feedback from the observed price level, the following equation, which is derived by combining (2.17), (2.18), (2.19) and the market-clearing conditions, gives the unique *fiscalist* equilibrium price level, P_1^F,

$$\frac{M_0 + (1+i_0)\,B_0}{P_1^F} = \tau_1 - g_1 + m_1 \tag{2.20}$$

where $m_1 = v'^{-1}\left[u'\left(y - g_1\right)\right]$. The FTPL's logic for this uniqueness result is, at first sight, simple. Any price different from P_1^F above will result in the violation of an individual optimal condition or/and a market clearing condition, for these are, in principle, the only ingredients included in the characterization of the fiscalist equilibrium (see proof of Proposition 2). In what follows I check the general validity of this argument. In doing so, I follow a simple strategy: pick up an arbitrary price $\overline{P}_1 \neq P_1^F$ and analyze how, if at all, such a price is inconsistent with the three equilibrium requirements listed in Definition 2: (i) fulfillment of government constraints, (ii) individual optimization problem and (iii) market-clearing. To clear the desk, it is convenient to recall the following observation: when dealing with a failure of the market clearing conditions, we can concentrate just on situations in which the markets for goods and for new bonds fail to clear since, given the fully elastic monetary rule considered here, we learn that the supply of dollars is always identical to the demand.

Firstly, let's consider several alternative interpretations about the violation of some of the equilibrium requirements listed above when considering an arbitrary price $\overline{P}_1 < P_1^F$. For the sake of clarity, it is convenient to consider the two central pieces of the intuitive fiscalist argument given above, *market clearing* and *individual optimization*, separately.

(Market clearing). Let's assume that the households never demand a positive quantity of government bonds, regardless of the price level, i.e. $B_1 = 0$ in every contingency. Notice that such a lending rule is always optimal. Given that lending rule, we can solve for the household's goods and dollars demand functions combining the budget constraint (2.15), holding as an equality after imposing a zero-final stock of desired bonds, and the first order condition (2.17). Nothing up to this point presumes an underlying non-optimal behavior of

the households. In face of $\overline{P}_1 < P_1^F$, we see from the government budget constraint (2.19) that the households refusal to purchase new bonds implies that the government cannot attain its fiscal objectives simultaneously, thus, unchaining a potential "crisis". A natural next step is to consider the possibility that the government activates a crisis-resolution device so as to render $\overline{P}_1$ a non-equilibrium outcome, hence, defeating the crisis. There are not many available options for a government that has already committed to some no price-contingent monetary and fiscal policies, apart from printing bonds and trying to sell them. This, assuming is a costless activity, is always feasible for the government. Then, one could think that a positive supply of government bonds, when coupled with a zero demand, creates a problem of excess of supply in the newly-issued bonds market or, alternatively, an excess of demand in the goods market[15]. Let's treat these two markets (bonds and goods) separately.

In the bonds market, the government, as a monopolist, may take two courses of action: it may fix the quantity of bonds, B_1^s, or it may announce the price at which it will meet the demand. This latter form of government intervention is, perhaps, the most popular in macro models, specially when assuming that the government chooses the price of a dollar as the reference-price at which it supplies its bonds[16], say by setting the price of one bond supplied today equal to the price of one dollar (i.e. $\overline{P}_1$; this is the convention followed in this paper). In this case there exists a unique market-clearing quantity consistent with a zero real demand for bonds, $B_1 = B_1^s = 0$. On the other hand, if the government chooses the quantity of bonds to be supplied at each bond-price, say $\frac{1}{Q_1}$, then there is a unique market clearing bond-price[17], $\frac{1}{Q_1} = 0$. Alternatively, we might think of the government offering contracts specifying a given quantity of bonds to be delivered at a particular dollar-price, of the kind (B_1^s, P_1^s), letting the "market forces" determine the

[15] This "disequilibrium" outcome becomes apparent by summing up the household's constraint (2.15), holding as a equality with $\frac{B_1}{P_1} = 0$ and the government's constraint (2.19) after imposing $\frac{B_1^s}{P_1} > 0$ and $M_1 = M_1^s$.

[16] The fact that the government is only choosing the price of reference at which it is willing to supply bonds and not choosing the exact value of that price directly, does not mean that the government is not choosing the supply-price. Private banks usually fix the interest rate at which they are willing to meet the demand for loans by using a reference rate (e.g. LIBOR), which, of course, cannot be understood as the private banks as having their hands free to simultaneously choose interest rates and transacted quantities.

[17] This view on the role of the government in the bonds market is in the same spirit of Cochrane (2005): "The government *auctions* new debt, and accepts whatever price results, as equity issuers auction bundles of contingent claims in Walrasian equilibrium" (italics as in original).

price of each contract. Still, the market for debt-contracts may clear at a zero real value for each contract offered. To put it in plain words, the market for a valueless asset (from the perspective of the potential buyers) may always "clear": either that asset is not traded at all or, if one wishes so, it is transacted at a zero real value.

Now, given that the household never wastes resources purchasing non-performing bonds when these are offered at a positive price, we learn that the government demand for consumption goods satisfies the following equality

$$g_1 = \tau_1 - \frac{M_0 + (1+i_0)\,B_0 - M_1}{\overline{P}_1} \tag{2.21}$$

In view of (2.21), when the government commits to a fixed τ_1, total wealth available for government consumption increases with the price level and vice versa, i.e. a commitment to a given g_1 is only possible if τ_1 varies (inversely) with the price so as to satisfy (2.21) as an identity[18]. Once total wealth available for government consumption is computed correctly, i.e. accounting for the fact that the households will not purchase non-performing bonds, the ability of the government to preclude any $\overline{P}_1 < P_1^F$ as an equilibrium outcome by printing bonds so as to induce an excess-of-demand in the goods market vanishes[19].

In sum, a government's plan to fight a crisis based upon its unlimited ability to print non-performing bonds will not preclude the crisis whenever the households behave in a contingent optimal behavior. Further, when printing bonds is the *unique*[20] alternative government plan, a crisis is unavoidable and the non-Ricardian set of policy commitments becomes non-credible.

(Individual optimization). We may assume that for $\overline{P}_1 < P_1^F$ the households demand

[18] One of Bassetto's (2002) main conclusions is in close connection with this observation: "[...] the strategy I outline above forces the government to increase its taxes in response to a debt crisis; in such an occurrence, not enough resources would be available to pursue the original plan". While he considers the occurrence of a crisis as the outcome of "any (possibly irrational) reason", the crisis I consider here is unambiguously compatible with an individual (contingent) rational behavior.

[19] This idea should be clear: I cannot influence the outcome of the market for diamonds by just saying that I want to buy many of them. As the rest of the agents in that market understand that my resources are not in line with my desires, one shouldn't expect any effect in the price of diamonds from my words. Properly speaking, I cannot demand any diamond at a positive price if I do not have a cent and nobody is (perhaps, rationally) lending me. The same is true for the government. This *demand* versus *desire* distinction is clearly captured in Cochrane's words: "you can't double your demand for Porsches, counting on the price to halve." (Cochrane (2005)).

[20] Whether there are alternative government's plans to prevent crisis and the conditions under which may be implemented is the subject of the next sections.

a strictly positive real amount of government bonds. Then, from the definition of equilibrium (condition (ii)) we learn that such a price-contingent individual behavior can not be compatible with the existence of an equilibrium at that $\overline{P}_1$. That is, any crisis-price inducing households to demand non-performing bonds will never lead to an observed crisis: it is self-defeated by a households' non-optimal behavior.

Still, the following are natural questions when analyzing an equilibrium selection device designed by the government: (i) what is the precise mechanism that renders most prices incompatible with the existence of an equilibrium?, and (ii) how is that mechanism implemented by government? In answering the first question, this "individual optimization" argument offers a simple resolution: when the households face a price $\overline{P}_1 < P_1^F$, they make a suboptimal lending decision, this behavior being the "true" equilibrium rejection mechanism. However, this argument remains silent with respect to the second query (*how is that mechanism implemented by government?*). All we know is that a positive demand for bonds inconsistent with the existence of equilibrium but we do not know the underlying process leading the household to demand useless bonds when the money-price is "too low" and, critically, we can never deduce a mechanism linking the set of feasible government actions to such a private decision for, as argued above, a rule prescribing a zero demand for government bonds is always feasible, no matter what the price level would happen to be. In this sense, a positive demand for bonds is not different from a situation in which the household's constraint (2.15) holds as a strict inequality: in either case an equilibrium (optimality) condition is violated but in neither case can one trace a cause-effect relationship between such a sub-optimal individual behavior and the foregoing money-price level. An equilibrium selection device that rejects some price vectors as potential equilibria by ex ante *assuming* rather than *proving* the existence of a conflict between individual rational behavior and prices cannot then be considered a policy-based device one.

Finally, the case of a potential crisis-price $\overline{P}_1$, such that $\overline{P}_1 > P_1^F$, can be handled exploiting the reasonings followed above for the opposite kind of deviations from the fiscalist unique equilibrium, with an added qualification. Now, when considering eventual violations of the transversality condition associated with positive private borrowing (i.e.

situations in which the households find a positive demand for their own bonds), one cannot invoke an argument based on sub-optimal household's lending decisions since, indeed, not taking advantage of an opportunity for selling bonds, maturing after death, at a positive price cannot be optimal. Further, if we were to allow the government to passively purchase *any* amount of bonds supplied by the households paying an strictly positive price, these would optimally supply infinite amounts of debt, thus precluding any economically meaningful equilibrium[21]. In view of these reflections, it seems natural to impose the no-Ponzi games condition $B_1 \geq 0$ and, similarly, $B_1^s \geq 0$, with both inequalities holding for *any* $\overline{P}_1$, so that there is never a market for privately-issued bonds. This modelling device simplifies the previous analysis to a great extent, as now we do not need to worry about potential violations of the bonds-market clearing condition along which the government tries to implement its original plan by demanding private bonds when faced with an arbitrary $\overline{P}_1 > P_1^F$: we are ruling out those violations *by assumption* and for *any* price level, not just for one, very much like in the Ricardian case in Proposition 1. As argued by Weil (2002), imposing $B_1 \geq 0$ ("*consumers are forbidden to die in debt*") amounts to impose a *feasibility* constraint, rather than an *optimality* constraint. But all we know about feasibility constraints is that they can never be violated, no matter what the price level is or whether such a price can be supported as an equilibrium or not. Hence, a mechanism designed by the government to rule out a crisis-price $\overline{P}_1 > P_1^F$ based upon the promise to transfer any resources above the (unique) amount compatible with the original fiscal plan to the private sector by purchasing private debt is unfeasible and, hence, non-credible.

In sum, the standard FTPL's interpretation of the government transversality condition as a separate equilibrium condition does not shed any light into the question of how the government may combine its fiscal instruments so as to implement a successful equilibrium selection device able to solve the classical nominal indeterminacy associated with an elastic money supply rule. A government strategy of "keeping on the original plan" in view of a crisis will imply, in most cases, a non-credible "threat", for a well-posed demand function cannot be conceived without respecting an underlying budget set. Altering

[21] This argument can be found in Woodford (1998).

the government budget set by printing non-performing bonds is not always possible, for someone else must voluntarily stand on the other side of the market ready to rescue a insolvent government plan. However, accepting such a possibility amounts to assume that the households behave in a non-optimal way when nothing precludes them from making the (contingent) optimal decisions. This is not a good axiom to start with for any theory claimed to be an equilibrium selection device founded on a "suitable" design of fiscal rules.

2.4 The limits of well-defined non-Ricardian policies

Using the simple one-period framework presented above, in this section I argue that a suitable re-design of the class of fiscal policies analyzed in the preceding section, which differs from a completely exogenous fiscal policy rule, can effectively rule out some potential equilibria, very much in the spirit of the standard FTPL. However, price and equilibrium uniqueness requires some extra conditions not considered by the standard FTPL. Specifically, a critical (and rather strong) necessary condition for the government to be able to implement an exogenously targeted equilibrium primary surplus while committing to a fully-elastic money-supply rule is that the fiscal plan must involve a non-positive level of seigniorage.

As shown below, implementation of "genuine" non-Ricardian policies (i.e. policies that create a link between prices and private wealth that break the Ricardian Equivalence Theorem) resolves only a part of the indeterminacy problem associated to an elastic supply of money, although this requires a fiscal policy that reacts at off-equilibrium prices, for the presence of non-Ricardian elements is not a sufficient condition for equilibrium uniqueness. Roughly speaking, genuine non-Ricardian fiscal policies solve "half of the problem", and they do so in a way compatible with the arguments given by the advocates of the FTPL, namely, by using fiscal policy to back money with valuable resources collected by the government, i.e. taxes. By contrast, the other half of the problem, which has been systematically neglected, is related to situations in which money injections should back an insufficient amount of taxes. A solution to this latter problem cannot be given by invoking non-Ricardian fiscal-effects. This requires a much more drastic assumption: the government must plan a fiscal policy sufficiently solvent so as to never need new positive, in net terms, monetary injections.

2.4.1 Ruling out government supersolvency

Below I argue that there is a simple policy rule which removes some potential equilibrium price sequences, namely those associated with prices satisfying $\overline{P}_1 > P_1^F$ in (2.20), which, otherwise, would force the government to increase its consumption, to reduce taxes or to pay a supersolvency premium on the initial stock of debt (i.e. redeeming bonds above par). I first show the basic insights of this argument by using a simplified version of the one-period model presented above in which the households do not have government bonds at the beginning of the period[22]. Then I extend the analysis to a more general case with a positive initial stock of government debt and, in section 2.6, I consider a multiple-period economy.

Consistently with the arguments given in the previous section, I assume that individuals never demand nor supply bonds at the end of the period, i.e. $B_1 = 0$, so that the argument does not give any special role to the transversality condition. Let's consider a price level, $\overline{P}_1$, such that $\overline{P}_1 > P_1^F$ (with P_1^F given in (2.20) after imposing $(1 + i_0)\, B_0 = 0$) and assume that the government sets some exogenous targets for g_1 and τ_1, labelling these targets as $\overline{g}_1$ and $\overline{\tau}_1$. As (2.20) holds as an identity, a price $\overline{P}_1 > P_1^F$ implies that the fiscal targets cannot be attained[23]. For example, if the government insists on consuming $\overline{g}_1$ regardless of the price level then, given that $B_1 = 0\ \forall \overline{P}_1$, taxes must adjust downwards so as to satisfy (2.20). The equilibrium associated with $\overline{P}_1$ would entail the following allocations:

$$g_1 = \overline{g}_1, \quad c_1 = y - \overline{g}_1, \quad M_1\left(\overline{P}_1\right) = v'^{-1}\left[u'\left(y - g_1\right)\right]\overline{P}_1, \quad B_1 = 0$$

with the equilibrium tax , τ_1, satisfying

$$\tau_1 = \overline{g}_1 - \frac{M_1\left(\overline{P}_1\right) - M_0}{\overline{P}_1}$$

[22] This is consistent with Woodford's (1995) analysis of an economy without initial nominal-bonds holdings.

[23] Henceforth, I assume that the households behave in a contingent optimal way, i.e. their demands for goods and dollars obey the optimality condition (2.17) for any price.

Labelling seigniorage as $s(P_1)$, we may express deviations of the actual tax from the target as

$$\overline{\tau}_1 - \tau_1 = s\left(\overline{P}_1\right) - s\left(P_1^F\right) > 0 \Leftrightarrow \overline{P}_1 > P_1^F$$

This adjustment is plainly consistent with the Ricardian argument in Proposition 1: as the government is receiving more seigniorage, $\frac{M_1 - M_0}{\overline{P}_1}$, than needed to implement its original consumption plan, its constraint forces a tax reduction, since, this is the only alternative for the government to dispose of its "extra" income from the Ricardian perspective. In what follows, I study the effects of allowing the fiscal branch of the government to consume that extra income through an alternative mechanism, along the lines of the following assumption.

Assumption 1. The fiscal branch of government can participate in the money market buying money and giving consumption goods in exchange.

I do not try to claim any realism in this assumption, for the aim here is just to uncover the necessary conditions under which fiscal policy may reduce the number of potential equilibria. The necessity of this assumption is discussed later. For the moment, it is important to notice that the action by the fiscal authority described in that assumption is not incompatible with the simultaneous central bank's commitment to supply as much money as demanded so it is not at odds with the view of the two authorities, fiscal and monetary, implementing their policy instruments in a decentralized fashion.

Now, suppose that in an eventual situation in which the fiscal branch of the government would receive an amount of real resources (i.e. taxes plus seigniorage) above the one compatible with its exogenous targets, it commits to inelastically supply a positive quantity of goods equal to the "excess" of resources in exchange for any strictly positive amount of dollars. Then any $\overline{P}_1 > P_1^F$ cannot be part of an equilibrium. The proof of this claim is based on a simple arbitrage argument. Let's suppose that when $s\left(\overline{P}_1\right) > s\left(P_1^F\right)$ the government keeps its targeted tax, so that its constraint now reads as

$$\overline{g}_1 + w_1 \equiv g_1 = \overline{\tau}_1 + s\left(\overline{P}_1\right) \tag{2.22}$$

where w_1 stands for the excess of budget available for government consumption (with respect to its target $\overline{g}_1$). The commitment to bid w_1 in exchange for any arbitrary strictly positive amount of dollars implies that any individual could buy, e.g., a single dollar from the central bank at a real cost[24] of $\frac{1}{\overline{P}_1}$ and then supply that dollar in exchange for the (strictly positive) amount of goods supplied by the fiscal authority, $w_1 + \frac{1}{\overline{P}_1}$. Such an operation gives that individual a windfall gain of w_1 units of consumption[25]. Clearly, the existence of such a free-lunch opportunity is not compatible with the existence of an equilibrium. Thus, given the commitment to dispose of any eventual excess of seigniorage, by accepting dollars in exchange for it, a new necessary condition must be added to the definition of an equilibrium for this economy: for $\overline{P}_1$ to be part of an equilibrium, $\overline{P}_1 - P_1^F$, cannot be positive.

It is worth noticing that this condition implies that at off-equilibrium prices government consumption, $\overline{g}_1 + w_1$, varies with the price, which, in turn, means that available private wealth and, hence, consumption demand, depend on the price, thus creating a link between prices and optimal decisions. In other words, this fiscal price-contingent strategy creates well-defined price-based non-Ricardian wealth effects[26].

It remains to check that such a government's commitment is a credible one. Clearly, this is the case here, as this commitment only implies that the government makes a "gift" to the households in case these were to pay a level of seigniorage above the (unique) one compatible with the fiscal targets. Further, as such a commitment can be made in a credible way regardless of the particular off-equilibrium price, it provides a "valid" extra condition which helps to rule out some equilibria.

The extension of the previous argument to the case in which the households have a positive stock of government debt at the beginning of the period is straightforward. In that case, the government does not only target a particular value for taxes and government consumption but also aims to redeem its existing debts at their contractual value.

[24] Let's assume, for simplicity, that this marginal increase in the total nominal demand for dollars does not have any significant effect on the price level, so that this additional dollar can be purchased at the original price, $\overline{P}_1$.

[25] Notice that the original "excess of seigniorage" in hands of the fiscal authority, $s(P_1) - s(P_1^F)$, is augmented by $\frac{1}{\overline{P}_1}$, i.e. the extra seigniorage received by the sale of the additional dollar used in this arbitrage operation.

[26] Notice that the assumption of an economy populated by identical households does not automatically implies the fulfillment of the Ricardian Equivalence Theorem, for such a result breaks down as long as government consumption is not constant (see, e.g., Blanchard and Fischer (1989)).

According to the arbitrage argument given above, any commitment to give up the excess of seigniorage with respect to the targeted level will be credible, where the targeted level of seigniorage now satisfies,

$$s\left(P_1^F\right) \equiv v'^{-1}\left[u'\left(y-\overline{g}_1\right)\right] - \frac{M_0}{P_1^F} = \frac{\left(1+i_0\right)B_0}{P_1^F} + \overline{g}_1 - \overline{\tau}_1$$

Hence, such a price-contingent fiscal policy will remove any potential equilibrium price in which the government would be forced either to pay a supersolvency-premium on its initial debt or to decrease the level of taxes, i.e. any $\overline{P}_1 > P_1^F$. However, notice that the wealth effect induced by this fiscal strategy does not arise directly from government bonds, rather, it is associated with dollar holdings: *government dollars are net wealth.*

Why is Assumption 1 necessary in this argument? In this economy, there are four ways for the government to dispose of excess "unwanted" resources: paying a supersolvency premium on the initial stock of debt (i.e. violating its commitment to redeem bonds at par), consuming above the target, reducing taxes below the target and purchasing money paying a price above the market one[27]. Except the last one, any other distribution policy will imply the violation of a fiscal commitment, however without necessarily violating any equilibrium condition, as in the Ricardian case. It is worth noticing that a policy that makes total private outside wealth dependant on the price level need not deliver price uniqueness (e.g. a passive adjustment of government consumption). In this sense breaking the Ricardian Equivalence result is not sufficient to yield a unique equilibrium, for that policy must also generate arbitrage opportunities[28].

2.4.2 The asymmetry between government default and supersolvency

Can we apply a similar arbitrage argument to rule out equilibrium prices, $\overline{P}_1$, such that $\overline{P}_1 < P_1^F$? The answer depends critically on the sign of $s\left(P_1^F\right)$, i.e. the targeted level of seigniorage required to avoid an upward adjustment in the primary surplus.

[27] In the argument developed here, the government does not commit directly to pay a particular price when redeeming money but, by committing to sell a given amount of consumption goods in exchange of any quantity of dollars, it is indirectly allowing for the possibility of selling goods at a zero net price.

[28] This observation resembles one of the conclusions of Cushing (1999). He allows for non-Ricardian households in an OLG model concluding that such a departure is not sufficient for equilibrium uniqueness under a pure interest rate peg.

Firstly, assume that $s\left(P_1^F\right) > 0$, and, as before, let's first consider the case with zero initial debt, so that a positive level of targeted seigniorage, $s\left(P_1^F\right)$, goes in hand with a targeted primary fiscal deficit, i.e. $\overline{g}_1 - \overline{\tau}_1 > 0$. Now, a price $\overline{P}_1 < P_1^F$ cannot be ruled out as a candidate equilibrium price exploiting the non-Ricardian argument outlined above. To see this, notice that when the price $\overline{P}_1 < P_1^F$ and the government consumes $\overline{g}_1$, the level of seigniorage actually paid by the households falls below the targeted one, i.e. $s\left(\overline{P}_1\right) - s\left(P_1^F\right) < 0$. The government can only meet its fiscal targets by inducing individuals to pay the targeted level of seigniorage which, in turn, implies that in face of $\overline{P}_1$, it should be able to engineer a policy aimed at depreciating the real value of money, something which is only possible if it manages to increase money supply. In the previous case, for a price $\overline{P}_1 > P_1^F$, the government was able to "defeat" the ongoing market price, $\overline{P}_1$, by inelastically supplying an amount of goods equal to $s\left(\overline{P}_1\right) - s\left(P_1^F\right) > 0$ in exchange for money. Symmetrically, to rule out a market price $\overline{P}_1 < P_1^F$, the government should be able to inelastically supply an amount of money equal to $M\left(P_1^F\right) - M\left(\overline{P}_1\right) > 0$, in exchange for consumption goods, where $M\left(P_1^F\right)$ and $M\left(\overline{P}_1\right)$ are the stocks of money satisfying (2.17) for P_1^F and $\overline{P}_1$, respectively. Such a policy announcement, however, is not consistent with the assumed commitment to meet the ongoing demand for nominal balances in a completely elastic fashion. Equivalently, a negative w_1 in (2.22), holding the target $\overline{g}_1$, can only be corrected by increasing τ_1 or abandoning the monetary rule.

Notice that the inability of the government to put upward pressure on the price level in the present case is not a consequence of the strict separation of competencies between the fiscal authority and the central bank, rather, it is a natural result arising from the specified money supply rule. Even if we think of a central bank fully subordinated to the dictates of a fiscal authority facing financial difficulties, there is not any arrangement capable of providing individuals with the right incentives to pay a higher level of seigniorage under this monetary rule. Therefore, an arbitrary price $\overline{P}_1 < P_1^F$ can be an equilibrium outcome, with the fiscal authority being forced to either increase taxes or decrease government consumption (or a combination of both). It follows that in this simple setting without initial debt holdings, the government cannot credibly commit to run an arbitrary primary deficit under any circumstance.

As before, extending this nominal indeterminacy result to the more general case in which there is a positive stock of initial government's debts is straightforward. When the standard fiscalist solution in (2.20) calls for a positive level of seigniorage, i.e. $s\left(P_1^F\right) > 0$, there is no guarantee that the government will always be able to avoid default if it insists on a particular level of primary surplus.

The reason for the asymmetry between the default and supersolvency cases when $s\left(P_1^F\right) > 0$ is very intuitive. Ruling out default would need a violation of the money supply rule, that is, a critical departure from the context within which the FTPL claims to yield price uniqueness. Ruling out a supersolvency premium only requires a commitment from the government to distribute any excess of seigniorage by purchasing dollars and, as argued before, such a commitment can always be made in a credible way. Moreover, such a fiscal strategy is not incompatible with the commitment to sell as many dollars as demanded at the market price. Put it other way, the ability of the government to rule out supersolvency is possible because for a price such that $\overline{P}_1 > P_1^F$, it enjoys a strong financial position, in the sense that it is collecting more resources than needed to go ahead with its original plans. On the other hand, when $\overline{P}_1 < P_1^F$, the government faces a weak financial position in that it will be unable to meet its plans at that price. For a particular agent, whether a household or an economic authority, to be able to "defeat" a potential market equilibrium outcome, that agent must be able to generate arbitrage opportunities at the ongoing market price, which is only possible if that agent is willing to face losses in trading with the different assets and goods. The later, using the above terminology, requires that agent to be in a strong financial position. In the next section, I provide further intuition on this asymmetry in the context of a private firm.

2.4.3 The necessity of convertible (non-fiat) money

In this simple one-period model, an obvious way to circumvent the multiplicity of equilibria problem arising when $s\left(P_1^F\right) > 0$ is to further restrict the fiscal-monetary program so that it satisfies $s\left(P_1^F\right) \leq 0$. This non-positive seigniorage condition can then be exploited to rule out prices satisfying $\overline{P}_1 < P_1^F$, as potential equilibria, provided the government sets an upper bound to the (negative) volume of seigniorage, as stated in the following assumption[29].

Assumption 2. The central bank commits to supply any amount of money at the ongoing market price level but does not commit to symmetrically purchase any amount of money at any market price.

Then, only if $s\left(P_1^F\right) \leq 0$ and Assumptions 1 and 2 hold, it follows that there is unique equilibrium price, which coincides with the fiscalist's standard solution, P_1^F in (2.20), provided the government commits to the appropriate price-contingent fiscal rule. The proof for this claim is simple given the previous results. Given that the government can credibly commit to distribute any excess of seigniorage by accepting dollars in exchange for consumption goods, any price $\overline{P}_1 > P_1^F$ can be ruled out as an equilibrium price. On the other hand, any price $\overline{P}_1 < P_1^F$ would be associated with a negative level of seigniorage higher, in absolute value, than the targeted one, $s\left(P_1^F\right)$. But given Assumption 2, for such a price to be part of an equilibrium, the households must expect a transfer or resources from the government greater than the one compatible with the fiscal objectives. As the government is not obliged to redeem money for an arbitrary amount of consumption goods, such individual's beliefs are not rational and, hence, prices consistent with those beliefs can be ruled out. The necessity of Assumption 2 for this uniqueness result is clear, as well.

Yet, it must be recognized that the latter class of deviations from the equilibrium price level (i.e. prices below P_1^F) are not corrected by means of a any non-Ricardian wealth effect. Indeed, government consumption may be kept at the targeted value along those deviations. Instead, the driving force to rule out those prices is a pure convertible-asset valuation argument. It is worth providing some intuition about the necessary condition for the above uniqueness result, $s\left(P_1^F\right) \leq 0$, relating it to the economic concepts of *convertible* and *fiat* money. Condition $s\left(P_1^F\right) \leq 0$ implies that, except in the particular case in which $s\left(P_1^F\right) = 0$, money must be a convertible asset, in the sense that the government is actually retiring money from circulation, i.e. $M_1\left(P_1^F\right) < M_0$, something which is only possible by giving up some valuable resources (i.e. consumption goods) in exchange

[29] Again, the aim here is not to claim the realism of this assumption but rather to impose a minimum set of conditions characterizing the institutional framework within which the FTPL's basic postulates are consistent. As argued later, this assumption is necessary for equilibrium uniqueness.

for it. Taken literally, that condition forces a reconsideration of the nature of money. When $s\left(P_1^F\right) < 0$, money is not longer a purely fiat asset whose real value is exclusively driven by individual expectations on the future acceptability of such an asset by other private agents, but rather, as the fiscal branch of the government repurchases money, its equilibrium real value must also reflect the value of those consumption goods being given in exchange for it. Further, this "dual-value of money" argument also applies in the limiting case in which the government "gives" zero units of consumption goods in exchange for a dollar (i.e. $s\left(P_1^F\right) = 0$ and money is neutral from a fiscal perspective). Notice that price uniqueness in this particular case follows from the assumption of no-speculative hyperinflations. If $\lim_{m \to 0} mv'(m) = 0$, then speculative hyperinflationary paths cannot be ruled out unless the fiscal authority commits to a price-contingent real backing scheme as in Obstfeld and Rogoff (1983), i.e. even in this case *money must be convertible when the Walrasian auctioneer were to dictate a infinite price level.* Such a commitment can be readily included in the fiscal authority's set of contingent actions without altering substantially the characterization of the unique equilibrium just described, since that commitment implies that convertibility never takes place *in equilibrium.*

The role of the necessary conditions for the uniqueness result (Assumptions 1 and 2) is a very intuitive one (as also discussed in the next section). Assumptions 1 and 2, when considered jointly, imply that turning money into a convertible asset is at the full discretion of the government through its fiscal choices. This condition is nothing but a reflection of the high degree of "fiscal dominance" that is necessary for a theory which tries to give fiscal policy a first order importance role in the determination of the price level, as the FTPL does.

The following proposition summarizes the main previous results.

Proposition 3 *In the one-period economy described above, when the government targets a particular primary surplus,* $\overline{\tau}_1 - \overline{g}_1$ *and aims to redeem the initial stock of debt at the contractual value, and simultaneously follows a completely elastic money supply rule, the equilibrium price,* P_1^e, *satisfies the following conditions:*

(i) If Assumption 1 holds and the government commits to inelastically supply any

excess of seigniorage, $s(P_1) - s(P_1^F) > 0$, *in exchange for money, then* P_1 *cannot be part of an equilibrium. Any equilibrium price must satisfy* $P_1^e \leq P_1^F$, *where* P_1^F *is defined as*

$$P_1^F = \frac{M_0 + (1 + i_0) B_0}{\overline{\tau}_1 - \overline{g}_1 + v'^{-1} \left[u' \left(y - \overline{g}_1 \right) \right]}$$

(ii) If the targeted level of seigniorage satisfies $s(P_1^F) > 0$ *and* $P_1^e < P_1^F$, *then the original fiscal-monetary plan cannot be implemented.*

(iii) If Assumptions 1 and 2 hold, the targeted level of seigniorage satisfies $s(P_1^F) \leq 0$ *and the government commits to inelastically supply any excess of seigniorage in exchange for money, then there is a unique equilibrium price,* P_1^e, *satisfying* $P_1^e = P_1^F$.

The message of this proposition is also useful to understand one of the center claims of the FTPL: the non-necessary cooperative behavior between the fiscal and monetary authorities. As only fiscal plans that involve a non-positive level of seigniorage are credible in face of a commitment to an elastic money supply rule, the fiscal authority does not need any financial help from the central banker, for it is sufficiently solvent so as to never need seigniorage or, perhaps, even to purchase (i.e. destroy) money.

2.5 Learning from the fiscalist stock-analogy

In this section, I discuss to what extent the arguments given in the previous sections force a reconsideration of the fiscalist stock-analogy mentioned earlier. The main claim here is that a strategy based on applying a standard asset valuation approach *a la* Lucas (1978) to determine the value of money through the "government valuation equation" (2.13) may be misleading, at least, for two reasons.

The first reason is an obvious one: the RHS of (2.13) is not a correct measure of the total discounted amount of resources, collected by the government, i.e. that term cannot be understood as the "profits" of the government. To see this, notice that net income gained by the government because of its monopolist production of money, according to the RHS of (2.13), is given by the so-called inflation-tax term, $\frac{i_t}{1+i_t} m_t$, which is not the correct measure of those monopolistic profits, as these correspond to the amount of seigniorage,

$\frac{M_t - M_{t-1}}{P_t}$. The inflation-tax referred above is a measure of the opportunity cost borne by an individual who accepts to hold a share of his wealth in the form of monetary balances rather than bonds. But such an opportunity cost cannot be thought as of being net income going to the government[30]. Logically, if the RHS (2.13) is not a correct measure of the firm's profits then the LHS of that equation cannot be thought as of being the number of outstanding shares of the firm, if one is still willing to accept an analogy between a firm and the government. Thus, when the "government valuation equation" is written so as to contain the total discounted value of the total net real government's income in its RHS, what appears on the LHS is the initial stock of government's debts, if any, i.e.,

$$\frac{(1+i_0)B_0}{P_1} = \sum_{t=1}^{\infty} \frac{\tau_t - g_t + \frac{M_t - M_{t-1}}{P_t}}{\prod_{s=1}^{t-1}(1+r_s)} \tag{2.23}$$

Equation (2.23) makes clear that the true "profits" depend on the initial period's price level, P_1, through the term $\frac{M_0}{P_1}$, so the firm's stock-analogy losses its attractiveness here, as we cannot longer assume that total profits are fully independent of the price level.

The second reason is related to a more fundamental observation. The standard FTPL argues that a policy mix resulting in a unique equilibrium sequence of discounted government's total surpluses (i.e. primary surpluses plus seigniorage) will result in a unique equilibrium whenever there is an initial stock of government's nominal obligations. Such an argument would be correct (put aside, for a while, the previous observation on the incorrect measure of total surpluses) whenever the only reason for holding government nominal bonds and, critically, money, is the expectation of redeeming these assets for government-provided valuable resources, i.e., according to the stock-analogy what gives value to money is the amount of resources that the government is going to employ in purchasing money or in distributing dividends on it. This simple and rather strong conclusion should not be surprising since, after all, this is the basic assumption underlying the standard model of equity valuation which the FTPL looks at: the only reason for holding

[30] For example, let's consider an infinite-horizon economy in which the level of private consumption, the real interest rate and the supply of money are constant. Constant money supply implies that both seigniorage and inflation (ruling out bubble-solutions) are zero but the inflation-tax will be positive as long as the real interest rate is positive. Identifying this last concept with seigniorage (i.e. the net effective gain of the central bank) only leads to an erroneous calculation of total government's surpluses.

equity, and what gives that equity a real value, is the expectation of receiving profits from the issuer firm via distributed dividends or share repurchases. As the government does not pay any dividend on money (i.e. zero nominal interest on money), the only way in which the stock-analogy can work here is by allowing the government to repurchase money. Not surprisingly, the class of policies studied in the previous section yield the uniqueness result supported by the stock analogy precisely when the policy-mix involves a non-positive level of seigniorage money, i.e. when money effectively gets a share of the government's "profits".

Nevertheless, the fiscalist stock-analogy is still useful to shed some light on several issues discussed before. The asymmetry between government default and supersolvency arising in the "modified" FTPL developed in the previous section is something natural in the context of a firm. To see this, let's write the following equation, reminiscent of the one-period economy government's budget constraint (2.19),

$$\frac{S_0}{Q_1} = \pi_1 + \frac{S_1}{Q_1}$$

where S_0 and S_1 are the initial and the final number of outstanding shares of the firm, respectively, π_1 is the net operational profit of the firm and $\frac{1}{Q_1}$ is the real value of each share (measured in units of profits). A non-zero terminal portfolio, $\frac{S_1}{Q_1}$, can be justified if the share-holders find this firm's paper useful even after the firm has been liquidated, i.e. these S-shares are "tasty" for the share-holders[31]. Let's also assume that the share-holders want to hold a particular terminal portfolio, measured in real terms, i.e. $\frac{S_1}{Q_1} = \kappa$, for some given (positive) κ, and that the firm commits to supply as many shares as demanded at the ongoing market price, Q_1.

Then, if $\pi_1 > 0$, the firm offers the investors the possibility of exchanging some shares for profits up to the point at which total profits are fully distributed. When the share-holders behave optimally (i.e. not leaving any valuable profit unexploited), the equilibrium price of each share satisfies

$$\frac{S_0}{Q_1^F} = \pi_1 + \kappa \tag{2.24}$$

[31]For a similar "tasty-share" story, see, e.g., Marimón (2001).

and the number of shares repurchased by the firm is $S_0 - Q_1^F \kappa$.

However, if the firm plans to suffer a loss, i.e. $\pi_1 < 0$, there is no mechanism to force investors to provide the firm with the required resources, notwithstanding the fact that investors may be willing to buy some new shares, just because they like them. But nothing guarantees that this later form of funding will be enough to allow the firm to go ahead with its plans, even if the RHS of (2.24) is strictly positive. The firm's commitment to consume no more than π_1 rules out any price $Q_1 > Q_1^F$, but there are multiple equilibria in which actual losses, in absolute value, are below π_1 and the equilibrium price level satisfies $Q_1 \leq Q_1^F$.

We can extend this simple analogy by introducing "nominal bonds", that is, a new type of shares that are not perceived as intrinsically useful by the holders and that are denominated in terms of the tasty ones (call them N-shares). Let's focus on the case in which the firm plans a positive level of operational profits, i.e. $\pi_1 > 0$, and tries to repurchase the outstanding no-tasty shares, N_0, at a price equal to Q_1^F in the following equation

$$\frac{S_0 + N_0}{Q_1^F} = \pi_1 + \kappa$$

Also, let's assume that S_0, N_0, π_1 and κ are such that the firm needs to sell new S-shares to meet its plans, i.e. $S_1 - S_0 > 0$. Then, at any price $Q_1 > Q_1^F$, the firm is not distributing all its profits even after having repurchased the entire initial stock of N-shares. A commitment to distribute any remaining profits via repurchases of S-shares renders such a price $Q_1 > Q_1^F$ a non-equilibrium outcome: not accepting such a repurchase offer amounts to give up a profitable opportunity for the investors. On the other hand, if $Q_1 < Q_1^F$, new net capital injections, as measured by $\kappa - \frac{S_0}{Q_1}$, are not sufficient to repurchase the outstanding N_0 shares at the ongoing price Q_1. What can the firm do? It cannot directly sell more newly printed S-shares, given the commitment to a completely elastic supply rule. It cannot increase its operational profits, as they are assumed to be given exogenously. There is nothing the firm can do to provide investors with the right incentives to purchase new S-shares, promising, at the same time, that this extra income will be rebated to those same investors through a higher redemption value for the N-

shares and, as a result, multiple equilibria along which each N-share takes a value below a S-share cannot be ruled out.

An interesting example shows up when assuming that the investors do not find S-shares useful *per se* anymore (i.e. $\kappa = 0 \; \forall Q_1$). Under this assumption, the initial claims on the firm's profits, S_0 and N_0, may always have a unique equilibrium common value since nothing may preclude the firm from committing to distribute its profits across the different types of shares evenly (i.e. redeeming both types of shares at the same value). Of course, if the only outstanding claims are N-shares ($S_0 = 0$), there will be a unique equilibrium price for those shares and, naturally, the price for a security that is not traded at all (S-shares) is not defined. Thus, what makes the problem of pricing S-shares a non-trivial one is the possibility of an existing demand for these securities which is not exclusively driven by the investors' expectations on the firm's stream of profits or losses (in this simple example, the existence of a positive portfolio κ, and in a monetary economy, the public's perception of money as an object which provides utility per se, helps to overcome a cash-in-advance constraint or reduces the cost of making transactions). In such a case, a new kind of profit, different form the operational one, emerges, for net sales of these shares increase the firm's net worth. Further, when the firm commits to an elastic supply of S-shares, the amount of this later form of profit will not be uniquely determined, unless the firm actually uses a portion of its operational profits to purchase those shares, in which case, in equilibrium, S-shares are not longer an additional source of (indeterminate) profits but a destination in the distribution process of the exogenous (and uniquely determined) operational gains. In the same fashion, what makes the problem of pricing money a non-trivial one and, potentially, a "difficult" one is the possibility that private individuals find money useful even if it is well understood that the government will never redeem that money for consumption goods, i.e. the possibility of money being a fiat asset.

Also, as the examples above show, the problem of default is linked to the existence of an underlying commitment to redeem the entire stock of N-shares at the price of each S-share. In this sense, default is a true possibility for certain firm's claims as it is a true possibility for government dollar-denominated bonds whenever the central bank follows its own autonomous monetary rule. If this commitment were absent (as in Bassetto (2002)), a concern on a possible debt-default would no longer be justified. If the monetary authority behaved as a servant of the fiscal authority (as in the fiscal-dominance regime studied by Sargent and Wallace (1981)) default on the initial stock of dollar-denominated bonds could always be avoided too.

Comparing the above examples with the economy analyzed in the previous section yields, indeed, an interesting analogy. In particular, Assumptions 1 and 2 presented above, which are necessary for equilibrium uniqueness, can be also seen as the necessary elements to go from the standard institutional framework of a firm to the one described in the preceding section. Notice that Assumption 2 implies that money, while being potentially a convertible asset, is just a residual claim to the eventual government surpluses, that is, there is nothing forcing the government to redeem any arbitrary quantity of dollars at any arbitrary price, very much like the S-shares above. Assumption 1 plays a crucial role in removing potential equilibria in which the government, having committed to an exogenous primary surplus, would be forced to redeem the existing debt at a value above the contractual one. Indeed, this assumption can be seen as of capturing the autonomy of a solvent firm in deciding how to distribute its profits across different classes of shares (money and bonds). Thus, once we assume that a firm's total profits may depend on the seigniorage gained by the sales of tasty shares, there is an analogy between the mechanism of determination of the equilibrium value of those tasty shares and the one governing the value of government-issued money when a correctly specified (i.e. credible) fiscal-monetary plan that involves a non-positive level of seigniorage is at work.

2.6 A multiple-period economy

In this section I extend the arguments developed before to an economy with multiple periods. This scenario allows for an explicit consideration of the nominal rate as a policy instrument. Before developing the formal arguments, it is useful to briefly discuss the nature of the commitment associated to an interest rate peg and its influence on private expectations. Proposition 1 states that, in a perfect-foresight economy, under a Ricardian policy the ability of the government to commit to a particular nominal return paid by its debt regardless of the initial price level implies that, while the total present value of surplus depends on the particular realization of P_1, the sequence of equilibrium inflation rates is independent of P_1. Thus, at least regarding the determination of inflation, Ricardian policies in a perfect foresight setting do not pose important difficulties. However, as stressed by Sargent and Wallace (1975), and more recently by Woodford (2003), as we consider a scenario with uncertainty, a policy of pegging the nominal interest rate will involve price-indeterminacy at each date, and hence, the *actual* inflation rate will be indeterminate even if it is unique *in expectation.* The basis for a formal argument is simple. Let's consider a linear version of a stochastic Fisher equation[32],

$$i_t = \widetilde{r}_t + E_t \pi_{t+1} \tag{2.25}$$

where $\widetilde{r}_t$ is the targeted real return paid by a bond issued at t and maturing at $t+1$, assumed to depend upon some fundamentals (e.g. preferences, endowments and government consumption) and $E_t \pi_{t+1}$ is the expectation as of t of the inflation rate prevailing between t and $t+1$. For given i_t (assumed to be set directly by the government) and $\widetilde{r}_t$, (2.25) can be solved for a unique $E_t \pi_{t+1}$. However, (2.25) does not have a unique finite solution for the stochastic process $\{\pi_{t+1}\}$, as any finite π_{t+1} satisfying

$$\pi_{t+1} = i_t - \widetilde{r}_t + \psi_{t+1} \tag{2.26}$$

is a solution of (2.25), provided ψ_{t+1} is not forecastable at t, i.e. $E_t \psi_{t+1} = 0$. We can not say much about ψ_{t+1}, apart from this latter condition, as a non-zero ψ_{t+1} could arise as the result of an underlying process linking prices to fundamentals but critically, as stressed by Woodford (2003; sec. 2.2.1), also as a pure speculative component in the process generating prices (*sunspot states*), totally unrelated to fundamentals. In a Ricardian world, the distinction of fundamental vs. non-fundamental elements in the solution for the realized inflation rate (i.e. in the price P_{t+1}) is unimportant regarding the private rational expectation formed one period earlier about P_{t+1}, for the public's perception that the government will adjust its fiscal instruments so as to honor its maturing debt at

[32] A detailed derivation of a similar equilibrium condition as a local approximation in an optimizing model can be found in Woodford (2003, C. 2).

the contractual value implies that for a given $\widetilde{r}_t$, $E_t\psi_{t+1} = 0$ must hold always. In this sense, fiscal commitment rules out arbitrary self-fulfilled prophecies on $E_t\psi_{t+1}$, and hence a policy of fixing the nominal interest rate paid by government bonds, coupled with a Ricardian fiscal rule will induce a unique inflation expectation. If, on the other hand, the public understands that such a fiscal response at $t+1$ will not be observed, say because the government announces a sequence of primary surpluses totally independent of the price level, the previous reasoning will not apply. Under such a fiscal rule, the announcement of a particular nominal interest may not be credible and arbitrary expectations on ψ_{t+1} consistent with individual optimization can not be ruled out in advance, even if there is not any other source of uncertainty beyond the random process that produces a particular expectation on ψ_{t+1}. In what follows I argue that such a possibility must not be disregarded, unless the fiscal plan involves the clear commitment to devote a fraction of the targeted primary surpluses to repurchase a fraction of the beginning-of-period stock of money, as in the one-period economy studied before.

To prove this claim in the simplest possible environment, let's consider a two-period economy, similar in every other respect to the one presented before. Along the lines of the non-Ricardian policy described in section 2.2, it is assumed that the government commits to implement a sequence of price-invariant primary surpluses. The government also announces the nominal interest rate prevailing between periods 1 and 2, i_1. Since the aim here is to show that such a plan involving sequences of exogenously targeted primary surpluses and nominal interest rates may not be always credible, it is convenient to distinguish between government's targeted values for its policy instruments and households' beliefs on those values. For this purpose I introduce a new notation to accommodate the eventual differences between government-announced and household-expected nominal interest rates, the former being denoted by i_1^a and the later by i_1^e. Thus, the difference between these rates can be understood as the presence of a sunspot component in the expectation on the actual rate (i.e. using the notation above, I am considering here $E_t\psi_{t+1} = \psi_{t+1} \neq 0$, where the equality is imposed to preserve the certainty-hypothesis[33]).

[33] Considering a non-degenerate distribution function for ψ_{t+1} will not alter the argument.

The household's budget constraints are now given by (2.15) and

$$c_2 \leq y_2 - \tau_2 + \frac{(1+i_1)\, B_1 + M_1 - M_2 - B_2}{P_2}$$

where i_1 is the observed (actual) interest rate paid by government bonds. As before, it is assumed that the household follows a bonds demand rule in the terminal period which is independent of the observed price level, i.e. $B_2 = 0$ for any P_2. Following a similar argument, the households' demand for bonds in the first period is assumed to be optimal given their beliefs about the government's resources backing its debt, i.e.,

$$\frac{B_1(i_1^e)}{P_1} = \frac{1}{1+r_1}\left[\overline{\tau}_2 - \overline{g}_2 + m_2 - \frac{m_1(i_1^e)P_1}{P_2}\right] \tag{2.27}$$

The LHS of (2.27) represents the demand for government bonds expressed in real terms and the RHS measures the discounted value of the resources available for debt redemption in the following period, i.e. the primary surplus plus seigniorage. The discount factor is defined, according to (2.6), as $\frac{1}{1+r_1} = \beta\frac{u'(c_2)}{u'(c_1)}$. In (2.27) I am representing the demand for real balances in the first period as $m_1(i_1^e)$ to explicitly account for the fact that it depends directly on the nominal interest rate the individuals expect (see (2.5)). As before, m_2, being the demand for real balances in the terminal period, satisfies a first order condition similar to (2.17).

First, let's suppose that $i_1^e > i_1^a$, i.e. households expect a nominal return on bonds higher than the one announced by the government. Then, using (2.6) we can express the level of seigniorage in period 2 for an interest rate i_1^e as

$$s_2\left(i_1^e\right) = m_2 - \frac{1+r_1}{1+i_1^e}m_1(i_1^e)$$

In order to learn whether those beliefs can be self-confirmed in equilibrium, I consider a vector of private consumption demand $\{c_1, c_2\}$ compatible with the goods-market clearing condition. Then, for $i_1^e > i_1^a$, the following inequality must hold

$$s_2\left(i_1^e\right) > s_2\left(i_1^a\right) \tag{2.28}$$

In face of (2.28), the government may commit to follow a policy of rebating the excess of resources, $s_2(i_1^e) - s_2(i_1^a)$, giving it in exchange for any arbitrarily positive amount of dollars (Assumption 1), thus creating an arbitrage opportunity which cannot be compatible with an equilibrium. As individuals anticipate that the government will distribute the "excess of profits" in exchange for dollars rather than for bonds, the expected effective return for bonds cannot be above the announced one. That is, if the government is determined to defend the announced nominal rate there cannot be any equilibrium with $i_1^e > i_1^a$.

Second, let's consider the case in which $i_1^e < i_1^a$, so that $s_2(i_1^e) < s_2(i_1^a)$. Then, as long as $s_2(i_1^a) > s_2(i_1^e) > 0$, we cannot rule out equilibria in which the government defaults. Given that the government commits to run an exogenous primary surplus, individuals correctly anticipate that the level of seigniorage in period 2 will be insufficient to redeem the stock of bonds issued one period before at their contractual value, increasing the relative demand for real balances and decreasing real lending to the government. Notice that in this economy the fact that (partial) nominal default is anticipated does not necessarily eliminate the incentives of the household to lend real resources to the government by purchasing bonds in period 1 at the price of one dollar. Formally, the relevant condition governing the optimal household's decision on how much to lend can be discomposed into two separate decisions. First, the decision of whether to lend or not at all. The household will be willing to lend some valuable resources, in a positive and finite amount, if the following first order condition holds,

$$(1+i_1^e)\frac{P_1}{P_2} = \frac{u'(c_1)}{\beta u'(c_2)} \tag{2.29}$$

Therefore, as long as the expected inflation rate, $\frac{P_2}{P_1}$, satisfies (2.29), the fact that $i_1^e < i_1^a$ is irrelevant for this decision. All that matters is that each unit of consumption goods invested in bonds at period 1 yields the required real return. Thus, as discussed above, the government's lack of commitment to respond to deviations of the effective nominal rate from the announced one allows for beliefs on expected inflation to become self-confirmed in equilibrium[34]. Second, as an expected positive rate of default is associated in equilibrium

[34]The trade-off between a standard Ricardian policy and the exogenous fiscal policy considered here

with a lower rate of inflation (see (2.29)) rational individuals will reduce the real amount of debt purchased in period 1. But this reduction is not a direct consequence of the rational expectation of nominal default, instead it is a reflection of the fact that seigniorage in period 2 and, hence, the amount of resources available to payback government's debt falls.

Finally, when $i_1^e < i_1^a$ and $s_2(i_1^e) < s_2(i_1^a) \leq 0$, under Assumptions 1 and 2 and an excess seigniorage-rebating policy, i_1^e cannot be a rational belief, so that the unique equilibrium effective nominal interest is the announced one, i_1^a, i.e. the standard fiscalist non-default assumption holds. Thus, the necessary condition for the whole non-Ricardian program to be credible in the second period derived here, $s_2(i_1^a) \leq 0$, is similar to the one derived before for the one-period economy (Proposition 3 (iii)). Notice, however, that the credibility of the whole fiscal-monetary program requires condition $s_t\left(i_{t-1}^a\right) \leq 0$ to hold for any t, and not just for the (eventual) terminal period. In the two-period economy analyzed here, we must impose $s_1(i_1^a) \leq 0$ and $s_2(i_1^a) \leq 0$ in order to obtain a unique equilibrium. For example, if $s_2(i_1^a) \leq 0$ holds, so that $i_1^e = i_1^a$, we learn from (2.27) that the household's demand for government bonds at the end of period 1, in equilibrium, is unique, given the credible government announcements for period 2. It follows that the problem faced by the household in period 1 is identical to the one faced in the one-period economy, as we only need to augment the end-of-period 1 household's equilibrium portfolio with that extra debt-term[35]. Thus, the set of fiscal announcements for period 1 (exogenously targeted primary surplus and redemption of the initial debt, if any, at par) will only be credible if $s_1 \leq 0$.

The recursive nature of the optimization problem faced the household implies that we can extend this argument to economies with longer (and infinite) horizons in a straightforward way. The following proposition summarizes these findings.

Proposition 4 *A fiscal-monetary plan in which the government commits to maintain exogenously targeted sequences of taxes, government consumption and nominal interest rates is only credible if Assumptions 1 and 2 hold and, at the targeted policy-instruments*

becomes clear. The former is successful in pinning down inflation expectations, however at the cost of leaving taxes indeterminate. The opposite is true for the latter.

[35]The fact that the expected amount of resources backing government bonds issued at the end of period 1 is given uniquely implies that households' demand for bonds cannot differ from that quantity when behaving optimally.

sequences, the following condition holds for every $t \geq 1$

$$M_t \leq M_{t-1} \tag{2.30}$$

Corollary *When Assumptions 1 and 2 and condition (2.30) hold and the government can commit to maintain its targets (i.e. the announced fiscal-monetary program is credible), the equilibrium is unique.*

2.7 Conclusions

In this paper, I first examine the off-equilibrium behavior of the economic agents prescribed by the FTPL, showing that the way in which non-Ricardian policies are usually defined by the advocates of the FTPL involves a non-credible commitment by the government whenever the assumed fiscal-monetary program contains the announcement for maintaining a completely exogenous sequence of government consumption and a fully elastic money supply rule consistent with the central bank announcing an arbitrary nominal interest rate[36], thus showing that the standard FTPL is never a policy-based equilibrium selection device. The main novelty of this criticism is that it is based on the same core assumptions maintained by the proponents of this theory: there is a positive stock of government-issued assets at the beginning of the history owned by the households, flow of funds constraints must be respected in every contingency, although transversality conditions may be violated at off-equilibrium prices.

Then, using a standard Walrasian model in which money is introduced as an argument in the household's utility function, I identify the minimum set of conditions guarantying the credibility of a fiscalist policy close in spirit to the standard FTPL's non-Ricardian policies. Firstly, I show that when the fiscal authority targets a particular (exogenously determined) sequence of primary surpluses, any equilibrium in which the government would be forced to increase the transfer of resources to the private sector above the targeted level can be ruled out only by assuming that the fiscal authority may participate in the money market purchasing dollars in exchange for real consumption goods. This

[36]The terms "exogenous" and "arbitrary" are used here in the precise sense of the corresponding policy-instruments not being chosen according to a Ricardian rule.

assumption can then be incorporated into a fiscal strategy that breaks down the Ricardian Equivalence Theorem. Further, it is shown that such a fiscal strategy must necessarily be price-contingent for it to be credible. Secondly, I argue that the previous equilibrium-rejection device is not always implementable to symmetrically rule out equilibria in which the government is forced to reduce the transfer of resources to the private sector below the targeted level, as such a device would be inconsistent with the assumed monetary policy rule. Thirdly, I show that a fiscal-monetary plan resulting in a unique equilibrium under a nominal interest rate peg is only credible if that plan involves a non-positive level of seigniorage. That is, the main arguments of this modified FTPL are only valid for non-fiat money. This is one of the most interesting conclusions of this paper.

Finally, I argue that the so-called stock-analogy exploited by some advocates of the FTPL, according to which there exist fiscal strategies that turn the government intertemporal constraint into a "government valuation equation" requires an important qualification for it to be a useful analogy to explain how the equilibrium value of money is determined, namely, that money must be a convertible asset, a result which is plainly consistent with the above arguments about the credibility of a non-Ricardian policy.

Chapter 3
On Fiscalist Divergent Price-Paths

3.1 Introduction[1]

The existence of multiple money-price equilibria is a well known result in many economic models and the issue has traditionally attracted much attention from economists. But, like many other basic aspects in monetary economics, the question of what are the "fundamental" determinants of the equilibrium value of money remains open. Firstly, modern government-issued money, being an intrinsically useless asset, posses an important modeling problem: what is the role to be given to an intrinsically useless asset? Although trying to give an answer to this question goes well beyond the aim of this paper, it is still useful in order to identify a first candidate to be an important determinant of the real value of money: the role played by money in the economy. Secondly, as pointed out by Sargent and Wallace (1981), since the monopolistic production of money may have effects on the balance-sheet of the issuer institution (in this paper, the government), public finance considerations seem to be another important candidate: the quantity of money and/or its value may be affected by fiscal factors.

In this paper I deal with two theoretical paradigms that share a common goal: combine the two broad sets of "determinants" given above so as to give an answer to the same question, i.e. where does the value of money come from? Apart from this common concern, everything else is conflicting in the two theories considered here: the traditional monetarist approach *à la* Sargent-Wallace and the recently developed Fiscal Theory of the Price Level (FTPL, henceforth).

[1]Footnotes present in the original version of this paper that parallel those in Chapter 2 are omitted here.

The monetarist paradigm is consistent with the following interpretation of the two determinants outlined above. The equilibrium price sequence depends positively on the sequence of money supplies. Of course, the equilibrium price level will also depend on the role played by money, i.e. it is also determined by those factors shaping the demand for money. Regarding the influence of fiscal variables, Sargent and Wallace argue that government deficits and debt are relevant for the price level as long as the government accommodates its monetary stance to meet some fiscal targets via seigniorage. With respect to the role of money in the economy, the degree of consensus within this paradigm is rather low. Still, many monetary models, different in their views about the demand for money, allow for a familiar monetarist result: when money is perceived by private individuals to be a purely fiat asset (i.e. non-convertible into intrinsically valuable goods or assets given by the government), speculative hyperinflationary paths along which the demand for real balances vanishes cannot be ruled out[2]. Also, the standard monetarist wisdom dictates that price-paths along which the demand for real balances grows without limit in a deflationary speculative fashion can be ruled out on the basis of individual optimality (see, e.g. Brock (1974, 1975) and Obstfeld and Rogoff (1983)).

The FTPL's view on the above particular issues is, basically, the opposite to the monetarist one. According to the FTPL, the price level is, fundamentally, a fiscal phenomenon. Fiscal variables like government surpluses/deficits, government debt (and its denomination, real or nominal) are thought to be the main determinants of the price level. Money supply and demand play a very marginal role. Indeed, money may be dropped from the economy, and the FTPL would still be able to say what the price level is, as long as the private sector holds some initial financial wealth denominated in dollars and some general sign conditions hold. The fiscal-monetary policy-coordination highlighted by Sargent and Wallace is not longer present in a fiscalist world. Fiscal and monetary policies can be designed and implemented in a fully uncoordinated fashion, implying, among other things, that an independent central bank does not longer guarantee inflation stability. Further, this theory is consistent with a rather counterintuitive result: the government, through an

[2]This result holds for models with money in the utility function (e.g. Obstfeld and Rogoff (1983)), overlapping generations models (e.g. Sargent and Wallace (1981)), random-matching models (e.g. Kiyotaki and Wright (1989)) and models in which money is explicitly modeled as the most liquid asset (e.g. Kiyotaki and Moore (2001)).

"appropriate" fiscal policy, may unchain a speculative deflation or hyperinflation. Thus, the FTPL is not only consistent with the general view that the actions of one agent (the government) can induce changes in the optimal actions of another agent (the private individual), but also is consistent with, e.g., the idea that government can act in a particular way that necessarily results in the private agents losing their "faith" in fiat money (i.e. causing a speculative hyperinflation).

A central objective of this paper is to shed some light on the current debate on the consistency of the fundamental postulates of the fiscal theory, confining the analysis to monetary rules based on a discretionary choice of the sequence of money supplies by the central banker, in which the term *discretionary* is referred to a monetary plan which is designed and executed in an autonomous way, i.e. free of any form of direct fiscal dominance and independently from the state of the economy. This is a natural terrain to study divergent price-paths, along which the price level explodes or implodes as time passes without a parallel change in the stock of money. Also, such a class of monetary rules has attracted much attention in the previous papers that analyze the internal consistency and/or the empirical plausibility of this theory. Yet the level of consensus within this branch of the literature is rather low. In a series of influential articles, Buiter (1999, 2001, 2002) argues that in a standard Sidrauski-Brock model with a finite number of periods, a monetary rule based on a discretionary choice of the sequence of money supplies coupled with a fully autonomous fiscal plan, as the ones considered by the FTPL, is likely to result in an over-determined system with more equations than unknowns. This is, to the best of my knowledge, one of the most clear-cut criticisms of the fiscal theory, for it is based on an uncontroversial simple mathematical result. However, Buiter argues that such an over-determinacy problem vanishes as one considers an infinite horizon economy, thus implicitly admitting the feasibility of fiscally-driven divergent price-paths. A central theme of the paper is to show that Buiter's arguments for the finite horizon case *necessarily* extend to the infinite horizon framework. This result is interesting on several grounds. First, because it naturally adds to the consistency of the current analysis of the internal consistency of the fiscal theory. Second, because many macro-monetary models are cast within the infinite-horizon framework. Moreover, it could be argued that the most appropriate time-horizon to deal with models of purely fiat money is, perhaps, the infinite one. The output obtained here regarding this question can be posed in plain words as follows: *totally uncoordinated fiscal-monetary plans, as those considered by Buiter, result in a problem of over-determination of the price level, regardless of the economy's time-horizon.*

The above observation pertains to the (in-)equilibrium prescriptions of this theory and, as such, should not be much controversial. I then turn the view to the off-equilibrium arguments exploited by the FTPL to justify the possibility of a fiscalist hyperinflation. An interesting insight arises from the analysis of this extreme possibility. Treating the government intertemporal budget constraint as a money-valuation-equation, as advocated by the FTPL's proponents, rather than as an identity, is not sufficient to sustain the FTPL's arguments. If one thinks of an economy in which money could be positively valued by private individuals even when it is publicly known that money will never be turned into intrinsically valuable consumption goods by the government (and the economy under study falls in this category), then one cannot explain an equilibrium in which money is valueless (i.e. the terminal situation along a hyperinflationary path) by drawing an analogy between money and a private firm's share. No matter how low is the present-value stream of fiscal surpluses (zero in the limit), a zero value for money would never be a necessary unique result. A particular form of the government *default versus supersolvency asymmetry* discussed in Chapter 2 applies here: the government can use its ability to raise taxes so as to avoid a speculative hyperinflation (as shown by Obstfeld and Rogoff (1983)) by committing to pay a price for money higher than that prevailing in the market, thus creating an arbitrage opportunity, but the government cannot force a speculative hyperinflation by "committing" to pay nothing for an asset (money) that, at the prevailing market-price, could be positively valued by private agents.

The paper is structured as follows. In Section 3.2, I develop a simple model that has been widely used in the previous literature and discuss the main implications regarding the equilibrium price-sequence determination under alternative monetarist and fiscalist formulations. In section 3.3, I show that fiscalist speculative deflationary price-paths can not be part of an equilibrium. In section 3.4, I first argue that a non-Ricardian plan involving a complete lack of fiscal and monetary coordination causes a problem of price over-determination, regardless of the time-horizon of the economy. I then discuss the fiscalist arguments concerning the occurrence of a divergent hyperinflationary path as a unique equilibrium outcome. Section 3.5 contains a summary of the main conclusions.

3.2 Monetarist discipline vs Fiscalist *laissez-faire*

In this section, I first present a simple set-up to frame the arguments of the two paradigms at stake, the monetarist and the fiscalist. The model developed below contains a continuum of identical households and the government. Each household tries to maximize its total discounted utility in a Walrasian competitive fashion and the government collects taxes, issues and purchases debt, consumes some real goods and prints money. Time is discrete, there is no uncertainty and households are assumed to form their expectations rationally.

Then, I introduce the definition of alternative policies, according to the fiscalist Ricardian versus non-Ricardian distinction and, using some particular examples of these two broad classes of policies, I characterize the Walrasian competitive equilibria arising under each policy specification. The general aim here is to provide a general picture of the implications of several Ricardian and non-Ricardian fiscal-monetary programs for the determination of the equilibrium price sequence as they are usually presented in the previous literature[3].

3.2.1 A simple model

In most of the cases analyzed in this paper, I study price-level determination using an infinite-horizon economy[4]. The representative household is assumed to enjoy utility from consuming real goods and from holding real balances and tries to maximize the following

[3]The examples presented below closely follow those contained in Sargent and Wallace (1981), for Ricardian policies, and in Woodford (1995), for the non-Ricardian fiscal-monetary mix.

[4]In section 3.4 I use a one-period version of this general model to facilitate the comparison with Buiter's results.

objective function[5]

$$\max_{\{c,M\}} \sum_{t=1}^{\infty} \beta^{t-1} \left[\frac{1}{1-\theta} c_t^{1-\theta} + \frac{1}{1-\theta} \left(\frac{M_t}{P_t} \right)^{1-\theta} \right] \tag{3.1}$$

where c_t represents the household's consumption, and $\frac{M_t}{P_t}$ corresponds to real money holdings, i.e., nominal balances, M_t, deflated by the general price level, P_t. The parameter $\beta \in (0,1)$ is the subjective discount factor. The parameter θ is strictly positive. As θ approaches unity, the felicity function converges to the natural-logarithmic form.

The household's flow of funds budget constraints are given by

$$b_t + \frac{B_t + M_t}{P_t} \leq y - \tau_t - c_t + (1 + r_{t-1})\, b_{t-1} + \frac{(1 + i_{t-1})\, B_{t-1} + M_{t-1}}{P_t} \quad (t \geq 1) \tag{3.2}$$

where $y > 0$ is the exogenous household's endowment of consumption goods, assumed to be constant over time, τ_t is a real-denominated lump-sum tax collected by the government; b_t and B_t are the amount of bonds denominated in real and monetary terms, respectively, demanded/supplied by the household at the beginning of period t. Bonds are assumed to mature one period after issued. Each real-denominated bond issued at $t-1$ is redeemed at a value equivalent to $1 + r_{t-1}$ consumption goods at t. Each dollar-denominated bond issued at $t-1$ is redeemed at a value equivalent to $1 + i_{t-1}$ dollars at t. Households are also endowed with some financial wealth at the beginning of the first period ($t = 1$), that may include government bonds, inclusive of interest, $(1 + i_0)\, B_0$ and $(1 + r_0)\, b_0$, and money balances, M_0. The face-value of this initial financial wealth is given exogenously and assumed to be strictly positive. Households are also constrained by the following

[5] The choice of this particular utility function is made for simplicity. To my knowledge, most of the previous papers about the FTPL in which real balances enter as an argument in the utility function assume consumption-money separability. Consideration of an isoelastic utility function adds to the simplicity of some mathematical derivations and allows for different classes of equilibrium price-paths just by assuming different values for the parameter θ, as discussed later.

no-Ponzi games condition[6]

$$\lim_{T\to\infty}\left(b_T+\frac{B_T+M_T}{P_T}\right)/\prod_{j=1}^{T-1}(1+r_j)\geq 0 \tag{3.3}$$

Combining (3.2) and (3.3), we learn that the household's intertemporal budget constraint at any period $t \geq 1$ is given by

$$(1+r_{t-1})\,b_{t-1}+\frac{(1+i_{t-1})\,B_{t-1}+M_{t-1}}{P_t}+\sum_{j=0}^{\infty}\frac{y-\tau_{t+j}}{\prod_{s=0}^{j-1}(1+r_{t+s})}\geq\sum_{j=0}^{\infty}\frac{c_{t+j}+\frac{i_{t+j}}{1+i_{t+j}}\frac{M_{t+j}}{P_{t+j}}}{\prod_{s=0}^{j-1}(1+r_{t+s})} \tag{3.4}$$

When the household behaves optimally, equations (3.2), (3.3) and (3.4) all hold as equalities. The remaining necessary conditions of the maximization problem faced by the representative household are

$$c_t^{-\theta}-\lambda_t=0 \tag{3.5}$$

$$\frac{m_t^{-\theta}-\lambda_t}{P_t}+\beta\frac{\lambda_{t+1}}{P_{t+1}}=0 \tag{3.6}$$

$$-\frac{\lambda_t}{P_t}+\beta\frac{(1+i_t)\,\lambda_{t+1}}{P_{t+1}}=0 \tag{3.7}$$

$$-\lambda_t+\beta\,(1+r_t)\,\lambda_{t+1}=0 \tag{3.8}$$

where the λ's are the familiar Lagrangian multipliers and $m_t \equiv \frac{M_t}{P_t}$.

The fiscal branch of the government sets the level of taxes and government consumption and manages the public debt, issuing, purchasing and redeeming bonds. The central bank chooses the sequence of money supplies independently of any other variable in the economy. The policy mix is assumed to be consistent with the following general restrictions: $M_t^s \geq 0$ and $g_t < y$, where g_t stands for government consumption. The consolidated

[6]This form for the no-Ponzi games condition, in which the present value of the *aggregate* long-run private financial wealth is required to be non-negative can be found, e.g. in Benhabib *et al.* (2002), Buiter (1999, 2001, 2002), Buiter and Sibert (2004), Canzoneri *et al.* (2001), Daniel (2004), Woodford (1995, 2001, 2003). By contrast, McCallum (2001, 2003) argues in favor of a separate non-negative limiting condition for each of the assets involved considered individually. I analyze the implications of these two different constraints in the following section.

government's flow of funds constraint is given by

$$b_t^s + \frac{B_t^s + M_t^s}{P_t} = g_t - \tau_t + (1 + r_{t-1})\, b_{t-1}^s + \frac{M_{t-1}^s + (1 + i_{t-1})\, B_{t-1}^s}{P_t} \quad (t \geq 1) \qquad (3.9)$$

where b_t^s, B_t^s and M_t^s are, respectively, the government's demand/supply of real-denominated and nominal bonds and the supply of money during period t. Combining the sequence of constraints in (3.9) with the following transversality condition

$$\lim_{T \to \infty} \left(b_T^s + \frac{B_T^s + M_T^s}{P_T} \right) / \prod_{j=1}^{T-1} (1 + r_j) = 0 \qquad (3.10)$$

gives the following government intertemporal constraint,

$$(1 + r_{t-1})\, b_{t-1} + \frac{(1 + i_{t-1})\, B_{t-1} + M_{t-1}}{P_t} = \sum_{j=0}^{\infty} \frac{\tau_{t+j} - g_{t+j} + \frac{i_{t+j}}{1+i_{t+j}} \frac{M_{t+j}^s}{P_{t+j}}}{\prod_{s=0}^{j-1} (1 + r_{t+s})} \qquad (3.11)$$

3.2.2 Ricardian vs non-Ricardian

In this subsection I introduce a standard definition of Ricardian and non-Ricardian policies and examine the implications of alternative fiscal-monetary programs for the equilibrium of the model. In particular, I will assume that the policy announcements from both economic authorities are taken as credible by the households, thus respecting the standard Ricardian and non-Ricardian arguments. The following definition contains a standard Ricardian versus non-Ricardian distinction.

Definition 1 *A government policy is Ricardian if it is formulated in such a way that the transversality condition (3.10) is satisfied for any price sequence* $\{P_t\}_{t=1}^{\infty}$. *It is non-Ricardian otherwise.*

The definition of a Walrasian competitive equilibrium for this economy is given below.

Definition 2 *A perfect foresight equilibrium in this economy is a set of allocations* $\{c_t\}$, $\{b_t\}$, $\{B_t\}$, *and* $\{M_t\}$,*a set of positive prices and interest rates sequences* $\{P_t\}$,$\{r_t\}$, $\{i_t\}$, *and a government policy such that the following conditions are satisfied:*

1. *Households maximize their utility subject to the constraints (3.2) and (3.3), the price and interest rates sequences and the government policy.*

2. *The government satisfies its budget constraint (3.9) as an equality in every single period and the transversality condition (3.10).*

3. *All markets clear in every period, i.e.,* $y = c_t + g_t$, $M_t = M_t^s$, $b_t = b_t^s$ *and* $B_t = B_t^s$ *for* $t \geq 1$.

A fiscal-dominance Ricardian regime Let's consider a fiscal plan involving an exogenous sequence of constant primary deficits, $\tau - g < 0$ for $t \geq 1$. Also, let's assume, for simplicity, that all debt is real-denominated[7]. As the fiscal authority moves first, the central banker must adjust its monetary policy so that the consolidated government intertemporal budget constraint (3.11) holds for any feasible price sequence, i.e. the sequence of seigniorage, $\left\{\frac{M_t - M_{t-1}}{P_t}\right\}_{t=1}^{\infty}$, adjusts endogenously. For example, if the monetary authority tries to collect a constant level of seigniorage, s, money supply must obey the following rule

$$M_t = P_t s + M_{t-1} \quad (t \geq 1) \tag{3.12}$$

where s satisfies

$$s = g - \tau + (1 - \beta)(1 + r_0) b_0 \tag{3.13}$$

Equation (3.13) is derived from the intertemporal constraint (3.11) and the individual optimization and market clearing conditions. Of course, as long as there is an upper limit for a constant level of seigniorage, say s^*, the fiscal authority is always limited in its choice of the primary deficit by the following constraint

$$g - \tau < s^* - (1 - \beta)(1 + r_0) b_0 \tag{3.14}$$

Further, depending on the properties of the utility function, a non-price contingent sequence of primary deficits may not be sustainable at all. The following examples illustrate

[7]This set-up resembles the one considered by Sargent and Wallace (1981; Appendix B) in which all government debt is indexed or real-denominated.

this observation. First, if $\theta \geq 1$[8], the following condition holds as a strict inequality

$$\lim_{m \to 0} m \frac{\partial \left[\frac{1}{1-\theta} m^{1-\theta}\right]}{\partial m} \geq 0 \tag{3.15}$$

In such a case we learn that there cannot be speculative hyperinflationary paths in equilibrium[9], so there is a unique equilibrium price sequence. It follows that a policy of constant seigniorage and primary deficits satisfying (3.13) and (3.14) is feasible. For example, in the limiting case in which θ approaches unity, so that the liquidity services function is represented in logarithmic form, the initial equilibrium price level satisfies

$$\frac{M_0}{P_1} = -\left(1 + r_0\right) b_0 + \frac{\tau - g + c}{1 - \beta} \tag{3.16}$$

The rest of the elements of the equilibrium price sequence can be solved uniquely combining the first order conditions (3.5), (3.6) and (3.8), the market clearing conditions and the money supply rule (3.12).

Second, if $\theta < 1$, condition (3.15) holds as an equality and, hence, speculative hyperinflationary paths along which the demand for real balances falls towards zero cannot be ruled out, unless the government commits to implement a price-contingent money-convertibility plan, as described by Obstfeld and Rogoff (1983) and Nicolini (1996). It follows, that in the absence of such a contingent convertibility plan, the commitment to run an exogenous sequence of primary surpluses may not be credible[10]. But even if paths with real balances disappearing are not observed in equilibrium, a feasible exogenous fiscal policy does not guarantee price uniqueness, as the same constant level of seigniorage may be collected for different inflation and price paths along a hump-shaped inflation-tax Laffer curve[11].

[8] The equality part is understood as $\theta \searrow 1$.

[9] See Obstfeld and Rogoff (1983).

[10] This is just a reflection of the fact that, at some point(s) in time, it may not be possible to extract any seigniorage from pure fiat money, for in that case, there may be equilibria along which the demand for real balances drops to zero.

[11] Both kinds of speculative inflation paths, with and without a sustainable constant level of seigniorage, are analyzed by Sargent and Wallace (1981, 1987).

A monetary-dominance Ricardian regime Let's now consider the opposite regime in Sargent and Wallace's "game of chicken", in which the monetary authority moves first, announcing a constant exogenous growth rate for the nominal stock of money, i.e. $M_t = (1 + \mu) M_{t-1}$, with $\mu > 0$ and the initial condition $M_0 > 0$ taken as given. As in the fiscal-dominance regime above, if condition (3.15) holds as a strict inequality there is a unique equilibrium price sequence which can be solved by combining the money and consumption-goods market clearing conditions and the first order condition (3.6). In this case, the only fiscal variable that affects the equilibrium price-sequence is the level of government expenditure since, in equilibrium, it determines the available resources for private consumption and, hence, the demand for real balances. On the other hand, if (3.15) is satisfied as an equality then we cannot rule out multiple price sequences along speculative hyperinflationary paths. In either case the fiscal authority must adjust the sequence of primary surpluses so as to satisfy (3.11) as an identity.

Summarizing, under both Ricardian regimes the price level and the inflation rate are purely monetary phenomena. Fiscal variables like debt and primary surpluses only influence directly the price sequence if the central bank accommodates its policy so as to collect a particular targeted level of seigniorage[12]. When condition (3.15) is satisfied as an equality, the most intuitive case according to Obstfeld and Rogoff (1983), the stronger version of the "game of chicken", according to which one of the economic authorities moves first, fixing exogenously its policy instruments, yields some "unpleasant" results. Under a monetary-dominance regime, nothing precludes a multiplicity of equilibrium price paths. Also, a fiscal-dominance regime may not be implementable unless there is a clear commitment from both fiscal and monetary authority to alter their original plans so as to implement a price-contingent money-convertibility plan. Overall, one of the central messages arising from the two policies analyzed here is that a certain degree of fiscal and monetary coordination is always needed, as dictated by the government intertemporal budget constrain (3.11).

[12] As noticed above, strictly speaking, government consumption influences the equilibrium price sequence through its effect on private consumption.

A non-Ricardian regime Here I describe the FTPL's arguments concerning price level determination using the simplest class of non-Ricardian policies, namely, policies characterized by a complete lack of fiscal and monetary coordination along which both authorities set their policy instruments without any feedback from the observed price level[13]. Let's assume that the central bank announces an exogenous sequence of money supplies, $\{M_t\}_{t=1}^{\infty}$. First, let's suppose that the fiscal authority targets an exogenous sequence of constant primary surpluses/deficits. Then, the FTPL suggests that the entire equilibrium price-sequence can be solved from the government intertemporal budget constraint (3.11), which must be respected in equilibrium. Whether the price sequence constructed in this way is unique or not depends on the properties of the utility function. Specifically, the sign of the relationship between the inflation-tax terms on the right hand side of (3.11), $\frac{i_{t+j}}{1+i_{t+j}}\frac{M^s_{t+j}}{P_{t+j}}$, and the inflation rate, $\frac{P_{t+j+1}}{P_{t+j}}$, depends on the value of the parameter θ according to the following condition[14]

$$\frac{\partial\left(\frac{i_{t+j}}{1+i_{t+j}}\frac{M^s_{t+j}}{P_{t+j}}\right)}{\partial\frac{P_{t+j+1}}{P_{t+j}}} > 0 \;\; (=0) \;\; [<0] \quad \text{if } \theta > 1 \;\; (\to 1) \;\; [<1]$$

Let's define Ω_t as the time-t discounted value of the current and future inflation-tax terms, i.e.

$$\Omega_t \equiv \sum_{j=0}^{\infty} \frac{\frac{i_{t+j}}{1+i_{t+j}}\frac{M^s_{t+j}}{P_{t+j}}}{\prod_{s=0}^{j-1}(1+r_{t+s})}$$

It can be readily verified that the following sign condition holds

$$\text{sign}\left\{\frac{\partial\Omega_1}{\partial P_1}\right\} = \text{sign}\left\{\frac{\partial\left(\frac{i_{t+j}}{1+i_{t+j}}\frac{M^s_{t+j}}{P_{t+j}}\right)}{\partial\frac{P_{t+j+1}}{P_{t+j}}}\right\}$$

Then, provided $\theta \geq 1$, we learn that if there is a price level P_1 that satisfies the intertemporal budget constraint (3.11), then it must be the unique one satisfying that condition. Of course, for such a price level to be part of an equilibrium, the following sign condition

[13] This generic class of non-Ricardian fiscal-monetary programs, in which both economic authorities set their policy instruments without any feedback from the price level, has been considered by a large number of authors, both defendants and opponents of the FTPL (see e.g. Buiter (1999, 2001, 2002), Cochrane (2005), Evans and Honkapohja (2004), Woodford (1995, 1998, 2001)).

[14] This condition is derived under the assumption that the corresponding individual first order conditions are satisfied.

must hold[15]

$$-\left(1+r_{t-1}\right)b_{t-1}+\sum_{j=0}^{\infty}\frac{\tau_{t+j}-g_{t+j}+\frac{i_{t+j}}{1+i_{t+j}}\frac{M^s_{t+j}}{P_{t+j}}}{\prod_{s=0}^{j-1}\left(1+r_{t+s}\right)}>0 \tag{3.17}$$

The case in which $\theta < 1$ requires a separate argument. As in this case both sides of (3.11) are decreasing in P_1, we cannot get a clean single-crossing result as before, as there may potentially be several initial price levels (and inflation sequences) compatible with that constraint. Generally, a similar argument applies to any utility function that yields a hump-shaped inflation-tax Laffer curve. In view of this potential multiplicity of equilibria, Woodford (1995) proposes the following tax policy

$$\tau_t=\overline{\tau}_t-\frac{i_t}{1+i_t}\frac{M_t}{P_t} \tag{3.18}$$

where $\overline{\tau}_t$ is set exogenously. Such a tax policy implies that the right hand side of the budget constraint (3.11) does not contain any price-dependent variable, i.e.,

$$\left(1+r_{t-1}\right)b_{t-1}+\frac{\left(1+i_{t-1}\right)B_{t-1}+M_{t-1}}{P_t}=\sum_{j=0}^{\infty}\frac{\overline{\tau}_{t+j}-g_{t+j}}{\prod_{s=0}^{j-1}\left(1+r_{t+s}\right)} \tag{3.19}$$

It follows that, as long as a sign condition similar to (3.17) holds, there is unique price level, P_1, such that (3.19) is satisfied. The characterization of the entire price-sequence constructed in that way is then straightforward. Once P_1 is uniquely determined, the net supply of government bonds can be computed uniquely from the flow of funds constraint (3.9), with the composition of the total end-of-period stock of debt being autonomously fixed by the issuing agent. Then, we can solve for a unique P_2 following the same argument as before, i.e. by computing (3.19) one period forward, and, in this fashion, the unique price sequence consistent with the foregoing fiscal-monetary non-Ricardian plan.

Thus, according to the FTPL, a fully uncoordinated fiscal-monetary plan like the one just presented works *de facto* as an equilibrium selection device, as pointed out by Kocherlakota and Phelan (1999). It is worth noticing that the unique fiscalist price path need not to coincide with the fundamental monetarist one, in the sense that, even in face of a constant money supply policy, the fiscal decisions may well unchain either

[15] Recall the assumption that the dollar-denominated household's initial financial wealth (i.e. dollars and nominal bonds inherited from period 0) is strictly positive.

a hyperinflation or a deflation. These issues, i.e. the degrees of freedom of the fiscal authority to implement a policy which is only consistent with a particular divergent hyperinflationary or deflationary path, and some potential "anomalies" implicit in the construction of the fiscalist price-sequence just described are analyzed in the following sections.

3.3 Fiscalist speculative deflations: An issue of rationality

Woodford (1995) argues that the only requirement to be imposed on the evolution of the stock of total financial wealth is the transversality condition (3.10), thus admitting the possibility that the total discounted value of each of the three terms in that condition (real balances and debt, nominal and real) differs from zero. That is, according to that argument the following constraint is not strictly necessary for equilibrium

$$\lim_{T\to\infty} \frac{b_T^s}{\prod_{j=1}^{T-1}(1+r_j)} = \lim_{T\to\infty} \frac{B_T^s/P_T}{\prod_{j=1}^{T-1}(1+r_j)} = \lim_{T\to\infty} \frac{M_T^s/P_T}{\prod_{j=1}^{T-1}(1+r_j)} = 0 \tag{3.20}$$

All that matters regarding the individual optimization problem is that the present value of the long-run aggregate financial wealth, i.e. the sum of the three terms above, converges to zero: "*Equilibria of the kind constructed above will, for some specifications of the monetary and fiscal sequences, imply explosive growth of real money balances to such an extent that (3.20) is violated. These solutions* [...] *involve the supply of government debt becoming negative - i.e., the government must become a net lender to the private sector.* [...] *Because their monetary assets are offset by debt, households are not over-accumulating wealth along such paths.*" (Woodford (1995), equation numbering adapted[16]).

A similar environment (i.e., non-Ricardian exogenous sequences of money supplies and primary surpluses/deficits in an infinite-horizon economy) has been analyzed by some other authors analyzing the FTPL. Buiter (1999, 2001, 2002) provides a lucid proof against the fiscalist arguments in a finite-horizon economy in which real balances enter as an argument in the household's utility function (to be discussed later in detail), however

[16] A similar argument can be found in Woodford (2003).

he explicitly recognizes that a symmetric argument cannot be applied to an infinite-horizon economy: "*In the simple 'money in the direct utility function' model of this paper, unbounded real balances do not violate the equilibrium conditions, because the nominal interest rate would go to zero, creating an unbounded equilibrium demand for real balances without consumption becoming unbounded.*". (Buiter (2001)). Marimón (2001) presents a model inspired in a firm's equity valuation problem, similar in most respects to the one employed here. He analyzes fiscalist deflationary paths like the ones described by Woodford in an infinite-horizon set-up. Some of the main conclusions in that article are the following: "*Nevertheless, the (fiscalist) determinacy result requires that proper present value calculations are made by all agents. In our deterministic context this is already difficult; in a stochastic context, [...] it may be close to impossible. [...] As existing experimental evidence suggests, it is unlikely that all the equilibrium paths determined by the fiscal theory will arise. This again, is an empirical matter* [...]" (Parentheses added).

With respect to the above criticisms, here I take a further step, showing that speculative deflationary paths associated with a violation of (3.20), in the sense described later, can not be part of an equilibrium for this economy. For the sake of the clarity, I first consider a non-monetary economy in which real bonds are the only available financial asset. Although such a scenario does not shed any light on the question of whether fiscalist deflationary paths are possible equilibrium outcomes, it provides some useful insights on the economic meaning of the transversality condition. Then, I introduce money along the lines of the model discussed in the preceding section to show that a long-run accumulation of real balances by the households inconsistent with (3.20) tantamounts to a violation of an individual optimization condition. Without loss of generality, for the remaining of this section it is assumed that the utility functions in (3.1) are logarithmic[17]. Also, to save on notation, it is assumed that government consumption is zero at every period and taxes are set at the same level across periods, i.e. $g_t = 0$ and $\tau_t = \tau \ \forall t \geq 1$.

[17]This assumption simplifies computations, as the equilibrium inflation-tax Laffer curve is independent of the inflation rate. Such an assumption is critical to rule out hyperinflationary paths, as discussed before, but it is irrelevant with respect to deflationary paths analyzed here.

3.3.1 A cash-less economy

Let's first think of an economy in which individuals maximize the following felicity function,

$$\max_{\{c,M\}} \sum_{t=1}^{\infty} \beta^{t-1} \log c_t$$

subject to the sequence of flow of funds constraints

$$b_t \leq y - \tau - c_t + (1 + r_{t-1})\, b_{t-1} \quad (t \geq 1)$$

Thus, money is not traded at all. Consistently, nominal bonds, whose real value is linked to money, are not transacted either. As before, households are assumed to hold some initial financial wealth in the form of real-denominated bonds, inclusive of interest, maturing at the beginning of period 1, $(1 + r_0)\, b_0 > 0$. As the stock of money and nominal bonds are identically equal to zero at each date, the equivalent to the transversality condition imposed previously, (3.3), would now read as follows

$$\lim_{T \to \infty} b_T / \prod_{j=1}^{T-1} (1 + r_j) \geq 0 \tag{3.21}$$

For logarithmic utility function, the Euler equation takes the form

$$\frac{c_{t+1}}{c_t} = \beta\, (1 + r_t) \tag{3.22}$$

The government budget constraint now becomes

$$b_t^s = -\tau + (1 + r_{t-1})\, b_{t-1}^s \quad (t \geq 1)$$

After imposing market clearing and individual optimization together with the no-Ponzi games condition (3.21) holding as an equality, the existence of equilibrium calls for a unique tax sequence for which, according to the assumption of constant tax rate, each element (denoted by τ^M) satisfies the following condition

$$\tau^M = (1 - \beta)\, (1 + r_0)\, b_0 \tag{3.23}$$

Although obvious, it is worth stressing the argument followed to label the tax rate satisfying (3.23) as the unique equilibrium one: implementation of any other (constant) tax policy requires a departure from some fundamental features of the economy we are studying. For example, a tax rate below τ^M, maintaining the assumption of zero g, could only be implemented by, e.g., allowing the government to break its commitment to redeem the initial stock of government bonds at its contractual value or to employ some coercive instruments to force individuals to purchase bonds that pay a real return below its opportunity cost, i.e. by violating (3.22). A special case of such a coercive action is that associated with the transversality condition (3.21) holding as a strict inequality, for in that case the households would be forced to accumulate government bonds whose present redemption value is zero. Likewise, a tax rate above τ^M would require the payment of a supersolvency premium on the initial stock of bonds or the commitment by the government to borrow (lend) at a real rate above (below) the one consistent with the Euler equation (3.22). An example of such a "generous" fiscal policy can be constructed by allowing the government to accumulate bonds issued by the households in an amount that violates the transversality condition (3.21). To see this, let's assume that the government implements a tax rate τ^H, such that $\tau^H > \tau^M$. Under this policy the government intertemporal budget constraint becomes

$$(1 + r_0)\, b_0 = \frac{\tau^H}{1 - \beta} + \lim_{T \to \infty} \beta^{T-1} b_T \tag{3.24}$$

with $\lim_{T \to \infty} \beta^{T-1} b_T < 0$. One readily recognizes that such a long-run accumulation of bonds plays the role of a mechanism to distribute resources from an otherwise supersolvent government to the private sector. Thus, the purchase of those bonds tantamounts to an outright transfer (equivalent to a reduction in the tax rate[18] from τ^H to τ^M), and not to a process of accumulation of *wealth*, for, indeed, the government is never going to recover those resources transferred in this way. In such a scenario, one could even argue that the last term in (3.24) does not represent a stock of properly defined bonds, since the buyer is expecting a future zero payoff from that financial investment.

[18]Indeed, the government's purchase of those bonds would require a direct individually-based form of allocation, e.g. rationing, very much as a tax reduction.

In sum, implementation of a tax policy different from the one for which the transversality condition (3.21) is not satisfied requires the incorporation of some strange elements into the definition of equilibrium for the canonical economy described here, like the ability of the government to sell bonds whose return is not in line with the marginal rate of substitution, as dictated by the first order condition (3.22), or to violate the limiting condition (3.21) in order to transfer resources to the private sector by purchasing "bonds" that are never going to be redeemed (henceforth, *non-performing bonds*).

3.3.2 Introducing money

Now I return to the general model with money described in the preceding section. To facilitate the comparison with the cash-less economy analyzed above, I maintain the same assumptions, i.e. logarithmic utility functions, time-invariant sequences of taxes and zero government expenditure. The numerical values of y, $(1+r_0)\, b_0$ and β are thought to be the same as in the cash-less example. For the sake of the clarity, I first focus on a simple monetary rule along which the central bank keeps the stock of money constant at its initial value, i.e. $M_t = M_0 > 0$, $t \geq 1$. As the aim here is to study the implications of a positive present-value of the long-run stock of real balances held by the households coupled with an equivalent accumulation of bonds by the government, and the denomination of those bonds is insubstantial for this issue, I focus on the simplest case in which the amount of nominal debt is negligible.

With the foregoing set of assumptions, the relevant equilibrium intertemporal government constraint computed at the initial period is

$$\frac{M_0}{P_1^F} + (1+r_0)\, b_0 = \frac{\tau + y}{1-\beta} \tag{3.25}$$

For the fiscalist price, P_1^F, to be well defined, the sequence of primary surpluses must be consistent with the following sign condition (similar to (3.17))

$$\frac{\tau + y}{1-\beta} - (1+r_0)\, b_0 \geq 0$$

Given the specification for the sequence of government consumption and the constant money supply rule, we learn that the fundamental monetarist equilibrium price level, i.e. the price that satisfies the first order condition (3.6) and is consistent with the transversality condition (3.20), call it P^f, is constant. Its value is given by

$$P^f = (1-\beta)\frac{M_0}{y} \tag{3.26}$$

As the stock of money is constant, seigniorage is identically equal to zero, so money is neutral from a fiscal perspective. Then, it can be readily verified that the unique equilibrium tax sequence consistent with that monetarist solution is τ^M, given above by (3.23).

In order to obtain a fiscalist deflationary price sequence consistent with (3.3) holding as an equality and with a simultaneous violation of (3.20), the tax sequence, denoting each element by τ^H as in the previous subsection, must satisfy the following (equivalent) sign conditions

$$\begin{aligned} \frac{\tau^H}{1-\beta} &> (1+r_0)\,b_0 \\ \tau^H &> \tau^M \end{aligned}$$

When these inequalities hold, according to the FTPL's arguments, the following inequalities are also satisfied

$$\begin{aligned} P_1^F &< (1-\beta)\frac{M_0}{y} \equiv P^f \\ P_{t+1}^F &< P_t^F \quad (t \geq 1) \end{aligned}$$

where P_t^F refers to the elements of the price sequence for which (3.6) is satisfied at every period, given the market clearing conditions. Then, the terms in (3.20), by construction, satisfy

$$-\lim_{T\to\infty}\frac{b_T}{\prod_{j=1}^{T-1}(1+r_j)} = \lim_{T\to\infty}\frac{M_T^s/P_T}{\prod_{j=1}^{T-1}(1+r_j)} = -(1+r_0)\,b_0 + \frac{\tau^H}{1-\beta} > 0$$

with $(1+r_t) = \beta^{-1}$ $(t \geq 1)$ and $\lim_{T\to\infty} \beta^{-1} \frac{P_{T+1}}{P_T} = 0$.

Thus, such a path is associated with a long-run accumulation of households' bonds by the government and, symmetrically, with a positive present-value stock of real balances in hands of the households, as described in Woodford's passage. A natural question arises: does this accumulation of bonds by the government hide an outright uncompensated transfer of resources, as in the cash-less example, or, on the contrary, the parallel accumulation of monetary wealth by the private sector is sufficient to compensate that transfer? To see how the inclusion of money might change the picture described in the cash-less economy, it is useful to rewrite the government intertemporal constraint (3.25) as follows,

$$(1+r_0)\, b_0 = \frac{\tau^H}{1-\beta} + \lim_{T\to\infty} \beta^{T-1} b_T \tag{3.27}$$

By comparing (3.24) and (3.27) we arrive at an uncontroversial result: adding money does not change anything. The same anomalies described in the cash-less economy are present here, i.e. the long-run stock of bonds accumulated by the government plays the role of a pure uncompensated transfer to the private sector and, hence, considering those "bonds" as "wealth" is a conceptual anomaly[19].

Then, what is special in the fiscalist wealth-offsetting effect? In the limit (as t becomes unboundedly large) we see individuals selling bonds to the government at a unit price of one consumption good and accumulating real balances through a money revaluation-process that, however, does not imply a symmetric flow of funds towards the government. The last term in (3.27) represents resources flowing from the government to the households, but a positive value of $\lim_{T\to\infty} \beta^{T-1} \frac{M_T}{P_T}$ does not represent any flow of resources from the households to the government. At its core, this accounting misspecification is a reflection of a flawed strategy based upon adding flow- and stock-variables together, which is "solved" by introducing a new policy instrument (non-performing bonds)[20].

[19] The analysis here differs from the one carried out by Marimón (2001) in this important respect. Marimón draws an analogy between the stock of money and the outstanding shares of a company, and labels a non-Ricardian policy violating (3.20) as a policy of permanently undistributed profits, in that the firm (i.e. the government) accumulates more profits (i.e. primary surpluses) than needed to redeem the initial stock of debt. Here I am sticking to the same interpretation as in the cash-less example: the extra "profits" are being distributed through the purchase of non-performing bonds, since, otherwise, the sequence of flow of funds *constraints* would be violated.

[20] In a recent paper Buiter and Sibert (2004) also justify the possibility of a deflationary bubble along

The above accounting inconsistency extends to the fiscalist explanation of a deflationary bubble in the price level. From the classical works by Brock (1974, 1975) and Obstfeld and Rogoff (1983), we learned that individual optimal behavior calls for a zero present-value for the long-run stock of real balances holdings, $\lim_{T\to\infty}\beta^{T-1}\frac{M_T}{P_T}$, since a strictly positive value tantamounts to the existence of an arbitrage opportunity: an individual could increase its total utility by reducing its monetary holdings and increasing its consumption. A direct implication of that argument is that, in equilibrium, the amount of valuable resources that the private sector is willing to devote to maintain a positive stock of real balances cannot be above that one consistent with the "monetarist-fundamental" price P^f defined in (3.26).

The FTPL is at odds with that argument, the reason being that the household's attempt to reduce her stock of real balances would cause an immediate default, through a violation of its no-Ponzi games condition (3.3). However, from the above analysis, we know that private default, i.e. the long-run accumulation by the government of non-performing private bonds, is unavoidable given the fiscal commitment to a tax sequence like τ^H, and no change in the price level will correct it. In a sense, Woodford's assertion that the households are not over-accumulating wealth is true, since the households, at the aggregate level, are indeed insolvent, for those bonds sold in the long-run are never being redeemed. Thus, if one really wishes to rule out private default as an equilibrium outcome, a necessary first step is to impose the limiting condition $\lim_{T\to\infty}\beta^{T-1}b_T = 0$ (which clearly forces a change in the tax plan). Once this condition is imposed, $\lim_{T\to\infty}\beta^{T-1}\frac{M_T}{P_T} = 0$ appears as an equilibrium condition driven by *individual optimization* and deflationary bubbles are then eliminated. If, on the other hand, one is willing to *assume* the possibility of private default in equilibrium, the former equality is not necessary for equilibrium, *by assumption*, but the latter will still bite, as long as the household, *by assumption*, maximizes her utility.

Thus, regardless of whether the government behaves like a private optimizing agent,

which (3.20) is violated. Although they do not refer to a non-Ricardian fiscal plan, the same accounting anomaly as the one analyzed here applies in their environment: an unbounded accumulation of real balances by the households does not offset an unbounded accumulation of (non-performing) bonds by the government and, hence, along a deflationary bubble constructed in this way the private sector becomes insolvent in the sense just described. A similar observation applies to some deflationary equilibria studied by Benhabib *et al.* (2002, see e.g. their Section VI).

by not purchasing non-performing bonds, or not, it is never optimal for the household to over-accumulate real balances in this economy and, hence, deflationary bubbles consistent with the violation of $\lim_{T\to\infty} \beta^{T-1} \frac{M_T}{P_T} = 0$ can only be based on non-optimal private portfolio choices. Offsetting a non-standard[21] government fiscal plan with a non-optimizing "equilibrium" individual behavior does not help much to understand the conditions under which a deflationary bubble could take place and the role, if any, that fiscal policy might play in unchaining it.

Extending the analysis to a money supply rule for which money is not fiscally neutral, in the sense that seigniorage is not identically equal to zero at each date, will not alter the main conclusion above. In this more general case, the government's intertemporal constraint becomes

$$(1 + r_0)\, b_0 = \frac{\tau}{1-\beta} + \sum_{t=1}^{\infty} \beta^{t-1} \frac{M_t - M_{t-1}}{P_t} + \lim_{T\to\infty} \beta^{T-1} b_T$$

As the last term in the right side is negative, the present-value of total net uncompensated inflows of resources, i.e. taxes and seigniorage, exceeds the total standard net uncompensated outflows, that is the initial stock of bonds. The gap is covered by a "non-standard" uncompensated outflow via purchases of no-performing bonds. But optimal individual choices still require the fulfillment of the limiting condition $\lim_{T\to\infty} \beta^{T-1} \frac{M_T}{P_T} = 0$.

3.4 Fiscalist non-deflationary paths: Not everything works

In this section I focus on fiscalist price paths along which the price level is not thought to decrease monotonically over time as in the case analyzed in the previous section. This general class of solutions includes both the fundamental monetarist solution and, possibly, multiple speculative hyperinflationary paths. The latter class of price-paths in the context of a Sidrauski-Brock model similar to the one in this paper have been extensively analyzed in the previous literature. Kocherlakota and Phelan (1999) consider a simple experiment consisting in a one-time decrease in the money supply under both a Ricardian-monetarist and a non-Ricardian-fiscalist policy: "*The monetarist device predicts a one-time decrease*

[21] By non-standard, here I mean the explicit consideration of a non-standard asset or, more precisely, a novel government-policy instrument (i.e. non-performing bonds).

in the price level [...] the fiscal theory device predicts a speculative hyperinflation. Which prediction seems more plausible? You decide." Similarly, McCallum (2001) argues that the standard (fundamental) monetarist price sequence "*is arguably more plausible* (than a fiscalist hyperinflationary one) *since it is the solution that is typically regarded as the bubble-free 'fundamentals' solution.*" (Parentheses added). McCallum (2003) also investigates the learnability of fiscalist hyperinflationary paths in a stochastic environment[22]: "[...] *the fiscalist hypothesis implies that, despite a constant money stock, the bond stock and the price level both explode as time passes - but without violating any optimally condition for private agents.* [...] *The crucial issue is, which of the two solutions provides the better guide to reality, i.e., to price level behavior in actual economies?* [...] *it is demonstrated that with the basic policy specification the traditional fundamentals solution is E-stable and therefore learnable, whereas the fiscal-theory bubble solution is not.*"

Thus, to the best of my knowledge, all the previous papers analyzing the FTPL under an exogenous money supply rule in an infinite-horizon economy in which the velocity of money-circulation depends on the nominal interest rate come to agree on the same general point: fiscalist hyperinflationary paths are counterintuitive, implausible, unrealistic or not learnable, yet theoretically consistent, in the sense that along such paths no necessary condition for equilibrium is violated. Much on the contrary, in the finite-horizon counterpart of the models studied by the authors mentioned above, Buiter contends that in the FTPL "*the initial price level is determined twice, once from the monetary equilibrium conditions and once from the government's intertemporal budget constraint. Except through a fluke, these two values of the initial price level will not be the same.*" (Buiter (2002; p. 474). In plain words, this statement implies, that an arbitrary combination of fiscal and monetary plans coupled with an exogenous initial stock of nominal-denominated assets leads directly to an implementation problem: one of the two economic authorities must blink. In the remaining I first describe Buiter's argument in a simple one-period economy and then show that the same implementation problem is also present in the infinite-horizon version of the model. Finally, I analyze in some detail the FTPL's arguments using an extreme example of a fiscalist hyperinflation, devoting special attention to the role of

[22]Evans and Honkapohja (2004) study fiscalist solutions along some learnibility criteria, as well.

transversality conditions, the fiscalist stock-analogy and the controversy on the origin of the initial stock of government liabilities.

3.4.1 A finite-horizon economy: Buiter's criticism

Let's think of a simple one-period economy, as a special case of the infinite-horizon one discussed so far. In such an economy, Buiter's criticism holds: a fully uncoordinated non-Ricardian fiscal-monetary plan is likely to be non-implementable. Let's assume that the representative household chooses the level of consumption, real money holdings and bonds so as to maximize

$$\frac{1}{1-\theta}c_1^{1-\theta} + \frac{1}{1-\theta}\left(\frac{M_1}{P_1}\right)^{1-\theta} \tag{3.28}$$

subject to the flow budget constraint

$$b_1 + \frac{B_1 + M_1}{P_1} \leq y - \tau_1 - c_1 + (1 + r_0)\, b_0 + \frac{(1 + i_0)\, B_0 + M_0}{P_1}$$

and the borrowing limit $\frac{B_1}{P_1} + b_1 \geq 0$. When optimizing, the household behaves according to the following conditions

$$\frac{m_1^{-\theta} - c_1^{-\theta}}{P_t} = 0 \tag{3.29}$$

$$c_1 = y - \tau_1 + (1 + r_0)\, b_0 + \frac{(1 + i_0)\, B_0 + M_0}{P_1} - \frac{M_1}{P_1}$$

The government budget constraint is given by

$$b_1^s + \frac{B_1^s + M_1^s}{P_1} = g_1 - \tau_1 + (1 + r_0)\, b_0 + \frac{M_0 + (1 + i_0)\, B_0}{P_1}$$

After imposing $\frac{B_1}{P_1} + b_1 = 0$ as an equilibrium requirement, the government's constraint reads as follows

$$\frac{M_0 + (1 + i_0)\, B_0}{P_1} = -\,(1 + r_0)\, b_0 + \tau_1 - g_1 + \frac{M_1^s}{P_1}$$

Let's assume that the central banker sets M_1^s and the fiscal authority chooses g_1, both being fixed independently of the observed price level, P_1. Then, given the market clearing conditions $c_1 = y - g_1$ and $M_1 = M_1^s$, we learn that there can be, at most, two alternative equilibrium price levels:

$$P_1^f = \frac{M_1}{y_1 - g_1} \text{ for any } \theta > 0 \tag{3.30}$$

$$P_1^h = \infty \text{ for any } \theta < 1 \tag{3.31}$$

Notice that when $\theta > 1$, there is a unique equilibrium price level (the fundamental one in (3.30)), as in such a case (3.15) is satisfied as a strict inequality. But even if (3.15) holds as an equality, we can characterize all the feasible equilibrium price-levels (two in this case, the fundamental and the hyperinflationary one) regardless of the tax policy at work, τ_1, and of the initial stock of financial wealth (i.e. independently of its magnitude and composition). It follows that the tax rate may take, at most, two possible values, τ_1^f and τ_1^h, each associated with one of the two possible prices consistent with the market clearing conditions and the optimality condition (3.29), P_1^f and P_1^h, respectively. These two feasible values are given below

$$\begin{aligned} \tau_1^f &= g_1 + (1+r_0)\, b_0 + \frac{M_0 + (1+i_0)\, B_0}{P_1^f} - \frac{M_1^s}{P_1^f}, \text{ for } P_1 = P_1^f \\ \tau_1^h &= g_1 + (1+r_0)\, b_0 \text{ for any } \theta < 1 \text{ for } P_1 = \infty \end{aligned} \tag{3.32}$$

Thus, any fiscal policy not in line with the above feasible tax rates, will result in an over-determined system, so an uncoordinated fiscal policy, in the sense just described, is not implementable.

3.4.2 An infinite-horizon economy

The above over-determinacy result should be uncontroversial. As pointed out by Buiter, it is just a question of counting equations and unknowns. However, Buiter fails to extend that argument to an infinite-horizon economy: "*Proposition 5, that under a non-Ricardian fiscal-financial-monetary programme with an exogenous rule for the nominal money stock the general price level is over-determined, now* (in the infinite horizon case) *only applies when the velocity of circulation of money does not depend on the nominal interest rate* *and, through that, on expected future price levels.*" (Buiter (2001), parentheses added). Here I argue that a similar over-determination problem arises in a with infinite-horizon model with real balances in the utility function, i.e. a model in which the velocity of circulation does depend on the nominal interest rate. I distinguish two cases, as before: $\theta \geq 1$ and $\theta < 1$.

First, when $\theta \geq 1$, we know from (3.15) that speculative hyperinflationary paths are not possible. Once the monetary authority has fixed the sequence of money supplies and the fiscal branch has set the sequence of government consumption, there is a unique equilibrium price sequence, as shown in the preceding section. For example, with logarithmic utility functions, if the central bank sets an exogenous money supply in period 1, M_1, and commits to increase it at a constant rate μ from that period on, then a constant tax rate sequence and the unique equilibrium price sequence must satisfy

$$\tau = \left[\frac{1-\beta}{1-\frac{\beta}{1+\mu}}\frac{(1+i_0)\,B_0+M_0}{M_1}-1\right](y-g)+g+(1-\beta)\,(1+r_0)\,b_0 \qquad (3.33)$$

$$P_t = \left(1-\frac{\beta}{1+\mu}\right)\frac{(1+\mu)^{t-1}\,M_1}{y-g}, \quad (t \geq 1) \qquad (3.34)$$

It is worth noticing that the equilibrium price sequence in (3.34) is completely independent of the tax policy and the initial stock of nominal financial wealth, $(1+r_0)\,b_0 + (1+i_0)\,B_0 + M_0$. On the contrary, the equality in (3.33) makes clear that the unique feasible tax policy depends on the initial volume of government obligations and on the time-path of the money supply and, hence, on the price sequence: a plain monetarist-Ricardian monetary-dominance regime at work.

Second, if $\theta < 1$, we know from the results discussed in section 3.2 that there are multiple equilibrium price-paths, one fundamental and, in an infinite horizon economy, an infinite number of speculative hyperinflationary paths consistent with the first order conditions of the household's optimization problem and the market clearing conditions. This potential multiplicity of equilibrium price-paths might suggest that the tax policy is not subject to a constraint like (3.33). Here I argue that even when $\theta < 1$, there are not a continuum of equilibrium price-sequences, in the sense that between two consecutive different feasible hyperinflationary price-sequences (to be defined later), there are a continuum of non-equilibrium price-sequences.

The proof of the above claim (namely, that there are not a continuum of equilibrium price-sequences) is based on a simple observation. From the one-period economy described above, we learn that in a n-period economy there are $n+1$ (potential) equilibrium price sequences, n speculative and one fundamental. Each of the n speculative prices-paths can be labelled by its initial element, P_1^j $(j = 1, ..., n)$, such that $P_1^j > P_1^{j-1} > P_1^f$, where P_1^f is the first element of the unique bubble-free equilibrium sequence. Let's denote the set containing the n (potential) hyperinflationary-equilibrium initial prices by Ψ_n. As $M_t > 0$, there is a continuum of initial non-feasible, thus out-of-equilibrium price levels, P_1^{NE}, not belonging to Ψ_n and satisfying $P_1^j > P_1^{NE} > P_1^{j-1}$. Hence, as the number of periods increases by one, the number of feasible equilibrium price sequences increases by one, but the number of non-feasible price sequences increases by a continuum (i.e. by an infinite measure). Obviously, the argument goes on as we increase n without limit. It follows that a tax sequence chosen arbitrarily need not to be implementable. Formally, let's consider the fiscalist solution discussed in section 3.2 with the added simplifying assumptions used above, i.e. constant money supply ($M_t = M_0$ for $t \geq 1$) and constant and exogenous government consumption. The equivalent to the government valuation equation (3.19) now is given by[23]

$$\frac{(1+i_0)\,B_0 + M_0}{P_1} = \frac{\overline{\tau} - g}{1-\beta} - (1+r_0)\,b_0 \tag{3.35}$$

In view of (3.35), the fiscalist proposition according to which "the path of the money supply does not matter" is plainly justified: the sequence of money supplies $\{M_t\}_{t=1}^{\infty}$ is completely irrelevant for the determination of P_1 and the price level is only affected by fiscal variables (put aside M_0, which need not to be strictly positive for the fiscalist argument as long as $B_0 > 0$). However, for an arbitrary $\overline{\tau}$, the price, P_1, satisfying (3.35) need not to belong to the set containing all P_1^j, even after taking into account that this set may contain an infinite number of elements. If we were to choose a particular $\overline{\tau}$ randomly, the system would be lead to a collapse of the price level with a probability 1.

[23]Recall that when $\theta < 1$, solving for the initial price level using the government intertemporal budget constraint may not give a unique solution. As pointed out before, a sufficient condition to preclude that possibility is to impose a tax policy like the one in (3.18). To keep notation simple, in writing (3.35) I assume a constant term $\overline{\tau}_t = \overline{\tau}$ $(t \geq 1)$.

Formally, if an arbitrary P_1 is not the fundamental one P_1^f, then it must satisfy $P_1 > P_1^f$, so it would be associated with a speculative hyperinflationary path. We also know that the last finite element of any sequence whose first element is in Ψ_n is uniquely given by $P^* = \frac{M}{y-g}$. As $P_1 \notin \Psi_n$, the last finite and positive element of the price sequence satisfying the individual optimization conditions and the market-clearing conditions associated with P_1, call it $\widetilde{P}$, must satisfy $\widetilde{P} > P^*$. But then, the first order condition (3.6) is violated, i.e. it satisfies

$$\frac{\left(\frac{M}{\widetilde{P}}\right)^{-\theta} - \left(\frac{M}{P^*}\right)^{-\theta}}{\widetilde{P}} = -\beta \frac{(y-g)^{-\theta}}{\widetilde{P}_{+1}} > 0 \tag{3.36}$$

where $\widetilde{P}_{+1}$ would be the next period (negative) price level.

Clearly, the tax sequence must satisfy a restriction similar to (3.33), although here, as the set of feasible prices contains an infinite number of elements, the set containing the potential constant taxation levels (i.e. those tax policies involving a term $\overline{\tau}$ that avoid the over-determination problem in (3.36)) includes an infinite number of elements, as well. Each of these elements must satisfy the following restriction

$$\begin{aligned} \overline{\tau}^f &= (1-\beta)\left[\frac{(1+i_0)\,B_0 + M_0}{P_1^f} + (1+r_0)\,b_0\right] + g \quad \text{for } P_1^f \\ \overline{\tau}^j &= (1-\beta)\left[\frac{(1+i_0)\,B_0 + M_0}{P_1^j} + (1+r_0)\,b_0\right] + g \quad \text{for } P_1^j \in \Psi_n \end{aligned} \tag{3.37}$$

It follows that, as P_1^f and P_1^j depend on the particular money-supply sequence $\{M_t\}_{t=1}^{\infty}$, so must depend the sequence of potential constant taxation levels, as well. And, as before, in characterizing the set of feasible equilibrium price-sequences the information on the initial stock of government liabilities is completely irrelevant. In this precise sense, *except through a fluke* a fully uncoordinated non-Ricardian fiscal-monetary program will not pass its preliminary test: consistency with the equilibrium conditions, among which, a non-negative price level should be listed as a fundamental one.

3.4.3 Speculative hyperinflations: A necessary result?

The previous result poses an important obstacle for the fiscalist interpretation of the government budget intertemporal budget constraint (3.35) as a government valuation equation, along which the equilibrium price of outstanding nominal-denominated assets is computed as the price of a private firm's equity, since, as just shown, an arbitrary stream of current and future profits (i.e. government surpluses) coupled with an arbitrary initial stock of firm's shares (i.e. initial nominal financial wealth, $(1 + i_0) B_0 + M_0$) will, in most cases, lead to a negative price for the firm's equity as time passes. The extension of this critique to a stochastic environment, in which the sequence of fiscal surpluses and/or money supplies are subject to purely exogenous random shocks is straightforward, as well.

Still one could argue that the restriction on the set of implementable tax sequences contained in (3.37) does not shed light on the question of which policy, monetary or fiscal, has the last word in the mechanism of equilibrium-prices determination. For example, the FTPL's arguments would be consistent with the idea that, either through a fluke or through a purposively fiscal-monetary coordinated mix, the government may unchain a speculative hyperinflationary path, as suggested by Kocherlakota and Phelan (1999), provided that individual preferences satisfy $\theta < 1$ and the foregoing fiscal-monetary plan is consistent with the implementation condition in (3.37)[24]. Although this question, in contrast to those analyzed up to this point, pertains to the off-equilibrium sphere, the starkness of a fiscally-induced speculative hyperinflation provides a natural terrain in which one can learn on the fundamental differences of the two paradigms in question.

The fact that an intrinsically useless asset, like modern government-supplied money, may have a positive real value in equilibrium is not just a well-known result, but it is, perhaps, the most distinctive feature of non-convertible money. However, as the real value of money is only rooted on the private expectations about the value of that durable asset, nothing precludes the possibility of a class of equilibria in which money does not have any positive value. When the faith of the households in the current or future value of money weakens, the current real value of money falls, perhaps, to zero. Such a possibility

[24] As just shown, both conditions are necessary along an equilibrium hyperinflationary path.

is a natural result due to the nature of modern money, as long as government-issued notes can be thought as of being an intrinsically useless asset. Moreover, this possible outcome is a "healthy" result in any model of (non-convertible) purely fiat money. As shown by Obstfeld and Rogoff (1983), in an economy identical in its basic aspects to the one studied here, when private faith threatens to vanish, the government, by committing to turn money into a claim for an intrinsically valuable good, may avoid a speculative hyperinflation, adding the "missing doses of private faith" in money. But how can the government purposively reduce the real value of money? According to the monetarist wisdom, this can only be done by announcing a policy of increasing money supplies. But this will result in a 'fundamental' hyperinflation and not necessarily in a speculative one, that is, the real value of money need not drop to zero[25]. Thus, the asymmetry between avoiding a speculative hyperinflation and unchaining it is clear: private faith in money can be reinforced by the government, but the government cannot eliminate it. The FTPL is at odds with this principle. The following simple example seeks to explain why.

For simplicity, I consider a version of the simple one-period economy analyzed above[26]. Let's initially think of an economy without government in which the representative household begins her life with a positive (and finite) stock of an intrinsically useless asset, called money and denoted by M, whose quantity can not be altered, so that the end-of-period aggregate stock of money must coincide with the initial one. As before, the household receives an endowment of consumption goods, y. These, consumption goods and money, are the only good/assets transacted in this economy. Preferences are given by (3.28) and $\theta < 1$ holds. The household-j budget constraint is

$$\frac{M^j}{P} \leq y - c^j + \frac{M}{P}$$

where $\frac{M^j}{P}$ and c^j are, respectively, the household's demand for real balances and consumption goods. The characterization of the two alternative equilibria is similar to the

[25] Along a non-specualtive hyperinflation the price level remains finite as long as the present and future money supplies remain finite as well.

[26] This assumption is made for the clarity of the exposition. Nonetheless one may think of this economy as the first period of a multi-period one, with finite or infinite horizon, in which the endowments of money and consumption goods remain constant and the government commits to a zero primary surplus from period 2 on, thus rolling over any stock of debt alive at the end of period 1.

model with an active government discussed above. Given individual preferences, we learn that in addition to the fundamental price, $P^f = \frac{M}{y}$, there is another equilibrium in which money is valueless. In either case, equilibrium requires $c^j = y$ and $\frac{M^j}{P} = \frac{M}{P}$.

Now, let's introduce another agent into the picture, the government. This new agent is special in that it has the exclusive right to tax individuals and to supply an asset whose possession does not yield any utility to the households *per se*. Let's refer to this asset as a bond and denote its supply by d, its demand by household-j by d^j and its price, measured in units of consumption goods, by q. Suppose, that at the beginning of the period, before any trade takes place, the government announces that it will make a transfer to the private sector, which involves the delivery of consumption goods and/or money, equivalent to the value of T monetary units. Thus, its constraint reads as follows

$$qb + \tau = \frac{T}{P} \tag{3.38}$$

The Ricardian approach would be consistent with the following idea. The transversality condition $qb = 0$ must hold *always*, for any vector of prices (P, q). As the flow of funds identity (3.38) must hold for any conceivable P, the government must raise resources from the private sector to pay the transfer whenever the price is finite, like P^f above and, trivially, $\tau = 0$ holds in the equilibrium in which money is valueless. The FTPL, on the contrary, allows the government to ex-ante commit to a non-contingent zero tax. As a fundamental piece in the fiscalist argument is that the government always honors its commitments, it follows that the unique equilibrium calls for a zero value for money. Why cannot P^f be now part of an equilibrium? Because, according to the fiscalist dictum, it would violate the government transversality condition and, with it, some individual optimality and/or market clearing condition. In the previous chapter, I have argued that it is always possible for the market for non-performing bonds to clear:

$$qb = q\sum_j b^j = 0 \tag{3.39}$$

That is, either both supply and demand are equal to zero or so it is the price level, q. This simple argument applies regardless of the value of P. Accepting the possibility that (3.39) *may* hold always and *assuming* that this is, indeed, the case tantamounts to impose the Ricardian hypothesis, under which P^f may well be an equilibrium outcome. Although to understand the lack of generality of the FTPL's arguments only the *may* part of the previous statement is necessary. The following remarks aim at clarifying some critical questions at this point.

i. Transversality conditions: identities or equilibrium conditions? Intertemporal government constraints or asset valuation equations? These questions are at the centre of the current debate on the FTPL. For some authors, like Buiter, transversality constraints must always hold, both at equilibrium and off-equilibrium prices. On the other hand, some proponents of this theory have provided some arguments for treating them as equilibrium conditions. Among those arguments, I find Cochrane's (2005) ones specially appealing, mainly for their clarity. According to him, one should view government bonds and money as shares whose payoff is given by the expected fiscal surpluses: "The government *auctions* new debt, and accepts whatever price results [...]" (italics as in original). Thus, a positive supply of bonds, $b > 0$, cannot be ruled out *a priori*. In our simple economy, the households could auction consumption goods and/or money in exchange for those bonds. Can we assume that in some contingencies the households supply a positive amount of goods and/or money? Yes, thus violating an optimality condition. The resulting price of a bond would not capture the stream of surpluses backing those bonds, i.e. zero in this simple one-period economy[27]. That price wouldn't be an equilibrium one. In this sense, it can be argued that treating transversality conditions as identities seems a "too strong" assumption. In the same way as the first order condition governing the optimal individual demand for money (3.6) may be seen as an asset valuation equation, which, somewhat tautologically, can be violated at off-equilibrium money-prices, the government intertemporal budget may be understood as a bond-valuation equation. But, can we assume that in some contingencies the households do not supply any goods and/or money in exchange for bonds? Yes, such an option is always feasible. In that case, the lending

[27]The argument clearly applies in the same manner to an infinite horizon economy, where for a given stream of future fiscal surpluses, determined independently from the price of a bond, and a given stock of government bonds being auctioned, the equilibrium bond-price is unique. The fact that in a finite horizon economy the equilibrium bond-price for bonds auctioned in the terminal period is zero only adds a numerical detail.

decision is the optimal one. The real value of a bond then drops to zero, in line with the future government's surpluses. Then, no matter what P happens to be, the government must abandon its target for a zero tax so as to satisfy $\tau = \frac{T}{P}$. Taking for granted that the government auctions new debt whenever P is finite, which assumption captures better the household's off-equilibrium lending behavior? This is a religious issue. Can we establish any link between P and the household's lending decision? No. Independently of the particular value of P, a household's strategy of a zero supply of goods and/or money in exchange for bonds is always feasible and optimal. Thus, regardless of whether one views transversality conditions as identities or as equilibrium conditions, there is *always* room for a Ricardian-like mechanism to operate, which, tautologically, means that the FTPL's non-Ricardian mechanism need not to operate.

Concerning the above two questions, the Ricardian hypothesis, takes a rather conservative approach: we know how to describe in an unambiguous way what the unique optimal lending decision is, but we do not have any guide to explain a non-optimal lending decision as a function of the value of money, hence we assume that the lending decision is the right one for any money-price. Such a hypothesis will not rule out equilibria in which money is not positively valued. Yet, as stressed above, that possibility *should* always arise in an economy in which money is a purely fiat asset. On the other hand, such a hypothesis will rule out the idea that money cannot be positively valued in equilibrium because, if that were the case, the households would feel an irrepressible temptation to demand (useless) government bonds and/or the market for money and/or consumption goods could not clear.

ii. Money as a private firm's stock. In Chapter 2, I have argued that the fiscalist stock-analogy can be interpreted as a useful tool to understand the limits of the FTPL as a theory capable to challenge the traditional monetarist propositions on the equilibrium value of fiat money. Here I take up the stock-analogy to argue that along a fiscalist hyperinflation the mechanism of price determination has no resemblance to the standard asset-valuation argument employed in solving for the equilibrium value of a firm's stock. The logic of this claim is simple. Following the fiscalist argument, let's think of the outstanding beginning-of-period stock of money and nominal bonds as residual claims to the government's fiscal surpluses. From the no-government case analyzed above, we know that there is an equilibrium in which the initial stock of money still has a positive value, simply because there is an underlying liquidity services function. This illustrates an important difference between money and the share of a firm. The former may be positively valued by the private sector even when its issuer commits to never redeem it in exchange for any intrinsically valuable good or to pay any dividend on it. Then, if money is still thought to be a "government share", it must be recognized that it may well be traded at a positive price even when the government is expected to make no profits. Then, how can we justify a zero value for money as a necessary unique equilibrium outcome? Clearly, not simply by arguing that the expected stream of profits backing the stock of money is zero, for those profits were already zero in the no-government case (by construction) and there was an equilibrium at which money was positively valued[28]. In this sense, the stock analogy does not provide a sufficient intellectual framework to justify a fiscalist hyperinflation, this being a reflection of the fact that treating transversality conditions as equilibrium conditions is not sufficient to justify the off-equilibrium link between money prices, debt prices, market clearing and individual optimality implicit in the fiscalist hypothesis.

iii. The initial stock of nominal assets. Where did it come from? Does it matter? The FTPL regards the existence of an initial stock of nominal-denominated assets (i.e. money and/or nominal bonds) as a necessary condition for a non-Ricardian fiscal-monetary to yield equilibrium uniqueness. Niepelt (2004) forcefully argues against this hypothesis: if individuals would had anticipated the future implementation of a non-Ricardian regime at the time of purchasing the initial stock of government-issued assets (not explicitly modeled), they wouldn't have bought as much nominal assets in the first place. In view of these reflections, he suggests that the non-Ricardian argument would work properly if instead of assuming that there exists a positive stock of assets which were voluntarily bought by the households in some past date, the government commits to make nominal

[28]At this point one could argue that the government, apart from its monopolistic right to levy taxes, may also change the stock of money, so that it could use the latter to always meet its commitment to transfer T units of money to the private sector. This is true, the government may always finance a nominal-denominated transfer by printing money, but such a strategy leads to a plain *monetarist* fiscal-dominance regime similar to the one described in section 3.2.2.

transfers and/or to levy taxes in a given nominal amount. The example constructed above clearly fulfills Niepelt's requirements to activate a fiscalist price-determination mechanism. On the one hand, the initial stock of money is explicitly treated as an exogenously (and freely) given endowment. One may clearly understand this as the government dropping (i.e. giving for free) that money evenly across households at the beginning of the history, before voluntary trade takes place. On the other hand, we have just checked in the above example that regarding T as a pure nominal-denominated transfer rather than as a bunch of maturing government nominal bonds does not make any difference regarding the ability of the government to "choose" a hyperinflationary path as a unique equilibrium. Something is special in the FTPL, but it has no connection with the *voluntary-trade versus coercive-transfer* distinction highlighted by Niepelt. By contrast, most of the analysis on the special features of the FTPL carried out in the paragraphs above is focused on the role of the transversality conditions. This seems a natural strategy after recognizing that the Ricardian *versus* non-Ricardian distinction is, indeed, driven by an assumption about transversality conditions (Definition 2), and *only* by that.

Summarizing, none of the arguments built up to support the validity of the FTPL listed above provide a lucid explanation of the question we are after: how can the government implement a fiscal policy that *necessarily* results in private individuals loosing their faith on money? Although such a possibility may be seen as a minor pitfall in this theory, confined to a counterintuitive, although remote, theoretical possibility, it must be recognized that the core mechanism of this theory works always in the same manner, regardless of whether the mechanism leads to the stark corner of a divergent hyperinflation or to a seemingly appealing explanation of some observed facts, and independently of the monetary rule at work (say, an interest rate peg or a nominal stock target).

3.5 Conclusions

In this final chapter, I dealt with a particular question raised in the recent debate on the validity of the Fiscal Theory of the Price Level (FTPL): the feasibility of fiscal-driven divergent price-paths in economies in which the monetary authority sets a nominal anchor by announcing an exogenous sequence of money supplies. In part, this question, as opposed to those tackled in the previous chapter, pertains to the behavior of the economic variables *in equilibrium*, and, as such, should not be much controversial. Yet, for a theory that seems to posses a special ability to attract confusion, the degree of consensus on that question is still low. In a series of influential papers, Buiter (1999, 2001, 2002) argues that uncoordinated monetary-fiscal programs as those proposed by the FTPL are generally non-implementable in finite-horizon economies, for they involve a mathematical over-determination problem. However, he denies the existence of that problem in infinite-horizon models in which the demand for real balances depends upon expected inflation. This observation has been overlooked in the subsequent work on the FTPL devoted to the study of fiscalist deflationary or hyperinflationary paths. This chapter is intended to fill this gap, by answering the following specific question: Do the implementation problems of non-Ricardian programs vanish in an infinite-horizon economy?

The answer to that question provided here is unambiguously negative. Using a standard model with money in the utility function, I first show that fiscalist divergent deflationary price-paths along which the private sector violates its solvency constraint are irrational bubbles, since, regardless of the government's fiscal plans it is never optimal for the household to accumulate an infinite amount of real balances. Second, I argue that, even when preferences allow for the possibility of a speculative hyperinflation, a fully uncoordinated fiscal non-Ricardian plan, as those considered by Buiter, will generally result in a negative price level. These are the two ways in which Buiter's findings for a finite-horizon economy emerge in the infinite counterpart. In this way, this work adds to a better understanding of some of the fundamental obstacles of this relatively new theory to offer a solid alternative to the traditional Ricardian-monetarist paradigm.

Also, I exploit the extreme case of a fiscalist hyperinflation to argue that the price-determination mechanism stressed by FTPL cannot be supported by just assuming that transversality conditions are equilibrium conditions (rather than identities) or by arguing that the equilibrium value of money can be determined following the same arguments traditionally employed in solving for the equilibrium value of a firm's equity.

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